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**OPTIMAL DECISION POLICY DEVELOPMENT FOR
HYBRID MTS-MTO SUPPLY CHAINS**

**WANG FENGYU
SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING
2014**

OPTIMAL DECISION POLICY DEVELOPMENT FOR HYBRID MTS-MTO SUPPLY CHAINS

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Abstract

The supply chain competition is increasingly characterized by high product variety, low volume and high service level. The challenge for the supply chain managers is to achieve responsiveness without sacrificing efficiency when dealing with high product mix. Relying on modular product design and manufacturing process standardization, more and more supply chains adopt a hybrid model that combines make-to-stock (MTS) and make-to-order (MTO) systems in sequence. Such hybrid systems can lower system cost by taking advantage of economies of scale of the MTS stage and satisfy the requirement of high product variety by taking advantage of flexibility of the MTO stage.

Some researchers have found that the performance of hybrid supply chains suffers when the system capacity is constrained. This issue is not well addressed in the literature as most research is focused on strategy, and there are hardly any solutions available to develop effective operational decision policies, aligned with the overall strategy, that ensure system performance. The lack of support from operational decision policy for control of hybrid supply chains is a key research gap.

This research proposes a joint admission-inventory control policy to bridge the gap that consists of a sequence of decisions made at the beginning of each (inventory) review period. The joint control policy makes the decisions to replenish the semi-finished module inventory at each review epoch, and sets the cap on the maximum number of orders that can be accepted during a period.

However, the development of optimal joint control policy is a challenge. For a single period, the search space of the joint control policy is a multiple of reorder point, order up-to-level, and the number of orders that can be admitted in that period. Over multi-period planning horizon, the development of joint control policy faces the “curse of dimensionality” precluding an exhaustive search for the optimal policy in polynomial time. In addition, the performance comparison of candidate joint control policies is an issue as the optimization needs to satisfy the dual objectives of minimum cost and on-time delivery rate. The main objective of this research is to develop a more effective approach for the development of joint admission-inventory control policy.

An innovative approach is proposed in this research to overcome the curse of dimensionality. By focusing on the codependency between admission control and inventory policies, a generic structure is formulated which can then be contextualized to develop specific joint control policies. The generic structure converts the optimization problem of joint control policy into identification of the optimal parameters of the generic structure, a combinatorial problem that can be solved by well-established methods.

The advantage and effectiveness of the innovative approach is demonstrated and evaluated by employing Response Surface Methodology. By fitting first-order regression models to 250 local regions and fitting second-order response surface model to 15 local regions that are near optimum, the inputs are identified for the generic structure. The simulation results show that compared to the benchmark joint control policy developed using Bellman equation, the new joint control policy achieves a better service level that is

closer to target on-time delivery. In general, the new policy derived from the generic structure performs very well.

This research takes the hybrid supply chain research to next level; it is a necessary step to complement the state-of-the-art in strategy-based research. By establishing an alignment between the strategy and operational decision policy, this research benefits companies adopting postponement strategy that want to gain and sustain competitive advantage in time-based competition.

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Chapter 1 Introduction

1.1. Challenges in time-based competition

The supply chain competition paradigm has been undergoing changes in the age of mass customization. Stalk (1988) first indicated that competition evolves from low wages to product variety wars, leading to the need for time-based competitive advantage. The competition for product variety is characterized by high product mix and low volume. In addition to product variety, leading companies are also developing strategies for creating flexible and rapid-response manufacturing systems and increasing innovation in order to move towards *time-based competition*.

The way leading companies manage time in production, new product development and introduction, sales and distribution represents the most powerful source of time-based competitive advantage. Stalk (1988) advocated time-based strategy with respect to time-based manufacturing, time-based sales and distribution, and time-based innovation, facilitating companies to gain and sustain the competitive advantage. It is clear that mass customization eventually aims to customize goods and services at the cost-efficiency level of mass production (Fisher, 1997; Hoek et al., 1998b).

However, product cost is highly sensitive to product variety. According to Stalk (1988), the variety-related cost, which reflects costs such as setup, materials handling, inventory, as well as other overhead costs, usually increases at a rate of 20% to 35% per unit each time variety doubles. By contrast, the scale-related cost can decline 15% to 25% per unit each time volume doubles. So, the first challenge in time-based competition is to

(re)design supply chains such that it can achieve responsiveness without sacrificing cost efficiency.

It is clear that moving towards time-based competition requires structural changes in organizations. Blackburn (1991) observed that leading companies pay more attention to entire value-delivery systems, transferring entire organizations into one that focuses on the total time required in product and/or service delivery.

Figure 1.1: The challenges and the requirements for supply chain restructuring in mass customization

The following observations and findings, as summarized in Figure 1.1, have been made with research of time-based competition:

- Firstly, not all products require the same level of responsiveness. Fine (1998) explained that different products and processes have their own *clock-speeds*, which are the rates of evolution or change in specific industries. For examples, airplanes may take decades to change, whereas mobile phone models may change every month. Fine (1998) further summarized a new principle in

supply chain dynamics, namely *the law of Clock-speed Amplification*—moving closer to the end customer in a supply chain, the clock-speed increases, sometimes, dramatically. In this respect, the supply chain design should cater for different time-related requirements of products.

- Secondly, there are many mismatches between products and the supply chains they rely on. Fisher (1997) observed that mismatches normally arise in two situations: (a) when a responsive supply chain is designed for functional products that actually require a highly efficient supply chain for predictable demand, stable delivery lead-time, long lifecycle, and low margin; (b) when a traditional high efficient supply chain is extended for delivery of innovative products that actually require responsive supply chains for unpredictable demand, short lifecycle, great variety, and high margin.
- Thirdly, it is not the fill rate of stock, but rather the delivery reliability within a time window that is critical in time-base competition. Andries and Gelders (1995) indicated that besides cost, supply chain leadership should be built through other competitive factors—quality, lead time, delivery reliability, product diversity, etc. Customers will accept longer lead time if delivery reliability is assured. Sox et al. (1997) observed that delivery within a short time interval and fill rate within the “service window” is more common in practice than “off-the-shelf”.

It is obvious that time-based advantage does not mean the faster the better. Instead, the rhythm of the supply chain delivery is the key. The second challenge, therefore, is to

manage supply chain's rhythm for ensuring delivery speed and reliability that matches the production requirements.

1.2. Postponement

1.2.1. Weaknesses of traditional supply chains

Traditional supply chain structures, i.e., make-to-stock (MTS) and make-to-order (MTO), have difficulty, as summarized in Figure 1.2, when dealing with the challenges of time-based competition (Figure 1.2). MTS systems are well studied in literature, which typically offer less expensive producer-specified products with low variety (Soman et al., 2004). The common problems with MTS systems are high inventory cost of carrying finished products and high risk of obsolescence in some industries. For MTO systems, the main issue is the uncertain and relatively long delivery time due to production complexity in customization.

Figure 1.2: The advantages & disadvantages of traditional MTS and MTO supply chains

1.2.2. Postponement and decoupling point

In practice, companies deal with the challenges of product variety and shorter lead time by employing *Postponement* or *Delayed Differentiation*, through increasing standardization degree in part and process design, whereby the product differentiation processes can be postponed downstream closer to the customer (Lee et al., 1993; Lee & Tang, 1997; Swaminathan and Lee, 2003; Yang and Burns, 2003; Yang et al., 2004a). Postponement as a principle was first introduced by Bucklin (1965) for individual organizations to shift the risk of owning goods to others. Over the last few decades, postponement has been generalized as a strategy for delaying the time of making decision and avoiding risk of uncertainty and is widely applied in distribution/logistics, manufacturing and supply chain contexts.

		Logistics	
		Speculation (Decentralized inventories)	Postponement (Centralized inventories and direct distribution)
Manufacturing	Speculation (make to inventory)	Full speculation strategy	The logistics postponement strategy
	Postponement (make to order)	The manufacturing postponement strategy	The full postponement strategy

Figure 1.3: The postponement strategies (Pagh & Cooper, 1998)

Zinn & Bowersox (1988) conducted the well known postponement study in supply chains and introduced five types of postponement in distribution and manufacturing—labeling, packing, assembly, manufacturing, and time postponement—by studying eight products’

physical and demand characteristics. By employing postponement and combining it with a holistic view of supply chain, Pagh & Cooper (1998) proposed four postponement strategies (Figure 1.3) in a manufacturing-logistics matrix: full speculation, logistics postponement, manufacturing postponement, and full postponement.

Hoek (1997, 1998a) proposed the conceptual model of “postponed manufacturing system.” As per the definition, the final (secondary) manufacturing activities which are performed closer to the customer are postponed until customer orders are received; whereas the primary (basic) manufacturing activities which are separated from the secondary manufacturing activities produce standard products or generic modules in large-scale quantity without customer demand. This separation frees primary manufacturing to focus on large economical runs and final manufacturing to focus on responding to customer demand. The global efficiency of postponed manufacturing systems can be enhanced through consolidation of activities in large-scale primary operations upstream, and the local responsiveness can be enhanced through decentralization of activities downstream.

The development of postponement is mainly driven by industry applications, and plenty of case studies have been reported in the literature. Donk (2001) noted that companies in food industry have to deliver a greater variety of products and meet higher logistical demands, while keeping cost as low as possible. Diversified customer orders have direct impact on production and distribution channel (e.g., different packaging and labeling), making pure MTS system infeasible. At the same time, constraints, such as short delivery time and frequent delivery, perishability of product, make pure MTO model unviable as well. Similar competitive environments are developing in other industries also, for

example, electronics, automotive, and apparel. More case studies and surveys about postponement can be found in Yang et al. (2004a, 2004b), and Perona et al. (2009).

For implementing postponement in a supply chain, a *postponement point* needs to be located, such that certain decisions or activities that take place before the point are carried out based on forecast or speculation, while others, after the point, are only triggered upon arrival of firmed orders. Cooper (1993) defined four postponement points in supply chains—bundle manufacturing at production plant, unicentric at central warehouse, deferred assembly, and deferred packaging at theatre warehouse—and managed to link them to five types of postponement by Zinn and Bowersox (1988).

The postponement point was later replaced by the term of *decoupling point*, which is also known as Order Penetration Point (OPP). Sharman (1984) defined OPP as the location where a product becomes earmarked for a particular customer, reflecting how far the customer order penetrates into the supply chain. Coincidentally, Sharman suggested that downstream of the OPP, customer orders drive material flows; whereas upstream, forecasts and plans do the drive.

To highlight the linkage between decoupling point and customer order, Olhager and Ostlund (1990) defined the point as Customer Order Point (COP). Rudberg and Wikner (2004) named the point as Customer Order Decoupling Point (CODP). The terms of OPP, COP, CODP are used interchangeably. In this study, the term decoupling point will be used throughout the report to represent this point.

Figure 1.4: Five Decoupling Points (Hoekstra & Romme, 1992)

Hoekstra & Romme (1992) broadly summarized five types of decoupling point (Figure 1.4):

- Make and ship to stock (DP1): Products are manufactured and distributed to stock points, which are spread out and located close to customer.
- Make to stock at central stock (DP2): Products are held in stock at the end of production, from where they are delivered to customers.
- Assembly to order (DP3): System elements or subsystems are held in manufacturing centre, and the final assembly takes place for a specific customer order.
- Make to order (DP4): Raw material and components are kept in stock, and each customer order is a specific project.
- Purchase and make to specific order (DP5): No stock is kept at all. Purchase and production take place upon receipt of a specific order.

Similarly, Hoek (1997, 2001) proposed seven decoupling points, namely, Engineer to order (as in construction), Purchase to order (as in high-end electronics), Make to order (as in restaurants), Manufacture/assemble to order (as in some PC-products), Package and Label to order (as in some packaged foods), Ship to order (as in retailers), and Final adaptation/customization in retail channel (as making of fresh salads). Hoek (1997, 2001) further mapped the seven decoupling points into four postponement strategies—bundle manufacturing at production plant, uni-centric at central warehouse, deferred assembly, and deferred packaging at theatre warehouse.

Yang and Burns (2003) found that postponement is closely linked with the decoupling point, and the levels of postponement can also be related to different decoupling points. They further suggested that postponement application may be a reasonable starting point for determining the location of the decoupling point.

1.2.3. Decoupling point positioning

Each supply chain is special and the determination of the precise location of the decoupling point at operational level is a challenge, as it is affected by a number of factors. Fundamentally, three types of factors affect decoupling point positioning—market, process, and product nature (Sharman, 1984; Pagh & Cooper, 1998; Hoek et al., 1998b; Olhager, 2003; Swaminathan and Lee, 2003).

- Market factors are related to customer demand (e.g., demand variance and demand correlation) and service requirements (e.g., delivery and customization requirements). A broad product range and a high level of customization make it impossible to locate the decoupling point very close to the customer (as typical

MTS system does) due to high holding cost for a wide range of finished products, whereas high service level requirements (relatively short delivery lead time) make it impossible to locate the decoupling point very far upstream (as typical MTO system does).

- Product factors are related to the design of the product, including the degree of component standardization, level of product modularity, as well as degree of which end-products can be substituted for each other. The physical characteristics of products also determine the viability of decoupling point by taking into account company policies or the fact that not all products are modular, such as tablets.
- Process factors are related to manufacturing processes and distribution networks in supply chains. New process standardization or process re-sequencing may result in decoupling point repositioning (Swaminathan and Lee, 2003), but the cost of process redesign could be a key obstacle to process standardization or re-sequencing.

In addition, the positioning of the decoupling point also needs to consider other related factors such as logistics cost and supply chain flexibility for reaching the best balance. From product and process design perspective, the higher the degree of standardization in product design, the further downstream the decoupling point could be located, and more flexible supply chains could be (Feitzinger and Lee, 1997). On the other hand, in order to achieve lower inventory holding cost and higher production effectiveness, decoupling point would not be located far downstream, even if the flexibility of process and product design allows.

1.3. Hybrid MTS-MTO supply chains

Hybrid MTS-MTO system is a new supply chain control strategy that supports the implementation of postponement at the operational level. The hybrid system combines MTS and MTO in a sequential manner and exhibits the characteristics inherited of both MTS and MTO systems. The new supply chain strategy is designed for products which share common semi-finished modules and can be customized to different finished products. The strategy is implemented in two stages: (1) standard semi-finished modules are produced (MTS stage) and stocked as intermediate inventory; 2) upon receipt of customer orders, semi-finished modules are assigned to orders for customization according to specific requirements (MTO stage). The boundary between the two production stages is the decoupling point, which allows the manufacturing of common semi-finished modules and the customization of finished products to occur independently.

Both MTS and MTO stages have their own dedicated resources for achieving their respective objectives:

- In the MTS stage, standard semi-finished modules (neutral items) are produced governed by inventory policies. As the semi-finished modules are standard and shared by different types of finished products, they can be produced in large quantities. By taking advantage of economies of scale, production during MTS stage can achieve high efficiency at low cost.
- In the MTO stage, upon receipt of customer orders, semi-finished modules are assigned to customer orders and flow through MTO production stage undergoing customization to deliver product variety. Because the finalization takes fewer

production steps compared to a pure MTO system, the entire system is more responsive. The consumption of semi-finished modules triggers the production at the MTS stage for replenishment.

The potential advantages of the hybrid supply chains are:

- (a) Lower supply chain cost by taking advantage of economies of scale in MTS stage, and
- (b) High product variety by taking advantage of flexibility in MTO stage.

Essentially, the strategic decision of decoupling point forms the basis for entire logistics in organizations, and it is also the foundation for planning and control of the products flows (Hoekstra and Romme, 1992); so the decoupling point positioning has been the main research focus in postponement.

However, there are hardly any studies that look at operational decision and policy making, which make the implementation of postponement in practice a challenge, considering the fact that the alignment of operational decisions and policies with strategic decisions is critical for the success of supply chains. This lack of operational decision-making in a hybrid supply chain is the motivation for this research.

1.4. Summary and report organization

Hybrid MTS-MTO is a new supply chain strategy suitable for dealing with the challenges of product variety and responsiveness in today's time-based competitive environment. Hybrid supply chains inherit two important characteristics—lower cost by taking advantage of economies of scale in the MTS stage for standard semi-finished modules

production, and at the same time, high product variety by taking advantage of flexibility in the MTO stage.

However, fully exploiting the potentials of new hybrid supply chains remains a challenge, which can be addressed strategically or operationally. This research addresses it from the operational decision perspective by proposing a new way to develop decision policy for controlling the performance of hybrid supply chains.

The rest of the chapters are organized as follows. Chapter 2 contains the review of the relevant literature in the development of hybrid MTS-MTO supply chains. Two research streams are identified: strategy-based decoupling point (re)positioning and operational decision policy development. This research falls into the second research stream and tries to bridge the research gap of lack support of operational decision policy in the domain of hybrid supply chain research. Chapter 3 describes stochastic processes of a hybrid supply chain; the system behavior reflected by the evolution of the stochastic processes is partly controlled by the joint control policy and partly random. A new joint admission-inventory control policy is proposed for ensuring system performance. However the development of joint control policy faces the “curse of dimensionality” as a comprehensive search of optimal joint control policy cannot be completed in polynomial time.

In chapter 4 it is proved that for given state space the optimal joint control policy developed by Bellman equation is unique optimal, which exhibits the optimal codependency between admission control and inventory policies. An experiment of Markov Decision Process (MDP) model is created for developing the optimal joint control policy. Through the analysis of the unique optimal policy, a generic structure and related structural properties are obtained, which are used for deriving new joint control

policies. The sensitivity study shows that the generic structure is valid under different operational conditions. The formalization of the generic structure is the key contribution of this research, which successfully converts the optimization problem of finding optimal joint control policy into one that identifies the optimal parameter settings of the generic structure.

The advantage and effectiveness of generic structure is demonstrated and evaluated in Chapter 5, in which a new joint control policy is developed by employing Response Surface Methodology (RSM) relying on the generic structure. The simulation result shows that compared to the benchmark policy developed by Bellman equation, the new policy achieves better service level that is closer to the target of 90% on-time delivery by allowing about nine additional late deliveries and accept about 20 more orders for a planning horizon of 52 weeks; as consequence, the new policy requires about additional two setups and incurs slightly higher inventory for catering to the additional orders. Finally, Chapter 6 summarizes the research outcomes and future research directions.

Chapter 2 Literature Review

2.1. System performance comparison

Supply chain performance is commonly measured by cost and service level: reasonable cost ensures participation in the market, and service level determines the survival of business. Traditionally, service level is measured differently for MTS and MTO systems: for the former fill rate and stock ready rate are two common indicators (Silver, 1985; Axsäter, 2006), whereas for the latter the rate of meeting *quoted* lead times (on-time-delivery rate) is the key indicator.

In comparison between the hybrid MTS-MTO and traditional MTS or MTO systems, cost and service level are still the key criteria; in addition, a third criterion, the flexibility of the supply chains is also examined. Two major types of research methodology are engaged for performance comparison: case study and mathematical modeling. Based on the results shown in Table 2.1, in general, the performance of hybrid system is superior.

In terms of cost saving Aviv and Federgruen (2001) reported that compared with traditional MTS systems, the hybrid system can achieve total cost reduction (in terms of inventory holding and backlogging) of 16% per month without seasonality and up to 51% with strong seasonality. The study by Atali and Ozer (2012) showed similar cost benefits of 12.38% to 22.69%, depending on production constraint and fluctuating demand patterns. Based on numerical results, Kaminsky and Kaya (2008) concluded, “The cost of the new combined strategy is significantly lower than the cost of either pure strategy.”

Table 2.1: Performance of hybrid MTS-MTO supply chain in comparison to pure MTS and MTO systems

Author	Research methodology		Performance criteria		
	Modeling	Case study	Cost saving	Service level	Flexibility
Olhager & Ostlund (1990)		Y	Improved	Improved	Improved
Aviv & Federgruen (2001)	Y		16% - 51%		Improved
Gupta & Benjaafar (2004)	Y		Improved	Improved	
Skipworth & Harrison (2006)		Y		Improved	
Harrison & Skipworth (2008)		Y		Improved	
Kaminsky & Kaya (2008)	Y		Improved		
Atali & Ozer (2012)	Y		12.4 – 22.7%		Improved

Olhager and Ostlund (1990) compared service satisfaction in a case study and concluded that the hybrid system can reduce manufacturing and delivery lead times, and improve delivery dependability. Skipworth and Harrison (2006) measured service satisfaction from three perspectives: order lead time, delivery reliability, and ex-stock availability (the proportion of orders and enquiries for which the correct stock is either finished or available), and proved two propositions from customer service perspective: (1) hybrid system, as an alternative to MTS system, can increase ex-stock availability. (2) hybrid system, as an alternative to MTO system, can reduce order lead-times and increase delivery reliability, but may introduce demand amplification.

Strictly speaking, service satisfaction of hybrid MTS-MTO supply chains is market oriented, which is determined by external forces (Andries and Gelders, 1995; Olhager, 2003; Yang and Burns, 2003). In other words, the customer service is a kind external

constraint rather than a pure performance measure, and should not be compromised; satisfying the constraint ensures the survival of the supply chain.

Although the hybrid MTS-MTO supply chain shows potentials, it also has limitations, especially when it is constrained in capacity. Gupta & Benjaafar (2004) showed following drawbacks of hybrid supply chains:

- When the utilization in MTS stage is high, the cost advantage of hybrid systems over pure MTO systems diminishes.
- When the utilization in MTO stage is sufficiently high, hybrid systems become inferior to pure MTS systems.
- The value (in terms of cost) of hybrid systems reduces when the requirement to service level is tight. The reduction is particularly significant when the utilization in MTO stage is high.

Aviv and Federgruen (2001), Kaminsky and Kaya (2008), and Atali and Ozer (2012) also reported similar result: the value of the hybrid system declines as the production becomes constrained. Skipworth and Harrison (2006) concluded, from manufacturing planning and scheduling perspective, that to respond to higher demand variability hybrid systems require excess capacity and high throughput efficiency.

2.2. Performance control of hybrid supply chains

Performance control is a key issue in adoption of hybrid supply chains. In literature, the issue is addressed by either decoupling point (re) positioning at strategic level or decision policy control at operational level.

2.2.1. Decoupling point (re)positioning

The first research stream in performance control of hybrid supply chains is strategy-based and tends to control system performance by (re) positioning the decoupling point. Cochran & Kim (1998) studied a multi-stage push/pull system where the boundary between push (upstream stage) and pull (downstream stage) is flexible. The research suggests that the system performance is improved by adjusting the decoupling point to reach a trade-off between delivery lead time and inventory level, and more importantly, proper decoupling point location can be determined by the ratio of delivery lead time and inventory replenishment lead time.

Due to market changes and technology development, the location of decoupling point needs to be reviewed periodically and repositioned, if necessary. There are two approaches for decoupling point (re)positioning: empirical and optimization-based.

2.2.1.1 Empirical approach

The empirical approach answers the strategic question of where to *shift* the decoupling point from its current location. Shifting of decoupling point is mainly driven by the need to strengthen competitive priority (Olhager and Ostlund, 1990; Olhager, 2003). Basically, the shift relocates the decoupling point away from its current location to achieve competitive advantage.

Rudberg and Wikner (2004) explained the rationale behind the shifting of decoupling point using Figure 2.1, in which two forces, supply chain productivity and flexibility, need to reach the best trade-off. The first force moves decoupling point downstream for increasing productivity (in terms of customization), at the same time making the system

more responsive to the customer. The second force, on the other hand, moves the decoupling point upstream for increasing flexibility, and reduces inventory cost and risk.

Figure 2.1: The trade-off between productivity and flexibility in decoupling point location (Rudberg and Wikner, 2004)

The most well known decoupling point repositioning study by Lee et al. (1993) looked at the supply chain of Hewlett-Packard Deskjet-Plus printer. In the case study the decoupling point was shifted upstream for increasing flexibility. Different versions of Deskjet-Plus printers, manufactured by Vancouver Division of HP, were assembled, tested and shipped (in anticipation of demand) to the distribution centers (DCs) in North America, Europe, and Far East with different power modules and manuals. HP redesigned the product such that the power supply module could be added later, and the activity was postponed to DCs. After the change, the Vancouver factory only produced generic Deskjet-Plus printer without power supply module and manual, i.e., standard semi-finished goods, and shipped generic printers to the DCs. The DCs localized the generic products to the specific options as needed. The new system brought significant performance improvement in Vancouver factory by minimizing re-setup and the actual customer service level in DCs increased considerably from 86% to 99.9%.

Vlist et al. (1997) presented a case in truck manufacturing with three key sequential manufacturing stages: (1) parts manufacturing, (2) components assembly at suppliers, and (3) truck assembly at the manufacturer. It was observed that over the years the decoupling point has shifted from after truck assembly (but before customer specific upgrade) to the start of truck assembly. With this change, firstly, the costly stock of manufactured trucks disappeared, since trucks are assembled only upon receipt of customer order and the variations are readily accommodated at the beginning of truck assembly. Secondly, the costly components such as fuel pumps are now stocked at the beginning of truck assembly process. The authors argued that by gradually shifting the decoupling point upstream to the component suppliers, the truck assembler can achieve virtually zero fuel pump inventory, provided the lead time is short enough.

Vorst et al. (2001) presented another case study in poultry (food) industry, and found that it is feasible to shift current decoupling point upstream between the processes of packaging and labeling, to increase flexibility in dealing with unpredictable demand and short delivery time.

These empirical case studies demonstrate how decoupling points can be shifted away from current positions and illustrate the corresponding rationale. Companies definitely require guidance for precisely positioning the decoupling point among their supply chains.

However, the empirical approach does have its drawbacks. Firstly, the shift may not take a global view of supply chain performance improvement, and thus may not be justifiable. Secondly, this approach is based on case studies, and without mathematical modeling the

approach does not lead to generic insights about the shift. Thirdly, the shift may not always be feasible, since it could be limited by high cost of product or process re-designs.

Further, it is fully understood that repositioning of decoupling point has significant impact on supply chain structure. From market's point of view, the decoupling point provides the interface between the market and the supply chain. The market uncertainty transmits into the supply chains via the decoupling point, and the supply chain needs to buffer the uncertainty in order to deliver the right products, in right quantity, at the right time. The location of decoupling point directly determines where and what types of semi-finished modules are kept for buffering the uncertainty. In most cases, as highlighted by Sharman (1984) the decoupling point is the last point at which inventory is held. Hoekstra and Romme (1992) also suggested that the decoupling point coincide with the main stock point, and in principle, no inventory should be stocked downstream (of this point) without strong economical justification.

2.2.1.2 Optimization-based approach

The optimization-based approach joint determines the best location of the decoupling point and the appropriate inventory level at the point disregarding current location. The optimization procedure normally employs mathematical modeling and other Operation Research (OR) techniques which pursue the lowest total cost, subject to service constraints.

The joint optimization procedure tries to address two strategic questions at the same time (Gupta and Benjaafar, 2004; Jewkes and Alfa, 2009):

- Location of the decoupling point, a strategic decision for determining supply chain structure.
- Optimal level of inventory to be held at the decoupling point, a strategic decision in inventory management.

Gupta and Benjaafar (2004) developed an optimization model for locating the decoupling point and determining semi-finished module inventory, taking into account the inventory holding cost and product and process re-design costs subject to an upper bound of the average lead time. The results show the interdependency between system performance (service level and resource utilization), the decoupling point location, and corresponding inventory level; if inventory holding cost or the product/process re-design cost is high, the system maintains high performance by locating the decoupling point upstream.

Jewkes & Alfa (2009) addressed the same problem, but with a new parameter of θ ($0 \leq \theta \leq 1$), the completion degree of semi-finished modules, for determining the location of decoupling point. The parameter θ represents the proportion of production completed by a supplier; $(1 - \theta)$ is the remaining process portion that is driven by customer order and is completed by the manufacturer. The optimization minimizes the total cost—comprising of order delay cost, semi-finished module inventory holding cost, disposal cost for incorrect stock of semi-finished modules, and storage capacity cost for semi-finished modules—by determining the optimal point of differentiation and optimal semi-finished goods buffer size. Extensive numerical results show that the optimal semi-finished module buffer size increases in both customer demand and the likelihood that semi-finished modules are not suitable for customization reflecting a more demanding market.

It is reversely proportional to the rate of which the supplier can provide suitable items to the manufacturer. The determination of optimal degree of delayed product differentiation (parameter θ) is more complex. It is lower if customers accept a wider range of product characteristics, but it is uncertain in customer demand level and the supplier's capacity to produce suitable items. The study helps understand how the degree of differentiation affects the tradeoff between customer order fulfillment delay and inventory risks when both MTS and MTO stages of production take non-negligible time and the production capacity is limited.

Garcia-Dastugue & Lambert (2007) studied a supply network comprising of two manufacturers, nine DCs, and multiple stores employing time-based postponement—the intentional delay of activities to a later time without changing the sequence of the activities. The optimization minimizes the holding cost of inventory by determining the locations of the decoupling point and the corresponding inventory levels without changing the sequence of activities in the supply chain. The manufacturers can either locate the decoupling point at the beginning of manufacturing and stock raw material, or locate the decoupling point at the end of manufacturing and stock finished product. By choosing the former, the manufacturers need to hold seven day of inventory of raw material to satisfy the immediate need in productions. Meanwhile, the DCs need to hold 11 days of safety stock of finished product considering the long delivery time from the manufacturer. On the other hand, if decoupling point is located at the end of manufacturing, the manufacturers hold 17 days of finished product inventory and zero raw material stock. Meanwhile, each DC holds just seven days of finished product instead of 11 days. So the second option results in inventory saving of a total of 36 days

at the nine DCs. The research clearly demonstrates and quantifies the trade-off between the location of the decoupling point and the inventory level in the supply network.

By employing OR techniques, these models provide a means for exactly positioning decoupling point and setting strategic inventory level; the intensive experiment and analysis give insights into the interrelationship between decoupling point location, costs, and the service constraints.

This optimization-based approach is also not without its drawbacks. The first drawback is that it determines the optimal decoupling point location irrespective of the current one, which could lead to drastic reengineering of supply chains. Secondly, it makes two strategic decisions—decoupling point location and inventory level determination—at the same time. This may give an incorrect perception that the two decisions should be made concurrently. The fact is that the decoupling point location determines entire supply chain structure, whereas inventory level is just one strategic decision subject to many other factors given a supply chain structure. Thirdly, because of the joint determination of two important decisions, the optimization becomes more complicated and can only be performed at an aggregate level. Last but not the least the optimization-based study is unable to tackle the major limitation of the hybrid MTS-MTO systems which is that the system performance declines as the production becomes more constrained.

2.2.2. Operational decision policy

Recently, some researchers have studied the development of operational decision policy for controlling the hybrid system's performance. Perona et al. (2009) studied a hybrid system for a manufacturing SME and proposed a framework to manage three groups of

finished products and two groups of intermediates. The proposed framework consists of two steps: (1) decide decoupling points for different product groups, and (2) directly assign traditional inventory policies to product groups. The framework basically combines strategic and operational decisions (i.e. inventory control policy) without taking into account admission control. At operational level the authors essentially address a classical inventory control problem.

Atali & Ozer (2012) studied a hybrid system where production capacities at both the stages are restricted within lower and upper limits, and a modified base-stock policy is used for maintaining the inventory of semi-finished modules. A close-to-optimal heuristic production policy was proposed to minimize the finite-horizon expected production and inventory costs. Still, the predetermined base-stock is *independent* of the lower and upper production bounds. As a result the controls of production and inventory are disconnected, and the decisions on inventory and production are made separately and independently.

Wang et al. (2011a) took one more step and connected the two controls (production and inventory) by proposing a joint admission-inventory control policy for a hybrid system. The joint control policy reviews and replenishes the inventory of semi-finished modules periodically, and rejects customer orders, during a review period, once a dynamically set admission cap is reached. However, the underlying principle of the connection between the two controls is not studied. Wang et al. (2011b) further identified that both reorder point and the maximum level of work-in-progress (WIP) affect system performance, and the impact on inventory level, production throughput and lead-time is interlinked. This forms the second research stream in hybrid supply chain which has started to consider operational decisions to address system performance.

2.3. Summary and research gaps

The system performance of hybrid supply chains, in general, is superior, but the performance declines as capacity becomes constrained. There are two research streams in the domain of hybrid supply chains that try to deal with this challenge. The first research stream looks at strategy-based decoupling point (re) positioning. The main contribution of strategy-based research is that it provides a way to (re) engineer the entire supply chain so that entire system can achieve better balance between system flexibility, cost, and responsiveness through (re) positioning the decoupling point. The (re) positioning, of course, is subject to market factors, product design, and process design; the complexity and cost incurred may, however, compromise the strategic decision.

The second research stream focuses on operational decision making to control system performance. The alignment between supply chain strategy and operational decision policy is critical since an optimal supply chain structure (including a decoupling point) still requires strong support from operational decision policy. The main contribution of the second stream is that it controls the hybrid system performance from an operational decision perspective and directly addresses the system performance issue. The second stream also complements the state-of-the-art of strategy-based research.

However the second research stream has not been developed well and there are hardly any solutions that can generate effective operational decision policies for ensuring target system performance. The lack of support from operational decision policy for control of hybrid supply chains is the main research gap.

To bridge the gap, this research develops a joint admission-inventory control policy as a means of system performance control. The joint control policy consists of a sequence of decisions over multiple periods; at each epoch the joint control policy answers the following two questions:

- (a) Should the semi-finished modules be replenished and to what level?
- (b) What is the maximum number of orders that can be accepted during the period?

There are a number of research questions this research seeks to answer:

- Does the optimal joint control policy exist and can it be identified?
- What are the challenges the development of joint control policy faces?
- How do the control mechanisms of admission control and inventory policies interact and influence each other in hybrid system performance control?
- Can the interaction be reflected and exploited in the development of joint control policy?
- Is there a more effective solution for identifying joint control policies?

By answering these questions, this research tries to achieve two objectives: (a) have a good understanding of the interaction between admission control and inventory policies to ensure system performance; and (b) develop a simple and more effective solution for joint control policy development.

This research takes hybrid supply chain research to next level by developing a hybrid inventory-admission control policy, which is a necessary step to complement the strategy-based research.

Chapter 3 Joint Admission-Inventory Control Policy

Hybrid supply chains are complex by nature; operational decision policy development requires understanding of the system behavior, modeling and analysis. This chapter studies the characteristics of the two stochastic processes representing inventory position and WIP, as well as their interaction. The clear interaction between the stochastic processes, revealed in Markov property exploration, leads this research to develop an optimal joint control policy exhibiting the best match between the two control mechanisms—admission control and inventory policy.

3.1. Hybrid supply chain: introduction

This research studies a hybrid MTS-MTO supply chain with lost sales (Figure 3.1) which is managed using two control mechanisms—periodic (s, S) inventory policy and admission control. The planning horizon is divided into N periods, N is an integer, and operational decision is made at the beginning of each period. The customer demands follow a Poisson process with parameter λ . The MTO stage is modeled as a single server with finite capacity; the processing time of product customization follows Exponential distribution (μ) . In the MTS stage, it is assumed that infinite capacity is available to produce the semi-finished modules within a pre-set replenishment lead-time. This assumption prevents the MTS stage from becoming the system bottleneck.

The decoupling point, the boundary between MTS and MTO stages, is predefined in the hybrid supply chain. The semi-finished module inventory at the decoupling point is the central point connecting and synchronizing the two production stages. On one side, MTO production continuously pulls semi-finished modules from the stock according to

customer orders; on the other side, governed by the inventory policy, MTS production replenishes the semi-finished module stock periodically.

Figure 3.1: The joint admission-inventory control for the MTS-MTO supply chain

The first control mechanism, admission control, determines the cap, or maximum number of customer orders that can be admitted during a period. Once the given admission cap is reached in a period, all subsequent orders are rejected; the cap maybe different from period to period.

The admission control is also the link between market and the hybrid supply chain, through which the uncertainty from market is transferred into the system. By rejecting customer orders under certain circumstances, admission control can prevent the supply chain from getting congested over the long run.

The second control mechanism of periodic review (s, S) policy helps control the replenishment of the semi-finished module inventory: When the inventory position (IP) level drops below the reorder point (s) , it is brought to the up-to-level (S) by placing a

production order. Under general assumptions concerning the demand pattern and cost factors involved, the periodic review (s, S) system produces the lowest total of replenishment, carrying, and shortage costs (Silver, 1985). Periodic review inventory policy is also a common practice for coordination of group ordering and reduction of investment in information collection (Maddah et al., 2004; Chiang, 2007). One disadvantage of the periodic (s, S) inventory policy is that it requires a relatively high safety stock, and therefore may increase carrying cost.

The two control mechanisms interact and influence each other. With limited capacity in the hybrid system, admission control determines workload in MTO stage and maintains service level, based on Little's law. The workload consumes semi-finished module stock directly, triggering its replenishment as and when the reorder point(s) is breached. In return, the level of inventory influences the decisions of admission control: If there is a shortage of inventory, the deliveries of accepted orders may be delayed, and MTO production would slow down the acceptance of new customer orders. On the other hand, if the inventory is high, the system is likely to admit more orders, as long as the service level can be maintained, leading to reduction in inventory carrying cost.

It is imperative to have a joint control policy to synchronize and coordinate the two control mechanisms for consistent decision-making and better system performance. In this chapter a joint control policy $\pi = (s, S, a_t)$ is proposed for controlling the hybrid system performance, where s is the reorder point, S is the up-to-level, and a_t is the admission cap for t^{th} period. The joint control policy consists of a sequence of decisions made over N periods.

Classical admission control is a *continuous* procedure for managing congestions. Naor (1969) considered a *critical-number* policy for an M/M/1 steady-state queueing model and proposed that the system can admit new jobs if and only if $i < \max$, where i is the number of jobs already in the system and $\max$ is the critical-number. The critical-number policy has been shown to be optimal for much more general problems than Naor's (Stidham, 1985).

To avoid potential congestion in the hybrid supply chain, which is reviewed periodically, the joint control policy still requires a critical number that is the maximum number of orders (Max_WIP) that can be accommodated in a period. More specifically, it is equivalent to the admission cap plus the accumulated WIP brought from previous period(s). The determination or optimization of the critical number (Max_WIP) needs to take into account the constraint of semi-finished module inventory, in addition to the capacity of the server in M/M/1 queue.

Other assumptions made for the hybrid supply chain include:

- One set of common semi-finished modules caters for different finished products.
- A single pre-defined service level is applicable to all customer orders.
- The server in MTO stage can process one job at a time.
- Admitted orders queue up in front of MTO server as WIP in a FIFO manner; no semi-finished modules are assigned to WIP until processing begins.
- Once the MTO server becomes available, the first order waiting at the WIP queue is assigned a semi-finished module and released into MTO stage for production.

3.2. Policy Development

There are two key challenges in the development of the joint control policy: first is the “curse of dimensionality”. The search space of the joint control policy is $R^3 = (R_1 \times R_2 \times R_3)$, where R_1 is the total number of potential reorder points (s), R_2 is the number of potential up-to-levels (S), and R_3 is the number of potential order admission caps. Even for a relatively small production system with 10 production lines, with each line’s WIP limited to 20 jobs, R_3 alone can be as large as 20^{10} . Obviously, the model cannot be solved in polynomial time.

The second challenge is that the policy has to satisfy multiple objectives of meeting service level and minimizing system cost.

It is apparent that inventory and admission control policies have different characteristics—the inventory policy (s and S) is stationary over the N -period horizon whereas the admission cap varies from period to period. To simplify, we can eliminate the joint control policies where the values of s and S are not stationary over the planning horizon. This significantly reduces the search space but the second challenge still remains: the optimal policy should satisfy multiple objectives—optimizing cost and providing on-time delivery. In practice, due to the large solution space, it is not only difficult to find optimal policies but the size of $(R_1 \times R_2)$ also makes the comparison between joint control policies very difficult and intractable. Thus, the identification of the optimal joint control policy remains a challenge.

3.3. Stochastic processes and properties

In order to deal with the challenges in joint control policy development, as a first step, the stochastic processes of $\{IP_t\}$ and $\{WIP_t\}$ (the index $t \in [0, N)$ is integer) should be studied. The stochastic process $\{IP_t\}$ represents the semi-finished module inventory position at review epochs, which is controlled by the inventory policy directly; the stochastic process $\{WIP_t\}$ represents the WIP at review epochs.

The two processes reflect the dynamic nature of the hybrid supply chain, and the system model can be built once a good understanding of the basic characteristics of the processes, as well as their interaction is established.

Proposition 3.1. Stochastic process $\{WIP_t\}$ possesses Markov property.

Markov property is also known as memory-less property. A stochastic process has the Markov property if the conditional probability distribution of future states of the process depends only upon the present state, and not on the sequence of processing events. Markov property makes stochastic process study simple and practical.

Proof. First of all, the number of jobs in classical M/M/1 queue follows a stochastic process that possesses Markov Property. The random variable can be described as a birth-and-death process with birth rate λ and death rate $1/\mu$. The birth-and-death process is a special case of continuous-time Markov process (Hillier and Lieberman, 2005).

Further, the number of orders admitted during t^{th} period is

$$d_t = \min\{\lambda_t, a_t, g(IP_t)\}, \quad (3.1)$$

where λ_t is the demand during t^{th} review period, a_t is the admission cap according to the joint control policy at t^{th} review period, and IP_t is the inventory position at t^{th} review epoch. The function $g(IP_t)$ indicates the impact of IP which may reduce d_t if there is inventory shortage. The birth rate of the new process is $\lambda' = E(d_t)$ whereas the death rate remains as $1/\mu$ as shown in Figure 3.2.

Let $\{\overline{WIP}_t\}$ denote the continuous-time stochastic process with birth rate of λ' . Then the state space is a subset of the classical M/M/1 queue due to the potential rejection of orders or potential shortage of inventory, and the Markov Property remains for the continuous process of $\{\overline{WIP}_t\}$.

Figure 3.2: $\{\overline{WIP}_t\}$ is a continuous Markov process with order rejection

The values of discrete stochastic process $\{WIP_t\}$, observed at each review epoch by consolidating all state transitions of the previous period, correspond to a random sequence of $\{\overline{WIP}_t\}$ Markov process. Over the long run, $\{WIP_t\}$ will reach all states of the Markov process of $\{\overline{WIP}_t\}$, so the states in Figure 3.2 are irreducible for $\{WIP_t\}$, and the Markov Property stays valid. $\square$

The stochastic process $\{IP_t\}$ is governed by the periodic (s, S) inventory policy. (s, S) policy has been proven optimal for classical inventory problem, but the determination of the optimal s and S is a challenge since analytical solutions are considered almost

impossible (Scheneider and Ringuest, 1990; Maddah et al., 2004). Veinott & Wagner (1965) proposed an approximate solution and developed the optimal policy by taking into account holding cost and shortage cost for a procurement cycle and determining the expected non-convex cost function of a review period. The optimal (s, S) policy is identified via iteration, in which bounds for s and S are established with the sequence of $\underline{s} \leq s \leq \bar{s} \leq \underline{S} \leq S \leq \bar{S}$. Since then many efforts have been made to improve the bound-based approximation solution (Archibald and Silver, 1978; Federgruen and Zipkin, 1984; Zheng and Federgruen, 1991).

In hybrid supply chain control, it is assumed that inventory review period is the same as inventory replenishment lead-time. Normally, in periodic review systems, the replenishment lead-time is considered to be an integer multiple, n , of the review period. Thus for $n > 1$, more than one replenishment orders can be placed during one replenishment cycle, and quantity of each order could be smaller. With high frequency of replenishment and small order sizes, the system can maintain a relatively low level of inventory. However, by setting ‘ n ’ equal to 1, the system reaches upper bound of inventory level.

Definition 3.1. A standard review period is defined as the duration between

- the beginning of t^{th} period, after the completion of replenishment, if any, and
- the beginning of $(t+1)^{\text{th}}$ period, after the completion of replenishment, if any.

Proposition 3.2. The level of IP of the stochastic process $\{IP_t\}$ falls within the range $[s+1, S]$.

Proof. As per Definition 3.1 at a review epoch, all replenishment orders should have been filled with no outstanding orders. A new order cycle starts with $IP = S$, after a replenishment. One order cycle may cut across multiple review periods. At the beginning of an order cycle, IP is guaranteed to be equal to S , and during one cycle no inventory position can be visited more than once.

For Poisson demand, the “average number of periods with $IP = j$ during an order cycle”, denoted as m_j , can be written as (Axsäter, 2006),

$$m_j = \frac{1}{(1 - g_0)} \sum_{k=j+1}^S m_k g_{k-j}, \quad j = s + 1, s + 2, \dots, S - 1, \quad (3.2)$$

where g_j is the probability that the period demand during a review period is j , $j = 0, 1, \dots$.

By applying (3.2) recursively for $j = S-1, S-2, \dots, s+1$, at a review epoch, the distribution of IP can be described as

$$P(IP = k) = \frac{m_k}{\sum_{j=s+1}^S m_j}, \quad k = s + 1, s + 2, \dots, S. \quad (3.3)$$

In the hybrid supply chain, inventory consumption is determined by the processing rate of the MTO server, which follows Poisson process, so (3.3) holds for the joint control policy.

□

With Proposition 3.2, it is clear that at a review epoch, there are two possibilities: (a) $IP = S$, or (b) IP falls within $[s+1, S-1]$.

Proposition 3.3. Stochastic process $\{IP_t\}$ possesses Markov property.

Proof. The variable IP_t in (s, S) policy is not uniformly distributed, but the probability of the variable can always be derived recursively from the highest value of S , since the probability of reaching S is always equal to 1 in an order cycle (Axsäter, 2006).

In a hybrid supply chain, the inventory consumption during a period is determined by lower of the MTO service rate of $1/\mu$ and the number of admitted but not yet delivered orders ($WIP_t + d_t$). Let λ'' denote the inventory consumption rate, then

$$\lambda_t'' = \min\left\{(WIP_t + d_t), \frac{1}{\mu}\right\}. \quad (3.4)$$

Since $\{1/\mu\}$ is a Poisson process having Markov Property, WIP_t represents *current* WIP level, and d_t represents a *future* decision, λ'' possesses Markov Property.

Finally, since inventory replenishment lead-time is equivalent to a review period, at t^{th} review epoch, all replenishment orders placed in previous periods would have arrived. Thus the expected value of IP_{t+1} is determined by the size of replenishment order placed, if any, at t^{th} review epoch only. So,

$$IP_{t+1} = IP_t + Q_t - \lambda_t'', \quad (3.5)$$

$$Q_t = \begin{cases} S - IP_t, & \text{if } IP_t \leq s. \\ 0, & \text{otherwise.} \end{cases} \quad (3.6)$$

From (3.5) and (3.6) IP_{t+1} is determined by the current status i.e., IP_t and Q_t only. Thus $\{IP_t\}$ possesses Markov Property. $\square$

It is clear that the processes $\{WIP_t\}$ and $\{IP_t\}$ are interlinked, so, a joint control policy $\pi = (s, S, a_t)$ is required to determine the two variables $(IP, WIP)_t$ simultaneously, instead of

computing them separately. An *optimal* joint control policy may control the combined stochastic process $\{(IP, WIP)_t\}$ more effectively if it can account for the implicit interactions between the two processes. So the stochastic process $\{(IP, WIP)_t\}$ is of the research interest as it exhibits the interaction of the two stochastic processes, and helps to explore the codependency between the two control mechanisms (admission control and inventory policy).

Lemma 3.1. $\{(IP, WIP)_t\}$ possesses Markov property.

Proof. Based on (3.4), (3.5) and (3.6) the probability of IP_{t+1} depends on many parameters including WIP_t , but not WIP_{t+1} . So variables IP_t and WIP_t are independent, and

$$\Pr(IP_t) = \Pr(IP_t | WIP_t). \quad (3.7)$$

Therefore,

$$\Pr(IP_t \cap WIP_t) = \Pr(IP_t) \Pr(WIP_t), \quad (3.8)$$

$$\Pr(IP_t \cap WIP_t) = \Pr(IP_t, WIP_t) = \Pr((IP, WIP)_t). \quad (3.9)$$

From (3.8) and (3.9),

$$\Pr((IP, WIP)_t) = \Pr(IP_t) \Pr(WIP_t). \quad (3.10)$$

Since $\{WIP_t\}$ and $\{IP_t\}$ possess Markov Property according to Proposition 3.1 and 3.3, thus, based on Equation (3.10) stochastic process $\{(IP, WIP)_t\}$ inherits the Markov Property. $\square$

Looking at the stochastic process $\{(IP, WIP)_t\}$, at t^{th} review epoch, the joint control policy $\pi = (s, S, a_t)$ assesses the combined variable $(IP, WIP)_t$ and makes two decisions:

- (1) Place a production order for replenishing the semi-finished module inventory with the quantity $(S - IP_t)$, if $IP_t \leq s$, where IP_t is the inventory position, and
- (2) Specify the admission cap, a_t , for the period.

3.4. Performance measures

The performance of hybrid supply chains is measured by cost and service level; reasonable cost ensures participation in the market, while service level affects the survival of the business. Performance measures are also the key optimization criteria in the development of optimal joint control policy.

3.4.1. Service level

Traditionally, in pure MTS systems, type 1 service level (i.e., probability of no stock out per stock replenishment cycle) and type 2 service level (i.e., fill rate, or the fraction of demand that can be satisfied from stock) are common performance measures (Silver, 1985; Axsäter, 2006). In contrast, in pure MTO systems, service level is measured by delivery time, which is proposed based on the order's special requirement and current workload. The setting of due dates normally requires external negotiation and internal coordination during the quotation phase, and the lead time cannot be predicted accurately at the time it is proposed to the customer (Corti et al., 2006; Zorzini et al., 2008).

Because of the special characteristics of hybrid MTS-MTO supply chains, traditional measurements are not suitable for hybrid supply chains. First of all, type 1 and type 2 service levels for traditional MTS system have no meaning in hybrid MTS-MTO supply chains as no order can be fulfilled immediately (due to customization required) and

customers have to accept some delay in receiving the finished products. Secondly, the availability or stock-out of the semi-finished module is not the only factor influencing the hybrid supply chain service level. Besides the service measurement of traditional MTO system cannot be applied directly due to other market considerations.

The competition hybrid supply chains face is more market oriented in that the customer service level is determined by market forces (Li, 1992; Andries and Gelders, 1995; Olhager, 2003; Yang and Burns, 2003). Customer satisfaction becomes a kind of external constraint that cannot be compromised in the development of the joint control policy. By taking this into account, Gupta and Benjaafar (2004) proposed that service levels for hybrid supply chains be measured in either of the following two ways:

- The upper bound on the average order-fulfillment time, or
- The upper bound on the probability of not filling customer orders within the promised lead time.

This research adopts the second measure and defines customer service level of the hybrid supply chain using order On-Time Delivery (OTD) rate, denoted as β , within a promised lead-time. The lead-time denoted as LT —the time from receiving the customer order to the time the finished product is ready for delivery—is determined by market forces. Note that OTD is calculated for accepted orders only in the lost sales system.

3.4.2. Cost structure

Minimization of cost, consisting of inventory and service costs, is one of the main objectives of the optimal joint control policy. Inventory-related costs comprise of holding and setup costs. Holding cost is the cost of carrying inventory on-hand represented by

inventory level (denoted as IL) for a period. Due to the stochastic nature of inventory consumption, it is not easy to trace IL_t directly. Instead, it is a common practice to estimate inventory level using IP and lead-time demand.

With the assumption made that inventory review period is the same as inventory replenishment lead-time, IL (*without* lost sales) can be calculated as

$$IL_{t+1} = IP_t - D_t, \quad (3.11)$$

where D_t is the lead-time demand, $E(D_t) = \lambda$ and IP is defined as the sum of IL and outstanding replenishment orders, if any.

If the number of orders accepted during a period is denoted as d_t , then $d_t = \min \{a_t, D_t\}$, and IL *with* lost sales can be calculated as

$$IL_{t+1} = IP_t - d_t. \quad (3.12)$$

Setup cost is incurred when MTS stage is setup to produce a new lot of semi-finished modules. According to (s, S) policy, at t^{th} review epoch, if IP_t is equal to or lower than reorder point (s), a new replenishment order with quantity $(S - IP_t)$ is placed. Let N_t^s denote the number of setups required in t^{th} period, then

$$N_t^s = \begin{cases} 1, & IP_t \leq s. \\ 0, & IP_t > s. \end{cases} \quad (3.13)$$

Service-related costs comprise of lost sales cost i.e., order rejection cost, and late delivery cost. During t^{th} period, after having admitted d_t customer orders, the admission control will reject all subsequent orders. The number of rejected orders is $(D_t - d_t)$, so the expected number of rejected orders N_t^r can be expressed as

$$N_t^r = \sum_{D_t=d_t+1}^N (D_t - d_t) \Pr(D_t, \lambda), \quad (3.14)$$

where $\Pr(D_t, \lambda)$ is the probability of demand being D_t .

Late delivery cost is the penalty incurred when an accepted customer order is not delivered within the promised delivery lead-time. Assuming that at any given time there are $(k - 1)$ accepted orders in the system (i.e. average $WIP = k - 1$), the time a newly accepted order spends in the system follows Erlang distribution (Hiller and Lieberman, 2005) with probability density function (pdf)

$$f(t) = \frac{(\mu k)^k}{(k - 1)!} t^{k-1} e^{-k\mu t}, \quad \text{for } t \geq 0.$$

For a given LT (variable t) and β (the area under the distribution when $t \leq LT$), Erlang distribution can be used to determine the maximum WIP level (variable k). Once the WIP level exceeds the maximum WIP or if the number of total jobs in the system goes past k , on-time delivery would start to get affected.

Denoting the average WIP level as k' , total expected late delivery can be calculated as

$$\begin{aligned} N_t^l &= \sum_{k=1}^{k'} \Pr\{t, k, \mu | t > LT\} \\ &= \sum_{k=1}^{k'} (1 - \Pr\{t, k, \mu | t \leq LT\}) \\ &= \sum_{k=1}^{k'} \left(1 - \int_0^{LT} f(t, k, \mu) dt \right), \end{aligned} \quad (3.15)$$

where $f(t)$ is Gamma distribution since k' is not necessarily an integer.

Because of the generality of the cost structure discussed above, the total cost for t^{th} review period can then be written as

$$\begin{aligned} c_t &= c_t^h + c_t^s + c_t^r + c_t^l. \\ &= (u^h \times IL_{t+1}) + (u^s \times N_t^s) + (u^r \times N_t^r) + (u^l \times N_t^l), \end{aligned} \quad (3.16)$$

where

- u^h represents the holding cost per unit per period,
- u^s is the setup cost per batch,
- u^r is the rejection cost per unit, and
- u^l denotes the order delivery delay cost per unit.

Notice that backlog cost is not modeled in hybrid MTS-MTO supply chain even though it is commonly included in traditional inventory models. Since shortage of semi-finished modules will result in late delivery of the order, the relevant cost is accounted for in the cost of late delivery.

Proposition 3.4. The expected system cost is bounded for the stochastic process $\{(WIP, IP)_t\}$.

Proof. First of all, one period cost can be computed by adding inventory holding cost, MTS setup cost, customer order rejection cost, and late delivery cost incurred in that period.

- a) The highest inventory level $IP_t \leq S \leq \bar{S}$, so IP_t is bounded. The inventory holding quantity in (3.12) is bounded, and the corresponding one period holding cost is bounded.

- b) The maximum number of setups is 1 per period, so the one period setup cost is bounded.
- c) From (3.14) the number of rejected customer orders is bounded, so the corresponding one period rejection cost is bounded.
- d) From (3.15) the number of late deliveries is bounded, so, the corresponding one period late delivery cost is bounded.

Therefore, the one period total system cost is bounded. Clearly, the cumulative system cost for arbitrary number of ' N ' periods is also bounded. $\square$

3.5. Summary

This research employs two control mechanisms—admission and inventory control—for managing the performance of hybrid supply chains. With limited capacity in the hybrid system, the two control mechanisms interact and influence each other, and need to be integrated into one joint admission-inventory control policy.

However, the identification of optimal joint control policy faces two key challenges: the “curse of dimensionality” and multi-objective optimization. In order to deal with the challenges, as a first step, characteristics of the two stochastic processes representing IP and WIP, as well as their interaction, are studied in this chapter; it is proved that the stochastic processes possess Markov property, which makes system modeling simple and practical. The clear interaction between the stochastic processes, revealed in Markov property study, leads this research to develop an *optimal* joint control policy.

An optimal joint control policy can control the two stochastic processes $\{WIP_t\}$ and $\{IP_t\}$ more effectively, by taking advantage of their implicit interaction. Therefore, the

stochastic process $\{(IP, WIP)_t\}$ is of the research interest, as it exhibits the interaction between the two stochastic processes, and helps to explore the codependency between the two control mechanisms in a structured way, a topic addressed in the next chapter.

New customer satisfaction requirements and the cost structure, critical optimization criteria important in system modeling and in identifying the optimal joint control policy, are also discussed and defined in this chapter.

Chapter 4 Generic Structure of Joint Control Policy

In the previous chapter, the interaction between admission control and inventory policies for the hybrid supply chain is discussed; it becomes imperative that the two control mechanisms be integrated as one joint admission-inventory control policy to control the two stochastic processes $\{WIP_t\}$ and $\{IP_t\}$ at the same time by reflecting on their implicit interaction. Over the long run, the status of stochastic process $\{(IP, WIP)_t\}$ is completely determined by the transition probabilities—the likelihood of transitioning between various states, regardless of where the process starts or what happens along the way—as it possesses the Markov property. Applications of different joint control policies, however, would result in different sets of transition probabilities.

Discrete Markov Decision Process (MDP) is employed in this chapter for system modeling, and the optimality of the joint control policy, developed by Bellman equation, is discussed, which exhibits the codependency, i.e., the best match, between admission control and inventory policies.

The codependency is examined further through experimental analysis; a generic structure of joint control policy is formed and the structural properties are discussed. The identification of the generic structure leads to a radical new approach in the development of joint control policy.

4.1. System modeling

The stochastic process $\{(IP, WIP)_t\}$ is modeled using MDP, a sequential decision model. A finite time horizon is considered and the inventory is reviewed every week. By making

decisions at each review epoch, the joint control policy affects the transition probabilities, and further influences the evolution of the stochastic process.

MDP model is a 4-tuple $\{U, A, P^x, C^x\}$, where:

- U represents the state space;
- A represents the decision space;
- P^x is a one-step transition probability matrix for a decision $a^x \in A$, indicating the probabilities of transiting from a state $u^i \in U$ to the next one, by taking the decision a^x ; and
- C^x is a one-step cost matrix indicating the costs associated with the corresponding transition probability P^x .

Let define a real number space U with size M ; over U is a non-empty set of objects representing the combinations of IP and WIP such that

$$u^i = (IP, WIP)^i, \quad i \in 1, \dots, M \quad (4.1)$$

The state space is finite, or M is bounded since: (a) for given (s, S) inventory policy, $s+1 \leq IP \leq S$, and (b) for given capacity and service level, according to Little's law (Hopp and Spearman, 2000), the level of WIP is finite, i.e., $0 \leq WIP \leq Max_WIP$. The state space is organized as a fixed state matrix of

$$U = [u^1, \dots, u^i, \dots, u^M]^T = \begin{bmatrix} u^1 \\ \dots \\ u^i \\ \dots \\ u^M \end{bmatrix}.$$

At t^{th} review epoch, the status of the system, u_t , could take any of the values in the state space U , i.e., $u_t = u^i$, where i is the index in the state space.

Property 4.1. At a review epoch, the state space U can be divided into two groups:

- i. Setup (inventory replenishment) is expected i.e., expected $IP = S$; or
- ii. No setup is expected, and the expected IPs falls in the range $[s + 1, S - 1]$.

Proof. Based on Proposition 3.2, at arbitrary t^{th} review epoch IP falls into $[s+1, S]$ for all states, i.e., $IP_t \in [s+1, S]$.

The expected inventory consumption rate per period (from equation 3.4) is

$$\lambda_t'' = \min \left\{ (WIP_t + d_t), \frac{1}{\mu} \right\},$$

where $(WIP_t + d_t)$ is the total number of orders accommodated during t^{th} period. Let $IP_t = IP^i$. Then at $(t+1)^{\text{th}}$ review epoch, $IP_{t+1} = IP_t - \lambda_t'' = IP^i - \lambda_t''$. At this point, there are two possibilities: if $IP_{t+1} \leq s$, then it is brought up to S immediately according to inventory policy, and $IP_{t+1} = S$; otherwise IP_{t+1} retains its value which falls within $[s + 1, S - 1]$. $\square$

Since there is no restriction placed on the class of allowable policies, there can be many ways of deriving the joint control policy. This chapter focuses on an important subclass of policies, namely the stationary policies. A policy is said to be stationary if it is non-randomized and the decision it chooses at time t (e.g., at t^{th} review epoch) only depends on the state of the process at time t (Ross 1983).

The decision space is denoted as $A = \{(s, S, a_t)\}$ where s is the reorder point, S is the up-to-level, and a_t is the admission cap during t^{th} period. Obviously the lowest value for the number admitted is zero. So, $a_t \in \{0, 1, 2, \dots, K\}$, where K is an integer indicating the

largest number of customer orders that can be admitted during a review period; $K \leq \text{Max_WIP}$, the critical number of the joint control policy. For stationary policies, the decisions made are dependent on states only, not the time periods, so, $A = \{(s, S, a_t)\} = \{(s, S, a^x)\}$, where $a^x \in \{0, 1, 2, \dots, K\}$.

In the MDP model, one-step transition matrices are defined for individual decisions in the decision space, so there are total of $(K+1)$ transition matrices. For a particular decision a^x , the probability that the system transits from state u^i to u^j forms one-step $(M \times M)$ transition matrix as in (4.2), where $i, j = 1, 2, \dots, M$, and M is the size of state space.

$$P^x = |p^{i,j}| = \begin{bmatrix} p^{1,1} & p^{1,2} & \dots & p^{1,M} \\ p^{2,1} & p^{2,2} & \dots & p^{2,M} \\ \dots & \dots & \dots & \dots \\ p^{M,1} & p^{M,2} & \dots & p^{M,M} \end{bmatrix}. \quad (4.2)$$

For any state u^i , the corresponding i^{th} row in the matrix has

$$\sum_{j=1}^M p^{i,j} = 1, \quad \text{for } j = 1, 2, \dots, M. \quad (4.3)$$

After taking the decision a^x at t^{th} review epoch, the system state transits from state $u_t = u^i$ to $u_{t+1} = u^j$ according to the one-step probability $p^{i,j}$ at the estimated cost of $c_t = c^{i,j}$, which is the total cost defined in (3.16) for the period.

Each decision a^x has its own one-step cost matrix

$$C^x = |c^{i,j}| = \begin{bmatrix} c^{1,1} & c^{1,2} & \dots & c^{1,M} \\ c^{2,1} & c^{2,2} & \dots & c^{2,M} \\ \dots & \dots & \dots & \dots \\ c^{M,1} & c^{M,2} & \dots & c^{M,M} \end{bmatrix}. \quad (4.4)$$

By solving the MDP model, subject to pre-defined on-time delivery, there are two promise outcomes:

- An optimal joint control policy π^* , and
- The expected total system cost following the optimal joint control policy π^* .

4.2. Identification of codependency

The codependency can be identified through the analysis of optimal joint control policies. The discussion of the optimization procedure for exploring the codependency is not to identify the optimal joint control policy, but to prove the uniqueness and optimality of the codependence for a specific state space.

4.2.1. Optimization function

Joint control policy is a sequence of decisions made over multiple periods by monitoring the status of the stochastic process $\{(IP, WIP)_t\}$; it is a mapping between the decision space A and the state space U of the stochastic process.

For the state space that results from an inventory policy in operation, the codependency can be found by identifying the optimal admission control that minimizes the expected total system cost, while satisfying the service level. The policy can be identified by solving the following optimization problem

$$\min_{\pi \in \Pi} E \left\{ \sum_{t=0}^N \gamma^x c_t^\pi \right\} \text{ s.t. service level,} \quad (4.5)$$

where γ is the discount factor, which is very close to 1, and Π is a set of decision policies, c_t^π is one period system cost following a joint control policy π .

One approach to solving the optimization problem is to create an optimal substructure, divide the problem into smaller sub-problems, and solve the sub-problems recursively using the same algorithm. Bellman (1957) developed Bellman Equation (also known as dynamic programming equation) for this purpose and specified the Principle of Optimality, which is that the Bellman equation is a necessary condition for the optimality of the original problem. The standard Bellman Equation (Powell, 2007) is

$$V_t(u_t) = \min_{a^x \in A} \left(C_t(u_t, a^x) + \gamma \sum_{u^j \in U} \Pr(u_{t+1} = u^j | u_t = u^i, a^x) V_{t+1}(u^j) \right). \quad (4.6)$$

In the backward algorithm, column vector V_{t+1} represents the expected minimum system cost from $(t+1)^{\text{th}}$ period onwards; column vector C_t is the expected one-step cost; $Pr(\cdot)$ is a transition matrix which gives the probability that a state transitions from u_t to u_{t+1} , where $u_t = u^i$, $u_{t+1} = u^j$, $i, j = 1, 2, \dots, M$, when action a^x is chosen. So, (4.6) is equivalent to

$$V_t = \begin{bmatrix} v_t(u^1) \\ \vdots \\ v_t(u^i) \\ \vdots \\ v_t(u^M) \end{bmatrix} = \min_{a^x \in \pi} \left(\begin{bmatrix} c_t^\pi(u^1) \\ \vdots \\ c_t^\pi(u^i) \\ \vdots \\ c_t^\pi(u^M) \end{bmatrix} + \gamma \begin{bmatrix} \ddots & & & \\ & \Pr(u^j | u^i) & & \\ & & \ddots & \\ & & & \ddots \end{bmatrix} \begin{bmatrix} \vdots \\ \vdots \\ v_{t+1}(u^j) \\ \vdots \\ \vdots \end{bmatrix} \right).$$

The expected cost vector V_{t+1} is *known* since it can always be calculated by backward recursion from any initial cost array. Based on Bellman's Principle of Optimality, regardless of the initial value, the remaining decisions must constitute an optimal policy. The principle is presented next.

Definition 4.1. Let

$$V(u^i) = \inf_{\pi} V_{\pi}(u^i) \quad (4.7)$$

A policy π^* is said to be γ -optimal if $V^{\pi^*}(U) = V(U)$, for any initial state.

Using a simple backward recursion, at $(t+1)^{\text{th}}$ review epoch, if the expected minimum cost (representing the total costs for all states from $(t+1)^{\text{th}}$ period onwards), denoted as column vector V_{t+1} , is known, and the smallest one-step cost C_t (a column vector representing the one period cost for all states moving from time t to $(t+1)$) can be obtained by evaluating each possible decision $a^x \in A$, then the expected minimum system cost V_t at t^{th} review epoch is simply V_{t+1} plus C_t . The same process can be carried out recursively to compute the expected minimum cost vector in any period.

4.2.2. The optimality of codependency

At this stage, the original N -period optimization problem can be replaced by the Bellman equation (which minimizes individual period cost over N periods) provided the original optimization problem can be converted to the Bellman Equation. So, it needs to prove the equivalence between the original objective function and its Bellman equation version.

Let column vector $F_t^*(u^i)$ represent the minimum expected total system cost from t^{th} period onwards. The original optimization problem in (4.5) can be rewritten as

$$F_t^*(u^i) = \min_{\pi \in \Pi} E \left\{ \sum_{t'=t}^N \gamma^{t'} C_{t'}^{\pi}(u^i, a^x | \pi) \right\}. \quad (4.8)$$

Proposition 4.1. For an arbitrary policy π ,

$$F_t^{\pi}(u^i) = V_t^{\pi}(u^i) \quad (4.9)$$

The Proof can be found in Appendix 4.1.

Theorem 4.1. The expected minimum cost from time t onward for the original problem (4.8) and the Bellman Equation (4.9) are equal.

$$F_t^*(u^i) = V_t(u^i) \quad (4.10)$$

The Proof can be found in Appendix 4.2.

From Theorem 4.1 the expected minimum cost for t^{th} period and beyond can be derived using Bellman Equation. Next it is proved that the solution derived is unique and optimal, and the admission control policy best matches with the given inventory policy; in other words, the codependency between the admission control and inventory policies is uniquely optimal.

Theorem 4.2. For bounded system cost, the value function of Bellman Equation converges to an optimal solution.

The Proof is provided in Appendix 4.3.

Proposition 4.2. The bounded cost solution of Bellman Equation $V(u^i)$ is the unique optimal solution.

The Proof is provided in Appendix 4.4.

Proposition 4.2 suggests that the solution derived using Bellman Equation is a uniquely optimal solution of the original optimization problem. In summary, for a specific state space, there is a unique joint control policy that exhibits the optimal codependency between inventory and admission control policies.

4.2.3. Discounted value-iteration algorithm

For solving the Bellman Equation in experiments, the classical dynamic programming algorithm, discounted value-iteration algorithm (in Table 4.1) is developed using MATLAB.

Table 4.1: Discounted value iteration algorithm

Step 1. initialization:

set cumulative cost vector $V_0(u^i) = 0$,

set tolerance parameter $\epsilon = 0.00001$ and discount $\gamma = 0.9999$.

Step 2. For $u^i \in U, i = 1, 2, \dots, M$, compute the system cost:

$$V_{t+1}(u^j) = \min_{a^x \in A} \left(C_{t+1}(u^j, a^x) + \gamma \sum_{u^j \in U} Pr(u_{t+1} = u^j | u_t = u^i, a^x) V_t(u^i) \right)$$

C_{t+1} is the one-step cost vector when state is $u_t = u^i$ and decision a^x is chose, where $a^x \in A, x = 0, 1, 2, \dots, K$; $Pr(\cdot)$ is the one-step transition probability when the system transits from state u^i to next state of $u^j, j = 1, \dots, M$, when a^x is chosen.

Step 3. If $\|V_{t+1}(u^j) - V_t(u^j)\| < \epsilon(1-\gamma)/2\gamma$, or $t \geq 9000$, stop;

else

update $\|V(u^i)\| = \min_{i=j} \|V(u^j)\|$,

set $t=t+1$ go to step 2.

The backward reversion algorithm starts with initial value $V_0 = 0$. Based on the Principle of Optimality, different initial values would not change the optimal policy, but they may

affect the speed of convergence. During the first period, all decisions in the decision space are evaluated for all states; the best decisions and the corresponding costs incurred by applying the decisions are recorded.

For subsequent periods (iterations) following evaluation procedure is repeated: if a smaller cost is found for a particular state, then its cost and corresponding decision are updated. Instead of decreasing from N to 0, the time subscript t in the algorithm actually starts from 0 and increases until the stopping criteria of $\epsilon(1-\gamma)/2\gamma$ is met or maximum of 9000 iterations is reached. The final mapping between states and decisions recorded is the optimal policy.

4.3. Experiment design of the MDP model

Experiment is designed for the benchmark scenario to identify the generic structure of the joint control policy, in which the arrival rate (Poisson) λ is set to 8, service time (Exponential) μ is set to 1/6, and the target delivery lead-time is set to 2 periods. With these parameters, the demand overwhelms the capacity but the admission control ensures the systems stays stable, over the long run, by rejecting customer orders as and when necessary.

The cost structure is determined by business environment or the market, and specifies the operational conditions under which the optimal joint control policies operate. The cost structure of the benchmark scenario is defined as: holding cost per unit per period u^h is \$2, setup cost per batch u^s is \$25, late delivery cost per unit u^l is \$3, and rejection cost per unit u^r is \$1.

To create the state space of the experiment, inventory policy needs to be generated first to respond to the cost structure. Compared to bound-based solutions in inventory management literature, Power Approximation developed by Ehrhardt (1997) is the most direct way to identify an approximate solution to the optimal (s, S) policy for a periodic system. The heuristic only requires knowledge of the mean (μ) and variance of demand (σ^2) , provided replenishment lead time (L) , setup cost of replenishment (K) , unit penalty cost (p) and unit holding cost (h) are fixed. Ehrhardt and Mosier (1984) revised the Power Approximation formulae because of convergence difficulties in the original formulae for some problems with same concepts and parameter setting as in the original formulae.

In the revised version, S_0 is denoted as

$$S_0 = (L + 1)\mu + v\sigma\sqrt{L + 1},$$

where $v = \frac{p}{p+h}$. Let $\mu_L = (L + 1)\mu$ and $\sigma_L = \sigma\sqrt{L + 1}$. Compute

$$D_p = 1.30\mu^{0.494} \left(\frac{K}{h}\right)^{0.506} \left(1 + \frac{\sigma_L^2}{\mu^2}\right)^{0.116},$$

$$z = [D_p/(\sigma_L p/h)]^{1/2},$$

$$s_p = 0.973\mu_L + \sigma_L \left(\frac{0.183}{z} + 1.063 - 2.192z\right).$$

If D_p/μ is greater than 1.5, let $s=s_p$ and $S=s_p+D_p$. Otherwise, let $s = \min\{s_p, S_0\}$ and $S = \min(s_p+D_p, S_0)$. S_p , D_p and S_0 are rounded to the nearest integer.

Note that the cost structure of joint control policy does not include unit stock-out penalty or backlog cost; instead, the relevant cost is accounted for by the cost of late delivery. So,

in the experiment, the parameter set to \$2 by assumption. Based on the Power Approximation algorithm, the inventory policy is calculated as $(s, S) = (6, 18)$.

The four elements of MDP model are defined in following sub-sections for the benchmark scenario.

4.3.1. State space

According to Proposition 3.2 the stochastic variable IP falls in the range $[s+1, S]$, so with the inventory policy $(6, 18)$, the variable IP is confined to the set of countable values $\{IP\} = \{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18\}$.

Assuming Erlang distribution, the WIP level and its corresponding expected on-time delivery rate β is shown in Table 4.2. When WIP level is smaller than or equal to 8, $\beta > 90\%$; when WIP level = 10, β declines to 75.76%. In order to make the order acceptance decision more flexible, the maximum level of WIP is set at 10. Correspondingly, the set containing the stochastic variable WIP has the following countable values $\{WIP\} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Table 4.2: WIP level and the corresponding late delivery probability

WIP	5	6	7	8	9	10	11	12	13
β	0.9924	0.9797	0.9542	<u>0.9105</u>	0.8450	<u>0.7576</u>	0.6528	0.5384	0.4240

Based on the ranges of IP and WIP, state space $U = \{(IP, WIP)\}$ is formed with a total of 132 members as shown in Table 4.3; each entry of the matrix represents a state.

4.3.2. Decision space

Decision space, A is represented as $\{(s, S, a^x)\}$, where $a^x \in \{0, 1, 2, \dots, K\}$. Since maximum WIP (Max_WIP) is set at 10, K , the maximum of admission cap is set to 10 also. Thus, the decision space $A = \{(6, 18, a^x)\}$, where $a^x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Table 4.3: The state space of the stochastic process $\{(IP, WIP)^i\}$ in benchmark scenario

IP	WIP										
	0	1	2	3	4	5	6	7	8	9	10
18	(18,0)	(18,1)	(18,2)	(18,3)	(18,4)	(18,5)	(18,6)	(18,7)	(18,8)	(18,9)	(18,10)
17	(17,0)	(17,1)	(17,2)	(17,3)	(17,4)	(17,5)	(17,6)	(17,7)	(17,8)	(17,9)	(17,10)
16	(16,0)	(16,1)	(16,2)	(16,3)	(16,4)	(16,5)	(16,6)	(16,7)	(16,8)	(16,9)	(16,10)
15	(15,0)	(15,1)	(15,2)	(15,3)	(15,4)	(15,5)	(15,6)	(15,7)	(15,8)	(15,9)	(15,10)
14	(14,0)	(14,1)	(14,2)	(14,3)	(14,4)	(14,5)	(14,6)	(14,7)	(14,8)	(14,9)	(14,10)
13	(13,0)	(13,1)	(13,2)	(13,3)	(13,4)	(13,5)	(13,6)	(13,7)	(13,8)	(13,9)	(13,10)
12	(12,0)	(12,1)	(12,2)	(12,3)	(12,4)	(12,5)	(12,6)	(12,7)	(12,8)	(12,9)	(12,10)
11	(11,0)	(11,1)	(11,2)	(11,3)	(11,4)	(11,5)	(11,6)	(11,7)	(11,8)	(11,9)	(11,10)
10	(10,0)	(10,1)	(10,2)	(10,3)	(10,4)	(10,5)	(10,6)	(10,7)	(10,8)	(10,9)	(10,10)
9	(9,0)	(9,1)	(9,2)	(9,3)	(9,4)	(9,5)	(9,6)	(9,7)	(9,8)	(9,9)	(9,10)
8	(8,0)	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)	(8,6)	(8,7)	(8,8)	(8,9)	(8,10)
7	(7,0)	(7,1)	(7,2)	(7,3)	(7,4)	(7,5)	(7,6)	(7,7)	(7,8)	(7,9)	(7,10)

4.3.3. One-step transition matrices

A one-step transition matrix specifies the probability that a decision a^x will lead the system from a state at time t to the next one at time $(t+1)$. Each decision has a one-step transition matrix, so there are a total of 11 one-step transition matrices.

Since the states before and after a transition are *known*, a transition probability is basically the probability of going from one known state to another known state as a result

of the decision. Of course, some of the transitions may not be feasible. A transition matrix in this experiment has the size of 132 x 132.

For the case of admission cap is 5, for example, the system may admit 0, 1, 2, 3, 4, or 5 orders. For each of these cases there is a probability matrix that specifies the transition probabilities. So, the one-step transition matrix for “admission cap = 5” is the sum of all the probability matrices. Table 4.4 shows a portion of the probability matrix, the corresponding inventory levels and setups incurred for the case of 5 received orders.

The table specifies the probabilities from states $u_t = u^i \in \{(18,0), (18,1), (18,2), \dots, (18,10)\}$ to states $u_{t+1} = u^j \in \{(18,0), (18,1), (18,2), \dots, (18,10)\}$ when five orders are admitted. When a transition is infeasible, the corresponding probability is marked with “x”. For example, it is not possible to transit from source state (18, 6) to target (18, 7) when five orders are admitted. For feasible transitions the probabilities of completion of jobs required for the transition are specified. For example when the system transitions from state (18, 8) to (18, 0), it needs to complete 13 jobs; thus transition probability = $Pr(\text{completed job} = 13 | \mu = 1/6)$.

4.3.4. One-step cost matrices

Similar to transition matrices, there are a total of 11 expected cost matrices with the size of 132x132, representing the corresponding estimated one-step total incurred cost. Each cost matrix comprises of four sub-matrices that specify inventory holding cost, setup cost, late delivery cost, and order rejection cost.

For the first three cost elements, i.e., holding cost, setup cost, and late delivery cost, the matrices for a decision $a^x \in A$ are obtained by adding the cost matrices for “received

orders = n ” where $n = 0, 1, \dots, a^x$, similar to the development of the transition matrix for the decision a^x .

The expected holding cost is based on the *inventory level* (stock on hand) at *next* review epoch, and expected inventory level is only available for those elements whose counterparts in the transition matrix have non-zero probability of transition. For example (Table 4.4) when the system state transitions from (18, 8) to (18, 0), the transition probability is “ex(13)”, and the expected inventory level equals to 6 ($= 18 + 1 - 13$, where 18 is the IP after replenishment, 1 is the number of jobs being processed in MTO server, and 13 is the expected number of completed jobs).

Setup cost is incurred only when $IP_{t+1} = S$ after a replenishment order is placed. Table 4.4 shows an example for “received orders = 5”, where nine states would incur setup cost.

In order to avoid double counting, the expected late delivery cost is only considered for newly admitted orders. If a newly admitted order cannot be completed by next review period, it incurs late delivery cost. For example, assume that the system starts with $WIP_t = 8$ and accepts five orders during a period. Since service rate is $1/\mu = 6$ jobs per period and jobs are produced in FIFS manner, it is expected that the last admitted order would likely not complete by the end of $(t+1)^{\text{th}}$ period (i.e., at the $(t+2)^{\text{th}}$ review epoch) and therefore incur late delivery cost.

Unlike other cost matrices, the expected rejection cost solely depends on the decision a^x made at a review epoch. The number of rejected orders is a single number which needs to be exploded into a 132x132 matrix so that it can be added to other matrices.

Table 4.4: Example of transition matrix, and corresponding holding inventory matrix and setup matrix for "received jobs = 5"

received=5		IP(t+1)=18												Expected IL(t+2)										Setup incurred										
	WIP	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10	0	1	2	3	4	5	6	7	8	9	10
IP(t)=18	0	x	x	x	x	x	x	x	x	x	x	x																						
	1	x	x	x	x	x	x	ex(0)	x	x	x	x					18																	
	2	x	x	x	x	x	x	x	ex(0)	x	x	x					18																	
	3	x	x	x	x	x	x	x	x	ex(0)	x	x									18													
	4	x	x	x	x	x	x	x	x	x	ex(0)	x									18													
	5	x	x	x	x	x	x	x	x	x	x	x	ex(0)									18												
	6	x	x	x	x	x	x	x	x	x	x	x	x																					
	7	x	x	x	x	x	x	x	x	x	x	x	x																					
	8	ex(13)	ex(12)	x	x	x	x	x	x	x	x	x	x	6	6									1	1									
	9	ex(14)	ex(13)	ex(12)	x	x	x	x	x	x	x	x	x	5	5	6								1	1	1								
10	ex(15)	ex(14)	ex(13)	ex(12)	x	x	x	x	x	x	x	x	4	4	5	6							1	1	1	1								

(x indicates infeasible transitions)

Finally, the four cost elements are summed up for a decision $a^x \in A$, thus constituting a one-step cost matrix.

4.4. Experimental scenarios

The cost factors of u^s , u^h , u^r and u^l in the cost structure are given as constants and pre-set for each of the scenarios. A total of nine scenarios (Table 4.5) are designed for experiments. The first scenario described in the previous section is the benchmark scenario. Scenarios 2 to 9 are used for sensitivity analysis to validate the generic structure. Compared to the benchmark scenario, the other scenarios differ from each other in only one parameter setting.

Table 4.5: Experimental scenarios

Scenario No	Inventory policy (s, S)	Setup cost / batch (u^s)	Holding cost / unit / period (u^h)	Late delivery cost / unit (u^l)	Rejection cost / unit (u^r)
*1	(6, 18)	25	2	3	1
2	(6, 18)	20	2	3	1
3	(6, 18)	15	2	3	1
4	(6, 18)	25	1	3	1
5	(6, 18)	25	3	3	1
6	(6, 18)	25	2	1	1
7	(6, 18)	25	2	5	1
8	(6, 18)	25	2	3	3
9	(6, 18)	25	2	3	5

(*: the benchmark scenario)

4.5. Numerical results

The discounted value iteration algorithm (Table 4.1) runs on a Intel® Quad 2.66 GHz PC running Windows 7 professional ® 32-bit operating system. The benchmark scenario

problem including 132 iterations takes less than 5 seconds to solve. The highest number of iterations (149) are required by scenario 4 and the lowest (82) by scenario 9.

4.5.1. Identification of the generic structure

The first joint control policy obtained for the benchmark scenario is shown in Figure 4.1. In the matrix, each cell represents an admission cap for the corresponding state u_t represented by an IP (row) and a WIP (column), where $6 (=s) < IP \leq 18 (=S)$, and $0 \leq WIP \leq 10$. For example, the state (17, 1) represents $IP_t = 17$ (row 2) and $WIP_t = 1$ (column 2), and corresponding decision in the joint control policy is $(s, S, a^x) = (6, 18, 9)$. There are total of 132 decisions that make up the joint control policy, mapping to 132 system states.

In Figure 4.1, there is a “setup-line” which divides the state group into two parts. Along the line, many states accept zero admissions for the period. For example for state (11, 5), there are five jobs as WIP, one is being processed in the MTO server and the other four are waiting to be processed. By accepting zero admissions and finishing processing of the four waiting jobs, the expected IP at next epoch would be seven ($11 - 4 = 7$), which is just higher than the reorder point of six.

So, by accepting no admission costly replenishment can be avoided. States above the setup-line (inclusive of those on the line) do not require setup, whereas states below the line—group $n1$ —require setups. This result reflects Property 4.1.

Figure 4.1: Division of state space by the “setup-line”

According to Property 4.1, there are multiple expected IP_{t+1} for states above the setup-line. In the benchmark scenario, states above the setup-line can be further divided into 3 groups (Figure 4.2)— $n2$, $n3$ and $n4$. The expected IP_{t+1} can be derived by

$$IP_{t+1} = IP_t - (WIP_t - Q) - a_t, \quad (4.11)$$

$$Q = \begin{cases} 0, & WIP_t = 0 \\ 1, & \text{otherwise} \end{cases}$$

where a_t represents the best decision for state $u_t = (IP, WIP)_t$, which can be found in the policy matrix in Figure 4.1. Since one job is running in MTO server, the requirement for inventory is $(WIP_t - 1)$ in (4.11). This observation leads to Property 4.2.

Property 4.2. For groups (of states) requiring no setup, the representative IP starts from $s+1$, and is increased by one for a new group (of states).

However the sizes of state groups may not be equal. For the result depicted in Figure 4.2 group $n2$ has 43 members (51.8% of total 83), and group $n3$ has 32 members (38.6%).

Groups $n2$ and $n3$ constitute the majority (more than 90% of total 83 members) of states not requiring a setup.

Figure 4.2: Division of no-setup state space into three groups by IP_{t+1} .

For states in group $n1$ the total number of jobs (Max_WIP) that the system can accommodate during a review period can be computed by adding the admission cap, a_t and WIP_t . The values shown in Figure 4.3 are distributed from 9 to 12. For a stable system, the critical number of Max_WIP should converge to a constant depending on the target delivery lead-time. By normalizing the quantity, a new parameter for the generic structure is identified. This observation leads to Property 4.3.

Property 4.3. For group $n1$ requiring setup, the maximum number of jobs (Max_WIP) that the hybrid system can accommodate for a review period can be normalized and represented by one figure.

Figures 4.1 to 4.3 demonstrate the generic structure of the joint control policy.

 Figure 4.3: Group $n1$ of the generic structure

4.5.2. Sensitivity analysis

The experimental results (Appendices 4.5) show that the generic structure holds for all the nine scenarios. In general, the state space can be divided into a maximum of four groups, with the first three groups $n1$, $n2$, and $n3$ occupying more than 90% of the total states. So the generic structure can be further simplified by merging group $n4$ with $n3$; this change results in more orders being admitted when IP level is higher.

The sensitivity analysis is conducted based on the simplified generic structure, with the following three groups:

- Group 1 ($n1$) consists of states that require setup.
- Group 2 ($n2$) consists of states that do not require setup and expected $IP_{t+1} = s+1$.
- Group 3 ($n3$) consists of states that do not require setup and expected $IP_{t+1} \geq s+2$.

The impact of cost factors on the state grouping is complicated; the detailed changes in group sizes are shown in Figure 4.4. The impact of setup cost per batch (u^s) on group size is minimal. When u^s increases, the size of group $n1$ gradually reduces from 53 to 49,

while group $n2$ reduces from 48 to 43. In contrast, the size of group $n3$, which keeps higher inventory, increases. This shows that as the setup cost increases, the joint control policy, driven by the codependency, becomes more conservative by avoiding setup for larger number of states with higher remaining inventory.

Figure 4.4: The state membership in group $n1$, $n2$ and $n3$ (for different cost factors)

For increasing in holding cost per unit (u^h), a key observation is that the membership in group $n3$ decreases considerably from 50 to 29; in contrast, the size of group $n2$ increases from 32 to 56. When the holding cost is high the joint control policy results in more states with lower inventory. In addition the size of group $n1$ is relatively stable implying that the holding cost has minimum impact on the setup for inventory replenishment.

When late delivery cost per unit (u^l) increases, the size of group $n3$ increases from 36 to 43, confirming that the joint control policy responds to the codependency by keeping

higher inventory to ensure there is sufficient inventory, thereby avoiding late delivery penalty.

The impact of rejection cost per unit (u') is also significant. The number of states requiring setup is 49 when rejection cost is \$1 but increases to 102 (or 77.3% of total 132 states) when the rejection cost increases to \$5. The joint control policy responds to the codependency by ensuring that enough semi-finished modules are available and avoid rejection in the following period. This could be attributed to the fact that when the capacity (μ) cannot cope with the demand (λ), the joint control policy tries to minimize the number of rejections for high rejection cost scenario.

In summary, the generic structure holds under different operational conditions. Besides the rejection cost, the changes in other three cost factors have relatively small impact on the size of group $n1$, but have considerable impact on $n2$ and $n3$ groups. With respect to the codependency existing between inventory policy and admission control, the joint control policy manages to adjust members of state space to minimize the total system cost.

4.6. Summary

In this chapter the stochastic process $\{(IP, WIP)_t\}$ is modeled using discrete MDP; after proving their equivalence, the multiple period optimization problem (for development of optimal joint control policy) is converted into a Bellman Equation. It is proved that the joint control policy developed by Bellman equation is uniquely optimal for a specific state space, which exhibits the codependency, i.e., the best match, between the two control mechanisms (admission control and inventory policy).

Through analysis of experimental results, a generic structure of joint control policy is obtained and the structural properties of the policy are observed for reflecting the codependency.

The structural properties hold for different operational conditions of the generic structure. Two key parameters of the generic structure are the reorder point (s) and order up-to-level (S) which define the state space; the third parameter Max_WIP is the maximum jobs that the system can accommodate in a review period.

Once the generic structure has been identified, the next important step is to formulate the optimization problem and determine the parameters of the generic structure. There are two key issues in the development of the generic structure. First is the determination of the number of groups that do not require setup. Based on the experiments, dividing the states into two groups, n_2 and n_3 is a good start. For complex set of conditions, further experiments may be required to sort out the grouping. Second issue is the determination of the number of states each group should have.

The development of the generic structure of joint admission-inventory control policies is a major contribution of this research; this essentially converts the optimization problem of finding the optimal joint control policy into one that identifies the optimal parameters of the generic structure. The latter is a typical combinatorial optimization problem with a limited search space; more importantly, there are many mature optimization techniques available for solving combinatorial optimization problems. By effectively overcoming the “curse of dimensionality”, this new approach significantly simplifies the development of the joint control policy.

In the next chapter, the optimal inputs of the generic structure are identified by Response Surface Methodology and a new joint control policy is developed. The effectiveness of the generic structure is demonstrated and evaluated through policy performance benchmarking.

Appendix 4.1

Proposition 4.1. For an arbitrary policy π ,

$$F_t^\pi(u^i) = V_t^\pi(u^i) \quad (4.9)$$

Proof. Induction is used in the proof. For *single* period problem, it is always true that

$$F_N^\pi(u^i) = V_N^\pi(u^i) = C_N^\pi,$$

where π is a arbitrary policy, C_N^π is one-step costs at time N . Let us assume (4.9) holds for multiple periods $t+1, t+2, \dots, N$. We now need to prove that (4.9) holds for t .

Since

$$\begin{aligned} V_t^\pi(u^i) &= C_t(u^i, a_t^x | \pi) + E[V_{t+1}^\pi(u_{t+1}) | u_t = u^i] \\ &= C_t(u^i, a_t^x | \pi) + E[F_{t+1}^\pi(u_{t+1}) | u_t = u^i] \\ &= C_t(u^i, a_t^x | \pi) + E \left[E \left[\left\{ \sum_{t'=t+1}^{N-1} C_{t'}(u_{t'}, a_{t'}^x | \pi) + C_T(u_T | \pi) \right\} \middle| u_{t+1} \right] \middle| u_t \right] \end{aligned}$$

By applying the *law of iterated expectations* that $E[E\{./u_{t+1}\} | u_t] = E[./u_t]$ (Powell 2007),

$$\begin{aligned} V_t^\pi(u^i) &= C_t(u^i, a_t^x | \pi) + E \left[\left\{ \sum_{t'=t+1}^{N-1} C_{t'}(u_{t'}, a_{t'}^x | \pi) + C_N(u_N | \pi) \right\} \middle| u_t \right] \\ &= E \left[C_t(u^i, a_t^x | \pi) + \sum_{t'=t+1}^{N-1} C_{t'}(u_{t'}, a_{t'}^x | \pi, u_t) + C_N(u_N | \pi, u_t) \right] \\ &= E \left[\sum_{t'=t}^{N-1} C_{t'}(u_{t'}, a_{t'}^x | \pi, u_t) + C_N(u_N | \pi, u_t) \right] \\ &= F_t^\pi(u_t). \quad \square \end{aligned}$$

Appendix 4.2

Theorem 4.1. The expected minimum costs time t and later for the original problem in (4.8) and the Bellman Equation in (4.9) are equal.

$$F_t^*(u^i) = V_t(u^i) \quad (4.10).$$

Proof. In order to prove (4.10), we need to prove that $V_t(u^i) \leq F_t^*(u^i)$, and vice versa.

Induction is used to prove the first part. Let π be any arbitrary policy for single period. It is clear that $V_t^\pi(u^i) = F_t^\pi(u^i) = C_N^\pi(u^i)$, where C_N^π is the minimum one-step cost vector.

Assume for $t' = t+1, t+2, \dots, N$, the following holds

$$V_{t'}^\pi(u^i) \leq F_{t'}^*(u^i).$$

According to the optimality equation in (4.6)

$$V_t(u^i) = \min_{a^x \in A} \left(C_t(u^i, a^x) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x) V_{t+1}(u^j) \right)$$

and (4.11) $V_{t+1}^\pi \leq F_{t+1}^*$, so

$$V_t(u^i) = \min_{a^x \in A} \left(C_t(u^i, a^x) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x) F_{t+1}^*(u^j) \right).$$

Since F^* is the optimal value for original problem, and of course, $F_{t+1}^* \leq F_{t+1}^\pi$ for an arbitrary policy π , so

$$V_t(u^i) \leq \min_{a^x \in A} \left(C_t(u^i, a^x) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x) F_{t+1}^\pi(u^j) \right).$$

$$\begin{aligned}
 &\leq C_t(u^i, a^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, \pi) F_{t+1}^\pi(u^j) \\
 &= C_t(u^i, a^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, \pi) V_{t+1}^\pi(u^j) \\
 &= V_t^\pi(u^i) = F_t^\pi(u^i)
 \end{aligned}$$

This implies, $V_t(u^i) \leq F_t^\pi(u^i)$ for all $\pi \in \Pi$, thus $V_t(u^i) \leq F_t^*(u^i)$.

Secondly, for proving the second part, let any $\epsilon > 0$, and prove that there exists a policy π satisfies

$$V_t(u^i) \geq F_t^\pi(u^i) - \epsilon. \quad (4.12)$$

This is true for single period T that $V_T^\pi(u^i) = F_T^\pi(u^i) = C_T^\pi(u^i)$. C_T^π is the minimum one-step cost vector, so (4.12) holds for single period.

Let assume (4.12) holds for $t' = t+1, t+2, \dots, T$, or

$$V_{t+1}(u^i) \geq F_{t+1}^\pi(u^i) - \epsilon. \quad (4.13)$$

Because

$$F_t^\pi(u^i) = C_t(u^i, a_t^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, \pi) F_{t+1}^\pi(u^j)$$

Due to (4.13)

$$\begin{aligned}
 F_t^\pi(u^i) &\leq C_t(u^i, a_t^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x \in \pi) (V_{t+1}(u^j) + \epsilon) \\
 &= C_t(u^i, a_t^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x \in \pi) V_{t+1}(u^j) + \gamma \epsilon
 \end{aligned}$$

$$= \left\{ C_t(u^i, a_t^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x \in \pi) V_{t+1}(u^j) - \epsilon \right\} + (\gamma\epsilon + \epsilon)$$

Knowing that $V_t(u^i)$ represents the minimum cost for the optimal policy, we can always design a policy π such that

$$V_t(u^i) \geq C_t(u^i, a_t^x | \pi) + \gamma \sum_{u^j \in U} \Pr(u^j | u_t = u^i, a^x \in \pi) V_{t+1}(u^j) - \epsilon.$$

Therefore,

$$F_t^*(u^i) \leq F_t^\pi(u^i) \leq V_t(u^i) + (\gamma\epsilon + \epsilon) = V_t(u^i) + (1 + \gamma)\epsilon,$$

$$F_t^*(u^i) \leq V_t(u^i) + (\gamma + 1)\epsilon.$$

So, (4.12) is proved. $\square$

Appendix 4.3

Theorem 4.2. For bounded system cost, the value function of Bellman Equation converges to an optimal solution.

The process status at next the decision-making moment still falls into the same state space as a result of applying the policy to uncertain conditions. Let P_{min} be the minimum operator which is acting on V_t to produce V_{t+1} , and denote one-step P_{min} mapping as:

$$P_{min}: V \rightarrow V,$$

and k-step mapping as

$$P_{min}^k: V \rightarrow V.$$

Definition 4.2. $P_{min}: V \rightarrow V$ is contraction mapping and has the property

$$\|P_{min}v - P_{min}u\| \leq \gamma \|v - u\|,$$

where $0 \leq \gamma < 1$, v and u are column vectors, $\|\cdot\|$ is sup-norm.

Definition 4.3. Based on triangle inequality,

$$\|v + u\| \leq \|v\| + \|u\|.$$

Proof. Definition 4.2 and 4.3 are required for the proof.

First, we prove the distance between v_n and v_{n+m} goes to zero for sufficiently large n .

$$\begin{aligned} v^{n+m} - v^n &= v^{n+m} - v^{n+m-1} + v^{n+m-1} - \dots - v^{n+1} + v^{n+1} - v^n \\ &= \sum_{k=0}^{m-1} (v^{n+k+1} - v^{n+k}). \end{aligned}$$

Taking sup-norms and applying triangle inequality, then

$$\begin{aligned}
 \|v^{n+m} - v^n\| &= \left\| \sum_{k=0}^{m-1} (v^{n+k+1} - v^{n+k}) \right\| \\
 &\leq \sum_{k=0}^{m-1} \|v^{n+k+1} - v^{n+k}\| \\
 &= \sum_{k=0}^{m-1} \| (P_{min}^{n+k} v^1 - P_{min}^{n+k} v^0) \| \\
 &\leq \sum_{k=0}^m \gamma^{n+k} \|v^1 - v^0\|
 \end{aligned}$$

Since $0 \leq \gamma < 1$, and v^1 and v^0 are deterministic, the value can be made arbitrarily small, and v^n has a limit point v^* :

$$\lim_{n \rightarrow \infty} v^n \rightarrow v^*.$$

Now we prove that v^* is a fixed point of the mapping P_{min} .

$$\begin{aligned}
 0 &\leq \|P_{min} v^* - v^*\| \\
 &= \|P_{min} v^* - v^n + v^n - v^*\| \\
 &\leq \|P_{min} v^* - v^n\| + \|v^n - v^*\| \\
 &= \|P_{min} v^* - P_{min} v^{n-1}\| + \|v^n - v^*\| \\
 &\leq \gamma \|v^* - v^{n-1}\| + \|v^n - v^*\|
 \end{aligned}$$

Since

$$\lim_{n \rightarrow \infty} v^n \rightarrow v^*,$$

So,

$$\lim_{n \rightarrow \infty} \|v^* - v^{n-1}\| = \lim_{n \rightarrow \infty} \|v^n - v^*\| = 0.$$

We have

$$0 \leq \|P_{min}v^* - v^*\| \leq 0,$$

so that

$$\|P_{min}v^* - v^*\| = 0,$$

or

$$P_{min}v^* = v^*.$$

Assuming there are two limit point v^* and u^* , based on the property of contraction mapping, then

$$\begin{aligned} \|v^* - u^*\| &= \|P_{min}v^* - P_{min}u^*\| \\ &\leq \gamma \|v^* - u^*\| \end{aligned}$$

Since $\gamma < 1$, it must be true that $v^* = u^*$. $\square$

Appendix 4.4

Proposition 4.2. The bounded cost solution of Bellman Equation $V(u^i)$ is the unique optimal solution.

Proof. Suppose $V_i(u^i)$ satisfies the Bellman Equation. Since the state transition of stationary system is independent of time, let's simplify the Bellman Equation as

$$V(u^i) = \min_{a^x \in A} \left(C(u^i, a^x) + \gamma \sum_{u^j \in U} \Pr(u^j | u^i, a^x) V(u^j) \right).$$

Assume π^* is the optimal policy for $V(u^i)$, so,

$$V(u^i) = V^{\pi^*}(u^i) = C(u^i, \pi^*) + \gamma \sum_{u^j \in U} \Pr(u^j | u^i, \pi^*) V(u^j)$$

Assume $G(u^i)$ also satisfies the optimality equation, but the policy π^* may not be the optimal for $G(u^i)$, then

$$G(u^i) \leq G^{\pi^*}(u^i).$$

Therefore,

$$\begin{aligned} G(u^i) - V^{\pi^*}(u^i) &\leq G^{\pi^*}(u^i) - V^{\pi^*}(u^i) \\ &= \left(C(u^i, \pi^*) + \gamma \sum_{u^j \in U} \Pr(u^i | u^j, \pi^*) G(u^j) \right) \\ &\quad - \left(C(u^i, \pi^*) + \gamma \sum_{u^j \in U} \Pr(u^j | u^i, \pi^*) V(u^j) \right) \\ &= \gamma \sum_{u^j \in U} \Pr(u^i | u^j, \pi^*) [G^{\pi^*}(u^j) - V(u^j)] \end{aligned}$$

$$\begin{aligned}
 &\leq \gamma \sum_{u^j \in U} \Pr(u^i | u^j, \pi^*) |G^{\pi^*}(u^j) - V(u^j)| \\
 &\leq \gamma \sum_{u^j \in U} \Pr(u^i | u^j, \pi^*) \max_{u^j} |G^{\pi^*}(u^j) - V(u^j)| \\
 &\leq \gamma \max_{u^j} |G^{\pi^*}(u^j) - V(u^j)| \\
 &= \gamma \max_{u^j} |G^{\pi^*}(u^j) - V^{\pi^*}(u^j)|.
 \end{aligned}$$

So,

$$|G(u^i) - V^{\pi^*}(u^i)| \leq \gamma \max_{u^j} |G^{\pi^*}(u^j) - V^{\pi^*}(u^j)|,$$

and

$$\max_{u^i \in U} |G(u^i) - V^{\pi^*}(u^i)| \leq \gamma \max_{u^i \in U} |G^{\pi^*}(u^i) - V^{\pi^*}(u^i)|$$

since $\gamma < 1$, meaning

$$\max_{u^i \in U} |G^{\pi^*}(u^i) - V^{\pi^*}(u^i)| = 0.$$

The policy π^* must be optimal for $G(u^i)$ and $G(u^j) = V(u^j)$. $\square$

Appendix 4.5 Generic structures of experimental scenarios

(1) Generic structure for scenario 2

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	9	8	7	6	5	4	3	3	2	1
17	9	9	8	7	6	5	4	3	2	1	1
16	8	8	7	6	5	4	3	2	2	1	1
15	7	7	6	5	4	3	3	2	1	1	2
14	7	6	6	5	4	3	2	1	2	2	2
13	6	6	5	4	3	2	1	2	3	2	2
12	5	5	4	3	2	1	0	4	3	3	2
11	4	4	3	2	1	0	5	4	3	3	2
10	3	3	2	1	0	6	5	4	3	2	2
9	2	2	1	0	7	6	4	3	1	1	1
8	1	1	0	8	7	5	4	3	1	1	1
7	10	0	9	7	6	5	3	2	1	1	0

Figure A4.5.1 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	9	9	9	9	9	9	9	8	8	8
17	8	8	8	8	8	8	8	8	8	8	7
16	8	8	8	8	8	8	8	8	7	7	6
15	8	8	8	8	8	8	7	7	7	6	4
14	7	8	7	7	7	7	7	7	5	4	3
13	7	7	7	7	7	7	7	5	3	3	2
12	7	7	7	7	7	7	7	2	2	1	1
11	7	7	7	7	7	7	1	1	1	0	0
10	7	7	7	7	7	0	0	0	0	0	-1
9	7	7	7	7	-1	-1	0	0	1	0	-1
8	7	7	7	-2	-2	-1	-1	-1	0	-1	-2
7	-3	-7	-3	-2	-2	-2	-1	-1	-1	-2	-2

Figure A4.5.1 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	10	10	10	10	10	10	10	11	11	11
17	9	10	10	10	10	10	10	10	10	10	11
16	8	9	9	9	9	9	9	9	10	10	11
15	7	8	8	8	8	8	9	9	9	10	12
14	7	7	8	8	8	8	8	8	10	11	12
13	6	7	7	7	7	7	7	9	11	11	12
12	5	6	6	6	6	6	6	11	11	12	12
11	4	5	5	5	5	5	11	11	11	12	12
10	3	4	4	4	4	11	11	11	11	11	12
9	2	3	3	3	11	11	10	10	9	10	11
8	1	2	2	11	11	10	10	10	9	10	11
7	10	1	11	10	10	10	9	9	9	10	10

Figure A4.5.1 (c): Group n1 of the generic structure

(2) Generic structure for scenario 3

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	9	8	7	6	5	4	4	3	2	1
17	9	9	8	7	6	5	4	3	2	2	1
16	8	8	7	6	5	4	3	2	2	1	2
15	8	7	6	5	5	4	3	2	1	2	2
14	7	7	6	5	4	3	2	1	2	2	2
13	6	6	5	4	3	2	1	3	3	2	2
12	5	5	4	3	2	1	4	4	3	3	2
11	4	4	3	2	1	5	5	4	3	3	2
10	3	3	2	1	0	6	5	4	3	2	2
9	2	2	1	0	7	6	4	3	1	1	1
8	1	1	0	8	6	5	4	3	1	1	1
7	10	10	8	7	6	4	3	2	1	1	0

Figure A4.5.2 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	9	9	9	9	9	9	8	8	8	8
17	8	8	8	8	8	8	8	8	8	7	7
16	8	8	8	8	8	8	8	8	7	7	5
15	7	8	8	8	7	7	7	7	7	5	4
14	7	7	7	7	7	7	7	7	5	4	3
13	7	7	7	7	7	7	7	4	3	3	2
12	7	7	7	7	7	7	3	2	2	1	1
11	7	7	7	7	7	2	1	1	1	0	0
10	7	7	7	7	7	0	0	0	0	0	-1
9	7	7	7	7	-1	-1	0	0	1	0	-1
8	7	7	7	-2	-1	-1	-1	-1	0	-1	-2
7	-3	-3	-2	-2	-2	-1	-1	-1	-1	-2	-2

Figure A4.5.2 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	10	10	10	10	10	10	11	11	11	11
17	9	10	10	10	10	10	10	10	10	11	11
16	8	9	9	9	9	9	9	9	10	10	12
15	8	8	8	8	9	9	9	9	9	11	12
14	7	8	8	8	8	8	8	8	10	11	12
13	6	7	7	7	7	7	7	10	11	11	12
12	5	6	6	6	6	6	10	11	11	12	12
11	4	5	5	5	5	10	11	11	11	12	12
10	3	4	4	4	4	11	11	11	11	11	12
9	2	3	3	3	11	11	10	10	9	10	11
8	1	2	2	11	10	10	10	10	9	10	11
7	10	11	10	10	10	9	9	9	9	10	10

Figure A4.5.2 (c): Group n1 of the generic structure

(3) Generic structure for scenario 4

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	8	7	6	5	4	4	3	2	2	1
17	8	7	6	6	5	4	3	3	2	1	1
16	7	7	6	5	4	4	3	2	1	1	1
15	6	6	5	5	4	3	2	2	1	1	2
14	6	6	5	4	3	2	2	1	2	2	2
13	5	5	4	3	2	2	1	2	3	2	2
12	5	4	3	3	2	1	0	3	3	2	2
11	4	4	3	2	1	0	4	4	3	2	2
10	3	3	2	1	0	6	5	4	3	2	2
9	2	2	1	0	7	6	4	3	2	1	1
8	1	1	0	8	7	5	4	2	1	1	0
7	10	0	9	7	6	4	3	2	1	1	0

Figure A4.5.3 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	10	10	10	10	10	9	9	9	8	8
17	9	10	10	9	9	9	9	8	8	8	7
16	9	9	9	9	9	8	8	8	8	7	6
15	9	9	9	8	8	8	8	7	7	6	4
14	8	8	8	8	8	8	7	7	5	4	3
13	8	8	8	8	8	7	7	5	3	3	2
12	7	8	8	7	7	7	7	3	2	2	1
11	7	7	7	7	7	7	2	1	1	1	0
10	7	7	7	7	7	0	0	0	0	0	-1
9	7	7	7	7	-1	-1	0	0	0	0	-1
8	7	7	7	-2	-2	-1	-1	0	0	-1	-1
7	-3	7	-3	-2	-2	-1	-1	-1	-1	-2	-2

Figure A4.5.3 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	9	9	9	9	9	10	10	10	11	11
17	8	8	8	9	9	9	9	10	10	10	11
16	7	8	8	8	8	9	9	9	9	10	11
15	6	7	7	8	8	8	8	9	9	10	12
14	6	7	7	7	7	7	8	8	10	11	12
13	5	6	6	6	6	7	7	9	11	11	12
12	5	5	5	6	6	6	6	10	11	11	12
11	4	5	5	5	5	5	10	11	11	11	12
10	3	4	4	4	4	11	11	11	11	11	12
9	2	3	3	3	11	11	10	10	10	10	11
8	1	2	2	11	11	10	10	9	9	10	10
7	10	1	11	10	10	9	9	9	9	10	10

Figure A4.5.3 (c): Group n1 of the generic structure

(4) Generic structure for scenario 5

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	10	9	8	7	6	5	4	3	2	1
17	9	9	8	7	6	5	4	3	2	1	1
16	8	8	7	6	5	4	3	2	2	1	1
15	8	8	7	6	5	4	3	2	1	0	2
14	7	7	6	5	4	3	2	1	0	2	2
13	6	6	5	4	3	2	1	0	3	3	2
12	5	5	4	3	2	1	0	4	3	3	2
11	4	4	3	2	1	0	5	5	4	3	2
10	3	3	2	1	0	8	6	4	3	2	2
9	2	2	1	0	9	7	5	4	1	2	1
8	1	1	0	9	7	6	4	3	1	1	1
7	10	0	9	8	6	5	3	2	1	1	1

Figure A4.5.4 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	8	8	8	8	8	8	8	8	8	8
17	8	8	8	8	8	8	8	8	8	8	7
16	8	8	8	8	8	8	8	8	7	7	6
15	7	7	7	7	7	7	7	7	7	7	4
14	7	7	7	7	7	7	7	7	7	4	3
13	7	7	7	7	7	7	7	7	3	2	2
12	7	7	7	7	7	7	7	2	2	1	1
11	7	7	7	7	7	7	1	0	0	0	0
10	7	7	7	7	7	-2	-1	0	0	0	-1
9	7	7	7	7	-3	-2	-1	-1	1	-1	-1
8	7	7	7	-3	-2	-2	-1	-1	0	-1	-2
7	-3	7	-3	-3	-2	-2	-1	-1	-1	-2	-3

Figure A4.5.4 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	11	11	11	11	11	11	11	11	11	11
17	9	10	10	10	10	10	10	10	10	10	11
16	8	9	9	9	9	9	9	9	10	10	11
15	8	9	9	9	9	9	9	9	9	9	12
14	7	8	8	8	8	8	8	8	8	11	12
13	6	7	7	7	7	7	7	7	11	12	12
12	5	6	6	6	6	6	6	11	11	12	12
11	4	5	5	5	5	5	11	12	12	12	12
10	3	4	4	4	4	13	12	11	11	11	12
9	2	3	3	3	13	12	11	11	9	11	11
8	1	2	2	12	11	11	10	10	9	10	11
7	10	1	11	11	10	10	9	9	9	10	11

Figure A4.5.4 (c): Group n1 of the generic structure

(5) Generic structure for scenario 6

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	10	9	8	7	6	5	4	3	2	2
17	9	9	8	7	6	5	4	3	2	2	2
16	8	8	7	6	5	4	3	2	2	1	2
15	7	7	6	5	4	3	3	2	1	2	2
14	6	6	5	5	4	3	2	1	3	3	9
13	6	6	5	4	3	2	1	3	9	10	10
12	5	5	4	3	2	1	10	10	10	10	10
11	4	4	3	2	1	10	10	10	10	10	10
10	3	3	2	1	10	10	10	10	10	10	10
9	2	2	1	0	10	10	10	10	10	10	10
8	1	1	0	10	10	10	10	4	3	10	10
7	10	0	10	10	10	8	5	4	1	10	9

Figure A4.5.5 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	8	8	8	8	8	8	8	8	8	8	7
17	8	8	8	8	8	8	8	8	8	7	6
16	8	8	8	8	8	8	8	8	7	7	5
15	8	8	8	8	8	8	7	7	7	5	4
14	8	8	8	7	7	7	7	7	4	3	-4
13	7	7	7	7	7	7	7	4	-3	-5	-6
12	7	7	7	7	7	7	-3	-4	-5	-6	-7
11	7	7	7	7	7	-3	-4	-5	-6	-7	-8
10	7	7	7	7	-3	-4	-5	-6	-7	-8	-9
9	7	7	7	7	-4	-5	-6	-7	-8	-9	-10
8	7	7	7	-4	-5	-6	-7	-2	-2	-10	-11
7	-3	7	-4	-5	-6	-5	-3	-3	-1	-11	-11

Figure A4.5.5 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	10	11	11	11	11	11	11	11	11	11	12
17	9	10	10	10	10	10	10	10	10	11	12
16	8	9	9	9	9	9	9	9	10	10	12
15	7	8	8	8	8	8	9	9	9	11	12
14	6	7	7	8	8	8	8	8	11	12	19
13	6	7	7	7	7	7	7	10	17	19	20
12	5	6	6	6	6	6	16	17	18	19	20
11	4	5	5	5	5	15	16	17	18	19	20
10	3	4	4	4	14	15	16	17	18	19	20
9	2	3	3	3	14	15	16	17	18	19	20
8	1	2	2	13	14	15	16	11	11	19	20
7	10	1	12	13	14	13	11	11	9	19	19

Figure A4.5.5 (c): Group n1 of the generic structure

(6) *Generic structure for scenario 7*

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	8	7	6	5	4	3	2	2	1
17	9	8	7	6	5	5	4	3	2	1	1
16	8	8	7	6	5	4	3	2	1	1	1
15	7	7	6	5	4	3	2	2	1	1	1
14	6	6	5	4	3	3	2	1	0	2	2
13	6	5	5	4	3	2	1	0	2	2	2
12	5	5	4	3	2	1	0	3	3	2	2
11	4	4	3	2	1	0	4	3	3	2	2
10	3	3	2	1	0	5	4	3	1	2	1
9	2	2	1	0	6	5	4	3	1	1	0
8	1	1	0	7	6	4	3	2	1	1	0
7	10	0	8	7	5	4	3	2	1	1	0

Figure A4.5.6 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	9	9	9	9	9	9	9	8	8
17	8	9	9	9	9	8	8	8	8	8	7
16	8	8	8	8	8	8	8	8	8	7	6
15	8	8	8	8	8	8	8	7	7	6	5
14	8	8	8	8	8	7	7	7	7	4	3
13	7	8	7	7	7	7	7	7	4	3	2
12	7	7	7	7	7	7	7	3	2	2	1
11	7	7	7	7	7	7	2	2	1	1	0
10	7	7	7	7	7	1	1	1	2	0	0
9	7	7	7	7	0	0	0	0	1	0	0
8	7	7	7	-1	-1	0	0	0	0	-1	-1
7	-3	-2	-2	-2	-1	-1	-1	-1	-1	-2	-2

Figure A4.5.6 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	10	10	10	10	10	10	10	10	11	11
17	9	9	9	9	9	10	10	10	10	10	11
16	8	9	9	9	9	9	9	9	9	10	11
15	7	8	8	8	8	8	8	8	9	10	11
14	6	7	7	7	7	8	8	8	8	11	12
13	6	6	7	7	7	7	7	7	10	11	12
12	5	6	6	6	6	6	6	10	11	11	12
11	4	5	5	5	5	5	10	10	11	11	12
10	3	4	4	4	4	10	10	10	9	11	11
9	2	3	3	3	10	10	10	10	9	10	10
8	1	2	2	10	10	9	9	9	9	10	10
7	10	1	10	10	9	9	9	9	9	10	10

Figure A4.5.6 (c): Group n1 of the generic structure

(7) Generic structure for scenario 8

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	8	7	6	5	5	4	4	3	3
17	9	8	7	7	6	5	5	4	3	3	3
16	8	8	7	6	6	5	4	3	3	3	3
15	8	8	7	6	5	4	3	3	3	3	3
14	7	7	6	5	4	3	3	3	3	3	3
13	6	6	5	4	3	4	4	3	3	3	7
12	5	5	4	3	2	4	4	4	4	4	6
11	4	4	3	2	6	5	5	5	4	4	4
10	3	3	2	7	7	7	6	5	4	3	2
9	10	10	10	10	9	8	6	4	3	2	2
8	10	10	10	9	8	6	5	4	3	2	1
7	10	10	9	8	7	6	4	3	2	2	1

Figure A4.5.7 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	9	9	9	9	8	8	7	7	6
17	8	9	9	8	8	8	7	7	7	6	5
16	8	8	8	8	7	7	7	7	6	5	4
15	7	7	7	7	7	7	7	6	5	4	3
14	7	7	7	7	7	7	6	5	4	3	2
13	7	7	7	7	7	5	4	4	3	2	-3
12	7	7	7	7	7	4	3	2	1	0	-3
11	7	7	7	7	2	2	1	0	0	-1	-2
10	7	7	7	1	0	-1	-1	-1	-1	-1	-1
9	-1	-1	-2	-3	-3	-3	-2	-1	-1	-1	-2
8	-2	-2	-3	-3	-3	-2	-2	-2	-2	-2	-2
7	-3	-3	-3	-3	-3	-3	-2	-2	-2	-3	-3

Figure A4.5.7 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	10	10	10	10	10	11	11	12	12	13
17	9	9	9	10	10	10	11	11	11	12	13
16	8	9	9	9	10	10	10	10	11	12	13
15	8	9	9	9	9	9	9	10	11	12	13
14	7	8	8	8	8	8	9	10	11	12	13
13	6	7	7	7	7	9	10	10	11	12	17
12	5	6	6	6	6	9	10	11	12	13	16
11	4	5	5	5	10	10	11	12	12	13	14
10	3	4	4	10	11	12	12	12	12	12	12
9	10	11	12	13	13	13	12	11	11	11	12
8	10	11	12	12	12	11	11	11	11	11	11
7	10	11	11	11	11	11	10	10	10	11	11

Figure A4.5.7 (c): Group n1 of the generic structure

(8) Generic structure for scenario 9

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	8	7	7	6	5	5	4	4	4
17	9	8	8	7	6	6	5	4	4	3	4
16	9	8	7	7	6	5	4	4	4	3	4
15	8	8	7	6	5	5	4	4	3	3	4
14	8	7	7	6	5	5	4	4	3	4	8
13	7	7	6	6	5	5	4	4	4	4	7
12	8	7	6	6	5	5	4	4	4	7	6
11	8	7	7	6	6	5	5	8	7	6	5
10	10	8	8	7	7	9	8	7	6	5	4
9	10	10	10	10	9	8	7	6	5	3	3
8	10	10	10	9	8	7	6	5	4	3	2
7	10	10	10	9	8	6	5	4	3	2	2

Figure A4.5.8 (a): Division of state space by the “setup-line”

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	9	9	9	8	8	8	7	7	6	5
17	8	9	8	8	8	7	7	7	6	6	4
16	7	8	8	7	7	7	7	6	5	5	3
15	7	7	7	7	7	6	6	5	5	4	2
14	6	7	6	6	6	5	5	4	4	2	-3
13	6	6	6	5	5	4	4	3	2	1	-3
12	4	5	5	4	4	3	3	2	1	-3	-3
11	3	4	3	3	2	2	1	-3	-3	-3	-3
10	0	2	1	1	0	-3	-3	-3	-3	-3	-3
9	-1	-1	-2	-3	-3	-3	-3	-3	-3	-2	-3
8	-2	-2	-3	-3	-3	-3	-3	-3	-3	-3	-3
7	-3	-3	-4	-4	-4	-3	-3	-3	-3	-3	-4

Figure A4.5.8 (b): Division of no-setup state space by IP

	WIP(t)										
IP (t)	0	1	2	3	4	5	6	7	8	9	10
18	9	10	10	10	11	11	11	12	12	13	14
17	9	9	10	10	10	11	11	11	12	12	14
16	9	9	9	10	10	10	10	11	12	12	14
15	8	9	9	9	9	10	10	11	11	12	14
14	8	8	9	9	9	10	10	11	11	13	18
13	7	8	8	9	9	10	10	11	12	13	17
12	8	8	8	9	9	10	10	11	12	16	16
11	8	8	9	9	10	10	11	15	15	15	15
10	10	9	10	10	11	14	14	14	14	14	14
9	10	11	12	13	13	13	13	13	13	12	13
8	10	11	12	12	12	12	12	12	12	12	12
7	10	11	12	12	12	11	11	11	11	11	12

Figure A4.5.8 (c): Group n1 of the generic structure

Chapter 5 Development of Joint Control Policy

Relying on the generic structure and structural properties explored in the previous chapter, in this chapter the optimal inputs of the generic structure are identified by employing Response Surface Methodology (RSM) and a new joint control policy is developed. The performance of the new policy is benchmarked with a policy developed using Bellman equation to demonstrate the advantage and effectiveness of the new approach.

5.1. Optimization problem

Based on the generic structure the proposed approach converts the optimization problem of finding a joint control policy into a combinatorial optimization problem of identifying three optimal controllable parameters, namely reorder point (*ROP* or *s*), order-up-to (*UPTO* or *S*), and maximum WIP for group *n1* (*CAP* or *Max_WIP*), for achieving optimal system performance in terms of service level and total cost.

However, under certain circumstances, such as low rejection cost per unit, joint control policies tend to reject more orders in order to maintain high on-time delivery rate. It is therefore required to measure and minimize the *number of orders rejected*, a new performance indicator of optimization. In the beginning, the interconnection between the newly imposed performance indicator and three controllable inputs is unclear.

The objective of optimization is to determine the settings of controllable parameters by searching through the decision space $X = \{(ROP, UPTO, CAP)\}$ in order to optimize on-time delivery rate $E(Y_1)$, system cost $E(Y_2)$, and number of orders rejected $E(Y_3)$

$$Y_i = f(X|Z), \quad (5.1)$$

where function f represents the hybrid supply chain model (Figure 5.2), and Z represents *uncontrollable* inputs such as operational condition parameters, as well as stochastic variables of the hybrid system. The stochastic nature of Z results in uncertainty of individual output y_i even for the same setting of input $(ROP, UPTO, CAP) \in X$.

The search space of the optimization process is constrained within the lower and upper bounds of the input parameters, and the deterministic optimization function can be expressed as

$$\text{Optimize } E[Y_i] = E[f(X|Z)] \text{ over the decision space } X, \quad (5.2)$$

subject to:

$$lb_{ROP} \leq ROP \leq ub_{ROP}; \quad (5.3)$$

$$lb_{UPTO} \leq UPTO \leq ub_{UPTO}; \quad (5.4)$$

$$lb_{CAP} \leq CAP \leq ub_{CAP}. \quad (5.5)$$

where $lb_{ROP} > 0$ and $ub_{ROP} \leq \bar{s}$, $lb_{UPTO} \geq \underline{s}$ and $ub_{UPTO} \leq \bar{S}$ considering that $0 < \underline{s} \leq s \leq \bar{s} \leq \underline{S} \leq S \leq \bar{S}$; $lb_{CAP} > 0$ and $ub_{CAP} \leq Max_WIP$. Equations (5.3), (5.4) and (5.5) define the decision space X .

5.2. Optimization process design

A closed-loop process is designed for solving the optimization problem as shown in Figure 5.1, in which the hybrid supply chain is represented by its simulation model.

A simulation model is more flexible and solves real problems without making too many restricting assumptions (Yalcinkaya and Bayhan, 2009). Analytic models are normally built for solving optimization problems and often involve conceptualizing underlying systems in certain predefined structures with some details omitted for yielding information about long term or steady state average behavior (Standridge, 2004); in contrast, simulation models do not have such limits.

Two groups of *uncontrollable* parameters are included in the simulation model: the first group consists of factors that specify operational conditions, which are out of control of the policy developer but influence the development of joint control policies, such as inventory holding cost per unit per period u^h , setup cost per batch u^s , order rejection cost u^r , and late delivery cost u^l . Whenever these operational parameters change, joint control policies need to be re-developed.

Figure 5.1: Optimization process design

The second group consists of stochastic parameters of hybrid supply chain model, such as demand rate λ , production speed μ and inventory replenishment lead-time.

For each set of controllable inputs ROP , $UPTO$, and CAP , the simulation model responses reflect system performance in terms of service level (on-time delivery rate, y_1), total cost (y_2), and number of orders rejected (y_3).

The optimization process establishes the cause-and-effect relationship between controllable input parameters (i.e., ROP , $UPTO$, and CAP) and responses (i.e., y_1 , y_2 , and y_3) in order to identify the optimal inputs. However, because of the stochastic nature of responses of the simulation system and the limited number of observations obtained, the cause-and-effect relationship is not straightforward. For such cases, only those optimization methods that do not exploit the structure of function f can be applied (Keys et al., 2002). Thus, RSM, a metamodeling technique, is employed as an optimizer to synthesize response surfaces that are used to replicate the function f based on the sample of responses.

RSM is a well-established optimization approach that fits low order response surface model or polynomial through response points. It is a collection of statistical techniques for developing, improving and optimizing processes. RSM was first proposed by Box and Wilson (1951) for real non-simulated systems, but also has been applied for random simulation models (Greewood et al., 1998; Keys et al., 2002; Tekin and Sabuncuoglu, 2004; Yalcinkaya and Bayhan 2009). The biggest advantage of RSM is that it uses well-known statistical tools, and compared to many gradient methods, is a relatively efficient method of simulation optimization (Tekin and Sabuncuoglu, 2004).

As a local optimization approach RSM has a number of advantages: (1) it easily provides a good fit over a small local region, and (2) it doesn't waste computational effort for capturing response surfaces in suboptimal regions. However, the effectiveness of RSM,

to a large extent, depends on the experimenter's ability to define a small enough search space (Keys et al., 2002), otherwise the computational requirements may be huge. So RSM is not well suited to large-scale problems with large search spaces (Beyer and Sendhoff, 2007). In addition, since model adequacy and accuracy tests are critical for estimating optimum values more precisely, RSM is a progressive trial-and-error process which is time consuming and may not eventually arrive at the right model (Anjum et al., 1997).

By designing input settings over a local region in the search space and obtaining system responses, RSM fits metamodels to the local region by determining the coefficients of metamodels. The optimization process continuously changes local regions and evaluates the performance of fitted response surfaces until the optimum region(s) are found.

5.2.1. Algorithm for deriving the joint control policy

The simulation model in the closed-loop optimization process is driven by joint control policy, so the algorithm (Table 5.1) is required to translate a set of given inputs of *ROP*, *UPTO*, and *CAP* into a joint control policy for running simulation and generating responses.

Based on structural properties, the state space is divided into a number of groups: n_1 , n_2 , ...; the first group n_1 , which requires setup, is separated from others that do not require setups, by a "setup line". The division of groups that do not require setups is not straightforward. For example, based on the numeric results of nine scenarios in the previous chapter, in which the state space is divided into three groups: n_1 , n_2 and n_3 , the ratio of group size n_2 over (n_2+n_3) varies from 39% to 68%. To simplify the grouping

process, a heuristic is applied and a proportion is allocated to each of the groups that do not require setups. The algorithm runs at time zero of simulation.

Table 5.1: The algorithm for the joint control policy

Step 1.	<p>Create a two-dimension matrix for given ROP, $UPTO$, and CAP</p> <ul style="list-style-type: none"> • X-dimension: variable WIP^i, where i is 0, 1, ... CAP. • Y-dimension: variable IP^j, where j is $(ROP+1)$, $(ROP+2)$, ..., $UPTO$.
Step 2.	<p>Identify “setup line”</p> <ul style="list-style-type: none"> • For each cell, calculate $Setup_threshold = IP^j - (ROP + 1) - WIP^i + 1$. • Cells with $Setup_threshold == 0$ form the “setup line”
Step 3.	<p>Determine members for groups which do not require setup</p> <ul style="list-style-type: none"> • Set parameter group_number = p, p is an integer <p>Create groups $n2$ to np</p> <ul style="list-style-type: none"> • Assign members to each group according to a given proportion. For the benchmark scenario, the heuristic used is that two third go to group $n2$ and one third to $n3$.
Step 4.	<p>Calculate admission cap for members in group ni, $i = 2 \dots, p$.</p> <ul style="list-style-type: none"> • For member in group ni, Admission cap = $IP^j - WIP_{waiting} - (ROP + i - 1)$, Where $WIP_{waiting} = (WIP^i - 1)$, if $(WIP^i > 1)$, $WIP_{waiting} = 0$, otherwise.
Step 5.	<p>Calculate admission cap for members in group $n1$ that require setup</p> <ul style="list-style-type: none"> • Admission cap = $CAP - WIP^i$

5.2.2. Simulation model

The Joint Admission-inventory Control Simulation Model (Figure 5.2) is developed using discrete event simulation package FlexSim[®], consisting of 12 key FlexSim[®] objects:

1. Periodic review timer (FlexSim[®] source object) triggers the “Review process” at review epochs.

2. Review process (FlexSim[®] processor object) performs three functions: (a) If the semi-finished module Inventory Position $IP \leq s$ (reorder point), it places a replenishment order with the quantity of $(IP - s)$; (b) It calculates the inventory holding cost incurred for the previous period; and (c) It determines the admission cap, a_t , for current period.
3. Customer order (FlexSim[®] source object) generates customer orders according to a Poisson process.
4. Order processing (FlexSim[®] processor object): If the maximum admission is reached, it rejects all subsequent orders for the period and records rejection cost incurred.
5. WIP (FlexSim[®] queue object) accommodates accepted orders that are waiting to be loaded into MTO production.
6. Raw material (FlexSim[®] source object) provides unlimited and immediately available raw material for MTS production.
7. MTS setup (FlexSim[®] combiner object) consolidates raw materials into a batch according to the indicated size $(IP - s)$ and loads into MTS production. Setup cost is incurred for a new batch.
8. MTS (FlexSim[®] processor object) represents MTS batch production and logistics; the processing time is a variable that follows Uniform distribution.
9. Receiving (FlexSim[®] separator object) unpacks the batch received from MTS.
10. SFG inventory (Flexim[®] queue object) accommodates semi-finished modules.
11. MTO (FlexSim[®] combiner object) customizes semi-finished modules into finished products at the rate of μ .

12. FG delivery (FlexSim[®] sink object): For a finished product, it checks if on-time delivery is satisfied; else, late delivery cost is computed.

Figure 5.2: The Joint admission-inventory control simulation model in FlexSim[®]

In the simulation model one simulation clock unit is equivalent to one hour in real world; system review is performed every 168 hours, or once a week. The total simulation run length is 10,416 hours, or 62 weeks. Data for the first 840 hours (5 weeks) is skipped (warm-up period), and the last 840 hours (5 weeks) is ignored so that incomplete orders are eliminated from data collection. The simulation system performance is calculated for a total of 52 weeks.

The simulation model serves to obtain system responses for performing metamodel fitting, and is also used as a validation mechanism for benchmarking the performance of the joint control policies developed.

The closed-loop optimization process needs to tackle different types of uncertainties. The first type is from the simulation model design, which include the stochastic variables in system modeling. The second type is from individual responses y_i (simulation outputs) due to the stochastic nature of simulation and measurement errors (round off errors). These two types of uncertainties are common in simulation model development and can be addressed through model verification and output analysis. The third type comes from metamodeling technique itself and is discussed in the next section.

5.3. Metamodel fitting

The purpose of metamodel fitting is to model the true function f (simulation model in Figure 5.2) using first or second-order polynomials. The fitted models are deterministic in nature and can provide insight into the cause-and-effect relationship between inputs and responses.

In most of the cases, the true response surface usually exhibits curvature near the optimum and a second-order response surface is capable of capturing such curvature because of its flexibility and a wide variety of functional forms. Box and Wilson (1951) suggested that a second-order response surface is sufficient for replicating the true function f under study. Normally, RSM is applied as a series of steps: it starts by fitting first-order linear regression model, continuously searches for the optimum region(s) within the solution space, and terminates by fitting second-order surface response model to estimate the optimum system performance.

5.3.1. Response variables

First-order model is a multiple linear regression model with k regressor variables; the response variable of observations is described as

$$y_i = \beta_{i,0} + \sum_{j=1}^k \beta_{i,j}x_j + \varepsilon_i, \quad (5.6)$$

where parameter $\beta_{i,j}$, $j = 0, 1, \dots, k$, are the regression coefficients, representing the expected change in response y_i per unit change in x_j when all the remaining independent variables x_m ($m \neq j$) are hold constant. In this research, $K = 3$, and x_1 , x_2 , and x_3 represent *ROP*, *UPTO*, and *CAP* respectively; the response variable y_1 is service level, y_2 total cost, and y_3 the number of orders rejected.

Let $b_0, b_1, \dots, b_k$ be the least squares estimators to $\beta_0, \beta_1, \dots, \beta_k$. Then as per Myers et al. (2009) the fitted regression model can be written as

$$\hat{y}_i = b_{i,0} + \sum_{j=1}^k b_{i,j}x_j. \quad (5.7)$$

A first-order regression model is fitted over a small *local* region of interest $\mathbb{R} \in X$ so that within $\mathbb{R}$ each response variable y_i is represented by a plane described by equation (5.7). Starting from an arbitrary local region, RSM allows the investigation to continuously move towards near optimal local regions systematically, based on the understanding of function f . Over the course of the investigation, the performance of response surfaces (e.g. on-time delivery rate, system cost and number of rejected orders) are evaluated continuously; the direction of movement is determined by steepest ascent/descent.

The typical second-order response surface variable can be written as (Myers et al., 2009)

$$y_i = \beta_{i,0} + \sum_{j=1}^k \beta_{i,j}x_j + \sum_{l=1}^k \beta_{i,l,l}x_l^2 + \sum_{j<l}^k \beta_{i,j,l}x_jx_l + \varepsilon_i, \quad (5.8)$$

where $K = 3$.

The corresponding fitted response surface model is of the form

$$\hat{y}_i = b_{i,0} + \sum_{j=1}^k \beta_{i,j}x_j + \sum_{l=1}^k b_{i,l,l}x_l^2 + \sum_{j<l}^k b_{i,j,l}x_jx_l, \quad (5.9)$$

where $b_{i,k}$ is the least squares estimator of $\beta_{i,k}$.

In both first-order regression and second-order response surface models, residual ε_i is the difference between the observation y_i and the fitted value $\hat{y}_i$. In order to use equations (5.7) and (5.9) as approximation to (5.6) and (5.8), an assumption is made that residual ε_i is normally and independently distributed with mean zero, or $\varepsilon \sim NID(0, \sigma^2)$, where $\sigma^2 = \frac{E(SS_E)}{(n-p)}$, $(n-p)$ is the degrees of freedom for n observations and p regressors. SS_E is the residual sum of squares or error for response y_i , which can be calculated as

$$SS_E(y_i) = \sum_{j=1}^n (y_{i,j} - \hat{y}_{i,j})^2. \quad (5.10)$$

The examination of residual plots (such as “Normal probability plot”, “Residuals vs. fits”, or “Histogram of residuals”) helps determine if the ordinary least squares assumptions are met. If these assumptions are satisfied, then ordinary least squares regression produces unbiased coefficient estimates with minimum variance.

5.3.2. Multiple-response optimization

The optimization problem in (5.2) is a multiple-response problem as three response variables are optimized simultaneously. The challenge of multiple-response problem is the mutual dependency of the response variables as a fixed set of inputs may not always be operational (Vining, 1998). In practice, it may not be possible to find optimal input settings for satisfying all the response targets. Therefore, the solution for a multiple-response problem is usually a compromise set of input combinations (Jiang et al., 2009).

The well-known Derringer-Suich desirability function (Derringer and Suich, 1980) is adopted in this research. The idea is to transform a multiple-response problem into a single response problem by means of mathematical transformations (Castillo et al., 1996). The desirability $d_i \in [0.0, 1.0]$ is calculated for each response y_i depending on whether it is single or double-sided, as shown in Equations (5.11) and (5.12), respectively

$$d_i = \begin{cases} 0 & \hat{y}_i \leq y_i^{\min} \\ \left[\frac{\hat{y}_i - y_i^{\min}}{y_i^{\max} - y_i^{\min}} \right]^r & y_i^{\min} \leq \hat{y}_i \leq y_i^{\max} \\ 1 & \hat{y}_i \geq y_i^{\max} \end{cases} \quad (5.11)$$

$$d_i = \begin{cases} \left[\frac{\hat{y}_i - y_i^{\min}}{target - y_i^{\min}} \right]^m & y_i^{\min} < \hat{y}_i < target \\ \left[\frac{target - \hat{y}_i}{target - y_i^{\max}} \right]^n & target < \hat{y}_i < y_i^{\max} \\ 0 & \hat{y}_i \leq y_i^{\min} \text{ or } \hat{y}_i \geq y_i^{\max} \end{cases} \quad (5.12)$$

where $y_i^{\min}$ and $y_i^{\max}$ are the minimum and maximum acceptable values of the i^{th} response, and $\hat{y}_i$ is the fitted value. The r , m and n are the *weights* for specifying desirability functions around the target value for a response. The higher the weight, the more the

importance assigned to the response in the optimization procedure. The concept is shown in Figure 5.6 for double-sided desirability function.

The composite desirability function, defined as the geometric mean of the individual desirability, is used in performance comparison between fitted models.

$$D = \sqrt[k]{d_1 \times d_2 \times \dots \times d_k} \quad (5.13)$$

Figure 5.3: The role of weight in double-sided desirability function

5.3.3. Adequacy and accuracy tests

In order to replicate function f , fitted models need to go through adequacy and accuracy tests. If the models fail the tests, they will likely lead to poor or misleading results (Myers et al., 2009).

The commonly used adequacy test proves that the fitted models can provide an adequate approximation to the true function f . According to Myers et al. (2009), the F -test for lack-of-fit is

$$F_0 = \frac{SS_{LOF}/(m - p)}{SS_{PE}/(n - m)}, \quad (5.14)$$

where SS_{LOF} is the lack-of-fit sum of squares, SS_{PE} is the pure-error sum of squares; $(m - p)$ and $(n - m)$ are the corresponding degrees of freedom

$$SS_{PE} = \sum_{i=1}^m \sum_{obs=1}^n (y_{i,obs} - \bar{y}_i)^2 \quad (5.15)$$

$$SS_{LOF} = \sum_{i=1}^m n(\bar{y}_i - \hat{y}_i)^2 \quad (5.16)$$

where $y_{i,obs}$ is i^{th} value in an observation, $\bar{y}_i$ is the average of observations, and $\hat{y}_i$ is predicted value of fitted model. A p -value of (lack-of-fit test) below 0.05 is proof that the polynomial does not adequately fit observations, and higher order terms are needed for the test.

The accuracy test checks how well the function f is replicated by the fitted models. The following two questions need to be answered (Keys et al., 2002):

- How well does the model fit to $E(Y_i)$?
- How well does the fitted model capture the essential features or variations of the response surface?

The first question is addressed by significance of regression of fitted models. The test determines whether inputs are significantly related to a response by using ANOVA. The null hypothesis is

$$H_0: \beta_0 = \beta_2 = \beta_3 \dots = \beta_k = 0. \quad (5.17)$$

The p -value is probability of obtaining F -test statistic F_0 if the null hypothesis H_0 is true, where

$$F_0 = \frac{SS_R/p}{SS_E/(n-p-1)}, \quad (5.18)$$

The regression sum of squares SS_R is derived by

$$SS_R(y_i) = \sum_{j=1}^n (\hat{y}_{i,j} - \bar{y}_i)^2, \quad (5.19)$$

If the p -value of a regressor is less than $\alpha = 0.05$ then the corresponding input is significantly related to the response. If there is at least one input that is significantly related to the response, then evidence exists that the null hypothesis can be rejected.

The second question is addressed by the coefficient of determination (R^2). It is used to describe how well a fitted model captures the essential feature of variations of the true response surface, or the proportion of total variation of outcomes explained by the fitted model. The coefficient of determination is defined as

$$R^2(y_i) = \frac{SS_R(y_i)}{SS_T(y_i)}, \quad (5.20)$$

where the total sum of squares SS_T is

$$SS_T(y_i) = \sum_{j=1}^n (y_{i,j} - \bar{y}_i)^2. \quad (5.21)$$

$R^2 \in [0,1]$ and the closer to 1 R^2 is, the better the fit. In order to eliminate the influence of number of coefficients, it is common to use adjusted coefficient of determination as

$$R_{adj}^2 = 1 - \frac{n-1}{n-p-1}(1-R^2), \quad (5.22)$$

where n is the number of observations, and p is the number of coefficients.

Lack-of-fit test, significance of regression, and coefficient of determination (R^2) are conducted for second-order response surface models in section 5.5.

5.4. Experiment design

The experiment is designed for the benchmark scenario discussed in the previous chapter. The scope of the optimization process is limited by the lower and upper bounds of the three optimal controllable parameters of the generic structure: ROP , $UPTO$, and CAP . By referring to the inventory policy $(s, S) = (ROP, UPTO) = (6, 18)$ generated in the previous chapter and considering that $0 < \underline{s} \leq s \leq \bar{s} \leq \underline{S} \leq S \leq \bar{S}$, the lower and upper bounds of the first two parameters of the generic structure are set as

- $lb_{ROP} = 2$ and $ub_{ROP} = 10$;
- $lb_{UPTO} = 14$ and $ub_{UPTO} = 27$.

Table 4.2 shows that when $WIP = 7$, the on-time delivery rate is as high as 95.4% and when $WIP = 13$, the on-time delivery rate drops to 42.4%. So the range for the third parameter is set as $lb_{CAP} = 7$ and $ub_{CAP} = 13$.

The optimization function for the benchmark scenario is

$$\text{Optimize } E[Y_i] = E[f(X|Z)], \quad (5.2)$$

subject to the decision space X as follows

$$2 \leq ROP \leq 10; \quad (5.23)$$

$$14 \leq UPTO \leq 27; \quad (5.24)$$

$$7 \leq CAP \leq 13. \quad (5.25)$$

5.4.1. Input data collection scheme

RSM performs metamodel fitting over individual local regions, so it needs a systematic way of defining local region before collecting inputs. The grid-oriented input collection scheme proposed by Greenwood et al. (1998) is chosen for forming local regions. Based on the scheme the input collection is carried out according to a three dimensional grid pattern (Figure 5.4), in which the grid-points are spaced one unit apart in each dimension such that $\Delta_{ROP} = \Delta_{UPTO} = \Delta_{CAP} = 1$. A typical local region is cubical and formed as $4\Delta_{ROP} \times 4\Delta_{UPTO} \times 2\Delta_{CAP}$; local regions R_i and R_j in Figure 5.4 illustrate two local regions.

Figure 5.4: Decision space cube and regions

Based on lower and upper bounds of the optimization problem specified in (5.23), (5.24) and (5.25), there are a total of $5 \times 10 \times 5 = 250$ local regions (Table 5.2) in the decision space X . The grid-points over a local region R_i are taken for metamodel fitting following Design of Experiments (DoE).

Table 5.2: Input ranges for metamodel fitting

No	$4\Delta_{ROP}$	$4\Delta_{UPTO}$	$2\Delta_{CAP}$
1	2 – 6	14 – 18	7 – 9
2	3 – 7	15 – 19	8 – 10
3	4 – 8	16 – 20	9 – 11
4	5 – 9	17 – 21	10 – 12
5	6 – 10	18 – 22	11 – 13
6		19 – 23	
7		20 – 24	
8		21 – 25	
9		22 – 26	
10		23 – 27	
Number of ranges	5	10	5

5.4.2. DoE for metamodel fitting

Two-level 2^3 full factorial design is employed for fitting first-order regression model; the design codes each input at two levels: low (coded as ‘-1’) and high (coded as ‘+1’).

Figure 5.5: 2^3 full factorial design of experiment

There are a total of eight input collection points (Figure 5.5) over a local region, each requiring two simulation runs; so a total of 16 simulation runs are made for fitting a first-order linear regression model. The coded design matrix (Table 5.3) is generic for all the local regions. By applying the coded design matrix to a specific local region, one can get eight sets of desired inputs for simulation runs over the region. For the example of region $R = (2 \sim 6) \times (19 \sim 23) \times (10 \sim 12)$, eight sets of uncoded simulation inputs are shown in Table 5.3.

Table 5.3: 2^3 factorial design matrix

Run	Generic Inputs (coded)			Inputs for example region (uncoded)		
	<i>ROP</i>	<i>UPTO</i>	<i>CAP</i>	<i>ROP</i>	<i>UPTO</i>	<i>CAP</i>
1	-1	-1	-1	2	19	10
2	+1	-1	-1	6	19	10
3	-1	+1	-1	2	23	10
4	+1	+1	-1	6	23	10
5	-1	-1	+1	2	19	12
6	+1	-1	+1	6	19	12
7	-1	+1	+1	2	22	12
8	+1	+1	+1	6	23	12

After the near optimum regions are found through the result analysis of first-order regression fitting, Box-Behken design (Figure 5.6) is employed for fitting second-order surface response.

Figure 5.6: $K = 3$ Box-Behnken design with a center point

Box-Behnken design with $k = 3$ is a spherical design, where the grid-points on edge are equi-distant ($= \sqrt{2}$) from the center point. The design is also more efficient and is an important alternative to the classical Central Composite Design (Myers et al., 2009).

The Box-Behnken design matrix is deployed based on the construction of balanced incomplete block designs in Table 5.4. In the first block, input *ROP* and *UPTO* are paired together in a 2^2 factorial design while third input *CAP* is fixed at the center ($CAP = 0$). The same applies to blocks 2 and 3. The last row in the design is a vector of center runs. Table 5.4 shows the generic coded inputs and uncoded Box-Behnken inputs for the example region of $R = (2 \sim 6) \times (19 \sim 23) \times (10 \sim 12)$.

To fit a second-order response surface, two simulation replications are carried out for each setting in blocks, and six replications are carried out for the centre point. So a total of 30 replications are required for fitting a second-order metamodel over a local region.

Table 5.4: Box-Behnken design matrix

	Run	Generic Inputs (coded)			Inputs for example region (uncoded)		
		<i>ROP</i>	<i>UPTO</i>	<i>CAP</i>	<i>ROP</i>	<i>UPTO</i>	<i>CAP</i>
Block 1	1	-1	-1	0	2	19	11
	2	-1	+1	0	2	23	11
	3	+1	-1	0	6	19	11
	4	+1	+1	0	6	23	11
Block 2	5	-1	0	-1	2	21	10
	6	-1	0	+1	2	21	12
	7	+1	0	-1	6	21	10
	8	+1	0	+1	6	21	12
Block 3	9	0	-1	-1	4	19	10
	10	0	-1	+1	4	19	12
	11	0	+1	-1	4	23	10
	12	0	+1	+1	4	23	12
-	13	0	0	0	4	21	11

5.4.3. Multiple-response target setting

By applying the first joint control policy (Figure 4.1, obtained using Bellman equation) to the simulation model, the following results are obtained:

- average service level (on-time delivery rate) $y_1 = 98.47\%$,
- average system cost $y_2 = \$1,403.17$, and
- average number of rejections $y_3 = 198.73$ (out of a total of 415, for a 48% rejection rate).

Apparently, the on-time delivery rate is very high. In order to achieve a more balanced result of performance, new performance measure targets for y_i (Table 5.5) are set. Among the responses, variable y_1 is the most important and is bound on both sides with weights $m_1 = 2$, and $n_1 = 2$; the target rate is set to 90% on-time delivery; response variable y_2 has a single-sided bound with the target total cost of \$1500; response variable y_3 also has a

single-sided bound with a new order rejection target of 150 or 36% rate. The weights are set at 1 for y_2 and y_3 .

Table 5.5: The target, minimum and maximum values set for response variables

Response	Lower	Target	Higher
On-time delivery (y_1)	70%	90%	100%
System cost (y_2)	-	1500	2000
No. of rejections (y_3)	-	150	220

The optimization process searches for the optimum settings of inputs that can achieve the best performance against the targets set in Table 5.5.

5.5. Joint control policy development

Metamodel is fitted by employing statistics software Minitab[®] 16; over a local region, three response variables are fitted simultaneously for predicting service level, total cost and number of orders rejected.

5.5.1. Optimal local regions

5.5.1.1 Preliminary selection of near optimum regions

Considering the number of local regions in this research is limited, the first-order metamodel is fitted for all the 250 local regions. Obviously, *composite desirability* is a good indicator reflecting the overall system response; by sorting composite desirability in decreasing order good candidates, which are closer to the optimum, can be found.

However, there are two considerations: (a) desirability for service level has a double-sided bound, and a high on-time delivery rate (much higher than 90%) would lower the

desirability of service level, resulting in low composite desirability; (b) for the unknown function f , high on-time delivery rate might not necessarily cause high cost and/or high order rejection.

So, individual desirability is still required in the selection procedure for some cases where composite desirability is low (due to high on-time delivery rate) but desirability for cost and/or order rejection is high. So if a local region meets the following criteria, it is considered a good candidate also:

- (Desirability for total cost > 0.8) & (Desirability for rejection > 0.8) & (On-time delivery rate $> 90\%$).

As a result, out of the 250 local regions a total of 65 are selected (Appendix 5.1), which meet the criteria of high composite desirability ($\geq 85\%$) for normal situations, or above criteria for special cases.

In next filtering step, eight local regions with the highest composite desirability and seven special local regions, which meet above criterion, are selected as good candidates. These outstanding local regions are listed in Table 5.6. According to first-order responses, service levels for identified local regions are all higher than 90%; while total cost varies from 1508.44 to 1625.62, and numbers of orders rejected vary from 152 to 172.

The column “Response of regress model” (Table 5.6) shows the best responses based on the fitted polynomials. Note that the corresponding inputs may not necessarily be integers as RSM searches through a continuous space.

The continuous spaces in Table 5.6 are the potential regions from where optimal joint control policies can be derived. To a certain extent the settings of *ROP*, *UPTO* and *CAP*

in each of the selected regions reflect the codependency between admission control and inventory control, whereas in the rest of the regions, the settings are mismatched. As a result regions with mismatched parameters are excluded from the optimization process.

5.5.1.2 Identification of optimum region

Second-order metamodel fitting is performed for the outstanding local regions. All 15 local regions (Table 5.7) satisfy the basic condition to be the best local region with service level response $\geq 90\%$. There are only three local regions whose composite desirability is lower than 0.80, but their service levels responses are 92.01%, 93.54%, and 94.86%, respectively and total costs and rejections of the three regions are *not* the highest, so they could still be the best local region(s).

Lack-of-fit test and coefficient of determination are used to evaluate and filter out the best region(s). Three local regions 845, 1029, and 1145 (Table 5.7) fail the lack-of-fit test with at least one p -value < 0.05 , evidence that the second-order polynomials do not accurately fit the observations; they require even higher order terms for test, so the three local regions are discarded as RSM ends with second-order fitting.

Coefficient of determination (R^2) tests model accuracy: the closer to 1 its value, the better the fitted model captures the essential feature of the variations of the response surface. By comparing the coefficients of determination R^2 for service level, total cost, and number of rejection among the remaining 12 local regions, regions 908 with search space $R_{908} = (2 \sim 6) * (21 \sim 25) * (9 \sim 11)$ and 1007 with search space $R_{1007} = (2 \sim 6) * (20 \sim 24) * (10 \sim 12)$ are superior to others with good and reliable responses.

Table 5.6: Selected near-optimum regions (based on first-order metamodel)

Local region ID	Local region space			Desirability				Response of regression model		
	ROP	UPTO	CAP	Composite	Service level	Total cost	Rejection	Service level	Total cost	Rejection
736	5~9	19~23	7~9	0.58	0.44	0.84	0.71	93.38%	1582.50	170.00
820	3~7	23~27	8~10	0.89	0.83	0.76	0.89	90.00%	1583.44	166.67
824	4~8	17~21	8~10	0.81	0.78	0.94	0.75	91.18%	1531.00	167.75
845	6~10	18~22	8~10	0.40	0.19	0.92	0.74	95.61%	1541.25	168.50
908	2~6	21~25	9~11	0.92	1.00	0.98	0.73	90.00%	1508.44	169.14
1007	2~6	20~24	10~12	0.89	1.00	0.92	0.69	90.00%	1539.40	171.68
1025	4~8	18~22	10~12	0.80	0.80	0.90	0.90	91.56%	1549.50	157.25
1029	4~8	22~26	10~12	0.90	1.00	0.75	0.89	90.00%	1625.62	157.76
1046	6~10	19~23	10~12	0.33	0.13	0.87	0.78	96.36%	1565.75	165.13
1123	4~8	16~20	11~13	0.89	1.00	0.91	0.69	90.00%	1543.62	171.41
1134	5~9	17~21	11~13	0.78	0.68	0.90	0.88	91.72%	1548.75	158.75
1137	5~9	20~24	11~13	0.89	1.00	0.88	0.72	90.00%	1561.07	169.44
1138	5~9	21~25	11~13	0.90	1.00	0.82	0.80	90.00%	1592.04	163.85
1139	5~9	22~26	11~13	0.92	1.00	0.75	0.97	90.00%	1623.69	152.22
1145	6~10	18~22	11~13	0.48	0.29	0.87	0.71	94.57%	1562.75	170.25
							Low	90.00%	1508.44	152.22
							High	96.36%	1625.62	171.68

Table 5.7: Second-order fitted models' performance, model adequacy and accuracy test results

Local region ID	Local region space			Service Level		System cost		Rejection		Desirability				Surface response		
	ROP	UPTO	CAP	LOF p-val	Adj R ²	LOF p-val	Adj R ²	LOF p-val	Adj R ²	Composite	Service	Total	Rejection	Service	Total cost	Rejection
736	5~9	19~23	7~9	0.526	32.72%	0.542	53.56%	0.798	16.64%	0.67	0.63771	0.686	0.727	92.01%	1657.15	169.08
820	3~7	23~27	8~10	0.506	59.92%	0.536	28.66%	0.792	7.59%	0.84	0.99997	0.658	0.769	90.00%	1671.06	166.17
824	4~8	17~21	8~10	0.152	51.22%	0.391	61.61%	0.377	70.20%	0.88	1	0.864	0.680	90.00%	1568.14	172.41
845	6~10	18~22	8~10	0.046	61.86%	0.005	79.09%	0.078	18.94%	0.55	0.41668	0.861	0.626	93.54%	1569.35	176.17
908	2~6	21~25	9~11	0.664	87.11%	0.335	72.96%	0.826	18.77%	0.91	1	0.835	0.824	90.00%	1582.44	162.35
1007	2~6	20~24	10~12	0.097	82.39%	0.152	73.98%	0.329	44.28%	0.85	0.99883	0.777	0.670	90.00%	1611.54	173.10
1025	4~8	18~22	10~12	0.684	42.23%	0.292	74.47%	0.707	42.46%	0.93	0.99919	0.800	0.956	90.00%	1600.08	153.06
1029	4~8	22~26	10~12	0.117	70.71%	0.518	62.19%	0.009	53.31%	0.90	0.99911	0.752	0.888	89.99%	1624.16	157.81
1046	6~10	19~23	10~12	0.539	48.61%	0.881	85.56%	0.45	19.15%	0.45	0.26425	0.719	0.784	94.86%	1640.31	165.15
1123	4~8	16~20	11~13	0.924	55.17%	0.958	56.05%	0.125	57.66%	0.90	1	0.935	0.712	90.00%	1532.27	170.13
1134	5~9	17~21	11~13	0.729	65.22%	0.421	76.81%	0.496	23.89%	0.86	1	0.846	0.653	90.00%	1577.23	174.27
1137	5~9	20~24	11~13	0.846	46.27%	0.911	50.15%	0.512	33.54%	0.88	1	0.619	0.965	90.00%	1690.34	152.47
1138	5~9	21~25	11~13	0.601	61.02%	0.665	58.41%	0.446	27.52%	0.90	1	0.874	0.749	90.00%	1562.83	167.55
1139	5~9	22~26	11~13	0.406	49.32%	0.726	55.94%	0.204	18.80%	0.84	1	0.701	0.723	90.00%	1649.36	169.40
1145	6~10	18~22	11~13	0.003	64.20%	0.023	75.01%	0.265	28.19%	0.83	1	0.606	0.767	90.00%	1696.95	166.32
				Low	32.72%		28.66%		7.59%					89.99%	1532.27	152.47
				High	87.11%		85.56%		70.20%					94.86%	1696.95	176.17

Note that the adjusted R^2 of order rejection for all surface response models is relatively low, probably because of the loose interconnection between the imposed response and controllable inputs.

5.5.1.3 Residual examination and ANOVA

In order to validate the two superior local regions, they need to satisfy normality assumptions and their significance of inputs tested through ANOVA.

Three second-order response surface models for local region 908 are fitted as described in (5.26), (5.27) and (5.28), where x_1 , x_2 , and x_3 represent *ROP*, *UPTO*, and *CAP*, respectively.

$$y_1 = 4.2724 + 0.0581x_1 + 0.2181x_2 + 0.5329x_3 - 0.0179x_1^2 - 0.003x_2^2 - 0.0176x_3^2 \\ + 0.0026x_1x_2 + 0.091x_1x_3 - 0.0103x_2x_3. \quad (5.26)$$

$$y_2 = 13216.2 - 34.9x_1 + 555.5x_2 - 1182.0x_3 + 30.4x_1^2 + 9.6x_2^2 + 44.0x_3^2 - 3.6x_1x_2 \\ - 18.8x_1x_3 + 18.3x_2x_3. \quad (5.27)$$

$$y_3 = 2759.03 - 17.25x_1 - 121.06x_2 - 225.25x_3 + 1.34x_1^2 + 1.16x_2^2 + 3.88x_3^2 \\ + 0.62x_1x_2 - 0.88x_1x_3 + 6.37x_2x_3. \quad (5.28)$$

Two residual plots, i.e., normal probability plot and “residual vs. fit” plot, are used to verify the normality assumption: the residuals are normally and independently distributed i.e., $\varepsilon \sim NID(0, \sigma^2)$.

In normal probability plot, the values of residual are plotted against its estimated cumulative probability; the scales are transformed as necessary so that the fitted distribution forms a straight line. In other words, if the residuals are normally distributed,

points in “normal probability plots” should generally form a straight line. Each of the normal probability plots in Appendix 5.2 (a), (c), and (e) forms a straight line for individual polynomials.

The points in the second plot, i.e., “residuals versus fits” or “residuals versus predicted response” plot, should display a random pattern on either side of the zero line, suggesting that the variance of the original observations is close to zero for all values of y_i . There should not be any recognizable pattern in the residual plot. In addition, if a point lies far from the majority of points, it may be an outlier. By observing the points in Appendix 5.2 (b), (d), and (f), in general, the residuals are scattered randomly around zero.

Analysis of Variance (ANOVA) gives a summary of the main effects, squared terms, and interactions for second-order polynomials at the level of $\alpha = 0.05$. If the p -value of an input (be it a linear or quadratic) is *less* than 0.05 then it is significant to the corresponding response. If there is at least one input that is significantly related to a response, then evidence exists that the null hypothesis (that none of the inputs is significant) can be rejected.

Based on ANOVA results for local region 908, for the second-order polynomial of service level (Appendix 5.2 (g)), the squared term $ROP*ROP$ ($p_value = 0.000$) is significant, but individual main effects are not significant at the level of $\alpha = 0.05$.

Similar results are found for the second-order polynomial of total cost (Appendix 5.2 (h)) in that squared term $ROP*ROP$ ($p_value = 0.001$) is significant but individual main effects and other second-order terms are not significant at the level of $\alpha = 0.05$.

As for ANOVA for the second-order polynomial of order rejection (Appendix 5.2 (i)), the main effects *UPTO* (p -value = 0.044) and *CAP* (p -value = 0.041) are significant, as is their interaction *UPTO*CAP* (p -value = 0.008). Based on above ANOVA results the null hypotheses can be rejected for the fitted models in local region 908.

Three second-order response surface models for local region 1007 are fitted as described in (5.29), (5.30) and (5.31).

$$\begin{aligned} y_1 = & -3.2969 - 0.0557x_1 + 0.0927x_2 + 0.6637x_3 - 0.0128x_1^2 + 0.0038x_2^2 \\ & - 0.0066x_3^2 + 0.0059x_1x_2 + 0.0091x_1x_3 \\ & - 0.0277x_2x_3. \end{aligned} \quad (5.29)$$

$$\begin{aligned} y_2 = & 9590.56 - 13.53x_1 - 162.91x_2 - 1313.00x_3 + 18.06x_1^2 - 7.56x_2^2 + 10.63x_3^2 \\ & - 5.50x_1x_2 - 8.06x_1x_3 + 54.19x_2x_3. \end{aligned} \quad (5.30)$$

$$\begin{aligned} y_3 = & -1134.02 + 15.57x_1 + 80.32x_2 + 84.71x_3 - 0.94x_1^2 - 1.76x_2^2 - 4.40x_3^2 \\ & - 2.03x_1x_2 + 3.50x_1x_3 - 0.12x_2x_3. \end{aligned} \quad (5.31)$$

Based on two residual plots generated for local region 1007 (Appendix 5.3 (a) to (f)), it can be seen that the normality assumptions are met for polynomials in (5.29), (5.30), and (5.31).

Based on ANOVA result for local region 1007, for the second-order polynomial of service level in Appendix 5.3 (g), the squared term *ROP*ROP* (p -value = 0.023) and interaction term *UPTO*CAP* (p -value = 0.011) are significant at the level of $\alpha = 0.05$ but not individual main effects.

Similar result is found for the second-order polynomial of total cost (Appendix 5.3 (h)) that squared terms $ROP*ROP$ ($p_value = 0.081$) and $UPTO*CAP$ ($p_value = 0.009$) are significant, but individual main effects are not significant at the level of $\alpha = 0.05$.

As for ANOVA for the second-order polynomial of order rejection in local region 1007 (Appendix 5.3 (i)), the interaction term $UPTO*CAP$ ($p_value = 0.062$) is significant. Based on above ANOVA results the null hypotheses can be rejected for the fitted models in local region 1007.

In summary, the second-order response surfaces fitted in both local regions 908 and 1107 meet residual normality assumptions and pass model adequacy and accuracy tests.

5.5.2. New joint control policy

5.5.2.1 Optimal inputs of the generic structure

RSM metamodel is fitted through a continuous search space, so the optimal inputs of best surface responses might not necessary be integers. However, the parameters ROP , $UPTO$ and CAP need to be integers to form the generic structure.

Minitab[®] 16 Response Optimizer tool is used to evaluate new responses interactively by changing inputs within a local region. The tool (Figure 5.7) reserves a column for each of the input parameter, and the curves in each column illustrate the effects of the input on different responses (rows) over the continuous space of the input.

Figure 5.7: Minitab 16® Response Optimizer for illustrating the affects of inputs.

The vertical line in a column represents current setting of the input. The numbers displayed at the top of a column show high, current and low input settings. The horizontal dotted lines and numbers are the corresponding responses of current inputs. The trend of plots reflects the affects of each input to responses, and the particular inputs can be adjusted accordingly. Minitab recalculates responses and desirability for every change made to the model.

Through the interactive evaluation, a total 12 sets of inputs (Table 5.8) are selected from local regions 908 and 1007 for good responses and high composite desirability. In order to distinguish the best inputs from others, all 12 sets of inputs are used in the simulation model for system performance comparison. The simulation results are shown on the right side of Table 5.8 and serve as the basis for selection of the best integer inputs.

Table 5.8: Surface responses and simulation performance for integer inputs

Local region ID	Local region space			Integer Input	Composite Desirability	Surface response			Simulation result (Mean)		
	ROP	UPTO	CAP			Service level	Total cost	Rejection	Service level	Total cost	Rejection
908	2~6	21~25	9~11	(4, 21, 11)	0.93	90.19%	1559.63	156.88	88.28%	1610.63	171.03
908	2~6	21~25	9~11	(4, 24, 9)	0.81	90.73%	1628.23	164.16	93.93%	1568.47	185.53
908	2~6	21~25	9~11	(4, 24, 10)	0.85	89.37%	1646.38	162.03	87.95%	1703.73	159.53
908	2~6	21~25	9~11	(4, 25, 9)	0.84	89.71%	1694.38	159.63	93.75%	1608.63	183.90
1007	2~6	20~24	10~12	(4, 20, 10)	0.85	89.77%	1549.81	177.56	91.94%	1509.47	179.23
1007	2~6	20~24	10~12	(4, 20, 11)	0.81	90.51%	1511.44	181.46	88.87%	1573.20	175.07
1007	2~6	20~24	10~12	(4, 20, 12)	0.88	89.94%	1494.31	176.56	86.35%	1587.63	174.93
1007	2~6	20~24	10~12	(4, 21, 10)	0.80	89.06%	1596.72	176.55	90.49%	1560.47	178.50
1007	2~6	20~24	10~12	(4, 22, 10)	0.81	89.09%	1628.50	172.02	87.95%	1642.77	172.40
1007	2~6	20~24	10~12	(4, 23, 10)	0.86	89.88%	1645.16	163.98	87.01%	1676.67	167.63
1007	2~6	20~24	10~12	(5, 21, 12)	0.81	89.83%	1586.38	181.73	88.81%	1621.17	174.93
1007	2~6	20~24	10~12	(5, 22, 11)	0.81	90.11%	1637.84	176.57	91.33%	1621.10	165.57
					Low	89.06%	1494.31	156.88	86.35%	1509.47	159.53
					High	90.73%	1694.38	181.73	93.93%	1703.73	185.53

Due to consistent performance in surface response and simulation, the set of integer inputs $ROP = 4$, $UPTO = 20$, and $CAP = 10$ in local region 1007 is finally selected to form the generic structure. By using the algorithm described in Table 5.1, the new joint control policy is developed and shown in Table 5.9.

Compared to the targets (Table 5.5) the simulation performance (Table 5.8) suggests that the new control policy performs fairly well: the mean of service level is 91.94%, the mean of total cost is 1509.47 and the mean number of orders rejected is 179.

Table 5.9: Joint control policy developed for $ROP = 4$, $UPTO = 20$, and $CAP = 10$

		WIP(t)										
		0	1	2	3	4	5	6	7	8	9	10
IP(t)	20	14	14	13	12	11	10	9	8	7	6	5
	19	13	13	12	11	10	9	8	7	6	5	4
	18	12	12	11	10	9	8	7	6	5	5	4
	17	11	11	10	9	8	7	6	6	5	4	3
	16	10	10	9	8	8	7	6	5	4	3	2
	15	10	10	9	8	7	6	5	4	3	2	1
	14	9	9	8	7	6	5	4	3	2	1	0
	13	8	8	7	6	5	4	3	2	1	1	0
	12	7	7	6	5	4	3	2	1	2	1	0
	11	6	6	5	4	3	2	1	0	2	1	0
	10	5	5	4	3	2	1	0	3	2	1	0
	9	4	4	3	2	1	0	4	3	2	1	0
	8	3	3	2	1	0	5	4	3	2	1	0
	7	2	2	1	0	6	5	4	3	2	1	0
	6	1	1	0	7	6	5	4	3	2	1	0
	5	10	0	8	7	6	5	4	3	2	1	0

5.5.2.2. New policy performance

The performance comparisons are made between the heuristic policy in Table 5.9, benchmark policy in Table 5.10, and traditional way of two independent controls, in which semi-finished module inventory replenishment and order admission are determined separately and independently.

The benchmark policy (Table 5.10) is developed by employing Bellman equation based on inventory policy $(s, S) = (ROP, UPTO) = (4, 20)$, which is optimal for the specific state space and exhibits the best codependency between admission control and inventory policy as discussed before.

Table 5.10: The benchmark joint control policy developed using Bellman equation

		WIP(t)										
		0	1	2	3	4	5	6	7	8	9	10
IP(t)	20	10	10	10	9	8	7	6	5	4	3	3
	19	10	10	9	8	7	6	5	4	4	3	2
	18	10	10	9	8	7	6	5	4	3	3	2
	17	10	9	8	7	6	5	5	4	3	2	2
	16	9	9	8	7	6	5	4	3	2	2	1
	15	8	8	7	6	5	4	4	3	2	1	1
	14	8	8	7	6	5	4	3	2	1	1	0
	13	7	7	6	5	4	3	2	2	1	0	2
	12	6	6	5	4	3	2	2	1	0	2	2
	11	5	5	4	3	2	2	1	0	2	3	2
	10	4	4	3	3	2	1	0	4	4	4	3
	9	4	4	3	2	1	0	6	5	4	3	3
	8	3	3	2	1	0	8	6	5	3	3	1
	7	2	2	1	0	8	7	5	4	1	1	1
	6	1	1	0	9	7	6	4	1	1	1	1
	5	10	0	9	7	6	5	1	1	1	1	1

In the development of independent controls, the optimal periodic inventory control policy is identified as $(s, S) = (6, 18)$ using Power Approximation algorithm (as discussed in Section 4.3). Through simulation test, when the critical number of Max_WIP is set to 9, the on-time delivery rate is close the target of 90%. By adopting this configuration, the hybrid system can admit new orders if and only if $WIP < Max_WIP = 9$.

All three control policies, i.e. the heuristic policy, the benchmark policy, and the traditional way of independent controls, are applied to the simulation model separately for performance comparison; the numerical results are showed in Table 5.11, in which the total cost is further broken down into individual costs: inventory holding cost, setup (replenishment) cost, order late delivery cost, and order rejection cost.

According to the results, in general, the performance of the independent controls is inferior to joint control policies. Due to the lack of interconnection between inventory and production decisions, the independent admission control admits more orders resulting in the highest late delivery cost of \$130.10. Meanwhile, the independent controls incur the highest setup (replenishment) cost of \$460.83 compared to \$315.83 and \$274.17 of other two policies, showing that the inventory policy could not cope with the demand as well as the joint control policies do.

Table 5.11: System performance comparison between the new policy and benchmark policy

		Simulation result (Mean)						
Optimization approach	Inputs	Service level	System cost (\$)					Number of Rejections
			Holding	Setup	Late delivery	Order rejections	Total cost	
<Independent inventory and admission controls>								
	(s, S) = (6, 18) MAX_WIP=10	81.02%	879.80	455.00	231.90	136.40	1703.10	136.40
<Joint admission-inventory control >								
RSM	ROP=4, UPTO=20, CAP=10	91.94%	914.00	315.83	100.00	179.23	1509.47	179.23
MDP	(s, S) = (4, 20) MAX_WIP=10	94.06%	875.27	274.17	74.20	201.63	1425.27	201.63
	Difference	-2.12%	38.73	41.66	25.80	-22.40	84.20	-22.40
	Difference (%)	-2.25%	4.24%	13.19%	25.80%	-11.11%	5.58%	-11.11%

In the comparison between the heuristic policy and benchmark policy, the numerical results show that the service level for the benchmark policy reaches as high as 94.06% whereas the service level for the new policy is 91.94% or 2.25% lower; the latter is actually closer to the target of 90% on-time delivery.

The time scale of on-time delivery for the two joint control policies are different: in MDP model it is measured in weekly basis, and an order can meet the target delivery lead-time of two weeks only if it is produced within two weeks (two time units) inclusive of the

week of arrival; whereas in RSM-based optimization, on-time delivery is measured on an hourly basis, and an order can meet the delivery target so long it is produced within 336 hours (two weeks) from the moment of the arrival. As result, the benchmark policy is more rigorous and conservative.

The numerical result also shows that the new heuristic policy incurs \$22.40 less on average in order rejection compared to the benchmark policy, by accepting extra $(\$22.40 / u^r) = (\$22.40 / \$1) \approx 22$ orders. The result is consistent with the holding cost in that additional \$38.73 are incurred by the heuristic policy for admitting extra $(\$38.73 / u^l) = (\$38.73 / \$2) \approx 20$ orders.

As a consequence of extra admissions, the heuristic policy requires additional $(\$41.66 / u^s) = (\$41.66 / \$25) \approx 2$ setups and causes additional $(\$25.80 / u^l) = (\$25.80 / \$3) = 9$ late deliveries. The total cost incurred by the new policy is 5.58% higher than incurred by the benchmark policy.

In summary, compared to the benchmark policy, which is more conservative, the heuristic policy developed using RSM performs fairly well with “better” on-time delivery, lower rejection cost, and slightly higher total cost.

5.6. Summary

This chapter proposes a closed-loop optimization procedure for identifying three optimal controllable inputs *ROP*, *UPTO* and *CAP* of the generic structure. RSM is employed as an optimization tool to synthesize response surfaces based on the sample of responses; a simulation model is developed which acts as a proxy for the hybrid system and helps

collect system responses, such as on-time delivery rate, total cost, and number of orders rejected.

The application of the methodology is then demonstrated using the generic policy parameters found in the previous chapter. The response variables are fitted simultaneously over small local regions individually. During first-order regression fitting, based on the selection criteria, 15 local regions are selected from a total of 250 candidate regions, which are near optimum. The settings of ROP , $UPTO$ and CAP in each of the selected regions reflect the codependency between admission and inventory control. Subsequently, second-order response surface models are fitted over the candidate regions. By comparing their performance, and adequacy and accuracy tests, eventually two local regions are selected: local region 908 with search space, $R_{908} = (2\sim6) \times (21\sim25) \times (9\sim11)$ and local region 1007 with search space, $R_{1007} = (2\sim6) \times (20\sim24) \times (10\sim12)$. The residual plots show that both the local regions meet the residual normality assumption, and the adequacy and accuracy tests suggest second-order polynomials over the two local regions fit the simulation observation well.

The search for the best integer inputs continues within the two local regions. By comparing surface responses and simulation performance of different sets of integer inputs, the inputs $ROP = 4$, $UPTO = 20$, and $CAP = 10$ are finally identified as the best integer inputs to form the generic structure, which is used to develop the new joint control policy.

Meanwhile, a benchmark policy is generated using Bellman equation based on inventory policy $(s, S) = (ROP, UPTO) = (4, 20)$. The numerical results suggest that compared to

the benchmark policy, which is more conservative, the new policy achieves a “better” service level (closer to target) but admits 20 more orders and incurs slightly higher total cost over a time horizon of 52 weeks.

The new approach to develop the joint control policy represents a more effective way that quickly reaches near optimal solutions. The advantage and effectiveness of the new approach, which relies on the identification of the optimal generic structure, are demonstrated in this chapter.

Appendix 5.1: First-order fitting results

Local region ID	Local region			Desirability				Optimum Prediction		
	ROP	UPTO	CAP	Composite	Service level	Total cost	Rejection	Service level	Total cost	Rejection
707	2~6	20~24	7~9	0.87	1	0.974	0.5775	90.00%	1512.99	179.58
718	3~7	21~25	7~9	0.85	0.99807	0.94346	0.56813	89.98%	1528.27	180.23
736	5~9	19~23	7~9	0.58	0.438435	0.835	0.71429	93.38%	1582.50	170.00
802	2~6	15~19	8~10	0.86	1	1	0.55212	90.00%	1490.01	181.35
807	2~6	20~24	8~10	0.87	1	0.88552	0.65184	90.00%	1557.24	174.37
815	3~7	18~22	8~10	0.86	1	1	0.5574	90.00%	1490.20	180.98
816	3~7	19~23	8~10	0.87	1	0.70523	0.80277	90.00%	1647.39	163.81
820	3~7	23~27	8~10	0.89	0.83312	0.7618	0.889123	90.00%	1583.44	166.67
824	4~8	17~21	8~10	0.81	0.778232	0.938	0.74643	91.18%	1531.00	167.75
827	4~8	20~24	8~10	0.85	1	0.65588	0.81391	90.00%	1672.06	163.03
845	6~10	18~22	8~10	0.40	0.192539	0.9175	0.73571	95.61%	1541.25	168.50
906	2~6	19~23	9~11	0.88	1	0.83586	0.71984	90.00%	1582.07	169.61
908	2~6	21~25	9~11	0.92	1	0.98313	0.72654	90.00%	1508.44	169.14
910	2~6	23~27	9~11	0.88	1	0.69743	0.85901	90.00%	1651.28	159.87
913	3~7	16~20	9~11	0.88	1	0.9691	0.62945	90.00%	1515.45	175.94
914	3~7	17~21	9~11	0.87	1	0.890745	0.63305	90.00%	1554.63	175.69
915	3~7	18~22	9~11	0.86	1	1	0.55108	90.00%	1430.96	181.42
916	3~7	19~23	9~11	0.87	1	0.91178	0.61602	90.00%	1544.11	176.88
925	4~8	18~22	9~11	0.87	1	0.83687	0.67201	90.00%	1581.57	172.96
926	4~8	19~23	9~11	0.86	1	0.86686	0.64532	90.00%	1566.57	174.83
928	4~8	21~25	9~11	0.88	0.99945	0.82396	0.73079	90.00%	1588.02	168.84
929	4~8	22~26	9~11	0.85	1	0.71243	0.74596	90.00%	1643.78	167.78
930	4~8	23~27	9~11	0.88	1	0.78966	0.75227	90.00%	1605.17	167.34
1004	2~6	17~21	10~12	0.89	0.99489	0.95508	0.65806	89.95%	1522.46	173.94
1006	2~6	19~23	10~12	0.87	1.00	0.89	0.63	90.00%	1554.04	175.66
1007	2~6	20~24	10~12	0.89	1.00	0.92	0.69	90.00%	1539.40	171.68
1014	3~7	17~21	10~12	0.88	1.00	0.90	0.66	90.00%	1552.04	173.60
1016	3~7	19~23	10~12	0.89	1.00	0.92	0.67	90.00%	1539.65	173.31
1020	3~7	23~27	10~12	0.87	1.00	0.63	0.91	90.00%	1683.18	156.07
1023	4~8	16~20	10~12	0.87	1.00	1.00	0.57	90.00%	1500.14	179.83

Appendix 5.1 (cont')

Local region ID	Local region			Desirability				Optimum Prediction		
	ROP	UPTO	CAP	Composite	Service level	Total cost	Rejection	Service level	Total cost	Rejection
1024	4~8	17~21	10~12	0.89	1.00	0.86	0.81	90.00%	1568.46	163.38
1025	4~8	18~22	10~12	0.80	0.80	0.90	0.90	91.56%	1549.50	157.25
1026	4~8	19~23	10~12	0.88	1.00	0.93	0.65	90.00%	1534.98	174.42
1028	4~8	21~25	10~12	0.87	1.00	0.70	0.81	90.00%	1651.12	163.61
1029	4~8	22~26	10~12	0.90	1.00	0.75	0.89	90.00%	1625.62	157.76
1037	5~9	20~24	10~12	0.89	1.00	0.72	0.86	90.00%	1638.64	159.66
1038	5~9	21~25	10~12	0.88	1.00	0.70	0.84	90.00%	1651.23	161.15
1039	5~9	22~26	10~12	0.85	1.00	0.73	0.73	90.00%	1636.15	168.83
1046	6~10	19~23	10~12	0.33	0.13	0.87	0.78	96.36%	1565.75	165.13
1102	2~6	15~19	11~13	0.86	1.00	0.96	0.57	90.00%	1518.71	180.44
1103	2~6	16~20	11~13	0.86	1.00	0.82	0.68	90.00%	1589.01	172.35
1107	2~6	20~24	11~13	0.88	1.00	0.87	0.67	90.00%	1562.84	173.03
1110	2~6	23~27	11~13	0.87	1.00	0.66	0.87	90.00%	1669.19	159.31
1114	3~7	17~21	11~13	0.88	1.00	0.82	0.73	90.00%	1589.82	169.24
1116	3~7	19~23	11~13	0.86	1.00	0.62	0.88	90.00%	1688.60	158.06
1119	3~7	22~26	11~13	0.88	1.00	0.77	0.77	90.00%	1616.15	165.89
1123	4~8	16~20	11~13	0.89	1.00	0.91	0.69	90.00%	1543.62	171.41
1125	4~8	18~22	11~13	0.85	1.00	1.00	0.53	90.00%	1492.59	183.05
1126	4~8	19~23	11~13	0.89	1.00	0.97	0.65	89.99%	1515.23	174.76
1127	4~8	20~24	11~13	0.88	1.00	0.78	0.76	90.00%	1610.49	166.97
1128	4~8	21~25	11~13	0.86	1.00	0.75	0.74	90.00%	1626.04	168.44
1134	5~9	17~21	11~13	0.78	0.68	0.90	0.88	91.72%	1548.75	158.75
1137	5~9	20~24	11~13	0.89	1.00	0.88	0.72	90.00%	1561.07	169.44
1138	5~9	21~25	11~13	0.90	1.00	0.82	0.80	90.00%	1592.04	163.85
1139	5~9	22~26	11~13	0.92	1.00	0.75	0.97	90.00%	1623.69	152.22
1145	6~10	18~22	11~13	0.48	0.29	0.87	0.71	94.57%	1562.75	170.25

A total of 65 local regions are identified which have good responses and high composite desirability.

Appendix 5.2: Residual examination & ANOVA for local region 908

(a) The residual normal probability plot and (b) residuals versus fits plot for service level over local region 908 (Minitab® 16)

(c) The residual normal probability plot and (d) residuals versus fits plot for total cost over local region 908 (Minitab® 16)

(e) The residual normal probability plot and (f) residuals versus fits plot for rejection over local region 908 (Minitab® 16)

Local region 908 Analysis of Variance for Service Level						
Source		DF	Seq SS	Adj SS	Adj MS	P
Regression		9	0.339	0.339	0.038	22.78
Linear		3	0.293	0.005	0.002	1.06
ROP		1	0.278	0.000	0.000	0.26
UPTO		1	0.009	0.002	0.002	1.36
CAP		1	0.007	0.004	0.004	2.41
Square		3	0.039	0.039	0.013	7.86
ROP*ROP		1	0.036	0.038	0.038	22.78
UPTO*UPTO		1	0.001	0.001	0.001	0.63
CAP*CAP		1	0.002	0.002	0.002	1.38
Interaction		3	0.007	0.007	0.002	1.4
ROP*UPTO		1	0.001	0.001	0.001	0.52
ROP*CAP		1	0.003	0.003	0.003	1.6
UPTO*CAP		1	0.003	0.003	0.003	2.07
Residual Error		20	0.033	0.033	0.002	
Lack-of-Fit		3	0.003	0.003	0.001	0.54
Pure Error		17	0.030	0.030	0.002	
Total		29	0.372			

(g) ANOVA for service level (Minitab® 16) over local region 908

Local region 908 Analysis of Variance for Total Cost						
Source		DF	Seq SS	Adj SS	Adj MS	P
Regression		9	626089.00	626089.00	69565.00	9.7
Linear		3	479015.00	27521.00	9174.00	1.28
ROP		1	250500.00	156.00	156.00	0.02
UPTO		1	197803.00	14575.00	14575.00	2.03
CAP		1	30713.00	19605.00	19605.00	2.73
Square		3	123410.00	123410.00	41137.00	5.73
ROP*ROP		1	99977.00	109201.00	109201.00	15.22
UPTO*UPTO		1	9150.00	10934.00	10934.00	1.52
CAP*CAP		1	14283.00	14283.00	14283.00	1.99
Interaction		3	23663.00	23663.00	7888.00	1.1
ROP*UPTO		1	1682.00	1682.00	1682.00	0.23
ROP*CAP		1	11250.00	11250.00	11250.00	1.57
UPTO*CAP		1	10731.00	10731.00	10731.00	1.5
Residual Error		20	143505.00	143505.00	7175.00	
Lack-of-Fit		3	25325.00	25325.00	8442.00	1.21
Pure Error		17	118180.00	118180.00	6952.00	
Total		29	769594.00			

(h) ANOVA for total cost (Minitab® 16) over local region 908

Local region 908 Analysis of Variance for Rejection							
Source		DF	Seq SS	Adj SS	Adj MS	F	P
Regression		9	2354.95	2354.95	261.66	1.74	0.144
Linear		3	560.25	1135.94	378.65	2.52	0.087
ROP		1	49.00	38.08	38.08	0.25	0.62
UPTO		1	169.00	692.16	692.16	4.61	0.044
CAP		1	342.25	712.00	712.00	4.75	0.041
Square		3	419.70	419.70	139.90	0.93	0.443
ROP*ROP		1	169.74	213.35	213.35	1.42	0.247
UPTO*UPTO		1	139.08	157.96	157.96	1.05	0.317
CAP*CAP		1	110.88	110.88	110.88	0.74	0.4
Interaction		3	1375.00	1375.00	458.33	3.06	0.052
ROP*UPTO		1	50.00	50.00	50.00	0.33	0.57
ROP*CAP		1	24.50	24.50	24.50	0.16	0.690
UPTO*CAP		1	1300.50	1300.50	1300.50	8.67	0.008
Residual Error		20	2999.75	2999.75	149.99		
Lack-of-Fit		3	150.25	150.25	50.08	0.3	0.826
Pure Error		17	2849.50	2849.50	167.62		
Total		29	5354.70				

(i)ANOVA for number of rejection (Minitab® 16) over local region 908

Appendix 5.3: Residual examination & ANOVA for local region 1007

(a) The residual normal probability plot and (b) residuals versus fits plot for service level over local region 1007 (Minitab® 16)

(c) The residual normal probability plot and (d) residuals versus fits plot for total cost over local region 1007 (Minitab® 16)

(e) The residual normal probability plot and (f) residuals versus fits plot for rejection over local region 1007 (Minitab® 16)

Local region 1007 Analysis of Variance for Service Level							
Source		DF	Seq SS	Adj SS	Adj MS	F	P
Regression		9	0.459	0.459	0.051	16.07	0.000
Linear		3	0.406	0.006	0.002	0.64	0.601
ROP		1	0.322	0.000	0.000	0.12	0.732
UPTO		1	0.036	0.000	0.000	0.13	0.719
CAP		1	0.048	0.005	0.005	1.71	0.206
Square		3	0.022	0.022	0.007	2.31	0.108
ROP*ROP		1	0.020	0.019	0.019	6.04	0.023
UPTO*UPTO		1	0.002	0.002	0.002	0.53	0.477
CAP*CAP		1	0.000	0.000	0.000	0.1	0.754
Interaction		3	0.032	0.032	0.011	3.32	0.041
ROP*UPTO		1	0.004	0.004	0.004	1.38	0.254
ROP*CAP		1	0.003	0.003	0.003	0.83	0.372
UPTO*CAP		1	0.025	0.025	0.025	7.75	0.011
Residual Error		20	0.064	0.064	0.003		
Lack-of-Fit		3	0.019	0.019	0.006	2.47	0.097
Pure Error		17	0.044	0.044	0.003		
Total		29	0.523				

(g) ANOVA for service level (Minitab® 16) over local region 1007

Local region 1007 Analysis of Variance for Total Cost							
Source		DF	Seq SS	Adj SS	Adj MS	F	P
Regression		9	1042454.00	1042454.00	115828.00	10.16	0.000
Linear		3	894034.00	21256.00	7085.00	0.62	0.609
ROP		1	396585.00	23.00	23.00	0	0.965
UPTO		1	393443.00	1307.00	1307.00	0.11	0.738
CAP		1	104006.00	21224.00	21224.00	1.86	0.188
Square		3	48507.00	48507.00	16169.00	1.42	0.267
ROP*ROP		1	40504.00	38548.00	38548.00	3.38	0.081
UPTO*UPTO		1	7170.00	6757.00	6757.00	0.59	0.45
CAP*CAP		1	834.00	834.00	834.00	0.07	0.79
Interaction		3	99913.00	99913.00	33304.00	2.92	0.059
ROP*UPTO		1	3872.00	3872.00	3872.00	0.34	0.566
ROP*CAP		1	2080.00	2080.00	2080.00	0.18	0.674
UPTO*CAP		1	93961.00	93961.00	93961.00	8.24	0.009
Residual Error		20	227929.00	227929.00	11396.00		
Lack-of-Fit		3	59508.00	59508.00	19836.00	2	0.152
Pure Error		17	168421.00	168421.00	9907.00		
Total		29	1270383.00				

(h) ANOVA for total cost (Minitab® 16) over local region 1007

Local region 1007 Analysis of Variance for Rejection							
Source		DF	Seq SS	Adj SS	Adj MS	F	P
Regression		9	4313.41	4313.41	479.27	3.56	0.009
Linear		3	2853.12	359.47	119.82	0.89	0.463
ROP		1	217.56	29.87	29.87	0.22	0.643
UPTO		1	2626.56	317.71	317.71	2.36	0.14
CAP		1	9.00	88.34	88.34	0.66	0.427
Square		3	539.66	539.66	179.89	1.34	0.291
ROP*ROP		1	65.21	105.00	105.00	0.78	0.388
UPTO*UPTO		1	331.75	364.00	364.00	2.7	0.116
CAP*CAP		1	142.70	142.70	142.70	1.06	0.315
Interaction		3	920.63	920.63	306.88	2.28	0.110
ROP*UPTO		1	528.13	528.13	528.13	3.92	0.062
ROP*CAP		1	392.00	392.00	392.00	2.91	0.103
UPTO*CAP		1	0.50	0.50	0.50	0	0.952
Residual Error		20	2691.96	2691.96	134.60		
Lack-of-Fit		3	480.13	480.13	160.04	1.23	0.329
Pure Error		17	2211.83	2211.83	130.11		
Total		29	7005.37				

(i)ANOVA for number of order rejection (Minitab® 16) over local region 1007

Chapter 6 Summary and Future Research

To tackle the challenges of high product variety and responsiveness in mass customization, many manufacturing organizations implement postponement strategy and restructure their supply chains by adopting a hybrid model consisting of MTS and MTO systems. Researchers have developed control policies for such hybrid systems which can lower system cost by taking advantage of economies of scale of the MTS stage and satisfy the requirement of high product variety by taking advantage of flexibility of the MTO stage. Some researchers have also noted that the performance of hybrid supply chains suffers when system capacity is constrained.

Literature review conducted in chapter 2 found that the state-of-the-art research in hybrid supply chain is mainly strategy-based: by (re)positioning the decoupling point, the research focuses on (re)engineering the entire supply chain so that the system can achieve better balance between system flexibility, cost, and responsiveness. But the strategy-based research cannot solve the performance issue appropriately due to lack of support from operational policies; there are hardly any solutions that can generate effective operational decision policies for ensuring target system performance. The lack of support from operational decision policy for control of hybrid supply chains is the gap addressed by this research.

In order to understand system behaviour, the characteristics of the stochastic processes representing the variables IP and WIP and their interaction are studied in chapter 3. It is found that with limited hybrid system capacity, the two control mechanisms interact and influence each other in controlling the stochastic processes; thus it becomes imperative

that the two control mechanisms be integrated into a joint admission-inventory control policy.

A joint admission-inventory control policy, proposed in this research, consists of a sequence of decisions over multiple periods. For a single period, the search space of the joint control policy is a multiple of reorder point, order up-to-level, and the maximum number of orders that can be admitted in the period. Over multi-period planning horizon, the development of joint control policy faces the “curse of dimensionality” precluding an exhaustive search for the optimal policy in polynomial time. In addition, the performance comparison of candidate joint control policies is also challenging as optimization needs to satisfy dual objectives of minimum cost and on-time delivery rate.

In the development of joint control policy, an *optimal* joint control policy is of immense research interest, as it should be able to control the two stochastic processes $\{WIP_t\}$ and $\{IP_t\}$ jointly by making use of their interaction.

In chapter 4, the interaction between the two stochastic processes $\{(IP, WIP)_t\}$ is studied for exhibiting and exploring the codependency between the two control mechanisms (admission control and inventory policy) in a structured way. The stochastic process is modeled using discrete MDP, and it is proved that the joint control policy developed is a uniquely optimal solution of the original optimization problem. In other words, for a specific state space, the unique joint control policy exhibits the optimal codependency of the best match between admission control and inventory policies.

A generic structure is then obtained through analysis of the unique optimal solution that reflects the codependency. The generic structure is formulated as a two-dimensional matrix with three parameters: the reorder point (ROP) and order up-to-level ($UPTO$)

define the y -axis, while the third parameter Max_WIP (CAP), defines the x -axis. The analysis of the generic structure leads to the following structural properties:

- (1) Members in the matrix can be divided into two categories based on a “setup-line”. Members in first category (group $n1$) require setup, while the rest do not.
- (2) Members that do not require setup (second category) can be further divided into multiple groups based on expected inventory position (IP_{t+1}) at the next review epoch. The division starts with members having $IP_{t+1} = (s+1)$ that are assigned to group $n2$, and a new group is formed when IP_{t+1} increases by one each time.
- (3) The maximum number of jobs during a period for group $n1$ (i.e., the maximum number of jobs the hybrid system can handle during a new inventory cycle after a setup) can be normalized into one number, represented by parameter CAP .

Relying on the structural properties, the development of the joint control policy is converted into identification of the optimal inputs of the generic structure.

In chapter 5, the application of the new approach is shown using the generic policy parameters. The optimal inputs ROP , $UPTO$ and CAP of the generic structure are identified through RSM and a new joint control policy is developed.

Input parameters ROP , $UPTO$ and CAP form a search space, and in order to explore and identify the optimal inputs three response variables (on-time delivery rate, total cost, and number of orders rejected) are fitted to first-order regression models and second-order response surface models over the search space. The numerical results show that, compared to the benchmark policy developed using Bellman equation, the new policy developed using the proposed approach achieves a “better” service level (closer to the target of 90% on-time delivery) but accepts about 20 additional orders and incurs a

slightly higher total system cost over a time horizon of 52 weeks. In the process of developing the new joint control policy, the advantage and effectiveness of the new approach to policy development, which relies on the identification of the optimal generic structure, is demonstrated.

This research successfully addresses the system performance issue of hybrid supply chains and overcomes the “curse of dimensionality” by proposing an innovative approach to the development of joint control policy. The key contributions to the literature are as follows:

- Firstly, by identifying the research gap (lack of support of operational decision policy) this work takes the hybrid supply chain research to the next level. The alignment between supply chain strategy and operational decision policy is critical since an optimal supply chain structure (including a decoupling point) still requires strong support from operational decision policy. So the development of operational decision policy is a necessary step to complement the state-of-the-art strategy-based research.
- Secondly, this research proposes a new joint admission-inventory control policy to bridge the research gap, successfully addressing the outstanding performance issue of hybrid MTS-MTO supply chains—the performance of the system declines as the production constraints become more severe—by integrating two control mechanisms of admission control and inventory into one decision process.
- Thirdly, this research proves that the joint control policy developed using Bellman equation is uniquely optimal for a specific state space, which exhibits the codependency, i.e., the best match, between the two control mechanisms

(admission control and inventory policy). This proof leads to the formulation of the generic structure.

- Fourthly, the creation of the generic structure is a major contribution of this research to the development of the joint control policy. This is achieved by converting the problem of the optimal joint control policy into identification of the optimal parameters of the generic structure. Latter is a typical combinatorial optimization problem with a constrained search space; more importantly, there are many mature optimization techniques available for solving combinatorial optimization problems. The creation of the generic structure leads to a radically new approach to the development of joint control policy that effectively overcomes the “curse of dimensionality”.
- Finally, this research demonstrates the advantage and effectiveness of the new approach with the help of extensive experiments.

Hybrid MTS-MTO supply chains are complex by nature, and there are a number of outstanding issues that can be addressed in the future. First of all, this research focuses on addressing the impact of limited capacity in MTO stage with the assumption of infinite capacity in MTS stage. According to Gupta & Benjaafar (2004), when the utilization in MTS stage is high, the cost advantage of new system over pure MTS system diminishes. The negative impact of restrictive capacity in MTS stage still needs to be addressed. The design of a control process that takes into account production at both MTS and MTO stages, as well as semi-finished inventory control, remains a challenge.

Secondly, current research studies a two-stage supply chain; however, in practice supply chains could present a more complex structure with multiple strategic inventory nodes for

multiple levels of product differentiation. The development of decision policy for supporting postponement under such circumstances remains to be tackled.

Thirdly, when optimizing the generic structure of large-scale problems the following need to be considered:

- (1) A systematic approach is required for division of state groups; and
- (2) A suitable global optimization or approximate solution approach, (meta-heuristics such as evolutionary algorithms), or some combination of available methodologies may need to be considered for exploring globally optimal inputs.

List of Publications

- Wang, F., Piplani, R., Lim, R., Lee, E.W., 2011. Development of an Optimal Decision Policy for MTS-MTO System, POMS 22nd Annual Conference, Reno, Nevada, U.S. 29 April to 2 May, 020-0427.
- Wang, F., Xu, L.X.X., Lim, R., Lee, E.W., Zarzycki, M., 2011. Simulation-Based Operational Decision Analysis at Decoupling Point in MTS-MTO System, IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), Singapore, 6-9 Dec, pp. 56-60.
- Wang, F., Piplani, R., Xu, L.X.X., Bhullar, A.S., 2012. MTS Lead Time Uncertainty Study in Periodic Review MTS-MTO system, IEEE International Conference on Industrial Engineering and Engineering Management (IEEM), Hong Kong, 10-13 Dec, pp. 1368-1372.
- Wang, F., Piplani, R., Xu, L.X.X., 2014. Joint Admission and Inventory Control Policy Development Using Response Surface Methodology, POMS Singapore 2014, Singapore. 21-23 July, 052-0083.
- Wang, F., Piplani, R., 2014. Development of Joint Admission-inventory Control Policy for Hybrid Supply Chains. European Journal of Operational Research, submitted.
- Wang, F., Piplani, R., 2015. Joint Admission-Inventory Control Policy Development for Hybrid Supply Chain Using Response Surface Methodology. Computers and Operations Research, in preparation.

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