Fiber based whispering gallery mode resonators

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**NANYANG
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Abstract

The ability to confine light in small volume is a key challenge in modern optics. Optical microresonator supporting whispering gallery mode (WGM) is able to achieve over 10^9 Q-factor with micron-scale modal volume. Due to the high-Q and small-mode-volume characteristics, WGM resonators have become the basis of a wide range of applications including sensing, filtering and switching devices; the studies of quantum electrodynamics (QED) and optical nonlinear phenomena have grown to rely on WGM as it provides quantum mechanical analogy and low power-threshold. Despite the continuous effort to optimize WGM resonators for high-Q performance, the problem of efficient power delivery with robust mechanical stability remains unsolved. This thesis investigates the fiber based WGM resonators to address all the above mentioned challenges.

Metallic diffraction grating as a coupler to the WGM resonator is proposed. It is the first time that such coupler is extended to the use of metallic material. With the employment of this unconventional structure, the new WGM resonator gives rise to interesting phenomena and performance enhancement. Unlike the fragile fiber taper coupler, the metallic diffraction grating is mounted on a much larger platform – the end facet of a single mode fiber (SMF) – so that the coupler has sound mechanical strength and is durable for repeated use. In contrast with dielectric grating, the metallic grating is able to provide higher diffraction efficiency and therefore higher coupling efficiency to the WGM resonator. The coupling utilizes the first order diffractions that bear transverse wavevector to align with the local wavevector of the WGM. Phase-matching conditions can therefore be tuned by the period of the grating.

Due to the interaction of multiple scattering processes in the metallic diffraction grating coupled WGM resonator, Fano resonances are observed in both reflected and transmitted spectra. The Fano resonance is found tunable by the incident polarization, grating thickness and filling factor. Hence, the grating coupler generates tunable resonance shapes and provides an entirely new degree of freedom for the realization of different applications. Moreover, the metallic grating is the only known coupler for WGM resonators that is able to excite and manipulate modes of two propagation directions simultaneously. Such complex resonance system is modelled for power

coupling calculation. The use of metal provides a reflection pathway so that single port coupling with tunable phase-matching design is realized for the first time. The grating coupled WGM resonator serves as a practical platform for study of linear and nonlinear applications that require mechanical robustness and simple configuration. Despite the use of a lossy material (gold), high Q-factor is observed for coupled silica microspheres. Moreover, four wave mixing (FWM) is demonstrated in the reflection spectra of a metallic grating coupled microsphere resonator at milliwatt power-threshold. With further optimization of the grating coupler (in terms of material and diffraction efficiency), the metallic diffraction grating coupled WGM resonators would give rise to single-probe sensing and switching device as well as low power-threshold fiber lasers and optical frequency comb.

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Glossary of acronyms

ASE	amplified spontaneous emission
CW	continuous wave
FDTD	finite difference time domain
FSR	free spectral range
FWM	four wave mixing
MIR	mid infrared
OSA	optical spectrum analyzer
PEC	perfect electrical conductor
QED	quantum electrodynamics
SEM	scanning electron microscope
SMF	single mode fiber
TE	transverse electric
TM	transverse magnetic
WGM	whispering gallery mode

Chapter 1 Introduction

1.1 Background

Optical resonators confine light both in space and time. The optical resonators are extremely important in modern optics, in that they play an indispensable role in almost all aspects of linear [1-3] and nonlinear optics [4, 5].

By methods of light confinement, optical resonators can be categorized into the Fabry-Perot, photonic crystal and whispering gallery types.

Light confinement with a pair of mirrors, or the Fabry-Perot principle, is the most straightforward cavity construction mechanism. However, due to the difficulty to find high-quality mirrors, miniaturization of the conventional Fabry-Perot cavity is either complicated or yields low quality factor (Q). One successful example of such type of resonator is the micropillar that utilizes AlAs/GaAs distributed-Bragg-reflected mirrors to confine light [6-8]. It serves as a semiconductor based single photon source when a single InAs or InGaAs quantum dot is embedded in the micropillar resonator. Based on this single photon source, the micropillar resonators have been investigated for applications including quantum key distribution [9], formation of polarization entanglement [10], and dual-rail teleportation [11].

Among the different types of optical resonators, photonic crystal cavities [12, 13] provide the smallest mode volume (V) [14, 15]. These are artificial dielectric structures with periodically modulated refractive indices at a structural dimension close to an optical wavelength. Because of this property, the photonic crystal cavities are usually called the nanocavities. The periodically

modulated refractive index alters the density of photonic states, allowing design of unique photonic band diagrams in terms of frequency and wavevector. Since the photon density in the nanocavities are generally very high, various optical nonlinear effects are able to occur at low pump energy, which enables the fabrication of practical all-optical switches [16] and logic gates [17] [13]. Specifically, all-optical switching at an input energy of less than 100 fJ has been demonstrated using a two-dimensional silicon photonic crystal nanocavity [16].

Unlike Fabry-Perot and photonics crystal, whispering gallery mode (WGM) resonator confines light by way of total internal reflections. The WGM resonators are usually axisymmetric so that light can travel around the perimeter of the resonator without experiencing considerable loss.

WGM was first discovered in sound waves by Lord Rayleigh, who studied the phenomenon based on the whispering gallery of the St Paul's Cathedral [18]. St Paul's cathedral is famous for its circular dome which is of both architectural ingenuity and scientific value. It is said that one is able to hear the whisper to the dome coming from a far distance if he held his ears to the dome wall – the sound waves travel along the perimeter of the dome with very little loss – hence the name “whispering gallery”.

To support optical waves, the optical version of whispering gallery must reduce its dimension to the order of millimeters or even micrometers. The optical confinement of a WGM resonator in the spatial domain is also described by the mode volume (V) while the temporal capability is represented by the quality factor (Q). A tradeoff between the mode volume and quality factor exists which has perplexed researchers for decades: high- Q resonators usually cover a large mode volume (dielectric resonators), while resonators of tight confinement always have low Q -factors (plasmonic resonators). WGM resonator is a good candidate to resolve, though not completely, the tradeoff between V and Q . Since WGM is confined inside a monolithic geometry and the fundamental mode always stays close to the surface boundary of the resonator, WGM resonators generally exhibit very high quality factors and relatively small mode volume.

Over the years, optical fibers have served the purpose of an important category of WGM resonators. Since the optical fibers are generally made of amorphous materials, circularly symmetric shapes would instantly form when proper heat is applied. Besides the advantage of easy-fabrication, optical fibers offer a wide range of materials for different applications. For

example, the silica optical fibers are generally used for its low-loss properties [19, 20] while chalcogenide fibers are currently explored for optical nonlinear effects and photo-induced properties [21, 22]. On the other hand, optical fibers are widely applied as power coupling devices in the microresonator system, regardless of whether the resonator itself is made of optical fibers or not [4, 21, 23-25]. Fiber-based WGM resonator systems are small and compact, low cost, easy to handle and are convenient to be incorporated into the already-mature fiber-optic system.

Efficient and robust power coupling to WGM is a major challenge to the practical realization of WGM resonators. Till now, most low-power-threshold optical nonlinear effects inside the WGM resonator are achieved by coupling with fiber taper [4, 24, 26, 27], which has been reported to achieve almost 100% coupling efficiency [28]. However, the fiber taper is fragile and easily contaminated, which prohibits its use in practical applications. The imminent task is to design a practical WGM coupler that is both efficient in power coupling and robust enough to withstand environmental instabilities.

1.2 Thesis outline

Chapter 2 reviews the basic properties of WGM resonator in the context of two resonator shapes: the spherical resonator and spheroidal resonator. Analytical derivation of the spherical dielectric resonator is briefly presented. Asymptotic solutions of the modes which bear significance for both determination of the resonance wavelength and design of the phase-matched coupling are discussed. Optical modes in spheroidal resonators are then investigated. Two categories of the spheroidal resonators are of interest: 1. Microbottle resonator that resembles a cylinder in geometry but exhibits adiabatic profile either in diameter or refractive index along the cylinder length; 2. Almost spherical resonator that induces eccentricity splitting along the polar direction. These resonators are categorized together as they both have rather large mode coverage and are related to the problem of light confinement in the polar direction. After a discussion of the quality factor, some typical applications of the WGM resonators are reviewed.

Of the many WGM coupling methods that are developed over the years, **Chapter 3** discusses four types of conventional techniques that are already widely used. The coupling mechanism of

the waveguide coupler (tapered optical fiber is considered as one type of waveguide) is first explained in details. The prism coupler utilizes total internal reflection to excite WGM. Based on a similar principle, the angle polished fiber tip can be considered as a miniaturized version of prism coupler. The tapered optical fiber is considered as the main-stream coupling device that offers almost 100% coupling efficiency at critical coupling. When the fiber is tapered down to a small waist diameter and breaks, a tapered fiber tip is generated. Such structure is able to couple WGM the same way as a fiber taper but it provides the reflection pathway as an additional output port. Lastly, diffraction grating couples to the WGM with the diffracted modes. However, the current diffraction gratings made of dielectric materials are extremely inefficient.

Chapter 4 proposes a new WGM coupler – the metallic diffraction grating. With the use of the metallic material, and the diffraction modes that come along with the grating, several interesting features of the coupled microresonator are unveiled. The basic coupling mechanism is first explained based on the finite difference time domain (FDTD) results. Phase-matching designs are then presented based on the conclusions reviewed in Section 2.1. Finally, fabrication process of the gold grating coupler is presented.

Since the grating coupler gives rise to more than one input modes, the metallic grating coupled WGM resonator constitutes a complex system that eventually produces asymmetric resonance line-shapes – the Fano resonances. As the phase-matching conditions can be controlled by the grating design, the possibility of different WGMs being selectively enhanced or suppressed is further explored. **Chapter 5** identifies the optical modes in the resonance system and discusses several degrees of mode tunability through polarization conditions and grating design. An interesting finding related to selective enhancement and suppression of different radial-order WGMs as well as their bi-directionality is then discussed.

However complex, the modes that are responsible for the excitation of WGM must be identified to explicitly calculate the coupling efficiency. Coupling efficiency is important as it directly affects the amount of input power to generate optical nonlinear effects. **Chapter 6** explains a new theoretical model to study the power coupling mechanism of the grating-resonator system. Based on the model, optimization of the grating coupler can be made for further enhancement of the coupling efficiency.

Having understood the power coupling efficiency of the metallic grating coupler, it is possible to assess the power threshold of some nonlinear effects. **Chapter 7** presents the experimental work of a dual-cavity laser that is enabled by a gold grating coupler. A WGM sphere resonator is used as high-Q filter while an external fiber loop cavity serves as the laser cavity. It is demonstrated that the laser constructed from such dual-cavity architecture is thermally stable. In addition, four wave mixing (FWM) is also observed in the coupled structure.

Chapter 8 concludes the whole thesis and gives several areas of future work.

Chapter 2 Optical modes in whispering gallery mode resonators

2.1 Optical modes in spherical resonators

Sphere is the simplest and most common geometry of WGM resonators. It can be fabricated from a large range of materials and in many methods. Generally, the spheres can be made from the melting process, the liquid droplets, or the sol-gel method [29].

Melting the tip of a glass fiber has been the most popular and easiest way [19, 22, 23]. The resonator hence made is usually attached with a fiber stem. Micro-droplet resonators [30, 31] are suited for solution or liquid based applications, but are not as stable in atmospheric conditions. The sol-gel process rids the resonator free of the stem to create a free-standing geometry. However, like some types of micro-droplet resonators, the sol-gel generated resonators are mainly liquid based, making precision coupling somewhat difficult. This is why instead of making solid resonators, sol-gel chemistry is often employed to realize a functional coating on the sphere surface [32].

As shown in Fig. 2-1, the WGM inside a sphere is determined by many parameters, which in turn reveal the information of field distribution along polar, azimuthal and radial directions. The fundamental mode always stays in the equatorial plane and has the lowest radial and azimuthal order. To understand the meaning of different mode indices and their implications on various

applications, analytical derivations are carried out to characterize the mode in a spherical resonator.

Fig. 2-1 Schematic diagram of a WGM (third radial order and higher azimuthal order) inside a perfect sphere resonator; field distribution on the right shows an example of waveguide coupled fundamental WGM in the equatorial plane of the sphere.

2.1.1 Analytical derivation

The theory of WGMs in spherical resonators is well-established [33-35]. The work of Oraevsky delivers a comprehensive overview of the problem [34]. For light incident on the surface of resonator at a grazing incidence, it can be contained within the circular trajectory of the resonator perimeter. Constructive interference among the propagating light waves occurs if the total distance of propagation is equal to integer multiples of the wavelength, i.e. the circumference $2\pi R = \ell\lambda$, where R is the radius of curvature, λ is the resonance wavelength and ℓ the polar mode number that is close to the number of wavelengths to fit into the circumference. For the WGMs that are adjacent to the resonator equatorial plane (which is generally the case for all WGMs in a large WGM resonator), the optical energy is mainly concentrated in the electric field components that can be treated as either transverse electric (TE, tangential) or transverse magnetic (TM,

normal to the surface) modes [36]. Hence, analytical solutions of the WGMs can be treated in a spherical coordinate for two polarizations by solving the scalar wave equations of either the electric field (E) or magnetic field (H). The electric field is often chosen. It can be expressed by the symbol ψ as a function of θ , φ and r (representations are labeled in Fig. 2-1). Converting Helmholtz equation into the spherical coordinate, and we obtain Equation (2.1).

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2(\theta)} \frac{\partial^2 \psi}{\partial \varphi^2} + k^2 \psi = 0 \quad (2.1)$$

Where n is the refractive index of the resonator and k is the wave-vector inside the sphere. By separation of variables as Equation (2.2) and substitution into Equation (2.1), the field can be separated into three equations that are related to φ , θ and r respectively.

$$\psi = \Theta(\theta)R(r)e^{\pm im\varphi} \quad (2.2)$$

$$\Theta'' - \cot(\theta) \Theta' + \left[\ell(\ell + 1) - \frac{m^2}{\sin^2(\theta)} \right] \Theta = 0 \quad (2.3)$$

$$R'' + \left[k^2 - \frac{\ell(\ell+1)}{r^2} \right] R = 0 \quad (2.4)$$

R represents the field component that varies in the radial direction and Θ is the component that changes along the polar angle direction. For the azimuthal direction, function of the field is expressed as the exponential term in Equation (2.2). Due to the cavity forming conditions discussed earlier (that the number of wavelengths fit into the circular perimeter must be an integer), phase is the only changing component along the round-trip circulation. Here, the azimuthal mode number (m) is introduced to represent the number of wavelengths along the circumference of resonator in the equatorial plane. Equation (2.3) and (2.4) can be solved for analytical solutions to yield field profiles along the polar and radial direction respectively. The mathematical derivation of Equation (2.3) and (2.4) generates another mode number ℓ that indicates the polar profiles of the field. In terms of physical meaning, the polar mode number ℓ is related to our earlier explanation of the cavity forming conditions where the symbol first appeared. Unlike m , ℓ counts the number of wavelengths that can be fit into the cavity which is located in any plane of the resonator. Hence, the azimuthal mode number can be considered as

bearing information of the wave projection onto the equatorial plane while the polar mode number describes the actual trajectories of wave propagation.

Solutions of Equation (2.3) can be represented by the adjoint Legendre polynomials which is indicated by $P_\ell^m(\cos(\theta))$. Equation (2.4) is reformulated as the Bessel equation and the solutions are given by Bessel functions.

$$\begin{aligned}\psi(r, \theta, \varphi) &= C_i P_\ell^m(\cos(\theta))(kr)^{1/2} J_\nu(kr) e^{\pm im\varphi} & r \leq a \\ \psi(r, \theta, \varphi) &= C_e P_\ell^m(\cos(\theta))(k_0 r)^{1/2} H_\nu^{(1)}(k_0 r) e^{\pm im\varphi} & r > a\end{aligned}\quad (2.5)$$

In Equation (2.5), C_i and C_e are two arbitrary constants that modify the internal and external field (relative to the resonator boundary) respectively. J_ν is the Bessel function and $H_\nu^{(1)}$ is the Hankel function of the first kind. The radius of curvature is represented by the letter a ; and k_0 is the wave number in vacuum. Boundary value condition requires that the tangential component of the field equals at the resonator boundary where refractive index differs by $n-1$ (assuming the resonator is placed in air). It then follows that C_i and C_e satisfies $C_i/C_e = \sqrt{k_0/k} H_\nu^{(1)}(k_0 a)/J_\nu(k_0 a)$. From the boundary value condition, we also have the characteristic equation for the TE-polarized waves as Equation (2.6). The TM-polarized waves can be worked out in the same way and is not discussed here.

$$\frac{[(ka)^{1/2} J_\nu(ka)]'}{(ka)^{1/2} J_\nu(ka)} = \left(\frac{\varepsilon}{\mu}\right)^{1/2} \frac{[(ka)^{1/2} H_\nu^{(1)}(k_0 a)]'}{(ka)^{1/2} H_\nu^{(1)}(k_0 a)} \quad (2.6)$$

While solving Equation (2.6), it is realized that the equation has many roots. Hence, it is necessary to introduce the third mode number q , the radial mode number that indicates number of maxima along the radial directions.

Going back to the schematic diagram shown in Fig. 2-1, meanings of the mode numbers can be summarized as follows: the radial mode number (q) represents the number of maxima along the radial direction (the capital “R” axis); the polar mode number (ℓ) represents the number of wavelengths that fit into the actual WGM trajectory; the azimuthal mode number (m) is the number of wavelengths that fit into the trajectory’s projection onto the equatorial plane; and the number of maxima along the polar direction (the “z” axis) is given by $\ell - |m| + 1$.

2.1.2 Spectral positions

Resonance wavelength can be determined by solving Equation (2.6). However, direct and exact solutions are almost impossible to obtain. The roots are usually approximated by asymptotic expansion of the characteristic equation in powers of $\ell^{-1/3}$. This approach was first formulated in the pioneering work of Keller [37] and then expanded to the power of $\ell^{-8/3}$ by Schiller [38]. The main result of Schiller's work is presented in Equation (2.7)

$$nka = v + \alpha_q \left(\frac{v}{2}\right)^{\frac{1}{3}} - \frac{P}{\sqrt{n^2-1}} + \frac{3\alpha_q^2}{20} \left(\frac{v}{2}\right)^{-\frac{1}{3}} + \frac{\alpha_q}{6} \frac{nP(2P^2-3n^2)}{(n^2-1)^{\frac{3}{2}}} \left(\frac{v}{2}\right)^{-\frac{2}{3}} + O(\ell^{-1}) \quad (2.7)$$

where α_q is the q^{th} (radial mode number) root of the Airy function, $v=\ell+1/2$ (polar mode number), and P is equal to n or $(1/n)$ for TE or TM modes. The resonance wavelength is related to the wave number by $k=2\pi/\lambda$. Hence, from asymptotic solutions of the spherical resonator, the spectral position of the resonances can be approximately found.

2.2 Optical modes in spheroidal resonators

Perfect spheres are hard to make in nature. As a result, most spherical resonators fabricated in laboratories are spheroids – either prolate or oblate. Almost all microspheres formed from heating glass fibers are not perfect. This is inevitable as the sphere is always supported by a fiber stem which then breaks the sphere symmetry. An important consequence of such asymmetry is the broken mode degeneracy of azimuthal WGMs [39], which in turn influences the spectral density of resonances, intermodal coupling, and Q-factor.

On the other hand, spheroidal shapes are sometimes purposely intended to create different features of the WGM confinement. An important category is the microbottle resonator, which resembles the cylindrical resonator in shape but has a slowly varying spatial distribution along the cylinder length [40]. A crucial feature that makes microbottle refreshingly interesting is its capability to form standing waves and confine light along the longitudinal direction.

2.2.1 Optical modes in microbottle resonators¹

Fig. 2-2 Schematic illustration of a fiber taper coupled whispering gallery bottle resonator in the cylindrical coordinates. The fiber tapers are placed at the two caustics of the bottle resonator so that light is coupled in and out most efficiently. The direction indicated for light propagation in output taper coupler is not unitary: light may propagate in the opposite direction when the output coupler is placed in a different position.

As compared to a majority of other geometries, bottle resonators are quite new to the WGM community. In 2004, Sumetsky first proposed that in analogy to magnetic bottles used in plasma fusion, tapered optical fiber is also able to create whispering gallery bottles (WGBs) that localizes optical waves within a small longitudinal length [41]. As depicted in Fig. 2-2, the WGB usually exhibits a radius profile along the z -axis, creating a bottle shape that is fat in the middle and slimming down towards two ends. WGM spirals down the bottle and moves back and forth inside it – light does not escape into the homogeneous length of the fiber. Due to its unique features, the microbottle resonators are especially useful in areas such as all-optical signal processing (including filtering and switching) [20, 42] and the study of quantum electrodynamics [43]. The microbottle resonators are found in different types of glass fibers (and other materials)

¹ Part of this work has appeared in “Whispering gallery resonator based on index profiling,” Photonics Global Conference (2012).

that are able to create a radius profile [20, 44] or refractive index profile [21]. The analytical theories of a microbottle resonator with parabolic radius-profile has been given in Louyer's work [43]. The theory of the index-profiled bottle resonator is less investigated and will be briefly treated in this section.

Fig. 2-3 A microbottle resonator with a refractive index profile along the longitudinal direction (z) being placed in the cylindrical coordinates.

As shown in Fig. 2-3, the structure investigated is a section of a fiber that exhibits both refractive index profile $n(z)$ and radius profile $\rho(z)$ along the z -axis (the radius profile is not manifested as it varies little along z -axis in practical cases). Similar to the sphere resonator being solved in spherical coordinates, the index-profiled bottle can be solved in the cylindrical coordinates. However, since the material is not homogeneous, the wave equation is related to the dielectric constant as Equation (2.8).

$$\nabla^2 \psi + \nabla \left(\frac{\psi \nabla \epsilon}{\epsilon} \right) + k^2 \psi = 0 \quad (2.8)$$

By decoupling Equation (2.8) in the z -direction, and through separation of variables by substituting the expression of Equation (2.9) into Equation (2.8), we obtain the wave equations of the radial (r) and longitudinal (z) direction.

$$\psi = R(r)Z(z)e^{\pm im\varphi} \quad (2.9)$$

$$R'' + \frac{R'}{r} + \left(\epsilon k_\varphi^2 - \frac{m^2}{r^2} \right) R = 0 \quad (2.10)$$

$$Z'' + Z'(\ln \epsilon)' + [(\ln \epsilon)'' + \epsilon k^2 - \epsilon k_\varphi^2] Z = 0 \quad (2.11)$$

Since WGM typically has huge angular momentum, the azimuthal dependence of the waves resembles that in the sphere and is represented by an exponential term with m as the azimuthal mode number. The dielectric constant is calculated as $\varepsilon(z) = n^2(z)$. In Equation (2.10) and (2.11), k_ϕ is the azimuthal wave number of the WGM. Angular momentum conservation requires that $n(z)\rho(z)k_\phi$ is a constant down the fiber length. At the two caustics ($z=z_c$), which are positions where forward-propagating WGM starts to go backward, the waves are considered to stop and k_z is equal to zero. Therefore, at this point, the overall wave vector $k^2 = k_r^2 + k_\phi^2$, where k_r vanishes since the WGM normally has no tendency of traveling along the radial axis. On the other hand, the total wave number follows the equation $kn(z_c)\rho(z_c) = m$. Hence, we have the azimuthal wave number as $k_\phi^2 = \frac{m^2}{n^2(z)\rho^2(z)}$.

Equation (2.10) is a Bessel function and the solutions are given in Equation (2.12).

$$\begin{aligned} \psi(r, \theta, \phi) &= C_i J_\nu(k_\phi r) e^{\pm im\phi} & r \leq \rho(z) \\ \psi(r, \theta, \phi) &= \left[C_e H_\nu^{(1)}(k_\phi r/n(z)) + H_\nu^{(2)}(k_\phi r/n(z)) \right] e^{\pm im\phi} & r > \rho(z) \end{aligned} \quad (2.12)$$

For adiabatic refractive index and radius profiles, Equation (2.11) can be simplified to the Schrödinger equation of a particle with mass M and pseudo-potential V_{eff} . If the index and radius profiles are expressed in the form $n(z) = n_0(1 - f(z))$ and $\rho(z) = \rho_0(1 - g(z))$, we have Equation (2.13) (where n_0 and ρ_0 are the values at $z=0$).

$$Z'' + \left[\frac{\rho_0^2 n_0^2 k^2 - m^2}{\rho_0^2} - \frac{2m^2}{\rho_0^2} (f(z) + g(z)) \right] Z = 0 \quad (2.13)$$

For parabolic profile, where we assume functions $f = \frac{u^2}{2}z^2$, $g = \frac{v^2}{2}z^2$ and the coefficient $h^2 = u^2 + v^2$, Equation (2.13) assumes a similar form as the harmonic oscillator which is characterized by the equation $\left(-\frac{\hbar^2}{2M} \frac{d^2}{dz^2} + \frac{1}{2} M \omega^2 z^2 \right) \psi = E \psi$. E represents the discrete energy levels allowed for the wave ψ . These energy levels are equidistant with $\Delta E = \hbar\omega$ and $E_0 = \hbar\omega/2$. If we identify the total energy as $\frac{\rho_0^2 n_0^2 k^2 - m^2}{\rho_0^2}$ and $\Delta E_m = 2mh/\rho_0$, we can solve for the eigenvalues and field distribution as Equation (2.14) and (2.15) respectively.

$$k_{mq} = \frac{1}{n_0} \sqrt{\frac{m^2}{\rho_0^2} + \left(p + \frac{1}{2}\right) \Delta E_m} \quad (2.14)$$

$$Z_{m,p} = C_{m,p} \exp\left(-\frac{mh}{2\rho_0} z^2\right) H_p\left(\sqrt{\frac{mh}{\rho_0}} z\right) \quad (2.15)$$

where p is introduced as the axial mode number to indicate the number of field maxima ($p+1$) along the z -axis, $C_{m,p}$ is a constant and H_p is the Hermite polynomial of order p . Unlike the sphere resonator, bottle resonator does not have a polar mode number, as the polar direction is replaced by the axial coordinate. Hence, microbottles are considered as WGM resonators that naturally break the azimuthal degeneracy. Fig. 2-4 shows three normalized field distributions in z direction of a bottle resonator with $h=0.001/\mu\text{m}$, $m=50$, $\rho_0=50\mu\text{m}$ and $n_0=1.45$.

Fig. 2-4 Normalized field distributions along z -axis of a bottle resonator with $h=0.001/\mu\text{m}$, $m=50$, $\rho_0=50\mu\text{m}$ and $n_0=1.45$ for (a) $p=1$; (b) $p=2$; (c) $p=4$.

It is demonstrated that even a very slowly varying longitudinal profile (either in geometry or optical properties) is able to confine light within a small volume. It is also found that however different the bottle resonator is from its spherical counterpart, mode properties in the radial direction are still the same. The bottle resonator creates field localization that is otherwise impossible for similar geometries. It also provides a new degree of freedom for tuning of a third dimension, free spectral range, and azimuthal degeneracy.

2.2.2 Eccentricity splitting in spheroids

In perfect spheres, modes are degenerate for different azimuthal orders: it makes no difference as to which plane is chosen for the mode to propagate because the optical paths are identical. The azimuthal mode number $|m|$ is then able to take any integer value between 1 and ℓ , with $m = \ell$ being the fundamental mode of the equatorial plane. Hence, we have $(2\ell+1)$ azimuthal degeneracy in a perfect spherical resonator. When the sphere is no longer perfect and becomes a spheroid, WGMs traveling in different planes turn different. As a result, those modes that are not contained in the equatorial plane appear at new frequencies to populate the optical spectrum. This phenomenon is usually called eccentricity splitting, which is inferred from the geometry term “eccentricity” that describes the difference between polar (r_p) and equatorial (r_e) radius. Eccentricity splitting is a common phenomenon that has been observed and studied by many authors [39, 45-48]. Due to the difference in radii, the spectral position of resonance inferred from Equation (2.7) is no longer accurate. Several efforts have been attempted to accurately identify the modes in spheroidal resonators [36, 41, 49-51]. Lai has carried out early work to comprehensively address this problem [51]. An important conclusion of Lai’s results is listed as Equation (2.16), which is useful in quick assessment of the eccentricity value from the output spectrum.

$$\Delta\omega_{m,\ell,q}^{\Delta m} = |\omega_{m,\ell,q} - \omega_{m-1,\ell,q}| \approx \omega_{m,\ell,q} \cdot \epsilon \frac{|m|+1/2}{\ell^2} \approx \epsilon \cdot \Delta\omega_{m,\ell,q}^{\Delta \ell} \quad (2.16)$$

where ϵ is the eccentricity and is given by $\epsilon = |r_p - r_e|/a$. Hence, from Equation (2.16), numerical value of the eccentricity can be roughly evaluated by comparing the azimuthal and polar free spectral range (FSR).

Although Lai's work provides a good explanation of the eccentricity splitting, the exact spectral position of an azimuthal WGM is more accurately derived by Gorodtesky in Equation (2.17) [36].

$$nk_{\ell,m,q}a = \ell + \alpha_q \left(\frac{\ell}{2}\right)^{\frac{1}{3}} + \frac{2(\ell-|m|)(r_p-r_e)+r_p}{2r_e} - \frac{\chi n}{\sqrt{n^2-1}} + \frac{3\alpha_q^2}{20} \left(\frac{\ell}{2}\right)^{-\frac{1}{3}} + \frac{\alpha_q}{12} \left[\frac{2(\ell-|m|)(r_p^3-r_e^3)+r_p^3}{r_e^3} + \frac{2n\chi(2\chi^2-3n^2)}{(n^2-1)^{\frac{3}{2}}} \right] \left(\frac{\ell}{2}\right)^{-\frac{2}{3}} + O(\ell^{-1}) \quad (2.17)$$

where parameter χ takes the value of 1 for TE-polarization and $1/n^2$ for TM polarization.

2.3 Quality factors

WGMs supported by different geometries have very high Q-factors. The Q-factor is defined by the resonator's capability to store energy at a specific frequency. It is also a quantification of the various losses in the resonator. The Q-factor can be measured in either frequency or time domain, as shown in Equation (2.18).

$$Q_0 = \omega \frac{E_{stored}}{P_{diss}} = \omega\tau = \frac{\omega}{\Delta\omega} \quad (2.18)$$

where Q_0 is the resonator's intrinsic Q-factor, ω is the resonance frequency, E_{stored} is the energy stored in the resonator, P_{diss} is the power dissipated from the resonator, τ is the photon life time (or cavity ring-down time), $\Delta\omega$ is the resonance linewidth as measured from the frequency spectrum. The intrinsic Q-factor of an independent WGM resonator is determined by several aspects as listed in Equation (2.19).

$$Q_0^{-1} = Q_{mat}^{-1} + Q_{rad}^{-1} + Q_{sca}^{-1} + Q_{cont}^{-1} \quad (2.19)$$

Material absorption and bulk Rayleigh scattering is the main contribution (Q_{mat}) that limits the Q-factor. Q_{mat} for undoped silica glass is evaluated as 10^{10} (determined by the bulk material only). However, the loaded Q-factor of the WGM resonator is only between 10^8 and 10^9 [19, 52, 53]. This is because the loss mechanism is affected by another contribution of the material properties: contamination (Q_{cont}). During the fabrication of silica microsphere for example, the material would adsorb atmospheric water which forms a nanolayer of molecules on the resonator surface. The direct result is that much of the mode radiation is lost in the nanolayer [52, 53]. On

the other hand, surface inhomogeneities create surface scattering losses (Q_{sca}) that could reduce the Q-factor. Resonators made of amorphous material from surface tension induced effects usually have defect free surfaces characterized by molecular-scale homogeneities. Hence, glass-based resonators generally have higher Q-factors. On the other hand, surface roughness also induces scattering that degrades the Q-factor. The surface roughness influences the Q_{sca} following Equation (2.20).

$$Q_{sca} \approx \frac{3\lambda^3 R}{8\pi^2 n_0 B^2 \sigma^2} \quad (2.20)$$

Where λ is the resonance wavelength, R is the radius of the resonator, n_0 is the refractive index of the resonator, σ is the surface roughness and B is the correlation length of the roughness [54]. For example, a surface roughness of 2 nm and correlation length of 10 nm at a resonance wavelength of 1550 nm for a BaF₂ disk resonator with radius of 6 mm yields a scattering Q-factor on the order of 10^{13} [54]. Generally, the surface roughness of silica glass is better than 2 nm [55, 56], which would give rise to higher quality factors.

The last contribution to the resonator loss is radiation (Q_{rad}), which is inevitable for all circularly shaped geometries. It is mentioned in Chapter 1 that the light waves propagate inside the resonator by total internal reflections. Yet, to be precise, “total internal reflection” never happens on curved surfaces as certain amount of radiation always escapes through the interface. The radiation factor is defined by the wave number as $Q_{rad} = Re(k_{m,l,q})/Im(k_{m,l,q})$. By semiclassical approximation, models can be developed to explain the behavior of Q-factor in relation to the surface radiation [57]. It is found that Q_{rad} increases as the polar mode number increases but decreases as the radial mode number increases. The implication is that larger resonators have higher Q-factors: the larger the size, the more the curvature resembles a flat surface. It is also found that TE-polarized modes have slightly higher Q-factors than the TM-polarized WGMs.

The silica glass resonators usually have high Q-factors as Q_{mat} and Q_{sca} are high. However, it is limited by Q_{cont} as atmospheric contamination is inevitable when the resonator is placed in free air. Another type of ultrahigh-Q resonators are made of crystalline materials from polishing

techniques. Q-factor exceeding 10^{11} with millimeter size have been achieved with fluoride crystalline materials [58].

2.4 Applications of WGM resonators

2.4.1 Sensors

Whispering gallery mode's ability to confine light for a long period of time inherently increases optical path length of the mode. One straightforward outcome is a significant increase in interaction path between light and its surrounding environment. This makes WGM resonator a good candidate for sensing applications. On the other hand, fundamental WGMs essentially propagate close to the surface of the resonator; one would then expect the mode to be a sensitive indicator of the resonator surface, e.g. change in molecules that are adsorbed to the resonator surface can be detected by the WGM resonance.

For bio/chemical sensor development, as the sensing is usually performed in liquid, it is necessary to integrate the resonator with microfluidics. A number of schemes have been implemented to approach this problem: from standalone solid microsphere in a fluidic cell [59, 60], to liquid droplets [30] or droplets immersed in oil [61]; planar ring resonators on chip have also been utilized for sensing applications [62, 63]. The advantage of a standalone solid microsphere resonator is that it possesses very high Q-factor; the disadvantage is that both fluidic integration and mass production are difficult. Liquid droplets that encapsulate the target analyte are convenient to use, but they usually suffer from fast evaporation. Planar ring resonators can be mass produced with good repeatability, but their Q-factors are usually several orders lower than the standalone microsphere resonator due to bad side-wall roughness.

The opto-fluidic ring resonator (OFRR) is another class of sensing platform that inherently integrates photonics with microfluidics [64-66]. The center piece of the OFRR is a capillary with very thin wall. The target analyte flows inside the capillary along its length while WGM is excited in the lateral surface of the capillary. It is then guaranteed that the WGM confined within the capillary wall is able to interact with the analyte and detect any changes in the environment. In conclusion, the OFRR is both a resonator and container of the sensing fluid. It has been broadly applied in applications such as refractometry [67], label-free biosensing [64], Raman

based sensing [68], microscale gas chromatographic vapor analysis [69], and microfluidic lasers [70]. The same principle can be applied to a bubble (a hollow fiber bulge) cavity to create microfluidic optomechanical resonator for surface sensing [71]. Currently, the major limitation of the OFRR device is the small modal penetration depth as opposed to thick capillary wall.

2.4.2 Lasers

The use of WGM resonators in the field of lasers is vast and can be categorized as the linear and nonlinear regime.

In the linear regime, WGM resonators are generally used as filters to stabilize the laser cavity. As WGM generally has very high Q-factors, it can be used as a good reference to force the laser to resonate at the WGM wavelength. Such forcing is usually done by optical feedback from the WGM scattering into the laser cavity [72]. On the other hand, high Q-factors gives rise to narrow linewidth resonance – the filtering function ensures that the laser lases at the WGM resonance which bears an intrinsic Q-factor of the microresonator. This naturally reduces the linewidth of the laser [48]. Nowadays, WGM resonator stabilized semiconductor lasers with wavelength range of 390-2900 nm and linewidth smaller than 1.8 kHz are already commercially available [73]. The narrowest linewidth laser (<40 mHz at $1.5\ \mu\text{m}$) to date is demonstrated with the resonator-stabilized principle using a silicon single-crystal optical cavity [74].

In the nonlinear regime, lasing is achieved either by nonlinear scattering processes or doping the resonator with gain medium.

Brillouin scattering and Raman scattering are two common nonlinear scattering phenomena in WGM resonators. Stimulated Brillouin scattering (SBS) has been demonstrated in liquid droplets decades ago [75, 76]. Recently, Brillouin lasers are increasingly studied with high-Q solid WGM resonators [77]. Continuous wave Raman lasers with low threshold and large tunability were made available on WGM resonators many years ago [78, 79]. In 2002, a Raman laser with ultra-threshold on a fiber taper coupled silica microsphere resonator is demonstrated for the first time [80]. The reason for the SBS and SRS to occur at very low power thresholds is due to the high Q/V ratio in WGM cavities. These nonlinear processes are made possible also because of efficient power coupling techniques that are constantly maturing over time.

Similarly, miniature lasers that rely on gain materials are made possible at much lower power threshold with WGM resonators. The first WGM lasers were realized in solid materials [81, 82]. Then they were made available in capillaries where laser dyes flowing through the capillary pump the localized WGM resonances [83, 84]. On the other hand, instead of doping the resonator with active material, it can also be coated with a thin layer of gain medium. For example, erbium-doped sol gel films were applied to the surface of silica microsphere resonators to create low-threshold WGM lasers [32].

2.4.3 Optical frequency comb and metrological standard

Optical frequency comb is basically a frequency spectrum that is consisted of equidistant frequency lines [85, 86]. It is an ultra-precise frequency standard that can be used to measure unknown optical frequencies. Such unique capability has made frequency comb a revolutionary tool in metrology and laser spectroscopy and, has enabled the development as well as practical use of time standards in the optical domain [85-88]. Conventionally, the optical frequency comb is generated from a femtosecond laser and the comb line spacing is locked to a mode-locked laser. In 2007, a new way of comb generation based on optical resonators is devised that significantly reduces the size and power requirements while bringing new flexibilities in terms of repetition rate and spectral range [89]. This approach is based on the third order nonlinearity parametric oscillation in ultra-high-Q optical resonators. The ultra-high Q-factor and small mode volume make the WGM resonators especially efficient in optical frequency conversion processes [90]. The first of such frequency comb utilizes a silica microtoroidal resonator that was pumped by a CW laser; it has achieved a 500-nm wide span at around 1.5 μm without relying on any external spectral broadening; it can also operate at previously unattainable repetition rates, which are useful in applications where access to individual comb lines is required [89]. Later, an optical frequency comb generated using a crystalline CaF_2 resonator with mode spacing as small as 12 GHz is demonstrated [91]. Currently, microresonator based frequency comb has yet to mature in order to compete with its conventional counterpart, but it has already seen potential applications in various fields including telecommunication, optical and microwave waveform synthesis [91, 92] and frequency comb spectroscopy [93, 94].

Chapter 3 Conventional coupling schemes of whispering gallery mode resonators

3.1 Introduction

Efficient and stable coupling of WGM is considered as the key challenge for applications based on optical resonators. The WGM coupling methods are varied and can be categorized as radiative and evanescent coupling. Radiative coupling, also known as free beam coupling, are much less efficient in transferring power [95]. Evanescent coupling methods, on the other hand, which utilize evanescent waves at optical interfaces, may achieve almost 100% coupling efficiency (for fiber taper). Quality of the evanescent coupling is assessed by two criteria: the phase-matching conditions and realization of critical coupling. This section explains the coupling phenomenon of evanescent coupler as well as some important existing coupling techniques. During the discussion of fiber taper coupler, mathematical representation of the coupling phenomenon is introduced, some results of which can be applied to other evanescent techniques. In addition, the term “cavity” is used to represent the inside of the resonator.

3.2 Waveguide based coupling

Waveguide coupler is the most common coupling device for WGM resonators. Tapered optical fiber, or “fiber taper”, is an important category of the waveguide couplers. It has been widely applied in the coupling of microsphere [23], microtoroid [96], and microbottle [20] resonators. Fiber taper is usually tapered from a single mode fiber down to a diameter of no more than 2 μm , so that the propagating fiber mode is able to leak as evanescent waves and overlap with the evanescent tail of the WGM. Taking the fiber taper (or more generally, waveguide) coupled resonator as an example, this subsection briefly describes the phase-matching conditions and critical coupling.

To transfer power efficiently from one device to another, the phase-matching conditions must first be satisfied. Simply put, optimum coupling demands that the phase mismatch between propagation constants of the coupler mode and the WGM be zero ($\Delta\beta = \beta_{\text{coupler}} - \beta_{\text{WGM}} = 0$). The propagation constant β is a numerical representation of the mode’s spatial oscillation period. As illustrated in Fig. 3-1, the spatial oscillation pattern of the waveguide (p_w) needs to be in phase with that of the WGM resonance (p_r) to achieve constructive interference. Since the resonator has a curved surface, it requires that the WGM projection onto the waveguide is in phase with that of the waveguide mode. The Gaussian envelope of the waveguide mode represents a distribution of the coupling coefficient (characterized by the coupling lifetime τ_c) as a result of the resonator curvature. The propagation constant of a WGM is obtained as the real part of the mode’s wave number which is in turn determined by Equation (2.7) or (2.17), depending on the resonator geometry. Hence, the β in the waveguide can be designed in a way that the two oscillation periods are equal.

Fig. 3-1 Illustration of phase-matching between resonator and waveguide that inputs continuous wave (CW) light. The curved WGM distribution is straightened as the mode projection onto the waveguide. The spatial period of the waveguide mode must match that of the straightened distribution for the WGM. The spatial oscillation periods of the WGM and waveguide mode are represented by p_r and p_w respectively.

When coupling is phase-matched, maximum power transfer might still not happen. It calls for a second requirement to achieve optimum coupling: the distance between the resonator and coupler needs to be just right in order to achieve the so-called “critical coupling”. To understand critical coupling in waveguide coupled resonator, temporal coupled mode theory is used to model the changing field amplitude of WGM both inside and outside the cavity [97]. A matrix formulation presented by Yariv gives identical results but is less emphasizing on the relationship

of coupling coefficients [98]. Fig. 3-2 illustrates a 2-D diagram of a waveguide coupled optical resonator, which is the subject of the investigation for the following content.

Fig. 3-2 Two-dimensional schematic of a waveguide coupled WGM resonator.

In Fig. 3-2, it is depicted that every time the waveguide inputs power into the cavity, it experiences intrinsic losses by the resonator and then sends back equal amount of energy to the waveguide. The dynamic behavior of the changing field inside the cavity can thus be expressed as Equation (3.1).

$$\frac{da}{dt} = j\omega_0 a - \left(\frac{1}{\tau_0} + \frac{1}{\tau_e} \right) a + \kappa \cdot s \quad (3.1)$$

Equation (3.1) describes the time-varying field amplitude of the WGM (a) inside the cavity. τ_0 and τ_e are the life time due to cavity intrinsic loss and energy exchange with the outside respectively; their inverses are the respective field decay rates. The resonance frequency is ω_0 , s is the field amplitude that is input into the waveguide and κ is the amplitude coupling coefficient (from waveguide to cavity). Since we are considering the time variation of field inside resonator

while the incident excitation is continuous wave, we have $|s|^2$ representing the power carried by the incident wave and $|a|^2$ as the energy inside the cavity. The temporal-steady expression of the cavity mode at frequency ω can be derived from Equation (3.1) as Equation (3.2).

$$a = \frac{\kappa s}{j(\omega - \omega_0) + \left(\frac{1}{\tau_0} + \frac{1}{\tau_e}\right)} \quad (3.2)$$

When input into the waveguide is $s=0$ and the resonator has no intrinsic loss ($1/\tau_0=0$), at $t=0$ we have finite amplitude of the WGM, while at $t>0$, the mode decays exponentially. Hence, Equation (3.3) is obtained as the following.

$$\frac{d|a|^2}{dt} = -\frac{2}{\tau_e} |a|^2 \quad (3.3)$$

Consider the time-reversed solution. At $t<0$, the wave grows exponentially. If we denote the cavity field amplitude at this time as a^* , Equation (3.4) can be obtained.

$$\frac{d|a^*|^2}{dt} = \frac{2}{\tau_e} |a^*|^2 \quad (3.4)$$

By feeding the drive frequency $\omega = \omega_0 - \frac{j}{\tau_e}$ into Equation (3.2) and keeping $1/\tau_0=0$, we have the time reversed expression of the cavity field amplitude as Equation (3.5).

$$a^* = \frac{\kappa \cdot s^*}{2/\tau_e} \quad (3.5)$$

From $t<0$ to $t=0$, the energy variation inside the cavity should be equal to the power input from the waveguide (since we assume that s is input at $t=0$). From Equation (3.4) and (3.5), an important conclusion can then be obtained.

$$|\kappa| = \sqrt{\frac{2}{\tau_e}} \quad (3.6)$$

Phase of the coupling coefficient can be neglected as the relative phase between a and s is arbitrary. Hence, Equation (3.1) and (3.2) can be rewritten as (3.7) and (3.8) respectively.

$$\frac{da}{dt} = j\omega_0 a - \left(\frac{1}{\tau_0} + \frac{1}{\tau_e}\right) a + \sqrt{\frac{2}{\tau_e}} \cdot s \quad (3.7)$$

$$a = \frac{\sqrt{\frac{2}{\tau_e}} s}{j(\omega - \omega_0) + \left(\frac{1}{\tau_0} + \frac{1}{\tau_e}\right)} \quad (3.8)$$

Waveguide transmission after coupling to the cavity is expressed by $t = c_s s + c_a a$, where $c_a = \sqrt{2/\tau_e}$ is straightforwardly derived from the case when waveguide input is zero. On the other hand, the net power flowing into the resonator is equal to the rate of energy buildup in the cavity less the resonator losses. And from Equation (3.7), we have:

$$\begin{aligned} |s|^2 - |t|^2 &= \frac{d|a|^2}{dt} + \frac{2}{\tau_0} |a|^2 = -2 \left(\frac{1}{\tau_0} + \frac{1}{\tau_e} \right) |a|^2 + \sqrt{\frac{2}{\tau_e}} (a^* \cdot s + a \cdot s^*) + \frac{2}{\tau_0} |a|^2 = \\ &= -\frac{2}{\tau_e} \left| \frac{t - c_s s}{c_a} \right|^2 + \sqrt{\frac{2}{\tau_e}} \left[\left(\frac{t - c_s s}{c_a} \right)^* s + \left(\frac{t - c_s s}{c_a} \right) s^* \right] \end{aligned} \quad (3.9)$$

Simplifying both sides of Equation (3.9), it can be obtained that $c_a = -1$. With this result, transmittance of the waveguide coupled resonator is then derived as Equation (3.10).

$$T = \left| \frac{t}{s} \right|^2 = \left| \frac{\frac{1}{\tau_0} + \frac{1}{\tau_e} - j(\omega - \omega_0)}{\frac{1}{\tau_0} + \frac{1}{\tau_e} + j(\omega - \omega_0)} \right|^2 \quad (3.10)$$

It is observed that at resonance frequency, output power from the waveguide varies with the value of coupling coefficients. For a specific resonator, the intrinsic life time is fixed and the amount of coupled power is largely determined by the value of $1/\tau_e$, which is influenced by the relative distance between the resonator and waveguide. When the coupler is positioned from the resonator so that $1/\tau_e = 1/\tau_0$, i.e., the cavity loss is just compensated by the amount of power delivered by the coupler, the transmittance hits zero and we call it the critical coupling. When coupler is farther away from the cavity, i.e., $1/\tau_e < 1/\tau_0$, the cavity loss cannot be completely compensated by the input power, transmittance does not go to zero – a scenario known as undercoupling. When the coupler is so close to the resonator that external power transfer over-compensates the cavity loss ($1/\tau_e > 1/\tau_0$), the cavity is said to be over-coupled and the transmittance at resonance frequency does not go to zero.

Being incredibly power efficient, the tapered optical fiber is the main stream coupling device for WGM resonators. However, due to its fragile and easy-to-contaminate nature, fiber taper in

practical device or optical integration has not been widely realized. Optical coupler with better mechanical robustness is in serious need.

3.3 Prism and angle polished tip based coupling

Prism couplers are suitable for coupling to WGM resonators that are relatively large in size [48, 99, 100]. It is considered as the oldest optical couplers [101] and have been extensively investigated by many authors [102-104]. The prism coupler relies on the frustrated total internal reflection to generate evanescent waves, the critical coupling of which is also tuned by the gap between resonator and coupler. Phase-matching conditions, on the other hand, can be adjusted by varying the incident angle. The output beam can be collected at the reflection side where sharp loss dips corresponding to the WGMs appear in the spectrum. Fig. 3-3(a) illustrates a simple schematic of the prism coupling configuration. Coupling efficiency of the prism depends heavily on the modal quality of the incident beam, which is difficult to control as the input mode is essentially a free beam. Up to date, the best coupling efficiency achieved with a prism coupler is around 80% [102].

Fig. 3-3 Schematic diagrams of (a) prism coupled resonator; (b) angle polished fiber tip coupled resonator.

The angle polished fiber tip as a WGM coupler was first proposed and demonstrated by Ichenko with a silica glass microsphere resonator [25]. As shown in Fig. 3-3(b), the tip of a single mode fiber is polished till the end facet inclines at the designed angled; the propagating fiber mode incident on the end facet would then undergo total internal reflection that generates evanescent waves. Such coupling mechanism is in principle identical to that of the prism, if we consider the angle polished fiber tip as a miniaturized prism coupler. Like prisms, the angled fiber tip is also suitable to work with large optical resonators. However, angled fiber tip makes improvement over the prism by containing the input beam within its core, resulting in improved mode quality.

Due to the conveniences they bring to geometry configuration and mechanical stability, both prism and angled fiber tip couplers have seen revived usage in the coupling of ultrahigh-Q crystalline resonators to create microresonator based optical frequency comb [105-108]. However, smaller resonators continue to be coupled by tapered optical fiber for higher coupling efficiency (but with poorer mechanical strength). Additionally, both prism and angled fiber tip are essentially input couplers: the output beam has to be collected in free space (or with the help of an additional output coupler). Such configuration increases complexity of the coupling system and might not be power efficient for some applications.

3.4 Tapered fiber tip based coupling²

Compared to other couplers, tapered fiber tip is a newcomer that has not been extensively used or studied [109]. As shown in the left part of the schematic illustration of Fig. 3-4, coupling mechanism of the tapered fiber tip resembles both the fiber taper and prism coupler (left tapered fiber tip). On one hand, it is essentially a section of fiber taper that supports the propagation of core mode which then leaks out as evanescent waves to couple with the cavity mode. On the other hand, the tip utilizes the taper transition region (which resembles the angle incident prism) to provide optical reflection. Because of the reflection, tapered fiber tip facilitates a single-port configuration which is useful to reduce the complexity of optical integrations.

² Part of this section has appeared in “Whispering gallery mode excitation and collection using fused-tapered fiber tips,” Photonics Global Conference (2012).

Fig. 3-4 Experimental setup of a microsphere resonator coupled by a pair of tapered fiber tips. Light is input from a tunable laser at telecommunication wavelength into a single mode fiber. The SMF has its tip tapered down to several hundred nanometers of diameter. It is then mounted on a three axis translational stage to control the tip's position from the microsphere resonator. Another tapered fiber loaded on another three axis translational stage is placed in the vicinity of the microsphere to serve as an output coupler. The output tip then sends light to a photodetector. After a data-processing program, the optical information is processed into the coupled spectra.

On the other hand, as shown in the schematic setup of Fig. 3-4, a second tapered tip serving as an output coupler can make a two-port coupling configuration that is free from the interference of reflection background [110]. However, one serious drawback of the tapered tip is that phase-matching designs are not precisely controlled in fabrication. The coupling efficiency is mainly dependent on the scattering characteristics of the tip. Hence, the evanescent fields as well as phase-matching conditions are determined by the exact shape of the tapered tip. Precise control of the shape cannot be achieved every time, as evidenced in Fig. 3-5, where two drastically different shapes are produced with almost the same tapering processes.

Fig. 3-5 SEM images of two tapered fiber tip couplers.

3.5 Dielectric grating based coupling

Diffraction grating made of dielectric material as a WGM coupler has been proposed some time ago [111]. Yet, the research community has seen very few of grating coupled optical resonators. Reference [108] engraves grating on the resonator for directional free-space coupling. However, such method scars the resonator and decreases Q .

Fig. 3-6 Schematic diagram of a diffraction grating coupled resonator.

Essentially, diffraction grating utilizes the diffracted beam to phase-match with the WGM (Fig. 3-6). In principle, coupling efficiency is determined by the diffraction efficiency of the grating: the more power diffracted to the desired angle, the more can be sent into the resonator. Gratings made of dielectric materials typically have low first order diffraction efficiency at normal incidence if the structure is to maintain the evanescent leakage for WGM coupling – most of the power is transmitted without being diffracted as the zeroth diffraction order. Hence, although the dielectric grating is mechanically sound as compared to tapered fiber based couplers, the power

Conventional coupling schemes of whispering gallery mode resonators

transfer performance is poor and the diffraction grating as a resonator coupler remains unexplored.

Chapter 4 Metallic grating coupler

Up till now, fiber taper provides the highest coupling efficiency and serves as the most common WGM coupler. However, due to its fragile and easy-to-contaminate nature, tapered fiber is not a good choice for integrated photonics applications. In this chapter, we propose a new WGM coupler based on both the diffraction and plasmonic effects of metallic nano-structures³.

Plasmonic effects have been introduced to enhance the coupling of microresonators in many works: it has been demonstrated that the high-Q performance can be used to enhance the sensitivity of plasmonic sensing [112]; vice versa, coupling enhancement of WGM by plasmonics is also possible with gold-film assisted prism [113], gold nanorods [114], gold nanowires [115] and metal clad planar waveguide [116], and 2D array of plasmonic nano-focusing structures [117]. However, the existing plasmonic assisted WGM couplers do not truly utilize the properties of metal to enhance coupling. Even worse, the loss associated with metallic structure on the WGM resonator always significantly reduces the high-Q performance of the resonator.

4.1 Phase-matching design with grating

A 2-D schematic of metallic grating coupled WGM resonator is depicted in Fig. 4-1. The grating is made of gold and is situated on a silica substrate. It has a filling factor (metal/period) of 50% and a thickness (h) of 100nm. The WGM resonator is treated as a microdisk that is also made of silica. Optical input is in the form of Gaussian waves and is incident from the substrate onto the

³ Section 3.1-3.4 has appeared in “Metallic diffraction grating enhanced coupling in whispering gallery resonator,” Optics Express 21, 8939 (2013).

grating. Transverse magnetic (TM) polarization is indicated for the incident wave depicted in this case. The resonator is placed in the vicinity of the grating so that they create between them an air gap of size d . The 2-D simulation of the WGM resonator is equivalent to the 3-D case as the fundamental polar WGM only propagates within a 2-D plane and the grating stretches only in one dimension. For the coupling of a sphere resonator, a fiber stem also extends from the end of the resonator and points in the z -axis (not shown in the schematic).

Fig. 4-1 Two-dimensional schematic illustration of a metallic grating coupled whispering gallery mode resonator.

The period of the diffraction grating (Λ) is designed to phase-match the first diffraction orders with the fundamental WGM in a silica microdisk (radius $a=4\mu\text{m}$). For the diffraction grating, effective refractive index of the diffracted mode can be calculated using the grating equation, which is rewritten as Equation (4.1)

$$n_t \sin \theta_q = n_i \sin \theta_i - q\lambda/\Lambda \quad (4.1)$$

where q is the diffraction order, n_t is the effective mode index of the diffraction order, θ_q is the diffraction angle of the q_{th} diffraction order, θ_i is the incident angle, n_i is the refractive index of the incident medium and λ is the incident wavelength. The first term on the right hand side of the equation represents the transverse component of the incident wave vector; the second term is the wave vector of the grating diffraction and the term on the left hand side of the equation represents the resultant transverse wave vector of the grating when light is incident onto the grating. At normal incidence, the grating equation associated with the negative 1st diffraction can be reduced to $n_{grating} = n_t \sin \theta_q = \lambda/\Lambda$, where $n_{grating}$ is the x-component of the effective mode index. In order to satisfy the phase-matching conditions, we calculate the effective mode index of the first radial order WGM (n_{WGM}) using Equation (2.7) to obtain the relationship $n_{grating} = n_{WGM}$. The grating period is then found to be around 440 nm for an incident wavelength of 550 nm. We choose this pitch size to couple with the spectral transmission ranging from 500 nm to 600 nm. By benchmarking with the dielectric grating proposed in Reference [111], we performed the calculations for both a gold grating and a silica grating of the same structural parameters using finite different time domain (FDTD) method.

In the simulation, the incident wave is TM-polarized to excite the surface plasmon resonance (SPR). The TM-polarization is defined as the x-direction while TE-polarization as piercing out of the x-y plane. Because of the bending effect of the grating diffraction, the transmitted light is able to align its wave vector in the tangential direction of the WGM. In addition, we found that the coupled field strength hardly varies with the Gaussian beam size when the beam is at least 4 grating periods wide. In this investigation, a beam width of 2 μm is used.

Fig. 4-2 (a) Normalized optical power spectra for coupled WGM in a silica microdisk of $4\mu\text{m}$ in radius by a gold grating ($d=133\text{ nm}$, $\Lambda=435\text{ nm}$, $h=100\text{ nm}$) and silica grating ($d=58\text{ nm}$, $\Lambda=435\text{ nm}$, $h=100\text{ nm}$) respectively. Insets: Field intensity distribution of the first and second radial-order WGMs, color scales are normalized to the respective images. Field intensity distributions (log scale) of the coupled WGM from (b) silica grating; (c) gold grating, before the light completes half round trip of the cavity. Color scales of (b) and (c) are normalized to that of (c).

Fig. 4-2(a) shows the normalized optical power coupled into the WGM microdisk by both a gold grating and a silica grating, where d is respectively optimized for maximum coupled field intensity. The power spectrum is obtained from a point monitor that is placed inside the cavity at a field intensity maximum. For the gold grating, normalized intensity distributions for the first and second radial orders are shown in the insets. The distribution images are normalized to their own field maxima. It is observed that the gold grating couples almost 6 times of the power with the silica grating. Fig. 4-2(b) and (c) compares the coupled field pattern before light travels one round trip inside the microdisk resonator. It is observed that the dielectric grating shoots out most of the light as the zero diffraction order (in the y-axis) while the gold grating diverts more light to the curvature of the microdisk.

Fig. 4-2(b) reveals the essential flaw of the dielectric grating coupler, i.e., the zeroth-order diffraction is much stronger than the other diffractions. The zeroth-order diffraction does not bend in the transverse direction, so it cannot be used in the coupling. However, grating made of rectangular grooves with dielectric material can be designed to suppress the zeroth-order diffraction by tuning the thickness value h (while maintaining the filling factor as 50%). An investigation of the coupled field strength with respect to different grating thicknesses for gratings made of silica and gold at two polarization states are presented below.

A silica microdisk resonator with radius 5 μm is coupled by grating with period 400 nm. The periodicity is designed for WGM resonance wavelength of around 552 nm for TE-polarized wave and 547 nm for TM-polarized wave respectively. Intracavity field intensity is taken as the magnitude of the Poynting vector at a spatial location of the corresponding WGM distribution where its value is the largest. The maximum intracavity field intensity occurs at an almost fixed spatial location inside the resonator, because both clockwise propagating and counter-clockwise propagating WGMs are equally excited to give rise to standing wave. The standing wave does not move or change its shape over time, which is in contrast with the directional waveguide coupled resonator (e.x. the fiber taper coupled microsphere). The field intensity of the mode with resonance wavelength of around 547 nm is taken at a point that is around 4.8 μm away from the center of the microdisk, while that of the resonance at 552 nm is taken at a point roughly 4.83 μm away. These distance values might change with different couplers; it is made sure that only the largest values within the considered spectral range are recorded for each data point.

When the incident wave is TM-polarized, surface plasmon resonance can be excited. The coupling performance of the grating is therefore affected by the plasmonic effect. Unlike the localized SPR in a metallic nanoparticle, the rectangular slabs that constitute the grating have much larger dimension, resulting in weaker plasmonic resonance. As shown in the electric field distribution in Fig. 4-3(b), the fields are mainly confined to the corners of the metallic slab: the stronger the resonance, the tighter the field confinement. In contrast, distribution of electric field around the grating grooves under TE-polarization [Fig. 4-3(a)] does not show any field localization on the surface or the corners. The electric field intensity profiles are taken at the respective resonance wavelengths of the bare gratings (gratings on substrate without resonators). The grating height and filling factor are both optimized for the coupling to TE-polarized

resonance at 552 nm and TM-polarized resonance at 547 nm respectively (optimization results are discussed in the following text). The field intensity shown in Fig. 4-3(a) and (b) are normalized to the same log scale.

Fig. 4-3 Electric field intensity distribution for gold grating when (a) TE-polarized light, $h=100$ nm, filling factor=50%, resonance wavelength~510 nm; (b) TM-polarized light, $h=250$ nm, filling factor=25%, resonance wavelength ~ 523 nm.

The structural dimension of the metallic slab determines the SPR and its effect on the overall coupling. A width of 100 nm (a filling factor of 25%) is found to be able to deliver good

coupling performance for the gold grating. Hence, the width is maintained for the comparison of TM-polarized coupling performance among different gratings. Fig. 4-4 shows the maximum coupled intracavity field intensity in the disk resonator for gratings made of gold, silica and perfect electrical conductor (PEC) when the incident light is TM-polarized. PEC is an ideal metal that does not exhibit any dissipative loss of the normal metals. The PEC is more power efficient than gold but cannot excite any plasmonic effect even under TM-polarization.

Fig. 4-4 Maximum intracavity field intensity in a silica microdisk resonator with respect to grating thickness (h) for TM-polarized light at a resonance wavelength of around 547 nm. Both the PEC and gold grating have a filling factor of 25%, silica grating as 50%.

The electric field intensity is normalized to the data of the light source and not to the value “1”, so the unit is taken as “arbitrary unit (a.u.)”. It is observed that when comparing the mode intensity at the target resonance wavelength, gold grating is no better than the silica grating, which in turn performs worse than the PEC grating. This means that the metallic material inherently couples more efficiently than the dielectric material when used as a grating coupler. It

is due to the lossy nature of the gold that the power coupled into the WGM resonator is not as much as designed. On the other hand, however, since the TM-polarized gold grating excites weak plasmonic effect, it is expected to see the optimized coupling at some spectral position that is pushed towards the SPR. Fig. 4-5 shows the local electric field intensity spectra for bare gratings when the thickness value is varied. All gratings have a filling factor of 25%. A point monitor placed 5 nm away from the corner of the rectangular slab is used to record the electric field intensity.

Fig. 4-5 Local electric field intensity spectra of bare gold grating under TM-polarization for three different thickness values. Filling factor is maintained as 25%.

The local electric field intensity spectra show that different grating thicknesses give rise to different resonance wavelengths and Q-factors. From the graph, it is observed that when $h=250$ nm (red, solid curve), the grating resonance (523 nm) has the highest Q-factor of the three. Fig. 4-6 shows the maximum intracavity field intensity of TM-polarized light at another resonance (~ 540 nm) for the same gratings of Fig. 4-4. The maximum mode intensity is blue shifted because

the SPR wavelength occurs at a shorter wavelength. The resultant maximum resonance does not overlap with the SPR peak because 1) the plasmonic resonance is not very strong (as manifested by the low Q-factor); 2) the grating coupler still has to balance the SPR enhancement with the material loss and phase-matching condition that is imposed by the grating period.

Fig. 4-6 Maximum intracavity field intensity in a silica microdisk resonator with respect to grating thickness (h) for TM-polarized light at a resonance wavelength of around 540 nm.

Both the PEC and gold grating have a filling factor of 25%, silica grating as 50%.

For the resonance that is enhanced by the SPR effect, it is seen that PEC and silica grating performs generally the same (PEC is still slightly better than silica grating as the metallic material inherently works better to bend light). Despite the lossless nature of PEC, the coupled field intensity of the gold grating significantly exceeds that of the PEC. Such result can only be attributed to the SPR enhancement. Moreover, the best coupling happens when the gold grating thickness is 250 nm. This is because with this thickness, the SPR has the largest Q-factor (Fig. 4-5). While the resonance wavelength does not shift much for gratings of different thicknesses, its SPR Q-factor plays a more important role in modifying the coupling performance.

Fig. 4-7 Maximum intracavity field intensity in a silica microdisk resonator with respect to grating thickness (h) for TM-polarized light at a resonance wavelength of around 552 nm.

For TE-polarization, there is no SPR effect; so 25% is no longer the optimized filling factor. As shown in Fig. 4-7, where coupled intracavity field intensity are shown for three different gratings – gold grating with 50% filling factor, gold grating with 25% filling factor, and silica grating with 50% filling factor – the gold grating can be designed to couple much more efficiently than the silica grating when their filling factors are the same. The gold grating with 25% of filling factor couples no better than the silica grating, because changing the slab width can no longer enhance SPR effect and only the grating design plays the major role in affecting the coupling performance.

In conclusion, for both TE and TM polarizations, the gold grating is a more efficient coupler than the silica grating. With design of the periodicity, filling factor and grating thickness, surface plasmon resonance can be utilized to enhance the coupling significantly. Especially, the metallic

grating is useful for sensing applications since TM-polarization generally has a higher sensitivity as compared to TE-polarization [118].

4.2 Critical coupling and spectral coverage

Aside from grating design, critical coupling needs to be achieved in order to maximize the coupled field intensity in the microdisk resonator. As introduced in Section 3.2, critical coupling occurs when the round trip loss equals the amount of power coupled from the waveguide. Round trip loss is obtained by calculating the cavity ring-down time of the same microdisk ($a=4\text{ }\mu\text{m}$, $n=1.5$) with an electric dipole source placed near the disk. For the resonance wavelength of 564.7 nm, the round trip loss is around 12.6%. The amount of power coupled into the WGR from the waveguide is designated by the coupling coefficient κ ⁴, which is defined as $\kappa(\omega) = \frac{\text{Power_coupled}(\omega)}{\text{Power_source}(\omega)}$ [119]. Hence, the κ that yields critical coupling (κ_c) for the abovementioned microdisk is 12.6%. However, this κ is calculated for a single mode coupling scenario. For a grating-coupled cavity, the incident light is diffracted into a series of modes, of which only a fraction are coupled to the WGM. Fig. 4-8 shows the coupled field intensity and κ for a range of gap sizes (d) at their respective resonance wavelengths. For both cases, the coupling coefficients decrease monotonically with d while the coupled field intensity first increases and then decreases with increasing d . When d is optimized to achieve maximum coupled field intensity, the corresponding coupling coefficients (κ_{max}) for the silica grating and gold grating are both much smaller than the κ_c calculated for waveguide-coupled WGM (12.6%). This indicates that those modes that can be coupled into the cavity fulfil the critical coupling requirements while the rest does not contribute to the coupling. We denote those modes that can be coupled to WGM as the 'useful modes'. When κ_{max} is achieved, the ratio of useful modes over total power can be calculated as $\eta=2\times\kappa_{\text{max}}/\kappa_c$, where the factor 2 is attributed to the fact that light is coupled into the cavity in both clockwise and counter-clockwise directions. For the silica grating coupler, only a maximum of 8.9% of the total power can be coupled to WGM; for the gold grating

⁴ For simplicity, the coupling coefficient defined in this context is normalized to power, which is different from the one discussed in Section 3.2 that is normalized to field amplitude.

coupler investigated here, a maximum of 57% can be coupled, suggesting a 6-fold efficiency enhancement provided by the metallic grating coupler.

Fig. 4-8 Electric field intensity and coupling coefficient (κ) as a function of the separation d for (a) silica grating; (b) gold grating. Each intensity data point is measured at the spatial maximum of the field for the respective resonance wavelengths. The insets show the respective field distribution of the coupled WGM when d is optimized for the maximum coupled field intensities (silica: $d=58$ nm; gold: $d=133$ nm). Both color scale and the left axis of the graphs are normalized to those of the gold grating.

As compared to dielectric materials, gold is a good reflector in both visible and NIR regimes. Hence, the reflectivity associated with metallic grating is much larger than its dielectric counterpart. Because of the plasmonic effect, gold grating is more efficient in coupling to the transverse propagating mode at the visible spectral range, so that the losses due to WGM in the reflection signal is more significant.

As Fig. 4-9(a) shows, the reflected power with a bare silica grating is no more than 0.5% of the launching power. With the microdisk resonator placed at $d=58$ nm from the grating, the reflection signal increases and the oscillation pattern is due to the Fabry-Perot cavity created by the zeroth diffraction order. The loss dips attributed to WGM are not clearly observed. In contrast, as Fig. 4-9(b) shows, the gold grating provides substantially stronger reflection with or

without the presence of cavity. The WGM resonances can be distinguished with good clarity while the Fabry-Perot oscillation is relatively weak. Compared with other fiber based couplers, the metallic grating offers the advantage of resonance monitoring with single port configuration, which is highly demanded in all practical applications.

Fig. 4-9 Reflected optical power spectrum with and without the whispering gallery microdisk for (a) silica grating ($d=30$ nm); (b) gold grating ($d=135$ nm).

While investigating the spectral response of the metallic grating coupler, the period of the gold grating is changed to $1.3\ \mu\text{m}$ to simulate the coupling in the NIR range. The coupled power spectrum is shown in Fig. 4-10(a) and the field distribution for a wavelength of $1.148\ \mu\text{m}$ is shown in the inset. As the wavelength increases, the free spectral range increases and the loaded Q-factor decreases. The power distribution demonstrates that it is still possible to effectively couple to WGM within the IR region where surface plasmon resonance (SPR) of nanoparticle/nanowire is significantly weaker. The reflected optical power spectra are shown in Fig. 4-10(b). The spectrum for which the resonator is present demonstrates unique features of the WGM resonances: for all the WGM dips within the spectral range investigated, it is immediately followed by a power peak, so that the spectrum is consisted of a great number of vertical lines. Such phenomenon results from the interplay of two resonance effects – the high-Q WGM and the rather low-Q grating reflection, which are combined to modify the shape of the resonances.

The resulted sharp resonance lines are Fano resonances which potentially hold even higher Q-factor [120]. Fano resonances in the whispering gallery resonator have triggered interesting physics and new applications. The next chapter will be dedicated to the discussion of Fano resonances in metallic grating coupled WGM resonator.

Fig. 4-10 (a) Normalized optical power spectrum of a gold grating coupled microdisk ($d=310$ nm; $h=100$ nm; $\Lambda=1300$ nm), inset: field distribution of a WGM at the resonance wavelength of 1.148 μm ; (b) Reflected optical power spectra of the gold grating with and without the presence of the microdisk resonator.

4.3 Fabrication of metallic diffraction grating

The gold grating discussed in this chapter is patterned by focused ion beam (FIB) on a homogeneous gold film. A layer of gold can be deposited on the end facet of an optical fiber so that the whole fiber serves as a coupling probe. The fabrication processes can be summarized as below.

1. Cleaving: normal cleave the SMF to make a smooth end facet;
2. Coating: deposition of gold film with E-beam evaporation or sputtering;
3. Patterning: focused ion beam (FIB).

Fig. 4-11 Microscope image of the end facet of a normal-cleaved SMF.

A piece of SMF is first cleaved with cleaver to yield a smooth surface. The SMF should have a pigtail length of at least 40 cm to make room for splicing with fiber couplers. The cleaved fiber then undergoes E-beam evaporation or sputtering for gold deposition of around 100 nm. A good surface quality is required for the cleaved end facet: it should not be slanted or cracked and should be kept clean all the time. Fig. 4-11 shows an example of a good cleave. Fig. 4-12 shows the end facets of a series of coated samples that are either fit for FIB patterning [Fig. 4-12(a)], slanted [Fig. 4-12(b)], cracked [Fig. 4-12(c)], or scattered with micro-dust [Fig. 4-12(d)]. When the cleaved surface is slanted, the incident angle of light onto the grating is no longer 0° . It would then lead to uneven diffractions that create different coupling performance (this phenomenon will be discussed in Section 5.4). When the surface is cracked, it is possible that the crack extends to the core area and degrades evanescent field of the grating. Even worse, the microresonator might get scarred by protrusions from the crack when they come close. Contamination must also be avoided to ensure unperturbed evanescent waves for coupling.

Fig. 4-12 Microscope images of sample end facets after gold film deposition (a) an ideal sample; (b) cleaved surface is slanted; (c) surface is cracked due to bad cleaving; (d) contaminated by dusts.

FIB patterning is the key process of the grating fabrication as it determines the quality of the evanescent waves. However, patterning on the end facet of a silica fiber is difficult as the fiber tip is small and silica is non-conductive. Fig. 4-13 shows examples of failed patterns due to charging effect during the FIB process. Because of the charging effect, the fiber would drift while being milled and the patterning sometimes does not cover the core area [Fig. 4-13(a)]. In this case, light in the fiber core is mostly reflected by the metal coating and coupling cannot happen. Sometimes, the drifting is so serious that each grating groove needs more milling time. Then the gold might be totally milled when the drifting speed exceeds the milling speed [Fig. 4-13(b)]. Drifting could also result in defected [Fig. 4-13(c)] and crooked [Fig. 4-13(d)] lines.

Although the gratings fabricated in Fig. 4-13(c-d) are still useable for coupling, they are not of the intended filling factor. Generally, such drifting effect can be weakened or avoided by firmly fixing the fiber ends to the wall of a conductive holder. As a result, examples of some successful patterning are shown in Fig. 4-14.

Fig. 4-13 (a) Microscope image of a patterned grating with the fiber core falling out of the patterned area; (b) microscope image of a patterned grating with almost zero filling factor; (c) SEM image of a grating with defected milling lines; (d) SEM image of a grating with thin deposition and crooked patterning lines.

Fig. 4-14 SEM images of (a) a patterned 1-D grating with pitch size of 1.2 μ m and filling factor of 50% (b) patterned grating with pitch size of 300 nm. The SMF used in this sample transmits light at the visible wavelength and hence has a core diameter of around 4 μ m.

Chapter 5 Optical modes in metallic grating coupled microsphere resonator

As described briefly in the last chapter, optical modes in the metallic grating coupled microsphere are varied⁵: there are diffraction modes of the grating, different families of the coupled WGMs, surface plasmon resonances for shorter wavelength and TM-polarized light. As will be revealed in the later section of this chapter, there is a weak Fabry-Perot cavity along the resonator diameter. The direct result of such conglomeration of modes is Fano resonance, which is described as asymmetric resonance line-shape originated from the interplay of two resonating events with different Q factors [120].

With its high-Q feature, WGM is prone to interact with any coherent low- or moderate-Q resonance to give Fano resonance. Fan first pointed out in his theoretical work [121], and then Liang demonstrated in experiments [122], that a side-coupled waveguide-WGM system could generate sharp asymmetric line-shapes in the transmission spectra. Later, Li demonstrated similar Fano features in microtoroid structures by manipulating interactions between two WGMs in one resonator and between two WGMs in two separate resonators respectively [123, 124]. Hu even exhibited Fano resonant features in a system where two coherent optical beams interfere with each other inside a single ring cavity [125].

⁵ Part of Section 5.1-5.4 has appeared in “Tunable Fano resonances in metallic grating coupled whispering gallery resonator,” *Applied Physics Letters* 103, 151108 (2013).
The work that appears in Section 5.5 has been submitted to *Optics Express* (2015).

However, till now, all the Fano resonances related to WGMs are demonstrated in the so-called side-coupling configurations. High-Q WGM resonators are typically coupled to optical fiber taper in the transverse directions. Such coupling method is usually fragile and unstable. Although more robust waveguide-WGM coupling can be implemented in planar waveguide chips, such as silicon-on-insulator wafers, the WGM resonators in planar waveguides have much lower Q-factors because of serious sidewall roughness. Lack of a robust and efficient coupling method becomes the main obstruction for further expanding applications of high-Q Fano resonances.

This chapter studies the Fano resonances of the metallic grating coupled resonator in the “vertical” coupling configuration.

It is found that the Fano resonances in the reflected spectra are the consequences of the interactions of WGMs and the direct reflections of grating. Therefore, it can be deduced instantly that the grating design plays an important role in generating different Fano shapes. Functional analysis of the grating parameters are given for the tunability of Fano resonances.

Additionally, selective excitation of different WGMs can be achieved with the grating coupler by purposely making the positive and negative diffractions asymmetric, which will be investigated in the last section. Bi-directionality of the propagating WGMs also bring about new and interesting phenomena.

5.1 Fano resonances in the metallic grating coupled microsphere

Fig. 5-1 Schematic illustration of the experimental setup for metallic grating coupled WGM microsphere, the insets show the SEM images of the fiber end-facet and details of the grating.

The WGM cavity used in this experiment is a silica microsphere formed by melting the end of a single mode fiber and the microsphere has a diameter of $L=182\text{ }\mu\text{m}$. The sphere is supported by a fiber stem that is not drawn in the figure. The fiber stem points out of the paper so that the WGM thus excited propagates in a plane that is not affected by the stem. We use a gold grating patterned at the end-facet of a single mode fiber (SMF) as the input coupler. The fiber coupler is mounted close to and aimed normally (incident angle $=0^\circ$) at the WGM microsphere (Fig. 5-1), namely, vertical coupling. The grating is fabricated by patterning an array of rectangular slits on a 100-nm-thick-gold film that is previously sputtered on the end facet of the SMF. The pitch size (period) of the grating is measured to be $\Lambda=1.1\text{ }\mu\text{m}$ and the filling factor is roughly 50/50 (cannot be precisely controlled). The grating is designed as such to phase match its first diffraction orders with the fundamental WGM in the microsphere. Hence, the diffracted light is able to couple to the WGM in the microsphere, resulting in sharp line-shapes in the reflection spectra. As shown in the illustration of experimental setup in Fig. 5-1, the light source is a tunable laser with

spectral range of 1500-1630 nm. Light coming out of the tunable laser is already linearly polarized. The polarization state is further controlled by a fiber pigtailed polarization controller. It is then passed through a circulator that separates input light from reflected light into two different ports. The input light is sent to the end-facet of single mode fiber that bears the metallic grating. Optical reflection from the grating is passed through the circulator to the photo-detector. A LabVIEW data collection module is used to collect and plot the coupled spectra.

Fig. 5-2 Reflection spectra of the gold grating coupled microsphere when the air gap between the resonator and grating coupler is (a) 175 nm; (b) 105 nm; (c) 35 nm. The incident wave is TE-polarized.

Fano resonances are observed in the reflection spectra of Fig. 5-2. The air gap between the grating and the microsphere decreases continuously. In Fig. 5-2(a), the Q-factor is measured to be on the order of 10^6 , while the peak-to-dip intensity difference is 19.5% of the background reflection. As the air gap decreases [Fig. 5-2(b)], the Q-factor maintains on the order of 10^6 ,

while the peak-to-dip intensity difference increases to 55% of the background reflection. When the air gap further decreases [Fig. 5-2(c)] and the coupling strength between grating and microsphere becomes stronger, the peak-to-dip intensity difference increases to 73% of the background reflection; but the Q-factor sacrificially decreases to the order of 10^4 . Note that the Q-factor measurement has reached our spectral resolution limit (1 pm) for the cases shown in (a) and (b). The resonances should have higher Q's than those measured in the current setup.

Fig. 5-3 Reflection spectra of a gold grating coupled microsphere with a diameter of around 80 μm . The FSR is determined and indicated in the graph as 6.5 nm. Background oscillations correspond to the weak Fabry-Perot cavity along the sphere resonator.

The sphere resonator creates a weak Fabry-Perot cavity along the propagation direction of the SMF. Such cavity generates periodic oscillations on the reflected background (Fig. 5-3). The spectrum shown in Fig. 5-3 is obtained by coupling a sphere with a diameter of 80 μm . The

spectrum of a smaller resonator is presented so that groups of higher-order azimuthal modes are clearly separated from one another. The free spectrum range (FSR) can then be easily evaluated.

Fig. 5-4 Reflection spectrum of a metallic grating coupled silica sphere resonator. The equatorial radius (r_e) is around 52 μ m. The azimuthal and polar FSRs are indicated in the graph.

As an example, the reflected spectrum of a larger resonator (diameter~104 μ m) is presented in Fig. 5-4 to showcase the polar FSR ($m=\ell$) and azimuthal FSR. In Fig. 5-4, the spectrum is packed with different azimuthal orders. Using Equation (2.16), the eccentricity of the sphere can be calculated based on the azimuthal and polar FSR. The eccentricity of the coupled geometry is then roughly estimated as 0.166.

Fig. 5-5 Reflection spectrum of a gold grating coupled sphere resonator (same structure and spectrum presented in Fig. 5-4) with the mode numbers calculated and labeled using Equation (2.17).

In Fig. 5-5, several resonances are zoomed in and Equation (2.17) is used to find the mode numbers of for each resonance. The mode numbers are labeled following the order of “polarization”, “radial mode number”, “polar mode number”, and the “azimuthal mode number”. It is concluded that although asymmetric line-shape is obtained in the grating coupled sphere resonator, the spectral properties of the resonances can still be identified using the results derived for symmetric line-shapes.

5.2 Mathematical description of the multi-resonance system

In order to understand the formation of Fano resonances in a complex coupling system, transfer matrix method (TMM) [121] is used to model the vertically coupled WGM resonator. Exact quantification as to how much power is coupled from the SMF to WGM is obscured in this model, as the focus is to analyze phase changes that arise from multiple resonances.

Fig. 5-6 Schematic diagram of the formulated transfer matrices; T_g and T_r stands for the transfer matrix of the grating and coupled WGM resonator respectively, the fourth layer is the free space beyond the resonator.

Similar to the Fano resonances excited in photonic crystal slabs investigated in Reference [126], the Fano resonances in our grating coupled resonators are results of two interfering pathways: direct reflection background from the grating and emission from the high-Q WGM resonances. The model is formulated as shown in Fig. 5-6 and the matrix formulation is expressed as Equation 5.1.

$$\begin{bmatrix} A_4 \\ 0 \end{bmatrix} = T_{total} \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} = T_r T_g \begin{bmatrix} A_1 \\ B_1 \end{bmatrix} \quad (5.1)$$

where A_4 stands for the amplitude of the outgoing field at the fourth layer; A_1 and B_1 are the incident and reflected field amplitudes in the first layer (SMF). The air gap between the WGM resonator and the grating is not taken into consideration since the gap size (around 100 nm) is much smaller than the incident wavelength. The overall transfer matrix is thus the product of two essential matrices that correspond to the metal grating (T_g) and the WGM resonator (T_r) respectively.

For the metallic grating, the transfer matrix can be expressed by Equation (5.2), where t_g and r_g are the direct reflection and transmission coefficients with the WGM resonator being removed.

$$T_g = \begin{bmatrix} \frac{1}{t_g^*} & -\frac{r_g^*}{t_g^*} \\ -\frac{r_g}{t_g} & \frac{1}{t_g} \end{bmatrix} \quad (5.2)$$

Under TE-polarization (the direction of which is defined as along the length of the gold strips), surface plasmon resonance (SPR) is not excited, while the reflection and transmission from this metal grating can be accounted for by two contributions: one component from the SMF/air interface segment and the other component from the SMF/gold film/air interface segment. Considering the filling factor of the grating (50/50), both two contributions occupy the proportions of 50%. We calculate the reflection and transmission coefficients using the formulae of Reference [127]. For the SMF/gold film/air segment, the metal grating is regarded as one homogeneous layer of gold, while for the SMF/air segment, the metal film is replaced by a homogeneous layer of air. The reflection and transmission coefficients under TE-polarization are thus expressed by Equation (5.3a-b).

$$r_{d,TE} = 0.5 \cdot \frac{(k_{z1}k_{z2} - k_{z2}k_{z3'})\cos(k_{z2}h) + j(k_{z1}k_{z3'} - k_{z2}^2)\sin(k_{z2}h)}{(k_{z1}k_{z2} + k_{z2}k_{z3'})\cos(k_{z2}h) + j(k_{z1}k_{z3'} + k_{z2}^2)\sin(k_{z2}h)} + (1-0.5) \cdot \frac{k_{z1} - k_{z3'}}{k_{z1} + k_{z3'}} \quad (5.3a)$$

$$t_{d,TE} = 0.5 \cdot \frac{2k_{z1}k_{z2}}{(k_{z1}k_{z2} + k_{z2}k_{z3'})\cos(k_{z2}h) + j(k_{z1}k_{z3'} + k_{z2}^2)\sin(k_{z2}h)} + (1-0.5) \cdot \frac{2k_{z1}}{k_{z1} + k_{z3'}} e^{-jk_{z3'}h} \quad (5.3b)$$

In Equation (5.3), h is the thickness of the metallic grating; k_{z1} , k_{z2} and $k_{z3'}$ represent the z -components of the wave vectors in SMF, gold film and air respectively (the symbol $3'$ suggests

the absence of the silica resonator). Each wave vector is calculated from the equation $k_{zi} = \sqrt{\varepsilon_i k_0^2 - k_x^2}$ ($i=1,2,3$), where ε_i is the permittivity of the i^{th} layer. Since we are considering the field in the zero diffraction direction, k_x vanishes and k_{zi} is solely determined by the refractive index of the material.

On the other hand, when the incident light is TM-polarized (which is defined in the x-direction of Fig. 5.4), SPR is excited in the gold strips. Besides the contributions from the SMF/gold film/air and the SMF/air interfaces, the SPR in gold strips constitutes the third component of field contribution, for which the phase and amplitude are both altered by the resonance. A simple formula can express the phase (θ) and amplitude (A) of this SPR contribution based on the Q-factor and the resonance wavelength [128]. Finite domain time difference (FDTD) simulations yield a Q-factor of 1.25 and a resonance wavelength of $1.8\mu\text{m}$ for a single gold strip. The operating wavelength ($1.55\mu\text{m}$) falls within the low-Q range of the SPR, while the additional phase shift due to the SPR is relatively constant over our observation range. The reflection and transmission coefficients of the gold grating under TM-polarization are hence expressed as Equations (5.4a-b).

$$r_{d,TM} = 0.5 \cdot \frac{(k_{z1}k_{z2} - k_{z2}k_{z3'})\cos(k_{z2}h) + j(k_{z1}k_{z3'} - k_{z2}^2)\sin(k_{z2}h)}{(k_{z1}k_{z2} + k_{z2}k_{z3'})\cos(k_{z2}h) + j(k_{z1}k_{z3'} + k_{z2}^2)\sin(k_{z2}h)} + (1-0.5) \cdot \frac{k_{z3'} - k_{z1}}{k_{z1} + k_{z3'}} + Ae^{j\theta} \quad (5.4a)$$

$$t_{d,TM} = t_{d,TE} + Ae^{j\theta} \quad (5.4b)$$

For the WGM resonator, or the 3rd layer in Fig. 5-6, the transfer matrix considers the effect of both WGM coupling and a Fabry-Perot cavity. The matrix that is associated with coupling can be derived from the results of Reference [129]. Details regarding coupling efficiencies and gap-size-dependences are not included in this model as they do little to modify the resonance shape. The matrix that is associated with Fabry-Perot cavity is constituted of a reflecting interface matrix and a propagation matrix. The Fabry-Perot cavity is confined between the gold grating and the right edge of the resonator (Fig. 5-4). Since the light coupled back to the grating through this passage way is very small as compared to the direct reflection and WGM-coupling paths, the Fabry-Perot modes have little influences on the shaping of Fano resonance (though they modify

a good deal of the background reflection). The transfer matrix for the resonator layer is expressed in Equation (5.5), where γ is used to represent the line width of cavity resonance (unit in Hz), ω_0 as the resonance frequency (unit in Hz), r as the amplitude reflectivity of the resonator's reflecting surface and $\varphi = k_{z3} L$ as the phase change of light after traversing one diameter length across the Fabry-Perot cavity.

$$T_r = T_{r,FP} T_{r,WGM} = \frac{1}{j\sqrt{1-r^2}} \begin{bmatrix} -1 & -r \\ r & 1 \end{bmatrix} \begin{bmatrix} e^{j\varphi} & 0 \\ 0 & e^{-j\varphi} \end{bmatrix} \begin{bmatrix} \frac{(\omega - \omega_0) - j\gamma}{(\omega - \omega_0)} & \frac{-j\gamma}{(\omega - \omega_0)} \\ \frac{j\gamma}{(\omega - \omega_0)} & \frac{(\omega - \omega_0) + j\gamma}{(\omega - \omega_0)} \end{bmatrix} \quad (5.5)$$

By summarizing Equation (5.1), (5.2) and (5.5), the overall amplitude reflectivity can be calculated. We take the line width of WGM resonance as $\gamma = 0.002(2\pi c)$ MHz, which is $\Delta\lambda \approx 4.8$ nm at a resonance wavelength of $\lambda_0 = 1.55 \mu\text{m}$ – a low-Q resonance to demonstrate details of the shape. To simulate the experimental conditions, we assume the thickness of the grating as $h = 100$ nm, the resonator diameter as $L = 182 \mu\text{m}$, and the amplitude reflectivity of the resonator reflecting interface as $r = 0.02$; using results of Equations (5.3) and (5.4), we obtain the reflection intensity spectra for both TE- and TM-polarizations, which are shown in Fig. 5-7(a) and (b) respectively. It is observed that the asymmetric line-shape of the Fano resonance goes from a peak to a dip for the TE-polarization (peak-dip resonance); while it reverses the sequence for the TM-polarization by going from a dip to a peak (dip-peak resonance).

Fig. 5-7 Theoretical reflection spectra of the grating coupled resonator for (a) TE-polarization; (b) TM-polarization. The resonance wavelength is $1.55 \mu\text{m}$, resonance linewidth

is around 4.8 nm, $A=1.1\ \mu\text{m}$, $h=100\ \text{nm}$, $L=182\ \mu\text{m}$. Coupling efficiencies for TE and TM-polarized modes are assumed to be 100% (at critical coupling).

5.3 Tunable resonance shape

The Fano resonances are very useful in WGM resonator based applications including sensing, switching and lasing. This section discusses the tunability of Fano resonance line shape by way of altering incident polarization and grating design. The resonance can be tuned either to have an asymmetric shape that exhibits an almost vertical edge in the center, or as nearly symmetric (Lorentzian) resonance to emerge as a total peak/dip. The sharp edge of an asymmetric Fano resonance can serve as a good sensing indicator because it changes very fast and can be easily discerned from the spectrum. The asymmetric line shape also gives an idea of optical switching devices where the on and off status can be controlled by the peak and dip features of the resonance. On the other hand, symmetric line shapes are useful when power maximization/minimization is needed for the resonance wavelength. Its most useful case is perhaps when constructing a laser cavity and the resonator is used as optical feedback: it is always desired that the reflected resonance is a peak, so that more power is sent back to the cavity. An application of the Fano resonance as a laser will be studied in detail in Chapter 7.

5.3.1 Polarization

Fig. 5-8 Simulated reflection spectrum for a silica microdisk with a diameter of $16\mu\text{m}$ and grating pitch size of $1.2\mu\text{m}$ when incident light is (a) TM- polarized; (b) TE-polarized; experimentally measured reflection spectrum for a silica microsphere with a diameter of $182\mu\text{m}$ and a gold grating pitch size of $1.1\mu\text{m}$ when the incident light is (c) TM-polarized; (d) TE-polarized. Higher azimuthal-order WGMs are observed. The insets zoom in on two Fano resonances to show an enlarged view.

As shown in Fig. 5-8, the same polarization-dependence derived in Section 5.2 is observed in both experiments and simulations. FDTD method is used to simulate the reflection spectrum of the grating coupled resonator under different incident polarizations. Due to limited computational power, a 2-D silica disk resonator with a diameter of $L=16\mu\text{m}$ is calculated. It is seen in Fig. 5-8 that, despite a striking size difference between the resonator used in simulation and experiment, the TE-polarization is always associated with peak-dip resonances while TM-polarization with dip-peak resonances. In relation to the results derived from transfer matrix method of the last section, Fig. 5-8(a) and (c) correspond to that of Fig. 5-7(b) while Fig. 5-8(b) and (d) correspond to that of Fig. 5-7(a). With a fixed grating thickness and filling factor, the

phase difference between TE and TM-polarization determines the on-off or off-on sequence of the Fano resonances, regardless of resonator size, coupling strength, input frequency or grating geometry.

5.3.2 Filling factor

Fig. 5-9 Simulated reflection spectrum for a silica microdisk with a diameter of $10\mu\text{m}$ and grating pitch size of $0.42\mu\text{m}$ when the filling factor is (a) 10%; (b) 27.5%; (c) 45%; (d) 62.5%. Grating thickness is 100 nm.

The effect of grating filling factor on the reflected resonance is investigated using FDTD method. To avoid dissipative losses in the metal, a grating made of perfect electrical conductor (PEC) is studied. In this case, SPR effect is absent, making the analysis simpler. For the results shown in Fig. 5-9, only TE polarization is explored. The trend of shifting symmetry is clearly observed as the filling factor of grating is increased: the reflected Fano resonances shift from a peak-dominated resonance to an almost absorption dip.

The filling factor clearly changes the diffraction pattern of the grating, hence changing the output phase of the resonance. Direct visualization of phase variation as a result of varying filling factor is hard to picture, but the trend can be roughly estimated with the TMM model. Using the modeling results of Section 5.2, reflected spectra with different filling factors can be plotted (Fig. 5-10). Since the model does not account for coupling details or the actual coupling efficiency of the metallic grating, the modeling results of Fig. 5-10 do not exactly agree with that of Fig. 5-9. Yet, it can be seen that the filling factor alters the resonance shape in the same manner as described by the FDTD results: the peak-dominated resonance becomes more and more dip-dominated as the filling factor is increased.

Fig. 5-10 Theoretical reflection spectra of the grating coupled resonator for a gold grating coupled silica microdisk resonator ($L=10\mu\text{m}$) when the grating filling factor is (a) 30%; (b) 60%; (c) 90%. The resonance is set at a wavelength of around $0.55\mu\text{m}$ and has a linewidth of around 3.2 nm . The grating has a period of $0.42\mu\text{m}$ and a thickness h of 100 nm .

5.3.3 Thickness

The grating depth bears a small distance of wave propagation that alters the phase of traversed waves. It is easy to understand that such phase alteration would give rise to different Fano resonances. Fig. 5-11 shows the simulated reflection spectra as the grating depth is increased from 50 nm to 200 nm . The resulted Fano resonances grow more and more symmetric as h increases. Since the grating depth is much smaller than the wavelength, the change in Fano shape

depicted in Fig. 5-11 is small as compared to that of Fig. 5-10. Hence, with a fixed h , the filling factor plays a crucial role in shaping the Fano resonance. This factor can be controlled in fabrication by the FIB technique.

Fig. 5-11 Simulated reflection spectrum for a silica microdisk with a diameter of $10\mu\text{m}$ and grating pitch size of $0.42\mu\text{m}$ when the grating thickness is (a) 50 nm; (b) 100 nm; (c) 150 nm; (d) 200 nm. Filling factor of the grating is 50%.

5.4 Higher radial-order modes in grating coupled resonator

5.4.1 Motivation

From Section 2.1, we know that the WGM families are large and varied. Depending on the different applications, higher-order modes in a WGM resonator need to be selectively suppressed or enhanced.

On one hand, a clean spectrum is often favored for filtering and many nonlinear applications where mode competition demands that power of the desired resonances are maximized. Usually, the higher-order azimuthal modes can be eliminated by optimizing resonator geometry, such as the microtorus [130] and microtoroid resonators [131]. Modes of a higher radial order ($q>1$) are difficult to eliminate without changing the resonator structure. Efforts have been made to clean up spectrum of a microbottle resonator by scarring the resonator surface [132]. Mode filtering on a disk resonator has been demonstrated with the help of a prism mode damper [133]. However, a non-destructive mechanism that does not rely on additional structures for suppression of unwanted resonances is still lacking.

On the other hand, high-order WGMs prove to be useful in several aspects. Bio-chemical sensing that is carried out in microfluidic samples and is confined by capillaries depends on the higher radial order WGMs for higher sensitivity [134]. The greatest challenge of such applications is the thick capillary wall against mode penetration: the wall thickness needs to be reduced in order for the mode to penetrate and interact with the sample [66]. In addition to wall thickness reduction, radially gradient refractive index profile is a new proposal of the solution [135, 136]. However, the gradient index profile is hard to achieve with a good resolution. Hence, higher radial order WGM for deeper penetration is considered as the most promising solution. In the domain of nonlinear optics, high-order modes are usually desired by applications that require rather large interaction volume and that are not so stringent on Q-factor requirement [137, 138]. Moreover, for WGM resonators in general, the higher radial order WGMs are sometimes desired to avoid surface defects of the resonator so that higher Q-factors can be achieved [136]; burying the mode well inside the resonator also bears the potential benefit of achieving critical coupling at mechanical contact [136].

In this section, an angle-incident subwavelength metallic grating coupler is investigated for selective enhancement or suppression of higher radial order WGMs. A small incident angle gives rise to an unequal excitation of two counter-propagating waves inside the resonator. By phase-matching designs, the diffraction grating is able to separate modes of different orders into different propagation directions.

5.4.2 Results and discussions

As depicted in the 3-D schematic of Fig. 5-12(a), the investigated structure consists of a one-dimensional gold grating patterned on the end facet of a single mode fiber (SMF) and a microsphere placed vertically above the grating. The SMF is angle-cleaved so that input beam is incident upon the grating at a small angle. The schematic shows a normal cleaved end facet with a tilted input beam, which is an equivalent but more convenient-to-calculate case of the angle-cleaved SMF. The grating is able to excite multiple diffraction orders that bear transverse momentum to couple to the WGM of the microsphere [16]. At 0° incident angle, the diffracted beams mainly consist of the zeroth and first orders while the higher orders are negligible. This is still the case when the incident angle is small ($\theta < 10^\circ$). Fig. 5-12(a) shows that, when the light is input at an angle θ , it would emerge from the grating as a tilted zeroth order diffraction that shoots through the sphere and the $\pm 1^{\text{st}}$ order diffractions that then curve inside the sphere as the counter-clockwise (CCW) and clockwise (CW) propagating WGMs. Since the 3-D WGMs of fundamental polar orders are equivalent to those of a 2-D disk, we use 2-D finite difference time domain (FDTD) method to study the WGMs coupled by a gold grating at a small incident angle. Fig. 5-12(b) shows the 2-D schematic of the configuration, which is the x-y cross section of Fig. 5-12(a).

Fig. 5-12 (a) 3-D schematic illustration of the metallic grating coupled silica microsphere with a small incident angle; (b) 2-D illustration of the x-y cross section of the 3-D structure – the same configuration investigated by the FDTD method.

Fig. 5-13(a) shows the power flux of light (termed the “power transmission through monitor”) integrated along a short line-monitor that is inserted near the surface of a silica microdisk (radius = 15 μm). The line monitor is able to detect the propagation direction of waves and indicates the CCW-propagating mode as positive transmission while CW-propagating waves as negative transmission. The incident angle is 7° and the grating pitch size is $\Lambda=1200$ nm. The filling factor (gold/air) is (50/50) and the thickness of the gold film is 100 nm. The microdisk is placed from the top of the grating to maintain an air gap of 100 nm, so that critical coupling for the fundamental WGM can be achieved [139, 140]. Only the transverse electric (TE) polarization is investigated (along the z-axis) given that the transverse magnetic (TM) waves behave similarly.

Fig. 5-13 (a) Power transmission through Monitor 1 that is inserted near the surface of the silica microdisk with a radius of $15\mu\text{m}$. The grating has a pitch of $\Lambda=1200\text{ nm}$ and an incident angle of 7° . Typical power density distribution of the (b) 1st radial-order WGM at 1515.4 nm ; (c) 2nd radial-order WGM at 1513 nm ; and (d) 3rd radial-order WGM at 1590.7 nm .

In Fig. 5-13(a), altogether three radial orders of WGM resonances (indicated by mode number q) are observed: the fundamental mode ($q=1$, or the 1st) is coupled by the negative first-order diffraction and as a result, travels in the CW direction; the second and third radial-order WGMs ($q=2, 3$, or the 2nd, 3rd) are coupled by the positive first-order diffraction and travels in the CCW direction. All of the resonances are indicated by their radial order in the graph and three power density distributions are shown in Fig. 5-13(b-d) for the typical 1st, 2nd, and 3rd radial-order WGMs at a wavelength of 1515.4 nm , 1513 nm , and 1590.7 nm respectively. Mode splitting is

observed for several resonances in the longer wavelength region, where a small amount of the 1st radial-order WGM is coupled by the positive first-order diffraction.

Fig. 5-14 Power spectrum of the CW- and CCW- propagating waves that are coupled out by a fiber taper and measured using Monitor 3 and 2 respectively.

To verify the traveling direction of the excited WGMs, a tapered silica fiber coupler is placed on top of the microdisk to measure the out-coming spectra [Fig. 5-12(b)]. The tapered fiber is set at a waist diameter of 1 μm and is placed to create an air gap of 300 nm between the taper and resonator. Line monitors 2 and 3 record the spectrum of modes that are propagating in the negative x (CCW) and positive x (CW) directions respectively. Fig. 5-14 shows the measured spectra, where the CCW-propagating waves are presented in negative values for clear recount of some spectral details. At the CW propagating end, only the resonances of the 1st radial-order are observed. At the other end that couples only the CCW propagating waves, the 2nd and 3rd radial-orders are presented along with a Fabry-Perot background generated by the zeroth-order diffraction.

Fig. 5-15 Calculated effective mode indices of the WGM and diffraction orders for the structure investigated in Fig. 5-13.

The probability of one WGM being coupled to a specific diffraction order is determined by the phase-matching conditions, which can usually be understood as the numerical matching of the effective mode indices (n_{eff}). The effective mode index of a WGM (n_{WGM}) is determined by Equation (2.17) for the case of spherical resonator. On the other hand, the effective index of the diffraction mode (its transverse component that couples the WGM) is given by the grating equation which is rewritten here as Equation (5.6).

$$n_k = \left| n_i \sin \theta - k \frac{\lambda}{\Lambda} \right| \quad (5.6)$$

where θ is the incident angle, λ is the wavelength, Λ is the pitch of the grating, n_i is the refractive index of the incident medium and k is the order of the diffraction. The effective mode indices of the resonances are found and presented in Fig. 5-15.

From Fig. 5-15, it is observed that the fundamental WGM is prone to couple with the negative first-order diffraction. The 2nd and the 3rd radial-order WGMs have smaller n_{eff} and are more likely to couple with the positive first-order diffraction. Hence, the WGM propagation directions agree with the results of Fig. 5-13. As the wavelength increases, the n_{eff} of the first-order diffractions increases while that of the WGMs decreases. In this case, the 1st radial-order WGM parts away from the negative diffraction while the 2nd and 3rd radial-orders grow closer to the positive diffraction. This is why in Fig. 5-14, the peak intensity of the 1st orders decreases and that of the 2nd and 3rd orders increase as the wavelength increases. Moreover, during this process, the n_{eff} of the 1st radial-order WGM grows closer to the positive first-order diffraction, making it more probable for the WGM to couple to the positive diffraction and mode splitting occurs. It is observed that the higher radial orders WGMs are not coupled by the negative first-order diffraction at a shorter wavelength even though their effective indices are closer. This is because the gap size between resonator and grating is optimized for the critical coupling of the fundamental modes. Since the higher radial order WGMs are buried deeper inside the resonator, achieving critical coupling for these modes requires that the gap size be reduced. Hence, when phase-matching conditions are approached, the lower orders are usually excited first (for the case when its mode index is not too different from that of the coupler).

Mode splitting is observed as a result of intermodal coupling between the CW- and CCW-propagating modes in small resonators where ratio of modal to resonator volume is large [141]. Due to the presence of different diffraction orders, waves of both propagation directions are excited. In this case, the occurrence of mode splitting at a particular wavelength is determined by the relative field strength of the counter-propagating waves, which is in turn controlled by the phase-matching conditions. Fig. 5-16(a-d) zooms in on four resonances of Fig. 5-13(a) to show the evolution of mode splitting. It is observed that splitting does not appear till the wavelength of 1.552 μ m in Fig. 5-16(b). As the wavelength increases, the field strength of the CCW-propagating 1st radial-order WGM increases and intermodal coupling grows stronger.

Fig. 5-16 Enlarged illustrations of four resonances shown in Fig. 5-13(a) at wavelengths of around (a) 1.534 μm ; (b) 1.552 μm ; (c) 1.570 μm ; 1.590 μm .

According to Equation (5.6), the probability of a specific WGM being coupled to one of the diffraction orders is mainly dependent on the incident angle and grating pitch size. The incident angle determines the separation between the negative and positive first order diffractions while the pitch size determines the slope of change. It is then possible to design the grating pitch to fulfill specific coupling scenarios where WGMs of different radial orders are selectively enhanced or suppressed. Fig. 5-17 shows the spectra coupled by a grating with 1125nm-pitch size and a fixed incident angle at 7° .

Fig. 5-17 (a) Calculated effective mode indices of the WGM and diffraction orders for an incident angle of 7° and a grating pitch size of 1125 nm; (b) Power spectrum of the CW and CCW travelling modes that are coupled out with a fiber taper and measured at the two ends respectively.

As Λ decreases, the slope of the grating index increases while the n_{eff} values are shifted upward for both positive and negative diffraction orders. Fig. 5-17(a) shows that, the n_{eff} of the negative first-order diffraction is so elevated that all n_{WGM} is far away. For the 3rd radial-order WGM, since its n_{eff} is parting with the first-order diffraction as the wavelength decreases, decreasing resonance peaks are observed in Fig. 5-17(b).

The 3rd radial-order resonance close to 1.515 μ m is unexpectedly weak because it coincides with a rather low-Q Fano resonance. Unlike the Fano resonance observed in Section 5.2 which is the result of interfering resonance and reflection background of the grating, this Fano resonance comes from the interference of a CCW-propagating 3rd radial-order WGM of a rather low-Q and a CW-propagating high-Q fundamental WGM that is backscattered at the taper-resonator coupling point.

For the 2nd radial-order WGM, the positive diffraction intersects with the n_{eff} of the resonance at around 1.57 μ m, yielding the highest peak intensity in the taper-coupled spectrum. The 1st radial-order WGM, on the other hand, is coupled by the negative diffraction throughout the CW-propagating spectrum and is weakly coupled by the positive diffraction for the last two resonances of the spectrum. Bi-directionality of the travelling waves gives rise to mode splitting, some examples of which are indicated by pairs of adjacent peaks appearing in the spectra of both propagation directions for the fundamental mode.

Hence, it is shown that, by varying the grating pitch size, 2nd radial-order WGM is selectively enhanced while the 3rd and fundamental modes are suppressed.

Fig. 5-18 (a) Calculated effective mode indices of the WGM and diffraction orders for an incident angle of 10° and a grating pitch size of 1200 nm; (b) Power spectrum of the CW and CCW travelling modes that are coupled out with a fiber taper and measured at the two ends respectively.

As illustrated in Fig. 5-18(a), the n_{eff} of the diffraction modes are separated further apart at an incident angle of 10° . All modes are more likely to couple with the positive diffraction as the wavelength increases. Since mode of a lower radial order has a greater probability to be coupled when the phase-matching conditions are approached, the 2nd and 3rd radial order resonances have comparable peak intensities [Fig. 5-18(b)] even when the 3rd order WGM is closer in fulfilling the phase-matching conditions.

On the other hand, considerable coupling of the negative diffraction to the 1st radial-order WGM is inevitable as the air gap is optimized for the fundamental mode. Similar to the case in Fig. 5-17(b), Fano resonances are observed in the first two resonances of Fig. 5-18(b). At longer wavelengths, the n_{eff} of the fundamental WGM grows closer to the positive diffraction and appears in the CCW-propagating waves.

Hence, it is demonstrated that by choosing the incident angle, WGM of even higher radial orders can be selectively enhanced. As it is difficult to optimize the gap size for the critical coupling of 3rd radial-order mode in such small resonator, the coupled resonances are relatively weak.

The incident angle induces uneven diffraction modes and makes design and separation of different radial order modes possible. Particularly, by maximizing the excited higher radial order WGMs, the resonator can have more power extending towards the geometry center. This could potentially solve the mode penetration problem in capillary sensing that is introduced in Section 2.4.1, where it is explained that the capillary wall is usually thick due to fabrication limitation and requirement on mechanical support. If higher order modes are maximized from the phase-matching design, the mode will be able to extend more into the liquid analyte and interact. This will improve the sensitivity and detection limit of the sensor.

On the other hand, the study of bi-directional WGMs in optical resonators could contribute to the construction of gyroscope, which is an indispensable part of global positioning system (GPS) and other navigation devices. The gyroscope must be able to detect even the slightest disturbance or rotational movement in order to keep the vehicle on course. It is essentially a sensing function that is generally realized by interference of two counter-propagating waves along the perimeter of a circular resonator [142]. When the resonator utilizes high-Q WGMs, the round-trip time difference between the counter-propagating waves can be more prominently expressed. Usually,

the WGM in a resonator can only be excited efficiently in one propagation direction. The other propagation direction is generally achieved by backscattering from small disturbance objects adsorbed on the resonator surface, which yields low coupling efficiency. The metallic grating coupler enables simultaneous excitation of bi-directional WGMs because as long as the incident angle is small, both the positive and negative diffractions will drive a separate mode in the resonator, resulting in two counter-propagating waves. By designing the incident angle and resonator geometry, the interference of the modes can be engineered. Hence, the metallic grating coupled WGM resonator can serve as a promising platform for modern day gyroscope sensor.

Chapter 6 Power transfer mechanism of metallic grating coupled resonator

As introduced in Chapter 4 & 5, the metallic grating is uniquely useful in applications where stable coupling or single-port configuration is necessary. Fano resonance and its shape variation with polarization are experimentally demonstrated in the reflection spectra of grating coupled microsphere resonator [140]. With the direct reflection pathway, the resonance has an asymmetric line-shape that is resulted from the interaction of multiple resonances and backgrounds. Currently, the coupling efficiency of the tapered fiber is evaluated in the transmitted power spectrum by the depth of the resonance dip [90]. Metallic grating, in contrast, brews a much more complicated resonance system, for which power transfer cannot be directly derived from the transmission/reflection spectra. This chapter analyzes the power coupling mechanism of the diffraction grating coupled WGM resonator, so as to facilitate understanding and endeavor of efficiency optimization⁶. In the last section, observations of parametric oscillations in a grating coupled sphere resonator are presented.

6.1 Mathematical model

To understand the power transfer mechanism of the many scattering processes of the structure, a mathematical model is set up to simulate the coupling phenomenon. A 3-D schematic of the modelled structure is presented in Fig. 6-1, where the silica microsphere resonator is aligned vertically above a one-dimensional gold grating patterned on the end facet of a SMF. Tunable laser (a sweeping source) inputs light at telecommunication wavelength from the SMF to excite WGM in the sphere. Reflection is

⁶ The work presented in Section 6.1-6.3 has been submitted to Optics Letters (2015).

observed and measured using a fiber based optical circulator. Transmitted light is captured by a free-space photodetector at the exiting end of the sphere.

Fig. 6-1 Schematic illustration describing the diffraction grating coupled microsphere resonator.

Before passing through the grating, light travels as a single mode, the field amplitude of which is denoted s_+ , where the “+” sign indicates power flow to the resonator. After the grating, the single mode in the fiber is supposedly diffracted to a zeroth-order and two symmetric first-order modes (all the other diffraction orders are negligible). However, since the grating has sub-wavelength pitch size, the diffracted modes are evanescent waves. When the microsphere is placed in the vicinity of the grating, WGM can be coupled to

the evanescent waves of the first-order diffractions that bear transverse wavevectors (along the x-axis). The coupling coefficient of the i^{th} input mode is defined as κ_i . At the exiting end of the sphere, amplitude of the field (s_i) that comes out of the resonator is indicated by the “-” sign.

Since the system involves two resonating structures as well as multiple scattering pathways, we use a two-stage cascaded model to characterize the power transfer mechanism. As described in Fig. 6-2, there are altogether eight scattering ports in which power exchange happens between different optical elements. The two resonating structures are the microsphere resonator and diffraction grating which are described in Fig. 6-2(a) and (b) respectively. Diffraction is characterized by Port 1-4 [Fig. 6-2(b)], with Port 1 aligning with the SMF, Port 2 and 3 in the transverse directions (x-axis) of the transmitted and reflected first-order diffractions respectively, and Port 4 as the transmitted zeroth-order diffraction that cannot be coupled to the WGM. It is assumed that the grating fulfills phase-matching conditions and all power delivered to Port 3 is able to couple into the cavity.

The dissipative loss induced by the metallic material is small as compared to the coupled power, so the grating is considered as lossless.

Light interaction with the microsphere resonator happens for Ports 5-8, where input of Port 5 and 6 are the output of Port 2 and 4 respectively (indicated by the dotted lines across figures). The WGM resonator system [Fig. 6-2(a)] can be viewed as two pairs of input-output ports that correspond to the coupled WGM (Port 5&7) and the uncoupled background (Port 6 &8). Unlike a waveguide coupler that has an output end to measure transmission, the grating's output port for the first order diffractions cannot be physically identified on its transmitting end. As a result, the output port (S_{7-}) is present everywhere around the resonator surface and it outputs radiation that is considered negligible in the model. Since the scattering system is symmetric and reciprocal, it is also possible to input from Port 7 (as indicated by the dotted arrow of s_{7+}) and output from Port 5.

Fig. 6-2 Schematic illustration of scattering system for (a) a microsphere resonator; and (b) a diffraction grating. The resonator system is cascaded after the grating system.

At zero incident angle, the positive and negative diffraction modes are symmetric – we can then group them together in Ports 2, 3 and 5. Furthermore, Ports 5 and 6 are placed inside the resonator because they are evanescent waves. When the resonator system is cascaded with the diffraction system, a 4-port network is obtained, with the end port numbers labeled as I, II, III, and IV that correspond to Port 1, 3, 7 and 8 respectively. Do take note that the Roman numerals represent the end ports of cascaded systems while the Arabic numerals indicate port labels of the two individual systems. The scattering matrix of this cascaded 4-port network is calculated from the independent scattering matrices of the resonator and grating.

First, the scattering matrix of the diffraction grating is found [Equation (6.1)].

$$\begin{pmatrix} s_{1-} \\ s_{2-} \\ s_{3-} \\ s_{4-} \end{pmatrix} = \exp(j\varphi) \begin{bmatrix} r_{11} & jt_{12} & r_{13} & jt_{14} \\ jt_{12} & r_{22} & jt_{23} & r_{24} \\ r_{13} & jt_{23} & r_{33} & jt_{34} \\ jt_{14} & r_{24} & jt_{34} & r_{44} \end{bmatrix} \begin{pmatrix} s_{1+} \\ s_{2+} \\ s_{3+} \\ s_{4+} \end{pmatrix} \quad (6.1)$$

where the amplitude scattering coefficient from one port to another is indicated by a pair of port labels in the subscripts: the first label as the input port and the second as the output port. For example, r_{24} is the amplitude response of Port 4 when power of amplitude 1 is input to Port 2; r_{22} is the amplitude reflection coefficient of Port 2. Since the system is lossless and reciprocal, the scattering coefficients remain the same when the input and output ports are exchanged, i.e., we have both the reflection and transmission coefficients satisfying $r_{ik}=r_{ki}$ and $t_{ik}=t_{ki}$, where i and k are any integer number between 1 and 4. The transmission coefficients are appended with the complex unit “ j ” to suggest $\pi/2$ phase change in the transmitted field. In reference with Section 5.3, the phase change φ is introduced to indicate light propagation through grating that is determined by the grating design.

Second, the corresponding scattering matrix for the WGM resonator coupling is studied. The coupling phenomenon of an optical resonator has been studied by many authors [98, 102, 143]. Based on the formulation of temporal coupled mode theory described in Section 3.2, the dynamic equations of the resonances can be written as Equation (6.2-6.3).

$$\frac{da}{dt} = j\omega_0 a - (\sigma_e + \sigma_0)a + \vec{\kappa}_i \cdot \vec{s}_{i+} \quad (6.2)$$

$$s_- = Cs_+ + d_i \cdot a \quad (6.3)$$

where a is the amplitude of the WGM resonance, ω_0 is the resonance frequency, σ_0 and σ_e are used to directly represent the real loss coefficients related to the cavity intrinsic loss and external coupling respectively. These coefficients are assumed to be constant for all frequencies considered in this problem. C is the scattering matrix of the direct pathway of the grating (Ports 6 & 8) [143] and is expressed by the reflection and transmission coefficient r , t , where $r^2+t^2=1$. Coupling coefficients of the i^{th} mode for coupler-to-resonator and resonator-to-outside are denoted κ_i and d_i respectively. Since the input modes are not singular and not all of them are able to couple to the WGM, we adopt the vector representation of the input field amplitude and coupling coefficient, where the amplitude vector assumes the direction of the input wavevector and the coupling coefficient aligns with the local direction of the WGM [102]. According to this rule, the coupling term vanishes for both Port 6 and 8 ($\kappa_6=d_6=\kappa_8=d_8=0$), as the zeroth-

order diffraction is orthogonal to the local wave vector of WGMs at the point of coupling. Since the resonator system excludes the details of the coupler, coupling at any geometrical location near the surface of the cavity is the same. Hence, the coupling system is considered as a symmetrical one, yielding $\kappa_5 = \kappa_7$ and $d_5 = d_7$.

Derivation of Equation (6.2-6.3) reveals $\kappa = d = \sqrt{2\sigma_e}$ [143] (same results for κ can be found in Equation (3.6)). Equation (6.3) can therefore be rewritten as Equation (6.4) by substituting α with the steady state field amplitude (similar derivations can be found in Equation (3.8)).

$$\frac{s_-}{s_+} = C + \frac{2\sigma_e}{j(\omega - \omega_0) + (\sigma_e + \sigma_0)} = C + f \cdot S \quad (6.4)$$

where $f = 2\sigma_e/[j(\omega - \omega_0) + (\sigma_e + \sigma_0)]$ is used to represent the spectral response of resonance and S is the scattering matrix corresponding to the resonance. According to Reference [143], the scattering matrix C and S are listed in Equation (6.5).

$$C = \begin{bmatrix} r & 0 & jt & 0 \\ 0 & r & 0 & jt \\ jt & 0 & r & 0 \\ 0 & jt & 0 & r \end{bmatrix} \quad (6.5a)$$

$$S = \begin{bmatrix} -(r \pm jt) & 0 & \mp(r + jt) & 0 \\ 0 & 0 & 0 & 0 \\ \mp(r + jt) & 0 & -(r \pm jt) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.5b)$$

where the r and t denotes reflection and transmission coefficients at the coupling interface. The “ $\pm$ ” sign denotes even (+) and odd (−) mode which can also be interpreted as different phase changes in the grating as a result of different polarization responses [140, 143]. Adding the elements of C and S together, we obtain the matrix for resonator coupling as Equation (6.6).

$$\begin{pmatrix} s_{5-} \\ s_{6-} \\ s_{7-} \\ s_{8-} \end{pmatrix} = \begin{bmatrix} \mp jf & 0 & j - jf & 0 \\ 0 & 0 & 0 & j \\ j - jf & 0 & \mp jf & 0 \\ 0 & j & 0 & 0 \end{bmatrix} \begin{pmatrix} s_{5+} \\ s_{6+} \\ s_{7+} \\ s_{8+} \end{pmatrix} \quad (6.6)$$

The direct reflection from the resonator surfaces are small enough to be regarded as negligible [140], so the reflection and transmission coefficients in this system are $r=0$, $t=1$ respectively. It is noted that the

scattering matrix of Equation (6.6) is only unitary when the coupling factor $K=\sigma_e/\sigma_0=1$, i.e., when critical coupling is achieved. When $K \neq 1$, the matrix is no longer lossless and the losses can be accounted for by resonator radiation at the point of coupling.

The overall scattering matrix of the cascaded system can be achieved by a series of mathematical derivation based on the matrices of Equation (6.1) and (6.6) [144].

First, the two scattering matrices are rewritten as an 8x8 matrix in Equation (6.7). Elements of the end ports are listed first, which are then followed by the connecting ports. For ports that have no scattering relation, the matrix elements are indicated by 0.

$$\begin{pmatrix} S_{1-} \\ S_{3-} \\ S_{7-} \\ S_{8-} \\ S_{2-} \\ S_{4-} \\ S_{5-} \\ S_{6-} \end{pmatrix} = \begin{bmatrix} r_{11}\exp(j\varphi) & r_{13}\exp(j\varphi) & 0 & 0 & jt_{12}\exp(j\varphi) & jt_{14}\exp(j\varphi) & 0 & 0 \\ r_{13}\exp(j\varphi) & r_{33}\exp(j\varphi) & 0 & 0 & jt_{23}\exp(j\varphi) & jt_{34}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & \mp jf & 0 & 0 & 0 & j - jf & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & j \\ jt_{12}\exp(j\varphi) & jt_{23}\exp(j\varphi) & 0 & 0 & r_{22}\exp(j\varphi) & r_{24}\exp(j\varphi) & 0 & 0 \\ jt_{14}\exp(j\varphi) & jt_{34}\exp(j\varphi) & 0 & 0 & r_{24}\exp(j\varphi) & r_{44}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & j - jf & 0 & 0 & 0 & \mp jf & 0 \\ 0 & 0 & 0 & j & 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} S_{1+} \\ S_{3+} \\ S_{7+} \\ S_{8+} \\ S_{2+} \\ S_{4+} \\ S_{5+} \\ S_{6+} \end{pmatrix} \quad (6.7)$$

From the scattering matrix of Equation (6.7), four intermediary matrices can be worked out as the following:

$$S_{ee} = \begin{bmatrix} r_{11}\exp(j\varphi) & r_{13}\exp(j\varphi) & 0 & 0 \\ r_{13}\exp(j\varphi) & r_{33}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & \mp jf & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.8)$$

$$S_{ei} = \begin{bmatrix} jt_{12}\exp(j\varphi) & jt_{14}\exp(j\varphi) & 0 & 0 \\ jt_{23}\exp(j\varphi) & jt_{34}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & j - jf & 0 \\ 0 & 0 & 0 & j \end{bmatrix} \quad (6.9)$$

$$S_{ie} = \begin{bmatrix} jt_{12}\exp(j\varphi) & jt_{23}\exp(j\varphi) & 0 & 0 \\ jt_{14}\exp(j\varphi) & jt_{34}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & j - jf & 0 \\ 0 & 0 & 0 & j \end{bmatrix} \quad (6.10)$$

$$S_{ii} = \begin{bmatrix} r_{22}\exp(j\varphi) & r_{24}\exp(j\varphi) & 0 & 0 \\ r_{24}\exp(j\varphi) & r_{44}\exp(j\varphi) & 0 & 0 \\ 0 & 0 & \bar{j}jf & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.11)$$

The relations between the two systems through Ports 2, 4, 5 and 6 are given by an identity matrix which is obtained from the derivation below:

$$\begin{pmatrix} S_{2-} \\ S_{4-} \\ S_{5-} \\ S_{6-} \end{pmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} S_{2+} \\ S_{4+} \\ S_{5+} \\ S_{6+} \end{pmatrix} \rightarrow \Gamma = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (6.12)$$

The final scattering matrix is calculated from $S_{coupling} = S_{ee} + S_{ei}(\Gamma - S_{ii})^{-1}S_{ie}$ [144]. This equation links the two individual scattering systems by the relation that the output of one system is the input of another. Hence, according to Equations (6.8-6.12), we have the overall scattering matrix relation as (6.13).

$$\begin{pmatrix} S_{1-} \\ S_{3-} \\ S_{7-} \\ S_{8-} \end{pmatrix} = \exp(j\varphi) S_{coupling} \begin{pmatrix} S_{1+} \\ S_{3+} \\ S_{7+} \\ S_{8+} \end{pmatrix} \quad (6.13a)$$

$$S_{\text{coupling}} =$$

$$\begin{bmatrix} r_{11} + \frac{f \cdot t_{12}^2 \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} & r_{13} + \frac{f \cdot t_{12} t_{23} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} & \frac{\mp t_{12}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} & -t_{14} + \frac{f \cdot r_{24} t_{12} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \\ r_{13} + \frac{f \cdot t_{12} t_{23} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} & r_{33} + \frac{f \cdot t_{23}^2 \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} & \frac{\mp t_{23}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} & -t_{34} + \frac{f \cdot r_{24} t_{23} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \\ \frac{\mp t_{12}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} & \frac{\mp t_{23}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} & \frac{\mp f \pm r_{22} \exp(j\varphi)(f-1)^2 j}{[f \cdot r_{22} \exp(j\varphi) \mp j] \exp(j\varphi)} & \frac{\mp r_{24}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} \\ -t_{14} + \frac{f \cdot r_{24} t_{12} \exp(j\varphi)}{f \cdot \exp(j\varphi) \mp j} & -t_{34} + \frac{f \cdot r_{24} t_{23} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} & \frac{\mp r_{24}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} & -r_{44} + \frac{f \cdot r_{24}^2 \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \end{bmatrix} \quad (6.13b)$$

Reflection and transmission coefficient for the four end ports are hence given by the first column of the scattering matrix of Equation (6.13b). Since the phase change through grating is small, the exponential term $\exp(j\varphi)$ can be considered as 1. Hence, the generalized reflectance and transmittance are listed in Equation (6.14-6.17).

$$R_I = \left| r_{11} + \frac{f \cdot t_{12}^2 \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \right|^2 = \frac{[\sigma(t_{12}^2 + r_{11}r_{22}) \pm r_{11}(\omega - \omega_0)]^2 + \sigma^2 r_{11}^2}{[\sigma r_{22} \pm (\omega - \omega_0)]^2 + \sigma^2} \quad (6.14)$$

$$R_{II} = \left| r_{13} + \frac{f \cdot t_{12} t_{23} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \right|^2 = \frac{[\sigma(t_{12} t_{23} + r_{13}r_{22}) \pm r_{13}(\omega - \omega_0)]^2 + \sigma^2 r_{13}^2}{[\sigma r_{22} \pm (\omega - \omega_0)]^2 + \sigma^2} \quad (6.15)$$

$$T_{III} = \left| \frac{\mp t_{12}(f-1)j}{f \cdot r_{22} \exp(j\varphi) \mp j} \right|^2 = \frac{t_{12}^2 (\omega - \omega_0)^2}{[\sigma r_{22} \pm (\omega - \omega_0)]^2 + \sigma^2} \quad (6.16)$$

$$T_{IV} = \left| -t_{14} + \frac{f \cdot r_{24} t_{12} \exp(j\varphi)}{f \cdot r_{22} \exp(j\varphi) \mp j} \right|^2 = \frac{[\sigma(r_{24} t_{12} - r_{22} t_{14}) \mp t_{14}(\omega - \omega_0)]^2 + \sigma^2 t_{14}^2}{[\sigma r_{22} \pm (\omega - \omega_0)]^2 + \sigma^2} \quad (6.17)$$

In experiment, only the reflection of Port I is collected while that of Port II is diffracted out of the fiber core and driven into free space. For transmission, since Port III and IV are physically inseparable, responses of both ports are captured by the free space detector. Hence, we have the reflectance $R = R_I$ and the transmittance $T = T_{III} + T_{IV}$.

6.2 Verification of cascaded model

In this section, the model set up in the previous section is put to test with the experimental results. The modeled spectra are put alongside those obtained in the experiment for close examination of power coupling and interaction among different scattering processes.

A patterned gold grating with pitch size $1.1\ \mu\text{m}$ is used to couple to a silica microsphere with diameter of around $130\ \mu\text{m}$. In order to inflict a small reflection coefficient, the grating has a thin gold deposition ($\sim 80\ \text{nm}$) and a small gold filling factor ($\sim 10\%$). A portion of the coupled transmission and reflection spectra are shown in Fig. 6-3(a-c) where the gap size between the grating and sphere is varied. Input light is TE-polarized (along the z -axis). For TM-polarized (along the x -axis) light, since it is inherently difficult to diffract light to the same direction of its polarization, and surface plasmon resonance is not excited with the current grating design, the coupling efficiency is not as good as the TE-polarized case.

In Fig. 6-3, two adjacent azimuthal WGMs (azimuthal mode number $m \neq$ polar mode number ℓ) are presented. Unlike the results of the previous work, the resonances have almost symmetric shapes due to low reflectivity of the gold grating.

From Fig. 6-3(a) to (c), the gap between the grating and sphere is decreased, resulting in decreasing Q but increasing peak-to-dip differences with respect to background (“peak-to-dip difference” for short) in both transmitted and reflected spectra. Due to a drastic decrease in gap size, it is observed in Fig. 6-3(c) that the transmitted power deviates significantly from the preceding two cases. The measurements are meaningful as each transmission dip and reflection peak are still in proportion with its own background power level. However, they do not represent the true ratio between transmitted and reflected power, as the transmission detector only catches part of the transmitted power and the reflection experiences insertion and propagation losses before reaching the detector. The Q -factors of the resonance at $1574.47\ \text{nm}$ are indicated in the graph while the dip to background difference for the transmitted resonance is calculated.

Unlike side coupling with the fiber tapers, it is observed that the intensity of the transmission dip drops closer to the corresponding reflection peak as the gap size decreases; and in general, the transmitted resonance always loses more power than what its corresponding reflection peak gains.

To model the consequence of varying gap size, power reflection coefficient in the SMF is first measured and calculated in experiment: after considering the losses in the optical network, the power reflection coefficient r_{11}^2 is evaluated as ~ 0.03 . The FDTD method is then used to estimate the other scattering coefficients of Equation (6.1). When the grating is placed in the vicinity of a very large resonator – large enough to treat the curved surface as a straight plane, the structure can be considered as symmetric with the grating sandwiched by two almost identical silica slabs. This would greatly simplify the numerical search of scattering coefficients, as it gives rise to almost symmetric coefficients for the two sides of the grating (+y and -y). A grating made of perfect electrical conductor (PEC) is modelled so that the effect of dissipative losses in metallic material is excluded. For a PEC grating with 10%-filling-factor, 80-nm-thickness and 1.1- μm -pitch-size, the reflection coefficients r_{22} and r_{24} are both found to be comparable with r_{11} ($r_{22} \approx r_{24} \approx r_{11}$). When the resonator is on-resonance, and after a conversion calculation the transmission coefficients t_{12} is found comparable with t_{14} ($t_{12} \approx t_{14}$). Based on these estimations and calculations, while converting frequency into wavelength, we modelled three transmission and reflection spectra for three coupling scenarios that are shown in Fig. 6-3(d-f).

Note that since the spectral resolution of our experimental demonstration is limited to 1pm, it is hard to accurately assess the resonance linewidth. Due to limited computing power, the resonance wavelength is directly set at 1574.47 nm in the model. For a clear presentation of the spectral feature, the resonance linewidth (which is numerically equal to σ_0) is set at 5.4 pm.

Fig. 6-3 (a) (b) (c) Transmission and reflection spectra of a gold grating coupled microsphere as the gap size decreases; Modeled reflection and transmission spectra using the results of Equation (6.14) , (6.16) and (6.17) when (d) $K=0.5$; (e) $K=1$; (f) $K=2$.

As discussed in Section 3.2, the coupling factor $K=\sigma_e/\sigma_0$, which is essentially determined by the gap size between resonator and its evanescent coupler, indicates how the resonator is coupled: $K=1$ denotes critical coupling where maximum power coupling is achieved, $K>1$ indicates over-coupling (gap is small) and $K<1$ as under-coupling (gap is large), where in both cases, the loss coefficients are mismatched and coupling is less efficient than the critical coupling⁷.

From the modeled spectra, it is observed that when the resonator is under-coupled [Fig. 6-3(d)], the peak-to-dip differences of the resonances are small, while Q-factor is high. The resonance shape measured in experiment seems almost symmetric due to a limited spectral resolution. At critical coupling [Fig. 6-3(e)], peak-to-dip differences are increased, and the Q-factor is maintained as the case with under-coupling. When the resonator is over-coupled [Fig. 6-3(f)], the peak-to-dip difference of the reflected resonance further increases while the Q-factor decreases. The peak-to-dip difference of the transmitted resonance is slightly reduced as compared to the critical coupling case. Moreover, the coupled resonances have more symmetric line-shapes. Such results are different from that of Fig. 6-3(c) due to the dissipative loss of metal. For under-coupling and critical coupling, the metallic grating is too far away from the resonator to modify the WGM resonance. When the grating is close to the microsphere and it becomes part of the resonator, the intrinsic loss coefficient (σ_0) increases to include the dissipative loss in metal. In this case, critical coupling might still happen as it requires a larger coupling coefficient (σ_c) to match with. Hence, the dip of the transmitted resonance is able to hit even deeper than Fig. 6-3(b) while the line-shapes can grow more symmetric as with critical or under-coupling cases.

On the other hand, Fig. 6-3 (d-f) also shows the same behavior of Fig. 6-3 (a-c), where the transmitted resonance always loses more than what the reflected resonance gains. The explanation is straightforward as the reflected power obtained in experiment is only part of the total reflection.

For extreme cases where all the reflection coefficients tend to zero, Equations (6.14-17) are reduced to the following for $\omega=\omega_0$:

$$R_I = t_{12}^4 + r_{11}^2 \quad (6.18)$$

$$R_{II} = t_{12}^2 t_{23}^2 + r_{13}^2 \quad (6.19)$$

$$T_{III} = 0 \quad (6.20)$$

$$T_{IV} = t_{14}^2 + r_{24}^2 t_{12}^2 \quad (6.21)$$

From Equation (6.18), it is found that the power scattered into Port I is consisted of direct grating reflection into the fiber core and scattered WGM from the cavity. The term t_{12}^4 indicates that the portion of coupled WGM being scattered back into the fiber core is t_{12}^2 while the amount of power coupled from the incident port to the WGM cavity is also t_{12}^2 . Equation (6.19) talks about

similar process where the feedback scattering from the cavity is given by $t_{12}^2 t_{23}^2$. Reflection of this port is lost in the experiment as it does not propagate in the core. Equation (6.20) reveals a symmetric Lorentzian line-shape that is characteristic of an optical resonance. Equation (6.21) indicates that Port IV receives both direct grating transmission t_{14}^2 and scattering from cavity mode $r_{24}^2 t_{12}^2$. Hence, except Port 3, power scattering in all the other ports is made of a direct grating scattering and a resonance feedback.

6.3 Evaluation of coupling efficiency

In contrary to fiber taper coupled WGM resonance that yields symmetric line-shape, the grating coupled microsphere involves multiple resonances and scattering processes that end up with asymmetric Fano resonances. Coupling efficiency cannot be directly assessed from an asymmetric resonance in the transmitted spectrum. However, it can be roughly evaluated in the case of almost symmetric resonance. When reflection coefficients are small, Equation (6.16-17) gives $T \approx t_{12}^2 + t_{14}^2$ for $|\omega - \omega_0| \gg 0$ and $T \approx t_{14}^2$ for $\omega = \omega_0$. In this case, the resonance is almost symmetric and the resonance dip almost coincides with the resonance frequency. Since the reflected power is small, $t_{12}^2 + t_{14}^2$ can be considered as the total power (the background power level in the spectrum). The coupling efficiency (t_{12}^2) can then be calculated by directly evaluating the ratio of resonance depth with the transmission background power.

From the results of Fig. 6-3 (a-c), it is verified that the gold grating used in this work gives almost symmetric resonance shapes. Hence, the true coupling efficiency of the grating can be evaluated at critical coupling by the depth of the transmitted resonance. From Fig. 6-3 (b), this efficiency is calculated as around 51%.

From a design point of view, the coupling efficiency to WGM resonator can be optimized by improving the diffraction efficiency of the first-order diffractions. This will improve the value of the transmission coefficient t_{12} . Suppression of the zeroth-order diffraction might help improve the efficiency, but small reflection coefficients in all ports need to be maintained at the same time. On the other hand, grating material is not limited to gold: optimization can be achieved by using less lossy material that is able to achieve higher diffraction efficiency with thinner film deposition. Silver might be a better candidate as it has less dissipative losses.

Chapter 7 Fiber laser enabled by Fano resonance in metallic grating coupled resonator

7.1 Introduction

Since its inception, increasing the efficiency of optical effects has been the most essential goal of optical resonators. Owing to the substantially prolonged photon life-time and small mode volume, WGM resonators have made significant contributions towards this goal. As reviewed in Section 2.4.2, WGM resonator plays an important role in the construction of miniaturized lasers, in both the linear and nonlinear regime. In the last 20 years, WGM resonator opens up new avenues to the narrow linewidth lasers due to its high Q-factor and small mode volume. However, small mode volume also makes the WGM resonator susceptible to thermal effect, as then the optical power has to dissipate through a small surface area. In this case thermal drift can easily induce a mismatch between the pump and resonance wavelengths, affecting the lasing performance. Thermal self-locking [145] has been proposed to solve this problem, but has proved ineffective for power fluctuation and usually needs a fixed pump power. Dual-cavity architectures [146-148] have also been utilized to stabilize the lasing feedback; but currently this method needs both a bus and a drop waveguide to form a loop – significantly increasing the system complexity. Fiber-based system on the other hand, heavily relies on the tapered fiber, which is both mechanically fragile and unable to provide optical feedback for the realization of single-port configuration [32]. Lack of a robust and efficient coupling method limits the application of WGM based narrow linewidth fiber laser.

On the other hand, lowering of threshold conditions for nonlinear effects in microsphere resonators have been investigated by authors early in the year of 1996 [149].

Spontaneous emission from dye-impregnated polystyrene or microdroplet resonators have been observed during some early investigations [150, 151]. In the past few decades, both stimulated Brillouin [75, 152] and Raman [4, 22, 26, 80, 153-155] scattering in various WGM resonator geometries have been investigated. Specifically, it is generally considered a milestone for the realization of ultralow-threshold Raman lasing in a silica microsphere which is demonstrated by the Vahala group in the year of 2002 [80]. The microsphere resonator was coupled by a fiber taper which is known as, up to date, the most efficient coupler. However, as mentioned many times in previous chapters, the fiber taper coupler is too fragile to serve the purpose of many practical applications.

Due to the inversion symmetry, silica is not able to perform second order nonlinear frequency conversion. The lowest nonlinearity with a silica based resonator is the third order. Hence, it is possible to generate optical parametric oscillation in a silica microresonator whereby two pump photons are converted to one signal photon and one idler photon. On the other hand, parametric oscillations have become uniquely useful in applications of switching [27, 42], lasing [156, 157], and the study of fundamental phenomena [158]. Specifically, optical parametric oscillations are essential in building microresonator based frequency comb that are superior in generating low power threshold, wide frequency range selections, and high repetition rates [5, 24, 105, 106, 159-161].

In contrast with Raman scattering that relies on the stimulated gain, the optical parametric oscillation in microresonators are hard to achieve. This is because the light field in the resonators has to fulfill both energy and momentum conservations, posing stringent phase-matching requirements on the resonator design. In the pioneering work of Kippenberg, degenerate four wave mixing (FWM) was demonstrated for the first time in a microtoroid resonator [26]. In his work, Kippenberg formulated the power threshold of the Kerr-nonlinearity optical parametric process by equating the parametric gain with the cavity loss. This expression is shown as Equation (7.1).

$$P_{th}^{Kerr} = \frac{\omega_0^2 Q_0^{-2} (1+K)^2 + (\Delta\omega/2)^2}{2\gamma\Delta\omega(c/n_{eff})} \frac{\pi^2 R n_{eff}}{C(\Gamma)\lambda_0} \frac{(K+1)^2}{Q_0 K} \quad (7.1)$$

where K is the coupling factor (same as in previous sections); Q_0 is the intrinsic quality factor of the cavity; $\Delta\omega$ is the frequency detuning; R is the resonator radius; n_{eff} is the effective mode index of the cavity mode and $C(\Gamma)$ is a correction factor in the range of 1 to 2 that accounts for the intermodal coupling losses; γ is the effective nonlinearity that is related to the nonlinear coefficient that is defined by Equation (7.2)

$$\gamma = \frac{n_2 \omega_0}{c A_{eff}} \quad (7.2)$$

Where n_2 is the nonlinear index of the material, ω_0 is the resonance frequency, c is the speed of light in vacuum and A_{eff} is the effective area of the mode [162].

Equation (7.1) explains the reason why optical parametric oscillations are much easier to happen in the WGM cavity. Since the power threshold is proportional to the cross-sectional modal area and inversely related to the Q-factor, modes with high quality factor and small modal area could easily exceed the parametric gain and emerge at new frequencies.

The microtoroid resonator has ultrahigh quality factor and very small cross-sectional modal area due to the tight confinement of modal energy in three dimensions. It is especially useful in studying the microresonator based nonlinear optical phenomena as it has rendered dispersion control possible by way of geometrical design.

Parametric oscillation is relatively difficult in spherical resonators as the 3-D geometry increases both design and fabrication complexity. To our best knowledge, FWM in silica sphere (or bubble) resonator has only been demonstrated by two groups around the world [163-165]. In addition, all of the parametric processes in spherical resonator mentioned above are coupled by a fiber taper.

7.2 Dual-cavity fiber laser enabled by optical feedback from metallic grating coupler

This section presents the design and experimental setup of a thermally self-stabilized fiber laser based on the Fano resonance of the sub-wavelength metallic grating coupled WGM microsphere resonator. First, the resonator is used in the linear regime to lock an external laser cavity. Then, a first observation of parametric oscillation in metallic grating coupled microsphere is presented.

7.2.1 The linear regime

The experimental setup of the dual cavity laser system is shown in Fig. 7-1. The metallic grating coupled resonator provides stable feedback that acts as a high-Q notch filter while an external fiber loop cavity eliminates thermal instability. In this experiment, the sphere is fabricated by melting the end of a SMF in the electric arc of a fusion splicer. The same grating coupler used in Section 6.3 is used here to couple the sphere resonator. The polarization controller (PC) and piezo nanonpositioning stage are tuned respectively to optimize the incident polarization and gap size. As demonstrated in Section 5.3, the Fano resonance has higher Q-factor when the resonator is TE-polarized as compared with the TM-polarized. Thus the polarization is kept at TE.

Fig. 7-1 Experimental setup of metallic grating enabled dual-cavity fiber laser. EDFA: preamplifier; TF: tunable filter; PC: polarization controller; ISO: polarized isolator; TL: tunable laser; C: 90/10 coupler; PD: photodetector; Cir: circulator; S: sphere resonator; G: metallic grating coupler on fiber probe; OSA: optical spectrum analyzer.

As a result of coherent interference between WGM and the background, Fano resonances are observed in the reflected spectrum as shown in Fig. 7-2. Since the gold diffraction grating used in this work has thin gold deposition and small filling factor, the coupled resonance line-shapes are almost symmetric, exhibiting a very high peak and only a shallow dip. When the gap size is optimized so that the resonator is almost critically coupled, the loaded Q-factor is measured to be at least the order of 10^6 (due to limited spectral resolution, the actual Q-factor is supposed to be higher than this measurement). It is observed that several higher azimuthal order WGMs are excited. This is inevitable due to the eccentricity splitting of the sphere, which is resulted in a spheroidal shape rather than a perfect sphere. The spectrum is presented in frequency domain (the bottom axis) and the polar FSR (associated with the polar mode number ℓ) is measured as 532.2 GHz.

Fig. 7-2 Reflected spectrum of the gold grating coupled silica microsphere (SEM image of which is shown in Fig. 7-1).

After the measurement of the linear spectra, the structure is disconnected from the tunable laser and inserted into a fiber loop cavity as depicted in Fig. 7-1. The incident polarization and gap size are fixed to ensure high-Q performance. The preamplifier provides the gain; narrow bandwidth filter (TF) (3dB bandwidth is 0.3 nm) is tuned to around 1553.375 nm, ensuring the single-longitudinal-mode operation in the laser cavity; the in-line polarized isolator (ISO) ensures that the light is line-polarized and no back reflection is directed to the light source; another polarization controller (PC) is used to optimize the incident power; and an OSA is used to measure the output spectrum. As a result, a dual-cavity configuration is achieved, with the microsphere as a high-Q cavity that ensures only eigenfrequencies of the resonator are amplified and the fiber loop as a laser cavity stabilizes the WGM resonances from the outside.

The measured lasing output is shown in the inset of Fig. 7-3. The lasing wavelength matches with that of the Fano resonance. To prove the thermal-stability feature, input power is increased step by step while five minutes are allowed for spectral stabilization at each step before the spectra are recorded. As shown in Fig. 7-3, when the pumping power increases, the lasing wavelength shifts to longer wavelengths, while the lasing intensity gradually increases. It means that the locking between the pumping and WGM resonance is independent of the pumping power. Although the thermal effect shifts the Fano resonance wavelength, only the resonance provides feedback to the lasing cavity that then amplifies the resonance with the EDFA. Hence, the pumping wavelength is determined by the Fano resonance while the locking between pumping and WGM resonance is automatically fulfilled.

Fig. 7-3 Peak wavelength of the dual-cavity laser versus the input power (calculated from the power measurement at the photodetector), the inset shows the measured spectra from the OSA.

7.2.2 The nonlinear regime

The self-locked Fano resonance determines the resonance wavelength of the lasing cavity, therefore eliminating the influence of both thermal and mechanical disturbance. It is then possible to further increase the input power to generate optical nonlinear effects. With the same experimental setup of Fig. 7-1, the input optical power is slowly increased by tuning the output gain of the preamplifier. The output spectra from the OSA are shown in Fig. 7-4.

Fig. 7-4 Reflected spectrum of a grating coupled silica microsphere when pump power is (a) 8.1 mW; (b) 8.5 mW; (c) 9.5 mW.

It is observed in Fig. 7-4 (a) that as the power level is low, the spectrum has only one peak at the pump wavelength. As the input power is increased to 8.5 mW, new peaks start to appear on both sides of the pump peak, with the signal on the left and idler on the right. The spectral distances from the new peaks to the pump frequency are both equal to the polar FSR of the microresonator measured in Fig. 7-2. As the input power is further increased to 9.5 mW, another pair of signal-idler frequencies appears in the spectrum. FWM is generally difficult to achieve since both energy and momentum conservation needs to be fulfilled. In the dual-cavity lasing system, the fiber loop cavity is far from efficient to reach the power threshold of optical parametric process in silica glass (nonlinear index $n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$ [166]). On the other hand, according to Equation (7.1), resonance with high Q-factors and small mode volume has inherently small power threshold. It is then possible for FWM to occur at the eigenfrequencies of the resonator. Additionally, as the microresonator is much smaller than the fiber loop cavity, the eigenfrequencies as well as the fundamental FSR are significantly different for the WGM resonator and fiber loop. It is hence concluded that the new frequencies generated by increasing input power of the pumping wavelength can only be the result of oscillation in the microresonator.

Generally, the fundamental WGM of a sphere resonator with diameter of a little over 100 μm has an effective mode area of around 10 μm^2 [57]. By Equation (7.2), we have the nonlinear coefficient of the idler frequency estimated as $\gamma_i = 1.94 \text{ W}^{-1}\text{km}^{-1}$. According to Equation (7.1), the low power threshold for optical Kerr effect in the silica WGM resonator is mainly attributed to the high Q-factor. This is why nonlinear optical phenomenon can happen with low input powers in a material such as silica that has a small nonlinear index.

According to the results presented above, the power threshold in the metallic grating coupled microsphere resonator is measured as 8.5 mW. New signal and idler frequencies emerge at places that correspond to the eigenfrequencies of the resonator. This is because the WGM resonance in the sphere resonator has a high quality factor ($>10^6$, measured linewidth is limited to spectral resolution). However, as compared to the results of Reference [26], the power threshold demonstrated in this work is much higher. The reason is that the pump mode is not a fundamental WGM and it has much larger cross-sectional modal area.

It is also noted that the excitation of FWM is against the conclusions of Reference [163] where it is stated that due to dispersion constraints, FWM can only happen for larger microsphere resonator with a diameter of no less than $150\mu\text{m}$. The polar diameter of the sphere resonator used in this work is $130\mu\text{m}$ at the largest. There are two reasons why FWM is able to happen for such a small sphere resonator. First, the resonator is coupled to a higher azimuthal order WGM that has different dispersion from the fundamental mode. Second, the resonator is good at storing energy for long photon lifetime, therefore inducing thermal effect that renders the dispersion conditions different from theoretical predictions.

In conclusion, the dual-cavity fiber laser enabled by a metallic grating coupler is demonstrated to be thermally stable and power efficient. If higher quality resonator is used (e.g. by placing the resonator in a nitrogen-purged-box to maintain high Q-factor), the laser could result in narrower linewidth for spectroscopy and notch filter applications. By further increasing the input power while optimizing the resonator for dispersion control, the FWM process would be more power efficient and result in widely-spanned frequency comb spectrum.

Chapter 8 Summary and future work

8.1 Summary

This thesis has investigated the optical phenomena of whispering gallery mode resonators. Systematic approaches from analytical modeling and numerical simulations to experimental verifications have been carried out.

Efficient power delivery to optical resonators is generally recognized as a key challenge. Several existing power coupling techniques are reviewed, with their advantages and weaknesses discussed under the context of different applications. Being mechanically stable, the dielectric grating serves as a good coupler for large and small resonators alike. However, poor power efficiency leaves the grating coupler almost unexplored.

For the first time, metallic grating is proposed as a new coupler for the WGM microresonators. As compared to the dielectric grating, the metallic material enables higher diffraction efficiency, or simply put, it bends more light to the transverse direction to couple with the WGM, owing to the diffractive nature of the structure. On the other hand, the grating provides optical reflection that bears feedback from the WGM resonances. Thus, single port coupling configuration with tunable phase-matching design is realized for the first time.

Because of the presence of the direct pathways, Fano resonances are observed in both transmitted and reflected spectra. The Fano resonance has asymmetric line-shape that is tunable by both grating parameters and input polarization. It is then possible to make grating designs that yield specific line-shapes to target different applications. For example, a resonance peak can be

engineered to appear in the reflected signal for the construction of fiber laser system. The metallic grating provides the simplest configuration to tune the shape of Fano resonance without the need of incorporating an additional optical element for phase variation.

While the metallic grating coupler gives rise to new and exciting phenomena, it is a complex coupling system. Transfer matrix model is set up to understand the phase changes of field through each optical element, so that shape tunability of the Fano resonance finds design references in theory.

On the other hand, the metallic grating is the only known coupler for WGM resonators that is able to excite and manipulate modes of two propagation directions simultaneously. Uneven positive and negative diffraction in the event of a non-zero incident angle gives rise to controlled excitation of higher radial-order WGMs. The grating can be designed to provide different phase-matching conditions so that WGMs of a specific order is selectively enhanced or suppressed. Modes of different orders are then separated by being selectively sent into two different propagating directions.

In order to assess the power coupling efficiency of the multi-resonance structure, a cascaded model is set up to study the power interaction of each scattering processes. The cascaded scattering matrix reveals the physical meaning of both reflected and transmitted spectra obtained in the experiment; results of the model point out a method of efficiency calculation for almost-symmetric line-shapes that can be engineered by the grating design.

The grating coupler provides an entirely new degree of freedom for the realization of a wide range of applications. For lasers, metallic grating coupled resonator provides single-port feedback so that external cavity controlled thermal stability and frequency locking is possible. In the domain of nonlinear optics, four wave mixing is observed in the gold grating coupled silica microsphere. The power threshold is measured to be on the level of milliwatts. While confirming the power efficiency of the metallic grating coupler, this is also the first time that optical parametric oscillations are observed in a metallic grating coupled sphere resonator.

8.2 Future work

Firstly, the grating can be further designed to improve diffraction efficiency so that more power is delivered to the cavity. The grating patterned on a silver film is found to offer higher coupling efficiency as silver has less dissipative loss. However, pure silver creates fabrication problem when it quickly oxidizes in air. To solve this problem, a layer of gold that is no more than 5 nm thick can be deposited onto the silver. This way, the silver layer is shielded from the exposure of the air and the thin gold film would not affect the coupling of the field.

Secondly, the grating coupled resonator discussed in this thesis serves as a useful platform for the realization of a wide range of applications. The resonator and coupling fiber can be packaged together to serve as a single probe for both sensing and lasing applications. The resultant probe can be used as a gas sensor, a solution based sensor, or an all-fiber laser system.

Thirdly, for applications of nonlinear optics, phenomena other than parametric oscillations are yet to be demonstrated. Different nonlinear effects pose different challenges. For example, stimulated Raman scattering requires higher power threshold and a smaller resonator volume. It then brings power coupling and fabrication difficulties. On the other hand, the parametric oscillations inside the grating coupled microsphere deserve more investigation into the details of efficiency improvement. Moreover, the tunable Fano resonance offers inspirations for both switching and filtering devices where the on-off sequence can be engineered by grating parameters and polarization.

Specifically, the following discusses the possibility of grating coupling to resonators of higher refractive index that are promising in generating new linear and nonlinear phenomena.

Microsphere resonators made from silica glass has been dominating in applications that utilize similar geometries. However, silica's transparency window does not extend beyond 2 μ m, limiting its use in the MIR regime. In recent years, both chalcogenide [21, 22, 167-169] and fluoride [170-172] glass fibers (jointly named as the "soft glass") have seen a growing number of popularity due to their MIR transparency. As the soft glasses generally have much lower glass transition temperatures than silica, power coupling to soft glass resonators needs additional control. In this case, the fiber taper is not a good coupling device as the taper is thin and tends to

be attracted by the high-index resonator when they come close. It is then possible for the resonator to come in contact with the coupler, resulting in deep over-coupling and a melted resonator [173, 174]. The metallic grating is situated on the end facet of a SMF, so it is not easily attracted by another high-index medium. Stronger mechanical strength means higher control precision. Coupling towards the soft glass microspheres using metallic grating can be carried out at finer steps for the realization of critical coupling. Moreover, different phase-matching conditions can be achieved by different grating periods. Hence, unlike fiber taper that is unable to make significant change to the fiber mode index for coupling of large index resonator, the diffraction grating is able to vary its design in order to suit the index of the resonator.

As an example, the fluoride, or more often known as the ZBLAN fiber, is used to fabricate a microsphere resonator. ZBLAN glass has a glass transition temperature of 262 °C [175], which is way lower than that of the silica glass. Hence, processing techniques that are generally meant for silica glass fibers cannot be applied to the ZBLAN fibers. We tried two methods to fabricate microsphere resonators from soft glass fibers – one with an in-house resistive heating system and the other with the commercial Vytran LFS-4100 fusion splicer. Fig. 8-1(a-d) shows examples of some soft glass microspheres using the resistive heating setup.

All of the samples shown in Fig. 8-1 have one or two defects. In Fig. 8-1(a), a greyish area appears at the tip of the sphere that suggests crystallization (352 °C). The crystallization temperature is much higher than that of the glass transition temperature. Hence, such failure is a result of improper temperature control or inhomogeneous heating. The sample shown in Fig. 8-1(b) traps significant amount of bubbles and contaminants that are visible through the transparent glass. Such effect might be due to uneven heating or sudden disturbance from the surrounding environment. The contaminants also suggest an unclean environment. Fig. 8-1(c) shows a fiber tip that has failed to form a sphere. This is because the fiber was not properly cleaved before heating. Fig. 8-1(d) shows a failed sphere made from a chalcogenide glass fiber (GLS, glass transition at 560 °C [176]), where the fiber bent before the melt material closes in to form the spherical shape. The reason for this to happen is either because the heating spot is not chosen correctly from the beginning or the relative position between the heating element and fiber shifts along the way.

Fig. 8-1 Microscope images of (a) a crystallized ZBLAN glass sphere; (b) a ZBLAN glass sphere that trapped bubbles and contamination inside during the heating process; (c) a ZBLAN bulge that failed to form a sphere; (d) chalcogenide sphere resonator with a bent stem.

According to our experiences with the above trial fabrications, a list of fabrication tips can be concluded as follows:

1. Set the correct temperature so that glass transition temperature is reached but crystallization temperature is still far away. Several trials have to be run to find a suitable temperature range, as the fiber samples are different and they might not be placed at the same distance from the heating element every time.
2. Keep the environment clean. Make sure that the amount of dust or other contaminations in the surrounding is not significant.

3. Try to heat the sphere with a spatial uniformity (usually by placing the fiber end inside a hollow cylindrical container). Gravity improves the symmetry of sphere, so it is better to put the fiber in a standing position with the heating element heating from the top.
4. Keep the heating stable over time. Sudden movement of the relative position between heating element and fiber must be avoided.
5. Cleave the fiber end before heating so that the end facet has a uniform geometry.

With all of the above concerns taken care of, a suitable microsphere made of ZBLAN glass is fabricated, the microscope image of which is shown in Fig. 8-2. It is observed that the sphere fabricated is still not perfect, as it has a small protrusion at the tip. However, such protrusion does not affect the coupling of WGMs which generally stay on the equatorial plane.

Fig. 8-2 Microscope image of a ZBLAN glass microsphere on the stem fabricated from an in-house resistive heating device.

Using the same experimental setup presented in Fig. 5-1, coupling between a gold grating and a large ZBLAN microsphere is demonstrated in Fig. 8-3(a). The microsphere used in this experiment is fabricated from a commercial fusion splicing machine [Fig. 8-3(b)].

Fig. 8-3 (a) Transmitted spectrum of a gold grating coupled sphere resonator made from ZBLAN glass; (b) microscope image of the coupled ZBLAN glass sphere that is fabricated by the Vytran LFS4100 Fusion Splicer.

Both an uncoupled background (when the air gap between resonator and grating is large) and a coupled spectrum (when the sphere resonator is over-coupled) are shown in Fig. 8-3(a). The uncoupled spectrum reveals a Fabry-Perot oscillation with a very small period, which is due to

the large dimension [Fig. 8-3(b)] and high refractive index of the resonator. The coupled spectrum shows that the sphere resonator gives higher order modes that are densely packed together, as eccentricity splitting is unavoidable in such a resonator with a supporting stem. New techniques need to be implemented to taper the ZBLAN fiber before the sphere is formed, such as the technique implemented in Reference [177].

Similar to the resonance spectrum shown in Section 6.2, the coupled resonance line-shape for the ZBLAN sphere is almost symmetric [Fig. 8-3(a)]. It is also observed that the resonance has a linewidth of around 6pm, which corresponds to a Q-factor on the order of 10^5 . Two factors have led to such low Q-factor: over-coupling and exposure to atmospheric water. The air gap can be further optimized to achieve greater Q; and to avoid water adsorption on the resonator surface, it can be placed in a vacuum environment or in a nitrogen-purging container.

Moreover, the coupling efficiency is expected to improve with a more suitable phase-matching design. For higher refractive index and larger sphere dimension, the grating needs a smaller pitch size to fulfill phase-matching requirements. There is still room for optimization with both the film thickness and metal filling factor.

Publication

Journal

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4. **Yanyan Zhou**, Di Zhu, Xia Yu, Feng Luan, “Metallic grating coupled whispering gallery mode resonator,” the 7th Conference on Materials for Advanced Technologies (ICMAT 2013).
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