3D surface characterization and localization of anatomical landmarks

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2006

Liu, X. (2006). 3D surface characterization and localization of anatomical landmarks.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/6554>

<https://doi.org/10.32657/10356/6554>

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3D Surface Characterization and Localization of Anatomical Landmarks

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A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2006

ABSTRACT

This study presents a method for extracting anatomical landmarks on the surface from measured data by way of local shape analysis. In order to delineate the distortion of the anatomical structure's surface, a 3D representation by differential geometry is used. The surface was scanned by a 3D non-contact laser scanner and curvatures were calculated from the acquired coordinates. The curvature maps were plotted and several manifest landmarks and structures were identified because of their pronounced curvature characteristics on the curvature maps. The accuracy and error of the curvature calculation were also studied. A novel approach of localizing anatomical landmarks was proposed.

The proposed method is applied in the following clinical areas: (a) measure the angle of tibial torsion objectively and reproducibly (b) predict tibial nail length required in the surgery objectively (c) monitor the depth, area, and volume changes of ulcers on the foot and leg.

Eighteen subjects' legs were scanned. By checking curvature maps the landmarks were characterized and localized. With these landmarks, tibial torsion could be calculated. The high reproducibility of the result and its coincidence with other studies' results showed the promise of this method. This method was proposed as a radiation-free, fast and objective way of measuring the tibial torsion.

Tibial tubercle-medial malleolar distance (TMD) has been proven as an easy, cheap, accurate and radiation-free method of determining the requisite length of an intramedullary nail for fracture fixation. However, presently the two anatomical landmarks are localized by palpation, and the distance is measured by rulers. This

makes the procedure subject to the examiners' experience and preference. This may cause errors in choosing the proper tibial nail. Based on the scanning of the subjects, the TMD can be measured objectively, which is easy, fast and radiation-free.

A patient's foot ulcer was scanned for ten times within three weeks. With the surface's 3D coordinates, curvature maps of the ulcer-ridden area were calculated. A boundary detection method was proposed, aiming to find those points on the edge of the ulcer by specific combination of Gaussian curvature and rim curvature. After smoothed by cubic spline interpolation, the envelope lines of those points were located as proper boundaries. To compare the curvature maps to the photographic image of the boundaries, we designed a special platform holding the camera and the scanner's wand. The camera's position in the world coordinate system and the nodal point of the camera was estimated by this platform. The curvature maps were projected through the nodal point on the Charge Couple Device (CCD, the image sensor on which the image is formed in a digital camera) plane and compared with digital image. The device could provide a clear guidance and verification of located boundary on curvature maps. Subsequently, the depth, area, and volume could be calculated after the boundary was located. The results indicate that the laser scanning and its curvature maps are found to be useful as convenient clinical tools for non-contact measurement of wounds.

Keywords: Foot surface, 3D non-contact laser scanner, anatomical landmark, principal curvatures, Gaussian curvature, mean curvature, Koenderink shape index, tibial torsion, tibial nail, non-contact wound measurement

ACKNOWLEDGEMENT

I owe a debt of gratitude to Professor Kim Wangdo. He has been an excellent supervisor and his guidance has been proven to be invaluable during this course of research. I am truly lucky to have him as my supervisor. Professor Burkhard Drerup of Münster Universitätsklinikum in Germany has also provided sound technical support and offered an outstanding professional conduct. Also I want to thank Prof. Fuss F. Konstantin, Prof. Frobin Wolfgang, Prof. John Tan, Dr. Ying Ning and Mr. Teu Koon Kiat. Numerous impromptu discussions with them have been very useful and have enriched my graduate experience greatly. Dr. Wong Yueshuen in Singapore General Hospital, Dr. Arjandas Mahadev in K.K. Hospital and Dr. Adriaan Erasmus in Tan Tock Seng Hospital have given me great support from the view point of medical doctors and provided me with a number of subjects for measurement. Thanks also should be given to those Ph.D. students, staff and technicians in the Mechanics of Machines Laboratory who have offered generous helps to me during the research.

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CHAPTER 2

INTRODUCTION

The background section starts with introduction of human anatomy that related to this study. With an example on human spine deformity detection, the definition of anatomical landmarks is given and their importance in biomedical application is described. Non-contact body surface measurement methods are discussed, among which the 3D laser scanner has unmatched advantages. The objectives are to develop a curvature extraction method suitable for 3D laser scanner data and apply it successfully on three medical applications: tibial torsion measurement, tibial nail estimation and wound documentation. At the end of this chapter, the scope and the frame of this study are depicted.

BACKGROUND

The human skeletal system, composed by axial skeleton and appendicular skeleton, including 206 bones, supports, protects and moves the human body. Together with the muscles and ligaments, the bones contribute to the human body shape and form. Observation or palpation of the bones beneath the skin helps the doctors to know what happened inside the human body. For example, the spine is formed from 26 irregular bones connected and reinforced by ligaments in such a way that a flexible, curved structure results (cervical curvature, thoracic curvature, lumbar curvature, sacrum and coccyx) [1]. By checking the abnormal spinal curvature, the physiologists know the deformation of the spine before a possible surgical treatment can be planned. In these cases, naturally, the vertebra prominens formed by the 3rd, 4th and 5th cervical vertebra

is taken as a reference point. All over the human body, many locations such as inner and outer malleoli, the tibial tubercle, the clavicle, the distal end of ulna resemble the vertebra prominens in a way that they have a prominent identified features. These points on the body surface with pronounced characteristics compared with other points in their vicinity are called anatomical landmarks. These points have raised people's great interests because they help to get a keen and accurate understanding of our inner body differences between healthy persons and patients. Orthopaedists can easily infer or know what happened to their patients by examining features of these landmarks before and after surgical procedure. Certainly, checking the relationship among anatomical landmarks and areas of interests can also help doctors to evaluate the progress of recovery. With at least three landmarks on a certain body surface, a body-fixed coordinate system could be built. This provides a reference system independent on individual's position for further measurement and analysis.

In the last two decades, besides direct palpation on the human body, numerous ways of non-contact surface measurement method have been proposed, aiming to give a more accurate description of surfaces. Many researchers have worked on the body surface by geometrical local shape analysis. A number of methods for surface measurements and evaluations have been developed as such: stereography, moiré topography of the shadow type or projection type and rasterstereography using cross rasters or line rasters. All these methods were based on the stereophotogrammetry, which might have the disadvantages of time consuming, low accuracy or complicated calibration procedures.

With the development of laser technology, the 3D non-contact scanners become an important and fast way of collecting data on body surface. It is desirable to develop a

suitable method for evaluation of the data collected from 3D non-contact scanner based on those methods for stereophotogrammetry. The method should allow the extraction of curvatures containing information for local shape analysis.

OBJECTIVE

In this study, a method for extraction of curvatures and evaluation of prominent features is developed. It aims to approximate a surface with those unevenly distributed data collected from 3D non-contact scanner. Those data were converted into a set of points situated on the intersections of a grid with constant grid space. At the same time, the curvatures at these points were calculated and plotted. By checking the map of Gaussian curvature, mean curvature or Koenderink shape index, some characteristic regions or points were also discriminated. This method was used on the foot, lower leg and wound surface and several reproducible landmarks were localized with a high precision. The angle formed by the anatomical landmarks gives a quantitative and objective measurement of tibial torsion. With the curvature maps, the tibial tubercle and the medial malleolus could be easily located, which avoids the subjective errors made by examiners' palpation. In this way, the tibial tubercle-medial malleolus distance (TMD) could be accurately and reproducibly decided for choosing the proper tibial nail. The objective characterization of skin surface also promises to be a convenient way of assessing and monitoring the wound healing process.

SCOPE

The emphasis is on extracting curvatures and looking for prominent landmarks as well as reconstructing foot and lower leg surface from unevenly distributed data. FastSCAN is the only tool used for scanning the surface. It scans the foot and lower

leg in the static position. Local surface fitting and curvature maps are the methods to characterize the measured surfaces. Three possible applications found (tibial torsion, tibial nail determination and wound measurement) are closely related to the foot and lower leg.

ORGANIZATION

The study slightly dips into non-contact surface measurement and its application but intensively reviews the previous works on the applications found later. Then it concentrates on the curvature calculation method suitable for the surface measured by FastSCAN. After characterizing the surface, the applications found are described in details.

Following the Introduction, Chapter One, Chapter Two gives a full literature review about the methods of surface measurement, surface fitting, curvature calculation and the measurement methods of tibial torsion, tibial nail and wound. Chapter Three performs the surface fitting and curvature calculation in a specific and detailed way and the accuracy is studied. Chapter Four describes the equipment used for surface scan and the way to localize the anatomical landmarks. The medical application of the method is shown in Chapter Five. Conclusion is drawn in Chapter Six and future works are outlined.

CHAPTER 3

LITERATURE REVIEW

An intensive literature review is given in this chapter. In section 2.1, the non-contact surface measurement methods are introduced, including their applications in biomedical areas. Then the current existing works done on tibial torsion measurement, tibial nail estimation and wound documentation have been studied in depth and their advantages and disadvantages are pointed out. The common disadvantage of the previous works is the intrinsic subjectivity causes a poor repeatability and reproducibility, which is overcome by introducing an objective measurement method by curvature maps. Thus, the necessity of this study is justified.

2.1 NON-CONTACT SURFACE MEASUREMENT AND ITS APPLICATION

One of the major problems that scientists and engineers face nowadays is the characterization of the topography and surface quality of 3D objects and the study of changes or transformations of that surface when it is exposed to external agents [2]. The non-contact surface measurement has long been proposed and applied with various purposes such as defect detection and medical examination. The measurements were done with different optical methods and techniques such as moiré topography [3], rasterstereography and stereophotography [4], which enabled us to quantitatively and qualitatively analyze opaque object surfaces.

The moiré method is an optical method suitable for non-contact surface shape analysis. The basic principle of moiré effect (Figure 3-1) lies in superposition of two

identical gratings G_1 and G_2 with a grid constant l . α is the angle between the two gratings. When they are exactly parallel, that is α is zero, no fringe happens. When there is a small angle α between gratings, the moiré pattern of fringe spacing T is emerging such as $T = \frac{l}{2\sin(\alpha/2)}$. A grating strips (lines) displacement on width l causes the perpendicular transposition of the moiré fringe pattern T . When one grating is projected to a second plane, the moiré fringes pattern also arises. When the image of first grating G_1 is distorted in the plane of second G_2 the moiré fringes is curved. It can be used for the evaluation of the surface form. Many have used the shadow moiré technique to detect features of the body surfaces such as back, leg and foot.

Moreno Yeras [5] tried to detect the spine illness on human back with the moiré topography, concluding that it might evaluate the asymmetry in patients in a more accurate way than the traditional visual evaluation and can be used by non-specialized staff. This structured illumination method was applied on the study of the sole of foot by Cortizo et al. [2]. During a walk, the deformation of the foot was measured by an experimental system consisting of a footbridge, an optical slide projector to project the Rochi grids and the CCD camera to capture a sequence of images. The topographies of the static or kinetic footprint could be recorded and compared among different cases, which could help in a medical diagnosis or following up the changes of the sole of the foot during a walk along a medical treatment. Although their precision is good enough for deformation measurement, the details of the sole of the foot might be omitted.

Figure 3-1: Basic principle of moiré effect (up) and scheme of moiré topographic system (bottom). The local deviation (ω) of surface M from plane P can be measured. [6]

Dunn et al. [7] summarized a method for measuring burned surface area without contact by structured light. A regular geometric pattern such as a uniform grid of intersected stripes was projected on the three-dimensional body surface and the illuminated surface was viewed from a camera of a different view angle. After the camera and projector were calibrated, the three-dimensional coordinates could be

determined by the intersection of two lines: one line was starting from the grid intersection on the camera's image plane and passing through the center of the camera's lens, and the other line was starting from the corresponding grid intersection on the slide and passing through the center of the projector's lens. A least-squares intersection was found in case of that the two lines did not meet each other because of inaccuracies caused by quantifying the digital image and determining the camera and projector's calibration matrices. In order to avoid the risk of exposing the eyes to the laser beam, the white light was used as the structured light source. Unfortunately, the poor contrast of white-light stripes caused problems for locating the corresponding grid intersections in camera image. Additional image processing including segmentation, line thinning, intersection detection and grid labeling was used to correctly match the grid intersections on the camera image with those intersections on the slide. Their experiment results showed that it was an effective and accurate way of three-dimensional body surface reconstruction to measure the burned body area for more accurate cure and to investigate face-swelling changes for evaluation of the efficacy of anti-inflammatory drugs. However, the calibration procedure and image processing is a bit complicated and time consuming.

An anatomical landmark is a point assigned by an expert that corresponds between organisms in some biological meaningful way. Anatomical landmarks on the body surface are either isolated points or well defined regions exhibiting a conspicuous shape compared with their vicinity. They are used to mark and separate anatomical regions. The particular shape of the landmark often reflects the underlying structures. In the case of the foot and ankle region the prominences of the malleoli are produced by underlying bony prominences and tendons. Thus, the lateral or outer malleolus

landmark indicates the underlying malleolus of the fibula. In the same way the medial or inner malleolus landmark indicates the prominence of the tibia.

In clinical examination of the lower extremity the malleoli play an important role: leg length measurement generally is referred to them and the axis definition of the upper ankle joint refers to the line connecting the two landmarks. The knee replacement surgery requires an accurate assessment of the rotation of the tibia while inserting the implant. Presently it is largely estimated by observing the rotation of the foot, but this is extremely arbitrary and based on surgeon's experience. An objective measure based on rotation of bimalleolar axis may be helpful.

In kinematic and kinetic analysis of the lower limb in standing and walking knowing the exact position of the ankle joint is of particular importance. In general this is achieved by the optical markers attached to the skin in close vicinity to the malleoli. Unfortunately due to skin movements (Figure 3-2), the reliability of this method is often called in question, which is proven by Cappozzo et al. [8].

Figure 3-2: The skin movement makes the attached optical landmarks less reliable.

A different, straightforward approach to this problem is to define and localize an anatomical landmark by its particular shape. As the shape mainly reflects the underlying bony structure and is only little affected by the skin – which is locally

thought of to be homogenous – this landmark definition is not affected by skin movements and is independent of the accidental circumstances of marker fixation. Hence, in this case a high reliability in the exact localization of the landmark is expected.

In order to investigate the shape of a given anatomical region the 3D surface data are necessary with high accuracy allowing mathematical analysis by methods of differential geometry – i.e. twofold differentiation. As a result of this approach curvature data are obtained providing an invariant shape description, where the term invariant means that it is neither dependent of the actual coordinate system nor dependent of surface parameterization [9].

Drerup and Hierholzer [10] described how to determine anatomical landmarks on body surface by data acquired by optical methods such as moiré topography or rasterstereography. The shape analysis was done by means of surface curvature. Gaussian curvature [11] was extracted from those data. They identified the location of vertebra prominens with the curvature analysis and tested the accuracy by comparing the results with the conventional method of palpation and marking. The mean deviation was in an order of 1 mm with a standard deviation of about 2.5 mm in the lateral and 6 mm in the longitudinal direction. Basing on their detection of the vertebra prominens landmark by surface curvature analysis, they concluded that this method was possible to apply on body surface to find anatomical landmarks with a high degree of accuracy.

Figure 3-3: Left: (a) stereophotography and (b) rasterstereography. Right: basic arrangement for rasterstereography [12].

Frobin and Hierholzer [4] outlined a real video rasterstereographic system for measurement of human body surface (Figure 3-3). The system was made up of optical system and image processing system. A projector projected a certain pattern of heavy and light lines on the body surface and a camera captured the lines. Then the video image was digitized and stored in the computer memory for image processing. The image processing was divided into several steps: peak detection, line search, line sequence analysis, line numbering and surface reconstruction. With their system, it took 90 seconds for image analysis and surface reconstruction of a medium-sized subject. The mean z coordinate (z is surface point's height (surface relief)) error of the reconstructed surface points was in the order of 0.5mm and was further reduced to 0.2mm by using a surface interpolation and smoothing algorithm. With more than

1200 video rasterstereographs reconstructed without a single error, the video rasterstereography was proved to be an accurate and efficient way to measure irregular surfaces.

Drerup and Hierholzer [13] presented a method for back shape measurement using video rasterstereography and three-dimensional reconstruction of spinal shape. The method comprised automatic back shape measurement and shape analysis that were suitable for the diagnosis of scoliosis. Based on a model, calculations of the spinal midline and of vertebral rotation determined from surface rotation were done within 5 minutes. Then the midline of the spine acquired by the rasterstereographic and radiographic methods were matched and compared. They found that the difference between the two curves expressed in the form of root mean square deviation was in the order of 4 mm. If vertebral rotation was inferred from surface rotation, the root mean square deviation was about 3°. In application, the video rasterstereography had the advantage in providing the researchers with a high sampling density of about 10 points/cm² on the back surface. It was understandable that the collected points with a high density will allow complicated and accurate shape analysis. The conventional parameters of scoliosis such as Cobb angle could also be achieved with a limited accuracy from the reconstructed midline. Although the video rasterstereography measurement followed by back shape analysis aiming to replace the radiographs was adequate in their clinical practices to examine scoliosis, it still needed improvement to give more accurate results for evaluation of those conventional parameters.

Figure 3-4: The experiment setup for spinal deformity detection [14].

Zubairi [14, 15] applied the rasterstereography aided by computer to detect spinal deformity (abnormal spinal curvatures) in the mass screening of school children. A digital camera, a projector and an ordinary PC were used to perform this task (Figure 3-4). A pattern of rasters was projected on the back of the subject and recorded. After the image of the distorted lines was digitized, the lines were detected and calculated by an algorithm based on thresholding and chain coding principle. The vertical deviations

of horizontal lines and the horizontal deviations of vertical lines were computed to detect any spinal deformity. Since the human back is symmetrical, any asymmetry on the back was reflected by the different deviation of the projected rasters on both sides of the central axis. This methodology is suitable for screening school children for spinal deformity because it is fast, simple and non-invasive.

Barros et al. [16] proposed a methodology for a three-dimensional surface reconstruction with digital video camcorders and slide projectors (Figure 3-5). Beginning with a structured light projected on the surface, a set of points with a relative large density was obtained without the placement of markers on the surface. Then by analyzing the images, the points were recognized, labeled and calculated automatically for their three-dimensional coordinates in the space. A function $z = f(u, v)$ was employed to represent the surface. The domain of the projection plane of the function $z = f(u, v)$ was defined by the two eigenvectors associating to the two larger eigenvalues of the variance and covariance matrix of the reconstructed points. The projection plane formed in this way gave the minimum deformation of the points projected on it. After that, the borders of the projected surface were identified and those points which did not belong to the effective body surface projected region were excluded by the Delaunay triangulation method (Figure 3-6). By way of Biharmonic Spline Interpolation, a surface in the form of $z = f(u, v)$ was fitted to the data. The analysis was carried out to reach various goals based on the need of different research. As an example, the back surface was reconstructed and analyzed. This methodology gained its advantage by simplicity and low cost for implementation due to widespread equipments such as digital video camcorders and slide projectors.

Drummond and Duffy [17] designed an apparatus to measure the surface changes of the chest and abdomen when the patients lied supinely and breathed spontaneously. The laser beam was spread by a cylindrical lens into a strip of light, which was moved accurately by a computer-controlled galvanometer mirror. Then the line image from a video camera was analyzed to estimate the height of the surface profiles, from which the shape and volume of objects could be calculated. Their optical system gave accurate and reproducible results in supine human subjects.

These methods utilized cameras, camcorders and projectors are only suitable to register a relatively larger surface such as human back surface. Without a high data density, they fail to detail small surface areas, which makes them difficult to characterize the regions such as inner and out malleoli.

Figure 3-5: One camera view of the subject's back with projected points [16].

Figure 3-6: Delaunay triangulation method applied to the projected points [16].

Recently, as a fast, convenient and accurate way of surface registration, laser scanner has been widely used in many areas such as medical monitoring, orthopaedics and surface area measurement.

Kondo et al. [18] developed a prototype laser scanner with only one single laser sensor to measure chest wall displacement during respiration. They performed experiment both in vitro and in vivo. However, based on their results, a prototype laser scanner with more than one laser sensor would have performed better to monitor respiration in spontaneously breathing subjects.

Bibb et al. [19] described the application of non-contact three-dimensional scanners in the design and manufacture of prostheses. In their practice, the surface data of a patient's right side face were obtained and used to reconstruct the left side face of the patient whose left eye together with the surrounding bones and soft tissues were excised because of tumor. The patient's chin was supported on a polystyrene block to

minimize involuntary movements to achieve a high accuracy. A laminated object manufacturing (LOM) model was made according to the data to provide a realistic visual appearance and guide for the prosthesis. But their accuracy for the scanned data was 0.05mm and their LOM model was estimated to be ± 0.1 mm which was only sufficient for prosthesis manufacture.

Tikuissis et al. [20] accurately studied the human body surface area with the aid of three-dimensional laser scanning technology and 3D modeling software. They first obtained the raw whole-body scans of 641 adults (395 men and 246 women). After patching and smoothing of the surface scans, they calculated the overall body surface area (2.03 m^2 for men and 1.73 m^2 for women). Their tests on various prediction equations for body surface area showed that although most equations predicted the human body surface area reasonably closely, over predictions were found with increasing body size. They finally gave their prediction equations for men and women with root mean square errors of around 1.3%.

Besides the above techniques, there are lots of other techniques to collect surface information. Since we are mainly interested in the data analysis from laser scanner scanned surface, various other non-contact surface registration techniques are out of our concern.

After the surface is registered by various non-contact tools, the next step is the surface reconstruction. Surface reconstruction includes changing unorganized data into regularly distributed points, which would be easier to interpret or further disclose other information such as surface curvature. There are number of surface reconstruction methods such as alpha shapes [21, 22], thin plate spline [23] and curvature estimation methods such as quadric surface patches [24], circular fitting [25, 26] etc.. We employ

a simple second order polynomial in the least square surface fitting to estimate surface curvatures. It is easy to understand and accurate enough for our study, which has been applied successfully on the back surface. Furthermore, it is easy to implement.

2.2 TIBIAL TORSION MEASUREMENT

Tibial torsion is the twisting of the tibia about its longitudinal axis, which changes as people grow up. Knowing the normal range of tibial torsion at different ages or tracking the changes is important before corrective surgery is taken. In order to reduce lower-limb rotational defects such as in-toeing and out-toeing in adults, the accurate determination of tibial torsion is necessary. Accurate measurement of tibial torsion also helps to study its relationship to the congenital clubfoot (also known as talipes equinovarus, the forefoot is turning inwards and the whole foot is turned to the side) [27, 28] and to the pathology of the knee such as patellofemoral instability or Osgood-Schlatter disease (traction apophysitis of the tibial tubercle) [29].

Since it was first described by Le Damany [30], tibial torsion has long been studied by various methods, resulting in different angles with a rather large variation.

Elftman [31] reported his measurements on the lower right extremities of 35 male cadavers. The mean malleolar torsion (angle between the frontal plane and the bisection line of both internal and external malleoli), different from tibial torsion (the angle between the frontal plane and the bisection line of the distal articular surface of tibia), is about $27.40^{\circ} \pm 1.25^{\circ}$. And in general the malleolar torsion is 5° larger than the tibial torsion. He also found no correlation between the tibial torsion and femur torsion.

He further pointed out that in the past, the tendency to correlate the femoral and tibial torsion was because of a misinterpretation of the changes happening in the femur and the tibia during fetal development. However, Kobylansky et al. [32] made measurement in 50 cadaveric femora and their corresponding tibiae by means of a parallelograph and found a significant correlation between the femoral and tibial torsion. Their finding substantiated the clinical finding of Fabry et al. [33] that increased femoral anteversion may be associated with a compensatory external torsion of the tibia. Further study should be made before the correlation of tibial and femoral torsion could be verified.

Hutter and Scott [34] measured the tibial torsion by tropometer on skeletal specimens and by roentgenogram on living subjects. With the pins inserted at the distal and proximal articulation, the angle between the two pins was measured. This was deemed as the most accurate way. For the estimation of the tibial torsion on the adult patients, they used similar reference lines. For children whose tibial torsion could not be estimated by roentgenogram because their cartilaginous epiphyses yielded unsatisfactory roentgenographic image, their tibial torsions were estimated by the physical examination alone, which in their viewpoint, was the most practical way. They concluded that normal adults had about 20° external tibial torsion and the positions and other attitudes of posture would produce rotational changes in the tibiae. During the growth period, the correction of the deformities was possible by a modified Denis Browne splint (a light aluminum splint applied to the lateral leg and foot) but Dietz [35] argued that no evidence had been shown on the resolution of internal tibial torsion. Furthermore, a poorly applied Denis Browne splint could be harmful if it caused a heel valgus to develop.

Figure 3-7: The sketch of a cross section through the upper and lower ends of an adult right tibiofibular unit. UM: Upper Medial. UL: Upper Lateral. LM: Lower Medial. LL: Lower lateral. [36]

Rosen and Sandick [36] chose 4 constant points as reference points to determine the tibiofibular torsion: the upper lateral point (intersection of the anterior and lateral aspects of the distal part of the fibular head), the upper medial point (the most prominent point of the medial aspect of the tibial condyle), the lower lateral point (the intersection of the lateral and posterior aspects of the lateral malleolus) and the lower medial point (the intersection of the anterior and medial aspects of the medial malleolus)(Figure 3-7). The two upper points and two lower points formed the proximal and distal reference line, respectively. Because the projection in the axis of the tibiofibular unit would get the obscure position of the four points due to the density of the bones, they extended the lower leg by 15° to 30° from the right angle to show the 4 points more clearly on the film. The error of this anterior projection was negligible as well as the non-axial projection in the frontal plane. They also varied the tibiofibular torsion by osteotomizing at the middle of the tibiofibular units and inserted

a radiolucent peg in the medullary canal of the tibia. The 15° to 30° anterior projection proved to be the most accurate among all degree of medial and lateral torsion tested.

Wynne-Davies [37] followed 84 patients with talipes equinovarus to examine the nature of the residual deformity and assess the function of the deformed foot. He also described the radiographic technique. The dysplasia of the whole limb was found in many cases. Almost half the cases were falsely corrected because the foot was 'broken' at the talo-navicular level, leaving the heel in inversion, although the forefoot was plantigrade. He mentioned that in the living person the accurate measurement of tibial torsion was hard because either the complicated radiographs were taken or the pin was driven into the bone. They measured the tibial torsion with their own instrument, resulting in similar figures to those of Le Damany. A 15° variation between left and right side was found in the normal individuals. No patients with talipes equinovarus had the tibial torsion out of the normal range. Tibial torsion was found not significantly different in the clubfoot from normal values.

Kharmosh et al. [38] measured tibial torsion in 230 normal newborn children up to 5 years of old in order to determine the normal values. With an instrument similar to that of Wynne-Davies, they carried out the measurement while the children's legs were extended on a table. They took the tibial tuberosity and the medial and lateral malleoli as the proximal and distal reference points. They found that most of the newborn babies had a mild external torsion of 2.2°. With a rate of 1.3° per year, the tibial torsion increased gradually with an apparently greater rate at 22-27 months age group. No significant differences between left and right tibiae were found.

Staheli [39] presented a simple clinical method by calculating the transmalleolar axis of 180 normal children and adults. The external rotation of the transmalleolar axis

increased with advancing age, being 5° during the first year, 10° during mid-childhood and 14° in older children and adults. No significant difference was found between male and female as well as between left and right.

Herold and Marcovich [27] studied 1000 normal legs using the tropometer, the caliper and X-rays to measure the tibial torsion. They found that the caliper gave the most reproducible results. The X-rays method had about the same reproducibility as the tropometer. They confirmed that the external tibial torsion was rather small in young children and did not increase after 4 years of age, which was doubted by the results of other researchers [40, 41]. There was no significant difference between the healthy children and those with clubfoot on the tibial torsion. Also they found there was no correlation between the tibial torsion and forefoot adduction or untreated clubfoot. They concluded that the rotation osteotomy of the leg had no place in the primary treatment of the clubfoot.

Ritter et al. [40] carried out a longitudinal examination on the tibiofibular rotation of the children at birth (1000 legs), 6 months (302 legs), 12 months (154 legs) and 24 months (105 legs). They found at birth the average external rotation was 4° and then was gradually increased to 11° at 24 months. They also inferred that if the study continued up to maturity, the bimalleolar rotation would be about 20°. The children's sex, race and age starting walking were not correlated to the external rotation.

Malekafzali and Wood [42] designed a simple goniometric device, made up of one limb and two rubberized malleolar cups to measure the angle between the transcondylar and transmalleolar axes. Although the apparatus achieved good correlation with the results from roentgenographic methods, the clinician needs to adjust the horizontal limb of the goniometer to stand for the transcondylar axis. This

would invite possible subjective errors. Furthermore, both the clinical and radiological methods for measuring tibial torsion gave only an approximation.

To achieve the same accuracy as the direct measurement on necropsy specimens, Jakob et al. [43] developed a new method to measure tibial torsion with Computer Tomography (CT). The proximal reference line was defined as the axis passing through the widest transverse condylar diameter and the distal reference line was defined as the axis bisecting the anteroposterior diameter of the lower tibia end and also passing through the anterior half of the lateral malleolus. They measured the inclination between the two axes and got a mean value of 30° on 45 cadavers. The results and those from Le Damany's measurement belonged to the same statistical population with a 95% confidence level. By using the tibia and fibula together rather than using tibia alone to determine the distal reference line, they got a tibial torsion 6° larger than those from Le Damany's. Both of them found no significant difference between the right and the left leg.

Jend et al. [44] measured tibial torsion by CT. The distal reference line was defined as the connection of the pilon center point and the midpoint of a line over the fibular notch. The proximal reference line was the tangent to the dorsal border of the tibia. They got an average external tibial torsion of $40^\circ \pm 9^\circ$ in 70 healthy limbs with the range from 14° to 56° . Considering the accuracy, the rotational differences of tibial torsion between the left and right legs of more than 8° were considered to be pathologic. From the results of 42 limbs with fracture of both legs, they found that the tibial maltorsion were more likely leading to arthrosis of the ankle joint.

Jend et al. [44] also clarified the defects of taking the bimalleolar axis as the distal line of reference. Because the fibula motion influenced the angle that should only be related to tibia, the bimalleolar axis leads to errors.

Turner and Smillie [29] analyzed 1672 readings from 836 patients measured by a tropometer. They found that there was a significant increase in lateral tibial torsion of the patients with either patellofemoral instability or Osgood-Schlatter disease while the patients with panarticular disease had a prominent reduction in lateral tibial torsion.

The clinical methods for measuring tibial torsion assumed that the proximal tibial tilt (the transverse axis relative to the frontal plane) is uniformly 0. Laasonen et al. [45] proved that this was not true in 38 legs measured by CT. No radiologic methods for tibial torsion were widely accepted. Different CT methods had given too wide a variation to be clinically useful. At that time, they concluded that no reliable method for measuring tibial torsion seemed to be available.

Staheli et al. [46] measured 1000 normal lower extremities to obtain the normal values for the rotational profiles. With the transmalleolar axis and the thigh axis drawn on the projected photographic negative, they got the normal range from 0 to 45° with a mean of 25°. The thigh-foot angle, which was a combined deformity of both the tibia and the hind part of the foot, roughly paralleled the angle of the transmalleolar axis but with a lower mean value.

Clementz [47] studied the difference of tibial torsion in legs with a mobile fluoroscopy in 100 randomly selected healthy adults. The proximal reference line was the normal to the dorsal tangent to the posterior contours of femoral condyles. The distal reference line was the tangent of the articular facet of medial malleolus. The

difference in the same object ranged from -11° to -15° with a mean of $+2.1^{\circ}$. A greater outward rotation of the right tibia was found in 68% of the subjects. 25% of the subjects had a difference larger than 6° and 30% smaller than 2° .

Clementz et al. [48] assessed the tibial torsion measured by fluoroscopy, CT and cryosectioning technique in 10 human necropsy specimens. They defined the tangent to the anterior and posterior contour of the inner surface of the medial malleolus as the distal reference line and found good agreement among different techniques employed indicating the line was reliable. The proximal reference line was defined as the tangent connecting the posterior contours of the femoral condyles. The angle was defined as the angle formed by the normal to the proximal reference line and the distal one in the transverse plane. The estimated standard error of a single determination in fluoroscopy was about 1.3° . The maximum difference between results obtained with different methods was 5.4° and the average difference varied from 1.0° to 1.5° . In their opinion, the fluoroscopic method presented good repeatability and convenience for routine use.

Valmassy et al. [41] studied the normal values of tibial torsion based on the measure of 281 children by Tractograph[®]. Estimated by the angle between the transmalleolar axis and the frontal plane, they found from 1.5 to 6 years old the children's tibial torsion had an average increase rate of $1.4^{\circ}/\text{year}$ from 5.5° at 18 months to 11.2° at 6 years of age.

Reikeras and Hoiseth [49] studied the torsion of the leg on 50 adults by computed tomography (CT). They found that the anatomy of the dorsal aspects of the femoral condyles gave more stable results than those of the tibial condyles. The inclination between the dorsal tangent to the femoral condyles and the transmalleolar axis was used to estimate the tibial torsion because in healthy adults the extended knee didn't

have any rotation. Estimated from their data, the mean value of the external torsion was about 39°.

Stuberg et al. [50] compared the results of goniometer and CT scan on tibiofibular torsion measured on 17 normal subjects. No significant difference between testers was found on the tibiofibular torsion measured goniometrically. But the means of the two methods were significantly different within testers. However, since the variability of the CT method to measure tibiofibular torsion was reported as $\pm 4^\circ$, the authors concluded that the geometric techniques could be used by the clinician with relative assurance of the results compared to CT. But the clinician should be cautious about the measure by goniometer because an error of 1° to 4° was expected. It was necessary to have a greater magnitude to declare a difference. They concluded that the goniometric method gave reproducible results within an acceptable range, which was suitable for screening purpose.

Butler-Manuel [51] proposed a new technique to measure the tibial torsion with ultrasound on 21 dry tibiae and 21 patients with anterior knee pain syndrome and compared the results with the direct measurement and the CT. Two points on the posterior tibial border were used as the distal reference points. At these two points the border swept sharply anteriorly to join the medial malleolus and the lateral tibial border. The line connecting these two points was defined as the distal reference line. The results from ultrasound technique demonstrated a good statistical agreement with those from CT scans.

Sayli et al. [52] carried CT tibial torsion measurement on 25 adults. Their tibial torsion was defined as the angle between the dorsal tangent to the tibial condyles and

the line connecting the centers of medial and lateral malleoli. They got 31.07° for the right and 30.02° for the left tibiae.

Eckhoff et al. [53] studied the effect of choosing different reference point on the measured tibial torsion with CT. They tried different proximal lines of reference (posterior tibial axis, transtibial axis and anterior tibial axis) with the same bimalleolar axis and tilt the proximal tibial cut with respect to the longitudinal axis of tibia, only to find that there was no significant influence on the angle of tibial torsion when the scanning was carried out within 2cm of the articular surface of the knee.

Isolated external tibial torsion is a rare cause of patellofemoral instability, which is confirmed by the biomechanical studies. Cameron et al. [54] studied 17 knees with increased quadriceps angles secondary to excessive isolated external tibial torsion and suggested the derotational tibial osteotomy should be used to achieve medialization in patients with recurrent patellar dislocation whose external tibial torsion required rotational correction of greater than 20° . For those patients having recurrent patellar dislocation plus anterior knee pain, a combined derotational and Maquet type osteotomy was used. 13 out of 17 knees after operation were graded as good to excellent. The poor outcomes were due to multiple operations before.

Schneider et al. [55] measured the femoral antetorsion and tibial torsion by MRI (Magnetic Resonance Imaging). The distal reference line was the line connecting the center of the circle fitted to pylon tibiale and the midpoint of the line passing through the fibular notch of the tibia. The proximal line was the dorsal tangent to the tibia at the level proximal to the head of fibula. From 98 healthy lower limbs, they got the average of 41.7° and 6.1° for tibial torsion and side difference, respectively.

Lang and Volpe [56] used a gravity goniometer to measure tibial torsion and studied the accuracy of the in vivo measurement. The results were compared with those from CT. They found good agreement between the two methods on the same limb. Their study doubted that some landmarks traditionally used in vivo measurement might not be correct. They further specified that using the tibial tubercle as a proximal reference point might not give a true measurement of tibial or tibiofibular torsion.

Milner et al. [57] performed the 4 existing in vivo methods (simple geometry with caliper [39], C-clamp [40], goniometer [42] and photographed geometry [46]) of measuring tibial torsion on 10 cadavers and tried to figure out which one was the most accurate with the highest repeatability. Each of the 4 indirect measurement [39, 40, 42, 46] techniques was repeated 10 times on both limbs on a random selection of 4 of the cadavers. A direct method described by Butler-Manuel et al. [51] was also used to measure the tibial torsion on the cadavers. Because of the different reference lines employed, there was a large variance in the mean values of the tibial torsion measured by different direct and indirect methods. None of the indirect methods had a strong correlation with the direct measurement. The 4 indirect methods were poorly repeatable both in cadaver and in the living, much lower than that claimed by the authors. The direct measurement, however, had an excellent repeatability. The authors further pointed out that the main problems of the 4 indirect tibial measurement methods were their subjective nature. These methods relied highly on individual judgement rather than objective determination. Besides, all the 4 indirect methods used the transmalleolar axis as the distal reference line, which may not give the accurate results due to the motion of the fibula. The authors concluded that the alternative methods must be developed since none of the 4 in vivo techniques appeared to be accurate and highly repeatable.

Reikeras et al. [28] employed CT scan to measure the external torsion of the leg on 24 children with congenital clubfoot and compared the results to those of 24 healthy boys and girls in a control group. The dorsal tangent to the femoral condyles and the transmalleolar axis were taken as the proximal and distal reference line, respectively. They found the external torsion of the patients with clubfoot was significantly lower while the range of tibial torsion was higher than those of the normal population.

Tamari et al. [58] gave a new way of measuring tibiofibular torsion by an electronic digital inclinometer. The tibiofibular torsion was defined as the angle between a tangential line of the medial surface of the tibia at the level of the inferior point of the tibial tuberosity and the transmalleolar axis. In their point of view, this definition might give more consistent results than those methods taking the horizontal line as the proximal reference line, which is influenced more by the examiners. Very good intrarater reliability was observed.

For comparison purpose, various tibial torsion measurement methods are tabulated in Table 3-1.

Each method has its own advantages and disadvantages. The simple clinical assessment depends largely on the examiner's preference and experience. Roentgenographic methods require trained personnel, present additional radiological expense and exposure and when applied to children, incomplete ossification makes interpretation difficult. Some find assessing tibial torsion using radiographs of the tibia is unhelpful [59]. The position of the subjects' leg and the different plane of the cut will introduce artifacts to the CT assessment. Laasonen et al. [45] doubted the clinical use of CT because of its wide variation. As a possible alternative of CT, measurements by MRI will also vary according to the plane of the image. Fluoroscopy

requires a stable knee joint as a pre-requisite. US can only obtain full cross sectional images of tibia with much difficulty. As such, a new method, different from Jend's established CT method [44] and by Butler-Manuel [51], has to be developed to measure tibial torsion.

The two reference lines that give the angle of tibial torsion also differ with different references. Many researchers took the transmalleolar axis as the distal reference line. For the proximal reference line, some researchers turned to the tibial tubercle when they measured tibial torsion [37] or when they tried to reproduce others' experiments for comparison purpose [27, 56]. However, in the viewpoint of Lang and Volpe [56], using the tibial tubercle as a proximal reference point might not give a true measurement of tibial or tibiofibular torsion. Some other researchers took the dorsal tangent to the tibial condyles [45, 51, 55] or the dorsal tangent to the femur condyles [60]. Whereas Reikeras and Hoiseth [49] found that the anatomy of the dorsal aspects of the femoral condyles gave more stable results than those of the tibial condyles. However, this might only be true for healthy subjects. Jend et al. [44] pointed out that by using dorsal condylar contour of the femur, errors were likely due to the axial rotation of the knee if the knee could not be straightened due to trauma or if excessive laxity exists.

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Year	Author et al.	Sample number	Methods	Distal	Proximal	Average degree (healthy)	Left and right difference
1945	Elftman	35 cadavers' right lower extremities	DM	Transmalleolar axis	Frontal plane	27.40±1.25	Not studied
1949	Hutter	40 adult skeletons 200 adults	Pin Roentgenogram Physical examination	Line parallel to the transmalleolar axis	Line passing through the centers of condylar articulating surface	20	Not mentioned
1955	Rosen	Adults and children	Roentgenograph	The line connecting the lower lateral point (the intersection of the lateral and posterior aspects of the lateral malleolus) to the lower medial point (the intersection of the anterior and medial aspects of the medial malleolus)	The line connecting the upper lateral point (intersection of the the anterior and lateral aspects of the distal part of the fibular head) to the upper medial point (the most prominent point of the medial aspect of the tibial condyle)		Not mentioned
1964	Wynne-Davies	110 legs with talipes equinovarus A control group	A special caliper	Transmalleolar axis The angle from the coronal plane	Tibial tubercle	Similar to Le Damany's 23.7	15 in normal subjects
1976	Herold	1000	Tropometer Caliper X-rays	Transmalleolar axis Bimalleolar width Transmalleolar axis	Tibial tuberosity A board on which leg rests Tibial plateau	<2 4±2 2-4 9±3 4-10 6±6 >10 16±3	Not mentioned
1976	Ritter	>100 legs	DM	Transmalleolar axis	Line perpendicular to the long axis of femur	4 at birth increased to 11 at 24 months	
1980	Jakob	63 cases	CT	the axis bisecting the anteroposterior diameter of the lower tibia end and also passing though the anterior half of the lateral malleolus	the axis passing through the widest transverse condylar diameter	30	Not significant
1984	Laasonen	38 legs	CT	Axis through the centers Axis through the centers	Transverse axis of tibia Dorsal tibial tangent	24.7 31.3	

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1989	Reikeras	50 adults	CT	Transmalleolar axis	Dorsal tangent to the tibial condyles Dorsal tangent to the femoral condyles	33.075 39	Not significant
1989	Valmassy	281 children	Tractograph	Transmalleolar axis	Frontal plane	5.5 at 18 months 11.2 at 6 years average increase rate: 1.4/year	No significant
1991	Stuberg	17 normal subjects	Goniometer CT	Transmalleolar axis	Transcondylar axis	24.8±6.3 24.1±7 25.3±7.5	No significant difference
1992	Butler-Manuel	21 dry tibiae 42 legs	US CT CT Jend DM	Line through 2 points at which the posterior tibial border sweeps sharply anterior to join the MM and LM	Dorsal tangent to tibial condyles	Dry tibiae: 37.5 Patients: 43.5	Dry tibiae: 0.4 for DM 3.2 for CT Jend Patients: 4.2 for CT 5.0 for US right greater
1997	Schneider	98 healthy lower limbs	MRI	The line connecting the center of the circle fitted to pilon tibiale and the midpoint of the line passing through the fibular notch of the tibia	The dorsal tangent to the tibia at the level proximal to the head of fibula	41.7±8.9	6.1±4.5
1998	Lang	14 legs	Gravity goniometer CT	Centers of malleoli	Tibial tuberosity	18.7	Not significant
2001	Kristiansen	78 subjects	CT	Transmalleolar axis	Dorsal tangent to femur condyles	28 at 4 years 38 at maturity	0.5±2 not significant
2003	Tamari	20	Electronic digital inclinometer	Transmalleolar axis	A tangential line to the medial surface of the tibia at the level of inferior point of tibial tuberosity	Test 1: 47.1±8.7 Test 2: 46.8±7.2 Test 3: 46.8±7.3	Not significant

Table 3-1: The summary of various methods of tibial torsion measurement

Different researchers measure this angle with different methods and reference lines, resulting in a huge variation in the reported normal ranges of tibial torsion. Each method has its own advantages and disadvantages and no conventional technique for routine assessment of tibial torsion has gained wide acceptance yet.

Measurement by simple apparatus, although convenient and inexpensive, is inaccurate due to subjectivity. The ideal means of quantification of tibial torsion should be simple, rapid, inexpensive, reliable and suitable for a busy clinical environment and minimize subjective variables [42]. Thus a new method is needed, which satisfies the above requirements and at the same time eliminates the subjectivity.

2.3 TIBIAL NAIL ESTIMATION

Extensive use of intramedullary nails in the surgical treatment on the tibial fracture necessitates the accurate and rapid estimation of the length of nail required in medullary nailing of tibial fracture. The proper nail length is a must for satisfactory recovery results. Too short of a nail length causes the loss of reduction or malreduction. Too long results in fracture distraction, impingement on the patellar tendon or violation of the ankle mortise [61]. Substitution of an improper nail increases the risk of loss of reduction and time of operation and anesthesia and cost.

Relatively few researches have been done on estimation of the length of tibial nail used in the surgery. With the records of the nail length used on 59 patients and their body height, Fischmeister et al. [62] tried to correlate the requisite nail length in tibial fractures with the body height. A regression equation between the nail length and the body height was derived and the prediction on 12 patients showed an average difference of -0.09cm (SD: 0.93cm) between actual and predicted nail length. They

stated that in most cases with the regression equation, the proper selection of the manufactured nail could be made before operation.

Some existing methods on determination of nail length rely on the conventional radiographic method [63, 64] thus their accuracy is dented by overexposure, underexposure, malrotation and variation in magnification. The patients are exposed to radiations. Colen et al. [65] proposed a prediction method by tibial tubercle-medial malleolar distance (TMD). The distance from the most prominent point on medial malleolus to the tibial tubercle was measured on 14 cadavers. The nail lengths predicted by TMD and three other methods (full-length scanograms, spotograms, acrylic template overlays) were compared. With the lowest false prediction rate of 29% (scanogram, 79%, spotograms, 43%, template 100%), the author claimed that the TMD is an easy, cheap and accurate method of determination on the requisite nail length.

2.4 WOUND MEASUREMENT

A lot of treatment modalities have been employed to facilitate the chronic wound healing of different ulcers, including wound dressing [66, 67], ultrasound [68], vacuum-assisted closure [69] and electrical stimulation [70]. To quantify or compare the effects of various therapeutic modalities, wound healing rate was defined in different ways such as absolute area healed per day, percentage of initial area healed per day and advance of wound margin toward the wound center per day. These wound healing rates are limited on comparing healing rates of different wound size and many researchers proposed other ways of calculating healing rate [71]. For various methods

of monitoring the wound healing, the accurate wound measurement such as wound area, perimeter and volume is a crucial precondition.

More often clinicians measured the wound quantitatively than just observed qualitatively the wounds' appearance by naked eye or by a certain tool such as spectroscopy [72]. Accurate measurement is an important part of wound assessment. For decades of years, clinicians and researchers have tried to find ways (contact and non-contact) to quantify the area and volume of wounds and extract useful information for wound assessment. Wounds vary considerably in their nature, shape and site. Many measurement techniques exist (Table 3-2). It is important to choose the correct technique for a specific wound and to understand the degree of accuracy obtained from the measurements [73].

Contact	Tracing, overlays and contour
	Depth gauges
	Volume: moulding or liquid
Non-contact	Photography
	Stereophotogrammetry
	Video-image analysis
	Structured light
	Laser triangulation

Table 3-2: Various techniques of wound measurement [73].

As early as 1966, Pories et al. [74] used Jeltrate to make the impression of the wound following excision of the pilonidal sinus. Then the Jeltrate impression was dropped into a graduated cylinder partially filled with water and the volume of displaced water was measured.

Similarly, Covinton et al. [75] described a technique using dental impression-making materials to provide a quantitative measurement of the wound volume. Although this method is feasible, it's time consuming (45 minutes for determining

wound volume). Also the reliability of this method should be checked on clinical trials.

With an isotonic solution, Berg et al. [76] tried to measure the volume of pressure sores. After the sore was covered with a transparent film that follows the skin as much as possible, the isotonic solution was injected through the film. The amount of fluid used to fill the sore was taken as the volume.

It is desirable to have a method for quantifying wound areas with reproducible healing rates at the same time by different examiners. Ahroni et al. [77] traced the ulcer contour onto transparent film, which was scanned later by a hand-held computer scanner, and studied its reproducibility. The standard deviation among the 50 sets of three tracings ranged from 0.19mm^2 to 12.07mm^2 . The computerized wound surface area determination was a reliable method. In their study on diabetic foot ulcer healing, Ahroni et al. [78] applied the wound contour tracings obtained over the four week study interval by two methods to calculate the wound healing rate: the estimated rate of change in wound area and the estimated rate of linear advance of the wound margins. They found that ulcer healing rates and incidence of infection during the four-week study period did not significantly differ with regard to moist versus dry dressing type.

Majeske [79] evaluated intratester and intertester reliability of four methods of measuring wound surface area from transparent film tracings on 31 subjects with venous stasis ulcers. The graph paper, planimeter and digitizer methods gave reliable results and the graph paper technique was recommended because it was low in cost and easy to use.

Bohannon and Pfaller [80] reported the clinical accuracy of three techniques (graph paper counting, planimetry or weighing technique) of determining wound areas after it was recorded on a clear plastic film. For conforming known area, all these techniques were accurate. But for research purpose, the technique's reliability and tolerance level should be determined before using it.

With the observations of venous stasis ulcers ($n = 36$) and decubitus ulcers ($n = 37$), Thomas & Wysocki [81] compared three wound area measurement methods: photographs, wound tracings and Kunding measuring device. Although analysis of the data using Pearson correlation revealed that all three methods were highly correlated, repeated measures analysis of variance among the three methods showed that they were all significantly different from each other. Any of the three methods can be used as a valid index of wound area, but acetate transparent film tracing is the most accurate measure of actual wound area.

Stacey et al. [82] measured the surface area of 99 leg ulcers from a scaled photograph by a computerized ultrasonic digitizer. This method has a good correlation to the method calculating the area by multiplying the two maximal perpendicular diameters. The initial ulcer size was not a convincing predictor of subsequent ulcer healing. The time required for complete ulcer healing also was poorly predicted by the healing rates of individual ulcers calculated over 1 month intervals from presentation. Their study also claimed that the reduction in ulcer size within a set time interval was a poor predictor of eventual ulcer healing. Although the product of the maximal dimensions of an ulcer was deemed by them to be an easy and accurate method of monitoring treatment, it could not give the exact area of wound.

Smith et al. [83] proposed a video image system to record the wound data and to analyze the wound area, volume and color. Although they tried to do the best to minimize the image distortion by manually position the wound in the same horizontal plane and remain perpendicular to the axis of lens, still there was difference caused by the wound position. If the video image were combined with the 3D coordinates, the analysis would be more accurate and more informative.

Liskay et al. [84] compared two techniques, a common paper tape measure and a plastic grid device measure on sixty leg ulcers. The plastic grid proved to be a reliable, valid and feasible method to quickly assess wounds in a busy outpatient setting and it was superior to the tape measure in assessing large and/or irregular ulcers.

Etris et al. [85] compared the measurements of 450 lower extremity ulcers on four-week following up by photo method and tracing method. The wound surface areas measured from both methods were similar and they concluded that both methods were useful when used to evaluate responsiveness to therapy.

Solomon et al. [86] proposed a video image recording method followed by computer analysis to study the size of skin wounds and ulceration. The limb convexity in a two-dimensional image was also considered and incorporated into the method. The accuracy and precision of the technique were better than acetate tracings and photographic methods. Once it was set up, its ongoing cost is minimal.

Goihman-Yahr [87] used CAD to calculate the area and perimeter of ulcers of 10 patients. A microtip pen traced the boundaries on polythene sheets placed over the ulcers. In our viewpoint, the subjective error has been made when the boundary is traced by a pen on the polythene sheets.

Kantor and Margolis [88] verified the correlation between the simple wound measurement and planimetric wound area on 260 patients' venous leg ulcers followed up at least 24 weeks. A tight correlation was found between the planimetric wound area and the simple wound measurements (width, length, width x length, perimeter, and area based on the formula for an ellipse) for those ulcers with an area less than 40 cm². They confidently suggested that the simple wound measurements be used as predictor of healing in clinical settings. Later, with digital planimetry on 104 patients' wound areas, they found the percentage change in area over the first 4 weeks represented a practical and predictive measure of wound healing within 24 weeks [89].

Rajbhandari et al. [90] compared the area traced from a flexigrid Opsite film and a digital photograph for thirty diabetic foot ulcers over ten weeks. The digital image had significantly less interobserver variation than the traditional method with a pen and a transparent film.

Lagan et al. [91] examined the two wound area measurement methods (planimetry and digitizing) on two frequently used tracings (raw tracing and photographic tracing) on 11 wounds. Based on their wound shape and size, they concluded that the direct tracing was an inexpensive clinical and research assessment tool and the digitizing method gave a better repeatability than planimetry in the calculation that followed the direct tracing.

Thawer et al. [92] compared the reliability and validity of a new computerized technique to acetate tracings and planimetry on measurements of 45 chronic human wounds and 38 surgical animal wounds. The computerized technique permitted reliable and valid assessment of the surface area of both human and animal wounds

and the average of three repeated measurement results could improve the precision of both techniques.

Although direct tracing is easy-implemented, the contact is inevitable, which might cause pain, damage or contamination. Simple photographs can only give a limited amount of information concerning the volume when measuring a surface curved in three dimensions. A single photograph is not enough to describe the ulcer quantitatively since a 3D object such as a leg ulcer can never be correctly geometrically represented on a plane—a shortcoming of photography highlighted as an error by Bohannon and Pfaller [80]. Number of researchers turned to get the 3D surfaces of wounds by various tools and techniques.

Eriksson et al. [93] described stereophotogrammetric measurements of leg ulcers. With two cameras mounted on a stable bar, the 3D coordinates of each point were computed, based on which the area and volume could also be derived. In this method, the subjective error could occur when deciding the edge points.

Bulstrode et al. [94] made a portable stereocamera linked to a computer to take photographs of the wound. The system was compared with the other two cutaneous healing rate measurement techniques: direct tracing and simple photography. The stereocamera had a precision of 2% for edge length and 3.4% for area, which was between five and ten times more precise than the other two systems. Only the stereophotogrammetry had errors consistently less than the change being measured. However, the way to measure volume has not been clarified. Later they [95] performed measurement weekly using the computer-linked stereocamera on thirty chronic leg ulcers until complete healing occurred. There was a significant difference in healing rate for the first two weeks between clean ulcers entering the trial directly

and ulcers admitted first for cleansing before joining the trial, which suggested that the weekly healing rate of an ulcer may take up to 2 weeks to respond to a new form of treatment. They found that percentage change in area in the third week was the parameter which gave the earliest close correlation with time to complete healing but absolute ulcer size, change in ulcer size and rate of epithelial migration did not correlate well with time to complete healing. With the percentage change in area, it was found that time to complete healing could be predicted to within one week for 50% of the ulcers, making this a simple and useful early predictor of treatment efficiency.

Hayward et al. [96] developed a new imaging process estimating decubitus ulcers' surface area and volume by Nuclear Magnetic Resonance (NMR) spectroscopy of a mold of the pressure ulcer and compared it to the volume of the mold by water displacement with 17 measurements. A strong correlation was observed between the measurement of the volume of the mold by the computer system and measurement of the volume by water displacement. Surface area of the pressure sore mold correlated strongly with volume of the mold raised to the two-thirds power. In this way, the dose of newly proposed topical treatments might be accurately determined.

Bhat and Smith [97] combined the laser scanner and sound sensor together to get more detailed information of wound surface. The laser scanner registered the surface relief and the sound sensor measured the texture and hardness of the wound surface. Measurements of volume were performed on various shapes of wound on different material simulating depth profiles and surface relief. Although their results seemed accurate, the measurement has not been verified on true skin wound.

Ozturk et al. [98] applied two image-processing methods to compare the effects of different dressings on pig skin wounds. In the first method, the healing process was monitored by area measurements calculated by pixel counting, which showed good correlation with manual area measurements. In the second method, projecting a structured light pattern consisting of colored dots onto the skin and the 3D coordinates could be calculated from 3D image reconstruction from a single camera. This approach mitigated the problem of motion in a subject, since all the coding information was contained in a single image.

Altmeyer et al. [99] applied an interferometry procedure to measure the surface, depth and volume of ulcerous lesions. Two thousand interferometric measurements of volume were performed on objects of known volume. The computer-aided planimetry measured the ulcer area as a flat area regardless of its surface relief but the interferometry considered the relief of the ulcer while measuring area. The interferometry is a non-invasive and non-contact method of area and volume measurement.

By projecting a sequence of parallel stripes onto the surface of lesion, Plassmann et al. [100] observed the image of wound by a camera and processed the image by a computer. After the wound surface was reconstructed by triangulation, the healthy wound surface (the original skin surface) was interpolated by cubic spline. The difference between these two surfaces (the wound surface and the original healthy skin surface) was the volume. If the ratio of volume to area was larger than 4mm, the structure light system could measure the volume of wound with a precision of 5%. Later they made the MAVIS- Measurement of Area and Volume Instrument System, which was based on the principle of colour coded structured light [101]. They

compared the performance of the instrument with three traditional wound measurement techniques using results obtained from a clinical trial involving 50 patients. The MAVIS gave a more reproducible result. However, if the wound was very large and stretched around a limb, the system could not combine multiple images. A very deep wound could not be measured either. Also, the ambient light would influence the measurement.

Schubert and Zander [102] studied the suitability of four different methods (volume, area, perimeter and depth) for measuring pressure ulcer size during 10 consecutive weeks in a group of 11 elderly patients. The methods used for measuring volume and area were more reliable than the methods used for measuring perimeter and depth. Thus the volume and area measurements method was highly recommended. They also found that volume measurement was best suited for deep ulcers and the area measurement was best suited for broad, irregularly shaped ulcers.

Gorin et al. [103] calculated change in area over mean perimeter as an index of wound healing rate because the change in area per day was significantly and independently influenced by initial area ($p < .0001$), perimeter ($p < .0001$), length ($p < .00055$) and width ($p < .0175$). Linear healing per day was not influenced by any geometric variable, including area, perimeter, length, width and ratio of width to length. They concluded that linear healing per day was a valid means of comparing wound healing rates in wounds of different dimensions. Linear healing per unit of time should be preferred to measurements of change in wound area to quantify wound healing rates in clinical trials.

Mayrovitz [104] examined the accuracy of area calculations based on elliptical and rectangular shapes with dimensions based on length and width of 83 plantar ulcers.

Both of these two methods had similar overall errors and he optimized the calculation model to minimize the overall errors.

Lagan et al. [105] compared 4 two-dimensional wound measurement techniques: linear length and width using a ruler, planimetry, computerized stereophotogrammetry (SPG) length and width, and computerized SPG area by 66 raters on three plaster Paris wound models. The measurement with smallest standard error was taken as the most accurate one. The computer area technique was the most accurate techniques because it had the smallest standard error of measurement. Only the SPG area measurements for single ratings were found reliable enough for clinical or research purposes.

Marjanovic et al. [106] captured the wound surface with a laser displacement sensor. For each scanned line, they used cubic splines to reconstruct the surface of wound from its neighborhood. Then the difference (the volume sandwiched between the wound surface and its previous healthy skin surface) was given as the volume of the wound. They showed the volume changes of two venous ulcers over time. However, how to locate the edge of the wound was not presented. Earlier Ibbett et al. [107] did similar measurements with a laser displacement sensor and reconstructed the healthy skin surface by cubic splines, in which the edge was localized manually in an image processing software. There were two distinct problems when calculating the volume of wound when measured by a non-contact tool such as a laser scanner [108]. First problem was to localize the edge, manually or automatically. Second one was the way to reconstruct the healthy skin surface and calculate the volume.

With a three-camera vision system, Boersma et al. [109] reconstructed the wound surface and studied its accuracy that was decided mainly by the calibration of the system. A precision of better than 0.7mm could be achieved in the digital surface

model generation using texture projection. But this system cannot deal with the wound surface with a steep edge. Furthermore, how to detect the edge of the wound is not specified.

Langemo et al. [110] compared 2 wound volume measurement techniques, the Kundin device and stereophotogrammetry, on an L-shaped and a pear-shaped plaster of Paris wound model. The interrater and intrarater reliability and the bias and standard error of measurement were assessed. Although both techniques had acceptable accuracy, stereophotogrammetry was more accurate and had more clinical applications.

Kroushop et al. [111] projected laser dots on the surface of the wound and took photographs of the wound. By summing areas associated with each laser dot within the wound, the wound area could be calculated. With a six-coefficient second order polynomial, the original skin surface was estimated so that the volume could be derived. But this estimation assumed that the surface was originally smooth and had only simple curvature in two orthogonal directions. It was inaccurate on the wounds with more complicated surfaces. Furthermore, the border of the wound was determined with the involvement of human judgement, which might induce subjective errors.

The most effective way of monitoring the progression to healing was to calculate the percentage reduction of wound area over time, particularly within the first four weeks [112]. Flanagan assessed the efficacy of current wound measurement techniques and found that in differentiation between wounds of responding to treatment, the true area measurement and percentage of area reduction of wounds were useful parameters [113]. Identifying the boundary of the wound was crucial in tracing

and calculating wound surface area, which accounted for the greatest errors made in healing rate determination. A 20% to 40% reduction of wound area in 2 and 4 weeks served well as a reliable predictive indicator of healing, especially for venous leg ulcers. He suggested the development of a standardized method for measuring wounds and reporting wound healing rates should be the focus of future research.

CHAPTER 4

CURVATURE CALCULATION AND ACCURACY

In section 3.1, after a brief description of principle curvatures, mean and Gaussian curvatures, the mathematical method of calculating curvature from unevenly distributed data is presented in details. The problems encountered by surface fitting method mentioned in section 3.1 and how the rotating surface fitting or normal direction surface fitting solve them are shown in section 3.2. The accuracy is studied in section 3.3 as well as how the measurement errors propagate to the curvature calculation and how to reduce the system error.

3.1 CURVATURE CALCULATION

Taylor's theorem gives

$$\begin{aligned}
 \mathbf{x}(u + du, v + dv) &= \mathbf{x}(u, v) + d\mathbf{x} + \frac{1}{2}d^2\mathbf{x} + o(du^2 + dv^2) \\
 &= \mathbf{x}(u, v) + \mathbf{x}_u du + \mathbf{x}_v dv + (\mathbf{x}_{uu} du^2 + 2\mathbf{x}_{uv} dudv + \mathbf{x}_{vv} dv^2)/2 + o(du^2 + dv^2) \quad (3-1) \\
 &= \mathbf{x}_{00} + \mathbf{x}_u du + \mathbf{x}_v dv + (\mathbf{x}_{uu} du^2 + 2\mathbf{x}_{uv} dudv + \mathbf{x}_{vv} dv^2)/2 + o(du^2 + dv^2)
 \end{aligned}$$

This can be used to approximate the surface in the neighborhood of $\mathbf{x}$ by omitting those after the quadratic terms. The coefficients $\mathbf{x}_u, \mathbf{x}_v, \mathbf{x}_{uu}, \mathbf{x}_{uv}, \mathbf{x}_{vv}$ are the first and second derivatives of $\mathbf{x}$ with respect to u, v at surface point $\mathbf{x}_{00} = \mathbf{x}(u, v)$. With these first and second derivatives, we can calculate the principal curvatures and their directions. These two principal curvatures, say κ_1 and κ_2 , in two perpendicular directions, are maximum and minimum values of normal curvature. These quantities

are well suited for an invariant shape analysis. The average of the κ_1, κ_2 is called the mean curvature (H). The product of κ_1, κ_2 is called the Gaussian curvature (K). K is a sign for different cases of shapes.

Figure 4-1: Different shapes according to K. (a) plane. (b) cylinder/parabolic areas. (c) sphere/elliptic areas. (d) saddle shape/hyperbolic areas.

From Figure 4-1 one can see that the sign of the Gaussian curvature enables elliptic areas to be distinguished from hyperbolic areas. Similarly in the case of parabolic and elliptic areas one can easily discriminate convexity from concavity by the sign of mean curvature. It is easy to calculate the Gaussian curvature and mean curvature once the analytical surface in form of $\mathbf{x}=\mathbf{x}(u,v)$ has been given. But from the experiment, we could only get some discrete data, (x, y, z) coordinates. Now we consider how to

derive those curvatures from randomly distributed data. We consider $z=f(x, y)$. Because the surface is only described at a finite number of collected data, interpolation is not very accurate. For this reason and for further analysis, a surface represented by a set of regular grid data is desirable. To extract a grid point from the scattered data, we need to consider fitting a small second order polynomial surface patch for this grid point and other points in the neighborhood. A set of regular grid points has the following form: $x_{ik} = g \cdot i$, $y_{ik} = g \cdot k$, $z_{ik} = f(x_{ik}, y_{ik})$, where i and k are grid indices and g is grid constant. We will transform our randomly distributed data into such kind of regular grid data by introducing a polynomial

$$z = c_1 + c_2p + c_3q + c_4p^2 + c_5pq + c_6q^2 \quad (3-2)$$

where

$$p = x - x_{ik} \quad q = y - y_{ik} \quad (3-3)$$

x_{ik} and y_{ik} is the x and y coordinate of the center point of the small patch. This means we employ the local coordinate system to perform our fitting procedure. For reasons of analytical convenience, the origin of the local coordinate is chosen to coincide with the center point of the small patch.

After measuring the surface, we get a set of data in the form of (x_n, y_n, z_n) , n ranging from 1 to N . For each data point in the patch, we have the measured z value z_n and the calculated value z'_n from equation (3-2).

$$z'_1 = c_1 + c_2p_1 + c_3q_1 + c_4p_1^2 + c_5p_1q_1 + c_6q_1^2 \quad (3-4)$$

$$z'_2 = c_1 + c_2p_2 + c_3q_2 + c_4p_2^2 + c_5p_2q_2 + c_6q_2^2 \quad (3-5)$$

$\vdots$

$$z'_n = c_1 + c_2 p_n + c_3 q_n + c_4 p_n^2 + c_5 p_n q_n + c_6 q_n^2 \quad (3-6)$$

$$\begin{Bmatrix} z'_1 \\ z'_2 \\ \vdots \\ z'_n \end{Bmatrix} = \begin{bmatrix} 1 & p_1 & q_1 & p_1^2 & p_1 q_1 & q_1^2 \\ 1 & p_2 & q_2 & p_2^2 & p_2 q_2 & q_2^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & p_n & q_n & p_n^2 & p_n q_n & q_n^2 \end{bmatrix} \begin{Bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \end{Bmatrix} \quad (3-7)$$

We aim to get those coefficients c_1 , c_2 , c_3 , c_4 , c_5 and c_6 in such a way that they make the discrepancy between the fitting surface patch and those discrete data minimum. That is equivalent to minimize the differences between z'_n (the calculated z value) and z_n (the measured z value). This is obtained by least squares fitting of polynomial surface. In our case, the surface is to be determined by equation (3-2). At each data point, the difference between the surface elevation and data's z coordinates is $z'_n - z_n$. We then adjust z'_n by choice of those coefficients to minimize

$$E(z) = \sum_{n=1}^N (z'_n - z_n)^2 = \sum_{n=1}^N (z(p_n, q_n) - z_n)^2 \quad (3-8)$$

where N is the number of the points within the small patch selected for patch fitting.

A weighting function is used to put weightings on those points with respect to their distances to the grid point (the center point). The weightings drop off with increasing distance from the center point thus the weightings of the points are not equal over the whole area. We introduce a diagonal weighting matrix $\mathbf{G}$ to meet the requirement.

$$\mathbf{G} = \text{Diag}(g_1, g_2, \dots, g_n) \quad (3-9)$$

$$g_i = f \cdot (\exp(-(p_i^2 + q_i^2)/d^2)) \quad i=1, 2, \dots, n \quad (3-10)$$

d is the grid spacing constant. f is the factor adjusted with respect to the point's distance to the center point.

Then equation (3-8) becomes

$$E(z) = \sum_{n=1}^N g_n (z(p_n, q_n) - z_n)^2 \quad (3-11)$$

The function E of c_1, c_2, c_3, c_4, c_5 and c_6 will have a minimum only when

$$\partial E / \partial c_i = 0, \text{ for } i=1, 2, 3, \dots, 6. \quad (3-12)$$

These conditions yield six linear equations in the unknowns c_1, c_2, c_3, c_4, c_5 and c_6 (the normal equations).

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) = 0 \quad (3-13)$$

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) p_n = 0 \quad (3-14)$$

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) q_n = 0 \quad (3-15)$$

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) p_n^2 = 0 \quad (3-16)$$

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) p_n q_n = 0 \quad (3-17)$$

$$\sum_{n=1}^N 2g_n (z(p_n, q_n) - z_n) q_n^2 = 0 \quad (3-18)$$

Rearranging these equations, we get:

$$\sum_{n=1}^N g_n z(p_n, q_n) = \sum_{n=1}^N g_n z_n \quad (3-19)$$

$$\sum_{n=1}^N g_n z(p_n, q_n) p_n = \sum_{n=1}^N g_n z_n p_n \quad (3-20)$$

$$\sum_{n=1}^N g_n z(p_n, q_n) q_n = \sum_{n=1}^N g_n z_n q_n \quad (3-21)$$

$$\sum_{n=1}^N g_n z(p_n, q_n) p_n^2 = \sum_{n=1}^N g_n z_n p_n^2 \quad (3-22)$$

$$\sum_{n=1}^N g_n z(p_n, q_n) p_n q_n = \sum_{n=1}^N g_n z_n p_n q_n \quad (3-23)$$

$$\sum_{n=1}^N g_n z(p_n, q_n) q_n^2 = \sum_{n=1}^N g_n z_n q_n^2 \quad (3-24)$$

Solving these six linear equations, we can get the six coefficients c_1 , c_2 , c_3 , c_4 , c_5 and c_6 .

The technique of normal equations can be performed conveniently in matrix-vector form

$$\mathbf{z} + \mathbf{r} = \mathbf{z}' \quad (3-25)$$

$$\mathbf{z}' = \mathbf{A}\mathbf{C} \quad (3-26)$$

where

$$\mathbf{z} = (z_1, z_2, \dots, z_n)^T \quad (3-27)$$

$$\mathbf{z}' = (z'_1, z'_2, \dots, z'_n)^T \quad (3-28)$$

$$\mathbf{r} = (r_1, r_2, \dots, r_n)^T \quad (3-29)$$

$$\mathbf{C} = (c_1, c_2, c_3, c_4, c_5, c_6)^T \quad (3-30)$$

$$\mathbf{A} = \begin{bmatrix} 1 & p_1 & q_1 & p_1^2 & p_1 q_1 & q_1^2 \\ 1 & p_2 & q_2 & p_2^2 & p_2 q_2 & q_2^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & p_n & q_n & p_n^2 & p_n q_n & q_n^2 \end{bmatrix} \quad (3-31)$$

are the vectors of the measured z coordinates of data, the calculated z coordinates, the residuals ($r_n = z'_n - z_n$), the polynomial coefficients and the matrix containing the x , y differences between the grid point and its neighborhood points respectively.

We can optimize the coefficients by minimizing the weighted residuals.

$$\mathbf{r}^T \mathbf{G} \mathbf{r} = \text{minimum} \quad (3-32)$$

From equation (3-25), we arrive at the normal equations

$$\mathbf{N} \mathbf{C} = \mathbf{A}^T \mathbf{G} \mathbf{z} \quad (3-33)$$

with $\mathbf{N}$ being the normal equation matrix

$$\mathbf{N} = \mathbf{A}^T \mathbf{G} \mathbf{A} \quad (3-34)$$

Solving equation (3-33) by multiplying the inverse matrix $\mathbf{N}^{-1}$ on both sides, we get

$$\mathbf{C} = \mathbf{N}^{-1} \mathbf{A}^T \mathbf{Gz} \quad (3-35)$$

Thus, we fit the second order polynomial surface patch in a small area around the center point by way of least squares method. At the same time the irregular data points have been transformed into points on the intersections of grid. The first component of $\mathbf{C}$ is the z value of the grid point. The other component of $\mathbf{C}$ can be interpreted as the coefficients x_p, x_q etc. which are the first and second derivatives of $\mathbf{x}$ with respect to p and q at surface point (x_{ik}, y_{ik}, z_{ik}) . With these partial derivatives, we can calculate the principal curvatures, the Gaussian curvature and the mean curvature [9, 11, 114].

Since the vector $\mathbf{C}$ has six components, at least six points within the neighborhood of the center point are needed to get those coefficients. And for the redundant reason in fitting the six-coefficient polynomial (3-2), the area surrounding the selected center point should contain at least 10 points (based on trial-and-error and the scanning point density). Otherwise the point selected as the center point is discarded. The area with a small number of points also causes the determinant of $\mathbf{N}$ to be near zero. Then the vector $\mathbf{C}$ calculated from equation (3-35) is not reliable since the calculation involves computing the inverse matrix of a nearly singular matrix. So a threshold with more than 10 points inside the calculation area must be set.

3.2 SURFACE FITTING

3.2.1 ROTATING SURFACE FITTING

The method introduced in section 3.1 has certain limitations when it comes to calculate the cylindraceous body surface such as ankles.

1. It could only fit the surface patch by patch and the patch normal should be almost in the z direction (that means the patch should be almost parallel to the x-y plane). If not, the patch would have to be first rotated to meet that requirement.

2. This patch fitting has problems on the edge (Figure 4-2) due to lack of enough points for surface fitting or due to the abrupt altitude changes (on the edge the patch normal is almost perpendicular to z axis). Thus, the possible landmark candidate on the edge would have been possibly eliminated as well while setting threshold to eliminate edge problems.

Figure 4-2: The reconstructed outer malleolus surface. Units: mm.

In Figure 4-2, the z axis is penetrating the outer malleolus from inside to outside. This position is ideal for surface fitting. Note that at the edge of the patch, there are some points with exceptional values of z coordinates due to an unbalanced point distribution and undetermined linear equations where the operator hasn't enough points.

3. After we get the cylindraceous surface data, when performing surface fitting, the z axis will penetrate the surface twice, causing problems to select vicinal points for the two penetrating points. For one penetrating point, it is possible to select the other penetrating point's neighbour points for calculation, which results in wrong surface fitting.

A rotating surface fitting is introduced to accomplish surface fitting on a cylindraceous surface (here, the ankle region). The ankle region is taken as a deformed cylinder. First the world coordinate system origin is moved into the ankle if the origin is not inside it. The coordinate system transformation will put the x axis approximately parallel to the axis of the deformed cylinder with the z axis penetrating the surface of the ankle region. For this position (the ideal position for surface fitting), the surface fitting is carried out to the small patches around the intersection with the z axis. Then the world coordinate system is rotated around the x axis inside the ankle region from 0 to 2π with a certain step. After that, the origin will move along the x axis to the next position and for each origin, the rotation around the x axis and the surface fitting are repeated. This procedure could be imagined as a moving point of light source (the origin) emitting a laser beam (the z axis). The point of light source is moving along the axis of a cylinder (the x axis) and at each position, the laser beam will rotate about the cylinder axis penetrating the cylinder (the area for surface fitting). Then the point of

light source moves to the next position and rotates again until the light covers the whole cylinder. After the whole ankle region is covered, we will get a map of surface curvatures.

3.2.2 SURFACE FITTING IN NORMAL DIRECTION

Besides the 3D coordinates of the surface point, if the laser scanner could estimate the normal direction at each surface point, the local surface fitting could be calculated at each randomly distributed point. This could achieve the maximum resolution since modern laser scanner could register a surface with a large point density. But the time of calculation would be increased.

The way to perform surface fitting in the normal direction is as follows:

First a surface point is located as the center of surface fitting.

Then the normal direction of that point is taken as the z axis of a local coordinate system. The x and y axes of the local coordinate system are built in an arbitrary way because we are just aiming to fit the surface in the normal direction. All the points in the world coordinate system would be transformed into the local coordinate system by a transformation matrix. The next step would be the same as we did before in section 3.2.1 by taking the local origin as the center and searching for its adjacent points.

3.3 ACCURACY AND ERROR STUDY

3.3.1 ACCURACY TEST OF DATA COLLECTION AND CURVATURE CALCULATION

A test model made of plastic was designed with the surface tolerance of $\pm 5 \mu\text{m}$ to find out the accuracy of the measurement. It provides a cylindrical concavity (radius 60 mm), a spherical concavity (radius 60 mm) and a plane (Figure 4-3) respectively within one body.

Figure 4-3: The test model with cylinder and sphere on the front and plane on the back. Units: mm. The surface tolerance is $\pm 5 \mu\text{m}$.

Since the accuracy of data collection varies according to different environment (light, position, distance and etc.), a plane defined by

$$z = c_0 + c_1x + c_2y \quad (3-36)$$

is fitted to the scattered points on the test model's plane to estimate the accuracy of data collection in our measurement. By adjusting the coefficients c_0 , c_1 , c_2 , the difference between the experimentally measured z coordinates and the calculated z coordinates is minimized. The sum of the difference is equal to

$$E = \sum_{i=1}^n (z_c - z_i)^2 = \sum_{i=1}^n (c_0 + c_1x_i + c_2y_i - z_i)^2 \quad (3-37)$$

where z_c is the calculated z coordinate and z_i is the experimentally measured z coordinate. n is the number of the points involved in the plane fitting.

The function E has a minimum value only when

$$\frac{\partial E}{\partial c_i} = 0, \quad i=0, 1, 2. \quad (3-38)$$

From this, we get three normal equations, yielding three coefficients

$$nc_0 + c_1 \sum_{i=1}^n x_i + c_2 \sum_{i=1}^n y_i = \sum_{i=1}^n z_i \quad (3-39)$$

$$c_0 \sum_{i=1}^n x_i + c_1 \sum_{i=1}^n x_i^2 + c_2 \sum_{i=1}^n x_i y_i = \sum_{i=1}^n x_i z_i \quad (3-40)$$

$$c_0 \sum_{i=1}^n y_i + c_1 \sum_{i=1}^n x_i y_i + c_2 \sum_{i=1}^n y_i^2 = \sum_{i=1}^n y_i z_i \quad (3-41)$$

Once the coefficients have been known, the sum of difference can be calculated.

Since we have 3 independent variables, the degree of freedom is $n-3$. The error in the z direction is defined as

$$\text{Err}_z = \sqrt{\frac{E}{n-3}} \quad (3-42)$$

And the error in the normal direction (Err_n) can be calculated from Err_z .

$$\text{Err}_n = \text{Err}_z \cos \theta \quad (3-43)$$

where θ is the angle between the normal vector of the plane and the z -axis of the global coordinate system defined.

We assume the measurement of the sphere and cylinder share the same accuracy with the plane. The accuracy of curvature calculation is evaluated by comparing the calculated curvatures with the ones derived from the known radii.

From all the representative points obtained from the curvature calculation, the difference between the known and calculated curvatures was evaluated by mean error and its standard deviation.

Fitting a plane to the planar measurement points results in a mean error of about 0.02 mm for 50 mm x 50 mm area (Table 4-1) in the normal direction, providing a figure of the overall measurement accuracy.

Area for plane fitting	Mean error (mm)
10mmx10mm	5.60×10^{-3}
20mmx20mm	1.03×10^{-2}
50mmx50mm	2.08×10^{-2}

Table 4-1: The mean error of measurement on a plane by FastSCAN

	Sphere		Cylinder		Plane	
	MC (mm ⁻¹)	GC (mm ⁻²)	MC (mm ⁻¹)	GC (mm ⁻²)	MC (mm ⁻¹)	GC (mm ⁻²)
Manufactured value	1.67x10 ⁻²	2.78x10 ⁻⁴	8.33x10 ⁻³	0	0	0
	Mean error ± standard deviation					
Point density 40 points/cm ²	2.44x10 ⁻⁴ ± 1.30x10 ⁻³	8.94x10 ⁻⁶ ± 4.40x10 ⁻⁵	3.71x10 ⁻⁴ ± 9.56x10 ⁻⁴	2.36x10 ⁻⁶ ± 1.71x10 ⁻⁵	5.57x10 ⁻⁵ ± 5.08x10 ⁻⁴	-2.43x10 ⁻⁸ ± 4.55x10 ⁻⁷

Table 4-2: Evaluation of the test model by comparing the calculated curvatures with the ones derived from known radii. The curvatures are calculated from the whole machined surface for the mean error and standard deviation. (MC: Mean Curvature, GC: Gaussian Curvature)

Accuracy figures for the mean and Gaussian curvature are listed in Table 4-2. The data are obtained by fitting measurements of the cylindrical and spherical test body under the same scanning environment that was used subsequently for the foot. The point density of the scanning is 40 points/cm². The curvatures are calculated on the sample points with a distance of 5mm among them. Then the mean of the errors and standard deviations are tabulated in Table 4-2. The mean error is at least an order of magnitude less than the manufactured curvature. The small mean error confirms the accuracy of our method.

3.3.2 ACCURACY TEST BETWEEN THIRD ORDER POLYNOMIAL VERSUS SECOND ORDER POLYNOMIAL

Examining the Taylor theorem gives us the instinct feeling that if we perform local surface fitting with a third order polynomial,

$$z = c_1 + c_2p + c_3q + c_4p^2 + c_5pq + c_6q^2 + c_7p^3 + c_8p^2q + c_9pq^2 + c_{10}q^3 \quad (3-44)$$

it would have been more accurate for the curvature evaluation because the error of surface fitting is the remaining fourth and higher terms. A third order polynomial

means more expanding items of Taylor theorem and omits fewer items than those omitted by a second order polynomial. The sphere on the test model is used in the comparison. All the surface points' Gaussian and mean curvature within a 40mmx40mm square whose center is the same as the apex of the sphere projected on the plane are calculated and compared with the manufactured value. The mean error and standard deviation are listed in Table 4-3. The table shows a smaller mean error and standard deviation from the second order polynomial surface fitting. The calculation time is almost the same.

Error	2 nd order		3 rd order	
	Gaussian curvature (2 nd) (mm ⁻²) x 10 ⁻⁵	Mean curvature (2 nd) (mm ⁻¹) x 10 ⁻⁴	Gaussian curvature (3 rd) (mm ⁻²) x 10 ⁻⁵	Mean curvature (3 rd) (mm ⁻¹) x 10 ⁻⁴
Mean	1.0797	3.2368	1.0855	3.2603
σ	4.4079	12.8100	4.4449	12.9244

Table 4-3: The mean error calculated from 2nd and 3rd order polynomial surface fitting.

Each point's Gaussian and mean curvature error are plotted in Figure 4-4. Red and blue stand for each point's error from second and third polynomial local surface fitting, respectively. t test with correction for correlated samples is performed on the points' errors to see whether there is any significant difference between the two means. There is no significant difference ($p=0.10$) in the error made by fitting the surface with two different order polynomials.

Second order polynomial fitting is used throughout the calculations in this method. This is not only because curvatures are calculated from second derivative but also when a surface fitting with third or higher order polynomial is performed, the noise might make the surface fitting error slightly larger because the noise is also well fitted. The third order polynomial fitting will not ensure a more accurate surface fitting and curvature calculation.

a)

b)

Figure 4-4: Error of Gaussian (a) and mean curvature (b) calculation for each point by 2^{nd} (red) and 3^{rd} (blue) polynomial surface fitting.

3.3.3 ERROR PROPAGATION

In order to create a linear error propagation model, the relation between the errors, the errors of coefficients (ΔC) and the errors of measurement on z coordinates (Δz), are modeled.

1) From z coordinate to the coefficients of the fitted surface

The error propagation from z to C is derived from equation (3-35)

$$\Delta C = N^{-1} A^T G \Delta z \quad (3-45)$$

Then the covariance matrix of C is

$$\text{Cov}(C) = E(\Delta C \Delta C^t) = E((N^{-1} A^T G \Delta z)(N^{-1} A^T G \Delta z)^t) = B E(\Delta z \Delta z^t) B^t = B \sigma_z^2 B^t \quad (3-46)$$

where

$$B = N^{-1} A^T G \quad (3-47)$$

and σ_z is the variance of z coordinates.

2) From the coefficients of the fitted surface to curvature (The first fundamental coefficients E , F , G and second fundamental coefficients L , M , N are calculated from partial derivatives from equation (3-2)).

$$E = 1 + c_2^2 \quad (3-48)$$

$$\Delta E = \frac{\partial E}{\partial c_2} \Delta c_2 = 2c_2 \Delta c_2 \quad (3-49)$$

$$F = c_2 c_3 \quad (3-50)$$

$$\Delta F = \frac{\partial F}{\partial c_2} \Delta c_2 + \frac{\partial F}{\partial c_3} \Delta c_3 = c_3 \Delta c_2 + c_2 \Delta c_3 \quad (3-51)$$

$$G = 1 + c_4^2 \quad (3-52)$$

$$\Delta G = \frac{\partial G}{\partial c_4} \Delta c_4 = 2c_4 \Delta c_4 \quad (3-53)$$

$$L = \frac{2c_4}{\sqrt{c_2^2 + c_3^2 + 1}} \quad (3-54)$$

$$\Delta L = \frac{\partial L}{\partial c_4} \Delta c_4 + \frac{\partial L}{\partial c_2} \Delta c_2 + \frac{\partial L}{\partial c_3} \Delta c_3 = \frac{2\Delta c_4}{\sqrt{c_2^2 + c_3^2 + 1}} - \frac{2c_4 c_2 \Delta c_2}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} - \frac{2c_4 c_3 \Delta c_3}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} \quad (3-55)$$

$$M = \frac{c_5}{\sqrt{c_2^2 + c_3^2 + 1}} \quad (3-56)$$

$$\Delta M = \frac{\partial M}{\partial c_5} \Delta c_5 + \frac{\partial M}{\partial c_2} \Delta c_2 + \frac{\partial M}{\partial c_3} \Delta c_3 = \frac{\Delta c_5}{\sqrt{c_2^2 + c_3^2 + 1}} - \frac{c_5 c_2 \Delta c_2}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} - \frac{c_5 c_3 \Delta c_3}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} \quad (3-57)$$

$$N = \frac{2c_6}{\sqrt{c_2^2 + c_3^2 + 1}} \quad (3-58)$$

$$\Delta N = \frac{\partial N}{\partial c_6} \Delta c_6 + \frac{\partial N}{\partial c_2} \Delta c_2 + \frac{\partial N}{\partial c_3} \Delta c_3 = \frac{2\Delta c_6}{\sqrt{c_2^2 + c_3^2 + 1}} - \frac{2c_6 c_2 \Delta c_2}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} - \frac{2c_6 c_3 \Delta c_3}{\sqrt{(c_2^2 + c_3^2 + 1)^3}} \quad (3-59)$$

$$K = \frac{LN - M^2}{EG - F^2} \quad (3-60)$$

$$\begin{aligned} \Delta K &= \frac{\partial K}{\partial L} \Delta L + \frac{\partial K}{\partial N} \Delta N + \frac{\partial K}{\partial M} \Delta M + \frac{\partial K}{\partial E} \Delta E + \frac{\partial K}{\partial G} \Delta G + \frac{\partial K}{\partial F} \Delta F \\ &= \frac{N}{EG - F^2} \Delta L + \frac{L}{EG - F^2} \Delta N - \frac{2M}{EG - F^2} \Delta M - \frac{GLN - GM^2}{(EG - F^2)^2} \Delta E \\ &\quad - \frac{ELN - EM^2}{(EG - F^2)^2} \Delta G + \frac{2FNL - 2FM^2}{(EG - F^2)^2} \Delta F \\ &= \mathbf{K}_c \Delta \mathbf{C} \end{aligned} \quad (3-61)$$

$$\sigma_K^2 = E(\Delta K \Delta K^t) = E[(\mathbf{K}_c \Delta \mathbf{C})(\mathbf{K}_c \Delta \mathbf{C})^t] = \mathbf{K}_c \mathbf{E}(\Delta \mathbf{C} \Delta \mathbf{C}^t) \mathbf{K}_c^t = \mathbf{K}_c \text{Cov}(\mathbf{C}) \mathbf{K}_c^t \quad (3-62)$$

Substituting $\text{Cov}(\mathbf{C})$ from equation (3-46) into equation (3-62) gives

$$\sigma_K^2 = \mathbf{K}_c \mathbf{B} \sigma_z^2 \mathbf{B}^t \mathbf{K}_c^t \quad (3-63)$$

σ_K is the variance of curvature. This shows how noise in the data contributes to the variance of the estimated curvature. The variance of the curvature depends upon not only noise variance of data but also partial derivatives of the polynomial.

3.3.4 SYSTEM ERROR AND ITS COMPENSATION

When fitting a second order polynomial to patches for the curvature, there is a bias between the calculated and ideal curvature.

Let us denote the ideal Gaussian curvature as K_I , calculated from the inverse of the radius of a sphere, which is assumed to be known. The Gaussian curvature calculated from the surface fitting is denoted by K_c . The resultant Gaussian curvature from the data with noise is denoted by K_r .

Then we have the following relationships:

$$K_c = K_I + \varepsilon_s \quad (3-64)$$

$$K_r = K_c + \varepsilon_n \quad (3-65)$$

Here ε_s and ε_n stand for systematic error caused by the surface polynomial fitting to noiseless data and the error caused by the data with noise respectively.

If we could estimate the systematic error ε_s caused by the method itself, we could eliminate its influence as much as possible [115].

Assuming ε_s is a linear combination of c_2 to c_6 ,

$$\varepsilon_s = \lambda_1 + \lambda_2 c_2 + \lambda_3 c_3 + \lambda_4 c_4 + \lambda_5 c_5 + \lambda_6 c_6 \quad (3-66)$$

then for j th data, we have

$$\lambda_1 + \lambda_2 c_{j2} + \lambda_3 c_{j3} + \lambda_4 c_{j4} + \lambda_5 c_{j5} + \lambda_6 c_{j6} = K_c - K_I \quad (3-67)$$

Then our aim is to minimize the energy function

$$E = \sum_{j=1}^n v_j^2 = \sum_{j=1}^n [\lambda_1 + \lambda_2 c_{j2} + \lambda_3 c_{j3} + \lambda_4 c_{j4} + \lambda_5 c_{j5} + \lambda_6 c_{j6} - (K_{jc} - K_I)]^2 = \mathbf{v}^T \mathbf{v} \quad (3-68)$$

by solving the equations

$$\frac{\partial E}{\partial \lambda_i} = 0, i=1, 2, 3, 4, 5, 6. \quad (3-69)$$

$$\boldsymbol{\lambda} = \mathbf{M}^{-1} \mathbf{T}^T \mathbf{y} \quad (3-70)$$

where

$$\lambda = [\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6]^T \quad (3-71)$$

$$\mathbf{M} = \mathbf{T}^T \mathbf{T} \quad (3-72)$$

$$\mathbf{T} = \begin{bmatrix} 1 & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ 1 & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & c_{j2} & c_{j3} & c_{j4} & c_{j5} & c_{j6} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & c_{n2} & c_{n3} & c_{n4} & c_{n5} & c_{n6} \end{bmatrix} \quad (3-73)$$

$$\mathbf{y} = [K_{1c}, K_{2c}, \dots, K_{jc}, \dots, K_{nc}]^T - K_I \quad (3-74)$$

Once we know the system error, we could calculate the curvatures again and subtract the system errors from the results. By this way, more accurate curvatures can be obtained. The system error varies for some reasons such as ways of data distribution and point density. So each time the system error for the specific scanning data should be calculated and subtracted to minimize the errors of curvature derivation.

The errors with and without compensation calculated from some sample data are shown in Figure 4-5. The sample data are a sphere surface of radius 10mm generated by computer, the scanning of sphere and cylinder on the test model, respectively. After subtraction of system error, the error is minimized to about zero and the curvature is much more closer to the ideal value.

In each sample surface, the Gaussian curvature is calculated and the system error (ϵ_s) is subtracted. A drastically decrease is reflected from the different values of error represented by the red and blue dots. The root mean squares of the errors of all these

points for each test surface are tabulated in Table 4-4. This further indicates the system error subtraction decreases the errors of curvature calculation.

Root mean squares of the errors	Without bias compensation (mm ⁻²)	With bias compensation (mm ⁻²)
Computer generated sphere	7.63×10^{-6}	1.93×10^{-6}
Scanned sphere model	4.48×10^{-4}	2.29×10^{-4}
Scanned cylinder model	0.0167	1.43×10^{-4}

Table 4-4: The root mean squares of the errors of all the points for different surfaces with and without bias compensation.

(a)

(b)

(c)

Figure 4-5: The comparison between the results with (red) and without (blue) compensation. (a): computer generated sphere surface. (b): scanned sphere surface. (c): scanned cylinder surface.

CHAPTER 5

MATERIALS AND EXPERIMENT

The laser scanner FastSCAN and its scanning principles are described in section 4.1.1 followed by the process of making the plaster foot and how the surface data is collected in section 4.1.2. After the surface is scanned, section 4.2 clarifies how the Matlab program processes the data and characterizes the surface. Finally, in section 4.3, the outer malleolus region is taken as an example and the characteristic shape configurations are analyzed. Different ways of localizing the anatomical landmarks are explained step by step, after which comparison is made between the method of center of gravity and Gaussian hat.

4.1 SURFACE REGISTRATION

4.1.1 FASTSCAN

Measurement was done by the FastSCAN Scanner (Polhemus, Colchester, Vermont, USA). FastSCAN (the specification is shown in Table 5-1) is a fast and convenient way to scan object surfaces that provides 3D coordinates for the surface points. The measurement is done simply by sweeping the FastSCAN Magic Wand slowly and smoothly over the surface, as if spray painting on it. The Magic Wand consists of a centrally mounted diode line laser generator and two miniature cameras. The laser is cast on the surface and the intersection is viewed by the cameras from two different angles (Figure 5-1). A tracker operates a transmitter and two receivers (only one receiver is shown in Figure 5-1 for simplification purpose) to calculate the position and orientation of the Magic Wand, thus allowing the computer to reconstruct the

surface. The three-dimensional image is displayed immediately on the computer screen and the data file can later be stored and exported using different formats.

Figure 5-1: The principles of measuring object's surface by FastSCAN.

In Figure 5-1, a laser beam is projected on the object's surface (intersect with object's surface) and viewed by two cameras from different angles. The two cameras' position has already been exactly known. With the method of triangulation, the beam's relative position to the cameras can be calculated. At the same time, with the transformation matrix T , the position of the receiver on the wand can be detected in a magnetic field generated by the transmitter (the world coordinate system). By knowing the relative position and orientation between the receiver and the two cameras, which

has already been calibrated by the manufacturer, the surface coordinates in the world coordinate system can be calculated at the same time when the surface is scanned.

Figure 5-2: The experimental set-ups and the position of the leg while scanning for tibial torsion and tibial nail. The leg is in the neutral position. The receiver is attached to the leg to eliminate the influence of involuntary movement.

FastSCAN	
Processing Unit	<ul style="list-style-type: none"> •Custom electronics video processor •Polhemus FASTRAK® magnetic tracker
Wand	<ul style="list-style-type: none"> •450 mm (18 in) in length •2 miniature monochrome cameras •1 laser diode: 670 nm, 1 mW, Class II •Tracker Receiver
Reference System	<ul style="list-style-type: none"> •Tracker Transmitter •Tracker Receiver for attachment to movable objects
Resolution	<ul style="list-style-type: none"> •Laser line length increases with Wand-

	<p>Object range, typically 150 mm at 200 mm range (6 in at 8 in)</p> <ul style="list-style-type: none"> •Resolution along the laser line depends on Wand-Object range, typically 0.5 mm at 200 mm range (0.02 in at 8 in) •Scanning speed is 50 lines/sec, line-to-line resolution depends on scanning speed, typically 1 mm at 50 mm/sec (0.04 in at 2 in/sec). Resolution
Range	<ul style="list-style-type: none"> •Up to 760 mm (30 in) Wand-Transmitter and Receiver-Transmitter range •75 mm to 680 mm (3 in to 27 in) Wand-Object range
Accuracy	<ul style="list-style-type: none"> •With Transmitter as Reference, accuracy depends on Wand -Object range, typically 1 mm at 200 mm range (0.04 in at 8 in) •With Receiver as Reference, accuracy also depends on Receiver-Object range
Environment	<ul style="list-style-type: none"> •Operation in the presence of metal objects or electromagnetic fields may interfere with the scanner's tracking and degrade performance. Providing the Transmitter, Wand and Receiver are kept sufficiently far from metal objects or fields they should not pose a problem. •Because scanning relies on the cameras seeing the laser line, some surfaces may not be suitable for laser scanning, e.g. translucent, reflective, dark or deeply convoluted surfaces.

Table 5-1: Specifications of FastSCAN [116]

4.1.2 PLASTER FABRICATION AND DATA COLLECTION

A healthy 26-year-old girl's right foot and a 40-year-old man's right foot were made into plaster casts (Figure 5-3) and were scanned by FastSCAN (Figure 5-4).

The plaster foot is made as follows:

- I) The interested subject's foot is wrapped with bandage and wet with a mixture of water (90%) and plaster of Paris (10%). This is to make the bandage strong enough to support the plaster paste to pour in later.
- II) After the bandage is dried, it is cut with knife to peel it from the subject's foot. The dry plaster of Paris powder (70%) is mixed with water (30%) and pour into this mold when it is still in the state of paste.
- III) After the paste is hardened to solid, peel the bandage off.

The first reason for using plaster cast instead of human foot is that it does not move during the scan. Secondly, the plaster cast can be placed arbitrarily for a convenient measurement. Thirdly, at the phase of methodology development, scanning on the plaster cast ensures the reproducibility. Fourthly, using two plaster feet instead of one is to see whether the similar curvature maps can be repeatedly observed on different feet. Later the scanning was done on human legs and feet after the preliminary experiment was done on the plaster feet. The data were saved in the form of text files. For the foot and leg scanned by FastSCAN, the sweep was repeated several times from different directions to obtain enough points density for later analysis. Compared to the points collected by the rasterstereography, the point density was dramatically higher. High point density means one can employ more complicated methods to process them and get more accurate results. Also one can concentrate on a smaller area without worrying about that there are not enough points in that area.

Another problem will emerge when the scanner casts light on the foot surface. Sometimes there is a shadow caused by the convexity of the lateral and medial malleoli. The shadow not covered by the light from the scanner results in a blank area on the reconstructed foot surface. This blank area is eliminated by several scans from

different angles. These individual scans are then combined into a whole coherent data set.

Figure 5-3: The plaster casts of a 40-year-old healthy man's right foot (left) and a 26-year-old healthy girl's right foot (right).

Figure 5-4: The foot image displayed on the screen.

Total eighteen subjects were scanned after they provided their informed consent. The subjects were all healthy at the time of the scan and no one had a history of lower extremity surgery. Before their legs were scanned, the subjects were seated comfortably on a chair with their legs in the neutral position, bent 90 degree to the thigh. During the scanning process, they tried their best to stay stationary. In order to minimize the influence of involuntary movement, one receiver was attached to the shank so that it moved with the leg. We scanned several views from different angles. The individual views were merged into a single file with the complete measurement of the whole lower leg. The data was then saved in the form of text files for further processing. The density of the measurement points was about 40 points/cm². The scanning time was less than a minute and the processing time was less than 5 minutes. These times can be further reduced, but at the expense of image resolution.

4.2 SURFACE CHARACTERIZATION

Figure 5-5: Foot surface points displayed in Matlab (units: mm).

After all the data on the foot surface have been obtained, MATLAB (The Mathworks, Natic, MA, USA, Ver. 6.5) is used to process the coordinates of those three-dimensional points (Figure 5-5). An operator is invited to perform selection and calculation of those data. The calculation starts from a starting point selected arbitrarily. The operator takes that point as the origin and finds those points in the vicinity of that point. The vicinal region should be chosen carefully depending on the point density because a too small neighborhood results in an undetermined linear equations and a too large one causes a long computing time. Equation (3-10) is used to put weighting to each point involved according to their distance to the origin. Those points close to the origin are weighted much more than those points far away from the origin. Then with the method described in 3.1, the coefficients vector $\mathbf{C}$ is obtained. With these coefficients, the principal curvature, the Gaussian curvature and the mean curvature at that point can be calculated. The operator then moves with a certain step to the next point and takes this new point as the new origin and performs similar selection and calculation. The steps are selected according to the desired accuracy. One also should notice that a high accuracy (a small step) leads to a long computing time. The operator moves the origin systematically throughout the whole area. Finally the maps of reconstructed surface, the principal curvatures, the Gaussian curvature, the mean curvature and the Koenderink shape index are drawn. The maps of Gaussian curvature and mean curvature should be referred to each other to separate convexity from concavity. To separate convexity from concavity in one curvature map, the Koenderink shape index [117] is calculated.

Because the Koenderink shape index maps all the shape patches to $[-1, +1]$, it has the advantage of preventing the deterioration of color contrast due to extraordinary large or small value resulting from undetermined linear equations (e.g. on an edge or

on a hole). The extraordinary large or small curvature values in Gaussian or mean curvature map make other different values appear in the same color or obscure the contrast among them. In Figure 5-6, because the hole formed by the first and second toes, the curvature calculation at the edge of the hole is not reliable, which obscure the other part's color contrast (the ulcer's edge can not be seen). But this will not happen in Koenderink shape index maps because at most they take the color of +1 or -1. In Figure 5-7, the hole does not influence the other part's contrast. The ulcer's edge can be clearly observed (indicated by arrows). The map from $[-\infty, +\infty]$ to $[-1, +1]$ ensures the other curvature values' colors are not influenced by the abnormal results from the edges or holes of the surface.

For simplicity, we categorize the shapes into three surface types instead of nine (Appendix A, Table A-1) provided by Koenderink shape index. Each type of shape's color changes its deepness according to the surface types. For example, when the surface changes from ridge via dome to spherical cap, the color on the colorbar changes from light red to deep red concomitantly. The white is taken as transition between different colors for clear differentiation of different surface types (concave, convex or saddle-shaped). As a result, the outer malleolus and inner malleolus emerge from their neighborhood (Figure 5-8). Besides these two expectable regions, on the underside of the foot, there are 4 possible landmarks: the convex region formed by the distal ends of first and second metatarsal, the distal end of fifth metatarsal, the concave region under the arch and the heel. Due to the movement of the toes, they are obliterated from the consideration of landmarks.

Figure 5-6: The photo (up) and Gaussian curvature map (bottom) of a foot's underside with an ulcer. Units: mm.

Figure 5-7: The Koenderink shape index map of the same part of the foot. Units: mm.

Figure 5-8: The 3D display of map of Koenderink shape index of the ankle area (from 3 different viewpoints) and the underside of the foot. Red: convex, green: saddle-shaped, blue: concave, white: transitions between these three different shapes.

4.3 LOCALIZATION OF ANATOMICAL LANDMARKS

The detailed strategy in localizing a landmark primarily depends on its anatomical peculiarities. The procedure is broken up into discrete steps. 1). Surface regions are scanned for shape characteristic of the landmark sought after and its vicinity. From this procedure a region of interest is obtained, within which the landmark is to be localized. 2). The region of interest is scanned for surface curvatures that are characteristic for the landmark. This procedure provides the coordinates of the landmark sought after. In order to secure robustness of the procedure, this may be followed by step 3). Starting from the landmark coordinates obtained in the previous step, a landmark is determined by fitting a geometrical model to the adjacent surface areas. After the model is fitted, the distinguished point on this model is extracted and taken as the final locus of the landmark.

Figure 5-9: Curvature maps of the inner malleolus and outer malleolus. In Gaussian curvature map: red: convex or concave, blue: saddle-shaped, white: transition. In mean curvature map: red: mainly convex, blue: mainly concave, white: transition.

Regarding the outer malleolus landmark (Figure 5-9), the scan for characteristic shape configurations as a first step should incorporate the following features:

- I) A clearly convex region - indicating the true outer malleolus landmark - is identified: It exhibits Gaussian curvature and thus it is either convex or concave, according to mean curvature convexity prevails.
- II) An oblong region adjacent in one direction to the malleolus candidate. It is characterized by a slightly saddle-shaped ridge (Gaussian curvature: saddle-shaped or parabolic, mean curvature: convex with the predominant principal curvature perpendicular to the long axis of that region.). Anatomically, the ridge indicates the fibula.
- III) In the opposite direction – beyond the malleolar convexity – lies the submalleolar region. It is conspicuous due to its clear saddle shape.

These three shape requirements in combination provide a clear instruction for localization of the region of the malleolar landmark, enabling the whole process to be performed automatically.

As the second step a tentative determination of the landmark coordinates follows. In the simplest case this is just the point with maximum convexity. However, scattering of measurement data has a profound effect to the curvature and thus the reliability of the resulting coordinates is deteriorated. In this study the following method has been accomplished: the point of maximum Gaussian curvature is taken as a starting point for the fitting of a geometrical model as described below.

As for the geometrical model, the malleolus landmark is regarded as a convex region surrounded by a saddle-shaped region. Therefore, a robust procedure largely

independent of the scatter of surface measurement data may consist in fitting an idealized geometrical model to these data.

This may be done by fitting a surface of revolution with Gaussian convolution ('Gaussian hat', see Appendix B, Figure B-1, for details) to the points with positive Gaussian curvatures on the malleolar region (Figure 5-10). The fitting procedure is done by:

- I) The point with maximum Gaussian curvature is located and taken as a starting point. At the same time, the inflectional point in radial direction (the zero-Gaussian-curvature point where Gaussian curvature changes from positive to negative) is found. The distance between these two points defines the neighborhood for the fitting.
- II) The grid points are transformed from the global coordinate system into the local coordinate system for easy calculation of the difference between the Gaussian hat and the scattered points. The local coordinate system with the z axis coincident with the normal vector at the starting point is built for local surface fitting.
- III) In the local coordinate system, the adapted expression
$$z = A e^{-\frac{[(x-x_0)^2 + (y-y_0)^2]}{2\sigma^2}} - A + c_1 x + c_2 y$$
 is used for the surface fitting. The difference in z coordinates between the measured points and the Gaussian hat is minimized in an iterative way by varying the unknown parameters (A, σ , x_0 , y_0 , c_1 , c_2). In the local coordinate system, the point of maximum Gaussian curvature is taken as the origin, and then the hat apex is allowed to move in the range of $-2 \leq x_0 \leq +2$ and $-2 \leq y_0 \leq +2$ (the grid constant is 2mm). At the same time,

A and σ also vary from 1mm to 20mm. c_1 and c_2 are varying in a small range around zero to slightly change the orientation of the Gaussian hat. The combination of these six parameters that makes the difference minimum is taken as the final expression of the Gaussian hat. Then the apex of the Gaussian hat is transformed back into the global coordinate system.

The apex of the fitted Gaussian hat is taken as the landmark – in a strict sense it is a pseudo landmark [118].

The inner malleolus landmark is localized by a similar procedure.

Figure 5-10: Outer malleolus with a Gaussian hat fitted to the region neighboring the point of highest Gaussian curvature.

In order to observe the noise influence on the final position of the landmarks found by Gaussian hat fitting, artificial random noise with standard deviation of 0.1, 0.3, 0.5 and 1 mm is added on the z coordinate of experimentally collected data. Because the surface fitting is in the direction of z axis, it is obvious that the noise in that direction would have greater influence. The shift is taken as the absolute discrepancy between the found point position with noise and the one under original data accuracy (without noise) (Figure 5-11).

In an alternative way, the landmarks are defined by the center of gravity

$(\frac{\sum_{i=1}^N x_i}{N}, \frac{\sum_{i=1}^N y_i}{N}, \frac{\sum_{i=1}^N z_i}{N})$ of those N points with Koenderink shape index larger or smaller

than a certain threshold. To see the influence of the noise on the final loci of landmarks, the scanned data are corrupted by adding noise with mean of zero and standard deviation up to 1mm. The simulation is repeated ten times for each noise, the mean position shifts from the original position without noise are charted in Figure 5-11.

The localization by center of gravity of 100 points is less influenced by the noise. Considering that fitting a Gaussian hat takes a relatively long time of calculation and is not suitable for landmarks on the other parts of body, we finally adopt the localization by center of gravity for landmarks on other parts of body. The point number used for localization changes according to the area of possible landmark and point density, which should be adjusted on the basis of trial-and-error. The methods of Gaussian hat and center of gravity perform almost the same when the noise is about 0.1mm. Since our laser scanner will not give such a large noise (in our laser scanner, noise is 0.02mm), both can be chosen to localize the malleoli.

Figure 5-11: The influence of noise on the final loci of landmarks on outer malleolus.

In Figure 5-11, the final loci of landmarks are defined by Gaussian hat and the center of gravity of those top 25, 50 and 100 points with positive Gaussian curvature. The scanned data are corrupted by adding noise of different standard deviation up to 1mm. The mean position shifts (the distances of landmarks from original position without noise) of loci of landmarks decrease when more points are involved in the calculation of the center of gravity.

To study the reproducibility of the localization of the anatomical landmarks, a 26-year-old healthy man's right foot was scanned at the ankle area when he sat on the chair. The distance between the outer and inner malleoli was calculated. Three different examiners scanned the man's right foot over three consecutive days. Each examiner scanned the malleoli eight times. The ANOVA is used to check whether there is any significant difference between the means of different groups.

The bimalleolar distances measured by the three different testers (Figure 5-12) had almost the same mean with a small standard deviation. The results of the ANOVA ($p=0.89$) indicated a good level of reproducibility when landmarks were localized by determining the points' centers of gravity. Certainly, the number of points (N) chosen has an influence on the final location of the landmarks. But as the number (N) is increased, this influence is minimized along with the effect of noise on the shifting of the final location. For comparison, the number of points selected for calculating the center of gravity should be consistent across subjects with the precondition that these points cover the area of interest. The number (N) varies with point density for the same region of interest. We find that 40 is the ideal number for our scanned area.

Figure 5-12: The box plot of the bimalleolar distance measured by three different testers. The box has lines at lower quartile, median and upper quartile values. The lines extending from each end of the box show the extent of the rest of the data.

CHAPTER 6

RESULTS

This chapter shows the applications of anatomical landmarks found on the curvature maps. The method proposed is successfully applied on tibial torsion measurement (section 5.1), tibial nail estimation (section 5.2) and wound documentation (section 5.3). The current methods for these applications share the same shortcomings: subjectivity resulting in poor repeatability and reproducibility. The measurement of them depends strongly on the doctor's experience and preference. With curvature maps and objectively localized anatomical landmarks, the proposed methods enjoy a high repeatability and reproducibility. In each section, the significance of the study is given and its limitation is discussed as well.

5.1 TIBIAL TORSION

5.1.1 INTRODUCTION

Tibial torsion is the twisting of the tibia about its longitudinal axis, which changes as people grow up. Knowing the normal range of tibial torsion at different ages or tracking the changes is important for the monitoring of tibial torsion especially, if there are derangements. The successful treatment of rotational deformities of the lower limb by derotation osteotomy must be based on accurate diagnosis. In order to reduce irreversible lower-limb rotational defects such as in-toeing and out-toeing in adults, the accurate determination of tibial torsion is necessary. Accurate measurement of tibial torsion also helps to study its relationship to the congenital clubfoot [27, 28] and to the

pathology of the knee such as patellofemoral instability or Osgood-Schlatter disease [29].

Since it was first described by Le Damany [30], tibial torsion has long been studied by various methods. The most accurate technique is measuring the angle formed by the pins passing through the transcondylar axis of the head of the tibia and the axis of the distal articular surface of the tibia but this method could not be used in vivo [43]. With the aid of mechanical apparatus such as caliper and goniometer, several simple and inexpensive methods have been used clinically. Staheli and Engel [39] presented a simple clinical method by calculating the transmalleolar axis of 180 normal children and adults. With a C-clamp, Ritter et al. [40] carried out a longitudinal examination on the tibiofibular rotation of the children at birth (1000 legs), 6 months (302 legs), 12 months (154 legs) and 24 months (105 legs). Malekafzali and Wood [42] designed a simple goniometric device, made up of one limb and two rubberized malleolar cups to measure the angle between the transcondylar and transmalleolar axes. In 1985, Staheli et al. [46] measured the angle between the line perpendicular to the transmalleolar axis and the thigh axis from the projected photographic negatives on 1000 lower limbs. Milner and Soames [57] compared these four in vivo methods [39, 40, 42, 46] of measuring tibial torsion, aiming for a most accurate one. However, as the repeatability of the indirect techniques was low both in cadavers and the living, alternative clinical measurement methods of tibial torsion needed to be developed.

Besides the above researchers, many others tried different methods such as caliper [37], tropometer [27], tractograph [41], electronic digital inclinometer [58], roengenograph [27, 34, 36], gravity goniometer [56], computer topography (CT) [43, 45, 49, 60], magnetic resonance image (MRI) [55], fluoroscopy [48] and ultrasound

(US) [51]. Each method has its own advantage and disadvantage and none of them gained wide acceptance.

By utilizing curvature maps the landmarks are characterized and localized. In this way, tibial torsion can be measured by surface curvature. Scanning the lower leg and extracting anatomical landmarks to calculate the angle provides an objective way of measuring tibial torsion compared to other methods, which are subjected to examiner's experience and preference [31, 40, 57].

5.1.2 METHOD AND EXPERIMENT

The Koenderink shape index map of the lower leg (Figure 6-1) shows several anatomical landmarks that are formed by the bone underlying the skin. Two lines are defined by the conspicuous anatomical landmarks: the two malleoli, the fibular head, the point anterior to lateral tibial condyle and the medial tibial condyle. The angle between the two lines is taken as a measurement of tibial torsion. In 3D space, if line 1 is defined by two points $\mathbf{x}_1$ and $\mathbf{x}_2$ and line 2 by $\mathbf{x}_3$ and $\mathbf{x}_4$ (Figure 6-2), then the angle θ between the two lines is calculated as

$$\cos \theta = \frac{(\mathbf{x}_2 - \mathbf{x}_1) \bullet (\mathbf{x}_4 - \mathbf{x}_3)}{|\mathbf{x}_2 - \mathbf{x}_1| |\mathbf{x}_4 - \mathbf{x}_3|} \quad (5-1)$$

Here $\mathbf{x}_2 - \mathbf{x}_1$ means vector of line passing through point $\mathbf{x}_2$ and $\mathbf{x}_1$. $\mathbf{x}_4 - \mathbf{x}_3$ means vector of line passing through point $\mathbf{x}_4$ and $\mathbf{x}_3$.

Figure 6-1: The reconstructed surface (left) and the Koenderink shape index map (right) of the lower leg (Red: convex, green: saddle-shaped, blue: concave, white: transitions between these three different shapes.).

In Figure 6-1, five anatomical landmarks (*) are localized to form the proximal and distal reference lines. A: the fibular head. B: the anterior aspect of the lateral tibial condyle. C: The medial tibial condyle. D: outer malleolus. E: inner malleolus. The proximal reference line (X-X) connects the mid point of A and B with C. The distal

reference line (Y-Y) is the transmalleolar axis. The angle between these two reference lines is calculated as tibial torsion.

Figure 6-2: The angle between two lines in 3d space. Here x_1 , x_2 , x_3 and x_4 are corresponding to C, mid point of A and B, E and D in Figure 6-1 respectively.

The reproducibility of tibial torsion measurement is investigated by three different examiners who scanned the right legs of two male subjects over three consecutive days. Each examiner scanned each man's right leg eight times. The ANOVA was again used to detect any significant difference in the right leg's tibial torsion as determined by different examiners.

The t test with correction for correlated samples was performed to detect whether there were any significant differences between the tibial torsions of left and right legs. The null hypothesis stated no significant difference.

5.1.3 RESULTS

Figure 6-3: The box plot of tibial torsion measured by three different testers on two subjects' right legs.

In Figure 6-3, the box has lines at lower quartile, median and upper quartile values. The lines extending from each end of the box show the extent of the rest of the data. No significant difference is found among the means of results from different testers on subject 1 (a) and 2 (b).

The reproducibility was good, judging from the results of the ANOVA (Figure 6-3, $p=0.74$ and $p=0.80$ for subject 1 and 2 respectively. p stands for probability value). The angle of tibial torsion determined by the two lines had a relatively large variance with contributions from all the errors caused by localizing the five anatomical landmarks. Still, no significant differences were found among the means of the results from different testers. Scanning the leg and determining the anatomical landmarks

from the curvature maps is thus an objective method of measuring tibial torsion that allows for the reproducibility of results.

Eighteen pairs of legs from adult males were scanned, and their tibial torsion measurement ranged from 10.26° to 27.86° with a mean of 19.36°. This indicated a wide variation in tibial torsion, even among normal subjects. This range, however, is in concurrence with the normal values of tibial torsion. The normal range is from 0° to 45° with a mean of 25° [46]. Looking at Figure 6-4, we found a difference between left and right tibial torsions. Most right tibial torsions were found to be greater. Although this was supported by other researchers [51, 55], we are prone to believe that the difference between left and right could have happened by chance alone as our t test for correlated samples showed that the t was not significant at $p=0.10$. Other researchers also claimed that no significant difference was found between the sides [41, 43, 49, 50]. The small sample size and the measurements of only 18 pairs of legs limit the generalization of the findings. More subjects are required in order to study the difference between left and right tibiae.

Figure 6-4: The tibial torsion of 18 subjects. Most of the subjects have a larger tibial torsion of the right leg.

5.1.4. DISCUSSION

The subjective nature of the available methods poses the most significant problem to researchers. This challenge seems unavoidable, as it is due intrinsically to the lack of suitable landmarks on the leg [57]. Tibial torsion is measured by the angular relationship of a line drawn between two points in the proximal ends of the bones with a line drawn between two points in the distal ends of the bone. Four fixed points need therefore be determined. Although choice of the two pairs of points is arbitrary, they must be constant [36]. The method presented here has characterized the lower leg and provided several stable anatomical landmarks that can be used objectively for the measurement of tibial torsion. The previously inconsistent localization of the malleoli by palpation resulted in subjective and inaccurate measurement. This is now replaced by the objective determination of their position on curvature maps. The anatomical landmarks selected as reference points are constant from bone to bone and study to study. This ensures high reproducibility in the measurement of tibial torsion. As such, accurate comparisons can be made.

This proposed method however, is not as accurate as the measurements obtained by CT and MRI since it does not obtain the measurement from the bone. The motion between tibia and fibula will introduce errors (the tibiofibular torsion is about 5° less than actual tibial torsion [41]). This cannot be avoided but it can be minimized by the study of the range of tibia-fibular motion [38]. As long as the technique produces reproducible angles, it ensures a consistent relationship to the direct measurement on which basis the true tibial torsion can be calculated.

To minimize the scanning time, in a busy clinic environment, only the areas relevant in the determination of tibial torsion are scanned, namely the area below the

knee and the medial and lateral malleoli. Scanning the whole leg is unnecessary. The scanned surfaces can be stored in a database so that the changes in the tibial torsion can be monitored during follow up and mapped. This is especially useful for the treatment of children who require rotational osteotomies because of abnormal tibial torsion. It can also be used to determine the post-operation correction objectively.

This method does not work however, on obese subjects, as the required anatomical landmarks on their legs are often not distinguishable from the surrounding subcutaneous fat. The solution to this problem might then be to look for other anatomical landmarks that are more obvious in obese subjects such as the tibial tuberosity or the medial surface of the tibia [58]. The angle can be calculated accordingly and the relationship with the CT and MRI measurements can subsequently be adjusted appropriately.

5.1.5 CONCLUSION

We have investigated the feasibility of a method to objectively measure tibial torsion. The total eighteen subjects were engaged for this effort. The proposed method is characterized by its objective determination of anatomical landmarks that allows for a high level of reproducibility. The method offers a fast, objective and non-invasive way of screening once its reproducibility and accuracy are further verified. This can be done in vitro using the most accurate method of passing pins through the articular surface of the tibia on necropsy specimens or it can be done in vivo by performing CT scans on a large number of subjects.

5.2 TIBIAL NAIL

5.2.1 INTRODUCTION

Closed intramedullary nailing has become the fixation of choice for many unstable fracture patterns of the tibial shaft. Proper nail length is imperative for satisfactory outcomes. If the chosen nails are too short they will cause malreduction. However, if inserting a nail that is too long will result in fracture distraction, impingement on the patellar tendon or violation of the ankle mortise [65]. Tibial tubercle-medial malleolar distance (TMD) is an easy, cheap, accurate and radiation-free method of determination on the requisite nail length, which has been proved. However, presently the two anatomical landmarks are localized by palpation and the distance is measured only by rulers, which is subjected to examiners' experiences and preferences. This might cause errors in choosing the proper tibial nail required.

The tibial tubercle and medial malleolus distinguish themselves clearly on the Koenderink shape index map, from which the TMD can be calculated after that the tibial nail length can be predicted. The objectiveness of this method ensures the high reproducibility of localizing the anatomical landmarks, which results in a highly reproducible determination of tibial tubercle-medial malleolar distance. This ensures that different doctors will choose the same tibial nail length needed in the surgery for undislocated tibial fracture.

5.2.2 METHOD

The scanning was done in the same way on the same patients as tibial torsion scanning. On the Koenderink shape index map of leg, tibial tubercle and medial

malleolus are spotted clearly. With these two loci of the two landmarks, the TMD could be easily calculated (Figure 6-5).

Figure 6-5: The Koenderink shape index map of lower leg. Dot A: Tibial tubercle. Dot B: Medial malleolus. The TMD is the distance between A and B.

To study the reproducibility of measurement of the length of TMD, three different examiners scanned two men's right legs over three consecutive days. Each examiner scanned the two subject's right leg eight times. In the same way, the ANOVA is used to check the reproducibility.

The t test with correction for correlated samples is performed to detect whether there is any significant difference between the TMDs of left and right legs. The null hypothesis stated no significant difference.

5.2.3 RESULTS

A high reproducibility is observed from the box plot of the TMD. In Figure 6-6, no significant difference is found among the means of results from different testers on subject 1 ($p=0.81$) (a) and 2 ($p=0.91$) (b). This shows that TMD measured by the curvature maps by different testers gives almost the same results on one patient, leading to the same decision on the selection of tibial nail needed in the surgical operation. Previously, the TMD is measured by the traditional method with ruler, the preference and experience will make different testers locate the tibial tubercle and medial malleolus differently, thus resulting in a much larger difference that could easily exceed 10mm.

Only small differences between left and right TMDs are observed in Figure 6-7, which might result from errors of locating the anatomical landmarks by the curvature map. It is also possible that the intrinsic difference between left and right legs causes the difference of TMDs. However, we believe the difference between left and right TMD could have happened by chance alone since t test for correlated samples showed that the t is not significant at the significant level of 0.10. Similarly, more subjects and further research are warranted in the study the difference between left and right TMD before generalizing these findings to a wider population.

Figure 6-6: The ANOVA results on the two legs.

Figure 6-7: The left and right TMD of 18 subjects. The left and right TMD are very close to each other.

5.2.4 DISCUSSION

The method presented here has characterized the leg and provided two stable anatomical landmarks that can be used objectively for the measurement of TMD. The anatomical landmarks selected as reference points are constant from bone to bone, and it has been demonstrated that TMD is suitable for predicting the tibial nail length. The localization of anatomical landmarks by palpation results in subjective and inaccurate measurement. This is now replaced by the objective determination of their position on curvature maps. This ensures high reproducibility in the measurement of TMD, thus giving a reliable prediction of the necessary tibial nail regardless of the experience level of the doctor.

In the clinical situation of an injured patient in pain, only the areas relevant in the determination of TMD are scanned, namely the area of tibial tubercle and the medial malleolus. This will minimize scanning time. Scanning the whole leg is unnecessary.

Based on the TMD of the patient's uninjured leg, the tibial nail length needed for the fractured leg can be predicted. However, this prediction is only confined to an undislocated tibial fracture. For a fractured tibia, which is usually dislocated, this method fails to deliver the correct length because it cannot detect the type of dislocation. In this case, on the radiograph, the exact length can be measured with an X-ray ruler and any dislocation can be taken into account and be corrected.

Similarly this method does not work on obese subjects, as the required anatomical landmarks on their legs are often not distinguishable from the surrounding subcutaneous fat. The solution to this problem might then be to look for other anatomical landmarks that give stable distance based on which the tibial nail length can be predicted. Otherwise the tibial nail length has to be decided by other methods such as conventional radiological methods.

5.2.5 CONCLUSION

The TMD determined by the curvature maps is objective and consistent, resulting in accurate prediction of required tibial nail length. However, this prediction is confined to special situations only as this method requires skinny patients, undislocated fracture and uninjured skin at the landmarks interested. It serves as an example for further comparable applications and needs further clinical verification.

5.3 WOUND MEASUREMENT

5.3.1 INTRODUCTION

Health care costs associated with wound care are significant and continue to rise [111]. Numerous treatment modalities are available to manage chronic wounds, including various wound dressings [66, 67], low-energy laser [119], ultrasound [68] etc.. A reliable, standardized wound measurement technique is needed to better assess the efficacy of wound care products and to implement an appropriate treatment algorithm based on healing progress.

Currently, various techniques are used to quantify wound dimensions. The most common technique relies on the length and width measurement of the wound obtained with a ruler. While this simple technique is readily implemented in various clinical settings, its accuracy and reliability can vary according to the subjective determination of the wound edges. A variant of this technique involves placement of plastic film over the wound and tracing the wound boundary using a microtip pen [120]. The wound area is then estimated using graph paper, planimeter, weighing technique, digitizer, CAD, or hand-held computer scanner [77, 79, 80, 84, 85, 87, 88, 90-92]. While this tracing technique works fairly well for wounds on flat surfaces, this becomes more challenging if wounds are located over a curved surface. Furthermore, wound volume is often estimated by measuring the amount of sterile normal saline used to fill up a given defect [74, 76]. Although such volumetric measurement can be performed with minimal instrumentation and training, it is often not practical for some wounds (for example, on the plantar aspect of the foot) due to difficulty in positioning the wound, especially on patients with morbidity or obesity. An ideal technique is to employ a method which does not require contact so that potential pain, disruption of the healing

process, and risk of contamination can be minimized. Simple photography or video imaging has been employed to observe the wound and to measure its area [82, 83]. Others included a standard calibration cube in the picture so that the camera to wound distance can be correctly accounted for. Nevertheless, two dimensional photography is prone to parallax error and fails to fully convey quantifiable 3D surface geometry [86]. A 3D non-contact method is therefore needed to accurately describe wound geometry.

Many 3D non-contact wound surface measurement devices and methods have been proposed. These include stereophotogrammetry [93-95, 109, 110], laser scanner [106, 107], structured light [98, 100, 111], and laser scanner plus sound sensor [97]. None of these has been widely accepted and applied by practitioners due to their respective shortcomings such as low accuracy, high expense, and complicated calibration procedure. Also, they require long data acquisition or analysis time. A fast, accurate, objective and non-contact wound surface measurement method is desirable.

We propose a novel method of 3D optical measurement and a subsequent representation of a curvature map to evaluate the status of wound healing.

5.3.2. METHOD

The foot ulcer of a middle-aged diabetic patient was scanned by Fastscan and the ulcer's rim curvature map was computed. Utilizing a specified rim curvature value (Appendix C), the wound edge was detected and processed via a cubic spline smoothing. Photographic images of the wound were transformed into the same coordinate system as the scanned image to compare the curvature maps of the wound with photos for a better understanding of the boundary locations. To estimate the depth and volume of wounds, the original surface was reconstructed by interpolating the

surface regions outside the wound by using a second order polynomial, which was fitted to the points outside the wound boundary. The average differences between the reconstructed surface and the scanned wound surface were taken as the measure of wound mean depth. The volume was estimated as a product of the area and its mean depth. As a clinical trial, a patient's foot ulcer was scanned and monitored ten times in three weeks.

5.3.2.1 Platform design and calibration procedure

A special platform (Figure 6-8) that accommodates the laser scanner wand and digital camera together was fabricated, fixing the relative position between them to make the scanned surface and the digital image comparable. In Figure 6-8, the platform holds the wand and camera firmly and gives a fixed relationship between the wand and the camera. The components are as follows: a: Cuboid block, longitudinal constraint for wand. b: Camera stand, support and fixation of camera. c: Tilt base, better viewpoint for camera. d: Triangular block, support for tilt base. e: Horizontal base, support for the whole platform. f: U-shaped block, lateral constraint for wand. Two triangular blocks (Figure 6-8, d) were placed on a horizontal base (Figure 6-8, e) to provide a tilt base (Figure 6-8, c) for the other elements. The tilt base provides a better viewpoint for the laser scanner and camera and adds convenience when measuring the foot ulcer. The two cuboid blocks (Figure 6-8, a) secure the laser scanner's wand on the platform. The two u-blocks (Figure 6-8, f) were also designed to prevent lateral movement. The camera is fixed on the camera stand (Figure 6-8, b) by its connector (not drawn here but shown in Figure 6-12, top, h).

Figure 6-8: The assembly drawing of the designed platform.

The way to locate the camera in the world coordinate system is shown in Figure 6-9. The transmitter provides a reference coordinate system for the wound surfaces. The position of the wand's receiver referenced to the transmitter's coordinate system is constantly being recorded as 4x4 homogeneous transformation matrix T_1 , which is integrated in the FastSCAN software. The relative position of Charge Couple Device (CCD) with respect to the wand is fixed after they are placed on the platform. A CCD coordinate system can be built arbitrarily on the CCD, but for calculation convenience it is better to have the same orientation as the wand receiver's coordinate. The 4x4 homogeneous transformation matrix T_2 between them is established by measuring the relative displacement of the camera CCD with respect to the receiver on the wand. Finally the CCD position referred to the world coordinate system is known as the product of T_1 and T_2 .

$$\begin{Bmatrix} {}^1x_3 \\ {}^1y_3 \\ {}^1z_3 \\ 1 \end{Bmatrix} = T_1 \cdot T_2 \cdot \begin{Bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{Bmatrix} \quad (5-2)$$

Figure 6-9: The illustration of the three involved coordinate systems of transmitter, wand's receiver and camera's CCD and their transformation matrices.

Now, the nodal point (estimated by one focal length before CCD), the CCD plane, and the 3D scanned surface are all referenced to the world coordinate system and the preparation for the projection of the 3D scanned surface through the nodal point is ready.

Since the coordinates of the surface point, nodal point, and the CCD position are known, a coordinate system can be built on the CCD image plane (Figure 6-10), which takes o' as the origin. With known unit direction vectors of $\mathbf{e}_1$, $\mathbf{e}_2$, and $\mathbf{e}_3$, and the

distance d from P to P' , a and b for the projected point P' on the CCD image plane can be obtained by supposing the origin of the world coordinate system is O

$$\mathbf{OP} + d \cdot \mathbf{e}_3 = \mathbf{Oo}' + a \cdot \mathbf{e}_1 + b \cdot \mathbf{e}_2 \quad (5-3)$$

Solving equation (5-3) we can get a and b , which show the position of P' on the CCD plane. We perform this calculation point by point to get the projection of the wound on the CCD plane.

Figure 6-10: The method of projecting the scanned surface through the nodal point to the CCD image plane. a and b are the coordinates of point P' , and d is the distance from surface point P to its corresponding point P' .

5.3.2.2 Experimental test: Comparison of photograph and curvature surfaces

Figure 6-11: The rim curvature map of the scanned wound.

From this map (Figure 6-11), each individual point is assigned a specific color by indexing its rim curvature on the color bar (from -0.1 to 0.1), thus the ulcer can be clearly observed. Based on this area of interest, the depth, area, and volume can be calculated.

After the ulcer's rim curvature map was drawn, the procedure to compare the curvature map to the photo were carried out as follows:

1. The relationship between the camera and the laser scanner wand remained fixed once they were placed on the platform (Figure 6-12, top and bottom left). In this manner (as described previously), the position of the camera in the world coordinate system can be established.
2. A photograph was taken using a Nikon Coolpix 995 with a known focal length. The nodal point was estimated at a focal length before CCD (the sensors of the camera), and the CCD was deemed to be at the center of the camera. The CCD size was 7.2mm x 5.3mm, which was verified by Nikon technical support.
3. The surface was scanned by the laser scanner (Figure 6-12, bottom right).
4. The curvature map was calculated.
5. The curvature map of the surface was projected through the nodal point of the camera to the imaging plane of CCD (Figure 6-10).
6. After projecting the CCD through the nodal point, the curvature map was compared to the photograph.

To check the accuracy of the scale between the curvature maps projected on the CCD and the digital image, a piece of paperboard with a punched-out 30mm circle was scanned and photographed (Figure 6-13, a). The diameters of the circles on both images were compared. Similarly, to check the accuracy of the image orientations, a circle with a triangle and a rectangle along its edge is cut from another piece of paperboard (Figure 6-13, c).

Figure 6-12: The experimental setup and procedure.

In Figure 6-12, the platform (top, shares the same labels as in Fig. 5-8 except that the connector h is shown here) holds the scanner and camera firmly in place (bottom left) and their relative position in the world coordinate system (the transmitter defining the world coordinate system is visible in the lower left part) is calculated from the parameters of the camera and the transformation matrix provided by the laser scanner system. With the nodal point located, the wound is then scanned (bottom right) into the world coordinate system. In the subsequent analysis, a projection of the curvature map of the wound is calculated, allowing a one-to-one comparison with the survey photo of the camera.

Figure 6-13: The paperboard image and its projected curvature maps.

In Figure 6-13, the pictures are as follows: a) The paperboard with a punched 30mm circle. b) The curvature maps projected through the nodal point. The black circle line indicates the boundary of the circle. c) and d) The digital image and projected curvature maps show the same orientation of the shape cut out on the paper. The curvature maps are projected through the nodal point to the region of 7.2mm x 5.3mm (the area of CCD of the digital camera) to compare with the digital images.

5.3.2.3 Boundary location, depth and volume calculation

The rim curvature map of the scanned wound (Figure 6-11) is taken as an example to illustrate the strategy for boundary location.

First, the blue belt in Figure 6-11 is selected by hand as points on the area are associating very small rim curvatures, ranging from -0.08mm^{-1} to -0.1mm^{-1} . By manually comparing the curvature map to the photo, we found they are not located within, but rather just outside the boundary of the wound. Nevertheless, these points are useful in locating the wound boundary. In order to estimate the normal direction of the ulcer, a plane defined as (3-36) is fit to the blue belt points. After the coefficients of a plane are determined, its normal direction is taken as the normal direction of the wound, which coincides with the z axis of a local coordinate system (Figure 6-14). For the purpose of an analytical convenience, all points are transformed into the local coordinate system, which takes the center of gravity of the blue-belt points as the origin.

Figure 6-14: The local coordinate system of the wound. Origin o is the center of gravity of the blue belt points, and the z axis is the normal direction of the wound, estimated from the normal direction of the fitted plane.

After the local coordinate system is properly assigned within the blue-belt, boundaries of the wound can be located as follows:

1. The points meeting certain standards are selected, which include:
 - a) The Gaussian curvature is near zero ($-0.002 \text{ mm}^{-2} \leq K \leq 0.002 \text{ mm}^{-2}$).
 - b) The rim curvature is within a certain range. This range differs for different ulcers, but for one ulcer this range remains stable. Here, for the ulcer monitored for three weeks and measured ten times, the points with rim curvatures larger than -0.005 mm^{-1} are chosen. The program records their coordinates.
2. The Cartesian coordinates of the selected points are changed into cylindrical coordinates with the x-y plane represented by polar coordinates. The interval $[-\pi, +\pi]$ is divided into 40 equal intervals. The points are allocated to one of these intervals according to their polar angle. In each small interval, the point with the largest radial coordinate is selected (dark circle points in Figure 6-15).
3. The line encompassed by these boundary points is drawn (red line in Figure 6-15), and then smoothed by cubic spline interpolation (blue line in Figure 6-15).

After the boundary is established, the original surface of the ulcer is reconstructed with the points outside the ulcer. The six coefficient second order polynomial (3-2) is fitted to these points. For each point inside the boundary, the mean distance from the ulcer surface points to the reconstructed surface in the normal direction (same as the z axis in the local coordinate system) is taken as the ulcer's depth. The area of the ulcer is the sum of the triangle areas, which are generated by Delaunay triangulation [121, 122] with the points inside the boundary in the xy plane in the local coordinate system. The product of depth and area is the volume of the ulcer.

Figure 6-15: The located boundary of the wound.

To verify this method's accuracy to calculate depth, area, and volume, a test spherical cap is manufactured on a cuboid and scanned (Figure 6-16). The de facto volume of the spherical has been calculated from the dimensions measured. The same procedure of wound boundary location has been performed to locate the boundary of the spherical cap. The calculated volume (the product of the area and the mean depth) is compared to the manufactured one.

(a)

(b)

Figure 6-16: The spherical cap test model (a) and its rim curvature map (b) with a located boundary (black line).

5.3.2.4 A clinical trial

The wound area of a patient with a round ulcer was scanned and photographed for three weeks. Before taking measurements, an informed consent was collected. A total of ten measurements were taken on ten non-consecutive days. The curvature maps were calculated and then compared with the photographs for the purpose of locating the boundary on the curvature maps more accurately. The surface of the wound was reconstructed by the neighboring points, but outside the wound. The volumes of the scanned wound surfaces were calculated and compared.

5.3.3 RESULTS

5.3.3.1 Experimental test results

The two pictures (Figure 6-13, a and b) are printed out on one A4 paper and the diameters on both pictures are measured by a ruler. The diameters on these two pictures are 23.7mm and 23.2mm respectively. This indicates that the scale between the projected images and the digital images is 1.02. The calculation on the 3D curvature maps before it is projected on the CCD image plane gives a diameter of 30.1mm, also with a scale of 0.996. This shows the estimation of the nodal point and the position of the CCD image plane is accurate. Also, the relative position between the camera and the wand is properly estimated.

The volume of the spherical cap calculated from the program is $3.8131 \times 10^4 \text{ mm}^3$. The volume calculated from the formula for a spherical cap is $3.8052 \times 10^4 \text{ mm}^3$. The method is accurate with a percentage error of 0.21%.

5.3.3.2 The clinical trial

Using exactly the same experimental setting as with the paperboard comparison, the ulcer is photographed and scanned, and the scale is almost 1. The digital image of the ulcer and the projected curvature map are compared with the boundaries drawn on these images. In Figure 6-17, the photograph (a) gives a clear guide on looking for the boundary on the curvature map (b). In Figure 6-17 (b), the thick black line is the boundary borrowed from the photograph by an experienced surgeon. The dashed red line is the boundary located by the boundary detection method. Placing the photograph of the ulcer in the same coordinate system as the curvature map helps to correctly locate the boundary.

Figure 6-17: The comparison of the boundary between the photo and the projected curvature map. a: photograph. b: curvature map.

In Figure 6-17 (b), one may notice that the boundaries found by the curvature map and by the photograph do not match each other perfectly. The reason for this might include the reflectivity and scattering of skin [123], the movement of patients during scanning, the estimation of the nodal point and the CCD position, and the error in finding the relative position between the camera and the wand. Seeking a better boundary detection method on the curvature map may minimize the difference.

In order to check the accuracy of the ulcer boundary location and the subsequent area calculation, during the last ulcer measurement, a sheet of graph paper with a cut-out square is placed on the ulcer and photographed in the direction perpendicular to the ulcer (Figure 6-18).

Figure 6-18: The photograph of the ulcer together with the graph paper. The area can be estimated on the photograph. The yellow line is the boundary established by a physician.

The area of this ulcer calculated by counting the small squares included in the boundary is 758.20mm^2 . The calculation from the curvature map gives a mean area of 774.07mm^2 . The difference is about 2%.

To gain insight into the time course of wound healing, ten measurements of the ulcer were collected in three weeks. The results of the depth, area, and volume of the ulcer are shown in Figure 6-19. Each measurement was repeated three times. The maximum, minimum, and mean values were plotted in Figure 6-19. The ulcer's depth and the volume tended to decrease, indicating the ulcer was healing. At the end of the monitoring period, the ulcer's depth was smaller, which was confirmed by clinical observation. But, the area was increasing. This might be caused by wound debridement, which was performed almost daily.

Although for some measurements there was a large scattering of the depth, area, and volume, the recovery of the ulcer was still reflected. For various reasons, we could not scan the patient's ulcer every day, which prevented us from getting a detailed recovery line. Future studies will monitor ulcers every day over a longer period of time using a refined method of boundary detection.

Figure 6-19: Calculated dimensions of the ulcer over three weeks. The depth and volume decrease with time, while the area increases slightly.

5.3.4. DISCUSSION

The depth and volume are more important than the area to monitor the size of ulcers. This is because during the recovery phase of an ulcer, changes are observed primarily in depth and volume, but not so much in area. Clinically, wound depth has

been mostly measured in a conventional way by simply placing a depth gauge to measure the deepest aspect of the wound (depending on the clinician's choice of the deepest wound point). Similarly, the volume of an ulcer is determined by the volume of saline or other material needed to fill the wound [75]. Some non-contact measurements of the wound, however, rely on interactions between a computer and a human to judge the boundary of the ulcer, either by observing the images [111] and region of interest or by producing a color-coded depth map [106], which are subjective and prone to error.

The method presented in this study can be taken as the first step of an automatic and objective way of measuring wound area. By taking photographs of the wound together with curvature maps, we can ascertain the boundary on the curvature maps and make the curvature maps more informative. The boundary is located by selecting the points with specific combinations of Gaussian curvature (around zero) and rim curvature (above a certain threshold and getting by a trial-and-error approach). Although at an early stage, the boundary detection combination needs to be adjusted so that it resembles the boundary on the photograph; once decided, it remains stable. This makes the subsequent detections of the same ulcer's boundary easier and needs less human intervention during the long term of wound monitoring.

The reflectivity and scattering of the laser beam by the skin [123] will lower the quality of scanning, which might be lessened by a newly-designed laser scanner with a blue laser beam. Also, the involuntary movement of the patient influences the quality of scanning, and can cause abrupt changes such as ridges and grooves on the surface. These defects can result in errors when the program locates the wound boundary.

FastSCAN provides an additional receiver, which can be attached to the limb to compensate for movement during scanning.

Figure 6-20: The transverse view of influence of surface fitting by choosing extra large neighborhood for the round ulcer.

The wound surface is reconstructed by fitting a second order polynomial to the points just outside the boundary, and it was assumed by Krouskop et al. [111] that the body area of the wound was originally smooth and had only simple curvature in two orthogonal directions. We applied the same assumption. During our surface fitting, we found that if the area around the wound extends too far from the wound, the surface fitting may not be accurate as shown in Figure 6-20. In Figure 6-20, the black line is the transverse view of the wound and its neighborhood. The blue line is the transverse view of the reconstructed surface resulting from appropriate fitting neighborhood. The red one is from over-fitting neighborhood. Obviously, the volume sandwiched between the properly reconstructed surface (blue) and the wound surface (black) is larger than that sandwiched between the improperly reconstructed surface (red) and the wound surface. The over-fitting wound area might underestimate the depth and volume of the wound if some complicated shapes are included in the surface fitting area. But on the curvature maps, these complicated surface shapes can be easily

spotted and excluded, which give a strong indication to find the appropriate wound area for surface fitting.

With the laser scanner and the curvature maps, if the ulcer can be located in an anatomical coordinate system built by anatomical landmarks, it's possible that the wound's original healthy skin surface can be reconstructed by the other corresponding surface in the symmetrical position of the ulcer if it is scanned, flipped and matched to the ulcer's neighborhooding surface area. In this way, instead of a surface with only simple curvatures, a more complex and accurate surface reconstruction might be achieved.

5.3.5 CONCLUSION

We have demonstrated the feasibility of using a laser scanner together with curvature maps and photographs as a potentially useful, non-contact method of wound measurement. Measuring the patient's ulcer on ten occasions over a three week period showed the ulcer was healing. Although some of the results of depth, area, and volume are scattered at times, the recovery trend is still clearly shown. With the objective standard of selecting the boundary points on curvature maps, automatic detection may be realized. Another advantage of this method is that laser scanners are becoming less expensive, and once purchased do not incur ongoing costs. This method promises to be a fast, accurate, and non-contact way of wound measurement and documentation.

CHAPTER 7

CONCLUSION AND FUTURE WORKS

This chapter draws a conclusion to the previous chapters in section 6.1 and points out several areas where the future efforts should be made in section 6.2. The publications related to this study are listed in section 6.3.

6.1 CONCLUSION

This study proposes some applications of curvature maps calculated from scanned surface. Accurate measurement of tibial torsion is needed before the corrective surgery can be carried out. Proper tibial nail required in the tibial fracture surgery must be selected based on the TMD measurement. Wounds also need a non-contact, easy and accurate way to be monitored during the treatment. With the objectively found landmarks, tibial torsion could be measured and tibial nail length could be predicted. The wound can also be detected on the curvature map and can be followed and monitored by calculating its depth, area and volume. The high reproducibility shows the feasibility of this method. The results of the pilot study justify a further study with more subjects involved.

6.2 FUTURE WORKS

If the scanned surface has a high point density, it needs a long time for computation. A small area such as 4 mm² will have hundreds of points involved in the computation for the representative point that is extracted by way of least squares method. Since we aim to find some anatomical landmarks on the surface with a high accuracy, the

computing time is sacrificed for a better result. Further efforts could be made to reduce the computing time without loss of accuracy.

Due to the least square surface fitting, the curvature estimation is biased. Although in our case, this will not influence on the characterization and localization of anatomical landmarks, we have tried to eliminate the system error by introducing the “bias correction function”. It would be better if the system error could be subtracted from the curvature calculation on the scanning data (e.g. foot), but the system error is not constant and changes with different shapes. Even for the same scanning including the surface of sphere and cylinder, their system errors are not the same. This prevents the subtraction of system error when it deals with curvature calculations of unknown shapes. Thus the bias could not be compensated. How to deduct the bias for the unknown surface to increase accuracy should be investigated in the future. More experiments should be done with other assumptions on the system error function than a linear combination of coefficients of the fitted second order polynomial.

Instead of the method we have used for surface fitting, there might be other ways of surface fitting. Although the least square fitting and its following curvature calculation from the coefficients serve well here, it does not mean it is the only one and the best one. The problem we are facing now is surface reconstruction from measured points. It can be stated as follows: given a set of points lying on an unknown surface $\mathbf{X}$, construct a surface $\mathbf{S}$ that approximates $\mathbf{X}$. Maybe there is a method that gives better approximation and more accurate curvature calculation.

Other definitions of geometric model of localizing landmarks other than Gaussian hat might perform better than the Gaussian hat and the center of gravity. The strategy of automatically localizing the anatomical landmarks has been proposed but how to

realize it has not been studied. The efforts in the future should be made in the direction of less human interference. Once the lower leg is scanned, within a rather short waiting period, the Koenderink shape index is drawn and the position of the desired anatomical landmarks is localized. In the same way, the tibial torsion angle and the TMD are given automatically.

The definition of tibial torsion needs to be verified or found the correlation with the results estimated from CT or MRI. This could be done either in vivo or in vitro with a large number of subjects involved. Once the relationship is known, the CT or MRI could be replaced by the curvature maps for the measurement of tibial torsion. But still, the CT or MRI could not be replaced totally for there are obese subjects whose landmarks could not be estimated by the curvature maps. The solution might be finding other definition of tibial torsion that can be used universally, even for the obese.

The objective tibial nail length prediction by curvature maps should be verified with medical doctors. It's the same for the wound monitor by curvature maps. More patients observed for a long term should be involved before a prototype of non-contact wound monitor can be designed.

This method is not suitable for a deep wound with a small orifice because the scanner will not be able to capture the wound's surface. Future study should include the intertester and intratester reliability study, reduced processing time (currently about 5 minutes to get the boundary after scanning), method limitation determination, more wound documentation with more subjects over a longer duration, and a refined boundary detection method.

Finding the exact nodal point of the camera and the CCD position will improve the estimation of the boundary on the curvature maps. The final errors of locating the boundary will exclude the errors introduced by the inaccurate estimation of the nodal point and the CCD image plane. Then we can concentrate only on the boundary locating method, aiming to get a more precise position of the boundary. The comparison results might be better and the two boundaries (one from photograph and one from curvature map) might match with each other more closely.

The same subject's foot and leg with different foot or leg positions will be scanned to study the reliability of found landmarks. If more than three landmarks could be found reproducibly and located accurately, a coordinate system could be built for the study of relative movement between foot and shank. But this needs a scanner fast enough to capture the surface of leg and foot during movement.

6.3 RELATED PUBLICATION LIST

Journal

1. Liu, X., Kim, W.D. and Drerup, B. 2004. 3D Characterization and Localization of Anatomical Landmarks of the Foot by Fastscan. *Real-time Imaging*, 10(4), 217-228.
2. Liu, X., Kim, W.D., Drerup, B. and Arjandas, M. 2005. Tibial Torsion Measurement by Surface Curvature. *Clinical Biomechanics*, 20(4), 443-450.
3. Liu, X., Kim, W.D. and Drerup, B. 2004. 3D Characterization and Localization of Anatomical Landmarks of the Foot. *The Journal of Orthopaedic and Sports Physical Therapy*, 34(9), A16-A17.

4. Liu, X., Kim, W.D., Schmidt, R. and Drerup, B. 2004. Wound Measurement by Curvature Maps. (Under review).
5. Liu, X., Kim, W.D., Wong, Y.S. and Drerup, B. 2004. Objective Tibial Nail Prediction By Curvature Maps. (Under review).

Conference

1. Liu, X., Kim, W.D. and Drerup, B. 2002. Objective Determination of Anatomical Landmarks on the Foot Surface from Surface Curvature. *In Proceedings of International Congress on Biological and Medical Engineering 2002*, 4 to 7 Dec, Singapore
2. Liu, X., Kim, W.D. and Drerup, B. 2003. Accuracy requirements and shape recognition strategies in the automatic localization of the malleol landmarks from surface measurements. *In Proceedings of Salford's 2nd International Conference on Biomechanics of the Lower Limb in Health, Disease and Rehabilitation 2003*, 1 to 3 Sep., Manchester, U.K.
3. Liu, X., Kim, W.D. and Drerup, B. 2003. 3D Characterization and Localization of Anatomical Landmarks of the foot. *In Proceedings of the Second International Conference on Computational Intelligence, Robotics and Autonomous Systems (CIRAS 2003)*, 15 to 18, Dec, 2003, Singapore.
4. Liu, X., Kim, W.D. and Drerup, B. 2004. 3D Characterization and Localization of Anatomical Landmarks of the Foot. *In Proceedings of The International Association of Science and Technology for Development (IASTED)*, February 16 – 18, 2004, Innsbruck, Austria
5. Liu, X., Kim, W.D. and Drerup, B. 2004. 3D Characterization and Localization of Anatomical Landmarks of the Foot. *In Proceedings of Foot*

and Ankle Retreat: Measurement of Foot Motion: Forward and Inverse Dynamic Models. University of Southern California, Dept of Biokinesiology and Physical Therapy, April 30-May 1, 2004.

6. Liu, X., Kim, W.D. and Drerup, B. 2004. Foot Characterization and Anatomical Landmarks Localization. *In Proceedings of 1st International Bioengineering Conference 2004*, Singapore, 8-10 September, 2004
7. Liu, X., Kim, W.D., Drerup, B. and Arjandas, M. 2005. Tibial Torsion Quantification by Surface Curvature. *In Proceedings of Salford's 3rd International Conference on Biomechanics of the Lower Limb in Health, Disease and Rehabilitation 2005*, 5 to 7 Sep., Manchester, U.K, 2005

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APPENDIX

APPENDIX A: FOUNDATION OF DIFFERENTIAL GEOMETRY

The rate at which the unit tangent changes directions as it moves along the curve in some way measures how much the curve is curved. In shape analysis, curvature plays an important role. It is evident that the characteristics of the local shape of surface may be represented by the type and amount of local deviation from a plane. The local deviation is directly related to curvature. If $\mathbf{x} = \mathbf{x}(s)$ is a natural representation of a regular curve $\mathbf{C}$, then the tangent direction at point $\mathbf{x}(s)$ is defined by its derivative:

$$\dot{\mathbf{x}}(s) = \frac{d\mathbf{x}}{ds} \quad (\text{A-1})$$

Supposing that $\mathbf{x} = \mathbf{x}(s)$ is a regular curve with the class more than 2, then the tangent vector $\mathbf{t} = \mathbf{t}(s) = \dot{\mathbf{x}}(s)$ is of class C^1 . Its derivative

$$\dot{\mathbf{t}}(s) = \frac{d\mathbf{t}}{ds} = \ddot{\mathbf{x}}(s) \quad (\text{A-2})$$

is called the curvature vector on $\mathbf{C}$ at the point $\mathbf{x}(s)$ and is denoted by

$$\mathbf{k} = \mathbf{k}(s) = \dot{\mathbf{t}}(s) \quad (\text{A-3})$$

The magnitude of the curvature vector is denoted by

$$|\kappa| = |\mathbf{k}(s)| \quad (\text{A-4})$$

and is called the curvature of $\mathbf{C}$ at $\mathbf{x}(s)$. The reciprocal of the curvature is denoted by

$$|\rho| = \frac{1}{|\kappa|} = \frac{1}{|\mathbf{k}(s)|} \quad (\text{A-5})$$

and is called the radius of curvature at $\mathbf{x}(s)$. Thus the curvature can also be taken as the inverse of the radius of a best-fitting circle.

Since a surface is a two-dimensional entity, two parameters called principal curvatures are needed for a complete description of local shape. These two principal curvatures say κ_1 and κ_2 , in two perpendicular directions, are maximum and minimum values of normal curvature. Any point on a two-dimensional surface can be denoted by a two-parametric position vector

$$\mathbf{x} = \mathbf{x}(u, v) = \begin{pmatrix} x(u, v) \\ y(u, v) \\ z(u, v) \end{pmatrix} \quad (\text{A-6})$$

whose components are functions of two parameters of u and v . We assume these functions are continuous and differentiable as often as necessary. In order to extract local shape characteristics from some points given in the form $\mathbf{x} = \mathbf{x}(u, v)$, we have to consider those points $\mathbf{x} + d\mathbf{x} = \mathbf{x}(u + du, v + dv)$ in the vicinity of $\mathbf{x}$.

Let $\mathbf{x} = \mathbf{x}(u, v)$ be a coordinate patch on a surface of class greater than one. The differential of the mapping $\mathbf{x} = \mathbf{x}(u, v)$ at (u, v) is a 1-1 linear mapping

$$d\mathbf{x} = \mathbf{x}_u du + \mathbf{x}_v dv \quad (\text{A-7})$$

of the vectors (du, dv) in the uv plane onto the vectors $\mathbf{x}_u du + \mathbf{x}_v dv$ parallel to the tangent plane at $\mathbf{x}(u, v)$. Further, $d\mathbf{x}$ has the property that

$$\mathbf{x}(u + du, v + dv) = \mathbf{x}(u, v) + d\mathbf{x} + \mathbf{o}((du^2 + dv^2)^{1/2}) \quad (\text{A-8})$$

Thus the vector $d\mathbf{x}$ is a first order approximation to the vector $\mathbf{x}(u+du, v+dv) - \mathbf{x}(u, v)$ from the point $\mathbf{x}(u, v)$ on the patch to the neighboring point $\mathbf{x}(u+du, v+dv)$. The quantity

$$\begin{aligned} I &= d\mathbf{x} \cdot d\mathbf{x} \\ &= (\mathbf{x}_u du + \mathbf{x}_v dv) \cdot (\mathbf{x}_u du + \mathbf{x}_v dv) \\ &= (\mathbf{x}_u \cdot \mathbf{x}_u) du^2 + 2(\mathbf{x}_u \cdot \mathbf{x}_v) dudv + (\mathbf{x}_v \cdot \mathbf{x}_v) dv^2 \\ &= Edu^2 + 2Fdudv + Gdv^2 \end{aligned} \quad (\text{A-9})$$

Here

$$E = \mathbf{x}_u \cdot \mathbf{x}_u \quad F = \mathbf{x}_u \cdot \mathbf{x}_v \quad G = \mathbf{x}_v \cdot \mathbf{x}_v \quad (\text{A-10})$$

The function (A-9) is called the first fundamental form of $\mathbf{x} = \mathbf{x}(u, v)$. It is a homogeneous function of second degree in du and dv with coefficients E , F , and G , called the first fundamental coefficients, which are functions of u and v and vary from point to point on the coordinate patch. Thus the first fundamental form I is the quadratic form defined on vectors (du, dv) in the uv plane by

$$I(du, dv) = Edu^2 + 2Fdudv + Gdv^2 \quad (\text{A-11})$$

The first fundamental form is positive definite. That is $I \geq 0$ and $I=0$ if and only if $du=0$ and $dv=0$. The reason is that $I = |d\mathbf{x}|^2 \geq 0$ and, since $\mathbf{x}_u, \mathbf{x}_v$ are independent,

$$I = |d\mathbf{x}|^2 = |\mathbf{x}_u du + \mathbf{x}_v dv|^2 = 0 \text{ if and only if } du=0 \text{ and } dv=0.$$

Since I is positive definite, its coefficients must satisfy $E > 0$, $G > 0$ and $EG - F^2 > 0$.

This can also be verified directly. Since $\mathbf{x}_u, \mathbf{x}_v$ are independent, $\mathbf{x}_u \neq 0, \mathbf{x}_v \neq 0$ and

hence $E = \mathbf{x}_u \bullet \mathbf{x}_u = |\mathbf{x}_u|^2 > 0$ and $G = \mathbf{x}_v \bullet \mathbf{x}_v = |\mathbf{x}_v|^2 > 0$. Also, we find that

$$\begin{aligned} EG - F^2 &= (\mathbf{x}_u \bullet \mathbf{x}_u)(\mathbf{x}_v \bullet \mathbf{x}_v) - (\mathbf{x}_u \bullet \mathbf{x}_v)(\mathbf{x}_u \bullet \mathbf{x}_v) \\ &= (\mathbf{x}_u \times \mathbf{x}_v) \bullet (\mathbf{x}_u \times \mathbf{x}_v) = |\mathbf{x}_u \times \mathbf{x}_v|^2 \end{aligned} \quad (\text{A-12})$$

But at every point $\mathbf{x}_u \times \mathbf{x}_v \neq 0$, so $EG - F^2 > 0$.

We now suppose $\mathbf{x} = \mathbf{x}(u, v)$ is a patch on a surface of class greater than two. Then at each point on the patch there is a unit normal

$$\mathbf{N} = \frac{\mathbf{x}_u \times \mathbf{x}_v}{|\mathbf{x}_u \times \mathbf{x}_v|} \quad (\text{A-13})$$

which is a function of u and v of class C^1 with differential

$$d\mathbf{N} = \mathbf{N}_u du + \mathbf{N}_v dv \quad (\text{A-14})$$

Now we consider the quantity

$$\begin{aligned} \Pi &= -d\mathbf{x} \bullet d\mathbf{N} = -(\mathbf{x}_u du + \mathbf{x}_v dv) \bullet (\mathbf{N}_u du + \mathbf{N}_v dv) \\ &= -\mathbf{x}_u \bullet \mathbf{N}_u du^2 - (\mathbf{x}_u \bullet \mathbf{N}_v + \mathbf{x}_v \bullet \mathbf{N}_u) du dv - \mathbf{x}_v \bullet \mathbf{N}_v dv^2 \\ &= L du^2 + 2M du dv + N dv^2 \end{aligned} \quad (\text{A-15})$$

Here

$$L = \mathbf{x}_{uu} \bullet \mathbf{N} \quad M = \mathbf{x}_{uv} \bullet \mathbf{N} \quad N = \mathbf{x}_{vv} \bullet \mathbf{N} \quad (\text{A-16})$$

Also we have

$$\begin{aligned}
 II &= Ldu^2 + 2Mdudv + Ndv^2 \\
 &= \mathbf{x}_{uu} \cdot \mathbf{N}du^2 + 2\mathbf{x}_{uv} \cdot \mathbf{N}dudv + \mathbf{x}_{vv} \cdot \mathbf{N}dv^2 = d^2\mathbf{x} \cdot \mathbf{N}
 \end{aligned}
 \tag{A-17}$$

The function (A-15) is called the second fundamental form of $\mathbf{x} = \mathbf{x}(u, v)$. Here again II is a homogeneous function of second degree in du and dv with the second fundamental coefficients L , M and N . These coefficients are continuous functions of u and v .

Suppose P is a point on a surface of class greater than two, Q is a point in the neighborhood of P and $\mathbf{x} = \mathbf{x}(u, v)$ is a patch containing P and Q . Let $d = \mathbf{PQ} \cdot \mathbf{N}$ be the projection of $\mathbf{PQ}$ onto the unit normal $\mathbf{N}$ at P , as shown in Figure A-1. Observe that d is positive or negative depending on whether Q is on one or the other side of the tangent plane at P and that $|d|$ is the perpendicular distance from Q to the tangent plane at P . Now suppose P and Q are the points $\mathbf{x}(u, v)$ and $\mathbf{x}(u+du, v+dv)$ respectively.

Figure A-1: The relationship between $|d|$, $\mathbf{PQ}$ and $\mathbf{N}$ [124]

From Taylor's theorem we can get

$$\mathbf{x}(u + du, v + dv) = \mathbf{x}(u, v) + d\mathbf{x} + \frac{1}{2}d^2\mathbf{x} + o(du^2 + dv^2)
 \tag{A-18}$$

Thus

$$\begin{aligned}
d &= \mathbf{PQ} \cdot \mathbf{N} = (\mathbf{x}(u + du, v + dv) - \mathbf{x}(u, v)) \cdot \mathbf{N} \\
&= [\mathbf{dx} + \frac{1}{2}d^2\mathbf{x} + \mathbf{o}(du^2 + dv^2)] \cdot \mathbf{N} \\
&= \mathbf{dx} \cdot \mathbf{N} + \frac{1}{2}d^2\mathbf{x} \cdot \mathbf{N} + \mathbf{o}(du^2 + dv^2)
\end{aligned} \tag{A-19}$$

But $\mathbf{dx} \cdot \mathbf{N} = 0$, since $\mathbf{dx}$ parallel to the tangent plane at P. Hence from (4-17),

$$d = \frac{1}{2}d^2\mathbf{x} \cdot \mathbf{N} + \mathbf{o}(du^2 + dv^2) = \frac{1}{2}\Pi + \mathbf{o}(du^2 + dv^2) \tag{A-20}$$

Thus Π is the principal part of twice the projection of $\mathbf{PQ}$ and $\mathbf{N}$ and $|\Pi|$ is the principal part of twice the perpendicular distance from Q onto the tangent plane at P.

The function

$$\delta = \frac{1}{2}\Pi = Ldu^2 + 2Mdudv + Ndv^2 \tag{A-21}$$

is called the osculating paraboloid at P. The nature of this paraboloid determines qualitatively the nature of the surface in the neighborhood of P, depending on the value of $LN - M^2$.

- (1) Elliptic case: A point is called an elliptic point if $LN - M^2 > 0$. In this case δ as a function of du and dv is an elliptic paraboloid as shown in Figure A-2. δ maintains the same sign for all (du, dv) . In the neighborhood of an elliptic point the surface lies on one side of the tangent plane at the point and is shaped like the figure.
- (2) Hyperbolic case: A point is called a hyperbolic point if $LN - M^2 < 0$. In this case δ as a function of du and dv is a hyperbolic paraboloid as shown in Figure A-2. Here there are two distinct lines in the tangent plane through P which

divide the tangent plane into four sections in which δ is alternatively positive and negative. On the two lines, $\delta = 0$. In the neighborhood of a hyperbolic point the surface lies on both sides of the tangent plane as in the figure.

- (3) Parabolic case: A point is called a parabolic point if $LN - M^2 = 0$ and $L^2 + M^2 + N^2 \neq 0$. In this case δ as a function of (du, dv) is a parabolic cylinder, as shown in the figure. There is a single line in the tangent plane through P along which $\delta = 0$, otherwise δ maintains the same sign.
- (4) Planar case: A point is called a planar point if $L=M=N=0$. Here $\delta = 0$ for all (du, dv) . In this case the degree of contact of the surface and the tangent plane is of higher order than in the preceding case.

Figure A-2: Different cases with respect to different $LN - M^2$ [124]

If $\mathbf{n}$ and $\mathbf{t}$ are unit normal vector and unit tangent vector to the surface at P respectively, then the plane containing $\mathbf{n}$ and $\mathbf{t}$ are orthogonal to the surface at P. The intersection of this plane and the surface is a curve C. The curvature of C is the curvature of the normal cross section of the surface in the direction of $\mathbf{t}$. This is called

the normal curvature of the surface at the point P in the direction $\mathbf{t}$. The normal curvature denoted by κ_n is the vector projection of the curvature vector $\mathbf{k}$ of $\mathbf{C}$ at P onto the normal $\mathbf{N}$ at P. Thus

$$\kappa_n = (\mathbf{k} \cdot \mathbf{N})\mathbf{N} \quad (\text{A-22})$$

From (A-22), we can get

$$\kappa_n = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2} = \frac{II}{I} \quad (\text{A-23})$$

Clearly the normal curvature depends on the direction of the cross section. To give some measurement of the “amount of curvature” of a surface at a given point, it would be obviously useful to know the maximum and minimum normal curvature at that point. Accordingly, there are two perpendicular directions for which the values of κ_n take on maximum and minimum values. These two directions are called principal directions and the corresponding normal curvatures κ_1, κ_2 are called the principal curvatures.

A real number κ is a principal curvature in the direction $du:dv$ if and only if κ, du, dv satisfy

$$\begin{aligned} (L - \kappa E)du + (M - \kappa F)dv &= 0 \\ (M - \kappa F)du + (N - \kappa G)dv &= 0 \end{aligned} \quad (\text{A-24})$$

where $du^2 + dv^2 \neq 0$.

The above is a homogeneous system of equations and will have a nontrivial solution du, dv if and only if

$$\det \begin{pmatrix} L - \kappa E & M - \kappa F \\ M - \kappa F & N - \kappa G \end{pmatrix} = 0 \quad (\text{A-25})$$

That is the two principal curvatures κ_1, κ_2 are solutions of the equation

$$(EG - F^2)\kappa^2 - (EN + GL - 2FM)\kappa + (LN - M^2) = 0 \quad (\text{A-26})$$

The average of the roots κ_1, κ_2

$$H = \frac{\kappa_1 + \kappa_2}{2} = \frac{EN + GL - 2FM}{2(EG - F^2)} \quad (\text{A-27})$$

is called the mean curvature. The product of κ_1, κ_2

$$K = \kappa_1 \kappa_2 = \frac{LN - M^2}{EG - F^2} \quad (\text{A-28})$$

is called the Gaussian curvature.

To get a more detailed classification of shapes, the shape index function is defined by Koenderink:

$$si = \frac{2}{\pi} \arctan \frac{\kappa_2 + \kappa_1}{\kappa_2 - \kappa_1} \quad (\kappa_1 \geq \kappa_2) \quad (\text{A-29})$$

The information contained in the Koenderink shape index are formally equivalent to either the two principal curvatures or to the Gaussian and mean curvature taken as a pair. This way, all the surface patches, except for plane patches, are mapped into $si \in [-1, +1]$ (Table A-1). This shape index function has advantages compared to the Gaussian or mean curvature alone:

1. The shape index is scale invariant. For example, different-radius spheres will not differ themselves on the shape index map.
2. The convexities and concavities are on the opposite sides of the shape index scale, separated by saddle-shape regions.
3. Two shapes of the same scale differ only in the sign of shape index will fit to each other as stamp and mold.

Surface type	Shape index range
Spherical cup	$[-1, -7/8)$
Trough	$[-7/8, -5/8)$
Rut	$[-5/8, -3/8)$
Saddle Rut	$[-3/8, -1/8)$
Saddle	$[-1/8, +1/8)$
Saddle Ridge	$[+1/8, +3/8)$
Ridge	$[+3/8, +5/8)$
Dome	$[+5/8, +7/8)$
Spherical Cap	$[+7/8, +1]$

Table A-1 Shape categories by Koenderink shape index

APPENDIX B: GAUSSIAN HAT

The Gaussian convolution is defined by $z = Ae^{-\frac{[(x-x_0)^2 + (y-y_0)^2]}{2\sigma^2}}$. A is the amplitude, σ is the standard deviation which decides the expansion of the Gaussian hat on the xy plane. x_0, y_0 is the center of the Gaussian hat on the xy plane. The Gaussian hat is shown in Figure B-1.

Figure B-1: 3D display of Gaussian hat from two viewing directions.

APPENDIX C: RIM CURVATURE

The rim curvature is defined as a symmetric function of the principle curvatures, which is the difference of their absolute values [125]. This function emphasizes those regions with pronounced difference of curvature in the principle directions. In order to distinguish between concave and convex regions a modified function is used:

$$DK = \kappa_{>} - \kappa_{<} \cdot \text{signum}(\kappa_1 \cdot \kappa_2) \quad (\text{C-1})$$

Here $\kappa_{>}$ is the principle curvature with greater absolute value and $\kappa_{<}$ is the principle curvature with smaller absolute value. $\text{signum}(\kappa_1 \cdot \kappa_2)$ is equal to +1 if the two principle curvatures have the same sign. Otherwise it is equal to -1.

The rim curvatures of some special shapes are tabulated in Table C-1.

Sphere	Cylinder	Saddle $ \kappa_1 = \kappa_2 $	Cylinder	Sphere

DK=0	DK= $\kappa_{>}$	DK=0	DK= $\kappa_{>}$	DK=0

Table C-1: The rim curvatures of special shapes.

APPENDIX D: AN INDICATOR OF LEG DEFORMITY

D.1 INTRODUCTION

The tibial torsion estimated in the previous way is not the true tibial torsion since the angle of the two lines in the space is calculated on the plane that parallel to both of the lines. The normal direction of this plane is not coincident with the longitudinal axis of the tibia. It deviates by an angle. So this method is just an estimation of the tibial torsion. But the gathered information may give a way of measuring leg deformity.

D.2 DEFINITION OF THE LEG DEFORMITY INDICATOR

In this study, two different leg deformity indicators (DI1 and DI2) have been defined to measure how much the leg is deformed. The reason of trying different definitions is to see which deformity indicator is more consistent and reasonable.

Five vectors are defined (Figure D-1), which gives two different definitions of DI. The first vector v_1 is from the middle point of A and B to point C. The second vector v_2 is from D to E. The third vector v_3 gives the estimation on the longitudinal axis of tibia. It passes through the middle point of B&C and the middle point of D&E. The fourth vector v_4 is the cross product of normalized v_1 and v_2 . The angle β is the angle between v_3 and v_4 , which shows how v_4 deviates from v_3 . The fifth vector v_5 is the cross product of normalized v_{1p} and v_{2p} , where v_{1p} and v_{2p} are the projection of v_1 and v_2 on the plane that is perpendicular to the longitudinal axis of the tibia (v_3). The leg deformity indicator is calculated based on the magnitude of v_4 (DI1) and v_5 (DI2), separately. v_4 and v_5 are both normalized in the calculations.

Figure D-1: The vectors in calculating the deformity indicator.

In Figure D-1, v_1 is from the middle point of A and B to point C. v_2 is from D to E. v_3 passes through the middle point of B&C and the middle point of D&E. v_4 is the cross product of v_1 and v_2 . The angle β is the angle between v_3 and v_4 , which shows how v_4 deviates from v_3 . v_{1p} and v_{2p} are the projections of v_1 and v_2 when they are projected to the plane that is perpendicular to the tibia's longitudinal axis. v_5 is the cross product of v_{1p} and v_{2p} . The angle θ between v_{1p} and v_{2p} shows how v_1 deviates from v_2 on the plane perpendicular to the longitudinal axis of tibia.

D.3 EXPERIMENT

Altogether 18 subjects' left and right legs were scanned. Based on the landmarks located, the vectors and DI were calculated. Also the distribution of the DIs is studied.

The t test with correction for correlated samples was performed to detect whether there were any significant differences between the DIs of left and right legs. The null hypothesis stated no significant difference.

D.4 RESULTS

D.4.1 First definition of deformity indicator (DI1)

Figure D-2: The leg deformity indicator (DI1) of 18 subjects.

Eighteen subjects' left and right leg deformity indicator is shown in Figure D-2. The left legs' DI ranges from 0.29 to 0.59 with a mean of 0.44. The right legs' DI ranges from 0.34 to 0.58 with a mean of 0.45.

The distribution of the DI1 is plotted in Figure D-3.

Figure D-3: The distribution of DI1.

Most of the DI1 values are in the range of 0.4-0.5.

Figure D-4: The deviating angles of v_4 from v_3 .

In Figure D-4, the left legs' deviating angles ranges from 22.61° to 68.12° with a mean of 48.20° . The right legs' deviating angles ranges from 28.60° to 71.56° with a mean of 44.47° .

The t test for correlated samples showed that the t was not significant at $p=0.10$.

D.4.2 Second definition of deformity indicator (DI2)

Figure D-5: The leg deformity indicator (DI2) of 18 subjects.

Eighteen subjects' left and right leg deformity indicator (DI2) is shown in Figure D-5. The left legs' DI ranges from 0.09 to 0.36 with a mean of 0.22. The right legs' DI ranges from 0.10 to 0.43 with a mean of 0.24.

The distribution of the DI2 is plotted in Figure D-6.

Figure D-6: The distribution of DI2.

Most of the DI2 values are in the range of 0.1-0.3.

Figure D-7: The angles of v_{lp} from v_{2p} .

Figure D-7 shows that the left legs' θ ranges from 4.81° to 20.72° with a mean of 12.52° . The right legs' θ ranges from 5.78° to 25.68° with a mean of 13.78° .

The t test for correlated samples showed that the t was not significant at $p=0.10$.

D.5 DISCUSSION

The deformity indicator is derived from the anatomical landmarks which have been extracted by way of surface curvature. The objective determination of the anatomical landmarks gives highly reproducible results. Consequently, the reproducibility of calculating the deformity indicator is ensured.

The deformity indicator on the 18 subjects varies among the subjects. For one subject, there is difference between his left and right leg's deformity indicator. It's the same for the angles. The difference may be the result of the following reasons:

- I) For one person, there is difference between his left and right leg. This might account for a larger part of the resultant difference.
- II) During the scanning, the movement of the subjects cannot be avoided totally.
- III) The anatomical landmarks localization also has a high accuracy, their final location still changes due to noise. And the vector direction is sensitive to the position changes, which will influence the final results.

The two different definitions of the DI give different values. The second definition of DI highly correlates the DI with the angle between the transcondylar and transmalleolar axes projected on the plane perpendicular to the longitudinal axis of tibia. The DI2 as well as the angle θ give a better indication of deformity of leg than DI1 and angle β . The angle β between v_4 and v_3 in the first definition is too large and too inconsistent. Based on these reasons, we finally take the second definition as our deformity indicators.

All of the deformity indicators fall in the range of 0 to 0.5. From Figure D-6, we may suggest that the normal people's DI values are distributed normally with a mean value from 0.1 to 0.3. This gives an idea of the normal people's deformity indicator. The results may suggest that there is a range for normal people. For those people with abnormal leg deformities, their deformity indicator may not in this range. But unfortunately, we did not have the data of subjects with abnormal leg deformities.

To show that DI2 is already normalized and not correlated to the height of the subjects or the length of the bone, DI2 is compared to the previously studied TMD (Figure 6-7) on the same subjects. There is no obvious correlation between the deformity indicators and the TMD.

The small sample size and the measurements of only 18 pairs of legs limit the generalization of the findings. More subjects are required in order to study the deformity difference between left and right legs.

D.6 CONCLUSION

We have proposed a deformity indicator to gives the degree of deforms of the leg. The total eighteen subjects were engaged for this effort. The proposed method is characterized by its objective determination of anatomical landmarks that allows for a high level of reproducibility on calculating the deformity indicator. The method offers a fast, objective and non-invasive way of screening because it discloses the information of the bones' deforms from the surface scanning. Experiment needs to be carried out on more subjects includes those with abnormal leg deformities. The radiograph may also be necessary to see whether the deformity indicator is proportional to the leg deformities.