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**NANYANG
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UNIVERSITY**

**UNCERTAINTIES IN THE ESTIMATION OF
UNSATURATED SHEAR STRENGTH FROM SOIL-WATER
CHARACTERISTIC CURVE**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

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**UNCERTAINTIES IN THE ESTIMATION OF
UNSATURATED SHEAR STRENGTH FROM SOIL-WATER
CHARACTERISTIC CURVE**

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ABSTRACT

The shear strength of a soil is an important engineering property that is required in various geotechnical analyses involving saturated and unsaturated soils. The behaviour of unsaturated soils is governed by the Soil-Water Characteristic Curve that relates water content to soil suction. The contribution of shear strength due to soil suction has been proven to be substantial for certain soils and needs therefore to be accounted for to fully characterise the shear strength behaviour of a soil.

Experimental procedures for the direct determination of the unsaturated shear strength in the laboratory are available. However, lengthy testing procedures, complexity of the equipment and the high level of expertise required are limiting the application for practical purposes. As a result, numerous prediction equations that utilise the Soil-Water Characteristic Curve to estimate the unsaturated shear strength have evolved over the past decades. Despite their advantage to swiftly produce the unsaturated shear strength, the estimated shear strength is underpinned by several assumptions and uncertainties that are unavoidable.

Prediction equations to estimate the shear strength of unsaturated soils are widely used in the fields of research. However, limited studies have been carried out to address and quantify the various uncertainties in prediction equations that result in variability in the unsaturated shear strength estimation.

This research focused on the estimation of the shear strength due to soil suction and the uncertainties associated with the prediction equations for the unsaturated shear strength. The shear strength behaviour for unsaturated soils was reinterpreted based on the typical desaturation behaviour of a soil along the Soil-Water Characteristic Curve and an equation to estimate the unsaturated shear strength was proposed. Uncertainties in the estimation of the unsaturated shear strength were assessed and subsequently quantified. Experimentally-obtained results from a soil tested in this research were used to illustrate the variability in the estimated shear strength of an unsaturated soil.

Abstract

The shear strength behaviour proposed in this research agreed well with the experimentally-obtained shear strength of the soil tested in this research. Additionally, based on an independent Database of unsaturated shear strength measurements, the proposed equation from this research indicated an improved predictive capability compared to existing equations for the estimation of the unsaturated shear strength.

Uncertainty analyses, performed on the soil tested in this study, indicated that the variability in the estimated shear strength at the cohesion intercept was of the same magnitude as the reported variability of the effective cohesion from direct measurements.

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LIST OF SYMBOLS

Symbols are defined where they occur in the thesis.

B	<i>Pore-water pressure parameter</i>
C_r	<i>Constant parameter in the Fredlund and Xing (1994) equation</i>
$C(x)$	<i>Covariance matrix</i>
$C(\psi)$	<i>Correction function for the Fredlund and Xing (1994) equation</i>
C_u	<i>Coefficient of uniformity</i>
C_c	<i>Coefficient of curvature</i>
G_s	<i>Specific gravity of soil solids</i>
J	<i>Parameter sensitivity or Jacobian matrix</i>
N	<i>Number of data</i>
P_{at}	<i>Atmospheric pressure (101.3 kPa)</i>
Q_1	<i>25th percentile</i>
Q_3	<i>75th percentile</i>
R	<i>Universal gas constant</i>
R_s	<i>Radius of curvature of meniscus</i>
R^2	<i>Coefficient of determination</i>
S	<i>Degree of saturation</i>
S_e	<i>Effective degree of saturation</i>
S_r	<i>Residual degree of saturation</i>
T	<i>Absolute temperature</i>

List of Symbols

T_s	<i>Surface tension of water</i>
V_v	<i>Volume of voids</i>
V_w	<i>Volume of pore-water</i>
a	<i>Fredlund and Xing (1994) fitting parameter</i>
b	<i>Constant parameter in Goh et al. (2010)</i>
c	<i>Total cohesion</i>
c'	<i>Effective cohesion</i>
$c_{v,x}$	<i>Coefficient of variation</i>
d_f	<i>Horizontal displacement at failure</i>
e	<i>Void ratio or Euler's number</i>
g	<i>Gravitational acceleration</i>
h_c	<i>Capillary height</i>
k	<i>Number of parameters</i>
m	<i>Fredlund and Xing (1994) fitting parameter</i>
$m_{\tau_{us}}$	<i>Model factor for the unsaturated shear strength prediction equation</i>
n	<i>Porosity or Fredlund and Xing (1994) fitting parameter</i>
r	<i>Radius of capillary tube</i>
r_s	<i>Pearson product-moment correlation coefficient</i>
s_x	<i>Estimated standard deviation</i>
s_x^2	<i>Estimated variance</i>
$s_{x,y}^2$	<i>Estimated covariance</i>
u_w	<i>Pore-water pressure</i>
w	<i>Gravimetric water content</i>

List of Symbols

w_{opt}	<i>Optimum gravimetric water content</i>
y	<i>Constant parameter in Goh et al. (2010)</i>
$(u_a - u_w)$	<i>Matric suction</i>
Δu_w	<i>Change in pore-water pressure</i>
$\Delta \sigma_3$	<i>Change in confining pressure</i>
θ	<i>Normalized volumetric water content</i>
α	<i>Contact angle</i>
θ	<i>Volumetric water content</i>
θ_r	<i>Residual volumetric water content</i>
θ_s	<i>Saturated volumetric water content</i>
κ	<i>Fitting parameter</i>
π	<i>Osmotic suction or mathematical constant</i>
ρ_d	<i>Dry density</i>
$\rho_{d,max}$	<i>Maximum dry density</i>
ρ_w	<i>Density of water</i>
σ	<i>Total normal stress</i>
σ_x^2	<i>Variance</i>
$\sigma_{x,y}^2$	<i>Covariance</i>
σ'	<i>Effective stress</i>
τ	<i>Shear strength</i>
τ_{us}	<i>Unsaturated shear strength</i>
v_{w0}	<i>Specific volume of water</i>
ϕ'	<i>Effective friction angle</i>

List of Symbols

ϕ^b	<i>Angle indicating an increase in shear strength with respect to soil suction</i>
χ	<i>Effective stress parameter</i>
ψ	<i>Soil suction</i>
ψ_r	<i>Residual soil suction</i>
ψ_{sa}	<i>Saturation suction</i>
$\frac{u_v}{u_{v0}}$	<i>Relative humidity</i>
ω_v	<i>Molecular mass of water vapour</i>
$(\sigma - u_a)$	<i>Net normal stress</i>
$(\sigma - u_w)$	<i>Effective stress</i>
$(\sigma_3 - u_a)$	<i>Net confining stress</i>

LIST OF ABBREVIATIONS

Abbreviations are defined where they occur in the thesis.

<i>AEV</i>	<i>Air-entry value</i>
<i>ARE</i>	<i>Average relative error</i>
<i>DVPC</i>	<i>Digital volume-pressure controller</i>
<i>HAED</i>	<i>High-air entry disk</i>
<i>IQR</i>	<i>inter-quartile range</i>
<i>LVDT</i>	<i>Linear variable differential transformer</i>
<i>PI</i>	<i>Plasticity index</i>
<i>RS</i>	<i>Residual state condition</i>
<i>SSE</i>	<i>Sum of squared errors</i>
<i>SST</i>	<i>Sum of squared totals</i>
<i>SWCC</i>	<i>Soil-Water Characteristic Curve</i>
<i>USDA</i>	<i>United States Department of Agriculture</i>

CHAPTER 1 INTRODUCTION

1.1 RESEARCH BACKGROUND

A common and successful practice to describe the shear strength of a saturated soil is by applying the Mohr-Coulomb theory and utilizing Terzaghi's effective stress concept. However, many soils are encountered above the water table and are primarily in an unsaturated state where the pore-water pressures are negative. The negative pore-water pressures are commonly referred to as soil suction and contribute to the shear strength of a soil. Therefore, geotechnical applications of these soils have to consider the contribution of soil suction to shear strength.

The unsaturated shear strength can be determined by experimental procedures in the laboratory using a modified direct shear apparatus or a modified triaxial apparatus. Experimental studies show that the unsaturated shear strength is non-linear (e.g., Escario and Saez, 1986; Gan et al., 1988; Lee et al., 2005; Melinda et al., 2004). The non-linearity in the unsaturated shear strength was directly related to the Soil–Water Characteristic Curve (SWCC). Evaluating the unsaturated shear strength through direct measurements in the laboratory is a time-consuming and expensive task and that is often not justified for practical purposes. As a result, prediction equations to estimate the unsaturated shear strength have been proposed by numerous researchers (e.g., Goh et al., 2010; Khalili and Khabbaz, 1998; Oberg and Sallfors, 1997; Tekinsoy et al., 2004; Vanapalli et al., 1996b). Additional to the saturated shear strength parameters, all these prediction equations for the unsaturated shear strength incorporate information from the SWCC. Even though the unsaturated shear strength can for some soils be estimated reasonably, there is no single prediction equation that is suitable for reliably estimating the unsaturated shear strength of various soils (Garven and Vanapalli, 2006).

Although prediction equations to estimate the unsaturated shear strength are widely used in the field of research, few studies have been carried out to assess the

uncertainty and variability of these prediction equations. Zapata (1999) stated that the variability of the unsaturated shear strength due to uncertainties in the procedural testing of the SWCC can be very high, especially for fine-grained soils when the soil becomes relatively dry. Phoon et al. (2010) proposed a probability model to estimate the SWCC and concluded that the uncertainties in the SWCC model have a significant impact on numerous soil properties including the unsaturated shear strength. However, none of these studies quantified the uncertainties in the estimated shear strength arising from various sources.

This study reinterprets the contribution of the shear strength due to soil suction and fills the gap in previous research works by examining and quantifying the uncertainties in the estimation of the unsaturated shear strength based on the saturated shear strength parameters and SWCC.

1.2 OBJECTIVE AND METHODOLOGY

The objective of this research was to propose an appropriate prediction equation to estimate the shear strength of unsaturated soils and to subsequently analyse and quantify the uncertainties associated in the estimation.

The research could be broadly divided into experimental and theoretical parts. The experimental part included testing of a suitable soil to investigate the unsaturated shear strength behaviour. The theoretical parts included the development of an appropriate prediction equation to estimate the unsaturated shear strength and the quantification of the uncertainties in the estimation of the unsaturated shear strength. The variability in the estimated unsaturated shear strength was shown using the soil that was experimentally tested in this research.

The representation and behaviour of the unsaturated shear strength was first reviewed and the factors controlling and affecting the unsaturated shear strength were studied. The contribution of shear strength due to soil suction was reinterpreted and a prediction equation to estimate the unsaturated shear strength was developed. The proposed prediction equation was validated and analysed on its performance.

Chapter 1 Introduction

Proposed and existing prediction equations to estimate the unsaturated shear strength were analysed on their performance and predictive capability using an independent Database.

A number of prediction equations were selected for analyses on their model uncertainties. Model uncertainty in the SWCC and model uncertainty in the selected prediction equations for the unsaturated shear strength were quantified.

A soil was selected following established criteria and then tested. Basic soil property tests and unsaturated soil testing including SWCC and unsaturated shear strength measurements were carried out. Soil-Water Characteristic Curve measurements were obtained under different confining stresses using Tempe cell, pressure plate apparatus and a modified triaxial equipment. Single staged unsaturated direct shear tests under consolidated and drained (CD) conditions were carried out to investigate the unsaturated shear strength behaviour of the soil.

The experimentally-obtained results were further used to evaluate the variability in the estimated unsaturated shear strength due to the quantified model uncertainty in the SWCC and the prediction equation for the unsaturated shear strength.

1.3 ORGANIZATION

The thesis is organized in seven Chapters:

Chapter 1 introduced the research background including the identification of research needed. The objective and methodology of the research were stated and the organization of this thesis is outlined.

Chapter 2 first reviews basic concepts of unsaturated soil mechanics related to this research. Past studies on the SWCC and the unsaturated shear strength are reviewed and existing prediction equations for the unsaturated shear strength are summarized.

Chapter 3 presents the theories applied in this research. The shear strength with respect to soil suction is reinterpreted and a prediction equation for the unsaturated shear strength is proposed and validated. Further, basic uncertainty concepts applied in this research are outlined.

Chapter 1 Introduction

Chapter 4 describes the research programme. The experimental programme involving saturated and unsaturated soil testing is presented first. The analytical programme includes performance analyses of the proposed and existing prediction equations for the unsaturated shear strength. Furthermore, the quantification of the uncertainties in the estimation of the unsaturated shear strength is also described in the analytical programme.

Chapter 5 presents the results of the research programme. Results from the basic soil property tests and the unsaturated soil testing including SWCC and unsaturated shear strength measurements are presented. The results of the performance analyses of the proposed and existing prediction equations for the unsaturated shear strength are presented. Quantified uncertainties in the SWCC and from prediction equations for the unsaturated shear strength are presented last.

Chapter 6 discusses the main results from this research.

Chapter 7 states the concluding remarks of this research and recommendations for possible further studies.

CHAPTER 2 LITERATURE REVIEW

2.1 INTRODUCTION

The review of related literature for this research is given in this Chapter. Basic concepts regarding unsaturated soils are first briefly introduced. Subsequently, the Soil-Water Characteristic Curve (SWCC) and the unsaturated shear strength representation and behaviour are reviewed and prediction equations for the unsaturated shear strength are summarized. Ultimately, uncertainty and variability in basic soil properties, the SWCC and in prediction equations are reviewed.

2.2 UNSATURATED SOIL

A soil profile typically consists of a saturated and an unsaturated zone. The water table usually separates the saturated from the unsaturated zone. While the soil below the water table is in a saturated condition with positive pore-water pressures, the soil above the water table is primarily in an unsaturated condition having negative pore-water pressures.

Equilibrium condition in the unsaturated zone under no-flow conditions is represented by static equilibrium with the water table (Line 1 in Figure 2-1). Evaporation through the atmosphere and transpiration through vegetation result in an upward flow of pore-water causing a drying of the unsaturated soil above the water table (Line 2 in Figure 2-1). Infiltration causes a downward flow of pore-water, resulting in a wetting of the unsaturated soil (Line 3 in Figure 2-1). The pore-water pressure distribution present in the unsaturated zone governs the mechanical behaviour of the unsaturated soil.

2.2.1 Soil suction

The pore-water pressure distribution in the unsaturated zone is more complex than in the saturated zone below the water table where the pore-water pressure distribution

follows a linear relationship. In the unsaturated zone, the pore-water pressures are lower than the air pressure (i.e., negative) and are referred to as soil suction.

Figure 2-1 Illustration of the unsaturated zone in a typical soil profile (from Fredlund and Rahardjo, 1993)

Soil suction or the free energy state of soil water can be expressed in terms of the partial vapour pressure using the following thermodynamic relationship known as Kelvin's equation (Fredlund and Rahardjo, 1993):

$$\psi = -\frac{RT}{v_{w0}\omega_v} \ln\left(\frac{u_v}{u_{v0}}\right) \quad \text{Equation 2-1}$$

where

ψ = soil suction

R = universal gas constant

T = absolute temperature

v_{w0} = specific volume of water

ω_v = molecular mass of water vapour

$\frac{u_v}{u_{v0}}$ = relative humidity

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Soil suction is commonly referred to as total soil suction and is quantified in terms of the relative humidity. Total suction consists of two components, namely the matric suction and the osmotic suction (Fredlund and Rahardjo, 1993).

$$\psi = (u_a - u_w) + \pi \quad \text{Equation 2-2}$$

where

ψ = soil suction

$(u_a - u_w)$ = matric suction

π = osmotic suction

Matric suction is associated with the capillary phenomena arising from the surface tension of water whereas osmotic suction is related to the dissolved salt content in the pore-water of a soil. Osmotic suction is present in both saturated and unsaturated soils. Most engineering problems involving unsaturated soils are caused by environmental changes (e.g., rainfall or evaporation) which result in a change in pore-water pressures and are therefore directly linked to the matric suction term of total suction (Fredlund and Rahardjo, 1993).

2.2.1.1 Capillarity

Capillarity in a soil mass occurs at the interface between soil particles, water and air. The capillary phenomenon in soils is associated with the matric suction component of soil suction and arises from the surface tension and its contact angle with the solid phase. Menisci are formed in the soil pore structure at the air-water interface in a similar manner as water in a capillary glass tube.

In a small diameter glass tube under atmospheric conditions, water rises up in the capillary tube as a result of the surface tension of water and the tendency of water to wet the surface of the glass tube. The surface tension acting on the perimeter of the tube is responsible for holding the weight of the water column at a capillary height at equilibrium condition (Figure 2-2). From vertical force equilibrium arises the following expression:

$$2r \pi T_s \cos\alpha = r^2 \pi h_c \rho_w g \quad \text{Equation 2-3}$$

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where

r = radius of capillary tube

π = mathematical constant

T_s = surface tension of water

α = contact angle

h_c = capillary height

ρ_w = density of water

g = gravitational acceleration

Figure 2-2 Capillary model (from Fredlund and Rahardjo, 1993)

By rearranging Equation 2-3 and substituting the radius of the capillary tube with the radius of curvature of the meniscus, the expression for the maximum height of water in the capillary tube can be found as:

$$h_c = \frac{2 T_s}{R_s \rho_w g} \quad \text{Equation 2-4}$$

where

R_s = radius of curvature of meniscus

As the air pressure is atmospheric at the air-water interface ($u_a = 0$) and the water-pressure is negative ($u_w = -h_c \rho_w g$), the rise in the capillary tube can be written in terms of matric suction, surface tension and the radius of curvature of the meniscus.

$$(u_a - u_w) = \frac{2 T_s}{R_s} \quad \text{Equation 2-5}$$

Even though equations like Equation 2-5 are often used to illustrate the capillary rise in an unsaturated soil, a soil is not as ideal as a capillary tube, which makes it difficult and impracticable to transfer these equations directly to unsaturated soil mechanics. In a soil, the diameters of the pores vary and the capillary channels build a network consisting of different radii describing the soil-pores that exhibit hysteresis.

2.2.1.2 Hysteresis

Hysteresis can be explained by the ‘ink-bottle phenomena’. Water inside an empty capillary tube placed in a water bath rises up until it reaches equilibrium following a wetting process. Same equilibrium height is achieved if the capillary tube initially filled with water is placed in the water bath as the water inside the tube drains out until equilibrium is reached corresponding to a drying process (Figure 2-3, a). However, if a capillary tube with a bulb, resembling soil pores in a soil, is considered, the height where equilibrium condition is achieved might be different, depending on the radius and location of the bulb. For a wetting process, the bulb with a radius larger than the radius of the capillary tube prevents the water from rising beyond the base of the bulb (Figure 2-3, b). During a drying process, the initially filled water tube including the bulb reaches equilibrium condition at the same height as in the capillary tube with a constant radius (Figure 2-3, c).

In addition, contact angle and radius of curvature are greater in advancing menisci as compared to that of receding menisci, resulting in a higher soil suction at a specific water content for a drying process than for a wetting process. Other reasons for hysteresis are the entrapment of air for a soil under wetting, resulting in a lower water content at a specific soil suction. Changes in the soil structure and soil fabric caused

by swelling and shrinkage of a soil might also cause hysteresis in a soil (Fredlund and Rahardjo, 1993).

Figure 2-3 Capillary height and radius effect on capillarity (from Fredlund and Rahardjo, 1993)

2.2.2 Stress state variables

Stress state variables are non-material variables which describe the stress condition of a system (Fredlund and Rahardjo, 1993). To describe the mechanical behaviour of a soil, constitutive relations consisting of appropriate stress state variables and material properties are applied (Fredlund and Rahardjo, 1993).

The mechanical behaviour in saturated soils is controlled by the effective stress. The effective stress concept for saturated soils was proposed by Terzaghi in 1936 and is well accepted and numerous proven by its successful applications in practice and in research. Terzaghi's effective stress is defined as the difference between total normal stress and pore-water pressure as:

$$\sigma' = \sigma - u_w \quad \text{Equation 2-6}$$

where

$\sigma' = \text{effective stress}$

$\sigma = \text{total normal stress}$

$u_w = \text{pore-water pressure}$

Both total normal stress and pore-water pressure can be directly measured or evaluated by the external forces and the body forces of the soil.

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Terzaghi's definition of the effective stress fails for unsaturated soils where the influence of soil suction becomes significant. Several researchers proposed effective stress equations for unsaturated soils by modifying Terzaghi's effective stress concept to account for forces arising from capillarity (e.g., Aitchison, 1961; Bishop, 1959; Croney et al., 1958; Jennings, 1961). In the context of unsaturated soil mechanics, the equation proposed by Bishop (1959) is probably the most recognized equation and is written as:

$$\sigma' = (\sigma - u_a) + \chi (u_a - u_w) \quad \text{Equation 2-7}$$

where

$(\sigma - u_a)$ = *net normal stress*

χ = *effective stress parameter*

For a first approximation, the effective stress parameter (χ) was suggested to vary with the degree of saturation from unity for saturated conditions to zero for completely dry conditions. However, later studies showed that the effective stress parameter also depends on wetting history, loading path, soil type and internal structure of the soil and may not be the same for volume change and shear strength problems (Fredlund and Rahardjo, 1993). The effective stress parameter relates net normal stress to soil suction and bears a constitutive behaviour rather than a variable for the stress state at equilibrium (Fredlund, 2006).

Theoretical basis and justification of using two independent stress state variables based on continuum mechanics was provided by Fredlund and Morgenstern (1977). Three possible combinations of stress state variables which form two independent stress tensors to describe the stress condition in an unsaturated soil were proposed:

$$(1) \quad (\sigma - u_a) \text{ and } (u_a - u_w)$$

$$(2) \quad (\sigma - u_w) \text{ and } (u_a - u_w)$$

$$(3) \quad (\sigma - u_a) \text{ and } (\sigma - u_w)$$

All three proposed stress state combinations were experimentally tested and proven. For most practical engineering problems, the pore-air pressure is atmospheric (i.e.,

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zero gauge pressure). Therefore, the first set of stress state variables is most suitable as the changes in total normal stress can be separated from the changes in pore-water pressure or soil suction (Fredlund and Rahardjo, 1993). The use of two independent stress state variables eliminates the need of a single-valued effective stress equation as constitutive relations can be used to describe the mechanical behaviour of an unsaturated soil.

2.3 SOIL-WATER CHARACTERISTIC CURVE

The primary and most important information of an unsaturated soil is the relationship between water volume in the soil pores and soil suction. This relationship is commonly referred to as Soil-Water Characteristic Curve (SWCC). Physically, the SWCC represents the water volume in the soil pores at a specific soil suction. As a measure of the water volume in the soil pores, gravimetric or volumetric water content or degree of saturation can be used (Fredlund, 2006). Figure 2-4 shows typical SWCCs for various soils using the relationship between soil suction and degree of saturation.

Irrespective of soil type, the soil is in a saturated condition at zero soil suction and in a completely dry condition at a soil suction of approximately 10^6 kPa. A terminal suction of 10^6 kPa for completely dry soils is supported by experimental data (Fredlund and Xing, 1994) and the fact that the relative humidity at oven dryness corresponds to approximately 10^6 kPa (Zapata, 1999). Thermodynamic principles also confirm a soil suction value of 10^6 kPa for soils in completely dry conditions (Richards, 1965). Due to the large soil suction range from zero to 10^6 kPa, the SWCC is often represented in a semi-logarithmic plot as shown in Figure 2-4.

Figure 2-4 Soil-Water Characteristic Curves for different soils (after Vanapalli et al., 1999)

2.3.1 Soil-Water Characteristic Curve variables

Two variables that are often identified on a SWCC are the air-entry value and the residual state condition. Air-entry value and/or residual state condition are often used in unsaturated soil property functions such as the unsaturated shear strength and the hydraulic conductivity. While the definition and procedure for the identification of the air-entry value seem to be consistent in the literature, different interpretations on the determination of residual state condition are available.

2.3.1.1 *Air-entry value*

The air-entry value is defined as the soil suction at which air penetrates the macropores of the soil (Brooks and Corey, 1964). It is usually determined as the soil suction at the intersection of the horizontal line through the saturated degree of saturation and the tangent line at the inflection point of the SWCC as illustrated in Figure 2-4. This technique has been extensively used by several researchers (e.g., Fredlund and Xing, 1994; Goh et al., 2010; Leong and Rahardjo, 1997; Tekinsoy et al., 2004; Vanapalli et al., 1996b; Yang et al., 2004).

2.3.1.2 *Residual state condition*

Residual state condition is generally referred to as the condition when the water phase in a soil becomes discontinuous and therefore immobile or when a large change in soil suction is required to further remove water from the soil (Brooks and Corey, 1964; Luckner et al., 1989; van Genuchten, 1980; Vanapalli et al., 1996b; White et al., 1970). Even though these definitions for residual state condition seem to be consistent in the literature, different procedures have been used in the literature to identify residual state condition.

Some researchers suggested soil suction ranges for different soils where residual state condition may occur (e.g., Fredlund and Xing, 1994; van Genuchten, 1980). A soil suction range between 0 to 200 kPa was suggested as the soil suction range where residual state condition for gravels, sands and silts may occur whereas higher soil suction ranges of 500 kPa to over 1,500 kPa were suggested for clays (Vanapalli et al., 1996b). Van Genuchten (1980) treated the residual water content as an additional fitting parameter in the SWCC equation without any physical meaning. Most

commonly, researchers used a construction method on a semi-logarithmic plot of the SWCC to identify residual state condition (e.g., Goh et al., 2010; Rassam and Williams, 1999; Yang et al., 2004; Zhai and Rahardjo, 2012). For the construction method, a tangent line is drawn at the inflection point of the SWCC and another line is approximated at high soil suction values. Residual state condition is identified at the intersection point of the two straight lines. However, the construction procedure on a semi-logarithmic plot fails to identify residual state condition for certain clays as no line at high soil suction values can be approximated as illustrated in Figure 2-4 (Regina clay).

The importance of residual state condition for unsaturated soil property functions was recognized by various researchers (e.g., Brooks and Corey, 1964; van Genuchten, 1980; Vanapalli et al., 1996b). However, the residual water content is often treated as an additional fitting parameter in unsaturated soil property functions without physical meaning.

2.3.2 Soil-Water Characteristic Curve measurement

Soil-Water Characteristic Curve measurements have been proven to be the most important test for a successful application of unsaturated soil mechanics in geotechnical engineering (Fredlund, 2006). Testing procedures to measure SWCCs include pressure plate extractors, hanging column, chilled mirror hydrometer, modified triaxial cell and modified oedometer, to name a few.

Soil-Water Characteristic Curve measurements are generally represented through empirical closed-form equations. Several equations to best-fit SWCC measurements have been proposed over the past decades (e.g., Brooks and Corey, 1964; Fredlund and Xing, 1994; van Genuchten, 1980). The SWCC equations consist either of two or three fitting parameters. For all the SWCC equations, one variable is related to the air-entry value and a second variable is related to the desaturation behaviour of the soil (Fredlund, 2006).

2.3.3 Factors affecting the Soil-Water Characteristic Curve

The shape of the SWCC is strongly controlled by the grain-size distribution of the soil. The slope of the grain-size distribution is related to the slope of the SWCC (Yang

et al., 2004). A steep slope in the grain-size distribution results in a steep slope in the SWCC. The slope of the SWCC is an indication of how fast a soil desaturates. Soil-Water Characteristic Curves of clayey materials generally have gentler slopes as compared to those of sands. A gentler slope results in a larger stored water volume within the soil over a larger soil suction range. The air-entry value of clayey soils occurs at higher soil suction as compared to sandy materials.

Several studies indicate that the shape of the SWCC is affected by a change in net normal stress as shown in Figure 2-5 (e.g., Kawai et al., 2000; Lee et al., 2005; Ng and Pang, 2000). This behaviour is in agreement with Fredlund and Rahardjo (1993) who stated that a reduction in water content can be caused either by an applied load under constant soil suction or an increase in soil suction under a constant applied load. Several studies show that the air-entry value of the soil increases with an increase in net normal stress as the macro-pores controlling the air-entry value are reduced causing smaller interconnected pores (e.g., Lee et al., 2005; Ng and Pang, 2000; Thu et al., 2007). Lee et al. (2005) obtained a linear relationship between net normal stress and air-entry value which confirmed the same observation made by Rassam and Williams (1999). The slope of the SWCC after the air-entry value becomes gentler as the net normal stress of a soil increases, increasing the capacity of the soil to retain water over a larger suction range (Lee et al., 2005; Ng and Pang, 2000).

Figure 2-5 Effect of stress state on the Soil-Water Characteristic Curve (from Lee et al., 2005)

2.4 SHEAR STRENGTH

The Mohr-Coulomb failure criterion incorporating Terzaghi's effective stress principle is often used for interpreting and predicting the shear strength of a saturated soil (Terzaghi, 1943).

$$\tau = c' + (\sigma - u_w) \tan \phi' \quad \text{Equation 2-8}$$

where

τ = shear strength

c' = effective cohesion

$(\sigma - u_w)$ = effective stress

ϕ' = effective friction angle

2.4.1 Unsaturated shear strength representation

Two general approaches are often adopted for the interpretation of the unsaturated shear strength of a soil. Bishop (1959) proposed an equation for the unsaturated shear strength by incorporating the modified effective stress equation for unsaturated soils into the classical Mohr-Coulomb failure criterion as:

$$\tau = c' + ((\sigma - u_a) + \chi (u_a - u_w)) \tan \phi' \quad \text{Equation 2-9}$$

where

$(\sigma - u_a)$ = net normal stress

χ = effective stress parameter

$(u_a - u_w)$ = matric suction

The effective stress parameter (χ) was assumed to be strongly related to the degree of saturation. Fredlund et al. (1978) proposed an expression to interpret the unsaturated shear strength of a soil based on two independent stress state variables as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad \text{Equation 2-10}$$

where

ϕ^b = angle indicating an increase in shear strength with respect to soil suction

Under fully saturated conditions, the soil pores are filled with water and the pore-air pressure is therefore equal to the pore-water pressure. As a result, the matric suction term disappears and the conventional shear strength expression for saturated soils is recovered for both expressions. These two general approaches by Bishop (1959) and Fredlund et al. (1978) provide the basic concept for many unsaturated shear strength equations.

2.4.2 Extended Mohr Coulomb failure surface

The two stress state variables, net normal stress and matric suction, can be adopted to describe the failure surface of unsaturated soils by plotting the Mohr circles in a three dimensional form as illustrated in Figure 2-6 in a planar form (Fredlund and Rahardjo, 1993). The Mohr circles are plotted at the respective net normal stress and matric suction. The three-dimensional plot is an extension of the two-dimensional Mohr-Coulomb failure envelope with the soil suction axis as the third dimension.

Figure 2-6 Extended Mohr-Coulomb failure surface for unsaturated soils (from Fredlund and Rahardjo, 1993)

The effective friction angle indicates the increase in shear strength with respect to net normal stress. For zero matric suction, the air-pressure is equal to the pore-water pressure and the shear stress versus the net normal stress plane represents the saturated condition. The shear stress with respect to matric suction at the zero net normal stress plane represents the total cohesion and can be expressed by the effective cohesion and the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) as:

$$c = c' + (u_a - u_w) \tan \phi^b \quad \text{Equation 2-11}$$

where

c = total cohesion

2.4.2.1 Warped shear strength surface

Noor and Anderson (2006) proposed a comprehensive model for the shear strength of saturated and unsaturated soils. The model considers a non-linear increase in shear strength with respect to soil suction. It was proposed that the increase in shear strength with respect to soil suction drops back to the saturated shear strength value at a point that was termed as 'ultimate suction'. A total of six equations are required to fully describe the warped failure envelope for the shear strength. Besides the saturated shear strength parameters, unsaturated shear strength measurements at low and high soil suctions are required. Validation of the model against measured shear strength data showed a good representation of the shear strength. According to Noor and Anderson (2006), the shear strength model is applicable to coarse- and fine-grained soils.

2.4.3 Unsaturated shear strength behaviour

Several researchers measured the unsaturated shear strength and some general statements about the unsaturated shear strength behaviour can be made (i.e., Cunningham et al., 2003; Escario and Juca, 1989; Escario and Saez, 1986; Hossain and Yin, 2010; Lee et al., 2005; Vanapalli et al., 1996a; Zhan and Ng, 2006):

- The shear strength with respect to soil suction increases for some soils considerably

- Linear failure envelopes for the shear strength with respect to net normal stress are obtained when soil suction is constant
- Non-linear failure envelopes for the shear strength with respect to soil suction are obtained when net normal stress is constant

2.4.3.1 *Linear failure envelope with respect to net normal stress*

Experimental studies show that shear strength with respect to net normal stress follows a linear relationship at constant soil suction. Some studies however indicate that for some soils, the effective friction angle increases slightly with increasing soil suction (Figure 2-7). An increase in the effective friction angle with respect to soil suction is often attributed to the higher dilation behaviour with increasing soil suction (Zhan and Ng, 2006). However, Vanapalli et al. (1996b) stated that the effective friction angle might be considered constant for most practical applications and for soil suction values up to 500 kPa.

2.4.3.2 *Non-linear failure envelope with respect to soil suction*

Experimental studies show a non-linear shear strength behaviour with respect to soil suction (i.e., Cunningham et al., 2003; Donald, 1956; Escario and Juca, 1989; Hossain and Yin, 2010) (Figure 2-7 and Figure 2-8). The unsaturated shear strength results of the study by Donald (1956) on sand samples showed that the shear strength with respect to soil suction can reach a plateau and even decrease (Figure 2-8). Also for cohesive soils, Escario and Juca (1989) observed a decrease in shear strength at high soil suctions. Escario and Saez (1986) stated that the shear strength due to soil suction in clean sands has to completely disappear at high soil suctions.

For soil suctions lower than the air-entry value, the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) is approximately equal to the effective friction angle. The ϕ^b angle becomes non-linear for soil suctions higher than the air-entry value (Gan et al., 1988). Several researchers related the non-linearity in the shear strength with respect to soil suction to the corresponding Soil-Water Characteristic Curve (SWCC) (i.e., Bao et al., 1998; Fredlund et al., 1996; Gan and Fredlund, 1996; Vanapalli et al., 1996b).

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(a) Madrid Grey Clay

(b) Red Clay of Guadalix de la Sierra

(c) Madrid Clayey Sand

Figure 2-7 Unsaturated direct shear test results from Escario and Saez (1986)

Figure 2-8 Unsaturated direct shear test results from Donald (1956)

2.5 UNSATURATED SHEAR STRENGTH PREDICTION

Unsaturated soil property functions, like the shear strength, can be obtained through various procedures as illustrated in Figure 2-9. The SWCC has been proven fundamental for the understanding of unsaturated soil mechanics (Fredlund and Houston, 2009). Estimation procedures for unsaturated soil property functions make use of the SWCC, which can either be obtained experimentally or through estimation procedures. This research focuses on the estimation of the unsaturated shear strength whereby the SWCC data are experimentally obtained.

Figure 2-9 Procedures to determine unsaturated soil property functions (from Fredlund and Houston, 2009)

2.5.1 Prediction equations for the unsaturated shear strength

Several equations to obtain the unsaturated shear strength have been proposed. These equations can be broadly classified in two main categories as ‘fitting equations’ and ‘prediction equations’. Fitting equations represent the unsaturated shear strength based on direct measurements which is expensive and time-consuming. On the other hand, prediction equations to estimate the unsaturated shear strength are based on parameters that are easier to measure and are therefore especially attractive.

The non-linear shear strength behaviour with respect to soil suction is directly related to the SWCC (Bao et al., 1998; Fredlund et al., 1996; Gan and Fredlund, 1996;

Vanapalli et al., 1996b). Prediction equations to estimate the unsaturated shear strength that make use of the SWCC are reviewed in this Section.

Lamborn (1986) proposed a prediction equation for the unsaturated shear strength by extending a micro-mechanical model using the principle of thermodynamics as:

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \theta \tan\phi' \quad \text{Equation 2-12}$$

where

τ = shear strength

c' = effective cohesion

$(\sigma - u_a)$ = net normal stress

ϕ' = effective friction angle

ψ = soil suction

θ = volumetric water content

Based on the assumption that the normalized area of water is related to the rate at which soil suction contributes to the unsaturated shear strength, two general equations to estimate the unsaturated shear strength were proposed by Vanapalli et al. (1996b). The first equation proposed by Vanapalli et al. (1996b) was introduced as:

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \theta^\kappa \tan\phi' \quad \text{Equation 2-13}$$

where

θ = normalized volumetric water content $\left(\theta/\theta_s\right)$

κ = fitting parameter

θ_s = saturated volumetric water content

Equation 2-13 was simultaneously proposed by Fredlund et al. (1996). Based on five statically compacted fine grained soils, Vanapalli and Fredlund (2000) established a relationship between the fitting parameter (κ) and the plasticity index of the soil that can be mathematically expressed as:

$$\kappa = -0.0080 (PI)^2 + 0.0801 (PI) + 1 \quad \text{Equation 2-14}$$

where

PI = plasticity index

Garven and Vanapalli (2006) modified the expression for the fitting parameter (κ) using ten compacted soils involving clays, silts and tills as:

$$\kappa = -0.0016 (PI)^2 + 0.0975 (PI) + 1 \quad \text{Equation 2-15}$$

The second equation for the unsaturated shear strength proposed by Vanapalli et al. (1996b) is based on the same concept as Equation 2-13 but without a fitting parameter as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + \psi \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \tan \phi' \quad \text{Equation 2-16}$$

where

θ_r = residual volumetric water content

Equation 2-16 can also be expressed in terms of degrees of saturation instead of volumetric water contents (Vanapalli et al., 1996b). The contribution of shear strength with respect to soil suction according to Equation 2-16 becomes negative for soil suctions beyond residual state condition.

Both equations proposed by Vanapalli et al. (1996b) (i.e., Equation 2-13 and Equation 2-16) are an extension of the unsaturated shear strength proposed by Fredlund et al. (1978) using two independent stress state variables. They both show the theoretical basis of using the SWCC for the estimation of the unsaturated shear strength over the entire soil suction range.

Oberg and Sallfors (1997) proposed a prediction equation for the unsaturated shear strength that is based on an analytical model considering spheres. The effective stress parameter in Bishop (1959)'s formulation (Equation 2-9) was treated as a simple scaling factor reflecting the fraction of the pore area that is occupied by water. Based on their analytical model, the effective stress parameter can be replaced with

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reasonable accuracy by the degree of saturation for non-clayey materials and for degrees of saturation higher than about 50%.

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi S \tan\phi' \quad \text{Equation 2-17}$$

where

S = degree of saturation

Bao et al. (1998) proposed a modification of Equation 2-16 as introduced by Vanapalli et al. (1996b) by assuming a linear variation between air-entry value and residual state condition on a semi-logarithmic plot of the SWCC.

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \left(\frac{\log(\psi_r) - \log(\psi)}{\log(\psi_r) - \log(AEV)} \right) \tan\phi' \quad \text{Equation 2-18}$$

where

ψ_r = residual soil suction

AEV = air-entry value

Equation 2-18 was proposed to estimate the unsaturated shear strength between the air-entry value and residual state condition.

An empirical relationship for Bishop (1959)'s effective stress state parameter was proposed by Khalili and Khabbaz (1998). Khalili and Khabbaz (1998) concluded that no correlation between the effective stress parameter and volumetric parameters such as degree of saturation and volumetric water content may be found as the effective stress parameter is strongly related to the soil structure.

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \chi \tan\phi' \quad \text{Equation 2-19}$$
$$\chi = \left(\frac{\psi}{AEV} \right)^{-0.55}$$

An analysis of 14 fine-grained soils was used to obtain the empirical relationship for Bishop (1959)'s effective stress state parameter. The equation proposed by Khalili and Khabbaz (1998) is valid for soil suctions higher than the air-entry value.

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Aubeny and Lytton (2003) proposed an extension of the equation by Lamborn (1986) (Equation 2-12) by implementing an additional factor. The additional factor is a function of the volumetric water content and is dependent on the degree of saturation. The additional factor takes into account that the water phase for an unsaturated soil is not acting over the entire particle surface.

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi f_1 \theta \tan\phi'$$

$$f_1 = \left(\frac{1}{\theta}\right)$$

$$\text{if } S = 100\%$$

$$f_1 = 1 + \left(\frac{S - 85}{15}\right) \left(\frac{1}{\theta} - 1\right)$$

Equation 2-20

$$\text{if } 85\% \leq S \leq 100\%$$

$$f_1 = 1$$

$$\text{if } S < 85\%$$

An empirical-analytical logarithmic function to estimate the increase in shear strength due to soil suction was proposed by Tekinsoy et al. (2004).

$$\tau = c' + (\sigma - u_a) \tan\phi' + (AEV + P_{at}) \ln\left(\frac{\psi + P_{at}}{P_{at}}\right) \tan\phi' \quad \text{Equation 2-21}$$

where

P_{at} = atmospheric pressure (101.3 kPa)

The air-entry value as the only parameter from the SWCC is needed to predict the increase in shear strength due to soil suction as proposed by Tekinsoy et al. (2004). The equation was reported to be most suitable for fine-grained soils (Tekinsoy et al., 2004).

Sheng et al. (2008) developed an elasto-plastic model for unsaturated soils which based on independent stress state variables. The shear strength equation is embodied

in the model in terms of apparent tensile strength. The shear strength equation can be written in terms of the extended Mohr-Coulomb criterion as:

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \tan\phi^b$$

$$\tan\phi^b = \tan\phi'$$

$$\text{if } \psi < \psi_{sa}$$

Equation 2-22

$$\tan\phi^b = \tan\phi' \left(\left(\frac{\psi_{sa}}{\psi} \right) + \left(\frac{\psi_{sa} + 1}{\psi} \right) \ln \left(\frac{\psi + 1}{\psi_{sa} + 1} \right) \right)$$

$$\text{if } \psi > \psi_{sa}$$

where

ψ_{sa} = saturation suction

ϕ^b = angle indicating an increase in shear strength with respect to soil suction

The equation uses a term introduced as ‘saturation suction’ as the only parameter to be evaluated from the SWCC. The saturation suction is defined as the maximum soil suction that corresponds to full saturation. For soils at drying, the saturation suction is the same as the air-entry value whereas for soils at wetting, the saturation suction is usually smaller than the air-entry value (Zhou and Sheng, 2009).

Goh et al. (2010) proposed a set of equations to predict the unsaturated shear strength at drying and wetting. The normalized volumetric water content was used as the controlling soil parameter as:

$$\tau = c' + (\sigma - u_a) \tan\phi' + \psi \tan\phi'$$

$$\text{if } \psi < AEV$$

$$\tau = c' + ((\sigma - u_a) + AEV) \tan\phi' + (\psi - AEV)b \theta^\kappa \tan\phi' \quad \text{Equation 2-23}$$

$$\text{if } \psi \geq AEV$$

$$\kappa = (\log(\psi) - \log(AEV))^\gamma$$

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where

b and y = constant parameters

At drying:

$$y_d = 0.502 \ln(PI + 2.7) - 0.387 \quad \text{Equation 2-24}$$

$$\begin{aligned} b_d = & -0.245 \left(\ln(n_d(PI + 4.4)) \right)^2 \\ & + 2.114 \left(\ln(n_d(PI + 4.4))(n_d(PI + 4.4)) \right) \\ & - 3.522 \end{aligned} \quad \text{Equation 2-25}$$

where

y_d = parameter 'y' for drying shear strength

b_d = parameter 'b' for drying shear strength

n_d = Fredlund and Xing (1994) fitting parameter 'n' for drying SWCC

At wetting:

$$y_w = 3.55y_d - 3.00 \quad \text{Equation 2-26}$$

$$b_w = 0.542b_d \left(\frac{n_d}{n_w} \right) + 0.389 \quad \text{Equation 2-27}$$

where

y_w = parameter 'y' for wetting shear strength

b_w = parameter 'b' for wetting shear strength

n_w = Fredlund and Xing (1994) fitting parameter 'n' for wetting SWCC

The set of equations to estimate the unsaturated shear strength by Goh et al. (2010) were developed using six sets of shear strength data consisting of coarse- and fine-grained soils.

The reviewed prediction equations for the unsaturated shear strength were mostly developed and validated on very limited soil data. Few studies (e.g., Garven and Vanapalli, 2006; Goh et al., 2010) analysed the performance and applicability of

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some of these prediction equations on independent data sets. These studies indicated that there was not a single prediction equation that reliably estimated the unsaturated shear strength reasonably for a wide range of soils. However, the prediction equations proposed by Vanapalli et al. (1996b), Oberg and Sallfors (1997) and Khalili and Khabbaz (1998) are the equations commonly adopted and referred to in research studies. Garven and Vanapalli (2006) established the most comprehensive relationship for the kappa parameter to be used with equation 2-15 proposed by Vanapalli et al. (1996) using several soil data sets.

2.6 UNCERTAINTY IN SOIL PROPERTIES

The evaluation of soil properties is a classical task in geotechnical engineering. Uncertainties in soil properties are unavoidable as a soil exhibits natural variations from location to location. Variation in soil properties can be a major contributor of uncertainties in geotechnical analyses (Christian et al., 1994). Quantification of uncertainties in soil properties is usually done within the framework of probability theory by modelling soil parameters as random variables. Laboratory tests on natural soils indicate the applicability of most soil properties as random variables complying with the normal or log-normal probability distribution (Lumb, 1966).

2.6.1 Sources and types of uncertainties

The total uncertainty in the evaluation of geotechnical soil properties can be related to two sources, namely the (1) natural variability of the soil and to (2) knowledge uncertainty as illustrated in Figure 2-10 (adapted from Baecher and Christian, 2005).

Figure 2-10 Main components contributing to the total uncertainty in the determination of soil properties (adapted from Baecher and Christian, 2005)

Most soils are formed over time by various natural processes and their properties will therefore neither be constant from a spatial nor a temporal point of view. A characteristic value of a soil property is in general assigned to a specific soil volume at a certain point in time. The natural variability is therefore associated with the inherent randomness of natural processes resulting in variability over space and/or time (Baecher and Christian, 2005). The majority of soil properties can be considered temporally constant for practical geotechnical engineering. Natural variability is

therefore assumed to be spatially random and temporally constant (Baecher and Christian, 2005).

Knowledge uncertainty is associated with a lack of data, information and with the inability to model the real world. Knowledge uncertainty in the evaluation of soil properties can be further divided into; (1) characterisation uncertainty, (2) model uncertainty and (3) transformation uncertainty. Characterisation uncertainty results from measurement errors and data uncertainty such as inconsistency of data, data handling and transcription errors and inadequate representativeness of data. Model uncertainty reflects the inability of a model to describe the true physical behaviour of reality. Transformation uncertainty arises when the parameters of interest are interpreted from measurement results.

Whereas knowledge uncertainty can be reduced by additional data, improved measurements and models, the natural variability of a soil is inherent and cannot be reduced by additional information.

2.6.2 Uncertainty in soil properties

Various researchers modelled the uncertainty in soil properties by separating between uncertainty caused by (1) data scatter and (2) systematic errors as illustrated in Figure 2-11 (e.g., Baecher and Ladd, 1997; Christian, 2004; Christian et al., 1994; Whitman, 2000).

Figure 2-11 Categories of uncertainty in soil properties (from Christian et al., 1994)

Uncertainty in data scatter was further divided into real spatial variation and random testing errors. A systematic error results from an error in the computed mean value

of the property due to the limited number of tests available leading to statistical uncertainty and error due to bias in the measurements or model (Christian et al., 1994).

Spatial variation can be seen as data scatter around the trend value of a soil property and systematic errors as uncertainty in the trend value itself (Baecher and Ladd, 1997). However, soil properties are often not measured directly and a model is used to relate a measured quantity to the respective soil property introducing model uncertainty as an additional source of uncertainties (Phoon and Kulhawy, 1999b). Therefore, the variability in the estimation of soil properties results from various sources of uncertainty as illustrated in Figure 2-12.

Figure 2-12 Uncertainty in soil properties estimates (adapted from Phoon and Kulhawy, 1999a)

The value of a soil property and its variation is best evaluated based on site-specific data. Sample values of a property in a representative soil volume may be referred to as a real world sample described by a probability distribution. However, evaluation of soil properties in a soil volume is limited in the number of test points and their distribution and the parameters can only be inferred. Limited site-specific data makes it often impossible to perform statistical analyses and guidelines on possible ranges of the variation in soil properties become useful as a first approximation.

Considerable research to produce statistics on the variability of soil index properties and saturated soil properties has been done and is available in the literature (e.g., Baecher and Christian, 2003; Harr, 1977; Phoon and Kulhawy, 1999a, b; Rahardjo et al., 2012). On the contrary, very limited studies were carried out on the variability of unsaturated soil properties. Information on the variability of the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) for residual soils was reported in Rahardjo et al. (2012). Typical measurement errors for some properties were reported in Phoon and Kulhawy (1999a). The normal or the log-normal distribution assessed from the sample mean and sample standard deviation is applicable for most soil properties (Lumb, 1966).

The coefficient of variation has been proven to be a useful relative measure to concisely describe soil variability (Harr, 1987; Phoon and Kulhawy, 1999a). Typical coefficients of variation for soil properties are summarized in the following Sections 2.6.2.1 and 2.6.2.2 for soil index properties and strength parameters, respectively. Section 2.6.2.3 summarizes studies carried out on the variability of the Soil-Water Characteristic Curve (SWCC).

2.6.2.1 Variability in soil index properties

Soil index properties show some of the lowest coefficients of variation for the inherent soil variability and measurement error (Appendix A). The coefficient of variation of measurement errors for soil index properties can be expected to be lower than 15 % (Phoon and Kulhawy, 1999a). Soil index properties were reported to be normally distributed (Lumb, 1966) (Appendix A).

2.6.2.2 Variability in strength properties

Phoon and Kulhawy (1999b) estimated for typical mean values of the effective friction angle a coefficient of variation between 5 and 15 % for both, the inherent variability and the measurement error (Appendix A). Reported coefficients of variation by other researchers confirm a range of 5-15 % for the inherent variability of the effective friction angle (e.g., Harr, 1977; Pinheiro Branco et al., 2014; Rahardjo et al., 2012).

Reported ranges for the coefficient of variation for the inherent variability of the effective cohesion are 10-70 %, which is considerably higher than those for the effective friction angle (Harr, 1987; Pinheiro Branco et al., 2014; Rahardjo et al., 2012).

The coefficient of variation of the inherent variability for the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) was reported as 25-40 % for residual soils in Singapore (Rahardjo et al., 2012). Several studies indicate that the strength properties follow a normal distribution (e.g., Lumb, 1966; Pinheiro Branco et al., 2014) (Appendix A).

2.6.2.3 *Variability in the Soil-Water Characteristic Curve*

Estimates for unsaturated soil property functions are either directly or indirectly dependent on the SWCC (Fredlund and Houston, 2009). Therefore, the reliability of estimated unsaturated soil property functions is directly related to the accuracy of the respective SWCC used in the estimation.

Due to a lack in sufficient direct measurements of the SWCC, several researchers used a first-order error analysis to approximate the variability in the fitting parameters of the SWCC equation (e.g., Kool et al., 1987; Mishra et al., 1989; Zapata et al., 2000; Zhai and Rahardjo, 2013). Beck and Arnold (1977) and Kool et al. (1987) indicated that the covariance matrix representing parameter uncertainties can be estimated by first-order error analysis. Based on the first-order error approximation of the covariance matrix of the fitting parameters of the SWCC equation, Zapata (1999) and Zhai and Rahardjo (2013) constructed confidence intervals of the SWCC. Zapata (1999) indicated that the variability in the SWCC for sand is highest around the air-entry value whereas the desaturation region of the SWCC exhibits the highest variability for clays. Zhai and Rahardjo (2013) found that the highest variability in the SWCC for a silty sand is between the air-entry value and the residual state. Figure 2-13 illustrates the best fitted SWCC including the confidence interval and the respective variability for El Paso Sand as evaluated by Zapata (1999).

(a) Best fitted Soil-Water Characteristic Curve including the confidence interval

(b) Soil-Water Characteristic Curve variability

Figure 2-13 Best fitted Soil-Water Characteristic Curve including the confidence interval and the respective variability of El Paso Sand (from Zapata 1999)

Statistical and probabilistic analyses of SWCC fitting parameters based on databases have been carried out by Sillers and Fredlund (2001) and Phoon et al. (2010). Sillers and Fredlund (2001) statistically assessed the fitting parameters for various SWCC models based on the United States Department of Agriculture (USDA) soil classification system. The coefficients of variation for the SWCC fitting parameters were found to be large for most fitting parameters of the respective models. Probabilistic analyses by Phoon et al. (2010) confirms the high coefficient of variation for some fitting parameters of the van Genuchten (1980) SWCC model based on soil classifications. The study by Phoon et al. (2010) further suggested that

the SWCC fitting parameters for the van Genuchten (1980) model are lognormal distributed and that the fitting parameters are correlated. The studies by Fredlund and Sillers (2001) and Phoon et al. (2010) both indicate that the uncertainties in the SWCC parameters might not be based on classification systems.

Even though analyses on the variability of the SWCC have been carried out, there is to date no quantitative statistics available regarding the uncertainty in the SWCC equations or the respective fitting parameters.

2.6.3 Uncertainty in the estimation of unsaturated soil property functions

The experimental effort required to directly determine unsaturated soil properties is often not feasible for most practical circumstances. Unsaturated soil property functions which estimate a value for the respective unsaturated soil property provides the key for the implementation of unsaturated soil mechanics in geotechnical engineering practice (Fredlund, 2000). Unsaturated soil property functions are generally based on the saturated soil properties and information of the SWCC. Therefore, the accuracy of these unsaturated soil property functions depends heavily on the SWCC and the empirical estimation equation of the unsaturated soil property function (Fredlund and Houston, 2009).

Some researchers (e.g., Kool and Parker, 1988; Mishra et al., 1989) estimated the parameters for unsaturated flow and assessed the respective uncertainties in the hydraulic conductivity function. Mishra et al. (1989) estimated the soil hydraulic properties from the grain-size distribution and quantified the uncertainties in these parameter estimates. A first-order approximation to the parameter covariance matrix was adopted to assess the uncertainty in the SWCC due to parameter estimation for the relative conductivity in unsaturated soils. It was concluded that the SWCC can be reasonably estimated, however the estimation of the relative conductivity showed a high discrepancy compared to the measured data which were attributed to the inaccurate estimation of the saturated conductivity (Figure 2-14).

(a) Measured and estimated Soil-Water Characteristic Curve (dashed lines indicate one standard deviation interval around the estimated value)

(b) Measured and estimated relative conductivity (dashed lines indicate one standard deviation interval around the estimated value)

Figure 2-14 Measured and estimated Soil-Water Characteristic Curve and relative conductivity (from Mishra et al., 1989)

Zapata (1999) assessed the variability in the estimated unsaturated shear strength based on the confidence intervals of the SWCC. The confidence intervals were evaluated based on a first-order error approximation for the SWCC fitting parameters. Zapata (1999) concluded that the variability in the estimation of the unsaturated shear strength due to uncertainties in the SWCC parameters generally increases with

increasing soil suction as shown in Figure 2-15. The relative error in the shear strength due to uncertainty in the SWCC fitting parameters was found to be less than 10 % for sand but as high as 40 % for silts and clays at higher soil suctions around 1,000 kPa (Figure 2-15).

Figure 2-15 Estimated relative error in the unsaturated shear strength due to uncertainties in the fitting parameters of the Soil-Water Characteristic Curve

Analyses of unsaturated soil property functions indicate that the adopted SWCC greatly impacts the estimation of the unsaturated soil property function. Past studies on the uncertainties in the estimation of unsaturated soil properties considered either uncertainty in the basic input parameter and the SWCC or only the SWCC. The uncertainty in the adopted estimation equation for the unsaturated soil property function were in past studies not considered.

2.7 SUMMARY

The unsaturated shear strength can be evaluated through direct measurements in the laboratory or estimated by prediction equations as proposed by several researchers. Prediction equations to estimate the highly non-linear unsaturated shear strength

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failure envelope are an attractive alternative to the time-consuming direct measurements of the unsaturated shear strength.

Several researchers demonstrated that the non-linear shear strength behaviour is directly related to the SWCC of the soil. Therefore, many proposed prediction equations for the unsaturated shear strength are based on information of the SWCC. Even though the SWCC has been investigated rigorously in the past, there are discrepancies in the literature regarding the evaluation of the residual state.

Although prediction equations for the unsaturated shear strength are widely used in the field of research, very limited studies have been carried out to assess and quantify the uncertainties involved in the estimation. Unlike saturated soil properties, unsaturated soil properties have been much less investigated on their uncertainties especially for the case when the unsaturated soil property function is estimated. Studies indicate that the impact of the variability in the SWCC on the estimated shear strength might be very high.

There is a need to investigate the concept of residual state and its possible effect on the estimation of the unsaturated shear strength from the SWCC. Further, the uncertainties involved in the estimation of the unsaturated shear strength need to be addressed and quantified in order to draw qualitative conclusions on the applicability of the unsaturated shear strength estimation. Therefore, this research focuses on the concept of residual state including its possible impact on the unsaturated shear strength prediction. This also includes the quantification of the uncertainties involved in the estimation of the unsaturated shear strength.

CHAPTER 3 THEORY

3.1 INTRODUCTION

The following Chapter presents the general theories applied throughout this study. A proposed conceptual model for the contribution of shear strength with respect to soil suction is presented and a prediction equation based on the presented concept is proposed. In addition, the theories used to quantify the uncertainty in the estimation of the unsaturated shear strength are presented.

3.2 SOIL-WATER CHARACTERISTIC CURVE

The Soil-Water Characteristic Curve (SWCC) defines the relationship between soil suction and water volume in a soil which primarily governs the engineering behaviour of unsaturated soils. Therefore, the SWCC becomes the essential information required for the successful application of unsaturated soil mechanics (Fredlund and Rahardjo, 1993).

The water volume in a soil can be expressed by the gravimetric or volumetric water content or by the degree of saturation. The degree of saturation however appears to be the variable most closely related to the unsaturated soil behaviour and defines the meaning of the air-entry value of the soil most clearly (Fredlund, 2006). The air-entry value is defined as the soil suction where air starts first to penetrate the soil pores. Therefore, the degree of saturation is used throughout this study as a measure of water volume in the soil pores. The degree of saturation is defined as the ratio of pore-water volume to void volume of the soil as follows:

$$S = \frac{V_w}{V_v} \quad \text{Equation 3-1}$$

where

S = degree of saturation

V_w = volume of pore-water

V_v = volume of voids

Soil-Water Characteristic Curves reported in literature related to soil science are often expressed in terms of volumetric water content. A simple volume-mass relationship converts volumetric water content to the respective degree of saturation.

$$S = \frac{\theta}{n} = \theta \frac{(1 + e)}{e} \quad \text{Equation 3-2}$$

where

θ = volumetric water content

n = porosity

e = void ratio

3.2.1 Representation of the Soil-Water Characteristic Curve

The SWCC is often embedded in constitutive relations to describe unsaturated soil property functions such as the unsaturated shear strength or the unsaturated permeability. Having a continuous function that represents experimental SWCC data is therefore of great advantage. Several empirical closed-form equations to represent experimental SWCC data with a continuous function have been proposed (e.g., Brooks and Corey, 1964; Fredlund and Xing, 1994; van Genuchten, 1980).

The SWCC equation proposed by Fredlund and Xing (1994) has been reported to provide the best fit between experimental and calculated data for a wide range of soils over the entire soil suction range from zero to 10^6 kPa (e.g., Leong and Rahardjo, 1997; Sillers and Fredlund, 2001; Zapata et al., 2000). The Fredlund and Xing (1994) equation was therefore adopted throughout this research for the representation of the SWCC. Fredlund and Xing (1994)'s equation is based on the pore-size distribution of the soil and consists of three fitting parameters. The Fredlund and Xing (1994) equation in terms of degree of saturation is given as:

$$S = C(\psi) \frac{1}{\left(\ln \left(e + \left(\frac{\psi}{a} \right)^n \right) \right)^m} \quad \text{Equation 3-3}$$

where

$C(\psi)$ = correction function

e = Euler's number

ψ = soil suction

a, n and m = fitting parameters

The correction function ensures that the SWCC reaches a terminal suction of 10^6 kPa in completely dry soil conditions as suggested by several researchers (e.g., Fredlund and Xing, 1994; Richards, 1965; Zapata, 1999). Fredlund and Xing (1994) proposed the following correction function:

$$C(\psi) = \left(1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right) \quad \text{Equation 3-4}$$

where

C_r = constant parameter

The continuous SWCC obtained by the Fredlund and Xing (1994) equation represents SWCC data over the entire soil suction range from zero to 10^6 kPa.

3.2.2 Fitting of the Soil-Water Characteristic Curve

The fitting parameters according to the Fredlund and Xing (1994) equation were in this study obtained by minimizing the sum of squared errors between experimental data and calculated values as follows:

$$SSE(a, m, n) = \sum_{i=1}^N (S_i - S(\psi_i, a, m, n))^2 \quad \text{Equation 3-5}$$

where

SSE = sum of squared errors

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$N = \text{number of data}$

The non-linear minimization problem was solved in a spreadsheet with initial values for the fitting parameters set to: $a = 1$; $n = 1$; $m = 1$; and $C_r = 1,500 \text{ kPa}$ (constant).

3.3 DEVELOPMENT OF PROPOSED PREDICTION EQUATION FOR THE ESTIMATION OF THE UNSATURATED SHEAR STRENGTH

The shear strength for unsaturated soils in terms of two independent stress state variables according to Fredlund et al. (1978) is written as:

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad \text{Equation 2-10}$$

where

τ = shear strength

c' = effective cohesion

$(\sigma - u_a)$ = net normal stress

ϕ' = effective friction angle

$(u_a - u_w)$ = matric suction

ϕ^b = angle indicating an increase in shear strength with respect to soil suction

A change in mechanical soil behaviour due to soil suction is directly related to a change in water volume in the soil. The SWCC that relates soil suction to water volume provides this essential information. Therefore, prediction equations to estimate the unsaturated shear strength are based on the saturated shear strength parameters and information of the SWCC (e.g., Garven and Vanapalli, 2006; Goh et al., 2010; Khalili and Khabbaz, 1998; Oberg and Sallfors, 1997; Vanapalli et al., 1996b). For prediction equations, the angle indicating a possible increase in shear strength with respect to soil suction (ϕ^b) can be expressed in a general form as:

$$\tan\phi^b = f(\text{SWCC}, x_i)\tan\phi' \quad \text{Equation 3-6}$$

where

$f(\text{SWCC}, x_i)$ = function controlling the shear strength with respect to soil suction

x_i = additional parameter(s)

3.3.1 Proposed behavioural model for the shear strength with respect to soil suction

The proposed behavioural model for the unsaturated shear strength as developed in this study is derived by using the typical desaturation behaviour of a soil along the SWCC. The concept of desaturation in a porous media was originally introduced by White et al. (1970) and adapted by various researchers (e.g., Bear, 1979; Luckner et al., 1989; Vanapalli et al., 1996b). These concepts are modified in this study in order to develop a prediction equation for the unsaturated shear strength.

3.3.1.1 Modified desaturation behaviour

Three distinct saturation zones can be identified along a drying branch of the SWCC as illustrated in Figure 3-1. They are herein referred to as ‘boundary effect zone’, ‘transition zone’ and ‘residual zone’. The air-entry value separates the boundary effect zone from the transition zone whereas the residual state condition separates the transition zone from the residual zone.

Figure 3-1 Desaturation stages of a soil on a Soil-Water Characteristic Curve (as presented in Schnellmann et al., 2015)

The soil pores in the boundary effect zone remain essentially filled with pore-water and an increase in soil suction has basically no effect on the pore-water volume in the soil. As a result, the area where the negative pore-water pressure (soil suction) acts on the soil particles is not reduced; increasing the particle contact in the soil matrix

similar to an increase in net normal stress. As no change in pore-water volume takes place in the boundary effect zone, the parameter controlling the mechanical behaviour of the soil has to remain constant, reflecting the area where the negative pore-water pressure (soil suction) acts on the soil particle. Air first penetrates the macro-pores of the soil as the radii of the air-water interface for larger pores are larger and therefore, the capillary force is weaker as compared to that in smaller pores.

Desaturation of the soil starts in the transition zone where pore-water volume is first drained out from the macro-pores. Consequently, the area where the negative pore-water pressure acts (soil suction) on the soil particles starts to decrease as the negative pore-water pressure increases. Desaturation continues as soil suction increases and pore-water volume continuously drains out of the soil pores. The pore-water volume significantly reduces with increasing soil suction. Therefore, the area where the negative pore-water pressure (soil suction) acts on the soil particles reduces constantly in the transition zone resulting in a non-linear mechanical behaviour of the soil.

Desaturation of the soil starts to slow down as residual state condition is approached and a large change in soil suction results in a relatively small reduction in water volume. The soil suction at residual state condition might be very high and remaining soil water might only be present as a thin water film around the soil particles.

The water phase in the residual zone is disconnected (except for a very thin film of water around the soil particle) and the air phase is continuous. In the residual zone, the water remaining in the soil is assumed to lose its capability to respond to hydraulic flow and the loss in water volume becomes negligible. As the water phase is disconnected in the residual zone, practically no pressure can be transmitted through the water phase (Bear, 1979). Therefore, the effect of capillarity on the mechanical behaviour of the soil becomes ineffective and the role of the parameter controlling the mechanical behaviour of the soil becomes negligible in the residual zone.

Although further desaturation might be achieved through oven drying, it is very unlikely that a soil in a natural environment achieves a completely dry condition as soil at high soil suction absorbs water from the atmosphere as a result of water vapour movement.

3.3.1.2 *Proposed failure envelope for the shear strength with respect to soil suction*

The shear strength due to soil suction as derived from the modified desaturation behaviour depends on the interaction between soil suction and pore-water volume in the soil (i.e., area where the negative pore water pressure acts on the soil particles). Therefore, an increase in shear strength due to soil suction might only occur in the boundary effect zone and in the transition zone as only in these two zones is negative pore-water volume available.

There is basically no change in area where the negative pore-water pressure acts on the soil particles in the boundary effect zone. Hence, the soil-pores remain essentially saturated and an increase in shear strength due to soil suction is in the boundary effect zone as effective as an increase in effective stress. The area where the negative pore-water pressure acts on the soil particles is continuously reduced in the transition zone. As a result, an increase in shear strength due to soil suction becomes less effective in the transition zone as compared to the boundary effect zone and the shear strength failure surface becomes non-linear. The increase in shear strength due to soil suction reaches a maximum value in the transition zone and decreases to zero (i.e., saturated shear strength) at residual state condition. The soil water in the residual zone is assumed to be absorbed to the soil particles and therefore loses the ability to contribute to shear strength.

The proposed shear strength behaviour as developed in this research is illustrated on an extended Mohr-Coulomb failure surface Figure 3-2 (as presented in Schnellmann et al. 2015). The shear strength due to soil suction increases linearly up to the air-entry value of the soil with the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) being equal to the effective friction angle (ϕ'). As the soil starts to desaturate beyond the air-entry value, ϕ^b decreases continuously until residual state condition where ϕ^b becomes zero and the increase in shear strength due to soil suction becomes zero.

Figure 3-2 Extended Mohr-Coulomb failure surface for the proposed behaviour of the shear strength with respect to soil suction for coarse-grained soils (as presented in Schnellmann et al., 2015)

The behavioural shear strength model proposed in this study addresses the effect of soil suction (or negative pore-water pressures) as the soil desaturates along the SWCC and therefore accounts for effects associated with capillarity. Effects of electro-chemical forces (van der Waals and electrical double layer), which may play an important role in clayey soils, are not considered.

Figure 3-3 illustrates upper bound magnitudes of inter-particle stresses corresponding to particle sizes. For particle sizes corresponding to sands, a change in the degree of saturation results only in a change in capillary attraction which can be explained by the desaturation behaviour of a soil along the SWCC. However, for particle sizes corresponding to clayey soils, electro-chemical forces might become of significant magnitude. Electro-chemical forces might increase or decrease the inter-particle stress corresponding to their nature. Effects of electro-chemical forces are not explained by the desaturation behaviour of a soil along the SWCC and are beyond the scope of this study.

Furthermore, the original desaturation behaviour of a porous media by White et al. (1970) was developed based on spheres of different diameters and is therefore more applicable to coarse-grained soils.

Figure 3-3 Upper bounds of inter-particle stress (from Lu and Likos, 2006)

Therefore, the proposed failure envelope for the shear strength with respect to soil suction is more appropriate for coarse-grained soils such as sands and silts. Especially at high soil suctions, clayey materials might behave differently as a considerable amount of water volume could still be available to transmit negative pore-water pressure to soil particles and hence to increase the shear strength.

3.3.2 Proposed controlling parameter for unsaturated soil property functions

The effective degree of saturation remains unity up to the point where the macropores of the soil start to desaturate and air first penetrates the soil matrix. A constant decrease in the effective degree of saturation takes place in the transition zone until the residual state condition is reached, where the effective degree of saturation becomes zero (Figure 3-1). The effective degree of saturation describes the behaviour of a soil progressing from a completely saturated condition to a residual state according to the proposed modified desaturation behaviour as described in Section 3.3.1.1.

It is therefore suggested in this research that the effective degree of saturation is used as the controlling parameter to account for the effect of soil suction in unsaturated

soil property functions (as presented in Schnellmann et al., 2014). The effective degree of saturation is defined as follows:

$$S_e = \left(\frac{S - S_r}{1 - S_r} \right), S_e \geq 0 \quad \text{Equation 3-7}$$

where

S_e = effective degree of saturation

S = degree of saturation

S_r = residual degree of saturation

To calculate the effective degree of saturation at a particular soil suction, the degree of saturation at residual state condition needs to be determined first. The selected residual degree of saturation greatly affects the effective degree of saturation and hence the performances of the unsaturated soil property function. It is therefore imperative to define and determine the residual degree of saturation properly in order to obtain meaningful results.

3.3.2.1 Proposed residual state concept

Based on the proposed modified desaturation behaviour of a soil as described in Section 3.3.1.1, residual state is defined as a condition where a change in water volume becomes negligible over a large soil suction range. This definition is consistent with definitions for residual state condition as proposed by several researchers (e.g., Brooks and Corey, 1964; Luckner et al., 1989; van Genuchten, 1980; Vanapalli et al., 1996b; White et al., 1970). The water phase in the soil at residual state condition is assumed to be discontinuous, meaning there is no connected network of water in the soil pores that transmits soil suction forces associated with capillarity.

It is suggested that the soil suction where residual state condition might occur is visually best identified on an arithmetic plot of the SWCC as illustrated in Figure 3-4. Compared to a semi-logarithmic plot usually adopted to represent the SWCC, an arithmetic plot allows the identification where the SWCC starts to level off, indicating residual state condition, since only a small change in water volume occurs beyond

this point. As illustrated in Figure 3-4, it is not possible to identify a soil suction range where the SWCC starts to level off on a semi-logarithmic plot of the SWCC.

^v obtained visually on an arithmetic plot of the SWCC (as proposed in this study); ^c obtained through the construction method on a semi-logarithmic plot of the SWCC

Figure 3-4 Proposed identification of residual state condition on a Soil-Water Characteristic Curve (as presented in Schnellmann et al., 2015)

Residual state condition is in the literature most commonly determined using the construction method on a semi-logarithmic plot of the SWCC (Goh et al., 2010; Rassam and Williams, 1999; Yang et al., 2004; Zhai and Rahardjo, 2012). However, the construction method to determine residual state condition can differ greatly from the residual state definition proposed by several researchers as a substantial amount of water volume might still be available (Figure 3-4). This becomes evident if the residual degree of saturation as obtained through the construction method is compared with the arithmetic plot of the SWCC as illustrated in Figure 3-4.

Based on the proposed modified desaturation behaviour (Section 3.3.1.1), the identification of residual state condition on an arithmetic plot of the SWCC is more meaningful since it is possible to identify residual state condition at a point where the SWCC starts to level off and a large change in soil suction only causes a negligible change in water volume. The proposed visual identification of residual state condition on an arithmetic plot of the SWCC might not yield a clearly defined value and might

be somewhat subjective. Therefore, a criterion to identify residual state condition analytically is needed to minimize the subjectivity of the proposed identification procedure using an arithmetic plot of the SWCC. In this case, residual state condition is defined as a condition where the rate of change in water volume becomes negligible. This point can be determined mathematically as the point where the second derivative of the SWCC with respect to soil suction becomes insignificant (as presented in Schnellmann et al., 2015):

$$\frac{\partial^2 S}{\partial \psi^2} \rightarrow 0 \quad \text{Equation 3-8}$$

where

S = degree of saturation

ψ = soil suction

The second derivative of the SWCC with respect to soil suction based on the Fredlund and Xing (1994) equation (Equation 3-3) is given in Appendix B.

3.3.3 Suggested prediction equation for the unsaturated shear strength

Based on the proposed modified desaturation behaviour presented in Section 3.3.1.1, the effective degree of saturation emerges as the controlling parameter for unsaturated soil property functions. Therefore, the following substitution for the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) in Equation 3-6 is made:

$$\tan \phi^b = \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi' \quad \text{Equation 3-9}$$

The prediction equation for the shear strength with respect to soil suction can therefore be written as:

$$\tau = c' + (\sigma - u_a) \tan \phi' + \psi \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi' \quad \text{Equation 3-10}$$

if $\psi < \psi_r$

$$\tau = c' + (\sigma - u_a) \tan \phi'$$

$$\text{if } \psi \geq \psi_r$$

where

$\psi_r = \text{residual soil suction}$

The suggested prediction equation for the unsaturated shear strength as derived in this study (Equation 3-10) was originally proposed by Vanapalli et al. (1996b). However, the behaviour of the shear strength with respect to soil suction at and beyond residual state condition as proposed in this study differs greatly from Vanapalli et al. (1996b)'s model. Vanapalli et al. (1996b) stated that beyond residual state condition, the unsaturated shear strength may increase, decrease or remain constant. Values between zero to 200 kPa and 500 kPa to over 1,500 kPa were suggested as reasonable soil suction values for the residual state condition for coarse- and fine-grained soils, respectively.

3.3.4 Validation of proposed concepts

Analyses were carried out to validate the applicability of the effective degree of saturation as the controlling parameter in unsaturated soil property functions and to validate the proposed residual state condition. For the validation analyses of the proposed concepts, only the possible increase in shear strength due to soil suction according to Equation 3-10 was considered.

$$\tau_{us} = \psi \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi' \quad \text{Equation 3-11}$$

$$\text{if } \psi < \psi_r$$

where

$\tau_{us} = \text{unsaturated shear strength}$

For the following analyses, the proposed prediction equation (Equation 3-11) for the shear strength with respect to soil suction was used as a fitting equation. Measured shear strength data were fitted with the proposed equation (Equation 3-11) using the residual degree of saturation as a fitting parameter. For the fitting procedure, the sum

of squared errors between measured and calculated shear strength values was minimized:

$$SSE = \sum_{i=1}^N \left(\tau_{us} - \left(\psi \left(\frac{S - S_r^*}{1 - S_r^*} \right) \tan \phi' \right) \right)^2 \quad \text{Equation 3-12}$$

where

SSE = sum of squared errors

N = number of data

τ_{us} = measured unsaturated shear strength

S_r^* = residual degree of saturation (used as fitting parameter)

3.3.4.1 Database 1

Soil data sets were used to validate the proposed concepts introduced in Section 3.3.1, Section 3.3.2 and Section 3.3.3. The soil data sets were extracted from literatures such as research papers and conference proceedings. In addition to unsaturated shear strength measurements, the extracted soil data sets were required to provide SWCC measurements and information on the saturated shear strength parameters. All SWCC and unsaturated shear strength data compiled in Database 1 were obtained on the drying path. The experimental shear strength data were obtained by unsaturated triaxial compression tests or by unsaturated direct shear tests. The validation analyses were carried out using the SWCC obtained under the same net normal stress at which the corresponding unsaturated shear strength was measured. For cases where the SWCC measurements were obtained under zero net normal stress, the corresponding unsaturated shear strength measurements were back-calculated to zero net normal stress by assuming a constant effective friction angle as illustrated in Figure 3-5. To avoid any confusion, back-calculated measured shear strength data are referred to as ‘measured shear strength data’ for brevity. The soil data sets used in the validation analyses are summarized in Table 3-1.

Figure 3-5 Back-calculation of measured unsaturated shear strength data

Table 3-1 Database 1

Soil No.	Soil	USCS	Reference
1-01	Decomposed Granite	SM	Hossain and Yin (2010)
1-02	Residual Clay	CH	Kayadelen et al. (2007)
1-03	Edosaki Sand	SM-SC	Gallage and Uchimura (2006)
1-04	Weathered Granite	SM	Lee et al. (2005)
1-05	Weathered Granite	SM	Lee et al. (2005)
1-06	Weathered Granite	SM	Lee et al. (2005)
1-07	Weathered Granite	SM	Lee et al. (2005)
1-08	Clay	CH	Miao et al. (2002)
1-09	Madrid Clay Sand	SG-SM	Escario and Juca (1989)
1-10	Madrid Gray Clay	CH	Escario and Juca (1989)
1-11	Red Silty Clay	CL	Escario and Juca (1989)

3.3.4.2 *Validation of the effective degree of saturation as the controlling parameter for the unsaturated shear strength*

A first analysis was carried out to validate the applicability of the effective degree of saturation as controlling parameter for the unsaturated shear strength according to Equation 3-11. Therefore, the suggested prediction equation (Equation 3-11) was used as a fitting equation (Equation 3-12) to assess the capability of the equation to represent measured shear strength data.

The performance capability was investigated by evaluating the coefficient of determination and the average relative error based on measured and calculated shear strength data. The coefficient of determination between measured and calculated data was determined based on the 1:1 line. The better the calculated data fits the 1:1 line, the closer the coefficient of determination is to one. The coefficient of determination is defined as:

$$R^2 = 1 - \frac{SSE}{SST} \quad \text{Equation 3-13}$$

$$SSE = \sum_{i=1}^N (\tau_{us,i} - \hat{\tau}_{us,i})^2 \quad \text{Equation 3-14}$$

$$SST = \sum_{i=1}^N (\hat{\tau}_{us,i} - \bar{\tau}_{us,i})^2 \quad \text{Equation 3-15}$$

where

R^2 = coefficient of determination

SSE = sum of squared errors

SST = sum of squared totals

τ_{us} = measured unsaturated shear strength

$\hat{\tau}_{us}$ = estimated unsaturated shear strength

$\bar{\tau}_{us}$ = average of the unsaturated estimated shear strength

The average relative error indicates the measure of curve match between measured and calculated data. The smaller the average relative error, the better is the descriptive capability of the equation. The average relative error is defined as:

$$ARE = \frac{1}{N} \sum_{i=1}^N \left| \frac{\tau_{us,i} - \hat{\tau}_{us,i}}{\tau_{us,i}} \right| \quad \text{Equation 3-16}$$

where

ARE = average relative error

Figure 3-6 shows a plot of measured versus calculated (i.e., fitted) shear strength data by employing the proposed prediction equation as fitting equation according to Equation 3-12. Comparison between measured and calculated shear strength data shows a very good agreement for the entire Database 1 (Figure 3-6). This is further confirmed by the high coefficients of determination of 0.97 and 0.95 for coarse- and fine-grained soils, respectively (Figure 3-6).

Figure 3-6 Measured versus calculated shear strength data of Database 1 (calculated according to Equation 3-12)

The average relative errors for the soil data sets in Database 1 are summarized in Table 3-2. The average relative error for the soil data sets varies between 0.02 and 0.13. The average relative error for coarse- and fine-grained soils is 0.10 and 0.08, respectively (Table 3-2).

Table 3-2 Average relative error between measured and calculated shear strength data

Soil No.	Soil	USCS	ARE (-)
1-01	Decomposed Granite	SM	0.11
1-02	Residual Clay	CH	0.05
1-03	Edosaki Sand	SM-SC	0.08
1-04	Weathered Granite	SM	0.09
1-05	Weathered Granite	SM	0.08
1-06	Weathered Granite	SM	0.08
1-07	Weathered Granite	SM	0.13
1-08	Clay	CH	0.02
1-09	Madrid Clay Sand	SG-SM	0.10
1-10	Madrid Gray Clay	CH	0.12
1-11	Red Silty Clay	CL	0.13
Entire Database 1			0.09
Coarse-grained soils of Database 1			0.10
Fine-grained soils of Database 1			0.08

Based on the coefficient of determination and the average relative error, the results of the evaluation indicate the potential that the effective degree of saturation can be successfully used as the controlling parameter for the unsaturated shear strength for both coarse- and fine-grained soils.

Additional plots of the calculated shear strength envelopes (i.e., fitted shear strength according to Equation 3-12) for the individual soil data sets in Database 1 are given in the Appendix B.

3.3.4.3 Validation of proposed residual state concept

A second analysis was carried out to validate the proposed residual state concept and its visual identification on an arithmetic plot of the SWCC as opposed to identification method on a semi-logarithmic plot of the SWCC.

In a first step, the residual state condition was identified visually on an arithmetic plot of the SWCC as proposed in this study. Corresponding to the identified residual state condition, residual degree of saturation and residual soil suction were determined. In a second step, the residual degree of saturation was back-calculated by employing the suggested prediction equation as a fitting type of equation (Equation 3-12). The back-calculated residual degree of saturation obtained through the fitting analysis was then compared with the residual degree of saturation obtained visually on the arithmetic plot of the SWCC.

Table 3-3 compares the visually obtained residual state condition with the one obtained through the back-calculated residual degrees of saturation according to Equation 3-12. For coarse-grained soils, the residual degrees of saturation identified visually on the arithmetic plot of the SWCC are comparable with the back-calculated residual degree of saturation obtained from the fitting analysis (soil no. 1-01 and 1-3 to 1-7 in Table 3-3).

The proposed mathematical criterion of the second derivative of the SWCC (with respect to soil suction) around a value of $1.0\text{E-}10 \text{ kPa}^{-2}$ appears to yield consistent back-calculated residual state conditions for all the coarse-grained soils in Database 1. The results indicate that the actual residual state condition for coarse-grained soils might occur at much higher soil suction values than those conventionally considered in the literature.

Some fine-grained soils show a considerable variation between visually-identified and back-calculated residual state condition. This might be attributed to the fact that electro-chemical forces might play a crucial role in the unsaturated shear strength of fine-grained soils as highlighted in Section 3.3.1.2. The results for the back-calculated residual state condition confirm that for fine-grained soils (soil no. 1-02 and 1-8 to 1-11 in Table 3-3) a residual state condition corresponding to a soil suction of 3,000 kPa as adopted by various researchers (e.g., Kim and Borden, 2011; Vanapalli et al., 1996b) might be a reasonable approximation for the estimation of the unsaturated shear strength.

Table 3-3 Residual state condition for soil data sets in Database 1

Soil No.	Residual state condition				
	Visually-identified on an arithmetic plot of the SWCC		Obtained through fitting		
	$S_r^v (-)$	$\psi_r^v (kPa)$	$S_r^* (-)$	$\psi_r^* (kPa)$	$\frac{\partial^2 S}{\partial \psi^2} (kPa^{-2})$
1-01	0.29-0.27	30,000-40,000	0.31	24,316	1.48E-10
1-02	0.32-0.29	40,000-60,000	0.44	13,150	6.01E-10
1-03	0.16-0.13	10,000-20,000	0.14	15,003	1.65E-10
1-04	0.15-0.12	10,000-20,000	0.13	14,609	1.80E-10
1-05	0.15-0.12	10,000-20,000	0.14	13,843	2.18E-10
1-06	0.14-0.12	10,000-20,000	0.14	12,026	3.09E-10
1-07	0.14-0.11	10,000-20,000	0.15	8,594	6.78E-10
1-08	0.11-0.09	20,000-30,000	0.31	2,395	3.98E-8
1-09	0.12-0.11	30,000-40,000	0.23	4,264	4.19E-9
1-10	0.23-0.19	40,000-60,000	0.60	2,462	1.91E-8
1-11	0.24-0.21	40,000-60,000	0.51	2,598	1.75E-8

^v obtained visually on an arithmetic plot of the SWCC (as proposed in this study); ^{*} obtained through fitting (i.e., Equation 3-12)

Based on the results shown in Table 3-3, residual state condition is visually best identified on an arithmetic plot of the SWCC as proposed in this research. To avoid any subjectivity of a visual identification, the second derivative of the SWCC with respect to soil suction at a value of $1.0E-10$ kPa⁻² might be used as a criterion to determine residual state condition. Plots of the SWCCs of soil Database 1 on a semi-logarithmic and on an arithmetic scale including the visually-identified residual state condition are given in the Appendix B.

3.3.4.4 Empirical correlation for residual state condition

The introduced concepts in Section 3.3.1, Section 3.3.2 and Section 3.3.3 are based on assumptions that are more applicable for coarse-grained soils. To provide a

practical approach to estimate the unsaturated shear strength for a wide range of soils, an empirical criterion for residual state condition to be used in combination with the suggested prediction equation (Equation 3-10) was established. The empirical criterion was developed by back-calculating the residual degrees of saturation according to Equation 3-12. To provide an empirical criterion that is applicable for a wide range of soils, the entire Database 1 consisting of coarse- and fine-grained soils was used in the analysis.

A correlation between the air-entry value of the soil and the soil suction corresponding to the back-calculated residual degrees of saturation was found (Figure 3-7). The tabulated values according to Figure 3-7 are summarized in the Appendix B.

Figure 3-7 Proposed empirical correlation for residual state condition

The following exponential function was used to describe the correlation between air-entry value and soil suction corresponding to the back-calculated residual degree of saturation:

$$\psi_r^e = 3,000 + 25,000^{-0.05 (AEV)} \quad \text{Equation 3-17}$$

where

ψ_r^e = empirical residual soil suction

AEV = air-entry value

The residual degree of saturation corresponding to the empirical criterion (Equation 3-17) is obtained by substituting the soil suction in the SWCC equation (Equation 3-3) with the obtained empirical correlation for the residual soil suction as follows:

$$S_r^e = C(\psi) \frac{1}{\left(\ln \left(e + \left(\frac{3,000 + 25,000^{-0.05 (AEV)}}{a} \right)^n \right) \right)^m} \quad \text{Equation 3-18}$$

where

S_r^e = empirical residual degree of saturation

$C(\psi)$ = correction function

e = Euler's number

a, n and m = Fredlund and Xing (1994) fitting parameters

3.3.5 Proposed prediction equation to estimate the unsaturated shear strength

The proposed prediction equation to estimate the unsaturated shear strength makes use of the effective degree of saturation as the controlling parameter for the contribution of shear strength due to soil suction. The basic form of the equation was first proposed by Vanapalli et al. (1996b) in terms of volumetric water content. However, the interpretation of residual state condition by Vanapalli et al. (1996b) differs greatly from the proposed residual state concept in this research.

The proposed prediction equation estimates the unsaturated shear strength based on the saturated shear strength parameters and the SWCC. Therefore, obtaining a reliable SWCC is crucial as the effective degree of saturation, determined from the SWCC, controls the contribution of shear strength due to soil suction.

The proposed equation to estimate the unsaturated shear strength as developed in this study is:

$$\tau = c' + (\sigma - u_a) \tan \phi' + \psi \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi' \quad \text{Equation 3-19}$$

if $\psi < \psi_r$

$$\tau = c' + (\sigma - u_a) \tan \phi'$$

$$\text{if } \psi \geq \psi_r$$

$$\psi_r^e = 3,000 + 25,000^{-0.05 (AEV)}$$

$$S_r^e = C(\psi) \frac{1}{\left(\ln \left(e + \left(\frac{3,000 + 25,000^{-0.05 (AEV)}}{a} \right)^n \right) \right)^m}$$

where

τ = shear strength

c' = effective cohesion

$(\sigma - u_a)$ = net normal stress

ϕ' = effective friction angle

ψ = soil suction

S = degree of saturation

S_r = residual degree of saturation

ψ_r = residual soil suction

ψ_r^e = proposed empirical residual soil suction

AEV = air-entry value

S_r^e = proposed empirical residual degree of saturation

$C(\psi)$ = correction function

e = Euler's number

a, n and m = Fredlund and Xing (1994) fitting parameters

3.4 UNCERTAINTY IN THE ESTIMATION OF THE UNSATURATED SHEAR STRENGTH

Unsaturated soil property functions are successfully estimated by using the Soil-Water Characteristic Curve (SWCC) together with the saturated soil properties. Therefore, the accuracy of an estimate of an unsaturated soil property function is dependent on the accuracy of the SWCC, the saturated soil properties and the respective prediction equation for the soil property. The following two Section 3.4.1 and Section 3.4.2 provide the basic theories applied in this research for the quantification of the uncertainty in the SWCC model (fitting equation) and for the uncertainty in the prediction equation for the unsaturated shear strength.

3.4.1 Uncertainty in fitting equations

Beside the numerical values obtained through a fitting process, it is desirable to have an assessment of the reliability of the fitted equation representing the actual values. The residual error between actual and fitted values indicates uncertainty in the fitting parameters of the equation used to represent the actual values. Information on the reliability of parameter estimates is contained in the parameter covariance matrix (Kool and Parker, 1988). The covariance matrix can be written in matrix form as:

$$C(x) = \begin{bmatrix} \sigma_{1,1}^2 & \cdots & \sigma_{1,i}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{i,1}^2 & \cdots & \sigma_{i,i}^2 \end{bmatrix} \quad \text{Equation 3-20}$$

where

$C(x)$ = covariance matrix

x = parameter vector

$\sigma_{x,y}^2$ = covariance

The diagonal of the covariance matrix represents the variance of the parameters of the vector and the matrix can be rewritten as:

$$C(x) = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1,i}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{i,1}^2 & \cdots & \sigma_i^2 \end{bmatrix} \quad \text{Equation 3-21}$$

where

$\sigma_x^2 = \text{variance}$

Based on first-order error analysis assuming small parameter perturbations around the mean and neglecting higher order derivatives, an approximation for the covariance matrix is given as (Kool and Parker, 1988):

$$C(x) = \sigma_x^2 (J^T J)^{-1} \quad \text{Equation 3-22}$$

where

$J = \text{parameter sensitivity or Jacobian matrix}$

The Jacobian matrix is defined by the partial derivatives of the equation with regard to its parameters and an unbiased estimate for the variance is given by:

$$s_x^2 = \frac{SSE}{N - k} \quad \text{Equation 3-23}$$

where

$s_x^2 = \text{estimated variance}$

$SSE = \text{sum of squared errors}$

$N = \text{number of data}$

$k = \text{number of parameters}$

Equation 3-22 and Equation 3-23 are from linear regression analysis and are only an approximation for non-linear problems. However, Equation 3-22 and Equation 3-23 yield reasonable approximations for the parameter variance when no parameter constraints are applied and the objective function (SSE) corresponds to a true minimum (Kool and Parker, 1988). Substituting Equation 3-23 into Equation 3-22 results in an approximation for the covariance matrix for non-linear regression problems as:

$$C(x) \approx \frac{SSE}{(N - k)(J^T J)} \quad \text{Equation 3-24}$$

Uncertainties in the Fredlund and Xing (1994) equation were quantified by the approximated covariance of the fitting parameters according to Equation 3-24.

3.4.2 Uncertainty in prediction equations

A common method to quantify the error in prediction equations is the model factor approach which was used by several researchers (e.g., Ang and Tang, 1984; Dithinde et al., 2011; Ronold and Bjerager, 1992). The model factor compensates for errors in the prediction equation due to a lack of knowledge in the modelling of the true state which can be represented as (Ang and Tang, 1984):

$$Y = M \hat{Y} \quad \text{Equation 3-25}$$

where

Y = true state

M = model factor

$\hat{Y}$ = estimated state

A model factor equal to one indicates an unbiased estimate of the true state whereas a value different from one indicates bias in the prediction equation. Therefore, the model factor accounts for the systematic errors in the prediction equation.

Experimental data from research studies are generally obtained under well-controlled laboratory conditions with a high level of expertise. The errors in such experimental data can be assumed to be of much lower magnitude compared to the errors in the model factor. Therefore, it is assumed that the experimental data represent the true state.

$$Y \approx Y_{exp} \quad \text{Equation 3-26}$$

where

Y_{exp} = experimentally obtained state

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Substituting Equation 3-26 into Equation 3-25 results in a ratio for the model factor which can be expressed as:

$$m_i = \frac{y_i}{\hat{y}_i} \quad \text{Equation 3-27}$$

where

m_i = model factor

y_i = experimental value

$\hat{y}_i$ = estimated value

The estimated value is assumed to be obtained from the mean values of the prediction parameters. A number of realizations for the model factor take on a range of values which implies that the model factor can be considered as a random variable following a probability distribution with mean and standard deviation.

A multiplicative model factor in the form of Equation 2-27 was used to quantify the uncertainty in the prediction equations for the unsaturated shear strength.

CHAPTER 4 RESEARCH PROGRAMME

4.1 INTRODUCTION

The research programme is divided into experimental and analytical parts. The experimental programme involved saturated and unsaturated laboratory testing of a suitable soil. In the analytical part, prediction equations for the unsaturated shear strength were first evaluated on their performance. Subsequently, uncertainties associated with a number of selected prediction equations were assessed and quantified. The quantified uncertainties were used in the estimation of the unsaturated shear strength of the soil tested in this study.

4.2 EXPERIMENTAL PROGRAMME

The experimental programme consists of testing a suitable soil to obtain the unsaturated shear strength behaviour. Besides unsaturated shear strength tests, basic soil property tests and Soil-Water Characteristic Curve (SWCC) tests were carried out.

4.2.1 Soil used in the research

The proposed behavioural model for the shear strength with respect to soil suction is more suitable for coarse-grained soils as compared to fine-grained soils as highlighted in Section 3.3.1.2. Furthermore, the selected soil needed to exhibit effects due to soil suction. Therefore, the soil used for the experimental programme was a sand which allowed analyses of the aptness of the proposed behavioural model and concepts introduced in Section 3.3.

4.2.2 Preparation of soil specimens

Static compaction was adopted to obtain homogenous soil specimens with a uniform density. In order to obtain identical specimens, the soil was statically compacted at a pre-selected water content and a target density. The prepared soil was carefully

compacted by hand with the aid of a plunger in the respective test equipment (or mould) to reduce sample disturbance.

The soil specimens were compacted in layers of 1 cm thickness. After compacting a layer, the soil surface was scratched before placing the prepared soil mixture for the next layer inside the test equipment (or mould) to ensure bonding between the layers. The procedure was repeated until the designed height of the soil specimen was achieved.

4.2.3 Testing for basic soil properties

Basic soil property tests were carried out to characterize the soil and to obtain the compaction characteristics. Conventional direct shear tests were conducted to obtain the saturated shear strength parameters. The basic soil property tests were conducted in accordance with the ASTM (2011b) Standards.

4.2.3.1 Soil index properties

Grain-size analysis, specific gravity and Atterberg limit tests were conducted to determine the index properties of the soil. The grain-size distribution was obtained by dry sieving and hydrometer analyses according to ASTM (2011b) D422-63. Specific gravity and Atterberg limit tests were conducted following ASTM (2011b) test method D854-10 and D4318-10, respectively.

4.2.3.2 Compaction characteristics

The compaction characteristics of the soil were obtained by the Standard Proctor compaction test given in ASTM (2011b) D698-07.

4.2.3.3 Saturated shear strength

Conventional direct shear test equipment was used to determine the effective shear strength parameters of the soil. With the soil in a saturated condition, single stage tests under consolidated and drained (CD) conditions were performed following the ASTM (2011b) D3080-04 test method. The failure envelope for the saturated shear strength was obtained by a series of CD tests under different net normal stresses.

4.2.4 Testing for unsaturated soil behaviour

Unsaturated soil testing included SWCC tests and CD unsaturated shear strength tests. Prior to the actual testing, the soil specimens were compacted, saturated and then consolidated under pre-determined net normal stress and soil suction. All the unsaturated soil testing was conducted on the drying path.

4.2.4.1 Axis-translation technique

Measuring negative water pressures in the laboratory has its limitations as water in measuring systems will start to cavitate when water pressures approach -1 atm (-101.3 kPa).

The axis-translation technique is commonly used to circumvent measuring high negative pore-water pressures in the laboratory. This procedure involves a translation of the air pressure, allowing the application of positive water pressures to avoid cavitation in the measuring system. The high-air entry disk (HAED) is the key element for a successful application of the axis-translation technique. The HAED needs to be in a saturated condition in order to function properly. The HAED separates the air from the water phase allowing independent control of water pressure and air pressure. The HAED allows a continuous flow of water but prevents the flow of air up to the designated air-entry value of the HAED.

The axis-translation technique was originally developed by Hilf (1956) and successfully applied in unsaturated soil testing by several researchers (e.g., Escario and Saez, 1986; Gan et al., 1988; Goh et al., 2010; Lee et al., 2005; Vanapalli et al., 1996a). The unsaturated soil testing in this research was conducted by applying the principles of the axis-translation technique. The procedures described in Fredlund and Rahardjo (1993) were adopted for the unsaturated soil testing.

4.2.4.2 Soil-Water Characteristic Curve tests

Soil-Water Characteristic Curve tests were conducted on the drying path under different net confining stresses. Tempe cell, pressure plate apparatus and a modified triaxial cell were used for the measurement of SWCC data.

4.2.4.2.1 Soil-Water Characteristic Curve under zero net confining stress

Soil-Water Characteristic Curve measurements under zero net confining stress were conducted using Tempe cell and pressure plate apparatus. Tempe cell consisting of a HAED with an air-entry value of 100 kPa were used for soil suction values below 100 kPa. For soil suction values above 100 kPa, the soil specimens were transferred into a pressure plate consisting of a HAED with an air-entry value of 500 kPa. Schematic illustration of a Tempe cell and a pressure plate apparatus are shown in Figure 4-1 and Figure 4-2, respectively. Tempe cell and pressure plate operate on the same principle by applying an air pressure to a soil specimen while maintaining a constant water pressure. The SWCC measurements were obtained in accordance with ASTM (2011a) D6836-02.

Figure 4-1 Schematic illustration of a Tempe cell

Figure 4-2 Schematic illustration of a Pressure plate apparatus

4.2.4.2.1.1 Testing procedure

The compacted soil specimen was placed inside the Tempe cell on the previously saturated HAED. The soil specimen was saturated in the Tempe cell from the bottom by applying a small pressure head.

The saturated soil specimens were subjected to a soil suction by pressurizing the Tempe cell with an applied air pressure. The water compartment below the HAED was open to atmospheric condition allowing water to drain out from the soil specimen through the HAED. The soil specimen was periodically weighed to record the mass and to evaluate the water volume in the soil specimen. Once equilibrium under an applied air pressure (corresponding to specific soil suction) was achieved, the air pressure was increased to impose the next soil suction stage.

Before the applied air pressure in the Tempe cell exceeded the air-entry value of the HAED (100 kPa for this research), the soil specimen was transferred to the pressure plate apparatus consisting of a HAED with an air-entry value of 500 kPa for the subsequent soil suction stages. The pressure plate apparatus was operated following the same principles as for the Tempe cell.

Following the last soil suction stage, mass and water content of the soil specimen were measured to back-calculate the final water contents of the soil at each suction stage.

4.2.4.2.2 Soil-Water Characteristic Curve under applied net confining stress

Soil-Water Characteristic Curve measurements under an applied net confining stress were conducted using a modified triaxial cell as illustrated in Figure 4-3. A modified triaxial cell allows the individual application of confining pressure, pore-air pressure and pore-water pressure. The base pedestal is equipped with a HAED with an air entry value of 500 kPa which is placed above a water compartment to control the pore-water pressure. An additional flushing line in the base pedestal allows for the periodic flushing out of diffused air accumulated below the HAED during testing. Pore-air pressure was applied through the top cap placed on the porous stone on top of the specimen. Confining pressure and pore-water pressure were controlled with a

digital volume-pressure controller (DVPC) and additionally measured by pressure transducers.

Figure 4-3 Schematic illustration of a modified triaxial cell

4.2.4.2.2.1 Testing procedure

The compacted soil specimen was placed on top of the previously saturated HAED and properly prepared for testing. The SWCC test in the modified triaxial cell consisted of three stages; (1) saturation, (3) consolidation, and (3) drying of the soil specimen.

Saturation of the soil specimen was achieved by applying a cell pressure and a pore-water pressure. Pore-water pressure to saturate the soil specimen was applied through the top cap. The pore-water pressure parameter was used to indicate the saturation of the soil specimen. The pore-water pressure parameter is defined as the ratio of change in pore-water pressure due to an incremental increase in confining pressure as follows:

$$B = \frac{\Delta u_w}{\Delta \sigma_3} \quad \text{Equation 4-1}$$

where

B = pore-water pressure parameter

Δu_w = change in pore-water pressure

$\Delta \sigma_3$ = change in confining pressure

The pressures were increased in 50 kPa intervals by maintaining a net confining pressure of 10 kPa. The saturation was assumed to be achieved by a pore-pressure parameter value of 0.95 or higher. Total volume change and pore-water volume change of the soil specimen were recorded by the two DVPCs used to maintain the cell pressure and pore-water pressure, respectively.

After the saturation was completed, the soil specimen was isotropically consolidated under a pre-determined net confining stress. Pore-water pressure during consolidation was applied from the bottom through the HAED installed in the base pedestal. Confining pressure and pore-water pressure were controlled by DVPCs and monitored by pressure transducers. Consolidation was assumed to be completed when the change in pore-water volume reached equilibrium.

Drying of the soil specimen was achieved by applying a pre-determined pore-air pressure through the top cap while confining pressure and pore-water pressure were kept constant. The application of pre-determined pore-air pressure and pore-water pressure imposes the designed soil suction on the soil specimen.

Total water volume change and pore-water volume change were measured by DVPC while pressure transducers measured the pore-water pressure and pore-air pressure. Equilibrium under specific soil suction was reached when there was no more change in pore-water volume as indicated by the DVPC. Once equilibrium under a soil suction was reached, pore-air pressure was decreased to impose a new soil suction while confining pressure and pore-water pressure were kept constant.

Following the last soil suction stage, mass and water content of the soil specimen were measured to back-calculate the final water contents of the soil at each soil suction stage.

4.2.4.2.3 *Fitting of Soil-Water Characteristic Curve measurements*

The SWCC measurements obtained through the experimental procedures explained in the previous Sections 4.2.4.2.1 and 4.2.4.2.2 were fitted with the Fredlund and Xing (1994) equation (Equation 3-3) to obtain a continuous SWCC. The continuous

SWCC obtained through the Fredlund and Xing (1994) equation is assumed to represent the SWCC measurements over the entire soil suction range from zero to 10^6 kPa. The respective theory for the SWCC including the fitting process is given in Section 3.2.

4.2.4.3 *Unsaturated shear strength tests*

A back-pressure shear box was used to perform single staged unsaturated shear strength tests under consolidated and drained (CD) conditions. In addition to a conventional saturated direct shear apparatus, the back-pressure shear box allowed control and measurement of pore-air pressure and pore-water pressure to impose a soil suction to the specimen. A schematic illustration of the back-pressure shear box is shown in Figure 4-4.

Figure 4-4 Schematic illustration of a back-pressure shear box

The shear box is placed in a pressure chamber allowing air pressure to be applied to the soil specimen. The lower half of the shear box consists of a water compartment and a HAED with an air-entry value of 500 kPa to control the pore-water pressure in the soil specimen through a DVPC. Pore-air pressure and pore-water pressure were additionally measured by pressure transducers to control the applied soil suction.

4.2.4.3.1 *Testing procedure*

The soil specimens were directly compacted in the shear box on the previously saturated HAED. The possible high soil suction within the soil specimen was relaxed by saturating the soil specimens prior to testing. The coarse porous stone and loading cap were placed on top of the soil specimens and the initial soil suction in the specimen was then relaxed by adding water on top of the specimen, allowing the specimen to saturate.

The saturated soil specimens were consolidated under pre-determined normal stress, air pressure and water pressure. Pressures were applied in the sequence of normal stress, air pressure and water pressure where care was taken so that the normal stress was greater than air pressure, which was larger than the water pressure. The amount of water drained out of the soil specimens and the overall volume change during consolidation was recorded by a DPVC and a linear variable differential transformer (LVDT), respectively. The consolidation was assumed to be completed when there was no more change in water volume and overall volume change as indicated by the DPVC and LVDT, respectively.

Once equilibrium condition was reached under the applied net normal stress and soil suction, the soil specimens were sheared under a constant shearing rate. Single stage direct shear tests were conducted to avoid any ambiguities in later stages.

4.2.4.3.2 *Selection of displacement rate*

For unsaturated shear strength tests under drained conditions, the selected displacement rate needs to be slow enough to ensure no build up of excess pore-water pressure in the soil specimen during shearing. In addition, the pore-water pressure is measured in the water compartment below the HAED. Therefore, a sufficiently slow shearing rate ensures that the pore-water pressure in the water compartment and in the soil specimen is similar (Han, 1997).

The appropriate displacement rate was evaluated by plotting the variation of the shear strength for different displacement rates. After a specific displacement rate, the variation in shear strength remains constant. Any rate equal or slower was considered suitable for unsaturated shear strength tests under drained conditions (Gan, 1986). The highest net normal stress and soil suction resulting in the lowest permeability in

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the soil specimen were selected to evaluate the appropriate displacement rate. Typical displacement rates for unsaturated shear strength tests under CD conditions used in several studies are summarized in Table 4-1.

Table 4-1 Displacement rates used in several studies for unsaturated direct shear tests

Soil	Direct shear test	Displacement rate (mm/min)	Reference
Madrid Gray Clay	CD	0.00168	Escario and Saez (1986)
Red Clay	CD	0.00168	Escario and Saez (1986)
Madrid Clayey Sand	CD	0.00168	Escario and Saez (1986)
Glacial Till	CD	0.0102	Gan (1986)
Glacial Till	CD	0.001875	Vanapalli et al. (1996a)
Residual Soil	CD	0.004	Melinda et al. (2004)
Expansive Clay	CD	0.0019	Zhan and Ng (2006)
Ankara Clay	CD	0.006	Cokca and Tilgen (2010)
Decomposed Granite	CD	0.004	Hossain and Yin (2010)

4.2.4.3.3 *Extended Mohr-Coulomb failure envelope*

The extended Mohr-Coulomb failure criterion was adopted to represent the failure envelopes of the unsaturated shear strength measurements. Failure was considered at the peak value where the shear stress peaked or started to remain fairly constant as observed from the experimental measurements. For tests where the shear stress versus horizontal displacement curve did not reach a peak or a constant value, failure was identified where the shear stress started to have an approximately constant slope.

4.2.5 **Programme for unsaturated soil testing**

The programme for the unsaturated soil testing was designed to obtain the shear strength behaviour and the SWCC behaviour of a coarse-grained soil. Soil-Water Characteristic Curve measurements and unsaturated shear strength measurements were carried out under various net confining stresses and soil suctions.

4.2.5.1 *Testing programme for the Soil-Water Characteristic Curves*

Soil-Water Characteristic Curve measurements were obtained under different net confining stresses. Tempe cell and pressure plate apparatus were used to obtain SWCC measurements under zero net normal. For SWCC measurements under net confining stress, a modified triaxial cell was used. Measurements were carried out for soil suctions up to 500 kPa. The testing programme for the SWCC measurements is presented in Table 4-2.

Table 4-2 Testing programme for the Soil-Water Characteristic Curves

Net normal stress ($\sigma - u_a$) (kPa)	Matric suction ($u_a - u_w$) (kPa)	Equipment
0	0-500	Tempe cell and Pressure plate
100	0-500	Modified triaxial cell
200	0-500	Modified triaxial cell
300	0-500	Modified triaxial cell

4.2.5.2 *Testing programme for the unsaturated shear strength*

Unsaturated direct shear tests were conducted under different net normal stresses and soil suctions. Tests were carried out under four different net normal stresses and five different soil suction values. Therefore, a total of 20 unsaturated shear strength tests according to Table 4-3 were conducted to study the unsaturated shear strength behaviour of the selected soil.

Table 4-3 Testing programme for the unsaturated shear strength

Net normal stress ($\sigma - u_a$) (kPa)	Matric suction ($u_a - u_w$) (kPa)				
50	20	50	100	200	450
100	20	50	100	200	450
200	20	50	100	200	450
300	20	50	100	200	450

4.3 ANALYTICAL PROGRAMME

In the first part of the analytical programme, the proposed prediction equation to estimate the unsaturated shear strength as developed in Section 3.3 is analysed on its performance in comparison to existing prediction equations. In the second part, uncertainties in the estimation of the unsaturated shear strength arising from the Soil-Water Characteristic Curve (SWCC) and from the prediction equation for the unsaturated shear strength were quantified. The analyses on the performance of the prediction equations and the quantification of the uncertainty were carried out on independent soil data sets compiled from the literature.

4.3.1 Prediction equations used in the performance analysis

Existing prediction equations for the unsaturated shear strength (as described in Section 2.5.1) and the proposed equation from this research (as developed Section 3.3) were analysed. The shear strength equations assessed in the performance analysis were all capable of estimating the unsaturated shear strength based on information of the SWCC together with the saturated shear strength parameters. Only the contribution of the shear strength due to soil suction in the respective prediction equations was considered in the analyses (Table 4-4). Definitions of symbols and information on usage of the prediction equations were elaborated in Section 2.5.1 and in Section 3.3.5 for the existing equations and the proposed equation, respectively.

Table 4-4 Prediction equations analysed on their performance

Author	Prediction equation	Soil	Equation
Lamborn (1986)	$\tau_{us} = \psi \theta \tan \phi'$	n.a.	4-2
Vanapalli et al. (1996b)	$\tau_{us} = \psi \theta^{\kappa} \tan \phi'$	n.a.	4-3
Vanapalli and Fredlund (2000)	$\kappa = -0.0080 (PI)^2 + 0.0801 (PI) + 1$	Fine-grained soils	2-14
Garven and Vanapalli (2006)	$\kappa = -0.0016(PI)^2 + 0.0975(PI) + 1$	Fine-grained soils	2-15

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Table 4-4 Continued

Vanapalli et al. (1996b)	$\tau_{us} = \psi \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right) \tan \phi'$	n.a.	4-4
Oberg and Sallfors (1997)	$\tau_{us} = \psi S \tan \phi'$	Sands, silts and non- clayey soils	4-5
Bao et al. (1998)	$\tau_{us} = \psi \left(\frac{\log(\psi_r) - \log(\psi)}{\log(\psi_r) - \log(AEV)} \right) \tan \phi'$	Fine-grained soils	4-6
Khalili and Khabbaz (1998)	$\tau_{us} = \psi \chi \tan \phi'$ $\chi = \left(\frac{\psi}{AEV} \right)^{-0.55}$	Clayey soils	4-7
Aubeny and Lytton (2003)	$\tau_{us} = \psi f_1 \theta \tan \phi'$ $f_1 = \left(\frac{1}{\theta} \right)$ <i>if</i> $S = 100\%$ $f_1 = 1 + \left(\frac{S - 85}{15} \right) \left(\frac{1}{\theta} - 1 \right)$ <i>if</i> $85\% \leq S \leq 100\%$ $f_1 = 1$ <i>if</i> $S \leq 85\%$	Fine-grained soils	4-8
Tekinsoy et al. (2004)	$\tau_{us} = (AEV + P_{at}) \ln \left(\frac{\psi + P_{at}}{P_{at}} \right) \tan \phi'$	Fine-grained soils	4-9
Sheng et al. (2008)	$\tau_{us} = \psi \tan \phi^b$ $\tan \phi^b = \tan \phi'$ <i>if</i> $\psi < \psi_{sa}$ $\tan \phi^b = \tan \phi' \left(\left(\frac{\psi_{sa}}{\psi} \right) + \left(\frac{\psi_{sa} + 1}{\psi} \right) \ln \left(\frac{\psi + 1}{\psi_{sa} + 1} \right) \right)$ <i>if</i> $\psi > \psi_{sa}$	n.a.	4-10

Table 4-4 Continued

Goh et al. (2010)	$\tau_{us} = \psi \tan \phi'$	Coarse-and fine-grained soils	4-11
	<i>if</i> $\psi < AEV$		
	$\tau_{us} = (AEV) \tan \phi' + (\psi - AEV)b \theta^\kappa \tan \phi'$		
	<i>if</i> $\psi \geq AEV$		
	$\kappa = (\log(\psi) - \log(AEV))^y$		
Proposed from this study	$\tau_{us} = \psi \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi'$	Coarse- and fine-grained soils	4-12
	<i>if</i> $\psi < \psi_r$		
	$\tau_{us} = 0$		
	<i>if</i> $\psi \geq \psi_r$		
	$\psi_r^e = 3,000 + 25,000^{-0.05 (AEV)}$		
	$S_r^e = C(\psi) \frac{1}{\left(\ln \left(e + \left(\frac{3,000 + 25,000^{-0.05 (AEV)}}{a} \right)^n \right) \right)^m}$		

In order to assess all the prediction equations consistently over the soil suction range where measured shear strength data were available, reasonable assumptions had to be made for some of the equations, which included:

- Linear increase in shear strength with respect to soil suction up to the air-entry value of the soil.

$$\tau_{us} = \psi \tan \phi'$$

$$\textit{if } \psi \leq AEV$$

Equation 4-13

where

τ_{us} = unsaturated shear strength

ψ = soil suction

ϕ' = effective friction angle

$AEV = \text{air-entry value}$

This assumption was necessary for the equations proposed by Bao et al. (1998) (Equation 4-6) and Khalili and Khabbaz (1998) (Equation 4-7) as these equations are not applicable for soil suctions lower than the air-entry value.

- Zero increase in shear strength with respect to soil suction if the calculated shear strength value becomes negative.

$$\tau_{us} = 0$$

Equation 4-14

$$\text{if } \tau_{us} < 0$$

This assumption was necessary for the equations proposed by Vanapalli et al. (1996b) (Equation 4-4) and Bao et al. (1998) (Equation 4-6) as the calculated shear strength values would otherwise become negative beyond residual state condition.

4.3.2 Database 2

Soil data sets were used to assess the performance of the prediction equations for the unsaturated shear strength and to quantify the uncertainties in the estimation of the unsaturated shear strength. The soil data sets in Database 2 were required to provide SWCC measurements and information on the saturated shear strength parameters as these information were needed to estimate the unsaturated shear strength according to the analysed equations. In addition, unsaturated shear strength measurements to compare and quantify the uncertainties in the estimated shear strength values were required for these data sets. Soil data were extracted from various sources such as research papers, conference proceedings and theses. All SWCC and unsaturated shear strength data compiled in Database 2 were obtained on the drying path. The experimental shear strength data were obtained by unsaturated triaxial compression tests or by unsaturated direct shear tests. The SWCC data in Database 2 were all obtained under zero net normal stress. Therefore, the corresponding measured unsaturated shear strength data were back-calculated to zero net normal stress plane by assuming the effective friction angle to be constant as illustrated in Figure 3-5. The back-calculated shear strength data based on experimentally-obtained

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measurements are referred to as ‘measured shear strength data’ for brevity. Table 4-5 summarizes the soil data sets used in Database 2.

Table 4-5 Database 2

Soil No.	Soil	USCS	Reference
2-01	Sand-Kaolin Mixture SK5	MH	Goh (2012)
2-02	Sand-Kaolin Mixture SK10	CL	Goh (2012)
2-03	Sand-Kaolin Mixture SK17	CL	Goh (2012)
2-04	Residual Soil	CL	Rahardjo et al. (2004)
2-05	Mixed Decomposed Granite	SW-SM	Zhao et al. (2013)
2-06	Indian Head Till (at optimum)	CL	Vanapalli et al. (1996b)
2-07	Indian Head Till (dry of optimum)	CL	Vanapalli et al. (1996b)
2-08	Indian Head Till (wet of optimum)	CL	Vanapalli et al. (1996b)
2-09	SJ10a	CL	Khalili et al. (2004)
2-10	SJ10b	CL	Khalili et al. (2004)
2-11	SJ11	CL	Khalili et al. (2004)
2-12	Tailings - 50 m	n.a.	Rassam and Williams (1999)
2-13	Tailings - 150 m	n.a.	Rassam and Williams (1999)
2-14	Silty Clay (recompacted)	n.a.	Zhan and Ng (2006)
2-15	Silty Clay (natural)	n.a.	Zhan and Ng (2006)

4.3.3 Performance of prediction equations for the unsaturated shear strength

Analysis and comparison between the measured shear strength from the literature and the estimated shear strength of the various prediction equations were made to assess the applicability of the prediction equations on independent soil data sets. The performance of the proposed prediction equation from this research in comparison to the existing prediction equations was of special interest.

The prediction equations summarized in Table 4-4 were reported to be applicable for either both coarse- and fine-grained soils, only for fine-grained soils or the information on the applicability was not specified. Database 2 consists mainly of fine-grained soils. Therefore, all the prediction equations for the unsaturated shear strength summarized in Table 4-4 were analysed using the entire Database 2 to assess their performance and to verify the applicability of the respective prediction equations.

Statistical analyses were performed to assess the predictive capability of the equations (Table 4-4). The coefficient of determination and the average relative error between experimental and estimated shear strength values were used as performance criteria as introduced previously in Section 3.3.4.2.

4.3.4 Uncertainties in the estimation of the unsaturated shear strength

The estimation of the unsaturated shear strength involves a considerable degree of uncertainties. These uncertainties can be broadly distinguished in the two main sources (1) model uncertainty and (2) soil parameter uncertainty. Model uncertainty arises from idealization and simplification made in any analytical model representing a physical process and is unavoidable. Soil parameter uncertainty arises from the inherent variability of the soil property itself and from systematic errors introduced by its evaluation.

This research focuses on the quantification of the uncertainty in the analytical models involved in the estimation of the unsaturated shear strength. This are namely the SWCC equation and the prediction equation for the unsaturated shear strength.

4.3.4.1 *Uncertainty in the Soil-Water Characteristic Curve equation*

The Fredlund and Xing (1994) equation was applied throughout this research to fit a continuous function to measured SWCC data. The continuous fit represents the measured SWCC data over the entire soil suction range from zero to 10^6 kPa.

The Fredlund and Xing (1994) fitting parameters were obtained by minimizing the sum of squared errors between experimental and calculated values as described in Section 3.2.2. The fitted values of the fitting parameters represent their average value. The residual error of the fitting process indicates uncertainty in the fitting parameters of the SWCC equation used to represent the measured data. The variances of the

individual fitting parameters were estimated by a first-order error approximation for the covariance matrix as described in the Theory Section 3.4.1 as:

$$C(x) \approx \frac{SSE}{(N-3) \sum_{i=1}^N \left(\left(\frac{\partial S_i}{\partial x} \right)^T \left(\frac{\partial S_i}{\partial x} \right) \right)} \quad \text{Equation 4-15}$$

where

$C(x)$ = covariance matrix

x = parameter vector $[a, n, m]$

SSE = sum of squared errors

N = number of data

S = degree of saturation

The partial derivatives of the Fredlund and Xing (1994) equation with regard to its fitting parameters are given in Appendix C.

The first-order error approximation of the covariance matrix of the SWCC fitting parameters was assumed to represent uncertainty due to the inability of the SWCC equation to accurately represent measured SWCC data.

The approximated covariance matrix obtained by Equation 4-15 results in an estimate of the variance and covariance of the respective parameters as follows:

$$C(x) \approx \begin{bmatrix} s_a^2 & s_{a,n}^2 & s_{a,m}^2 \\ s_{n,a}^2 & s_n^2 & s_{n,m}^2 \\ s_{m,a}^2 & s_{m,n}^2 & s_m^2 \end{bmatrix} \quad \text{Equation 4-16}$$

where

s_x^2 = estimated variance

$s_{x,y}^2$ = estimated covariance

Non-zero estimates for the covariance between the fitting parameters ($s_{a,n}^2$, $s_{a,m}^2$ and $s_{n,m}^2$) indicates interdependence between the respective Fredlund and Xing (1994) fitting parameters. The covariance matrix for the Fredlund and Xing (1994) fitting

parameters according to Equation 4-15 was calculated for the soil data sets in Database 2.

First-order error analysis was adopted to evaluate the propagation of the uncertainty in the fitting parameters through the Fredlund and Xing (1994) equation. The standard deviation in the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) equation without considering dependencies between the fitting parameters was estimated as:

$$s_S^{SWCC} = \sqrt{\sum_{i=1}^k \left(\frac{\partial S}{\partial x_i} s_{x_i} \right)^2} \quad \text{Equation 4-17}$$

where

s_S^{SWCC} = estimated standard deviation in the degree of saturation due to model uncertainty in the SWCC equation

k = number of parameters

s_x = estimated standard deviation in the fitting parameter

The standard deviation in the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) equation considering dependencies between the fitting parameters was estimated as:

$$s_S^{SWCC} \leq \sum_{i=1}^k \left| \frac{\partial S}{\partial x_i} s_{x_i} \right| \quad \text{Equation 4-18}$$

The partial derivatives for the Fredlund and Xing (1994) equation with regard to its fitting parameters are given in Appendix C.

4.3.4.2 Uncertainty in prediction equations for the unsaturated shear strength

Besides the proposed prediction equation for the estimation of the unsaturated shear strength, two well-known and frequently adopted prediction equations for the unsaturated shear strength were assessed on their uncertainties. The equations proposed by Oberg and Sallfors (1997), Khalili and Khabbaz (1998) and Garven and

Vanapalli (2006) are often referred to and applied in many research works. In addition, these equations were analysed in the limited research works on independent soil data sets. Unlike the equations proposed by Oberg and Sallfors (1997), and the prediction equation proposed in this research, the equation proposed by Khalili and Khabbaz (1998) uses only one parameter of the SWCC whereas the other equations utilize the entire SWCC in the estimation of the unsaturated shear strength. Therefore, the equations proposed by Oberg and Sallfors (1997) and Garven and Vanapalli (2006) were further assessed on their model uncertainties in addition to the prediction equation proposed in this research as summarized in Table 4-6. These equations were reported to be applicable for fine- and coarse grained soils.

Table 4-6 Prediction equations analysed on uncertainty

Author	Prediction equation	Soil	Equation
Oberg and Sallfors (1997)	$\tau_{us} = \psi S \tan \phi'$	Sands, silts and non-clayey soils	4-5
Garven and Vanapalli (2006)	$\tau_{us} = \psi \theta^{\kappa} \tan \phi'$	n.a.	4-3
	$\kappa = -0.0016(PI)^2 + 0.0975(PI) + 1$	Fine-grained soils	2-15
Proposed from this research	$\tau_{us} = \psi \left(\frac{S - S_r}{1 - S_r} \right) \tan \phi'$ $\text{if } \psi < \psi_r$ $\tau_{us} = 0$ $\text{if } \psi \geq \psi_r$ $\psi_r^e = 3,000 + 25,000^{-0.05 (AEV)}$ $S_r^e = C(\psi) \frac{1}{\left(\ln \left(e + \left(\frac{3,000 + 25,000^{-0.05 (AEV)}}{a} \right)^n \right) \right)^m}$	Coarse-and fine-grained soils	4-12

The uncertainty in the selected prediction equations was assessed by a model factor as introduced in the Section 3.4.2. The model factor between measured and estimated shear strength values for a number of realizations was evaluated as:

$$m_{\tau_{us}} = \frac{1}{N} \sum_{i=1}^N \frac{\tau_{us,i}}{\hat{\tau}_{us,i}} \quad \text{Equation 4-19}$$

where

$m_{\tau_{us}}$ = model factor for the unsaturated shear strength prediction equation

τ_{us} = measured unsaturated shear strength

$\hat{\tau}_{us}$ = estimated unsaturated shear strength

The corresponding standard deviation of the model factor was calculated as:

$$s_{m_{\tau_{us}}} = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (m_{\tau_{us},i} - m_{\tau_{us}})^2} \quad \text{Equation 4-20}$$

where

$s_{m_{\tau_{us}}}$ = standard deviation of the model factor for the unsaturated shear strength prediction equation

Database 2 consisting of 15 soils with a total of 42 measured shear strength data was used to determine the statistics of the model factor of the selected prediction equations for the unsaturated shear strength. A sample size of 42 data was considered sufficiently large to draw statistical inference of the model factor of the selected prediction equations for the unsaturated shear strength.

A number of steps were carried out to establish the respective statistics of the model factor. These steps included detection of data outliers, exploratory data analysis and the verification of randomness of the model factor. Outliers were first identified and excluded from the sample size. The exploratory data analysis was carried out on the sample size corrected for outliers and included descriptive statistics of the model factor and the identification of an appropriate probability distribution.

4.3.4.2.1 Detection of data outliers

Outliers are observations which deviate markedly from the main body or a random sample of a population and may greatly influence the statistics resulting in biased

conclusions. The soil data sets on which the uncertainties in this study were quantified were compiled from published research studies such as research papers, conference proceedings and theses. The information usually given in such studies are often limited and it is not possible to analyse such compiled data as detailed as it would be necessary to conclude on possible abnormalities to justify the data being termed as erroneous and excluded from the sample size. Therefore, the data were analysed by the box plot method and data which fell outside the interval of 1.5 multiplied with inter quartile range of the upper and lower quartile were identified as outliers and excluded.

$$Q_1 - 1.5(IQR) > outliers > Q_3 + 1.5(IQR) \quad \text{Equation 4-21}$$

where

$IQR = \text{inter-quartile range};$

$Q_1 = 25^{\text{th}} \text{ percentile};$

$Q_3 = 75^{\text{th}} \text{ percentile}$

$$IQR = Q_3 - Q_1 \quad \text{Equation 4-22}$$

4.3.4.2.2 *Exploratory data analysis*

The exploratory data analysis was based on the sample size corrected for outliers identified by the box plot method. The range of data and sample moments (i.e., mean, standard deviation, median, skewness and kurtosis) were evaluated and an appropriate distribution type was identified. The normal and the log-normal distributions often offer an adequate representation for engineering applications with limited data and were therefore considered first as possible distribution types.

The Shapiro-Wilk and the Kolmogorov-Smirnov test were used to test for normality or log normality of the model factor. Additionally, the normality plot was applied to further assess the correctness of the distribution. A nearly straight line in the normality plot suggests that the data are normally distributed. Log-normality was tested on the principle that if a random variable is log-normally distributed, its natural logarithm is normally distributed. To test for log-normality, the Shapiro-Wilk test

together with the normality plot was performed on the natural logarithm of the model factor. The null hypothesis of normality or log-normality for the model factor was rejected if the p-value of the Shapiro-Wilk test was less than 0.05.

4.3.4.2.3 *Verification of randomness of the model factor*

The model factor is considered as a random variable which is an appropriate assumption when the variation of the model factor is independent from variations in the database. The randomness of the model factor was assessed by the Pearson product-moment correlation coefficient. The correlation between the model factor and the basic input parameters was evaluated by:

$$r_s = \frac{\sum_{i=1}^N (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^N (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^N (Y_i - \bar{Y})^2}} \quad \text{Equation 4-23}$$

where

r_s = Pearson product-moment correlation coefficient

X_i = value of parameter X

$\bar{X}$ = mean of parameter X

Y_i = value of parameter Y

$\bar{Y}$ = mean of parameter Y

The Pearson product-moment correlation varies between minus one to plus one. The closer the value of the correlation coefficient is to zero the weaker is the correlation between the respective variables. Significance tests for the null hypothesis of no correlation were performed to assess possible correlation between the model factor and the various input parameters of the respective prediction equations.

4.3.5 **Application of the uncertainties in the estimation of the unsaturated shear strength**

In order to provide a comprehensive uncertainty analysis for the estimation of the unsaturated shear strength, uncertainty in the soil properties due to inherent soil variability and model uncertainties were considered. To demonstrate the propagation

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of the uncertainties in the estimated unsaturated shear strength, the soil experimentally tested in this research was used in the analyses. The model uncertainties quantified in this research (i.e., uncertainties in the SWCC equation and uncertainties in the respective prediction equation for the unsaturated shear strength) and typical values for the soil variability as reported in the literature were adopted in the analyses.

The prediction equations for the unsaturated shear strength analysed on their uncertainty in this research (i.e., Oberg and Sallfors, 1997; Garven and Vanapalli, 2006; and the proposed equation) can be written in a general form as:

$$\tau_{us} = m_{\tau_{us}} \psi f(SWCC, x) \tan(\phi') \quad \text{Equation 4-24}$$

where

τ_{us} = unsaturated shear strength

$m_{\tau_{us}}$ = model factor for the unsaturated shear strength prediction equation

ψ = soil suction

$SWCC$ = Soil-Water Characteristic Curve

x = additional parameter(s)

ϕ' = effective friction angle

The unsaturated shear strength is determined for a selected soil suction. The proposed prediction equation and the prediction equation proposed by Garven and Vanapalli, (2006) use an empirical parameter in the estimation of the unsaturated shear strength. However, the uncertainty of the empirical parameter is already accounted for in the model factor.

Uncertainty in the degree of saturation arises due to uncertainty in the SWCC equation and due to inherent soil variability. The total uncertainty in the degree of saturation was calculated as:

$$s_s = \sqrt{(s_s^{SWCC})^2 + (s_s^P)^2} \quad \text{Equation 4-25}$$

where

s_s = estimated standard deviation in the degree of saturation

s_s^{SWCC} = estimated standard deviation in the degree of saturation due to model uncertainty in the SWCC model

s_s^P = estimated standard deviation in the degree of saturation due to inherent soil variability

Uncertainties in soil properties are reviewed in Section 2.6.2 and typical values in terms of the coefficient of variation are given in Appendix A.

First-order error analysis and Monte-Carlo simulations were carried out to evaluate the propagation of the uncertainties in the estimation of the unsaturated shear strength by considering the input parameters as random variables.

The standard deviation in the estimation of the unsaturated shear strength following the first-order error analysis was evaluated as:

$$s_{\tau_{us}} = \sqrt{\sum_{i=1}^k \left(\frac{\partial \tau_{us}}{\partial x_i} s_{x_i} \right)^2} \quad \text{Equation 4-26}$$

where

$s_{\tau_{us}}$ = estimated standard deviation in the unsaturated shear strength estimation

s_x = estimated standard deviation in the respective parameter

The partial derivatives of the prediction equations for the unsaturated shear strength analysed on their uncertainty are given in Appendix C.

Monte-Carlo simulations require a probability distribution for the input parameters. Studies indicate that the degree of saturation and the effective friction angle are normally distributed (Appendix A). Therefore, the normal distribution was adopted for the degree saturation and the effective friction angle in the Monte-Carlo simulations. The probability distribution for the model factor of the selected prediction equations for the unsaturated shear strength was found to be normally distributed (Section 5.3.2.2.2). The number of trials for the Monte-Carlo simulations

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was assumed to be sufficiently large when the mean value and the standard deviation of the output (i.e., estimated shear strength) started to become constant with an increasing number of trials.

CHAPTER 5 PRESENTATION OF RESULTS

5.1 INTRODUCTION

This Chapter presents the experimental and analytical results of the research programme. The results of the experimental programme include basic soil property tests, Soil-Water Characteristic Curve (SWCC) tests and unsaturated shear strength tests. Subsequently, the results of the analytical programme are presented. The results of the performance analysis of the various prediction equations for the unsaturated shear strength are presented. Following, the results of the quantification of the uncertainties in the SWCC and in selected prediction equations for the unsaturated shear strength are presented. The uncertainties involved in the estimation of the unsaturated shear strength are propagated on the soil experimentally tested in this research. These results are presented at the end of this Chapter.

5.2 EXPERIMENTAL RESULTS

This Section presents the results of the experimental programme. The results of the basic soil properties test such as the index properties, compaction characteristics and the saturated direct shear test results are presented first. Subsequently, the results of the unsaturated soil testing which involved SWCC tests and unsaturated direct shear tests are presented.

5.2.1 Soil used in the research

The soil used in this research is a sand obtained from a gravel pit in Eschenbach SG, Switzerland. The sand, from here on referred to as ‘Sand vom Stücken’, was previously used in studies at the Hochschule für Technik Rapperswil (HSR), Switzerland (Schnellmann et al., 2010).

The Sand vom Stücken is a coarse-grained sand that shows effects due to soil suction and is therefore suitable for the purposes of this research. Prior to testing, the soil was oven-dried and then sieved to remove soil particles greater than 4 mm.

5.2.2 Testing for basic soil properties

5.2.2.1 Soil index properties

The index properties of the Sand vom Stücken are presented in Table 5-1. Figure 5-1 shows the grain-size distribution which was obtained by dry sieving and hydrometer tests. The soil comprises 90 % sand and 10 % of non-plastic fines from which 8 % are silt and 2 % are clay. The specific gravity of the sand is 2.72. The soil is classified as a well graded sand with silt (SW-SM), according to the Unified Soil Classification System (USCS).

Table 5-1 Soil index properties of the Sand vom Stücken

Soil index properties	Sand vom Stücken		
Specific gravity of soil solids	G_s	(-)	2.72
Plasticity Index	PI	(-)	NP
Grain-size distribution	D_{60}	(mm)	0.47
	D_{30}	(mm)	0.21
	D_{10}	(mm)	0.07
Coefficient of uniformity	C_u	(-)	6.57
Coefficient of curvature	C_c	(-)	1.38
Gravel content (> 4.75 mm)		(%)	-
Sand content		(%)	90
Fines content (< 0.075 mm)		(%)	10
Unified Soil Classification System	$USCS$	(-)	SW-SM

Figure 5-1 Grain size distribution of the Sand vom Stücken

5.2.2.2 *Compaction characteristic*

The compaction characteristic of the soil was obtained by the Standard Proctor compaction test as shown in Figure 5-2. The maximum dry density and the optimum water content of the Sand vom Stücken were according to the Standard Proctor compaction test determined as 1.88 Mg/m³ and 10.7 %, respectively.

Figure 5-2 Standard Proctor compaction curve of the Sand vom Stücken

Previous studies carried out at HSR on the Sand vom Stücken indicated that the sand might exhibit a saturation collapse under certain conditions. However, tests on the collapse potential showed that for an initial void ratio of 0.65 ($\rho_d = 1.65 \text{ Mg/m}^3$) and a water content of 8 %, there is no saturation collapse for vertical stresses of 200 kPa and 800 kPa. Therefore, all the specimens were statically compacted at a water content of 8 % to a dry density of 1.65 Mg/m^3 .

5.2.2.3 Saturated shear strength

Saturated direct shear tests under consolidated and drained (CD) conditions were conducted in a conventional direct shear apparatus to obtain the effective shear strength parameters. The soil was tested under three different net normal stresses to obtain the saturated shear strength parameters at the peak value. A shearing rate of 0.0015 mm/min was found to be appropriate for the unsaturated shear strength testing (Section 5.2.3.2.1). It was decided to use the same shearing rate of 0.0015 mm/min for the saturated shear strength tests. The saturated CD direct shear test results at the peak value are presented in Figure 5-3 and summarized in Table 5-2 (as presented in Schnellmann et al., 2013). For the tested effective stress range, the effective friction angle and the effective cohesion of the soil were determined as 33.4° and 0 kPa, respectively.

Figure 5-3 Results of the CD saturated direct shear tests of the Sand vom Stücken

Table 5-2 Peak shear stress and horizontal displacement at failure for the Sand vom Stücken obtained from saturated CD direct shear tests

Test	Effective stress $(\sigma - u_w), (kPa)$	Shear stress $\tau, (kPa)$	Horizontal displacement at failure $d_f, (mm)$
1	53	37	2.40
2	176	112	6.10
3	360	239	7.00

5.2.3 Testing for unsaturated soil behaviour

5.2.3.1 Soil-Water Characteristic Curve tests

Soil-Water Characteristic Curve tests were conducted on the drying path to obtain SWCC measurements of the soil under zero net normal stress and applied net confining stress as described in Section 4.2.4.2. The soil was tested under different net confining stresses using Tempe cell, pressure plate apparatus and a modified triaxial cell. The SWCC measurements were fitted using the Fredlund and Xing (1994) equation as described in Section 3.2.2. The SWCCs are denoted as ‘SWCC-x’, where ‘x’ indicates the stress state under which the SWCC was obtained.

Figure 5-4 to Figure 5-7 shows the SWCCs for the recompacted soil specimen of the Sand vom Stücken obtained under net confining stresses of 0 kPa, 100 kPa, 200 kPa and 300 kPa. The SWCCs are illustrated on a semi-logarithmic and on an arithmetic plot. The Fredlund and Xing (1994) fitting parameters obtained through the fitting process are summarized in Table 5-3.

It can be observed that the sand starts to desaturate significantly when the soil suction exceeds the air-entry value. The air-entry values for the SWCCs under stress states of 0 kPa, 100 kPa, 200 kPa and 300 kPa were evaluated as 1.0 kPa, 1.0 kPa, 1.2 kPa and 1.2 kPa, respectively.

Figure 5-4 Soil-Water Characteristic Curve of the Sand vom Stücken under zero net normal stress

Figure 5-5 Soil-Water Characteristic Curve of the Sand vom Stücken under 100 kPa net confining stress

Figure 5-6 Soil-Water Characteristic Curve of the Sand vom Stücken under 200 kPa net confining stress

Figure 5-7 Soil-Water Characteristic Curve of the Sand vom Stücken under 300 kPa net confining stress

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Table 5-3 Fredlund and Xing (1994) fitting parameters and empirical constant for the Sand vom Stücken

Net confining stress ($\sigma_3 - u_a$), (kPa)	Fredlund and Xing (1994) fitting parameters			Constant
	a	n	m	C_r
0	1.461	6.654	0.421	1,500
100	1.451	6.734	0.413	1,500
200	1.641	7.772	0.360	1,500
300	1.597	12.492	0.337	1,500

The residual state condition for the Sand vom Stücken was identified visually on an arithmetic plot of the SWCCs as proposed in this research. Figure 5-8 shows the identified residual state conditions based on the fitted SWCCs. Table 5-4 summarises the residual state conditions including the air-entry values for the SWCCs. The degrees of saturation at a specific soil suction vary minimally for SWCCs obtained under different net confining stresses as demonstrated in Figure 5-8 on an arithmetic and semi-logarithmic plot.

Figure 5-8 Identified residual state condition for the Sand vom Stücken

Table 5-4 Air-entry value and residual state condition for the Sand vom Stücken

SWCC	Air-entry value	Residual state condition	
		Visually-identified on an arithmetic plot of the SWCC	
		AEV (kPa)	S_r (–) ψ_r (kPa)
SWCC-0	1.0		0.12-0.10 10,000-20,000
SWCC-100	1.0		0.13-0.11 10,000-20,000
SWCC-200	1.2		0.15-0.13 10,000-20,000
SWCC-300	1.2		0.14-0.12 10,000-20,000

5.2.3.2 *Unsaturated shear strength*

Single staged unsaturated direct shear tests under consolidated and drained conditions (CD) were conducted on statically compacted soil specimen using a back-pressure shear box as described in the Section 4.2.4.3. The testing programme was designed to obtain the shear strength of the soil specimens under different net normal stresses and soil suctions. The tests were conducted under pre-determined net normal stresses and soil suctions on a drying path following the testing programme presented in Section 4.2.5.2. A name convention using ‘CD x-y’ was adopted to indicate the soil specimens. Whereas ‘CD’ indicates that the shear strength was obtained under consolidated and drained conditions, ‘x’ and ‘z’ stands for the net normal stress and soil suction under which the specimens were tested.

5.2.3.2.1 *Selection of displacement rate*

Tests at high net normal stress and soil suction were first carried out under varying displacement rates to evaluate the appropriate displacement rate for use in the unsaturated direct shear testing as described in Section 4.2.4.3.2. The results of the unsaturated shearing behaviour using different displacement rates are shown in Figure 5-9. The variation in peak shear strength for different displacement rates was found to be small. Therefore, a displacement rate of 0.0015 mm/min was found appropriate for the unsaturated direct shear testing. Similar displacement rates for unsaturated direct shear tests were adopted by Escario and Saez (1986) (i.e., 0.00168

mm/min) and Vanapalli et al. (1996a) (i.e., 0.001875 mm/min) for a silty sand and a glacial till, respectively (Table 4-1 in Section 4.2.4.3.2).

Figure 5-9 Shear stress versus horizontal displacement for different displacement rates for the Sand vom Stücken

5.2.3.2.2 Unsaturated shear strength behaviour

Unsaturated direct shear testing was carried out for the specimens under four different net normal stresses (i.e., 50 kPa, 100 kPa, 230 kPa and 320 kPa) by applying individually five different soil suctions (i.e., 15 kPa, 45 kPa, 95 kPa, 195 kPa and 445 kPa) to obtain the shear strength behaviour of the Sand vom Stücken. As determined earlier, the appropriate displacement rate of 0.0015 mm/min was used for these tests.

5.2.3.2.2.1 Constant soil suction

The relationship between shear stress and horizontal displacement and between vertical displacement and horizontal displacement for constant soil suctions at different net normal stresses are shown in Figure 5-10 to Figure 5-14 (as presented in Schnellmann et al., 2013).

The results for the unsaturated direct shear tests obtained under a constant soil suction of 15 kPa are shown in Figure 5-10. The shearing behaviour for net normal stresses of 50 kPa and 100 kPa shows a peak shear stress followed by a strain softening behaviour. For the net normal stresses of 230 kPa and 320 kPa, the shear stress curves versus horizontal displacement show a strain-hardening behaviour. The curve of the vertical displacement versus horizontal displacement shows a gradual change from

shear dilatant behaviour for a net normal stress of 50 kPa to a to a shear compression behaviour for the higher net normal stresses.

The results of the shearing behaviour for soil suctions of 45 kPa and 95 kPa are shown in Figure 5-11 and Figure 5-12, respectively. The shear stress curves versus horizontal displacement at net normal stresses of 50 kPa, 100 kPa and 230 kPa show a peak value followed by a strain-softening behaviour. The shear stress curve versus horizontal displacement for 320 kPa net normal stress shows a strain-hardening behaviour for both soil suctions of 45 kPa and 95 kPa. The corresponding vertical displacement versus horizontal displacement curves show shear dilation behaviour at a net normal stress of 50 kPa, gradually changing to shear compression behaviour at higher net normal stresses.

For constant soil suctions of 195 kPa and 445 kPa, the shear stress curves versus horizontal displacement all show a peak shear stress followed by a strain-softening behaviour as shown in Figure 5-13 and Figure 5-14. The corresponding vertical displacement versus horizontal displacement curves show shear dilation for all net normal stresses with the highest shear dilation corresponding to the lowest net normal stress.

Figure 5-10 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement under 15 kPa soil suction

Figure 5-11 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement under 45 kPa soil suction

Figure 5-12 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement under 95 kPa soil suction

Figure 5-13 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement under 195 kPa soil suction

Figure 5-14 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement under 445 kPa soil suction

5.2.3.2.2.2 *Constant net normal stress*

Figure 5-15 to Figure 5-18 show the shearing behaviour plotted for the same net normal stresses at different soil suctions (as presented in Schnellmann et al., 2013).

For net normal stresses of 50 kPa and 100 kPa, as shown in Figure 5-15 and Figure 5-16, the shear stress curves versus horizontal displacement show a peak shear stress followed by strain-softening behaviour for all soil suctions. The vertical displacement versus horizontal displacement curves shows all shear dilation behaviour.

Shown in Figure 5-17 and Figure 5-18 are the shear stress curves versus horizontal displacement and vertical displacement versus horizontal displacement curves for net normal stresses of 230 kPa and 320 kPa, respectively. The shear stress curves versus horizontal displacement change gradually from a hardening behaviour to a peak followed by strain-softening behaviour with increasing soil suctions. The corresponding vertical displacement versus horizontal displacement curve shows a gradual change from shear compression to shear dilation behaviour with increasing soil suction.

Figure 5-15 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement for 50 kPa net normal stress

Figure 5-16 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement for 100 kPa net normal stress

Figure 5-17 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement for 230 kPa net normal stress

Figure 5-18 Shear stress versus horizontal displacement and vertical displacement versus horizontal displacement for 320 kPa net normal stress

5.2.3.2.2.3 *General behaviour*

For all shear stress curves versus horizontal displacement, the shear stress increases with an increase in net normal stress under a constant soil suction value. At a given net normal stress, a higher shear stress can be observed for higher soil suctions. In short, both a higher net normal stress under a constant soil suction and a higher soil suction under a constant net normal stress increase the shear strength of the soil. Also the stiffness of the soil increases with an increase in net normal stress or an increase in soil suction. For the tests where a peak shear stress followed by a strain-softening behaviour is observed, the peak shear strength corresponds approximately to the point of the maximum dilatancy rate. However, for the tests where a strain-hardening behaviour is observed, the peak shear strength does not correspond with the point of the maximum dilatancy rate.

5.2.3.2.3 *Extended Mohr-Coulomb failure envelope*

The extended Mohr-Coulomb failure envelopes of the recompacted specimens of the Sand vom Stücken are presented in Figure 5-19 and Figure 5-20 (as presented in Schnellmann et al., 2013). Failure was considered where the shear stress peaked or started to remain constant as obtained from the experimental data presented in the previous Section 5.2.3.2.2. The numerical values of the peak shear stresses are tabulated in Appendix D.

The shear strength envelopes at constant soil suctions show a linear increase in shear stress with respect to net normal stress as presented in Figure 5-19. The cohesion intercept at zero net normal stress and the slope at constant soil suctions as summarized in Table 5-5 were obtained by assuming a linear relationship between net normal stress and shear stress as illustrated in Figure 5-19. The high coefficient of determinations at the various soil suctions confirms the linear relationship of the shear stress with respect to net normal stress (Table 5-5).

Figure 5-19 Failure envelopes corresponding to different soil suction values for the Sand vom Stücken (as presented in Schnellmann et al., 2013)

Table 5-5 Summary of the friction angle and total cohesion for the Sand vom Stücken (as presented in Schnellmann et al., 2013)

Soil suction	Slope	Coefficient of Determination	Friction angle	Cohesion intercept
$(u_a - u_w)(kPa)$		$R^2(-)$	$\phi' (^{\circ})$	$c (kPa)$
15	0.668	1.00	33.7	5
45	0.684	0.999	34.4	10
95	0.693	0.999	34.7	15
195	0.692	0.996	34.7	24
445	0.677	0.997	34.1	31

The variation of shear stress with respect to soil suction at constant net normal stresses shows a non-linear relationship with shear stresses being higher for higher net normal stresses as illustrated in Figure 5-20. Irrespective of net normal stress, the failure envelopes with respect to soil suction indicate that the shear stress reaches a plateau around a soil suction of 200 kPa.

The increase in shear strength with respect to soil suction can be represented by the angle indicating an increase in shear strength with respect to soil suction (ϕ^b) as demonstrated in Figure 5-21. The ϕ^b angle decreases with increasing soil suction reflecting the non-linear shear stress behaviour with respect to soil suction. For soil suctions up to 100 kPa, the ϕ^b angle slightly increases with increasing net normal stress. For soil suctions higher than 200 kPa, the ϕ^b angle varies minimally with increasing net normal stress.

Figure 5-20 Failure envelopes corresponding to different net normal stresses for the Sand vom Stücken (as presented in Schnellmann et al., 2013)

Figure 5-21 Angle indicating an increase in shear strength with respect to soil suction for the Sand vom Stücken

5.3 ANALYTICAL RESULTS

The development of the prediction equation to estimate the unsaturated shear strength as proposed from this study is presented in Section 3.3. The performance of the proposed prediction equation in comparison to the existing prediction equations for the unsaturated shear strength is presented first in this Section. Subsequently, the uncertainties in the selected prediction equations for the unsaturated shear strength are presented. The variability in the estimated unsaturated shear strength due to model and parameter uncertainty is illustrated on the soil experimentally tested in this research. These results are presented at the end of this Chapter.

5.3.1 Performance of prediction equations to estimate the unsaturated shear strength

This Section presents the performance analysis of the proposed prediction equation to estimate the unsaturated shear strength (as proposed in Section 3.3.5) in comparison to the existing prediction equations. Only the contribution in shear strength with respect to soil suction was considered in the analyses (as listed in Table 4-4 in Section 4.3.1). Database 2 consisting of a wide range of soils was used to independently assess the performance of the prediction equations for the unsaturated shear strength. The soil data sets in Database 2 were not used in the development of the proposed prediction equation from this research.

Statistical analysis using the coefficient of determination and the average relative error as described in Section 3.3.4.2 was used to compare the performance of the prediction equations and to assess the predictive capability of these equations. The coefficient of determination was evaluated based on the 1:1 line. Data points coinciding with the 1:1 line would indicate a perfect match between measured and estimated shear strength. The coefficient of determination results in a maximum value of one with values closer to one representing a better match between measured and predicted values. The average relative error varies between zero and infinity and indicates a measure of curve match between measured and predicted shear strength with a smaller value representing a better prediction.

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Table 5-6 summarizes the coefficient of determinations and the average relative error from the statistical analysis of the proposed and existing prediction equations using Database 2. Figure 5-22 illustrates measured versus predicted shear strength for all the prediction equations used in the analysis using Database 2. The average relative error and plots of comparisons between predicted and measured shear strength with respect to soil suction for each soil data set in Database 2 is summarized in Appendix D.

Table 5-6 Coefficient of determination and average relative error for the analysed equations using Database 2

Author	Equation	Coefficient of determination	Average relative error
		$R^2(-)$	$ARE(-)$
Lamborn (1986)	4-2	-3.97	0.58
Vanapalli et al. (1996b)	4-4	0.71	0.27
Oberg and Sallfors (1997)	4-5	0.73	0.30
Bao et al. (1998)	4-6	-4.12	0.43
Khalili and Khabbaz (1998)	4-7	0.42	0.40
Vanapalli and Fredlund (2000)	4-3/2-14	0.65	0.32
Aubeny and Lytton (2003)	4-8	-2.76	0.49
Tekinsoy et al. (2004)	4-9	0.26	0.71
Garven and Vanapalli (2006)	4-3/2-15	0.64	0.32
Sheng et al. (2008)	4-10	0.62	0.34
Goh et al. (2010)	4-11	0.55	0.36
Proposed from this research	4-12	0.78	0.25

Figure 5-22 Measured versus estimated shear strength for the analysed equations using Database 2

Figure 5-22 Continued

The prediction equations proposed by Lamborn (1986), Bao et al. (1998) and Aubeny and Lytton (2003) result in negative values for the coefficient of determination, indicating that the estimated values considerably underestimate the measured data.

Low values of less than 0.50 for the coefficient of determination were calculated for the prediction equations proposed by Khalili and Khabbaz (1998) and Tekinsoy et al. (2004). These prediction equations (i.e., Aubeny and Lytton, 2004; Bao et al., 1998; Khalili and Khabbaz, 1998; Lamborn, 1986; Tekinsoy et al., 2004) also result in the highest average relative errors among the prediction equations analysed in this research.

The prediction equation by Goh et al. (2010) provides a coefficient of determination of 0.55 and an average relative error of 0.36; a slightly better prediction than the aforementioned equations. Similar predictive capabilities based on the coefficient of determination and the average relative error are provided by the equations proposed by Vanapalli and Fredlund (2000), Garven and Vanapalli (2006) and Sheng et al. (2008). The coefficient of variation and the average relative error for those equations are between 0.62-0.65 and 0.32-0.34, respectively.

Additionally, values of slightly above 0.70 for the coefficient of variation and around 0.3 for the average relative error were evaluated for the prediction equations by Vanapalli et al. (1996b) and Oberg and Sallfors (1997).

Comparatively, the prediction equation proposed from this research results in the highest value of coefficient of determination of 0.78 and in the lowest average relative error of 0.25 among all the prediction equations analysed. Therefore, the proposed prediction equation developed in this research indicates the best predictive capability among all the equations analysed.

5.3.2 Uncertainties in the estimation of the unsaturated shear strength

The equations used in the estimation of the unsaturated shear strength are the SWCC equation to obtain the relationship between water content and soil suction, and the shear strength prediction equation. This research focused on the model uncertainties of the equations involved in the estimation of the unsaturated shear strength. Therefore, uncertainties in the SWCC equation and the prediction equation for the estimation of the unsaturated shear strength were quantified based on Database 2. Soil data sets in Database 2 were not used in the development of the proposed prediction equation from this research. The uncertainties in the estimation of the unsaturated shear strength were quantified on the prediction equation proposed from

this research, the equation by Oberg and Sallfors (1997) and the equation by Garven and Vanapalli (2006) as explained in Section 4.3.4.2.

5.3.2.1 Uncertainty in the Soil-Water Characteristic Curve

The Fredlund and Xing (1994) equation was applied throughout this research to best fit a continuous function to measured SWCC data. The continuous function represents the measured SWCC data over the entire soil suction range from zero to 10^6 kPa, describing the relationship between degree of saturation and soil suction.

The model uncertainties in the Fredlund and Xing (1994) equation were quantified by its fitting parameters (a, n and m). The variability in the degree of saturation due to uncertainties in the Fredlund and Xing (1994) fitting parameters, indicating model uncertainty in the SWCC, was evaluated by first-order error analysis.

5.3.2.1.1 Uncertainty in the Fredlund and Xing (1994) fitting parameters

The mean values of the Fredlund and Xing (1994) fitting parameters (a, n and m) were obtained by minimizing the sum of squared errors between measured and calculated data as described in Section 3.2.2. The uncertainties in the Fredlund and Xing (1994) equation were quantified an approximation of the covariance matrix as introduced in Section 3.4.1 and further explained in Section 4.3.4.1. The approximation of the covariance matrix results in an estimate of the variance of the Fredlund and Xing (1994) fitting parameters indicating uncertainty in SWCC equation.

Shown in Figure 5-23 and summarized in Table 5-7 are the estimated mean and the standard deviation of the Fredlund and Xing (1994) fitting parameters. The standard deviation for the fitting parameters ranges from 0.07-25.12, 0.01-0.15 and 0.00-0.02 for the fitting parameters 'a', 'n' and 'm', respectively. Shown in Figure 5-24 and summarized in Table 5-8 are the uncertainties in the Fredlund and Xing (1994) fitting parameters in terms of the coefficient of variation. The ranges for the coefficient of variation vary between 2-8 %, 1-6 % and 1-3 % for the fitting parameters 'a', 'n' and 'm', respectively.

The fitted mean including the approximated covariance matrix for the Fredlund and Xing (1994) fitting parameters (a, n and m) for the soil data sets in Database 2 is

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summarized in Appendix D. The approximated covariance for all the soil data sets results in a non-zero covariance for the fitting parameters ($s_{a,n}$, $s_{a,m}$ and $s_{n,m}$), indicating that the Fredlund and Xing (1994) fitting parameters are interdependent (Appendix D).

Figure 5-23 Standard deviation of the Fredlund and Xing (1994) fitting parameters due to model uncertainty for Database 2

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Table 5-7 Mean and standard deviation of the Fredlund and Xing (1994) fitting parameters for Database 2

Soil No.	Soil	Mean			Standard deviation		
		a	n	m	s_a	s_n	s_m
2-01	Sand-Kaolin Mixture SK5	66.47	1.29	0.79	1.41	0.02	0.01
2-02	Sand-Kaolin Mixture SK10	68.35	1.37	0.33	1.31	0.02	0.00
2-03	Sand-Kaolin Mixture SK17	74.79	1.22	0.81	2.57	0.04	0.02
2-04	Residual Soil	108.26	3.58	0.72	2.60	0.15	0.02
2-05	Mixed Decomposed Granite	72.80	1.91	0.42	2.81	0.07	0.01
2-06	Indian Head Till (at opt.)	152.67	0.79	0.58	5.27	0.02	0.01
2-07	Indian Head Till (dry of opt.)	18.11	0.73	0.51	1.25	0.02	0.01
2-08	Indian Head Till (wet of optimum)	313.70	0.91	0.55	25.12	0.05	0.02
2-09	SJ10a	78.90	1.14	0.11	4.22	0.03	0.00
2-10	SJ10b	78.90	1.14	0.11	4.22	0.03	0.00
2-11	SJ11	286.39	0.71	0.48	7.19	0.02	0.01
2-12	Tailings - 50 m	3.68	3.24	0.78	0.07	0.07	0.01
2-13	Tailings - 150 m	13.96	1.87	1.03	0.35	0.04	0.02
2-14	Silty Clay (recompacted)	32.02	3.24	0.14	0.61	0.05	0.00
2-15	Silty Clay (natural)	3.25	0.61	0.14	0.20	0.01	0.00

Figure 5-24 Coefficient of variation for the Fredlund and Xing (1994) fitting parameters due to model uncertainty for Database 2

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Table 5-8 Mean and coefficient of variation of the Fredlund and Xing (1994) fitting parameters for Database 2

Soil No.	Soil	Mean			Coefficient of variation		
		<i>a</i>	<i>n</i>	<i>m</i>	<i>c_{v,a}</i> (%)	<i>c_{v,n}</i> (%)	<i>c_{v,m}</i> (%)
2-01	Sand-Kaolin Mixture SK5	66.47	1.29	0.79	2.12	1.81	1.27
2-02	Sand-Kaolin Mixture SK10	68.35	1.37	0.33	1.91	1.36	0.95
2-03	Sand-Kaolin Mixture SK17	74.79	1.22	0.81	3.43	3.04	2.07
2-04	Residual Soil	108.26	3.58	0.72	2.40	4.19	2.79
2-05	Mixed Decomposed Granite	72.80	1.91	0.42	3.85	3.77	2.65
2-06	Indian Head Till (at optimum)	152.67	0.79	0.58	3.45	2.85	1.53
2-07	Indian Head Till (dry of optimum)	18.11	0.73	0.51	6.88	2.66	1.87
2-08	Indian Head Till (wet of optimum)	313.70	0.91	0.55	8.01	5.78	3.48
2-09	SJ10a	78.90	1.14	0.11	5.34	3.00	2.16
2-10	SJ10b	78.90	1.14	0.11	5.34	3.00	2.16
2-11	SJ11	286.39	0.71	0.48	2.51	2.15	1.15
2-12	Tailings - 50 m	3.68	3.24	0.78	1.79	2.08	1.12
2-13	Tailings - 150 m	13.96	1.87	1.03	2.51	2.40	1.62
2-14	Silty Clay (recompacted)	32.02	3.24	0.14	1.91	1.65	0.90
2-15	Silty Clay (natural)	3.25	0.61	0.14	6.21	1.90	1.37

5.3.2.1.2 *Uncertainty propagation of the Fredlund and Xing (1994) fitting parameters*

The propagation of the uncertainty in the Fredlund and Xing (1994) fitting parameters resulting in uncertainty in the degree of saturation was evaluated using first-order error analysis as described in Section 4.3.4.1. The variation in the degree of saturation due to parameter uncertainty was calculated by considering the fitting parameters as independent of each other and for the case of dependency between the fitting parameters. The evaluated variation in the degree of saturation represents model uncertainty in the Fredlund and Xing (1994) equation.

The uncertainty in the degree of saturation shows a similar pattern for all soil data sets in Database 2. Typical plots of the uncertainty in the degree of saturation due to parameter uncertainty in the Fredlund and Xing (1994) fitting parameters are shown for soil data set 2-07 in Figure 5-25 and Figure 5-26. The uncertainty in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters is plotted with respect to the degree of saturation (Figure 5-25) and with respect to soil suction (Figure 5-26) in terms of the standard deviation and in terms of the coefficient of variation. The plots for all the soil data sets in Database 2 are given in Appendix D.

The standard deviation in the degree of saturation increases with increasing soil suction (or decreases with reducing degrees of saturation) and peaks in the transition zone between air-entry value and residual state condition. With increasing soil suction (or decreasing degrees of saturation) the standard deviation in the degree of saturation reduces to zero at a terminal suction of 10^6 kPa. The coefficient of variation of the degree of saturation consistently increases with decreasing degrees of saturation. The coefficient of variation starts to increase considerably around the air-entry value of the soil and starts to flatten out in the transition zone. The highest coefficient of variation is reached at low degrees of saturation where the soil approaches a terminal soil suction of 10^6 kPa.

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Figure 5-25 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-07

Figure 5-26 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-07

Table 5-9 summarizes the maximum standard deviation and coefficient of variation for the degree of saturation for each soil data set in Database 2. The standard deviation in the degree of saturation considering the Fredlund and Xing (1994) fitting parameters as independent and dependent of each other varies between 0.003-0.017

and between 0.005-0.030, respectively. This results in a range for the coefficient of variation of 0.5-7.6 % and 0.7-10.2 % for the degree of saturation considering the Fredlund and Xing (1994) fitting parameters as independent and dependent of each other, respectively.

Table 5-9 Maximum variation in the degree of saturation of the soil data sets in Database 2

Soil No.	Soil	Standard deviation		Coefficient of variation	
		s_s		$c_{v,s}$ (%)	
		Independent	Dependent	Independent	Dependent
2-01	Sand-Kaolin Mixture SK5	0.007	0.012	2.89	4.11
2-02	Sand-Kaolin Mixture SK10	0.004	0.006	0.93	1.33
2-03	Sand-Kaolin Mixture SK17	0.012	0.020	4.79	6.85
2-04	Residual Soil	0.017	0.030	7.63	10.21
2-05	Mixed Decomposed Granite	0.012	0.021	3.63	5.03
2-06	Indian Head Till (at optimum)	0.007	0.012	2.39	3.59
2-07	Indian Head Till (dry of optimum)	0.009	0.016	2.40	3.63
2-08	Indian Head Till (wet of optimum)	0.015	0.026	4.99	7.53
2-09	SJ10a	0.004	0.006	0.67	0.97
2-10	SJ10b	0.004	0.006	0.67	0.97
2-11	SJ11	0.004	0.007	1.43	2.15
2-12	Tailings - 50 m	0.010	0.017	3.62	4.97
2-13	Tailings - 150 m	0.011	0.019	5.67	7.81
2-14	Silty Clay (recompacted)	0.003	0.005	0.50	0.70
2-15	Silty Clay (natural)	0.003	0.005	0.48	0.73

5.3.2.2 *Uncertainty in prediction equations for the unsaturated shear strength*

The uncertainty in the prediction equations for the unsaturated shear strength was quantified by a multiplicative model factor as described in Section 4.3.4.2. The model factor and its relevant statistics were evaluated for the prediction equation by Oberg and Sallfors (1997), the prediction equation by Garven and Vanapalli (2006) and the prediction equation proposed from this research as summarized in Table 4-6 in Section 4.3.4.2. Database 2, consisting of 41 shear strength measurements, was used to evaluate the relevant statistics of the model factor for the three selected prediction equations for unsaturated shear strength.

5.3.2.2.1 *Identification of data outliers*

Outliers were identified as values being outside 1.5 times the inter-quartile range as described in Section 4.3.4.2.1. Figure 5-27 shows the box plots of the model factor for the respective prediction equations.

Figure 5-27 Box plots of the model factor for the selected prediction equations

For the prediction equation proposed by Oberg and Sallfors (1997) and the prediction equation proposed from this research, the observations 35, 36 and 37 were identified as outliers. Observations 35, 36 and 37 all belong to soil data set 2-12. For the

prediction equation proposed by Garven and Vanapalli (2006), the observations 12 and 35 were identified as outliers. Observation 12 belongs to the soil data set 2-4 whereas observation 35 belongs to soil data set 2-12. Therefore, the soil data sets 2-4 and 2-12 were not considered for further analyses from hereon.

5.3.2.2.2 *Exploratory data analysis*

The exploratory data analysis for the model factor of the three selected prediction equations was calculated based on Database 2 corrected for outliers, resulting in a sample size of 41 observations. Table 5-10 summarizes the sample moments (i.e., mean, standard deviation, coefficient of variation, median, skewness and kurtosis) of the model factor for the three selected prediction equations for the unsaturated shear strength. The model factor for the prediction equations proposed by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the proposed prediction equation from this research result in mean values of 0.95, 1.21 and 1.15, respectively.

The standard deviation of the prediction equation by Oberg and Sallfors (1997) and the proposed prediction equation from this research are similar with values of 0.28 and 0.29, respectively. The prediction equation proposed by Garven and Vanapalli (2006) results in a standard deviation of 0.39.

The coefficient of variation of the model factors are 0.29, 0.32 and 0.25 for the equations by Oberg and Sallfors (1997), Graven and Vanapalli (2006) and the proposed equation from this research, respectively.

All the model factors for the prediction equations are positive skewed with values of 0.34, 0.37 and 0.39 for the prediction equation by Oberg and Sallfors (1997) Garven and Vanapalli (2006) and the proposed equation form this research, respectively. A small kurtosis (excess) for the prediction equation by Oberg and Sallfors (1997) and the proposed equation from this research of 0.16 and 0.03 were calculated. The prediction equation proposed by Garven and Vanapalli (2006) results in a kurtosis (excess) which is with 1.27 considerably higher than the one of a normal distribution.

The results of the normality tests of the model factor for the three analysed prediction equations are presented in Table 5-11. All the p-values for the Shapiro-Wilk and the Kolmogorov-Smirnov tests are greater than 0.05 for the model factor of the three

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prediction equations. Therefore, the null hypothesis of normality for the model factor cannot be rejected at a significance level of 5 %. Additional normality plots for the model factor of the analysed prediction equations are given in Appendix D.

Table 5-10 Descriptive statistics of the model factor for the three selected prediction equations for the unsaturated shear strength

Statistic		Author		
		Oberg and Sallfors (1997)	Garven and Vanapalli (2006)	Proposed from this research
		4-5	4-3/2-15	4-12
Sample size	N	41	41	41
Mean	$m_{\tau_{us}}$	0.95	1.21	1.15
Standard deviation	s	0.28	0.39	0.29
Coefficient of variation	c_v (%)	29	32	25
Median	-	0.99	1.18	1.11
Skewness	-	0.34	0.70	0.39
Kurtosis	-	0.16	1.27	0.03

Table 5-11 Normality test for the model factor for the three selected prediction equation for the unsaturated shear strength

Author	Equation	Shapiro-Wilk test		Kolmogorov-Smirnov test		Probability distribution
		W	$Prob.$	$Dist.$	$Prob.$	
Oberg and Sallfors (1997)	4-5	0.972	0.386	0.0752	0.736	Normal
Garven and Vanapalli (2006)	4-3/2-15	0.968	0.299	0.0628	0.859	Normal
Proposed from this research	4-12	0.981	0.731	0.0916	0.49	Normal

5.3.2.2.3 *Randomness for the model factor*

The randomness of the model factor was partially evaluated based on the Pearson product-moment correlation coefficient. Table 5-12 summarizes the Pearson product-moment correlation between the model factor of the three selected prediction equations for the unsaturated shear strength and the basic input parameters. The most p-values of the Pearson product-moment correlation are much greater than 0.05 and the null hypothesis of zero correlation cannot be rejected at the 5 % significance level. This indicates that there are no statistical dependencies between the model factor and the basic input parameters for the three selected prediction equations.

Table 5-12 Correlations between the model factor of the selected prediction equations for the unsaturated shear strength and the basic input parameters

Author	Equation	Parameter		Pearson product-moment correlation	
				<i>r</i>	<i>p – value</i>
Oberg and Sallfors (1997)	4-5	Fredlund and Xing (1994) fitting parameter	<i>a</i>	-0.01	0.95
			<i>n</i>	0.10	0.55
			<i>m</i>	0.21	0.20
		Effective friction angle	ϕ'	-0.02	0.91
Garven and Vanapalli (2006)	4-3/2-15	Fredlund and Xing (1994) fitting parameter	<i>a</i>	-0.15	0.36
			<i>n</i>	0.05	0.77
			<i>m</i>	0.24	0.13
		Kappa	κ	0.32	0.05
		Effective friction angle	ϕ'	-0.24	0.13
Proposed from this research	4-12	Fredlund and Xing (1994) fitting parameter	<i>a</i>	-0.10	0.53
			<i>n</i>	-0.03	0.83
			<i>m</i>	0.02	0.89
		Residual degree of saturation	S_r^e	-0.22	0.16
		Effective friction angle	ϕ'	-0.16	0.31

5.3.3 Application of the uncertainties in the estimation of the unsaturated shear strength

The quantified model uncertainties in the Fredlund and Xing (1994) SWCC equation and the prediction equations of the unsaturated shear strength proposed by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the proposed equation from this research were applied on the soil experimentally tested in this research. The inherent variability in the degree of saturation and the effective friction angle were considered on top of the model uncertainties quantified in this research.

Table 5-13 summarizes the input parameters including their uncertainties, in terms of the coefficient of variation, applied in the analysis. The propagation of the uncertainty in the estimated unsaturated shear strength were evaluated using first-order error analysis and Monte-Carlo simulations considering the input parameters as random variables as explained in Section 4.3.5.

Table 5-13 Input parameters and their uncertainty adopted in the analysis

Parameter	Equation	Type of uncertainty		Mean	Coefficient of variation	Distribution
				(−)	c_v (%)	
Model factor						
Oberg and Sallfors (1997)	4-5			0.95 ^a	29 ^a	
Garven and Vanapalli (2006)	4-3/2-15	Model	$m_{\tau_{us}}$	1.21 ^a	32 ^a	Normal ^a
Proposed from this research	4-12			1.15 ^a	25 ^a	
Degree of saturation	3-3	Model	S^{SWCC}	var. ^a	10 ^a	Normal ^a
	-	Inherent	S^P	var.	10	Normal
Effective friction angle	-	Inherent	$\tan(\phi')$	0.664	10	Normal

^a quantified in this study

5.3.3.1 Performance

Figure 5-28 shows the measured and estimated shear strength with respect to soil suction for the Sand vom Stücken as evaluated deterministically and as obtained through Monte-Carlo simulations. Deterministic evaluation and the mean value of the Monte-Carlo simulations result in almost identical shear strength failure envelopes for the prediction equations used in the analysis. It was found that for the Monte-Carlo simulations, a number of 1000 trials were appropriate as the average shear strength and the standard deviation remained constant for a higher number of trials as shown in the Appendix D.

Figure 5-28 Measured and estimated unsaturated shear strength

Summarized in Table 5-14 are the coefficient of determination and the average relative error between measured and estimated shear strength of the Sand vom Stücken as obtained deterministically and through Monte-Carlo simulations. Deterministic evaluation and Monte-Carlo simulations yield almost identical values for the coefficient of determination and the average relative error. Figure 5-29 shows the measured versus estimated shear strength values as obtained deterministically and obtained through Monte-Carlo simulations.

The proposed equation from this research results in the highest coefficient of determination of 0.86 and in the lowest average relative error of 0.23. The equation proposed by Oberg and Sallfors (1997) yields a similarly low average relative error as the equation proposed from this research but in a significantly lower coefficient of determination of 0.64. The equation proposed by Garven and Vanapalli (2006) yields

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a low coefficient of determination of 0.40 and a corresponding high average relative error of 0.45.

Table 5-14 Coefficient of determination and average relative error for the Sand vom Stücken

Author	Equation	Coefficient of determination		Average relative error	
		R^2 (–)		ARE (–)	
		Deterministic	Mean of Monte-Carlo simulation	Deterministic	Mean of Monte-Carlo simulation
Oberg and Sallfors (1997)	4-5	0.64	0.63	0.26	0.26
Garven and Vanapalli (2006)	4-3/2-15	0.40	0.40	0.45	0.45
Proposed from this research	4-12	0.86	0.87	0.23	0.23

Figure 5-29 Measured versus estimated shear strength for the Sand vom Stücken

5.3.3.2 Propagation of uncertainty

The uncertainty in the estimated shear strength of the selected prediction equations is shown, in terms of standard deviation and coefficient of variation, in Figure 5-30 and Figure 5-31, respectively. First-order error analysis and Monte-Carlo simulations yield almost identical results for both the standard deviation and the coefficient of variation in the estimated shear strength. Whereas the standard deviation in the estimated shear strength increases with increasing soil suction, the coefficient of variation however remains relatively constant. The coefficient of variation in the estimated shear strength yields a value of about 35 % for all the prediction equations used in the analysis.

Figure 5-30 Standard deviation in the estimated unsaturated shear strength of the selected prediction equations

Figure 5-31 Coefficient of variation of the unsaturated shear strength for the selected prediction equations

Figure 5-32 illustrates the uncertainty contribution of each parameter for the analysed equations.

For the equations proposed by Oberg and Sallfors (1997) and Garven and Vanapalli (2006), the uncertainty contribution for each parameter remains constant with respect to soil suction. The uncertainty contribution for these two equations is highest for the model factor in the prediction equation of the unsaturated shear strength with a value close to 80% for both equations. The uncertainty contribution of the degree of saturation and the effective friction angle is considerably smaller.

Comparatively, the uncertainty contribution for the proposed prediction equation from this research shows a different pattern as compared to the equations proposed by Oberg and Sallfors (1997) and Garven and Vanapalli (2006). While the uncertainty contribution of the effective friction angle remains relatively constant with a similar percentage as the equations proposed by Oberg and Sallfors (1997) and Garven and Vanapalli (2006), the uncertainty contributions in the model factor in the prediction equation for the unsaturated shear strength and the uncertainty contribution in the degree of saturation varies with respect to soil suction. While the uncertainty contribution in the model factor decreases with increasing soil suction, the uncertainty contribution in the degree of saturation increases with higher soil suctions. The uncertainty contribution in the model factor is higher than the uncertainty contribution in the degree of saturation up to soil suction of 300 kPa where the uncertainty contribution of the degree of saturation is the major uncertainty contribution.

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Figure 5-32 Uncertainty contribution for the analysed prediction equations for the unsaturated shear strength

CHAPTER 6 DISCUSSION OF RESULTS

6.1 INTRODUCTION

The main results of this research are discussed in the following Sections. The experimental results of the Soil-Water Characteristic Curve (SWCC) and the shear strength are discussed first. In the subsequent Sections, the results of the performance analysis of the prediction equations for the unsaturated shear strength and the quantified uncertainty in the SWCC and in the three selected prediction equations for the unsaturated shear strength are discussed. The last Section in this Chapter discusses the results for the estimated unsaturated shear strength and its uncertainties as applied to the soil tested from this research.

6.2 EXPERIMENTAL RESULTS

6.2.1 Soil-Water Characteristic Curve

The Sand vom Stücken shows a typical SWCC behaviour of a sand. The soil exhibits a low air-entry value after which the soil desaturates relatively fast as indicated by the steep slope of the SWCC (Figure 5-4 to Figure 5-7 in Section 5.2.3.1).

The degrees of saturation at a specific soil suction vary minimally for the SWCCs obtained under different net confining stresses and therefore indicate that the net confining stress has little effect on the SWCC of the Sand vom Stücken. The air-entry value of the soil tends to increase slightly for a higher net confining stress. However, an increase of 0.2 kPa between the SWCC obtained under zero net normal stress and the SWCC obtained at a confining stress of 320 kPa is very small (Table 6-1). As mentioned in Section 5.2.3.1, the residual state condition of the Sand vom Stücken was identified visually on an arithmetic plot of the SWCC. It was observed that residual state condition occurs at a soil suction between 10,000-20,000 kPa as summarized in Table 6-1. The corresponding residual degree of saturation varies between 0.1-0.15 for the SWCCs. Similarly as observed by the air-entry value, the

residual degree of saturation seems to increase slightly for SWCCs obtained at a higher net confining stress (Table 6-1). The soil specimens of the Sand vom Stücken were compacted to a relatively low void ratio to ensure that no saturation collapse occurs and this might partly explain the small effect of net confining stress on the SWCC.

It was suggested in Section 3.3.4.3 that residual state condition might be identified mathematically where the second derivative reaches a value of $1\text{E-}10 \text{ kPa}^{-2}$. The degrees of saturation and soil suction values obtained mathematically where the second derivative of the SWCC reaches a value $1\text{E-}10 \text{ kPa}^{-2}$ are summarised in Table 6-1. The comparison between residual state condition visually-obtained on a arithmetic plot and mathematically where the second derivative of the SWCC reaches a value of $1\text{E-}10 \text{ kPa}^{-2}$ shows good agreement. The residual degrees of saturation and the residual soil suction obtained mathematically are very similar to the ones obtained visually. This confirms that residual state condition might be identified where the second derivative of the SWCC reaches a value of $1\text{E-}10 \text{ kPa}^{-2}$.

Table 6-1 Air-entry value and residual state condition for the Sand vom Stücken

SWCC	Air-entry value	Residual state condition			
		Visually-identified on an arithmetic plot of the SWCC		Mathematically at $\frac{\partial^2 s^*}{\partial \psi^2} = 1\text{E-}10 \text{ kPa}^{-2}$	
		$S_r(-)$	$\psi_r(kPa)$	$S_r(-)$	$\psi_r(kPa)$
SWCC-0	1.0	0.12-0.10	10,000-20,000	0.10	17,000
SWCC-100	1.0	0.13-0.11	10,000-20,000	0.11	18,000
SWCC-200	1.2	0.15-0.13	10,000-20,000	0.13	19,000
SWCC-300	1.2	0.14-0.12	10,000-20,000	0.12	18,000

6.2.2 Shear strength

The saturated direct shear test results under consolidated and drained (CD) conditions indicate a linear failure envelope for the Sand vom Stücken. The effective friction angle (ϕ') and effective cohesion (c') were evaluated as 33.4° and 0 kPa, respectively. The failure envelopes from the CD unsaturated direct shear tests at constant soil suction values exhibit linear relationships with respect to net normal stress (Figure 5-19 in Section 5.2.3.2.3). The ϕ' angle obtained from the unsaturated shear strength results at constant soil suction values varies between 33.7° to 34.7° and agree well with the ϕ' angle of 33.4° from the saturated direct shear test results. Therefore, the ϕ' angle for the Sand vom Stücken might be considered constant for the tested soil suction range as indicated in Figure 6-1.

Figure 6-1 Friction angle with respect to soil suction for the Sand vom Stücken

The variation of shear strength with respect to soil suction shows a non-linear relationship as demonstrated in Figure 5-20 in Section 5.2.3.2.3. This observation agrees with those made by various researchers (e.g., Escario and Juca, 1989; Gan et al., 1988; Hossain and Yin, 2010; Zhan and Ng, 2006). The angle indicating an increase in shear strength with respect to soil suction (ϕ^b) was reported to increase according to the effective friction angle (ϕ') for soil suction values up to the air-entry value (Gan et al., 1988). As the air-entry value of the soil tested in this research is very low (i.e., around 1 kPa), this linear portion below the air-entry value is not captured. However, the shear strength with respect to soil suction seems to increase at low soil suction values according to ϕ' . This is demonstrated in Figure 6-2 for the

failure envelope at the cohesion intercept at zero net normal stress. The corresponding behaviour of the ϕ^b angle is shown in Figure 6-3. The ϕ^b angle seems to start at a value equal to the effective friction angle and decreases significantly in the range from 0 kPa to 100 kPa soil suction.

Figure 6-2 Failure envelope with respect to soil suction of the Sand vom Stücken (at the cohesion intercept)

Figure 6-3 Angle indicating an increase in shear strength with respect to soil suction of the Sand vom Stücken (at the cohesion intercept)

The experimental results from the saturated and unsaturated direct shear tests agree well with the conceptual shear strength behaviour for coarse-grained soils as proposed in this research. The shear strength with respect to net normal stress at constant soil suction values shows a linear relationship with an angle equal to the effective friction angle (ϕ'). The shear strength with respect to soil suction is non-

linear. The experimental results also seem to confirm that the shear strength with respect to soil suction increases up to the air-entry value according to the effective friction angle and decreases continuously with increasing soil suction as demonstrated in Figure 6-2.

The failure envelopes with respect to soil suction show a fairly similar pattern for all the tested net normal stresses as shown in Figure 5-20 in Section 5.2.3.2.3. This is more clearly demonstrated in Figure 6-4 where only the contribution of shear strength due to soil suction is plotted. The contribution of shear strength due to soil suction for net normal stresses of 100 kPa, 230 kPa and 320 kPa varies by maximal 5 kPa. This indicates that the net confining stress does not affect the shape of the non-linear failure envelope considerably for the Sand vom Stücken. This is consistent with the results of the SWCCs and reflects the strong relationship between the SWCC and the unsaturated shear strength.

Figure 6-4 Contribution of shear strength due to soil suction

6.3 ANALYTICAL RESULTS

6.3.1 Performance of prediction equations to estimate the unsaturated shear strength

The prediction equations by Lamborn (1986), Bao et al. (1998) and Aubeny and Lytton (2003) all result in a negative value for the coefficient of determination, indicating that these equations generally underestimate the measured shear strength substantially. These prediction equations also result in an accordingly high average relative error. This is especially the case for the equation proposed by Lamborn (1986), which underestimates all the measured shear strength data in Database 2.

The equations by Khalili and Khabbaz (1998), Tekinsoy et al. (2004) and Sheng et al. (2008) (for the shear strength at drying) use the air-entry value as the only parameter from the SWCC in the estimation of the unsaturated shear strength. Therefore, these equations are sensitive to the identified air-entry value. The equations by Khalili and Khabbaz (1998) and Tekinsoy et al. (2004) result in low coefficient of determinations of 0.42 and 0.26, respectively. Even though some of the soil data sets were estimated reasonably by these two equations, the predicted shear strength for the most soil data sets deviates markedly from the measured shear strength for both coarse- and fine-grained soils. The equation by Tekinsoy et al. (2004) generally overestimates, for some soil data sets considerably, the measured shear strength. A better predictive capability shows the prediction equation by Sheng et al. (2008) with a coefficient of determination of 0.62 and an average relative error of 0.34.

The equation proposed by Goh et al. (2010) results in a similar average relative error of 0.36 as the equation by Sheng et al. (2008) but with a lower coefficient of determination. The equation by Goh et al. (2010) uses the normalized volumetric water content, as the controlling parameter, including the air-entry value, the Fredlund and Xing (1994) fitting parameter 'n' and the plasticity index in the estimation of the unsaturated shear strength. For the same SWCC, different combinations of values for the fitting parameters are possible. Hence, depending on the fitting procedure, different values for the fitting parameter 'n' might be found and consequently impact the equation by Goh et al. (2010).

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The equations proposed by Vanapalli and Fredlund (2000) and Garven and Vanapalli (2006) both use the normalized volumetric water content by the power of a parameter (κ) as the controlling parameter for the estimation of the unsaturated shear strength. The two equations differ only in a slightly different κ relationship, which is for both equations a function of the plasticity index. For soils without plasticity, κ yields one for both of the equations. It is therefore not surprising that these two equations yield almost identical results for the coefficient of variation and the average relative error. As the normalized volumetric water content is equal to the degree of saturation, the equations by Vanapalli and Fredlund (2000) and Garven and Vanapalli (2006) yield for soils without plasticity the same results as the equation proposed by Oberg and Sallfors (1997), which uses the degree of saturation as the controlling parameter.

The prediction equation proposed by Oberg and Sallfors (1997) was reported to be applicable in the estimation of the shear strength of coarse-grained soils such as sands and silts. However, with a coefficient of determination of 0.72 and an average relative error of 0.30 for the entire Dataset 2, consisting of coarse- and fine-grained soils, the equation by Oberg and Sallfors (1997) indicates a better predictive capability than most prediction equations analysed.

The prediction equation proposed by Vanapalli et al. (1996b) results in a coefficient of determination of 0.70 and an average relative error of 0.27 which indicates a similar predictive capability as the equation by Oberg and Sallfors (1997). While the equation by Oberg and Sallfors (1997) generally overestimates the measured shear strength, the equation proposed by Vanapalli et al. (1996b) underestimates most measured shear strength data.

The equation proposed from this research to estimate the unsaturated shear strength indicates improved predictive capability compared to the existing prediction equations. The proposed equation yields in a considerably higher coefficient of determination of 0.78 as compared to all the other equations analysed in this research. Some of the lowest average relative errors were obtained by the proposed equation, resulting in the lowest average relative error of 0.25 among all the equations analysed. Generally, the proposed prediction equation slightly underestimates the measured shear strength. The proposed prediction equation results in an average relative error

higher than 0.50 only for the soil data set 2-12. However, all the analysed equations result in a high average relative error for soil data set 2-12, indicating that the particular soil data set might somehow deviate from the other soil data sets in Database 2.

The equations that indicate better predictive capabilities for the unsaturated shear strength (i.e., high coefficient of determination and low average relative error) use the entire SWCC in their estimation. This partly confirms the importance of using the entire SWCC in the evaluation of unsaturated soil property functions.

6.3.2 Uncertainties in the estimation of the unsaturated shear strength

The uncertainties in the estimation of the unsaturated shear strength were analysed for the prediction equations proposed in this study, the equations by Oberg and Sallfors (1997) and Garven and Vanapalli (1998). These prediction equations all make use of the entire SWCC to estimate the unsaturated shear strength. Therefore, the model uncertainty in the SWCC equation and the respective prediction equation for the unsaturated shear strength were quantified in this research.

6.3.2.1 Uncertainty in the Soil-Water Characteristic Curve

The uncertainty in the SWCC was quantified through the uncertainty in the Fredlund and Xing (1994) fitting parameters (a , n and m). The uncertainty in the Fredlund and Xing (1994) fitting parameters were evaluated by a first-order error analysis resulting in an approximation of the covariance matrix of the fitting parameters.

6.3.2.1.1 Uncertainty in the Fredlund and Xing (1994) fitting parameters

While the standard deviation for the Fredlund and Xing (1994) fitting parameters (a , n and m) tends to increase with an increasing mean value in the respective fitting parameter, the coefficient of variation for the Fredlund and Xing (1994) fitting parameters remains relatively constant with respect to its mean values as illustrated in Figure 5-24 in Section 5.3.2.1.1.

The individual coefficients of variation in the Fredlund and Xing (1994) fitting parameters are relatively small, indicating that the Fredlund and Xing (1994) equation represents measured SWCC data well. The average values in the coefficient of

variation in the Fredlund and Xing (1994) fitting parameters 'a', 'n' and 'm' due to model uncertainty are 4 %, 3 % and 2 %, respectively.

The soil data sets in Database 2 used to determine the uncertainty consist of a wide range of soils including sands, silts and clays and the respective fitting parameters cover a wide range of values. It is therefore assumed that the evaluated coefficients of variations for the fitting parameters, indicating model uncertainty in the Fredlund and Xing (1994) SWCC equation, are representative for a wide range of soils.

6.3.2.1.2 Uncertainty in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters

The uncertainty in the degree of saturation due to the propagation of uncertainty in the Fredlund and Xing (1994) parameters was calculated using first-order error analysis. The propagation of the parameter uncertainty was evaluated for each soil data set in Database 2 using the respective standard deviation of the fitting parameters. Zhai and Rahardjo (2013) and Phoon et al. (2010) indicated that the fitting parameters of the SWCC might be dependent on each other. The approximation of the covariance matrix for the Fredlund and Xing (1994) fitting parameters obtained in this research results in non-zero values for the covariance of the fitting parameters, confirming possible dependence between the fitting parameters in the SWCC. Therefore, the propagation of the uncertainty in the fitting parameters was evaluated considering the fitting parameters as dependent and as independent of each other. This results in an upper and a lower value for the uncertainty in the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) fitting equation.

The uncertainty (i.e., standard deviation and coefficient of variation) in the degree of saturation due to uncertainty in the fitting parameters varies with respect to degree of saturation and with respect to soil suction as representatively shown for soil data set 2-07 in Figure 5-25 and Figure 5-26 in Section 5.3.2.1.2. The standard deviation in the degree of saturation shows a peak value with relatively low standard deviations for soil suctions below the air-entry value and for soil suctions which are higher than the residual condition. In terms of the coefficient of variation, the uncertainty remains relatively small below the air-entry value and starts to increase significantly in the

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transition zone. The coefficient of variation starts to somewhat flatten out reaching an almost constant value around residual state condition.

Even though the coefficient of variation in the degree of saturation varies with respect to soil suction and with respect to the degree of saturation, the coefficient of variation shows a more uniform behaviour than the standard deviation. Therefore, the uncertainty in the degree of saturation due to model uncertainties in the Fredlund and Xing (1994) equation is further discussed in terms of the coefficient of variation as shown in Figure 6-5. The coefficient of variation in the degree of saturation due to parameter uncertainty in Fredlund and Xing (1994) equation were found to vary between 0.5-8 % when considering the fitting parameters independent of each other and 0.7-10 % when considering the fitting parameters as dependent. The average coefficient of variation for the entire Dataset 2 was found to be 3 % and 4 % for the fitting parameters being considered as independent and dependent, respectively.

The highest coefficients of variations occur at high soil suctions. However, the range of interest for the application in unsaturated soils is generally at much lower soil suctions. Therefore, a coefficient of variation of 5-10 % for the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) equation seems reasonable.

Figure 6-5 Coefficients of variations for the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) fitting parameters

6.3.2.2 Uncertainty in prediction equations for the unsaturated shear strength

The uncertainty in the prediction equations for the unsaturated shear strength proposed by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the prediction equation proposed from this research were quantified by a multiplicative model factor. The results of the model factor for the three prediction equations are summarized in Table 6-2.

Table 6-2 Uncertainty in the selected prediction equations for the unsaturated shear strength

Author	Equation	Mean	Coefficient of variation	Probability distribution
		$m_{\tau_{us}}$	c_v (%)	
Oberg and Sallfors (1997)	4-5	0.95	0.29	Normal
Garven and Vanapalli (2006)	4-3/2-15	1.21	0.32	Normal
Proposed from this research	4-12	1.15	0.25	Normal

The prediction equation proposed by Oberg and Sallfors (1997) results in a model factor of 0.95, indicating that the equation generally overestimates the measured shear strength. The model factor for the prediction equations proposed by Garven and Vanapalli (2006) and the proposed prediction equation from this research are 1.21 and 1.15, respectively. A value of more than one indicates that these two prediction equations generally underestimate the measured shear strength.

The coefficient of variations for the model factors are 29 %, 32 % and 25 % for the equations by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the proposed equation from this research, respectively. A critical value for the coefficient of variation for the model factor is 33 %. A value of less than 1/3 for the coefficient of variation indicates that for a normal distribution almost all (i.e., 99.73 %) of the data are expected within three standard deviations of the mean. Hence, negative values for the model factor, which per definition are not possible, are very unlikely based on a normal distribution. The coefficients of variation for the model factor of the analysed equations are below that critical value of 33 % suggesting that the normal distribution is appropriate to describe the model factor.

Normality tests (i.e., Shapiro-Wilk and Kolmogorov-Smirnoff) and normality plots further confirm that the normal distribution is an appropriate probability model for the prediction equations by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the proposed equation from this research.

6.3.3 Application of the uncertainties in the estimation of the unsaturated shear strength

The estimated unsaturated shear strength failure envelopes of the Sand vom Stücken obtained deterministically and with the mean of the Monte-Carlo simulations result in almost identical values. Besides the inherent soil variability, the model uncertainty in the SWCC equation and the model uncertainty in the prediction equation for the unsaturated shear strength as quantified in this research were considered in the analysis. The uncertainty (coefficient of variation or standard deviation) in the estimated shear strength obtained by first-order error analysis and by Monte-Carlo simulations yield in almost the same values. These results indicate that the Monte-Carlo simulations can be successfully applied to estimate the unsaturated shear strength.

The proposed prediction equation from this research results in an average relative error of 23 % which is slightly lower as compared to that of the equation by Oberg and Sallfors (1997). The average relative error for the equation by Garven and Vanapalli (2006) is 45 % which is much higher compared to that for the equation by Oberg and Sallfors (1997) and the proposed prediction equation. The coefficient of determination for the proposed prediction equation is 86 % which is much higher as compared to those for the equations by Oberg and Sallfors (1997) and Garven and Vanapalli (2006) which are 64 % and 40 %, respectively. This demonstrates the superior performance of the unsaturated shear strength for the Sand vom Stücken by the proposed prediction equation from this research as compared to the equations by Oberg and Sallfors (1997) and Garven and Vanapalli (2006). Even though these results are only from one soil data set and are evaluated by applying the model factor for the prediction equations, it further confirms the improved predictive capability of the proposed shear strength equation as demonstrated on an extensive Database in the performance analyses.

Chapter 6 Discussion of Results

The uncertainty in the estimated shear strength for the analysed equations by Oberg and Sallfors (1997), Garven and Vanapalli (2006) and the proposed equation from this research are comparable. For all three prediction equations, the variability in the estimated unsaturated shear strength at the cohesion intercept, due to model uncertainty and inherent soil variability, of the tested soil results in a coefficient of variation of 35 %. This is in the same magnitude as reported coefficients of variation of the effective cohesion (c') obtained through direct measurements.

The major uncertainty contribution in the estimated shear strength by Oberg and Sallfors (1997) and Garven and Vanapalli (2006) is the uncertainty in the prediction equations, which accounts for almost 80 % of the total uncertainty. The uncertainty in the degree of saturation (model uncertainty in the SWCC and inherent soil variability) and the effective friction angle (inherent soil variability) are considerably smaller. This was expected as the unsaturated shear strength equations are prediction equations and the quantified uncertainties are therefore comparatively high.

However, it is rather surprising to note that the uncertainty contribution of the proposed prediction equation shows a different behaviour. In the range from 0 to 200 kPa soil suction, the major uncertainty contribution in the estimated shear strength as proposed in this research is the uncertainty in the prediction equation for the unsaturated shear strength. For higher soil suction values, the uncertainty contribution from the degree of saturation (model uncertainty in the SWCC and inherent soil variability) is slightly higher than the uncertainty contribution from the prediction equation. The uncertainty contribution from the effective friction angle remains constant with respect to soil suction and only contributes little to the total uncertainty.

CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS

7.1 INTRODUCTION

This Chapter summarizes the conclusions drawn from this research and provides recommendations for further studies.

7.2 SUMMARY

7.2.1 General

The unsaturated shear strength has been extensively studied in previous research studies. However, there is to date no single prediction equation that reliably estimates the unsaturated shear strength for a wide range of soils. Furthermore, uncertainties in the estimation of the unsaturated shear strength have not been addressed in previous research studies. By extending basic concepts, this research makes an effort to improve the predictive capability of the unsaturated shear strength estimation and make an attempt to fill the gap in the literature by quantifying the uncertainties in the estimation of the unsaturated shear strength.

In this study, the shear strength due to soil suction was reinterpreted based on the typical desaturation behaviour of a soil along the Soil-Water Characteristic Curve (SWCC). Based on the typical desaturation behaviour of a soil along the SWCC, the evaluation of the residual state condition was revised. Based on the reinterpreted behavioural concept of the shear strength due to soil suction along the SWCC, a prediction equation that estimates the unsaturated shear strength was proposed. Beside the saturated shear strength parameters, the SWCC needs to be available to estimate the unsaturated shear strength. Performance analyses were carried out to investigate the predictive capability of the proposed prediction equation in comparison to existing prediction equations. Model uncertainties arising in the estimation of the unsaturated shear strength were quantified. Model uncertainty in the SWCC was quantified directly by the fitting parameters of the SWCC equation

whereas the uncertainty in prediction equations for the unsaturated shear strength was quantified by a multiplicative model factor.

A suitable soil was experimentally tested in the laboratory to obtain and study its SWCC and its unsaturated shear strength behaviour. Finally, the variability in the estimated unsaturated shear strength due to the quantified model uncertainty and inherent soil variability was demonstrated on the soil that was experimentally tested in this research. The propagation of the uncertainty in the estimation of the unsaturated shear strength was investigated by first-order error analyses and Monte-Carlo simulations.

7.2.2 Original contributions and limitations

The original contributions from this research include a revised concept to evaluate residual state for unsaturated soils that results in an improved prediction equation for the unsaturated shear strength. The original contribution regarding the identification of residual state and its application to the unsaturated shear strength has been published in the Canadian Geotechnical Journal (Schnellmann et al., 2015).

An empirical relationship for the evaluation of the residual degree of saturation for use in the estimation in the unsaturated shear strength has been proposed. The empirical relationship to evaluate the residual degree of saturation was established based on eleven soil data sets consisting of fine- and coarse-grained soils with air-entry values between 2.5 kPa and 120 kPa. The Fredlund and Xing (1994) SWCC equation together with the proposed prediction equation for the unsaturated shear strength were applied in the development of the empirical relationship. The proposed equation is therefore applicable for both fine- and coarse-grained soils with an air-entry value up to about 120 kPa. However, for sensitive and expansive soils where soil structure and electro-chemical processes need to be investigated, the proposed prediction equation is not suitable. Furthermore, the uncertainties in the models required in the estimation of the unsaturated shear strength were quantified based on the existing theories. Previous studies indicated that first-order error analysis could be adopted to approximate the covariance matrix for the fitting parameters of non-linear problems. Thus, the parameter variance of the Fredlund and Xing (1994) SWCC equation based on a single measured SWCC was approximated by first-order

error analysis. The uncertainties associated with three prediction equations (i.e., Oberg and Sallfors, 1997, Garven and Vanapalli, 2006 and the proposed equation from this research) for the unsaturated shear strength were quantified by a multiplicative model factor.

The applied concepts are applicable for soils on the drying and wetting paths. However, due to the limited soil data sets available on the wetting path, the empirical relationship for the residual degree of saturation and the model factor for the prediction equations for the unsaturated shear strength were established on soil data sets obtained on the drying path. Furthermore, the shear strength data in the used soil data sets were obtained from unsaturated triaxial compression tests or from unsaturated direct shear tests.

In addition to the analytical contributions, single-staged, consolidated and drained direct shear tests and SWCC tests under various net normal stresses were carried out on a coarse-grained soil. Quality experimental results are generally a valuable addition in the literature, especially for time-consuming measurements as encountered in the direct evaluation of unsaturated soil property functions. The unsaturated shear strength results have been published in Engineering Geology (Schnellmann et al., 2013).

The variability in the estimated unsaturated shear strength due to soil variability and model uncertainty is demonstrated on the coarse-grained soil tested in this research. First-order error propagation and Monte-Carlo simulations were adopted to evaluate the variability in the estimated shear strength due to soil variability and model uncertainties. The evaluated variability for the coarse-grained soil indicates a coefficient of variation in the same order of magnitude as values reported for the effective cohesion obtained through direct measurements.

7.3 CONCLUSIONS

Based on the findings from this research, the following conclusions can be made:

- 1) The experimentally-obtained SWCCs indicated that the net confining stress has little effect on the SWCC of the sand tested in this study.

- 2) The experimentally-obtained shear strength results showed that soil suction has a significant influence on the shear strength behaviour of the sand tested in this research.
 - The soil showed a peak value followed by a softening behaviour at low net normal stresses. For high net normal stresses, a strain hardening behaviour was observed.
 - The effective friction angle (ϕ') showed a linear relationship with respect to net normal stress at a constant soil suction.
 - The increase in shear strength with respect to soil suction showed a non-linear behaviour. The angle indicating an increase in shear strength with respect to soil suction (ϕ^b) continually decreased beyond the air-entry value.
- 3) Based on the typical desaturation behaviour of a soil along the SWCC, the residual state condition was reinterpreted.
 - It was suggested that residual state condition is visually best identified on an arithmetic plot of the SWCC.
 - To avoid the subjectivity of the visual identification of residual state condition, the second derivative of the SWCC with respect to soil suction could be used as a criterion to determine residual state condition mathematically.
 - Based on the reinterpreted residual state condition, residual state condition occurs at much higher soil suction values as compared to previously reported values.
- 4) The unsaturated shear strength of a soil can be reasonably estimated.
 - The proposed prediction equation to estimate the unsaturated shear strength of a soil shows improved predictive capabilities compared to existing equations. The proposed prediction equation shows a good agreement to measured data for both coarse- and fine-grained soils.
- 5) Model uncertainty in the estimation of the unsaturated shear strength arising from the SWCC and the prediction equation for the unsaturated shear strength were quantified.

- The uncertainty in the SWCC equation by Fredlund and Xing (1994) was quantified by its fitting parameters using an approximation for the covariance matrix. The uncertainties in the fitting parameters reflect the ability of the Fredlund and Xing (1994) equation to represent measured SWCC data.
 - Uncertainties in prediction equations for the unsaturated shear strength were quantified by a multiplicative model factor.
- 6) The variability in the estimated unsaturated shear strength at the cohesion intercept, due to model uncertainty and inherent soil variability, of the tested soil results in a coefficient of variation which is in the same magnitude as the coefficient of variation of the effective cohesion (c') obtained through direct measurements.
- Monte-Carlo simulations were successfully applied in estimating the unsaturated shear strength including its variability.
 - The model factor in the prediction equations for the unsaturated shear strength contributed most to the overall uncertainties.

7.4 RECOMMENDATIONS

The following recommendations can be made for further research:

- 1) The unsaturated shear strength is generally measured in the laboratory for a soil suction range between zero to a few hundred kilopascals. This is typically the range of interest for geotechnical applications. However, to further study and investigate the unsaturated shear strength behaviour, shear strength measurements at higher soil suction values would be valuable.
- 2) The proposed prediction equation addresses the possible increase in shear strength due to soil suction. For sensitive and expansive soils where soil structure and electro-chemical effects need to be investigated, the proposed prediction equation is not suitable. Future studies could extend the presented concepts to consider soil-structure effects and electro-chemical forces in the estimation of the unsaturated shear strength.
- 3) The model uncertainty in the SWCC were quantified in this research on a single measured SWCC using an approximation for the covariance matrix. To

complement the results from this study, the uncertainties in the SWCC could be quantified directly by repeatedly testing a soil under the same conditions.

- 4) The uncertainties in the prediction equations of the unsaturated shear strength were quantified by a multiplicative model factor assuming that laboratory experiments represent the true state. A scatter plot to examine the error magnitude of experimentally obtained unsaturated shear strength data would complement this study. However, this would require multiple unsaturated shear strength measurements.

7.5 PUBLICATIONS FROM THIS RESEARCH

During the course of this research, two journal papers were submitted and accepted. These papers are:

- Schnellmann, R., Rahardjo, H. and Schneider, H. R. (2013) “Unsaturated shear strength of a silty sand”, Engineering Geology, Vol. 162, pp. 88-96.
- Schnellmann, R., Rahardjo, H. and Schneider, H. R. (2015) “Controlling parameter for unsaturated soil property functions: validated on the unsaturated shear strength”, Canadian Geotechnical Journal, Vol. 52, pp 374-381.

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Appendices

APPENDICES

Appendix A

Appendix A

Reported coefficients of variation

The following section presents tabulated coefficients of variation for laboratory measured index and strength properties from several studies (e.g., Baecher and Christian, 2003; Harr, 1977, 1987; Phoon and Kulhawy, 1999a; Pinheiro Branco et al., 2014; Rahardjo et al., 2012). Also summarized are the probability density functions which were reported to describe the respective soil parameter.

Soil index properties

Table A-1 Reported coefficients of variation of inherent soil variability for soil index properties

Property ^a	Soil type	Property value	c_v (%)	Source
w_n (%)	Fine-grained	13-105	7-46	Phoon and Kulhawy (1999a)
	Silty clay	n.a.	20 ^b	Harr (1987)
	Clay	n.a.	13 ^b	Harr (1987)
w_L (%)	Residual soil	41-45	5-33 ^b	Rahardjo et al. (2012)
		41-51	15-32 ^b	Rahardjo et al. (2012)
		56-58	13-20 ^b	Rahardjo et al. (2012)
	Fine-grained	27-89	7-39	Phoon and Kulhawy (1999a)
w_P (%)	Residual soil	20-22	17-30 ^b	Rahardjo et al. (2012)
		32-37	24-32 ^b	Rahardjo et al. (2012)
		36-41	21-32 ^b	Rahardjo et al. (2012)
	Fine-grained	14-27	6-34	Phoon and Kulhawy (1999a)
PI (%)	Fine-grained	12-44	9-57	Phoon and Kulhawy (1999a)
G_s	Silt	2.56	1 ^b	Harr (1987)
	Silty clay	2.66	2 ^b	Harr (1987)

Appendix A

Table A-1 Continued

γ (kN/m^3)	-	20	1.8 ^b	Pinheiro Branco et al. (2014)
γ_d (kN/m^3)	Fine-grained	14-20	3-20	Phoon and Kulhawy (1999a)
		-	3 ^b	Harr (1987)
γ_s (kN/m^3)	Residual granite	16.4	3.5 ^b	Pinheiro Branco et al. (2014)
	Fine-grained	13-18	2-13	Phoon and Kulhawy (1999a)
D_r (%)	Sand	30-70	11-36 ^c	Phoon and Kulhawy (1999a)
S (-)	Silt	0.8	9 ^b	Harr (1987)
	Clay	0.9-1.0	9-13 ^b	Fredlund and Dahlman (1971)

^a w_n , natural water content; w_L , liquid limit; w_P , plastic limit; PI , plasticity index; G_s , specific gravity of soil solids; γ , total unit weight; γ_d , dry unit weight; D_r , relative density; S , degree of saturation

^b No comments made on whether measurement variability was included

^c Total variability for direct method of determination

Appendix A

Table A-2 Reported coefficients of variation of total measurement error for soil index properties

Property ^a	Soil type	Property value	c _v (%)	Source
w_n (%)	Fine-grained	16-21	6-12	Phoon and Kulhawy (1999a)
w_L (%)	Fine-grained	17-113	3-11	Phoon and Kulhawy (1999a)
w_p (%)	Fine-grained	2-35	7-18	Phoon and Kulhawy (1999a)
PI (%)	Fine-grained	4-44	5-51	Phoon and Kulhawy (1999a)
γ (kN/m ³)	Fine-grained	16-17	1-2	Phoon and Kulhawy (1999a)

^a w_n , natural water content; w_L , liquid limit; w_p , plastic limit; PI , plasticity index; γ , total unit weight

Appendix A

Table A-3 Reported probability density functions for some soil index properties

Property ^a	Soil type	Probability function	Source
w_L (%)	Marine clay	Normal	Lumb (1966)
	Alluvial sandy clay		Lumb (1966)
	Residual silty sand		Lumb (1966)
	Residual clayey silt		Lumb (1966)
w_P (%)	Marine clay	Normal	Lumb (1966)
	Alluvial sandy clay		Lumb (1966)
PI (%)	Marine clay	Normal	Lumb (1966)
	Alluvial Sandy clay		Lumb (1966)
γ_s (kN/m^3)	Residual granite	Normal	Pinheiro Branco et al. (2014)
γ_d (kN/m^3)	Residual granite	Normal	Pinheiro Branco et al. (2014)
S (—)	Clay	Normal	Fredlund and Dahlman (1971)

^a w_L , liquid limit; w_P , plastic limit; PI , plasticity index; γ_s , saturated unit weight; γ_d , dry unit weight

Appendix A

Strength properties

Table A-4 Reported coefficients of variation of inherent variability for strength properties

Property ^a	Soil type	Property value	c _v (%)	Source
ϕ' (°)	Residual granite	40.3	7.9 ^b	Pinheiro Branco et al. (2014)
		29-35	4-14 ^b	Rahardjo et al. (2012)
	Residual soil	33-37	8-15 ^b	Rahardjo et al. (2012)
		36-37	8-15 ^b	Rahardjo et al. (2012)
	Gravel	36	6	Phoon and Kulhawy (1999a)
	Sand	35-41	5-11	Phoon and Kulhawy (1999a)
		39-41	7-11	Phoon and Kulhawy (1999a)
		9-33	10-50	Phoon and Kulhawy (1999a)
	Clay, silt	17-41	4-12	Phoon and Kulhawy (1999a)
	Gravelly sand	37	5.3	Phoon and Kulhawy (1999a)
	Sand loose	-	14	Phoon and Kulhawy (1999a)
	Sand dense	-	12	Phoon and Kulhawy (1999a)
$\tan\phi'$ (TC)	Clay, silt	0.24-0.69	6-46	Phoon and Kulhawy (1999a)
$\tan\phi'$ (DS)		-	6-46	Phoon and Kulhawy (1999a)
		0.65-0.92	5-14	Phoon and Kulhawy (1999a)
$\tan\phi'$	Sand	0.762	7.3	Phoon and Kulhawy (1999a)
		0.717	13	Phoon and Kulhawy (1999a)

Appendix A

Table A-4 Continued

	Residual granite	9.3	68	Pinheiro Branco et al. (2014)
	Sand	-	40	Fredlund and Dahlman (1971)
c' (kPa)		11-14	18-48 ^b	Rahardjo et al. (2012)
	Residual soil	8-12	36-56 ^b	Rahardjo et al. (2012)
		18-23	36-54 ^b	Rahardjo et al. (2012)
		25-31	24-44 ^b	Rahardjo et al. (2012)
ϕ^b (°)	Residual soil	28-32	30-39 ^b	Rahardjo et al. (2012)
		30-32	22-34 ^b	Rahardjo et al. (2012)

^a ϕ' , effective friction angle; ϕ^b , angle indicating an increase in shear strength with respect to soil suction; TC, triaxial compression test; DS, direct shear test

Appendix A

Table A-5 Reported coefficients of variation of total measurement error for strength properties

Property ^a	Soil type	Property value	c _v (%)	Source
ϕ' (TC) (°)	Clay, silt	2-27	7-56	Phoon and Kulhawy (1999a)
ϕ' (DS) (°)	Sand	30-35	13-14	Phoon and Kulhawy (1999a)
	Clay, silt	24-40	3-29	Phoon and Kulhawy (1999a)
$\tan\phi'$ (TC)	Sand, silt	-	2-22	Phoon and Kulhawy (1999a)
$\tan\phi'$ (DS)	Clay	-	6-22	Phoon and Kulhawy (1999a)

^a ϕ' , effective friction angle; TC, triaxial compression test; DS, direct shear test

Appendix A

Table A-6 Reported probability density functions for some strength properties

Property ^a	Soil type	Probability function	Reference
ϕ' (kPa)	Residual granite	Normal	Pinheiro Branco et al. (2014)
	Residual silty sand	Normal	Lumb (1966)
	Residual clayey silt	Normal	Lumb (1966)
$\tan\phi'$	Residual silty sand	Normal	Lumb (1966)
	Residual clayey silt	Normal	Lumb (1966)
c' (°)	Residual clayey silt	Normal	Lumb (1966)

^a ϕ' , effective friction angle; c' , effective cohesion

Appendix B

The second derivative of the Fredlund and Xing (1994) equation is given below.

Equation B-1

$$\begin{aligned}
 \frac{\partial^2 S}{\partial \psi^2} &= \frac{\partial^2}{\partial \psi^2} \left(1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right) \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^m \\
 &= \left(1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right) \left(\frac{m n^2 \left(\frac{\psi}{a}\right)^{2n-2} \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^{m+1}}{a^2 \left(e + \left(\frac{\psi}{a}\right)^n\right)^2} + \frac{m(m+1)n^2 \left(\frac{\psi}{a}\right)^{2n-2} \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^{m+2}}{a^2 \left(e + \left(\frac{\psi}{a}\right)^n\right)^2} \right) \\
 &\quad - \frac{m(n-1)n \left(\frac{\psi}{a}\right)^{n-2} \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^{m+1}}{a^2 \left(e + \left(\frac{\psi}{a}\right)^n\right)^2} \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^m + \frac{C_r^2 \left(1 + \frac{\psi}{C_r}\right)^2 \ln\left(1 + \frac{10^6}{C_r}\right)}{C_r} \\
 &\quad + \frac{2 m n \left(\frac{\psi}{a}\right)^{n-1} \left(\frac{1}{\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)} \right)^{m+1}}{a C_r \left(1 + \frac{\psi}{C_r}\right) \ln\left(1 + \frac{10^6}{C_r}\right) \left(e + \left(\frac{\psi}{a}\right)^n\right)}
 \end{aligned}$$

Appendix B

Measured and fitted shear strength (according to Equation 3-12) with respect to soil suction for the soil data sets in Database 1 are given below.

Figure B-1 Measured and fitted shear strength with respect to soil suction for soil data set 1-01

Figure B-2 Measured and fitted shear strength with respect to soil suction for soil data set 1-02

Figure B-3 Measured and fitted shear strength with respect to soil suction for soil data set 1-03

Appendix B

Figure B-4 Measured and fitted shear strength with respect to soil suction for soil data set 1-04

Figure B-5 Measured and fitted shear strength with respect to soil suction for soil data set 1-05

Figure B-6 Measured and fitted shear strength with respect to soil suction for soil data set 1-06

Appendix B

Figure B-7 Measured and fitted shear strength with respect to soil suction for soil data set 1-07

Figure B-8 Measured and fitted shear strength with respect to soil suction for soil data set 1-08

Figure B-9 Measured and fitted shear strength with respect to soil suction for soil data set 1-09

Appendix B

Figure B-10 Measured and fitted shear strength with respect to soil suction for soil data set 1-10

Figure B-11 Measured and fitted shear strength with respect to soil suction for soil data set 1-11

Appendix B

Soil-Water Characteristic Curves for the soil data sets in Database 1 are given below.

Figure B-12 Soil-Water Characteristic Curve for soil data set 1-01

Figure B-13 Soil-Water Characteristic Curve for soil data set 1-02

Figure B-14 Soil-Water Characteristic Curve for soil data set 1-03

Appendix B

Figure B-15 Soil-Water Characteristic Curve for soil data set 1-04

Figure B-16 Soil-Water Characteristic Curve for soil data set 1-05

Figure B-17 Soil-Water Characteristic Curve for soil data set 1-06

Appendix B

Figure B-18 Soil-Water Characteristic Curve for soil data set 1-07

Figure B-19 Soil-Water Characteristic Curve for soil data set 1-08

Figure B-20 Soil-Water Characteristic Curve for soil data set 1-09

Appendix B

Figure B-21 Soil-Water Characteristic Curve for soil data set 1-10

Figure B-22 Soil-Water Characteristic Curve for soil data set 1-11

Appendix B

Table B-1 Best fitted residual degree of saturation according to Equation 3-12 and corresponding residual soil suction for Database 1

Soil No.	Air-entry value	Residual state	
		Best fitted residual degree of saturation	Corresponding residual soil suction
		$S_r^* (-)$	$\psi_r^* (kPa)$
1-01	15	0.31	24,316
1-02	52	0.44	13,150
1-03	4	0.14	15,003
1-04	2.5	0.13	14,609
1-05	5	0.14	13,843
1-06	8	0.14	12,026
1-07	15	0.15	8,594
1-08	100	0.31	2,395
1-09	32	0.23	4,264
1-10	120	0.60	2,462
1-11	42	0.51	2,598

Appendix C

Appendix C

The partial derivatives of the SWCC with regard to the Fredlund and Xing (1994) fitting parameters are given as follows:

$$\frac{\partial S}{\partial a} = C(\psi) \frac{m n S \left(\frac{\psi}{a}\right)^n}{a \left(\ln \left(e + \left(\frac{\psi}{a}\right)^n\right)\right)^{m+1} \left(e + \left(\frac{\psi}{a}\right)^n\right)} \quad \text{Equation C-1}$$

$$\frac{\partial S}{\partial n} = C(\psi) \frac{-m S \left(\frac{\psi}{a}\right)^n \ln \left(\frac{\psi}{a}\right)}{\left(\ln \left(e + \left(\frac{\psi}{a}\right)^n\right)\right)^{m+1} \left(e + \left(\frac{\psi}{a}\right)^n\right)} \quad \text{Equation C-2}$$

$$\frac{\partial S}{\partial m} = C(\psi) \frac{-S \ln \left(\ln \left(e + \left(\frac{\psi}{a}\right)^n\right)\right)}{\left(\ln \left(e + \left(\frac{\psi}{a}\right)^n\right)\right)^m} \quad \text{Equation C-3}$$

Appendix C

The partial derivatives of the prediction equations analysed on their uncertainty are given below. Only the increase in shear strength due to soil suction was considered in the analysis.

Oberg and Sallfors (1997) (Equation 4-5)

$$\frac{\partial}{\partial m}(m \psi S \tan \phi') = \psi S \tan \phi' \quad \text{Equation C-4}$$

$$\frac{\partial}{\partial S}(m \psi S \tan \phi') = m \psi \tan \phi' \quad \text{Equation C-5}$$

$$\frac{\partial}{\partial \tan \phi'}(m \psi S \tan \phi') = m \psi S \quad \text{Equation C-6}$$

Garven and Vanapalli (2006) (Equations 4-3/2-15)

$$\frac{\partial}{\partial m}(m \psi S^{\kappa} \tan \phi') = \psi S^{\kappa} \tan \phi' \quad \text{Equation C-7}$$

$$\frac{\partial}{\partial S}(m \psi S^{\kappa} \tan \phi') = m \psi \kappa S^{\kappa-1} \tan \phi' \quad \text{Equation C-8}$$

$$\frac{\partial}{\partial \tan \phi'}(m \psi S^{\kappa} \tan \phi') = m \psi S^{\kappa} \quad \text{Equation C-9}$$

Proposed prediction equation (Equation 4-12)

$$\frac{\partial}{\partial m}\left(m \psi \frac{S - S_r}{1 - S_r} \tan \phi'\right) = \psi \frac{S_r - S}{S_r - 1} \tan \phi' \quad \text{Equation C-10}$$

$$\frac{\partial}{\partial S}\left(m \psi \frac{S - S_r}{1 - S_r} \tan \phi'\right) = m \psi \frac{1}{1 - S_r} \tan \phi' \quad \text{Equation C-11}$$

$$\frac{\partial}{\partial \tan \phi'}\left(m \psi \frac{S - S_r}{1 - S_r} \tan \phi'\right) = m \psi \frac{S_r - S}{S_r - 1} \quad \text{Equation C-12}$$

Appendix D

Appendix D

Tabulated peak shear stress results obtained from the unsaturated CD direct shear tests.

Table D-1 Peak shear stress obtained from unsaturated CD direct shear tests for the Sand vom Stücken

Net normal stress $(\sigma - u_a), (kPa)$	Soil suction $(u_a - u_w), (kPa)$				
	15	45	95	195	445
50	35	43	48	53	60
100	73	81	88	99	104
230	159	165	172	187	190
320	218	230	238	242	245

Table D-2 Average relative error for the analysed equations for soil data sets in Database 2

Author	Equation	Average relative error (ARE) (-)														
		Soil data set														
		2-01	2-02	2-03	2-04	2-05	2-06	2-07	2-08	2-09	2-10	2-11	2-12	2-13	2-14	2-15
Lamborn (1986)	4-2	0.55	0.57	0.62	0.67	0.34	0.59	0.47	0.59	0.55	0.37	0.69	0.84	0.68	0.57	0.60
Vanapalli et al. (1996b)	4-4	0.29	0.27	0.16	0.37	0.33	0.09	0.15	0.14	0.11	0.24	0.24	0.73	0.29	0.24	0.32
Oberg and Sallfors (1997)	4-5	0.10	0.09	0.11	0.30	0.71	0.20	0.49	0.32	0.33	0.67	0.13	0.58	0.17	0.04	0.29
Bao et al. (1998)	4-6	0.48	0.58	0.42	0.46	0.16	0.29	0.32	0.58	0.46	0.38	0.64	0.25	0.19	0.48	0.52
Khalili and Khabbaz (1998)	4-7	0.49	0.41	0.43	0.28	0.11	0.34	0.04	0.41	0.43	0.73	0.13	0.59	0.31	0.37	0.90
Vanapalli et al. (2000)	4-3/2-14	0.52	0.28	0.31	0.50	0.72	0.09	0.19	0.15	0.22	0.46	0.12	0.57	0.16	0.23	0.30
Aubeny and Lytton (2003)	4-8	0.55	0.48	0.62	0.65	0.27	0.59	0.47	0.47	0.03	0.36	0.54	0.84	0.68	0.38	0.47
Tekinsoy et al. (2004)	4-9	0.07	0.10	0.18	0.48	0.90	0.16	0.65	0.12	2.47	3.02	0.47	0.78	0.88	0.10	0.23
Garven and Vanapalli (2006)	4-3/2-15	0.52	0.28	0.34	0.51	0.71	0.10	0.18	0.14	0.22	0.45	0.12	0.58	0.17	0.21	0.30
Sheng et al. (2008)	4-10	0.37	0.29	0.30	0.16	0.37	0.17	0.18	0.29	0.43	0.80	0.04	0.50	0.14	0.27	0.88
Goh et al. (2010)	4-11	0.11	0.09	0.16	0.57	0.52	0.10	0.15	0.13	0.43	0.75	0.35	0.75	0.82	0.03	0.48
Proposed from this research	4-12	0.21	0.23	0.07	0.36	0.42	0.02	0.18	0.17	0.12	0.33	0.24	0.66	0.23	0.16	0.32

Appendix D

The measured and estimated shear strength with respect to soil suction for the soil data sets in Database 2 are given below.

Figure D-1 Measured and predicted shear strength with respect to soil suction for soil data set 2-01

Appendix D

Figure D-2 Measured and predicted shear strength with respect to soil suction for soil data set 2-02

Appendix D

Figure D-3 Measured and predicted shear strength with respect to soil suction for soil data set 2-03

Appendix D

Figure D-4 Measured and predicted shear strength with respect to soil suction for soil data set 2-04

Appendix D

Figure D-5 Measured and predicted shear strength with respect to soil suction for soil data set 2-05

Appendix D

Figure D-6 Measured and predicted shear strength with respect to soil suction for soil data set 2-06

Appendix D

Figure D-7 Measured and predicted shear strength with respect to soil suction for soil data set 2-07

Appendix D

Figure D-8 Measured and predicted shear strength with respect to soil suction for soil data set 2-08

Appendix D

Figure D-9 Measured and predicted shear strength with respect to soil suction for soil data set 2-09

Appendix D

Figure D-10 Measured and predicted shear strength with respect to soil suction for soil data set 2-10

Appendix D

Figure D-11 Measured and predicted shear strength with respect to soil suction for soil data set 2-11

Appendix D

Figure D-12 Measured and predicted shear strength with respect to soil suction for soil data set 2-12

Appendix D

Figure D-13 Measured and predicted shear strength with respect to soil suction for soil data set 2-13

Appendix D

Figure D-14 Measured and predicted shear strength with respect to soil suction for soil data set 2-14

Appendix D

Figure D-15 Measured and predicted shear strength with respect to soil suction for soil data set 2-15

Appendix D

Table D-3 Fredlund and Xing (1994) SWCC fitting parameters and approximated covariance matrix

Soil No.	Soil	Mean	Approximated covariance matrix		
			<i>a</i>	<i>n</i>	<i>m</i>
2-01	Sand-Kaolin Mixture SK5	<i>a</i>	66.47	1.99E+00	-4.70E-02
		<i>n</i>	1.29	5.47E-04	2.65E-04
		<i>m</i>	0.79		9.91E-05
2-02	Sand-Kaolin Mixture SK10	<i>a</i>	68.35	1.71E+00	-3.20E-02
		<i>n</i>	1.37	3.48E-04	6.19E-05
		<i>m</i>	0.33		9.96E-06
2-03	Sand-Kaolin Mixture SK17	<i>a</i>	74.79	6.59E+00	-1.39E-01
		<i>n</i>	1.22	1.38E-03	7.28E-04
		<i>m</i>	0.81		2.80E-04
2-04	Residual Soil	<i>a</i>	108.26	6.77E+00	-5.07E-01
		<i>n</i>	3.58	2.25E-02	3.11E-03
		<i>m</i>	0.72		4.03E-04
2-05	Mixed Decomposed Granite	<i>a</i>	72.80	7.87E+00	-3.23E-01
		<i>n</i>	1.91	5.16E-03	8.95E-04
		<i>m</i>	0.42		1.26E-04
2-06	Indian Head Till (at optimum)	<i>a</i>	152.67	2.78E+01	-1.97E-01
		<i>n</i>	0.79	5.07E-04	2.49E-04
		<i>m</i>	0.58		7.85E-05

Appendix D

Table D-3 Continued

2-07	Indian Head Till (dry of optimum)	<i>a</i>	18.11	1.55E+00	-2.76E-02	-1.27E-02
		<i>n</i>	0.73		3.77E-04	1.87E-04
		<i>m</i>	0.51			8.94E-05
2-08	Indian Head Till (wet of optimum)	<i>a</i>	313.70	6.31E+02	2.65E+00	-5.56E-01
		<i>n</i>	0.91		2.74E-03	1.24E-03
		<i>m</i>	0.55			3.67E-04
2-09	SJ10a	<i>a</i>	78.90	1.78E+01	-1.80E-01	-1.10E-02
		<i>n</i>	1.14		1.17E-03	8.63E-05
		<i>m</i>	0.11			5.85E-06
2-10	SJ10b	<i>a</i>	78.90	1.78E+01	-1.80E-01	-1.10E-02
		<i>n</i>	1.14		1.17E-03	8.63E-05
		<i>m</i>	0.11			5.85E-06
2-11	SJ11	<i>a</i>	286.39	5.17E+01	-2.55E-01	-4.05E-02
		<i>n</i>	0.71		2.36E-04	1.50E-04
		<i>m</i>	0.48			3.08E-05
2-12	Tailings - 50 m	<i>a</i>	3.68	4.35E-03	-6.72E-03	-8.80E-04
		<i>n</i>	3.24		4.56E-03	6.22E-04
		<i>m</i>	0.78			7.64E-05
2-13	Tailings - 150 m	<i>a</i>	13.96	1.23E-01	-2.36E-02	-6.91E-03
		<i>n</i>	1.87		2.02E-03	8.18E-04
		<i>m</i>	1.03			2.81E-04

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Table D-3 Continued

2-14	Silty Clay (recompacted)	<i>a</i>	32.02	3.76E-01	-4.27E-02	-1.06E-03
		<i>n</i>	3.24		2.87E-03	6.92E-05
		<i>m</i>	0.14			1.61E-06
2-15	Silty Clay (natural)	<i>a</i>	3.25	4.08E-02	-2.61E-03	-4.11E-04
		<i>n</i>	0.61		1.32E-04	2.25E-05
		<i>m</i>	0.14			3.72E-06

Appendix D

The plots of the uncertainty in the degree of saturation due to model uncertainty in the Fredlund and Xing (1994) equation for the soil data sets in Database 2 are given below.

Figure D-16 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-01

Figure D-17 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-01

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Figure D-18 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-02

Figure D-19 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-02

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Figure D-20 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-03

Figure D-21 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-03

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Figure D-22 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-04

Figure D-23 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-04

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Figure D-24 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-05

Figure D-25 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-05

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Figure D-26 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-06

Figure D-27 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-06

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Figure D-28 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-07

Figure D-29 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-07

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Figure D-30 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-08

Figure D-31 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-08

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Figure D-32 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-09

Figure D-33 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-09

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Figure D-34 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-10

Figure D-35 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-10

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Figure D-36 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-11

Figure D-37 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-11

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Figure D-38 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-12

Figure D-39 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-12

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Figure D-40 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-13

Figure D-41 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-13

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Figure D-42 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-14

Figure D-43 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-14

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Figure D-44 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to the degree of saturation for soil data set 2-15

Figure D-45 Standard deviation and coefficient of variation in the degree of saturation due to uncertainty in the Fredlund and Xing (1994) fitting parameters with respect to soil suction for soil data set 2-15

Appendix D

The normality plots of the model factor of the selected prediction equations analysed on their uncertainty are given below.

Figure D-46 Normality plots for the model factor in the prediction equation for the unsaturated shear strength for Database 2

Appendix D

Number of Monte-Carlo trials.

Figure D-47 Influence of the number of trials in the Monte-Carlo simulations in the estimation of the unsaturated shear strength of the Sand vom Stücken