Safety and risk assessment of damaged multi-planar square hollow section (SHS) TT-, YT- and KT-joints

Sukumara Pillai, Vipin

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**Safety and Risk Assessment of Damaged Multi-
planar Square Hollow Section (SHS) TT-, YT-
and KT-joints**

Vipin Sukumara Pillai

School of Civil & Environmental Engineering

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Vipin Sukumara Pillai

School of Civil & Environmental Engineering

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Summary

Cracks are commonly discovered at the hot spot location of any tubular joint; it can be due to fatigue, accidental damage or earthquake. As a consequence, the plastic collapse load (P_c) of the tubular joints is reduced, and hence it is necessary to produce design guidance which can safely be used to estimate the static residual strength of cracked tubular structures in practice. The BS7910 (2005) and RP579/ASME FFS-1 (2007) have a provision for engineers to assess and predict quantitatively the instability loads due to cracks and defects in tubular welded structures. However, the recommendation is strictly applicable only for uni-planar tubular circular hollow section (CHS) joints, and it is strictly not applicable for cracked square hollow section (SHS) joints. There are many existing structures such as bridges, buildings, giant “Ferris” wheels and others which are fabricated using SHS members and these structures are multi-planar in nature. Therefore, there is a need to extend the above procedures for estimating the static residual strength of cracked multi-planar tubular SHS joints.

In order to achieve the research objectives, a completely new and robust finite element (FE) mesh generator is developed in this study. The newly developed automatic FE mesh generator is validated using the previous experimental test data as well as with the existing commercial software results. The new FE mesh generator is able to overcome the drawbacks of the previous mesh generator approaches by demonstrating the capability of achieving convergence of solutions even at a very high plastic deformation. The new FE mesh generator is versatile enough to model cracks of any arbitrary dimensions. The concept of using key-hole for elastic-plastic analysis is effectively incorporated in the new FE mesh generator. The “spider web” configuration which is the most efficient mesh design for the crack tip region enables a smooth conversion from a fine mesh to a coarser mesh away from the high stress concentration region. Mesh refinement and zoning techniques are extensively used in the new FE mesh generator so that it is robust to model different types of uni-planar and multi-planar SHS welded joints with and without cracks.

The new FE mesh generator is further used in the extensive parametric studies carried out on rectangular cracked plate with and without attachments. Subsequently, a new set of

equations are proposed for determining M_k , the weld toe magnification factor of cracked double-sided T-butt joints and cruciform X-joints subjected to both axial and bending loading. The additional fillet weld in X-joint is found to restrain the upward and opening movement of the crack.

The new FE mesh generator is subsequently used to study the plastic collapse and fracture behaviour of both cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The effect of the additional brace on uni-planar joints is investigated as well. Extensive parametric study is carried out for uni-planar SHS T-, Y- and K-joints and multi-planar SHS TT-, YT- and KT-joints subjected to axial tensile loading at the brace end. Subsequently, new equations for determining the reduction factor (F_{AR}) of six types of analysed SHS joints are proposed. The F_{AR} when multiplied with the P_c load of uncracked SHS joints, gives P_c load of cracked SHS joints. The results reveal that the crack area and the brace to chord width ratio (β) have a significant influence on the P_c load of the cracked SHS joints. It is also concluded that the additional multi-planar brace increases the P_c load of cracked SHS joints.

The fracture behaviour of both cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints is also investigated by using the failure assessment diagram (FAD) approach in the present study. The highest level, Level 3C FAD curve of cracked SHS joints is constructed using the full numerical results of the elastic J -integral (J_e), elastic-plastic J -integral (J_{ep}) values and P_c load. The constructed Level 3C FAD curves are further used to validate the standard Level 2A curve of the BS7910 (2005). It is found that standard Level 2A curve of the BS7910 (2005) is not always safe in assessing the safety and integrity of cracked SHS joints. Therefore, penalty factors of 1.2 and 1.1 for P_c load are recommended to move all the constructed Level 3C FAD curves above the standard Level 2A curve for cracked uni-planar and multi-planar SHS joints, respectively. These new Level 3C FAD curves are found to produce optimal solutions and they can be used to assess the safety and integrity of cracked uni-planar SHS T-, Y-, K-joints and multi-planar SHS TT-, YT-, KT-joints.

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List of Symbols

The nomenclatures used in the thesis are listed in alphabetical order in the following list. A symbol whose usage is restricted to only a specific analysis is not mentioned here, and it is explained at the appropriate area in the thesis.

a	= crack depth
A_c	= crack area
b_0	= chord width
b_1, b_2	= brace width
B	= plate width
c	= crack half length
E	= Young's modulus
F_{AR}	= reduction factor
g	= the gap distance of the gaped K-joint
h	= attachment height in plate-to-plate joints
h_0	= chord height
h_1, h_2	= brace height
h_w	= weld height
J	= J -integral
J_e	= elastic J -integral
J_{ep}	= elastic-plastic J -integral
J_{mat}	= J -integral fracture toughness
K	= stress intensity factor
K_I	= Mode-I stress intensity factor
K_{II}	= Mode-II stress intensity factor
K_{III}	= Mode-III stress intensity factor

K_{eff}	= effective stress intensity factor
K_{IC}	= plane strain stress intensity factor fracture toughness
K_{mat}	= stress intensity factor fracture toughness
K_r	= fracture ratio using stress intensity factor
l_c	= chord length
l_{c1}	= crack length along the weld toe parallel to the chord side wall
l_{c2}	= crack length along the weld toe perpendicular to the chord side wall
L	= attachment footprint width
L_r	= ratio of applied load to plastic collapse load
M_k	= weld toe magnification factor
P	= applied load
P_c	= plastic collapse load
t	= attachment thickness in plate-to-plate joints
t_0	= chord wall thickness
t_1, t_2	= brace wall thickness
t_w	= weld thickness
T	= main plate thickness
Y	= shape factor
α	= chord length to half chord width ratio
β	= brace to chord width ratio
2γ	= chord width to chord wall thickness ratio
η	= brace depth to chord width ratio
δ_I	= crack tip opening displacement (CTOD)
δ_{mat}	= crack tip opening displacement fracture toughness
ε	= principle strain
ψ	= dihedral angle of chord and brace
ν	= Poisson's ratio
θ	= weld angle
θ_1, θ_2	= brace to chord angle

σ	= applied stress
σ_y	= yield stress
τ	= brace wall thickness to chord wall thickness ratio

Subscripts

a	= deepest point
b	= bending loading
c	= crack ends
m	= axial loading

Chapter 1

Introduction

1.1 Background

In the past decades, an increase in use of the tubular hollow sections has taken place due to its admirable material and engineering properties. Two types of tubular hollow sections are predominantly observed in practise; one being circular hollow section (CHS) and the other being square and rectangular hollow section (SHS and RHS). CHS joints are aesthetically more pleasing and possess well-organized distribution of steel about the centroidal axes (Packer et al., 2009). Moreover, it offers minimum resistance to fluid flow. However, the disadvantage of CHS joints is that the fabrication is more difficult due to the need of specialized profiling when assembling the circular shapes together. As a result, SHS and RHS joints have emerged as an alternative. It provides easy connections to the flat face and is very much in demand for columns and trusses (Packer et al., 2009). They are used in wide areas such as ship building, nuclear safety, liquefied gas storage tanks, pipelines, double chord truss, dry dock maintenance facility, bridges, cranes, masts, underwater pipeline and traffic sign structures (Mashiri et al., 1998). SHS joints used in a truss of a footbridge and in a crane are illustrated in Figs 1.1 and 1.2, respectively.

The past works for determining the ultimate strength or plastic collapse load (P_c) of tubular joints focussed on uncracked CHS joints (Yura et al., 1980). Comparatively, less attention was given to research on the SHS joints; as it was relatively younger member in the family of tubular steel sections. Even the research on SHS joints overwhelmingly dealt with uni-planar joints. There are many existing structures such as bridges, buildings, giant “Ferris” wheels and others which are fabricated using SHS members and these structures are multi-planar in nature. Therefore, there is a need to extend the research for estimating the P_c load of cracked multi-planar tubular SHS joints.

In the multi-planar tubular SHS joints, cracks can occur during fabrication or in service. It may be due to fatigue, accidental damage or earthquake (Fig 1.3). Generally, engineers have 3 options, (1) to repair the cracked structures, (2) to replace the damaged structures, and (3)

to take no action until catastrophic failure occurs. In terms of economic as well as human costs, the consequences and risks associated with the latest type of failure are costly (Figs 1.4 to 1.6). Hence, there is a need to develop a quantitative engineering assessment to define the acceptability and safety of these cracks found in these welded structures. The standard BS7910 (2005) recommends the procedures for the acceptability of defects in welded structures. However, these procedures are envisioned for general applications and may not essentially provide optimal solutions for all types of structures such as cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints.

Yang (2005) developed a numerical model to simulate the plastic collapse and fracture behaviour of SHS and RHS joints containing cracks, and then to investigate the failure of SHS T-joints containing surface cracks under brace end axial loads. Using these data, the P_c loads of cracked SHS T-joints under brace end axial loads were derived and used to validate the usage of standard BS7910 (2005) Level 2A failure assessment diagram (FAD) for damaged SHS T-joints. However, the recommendation is strictly applicable only for uni-planar tubular SHS T-, Y- and K-joints where the chord and braces lie on the same plane. Even for uni-planar joints, there are significant convergent issues for elastic-plastic analyses in case of large plastic deformation. Early termination of FE analysis was observed especially while calculating the elastic-plastic J -integral of cracked SHS joints. The elements at the crack tip are observed to collapse during elastic-plastic analyses (crack tip blunting). Moreover, the previous approach (Yang, 2005) employed 5 different types of elements for finite element (FE) mesh modelling. Element shapes such as prism, tetrahedron and pyramid were used near the crack front with very sharp angles. The prior mesh generator used is also not flexible enough to account for the varying location of crack front position. It is due to the approach used in the mesh generator whereby, the cracked block is inserted to the previously generated uncracked SHS joints. In the case of cracked SHS joints, the crack shape is asymmetrical and it is vital for the mesh generator to be flexible to accommodate the varying nature of crack end position. The detailed description of drawbacks of previous approaches is covered in Chapter 4. And hence, it won't be further highlighted here.

The current research work aims to address the drawbacks of previous approaches by developing a completely new and robust FE mesh generator; the features of which will be

described in later chapters. Then the new FE mesh generator will be further used to investigate the cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints by undertaking extensive parametric study. The study of cracked multi-planar SHS TT-, YT- and KT-joints has not been reported so far.

1.2 Scope of Research Works

In this research work, the three most important areas of interest include

1. Development of a completely new and robust FE mesh generator to model various types of cracked structures such as uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints.
2. Extensive parametric study using the validated FE mesh generator to develop new equations for weld toe magnification factor of cracked double-sided T-butt joints and cruciform X-joints.
3. Calculation of both plastic collapse and fracture analyses of cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The research work also comprises the validation of the FE models with the other existing software's as well as with the previous experimental test results (Lie et al., 2006; Lie and Yang, 2007).

Based on the previous experiment test results (Lie et al., 2006; Lie and Yang, 2007), the fatigue crack is modelled in the numerical model and they are introduced at the corner of the weld toe which is the location of hot spot stresses. Parametric study is carried out for both cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints subjected to axial loading at the brace end and new expressions for determining the reduction factor (F_{AR}) of above mentioned joints are proposed. The effect of the additional brace on uni-planar joints is also investigated in the study.

J -integral results obtained from the FE analyses are used to construct the failure assessment diagrams (FADs) for cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The FADs are constructed in order to validate the standard Level 2A FAD curve of BS7910 (2005). A penalty factor for P_c load is recommended to move all the constructed FAD curves above the standard level 2A curve in order to optimize and carry out

safe assessment of cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints.

1.3 Objectives of the Research Works

The main objectives of this research work are as follows:

1. To develop a robust FE mesh generator for modelling uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints with and without cracks.
2. To conduct extensive parametric numerical study on cracked plate-to-plate welded joints such as cruciform X-joints and double-sided T-butt joints to study the effect of additional fillet weld.
3. To propose new equations for determining the weld toe magnification factor of cracked cruciform X-joints and double-sided T-butt joints.
4. To estimate the P_c load of cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints.
5. To conduct extensive parametric study on cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints and propose new reduction factor (F_{AR}) equations for the corresponding joints subjected to axial tensile loading at the brace end.
6. To validate the standard Level 2A failure assessment diagram (FAD) curve of BS7910 (2005).
7. To construct the Level 3C FAD curves for cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints and to recommend optimal solutions for the same by proposing new penalty factors for P_c load.
8. To investigate the effect of additional brace in cracked multi-planar SHS TT-, YT- and KT-joints.
9. To carry out a comprehensive study of fracture failure mode for cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints.

1.4 Originality of Research Works

1. Development of a completely new and robust FE mesh generator to model various geometries with and without cracks. Geometrical analyses, modelling and mesh generation of cracked multi-planar SHS TT-, YT- and KT-joints are carried out for the first time.
2. Proposal of new and separate equations for determining weld toe magnification factor for cracked double-sided T-butt joints and cruciform X-joints and the detailed study on effect of additional fillet weld on cracked plate-to-plate welded joints.
3. Estimation of the fully P_c load of cracked multi-planar SHS TT-, YT- and KT-joints.
4. Proposal of the new reduction factor (F_{AR}) equations for cracked uni-planar SHS T-, Y- and K-joints subjected to axial loading at the brace end.
5. Proposal of a penalty factor for P_c load to move all the constructed FAD curves of cracked uni-planar SHS T-, Y- and K-joints and multi-planar SHS TT-, YT- and KT-joints above the standard Level 2A failure assessment diagram (FAD) curve of BS7910 (2005) in order to carry out safe assessment of flawed structures.
6. Study of the effect of additional brace in cracked multi-planar planar SHS TT-, YT- and KT-joints.
7. Proposal of the new reduction factor (F_{AR}) equations for cracked multi-planar SHS TT-, YT- and KT-joints subjected to axial loading at the brace end.

1.5 Synopsis of the Thesis

The synopsis of this thesis is arranged as following:

- Chapter 2 includes the literature review. The significant previous works on cracked tubular structures is reviewed and the background of various concepts used in subsequent chapters is presented.
- Chapter 3 explains the geometry used for modelling cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The modelling of welds and the geometrical equations used for the same are described.
- Chapter 4 details the step-by-step mesh generation procedure to generate various cracked geometries using the new FE mesh generator. The different types of crack tips used, the zoning approach and the unique features of new FE mesh generator are highlighted.
- Chapter 5 focus on the numerical analyses and validation of cracked plate-to-plate welded joints modelled using the new FE mesh generator. It covers the extensive parametric study on different types of cracked plate-to-plate joints like cracked double-sided T-butt joints and cruciform X-joints. New weld toe magnification factor (M_k) equations for cracked double-sided T-butt joints and cruciform X-joints are presented. The effect of additional attachments on plate-to-plate joints is described.
- Chapter 6 concentrate on study of plastic collapse and fracture behaviour of cracked uni-planar SHS T-, Y- and K-joints. The numerical results from the new FE mesh generator are validated with the previous experimental test results (Lie et al., 2006; Lie and Yang, 2007) as well as with the existing commercial software results. The new reduction factor (F_{AR}) expressions and recommended penalty factors for P_c load for cracked uni-planar SHS T-, Y- and K-joints are also presented. Level 3C FADs constructed for cracked uni-planar SHS T-, Y- and K-joints using the elastic (J_e) and elastic-plastic J -integral (J_{ep}) values and P_c load are finally described.

- Chapter 7 covers the numerical analyses of cracked multi-planar SHS TT-, YT- and KT-joints generated using the new FE mesh generator. Novelty factor of present numerical study of cracked multi-planar SHS TT-, YT- and KT-joints is emphasized. Comparison of numerical analyses results for cracked multi-planar and uni-planar SHS joints and the general plastic collapse behaviour of cracked multi-planar SHS TT-, YT- and KT-joints are described. The effect of the additional brace on uni-planar T-, Y- and K-joints is also explained. The new reduction factor (F_{AR}) expressions and recommended penalty factors for P_c load for cracked multi-planar SHS TT-, YT- and KT-joints are also proposed. Level 3C FADs constructed for cracked multi-planar SHS TT-, YT- and KT-joints using the elastic (J_e) and elastic-plastic J -integral (J_{ep}) values and P_c load are finally presented.
- Chapter 8 list out the conclusions of the research work. Important research findings are enlisted and subsequently, recommendations for future research works are given.

Fig. 1.1 Square hollow sections used in the truss of a footbridge (Packer et al., 2009)

Fig. 1.2 Square hollow sections used in a crane (Packer et al., 2009)

Fig. 1.3 Broken gate of a dam chamber (Bernasovsky, 2009)

Fig. 1.4 Fatigue and ductile tearing of metallic sheet, for a Boeing KC-135 fuselage (Ahmadi and Yaghin, 2013)

Fig. 1.5 Fracture of a railway wheel: (a) broken wheel tyre and (b) fracture surface with extended fatigue crack growth (Richard et al., 2008)

Fig. 1.6 Detachment on a railway wheel caused by fatigue (Richard et al., 2008)

Chapter 2

Literature Review

2.1 Introduction

Over the past decades, there has been manifold increase in usage of steel tubular joints in various areas such as offshore platforms, pressure vessels, jibs, towers, railroad rails to name a few. As a result, extensive research on various tubular joints has been carried out throughout the world. The current chapter covers the literature review carried out for the present study. The significant previous works on cracked tubular structures is reviewed and the background of various concepts used in subsequent chapters is presented.

In general, welding is used to connect the steel frames made up of tubular members. The welding is performed at the joints or “nodes”. In a joint, the main member is labelled as the chord, while the other members which intersect with the chord, are called the braces (Yang, 2005). These nodal points represent the weakest region of the welded structure and therefore, research on the plastic collapse load (P_c) of these joints is essential (Fig. 2.1). According to CIDECT (Packer et al., 2009) design guides, the uni-planar tubular joints are called as T(Y)-, X- and K-joints, while the multi-planar tubular joints are identified as TT-, XX- and KK-joints (Fig. 2.2). The geometry of SHS and RHS joints is usually described by some standard variables (Packer et al., 2009), namely,

$\beta = b_1 / b_0$, ratio of brace width to chord width;

$\eta = h_1 / b_0$, ratio of brace depth to chord width;

$\tau = t_1 / t_0$, ratio of brace wall thickness to chord wall thickness;

and $2\gamma = b_0 / t_0$, ratio of chord width to chord wall thickness.

As mentioned before in Chapter 1, two types of tubular hollow sections are predominantly observed in practise; one being circular hollow section (CHS) and the other being square and rectangular hollow section (SHS and RHS). The detailed comparison of CHS and SHS joints

is carried out and is described in Section 2.2. The previous research work on uncracked multi-planar tubular joints is covered subsequently in Section 2.3.

It is very important to clearly understand the failure modes of SHS joints before carrying out the parametric study of the same. Different failure modes observed for SHS joints is summarized in Section 2.4. Finite element (FE) as evolved as the most widely used tool for studying the various characteristics of tubular joints. It allows engineers to see the behaviour of structures that is not detectable by human eye, such as the stresses in structures (Young et al., 2012). Pivotal previous works on FE modelling of tubular structures are summarized in Section 2.5.

The most essential review covered in this chapter relates to the previous works done on cracked tubular joints with special attention being paid to cracked SHS joints. All the previous works carried out by researchers on cracked SHS joints focussed on uni-planar joints where the brace and chord lie in the same plane and it is covered in Section 2.6.

The behaviour of cracked tubular joints is governed by plastic collapse (yielding) and/or crack extension (fracture). The approach for determining the P_c load is summarized in Section 2.7. Detailed fracture assessment procedures are reviewed in Section 2.8. Various fracture mechanics parameters such as stress intensity factor (K), proposed by Irwin (1957), the crack tip opening displacement (CTOD), developed by Wells (1961) and Cottrell (1961), and a line contour integral (J), proposed by Rice (1968) are introduced. Finally, failure assessment of structures containing cracks using approach mentioned in BS7910 (2005) is covered in detail.

2.2 Comparison of SHS with CHS

Engineers have realized the structural advantages of hollow sections, especially for structural members subjected to tension or compression. CHS has a more efficient shape and lower drag coefficients resulting in lower forces due to wave loading. Hence, most offshore structures are fabricated using CHS, and research in the field of CHS joints is very extensive (Berge and Tubby, 1994; Stacey et al., 1996). SHS and RHS are the youngest members in the family of steel sections, and they have evolved into a reputable member of the steel society under the endeavour of researchers (Wardenier, 2002). Their industrial production started as late as in 1959 in England. They possess practically all the favourable structural

qualities of CHS in various degrees, such as high radii of gyration about both axes, low surface, lack of ledges and edges, closed aerodynamic shape and aesthetic appearance. However, the distinct advantage of applying SHS in lieu of CHS emerges in the field of fabrication, where the facility of end preparation with flat cuts for SHS is significantly easier and more economical than that with profile cuts for CHS. For CHS joints, when the circular tubes are to be fitted directly together, the tube ends need to be profile cut. Semi-automatic or manual profile cutting needs to be used wherever full automated equipment is not available for profile cut. This tends to be more expensive when compared to straight bevel cuts on SHS joints. Moreover for some structures, a deck or panelling is laid on the top chord of trusses. For these types of structures, the surface of SHS is better for attaching and supporting the deck. They are therefore well suited for application on topside structure of offshore platforms as well as for onshore uses (Berge et al., 1995), such as trusses (Fig. 2.3), space frames (Fig. 2.4) and bridges (Fig. 2.5).

Finally, material handling is known to be the most expensive process in the hollow section industry (Packer et al., 2009). Handling and stacking are relatively easier for SHS compared to CHS. So it can be seen from above points that SHS offers excellent structural properties and this has led to its many applications in buildings, bridges, towers, etc. (Eekhout, 1996; Wardenier, 2002).

2.3 Review of Previous Works on Uncracked Multi-planar Tubular Joints

Initially, the works carried out to study the plastic collapse and fatigue strength of tubular joints predominantly focussed on uni-planar joints. The study of multi-planar tubular joints gathered a momentum in 80's and 90's with various work done by researchers throughout the world (Coutie et al., 1983; Bauer and Redwood, 1988; Yu et al., 1993; Liu, et al., 1993; Shahi et al., 1993; Mang et al., 1993; Herion et al., 1993; Soh et al., 1996). The salient observations made by above researchers on uncracked multi-planar tubular joints are summarized in this sub-section.

Preliminary tests by Coutie et al. (1983) on RHS multi-planar KK-joints noticed a slight decrease in strength compared to that of the uni-planar K-joint. For multi-planar KK-joints, Bauer and Redwood (1988) inferred that there is minute interactive effect caused by similar loading on a nearby wall of the chord. Packer et al. (1992) suggested that the range of

parameters studied by Coutie et al. (1983) and Bauer and Redwood (1988) may not sufficiently represent all the failure modes observed on multi-planar KK-joints. As a result, a reduction factor of 0.9 was recommended to use along with the uni-planar K-joint design formulae in the 1st edition of the CIDECT Design Guide (Packer et al., 1992). This recommendation was applicable only for multi-planar KK-joints with no eccentricity, which further pointed the need to carry out even more study on various types of multi-planar joints.

Yu et al. (1993) carried out numerical investigation into study the plastic collapse behaviour of multi-planar welded RHS T-joints. The geometrical parameters were not varied and kept at $\beta = 0.6$, $\gamma = 11.09$ and $\tau = 1.0$. 8-noded shell elements were used to model the joints. Compression loads were applied axially at the brace ends. It was observed that the load capacity of the multi-planar joint increased by up to 26% when compared to uni-planar joint. They recommended that further detailed parametric study is required in order to quantify the multi-planar effect of the joints in the broad range of geometrical parameters.

Liu et al. (1993) conducted experimental and numerical investigation into the plastic collapse behaviour of uncracked multi-planar RHS DX-joints with β ratio of 0.6. They observed that out-of-plane multi-planar brace increased the P_c load of joints when compared to uni-planar joints. Shahi et al. (1993) conducted study on uncracked multi-planar KK-SHS joints and they also pointed out that out-of-plane multi-planar brace increased the P_c load of joints when compared to uni-planar joints.

Mang et al. (1993) and Herion et al. (1994) conducted a parametric study on multi-planar K-joints with gap made of RHS sections. They proposed formulae for the calculation of strain concentration factors. Soh et al. (1996) observed that the strength ratio .i.e. multi-planar joint strength over uni-planar joint strength for uncracked X-joint varies significantly with variation of β but not of γ .

In the currently used codes of practises such as Eurocode 3 (2005), IIW (2005) and CIDECT (2009), it is recommended to use correction factors to uni-planar joint strength which is equivalent to the P_c load to determine the P_c load of corresponding uncracked multi-planar joints. The correction factors recommended by the codes of practises are described in Chapter 7 and hence it won't be covered here.

2.4 Failure Modes for SHS Joints

In a tubular joint, the loads can be applied either at the brace end or at the chord end. Basic load conditions are labelled as axial compression and tension (AX), in-plane bending (IPB) and out-of-plane bending (OPB), as depicted in Fig. 2.6. A typical load-displacement curve for a SHS T-joint is illustrated in Fig. 2.7. Generally, three possible failure criteria can be observed. They are P_c load, load at a certain deformation limit, and load at the observation of the first crack for uncracked joints or load at the beginning of crack propagation for cracked joints (Yang, 2005). The broadly acknowledged criterion of failure is the P_c load. Yura et al. (1980) recommended that the minimum loads associated with the three possible failure criteria should be used to define the failure of the joint.

Various studies have been carried out by researchers in the past on the failure mode of tubular joints. Previous experimental works on RHS welded joints has shown that the failure modes depend on the joint type, the geometric variables and the loading conditions (Wardenier, 1982). The possible failure modes of RHS joints are described in Fig. 2.8, where Mode A is a plastic failure of the chord face, Mode B is a ductile shear failure of the chord wall, Mode C is a failure at the weld at the base of the tension brace, Mode D is a local buckling of compressive areas in the bracing, Mode E is a local buckling of the chord walls, Mode F is a shear failure of the chord between the adjacent braces, and Mode G is a local buckling of the chord face.

For RHS or SHS T-joints subjected to compression loading at the brace ends, chord face plastification and punching shear are the most common modes of failure (Lalani, 1993). Local buckling of the chord usually occurs only for high γ values. This trend is more prominent for joints subjected to loading at the chord ends. For SHS T-joints subjected to brace end loading, the chord yields around the brace and chord intersection and the cross-section distorts at the top chord surface (chord face plastification). This mode is observed for joints where the brace width to chord width ratio β is less than 0.85. If the load increases further, a crack may be initiated, and this eventually leads to rupture of the brace from the chord (Fig. 2.9). For joints with very high β ratio, local buckling of the chord side wall is observed (Fig. 2.10). For the cracked SHS T-joints, there is a chance of the brittle fracture or

ductile tearing before the chord failure (Yang, 2005). Therefore, it is very important to consider the fracture failure of the cracked joints along with the plastic collapse failure.

There have been other works carried out by researchers on the failure modes of different types of tubular joints. Soh et al. (1996) observed that for CHS X-joints, the failure mode is punching shear for lower β ratio and chord face plastification for higher β ratio. Chan et al. (1998) carried an experiment on uncracked multi-planar CHS XT-joint for a specimen of geometrical parameters $\alpha = 21.86$, $\beta = 0.6$, $\gamma = 24.07$ and $\tau = 0.98$. Axial tensile loading was applied to brace. They observed that failure mode is chord face plastification for the joint. Feng and Young (2006) observed that for RHS X-joints, failure mode for β above 0.5 is chord face plastification. They recommended that further research is required to define the failure modes based on appropriate β value for RHS X-joints. For the present study, extensive parametric study is carried out to understand the failure modes of both cracked and uncracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The observations from the extensive parametric study on the failure modes of these joints are described in Chapters 6 and 7.

2.5 Finite Element Modelling of Tubular Joints

In structural engineering, the FE method has emerged as the most popular modelling tool for the prediction of responses of engineering problems (Lee et al., 2007). FE modelling has enabled the engineers to predict the responses of structures under difficult loading and boundary conditions, to compare different alternate designs and to predict the responses of flawed structures through safety assessment. For the specific case of tubular joints, FE modelling has helped the user to analyse and predict the location of hot spot stress, to carry out P_c load analysis, to carry out parametric study to understand the general behaviour of commonly used joint types and to complement experimental analyses. For the fatigue analysis of tubular joints, the advantages of FE modelling include fast modelling speed, inexpensive modelling cost, robustness and wider accessibility.

Even though the FE method has many advantages as mentioned in the above paragraph, there are various challenges faced by researchers in modelling complex geometries such as cracked SHS joints as well as issues associated with analysis for joints subjected to large deformation. For example, non-linear geometry FE analysis may result in severe

convergence problems, especially for higher plastic deformation (Du and Hancock, 1989; Kristiansen et al., 1992; Kristiansen and Turner, 1993). As a result J -integral values obtained especially at the crack tip may not be reliable. Anderson (2005) recommended using a key-hole for modelling the crack tip region for joints used for elastic-plastic analysis. The concept of using a key-hole for modelling a cracked SHS joints subjected to elastic-plastic analysis is performed for the first time in the present study. The detailed description of using key-hole for elastic-plastic modelling is covered in Chapter 4 and henceforth, it won't be explained much further in this Chapter.

It is very difficult to manually create the FE model of cracked tubular joints. Complex 3D geometry, non-linear material behaviour and large displacements have caused numerous problems over the years for researchers working on cracked tubular joints (Laham, 1993). Apart from above mentioned issues, the choice of elements used to model the joints also play a pivotal part in both accuracy and time saving for analyses. Over the years, researchers used mainly two types of elements for FE modelling, namely, shell elements and solid elements. Even though the computational effort for shell elements is less than the same for solid elements, solid elements are more accurate in representation of certain geometric shape including weld profile, loading and boundary conditions (Qian et al., 2002). Furthermore, even the time difference for analysis is within reasonable limit for both shell and solid elements. In the work published by Qian et al. (2002), it shows that the difference between total analysis times for both types of elements is around 6 minutes. For their study, HP B2000 workstation with 256 Mb memory was used to compare analyses time for both types of elements. For model with solid elements without weld, 550 elements consumed 467 seconds of CPU time. For model with shell elements without weld, 341 elements consumed 101 seconds of CPU time. Advances in computing hardware in recent times have considerably reduced the analyses time for the different types of models.

The generation of FE meshes manually consume lot of time. It is certainly not practical while carrying out any type of parametric study on complex geometries such as cracked SHS joints. As an alternative, researchers develop automatic FE mesh generating tools. The user needs to specify the basic parameters such as length and depth of the crack required and basic dimensions of geometry such as length, width and thickness of brace and chord. The

mesh generation tool thus saves valuable time required for meshing and it further enables the users to carry out extensive parametric study on various types of joints.

Yang (2005) developed a FE mesh generator that could be used to model cracked uni-planar SHS T-, Y- and K-joints. Even though it has produced accurate results for joints subjected to elastic analysis, there are various drawbacks of using the FE mesh generator. For example, early termination of FE analysis is observed for very large deformation. The limitations of FE mesh generator developed by Yang (2005) are covered extensively in Chapter 4 and therefore, it won't be summarized here. The present study develops a completely new and robust FE mesh generator that can be used to model cracked uni-planar SHS T-, Y- and K-joints, and cracked multi-planar SHS TT-, YT- and KT-joints. In addition, it can be used to model relatively simple geometries such as cracked plate-to-plate joints. The detailed geometric modelling and FE mesh generation procedure for the new FE mesh generator is covered in Chapters 3 and 4, respectively.

2.6 Past and Present Research Works on Cracked SHS Tubular Joints

The tubular joints such as CHS and SHS joints are subjected to various types of loading throughout their lifespan. Cracks appear at the hot spot location of these joints and it can be due to fatigue, damage from impacts and dropped objects, possible under design, corrosion damage and earthquake (Sharp, 1992). As mentioned in previous section, the challenging issue faced by the researchers is to accurately model the crack region of any tubular joint. Though various research works have been conducted throughout the world in last 2 decades on cracked uni-planar tubular joints, any published works on cracked multi-planar SHS TT-, YT- and KT-joints are not available till date.

In the 90's various research works were carried out in University of Manchester Institute of Science and Technology (UMIST), Manchester, U.K. Experimental and FE studies into the P_c load behaviour of cracked RHS K-joints with gap were carried out (Burdekin et al., 1992; Cheaitani, 1992; Burdekin and Laham, 1993; Laham, 1993; Cheaitani and Burdekin, 1994; Laham and Burdekin, 1995; Burdekin and Yang, 1997; Hadley et al., 1998). Non-linear FE techniques were employed to calculate the P_c load of uncracked and cracked K-joints subjected to axial loadings at the brace end. A total of 16 different geometries were analysed (Burdekin and Laham, 1993). They observed that the P_c load reduces by 6% after

introducing a surface crack having a crack-depth ratio (a/T) of 0.51. For the same joint, the P_c load reduces by 9% when a/T ratio becomes 0.63. Thus it can clearly be inferred from findings that presence of surface crack reduces the P_c load of RHS K-joint. They also observed that FE results showed an increase in P_c load value with an increasing in β ratio for a fixed τ ratio. They further inferred that the small discrepancies between experimental and numerical results observed were due to differences in the actual material stress-strain curves for the two cases. Laham (1993) recommended that more work is needed to check the effect of surface cracks on P_c load on a whole range of geometries.

Cheaitani and Burdekin (1994) carried out the assessment of the effects of cracks on the P_c load and fracture behaviour of CHS K-joints. They recommended that conservative estimate of plastic collapse strength of cracked CHS K-joints can be obtained by multiplying the strength of the uncracked joints by a reduction factor (F_{AR}). The F_{AR} expression takes care of the decrease in P_c load brought about by the presence of cracks. They have proposed the following expression for F_{AR} which is also been incorporated in BS7910 (2005), and it is expressed as

$$F_{AR} = \left[1 - \frac{\text{crack area}}{\text{weld length} * \text{chord thickness}} \right] \left[\frac{1}{Q_\beta} \right]^{m_q} \quad (2.1)$$

where Q_β is the geometrical modifier which depends on brace to chord width ratio (β) values, and it is expressed as

$$Q_\beta = 1, \text{ for } \beta \leq 0.6 \quad (2.2)$$

$$Q_\beta = \frac{0.3}{[\beta(1-0.833\beta)]}, \text{ for } \beta > 0.6 \quad (2.3)$$

For tubular joints containing part-thickness flaws, $m_q = 0$. So the existing equation for F_{AR} was developed from parametric study carried out for CHS joints and it may not be applicable for SHS joints. Even for CHS X-joints, Wang et al. (2011) proposed the following equation for determining F_{AR} of cracked CHS X-joints;

$$F_{AR} = \left(1 - \frac{A_c}{l_w * t_0} \right) * 0.8 - 0.28 \quad (2.4)$$

where A_c is the crack area, l_w is the weld length and t_0 is the chord thickness.

Furthermore, for CHS X- and K-joints Qian (2013) observed that F_{AR} has a significant effect on the failure assessment curve.

From the above equations, it can be seen that there is a need to optimize the F_{AR} equations for cracked SHS joints. The works done in UMIST also pointed to the need of carrying a wider extensive parametric study covering various types of joints as well as a broader set of parameters. Since their works were focussed only on uni-planar joints, it is important to carry out research on cracked multi-planar joints as well. So in order to optimize the equation for SHS joints, the present study carries out extensive parametric study for cracked uni-planar SHS T-, Y- and K-joints, and cracked multi-planar SHS TT-, YT- and KT-joints to propose new F_{AR} equations for these joints. The new equations for cracked uni-planar T-joints are already published (Lie et al., 2015).

The previous research works on SHS joints overwhelmingly focussed on uncracked joints (Rasmussen and Young, 1994; Rasmussen and Young, 2001; Feng and Young, 2011). From the last decade, there has been extensive research work on different types of cracked tubular joints undertaken in Nanyang Technological University (NTU), Singapore. Experimental tests using test rigs and numerical validations are part of on-going research works in NTU, Singapore (Lee et al., 1999; Lie et al., 2001; Chiew et al., 2003; Lie et al., 2003; Chiew et al., 2004; Yang, 2005; Lie et al., 2006; Chiew et al., 2007; Lee et al., 2007; Lie and Yang, 2007; Yang et al., 2007; Chiew et al., 2009; Lie et al., 2009; Lee et al., 2010; Lee et al., 2011; Lie and Yang, 2011; Lie and Yang, 2011; Lie et al., 2012; Lie et al., 2014; Lie et al., 2015; Lie et al., 2015). Both CHS and SHS tubular joints are researched at NTU, Singapore. Since the scope of the present study is on SHS joints, major observations from works on cracked SHS joints is summarized below.

In the experimental study carried out by Chiew et al. (2003), four cracked SHS T-joints were subjected to both plastic collapse and fatigue tests. The plastic collapse tests revealed that maximum value of hot spot stress (HSS) always occurs at the corner point of the joint. The

fatigue tests showed that under cyclic loading, cracks originate at the corners of the brace-chord intersection and further propagate in a direction parallel to the chord wall. Chiew et al. (2004) observed that the crack shape in a SHS joint is more complicated than in a CHS joint. The crack is observed to be unsymmetrical with respect to the deepest point and at any cross section; the profile of the crack is a curve.

Full scale static experimental tests were carried out for cracked uni-planar SHS T-, Y- and K-joints by Yang (2005), Lie et al. (2006), Lie and Yang (2007) and Yang et al. (2007). The details of the test rig, loading, boundary conditions and material properties used for the above mentioned experimental tests is covered in detail in Chapter 6. In their works, 3-D SHS T-, Y- and K-joint models with and without cracks was developed. The shape of the cracks was assumed to be two halves of semi-ellipses. Based on the experimental works, Yang (2005) developed a FE mesh generator that could be used to model cracked uni-planar SHS T-, Y- and K-joints. The P_c load was calculated using twice elastic compliance criterion for all the works. The background on twice elastic compliance criterion is covered in next section. The failure assessment diagrams (FADs) of 4 series SHS T-joints under brace end axial loading were constructed using the J -integral method. It was found that the shoulder part in most of the specified FADs fall inside the standard BS7910 (1999) Level 3A FAD curve. Therefore a conservative P_c load is needed. A penalty factor of 1.2 was introduced to estimate the P_c load. New set of FADs for T-, Y- and K-joints with newly introduced penalty factor of 1.2 were created. It was found that all of these new FADs fall outside the standard BS7910 (2005) Level 2A FAD curve.

There are various limitations of using the FE mesh generator developed by Yang (2005) such as early termination of FE analysis especially for very large deformation, usage of diverse types of elements to model the cracked SHS joints and the lack of robustness of the overall mesh design approach used for cracked SHS models. These limitations are covered in very detail in Chapter 4. Furthermore, the previous research works have not optimized the F_{AR} equations for cracked uni-planar SHS T-, Y- and K-joints. More importantly, there has not been any previously published research on cracked multi-planar SHS joints anywhere in the world. The present study addresses all the above mentioned research gaps; the details of which can be found in subsequent chapters.

2.7 Determination of Plastic Collapse Loads of SHS Joints

Tubular joints under axial tension often sustain monotonically increasing loads until the joint resistance is limited by the ductility or the plasticity in the chord material (Qian et al., 2013). Furthermore, when a tubular structure of a ductile material experiences excessive static loading, gross plastic deformation occurs (Muscat et al., 2003). This subsequently leads to plastic collapse or rupture of the structure. In practice, two types of stress analysis are used to protect the structure against gross plastic deformation, namely, elastic analysis and elastic-plastic analysis (Fig. 2.11). There are numerous difficulties associated with elastic analyses, such as complicated stress categorization. In elastic analysis, the allowable load is estimated indirectly by segregating the elastic stress into primary, secondary and peak categories (Yang, 2005). However in case of elastic-plastic analysis, the allowable load is estimated directly from the elastic-plastic response of the welded tubular structure. Two types of elastic-plastic analysis are used to describe the structural behaviour, namely, limit analysis and plastic analysis.

Among the two types of elastic-plastic analysis mentioned above, plastic analysis is more widely used due to its more accurate representation of steel structural behaviour. For limit analysis, there are two assumptions made, namely, the material is assumed to be an ideal elastic-perfectly plastic model and the deformation is assumed to be small. Stress-strain behavioural pattern of an elastic-perfectly material is shown in Fig. 2.12(a) and a sample load-deformation curve for the same is shown in Fig. 2.12(b). It can be clearly observed that, the structural response is linear initially. The plastic strain develops when the yield is exceeded and the structural response becomes non-linear. The load-deformation curve becomes horizontal at the limit load. Unrestricted plastic flow follows and the structure fails by loss of equilibrium at the limit load of the structure. In limit analysis, this limit load is taken as the load causing gross plastic deformation. This is a conservative evaluation and the post-yield strain hardening and large deformations seen in real world steel structures are not taken into consideration in limit analysis. Hence as an improvement to limit analysis, plastic analysis is used.

Unlike the trend shown in Fig. 2.12(a), the steel materials display linear elastic behaviour till yield and thereafter, the variation of stress with respect to strain is non-linear as depicted in Fig. 2.13(a). This post-yield phenomenon is labelled as strain hardening. The plastic

deformation continues even after the limit load specified in limit analysis is reached. Plastic analysis encompasses both post-yield strain hardening and large deformations (Fig. 2.13(b)). The main challenge in plastic analysis is defining the P_c load. There are various criteria to determine the P_c load and the method of determining P_c load is very important in the case of tubular joints (O'Connor, 1993). In the past, various researchers used different criteria to estimate the P_c load (Yura et al., 1980; Lu et al., 1994; Laham and Burdekin, 1995; Muscat et al., 2003). The various criteria predominantly focus to describe a load limit at which plastic deformation inclines to be large prior to actual physical collapse (Lie et al., 2009). In the present study, the twice-elastic compliance (TEC) criterion is used to determine the deformation limits. Past studies have used the TEC criterion to determine the P_c load of cracked tubular joints (Stacey et al., 1996; Muscat et al., 2003; Lie and Yang, 2011). This plastic criterion is defined in ASME VIII Division 2 (1998) as the twice-elastic slope (TES) criterion. The TEC criterion is explained in detail in Chapter 6 and therefore, it won't be highlighted here.

2.8 Fracture Assessment Procedures

The most common problem faced by designers of offshore structures concerns the joints between the tubular members which must be assessed for plastic collapse, fatigue and fracture (Smedley and Bolt, 1994). As mentioned before, the behaviour of cracked tubular joints is governed by plastic collapse (yielding) and/or crack extension (fracture). The current section summarizes important concepts in fracture. Fracture refers to the unstable propagation of a crack. Once the crack starts to move, the loading system is such that it produces fast accelerating growth. This catastrophic failure is usually described as brittle fracture.

Brittle fracture needs to be thoroughly researched because it can lead to catastrophic failure of structures. It always occurs at average stress levels far below those of general yielding and without prior warning and at high speeds as high as 7000 ft/s (7680 km/h) in steel. The crack surface is rough and it does not have any beach markings on it. Brittle fracture can be understood through the study of fracture mechanics.

Fracture mechanics is a method of describing the fracture and fatigue behaviour of cracked (flawed) structural members in expressions that can be used by the engineer, namely stress

(σ), crack size (a) and material properties (K_{IC}). In other words, it seeks to define the local conditions of stress and strain around a crack, in terms of the global parameters of loads, geometry and material properties. The engineers undertake a fracture mechanics analysis (a) to prevent catastrophic failure - fracture is dangerous as it is often manifested by catastrophic failure, and (b) to reduce loss of production costs- when a crack is discovered during inspection and maintenance, a decision has to be made whether the damaged structure or component should be repaired or replaced without any significant loss of production costs or without running the risks associated with catastrophic failure. There are various parameters and assessment procedures associated with fracture and it is covered in following sub-sections.

2.8.1. Fracture mechanics parameters

Three parameters have dominated the application of fracture mechanics to metal structures in the recent decades. These are the stress intensity factor (K), developed by Irwin (1957), the crack tip opening displacement (CTOD), proposed by Wells (1961) and Cottrell (1961), and a particular line contour integral (J), published by Rice (1968). The intensity of the stress field at the crack tip is called the stress intensity factor (K). It is used to assess the fatigue and fracture resistance of cracked welded structures (Zhang et al., 2002). It is applicable when the structure is subjected to linear elastic stresses. Generally, the K is given by following equation

$$K_e = f_g f_s f_i f_\theta \sigma_n \sqrt{\pi a} \quad (2.5)$$

where, f_g is the joint geometry factor, f_s is the crack size factor, f_i is the joint and crack coupling factor, f_θ is the influence factor of the dihedral angle between the brace and the chord, σ_n is the nominal stress. When K becomes equal to fracture toughness of the material (K_{mat}), the unstable crack propagation starts.

As mentioned in the previous paragraph, K is applicable when the structure is subjected to linear elastic stresses and therefore, the principles of linear-elastic fracture mechanics (LEFM) are used to analyse the structure. LEFM is applicable only when the nonlinear

effects and amount of inelastic deformation are minute compared to the actual crack size. However, plastic deformation occurs in most metallic structures prior to the crack extension. Crack tip blunting and crack tip opening are caused due to this plastic deformation (Fig. 2.14). The principle of LEFM is no longer sufficient to accurately represent the stress distribution of the cracked body and hence, principles of elastic-plastic fracture mechanics (EPFM) are used. The commonly used fracture parameter in EPFM is crack tip opening displacement (CTOD). It was developed by Wells (1961) and Cottrell (1961) to measure the crack extension. Burdekin and Stone (1966) proposed the following expression for CTOD for an infinite flat plate with a central crack under tensile loading (Fig. 2.15),

$$\delta_I = \left(\frac{8\sigma_y a}{\pi E} \right) \left[\ln \sec\left(\frac{\pi\sigma}{2\sigma_y} \right) \right] \quad (2.6)$$

where σ is the applied stress, σ_y is the yield stress and E is Young's modulus. Subsequently, the important values of stress and crack size for safe crack extension is calculated from

$$\delta_I = \delta_{mat} \quad (2.7)$$

As a consequence of these pioneering works by Burdekin and Stone (1966), many other researchers produced numerical, experimental and analytical solutions for CTOD covering various geometries (Ewalds and Wanhill, 1984; Berge et. al., 1988; Anderson and Dodds, 1991; Fujikubo et. al., 1991a, 1991b, 1991c). In practise, the engineers analyse the material behaviour by assuming the linear and the non-linear variation. This invariably led to use of line and surface integrals to accurately represent the amplitude of stress and strain singularity in the vicinity of a crack tip. Rice (1968) proposed a path independent line contour integral named J -integral which is expressed by

$$J_I = \int_{\Gamma} \left[w dy - T_i \frac{\partial u_i}{\partial x} ds \right] \quad (2.8)$$

where Γ is the path of the integral that encloses the crack tip, w is the strain energy density for an elastic body, T_i is the traction vector described according to the outward normal along Γ , and u_i is the displacement vector (Fig. 2.16).

In simple terms, J -integral is defined as the parameter which describes the state of affairs in the crack tip region. The J -integral becomes equal to J_c during the crack initiation, where J_c is a material property for a given thickness under specified environmental conditions. For plane-strain conditions, J_c is related to K_{IC} by the following equation

$$J_{IC} = \frac{1 - \nu^2}{E} K_{IC}^2 \quad (2.9)$$

where ν is the Poisson's ratio of the material and E is Young's modulus.

2.8.2. Failure assessment diagrams (FADs)

After summarizing the fracture parameters in the previous section, the current section aims to elaborate the commonly used fracture assessment procedure namely, the failure assessment diagram (FAD) approach. Several assessment methods based on fracture mechanics have been developed during the last 40 years. The widely used assessment technique that is followed in recent times is FAD approach. The basis of the FAD approach was originally developed by Dowling and Townley (1975). According to this approach, a structure can fail by either of two mechanisms, namely, brittle fracture and plastic collapse. A design curve labelled as failure assessment curve is used to interpolate between the two failure criteria as shown in Fig 2.17. The integrity of the structure is determined by the relative position of the assessment point on the FAD. The relative position is derived from two separate set of calculations covering both brittle fracture and plastic collapse. The final assessment procedure is quite straightforward .i.e. if the relative position of the point is inside the FAD curve, the structure is considered to be safe. However, if the relative position of the point is outside the FAD curve, the structure is considered to be unsafe. The reliability of the result produced by FAD approach depends on the accuracy by which the failure assessment curve is plotted which in turn depends upon the structural geometry, nature of loading and the crack size.

The BSI published document (BS7910, 2005) specifies three levels of assessment (Fig 2.18). The three levels are Level 1, Level 2 and Level 3. Level 1 is simplified and conservative assessment route and is aimed at inspectors for use on site for quick decisions with the minimum of data and calculation. In Fig 2.18, two parameters, namely, K_r and L_r are used to estimate the relative position of the assessment point. They are obtained by the following equations

$$K_r = \frac{\text{Stress intensity factor}}{\text{Fracture toughness}} \quad (2.10)$$

$$L_r = \frac{\text{Applied load}}{\text{Plastic collapse load}} \quad (2.11)$$

In Level 1 approach, the primary investigation for a known crack is carried out by estimating K_r and L_r . The crack is deemed to be safe if K_r is less than 0.707 and L_r is less than 0.80.

Unlike Level 1, Level 2 is intended for qualified engineers and requires simple data and analysis (Fig 2.18). Level 2 is more accurate than Level 1 with respect to flaws in stress gradients and the treatment of residual stresses. Level 2 is divided into two types, namely, Level 2A and Level 2B. The general Level 2A FAD curve is explained in detail in Chapter 6 and hence, it won't be highlighted here. The criteria for choosing between Level 2A and Level 2B depend on the type of stress-strain data present for the material. If the data is not available, Level 2A approach is chosen. The drawback with Level 2 is that its scope is restricted to limited ductile tearing. Therefore, to give more accurate predictions of structural behaviour, Level 3 assessment is used.

Level 3 is an advanced assessment approach requiring detailed data, computer analysis and considerable technical knowledge and expertise in assessment procedures. In Level 3 assessments, FAD curves are constructed based on the detailed information of stress-strain curves of the materials. It is more complex than Level 2. It is significantly relevant to materials having a high strain-hardening capacity and where an analysis of stable crack extension is needed. It is suitable for ductile materials that exhibit stable tearing, e.g. austenitic steels and ferritic steels on the upper shelf (Yang, 2005). There are 3 assessment methods in Level 3 approach, namely, Level 3A, 3B and 3C. The use of level 3A and 3B are similar to Level 2 in the sense that they depend on the type of stress-strain data that is

available. The highest level, namely, Level 3C FAD curve gives the most accurate representation of the structural behaviour of a component. The existing Level 2A FAD curve in BS7910 (2005) is intended for all types of structures and may not necessarily give optimal solutions for all types of structures such as cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The present study constructs the highest level .i.e. the Level 3C FADs for the above mentioned joints and they are described in detail Chapters 6 and 7. Suitable recommendations are subsequently made and it is worth noting that, for the first time, FAD curves for cracked multi-planar SHS TT-, YT- and KT-joints are constructed.

Fig. 2.1 Profile of RHS joint

T-joint

K-joint

TT-joint

X-joint

XX-joint

KK-joint

Fig. 2.2 Uni-planar and multi-planar joint classifications (Packer et al., 2009)

Fig. 2.3 A truss of SHS used to raise the submarine from the ocean floor (Packer et al., 2009)

Fig. 2.4 Lattice girders of SHS in a roof structure (Packer et al., 2009)

Fig. 2.5 Cycle path bridge (Packer et al., 2009)

Fig. 2.6 Basic loading modes

Fig. 2.7 A typical load-displacement curve

Fig. 2.8 Failure modes of RHS joints (Wardenier, 1982)

Fig. 2.9 Punching shear model (Wardenier, 1982)

Fig. 2.10 Chord side wall bearing model (Wardenier, 1982)

Fig. 2.11 Hierarchical chart of gross plastic deformation analysis

Fig. 2.12 Material model and structural response of elastic-perfectly plastic material

(a)

(b)

Fig. 2.13 Material model and structural response of elastic-plastic material

Fig. 2.14 CTOD under elastic-plastic fracture mechanics conditions

Fig. 2.15 An infinite flat plate with a central crack under tensile loading

Fig. 2.16 Contour path for determination of the J -integral

Fig. 2.17 Flaw assessment using FAD approach

Fig. 2.18 FADs in BS7910 (2005)

Chapter 3

Geometric Modelling of Cracked SHS Joints

3.1 Introduction

The principal objective of the present research work is the development of a completely new and robust finite element (FE) mesh generator that can be used to model cracked uni-planar square hollow section (SHS) T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints. The generated models using the new FE mesh generator are then analysed using ABAQUS (2011) FE analysis software. Therefore, the reliability of the results obtained using the ABAQUS (2011) software is very much related to the accuracy of the models produced using the new FE mesh generator. This chapter covers the geometric modelling of cracked SHS joints which are modelled using the new FE mesh generator. The step-by-step FE mesh generation procedure to generate various cracked geometries is described in the next chapter.

The present research work uses the specifications mentioned in various existing codes of practise such as AWS (2000), IIW (2005), BS EN10210-2 (2006) and CIDECT (2009) for precise geometrical modelling of the cracked SHS joints. The geometry of the SHS joints is introduced in Section 3.2. The detailed weld toe modelling of SHS T-, Y- and K-joints are also covered in Section 3.3. The modelling of inner, middle and outer brace corner of SHS T-, Y- and K-joints are explained in Section 3.4. Finally, the geometry of cracks observed in SHS T-, Y- and K-joints is described in Section 3.5.

3.2 Geometry of Square Hollow Section Joints

For modelling the SHS tubular joints, the basic dimensions of joints are defined in Fig. 3.1 according to CIDECT (2009) and IIW (2005) specifications. The SHS tubular joints are defined by some standard variables in order to clearly describe the geometry of the joints. The following variables are adopted based on dimensions given in Fig. 3.1 according to

CIDECT (2009) specifications, $\beta = b_1 / b_0$, $\tau = t_1 / t_0$, $2\gamma = b_0 / t_0$, $\varsigma = g / t_0$, $\alpha = 2L / b_0$, where the descriptions for the symbols used are listed in ‘List of symbols’ section.

A typical cross-section of a SHS tubular connection is shown in Fig. 3.2. For hot finished SHS joints, BS EN 10210-2:2006 recommends the external corner radius value (r_o) as $1.5t_1$ and internal corner radius (r_i) as $1.0t_1$, where t_1 is the specified thickness of the brace. For cold finished SHS joints, BS EN 10219-1:2006 recommends the external corner radius value (r_o) as $2.0t_1$ and internal corner radius as $1.0t_1$.

3.3 Modelling of Welds

Various works have been carried out in the past to study the effect of weld on the P_c load of cracked tubular joints (Burdekin and Laham, 1993; Lee and Wilmshurst, 1995; Lee, 1999). For some cases, it was observed that the strength of the joint was underestimated by even as high as 20% compared to the experimental data without considering the weld (Lee and Wilmshurst, 1995). For SHS tubular joints, accurate modelling of the weld is essential to estimate the P_c load, stress concentration factor (SCF) and stress intensity factor (SIF). Fig. 3.3 shows the weld of a SHS tubular joint showing the different parts of the weld at the intersection region (AWS, 2000). These parts are defined as follows,

Side – the straight part parallel to the chord wall where the dihedral is a right angle;

Toe – the straight part perpendicular to the chord wall where the dihedral is an obtuse angle;

Heel – the straight part perpendicular to the chord wall where the dihedral is an acute angle;

Corner – the curve part along the brace corner.

The detailed weld toe modelling used in the present study is described in following subsection 3.3.1.

3.3.1. Weld toe modelling

Weld toe modelling consists of clearly defining the radius of the weld toe, height of the weld and length at the weld toe. The radius of weld toe is defined first. The basic configuration of a weld toe for a SHS T-joint is shown in Fig. 3.4. The global co-ordinate system (X-Y-Z) of the SHS joint is defined with the referencing point (origin) located at O. It is located at the intersection of chord face and brace centroid. As illustrated in Fig. 3.4, the four corners are

labelled as C1, C2, C3 and C4. The corner C1 is enlarged to describe the detailed weld toe modelling. The brace width is denoted by h_b . Furthermore,

$$|O_o M_o| = |O_o N_o| = r_o \text{ (outer radius = } 1.5t_1 \text{)}$$

$$\left. \begin{aligned} \angle M_o O_o T &= (\pi/4) - (\phi/2) \\ |O_o T| &= r_o \cos [(\pi/4) - (\phi/2)] \\ |M_o T| &= r_o \sin [(\pi/4) - (\phi/2)] \end{aligned} \right\} \quad (3.1)$$

Therefore, the co-ordinates of O_o (X_o, Y_o, Z_o), are given by

$$\left. \begin{aligned} X_o &= 0 \\ Y_o &= h_b/2 - r_o \cos [(\pi/4) - (\phi/2)] \\ Z_o &= h_b/2 - r_o \cos [(\pi/4) - (\phi/2)] \end{aligned} \right\} \quad (3.2)$$

In practice, the weld toe is normally a smooth curve and it bears a right angle around the brace corner (Ji, 2006). For the present research study, a right angle arc M'N' is chosen with center at P_o . From Fig. 3.4,

$$\left. \begin{aligned} |P_o M_o| &= |O_o T| - |M_o T| = r_o \cos [(\pi/4) - (\phi/2)] - r_o \sin [(\pi/4) - (\phi/2)] = \sqrt{2} r_o \sin (\phi/2) \\ |P_o M'| &= |P_o M_o| + |M_o M'| = \sqrt{2} r_o \sin (\phi/2) + l_w \end{aligned} \right\} \quad (3.3)$$

Therefore, the radius of the weld toe ($|P_o M'|$) is obtained.

The coordinates of centre (P_o) are subsequently given by

$$\left. \begin{aligned} X_{P_o} &= 0 \\ Y_{P_o} &= h_b/2 - \sqrt{2} r_o \sin (\phi/2) \\ Z_{P_o} &= h_b/2 - \sqrt{2} r_o \sin (\phi/2) \end{aligned} \right\} \quad (3.4)$$

Eventually, the equations of the weld toe for the quarter part W_1 -C1 corner- W_2 are derived from Eqn. (3.3) and Eqn. (3.4), and are expressed as

$$\left. \begin{aligned} \text{from } W_1 \text{ to } N': Z + h_b/2 + l_w &= X = 0 \\ \text{from } N' \text{ to } M' \text{ (corner C1): } (Y - Y_{P_o})^2 + (Z - Z_{P_o})^2 - (\sqrt{2} r_o \sin (\phi/2) + l_w)^2 &= X = 0 \\ \text{from } M' \text{ to } W_2: Y + h_b/2 + l_w &= X = 0 \end{aligned} \right\} (3.5)$$

The equations for the other 3 quarters of the SHS T-joint is obtained similar to the above mentioned Eqn. (3.5) for the quarter part W_1 -C1 corner - W_2 and hence is not further repeated here.

The height of the weld (h_w) is depicted in Fig. 3.5 and the value is constant throughout the perimeter of brace-chord intersection. After defining the radius of the weld toe and the height of the weld, the next step is to define the length (L) at the weld toe and heel (in Fig. 3.5).

In the toe and heel,

$$\left. \begin{aligned} L &= \frac{t_1}{\sin(\psi)} \text{ and less than } 1.75 t_1 \text{ for } \psi > 135^\circ, \\ L &= \frac{(t_1 + l_w)}{\sin(\psi)} \text{ for } 90^\circ \leq \psi \leq 135^\circ, \text{ and } l_w = \frac{t_1}{2} \cdot \frac{(135^\circ - \psi)}{45^\circ}, \\ L &= \frac{t_1}{2} + \frac{t_1}{\sin(\psi)} \text{ for } \psi < 90^\circ \end{aligned} \right\} (3.6)$$

For the case of SHS Y- and K-joints, the weld toe profile is shown in Fig. 3.6. As described earlier, both the corners of the weld toe for SHS T-joint takes the form of a quarter-circle shape. In case of SHS Y- and K-joints, one corner of the weld toe retains the quarter circle shape and the expression for radius of the weld toe is same as that mentioned in Eqn. (3.3). The other corner of the weld toe takes the form of a quarter-ellipse (Fig. 3.6) and the expression for radius of the weld toe is given by;

$$|P_o N'| = |P_o N_o| + |N_o N'| = [\sqrt{2} r_o \sin (\phi/2)] / \sin (\psi) + l_w \quad (3.7)$$

Thus, the radius of the weld toe ($|P_oM'|$, $|P_oN'|$), height of the weld (h_w) and length at the weld toe (L) for SHS T-, Y- and K-joints are defined.

3.4 Modelling of Inner, Middle and Outer Brace Corner

The inner, middle and outer corner profile of brace is modelled according to the specifications given in BS EN 10210-2:2006 code of practice. The basic configuration of brace corner for a SHS T-joint is shown in Fig. 3.7. As mentioned before, for hot finished SHS joint, BS EN 10210-2:2006 recommends the external corner radius value (r_o) as $1.5t_1$ and internal corner radius (r_i) as $1.0t_1$, where t_1 is the specified thickness of the brace. The co-ordinates of centre of the arc for outer most curve part of brace, O_o (X_o , Y_o , Z_o) is already given in Eqn. (3.2). For the middle part, the corner radius (r_m) is chosen as the average of the external corner radius value (r_o) and internal corner radius; the value being $1.25t_1$.

Similar to case for outer most curve (Figs. 3.4 and 3.7), the co-ordinates of centre of the arc for middle curve part of brace, O_m (X_m , Y_m , Z_m) is obtained as

$$\left. \begin{aligned} X_m &= 0 \\ Y_m &= h_b/2 - t_1/2 - r_o \cos [(\pi/4) - (\phi/2)] \\ Z_m &= h_b/2 - t_1/2 - r_o \cos [(\pi/4) - (\phi/2)] \end{aligned} \right\} \quad (3.8)$$

Finally, the co-ordinates of centre of the arc for inner curve part of brace, O_i (X_i , Y_i , Z_i) is obtained as

$$\left. \begin{aligned} X_i &= 0 \\ Y_i &= h_b/2 - t_1 - r_o \cos [(\pi/4) - (\phi/2)] \\ Z_i &= h_b/2 - t_1 - r_o \cos [(\pi/4) - (\phi/2)] \end{aligned} \right\} \quad (3.9)$$

BS EN 10210-2:2006 further recommends the maximum value of $3.0t_1$ for external corner profile (cl_1 and cl_2 in Fig. 3.7). From the fig's 3.4 and 3.7,

$$|P_oM_o| = |P_oN_o| = |cl_1| = |cl_2| \quad (3.10)$$

From Eqn. (3.3), $|P_o M_o|$ is obtained as $\sqrt{2} r_o \sin (\phi/2)$. Since the external corner radius value (r_o) as $1.5t_1$ and value of ϕ is 60° , the external corner distance (cl_1) becomes

$$|cl_1| = |cl_2| = 1.06 t_1 \quad (3.11)$$

From Eqn. (3.11) it can clearly be seen that, the value of $|cl_1|$ is less than the maximum recommended value of $3.0t_1$ and therefore the present research works conforms to the BS EN 10210-2:2006 recommendations for modeling the corner profile of SHS joints.

3.5 Geometry of Cracks in SHS Joint

The previous research works on geometry of fatigue crack models in tubular joints predominantly focussed on CHS joints (Wu, 1985; Sigurdsson and Torhaug, 1993; Bowness and Lee, 1995; Dover et al., 1995; Cao et al., 1998 and Lie et al., 2001). For the CHS joints, the shape of the crack is modelled as a semi-ellipse (Rhee and Salama, 1987; Pook, 1993; and Lie et. al., 2003). In the case of SHS joints, the fatigue crack follows a different curvature due to dissimilarity of geometry with CHS joints. The stiffness distribution at the flat sidewalls is very different to that of CHS joints. The stress is observed to be highest in the near the corners of the brace, either at the brace or chord weld toe (Lie and Yang, 2007). The typical stress distribution pattern for the SHS T-, Y and K-joints are shown in Fig's 3.8 and 3.9 (Lie and Yang, 2007). It can be observed that the crack always initiates and propagate from the hot spot stress location at the chord corner (Fig. 3.10), and the shape of the crack is not symmetrical (Lie et al., 2006; Lie and Yang, 2007). Yang (2005) approximated the crack shape as two half semi-elliptical cracks, and therefore, it is also adopted in the present study (Fig's. 3.11 and 3.12).

The two parameters used to define the crack in a SHS joint are crack depth (a) and crack length (lc_1 and lc_2). As mentioned before, the crack shape is two half semi-ellipses and the length of each semi-ellipse is kept as a variable in the new FE mesh generator. In general, the length of the crack at the corners varies and it can easily be measured in engineering practise (Rhee, 1989; Shao, 2004). The parameters a , lc_1 and lc_2 are specified In the new FE mesh generator through dimensionless crack depth ratio (a/t_0) and crack length ratio (a/lc_1 , a/lc_2). The new FE mesh generator is versatile in nature to model cracked geometries with a broad range of validity .i.e. from very shallow cracks to very deep cracks as well as short

crack lengths to long crack lengths. These unique features and the detailed procedure for FE mesh generation of cracked SHS joints is covered in the next chapter and the basic sequence of modelling used in the present study is depicted in Fig. 3.13.

Fig. 3.1 Dimensional notations used by IIW (2005) and CIDECT (2009)

Fig. 3.2 Cross section of SHS tubular joints

Fig. 3.3 Weld of SHS tubular joints (AWS, 2000)

Fig. 3.4 Weld toe modelling for SHS T-joints

Fig. 3.5(a) Dimensions of weld ($\psi < 90^\circ$)

Fig. 3.5(b) Dimensions of weld ($\psi > 90^\circ$)

Fig. 3.6 Weld toe profile of SHS Y- and K-joints

Fig. 3.7 Modelling of inner, middle and outer brace corner for SHS T-joints

Fig. 3.8 Stress distribution of SHS T- and Y-joints under axial loading (Lie and Yang, 2007)

Fig. 3.9 Stress distribution of SHS K-joint under axial loading (Lie and Yang, 2007)

Fig. 3.10 Surface crack initiating from hot spot stress locations (Lie et al. 2006)

Fig. 3.11 Parameters used to define the surface crack

Fig. 3.12 Fatigue crack shape represented by two half semi-elliptical cracks

Fig. 3.13 Basic sequence of modelling used in the study

Chapter 4

Finite Element Mesh Generation of Cracked SHS Joints

4.1 Introduction

One of the important aspect of present research work is the development of a completely new and robust finite element (FE) mesh generator that can be used to model cracked uni-planar square hollow section (SHS) T-, Y- and K-joints, and cracked multi-planar SHS TT-, YT- and KT-joints. In fact, the cracked multi-planar SHS TT-, YT- and KT-joints are modelled for the first time and there are no published works for the same. The previous chapter focussed on the precise geometrical modelling of the above mentioned cracked SHS joints. The step-by-step FE mesh generation procedure to generate various cracked geometries using the new FE mesh generator is described in this chapter. The generated FE models are analysed using the ABAQUS (2011) software.

Complex three-dimensional geometry makes it very difficult to create the FE mesh model of a cracked SHS joint by using any commercial FE software. As already described in detail in Chapter 2, very little information about SHS joints with cracks have been published so far. Burdekin and AL Laham (1993) investigated the rectangular hollow section (RHS) fillet welded K-joints with gap experimentally and numerically. They recommended that more experimental work is required in order to check the influence of surface cracks on the P_c load for the whole range of geometries at room and low temperature. A series of research works are on-going for both cracked circular hollow section (CHS) and cracked SHS tubular joints at Nanyang Technological University, Singapore (Lee et al., 1999; Lie et al., 2001; Chiew et al., 2003; Lie et al., 2003; Chiew et al., 2004; Yang, 2005; Lie et al., 2006; Chiew et al., 2007; Lee et al., 2007; Lie and Yang, 2007; Yang et al., 2007; Chiew et al., 2009; Lie et al., 2009; Lee et al., 2010; Lee et al., 2011; Lie and Yang, 2011; Lie and Yang, 2011; Lie et al., 2012; Lie et al., 2014; Lie et al., 2015; Lie et al., 2015). The relevant details of above mentioned works are given in Chapter 2 and therefore it will not be repeated here. Coming to the particular case of the cracked SHS joints, Yang (2005) developed a FE mesh generator that could be used to model cracked uni-planar SHS T-, Y- and K-joints. One of the major

drawbacks in the FE mesh generator (Yang, 2005) was the early termination of FE analysis especially for very large deformation. This tendency was predominantly observed for elastic-plastic analyses during the calculation of J -integral. The elements at the crack tip are observed to collapse during elastic-plastic analyses and therefore, appropriate convergence of solutions is not achieved (Fig. 4.1). The new FE mesh generator addresses this issue by using a key-hole for the modelling the crack tip used in elastic-plastic analyses, and is described in Section 4.2. Separate types of crack tips are used for elastic and elastic-plastic analyses in the new FE mesh generator.

Another aspect of the mesh generator developed by Yang (2005) is the usage of different types of elements to model the cracked SHS joints. Different types of elements such as hexahedral elements with 20 nodes, pyramid elements with 20 nodes, Prism elements with 15 nodes and tetrahedron elements with 10 nodes are used (Fig. 4.2). Furthermore, to model the crack front, collapsed prism element with 15 nodes is used. The usage of different types of elements resulted in generation of elements with very sharp angles (Fig. 4.1). Therefore, the convergence of solutions is not achieved whenever large strain values are involved. The new FE mesh generator uses one type of element .i.e. 20 nodes hexahedral element to model the cracked SHS joints. Unlike the previous approach (Yang, 2005), the new FE mesh generator uses separate modelling techniques at crack front region for elastic and elastic-plastic analyses and this further explained in Section 4.3.

One another major limitation of the mesh generator developed by Yang (2005) relates to the overall mesh design approach used for cracked SHS models. The crack block is inserted into the previously generated uncracked SHS model. The border segments of the inserted crack block should exactly coincide with the mesh of uncracked SHS models. This requirement restricts the crack end position or in other words, the mesh generator is not flexible enough to account for cracks with varying lengths. In the new FE mesh generator, crack block is embedded in the SHS model. Separate FE mesh models are created for both uncracked and cracked SHS joints. The new FE mesh generator is versatile enough to account for cracks of any arbitrary length and is also flexible for modelling various combinations of unsymmetrical cracks having different depths.

In the mesh generation procedure, the crack tube and crack block of a flat plate are modelled first and subsequently, transition zones are used to connect the parent crack block with the other parts of the flat plate. The different types of crack tip used to model the cracked SHS models are described in Section 4.2. The mesh generation of the crack block is covered in Section 4.3 while the transition mesh zones are explained in Section 4.4. Consequently, the flat plate is converted to T-butt geometry by adding weld attachments and vertical plate. In this study, mapping techniques are used to transform 2-D crack front of the parent T-butt geometry into 3-D crack front of the SHS joints. Finally, the remaining parts of brace and chord are generated to complete the modelling of both cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints and this modelling part is described in Section 4.5. Section 4.6 highlights the salient features of the new FE mesh generator as well as list out the uniqueness and advantages of using the same when compared to previous modelling techniques.

4.2 Types of Crack Tip

The nature of mesh geometry around the crack tip is the most pivotal feature in the numerical modelling of flawed structures (Lo and Lee, 1992). In elastic analyses, all nodes at the crack tip are normally tied and the mid-side node of the crack element is moved to the quarter ($1/4$) point position to produce a $1/r^{1/2}$ strain singularity. A typical construction of the crack tip elements used for elastic analyses is shown in Fig. 4.3.a. In case of elastic-plastic analyses, a small plastic zone is formed in the crack tip region. Therefore, the $1/r^{1/2}$ strain singularity is no longer applicable at the crack tip region. In such cases, all elements surrounding the crack tip region produce a $1/r$ strain singularity, which corresponds to the actual crack tip strain field for a fully plastic material (Lie et al. 2014).

In general, there are two methods to construct crack tip elements for elastic-plastic analyses. Fig. 4.3.b shows the first method where the crack tip nodes are coincident and untied. This method results in the development of crack tip blunting due to large strains formed during the analyses. If the deformation at the crack is relatively small, it can still generate accurate elastic-plastic J -integral (J_{ep}) results (Lie et al. 2014). However, if the deformation is very large, the crack tip elements may collapse, producing invalid elastic-plastic J -integral (J_{ep}) results (Fig. 4.4). For a very large plastic deformation, it is frequently encountered that the

solution cannot converge. In order to overcome this difficulty, Anderson (2005) recommended using a key-hole, shown in Fig. 4.3.c, having a finite radius within the plastic zone at the crack tip. Therefore, crack tip elements having a finite radius key-hole are adopted in the present study (Fig. 4.3.c). This helps greatly in avoiding the elements enclosing the crack tip from collapsing and ensuring convergence even at a very large plastic deformation. The selection of finite radius needs to be carefully chosen. If the finite radius is too small, the elements around the crack tip may still collapse, while if the finite radius is too large, it may affect the accuracy of the results because the finite radius of key-hole should lie within the plastic zone (Lie et al., 2012). The value of key-hole radius is adopted in accordance with the recommendations given in API RP579 (2007), i.e. it should be at least five times smaller than tip radius in the deformed state. In the new FE mesh generator, the finite radius of the key-hole is set as a variable so that an appropriate value can be chosen by the users.

4.3 Mesh Generation of Crack Block

From the previous experimental observation of cracked SHS welded joints, it is observed that the cracks always initiate and propagate from the hot spot stress location at the chord corner, and the shape of the crack is not symmetrical (Lie et al. 2006a). Yang (2005) approximated the crack shape as two half semi-elliptical cracks, and therefore, it is also adopted in the present study. It should be noted that the un-symmetrical shape of crack is different to the symmetrical shapes that are present in CHS joints. The detailed description of geometry of cracks used in SHS joints is already described in Chapter 3 and henceforth, it will not be repeated here.

In order to generate the crack block for elastic analysis, a local coordinate UV is defined as shown in Fig. 4.5. A cross-section of a crack tube is described first by defining all the nodes in the UV coordinate plane. Fig. 4.5 shows the first cross-section of a crack tube with all the nodes in it, while Fig. 4.6 shows the second cross-section which is connected to the first one. Several variables and notations are used in the modelling of crack tube. '*Thcra*' is a variable, and it is used to define the geometry of the rectangular cross-section of a crack tube. For different crack shape and geometries, values of *Thcra* are related to the thickness of the chord member. N2X represent the number of rings in a cross section and is set as a variable

used to control rings of elements of the circular crack tube. N2X can be used to refine the mesh of the crack tube and to check the path independence of the J -integral. N2Y is another variable used to define the number of segments/divisions along the crack tube, and its value is the number of the cross-section in a crack tube. Therefore, the number of segments/divisions along the crack tube is $N2Y - 1$.

As shown in Figs. 4.5 and 4.6, the first 18 nodes are defined in the first cross-section, but the second 18 nodes are defined in the second cross-section. After defining all the collapsed prism elements, the next set of the nodes and the elements located in the first cross-section are defined and their numbers are $(N2Y+1)*18+1$ and $N2Y*16+1$, respectively. The geometry of each cross-section in a crack tube is not the same and a reduction factor 'n', depicted in Fig. 4.6, is used to control the reduction in cross-sectional area.

For generating a crack tube used in elastic analysis, the first ring of elements along the crack front is generated first. This is based on two considerations. Firstly, the first ring of elements along the crack front will degenerate from 20 nodes hexahedral element (Fig. 4.2) to 15 nodes collapsed prism element (Fig. 4.7); secondly, as mentioned earlier, in elastic analysis, in order to produce a $1/r^{1/2}$ strain singularity and to enhance the accuracy of calculation, the mid-side node of all the collapsed prism elements must be moved to the $1/4$ point position. Therefore, in order to locate all these collapsed prism elements easily, it is convenient to number the nodes of all the collapsed prism elements continuously and generate the ring of elements along the crack front first.

For the case of elastic-plastic analysis, crack tip elements having a finite radius key-hole are adopted in the present study (Fig. 4.8). As mentioned earlier, the key-hole is critical when there is large deformation of the material, especially when it is near the crack tip. It avoids the elements near the crack tip from collapsing which may give invalid J -integral results. Fig. 4.8 show the first cross-section of a surface crack block used in elastic-plastic analysis. It can be seen that the node numberings in this cross-section is continuous. Fig. 4.9 shows the second cross-section of a crack block used in elastic-plastic analysis which is connected to the first one. Similar to model used for elastic analyses, geometry of each cross-section is not the same and a reduction factor 'n', depicted in Fig. 4.9, is used to control this geometry. Figs. 4.10 and Fig. 4.11 depict the final generated cross sections of crack block for elastic and elastic-plastic analyses, respectively. It is worth mentioning that all the elements are 8

noded linear elements when they are generated first, and then they are modified to 20 nodes hexahedral element by a subroutine used for inserting the mid-side node.

The inner part of the crack tube is depicted in Fig. 4.12. By using the sweep technique, the full crack tube and the rectangular transition zone is generated as shown in Fig. 4.13. For the model shown, it can be observed that the number of all the cross-sections (N2Y) is 30, which means the crack tube is divided into 29 segments. Fig. 4.14 shows the enlarged view of transition zone and it is used to connect the crack tube with Part A and Part B of the parent crack block.

Part A of the crack block (the bottom part) is depicted in Fig. 4.15. The last step of generating the parent crack block is to generate Part B (the upper part). The entire mesh of a parent crack block (Mesh3) generated by this approach is shown in Figs. 4.16 and Fig. 4.17. The detailed cross-sectional view of the mesh design pattern at crack front is illustrated in Fig. 4.18.

4.4 Transition Zones

There are obvious advantages in using transition zones in any FE mesh generator. Firstly, it helps to reduce the number of elements in the FE mesh model. This will in turn reduce the analysis time of the FE mesh models. The pivotal regions such as crack front can be modelled with FE meshes of high mesh density and the regions that are further away from pivotal regions can be modelled with FE meshes of low mesh density. Secondly, it helps greatly in source code debugging. Errors in models can be spotted and rectified locally due to usage of different sub-zones. Furthermore, transition zones enable to connect parent crack block (Mesh3) with adjacent mesh blocks. Several transition zones are used to connect the parent crack block (Mesh3) with the other parts of the cracked structure. The first transition zone is named as Mesh4 (Fig. 4.19). Mesh 4 is the closest transition zone to parent crack block. The purpose of setting Mesh4 is demonstrated in Fig. 4.20, which shows the mesh details of Mesh4. Mesh4 is used to transform 4 elements to 2 elements at the position corresponding to the crack tube.

The adjacent transition zone connected to Mesh4 is named as Mesh5 (Figs. 4.21 and 4.22). There are two purposes for generating Mesh5. Firstly, it is used to transform 3 elements to 1 element at the position corresponding to Part B (upper part of parent crack block zone).

Secondly, it is used to transform 2 elements to 1 element at the position corresponding to Part A (bottom part of parent crack block zone).

The next transition zone generated is named Mesh6 (Fig. 4.23) which is connected to Mesh5. Mesh6 is used to transform 4 layers of elements to 1 layer of element.

The next step in transition zone generation is the generation of two zones which are oriented to the side direction of the parent crack block (Mesh3). They are named as Mesh11 (Fig. 4.24) and Mesh12 (Fig. 4.25). Mesh11 is symmetrical with Mesh12 and is used to align with the parent crack block.

The final step in FE mesh generation for a cracked plate is to generate the symmetrical mesh blocks Mesh13 and Mesh14 (Fig's. 4.26 and 4.27). Mesh13 and Mesh14 are merged with the previously generated mesh zones to form the cracked plate model. Figs. 4.28 to 4.30 illustrate the generated mesh zones of a plate with surface crack. The enlarged view of plate model can be seen from Figs. 4.31 and 4.32. The generated cracked plate model (Fig. 4.33) is subsequently transformed to a cracked T-butt joint by adding the vertical plate and the weld components (Fig. 4.34).

The cracked T-butt joint generated using above steps is curved to form the brace part of the cracked SHS joint. The basic framework used in the new FE mesh generator is illustrated in Fig. 4.35. The cracked T-butt joint is curved to form the shape of a SHS brace by keeping one axis coordinate constant and varying the other two axis coordinate positions. For the illustrated figure, the vertical coordinate axis (X-axis) of the cracked T-butt joint is kept constant while the other two axes (Y- and Z-axis) are transformed using the equations mentioned in Chapter 3 to form the brace part of cracked SHS joint. Regarding the crack block, the 2-D crack block is curved to a 3-D crack block shape by using mapping techniques (Fig. 4.36). The geometrical equations used for the 3-D mapping of crack block are already described in Chapter 3 and hence, it is not further repeated here.

4.5 Uni-planar to Multi-planar Transformation

The parent brace part generated in the previous section is transformed to a complete cracked uni-planar SHS T-joint by adding the chord parts on both sides (Fig. 4.37). The number of divisions in both chord and brace parts is kept as a variable in the new FE mesh generator so that the user can conveniently decide the extent of mesh-refinement needed. The final step in

numerical modelling using the new FE mesh generator is the creation of cracked multi-planar SHS TT-joints. This is achieved by adding the out-of-plane multi-planar brace to the cracked uni-planar SHS T-joint model (Fig. 4.37). The out-of-plane brace is oriented in a direction perpendicular to the cracked uni-planar SHS T-joint model. The geometrical dimensions of the out-of-plane brace are kept identical to the parent brace part of uni-planar model for comparison purposes. The steps involved in the uni-planar to multi-planar transformation for SHS Y- and K-joints are exactly similar to SHS T-joints. Figs. 4.38 to 4.40 depict the model of the completed mesh of typical cracked uni-planar SHS T-, Y- and K-joints, respectively. The angle of inclination of the brace part for uni-planar SHS Y- and K-joints is kept as a variable in the new FE mesh generator. For the depicted models (Figs. 4.39 and 4.40), the angle is 45^0 . The final completed mesh of a typical cracked multi-planar SHS TT-, YT- and KT-joints are illustrated in Figs. 4.41 to 4.43, respectively.

4.6 Important Features of New FE Mesh Generator

The last section of the current Chapter highlights some important features of the new FE mesh generator. First feature is regarding the versatility of the new FE mesh generator to model various types of uncracked and cracked structures. It can be effectively used to generate FE mesh models such as plate joints with both symmetrical and unsymmetrical cracks, plate-to-plate welded joints such as T-butt joints with single-sided and double-sided welds and cruciform X-joints. Regarding hollow section joints, it can be used to model cracked uni-planar SHS T-, Y- and K-joints, and cracked multi-planar SHS TT-, YT- and KT-joints. This versatility of the new FE mesh generator enables the author to carry out extensive parametric study of above mentioned joints which will be described in subsequent chapters. Furthermore, the step-by-step mesh generation technique enables the new FE mesh generator amenable to model other types of cracked structures such as uni-planar SHS X-joints and multi-planar KK-SHS joints; to name a few, in future.

The second feature of the new FE mesh generator is regarding the “spider web” configuration used to design the crack tip region. As mentioned before, the most vital area in FE modelling of any cracked structure is the crack tip region. The most efficient mesh design for the crack tip region has proven to be “spider web” configuration, which consist of concentric rings of quadrilateral elements that are focused towards the crack tip (Fig. 4.44).

The innermost ring is degenerated to triangular elements. The amount of mesh refinement needs to be more in the crack tip region since the region is characterized by steep stress and strain gradients. The “spider web” design helps in a smooth transition from a fine mesh at the crack tip region to coarser mesh further away from the crack tip region. Fig. 4.44 shows the sectional view of the mesh design pattern used at the crack tip region for FEACrackTM (2010) software and for new FE mesh generator. It can clearly be seen that the “spider web” configuration is strictly followed for modelling the crack tip region for both cases.

The third feature of new FE mesh generator is the provision for robust mesh refinement. The dense or highly refined FE mesh is used for important regions such as crack tip, crack block and high stress concentration spots. The coarse FE mesh is used in the regions that are further away from high stress concentration spots. *h*-refinement or local refinement is well incorporated in the new FE mesh generator so that element sizes can be reduced at will to obtain accurate analyses results (Fig. 4.45). For crack tube, mesh refinement is done by controlling the number of divisions along the crack tube. Fig. 4.46 clearly illustrates some typical cases of crack tube refinement used in the new FE mesh generator. The *h*-refinement is distinctly seen in crack tube with 14 divisions (Fig. 4.46). In this case, the meshes close to crack tip region are dense whereas the meshes that are away from crack tip region are coarse. Similarly, the crack tube can be further refined into 20, 26, 32 and 38 divisions as shown in Fig. 4.46. This type of local refining of FE meshes allows the user to be flexible while deciding on the number of elements to be used for a particular area of the model. Therefore, the new approach is versatile enough to account for cracks of any arbitrary length and is also flexible for modelling various combinations of unsymmetrical cracks having different depths.

As mentioned before, for the parts that are further away from regions of high stress concentration even more coarse mesh can be used. Fig. 4.47 illustrates a typical mesh refinement pattern used for a chord part with 5, 10 and 20 divisions along the length of the chord. Similar mesh refinement pattern is used in brace regions and hence it is not shown in this Chapter. For having a better understanding of extend of mesh refinement and choice of mesh size, load-displacement curves and *J*-integral values are plotted and compared for models with varying mesh density. A cracked uni-planar SHS T-joint with chord width of 350 mm, brace width of 150 mm, chord and brace thickness of 16 mm and weld thickness of

8 mm is modelled with varying mesh density. The crack is located at the corner of the brace, and the crack depth and length are 6.4 mm and 116 mm respectively. Since the crack is unsymmetrical in nature for SHS joints, first part of crack length is 33 mm and the second part of the crack length is 83 mm, thus making the total crack length as 116 mm.

The load-displacement curves obtained for models with coarse mesh and refined mesh are depicted in Fig. 4.48. The number of elements used for coarse mesh, refined mesh-01 and refined mesh-02 models are 12,040, 16,070 and 20,030 respectively. The cracked uni-planar SHS T-joints were subjected to brace end axial tensile loading and it can be seen from Fig. 4.48 that the meshes for all the models have converged. This further reinforces mesh refining capability of the new FE mesh generator.

The J -integral values for the above models are also plotted and presented in Fig's. 4.49 and 4.50. Yang (1996) recommended that at least three rings of elements should be used at the crack tip region so that J -integral values can be estimated from adequate contours to check the path independence of the results. In the new FE mesh generator, minimum four rings of elements are used while calculating the elastic J -integral, J_e , and the elastic-plastic J -integral, J_{ep} . The plots (Fig's. 4.49 and 4.50) reveal the convergence of solutions for mesh models with different mesh density. For the present study, the actual mesh size chosen is a compromise between achieving acceptable accuracy and reducing the computer cost of the analysis.

The fourth feature of the new FE mesh generator is the fast generation of different types of cracked and uncracked models. For example, a typical cracked multi-planar SHS YT-joint with 20000 elements can be generated within 4 minutes.

Other salient features of new FE mesh generator such as usage of single types of element .i.e. 20 nodes hexahedral element, usage of key-hole for modelling the crack tip for elastic-plastic analysis and zoning techniques are already described in detail in this Chapter and hence, it won't be repeated here.

This Chapter focus exclusively on the mesh generation procedure using the new FE mesh generator. Extensive validations for the same will be detailed in subsequent chapters. The validations for plate-to-plate welded joints generated using new FE mesh generator is covered in Chapter 5. The new FE mesh generator is validated with the FEACrackTM (2010) software as well as with the previously published results of Bowness and Lee (2000) for

single-sided T-butt joints. The stress intensity factors (K_I , K_{II} and K_{III}) and weld toe magnification factor (M_k) generated using the new FE mesh generator are validated for plate-to-plate welded joints.

The validation of cracked SHS T-, Y- and K-joints produced using the new FE mesh generator is carried out with previous experimental test results (Lie et al., 2006; Lie and Yang, 2007) and is described in Chapter 6. The load-displacement curves, elastic J -integral, J_e , and the elastic-plastic J -integral, J_{ep} and failure assessment diagrams (FADs) of cracked SHS T-, Y- and K-joints are further compared with previously published works (Yang, 2005; Lie et al., 2006; Lie and Yang, 2007; Lie et al., 2009; Lie and Yang, 2011) and are described in Chapter 6.

Fig. 4.1 FE mesh of the crack tip used by Yang, 2005

(a) Hexahedral
(20 nodes)

(b) Prism
(15 nodes)

(d) Tetrahedron
(10 nodes)

(c) Pyramid
(20 nodes)

Fig. 4.2 Different types of elements used by Yang, 2005

Fig. 4.3 Three different methods used to model the crack tip

Fig. 4.4 Distorted shape of plastic singularity elements

Fig. 4.5 The first cross-section of crack block for elastic analysis

Fig. 4.6 The second cross-section of crack block for elastic analysis

Fig. 4.7 Element types used at the crack tip

Fig. 4.8 The first cross-section of crack block for elastic-plastic analysis

Fig. 4.9 The second cross-section of crack block for elastic-plastic analysis

Fig. 4.10 Elastic crack tip

Fig. 4.11 Elastic-plastic crack tip

Fig. 4.12 Inner part of crack tube

Fig. 4.13 Generated unsymmetrical crack tube for SHS joints

Fig. 4.14 Enlarged view of transition zone of a crack tube

Fig. 4.15 Part A of the crack block

Fig. 4.16 Mesh3 block

Fig. 4.17 Mesh3 block - skeletal view

(a) Skeletal view

(b) Solid view

Fig. 4.18 Sectional view of the mesh design of the crack front

Fig. 4.19 Transition zone - Mesh4

Fig. 4.20 Enlarged view of Mesh4

Fig. 4.21 Transition zone - Mesh5

Fig. 4.22 Transition zone -Mesh5 - skeletal view

Fig. 4.23 Transition zone - Mesh6

Fig. 4.24 Sub-zone of Mesh11

Fig. 4.25 Sub-zone of Mesh12 - skeletal view

Fig. 4.26 Sub-zone of Mesh13

Fig. 4.27 Sub-zone of Mesh14 - skeletal view

Fig. 4.28 Generated mesh zones of a plate with a surface crack

Fig. 4.29 Generated mesh zones of a plate with a surface crack – skeletal view

Fig. 4.30 Mesh zones of a plate including Mesh13 and mesh14 – skeletal view

Fig. 4.31 Enlarged view of the plate model

Fig. 4.32 Enlarged view of the plate model –skeletal view

Fig. 4.33 Completed mesh of the plate model containing a surface crack

Fig. 4.34 Completed mesh of the T-butt joint containing a surface crack

Fig. 4.35 Sequence to form the parent brace of SHS T-joint

Fig. 4.36 Mapping of a 2D crack block to a 3D curved crack block

Fig. 4.37 Uni-planar to multi-planar transformation

Fig. 4.38 The completed mesh of a cracked uni-planar SHS T-joint

Fig. 4.39 The completed mesh of a cracked uni-planar SHS Y-joint

Fig. 4.40 The completed mesh of a cracked uni-planar SHS K-joint

Fig. 4.41 The completed mesh of a cracked multi-planar SHS TT-joint

Fig. 4.42 The completed mesh of a cracked multi-planar SHS YT-joint

Fig. 4.43 The completed mesh of a cracked multi-planar SHS KT-joint

Fig. 4.44 Spider-web configuration for FE mesh design

Fig. 4.45 h -refinement for typical FE mesh

Fig. 4.46 Mesh refinement for crack tube

Fig. 4.47 Mesh refinement for a typical chord part

Fig. 4.48 Load-displacement curves obtained using different meshes

Fig. 4.49 Elastic J -integral values calculated using different meshes

Fig. 4.50 Elastic-plastic J -integral values calculated using different meshes

Chapter 5

Extensive Parametric Study of Cracked Plate-to-Plate Welded Joints

5.1 Introduction

The new finite element (FE) mesh generator is used in the extensive parametric study of cracked plate-to-plate welded joints before it is used to analyse more complex geometries like cracked uni-planar and multi-planar square hollow section (SHS) joints. This chapter covers the parametric study of cracked plate-to-plate welded joints. The parametric study of cracked uni-planar and multi-planar SHS joints is described in subsequent chapters. Parametric studies are carried out on different types of cracked plate-to-plate welded joints such as double-sided T-butt joints and cruciform X-joints. It covers a wide range of weld and crack parameters. The FE results are then used to develop new weld toe magnification factor (M_k) equations for cracked double-sided T-butt joints and cruciform X-joints through multiple regression analyses.

The slag intrusion due to welding process act as a crack initiation at the weld toe of any plate-to-plate welded joint. Therefore, the study of fatigue strength of the welded joint is focused around this high stress concentration region. When the welded joint is subjected to long term cyclic loading, the crack will propagate, resembling a semi-elliptical shape. In the field of linear elastic fracture mechanics, the intensity of the stress field at the crack tip is called the stress intensity factor (K). It is used to evaluate the fatigue and fracture resistance of cracked welded structures (Zhang et al., 2002). For weld toe cracks, the K solutions are frequently expressed in terms of the M_k , which in turn is given by

$$M_k = \frac{K_{(\text{in plate with attachment})}}{K_{(\text{in same plate but without attachment})}} \quad (5.1)$$

The M_k concept pioneered by Maddox (1975) quantifies the change in K as an outcome of the presence of the weld and the attachment. It is used along with the K solutions for the same crack in a plain plate, without the welded joint. Thus, the K for a plate with attachment can be determined using the following equation:

$$K_{\text{plate with attachment}} = M_k Y \sigma \sqrt{\pi a} \quad (5.2)$$

The 2-D FE solution to M_k is restricted to edge crack at the weld toe, and they were found to overestimate the actual crack growth rates (PD 6493, 1991). Fu et al. (1993) conducted a number of FE analyses on 3-D T-butt joints with a 45° weld angle. They subsequently ignored their crack end solutions, as the path independence criteria for J -integral was not met. Lie et al. (2000) conferred that the two-dimensional M_k factors over-estimated and under-estimated the three-dimensional M_k factors of X-joints at both the deepest and crack end points by approximately 15% and 65%, respectively. The most comprehensive study of M_k solutions for 3-D T-butt joints was carried out by Bowness and Lee (2000). The equations proposed by them have been incorporated in British Standard guidance to methods for assessing the acceptability of flaws in metallic structures (BS7910, 2005). However, BS7910 (2005) uses a single set of equations for M_k determination of various cracked welded geometries such as T-butt geometry with one-side weld, T-butt geometry with double-sided weld and non-load carrying cruciform X-joint. The present study observed that M_k values for non-load-carrying cruciform X-joint notably differs with M_k values of T-butt geometry especially when the corresponding profile is subjected to bending loading. The detailed results of above mentioned trends are described in the subsequent sections.

In this chapter, the validation of the new FE mesh generator is carried out with the FEACrackTM (2010) software results and is described in Section 5.2. Upon validation, extensive parametric study is carried out. The scope of parametric study is explained in Section 5.3. The FE modelling part is subsequently covered in Section 5.4. The detailed discussion of results from parametric study is described in Section 5.5. Finally, multiple regression analyses are used to develop new M_k equations for cracked double-sided T-butt joints and cruciform X-joints and it is described in Section 5.6.

5.2 Validation of the New FE Mesh Generator for Plate-to-Plate Joints

The new FE mesh generator is validated with the FEACrackTM (2010) software; before using it for parametric study of cracked plate-to-plate welded joints. The comparison of stress intensity factors (K_I , K_{II} and K_{III}) generated using the new FE mesh generator and FEACrackTM (2010) software is illustrated in Fig. 5.1. The single-sided T-butt joint models

are used for comparison purpose with geometric parameters of $a/T=0.5$, $a/c=0.4$, $L/T=1.25$ and $\theta=45^\circ$. The observed mode of crack opening is mode I due to the tensile load applied to the considered model. It is observed that the maximum percentage difference of K_I at the deepest point is 1.94% while at the crack ends, it is 0.25%. It shows that the trend of K_I and K_{II} is symmetrical, while K_{III} is anti-symmetrical. Furthermore, the K_{II} and K_{III} values are almost zero due to the predominant nature of crack opening mode. The results covering all the three stress intensity factors namely K_I , K_{II} and K_{III} demonstrate good agreement of values between FEACrackTM (2010) software and the new FE mesh generator. Hence, the new FE mesh generator is suitable to be used in the parametric study of cracked plate-to-plate joints. For further verification, the M_k results of the single-sided T-butt joints produced by the new FE mesh generator are compared with M_k results of Bowness and Lee (2000), and it is explained in Section 5.5.1.

5.3 Scope of Parametric Study

Upon validation of the new FE mesh generator for cracked plate-to-plate welded joints, extensive parametric study is carried out to propose a set of new M_k equations for cracked double-sided T-butt joint and for cruciform X-joint. The various parameters used in this study are chosen in accordance with the works done by Bowness and Lee (2000). The parameters take into account the wide range of crack and weld dimensions associated with double-sided T-butt joints and X-joints. Extensive numerical analyses are able to be carried out because of the advances in computing power and the use of robust new FE mesh generator. For the double-sided T-butt joints and X-joints, this study represents the most comprehensive analyses for the M_k determination till date. The crack and weld parameters are covered in Sections 5.3.1 and 5.3.2, respectively. The details of table of analyses are described in Section 5.3.3.

5.3.1. Details of crack parameters

Weld toe cracks associated with fatigue of steel structures commonly resemble a semi-elliptical shape (Zhang et al., 2002). 11 different crack depth ratios (a/T) are analysed, which vary from a very shallow crack ratio of 0.05 to a deep crack ratio of 0.9 (Fig. 5.2). For all the analyses, the plate thickness (T) is kept constant and the crack depth (a) is adjusted, so as to

get the required crack depth ratio. The various combinations used for the analyses are illustrated in Table 5.1. The deep crack depth ratio of 0.9 accounts for the deepest crack depth ratio observed in practice. Crack aspect ratio (a/c) values vary from 0.1 to 1. This wide range practically covers the whole spectrum of crack aspect ratios that are frequently encountered in practice.

5.3.2. Details of welded joint parameters

For conducting the extensive numerical analysis of cracked double-sided T-butt joints and X-joints, four basic parameters namely crack depth ratio (a/T), crack aspect ratio (a/c), attachment footprint width ratio (L/T) and weld angle (θ) are considered in this study. The first two parameters are paraphrased in the preceding section, while the last two parameters are covered in this section. For standard fillet weld profiles, attachment footprint width ratios of 0.5, 1.25, 2.0 and 2.75 are investigated. The selected values conforms to the AWS D1.1-2000 (2000) recommendations. The attachment footprint width ratio covers main plate thickness, attachment thickness and length of weld in one parameter and is illustrated in Fig. 5.2. The X-joint has an additional attachment compared to the double-sided T-butt joint and it is represented using dashed lines in Fig. 5.2. The last parameter is the weld angle (θ). For the parametric study, weld angles of 30° , 45° , 60° and 75° are used. The equations for M_k mentioned in the BS7910 (2005) are valid only for the weld angle of 45° and for a sharp weld toe where the weld toe radius is taken to be zero. Lazzarin and Livieri (2001) proposed that, the sharp weld toe cases are commonly observed and they cause more severe stress concentration effect, and therefore, the present study focuses solely on the sharp weld toe for the axial and bending loading cases.

5.3.3. Details of table of analyses

In the present study, around 5000 cases of different geometries of single-sided and double-sided T-butt joints, non-load carrying cruciform X-joints and plain plates are analysed. The parameters and the loading pattern used are presented in Table 5.1. The relevant aspects concerning the table of analyses are given as follows:

- Four weld angles 30° , 45° , 60° and 75° are investigated for attachment footprint width ratio (L/T) values of 0.5 and 1.25.

- Largest weld angle of $\theta = 75^\circ$ is excluded for attachment footprint width ratio (L/T) values of 2.0 and 2.75. This is because the large weld angle is not compatible with the high value of attachment footprint width ratios (Bowness and Lee, 2000). Moreover, the current parametric study follows the parameters which were used by Bowness and Lee (2000) for developing the M_k equations for T-butt geometry with one-side weld.
- FE models for crack depth ratio (a/T) values 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9 are modelled using FEACrackTM software (2010).
- FEACrackTM software (2010) is not capable of generating the mesh for very shallow cracks with crack depth ratio (a/T) values of 0.05, 0.1 and 0.15. Hence, the newly developed FE mesh generator is used; which is capable of modelling very shallow cracks. It has already been validated for models with very shallow cracks (Lie et al., 2012; Lie et al., 2014).
- After the parametric study, a new set of equations are proposed for the deepest point as well as for the crack ends. The axial (membrane) loading is denoted by the subscript 'm', while the bending loading is denoted by the subscript 'b'. Furthermore, the deepest point of the crack is denoted by the subscript 'a' and the crack ends by the subscript 'c'.

5.4 Finite Element Modelling

The availability of numerous data improves the statistical accuracy of any regression analysis. In order to satisfy this criterion, adequate FE models are analysed in the present study to establish the relationship among the dependent parameter which is M_k and the independent parameters which are a/T , a/c , L/T and θ . The input variables of the mesh generator are crack depth (a), crack length ($2c$), main plate thickness (T), attachment thickness (t), weld height (h_w), weld length (l_w), weld angle (θ), attachment height (h), and the length and width of the main plate. In the parametric study, the external dimensions and thickness of the main plate are kept constant. The FEACrackTM (2010) software and the new FE mesh generator creates the mesh models and input files for ABAQUS (2011) software. On completion of the FE analyses using the ABAQUS (2011) software, the stress intensity factors are obtained as outputs which are subsequently used to calculate the M_k values of the double-sided T-butt joints and the X-joints. Fig. 5.3 shows the mesh design of a crack section region ($a/T=0.5$, $a/c=0.2$), which is exactly similar for both double-sided T-butt

joints and X-joints. Fig. 5.4 and 5.5 shows the full mesh model of a typical double-sided T-butt joint and X-joint, respectively.

In the analyses, double-sided T-butt joints and X-joints are subjected to both axial and bending loading (Fig. 5.6). Fig. 5.6 shows the loading and boundary conditions of a typical X-joint. The same for double-sided T-butt joints is not shown since the loading pattern and boundary conditions are similar to X-joint. Bending loading is applied in such a way that the attachment is loaded and the ends of the main plate parallel to crack face are restrained. In Fig. 5.6, it can be seen that the side perpendicular to crack face is restrained and it is due to symmetric nature of models used. For the case of plain plate, bending loading is achieved through pure bending. For axial loading cases, one end of the main plate parallel to crack face is restrained and a uniform stress is applied on the other end of the main plate. The Young's modulus and Poisson's ratio of the material used are 210 kN/mm^2 and 0.30 respectively.

5.5 Results and Discussion

This section covers (1) the comparison of M_k results between the X-joints with the T-butt joints subjected to axial and bending loading and (2) the influence of various parameters on M_k results. Fig. 5.7 illustrates the enlarged view of deformed crack area of a typical double-sided T-butt joint under axial loading. Similar deformation pattern is observed for a typical X-joint and hence it is not shown in this chapter. Fig. 5.8 and 5.9 depicts the typical deformed FE mesh of double-sided T-butt joint and X-joint under bending loading, respectively. It clearly indicates the deformation of the main plate under the action of bending loading.

In this section, the comparison of M_k results for T-butt joints with X-joints is carried out first and is described in Section 5.5.1. Thereafter, the effect of various parameters such as crack aspect ratio (a/c), weld angle (θ) and attachment footprint width ratio (L/T) on M_k results are covered in Sections 5.5.2, 5.5.3 and 5.5.4, respectively.

5.5.1. Comparison of M_k results for T-butt joints with X-joints

M_k vs a/T plots comparison between the X-joints with the T-butt joints under axial and bending loading are illustrated in Figs. 5.10 to 5.13. The representative samples depicted,

cover varying crack depth ratios for $a/c=0.2$, $L/T=1.25$, $\theta=45^\circ$. For validation purpose, the M_k results of single-sided T-butt joints from parametric study are compared with values produced by Bowness and Lee (2000). The results demonstrate good agreement of values between the two cases for single-sided T-butt joints.

For deepest point under axial loading cases, the M_k values for shallower cracks are higher for the X-joints when compared to the T-butt joints, and the reverse trend is observed for deeper cracks (Fig. 5.11). This phenomenon is mainly due to the presence of notch stress which magnifies the stress intensity factor for shallower cracks. In addition, as the crack opens, upward deformation of the plain plate takes place. However, for deeper cracks with a/T ratios greater than 0.2, the upward movement of the crack is restrained by the presence of the two attachments. Since the X-joint has an additional attachment, the M_k values tend to fall below that of the T-butt joint for deeper cracks. Similar trend is observed at the crack end regions (Fig. 5.10).

In general, it is observed that variation of M_k values is more pronounced when the geometry is subjected to bending loading as compared to axial loading as shown in Figs. 5.12 to 5.13. This phenomenon is attributed to the fact that, the loading region under the bending condition is relatively adjacent to the attachments as compared to the axial loading. All the plots distinctly establish the need for having separate M_k equations for the double-sided T-butt joint and X-joint.

For the comparison purposes, all the M_k results of double-sided T-butt joints from parametric study are compared with those of single-sided T-butt joints by Bowness and Lee (2000). For each case, there are more than 600 M_k results and the largest percentage difference obtained after comparing all M_k results is detailed below. In the case of axial loading, the largest percentage difference in M_k values is 55% at the deepest point ($a/c=0.1$, $L/T=2.75$, $\theta=60^\circ$). For the above mentioned case of largest percentage difference, the corresponding M_k values for double and single-sided T-butt joints are 0.5707 and 0.8847, respectively. The high value of largest percentage difference is attributed to the small nature of values and hence when the percentage difference is calculated, the resulting value obtained is large. Similarly, at the crack ends, the largest percentage difference in M_k values is 92% ($a/c=0.1$, $L/T=2.75$, $\theta=60^\circ$). The corresponding M_k values for double and single-sided T-butt joints are 0.7417 and 1.4275, respectively. For bending loading cases, the largest percentage difference in M_k

values is 127% at the deepest point ($a/c=0.1$, $L/T=2.75$, $\theta=60^\circ$). The corresponding M_k values for double and single-sided T-butt joints are 1.2299 and 0.5399, respectively. Similarly, at the crack ends, the largest percentage difference in M_k values is 158% ($a/c=0.1$, $L/T=2.75$, $\theta=45^\circ$). The corresponding M_k values for double and single-sided T-butt joints are 3.7780 and 1.4608, respectively. Another important point to be noted is that all the largest percentage difference values for the above cases are observed for geometries with very deep a/T value of 0.9 and with largest L/T value of 2.75. This geometry represents one of the extreme cases considered in the parametric study.

The M_k results of double-sided T-butt joints from parametric study are further compared with those of X-joints. In this comparison too, more than 600 M_k results are compared for each case and the largest percentage difference obtained is given below. In the case of axial loading, the largest percentage difference in M_k values is 136% at the deepest point (for $a/c=1.0$, $L/T=2.75$, $\theta=60^\circ$). For the above mentioned case of largest percentage difference, the corresponding M_k values for double-sided T-butt joints and X-joints are 1.8384 and 0.7770, respectively. The reason for high value of largest percentage difference is already explained above and hence, it is not repeated here. Similarly, at the crack ends, the largest percentage difference in M_k values is 46% (for $a/c=1.0$, $L/T=2.75$, $\theta=45^\circ$). The corresponding M_k values for double-sided T-butt joints and X-joints are 1.1386 and 0.7767, respectively. For bending loading cases, the largest percentage difference in M_k values is 62% at the deepest point (for $a/c=0.4$, $L/T=2.75$, $\theta=45^\circ$). The corresponding M_k values for double-sided T-butt joints and X-joints are 1.8544 and 3.0042, respectively. Similarly, at the crack ends, the largest percentage difference in M_k values is 76% (for $a/c=0.2$, $L/T=0.5$, $\theta=45^\circ$). The corresponding M_k values for double-sided T-butt joints and X-joints are 1.0273 and 1.8080, respectively. In this comparison too, the largest percentage difference values are observed for extreme geometries considered in the parametric study such as with very deep a/T value of 0.9 and with L/T values of 0.5 and 2.75.

5.5.2. Effect of crack aspect ratio on M_k results

The parametric study consists of various parameters which have influence on M_k values, and the prominent trends observed due to effect of the parameters are discussed in this section. Figs. 5.14 to 5.18 illustrate the representative plots depicting the variation of M_k results over

crack depth ratio (a/T) for different values of crack aspect ratio (a/c), weld angle (θ) and attachment footprint width ratio (L/T) for double-sided T-butt joints. Figs. 5.19 to 5.23 cover the same for X-joints.

Figs. 5.14 and 5.15 represent the effect of crack aspect ratio on M_k values at the deepest point and at the crack ends for axial loading. A similar trend is observed for bending loading cases, and hence only the axial loading cases are presented here. For cracks with higher a/c values (cracks with round shape), it is found that the M_k values are lower when compared to those for cracks with lower a/c values. This tendency is applicable at the crack ends (Figs. 5.15, 5.20), and is associated to restricted ability of the attachment to deform backwards which subsequently affects the opening of crack ends.

The study also observes that the M_k values decrease with increasing a/T ratio, thereby reporting higher M_k values for shallower cracks and lower M_k values for deeper cracks. The exception to this trend is at the deepest point for a/T ratios of 0.8 and 0.9 (under both axial and bending loading). This is due to negative output of stress intensity factors which in turn implies a rare condition of crack faces penetrating each other (Bowness and Lee, 2000). Moreover, the contact formulation is not defined in the current parametric study to account for the penetration of crack surfaces and subsequently, for the regression analyses, the very deep crack a/T ratios of 0.8 and 0.9 are omitted for deepest point of the crack under axial and bending loading.

5.5.3. Effect of weld angle on M_k results

The effect of varying weld angle (θ) on M_k values for double-sided T-butt joint is shown in Figs. 5.16 and 5.17, and it can be observed that M_k values increase with the increasing steepness of the weld angle (θ). The steepness raises the stress concentration effect especially for shallower cracks where the fatigue life of the cracked structure is most consumed. Influences of weld angle (θ) on M_k values for the X-joints are identical to the trend observed for the double-sided T-butt joints (Figs. 5.21 and 5.22).

5.5.4. Effect of attachment footprint width ratio on M_k results

The most salient observation associated with the effect of attachment footprint width ratio (L/T) is the occurrence of higher M_k values for L/T value of 1.25 as compared to L/T value

of 0.5 (Figs. 5.18 and 5.23). In cases of L/T value beyond 1.25, the tendency becomes marginal due to attachment base stiffening the top surface of the main plate (Bowness and Lee, 2000). The increase in weld toe notch stress due to this stiffening also plays a pivotal part in the observed trend. The above mentioned trend is observed for different loading cases as well as at different points of the crack.

5.6 Multiple Regression Analyses

Multiple regression analyses are used to develop the new set of equations of M_k for double-sided T-butt joints and for X-joints. The data from the parametric study are incorporated in the regression analyses with the exception of the following:

- Deepest point of the crack for axial and bending loading - 11 different crack depth ratios (a/T) are analysed for the study ranging from 0.05 for very shallow crack to 0.9 for very deep crack. However, for the regression analyses, the very deep crack a/T ratios of 0.8 and 0.9 are omitted. The rationale for the above omission is already explained in last paragraph of Section 5.5.2.

The data regression procedure for the development of new M_k equations for cracked double-sided T-butt joints and cruciform X-joints are covered in Section 5.6.1. The goodness of fit and validity ranges for the proposed equations are described in Sections 5.6.2 and 5.6.3, respectively.

5.6.1. Data regression

The equations are progressively developed in stages considering the effect of various parameters used in the parametric study. The primary development stage of the equations includes data concerning cases for constant weld angle (θ) and attachment footprint ratio (L/T). The secondary development stage involves the effect of varying weld angle (θ). The final development stage considers the varying attachment footprint ratio (L/T), thereby covering the whole set of parameters that influence the regression of M_k equations. The new parametric M_k equations are made up, in stages, from the following four basic functions:

- The crack depth to a power $C_1\left(\frac{a}{T}\right)^{C_2}$
- One minus the crack depth to a power $C_1\left[1 - \left(\frac{a}{T}\right)\right]^{C_2}$

- Exponent of the crack depth to a power $C_1 \exp[(\frac{a}{T})^{C_2}]$
- Polynomial functions $C_1 + C_2(\frac{a}{T})^1 + C_3(\frac{a}{T})^2 + \dots$

where C_1, C_2, C_3 etc. are coefficients. The computed data is modelled by a function which is a nonlinear combination of the model parameters and depends on one or more independent variables. For the present study, the dependent variable is M_k whereas the independent variables are $a/T, a/c, L/T$ and θ .

5.6.2. Goodness of fit of proposed new equations

The newly proposed equations of M_k are similar in form to the M_k equations described in BS7910 (2005). The equations specified in BS7910 (2005) are complex in nature due to the number of influencing parameters as well as the large amount of data used (Bowness and Lee, 2000). Histograms describing the error frequency distributions of double-sided T-butt joints are illustrated from Figs. 5.24 to 5.27. The percentage error is calculated using the following expression:

$$\% \text{ Error} = \frac{[M_{k(\text{equation})} - M_{k(\text{FEA})}]}{M_{k(\text{FEA})}} \times 100\% \quad (5.3)$$

In the histograms, the percentage error is plotted along the abscissa while the percentage of values that belong to a particular range is plotted along the ordinate. For the axial loading cases, the maximum percentage of over-prediction is 5.4% and the maximum percentage of under-prediction is 7.9%. For the bending loading case, the maximum percentage of over-prediction is 7.4% and the maximum percentage of under-prediction is 7.6%.

Histograms describing the error frequency distributions of X-joints are illustrated from Figs. 5.28 to 5.31. In the case of X-joints, for axial loading, the maximum percentage of over-prediction is 6.8% and the maximum percentage of under-prediction is 7.6%. For the bending loading cases, the maximum percentage of over-prediction is 6.8% and the maximum percentage of under-prediction is 6.9%. All the histograms endorse the goodness of fit of the proposed equations to the extensive data from which they are derived. For verifying the proposed equations further, the M_k values from the original data and the M_k

values obtained from the regression equations are plotted and compared as shown in Figs. 5.32 to 5.35 for double-sided T-butt joints and in Figs. 5.36 to 5.39 for X-joints. The plots reaffirm the accuracy of the proposed regression equations as the trends for both the considered cases exhibit analogous variation of M_k values.

5.6.3. Validity ranges of proposed equations

The new sets of M_k equations are proposed and presented in Appendix A and Appendix B for double-sided T-butt joint and X-joint, respectively. They cover M_k equations at the deepest point and at the crack ends of joints subjected to axial as well as bending loading. The weld angles (θ) in proposed equations are measured in radians. The validity range for crack depth ratio (a/T) is from 0.05 to 0.7 at the deepest point and from 0.05 to 0.9 at the crack ends. For cases of crack ends under bending loading, the proposed M_k equations are valid only for weld angle (θ) of 45° . These new M_k equations are already published in International Journal of Fatigue (Lie et al., 2015).

Fig. 5.1 Comparison of stress intensity factor (K) values from the new FE mesh generator with the same from FEACrack™ (2010) software

Fig. 5.2 Nomenclature of parameters in double-sided T-butt joint and X-joint

Fig. 5.3 Typical mesh detail for crack section region

Fig. 5.4 Typical FE mesh of double-sided T-butt joint

Fig. 5.5 Typical FE mesh of X-joint

Fig. 5.6 Loading and boundary conditions (a) axial loading; (b) bending loading

Fig. 5.7 Enlarged view of deformed crack area of double-sided T-butt joint under axial loading

Fig. 5.8 Typical deformed mesh of double-sided T-butt joint under bending loading

Fig. 5.9 Typical deformed mesh of X-joint under bending loading

Fig. 5.10 M_k vs a/T plot comparison b/w X-joint with T-butt joints- For axial loading at crack ends ($a/c=0.2$, $L/T=1.25$, $\theta=45^\circ$)

Fig. 5.11 M_k vs a/T plot comparison b/w X-joint with T-butt joints- For axial loading at deepest point ($a/c=0.2$, $L/T=1.25$, $\theta=45^\circ$)

Fig. 5.12 M_k vs a/T plot comparison b/w X-joint with T-butt joints- For bending loading at deepest point ($a/c=0.2$, $L/T=1.25$, $\theta=45^\circ$)

Fig. 5.13 M_k vs a/T plot comparison b/w X-joint with T-butt joints- For bending loading at crack ends ($a/c=0.2$, $L/T=1.25$, $\theta=45^\circ$)

Fig. 5.14 Effect of crack aspect ratio on M_k values - For axial loading at deepest point ($L/T=1.25$, $\theta=45^\circ$) – double-sided T-butt joint

Fig. 5.15 Effect of crack aspect ratio on M_k values - For axial loading at crack ends ($L/T=1.25$, $\theta=45^\circ$) – double-sided T-butt joint

Fig. 5.16 Effect of weld angle M_k values - For axial loading at deepest point ($a/c=0.4$, $L/T=1.25$) – double-sided T-butt joint

Fig. 5.17 Effect of weld angle M_k values - For axial loading at crack ends ($a/c=0.4$, $L/T=1.25$) – double-sided T-butt joint

Fig. 5.18 Effect of attachment footprint ratio on M_k values - For axial loading at crack ends ($a/c=0.2$, $\theta=45^\circ$) – double-sided T-butt joint

Fig. 5.19 Effect of crack aspect ratio on M_k values - For axial loading at deepest point ($L/T=1.25$, $\theta=45^\circ$) – X-joint

Fig. 5.20 Effect of crack aspect ratio on M_k values - For axial loading at crack ends ($L/T=1.25$, $\theta=45^\circ$) - X-joint

Fig. 5.21 Effect of weld angle M_k values - For axial loading at deepest point ($a/c=0.4$, $L/T=1.25$) - X-joint

Fig. 5.22 Effect of weld angle M_k values - For axial loading at crack ends ($a/c=0.4$, $L/T=1.25$) - X-joint

Fig. 5.23 Effect of attachment footprint ratio on M_k values - For axial loading at crack ends ($a/c=0.2$, $\theta=45^\circ$) - X-joint

Fig. 5.24 Histogram ba – double-sided T-butt joint

Fig. 5.25 Histogram bc – double-sided T-butt joint

Fig. 5.26 Histogram ma – double-sided T-butt joint

Fig. 5.27 Histogram mc – double-sided T-butt joint

Fig. 5.28 Histogram ma – X-joint

Fig. 5.29 Histogram mc – X-joint

Fig. 5.30 Histogram ba – X-joint

Fig. 5.31 Histogram bc – X-joint

Fig. 5.32 Comparison of equation predictions with the original data for axial loading-deepest point – double-sided T-butt joint

Fig. 5.33 Comparison of equation predictions with the original data for bending loading-deepest point – double-sided T-butt joint

Fig. 5.34 Comparison of equation predictions with the original data for axial loading-crack ends – double-sided T-butt joint

Fig. 5.35 Comparison of equation predictions with the original data for bending loading-crack ends – double-sided T-butt joint

Fig. 5.36 Comparison of equation predictions with the original data for axial loading-deepest point – X-joint

Fig. 5.37 Comparison of equation predictions with the original data for axial loading-crack ends – X-joint

Fig. 5.38 Comparison of equation predictions with the original data for bending loading-deepest point – X-joint

Fig. 5.39 Comparison of equation predictions with the original data for bending loading-crack ends – X-joint

Table 5.1 Table of analyses for parametric study

Loading	L/T	$\theta(\text{degree})$	a/T	a/c
m=axial, b=bending	0.5	$30^0, 45^0, 60^0,$ 75^0	0.05,0.1,0.15, 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m=axial, b=bending	1.25	$30^0, 45^0, 60^0,$ 75^0	0.05,0.1,0.15, 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m=axial, b=bending	2	$30^0, 45^0, 60^0,$ 75^0	0.05,0.1,0.15, 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0
m=axial, b=bending	2.75	$30^0, 45^0, 60^0,$ 75^0	0.05,0.1,0.15, 0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9	0.1,0.2,0.4,1.0

Chapter 6

Numerical Analyses and Validation of Cracked Uni-planar SHS Joints

6.1 Introduction

The past works for determining the plastic collapse load (P_c) of tubular joints were carried out originally for uncracked circular hollow section (CHS) joints (Yura et al., 1980). In the case of cracked CHS joints, Burdekin et al. (1992) proposed a reduction factor (F_{AR}) which when multiplied with the P_c load for uncracked CHS joints, gives the P_c load of cracked CHS joints. This is a convenient way for design engineers to estimate the P_c load of cracked CHS joints. For square hollow section (SHS) joints, the equations for estimating the design resistance of uncracked joints can be found in various existing codes of practise (Eurocode 3, 2005; IIW, 2005; CIDECT, 2009). However, there are no such specific equations for determining the P_c load of cracked SHS joints. Henceforth, there is a need to carry out parametric study to derive new F_{AR} expressions for cracked SHS joints.

In this study, a new mesh generator is developed which is capable of generating finite element (FE) mesh models of cracked uni-planar and multi-planar SHS joints. This chapter focuses on the study of cracked uni-planar SHS T-, Y- and K-joints. The numerical analyses results of cracked uni-planar SHS T-, Y- and K-joints produced using the new FE mesh generator is first validated with the previous experimental test results (Lie and Yang, 2007). The validated new FE mesh generator is then used to carry out extensive parametric study of cracked uni-planar SHS T-, Y- and K-joints. A set of new reduction factor (F_{AR}) equations are proposed for cracked SHS T-, Y- and K-joints subsequently.

For cracked CHS and SHS tubular joints, the P_c load check may not be sufficient for a complete integrity assessment of the flawed structures. This is especially relevant in the case of CHS and SHS joints where ductile extension of fatigue-induced surface cracks or fabrication defects creates critical threat to the safety of structures in the event of an

environmental overload caused by a 100-year return wave (Qian et al., 2010). Therefore, it is vital to develop engineering procedures, taking care of both P_c load of the tubular joint and fracture toughness of the material. The fracture analyses of cracked uni-planar SHS T-, Y- and K-joints are carried out with the help of models generated using the new FE mesh generator. The J -integral results obtained using the new FE mesh generator is validated against the results obtained from FEACrackTM (2010) software. The validations of standard Level 2A failure assessment diagram (FAD) curve of BS7910 (2005) as well as the new recommended penalty factors for P_c load are also presented in this chapter.

6.2 Validation of Finite Element Mesh Generator

The new FE mesh generator has to be validated first against the available experimental test results before it can be confidently used to carry out extensive parametric study of cracked uni-planar SHS T-, Y- and K-joints. Full-scale SHS T-, Y- and K-joints experimental tests were previously tested to failures in the industry sponsored project (Lie et al., 2006; Lie and Yang, 2007). The above mentioned experimental test results are important to check the validity and accuracy of the new FE mesh generator.

6.2.1. Test rig and loading system used for experimental tests

A specifically designed test rig shown in Fig. 6.1 was used to test the cracked SHS T-joints. The rig comprises of two servo-hydraulic actuators which can produce ± 1000 kN load and ± 125 mm displacement capacity respectively. The loads are applied to the two ends of the spread beam by the actuators. Subsequently, the loads are transferred to the brace end via the 6 bolts stiffened to the spread beam as depicted in Fig. 6.2. As a result, a total maximum axial load of 2000 kN can be applied to the specimen. Instron 8500 controller is used to control the actuators. The controller can be set either in load control or in displacement control. Regarding the support condition at both chord ends, pinned end condition were created by using a pin and a roller placed on top of 600 mm $\times$ 600 mm concrete supports as illustrated in Fig. 6.2 (Lie et al., 2006).

For SHS Y- and K-joints, another test rig was used for the static tests as shown in Fig. 6.3. The ends of the chord were fixed with the yellow rig as shown in Figs. 6.4(a) and 6.4(b) for

Y- and K-joints respectively. Two hydraulic jackets were used to apply the loads on the end of the brace. Each hydraulic jacket can produce 1000 kN load.

6.2.2. Material properties

The specimens were fabricated using BS4360 structural steel of grade 50D. The dimensions of SHS T-, Y- and K-joints used are given in Table 6.1. Each specimen contains one fatigue surface crack below the weld toe. The weld and fatigue crack details of SHS T-, Y- and K-joints used are listed in Table 6.2. The mechanical properties of the BS4360-50D steel supplied by the manufacturer, Corus, in UK, are given in Table 6.3. American Welding Society (AWS) Structure Welding Code D1.1-2000 (2000) specifications were used for weld profile. Ultrasonic inspection was used to guarantee the welds quality. The welding electrodes were E70XX type conforming to AWS D1.1-2000 (2000). The mechanical properties of the weld are tabulated in Table 6.4. The comparison of the yield stress of the weld material with the parent material yield stress of 380.3 MPa revealed only 5% difference. Hence, the weld material was found to match the parent material.

6.2.3. Load-displacement curves for validation purpose

The same dimensions of SHS T-, Y- and K-joints used in the experimental tests are modelled using the new FE mesh generator and are analysed using ABAQUS (2011) FE analysis software. Brace end axial loadings are applied to the numerical models as depicted in schematic representations in Figs. 6.5 and 6.6. The boundary conditions are identical to those used in experimental study. Fig. 6.7 shows the load-displacement curves for the P_c load of the SHS T-joint specimen. Experimental load-displacement curve illustrate a linear relationship in the elastic region up to a load of 680 kN. Then the slope of this curve changes to another linear region up to 899 kN. The curve then drops down to 805 kN after the peak load, then they increase slightly up to 825 kN before it drop down dramatically.

Fig. 6.8 shows the load-displacement curves for the P_c load of the SHS Y-joint specimen. The elastic load for the experimental curve is up to 520 kN, and then the curve turn into plastic region which is longer than the same for T-joint. Notable plastic deformation is observed for experimental curve. The same trend is not observed for numerical model due to absence of crack ductile tear modelling. From the recorded ACPD readings for the

experimental model, it was observed that the crack showed a clear ductile tearing until the load reached 1290 kN.

Fig. 6.9 displays the load-displacement curves for the P_c load of the SHS K-joint specimen. The elastic region for the experimental curve is up to 600 kN. The plastic region for the K-joint is shorter than the same for Y-joint. The crack showed an obvious ductile tearing until the load reached 1115 kN, and then the crack propagated through the chord wall thickness. The tendency of the load-displacement curve obtained from numerical analyses being slightly above the experimental curve is most prominent in the case of SHS K-joint. It is attributed to the fact that the numerical model is assumed to be perfect, and the stress-strain curve is obtained from the uni-axial tensile test which is not same as the actual condition in the full-scale tests. Moreover, K-joint has an additional brace component when compared to SHS T- and Y-joints; thereby the above mentioned reason is all the more valid. The load-displacement curves for SHS T-, Y- and K-joints clearly reveal the analogous trends between experimental and numerical models. Hence, the numerical models created using the new FE mesh generator are accurate and slightly conservative, and it can be used to carry out extensive parametric study of cracked SHS T-, Y- and K-joints.

6.2.4. Stress distribution and deformation at the brace-chord intersection zone

Before carrying out the parametric study to determine the reduction factor (F_{AR}) of the cracked SHS T-, Y- and K-joints, the stress distribution and deformation at the brace-chord intersection is investigated first. The enlarged view of the deformed crack for SHS T-joint is illustrated in Fig. 6.10. The depicted deformation of the cracked SHS T-joint is for the P_c load value of 786.09 kN. The failure mode observed for the model which has the brace to chord width ratio (β) equal to 0.57 is chord face plastification. The von Mises stress distributions are illustrated in Figs. 6.11 and 6.12. Fig. 6.11 represents the von Mises failure stress distributions for an applied load which is 40% of P_c load. It is clearly observed that the stress at the corners of the brace is the highest compared to other regions of the SHS T-joint. The final plastic collapse deformation of the SHS T-joint is illustrated in Fig. 6.12. At this stage, it is observed that yielding has spread throughout the brace-chord intersection as well as through the chord wall thickness.

The enlarged view of the deformed crack for SHS Y-joint is illustrated in Fig. 6.13. The P_c load of cracked SHS Y-joint is 853.05 kN. Chord face plastification failure mode is observed for the joint which has the brace to chord width ratio (β) equal to 0.50. Fig. 6.14 represents the von Mises stress distributions for an applied load which is 40% of the P_c load value. Similar to the trend observed for SHS T-joint, the stress at the brace corners is found to be the highest. For the final failure at the P_c load, yielding is found to spread throughout the brace-chord intersection of the SHS Y-joint as shown in Fig. 6.15.

The enlarged view of the deformed crack for SHS K-joint is illustrated in Fig. 6.16. The P_c load of cracked SHS K-joint is 971.80 kN. Chord face plastification failure mode is observed for the joint which has the brace to chord width ratio (β) equal to 0.50. Fig. 6.17 represents the von Mises stress distributions of SHS K-joint for an applied load which is 40% of the P_c load. It is observed that yielding initiates from the crack tip region as well as from the brace corners of the SHS T-, Y- and K-joints. The final plastic collapse deformation of the SHS K-joint is illustrated in Fig. 6.18. The stress close to crack region is observed to be higher when compared to the regions that are further away from the crack region.

6.3 Plastic Collapse of Cracked SHS Joints

The previous section justify that the new FE mesh generator is reliable and can be used for analysing the P_c load behaviour of cracked SHS T-, Y- and K-joints. It is used extensively in the parametric study. The behaviour of cracked tubular joints is governed by plastic collapse (yielding) and/or crack extension (fracture). The current section deals with the plastic collapse behaviour of uni-planar SHS T-, Y- and K-joints. The main highlights of this section are to (1) study the plastic collapse behaviour of uni-planar SHS T-, Y- and K-joints by undertaking the numerical parametric study, (2) to discuss the usage of reduction factor (F_{AR}) to estimate the plastic collapse of cracked SHS T-, Y- and K-joints, and (3) to propose a set of new reduction factor (F_{AR}) equations that can be used in practise.

6.3.1. Reduction factor

As described earlier, there is a need to propose a set of new reduction factor (F_{AR}) which can be used for specific geometries of cracked SHS T-, Y- and K-joints. The reduction factor

(F_{AR}) expression for circular hollow section (CHS) tubular joints proposed by Burdekin et al. (1992) has been incorporated in BS7910 (2005), and it is expressed as

$$F_{AR} = \left[1 - \frac{\text{crack area}}{\text{weld length} * \text{chord thickness}} \right] \left[\frac{1}{Q_{\beta}} \right]^{m_q} \quad (6.1)$$

where Q_{β} is the geometrical modifier which depends on brace to chord width ratio (β) values, and it is expressed as

$$Q_{\beta} = 1, \text{ for } \beta \leq 0.6 \quad (6.2)$$

$$Q_{\beta} = \frac{0.3}{[\beta(1-0.833\beta)]}, \text{ for } \beta > 0.6 \quad (6.3)$$

For tubular joints containing part-thickness flaws, $m_q = 0$ and for tubular joints containing through-thickness flaws, $m_q = 1$. The scope of using Eqn. (6.1) to calculate F_{AR} is limited in nature due to predominantly two reasons. Firstly, the numerical study and experimental validation used to derive the reduction factor (F_{AR}) was based solely for CHS joints (Burdekin et al., 1992). Eqn. (6.1) is intended for CHS joints, and therefore, it may not give optimal solutions for other types of tubular structures such as cracked SHS joints. Henceforth, there is a need to carryout parametric study to derive a set of new F_{AR} expressions for cracked SHS joints.

Secondly, it is specified in the BS7910 (2005) that for CHS tubular joints containing part-thickness flaws (surface cracks), $m_q = 0$. In that case, the $(\frac{1}{Q_{\beta}})^{m_q}$ term from Eqn. (6.1) will become one, indicating that the brace to chord width ratio (β) has no effect in the estimation of reduction factor (F_{AR}) for P_c loads of cracked CHS tubular joints. However, the current numerical study shows that the parameter β has an influence on the reduction factor (F_{AR}) values for SHS joints containing a surface crack; the results of which will be described in the subsequent sub-sections.

6.3.2. Parameters used in the parametric study

Extensive study is carried out to investigate the effect of two parameters, brace to chord width ratio (β) and crack area, on the reduction factor (F_{AR}) of cracked SHS T-, Y- and K-joints. For chord face yielding condition, the P_c load formulae for the uncracked SHS joints are given by Eurocode 3 (2005).

- For T- and Y-joints:

$$N_{1, RD} = \frac{k_n f_{y0} t_0^2}{(1-\beta) \sin \theta_1} \left(\frac{2\beta}{\sin \theta_1} + 4(1-\beta)^{0.5} \right) / \gamma_{M5} \quad (6.4)$$

- For K -joints:

$$N_{1, RD} = \frac{8.9 \beta \gamma^{0.5} k_n f_{y0} t_0^2}{\sin \theta_1} / \gamma_{M5} \quad (6.5)$$

where

$N_{1, RD}$ = design axial resistance of the welded joint

$$k_n = 1.0 \quad \text{for } n \leq 0 \text{ (tension)} \quad (6.6)$$

$$k_n = 1.3 - \frac{0.4}{\beta} n \quad \text{for } n > 0 \text{ (compression) but } k_n \leq 1.0 \quad (6.7)$$

f_{y0} = specified design yield strength

θ_1 = the angle between brace member and the chord

γ_{M5} = resistance of joints in hollow section lattice girder = 1.0

Eqns. (6.4) and (6.5) take into consideration of the effect of β and hence, due importance is given for the same in the current parametric study. It has already been explained in Chapter 4 that the crack shape is semi-elliptical in nature. The crack area refers to the cross-sectional area of the crack and it is given by the sum of the areas of two half semi-ellipses. The parameter ‘crack area factor’ represents the percentage of crack area with respect to the total cross-sectional area. From Eqn. (6.1), it can be seen that the total cross-sectional area is obtained by multiplying total weld length with the chord thickness. For example, a ‘crack

area factor' of 10% implies that the value of $\left(\frac{\text{crack area}}{\text{weld length} * \text{chord thickness}} \right) * 100\%$

is equal to 10%.

6.3.3. Geometry, loading and boundary conditions

The details of various geometries used in the parametric study are given in Table 6.5. 11 different values of β are analyzed for percentage crack area values of 5.8%, 10%, 20% and 0%. The values of β vary from a lower limit of 0.25 to a higher limit of 0.75. Stacey et al. (1996) recommended that for eliminating the chord length to half chord width ratio (α) effect, a minimum α value of 8 should be used. It was also recommended that the chord length should be at least four times the chord diameter (Stacey et al., 1996). The present parametric study satisfies both the above recommendations, and hence, the effect of α can be neglected. It was further recommended that, chord half width to thickness ratio (γ) does not have a significant effect on the P_c load for β up to 0.8. Therefore, the focus of present parametric study is towards the effect of β and the crack area on the reduction factor (F_{AR}) for cracked SHS T-, Y- and K-joints.

The material used in the present study is BS4360 structural steel of grade 50D, and the stress-strain curve of the material is plotted in Fig. 6.19. The loads are applied using displacement control. The Young's modulus and Poisson's ratio of the material used are 210 kN/mm² and 0.3 respectively. The cracked/uncracked SHS T-, Y- and K-joints are subjected to axial loading at the brace end. Regarding the boundary condition, the nodes at the brace and the chord ends are free and pinned, respectively. For all the cases considered in the present study, the chord length is kept constant at a value of 3000 mm. The longer value of chord length used ensures that the effect of chord end boundary conditions on the failure of damaged joints is negligible (Yang, 2005).

6.3.4. Determination of plastic collapse loads

In general, the joint assessments are carried out in order to avert possible failure mechanisms such as the gross plastic deformation (Lie and Yang, 2011). It is performed to guarantee that the design of service loads, stresses, and strains confine to the specified limits defined in the

codes of practise such as BS7910 (2005). Steel materials exhibit linear elastic behaviour up to yield and non-linear behaviour after that. The non-linear behaviour (strain hardening) allows the materials to support higher stress beyond yielding. A typical load-displacement curve obtained from the present study for SHS T-joint is depicted in Fig. 6.20. It shows the load-displacement behaviour for both cracked and uncracked SHS T-joints ($\beta = 0.5$, crack area = 5.8%). The presence of crack reduces the load-carrying capacity of the SHS T-joint. The main challenge in the FE analysis is to define a P_c load for calculating the allowable static load of the structures (Lie and Yang, 2011). It is usually done by employing a criterion for plastic collapse analysis. This criterion describes a load limit at which plastic deformation becomes excessive preceding the physical collapse (Lie et al., 2009). In the present study, the twice-elastic compliance (TEC) criterion is used to determine the deformation limits. Past studies have used the TEC criterion to determine the P_c load of cracked tubular joints (Stacey et al., 1996; Muscat et al., 2003; Yang, 2005; Lie and Yang, 2011). This plastic criterion is described in ASME VIII Division 2 (1998) as the twice-elastic slope (TES) criterion.

The TEC criterion is established on the load-deformation response of a structure in the plastic analysis as shown in Fig. 6.21. The P_c load is the load corresponding to the intersection of the load-deformation curve and the twice elastic compliance line. The twice elastic compliance line start from the origin of the load-deformation curve and has twice the slope of the initial elastic response (Lie and Yang, 2011), which is obtained using the equation

$$\tan \phi = 2 \tan \theta \quad (6.8)$$

where ϕ and θ are the angles measured from the load axis as shown in Fig. 6.21.

Load-displacement curves for uni-planar SHS T-joints obtained from parametric study are represented in Figs. 6.22 to 6.24. Brace end axial loading values are plotted against brace end displacement for crack area of 5.8%, 10% and 20%. In the curves, results from lower and higher β values are plotted, and it is observed that the characteristic strength of the uni-planar SHS T-joints increases with increasing value of β . It is also noted that the characteristic strength decreases with an increase in crack area value. Similar variation is

observed for the load vs displacement curves for SHS Y- and K-joints, and hence, they are not depicted in this chapter.

6.3.5. Effect of brace to chord width ratio (β) on reduction factor

In the FE analyses, the new FE mesh generator produce the input files automatically, which are then analysed using the ABAQUS (2011) software. Using the analyses results, the load-displacement relationship of the SHS T-, Y- and K-joints are plotted to obtain the P_c load. The reduction factor (F_{AR}) is calculated using the following relationship:

$$F_{AR} = \frac{\text{Plastic collapse load of cracked SHS joint}}{\text{Plastic collapse load of uncracked SHS joint}} \quad (6.9)$$

Tables 6.6 to 6.14 list out the P_c load of both uncracked and cracked SHS T-, Y- and K-joints. The reduction factor (F_{AR}) for the respective joints covering all the brace to chord width ratio (β) values are also given the corresponding tables.

Contrary to reduction factor (F_{AR}) of CHS joints containing part-thickness flaws (BS7910, 2005), the present study shows that β has an influence in F_{AR} for cracked SHS joints (Figs. 6.25, 6.27, 6.28). For a particular crack area, F_{AR} varies from 1.5% to 3.3% for different values of β . Furthermore, comparing the F_{AR} values for SHS joints with values obtained from BS7910 (2005); it shows that the latter is conservative. The BS7910 (2005) equation is conservative up to 6% for crack area of 5.8%, and up to 22.7% for crack area of 20%.

Another important observation is the mode of failure for cracked SHS joints. For low β values, the failure mode observed is punching shear, and for high β values, the failure mode observed is chord face plastification (Fig. 6.26). For the punching shear failure mode, deformation/bending of chord face is not observed; while for the chord face plastification failure mode, there is an obvious bending of the chord face (Fig. 6.26). For the chord face plastification mode, an increase of F_{AR} is observed with increasing β values (Figs. 6.25, 6.27, 6.28). This trend is different to the trend observed for F_{AR} calculated using the BS7910 (2005) equation, where the F_{AR} is independent of β value (Fig. 6.29).

6.3.6. Effect of crack area on reduction factor

The crack area has the most significant influence on F_{AR} of cracked SHS joints. The variation of F_{AR} with crack area for different β values is depicted in Figs. 6.30 to 6.35. Figs. 6.30, 6.32 and 6.34 shows the variation of F_{AR} with the crack area for punching shear failure mode; while Figs. 6.31, 6.33 and 6.35 shows the same for chord face plastification failure mode. The parametric study reveals that for cracked SHS joints, F_{AR} varies from 1.9% to 5.3% for different values of crack area (for a particular β value). The conservative nature of existing equation for CHS tubular joints (BS7910, 2005) is already explained in Section 6.3.5, and therefore, the similar trend observed for crack area is not described here.

6.3.7. New reduction factor equations

It is shown in the above that F_{AR} varies non-linearly with β and the crack area for SHS T-, Y- and K-joints. Hence, non-linear regression technique is used to curve fit the numerical data obtained from the parametric study. As discussed earlier in Section 6.3.5., two failure modes are observed for SHS joints, and hence, two set of equations are proposed in this study.

The proposed equations for T- and Y-joints are as follows:

$$\text{For } \beta \geq 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.23} * \beta^{0.03} * 1.02 \quad (6.10)$$

$$\text{For } \beta < 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.09} * \beta^{-0.03} * 0.97 \quad (6.11)$$

The proposed equations for K-joints are as follows:

$$\text{For } \beta \geq 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.21} * \beta^{0.03} * 1.02 \quad (6.12)$$

$$\text{For } \beta < 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.10} * \beta^{-0.03} * 0.97 \quad (6.13)$$

where

A_c is the crack area, l_w is the weld length and t_0 is the chord thickness.

The coefficient of determination values for T- and Y- joints are 0.982 and 0.935 and for K-joints are 0.980 and 0.942. These high values for the non-linear regression justify the statistical accuracy of the proposed equations. To ensure continuity of the proposed equations, plots are made showing the variation of F_{AR} for β at 0.49 and at 0.50. The plotted trends are shown in Fig's 6.36 and 6.37 and the plots reveal that there is good continuity for the proposed equations.

6.4 Fracture Analyses of Cracked Uni-planar SHS Joints

For an integrity assessment of any flawed structure, fracture analysis has to be performed along with the P_c load check. The previous section focuses on the P_c load while the current section focuses on the fracture behaviour of cracked uni-planar SHS T-, Y- and K-joints. Fracture control is intended to prevent fracture due to presence of defect or crack at the maximum loads experienced during operational service. The new FE mesh generator is used to create cracked uni-planar SHS T-, Y- and K-joints which are then used for parametric study to understand the fracture behaviour. Both elastic and elastic-plastic analyses for cracked uni-planar SHS T-, Y- and K-joints are carried out. The main highlights of this section are (1) to validate the J -integral results obtained using the new FE mesh generator against the FEACrackTM (2010) software, (2) to study the effect of crack depth ratio (a/T) and crack length ratio (c_2/c_1) on J -integral values, and (3) to validate the standard Level 2A failure assessment diagram (FAD) curve of the BS7910 (2005) (4) To construct the highest level, namely the Level 3C FAD curves of cracked uni-planar SHS T-, Y- and K-joints using the elastic J -integral (J_e), elastic-plastic J -integral (J_{ep}) and P_c load as well as to propose a new penalty factor for P_c load.

6.4.1. Brittle fracture of tubular joints

The brace-to-chord intersection region in any cracked tubular joint is the critical region where brittle fracture is most prominent (Lie and Yang, 2011). The rate of crack growth is fastest in this region. Figs. 6.38 to 6.40 illustrate the crack surfaces of the specimens produced from the previous experimental tests (Lie et al., 2006; Lie and Yang, 2007). The ductile tearing, fatigue surface and brittle fracture surface are clearly demarcated in the depicted plots. It reveals that the ductile tearing has occurred near a small region adjacent to the crack tips. The shiny and smooth natures of the fracture surfaces around the deepest point of the cracks indicate the occurrence of brittle cleavage fracture. These trends reaffirm the need to carry out numerical fracture analysis along with the plastic collapse analysis and to have a more comprehensive study of cracked tubular joints.

The new FE mesh generator needs to be validated first before it can be used for fracture analysis of cracked uni-planar SHS T-, Y- and K-joints. In order to meet the validation requirement, the J -integral obtained using the new FE mesh generator is compared with that of FEACrackTM (2010) software, and the results are described in Sub-section 6.4.2. Subsequently, the FE mesh generator is used to construct the FAD for cracked uni-planar SHS T-, Y- and K-joints. The same numerical models used in the plastic collapse behaviour study are also used in the fracture analyses. The standard BS7910 (2005) Level 2A/3A FAD is described in Sub-section 6.4.5 which recommends the procedures for the acceptability of defects in welded structures. However, these procedures are envisioned for general applications and may not necessarily give optimal solutions for all types of structures such as cracked uni-planar SHS T-, Y- and K-joints. The complexities associated with the tubular joint geometries like uncertainties with crucial fracture parameter solutions necessitate the validation of FADs for cracked uni-planar SHS T-, Y- and K-joints. The Level 3C FADs constructed for cracked uni-planar SHS T-, Y- and K-joints and the newly recommended penalty factor for plastic collapse load are given in Sub-section 6.4.5.

6.4.2. Calculation of J -integral

Fracture toughness represents the toughness of a material by its resistance to crack initiation and/or crack propagation. In the case of brittle materials, less energy is required for crack initiation and propagation. It is because, as soon as a crack initiates in a brittle material, it

will zip through the material. The concept of linear elastic fracture mechanics (LEFM) using theory of elasticity is sufficient to study/represent the crack initiation/propagation behaviour in brittle materials. However for ductile materials, the above approach using elastic theory may not be sufficient to adequately study the crack initiation/propagation behaviour. It is attributed to the fact that the ductile materials absorb more energy prior to crack initiation and its subsequent growth. Therefore, it becomes necessary to employ other approaches such as the use of plasticity theory to measure the energy required for crack initiation and propagation. J -integral is a volume integral formulation which is frequently employed for the elastic-plastic analyses of ductile materials. For the current study, the J -integral is calculated using the virtual crack extension (VCE) technique which is built in ABAQUS (2011) FE program. ABAQUS (2011) interprets the J -integral as a function of location along the crack front which provides the decrease in total potential energy of the loaded structure caused by an increase in the crack opening area at that same position on the crack front.

ABAQUS (2011) formulates this as follows:

$$-\delta P = \int_s J(s) \cdot \delta l(s) ds \quad (6.14)$$

where δP is the change in the structure's total potential energy, s is the coordinate which identifies position along the crack front as shown in Fig. 6.41, $J(s)$ is the J -integral value, and $\delta l(s)$ is the advance of the crack front normal to itself and in the local plane of the crack at this point.

The value of $J(s)$ and the VCE are then interpolated from the value at crack front nodal position using the same order of interpolation as used in the elements adjoining the crack front. Thus, equation (6.14) becomes, in the FE model, a set of equations which relate the set of total potential energy changes produced by crack advances at each crack front nodal position to the J -integral values of the same crack front nodal positions. Once these values are determined, the equations can be solved for these nodal positions values of J . The method is specifically suitable because it is simple to use, adds little to the cost of the analysis, and provides good accuracy (Aliabadi and Rooke, 1991).

The J -integral result values are assessed by examining the contour integral path dependence index. The contour integral path dependence index, d , is calculated using

$$d = \frac{(J_{\max} - J_{\min})}{2 J_{\text{avg}}} \quad (6.15)$$

where $J_{\max}$ is the largest J -integral contour value, $J_{\min}$ is the smallest J -integral contour value, and J_{avg} is the average of the contour values. The first contour is rejected while computing the path dependence index. Using 4 or 5 contours of the FE mesh is recommended in FEACrackTM (2010). Path dependence index values below 0.05 are identified to confirm low path dependence in the J -integral results (FEACrackTM, 2010). The typical contour path dependent index values obtained using the new FE mesh generator are in the 0.004-0.005 value range which is much lower than the maximum recommended limit of 0.05; this is to ensure that the path independence criterion is met for all the analyses. A sensitivity investigation is carried out on the number of rings to be used for elastic-plastic analysis. Cracked SHS T-joints with 4, 5 and 6 rings of elements around the crack tip are chosen as shown in Fig 6.42. The elastic-plastic J -integral (J_{ep}) are plotted against the load applied as shown in Fig 6.43. The results indicate that even 4 rings of elements might be sufficient even for the elastic-plastic analyses as the results from different cases exhibit good convergence of values. For the current study, 5 rings are used around the crack tip for elastic-plastic analyses.

The J -integral results produced using the new FE mesh generator is validated with the solutions obtained from the mesh generated using FEACrackTM (2010) software. As an initial study, an elastic-plastic analysis of a cracked flat rectangular plate model is carried out first. The loading and boundary conditions adopted are shown in Fig. 6.44 where the nodes are free at one end and are fixed at other end of the cracked plate. The loads are applied at the free end of the cracked plate as shown in Fig. 6.44. Corresponding to the previous experimental tests (Yang, 2005), the material used is BS4360 structural steel of Grade 50D, and the true stress-strain curves were obtained from 6 coupon tests. In the FE analysis, the PLASTIC parameter in ABAQUS (2011) is used to specify the plastic part of the material model. A cracked plate with the identical dimensions to that used in the present study is

modelled using FEACrackTM (2010) software and an elastic-plastic analysis is carried out accordingly. Fig. 6.45 illustrates the comparison of J -integral values obtained from FEACrackTM (2010) software as well as from the new FE mesh generator. The maximum percentage difference between both cases is 0.09% at the deepest point of the crack and 7.89% at the ends of the crack. The largest J -integral values for both cases are located at the deepest point of the crack. The pattern of variation of J -integral values with crack front angle values is found to be complementary for both cases. Moreover, there is convergence of values for both cases which confirms the reliability of using the new FE mesh generator.

6.4.3. Relationship between J -integral and crack front angle for different crack depth ratio (a/T)

The new FE mesh generator is also used to study the effect of crack depth ratio (a/T) and crack length ratio (c_2/c_1) on the J -integral values. Three cases with different values of a/T are analysed for the present study. Fig. 6.46 illustrate the relationship between J -integral and crack front angle for $a/T=0.3$, $a/T=0.5$ and $a/T=0.7$ for a crack length ratio (c_2/c_1) of 2. The pattern of variation of J -integral values with crack front angle for various cases of a/T values shows that the J -integral values increase with increasing crack depth ratio. The percentage difference at the deepest point of the crack between $a/T=0.3$ and $a/T=0.7$ is 18.31%. The percentage difference at the crack ends between $a/T=0.3$ and $a/T=0.7$ is 60.43%. The large percentage difference is due to the nature of small J -integral values of 0.118 and 0.298 for $a/T=0.3$ and $a/T=0.7$, respectively. The versatility of new FE mesh generator is reaffirmed by the results obtained from performing the analysis for various cases of crack depth ratio (a/T) values.

6.4.4. Relationship between J -integral and crack front angle for different crack length ratio (c_2/c_1)

Four cases with different values of c_2/c_1 are analysed in the present study. Fig. 6.47 illustrate the relationship between J -integral and crack front angle for $c_2/c_1=2$, $c_2/c_1=2.5$, $c_2/c_1=3$ and $c_2/c_1=4$ for a crack depth ratio of 0.5. The flexibility of the new FE mesh generator is illustrated by the pattern of results obtained for different values of c_2/c_1 . It is observed that the J -integral values are found to be higher for longer crack length. The trend of J -integral

values being higher for $c_2/c_1=4$ when compared to those of $c_2/c_1=2$ reaffirms that the crack length has an influence in the determination of J -integral values. The percentage difference at the deepest point of the crack between $c_2/c_1=2$ and $c_2/c_1=4$ is 20.31%. The percentage difference at the crack ends between $c_2/c_1=2$ and $c_2/c_1=4$ is 17.65%. The robustness of the new FE mesh generator can be reinforced from the successful adaptation of producing J -integral values for various unsymmetrical crack front shapes.

6.4.5. FADs of cracked SHS T-, Y- and K-joints

Upon validation of J -integral values produced from the new FE mesh generator, it can further be used for fracture analysis of cracked uni-planar SHS T-, Y- and K-joints. As mentioned elaborately in Chapter 2, FADs can be used for the fracture analysis of welded tubular joints. BS7910 (2005) specifies the guidance for assessing the acceptability of defects in welded structures based on FAD method. The general Level 2A FAD curve is given by

$$K_r = (1 - 0.14L_r^2)[0.3 + 0.7 \exp(-0.65L_r^6)] \quad (6.16)$$

where

$$K_r = \frac{K_I}{K_{IC}} \quad (6.17)$$

$$L_r = \frac{\text{applied load to the flawed structure}}{\text{plastic-collapse load of the flawed structure}} \quad (6.18)$$

The value of L_r is related to a reference stress through

$$L_r = \frac{\sigma_{ref}}{\sigma_y} \quad (6.19)$$

The K_r parameter of the FAD make use of linear elastic stress intensity factor with no provision for the effect of plasticity on the crack driving force (Lie et al., 2009). With the increasing value of L_r , the effective crack tip driving force increases along with the plasticity effect. The fracture occurs when the elastic-plastic value of crack tip driving force reaches a

critical value corresponding to the fracture toughness and is given by $\sqrt{EJ_{ep}} = K_{IC}$ (Zerbst et al., 2000). Therefore, Eqn. (6.16) can be modified as

$$K_r = \frac{K_I}{K_{IC}} = \sqrt{\frac{J_e}{J_{ep}}} \quad (6.20)$$

With the increase in plasticity, the ratio of elastic J -integral to elastic-plastic J -integral decreases, whereby the FAD curves reduce in height and shift to the right-side along the abscissa.

Eqn. (6.15) is intended for general applications and is independent of the structural geometry and the material stress-strain curve (Qian et al., 2010). It was developed from extensive experimental databases of laboratory specimens (Milne et al., 1988). Therefore, FAD curves are constructed for cracked uni-planar SHS T-, Y- and K-joints in this study to validate the standard BS7910 (2005) Level 2A/3A curve and to recommend penalty factor for the P_c load. The generated mesh models are analysed using ABAQUS (2011) software to obtain the elastic J -integral, J_e , and the elastic-plastic J -integral, J_{ep} , values. Figs. 6.48 and 6.49 represent typical variation of elastic and elastic-plastic J -integral values for axial loading case. The concept of using key-hole for elastic-plastic analysis is effectively incorporated in the new FE mesh generator and it enables to carry out elastic-plastic analyses without encountering convergence problems even at a very high plastic deformation. As a result, the accuracy of the elastic-plastic J -integral values obtained is enhanced.

For cracked uni-planar SHS T-, Y- and K-joints, FAD curves are constructed using the new FE mesh generator and are further compared with the FAD curves produced by Lie et al. (2009). Figs. 6.50 to 6.52 represent the constructed FAD curves for SHS T-, Y- and K-joints, respectively. The comparison with the FAD curves of Lie et al. (2009) reveals close similarity and it further verifies the suitability of using the new FE mesh generator for carrying out the parametric study for fracture analyses.

The highest level, namely the Level 3C FADs are constructed for cracked uni-planar SHS T-, Y- and K-joint models. The geometry and the material properties used are similar to the ones used for parametric study in Section 6.3 and can be found out in Table 6.5. Figs. 6.53 to 6.55 represent the constructed FAD curves for cracked uni-planar SHS T-, Y- and K-joints, respectively. It is observed that some of the constructed curves tend to fall below the

standard BS7910 (2005) Level 2A FAD curve. This is especially prominent towards the right side of the FAD curves which is near to the plastic-collapse region in the standard BS7910 (2005) Level 2A FAD curve. The results also reveal that the constructed FAD curves for cracked uni-planar SHS T-, Y- and K-joints with low brace to chord width ratio (β) appears to fall below the standard BS7910 (2005) Level 2A FAD curve when compared to the models with higher β ratio.

In order to achieve a conservative safe prediction, a penalty factor of 1.20 is recommended for cracked uni-planar SHS T-, Y- and K-joints. The newly proposed penalty factor is introduced to Eqn. (6.17) whereby the P_c load of the flawed structure is divided by a factor of 1.20 so that a conservative value of the P_c load is used to construct the FAD curves. Figs. 6.56 to 6.58 represent the constructed FAD curves for cracked uni-planar SHS T-, Y- and K-joints with the newly introduced penalty factor of 1.20. It is clearly observed that all the newly constructed FAD curves lie above the standard BS7910 (2005) Level 2A FAD curve, and therefore, the penalty factor of 1.20 is sufficient for a safe prediction the integrity assessment of cracked uni-planar SHS T-, Y- and K-joints.

Fig. 6.1 Test rig for SHS T-joint (Lie and Yang, 2007)

Fig. 6.2 Setup of the specimen (Lie and Yang, 2007)

Fig. 6.3 Test rig for Y- and K-joints (Lie and Yang, 2007)

Fig. 6.4 Schematic view of the test rig for Y- and K-joints (Lie and Yang, 2007)

Fig. 6.5 Loading and boundary conditions for SHS T-joint

Fig. 6.6 Loading and boundary conditions for SHS Y- and K-joints

Fig. 6.7 Comparison of numerical and experimental results for SHS T-joint

Fig. 6.8 Comparison of numerical and experimental results for SHS Y-joint

Fig. 6.9 Comparison of numerical and experimental results for SHS K-joint

Fig. 6.10 Enlarged view of crack at P_c load of SHS T-joint

Fig. 6.11 The von Mises stress distributions of SHS T-joint at 40% of P_c load

Fig. 6.12 The von Mises stress distributions of SHS T-joint at 100% of P_c load

Fig. 6.13 Enlarged view of crack at P_c load of SHS Y-joint

Fig. 6.14 The von Mises stress distributions of SHS Y-joint at 40% of P_c load

Fig. 6.15 The von Mises stress distributions of SHS Y-joint at 100% of P_c load

Fig. 6.16 Enlarged view of crack at P_c load of SHS K-joint

Fig. 6.17 The von Mises stress distributions of SHS K-joint at 40% of P_c load

Fig. 6.18 The von Mises stress distributions of SHS K-joint at 100% of P_c load

Fig. 6.19 Stress-strain curve of the BS4360-50D structural steel

Fig. 6.20 Sample load vs displacement curve for crack area 5.8%

Fig. 6.21 Twice elastic compliance criterion

Fig. 6.22 Load vs displacement curve for crack area 5.8% for uni-planar T-joints

Fig. 6.23 Load vs displacement curve for crack area 10% for uni-planar T-joints

Fig. 6.24 Load vs displacement curve for crack area 20% for uni-planar T-joints

Fig. 6.25 F_{AR} vs β plot comparison for uni-planar SHS T-joints

Fig. 6.26 Two modes of failure for SHS joints: (a) punching shear ($\beta=0.25$);
(b) chord face plastification ($\beta=0.75$)

Fig. 6.27 F_{AR} vs β plot comparison for uni-planar SHS Y-joints

Fig. 6.28 F_{AR} vs β plot comparison for uni-planar SHS K-joints

Fig. 6.29 Comparison of F_{AR} vs β plot between SHS and CHS joints

Fig. 6.30 Comparison of F_{AR} vs crack area plot for uni-planar SHS T-joints (for $\beta < 0.5$)

Fig. 6.31 Comparison of F_{AR} vs crack area plot for uni-planar SHS T-joints (for $\beta \geq 0.5$)

Fig. 6.32 Comparison of F_{AR} vs crack area plot for uni-planar SHS Y-joints (for $\beta < 0.5$)

Fig. 6.33 Comparison of F_{AR} vs crack area plot for uni-planar SHS Y-joints (for $\beta \geq 0.5$)

Fig. 6.34 Comparison of F_{AR} vs crack area plot for uni-planar SHS K-joints (for $\beta < 0.5$)

Fig. 6.35 Comparison of F_{AR} vs crack area plot for uni-planar SHS K-joints (for $\beta \geq 0.5$)

Fig. 6.36 F_{AR} vs β plot comparison for SHS T- and Y-joints equation

Fig. 6.37 F_{AR} vs β plot comparison for SHS K-joints equation

Fig. 6.38 Fracture surface of T-joint specimen after the tests (Lie and Yang, 2007)

Fig. 6.39 Fracture surface of Y-joint specimen after the tests (Lie and Yang, 2007)

Fig. 6.40 Fracture surface of K-joint specimen after the tests (Lie and Yang, 2007)

Fig. 6.41 Virtual crack extension

Fig. 6.42 Different number of rings used for sensitivity investigation

Fig. 6.43 Sensitivity test plot

Fig. 6.44 Plate mesh model used in the elastic-plastic analysis

Fig. 6.45 Comparison of J -integral values obtained using new FE mesh generator and FEACrackTM (2010)

Fig. 6.46 Variation of J -integral values with crack depth ratio

Fig. 6.47 Variation of J -integral values with crack length ratio

Fig. 6.48 Typical variation of elastic J -integral values for axial loading cases

Fig. 6.49 Typical variation of elastic-plastic J -integral values for axial loading cases

Fig. 6.50 Comparison of FAD curves of cracked uni-planar SHS T-joints

Fig. 6.51 Comparison of FAD curves of cracked uni-planar SHS Y-joints

Fig. 6.52 Comparison of FAD curves of cracked uni-planar SHS K-joints

Fig. 6.53 FAD curves of cracked uni-planar SHS T-joints

Fig. 6.54 FAD curves of cracked uni-planar SHS Y-joints

Fig. 6.55 FAD curves of cracked uni-planar SHS K-joints

Fig. 6.56 FAD curves of cracked uni-planar SHS T-joints with a penalty factor of 1.20

Fig. 6.57 FAD curves of cracked uni-planar SHS Y-joints with a penalty factor of 1.20

Fig. 6.58 FAD curves of cracked uni-planar SHS K-joints with a penalty factor of 1.20

Table 6.1 Dimensions of the specimens used for experimental validation

Specimen	b_0 (mm)	h_0 (mm)	t_0 (mm)	b_1 (mm)	h_1 (mm)	t_1 (mm)
T	350	350	15	200	200	12
Y	300	300	12.5	150	150	10
K	300	300	12.5	150	150	10

Table 6.2 The weld and fatigue crack details

Specimen	Crack Length l (mm)	l_{c1} (mm)	l_{c2} (mm)	Crack Depth a (mm)	Weld Width t_w (mm)
T	264	77	187	13.6	12
Y	153	45	108	7.2	8
K	125	37	88	4.9	8

Table 6.3 Mechanical properties of the material

Material	Yield Strength	Tensile Strength	Elongation	Charpy V-notch Temperature	Charpy V-notch Energy
	(N/mm ²)	(N/mm ²)	(A%)	(°C)	(J)
BS4360-50D	380.3	529	33	-20	170.6

Table 6.4 Mechanical properties of the weld (AWS, 2000)

Electrodes	Yield Strength	Tensile Strength
	(N/mm ²)	(N/mm ²)
E70XX	399.9	482.6

Table 6.5 Dimensions of specimens for crack area of 5.8%, 10%, 20 % and 0%

b_1	t_0	t_1	l_c	β	b_0	γ	α	τ
150.0	16	16	3000	0.25	600.0	18.8	10.0	1
150.0	16	16	3000	0.30	500.0	15.6	12.0	1
150.0	16	16	3000	0.35	428.6	13.4	14.0	1
150.0	16	16	3000	0.40	375.0	11.7	16.0	1
150.0	16	16	3000	0.45	333.3	10.4	18.0	1
150.0	16	16	3000	0.50	300.0	9.4	20.0	1
150.0	16	16	3000	0.55	272.7	8.5	22.0	1
150.0	16	16	3000	0.60	250.0	7.8	24.0	1
150.0	16	16	3000	0.65	230.8	7.2	26.0	1
150.0	16	16	3000	0.70	214.3	6.7	28.0	1
150.0	16	16	3000	0.75	200.0	6.3	30.0	1

Table 6.6 Plastic collapse load values for crack area 5.8% of SHS T-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	556.324	559.238	0.995
0.30	581.259	586.012	0.992
0.35	603.236	609.957	0.989
0.40	616.164	625.522	0.985
0.45	639.224	650.285	0.983
0.50	678.253	688.610	0.985
0.55	718.357	726.486	0.989
0.60	751.236	757.531	0.992
0.65	774.238	778.457	0.995
0.70	798.265	801.213	0.996
0.75	828.536	830.363	0.998

Table 6.7 Plastic collapse load values for crack area 10% of SHS T-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	553.343	559.238	0.989
0.30	578.095	586.012	0.986
0.35	600.125	609.957	0.984
0.40	612.148	625.522	0.979
0.45	633.696	650.285	0.974
0.50	668.668	688.610	0.971
0.55	707.997	726.486	0.975
0.60	740.168	757.531	0.977
0.65	762.584	778.457	0.980
0.70	786.038	801.213	0.981
0.75	815.499	830.363	0.982

Table 6.8 Plastic collapse load values for crack area 20% of SHS T-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	548.869	559.238	0.981
0.30	573.354	586.012	0.978
0.35	595.459	609.957	0.976
0.40	606.118	625.522	0.969
0.45	625.399	650.285	0.962
0.50	654.289	688.610	0.950
0.55	692.458	726.486	0.953
0.60	723.563	757.531	0.955
0.65	745.108	778.457	0.957
0.70	767.691	801.213	0.958
0.75	795.953	830.363	0.959

Table 6.9 Plastic collapse load values for crack area 5.8% of SHS Y-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	552.016	555.348	0.994
0.30	576.699	581.414	0.992
0.35	598.990	606.265	0.988
0.40	610.329	620.253	0.984
0.45	633.535	645.148	0.982
0.50	675.937	686.258	0.985
0.55	713.481	722.147	0.988
0.60	746.381	753.160	0.991
0.65	770.050	774.698	0.994
0.70	793.038	796.223	0.996
0.75	822.684	825.159	0.997

Table 6.10 Plastic collapse load values for crack area 10% of SHS Y-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	548.684	555.348	0.988
0.30	572.693	581.414	0.985
0.35	595.352	606.265	0.982
0.40	606.607	620.253	0.978
0.45	628.374	645.148	0.974
0.50	666.357	686.258	0.971
0.55	703.371	722.147	0.974
0.60	735.084	753.160	0.976
0.65	758.429	774.698	0.979
0.70	780.299	796.223	0.980
0.75	809.481	825.159	0.981

Table 6.11 Plastic collapse load values for crack area 20% of SHS Y-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	544.796	555.348	0.981
0.30	568.041	581.414	0.977
0.35	591.108	606.265	0.975
0.40	600.404	620.253	0.968
0.45	619.987	645.148	0.961
0.50	651.259	686.258	0.949
0.55	687.484	722.147	0.952
0.60	718.514	753.160	0.954
0.65	740.611	774.698	0.956
0.70	761.985	796.223	0.957
0.75	790.502	825.159	0.958

Table 6.12 Plastic collapse load values for crack area 5.8% of SHS K-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	585.493	587.255	0.997
0.30	612.400	615.477	0.995
0.35	629.549	634.626	0.992
0.40	650.887	658.793	0.988
0.45	666.998	676.469	0.986
0.50	707.045	714.909	0.989
0.55	747.134	753.159	0.992
0.60	781.765	786.484	0.994
0.65	803.253	806.479	0.996
0.70	823.505	825.155	0.998
0.75	852.345	853.198	0.999

Table 6.13 Plastic collapse load values for crack area 10% of SHS K-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	581.970	587.255	0.991
0.30	608.707	615.477	0.989
0.35	625.741	634.626	0.986
0.40	646.276	658.793	0.981
0.45	660.910	676.469	0.977
0.50	696.321	714.909	0.974
0.55	735.836	753.159	0.977
0.60	770.754	786.484	0.980
0.65	791.962	806.479	0.982
0.70	811.953	825.155	0.984
0.75	841.253	853.198	0.986

Table 6.14 Plastic collapse load values for crack area 20% of SHS K-joints

β	P_c load (kN) Cracked	P_c load (kN) Uncracked	F_{AR}
0.25	577.859	587.255	0.984
0.30	603.167	615.477	0.980
0.35	620.030	634.626	0.977
0.40	640.347	658.793	0.972
0.45	652.793	676.469	0.965
0.50	682.023	714.909	0.954
0.55	719.267	753.159	0.955
0.60	753.452	786.484	0.958
0.65	775.026	806.479	0.961
0.70	794.624	825.155	0.963
0.75	823.336	853.198	0.965

Chapter 7

Numerical Analyses of Cracked Multi-planar SHS Joints

7.1 Introduction

One of the most significant originality of the current research work is the plastic collapse and fracture analyses of cracked multi-planar square hollow section (SHS) TT-, YT- and KT-joints. This chapter covers the numerical analyses of these cracked joints generated using the new finite element (FE) mesh generator. There are no published works on plastic collapse and fracture behaviour of cracked multi-planar SHS TT-, YT- and KT-joints so far, and hence the novelty of the present study is emphasized in the current chapter.

In this chapter, the general plastic collapse behaviour of cracked multi-planar SHS TT-, YT- and KT-joints is elaborated first. Load-displacement curves of uni-planar and multi-planar SHS joints are compared so that the effect of the additional brace in cracked multi-planar joints is investigated. A parametric study is then carried out to investigate the effect of β and the crack area on the P_c load of multi-planar TT-, YT- and KT-joints. Subsequently, the reduction factor (F_{AR}) equations for cracked multi-planar SHS TT-, YT- and KT-joints are proposed. Finally, the fracture behaviour of cracked multi-planar SHS TT-, YT- and KT-joints is studied by using failure assessment diagram (FAD) method. The highest level, namely the Level 3C FAD curves of cracked multi-planar SHS TT-, YT- and KT-joints are constructed using the elastic J -integral (J_e), elastic-plastic J -integral (J_{ep}) and P_c load. The constructed Level 3C FAD curves are further used to validate the standard Level 2A FAD curve of the BS7910 (2005). It is found that the standard Level 2A FAD curve is not always safe in assessing the safety and integrity of cracked multi-planar SHS TT-, YT- and KT-joints. Therefore, a penalty factor working on the P_c load is presented to move all the constructed Level 3C FAD curves just outside of the standard Level 2A curve.

7.2 Plastic Collapse Behaviour of Cracked Multi-planar SHS TT-, YT- and KT- joints

The previous works done on multi-planar tubular joints focus purely on uncracked models (O'Connor, 1993; Shahi et al., 1993; Yu et al., 1993; Herion et al., 1994; Lee and Wilmschurst, 1995; Vegte and Wardenier, 1998). Yu et al. (1993) observed that the load capacity for uncracked multi-planar rectangular hollow section (RHS) TT-joints at plastic collapse limit is increased by up to 26% more than that of uncracked uni-planar RHS T-joint. However, their work did not vary the geometric parameters such as β and γ , and they recommended that further parametric studies are required to quantify the multi-planar effect of the joints covering the whole range of geometrical parameters (Yu et al., 1993). The design resistance or P_c load for each relevant plane of an uncracked multi-planar joint is determined by applying an appropriate multi-planar factor to the strength of the corresponding uncracked uni-planar joint (CIDECT, 2009). All the previous research works done on multi-planar tubular joints focus exclusively on uncracked models and it can be inferred that, there is a need to extend the study to cracked multi-planar tubular joints as well. Cracked multi-planar SHS TT-, YT- and KT-joints are developed using the new FE mesh generator and are analysed using ABAQUS (2011) FE analysis software. The following sub-sections describe the various observations made from the present study. The geometry, loading and boundary conditions used for the numerical study are described in Section 7.2.1. Stress distribution and deformation at the brace-chord intersection zone of cracked multi-planar SHS TT-, YT- and KT-joints under axial tensile loading is covered in Section 7.2.2. Finally, the effect of additional brace on the plastic collapse behaviour of cracked multi-planar SHS TT-, YT- and KT-joints is explained in Section 7.2.3.

7.2.1. Geometry, loading and boundary conditions

In order to understand the general plastic collapse behaviour of cracked multi-planar SHS TT-, YT- and KT-joints, FE models using the new FE mesh generator are created first in the numerical study. Additional braces are added to the cracked uni-planar SHS T-, Y- and K-joints, in order to extend them into cracked multi-planar SHS TT-, YT- and KT-joints, respectively. The newly added braces are positioned in a plane perpendicular to the cracked uni-planar SHS T-, Y- and K-joints. The dimensions of multi-planar SHS TT-, YT- and KT-joints used are given in Table 7.1. In the case of multi-planar SHS YT- and KT-joints, the

braces angle are modelled at an inclination of 45^0 . Similar to the case of cracked uni-planar joints, each specimen contains one surface crack under the weld toe and located at the hot spot stress location of the joints. The weld and crack details used are listed in Table 7.2. The specimens were produced using BS4360 structural steel of grade 50D (Lie and Yang, 2007). The mechanical properties of the material and the weld used are listed out in Table 7.3 and Table 7.4, respectively.

Brace end axial loadings are applied to the models as depicted in schematic diagrams shown in Figs. 7.1 and 7.2. The boundary conditions for cracked multi-planar SHS TT-joint are shown in Fig. 7.1, whereby the nodes are free at the brace end where the axial loading is applied and are pinned at the chord ends as well as at the new out-of-plane multi-planar brace end. The length of the chord is adequately long so that the effect of boundary conditions will be minimal. The boundary conditions for cracked multi-planar SHS YT- and KT-joints are shown in Fig. 7.2, showing that the nodes are free at the brace end where the axial loading is applied, and they are fixed at the chord ends and at the other brace ends.

7.2.2. Stress distribution and deformation at the brace-chord intersection zone

Before carrying out the parametric study to determine the reduction factor (F_{AR}) of the above multi-planar joints, the stress distribution and deformation at the brace-chord intersection is investigated first. The failure mode of the cracked multi-planar SHS TT-, YT- and KT-joints can be determined from the stress distribution and deformation described in the current subsection. The enlarged view of the deformed crack for SHS TT-joint is illustrated in Fig. 7.3 for P_c load of 934.39 kN. The observed failure mode for this model which has the brace to chord width ratio (β) equal to 0.57 is chord face plastification. The von Mises stress distributions are illustrated in Figs. 7.4 and 7.5. Fig. 7.4 represents the von Mises stress distribution for an applied load which is 40% of the P_c load. It is clearly observed that the stress at the corner is the highest compared to other regions of the SHS TT-joint. It is also noted that the spread of yielding is more prominent at the crack side of the brace-chord intersection compared to the uncracked side for the same. The final plastic collapse deformation of the SHS TT-joint is illustrated in Fig. 7.5. At this stage, it is observed that yielding has spread throughout the brace-chord intersection as well as through the chord wall thickness.

The enlarged view of the deformed crack for SHS YT-joint is illustrated in Fig. 7.6 for P_c load of 1161.33 kN. The von Mises stress distributions for SHS YT-joint are illustrated in Figs. 7.7 and 7.8. Chord face plastification failure mode is observed for the joint which has the brace to chord width ratio (β) equal to 0.50. Fig. 7.7 represents the von Mises stress distributions for an applied load which is 40% of the P_c load. Similar to the trend observed for SHS TT-joint, the spread of yielding is more perceptible at the crack side of the brace-chord intersection compared to the uncracked side for the same. For the final failure at the P_c load, yielding is found to spread throughout the brace-chord intersection of the SHS YT-joint (Fig. 7.8).

The enlarged view of the deformed crack for SHS KT-joint is illustrated in Fig. 7.9 for P_c load of 1074.25 kN. The von Mises stress distributions for SHS KT-joint are illustrated in Figs. 7.10 and 7.11. Chord face plastification failure mode is observed for the joint which has the brace to chord width ratio (β) equal to 0.50. Fig. 7.10 represents the von Mises stress distributions of SHS KT-joint for an applied load which is 40% of the P_c load. The final plastic collapse deformation of the SHS KT-joint is illustrated in Fig. 7.11. It is obvious that the spread of yielding has extended to more regions particularly around the brace-chord intersection. Out of the three brace-chord intersections in the cracked multi-planar SHS KT-joint, the spread of yielding is most prominent for the intersection corresponding to the axially loaded brace. The stress close to the crack is observed to be higher when compared to other regions that are further away from the hot spot location.

7.2.3. Comparison of cracked uni-planar and multi-planar SHS joints

The comparison of plastic collapse behaviour between cracked uni-planar and multi-planar SHS joints is described in the current sub-section. Fig. 7.12 shows the comparison of load-displacement curves for the uni-planar and multi-planar SHS T-joints. The P_c load of 934.39 kN for the cracked multi-planar SHS TT-joint reveals a 15.87% increase of load when compared to the same cracked uni-planar SHS T-joint where the P_c load is 786.09 kN. This phenomenon is mainly attributed to the increase of stiffening effect caused by the additional multi-planar brace. Similarly, in the case of load-displacement curves for the cracked uni-planar and multi-planar SHS Y-joints, the P_c load is found to increase by 26.55% for cracked multi-planar SHS YT-joint (Fig. 7.13). The P_c load values for cracked uni-planar and multi-

planar SHS Y-joints are 853.65 kN and 1161.33 kN, respectively. In the case of load-displacement curves for the cracked uni-planar and multi-planar SHS K-joints, the P_c load is found to increase by 9.53% for cracked multi-planar SHS KT-joint (Fig. 7.14). The P_c load values for cracked uni-planar and multi-planar SHS K-joints are 971.80 kN and 1074.25 kN, respectively. It can be concluded from the comparison of all the three types of joints that the additional out-of-plane brace increases the P_c load of the cracked uni-planar SHS T-, Y- and K-joints. It further leads to stiffening of cracked uni-planar SHS T-, Y- and K-joints.

Another significant observation from the comparison of load-displacement curves is that the effect of additional out-of-plane brace is more prominent for the Y-joint when compared to the T- and K-joints. The trend is established by the increase in the P_c load by 9.53%, 15.87% and 26.55% for cracked SHS K-, T- and Y-joints, respectively. This phenomenon is due to the structurally symmetric geometric nature of the K-joint when compared to the Y-joint. The presence of an extra brace in uni-planar K-joint when compared to uni-planar Y-joint brings about additional structural safety, thereby increasing the P_c load of the uni-planar K-joint. As a consequence, the addition of out-of-plane brace does not bring as prominent increase in P_c load of the K-joint when compared to the Y-joint. In conclusion, it can be established from the above results that the stiffening effect due to the out-of-plane brace is least for cracked SHS K-joint when compared to cracked SHS T- and Y-joints.

7.3 Plastic Collapse Load of Cracked Multi-planar SHS TT-, YT- and KT-joints

As mentioned earlier, the two criteria used to determine failure of tubular joints with cracks are plastic collapse and fracture. P_c load of cracked multi-planar SHS TT-, YT- and KT-joints is covered in the current section. Similar to equations for P_c load of uncracked uni-planar joints, several codes of practises recommend a correction factor for determining the P_c load of uncracked multi-planar joints and it is mentioned in Section 7.3.1. The parametric study is carried out to obtain the reduction factor (F_{AR}) equations which when multiplied with the equations for uncracked multi-planar SHS TT-, YT- and KT-joints, gives the P_c load of cracked multi-planar SHS TT-, YT- and KT-joints. The details of parametric study are covered in Section 7.3.2, and the effect of the brace to chord width ratio (β) and the crack area on the reduction factor (F_{AR}) are subsequently covered in Sections 7.3.3 and 7.3.4.

Finally, the proposed equations for the reduction factor (F_{AR}) of cracked multi-planar SHS TT-, YT- and KT-joints are presented in Section 7.3.5.

7.3.1. Plastic collapse load for uncracked multi-planar SHS joints

The various codes of practises recommend a correction factor to uni-planar joint strength which is equivalent to the P_c load to determine the P_c load of corresponding uncracked multi-planar SHS joints (Eurocode 3, 2005; IIW, 2005; CIDECT, 2009). The correction factor recommended are 1.0 in IIW (2005) and CIDECT (2009) design guides and 0.9 in Eurocode 3 (2005). The expression for uni-planar joint strength is already described in Chapter 6, and henceforth, it will not be further repeated here.

As mentioned before, the reduction factor (F_{AR}) when multiplied with the P_c load of uncracked multi-planar SHS joints, gives the P_c load of cracked SHS joints. For the present parametric study, the twice-elastic compliance (TEC) criterion is used to determine the P_c load of both uncracked and cracked multi-planar SHS TT-, YT- and KT-joints. The twice-elastic compliance (TEC) criterion is already described in detail in Chapter 6, and henceforth it will not be repeated here. After determining the P_c load, the reduction factor (F_{AR}) for multi-planar SHS TT-, YT- and KT-joints is calculated using the following relationship:

$$F_{AR} = \frac{\text{Plastic collapse load of cracked SHS multi-planar joint}}{\text{Plastic collapse load of uncracked SHS multi-planar joint}} \quad (7.1)$$

7.3.2. Details of the parametric study

The details of various geometries used in the parametric study are given in Table 7.5. 11 different values of β are analyzed for crack area values of 5.8%, 10%, 20% and 0%. The dimensions of the new multi-planar brace like the length and width of the brace as well as the weld details are kept exactly identical to the brace of uni-planar joints. The brace thickness to chord thickness ratio (τ) is kept constant throughout the parametric study. The thickness for the brace and chord used is 16.0 mm. All the chord length to half chord width ratio (α) values used in the parametric study are above 8.0 so that the effect of α on estimating the P_c load of multi-planar SHS TT-, YT- and KT-joints is eliminated (Stacey et al., 1996). For all the cases used in the present study, the chord length is kept at a constant value of 3000 mm.

The material used in the present study for cracked multi-planar SHS TT-, YT- and KT-joints is the same as that used in uni-planar joints which is BS4360 structural steel of grade 50D. The axial tensile loads are applied at the brace ends using displacement control. The Young's modulus and Poisson's ratio of the material used are 210 kN/mm^2 and 0.3, respectively.

Extensive study carried out by Yu (1997) on uncracked multi-planar circular hollow section (CHS) TT-joints showed that the multi-planar effect is due to geometric and loading conditions. The geometric effect is a function of the brace to chord width ratio (β) and the chord half width to thickness ratio (γ), with β as the most significant influence. For the present case of cracked multi-planar SHS TT-, YT- and KT-joints, crack area also has a significant influence in determining the P_c load. Therefore, parametric study is carried out to investigate the effect of two parameters, brace to chord width ratio (β) and crack area, on the reduction factor (F_{AR}) of cracked multi-planar SHS TT-, YT- and KT-joints.

7.3.3. Effect of brace to chord width ratio (β) on reduction factor

The load-displacement relationships of the multi-planar SHS TT-, YT- and KT-joints are plotted using the analysed results to obtain the P_c load values. Tables 7.6 to 7.14 list out the P_c loads of both cracked and uncracked multi-planar SHS TT-, YT- and KT-joints for crack area values of 5.8%, 10% and 20%, respectively. The reduction factor (F_{AR}) for the respective joints covering all the brace to chord width ratio (β) values are also given in the corresponding tables. The FE mesh model of a typical multi-planar SHS TT-joint used in the parametric study is depicted in Fig. 7.15. The shape of model before and after the analysis is illustrated, and it can be observed that a considerable yielding has occurred particularly around the brace-chord intersection which is the high stress concentration region. The load-displacement curves for the multi-planar SHS TT-joints are represented in Figs. 7.16 to 7.18. Brace end axial loading values are plotted against brace end displacement for crack area values of 5.8%, 10% and 20%. In the curves, results from lower and higher β values are plotted, and it is observed that the P_c load of the multi-planar SHS TT-joints increases with increasing value of β . Similar variation is observed for the load versus displacement curves for SHS YT- and KT-joints, and hence, they are not shown in this chapter.

Similar to the observation made for cracked uni-planar SHS joints, the present study shows that β has an influence in reduction factor (F_{AR}) for cracked multi-planar SHS joints (Figs. 7.19 to 7.21). For a particular value of crack area, F_{AR} vary from 0.31% to 2.71% for different values of β . It is observed that the influence of β value in F_{AR} estimation for multi-planar joints is less when compared to uni-planar joints. In other words, F_{AR} obtained for multi-planar joints are higher when compared to the same for uni-planar joints. This is expected since the presence of an additional brace in multi-planar joints increases the P_c load, and thereby, increasing the stiffness of the brace-chord intersection zone in multi-planar joints.

The failure modes for cracked multi-planar SHS TT-joints are depicted in Figs. 7.22 to 7.24. For low brace to chord width ratio (β) value, the failure mode observed is punching shear, and it is observed for a β value up to 0.50. In the case of higher β values, the failure mode observed is chord face plastification. In chord face plastification failure mode, clear bending of the chord face is observed.

7.3.4. Effect of crack area on reduction factor

The reduction factor (F_{AR}) is primarily introduced to account for the reduction in P_c load due to the presence of cracks in tubular joints. Therefore, the crack area has the most significant influence on F_{AR} of cracked multi-planar SHS TT-, YT- and KT-joints. Figs. 7.25, 7.27 and 7.29 depict the variation in the F_{AR} with the crack area for punching shear failure mode; while Figs. 7.26, 7.28 and 7.30 depict the same for chord face plastification failure mode. The parametric study reveals that for cracked multi-planar SHS joints, F_{AR} varies from 1.52% to 4.37% for different values of crack area. For the uni-planar SHS joints, the variation range from is 1.9% to 5.3%. Generally, it is observed that F_{AR} decreases with an increase in crack area value. As the crack size increases, the P_c load of the cracked multi-planar SHS TT-, YT- and KT-joints decreases, thereby, weakening the joints. Therefore, a lower value of F_{AR} which is calculated as per Eqn. (7.1) is obtained.

The parametric study shows that the F_{AR} decreases to 4.37% in the case of cracked multi-planar SHS TT- and YT-joints and 4.15% in the case of cracked multi-planar SHS KT-joints for crack area of 20%. It is worth to mention the above smaller percentage reduction value

for KT-joints relative to TT- and YT-joints. The presence of an additional brace in KT-joints increases the local stiffness of the joint, and thereby, increasing the bearing capacity to withstand the effect of cracks.

Figs. 7.31 to 7.36 illustrate comparison of F_{AR} values of uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints for selected values of brace to chord width ratio (β). It is noted that the effect of crack area has a lesser effect on the P_c load of multi-planar TT-, YT- and KT-joints when compared to uni-planar T-, Y- and K-joints. This phenomenon can be attributed to an increase in strength affected by the out-of-plane brace. Coming to the individual variation amongst cracked multi-planar SHS TT-, YT- and KT-joints, the percentage variation is 1.35% for SHS TT- and YT-joints and 1.14% for SHS KT-joints. The reason for smaller value for KT-joints relative to TT- and YT-joints has already been explained in previous paragraph, and therefore, it is not repeated here.

7.3.5. Proposed reduction factor equations

It is shown in the above sub-sections that F_{AR} varies non-linearly with β and crack area for SHS TT-, YT- and KT-joints. This non-linear variation necessitates the use of non-linear regression techniques to curve fit the numerical data obtained from the parametric study. Hence, for the two failure modes observed for cracked multi-planar SHS TT-, YT- and KT-joints, two set of equations are proposed after carrying out the non-linear regression.

The proposed equations are as follows:

The proposed equations for TT- and YT-joints are as follows:

$$\text{For } \beta \geq 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.23} * \beta^{0.01} * 1.02 \quad (7.2)$$

$$\text{For } \beta < 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.11} * \beta^{-0.02} * 0.98 \quad (7.3)$$

The proposed equations for KT-joints are as follows:

$$\text{For } \beta \geq 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.22} * \beta^{0.01} * 1.01 \quad (7.4)$$

$$\text{For } \beta < 0.5; F_{AR} = \left(1 - \frac{A_c}{l_w * t_0}\right)^{0.10} * \beta^{-0.02} * 0.98 \quad (7.5)$$

where

A_c is the crack area, l_w is the weld length and t_0 is the chord thickness.

The coefficient of determination values for TT- and YT- joints are 0.957 and 0.903, and for KT-joints are 0.959 and 0.912. These high values for the non-linear regression justify the statistical accuracy of the proposed equations. For verifying the proposed equations further, the reduction factor (F_{AR}) values from the numerical data and the F_{AR} values obtained from the regression equations are plotted and compared as shown in Figs. 7.37 to 7.42. The plots reconfirm the accuracy of the proposed regression equations as the values for both the considered cases exhibit good agreement. To ensure continuity of the proposed equations, plots are made showing the variation of F_{AR} for β at 0.49 and at 0.50. The plotted trends are shown in Fig's 7.43 and 7.44 and the plots reveal that there is good continuity for the proposed equations.

7.4 Fracture Analyses of Cracked Multi-planar SHS Joints

For a more comprehensive understanding of the failure behaviour of cracked multi-planar SHS TT-, YT- and KT-joints, fracture behaviour needs to be studied in addition to the plastic collapse behaviour. The current section deals with the fracture behaviour of cracked multi-planar SHS TT-, YT- and KT-joints by using the failure assessment diagram (FAD) method. As mentioned before, there has not been any fracture analyses of cracked multi-planar SHS TT-, YT- and KT-joints carried out by researchers, and therefore, this study brings out useful observations on cracked multi-planar SHS TT-, YT- and KT-joints. Fracture control is intended to prevent fracture due to presence of defect or crack at the

maximum loads experienced during operational service. The new FE mesh generator is used to create cracked multi-planar SHS TT-, YT- and KT-joints which are then used in the parametric study to understand the fracture behaviour. Both elastic and elastic-plastic analyses for cracked multi-planar SHS TT-, YT- and KT-joints are carried out. J -integral values are obtained for cracked multi-planar SHS TT-, YT- and KT-joints subjected to axial tensile loading at the brace end. Figs. 7.45 and 7.46 represent the typical variation of elastic (J_e) and elastic-plastic J -integral (J_{ep}) values with the axial loading case. The pattern of variation for multi-planar TT-, YT- and KT-joints is found to be similar to uni-planar T-, Y- and K-joints. The highest level, namely, the Level 3C FAD curve of cracked multi-planar SHS TT-, YT- and KT-joints are constructed using the elastic J -integral (J_e), elastic-plastic J -integral (J_{ep}) values and P_c load, and these FAD curves are described in Section 7.4.1.

7.4.1. Failure assessment diagrams (FADs) of cracked multi-planar SHS joints

The main objectives of the fracture analysis of cracked multi-planar SHS TT-, YT- and KT-joints are (1) to validate the standard Level 2A failure assessment diagram (FAD) curve of the BS7910 (2005), and (2) to recommend optimal solutions that can be safely used to assess cracked multi-planar SHS TT-, YT- and KT-joints. The general Level 2A failure assessment diagram (FAD) curve is already described in Chapter 6, and therefore, it is not repeated in this chapter. For the fracture analyses of cracked multi-planar SHS TT-, YT- and KT-joints, the same models used in the parametric study for plastic-collapse behaviour in Section 7.3 are selected (Table 7.5). The material properties and loading conditions are kept exactly the same in the parametric study for plastic-collapse analyses. On completion of both elastic and elastic-plastic analyses using the ABAQUS (2011) general purpose FE software, elastic (J_e) and elastic-plastic J -integral (J_{ep}) values are obtained, respectively. Level 3C FAD curves are constructed for cracked multi-planar SHS TT-, YT- and KT-joints using the elastic (J_e) and elastic-plastic J -integral (J_{ep}) values and P_c load. Figs. 7.47 to 7.49 represent the constructed FAD curves for cracked multi-planar SHS TT-, YT- and KT-joints, respectively. It is observed that some of the constructed curves tend to fall below the standard BS7910 (2005) Level 2A FAD curve. Similar to the trend observed for cracked uni-planar SHS T-, Y- and K-joints, the above observation is evident towards the right side of the FAD curves which is near to the plastic-collapse region in the standard BS7910 (2005) Level 2A FAD

curve. The results also reveal that the constructed FAD curves for cracked multi-planar SHS TT-, YT- and KT-joints with low brace to chord width ratio (β) appears to fall below the standard BS7910 (2005) Level 2A FAD curve when compared to the models with higher β ratio value. This is attributed to increasing P_c load of the multi-planar SHS TT-, YT- and KT-joints with increasing value of β .

In order to obtain a conservative and safe prediction, a penalty factor of 1.10 is recommended for cracked multi-planar SHS TT-, YT- and KT-joints. Figs. 7.50 to 7.52 represent the constructed FAD curves for cracked multi-planar SHS TT-, YT- and KT-joints with the newly introduced penalty factor of 1.10. It can be seen that all the newly constructed FAD curves lie just above the standard BS7910 (2005) Level 2A FAD curve. These new Level 3C FAD curves are found to produce optimal solutions, and they can be used to assess the safety and integrity of cracked multi-planar SHS TT-, YT- and KT-joints in practice.

Fig. 7.1 Loading and boundary conditions for SHS TT-joint

Fig. 7.2 Loading and boundary conditions for SHS YT- and KT-joints

Fig. 7.3 Enlarged view of crack at plastic collapse load (P_c) of SHS TT-joint

Fig. 7.4 The von Mises stress distributions of SHS TT-joint at 40% of P_c load

Fig. 7.5 The von Mises stress distributions of SHS TT-joint at 100% of P_c load

Fig. 7.6 Enlarged view of crack at plastic collapse load (P_c) of SHS YT-joint

Fig. 7.7 The von Mises stress distributions of SHS YT-joint at 40% of P_c load

Fig. 7.8 The von Mises stress distributions of SHS YT-joint at 100% of P_c load

Fig. 7.9 Enlarged view of crack at plastic collapse load (P_c) of SHS KT-joint

Fig. 7.10 The von Mises stress distributions of SHS KT-joint at 40% of P_c load

Fig. 7.11 The von-Mises stress distributions of SHS KT-joint at 100% of P_c load

Fig. 7.12 Comparison of load-displacement curves of uni-planar and multi-planar SHS T-joints

Fig. 7.13 Comparison of load-displacement curves of uni-planar and multi-planar SHS Y-joints

Fig. 7.14 Comparison of load-displacement curves of uni-planar and multi-planar SHS K-joints

(a)

(b)

Fig. 7.15 Typical multi-planar SHS models: (a) before FE analysis; (b) after FE analysis, with deformation plotted using a scale of 1.5:1

Fig. 7.16 Load vs displacement curve for crack area 5.8% for multi-planar TT-joints

Fig. 7.17 Load vs displacement curve for crack area 10% for multi-planar TT-joints

Fig. 7.18 Load vs displacement curve for crack area 20% for multi-planar TT-joints

Fig. 7.19 F_{AR} vs β plot comparison for multi-planar SHS TT-joints

Fig. 7.20 F_{AR} vs β plot comparison for multi-planar SHS YT-joints

Fig. 7.21 F_{AR} vs β plot comparison for multi-planar SHS KT-joints

(a)

(b)

Fig. 7.22 Mode of failure (punching shear) for SHS joints with low brace to chord width ratio (β); (a) $\beta=0.25$; (b) $\beta=0.35$, with deformation plotted using a scale of 1.5:1

Fig. 7.23 Two modes of failure for SHS joints: (a) punching shear ($\beta=0.50$);
(b) chord face plastification ($\beta=0.60$), with deformation plotted using a scale of 1.5:1

Fig. 7.24 Mode of failure (chord face plastification) for SHS joints with high brace to chord width ratio (β) of 0.70, with deformation plotted using a scale of 1.5:1

Fig. 7.25 Comparison of F_{AR} vs crack area plot of multi-planar SHS TT-joints for $\beta < 0.5$

Fig. 7.26 Comparison of F_{AR} vs crack area plot of multi-planar SHS TT-joints for $\beta \geq 0.5$

Fig. 7.27 Comparison of F_{AR} vs crack area plot of multi-planar SHS YT-joints for $\beta < 0.5$

Fig. 7.28 Comparison of F_{AR} vs crack area plot of multi-planar SHS YT-joints for $\beta \geq 0.5$

Fig. 7.29 Comparison of F_{AR} vs crack area plot of multi-planar SHS KT-joints for $\beta < 0.5$

Fig. 7.30 Comparison of F_{AR} vs crack area plot of multi-planar SHS KT-joints for $\beta \geq 0.5$

Fig. 7.31 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS TT-joint for $\beta < 0.5$

Fig. 7.32 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS TT-joint for $\beta \geq 0.5$

Fig. 7.33 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS YT-joint for $\beta < 0.5$

Fig. 7.34 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS YT-joint for $\beta \geq 0.5$

Fig. 7.35 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS KT-joint for $\beta < 0.5$

Fig. 7.36 Comparison of F_{AR} vs crack area plot b/w uni- and multi-planar SHS KT-joint for $\beta \geq 0.5$

Fig. 7.37 Comparison of equation predictions with the numerical data for multi-planar SHS TT-joints for $\beta \geq 0.5$

Fig. 7.38 Comparison of equation predictions with the numerical data for multi-planar SHS TT-joints for $\beta < 0.5$

Fig. 7.39 Comparison of equation predictions with the numerical data for multi-planar SHS YT-joints for $\beta \geq 0.5$

Fig. 7.40 Comparison of equation predictions with the numerical data for multi-planar SHS YT-joints for $\beta < 0.5$

Fig. 7.41 Comparison of equation predictions with the numerical data for multi-planar SHS KT-joints for $\beta \geq 0.5$

Fig. 7.42 Comparison of equation predictions with the numerical data for multi-planar SHS KT-joints for $\beta < 0.5$

Fig. 7.43 F_{AR} vs β plot comparison for SHS TT- and YT-joints equation

Fig. 7.44 F_{AR} vs β plot comparison for SHS KT-joints equation

Fig. 7.45 Typical variation of elastic J -integral values under axial loading

Fig. 7.46 Typical variation of elastic-plastic J -integral values under axial loading

Fig. 7.47 FAD curves of cracked multi-planar SHS TT-joints

Fig. 7.48 FAD curves of cracked multi-planar SHS YT-joints

Fig. 7.49 FAD curves of cracked multi-planar SHS KT-joints

Fig. 7.50 FAD curves of cracked multi-planar SHS TT-joints with a penalty factor of 1.10

Fig. 7.51 FAD curves of cracked multi-planar SHS YT-joints with a penalty factor of 1.10

Fig. 7.52 FAD curves of cracked multi-planar SHS KT-joints with a penalty factor of 1.10

Table 7.1 Dimensions of the specimens

Specimen	b_0 (mm)	h_0 (mm)	t_0 (mm)	b_1 (mm)	h_1 (mm)	t_1 (mm)	b_2 (mm)	h_2 (mm)	t_2 (mm)
TT	350	350	15	200	200	12	200	200	12
YT	300	300	12.5	150	150	10	150	150	10
KT	300	300	12.5	150	150	10	150	150	10

Table 7.2 The weld and fatigue crack details

Specimen	Crack Length l (mm)	l_{c1} (mm)	l_{c2} (mm)	Crack Depth a (mm)	Weld Width t_w (mm)
TT	264	77	187	13.6	12
YT	153	45	108	7.2	8
KT	125	37	88	4.9	8

Table 7.3 Mechanical properties of the material

Material	Yield Strength	Tensile Strength	Elongation	Charpy V-notch Temperature	Charpy V-notch Energy
	(N/mm ²)	(N/mm ²)	(A%)	(°C)	(J)
BS4360-50D	380.3	529	33	-20	170.6

Table 7.4 Mechanical properties of the weld (AWS, 2000)

Electrodes	Yield Strength	Tensile Strength
	(N/mm ²)	(N/mm ²)
E70XX	399.9	482.6

Table 7.5 Dimensions of specimens for crack area of 5.8%, 10%, 20 % and 0%

b_1	b_2	t_0	t_1	l_c	β	b_0	γ	α	τ
150.0	150.0	16	16	3000	0.25	600.0	18.8	10.0	1
150.0	150.0	16	16	3000	0.30	500.0	15.6	12.0	1
150.0	150.0	16	16	3000	0.35	428.6	13.4	14.0	1
150.0	150.0	16	16	3000	0.40	375.0	11.7	16.0	1
150.0	150.0	16	16	3000	0.45	333.3	10.4	18.0	1
150.0	150.0	16	16	3000	0.50	300.0	9.4	20.0	1
150.0	150.0	16	16	3000	0.55	272.7	8.5	22.0	1
150.0	150.0	16	16	3000	0.60	250.0	7.8	24.0	1
150.0	150.0	16	16	3000	0.65	230.8	7.2	26.0	1
150.0	150.0	16	16	3000	0.70	214.3	6.7	28.0	1
150.0	150.0	16	16	3000	0.75	200.0	6.3	30.0	1

Table 7.6 Plastic collapse load values of SHS TT-joints for crack area 5.8%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	694.120	694.982	0.999
0.30	715.258	716.469	0.998
0.35	732.866	734.445	0.998
0.40	749.035	751.259	0.997
0.45	778.324	781.269	0.996
0.50	827.356	830.395	0.996
0.55	875.163	877.928	0.997
0.60	947.107	949.204	0.998
0.65	996.325	997.183	0.999
0.70	1075.257	1075.827	0.999
0.75	1197.158	1197.793	0.999

Table 7.7 Plastic collapse load values of SHS TT-joints for crack area 10%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	689.193	694.982	0.992
0.30	709.419	716.469	0.990
0.35	725.058	734.445	0.987
0.40	737.751	751.259	0.982
0.45	765.730	781.269	0.980
0.50	811.927	830.395	0.978
0.55	858.816	877.928	0.978
0.60	930.135	949.204	0.980
0.65	979.393	997.183	0.982
0.70	1058.098	1075.827	0.984
0.75	1180.880	1197.793	0.986

Table 7.8 Plastic collapse load values of SHS TT-joints for crack area 20%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	684.272	694.982	0.985
0.30	705.012	716.469	0.984
0.35	720.189	734.445	0.981
0.40	732.477	751.259	0.975
0.45	757.831	781.269	0.970
0.50	796.490	830.395	0.959
0.55	842.478	877.928	0.960
0.60	911.255	949.204	0.960
0.65	958.462	997.183	0.961
0.70	1034.494	1075.827	0.962
0.75	1152.229	1197.793	0.962

Table 7.9 Plastic collapse load values of SHS YT-joints for crack area 5.8%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	689.717	691.265	0.998
0.30	710.671	712.588	0.997
0.35	728.944	731.247	0.997
0.40	744.409	747.369	0.996
0.45	773.505	777.212	0.995
0.50	822.746	826.598	0.995
0.55	869.360	872.983	0.996
0.60	942.185	945.219	0.997
0.65	993.515	995.366	0.998
0.70	1068.522	1070.159	0.998
0.75	1192.755	1194.583	0.998

Table 7.10 Plastic collapse load values of SHS YT-joints for crack area 10%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	684.815	691.265	0.991
0.30	704.864	712.588	0.989
0.35	721.170	731.247	0.986
0.40	733.184	747.369	0.981
0.45	760.976	777.212	0.979
0.50	807.388	826.598	0.977
0.55	853.105	872.983	0.977
0.60	925.284	945.219	0.979
0.65	976.613	995.366	0.981
0.70	1051.453	1070.159	0.983
0.75	1176.521	1194.583	0.985

Table 7.11 Plastic collapse load values of SHS YT-joints for crack area 20%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	679.921	691.265	0.984
0.30	700.481	712.588	0.983
0.35	716.322	731.247	0.980
0.40	727.937	747.369	0.974
0.45	753.118	777.212	0.969
0.50	792.021	826.598	0.958
0.55	836.859	872.983	0.959
0.60	906.484	945.219	0.959
0.65	955.721	995.366	0.960
0.70	1027.973	1070.159	0.961
0.75	1147.946	1194.583	0.961

Table 7.12 Plastic collapse load values of SHS KT-joints for crack area 5.8%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	695.867	696.654	0.999
0.30	717.414	718.557	0.998
0.35	736.358	737.871	0.998
0.40	752.890	754.369	0.998
0.45	780.092	782.259	0.997
0.50	832.040	834.259	0.997
0.55	877.236	879.126	0.998
0.60	951.245	952.397	0.999
0.65	998.498	999.257	0.999
0.70	1077.398	1077.959	0.999
0.75	1198.716	1199.328	0.999

Table 7.13 Plastic collapse load values of SHS KT-joints for crack area 10%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	691.548	696.654	0.993
0.30	712.205	718.557	0.991
0.35	729.179	737.871	0.988
0.40	741.560	754.369	0.983
0.45	767.482	782.259	0.981
0.50	816.539	834.259	0.979
0.55	860.867	879.126	0.979
0.60	934.216	952.397	0.981
0.65	982.430	999.257	0.983
0.70	1061.272	1077.959	0.985
0.75	1183.593	1199.328	0.987

Table 7.14 Plastic collapse load values of SHS KT-joints for crack area 20%

β	P_c Load (kN) Cracked	P_c Load (kN) Uncracked	F_{AR}
0.25	686.615	696.654	0.986
0.30	707.786	718.557	0.985
0.35	724.287	737.871	0.982
0.40	736.264	754.369	0.976
0.45	759.573	782.259	0.971
0.50	801.030	834.259	0.960
0.55	844.506	879.126	0.961
0.60	915.273	952.397	0.961
0.65	961.455	999.257	0.962
0.70	1037.622	1077.959	0.963
0.75	1154.905	1199.328	0.963

Chapter 8

Conclusions and Recommendations

8.1 Conclusions

It is necessary to produce design guidance which can safely be used to estimate the P_c load of cracked tubular structures in practice. It is more relevant in the case of both cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints, for which adequate research works have not been done till date. Since these structures are widely used in practise, it becomes all the more necessary to provide the design guidelines for these joints. In practice, non-validated design and inadequate maintenance can result in the untimely failure of these components which necessitates an earlier replacement of these components than anticipated. Fitness for service assessment is a quantitative engineering evaluation which is performed to demonstrate the structural integrity of a component containing a flaw discovered during pre-service or in-service inspections (Wells, 1981). The recent RP579/ASME FFS-1 (2007) and BS7910–Amendment 1 (2005) have a provision for engineers to assess quantitatively and to predict the instability loads due to cracks and defects in tubular welded structures. This approach is based on the failure assessment diagram (FAD) method. However, there is a need to extend and optimize the FAD for multi-planar tubular SHS joints.

In order to assist in risk assessment of damaged multi-planar square hollow section (SHS) welded joints, the present study develops a completely new and robust finite element (FE) mesh generator. The new FE mesh generator is able to overcome the drawbacks of the previous mesh generator approaches. The new FE mesh generator is validated using the previous experimental test data as well as with the existing commercial software results. Subsequently, extensive parametric studies are carried out on geometries such as rectangular cracked plate with and without attachments, uncracked and cracked uni-planar SHS T-, Y- and K-joints and uncracked and cracked multi-planar SHS TT-, YT- and KT-joints. Several conclusions from the present research work can be summarized as follows:

1. Literature review of significant previous works on cracked tubular structures is carried out and the background of related concepts used in present research study is summarized. The comparison of CHS and SHS joints is described as well as previous research on uncracked multi-planar tubular joints is highlighted. Subsequently, failure modes observed for SHS joints as well as previous works on FE modelling of tubular joints is highlighted. Past and present research works on cracked SHS joints carried out at different parts of the world is described in detail. Finally, the concepts behind plastic collapse (yielding) and crack extension (fracture) are presented. Literature review revealed that there exists considerable research gaps such as need to optimize the reduction factor (F_{AR}) expressions for cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints and to optimize Level 3C FAD curves for above mentioned joints. It also revealed the need for developing a new robust FE mesh generator that could model the various cracked SHS joints.
2. The geometric modelling of cracked SHS joints using the new FE mesh generator is carried out. The present research work uses the specifications mentioned in various existing codes of practise such as AWS (2000), IIW (2005), BS EN10210-2 (2006) and CIDECT (2009) for precise geometrical modelling of the cracked SHS joints. It is done so because the reliability of the results obtained using the FE software (ABAQUS, 2011) is very much related to the accuracy of the models produced using the new FE mesh generator. For hot finished SHS joints, BS EN 10210-2:2006 recommends the external corner radius value (r_o) as $1.5t_1$ and internal corner radius (r_i) as $1.0t_1$, where t_1 is the specified thickness of the brace. Regarding the shape and location of crack, it was observed from previously published works that the crack always initiates and propagates from the hot spot stress location at the chord corner and the shape of the crack is not symmetrical (Lie et al., 2006; Lie and Yang, 2007). Yang (2005) approximated the crack shape as two half semi-elliptical cracks, and therefore, it is also adopted in the present study.
3. A completely new and robust FE mesh generator is developed in this study. It can be effectively used to generate FE mesh models such as plate joints with both symmetrical and unsymmetrical cracks, plate-to-plate welded joints such as T-butt

joints with single-sided and double-sided welds and cruciform X-joints. Regarding hollow section joints, it can be used to model uncracked and cracked uni-planar SHS T-, Y- and K-joints, and uncracked and cracked multi-planar SHS TT-, YT- and KT-joints. The cracked multi-planar SHS TT-, YT- and KT-joints are modelled for the first time in the present study. The newly developed automatic FE mesh generator is validated using the previous experimental test data as well as with the existing commercial software results. The versatility of the new FE mesh generator facilitates the extensive parametric study of above mentioned joints.

4. The new FE mesh generator is able to overcome the drawbacks of the previous mesh generating approaches by demonstrating the capability of achieving convergence of solutions even at a very high plastic deformation. The new FE mesh generator is versatile enough to model cracks of any arbitrary dimensions. The concept of using key-hole for elastic-plastic analysis is effectively incorporated in the new FE mesh generator. The usage of only one type of element .i.e. 20 nodes hexahedral element eliminates the compatibility issues associated with mesh modelling. The “spider web” configuration which is the most efficient mesh design for the crack tip region enables a smooth conversion from a fine mesh at the crack tip region to a coarser mesh region away from high stress concentration region. Mesh refinement and zoning techniques are extensively used in the new FE mesh generator so that it is robust to model different types of uni-planar and multi-planar SHS welded joints with and without cracks. Another feature of the new FE mesh generator is the fast generation of different types of cracked and uncracked models. For example, a typical cracked multi-planar SHS YT-joint with 20000 elements can be generated within 4 minutes.
5. There is adequate provision for robust mesh refinement in the new FE mesh generator. The dense or highly refined FE mesh is used for important regions such as crack tip, crack block and high stress concentration spots. The coarse FE mesh is used in the regions that are further away from high stress concentration spots. *h*-refinement or local refinement is well incorporated in the new FE mesh generator so that element sizes can be reduced at will to obtain accurate analyses results. The load-displacement curves and *J*-integral values are plotted and compared for models with varying mesh density. The number of elements for mesh convergence test for coarse mesh, refined

mesh-01 and refined mesh-02 models are 12,040, 16,070 and 20,030 respectively. The cracked uni-planar SHS T-joints were subjected to brace end axial loading and it is observed that the meshes for all the models have converged for both load-displacement curves and J -integral value comparison plots.

6. The new FE mesh generator is validated with the FEACrackTM (2010) software as well as with the results produced by Bowness and Lee (2000); before using it for parametric study of cracked plate-to-plate welded joints. The comparison of stress intensity factors (K_I , K_{II} and K_{III}) generated using the new FE mesh generator and FEACrackTM (2010) software is carried out. The single-sided T-butt joint models are used for comparison purpose with geometric parameters of $a/T=0.5$, $a/c=0.4$, $L/T=1.25$ and $\theta=45^\circ$. The observed mode of crack opening is mode I due to the tensile load applied to the considered model. It is observed that the maximum percentage difference of K_I at the deepest point is 1.94% while at the crack ends, it is 0.25%. It shows that the trend of K_I and K_{II} is symmetrical, while K_{III} is anti-symmetrical. Furthermore, the K_{II} and K_{III} values are almost zero due to the predominant nature of crack opening mode. The results covering all the three stress intensity factors namely K_I , K_{II} and K_{III} demonstrate good agreement for both cases. The M_k results of single-sided T-butt joints from parametric study are compared with values produced by Bowness and Lee (2000). The results demonstrate good agreement of values between the two cases for single-sided T-butt joints.
7. In the present study, around 5000 cases of different geometries such as single-sided and double-sided T-butt joints, non-load carrying cruciform X-joints and plain plates are analysed in the parametric study. It showed that M_k results for cruciform X-joint differ with those of double-sided T-butt joint with percentage difference in values being as high as 136% and 76% for axial and bending loading cases, respectively. The percentage differences of M_k values are more discernible for shallower cracks; where the fatigue life of the cracked structure is most consumed. In the case of axial loading at the deepest point, the largest percentage difference in M_k values is 136% (for $a/c=1.0$, $L/T=2.75$, $\theta=60^\circ$). Similarly, at the crack ends, the largest percentage difference in M_k values is 46% (for $a/c=1.0$, $L/T=2.75$, $\theta=45^\circ$). The high value of largest percentage difference is attributed to the small nature of values, and hence

when the percentage difference is calculated, the resulting value obtained is large. For bending loading at the deepest point, the largest percentage difference in M_k values is 62% (for $a/c=0.4$, $L/T=2.75$, $\theta=45^\circ$). Similarly, at the crack ends, the largest percentage difference in M_k values is 76% (for $a/c=0.2$, $L/T=0.5$, $\theta=45^\circ$). All these results confirmed the need for having separate set of M_k equations for cracked double-sided T-butt joint and X-joint.

8. The new sets of M_k equations are proposed and presented in Appendix A and Appendix B for double-sided T-butt joint and X-joint, respectively. These new equations are already published (Lie et al., 2015). They cover M_k equations at the deepest point and at the crack ends of joints subjected to axial as well as bending loading. The weld angles (θ) in proposed equations are measured in radians. The validity range for crack depth ratio (a/T) is from 0.05 to 0.7 at the deepest point and from 0.05 to 0.9 at the crack ends. For cases of crack ends under bending loading, the proposed M_k equations are valid only for weld angle (θ) of 45° . The regression analyses further establish a good correlation between the FE results and the predicted equations, thereby corroborating the newly proposed equations.
9. The parameters such as crack aspect ratio (a/c), weld angle (θ) and attachment footprint width ratio (L/T) have significant influence on the M_k values. The M_k values are lower for cracks with higher a/c values (cracks with round shape) when compared to those for cracks with lower a/c values. This tendency is more noticeable at the crack ends, and is associated to restricted ability of the attachment to deform backwards which subsequently affects the opening of crack ends. The study also observes that the M_k values decrease with increasing a/T ratio, thereby reporting higher M_k values for shallower cracks and lower M_k values for deeper cracks. It can be observed that M_k values increase with the increasing steepness of the weld angle (θ). The steepness raises the stress concentration effect especially for shallower cracks where the fatigue life of the cracked structure is most consumed. The most salient conclusion associated with the effect of attachment footprint width ratio (L/T) is the occurrence of higher M_k values for L/T value of 1.25 as compared to L/T value of 0.5. In cases of L/T value beyond 1.25, the tendency becomes marginal due to attachment base stiffening the top surface of the main plate.

10. The numerical analyses results of cracked uni-planar SHS T-, Y- and K-joints produced using the new FE mesh generator is validated with the previous experimental test results (Lie and Yang, 2007). The specimens were fabricated using BS4360 structural steel of grade 50D. Each specimen contains one fatigue surface crack under the weld toe. The load-displacement curves for SHS T-, Y- and K-joints clearly reveal the analogous trends between experimental and numerical models. Hence, the numerical models created using the new FE mesh generator are accurate and it can be used to carry out extensive parametric study of cracked SHS T-, Y- and K-joints.
11. Parametric study is carried out to investigate the effect of two parameters, brace to chord width ratio (β) and crack area, on the F_{AR} of cracked uni-planar SHS T-, Y- and K-joints. Contrary to F_{AR} of CHS joints containing part-thickness flaws (BS7910, 2005), the present study shows that β has an influence in F_{AR} for cracked SHS T-, Y- and K-joints. For a particular crack area, F_{AR} varies from 1.5% to 3.3% for different values of β . Furthermore, comparing the F_{AR} values for SHS joints with values obtained from BS7910 (2005); it shows that the latter is conservative. The BS7910 (2005) equation is conservative up to 6% for crack area of 5.8%, and up to 22.7% for crack area of 20%. The study also shows that the crack area has the most significant influence on F_{AR} of cracked SHS joints. The parametric study reveals that for cracked SHS joints, F_{AR} varies from 1.9% to 5.3% for different values of crack area.
12. New equations for F_{AR} of cracked uni-planar SHS T-, Y- and K-joints are proposed. These new equations are already published (Lie et al., 2015). Thus, optimal solutions for F_{AR} estimation for cracked uni-planar SHS T-, Y- and K-joints are presented.
13. The new FE mesh generator is also used to study the effect of crack depth ratio (a/T) and crack length ratio (c_2/c_1) on the J -integral values. Four cases with different values of c_2/c_1 are analysed in the present study. The relationship between J -integral and crack front angle for $c_2/c_1=2$, $c_2/c_1=2.5$, $c_2/c_1=3$ and $c_2/c_1=4$ for a crack depth ratio of 0.5 is chosen. The flexibility of the new FE mesh generator is illustrated by the pattern of results obtained for different values of c_2/c_1 . It is observed that the J -integral values are found to be higher for longer crack length. The trend of J -integral values being higher for $c_2/c_1=4$ when compared to those of $c_2/c_1=2$ reaffirms that the crack length has an

influence in the determination of J -integral values. Coming to relationship between a/T and J -integral values, the latter increase with increasing a/T value. The percentage difference at the deepest point of the crack between $a/T=0.3$ and $a/T=0.7$ is 18.31%. The percentage difference at the crack ends between $a/T=0.3$ and $a/T=0.7$ is 60.43%. The large percentage difference is due to the nature of small J -integral values of 0.118 and 0.298 for $a/T=0.3$ and $a/T=0.7$, respectively. The versatility of new FE mesh generator is reaffirmed by the results obtained from performing the analysis for various cases of a/T and c_2/c_1 values.

14. The fracture behaviour of cracked uni-planar SHS T-, Y- and K-joints is also investigated by using the FAD approach in the present study. The highest level, Level 3C FAD curve of cracked SHS joints is constructed using the full numerical results of the elastic J -integral (J_e), elastic-plastic J -integral (J_{ep}) values and P_c load. The constructed Level 3C FAD curves are further used to validate the standard Level 2A curve of the BS7910 (2005). It is found that standard Level 2A curve of the BS7910 (2005) is not always safe in assessing the safety and integrity of cracked uni-planar SHS T-, Y- and K-joints. For obtaining a conservative safe prediction, a penalty factor of 1.20 is recommended for cracked uni-planar SHS T-, Y- and K-joints. It can be seen that all the newly constructed FAD curves using the penalty factor lie just above the standard BS7910 (2005) Level 2A FAD curve.
15. Plastic collapse and fracture analyses of cracked multi-planar SHS TT-, YT- and KT-joints are performed for the first time in the present study. The stress distribution and deformation at the brace-chord intersection is investigated first. It is clearly observed that the stress at the corner is the highest compared to other regions of the SHS TT-, YT- and KT-joints. It is also noted that the spread of yielding is more prominent at the crack side of the brace-chord intersection compared to the uncracked side for the same. At the plastic collapse stage, it is observed that yielding has spread throughout the brace-chord intersection as well as through the chord wall thickness. Load-displacement curves of uni-planar and multi-planar SHS joints are also compared so that the effect of the additional brace in cracked multi-planar joints is investigated. The P_c of 934.39 kN for the cracked multi-planar SHS TT-joint reveals a 15.87% increase of load when compared to the same cracked uni-planar SHS T-joint where the P_c is

- 786.09 kN. This phenomenon is mainly attributed to the increase of stiffening effect caused by the additional out-of-plane multi-planar brace. Similarly, in the case of load-displacement curves for the cracked uni-planar and multi-planar SHS Y-joints, the P_c is found to increase by 26.55% for cracked multi-planar SHS YT-joint. For the case of K-joints, the P_c is found to increase by 9.53% for cracked multi-planar SHS KT-joint.
16. Similar to uni-planar joints, parametric study is carried out to investigate the effect of β and crack area, on the F_{AR} of cracked multi-planar SHS TT-, YT- and KT-joints. It is observed that the influence of β value in F_{AR} estimation for multi-planar joints is less when compared to uni-planar joints. In other words, F_{AR} obtained for multi-planar joints are higher when compared to the same for uni-planar joints. This is expected since the presence of an additional brace in multi-planar joints increases the P_c load, and thereby, increasing the stiffness of the brace-chord intersection zone in multi-planar joints. The crack area has the most significant influence on F_{AR} of cracked multi-planar SHS TT-, YT- and KT-joints. The study reveals that the F_{AR} decreases to 4.37% in the case of cracked multi-planar SHS TT- and YT-joints and 4.15% in the case of cracked multi-planar SHS KT-joints for crack area of 20%.
 17. New equations for F_{AR} of cracked multi-planar SHS TT-, YT- and KT-joints are proposed. Non-linear regression techniques are used to curve fit the numerical data obtained from the parametric study. The coefficient of determination values for TT- and YT- joints are 0.957 and 0.903, and for KT-joints are 0.959 and 0.912. These high values for the non-linear regression justify the statistical accuracy of the proposed equations. For verifying the proposed equations further, the F_{AR} values from the numerical data and the F_{AR} values obtained from the regression equations are plotted and compared. The plots reconfirm the accuracy of the proposed regression equations as the values for both the considered cases exhibit good agreement.
 18. The fracture behaviour of cracked multi-planar SHS TT-, YT- and KT-joints is studied by using the FAD method. The highest level, namely, the Level 3C FAD curve of cracked SHS joints are constructed using the J_e , J_{ep} and P_c values. It is observed that some of the constructed curves tend to fall below the standard BS7910 (2005) Level 2A FAD curve. In order to obtain a conservative and safe prediction, a penalty factor

of 1.10 is recommended for cracked multi-planar SHS TT-, YT- and KT-joints. The new FAD curves constructed using penalty factor of 1.10 lie above the standard BS7910 (2005) Level 2A FAD curve, thereby providing optimal solutions for cracked multi-planar SHS TT-, YT- and KT-joints.

8.2 Recommendations for Future Research Work

There is sufficient scope for further research works in both cracked uni-planar and multi-planar SHS joints. The following points can be explored for research in future,

1. The present research work studied various types of joints such as cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints subjected to axial tensile loading at the brace end. In future, research work can be carried out for above joints subjected to in-plane as well as out-of-plane bending loading. Subsequently, new equations for F_{AR} of joints subjected to in-plane as well as out-of-plane bending loading can be obtained after carrying out the parametric study. In addition, multi-planar joint behaviour during axial compression loading can also be researched in future.
2. The current research works focussed only on joints which contain a single surface crack. However, there is a possibility that a joint has multiple cracks. The newly developed FE mesh generator needs to be modified in order to model multiple cracks in the SHS tubular joint.
3. The new FE mesh generator has its scope on surface cracks. However, through thickness cracks is a potential type of crack that can be considered to model and analysed in future, especially for cracked multi-planar SHS joints.
4. The new FE mesh generator creates geometries such as cracked uni-planar SHS T-, Y- and K-joints, and multi-planar SHS TT-, YT- and KT-joints as well as different types of plate-to-plate joints. The FE analysis is carried out using ABAQUS (2011) FE analysis software. The material stress-strain curve is specified by the user. The present research work has its scope in steel structures. However, the new FE mesh generator can be used to analyse and understand the structural behaviour of SHS sections made of other materials such as aluminium.

5. For current research works, the dimensions of out-of-plane multi-planar brace were kept identical to the dimensions of uni-planar brace. It was done for comparison purposes such as comparing the increase in P_c load of multi-planar SHS joints with respect that of uni-planar SHS joints. In future, out-of-plane multi-planar brace can be created with different dimensions to those from uni-planar brace and can be analysed to understand the structural behaviour of the joints.
6. Another broad area that can be explored by researchers relate to standardization of procedures for FE modelling. The existing codes of practises do not have clear guidelines in place to model various types of tubular joints. Guidelines can be made for areas such as the range of aspect ratio that can be used, minimum number of elements around pivotal regions such as crack tip area and so on. Currently, very few researchers work on cracked multi-planar tubular joints. So there is considerable scope of collaboration amongst various researchers throughout the world so that a set of guidelines can be developed as benchmark for modelling and analysis of both uncracked and cracked multi-planar tubular joints.

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Appendix A

Double-sided T-butt joint

Deepest point and axial loading

$$M_{k,ma} = f_1 \left(\frac{a}{T}, \frac{a}{c} \right) + f_2 \left(\frac{a}{T}, \theta \right) + f_3 \left(\frac{a}{T}, \theta, \frac{L}{T} \right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{a}{c} \right) = 0.060551 \left(\frac{a}{T} \right)^{A_1 + \left(A_2 \left(\frac{a}{T} \right) \right)^{A_3}} + 0.201338 \exp \left[\left(\frac{a}{T} \right)^{-0.152926} \right] + A_4$$

$$A_1 = -3.713269 \left(\frac{a}{c} \right)^2 + 4.815578 \left(\frac{a}{c} \right) - 0.628093$$

$$A_2 = 2.667204 \left(\frac{a}{c} \right)^{-0.059427}$$

$$A_3 = 0.028653 \left(\frac{a}{c} \right) + 3.587871$$

$$A_4 = -0.566177 \left(\frac{a}{c} \right)^3 + 1.007268 \left(\frac{a}{c} \right)^2 - 0.446218 \left(\frac{a}{c} \right) + 4.828197$$

$$f_2 \left(\frac{a}{T}, \theta \right) = A_5 \left[1 - \left(\frac{a}{T} \right)^{A_6} \right] + A_7 \left(\frac{a}{T} \right)^{0.172879 \left(\frac{a}{T} \right)}$$

$$A_5 = 0.126832(\theta)^2 + 0.760867(\theta) - 0.655510$$

$$A_6 = 0.240864(\theta) + 176.227945$$

$$A_7 = -0.478440(\theta) + 3.477915$$

$$f_3 \left(\frac{a}{T}, \theta, \frac{L}{T} \right) = A_8 \left(\frac{a}{T} \right)^{(A_9 \theta^2 + A_{10} \theta + A_{11})} + A_{12} \left(\frac{a}{T} \right)^{A_{13}} + A_{14} \left(\frac{a}{T} \right)^2 + A_{15} \left(\frac{a}{T} \right) + A_{16}$$

$$A_8 = 0.202513(\theta)^2 - 0.783236(\theta) + 1.378119$$

$$A_9 = 0.011564 \left(\frac{L}{T} \right)^3 - 0.050483 \left(\frac{L}{T} \right)^2 + 0.010365 \left(\frac{L}{T} \right) - 0.010172$$

$$A_{10} = 0.049639 \left(\frac{L}{T} \right)^3 - 0.157879 \left(\frac{L}{T} \right)^2 + 0.270526 \left(\frac{L}{T} \right) - 0.215981$$

$$A_{11} = 0.111072 \left(\frac{L}{T} \right)^2 - 0.280909 \left(\frac{L}{T} \right) + 0.119648$$

$$A_{12} = -0.159242(\theta)^2 + 1.125374(\theta) + 2.169380$$

$$A_{13} = 0.021376(\theta)^2 - 0.031318(\theta) + 0.087091$$

$$A_{14} = 0.083413 \left(\frac{L}{T}\right)^2 + 0.045886 \left(\frac{L}{T}\right) - 0.720037$$

$$A_{15} = -0.326816 \left(\frac{L}{T}\right)^2 + 0.372978 \left(\frac{L}{T}\right) + 0.424248$$

$$A_{16} = 0.261805 \left(\frac{L}{T}\right)^2 - 0.513702 \left(\frac{L}{T}\right) - 11.040943$$

Deepest point and bending loading

$$M_{k,ba} = f_1\left(\frac{a}{T}, \frac{a}{c}\right) + f_2\left(\frac{a}{T}, \theta\right) + f_3\left(\frac{a}{T}, \theta, \frac{L}{T}\right)$$

where

$$f_1\left(\frac{a}{T}, \frac{a}{c}\right) = 0.252514 \left(\frac{a}{T}\right)^{\left[A_1 + \left(A_2\left(\frac{a}{T}\right)\right)^{A_3}\right]} + 1.142999 \exp\left[\left(\frac{a}{T}\right)^{-0.218212}\right] + A_4$$

$$A_1 = -18.718517 \left(\frac{a}{c}\right)^2 + 11.763778 \left(\frac{a}{c}\right) - 1.379805$$

$$A_2 = 0.008308 \left(\frac{a}{c}\right)^{-6.096508}$$

$$A_3 = -2.966208 \left(\frac{a}{c}\right) - 3.294276$$

$$A_4 = -0.305848 \left(\frac{a}{c}\right)^3 + 0.007410 \left(\frac{a}{c}\right)^2 + 0.594054 \left(\frac{a}{c}\right) - 310.904315$$

$$f_2\left(\frac{a}{T}, \theta\right) = A_5 \left[1 - \left(\frac{a}{T}\right)^{A_6}\right] + A_7 \left(\frac{a}{T}\right)^{A_8}$$

$$A_5 = -1.242680(\theta)^2 + 2.774413(\theta) - 1.502169$$

$$A_6 = -13.842458(\theta)^2 + 14.629413(\theta) - 1.289560$$

$$A_7 = -3.709870(\theta)^2 + 5.635816(\theta) - 4.051851$$

$$A_8 = -8.154933 \left(\frac{a}{T}\right)^2 + 7.214484 \left(\frac{a}{T}\right) - 0.450525$$

$$f_3\left(\frac{a}{T}, \theta, \frac{L}{T}\right) = A_9 \left(\frac{a}{T}\right)^{(A_{10}\theta^2 + A_{11}\theta + A_{12})} + A_{13} \left(\frac{a}{T}\right)^{A_{14}} + A_{15} \left(\frac{a}{T}\right)^2 + A_{16} \left(\frac{a}{T}\right) + A_{17}$$

$$A_9 = 6.398997(\theta)^2 - 11.731149(\theta) + 4.388072$$

$$A_{10} = -1.269876 \left(\frac{L}{T}\right)^3 + 1.129879 \left(\frac{L}{T}\right)^2 - 3.056864 \left(\frac{L}{T}\right) - 3.632328$$

$$A_{11} = 0.434430 \left(\frac{L}{T}\right)^3 + 5.390495 \left(\frac{L}{T}\right)^2 - 0.789681 \left(\frac{L}{T}\right) + 3.753489$$

$$A_{12} = -2.574224 \left(\frac{L}{T}\right)^2 - 1.316737 \left(\frac{L}{T}\right) + 2.339313$$

$$A_{13} = -1.717647(\theta)^2 + 4.049350(\theta) - 2.357244$$

$$A_{14} = -1.396433(\theta)^2 - 2.019159(\theta) + 2.182500$$

$$A_{15} = -0.495607 \left(\frac{L}{T}\right)^2 + 1.661520 \left(\frac{L}{T}\right) - 0.546179$$

$$A_{16} = 0.800884 \left(\frac{L}{T}\right)^2 - 3.350862 \left(\frac{L}{T}\right) + 5.669418$$

$$A_{17} = -0.513969 \left(\frac{L}{T}\right)^2 + 2.605744 \left(\frac{L}{T}\right) + 306.001860$$

Crack ends and axial loading

$$M_{k,mc} = f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) f_2 \left(\frac{a}{T}, \frac{a}{c}, \theta\right) f_3 \left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) = A_1 \left(\frac{a}{T}\right)^{\left(A_2 \left(\frac{c}{a}\right)^2 + A_3 \left(\frac{c}{a}\right) + A_4\right)} + A_5 \left[1 - \left(\frac{a}{T}\right)\right]^{\left(A_6 \left(\frac{c}{a}\right)^2 + A_7 \left(\frac{c}{a}\right) + A_8\right)}$$

$$A_1 = 0.025765 \left(\frac{c}{a}\right)^2 + 0.782747 \left(\frac{c}{a}\right) + 3.771572$$

$$A_2 = -0.000118 \left(\frac{L}{T}\right)^2 + 0.000447 \left(\frac{L}{T}\right) + 0.000546$$

$$A_3 = 0.001721 \left(\frac{L}{T}\right)^2 - 0.006908 \left(\frac{L}{T}\right) - 0.010310$$

$$A_4 = 0.002552 \left(\frac{L}{T}\right)^2 - 0.009201 \left(\frac{L}{T}\right) - 0.002318$$

$$A_5 = -0.028648 \left(\frac{c}{a}\right)^2 - 0.726788 \left(\frac{c}{a}\right) - 3.742419$$

$$A_6 = 0.000058 \left(\frac{L}{T}\right)^2 + 0.000434 \left(\frac{L}{T}\right) - 0.001935$$

$$A_7 = -0.004599 \left(\frac{L}{T}\right)^2 + 0.011298 \left(\frac{L}{T}\right) + 0.028994$$

$$A_8 = -0.008625 \left(\frac{L}{T}\right)^3 + 0.040219 \left(\frac{L}{T}\right)^2 - 0.050949 \left(\frac{L}{T}\right) + 0.041654$$

$$f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) = \left[A_9 \left(\frac{a}{c}\right)^2 + A_{10} \left(\frac{a}{c}\right) + A_{11}\right] \left(\frac{a}{T}\right)^{A_{12}} + A_{13} \left[1 - \left(\frac{a}{T}\right)\right]^{A_{14}}$$

$$A_9 = 0.116234(\theta)^2 - 0.374263(\theta) - 1.242237$$

$$A_{10} = 1.337391(\theta)^2 + 0.510892(\theta) + 5.074312$$

$$A_{11} = 0.045274(\theta)^2 + 0.269000(\theta) - 0.060005$$

$$A_{12} = -0.172516 \left(\frac{a}{c}\right)^2 + 0.158428 \left(\frac{a}{c}\right) + 0.119272$$

$$A_{13} = -0.114599(\theta)^2 + 0.729533(\theta) - 0.226984$$

$$A_{14} = 37.422998 \left(\frac{a}{c}\right)^2 - 15.741006 \left(\frac{a}{c}\right) + 64.902988$$

$$f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right) = [A_{15}(\theta)^2 + A_{16}(\theta) + A_{17}] \left(\frac{a}{T}\right)^{A_{18}} + [A_{19}(\theta)^2 + A_{20}(\theta) + A_{21}] \exp\left(\frac{a}{T}\right)^{(A_{22}\theta^2 + A_{23}\theta + A_{24})}$$

$$A_{15} = -0.021667 \left(\frac{L}{T}\right)^2 + 0.288754 \left(\frac{L}{T}\right) - 0.538860$$

$$A_{16} = 0.170807 \left(\frac{L}{T}\right)^2 - 0.940051 \left(\frac{L}{T}\right) + 2.450643$$

$$A_{17} = -0.004130 \left(\frac{L}{T}\right)^3 - 0.311205 \left(\frac{L}{T}\right)^2 + 1.333145 \left(\frac{L}{T}\right) - 4.378479$$

$$A_{18} = 1.936338(\theta)^2 - 3.500136(\theta) + 2.549469$$

$$A_{19} = 0.009332 \left(\frac{L}{T}\right)^2 - 0.098144 \left(\frac{L}{T}\right) + 0.212162$$

$$A_{20} = -0.065609 \left(\frac{L}{T}\right)^2 + 0.331169 \left(\frac{L}{T}\right) - 0.968689$$

$$A_{21} = 0.016432 \left(\frac{L}{T}\right)^3 + 0.054013 \left(\frac{L}{T}\right)^2 - 0.420500 \left(\frac{L}{T}\right) + 1.720281$$

$$A_{22} = 0.643432 \left(\frac{a}{c}\right)^2 - 0.679865 \left(\frac{a}{c}\right) + 2.085410$$

$$A_{23} = -1.101930 \left(\frac{a}{c}\right)^2 + 1.208362 \left(\frac{a}{c}\right) - 3.738419$$

$$A_{24} = 0.571728 \left(\frac{a}{c}\right)^2 - 0.612597 \left(\frac{a}{c}\right) + 2.187432$$

Crack ends and bending loading

$$M_{k,bc} = f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T} \right) f_2 \left(\frac{a}{T}, \frac{a}{c} \right) f_3 \left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T} \right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T} \right) = A_1 \left(\frac{a}{T} \right)^{\left(A_2 \left(\frac{c}{a} \right)^2 + A_3 \left(\frac{c}{a} \right) + A_4 \right)} + A_5 \left[1 - \left(\frac{a}{T} \right) \right]^{\left(A_6 \left(\frac{c}{a} \right)^2 + A_7 \left(\frac{c}{a} \right) + A_8 \right)} + A_9$$

$$A_1 = -0.022318 \left(\frac{c}{a} \right)^2 + 0.282678 \left(\frac{c}{a} \right) + 6.992863$$

$$A_2 = 0.000270 \left(\frac{L}{T} \right)^2 - 0.000573 \left(\frac{L}{T} \right) - 0.001755$$

$$A_3 = -0.005380 \left(\frac{L}{T} \right)^2 + 0.012960 \left(\frac{L}{T} \right) + 0.014097$$

$$A_4 = 0.000309 \left(\frac{L}{T} \right)^2 - 0.004075 \left(\frac{L}{T} \right) - 0.102939$$

$$A_5 = -0.033629 \left(\frac{c}{a} \right)^2 + 0.483688 \left(\frac{c}{a} \right) - 8.719628$$

$$A_6 = 0.001219 \left(\frac{L}{T} \right)^2 - 0.002840 \left(\frac{L}{T} \right) + 0.011583$$

$$A_7 = -0.016670 \left(\frac{L}{T} \right)^2 + 0.043512 \left(\frac{L}{T} \right) - 0.161590$$

$$A_8 = 0.005082 \left(\frac{L}{T} \right)^3 + 0.059404 \left(\frac{L}{T} \right)^2 - 0.203734 \left(\frac{L}{T} \right) + 1.388345$$

$$A_9 = 0.721096 \left(\frac{a}{T} \right)^3 - 3.006464 \left(\frac{a}{T} \right)^2 - 4.985194 \left(\frac{a}{T} \right) + 0.062821$$

$$f_2 \left(\frac{a}{T}, \frac{a}{c} \right) = \left[0.798104 \left(\frac{a}{c} \right)^2 + 0.779694 \left(\frac{a}{c} \right) + 2.664297 \right] \left(\frac{a}{T} \right)^{A_{10}} + 0.011544 \left[1 - \left(\frac{a}{T} \right) \right]^{A_{11}}$$

$$A_{10} = 0.250560 \left(\frac{a}{c} \right)^2 - 0.413796 \left(\frac{a}{c} \right) + 0.339317$$

$$A_{11} = 0.940891 \left(\frac{a}{c} \right)^2 + 55.754444 \left(\frac{a}{c} \right) + 35.620650$$

$$f_3 \left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T} \right) = A_{12} \left(\frac{a}{T} \right)^{1.120475} + A_{13} \exp \left(\frac{a}{T} \right)^{A_{14}}$$

$$A_{12} = -0.004327 \left(\frac{L}{T} \right)^3 + 0.663760 \left(\frac{L}{T} \right)^2 - 2.643659 \left(\frac{L}{T} \right) + 0.259793$$

$$A_{13} = -0.101419 \left(\frac{L}{T}\right)^3 - 0.072900 \left(\frac{L}{T}\right)^2 + 1.583161 \left(\frac{L}{T}\right) + 0.019819$$

$$A_{14} = 0.856995 \left(\frac{a}{c}\right)^2 + 0.497396 \left(\frac{a}{c}\right) + 1.403672$$

Appendix B

X-joint

Deepest point and axial loading

$$M_{k,ma} = f_1 \left(\frac{a}{T}, \frac{a}{c} \right) + f_2 \left(\frac{a}{T}, \theta \right) + f_3 \left(\frac{a}{T}, \theta, \frac{L}{T} \right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{a}{c} \right) = -4.492610 \left(\frac{a}{T} \right)^{A_1 + \left(A_2 \left(\frac{a}{T} \right) \right)^{A_3}} + 0.773116 \exp \left[\left(\frac{a}{T} \right)^{0.087319} \right] + A_4$$

$$A_1 = 0.415296 \left(\frac{a}{c} \right)^2 - 0.148928 \left(\frac{a}{c} \right) + 0.989395$$

$$A_2 = 3.330545 \left(\frac{a}{c} \right)^{-0.147441}$$

$$A_3 = 0.444042 \left(\frac{a}{c} \right) + 1.896029$$

$$A_4 = 0.063350 \left(\frac{a}{c} \right)^3 - 0.324714 \left(\frac{a}{c} \right)^2 + 0.462540 \left(\frac{a}{c} \right) + 3.632629$$

$$f_2 \left(\frac{a}{T}, \theta \right) = A_5 \left[1 - \left(\frac{a}{T} \right)^{A_6} \right] + A_7 \left(\frac{a}{T} \right)^{1.113959 \left(\frac{a}{T} \right)}$$

$$A_5 = -0.087920(\theta)^2 + 0.572374(\theta) - 0.791248$$

$$A_6 = 0.241428(\theta) + 176.229552$$

$$A_7 = 0.108098(\theta) + 3.725823$$

$$f_3 \left(\frac{a}{T}, \theta, \frac{L}{T} \right) = A_8 \left(\frac{a}{T} \right)^{A_9 \theta^2 + A_{10} \theta + A_{11}} + A_{12} \left(\frac{a}{T} \right)^{A_{13}} + A_{14} \left(\frac{a}{T} \right)^2 + A_{15} \left(\frac{a}{T} \right) + A_{16}$$

$$A_8 = -0.128622(\theta)^2 - 0.198147(\theta) + 3.973462$$

$$A_9 = 0.025010 \left(\frac{L}{T} \right)^3 - 0.102770 \left(\frac{L}{T} \right)^2 + 0.114308 \left(\frac{L}{T} \right) + 0.012888$$

$$A_{10} = -0.025717 \left(\frac{L}{T} \right)^3 + 0.099637 \left(\frac{L}{T} \right)^2 - 0.102692 \left(\frac{L}{T} \right) - 0.144130$$

$$A_{11} = 0.031744 \left(\frac{L}{T} \right)^2 - 0.150212 \left(\frac{L}{T} \right) + 0.208836$$

$$A_{12} = 0.265321(\theta)^2 - 0.176888(\theta) + 2.742100$$

$$A_{13} = -0.044780(\theta)^2 + 0.196766(\theta) - 0.045390$$

$$A_{14} = -0.378266 \left(\frac{L}{T}\right)^2 + 1.000362 \left(\frac{L}{T}\right) - 0.798648$$

$$A_{15} = 0.255737 \left(\frac{L}{T}\right)^2 - 0.232654 \left(\frac{L}{T}\right) - 1.726004$$

$$A_{16} = 0.066961 \left(\frac{L}{T}\right)^2 - 0.656739 \left(\frac{L}{T}\right) - 12.236472$$

Deepest point and bending loading

$$M_{k,ba} = f_1 \left(\frac{a}{T}, \frac{a}{c}\right) + f_2 \left(\frac{a}{T}, \theta\right) + f_3 \left(\frac{a}{T}, \theta, \frac{L}{T}\right)$$

Where

$$f_1 \left(\frac{a}{T}, \frac{a}{c}\right) = 6.988220 \left(\frac{a}{T}\right)^{A_1 + \left(A_2 \left(\frac{a}{T}\right)\right)^{A_3}} + 0.000005 \exp \left[\left(\frac{a}{T}\right)^{-1.445040}\right] + A_4$$

$$A_1 = -2.032183 \left(\frac{a}{c}\right)^2 + 4.704754 \left(\frac{a}{c}\right) - 0.838850$$

$$A_2 = 0.811466 \left(\frac{a}{c}\right)^{-0.076785}$$

$$A_3 = 2.413184 \left(\frac{a}{c}\right) - 0.078940$$

$$A_4 = -7.499430 \left(\frac{a}{c}\right)^3 + 2.148718 \left(\frac{a}{c}\right)^2 + 7.416217 \left(\frac{a}{c}\right) - 67.180492$$

$$f_2 \left(\frac{a}{T}, \theta\right) = A_5 \left[1 - \left(\frac{a}{T}\right)^{A_6}\right] + A_7 \left(\frac{a}{T}\right)^{A_8}$$

$$A_5 = 0.268663(\theta)^2 - 0.619803(\theta) + 0.350971$$

$$A_6 = -16.721638(\theta)^2 + 15.981321(\theta) - 0.750452$$

$$A_7 = 1.311017(\theta)^2 - 1.725607(\theta) + 1.583463$$

$$A_8 = -16.2736 \left(\frac{a}{T}\right)^2 + 14.072986 \left(\frac{a}{T}\right) + 2.894084$$

$$f_3 \left(\frac{a}{T}, \theta, \frac{L}{T}\right) = A_9 \left(\frac{a}{T}\right)^{(A_{10}\theta^2 + A_{11}\theta + A_{12})} + A_{13} \left(\frac{a}{T}\right)^{A_{14}} + A_{15} \left(\frac{a}{T}\right)^2 + A_{16} \left(\frac{a}{T}\right) + A_{17}$$

$$A_9 = 4.809980(\theta)^2 - 6.646564(\theta) + 1.995096$$

$$A_{10} = -8.136387 \left(\frac{L}{T}\right)^3 - 6.806707 \left(\frac{L}{T}\right)^2 + 1.633523 \left(\frac{L}{T}\right) + 3.589077$$

$$A_{11} = 10.500346 \left(\frac{L}{T}\right)^3 + 4.395037 \left(\frac{L}{T}\right)^2 + 4.051203 \left(\frac{L}{T}\right) + 6.102024$$

$$A_{12} = -4.044141 \left(\frac{L}{T}\right)^2 - 4.610411 \left(\frac{L}{T}\right) - 2.665313$$

$$A_{13} = 0.006774(\theta)^2 - 0.449076(\theta) - 1.721500$$

$$A_{14} = 1.188501(\theta)^2 - 2.494186(\theta) + 2.454850$$

$$A_{15} = -0.550491 \left(\frac{L}{T}\right)^2 + 1.484763 \left(\frac{L}{T}\right) - 5.043941$$

$$A_{16} = 0.706771 \left(\frac{L}{T}\right)^2 - 2.929346 \left(\frac{L}{T}\right) + 2.372367$$

$$A_{17} = -0.530458 \left(\frac{L}{T}\right)^2 + 2.788328 \left(\frac{L}{T}\right) + 63.027519$$

Crack ends and axial loading

$$M_{k,mc} = f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) f_2 \left(\frac{a}{T}, \frac{a}{c}, \theta\right) f_3 \left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) = A_1 \left(\frac{a}{T}\right)^{\left(A_2 \left(\frac{c}{a}\right)^2 + A_3 \left(\frac{c}{a}\right) + A_4\right)} + A_5 \left[1 - \left(\frac{a}{T}\right)\right]^{\left(A_6 \left(\frac{c}{a}\right)^2 + A_7 \left(\frac{c}{a}\right) + A_8\right)}$$

$$A_1 = 0.009488 \left(\frac{c}{a}\right)^2 - 0.032470 \left(\frac{c}{a}\right) + 1.559611$$

$$A_2 = -0.000655 \left(\frac{L}{T}\right)^2 + 0.003219 \left(\frac{L}{T}\right) + 0.000537$$

$$A_3 = 0.005829 \left(\frac{L}{T}\right)^2 - 0.029824 \left(\frac{L}{T}\right) + 0.012945$$

$$A_4 = 0.004030 \left(\frac{L}{T}\right)^2 - 0.037447 \left(\frac{L}{T}\right) - 0.245743$$

$$A_5 = 0.003083 \left(\frac{c}{a}\right)^2 - 0.000877 \left(\frac{c}{a}\right) - 1.314918$$

$$A_6 = -0.003947 \left(\frac{L}{T}\right)^2 + 0.021370 \left(\frac{L}{T}\right) - 0.034986$$

$$A_7 = 0.016642 \left(\frac{L}{T}\right)^2 - 0.074120 \left(\frac{L}{T}\right) + 0.195741$$

$$A_8 = -0.048064 \left(\frac{L}{T}\right)^3 + 0.296542 \left(\frac{L}{T}\right)^2 - 0.671992 \left(\frac{L}{T}\right) + 1.618320$$

$$f_2\left(\frac{a}{T}, \frac{a}{c}, \theta\right) = \left[A_9 \left(\frac{a}{c}\right)^2 + A_{10} \left(\frac{a}{c}\right) + A_{11}\right] \left(\frac{a}{T}\right)^{A_{12}} + A_{13} \left[1 - \left(\frac{a}{T}\right)\right]^{A_{14}}$$

$$A_9 = 0.145204(\theta)^2 - 0.069587(\theta) - 0.475872$$

$$A_{10} = -0.178222(\theta)^2 - 0.047556(\theta) + 0.360921$$

$$A_{11} = -0.780757(\theta)^2 + 1.892267(\theta) + 0.273826$$

$$A_{12} = -0.310267 \left(\frac{a}{c}\right)^2 + 0.414930 \left(\frac{a}{c}\right) - 0.000607$$

$$A_{13} = 0.048005(\theta)^2 + 0.744457(\theta) - 0.176785$$

$$A_{14} = 51.751928 \left(\frac{a}{c}\right)^2 - 24.351576 \left(\frac{a}{c}\right) + 66.134014$$

$$f_3\left(\frac{a}{T}, \frac{a}{c}, \theta, \frac{L}{T}\right) = [A_{15}(\theta)^2 + A_{16}(\theta) + A_{17}] \left(\frac{a}{T}\right)^{A_{18}} + [A_{19}(\theta)^2 + A_{20}(\theta) + A_{21}] \exp\left(\frac{a}{T}\right)^{(A_{22}\theta^2 + A_{23}\theta + A_{24})}$$

$$A_{15} = -0.046420 \left(\frac{L}{T}\right)^2 + 0.813271 \left(\frac{L}{T}\right) - 0.219367$$

$$A_{16} = 0.048053 \left(\frac{L}{T}\right)^2 - 1.357723 \left(\frac{L}{T}\right) + 1.134210$$

$$A_{17} = -0.133030 \left(\frac{L}{T}\right)^3 + 0.666636 \left(\frac{L}{T}\right)^2 - 0.318879 \left(\frac{L}{T}\right) - 2.241793$$

$$A_{18} = 1.834710(\theta)^2 - 4.457370(\theta) + 3.103834$$

$$A_{19} = 0.035895 \left(\frac{L}{T}\right)^2 - 0.327775 \left(\frac{L}{T}\right) + 0.205291$$

$$A_{20} = -0.049037 \left(\frac{L}{T}\right)^2 + 0.545699 \left(\frac{L}{T}\right) - 0.711685$$

$$A_{21} = 0.049655 \left(\frac{L}{T}\right)^3 - 0.237000 \left(\frac{L}{T}\right)^2 + 0.106694 \left(\frac{L}{T}\right) + 1.126079$$

$$A_{22} = -0.240972 \left(\frac{a}{c}\right)^2 - 0.089663 \left(\frac{a}{c}\right) + 1.597577$$

$$A_{23} = -0.059767 \left(\frac{a}{c}\right)^2 + 0.423512 \left(\frac{a}{c}\right) - 3.986922$$

$$A_{24} = -0.191299 \left(\frac{a}{c}\right)^2 + 0.013854 \left(\frac{a}{c}\right) + 2.601067$$

Crack ends and bending loading

$$M_{k,bc} = f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) f_2 \left(\frac{a}{T}, \frac{a}{c}\right) f_3 \left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right)$$

where

$$f_1 \left(\frac{a}{T}, \frac{c}{a}, \frac{L}{T}\right) = A_1 \left(\frac{a}{T}\right)^{\left(A_2 \left(\frac{c}{a}\right)^2 + A_3 \left(\frac{c}{a}\right) + A_4\right)} + A_5 \left[1 - \left(\frac{a}{T}\right)\right]^{\left(A_6 \left(\frac{c}{a}\right)^2 + A_7 \left(\frac{c}{a}\right) + A_8\right)} + A_9$$

$$A_1 = -0.007850 \left(\frac{c}{a}\right)^2 + 0.098754 \left(\frac{c}{a}\right) + 4.978007$$

$$A_2 = 0.000158 \left(\frac{L}{T}\right)^2 - 0.000366 \left(\frac{L}{T}\right) - 0.001211$$

$$A_3 = -0.000924 \left(\frac{L}{T}\right)^2 + 0.002730 \left(\frac{L}{T}\right) + 0.016316$$

$$A_4 = 0.003248 \left(\frac{L}{T}\right)^2 - 0.009972 \left(\frac{L}{T}\right) - 0.096668$$

$$A_5 = -0.012128 \left(\frac{c}{a}\right)^2 + 0.146017 \left(\frac{c}{a}\right) - 6.320828$$

$$A_6 = 0.000541 \left(\frac{L}{T}\right)^2 - 0.001556 \left(\frac{L}{T}\right) + 0.005504$$

$$A_7 = -0.002041 \left(\frac{L}{T}\right)^2 + 0.004307 \left(\frac{L}{T}\right) - 0.054882$$

$$A_8 = 0.003671 \left(\frac{L}{T}\right)^3 + 0.041239 \left(\frac{L}{T}\right)^2 - 0.157534 \left(\frac{L}{T}\right) + 1.174081$$

$$A_9 = -0.719315 \left(\frac{a}{T}\right)^3 - 1.055583 \left(\frac{a}{T}\right)^2 - 3.156014 \left(\frac{a}{T}\right) - 0.161323$$

$$f_2 \left(\frac{a}{T}, \frac{a}{c}\right) = \left[1.633304 \left(\frac{a}{c}\right)^2 + 2.763987 \left(\frac{a}{c}\right) + 4.273512\right] \left(\frac{a}{T}\right)^{A_{10}} - 14.120012 \left[1 - \left(\frac{a}{T}\right)\right]^{A_{11}}$$

$$A_{10} = -0.116568 \left(\frac{a}{c}\right)^2 + 0.503130 \left(\frac{a}{c}\right) - 1.260482$$

$$A_{11} = 5.017833 \left(\frac{a}{c}\right)^2 + 55.811596 \left(\frac{a}{c}\right) + 48.126172$$

$$f_3\left(\frac{a}{T}, \frac{a}{c}, \frac{L}{T}\right) = A_{12} \left(\frac{a}{T}\right)^{0.808994} + A_{13} \exp\left(\frac{a}{T}\right)^{A_{14}}$$

$$A_{12} = 1.484184 \left(\frac{L}{T}\right)^3 - 4.154186 \left(\frac{L}{T}\right)^2 - 0.65077 \left(\frac{L}{T}\right) + 0.179447$$

$$A_{13} = -0.749532 \left(\frac{L}{T}\right)^3 + 1.990338 \left(\frac{L}{T}\right)^2 + 0.76720 \left(\frac{L}{T}\right) - 0.142959$$

$$A_{14} = 0.116865 \left(\frac{a}{c}\right)^2 + 0.474747 \left(\frac{a}{c}\right) + 1.653676$$