Numerical studies of turbulent flows

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Signature: _____

*“What we observe is not nature itself,
but nature exposed to our method of questioning.”*

WERNER HEISENBERG

Abstract

Turbulent flows are investigated using numerical simulations. Three aspects of turbulent flows are the focus points for the present work. Firstly, turbulent drag reduction under the effect of spanwise wall oscillations for a boundary layer has been studied using direct numerical simulations. Previous studies focused on channel or pipe flows which are numerically easier due to periodic boundary conditions. Here, we use a novel technique with discontinuous zone of oscillation to study the effect of starting and ending oscillations on boundary layer flows. Spatial transient of Reynolds stresses have been reported for temporal forcing. A predictive relation for deprecation of drag reduction performance with Reynolds number has been proposed. Steady spatial half-square waves have been shown to be energetically more efficient due to lesser power consumption. Optimization of parameters yield $\sim 18\%$ net energy savings.

Secondly, the effect of wall oscillations on low speed streaks and transition region are investigated. Previous studies have used linear approximation to study wall oscillation effects on streaks whereas in the current work, non-linear interaction terms are included which show a marked difference in results. The wall oscillations are shown to reduce the skin friction which drops below the laminar Blasius flow value (without the presence of streaks) for certain cases of wall oscillations. Influence of wall oscillations on bypass transition has been investigated with both spatial and temporal forcing. A more pronounced delay has been observed with spatial oscillations. Effect of starting position

of oscillation, wavenumber and turbulence intensity on skin friction coefficient has been studied.

Thirdly, for multi-scale analysis of turbulent flows, a new methodology based on novel optimization techniques for scale decomposition is introduced. The optimization procedure leads to a band-pass filter with prescribed properties in the Kolmogorov energy spectrum. With this filter, scale decomposition using Fourier transform can be performed very efficiently while adequately suppressing Gibbs ringing artifacts. Continuous scale decomposition is particularly important to understand the dynamics of eddies across different wavenumbers. Compared with previous work on multi-scale turbulent flow analysis and visualization, the proposed method enables flexible and efficient turbulence decomposition in a continuous manner.

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Anyone who decides to do a PhD is often warned by peers about the dire straits it throws you into. And a certain optimism just gives you the required impetus that your experience would not be as distasteful as others. Mine was not an exception either. With all kinds of restraint as constant companions, my doctoral journey has been quite a roller-coaster ride. And I am extremely indebted to lots of people in this journey.

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NOMENCLATURE

Symbol	Description
DR	Drag Reduction
DNS	Direct Numerical Simulation
U_∞	free-stream velocity
ν	kinematic viscosity of the fluid
δ	Boundary layer thickness, i.e. y location where U is 99% of U_∞
δ^*	Displacement thickness
Θ	Boundary layer momentum thickness,
Re_{δ^*}	Reynolds number based on displacement thickness ($= U_\infty \delta^* / \nu$)
Re_Θ	Reynolds number based on momentum thickness ($= U_\infty \Theta / \nu$)
+	indicates quantity normalized by inner variables, i.e. by u_τ and ν
u_τ	friction velocity for stationary wall configuration ($= \sqrt{\tau_w / \rho}$)
$u_{\tau,o}$	friction velocity for oscillating wall configuration ($= \sqrt{\tau_{w,o} / \rho}$)
U, V, W	mean streamwise, wall-normal and spanwise velocity
u, v, w	r.m.s. of streamwise, wall-normal and spanwise velocity fluctuations
$\mathbf{U}$	velocity vector
x, y, z	mean streamwise, wall-normal and spanwise coordinate
y^+	y coordinate normalized with inner variables ($= y u_\tau / \nu$)
u^+	u normalized with inner variables ($= u / u_\tau$)
C_f	skin-friction coefficient ($= 2\tau_w / (\rho U_\infty^2)$)
D_m	maximum wall displacement
W_m	maximum wall speed

T	period of wall oscillation
T^+	period of wall oscillation normalized with inner variables ($= Tu_\tau^2/\nu$)
ω	frequency of wall oscillation ($= 2\pi/T^+$)
λ_x	wavelength of wall oscillation
λ_x^+	wavelength of wall oscillation normalized with inner variables ($= \lambda_x u_\tau/\nu$)
κ	wavenumber of wall oscillation ($= 2\pi/\lambda_x$)

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Preface

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M. Skote, M. Mishra, P. S. Negi, W. Yanhua, H. M. Lee and P. Schlatter. Wall oscillation induced drag reduction of turbulent boundary layers. *Progress in Turbulence VI*, 2014.

M. Mishra and M. Skote. Wall oscillation induced drag reduction of turbulent boundary layers. Under review in *International Journal of Heat & Fluid Flow*.

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X. Liu, M. Mishra, C. W. Fu and M. Skote. Optimization-based Flow Decomposition for Visualizing Turbulence Structures. Under review in *IEEE Transactions on Visualization and Computer Graphics.*, 2014.

P. S. Negi, M. Mishra and M. Skote. DNS of a single low speed streak under spanwise oscillations. To appear in *Flow, Turbulence and Combustion*, 2014.

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Chapter 1

Introduction

Turbulence has been one of the greatest intractable physical phenomena which remains unsolved in modern times. The sheer difficulty of the problem can be estimated with this amusing quote from Horace Lamb in 1932

“I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.”

And decades since, with all the technological breakthroughs and research, we still are unable to comprehend the complexity of turbulent flows and understand the exact physics behind it. The multi-scale phenomena as shown by Kolmogorov’s hypothesis [50] characterizes turbulent flows ranging from microscales to megascales. This fractal like resemblance is both captivating and perplexing.

As interesting and intriguing turbulent flows are from physics point of view, equally important are its practical implications as well. With the energy and transport sector still

having a strong dependence on fossil fuels, sustainable solutions are the need of the hour. To put things in perspective, turbulent drag reduction is a major obstacle when it comes to meeting sustainability. Even a small decrease in drag can have a massive impact in the overall savings. For eg: a $\sim 1\%$ drag reduction can lead to a decrease of about 0.2% of direct operating cost for commercial flight vehicles. As a result of this drag reduction, total weight of fuel aboard is also lesser and so as much as 10 extra passengers can be accommodated with 1% drag reduction[72].

1.1 Drag Reduction

Drag or fluid resistance refers to the force which a solid experiences in the direction of the relative flow velocity. Drag forces are unavoidable but it is certainly possible to reduce them through some modifications based on the kind of flow regime. Drag due to the action of viscous stresses on the surface of the body is often referred to as *skin friction drag*. Various strategies have been explored for skin friction drag reduction. A few attempts can be classified as follows based on flow regime (laminar or turbulent):

1.1.1 Laminar Drag Reduction

Laminar flow control Laminar Flow Control (LFC) are active flow control technique of boundary layer in order to maintain the laminar state till a specific Re beyond which otherwise it would be characterized as in transition or turbulence without such a device . They often employ suction through slots, porous surface or perforations to keep the flow laminar. Laminar flow extent is also explored with the impact of surface tolerances such as wall roughness, waviness, steps, gaps etc. Natural

laminar-flow (NLF) concepts have also been explored which employs a favorable pressure gradient to delay the transition process [16].

Active wave control This technique primarily works on delaying the onset of turbulence by inhibition or delayed growth of Tollmien-Schlichting (T-S) waves. These T-S waves are supposed to be growing in space and time and eventually result in chaotic motion downstream. Some methods which have been explored to inhibit their growth are pressure-gradient control, wall suction and boundary layer heating and cooling.

1.1.2 Turbulent Drag Reduction

1.1.2.1 Passive Mechanisms

Riblets Longitudinally grooved surfaces or riblets first came into the picture with a 1937 German patent application from Kramer but without any proof of concept. Later in 1976, they came into light when Langley initiated research on modification of low speed streak structures. It was known from the work of Klebanoff [48] that 50% of the turbulence energy is generated in the lower 5% of the boundary layer as shown in Figure 1.1. The diameter of the low speed streaks are on the order of 30-40 wall units and the spacings of the near wall structures is kept at 100 wall units and the height was used as $0.01 - 0.02\delta$ where δ is the boundary layer thickness. Different geometries of riblets have been investigated with their orientation in streamwise direction such as a V-groove riblet, circular, trapezoidal and L-shaped riblets etc as shown in a cross sectional view in Figure 1.2. With the use of riblets, a drag reduction of $6 \pm 2\%$ was observed [100]. However, they are plagued with certain

disadvantages as dust or ice settles in the grooves and they are rendered ineffective.

Figure 1.1: Turbulence energy production in a turbulent boundary layer from Klebanoff [48]

Large Eddy Break Up devices Large Eddy Break-Up (LEBU) devices aim to attenuate large-scale structures in the outer regions of the boundary layer and thereby lead to a reduction in the Reynolds stresses near the wall. These consist, variously, of perforated screens, grids, 'fences' and airfoils mounted on the wall or within the flow. They can produce significant reductions in skin friction over extensive regions downstream of the device. [39]

Figure 1.2: Different riblet geometries; Cross sectional view showing V-groove, circular and L-shaped riblets respectively

1.1.2.2 Active Mechanisms

Magnetohydrodynamic (MHD) flow drag reduction An electrically conducting fluid in the presence of an electromagnetic field experiences a body force known as Lorentz force

$$\rho \mathbf{F} = \mathbf{j} \times \mathbf{B} \quad (1.1.1)$$

where $\mathbf{B}$ is the magnetic field and $\mathbf{j}$ is the electric current density induced by an external electric field or by the electric field induced by the fluid motion in the presence of magnetic field. Magnetohydrodynamic (MHD) manipulation can be used by changing orientation, geometry and by varying the properties of the field. They drastically change the structure of turbulence in the flow and hence reduce the skin friction. They also increase the stability of the flows.

Polymers and Surfactants Extremely dilute solutions of high molecular weight polymers or surfactants which form 'micelles' exhibit frictional resistance much lower than the pure solvent. Experimentally, it has been found that concentrations on the order of few parts per million (ppm) by weight can reduce the frictional resistance of solvent to as low as one-fourth of the original value. Suspensions of fibers such as wood pulp, nylon, sand also reportedly give good drag reduction. At 10 ppm, one can easily achieve drag reductions up to 50%. Drag reduction (DR) with microbubbles is also getting increasing attention particularly with its applications in marine vehicles. The injection of gas into a liquid turbulent boundary layer to form bubbles reduces skin friction drag locally by as much as 80 %. Although the effectiveness of microbubbles has been demonstrated and the bubble sizes have been found to be one of the important factors affecting the DR, the overall mechanism that leads to

this reduction is only poorly understood. Injection of particles into the freestream flow has also been found to have dramatic effects on drag reduction. Gas-particle suspensions, Liquid-fiber suspensions have been found experimentally to reduce the skin friction. However, a concrete mechanism is still not understood fully.

Active Wall Turbulence Control This methodology is based on the observation of turbulent coherent structures as observed in experiments and to control their progenitors using a wall motion or applying some feedback mechanism to modify these structures to reduce the drag. Blowing/suction for active control has also been widely studied. The primary focus of this work is the *open-loop wall oscillation* technique which has received a lot of attention in recent times. Two forms of oscillations have been explored here: *temporal* and *spatial* with waveforms in time and space respectively.

1.1.3 Effect of wall oscillations on transition delay

With regards to reducing turbulent drag, a lot of research work has been dedicated to delay the process of transition from laminar to a turbulent state and in some cases, even relaminarization has been shown to be possible. Two primary routes for transition to turbulence have been generally accepted. In the first scenario, breakdown occurs due to small amplitude velocity perturbations which lead to exponential amplification of Tollmien-Schlichting (TS) wavepackets and eventually, breakdown. In order to stall this transition process [5], various techniques have focused on reducing the low amplitude disturbances soon after they start to appear in a laminar boundary layer flow to prevent the amplification process of TS wavepackets. A second class of breakdown occurs in the form

of bypass transition which is characterized by free-stream vortical disturbances entering into the boundary layer and causing breakdown [28]. Surface roughness, acoustical waves etc are also generally attributed to assist the entry of these perturbations to the boundary layer. Other class of methods and techniques try to prevent this entry.

The other form of classification is in the form of active and passive techniques. Some active techniques use backward and feedforward with an array of sensors and actuators using reactive control to delay the laminar-turbulent transition. The general approach of these techniques is to attenuate the perturbation evolution process. Among passive methods, contouring the surface of an airfoil to maintain a favorable pressure gradient is a passive technique which is more pragmatic.

Du and Karniadakis [26] showed that with a traveling wave, the near wall streaks can be completely eliminated with weakening of low speed streaks. A recent work by Hack and Zaki [36] explores temporal forcings and their effect on bypass transition. Both temporal and spatial oscillations have been explored in this work. In figure 1.3, we see the lower skin friction levels possible with temporal wall forcing. The dashed line shows skin friction coefficient for the reference case where the transition process takes place between $400 < x < 800$ and skin friction reaches its maximum levels at $x = 800$. However, in the case of control being applied in the form of wall oscillations at $x = 400$, the transition process is delayed and the maximum occurs at $x = 1000$ which results in energy savings. This is pretty desirable from energy savings point of view and this alone can reduce 10-12% of the fuel consumption for commercial airliners. In addition, this gives us a window of understanding and clues to explain the exact mechanism of drag reduction process. A physical process to delay the transition process is to stabilize the near wall streaks to prevent their breakdown and eventually result as fully developed turbulence. More details

Figure 1.3: Representative figure for showing the effect of transition delay on skin friction.

have been included in the subsequent chapters.

1.2 Multiscale decomposition of turbulent flows

Multi-scale analysis is widely adopted in turbulence research for studying flow structures corresponding to specific length scales in the Kolmogorov spectrum. Due to complexities in multi-scale structures, turbulent flows are usually difficult to analyze and visualize. Universality of energy spectrum for isotropic turbulence is used to define a range of length scale for flows to characterize the properties of energy contained in eddies and their evolution over different length scales. This can be pivotal in understanding the morphology and dynamics of turbulent flow structures. An exploration in this direction

can be motivated by whether there are universal structures across different scales [29]. For example, finer-scale flow structures tend to be more sheet-like when compared to coarser-scale structures [11], but such a phenomenon is not obvious when directly looking at the original flow, which aggregates all different scales.

A timeline summary of major works and breakthroughs in multi-scale decomposition for turbulent flows has been shown in Fig. 1.4. Different from conventional vector field decomposition, turbulent flow decomposition often employs the Kolmogorov energy spectrum [59, 108] to characterize, or guide, the decomposition process [79]. The use of wavelets and curvelets have dominated this area which are naturally suited for such decomposition because of locality of their basis functions. However, they provide only dyadic or discrete scale decompositions instead of a continuous one which would be more desirable. The evolution of eddy structures across length scales is rather obscure at the moment and such decomposition techniques are helpful to provide insights into the physical mechanism. More details regarding the literature are provided in the introduction section of chapters 6 and 7.

1.3 Organization of thesis

This work specifically delves into three aspects of turbulent flows. Drag reduction due to spanwise wall oscillation is explored in **Chapters 2 & 3**. Two different forms of wall oscillation have been studied i.e. temporal and spatial wall forcings on a flat-plate boundary layer respectively. **Chapters 4 & 5** focus on the transition region with effect of wall oscillations studied on laminar streaks and its influence on transition delay process. **Chapters 6 & 7** summarizes multi-scale analysis techniques for studying turbulent flow

Figure 1.4: Major works in multi-scale decomposition of turbulent flows.

datasets with data-dependent and data-independent approaches are explained. They have been rigorously tested on various DNS datasets both qualitatively and quantitatively. And finally, the thesis ends with general conclusions on various aspects of turbulent flows.

Each chapter is complete in itself with introduction and literature review accompanying them in the beginning. A variety of numerical tweaks have been used on top of a spectral solver for Navier-Stokes equations which have been described in their respective chapters. Results are thereafter discussed with summary and conclusions in the end.

1.4 Contributions

- DNS results for turbulent boundary layer with discontinuous wall oscillation zone have been presented. Most of the previous DNS studies deal with wall-bounded channel flows with periodic boundary condition which makes it simpler from numerical and simulations point of view.
- A predictive relation for the effect of Re on drag reduction has been proposed and speculated based on the available data.
- Downstream variation of turbulent flow statistics has been reported with emphasis on starting point and end of oscillation.
- Steady spatial wall oscillations with half-square waves has been presented for the first time which shows promising results in terms of net power budget.
- Effect of oscillations on low speed streaks were reported. The results show consistency with linear theory but break away from the linearized results (previously published) once the streak is perturbed, setting in non-linear effects leading to flow

transition. This eventually leads to a drastic change in the performance of the drag reducing mechanism.

- Effectiveness of spatial oscillations in delaying bypass transition has been presented for the first time.
- A methodology for multi-scale decomposition (KoSCO) of flow data is developed, which is consistent with band-pass filtering in the Kolmogorov spectrum. It provides an alternative to curvelets and tries to overcome some of their inherent disadvantages. Optimization methods have been used for the first time to design such decomposition procedure.
- In addition to KoSCO, an analytical optimization model which is data independent and exploits the generality of Kolmogorov spectrum for turbulent flows has been developed with potential of real-time interactive decomposition owing to the shorter computing time.

Chapter 2

Wall oscillation induced drag

reduction of turbulent boundary

layers [†]

2.1 Introduction

Turbulence is a flow phenomenon with many technological and engineering applications and the plethora of length and time scales which ranges from large planetary scales to the smallest scales in the order of a micron make analyses challenging. Nevertheless, turbulence control with the objective of reducing skin friction drag has been a subject of intense research in the recent times because of the implied energy and cost savings. Wall motion is one such technique which reduces the skin friction and has been investigated in the past [47] with differing degree of complicated control mechanisms. Also, in the past years, experiments and numerical studies have explored and demonstrated that a

[†]M. Mishra, M. Skote. Submitted to *Intl. J. Heat & Fluid Flow*.

simple wall oscillation can significantly attenuate turbulence. However, even after years of research the exact mechanism for the phenomenon is still largely unknown.

Turbulent flow in a channel with wall oscillation was studied for the first time using direct numerical simulation (DNS) by Jung et al.[46]. Subsequently, Baron and Quadrio [7] reported the effect of the oscillations on the turbulent energy budget. DNS with oscillating walls of both channel and pipe flow were performed by Choi et al.[20] who also proposed a mechanism for drag reduction (DR). They provided a phenomenological explanation related to high speed fluid entering underneath the low-speed streaks and their effects on streamwise vortices to be the prime cause for DR. The transient behaviour of the flow with first few cycles of wall oscillations were analysed by Quadrio and Ricco[67] using DNS of channel flow. They observed that the analytical solution of second Stokes problem matched spanwise velocity profile from the simulation results. In another work by Quadrio and Ricco[68], only lower amplitudes of oscillation were shown to have net energy savings. Toubert and Leschziner [94] focused on performing DNS with sub-optimal oscillation parameters and showed the importance of phase-wise turbulence statistics.

Various attempts to explain the DR phenomenon have been published in last ten years. Xu and Huang[106] offers an explanation of the DR via the Reynolds stress transport equations. Ricco and Quadrio[76] derived a Stokes layer based parameter, which formed a linear relation with the resulting DR. Lately, Ricco et al.[75] have provided a plausible explanation of the DR from an analysis of enstrophy transport in a channel flow with the aid of DNS where they claim that enstrophy is increased due to the Stokes motion and this increase in both enstrophy and dissipation leads to drag reduction. However, this has been contradicted in a recent work by Agostini et al. [1].

Laadhari et al.[53] confirmed that the DR technique also applied to the boundary layer

flow. Subsequently, a lot of the experimental efforts have been focused on the turbulent boundary layer. Choi[21] conducted experiments to study the dynamics of the near wall structures. He described the bending of low speed streaks which was later confirmed by Ricco[73] who also reported the proportionality of the bending angle of the streaks and DR.

Spatial development of the DR was explored in the experimental work by Choi et al.[23] who reported a DR upstream of the start of oscillating portion of the wall. Also, Ricco and Wu[77] investigated the downstream development of the drag reduction but no upstream influence could be detected. In addition, the recent DNSs by Skote[91] and Yudhistira and Skote[109] of a boundary layer with similar oscillating parameters did not indicate any upstream influence.

In the past, the attenuation of Reynolds stresses in boundary layer flows in the zone of wall oscillation has been quantified [53, 97, 22, 77, 109, 91, 92]. The common trend observed is that the Reynolds shear stress diminishes more than the streamwise and normal velocity fluctuations, which is manifested in the attenuation of the correlation function close to the wall, and is confirmed by the present study. The spatial transients of the DR was investigated by Skote[91] and comparison with temporal transients in channel flow pointed towards several similarities. In the present work, we will focus on the spatial transients of the Reynolds stresses.

The influence of Reynolds number on DR is an important question for practical implementation aspects. This is still an open question and a clear understanding of the Reynolds number dependence of DR is presently lacking. In most previous channel flow

investigations though, a consensus seems to be that a relation

$$DR \sim Re_\tau^{-0.2}, \quad (2.1.1)$$

where $Re_\tau = hu_\tau/\nu$ is the Reynolds number based on channel half width (h), friction velocity (u_τ) and kinematic viscosity (ν). However, as indicated by Gatti and Quadrio [32], the value of the exponent in the relation (2.1.1) is most probably dependent on the oscillation parameters, thus a universal relation will be unattainable. In addition, in the study of the Reynolds number averaged Navier-Stokes equations using a perturbation analysis by Belan and Quadrio [9], it is found that a relation of the type

$$DR \sim \gamma + Re_\tau^\alpha \quad (2.1.2)$$

match the data better. Hence, the DR approaches a constant value as $Re_\tau \rightarrow \infty$.

A DNS simulation with same oscillation parameters (but lower Reynolds number) as in Ricco and Wu[77] was performed by Skote[91]. It confirmed that the resulting higher DR in DNS when compared with the experiments, is due to the lower Reynolds number in DNS. In the simulations presented here, the oscillation parameters remain identical. However, the Reynolds number is chosen to exactly match the one in the experiments [77]. Furthermore, two additional simulations at lower Reynolds numbers are presented while retaining the other parameters constant, in order to elucidate the dependency on Reynolds number.

Most of the previous DNS studies have focused on channel flow with the benefits of periodic boundary conditions which reduce the computational costs. However, not much work has been done to explore the effects on a turbulent boundary layer using numerical simulations. Note that the local drag reduction decreases downstream for a spatially growing boundary layer flow in contrast to the channel flow where drag reduction remains

constant in both time and space after attaining an equilibrium value. The same numerical code used for the present investigation has previously been used for both temporal and spatial oscillations of the wall under a turbulent boundary layer[91, 90, 92, 109]. Recently, Lardeau and Leschziner[54] reported their findings for boundary layer flows using DNS where they focused on the transition of the skin friction to a steady low-drag state and perform phase-averaged analysis of turbulent statistics.

In the light of the above discussion of previous works we enlist the aims of the present work. To entrust the fidelity of the results presented, we compare with a previous experiment by replicating the experimental setup. The new contributions to the understanding of DR of turbulent boundary layers with wall oscillations are three-fold. First, the spatial transient of key turbulence statistics are explored for the oscillated and unoscillated (reference) cases. Furthermore, several characteristic differences between the channel and turbulent boundary layer case with wall oscillations are pointed out in this context. Second, based on the current and previous simulations and experiments, a predictive relation for DR at high Re is provided, which differs from the general conclusion of a Reynolds number dependency according to Eq. (2.1.1). Third, spatial transients occurring downstream of the oscillating part of the wall are presented which have never previously been observed due to the limitations in computational box sizes in other boundary layer DNSs.

2.2 Numerical method and simulation parameters

The numerical code and grid resolution is used as previous simulations of oscillating turbulent boundary layer performed by Yudhishtira and Skote[109] and Skote[91]. The numerical code used, SIMSON (A Pseudo-Spectral Solver for Incompressible Boundary

Layer Flows) was originally developed at KTH, Stockholm [19].

An outline of the numerical scheme is presented in section 2.2.1, the various parameters used and the resolution are presented next in section 2.2.2, where also the implementation of the wall motion is discussed.

2.2.1 Numerical Scheme

A pseudo-spectral method with Fourier discretization in the streamwise and spanwise directions, and Chebyshev polynomials in the wall-normal direction has been used. The simulations are started with a laminar boundary layer at the inflow. A random volume force near the wall at the beginning of the computational domain is used to trigger the flow to transition.

At the end of the domain, a fringe region is added, to enable simulations of spatially developing flows. The flow in this region is forced from the outflow of the physical domain to the inflow. In this way the physical domain and the fringe region together satisfy periodic boundary conditions. The implementation is done by adding a volume force

$$F_i = \lambda(x)(\tilde{u}_i - u_i) \quad (2.2.1)$$

to the Navier-Stokes equations,

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} + F_i \quad (2.2.2)$$

The force F_i applies to the fringe region only where $\lambda(x)$ is the strength of the forcing and $\tilde{u}_i$ is the laminar inflow velocity profile the solution u_i is forced to. The fringe function is required to have minimum upstream influence and is designed as

$$\lambda(x) = \lambda_{max} f(x) \quad (2.2.3)$$

Figure 2.1: Function $f(x)$ with $\Delta x_{rise} = 3$ and $\Delta x_{fall} = 3$.

with

$$f(x) = S\left(\frac{x - x_{start}}{\Delta x_{rise}}\right) - S\left(\frac{x - x_{end}}{\Delta x_{fall}} + 1\right). \quad (2.2.4)$$

Here λ_{max} is the maximum strength of the fringe, x_{start} and x_{end} denotes the spatial extent of the region where the fringe is non-zero, Δx_{rise} and Δx_{fall} are the rise and fall distance of the fringe function respectively. Fig. 2.1 shows how the fringe function varies with x with $\Delta x_{rise} = 3$ and $\Delta x_{fall} = 3$. $S(\eta)$ is a continuous step function that varies from zero for $\eta \leq 0$ to unity for $\eta \geq 1$, and is given by,

$$S(\eta) = \begin{cases} 0, & \eta \leq 0, \\ 1/(1 + e^{(1/(\eta-1)+1/\eta)}), & 0 < \eta < 1, \\ 1, & \eta \geq 1. \end{cases} \quad (2.2.5)$$

The advantage of this expression is that $S(\eta)$ has continuous derivatives of all orders.

The function $f(x)$ is also used in enforcing the wall oscillation boundary condition as described in section 2.2.2 below.

The time integration is performed using a third-order Runge-Kutta-scheme for the non-linear terms and a second-order Crank-Nicolson method for the linear terms. A 3/2-

Figure 2.2: Illustration for the problem setup.

rule is applied to remove aliasing errors from the evaluation of the non-linear terms when calculating FFTs in the wall parallel plane.

2.2.2 Numerical parameters

All quantities are non-dimensionalized by the free-stream velocity (U_∞) and the displacement thickness (δ^*) at the starting position of the simulation ($x = 0$), where the flow is laminar. The Reynolds number is set by specifying $Re_{\delta^*} = U_\infty \delta^* / \nu$ at the laminar inlet ($x = 0$).

Wall oscillation is imposed at different locations to examine the influence of Re_Θ ($Re_\Theta = U\Theta/\nu$, Θ being the momentum thickness) on DR. Three different Re_Θ were

tested at 375, 505 and 1400. These Re_{Θ} values refer to the Reynolds number at which the oscillation starts. The streamwise location at which the oscillation was introduced corresponds to $x = 140$ (case 1) , $x = 250$ (case 2) and $x = 939$ (case 3). The third case has been setup to exactly replicate the conditions used by Ricco and Wu[77] in their water tunnel experiments. An additional simulation (case 4) has been setup to answer some questions which would be posed later in sections 2.3.5. The simulation denoted as case 2 has been presented earlier by Skote[91]. Since cases 1 and 2 are performed at a lower Re , the computational box is in these cases smaller. For case 3, a much longer box is used to accommodate the growth of Re before the oscillations commence. Table 2.1 gives the different parameters chosen for the computational box and grid. The length of the computational box is given in simulation length units ($\delta_{x=0}^*$). Note that cases 1 and 2 do not include the recovery region downstream of the point where oscillations cease.

An illustration for the current problem setup is shown in Fig. 2.2. After an initial trip of the flow, the flow undergoes transition to fully turbulent flow. A section of this turbulent flow is then subjected to wall oscillations. At the end of the domain, there is a fringe region to satisfy periodic boundary condition as discussed in the previous section.

Due to the fringe region, the results from the end 150 simulation units for case 1 and case 2, and 400 units for case 3 have been discarded to ensure that there are no upstream effects. The transition region is between $x = 5$ and $x = 100$. Thus, the region of a fully developed turbulent boundary layer, free from any influence of the numerical method, is $x = 100 - 450$ for case 1 and 2 and $x = 100 - 2000$ for case 3.

The implementation of the oscillating spanwise wall-velocity is identical to the one used by Skote[91]. The wall oscillation is applied in the spanwise direction at a particular region in streamwise direction. Therefore, a profile function $f(x)$ is utilized to define

the domain where the oscillation occurs. Furthermore, the gradual increase of the profile function prevents the spurious oscillations (Gibb's phenomena) which might otherwise be introduced due to a discontinuous jump in the velocity around the starting point of wall forcing.

The form of the wall velocity boundary condition is given by

$$W|_{y=0} = f(x)W_m \sin(\omega t) \quad (2.2.6)$$

where $f(x)$ is the same profile function as used for fringe region, see Eq. (2.2.4), and W_m is the maximum wall velocity. The parameter ω is the angular frequency of the wall oscillation, which is related to the period through $\omega = 2\pi/T$.

The resolution used for the simulations are summarised in Table 2.1 as shown in + units. Note that unless otherwise stated, the + superscript indicates that the quantity is made non-dimensional with the friction velocity of the unmanipulated boundary layer (the reference case), denoted u_τ^0 , and the kinematic viscosity (ν).

The sampling time for the reference case was 16800 in time units ($\delta_{x=0}^*/U_\infty$), and started only after a stationary flow (in the statistical sense) was reached. In the cases with wall forcing, the reference case was used as the initialized flow field and an additional 4000 time units was simulated before statistics was sampled during another 12000 time units, corresponding to 180 periods of oscillation.

In the simulations presented here, the angular frequency (ω) of the wall oscillation is set to 0.0909 and maximum wall velocity (W_m) is set as 0.5028; which in wall units corresponds to $T^+ = 67$ and $W_m^+ = 11.3$ based on u_τ^0 . Note that the value of W_m^+ and T^+ changes with streamwise position because local u_τ varies whereas W_m and T are kept fixed.

Case	L_x	L_y	L_z	N_x	N_y	N_z	Δx^+	Δy_{min}^+	Δz^+	W_m^+	T^+	x_{start}	x_{end}	$Re_{\Theta start}$
1	600	30	34	800	201	144	16	0.04	5.1	11.3	67	140	450	375
2	600	30	34	800	201	144	16	0.04	5.1	11.3	67	250	450	505
3	2400	75	34	3200	301	144	20	0.04	5.1	11.3	67	939	1165	1400
4	2400	75	34	3200	301	144	20	0.04	5.1	11.3	67	500	1500	963

Table 2.1: Numerical Parameters for the simulations presented.

2.3 Results

The results from the reference case are compared with the data from Schlatter and Örlü[84] who performed extensive DNS of flow over flat plate and compared with several experiments, and can now be considered as the standard database for comparison of turbulent statistics. Fig. 2.3 shows a comparison for the skin friction coefficient at different Re_{Θ} for the data from Schlatter and Örlü[84] with our current DNS results. The results show a good match and hence, for comparison of drag reduction, where c_f which is a significant quantity can be assumed to be matching well with other databases.

In the following sections 2.3.1 - 2.3.4, results from case 3 are presented while all three cases are used for the analysis in sections 2.3.5 - 2.3.7.

2.3.1 Comparison of DR with experiments

This section discusses the comparison of results with experiments on temporal wall oscillations performed in water tunnel[77]. The oscillation parameters used in the experiment are $T^+ = 67$, $W_m^+ = 11.3$ at $Re_{\Theta} = 1400$. Temporal oscillations were imposed on the wall for a region of 10δ in the streamwise direction where δ is the boundary layer thickness at the start of the oscillation. The present simulation (case 3) has been setup to exactly

Figure 2.3: Skin friction coefficient, c_f from (—) present DNS and (o) from Ref. [84].

match the experiments by starting the wall oscillation at the streamwise location where $Re_\theta = 1400$ and for a region of 10δ which corresponds to $x = 939 - 1165$ where the wall boundary condition is modified from the no-slip boundary condition according to Eq. (2.2.6).

The data for the first 50 oscillations is discarded to reach stable drag reduction values. In order to verify the convergence, statistics were compared for two time windows t_1 and t_2 , each consisting of 90 oscillations. Reynolds shear stress $(\overline{u'v'})$ was compared for these two time windows which is the most sensitive statistical quantity[109] with respect to the total sampling time, and was found to have reached a stationary state. The data presented here has been averaged for time interval $t_1 + t_2$ equivalent to 180 oscillations.

The resulting drag reduction (DR) is calculated from

$$DR(\%) = 100 \frac{c_f^0 - c_f}{c_f^0}, \quad (2.3.1)$$

where c_f^0 is the skin friction of the reference case. Variation of $DR(\%)$ in the streamwise

direction is shown in Fig. 2.4 and is compared with experimental data provided by Ricco and Wu[77]. There is a strong response to wall oscillations introduced at $x/\delta = 0$ in the first 1δ region. The experimental data exhibit a slightly more rapid increment of DR than the simulation results, which can be attributed to the slower response in the DNS due to the profile function when implementing the wall oscillation (as shown in Eq. (2.2.6)). After the initial spike in DR values, the increase is gradual from 1δ to 3δ reaching an equilibrium value range from 23% – 25% which until 10δ remains slightly higher ($\approx 1\%$) than those reported from the experiment. Thereafter, the oscillation is stopped at 10δ and the DR decays with a response that matches the experimental values quite remarkably, and suggests the high fidelity of the current simulations. Note also that the measurement data after the oscillation has ceased are more accurate since they are more straight forwardly obtained there (Pierre Ricco, private communication). In addition, the present results also match a similar experiment by Trujillo et al.[97] as shown in Fig. 2.4 (filled squares).

The attenuation of DR once oscillation is stopped shows a faster response compared to the corresponding spatial transient where the oscillation is introduced. In addition, after the oscillation has been stopped, drag increase (DI) is observed for a region of 2δ but the precise shape of the downstream DI is not conclusive. The DR is calculated through the ratio between skin frictions (see Eq. (2.3.1)) which are highly fluctuating quantities and the variations are enhanced when the ratio is calculated. Very small perturbation of the values may be amplified when calculating the DR according to Eq. (2.3.1). As can be seen in Fig. 2.5, where the skin friction is presented, the two values are not very far apart once the oscillation is stopped. Thus, in order to obtain the complete detailed picture of the DI development after the oscillation stops, simulation would need to be continued

Figure 2.4: DR(%) as a function of streamwise coordinate for $T^+ = 67$, $W_m^+ = 11.3$, (—) present DNS results as compared to experimental results from (o) from Ref. [77] , (■) from Ref. [97].

for a very long time that present resources do not permit. On the other hand, the effect of the eventual DI is small and would not affect the energy budget (see §2.3.7) in any considerable way.

2.3.2 Turbulence Statistics

In the presentation of results in this section, the scaling is done using the reference friction velocity u_τ^0 at the streamwise location from which profiles are presented. Also, an alternative scaling using the local friction velocity u_τ is presented and mentioned where used.

Mean streamwise velocity profile at $x/\delta = 6.4$ is compared with the experimental data in Fig. 2.6. This position was chosen by Ricco and Wu[77] to avoid any transitional effects, and the same location has been chosen here for the sake of comparison. However, the experimental results are for a different set of parameters at $W_m^+ = 9$, $T^+ = 83$ which

Figure 2.5: Skin friction coefficient for (—) unmanipulated boundary layer and (---) case 3 with wall oscillation.

accounts for some of the differences in the log region of the turbulent boundary layer. As observed in Fig. 2.6a, the mean velocity profiles from the oscillating case almost collapse to the unmanipulated case when normalized with u_τ^0 . Note that profiles in this case do not collapse onto $U^+ = y^+$ near the wall. When normalized with local u_τ which has a lower magnitude due to DR, it can be seen in Fig. 2.6b that the log layer shifts in the wall-normal direction which indicates the upward shift of turbulent fluctuations from the near wall region resulting in a diminished skin friction coefficient. The log layer from experiments falls on top of the present DNS results. Some differences can be seen for $y^+ < 20$ because for this set of experimental parameters, the DR is slightly higher than for $W_m^+ = 11.3$, $T^+ = 67$. These two scaling parameters (u_τ, u_τ^0) cannot provide the full picture and obviously, a third scaling which pertains to the logarithmic region might be necessary to elucidate the effects of oscillation on velocity profile. As discussed by Quadrio[66], using the u_τ^0 as velocity scale is equivalent to scaling using outer variables. On the other hand, using the local scale (u_τ) does provide a proper scaling in the viscous

Figure 2.6: Comparison of mean velocity profile at a streamwise location of $x/\delta = 6.4$ scaled with u_τ . (—) Results from DNS for unoscillated case, (---) Results from DNS for $W_m^+ = 11.3$, $T^+ = 67$, (o) Results from experiments[77] for $W_m^+ = 9$, $T^+ = 83$.

sublayer as the equation for the near-wall region remains unchanged for the oscillating boundary layer. However, in the log region, the local u_τ does not yield a collapse of velocity profiles.

Root mean square (rms) of streamwise velocity fluctuations (u_{rms}) are compared in Fig. 2.7a and 2.7b with the two different scalings. When scaled with u_τ^0 , the peak value decreases by 17% while it is reduced to 3.5% when scaled with u_τ . Both of these profiles are in good agreement with the experiments with a nearby set of parameters. u_τ^0 is obviously larger than u_τ and hence, the reduced decrease in u_{rms} with change of scaling.

Likewise, r.m.s. of wall-normal velocity fluctuations are shown in Fig. 2.7c and Reynolds shear stress, $\overline{u'v'}$ in Fig. 2.7e. As one would expect, v_{rms} and $\overline{u'v'}$ have reduced peaks by 7.5% and 21% respectively when scaled with u_τ^0 from reference case. Considerably lower values compared to the unoscillated case are observed for $\overline{u'v'}$ and is one of the principle reason for DR phenomenon. There is a small difference in the

Figure 2.7: R.m.s. of velocity fluctuations and Reynolds Shear stress at $x/\delta = 6.4$. (—) Results from DNS for unoscillated case, (---) Results from DNS for $W_m^+ = 11.3$, $T^+ = 67$, ($\circ$) Results from experiments by Ricco and Wu [77] for $W_m^+ = 9$, $T^+ = 83$. Note: For (d) and (f), the experimental u_{τ} values have been approximated by using (a) and (b) .

peak values with experiments which can be attributed to the fact that the oscillation parameters are different and result in slightly different DR. However, similar trends are

observed in the DNS and experimental profiles, e.g. the increase in values just before the rapid descent to zero at the edge of the boundary layer as seen in Fig. 2.7e. The peak reduction in the Reynolds shear stress found using DNS here is $\sim 21\%$ whereas Ricco and Wu[77] reported it as 25% at $W_m^+ = 9$, $T^+ = 83$. Data by Trujillo et al.[97] shows a peak reduction of $\sim 20\%$ which is closer to the present DNS results.

On the other hand, when local scaling is used, quite unexpectedly, the peak values for both v_{rms} and $\overline{u'v'}$ show an increase by 8% and 6% respectively as shown in figures 2.7d and 2.7f. The experimental results were not available with normalization by u_τ and it has been approximated based on the u_{rms} values for both scaling from figures 2.7a and 2.7b. With the interpolated experimental values, one may observe the increased fluctuations. The effect is more exaggerated in the present DNS results due to higher peak values than from experimental set of chosen parameters. More discussion on this phenomena will be discussed in section 2.3.5 where the peak values of these fluctuations are studied for varying Re_Θ .

For all these measures of turbulent fluctuations and shear stresses, the peak value occurs at a higher y^+ than for the reference case which indicates the wall-normal shift of disturbances and hence, the reduction of near wall turbulence. In addition, there is a clear pattern of reduction in fluctuations for $y^+ < 10$ indicating attenuation of near wall turbulence.

A correlation coefficient for a turbulent flow which quantifies the relative magnitude of the shear stress with respect to velocity fluctuations is given by

$$R_{uv} = \frac{\overline{u'v'}}{u_{rms}v_{rms}} \quad (2.3.2)$$

As can be seen from Fig. 2.8, there is a significant reduction in the correlation coefficient

Figure 2.8: Correlation coefficient R_{uv} at a streamwise location of $x/\delta = 6.4$. (—) DNS of the unoscillated case, (---) DNS with $W_m^+ = 11.3$, $T^+ = 67$, (o) Experiments by Ricco and Wu[77] for $W_m^+ = 9$, $T^+ = 83$. The vertical line represents $y^+ = 29$ at the edge of the Stokes layer.

near the wall. The thickness of a Stokes shear layer is given by $\delta_s^+ = \sqrt{4\pi T^+} = 29$ which is indicated in Fig. 2.8 with a vertical line. It can be clearly seen that for the oscillating wall, there is greater reduction of Reynolds shear stress than the velocity fluctuations below the edge of Stokes shear layer, above which it falls to the same values as the unmanipulated case. These findings match well with the experiments which are also shown in Fig. 2.8. Furthermore, it is important to note that the ‘bump’ occurring at $y^+ \sim 10$ for the reference case is absent in both experimental and simulation oscillation results. This can be attributed to the drastic reduction in Reynolds shear stress, $\overline{u'v'}$ which shows a considerable deviation at $y^+ = 10$ between the unmanipulated and manipulated cases respectively. In addition, due to the wall oscillation, the coherence between the two components of velocity is broken, and there is greater relative reduction of peak values for $\overline{u'v'}$ compared to u_{rms} and v_{rms} .

2.3.3 Streak Visualization

The bending of low speed streaks which was first reported by Choi[21] was studied extensively by Ricco[73] for turbulent boundary layer flows using experiments. Bandyopadhyay[6] proposed a drag reduction mechanism based on the reorientation of vorticity and correlated it to Stokes second problem. Recently, Toubert and Leschziner [94] presented a detailed analysis of streak modification in a channel flow and reported periodic streak reorientation in the near wall region based on the phase of the oscillation. Fig. 2.9 shows the results from the visualization experiments [73] on a boundary layer flow in the streamwise-spanwise ($x - z$) plane, where the flow was visualized by hydrogen bubbles generated by hydrolysis of water obtained by means of an energized platinum wire at $y^+ = 5$. Low speed streaks were recognized by identifiable single longitudinal bubble structures. As can be seen in Fig. 2.9a, the low speed streaks (the stretched bubble) remain stable for some distance and then burst. Fig. 2.9b shows that these streaks are still present once the oscillation is applied. Furthermore, the streaks are oblique in the region where oscillation takes place. A similar visualization was done based on the present DNS results and are shown in Fig. 2.9c-f. For the unoscillating section of the flow, the low speed streaks are clearly visible as seen in Fig. 2.9c and maintain the typical spanwise spacing of 100 wall units.

After the oscillation for the wall is introduced, the streaky structures can be identified as shown in figures 2.9d and their characteristic bending within the region of oscillation is illustrated in Fig. 2.9e. The inclination of the streaks remains constant in the zone of oscillation. Once the oscillation is stopped (Fig. 2.9f), the angle of streaks goes back to the streamwise direction. It is worthwhile to note that the flow shows a fast response by

Figure 2.9: For all the above figures, flow is from left to right (a) Low speed streak visualization for unoscillated flat plate by Ricco[73], (b) Inclination of low speed streak on applying wall oscillation [73], (c) Low speed streaks (darker regions) from present DNS results for the unoscillated flat plate, (d) Wall oscillation which begins from the solid line shows the bending of low speed streaks, (e) Within the zone of oscillation the streaks remain inclined throughout, (f) Solid line denotes the end of wall oscillation after which the streaks again align themselves to the streamwise direction.

Figures (a) and (b) reprinted from Modification of near-wall turbulence due to spanwise wall oscillations, Pierre Ricco, Journal of Turbulence, 1 July 2004, by permission of Taylor & Francis Ltd, <http://www.tandf.co.uk/journals>

Figure 2.10: Illustration for mechanism behind bending of low speed streaks.

streak reorientation both when introducing the oscillation and when it ceases.

The angle of inclination of streaks (γ) is a consequence of that while the freestream tries to convect the low speed streaks in the streamwise direction, there is at the same time a spanwise component of velocity due to the wall oscillation. This results into a vector addition of the two velocity components which leads to the bending of streaks as illustrated in Fig. 2.10. By utilizing the Stokes solution for the second problem, the maximum deflection angle of streak with respect to streamwise direction can be calculated [73]

$$\gamma = \tan^{-1} \left(\frac{W_m^+ \exp(-y^+ \sqrt{\pi/T^+})}{U_\infty^+} \right) \quad (2.3.3)$$

Ricco[73] mentioned that DR is proportional to γ . For $DR \sim 25\%$, he reported a $\gamma = 27^\circ$ while for the present DNS data we obtain $\gamma = 25^\circ$. In addition, from Fig. 2.9c and 2.9e, one can observe that the relative spacing between the streaks has increased. This increased spacing was also observed by Ricco[73] for DR values higher than 25%.

Furthermore, from these figures one may observe the reduced lengths of the streaks. This restructuring and reorientation of low speed streaks could be one plausible explanation for the skin friction attenuation.

2.3.4 Comparison with analytical solutions

For laminar flow produced by an oscillating wall has an analytical solution[8] (Stokes second problem) which gives the boundary layer profile:

$$W(y, t) = W_m \exp(-\eta) \sin(\omega t - \eta) \quad (2.3.4)$$

where $\eta = y\sqrt{\omega/2\nu}$.

The profile (2.3.4) is not an exact expression of W in this case, since the spatially developing boundary layer has been approximated with a parallel flow (hence, the normal velocity is zero and W is independent of x in the spanwise momentum equation). Furthermore, the Stokes solution does not take into account the spatial transient of the velocity profile after the introduction of wall oscillation, or in other words, the classical Stokes solution is independent of the streamwise location. An analytical solution for the spatially developing flow with half-plane motion has been derived by Zeng and Weinbaum[111]. This solution is valid for an identical schematic setup as the present as shown in Fig. 2.2 and the profiles are given as a function of the streamwise coordinate to show the spatial transients involved.

Dimensionless variables are defined as $\tau = \omega t$, $\xi = x/(2\nu/\omega)^{1/2}$, $\eta = y/(2\nu/\omega)^{1/2}$ where $(2\nu/\omega)^{1/2}$ is the penetration depth for Stokes first problem. Within the Stokes layer, the dimensionless governing equation and boundary conditions can be written as

Figure 2.11: (a) $W(\eta)$ at $\xi = 0.5$ (---) Eq. (2.3.8) and (—) Stokes solution (2.3.4) ; (b) (—) Stokes solution and (---) DNS results for case 3; The four set of curves are at time instants with maximum, minimum and zero wall velocity.

$$\frac{\partial W}{\partial t} = 2\nabla^2 W \quad (2.3.5)$$

$$W = 0 \quad \text{at} \quad \eta = 0, \quad \xi < 0 \quad (2.3.6)$$

$$W = \sin(\tau) \quad \text{at} \quad \eta = 0, \quad \xi > 0 \quad (2.3.7)$$

The solution given by Zeng and Weinbaum[111] is,

$$W(\xi, \eta, \tau) = \Re \left\{ \exp(i\tau) \left[\frac{1}{2} \exp(-(1+i)|\eta|) + \frac{1+i}{\pi} \int_0^\xi \frac{|\eta|}{(x'^2 + \eta^2)^{1/2}} K_1((1+i)(x'^2 + \eta^2)^{1/2}) dx' \right] \right\} \quad (2.3.8)$$

where K_1 is a modified Bessel function of the second kind and the solution is forced to be symmetric about $y = 0$ by using $||$.

A spatial transient exists in the velocity profiles just as oscillation is imposed with half-plane oscillations and this effect is visible in the range $0 < \xi < 1$. Fig. 2.11a shows the difference between Stokes solution and the spatial dependence of the velocity profile

due to half-plane oscillation at $\xi = 0.5$. However, this spatial transient disappears for $\xi > 1$ after which the profile reduces to the one given by Stokes solution (2.3.4). The streamwise coordinate at $\xi = 1$ is $(2\nu/\omega)^{1/2} = 0.2036$ beyond which the spatial transient cannot be observed. Due to the use of the step function (Eq. (2.2.6), for case 3, between $939 < x < 949$, corresponding to $0 < \xi < 49$) in the implementation of wall oscillation, it is hard to compare such spatial transients in the velocity profiles for the current simulations.

Comparison of DNS profiles from case 3 match reasonably well with the classical Stokes solution as shown in Fig. 2.11b. The instantaneous DNS profiles have been taken after a sufficiently long time (after ~ 180 oscillation cycles) at four different instants when the wall velocity is at its maximum, minimum and zero value. The profiles are shown for streamwise location where there are no spatial gradients present ($x = 1100$). Note that the profiles have been averaged only in the spanwise direction. See Appendix A for more.

2.3.5 Drag Reduction and dependency on the Reynolds number

In this section, all four cases with different streamwise locations for introduction of wall oscillation are investigated with respect to the Re_Θ . The influence on DR is shown in Fig. 2.12. With this presentation of the DR versus Reynolds number, it seems plausible that the decrease in DR in the streamwise direction for each of the three cases is decoupled from the reduction of maximum DR with Reynolds number at which the oscillation starts (or equivalently, the Reynolds number dependency of the DR in channel flow configuration). For channel flow geometry, there is no variation in Reynolds number, and hence only one (constant) value of DR is obtained. Hence, the DR in a boundary layer is a much more complicated affair than in a channel flow, and we cannot simply perform a high-Reynolds number channel flow simulation or experiment, and trust that the energy saving

Figure 2.12: $DR(\%)$ as a function of Re_Θ with wall oscillation introduced at (---) $Re_\Theta = 375$ (case 1), (—) $Re_\Theta = 505$ (case 2), (— · —) $Re_\Theta = 1400$ (case 3), (o) from Ref. [77].

will remain identical for a boundary layer. On the other hand, the temporal transients in the case of channel flow possess similar characteristics as the spatial transients in the boundary layer, as shown by Skote [91] using the DNS data denoted as case 2 in the present paper. However, the spatial transients has not disappeared yet as is clearly indicated in Fig. 2.12. Thus, even though the maximum DR at the beginning of the simulation is equal to the corresponding channel flow case, the spatial development of the DR in the boundary layer constitutes a fundamental difference between the two flow geometries.

Note that the W^+ value actually increases slightly downstream in these simulations since the amplitude (W) itself is kept constant. To use a truly constant W^+ would not produce much different results since u_τ is varying very little over the downstream region where the oscillation is applied. In fact, the reduction in DR would be even greater in such a simulation since the amplitude (W) would have to decrease downstream in that

Figure 2.13: DR(%) as a function of Re_θ with wall oscillation introduced at (---) $Re_\theta = 375$ (case 1), (—) $Re_\theta = 505$ (case 2), (— · —) $Re_\theta = 1400$ (case 3), (····) $Re_\theta = 950$ (case 4), (o) from Ref. [77].

case.

There can be two possible directions for the falling DR values as shown by the two arrows in Fig. 2.12. The DR would eventually reach a significantly smaller value if the trend in Fig. 2.12 is equally strong for all downstream positions and is shown by the solid arrow. On the other hand, if the fall of DR saturates beyond some Re_θ , then the dashed arrow indicates an alternative trend which suggests higher DR values for large Re_θ , and would also overlap with the DR levels for case 3 where the introduction of oscillations starts further downstream. Thus, one concern about the boundary layer yet to be determined is if the downstream variation of DR, after the initial spatial transients, settles to an identical relation $DR \sim Re^\alpha$ as found in channel flow case, or if the DR variation in the downstream direction is completely different (as indicated by the solid line in Fig. 2.12).

Figure 2.14: $\max(DR(\%))$ as a function of Re_Θ , (o) shows results for present DNS with wall oscillation; (x) for channel flow from Ref. [68], (+) for boundary layer experiments from Ref. [77] and (*) for channel flow simulations from Ref. [20]; (o) DNS for channel from Ref. [94]; (---) shows the curve $5068/Re_\Theta + 27.66$; (—) shows the curve $2884/Re_\Theta + 23.44$; (...) shows the curve $2679/Re_\Theta + 20.12$; (-.-) shows the variation of Re_Θ^α where $\alpha = -0.1686$.

In order to answer the question posed in figure 2.12, we overlap this result with case 4 results. As shown in figure 2.13, the route which DR takes is not that of rapid deprecation as shown by the simulations at lower Re (shown with solid arrow in figure 2.12). So it is clear from case 4 results that the actual route taken would be closer to the dashed arrow. However, it raises several questions. Case 4 results do not match up with case 3 results for the same Re_Θ . This indicates there seems to be a dependence on the starting position of oscillation to the streamwise variation. This might be critical since the dependence of DR on $Re_{\Theta_{start}}$ could be an important factor in determining the drag reduction levels for practical usage.

The precise variation of the DR with Re_Θ cannot be determined by the short simula-

tions presented here, and a much longer computational box with oscillations applied over the entire wall must be performed. Note that the reduction of DR with local Re_Θ shown in Fig. 2.12 is more severe than the decrease of maximum DR with Re_Θ . Hence, the slope of the DR variation downstream of maximum DR for each of the three cases in Fig. 2.12 is steeper than a line connecting the maximum of DR (this line is plotted in Fig. 2.14). For the remainder of this section, we will focus on the maximum DR and its dependency on Reynolds number where the oscillation starts. There are more data to compare with since the DR from the channel flow can be utilized for the analysis in this case.

Choi et al.[20] have provided results for DR in a channel for different Re_τ with $W_m^+ = 10$ and $T^+ = 50$. For comparison w.r.t Re_Θ , the relation $Re_\tau = 1.13Re_\Theta^{0.843}$ as given by Schlatter and Orlu [84] has been used for conversion. As can be seen in Fig. 2.14, the DR values decrease with Re_Θ for both DNS and experimental data. In addition, the present DNS shows good agreement with other channel and boundary layer flow experiments. By using curve fitting the following relation is arrived at:

$$\max(\text{DR}(\%)) = A/Re_\Theta + B \quad (2.3.9)$$

where $A = 2884$ and $B = 23.44$. This relation suggests that as Re_Θ becomes larger, DR approaches a constant value of 23.44% for this particular set of oscillation parameters ($W_m^+ = 11.3$, $T^+ = 67$). For the data from Ref. [20] with $W_m^+ = 10$ and $T^+ = 50$, the analysis yields $A = 2679$ and $B = 20.12$ in the expression (2.3.9). Also, recent results from Toubert and Leschziner[94] at a higher Re_Θ for channel follow a similar trend with $A = 5068$ and $B = 27.66$ respectively. That the coefficients in (2.3.9) depend on the oscillation parameters is consistent with the investigation of the power law relating DR with Re for channel flow. Furthermore, the analysis by Belan and Quadrio[9] indicated

clearly that a constant DR is obtained in the limit of very high Re , which is also consistent with findings of Iwamoto et al.[40].

On the other hand, a power law relation was proposed by Choi et al.[20] according to Eq. (2.1.1). Using the relation between Re_Θ and Re_τ above, we can derive $DR \sim Re_\Theta^{-0.1686}$. This relation is shown with dash-dot line in Fig. 2.14. Note that this power law does not fit the trend of the available data well. Although the rapid decay of DR at low Re is captured by the power law, the trend at higher Re where the values seem to reach a constant value has no correspondence to the power law which indicate a continuous decaying DR at high Re . In addition, the power law suggests that the DR approaches zero in the limit of infinite Reynolds number, which one might find peculiar. However, this is consistent with other consequences of $Re \rightarrow \infty$. For example, the logarithmic friction law yields that the friction velocity goes to zero ($u_\tau \rightarrow 0$), hence W_m also approaches zero in order to keep W_m^+ constant. Furthermore, since $u_\tau \rightarrow 0$, so does the skin friction ($c_f \rightarrow 0$) and thus there is no viscous drag to reduce. In short, $DR \rightarrow 0$ because there is no drag to reduce when $Re \rightarrow \infty$. Letting $Re \rightarrow \infty$ is in effect equivalent to removing the boundary layer completely. Indeed, $Re \rightarrow \infty$ can easily be accomplished by letting viscosity vanish.

2.3.6 Downstream development of Reynolds stresses

A comparative illustration of the effect of u_{rms} , v_{rms} and $\overline{u'v'}$ by plotting their maximum values at different Re_Θ (or different x positions) for case 1, 2 and 3 is presented. In Fig. 2.15, the quantities have been scaled with u_τ^0 from the reference case. Values are extracted from locations in the zone of oscillation with the first one being at the starting position for oscillation. Solid vertical line denotes the end of oscillation. Previously, in Fig. 2.7a,c,e, variation of these quantities in wall-normal direction was shown at $x/\delta = 6.4$

Figure 2.15: (Color online) Unmanipulated case (in black) with $(-)$ $\max(u_{rms})$, $(-)$ $\max(v_{rms})$ and $(--)$ $\max(-\overline{u'v'})$; For case 1 (in green), case 2 (in red) and case 3 (in blue). Solid vertical line marks the end of oscillation. Dashed vertical line shows the location of $x/\delta = 6.4$ used in Fig. 2.7. All scaling is done with u_τ^0 from the reference case.

and this position is marked by a dashed vertical line in Fig. 2.15. Streamwise velocity fluctuations shows more drastic decrease in its maximum value as soon as wall oscillation is implemented compared to the wall-normal and Reynolds shear stress values. This can be attributed to the fact that the bending of the low speed streaks which was shown to be instantaneously changing direction in figures 2.10b,d. The reorientation of the longitudinal vortices could be the reason for this local dip in peak values of u_{rms} while v_{rms} and $\overline{u'v'}$ are less sensitive initially to the introduction of wall oscillation. However, $\overline{u'v'}$ experiences

the maximum reduction in terms of relative change from the unmanipulated case. The peak drop in values for u_{rms} , v_{rms} and $\overline{u'v'}$ were 26%, 19% and 35% respectively and they rise further downstream to reach values of 16%, 17% and 29% respectively for case 3. For case 1 and 2, the peak values of v_{rms} and $\overline{u'v'}$ exactly coincide while a small difference in u_{rms} exists. The DR values also coincide at these points ($Re_\Theta > 650$) as can be seen in Fig. 2.12.

Furthermore, spatial transients of the Reynolds stresses once wall oscillation is introduced can be observed in Fig. 2.15. This is more pronounced for lower Re cases. There is a sudden drop in the peak values after which a non-monotonic variation follows before they reach a constant value. A similar observation was made for the temporal transients in channel flow by Quadrio and Ricco [67]. In their case, the non-monotonic behaviour was recorded for increasing number of cycles of oscillation. In addition, they reported a slight local maximum in the peak values before they reach the steady values. This additional transient effect has no spatial correspondence in the present boundary layer flow. On the other hand, we observe a peak for u_{rms} and $\overline{u'v'}$ slightly downstream of where the oscillation is stopped. This can only be detected for case 3 (see Fig. 2.15) as only for this case the simulation continues downstream of the oscillating wall. Also in previous channel flow simulations where the complete wall is forced into oscillation (due to stream-wise periodicity in those simulations [54]) these spatial effects do not exist. However, the peaks observed in the present boundary layer downstream of the point where oscillations cease, could be interesting to compare with channel flow temporal transients which could only be revealed if the oscillations were to be abruptly stopped during the simulation.

When the scaling is turned to local u_τ , a different pattern emerges and is shown in Fig. 2.16. Here again, end of the oscillation zone has been marked by a solid line and a dashed

Figure 2.16: (Color online) Unmanipulated case (in black) with (—) $\max(u_{rms})$, (---) $\max(v_{rms})$ and (- - -) $\max(-\overline{u'v'})$; For case 1 (in green), case 2 (in red) and case 3 (in blue). Solid vertical line marks the end of oscillation. Dashed vertical line shows the location of $x/\delta = 6.4$ used in Fig. 2.7. All scaling done with local u_τ from the respective cases.

line shows the location of $x/\delta = 6.4$. There is not much difference in how u_{rms} behaves except that the peak value increases and is still lower than the reference case. However, v_{rms} and $\overline{u'v'}$ show increase in values above the reference case which was also observed from the profiles in Fig. 2.7d,f. This indicates that the relative decrease in the *rms* values is lesser than the decrease in the u_τ , which in turn manifests itself when changing the normalizing parameter. Although u_{rms} and $\overline{u'v'}$ peak values seem to be stabilized after $x/\delta = 6.4$, v_{rms} still has a decreasing trend for case 3.

Figure 2.17: Ratio of maximum of turbulent statistics for reference and case 3, (· · ·) u_{rms} , (—) v_{rms} and (— —) $\overline{u'v'}$. Shaded region marks the zone of oscillation.

For case 1 and 2, u_{rms} does not reach the values of the reference case since the simulation domain is short and the recovery is not included. For case 3, on the other hand, the local maximum in the recovery zone is seen more clearly in this scaling compared to previously in Fig. 2.15. This post-oscillation peak is clearly visible for u_{rms} and $\overline{u'v'}$ whereas v_{rms} shows a local minimum at the same location. A slight drag increase is observed at this point of local Reynolds stress maximum/minimum as can be seen in Fig. 2.4. This has not been reported before due to the limitation of channel flow cases where the entire stretch of the wall undergoes oscillation. A discontinuous zone of oscillation in the streamwise direction, as in the present DNS (case 3), provides this new insight into the spatial transient of peak turbulent fluctuations downstream of the termination of oscillations.

In order to investigate the spatial transient of Reynolds stresses independent of the choice of scaling, we calculate the ratio between the peak values from the reference case and case 3. In Fig. 2.17, this ratio is shown for u_{rms} , v_{rms} and $\overline{u'v'}$. This clearly reveals comparative difference in response of the three statistical quantities with the advent of oscillation. While u_{rms} and $\overline{u'v'}$ have reached their minimum values within the zone of oscillation, v_{rms} is the only quantity which is still decreasing at the point where oscillation stops. Even after the termination of oscillation, it continues to decrease for a short distance downstream. This results in a delayed increase up to the reference value. The delay is augmented by the slower rate of increase compared to the other two components. $\overline{u'v'}$ is the only one showing an overshoot after the oscillations, but it rapidly goes back to the reference level. Hence, the Reynolds shear stress is the only component consistent with the small DI observed after the oscillations stop. Note that there is a small spatial lag between $\overline{u'v'}$ and u_{rms} during the decay after the oscillations. u_{rms} has the most immediate recovery exactly where the oscillation stops which indicates that the streaks are quickly regenerated. In Fig. 2.17, it looks as if u_{rms} starts decaying upstream of the point of oscillation cessation but it is the effect of the profile function (2.2.4) which gradually decrease the wall velocity starting slightly upstream. Hence, u_{rms} is the component most directly connected to the wall oscillation.

2.3.7 Energy Saving

To compute the net energy savings, we need to take into account both the energy required for wall oscillation as compared to the savings due to DR. The derivation of these terms was given for channel flow by Quadrio and Ricco[68] which was extended to the boundary layer case by Skote[91].

Case	Re_Θ	$P_{sav}(\%)$	$P_{req}(\%)$	$P_{net}(\%)$
1	375	27.57	61.13	-33.56
2	505	25.02	55.25	-30.23
3	1400	22.27	47.54	-25.27

Table 2.2: Power budget

In order to compute the saved power $P_{sav}(\%)$, DR (as percentage of ratio of skin-friction coefficients from reference and oscillated cases based on Eq. (2.3.1)) is integrated for a region with approximately constant value. The total saved power can be written as,

$$P_{sav}(\%) = \frac{1}{L} \int_{x_i}^{x_f} DR(\%) dx \quad (2.3.10)$$

where x_i denotes the position at which the initial transients disappear, x_f denotes the endpoint for oscillation and $L = x_f - x_i$.

Similarly, the wall oscillation requires power input which can also be defined in terms of the friction power of the reference flow [91] and can be written as:

$$P_{req}(\%) = 100 \frac{W_m^2}{2} \sqrt{\nu\omega/2} \bigg/ \frac{U_\infty}{L} \int_{x_i}^{x_f} (u_\tau^0)^2 dx. \quad (2.3.11)$$

The net saved power is then defined as $P_{net} = P_{sav} - P_{req}$. If P_{net} is negative it indicates that the input power required to oscillate the wall is greater than the saved power due to streamwise DR. However, it is possible that for an optimized set of oscillation parameters, one may have positive energy budget as reported by other researchers.

The required power to drive the wall oscillations are compared with the energy saving due to DR in Table 2.2. One important observation from these results is that the net energy saving is increasing with Re_Θ while keeping the amplitude of oscillation W_m and

T constant, which means that positive net energy saving might be possible at higher Re_Θ . This is in contrast to what is observed for channel flow where the maximum net energy saving decreases slightly with Re_τ [76].

For high (but finite) Re in practical applications, the power saving would approach a fixed value as represented by the constant term in Eq. (2.3.9), while the required power for oscillating the wall decreases with Re . Because of heavy computational requirements for conducting DNS ($\sim Re^{9/4}$), and limited resources, it is difficult for us to test at higher values of Re_Θ . There have been studies based on spatial oscillation and traveling waves showing a net positive energy saving [99, 69, 90]. Also, it is worthwhile to note that previously published experimental and DNS studies provide results at low Re and therefore, the net power saving may be lesser for real high Re engineering problems.

2.4 Conclusions

Direct numerical simulations have been performed to study the effect of wall oscillation on the turbulent boundary layer at different Re for same wall velocity and frequency. The resulting drag reduction and its variation in the streamwise direction is in excellent agreement with findings from experiments conducted by Ricco and Wu[77]. Mean velocity profiles exhibit the shifting of log layer away from the wall. Two different scaling parameters were used to elucidate the effect of wall oscillation. The attenuation of turbulence is indicated by reduced values of velocity fluctuations, correlation coefficient and Reynolds shear stress together with their simultaneous shift in the wall-normal direction.

The dependency of peak drag reduction values on the Re at which the oscillation commences, is similar to that of channel flows although a conclusive relation between

DR and Re seems elusive as the relation most probably varies for different oscillation parameters. On the other hand, we have in the present work diversified the analysis and speculate that an alternative relation of type (2.3.9) can describe the current set of available data. The relation given here suggests that drag reduction attains a constant value at higher Re . Separated from the relation between DR and Re (where DR refers to the uniquely determined DR in channel flow, or the maximum DR obtained in the boundary layer flow) is the spatial development of DR in a boundary layer. The main question that needs to be answered is if drag reduction obtained at a certain Reynolds number depends on the location at which the oscillations start?

Various turbulence statistics are compared at different Re and their response to the oscillations is described. Only the longitudinal velocity fluctuations exhibit a non-monotonic response, while the Reynolds shear stress obtains the most reduced relative value. Spatial transients are observed downstream of the oscillating part of the wall are reported for the first time. A small drag increase is observed after oscillations are terminated, which corresponds to an overshoot of the Reynolds shear stress. The increase of the normal Reynolds shear stress back to the reference case level is much slower than for the longitudinal component, which is the component that most rapidly responds to the wall forcing.

Streaks are visualized and their bending as a result of the oscillating wall is demonstrated. In addition, other characteristics such as increased streak spacing is observed when oscillations are applied. Energy calculations show negative net energy budget for the chosen set of parameters for oscillation. However, based on the observations of Reynolds number dependency, the prediction of the viability of wall oscillations as means of drag reduction technique in real applications can be derived.

Chapter 3

Drag reduction in turbulent boundary layers with half wave wall oscillations[†]

3.1 Introduction

Most of the previous studies have dealt with temporal form of wall oscillations which are specified as

$$w(t)_{y=0} = W_m \sin(\omega t) \quad (3.1.1)$$

where W_m is the amplitude and ω is the frequency of the imposed oscillations.

A few studies have been devoted to explore the spatial oscillations and its impact on reducing skin friction has been found to be greater than temporal oscillations ([92, 90, 99,

[†]M. Mishra, M. Skote. To appear in *Mathematical Problems in Engineering*, 2015.

69)). Spatial oscillations can be realized by enforcing the following boundary condition

$$w(x)_{y=0} = W_m \sin(kx) \quad (3.1.2)$$

where k is the spatial frequency of oscillation and is related to the wavelength (λ) as $k = 2\pi/\lambda$.

Spatial wall oscillation technique has its advantages and disadvantages. It can have greater drag reduction as compared to temporal oscillations and hence, there are higher net energy savings. It is an open-loop method so we do not require an array of distributed sensors on the surface. However, the implementation remains a challenge as it requires numerous moving parts which makes it impractical. Although there have been advances in the field of material science research to realize such waveforms in practical situations, but the physical realization still remains elusive with the current technology.

Almost all the previous works have implemented oscillation waveforms using sinusoidal functions. A recent study by Cimarelli et al. [24] explored different temporal waveforms. In this work, we would like to explore the possibility of using spatial square waves for drag reduction. In order to reduce the power required to incorporate these oscillations, we consider only the positive cycles of these oscillations. One of the ways to realize spatial oscillations can be via pulsed jets in spanwise direction for the near wall region. Another possibility is to actively manipulate wall roughness optimally distributed along the surface. The main contribution of the present work is to illustrate the use of smooth step functions to approximate the square waves which otherwise may give rise to Gibbs phenomenon when using spectral methods.

3.2 Numerical Setup

3.2.1 Governing Equations

The governing equations which are used for the simulations here are the Navier-Stokes equations which are formulated in terms of velocity-vorticity and written in tensor notation as,

$$\frac{\partial u_i}{\partial t} = -\frac{\partial p}{\partial x_i} + \epsilon_{ijk} u_j \omega_k - \frac{\partial}{\partial x_i} \left(\frac{1}{2} u_j u_j \right) + \frac{1}{Re} \nabla^2 u_i + F_i, \quad (3.2.1)$$

$$\frac{\partial u_i}{\partial x_i} = 0, \quad (3.2.2)$$

where u_i are the velocity components, ω_i are the vorticity components, p denotes pressure and F_i is the body force. The non-dimensional constant $Re = U_\infty \delta^* / \nu$ is the Reynolds number with U_∞ being the streamwise freestream velocity, δ^* is the displacement thickness at $x = 0$ and ν is the kinematic viscosity. x_i represents the coordinate system with (x, y, z) as streamwise, wall-normal and spanwise coordinates and t denotes time.

These equations are solved using a pseudo-spectral method with appropriate boundary conditions. The basic idea with spectral methods is to express the solution as sum of *basis functions* and then compute their coefficients such that they satisfy the governing partial differential equations satisfying the boundary conditions.

A third-order Runge-Kutta-scheme is used to perform time integration for the non-linear terms. A second-order Crank-Nicolson method is used for the linear terms. For removing aliasing errors, a 3/2-rule is applied to the evaluation of the non-linear terms when calculating Fourier transforms in the wall parallel $(x - z)$ plane. The numerical

code (SIMSON [19]) used for the simulations in this work have been developed at KTH, Stockholm.

3.2.2 Numerical Setup

Since we are trying to simulate a turbulent boundary layer with a spatially growing boundary layer, we need to choose our basis functions accordingly. A basic sketch for the computational setup is shown in figure 3.1. For the discretization in $(x - z)$ plane or the streamwise-spanwise plane, Fourier basis is chosen assuming the solutions are periodic in these directions. However, for the wall normal direction, periodicity does not apply and Chebyshev polynomials are instead used as basis functions for the y (wall-normal) direction.

For initializing the simulations, a laminar base flow is required and is given as the Blasius similarity solution [86]. A trip forcing using a random volume force is then incorporated at $x = 5$ for the flow to undergo transition and thereafter we have turbulent flow regime. Figure 3.1 depicts this scheme.

Particular attention must be given to the streamwise direction for which since we have a spatially growing boundary layer, and hence no natural periodicity exists. Instead, for the purpose of artificially creating a periodic computational domain, a fringe region is introduced at the end to achieve this. The purpose of this fringe region is to dampen the velocity fluctuations to zero and bring the velocity field back to the laminar Blasius solution such that there are minimum upstream effects [13]. This is achieved by introducing the volume forcing F_i in equation 3.2.1,

$$F_i = \lambda(x)(\tilde{u}_i - u_i) \quad (3.2.3)$$

Figure 3.1: Computational box as seen from the negative z direction with the growth of boundary layer illustrated.

Fringe region forces the solution back to the prescribed laminar inflow and thus, enforcing periodic boundary condition. The lower part illustrates the spanwise velocity forcing which is applied in a part of the wall under the turbulent boundary layer. Half square waves are used in the present study as shown in the bottom figure.

where $\lambda(x)$ is the strength of the forcing and $\tilde{u}_i$ is the laminar inflow velocity profile. The function λ is defined as

$$\lambda(x) = \lambda_{max} f(x) \quad (3.2.4)$$

Figure 3.2: Schematic picture of the fringe region.

Figure 3.3: Function approximations using finite Fourier series terms.

with

$$f(x) = S\left(\frac{x - x_{start}}{\Delta x_{rise}}\right) - S\left(\frac{x - x_{end}}{\Delta x_{fall}} + 1\right). \quad (3.2.5)$$

Here λ_{max} is the maximum strength of the fringe, x_{start} and x_{end} denotes the spatial extent of the region where the fringe is non-zero, Δx_{rise} and Δx_{fall} are the rise and fall distance of the fringe function respectively. Figure 3.2 shows a schematic of how the fringe function varies. $S(\eta)$ is a continuous step function that varies from zero for $\eta \leq 0$ to unity for $\eta \geq 1$, and is given by,

$$S(\eta) = \begin{cases} 0, & \eta \leq 0, \\ 1/(1 + e^{(1/(\eta-1)+1/\eta)}), & 0 < \eta < 1, \\ 1, & \eta \geq 1. \end{cases} \quad (3.2.6)$$

3.2.3 Wall oscillation implementation

The form of wall oscillation implemented here is a spatial square wave with only positive forcing to reduce power consumption. However, there are numerical challenges in

implementing this using pseudo-spectral method. A square wave when represented using Fourier basis gives rise to Gibbs phenomenon which is shown in Figure 3.3a. When we try to approximate the strong discontinuity in the square wave, it results in strong oscillations at the edges. These result in spurious values causing numerical instability and large computational errors. Increasing the number of terms in the Fourier series approximation does reduce the oscillation but it does not eliminate it completely.

In order to avoid Gibbs phenomenon, we utilize the same step function as we used for fringe region (equation 3.2.6). Using the step function is advantageous as it has continuous derivatives at all points and does not exhibit the spurious ringing phenomenon. Figure 3.3b shows the use of $f(x)$ in implementing the wall boundary condition for the present simulations. By including only a few Fourier coefficients, we can approximate the function quite accurately and eliminate Gibbs rings.

Spatial wall oscillation can be incorporated with the following boundary condition

$$w(x)_{y=0} = W_m f(x) \quad (3.2.7)$$

where $f(x)$ is the same profile function as used for fringe region, (see equation 3.2.5), and W_m is the amplitude of the spatial oscillations.

3.2.4 Numerical parameters

All quantities are non-dimensionalized by the free-stream velocity (U_∞) and the displacement thickness (δ^*) at the starting position of the simulation ($x = 0$), where the flow is laminar. The Reynolds number is set by specifying $Re_{\delta^*} = U_\infty \delta^* / \nu$ at the laminar inlet ($x = 0$). Note that unless otherwise stated, the + superscript indicates that the quantity

is made non-dimensional with the friction velocity of the unmanipulated boundary layer (the reference case), denoted u_τ^0 , and the kinematic viscosity (ν).

A computational domain with $L_x = 600$, $L_y = 30$ and $L_z = 34$ is chosen with a mesh resolution of $800 \times 201 \times 144$ respectively. The resolution of these simulations in wall units are $\Delta x^+ = 16$, $\Delta y_{min}^+ = 0.04$ and $\Delta z^+ = 5.1$. All scalings are done based on u_τ^0 from reference or unmanipulated case at the starting position of wall forcing ($x = 200$). Wall oscillation boundary conditions are employed between $x_{start} = 200$ and $x_{end} = 450$ once it is ascertained that the flow has become fully turbulent. The Reynolds number based on momentum thickness varies between $450 < Re_\theta < 715$ in the control region.

Table 3.1 summarizes the parameters chosen for the steady spatial oscillation in the present work. Only the positive forcing has been employed for these simulation setups as shown in figure 3.3b. The spatial frequencies have been doubled and halved with respect to PS2 to see its impact on drag reduction performance. Also, the amplitude of oscillations is varied to understand its impact on drag reduction. The aim is to observe the effect of removing the negative forcing of the wall boundary, and its effects on power budget and net energy savings. Figure 3.4 shows the wall boundary condition for the three parameter sets at $W_m = 0.5$.

3.3 Results & Discussion

In this section, we look into two aspects of the results obtained from our numerical simulations. First, we look into attenuation of c_f values with respect to the reference case. Subsequently, we present the power budget based on the different forcings.

Parameter Set (PS)	W_m	k
PS1	(0.1, 0.25, 0.4, 0.5, 0.6, 0.75, 1)	0.0628
PS2	(0.1, 0.25, 0.4, 0.5, 0.6, 0.75, 1)	0.1256
PS3	(0.1, 0.25, 0.4, 0.5, 0.6, 0.75, 1)	0.2512

Table 3.1: Oscillation parameters for the simulations presented.

3.3.1 Skin friction attenuation

We compare skin friction from the unoscillated or the reference case with the oscillated cases. Skin friction coefficient for turbulent flows is defined as

$$c_f = 2 \left(\frac{u_\tau}{U_\infty} \right)^2 \quad (3.3.1)$$

where u_τ is the friction velocity and is computed based on mean streamwise velocity gradient at the wall

$$u_\tau = \sqrt{\nu \left. \frac{\partial u}{\partial y} \right|_{y=0}} \quad (3.3.2)$$

The resulting drag reduction (DR) is then calculated from

$$\text{DR}(\%) = 100 \frac{c_f^0 - c_f}{c_f^0}, \quad (3.3.3)$$

where c_f^0 is the skin friction of the reference case. Figure 3.5 shows the results for the skin friction variation along streamwise direction. All three cases show skin friction attenuation. As soon as wall oscillation is applied at $x_{start} = 200$, we see a strong gradient which marks the spatial transient for c_f . For PS1, we have a longer wavelength and due to discontinuous half waves, we observe recovery of c_f back towards the reference case.

Figure 3.4: Wall boundary condition set for spanwise velocity component for PS1, PS2 and PS3 at $W_m = 0.5$.

Figure 3.5: Spatial development of skin friction along streamwise direction at $W_m = 1$.

Figure 3.6: Power required for wall oscillation.

Figure 3.7: Power saved for the three chosen parameter sets.

However, for PS2 and PS3, due to smaller wavelength, this recovery process is weaker. This is a crucial observation as it indicates that with positive forcing itself, we can get drag reduction of a similar order of magnitude as with a full cycle of wall oscillation. This would reduce the power required for forcing the wall oscillation and this increases our net power saving as we would see later. At $x_{end} = 450$, the oscillations are stopped and skin friction attains the reference case values.

3.3.2 Power Budget

To compute the net energy savings, we need to take into account both the energy required for wall oscillation as compared to the savings due to drag reduction. The derivation of these terms was given for channel flow by Quadrio and Ricco[68] which was extended to the boundary layer case by Skote[91].

In order to compute the saved power $P_{sav}(\%)$, DR (as percentage of ratio of skin-friction coefficients from reference and oscillated cases, see equation 3.3.3) is integrated for the region with wall oscillation. The total saved power can be written as,

$$P_{sav}(\%) = \frac{1}{L} \int_{x_{start}}^{x_{end}} DR(\%) dx \quad (3.3.4)$$

where x_{start} denotes the position at which the wall oscillation is started, x_{end} denotes the endpoint for oscillation and $L = x_{start} - x_{end}$.

Similarly, the wall oscillation requires power input which can also be defined in terms of the friction power of the reference flow [91] and can be written as:

$$P_{req}(\%) = \int_{x_{start}}^{x_{end}} \nu \frac{\partial w}{\partial y} \bigg|_{y=0} W dx \bigg/ \int_{x_{start}}^{x_{end}} (u_\tau^0)^2 U_\infty dx. \quad (3.3.5)$$

The net saved power is then defined as $P_{net} = P_{sav} - P_{req}$. If P_{net} is negative it indicates that the input power required to oscillate the wall is greater than the saved power due to streamwise DR. However, it is possible that for an optimized set of oscillation parameters, one may have positive energy budget as reported by other researchers.

Power required for the three oscillation cases is shown in figure 3.6. For lower amplitudes of forcing, we require lesser power and it grows exponentially for larger amplitudes. For different spatial frequencies, there is not much difference in input power required. Figure 3.7 shows the power saved based on equation 3.3.4. Here, we see that with increasing amplitude, the power saving saturates after a limit. The effect of spatial frequency is rather interesting as we see that the power saved for PS2 and PS3 is almost the same. PS1 has a lower power saving and that can be attributed to the recovery to unoscillated skin friction values discussed previously in section 3.3.1.

Figure 3.8 shows the net power savings for the different parameter sets. PS2 with $W_m = 0.5$ gives us the maximum net power saving ($\sim 18\%$) amidst our chosen parameter space. Performance quickly deteriorates for larger amplitudes which show negative net

Figure 3.8: Net power savings for the three chosen parameter sets together with a quadratic curve fit.

power savings indicating that we spend more power in oscillating the wall as compared to the power savings. No complete description of the drag reduction mechanism exists to date. Thus, the influence of the parameters on the drag reduction remains largely unexplained. The parameter space explored in the current work is definitely not exhaustive. The results still remain sub-optimal as the computational costs involved are quite high for a larger parameter space.

Although the results look promising in terms of the net power savings, one of the drawbacks of the proposed methodology is that it induces cross flow which might be undesirable in certain situations. In order to illustrate the phenomena, a slice of the flow at $y^+ = 10$ in the $x - z$ plane is shown in figure 9. The figure has been compressed 4 times in the streamwise direction for better visualization. From the figure, it can be seen

Figure 3.9: Half wave oscillations inducing crossflow in the spanwise direction.

that after the first oscillation stops, the streaks reorient themselves to the streamwise direction. However, a spanwise cross-flow manifests itself as can be seen by the oblique streaks in the regions after the second and third periods of forcing.

3.4 Conclusion

A new form of steady spatial wall oscillation technique in the form of square waves with positive forcing has been presented with promising results for developing an active drag reduction technique. Spectral methods were used to solve the governing equations and the use of a smooth step has been demonstrated to approximate a square wave to overcome Gibbs phenomenon and avoid sharp discontinuities. Downstream development of skin friction and power budget for different oscillation parameters have been presented. An optimal set of wall oscillation parameters for the current parameter space was found to have $\sim 18\%$ net energy savings.

Chapter 4

DNS of a single low speed streak subject to spanwise wall oscillations[†]

4.1 Introduction

More recently the idea of spanwise oscillation has been applied to the suppression of growth of low-speed streaks in the laminar boundary layer. These streaks occur due to the growth of disturbances in the laminar boundary layer [2] which can become sufficiently strong to trigger transition to turbulence [14, 3, 43, 82] by-passing the classical route of exponentially growing unstable TS waves. Some of the earlier work can be found in the PhD thesis of Berlin [10] where temporal spanwise oscillations are used to delay transition due to oblique waves. The work followed soon after the initial works of Jung et. al. [46] on turbulent flows. Ricco [74] studied the evolution of laminar streaks subjected to steady spanwise wall oscillations using the linearized unsteady boundary region equations (LUBR). Under the linearized conditions he showed substantial reduction of

[†]P. S. Negi, M. Mishra and M. Skote. To appear in *Flow, Turbulence and Combustion*, 2014.

the velocity fluctuations. Hack and Zaki [35] studied the effect on Orr-Sommerfeld modes in the laminar boundary layer under time-harmonic shear at the lower wall, showing substantial reduction of the free-stream disturbance entering the boundary layer. Jovanovic [45] used the linearized Navier Stokes equations to show large reduction in the ensemble energy densities of transitional Couette and Poiseuille flows under small amplitude transverse oscillations. Rabin et. al.[70] used a newly developed non-linear variational method to show that the minimum initial disturbance to trigger transition to turbulence is substantially increased in plane Couette flow in the presence of temporal spanwise wall oscillations. Despite these studies a fully non-linear simulation of transitional flows under wall oscillation control remains lacking. Furthermore, the effectiveness of the spanwise oscillations during the transition stage remains an open question. It is yet unknown if the transition process can be suppressed with the oscillation mechanism. Consistency of the parameters across different flow regimes (laminar, transition and fully turbulent), is another question which yet remains to be addressed.

In the current study the effect of steady spanwise oscillations on laminar streaks is studied using direct numerical simulations. Asai et al. [4] in their experimental study generated a single low speed streak to study its stability characteristics. The streak is generated by the use of a small screen set normal to the wall around the middle (spanwise) of the boundary layer plate and downstream of the leading edge. The entire experimental setup was numerically replicated by Brandt [52]. We have used a similar methodology to numerically generate a low speed streak and study the effect of spatial wall oscillation on the generated streak. The effect of oscillations is further studied during transition region which is artificially triggered to turbulence using time periodic blowing and suction at the wall with the aim of investigating the possibility of stabilizing a perturbed streak

or delaying the transition process of the perturbed streak by using an open loop control method which has proven efficient for turbulence suppression.

4.2 Numerical Setup

The DNS code used in this study was developed at KTH, Stockholm [19]. It is based on spectral method to solve the three-dimensional, time dependent, incompressible Navier-Stokes equations. The algorithm uses Fourier representation in the streamwise and the spanwise directions while uses Chebyshev polynomials in the wall-normal direction. The algorithm is based on a pseudo-spectral treatment of the non-linear terms with multiplications of those terms calculated in physical space to avoid the sum of convolution terms. Fast Fourier Transform (FFT) is used for the transformation between physical and spectral space. For the time advancement of the nonlinear terms, a four-step, low storage third order Runge-Kutta method is used while a second order Crank-Nicolson method is used for the advancement of the linear terms. Aliasing error from the evaluation of non-linear terms are removed by the 3/2 rule for FFT calculations in wall-parallel planes while in the wall-normal direction, increasing spatial resolution has been found to be more efficient than de-aliasing. In order to account for the downstream boundary layer growth, a spatial technique is found to be necessary. The requirement is combined with the periodic boundary condition in the streamwise direction by the use of a *fringe region*. This region is implemented at the downstream end of the computational domain, where a volume forcing is added to the flow using a function $\lambda(x)$, which is smoothly raised from zero such that the flow is forced to a desired solution v . The volume forcing term is:

$$F = \lambda(x)(v - u) \quad (4.2.1)$$

where v is the laminar boundary layer to which the solution is forced to. The forcing vector is smoothly changed from a laminar boundary layer profile at the beginning of the fringe region to the desired outflow conditions at the end of the fringe region. This laminar profile is identical to the inflow condition at the beginning of the computational region, hence periodic boundary condition in the streamwise direction applies. The fringe function used is of the kind:

$$\lambda(x) = \lambda_{max} \left[S \left(\frac{x - x_{start}}{\Delta_{rise}} \right) - S \left(\frac{x - x_{end}}{\Delta_{fall}} + 1 \right) \right] \quad (4.2.2)$$

where $S(a)$ is a smooth step function defined as:

$$S(a) = \begin{cases} 0, & a \leq 0 \\ 1 / \left[1 + \exp \left(\frac{1}{a-1} + \frac{1}{a} \right) \right], & 0 < a < 1 \\ 1, & a \geq 1 \end{cases} \quad (4.2.3)$$

The length scale used for the normalization is based on the inlet displacement thickness, δ^* , the velocity scale used is the inlet free stream velocity, U_∞ , and time is normalized using δ^*/U_∞ .

Asai et. al.[4], in their experimental setup study the flow over a boundary layer plate placed parallel to the oncoming flow of the wind tunnel test section. In order to create a single low speed streak, a 40-mesh wire-gauze screen is placed 500mm downstream of the elliptical leading edge of the plate. The wire-gauze screen has a porosity of 0.7 is set normal to the plate. The height is kept close to the displacement thickness of the laminar boundary layer without the screen. Further details of the setup is given in [4].

In order to simulate the screen in the current study, a momentum loss is induced in the near wall region of the Blasius boundary layer using a localized volume forcing, added to the streamwise component of the momentum equation. The forcing of the form:

$$F(x, y, z) = A_x S\left(\frac{y_{loc} + y_{scale} - y}{y_{scale}}\right) \left(\frac{y}{y_{loc}} - \frac{u}{u_f}\right) u_f S\left(\frac{t}{t_{scale}}\right) g(x, z), \quad (4.2.4)$$

with

$$g(x, z) = \exp\left[-\left(\frac{x - x_{loc}}{x_{scale}}\right)^2\right] \times \left[S\left(\frac{z + z_{loc} + z_{scale}/2}{z_{scale}}\right) - S\left(\frac{z - z_{loc} + z_{scale}/2}{z_{scale}}\right)\right] \quad (4.2.5)$$

where $S(a)$ is the same step function as defined for the fringe region. u_f is the value of u attained at a distance y_{loc} from the wall where the forcing starts decaying to zero in y_{scale} . The forcing location is around x_{loc} corresponding to the location of the screen in the experiment and has an extent of z_{loc} . z_{scale} is used to smoothly raise or reduce the forcing so that a higher grid resolution is not required to resolve the solution. Table 4.1 below shows the values of the parameters used to create the low speed streak. In the numerical study by Brandt [14], there are two values for z_{loc} which correspond to the 7.5mm and the 5mm widescreens. In our study we have only considered the case for the 7.5mm screen and hence only one value of z_{loc} has been used.

The displacement thickness based Reynolds number of 549.35 at the inlet has been used to match the Reynolds number for the experimental study [4] at the location of the screen (taking into account its downstream growth between the inlet and x_{loc}). The computational domain for the simulation was set at 450, 15 and 9 units in the streamwise, spanwise and wall-normal directions based on the inlet displacement thickness δ^* . The

Table 4.1: Parameter Values

Parameter	value	Parameter	value	Parameter	value
y_{loc}	1	z_{loc}	1.82	x_{scale}	1.4
y_{scale}	0.4	z_{scale}	0.35	u_f	0.07
x_{loc}	57.19	A_x	0.8	t_{scale}	300

fringe region was set to $50\delta^*$ units right before the end of the computational domain. The resolution of the study was set to $512 \times 73 \times 96$ (RES1) Fourier modes in the streamwise, spanwise and wall-normal directions respectively. The simulation was checked against two more cases run with $612 \times 73 \times 128$ (RES2) and $800 \times 91 \times 128$ (RES3) Fourier modes. There was no appreciable difference found in the statistics of the converged flow. All subsequent laminar streak cases were run with RES1 as that proved to be sufficient to resolve the flow. Flow convergence was carefully checked manually by comparing the averaged statistics for different time intervals. Oscillations with higher amplitudes were found to take longer for statistical convergence. For high amplitude cases the flow was found to be statistically stable by 8000 time units. The simulation was further translated to 10000 time units and all statistics from the final 2000 time units are reported herein. All statistical quantities presented are spanwise averages unless specifically mentioned otherwise.

The streak generated by the simulated screen represents a stable streak which undergoes substantial viscous decay and thus no natural transition is observed. In order to trigger a transition of the streak, secondary instabilities are generated by localized time-periodic blowing and suction at the wall. The blowing and suction are implemented by applying a time harmonic wall-normal velocity at the wall, the extent of which has been localized to a small region by using an exponential decay in the x and z directions for the

amplitude of the blowing and suction. The function for the wall-normal velocity is:

$$v_{var}(x, 0, z) = v_{wall} \exp \left[-((x - x_{sb})/x_{sc})^2 \right] \exp \left[-(z/z_{sc})^2 \right] \sin(\omega t), \quad (4.2.6)$$

where v_{var} is the wall-normal velocity having a maximum amplitude of $v_{wall} = 0.035$ at the wall at $x = x_{sb} = 110$ and $z = 0$ (midpoint of the plane). $x_{sc} = 2.5$ and $z_{sc} = 0.5$ represent the extent of the wall-normal velocity region of blowing and suction and $\omega = 0.36$ is the frequency of the blowing and suction. All transition cases were run at the highest resolution with 800x91x128 (RES3) fourier modes.

4.3 Streak validation

The numerically generated streak has been compared with the experimental values obtained by Asai et al. [4]. Figure 4.1 shows the spanwise distribution of the streamwise velocity at $x=81$ as obtained in the current study. The dots represent experimental values obtained in [4]. The streamwise velocity patterns shows the characteristic profile observed in laminar streaks where the low speed fluid is uplifted to the upper part of the boundary layer and high speed fluid is in turn displaced to the lower portion of the boundary layer. Similar results were obtained by Brandt [52]. All features of the experimental setup [4] and the numerical study [52] have been successfully reproduced in the current simulation work.

The plot of u_{rms}/u_{rms}^{max} at different downstream locations against the wall-normal coordinate normalized by the displacement thickness (Figure 4.2) shows that the maxima of the streamwise velocity fluctuations lies very close to $1.3\delta^*$ which is characteristic of streaks in a laminar boundary layer as shown in earlier DNS [2, 57] as well as in experi-

Figure 4.1: Spanwise modulation of streamwise velocity at $x = 81$. Lines represent the values obtained in the current study and the dots show the corresponding experimental values in [4]. The curves from bottom to top represent mean streamwise velocity values at $y = 0.5, 1, 1.5, 2$ and 2.5 respectively.

ments [102, 60]. Overall, the numerical technique is successful in generating a low speed streak which is similar to the one generated by the penetration of free-stream disturbances into the laminar boundary layer.

4.4 Streak response to wall oscillations

The oscillation at the wall assumes a simple trigonometric form for the spanwise velocity according to equation (3.1.2).

The wall oscillations was implemented in the domain from $x = 90$ to $x = 300$. The region was selected such that the start of the oscillation is sufficiently downstream of the simulated screen and the end point of the oscillations is sufficiently upstream of fringe region. A schematic of the wall oscillation is shown in Figure 4.3.

A parametric study of the effect of changing wavenumber (k) and maximum spanwise velocity (W_m) has been performed. The parameters are varied from $0.03 < k < 0.75$ and $0.16 < W_m < 0.6$. The wall-normal stokes layer profile at different locations is first

Figure 4.2: Wall-normal profile of streamwise fluctuations (normalized by their peak values) obtained which is characteristic of streaks in a laminar boundary layer

Figure 4.3: Schematic of wall oscillation implementation

depicted in Figure 4.4 for $W_m = 0.16$ and $k = 0.12$. The locations correspond to the wall phase of $\phi = \pi/6, \pi/2, 7\pi/6$ and $3\pi/2$ in the third wavelength of oscillation. The Stokes' layer stays confined within the laminar boundary layer which, at the start of the oscillation rises to $\delta_{99} = 3.3$. Stokes layer height becomes smaller with higher wavenumbers and thus confining it close to the wall region. Even for the lowest wavenumber used in the parametric study, the Stokes layer remains confined within the boundary layer. The Stokes layer was also checked for the case of flow without a streak and the profile remains nearly identical to the one with the streak.

Figure 4.4: Stokes profile for $W_m = 0.16$ and $k = 0.12$

Figure 4.5: The reduction of C_f due to spanwise wall oscillations

In previous studies [68, 99, 7] on turbulent flow under wall oscillation it was observed that drag reduction followed a monotonic behaviour with the maximum oscillation amplitude. On the other hand, the frequency of temporal oscillations had an optimum value beyond which the effectiveness of the mechanism diminished. The analogous case of spatial oscillations also displayed an optimum wavenumber for peak drag reduction. Similar behavior was reported by Ricco[74] in the linearized study of streaks under oscillation. In order to find the optimum wavenumber in the current study, the oscillation amplitude was kept constant at $W_m = 0.16$ and the wavenumber k of wall oscillation was varied from 0.03 to 0.75.

Figure 4.6: Streamwise velocity profile for $W_m = 0.16$ and $k = 0.03$

For the case of laminar streaks, the effect of oscillation on the wall drag remains small. Figure 4.5 shows the trend of the skin friction coefficient (C_f) values under different oscillation cases. While there is a slight reduction of the wall shear stress from the reference value (without oscillations), the difference is small. A closer view (not shown) reveals that lower wavenumbers create a relatively larger reduction in C_f . However, the relative difference between different oscillation cases is small. Interestingly, for $k = 0.03$ and 0.12 the skin friction coefficient drops below the two dimensional laminar blasius flow which is given by $C_f = 0.664/\sqrt{Re_x}$.

As can be expected from the results of the C_f curves, the effect of oscillation on the base flow is extremely small. Figure 4.6 shows the mean streamwise velocity profile for one of the cases at $x = 195$, which is within the control region. The profiles of the controlled and the uncontrolled flows are almost indistinguishable, even for the wavenumber that showed the most promising C_f reduction. Only one position is shown in the figure for clarity, however multiple positions and different wavenumber cases were investigated and for all cases the change in the base flow was found to be minor as depicted by Figure 4.6.

On the other hand, the peak values of streamwise velocity fluctuations are substan-

Figure 4.7: Peak u_{rms} values along the streamwise coordinate for the reference case (without oscillation) and the oscillated cases with $k = 0.03, 0.12, 0.3$ and 0.75

tially suppressed (Figure 4.7), reducing to about 25% of their original values. The trend is monotonic with the lower wavenumbers showing greater reduction in fluctuations. To make the comparison with the reference case clearer, figure 4.8 shows the trend of $\tilde{u}^{max}$, where we have defined $\tilde{u}^{max}$ as the ratio of the peak streamwise velocity fluctuations for the oscillated and non-oscillated cases. The reduction to 25% for $k = 0.03$ is clearly visible from the graph. An optimum wavenumber was however not obtained within the current range of wavenumbers. It was not possible to check for lower wavenumbers due to the limitation of the computational box size. The lowest wavenumber with $k=0.03$ corresponds to a streamwise wavelength of 210 which covers nearly 50% of the computational domain.

Along with the reduction in fluctuations the locations of the peak fluctuations are shifted away from the wall and the trend shows a monotonic behaviour with the shift being the highest for the lowest wavenumber studied (Figure 4.9). The exception being for the highest wavenumber case ($k = 0.75$) for which the shift of peak locations was higher than for the case of $k = 0.30$. Jung et al. [46] had reported similar shift of peak values away from the wall towards the center of the channel for turbulent channel

Figure 4.8: $\tilde{u}^{max}$ for $k = 0.03, 0.12, 0.30$ and 0.75 at $W_m = 0.16$

Figure 4.9: Outward shift in the wall-normal location of peak streamwise fluctuations

flows. An upward shift of the log-law layer has also been reported in turbulent flow cases attributed to the thickening of the viscous sub-layer[7, 99].

Higher amplitude oscillations were studied for $k = 0.03$ which showed a trend of monotonic rise in velocity fluctuation reduction with peak wall velocity (Figure 4.10). The trend was confirmed for different forcing wavenumber ($k = 0.12$) at multiple amplitudes (Figure 4.11), although the reduction is less pronounced as compared to the case with $k = 0.03$. The highest reduction in streamwise velocity fluctuations was expectedly seen for $W_m = 0.6$ and $k = 0.03$ which reduced the fluctuations to nearly 10% of the reference values.

Figure 4.10: $\hat{u}^{max}$ for different forcing amplitudes - $W_m = 0.16, 0.3, 0.6$ with $k = 0.03$

Figure 4.11: $\hat{u}^{max}$ for different forcing amplitudes - $W_m = 0.16, 0.3, 0.6$ with $k = 0.12$

While an optimum value for oscillation wavelength was not reached a substantial reduction in fluctuations was achieved with $k = 0.03$ with fluctuations reducing by 90% of their reference values. The turbulent kinetic energy within the volumetric box bounded by the oscillation zone was reduced by around 80% for higher amplitude cases. At an amplitude of 0.36, Ricco [74] reports total energy reduction of streamwise fluctuations of close to 80%. The energy reduction in the current study for $k = 0.03$ and $W_m = 0.36$ is about 74% suggesting that the lowest wavenumber in the study should be close to the optimum wavenumber as seen in [74].

A special case of oscillation control was found to yield some curious results at first

glance. The control was implemented for the case of Blasius flow without the presence of the constant volume forcing and thereby without the presence of the low speed streak. As seen in Figure 4.12, there is no change in the value of C_f between the Blasius flow case and the control case when the streak is absent and the two C_f curves overlap. In the light of this observed overlap, the results of Figure 4.5 become interesting, as the control appears to reduce the skin friction below the 2D Blasius flow value in the presence of the streak, but does not seem to do so when the streak is absent. These rather contradictory results can be reconciled when the spatial development of C_f is plotted against Re_{δ^*} as in Figure 4.13. The figure shows C_f values for two types of flows - laminar flow with and without the streak. And also their corresponding counterparts with spanwise flow control. The Re_{δ^*} values correspond approximately to the start and end of control region. C_f for cases without the streak expectedly overlap. The case of streak under wall control also converges to these values. On hindsight this is expected. The laminar flow with control remains largely two dimensional. The coupling of streamwise flow with its spanwise counterpart happens through the fluctuating terms. Hence, in the absence of disturbances (streak), the mean streamwise flow (and hence the C_f) is unaffected by the spanwise motion of the wall.

4.5 Transition

It remains to be seen if the optimum wavenumber found for the streak continues to be effective for turbulence suppression should the flow undergo transition. In order to answer this question, time harmonic blowing and suction is used to trigger the transition to turbulence. The blowing and suction is located within the oscillation zone to see if the

Figure 4.12: Wall control ($k = 0.03$) with and without the streak.

Figure 4.13: Spatial development of C_f with Re_δ^* .

oscillations are able to effectively damp out the initial growth of disturbances and if the flow is able to maintain its laminar state. The amplitude of wall oscillations was kept constant at $W_m = 0.5$ and the wavenumber was varied. In order to obtain a smoother discretization of the oscillations, the highest wavenumber studied was reduced slightly to $k = 0.6$. The wavenumber variation was thus from $k = 0.03$ to $k = 0.6$. We looked at the sharp jump in C_f which is used as a standard indicator for flow transition. Figure 4.14 shows the C_f values for different simulations. The dotted line indicates the case for transition without any oscillations which exhibits the sharp rise as expected.

For the case of oscillation during transition, the lowest wavenumber case ($k = 0.03$)

which exhibited the maximum reduction in C_f for the laminar streak appeared to only marginally affect the wall shear stress. The friction coefficient remained oscillatory for the lowest wavenumber with its mean being close to the reference values (without oscillation). Thus, the wall oscillations with $k = 0.03$ do not suppress the viscous wall friction despite being the optimal wavenumber for diminishing the velocity fluctuations in the laminar streak flow. On the other hand, considerable reduction in the peak streamwise fluctuations was obtained as shown in Figure 4.15.

The parametric study of wavenumbers in the transition region reveals a trend in C_f reduction, with higher wavenumbers (at least up $k = 0.3$ as indicated in Figure 4.14) to causing a greater skin friction reduction. This is contrary to what was noticed during simulations of laminar streaks under oscillation where greater effectiveness appeared for lower wavenumbers. While we did not achieve an optimum wavenumber for the laminar streak cases, there appears to be a maxima reached with regards to skin friction reduction close to $k = 0.3$. Further increase in wavenumber ($k = 0.6$) reduced the effectiveness of the mechanism.

The streamwise velocity fluctuations (shown in Figure 4.15) were found to reduce during the transition period, although unlike the laminar streak cases, the reduction does not appear to follow a clear trend with the reduction being oscillatory for some cases and more sustained in others.

The peak values for streamwise fluctuations exhibit oscillations in the streamwise direction for $k = 0.03, 0.12$ and 0.30 . The oscillations are most distinctly visible for the case of $k = 0.12$. These oscillations can be explained on the basis of the streamwise variation of the spanwise velocity gradient. The maximum suppression of turbulence occurs at location with maximum spanwise gradient. Due to the spatial oscillations of the

Figure 4.14: C_f during transition under oscillation

Figure 4.15: Peak streamwise fluctuations during transition under oscillation

spanwise velocity, there are alternating regions of flow where the streamwise gradient of the spanwise velocity decreases to zero. In these regions the turbulence starts regenerating again in accordance with the new flow conditions. However, the sinusoidal nature of the wall forcing implies that the zero gradient region is flanked by regions of rapidly changing spanwise velocity. These regions again create the spanwise unsteadiness in the flow causing turbulence suppression. What emerges are alternating regions of turbulence suppression and regeneration. This can be verified by measuring the streamwise distance of the peaks observed in figure 4.16 which shows the oscillation region for transition flow under wall forcing with $k = 0.12$. For this case the streamwise distance between the

Figure 4.16: Oscillatory nature of peak streamwise fluctuations for $k = 0.12$. Streamwise distance between the peaks (circles) matches the forcing wavelength

consecutive peaks (marked by circles) is found to be equal to $\Delta x = 27$ which is very close to the half wavelength of the forcing ($\lambda_{forcing}/2 = 26.4$) which corresponds to the distance between two consecutive regions of low streamwise gradient of spanwise velocity. For $k = 0.30$ the oscillation amplitude is small and visible only under a magnified view (not shown). Again the streamwise separation of the peaks ($\Delta x = 10.6$) matches with the half wavelength of forcing ($\lambda_{forcing}/2 = 10.5$). This phase dependence of the streamwise fluctuations appears to reduce with higher wavenumbers. There are two reasons for this apparent reduction. Firstly, with higher wavenumbers, the streamwise distance between consecutive high gradient regions is reduced. Hence, there is lesser time for turbulence to regenerate. Secondly, as the wavenumber is increased the height of the Stokes layer decreases. For all oscillation cases except $k = 0.03$, the location of the peak fluctuations is near the edge of the Stokes layer height where the amplitude of oscillations is highly diminished. The direct effect of reduced amplitude oscillations at these higher wall normal locations will be considerably small. Instead, the reduced peak values reflect the lower intensity of turbulent fluctuations which are being convected away from the near wall

Figure 4.17: Streamwise velocity fluctuations at $y = 0.5$. The phase dependence of fluctuations is more prominent at lower wall-normal locations

region, where the Stokes layer has its maximum effect. This lower intensity in turn leads to a reduced turbulent production, and thus lower peak values. This can be seen by looking at the streamwise fluctuations close to the wall instead of focusing on the peak values. Figure 4.17 shows the u_{rms} values at $y = 0.5$ for all the cases. The phase dependence of the streamwise fluctuations is prominently evident even for $k = 0.30$. Again for an even higher wavenumber case ($k = 0.60$), the phase dependence is barely visible due to the reasons explained above, although the short wavelength oscillations can actually be observed in Figure 4.17.

To explore this further, the u_{rms} oscillations at different wall-normal heights are studied for their phase relationship with the spanwise velocity. Figure 4.18 shows the u_{rms} values at different y locations within the control region for the case of $k = 0.3$. A Savitzky-Golay spatial filter is used to obtain the filtered signal free from the oscillations in the streamwise direction which uses a polynomial expression to fit successive subsets of the data with a linear least square method. A polynomial of order one and with a smoothing window subset of half the wavelength of wall oscillations was found sufficient to filter

Figure 4.18: Streamwise velocity fluctuations at $y = 0.1, 0.2, 0.3$ (From bottom to top) for $k = 0.3$. The filtered signal is shown with a dotted line.

out the oscillations present in the signal. The dotted lines in Figure 4.18 represent the smoothed signal obtained from the filter. The oscillating part of the signal obtained from subtracting the filtered signal from the original one is plotted against the wall-phase of the spanwise velocity as a polar plot in Figure 4.19. The positive and negative values of the fluctuating part of the signal are represented by solid and dashed lines respectively. It is evident that there is no unique relationship between the fluctuations and the wall phase of the spanwise velocity. The phase relation appears to be rotating in the counter clockwise direction as we move higher in the wall-normal direction. The phase plots however nearly collapse when they are plotted with the phase of the spanwise velocity at their corresponding wall-normal locations (Figure 4.20).

The peak wall-normal fluctuations (v_{rms}^{max}) are shown in Figure 4.21 which show substantial reduction as well. Comparing the wall-normal fluctuations for the different oscillation cases with the C_f values in Figure 4.14, a striking qualitative correlation can be seen. The lowest wavenumber, $k = 0.03$, seems to only marginally affect both the skin friction and the peak wall-normal fluctuations with the values of both remaining close to

Figure 4.19: Wall phase relationship of u_{rms} for $y = 0.1$ (blue), 0.2 (red), 0.3 (black). Solid lines represent the positive values while dashed line represents negative values.

Figure 4.20: Local phase relationship of u_{rms} .

the reference case (transition without oscillation) values. The largest reduction in both is observed for $k = 0.3$. Further increase in wavenumber ($k = 0.60$) yields rising values of both C_f and v_{rms}^{max} (although still remaining below the reference case values). Furthermore, both skin friction and peak wall-normal fluctuations for $k = 0.60$ are initially lower than the corresponding values for $k = 0.12$. However this trend is reversed further downstream with the case having $k = 0.12$ experiences greater reductions downstream in both skin friction and peak wall-normal fluctuations. The qualitative correlation between the two quantities (C_f and v_{rms}^{max}) is striking given the lack of similar qualitative correlation

Figure 4.21: Peak wall-normal fluctuations during transition under oscillation

between skin friction and the longitudinal velocity fluctuations.

The results of the current study can be seen in the light of a similar study done in [74] using linearized theory. The prediction of linear theory on the effect of wall oscillation is consistent with the non-linear results. This was to an extent expected. The present study extends the control region to a transitional flow case and finds that the forcing effectiveness changes and the optimum parameters for the two flow regions are an order of magnitude apart. It becomes important to consider the results of the laminar and transition regions together as it reminds of the obvious drawback of linear theory in its dependence of the fixed base state. In all scenarios where the flow undergoes a change in base state, as in by-pass transition to name one example, linear theory ceases to be accurate and its predictions will likely fail to some degree. A recent study [27] used the linear Navier Stokes equations for the traveling wave type wall forcing and created a phase space for the turbulent energy in the domain. The results qualitatively match the drag reduction map of [69] with the same parameters. This is certainly an interesting result and points to the dominant role played by the linear equations even in some fully turbulent flow cases. However, the linear study reported an explosive growth of solutions in regions

which correspond to drag increase. While in this case, the growth of disturbances appears to be modeled to an extent within the linear equations, predictions made solely on the basis of linear theory would probably be highly off the mark due to the change of base state that would follow. The order of magnitude difference in the optimum parameters of the wall forcing in two different regimes becomes a case in point. While the current study confirms the results of linear theory for laminar low speed streak, the slight perturbation leads to a change in base state and completely alters the effectiveness of the forcing in this new base state.

4.6 Conclusion

A parametric study of the effect of streamwise oscillations of spanwise wall velocity on a single-low speed streak has been performed showing large suppression in velocity fluctuations. The wall shear stress for the laminar streak cases was reduced slightly for all wavenumbers simulated and in two of the cases ($k = 0.03$ and $k = 0.12$) the value dropped below the laminar Blasius flow without streaks. An optimum wavelength for suppression could not be reached within the allowable oscillation parameters but the trend showed higher reduction of velocity fluctuations at lower wavenumbers. A comparison with the energy suppression values obtained by Ricco [74] suggests that the lowest wavenumber in the study ($k = 0.03$) should be very close to the optimum wavenumber. Changing the amplitude of oscillations showed a monotonic increase in velocity fluctuation reduction. The optimum wavenumber for oscillation changes, however, as the flow undergoes transition, with the new optimum wavenumber being approximately an order of magnitude larger (i.e. $k = 0.3$) than the near optimum observed for laminar streak cases. The results

therefore show consistency with linear theory but break away from the linearized results once the streak is perturbed, setting in non-linear effects leading to flow transition. This eventually leads to a drastic change in the performance of the drag reducing mechanism. In the transitional region, low wavenumber cases seemed to have only a marginal effect on the skin friction for flow undergoing transition, even though the streamwise velocity fluctuations are reduced for all cases. Oscillations are observed in the peak values of streamwise fluctuations which are explained on the basis of spatially alternating regions of high turbulence suppression and turbulence regeneration in the temporary absence of the velocity gradient. The oscillations in the streamwise velocity fluctuations at different wall normal locations have a strong phase relationship with the local phase of the spanwise velocity at the corresponding wall-normal location. A striking qualitative correlation is seen between the trends for skin friction and wall-normal fluctuations, one that merits further studies. In the present investigation, the flow was artificially disturbed to trigger the transition to turbulence, hence it yet does not answer whether the spanwise oscillations can prevent bypass transition. However, it points towards a drastic change in efficiency of the mechanism (for the same parameters) between the laminar and transition region which would be useful to keep in mind for any practical implementation.

Chapter 5

The effect of spanwise oscillations on bypass transition [†]

5.1 Introduction

Given the efficacy of spanwise oscillations to suppress turbulence, a question arises if they can be used to delay or even prevent transition. One route of transition from laminar to turbulent flows, called bypass transition, involves the break up of near wall boundary layer streaks [43, 14, 110, 82], which exist in laminar flows. Since the oscillating wall forcing attenuates the near wall streaks in a turbulent region, a similar effect on laminar streaks could hinder their growth, and thus causing a delay or even prevention of transition to turbulence. There has thus, been an inception of the idea of using the same open loop wall control technique for pre-transitional boundary layers. Transition to turbulence via the bypass route is known to occur due to the non-linear interaction of the continuous modes of the Orr-Sommerfeld spectrum [43, 82], which form a basis for the description of

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free stream turbulence [34]. These modes enter the boundary layer through a process of shear filtering [42] which allows the low frequency modes of the continuous spectrum to enter into the boundary layer while damping out the high frequency modes. Andersson et al. [2] and Luchini [57] calculated that the optimal growth within the boundary layer occurs for streamwise aligned vortices which develop into streamwise streaks. Two possible breakdown mechanisms were shown by Andersson et al. [3]. Brandt et al. [14] showed the dependence of transition on the spectral content of the free stream turbulence. Zaki and Durbin [110] showed that both high and low frequency modes are necessary for bypass transition. However only two modes (one high and one low frequency) suffice to create a transition scenario. The two mode model [110] was confirmed by Schlatter et. al.[82]. The same authors also go on to show the existence of two possible breakdown mechanisms as elucidated in [3]. A review of bypass transition is presented by Durbin and Wu [28].

While the quantitative picture of bypass transition remains incomplete, the qualitative picture puts the focus on the break-up of low-speed streaks in laminar boundary layer. With spanwise oscillations suppressing streak formation, their effect was then studied in pre-transitional boundary layers. Ricco [74], in a linearized study showed large attenuation of laminar streaks under the effect of steady spatial oscillations. Hack and Zaki [35] show an enhancement of the shear filtering process in another linearized study with time harmonic wall oscillations. The shear filtering process also assumed a time dependence due to the temporal variation of the base flow [35]. Jovanovic [45] studied small amplitude oscillations in transitional Couette and Poiseuille flows and showed large ensemble energy density reduction. Using a newly developed non-linear variational method, Rabin et. al. [70] show that the minimum initial disturbance to trigger transition to turbulence is substantially increased in plane Couette flow in the presence of temporal spanwise wall

oscillations. Stellan Berlin's PhD work [10] predates all these studies in which he studies temporal wall oscillations on oblique wave transition as well as transition due to random noise and shows that large transition delay is possible. Hack and Zaki [36] performed a DNS study with temporal oscillations on pre-transitional boundary layers and showed transition delay for flows with free stream turbulence.

The recent studies have shown promising signs of creating large transition delay using a simple open loop technique. In turbulent flows spatial and temporal oscillations have been analogous with spatial oscillations having a better energetic performance. In the current study, Large Eddy Simulations (LES) have been performed to further investigate the effectiveness of spanwise oscillations in delaying bypass transition with a focus on spatial oscillations. Analogous case of temporal oscillations have been performed to compare the two forms of wall forcing in their effectiveness and creating transition delay. The spatial extent and location of the wall control region have also been explored. The wall oscillations implemented were of the same type as in equation 3.1.1 and equation 3.1.2.

5.2 Numerical Setup

The code used in this study was developed at KTH, Stockholm [19]. It is based on spectral method to solve the three-dimensional, time dependent, incompressible Navier-Stokes equations. The algorithm uses Fourier representation in the streamwise and the spanwise directions while uses Chebyshev polynomials in the wall-normal direction. The algorithm is based on a pseudo-spectral treatment of the non-linear terms with multiplications of those terms calculated in physical space to avoid the sum of convolution terms. Fast Fourier Transform (FFT) is used for the transformation between physical and spectral

space. For the time advancement of the nonlinear terms, a four-step, low storage third order Runge-Kutta method is used while a second order Crank-Nicolson method is used for the advancement of the linear terms. Aliasing error from the evaluation of non-linear terms are removed by the 3/2 rule for FFT calculations in wall-parallel planes while in the wall-normal direction, increasing spatial resolution has been found to be more efficient than dealiasing. LES simulations have been performed using the ADM-RT (Approximate Deconvolution Model with Relaxation Term) model to approximate the sub-grid scales. In order to account for the downstream boundary layer growth, a spatial technique is found to be necessary. The requirement is combined with the periodic boundary condition in the streamwise direction by the use of a fringe region. This region is implemented at the downstream end of the computational domain, where a volume forcing is added to the flow such that the flow is forced to a desired solution inlet solution.

The length scale used for the normalization is based on the inlet displacement thickness, δ^* , the velocity scale used is the inlet free stream velocity, U_∞ , and time is normalized using δ^*/U_∞ .

The displacement thickness based Reynolds number of 300 at the inlet has been used. The computational domain for the simulation was set at 2000, 180 and 60 units in the streamwise, spanwise and wall-normal directions based on the inlet displacement thickness δ^* . The fringe region was set to $100\delta^*$ units right before the end of the computational domain. The resolution of the study was set to 512, 128 Fourier modes in the streamwise and spanwise directions respectively with 121 points in the wall-normal direction. The effect of free stream was added as a superposition of the continuous modes of the Orr-Sommerfeld spectrum. In order to discretize the free stream turbulence spectrum the wavenumber space is divided into concentric spherical shells with points selected from

each shell such that the equivalent spectrum is isotropic. Care is taken to eliminate special cases such as standing waves etc. The selection is performed for 20 discrete shells amounting to a total of 200 spectral modes for the free stream turbulence. The energy distribution of the spectrum is done in accordance with the von-Karman spectrum for homogeneous turbulence which is of the form:

$$E(k) \propto \frac{k^4}{(C + k^2)^{17/6}} \quad (5.2.1)$$

The spectrum is normalized to create a turbulence intensity of 4.7% at the inlet for almost all cases. For further details on the calculation of free stream turbulence spectrum refer to [14].

The simulation was run for 15000 time steps. All statistics up till 4000 time steps were discarded and subsequent values were used to compute all averaged quantities. The statistical quantities presented herein are spanwise averages unless specifically mentioned otherwise.

A reference case for transition due to free stream turbulence without any wall control was setup. The Orr-Sommerfeld spectrum was calculated with a turbulence intensity (Ti) of 4.7%. The same spectrum was used as the inlet condition for all subsequent cases with wall control. Comparisons of all wall control cases are made with this reference case.

5.3 Spatial Oscillations

Steady spatial wall oscillations are implemented as described by equation 3.1.2 with the start of wall control at $x_{start} = 400$. The extent of wall control is kept till $x_{end} = 800$. The wavenumber for oscillation was set as $k = 0.314$ with the maximum amplitude of wall oscillation $W_m = 0.9$.

Figure 5.1: Transition delay induced due to oscillating wall control.

The comparison of the coefficient of skin friction (C_f) for the wall control and references cases show a clear downstream shift of the rise in C_f curve (Figure 5.1), which is characteristic of transition. The minima of the C_f curve for the reference case occurs at $x = 390$ while for the wall control case it occurs at $x = 558$. The values correspond to $Re_{\delta^*} = 685, 774$ respectively, signifying a transition delay of $\Delta Re_{\delta^*} = 89$. Visually, transition region appears to have moved downstream of the control region. This transition delay however is not sustained as the extent of the control region is increased. Moving the end of wall control further downstream to $x = 1200$, causes no change in transition delay. The C_f curve in Figure 5.2 exhibits a sharp rise within the control region, and then stabilizes with its values remaining below the corresponding values of the reference case. An even further downstream shift of the end of the control region ($x_{end} = 1800$) further confirms this saturation of transition delay.

Figure 5.2: Transition delay in a longer oscillation region

5.3.1 Effect of starting position of oscillation

The wall control mechanism was further tested by changing the start location (x_{start}) of the oscillation. The start was shifted sequentially upstream to $x_{start} = 400, 300, 200, 0$, where $x_{start} = 0$ signifies the beginning of the computational domain. Figure 5.3 shows the effect of x_{start} on transition delay. An upstream movement of the wall control start increased the transition delay that could be achieved. The minima of the C_f curve occurred at $Re_{\delta^*} = 771, 837$ and 861 for $x_{start} = 400, 300$ and 200 respectively. The corresponding ΔRe_{δ^*} values being $85, 152$ and 176 . Transition delay appeared to saturate for values upstream of $x = 200$ and the transition delay for $x_{start} = 200, 0$ is nearly identical. Thus, the maximum transition delay achieved, when calculated from the shift of the minima of the C_f curve, was $\Delta Re_{\delta^*} = 176$.

The evolution of flow in the case of wall control in the transitional region can be compared with the case of flow control in fully turbulent flows. Figure 5.4 shows the skin

Figure 5.3: Transition delay for different start points of oscillation ($x_{start} = 400, 300, 200, 0$)

friction coefficient development in the two cases of pre ($x_{start} = 200$) and post ($x_{start} = 1000$) transition wall control, plotted against Re_δ^* . For the post transition case, the fully turbulent nature of flow is confirmed by the existence of log region for the mean streamwise velocity (not shown). The C_f curve for the two cases almost converge. Thus, the flow eventually converges to the same state independent of the starting location of wall control.

5.3.2 Effect of wavenumber

Different oscillation wavenumbers were tested for transition delay, while keeping the oscillation amplitude constant at $W_m = 0.9$. The start of wall control is kept at $x = 300$ which is upstream of the minima of the C_f curve in the reference case. The end is kept at $x = 1800$ which is slightly upstream of the fringe region. Figure 5.5 shows the skin friction development for $k = 0.0125, 0.0314, 0.628$. The initial change of C_f is almost the

Figure 5.4: Coefficient of skin friction development for pre and post transitional wall control.

same for all cases, with the values reducing at the onset of wall oscillation. A visible delay in transition is observed for all cases, the trend however is non-monotonic. The lowest wavenumber, $k = 0.0628$, creates the least delay in transition with $\Delta Re_{\delta^*} = 125$. This also has the least suppression of turbulence in the fully turbulent regime. $k = 0.0314$ causes the largest transition delay of $\Delta Re_{\delta^*} = 176$ while $k = 0.0628$ creates a delay of $\Delta Re_{\delta^*} = 150$. It is worth noting that the drag reduction in the fully turbulent regime, for the wall control case of $k = 0.0314$ is lower than that for $k = 0.0628$. For transition delay however the trend between the two cases is reversed. Hence, the effectiveness for drag reduction in fully turbulent cases and the transition delay caused with the same set of parameters is not well correlated.

Figure 5.5: Transition delay for different wavenumbers ($k = 0.0125, 0.0314, 0.0628$)

5.3.3 Effect of turbulence intensity

The wall control is implemented for a case of different turbulence intensity. The inlet turbulence spectrum is normalized to a turbulent intensity of 3.5% and wall control is implemented from $x = 400$. Figure 5.6 compares the evolution of C_f for the two different turbulent intensities and their respective wall control cases. Expectedly, transition delay is seen even for the low intensity case. Strikingly however, the delay in transition in both case is very similar with $\Delta Re_{\delta^*} = 94$ for $ti = 3.5\%$, while $\Delta Re_{\delta^*} = 89$ for the higher turbulence intensity ($ti = 4.7\%$) case.

Qualitatively, bypass transition is known to occur due to the non-linear interaction of continuous modes of the Orr-Sommerfeld spectrum [43, 14, 110, 82]. Wall oscillations act to further dampen the amplitude of the continuous modes within the Stokes layer [35, 74] in the laminar boundary layer. The wall control technique then, should theoretically create a transition delay if its implementation is just upstream of the transition location.

Figure 5.6: Transition delay for different turbulent intensities ($k = 0.0314$, $x_{start} = 400$)

With this in mind, the oscillation was implemented for a short region ($x = 200 - 400$) right before the minima of the C_f curve of the reference case. Figure 5.7 shows the C_f change in case of a short pre-transitional control region, comparing it with a case of an extended control region. Again, calculating delay using the minima of the C_f curve, a relatively large amount of transition delay ($\Delta Re_{\delta^*} = 99$) can be achieved with a short pre-transitional wall control.

5.4 Temporal Oscillations

Temporal and spatial oscillations are seen to be largely analogous in turbulent flows in terms of their efficacy to create drag reduction. Indeed, the optimum wavenumbers for spatial oscillations and the optimum angular frequency for temporal oscillations have been shown to be the same under space time conversion [99]. Temporal oscillations are

Figure 5.7: Transition delay with a short pre-transitional wall control ($k = 0.0314$)

therefore tested for their ability to create transition delays as seen in the spatial cases. Figure 5.8 shows the C_f curve for the case of wall control implemented for two different start locations ($x = 400, 300$) to $x = 1200$, keeping the maximum oscillation amplitude, $W_m = 0.9$. The oscillation frequency was set at $\omega = 0.14$. While temporal oscillations appear to cause a delay in transition as well, the spatial delay is not as high as in the case of spatial oscillations. The trend observed by moving x_{start} is reverse for temporal oscillations, with $\Delta Re_{\delta^*} = 41$ for $x_{start} = 400$, but reducing to $\Delta Re_{\delta^*} = 13$ when x_{start} is moved upstream to 300.

Changing the oscillation frequencies did not appear to have a large difference in transition delay. Three different oscillation frequencies were used for wall control with $x_{start} = 300$, $x_{end} = 1200$ and $W_m = 0.9$. $\omega = 0.05$ caused the largest delay in transition which amounted to $\Delta Re_{\delta^*} = 31$. Subsequent higher frequencies showed a much lower capacity transition delay (Figure 5.9).

Figure 5.8: Transition delay for temporal oscillation cases ($W_m = 0.9$, $\omega = 0.14$)

Figure 5.9: Transition delay for temporal oscillation cases ($W_m = 0.9$, $\omega = 0.14$)

A direct comparison can be seen between a temporal and spatial oscillation in Figure 5.10 for $\omega = 0.05$ and $k = 0.0314$. The suppression of turbulence for the two cases is

Figure 5.10: Transition delay for temporal oscillation cases ($W_m = 0.9$, $\omega = 0.14$)

very similar in the fully turbulent regime, with both cases displaying similar levels drag reduction post transition. However, there is a marked difference between the amount of transition delay caused by the spatial and temporal oscillation cases with the spatial case displaying a transition delay of $\Delta Re_{\delta^*} = 151$ while the delay for the temporal case was only $\Delta Re_{\delta^*} = 31$.

5.5 Conclusion

Effect of wall oscillations on the transition region has been explored in the present work. It remains inconclusive if the change represents a shift in the actual point of transition or if these oscillations merely retard the spatial development of flow post transition. Spatial oscillations definitely seem to be superior to temporal ones and are characterized by different phenomena. The downstream shift is affected by the starting point of the

oscillation region, but such a downstream shift saturates quickly. On the other hand transition under temporal oscillations exhibits a different character. While a small downstream shift is seen in the rise in C_f values as well, the behavior is not consistent when the start of the oscillations is shifted upstream. Temporal and spatial oscillations thus, do not show analogous character during transition as is seen for fully developed turbulent flows. Starting point of oscillations does not affect the final state of the flow as they all converge to the same state as seen for the spatial oscillations. A large delay in transition can be achieved with a small pre-transitional wall oscillation region.

Chapter 6

Kolmogorov spectrum consistent optimization for multi-scale flow decomposition[†]

6.1 Introduction

Turbulent flow consists of self-similar structures with a wide range of length scales. This self-similarity has led to the idea of energy cascade [78, 50, 51], which governs turbulent flows universally. Over the past few years, multi-scale decomposition has gained increasing interest in turbulence community for both modeling and analysis[79], and has proven to be useful for understanding the evolution of eddies and the interaction between turbulent flow structures at different scales. This can be pivotal in understanding the morphology and dynamics of turbulent flow structures. An exploration in this direction can be motivated by the question of whether there are universal structures across different scales [29].

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The energy cascade phenomenon for a turbulent flow is characterized by the Kolmogorov spectrum which gives the variation of energy $E(k)$ contained by all eddies with different length scales (k) in a log-transformed Fourier space. A *scale* for a turbulent flow is usually referred to as a range of wavenumbers that can be obtained via a perfect band-pass filter (BPF) in the Kolmogorov spectrum, which also corresponds to a perfect BPF in Fourier space. However, it is well known that Fourier transform with perfect BPF usually produces strong Gibbs ringing artifacts [33] due to insufficient basis.

This phenomenon can generate spurious structures, leading to error-prone conclusions regarding the flow characteristics. The drawback of using perfect BPF in Fourier space has motivated the research in the direction of utilizing local-support basis multi-resolution methods such as *wavelets* [29, 87] and *curvelets* [17]. Wavelets are based on a symmetric local basis while curvelets, as an extension of wavelets, have an extended dimension of localized orientation with finer-scale ridge-shaped basis functions.

In recent years, curvelets have found increasing use among various research groups for scale decomposition studies. Bermejo-Moreno and Pullin[11] presented multi-scale geometrical decomposition for isotropic flow and performed characterization of the flow structures based on direct numerical simulation (DNS) data. Geometry of enstrophy and dissipation structures were shown by Bermejo-Moreno et al[12]. Ma et al[58, 59] also used curvelets to provide a geometric analysis of flow structures. Using a similar methodology, Yang et al[108] showed the evolutionary geometry of the Lagrangian scalar field for stationary isotropic turbulence. Yang and Pullin[107] reported results for anisotropic channel flow with a study of the geometry of Lagrangian and Eulerian structures.

In addition to the wavelets and curvelets, Leung et al.[55] also presented multi-scale decomposition based on spatial filtering. However, the effect of spatial filtering in the

Fourier space does not characteristically confine the results to a sharp band and hence a large overlap of different scales results as a consequence. Due to the large overlap, the flow structures cannot be uniquely identified as belonging to different energetic bands.

With the use of curvelets/wavelets, one can significantly reduce Gibbs rings due to band-limiting basis in comparison to the Fourier transform. However, a severe drawback with those methods is that they impose a strong restriction on the selection of scale location and bandwidth, leading to dyadic discontinuous scales with fixed bandwidth.

In fact, curvelets/wavelets were not originally designed for scale decomposition purpose. Their advantage of using a localized basis suits better representation of sharp edges and hence, is a better way for data representation and compression. However, in the case of turbulence, scale decomposition is a global operation, and Kolmogorov spectrum is nothing but a symmetric extension of the Fourier transform. Thus, it is more discernible to perform scale decomposition directly in Fourier space. Moreover, by carefully examining the curvelets/wavelets scale decomposition results, they are no different from a BPF in Kolmogorov spectrum with a certain fall-off. This motivates us to develop a scale decomposition technique directly with Fourier transform. The distinct advantage of methods in Fourier space, e.g. *scale selection* and *bandwidth*, can be better exploited using this approach, as it enables us to have a continuous scale decomposition which is very important for more comprehensive study of coherent structures across different scales in turbulence.

The only hurdle which prevents most researchers from using this naive approach is the ringing artifacts due to Gibbs phenomena. However, if nearby scales are introduced using the global-support Fourier basis, it can significantly reduce Gibbs rings to similar levels as methods using a local-support basis. Considering the drawbacks of present methodologies,

we propose an alternative to curvelets/wavelets, which in essence is an optimization-based approach to retaining the benefits of using Fourier transform and reducing Gibbs phenomena without having to confine the scale numbers to be only dyadic. Therefore, a filter in Fourier space can be directly designed to achieve multi-scale decomposition similar to ones using a local-support basis. The filtered data is then converted back to the physical domain, which gives us the scale decomposition results. Guided by the previous discussion on the advantages of using Fourier-based methods and drawbacks of curvelets/wavelets, our proposed method provides the following contributions to the field of multi-scale flow analysis:

- We developed a new scale decomposition method, Kolmogorov Spectrum Consistent Optimization (KoSCO), where the filter is designed directly in the Kolmogorov spectrum consistent space.
- We developed a novel optimization-based framework where an objective function is designed and its minimization gives the desired filter shape for scale decomposition.
- More flexible control over the band location and bandwidth in Kolmogorov spectrum is achieved with the proposed method. This unique feature of having continuous scales, which is the deficiency of the current methods based on curvelets/wavelets, can be beneficial for turbulence research as it greatly enhances the capability to track the evolution of structures across different continuous scales.

This paper is organized as follows. In §6.2, a formal derivation of the proposed scale separation method and the solution procedure are given. In §6.3, the method is verified for different test cases and the results are further discussed. Lastly, in §6.4, we draw the conclusion and discuss the future work.

6.2 Kolmogorov spectrum consistent optimization

In the following sections, we point out some key observations which led us to the proposed method in this paper.

6.2.1 Filter design

First, we define the *scale* for a turbulent flow suitable for decomposition purposes. The Kolmogorov spectrum, which illustrates the energy cascade governing turbulent flows, consists of the total energy carried by different sizes of eddies. Ideally, a *scale* is referred to as being a perfect band-pass filter (BPF) in Kolmogorov spectrum, which is illustrated in Fig. 6.1a as the solid line. However, a perfect BPF exhibits strong Gibbs ringing artifacts, which will be further discussed in detail in §6.3.1.

Since Gibbs ringing artifacts are generated due to insufficient basis, they can be suppressed by adding a few neighboring scales. On the other hand, the number of neighboring scales should be kept at minimum in order to preserve the sharpness of the filter. Thus, by introducing a certain amount of nearby scales, the Gibbs rings can be suppressed while the extent of the deviation from a perfect BPF is limited. The filtered results using such filter could therefore be much more meaningful than either a perfect BPF or a filter that includes an excessive amount of other scales.

Hence, the filter can be designed such that it has a fall-off from a perfect BPF to include some nearby scales. A fall-off is a non-zero extended region from the perfect BPF, which should be monotonic. Such monotonicity criterion is to ensure we do not introduce falsely magnified nearby scales; otherwise some spurious oscillating structures

Figure 6.1: Constraints for the filter design process.

may appear. Thus, we model the filter with the fall-off as a Gaussian function:

$$\hat{G}(k) = \begin{cases} 1 & \text{if } k \in \beta \\ e^{-r^2/2\sigma^2} & \text{if } k \notin \beta \end{cases}, \quad (6.2.1)$$

where β is the scale bandwidth; σ is a parameter to control the extent of the fall-off; and r is the distance from the perfect BPF. Fig. 6.1a illustrates the shape of the designed filter with a dashed line. This filter is applied to a Kolmogorov spectrum consistent space, which is obtained by using the similar procedure as in Kolmogorov spectrum calculation. This involves reduction of an N-dimensional field to a scalar (i.e. energy) in the Fourier space at different wavenumbers.

Now the problem is on how to specify an appropriate value of σ such that Gibbs ringing artifacts can be greatly suppressed while introducing a minimum of nearby scales. If σ is too small, Gibbs ringing artefacts will not be suppressed, whereas the filter remains very sharp. On the other hand, if σ is large, a wide range of nearby scales are added; although Gibbs ringing is suppressed, the notion of a sharp band-pass filter is lost. Thus,

an optimization problem can be defined where we try to balance the preservation of the sharp filter fall-off and the removal of Gibbs rings by finding a suitable σ . We refer to such optimization as Kolmogorov Spectrum Consistent Optimization (KoSCO) and we entail further details in the following.

6.2.2 Objective function definition

Let $\mathcal{G}$ be the desired filter and $\mathcal{G}_0$ be a perfect BPF ($\sigma \rightarrow 0$) defined using Eq. (6.2.1) in a Kolmogorov spectrum consistent space. Then, we can perform an inverse log transform and symmetric dimension extension to get the desired filter in Fourier space. In the following, u denotes the magnitude of the velocity vector $\mathbf{u}$. All quantities with a hat ($\hat{\cdot}$) are Fourier transform of the respective quantities.

Our objective function consists of two terms. We define the first term as the difference to measure the closeness between the result from a perfect BPF and the result from the designed filter. This is to ensure that the desired filter has a sharp fall-off. Mathematically, this term can be written as

$$E_k = \psi \int \left(\hat{\mathcal{G}}^2 - \hat{\mathcal{G}}_0^2 \right) |\hat{u}|^2 dk \quad \text{and} \quad \psi = \left(\int \hat{\mathcal{G}}_0^2 |\hat{u}|^2 dk \right)^{-1}, \quad (6.2.2)$$

where ψ is a normalization constant. E_k is a decreasing function (see Fig. 6.1b) with respect to the filter parameter σ defined in Eq. (6.2.1), since as σ decreases, the desired filter approaches a perfect BPF.

Next, we try to model the Gibbs rings or oscillation artifacts from which most of the decomposition methods (including wavelets and curvelets) suffer inherently. An example of this phenomenon is shown in Fig. 6.2 (which will be described further in section 6.3.1). Applying a naive band-pass filter using FFT results in spurious rings which contaminates

Figure 6.2: Gibbs ringing phenomena.

the results and is a known problem due to which Fourier based methods are not generally utilized for such decomposition purpose. We define the second term in our objective function as a measure of these spurious structures. Quantifying the Gibbs rings in the objective function allows us to reduce them as much as possible through our optimization process. This is the key to utilizing Fourier based methods and their superior advantages of continuous scale location and bandwidth. One implicit condition which the decomposition method must obey, and which is also physically intuitive, is that a region which is originally smooth (small gradients with low frequency) should also be smooth in the resulting decomposition. Based on this, we define the second term of our objective term as the *regularization* term for the measurement of the strength of the Gibbs rings, which measures the difference of the gradients in the original field and the decomposed result as a weighted sum of the gradients where the weight is inversely proportional to the strength of gradient and can be mathematically written as

$$E_r = \phi \sum_{\alpha} \int |w_{\alpha} : \partial_{\alpha} * u(1 - \mathcal{G})|^2 \quad \text{and} \quad \phi = \left(\sum_{\alpha} \int |w_{\alpha} : (\partial_{\alpha} * u)|^2 dk \right)^{-1}, \quad (6.2.3)$$

where $*$ is the convolution operator; α denotes any derivative direction; $:$ represents element wise product; ϕ is the normalization constant, and $w_{\alpha} = e^{-(\partial_{\alpha} u)^2 / 2\theta^2}$ is a Gaussian weight with respect to gradients of the original field. The parameter θ is the standard

deviation of the gradients of the original field (calculated by comparing the value at each point with the mean gradient over the field). Although a Gaussian weight has been chosen for the present case, any function with a monotonic fall-off can be used to define the weights. We use the weight at different locations to control the influence of the regularization term, where regions with large gradients will have less impact on the measurement and vice versa. This will enforce a strong constraint only in smooth regions. Since the Fourier transform has global support basis, the constraint in smooth regions will also give the constraint in the surrounding non-smooth regions. Therefore, this term can effectively measure the overall strength of ringing artifacts. As shown in Fig. 6.1b, with lower σ , E_r increases, which indicates that the result contains more rings. Note that the choice of the weight w_α only alters the objective function, and thus does not violate the invariance of spectral cascade rate.

Note that the two terms E_k and E_r are defined in two different domains: E_k in wavenumber domain while E_r in spatial domain which is inefficient for optimization. Thus, we unify the two domains by transforming term E_r into wavenumber domain using Plancherel's theorem, which gives:

$$E_g = \phi \sum_{\alpha} \int \left| \hat{w}_\alpha * [\hat{\partial}_\alpha \hat{u}(1 - \hat{\mathcal{G}})] \right|^2 dk. \quad (6.2.4)$$

E_r and E_g have the same value; they only differ in space for optimization purpose. To enhance the order of magnitude of the difference measurement, both E_k and E_g are raised to an exponent m . In the remainder, m is chosen to be around 5 in the results presented here.

The total objective function can be defined as a linear combination of E_k and E_g as:

$$E = (1 - \lambda)E_k^m + \lambda E_g^m, \quad (6.2.5)$$

where λ is a regularization parameter in the range $[0, 1]$ to control the relative importance of E_k and E_g . Not all λ values yield an optimizable problem, i.e. a minimizable function E . See Fig. 6.1b for an illustration of E_k , E_g and E respectively. An appropriate λ can be calculated by enforcing the *optimizability constraint*.

By increasing λ , the impact of E_g is increased, which has the effect of introducing more nearby scales to suppress rings for the final decomposition result. There exists a constraint such that rings are suppressed by introducing minimum amount of E_g , which makes the objective function locally optimizable, as shown by the solid line in Fig. 6.1b. We call this constraint the *optimizability constraint* and it requires that the objective function has small gradients for small σ . This allows us to automatically calculate the most appropriate λ .

Given a relatively small value of σ , and denoting the derivatives for E_k and E_g at this value as E'_k and E'_g , λ can be calculated as:

$$\lambda = \frac{\epsilon - mE_k^{m-1}E'_k}{m(E_g^{m-1}E'_g - E_k^{m-1}E'_k)}, \quad (6.2.6)$$

where ϵ is a small gradient tolerance value which is selected to be around 0.001 in our experiments.

6.2.3 Solution procedure

With λ calculated, we can search for the filter parameter σ by minimizing the objective function (6.2.5). Note that by automatically computing λ , we ensure that the objective is always optimizable. Thus, the minimization can be quickly solved using Brent's method [15], which makes our optimization very stable and ensures that it always yields a solution. Once we obtain an optimized σ , we can perform fast Fourier transform (FFT) filtering to

efficiently obtain scale decomposition results.

In order to avoid the wave reflection effects due to non-periodic boundary, a periodic reconnection by mirror extension of the data is performed, which is also required in curvelets [107].

The procedure for KoSCO algorithm can be summarized as follows:

- A filter is designed in Kolmogorov spectrum consistent space with an unknown parameter σ , and is subsequently transformed to Fourier space.
- Based on this transformed filter in Fourier space, an objective function is constructed with Eq. (6.2.5).
- With the derivatives of E_k and E_g , λ is computed according to Eq. (6.2.6).
- Using the computed λ and given an initial σ (e.g., $\sigma_0 = 1.0$), the Brent's algorithm is employed to efficiently find the optimal filter parameter σ_m .
- Finally, FFT filtering is performed with parameter σ_m to get the scale decomposition result.

6.3 Results and discussions

6.3.1 Gibbs ringing artefacts

We use a circle image (see Fig. 6.2a) as a test case to demonstrate the performance of KoSCO compared to other existing methods with the same scale parameters (the location and bandwidth of the scale). The reason we chose this image is that it is simple with sharp edges, and hence may have strong Gibbs rings around the edge if inappropriately

decomposed. Fig. 6.2b shows the scale decomposition result from a perfect BPF in Fourier space, which introduces strong ringing artifacts. Due to the global-support Fourier basis, a perfect BPF in Fourier space lacks suitable basis functions to completely represent structures, making the result contaminated by oscillating patterns.

Curvelets use local-support basis to avoid ringing artifacts. Fig. 6.2c shows scale decomposition result using curvelets (DCuT) [17] which is the technique used by several groups recently. We can easily notice that it has fewer rings than perfect BPF in Fourier space. However, the rings are still not adequately suppressed.

In Fig. 6.2d, we show the scale decomposition result from KoSCO. Clearly, the Gibbs rings are much more suppressed. Although we are unable to completely remove the Gibbs rings, KoSCO is still at an advantage when compared with the state-of-the-art. Eliminating rings when performing a multiscale decomposition is crucial since they can lead to spurious structures which may not physically exist. Turbulent flows are highly fluctuating and it is difficult to distinguish the presence of such rings from the true data.

6.3.2 Multi-scale diagnostics using KoSCO

In order to demonstrate the multi-scale decomposition capabilities of KoSCO, tests were conducted on a fractal image. Fractals can provide a very good example for multi-scale phenomena exhibiting the characteristic self-similarity. In Fig. 6.3, we show multi-scale decomposition results for the same fractal image (512×512) used by Bermejo-Moreno and Pullin[11]. Perfect BPF produces spurious structures due to Gibbs ringing which makes the multi-scale decomposition contaminated by artifacts as can be seen in the first row. Using curvelets, one can get a maximum of 6 scales. The results by Bermejo-Moreno & Pullin[11] can be seen in the second row of Fig. 6.3. We ignore the largest and the

Figure 6.3: Comparison of multiscale decomposition results using perfect BPF, curvelets and KoSCO for a Julia fractal. Second row reprinted with permission from I. Bermejo-Moreno and D. I. Pullin, *J. Fluid. Mech.* 603, 101(2008). Copyright 2008, Cambridge University Press.

smallest scales because they do not convey meaningful structures for comparison. The largest scale is too coherent and shows almost the mean value while the smallest scale is too incoherent and shows only noise.

Based on the spectrum of the results from curvelets, we estimate their locations and bandwidths and perform the decomposition again using KoSCO. Very similar decomposition results can be obtained as shown in the third row of Fig. 6.3. Even though we rely on Fourier transform, we have successfully suppressed Gibbs ringing artifacts. By observation, length scales from the decomposed results are decreasing in geometrical size as one goes from larger to smaller scales. In comparison to curvelets, our results show more continuous features and better preservation of original structures. For scale 2 in

Figure 6.4: 2D results for isotropic turbulence, (top left) Original flow, increasing scales from left to right and top to bottom (Red shows higher values and blue shows smaller values of the normalized velocity magnitude).

Fig. 6.3, at the center of the fractal, we see intermittent features for curvelets results whereas for KoSCO, the features are smoother and therefore, constitute a better repre-

Figure 6.5: Spectrum for decomposition results. Black solid: original; light grey: curvelets. For (a), dark grey: KoSCO; for (b) and (c), dark grey dashed: odd scales using KoSCO; dark grey solid: even scales using KoSCO.

resentation of the structures in the original image. Each decomposed scale is represented in the spectrum correspondingly as shown in Fig. 6.5a. The results from KoSCO have sharper fall-offs than the results from curvelets and hence are better representations of the scales. With the help of optimization, we obtain optimal selection of fall-offs with suppressed Gibbs rings. The reduction of fall-off ensures that the decomposition results belong to an individual scale and reduce the inclusion of other scales. This can be attributed to the monotonicity criterion used while designing our filter (see §6.2.1) whereas in curvelets decomposition, the filter fall-offs tend to be fluctuating. Such fluctuating filters may result in intermittent features and may not preserve the scale structures well. On the other hand, our constraint on monotonicity helps to obtain more reliable results. Note that while curvelets is capable of producing only 6 scales, KoSCO has the capability to yield infinite number of scales.

6.3.3 Application to DNS data

We use forced isotropic turbulence data provided by John Hopkins University (JHU) turbulence data cluster [56] at $Re_\lambda = 433$ to obtain scale decomposition results using KoSCO. The domain is a cube of length 2π with periodic boundary conditions having 1024^3 grid points. We choose a portion of this data with 512^3 points for our testing purposes.

For the 2D case, we use a cross section of this data at the mid-point of the Z -plane. Results for 8 different scales of velocity magnitude are shown in Fig. 6.4, and their corresponding spectrums are illustrated in Fig. 6.5b. The different scales educe structures corresponding to different length scales.

Next, we decompose the 3D flow into 15 scales to demonstrate the ability to have

Figure 6.7: Video showing multi-scale turbulence structures for 3D isotropic case using KoSCO (enhanced online).

Figure 6.8: Probability distribution of velocity magnitude for different scales.

continuous scale decomposition using KoSCO. The corresponding energy spectrum can be seen in Fig. 6.5c. In order to visualize this volumetric data, iso-contouring technique is used. Iso-surfaces for this 3D decomposition can be seen in Fig. 6.6 where we show odd numbered scales for the sake of clarity. The iso-contour value used for visualizing the volumetric data has been chosen as the mean plus 1.5 times the standard deviation [11, 55]. It can be clearly seen that structures with reduced sizes are educed with increasing scale numbers.

Scales 1-3 are in the forcing range with the energy containing scales which comprise the large scale structures. Ellipsoidal-shaped structures can be observed for this scale. Scales 4-8 correspond to the inertial range where tube-like structures are pre-dominant. The tubes become thinner as we go to smaller scales indicating a stretching process in the inertial region. For greater clarity of structures, readers are kindly suggested to look at video in Fig. 6.7 (enhanced online). The dissipative range of scales can be seen in Scale 9-15 where structures are quite small and not visually recognizable.

The distribution of velocity magnitude at different scales normalized with its standard deviation is shown in Fig. 6.8. We only show odd-numbered scales to avoid confusion. It gives us the insight about the distribution for each of these scales in comparison to the original velocity field. Scales 4-8 lie in the inertial range and collapse on top of each other. For the dissipative range, i.e. Scales 9-15, the distribution function shows a reduced range. Hence, we can clearly distinguish between the inertial and dissipative range and these observations are inline with those reported by Bermejo-Moreno and Pullin[11]. Note that curvelets can provide only 6 scales for this data size.

Some characteristic velocity and length scales for isotropic turbulence can be computed based on the following:

$$\overline{u_i^2} = \frac{2}{3} \int_0^\infty E_i(k) dk, \quad (6.3.1)$$

$$L_i = \frac{\pi}{2\overline{u_i^2}} \int_0^\infty \frac{E_i(k)}{k} dk, \quad (6.3.2)$$

$$L'_i = \frac{\pi}{2\overline{u_i^2}} \int_0^\infty \frac{E_i(k)}{k} dk, \quad (6.3.3)$$

where $E_i(k)$ and $\overline{u_i^2}$ are energy spectrum and squared characteristic integral velocity respectively for scale i , and η is the Kolmogorov length scale. Also, the following relations hold for the original velocity field $E(k)$ and $\overline{u^2}$:

$$E(k) = \sum_i E_i(k), \quad (6.3.4)$$

$$\overline{u^2} = \sum_i \overline{u_i^2}. \quad (6.3.5)$$

which simply describes that after summing up the individual scales, we recover the original velocity field. The corresponding values are shown in table 6.1 for both the original velocity field and for different scales. This breakdown of scales show reasonably correct trends with reducing length and velocity scales as one goes from the mean and inertial to the dissipative ranges and how they are compared with the original velocity field.

We also perform a study on the impact of different resolutions on the estimation of the filter shape for 3D isotropic case

We also perform a study on the influence of different resolutions on the estimation of the filter shape for 3D isotropic case as shown in Table 6.2. The start of the band and the bandwidth along with parameter σ used for filter design (see Eq. (6.2.1)) is enlisted. As can be seen, the optimal σ value does not have any direct dependence on the resolution, scale location and scale bandwidth, and should be computed independently given any dataset.

The complete solution process for each scale requires 10-20 iterations for the optimization procedure. For a single scale decomposition of 512^3 dataset, it takes about 20 minutes on a desktop PC with Intel i7 processor. This is faster in comparison to curvelets with regards to the processing time. In addition, curvelets requires storage of extra dimensions such as direction and orientation which can pose immense memory problems for high resolution DNS datasets. Hence, KoSCO performs scale decompositions using lesser computational resources as compared to curvelets.

6.4 Conclusion

A methodology for multi-scale decomposition (KoSCO) of flow data is developed, which is consistent with band-pass filtering in the Kolmogorov spectrum. It provides an alternative to curvelets and tries to overcome some of their inherent disadvantages. Optimization methods have been used for the first time to design such decomposition procedure. Unlike previous methods, the size and location of the various scales can be easily controlled. The method has been proven to be capable of adequately suppressing Gibbs ringing artifacts while preserving sharp band-pass filter properties. The scale decomposition results for Julia fractal and turbulent flow fields have been presented together with their spectrum and probability distribution function for different scales. KoSCO can provide a framework for multi-scale analysis for geometric structure identification with specific applications to turbulent flows, atmospheric flows, multiphase flows, etc. It can be easily extended to any N-dimensional data with multi-scale phenomena where scales are defined with respect to a certain spectrum. In addition, KoSCO can also support scale decomposition for both isotropic and anisotropic data since we operate in Fourier space rather than the spatial domain.

Scale	$\overline{u_i^2}/\overline{u^2}$	L_i/η	L'_i/η
Original	1	122.5405	122.5405
1	0.333676	56.9197	170.5839
2	0.251062	31.0462	123.6593
3	0.182859	15.5049	84.7914
4	0.140593	8.5200	60.6002
5	0.122404	5.3627	43.8115
6	0.105685	3.3913	32.0890
7	0.087093	2.0499	23.5373
8	0.067492	1.1749	17.4077
9	0.046771	0.6041	12.9155
10	0.028451	0.2734	9.6111
11	0.015230	0.1087	7.1350
12	0.007394	0.0388	5.2500
13	0.003910	0.0150	3.8262
14	0.002433	0.0068	2.7978
15	0.000918	0.0022	2.3794

Table 6.1: Characteristic integral velocity and length for different scales

Resolution			32 ³	64 ³	128 ³	256 ³	384 ³	512 ³
Scale	Band-start	Bandwidth	σ	σ	σ	σ	σ	σ
1	0.2	0.05	0.03463	0.0427	0.02801	0.02298	0.02409	0.02409
2	0.25	0.05	0.05149	0.02409	0.02409	0.03909	0.02298	0.02298
3	0.3	0.05	0.02409	0.02409	0.03075	0.03480	0.04270	0.02298
4	0.35	0.05	0.05149	0.02409	0.02967	0.02801	0.03084	0.03480
5	0.4	0.05	0.03123	0.02999	0.02801	0.02801	0.02801	0.02967
6	0.45	0.05	0.02801	0.03008	0.02801	0.02801	0.02801	0.02801
7	0.5	0.05	0.02801	0.02801	0.02965	0.02801	0.02801	0.02801
8	0.55	0.05	0.02801	0.02801	0.02964	0.02801	0.02801	0.02801
9	0.6	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
10	0.65	0.05	0.02539	0.02801	0.02801	0.02801	0.02801	0.02968
11	0.7	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
12	0.75	0.05	0.02801	0.02801	0.02801	0.02801	0.02801	0.02801
13	0.8	0.05	0.02801	0.02801	0.03490	0.02801	0.02801	0.02801
14	0.85	0.05	0.03007	0.03219	0.02409	0.04346	0.03293	0.03092
15	0.9	0.05	0.02801	0.02801	0.02409	0.02409	0.02409	0.02409

Table 6.2: Filter parameters for a 3D isotropic case with different resolutions

Chapter 7

Optimization-based Flow

Decomposition for Continuous

Turbulence Structure Visualization[†]

7.1 Introduction

Turbulence dynamics is multi-scale in nature. As revealed by both experimental observations and numerical simulations [65], the vortical structures in turbulent flows can span a wide and continuous range of spatial scales: the ratio between the largest and smallest scales can be in the order of a thousand for high Reynolds number flows. Since vortical structures of many different scales co-exist collectively in a turbulent flow, it is thus complicated to analyze and visualize the physical process of turbulence dynamics directly within a composed flow.

[†]X. Liu, M. Mishra, C. W. Fu and M. Skote. Under review in *IEEE Transactions on Visualization and Computer Graphics.*, 2014.

This motivates the development of multi-scale methods [29, 11] to analyze turbulent flows by scale decomposition. For example, finer-scale flow structures tend to be more stretched compared to coarser-scale structures [11], but such a phenomenon is not obvious when directly looking at the original flow, which aggregates all different scales. Moreover, unveiling the inter-scale transition and interaction of flow structures by multi-scale decomposition can enrich our understanding on turbulence dynamics, especially for showing the transition from laminar to turbulent flows in boundary layers [107].

Different from conventional vector field decomposition, turbulent flow decomposition often employs the Kolmogorov energy spectrum [59, 108] to characterize, or guide, the decomposition process [79], since such spectrum indicates the variation of turbulent energy in different length scales in a contracted Fourier space. A *scale* in the decomposition is usually referred to as a band of wavenumbers in the Kolmogorov energy spectrum, and in principle, this should be determined by employing a perfect band-pass filter (BPF) in the Kolmogorov energy spectrum. However, such a straightforward decomposition usually leads to strong ringing artifacts [33], which contaminates the decomposition and disturbs the real turbulence structures in the decomposed results.

To relieve this issue, turbulence researchers often employ multi-scale decomposition methods such as wavelets [29, 98] and curvelets [11, 59]. Wavelets-based methods can help reduce the ringing artifacts while maintaining a sharp BPF, whereas curvelets-based methods can further improve the decomposition quality, particularly for stretched structures.

However, since wavelets and curvelets are not originally designed for flow decomposition, they exhibit a number of limitations. First, they restrict the number of decomposed scales due to their discrete nature, so they cannot allow continuous scale decomposition

even though this relates to the physical fact that scales in turbulent flows are continuous, i.e., scales in a transitional flow can smoothly evolve from one to another. Second, they are less accurate in tracking flow structures compared to continuous scale decomposition, particularly for boundary layer flows with transition regions. Lastly, to manage the additional dimensions, wavelets & curvelets based methods require large amount of memory during the computation.

Our Method. We present a new optimization-based method for flow decomposition, aiming at improving the visualization of scale structures in a turbulent flow. Since we derive our method based on the Kolmogorov energy spectrum, it is continuous in nature, and can overcome the various drawbacks of wavelets and curvelets. Moreover, by means of optimization, our method can greatly improve the decomposition accuracy and effectively suppress the ringing artifacts while maintaining a sharp BPF shape. Furthermore, we devise a novel analytical model for the objective function in the optimization, thereby making the optimization highly efficient to compute.

In detail, we address the turbulent flow decomposition problem by formulating a novel optimization framework, which is devised based on the following key observation:

Our optimization goal is to compute a BPF in the Kolmogorov energy spectrum, such that it takes appropriate amount of nearby scales to suppress ringing artifacts while maintaining sharp filter shape.

To realize this novel optimization approach, we design and formulate a parameterized BPF with extended fall-off region in the Kolmogorov energy spectrum, and devise an objective function to measure its closeness to a perfect BPF as well as the amount of ringing artifacts. Moreover, we further derive a new analytical form of the objective func-

tion by performing domain transformation and analytical approximation on the original objective function. Hence, we can efficiently solve for an optimized BPF in a very short time (around 0.02s for each scale), and decompose a turbulent flow directly by performing Fourier-space filtering with the optimized BPF. Compared with recent works in turbulent flow decomposition, our method improves several orders of magnitude in computational efficiency while achieving high-quality scale extraction. Rather than focusing on the presentation of single-scale local/global features as in most existing work [89, 80], we present new continuous scale visualization results for various turbulent flows, i.e., isotropic and boundary layer turbulent flows.

Contributions. The key technical contribution of this paper is a new *analytical optimization model* that can efficiently and continuously decompose turbulence scales from a given flow field according to the Kolmogorov energy spectrum. *Continuous flow decomposition* is a highly desirable and unique characteristic of this work, which is not feasible with existing wavelets/curvelets-based approaches. Moreover, our decomposition method is highly *efficient*, requiring less memory and computation while facilitating *continuous visualization* of length-scale structures for investigating turbulence scale properties such as scale correlation and transformation. Continuous visualization is highly beneficial for the scientific study of turbulent flows, and has not been explored and presented in any previous flow visualization work. Furthermore, users are free from tedious parameter tuning, and yet can obtain sharp and ringing-minimized decomposition results. Lastly, experts in turbulence research are also involved as collaborators to support the research with domain knowledge, and to evaluate and enhance the visualization results.

7.2 Related Work

This section reviews and discusses the following three areas of research related to turbulent flow simulation, analysis, and visualization. Note that due to space limit, we did not include research works in general flow simulation and visualization.

7.2.1 Direct Numerical Simulation (DNS)

DNS is an accurate solution of flow equations, capable of capturing a full range of scales without incurring any turbulence model [65]. Thus, DNS has been a highly important tool in turbulence research [62] for producing high-quality flow simulations that imitate the real flow dynamics; and multi-scale flow analysis is often conducted on the DNS data to help reveal the underlying physical process. This is useful, e.g., studying near-wall flow structures [105], and developing new turbulence models based on DNS analysis [71], where accurate experimental measurements are still difficult to obtain. Moreover, control of turbulence with methods yet to be realized in a laboratory can be investigated by using DNS [92].

7.2.2 Multi-scale Analysis of Turbulence

One key approach to turbulence analysis is multi-scale flow decomposition. Early methods [29, 18] are mainly wavelet-based since wavelets are multi-scale in nature and can help reduce the ringing artifacts. However, wavelets have a number of shortcomings; e.g., Candes and Donoho [18] showed that due to the isotropic basis, wavelets are not effective in representing stretched structures in a turbulent flow. Hence, curvelets, which are extensions of wavelets with elongated basis, were proposed [17] for scale decomposition

with highly-stretched structures. Bermejo-Moreno and Pullin [11] presented a multi-scale geometrical decomposition method based on curvelets for isotropic flows while Yang et al. [108] showed the evolutionary geometry of the Lagrangian scalar field for stationary isotropic turbulence. Other than wavelets and curvelets, Leung et al. [55] used spatial filter for flow decomposition, but since the filter is too wide, the decomposed flow may excessively include too many nearby scales. Most recently, Mishra et al. [61] developed an optimization method to construct Fourier-space filters, but their method is inefficient and computationally less reliable for continuous flow decomposition.

This paper presents a new continuous scale decomposition method based on computational optimization. Compared to the above approaches, our method is based on a novel analytical model for the objective and allows exceedingly-high efficient and unconditionally stable continuous scale decomposition with more independent turbulence scales. Thus, it can lead to more effective turbulence visualization and analysis.

7.2.3 Turbulence Visualization

There are three major research directions in turbulence visualization: i) improving the data processing efficiency, particularly for interactive visualization; ii) developing visualizations targeted for specific applications; and iii) enhancing the visualization by identifying structures/features in the flow data.

Among various works in the first research direction, Gregory et al. [41] developed a system that optimizes the data management and caches the computations, thereby enabling interactive visualization of terabyte-sized flow data. Treib et al. [95] presented a GPU-based system design for feature-based turbulence visualization; their method works on a flow field with compressed representation, and can efficiently deliver high-resolution

visualization on a desktop computer.

The second research direction is application-oriented. Wiebe et al. [103] employed the footprint of vortices induced from boundary walls to form a new type of streak line visualization. Williams et al. [104] used a reference model of an ideal vortex to model and identify real vortex cores for geophysics. Wei et al. [101] introduced a dual-space method to analyze particle data from turbulent combustion simulation using model-based clustering. Koehler et al. [49] developed visual analysis of vortices produced from the deformable flapping wings of a dragonfly. More recently, Shafii et al. [88] extracted vortices in wind farms, and visualized and analyzed the interplay between these vortices and the forces on the wind turbine's blades.

The third direction focuses on extracting turbulent features for turbulence visualization, where many criteria have been explored, e.g., the Q and λ_2 criteria [44]. Silver and Wang [25] isolated and tracked the local volume-based features in the form of regions-of-interest in time-varying 3D fluid data. Stegmaier et al. [89] combined vortex core line detection and the λ_2 method to visualize and analyze turbulent flows. Helgeland et al. [37] visualized the energetic structures in a turbulent flow field by using structure-based tensors. Later, Helgeland et al. [38] developed a vorticity field line approach with specialized particle advection and a seeding strategy. Schafhitzel et al. [81] improved the vortex core line detection by constructing curves that connect λ_2 minima. Pobitzer et al. [64] employed proper orthogonal decomposition (POD) to separate the energy scale in a turbulent flow.

Coherent structures are another useful features for enhancing turbulence visualization. Garth et al. [31] characterized and visualized coherent Lagrangian structures by adaptive computation of finite-time Lyapunov exponent fields. Schafhitzel et al. [80] visualized

Figure 7.1: The Kolmogorov spectrum (left) of a 512^3 DNS data (right) of isotropic turbulence.

and tracked coherent structures based on shear stress, so that we can identify and track both vortices and shear layers, while Gaither et al. [30] detected and visualized critical structures in a massive turbulent-flow simulation.

This work proposes a continuous and highly efficient scale decomposition method to aid turbulent flow visualization. Compared to the previous works in turbulent flow visualization and analysis, we devise a novel optimization-based technique in the Kolmogorov energy spectrum. Due to our analytical formulation of the optimization model, the turbulence scales can be efficiently solved with high performance. By this, we can efficiently optimize the separation of turbulence scale structures, and then extract and visualize meaningful features relevant to the Kolmogorov energy spectrum. This is a new form of decomposition-based visualizations we developed with domain experts in turbulence research, and several data sets including high-resolution isotropic and boundary layer flow turbulence are also explored.

Figure 7.2: Blue line: perfect band-pass filter (BPF). Red lines: our parameterized filter with two extended fall-off regions.

7.3 Continuous Scale Decomposition

This section presents our novel optimization formulation to decompose a turbulent flow field based on the Kolmogorov energy spectrum. Such spectrum is computed by the logarithmic contraction of the Fourier transform of a turbulent flow field, see Fig. 7.1, and is one-dimensional by nature [65].

To achieve our goal, we propose the followings: First, we design a parameterized filter shape with fall-off regions in the Kolmogorov energy spectrum domain (subsection 7.3.1), and formulate an objective function to measure the filter sharpness and the amount of ringing (subsection 7.3.2). Then, we derive an efficient analytical model of the objective function (subsection 7.3.3), and further solve the optimization to obtain appropriate filters for the decomposition (subsection 7.3.4).

7.3.1 Parameterized Band-Pass Filter

We build our parameterized band-pass filter (BPF) by attaching two extended fall-off regions to a perfect BPF, see Fig. 7.2. Note that to avoid bias, the two fall-off regions

should be symmetric. Moreover, they should be monotonically attenuating in both spatial and wavenumber domains, i.e., the filter value should gradually drop to zero, e.g., see the red curves in Fig. 7.2.

Here we use the Gaussian function to model the fall-off regions in our parameterized filter

$$\mathcal{G}(k) = \begin{cases} 1 & \text{if } k \in [k_1, k_2] \\ e^{-\theta(k-k_1)^2} & \text{if } k < k_1 \\ e^{-\theta(k-k_2)^2} & \text{if } k > k_2 \end{cases}, \quad (7.3.1)$$

where $\mathcal{G}(k)$ is defined in wavenumber domain (k); k_1 and k_2 are lower and upper bounds of the filter band, respectively; and θ is a parameter to control the extent of the fall-off. By varying θ , we can introduce different amount of nearby scales into the decomposition to suppress the ringing.

The reason why we choose the Gaussian function is due to the minimization property of the uncertainty principle [93]: When the support range of a Gaussian in the spatial domain is squeezed for suppressing the ringing, its spreading in the wavenumber domain is simultaneously minimized. Thus, we can employ a Gaussian to look for a balance between the support in the spatial domain and the filter sharpness in the wavenumber domain. Since $\mathcal{G}(k)$ is defined in the Kolmogorov energy spectrum, it is equivalent to a filter in the Fourier space by an inverse logarithmic transform together with a symmetric dimension extension.

7.3.2 The Objective Function

Next, we formulate our objective function by measuring and balancing the *filter sharpness* and the *amount of ringing*. To facilitate the discussion, we define:

- $\mathbf{u}$ and u : the flow velocity field, and its magnitude;
- $\mathbf{k}$ and k : the wave vector and its magnitude;
- $\mathcal{G}$: the desired filter as defined by Eq. 7.3.1;
- $\mathcal{G}_0$: a perfect BPF ($\theta \rightarrow \infty$ in Eq. 7.3.1);
- $\hat{(\cdot)}$: Fourier transform of respective quantity;
- θ_l and θ_u : the lower and upper bounds of θ , respectively, during the optimization process; and
- θ_m : the optimal θ value after the optimization.

Note that in all the discussions below and also in our implementation, we normalize k to the range $[0, 1]$.

Filter Sharpness. The first term of the objective function E_d evaluates the filter sharpness by measuring how close the decomposition results by $\mathcal{G}$ and $\mathcal{G}_0$ are:

$$E_d = \psi_d \int_0^1 \left(\hat{\mathcal{G}}(\theta)^2 - \hat{\mathcal{G}}_0^2 \right) |\hat{u}|^2 dk, \quad (7.3.2)$$

where ψ_d is formulated as $\psi_d = \left(\int_0^1 [\hat{\mathcal{G}}(\theta_l)^2 - \hat{\mathcal{G}}_0^2] |\hat{u}|^2 dk \right)^{-1}$, which normalizes E_d to $[0, 1]$.

Amount of Ringing. Recall that the Fourier basis has global support, and it propagates wave-like rings from high-frequency fluctuations, thus contaminating surrounding smooth regions. Hence, we estimate the amount of ringing to constrain the flow decomposition by examining the difference between the given data and the decomposed data in originally-smooth regions:

$$E_r = \psi_r \int_0^1 |w \nabla [u(1 - \mathcal{G}(\theta))]|^2 dk, \quad (7.3.3)$$

where $\psi_r = \left(\int_0^1 |w \nabla [u(1 - \mathcal{G}(\theta_u))]|^2 dk \right)^{-1}$ is a normalization factor similar to ψ_d ; $w = e^{-|\nabla u|^2/2\sigma^2}$ is a Gaussian weight with respect to the gradients of the original field, helping to enforce a strong constraint in the smooth regions; and σ is the standard deviation of the gradients in the entire original flow field.

Eqs. 7.3.2 and 7.3.3 can be used to construct an objective function, but they are not efficient for the computation. Moreover, their current formulations cannot lead to effective parameter fine-tuning for creating a sharp fall-off filter. Hence, we enhance the order of magnitude of their measurements, and raise both E_d and E_r by an exponent m , which is set to be 5 in our implementation.

Overall Objective Function. We define the overall objective function E as a linear combination of E_d and E_r :

$$E = E_d^m + \lambda E_r^m, \quad (7.3.4)$$

where $\lambda > 0$ is a parameter to control the influence of E_r to E . Having a larger value of λ introduces more nearby scales to the decomposition result, so the ringing artifacts can be more suppressed, but in return, the filter shape ($\mathcal{G}$) is further deviated from the perfect band-pass filter ($\mathcal{G}_0$).

7.3.3 Deriving an Efficient Analytical Form

Evaluating Eq. 7.3.4 is computationally very expensive since it requires numerical integration over the entire data space, especially when we need a number of iterations to complete the optimization, making Eq. 7.3.4 very slow for processing high-resolution 3D DNS data. Noting that the model can be re-formulated by considering domain transformation and analytical approximation, we eventually reformulate Eq. 7.3.4 to form an

analytical objective function, which can be evaluated with exceedingly high performance.

Domain Consistency. Since E_d and E_r are defined in two different domains (E_d in wavenumber domain and E_r in spatial domain), their optimization becomes inefficient due to the time-consuming integral transforms. Hence, we propose to unify them by transforming E_r into the wavenumber domain using the Plancherel's theorem [63]:

$$E_r = \psi_r \int_0^1 |\hat{w} * i\mathbf{k}\hat{u}(1 - \hat{\mathcal{G}}(\theta))|^2 dk, \quad (7.3.5)$$

where $\psi_r = \left(\int_0^1 |\hat{w} * i\mathbf{k}\hat{u}(1 - \hat{\mathcal{G}}(\theta_u))|^2 dk \right)^{-1}$.

Re-formulating E_d . Since E_d is primarily for measuring how close the decomposition results by $\hat{\mathcal{G}}$ and $\hat{\mathcal{G}}_0$ are, we propose to discard the term $|\hat{u}|^2$ and simplify E_d as

$$\tilde{E}_d = \tilde{\psi}_d \int_0^1 [\hat{\mathcal{G}}(\theta)^2 - \hat{\mathcal{G}}_0^2] dk, \quad (7.3.6)$$

where $\tilde{\psi}_d = (\int_0^1 [\hat{\mathcal{G}}(\theta_l)^2 - \hat{\mathcal{G}}_0^2] dk)^{-1}$. Inserting the specific form of $\hat{\mathcal{G}}$ into Eq. 7.3.6 with some integral approximations (see Appendix B for details), we can re-formulate E_d as

$$\tilde{E}_d = \sqrt{\frac{\theta_l}{\theta}}, \quad \theta > 0. \quad (7.3.7)$$

Obviously, $\tilde{E}_d$ is an asymptotically-decreasing function that approaches 0 when $\theta \rightarrow \infty$, see Fig. 7.3 for an illustration.

Re-formulating E_r . Since w (or $\hat{w}$) and u (or $\hat{u}$) are independent of θ , they can be taken out of the integral as scaling parameters and then absorbed into λ to form a new parameter $\tilde{\lambda}$. Thus, E_r can be re-formulated as:

$$\tilde{E}_r = \tilde{\psi}_r \int_0^1 k^2 (1 - \hat{\mathcal{G}}(\theta))^2 dk, \quad (7.3.8)$$

where $\tilde{\psi}_r = \left(\int_0^1 k^2 (1 - \hat{\mathcal{G}}(\theta_u))^2 dk \right)^{-1}$. Inserting the specific form of $\hat{\mathcal{G}}$ into Eq. 7.3.8 together with some integral approximation and further derivation (see Appendix B for

Figure 7.3: Objective function $\tilde{E}$ and its constituting components $\tilde{E}_d$ and $\tilde{E}_r$. Note that both axes are in log-scale.

details), we obtain an analytical expression for $\tilde{E}_r = \tilde{\psi}_r H(\theta)$, where

$$H(\theta) = \frac{\gamma}{3} - \frac{2(2^{3/2} - 1)\sqrt{\pi}\alpha\theta + 3\sqrt{2}\beta\sqrt{\theta} + \eta}{2^{5/2}\theta^{3/2}}, \quad (7.3.9)$$

$\tilde{\psi}_r = H(\theta_u)^{-1}$; $\alpha = k_1^2 + k_2^2$, $\beta = 2(k_2 - k_1)$; $\gamma = 1 + k_1^3 - k_2^3$; and $\eta = (2^{5/2} - 1)\sqrt{\pi}$. Since $\tilde{E}_r$ increases with θ , it is consistent to the observation that when θ increases, the filter is narrowed and the ringing effect becomes more apparent.

Analytical Objective Function. Lastly, we re-formulate our objective function by combining $\tilde{E}_d$, $\tilde{E}_r$, and $\tilde{\lambda}$:

$$\tilde{E} = \tilde{E}_d^m + \tilde{\lambda}\tilde{E}_r^m = \left(\frac{\theta_l}{\theta}\right)^{\frac{m}{2}} + \tilde{\lambda} \left[\frac{H(\theta)}{H(\theta_u)}\right]^m. \quad (7.3.10)$$

Originally, $\tilde{\lambda}$ is a user parameter for regulating the ringing: If we increase $\tilde{\lambda}$, more nearby scales are taken into the decomposition to suppress the ringing. However, not all $\tilde{\lambda}$ values are acceptable. Hence, we need to determine appropriate $\tilde{\lambda}$ values, preferably in

Figure 7.4: Continuous scale decomposition on a 512×512 isotropic DNS turbulent flow data. Red and blue colors indicate high and low velocities, respectively.

an automatic manner, which is described next.

Determination of $\tilde{\lambda}$. Experimentally, we found that to yield an optimizable problem, i.e., a minimizable $\tilde{E}$, $\tilde{\lambda}$ should be larger than a certain critical value. This forms the *optimizability constraint*, which makes $\tilde{E}$ to be almost flat beyond a certain θ , see Fig. 7.3. Such a constraint requires $\tilde{E}$ to have small gradients for all its values beyond the minimum of $\tilde{E}$, thus allowing us to automatically determine an appropriate $\tilde{\lambda}$.

Given θ_u as the upper limit of θ and denoting the derivatives of $\tilde{E}_d$ and $\tilde{E}_r$ at θ_u as $\tilde{E}'_d$ and $\tilde{E}'_r$, respectively, we compute $\tilde{\lambda}$ by solving the equation $\partial_\theta \tilde{E}(\theta_u) = \epsilon$, which gives:

$$\tilde{\lambda} = \frac{\epsilon - m\tilde{E}_d^{m-1}\tilde{E}'_d}{m\tilde{E}_r^{m-1}\tilde{E}'_r}, \quad (7.3.11)$$

where parameter ϵ is a fixed small gradient value empirically chosen to be 5×10^{-8} in all our experiments.

Figure 7.5: We compare the scale decomposition quality of our method against the state-of-the-art curvelets method and perfect BPF using a fractal image (Julia fractal image), which exhibits spatial structures of varying scales; here we follow Bermejo-Moreno and Pullin [11], who conducted experiments on curvelets-based method by analyzing multi-scale features in this fractal image. Visual comparison above shows that our results are comparable to the curvelets results. See the intermittencies illustrated in the red box of the curvelets result. Our method can, however, mostly avoid them with better structure preservation.

7.3.4 Solving the Optimization and Decomposing the Flow

After computing $\tilde{\lambda}$, we employ the Brent's algorithm [15] to quickly search for an appropriate θ to minimize the objective function (Eq. 7.3.10). By the automatic method to calculate $\tilde{\lambda}$ (see Eq. 7.3.11), our optimization model is ensured to be unconditionally stable; it is because the automatic calculation of $\tilde{\lambda}$ guarantees that the objective function always attain the minimal value within the search range. The experiments to be presented later on in Sections 7.4 & 7.5 also show that our method always converges effectively to the desired solutions for a variety of turbulent flow data we have worked with in this research. Fig. 7.5 presents a comparison of our decomposition results against those from a

perfect BPF (without any optimization and with our estimated optimal filter parameter) and from the state-of-the-art curvelets method. It is apparent that the decomposition result from perfect BPF may introduce unexpected structures, which are incorrect. Comparing with curvelets, our optimization also demonstrates closeness to the corresponding spectrum shape in the results, particularly, for preserving structures appropriately for corresponding scales.

To avoid potential ringing artifacts resulted from the boundary reflection of non-periodic data sets, we create a mirror extension of the data to enforce periodicity in the computation, which is also required in other methods, e.g., curvelets, see Yang et al. [107]. Fig. 7.4 shows one of our 2D flow decomposition results, and the following summarizes our procedure.

[Step 1]: First, we define a scale-decomposition filter (Eq.7.3.1) in the Kolmogorov energy spectrum with an unknown parameter θ , and then transform it to Fourier space.

[Step 2]: We construct the objective function (Eq. 7.3.10) in Fourier space.

[Step 3]: By calculating the derivatives of $\tilde{E}_d$ and $\tilde{E}_r$ at θ_u , we compute $\tilde{\lambda}$ according to Eq. 7.3.11.

[Step 4]: Given $\tilde{\lambda}$ and an initial θ , which is $(\theta_l + \theta_u)/2$, we employ the Brent's algorithm to efficiently find the optimal filter parameter θ_m .

[Step 5]: Lastly, we perform an inverse Fourier transform with the optimized filter and decompose the flow.

7.4 Continuous Scale Visualization

7.4.1 Isosurface Visualization

After the flow decomposition, turbulence structures hidden in different scales of the input flow field can be revealed through visualization. Here we employ isosurface rendering to produce 3D visualizations since isosurface rendering is commonly adopted by scientists in turbulence research. Following the convention used in the turbulence community [11],

the velocity magnitude of the decomposed flow field is assumed to conform to a Gaussian distribution since turbulent flows are chaotic by nature. Hence, we follow [11] to use $\mu + 1.5\sigma$ as the isovalue to produce the isosurfaces, where μ and σ are the mean and standard deviation of the flow velocity field, respectively.

Unlike the conventional isosurface rendering, which shades isosurfaces with a constant color, in our visualization, we texture the isosurfaces of the decomposed flow fields with the input flow velocity, e.g., see Fig. 7.6, where the color coding indicates the magnitude of the flow velocity. This color-coding scheme helps reveal and relate the distribution of turbulence scale structures with respect to the input flow. This is also similar to the existing λ_2 flow visualization [83], where the isosurfaces of λ_2 structures are colored by the velocity field.

In our decomposition framework, we can flexibly decompose an input flow field using continuous band ranges (scales), and produce *continuous scale visualization* to smoothly reveal the variation of length scales as distributed over the input flow field. This is particularly useful for showing fractal-like turbulence structures. See Figs. 7.6, 7.9, and 7.11 for our continuous scale visualization results. It is worth noting that conventional flow decomposition methods with wavelets and curvelets can only produce *discrete* rather than *continuous* decomposition. Continuous visualization of length-scale structures is highly desirable for scientific study of turbulent flows, especially in transitional regions of the boundary layer flow. However, it has not been explored and presented in any of the existing flow visualization work we aware of.

In the following subsections, we present case studies of visualizing a variety of turbulent flows with our method. Note that turbulent flows can generally be classified into two classes: isotropic flows and sheared flows [96]. For isotropic flows, we consider a typical

Figure 7.7: Fourteen selected representative filters that are roughly evenly distributed in the Kolmogorov energy spectrum; the first eight filters starting from wavenumber 0.3 were used to produce the eight visualizations (scales 1-8) shown in Fig. 7.6. Note that the solid and dotted lines indicate even-numbered and odd-numbered scales, respectively.

forced isotropic flow in a periodic cube, whereas for sheared flows, we consider a boundary layer flow on a flat plate. They are both typical representatives of the two basic flow classes. Thus, we use DNS flow data sets of them, particularly with different parameters such as variant Reynolds numbers in the boundary layer flows, for visual examination of scale structures in turbulent flow fields. Note also that the analysis below is done with the involvement of experts in turbulence research.

7.4.2 Isotropic Turbulent Flow

The first data set we experimented with is an isotropic DNS turbulence data obtained from John Hopkins University (JHU) turbulence database cluster [56] at $Re_\lambda = 433$. The

simulation was done in a cube with a grid resolution of 1024^3 .

Fig. 7.6 shows our visualization results. Here we use a bandwidth of size 0.05 (i.e., $\delta k = k_2 - k_1 = 0.05$ in Fig. 7.2) in the Kolmogorov energy spectrum, smoothly shift the band range like a sliding window, and then optimize the related filters to produce the continuous scale visualization. Again, since the figure can only present certain image instances of the continuous scale visualization, Fig. 7.6 shows only the visualizations of the eight larger-scale band ranges as the representatives. See Fig. 7.7 for the corresponding optimized filters in the Kolmogorov energy spectrum, where we pick 14 (roughly evenly distributed) band ranges from the continuous filter space. Note also that if we use curvelets to decompose this data set for visualization, only six discrete scales can be obtained due to the dyadic (powers-of-two) scale limit, but using our decomposition framework, we can flexibly specify a band range and explore turbulent structures continuously in the Kolmogorov energy spectrum.

The first three band ranges with larger-scale structures (i.e., scales 1 to 3 in Fig. 7.6) are in the forcing range of the Kolmogorov energy spectrum. They contain most of the turbulent energy and form the larger-scale structures. In these scales, thick tubes or blob-like structures can be observed. The rest of the band ranges with smaller-scale structures (i.e., scales 4 to 8 in Fig. 7.6) correspond to the inertial range, where thinner tube-like structures dominate. The tubes become even thinner and more stretched as we continuously move to smaller scales, indicating a turbulent stretching process. This dissipative range of decomposed scales are not presented in Fig. 7.6 because the related structures are too small to be recognizable if we show their visualizations in the same size like the visualizations of scales 1-8 in Fig. 7.6. Hence, we use a separate zoom-in figure, i.e., Fig. 7.8, to show one band range sample in the dissipative range.

Figure 7.8: A zoom-in view to show the smaller-scale dissipative range: band range $[0.80, 0.85]$ in the Kolmogorov energy spectrum. The yellow boxes above highlight some of the sheet-like structures in this smaller-scale isotropic turbulence.

Our visualization results are consistent with previous work on multi-scale (discrete) turbulence analysis. For example, Moreno et al. [11] also observed blob-like structures in larger-scale energy-containing range. This is consistent to our decomposition results in larger-scale band range, i.e., scales 1 to 3 in Fig. 7.6. Moreover, they also observed sheet-like structures in the dissipative range, and this can also be seen in our decomposition results, see the boxed regions in Fig. 7.8 for examples of sheet-like structures in isotropic turbulence.

7.4.3 Turbulent Boundary Layer Flow

A classical example of anisotropic turbulence is the turbulent boundary layer flow over a flat plate. Here we use two DNS boundary layer turbulence data sets obtained from Royal Institute of Technology (KTH) Sweden at $Re_\tau = 1000$ and $Re_\tau = 4000$ [85] to explore the capability of our method for anisotropic turbulence. The simulations that produced these

Figure 7.9: Continuous scale visualization of boundary layer turbulence at low Reynolds number. The image on top left shows the input flow velocity field, which is visualized using standard λ_2 isosurface visualization method.

Figure 7.10: Hairpin structures observed in different visualizations: (a) the hairpin structure in standard λ_2 visualization of the original flow; and (b) to (d) hairpin structures observed in our visualizations, as continuously distributed over nearby band ranges. As we go across band ranges, we may continuously visualize the formation, evolution, and splitting of these structures.

Figure 7.11: Continuous scale visualization of boundary layer turbulence at high Reynolds number. Again, the image on top left shows the input flow velocity field, which is visualized using standard λ_2 isosurface visualization method.

data sets were done in a very high resolution rectangular domain, in which we cropped a $1000 \times 257 \times 1024$ region for visualization and analysis. Note that unlike isotropic turbulence, the grid along the wall normal direction (+Y) is stretched to account for the wall effects, see Fig. 7.9 (top left).

Decomposing turbulent scale structures in boundary layer flows requires special treatment since the scales for a turbulent boundary layer flow along the wall normal direction is not well defined in terms of the Kolmogorov energy spectrum due to the data non-

periodicity and the stretching of the simulation coordinates along wall normal direction, where the Fourier transform is not applicable. Hence, we can only decompose the flow field over the X-Z plane, i.e., the streamwise and spanwise directions, see again Fig. 7.9 (top left). As a result, we perform our decomposition 2D slice by 2D slice along the Y axis, and then stack the 2D decomposed results to form the overall flow decomposition. Figs. 7.9 and 7.11 show our continuous scale decomposition results of the turbulent boundary layer flows at low and high Reynolds numbers. Like isotropic turbulence, we present mainly the larger-scale band ranges in these figures.

Low Reynolds number boundary layer flow. If we zoom into the visualizations in Fig. 7.9, we can find some important Ω -shaped structures known as the hairpin structures, see Fig. 7.10. These structures were initially observed in the isosurface visualization of λ_2 features, see Fig. 7.10(a), but from our visualization results, see Fig. 7.10(b-d), we can see that our method can also capture and present these structures, but at relatively smaller scales, indicating that hairpin structures are usually formed with small-scale structures. Across scales, we can see their formation, evolution, and splitting.

Moreover, unlike the λ_2 hairpin structures, which cannot tell what band range (scale) the structures associate with in the Kolmogorov energy spectrum, our method can pinpoint the band ranges for the structures in a continuous manner, see again Fig. 7.10. Hence, we can enable more precise length-scale analysis of hairpin structures, where λ_2 visualization cannot offer. Note also that such result has not been achieved in any existing turbulence research.

High Reynolds number boundary layer flow. Fig. 7.11 presents isosurface visualizations of some larger-scale band ranges selected from our continuous scale visualization.

From the visualizations, we can find that most larger-scale structures are further away from the wall on the bottom while at smaller-scale band ranges, smaller-scale structures are relatively closer to the wall. These visualizations are consistent with the physical property of boundary layer turbulent flows, which are known to consist of two layers: an outer layer, which tends to have larger length scales, and an inner layer, which tends to possess smaller length scales.

7.5 Discussions

7.5.1 Implementation

We implement our scale decomposition method and its related visualization on a workstation with Intel Core (TM) i7-3930K CPU@3.20GHz, 28GB RAM and 2TB hard drive. Since we base our optimization framework on an analytic objective, it takes only around 0.02 sec. (experimentally over different band ranges) to optimize the filter shape for a given band range (scale). Note that the filter shape optimization is independent of the data sets since the ringing artifacts are mainly resulted from the oscillatory property of the filter in the data transformation, rather than the data set itself. This can also be verified by referring to our optimization formulation that the filter parameter does not actually depend on any data value.

After we obtain the filter shape for a given band range, we then need to extract the turbulence structures related to the band range by a Fourier space filtering, which depends on the size (resolution) of the data set. For the 2D data sets with a resolution of 512^2 , it takes around 0.3 sec. for the filtering, while for the 3D isotropic turbulence data sets with a resolution of 512^3 and the 3D boundary layer turbulence data sets with a resolution of

Figure 7.12: Comparison of ringing. (a) input image; (b) scale decomposition with a perfect band-pass filter in Fourier space; (c) scale decomposition with curvelets method; and (d) scale decomposition with our method.

$1000 \times 257 \times 1024$, it takes around 5 and 2 min., respectively, for the filtering.

7.5.2 Filter Parameter

Our optimization process starts with a given band range, i.e., k_1 and k_2 , and optimizes the related filter parameter, see Fig. 7.2. Table 7.1 shows the estimated filter parameters for different given band ranges, where $\sigma_m = \sqrt{1/(2\theta_m)}$ is the standard deviation of the Gaussian function, indicating the sharpness of the fall-off regions; a larger θ_m indicates a

sharper fall-off.

From Table 7.1, we can see that as we move from larger to smaller scales (top to bottom in the table), θ_m increases; hence, it shows an increase in the sharpness in the fall-off region defined in the log-space of the Kolmogorov energy spectrum. However, it does not mean that there is an increase in the sharpness of the fall-off region in the Fourier space since there is a log-transformation, and the filters for smaller scales are more compressed. In fact, as we go to smaller scales in the Fourier domain, the sharpness of the fall-off region decreases. This indicates stronger ringing artifacts at smaller scales, thus requiring more nearby scales to suppress them.

7.5.3 Ringing Artifacts

In our flow decomposition framework, we aim to maintain a sharp filter while minimizing the ringing artifacts. To show that our method can still effectively minimize the ringing artifacts while maintaining a sharp filter, we compare results from our method with the existing state-of-the-art method, which is the curvelets scale decomposition method [11].

Since turbulent flows are chaotic by nature, it is hard to visually examine the amount of ringing by directly looking at them. Hence, we use an image of a solid square characterized by sharp edges, see Fig. 7.12 (a), which will unavoidably incur ringing, particularly for a perfect band-pass filter. Fig. 7.12 (b-d) compares the ringing artifacts resulted from a perfect band-pass filter, the curvelets method, and our method. From Fig. 7.12 (b), we can clearly see that since the Fourier basis has only global support, a perfect band-pass filter may discard necessary basis functions, and thereby produce oscillatory ringing structures in the decomposed results. From Fig. 7.12 (c), we can see that although the state-of-the-art curvelets method can suppress the ringing problem with local-support

Figure 7.13: Comparison with curvelets-based decomposition method. It is noted that although we employ completely different approach, our result is similar to curvelets-based method, indicating ring minimization property of our method.

basis, certain amount of ringing artifacts can still be seen.

In our case, we generate our decomposition results by constructing an optimized filter shape from each band range used in the curvelets method, see Fig. 7.12(d). Although ringing artifacts cannot be completely avoided, by comparing Fig. 7.12 (c) and (d), we can see that our method, while striving to maintain a sharp filter shape, can also effectively minimize the ringing artifacts with a quality that is compelling (perhaps more superior) to that of the curvelets method.

7.5.4 Evaluation: Energy Spectrum Distribution

To verify our decomposition results, we compare the energy spectrum distribution of our results to that from curvelets, which have been widely used in turbulence research community. To prepare for this experiment, we use the fractal image (see Fig. 7.5 (middle left)) from Bermejo-Moreno and Pullin [11], who conducted experiments with curvelets decomposition methods and considered the fractal image since it exhibits multi-scale features. Moreover, we estimate the extent of the band ranges in the energy spectrum from the curvelets decomposition result, and then use them as the input parameters to our method for obtaining our filters.

Fig. 7.5 shows the decomposition results from our method and also curvelets method, while Fig. 7.13 presents their corresponding energy spectrum distributions. From Fig. 7.5, we can see that our decomposition results are similar to those from curvelets although we develop a novel approach to decompose turbulence scales. From Fig. 7.13, we can also see that the two spectrums are similar in shape, but the spectrums for curvelets have more fluctuations and jerkiness, which may lead to artificial intermittency structures, e.g., see the red box in Fig. 7.5. On the contrary, due to our optimization formulation, our method can better preserve the structures (see again Fig. 7.5). Furthermore, our method allows the specification of band ranges continuously spreading over the Kolmogorov energy spectrum (for continuous scale visualization), while curvelets methods can only provide a maximum of six scales (for 512×512 images) due to dyadic scaling.

7.6 Conclusion

In this paper, we present a novel optimization-based technique to decompose a turbulent flow field into scale (band range) components in the Kolmogorov energy spectrum. The band ranges for the decomposed scales can be continuously specified, and our method can enable us to deliver *continuous scale decomposition* and *continuous scale visualization* of decomposed turbulence structures.

Our method is derived from an analytical optimization model, which can be solved with particularly high efficiency to produce an optimal filter shape that maintains a sharp filter shape while reducing the amount of ringing. By this, we can produce high-quality scale decomposition that is comparable (and sometimes superior) to the state-of-the-art methods. Moreover, our computing time is much shorter, making our method suitable for processing large volume of data sets with higher resolutions and dimensions, e.g., time-varying flow data sets. We also experimented our methods with DNS data sets, including isotropic and boundary layer turbulent flows; the results show that our visualization can unveil hidden turbulence structures such as blob-like and sheet-like structures in different band ranges in a continuous manner.

Scale	k_1	k_2	θ_m	$\sigma_m = \sqrt{1/(2\theta_m)}$
1	0.10	0.15	770.17	0.02547
2	0.15	0.20	789.41	0.02517
3	0.20	0.25	803.81	0.02494
4	0.25	0.30	818.87	0.02471
5	0.30	0.35	835.51	0.02446
6	0.35	0.40	853.09	0.02421
7	0.40	0.45	872.61	0.02397
8	0.45	0.50	893.75	0.02365
9	0.50	0.55	915.73	0.02337
10	0.55	0.60	942.51	0.02303
11	0.60	0.65	968.29	0.02272
12	0.65	0.70	996.53	0.02239
13	0.70	0.75	1023.12	0.02211
14	0.75	0.80	1054.54	0.02177
15	0.80	0.85	1088.58	0.02143
16	0.85	0.90	1122.53	0.02111
17	0.90	0.95	1158.62	0.02077
18	0.95	1.00	1195.77	0.02045

Table 7.1: Filter parameters estimated for different band ranges in the Kolmogorov spectrum; σ_m is the standard deviation of Gaussian to measure the sharpness of the filter fall-off.

Chapter 8

Conclusion & Future Work

A summary of the main results is reported hereafter. Few possible future directions of work have been proposed.

8.1 Turbulent drag reduction due to wall oscillations

Effect of both temporal and spatial wall oscillations on turbulent boundary layer has been studied with focus on skin friction attenuation using direct numerical simulations. The variation of drag reduction in the streamwise direction is in excellent agreement with a similar experimental study showing high fidelity of the current simulation setup and results. The wall normal shift of log layer of mean velocity profile is clearly seen for the simulations performed. Attenuation of turbulence is quantified with reduced values of velocity fluctuations, correlation coefficient and Reynolds shear stress together and their simultaneous shift in the wall-normal direction. Reynolds number and its diminishing effect on peak drag reduction values from various available results from DNS studies and experiments has been compared and relation has been hypothesized. The relation

suggests that we reach constant drag reduction performance for higher Reynolds numbers. However, its inconclusive due to inadequate number of data points and can best be seen as a speculation.

Various turbulence statistics are compared at different Re and their response to the oscillations is described. While the Reynolds shear stress obtains the most reduced value, the longitudinal velocity fluctuations are the only one to exhibit a non-monotonic response. For the first time spatial transients for the downstream region of the oscillating part of the wall has been reported. A small drag increase is observed after oscillations are terminated, which corresponds to an overshoot of the Reynolds shear stress. Bending of low speed streaks has been visualized along with other characteristics such as increased spacing. Energy calculations show negative net energy budget for the chosen set of parameters for oscillation. However, based on the observations of Reynolds number dependency, the prediction of the viability of wall oscillations as means of drag reduction technique in real applications can be derived.

A new form of steady spatial wall oscillation technique in the form of half square waves with positive forcing has been presented with promising results for developing an active drag reduction technique. An optimal set of wall oscillation parameters for the current parameter space was found to have $\sim 18\%$ net energy savings.

8.2 Transition region with wall oscillation

Effect of spanwise oscillations on a single low speed streak has been studied showing large suppression of velocity fluctuations. The wall shear stress reduced slightly for all wavenumbers simulated. An optimal wavelength for suppression could not be reached

within the oscillation parameter space although a clear trend showing higher reduction of velocity fluctuations at lower wavenumbers was observed. As the flow undergoes transition, the optimal wavenumber for oscillation changes, with the new optimum wavenumber being approximately an order of magnitude larger than the near optimum observed for laminar streak cases. The results show consistency with previous linearized studies but differs once the non-linear terms start to play a role during transition. The oscillations in the streamwise velocity fluctuations at different wall normal locations have a strong phase relationship with the local phase of the spanwise velocity at the corresponding wall-normal location.

When oscillations are applied in the transition region, a shift is observed in the actual point of transition. However, it remains inconclusive whether these oscillations merely retard the spatial growth of flow post transition. The downstream shift is affected by the starting point of the oscillation region, but such a downstream shift saturates quickly. Judging by peak streamwise velocity fluctuations is more ambiguous since the rise in its amplitude is smoother, lacking the sharp change in magnitude which is seen in peak wall-normal fluctuations.

Spatial forcing when compared with temporal oscillations exhibits a different character. While a small downstream shift is seen in the rise in skin friction values as well as in the location of v_{rms} profile, the behavior is not consistent when the start of the oscillations is shifted upstream. In fact upstream shift causes the kink location to also be shifted upstream. Temporal and spatial oscillations then do not show analogous character during transition as is seen for fully developed turbulent flows.

8.3 Multiscale decomposition of turbulent flows

Multiscale decomposition methodology (KoSCO) for turbulent flows has been developed for 3D DNS datasets. The technique consistent with band-pass filtering in the Kolmogorov spectrum. It provides an alternative to state-of-the-art method, curvelets and tries to overcome some of their inherent disadvantages. For the first time, optimization methods have been used to design such decomposition procedure. Unlike previous methods, the size and location of the various scales can be easily controlled and provides continuous scale decomposition. Gibbs rings are substantially suppressed and has been demonstrated using several examples. The validity of the confined bandwidth scales has been quantified using the probability distribution function for various scales. In addition, we have developed a data-independent approach using analytical optimization. This has unique advantages of near real-time tracking of turbulence structures since the computational time is very low and is suitable for exploring large unsteady DNS datasets. With this, a framework has been provided for multi-scale analysis for geometric structure identification with specific applications to turbulent flows, atmospheric flows, multiphase flows, etc.

8.4 Future Work

Possible future work on the lines of work presented in this thesis can be as follows. Some open questions remain regarding the spatial transients for effect of wall oscillations on turbulent flows. Why does the drag reduction decrease downstream more than expected once the oscillation has started? Will the drag reduction obtained at a certain Reynolds number depend on the location at which the oscillations start? What is the explanation for the slight DI once oscillation stops? Also, looking at a spectrum of parameters for a

Figure 8.1: Travelling wave experimental setup.

traveling wave like oscillation ($W_m \sin(\kappa x - \omega t)$) pattern could yield much higher energy savings and give a better and more robust predictive relation to quantify drag reduction performance degradation with Reynolds number. Performing experiments on a moving flat plate are a challenging task and is still a work in progress at fluid mechanics lab in our group. Figure 8.1 illustrates the experimental setup where linear motors and controllers would be used to emulate strips of flat-plate which move like a traveling wave in spanwise direction.

Exploration of time-varying DNS turbulence data sets in order to track turbulence scale structures for further analysis can be undertaken. This is particularly useful for turbulent boundary layer flows to understand the dynamics of hairpin structures: their birth, evolution, merge or split. Since the proposed method is highly efficient for scale decomposition, it can help to enable structure tracking in a continuous scale manner.

Appendix A

Stokes problem for boundary layer with moving half planes

Stokes solution for the second problem has the laminar solution for an infinite plate as[8],

$$W(y, t) = W_m \exp(-\eta) \sin(\omega t - \eta) \quad (\text{A.0.1})$$

where $\eta = y\sqrt{\omega/2\nu}$.

The profile (A.0.1) is not an exact expression of W in this case, since the spatially developing boundary layer has been approximated with a parallel flow (hence, the normal velocity is zero and W is independent of x in the spanwise momentum equation), leading to the solution (A.0.1). The Stokes solution does not take into account the spatial transient of the velocity profile after the introduction of wall oscillation or in other words, the classical Stokes solution is independent of the streamwise location. An analytical solution for spatially developing boundary layer with half-plane motion has been derived by Zeng and Weinbaum[111]. This solution is valid for an identical schematic setup as the present as shown in figure 2.2 and the profiles are given as a function of the streamwise coordinate

to show the spatial transients involved.

Dimensionless variables are defined as $\tau = \omega t$, $\xi = x/(2\nu/\omega)^{1/2}$, $\eta = y/(2\nu/\omega)^{1/2}$ where $(2\nu/\omega)^{1/2}$ is the penetration depth for Stokes first problem. Within the Stokes layer, the dimensionless governing equation and boundary conditions can be written as

$$\frac{\partial W}{\partial t} = 2\nabla^2 W \quad (\text{A.0.2})$$

$$W = 0 \quad \text{at} \quad \eta = 0, \quad \xi < 0 \quad (\text{A.0.3})$$

$$W = \sin(\tau) \quad \text{at} \quad \eta = 0, \quad \xi > 0 \quad (\text{A.0.4})$$

The solution given by Zeng and Weinbaum[111] is,

$$W(\xi, \eta, \tau) = \Re \left\{ \exp(i\tau) \left[\frac{1}{2} \exp(-(1+i)|\eta|) + \frac{1+i}{\pi} \int_0^\xi \frac{|\eta|}{(x'^2 + \eta^2)^{1/2}} K_1((1+i)(x'^2 + \eta^2)^{1/2}) dx' \right] \right\} \quad (\text{A.0.5})$$

where K_1 is a modified Bessel function of the second kind and the solution is forced to be symmetric about $y = 0$ by using $||$.

A spatial transient exists in the velocity profiles just as oscillation is imposed with half-plane oscillations and this effect is visible in the range $0 < \xi < 1$. Figure A.1a shows the difference between Stokes solution and the spatial dependence of the velocity profile due to half-plane oscillation at $\xi = 0.5$ from (A.0.5). However, this spatial transient disappears for $\xi > 1$ after which the profile reduces to the one given by Stokes solution (A.0.1). The streamwise coordinate at $\xi = 1$ is $(2\nu/\omega)^{1/2} = 0.2036$ beyond which the spatial transient cannot be observed. Due to the use of the step function in the implementation of wall oscillation, it is hard to compare such spatial transients in the velocity profiles for the current simulations.

(a) $W(\eta)$ at $\xi = 0.5$ (---) Eq. A.0.5 and (—) Stokes solution (A.0.1) ; (b) (—) Stokes solution and (---) DNS results for case 2; The four set of curves are at time instants with maximum, minimum and zero wall velocity.

Comparison of DNS profiles from case 2 match reasonably well with the classical Stokes solution as shown in figure A.1b. The instantaneous DNS profiles have been taken after a sufficiently long time (after ~ 250 oscillation cycles) at four different instants when the wall velocity is at its maximum, minimum and zero value. The profiles are shown for streamwise location where there are no spatial gradients present ($x = 350$). Note that the profiles have been averaged only in the spanwise direction.

A.1 Analytical solution for spatial oscillation

Due to spanwise symmetry, the derivatives in z direction are dropped and therefore, following steady state Navier-Stokes equations can be derived:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (\text{A.1.1})$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (\text{A.1.2})$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right), \quad (\text{A.1.3})$$

which shows that eq. A.1.3 is independent where $w(x, y)$ can be solved in the domain $y \in (0, \infty)$ with the following boundary condition,

$$w(x, 0) = A \sin(\kappa x)$$

We assume that at steady state, $v \sim 0$ and the flow has settled which leads us to

$$u \frac{\partial w}{\partial x} = \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \quad (\text{A.1.4})$$

We then reduce our equations to account for boundary layer approximations as shown by Viotti et al. [99] and we write our governing equations as:

$$y \frac{\partial w}{\partial x} = \frac{\partial^2 w}{\partial y^2}$$

with boundary conditions:

$$w(x, 0) = \sin(x)$$

$$\lim_{y \rightarrow \infty} w(x, y) = 0.$$

Using method of separation of variables, we assume solution to be of the form:

$$w(x, y) = X(x).Y(y)$$

Inserting this to the pde, we get:

$$yX'Y = XY''$$

$$\frac{X'}{X} = \frac{Y''}{yY} = -\lambda$$

Now we get the following ode's

$$X' + \lambda X = 0$$

Comparison of temporal and spatial velocity profiles, solid line shows classical Stokes solution for temporal oscillation, dashed line shows velocity profile for spatial oscillation, (o) solution by Viotti et al. [99]; at maximum, minimum and zero wall velocities

$$Y'' + \lambda y Y = 0$$

from which we get two general solutions of the following form:

$$X(x) = c_1 \exp(-\lambda x) \quad Y(y) = c_2 Ai(\lambda^{1/3} y) + c_3 Bi(\lambda^{1/3} y)$$

where Ai and Bi are Airy's functions and the general solution can be expressed as:

$$w(x, y) = c_1 \exp(-\lambda x) \cdot (c_2 Ai(\lambda^{1/3} y) + c_3 Bi(\lambda^{1/3} y))$$

One of the constants can be dropped and we re-write our expression as

$$w(x, y) = \exp(-\lambda x) \cdot (C_2 Ai(\lambda^{1/3} y) + C_3 Bi(\lambda^{1/3} y))$$

where $C_2 = c_1 c_2$ and $C_3 = c_1 c_3$. The final solution satisfying the boundary conditions can be derived as,

$$w(x, y) = \frac{\Gamma(2/3)3^{2/3}}{8} \left(\begin{aligned} & (-\sqrt{3} + i)e^{-ix} Ai(-(\sqrt{3} + i)y/2) \\ & - (\sqrt{3} + i)e^{ix} Ai(-(\sqrt{3} - i)y/2) \\ & + (1 + \sqrt{3}i)e^{-ix} Bi(-(\sqrt{3} + i)y/2) \\ & + (1 - \sqrt{3}i)e^{ix} Bi(-(\sqrt{3} - i)y/2) \end{aligned} \right)$$

Appendix B

Analytical derivation of the filter formulation

B.1 Derivation of Analytical Objective

The filter shape employed in our continuous scale flow decomposition method is a band-pass filter (BPF), which is constructed by attaching two symmetric fall-off extensions to a perfect BPF. Here we derive the analytical form of the objective that automatically finds the optimal filter parameter θ_m based on the optimization criteria we presented in the paper.

We construct our filter in the Kolmogorov energy spectrum as

$$\hat{\mathcal{G}}(k; \theta) = \begin{cases} 1 & \text{if } k_1 \leq k \leq k_2 \\ \hat{g}(k - k_1; \theta) & \text{if } k \leq k_1 \\ \hat{g}(k - k_2; \theta) & \text{if } k \geq k_2, \end{cases} \quad (\text{B.1.1})$$

where $\hat{g}(k; \theta) = e^{-\theta k^2}$ is the Gaussian function that represents the fall-off extensions, in order to reduce the ringing artifacts; θ is a filter shape parameter that controls the amount

of fall-off; k_1 and k_2 represent the lower and upper band range, respectively ($k_2 > k_1 \geq 0$); and the bandwidth is $\delta k = k_2 - k_1$.

To obtain an optimal filter parameter θ_m , we need to solve an optimization problem with a certain objective. Our objective function has two terms: a data term E_d , which measures the closeness of our filter to a perfect band-pass filter, and a ringing term E_r , which measures the amount of ringing.

(i) First, the data term is constructed as:

$$\begin{aligned} E_d &= \psi_d \int_0^1 \left[\hat{\mathcal{G}}(k; \theta)^2 - \hat{\mathcal{G}}_0^2(k) \right] |\hat{u}|^2 dk \\ &\simeq \psi_d \int_{-\infty}^{+\infty} \left[\hat{\mathcal{G}}(k; \theta)^2 - \hat{\mathcal{G}}_0^2(k) \right] |\hat{u}|^2 dk, \end{aligned} \quad (\text{B.1.2})$$

where $\hat{\mathcal{G}}_0(\mathbf{k})$ is the perfect band-pass filter in the Kolmogorov energy spectrum. Since E_d is primarily for measuring how close the decomposition results by $\hat{\mathcal{G}}$ and $\hat{\mathcal{G}}_0$ are, it can be simplified by discarding $|\hat{u}|^2$ as:

$$\tilde{E}_d = \tilde{\psi}_d \int_{-\infty}^{+\infty} \left[\hat{\mathcal{G}}(k; \theta)^2 - \hat{\mathcal{G}}_0^2(k) \right] dk, \quad (\text{B.1.3})$$

where $\tilde{\psi}_d = (\int_0^1 [\hat{\mathcal{G}}(\theta_l)^2 - \hat{\mathcal{G}}_0^2] dk)^{-1} \simeq (\int_{-\infty}^{+\infty} [\hat{\mathcal{G}}(\theta_l)^2 - \hat{\mathcal{G}}_0^2] dk)^{-1}$. By inserting the specific form of $\hat{\mathcal{G}}$ into the equation above, we can rewrite $\tilde{E}_d$ as:

$$\begin{aligned} \tilde{E}_d &= \tilde{\psi}_d \int_{-\infty}^{k_1} e^{-2\theta(k-k_1)^2} dk + \tilde{\psi}_d \int_{k_2}^{+\infty} e^{-2\theta(k-k_2)^2} dk \\ &= \tilde{\psi}_d \sqrt{\frac{\pi}{2\theta}}, \quad \theta > 0, \end{aligned} \quad (\text{B.1.4})$$

and

$$\begin{aligned} \tilde{\psi}_d &= \left(\int_{-\infty}^{k_1} e^{-2\theta_l(k-k_1)^2} dk + \int_{k_2}^{+\infty} e^{-2\theta_l(k-k_2)^2} dk \right)^{-1} \\ &= \sqrt{\frac{2\theta_l}{\pi}}, \quad \theta_l > 0. \end{aligned} \quad (\text{B.1.5})$$

Thus, we obtain the analytical form of E_d :

$$E_d = \sqrt{\frac{\theta_l}{\theta}}. \quad (\text{B.1.6})$$

(ii) The ringing term can be written as a weighted measure between the filtered and the original data:

$$E_r = \psi_r \int_0^1 |w \nabla [u(1 - \mathcal{G}(k; \theta))]|^2 dk, \quad (\text{B.1.7})$$

where

$$\psi_r = \left(\int_0^1 |w \nabla [u(1 - \mathcal{G}(k; \theta_u))]|^2 dk \right)^{-1}.$$

After transformation by the Plancherel's theorem, we obtain

$$E_r = \psi_r \int_0^1 |\hat{w} * i k \hat{u} (1 - \hat{\mathcal{G}}(k; \theta))|^2 dk, \quad (\text{B.1.8})$$

where $\psi_r = \left(\int_0^1 |\hat{w} * i k \hat{u} (1 - \hat{\mathcal{G}}(k; \theta_u))|^2 dk \right)^{-1}$. Since $\hat{w}$ and $\hat{u}$ are independent of θ , they can be taken out of the integral and absorbed into λ . So, we reduce E_r to

$$\tilde{E}_r = \tilde{\psi}_r \int_0^1 k^2 [1 - \hat{\mathcal{G}}(k; \theta)]^2 dk = \tilde{\psi}_r H(\theta). \quad (\text{B.1.9})$$

where $\tilde{\psi}_r = \left(\int_0^1 k^2 (1 - \hat{\mathcal{G}}(k; \theta_u))^2 dk \right)^{-1}$.

Now, by inserting the specific form of $\hat{\mathcal{G}}(k; \theta)$ into $H(\theta)$, we can obtain

$$H(\theta) = \int_0^{k_1} k^2 [1 - e^{-\theta(k-k_1)^2}]^2 dk + \int_{k_2}^1 k^2 [1 - e^{-\theta(k-k_2)^2}]^2 dk. \quad (\text{B.1.10})$$

With integral approximation,

$$\begin{aligned} H(\theta) &= \int_0^{k_1} k^2 dk + \int_{k_2}^1 k^2 dk \\ &- 2 \left(\int_{-\infty}^{k_1} k^2 e^{-\theta(k-k_1)^2} dk + \int_{k_2}^{+\infty} k^2 e^{-\theta(k-k_2)^2} dk \right) \\ &+ \left(\int_{-\infty}^{k_1} k^2 e^{-2\theta(k-k_1)^2} dk + \int_{k_2}^{+\infty} k^2 e^{-2\theta(k-k_2)^2} dk \right). \end{aligned} \quad (\text{B.1.11})$$

Note that a general definite integral is

$$\begin{aligned} \int_a^b x^2 e^{-\theta(x-\alpha)^2} dx &= \frac{1}{4\theta^{3/2}} (2\sqrt{\theta}(a+\alpha)e^{-\theta(a-\alpha)^2} \\ &- \sqrt{\pi}(2\alpha^2\theta+1)\text{erf}(\sqrt{\theta}(a-\alpha)) \\ &- 2\sqrt{\theta}(\alpha+b)e^{-\theta(b-\alpha)^2} \\ &+ \sqrt{\pi}(2\alpha^2\theta+1)\text{erf}(\sqrt{\theta}(b-\alpha))). \end{aligned} \quad (\text{B.1.12})$$

Thus,

$$\begin{aligned} \int_{-\infty}^{k_1} k^2 e^{-\theta(k-k_1)^2} dk + \int_{k_2}^{+\infty} k^2 e^{-\theta(k-k_2)^2} dk = \\ \frac{1}{2\theta^{3/2}} [\sqrt{\pi}(\theta(k_1^2 + k_2^2) + 1) - 2\sqrt{\theta}(k_1 - k_2)]. \end{aligned} \quad (\text{B.1.13})$$

Similarly,

$$\begin{aligned} \int_{-\infty}^{k_1} k^2 e^{-2\theta(k-k_1)^2} dk + \int_{k_2}^{+\infty} k^2 e^{-2\theta(k-k_2)^2} dk = \\ \frac{1}{2^{5/2}\theta^{3/2}} [\sqrt{\pi}(2\theta(k_1^2 + k_2^2) + 1) - 2\sqrt{2\theta}(k_1 - k_2)]. \end{aligned} \quad (\text{B.1.14})$$

Therefore, we have

$$\begin{aligned} H(\theta) &= \frac{1}{3} (1 + k_1^3 - k_2^3) \\ &- \frac{1}{\theta^{3/2}} [\sqrt{\pi}(\theta(k_1^2 + k_2^2) + 1) - 2\sqrt{\theta}(k_1 - k_2)] \\ &+ \frac{1}{2^{5/2}\theta^{3/2}} [\sqrt{\pi}(2\theta(k_1^2 + k_2^2) + 1) - 2\sqrt{2\theta}(k_1 - k_2)]. \end{aligned} \quad (\text{B.1.15})$$

If we put $\alpha = k_1^2 + k_2^2$, $\beta = 2(k_2 - k_1)$; $\gamma = 1 + k_1^3 - k_2^3$; and $\eta = (2^{5/2} - 1)\sqrt{\pi}$, we can then obtain

$$H(\theta) = \frac{\gamma}{3} - \frac{2(2^{3/2} - 1)\sqrt{\pi}\alpha\theta + 3\sqrt{2}\beta\sqrt{\theta} + \eta}{2^{5/2}\theta^{3/2}}, \quad (\text{B.1.16})$$

and $\tilde{\psi}_r = H(\theta_u)^{-1}$.

(iii) By combining the above deviations with a weight parameter $\tilde{\lambda}$, we can construct the overall analytical form for our objective, i.e.,

$$\begin{aligned} \tilde{E} &= \tilde{E}_d^m + \tilde{\lambda}\tilde{E}_r^m \\ &= \left(\frac{\theta_l}{\theta}\right)^{\frac{m}{2}} + \tilde{\lambda} \left[\frac{H(\theta)}{H(\theta_u)}\right]^m. \end{aligned} \quad (\text{B.1.17})$$

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