Dynamics of current-driven magnetic domain walls and skyrmions

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NANYANG
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**DYNAMICS OF CURRENT-DRIVEN MAGNETIC
DOMAIN WALLS AND SKYRMIONS**

INDRA PURNAMA

SCHOOL OF PHYSICAL AND MATHEMATICAL SCIENCES

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Abstract

The dynamics of current-driven magnetic domain walls and skyrmions are both topics of high interest in the magnetism community due to the possibility of employing them in novel high-density non-volatile memory devices. One such device is the well-known racetrack memory, where the information is stored in a ferromagnetic nanowire as compared to the conventional harddisk memory, while the operation is mediated via the injection of current to the nanowire. In a domain wall-based racetrack memory, the information is represented by magnetic domains that are separated by domain walls, while in a skyrmion-based racetrack memory, the information is represented by the presence of the skyrmions themselves. However, to be able to realize such devices, it is very important to have a complete understanding of both the domain wall and dynamics under the application of current. At present, the investigation of domain wall dynamics is mainly focused on finding additional driving mechanisms to improve its mobility. In the case of skyrmion dynamics, the research is still in the very early stage, although it has been shown that it is possible to inject individual skyrmions individually which brings skyrmion-based devices closer to realization.

In this thesis, the dynamics of current driven magnetic domain walls and skyrmions in various nanostructures are investigated. We show by micromagnetic simulations that it is possible to drive multiple domain walls in a multi-nanowire system by just applying current to one of the nanowires. The phenomenon is made possible due to the magnetostatic coupling between the domain walls. When materials with in-plane anisotropy are considered, the coupling is realized when the nanowires are placed parallel to each other, and the technique can be further utilized to drive several domain walls in the current-free nanowire. When materials with strong perpendicular magnetization anisotropy are used, the phenomenon is realized by stacking the nanowires vertically, and the coupling between the

domain walls is found to be strong enough to change the domain wall configurations and increase their current-driven speed.

As compared to the domain wall, a current-driven skyrmion is found to move at an angle with respect to the intended conduction electron flow direction due to the presence of the Magnus force. Here, we show that it is possible to guide the movement of a skyrmion by surrounding and compressing the skyrmion with strong local potential barriers. The compressed skyrmion also receives higher torque contribution from the conduction electrons, which results in a significant increase of the skyrmion speed.

Our research work can then be said to contribute to three areas. First, our work on a system of multiple nanowires with in-plane magnetization anisotropy has made it possible to design a memory device where the data is remotely driven instead of being directly driven by current injection. Such device will then benefit from increased data retention as the data will experience no heating due to the absence of direct current injection. From the physics point of view, our work on the multiple nanowire system has also shed a light on the spring-like nature of the Domain Wall- Domain Wall (DW-DW) interaction, which can potentially be used as a framework to investigate the mass property, i.e. the inertia, of DWs in ferromagnetic systems. The second contributions can be found in our work on the coupled DW system in vertically-stacked nanowires with perpendicular magnetization anisotropy (PMA). Our results shows it is possible to optimize the speed of the DWs in PMA nanowires by locking the DWs in a configuration that is least hindered when driven by current. Lastly, our work on the skyrmion research provides a framework to solve the issue of skyrmion annihilations. We show that any skyrmion-based device that are designed in the future will require the inclusion of artificial potential barriers at the edges of the device to prevent the skyrmion to be annihilated, which we accomplished by introducing curbs to the design of the skyrmion device.

Chapter 1

Introduction

Recent advances in nanostructure fabrication have led to rapid growth in the performance and the miniaturization of semiconductor-based electronic devices. However, as the device size starts to approach its fundamental limits, several issues such as high leakage current start to arise, prompting the search for alternative technologies [1, 2].

Spintronics is an emerging technology that is based on ferromagnetic materials which aim to provide an alternative to semiconductor-based devices [3 -5]. As compared to semiconductor-based electronics, the data in spintronics devices are represented by the direction of the electron spins within the ferromagnetic material. Therefore, spintronics devices are non-volatile by nature due to the spins direction being preserved by the exchange interaction between the electrons in the ferromagnetic materials.

Currently, research in spintronics is focused on understanding the dynamics of magnetic domain walls (DW) and skyrmions under the application of a current. The motions of the DWs and the skyrmions in such situation is accomplished by the transfer of angular momentum between the conduction electrons and the local magnetizations of the ferromagnetic materials [6-15].

This thesis covers research on the dynamics of current-driven magnetic DWs and skyrmions via micromagnetic simulations. Several designs of DW- and skyrmion-based memory devices are discussed and proposed.

1.1 Magnetic domain walls

Magnetic domain walls (DWs) are regions with finite widths in magnetic materials which separate domains with distinct magnetization directions. DWs are formed as a result of the energy minimization of the ferromagnetic system. In our work, we shall focus on ferromagnetic nanowires system with either in-plane or perpendicular magnetization anisotropy (PMA). The in-plane anisotropy and the perpendicular anisotropy refers to the directional dependence of the ferromagnet's magnetizations. In materials with in-plane anisotropy, the energy of the system favors the magnetizations to lie in the in-plane direction, while in materials with perpendicular anisotropy, the system prefers the magnetizations to point perpendicularly. In nanowires with in-plane anisotropy, the DWs are encountered at points where the magnetization strongly diverge/converge, whereas in nanowires with perpendicular anisotropy, the DWs can be found in the region that separates the up/down magnetization domains. Fig. 1.1 shows the Atomic Force Microscopy (AFM) image as well as the Magnetic Force Microscopy (MFM) image of a DW (dark contrast) in a nanowire with in-plane magnetization anisotropy:

Figure 1.1. (a) Atomic Force Microscopy (AFM) image of the corresponding nanowire, which shows the physical form of the nanowire. (b) Magnetic

Force Microscopy (MFM) image of a DW (dark contrast) in a patterned nanowire with in-plane magnetization anisotropy. In nanowires with in-plane magnetization anisotropy, the magnetizations of the ferromagnetic nanowire prefer to lie parallel to surface of the nanowire. Additionally, due to the dimension of the nanowire here, the magnetizations can only point to two directions, which are shown by the two arrows in the image. The magnetization configuration of the nanowire is represented by the arrows.

In Fig. 1.1 (b), the dark contrast shows that the magnetic tip of the MFM is attracted by a local stray magnetic field at the center of the nanowire, while the two bright contrasts at the end of the nanowire show that the MFM tip is repelled by a stray magnetic field which is directed opposite to the one at the center of the nanowire. The dark/bright contrasts represent how the magnetization of the nanowire strongly converges/diverges at that point, which we can use to deduce the magnetization configuration.

The idea of electrically manipulating magnetic domain states is particularly attractive due to the possibility of creating DW-based magnetic memory device. Fig. 1.2 shows a schematic of our proposed DW-based memory device [5]. Here, the data bits are represented by the magnetic domains within the nanowires, while the DWs serve to separate them. The data can then be read or written by shifting the DWs through the application of current to the nanowires. The operating speed of the data storage in such device depends on how fast the DWs can be moved within the nanowires, while the density is determined by how close the nanowires can be placed to each other. This is because as the nanowires are placed closer to each other, the DW-DW interaction starts to become prominent.

Figure 1.2. Schematic of our proposed DW-based memory device. The bits are carried to the read/write sensor by applying current to the nanowire

The investigations of DW-DW interaction initially were carried on systems with closely-spaced nanostructures mainly due to the ease of DW generation [20-22]. Fig. 1.3 shows the magnetic configuration of a pair of ferromagnetic nanorings that are obtained by combining the element sensitivity of time resolved X-ray Magnetic Circular Dichroism (XMCD) with the spatial resolution of Photoemission Electron Microscopy (PEEM). The image shows that the DWs that are facing each other have a transverse shape, while the lone DW has a vortex shape. Fig. 1.3 (b) and (c) show an array of such rings with separation of 40 nm and 500 nm, respectively. The two images show that the transverse DWs are only observed when the rings are closely-spaced, which implies that the DW-DW interaction is strong enough to change the shape of DW and therefore should not be neglected in the studies of high-density ferromagnetic devices. Inset of Fig. 1.3 (d) shows the electron holography of the magnetization configuration of a transverse DW in one of the rings, with the black lines represent the stray magnetic fields from the ring. The plot that is shown in Fig. 1.3 (d) represents the strength of the stray magnetic field as a function of distance from the edge of the ring. The results

revealed that magnetic DWs behave much like magnetic charges as they act as magnetic field sources/sinks. By considering the magnetization configuration of the ferromagnetic system, it is then possible to assign positive/ negative magnetic charges to the DWs. For DW where the magnetizations around it converge/diverge, it is designated a positive/negative magnetic charge, respectively.

Figure 1.3 (a) High resolution XMCD-PEEM image of two rings in the onion state. A noninteracting vortex wall (top) and three interacting transverse walls are visible. (b) an image of an array of the rings which shows the strong DW-DW interaction when the rings are closely spaced. (c) a similar

array of rings but with much higher separation, which shows no DW-DW interaction. (d) electron holography of the stray magnetic field [21].

The focus of DW-DW interaction studies were then shifted to systems of closely-spaced nanowires [23-45]. Fig. 1.4 (a) shows an SEM image of a pair of nanowires that are separated by 100 nm. In such system, a pair of DWs with different magnetic charges can be created at A1 and A2 simply by applying an oblique external field.

Figure 1.4 SEM image of a typical two-nanowire structure that is used to investigate DW-DW interaction (b) Schematic of oppositely charged Transverse Domain Walls (TDWs) created in “A” corners moving, under field H, along wires separated by d. (c) Open circles: External magnetic field that was needed to break the attraction coupling of the two DWs. The lines represent the models that are used to describe the DW-DW

interaction. Inset: Example MOKE observed hysteresis loop of a nanowire [26].

The DW-DW interaction was then investigated by applying an external field along the direction of the nanowire to bring the two DWs closer to each other and couple them via their stray field interaction, as shown by Fig. 1.4 (b). Fig. 1.5 (c) shows the magnetic field strength that was needed to break the coupling between the two DWs as a function of nanowire separation, which revealed the attraction nature of DWs with opposite charges.

The magnetic charge-like nature of the DWs also affects their dynamics in a single nanowire system. Fig. 1.5 (a) shows the micromagnetic simulations of a pair of vortex DWs with opposite magnetic charges in a single nanowire. The results show that, the two DWs in the single nanowire system are attracted to each other due to their stray field interaction. Interestingly, the DW-DW interaction in a single nanowire system also revealed the topological nature of the DWs [46-51].

Figure 1.5 (a) micromagnetic simulations showing the DW-DW interaction in a single nanowire system [48]. (b) various magnetization configurations with their respective topological charges [46].

As Fig. 1.5 shows us, it is possible to assign topological charges to parts of the DWs. Note that the topological charges concept is different than the magnetic charges concept that has been introduced earlier. The magnetic charges were introduced to represent the attracting/ repelling stray magnetic fields that the DWs create, while the topological charges are mathematical numbers that are assigned to specific points within the DWs to show how the magnetizations around it curl around that particular points [46]. The topological charges do not emit any kind of magnetic fields, and they are used in the study of magnetism mainly to illustrate how DWs with similar topological numbers are protected from annihilating each other.

For each vortex DW, a half-negative topological charge can be assigned to both sides of the DWs, whereas the vortex core can be assigned a single positive topological charge. The micromagnetic simulations show that as the two DWs collide with each other, they form a bound state instead of get annihilated simply because the two DWs possess similar topological charges at each side of the nanowire.

As we have discussed before, aside from investigating the DW-DW interaction to determine the device density, it is also very important to investigate how the DW speed can be increased as that will determine the operating speed of all DW-based devices. In general, it has been shown that the speed of a current-driven DW can be increased by either improving the driving force from the current or by enhancing the DW fidelity (i.e. its rigidity) [52-61], which is a measure of how strong the DW can resist a structural change upon the application of external stimuli.

Figure 1.6 (a) DW velocity as a function of an external transverse field. (b) DW configuration for various strength of external transverse field. The DW with the biggest shape was given the strongest transverse field. [58]

This is because the application of current induces a tangential torque to the DW which breaks the DW shape beyond a critical current density value. Hence when the DW fidelity can be improved, the DW will be able to withstand higher current density and gets driven with higher velocity. For instance, Fig. 1.6 (a) shows that the current-driven speed of a DW in a nanowire with in-plane magnetization anisotropy can be increased by employing a transverse external field to improve the fidelity of the current-driven DW. Fig. 1.6 (b) shows how the transverse component of the DW and its fidelity are increased by the transverse field.

Figure 1.7 (a) Optical micrograph of a Pt/Co/AlO_x wire. To better distinguish from the magnetic contrast in the following images, the micrograph is given an artificial height profile. (b) Differential Kerr microscopy image of the domain structure after application of a magnetic field (blue arrow). The bright contrast represents magnetizations that are pointing down, while the dark contrast (between A and B) represents magnetizations that are pointing up. (c) Under the effect of current (red arrow), the domain walls move in the same direction (from A to A' and B to B'). As one of the domains previously created by field grows while the other shrinks, the differential image contains both bright and dark contrast. (d) Schematic representation of a domain wall. The black arrows symbolize the magnetization, continuously rotating between the two domains. (e) Schematic representation of the torque created by an easy-axis magnetic field (thick blue arrow) on the domain wall. In a simplified domain-wall structure (a region of in-plane uniform magnetization) the torque (thin blue arrow) points in-plane, perpendicular to the magnetization. (f) Instead of a single torque, the electric current produces two torques (red arrows). One points in-plane (T_{NA}), whereas the other is perpendicular to the plane (T_A). In the Pt/Co/ samples, besides producing Spin Transfer Torque (STT), the current generates a strong in-plane Rashba effective field (H_R) [55].

In the case of nanowires with perpendicular magnetization anisotropy (PMA), it is possible to create a system whereby the fidelity- enhancement field is produced internally. Fig. 1. 7 shows the optical micrograph of a trilayer nanowire with structural

inversion symmetry (SIA). In such structure, the inversion symmetry was found to improve the non-adiabatic torque from the conduction electron as well as creating a Rashba field, which is an effective magnetic field that is produced by the spin orbit interaction to the conduction electrons, to the DW-carrying nanowire. The DW fidelity in the trilayer is then improved due to the presence of the Rashba field. Consequently, the DW in the PMA nanowire can now be driven with high current density to achieve a speed of 400 m/s.

Recent progress in DW research has been focused on understanding the details of the current-driven DW motion in such system with spin-orbit couplings. Fig. 1.8 shows the schematic of a work to investigate the motion of DW under the application of current which is in the same line as the DW [132]. The results shown there further corroborates the fact that the presence of spin-orbit coupling induces additional torques in two different directions to the DW. Interestingly, each of the spin-orbit torques lies in the same axis as the adiabatic and the non-adiabatic torques of the spin-transfer torques.

Figure 1.8 Green arrows represent the magnetic moments. (a) When current flows through the Ta in x direction (electrons in $-x$ direction) electrons with spin polarization in $-y$ direction accumulate at the interface of the Ta and CoFeB. This results in the transfer of spin orbit torque to the domain wall in CoFeB. The green parallelogram represents the plane of the sample and the three arrows show the directions of x, y and z-axes. ($\otimes$). (b) The MOKE image of the 20 microns wide magnetic bar shows the formation of a longitudinal wall along it when current flows in the $+x$ direction. The longitudinal domain wall at $y = 0$ separates an “out of the plane” ($\odot$) polarized domain for $y < 0$ and an “into the plane” ($\otimes$) polarized domain for $y > 0$. (c) MOKE images of the magnetic bar after a positive current pulse of different magnitude is applied on a longitudinal domain wall in the presence of an in-plane field (H_{in}) in $-x$ and $+x$ direction, which shows that the DW moves perpendicular to the current direction.

Aside from the typical ferromagnets, there are also efforts to realize DW memory with Synthetic AntiFerromagnetic (SAF) nanowires [133]. The argument here is that, by using SAF systems, less stray magnetic fields will be produced which may lead to a higher data density as the DWs in the same nanowire will not affect each other. Fig 1.9 shows the Kerr microscopy image of the current-driven DW motion in the SAF system. The experimental results also show that the DW in the SAF nanowire is able to move faster than a DW in typical ferromagnetic system due to the fact that the DW in the SAF system is more rigid.

Figure 1.9 (a,b) Kerr microscope images of a single, DW moving along a nanowire formed from 20 TaN/15 Pt/3 Co/7 Ni/1.5 Co/tRu Ru/1.5 Co/7 Ni/1.5 Co/50 TaN with $t_{Ru}=8$ (SAF) (b). Each image (left to right) is taken after applying a train of current pulses (current density $J \approx 3.0 \times 10^8 \text{ A cm}^{-2}$;) composed of $4 \times 5 \text{ ns}$ (a) and $2 \times 5 \text{ ns}$ (b) pulses. Blue and orange arrows represent the direction of current flow over the 100 ns integrated pulse time span. Light and dark contrast corresponds to $\uparrow$ and $\downarrow$ magnetization directions, respectively.”

Other multilayer structures other than the SIA have also been investigated in order to find the most suitable candidate for high DW speed [62-74]. It was shown that in a bilayer PMA structure, that there is a possibility that the DWs are aligned oppositely in the Néel configuration. The opposite orientation is attributed to the stray field coupling between them, which also results in a more rigid DW structure. Such magnetization configuration shall prove to be suitable for DW-based devices, as will be shown later in this thesis.

Figure 1.10 (a) MFM phase image of a Co/Au/Co film (b) various possible magnetization configurations which correspond to the Co bilayer structure. [66]

1.2. Magnetic Skyrmion

Figure 1.11. A magnetic skyrmion on a ferromagnetic thin film, as simulated by micromagnetic simulations.

Skyrmions, which were initially used to describe baryons in particle physics [80], have recently been found in extended lattices of bulk non-centrosymmetric magnetic materials such as MnSi, (FeCo)Si, and FeGe [81- 84]. The skyrmions have also been observed via spin-polarized scanning tunneling microscopy (STM) at zero applied fields in Fe monolayer that is grown on Ir (111) [85]. The presence of skyrmions in such an ultrathin film is attributed to the Dzyaloshinskii–Moriya Interaction (DMI) [86, 87], which arises from a three-site indirect exchange mechanism between two spins and a neighbor atom with a large spin-orbit coupling. Due to the DMI, the magnetization in the ferromagnets now favor spin canting, which then allows the skyrmion to be created.

Due to its topological configuration, the skyrmions have been shown to be far less hindered by defects when driven by current [88, 89] as compared to the magnetic DWs. This is due to the fact that the skyrmion is topologically protected, and therefore when defects are present, the skyrmion is able to deform its shape to skirt around the defects without being annihilated. However, the skyrmion is also shown to be deflected from the intended direction and annihilated at the film edges due to the presence of the

Magnus force [90, 91], which is a gyroscopic force that is felt by the skyrmion as a reaction to the deflection of the conduction electrons within the skyrmion vicinity.

Figure. 1.12 (a) Skyrmion deflection and annihilation at the edge of the nanowire during the application of current. (b) Schematic showing the deflection of the conduction electron which becomes the source of the skyrmion deflection [130].

Fig. 1.12 (a) shows the skyrmion deflection due to the Magnus force. As the micromagnetic simulation showed us, the skyrmion is pushed to the side in addition to its forward movement due to the application of current. The skyrmion is then annihilated at the side of the nanowire when the applied current density has exceeded the critical value.

To describe the skyrmion deflection phenomenon, one can consider the double – exchange model where the conduction electrons and the local magnetizations are coupled ferromagnetically. In this model, the electrons are viewed to hop between atoms during conduction, and the path is to be determined by the magnetization configuration of the ferromagnet. When the subsequent local magnetizations are aligned in the same

direction, the electrons are free to ‘hop’ ahead in a fixed direction. However, when the magnetization configuration is not a constant, the electron path will change such that it experiences the least magnetization change. When a skyrmion is present in the ferromagnetic structure, the magnetization configuration of the ferromagnetic layer is locally changed, and as a result, the conduction electrons are deflected in a specific direction. The skyrmion is then pushed to the opposite direction as a reaction to the deflected electrons, as shown by Fig. 1. 12 (b).

The skyrmion annihilation at the edge of the nanowire due to the Magnus force happens when the applied current density is increased beyond the critical value. Below this critical value, the skyrmion is prevented from annihilation by the magnetization edge barrier.

Figure 1. 13 (a) The magnetization configuration across the width of a ferromagnetic layer with DMI included [125]. (b) Schematic showing the magnetization configuration of a ferromagnetic layer with DMI [126].

Fig. 1. 13 (a) shows the magnetization configuration of the ferromagnetic layer across the width of the nanostructure. At the center of the nanostructure, the magnetization is shown to point mostly up (+ z direction) due to the strong exchange interaction between the local magnetizations. However, at the sides of the nanostructure, there is a lack of magnetization and hence the DMI energy starts to become more relevant. Consequently,

the magnetization of the ferromagnetic layer is twisted in the in-plane direction at the sides of the nanostructure. The magnetization twisting in return becomes a weak potential barrier that prevents the skyrmion to be annihilated at the sides of the nanostructure during the application of low current density. Fig. 1. 13 (b) shows a schematic of the magnetization configuration of a skyrmion within a patterned ferromagnetic structure. The magnetizations at the sides are twisted due to the lack of exchange interaction there.

Due to the small size of the skyrmions, they have been promoted as a possible candidate to represent data bits in the future [92 -126]. The skyrmions have also been shown to be highly interchangeable with DWs in a ferromagnetic structure. Fig. 1. 14 shows how a skyrmion can be generated when two DWs are injected to a nanowire with a bigger width. In the DW-skyrmion conversion, it is necessary to have two DWs in the initial state so that a single magnetic domain is present in the narrow nanowire. This is because a magnetic skyrmion is a single domain by itself, and therefore to create a skyrmion, two DWs are needed to create the initial magnetic domain in the narrow nanowire.

Figure 1. 14 (a)–(f) Micromagnetic simulations showing the transformation of a pair of current driven DWs to a skyrmion [128].

Recent experimental work has shown that the skyrmion injection method via DW conversion is indeed possible, as shown in Fig. 1.15 [134]. They showed that the presence of the geometrical constriction introduces a non-uniform current distribution which further helps the conversion from DWs to skyrmions.

Fig. 1.15. Experimental generation of magnetic skyrmions. (A) Sparse irregular domain structures are observed at both sides of the device at a perpendicular magnetic field of $B = 5\text{mT}$. (B) Upon passing a current of $j_e = +5 \times 10^5 \text{ A/cm}^2$ through the device, the left side of the device develops predominantly elongated stripe domains, whereas the right side converts into dense skyrmion bubbles

Another method for skyrmion injection is to use a strong current injection through the tip of Scanning Tunneling Microscope (STM) [94]. They showed that by it is possible to locally flip the magnetizations of the ferromagnet to generate a skyrmion individually.

Figure 1.16 (a) Successive population of the island with skyrmions by injecting higher-energy electrons through local voltage sweeps. (b) Skyrmion probability decreases with increasing B field. About 100 mT are needed to compensate for different current directions at $U = +600$ and -600 mV (data points and fitted hyperbolic tangent functions, $I = 100$ nA.)

However, as we mentioned it earlier, the magnetic skyrmion is prone to annihilation due to the presence of the Magnus force. A possible solution to mitigate the Magnus force will be presented in this thesis.

1.3. Objective of this thesis

The focus of this thesis revolves around the dynamics of DWs in multi nanowire system as well as the dynamics of skyrmion under the application of external current. Specific objectives of this thesis are given below:

- i. To study the dynamics of current-driven DWs in a parallel multi-nanowire system with in-plane magnetization anisotropy.
- ii. To study the dynamics of current-driven DWs in a vertically-stacked multi-nanowire system with perpendicular magnetization anisotropy.
- iii. To study the dynamics of current-driven skyrmion in a patterned nanowire

1.4. Organization of the thesis

Chapter 1 presents a brief introduction as well as the motivations of the overall research work.

In Chapter 2, the theoretical model for the domain wall and skyrmion dynamics is described.

In Chapter 3, the dynamics of coupled DW in parallel multi-nanowire system with in-plane magnetization anisotropy is discussed.

In Chapter 4, the dynamics of coupled DW in sandwich nanowire system with perpendicular magnetization anisotropy is discussed.

In Chapter 5, the dynamics of skyrmion in a patterned nanowire is discussed, a solution to counter the Magnus force is also presented.

Chapter 2

Model of magnetization dynamics

2.1 Landau-Lifshitz-Gilbert equation

The magnetization dynamics under zero applied current can be expressed by the phenomenological Landau-Lifshitz-Gilbert equation (LLG), which reads:

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M} \times \mathbf{H}_{eff}) - \frac{\lambda}{M_s}(\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{eff})) \quad (2.1),$$

with γ' as the gyromagnetic ratio of the conduction electron, $\mathbf{M}$ as the local magnetization vector, $\mathbf{H}_{eff}$ as the effective magnetic field, M_s as the saturation magnetization, and λ as the Gilbert damping coefficient. The first term describes the precession motion of the magnetization vector along the effective field, while the second term describes the damping term that will bring the magnetization vector to the direction of the effective field. Fig. 2.1 shows a schematic of the LLG equation:

Figure 2.1 Vector representation of the LLG equation, the damping term brings the magnetization closer to the direction of the effective field $\mathbf{H}_{eff}$, while the precession term rotates the local magnetization around $\mathbf{H}_{eff}$.

The effective field represents the contribution from the energies of the system, and it is defined as $H_{eff} = \frac{1}{M_s} \frac{\delta w}{\delta \mathbf{m}}$, where w is the total energy density of the system, and $\mathbf{m} = \mathbf{M}/M_s$. For our ferromagnetic system, the total energy is a sum of the exchange energy, demagnetization energy, magnetocrystalline energy, Zeeman energy, and the Dzyaloshinskii-Moriya-Interaction (DMI) energy.

- Exchange Energy

The exchange energy (E_x) comes from the quantum-mechanical exchange interaction between the local magnetizations. It is minimized when the local magnetizations are aligned in the same direction. Mathematically, it can be expressed as:

$$E_x = A(\nabla \mathbf{M})^2 \quad (2.2),$$

Where A is the exchange constant of the material.

- Magnetocrystalline Anisotropy Energy

The crystalline energy (E_{ani}) comes from spin-orbit interaction between the local magnetization and the crystal electric field. It is minimized when the local magnetization is aligned with the easy axis of the crystal structure. Mathematically, it is expressed as:

$$E_{ani} = K \sin^2 \theta \quad (2.3),$$

where K is the anisotropy constant of the material, and θ is the angle between the local magnetization and the easy axis.

- Zeeman Energy

The Zeeman energy (E_{zeeman}) comes from the interaction between the local magnetization and the external applied field. It is minimized when the local magnetization points in the same direction as the external field ($\mathbf{H}_{ext}$). Mathematically, it can be expressed as:

$$E_{zeeman} = -\mathbf{M} \cdot \mathbf{H}_{ext} \quad (2.4),$$

- Demagnetization Energy

The demagnetization energy (E_d) comes from the interaction between the local magnetization and local stray field from the system itself. It is minimized when the magnetization configuration of the system creates a closed loop. Mathematically, it can be expressed as:

$$E_d = \frac{1}{2} \mathbf{H}_d \cdot \mathbf{M} \quad (2.5),$$

where $\mathbf{H}_d$ is the local stray field from the system, which is mostly referred as the demagnetizing field.

- DMI Energy

The DMI energy (E_{DMI}) arises from a three-site indirect exchange mechanism between two local magnetizations and a neighbour atom with a large spin-orbit coupling. It is minimized when the local magnetizations are aligned perpendicular to each other. Mathematically, it can be expressed as:

$$E_{DMI} = -\mathbf{D} \cdot (\mathbf{M}_1 \times \mathbf{M}_2) \quad (2.6),$$

where $\mathbf{M}_1$ and $\mathbf{M}_2$ are the vectors of the two local magnetizations while $\mathbf{D}$ is the DMI vector. $\mathbf{D}$ lies on the same plane with the line that joins $\mathbf{M}_1$ and $\mathbf{M}_2$, as shown in Fig. 2.2:

Figure 2.2 Schematic showing the presence of DMI on the interface between a ferromagnetic layer and a material with high spin orbit coupling.

2.2 Spin- Transfer Torque

The LLG equation discussed above does not take into account the contribution from spin-polarized current to the magnetization dynamics. When current is applied to the ferromagnetic layer, the conduction electrons will try to align their spins to the same direction as the local magnetizations. Thus, when local magnetization configuration is not uniform, there is a continuous transfer of angular momentum to the conduction electrons from local magnetizations so that the conduction electron spins can be aligned in the same direction as the local magnetization. Fig. 2.3 illustrates the angular momentum transfer phenomenon between the conduction electron and the local magnetization.

a)

Figure 2.3 Angular momentum transfer phenomenon between the conduction electron (yellow sphere with black arrow) and the local magnetizations (red arrows) within the ferromagnet. The conduction electron here is flown from left to right ($t_3 > t_2 > t_1$).

The torque that the local magnetizations receive from the spin-polarized electron due to the angular momentum transfer phenomenon is termed as Spin-Transfer Torque (STT), and its contribution to the magnetization dynamics can be represented by adding two extra terms to the LLG equation as follow [6-14]:

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M} \times \mathbf{H}_{eff}) - \frac{\lambda}{M_s}(\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{eff})) - (\mathbf{u} \cdot \nabla)\mathbf{M} + \frac{\beta}{M_s}\mathbf{M} \times (\mathbf{u} \cdot \nabla)\mathbf{M} \quad (2.7),$$

$$\mathbf{u} = -\frac{g\mu_B P}{2eM_s}\mathbf{J} \quad (2.8),$$

Here g , μ_B , and e are the Landè factor, Bohr magneton, and electron charge respectively, M_s is the saturation magnetization, P is the conduction electron spin polarization, and $\mathbf{J}$ is the current density vector. With the above formulism, the third term on the right hand side is called as the adiabatic STT, which represents the polarizing process of the conduction electron, while the last term corresponds to the non-adiabatic STT, which represents the contribution from the conduction electrons that deviate from the polarizing process of the angular momentum transfer phenomenon. Note that the change in the direction of the local magnetization in Fig. 2.3 has been exaggerated to assist in understanding the angular momentum transfer phenomenon; in practice, the torque that the local magnetization receives from individual conduction electron is very small, and it typically requires an applied current density in the order of $> 10^9$ A/m² for the STT to be significant.

2.3. DW configuration

It is possible to employ eq. 2.7 to obtain the equation of motion of a magnetic domain wall (DW) under the application of external field or current [141-143]. To do that, first we have to obtain the mathematical expression of the DW configuration in a nanowire to be used as an input. As discussed in Chapter 1, magnetic domain wall (DW) is created in a ferromagnetic material to minimize the total energy of the system. Therefore, it is possible to obtain the magnetization configuration of a DW by minimizing its energy function.

Under zero external fields, the magnetic moments within the ferromagnetic nanowire will point closer to the anisotropy easy axis, additionally, the exchange interaction will try to keep the magnetizations parallel to each other, which results in minimum demagnetization energy. The DW energy (w_{DW}) then becomes:

$$w_{DW} = \int \left(A(\nabla \mathbf{M})^2 + K \sin^2 \theta \right) dV \quad (2.9).$$

Variational calculus can then be applied to obtain the equation for the DW configuration within the nanowire. To do so, the function $\mathbf{M}$ can be simplified by taking into account several approximations. First, we assume that the magnetization component of the DW only varies along one axis. This approximation is allowed in our study because we are mainly investigating the movement of a DW along a nanowire, which is typically patterned in one direction. Secondly, we assume that the magnetization component of the system along the transverse direction is constant across the length of the nanowire. Function $\mathbf{M}$ can then be written as:

$$\mathbf{M} = M\hat{\mathbf{r}} + \theta(y)\hat{\boldsymbol{\theta}} + \varphi\hat{\boldsymbol{\phi}} \quad (2.10),$$

$$\frac{\partial \varphi}{\partial x} = \frac{\partial \varphi}{\partial y} = \frac{\partial \varphi}{\partial z} = 0 \quad (2.11),$$

The DW energy function can then be rewritten as:

$$w_{DW} = \int \left(A \left(\frac{\partial \theta}{\partial y} \right)^2 + K \sin^2 \theta \right) dV \quad (2.12),$$

Variational calculus applied on eq. 2.10 gives us:

$$2K \sin \theta \cos \theta = 2A \frac{d}{dy} \frac{\partial \theta}{\partial y} \quad (2.13),$$

which, after multiplication by $\frac{\partial \theta}{\partial y}$ on both side and integrating them, leads to:

$$K \sin^2 \theta = A \left(\frac{\partial \theta}{\partial y} \right)^2 \quad (2.14).$$

The exact DW configuration can then be obtained by performing a final integration:

$$\sqrt{A/K} \int \frac{d\theta}{\sin \theta} = \int dy \quad (2.15),$$

which gives us:

$$\theta(y,t) = 2 \arctan(\exp[\frac{y}{\Delta}]) \quad (2.16),$$

where $\Delta = (A/K)^{1/2}$.

Eq. 2.16 can be used to represent the magnetization configuration of a DW in both material with perpendicular magnetization anisotropy (PMA) or in-plane anisotropy. For PMA material, K represents the crystalline anisotropy constant of the material, and θ represents the angle between the magnetization and the z axis. For materials with in-plane anisotropy, K is related to demagnetization field, and θ represents the angle between the magnetization and the y axis. Fig. 2.4 shows a schematic of the DW configuration in both nanowires with PMA and in-plane anisotropy:

Figure 2.4 Schematic of the DW configuration in (a) PMA material and (b) material with in-plane anisotropy. Such DW configuration in materials with in-plane anisotropy is also known as the ‘transverse’ configuration.

2.4. Domain wall dynamics equation

The DW dynamics are obtained by making use of eq. 2.16 together with the modified LLG equation. First, the DW configuration functions are modified to include the DW position $q(t)$, we also allow the tilting angle, φ to change over time:

$$\theta(y, t) = 2 \arctan(\exp[\frac{y-q(t)}{\Delta}]) \quad (2.17),$$

$$\varphi = \psi(t) \quad (2.18).$$

Several useful relations can be derived from eq. 2.17:

$$\begin{aligned} \frac{\partial \theta}{\partial y} &= \frac{\sin \theta}{\Delta}, & \dot{\theta} &= -\dot{q} \frac{\sin \theta}{\Delta}, \\ \delta \theta &= -\left(\frac{\partial \theta}{\partial y}\right) dq = -\frac{\sin \theta}{\Delta} dq & \delta \varphi &= d\psi \end{aligned} \quad (2.19),$$

By representing the DW configuration with eq. 2.17, the energy functions of the system can be rewritten as:

$$w_{EX} = A(\nabla M)^2 = A\left(\frac{\partial \theta}{\partial y}\right)^2 = A \frac{\sin^2 \theta}{\Delta^2} = \frac{A}{\Delta^2} \sec^2 h^2 \left(\frac{y-q}{\Delta} \right) \quad (2.20),$$

$$w_{DEMAC} = \frac{1}{2} H_d \square M = \frac{1}{2} M^2 N_z \sin^2 \theta \sin^2 \varphi = \frac{1}{2} M^2 N_z \sin^2 \varphi \sec^2 h^2 \left(\frac{y-q}{\Delta} \right) \quad (2.21),$$

$$w_{ANI} = K \sin^2 \theta = K \sec^2 h^2 \left(\frac{y-q}{\Delta} \right) \quad (2.22),$$

$$\begin{aligned} w_{ZEEMAN} &= -M \square H_{EXT} = -MH_z \cos \theta - MH_y \sin \theta \sin \varphi - MH_x \sin \theta \cos \varphi \\ &= -MH_z \tanh \left(\frac{q-y}{\Delta} \right) - MH_y \sec h \left(\frac{y-q}{\Delta} \right) \sin \varphi - MH_x \sec h \left(\frac{y-q}{\Delta} \right) \cos \varphi \end{aligned} \quad (2.23),$$

With the total energy of the system (w_{total}) being the summation of all the energy terms:

$$w_{total} = w_{EX} + w_{DEMAC} + w_{ANI} + w_{ZEEMAN} \quad (2.24),$$

We then proceed to calculate the areal energy density σ , so that only the DW position (q) and the tilting angle (φ) are the only variables left.

$$\sigma = \int_{-\infty}^{\infty} w_{total} dy \quad (2.25).$$

This gives us:

$$\sigma = \frac{2A}{\Delta} + 2K\Delta + \Delta M^2 N_z \sin^2 \psi - \pi M \Delta (H_y \sin \psi + H_x \cos \psi) - 2qMH_z \quad (2.26),$$

with the partial derivatives being:

$$\frac{\partial \sigma}{\partial \psi} = 2\Delta M^2 N_z \sin \psi \cos \psi - \pi M \Delta (H_y \cos \psi - H_x \sin \psi) \quad (2.27),$$

$$\frac{\partial \sigma}{\partial q} = -2MH_z \quad (2.28).$$

We then proceed to find equivalent expressions for the left-hand sides of eq. 2.27 and eq. 2.28 from the LLG equation. In the spherical coordinate, the LLG equation can be written as:

$$\frac{\delta w}{\delta \theta} = \frac{M_s}{\gamma} \left(\dot{\phi} \sin \theta - \alpha \dot{\theta} - \beta u \frac{\partial \theta}{\partial y} \right) \quad (2.29),$$

$$\frac{\delta w}{\delta \varphi} = -\frac{M_s}{\gamma} \left(\dot{\theta} + \alpha \dot{\phi} \sin \theta + u \frac{\partial \theta}{\partial y} \right) \quad (2.30).$$

While the areal energy density becomes:

$$d\sigma = \int_{-\infty}^{+\infty} \delta w dy = \int \left(\left(\frac{\delta w}{\delta \theta} \right) \delta \theta + \sin \theta \left(\frac{\delta w}{\delta \varphi} \right) \delta \varphi \right) dy \quad (2.31),$$

$$d\sigma = -\frac{2M_s}{\gamma} \left(\frac{\alpha \dot{q}}{\Delta} + \dot{\psi} - \frac{\beta u}{\Delta} \right) dq + \frac{2M_s}{\gamma} (\dot{q} - \alpha \dot{\psi} \Delta - u) d\psi \quad (2.32),$$

which gives us

$$\frac{\partial \sigma}{\partial q} = -\frac{2M_s}{\gamma} \left(\frac{\alpha \dot{q}}{\Delta} + \dot{\psi} - \frac{\beta u}{\Delta} \right) \quad (2.33),$$

$$\frac{\partial \sigma}{\partial \psi} = \frac{2M_s}{\gamma} (\dot{q} - \alpha \dot{\psi} \Delta - u) \quad (2.34),$$

Which can be rearranged into:

$$(1 + \alpha^2) \dot{q} = -\frac{\gamma}{2M_s} \left[\alpha \Delta \left(\frac{\partial \sigma}{\partial q} \right) - \left(\frac{\partial \sigma}{\partial \psi} \right) \right] + (1 + \alpha \beta) u \quad (2.35),$$

$$(1 + \alpha^2) \dot{\psi} = -\frac{\gamma}{2M_s} \left[\left(\frac{\partial \sigma}{\partial q} \right) + \frac{\alpha}{\Delta} \left(\frac{\partial \sigma}{\partial \psi} \right) \right] + \left(\frac{\beta - \alpha}{\Delta} \right) u \quad (2.36),$$

Finally, after using eq 27 and 28 to replace the energy terms in eq. 2.35 and 2.36, we obtain the equation of motion for the DW:

$$(1 + \alpha^2) \frac{dq}{dt} = \gamma \Delta \left[\alpha H_z - \frac{\pi}{2} (H_y \cos \psi - H_x \sin \psi) \right] + \frac{\gamma}{2} \Delta M N_z \sin 2\psi + (1 + \alpha \beta) u \quad (2.38),$$

$$(1 + \alpha^2) \dot{\psi} = \gamma \left[H_z + \alpha \frac{\pi}{2} (H_y \cos \psi - H_x \sin \psi) \right] - \frac{\gamma}{2} \alpha M N_z \sin 2\psi + \left(\frac{\beta - \alpha}{\Delta} \right) u \quad (2.39),$$

Where:

- dq/dt represents the speed of DW
- $d\psi/dt$ represents the rate of change of the tilting angle
- H_z, H_y, H_x represent the field applied in the corresponding axis
- N represents the demag term
- u represent the spin-transfer contribution

The above equation of motions are derived for PMA DWs. For DWs in nanowires with in-plane anisotropy, the magnetic fields are interchanged:

$$\frac{dq}{dt}_{PMA} (H_z, H_y, H_x) \leftrightarrow \frac{dq}{dt}_{in-plane} (H_y, H_z, H_x) \quad (2.40),$$

Under zero external field, the DW dynamic equations are reduced to:

$$(1 + \alpha^2) \frac{dq}{dt} = \frac{\gamma}{2} \Delta M N_z \sin 2\psi + (1 + \alpha\beta)u \quad (2.41),$$

$$(1 + \alpha^2) \dot{\psi} = -\frac{\gamma}{2} \alpha M N_z \sin 2\psi + \left(\frac{\beta - \alpha}{\Delta} \right) u \quad (2.42),$$

The effect of the STT to the DW dynamics can then be summarized into two: the first is to propagate the DW along the direction of the conduction electron flow, and the second is to rotate the DW along the ψ direction.

The DW motions are then simulated by via Object Oriented MicroMagnetic Framework (OOMMF), with the material parameters for the simulations set to correspond to Permalloy:

- Exchange energy constant (A) = 13×10^{-12} J/m and
- Saturation magnetization (M_s) = 860 kA/m.
- Nanowire width = 100 nm
- Nanowire thickness = 10 nm
- Gilbert damping constant, $\alpha = 0.1$
- Nonadiabatic constant, $\beta = 0.4$
- Simulation mesh size = 5 nm x 5 nm x 5 nm

The nanowire dimension was chosen to be 100 nm x 10 nm to ensure that the DW transverse shape is stable. To generate a DW in the micromagnetic simulation, we set the magnetization components as:

$$\triangleright m_x = e^{\tanh\left(\frac{x-q}{\Delta}\right)}$$

$$\triangleright m_y = -0.1$$

$$\triangleright m_z = 0$$

To generate a DW in PMA nanowires, we simply change the input order: $m_x \rightarrow m_z$. The above setting will generate a DW in materials with in-plane anisotropy as shown in Fig 2.5, which will eventually come to a stable equilibrium state.

Figure 2.5 Snapshots of the DW generation in the micromagnetic simulations.

The stable state of the DW is the one that is used in the subsequent studies and investigations. To change the orientation of the magnetizations within the DW, it can be done by changing the sign of m_y .

To measure the speed, we calculate the change in the overall magnetization value of the nanowire, as shown in Fig. 2.6

Fig. 2.6 The domain wall position as a function of time for current density of $5 \times 10^{11} \text{ A/m}^2$

Fig. 2.7 shows the average speed of the DW as a function of the applied current density. The result shows that the dynamics of transverse DWs can be divided into two regions; the low current density region and the high current density region. At low current density, the rotational torque is suppressed by the demag term and the DW tilting angle becomes fixed (ψ_f):

$$\frac{\gamma}{2} \alpha M N_z \sin 2\psi_f = \left(\frac{\beta - \alpha}{\Delta} \right) u \quad (2.43).$$

Substituting eq. 2.43 to eq. 2.41 gives us:

$$v = \frac{\beta}{\alpha} u \quad (2.44),$$

where v is the speed of the DW at low current density.

Figure 2.7. The speed of a single DW as a function of applied current density obtained by OOMMF using the above mentioned parameters. The red line is used to show the onset of the high current density region, i.e. the Walker breakdown current density limit.

The general DW behaviour for various values of α and β is shown in Fig. 2.8 [7]. The results show that for various α and β , eq. 2.44 is generally followed, while for $\beta=0$, the DW shows an absence of movement in the low current density region. Additionally, it has also been shown that the two constants, α and β , are physically not related to each other; α is related to the magnetization evolution that takes into account the equilibrium local spin density at each moment in time, while β is included in the magnetization

dynamics equation to take into account the contribution of the non-equilibrium spin density [7].

Figure 2.8 Steady velocity computed for a transverse domain wall by micromagnetics in a $120 \times 5\text{nm}^2$ wire as a function of the velocity u representing the spin-polarized current density, with the relative weight (β) of the exchange field term as a parameter. Open symbols denote vortices nucleation. The shaded area indicates the available experimental range for u .

In the high current density region, the average speed of the DW drops appreciably compared to the low current density region for $\beta \neq 0$. Here, the rotational torque is much stronger than the demagnetization term, and therefore the DW rotates continuously in the ψ direction. However, as the DW rotates, it has to adjust its shape continuously to follow the shape of the nanowire, which results in a cyclic breakdown in the DW shape and a drastic drop in the average velocity, as shown in Fig. 2.9. The phenomenon is also known as the Walker breakdown phenomenon.

Figure 2.9 Plot showing the position of a current-driven DW together with the change in the DW magnetization component (y axis) as functions of time under the application of high current density. Inset is the corresponding DW shape.

The simulation result shows that in one cycle of rotation, the DW moves forward twice and moves backward twice. The DW moves forward when the transverse component of the DW ($\mathbf{M}_x + \mathbf{M}_y$) is mostly pointing to the side of the nanowire. Conversely, the DW moves backward when the transverse component of the DW is mostly pointing perpendicular to the nanowire. The result also shows that the DW undergoes a shape breakdown during the backward motion, which is identified by the nucleation of anti-vortex within the DW. For $\beta=0$, the simulation results show that the DW is able to move with an increased speed as a function of the applied current density, as shown in Fig. 2.6 [7].

2.5 Skyrmion dynamics equation

As compared to the DW, a skyrmion is not restricted to move to follow the nanowire shape. The skyrmion dynamics can then be described by the Thiele's equation, which was first used to describe the dynamics of magnetic bubbles [123]. The Thiele equation can be derived from the LLG equation by mapping it to the translational mode [90]. The Thiele equation reads:

$$\mathbf{G} \times (\mathbf{v}_s - \mathbf{v}_d) + D(\beta \mathbf{v}_s - \alpha \mathbf{v}_d) + \mathbf{F}_{pin} = 0 \quad (2.45),$$

where $\mathbf{v}_s$ is the velocity of the conduction electron and $\mathbf{v}_d$ is the velocity of the skyrmion.

The gyromagnetic coupling vector $\mathbf{G} = (0, 0, G)$ is given by:

$$G = \int_{unitcell} d^2r \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) \cdot \mathbf{m} = 4\pi \quad (2.46).$$

While the dissipative force tensor D is given by:

$$D_{ij} = \int_{unitcell} d^2r \partial_i \mathbf{m} \cdot \partial_j \mathbf{m} = \begin{cases} D & \text{for } (i, j) = (x, x) \text{ and } (y, y) \\ 0 & \text{otherwise} \end{cases} \quad (2.47).$$

The first term in the Thiele equation represents the Magnus force, i.e. the overall force that the skyrmion receives from the precession of the magnetizations. The second term in the Thiele equation represents the dissipative force from the LLG. The third term represents the random pinning force from impurities.

In the absence of impurities ($\mathbf{F}_{pin} = 0$), the skyrmion speed can be expressed as:

$$\mathbf{v}_d = \mathbf{v}_{\parallel} + \mathbf{v}_{\perp} \quad (2.48),$$

$$\mathbf{v}_{\parallel} = \left(\frac{\beta}{\alpha} + \frac{\alpha - \beta}{\alpha^3 (D/G)^2 + \alpha} \right) \mathbf{v}_s \quad (2.49),$$

$$\mathbf{v}_{\perp} = \left(\frac{\alpha - \beta}{\alpha^2 (D/G)^2 + 1} \right) (\hat{z} \times \mathbf{v}_s) \quad (2.50).$$

For the skyrmion injection, we use the following input for the magnetization configuration:

$$\triangleright m_z = 2P \left(e^{\frac{r^2}{w^2}} - 0.5 \right)$$

$$\triangleright m_x = \frac{x}{r} (1 - |m_z|)$$

$$\triangleright m_y = \frac{y}{r} (1 - |m_z|)$$

Similarly, the skyrmion will eventually come to a stable equilibrium state. To calculate the speed, we measure how far the skyrmion has moved in the nanowire over a specific period in time.

To improve the precision of the simulation results, we chose the mesh size that is not bigger than the exchange length of the materials that are considered, which is approximately 5 nm [135].

Chapter 3

Dynamics of coupled domain wall in nanowires with in-plane anisotropy

3.1. Chapter Introduction

The basic operating mechanism of DW-based magnetic devices lies on the controlled motion of the DWs within the ferromagnetic structure. However, high data density design implies that multiple DWs within multiple nanowires have to be placed very close to each other. Hence it is important to investigate how the dynamics of the DWs are affected by the inter-wire and the intra-wire interactions between the DWs. In this chapter, the dynamics of coupled magnetic domain walls (DWs) in a multi-nanowire system with in-plane anisotropy is investigated. The results show that in a system with multiple parallel nanowires, the DWs are coupled to each other through their stray field interactions [23]. When spin-polarized current is applied to any of the nanowires, the coupled DWs are found to move together at the same time. Here, the remote driving of the DW in the current-free nanowire is accommodated by its magnetostatic coupling with the current-driven DW. As the current density is increased, the coupling is shown to be strong enough to induce DW chirality reversal in the current-free nanowire. Coupling breaking is observed at a critical current density which varies in a non-linear manner with respect to the interwire spacing. When current is applied to both nanowires in the opposite direction, the coupled domain walls are shown to undergo oscillatory motion, and it is possible to deduce the coupled DW mass by analysing the oscillatory motion [24].

The intra-wire interaction between multiple DWs in a two-nanowire system is also investigated. The intra-wire interaction is encountered when a remote-driven DW collides with another set of DWs of similar topological structure within the current-free nanowire. The intra-wire interaction between the DWs is repulsive in nature when the two DWs have identical topological structure, and it allows for multiple DWs in the current-free nanowire to move together with the remote-driven DW. By analyzing the structure of the DWs, the repulsive force is found to be related to the exchange energy contribution in the magnetization dynamics [25].

3.2. Domain wall coupling

We first start by investigating the stray field around a transverse DW in nanowires with in-plane magnetization anisotropy. As discussed in Chapter 2, the DWs in thin and narrow nanowires with in-plane magnetization anisotropy are stabilized in a transverse configuration to minimize the energy of the system [18-19]. However, due to the significant size of the DW, magnetic charges are then accumulated at the sides of the DW, which leads to the emergence of stray magnetic fields around the DW [26-45]. Fig. 3.1 shows the stray magnetic field 100 nm away from a Head-to-Head transverse DW, as calculated by COMSOL [78]. COMSOL is a general-purpose software platform, based on advanced numerical methods for modelling and simulating physics-based problems

As can be seen in Fig. 3.1, the magnitude of the stray magnetic field in the x direction is in the order of ~ 30 Oe, which is strong enough to move a DW in an adjacent nanowire. The distribution and the direction of the stray magnetic field are determined by the chirality and the type of the DW. Fig. 3.2 shows the 4 types of transverse DW that can be found in the nanowires with their respective magnetic charge representation.

In general, a DW can be represented as a pair of magnetic charges, i.e. a dipole, with the magnetic charge at the base of the triangular shape to be approximately ~3 times stronger than the magnetic charge at the peak of the DW. Here, the magnetic charge is calculated as $Q = \int_V \rho_M dV$ to take into account the magnetization divergence within the nanowire [29].

Figure 3.1 The stray field magnetic field created by a DW. The measurement is taken 100 nm away from both sides of the DW. (a) The flux density of the stray magnetic field in x direction, as measured near to the base of the triangular shape of the DW. The maximum magnitude of the flux density is approximately 3.2 mT. (b) The flux density of the stray magnetic field in y direction, as measured near to the base of the triangular shape of the DW. The maximum magnitude of the flux density is approximately 9

mT . (c) The flux density of the stray magnetic field in y direction, as measured near to the apex of the triangular shape of the DW. The maximum magnitude of the flux density is approximately $1.4 mT$. (d) The flux density of the stray magnetic field in x direction, as measured near to the apex of the triangular shape of the DW. The maximum magnitude of the flux density is approximately $0.4 mT$.

Figure 3.2. Magnetic charge representations of transverse DWs. Up Head-to-Head (UHH) DW represented by a strong positive magnetic charge at the base of the triangular shape and a weak negative magnetic charge at the apex of the triangular shape. The same can be found for Down Head-to-Head (DHH) DW, only now we reverse the position of the base and the apex. For Up Tail-to-Tail (UTT) DW, the prominent charge is the negative magnetic charge, which can be found at the base of the triangular shape. The same can be found for Down Tail-to-Tail (DTT) DW, only now we reverse the position of the base and the apex.

By using the dipole representation, we can see that Head-to-Head (HH) DWs are always attracted to Tail-to-Tail (TT) DWs. This is because HH DW has a strong positive charge while TT DW has a strong negative charge [131]. In a multi-nanowire system, it

is then possible for a HH DW to attract a TT DW in an adjacent nanowire, and vice versa, via their magnetostatic interaction whose strength depends on the DW orientation. OOMMF micromagnetic simulations are then employed to investigate the dynamics of the DWs in such cases [77]. The material parameters for the simulations are set to correspond to Permalloy:

- Exchange energy constant (A) = 13×10^{-12} J/m and
- Saturation magnetization (M_s) = 860 kA/m.
- Nanowire width = 100 nm
- Nanowire thickness = 10 nm
- Gilbert damping constant, $\alpha = 0.1$
- Nonadiabatic constant, $\beta = 0.4$
- Simulation mesh size = 5 nm x 5 nm x 5 nm
- The separation between the nanowires is set to 100 nm.

Figure 3.3 Snapshots of micromagnetic simulations showing the remanent state of a two-DW system (top view). Top image: the initial state of the system, the two DWs are generated at a distance away from each other. Bottom image; the state of the system after approximately 1 ns of simulation time has passed.

Fig. 3.3 shows the snapshots of the micromagnetic simulations of the two-nanowire system (top view). Initially, a HH DW and a TT DW are generated at different position on their respective nanowire. However, after a brief period of time, the two DWs are found to move towards each other even without the application of external field. Thus, the movements of the two DWs here can be attributed to the magnetostatic interaction between them, and the two DWs can be considered to be coupled to each other.

As the magnetic charge distribution of a DW depends on its chirality, therefore we can expect the coupling strength between the HH DW and the TT DW to also depend on their chiralities. Shown in Fig. 3.4 are the values of the energy terms of the 8 combinations that are possible. The two combinations where the DWs have their dominant charges next to each other are shown to have the lowest demagnetization and exchange energies, giving them the lowest total energy. These two combinations are then the most stable combinations out of the 8 possible combinations. When the dominant charges are situated at the furthest sides of the two nanowires, the total energy of the system is the highest, which attributes them as the least stable combinations. The cases where only one of the DWs has its dominant charge facing the other DW are the intermediate states, which give them higher stability than the least stable combinations.

Figure 3.4. The demagnetization energy and the exchange energy of the 8 attraction cases. A and B are the combinations that have the lowest total energy, G and H are the combinations with highest total energy, C~F are the combinations with intermediate total energy value.

3.3. Remote domain wall driving

Spin-polarized current is then applied to move the nanowire with the TT DW. Fig. 3.5 shows that as the TT DW moves, the HH DW in the adjacent nanowire also moves in the same direction. Similar phenomenon is observed when spin-polarized current is applied only to the wire with the HH DW.

Figure 3.5. The simultaneous motions of the two DWs in CDWS, here current is only applied to the second nanowire

In this case, the TT DW moves in the same direction as the HH DW, which means that the TT DW is coupled to the HH DW and vice versa. Both cases reveal that coupling between the two DWs is strong enough to induce DW motion within nanowires where spin-polarized current is not applied. The two DWs system can then be considered as a coupled domain wall system (CDWS), and the DW in the current-free nanowire can be said to be remote-driven.

Shown in Fig. 3.6 is the displacement of the CDWS as a function of time for various current densities. For current densities $J \leq J_a$ where $J_a = 2.755 \times 10^{12} \text{ A/m}^2$, the CDWS move with a constant speed that along the nanowire. In this range of applied current, the speed of the CDWS is found to depend linearly on the applied current density.

Figure 3.6. The displacement of the CDWS as a function of time for various current density values. Inset is the remanent state of the CDWS.

For current density higher than J_a , the CDWS accelerates and decelerates in a periodic behaviour. Here in Fig. 3.6 we show the displacement of the CDWS as a function time for $J = 3.179 \times 10^{12} \text{ A/m}^2$ and $J = 3.391 \times 10^{12} \text{ A/m}^2$. The average speeds are 163.21 m/s and 118.73 m/s respectively. The results show that when the current density is increased beyond J_a , the average speed of the CDWS drops drastically. Thus J_a can then be said to be the Walker breakdown limit of the CDWS. It is higher than the Walker breakdown limit of a single nanowire which in our simulation was found to be $J_b = 1.696 \times 10^{12} \text{ A/m}^2$. The shift in the Walker breakdown limit of the CDWS implies that the magnetostatic coupling between the two DWs have increased their stability.

Figure 3.7. The normalized transverse component of DW the magnetization as functions of time for a CDWS and a single nanowire system. The applied current density is $J = 3.179 \times 10^{12} \text{ A/m}^2$

Shown in Fig. 3.7 is the transverse component of the magnetization of a CDWS and a single DW as a function of time. The applied current density is $J = 3.179 \times 10^{12} \text{ A/m}^2$, which is well above Walker breakdown current density limit for both cases. The maximas and the minimas of the graph represent the times where transverse DWs are observed. The increase and decrease represent the chirality flipping of the DWs. The timeframe where a DW retains its transverse shape in CDWS is found to be extended compared to the single nanowire case, which further shows the increased stability of the DW in the CDWS.

Figure 3.8. The speed of CDWS and a single DW as a function of applied current density. The dimension of the single nanowire system is equal to the nanowires in the CDWS.

However, as shown in Fig. 3.8, below their respective Walker breakdown current density limit, a CDWS move through the nanowires with speed approximately a third of what a single DW can achieve in a single nanowire. In CDWs, the current-driven DW has to reduce its propagation speed as it is dragging along the remote-driven DW.

3.4. Spring-like Interaction of Coupled Domain Walls

To understand the characteristics of the coupling in the CDWS, spin polarized current is applied to the TTDW along $+x$ direction and to the HH DW along $-x$ direction.

The applied current causes the two DWs to move in the opposite direction to each other, while the coupling between the two DWs acts against the applied current by attracting the two DWs to each other. The resulting motion of the DWs is a result of the competition between the two forces. Shown in Figure 3.9 is the separation between the two DWs along the horizontal direction as a function of simulation time. The distance between the two DWs increases and decreases until it reaches a certain equilibrium position. At any time, the velocities of the two DWs are equal in magnitude but opposite in direction. Mathematically, this can be described by a damped oscillation equation:

$$x(t) = Ae^{-\zeta\omega_0 t} \sin\left(\sqrt{1-\zeta^2}\omega_0 t\right) \quad (3.1)$$

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2 x = \frac{F_s(J)}{m} \quad (3.2)$$

with x being the separation between the two DWs, ω_0 the natural frequency, ζ the damped oscillation's damping parameter, which is different from the Gilbert damping parameter α . Here we use the concept of DW mass [16-17] to represent the DWs. $F_s(J)$ is the force exerted by the spin-polarized current to the DWs. In the 1-D model, the force exerted by a spin-polarized current to a DW is a linear function of current density [7]

$$F(J) = \gamma J \quad (3.3),$$

where γ is the constant relating the current density to the magnitude of the force. Fig. 3.9 shows that after 10 ns, the oscillation amplitude can be considered negligible and the system can be said to reach equilibrium.

Figure 3.9 Separation between the two DWs as a function of simulation time. Inset shows the directions of the applied current on both nanowires. The magnitude of the applied current is equal at anytime

The two DWs stop at equilibrium and are displaced by an equal distance from the initial position. Thus by setting $\ddot{x} = 0$ and $\dot{x} = 0$ in equation (4) gives us

$$x_f = \frac{\gamma}{\omega_0^2 m} J \quad (3.4).$$

The final separation (x_f) between the two DWs increases linearly with respect to linear increase of current density as shown in Fig. 3.10. The natural frequency (ω_0) can be approximated to be independent of the current density. In equilibrium, the force from the spin-polarized current (F_s) is balanced by the attraction force from the coupling (F_c)

$$F_c = F_s \quad (3.5).$$

Figure 3.10. The final separation between the two DWs as a function of current density.

Thus we have the interaction force to be a linear function of the separation between the DWs:

$$F_c = m\omega_0^2 x \quad (3.6).$$

The CDWS can then be modelled as two masses connected by a spring.

The value of the spring constant will give us more understanding to the coupling between the two DWs, and also give us the information on the ‘reduced mass’ of the CDWS. To obtain the spring constant, we look at how the energy of the system evolves.

Figure 3.11. The demagnetization energy of the system as a function of the final separation between the DWs in the equilibrium states.

The total energy of the system is a sum of its demagnetization energy and exchange energy. The exchange energy represents the shape of the DWs and in this case where current is applied to both of the nanowire, the shapes of the two DWs remain constant thus giving no change to the total energy. The evolution of the total energy of the system comes mainly from the evolution of the demagnetization energy. Fitting the demagnetization energy as a function of the equilibrium positions obtained in Fig. 3.11 to a quadratic function will give us the spring constant of the system.

Figure 3.12 The period of the oscillation and the spring constant of the CDWS as a function of interwire spacing

Shown in Fig. 3.12 are the calculated spring constant and the oscillation period of the CDWS as a function of the interwire spacing. The spring constant is found to be decreasing as the distance between the nanowires is increased. This shows that the coupling between the two DWs is weaker for higher interwire spacing. The oscillation period increases as the distance is increased. In a simple harmonic motion, the natural frequency is related to the mass by

$$\omega_0^2 = \frac{k}{m} \quad (3.7).$$

Figure 3.13. The mass of the DWs in CDWS as a function of interwire spacing.

We use $\omega_0 = 2\pi/T$ to relate the natural frequency to the oscillation period, thus we have

$$m = \frac{kT^2}{4\pi^2} \quad (3.8).$$

The obtained mass shown in Fig. 3.13 is of the order 10^{-24} kg, which is in a good agreement with the values reported by others [16, 17].

In the case where current is applied to both nanowires but in opposite direction as being discussed here, coupling between the two DWs is not observed when the current density is increased beyond a certain critical value. The coupling is broken and the two DWs move irrespective of each other. Shown in Fig. 3.14 is the critical current density as a function of interwire spacing. The critical current density decreases

proportionally to the square root of the distance between the wires. This shows that the coupling is weaker for large interwire spacing.

Figure 3.14. The critical current density as a function of interwire spacing in CDWS.

The fitted line is proportional to $\frac{1}{\sqrt{d}}$ where d is the interwire spacing. Inset is the direction of the applied current in both of the nanowires

3.5. Multiple remote driving via exchange interaction

Here we investigate the intra-wire interactions between multiple transverse DWs in a two-nanowire system by micromagnetic simulations. The intra-wire interaction is encountered when a remote-driven DW collides with multiple DWs of similar topological structure within the current-free nanowire. As discussed previously, HH DW can be considered as a positive magnetic charge while TT DW can be considered

as negative magnetic charge. Therefore when a HH DW and a TT DW are present in a single nanowire, the two DWs are attracted to each other by their opposite magnetic charge, where they eventually collide. If the two DWs have the opposite topological structure, this attraction is unopposed which leads to DW annihilations, as shown in Fig. 3.15.

Figure 3.15 Schematic showing the winding numbers of transverse DWs. In the case where they have the same winding numbers, the two DWs are compressed upon collision.

However, when the two DWs have similar topological structure, the two DWs are compressed upon collision and the magnetostatic interaction is stabilized by the exchange repulsion force from their topological defects interaction [25, 46-51]. Here, the topological structure of the DWs can be identified by using the winding numbers. For a transverse DW, a $+1/2$ winding number can be assigned to the base of its triangular shape, and a $-1/2$ winding number can be assigned to the peak of its triangular shape.

Figure 3.16 The magnetization component in the nanowire direction of an uncompressed DW (DW_λ) and a compressed DW (DW_δ). Inset is the illustration of the compressed DWs.

Fig. 3. 16. shows a diagram of the magnetization configurations of two DWs with similar winding numbers. The calculated half width, both the uncompressed (λ_{rem}) and the compressed (δ_{rem}) in remanance are shown in the inset of Fig.3.17.

Figure 3.17 The exchange energy of unperturbed (E_λ) and compressed (E_δ) DWs as a function of wire width. Inset is the half-width of the unperturbed DW and the compressed DW.

To obtain the repulsion force, we find the derivative of the exchange energy with respect to the DW compression, δ . The DWs are compressed at the sides that are facing each other ($\delta < \lambda$). The compression is the physical representation of the repulsion force that they exert to each other. The magnetization components ($\mathbf{M}$) of a relaxed DW can be approximated as:

$$\overline{\mathbf{M}} = M_s (\cos \theta \hat{\mathbf{e}}_x + \sin \theta \hat{\mathbf{e}}_y) \quad (1),$$

$$\cos \theta = -\tanh\left(\frac{l}{\lambda}\right) \quad (2),$$

for ($0 < l < 2\lambda$), with 2λ as the full width of a transverse DW [8]. $M_s = 860,000$ A/m is the saturation magnetization of the permalloy. $\hat{\mathbf{e}}_x$ and $\hat{\mathbf{e}}_y$ are the unit vectors in the x and y directions, respectively. The *cosine* function is equal to a negative *tanh* function for

HH DW and positive function for TT DW. The exchange energy (E_{exc}) of a single DW is therefore equal to:

$$E_{exc} = Awt \int_{-\lambda}^{+\lambda} |\nabla \mathbf{M}|^2 dl \approx 2 \frac{Awt}{\lambda} \quad (3),$$

with w and t as the width and the thickness of the nanowire, respectively. $A = 13 \times 10^{-12}$ J/m is the exchange interaction constant of the permalloy. The expression shows that the exchange energy increases rapidly as the half-width of the transverse DW (λ) is reduced. The energy calculation above is applicable when the DW has width of 2λ ; when the domain wall is compressed, the energy of each side of the DW is calculated separately.

$$E_{exc} = E_{exc_left} + E_{exc_right} = \frac{Awt}{\delta_{left}} + \frac{Awt}{\delta_{right}} \quad (4).$$

Where E_{exc_left} and E_{exc_right} are the exchange energy of the left and the right side of the DW, respectively. δ_{left} and δ_{right} are the half-width of each side of the DW.

The increased exchange energy of the compressed DWs as compared to the relaxed DW can be considered as a result of the work done by the system to oppose the repulsion force, F_{exc} , that the two DWs exert to each other during the compression. To obtain the repulsion force, we find the derivative of the exchange energy with respect to the DW compression, δ :

$$F_{exc} = - \left. \frac{\partial E_{exc_left}}{\partial \lambda} \right|_{\lambda=\delta} = \frac{Awt}{\delta^2} \quad (5).$$

For nanowire with width $w = 100$ nm and thickness $t = 6$ nm, and DW compression of $\delta_c = 15.1$ nm, the equation gives $F_{exc} = 3.42 \times 10^{-11}$ kg.m/s². The simulation results here was obtained with mesh size of 5 nm x 5 nm x 5nm. The magnitude of the exchange

repulsion force is comparable to the driving force that is exerted by the spin-polarized and the intrawire magnetostatic force [8, 51]. Here we use thinner nanowire to ensure that the topological interaction is present.

In equilibrium, with zero external field and current, the repulsion force prevents the two DWs to annihilate under their own magnetostatic interaction. When current is not applied to the system, the exchange repulsion force (F_{ex}) is balanced by the intra-wire magnetostatic interaction (F_{mag}). The magnetostatic interaction when the two DWs are compressed can be calculated as:

$$F_{mag} = -\frac{1}{4\pi\mu_0} \frac{Q^2}{r^2} = -\frac{\mu_0}{4\pi} \frac{(CMwt(\tanh 1 + \tanh \frac{\delta}{\lambda}))^2}{(2\delta)^2} \quad (6).$$

Where μ_0 is the vacuum permeability constant, Q is the magnetic charge of the DW ($Q = -\mu_0 C \int_{-\lambda}^{\delta} \nabla \cdot \mathbf{M} dV$), r is the distance between the two DWs, and C is the proportionality constant which relate the DW shape and magnetic charge representation. The total force that acts on the DW as a function of the compression is then equal to:

$$F_{tot} = F_{exc} - F_{mag} = \frac{Awt}{(\delta)^2} - \frac{\mu_0}{16\pi} \frac{(CMwt(\tanh 1 + \tanh \frac{\delta}{\lambda}))^2}{(\delta)^2} \quad (7).$$

At equilibrium, with no current applied to the system, the exchange repulsion force is balanced with the magnetostatic attraction force ($F_{exc}(\delta_{remanance}) = F_{mag}(\delta_{remanance})$), which gives us:

$$C = \frac{4}{M(\tanh 1 + \tanh \frac{\delta_{rem}}{\lambda_{rem}})} \sqrt{\frac{\pi A}{\mu_0 wt}} \quad (8).$$

For nanowire of width $w = 100$ nm and $t = 6$ nm, the proportionality constant is equal to $C_{w=100,t=6} \approx 0.83$. The total force ($F_{\text{tot}} = F_{\text{exc}} + F_{\text{mag}}$) as a function of δ is plotted in Fig. 3.18. The graph shows that the total force that acts to the compressed DW is an attractive force until the half-widths of the two DWs reach the remanance value, whereby further compression will change the total force into a repulsive force.

Figure 3.18 The calculated total force (F_{tot}) as a function of DW compression length (δ) in a nanowire with width (w) of 100 nm and thickness (t) of 6 nm. Shown in the upper right inset is the schematic diagram of the simulation model. The two nanowires are separated 50 nm away.

To explore the role of the exchange repulsion force in DW dynamics, we consider the case of one-directional collision between two DWs. In the one-directional collision, a single DW is driven by a local driving force and is allowed to collide with another DW. To create a local driving force for the first DW, the remote driving

technique is employed. In the remote driving technique, another nanowire is placed next to our nanowire of interest. The first nanowire, or the nanowire of interest, is called as the active nanowire, while the second nanowire is called as the driver nanowire. A single DW, which shall be called as the driver DW, is generated in the driver nanowire to be used as a stray magnetic field generator. The driver DW and the first DW (DW1) in the active nanowire are coupled, and their dynamics are related to each other [23-25]. Shown in upper-right inset of Fig. 3.17 is the diagram of the simulations. The driver DW and DW1 are oriented such that the triangle bases face each other to assure the strongest magnetostatic attraction between them [29]. To ensure that the topological repulsion is present, the next DWs in the active nanowire need to have similar winding numbers placement, this is achieved by aligning all of the DWs in the active nanowire such that the bases of their triangular shapes face the driver nanowire. To inject the DWs into the nanowires, one possible solution is to fabricate an additional copper line to be used to create a fringing field [4]. Here, we incorporate a copper line that is aligned in an angle with the nanowire to create a fringing field directed at an oblique angle, as shown in Fig.3.19.

With the following design, the copper line will generate an Oersted field (H_g) at an angle (α) with respect to the nanowire. In our experiments [136], we found that the current density that is needed to perform this method is in the order of 10^{12} A/m², for both nanowires with in-plane and perpendicular anisotropy. The x component of the field (H_{gx}) serves to switch the magnetization of the active nanowire and create 2 DWs at the same time, while the y component (H_{gy}), will determine the chirality of the DWs. For instance, when a positive current is applied to the copper line, two DWs with Up chirality will be created. Similarly, when negative current is applied, two DWs with Down chirality will be created, as shown in Fig. 3.20.

Figure 3.19. Schematic of the DW-generator copper line (top view). Inset is the direction of the generated Oersted field when positive current is applied to the copper line.

Figure 3.20. The magnetization state of the nanowire after the application of (a) positive current and (b) negative current through the copper line.

After the DWs are created, we apply a field in the x direction to send DW_B into $+x$ direction and DW_A to the $-x$ direction. Eventually DW_A will be annihilated at the left end and we are left with a single DW with its base at the lower edge of the nanowire, as shown in Fig. 3.21.

Figure 3.21. The magnetization state of the nanowire after the application of (a) negative field and (b) positive field through the copper line.

Fig. 3.22 shows that the methods above can be repeated multiple times to create our DWs in the active nanowire.

Figure 3.22. The magnetization state of the active nanowire after multiple DW generations and injections processes.

Spin polarized current is then applied to the driver nanowire. As the driver DW moves, it interacts with DW1 in the active nanowire due to their stray magnetic field interaction. Due to the magnetostatic interaction, DW1 is remote-driven in the same direction as the driver DW. Eventually DW1 collides with the next DW (DW2) in the active nanowire. While the remote driving technique serves as the driving force for DW1 in the active nanowire, it is the exchange repulsion that continuously drives DW2 ahead. Here, DW2 and DW1 are compressed further beyond the half-width remanance ($\delta_{\text{remanance}}$); the interaction between DW1 and DW2 then becomes repulsive, which then serves to reduce the speed of DW1 and at the same time becomes the driving source for

DW2. Shown in Fig. 3.23 are the snapshots of the simulation to show the motion of the DWs. The width (w) and the thickness (t) of the nanowire is 100 nm and 6 nm, respectively. The current density applied to the driver DW is $J = 1.06 \times 10^{12}$ A/m². Initially (I), the driver DW and DW1 move together with an equal velocity of $v_I \approx 90$ m/s. For a short period of time (II) between $t \approx 6$ ns and $t \approx 8$ ns, the magnetostatic attraction between DW1 and DW2 propels DW1 to move forward, increasing its velocity to $v_2 \approx 130$ m/s. The attraction also pulls DW2 to move backward with a velocity of $v_3 \approx -40$ m/s. Shown in the inset is an illustration of the magnetostatic attraction between DWs of opposite magnetic charge. At $t \approx 8$ ns, the collision occurs and the topological repulsion starts to become prominent. The two DWs start to get compressed (III). As a result (IV), DW2 is pushed by DW1 to move in the same direction with a velocity of $v_6 \approx 50$ m/s. Shown in Fig. 3.24 is the velocities of DW1 and DW2 as a function of time. From $t \approx 8$ ns to $t \approx 14$ ns, the two DWs are pushing each other by gradually changing their shapes until they arrive at an equal velocity [23, 24]. Because the original velocity of the driver DW (v_I) is higher than the velocity of the two DWs in the active nanowire, the driver DW is able to move closer to DW1. However, this results in the increase in the magnetostatic interaction. The driver DW is decelerated, and in equilibrium it moves with the same velocity as the DWs in the active nanowire (v_6).

Figure 3.23 Multiple remote DW driving (a) Snapshot of the simulations showing the collision between DW1 and DW2 in the active nanowire.

Figure 3.24 The velocity of the DWs as a function of simulation time. A time span of approximately $t \approx 6$ ns is needed for the two DWs to reach equilibrium starting from the moment of impact.

It is possible to drive multiple DWs in the active nanowire by utilizing multiple collision processes. For instance, when a third DW (DW3) is present in the active nanowire, it will collide with DW2. The collision will compress both of the DWs, and as a result, DW3 will move together with DW1 and DW2. We have extended the method to drive up to 4 DWs in the active nanowire at the same time, the DW group moves with a velocity of $v \approx 30$ m/s under applied current density of $J = 1.06 \times 10^{12}$ A/m². In the equilibrium, all of the DWs move with equal velocity, with the driving force being balanced by the damping force from the energy dissipation.

Figure 3.25 The speed of the DW group as a function of total number of DWs that is being driven at the same time in the two-nanowire system.

Shown in Fig. 3.25 is the speed of the collective DWs as a function of the total number of DWs (N) in the system. The current density applied to the driver DW is $J = 1.06 \times 10^{12} \text{ A/m}^2$. The first point ($N = 1$) refers to the case where we are only driving the driver DW.

As described before, the motions of DW2 and the subsequent DWs are the results of collisions with the remote-driven DW. However, as can be seen from Fig. 3.23, the process is not instantaneous; approximately $t \approx 6 \text{ ns}$ is needed for the DWs that are involved in the collisions to reach an equilibrium velocity. Therefore, it is possible for a driver DW that is moving with a high velocity to 'bypass' a group of DWs in the active nanowire when there is not enough time for the DW group to reach equilibrium. Shown in Fig. 3.26 is the snapshots of the simulation with current density of $4.24 \times 10^{12} \text{ A/m}^2$ applied to the driver nanowire. We expect no structural breakdown to occur within the

DWs as the Walker breakdown limit of the two-nanowire system is higher than that of the single nanowire system [23, 24].

Figure 3.26 The selective remote driving phenomenon. The driver DW is shown to bypass a group of DWs in the active nanowire. Due to the high velocity motion of the driver DW, it is able to move pass the DW group in the active nanowire. The applied current density is 4.24×10^{12} A/m².

DW1 and DW2 are initially placed very close to each other and far away from the driver DW. Due to the high propagation velocity of the driver DW, the interaction time between the driver DW and DW1 is very short. Consequently, there is not enough time to decelerate the driver DW and accelerate DW2 to reach equal velocity. As a result, the driver DW is able to move pass DW1 and DW2 and continues its motion without any coupling to the DWs in the active nanowire. In other words, by adjusting the applied current density to the driver nanowire, we will be able to control the coupling between the DWs in the active and the driver nanowire.

In short, we have shown that the DWs in a multi-nanowire system with in-plane anisotropy are coupled to each other through their stray magnetic field interaction. This makes it possible to design a memory device where the data is driven through remote coupling instead of by direct current injection to the data nanowire. This is beneficial for the data as we can expect virtually no heating in the data nanowire, which translates to a much better data retention. Additionally, the ability to break the coupling also grants extra flexibility in the design of the memory device. The ability to break the coupling also gives the device designer the option to ‘skip’ several data in the data nanowire, which might translates to faster device operation. However, the implementation of such option will require advanced algorithm to ensure that the coupling breaking does not skip the critical data, and development of such algorithm is beyond the scope of our work.

Chapter 4

Coupled domain wall dynamics in nanowires with perpendicular magnetization anisotropy

4.1 Chapter Introduction

As discussed previously in Chapter 2, the propagation of a DW is typically followed by a precession around the in-plane direction. The DW speed at any moment is thus related to the angle that the DW makes in the in-plane direction, as shown by eq. 2.38. At low current density, the DW is only rotated by less than 90° degree before the precession stops, while the DW speed increases linearly with respect to the applied current density, as shown by eq 2.43. and eq 2.44. However, at high current density, the DW maintains a constant precessions which results in the Walker breakdown phenomenon and overall drop in the average speed of the DW. In this chapter, we present a method to suppress the Walker breakdown phenomenon for DWs in Perpendicular Magnetization Anisotropy (PMA) nanowires by making use of a sandwich structure design, where two PMA nanowires that are separated by a spacer layer are stacked vertically. In such design, we show by micromagnetic simulation that the DWs are coupled to each other in the Néel configuration due to the magnetostatic coupling. We further show that under equal applied current density, the coupled DWs from the sandwich structure are able to move faster than individual DWs from a single PMA nanowire due to Walker breakdown suppression. By analyzing the magnetization configurations of the sandwich structure, we found that the magnetic domains in the

sandwich structure are curled which gives rise to an effective magnetic field that eventually suppresses the Walker breakdown.

The DW dynamics in the PMA sandwich structure are investigated by using the OOMMF [77] micromagnetic simulation program, with the addition of the spin-transfer torque term to the Landau-Lifshitz-Gilbert (LLG) equation. Fig. 4.1 shows the schematic diagram of the simulation model, whereby two PMA nanowires were stacked vertically with non-magnetic insulating spacer layers between them. The simulation parameters are: [63-65]

- Exchange energy constant (A) = 13×10^{-12} J/m and
- Saturation magnetization (M_s) = 6×10^5 kA/m.
- Magnetocrystalline anisotropy (K) = 4×10^5 J/m³
- Nanowire width = 40 nm
- Nanowire thickness = 6 nm
- Spacer layer thickness = 2 nm
- Gilbert damping constant, $\alpha = 0.1$
- Nonadiabatic constant, $\beta = 0.4$
- Simulation mesh size = 5 nm x 5 nm x 3 nm

Figure 4.1 Schematic of the simulation model that is employed to investigate the DW dynamics in a sandwich PMA structure.

4.2 Coupled Néel Domain Walls in Two-Nanowire System

First, the magnetization configurations of the DWs in the sandwich structure are investigated. Shown in Fig. 4.2 are the magnetization configurations of the two PMA nanowires as functions of their position along the y axis ($x = 5$ nm).

Figure 4.2 The magnetization configuration of the two-nanowire system (left-edge of nanowire, $x = 5$ nm). Insets are the schematics of the DW magnetization configuration at each nanowire (top view).

As shown in Fig. 4.2 and the respective insets (top view), the DWs in the two nanowire system are aligned along the y axis. In other words, the sandwich setup has forced the DWs to adopt the Néel configuration.

Figure 4.3 Schematic showing the magnetization of the domains at two cross-sections: before and after the DW

Furthermore, we also see that the two DWs are aligned in the opposite direction, which suggests that they are coupled to each other through their magnetostatic interaction. Fig. 4.3 shows the cross-sections of the magnetization configuration of the domains. The domains around the DWs are also found to be slightly curled towards the x axis ($M_x \approx \pm 3.6 \times 10^4$ A/m depending on the position) as shown in the cross-sections of the nanowires in Fig. 4.3 [75-76].

We proceed to investigate the dynamics of the two DWs under the application of current. Initially, a DW is generated in each nanowire at the same position with respect to the y axis. Spin-polarized current is then applied to the bottom PMA nanowire. Fig. 4.4 shows the snapshots of the simulations (side-view) for different applied current density. The DWs in the sandwich structure are found to behave differently depending on the applied current density.

Figure 4.4 Snapshots of the two-nanowire simulations for various range of applied current density (side-view). Current is applied to the bottom nanowire.

For low applied current density, the bottom DW is pinned directly below the upper DW, which implies that a local pinning site is induced within the bottom nanowire and vice versa. The intrinsic pinning that the coupled DW experiences in the sandwich system can be attributed to the additional demagnetization energy that the curled magnetic domains impose to the DW. As mentioned by Ono et al. [137], the demagnetization energy of the DW depends on its relative orientation with respect to the domains. And now, as the domains are curled in the sandwich structure, it creates a small demagnetizing field that becomes the source of the intrinsic pinning. The bottom DW remains stationary (pinned at the initial position) until the applied current density is increased to $J_a = 0.96 \times 10^{12} \text{ A/m}^2$, where it starts to propagate along the bottom PMA nanowire. However, the upper DW is also shown to be driven at the same instance in the same direction and speed as the bottom DW, even though there is no current applied to the upper PMA nanowire. The remote-driving of the upper DW in the absence of direct current shows that the two DWs are coupled to each other [23, 24].

The relation between the speed of the coupled DWs of the two-nanowire system and the applied current is shown in Fig. 4.5, the speed plot of a single DW is also included for comparison. As compared to the single nanowire system, the coupled DWs of the sandwich structure move with a noticeably higher speed. For instance, with applied current density of $J = 2.68 \times 10^{12} \text{ A/m}^2$, the coupled DWs move with a speed of $\approx 140 \text{ m/s}$, which is approximately 50% faster than the single DW.

Figure 4.5 Plot showing the relation between the speed of the coupled DWs and the applied current density. The speed of a single DW is included as comparison.

Increasing the applied current density beyond $J_b = 3.7 \times 10^{12} \text{ A/m}^2$ results in the breaking of the coupling, which allows the bottom DW to move independently from the upper DW. Similar results are obtained when current is applied to the upper nanowire.

The increased speed of the coupled DWs of the sandwich structure can be attributed to the suppression of the rotation of the tilting angle (φ) [7]. Due to curling domains, the magnetization of the nanowire can be written as:

$$\theta(y, t) = 2 \arctan(\exp[\frac{y - q(t)}{\Delta}]) \quad (4.1),$$

$$\varphi(t) = \psi(t) + \gamma(y) \quad (4.2),$$

$$\gamma(y) = 2C \tan^{-1}(\exp(\frac{(y - q(t))}{\Delta})) \quad (4.3),$$

where q is the position of the DW along the nanowire, and φ is the tilt angle of the nanowire. The second term in the tilt angle equation is the additional term that comes from the curling domains. C is a constant that represents the degree of curling ($C < 1$).

The energy of the system can then be calculated as:

$$\begin{aligned} w_{DEMAG} &= \frac{1}{2} H_d \bullet M = \frac{1}{2} M^2 N_z \sin^2 \theta \sin^2 \varphi \\ &\approx \frac{1}{2} M^2 N \sin^2 \theta (\sin^2 \psi + \sin^2 \gamma \cos^2 \psi) \end{aligned} \quad (4.4),$$

$$w_{TOT} = A \left(\frac{\partial \theta}{\partial y} \right)^2 + K \sin^2 \theta + \frac{1}{2} M^2 N \sin^2 \theta (\sin^2 \psi + \sin^2 \gamma \cos^2 \psi) \quad (4.5).$$

The derivatives of the surfaces energy then become:

$$\frac{\partial \sigma}{\partial \psi} = (MN - B) \Delta M \sin 2\psi, \quad \frac{\partial \sigma}{\partial q} = 0 \quad (4.6),$$

$$B = \frac{MN}{2\Delta} \int_{-\infty}^{\infty} \sin^2 \gamma \sin^2 \theta dy \quad (4.7).$$

If we consider the Néel state of the DW in the sandwich structure ($\psi = \pi/2 + \delta$), the coupled DW dynamics then becomes:

$$(1 + \alpha^2) \dot{q} = (B - MN) \frac{\gamma}{2} \Delta \sin 2\delta + (1 + \alpha\beta) u \quad (4.8),$$

$$(1 + \alpha^2) \dot{\delta} = (MN - B) \frac{\gamma}{2} \alpha \sin 2\delta + \left(\frac{\beta - \alpha}{\Delta} \right) u \quad (4.9).$$

Where q is the position of the DW along the nanowire, γ is the gyromagnetic constant, N is the demagnetization factor of the nanowire, and ψ is the tilting angle of the DW in

the sandwich structure. B is the contribution from the curled magnetic. It can be seen that the presence of the curled magnetic domains (B) suppresses the change in the tilting angle of the DW, as well as increases to the DW velocity (dq/dt). Here, the B constant is approximately equal to $\mu_0 H = 100$ mT.

Figure 4.6 The magnetization component of the DWs in the bottom and upper nanowires as functions of simulation time.

Fig. 4.6 shows the magnetizations of the upper and bottom nanowires of the sandwich structure as functions of simulation time with current $J = 2.68 \times 10^{12}$ A/m² applied to the bottom nanowire. The result shows that due to the application of current, the tilting angle of the coupled DW gradually changes as functions of time, as suggested by Eq 4.9.

Shown in Fig. 4.7 are magnetization components of the two DWs as a function of the applied current density. The result shows that the magnetizations of the two DWs move closer to the x axis for higher current density, which shows that the two DWs

gradually change from the Néel configuration to the Bloch configuration as the applied current density is increased. The coupling between the two DWs of the sandwich structure is broken when the upper DW has changed back to the Bloch configuration.

Figure 4.7 Plot showing the change in the tilting angle of each DW in the sandwich system.

We have performed additional simulations to investigate the dynamics of the DWs in the sandwich structure for different material parameters. Fig. 4.8 shows the DW dynamics for various values of saturation magnetization. The simulation results show that the coupled DW dynamics remain the same even with different values for the saturation magnetization. The change in the value of the saturation magnetization only affects the speed of the coupled DW upon the application of current.

Figure 4.8 The coupled DW position as a function of simulation time for different values of saturation magnetization.

Fig. 4.9 shows the DW position vs time plots of the coupled DWs in the sandwich structure for various values of the non-adiabatic constant.

The speed of the coupled DWs is shown to increase with increasing value for the non-adiabatic spin transfer constant. Fig. 4.10 shows the corresponding dynamics of the DW in the bottom nanowire. Fig. 4.9 and 4.10 show that the change in the nonadiabatic constant of the spin-transfer-torque equation results in the increase of the propagation speed and also in the tilting angle of the DW, which both agree with Eq 4.8 and Eq. 4.9.

Figure 4.9 The speed of the coupled DWs in the sandwich structure for different values of the non-adiabatic spin transfer constant.

Figure 4.10 The magnetization component along the x direction of the DW in the bottom nanowire. The vibration that is shown in the figure represents the actual DW vibration in the simulations, which shows how the current-driven DW does not immediately reaches equilibrium upon the application of current to the system.

The current that is needed to drive the coupled DWs in the sandwich structure depends on the local pinning that is induced by the curling magnetic domains. Fig. 4.11 (a) shows the change in the threshold current in sandwich structure with two nanowires with respect to the nanowire thickness. The results show that it is possible to lower the threshold current by making the PMA nanowire thinner.

Figure 4.11 (a) Threshold current as a function of nanowire thickness. The spacer layer is maintained at 2 nm. (b) The coupled DWs position as function of time for different sandwich structure width.

For instance, the threshold current is reduced to $J = 1.5 \times 10^{11} \text{ A/m}^2$ when the thickness of both the top and the bottom nanowires of the sandwich structure is reduced to 1 nm. In contrary, we also found that the coupled DW dynamics of the sandwich structure is relatively independent of the device width, as shown in Fig. 4.11 (b).

Figure 4.12 (a) Schematic of the simulation that is used to investigate the pinning in the sandwich structure. (b) The speed of the DWs in the bottom and the top of the nanowires with pinning sites as functions of time. (c) The magnetization component along the x direction of the DWs in the bottom and the top nanowires as functions of time. (d) The magnetization component of the DWs after the coupling breaking.

We have also performed additional simulations to investigate the effect of extrinsic pinning to the dynamics of the coupled DW in the sandwich structure. Two pinning sites are created in both nanowires of the sandwich structure as shown below in

Fig. 4.12 (a). Current is only applied to the bottom nanowire. The width (w_n) and the depth (d_n) of the pinning sites are 10 nm and 10 nm, respectively. The results show that the pinning sites affect the DWs in the top and bottom nanowires individually. Shown in above in Fig 4.12 (b) are the dynamics of the two DWs when a $J = 2.68 \times 10^{12}$ A/m² current is applied to the bottom nanowire. Initially the two DWs are able to move together with nearly a constant speed and a constant tilting angle up to $t = 15$ ns where they finally arrive at the pinning sites. The DW in the top nanowire is shown to be stopped by the pinning sites, while the DW in the bottom nanowire is shown to be able to continue its movement. Fig. 4.12 (c) shows that DWs start to change their behaviour after the coupling is broken. Fig 4.12 (d) shows that the DW in the top nanowire starts to come to equilibrium as it oscillates around the x axis with the natural frequency of the spin precession. At the same time, the bottom nanowire starts to rotate in the xy plane which signals that the DW has experienced the Walker breakdown due to the high applied current density.

4.3 Coupled Domain Walls in Three-Nanowire System

We proceed to investigate the coupling and the dynamics of the DWs in a sandwich structure with three PMA nanowires. As compared to the two-nanowire system, the behavior of the three-nanowire system is determined not only by the applied current density but also by which nanowire the current is applied to. When current is applied to the bottom nanowire, all three DWs of the three nanowires remains stationary (pinned), this is similar to the case of the two-nanowire system when low current density is applied. However, the three DWs remain stationary even under the application of high current density, as shown in the first image of Fig. 4.13 (side-view). The coupling is only broken when the applied current density is increased beyond $J_c = 8.03 \times 10^{12}$ A/m²

in which the bottom DW moves independently from the other DWs, as shown in the second image of Fig. 4.13.

Figure 4.13 Snapshots of the three-nanowire simulations for various range of applied current density (side-view). Current is applied to the bottom nanowire, the coupled DWs are pinned below a critical current density of $J_c = 8.03 \times 10^{12} \text{ A/m}^2$.

When current is applied to the middle nanowire, the behavior of the three-nanowire system becomes similar to that of the two-nanowire system; i.e. the three DWs are able to move as a group when the applied current density is increased beyond a critical value, as shown in the first image of Fig. 4.14. The critical current density that is needed to move the three DWs is $J_d = 0.4 \times 10^{12} \text{ A/m}^2$, which is lower than the minimum current density that is needed to move the two-DW group (J_a). During the motion, the three DWs move with a speed that is comparable to a single DW, as shown in Fig. 4.15 This result can be attributed to the uncurled domain of the middle nanowire, as explained latter. The coupling between the three DWs is again broken when the applied current density is increased beyond $J_e = 4.8 \times 10^{12} \text{ A/m}^2$, which allows the middle DW to move independently from the other DWs, as shown in the second image of Fig. 4.14.

Figure 4.14 The coupled DWs are able to move together when the current is applied to the middle nanowire and the current density is higher than $J_d = 0.4 \times 10^{12} \text{ A/m}^2$ but lower than $J_e = 4.8 \times 10^{12} \text{ A/m}^2$. The coupling is broken when the current that is applied to the middle nanowire has exceeded J_e .

The magnetization configurations of all three nanowires are shown in Fig. 4.16 (left-edge of nanowire, $x = 5 \text{ nm}$). Current is applied at the middle nanowire, and the magnetization configurations are taken at $t = 20 \text{ ns}$ after the start of the simulation. The applied current density is $J = 2.68 \times 10^{12} \text{ A/m}^2$. The DW in the middle nanowire behaves as if it is in a single nanowire system; as shown in the corresponding inset (top view).

Figure 4.15 Plot showing the relation between the speed of the coupled DWs and the applied current density.

The middle DW is free to rotate along the xy plane as it propagates within the middle nanowire. The behaviour of the middle DW can be attributed to the unique magnetization configuration of the middle nanowire. As shown in Fig. 15, the domains in the middle nanowire are aligned perfectly along the perpendicular z axis, while the domains of the top and bottom nanowires are slightly curled along the x axis similar to the top and bottom nanowires of the two-nanowire sandwich structure.

Figure 4.16 The magnetization configuration of the three-nanowire system (left-edge of nanowire, $x = 5 \text{ nm}$). Insets are the schematics of the DW magnetization configuration at each nanowire (top view).

The magnetization configuration of the middle nanowire then becomes similar to the magnetization configuration of a single nanowire system. This in turn allows the middle DW in the three-nanowire system to rotate in the xy plane and achieve a speed that is similar to a DW in a single nanowire system, as shown in Fig. 4.17.

Figure 4.17 Plots showing the dynamics of the components of the DWs in the three-nanowire sandwich structure.

4.4 Device designs

To ensure that the current only flows in one nanowire, the gold contact pads for the current injection should only touch one of the nanowires. This can be done by doing lithography twice: first is for the deposition of the insulator and the second is for the gold contact. The insulator and the gold are patterned in different position so that the gold is able to be connected with the upper nanowire without touching the bottom nanowire. Fig. 4.18 shows a schematic of the process:

Figure 4.18 (a) Initial sandwich structure after the ion milling, top view. (b) Resist deposition (c) Resist after ion milling. Two open areas are created for the insulator deposition. (d) The insulator is deposited and the resist is stripped. (e) Second round of lithography, another pair of open areas is created for the gold deposition (f) Gold pads deposition.

Figure 4.19. A design of the DW device with sandwich structure

With the above design, the second PMA nanowires at the bottom are isolated and left as a floating ground. With this design, the current is expected to flow only in the top layer of the sandwich structure, which mimics the results shown in our simulation. We have also performed additional simulations to investigate how thick the insulator layer at the middle can be made while still maintaining the coupling between the DWs. We found that the DWs can still be coupled with insulator layer of 10 nm, which are thick enough to ensure that there is no current that flows in the lower nanowire.

To generate the coupled DWs, current can be applied to the injection line:

The DWs at the ends of the domains are coupled to each other

Figure 4.20. (sideview) (a) Coupled DW injection process, current is passed through the DW Injection line. A magnetic field is then created around the DW injection line. (b) A new magnetic domain is created in each layers of the sandwich structure due to the magnetic field. The DWs at the sides of the magnetic domains are coupled to each other due to their own stray magnetic fields.

For the three-nanowire sandwich system, the above fabrication method can be extended to add the third nanowire, as shown below:

Figure 4.21. A design of the three-nanowire structure. The current is expected to flow only in the middle nanowire. The DW injector shall inject DWs into all the nanowires in similar fashion to the two-nanowire structure.

Chapter 5

Guiding current-induced skyrmion motion

5.1 Chapter Introduction

Magnetic skyrmion is a unique magnetization field configuration with integer topological number that has recently been observed in ultrathin ferromagnetic film that is interfaced with another material with strong spin-orbit coupling [82, 83]. The topological nature of the skyrmions allow them to circumvent around random pinning sites or impurities as they move within the magnetic layer, which makes them interesting as information carriers in memory devices [86, 87]. However, when the skyrmion is driven by a current, a Magnus force is generated which leads to the skyrmion moving away from the intended direction [88, 115]. The deflection poses a serious problem to the realization of skyrmion-based devices, as it leads to skyrmion annihilation at the film edges [89].

In this chapter, we investigate a method to prevent the skyrmion to be annihilated at the edges by introducing kerbs to the nanostructure to act as a pair of strong potential barriers. We also show that the speed of the current-driven skyrmion can be increased significantly by narrowing the space between the potential barriers and compressing the skyrmion at the same time. We later show that the increase in speed can be attributed to the increased effectiveness of the spin-transfer torque (STT) that the compressed skyrmion receives from the conduction electron.

5.2 Skyrmion motion on a wide plane

The investigation is carried out by performing micromagnetic simulations using Mumax [79, 127, and 128] and OOMMF [77]. First, we simulate the skyrmion dynamics on a wide plane to observe the skyrmion deflection due to the Magnus force. The material parameters that were chosen for the simulations correspond to that of Co/Pt multilayers [129], and the values are as follow: exchange stiffness = 1.5×10^{-11} J/m, saturation magnetization = 5.8×10^5 A/m, Gilbert damping = 0.1, non-adiabaticity of STT = 0.35, magnetocrystalline anisotropy constant = 6×10^5 J/m², spin polarization = 0.7. The ferromagnetic layer has thickness of 0.8 nm. DMI strength = 3×10^{-3} J/m³, which is strength needed to stabilize a skyrmion in Co/Pt simulation [129, 144].

Figure 5.1 The change in the propagation angle of the skyrmion as a function of the non-adiabatic constant. Top inset is a schematic of a setup for skyrmion driving with in-plane current. Bottom inset is a snapshot of a skyrmion driving simulation with in-plane current.

Inset of Fig. 5.1 (a) shows the movement of a skyrmion under the application of an in-plane current for various Gilbert damping (α) and non-adiabatic STT constants (β). In the in-plane driving case, the magnetization dynamics is expressed Eq. 2.7, and the skyrmion is driven mostly by the field-like torque from the STT [7].

The simulation results show that under the application of in-plane current, the skyrmion is deflected by the Magnus force and moves at an angle with respect to the conduction electron flow when $\alpha \neq \xi$ [128]. Fig. 5.1 shows the change in the propagation angle as a function of the non-adiabatic constant. The equation for the drift velocity of the skyrmion can be obtained by mapping the LLG equation onto the translational mode in the continuum limit, while assuming the rigidity of the spin textures during the skyrmion motion, as shown in eq.2.45 of Chapter 2. In the absence of impurities, the drift velocity of the skyrmion is expressed as:

$$\mathbf{v}_d = \mathbf{v}_\parallel + \mathbf{v}_\perp \quad (1),$$

$$\mathbf{v}_\parallel = \left(\frac{\beta}{\alpha} + \frac{\alpha - \beta}{\alpha^3 (D/G)^2 + \alpha} \right) \mathbf{v}_s \quad (2),$$

$$\mathbf{v}_\perp = \left(\frac{\alpha - \beta}{\alpha^2 (D/G)^2 + 1} \right) (\hat{z} \times \mathbf{v}_s) \quad (3),$$

where $\mathbf{v}_\parallel$ and $\mathbf{v}_\perp$ are the components of $\mathbf{v}$ parallel and transverse to $\mathbf{v}_s$, respectively. D and G represent the strength of the Magnus force and the dissipative force, respectively.

5.3 Skyrmion motion on a track

The micromagnetic simulations have shown that, when the skyrmion is driven by current, it deviates from the intended propagation direction after a short travelling distance due to the Magnus force. This implies that if the film is patterned into a track

shape, the skyrmion will be pushed to the edge by the Magnus force. The deflection towards the edge of the track eventually leads to skyrmion annihilation, as shown in Fig. 5.2.

Figure 5.2 Snapshot of the skyrmion driving simulation on a track with high applied current density which results in skyrmion annihilation. The conduction electrons flow from left to right, and the track has a width (w) of 60 nm.

The deviation of the skyrmion motion from the conduction electron flow also prevents reversible two-way operation, as the skyrmion tend to move to the opposite edge of the track when the applied current is reversed. One way to overcome the deflection is by applying low current density and using the default potential barrier from the edge magnetization to counter the Magnus force. As discussed in Chapter 2, the magnetizations of the nanowire are bent at the edges due to the lack of exchange energy

to balance the DMI. The bent magnetization then serves as a potential barrier to counter the Magnus force, up to applied current density in the order of $\sim 10^{11}$ A/m².

As discussed above, to prevent the skyrmion to be annihilated on a track, a strong potential barrier is needed at the edge to counter the Magnus force. As the Magnus force is proportional to the applied current density, the strength of the potential barrier then determines the maximum current density that can be applied as well as the maximum skyrmion speed. Fig. 5.3 shows the schematic of our proposed kerbed track that shall impose a stronger edge potential barrier on the skyrmion, thus allowing the skyrmion to be driven with higher speed by applying higher current density. The kerbed track is formed by creating a rectangular groove on the surface of the perpendicular magnetization anisotropy (PMA) track, leaving the two edges of the track with limited height and width. The skyrmion shall then be confined within the track by the two kerbs.

Figure 5.3 Schematic of the proposed kerbed skyrmion track.

Fig. 5.4 shows snapshots of the simulations of the current-driven skyrmion motion within the curbed track. As compared to the uncurbed track, the result shows that the skyrmion in the curbed track is able to be driven to the end of the track without being annihilated at the edge, owing to the strong edge potential barriers from the curbs.

The result also shows that the skyrmion size is compressed to fit the area that is not covered by the curbs. In the case when the skyrmion is surrounded by a curb of $s = 15$ nm wide on each side, the skyrmion diameter (d_s) is reduced to $d_s = w - 2s = (60 - 2 \times 15) = 30$ nm. With respect to the skyrmion confinement, it is preferable for DMI effect to be absent in the curbs to increase the contribution of the perpendicular anisotropy to the magnetization configuration within the curbs. This in turn shall create a stronger potential barrier to keep the skyrmion within the non-curbed area. For applied current density in the range of 10^{11} - 10^{12} A/m². Our simulation results show that the curbs are able to confine the skyrmion with or without the presence of DMI in the curbs.

Figure 5.4 Snapshot of the skyrmion driving simulation on a kered track which results in fast skyrmion motion.

Fig. 5.5 shows the speed of the skyrmion in the proposed kered device. The results show that the skyrmion speed is increased due to the inclusion of the kerbs, for both in-plane driving setup and also perpendicular driving setup. For instance, when the

skyrmion is driven in the in-plane driving setup with a current density of $J = 10 \times 10^{11}$ A/m² and kerbs of $s = 15$ nm wide, the speed of the skyrmion is found to be increased by almost 50%, from ~ 75 m/s to ~ 110 m/s. The speed increment is found to be more prevalent in the case of the perpendicular driving setup.

Figure 5.5 The skyrmion speed as a function of the curb width for various applied current density. Inset is a cross-sectional plot of the kerbed track to show the stray field that is generated by the kerbs.

Inset of Fig. 5.5 shows the stray magnetic field that the kerbs exert to the skyrmion. The stray field from the kerbs is directed in the same direction as the skyrmion magnetization, hence the stray field acts to increase the stability of the skyrmion against thermal fluctuation.

The increase in skyrmion driving speeds can be attributed to the increased STT that the skyrmion receives from the conduction electrons. Fig. 5.6 (a) shows a snapshot of a skyrmion that is compressed by the kerbs at the two edges. The skyrmion size is shown to be reduced to fit the area that is not covered by the kerbs.

Figure 5.6 (a) Compressed skyrmion (b) Spin-Transfer Torque for in-plane current injection

5.6 (a) Snapshot of the compressed skyrmion on the kered track. (b) The associated spin-transfer torque that is acting on the skyrmion when an in-plane current is applied.

As a result, the skyrmions become elliptical and elongated along the nanotrack axis. Fig. 5.6 (b) shows the STT that acts on a skyrmion in the case of in-plane driving. The calculated torque is shown to be most prominent at the front and the back sides of the skyrmion, where the front torque is directed at $-z$ while the back torque is directed at $+z$. The two torques are equal in values with a maximum value of $\tau_{\text{drive}} = 6.5 \times 10^{-3} \text{ s}^{-1}$ when the skyrmion is driven on an unkerbed track.

Fig. 5.7 shows the ellipticity of the skyrmion and the maximum torque (τ_{drive}) as functions of the kerb width (s). The maximum torque that acts on the skyrmion is found

to increase rapidly with increasing kerb width, which results in the increased skyrmion speed. The increased torque can be attributed to the fact that the compressed skyrmion possesses a higher magnetization divergence, which directly affects the STT, as shown in Eq. 3.

Figure 5.7 The ellipticity of the skyrmion and the maximum torque that the skyrmion receives as functions of kerb width.

It is possible to extend the skyrmion guiding technique to drive the skyrmion on a curved track. Fig. 5.8 shows the snapshots of the simulations when the skyrmion is driven on a curved track which has a 180° turning angle. The results show that without the kerb (i), the skyrmion is annihilated within the curvature even though the current is distributed to follow the shape of the track. However, with the inclusion of the kerbs to the track (ii), the annihilation is prevented and the skyrmion is guided to make the 180° turn.

5.4 Guiding skyrmion motion

Figure 5.8 Skyrmion motion on a curved track (i) without kerbs and (ii) with kerbs.

Fig. 5.9 shows the position of the skyrmion within the kerbed curved track as a function of time in the case of clockwise and anti-clockwise driving. In contrast to the straight track where the skyrmion maintains a constant velocity throughout its motion, the skyrmion on the curved track is shown to be slowed down with different deceleration depending on the direction of the applied current. The slowing down of the skyrmion can be attributed to the uneven current distribution that the skyrmion receives within the curved track, while the different in the behavior between the clockwise and anti-clockwise driving can be attributed to the Magnus force. As discussed before, during the application of current, the skyrmion is pushed to one side of the track due to the Magnus force. Therefore, with the application of clockwise and anti-clockwise currents,

the skyrmion is pushed to the outer and inner arc, respectively. Since the turning radius for the inner arc is much smaller, it is more difficult for the skyrmion to complete the turn when it is driven in the anti-clockwise direction.

Figure 5.9 Skyrmion position along the y axis as a function of time for both clockwise driving and anti-clockwise driving.

The reduced size of a skyrmion can also confer a significant increase in data storage density to a potential skyrmion-based memory device. As skyrmions in a magnetic thin film are of the same topological charge, two skyrmions experience repulsion from each other, and the strength of the repulsion can be related to the size of the skyrmions, as shown by Fig. 5.10. The minimum inter-skyrmion distance whereby two skyrmions can remain undisturbed from each other therefore shall also depend on the skyrmion size. The inter-skyrmion distance (d_e) and the radius of the skyrmion (r_s) are plotted in Fig. 5.11, and they are shown to vary almost linearly. This means that the kerbed track can also be utilized to pack the skyrmions closer to each other. At kerb width of around 5 nm, we observed that the skyrmion is unexpectedly enlarged, which can be attributed to the presence of the stray magnetic field from the kerb. However, the size increment from

the stray magnetic field is immediately overcome by the skyrmion compression from the kerbs as the distance between the kerbs is decreased.

Figure 5.10 The stray magnetic fields that are created around two interacting skyrmions.

Figure 5.11 Plot showing the relation between the equilibrium distance (d_e), the skyrmion radius, and the kerb width.

Conclusions

In conclusion, we have investigated the magnetostatic coupling in multi-nanowire systems for both in-plane and perpendicular materials. For the in-plane material, we have shown how the current-driven DW motion is affected when the DW is coupled to adjacent DW of opposite polarity. The coupled DW within the adjacent nanowires are induced to move in the same direction as the current-driven DW. In the coupled DW system, the Walker breakdown is shifted to higher current density limit. It is interesting to see that the chirality flipping is observed on both nanowires, even though spin-polarized current is only applied to one of the nanowires. Coupling two DWs or more can also be an alternative method to move DWs with only applying spin polarized current to specific wires. When current is applied to both nanowires in opposite direction, the two DWs undergo a damped oscillation motion, revealing the spring-like nature of the magnetostatic coupling. We have also investigated the dynamics of multiple transverse DWs collision in two-nanowire system. During the collision, the DWs are compressed and the DWs push each other via the exchange repulsion force. This phenomenon is encountered only when transverse DWs with equal winding numbers collide.

For DWs in PMA materials, we have shown that coupled DWs in PMA sandwich structure are able to form the Néel configuration under zero external field or current. When current is applied to one of the nanowires, the coupled DWs of a two-nanowire system are able to move faster than a single DW for a specific range of applied current due to the Walker breakdown suppression. When the stacking of the sandwich structure is increased to include three nanowires, the simulations show that the DWs are still coupled to each other and they are still able to move together when current is applied to the middle nanowire. However, the magnetic domains of the middle nanowire are not

curled anymore, and thus the speed of the coupled DW in the three-nanowire system is similar to that of the single nanowire system.

We have also shown that it is possible to guide the skyrmion motion and prevents the skyrmion to be annihilated by creating kerbs around the skyrmion to counter the Magnus force. We also show that the speed of a current-driven skyrmion is increased significantly when the skyrmion is compressed by the kerbs. The increase in the current-driven speed can be attributed to the higher torque that the compressed skyrmion receives from the conduction electron.

Future Work

At present, most research on magnetic DWs are focused on increasing its current-driven propagation speed in 2D patterned nanowires, as we have mentioned in the early part of this thesis. However, there is a possibility in the near future that the memory device has to accommodate a 3D design in order to increase its density even further. Hence, there is merit in investigating the DW dynamics in other ferromagnetic systems to find the most suitable candidate for the 3D DW-based memory device.

One such candidate is the cylindrical nanowire. It is possible to design a 3D memory by using vertically aligned nanowires instead of the typical 2D patterned nanowires. By using such design, we can then have more data density as now there is an additional degree of freedom. On this topic, we have performed a preliminary work to investigate the possibility of inducing a current-driven DW motion in such nanowire. Fig. R1 (a) shows the magnetoresistance reading of the current-driven DW experiment in cylindrical nanowires, and fig. R1 (b) shows the Scanning Electron Microscope image of our setup.

Fig. R1 (a) the magnetoresistance reading of the cylindrical nanowire (I) before the injection of current and (II) after the injection of current. (b) the setup of the measurement.

The cylindrical nanowire was created by using the template-assisted electrodeposition technique [138]. Before the current injection, we performed a field sweep to nucleate a DW in the center of the nanowire. Upon the injection of current to the nanowire, the DW resistance dropped abruptly which implies that the DW had been driven to the end of the nanowire. This result means that it is possible to obtain a current-driven DW motion in a cylindrical nanowire, which makes cylindrical nanowires a viable alternative to the 2D patterned nanowires. Micromagnetic simulations have predicted several interesting features of DWs in cylindrical nanowires [139], such as the lack of intrinsic pinning as well as the lack of Walker breakdown, which further emphasizes the need of more investigations in this topic.

On the field of skyrmion research, one investigation that is accessible to everyone and yet is highly sought is the theoretical understanding of its current-driven motion with spin-orbit coupling. It has been shown that the inclusion of spin-orbit coupling effects can increase the speed of a current-driven DW, however its effect on a current-driven skyrmion is generally unknown.

Two possible torques can be expected to act on the magnetic moments of the ferromagnets when the systems with spin-orbit coupling are considered [140]:

$$\tau_1 = T_{SHE} (\mathbf{M} \times (\hat{\mathbf{j}} \times \hat{\mathbf{z}}) \times \mathbf{M}) \quad (\text{S1}),$$

$$\tau_2 = T_{Rashba} (\mathbf{M} \times (\hat{\mathbf{j}} \times \hat{\mathbf{z}})) \quad (\text{S2}),$$

T_{SHE} represents the torque magnitude from the Spin Hall Effect (SHE), while T_{Rashba} represents the torque magnitude from the Rashba effect. $\hat{\mathbf{j}}$ and $\hat{\mathbf{z}}$ are unit vectors in the current and out-of-plane direction.

We have performed a preliminary work on this topic. As we can see, compared to the STT torques, there is no time component in the two torques. Therefore, the two torques can be introduced to the Thiele equation as two independent vectors ($\mathbf{F}_1$ and $\mathbf{F}_2$):

$$\mathbf{G} \times (\mathbf{v}_s - \mathbf{v}_d) + D(\beta \mathbf{v}_s - \alpha \mathbf{v}_d) + \mathbf{F}_1 + \mathbf{F}_2 \quad (\text{S3})$$

The result implies that the two torques will affect the skyrmion motion by changing both its direction as well as its current-driven speed. However, further investigations are still needed to fully understand how the spin-orbit torques affect the skyrmion dynamics.

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List of Publications

Papers

1. **I. Purnama**, I. S. Kerk, G. J. Lim, and W.S. Lew, "Coupled Néel domain wall motion in sandwiched perpendicular magnetic anisotropy nanowires", Scientific Reports 5, 8754 (2015).
2. **I. Purnama**, W. L. Gan, D. W. Wong, and W. S. Lew, "Guided current-induced skyrmion motion in 1D potential well", Scientific Reports, accepted.
3. **I. Purnama**, M.C. Sekhar, W.S. Lew, T. Ono, "Remote driving of multiple magnetic domain walls due to topological interaction", Applied Physics Letters, 104, 092414 (2014).
4. **I. Purnama**, M.C. Sekhar, S. Goolaup, W.S. Lew, "Current-induced coupled domain wall motions in a two-nanowire system", Applied Physics Letters, 99, 152501 (2011).
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6. H. T. Fook, W.L. Gan, **I. Purnama**, and W.S. Lew, "Mitigation of magnus force in current-induced skyrmion dynamics", IEEE Transactions on Magnetics, under revision.
7. S. Krishnia, **I. Purnama**, and W.S. Lew*, "Direct observation of ice-rule violation and monopole dynamics in geometrically frustrated artificial honeycomb spin ice", submitted.

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13. M.C. Sekhar, **I. Purnama**, S. Goolaup, W.S. Lew, "Depinning assisted by domain wall deformation in cylindrical NiFe nanowires", *Journal of Applied Physics*, 115, 083913 (2014).
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Conference presentation

1. “Guiding current-induced Skyrmion motion” (invited), **I. Purnama**, W. L. Gan, D. W. Wong, and W. S. Lew, *The 9th International Conference on Computational Physics* 2015, Singapore.
2. “Multi-vortex magnetic nanoparticles: A twin-functionalized agent for magnetomechanical cancer-cell destruction and hyperthermia”, D. W. Wong, N. Liu, C. B. Tan, W. L. Gan, , **I. Purnama**, and W. S. Lew, *20th International Conference on Magnetism, Barcelona, Spain*
3. “Skyrmion pinning dynamics in nanostructures for diode and symmetric operations”, H. T. Fook, C. C. I. Ang, W.L. Gan, **I. Purnama**, and W.S. Lew, *20th International Conference on Magnetism, Barcelona, Spain*

4. “Ice-rule violation and magnetic monopoles hopping in artificial spin ice”, S. Krishna, **I. Purnama**, C. S. Soh, and W. S. Lew, *20th International Conference on Magnetism, Barcelona, Spain*
5. “Quantized energy states in spin ice due to magnetic coupling”, C.S. Soh, **I. Purnama**, S. Krishnia, and W.S. Lew, *20th International Conference on Magnetism, Barcelona, Spain*
6. “Determining pseudo-particle properties of skyrmion from its response to applied alternating current” S. G. Wu, W. L. Gan, **I. Purnama**, W. S. Lew, *INTERMAG 2015, Beijing, PR China*
7. “2D transport of superparamagnetic microbeads on a ferromagnetic hexagonal nanolattice”, J.X. Chen, W. L. Gan, D. W. Wong, **I. Purnama**, W. S. Lew, *INTERMAG 2015, Beijing, PR China*
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11. “Magnetic nanoparticles for magnetomechanical cell destruction and magnetic hyperthermia agents”, D. W. Wong, Y. Yang, W. L. Gan, N. Liu, **I. Purnama**, M. Chandra Sekhar, S. Y. Wong, J. Ding and W. S. Lew, *INTERMAG 2015, Beijing, PR China*

12. “Emerging chirality combination for remote Walker breakdown in multiple nanowire system”, S. Krishnia, **I. Purnama**, and W.S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
13. “Topological charge as a diagnostic of magnetic Skyrmion configurations”, M Jalil, S.G Tan, Z.B. Siu, W.L. Gan, **I. Purnama**, and W. S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
14. “Current-driven Skyrmion dynamics in a memory device”, **I. Purnama**, W. L. Gan, D. W. Wong, and W. S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
15. “Magnetic nanoparticles with triple-vortex state”, W. L. Gan, D. W. Wong, Yong Yang, M. Chandra Sekhar, **I. Purnama**, Jun Ding and W. S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
16. “Defects and magnetization reversal process in geometrical frustrated artificial spin ice lattice”, S. Krishnia, P. Sethi, C. Murapaka, **I. Purnama**, and W.S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
17. “Dynamics of helical domain wall in cylindrical nanowires”, D. W. Wong, M. Chandra Sekhar, W. L. Gan, **I. Purnama**, Chinkhanlun Guite and W. S. Lew, *59th Annual Magnetism and Magnetic Materials Conference, Hawaii, US*.
18. “Domain wall remote driving in coupled nanowire system for memory applications”, S. Krishnia, **I. Purnama**, C. S. Murapaka, W. S. Lew, *IEEE Magnetics Symposium 2014, 50th Anniversary of IEEE Magnetics Society, Singapore*.
19. “Magnetic nanoparticles for cancer treatment”, D. W. Wong, W. L. Gan, Yong Yang, M. Chandra Sekhar, **I. Purnama**, Jun Ding and W. S. Lew, *IEEE*

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20. “High Speed Transport of 1D Skyrmion Gas”, W. L. Gan, **I. Purnama**, D. W. Wong, C. I. Ang, W. S. Lew, *IEEE Magnetics Symposium 2014, 50th Anniversary of IEEE Magnetics Society, Singapore.*
 21. “Néel coupling in sandwiched perpendicular magnetic anisotropy nanowires”, **I. Purnama**, I. S. Kerk, G. J. Lim and W.S. Lew, *4th International Conference on Superconductivity and Magnetism 2014, Antalya, Turkey.*
 22. “Driving multiple magnetic domain wall remotely via topological interaction”, **I. Purnama**, I.S. Kerk, C. Guite and W.S. Lew, *4th International Conference on Superconductivity and Magnetism 2014, Antalya, Turkey.*
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28. “Anomalous domain wall depinning behaviour in cylindrical nanowires”, M. Chandra Sekhar, **I. Purnama**, S. Goolaup and W.S. Lew, *Joint European Magnetic Symposia (JEMS) 2013, Rhodes Island, Greece*.
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30. “Low current domain-wall oscillators”, M. Chandra Sekhar, **I. Purnama**, and W.S. Lew, *ISAMMA 2013, Taichung, Taiwan*.
31. “Spin wave generation due to domain wall collision”, R. Maddu, M. Chandra Sekhar, **I. Purnama**, and W.S. Lew, *ISAMMA 2013, Taichung, Taiwan*.
32. “Multiple remote domain wall motion due to exchange interaction”, **I. Purnama**, M. Chandra Sekhar, and W.S. Lew, *ISAMMA 2013, Taichung, Taiwan*.
33. “Coupled vortex domain wall transformation and selective motion”, **I. Purnama**, R. Shiina, M. Chandra Sekhar, W.S. Lew and T. Taniyama, *ISAMMA 2013, Taichung, Taiwan*.
34. “Vortex-to-transverse coupled transformation in two-nanowire system”, **I. Purnama**, M. Chandra Sekhar, W.S. Lew, *International Magnetic Conference (Intermag) 2012, Vancouver, Canada*.
35. “Striped cylindrical nanowire as a magnetic domain wall nanobarcode”, **I. Purnama**, M. Chandra Sekhar, W.S. Lew, *International Magnetic Conference (Intermag) 2012, Vancouver, Canada*.

36. “Remote barrier and trap induced by neighboring domain walls”, **I. Purnama**, M. Chandra Sekhar, and W.S. Lew, *International Magnetic Conference (Intermag) 2012, Vancouver, Canada*.
37. “Field and current driven domain wall depinning at the notch and anti-notch in cylindrical nanowires”, M. Chandra Sekhar, **I. Purnama**, and W.S. Lew, *International Magnetic Conference (Intermag) 2012, Vancouver, Canada*.
38. “Coupled domain wall oscillations in closely spaced cylindrical nanowires”, **I. Purnama**, M. Chandra Sekhar, and W.S. Lew, *International Magnetic Conference (Intermag) 2012, Vancouver, Canada*.
39. “Smaller domain wall mass of coupled domain wall system” **I. Purnama**, M. Chandra Sekhar, S. Goolaup and W.S. Lew, *ICMAT 2011, Singapore*.
40. “Crossover in domain wall potential polarity as a function of anti-notch geometry”, M. Chandra Sekhar, **I. Purnama**, S. Goolaup and W.S. Lew, *ICMAT 2011, Singapore*.
41. “Collective motions assisted by magnetostatic interactions in coupled domain wall system”, **I. Purnama**, M. Chandra Sekhar, S. Goolaup and W.S. Lew, *Intermag 2011, Taipei*.
42. “Field-driven domain wall movements in asymmetric nanorings”, X. H. Wang, **I. Purnama**, S. Goolaup and W.S. Lew, *International symposium on advanced magnetic materials and applications (ISAMMA) 2010, Sendai, Japan*.

Intellectual Property

1. “Non-volatile logic device”, W.S. Lew, M. Chandra Sekhar, **I. Purnama**, S. Goolaup, P. Sethi, and C. Guite, Patent Cooperation Treaty (PCT) Application No: PCT/SG2014/000218.
2. “A memory device”, W.S. Lew, **I. Purnama** and M. Chandra Sekhar, Patent Cooperation Treaty (PCT) Application No: PCT/SG2014/000002.
3. “Non-volatile memory architecture based on domain wall remote driving sandwiched perpendicular magnetic anisotropy nanowires”, W.S. Lew, **I. Purnama**, I.S. Kerk, and G.J. Lim, Technology Disclosure, NTU ref TD/268/13 (2013).
4. “Reconfigurable magnetic logic”, W.S. Lew, M. Chandra Sekhar, and **I. Purnama**, US Provisional Patent, 61/824,738 (2013).
5. “Coupled domain wall oscillator as microwave generator”, W.S. Lew, M. Chandra Sekhar, and **I. Purnama**, US Provisional Patent 61/845,097 (2013).
6. “Logic operation based on magnetic domain wall profile”, W.S. Lew, M. Chandra Sekhar, and **I. Purnama**, US Provisional Patent 61/748,337 (2012).
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