Modelling, analysis, and optimization in resource planning for outpatient clinics

Nguyen Thi Thu Ba

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**MODELLING, ANALYSIS, AND OPTIMIZATION IN
RESOURCE PLANNING FOR OUTPATIENT CLINICS**

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NANYANG TECHNOLOGICAL UNIVERSITY

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RESOURCE PLANNING FOR OUTPATIENT CLINICS**

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FOR THE DEGREE OF DOCTOR OF PHILOSOPHY
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2014

DECLARATION

I hereby declare that this thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously

Nguyen Thi Thu Ba
10 December 2014

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ABSTRACT

Outpatient services in Singapore have been facing an increasing and lengthy appointment lead-time, which is regarded as being unacceptable for healthcare services. Specifically, Tan Tock Seng Hospital (TTSH) has been under the great pressure in reducing the outpatients' long appointment lead-time to meet the guidelines of Singapore Ministry of Health (MOH) in terms of the median, 95th percentile, 100th percentile. This brings various challenges to TTSH in operating its re-entry outpatient appointment system in which patients may need to re-visit for follow-up treatments.

The thesis focuses on both the strategic and tactical levels for the optimal capacity planning, and on the operational level for every individual appointment scheduling. Mathematical models of capacity planning and appointment scheduling rules with quantitative admission constraints are developed to achieve the MOH's appointment lead-time targets.

The capacity planning approach aims to match the capacity of healthcare provider to patients' demands restricted by the patients' appointment lead-time targets. The capacity planning approach contains two parts: capacity design and admission management problems. Mixed integer programming models are proposed with the minimization of maximum required resource and the maximum number of appointed patients as objectives of the capacity design and the admission management problems, respectively. In the models, a finite planning horizon, multiple types of patients, identical physicians, dependent demands between types of patients, and constant discharge rates are assumed. Initially, deterministic new arrival demand is assumed; however, the assumption is later expanded to be stochastic. Under uncertainty of arrival

demands, safe and tractable deterministic equivalents are derived. Branch and Cut algorithm are used to solve deterministic models and deterministic equivalents. The capacity designs and admission management models are tested with numerical experiments using real data of the Urology specialty. The results show the feasibility and efficiency of the proposed models. Finally, sensitivity analyses are carried out, which provides some deeper insights into the findings.

The appointment scheduling rule approach is to determine either the optimal required capacity or the optimal admission schemes, at the operational level. The approach entails decision making for every individual patient whose arrival information is unknown. Two different rules, the exact and relaxed rules, are derived for scheduling patients' appointments with the maximization of resource utilized as their objective. In addition, the rules account for the need to maintain the MOH's appointment lead-time targets. The appointment scheduling rules are then simulated using the real data from the Urology specialty and optimal capacity planning scheme. Simulation results show the efficacy of the proposed policies.

The capacity planning models and the appointment scheduling rules can decrease the probability of failure to satisfy the guidelines of MOH's appointment lead-time. Using the proposed models and rules helps TTSH to cope with challenges that it encounters.

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LIST OF SYMBOLS

CDF	: Cumulative distribution function
EASR	: Evaluating appointment scheduling rule
EPF	: Evaluating patient's flow
FIFO	: First-in-first-out
FILO	: First-in-last-out
FV	: First-visit
MOH	: Ministry of health
OAS	: Optimizing appointment scheduling
RV	: Re-visit

CHAPTER 1: INTRODUCTION

Outpatient services have been facing an increasing and lengthy appointment lead-time (refer to Figure 1.1) for a wide range of healthcare services throughout the world. The outpatient service of Singapore is not an exception. Thus, the Singapore Ministry of Health (MOH) has set guidelines for its hospitals in managing their performance in terms of median, 95th percentile, and 100th percentile targets of appointment lead-times. Tan Tock Seng Hospital (TTSH) is amongst hospitals that are under pressure of reducing patients' appointment lead-time. Although TTSH has successfully shortened patients' appointment lead-time, achieving the MOH's targets poses challenges to TTSH in both planning capacity and scheduling patients' appointments.

The achievement of MOH's appointment lead-time targets can be addressed in three different decision levels. The highest decision level, capacity design problem, strategic decisions have to be made on the required capacity of services or the optimal required capacity. These strategic decisions provide insights between established appointment lead-time targets and service requirements, which helps hospitals to consider the future resource investments. In the next level, admission management problem, tactical decisions are made for the optimal admission scheme. These tactical decisions concentrate on managing the admission of patients' requests in order to meet the MOH's appointment lead-time targets. At the lowest level, operational decisions aim to determine each individual patient's appointment. Appointment scheduling rules with quantitative admission constraints are proposed to make decisions on patients' appointment date.

Given an appointment lead-time policy, the thesis addresses the three stated levels of decisions making that determine the required capacity, the admission scheme, and the individual patient's appointment. Mathematical models are created to assist hospitals to plan capacity. The capacity planning models, which adopt a network flow approach to tackle the capacity design and the admission management problems, are proposed. The models are initially investigated with the assumption of deterministic new demands. Subsequently, the assumption is relaxed to allow uncertainties of new arrival demands. In addition, appointment scheduling rules are established as operational guidelines to administer assignments of patients' requests.

1.1. Definitions of terminologies

a. Appointment lead-time

Appointment lead-time is the duration from a patient's request date to his/her actual appointment date.

Figure 1.1: Definition of appointment lead-time

b. Outpatient versus Inpatient

Outpatient is a non-admitted patient. The outpatient leaves the hospital, medical center, or clinic after receiving the consultation or treatment. *Inpatient* is an admitted patient. The inpatient needs to remain in the hospital or clinic at least one overnight for treatments.

c. Capacity design

Capacity design is defined as a determination of the total number of physicians needed to meet arrival demands. The objective of the capacity design problem is to minimize the number of physicians that are needed to satisfy the appointment requests for all patients.

d. Admission management

Admission management is the optimization of the number of appointment requests that can be scheduled, matching the available provided resources. The goal is to maximize the number of scheduled patients without violating the appointment lead-time targets. In order to obtain the optimal admission scheme, the physician allocations amongst types of demands will be simultaneously optimized.

e. Appointment scheduling policy

Appointment scheduling policy makes decisions on scheduling appointments for each patient's requests. The objective is to determine whether a request for an appointment is scheduled or denied, and to assign the appointment date for the scheduled patient.

f. Deterministic versus stochastic demand

Deterministic demand is one in which the appointment requests are exactly given for the entire planning horizon. The models generate a deterministic output for a given deterministic demand with given initial conditions. Conversely, *stochastic demand* is one in which the appointment requests are probabilistic, based on a probability distribution. The models with uncertain demands will yield a probabilistic output with given initial conditions.

g. First-visit patient

A *first-visit (FV) patient* or a new patient is the patient who requests an appointment for the first time or who was discharged from his/her last visit.

h. Re-visit patient

A *re-visit (RV) patient* is patient with follow-up appointment/s for returning to the clinic as an outpatient for further treatment/s.

i. Appointment interval

An appointment interval or slot interval, denoted Δt , is the time interval between two appointment times. The appointment time can be fixed or varied. For the fixed appointment time problem, the appointment time is fixed and the appointment interval is predetermined; however the appointment interval may differ for different types of patients. The varied appointment time problem aims to determine both the beginning and the ending appointment time of the patients. The illustrations of the mentioned three categories (a fixed and equal,

a fixed and unequal, and a varied slot interval) are presented in Figures 1.2, 1.3, and 1.4.

Figure 1.2: A fixed and equal slot interval demonstration

Figure 1.3: A fixed and unequal slot interval demonstration

Figure 1.4: A varied slot interval demonstration

j. Network flow

A *network flow* (Bertsimas and Tsitsiklis, 1997) is specified for a directed graph $G=(N,A)$, where N denotes the set of nodes and A denotes the set of arcs, together with some additional numerical information (Figure 1.5), such as

the external supply to each node $i \in \mathcal{N}$, the non-negative capacity of each arc $(i, j) \in \mathcal{A}$, and the cost per unit of flow along arc (i, j) .

Figure 1.5: A directed graph $G = (\mathcal{N}, \mathcal{A})$ with $\mathcal{N} = \{1, 2, 3, 4\}$ and $\mathcal{A} = \{(1, 2), (1, 4), (4, 1), (1, 3), (4, 3)\}$

Three conditions on the flow network are required. First, the flow needs to be conserved as shown in the Equation 1.1. For each node i , the total amount of in-flows (the flow that enters into node i) must be the same as the total amount of out-flows (the flow that leaves node i). Second, the amount of flow through an arc must be non-negative as shown in Equation 1.2. Finally, Equation 1.3 needs to be satisfied; that is the amount of flow is less than or equal to the capacity of the arc. This approach can be used to assign supplies to demands.

$$\text{Total in-flows at } i = \text{total out-flows at } i, \quad \forall i \in \mathcal{N} \quad (1.1)$$

$$\text{The amount of flow through } (i, j) \geq 0, \quad \forall (i, j) \in \mathcal{A} \quad (1.2)$$

$$\text{The amount of flow through } (i, j) \leq \text{the capacity of } (i, j), \quad \forall (i, j) \in \mathcal{A} \quad (1.3)$$

The special structure of the network flow provides advantages of developing wide applications of optimization problems (i.e., assignment problems which assign patients' requests to service providers' capacity or vice

versa). There are a large number of algorithms, which are developed to solve network flow optimization problems. For instance, a problem considers modeling patients as integer flows, and allowing arrival demands to be uncertain. These considerations can be modeled with a network flow by forcing relevant constraint to be integer and to be linearly certain, respectively. The above advantages of the network flow will be taken in modeling the thesis's problems.

1.2. Operations research in Healthcare Management

More than a half century, many studies have been addressing quantitative methods, such as queuing theory (Bailey, 1952; Hassin and Mendel, 2008; etc.), mathematical programming (Fries and Marathe, 1981; Vanden Bosche et al., 1999; Chakraborty et al., 2010; etc.), simulation (Rising et al., 1973; LaGanga and Lawrence, 2007; etc.), to analyze and optimally manage healthcare system. These studies aim to increase the efficiency and effectiveness of healthcare delivery.

A great number of the studies focus on ways to find better appointment scheduling policies, either to reduce healthcare provider's idle time, healthcare provider's overtime, and patients' waiting time; or to increase the number of patients being served. The studies concentrate on planning and controlling patients' timing and flow as well as patient volumes and the lengths of waiting lists (Frost, 1980; Worthington, 1991; etc.). One method that has been used is to optimize the available resources via admission management (Qu et al., 2007), to determine the required capacity, which defines as capacity design (Ittig, 1985; Demir et al., 2012).

Recently, patients' appointment lead-time has received more attention because the longer appointment lead-time, the worse the patients' health conditions would become. One indication of this is the crucial concerns of MOH of Singapore on the patients' appointment lead-time. Hence, problems which focus on the patients' appointment lead-time are an interesting and challenging field to study on.

1.3. Motivation and research objectives

The Department of Urology at TTSH in the outpatient clinic, named as B1B, has been facing an increased patients' appointment lead-time and has been unable to achieve the MOH's appointment lead-time targets, even given that the new arrival demands were stable during 2007 to 2009 periods. The length of appointment lead-time at the Department of Urology was almost twice as long as the MOH's appointment lead-time guidelines. Table 1.1 illustrates the comparison between MOH's appointment lead-time targets and TTSH's performance.

Table 1.1: MOH's appointment lead-time targets vs. actual performance

MOH's appointment lead-time targets		Urology's performance of 2009
Median	14 days	30 days
95th percentile	42 days	84 days
100%	60 days	100 days

For this study on the chosen specialty, a new patient (FV patient) needs to call a contact center to book an appointment. Then, the patient has to virtually wait until his/her appointment date. A reminder of the appointment is sent in order to reduce the rate of no-show (Kourany et al., 1990). After the first

visit, a patient typically continues to return to the clinic at regular intervals as part of a treatment plan to assure continuity of care (Sudhakar-Krishnan and Rudolf, 2007). The patient then becomes a re-visit (RV) patient.

There is a restricted range of the appointment lead-time for the RV patients. In addition, the patient may need to return as an RV patient several times, which depends on treatment protocol. This re-entry system (Figure 1.6) challenges the service providers in planning their capacity because any new arrival leads to additional RV arrivals into the system.

Figure 1.6: Patient flow through the re-entry system

Currently, the clinic plans capacity by allocating the available patient slots into blocks dedicated for FV or RV patients. These allocations are guided by the preferences of each physician. Besides, the scheduling of patients' requests follows a first-come-first-out (FIFO) or first-come-first-served rule. The current practice leads to a long appointment lead-time due to mismatch between the planned capacity of the healthcare providers and patients' demands, as well as non-optimal capacity planning scheme. Moreover, the first-come-first-served appointment policy may block the arrival of the urgent

cases which may jeopardize patient's life and cause un-achievement of MOH's appointment lead-time targets. Therefore, a systematic way for a capacity planning and effective appointment scheduling rules is essential.

Although Iversen (1993), Elkhuizen et al. (2007), and Bowers (2010) investigated the mean and 95th percentile appointment lead-times of patients in their studies, neither the re-entry characteristic nor the specific methodology to solve the problem was considered. The neglect of the re-entry characteristic in the study may cause under-estimation of required capacity or non-optimal admission scheme. In addition, although many studies explored appointment scheduling rules for different types of patients such as urgent or less urgent, their objectives were to maximize the profits including a cost for patients' waiting times; however, these studies do not account for the patients' appointment lead-time. The disregard of patients' appointment lead-time could induce not only inappropriate decisions on appointment lead-times for urgent cases versus less urgent cases, but also unacceptable long appointment lead-times for all patients.

The pressure of reducing patients' appointment lead-time and research gaps on healthcare system (refer to Chapter 2 for the detailed research gaps) serve as motivations to develop capacity planning models to determine the optimal required physicians and the optimal admissions scheme. They also serve as the motivation to create a set of appointment scheduling policies for making decisions on patients' appointment date. Both the capacity planning models and appointment scheduling policies consider the MOH's service targets for median, 95th percentile, and 100th percentile patients' appointment

lead-time. The study begins with an assumption of deterministic new arrival demands. Then, this assumption is relaxed to be uncertain.

1.4. Outline of the thesis

This thesis aims to construct mathematical models for planning the required capacity and for managing patients' admission, and to develop scheduling policies for scheduling a request, which are organized into six chapters. Figure 1.7 illustrates the organization and relationship amongst the chapters.

Figure 1.7: Organization of the thesis

An intensive review of literatures for healthcare systems that have emerged in the past 60 years is presented in Chapter 2. The literature review is classified in two main groups: the review of outpatient appointment systems and the review of patients' appointment lead-time. The goals of this review of

the two main groups of literature are to reveal the research gaps of the capacity planning and appointment scheduling problems for the re-entry system.

The revealed research gaps inspire capacity planning models which are shown in Chapter 3. The capacity planning models are proposed to determine an optimal required capacity (from the capacity design model) and also to obtain an optimal admission scheme (from the admission management model) that address deterministic states of new arrival demands. The capacity design model aims to minimize the maximum required physicians, assuming that all requests must be accepted. On the other hand, the admission management model focuses on maximizing total number of patients admitted from the provided available capacities. The network flow approach is introduced to construct the models. Computational results and sensitivity analysis are executed to show the efficacy of the proposed models and to find the relationship between the objectives and the service's targets. Based on the analysis, important insights are summarized for service providers to make investments of scarce resources and to manage the admissions.

Once the optimal capacity planning is obtained from Chapter 3, the concern now is how to operate the system with the given optimal plan. This concern is tackled by developing appointment scheduling policies in Chapter 4. In this chapter, different appointment scheduling rules, which account for the restrictions of MOH's appointment lead-time targets, are investigated. The maximum utilization of the scarce resources is considered as their objective. Numerical simulation experiments are implemented to examine the efficiency and effectiveness of the proposed scheduling policies. The summary of implications for scheduling patients' requests is also presented.

In Chapter 5, the capacity planning models are explored with the uncertainty of new arrival demands. Hence, deterministic equivalents of the stochastic models are established to address both known and unknown distributional information of the uncertain demands. Insights of the derived models on planning the required capacities and on managing the admission of patients are summarized. The impact of distributional information of the uncertainty to the efficiency and the effectiveness of the proposed models is also investigated in this chapter. Finally, contributions of this research and possible future research works are summarized in Chapter 6.

CHAPTER 2: LITERATURE REVIEW

The thesis aims to propose planning solutions and appointment scheduling rules that can help outpatient systems to achieve a given appointment lead-time policy. Hence, the literature is reviewed in two areas: outpatient appointment systems and the patients' appointment lead-time. Figure 2.1 illustrates the structure of the literature review.

Figure 2.1: Structure of the literature review

The appointment systems of outpatient clinics have been studied for more than a half of century. One of the earliest researches was studied in early of the 1950s by Bailey (1952) that adopted a queuing theory to investigate relationships between patients' waiting time and physicians' idle time. The author analyzed impacts of patients' arriving to a clinic, an appointment interval, a number of patients per session, and a patients' arrival time onto the clinic performance (i.e., patients' waiting time and physicians' idle time). Many further studies investigated how to improve service performances of outpatient

appointment systems. The investigations are classified in capacity planning and appointment scheduling approaches.

Patient's appointment lead-time has also been a key concern raised by healthcare service management board due to its important performance role in improving the quality of population's life. Many studies focused on ways to reduce patients' appointment lead-time, such as Culyer and Cullis (1976) and Worthington (1987). The authors explored patients' appointment lead-time via evaluating waiting list behaviors to assess management options. Different admission criteria were developed in these studies.

The following is the outline of the literature reviews. Section 2.1 discusses about outpatient appointment system, which is followed by the review of patients' appointment lead-time, Section 2.2. Finally, research gaps of appointment systems of outpatient clinics are discussed in Section 2.3.

2.1. Outpatient appointment system

The review of outpatient appointment system involves literatures of the capacity planning and appointment scheduling approaches. The capacity planning aims to match the service provider's resource with arrival demands. On the other hand, the appointment scheduling focuses on providing guidelines of patients' arrival time to utilize the resource.

2.1.1. Capacity planning approach

Capacity planning of outpatient appointment systems can be categorized into capacity design and admission management problems. The capacity design studies aim to find an upper bound of required resources that links to the established appointment lead-time targets and vice versa. The optimal upper

bound provides a planning driver for healthcare service providers in plotting the optimal required capacity to meet the appointment requirements. In contrast, admission management literatures concentrate on managing available capacities as well as appointment requests. The Admission management literatures show its interest in obtaining the best performance, in terms of the maximum number of appointment requests that can be scheduled with the limited available resources.

a. Capacity design studies of outpatient appointment systems

Capacity design studies include “*directed capacity design*” and “*non-directed capacity design*” problems. The *directed capacity design* problem explores analytical, simulation, or queuing theory methods to design the required capacity level. On the other hand, the *non-directed capacity design* problem predicts the arrival demands. Summaries of *directed capacity design* and *non-directed capacity design* are presented in Appendices A1.1 and A1.2.

The *directed capacity design* problem was investigated by using a linear programming in Ittig (1985), analytical methods in Thomas et al. (2001), Bowers et al. (2005), Qu and Shi (2009), a simulation method in Elkhuisen et al. (2007), and a queuing theory in Green and Savin (2008). Ittig (1985) studied the capacity level to maximize profit while considering both linearly and exponentially dependent appointment lead-times. The profit is a function of patients’ revenue, additional capacity, and patients’ cost. The capacity level is subjected to constraints of arrival demands, which is a function of appointment lead-times. This study provided a trade-off between patients’ appointment lead-time and service capacity.

Thomas et al. (2001) suggested an estimated capacity function. The estimation of the capacity depends on a mean number of arrivals and a percentage of patients who will be seen within the lead-time targets. The study was to determine a surplus capacity to avoid long appointment lead-times of patients. The developed capacity function applies only for cases in which an absence of a waiting list is presented.

Bowers et al. (2005) provided a simple function to analyze a number of clinics required in each specialty from information of arrival demands, no-show rates, and an expected utilization of the clinic. The authors considered follow-up rates in the study. However, the authors assumed that patients only follow-up once. Moreover, though consultation times were assumed to be deterministic, it might be different amongst different groups of patients.

Qu and Shi (2009) derived a function of demands and no-show rates for two different types of patients (e.g., the patients with fixed appointments need to book in advance and those with open appointments are walk-in patients) to determine the expected number of scheduled patients and the variance of the number of consulted patients for each type. As the results, the required capacity was determined. Subsequently, different rules for determining the required capacity were proposed. Analyses of experimental results from the proposed rules showed impacts of no-show rates and patients' demand on both the number of patients consulted and the percentage of each type of patients.

Elkhuizen et al. (2007) briefly suggested a simulation approach to predict a capacity level with considering a mean and 95th percentile of patients' appointment lead-time. This study aims to provide a guideline of capacity

analyses for a steady state. Arrival demands and consultation times are assumed to be a Poisson distribution and a deterministic value, respectively.

Queuing theory has been employed extensively to model and analyze different healthcare systems (see Lakshmi and Sivakumar, 2013; for an overview). However, only Green and Savin (2008) investigated the *directed capacity design* problem of an outpatient appointment system supported by a queuing model. The derived queuing model helps to determine a capacity requirement for the same day appointment system. The study focused on identifying an appropriate compromise between physician capacity level and patient backlog. The developed models were carried out with assumptions of deterministic and exponential consultation times. The authors assumed a Poisson distribution for arrival demands.

The *non-directed capacity design* (Appendix A1.2) problem has recently received a great attention such as Leitch et al. (1990), Bennett and Worthington (1998), Porta-Sales et al. (2005), Bamford and Chatziaslan (2009), and Demir et al. (2012). A comparison of the studies is summarized in Appendix A1.2. Leitch et al. (1990) used a statistical analysis to decide whether a patient should be assigned a follow-up appointment or discharged. The study intends to eliminate unnecessary assignments of follow-up cases that could help to better plan for the required capacity.

A part of the study in Bennett and Worthington (1998) recommended additional required capacity decisions using a simple calculation on spreadsheet to simulate the result. The study suggests that appointment lead-times of follow-up cases should be considered in planning sizes of clinics, since the number of appointments saved is positively correlated with the length of

patients' appointment lead-time. The scheme of the required capacity is, therefore, optimized from the saved demands.

Porta-Sales et al. (2005) aimed to identify reasons of follow-up non-compliance such as becoming an inpatient, or passing away, or following-up somewhere else. The study proposes an equation to predict potential non-compliers to follow-up. Using the result, the predicted requirement of capacity is more accurate due to a low risk of non-compliances.

In Bamford and Chatziaslan (2009), the study introduced a survey method to measure an available capacity which helps to obtain realistic measure on the performance. The authors aimed to reveal a capacity measurement at the operational level, which considers the variation of available capacity throughout a year. From the finding, the human resources can plan the required personnel to match the demand.

Demir et al. (2012) suggested a model to predict a future demand, assuming that the demand linearly increases with time. The authors considered both independent and dependent demands between specialties. The study firstly estimated the current demand using confidence interval. Then, a linear regression method was applied to predict the future demand. In the study, an assumption of constant and same consultation time was made.

b. Admission management studies of outpatient appointment systems

Admission management problem is explored when there is an incapability of matching a service provider's capacity to the arrival demands. Admission management studies concentrate on maximizing the throughput or the number of patients seen. As a result, a resource allocation method is addressed to

manage optimally arrival demands for each type of patients, which brings service providers the greatest number of patients seen.

One of the studies addressing admission management problem is Qu et al. (2007). Qu et al. (2007) paid attention to optimizing the percentage of appointments held open per session. The study is to maximize the expected number of patients consulted. This study developed a mathematical function of the number of patients consulted, in which different no-show rates amongst types of patients are explored. The authors assumed that the distributions of arrival demands are known, and the number of scheduled patients is predetermined. However, only single horizon for the open appointments was investigated. Subsequently, the study was expanded to the hybrid horizon in Qu et al. (2011). All assumptions of the single horizon approach study were remained the same in the latter investigation.

2.1.2. Appointment scheduling approach

The appointment scheduling problem in the literatures focuses on optimizing patients' waiting time, physicians' idle time, and physicians' overtime. The appointment lead-times of patients are not considered in these studies. The studies of appointment scheduling are divided into the following three categories: queuing theory, simulation, and mathematical methods.

a. Queuing theory studies of the appointment scheduling problem

Studies exploring queuing theory (refer to Appendix A1.3) aim to find an optimal schedule, considering the trade-off between patients' waiting time and physicians' idle time as a measurement of service's performance. The studies investigated impacts of various factors such as the impact of no-show in

Brahimi and Worthington (1991), Liu and Liu (1998), and Hassin and Mendel (2008); the impact of doctors' punctuality in Liu and Liu (1998); and the impact of length of appointment intervals/slot intervals in Pegden and Rosenshine (1990) and Hassin and Mendel (2008). Brahimi and Worthington (1991), and Liu and Liu (1998) considered a deterministic appointment interval; while Pegden and Rosenshine (1990), and Hassin and Mendel (2008) tried to determine the optimal appointment intervals. In addition, these papers made various assumptions on a service time distribution. The stochastic service system was investigated with an exponential distribution in Hassin and Mendel (2008) and Pegden and Rosenshine (1990); and a general distribution in Brahimi and Worthington (1991) and Liu and Liu (1998). The punctuality of patient was assumed in the above studies.

b. Simulation studies of the appointment scheduling problem

Simulation studies for the appointment scheduling problem focus on *optimizing appointment scheduling, evaluating appointment scheduling rules, or evaluating patients' flow*. A summary of the studies for the above three groups of the appointment scheduling problem is presented in Appendix A1.4.

Firstly, the *optimizing appointment scheduling* studies focused on scheduling physicians' time and suggesting appointment scheduling policies in order to minimize patients' waiting time and physicians' idle time (Bailey, 1952, and Rising et al., 1973). The researchers investigated the relationship between patients' waiting time and physicians' idle time in these studies. The *optimizing appointment scheduling* studies aimed to determine the number of patients attending clinics and to identify the number of appointments per slot

which resulted an optimal appointment scheduling. Bailey (1952) did not include walk-in patients in the model whilst Rising et al. (1973) assumed a negative exponential arrival time for walk-in patients. Fixed and equal slot intervals were assumed in these studies.

Unlike the *optimizing appointment scheduling* studies, the *evaluating appointment scheduling rules* studies evaluated the different appointment rules under various scenarios, for example in Blanco White and Pike (1964), Vissers (1979), Vissers and Wijngaard (1979), Ho and Lau (1992), Klassen and Rohleder (1996), Su and Shih (2003), and LaGanga and Lawrence (2007). The combined objectives of either patients' waiting time or physicians' idle time or physicians' overtime were developed in these studies. Su and Shih (2003) paid extra attention to patients' throughput time and a utilization rate of the service provider. Patients' punctuality was assumed in Ho and Lau (1992), Klassen and Rohleder (1996), Su and Shih (2003), and LaGanga and Lawrence (2007). Likewise, Blanco White and Pike (1964), and Vissers (1979) considered the lateness of patients while Vissers and Wijngaard (1979) focused on early arrivals of patients.

With the *evaluating appointment scheduling rules* studies, the no-show rate is predetermined and included in the studies except in Su and Shih (2003). Impacts of physicians' lateness were examined in Blanco White and Pike (1964), Vissers (1979), and Vissers and Wijngaard (1979). An assumption of allowing walk-in patients was not made in Ho and Lau (1992) and in LaGanga and Lawrence (2007), but in Klassen and Rohleder (1996) and in Su and Shih (2003). Different assumptions of the consultation times were used amongst the studies.

Finally, *evaluating patient's flow* studies explored the effect of patients' flow and the impact of appointment scheduling on the stations' performance of a clinic. Harper and Gamlin (2003), and Wijewickrama and Takakuwa (2005) considered different priorities amongst categories of patients, while Cote (1999) gave the same priority to all patients. Cote (1999) measured utilization, patients' queue length, probability of full occupied stations, and patients' flow time. Harper and Gamlin (2003) considered the objectives of patients' waiting from their appointment time till their first time served, the percentage of patients who wait more than 30 minutes, and the mean time of patients spent in the clinic. Wijewickrama and Takakuwa (2005) examined the trade-off between the patients' waiting time and the doctors' idle time.

c. Mathematical studies of the appointment scheduling problem

The other studies address mathematical methods (Appendix A1.5) for scheduling patients' appointments. Most early studies focused on exploring the appointment scheduling for a known number of patients into equal appointment interval or optimizing the patients' appointment time, while the recent studies focused on maximizing the number of appointments scheduled along with the patients' sequence.

Firstly, the studies of the appointment scheduling problem for a given number of patients into equal slot interval examined combinations amongst either of patients' waiting time, physicians' idle time, or physicians' overtime. Fries and Marathe (1981) derived a dynamic programming for solving a D/M/1 system with an assumption of negative exponential service time. Vanden Bosch and Dietz (2000, 2001) proposed two heuristic algorithms for a D/G/1 problem.

The studies investigated a distinct truncated erlang and a gamma distribution of service times in Vanden Bosch and Deitz (2000), and a general distribution of service times followed by the examination of the solutions with an erlang distribution of service times in Vanden Bosch and Deitz (2001). Baker and Atherill (2002) introduced a heuristic discrete algorithm for a G/G/1 queue problem. The service time was assumed to be a gamma distribution. Kaandorp and Koole (2007) approached a condition of the multi-modular objective for D/M/1 queue problem, and then proposed a heuristic algorithm to obtain the optimal solution.

Fries and Marathe (1981) did not consider no-show characteristics in their studies, while the rest of authors assumed similar no-show rates (Vanden Bosch and Dietz, 2001; and Kaandorp and Koole, 2007) or different no-show rates amongst types of patients (Vanden Bosch and Dietz, 2000; and Baker and Atherill, 2002). However, only Baker and Atherill (2002) evaluated the impact of the patients' punctuality to the performance.

Secondly, the mathematical studies that optimize the patients' appointment times for a known number of patients were introduced in Wang (1993, 1997), Vanden Bosch (1999), and Denton. and Gupta (2003). Wang (1993, 1997) measured the physicians' completion time and patients' flow-time accounting the detail of patients' flow. The authors investigated a $S(n)/M/1$ queue in Wang (1993) and a D/G/1 queue in Wang (1997). The service time was assumed to be exponential in Wang (1993) and to be Coxian-type and in Wang (1997). Vanden Bosch (1999) and Denton and Gupta (2003) proposed heuristic algorithms to solve problems with an Erlang and a general service time, respectively. The authors considered the trade-off of the combinations

amongst either patients' waiting time or physicians' idle time, or physicians' overtime. The above studies assumed that the walk-in patients were not allowed and the scheduled patients always arrive on time.

Lastly, the studies addressing the maximum number of patients scheduled and the patients' sequence were explored in Muthuraman & Lawley (2008), Chakraborty et al. (2010), and Zeng et al. (2010). In these studies, heuristic algorithms with stopping criteria were derived to schedule patients' appointments into provided slots. The length of the provided slots might be different but the slots were predetermined. Muthuraman and Lawley (2008) and Chakraborty et al. (2010) provided heuristic algorithms to find the optimal solutions of unimodal problems. However, the former study paid attention to an exponential distributed service time while the latter study assumed a general distributed service time. Zeng et al. (2010) offered two sequential scheduling procedures to solve a multi-modular problem with an exponential distribution of service times. All of the above studies considered distinct no-show rates but excluded the lateness of patients in their models. Fixed and equal slot intervals were assumed in Muthuraman and Lawley (2008) and Chakraborty et al. (2010), while fixed but different slot lengths were explored in Zeng et al. (2010).

2.2. Appointment lead-time problem

The patients' appointment lead-time studies focused on three different categories: a capacity management, an admission management, and an appointment lead-time measurement. The capacity management and the admission management literatures made decisions on investments of service

rates and develop criteria to assign patients' appointments, respectively. The studies of appointment lead-time measurement proposed measurements to determine factors which impact on patients' appointment lead-times.

2.2.1. *Capacity management studies*

The capacity management studies (Appendix A1.6) explored relationships between investments of the scarce resource and patients' appointment lead-times at either the hospital level or the government level. In these studies, the authors measured the appointment lead-times directly or indirectly from a waiting list size.

Forst (1980) pursued a regression method to identify an appropriate percentage increase in the number of consultants to obtain an equilibrium waiting list of an inpatient system. The arrival demand was determined from the total number of patients in the waiting list. The service rate was determined from the number of consultants, the waiting list size, and the mean of appointment lead-times. A trade-off between additional consultants and patients' appointment lead-times provides an insight of resource investment.

Ittig (1985) derived a linear programming model to design an appropriate capacity level with a constraint of demand function and a queuing relationship between a demand, a service capacity, and a mean appointment lead-time. The study investigated both a linear and an exponential demand function of the mean appointment lead-time. An application of hospital outpatient facility was illustrated in the paper.

Worthington (1987) probed a queuing theory for an intuitive management action on the number of beds, mean of length of stay, feedback

policies etc. to govern a waiting list size. A linearly decreasing of arrival rates was assumed. A mean and a coefficient of variation for service rate were used in developing the model. The expected anticipated appointment lead-time was determined from the waiting list size.

Worthington (1991) used a projection method to evaluate different capacity planning alternatives considering routines of patients. The goal of the study was to predict the future waiting list size as well as the mean appointment lead-time. The author restricted the appointment lead-times of RV patients, who re-visit as an outpatient or as an inpatient. The restricted appointment lead-time lengths were set to be within 2 weeks, 2 months, and 6 months depending on a priority of patient groups.

Iversen (1993) provided a stochastic non-linear programming model to help a government in funding a budget to a hospital. The funded budget was subjected to an estimated optimal appointment lead-time provided from the hospital. In this study, the author assumed an exponential distribution time for subsequent arrivals and an exponential service rate. Only a general model was proposed and none of methodologies was suggested to solve the problem. In addition, Iversen (1993) also provided a model at the hospital level which can help the service provider to control patients' admission. However, this perspective of the study will not be discussed until the next review of admission management literatures.

In Street and Duckett (1996), an empirical method was proposed to recognize a financial incentive structure which relates to the waiting list. The empirical results from a case study of surgical service in Australia showed that the waiting list can be resolved by introducing a funding-reward policy

according to the service provider workload. Rewards of different categories of patients were suggested to consider a funding policy.

Bowers (2010) proposed a predictive model for an additional resource. Proposed changes of appointment lead-times to the investment were simulated. Besides, analysis of different scenarios was also presented. The author aimed to provide a better intuition of information needed to achieve the targets of patients' appointment lead-time.

2.2.2. *Admission management studies*

Likewise, Culyer and Cullis (1976), Leitch et al. (1990), Ratcliffe et al. (2001), Kopach et al. (2007), Bibi et al. (2007), Goddard and Tavakoli (2008), and Dixon and Siciliani (2009) concentrated on controlling admissions of patients, accounting for the patients' appointment lead-time. Culyer and Cullis (1976), Leitch et al. (1990), Ratcliffe et al. (2001), and Goddard and Tavakoli (2008) developed priority admission criteria. On the other hand, Bibi et al. (2007) explored the impacts of the centralization and over-booking policies onto the number of scheduled patients and the mean appointment lead-time, while Dixon and Siciliani (2009) focused on a mathematical link between the waiting list and patients' appointment lead-times. Summary of the mentioned study is presented in Appendix A1.7.

Culyer and Cullis (1976) categorized the criteria of patients to schedule their admissions. A priority index of admission included a patient's spent time in the waiting list, an expected rate of deterioration, a degree of disability, and social factors. The public interest of fairness and efficiency factors was embodied in the study. The appointment lead-time was regulated by the number

available beds, a length of stay, an operating theater capacity etc. The authors proposed a maximal reduction in need out of given resources and of an obtained balance amongst departments as the study's objective.

Leitch et al. (1990) controlled admissions by evaluating the need of follow-up cases in an outpatient clinic. With this approach, criteria of discharged patients are discussed via reviewing the entry of the trial in one year. The morbidity of patients was considered when evaluating the standard for being discharged.

Ratchiffe et al. (2001) considered an incremental cost effectiveness ratio between a mean of patients' costs and a mean of survival lengths. The authors aimed to match criteria of donors and recipients. Simulated results of this modeling were analyzed to suggest trends of allocation policy for transplantation program. A median and a range of inter-arrival times and those of service times were used in the simulation.

A part of the study in Kopach et al. (2007), the authors examined effects of different variables on patients' preference for selections of physicians and on the patient throughput. A fraction of open-access patients and a scheduling horizon (considered as maximum appointment lead-time) were examined. Besides, the authors accounted for the flow of patients at clinics as well. The percentage of patients who are able to visit the same physician was measured to evaluate the satisfaction of patients. Findings showed that either the increase of patients' appointment lead-time or the decrease of open-access percentage leads to a low throughput.

Bibi et al. (2007) reported insights of an intervention program onto the appointment lead-time. An overbooking practice and a centralization policy

were examined in the intervention. Performances between the non-intervention and the intervention programs were compared to prove the necessity of the intervention program. Measurements of the performances were a number of scheduled appointments, an average appointment lead-time, a service provider's capacity, a satisfaction rate, and no-show rates.

Goddard and Tavakoli (2008) investigated an adjustment of patients' demand via priorities of patients' access. The authors approached a queuing theory method to determine a priority index for an admission. The study aimed to optimize the proportion of each daily cohort for sick patients, the mean expected duration of wait, and the expected duration of wait for the most marginal patient. The expected time to clear a waiting list and a maximum appointment lead-time were considered as achievable targets in the model. The same no-show rates regarded as withdraw rates are assumed in the study.

Dixon and Siciliani (2009) derived a mathematical method to compare and to link a distribution of patients in the waiting list with a distribution of patients' appointment lead-times. The authors showed that the appointment lead-time can be calculated from the number of patients in the list, and vice versa. A density function and a cumulative distribution were used to calculate one performance from the other.

2.2.3. *Appointment lead-time measurement studies*

The studies of the appointment lead-time measurement (Appendix A1.8) provide waiting list statistics which pay attention to the patients' appointment lead-time. Don et al. (1987) analyzed the difference between the prevalence and the incidence measurements of the patients' appointment lead-time. From

the findings, the authors emphasized key considerations (the time between placement on the waiting list and admission, inclusion/exclusion of the patients who will never be admitted from the waiting list due to occurrences of elective admission or dying before being admitted, etc.) when using data to calculate lead-times.

Dixon (2004) continued to identify factors that cause the difference between appointment lead-time measurements. The measurement of patient's appointment lead-time was implemented from two different sources of data. The detailed differences of mean and median wait were presented. This report was useful for observing patients' experience on the waiting list for a particular date.

Torkki et al. (2002) reported discrepancies in the waiting list management at a surgery department. Findings implied that the accuracy of a waiting list measurement is obtained from the best queue discipline since an accurate measurement of the waiting list provides a good balance between arrival demands and departure rates. The method can be used to disclose the performance of the waiting list management.

Tsang and Resneck (2006) presented findings of patients' appointment lead-time when patients switch a service mode. The authors investigated impacts of different patient categories, i.e., Medicare versus Cash-paying patients, on the patients' access and patients' appointment lead-time. Besides, the relationship between a physician extender and the patient appointment lead-time was also revealed.

Dimakou et al. (2009) reported impacts of appointment lead-time targets to the actual appointment lead-times. The appointment lead-time targets

were established by a government. The relationship between the appointment lead-times and the rates of survival and hazard was explored. According to the findings, suggestions of admission tactics can be initiated.

2.3. Research gaps

The studies, reviewed in Sections 2.1 and 2.2, propose various approaches for solving the problems of outpatient appointment systems. To facilitate the understanding of the contributions of the thesis, summary of the gaps between the literatures and TTSH problem is presented in Table 2.1.

The studies of the outpatient appointment system have explored either the capacity planning or the appointment scheduling approach for solving the problem. Although Bowers et al. (2005) considered follow-up demands in planning capacity, the study assumed that patients follow-up only once. Besides, the study did not consider the patients' appointment lead-time. In contrast to the study of Bowers et al. (2005), Ittig (1985), Thomas et al. (2001), and Elkhuizen et al. (2007) addressed the appointment lead-time characteristics. Ittig (1985) paid attention to the appointment lead-time, in terms of maximum value, in depth. However, the author developed a mathematical model for a non-reentry system. Thomas et al. (2001) developed an analytical function to estimate the surplus capacity. The estimated surplus capacity is to serve a certain percentage of patients who will be seen within the lead-time targets. The study was still for a non-reentry system. Elkhuizen et al. (2007) briefly suggested a simulation method to evaluate the 95th percentile appointment lead-time of patients for a non-reentry system. Neither the re-entry

characteristic nor appointment lead-time targets were considered in the studies of outpatient appointment systems.

Table 2.1: Summary of the research gaps

No.	Up-to-date researches	TTSH's outpatient appointment system
1	<ul style="list-style-type: none"> • None re-entry systems • Semi re-entry systems 	A re-entry system
2	<ul style="list-style-type: none"> • Patients' waiting time • Physicians' idle and/or overtime. 	Patients' appointment lead-times
3	<p>Investigate:</p> <ul style="list-style-type: none"> • Capacity design/ management • Admission management via a waiting list size. • Appointment scheduling rules (e.g. patients' slot interval) 	<p>Needs:</p> <ul style="list-style-type: none"> • Capacity design/ management • Admission management via a waiting list size. • Appointment scheduling rules (e.g. patients' appointment date)

Furthermore, the studies addressing patients' appointment lead-times aimed to deploy the capacity management, to develop constraints for controlling patients' admission, or to introduce measurements for a report of patients' appointment lead-time. Firstly, the studies of the capacity management examined the patients' appointment lead-times via the waiting list size to make decision on a resource investment. The authors ignored the re-entry characteristic in their investigations. Secondly, the studies of the demand management paid attention to the re-entry characteristic in deriving the constraints to schedule patients' admissions. However, the patients' appointment lead-times were neglected in developing the admission constraints. Finally, the studies of the appointment lead-time measurements only aim to reveal the differences of the appointment lead-time value from different measurements.

Figure 2.2: Summary of the literature review and content of the thesis

Figure 2.2 shows the content of the thesis among the reviewed literatures. The studies in this thesis are highlighted in the mentioned figure. In general, none of the studies has researched the appointment lead-times for the re-entry outpatient appointment system as that is currently implemented at TTSH. Therefore, the further development of the capacity planning as well as the appointment scheduling approaches for the re-entry system is necessary to address the research gaps. The achievement of patients' appointment lead-time targets is compulsory in both approaches.

Although re-entrant manufacturing systems have been an active research topic in production, the instantaneous re-entrant characteristics of production lines are a focus. For example, Narahari and Khan (1996b), Choi and Ko (2009), and Fattahi et al. (2010) explored a scheduling policy to maximize the system throughput; while Chen et al. (2008) and Yang et al. (2008) proposed heuristic searches to minimize the items' completion time. Narahari and Khan (1996a), and Choi and Ko (2009) focused on allocating workstations for the maximization of the system throughput. In particular, Narahari and Khan (1996a) determined the number of inspection stations which considers the possibilities of an acceptance, a rejection, or a rework at some previous stages. Choi and Ko (2009) addressed simulation based approach for an allocation of the workstation process and buffering capability. Koole and Pot (2006) investigated a scheduling policy to balance the workload in the re-entrant manufacturing system. The authors aim to minimize the total average expected workload of the system. In general, no studies of the re-entrant manufacturing systems have considered the restriction of the items'

lead-time in their studies. Hence, it is a great opportunity but challenge to explore the promising field of research.

In the next three chapters, mathematical models for capacity planning and algorithms for the appointment scheduling problems are introduced. The capacity planning models aim to make decisions on the optimal required capacity or the optimal admission schemes. The patients' appointment lead-times are the main constraints in the models. The study is begun with the assumption of deterministic arrival demand, and later relaxed to be stochastic. The studies concentrate on the strategic and tactical levels. In addition, the appointment scheduling algorithms execute the optimal capacity planning scheme at the operational level. The derived algorithms with quantitative admission constraints are to schedule every individual patient's appointment to utilize the resource. The achievement of appointment lead-times targets is a compulsory constraint when carrying out the algorithms.

CHAPTER 3: CAPACITY PLANNING MODELS OF RE-ENTRY APPOINTMENT SYSTEMS WITH DETERMINISTIC DEMAND

3.1. Problem statement

Since a long appointment lead-time is considered as unacceptable for healthcare services, TTSH has been pressurized to shorten the patients' appointment lead-time and to achieve the MOH's guidelines on patients' appointment lead-times for median, 95th percentile, and 100th percentile. A systematic way with a mathematical method for an optimal capacity planning is useful to improve the patients' appointment lead-time. The capacity planning optimization can help to increase the access of patients in terms of the appointment lead-time. However, any arrival of a new patient may lead to an increased demand from follow-up cases to assure the continuity of care. Hence, the successful implementation of the capacity planning needs to consider the contribution of RV demands.

In order to match the healthcare demand and supply, the hospital needs to make the capacity planning decisions at the strategic and tactical levels. The decisions made at the strategic level include an optimal requirement of physicians and appropriate allocations of the required physicians to each type of patients. The capacity design that accounts for the healthcare demand is needed for the future investment of required resource. On the other hand, the tactical level concentrates on optimizing the capacity planning problem in which it is not possible to increase capacity. As a result, an admission

management decision is needed to achieve the MOH's appointment lead-time targets. This level decides on the optimal allocation of a given capacity to allow the greatest number of patients to be seen.

According to the literature review in Chapter 2, although many studies approached the capacity planning problems, neither the re-entry characteristic nor the patients' appointment lead-time was included. To narrow the research gap, the thesis explores the following questions in this chapter.

- (1) What is the optimal required resource level for the re-entry system to achieve the service targets?
- (2) What is the optimal admission policy for the re-entry system for a given capacity level and subject to established appointment lead-time targets?

Planning capacity for the re-entry system is considered for a finite horizon, deterministic arrivals of FV patients, and dependent arrivals of RV patients. An actual patient can generate more than one patient-visit is highlighted. Therefore, "patient" will represent "patient-visit" throughout the thesis. The FV patients' appointment lead-time targets and the range of the RV patients' appointment lead-times are provided.

A network flow model is adopted to develop the capacity planning models; however, only the deterministic arrival demands are considered in this chapter. This chapter has four sections. In Section 3.2, the network flow model is developed. Then, the mathematical modeling formulations of the two different problems, the capacity design and the admission management problems, are presented in Sections 3.3 and 3.4, respectively. Finally, we provide a summary in Section 3.5.

3.2. Model development

Figure 3.1 demonstrates a network flow model for the capacity planning problem. Let u , v , and w correspondingly be a median, p^{th} percentile, and 100^{th} percentile of FV appointment lead-time targets; for instance, patients' appointment lead-time targets might be two weeks for the median, six weeks for the 95th percentile and ten weeks for the 100th percentile. Furthermore, $[a, b]$ and $\bar{a}$ denote the restrictions on the RV appointment lead-time range and on the RV's mean appointment lead-time, respectively; for instance, it might require that a RV patient needs to be scheduled between two and ten weeks, with a mean of seven weeks. Summary of notation is listed in Table 3.1

Figure 3.1: A network flow model for planning capacity of the re-entry appointment system

The first S time-units are termed to be the arrival horizon. The study assumes that FV demands are known for S time-units into the future. However, due to the patient dynamics and the inherent appointment lead-times, the model horizon is extended to T time-units; T needs to be significantly long to cover the maximum appointment lead-time allowed for

the last arriving FV patient. Moreover, the objective of the problem is the minimization of the maximum required capacity. Therefore, the maximum appointment lead-time for RV patients needs to be also considered when setting T . In other words, $T = \text{Max}[S + w; S + b]$ so as to cover the maximum possible appointment lead times for both FV and RV patients that arrive within the arrival horizon.

Table 3.1: List of notation for the capacity planning problem

	Notation	Descriptions
Decision variables	z_{ij}	Number of FV patients who make a request in the i^{th} time-unit and have their appointment scheduled in the j^{th} time-unit.
	x_{ij}	The number of FV patients who request in the i^{th} time-unit and have appointment in the j^{th} time-unit, and still remain in the system as RV patients after their appointment in the j^{th} time-unit.
	y_{ij}	The number of RV patients who have an appointment in the i^{th} time-unit and have their next appointment in the j^{th} time-unit, and still remain as RV patients after the j^{th} time-unit.
	d_i^f	The number of FV patients who make a request in the i^{th} time-unit, and are discharged after their first appointment.
	d_j^r	The number of RV patients who are discharged after their appointment in time-unit j .
	f_i^a	The number of admitted FV patients who make a request in the i^{th} time-unit.
	f_i^d	The number of denied FV patients who make a request in the i^{th} time-unit.
	f^d	Total number of denied requests
	c_i^f	Capacity for FV patients in the i^{th} time-unit, minutes.
	c_j^r	Capacity for RV patients in the j^{th} time-unit, minutes.
	c_i	Total capacity in the i^{th} time-unit, minutes.
	q	The maximum required capacity per time-unit, minutes.

Table 3.1: List of notation for the capacity planning problem (cont')

	Notation	Descriptions
Definition of sets	N	Index set for the arrival horizon.
	Z	Set of all $z_{ij}: j - i \geq 0$.
	L^m, L^p	Set of all $z_{ij} \in Z$ that have $j - i \leq u$ and set of all $z_{ij} \in Z$ that have $j - i \leq v$, respectively.
Inputs	u, v, w	Appointment lead-time targets for median, p^{th} percentile, and 100 th percentile respectively, time-units; ($u < v < w$).
	p	A p^{th} percentile of a given appointment lead-time target v ; ($50 < p < 100$).
	τ^f, τ^r	Consultation times for FV and RV patients, in minutes. These times are specified by the doctors.
	α, β	Discharge rates for FV and RV patients, respectively ($0 < \alpha, \beta < 1$)
	f_i	The number of FV patients' requests that arrive in the i^{th} time-unit.
	r_j^f, r_j^r	The number of pre-scheduled FV and RV patients with appointments in time-unit j , who still remain as RV patients after their appointments.
	$[a, b]$	Restricted appointment lead-time of RV patients or range of RV appointment lead-time, time-units.
	$\bar{a}$	RV's mean appointment lead-time, time-units.
Other notation	S	The number of time units in the arrival horizon
	T	The number of time-units in the planning horizon, $T = \text{Max}(S + b, S + w)$.
	i, j	Indices for time-units.

The above description leads to design of S and T nodes that represent FV and RV patient nodes (or FV-nodes and RV-nodes), respectively. FV-nodes are numbered F1, F2, up to FS, and RV-nodes R1, R2, up to RT. Assumptions are summarized in Table 3.2.

Table 3.2: List of assumptions for the capacity planning problem

No.	Assumptions
1	Patient requests cannot be rejected.
2	The designed capacity needs to be sufficient for both FV and RV patients, who are already in the system at the start.
3	The model considers a single type of FV patients and a single type of RV patients in terms of consultation times, appointment lead-times, and discharge rates.
4	Physicians are identical in terms of consultation times
5	A patient's appointment can be scheduled in the same time-unit as the patient's request.
6	There is no uncertainty in the rate at which patients arrive for their appointment requests.
7	Patients continue to be RV patients until they are discharged from the system.
8	An FV patient is a patient who calls to request an initial appointment. After the first appointment the FV patient might be discharged; if not, the FV patient becomes a RV patient and requests a re-visit appointment.
9	The model does not consider discharges after the arrival horizon.
10	There are no new requests after the arrival horizon.
11	There are constant discharge rates α, β for FV and RV patients respectively.

The number of admitted FV patients at each time-unit, f_i^a , is signified by the black thick solid arrows that go into the FV-nodes. The number of scheduled FV patients is decided from the deterministic arrivals of new demands at the corresponding time-unit i , which is defined as f_i . The grey thick solid arrows that go into RV-nodes represent the total number of pre-assigned FV patients, r_j^f , and pre-assigned RV patients, r_j^r . The FV pre-assigned patients are patients who made their requests before the start of the

arrival horizon and their appointment dates are within the arrival horizon. The RV pre-assigned patients are patients whose last appointment was before the start of the arrival horizon but their following appointment is within the arrival horizon.

The impact of discharge rates is considered on designing the clinic capacity. The discharge rates may be different following each visit, but for the sake of model simplification, constant discharge rates α, β are assumed for FV and RV patients respectively. Since the discharge rates are different, it is necessary for the model to track whether a patient visit is a first visit or a return visit.

The thick dashed arrows that go out from FV-nodes represent the total number of FV discharges, d_i^f , namely the number of FV patients that are discharged after their first appointment. For example, the thick dashed arrow that goes out from node F1 represents the total number of discharges for FV patients, who make requests for appointments in the first time-unit. However, the schedule for their appointments may be at another time. Some patient's appointment dates could be on the first time-unit, or on the second time-unit or up to the T^{th} time-unit.

The thick dashed arrows that go out from RV-nodes represent total number of RV discharges, d_j^r , for patients that are discharged as RV patients. For example, the thick dashed arrow that goes out from node R1 represents the total number of patients that will be discharged after their next appointment. The time until the next appointment is between a to b time-units from the 1st time-unit.

The thin solid lines that connect FV-nodes and RV-nodes show the number of FV patients who make their request in time-unit i and are assigned appointments in time-unit j , and that will remain in the system after their consultation in time-unit j . It is denoted by x_{ij} . The patients' appointment date is allowed to be the same as their request date. As mentioned above, the constant FV discharge rate, α , is assumed. Therefore, the thin solid arrows that connect FV-nodes and RV-nodes can also be used to relate to the assignments of appointment requests for appointed FV patients.

The dashed arrows that connect two RV-nodes signify the number of RV patients with appointments in time-unit i that have their next appointment in time-unit j , and that will not be discharged at time-unit j . It is denoted by y_{ij} . The randomness in the no-show is not modeled. Effectively, it is assumed that all patients will arrive; alternatively it could be assumed that there is a fixed fraction that does not arrive. Furthermore, assumption of identical physicians, both in the eyes of the patients and in their consultation time, is made. Hence, patients exercise no choice as to which physician to see, or the timing of their appointment. The assumptions simplify the model, and it is believed that the assumptions are acceptable given the intent to use the model for tactical planning. Finally, patients remain as a RV patient until he/she is discharged.

A mixed integer programming model is adopted for solving the capacity planning problem (refer Section 1.1 for its advantage and Figure 2.2 for the gap of the studied methodologies). Planning the capacity of the service provider to the patients' demands is explored via examining the optimal required capacity and the optimal admission schemes. The study of an optimal

required capacity policy is obtained from the capacity design approach in Section 3.3. Subsequently, the study of an optimal admission policy is acquired from the admission management approach in Section 3.4. Both the capacity design and the admission management problem need to preserve the conservation of flow at every node and the constrained capacity through every node.

3.3. Capacity design approach for an optimal required physicians

The rejection of the requests of patients for treatments is considered unacceptable for healthcare services where all sick patients must be given treatments. The long-term decision of future investments on the service provider's resource is needed. Hence, an efficient capacity design approach is proposed to handle this problem. This approach aims to determine the optimal number of required physicians to serve all the arrival demands without violating the appointment lead-time targets of MOH.

3.3.1. *Mathematical modeling for the capacity design approach*

The objective of this problem is to minimize the required physicians assuming that all appointment requests must be scheduled. Moreover, although only the arrival demand for the first S periods is considered, the designed capacity needs to be sufficient for both FV and RV patients who are already in the system after the S^{th} arrival horizon. The mixed integer program model for the capacity design problem (P1) is presented as following:

[P1]

$$\text{Min } q \quad (3.1')$$

Subject to:

$$q \geq c_j, \forall j \in N, \quad (3.2')$$

$$\sum_{j=i}^T z_{ij} = f_i, \forall i \in N, \quad (3.3')$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (3.4')$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (3.5')$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (3.6')$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (3.7')$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (3.8')$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (3.9')$$

$$y_{ij} = 0, \forall j \geq T + 1, \forall i \leq S, \quad (3.10')$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (3.11')$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2}(\sum_{i=1}^S \sum_{j=i}^T z_{ij}) + 1, \quad (3.12')$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100}(\sum_{i=1}^S \sum_{j=i}^T z_{ij}), \quad (3.13')$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (3.14')$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (3.15')$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \forall j \in N, \quad (3.16')$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \forall j \in N, \quad (3.17')$$

$$c_j - (c_j^f + c_j^r) = 0, \forall j \in N, \quad (3.18')$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r \geq 0, \forall i, j \in N, \quad (3.19')$$

$$z_{ij} \text{ integer}, \forall i, j \in N. \quad (3.20')$$

The objective (3.1') minimizes the required capacity, per time-unit, to achieve the appointment lead-time targets. The required capacity for the clinic is shown in constraint (3.2'). The above objective leads to the optimal allocated capacity between FV and RV patients as well, in each time-unit. The conservation flow at FV-nodes is represented in constraints (3.3'), (3.4'), and (3.5'). Constraint (3.3') schedules appointments for all requests. Constraint (3.4') determines the RV patients who remain after the first visit. The number of FV patients who are discharged after their first visit is specified in constraint (3.5').

The conservation of flow at RV-nodes is modeled in constraints (3.6') and (3.7'). The first bracket of constraint (3.6') is the total number of pre-assigned FV and RV patients plus the total number of FV patients, plus the total number of RV patients, all with appointments in time-unit j . The second bracket of constraint (3.6') is the number of RV patients discharged after their visit in time-unit j plus the number of RV patients remaining with a subsequent appointment (the flows that go out from the same time-unit j). Constraint (3.7') specifies the number of patients discharged after their visit in time-unit j .

Constraints (3.8') and (3.9') assure that no RV patients will have appointment lead-times falling outside the restricted range $[a, b]$. We accomplish this by defining y_{ij} to be zero if the time interval between i and j falls outside the restricted range. Constraint (3.10') prohibits RV patients' appointments to be made after last date of the model horizon T . Constraint (3.11') specifies the number of RV patients discharged after their visit in time-unit i . Constraints (3.12') and (3.13') assure that the targets of the appointment

lead-time for the median and p^{th} percentile are satisfied, respectively. Constraint (3.14') assures a realistic appointment date and does not allow appointments that go beyond the 100th percentile of limit. The restriction on the RV mean lead-time is represented in constraint (3.15').

The required capacities of FV and RV patients in time-unit j are determined in constraints (3.16') and (3.17'), respectively. The total required capacity in time-unit j is presented in constraint (3.18'). The required capacity for either FV or RV patients includes the required capacity of pre-assigned patients and that of the new arrivals. Constraint (3.19') requires non-negative variables. Finally, constraint (3.20') requires the integer assignment of FV patients.

3.3.2. *Numerical experiments*

In this part, numerical experiments are performed to investigate how the proposed mathematical model sets the appointments for patients. We report on four performance metrics: the maximum required capacity, the mean required capacity, the FV's mean appointment lead-time, and the RV's mean appointment lead-time.

a. Experimental design

Data from years 2008, 2009, and 2010 were obtained from the Urology specialty. The data from year 2008 is used to determine the number of pre-assigned FV and RV patients in the model. The data from years 2009 and 2010 are used to determine the inputs on the arrival of FV patients, on the discharge rates of FV and RV, and on the restricted appointment lead-times of RV patients. The design of data is due to the fact that the FV and RV pre-

assigned patients of year 2009 came from requests in 2008, and the FV demands of year 2009 were scheduled with appointments in years 2009 and 2010. The length of the RV mean appointment lead-time is currently not restricted. Hence, the optimization model, as formulated, would set the appointment lead-time for RV patients to be unrealistically long. To overcome this, the range for the RV appointment lead-times is set based on expert judgment; a constraint on the mean RV appointment lead-time is also imposed. The detailed inputs are listed in Appendices A2.1 and A2.2.

The numerical experiments were implemented with the year 2009 as the arrival horizon. The planning horizon extends into 2010. The total number of pre-assigned FV and RV patients are 1,277 and 10,970 respectively. As the arrival horizon is 2009, pre-assigned patients are any patient who made an appointment request before Jan 1st 2009 and had their appointment scheduled on or after Jan 1st 2009. In addition, the number of total new FV requests in 2009 is 9,760 patients. The model is set up to plan the required capacity on a weekly basis.

Due to the integer variables and the desire to find a global optimal solution, the Branch and Cut algorithm (Bertsimas and Tsitsiklis, 1997) a variant of the Branch and Bound algorithm, is chosen to solve the problem. Furthermore, the chosen algorithm can provide faster computation time to find the optimal solution/s than the Branch and Bound algorithm does, due to the advantages from running the Branch and Bound algorithm with cutting planes.

The computation time to find the optimal solutions to all numerical examples is less than four minutes, implemented in IBM ILOG CPLEX

Optimization Studio V12.4 on HP Pavilion G6 Notebook 1.9GHz and 4GB RAM personal computer with the Windows 7 operating system.

b. Experimental results

Given inputs as mentioned in the experimental design part above (detailed inputs are in Appendix A2), Figures 3.2, 3.3 and 3.4 present a comparison between the historical and the model's performance in terms of the cumulative required capacity, the mean required capacity per week, and the maximum required capacity per week, respectively.

Figure 3.2: The cumulative capacity of actual vs. model for the capacity design problem

The actual cumulative capacity (Figure 3.2) for FV patients is always less than what should have been provided according to the model. However, in terms of RV patients, the actual cumulative capacity is above what is obtained

from the model. The implication of the result is that the hospital could achieve the lead-time targets by providing more capacity to FV patients and less capacity to RV patients while maintaining the same total capacity. In other words, the proposed model provides the most efficient and effective planned capacity by systematically pushing and pulling the RV and FV patients' appointments so that the clinic can achieve the FV's appointment lead-time targets with the least maximum required capacity.

Figure 3.3: The mean required capacity of actual vs. model for the capacity design problem

A further look at the results, the mean required capacity, the maximum required capacity, and the mean appointment lead-time of the proposed model and the actual plan are compared. Figure 3.3 shows that for the FV patients the mean number of the required slots (134.5 slots per week) from the model is slightly more than that of the actual provided slots (131.6 slots per week). However, for the RV patients the mean number of the required slots (352 slots per week) from the model is much less than that was provided (394 slots per week) from the actual plan. The mean total number of the required slots from

the model (487 slots per week) is less than that of the provided slots from the actual plan (525 slots per week) for both FV and RV patients. In general, we can achieve the appointment lead-time targets without requiring additional capacity.

Furthermore, Figure 3.4 demonstrates that the maximum required capacity obtained from the model is only 680 slots per week (14.5 full-time equivalent doctors) for both FV and RV patients. This is less than values extracted from the actual plan, 751 slots per week (17.8 full-time equivalent doctors). This is evidence of the potential efficacy of the model for capacity planning. Since physicians are a scarce resource, the reduction in required resources is significant.

Figure 3.4: The maximum required capacity of actual vs. model for the capacity design problem

The drivers for this improvement are investigated with a detailed study on the maximum required capacity for each FV or RV patient type. The results (Figure 3.4) illustrate that the maximum required capacities for each FV and RV patient type from the proposed model are actually 2 and 1.3 times,

respectively, as much as those from the actual plan. This suggests that reasonable assignments or allocations of these resources might vary from week to week to accommodate the appointment schedules and to achieve the appointment targets. The suggested model can help to do this, by proactively adjusting the appointments allocations for RV and FV patients to minimize the maximum required number of physicians.

Finally, the results show that the model produces shorter appointment lead-times for the newly arriving patients while still being able to maintain the continuity of treatments for RV patients. The FV's mean appointment lead-time of the proposed model is 4.2 weeks. This is 35% lower than the FV's mean appointment lead-time of the actual plan (6.4 weeks). A consequence is that the appointment lead-times of RV's patients are increased. The mean lead-times of RV patients from the model and the actual plan are 23 and 16 weeks respectively. However, this increase in lead-time does not violate the time constraints between treatments, and the continuity of care is maintained.

From the above results, we see that the proposed model provides a systematic view to optimize the required capacity via the push-pull mechanism, and can produce better capacity plans than the actual practice. The push part of the mechanism is to delay the appointments of the RV patients, while the pull part of the mechanism is to bring the appointments of the FV patients forward to be within the MOH's appointment lead-time targets.

3.3.3. Sensitivity analysis

This subsection examines the sensitivity of the model's objective to changes of the appointment lead-time targets, of the restriction on RV's mean appointment lead-time, and of the discharge rates. Table 3.3 lists terminology conventions for displaying and discussing the findings.

3.3.3.a. Sensitivity to changes of the appointment lead-times for the capacity design problem

This part examines the sensitivity of the maximum required capacity, the mean required capacity, FV's mean appointment lead-time, and RV's mean appointment lead-time when changing the FV appointment lead-time targets as well as the restricted length of RV's mean appointment lead-time. The changes of the FV appointment lead-time targets refer to changes of the median, or the 95th percentile, or the 100th percentile of appointment lead-time target.

Table 3.3: Definition of signs for sensitivity analysis

No.	Sign	Descriptions
1	-	A negative correlation between the lead-time target, under a specific lead-time target category, and the measurement.
2	+	A positive correlation between the lead-time target, under a specific lead-time target category, and the measurement.
3	0	The measurement's value remains the same while changing the lead-time target that belongs to the same category.
4	1	An indeterminate correlation.
5	Inf.	Infeasible solution/s

Table 3.4: Summary of sensitivity analysis to changes of appointment lead-times for the capacity design problem

RV's mean appointment lead-time (weeks)			Maximum required capacity					Mean required capacity					FV's mean appointment lead-time					RV's mean appointment lead-time					
	FV's discharge rate		0.38		0.38*	0.5	0.7	0.38		0.38*	0.5	0.7	0.38		0.38*	0.5	0.7	0.38		0.38*	0.5	0.7	
	RV's discharge rate		0.1	0.2	0.32*	0.32		0.1	0.2	0.32*	0.32		0.1	0.2	0.32*	0.32		0.1	0.2	0.32*	0.32		
5	Lead-time target	Median	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	0	0	0	0	0	
		95 th percentile	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	0	0	0	0	0
		100 th percentile	-	-	-	-	-	1	1	1	1	1	1	+	+	+	+	+	0	0	0	0	0
10		Median	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	0	0	0	0	0	
		95 th percentile	-	-	-	-	-	-	-	-	-	1	+	+	+	+	+	0	0	0	0	0	
		100 th percentile	-	-	-	-	-	-	1	-	1	1	+	+	+	+	+	0	0	0	0	0	
16*		Median	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	0	0	0	0	0	
		95 th percentile	-	-	-	-	-	-	-	-	-	1	+	+	+	+	+	0	0	0	0	0	
		100 th percentile	-	-	-	-	-	1	1	-	1	1	+	+	+	+	+	0	0	0	0	0	
20		Median	-	-	-	-	0	-	-	-	-	1	+	+	+	+	1	0	0	0	0	0	
		95 th percentile	-	-	-	-	0	1	-	-	1	1	+	+	+	+	+	0	0	0	0	0	
		100 th percentile	-	-	-	-	0	1	-	1	1	1	+	+	+	+	1	0	0	0	0	0	
25		Median	-	0	0	0	0	-	1	1	1	1	+	+	+	1	1	0	0	-	1	1	
		95 th percentile	-	-	0	0	0	1	1	1	1	1	+	+	+	+	+	0	0	1	1	1	
		100 th percentile	-	0	0	0	0	-	1	1	1	1	+	1	1	+	1	0	0	1	1	1	
30		Median	0	0	0	0	0	0	-	-	1	1	1	+	+	+	1	1	1	-	1	1	
		95 th percentile	-	0	0	0	0	1	1	+	1	1	+	+	+	+	1	1	1	1	1	1	
		100 th percentile	-	0	0	0	0	-	1	1	1	1	+	1	1	1	1	1	1	1	1	1	

* These are the historical RV's mean appointment lead-time and discharge rates at TTSH

Numerical experiments are performed for 5 different sets of discharge rates, for 6 levels of RV's mean lead-time, and 15 different sets of appointment lead-time targets (refer to Appendix 3 for the combinations that were considered). Thus, $5 \times 6 \times 15 = 450$ test cases were set up and solved.

Table 3.4 presents a summary of the sensitivity analysis to the changes of appointment lead-times. The details of those inputs and results are presented in Appendices A3.1 to A3.7.

The results show that the maximum required capacity decreases as an appointment lead-time target for FV patients increases; this is not surprising, but as expected. This finding is consistent for all considered pairs of discharge rates for the FV and RV patients. Similarly, the maximum required capacity decreases as the RV's mean appointment lead-time increases. However, the maximum required capacity remains unchanged with the change of the appointment lead-time targets when the RV's mean appointment lead-time is significantly large.

The mean required capacity is generally negatively correlated or indeterminate with the appointment lead-time targets, but not for all cases considered. When the RV's mean appointment lead-time is significantly large, there is not a definitive correlation pattern.

The mean appointment lead-time of the FV patients is positively correlated to median, 95th percentile, and 100th percentile of appointment lead-time targets in most of the cases. This trend may not exist when the restriction of mean lead-time for RV patients is large, as then the model can have alternative optimal solutions, which have different mean lead-times for FV patients. Besides, increasing the RV's mean appointment lead-time decreases

the FV's mean appointment lead-time; this trend is consistent for all settings of the discharge rates.

The results also show that FV's mean appointment lead-time measured at every time-unit is not greater than a maximum FV's mean appointment lead-time, $\bar{l}$, estimated from the median, 95th percentile, and 100th percentile appointment lead-time (u , v , and w). Equation (3.21') provides the model with $\bar{l}$ being the estimate for the maximum FV's mean appointment lead-time. The first term of Equation (3.21') is the maximum value of the FV's mean appointment lead-time obtained from the patients whose appointment lead-times are not greater than the median appointment lead-time target, u . The second term of Equation (3.21') is the maximum value of the FV's mean appointment lead-time obtained from the patients whose appointment lead-times are greater than the median lead-time target, u , but not greater than 95th percentile appointment lead-time target, v . The last term of Equation (3.21') is the maximum value of the FV's mean appointment lead-time obtained from the patients whose appointment lead-times are greater than the 95th percentile appointment lead-time target, v . The maximum appointment lead-time target, w , must not be violated.

$$\bar{l} = 0.5u + 0.45v + 0.05w \quad (3.21')$$

The mean appointment lead-time of RV patient tends to be positively correlated with the change of the restricted length for the RV's appointment lead-time. However, it remains unchanged when the limit on the RV's mean appointment lead-time is small. This reinforces the push-pull mechanism in assigning the FV and RV appointments.

3.3.3.b. Sensitivity to changes of the discharge rates for the capacity design problem

This part analyzes the sensitivity of the maximum required capacity, the mean required capacity, the FV's mean appointment lead-time, and the RV's mean appointment lead-time to the changes of discharge rates (the detailed results are in Appendices A3.6 and A3.7). The maximum required capacity decreases when the discharge rate increases. This pattern exists for changes of both FV's and RV's discharge rates. However, this negative correlation is not always true when measuring the mean required capacity. The mean required capacity is not minimized by the model; hence, when the discharge rate increases, in order for the model to decrease the maximum required capacity, it might need to increase the mean required capacity.

In terms of appointment lead-time, the change of either FV's or RV's discharge rates does not impact the general way of providing appointments to patients, specifically FV patients. From the numerical results, the FV's and RV's mean appointment lead-times remain unchanged with changes of discharge rates.

3.3.4. Pattern of scheduling an appointment

Numerical experiments are implemented for all test cases, which are combined from different levels of the discharge rates, the RV's mean lead-time, and the appointment lead-time targets (mentioned in Sub-section 3.3.3). The obtained results show that the model solution tends to set the appointment lead-times of the FV patients to one of the appointment lead-time targets. That is, the optimal solution plans appointments for the FV patients to be with lead

times of u time-units (the median target), or v time-units (the 95th percentile target) or w time-units (the 100th percentile target). Table 3.5 reports the percent of the FV patients whose appointments are set to one of the appointment lead-time targets. From 450 cases experimented, a minimum 3%, mean 39%, median 47%, and maximum 50% of the FV demands are assigned the median appointment lead-time. Similarly, a minimum 3%, mean 37.4%, and maximum 45% of the total number of FV demands are assigned the 95th percentile appointment lead-time.

Table 3.5: Percentage of patients whose appointment lead-times are equal to one of appointment lead-time targets for the capacity design approach

Targets of patients' appointment lead-time	Percentage of patients (%)			
	Min	Median	Max	Mean
Median	3.0	47.0	50.0	39.0
95th percentile	3.0	45.0	45.0	37.4
100th percentile	0.0	5.0	5.0	4.4

Furthermore, the estimated value of FV's mean appointment lead-time, $\bar{l}$, should correspond to the MOH's targets in any time-unit during the model's period, when providing appointments to FV patients. Hence, the scheduling of patients' requests needs not only to pay attention to maintaining the satisfaction of the MOH's appointment lead-time targets but also to prevent the risk of violation of the MOH's appointment lead-time targets in the long term.

This finding implies that to achieve the appointment lead-time targets, a scheduling policy may focus just on these three appointment lead-time targets and the estimated value of mean appointment lead-time for FV patients

(from Equation (3.21')). This is an interesting way of scheduling because it not only enables easy control of appointment lead-time targets to plan capacity, but also gives more fairness to all patients making requests (for example: patients may receive median or 95th percentile or maximum appointment lead-time target, majority of patients are divided into 3 groups of the appointment lead-time measurements. Within each group, almost patients would receive similar appointment lead-time which brings the fair postponement to patients before seeing doctor. As the result, the fairness is manageable). Hence, this scheduling approach should be implemented in providing appointment dates to patients.

3.3.5. Discussions

A mixed integer model has been proposed to design the maximum required capacity for the re-entry system, with a specified accounting for the appointment lead-time characteristics. The proposed model minimizes the maximum required capacity while achieving service targets on the median, 95th percentile, and 100th percentile for the appointment lead-times for FV patients and satisfying the constraints on the re-visit appointment lead-times.

With numerical experiments, we found that the proposed model can provide systematic capacity planning for the re-entry system at the tactical level. The proposed model pushes and pulls the appointments to minimize the maximum required capacity. This push-pull mechanism of the proposed model is to delay the appointment to patients as long as possible without violating the FV's appointment lead-time targets and the RV's appointment lead-time constraints. Regardless of how the system pushes and pulls the appointments

to optimize the required capacity, one of the appointment lead-time targets should be the appointment lead-time for FV patients' requests and the estimated value of FV's mean appointment lead-time should be within the MOH targets in any time-unit during the model's period. However, there is no specific trend in scheduling the appointments of RV patients. Therefore, a deeper investigation on the scheduling of RV patients is encouraged.

The sensitivity analysis was done to understand how the maximum required capacity, the mean required capacity, the FV's mean appointment lead-time, and the RV's mean appointment lead-time are affected by changes to the appointment lead-time targets, the discharge rates, and the restriction of RV's mean appointment lead-time.

- The maximum required capacity is negatively correlated with the appointment lead-time targets, the discharge rates, and the restriction of RV's mean appointment lead-time. However, it is not always true for the mean required capacity measure.
- The FV's mean appointment lead-time is positively correlated with the FV's appointment lead-time targets. However, the RV's mean appointment lead-time generally increases with increases in the restriction on the RV's mean appointment lead-time, until the latter becomes very large.
- Changing the discharge rates does not have much impact on the determination of the FV patient's appointment.

The model finds the capacity level needed to achieve the appointment lead-time targets assuming all requests will be accepted. However, the proposed capacity may not be feasible this due to limited availability of

physicians. This limitation leads to unachievable appointment lead-time targets. Hence, if it is mandatory to achieve the appointment lead-time targets, there must be consideration for denying patient requests. A question is that: “For a given capacity plan, what is the minimum number of denied requests or the optimal admission scheme to achieve the appointment lead-time targets?” In Section 3.4, a mathematical model of admission management problem is presented to tackle this issue.

3.4. Admission management approach

As discussed previously (in Sub-section 3.3.5), the available capacity may not be able to match the designed plan. Hence, an approach to manage the admissions to achieve the appointment lead-time targets is necessary. The admission management approach aims to minimize the number of denied FV patients. A mathematical model of this problem can help to allocate the available capacity to each type of patients, i.e., FV and RV patients, where the demand of RV patients depends on the number of appointed FV patients properly.

3.4.1. Mathematical modeling for the admission management approach

The objective of this problem is to minimize the total number of denied FV patients. Although only the arrival demands for the first S periods are considered, the planning horizon needs to be sufficient for both FV and RV patients who are scheduled in the last time-unit of the arrival horizon, S . A mixed integer program model again is proposed for this admission management problem (P2). Most of the constraints of the capacity design

problem (in Section 3.3) remain unchanged, except for the objective function and the constraints related to the FV patients' scheduling and the total required capacity.

[P2]:

$$\text{Min } f^d = \sum_{i=1}^S f_i^d \quad (3.1'')$$

Subject to:

$$f_i^a + f_i^d = f_i, \quad \forall i \in N, \quad (3.2'')$$

$$\sum_{j=i}^T z_{ij} = f_i^a, \quad \forall i \in N, \quad (3.3'')$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \quad \forall i, j \in N, \quad (3.4'')$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \quad \forall i \in N, \quad (3.5'')$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \quad \forall j \in N, \quad (3.6'')$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \quad \forall j \in N, \quad (3.7'')$$

$$y_{ij} = 0, \quad \forall j - i < a, \quad \forall i, j \in N, \quad (3.8'')$$

$$y_{ij} = 0, \quad \forall j - i > b, \quad \forall i, j \in N, \quad (3.9'')$$

$$y_{ij} = 0, \quad \forall j \geq T + 1, \quad \forall i \leq S, \quad (3.10'')$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (3.11'')$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij}, \quad (3.12'')$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} \left(\sum_{i=1}^S \sum_{j=i}^T z_{ij} \right) \quad (3.13'')$$

$$z_{ij} = 0, \quad \forall j - i > w, \quad \forall j - i < 0, \quad (3.14'')$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (3.15'')$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \quad \forall j \in N, \quad (3.16'')$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \quad \forall j \in N, \quad (3.17'')$$

$$c_j - (c_j^f + c_j^r) \geq 0, \forall j \in N, \quad (3.18'')$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r, f_i^a, f_i^d \geq 0, \forall i, j \in N, \quad (3.19'')$$

$$z_{ij} \text{ integer}, \forall i, j \in N. \quad (3.20'')$$

The objective (3.1'') minimizes the total number of denied FV patients to achieve the appointment lead-time targets. The number of denied FV patients, per time-unit, is shown in constraint (3.2''). The conservation flow at FV-nodes is represented in constraints (3.3''), (3.4''), and (3.5''). Constraint (3.3'') schedules appointments for all admitted requests. Constraint (3.4'') determines the RV patients who remain after the first visit. The number of FV patients who are discharged after their first visit is specified in constraint (3.5'').

The conservation of flow at RV-nodes is modeled in constraints (3.6'') and (3.7''). The first bracket of constraint (3.6'') is the total number of pre-assigned FV and RV patients plus the total number of FV patients, plus the total number of RV patients, all with appointments in time-unit j . The second bracket of constraint (3.6'') is the number of RV patients discharged after their visit in time-unit j plus the number of RV patients remaining with a subsequent appointment (the flows that go out from the same time-unit j). Constraint (3.7'') specifies the number of patients discharged after their visit in time-unit j .

Constraints (3.8'') and (3.9'') assure that no RV patients will have appointment lead-times falling outside the restricted range $[a, b]$. We accomplish this by defining y_{ij} to be zero if the time interval between i and j falls outside the restricted range. Constraint (3.10'') prohibits RV patients'

appointments to be made after last date of the model horizon T . Constraint (3.11'') specifies the number of RV patients discharged after their visit in time-unit i . Constraints (3.12'') and (3.13'') assure that the targets of the appointment lead-time for the median and p^{th} percentile are satisfied, respectively. Constraint (3.14'') assures a realistic appointment date and does not allow appointments that go beyond the 100th percentile of limit. The restriction on the RV mean lead-time is represented in constraint (3.15'').

The required capacities of FV and RV patients in time-unit j are determined in constraints (3.16'') and (3.17''), respectively. The total required capacity in time-unit j is presented in constraint (3.18''). The total required capacity must not be greater than the available capacity, each time-unit, which are now input parameters. Furthermore, the required capacity for either FV or RV patients includes the required capacity of pre-assigned patients and that of the newly appointed arrivals. Constraint (3.19'') requires non-negative variables. Finally, constraint (3.20'') requires the integer assignment of FV patients

3.4.2. Numerical experiments

In this part, implementations of numerical experiments are done for available physicians (Appendix A2.3) plus the given inputs, as mentioned for the capacity design approach (mentioned in Section 3.3). Detailed analyses are executed via the comparison between the historical and proposed model's performance, in terms of the cumulative appointments, the maximum number of appointments, and the mean number of appointments per week.

The model is set up to plan the admission decisions on a weekly basis. The Branch and Cut algorithm (mentioned in Sub-section 3.3.2) is used to solve the admission management model. The computation time to find the optimal admission scheme to all numerical examples is less than three minutes, implemented in IBM ILOG CPLEX Optimization Studio V12.4 on HP Pavilion G6 Notebook 1.9GHz PC with the Windows 7 operating system

For the real test case, the model achieves the appointment lead-time targets without denying any patient request. Figures 3.5, 3.6 and 3.7 illustrate the comparisons of actual and model performances for the cumulative, the mean number of appointments, and the maximum number of appointments, respectively.

Figure 3.5: The cumulative appointments from actual vs. model of the admission management problem

The actual cumulative number of appointments (Figure 3.5) for the FV patient type is always less than what is obtained from the proposed model. For RV patients, the cumulative number of appointments from the proposed model is generally less than the actual number. Although this result is not always true throughout the planning horizon, the final cumulative number of appointments from the model is less than that from the actual. The results imply that the model systematically pushes and pulls the RV and FV patients' appointments to make appropriate physician allocations so that the achievement of the appointment lead-time targets can be obtained with the least number of denied patients.

Figure 3.6: The mean number of appointments from actual vs. model of the admission management problem

The comparison of actual and model performance in terms of the mean number of appointments (Figure 3.6) reveals a better understanding on the above finding. The mean number of seen FV and RV patients is not much

different between the proposed model's result (524.2 patients per week) and the actual plan (525 patients per week); but the appointment lead-time targets can now be met. The achievement of the appointment lead-time targets results from providing 3 more FV appointments per week (134.5 appointments per week from the proposed model versus 131.5 appointments per week from the actual plan). As a result, there are fewer appointments assigned to RV patients when the capacity is pre-determined and fixed. The mean number of appointments for RV patients obtained from the proposed model (389.6 appointments per week) is reduced by 3.7 appointments per week in comparison with the same measurement from the actual plan. This explains an important role of the model in providing an effective and efficient admission scheme which helps to achieve the appointment lead-time targets by pushing and pulling the RV and FV patient appointments appropriately.

Figure 3.7: The maximum number of appointments from actual vs. model of the admission management problem

Furthermore, Figure 3.7 shows the maximum number of slots for both the FV and RV categories. The proposed model provides 20% more appointments than that from the historical data (899 patients from the model versus 751 patients from the actual practice). This is because the proposed model allows scheduling only the RV patients in some weeks of the planning horizon. As the results, the plan can have more appointment slots because the RV patients have shorter consultation time. Besides, the maximum number of appointments for the FV patients (429 patients) from the proposed model is about 1.7 times as much as that is from the actual plan (248 patients). Similarly, the maximum number of appointments for the RV patients (899 patients) from the proposed model is about 1.7 times as much as that is from the actual plan (545 patients). The finding implies that the allocated capacity between different types of patients may differ in each time-unit to regulate the appointment schedules and to satisfy the appointment lead-time targets with a minimal rejection of patients.

The proposed model only accepts a new arrival when the appointment lead-time targets are satisfied and the continuity of care for RV patients is assured. In this case, none of the new requests is rejected but the appointment lead-time targets are still achieved. The FV's mean appointment lead-time from the proposed model is 2.9 weeks. This is 55% lower than that of the actual plan (6.4 weeks). As a consequence, the mean appointment lead-time of RV patients of the proposed model (22.7 weeks) is longer than that of the actual plan (16 weeks). It is 43% increase of the RV's expected appointment lead-time, when all new arrivals are scheduled, continuity of care is not violated, and MOH's targets are satisfied.

The above analyses demonstrate the efficiency of the proposed model in providing a systematic push-pull to generate an optimal FV admission scheme subject to the given capacity. This way of planning is difficult to implement manually for the re-entry system, but easily executed via the proposed model. Furthermore, the proposed model can also help to reveal infeasibility for an available capacity level. The infeasibility happens when all new arrivals are rejected but the given capacity still causes the discontinuity of care of the pre-assigned FV and RV patients. This infeasibility will be discussed in the coming sensitivity analysis, Section 3.4.3.

3.4.3. *Sensitivity analysis*

The sensitivity of the model's objective (the total number of denied requests) to changes of key factors (i.e., the FV's appointment lead-time targets, the restriction on RV's mean appointment lead-time, and the discharge rates) is examined. A further look at the impact of the key factors on the total unused capacity, on the FV's mean appointment lead-time, and on the RV's mean appointment lead-time are also probed. The numerical experiments are implemented with the similar combinations of the capacity design problem, which involve 5 different sets of the discharge rates, 6 levels of the RV's mean lead-time, and 15 different sets of the FV's appointment lead-time targets.

The terminology convention in Table 3.3 is also used for displaying the findings in Table 3.6, which shows a summary of the sensitivity analysis. The detailed inputs and results are presented in Appendices A4.1 to A4.7.

The result shows that infeasibility might exist in the cases in which the restricted RV's mean lead-time is short or the discharge rates are low (refer to

Table 3.6). This implies the fact that either a short restricted RV's mean lead-time or a low discharge rate causes a large number of appointments that need to be scheduled. Hence, the given capacity might be insufficient to serve only pre-assigned FV and RV patients with no new patient arrivals. The infeasible cases do not reveal any pattern of admission management schemes, but only the inability of serving the pre-assigned patients. Consequently, we analyze only the feasible cases to see how the number of denied patients changes with the appointment lead-time targets as well as the discharge rates.

3.4.3.a. Sensitivity to changes of the appointment lead-times for the admission management problem

The sensitivity of the total number of denied FV requests, total unused capacity, FV's and RV's mean appointment lead-times to changes of the FV appointment lead-time targets the RV's mean appointment lead-time are analyzed. The summary of the sensitivity analysis is shown in Table 3.6.

The longer are the FV appointment lead-time targets, the fewer are the denied FV patients. The negative correlation is consistent in all feasible cases until all patients' requests are admitted. The admittance of the patients' request happens when either there is a wide range for assigning RV's appointments or high discharge rates cases. In other words, the total number of denied FV requests remains unchanged when the given capacity is well planned and sufficient for the total number of required appointments.

Table 3.6: Summary of sensitivity analysis to changes of the appointment lead-times for the admission management problem

RV's mean lead-time (weeks)		Number of denied patients					Total unused capacity					FV's mean lead time (weeks)					RV's mean lead time (weeks)				
	FV's discharge	0.38		0.38*	0.5	0.7	0.38		0.38	0.5	0.7	0.38		0.38	0.5	0.7	0.38		0.38	0.5	0.7
	RV's discharge	0.1	0.32	0.32*	0.32		0.1	0.2	0.32	0.32		0.1	0.2	0.32	0.32		0.1	0.32	0.32	0.32	
	Lead-time target																				
5	Median	Inf.	Inf.	-	-	-	Inf.	Inf.	-	-	-	Inf.	Inf.	+	+	+	Inf.	Inf.	0	0	0
	95 th percentile	Inf.	Inf.	-	-	-	Inf.	Inf.	-	1	-	Inf.	Inf.	+	+	+	Inf.	Inf.	0	0	0
	100 th percentile	Inf.	Inf.	-	-	-	Inf.	Inf.	-	1	1	Inf.	Inf.	+	+	+	Inf.	Inf.	0	0	0
10	Median	Inf.	-	-	-	-	Inf.	1	1	1	-	Inf.	+	+	+	+	Inf.	0	0	0	0
	95 th percentile	Inf.	-	-	-	-	Inf.	-	-	-	-	Inf.	+	+	+	+	Inf.	0	0	0	0
	100 th percentile	Inf.	-	-	-	-	Inf.	1	-	1	1	Inf.	+	+	+	+	Inf.	0	0	0	0
16*	Median	-	-	-	-	-	1	1	-	-	1	+	+	+	+	+	0	0	0	0	0
	95 th percentile	-	-	-	-	-	-	-	-	-	1	+	+	+	+	+	0	0	0	0	0
	100 th percentile	-	-	-	-	-	1	-	1	-	-	+	+	+	+	+	0	0	0	0	0
20	Median	-	-	-	0	0	-	-	1	1	1	+	+	+	+	+	0	0	0	0	0
	95 th percentile	-	-	-	-	-	-	-	1	1	1	+	+	+	+	+	0	0	0	0	0
	100 th percentile	-	-	-	0	0	-	1	-	1	1	+	+	+	1	1	0	0	0	0	0
25	Median	0	0	0	0	0	1	1	-	-	1	+	+	+	+	+	0	0	-	-	-
	95 th percentile	-	-	-	-	-	1	1	1	1	1	1	+	+	+	+	0	0	-	-	-
	100 th percentile	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1
30	Median	0	0	0	0	0	1	-	-	-	1	+	+	+	+	+	-	1	1	1	-
	95 th percentile	-	-	-	-	-	1	1	1	1	1	+	+	+	+	+	-	-	1	1	1
	100 th percentile	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

* These are the historical RV's mean appointment lead-time and discharge rates at TTSH.

The negative correlation between the total number of denied patients and the FV appointment lead-time targets does not lead to the negative correlation between the total unused capacity and the FV appointment lead-time targets. The indeterminate correlation between total unused capacity and the FV appointment lead-time targets is due to not only the number of denied patients, but also the appointment lead-time of scheduled FV patients. Particularly, even the number of denied patients decreases, but the long FV's appointment lead-time leads to decreasing the demand of the RV patients. Furthermore, if the decrease of the used capacity from the RV patients is less than that from the FV patients (or from the number of appointed FV patients), there is an increase of the total unused capacity.

The FV's mean lead-time is not always positively correlated to median, 95th percentile, and 100th percentile of the appointment lead-time targets. The positive correlation is maintained when either the restriction of the RV's mean lead-time is tight or the discharge rates are sufficiently low. Otherwise, no specific trend occurs. This indeterminate correlation strengthens the relationship between the FV appointment lead-time targets and the unused capacity. Furthermore, in any time-unit of the model horizon beyond the effect of pre-assignments, the obtained FV mean lead-time is generally less than the estimate of the FV mean lead-time (Equation 3.21'). The finding implies that the satisfaction of appointment lead-time targets should be maintained throughout the model horizon.

The RV's mean lead-time tends to remain unchanged to the change of restricted RV's lead-time in most cases. This trend does not exist when either the restriction of the RV's mean lead-time or the discharge rates are

sufficiently wide or large, respectively. Moreover, the RV's mean appointment lead-time obtained from the proposed model tends to be equal to the restricted RV's lead-time with the unchanged FV appointment lead-time targets.

3.4.3.b. Sensitivity to changes of the discharge rates for the admission management problem

This part analyzes changes of the discharge rates to trends of the objective (the total number of denied FV requests), the FV's mean lead-time, the RV's mean lead-time, and the total unused capacity.

The higher the discharge rates are, the lower the total number of denied FV patients is. This pattern holds for changes of either FV or RV discharge rates. This trend also exists when analyzing the relationship between the total unused capacity and the discharge rates. The more patients are discharged, the less capacity need to be invested. This finding illustrates the fact that more patients discharged leads to more space to accept new arrivals. Furthermore, the numerical results illustrate that the FV's and RV's mean appointment lead-times remain unchanged with changes of the discharge rates. This finding could help the hospital to include the discharge rates in planning its resources.

3.4.4. Pattern of scheduling an appointment

There is a similarity for scheduling an appointment between the capacity design and the admission management approaches. From the computational results, it has also been found that the appointment of patients tends to be at the median appointment date, the 95th percentile appointment date, or the 100th percentile appointment date (refer to Table 3.7). The results report the percent of the FV patients whose appointments are set to one of the appointment lead-

time targets. From the experimental cases, a minimum 3%, mean 34%, median 50%, and maximum 50% of the FV demands are assigned a median appointment lead-time. Similarly, a minimum 2%, mean 31.7%, median 45% and maximum 45% of the total number of FV demands are assigned a 95th percentile appointment lead-time.

Table 3.7: Percentage of patients whose appointment lead-time is equal to one of appointment lead-time targets for the admission management approach

Targets of patients' appointment lead-time	Percentage of patients (%)			
	Min	Median	Max	Mean
Median	3.0	50.0	50.0	34.0
95th percentile	2.0	45.0	45.0	31.7
100th percentile	0.0	5.0	5.0	4.4

Similar to the capacity design approach, the correspondence of the estimated value of the FV mean appointment lead-time, $\bar{l}$, to the FV appointment lead-time targets in each time-unit should be considered when providing appointments to the FV patients in the admission management approach. Considering this correspondence would maintain the satisfaction of the FV appointment lead-time targets and avoid the violation of the targets as well.

3.4.5. Discussions

The mixed integer programming model is proposed to manage the admission of FV patients to maintain the continuity of care for the scheduled patients of the re-entry system. The proposed model can provide the best resource allocation or the optimal admission scheme in order to achieve the

highest number of new patients scheduled within the MOH's targets. The numerical experiments reveal that the appointments of new patients should be the median, the 95th percentile, or the 100th percentile appointment date. This way of scheduling can help not only to optimize the number of new patients appointed, but also to manage the admission of patients. In addition, the estimate of the FV mean appointment lead-time should be the upper bound for scheduling FV patients' appointment.

However, scheduling the appointment of RV patients is indeterminate. The preliminary suggestion to the doctor for making decision on the follow-up appointment is that non-serious cases should be postponed to leave the space for more serious RV cases. By doing this, the restriction of the mean appointment lead-time for RV can be easily maintained while the continuity of care of RV patients is not broken.

The sensitivity analysis was done to explore the trends for the number of denied patients, the total unused capacity, the FV's mean appointment lead-time, and the RV's mean appointment lead-time to changes of the appointment lead-time targets, the discharge rates, and the restriction of the RV's mean appointment lead-time. Some conclusions are as follows.

- The total number of denied FV patients is negatively correlated with the appointment lead-time targets, the restricted value of the RV's mean appointment lead-time, and the discharge rates.
- The FV's mean appointment lead-time tends to be positively correlated with the appointment lead-time targets of FV patients.

- The higher the discharge rate is, the lower the number of denied patients is. This trend is true for the change of either FV or RV discharge rates.

3.5. Summary

In this chapter, the mixed integer programming models are introduced to solve the capacity design and the admission management problems for the re-entry systems. There are common findings between the two approaches in scheduling patient appointments. The push-pull mechanism is suggested for appointing patients' request. The push part of the mechanism is to delay the appointments of the RV patients, while the pull part of the mechanism is to bring the appointments of the FV patients forward to be within the MOH's appointment lead-time targets. The push-pull mechanism can help to minimize the maximum required capacity and the number of denied requests, respectively, for the capacity design and the admission management approaches.

The study found that there is a strong relationship between service targets and the trend of scheduling FV patients' request. The expected mean appointment lead-time should not be violated at any time-unit when providing appointments to patients. Moreover, the patient's appointment should be the median, 95th percentile, and 100th percentile appointment date.

The sensitivity analyses are conducted for both capacity design and admission management approaches to investigate the sensitivity of the performances to the changes of service targets, the discharge rates, and the restricted length of RV patients' appointment lead-time.

a. Sensitivity analysis for the capacity design problem

- The maximum required capacity is negatively correlated with the appointment lead-time targets, the discharge rates, and the restriction of the RV mean appointment lead-time. However, it is not always true for the mean required capacity cases.
- The FV mean appointment lead-time is positively correlated with the FV appointment lead-time targets. However, the positive correlation between RV mean appointment lead-time and the restriction of the RV mean appointment lead-time is indeterminate when the latter part is large.
- Changing the discharge rates does not impact on the general way of assigning the appointments of the FV patients.

b. Sensitivity analysis for the admission management problem

- The total number of denied FV patients is negatively correlated with the appointment lead-time targets, the restricted value of the RV mean appointment lead-time, and the discharge rates.
- The FV mean appointment lead-time tends to be positively correlated with the appointment lead-time targets for FV patients.
- The higher the discharge rate is, the lower the number of denied patients is. This trend is true for the change of either FV or RV discharge rates.

The proposed models help to plan the capacity for each type of the patients throughout the periods to minimize the maximum required capacity or the number of denied patients. The results from the proposed models illustrate the tendency of scheduling the patients' requests on the median, 95th

percentile, and 100th percentile appointment date, and the estimate of mean appointment lead-time for the FV patients; nevertheless, the quantitative constraints to decide on scheduling the appointment are not clear. Therefore, to answer the question “How should operators implement the planned capacity to optimize the resource utilization?”, the next chapter will propose the admission appointment scheduling policies to schedule every individual appointment of patients subject to the optimal planned capacity. The proposed policies provide the quantitative scheduling constraints as a guideline to decide the patients' appointment date.

CHAPTER 4: ADMISSION SCHEDULING POLICIES OF APPOINTMENT REQUESTS FOR RE-ENTRY SYSTEMS

In the previous chapter, mathematical models for determining the optimal required capacity and the optimal admission plan were proposed. In this chapter, the scheduling rules with quantitative admission constraints are established to determine a specific appointment date for patients. Section 4.1 presents the problem definition. Section 4.2 derives patients' admission constraints; this is followed by appointment scheduling rules for making decisions on the patients' appointment in Section 4.3. In Section 4.4, numerical experiments are examined for validation of the proposed rules. Finally, a summary of the chapter is presented in Section 4.5.

4.1. Definition of the appointment scheduling problem

In the outpatient appointment system that requires FV patients to book an appointment in advance, the operation staffs currently use the first-in-first-out (FIFO) rule to provide an appointment to a request of each patient until none of the designated capacity is available. This way of assigning patient appointments intuitively aims to minimize the unused resources in terms of empty slots or unscheduled slots; however, the satisfaction of the appointment lead-time targets is usually not considered when deciding on the patients' appointment. As a result, it might violate the MOH's appointment lead-time targets. Furthermore, patients seem to reserve a suggested appointment slot regardless of the appointment lead-time length and the future decision on his/her arrival. The continual offering of an appointment slot to patients and

patients' behavior on reserving appointment slots could result in unnecessarily long appointment lead-times (The poor appointment lead-time performances resulting from the current appointment scheduling practice was previously shown in Table 1.1). Therefore, a guideline for scheduling patients' appointments is necessary to manage the patients' appointment lead-time.

Extending the capacity planning models in chapter 3, an optimal planned capacity based on information of the estimated future demands is obtained. However, the actual arrival demands often differ from the forecast and the operation staffs do not have full information of the future demands. To facilitate the assignments of every individual FV patient's request with given median, 95th percentile, and 100th percentile appointment lead-time targets, the development of appointment scheduling rules considering quantitative constraints for making decision of the patients' appointment is introduced. The quantitative admission constraints need to involve the MOH's median, 95th percentile, and 100th percentile appointment lead-time targets as prerequisites.

The development of the appointment scheduling rules needs to assume that the planned capacity for a specific type of patients cannot be used by the other type of patients. Thus, the optimal planned capacity of RV patients needs to be sufficient to allow the continuity of care for all admitted FV patients regardless of the accuracy of the forecast FV demands. To enforce the patients' continuity of care condition, overbooking is not allowed. The second assumption implies that the violation of RV continuity of care (or the violation of the RV patients' appointment lead-times) should not depend on the accuracy of the estimated FV demands. Furthermore, since the RV patients' appointment depends on the patients' health condition and on the decision of

physicians, the constraints for setting the RV patients' appointment date are not addressed in this thesis. Therefore, only the appointment scheduling rules for determining the FV patients' appointment is developed. The assumptions are summarized in Table 4.1.

Table 4.1: List of assumptions for the appointment scheduling rules

No.	Assumptions
1	The planned capacity for a specific type of patient cannot be used by the other type of patient.
2	The rejection of patient-requests may happen.
3	The appointment scheduling rules are only applied for the FV patient-requests.
4	Patients' preference on either physician or appointment time-units is not considered.
5	A patient's appointment can be scheduled in the same time-unit as the patient's request.
6	All scheduled patients show up
7	The appointment lead-time targets are always maintained.
8	Future arrival demands are unknown
9	One slot can only be assigned to one patient

In the previous chapter (chapter 3), the preliminary findings show that the targets of the FV patients' appointment lead-time do drive the decision of the patients' appointment date. The mean appointment lead-time of the FV patients is less than the estimate of the FV mean appointment lead-time (refers to Equation 3.21). In addition, appointments of the FV patients tend to be on the median, 95th percentile, or 100th percentile appointment date. Moreover, the achievement of the appointment lead-time targets should be maintained at each time-unit throughout the planning horizon. The above findings will be

used in this chapter to derive the appointment scheduling rule and its sets of constraints, which decide the FV patients' appointment date. The derived appointment scheduling rule and its constraints will consider a trade-off between the achievement of appointment lead-time targets and the number of unscheduled slots or unused slots from the optimal required capacity or the optimal admission scheme. Therefore, the rejection of the patients' request will be allowed in scheduling individual patient's request. The decision of rejection happens when there would be a violation of one of more of the appointment lead-time targets.

4.2. Patients' admission constraints

In this section, the quantitative admission constraints for scheduling the FV patients' appointment date are developed. The objective is to minimize the number of un-scheduled slots subject to the achievement of MOH's appointment lead-time targets. Since the rules aim to maintain the MOH's appointment lead-time targets throughout the arrival horizon, the percent of appointments whose appointment lead-time are not greater than median and p^{th} percentile must be greater than or equal to 0.5, and $\frac{p}{100}$ times of the total number of appointments at every time-unit. In other words, at every time-unit, at most 50% and $(100-p)\%$ of the patients have appointment lead-times greater than the median and p^{th} percentile appointment lead-time targets, respectively. In addition, none of the appointments has a lead-time beyond the 100th percentile lead-time target.

Table 4.2: List of notation for the development of the admission constraints

No.	Notation	Description
1	A_i^m	The cumulative number of admitted FV patients whose appointment dates are scheduled within the median lead-time target, as of time-unit i .
2	A_i^p	The cumulative number of admitted FV patients whose appointment dates are scheduled beyond the median lead-time target but within the p^{th} percentile lead-time target, as of time-unit i .
3	A_i^{100}	The cumulative number of admitted FV patients whose appointment dates are scheduled beyond the p^{th} percentile lead-time target but within the 100 th percentile lead-time target, as of time-unit i .
4	a_i^m	The number of admitted FV patients who arrive at time-unit i and are scheduled so that their appointment lead-times are equal to or less than the median lead-time target.
5	a_i^p	The number of admitted FV patients who arrive at time-unit i and are scheduled so that their appointment lead-times are limited by the p^{th} percentile lead-time target.
6	a_i^{100}	The number of admitted FV patients who arrive at time-unit i and are scheduled so that their appointment lead-times are limited by the 100 th percentile lead-time target.
7	b_{ij}	The number of available FV slots in time-unit j , measured at time-unit i .
8	c_i^m	A median-constraint at time-unit i .
9	c_i^p	A p^{th} -percentile-constraint at time-unit i .
10	c_i^{100}	A 100 th -percentile-constraint at time-unit i .

The constraints for scheduling patient appointments, that are restricted by the median, the p^{th} percentile, and the 100th percentile, are defined as the median-constraint, the p^{th} -percentile-constraint, and the 100th-percentile-constraint, respectively. These constraints are dynamically executed at each

time-unit i of the arrival horizon. The notation for development of the admission constraints in the coming derived appointment scheduling rule is shown in Table 4.2.

4.2.1 *Exact-admission constraints set*

In the previous chapter 3, the preliminary findings revealed that the targets of the FV patients' appointment lead-time do drive the decision of the patients' appointment date. The appointments of the FV patients tend to be on the median, p^{th} percentile, or 100^{th} percentile appointment date. Hence, the constraints of the patients' appointment should first be restricted to the median target, otherwise, the exact established p^{th} percentile or 100^{th} percentile appointment lead-time targets would be addressed. Allowing the admissions to be scheduled within the median lead-time target increases the number of patients' appointment satisfying the median target and raises the total number of the admitted patients.

The development of the admission constraints, in which the p^{th} -percentile-constraint and the 100^{th} -percentile-constraint address the exact p^{th} percentile and 100^{th} percentile lead-time targets respectively, are shown as Equations (4.1) to (4.6) of the “*exact-admission constraints set*”.

[Exact-admission constraints set]:

$$c_i^m = \sum_{j=i}^{j=i+u} b_{ij}, \forall i, \quad (4.1)$$

$$c_i^p = \text{Min}_{k=i+v} \left\{ b_{ik}, \left(\frac{p}{50} - 1 \right) A_i^m - A_{i-1}^p \right\}, \forall i, \quad (4.2)$$

$$c_i^{100} = \text{Min}_{k=i+w} \left\{ b_{ik}, \frac{(100-p)}{p} (A_i^m + A_i^p) - A_{i-1}^{100} \right\}, \forall i, \quad (4.3)$$

where:

$$A_i^m = A_{i-1}^m + a_i^m, \forall i, \quad (4.4)$$

$$A_i^p = A_{i-1}^p + a_i^p, \forall i, \quad (4.5)$$

$$A_i^{100} = A_{i-1}^{100} + a_i^{100}, \forall i. \quad (4.6)$$

Equation (4.1) shows the median-constraint in time-unit i . It determines the maximum number of patients that arrives in time-unit i and can be scheduled within the median appointment lead-time target (u). This median-constraint aims to utilize the planned resource, while maintaining the median lead-time target in every time-unit i .

Equation (4.2) determines the p^{th} -percentile-constraint in time-unit i . It is the maximum number of patients that arrives in time-unit i and that can be scheduled with appointment lead times equal to the p^{th} percentile appointment lead-time target (v). On the right hand side of equation (4.2), the first term is the maximum number of available slots in which patients' appointment lead-times are at the p^{th} percentile lead-time target. Furthermore, the achievement of the p^{th} percentile lead-time target in time-unit i must assure the maintenance of the median lead-time target in that time-unit, therefore, $\frac{A_i^p}{A_i^m} \leq \frac{(p-50)}{50}$. Since $A_i^p = A_{i-1}^p + a_i^p$, the inequality becomes $A_{i-1}^p + a_i^p \leq \left(\frac{p}{50} - 1\right) A_i^m$. Then, $a_i^p \leq \left(\frac{p}{50} - 1\right) A_i^m - A_{i-1}^p$. As the result, the second term on the right hand side of equation (4.2) is the maximum number of the patients (a_i^p) that can be scheduled to satisfy the median and the p^{th} percentile lead-time target in time-unit i .

Similarly, Equation (4.3) illustrates the 100^{th} -percentile-constraint in time-unit i . It determines the maximum number of patients that arrives in time-

unit i and that can be scheduled with an appointment lead-time equal to the maximum lead-time target (w). On the right hand side of equation (4.3), the former term is the maximum number of available slots in which patients' appointment lead-time is within the maximum lead-time target. Furthermore, the achievement of the maximum lead-time target in time-unit i must assure the maintenance of the p^{th} percentile lead-time target in that time-unit, therefore, $\frac{A_i^{100}}{A_i^m + A_i^p} \leq \frac{(100-p)}{p}$. Since $A_i^{100} = A_{i-1}^{100} + a_i^{100}$, the inequality becomes $A_{i-1}^{100} + a_i^{100} \leq \frac{(100-p)}{p} (A_i^m + A_i^p)$. It leads to $a_i^{100} \leq \frac{(100-p)}{p} (A_i^m + A_i^p) - A_{i-1}^{100}$. Then, the latter term on the right hand side of equation (4.3) is the maximum number of patients (a_i^{100}) that should be scheduled to maintain the MOH's lead-time targets in time-unit i .

Equations (4.4), (4.5), and (4.6) determine the cumulative number of scheduled patients in time-unit i , whose lead-times are within the targets of median, at the p^{th} percentile, and at the 100^{th} percentile appointment lead-time, respectively. The cumulative number of the scheduled patients, in time-unit i , is the cumulative number of the scheduled patients in the previous time-unit ($i-1$) and the number of the scheduled patients in time-unit i . The number of each type of the scheduled patients in each time-unit i (a_i^m , a_i^p , or a_i^{100}) is determined at the end of each time-unit. In general, at least 50% of scheduled patients need to have lead-times shorter than the median u , and a maximum of $(p-50)\%$ and $(100-p)\%$ of the scheduled patients have lead-times less than the p^{th} percentile, and maximum lead-time targets, v and w , respectively.

4.2.2 Relaxed-admission constraints set

Although the above “*exact-admission constraints set*” helps the staff to decide on patients’ appointment, would it cause low utilization of the planned capacity? To answer the question, this section develops the relaxation of the set of exact-admission constraints, named as the “*relaxed-admission constraints set*”.

The relaxed-admission constraints set aims to provide appointments to patients whose appointment lead-time is within the median, the p^{th} percentile, or not exceeding the maximum lead-time target. The MOH’s targets for the median, the p^{th} percentile, and the maximum lead-time throughout the planning horizon is also maintained when deriving this relaxed-admission constraints set. The patients are allowed to have appointments within the p^{th} percentile or within the maximum lead-time target (if they cannot get the appointments within the median or p^{th} percentile lead-time target, respectively). In other words, with the set of the relaxed-admission constraints, at least 50%, $p\%$, and 100% of scheduled patients’ lead-time is shorter than the median, p^{th} percentile, and maximum lead-time targets, u , v and w , respectively.

[Relaxed-admission constraints set]:

$$c_i^m = \sum_{j=i}^{j=i+u} b_{ij}, \forall i, \quad (4.1')$$

$$c_i^p = \text{Min} \left\{ \sum_{k=i+u+1}^{i+v} b_{ik}, \left(\frac{p}{50} - 1 \right) A_i^m - A_{i-1}^p \right\}, \forall i, \quad (4.2')$$

$$c_i^{100} = \text{Min} \left\{ \sum_{k=i+v+1}^{i+w} b_{ik}, \frac{(100-p)}{p} (A_i^m + A_i^p) - A_{i-1}^{100} \right\}, \forall i, \quad (4.3')$$

where:

$$A_i^m = A_{i-1}^m + a_i^m, \forall i, \quad (4.4')$$

$$A_i^p = A_{i-1}^p + a_i^p, \forall i, \quad (4.5')$$

$$A_i^{100} = A_{i-1}^{100} + a_i^{100}, \forall i. \quad (4.6')$$

Equations (4.1') to (4.6') represent the set of the relaxed-admission constraints. The development of the *relaxed-admission constraints* set is similar to that of the *exact-admission constraints* set (Section 4.2.1).

4.3. Appointment scheduling rule

The appointment scheduling rule determines the patients' appointment time from the calculated admission constraints (median, p^{th} percentile, and 100^{th} percentile constraints) to maintain the satisfaction of the MOH's appointment lead-time targets at any time-unit. It is myopic in that it admits and schedules the patients without considering future arrival information and assuming that the patients' appointment cannot be changed. Furthermore, it will not assign a patient, if this assignment would violate any of the three constraints (median-constraint, p^{th} -percentile-constraint, and 100^{th} -percentile-constraint). When scheduling a patient, the scheduling rule prioritizes the constraints in the order of median-constraint, p^{th} -percentile-constraint, and 100^{th} -percentile-constraint. The appointment scheduling rule (R1) is built as follows:

[R1]:

Step 1: Given i . Set: $a_i^m = 0$; $a_i^p = 0$; $a_i^{100} = 0$.

Step 2:

- Determine the potential appointment dates k so that the patients' appointment lead-time satisfies the median lead-time target (u).
- Determine the median- constraint c_i^m corresponding to k .

Step 3:

Schedule the patients' requests until the constraint c_i^m is binding or all the requests are admitted given the constraint c_i^m . If all the requests are scheduled, go to step 8. Otherwise, update a_i^m and go to step 4.

Step 4:

- Determine the potential appointment dates k at which the patients' appointment lead-time satisfies the p^{th} percentile lead-time target (v).
- Determine the p^{th} -percentile-constraint c_i^p corresponding to k .

Step 5:

Schedule the patients' request until the constraint c_i^p is binding or all the requests are admitted given the constraint c_i^p . If all the requests are scheduled, go to step 8. Otherwise, update a_i^p and go to step 6.

Step 6:

- Determine the potential appointment dates k at which the patients' appointment lead-time is at the maximum lead-time target (w).
- Determine the 100th-percentile-constraint c_i^{100} corresponding to k .

Step 7:

Schedule the patients' request until the constraint c_i^{100} is binding or all the requests are appointed given the constraint c_i^{100} . Go to step 8.

Figure 4.1: Flow chart of the appointment scheduling rules

Step 8:

- Update: a_i^m , a_i^p , and a_i^{100} . Set $i = i + 1$.
- If i is greater the number of the maximum arrival horizon S , then stop and all the coming requests will be rejected. Otherwise, go to step 1.

Figure 4.1 summaries the flow of the proposed appointment scheduling rule. The first column describes the scheduling conditions subject to median constraints (steps 3 and 4 of the rule R1). Similarly, the second and third columns present the scheduling conditions subject to p^{th} -percentile-constraints and 100^{th} -percentile-constraints, respectively. The last column is to reject all the remaining patients' request when no available slot of the planning horizon remains. The appointment scheduling rule R1 is called the “*exact rule*” if the exact-admission constraints set is used to admit the patients. Otherwise, it is called the “*relaxed rule*”, if the relaxed-admission constraints set is employed to schedule the patients' appointment.

With the *exact rule*, the first-in-first-out (FIFO) sub-rule is executed for the patients who can get appointments within the median lead-time target. In other words, each patient's appointment date is scheduled in the earliest available slot which is within the median lead-time target (u). The FIFO aims to minimize the number of unused slots. If patients cannot get the appointments within median lead-time target, the p^{th} percentile appointment date will be considered before considering the 100^{th} percentile appointment date. The procedure is repeatedly performed until the arrival horizon S is exceeded.

With the *relaxed rule*, if a patient cannot get the appointment within the median lead-time target, he/she will get an available slot within the p^{th} percentile lead-time target. Admitting the patient to an available slot subject to the 100th-percentile-constraint is the last choice. The procedure is repeatedly done until the arrival horizon S is exceeded. For the patients that are scheduled within the median target, the FIFO sub-rule is executed to schedule the patients' appointment. The FIFO aims to minimize the unused resource or unscheduled slots. However, for appointments restricted by either the p^{th} -percentile-constraint or the 100th-percentile-constraint, the first-in-last-out (FILO) sub-rule is implemented. For each patient that cannot be admitted within the median lead-time target, their admission should be scheduled as late as possible but within the p^{th} -percentile or 100th-percentile lead-time target. The FILO aims to allow more patients to be scheduled subject to higher priority constraint in the future, which leads to the greater number of admitted patients or the fewer number of unused slots. In other words, the more scheduled patients that satisfy the median lead-time target in the future, the higher the number of patients can be admitted to satisfy the p^{th} -percentile-constraint. For example: in time-unit i , the usage of the FILO for the p^{th} -percentile-constraint allows more patients to be scheduled under the median-constraint in time-unit $(i+1)$. This then allows for more patients to be scheduled within the p^{th} -percentile-constraint in time-unit $(i+1)$, that leads to the higher total number of patients would be scheduled in time-unit $(i+1)$ and so on.

Table 4.3 summarizes the sub-rules corresponding to the three constraints (median-constraint, p^{th} -percentile-constraint, and 100th-percentile-constraint) for the exact and relaxed rule.

Table 4.3: Summary of the sub-rules of the exact and relaxed rules

Constraint	Decision on the patients' appointment	
	<i>The exact rule</i>	<i>The relaxed rule</i>
Median-constraint	The FIFO sub-rule	The FIFO sub-rule
p^{th}-percentile-constraint	Schedule exactly at the p^{th} -percentile appointment lead-time v .	The FILO sub-rule
100th-percentile-constraint	Schedule exactly at the 100 th -percentile appointment lead-time w .	The FILO sub-rule

4.4. Numerical experiments

This section conducts numerical experiments to test the effectiveness of the proposed appointment scheduling rules. The rules are evaluated based on either the percentage of the unused slots or the percentage of the rejected requests. The evaluation is implemented for the optimal planned capacity from both the capacity design and the admission management approaches.

4.4.1. *Experimental design*

Data from years 2009 and 2010 of the Urology specialty at TTSH is used, similar to what was done in chapter 3. This data is used to determine the inputs on the arrival demands as well as the actual provided capacity. The median, 95th percentile, and 100th percentile appointment lead-time targets are 2 weeks, 6 weeks, and 9 weeks, respectively.

Furthermore, the 1140 optimal planned capacity scenarios for the FV patient were examined. The mentioned scenarios were obtained from the capacity design and admission management problems in Chapter 3 (Sections 3.3 and 3.4). The optimal planned capacity of the FV patients exclude that of the pre-assigned FV patients who arrive in 2008 but are scheduled on or after 1st Jan 2009. The demand of the FV patients, who arrive in 2009 but are scheduled for an appointment in 2010, is included in finding the optimal solution. Those 1140 test cases are designed from different levels of the discharge rates, the RV's mean appointment lead-time, and sets of the FV's appointment lead-time targets (Appendices 5 and 6).

With each appointment scheduling rule, 1141 test cases are examined. These test cases correspond to 1140 different optimal planned capacity scenarios and 1 actual provided capacity from the studied specialty. As a result, there are a total of 2282 runs that are executed to evaluate the performance of the proposed appointment scheduling rules. The scheduling rules are set-up to admit each individual patient's request into either the actual planned capacity or the optimal planned capacity, which is measured on a weekly basis. Detail of the inputs can be found in Appendices A5.1 to A5.14 and in Appendices A6.1 to A6.14. The computation time to simulate the rules which are built in spreadsheets with Visual Basic Application is less than 30 seconds for each test case.

4.4.2. *Experimental results*

Given the actual arrival demand, actual capacity, and actual lead-time targets (described in Section 4.3.1), both the *exact rule* and *relaxed rule* need

to reject 8% of the arrival demands to achieve the three appointment lead-time targets; likewise the studied hospital's median, 95th percentile and 100th percentile appointment lead-times were far away from the established targets. Table 1.1 shows that the actual lead-times for the studied specialty for median, 95th percentile, and 100th percentile are 2.1, 2, 1.7 times as much as those from the MOH's guideline, respectively. The patient rejections happen to arrivals in the first 8 weeks. During the first 8 weeks of 2009, the total number of arrivals was 1,382 patients; while the total number of available slots was only 487 slots during the first 10 weeks of 2009. However, the high number of slots was planned from week 11 onward (for example: a mean 5% over-provided capacity to the arrival demands). The duration of the first 10 weeks is used to compare, because the median appointment lead-time needs to be considered for the last arrival in week 8th of arrival horizon. Therefore, to assure the achievement of the established lead-time targets at every time-unit, the rejection of the patients' request cannot be avoided; then, the unused slots occur at a later time. In other words, the proposed scheduling rules provide efficient and effective quantitative constraints as well as systematic way to admit the patients so that the clinic can meet the established lead-time targets with maximum utilization of the provided resource. Comparing to the *relaxed rule*, the *exact rule* can help the staff in providing appointment to the patients with similar utilization level of the available capacity.

Furthermore, given the inputs as mentioned in the experimental design (sub-section 4.3.1), a comparison of the proposed appointment scheduling rules on the cumulative frequency of the percentage of the unused slots is

made. The comparison is carried out for both the planned capacity of the capacity design and the admission management approaches.

Figure 4.2 shows the comparison of the cumulative frequency of the percentage of the unused slots for the optimal required capacity of the capacity design approach. The relaxed appointment scheduling rule always has fewer unused slots than the exact one does. Furthermore, the mean percentage of the unused slots; which is measured over 570 test cases (shown in Table 4.4), from the relaxed rule (1%) is also slightly less than that from the exact rule (1.5%). However, the maximum percentage of the unused slots from both the exact and relaxed scheduling rules is similar (7%). The findings show that although the relaxation of the appointment scheduling rule improves the utilization of the planned resource, the improvement is negligible. Therefore, to help the scheduling achieve the lead-time targets, the exact rule should be used to determine the patients' appointment. The relaxed rule should be considered when the patients' appointment needs to be admitted urgently and cannot be scheduled using the exact rule; for example a severe case that gets an approval from a physician but no slot identified from the exact rule is available.

Table 4.4: Performance of the proposed appointment scheduling rules

Problem	Policy	Percentage of the unused slots (%)					
		Min	Median	90 th percentile	95 th percentile	Max	Mean
Capacity design	<i>Exact</i>	0	1	4	5	7	1.5
	<i>Relaxed</i>	0	0	3	3	7	1
Admission management	<i>Exact</i>	0	4	7	8	20	4
	<i>Relaxed</i>	0	3	7	10	24	3.8

Figure 4.2: Cumulative frequency of the percentage of the unused slots for the capacity design problem

Figure 4.3 compares the cumulative frequency of the percentage of the unused slots for the optimal allocated capacity of the admission management approach. There is an indeterminate trend for the performance of the two proposed appointment scheduling rules. Table 4.4 reveals detailed measures of percentage of the unused slots. The relaxed appointment scheduling rule gives an average 3.8% of the unused slots, while the exact appointment scheduling rule grants an average 4% of the unused slots. The mean percentage of the unused slots seems similar between the rules. However, the maximum percentage of the unused slots from the exact rule is 4% less than that from the relaxed rule. The findings signify that the exact appointment scheduling rule can achieve a good utilization of the planned capacity.

Figure 4.3: Cumulative frequency of the percentage of the unused slots for the admission management problem

The results (shown in Table 4.4 previously) show that the proposed rules give a mean 4% and a maximum 24% of the unused resource for the optimal allocated capacity obtained from the admission management approach. However, for the optimal required capacity from the capacity design approach, the proposed appointment scheduling rules have fewer unused slots. A high percentage of the unused slots is due to the long initial delay of the planned capacity; for example, the first available slot at which the patients' request can be scheduled is in week (or time-unit) 12th of the planning horizon.

These first 12 weeks from the start of the planning horizon can be defined as the “*blocked periods*” in which the resource is fully occupied by pre-assigned patients. This blocked period (up to time-unit 12th) leads to no requests that can be scheduled within the median target, i.e., $c_i^m = 0$. Subsequently, the rejection of the requests will happen for the 95th-percentile-constraint ($c_i^p = 0$) and 100th-percentile-constraint ($c_i^{100} = 0$). The number of the unused slots tends to be zero in the latter time-units of the model horizon when the median-constraint starts to be positive.

In Chapter 3, the network flow models have full information of the future demands, thus the optimal results for the capacity design problem allow some violations of MOH’s lead-time targets in the initial time periods, due to the pre-assignments. Then, in the later time periods, patients can be scheduled so that the performance constraints can be satisfied. In this chapter, the scheduling of the patients’ appointment is myopic and it is assumed that the MOH’s lead-time targets need to be maintained at all times. The different assumptions of the achievement of MOH’s targets, for the arrival horizon at the strategic level versus throughout the arrival horizon at the operational level, cause the possibility of the unused slots. The proposed rules perform well with the optimal required capacity from the capacity design approach due to good capacity planning. However, the efficacy of the proposed rules seems to decrease when addressing the optimal allocated capacity from the admission management approach. This decreased efficacy is due to having less capacity, which extends the period from the time-unit of the first new demand to that of the first available capacity due to the larger number of the pre-assignments.

The numerical experiments reveal that the proposed appointment scheduling rules are effective and efficient in guiding the assignments of the patients' requests. The proposed appointment scheduling rules help to utilize the resource in which the arrivals of FV patients are unknown but the MOH's appointment lead-time targets need to be maintained. The analysis signifies that patients should have the 95th percentile or the 100th percentile appointment date if they cannot get appointments within the median target. This result is coincident with the trend found in Chapter 3. However, the relaxation of the patients' appointment dates restricted by the 95th percentile or the 100th percentile target may be considered only as it is necessary (i.e., special cases that get approval from physician/s). This relaxation has a negligible impact on the performance.

In addition, we find an interesting implication for prioritizing patients. If physicians expect to give higher priority to some patients, the MOH's targets for the median, the 95th percentile, and the 100th percentile should guide the prioritization. For example, the prioritized patients' lead-time should not be longer than the median lead-time targets, while the 95th percentile or the maximum lead-time targets is set for the less severe or ordinary patients respectively. The duration should not exceed the median lead-time target to avoid the impact of the pre-assignments onto the appointment rules' performance. In other words, this way of specialization prevents the unexpected rejection of the patients' requests. Subsequently, it helps to increase the utilization of resource and to obtain a controllable appointment system

4.4.3. *Sensitivity analysis*

Sensitivity of the rules' performance to changes of the appointment lead-time targets are examined for both the capacity design and admission management approaches. Tables 4.5 and 4.6 summarize the analysis of 2280 test cases mentioned in the experimental design section, which excludes the actual capacity sample.

Regardless of the capacity design or admission management approach, the findings show that the percentage of the unused slots does not depend on the length of the established appointment lead-time targets; this is illustrated to be indeterminate to the changes of the lead-time targets (Tables 4.5 and 4.6). This finding implies that any change of the guideline will impact directly on the patients' lead-time, but indeterminately affect the utilization of the systems. The patients' appointment lead-time for median, 95th percentile, and 100th percentile will be equal to the established service targets due to the way that the admission constraints were developed; hence, any change of the lead-time targets will lead to the change of the patients' lead-time in practice. In other words, a commitment of guided appointment scheduling rules to admit patients will lead to the achievement of the established lead-time targets, while the planned resource promises to be highly utilized.

Table 4.5: Sensitivity analysis to the changes of the appointment lead-time targets for the *capacity design* approach

RV's mean lead-time		Percentage of the unused slots									
	FV discharge rate	0.38	0.38	0.38	0.5	0.7	0.38	0.38	0.38	0.5	0.7
	RV discharge rate	0.1	0.2	0.32	0.32	0.32	0.1	0.2	0.32	0.32	0.32
	FV lead-time	<i>Exact rule</i>					<i>Relaxed rule</i>				
5	Median	1	0	1	1	1	1	0	1	1	1
	95 th percentile	1	-	1	1	1	1	1	1	1	1
	100 th percentile	1	+	1	+	-	1	+	1	1	1
10	Median	1	1	1	1	1	1	1	1	1	1
	95 th percentile	-	1	1	1	1	1	1	1	1	1
	100 th percentile	1	1	+	1	1	1	1	+	1	1
16	Median	1	0	1	1	1	1	0	1	1	1
	95 th percentile	1	1	1	1	1	1	1	1	1	1
	100 th percentile	1	0	1	1	1	1	0	1	1	1
20	Median	1	1	1	-	-	1	1	1	1	1
	95 th percentile	1	1	1	1	1	1	1	1	1	1
	100 th percentile	1	-	1	1	1	1	1	1	1	1
25	Median	1	-	1	1	-	1	-	1	1	-
	95 th percentile	1	1	1	1	1	1	-	-	1	1
	100 th percentile	1	1	1	1	1	1	0	0	1	1
30	Median	1	1	-	-	-	1	-	1	-	-
	95 th percentile	1	1	1	1	1	1	1	-	1	1
	100 th percentile	+	1	1	1	1	+	0	0	1	1

Notes:

Sign	Description
-	A negative correlation
+	A positive correlation
0	A similar performance
1	An indeterminate correlation
Inf.	An infeasible solution

Table 4.6: Sensitivity analysis to the changes of the appointment lead-time targets for the *admission management* approach

RV's mean lead-time		Percentage of the unused slots									
	FV discharge rate	0.38	0.38	0.38	0.5	0.7	0.38	0.38	0.38	0.5	0.7
	RV discharge rate	0.1	0.2	0.32	0.32	0.32	0.1	0.2	0.32	0.32	0.32
	FV Lead-time	<i>Exact rule</i>					<i>relaxed rule</i>				
5	Median	Inf.	Inf.	1	1	1	Inf.	Inf.	1	1	1
	95th Percentile	Inf.	Inf.	1	1	1	Inf.	Inf.	1	1	1
	100th percentile	Inf.	Inf.	1	1	1	Inf.	Inf.	+	1	1
10	Median	Inf.	1	1	1	1	Inf.	1	1	1	1
	95th Percentile	Inf.	1	1	1	1	Inf.	1	1	1	1
	100th percentile	Inf.	1	1	1	1	Inf.	1	1	1	1
16	Median	1	1	1	1	-	1	1	1	1	-
	95th Percentile	1	1	1	1	1	1	1	1	1	1
	100th percentile	1	1	1	+	1	1	1	1	1	1
20	Median	-	1	1	-	-	1	1	-	-	-
	95th Percentile	1	1	1	1	1	1	1	1	-	-
	100th percentile	1	1	1	1	1	1	1	1	1	-
25	Median	1	-	-	-	1	1	1	-	1	-
	95th Percentile	1	1	1	-	1	1	1	1	1	-
	100th percentile	1	1	1	1	1	1	1	1	1	1
30	Median	1	-	1	-	1	-	1	1	-	-
	95th Percentile	1	1	1	1	1	-	-	1	-	-
	100th percentile	1	1	1	1	1	-	-	1	1	-

Notes:

<i>sign</i>	<i>Description</i>
-	A negative correlation
+	A positive correlation
0	A similar performance
1	An indeterminate correlation
Inf.	An infeasible solution

4.5. Summary

In this chapter, two appointment scheduling rules are proposed for making decisions on the patients' appointments. The exact appointment scheduling rule aims to schedule the patients onto the p^{th} percentile or the 100th percentile appointment date if the patients cannot get appointments within the median target. The relaxed appointment scheduling rule allows the appointment dates of patients to be more flexible. If patients have to be assigned an appointment longer than the median appointment lead-time target, then the rule will attempt to schedule the patients so that the lead-time is within the p^{th} percentile or the 100th percentile lead-time target.

The investigation for the effectiveness of the proposed rules shows that the exact appointment scheduling rule should be considered to make decisions for the patient appointment date, since this rule grants the lowest maximum percentage of the unused slots while the mean percentage of the unused slots is similar. However, the relaxation of the patient appointment dates restricted by the p^{th} percentile or the 100th percentile target may be also considered only in necessary scenarios (e.g., urgent cases).

In addition, the MOH's median appointment lead-time target should be considered to increase the utilization of planned slots when doing the specialization in types of patients. The period of implementing the specialization should not be longer than the median appointment lead-time target to avoid the unexpected rejections of patients' request. Furthermore, the guided appointment scheduling rules needs to be committed not only to satisfy the service target but to utilize the provided resource as well.

CHAPTER 5: APPROXIMATE PLANNING MODELS OF RE-ENTRY APPOINTMENT SYSTEMS WITH STOCHASTIC DEMAND

In Chapter 3, the network flow models for the capacity planning problems were proposed. The optimal required capacity and the optimal admission schemes are obtained based on the assumption of deterministic arrival demands. Subsequently, in Chapter 4, the derivation of the appointment scheduling rules to make decisions on usage of the optimal planned capacity was addressed. The proposed rules provide guidance on patients' appointment dates under the assumption that schedulers do not know the future arrival demands.

Although the proposed appointment scheduling rules focus on the uncertainty of the arrival demands, the input of the optimal planned capacity is obtained from the deterministic capacity planning models. As a result, the assumption of no rejections of patients' requests in the capacity design approach may not be reasonable. Besides, the optimal required capacity or the optimal admission scheme is not robust. Hence, in order to make capacity planning more robust, this chapter examines the study of capacity planning problems with stochastic demand arrivals. The model needs to allow the possible violation of constraints that contain uncertain elements, in order to build robust models for optimizing the capacity planning problem.

The stochastic programming and robust optimization techniques are the standard approaches in the literature to cope with uncertainty within an

optimization. The stochastic programming technique is to solve the problems that explicitly account for the uncertainty. The robust optimization technique focuses on solving the stochastic problems in which the realizations of random variables are modeled by an uncertainty set rather than by a probabilistic distribution. A stochastic problem can be approximated by a deterministic version by replacing random variables with their expectations. Another approximation is based on using probabilistic constraints, whereby each constraint is expressed so that it can be achieved with some specified probability.

Charnes et al. (1958) introduce the notion of a chance constraint as a way to limit the likelihood of a constraint violation. The authors proposed a procedure for determining deterministic equivalents due to the original stochastic problems being computationally intractable. The deterministic equivalent concepts were explored by Charnes and Cooper (1963). The study is to introduce deterministic equivalents for chance constrained programming problems. The deterministic equivalent of a chance constraint has been prominently focused to study due to a tractable approximation, and it is proved to be a second-order cone representation in Calafiore and Ghaoui (2006). In addition, the approximated deterministic equivalents from joint chance constraints were further investigated in Nemirovski and Shapiro (2006). The study aims to decompose the joint chance constraints into a number of chance constraints which can be converted into the deterministic equivalents in Calafiore and Ghaoui (2006). With the advantages of a robust linear programming model, safe and tractable equivalents for the capacity planning models from original joint chance constraints problem will be developed. The

approximation is for both full and partial distributional information of the uncertain arrival demands. The chapter is organized in the next four main sections. Section 5.1 presents the approximation of a stochastic capacity design model. Section 5.2 provides the approximation of a stochastic admission management model. The numerical experiments are shown in Section 5.3. Finally, Section 5.4 summarizes the findings obtained in the chapter.

5.1. Approximation of the stochastic capacity design model

5.1.1. *The stochastic capacity design model*

In this part, the deterministic linear programming problem (P1) in Chapter 3 is reformulated as a joint chance-constraint programming problem (denoted by JCC-P1), to model the uncertain arrival demands and to set a violation risk for the constraints related to the uncertain arrival demands. The uncertainty of the arrival demands is only for the FV patients. Hence, the joint chance-constraint- problem introduces a probability constraint for equation (3.3'). Since (P1) is a minimization problem, constraint (3.3') is re-written by changing the equality into an inequality as in inequality (5.1) and equation (5.2).

$$\sum_{j=i}^T z_{ij} \geq f_i, \forall i \in N_1, \quad (5.1)$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2, \quad (5.2)$$

where $N_1 = 1, \dots, S$ and $N_2 = S + 1, \dots, T$.

Due to the certainty of constraint (5.2), the probability constraint is unnecessary for this constraint. We now replace f_i in constraint (5.1) by a

random variable $\tilde{f}_i$ and thus model the arrivals to be uncertain. Then, constraint (5.1) is re-formulated as a joint chance constraint with a risk of constraint's violation $\varepsilon \in (0,1)$. The risk of violation, ε , represents the allowable probability that a set of constraints is violated; namely, this is the maximum probability that arriving patients cannot be scheduled in all periods in the capacity design problem.

The deterministic linear programming (P1) becomes a joint chance constraints programming (JCC-P1) as follows.

[JCC-P1]:

$$\text{Min } q \quad (5.1')$$

Subject to:

$$q \geq c_j, \forall j \in N, \quad (5.2')$$

$$Pr\left(\sum_{j=i}^T z_{ij} \geq \tilde{f}_i; \forall i \in N_1\right) \geq 1 - \varepsilon, \quad (5.3')$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2, \quad (5.4')$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (5.5')$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (5.6')$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (5.7')$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (5.8')$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (5.9')$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (5.10')$$

$$y_{ij} = 0, \forall j \geq T + 1, \forall i \leq S, \quad (5.11')$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.12')$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \left(\frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij}\right) + 1, \quad (5.13')$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} (\sum_{i=1}^S \sum_{j=i}^T z_{ij}) \quad (5.14')$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (5.15')$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.16')$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \quad \forall j \in N, \quad (5.17')$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \quad \forall j \in N, \quad (5.18')$$

$$c_j - (c_j^f + c_j^r) = 0, \quad \forall j \in N, \quad (5.19')$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r \geq 0, \quad \forall i, j \in N, \quad (5.20')$$

$$z_{ij} \text{ integer}, \quad \forall i, j \in N. \quad (5.21')$$

where $Pr(A)$ represents probability of an event A.

The joint chance constraint (5.3') requires that the probability that all patients can be scheduled is greater than the value $(1 - \varepsilon)$. The other constraints remain unchanged with the original problem (P1) in Section 3.3.

5.1.2. *Deterministic equivalents for the stochastic capacity design model*

An approximation of the problem is formed by decomposing the joint chance constraints of problem (JCC-P1) into a number of individual chance constraints. The decomposition is implemented by using a union bound or Boole's inequality: $Pr(\cup_i A_i) \leq \sum_i Pr(A_i)$ (Ross, 1993). The inequality represents the probability that at least one of the events happens is not greater than the sum of the probabilities of the individual events.

First, the joint chance-constraints are decomposed into S individual chance constraints. The decomposition is implemented by using Boole's

inequality. The constraint (5.3') requires that all the inequalities are to be satisfied with a minimum probability $(1 - \varepsilon)$, hence, it is reformulated using the intersection of the inequalities as shown in constraint (5.22') :

$$Pr(\cap_{i \in N_1} \{\sum_{j=i}^T z_{ij} - \tilde{f}_i \geq 0\}) \geq 1 - \varepsilon. \quad (5.22')$$

It can be rewritten by using $Pr(\cap_i A_i) = 1 - Pr(\cup_i (A_i)^c)$, where $(A_i)^c$ is a complement of A_i . Therefore, $1 - Pr(\cup_{i \in N_1} \{\sum_{j=i}^T z_{ij} - \tilde{f}_i \geq 0\}^c) \geq 1 - \varepsilon$; which is equivalent to constraint (5.23')

$$Pr(\cup_{i \in N_1} \{\sum_{j=i}^T z_{ij} - \tilde{f}_i < 0\}) \leq \varepsilon. \quad (5.23')$$

By using a union bound or Boole's inequality for the constraint (5.23'), the constraint becomes:

$$Pr(\cup_{i \in N_1} \{\sum_{j=i}^T z_{ij} - \tilde{f}_i < 0\}) \leq \sum_{i \in N_1} Pr(\sum_{j=i}^T z_{ij} - \tilde{f}_i < 0). \quad (5.24')$$

Given $Pr(\sum_{j=i}^T z_{ij} - \tilde{f}_i < 0) \leq \varepsilon_i$ and $\sum_{i \in N_1} \varepsilon_i \leq \varepsilon$, the following individual chance constraints in inequality (5.25') represent a conservative approximation of the constraints (5.3').

$$Pr(\sum_{j=i}^T z_{ij} - \tilde{f}_i < 0) \leq \varepsilon_i, \quad \forall i \in N_1. \quad (5.25')$$

The equivalent of inequality (5.25') is shown in inequality (5.26').

$$Pr(\sum_{j=i}^T z_{ij} - \tilde{f}_i \geq 0) \geq 1 - \varepsilon_i, \quad \forall i \in N_1. \quad (5.26')$$

The inequality (5.26') can be rewritten as in inequality (5.27').

$$Pr(\tilde{f}_i - \sum_{j=i}^T z_{ij} \leq 0) \geq 1 - \varepsilon_i, \quad \forall i \in N_1. \quad (5.27')$$

Designating ε_i as a variable destroys the convexity of the problem after approximation. This issue can be resolved by letting ε_i be a parameter and setting $\varepsilon_i = \frac{\varepsilon}{S}$ (refer to Nemirovski and Shapiro, 2006). Then, the equivalent of inequality (5.27') is given by (5.28'):

$$Pr(\tilde{f}_i - \sum_{j=i}^T z_{ij} \leq 0) \geq 1 - \frac{\varepsilon}{S}, \quad \forall i \in N_1. \quad (5.28')$$

5.1.2.a. Known exact probability distribution of the uncertain arrival demands

This part carries out the approximation for the capacity design approach. The deterministic equivalent is proposed given the distribution for the uncertain demand arrivals.

Assume that the probability distribution of the uncertain arrival demands $\tilde{f}_i$ is known. The cumulative distribution function (CDF) of $\tilde{f}_i$ can be applied. The left hand side of the constraint (5.28') can be presented as the cumulative distribution function of a real value random variable $\tilde{f}_i$ as follows:

$$F_{\tilde{f}_i}(\sum_{j=i}^T z_{ij}) = Pr(\tilde{f}_i \leq \sum_{j=i}^T z_{ij}), \forall i \in N_1. \quad (5.29')$$

Then, the constraint (5.28') can be rewritten as:

$$F_{\tilde{f}_i}(\sum_{j=i}^T z_{ij}) \geq 1 - \frac{\varepsilon}{S}, \forall i \in N_1. \quad (5.30')$$

Thus, inequality (5.30') can be replaced by (5.31') which shows an inverse distribution function of a real value random variable $\tilde{f}_i$.

$$\sum_{j=i}^T z_{ij} \geq F_{\tilde{f}_i}^{-1}\left(1 - \frac{\varepsilon}{S}\right), \forall i \in N_1. \quad (5.31')$$

After the simplification, we replace (5.3') by the set of constraints (5.31'), which guarantee that (5.3') holds; however, (5.31') is a tighter constraint than (5.3'). Every individual inequality of (5.31') is a computationally tractable deterministic equivalent. Then, the deterministic equivalent model (DE-P1^{*}) of the joint chance constraints problem (JCC-P1) is reformulated as follows:

[DE-P1^{*}]:

$$\text{Min } q \quad (5.1'^*)$$

Subject to:

$$q \geq c_j, \forall j \in N, \quad (5.2',*)$$

$$\sum_{j=i}^T z_{ij} \geq F_{\tilde{f}_i}^{-1} \left(1 - \frac{\varepsilon}{S} \right), \forall i \in N_1, \quad (5.3',*)$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2, \quad (5.4',*)$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (5.5',*)$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (5.6',*)$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (5.7',*)$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (5.8',*)$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (5.9',*)$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (5.10',*)$$

$$y_{ij} = 0, \forall j \geq T + 1, \forall i \leq S, \quad (5.11',*)$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.12',*)$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \left(\frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij} \right) + 1, \quad (5.13',*)$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} \left(\sum_{i=1}^S \sum_{j=i}^T z_{ij} \right) \quad (5.14',*)$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (5.15',*)$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.16',*)$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \forall j \in N, \quad (5.17',*)$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \forall j \in N, \quad (5.18',*)$$

$$c_j - (c_j^f + c_j^r) = 0, \forall j \in N, \quad (5.19',*)$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r \geq 0, \forall i, j \in N, \quad (5.20',*)$$

$$z_{ij} \text{ integer}, \forall i, j \in N. \quad (5.21',*)$$

5.1.2.b. Known mean and variance of the uncertain arrival demands

This part continues the approximation for the capacity design approach, where we assume that only mean (μ) and standard deviation (σ) of the arrival demand $\tilde{f}_i$ are known, but we do not know the distribution. We can replace the set of individual constraints (5.28') of capacity design problem with the following set of deterministic convex constraints, which is conservatively approximated and tractable.

$$\mu + \sigma \sqrt{\frac{s}{\varepsilon} - 1} - \sum_{j=i}^T z_{ij} \leq 0, \forall i \in N_1. \quad (5.32')$$

Proof:

The proof is done for each individual constraint in the set of constraints (5.28'). Given constraint i of the set of constraints (5.28'), let the decision vector be $\mathbf{z}_i = [z_{i1}, \dots, z_{iT}]' \in R_+^T$, and the coefficient vector be $\mathbf{k}_i = [k_1, \dots, k_T]' \in R^T$ and other vectors' definition as follows:

$$\begin{aligned} \tilde{\mathbf{z}}_i &= [(\mathbf{z}_i)', 1]' \in R_+^{T+1}, \\ \tilde{\mathbf{d}}_i &= [(\mathbf{k}_i)', \tilde{f}_i]' \in R^{T+1}, \\ \hat{\mathbf{d}} &= [(\widehat{\mathbf{k}})', \hat{f}_i] \in R^{T+1}. \end{aligned}$$

Given the definition of the scalar quantity $\varphi_i(\mathbf{z}_i) = (\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \in R$, let the scalar quantity be as shown in equation (5.33'):

$$(\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i = -\sum_{j=i}^T z_{ij} + \tilde{f}_i. \quad (5.33')$$

Then, constraint (5.28') can be rewritten as (5.34').

$$Pr \left((\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \leq 0 \right) \geq 1 - \frac{\varepsilon}{s}, \quad \forall i \in N_1. \quad (5.34')$$

It can be enforced robustly by the robust chance constraint with the infimum restriction:

$$\inf_{\tilde{\mathbf{d}}_i \sim (\hat{\mathbf{d}}_i, \text{var}(\tilde{\mathbf{d}}_i))} \Pr \left((\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \leq 0 \right) \geq 1 - \frac{\varepsilon}{S}, \quad \forall i \in N_1. \quad (5.35')$$

As the result, the constraint (5.35') can be illustrated as (5.36') by designating

$\varepsilon_i = \frac{\varepsilon}{S}$ (refer to Nemirovski and Shapiro, 2006).

$$\inf_{\tilde{\mathbf{d}}_i \sim (\hat{\mathbf{d}}_i, \text{var}(\tilde{\mathbf{d}}_i))} \Pr \left((\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \leq 0 \right) \geq 1 - \varepsilon_i, \quad \forall i \in N_1. \quad (5.36')$$

Therefore, constraint (5.34') is equivalent to the following constraints (5.37')

or (5.38') (refer to the theorem 3.1 in Calafiore and Ghaoui, 2006):

$$\hat{\varphi}_i(\mathbf{z}_i) + \left(\sqrt{\frac{S}{\varepsilon}} - 1 \right) \sigma_i(\mathbf{z}_i) \leq 0, \quad \forall i \in N_1, \quad (5.37')$$

or

$$\hat{\varphi}_i(\mathbf{z}_i) + k_{\varepsilon_i} \sigma_i(\mathbf{z}_i) \leq 0, \quad \forall i \in N_1, \quad (5.38')$$

where: $k_{\varepsilon_i} = \sqrt{\frac{1-\varepsilon_i}{\varepsilon_i}}$ and $\varepsilon_i = \frac{\varepsilon}{S}$.

Moreover, $\hat{\varphi}_i(\mathbf{z}_i)$ and $\sigma_i(\mathbf{z}_i)$, which are the expected value and standard deviation of the scalar quantity $\varphi_i(\mathbf{z}_i) = (\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i$, respectively, are obtained as follows:

$$\hat{\varphi}_i(\mathbf{z}_i) = E[\varphi_i(\mathbf{z}_i)] = E \left[(\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \right] = E \left[(\tilde{\mathbf{d}}_i)' \right] \cdot \tilde{\mathbf{z}}_i = \hat{\mathbf{d}}_i \cdot \tilde{\mathbf{z}}_i$$

$$\sigma_i(\mathbf{z}_i) = \sqrt{\text{var}(\varphi_i(\mathbf{z}_i))} = \sqrt{\text{var} \left((\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \right)} = \sqrt{(\tilde{\mathbf{z}}_i)' \cdot \text{var}(\tilde{\mathbf{d}}_i) \cdot \tilde{\mathbf{z}}_i}$$

In the rest steps of the proof, the expected value and covariance of the random vector $\tilde{\mathbf{d}}_i$, which are denoted by the $\hat{\mathbf{d}}_i \in R^{T+1}$ and $\text{var}(\tilde{\mathbf{d}}) \in R^{T+1, T+1}$, need to be acquired in order to calculate $\hat{\varphi}_i(\mathbf{z}_i)$ and $\sigma_i(\mathbf{z}_i)$ value from the known mean (μ) and variance (σ^2) of the arrival demands $\tilde{f}_i$. Equation (5.39') represents the calculation of the expected value of the random vector $\tilde{\mathbf{d}}_i$.

$$\hat{\mathbf{d}}_i = E(\tilde{\mathbf{d}}_i) = E\left([\mathbf{k}_i)', \tilde{f}_i\right]', \forall i \in N_1. \quad (5.39')$$

Since $\mathbf{k}_i$ is a known coefficient vector, the equation (5.39') is equivalent to the following equation (5.40') in which the expected value of the random arrival demand $(\tilde{f}_i)$ is $E(\tilde{f}_i) = \mu$.

$$\hat{\mathbf{d}}_i = [\mathbf{k}_i)', E(\tilde{f}_i)]' = [\mathbf{k}_i)', \mu]', \forall i \in N_1. \quad (5.40')$$

Similarly, the covariance of the random vector $\tilde{\mathbf{d}}_i$ is calculated in the following equation (5.41').

$$var(\tilde{\mathbf{d}}_i) = var\left([\mathbf{k}_i)', \tilde{f}_i\right)' = \begin{bmatrix} \mathbf{0}^{T \times T} & \mathbf{0}^T \\ (\mathbf{0}^T)' & \sigma^2 \end{bmatrix} \geq 0, \forall i \in N_1. \quad (5.41')$$

Where: $\mathbf{0}^{T \times T}$ is a T -by- T matrix in which all the elements are equal to zero, and $\mathbf{0}^T$ is a vector of size T in which all the elements are equal to zero.

From the equations (5.40') and (5.41'), the expected value $\hat{\varphi}_i(\mathbf{z}_i)$ and standard deviation $\sigma_i(\mathbf{z}_i)$ are calculated in the following equations (5.42') and (5.43'), respectively.

$$\hat{\varphi}_i(\mathbf{z}_i) = \hat{\mathbf{d}}_i \cdot \tilde{\mathbf{z}}_i = \mu - \sum_{j=i}^T z_{ij}, \forall i \in N_1, \quad (5.42')$$

$$\sigma_i(\mathbf{z}_i) = \sqrt{var\left([\mathbf{k}_i)', \tilde{f}_i\right)' \cdot \tilde{\mathbf{z}}_i} = \sigma, \forall i \in N_1. \quad (5.43')$$

Given equations (5.42') and (5.43'), constraint (5.37') can be rewritten as follows:

$$\mu + \sigma \sqrt{\frac{s}{\varepsilon} - 1} - \sum_{j=i}^T z_{ij} \leq 0, \forall i \in N_1.$$

Thus, constraint (5.32') robustly represents constraint (5.28'). $\square$

After the simplification, constraint (5.32') is equivalent to constraint (5.3'). Then, the deterministic equivalent model (DE-P1^{**}) of the joint chance constraint problem (JCC-P1) is reformulated as follows.

[DE-P1^{**}]:

$$\text{Min } q \quad (5.1^{**})$$

Subject to:

$$q \geq c_j, \forall j \in N, \quad (5.2^{**})$$

$$\sum_{j=i}^T z_{ij} \geq \mu + \sigma \sqrt{\frac{S}{\varepsilon} - 1}, \forall i \in N_1, \quad (5.3^{**})$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2 \quad (5.4^{**})$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (5.5^{**})$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (5.6^{**})$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (5.7^{**})$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (5.8^{**})$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (5.9^{**})$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (5.10^{**})$$

$$y_{ij} = 0, \forall j \geq T + 1, \forall i \leq S, \quad (5.11^{**})$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.12^{**})$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2}(\sum_{i=1}^S \sum_{j=i}^T z_{ij}) + 1, \quad (5.13^{**})$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100}(\sum_{i=1}^S \sum_{j=i}^T z_{ij}), \quad (5.14^{**})$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (5.15^{**})$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.16^{**})$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \forall j \in N, \quad (5.17^{**})$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \forall j \in N, \quad (5.18^{**})$$

$$c_j - (c_j^f + c_j^r) = 0, \forall j \in N, \quad (5.19^{**})$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r \geq 0, \forall i, j \in N, \quad (5.20^{**})$$

$$z_{ij} \text{ integer, } \forall i, j \in N. \quad (5.21^{**})$$

5.2. Approximation of stochastic admission management model

5.2.1. A stochastic admission management model

Like the approximation for the capacity design approach, the deterministic linear programming problem (P2) of the admission management approach in Chapter 3 (Section 3.4.1) can also be reformulated as a joint chance-constraint programming problem (JCC-P2). In the approximation of this admission management problem, the constraints (3.2'') and (3.3'') of the problem (P2) are necessarily combined into constraint (5.3) as follows:

$$\sum_{j=i}^T z_{ij} + f_i^d = f_i, \quad \forall i \in N,. \quad (5.3)$$

Assuming that the uncertainty of the arrival demands is only for the FV patients $\tilde{f}_i$, the joint chance-constraint problem becomes the problem with a chance constraint for constraint (5.3). Similar to the steps in Section 5.1, since (P2) is a minimization problem, constraint (5.3) is rewritten by changing the equality into an inequality as in (5.4) and (5.5).

$$\sum_{j=i}^T z_{ij} + f_i^d \geq f_i, \quad \forall i \in N_1, \quad (5.4)$$

$$\sum_{j=i}^T z_{ij} = 0, \quad \forall i \in N_2, \quad (5.5)$$

Where: $N_1 = 1, \dots, S$ and $N_2 = S + 1, \dots, T$.

f_i in constraint (5.4) is replaced by random variable $\tilde{f}_i$. Constraint (5.4) needs to be reformulated into a joint chance constraint with a risk of violation $\varepsilon \in (0,1)$. The risk of violation indicates how frequently the constraint can be violated.

The deterministic linear programming problem (P2) becomes a joint chance constraints programming problem (JCC-P2) as follows:

[JCC-P2]:

$$\text{Min } Q = \sum_{i=1}^S f_i^d \quad (5.1'')$$

Subject to:

$$Pr\left(\sum_{j=i}^T z_{ij} + f_i^d \geq \tilde{f}_i, \forall i \in N_1\right) \geq 1 - \varepsilon, \quad (5.2'')$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2, \quad (5.3'')$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (5.4'')$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (5.5'')$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (5.6'')$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (5.7'')$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (5.8'')$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (5.9'')$$

$$y_{ij} = 0, \forall j \geq T + 1, \forall i \leq S, \quad (5.10'')$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.11'')$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij}, \quad (5.12'')$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} \left(\sum_{i=1}^S \sum_{j=i}^T z_{ij} \right), \quad (5.13'')$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (5.14'')$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.15'')$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \forall j \in N, \quad (5.16'')$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \forall j \in N, \quad (5.17'')$$

$$c_j - (c_j^f + c_j^r) \geq 0, \forall j \in N, \quad (5.18'')$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r, f_i^d \geq 0, \forall i, j \in N, \quad (5.19'')$$

$$z_{ij} \text{ integer}, \forall i, j \in N. \quad (5.20'')$$

The joint chance constraint (5.2'') requires that the probability that the actual number of denied patients is less than the planned number is greater than the value $(1 - \varepsilon)$. The other constraints remain unchanged with the original problem (P2) in Section 3.4.

5.2.2. *Deterministic equivalents for the stochastic admission management model*

An approximation from decomposing the joint chance constraints of problem (JCC-P2) into a number of individual chance constraints will be carried out. The decomposition is implemented by using a union bound or Boole's inequality. The joint chance constraints problem will be decomposed into S individual chance constraints problem. The requirement of all the inequalities in the left hand side of constraint (5.2'') needs to be satisfied with a minimum probability $(1 - \varepsilon)$, the constraint (5.2'') becomes:

$$Pr(\cap_{i \in N_1} \{\sum_{j=i}^T z_{ij} + f_i^d - \tilde{f}_i \geq 0\}) \geq 1 - \varepsilon. \quad (5.21'')$$

A similar transforming procedure for inequality (5.21'') is executed (refer to Section 5.1.2). Then, an equivalent of inequality (5.21'') is obtained as shown in (5.22'').

$$Pr(\tilde{f}_i - \sum_{j=i}^T z_{ij} - f_i^d \leq 0) \geq 1 - \frac{\varepsilon}{S}, \forall i \in N_1. \quad (5.22'')$$

5.2.2.a. Known exact probability distribution of the uncertain arrival demands

This part implements the approximation for the admission management approach.

Assuming that the exact probability distribution of the uncertain arrival demands $\tilde{f}_i$ is known, the set of individual constraints (5.22'') of admission management model can be replaced by the set of deterministic equivalent constraints in inequality (5.23''), which is tractable. The developed steps are implemented similarly with those in sub-section 5.1.2.a.

$$\sum_{j=i}^T z_{ij} + f_i^d \geq F_{\tilde{f}_i}^{-1} \left(1 - \frac{\varepsilon}{S} \right), \forall i \in N_1. \quad (5.23'')$$

Then, the equivalent model (DE-P2^{*}) of the joint chance constraints problem (JCC-P2) is established as follows.

[DE-P2^{*}]:

$$\text{Min } Q = \sum_{i=1}^S f_i^d \quad (5.1''^*)$$

Subject to:

$$\sum_{j=i}^T z_{ij} + f_i^d \geq F_{\tilde{f}_i}^{-1} \left(1 - \frac{\varepsilon}{S} \right), \forall i \in N_1, \quad (5.2''^*)$$

$$\sum_{j=i}^T z_{ij} = 0, \forall i \in N_2, \quad (5.3''^*)$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \forall i, j \in N, \quad (5.4''^*)$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \forall i \in N, \quad (5.5''^*)$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \forall j \in N, \quad (5.6''^*)$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \forall j \in N, \quad (5.7''^*)$$

$$y_{ij} = 0, \forall j - i < a, \forall i, j \in N, \quad (5.8''^*)$$

$$y_{ij} = 0, \forall j - i > b, \forall i, j \in N, \quad (5.9'')^*$$

$$y_{ij} = 0, \quad \forall j \geq T + 1, \forall i \leq S, \quad (5.10'')^*$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.11'')^*$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij}, \quad (5.12'')^*$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} \left(\sum_{i=1}^S \sum_{j=i}^T z_{ij} \right) \quad (5.13'')^*$$

$$z_{ij} = 0, \forall j - i > w, \forall j - i < 0, \quad (5.14'')^*$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.15'')^*$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \quad \forall j \in N, \quad (5.16'')^*$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \quad \forall j \in N, \quad (5.17'')^*$$

$$c_j - (c_j^f + c_j^r) \geq 0, \quad \forall j \in N, \quad (5.18'')^*$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r, f_i^d \geq 0, \quad \forall i, j \in N, \quad (5.19'')^*$$

$$z_{ij} \text{ integer}, \quad \forall i, j \in N. \quad (5.20'')^*$$

5.2.2.b. Known mean and variance of the uncertain arrival demands

In this part, the approximation is also implemented for the admission management approach. However, the deterministic equivalent is proposed for the case when the mean (μ) and variance (σ) of the arrival demand $\tilde{f}_i$ are known but its distribution is unknown. The set of individual constraints (5.22'') of admission management model can be approximated with the following set of deterministic convex constraints, which is conservatively approximated and tractable.

$$\mu + \sigma \sqrt{\frac{S}{\varepsilon} - 1} - \sum_{j=i}^T z_{ij} - f_i^d \leq 0, \quad \forall i \in N_1. \quad (5.24'')$$

Proof:

The proof is done for every individual constraint in the set of constraints (5.22''). Given constraint i of the set of constraints (5.22''), let the decision vector to be $\mathbf{z}_i = [z_{i1}, \dots, z_{iT}, f_i^d]' \in \mathbf{R}_+^{T+1}$, the coefficient vector to be $\mathbf{k}_i = [k_1, \dots, k_{T+1}]' \in \mathbf{R}^{T+1}$ and other vectors' definition as follows:

$$\tilde{\mathbf{z}}_i = [(\mathbf{z}_i)', 1]' \in \mathbf{R}_+^{T+2},$$

$$\tilde{\mathbf{d}}_i = [(\mathbf{k}_i)', \tilde{f}_i]' \in \mathbf{R}^{T+2},$$

$$\hat{\mathbf{d}} = [(\widehat{\mathbf{k}})', \hat{f}_i] \in \mathbf{R}^{T+2}.$$

Scalar quantity:

$$\varphi_i(\mathbf{z}_i) = (\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i \in \mathbf{R},$$

and

$$\hat{\varphi}_i(\mathbf{z}_i) = \mathbb{E}[(\tilde{\mathbf{d}}_i)' \cdot \tilde{\mathbf{z}}_i] = \mathbb{E}[(\tilde{\mathbf{d}}_i)'] \cdot \tilde{\mathbf{z}}_i = \hat{\mathbf{d}}_i \cdot \tilde{\mathbf{z}}_i,$$

$$\sigma_i(\mathbf{z}_i) = \sqrt{\text{var}(\varphi_i(\mathbf{z}_i))} = \sqrt{(\tilde{\mathbf{z}}_i)' \cdot \text{var}(\tilde{\mathbf{d}}_i) \cdot \tilde{\mathbf{z}}_i}.$$

The rest steps of this proof procedure are similar as in the proof of sub-section 5.1.2.b. □

Then, the equivalent model (DE-P2^{**}) of the joint chance constraints problem (JCC-P2) is obtained as follows:

[DE-P2^{}]:**

$$\mathbf{Min} \ Q = \sum_{i=1}^S f_i^d \tag{5.1''**}$$

Subject to:

$$\sum_{j=i}^T z_{ij} + f_i^d \geq \mu + \sigma \sqrt{\frac{S}{\varepsilon} - 1}, \ \forall i \in N_1, \tag{5.2''**}$$

$$\sum_{j=i}^T z_{ij} = 0, \ \forall i \in N_2, \tag{5.3''**}$$

$$x_{ij} - (1 - \alpha)z_{ij} = 0, \quad \forall i, j \in N, \quad (5.4'')^*$$

$$d_i^f = \alpha \sum_{j=i}^T z_{ij}, \quad \forall i \in N, \quad (5.5'')^*$$

$$(r_j^f + r_j^r + \sum_{i=1}^j x_{ij} + \sum_{i=1}^j y_{ij}) - (d_j^r + \sum_{i=j}^{T+1} y_{ji}) = 0, \quad \forall j \in N, \quad (5.6'')^*$$

$$d_j^r - \beta(r_j^f + r_j^r + \sum_{i=1}^j y_{ij} + \sum_{i=1}^j x_{ij}) = 0, \quad \forall j \in N, \quad (5.7'')^*$$

$$y_{ij} = 0, \quad \forall j - i < a, \quad \forall i, j \in N, \quad (5.8'')^*$$

$$y_{ij} = 0, \quad \forall j - i > b, \quad \forall i, j \in N, \quad (5.9'')^*$$

$$y_{ij} = 0, \quad \forall j \geq T + 1, \quad \forall i \leq S, \quad (5.10'')^*$$

$$\sum_{i=S+1}^T d_i^r = 0, \quad (5.11'')^*$$

$$\sum_{z_{ij} \in L^m} z_{ij} \geq \frac{1}{2} \sum_{i=1}^S \sum_{j=i}^T z_{ij}, \quad (5.12'')^*$$

$$\sum_{z_{ij} \in L^p} z_{ij} \geq \frac{p}{100} \left(\sum_{i=1}^S \sum_{j=i}^T z_{ij} \right), \quad (5.13'')^*$$

$$z_{ij} = 0, \quad \forall j - i > w, \quad \forall j - i < 0, \quad (5.14'')^*$$

$$\sum_{j=1}^T \sum_{i=1}^j (j - i) y_{ij} - \bar{a} \sum_{j=1}^T \sum_{i=1}^j y_{ij} \leq 0, \quad (5.15'')^*$$

$$c_j^f - \left(\frac{1}{1-\alpha} \tau^f r_j^f + \tau^f \sum_{i=1}^j z_{ij} \right) = 0, \quad \forall j \in N, \quad (5.16'')^*$$

$$c_j^r - \left(\frac{1}{1-\beta} \tau^r r_j^r + \frac{1}{1-\beta} \tau^r \sum_{i=1}^j y_{ij} \right) = 0, \quad \forall j \in N, \quad (5.17'')^*$$

$$c_j - (c_j^f + c_j^r) \geq 0, \quad \forall j \in N, \quad (5.18'')^*$$

$$x_{ij}, y_{ij}, z_{ij}, c_i^f, c_j^r, d_i^f, d_j^r, f_i^d \geq 0, \quad \forall i, j \in N, \quad (5.19'')^*$$

$$z_{ij} \text{ integer}, \quad \forall i, j \in N. \quad (5.20'')^*$$

5.3. Numerical experiments

This section solves the chance-constrained program for a full or a partial information of the uncertainty distribution on a set of test problems. It requires either the distributional information, or the mean and variance of the demand arrival uncertainty. The purpose of the numerical experiments is to illustrate

the impact of assumptions of the uncertain demands on the performance, and the advantages of the proposed approximation. The performance measurement is the optimal required capacity or the optimal admission scheme.

The similar design of inputs mentioned in Chapter 3 (Section 3.3.2) is used. In addition, it requires to set the risk of violation, ε . Six levels of the violation risk, $\varepsilon = \{0.01; 0.05; 0.10; 0.20; 0.30; 0.40\}$, are examined. The arrival demands of the FV patients are assumed to be from a normal distribution or from an unknown distribution with given mean and standard deviation. For the assumption of unknown distribution, the mean and standard deviation are equal to 187.7 and 29 arrivals per week, respectively. For the known distribution of the uncertainty, we assume a normal distribution with the same mean and standard deviation above for the FV's arrival demands.

5.3.1. Numerical experiments of the deterministic equivalents for the capacity design problem

This part conducts numerical experiments for the same given inputs as in Section 3.2, except the changes mentioned above for the deterministic equivalents of the capacity design problem. Figures 5.1, 5.2, and 5.3 show the comparison results of the performance between the normal and the unknown distribution approaches, in terms of the maximum and the mean of required slots for both FV and RV, for only FV, and for only RV cases. The Appendix A7.1 presents simulation results of the problem.

Figure 5.1: Capacity design of the total number of FV and RV under the uncertainty for the normal and unknown distributions.

The comparison considering the total number of FV and RV slots (Figure 5.1) shows that increasing the allowed violation risk (ε) reduces the required capacity. This pattern is consistent for both maximum and mean measures. The minimum required slots from either the normal or the unknown distribution is equal to zero due to the restriction of the FV and RV lead-times; this causes no assignment at last time-units of the planning horizon. Furthermore, knowing the distributional information of the arrival demands (i.e., normal distribution) significantly reduces the number of planned resources for either the maximum or the mean required capacity; for example, given $\varepsilon = 0.01$, the maximum number of required slots for the partial distributional information (unknown distribution) case is 4,077 slots per week; for the normal distribution case, only 825 slots per week are needed. In other words, the fully distributional information of the FV demands gives a lower

investment cost, in terms of the required resource, in the future than the partial information of the new arrival demands does.

Figure 5.2: Capacity design for the FV patients under the uncertainty for the normal and unknown distributions.

Figure 5.3: Capacity design of the RV requests under the uncertainty for the normal and unknown distributions

From the performance measurements for only FV (Figure 5.2) or only RV (Figure 5.3), the results show that increasing the violation risk decreases the required capacity for both the maximum and the mean values. Similarly, knowing exact distributional information contributes a low investment cost into planning the required capacity.

Finally, this section also examines the comparison of the performance of the deterministic equivalents and the deterministic models, which are obtained in Section 3.3 of Chapter 3 (Appendix A7.1). The results show that the deterministic equivalents or approximated models are not only tractable but also safe, in terms of accounting for the demand arrival uncertainty. With the deterministic models, the risk of violation is greater than 40%; for example, either the obtained maximum or mean required capacity from the deterministic models (683 and 521 slots per week, respectively) is less than the necessary maximum and mean required slots from the approximated models. Thus, the approximate models assure that the risk of violation to be within 40% (749 and 580 slots per week, respectively, for normal distribution of the arrival demands; and 1,175 and 873 slots per week, respectively, for partial distribution of the arrival demands). The finding implies that the proposed deterministic equivalents assure the achievement of the MOH's lead-time targets with a low and manageable risk of patients rejected.

The above analyses demonstrate safety and tractability of the deterministic equivalents for the capacity design problem of the re-entry system. The proposed approximations produce a stable and robust solution which works well under the worst case or the highest acceptable risk of patients rejected. Besides, identifying the full distributional information of the

uncertainty plays an important role in preventing unnecessary investments of the required resource.

Corollary 5.1: If the arrival horizon is sufficient large in the capacity design problem, the deterministic equivalent with normal distribution requires less required capacity than that with only mean and variance information.

Proof:

For fully distributed information (e.g., normal distribution) of the uncertainty, the right hand side of constraint (5.3^{*}) (shown in Section 5.1.2.a) is determined by

$$F_{\tilde{f}_i}^{-1}\left(1 - \frac{\varepsilon}{S}\right) = \mu + \sigma z, \quad \forall i \in N_1, \quad (5.6)$$

Where: z is a score of a normal distribution.

For partially distributed information (e.g., known mean and standard deviation) of the uncertainty, the right hand side of constraint (5.3^{**}) (shown in Section 5.1.2.b) is $\mu + \sigma \sqrt{\frac{S}{\varepsilon} - 1}$. It is clear that eventually $\sqrt{\frac{S}{\varepsilon} - 1} > z$ when the arrival horizon S is sufficient large. As the result, the partially distributional information of the arrival demands leads to a higher predicted arrival demands than the fully distributed information does. The greater the number of predicted arrival demands is, the more required resource needs to plan. Thus, it concludes the corollary 5.1. $\square$

5.3.2. Numerical experiments of the deterministic equivalents for the admission management problem

This sub-section shows the conducted numerical experiments for the same given inputs as in Section 3.3 along with additional inputs mentioned in Section 5.3 for the deterministic equivalents of the admission management problem.

Figure 5.4: Admission management of the FV requests under the uncertainty for the normal and unknown distributions.

Figure 5.4 shows the comparison of the performance between a normal distribution and an unknown distribution. The performance measures the maximum number of admitted FV patients or the maximum number of the scheduled FV slots. The higher is the risk of the violation, the lower is the maximum number of admitted FV patients. This trend is consistent for both the full and the partial distributional information of the uncertain arrival demands. The decrease in the maximum number of admitted FV patients is

due to the decrease in the predicted FV arrival demands. Therefore, a lower risk of violation should be considered in order to maximize the number of scheduled patients and to highly utilize the provided resource.

Furthermore, using only partial information of the FV distribution to allocate the provided resource can allow the maximum number of admitted FV patients. As a result, it helps to improve the access of new patients and the utilization of the provided capacity, while the care continuity for the RV patients is maintained. In addition, the maximum number of admitted FV patients obtained from the deterministic equivalents or the approximated models is higher than that gained from the deterministic models, while the total provided capacity is the same. The approximate models addressing the uncertainty of the FV demands do help to eliminate the un-expected rejections of the FV requests. Subsequently, it is efficient and effective in increasing the utilization of the provided resource.

In conclusion, the findings demonstrate that the deterministic equivalents not only are safe and tractable, but also allow more new patients to be scheduled or higher utilization of the provided resource. The partially distributional information of the FV demands can maximize the number of the admitted patients which leads to the maximum utilization of the provided resource. Therefore, the deterministic equivalent using partially distributional information of the new arrival is highly encouraged, because it not only maximizes the resource utilization but also requires a low cost of data analysis. Furthermore, a low risk of violation should be considered to improve the access of new patients.

Corollary 5.2: For the admission management problem, if the arrival horizon is sufficient large, the deterministic equivalent with partially distributional information (e.g., known mean and standard deviation) provides a greater maximum number of admitted FV patients and resource utilization than that with normal distribution.

Proof:

Similar to the derivation of proof of corollary 5.1), comparison of the right hand sides of constraints (5.2'')* and (5.2'')**, which are shown in Sections 5.2.2.a and 5.2.2.b respectively, is carried out. The right hand side of constraint (5.2'')** is larger than that of constraint (5.2'')*. It implies that the deterministic equivalent with partially distributional information expects a greater number of new arrival demands than that with fully distributional information, when the arrival horizon is sufficient large. The higher predicted demands may lead to greater the maximum number of appointed patients. This concludes the corollary 5.2. □

5.3.3. Pattern of scheduling an appointment

The implementation of numerical experiments was carried out for 6 levels of the violation risk $\epsilon = \{0.01; 0.05; 0.10; 0.20; 0.30; 0.40\}$, for each of 450 test cases (which is described in Section 3.3.3). Thus, $6 \times 450 = 2,700$ runs were set up and solved for the individual capacity design and admission management problems. The results show similar trends for scheduling the FV patients which is discussed in Sections 3.3.4 and 3.4.4 of Chapter 3: the patients' appointment should also be on the median, the 95th percentile, or the 100th percentile appointment date, for the problems considering the uncertainty

of the FV demands. Tables 5.1 and 5.2 report the percentage of the FV patients whose appointment are set to one of the appointment lead-time targets for the capacity design and the admission management problems, respectively. Furthermore, the numerical experiments reveal that the mentioned trend of scheduling the FV patients is consistent regardless of the information of the FV arrival demand.

Table 5.1: Percentage of patients whose lead-time is equal to one of appointment lead-time targets for the capacity design with uncertain demands

Targets of patients' appointment lead-time	Percentage of patients (%)			
	Min	Median	Max	Mean
Median	4	36.5	46	31.5
95 th percentile	15	22	33	24.5
100 th percentile	3	5	5	4.8

Table 5.2: Percentage of patients whose lead-time is equal to one of appointment lead-time targets for the admission management with uncertain demands

Targets of patients' appointment lead-time	Percentage of patients (%)			
	Min	Median	Max	Mean
Median	14	38.5	52	34.5
95 th percentile	11	16.5	28	19.4
100 th percentile	3	5	5	4.6

The outcomes of the capacity design problem show a mean of 31.5%, 24.5%, and 4.8% of the FV demands that are assigned the median, 95th percentile, and 100th percentile appointment lead-time targets (Table 5.1);

while those of the admission management problem illustrate a mean of 34.5%, 19.4%, and 4.6% of the FV patients who are scheduled the median, 95th percentile, and maximum appointment lead-time targets (Table 5.2).

However, no specific way of scheduling RV patients has been found. The preliminary suggestion of decisions on the RV appointments is that less urgent cases should be postponed to allow more urgent cases to access the service provider quickly. Then, the restriction of the mean RV appointment lead-time could maintain.

5.4. Summary

This chapter formulates safe and tractable deterministic equivalents for the capacity planning models, based on using chance constraints. The approximation is implemented for both the capacity design and the admission management problems. An acceptable risk of violation needs to be established for a robust performance of the optimal capacity planning. The approximation considers both the case with the distribution of uncertain arrival demand is known and the case when only the first two moments of the distribution are known.

For the capacity design problem, the study shows that a higher risk of violation may lead to a lower requirement of planned resource. This pattern is consistent for either fully or partially distributed information of the uncertainty. Furthermore, knowing exact distribution can help to save the unnecessary requirement of resource. Therefore, knowledge of the distribution plays an important role and needs to be carefully considered in deciding an investment of the scarce resource.

For the admission management problem, the lower the risk of violation is, the greater the maximum number of admitted FV patients is. The trend holds regardless of the distributional information of the uncertainty. Increasing the maximum number of scheduled new patients is due to an increase in the expected new arrival demands. Therefore, only partial information of the FV demand's distributions (known mean and variance) is needed to determine an optimal admission scheme because it can provide a low cost of data collection and analysis, but highly utilize the scarce resource.

Furthermore, FV patients' appointment date should be the median, the 95th percentile, or the 100th percentile appointment date. This appointment scheduling for FV patients is independent from the information of the FV arrival demands distribution. In other words, regardless of having either fully or partly distributional information of the FV demands, the patients' appointment lead-time should be the median, or 95th percentile, or 100th percentile appointment lead-time targets.

On the other hand, no specific way for scheduling the appointment of RV patients has been found. The preliminary suggestion to the physicians for making decision on the follow-up appointment is that less severe cases should be postponed to leave space for more urgent cases. Subsequently, the restriction of the RV's mean appointment lead-time can be easily maintained throughout the planning horizon.

CHAPTER 6: CONCLUSIONS AND FUTURE RESEARCH

In the thesis, network flow models have been introduced for capacity planning and appointment scheduling approaches of the re-entry outpatient appointment system. The finite horizon is assumed in developing the problems. The studies explore the FV patients' appointment lead-time targets and the restrictions of the RV patients' appointment lead-time. According to the literature review, we did not find any study that investigates the patients' appointment lead-times for the re-entry system. Hence, the thesis contributes by reducing the research gap. The thesis starts the development of the capacity planning models with the deterministic assumption and later relaxes this to allow modeling of stochastic patient arrivals. Appointment scheduling policies to schedule patients' appointment date are also provided.

This chapter concludes the findings and contributions, as well as the draw-backs of the thesis. The thesis's findings and contributions are highlighted in Section 6.1. Then, possible future researches as well as the draw-backs of the thesis are discussed in Section 6.2.

6.1. Conclusions

This thesis focuses on developing mathematical models for capacity planning, which the thesis partitions into capacity design and admission management problems, and on deriving appointment scheduling rules to make decisions on patients' appointment. The capacity planning aims to determine the required

capacity and to manage the admission of patients' requests at strategic and tactical levels, respectively. Given established lead-time targets, the capacity design problem focuses on minimizing the future investment on the scarce resource, while the admission management problem is to maximize the number of admitted patients. The achievement of the MOH's appointment lead-time targets is compulsory and the continuity of care needs to be maintained in both problems.

The appointment scheduling rules establish admission constraints to schedule every individual patient's request at the operational level. The scheduling rules assist service providers to facilitate the decisions of the patients' appointment and to utilize the resource. The constructed admission constraints aim to achieve the MOH's targets of median, 95th percentile, and 100th percentile appointment lead-times throughout the model horizon.

The numerical experiments reveal the efficacy and the sufficiency of the developed capacity planning models as well as of the proposed appointment scheduling rules. The models and rules provide a systematic capacity planning and a systematic admission scheduling for the re-entry system under the restrictions of the appointment lead-time targets. Moreover, a short computation time of the models to obtain the optimal solutions brings a great value to the contributions of the thesis. These contributions are summarized in the following sub-sections 6.1.1, and 6.1.2.

6.1.1. Contributions of the capacity planning models

The capacity planning approach is divided into capacity design and admission management problems. The thesis formulated models for both

problems to initially address the deterministic FV demands. Then, the models are extended to cope with the stochastic FV demands. The models optimize the required resource and the admission schemes for the capacity design and the admission management, respectively. The optimal solutions from the proposed models must consider the achievement of MOH's service targets for the median, the 95th percentile, and the 100th percentile appointment lead-time. The study is to match the healthcare provider to patients' demand subject to restrictions on appointment lead-times of patients. Mixed-integer linear programming models minimize the maximum required capacity and maximize the number of admitted patients as objectives of the capacity design and admission management problems, respectively.

Key assumptions are a finite planning horizon, multiple types of patients, identical physicians, dependent demands between types of patients, and constant discharge rates. The models are then solved by a Branch and Cut algorithm. The results are examined using real data of the studied hospital.

The study of deterministic models (Chapter 3) illustrates that the proposed models systematically push and pull the appointments to obtain the optimal required resource and the optimal admission schemes. Regardless of how the push-pull mechanism is implemented, the scheduling of FV patient appointments should be at one of the appointment lead-time targets and the MOH's targets should be maintained throughout the planning horizon. However, no specific way in scheduling RV appointments has been found.

The sensitivity analyses (in Sub-sections 3.3.3 and 3.3.4) investigate the sensitivity of the models' performances to the change of a particular input.

❖ *Sensitivity analysis for the capacity design problem*

- The maximum required capacity is negatively correlated with the appointment lead-time targets, the discharge rates, and the restriction of RV's mean appointment lead-time. However, it is not always true for the mean required capacity cases.
- The FV's mean appointment lead-time is positively correlated with the FV's appointment lead-time targets. However, the positive correlation between RV's mean appointment lead-time and the restriction of the RV's mean appointment lead-time is not specific when the latter part is large.
- Changing the discharge rates does not impact on the general way of assigning FV patients' appointment.

❖ *Sensitivity analysis for the admission management problem*

- The total number of denied FV patients is negatively correlated with the appointment lead-time targets, the restricted value of the RV's mean appointment lead-time, and the discharge rates.
- The FV's mean appointment lead-time tends to be positively correlated with the appointment lead-time targets for FV patients.
- The higher the discharge rate is, the lower the number of denied patients is. This trend is true for the change of either FV or RV discharge rates.

The study of stochastic models (Chapter 5) concentrates on developing safe and tractable equivalents for the capacity planning models from original joint chance constraints problems. This chapter derived robust linear programming models or deterministic equivalents, which are approximations

of the stochastic models. The deterministic equivalents assure a robust performance of the obtained optimal solutions within the risk of violation ε . Furthermore, the study carries out the approximation for both fully and partially distributional information of the uncertain FV demands. The findings revealed a guideline on the necessary distributional information of FV demands to be used in solving a capacity design or admission management problem.

The results of the capacity design problem show that the higher risk of violation, ε , may lead to the lower optimal required resource. This pattern is consistent for both full and partial distributional information of the uncertainty. Moreover, knowing an accurate distribution of the uncertainty helps to save an unnecessary investment of the scarce resource.

On the other hand, the outcomes of the admission management problem reveal that a low risk of violation needs to be used to allow a high total number of admitted patients. In contrast to the capacity design problem, only the mean and variance of the FV demands, is needed in optimizing the admission scheme. This is an interesting finding because partially distributional information of the uncertainty not only allows high total number of appointed patients, but also requires less effort in data collection and analysis.

Furthermore, the numerical experiments show that regardless of the distribution of the FV demands, the patients' appointment lead-time should be the median, the 95th percentile, or the 100th percentile appointment lead-time targets. This independency of appointment scheduling decision with FV

demands information is consistent for both the capacity design and the admission management problems.

6.1.2. Contributions of the appointment scheduling policies

The proposed capacity planning models help to plan the capacity for each type of the patients for a finite arrival horizon to optimize the required capacity or the admission scheme. The models provide the optimal solutions based on information of the estimated future demands. However, the information of every individual arrival is unknown, which may lead to inappropriate scheduling of FV patients' appointment. The inappropriate appointments scheduling may cause low resource utilization and a violation of MOH's appointment lead-time targets. Therefore, the admission scheduling policies with quantitative admission constraints are developed to address the above concerns (chapter 4). The policies derive admission constraints to make decisions on patients' appointment in which the achievement of MOH's service targets is compulsory throughout the planning horizon.

Two different appointment scheduling rules, the exact and relaxed rules, are proposed in this thesis. The exact appointment scheduling rule aims to schedule appointments of patients onto the 95th percentile or the 100th percentile appointment date if the patients cannot get appointments within the median target. With the relaxed rule, if patients have to get appointments longer than the median target, the patients are allowed to get appointments in which their lead-time is within the 95th percentile or the 100th percentile lead-time target. However, first-in-last-out rule is used to decide appointment of

patients whose appointment lead-time is longer than the median lead-time target.

The numerical experiments show that the exact appointment scheduling rule should be considered to make decisions on the patients' appointment date, since the rule grants the lowest maximum percentage of unused slots while the mean percentage of the unused slots is similar. However, the relaxed rule may be considered for certain scenarios (e.g., urgent cases). Besides, the period of specialization (for example, seeing a certain type of patient within a specific number of days) should not be longer than the median lead-time target. This way of doing specialization can help to avoid unexpected rejection of patients' requests. In other words, it improves the utilization of the scarce resource.

Furthermore, any change of the lead-time targets will lead to the change of the patients' lead-time in practice; and the commitment of guided appointment scheduling rules to admit patients will lead to the achievement of the established lead-time targets, while the planned resource promises to be highly utilized

6.2. Limitation of the thesis and future research

The thesis investigated the capacity planning models for uniform re-entry systems that assumes a single category of patients; all of whom have uniform or identical consultation times, FV appointment lead-time targets, restrictions of RV appointment lead-time, and discharge rate factors. Hence, for systems in which there are multiple types or classes of patients, an extension of models should be developed.

Next, the development of the capacity planning models and the appointment scheduling rules implies that patients' no-show rates are independent from their appointment lead-times. This simplification allows the omission of the patients' no-show rates in the proposed models and the derived rules. An extension of the capacity planning models and appointment scheduling rules could be developed for a case where the patients' no-show rates depend on the appointment lead-time.

LIST OF PUBLICATIONS

- Thu Ba T. NGUYEN, Appa Iyer SIVAKUMAR, and Stephen C. GRAVES, (2014) *A Network Flow Approach to a Tactical Resource Planning in Outpatient Clinics*, Submitted to Health Care Management Science Journal, DOI: 10.1007/s10729-014-9284-0 (Accepted to publish).
- Thu Ba T. NGUYEN, Appa Iyer SIVAKUMAR, and Stephen C. GRAVES (2013), *An Approach for Reducing Access Time to Outpatient Clinic*, Proceedings of 7th IMA Conference on Quantitative Modelling in the Management of Health and Social Care, 25th – 27th March 2013, London, United Kingdom.
- Thu Ba T. NGUYEN, Appa Iyer SIVAKUMAR, and Stephen C. GRAVES (2013), *A Capacity Planning Model for Outpatient Clinic*, Proceedings of Singapore – MIT Alliance Symposium, 14th – 16th January 2013, Singapore.

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APPENDICES

APPENDIX 1: SUMMARIES OF THE LITERATURE REVIEW

Appendix A1.1: Directed capacity design studies for outpatient appointment systems

Study	Approach	Decision variable	Performance measures	Assumptions			
				Arrival demand	Service time	Appointment lead-time	Re-visit patient
Ittig (1985)	Linear programming	+ Capacity level	+ Patients' fixed cost + Patients' profit	Linearly and exponentially dependent on appointment lead-time	None	Compromised with capacity level	None
Thomas et al. (2001)	Analytical	Surplus capacity	Patients' appointment lead-time	+ Poisson distribution + Mean number of referrals	None	2 weeks	None
Bowers et al. (2005)	Analytical approach	+ Number of clinics required	+ clinics' utilization	A given estimation	Deterministic	None	2nd visit only
Qu and Shi (2009)		+ Expected number of patients scheduled + Percentage of each type slot	+ Probabilities of granting requests for fixed and open appointments. + Expectation and variance of the number of patient consulted	A given estimation	None	None	None
Elkhuizen et al.(2007)	Simulation: M/D/1	+ Capacity reserve for new patients	+ Mean and 95 th percentile of access time	Poisson	Deterministic	Given	None
Green and Savin (2008)	Queuing theory: + M/D/1/K + M/M/1/K	+ Panel size N	+ Expected patient backlog + Probability of same-day appointments	Poisson	Deterministic Exponential	Probability of walk-ins	None

Appendix A1.2: Non-directed capacity design studies for outpatient appointment systems

Study	Approach	Decision variable	Performance measures	Assumptions			
				Arrival demand	Service time	Appointment lead-time	Re-visit patient
Leitch et al. (1990)	Statistical analysis	Group of stable regular patients that should be discharged	Criteria of morbidity	None	None	3 and 12 monthly follow-up	Yes
Bennett and Worthington (1998)	Simulation	Additional doctors	Annual number of appointments saved	Average number of patients	Estimated service-time distribution	None	Yes
Porta-Sales et al. (2005)	+ Statistical analysis + Regression predictive model	+ Non-comply or comply to the scheduled follow-up	Causes of non-compliance with the follow-up appointment	None	None	None	Yes
Bamford & Chatziaslan (2009)	Survey	Resource and actual ability	Utilization	Steady decline	None	None	None
Demir et al. (2012)	+ Linear regression + Estimation using confident interval	Number of attendances	+ Expected future demands + 95% confidence interval of demands	Normal distribution	None	None	Yes

Appendix A1.3: Queuing studies of the appointment scheduling policies for outpatient systems

Study	Queuing model	Decision variable	Performance measures	Assumptions				
				Service time	No-show rate (%)	Slot interval	Patient's Punctuality	Physician's lateness
Pegden & Rosenshine (1990)*	$S(n)/M/1$	Patients' appointment time	+ Patients' waiting cost + Physicians' availability cost	Exponential distribution	None	Varied	Punctual	None
Brahimi & Worthington (1991)*	$M(t)/G/S$	Mean and a possible range of the number of patients attends clinics	+ Patients' waiting time + Probability that a physician is busy	+ Continuous + Approximated to be a discrete distribution	0.10	Fixed	Punctual	None
Liu & Liu (1998)*	$DX_n/G/m(t)$	Number of patients, X_n , to each block	+ Patients' waiting cost + Physicians' idle cost	+ Uniform (0,2) + Exponential (1) + Weibull(0.5,0.5)	> 0	Fixed	Punctual	Uniform distribution $[0, T_a]$
Hassin & Mendel (2008)	$S(n,p)/M/1$	Patients' appointment time of given n patients	+ Patients' waiting cost + Physicians' availability cost	Exponential distribution	$1-p$	Varied	Punctual	None

*These are revised from Cayirli and Veral (2003)

Appendix A1.4: Simulation studies of the appointment scheduling policies for outpatient systems

Study	Type of problem	Decision variable	Performance measures	Assumptions					
				Service time	No-show rate (%)	Slot interval	Walk-in	Patient's Punctuality	Physician's lateness
Bailey (1952)*	OAS	Number of attended patients	Patients' and physicians' waiting time	Pearson Type III	None	+ Fixed + Equal	None	Punctual	None
Rising et al. (1973)*	OAS	Number of appointments per slot	+ Physicians' utilization and overtime + Patients' waiting time	Normal: + (12.7; 9.5) + (9.6; 7.4) + (6.4; 5.4)	None	+ Fixed + Equal	Negative exponential arrival time	None	None
Blanco White & Pike (1964)*	EASR	+ Number of patients consulted + Length of the clinic	+ Patients' waiting time + Physicians' idle time + Percentage of patients seen within 30 minutes	+ Pearson Type III	5 & 10	None	None	+ Pearson Type VII	+ None + 5 and 10 minutes
Vissers (1979)*	EASR	+ The number of patients per slot + The slot interval	Mean of + Patients' waiting time + Physicians' idle time	+ Mean + Coefficient of variation	15	Varied	None	+ Mean + Variance	Mean value
Vissers & Wijngaard (1979)	EASR	+ The number of appointments	+ Patients' waiting time + Physicians' idle time	Mean	None	+ Fixed + Equal + In-equal	None	Earliness	Mean & standard error
Ho & Lau (1992)*	EASR	Scheduling rule	+ Patients' waiting cost + Physicians' idle cost	+ Uniform + Exponential	0, 10, and 20		None	Punctual	None
Klassen & Rohleder (1996)*	EASR	+ Scheduling rule + Position of urgent call slots		+ Exponential + Uniform + Lognormal	0.5, 1, and 2		Urgent cases	Punctual	None

*These are revised from Cayirli and Veral OAS : Optimizing appointment scheduling (2003)EPF: evaluating patient's flow.
EASR: Evaluating appointment scheduling rules

Appendix A1.4: Simulation studies of the appointment scheduling policies for outpatient systems (cont')

Study	Type of problem	Decision variable	Performance measures	Assumptions					
				Service time	No-show rate (%)	Slot interval	Walk-in	Patient's punctuality	Physician's lateness
Su & Shih (2003)	EASR	Scheduling rule	+ Patients' waiting time + Patients' throughput time + Utilization rate	+ Gamma + Log-normal + Pearson + Weibull	None	3, 5, 7, 9, & 11 minutes	+ Poisson + Beta + Negative binomial	Punctual	None
LaGanga & Lawrence (2007)	EASR	+ Number of overbooked patients + Overbooking policy	+ Patients' waiting time + Physicians' overtime	+ Constant + Uncertain	$R > 0$	Fixed & equal	None	Punctual	None
Harper & Gamlin (2003)	EASR EPF	Scheduling rule	+ Patients' flow time + Percentage of patients' waiting time is more than 30 minutes + Patients' waiting time	+ Min & max + Mean & variance + 95 th confidence interval	None	+ Fixed + Unequal	None	None	Yes
Wijewickrama & Takakuwa (2005)	EASR EPF	Scheduling rule	+ Patients' waiting time + Physicians' idle time	+ Exponential + Triangular + Uniform	> 0	-	> 0	None	None
Cote (1999)	EPF	+ Physician's practice + Number of examination rooms	+ Examination room's utilization, queue length + Probability of full occupancy + Patients' flow time	+ Weibull + Gamma	None	None	None	Non-homogeneous Poisson	None

*These are revised from Cayirli and Veral (2003)
EPF: evaluating patient's flow.

OAS : Optimizing appointment scheduling
EASR: Evaluating appointment scheduling rules

Appendix A1.5: Mathematical studies of the appointment scheduling policies for outpatient systems

Study	Approach	Decision variable	Performance measures	Assumptions					
				Service time	No-show rates	Slot interval	Walk-in	Patient's punctuality	Number of patients scheduled
Fries & Marathe (1981)*	+ Linear combination objective + D/M/1 + Dynamic programming	Number of patients in each slot	+ Patients' waiting time + Physicians' idle time and overtime	Negative exponential	None	Fixed & equal	None	Punctual	Given
Vanden Bosch & Dietz (2000)*	+ Distinct rate and convex combination objective + D/G/1 + Two heuristic algorithms		+ Patients' waiting time + Physicians' overtime	Distinct truncated Erlang or Gamma	> 0 & Various		None	Punctual	Given
Vanden Bosch & Dietz (2001)*	+ Distinct rate and convex combination objective + D/G /1 + Two heuristic algorithms			+ General + Erlang	> 0 & same		None	Punctual	Given
Baker & Atherill (2002)	+ Linear combination objective + G/G/1 + Heuristic discrete optimization		+ Patients' waiting time + Servers' idle time	Gamma	> 0 & Various		None	Normal (8.3, 7.5) minutes	Given
Kaandorp & Koole (2007)	+ Multi-modular objective + D/M/1 + Stochastic non-linear optimization + Heuristic algorithm		+ Patients' waiting time + Physicians' idle time and overtime	Exponential	> 0 & same		None	Punctual	Given

*These are revised from Cayirli and Veral (2003)

Appendix A1.5: Mathematical studies of the appointment scheduling policies for outpatient systems (cont')

Study	Approach	Decision variable	Performance measures	Assumptions					
				Service time	No-show rates	Slot interval	Walk-in	Patient's punctuality	Number of patients scheduled
Wang (1993)*	+ Linear combination objective + $S(n)/M/1$ + Non-linear optimization	Optimal appointment time	+ Patients' delay time (waiting time plus service time) + Physicians' completion time	Exponential	None	Varied	None	Punctual	Given
Wang (1997)*	+ Linear combination objective + $D/G/1$ + Stochastic non-linear optimization		+ Patients' cost of flow time + Physicians' cost of completion time	Coxian-type distribution	None	Varied	None	Punctual	Given
Vanden Bosch (1999)*	+ Fixed rate & linear combination objective + $D/E/1$ + Heuristic algorithm		+ Patients' waiting time + Physicians' overtime	Erlang	None	Varied	None	Punctual	Given
Denton & Gupta (2003)*	+ Linear combination objective + $D/G/1$ + Stochastic linear programming + Heuristic algorithm		+ Patients' waiting time + Physicians' idle time + Physicians' overtime	General + Uniform + Gamma + Normal	None	Varied	None	Punctual	Given

*These are revised from Cayirli and Veral (2003)

Appendix A1.5: Mathematical studies of the appointment scheduling policies for outpatient systems (cont')

Study	Approach	Decision variable	Performance measures	Assumptions					
				Service time	No-show rates	Slot interval	Walk-in	Patient's punctuality	Number of patients scheduled
Muthuraman & Lawley (2008)	+ Unimodal objective + M/M/1 + Stochastic non-linear optimization + Heuristic algorithm	+ Number of patients in each slot + Number of scheduled patient	+ Patients' waiting time + Patients' revenue + Physicians' overtime + Patients' waiting	Exponential	> 0 & Various	Fixed & equal	None	Punctual	Optimized
Chakraborty et al. (2010)	+ Unimodal objective + M/G/1 + Stochastic non-linear optimization + Heuristic algorithm			+ General + Gamma			None	Punctual	Optimized
Zeng et al. (2010)	+ Multi-modular objective + M/M/1 + Stochastic non-linear optimization + Two sequential scheduling procedures			Exponential		Fixed & unequal	None	Punctual	Optimized

Appendix A1.6: Capacity management studies that consider the appointment lead-time factor

Study	Approach	Decision variable	Performance measures	Assumptions				
				Arrival demand	Service rate	No-show rates	Appointment lead-time	Type of clinic
Frost (1980)	Regression method	+ Percentage the number of consultants increased + Total number of consultants	Equilibrium waiting lists or mean appointment lead-time	Total number of patient in waiting list	+ Number of consultants + Waiting list size + Mean appointment lead-time	None	Mean	Inpatient
Ittig (1985)	Linear programming	Optimal capacity level	+ Patients' fixed cost + Patients' profit	Linearly dependent & exponentially dependent on appointment lead-time	None	None	Compromised	Outpatient
Worthington (1987)	Queuing method: $M(\lambda_q)/G/S$	Management actions: + Number of beds + Mean service time + Waiting lists combination + Feedback policies	Waiting list size	Linearly decreasing (λ_q)	+ Mean + Coefficient of variation	Discouraged from joining the waiting list	Via waiting list size	Inpatient

Appendix A1.6: Capacity management studies that consider the appointment lead-time factor (cont')

Study	Approach	Decision variable	Performance measures	Assumptions				
				Arrival demand	Service rate	No-show rates	Appointment lead-time	Type of clinic
Worthington (1991)	Projection method	Capacity planning alternatives	+Future waiting list size + Projected average appointment lead-time	Referral rate & re-visit proportions	None	None	2 weeks, 2 & 6 months for both outpatient & inpatient	Revisit patient: an outpatient or an inpatient
Iversen (1993)	Stochastic non-linear programming	Government level: + Level of resources (s) Hospital level: + Expected lead-time + Number of patients in the waiting list during a period	Government: patients' lead-time announced by the hospital: + Hospital's utility + Government's reaction	Exponential distribution of time for subsequent arrivals	Exponential	None	$t^*(s)$	Inpatient
Street & Duckett (1996)	+ Empirical method + Case study	Incentive structure: + Additional revenue for additional output + Addition marginal resources	+ Recorded numbers of patients waiting	None	None	None	+ Category 1: 30 days + Category 2: 90 days	Inpatient
Bowers (2010)	+ Simulation + Predictive model	+ Reduce or increase the waiting list size + Evaluate an investment in additional resources	+ Mean patients' lead-time + 95% confidence interval of patients' lead-time	Empirical	Empirical	None	100% within 18 weeks	Re-visit patient considered as inpatient

Appendix A1.7: Admission management studies that consider the appointment lead-time factor

Study	Approach	Decision variable	Performance measures	Assumptions				
				Arrival demand	Service rate	No-show rates	Appointment lead-time	Type of clinic
Culyer & Cullis (1976)	Empirical method	Priority index for admission: + Time already spent on the waiting list + The expected rate of deterioration + The degree of disability + Social factors (wage earners etc.)	+ Maximum reduction in need out of given resources + Balance amongst departments, hospitals, and regions	None	Available beds & length of stay	None	A part of priority index	Inpatient
Leitch et al. (1990)	Statistical analysis	Group of stable regular patients that should be discharged	Criteria of morbidity	None	None	None	3 and 12 monthly follow-up	Outpatient
Ratcliffe et al. (2001)	Simulation method	Allocation policies: + Recipient's criteria + Donor's criteria	Incremental cost effectiveness ratio: + Mean patients' costs + Mean length of survival	Median & range of inter-arrival time	Median & range	None	Depend on allocation policy	Inpatient
Kopach et al. (2007)	Simulation method	+ Fraction of open-access patients + Scheduling horizon	+ Patients' satisfaction + Patients' throughput	Observed inter-arrival time	Recorded data at every states	> 0	Scheduling horizon	Outpatient

Appendix A1.7: Admission management studies that consider the appointment lead-time factor (cont')

Study	Approach	Decision variable	Performance measures	Assumptions				
				Arrival demand	Service rate	No-show rates	Appointment lead-time	Type of clinic
Bibi et al. (2007)	Report paper	Intervention program: + Overbooking practice + Centralization policy	+ Mean appointment lead-times (days) + Average doctors working hours or time utilization + None-attendance proportion (%) + Satisfaction rating	Recorded data	None	> 0	Recorded appointment lead-time	Outpatient
Goddard & Tavakoli (2008)	Queuing theory: G/G/k	Adjustment of patients' demand: + Criteria of accessing to the waiting list + Patients' order in the waiting list	+ Proportion of each daily cohort of sick patients. + Mean expected duration of wait. + Expected duration of wait for the most marginal patient	Arrival rate at each state	Leaving rate at each state	Same withdraw rates	+ Expected time to clear the waiting list + Maximum appointment lead-time	Inpatient
Dixon & Siciliani (2009)	Mathematical method	+ Distribution of patients treated + Distribution of patients' appointment lead-time	Comparison of appointment lead-time measures from: + Treated patients + Waiting patients	Recorded data	None	None	Maximum	Inpatient & Outpatient

Appendix A1.8: Studies on the appointment lead-time measurements

Study	Approach	Findings	Performance measures	Assumptions				
				Arrival demand	Service rate	No-show rates	Appointment lead-time	Type of clinic
Don et al. (1987)	Report paper	Factors considered when using different measurements	Patients' appointment lead-time	Recorded data	None	Yes	Measured	Unspecific
Dixon (2004)		Factors cause the difference between appointment lead-time measurements	Median appointment lead-time	Recorded data	None	None	Measured	Unspecific
Torkki et al. (2002)	Empirical & report paper	Discrepancies in waiting list management	+ Mean appointment lead-time + Distribution of an individual appointment lead-time	$\alpha(t)$	$\delta(t)$	None	Recorded	Inpatient
Tsang & Resneck (2006)	Survey & empirical method	Reasons of long appointment lead-time when changing moles	Recorded mean & median appointment lead-time	Recorded data	None	None	Recorded	Outpatient
Dimakou et al. (2009)	Statistical & analysis report	Appointment lead-time target	Survival and hazard rates	Recorded data	None	None	National target: 12 & 15 months	Inpatient

**APPENDIX 2: THE INPUTS FOR CAPACITY PLANNING
 PROBLEMS**

Appendix A2.1: List of inputs 1

No.	Descriptions		Uro	Time-unit/s
1	Appointment lead-time target	Median (u)	2	Weeks
2		95 th percentile(v)	6	Weeks
3		100 percentile(w)	9	Weeks
4	Consultation time slots	FV (τ^f)	15	Minutes
5		RV (τ^r)	10	Minutes
6	Discharge rates	FV (α)	38	(%)
7		RV (β)	32	(%)
8	Appointment lead-time of RV	Min (a)	2	Weeks
9		Max (b)	30	Weeks
10		Average ($\bar{a}$)	30	Weeks

Appendix A2.2 List of inputs 2

Week	Number of pre-assigned patients		FV demand	Week	Number of pre-assigned patients		FV demand
	FV	RV			FV	RV	
1	138	376	180	27	0	166	132
2	172	425	169	28	0	161	177
3	161	399	200	29	0	159	215
4	91	213	114	30	0	158	195
5	124	396	162	31	0	161	207
6	113	396	179	32	0	129	146
7	100	352	169	33	0	151	207
8	98	388	209	34	0	143	248
9	82	271	179	35	0	133	187
10	85	210	164	36	0	172	229
11	60	296	208	37	0	153	222
12	43	301	206	38	0	118	151
13	4	254	188	39	0	149	245
14	2	239	165	40	0	165	190
15	1	186	162	41	0	173	188
16	0	308	153	42	1	168	212
17	0	254	219	43	0	144	245
18	0	206	156	44	0	137	180
19	0	237	150	45	1	166	171
20	1	228	210	46	0	140	196
21	0	203	181	47	0	203	206
22	0	205	207	48	0	110	172
23	0	198	181	49	0	172	217
24	0	199	186	50	0	154	155
25	0	141	227	51	0	149	211
26	0	173	163	52	0	82	169

Appendix A2.3: Provided capacity for the admission management problem

Week	Provided capacity (minutes)	Week	Provided capacity (minutes)	Week	Provided capacity (minutes)	Week	Provided capacity (minutes)
1	5980	26	7015	51	7735	76	7585
2	7210	27	7560	52	5000	77	7690
3	7335	28	7125	53	5395	78	7510
4	3900	29	7165	54	8230	79	7410
5	7130	30	6855	55	7965	80	6870
6	7105	31	7085	56	7240	81	7345
7	6705	32	6280	57	8200	82	7085
8	7465	33	7565	58	7460		
9	5730	34	7875	59	4405		
10	5135	35	7325	60	7440		
11	7760	36	7965	61	8350		
12	7135	37	8065	62	6660		
13	7145	38	6715	63	8045		
14	6890	39	7880	64	7650		
15	6080	40	8200	65	8075		
16	8080	41	8715	66	6475		
17	7395	42	8680	67	7740		
18	5925	43	8850	68	7730		
19	7490	44	7635	69	7900		
20	7265	45	6950	70	7690		
21	7280	46	6805	71	7285		
22	6720	47	8915	72	8010		
23	6840	48	5925	73	8985		
24	6830	49	7900	74	5835		
25	6625	50	7480	75	7340		

**APPENDIX 3: SENSITIVITY ANALYSIS CHO CAPACITY
DESIGN PROBLEM**

Appendix A3.1: Sensitivity analysis for the capacity design problem, FV discharge rate = 0.38 and RV discharge rate = 0.32

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)												Restricted RV average lead time ($\bar{a}$)											
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	185.5	152.0	129.2	118.8	113.8	113.8	129.0	114.9	105.3	100.7	98.2	98.2	3.6	3.6	3.6	3.6	2.5	2.6	5	10	16	20	24.4	23.6
	2	6	9	184.3	151.2	128.5	118.2	113.8	113.8	128.5	114.6	104.9	100.5	98.2	98.2	4.1	4.1	4.1	4.1	2.7	2.9	5	10	16	20	24.4	23.1
	3	6	9	183.0	150.3	127.9	117.6	113.8	113.8	128.0	114.2	104.6	100.2	98.2	98.2	4.5	4.5	4.6	4.6	3.2	3.4	5	10	16	20	24.0	23.8
	4	6	9	181.8	149.4	127.3	117.1	113.8	113.8	127.4	113.8	104.2	100.0	98.0	98.2	5.0	5.0	5.1	5.1	4.0	3.9	5	10	16	20	23.6	23.0
	5	6	9	180.6	148.6	126.6	116.5	113.8	113.8	126.9	113.3	103.9	99.7	98.2	97.7	5.4	5.5	5.6	5.6	4.4	4.6	5	10	16	20	23.3	23.0
95 th Percentile	2	3	9	188.2	153.8	130.5	119.9	113.8	113.8	129.9	115.3	105.3	100.7	97.4	97.7	2.7	2.7	2.8	2.8	2.4	2.3	5	10	16	20	25.0	23.7
	2	4	9	186.8	152.9	129.8	119.3	113.8	113.8	129.5	115.0	105.3	100.7	97.9	98.0	3.2	3.2	3.2	3.2	2.4	2.4	5	10	16	20	24.6	24.1
	2	5	9	185.5	152.0	129.2	118.8	113.8	113.8	128.9	114.8	105.1	100.6	98.0	98.0	3.6	3.6	3.7	3.7	2.7	2.6	5	10	16	20	24.3	24.5
	2	6	9	184.3	151.2	128.5	118.2	113.8	113.8	128.5	114.6	104.9	100.5	98.2	98.2	4.1	4.1	4.1	4.1	2.7	2.9	5	10	16	20	24.4	23.1
	2	7	9	183.0	150.3	127.9	117.6	113.8	113.8	128.1	114.3	104.7	100.3	98.2	98.3	4.5	4.5	4.6	4.6	3.1	3.2	5	10	16	20	24.0	23.0
	2	8	9	181.8	149.4	127.2	117.1	113.8	113.8	127.6	114.1	104.5	100.0	98.1	98.5	5.0	5.0	5.0	5.0	3.1	3.6	5	10	16	20	24.2	23.1
100 th percentile	2	6	7	184.6	151.4	128.7	118.4	113.8	113.8	128.7	114.7	105.0	100.3	97.3	98.0	4.0	4.0	4.0	4.0	2.9	3.0	5	10	16	20	25.0	23.6
	2	6	8	184.5	151.3	128.6	118.3	113.8	113.8	128.6	114.6	104.9	100.4	98.0	98.0	4.0	4.0	4.1	4.1	2.7	2.9	5	10	16	20	24.3	24.0
	2	6	9	184.3	151.2	128.5	118.2	113.8	113.8	128.5	114.6	104.9	100.5	98.2	98.2	4.1	4.1	4.1	4.1	2.7	2.9	5	10	16	20	24.4	23.1
	2	6	10	184.1	151.0	128.4	118.1	113.8	113.8	128.5	114.5	104.9	100.4	97.4	98.1	4.1	4.1	4.2	4.2	2.7	2.8	5	10	16	20	25.0	23.6
	2	6	11	183.9	150.9	128.4	118.0	113.8	113.8	128.6	114.4	104.8	100.4	98.3	98.2	4.1	4.2	4.2	4.2	2.8	3.0	5	10	16	20	24.2	23.4
	2	6	12	183.8	150.8	128.3	118.0	113.8	113.8	128.5	114.4	104.8	100.4	98.2	97.6	4.2	4.2	4.3	4.3	2.9	2.9	5	10	16	20	24.2	24.0

Appendix A3.2: Sensitivity analysis for the capacity design problem, FV discharge rate = 0.38 and RV discharge rate = 0.1

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)												Restricted RV average lead time ($\bar{a}$)											
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	291.2	196.7	150.4	132.9	117.7	114.1	204.9	154.0	128.1	118.4	110.1	107.7	3.6	3.6	3.6	3.6	3.6	2.6	5	10	16	20	25	27.0
	2	6	9	289.9	195.9	149.8	132.5	117.4	114.1	204.2	153.6	127.9	118.3	109.9	107.7	4.0	4.1	4.1	4.1	4.1	2.9	5	10	16	20	25	27.0
	3	6	9	288.5	195.1	149.3	132.0	117.0	114.1	203.4	153.0	127.6	118.0	109.7	107.7	4.4	4.5	4.6	4.6	4.6	3.1	5	10	16	20	25	27.1
	4	6	9	287.2	194.2	148.8	131.6	116.7	114.1	202.7	152.4	127.2	117.8	109.2	107.7	4.8	5.0	5.0	5.1	5.1	2.7	5	10	16	20	25	27.1
	5	6	9	285.8	193.5	148.3	131.2	116.5	114.1	202.0	152.3	127.0	117.5	109.2	107.7	5.2	5.4	5.5	5.5	5.4	3.3	5	10	16	20	25	26.9
95th Percentile	2	3	9	294.6	198.3	151.5	133.8	118.5	116.2	206.4	154.3	128.3	118.5	109.9	108.5	2.6	2.7	2.7	2.8	2.8	1.8	5	10	16	20	25	26.6
	2	4	9	293.1	197.5	150.9	133.3	118.1	115.5	205.8	153.9	128.2	118.5	110.0	108.2	3.1	3.2	3.2	3.2	3.2	2.2	5	10	16	20	25	26.6
	2	5	9	291.5	196.7	150.4	132.9	117.8	114.9	204.9	153.8	128.0	118.1	109.8	108.0	3.5	3.6	3.6	3.7	3.7	2.7	5	10	16	20	25	26.7
	2	6	9	289.9	195.9	149.8	132.5	117.4	114.1	204.2	153.6	127.9	118.3	109.9	107.7	4.0	4.1	4.1	4.1	4.1	2.9	5	10	16	20	25	27.0
	2	7	9	288.2	195.0	149.3	132.0	117.0	113.8	203.6	153.0	127.9	118.3	109.8	107.5	4.4	4.5	4.5	4.6	4.6	3.3	5	10	16	20	25	27.0
	2	8	9	286.7	194.1	148.7	131.5	116.6	113.8	203.1	152.9	127.6	118.0	109.7	108.1	4.9	5.0	5.0	5.0	5.0	3.7	5	10	16	20	25	26.7
100th percentile	2	6	7	290.5	196.2	150.1	132.6	117.5	115.5	204.5	153.8	128.1	118.4	110.0	108.3	3.9	4.0	4.0	4.0	4.0	2.7	5	10	16	20	25	26.5
	2	6	8	290.2	196.0	150.0	132.5	117.5	114.5	204.1	153.7	128.0	118.3	110.0	107.8	3.9	4.0	4.0	4.1	4.1	2.9	5	10	16	20	25	26.9
	2	6	9	289.9	195.9	149.8	132.5	117.4	114.1	204.2	153.6	127.9	118.3	109.9	107.7	4.0	4.1	4.1	4.1	4.1	2.9	5	10	16	20	25	27.0
	2	6	10	289.6	195.7	149.7	132.4	117.3	113.9	204.1	153.5	127.7	118.2	109.8	107.6	4.0	4.1	4.1	4.2	4.2	3.1	5	10	16	20	25	26.9
	2	6	11	289.4	195.5	149.6	132.3	117.2	113.8	204.4	153.3	127.8	117.9	109.8	107.6	4.1	4.2	4.2	4.2	4.2	3.1	5	10	16	20	25	26.9
	2	6	12	289.1	195.3	149.5	132.2	117.2	113.8	204.4	153.1	127.5	118.1	109.7	107.5	4.1	4.2	4.2	4.3	4.3	3.3	5	10	16	20	25	27.0

Appendix A3.3: Sensitivity analysis for the capacity design problem, FV discharge rate = 0.38 and RV discharge rate = 0.2

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)												Restricted RV average lead time ($\bar{a}$)											
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	228.3	172.3	139.3	125.6	113.8	113.8	160.0	132.8	116.6	109.7	103.4	103.2	3.6	3.6	3.6	3.6	3.0	2.5	5	10	16	20	25	25.5
	2	6	9	226.9	171.4	138.7	125.2	113.8	113.8	159.5	132.4	116.0	109.3	103.0	103.2	4.0	4.1	4.1	4.1	3.1	3.1	5	10	16	20	25	25.4
	3	6	9	225.6	170.6	138.1	124.7	113.8	113.8	158.9	131.7	115.7	109.1	103.1	103.0	4.5	4.5	4.6	4.6	3.3	3.2	5	10	16	20	25	25.4
	4	6	9	224.3	169.7	137.5	124.2	113.8	113.8	158.2	131.5	115.4	108.8	103.3	102.9	4.9	5.0	5.0	5.1	3.4	3.2	5	10	16	20	25	25.4
	5	6	9	223.0	168.9	136.9	123.7	113.8	113.8	157.5	130.9	115.1	108.4	103.3	102.8	5.3	5.5	5.5	5.6	3.4	3.3	5	10	16	20	25	25.3
95th Percentile	2	3	9	231.4	174.0	140.4	126.7	114.1	113.8	161.3	133.0	116.5	109.5	103.1	102.7	2.7	2.7	2.7	2.8	2.8	1.8	5	10	16	20	25	26.1
	2	4	9	229.9	173.2	139.9	126.2	113.8	113.8	160.8	132.8	116.2	109.5	103.3	103.2	3.1	3.2	3.2	3.2	3.0	2.4	5	10	16	20	25	25.7
	2	5	9	228.4	172.3	139.3	125.7	113.8	113.8	160.1	132.4	116.3	109.4	103.2	102.8	3.6	3.6	3.6	3.7	3.0	2.8	5	10	16	20	25	25.5
	2	6	9	226.9	171.4	138.7	125.2	113.8	113.8	159.5	132.4	116.0	109.3	103.0	103.2	4.0	4.1	4.1	4.1	3.1	3.1	5	10	16	20	25	25.4
	2	7	9	225.5	170.5	138.1	124.6	113.8	113.8	158.8	132.1	115.9	109.2	103.5	102.9	4.5	4.5	4.5	4.6	3.3	3.3	5	10	16	20	25	25.5
	2	8	9	224.2	169.6	137.4	124.1	113.8	113.8	158.6	131.8	115.6	109.0	103.4	103.3	4.9	5.0	5.0	5.0	3.6	3.5	5	10	16	20	25	25.5
100th percentile	2	6	7	227.4	171.7	138.9	125.3	113.8	113.8	159.6	132.6	116.3	109.4	102.9	103.0	3.9	4.0	4.0	4.0	3.1	3.0	5	10	16	20	25	25.6
	2	6	8	227.2	171.6	138.8	125.2	113.8	113.8	159.6	132.3	116.2	109.4	103.5	103.2	4.0	4.0	4.0	4.1	3.1	2.9	5	10	16	20	25	25.7
	2	6	9	226.9	171.4	138.7	125.2	113.8	113.8	159.5	132.4	116.0	109.3	103.0	103.2	4.0	4.1	4.1	4.1	3.1	3.1	5	10	16	20	25	25.4
	2	6	10	226.7	171.3	138.6	125.1	113.8	113.8	159.5	132.3	116.0	109.3	103.4	102.9	4.1	4.1	4.1	4.2	3.2	2.9	5	10	16	20	25	25.3
	2	6	11	226.5	171.1	138.5	125.0	113.8	113.8	159.6	132.0	115.9	109.2	103.4	103.2	4.1	4.2	4.2	4.2	3.1	3.2	5	10	16	20	25	25.2
	2	6	12	226.4	171.0	138.4	124.9	113.8	113.8	159.5	132.0	116.0	109.2	103.3	103.2	4.2	4.2	4.2	4.3	3.2	3.0	5	10	16	20	25	25.6

Appendix A3.4: Sensitivity analysis for the capacity design problem, FV discharge rate = 0.5 and RV discharge rate = 0.32

Lead-time	Median	95th	100th	Restricted RV average lead time ($\bar{a}$)												Restricted RV average lead time ($\bar{a}$)											
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	177.2	146.4	125.3	115.6	113.8	113.8	123.1	110.2	101.3	97.0	94.9	95.2	3.6	3.6	3.6	3.6	2.3	2.5	5	10	16	20	23.7	22.7
	2	6	9	176.1	145.6	124.7	115.1	113.8	113.8	122.6	109.6	101.0	96.8	94.6	95.1	4.1	4.1	4.1	4.1	3.0	2.9	5	10	16	20	23.9	22.7
	3	6	9	175.0	144.8	124.2	114.6	113.8	113.8	122.2	109.2	100.6	96.6	94.5	95.0	4.5	4.6	4.6	4.6	3.1	3.0	5	10	16	20	24.0	22.3
	4	6	9	173.8	144.0	123.6	114.1	113.8	113.8	121.7	108.9	100.4	96.4	93.7	94.9	5.0	5.0	5.1	5.1	2.9	2.9	5	10	16	20	25.0	22.7
	5	6	9	172.7	143.3	123.0	113.8	113.8	113.8	121.2	108.6	100.1	95.8	94.4	95.1	5.4	5.5	5.6	5.4	3.0	3.0	5	10	16	20	23.9	22.9
95th Percentile	2	3	9	179.6	148.1	126.6	116.8	113.8	113.8	123.8	110.3	101.2	96.8	95.0	95.1	2.7	2.7	2.8	2.8	1.6	1.9	5	10	16	20	24.1	22.8
	2	4	9	178.4	147.3	126.0	116.2	113.8	113.8	123.5	110.2	101.1	96.9	95.0	94.8	3.2	3.2	3.2	3.2	1.9	1.9	5	10	16	20	24.0	23.5
	2	5	9	177.2	146.4	125.4	115.7	113.8	113.8	123.0	109.9	101.0	96.8	95.2	95.0	3.6	3.6	3.7	3.7	2.0	2.4	5	10	16	20	23.8	23.2
	2	6	9	176.1	145.6	124.7	115.1	113.8	113.8	122.6	109.6	101.0	96.8	94.6	95.1	4.1	4.1	4.1	4.1	3.0	2.9	5	10	16	20	23.9	22.7
	2	7	9	175.0	144.8	124.1	114.6	113.8	113.8	122.2	109.6	100.8	96.6	94.5	95.2	4.5	4.5	4.6	4.6	3.2	3.0	5	10	16	20	24.0	22.6
	2	8	9	173.9	143.9	123.5	114.1	113.8	113.8	121.9	109.3	100.5	96.4	94.7	95.0	4.9	5.0	5.0	5.0	3.3	3.4	5	10	16	20	23.8	23.0
100th percentile	2	6	7	176.4	145.9	124.9	115.3	113.8	113.8	122.6	109.8	100.8	96.6	94.5	95.0	4.0	4.0	4.0	4.0	2.6	3.0	5	10	16	20	23.9	22.3
	2	6	8	176.2	145.7	124.8	115.2	113.8	113.8	122.7	109.7	100.9	96.7	94.2	94.8	4.0	4.0	4.1	4.1	3.0	3.0	5	10	16	20	23.9	23.1
	2	6	9	176.1	145.6	124.7	115.1	113.8	113.8	122.6	109.6	101.0	96.8	94.6	95.1	4.1	4.1	4.1	4.1	3.0	2.9	5	10	16	20	23.9	22.7
	2	6	10	175.9	145.5	124.6	115.0	113.8	113.8	122.7	109.5	100.8	96.8	95.0	95.1	4.1	4.1	4.2	4.2	3.0	2.9	5	10	16	20	23.4	22.3
	2	6	11	175.7	145.3	124.6	115.0	113.8	113.8	122.8	109.5	100.8	96.7	94.8	94.9	4.2	4.2	4.2	4.2	2.9	2.8	5	10	16	20	23.9	22.7
	2	6	12	175.6	145.2	124.5	114.9	113.8	113.8	122.8	109.6	100.8	96.7	94.9	94.3	4.2	4.2	4.3	4.3	3.0	2.9	5	10	16	20	23.5	22.9

Appendix A3.5: Sensitivity analysis for the capacity design problem, FV discharge rate = 0.7 and RV discharge rate = 0.32

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)												Restricted RV average lead time ($\bar{a}$)											
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	163.3	137.2	119.0	113.8	113.8	113.8	113.4	102.1	94.6	90.5	88.5	90.2	3.6	3.6	3.6	2.4	2.3	2.5	5	10	16	20	24.0	22.2
	2	6	9	162.4	136.6	118.5	113.8	113.8	113.8	113.0	101.9	94.4	90.8	89.5	90.3	4.1	4.1	4.1	2.9	3.0	3.0	5	10	16	20	23.5	21.8
	3	6	9	161.5	135.9	118.0	113.8	113.8	113.8	112.6	101.6	94.2	90.6	88.6	89.5	4.5	4.6	4.6	3.1	3.0	2.8	5	10	16	20	24.0	22.2
	4	6	9	160.6	135.3	117.5	113.8	113.8	113.8	112.2	101.3	94.0	90.5	88.2	89.5	5.0	5.1	5.1	3.0	2.9	2.9	5	10	16	20	24.2	22.5
	5	6	9	159.7	134.7	117.0	113.8	113.8	113.8	111.8	101.0	93.8	90.5	88.3	89.4	5.4	5.5	5.6	2.9	3.0	2.9	5	10	16	20	24.1	22.2
95 th Percentile	2	3	9	165.4	138.6	120.2	113.8	113.8	113.8	113.8	102.2	94.3	90.8	89.2	89.5	2.7	2.7	2.8	1.6	1.4	1.6	5	10	16	20	23.5	22.5
	2	4	9	164.4	137.9	119.6	113.8	113.8	113.8	113.6	102.2	94.3	90.9	88.8	88.8	3.2	3.2	3.2	1.8	1.8	2.0	5	10	16	20	23.8	22.5
	2	5	9	163.3	137.3	119.0	113.8	113.8	113.8	113.3	102.0	94.3	90.6	88.8	89.5	3.6	3.7	3.7	2.0	2.0	2.4	5	10	16	20	23.7	22.4
	2	6	9	162.4	136.6	118.5	113.8	113.8	113.8	113.0	101.9	94.4	90.8	89.5	90.3	4.1	4.1	4.1	2.9	3.0	3.0	5	10	16	20	23.5	21.8
	2	7	9	161.5	135.8	117.9	113.8	113.8	113.8	112.8	102.0	94.3	90.8	88.5	89.2	4.5	4.6	4.6	3.2	3.1	2.9	5	10	16	20	24.3	22.6
	2	8	9	160.6	135.1	117.4	113.8	113.8	113.8	112.6	101.7	94.0	90.8	89.0	89.4	4.9	5.0	5.0	3.3	3.4	3.3	5	10	16	20	23.7	22.5
100 th percentile	2	6	7	162.8	136.8	118.7	113.8	113.8	113.8	113.1	102.0	94.1	90.6	88.4	89.2	4.0	4.0	4.0	2.8	2.7	2.8	5	10	16	20	24.1	22.0
	2	6	8	162.6	136.7	118.6	113.8	113.8	113.8	113.0	101.9	94.3	90.6	88.4	89.2	4.0	4.1	4.1	3.0	2.9	2.9	5	10	16	20	24.0	22.7
	2	6	9	162.4	136.6	118.5	113.8	113.8	113.8	113.0	101.9	94.4	90.8	89.5	90.3	4.1	4.1	4.1	2.9	3.0	3.0	5	10	16	20	23.5	21.8
	2	6	10	162.2	136.4	118.4	113.8	113.8	113.8	113.2	101.8	94.4	90.4	89.4	89.5	4.1	4.2	4.2	2.9	3.0	3.0	5	10	16	20	23.2	21.7
	2	6	11	162.1	136.3	118.3	113.8	113.8	113.8	113.2	101.9	94.3	90.8	89.4	89.8	4.2	4.2	4.2	2.9	2.9	2.8	5	10	16	20	23.4	22.0
	2	6	12	162.0	136.2	118.2	113.8	113.8	113.8	113.2	101.7	94.3	90.6	89.3	89.4	4.2	4.3	4.3	3.1	3.0	3.0	5	10	16	20	23.1	22.4

Appendix A3.6: Sensitivity analysis for the capacity design problem, $\bar{a} = 16$ and FV discharge rate changed

Lead-time	FV discharge rate (1)			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7
	RV discharge rate (2)			0.32							0.32							0.32						
	Ratio= $\frac{(1)}{(2)}$			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2
	Median	95 th	100 th	Max capacity (hours/week)							Average capacity (hours/week)							FV's average lead time (weeks)						
Median	1	6	9	129.2	126.9	125.3	123.7	122.1	120.6	119.0	105.3	102.8	101.3	99.5	97.9	96.3	94.6	3.6	3.6	3.6	3.6	3.6	3.6	3.6
	2	6	9	128.5	126.3	124.7	123.2	121.6	120.0	118.5	104.9	102.7	101.0	99.3	97.7	96.1	94.4	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	3	6	9	127.9	125.7	124.2	122.6	121.1	119.5	118.0	104.6	102.3	100.6	99.0	97.3	95.8	94.2	4.6	4.6	4.6	4.6	4.6	4.6	4.6
	4	6	9	127.3	125.1	123.6	122.0	120.5	119.0	117.5	104.2	101.9	100.4	98.7	97.1	95.6	94.0	5.1	5.1	5.1	5.1	5.1	5.1	5.1
	5	6	9	126.6	124.5	123.0	121.5	120.0	118.5	117.0	103.9	101.6	100.1	98.5	96.8	95.4	93.8	5.6	5.6	5.6	5.6	5.6	5.6	5.6
95 th Percentile	2	3	9	130.5	128.2	126.6	125.0	123.3	121.7	120.2	105.3	102.9	101.2	99.4	97.7	96.0	94.3	2.8	2.8	2.8	2.8	2.8	2.8	2.8
	2	4	9	129.8	127.6	126.0	124.4	122.8	121.2	119.6	105.3	102.7	101.1	99.4	97.7	96.0	94.3	3.2	3.2	3.2	3.2	3.2	3.2	3.2
	2	5	9	129.2	126.9	125.4	123.8	122.2	120.6	119.0	105.1	102.7	101.0	99.3	97.7	96.0	94.3	3.7	3.7	3.7	3.7	3.7	3.7	3.7
	2	6	9	128.5	126.3	124.7	123.2	121.6	120.0	118.5	104.9	102.7	101.0	99.3	97.7	96.1	94.4	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	7	9	127.9	125.7	124.1	122.6	121.0	119.5	117.9	104.7	102.4	100.8	99.2	97.5	95.9	94.3	4.6	4.6	4.6	4.6	4.6	4.6	4.6
	2	8	9	127.2	125.0	123.5	122.0	120.4	118.9	117.4	104.5	102.2	100.5	98.9	97.3	95.6	94.0	5.0	5.0	5.0	5.0	5.0	5.0	5.0
100 th percentile	2	6	7	128.7	126.5	124.9	123.4	121.8	120.3	118.7	105.0	102.5	100.8	99.1	97.4	95.8	94.1	4.0	4.0	4.0	4.0	4.0	4.0	4.0
	2	6	8	128.6	126.4	124.8	123.3	121.7	120.1	118.6	104.9	102.6	100.9	99.3	97.6	95.9	94.3	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	9	128.5	126.3	124.7	123.2	121.6	120.0	118.5	104.9	102.7	101.0	99.3	97.7	96.1	94.4	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	10	128.4	126.2	124.6	123.1	121.5	119.9	118.4	104.9	102.5	100.8	99.2	97.5	96.0	94.4	4.2	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	11	128.4	126.1	124.6	123.0	121.4	119.8	118.3	104.8	102.5	100.8	99.1	97.5	96.0	94.3	4.2	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	12	128.3	126.0	124.5	122.9	121.3	119.8	118.2	104.8	102.4	100.8	99.1	97.4	95.9	94.3	4.3	4.3	4.3	4.3	4.3	4.3	4.3

Appendix A3.7: Sensitivity analysis for the capacity design problem, $\bar{a} = 16$ and RV discharge rate changed

Lead-time	FV discharge rate (1)			0.38						0.38						0.38					
	RV discharge rate (2)			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio= $\frac{(1)}{(2)}$			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95 th	100 th	Max capacity (hours/week)						Average capacity (hours/week)						FV's average lead time (Weeks)					
Median	1	6	9	150.4	144.4	139.3	134.7	129.2	127.1	128.1	122.1	116.6	111.4	105.3	102.7	3.6	3.6	3.6	3.6	3.6	3.6
	2	6	9	149.8	143.9	138.7	134.1	128.5	126.4	127.9	121.8	116.0	110.9	104.9	102.4	4.1	4.1	4.1	4.1	4.1	4.1
	3	6	9	149.3	143.3	138.1	133.5	127.9	125.7	127.6	121.4	115.7	110.8	104.6	102.1	4.6	4.6	4.6	4.6	4.6	4.6
	4	6	9	148.8	142.7	137.5	132.9	127.3	125.1	127.2	121.2	115.4	110.3	104.2	101.9	5.0	5.0	5.0	5.1	5.1	5.1
	5	6	9	148.3	142.2	136.9	132.3	126.6	124.4	127.0	120.7	115.1	110.2	103.9	101.5	5.5	5.5	5.5	5.5	5.6	5.6
95 th Percentile	2	3	9	151.5	145.6	140.4	136.0	130.5	128.4	128.3	122.0	116.5	111.4	105.3	102.9	2.7	2.7	2.7	2.8	2.8	2.8
	2	4	9	150.9	145.0	139.9	135.3	129.8	127.7	128.2	121.9	116.2	111.4	105.3	102.7	3.2	3.2	3.2	3.2	3.2	3.2
	2	5	9	150.4	144.5	139.3	134.7	129.2	127.1	128.0	121.6	116.3	111.2	105.1	102.6	3.6	3.6	3.6	3.7	3.7	3.7
	2	6	9	149.8	143.9	138.7	134.1	128.5	126.4	127.9	121.8	116.0	110.9	104.9	102.4	4.1	4.1	4.1	4.1	4.1	4.1
	2	7	9	149.3	143.3	138.1	133.5	127.9	125.7	127.9	121.5	115.9	111.0	104.7	102.3	4.5	4.5	4.5	4.6	4.6	4.6
	2	8	9	148.7	142.7	137.4	132.8	127.2	125.1	127.6	121.5	115.6	110.7	104.5	102.0	5.0	5.0	5.0	5.0	5.0	5.0
100 th percentile	2	6	7	150.1	144.1	138.9	134.3	128.7	126.6	128.1	121.9	116.3	111.2	105.0	102.4	4.0	4.0	4.0	4.0	4.0	4.0
	2	6	8	150.0	144.0	138.8	134.2	128.6	126.5	128.0	121.8	116.2	111.0	104.9	102.5	4.0	4.0	4.0	4.1	4.1	4.1
	2	6	9	149.8	143.9	138.7	134.1	128.5	126.4	127.9	121.8	116.0	110.9	104.9	102.4	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	10	149.7	143.8	138.6	134.0	128.4	126.3	127.7	121.7	116.0	110.9	104.9	102.4	4.1	4.1	4.1	4.2	4.2	4.2
	2	6	11	149.6	143.7	138.5	133.9	128.4	126.2	127.8	121.7	115.9	110.8	104.8	102.3	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	12	149.5	143.6	138.4	133.8	128.3	126.1	127.5	121.3	116.0	110.8	104.8	102.3	4.2	4.2	4.2	4.3	4.3	4.3

**APPENDIX 4: SENSITIVITY ANALYSIS CHO ADMISSION
MANAGEMENT PROBLEM**

Appendix A4.1: Sensitivity analysis for the admission management problem, FV discharge rate = 0.38, and RV discharge rate = 0.32

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)																							
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Number of denied patients						Total unused capacity (hours)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	7475	3669	1297	98	0	0	3118.2	2414.5	1966.7	1729.1	1784.1	1786.7	3.6	3.6	3.6	3.6	2.5	2.5	5	10	16	20	22.4	22.5
	2	6	9	7394	3597	1227	30	0	0	3103.4	2415.3	1950.7	1715.1	1777.9	1782.6	4.1	4.1	4.1	4.1	2.6	2.8	5	10	16	20	22.1	22.7
	3	6	9	7307	3522	1157	0	0	0	3087.4	2387.6	1934.0	1734.8	1770.0	1763.3	4.6	4.6	4.6	4.4	3.3	3.2	5	10	16	20	21.6	22.4
	4	6	9	7217	3447	1086	0	0	0	3070.9	2373.8	1927.9	1751.0	1769.9	1763.3	5.1	5.1	5.1	4.5	3.3	3.2	5	10	16	20	21.6	22.4
	5	6	9	7129	3371	1015	0	0	0	3054.8	2373.7	1912.7	1751.2	1769.9	1763.3	5.6	5.6	5.6	4.8	3.3	3.2	5	10	16	20	21.6	22.4
95 th Percentile	2	3	9	7654	3805	1429	239	14	14	3183.7	2489.5	2017.7	1802.9	1796.7	1795.5	2.8	2.8	2.8	2.8	1.8	2.1	5	10	16	20	22.6	22.6
	2	4	9	7568	3738	1362	167	0	0	3153.2	2457.9	2000.5	1768.6	1783.0	1778.4	3.2	3.2	3.2	3.2	2.2	2.3	5	10	16	20	22.6	22.5
	2	5	9	7477	3668	1296	98	0	0	3132.3	2428.1	1971.6	1741.5	1783.4	1792.6	3.7	3.7	3.7	3.7	2.6	2.6	5	10	16	20	22.4	22.5
	2	6	9	7394	3597	1227	30	0	0	3103.4	2415.3	1950.7	1715.1	1777.9	1782.6	4.1	4.1	4.1	4.1	2.6	2.8	5	10	16	20	22.1	22.7
	2	7	9	7314	3522	1157	0	0	0	3075.6	2374.6	1923.5	1736.1	1782.9	1767.2	4.5	4.6	4.6	4.4	2.9	3.0	5	10	16	20	22.1	22.0
	2	8	9	7231	3446	1086	0	0	0	3049.0	2360.6	1904.7	1739.2	1773.5	1772.6	5.0	5.0	5.0	4.5	3.1	3.1	5	10	16	20	21.9	22.0
100 th percentile	2	6	7	7416	3617	1246	51	0	0	3107.4	2418.9	1957.2	1738.2	1767.7	1759.9	4.0	4.0	4.0	4.0	2.5	2.6	5	10	16	20	22.0	22.4
	2	6	8	7405	3607	1237	40	0	0	3105.4	2418.9	1955.7	1724.8	1780.5	1766.5	4.0	4.1	4.1	4.1	2.7	2.6	5	10	16	20	22.4	21.9
	2	6	9	7394	3597	1227	30	0	0	3103.4	2415.3	1950.7	1715.1	1777.9	1782.6	4.1	4.1	4.1	4.1	2.6	2.8	5	10	16	20	22.1	22.7
	2	6	10	7384	3587	1218	20	0	0	3100.7	2408.7	1952.2	1713.1	1783.3	1783.8	4.2	4.2	4.2	4.2	3.1	2.8	5	10	16	20	22.0	22.5
	2	6	11	7372	3576	1208	11	0	0	3091.4	2399.6	1938.9	1711.5	1781.3	1773.5	4.2	4.2	4.2	4.2	2.8	3.0	5	10	16	20	22.3	22.0
	2	6	12	7364	3566	1198	2	0	0	3089.9	2397.5	1948.4	1710.0	1783.1	1762.2	4.2	4.3	4.3	4.3	3.2	2.9	5	10	16	20	22.2	22.2

Appendix A4.2: Sensitivity analysis for the admission management problem, FV discharge rate = 0.38 and RV discharge rate = 0.1

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)																							
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Number of denied patients				Total unused capacity (hours)				FV's average lead time (weeks)				RV's average lead time (weeks)											
Median	1	6	9	I	I	4011	1859	0	0	I	I	1482.6	1167.2	994.9	989.5	I	I	3.6	3.6	2.8	2.7	I	I	16	20	25	25.4
	2	6	9	I	I	3962	1808	0	0	I	I	1483.1	1160.7	962.1	1000.8	I	I	4.1	4.1	3.0	3.1	I	I	16	20	25	25.2
	3	6	9	I	I	3913	1756	0	0	I	I	1479.5	1154.0	941.5	958.8	I	I	4.6	4.6	3.6	3.6	I	I	16	20	25	25.0
	4	6	9	I	I	3864	1704	0	0	I	I	1470.7	1147.5	941.9	934.1	I	I	5.1	5.1	3.6	4.2	I	I	16	20	25	24.8
	5	6	9	I	I	3817	1651	0	0	I	I	1443.8	1140.6	941.9	934.1	I	I	5.6	5.6	3.6	4.2	I	I	16	20	25	24.8
95 th Percentile	2	3	9	I	I	4108	1961	14	14	I	I	1547.2	1235.0	1003.9	1010.2	I	I	2.8	2.8	2.5	2.2	I	I	16	20	25	25.5
	2	4	9	I	I	4063	1911	0	0	I	I	1522.6	1219.9	965.1	1015.6	I	I	3.2	3.2	2.5	2.6	I	I	16	20	25	25.4
	2	5	9	I	I	4014	1861	0	0	I	I	1515.5	1192.6	950.2	1005.7	I	I	3.7	3.7	2.6	2.6	I	I	16	20	25	25.4
	2	6	9	I	I	3962	1808	0	0	I	I	1483.1	1160.7	962.1	1000.8	I	I	4.1	4.1	3.0	3.1	I	I	16	20	25	25.2
	2	7	9	I	I	3910	1752	0	0	I	I	1469.1	1129.8	963.2	999.0	I	I	4.6	4.6	3.2	3.1	I	I	16	20	25	25.2
	2	8	9	I	I	3855	1693	0	0	I	I	1442.7	1101.4	947.2	984.1	I	I	5.0	5.0	3.3	3.5	I	I	16	20	25	25.0
100 th percentile	2	6	7	I	I	3982	1828	0	0	I	I	1483.1	1163.4	959.5	966.5	I	I	4.0	4.0	2.8	2.5	I	I	16	20	25	25.4
	2	6	8	I	I	3972	1818	0	0	I	I	1482.8	1162.1	971.8	998.9	I	I	4.1	4.1	2.9	2.8	I	I	16	20	25	25.4
	2	6	9	I	I	3962	1808	0	0	I	I	1483.1	1160.7	962.1	1000.8	I	I	4.1	4.1	3.0	3.1	I	I	16	20	25	25.2
	2	6	10	I	I	3953	1798	0	0	I	I	1482.9	1159.6	954.0	1003.1	I	I	4.2	4.2	3.2	2.9	I	I	16	20	25	25.5
	2	6	11	I	I	3943	1788	0	0	I	I	1480.6	1158.3	936.9	947.9	I	I	4.2	4.2	2.9	3.1	I	I	16	20	25	25.2
	2	6	12	I	I	3933	1777	0	0	I	I	1479.3	1156.6	966.4	980.9	I	I	4.3	4.3	3.3	3.2	I	I	16	20	25	25.1

Appendix A4.3: Sensitivity analysis for the admission management problem, FV discharge rate = 0.38 and RV discharge rate = 0.2

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)																							
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Number of denied patients						Total unused capacity (hours)						FV's average lead time (weeks)						RV's average lead time (weeks)					
Median	1	6	9	I	6290	2532	912	0	0	I	2293.8	1706.9	1419.3	1358.5	1361.9	I	3.6	3.6	3.6	2.8	2.7	I	10	16	20	24	24.3
	2	6	9	I	6234	2470	852	0	0	I	2297.8	1677.1	1406.0	1351.1	1361.5	I	4.1	4.1	4.1	3.1	2.7	I	10	16	20	24	24.4
	3	6	9	I	6177	2411	791	0	0	I	2276.0	1688.1	1400.7	1352.0	1333.3	I	4.6	4.6	4.6	3.3	3.5	I	10	16	20	24	24.0
	4	6	9	I	6120	2350	730	0	0	I	2263.7	1659.2	1391.2	1336.4	1327.4	I	5.1	5.1	5.1	3.8	3.7	I	10	16	20	23	23.8
	5	6	9	I	6060	2289	669	0	0	I	2254.4	1649.8	1381.9	1336.4	1327.4	I	5.6	5.6	5.6	3.8	3.7	I	10	16	20	23	23.8
95 th Percentile	2	3	9	I	6404	2649	1031	14	14	I	2359.9	1762.8	1482.8	1373.6	1382.7	I	2.8	2.8	2.8	1.9	2.3	I	10	16	20	24	24.6
	2	4	9	I	6352	2590	973	0	0	I	2336.8	1743.4	1467.4	1378.4	1376.2	I	3.2	3.2	3.2	2.6	2.4	I	10	16	20	24	24.6
	2	5	9	I	6296	2532	914	0	0	I	2307.3	1706.8	1438.5	1356.3	1371.5	I	3.7	3.7	3.7	2.8	2.6	I	10	16	20	24	24.5
	2	6	9	I	6234	2470	852	0	0	I	2297.8	1677.1	1406.0	1351.1	1361.5	I	4.1	4.1	4.1	3.1	2.7	I	10	16	20	24	24.4
	2	7	9	I	6168	2409	788	0	0	I	2251.6	1649.0	1382.1	1352.9	1340.0	I	4.6	4.6	4.6	3.2	3.0	I	10	16	20	24	24.3
	2	8	9	I	6100	2342	723	0	0	I	2240.8	1622.0	1361.4	1351.0	1355.2	I	5.0	5.0	5.0	3.4	3.2	I	10	16	20	24	24.1
100 th percentile	2	6	7	I	6252	2490	872	0	0	I	2283.8	1680.2	1413.2	1325.4	1348.4	I	4.0	4.0	4.0	2.7	2.7	I	10	16	20	24	23.8
	2	6	8	I	6243	2480	862	0	0	I	2283.8	1678.7	1411.7	1352.2	1365.9	I	4.1	4.1	4.1	2.9	2.5	I	10	16	20	24	24.2
	2	6	9	I	6234	2470	852	0	0	I	2297.8	1677.1	1406.0	1351.1	1361.5	I	4.1	4.1	4.1	3.1	2.7	I	10	16	20	24	24.4
	2	6	10	I	6225	2460	842	0	0	I	2283.4	1675.7	1408.5	1357.0	1356.8	I	4.2	4.2	4.2	2.9	2.9	I	10	16	20	24	23.9
	2	6	11	I	6215	2451	832	0	0	I	2294.8	1674.3	1407.0	1314.1	1317.2	I	4.2	4.2	4.2	2.8	3.1	I	10	16	20	24	23.6
	2	6	12	I	6206	2440	822	0	0	I	2276.8	1672.4	1405.4	1331.6	1356.4	I	4.3	4.3	4.3	3.1	3.0	I	10	16	20	24	23.8

Appendix A4.4: Sensitivity analysis for the admission management problem, FV discharge rate = 0.5 and RV discharge rate = 0.32

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)																							
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
				Number of denied patients						Total unused capacity (hours)						FV's average lead time (weeks)						RV's average lead time (Weeks)					
Median	1	6	9	6971	3280	910	0	0	0	3078.4	2477.8	2056.8	1988.4	1998.4	1996.3	3.6	3.6	3.6	2.7	2.4	2.3	5	10	16	20	21.4	21.6
	2	6	9	6890	3209	842	0	0	0	3071.3	2482.2	2045.8	1971.9	1983.5	1993.1	4.1	4.1	4.1	2.9	2.6	2.8	5	10	16	20	21.1	21.2
	3	6	9	6808	3136	774	0	0	0	3045.2	2452.3	2034.9	1978.0	1978.3	2006.9	4.6	4.6	4.6	3.5	3.2	2.8	5	10	16	20	20.7	21.3
	4	6	9	6723	3064	704	0	0	0	3044.0	2458.5	2023.4	1978.0	1978.3	2006.9	5.1	5.1	5.1	3.5	3.2	2.8	5	10	16	20	20.7	21.3
	5	6	9	6638	2990	633	0	0	0	3030.1	2428.6	2011.7	1978.0	1978.3	2006.9	5.6	5.6	5.6	3.5	3.2	2.8	5	10	16	20	20.7	21.3
95 th Percentile	2	3	9	7163	3423	1050	14	14	14	3075.4	2546.0	2125.6	2008.4	1993.2	1995.0	2.8	2.8	2.8	2.4	1.8	2.0	5	10	16	20	22.0	21.6
	2	4	9	7072	3356	982	0	0	0	3066.5	2521.8	2100.7	1974.7	1971.3	1988.2	3.2	3.2	3.2	2.6	2.4	2.2	5	10	16	20	21.9	21.3
	2	5	9	6978	3280	913	0	0	0	3071.3	2493.8	2072.3	1979.8	1983.5	1993.6	3.7	3.7	3.7	2.6	2.6	2.4	5	10	16	20	21.3	21.2
	2	6	9	6890	3209	842	0	0	0	3069.3	2482.2	2045.8	1971.9	1999.0	1993.1	4.1	4.1	4.1	2.9	2.6	2.8	5	10	16	20	21.1	21.2
	2	7	9	6810	3133	769	0	0	0	3054.7	2434.7	2018.2	1967.5	2018.8	1971.3	4.6	4.6	4.6	3.2	2.8	2.9	5	10	16	20	21.2	21.3
	2	8	9	6731	3057	697	0	0	0	3053.2	2407.9	2006.6	1959.5	1992.7	2002.3	5.0	5.0	5.0	3.2	3.2	3.1	5	10	16	20	20.4	21.4
100 th percentile	2	6	7	6915	3231	866	0	0	0	3075.4	2485.8	2064.8	1998.9	1993.2	2001.0	4.0	4.0	4.0	2.9	2.5	2.5	5	10	16	20	21.1	21.5
	2	6	8	6903	3220	853	0	0	0	3066.5	2466.2	2047.7	1983.7	1971.3	2027.3	4.1	4.1	4.1	2.7	2.9	2.5	5	10	16	20	21.8	21.4
	2	6	9	6890	3209	842	0	0	0	3071.3	2482.2	2045.8	1971.9	1983.5	1993.1	4.1	4.1	4.1	2.9	2.6	2.8	5	10	16	20	21.1	21.2
	2	6	10	6878	3197	832	0	0	0	3069.3	2480.1	2044.3	1985.6	1999.0	2030.8	4.2	4.2	4.2	3.1	3.1	2.8	5	10	16	20	20.6	21.6
	2	6	11	6865	3187	821	0	0	0	3054.7	2461.3	2042.4	1987.7	2018.8	2012.6	4.2	4.2	4.2	2.9	2.8	2.9	5	10	16	20	21.2	21.5
	2	6	12	6856	3176	810	0	0	0	3053.2	2459.0	2040.6	1975.4	1992.7	2007.7	4.3	4.3	4.3	3.3	3.0	2.9	5	10	16	20	21.6	21.5

Appendix A4.5: Sensitivity analysis for the admission management problem, FV discharge rate = 0.7 and RV discharge rate = 0.32

Lead-time	Median	95 th	100 th	Restricted RV average lead time ($\bar{a}$)																									
				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30		
				Number of denied patients						Total unused capacity (hours)						FV's average lead time (weeks)						RV's average lead time (weeks)							
Median	1	6	9	6106	2534	177	0	0	0	3054.1	2600.0	2235.9	2427.0	2407.0	2422.0	3.6	3.6	3.6	2.6	2.5	2.4	5	10	16	20	20.0	19.5		
	2	6	9	6028	2464	112	0	0	0	3048.0	2590.7	2227.1	2378.6	2352.3	2376.7	4.1	4.1	4.1	2.8	2.7	2.6	5	10	16	20	19.7	19.4		
	3	6	9	5949	2396	47	0	0	0	3043.7	2581.9	2218.3	2431.2	2369.5	2382.2	4.6	4.6	4.6	3.1	3.0	2.9	5	10	16	20	19.7	19.1		
	4	6	9	5868	2325	0	0	0	0	3034.2	2572.6	2236.0	2431.2	2369.5	2382.2	5.1	5.1	5.1	3.1	3.0	2.9	5	10	16	20	19.7	19.1		
	5	6	9	5788	2254	0	0	0	0	3022.9	2563.4	2246.4	2431.2	2369.5	2382.2	5.6	5.6	5.2	3.1	3.0	2.9	5	10	16	20	19.7	19.1		
95th Percentile	2	3	9	6316	2688	339	14	14	14	3142.1	2665.2	2333.9	2422.1	2442.5	2452.8	2.8	2.8	2.8	2.0	1.8	1.8	5	10	16	20	20.5	20.4		
	2	4	9	6219	2614	262	0	0	0	3102.8	2632.9	1483.1	2385.4	2377.4	2440.2	3.2	3.2	3.2	2.4	2.3	2.2	5	10	16	20	20.3	20.7		
	2	5	9	6122	2540	188	0	0	0	3089.7	2602.1	2262.1	2425.7	2403.4	2394.1	3.7	3.7	3.7	2.6	2.6	2.6	5	10	16	20	19.7	20.0		
	2	6	9	6028	2464	112	0	0	0	3048.0	2590.7	2227.1	2378.6	2352.3	2376.7	4.1	4.1	4.1	2.8	2.7	2.6	5	10	16	20	19.7	19.4		
	2	7	9	5940	2384	38	0	0	0	3018.9	2556.6	2209.7	2363.7	2330.2	2324.6	4.6	4.6	4.6	3.0	2.7	3.0	5	10	16	20	19.3	19.4		
	2	8	9	5858	2301	0	0	0	0	2987.7	2525.5	2204.7	2340.7	2332.8	2342.9	5.0	5.0	4.8	3.2	2.9	3.2	5	10	16	19	19.3	19.5		
100th percentile	2	6	7	6058	2492	144	0	0	0	3057.8	2575.8	2265.4	2399.4	2467.0	2371.4	4.0	4.0	4.0	2.6	2.5	2.5	5	10	16	20	20.5	20.3		
	2	6	8	6044	2478	126	0	0	0	3035.9	2569.1	2242.7	2308.1	2548.3	2370.8	4.1	4.1	4.1	2.8	2.8	2.9	5	10	16	19	20.9	19.4		
	2	6	9	6028	2464	112	0	0	0	3048.0	2590.7	2227.1	2378.6	2352.3	2376.7	4.1	4.1	4.1	2.8	2.7	2.6	5	10	16	20	19.7	19.4		
	2	6	10	6013	2451	99	0	0	0	3034.3	2589.0	2225.2	2418.6	2424.0	2435.8	4.2	4.2	4.2	3.0	3.0	2.7	5	10	16	20	19.1	21.3		
	2	6	11	5998	2437	87	0	0	0	3029.6	2587.1	2223.7	2433.0	2438.6	2367.8	4.2	4.2	4.2	3.0	2.3	3.0	5	10	16	20	20.3	19.9		
	2	6	12	5990	2425	74	0	0	0	3026.6	2577.0	2221.8	2498.8	2355.2	2419.4	4.3	4.3	4.3	2.7	2.7	3.1	5	10	16	20	19.7	19.6		

Appendix A4.6: Sensitivity analysis for the admission management problem, $\bar{a} = 16$ and FV discharge rate changed

Lead-time	FV discharge rate (1)			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7
	RV discharge rate (2)			0.32							0.32							0.32						
	Ratio= $\frac{(1)}{(2)}$			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2
	Median	95 th	100 th	Number of denied patients							Total unused capacity (hours)							FV's average lead time (weeks)						
Median	1	6	9	1297	1075	910	738	559	374	177	1966.7	2013.4	2056.8	2102.2	2139.6	2187.3	2235.9	3.6	3.6	3.6	3.6	3.6	3.6	3.6
	2	6	9	1227	1005	842	670	492	307	112	1950.7	2001.2	2045.8	2091.5	2129.4	2177.4	2227.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	3	6	9	1157	936	774	603	425	242	47	1934.0	2000.3	2034.9	2081.3	2119.3	2168.3	2218.3	4.6	4.6	4.6	4.6	4.6	4.6	4.6
	4	6	9	1086	868	704	534	358	174	0	1927.9	1991.4	2023.4	2070.5	2109.3	2158.3	2236.0	5.1	5.1	5.1	5.1	5.1	5.1	5.1
	5	6	9	1015	797	633	467	289	108	0	1912.7	1979.0	2011.7	2060.3	2098.7	2149.0	2246.4	5.6	5.6	5.6	5.6	5.6	5.6	5.2
95 th Percentile	2	3	9	1429	1213	1050	882	706	524	339	2017.7	2082.0	2125.6	2173.7	2223.5	2275.2	2333.9	2.8	2.8	2.8	2.8	2.8	2.8	2.8
	2	4	9	1362	1145	982	813	638	452	262	2000.5	2054.2	2100.7	2142.2	2191.0	2240.7	1483.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2
	2	5	9	1296	1076	913	742	566	381	188	1971.6	2026.9	2072.3	2114.4	2162.1	2211.2	2262.1	3.7	3.7	3.7	3.7	3.7	3.7	3.7
	2	6	9	1227	1005	842	670	492	307	112	1950.7	2001.2	2045.8	2091.5	2129.4	2177.4	2227.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	7	9	1157	935	769	598	418	233	38	1923.5	1989.3	2018.2	2063.5	2109.9	2158.8	2209.7	4.6	4.6	4.6	4.6	4.6	4.6	4.6
	2	8	9	1086	864	697	526	345	159	0	1904.7	1963.0	2006.6	2052.3	2099.3	2148.7	2204.7	5.0	5.0	5.0	5.0	5.0	5.0	4.8
100 th percentile	2	6	7	1246	1028	866	694	520	336	144	1957.2	2018.8	2064.8	2111.5	2161.2	2212.3	2265.4	4.0	4.0	4.0	4.0	4.0	4.0	4.0
	2	6	8	1237	1016	853	682	506	321	126	1955.7	2003.2	2047.7	2093.5	2141.7	2191.4	2242.7	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	9	1227	1005	842	670	492	307	112	1950.7	2001.2	2045.8	2091.5	2129.4	2177.4	2227.1	4.1	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	10	1218	995	832	660	482	295	99	1952.2	2000.3	2044.3	2090.2	2128.3	2175.7	2225.2	4.2	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	11	1208	986	821	649	470	284	87	1938.9	2000.7	2042.4	2088.4	2126.3	2174.2	2223.7	4.2	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	12	1198	975	810	638	458	272	74	1948.4	2000.5	2040.6	2086.8	2124.2	2172.6	2221.8	4.3	4.3	4.3	4.3	4.3	4.3	4.3

Appendix A4.7: Sensitivity analysis for the admission management problem, $\bar{a} = 16$ and RV discharge rate changed

Lead-time	FV discharge rate (1)			0.38						0.38						0.38					
	RV discharge rate (2)			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio= $\frac{(1)}{(2)}$			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95 th	100 th	Number of denied patients						Total unused capacity (hours)						FV's average lead time (weeks)					
Median	1	6	9	4011	3206	2532	1960	1297	1053	1482.6	1587.9	1706.9	1808.3	1966.7	2032.1	3.6	3.6	3.6	3.6	3.6	3.6
	2	6	9	3962	3150	2470	1897	1227	980	1483.1	1582.6	1677.1	1795.7	1950.7	2013.9	4.1	4.1	4.1	4.1	4.1	4.1
	3	6	9	3913	3095	2411	1832	1157	912	1479.5	1571.9	1688.1	1787.3	1934.0	2005.8	4.6	4.6	4.6	4.6	4.6	4.6
	4	6	9	3864	3040	2350	1766	1086	836	1470.7	1564.4	1659.2	1776.1	1927.9	1996.2	5.1	5.1	5.1	5.1	5.1	5.1
	5	6	9	3817	2984	2289	1698	1015	762	1443.8	1556.5	1649.8	1748.2	1912.7	1981.7	5.6	5.6	5.6	5.6	5.6	5.6
95 th Percentile	2	3	9	4108	3312	2649	2083	1429	1190	1547.2	1661.7	1762.8	1871.8	2017.7	2092.4	2.8	2.8	2.8	2.8	2.8	2.8
	2	4	9	4063	3260	2590	2022	1362	1120	1522.6	1613.9	1743.4	1850.3	2000.5	2066.5	3.2	3.2	3.2	3.2	3.2	3.2
	2	5	9	4014	3205	2532	1960	1296	1052	1515.5	1606.1	1706.8	1823.8	1971.6	2042.1	3.7	3.7	3.7	3.7	3.7	3.7
	2	6	9	3962	3150	2470	1897	1227	980	1483.1	1582.6	1677.1	1795.7	1950.7	2013.9	4.1	4.1	4.1	4.1	4.1	4.1
	2	7	9	3910	3092	2409	1829	1157	911	1469.1	1548.9	1649.0	1770.0	1923.5	2001.0	4.6	4.6	4.6	4.6	4.6	4.6
	2	8	9	3855	3031	2342	1760	1086	840	1442.7	1519.0	1622.0	1744.1	1904.7	1981.4	5.0	5.0	5.0	5.0	5.0	5.0
100 th percentile	2	6	7	3982	3172	2490	1916	1246	1000	1483.1	1583.4	1680.2	1801.0	1957.2	2032.1	4.0	4.0	4.0	4.0	4.0	4.0
	2	6	8	3972	3160	2480	1908	1237	992	1482.8	1599.8	1678.7	1785.1	1955.7	2020.8	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	9	3962	3150	2470	1897	1227	980	1483.1	1582.6	1677.1	1795.7	1950.7	2013.9	4.1	4.1	4.1	4.1	4.1	4.1
	2	6	10	3953	3140	2460	1888	1218	972	1482.9	1578.2	1675.7	1795.4	1952.2	2016.8	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	11	3943	3132	2451	1878	1208	961	1480.6	1577.6	1674.3	1790.9	1938.9	2014.5	4.2	4.2	4.2	4.2	4.2	4.2
	2	6	12	3933	3123	2440	1868	1198	956	1479.3	1576.4	1672.4	1793.1	1948.4	2014.4	4.3	4.3	4.3	4.3	4.3	4.3

**APPENDIX 5: RESULTS FOR THE EXACT APPOINTMENT
 SCHEDULING POLICY**

Appendix A5.1: Results of exact appointment scheduling policy for the capacity design problem, FV discharge = 0.38 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	40	0	40	126	386	495	0	0	0	1	4	5
	2	6	9	621	9	214	215	337	360	6	0	2	2	3	4
	3	6	9	232	0	270	344	513	413	2	0	3	4	5	4
	4	6	9	285	270	250	284	121	97	3	3	3	3	1	1
	5	6	9	133	205	102	196	0	90	1	2	1	2	0	1
95 th Percentile	2	3	9	172	122	233	80	257	257	2	1	2	1	3	3
	2	4	9	211	147	321	181	166	140	2	2	3	2	2	1
	2	5	9	40	0	40	49	245	135	0	0	0	1	3	1
	2	6	9	621	9	214	215	337	360	6	0	2	2	3	4
	2	7	9	423	9	9	9	389	423	4	0	0	0	4	4
	2	8	9	456	9	9	9	318	489	5	0	0	0	3	5
100 th percentile	2	6	7	40	0	0	0	176	286	0	0	0	0	2	3
	2	6	8	276	0	9	251	132	175	3	0	0	3	1	2
	2	6	9	621	9	214	215	337	360	6	0	2	2	3	4
	2	6	10	218	29	40	118	237	452	2	0	0	1	2	5
	2	6	11	223	129	40	60	156	344	2	1	0	1	2	4
	2	6	12	70	182	245	292	280	353	1	2	3	3	3	4

Appendix A5.2: Results of exact appointment scheduling policy for the capacity design problem, FV discharge = 0.38 & RV discharge = 0.1

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	20	9	271	19	114	271	0	0	3	0	1	3
	2	6	9	370	135	124	345	294	124	4	1	1	4	3	1
	3	6	9	20	197	9	9	9	9	0	2	0	0	0	0
	4	6	9	20	209	32	8	2	32	0	2	0	0	0	0
	5	6	9	134	68	78	9	25	78	1	1	1	0	0	1
95 th Percentile	2	3	9	9	156	160	59	78	160	0	2	2	1	1	2
	2	4	9	20	95	215	9	28	215	0	1	2	0	0	2
	2	5	9	20	95	9	0	71	9	0	1	0	0	1	0
	2	6	9	370	135	124	345	294	124	4	1	1	4	3	1
	2	7	9	20	139	9	9	287	9	0	1	0	0	3	0
	2	8	9	20	9	9	19	19	9	0	0	0	0	0	0
100 th percentile	2	6	7	9	9	9	9	206	9	0	0	0	0	2	0
	2	6	8	238	9	34	164	192	34	2	0	0	2	2	0
	2	6	9	370	135	124	345	294	124	4	1	1	4	3	1
	2	6	10	29	314	70	66	0	70	0	3	1	1	0	1
	2	6	11	40	2	230	3	341	230	0	0	2	0	3	2
	2	6	12	86	190	365	20	107	365	1	2	4	0	1	4

Appendix A5.3: Results of exact appointment scheduling policy for the capacity design problem, FV discharge = 0.38 & RV discharge = 0.2

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	29	54	9	9	524	504	0	1	0	0	5	5
	2	6	9	29	91	0	200	490	193	0	1	0	2	5	2
	3	6	9	29	35	29	20	292	252	0	0	0	0	3	3
	4	6	9	29	152	9	22	152	156	0	2	0	0	2	2
	5	6	9	29	138	9	20	0	46	0	1	0	0	0	0
95 th Percentile	2	3	9	149	122	121	35	229	439	2	1	1	0	2	4
	2	4	9	29	9	81	131	183	510	0	0	1	1	2	5
	2	5	9	29	58	171	22	223	108	0	1	2	0	2	1
	2	6	9	29	91	0	200	490	193	0	1	0	2	5	2
	2	7	9	29	9	9	9	413	300	0	0	0	0	4	3
	2	8	9	40	9	9	9	451	443	0	0	0	0	5	5
100 th percentile	2	6	7	9	9	9	294	615	126	0	0	0	3	6	1
	2	6	8	20	53	8	155	199	444	0	1	0	2	2	5
	2	6	9	29	91	0	200	490	193	0	1	0	2	5	2
	2	6	10	40	8	20	20	305	366	0	0	0	0	3	4
	2	6	11	49	224	9	29	219	519	1	2	0	0	2	5
	2	6	12	121	270	29	0	433	277	1	3	0	0	4	3

Appendix A5.4: Results of exact appointment scheduling policy for the capacity design problem, FV discharge = 0.5 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	40	21	60	156	586	394	0	0	1	2	6	4
	2	6	9	54	236	270	226	352	206	1	2	3	2	4	2
	3	6	9	40	205	113	149	229	177	0	2	1	2	2	2
	4	6	9	193	29	19	99	76	0	2	0	0	1	1	0
	5	6	9	82	99	22	144	0	0	1	1	0	1	0	0
95 th Percentile	2	3	9	290	22	29	181	257	266	3	0	0	2	3	3
	2	4	9	40	156	168	100	197	136	0	2	2	1	2	1
	2	5	9	59	387	167	114	157	11	1	4	2	1	2	0
	2	6	9	54	236	270	226	352	206	1	2	3	2	4	2
	2	7	9	491	20	29	159	220	126	5	0	0	2	2	1
	2	8	9	251	9	9	57	170	231	3	0	0	1	2	2
100 th percentile	2	6	7	27	9	64	506	45	125	0	0	1	5	0	1
	2	6	8	29	206	275	227	246	136	0	2	3	2	3	1
	2	6	9	54	236	270	226	352	206	1	2	3	2	4	2
	2	6	10	49	8	20	62	97	265	1	0	0	1	1	3
	2	6	11	60	501	29	219	149	346	1	5	0	2	2	4
	2	6	12	103	20	29	236	441	327	1	0	0	2	5	3

Appendix A5.5: Results of exact appointment scheduling policy for the capacity design problem, FV discharge = 0.7 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	49	29	103	371	463	415	1	0	1	4	5	4
	2	6	9	49	437	158	349	454	397	1	4	2	4	5	4
	3	6	9	40	178	252	172	172	172	0	2	3	2	2	2
	4	6	9	193	117	60	0	0	0	2	1	1	0	0	0
	5	6	9	189	65	60	0	0	1	2	1	1	0	0	0
95 th Percentile	2	3	9	112	20	78	257	257	257	1	0	1	3	3	3
	2	4	9	153	29	96	191	198	145	2	0	1	2	2	1
	2	5	9	40	29	49	41	16	210	0	0	1	0	0	2
	2	6	9	49	437	158	349	454	397	1	4	2	4	5	4
	2	7	9	380	20	29	191	236	88	4	0	0	2	2	1
	2	8	9	611	9	9	78	125	125	6	0	0	1	1	1
100 th percentile	2	6	7	295	9	552	196	75	152	3	0	6	2	1	2
	2	6	8	164	0	255	261	64	88	2	0	3	3	1	1
	2	6	9	49	437	158	349	454	397	1	4	2	4	5	4
	2	6	10	49	9	60	75	243	161	1	0	1	1	2	2
	2	6	11	74	365	138	272	344	174	1	4	1	3	4	2
	2	6	12	70	40	146	241	339	242	1	0	1	2	3	2

Appendix A5.6: Results of exact appointment scheduling policy for the capacity design problem, $\bar{a} = 16$ & FV discharge rate changed

Lead-time	FV discharge rate (1)			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7
	RV discharge rate (2)			0.32							0.32						
	Ratio= $\frac{(1)}{(2)}$			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2
	Median	95 th percentile	100 th percentile	Number of wasted slots							Percent of the wasted slots (%)						
Median	1	6	9	40	9	60	29	60	92	103	0	0	1	0	1	1	1
	2	6	9	214	194	270	177	232	322	158	2	2	3	2	2	3	2
	3	6	9	270	39	113	30	87	60	252	3	0	1	0	1	1	3
	4	6	9	250	20	19	19	20	60	60	3	0	0	0	0	1	1
	5	6	9	102	20	22	20	20	60	60	1	0	0	0	0	1	1
95 th Percentile	2	3	9	233	29	29	56	29	68	78	2	0	0	1	0	1	1
	2	4	9	321	20	168	40	40	40	96	3	0	2	0	0	0	1
	2	5	9	40	312	167	36	49	49	49	0	3	2	0	1	1	1
	2	6	9	214	194	270	177	232	322	158	2	2	3	2	2	3	2
	2	7	9	9	29	29	28	29	29	29	0	0	0	0	0	0	0
	2	8	9	9	20	9	18	20	170	9	0	0	0	0	0	2	0
100 th percentile	2	6	7	0	9	64	678	528	96	552	0	0	1	7	5	1	6
	2	6	8	9	98	275	163	121	167	255	0	1	3	2	1	2	3
	2	6	9	214	194	270	177	232	322	158	2	2	3	2	2	3	2
	2	6	10	40	20	20	20	20	60	60	0	0	0	0	0	1	1
	2	6	11	40	29	29	31	50	60	138	0	0	0	0	1	1	1
	2	6	12	245	29	29	81	29	60	146	3	0	0	1	0	1	1

Appendix A5.7: Results of exact appointment scheduling policy for the capacity design problem, $\bar{a} = 16$ & RV discharge rate changed

Lead-time	FV discharge rate (1)			0.38											
	RV discharge rate (2)			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio= $\frac{(1)}{(2)}$			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	271	29	9	9	40	20	3	0	0	0	0	0
	2	6	9	124	198	0	0	214	40	1	2	0	0	2	0
	3	6	9	9	20	29	29	270	107	0	0	0	0	3	1
	4	6	9	32	77	9	29	250	40	0	1	0	0	3	0
	5	6	9	78	207	9	186	102	134	1	2	0	2	1	1
95th Percentile	2	3	9	160	103	121	82	233	29	2	1	1	1	2	0
	2	4	9	215	38	81	9	321	61	2	0	1	0	3	1
	2	5	9	9	0	171	9	40	9	0	0	2	0	0	0
	2	6	9	124	198	0	0	214	40	1	2	0	0	2	0
	2	7	9	9	29	9	12	9	29	0	0	0	0	0	0
	2	8	9	9	9	9	9	9	8	0	0	0	0	0	0
100th percentile	2	6	7	9	9	9	9	0	8	0	0	0	0	0	0
	2	6	8	34	179	8	312	9	89	0	2	0	3	0	1
	2	6	9	124	198	0	0	214	40	1	2	0	0	2	0
	2	6	10	70	11	20	20	40	20	1	0	0	0	0	0
	2	6	11	230	9	9	26	40	29	2	0	0	0	0	0
	2	6	12	365	9	29	29	245	0	4	0	0	0	3	0

Appendix A5.8: Results of exact appointment scheduling policy for the admission management problem

FV discharge = 0.38 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	2285	6091	8463	9662	9760	9760	98	95	303	218	628	511	4	2	4	2	6	5
	2	6	9	2366	6163	8533	9730	9760	9760	259	57	122	272	213	582	11	1	1	3	2	6
	3	6	9	2453	6238	8603	9760	9760	9760	57	132	160	288	240	424	2	2	2	3	2	4
	4	6	9	2543	6313	8674	9760	9760	9760	80	75	187	122	117	78	3	1	2	1	1	1
	5	6	9	2631	6389	8745	9760	9760	9760	60	90	116	0	117	0	2	1	1	0	1	0
95 th Percentile	2	3	9	2106	5955	8331	9521	9746	9746	137	121	131	266	491	491	7	2	2	3	5	5
	2	4	9	2192	6022	8398	9593	9760	9760	246	266	231	222	558	623	11	4	3	2	6	6
	2	5	9	2283	6092	8464	9662	9760	9760	106	179	35	326	493	514	5	3	0	3	5	5
	2	6	9	2366	6163	8533	9730	9760	9760	259	57	122	272	213	582	11	1	1	3	2	6
	2	7	9	2446	6238	8603	9760	9760	9760	477	135	185	445	211	495	20	2	2	5	2	5
	2	8	9	2529	6314	8674	9760	9760	9760	401	67	67	336	464	615	16	1	1	3	5	6
100 th percentile	2	6	7	2344	6143	8514	9709	9760	9760	211	392	410	196	484	333	9	6	5	2	5	3
	2	6	8	2355	6153	8523	9720	9760	9760	274	86	77	327	383	563	12	1	1	3	4	6
	2	6	9	2366	6163	8533	9730	9760	9760	259	57	122	272	213	582	11	1	1	3	2	6
	2	6	10	2376	6173	8542	9740	9760	9760	134	275	115	185	640	377	6	4	1	2	7	4
	2	6	11	2388	6184	8552	9749	9760	9760	171	37	110	315	664	262	7	1	1	3	7	3
	2	6	12	2396	6194	8562	9758	9760	9760	241	73	286	263	293	302	10	1	3	3	3	3

Appendix A5.9: Results of exact appointment scheduling policy for the admission management problem
FV discharge = 0.38 & RV discharge = 0.1

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	Inf.	Inf.	5749	7901	9760	9760	Inf.	Inf.	76	211	806	563	Inf.	Inf.	1	3	8	6
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	381	188	510	212	Inf.	Inf.	7	2	5	2
	3	6	9	Inf.	Inf.	5847	8004	9760	9760	Inf.	Inf.	142	159	622	272	Inf.	Inf.	2	2	6	3
	4	6	9	Inf.	Inf.	5896	8056	9760	9760	Inf.	Inf.	196	183	223	21	Inf.	Inf.	3	2	2	0
	5	6	9	Inf.	Inf.	5943	8109	9760	9760	Inf.	Inf.	114	5	20	21	Inf.	Inf.	2	0	0	0
95 th Percentile	2	3	9	Inf.	Inf.	5652	7799	9746	9746	Inf.	Inf.	127	111	491	491	Inf.	Inf.	2	1	5	5
	2	4	9	Inf.	Inf.	5697	7849	9760	9760	Inf.	Inf.	270	123	585	446	Inf.	Inf.	5	2	6	5
	2	5	9	Inf.	Inf.	5746	7899	9760	9760	Inf.	Inf.	419	20	565	605	Inf.	Inf.	7	0	6	6
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	381	188	510	212	Inf.	Inf.	7	2	5	2
	2	7	9	Inf.	Inf.	5850	8008	9760	9760	Inf.	Inf.	448	723	624	535	Inf.	Inf.	8	9	6	5
	2	8	9	Inf.	Inf.	5905	8067	9760	9760	Inf.	Inf.	98	99	532	374	Inf.	Inf.	2	1	5	4
100 th percentile	2	6	7	Inf.	Inf.	5778	7932	9760	9760	Inf.	Inf.	155	36	561	395	Inf.	Inf.	3	0	6	4
	2	6	8	Inf.	Inf.	5788	7942	9760	9760	Inf.	Inf.	213	477	489	405	Inf.	Inf.	4	6	5	4
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	381	188	510	212	Inf.	Inf.	7	2	5	2
	2	6	10	Inf.	Inf.	5807	7962	9760	9760	Inf.	Inf.	97	153	559	648	Inf.	Inf.	2	2	6	7
	2	6	11	Inf.	Inf.	5817	7972	9760	9760	Inf.	Inf.	113	381	543	346	Inf.	Inf.	2	5	6	4
	2	6	12	Inf.	Inf.	5827	7983	9760	9760	Inf.	Inf.	519	241	612	464	Inf.	Inf.	9	3	6	5

Appendix A5.10: Results of exact appointment scheduling policy for the admission management problem

FV discharge = 0.38 & RV discharge = 0.2

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	Inf.	3470	7228	8848	9760	9760	Inf.	117	206	245	745	584	Inf.	3	3	3	8	6
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	80	1047	372	446	Inf.	4	1	12	4	5
	3	6	9	Inf.	3583	7349	8969	9760	9760	Inf.	151	88	193	286	286	Inf.	4	1	2	3	3
	4	6	9	Inf.	3640	7410	9030	9760	9760	Inf.	177	234	85	251	230	Inf.	5	3	1	3	2
	5	6	9	Inf.	3700	7471	9091	9760	9760	Inf.	32	134	128	49	0	Inf.	1	2	1	1	0
95 th Percentile	2	3	9	Inf.	3356	7111	8729	9746	9746	Inf.	226	176	118	491	491	Inf.	7	2	1	5	5
	2	4	9	Inf.	3408	7170	8787	9760	9760	Inf.	403	229	285	389	641	Inf.	12	3	3	4	7
	2	5	9	Inf.	3464	7228	8846	9760	9760	Inf.	319	53	354	516	512	Inf.	9	1	4	5	5
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	80	1047	372	446	Inf.	4	1	12	4	5
	2	7	9	Inf.	3592	7351	8972	9760	9760	Inf.	186	116	487	279	372	Inf.	5	2	5	3	4
	2	8	9	Inf.	3660	7418	9037	9760	9760	Inf.	230	607	288	371	528	Inf.	6	8	3	4	5
100 th percentile	2	6	7	Inf.	3508	7270	8888	9760	9760	Inf.	248	96	63	367	419	Inf.	7	1	1	4	4
	2	6	8	Inf.	3517	7280	8898	9760	9760	Inf.	387	71	226	600	289	Inf.	11	1	3	6	3
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	80	1047	372	446	Inf.	4	1	12	4	5
	2	6	10	Inf.	3535	7300	8918	9760	9760	Inf.	274	129	320	519	388	Inf.	8	2	4	5	4
	2	6	11	Inf.	3545	7309	8928	9760	9760	Inf.	161	27	367	526	580	Inf.	5	0	4	5	6
	2	6	12	Inf.	3554	7320	8938	9760	9760	Inf.	223	224	113	564	470	Inf.	6	3	1	6	5

Appendix A5.11: Results of exact appointment scheduling policy for the admission management problem

FV discharge = 0.5 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	2789	6480	8850	9760	9760	9760	109	208	39	612	404	758	4	3	0	6	4	8
	2	6	9	2870	6551	8918	9760	9760	9760	170	354	176	397	287	524	6	5	2	4	3	5
	3	6	9	2952	6624	8986	9760	9760	9760	181	262	161	182	330	414	6	4	2	2	3	4
	4	6	9	3037	6696	9056	9760	9760	9760	149	129	131	0	106	0	5	2	1	0	1	0
	5	6	9	3122	6770	9127	9760	9760	9760	56	100	93	0	29	0	2	1	1	0	0	0
95 th Percentile	2	3	9	2597	6337	8710	9746	9746	9746	129	50	54	495	491	491	5	1	1	5	5	5
	2	4	9	2688	6404	8778	9746	9760	9760	73	298	26	527	547	555	3	5	0	5	6	6
	2	5	9	2782	6480	8847	9760	9760	9760	135	242	520	511	427	438	5	4	6	5	4	4
	2	6	9	2870	6551	8918	9760	9760	9760	170	354	176	397	287	524	6	5	2	4	3	5
	2	7	9	2950	6627	8991	9760	9760	9760	441	181	50	337	271	506	15	3	1	3	3	5
	2	8	9	3029	6703	9063	9760	9760	9760	519	297	179	365	290	516	17	4	2	4	3	5
100 th percentile	2	6	7	2845	6529	8894	9760	9760	9760	158	303	100	247	327	439	6	5	1	3	3	4
	2	6	8	2857	6540	8907	9760	9760	9760	228	43	82	455	322	575	8	1	1	5	3	6
	2	6	9	2870	6551	8918	9760	9760	9760	170	354	176	397	287	524	6	5	2	4	3	5
	2	6	10	2882	6563	8928	9760	9760	9760	180	439	188	628	256	380	6	7	2	6	3	4
	2	6	11	2895	6573	8939	9760	9760	9760	103	504	166	340	505	415	4	8	2	3	5	4
	2	6	12	2904	6584	8950	9760	9760	9760	111	241	360	428	362	448	4	4	4	4	4	5

Appendix A5.12: Results of exact appointment scheduling policy for the admission management problem

FV discharge = 0.7 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	3654	7226	9583	9760	9760	9760	214	204	371	647	590	672	6	3	4	7	6	7
	2	6	9	3732	7296	9648	9760	9760	9760	550	253	366	299	324	395	15	3	4	3	3	4
	3	6	9	3811	7364	9713	9760	9760	9760	116	289	223	200	552	584	3	4	2	2	6	6
	4	6	9	3892	7435	9760	9760	9760	9760	127	420	131	0	57	217	3	6	1	0	1	2
	5	6	9	3972	7506	9760	9760	9760	9760	131	80	121	0	0	217	3	1	1	0	0	2
95 th Percentile	2	3	9	3444	7072	9421	9746	9746	9746	172	130	166	491	491	491	5	2	2	5	5	5
	2	4	9	3541	7146	9498	9760	9760	9760	469	161	269	389	392	558	13	2	3	4	4	6
	2	5	9	3638	7220	9572	9760	9760	9760	161	45	382	345	345	392	4	1	4	4	4	4
	2	6	9	3732	7296	9648	9760	9760	9760	550	253	366	299	324	395	15	3	4	3	3	4
	2	7	9	3820	7376	9722	9760	9760	9760	213	74	266	229	361	569	6	1	3	2	4	6
	2	8	9	3902	7459	9760	9760	9760	9760	326	236	384	513	348	678	8	3	4	5	4	7
100 th percentile	2	6	7	3702	7268	9616	9760	9760	9760	156	138	329	383	268	378	4	2	3	4	3	4
	2	6	8	3716	7282	9634	9760	9760	9760	149	731	329	391	364	278	4	10	3	4	4	3
	2	6	9	3732	7296	9648	9760	9760	9760	550	253	366	299	324	395	15	3	4	3	3	4
	2	6	10	3747	7309	9661	9760	9760	9760	73	332	258	339	563	226	2	5	3	3	6	2
	2	6	11	3762	7323	9673	9760	9760	9760	517	177	352	236	429	205	14	2	4	2	4	2
	2	6	12	3770	7335	9686	9760	9760	9760	646	136	333	380	458	293	17	2	3	4	5	3

Appendix A5.13: Results of exact appointment scheduling policy for the admission management problem,
 $\bar{a} = 16$ & FV discharge rate changed

Lead-time	FV discharge Rate			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7			
	RV discharge Rate			0.32								0.32								0.32							
	Ratio			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2			
	Median	95 th	100 th	Model's appointed patients (number of patients)								Number of wasted slots								Percent of the wasted slots (%)							
Median	1	6	9	8463	8685	8850	9022	9201	9386	9583	303	174	39	130	201	136	371	4	2	0	1	2	1	4			
	2	6	9	8533	8755	8918	9090	9268	9453	9648	122	139	176	516	185	151	366	1	2	2	6	2	2	4			
	3	6	9	8603	8824	8986	9157	9335	9518	9713	160	129	161	202	315	211	223	2	1	2	2	3	2	2			
	4	6	9	8674	8892	9056	9226	9402	9586	9760	187	67	131	522	201	174	131	2	1	1	6	2	2	1			
	5	6	9	8745	8963	9127	9293	9471	9652	9760	116	150	93	96	85	88	121	1	2	1	1	1	1	1			
95th Percentile	2	3	9	8331	8547	8710	8878	9054	9236	9421	131	77	54	89	76	114	166	2	1	1	1	1	1	2			
	2	4	9	8398	8615	8778	8947	9122	9308	9498	231	40	26	340	173	174	269	3	0	0	4	2	2	3			
	2	5	9	8464	8684	8847	9018	9194	9379	9572	35	82	520	182	261	181	382	0	1	6	2	3	2	4			
	2	6	9	8533	8755	8918	9090	9268	9453	9648	122	139	176	516	185	151	366	1	2	2	6	2	2	4			
	2	7	9	8603	8825	8991	9162	9342	9527	9722	185	287	50	255	209	378	266	2	3	1	3	2	4	3			
	2	8	9	8674	8896	9063	9234	9415	9601	9760	67	120	179	516	400	241	384	1	1	2	6	4	3	4			
100th percentile	2	6	7	8514	8732	8894	9066	9240	9424	9616	410	81	100	104	529	187	329	5	1	1	1	6	2	3			
	2	6	8	8523	8744	8907	9078	9254	9439	9634	77	248	82	146	188	87	329	1	3	1	2	2	1	3			
	2	6	9	8533	8755	8918	9090	9268	9453	9648	122	139	176	516	185	151	366	1	2	2	6	2	2	4			
	2	6	10	8542	8765	8928	9100	9278	9465	9661	115	209	188	192	110	335	258	1	2	2	2	1	4	3			
	2	6	11	8552	8774	8939	9111	9290	9476	9673	110	107	166	51	131	138	352	1	1	2	1	1	1	4			
	2	6	12	8562	8785	8950	9122	9302	9488	9686	286	323	360	472	212	369	333	3	4	4	5	2	4	3			

Appendix A5.14: Results of exact appointment scheduling policy for the admission management problem,
 $\bar{a} = 16$, &RV discharge rate changed

Lead-time	FV discharge Rate			0.38						0.38						0.38					
	RV discharge Rate			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	5749	6554	7228	7800	8463	8707	76	249	206	741	303	99	1	4	3	10	4	1
	2	6	9	5798	6610	7290	7863	8533	8780	381	325	80	330	122	157	7	5	1	4	1	2
	3	6	9	5847	6665	7349	7928	8603	8848	142	249	88	354	160	269	2	4	1	4	2	3
	4	6	9	5896	6720	7410	7994	8674	8924	196	113	234	99	187	99	3	2	3	1	2	1
	5	6	9	5943	6776	7471	8062	8745	8998	114	78	134	157	116	90	2	1	2	2	1	1
95th Percentile	2	3	9	5652	6448	7111	7677	8331	8570	127	106	176	149	131	289	2	2	2	2	2	3
	2	4	9	5697	6500	7170	7738	8398	8640	270	103	229	136	231	64	5	2	3	2	3	1
	2	5	9	5746	6555	7228	7800	8464	8708	419	353	53	192	35	162	7	5	1	2	0	2
	2	6	9	5798	6610	7290	7863	8533	8780	381	325	80	330	122	157	7	5	1	4	1	2
	2	7	9	5850	6668	7351	7931	8603	8849	448	159	116	94	185	43	8	2	2	1	2	0
	2	8	9	5905	6729	7418	8000	8674	8920	98	304	607	153	67	102	2	5	8	2	1	1
100th percentile	2	6	7	5778	6588	7270	7844	8514	8760	155	301	96	356	410	86	3	5	1	5	5	1
	2	6	8	5788	6600	7280	7852	8523	8768	213	549	71	350	77	68	4	8	1	4	1	1
	2	6	9	5798	6610	7290	7863	8533	8780	381	325	80	330	122	157	7	5	1	4	1	2
	2	6	10	5807	6620	7300	7872	8542	8788	97	221	129	182	115	423	2	3	2	2	1	5
	2	6	11	5817	6628	7309	7882	8552	8799	113	68	27	263	110	160	2	1	0	3	1	2
	2	6	12	5827	6637	7320	7892	8562	8804	519	119	224	79	286	145	9	2	3	1	3	2

**APPENDIX 6: RESULTS FOR THE RELAXED APPOINTMENT
 SCHEDULING POLICY**

Appendix A6.1: Results of relaxed appointment scheduling policy for the capacity design problem, FV discharge =0.38 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	40	0	40	76	278	173	0	0	0	1	3	2
	2	6	9	603	9	78	218	0	0	6	0	1	2	0	0
	3	6	9	232	0	184	344	0	0	2	0	2	4	0	0
	4	6	9	285	270	250	287	212	255	3	3	3	3	2	3
	5	6	9	133	205	102	203	0	90	1	2	1	2	0	1
95 th Percentile	2	3	9	151	122	233	80	257	257	2	1	2	1	3	3
	2	4	9	151	147	249	181	246	246	2	2	3	2	3	3
	2	5	9	40	0	40	49	0	0	0	0	0	1	0	0
	2	6	9	603	9	78	218	0	0	6	0	1	2	0	0
	2	7	9	440	9	9	9	0	0	5	0	0	0	0	0
	2	8	9	283	9	9	9	0	0	3	0	0	0	0	0
100 th percentile	2	6	7	40	0	0	0	0	0	0	0	0	0	0	0
	2	6	8	78	0	9	76	0	0	1	0	0	1	0	0
	2	6	9	603	9	78	218	0	0	6	0	1	2	0	0
	2	6	10	49	29	40	118	0	0	1	0	0	1	0	0
	2	6	11	60	124	40	60	0	0	1	1	0	1	0	0
	2	6	12	70	182	195	261	0	0	1	2	2	3	0	0

Appendix A6.2: Results of relaxed appointment scheduling policy for the capacity design problem, FV discharge =0.38 & RV discharge = 0.1

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	20	9	235	19	114	235	0	0	2	0	1	2
	2	6	9	370	135	105	242	294	105	4	1	1	2	3	1
	3	6	9	20	277	9	9	9	9	0	3	0	0	0	0
	4	6	9	20	388	55	7	2	55	0	4	1	0	0	1
	5	6	9	134	138	78	9	62	78	1	1	1	0	1	1
95 th Percentile	2	3	9	9	156	169	59	78	169	0	2	2	1	1	2
	2	4	9	20	99	236	9	28	236	0	1	2	0	0	2
	2	5	9	20	104	9	0	73	9	0	1	0	0	1	0
	2	6	9	370	135	105	242	294	105	4	1	1	2	3	1
	2	7	9	20	117	9	9	287	9	0	1	0	0	3	0
	2	8	9	20	9	9	36	19	9	0	0	0	0	0	0
100 th percentile	2	6	7	9	9	9	9	206	9	0	0	0	0	2	0
	2	6	8	93	9	34	153	192	34	1	0	0	2	2	0
	2	6	9	370	135	105	242	294	105	4	1	1	2	3	1
	2	6	10	29	207	70	66	0	70	0	2	1	1	0	1
	2	6	11	40	2	120	3	341	120	0	0	1	0	3	1
	2	6	12	40	286	365	20	107	365	0	3	4	0	1	4

Appendix A6.3: Results of relaxed appointment scheduling policy for the capacity design problem, FV discharge =0.38 & RV discharge = 0.2

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	29	66	9	9	197	228	0	1	0	0	2	2
	2	6	9	29	116	0	200	1	0	0	1	0	2	0	0
	3	6	9	29	62	29	20	1	1	0	1	0	0	0	0
	4	6	9	29	152	9	22	0	1	0	2	0	0	0	0
	5	6	9	29	138	9	20	0	1	0	1	0	0	0	0
95 th Percentile	2	3	9	29	138	137	142	229	257	0	1	1	1	2	3
	2	4	9	60	9	60	167	239	248	1	0	1	2	2	3
	2	5	9	29	70	171	22	60	0	0	1	2	0	1	0
	2	6	9	29	116	0	200	1	0	0	1	0	2	0	0
	2	7	9	29	9	9	9	16	125	0	0	0	0	0	1
	2	8	9	40	9	9	9	1	125	0	0	0	0	0	1
100 th percentile	2	6	7	9	9	9	294	9	1	0	0	0	3	0	0
	2	6	8	20	145	8	123	0	2	0	1	0	1	0	0
	2	6	9	29	116	0	200	1	0	0	1	0	2	0	0
	2	6	10	40	8	20	20	1	0	0	0	0	0	0	0
	2	6	11	49	306	9	29	0	1	1	3	0	0	0	0
	2	6	12	209	270	29	0	0	0	2	3	0	0	0	0

Appendix A6.4: Results of relaxed appointment scheduling policy for the capacity design problem, FV discharge =0.5 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	40	21	60	160	169	160	0	0	1	2	2	2
	2	6	9	60	175	270	226	125	0	1	2	3	2	1	0
	3	6	9	40	117	87	149	0	0	0	1	1	2	0	0
	4	6	9	193	29	18	99	0	0	2	0	0	1	0	0
	5	6	9	82	99	20	0	0	0	1	1	0	0	0	0
95 th Percentile	2	3	9	235	20	29	181	257	257	2	0	0	2	3	3
	2	4	9	40	170	116	100	241	240	0	2	1	1	2	2
	2	5	9	66	387	129	112	2	0	1	4	1	1	0	0
	2	6	9	60	175	270	226	125	0	1	2	3	2	1	0
	2	7	9	491	20	29	159	0	0	5	0	0	2	0	0
	2	8	9	535	9	9	43	125	125	5	0	0	0	1	1
100 th percentile	2	6	7	42	9	62	552	0	0	0	0	1	6	0	0
	2	6	8	29	125	117	85	62	125	0	1	1	1	1	1
	2	6	9	60	175	270	226	125	0	1	2	3	2	1	0
	2	6	10	49	8	20	62	0	0	1	0	0	1	0	0
	2	6	11	107	248	28	129	125	125	1	3	0	1	1	1
	2	6	12	229	20	29	236	0	75	2	0	0	2	0	1

Appendix A6.5: Results of relaxed appointment scheduling policy for the capacity design problem, FV discharge =0.7 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	49	29	76	160	166	168	1	0	1	2	2	2
	2	6	9	49	437	158	125	125	125	1	4	2	1	1	1
	3	6	9	40	104	308	0	0	0	0	1	3	0	0	0
	4	6	9	193	119	60	0	0	0	2	1	1	0	0	0
	5	6	9	189	136	60	0	0	1	2	1	1	0	0	0
95 th Percentile	2	3	9	46	20	78	257	257	257	0	0	1	3	3	3
	2	4	9	139	29	96	239	241	241	1	0	1	2	2	2
	2	5	9	40	29	49	1	1	1	0	0	1	0	0	0
	2	6	9	49	437	158	125	125	125	1	4	2	1	1	1
	2	7	9	189	20	29	0	1	13	2	0	0	0	0	0
	2	8	9	611	9	9	0	125	125	6	0	0	0	1	1
100 th percentile	2	6	7	295	9	622	1	0	0	3	0	6	0	0	0
	2	6	8	34	0	233	0	62	1	0	0	2	0	1	0
	2	6	9	49	437	158	125	125	125	1	4	2	1	1	1
	2	6	10	49	9	60	40	79	125	1	0	1	0	1	1
	2	6	11	60	216	138	125	125	125	1	2	1	1	1	1
	2	6	12	395	40	103	0	0	0	4	0	1	0	0	0

Appendix A6.6: Results of relaxed appointment scheduling policy for the capacity design problem, $\bar{a} = 16$ & FV discharge rate changed

Lead-time	FV discharge rate (1)			0.38											
	RV discharge rate (2)			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio= $\frac{(1)}{(2)}$			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95 th percentile	100 th percentile	Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	40	9	60	29	60	75	76	0	0	1	0	1
	2	6	9	78	145	270	148	143	322	158	1	1	3	2	1
	3	6	9	184	33	87	30	86	60	308	2	0	1	0	1
	4	6	9	250	20	18	18	20	60	60	3	0	0	0	0
	5	6	9	102	20	20	20	20	60	60	1	0	0	0	0
95th Percentile	2	3	9	233	29	29	37	29	68	78	2	0	0	0	0
	2	4	9	249	20	116	40	40	40	96	3	0	1	0	0
	2	5	9	40	351	129	36	49	49	49	0	4	1	0	1
	2	6	9	78	145	270	148	143	322	158	1	1	3	2	1
	2	7	9	9	29	29	27	29	29	29	0	0	0	0	0
	2	8	9	9	20	9	21	20	50	9	0	0	0	0	0
100th percentile	2	6	7	0	9	62	678	528	70	622	0	0	1	7	5
	2	6	8	9	47	117	74	67	104	233	0	0	1	1	1
	2	6	9	78	145	270	148	143	322	158	1	1	3	2	1
	2	6	10	40	20	20	20	20	60	60	0	0	0	0	0
	2	6	11	40	29	28	29	50	60	138	0	0	0	0	1
	2	6	12	195	29	29	55	29	60	103	2	0	0	1	0

Appendix A6.7: Results of relaxed appointment scheduling policy for the capacity design problem, $\bar{a} = 16$ & RV discharge rate changed

Lead-time	FV discharge rate (1)			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7
	RV discharge rate (2)			0.32							0.32						
	Ratio= $\frac{(1)}{(2)}$			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2
	Median	95 th percentile	100 th percentile	Number of wasted slots							Percent of the wasted slots (%)						
Median	1	6	9	235	29	9	7	40	20	2	0	0	0	0	0	235	29
	2	6	9	105	198	0	0	78	40	1	2	0	0	1	0	105	198
	3	6	9	9	20	29	29	184	107	0	0	0	0	2	1	9	20
	4	6	9	55	29	9	29	250	40	1	0	0	0	3	0	55	29
	5	6	9	78	207	9	186	102	134	1	2	0	2	1	1	78	207
95th Percentile	2	3	9	169	103	137	77	233	29	2	1	1	1	2	0	169	103
	2	4	9	236	38	60	9	249	61	2	0	1	0	3	1	236	38
	2	5	9	9	0	171	9	40	9	0	0	2	0	0	0	9	0
	2	6	9	105	198	0	0	78	40	1	2	0	0	1	0	105	198
	2	7	9	9	29	9	12	9	29	0	0	0	0	0	0	9	29
	2	8	9	9	9	9	9	9	6	0	0	0	0	0	0	9	9
100th percentile	2	6	7	9	9	9	9	0	8	0	0	0	0	0	0	9	9
	2	6	8	34	143	8	312	9	43	0	1	0	3	0	0	34	143
	2	6	9	105	198	0	0	78	40	1	2	0	0	1	0	105	198
	2	6	10	70	9	20	20	40	18	1	0	0	0	0	0	70	9
	2	6	11	120	9	9	26	40	29	1	0	0	0	0	0	120	9
	2	6	12	365	9	29	29	195	0	4	0	0	0	2	0	365	9

Appendix A6.8: Results of relaxed appointment scheduling policy for the admission management problem
FV discharge =0.38 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	2285	6091	8463	9662	9760	9760	98	222	303	218	311	332	4	4	4	2	3	3
	2	6	9	2366	6163	8533	9730	9760	9760	186	57	86	175	205	515	8	1	1	2	2	5
	3	6	9	2453	6238	8603	9760	9760	9760	55	132	156	182	240	448	2	2	2	2	2	5
	4	6	9	2543	6313	8674	9760	9760	9760	76	75	187	0	0	0	3	1	2	0	0	0
	5	6	9	2631	6389	8745	9760	9760	9760	63	90	116	0	0	0	2	1	1	0	0	0
95 th Percentile	2	3	9	2106	5955	8331	9521	9746	9746	137	121	0	266	491	491	7	2	0	3	5	5
	2	4	9	2192	6022	8398	9593	9760	9760	381	300	57	222	389	389	17	5	1	2	4	4
	2	5	9	2283	6092	8464	9662	9760	9760	95	231	799	309	345	345	4	4	9	3	4	4
	2	6	9	2366	6163	8533	9730	9760	9760	186	57	86	175	205	515	8	1	1	2	2	5
	2	7	9	2446	6238	8603	9760	9760	9760	469	651	636	169	169	404	19	10	7	2	2	4
	2	8	9	2529	6314	8674	9760	9760	9760	408	30	58	128	302	389	16	0	1	1	3	4
100 th percentile	2	6	7	2344	6143	8514	9709	9760	9760	121	462	410	196	331	247	5	8	5	2	3	3
	2	6	8	2355	6153	8523	9720	9760	9760	155	86	76	393	231	370	7	1	1	4	2	4
	2	6	9	2366	6163	8533	9730	9760	9760	186	57	86	175	205	515	8	1	1	2	2	5
	2	6	10	2376	6173	8542	9740	9760	9760	217	274	119	185	487	205	9	4	1	2	5	2
	2	6	11	2388	6184	8552	9749	9760	9760	253	207	105	315	487	205	11	3	1	3	5	2
	2	6	12	2396	6194	8562	9758	9760	9760	521	189	286	265	205	205	22	3	3	3	2	2

Appendix A6.9: Results of relaxed appointment scheduling policy for the admission management problem
FV discharge =0.38 & RV discharge = 0.1

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	Inf.	Inf.	5749	7901	9760	9760	Inf.	Inf.	108	210	468	303	Inf.	Inf.	2	3	5	3
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	545	188	564	205	Inf.	Inf.	9	2	6	2
	3	6	9	Inf.	Inf.	5847	8004	9760	9760	Inf.	Inf.	150	449	539	192	Inf.	Inf.	3	6	6	2
	4	6	9	Inf.	Inf.	5896	8056	9760	9760	Inf.	Inf.	329	160	247	0	Inf.	Inf.	6	2	3	0
	5	6	9	Inf.	Inf.	5943	8109	9760	9760	Inf.	Inf.	114	5	0	0	Inf.	Inf.	2	0	0	0
95 th Percentile	2	3	9	Inf.	Inf.	5652	7799	9746	9746	Inf.	Inf.	127	109	491	491	Inf.	Inf.	2	1	5	5
	2	4	9	Inf.	Inf.	5697	7849	9760	9760	Inf.	Inf.	167	252	389	389	Inf.	Inf.	3	3	4	4
	2	5	9	Inf.	Inf.	5746	7899	9760	9760	Inf.	Inf.	341	26	347	345	Inf.	Inf.	6	0	4	4
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	545	188	564	205	Inf.	Inf.	9	2	6	2
	2	7	9	Inf.	Inf.	5850	8008	9760	9760	Inf.	Inf.	691	337	356	179	Inf.	Inf.	12	4	4	2
	2	8	9	Inf.	Inf.	5905	8067	9760	9760	Inf.	Inf.	167	27	348	131	Inf.	Inf.	3	0	4	1
100 th percentile	2	6	7	Inf.	Inf.	5778	7932	9760	9760	Inf.	Inf.	213	36	247	247	Inf.	Inf.	4	0	3	3
	2	6	8	Inf.	Inf.	5788	7942	9760	9760	Inf.	Inf.	289	769	229	229	Inf.	Inf.	5	10	2	2
	2	6	9	Inf.	Inf.	5798	7952	9760	9760	Inf.	Inf.	545	188	564	205	Inf.	Inf.	9	2	6	2
	2	6	10	Inf.	Inf.	5807	7962	9760	9760	Inf.	Inf.	89	77	280	229	Inf.	Inf.	2	1	3	2
	2	6	11	Inf.	Inf.	5817	7972	9760	9760	Inf.	Inf.	312	499	369	205	Inf.	Inf.	5	6	4	2
	2	6	12	Inf.	Inf.	5827	7983	9760	9760	Inf.	Inf.	392	421	435	228	Inf.	Inf.	7	5	4	2

Appendix A6.10: Results of relaxed appointment scheduling policy for the admission management problem
FV discharge =0.38 & RV discharge = 0.2

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	Inf.	3470	7228	8848	9760	9760	Inf.	111	206	246	395	343	Inf.	3	3	3	4	4
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	79	837	205	229	Inf.	4	1	9	2	2
	3	6	9	Inf.	3583	7349	8969	9760	9760	Inf.	151	67	235	286	276	Inf.	4	1	3	3	3
	4	6	9	Inf.	3640	7410	9030	9760	9760	Inf.	213	234	136	251	209	Inf.	6	3	2	3	2
	5	6	9	Inf.	3700	7471	9091	9760	9760	Inf.	32	134	122	0	0	Inf.	1	2	1	0	0
95 th Percentile	2	3	9	Inf.	3356	7111	8729	9746	9746	Inf.	226	184	117	491	491	Inf.	7	3	1	5	5
	2	4	9	Inf.	3408	7170	8787	9760	9760	Inf.	403	137	283	389	389	Inf.	12	2	3	4	4
	2	5	9	Inf.	3464	7228	8846	9760	9760	Inf.	319	53	400	345	345	Inf.	9	1	5	4	4
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	79	837	205	229	Inf.	4	1	9	2	2
	2	7	9	Inf.	3592	7351	8962	9760	9760	Inf.	261	114	246	169	177	Inf.	7	2	3	2	2
	2	8	9	Inf.	3660	7418	9037	9760	9760	Inf.	345	761	233	254	202	Inf.	9	10	3	3	2
100 th percentile	2	6	7	Inf.	3508	7270	8888	9760	9760	Inf.	248	95	63	247	247	Inf.	7	1	1	3	3
	2	6	8	Inf.	3517	7280	8898	9760	9760	Inf.	387	42	228	229	229	Inf.	11	1	3	2	2
	2	6	9	Inf.	3526	7290	8908	9760	9760	Inf.	156	79	837	205	229	Inf.	4	1	9	2	2
	2	6	10	Inf.	3535	7300	8918	9760	9760	Inf.	274	129	320	361	229	Inf.	8	2	4	4	2
	2	6	11	Inf.	3545	7309	8928	9760	9760	Inf.	86	27	367	205	205	Inf.	2	0	4	2	2
	2	6	12	Inf.	3554	7320	8938	9760	9760	Inf.	219	207	113	207	207	Inf.	6	3	1	2	2

Appendix A6.11: Results of relaxed appointment scheduling policy for the admission management problem
FV discharge =0.5 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	2789	6480	8850	9760	9760	9760	155	263	25	308	302	532	6	4	0	3	3	5
	2	6	9	2870	6551	8918	9760	9760	9760	174	354	176	205	379	355	6	5	2	2	4	4
	3	6	9	2952	6624	8986	9760	9760	9760	134	261	161	182	504	362	5	4	2	2	5	4
	4	6	9	3037	6696	9056	9760	9760	9760	404	146	131	0	161	0	13	2	1	0	2	0
	5	6	9	3122	6770	9127	9760	9760	9760	65	100	93	0	29	0	2	1	1	0	0	0
95 th Percentile	2	3	9	2597	6337	8710	9746	9746	9746	106	50	54	491	491	491	4	1	1	5	5	5
	2	4	9	2688	6404	8778	9760	9760	9760	379	298	23	399	389	389	14	5	0	4	4	4
	2	5	9	2782	6480	8847	9760	9760	9760	163	242	560	345	545	345	6	4	6	4	6	4
	2	6	9	2870	6551	8918	9760	9760	9760	174	354	176	205	379	355	6	5	2	2	4	4
	2	7	9	2950	6627	8991	9760	9760	9760	501	143	15	169	237	404	17	2	0	2	2	4
	2	8	9	3029	6703	9063	9760	9760	9760	713	184	167	128	187	247	24	3	2	1	2	3
100 th percentile	2	6	7	2845	6529	8894	9760	9760	9760	271	303	100	247	390	247	10	5	1	3	4	3
	2	6	8	2857	6540	8907	9760	9760	9760	213	43	83	229	229	406	7	1	1	2	2	4
	2	6	9	2870	6551	8918	9760	9760	9760	174	354	176	205	379	355	6	5	2	2	4	4
	2	6	10	2882	6563	8928	9760	9760	9760	119	567	240	521	205	205	4	9	3	5	2	2
	2	6	11	2895	6573	8939	9760	9760	9760	103	452	271	205	382	205	4	7	3	2	4	2
	2	6	12	2904	6584	8950	9760	9760	9760	307	210	360	205	205	284	11	3	4	2	2	3

Appendix A6.12: Results of relaxed appointment scheduling policy for the admission management problem
FV discharge =0.7 & RV discharge = 0.32

Average RV's lead time ($\bar{a}$)				5	10	16	20	25	30	5	10	16	20	25	30	5	10	16	20	25	30
Lead-time	Median	95 th	100 th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	3654	7226	9583	9760	9760	9760	189	204	371	447	344	417	5	3	4	5	4	4
	2	6	9	3732	7296	9648	9760	9760	9760	862	553	366	205	324	205	23	8	4	2	3	2
	3	6	9	3811	7364	9713	9760	9760	9760	116	60	223	182	300	182	3	1	2	2	3	2
	4	6	9	3892	7435	9760	9760	9760	9760	130	453	91	0	0	0	3	6	1	0	0	0
	5	6	9	3972	7506	9760	9760	9760	9760	131	61	0	0	0	0	3	1	0	0	0	0
95 th Percentile	2	3	9	3444	7072	9421	9746	9746	9746	96	129	166	491	491	491	3	2	2	5	5	5
	2	4	9	3541	7146	9498	9760	9760	9760	385	359	269	389	389	389	11	5	3	4	4	4
	2	5	9	3638	7220	9572	9760	9760	9760	207	45	420	345	345	345	6	1	4	4	4	4
	2	6	9	3732	7296	9648	9760	9760	9760	862	553	366	205	324	205	23	8	4	2	3	2
	2	7	9	3820	7376	9722	9760	9760	9760	268	74	266	169	287	169	7	1	3	2	3	2
	2	8	9	3902	7459	9760	9760	9760	9760	89	236	128	128	131	128	2	3	1	1	1	1
100 th percentile	2	6	7	3702	7268	9616	9760	9760	9760	215	93	393	247	247	247	6	1	4	3	3	3
	2	6	8	3716	7282	9634	9760	9760	9760	336	760	375	229	229	229	9	10	4	2	2	2
	2	6	9	3732	7296	9648	9760	9760	9760	862	553	366	205	324	205	23	8	4	2	3	2
	2	6	10	3747	7309	9661	9760	9760	9760	72	548	258	205	205	205	2	7	3	2	2	2
	2	6	11	3762	7323	9673	9760	9760	9760	601	177	318	236	271	205	16	2	3	2	3	2
	2	6	12	3770	7335	9686	9760	9760	9760	765	157	341	205	445	205	20	2	4	2	5	2

Appendix A6.13: Results of relaxed appointment scheduling policy for the admission management problem,

$\bar{a} = 16$ & FV discharge rate changed

Lead-time	FV discharge Rate			0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7	0.38	0.45	0.5	0.55	0.6	0.65	0.7
	RV discharge Rate			0.32							0.32							0.32						
	Ratio			1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2	1.2	1.4	1.6	1.7	1.9	2.0	2.2
	Median	95 th	100 th	Model's appointed patients (number of patients)							Number of wasted slots							Percent of the wasted slots (%)						
Median	1	6	9	8463	8685	8850	9022	9201	9386	9583	303	174	25	200	281	136	371	4	2	0	2	3	1	4
	2	6	9	8533	8755	8918	9090	9268	9453	9648	86	139	176	520	185	244	366	1	2	2	6	2	3	4
	3	6	9	8603	8824	8986	9157	9335	9518	9713	156	116	161	154	315	290	223	2	1	2	2	3	3	2
	4	6	9	8674	8892	9056	9226	9402	9586	9760	187	67	131	606	201	214	91	2	1	1	7	2	2	1
	5	6	9	8745	8963	9127	9293	9471	9652	9760	116	150	93	126	85	88	0	1	2	1	1	1	1	0
95th Percentile	2	3	9	8331	8547	8710	8878	9054	9236	9421	0	69	54	89	76	107	166	0	1	1	1	1	1	2
	2	4	9	8398	8615	8778	8947	9122	9308	9498	57	54	23	335	404	231	269	1	1	0	4	4	2	3
	2	5	9	8464	8684	8847	9018	9194	9379	9572	799	86	560	170	261	212	420	9	1	6	2	3	2	4
	2	6	9	8533	8755	8918	9090	9268	9453	9648	86	139	176	520	185	244	366	1	2	2	6	2	3	4
	2	7	9	8603	8825	8991	9162	9342	9527	9722	636	287	15	272	209	378	266	7	3	0	3	2	4	3
	2	8	9	8674	8896	9063	9234	9415	9601	9760	58	92	167	603	421	177	128	1	1	2	7	4	2	1
100th percentile	2	6	7	8514	8732	8894	9066	9240	9424	9616	410	76	100	86	479	33	393	5	1	1	1	5	0	4
	2	6	8	8523	8744	8907	9078	9254	9439	9634	76	248	83	150	263	49	375	1	3	1	2	3	1	4
	2	6	9	8533	8755	8918	9090	9268	9453	9648	86	139	176	520	185	244	366	1	2	2	6	2	3	4
	2	6	10	8542	8765	8928	9100	9278	9465	9661	119	323	240	302	111	375	258	1	4	3	3	1	4	3
	2	6	11	8552	8774	8939	9111	9290	9476	9673	105	106	271	37	76	155	318	1	1	3	0	1	2	3
	2	6	12	8562	8785	8950	9122	9302	9488	9686	286	323	360	472	193	432	341	3	4	4	5	2	5	4

Appendix A6.14: Results of relaxed appointment scheduling policy for the admission management problem,
 $\bar{a} = 16$ & RV discharge rate changed

Lead-time	FV discharge rate (1)			0.38						0.38						0.38					
	RV discharge rate (2)			0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35	0.1	0.15	0.2	0.25	0.32	0.35
	Ratio= $\frac{(1)}{(2)}$			3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1	3.8	2.5	1.9	1.5	1.2	1.1
	Median	95th	100th	Model's appointed patients (number of patients)						Number of wasted slots						Percent of the wasted slots (%)					
Median	1	6	9	5749	6554	7228	7800	8463	8707	108	249	206	628	303	73	2	4	3	8	4	1
	2	6	9	5798	6610	7290	7863	8533	8780	545	325	79	488	86	159	9	5	1	6	1	2
	3	6	9	5847	6665	7349	7928	8603	8848	150	249	67	354	156	269	3	4	1	4	2	3
	4	6	9	5896	6720	7410	7994	8674	8924	329	191	234	98	187	99	6	3	3	1	2	1
	5	6	9	5943	6776	7471	8062	8745	8998	114	78	134	196	116	56	2	1	2	2	1	1
95th Percentile	2	3	9	5652	6448	7111	7677	8331	8570	127	106	184	241	0	292	2	2	3	3	0	3
	2	4	9	5697	6500	7170	7738	8398	8640	167	103	137	136	57	48	3	2	2	2	1	1
	2	5	9	5746	6555	7228	7800	8464	8708	341	353	53	195	799	179	6	5	1	3	9	2
	2	6	9	5798	6610	7290	7863	8533	8780	545	325	79	488	86	159	9	5	1	6	1	2
	2	7	9	5850	6668	7351	7931	8603	8849	691	125	114	535	636	36	12	2	2	7	7	0
	2	8	9	5905	6729	7418	8000	8674	8920	167	304	761	771	58	33	3	5	10	10	1	0
100th percentile	2	6	7	5778	6588	7270	7844	8514	8760	213	301	95	329	410	182	4	5	1	4	5	2
	2	6	8	5788	6600	7280	7852	8523	8768	289	549	42	350	76	68	5	8	1	4	1	1
	2	6	9	5798	6610	7290	7863	8533	8780	545	325	79	488	86	159	9	5	1	6	1	2
	2	6	10	5807	6620	7300	7872	8542	8788	89	221	129	182	119	706	2	3	2	2	1	8
	2	6	11	5817	6628	7309	7882	8552	8799	312	153	27	262	105	160	5	2	0	3	1	2
	2	6	12	5827	6637	7320	7892	8562	8804	392	184	207	81	286	144	7	3	3	1	3	2

**APPENDIX 7: RESULTS FOR CAPACITY PLANNING WITH
 STOCHASTIC DEMANDS**

Appendix A7.1: Simulation results for deterministic equivalents of capacity design model

Descriptions	Measurement (number of slots per week)	Distribution	ϵ						Deterministic model
			0.01	0.05	0.1	0.2	0.3	0.4	
Both FV & RV	Max	Normal	825	781	771	760	754	749	683
		Unknown	4077	2171	1719	1399	1259	1175	
	Average	Normal	632	602	595	587	583	580	521
		Unknown	2870	1560	1248	1028	931	873	
	Min	Normal	0	0	0	0	0	0	0
		Unknown	0	0	0	0	0	0	
FV	Max	Normal	467	427	420	413	408	405	386
		Unknown	2663	1354	1053	840	746	690	
	Average	Normal	212	194	190	186	183	181	136
		Unknown	1483	739	562	437	382	349	
	Min	Normal	0	0	0	0	0	0	0
		Unknown	0	0	0	0	0	0	
RV	Max	Normal	825	781	771	760	754	749	683
		Unknown	4077	2171	1719	1399	1259	1175	
	Average	Normal	421	408	405	402	400	398	385
		Unknown	1387	821	686	591	549	525	
	Min	Normal	0	0	0	0	0	0	0
		Unknown	0	0	0	0	0	0	

Appendix A7.2: Simulation results for deterministic equivalents of admission management model

Descriptions		Measurement (Number of slots per week)	Distribution	ε					Deterministic model	
				0.01	0.05	0.1	0.2	0.3		0.4
		Total number of accepted patients	Normal	12753	12689	12675	12648	12640	12642	9760
	Unknown	13116	13116	13116	13114	13110	13085			
Capacity allocation	Both FV & RV	Max	Normal	899	899	899	899	899	899	899
			Unknown	899	899	899	899	899	899	
		Average	Normal	560	559	559	558	558	558	531
			Unknown	569	567	567	566	565	565	
		Min	Normal	0	0	0	0	0	0	0
			Unknown	0	0	0	0	0	0	
	FV	Max	Normal	459	459	426	393	461	467	429
			Unknown	556	548	548	556	556	494	
		Average	Normal	173	172	172	172	172	172	136
			Unknown	178	178	178	178	178	177	
		Min	Normal	0	0	0	0	0	0	0
			Unknown	0	0	0	0	0	0	
	RV	Max	Normal	899	899	899	899	899	899	899
			Unknown	899	899	899	899	899	899	
		Average	Normal	386	386	386	386	386	386	394
			Unknown	391	389	389	389	388	388	
		Min	Normal	0	0	0	0	0	0	0
			Unknown	0	0	0	0	0	0	