Design of wireless power transfer coils to minimise capacitor stress and AC resistance

Tanzania, Robin

2016

Tanzania, R. (2016). Design of wireless power transfer coils to minimise capacitor stress and AC resistance. Master's thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/66032>

<https://doi.org/10.32657/10356/66032>

NANYANG
TECHNOLOGICAL
UNIVERSITY

**DESIGN OF WIRELESS POWER TRANSFER
COILS TO MINIMISE CAPACITOR STRESS AND
AC RESISTANCE**

ROBIN TANZANIA
**SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING**
2016

**DESIGN OF WIRELESS POWER TRANSFER
COILS TO MINIMISE CAPACITOR STRESS AND
AC RESISTANCE**

ROBIN TANZANIA

**SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING**

A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
Master of Engineering

2016

Acknowledgements

I would like to thank my supervisor Assoc. Prof. Siek Liter for his guidance and support. He has always given me the motivation and encouragement to keep advancing in my research.

I would like to express my gratitude to my co-supervisor Choo Fook Hoong, Dr. Foo Chek Fook, and Dr. Tan Yen Kheng for their invaluable ideas, guidance and encouragement throughout my research.

Deep thanks to Aaron Pereira, the team-mate who built the high-frequency and high-power inverter, without which the experimental validations would not have been possible. Also thanks to Yap Henggoh, who played an instrumental role in the early stages of the project. Also thanks to friends and fellow researchers who have directly or indirectly contributed to this wireless charging project, in ideas, knowledge and encouragement: Dr. Arie Nawawi, Dr. Tong Chinfoong, Bai Xiaoyin, Aaron Cai Qingwei, Andrew Ong Chuanen, Jayathurathnage Prasad Kumara Sampath, Rudy Tjandra and many others.

I feel very grateful to my family and friends for their love and support. Especially to my girlfriend, Lan Lan, who has seen me through my most difficult moments.

This project is supported by Energy Research Institute @NTU (ERI@N), and had previously received support from TUM-CREATE and WIPO. I would like to acknowledge the support from all parties.

Table of Contents

Acknowledgements.....	i
Summary.....	v
List of Figures.....	vi
List of Tables.....	viii
List of Abbreviations.....	ix
List of Symbols.....	x
Chapter 1 Introduction.....	1
1.1 Background.....	1
1.2 Objective.....	2
1.3 Scope.....	4
1.4 Organisation of the report.....	5
Chapter 2 Existing Coil Designs.....	6
2.1 Korea Advanced Institute of Science and Technology.....	8
2.2 University of Auckland.....	10
2.3 Saitama University.....	11
2.4 University of Zaragoza.....	12
2.5 Oak Ridge National Laboratory.....	13
2.6 Swiss Federal Institute of Technology in Zurich.....	13
2.7 Conductix-Wampfler, Bombardier, Qualcomm and Plugless Power.....	14
2.8 Conclusion.....	16
Chapter 3 Minimum Stress Condition.....	17
3.1 The four WPT topologies.....	17
3.2 Electrical parameters of circuit components.....	20
3.3 Equal stress condition.....	21
3.4 Simulation.....	26
Chapter 4 Coil Resistance.....	29
4.1 Geometry for maximum coupling.....	30
4.2 Estimation of the number of turns.....	32
4.3 Geometry for minimum wire length.....	33
4.4 AC resistance of Litz wire in a winding.....	35
4.5 Conclusion.....	38
Chapter 5 Experimental Validation.....	41
5.1 Initial design.....	41
5.2 AC resistance from power dissipation.....	42
5.3 AC resistance of capacitors.....	43

5.4 Results.....	44
5.5 A 10 kW test.....	46
Chapter 6 Displacement Sensitivity.....	50
6.1 Measurements and surface-fitting.....	51
6.2 Constant input voltage.....	54
6.3 Constant input current.....	58
6.4 Constant output voltage.....	62
6.5 Effect of angular displacement.....	67
6.6 Conclusion.....	68
Chapter 7 Conclusion and Future Work.....	70
7.1 Conclusion.....	70
7.2 Future Work.....	71
Publications.....	73
Bibliography.....	74
Appendices.....	78
Appendix A: MATLAB Code for Observing Capacitor Stress Behaviour.....	78
Appendix B: MATLAB Code for Calculating Power Dissipation by Natural Convection..	80
Appendix C: MATLAB Code for Surface Fitting.....	81
Appendix D: MATLAB Code for Solving Circuit Equations.....	83
Appendix E: MATLAB Code for Calculating Parameter Change due to Displacement.....	84

Summary

A Wireless Power Transfer system for charging electric vehicles must have high efficiency to compete with wired charging. Although efficiency can be increased using a number of ways, in the lumped circuit model the only loss comes from parasitic resistances of the system. In a WPT system, AC resistance of power transfer coils is the most dominant source of power loss and it increases considerably with frequency. Therefore AC resistance of coils is the most important factor of efficiency. Additionally, most WPT systems work using resonant coupling. Due to resonance, reactive voltage in a series resonant tank and current in a parallel resonant tank can be several times the input and output voltages and currents, depending on the topology used. This work presents a design procedure that attempts to address those two problems in the design of a WPT system. First is to minimise capacitor stress by quantifying the voltage and current stresses in relation to the operating conditions of the WPT system and second to minimise the absolute resistance of a WPT coil. This design procedure results in the lightest coil designs that can be chosen from commercially available Litz wires. Experimental validation has achieved 10 kW wireless power transfer using 0.6 kg coils with 18 cm air-gap, 98.6 % coil-to-coil efficiency and 94.8 % DC-to-DC efficiency, with the measured values agreeing well with the design calculations. Sensitivity analysis has also been done to explore the effects of coupling factor in the system.

List of Figures

Figure 1: An illustration of an EV wireless charging system.....	2
Figure 2: Stresses in resonant circuits.....	3
Figure 3: Power loss due to a current passing through a resistance.....	4
Figure 4: The first electrical resonance curves published by Hertz in 1887, showing the greatest length of spark obtainable in his detecting loop for various lengths of wire in the loop [11].....	7
Figure 5: Resonant transformer.....	8
Figure 6: Diagram of one of Tesla's wireless power experiments [12].....	8
Figure 7: A summary on the developments of WPT for OLEV [13].....	9
Figure 8: (a) power supply module and (b) pick up module used for 500 kW WPT [14].....	9
Figure 9: Magnetic coupler designs: (a) Circular, (b) DD, (c) DDQ and (d) BP [2].....	10
Figure 10: (a) 3 kW transformer, (b) 10 kW transformer and (c) core without winding showing cooling structure [16].....	11
Figure 11: 2 kW rectangular coils: (a) SS topology and (b) SPS topology [17].....	12
Figure 12: Distributed winding: (a) 7 turns for transmit coil, (a) 5 turns for receive coil [18]..	13
Figure 13: 5 kW system: (a) inverter with SiC MOSFETs, (b) coil, core and carrier [19].....	14
Figure 14: The commercial and development projects for RPEV by Bombardier [13].....	15
Figure 15: Plugless Power inductive charging system components: (a) control panel, (b) ground pad, and (c) vehicle adapter [23].....	16
Figure 16: Circuit diagrams of the four WPT topologies.....	18
Figure 17: Capacitor stress behaviour of the four WPT topologies.....	23
Figure 18: Simulation scenario.....	26
Figure 19: Simulation to verify minimum stress condition.....	28
Figure 20: A coil with circular spiral structure.....	30
Figure 21: Coupling factor k as a function of spacing for various coil configurations for identical primary and secondary coils [25].....	31
Figure 22: On-chip inductor realisations: (a) square, (b) hexagonal (c) octagonal, and (d)	

circular [26].....	33
Figure 23: Coefficients for current sheet expression [26].....	33
Figure 24: Relationship between the wire length and the average radius.....	35
Figure 25: Litz wire construction types [28].....	36
Figure 26: Error between the DC resistance and the exact resistance as a function of strand diameter and skin depth ratio [30].....	37
Figure 27: Numeric values of η for coils with different cross sections. b and t are the coil's width and thickness, as shown in Figure 20 [8].....	38
Figure 28: to find AC resistance.....	39
Figure 29: Spreadsheet used for designing SS-topology coils used in the experiment.....	40
Figure 30: Primary and secondary coils used for testing.....	44
Figure 31: Input and output voltages, currents and power of the 1.7 kW test.....	45
Figure 32: 60 parallel MLCCs in series with another 60 parallel MLCCs.....	47
Figure 33: Input and output voltages, currents and power of the 10 kW test.....	48
Figure 34: Coordinate system for measurement of coupling and positioning.....	51
Figure 35: Coupling map with estimated surface and measured data.....	53
Figure 36: Parameter change due to horizontal displacement with constant input voltage.....	55
Figure 37: Parameter change due to vertical displacement with constant input voltage.....	57
Figure 38: Parameter change due to horizontal displacement with constant input current.....	60
Figure 39: Parameter change due to vertical displacement with constant input current.....	62
Figure 40: Parameter change due to horizontal displacement with constant output voltage.....	64
Figure 41: Parameter change due to vertical displacement with constant output voltage.....	66
Figure 42: Angle due to ride height difference.....	67
Figure 43: Coordinate system for measurement of coupling and angle.....	67
Figure 44: Coupling comparison for the two angles.....	68

List of Tables

Table 1: Electrical parameters of circuit components.....	20
Table 2: Capacitor stresses for Figure 17.....	24
Table 3: Parameter list for minimum stress condition.....	25
Table 4: Capacitor stress comparison between calculations and simulation.....	28
Table 5: Initial design results for 38AWG-strand wires.....	41
Table 6: AC resistance of coils with 38AWG strands.....	42
Table 7: Comparison between design and experimental setup.....	45
Table 8: AC results comparison for 10 kW test.....	48
Table 9: DC results comparison for 10 kW test.....	49

List of Abbreviations

AC	Alternating Current
AWG	American Wire Gauge
BP	Bipolar
DC	Direct Current
DD	Double-D
DDQ	Double-D Quadrature
ETH	Eidgenössische Technische Hochschule
ESC	Equal Stress Condition
EV	Electric Vehicle
FE	Finite Element
FHA	First Harmonics Approximation
KAIST	Korea Advanced Institute of Science and Technology
LMD	Linear Mass Density
MLCC	Multi-Layer Ceramic Capacitor
OLEV	On-Line Electric Vehicles
ORNL	Oak Ridge National Laboratory
PP	Parallel-Parallel
PS	Parallel-Series
Q-factor	Quality factor
SP	Series-Parallel
SPS	Series-Parallel-Series
SS	Series-Series
VAC	Voltage Alternating Current
VDC	Voltage Direct Current
Vrms	Voltage root mean square
w.AWG	Wire AWG
WPT	Wireless Power Transfer

List of Symbols

a	Input-output voltage ratio
a_{max}	Outer radius of coil
a_{min}	Inner radius of coil
A	Convection area
A_w	Conduction area of wire
b	Width of coil
c_1	Coefficient for current sheet expression
c_2	Coefficient for current sheet expression
c_3	Coefficient for current sheet expression
c_4	Coefficient for current sheet expression
C	Capacitance
C_1	Primary capacitor
C_2	Secondary capacitor
d_{in}	Inner diameter of coil
d_{out}	Outer diameter of coil
D_s	Strand diameter
D_w	Wire diameter
e_{1n}	Coefficient for exponential function
e_{2n}	Coefficient for exponential function
e_{3n}	Coefficient for exponential function
f	Frequency
f_0	Resonant frequency
f_f	Simplified circular spiral coil factor
F	Fill factor
I	Current
I_1	Source current
I_2	Load current
I_C	Capacitor current
I_{C1}	Primary capacitor current
I_{C2}	Secondary capacitor current

I_{dc1}	DC source current
I_{dc2}	DC load current
I_L	Coil current
I_{L1}	Primary coil current
I_{L2}	Secondary coil current
k	Coupling factor
l	Wire length in a coil
L	Inductance
L_1	Primary coil
L_2	Secondary coil
M	Mutual inductance
n	Coefficient numbering
N_t	Number of turns
N_s	Number of strands in the wire
NB	Number of bunching operations
NC	Number of cabling operations
p	Convection perimeter
p_1	Coefficient for polynomial function
p_2	Coefficient for polynomial function
p_3	Coefficient for polynomial function
P_{dc1}	DC source power
P_{dc2}	DC load power
q	Power dissipated
r	Average radius of coil
r_{in}	Inner radius of coil
r_{out}	Outer radius of coil
r_r	Ratio of average radius to outer radius
R	Resistance
R_1	Primary parasitic resistance
R_2	Secondary parasitic resistance
R_{ac}	AC resistance of coil
R_{cap}	AC resistance of capacitor
R_{C1}	Primary capacitor resistance
R_{C2}	Secondary capacitor resistance
R_{dc}	DC resistance of coil

R_L	Load resistance
R_{L1}	Primary coil resistance
R_{L2}	Secondary coil resistance
R_{Ldc}	DC load resistance
t	Thickness of coil
T_a	Ambient temperature
T_p	Coil temperature
V	Voltage
V_1	Source voltage
V_2	Load voltage
V_C	Capacitor voltage
V_{C1}	Primary capacitor voltage
V_{C2}	Secondary capacitor voltage
V_{dc1}	DC source voltage
V_{dc2}	DC load voltage
V_L	Coil voltage
V_{L1}	Primary coil voltage
V_{L2}	Secondary coil voltage
x	Horizontal displacement between primary and secondary coil
y	Vertical displacement between primary and secondary coil
Z_{eq}	Equivalent impedance seen by source
β	Conduction area to coil area ratio
δ	Skin depth
η	Value corresponding to width-thickness ratio of coil
θ	Angle between primary and secondary coil
μ	Permeability of wire
ρ	Resistivity of wire
σ	Conductivity of wire
ω	Angular frequency
ω_0	Resonant angular frequency

Chapter 1

Introduction

1.1 Background

One of the challenges in using electric vehicles (EVs) is refuelling. A conventional petrol-fuelled passenger car, for example, would only need a few minutes to pump into the car a tank full of petrol, which would enable the car to run for a few hundred kilometres. In contrast, a battery-powered car would need to charge for several hours, usually overnight, and yet the range would still be less than a petrol-fuelled car [1].

Wireless Power Transfer (WPT) technology can be used to charge EVs without the use of wires, which has a number of advantages as opposed to wired charging. The charging process can be automated more easily as there is no wire to move and no connection to be made, especially together with the rapid advancement in driverless vehicles occurring currently. It could be safer as users do not handle high power cables. It could be less prone to wear as there is no connector to plug and unplug. It could also be less prone to damage from weather conditions and vandalism due to all components being embedded/concealed. These characteristics means wireless charging could be more robust and easier to deploy in dirty and wet environments, compared to wired charging.

Due to these advantages, WPT also offers the possibility to charge an EV on the road, whether stopped or moving, by embedding the charging transmitter under the road surface. A long track or a series of coils maybe used for charging moving vehicles. In comparison, charging vehicles that are briefly stopped requires less material and space. On the road charging will in turn allow electric vehicles to carry a smaller battery or extend their driving range [2].

A WPT system for charging EVs usually employs the induction of electricity by a changing magnetic field. In a typical setup, a transmitter coil is placed on/in the ground, powered by a high frequency inverter to generate an alternating magnetic

field. A receiver coil, mounted on the bottom of the vehicle, is then positioned above the transmitter coil such that the alternating magnetic field induces electricity in the receiver coil. Power is then transferred via a rectifier to the battery charger to charge the battery of the vehicle. This is illustrated in Figure 1.

Figure 1: An illustration of an EV wireless charging system

1.2 Objective

Firstly, the specific method that is used for most WPT systems is based on resonant inductive coupling [3]. But currently, analysis of capacitor stresses in a resonant WPT system is largely absent in the literature.

Consider a series LC tank as shown in Figure 2(a) and a parallel LC tank in Figure

2(b). When these circuits are operated at their LC resonant frequency, the reactive voltage in the series resonant tank will be much higher than the source voltage, which should be close to zero as the source sees a short circuit at resonant. Similarly, the reactive current in the parallel resonant tank will be much higher than the source current, which should be close to zero as the source sees an open circuit at resonant.

Figure 2: Stresses in resonant circuits

Due to resonance, reactive voltage in a series resonant tank and current in a parallel resonant tank can be several times the input and output voltages and currents. For example, in [4], a resonant current of 60 A is needed to deliver 10 A to a 300 W load. Capacitor voltage stress has also been identified as a limiting factor [3]. [5] has used split series capacitor as a measure to mitigate the effect of high voltage. Components and isolation that can withstand the stresses are not easy to obtain and will add cost, so minimizing the voltage and current stresses is an important part of a practical design. Furthermore, large values of voltage and current will be a concern in terms of safety for consumer products.

Secondly, a WPT system for charging electric vehicles must have high efficiency to compete with wired charging. Some efforts to increase efficiency in a WPT design can be found in the literature [2][6][7][8]. [6] has proposed a design procedure to minimise copper mass, but has ignored AC resistances. [7] and [8] took AC resistance into account, but has proposed a coil design procedure that maximises Q-factor, instead of directly minimising the parasitic resistance. The need for higher Q-factor coils to increase efficiency has also been suggested [2][5] [9], but as with all other efforts that analysed efficiency in terms of Q-factor,

loading, inductance and frequency are considered, while AC resistance is neglected.

As WPT operates in near-field, lumped circuit model is accurate. In the lumped circuit model, there is no energy loss, i.e. power from source are completely sunk into the resistive elements in the circuit. On the other hand, if the circuit operates using far-field EM radiation, some energy will be lost to space and thus the circuit diagram will not fully represent the physical system.

Thus in near-field operation, the only power loss is from parasitic resistance. In this case, radiated power in the far-field can be found from the H-field equation of a loop antenna and can be shown to be negligible. Thus minimizing the absolute resistance of a coil will have the greatest impact on efficiency.

Figure 3: Power loss due to a current passing through a resistance

This report presents a design procedure that attempts to address those two problems in the design of a WPT system, first is to minimise the capacitor stress by quantifying the voltage and current stresses in relation to the operating conditions of the WPT system and second to minimise the absolute resistance of a WPT coil.

1.3 Scope

The study is based on the application of the near-field resonant inductive coupling technique for high power EV charging. For the winding, the conductor material used is Litz wire, as it is the most common for this application [2].

The dimensions under consideration in this study are approximated to be quite realistic for this application: coil diameter of 60 cm and air gap of 15 cm. And the power is in the range of kilowatts. Lastly, the circuits analysed are those commonly used for this application: the 2-coil system using the four basic resonant circuit

topologies, i.e. Series-Series (SS), Series-Parallel (SP), Parallel-Series (PS) and Parallel-Parallel (PP).

1.4 Organisation of the report

The report is organised as follows.

Chapter 1 introduces the concept of wireless charging for electric vehicle and the gap in the literature and the focus of the study.

Chapter 2 presents the review of existing coil designs to note their research contributions and gaps.

Chapter 3 presents the derivation of the circuit parameters of each topology and the minimum stress condition.

Chapter 4 presents the relationships between the geometry, coupling and inductance of a pair of coils, which are used with the minimum stress condition to derive AC resistance of coil.

Chapter 5 presents an experiment that has been conducted for SS topology to verify the design calculations.

Chapter 6 presents some analysis to assess the change in the circuit parameters due to the change in coupling.

Chapter 7 presents the conclusions and future work.

Chapter 2

Existing Coil Designs

There are already a wealth of publications in the field of Wireless Power Transfer [10]. The study on this topic can be traced back to the beginning of electromagnetism where the ability to transfer power using an oscillating magnetic field was simultaneously observed along with the phenomenon of electrical resonance [11].

In 1826, electrical resonance was first observed by Felix Savary, when he saw the oscillating magnetization of a steel needle by the current produced from discharging a Leyden jar, an early capacitor. 15 years later, Joseph Henry observed the same oscillatory behaviour when he used the current from a Leyden jar discharge to induce a current in another conductor. In 1868, Clerk Maxwell established the mathematical expression for resonance in an LCR circuit. By 1887, Heinrich Hertz had demonstrated the existence electromagnetic waves in space, using a loop of wire that was in resonance with a high frequency oscillator. In his experiments, when a resonating circuit induced a current in another circuit with the same resonant frequency, the inductive effect was much stronger than in a circuit with a different resonant frequency. He described it as "the symphonic relations between the mutually reacting circuits". The secondary circuit had an adjustable width spark gap and its resonant frequency was tuned by changing the length of the wire in the loop. The length of the wire was then graphed against the greatest length of spark obtainable in the secondary circuit, shown in Figure 4. This was the first published resonance curve. It effectively showed that the greatest energy transfer happened when the resonant frequency of the secondary circuit matched that of the primary. [11]

Figure 4: The first electrical resonance curves published by Hertz in 1887, showing the greatest length of spark obtainable in his detecting loop for various lengths of wire in the loop [11]

Nikola Tesla in particular has been regarded as the pioneer who brought forward modern ideas for using near-field resonant inductive coupling as a technique for Wireless Power Transfer [12]. Referring to Figure 5 for an example of this resonant transformer, this technique usually employs a primary (L1) and a secondary (L2) coils and their compensation capacitors (C1, C2). Primary (L1) and secondary (L2) coils function similar to a transformer with a weak coupling. Compensation capacitors (C1, C2) function to eliminate reactive power when operated at resonant frequency. It is essentially identical in principle and construction to Tesla's wireless power experiments shown in the diagram in Figure 6.

Figure 5: Resonant transformer

Figure 6: Diagram of one of Tesla's wireless power experiments [12]

Today, much progress has been made in the field of WPT using the near-field resonant inductive coupling technique, especially in the application for charging EVs. In this chapter, WPT coil designs for this application by other research groups will be explored, outlining their research focus and achievements.

2.1 Korea Advanced Institute of Science and Technology

The team at the Korea Advanced Institute of Science and Technology (KAIST) has published some work on WPT coil design. They mostly focused on charging of moving vehicles, which are named On-Line Electric Vehicles (OLEV). Their research focus and main contributions are the design of the coils to achieve increasing power level, gap distance, and the width of the primary coil, using multiple receivers. [13] has summarised their progress as shown in Figure 7.

	1 G (Car)	2 G (Bus)	3 G (SUV)	3+ G (Bus)	3+G (Train)	4 G (Bus)
Date	Feb. 27, 2009	July 14, 2009	Aug. 14, 2009	Jan. 31, 2010	Mar. 9, 2010	2010~ (under development)
Vehicle
System Spec.	air-gap= 1cm efficiency= 80%	air-gap= 17cm efficiency= 72%	air-gap= 17cm efficiency= 71%	air-gap= 20cm Efficiency=83%	air-gap= 12cm efficiency= 74%	air-gap= 20cm, efficiency= 80%
EMF	10mG	51mG	50mG	50mG	50mG	<10mG
Power Rail (width)	20cm	140cm	80cm	80cm	80cm	10cm
Pick-up
Power	3kW / pick-up	6kW / pick-up	15kW / pick-up	15kW / pick-up	15kW / pick-up	25kW / pick-up
Weight	20kg	80kg	110kg	110kg	110kg	80kg
Size	55x18x4 cm ³	160x60x11 cm ³	170x80x8 cm ³	170x80x8 cm ³	170x80x8 cm ³	80x100x8 cm ³

Figure 7: A summary on the developments of WPT for OLEV [13]

Figure 8: (a) power supply module and (b) pick up module used for 500 kW WPT [14]

Their latest achievement is 500 kW power transfer with 90.9% efficiency using 20

kHz frequency. 3 inverters, 6 power supply modules and 6 pick up modules were used in parallel to obtain the total power. A power supply module and a pick up module were shown in Figure 8 [14]. The air gap between the power supply module and the pick up module is 11 cm.

2.2 University of Auckland

Figure 9: Magnetic coupler designs: (a) Circular, (b) DD, (c) DDQ, and (d) BP [2]

The team at the University of Auckland has been researching WPT for more than 20 years. Early research was in developing wireless power supplies for industrial applications, such as automated guided vehicles for clean factory automation, focusing on converter and controller topologies [15]. For EV charging applications,

they mostly designed coils for charging of stationary vehicles, starting from circular coils. Then, in order to provide greater coupling, polarised coils were designed in the form of double-D-shaped (DD) coils. To provide better flux distribution and increase lateral tolerance, a quadrature coil was added to the DD coil to become a DDQ coil. Finally, the design was simplified into a bipolar (BP) coil that consisted of two magnetically independent coils [2]. These coils are shown in Figure 9. Higher Q was suggested for higher efficiency. The frequency used was 20 kHz.

2.3 Saitama University

(a)

(b)

(c)

Figure 10: (a) 3 kW transformer, (b) 10 kW transformer and (c) core without winding showing cooling structure [16]

The team at the Saitama University focused on compacting the design of their coils using H-shaped core. One contribution was also the use of split winding and split

capacitor in series to mitigate the effects of high voltage. Their 3 kW coils achieved 92.9% efficiency using 50 kHz frequency [5]. Their 10 kW coils had a core with aluminium cooling structure and achieved 95.1% efficiency using 30 kHz frequency [16]. These coils are shown in Figure 10.

2.4 University of Zaragoza

Figure 11: 2 kW rectangular coils: (a) SS topology and (b) SPS topology [17]

The team at the University of Zaragoza developed a coil design procedure that minimised copper mass [6]. This procedure was verified using a pair of rectangular spiral coils using the SS topology and achieved 2 kW with 82% efficiency using 20.1 kHz. It was later improved to 89%. Then, using similar rectangular spiral

coils, they also developed a high-misalignment tolerant compensation topology, that was named Series-Parallel-Series (SPS) topology. This achieved 2 kW with 92% efficiency using 19.6 kHz [17]. These coils are shown in Figure 11. In both cases, their experimental efficiencies were always lower than the theoretical ones as the effect of high frequency AC on wire resistance was not considered.

2.5 Oak Ridge National Laboratory

The team at the Oak Ridge National Laboratory (ORNL) investigated the use of spiral circular coils with distributed winding of Litz wire for in-motion vehicle charging. They achieved 2 kW with about 90% efficiency using 23 kHz. [18]

Figure 12: Distributed winding: (a) 7 turns for transmit coil, (a) 5 turns for receive coil [18]

2.6 Swiss Federal Institute of Technology in Zurich

The team at the Swiss Federal Institute of Technology in Zurich (ETH Zurich) presented an experimental verification of the finite element (FE) calculations used in the design of the transmission coils of a 5 kW WPT system, which achieves more than 96% DC-DC efficiency with 210 mm coil diameter and 52 mm air gap at 100 kHz [19]. Even though very high efficiency was achieved, there was no design method presented, only verification their FE model.

Figure 13: 5 kW system: (a) inverter with SiC MOSFETs, (b) coil, core and carrier [19]

2.7 Conductix-Wampfler, Bombardier, Qualcomm and Plugless Power

Private companies have also been developing some WPT systems that are deployed both commercially and experimentally.

Conductix-Wampfler is a manufacturer of WPT systems for large-scale industrial

applications, such as for automation and manufacturing facilities. Since 2002, the company has been demonstrating WPT for charging public electric buses in Genoa and Turin. This bus charging system transfers 60 kW or 120 kW power over a 40 mm air gap and achieves 95% grid-to-battery efficiency. [20]

Bombardier has WPT projects for trams and buses, mostly in Germany. Their PRIMOV trams has achieved power transfer of 250 kW in Augsburg, Germany. Figure 14 shows the commercial and development projects for RPEV by Bombardier. [13]

Figure 14: The commercial and development projects for RPEV by Bombardier [13]

Qualcomm has partnered with FIA to install wireless charging for the course cars of the Formula E Championship. Formula E is a racing championship starting in September 2014 with all-electric race cars. In the future, they expect to also install wireless charging for all the race cars. [21]

Plugless Power is selling wireless charging kits for Nissan Leaf, Chevrolet Volt and Cadillac ELR [22]. The system consists of a control panel, a ground pad, and a vehicle adapter that is mounted under the vehicle, as shown in Figure 15. It transfers 3.3 kW at 19.6 kHz with 89% efficiency. It has been tested by The Idaho National Laboratory (INL) for the power, efficiency, magnetic and electric field strengths at various gap and offset positions [23].

Figure 15: Plugless Power inductive charging system components: (a) control panel, (b) ground pad, and (c) vehicle adapter [23]

In these developments by private companies, there are minimal available details regarding the design of their coils.

2.8 Conclusion

Many of the coil designs resulted in high enough efficiencies, although they did not explicitly design to reduce AC resistance. Coil designs were mostly focused on obtaining high coupling in compact sizes. Design methodologies were rarely presented. Furthermore, most of the time, the reasons behind the inductance and capacitance values chosen were not explained at all.

In most works, regarding the efficiency of WPT systems, high Q and high coupling are often suggested. High Q is usually realised by manipulating inductance, frequency and load. High coupling is usually realised by modifying coil structure and using ferrite cores. [12]

There is a clear lack of literature addressing the high reactive voltages and currents in a resonant inductive WPT system, as well as a coil design methodology that includes the AC resistance.

Chapter 3

Minimum Stress Condition

The investigation into the minimum stress condition begins with the analysis of the equivalent circuits of the four basic compensation topologies.

The secondary compensation is used to form the secondary resonant tank and the system is to be operated at secondary resonant frequency. The primary compensation is used to form the primary resonant tank and eliminate reactance such that the source only see a resistive load. The compensation on each primary side and secondary side can be connected to either form a series or a parallel resonant tank. Therefore, there are four combinations that become the four topologies. [24]

In this chapter, the four topologies are first explained and their circuit equations are written down. Then the electrical parameters of circuit components are derived to establish the minimum stress condition.

3.1 The four WPT topologies

Figure 16 shows the circuit diagram of the four topologies. Figure 16(a) is the Series-Series topology, Figure 16(b) is the Series-Parallel topology, Figure 16(c) is the Parallel-Series topology, Figure 16(d) is the Parallel-Parallel topology.

In the circuit diagrams, V_1 is the sinusoidal AC source, and R_L is the load of the system. $R_1 = R_{L1} + R_{C1}$ is the combined equivalent resistance from the coil L_1 and the capacitor C_1 , whereas $R_2 = R_{L2} + R_{C2}$ is the combined equivalent resistance from L_2 and C_2 . All circuit components in this report refer to the same components used in the diagrams.

Figure 16: Circuit diagrams of the four WPT topologies

The equations for these circuits have been derived in various forms in the literature and can be found in [6][24], among others. Below, they are re-derived in a form that is useful for this analysis. They have also been checked to be equivalent to the published ones.

3.1.1 Series-Series

The circuit equations for SS, where $I_1 = I_{L1}$, $I_2 = I_{L2}$, $R_1 = R_{L1} + R_{C1}$ and $R_2 = R_{L2} + R_{C2}$

$$\begin{aligned} V_1 - \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) I_1 + j\omega M I_{L2} &= 0 \\ j\omega M I_{L1} - \left(R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \right) I_2 - R_L I_2 &= 0 \\ V_2 &= R_L I_2 \end{aligned} \quad (1)$$

3.1.2 Series-Parallel

The circuit equations for SP, where $I_1 = I_{L1}$, $R_1 = R_{L1} + R_{C1}$

$$\begin{aligned}
V_1 - \left(R_1 + j\omega L_1 + \frac{1}{j\omega C_1} \right) I_1 + j\omega M I_{L2} &= 0 \\
j\omega M I_{L1} - (R_{L2} + j\omega L_2) I_{L2} - \left(R_{C2} + \frac{1}{j\omega C_2} \right) (I_{L2} - I_2) &= 0 \\
- \left(R_{C2} + \frac{1}{j\omega C_2} \right) (I_2 - I_{L2}) - R_L I_2 &= 0 \\
V_2 &= R_L I_2
\end{aligned} \tag{2}$$

3.1.3 Parallel-Series

The circuit equations for PS, where $I_2 = I_{L2}$, $R_1 = R_{L2} + R_{C2}$

$$\begin{aligned}
V_1 - \left(R_{C1} + \frac{1}{j\omega C_1} \right) (I_1 - I_{L1}) &= 0 \\
- \left(R_{C1} + \frac{1}{j\omega C_1} \right) (I_{L1} - I_1) - (R_{L1} + j\omega L_1) I_{L1} + j\omega M I_{L2} &= 0 \\
j\omega M I_{L1} - \left(R_2 + j\omega L_2 + \frac{1}{j\omega C_2} \right) I_2 - R_L I_2 &= 0 \\
V_2 &= R_L I_2
\end{aligned} \tag{3}$$

3.1.4 Parallel-Parallel

The circuit equations for PP,

$$\begin{aligned}
V_1 - \left(R_{C1} + \frac{1}{j\omega C_1} \right) (I_1 - I_{L1}) &= 0 \\
- \left(R_{C1} + \frac{1}{j\omega C_1} \right) (I_{L1} - I_1) - (R_{L1} + j\omega L_1) I_{L1} + j\omega M I_{L2} &= 0 \\
j\omega M I_{L1} - (R_{L2} + j\omega L_2) I_{L2} - \left(R_{C2} + \frac{1}{j\omega C_2} \right) (I_{L2} - I_2) &= 0 \\
- \left(R_{C2} + \frac{1}{j\omega C_2} \right) (I_2 - I_{L2}) - R_L I_2 &= 0 \\
V_2 &= R_L I_2
\end{aligned} \tag{4}$$

3.2 Electrical parameters of circuit components

From the circuit equations in Equation (1) to Equation (4), the electrical parameters for each circuit component are derived with respect to the output voltage, V_2 and the output current, I_2 . The parameters include source voltage and current, primary capacitance and inductance values, and voltage and current values at each capacitor and inductor which are related by the following equations:

$$V_C = \frac{1}{j\omega C} I_C \quad (5)$$

$$V_L = j\omega L I_L \quad (6)$$

This derivation has several assumptions:

- there is no loss, i.e. $R_{L1} = R_{C1} = R_{L2} = R_{C2} = 0$
- the system operates at secondary resonant frequency, i.e.
 $\omega = \omega_0 = 2\pi f_0 = 1/\sqrt{L_2 C_2}$
- C_1 has been derived to eliminate reactance at primary side, such that the source only sees a resistive impedance.

After reducing to as few dependent variables as possible, the derived electrical parameters are summarised in Table 1, where Z_{eq} is the impedance seen by the source V_1 .

Table 1: Electrical parameters of circuit components

	SS	SP	PS	PP
C_1	$\frac{1}{\omega_0^2 L_1}$	$\frac{1}{\omega_0^2 L_1 (1-k^2)}$	$\frac{1}{\omega_0^2 L_1 \left(1 + \frac{\omega_0^2 k^4 L_2^2}{R_L^2}\right)}$	$\frac{1}{\omega_0^2 L_1 (1-k^2) + \frac{k^4 L_1 R_L^2}{(1-k^2) L_2^2}}$
L_1	$\frac{V_1^2}{I_2^2} \frac{1}{k^2 \omega_0^2 L_2}$	$\frac{V_1^2}{V_2^2} \frac{L_2}{k^2}$	$\frac{V_1^2}{V_2^2} \frac{1}{\left(\frac{1}{k^2 L_2} + \frac{k^2 \omega_0^2 L_2}{R_L^2}\right)}$	$\frac{V_1^2}{V_2^2} \frac{1}{\left(\frac{k^2}{L_2} + \frac{(1-k^2)^2 \omega_0^2 L_2}{k^2 R_L^2}\right)}$
V_1	$-j\omega_0 M I_2$	$\frac{M}{L_2} V_2$	$\left(\frac{L_1}{M} - j\frac{\omega_0 M}{R_L}\right) V_2$	$\left(\frac{M}{L_2} + j\frac{\omega_0 L_1 L_2}{M R_L} (1-k^2)\right) V_2$

	SS	SP	PS	PP
I_1	$\frac{-j}{\omega_0 M} V_2$	$\frac{L_2}{M} I_2$	$\frac{1}{\frac{L_1}{M} + j \frac{\omega_0 M}{R_L}} I_2$	$\frac{1}{\frac{M}{L_2} - j \frac{\omega_0 L_1 L_2}{M R_L} (1-k^2)} I_2$
Z_{eq}	$\frac{\omega_0^2 M^2}{R_L}$	$\frac{M^2}{L_2} R_L$	$\frac{L_1^2}{M^2} R_L + \frac{\omega_0^2 M^2}{R_L}$	$\frac{M^2}{L_2^2} R_L + \frac{\omega_0^2 L_1^2 L_2^2}{M^2 R_L} (1-k^2)^2$
V_{L2}	$j \omega_0 L_2 I_2$	$(-R_L + j \omega_0 L_2) I_2$	$j \omega_0 L_2 I_2$	$(-R_L + j \omega_0 L_2) I_2$
I_{L2}	I_2	$\left(1 + j \frac{R_L}{\omega_0 L_2}\right) I_2$	I_2	$\left(1 + j \frac{R_L}{\omega_0 L_2}\right) I_2$
V_{C2}	$-j \omega_0 L_2 I_2$	V_2	$-j \omega_0 L_2 I_2$	V_2
I_{C2}	I_2	$\frac{j}{\omega_0 L_2} V_2$	I_2	$\frac{j}{\omega_0 L_2} V_2$
V_{L1}	$\frac{L_1}{M} V_2$	$j \frac{\omega_0 L_1 L_2}{M} I_2$	$\frac{L_1}{M} V_2$	$j \frac{\omega_0 L_1 L_2}{M} I_2$
I_{L1}	$\frac{-j}{\omega_0 M} V_2$	$\frac{L_2}{M} I_2$	$\frac{-j}{\omega_0 M} V_2$	$\frac{L_2}{M} I_2$
V_{C1}	$\frac{-L_1}{M} V_2$	$-j \frac{\omega_0 L_1 L_2}{M} (1-k^2) I_2$	$\left(\frac{L_1}{M} - j \frac{\omega_0 M}{R_L}\right) V_2$	$\left(\frac{M}{L_2} + j \frac{\omega_0 L_1 L_2}{M R_L} (1-k^2)\right) V_2$
I_{C1}	$\frac{-j}{\omega_0 M} V_2$	$\frac{L_2}{M} I_2$	$\left(\omega_0 M + j \frac{L_1}{M} R_L\right) \omega_0 C_1 I_2$	$\left(\frac{-\omega_0 L_1 L_2}{M} (1-k^2) + j \frac{M}{L_2} R_L\right) \omega_0 C_1 I_2$

3.3 Equal stress condition

To observe the behaviour of the capacitor stresses, the parameters in Table 1 were used in a MATLAB code to calculate capacitor stresses (Appendix A). The input and output V_1 , I_1 , V_2 , I_2 were normalised at 1 while varying the secondary inductance L_2 from 1 μ H to 1 mH, resonant frequency f_0 from 5 kHz to 500 kHz, and coupling factor k from 0.05 to 0.5. As an example, the resulting capacitor stresses at the coupling factor $k=0.1$ are plotted and shown in Figure 17.

(a)

(b)

Figure 17: Capacitor stress behaviour of the four WPT topologies

Each of Figures 17(a) to 17(d) shows two intersecting planes. One plane is the primary capacitor stress (yellow) and the other is the secondary capacitor stress

(blue). The forms of the capacitor stresses can be inferred from the z-axis in each figure and are summarised in Table 2.

Table 2: Capacitor stresses for Figure 17

Side (colour)	Form of stress			
	17(a) SS	17(b) SP	17(c) PS	17(d) PP
Primary (yellow)	voltage	voltage	current	current
Secondary (blue)	voltage	current	voltage	current

There is an inverse relation between the primary and secondary capacitor stresses in all the topologies, where a slight decrease on one side corresponds to a large increase in the other. Therefore, the best compromise happens when the stress on both sides are equal, and the system can be said to have achieved the minimum stress. Additionally, there is also an inverse relation between all of the stress factors and the coupling factor, where the stresses decrease in proportion to the increase in the coupling factor.

Equal stress would mean differently for different topologies. With a fixed supply and load, the current is clamped for a series compensated tank, while the voltage is clamped for a parallel compensated tank. Therefore, for SS topology, the capacitor stresses are voltage stresses. And when these voltages are equal, the stress condition is optimal. Likewise for PP topology, the capacitor stresses are current stresses. And when these currents are equal, the stress condition is optimal.

For SP and PS however, capacitor stresses are not directly comparable because the parallel compensated side experiences current stress and the series compensated side experiences voltage stress.

The observation on the voltage stress relationship in SS is that equal stress happens when the ratio of the primary capacitor voltage to the output voltage is equal to the ratio of the secondary capacitor voltage to the output voltage. Similarly, the observation on the current stress relationship in PP is that equal stress happens when the ratio of the primary capacitor current to the output current is equal to the ratio of the secondary capacitor current to the output current.

Therefore, a conclusion is drawn that the minimum stress condition for SP and PS

occurs when the ratio of the capacitor voltage (on the series compensated side) to the output voltage is equal to the ratio of current stress (on the parallel compensated side) to the output current. The mathematical expressions for these relationships are written down in Table 3 as the equal stress condition (ESC).

To derive the system operating point with the minimum stress condition, the electrical parameter equations are then rearranged such that $V_1 = aV_2$ and $I_1 = I_2/a$, then adding the equal stress condition (ESC). Taking the magnitude of the resulting electrical parameters of the circuit components, a parameter list is obtained as in Table 3.

Table 3: Parameter list for minimum stress condition

	SS	SP	PS	PP
ESC	$ V_{C2} = V_{C1} $	$\frac{ V_{C1} }{V_2} = \frac{ I_{C2} }{I_2}$	$\frac{ V_{C2} }{V_2} = \frac{ I_{C1} }{I_2}$	$ I_{C2} = I_{C1} $
L_2	$\sqrt{a} \frac{R_L}{\omega_0 k}$	$\frac{R_L}{\sqrt{a} \omega_0 \sqrt{\frac{1}{k^2} - 1}}$	$\frac{R_L}{\sqrt{a} \omega_0 k}$	$\sqrt{a} \frac{R_L}{\omega_0} \frac{k}{\sqrt{1 - k^2}}$
L_1	$a L_2$	$a^2 \frac{L_2}{k^2}$	$\frac{a^3 k^2 L_2}{a + k^2}$	$\frac{a^2 L_2}{a + k^2 (1 - a)}$
V_{L2}	$\sqrt{a} \frac{V_2}{k}$	$V_2 \sqrt{1 + \frac{1}{a \left(\frac{1}{k^2} - 1 \right)}}$	$\frac{V_2}{\sqrt{a} k}$	$V_2 \sqrt{\frac{1 + k^2 (a - 1)}{1 - k^2}}$
I_{L2}	I_2	$I_2 \sqrt{1 + a \left(\frac{1}{k^2} - 1 \right)}$	I_2	$\frac{I_2}{\sqrt{a} k} \sqrt{1 + k^2 (a - 1)}$
V_{C2}	$\sqrt{a} \frac{V_2}{k}$	V_2	$\frac{V_2}{\sqrt{a} k}$	V_2
I_{C2}	I_2	$\sqrt{a} \frac{I_2}{k} \sqrt{1 - k^2}$	I_2	$\frac{I_2}{\sqrt{a} k} \sqrt{1 - k^2}$
V_{L1}	$\sqrt{a} \frac{V_2}{k}$	$\frac{\sqrt{a} V_2}{k \sqrt{1 - k^2}}$	$V_2 \sqrt{\frac{a^3}{a + k^2}}$	$V_2 \sqrt{\frac{a^3}{(a + k^2 (1 - a)) (1 - k^2)}}$
I_{L1}	$\frac{I_2}{a}$	$\frac{I_2}{a}$	$\frac{I_2}{a k} \sqrt{a + k^2}$	$\frac{I_2}{\sqrt{a} k} \sqrt{1 + k^2 \left(\frac{1}{a} - 1 \right)}$
V_{C1}	$\sqrt{a} \frac{V_2}{k}$	$\sqrt{a} \frac{V_2}{k} \sqrt{1 - k^2}$	$a V_2$	$a V_2$

	SS	SP	PS	PP
I_{C1}	$\frac{I_2}{a}$	$\frac{I_2}{a}$	$\frac{I_2}{\sqrt{a}k}$	$\frac{I_2}{\sqrt{a}k} \sqrt{1-k^2}$

If the list is simplified by substituting $a=1$, it will readily be observed that the capacitor stresses are only influenced by the coupling factor and the output voltage and current. This perspective shows that for a minimum stress condition, the magnitudes of the stresses don't depend on the values of the components, nor the frequency, instead they depend on the values of the input and output voltages and currents and the coupling factor.

3.4 Simulation

Figure 18: Simulation scenario

A simulation has been set up with the following scenario: power is drawn from a 3-phase 230 VAC mains supply which is rectified into 560 VDC, and desired output is 400 VDC, feeding an EV battery charger that has an equivalent load of 16 ohms for a 10 kW charging power. A full-bridge inverter powers the primary coil by converting 560 VDC into a square-wave that is equivalent to 504 Vrms sinusoidal AC when calculated using First Harmonics Approximation (FHA). Similarly, a 400 VDC output is approximated as 360 Vrms sinusoidal AC at the secondary side before rectification, with equivalent load of 13 ohms. This scenario is illustrated in Figure 18.

Using the INCA software [29], the coupling factor is estimated for a pair of 60 cm diameter coils with 15 cm gap, which is $k=0.29$. Lastly, the resonant frequency is set at 352.5 kHz.

The simulation used LTspice and was based on the circuit diagrams in Figure 16. The equations in Table 1 and Table 3 were used to calculate inductance and

capacitance values for every topology that are used in the simulation. These values are shown in Figure 19 with the simulation circuits. In Table 4, a comparison of capacitor stresses is made between calculations based on derived equations and simulation results. They are found to match well. The 360 Vrms output across the 13 ohms resistance is also obtained for all topologies.

(a)

(b)

(c)

(d)

Figure 19: Simulation to verify minimum stress condition

Table 4: Capacitor stress comparison between calculations and simulation

SS	Calculated	Simulated	SP	Calculated	Simulated
V_{C1} (V)	1469	1469	V_{C1} (V)	1406	1400
V_{C2} (V)	1469	1466	I_{C2} (A)	108.5	108.6
PS	Calculated	Simulated	PP	Calculated	Simulated
I_{C1} (A)	81.00	81.03	I_{C1} (A)	77.50	77.55
V_{C2} (V)	1049	1046	I_{C2} (A)	77.50	77.81

Chapter 4

Coil Resistance

Among many applications of WPT, charging EVs is possibly the largest in terms of magnitude of power transferred, such that even a system with a high efficiency percentage-wise will have a large power loss magnitude-wise. Therefore it is important to minimise as much as possible sources of power loss.

In the SS topology shown in Figure 16(a), the parasitic resistances, shown as R_1 and R_2 in Figure 16(a), are the only source of power loss in the system. With R_L as the load resistance, the efficiency of the system can be written as

$$Efficiency = \frac{I_2^2 R_L}{I_{L1}^2 R_1 + I_{L2}^2 R_2 + I_2^2 R_L} \quad (7)$$

These parasitic resistances are combinations of both the internal resistances of the capacitors and coils. Very little can be done to reduce the internal resistance of the capacitors, except by choosing the right type of capacitors. This is because for power and resonance purposes, commercially available capacitors are already manufactured using well-proven technologies. A custom made capacitor will probably not outperform a mass-manufactured one. Thus commercially available capacitors are adequate. On the other hand, there is no such mass-manufacturing of coils for WPT purposes, as WPT is still very much in the research domain and not yet in the commercial domain. Therefore, coil designs have varied greatly across various research on various applications, with focus on different aspects of the coil designs. In this chapter, a coil design procedure with the focus to minimise the resistance is described.

The coil resistance is affected by three main factors:

- the DC resistance, affected by conductor length and cross-sectional area;
- the skin effect, affected by the frequency and thickness of individual conductor;
- and the proximity effect, affected by the frequency and nearby conductors.

This means the coil dimensions, inductance, operating frequency and the construction of the Litz wire used for the coil all factor into the resistance considerations.

The inductance is already constrained by the minimum stress condition, as shown in the previous chapter. As only the inductance value appears in the equations, the physical construction of the coil can be of any type. In this study, circular spiral coil design is used, which has established coupling and inductance calculations [25][26] and has proven to work well in WPT applications [27].

This chapter discusses the optimisation of two other factors: geometrical optimisation for maximum coupling and shortest wire length, and maximum frequency for a given strand diameter of the Litz wire. Then the analysis continues until the AC resistance can be found from the data given in the Litz wire catalogue [28].

4.1 Geometry for maximum coupling

The space under a car available for a coil installation can physically restrict the maximum coil size. The car-to-ground gap is also usually quite constant for a given type of vehicles. Coil size and air gap will influence the coupling factor, which is in turn restricted by the maximum capacitor stress rating. Therefore, it is important to maximise coupling for any given coil size and air gap.

A circular spiral coil can be drawn as in Figure 20.

Figure 20: A coil with circular spiral structure

[25] calculated the mutual inductance and coupling factor of coils made of concentric circular rings. Figure 21 shows the plot of coupling factors of two identical coils made up of concentric circular turns, where a_{max} is the outer radius, a_{min} is the inner radius and -0.025 is the radius increment of each turn. In the plot, the coupling factor increases as the coil distance decreases. The coil distance is expressed as a ratio of the air gap to the coil outer radius.

Figure 21: Coupling factor k as a function of spacing for various coil configurations for identical primary and secondary coils [25]

The conclusions of this paper are that the coupling factor of two identical coils:

- increases with smaller inner radius up to $0.4a_{max}$
- is not much affected by variations of number of turns, when a_{max} and a_{min} are fixed

Therefore the maximum coupling factor is obtained when the inner radius is about half the outer radius. Additionally, the coupling factor is not affected much by the

number of turns. So using a circular spiral coil with average radius r , inner radius r_{in} , outer radius r_{out} , width b and thickness t , as shown in Figure 20, the maximum coupling can be obtained when

$$r_{in} = \frac{1}{2}r_{out} \text{ or } r = \frac{3}{2}r_{in} = \frac{3}{4}r_{out} \text{ or } t = \frac{2}{3}r \quad (8)$$

4.2 Estimation of the number of turns

[26] provided a simple expression to estimate inductance for on-chip inductors using planar spiral coils:

$$L = N_t^2 \mu r c_1 \left(\ln \frac{c_2}{F} + c_3 F + c_4 F^2 \right) \quad (9)$$

Thus the number of turns can be estimated as

$$N_t = \sqrt{\frac{L}{\mu r c_1 \left(\ln \frac{c_2}{F} + c_3 F + c_4 F^2 \right)}} \quad (10)$$

where the fill factor, $F = (r_{out} - r_{in}) / (r_{out} + r_{in})$, ranges from 0 to 1 for any given coil radius.

For larger planar spiral coils, such as one used in WPT system for EV charging, the estimate from this expression has been found to be fairly accurate and comparable to other more complex methods of estimation, such as Maxwell's method, which is used in the INCA software [29].

Figure 22 shows the shape and parameters of the spiral coils, where $d_{in} = 2r_{in}$ and $d_{out} = 2r_{out}$. Figure 23 shows the shape dependent coefficients for the expression, so for a circular spiral coil as in Figure 22(d), its inductance can be estimated using $c_1 = 1$, $c_2 = 2.46$, $c_3 = 0$ and $c_4 = 0.2$.

Figure 22: On-chip inductor realisations: (a) square, (b) hexagonal (c) octagonal, and (d) circular [26]

Layout	c_1	c_2	c_3	c_4
Square	1.27	2.07	0.18	0.13
Hexagonal	1.09	2.23	0.00	0.17
Octagonal	1.07	2.29	0.00	0.19
Circle	1.00	2.46	0.00	0.20

Figure 23: Coefficients for current sheet expression [26]

4.3 Geometry for minimum wire length

The length of a wire is directly proportional to the mass and the DC resistance of the wire, which is why it is important that a coil design should seek to minimise the wire length. The length of a wire in a circular spiral coil can be approximated to be equal to the sum of the circumferences of a series of circular concentric circles with the same number of turns that are evenly spaced between the outer radius and

the inner radius. It is estimated as:

$$l = N_t 2 \pi r \quad (11)$$

Using Equation (10) and simplifying the circular spiral coil factor as

$$f_f = c_1 \left(\ln \frac{c_2}{F} + c_3 F + c_4 F^2 \right) \quad (12)$$

then Equation (11) can be rewritten as:

$$l = 2 \pi \sqrt{\frac{L}{\mu}} \sqrt{\frac{r}{f_f}} \quad (13)$$

Usually r_{out} will be constrained by the available space for the coil, so in order to obtain the minimum length for a given r_{out} , the average radius will be varied relative to the outer radius by using the term $r_r = r/r_{out}$, the ratio of average radius to outer radius. This means that when $r_{in} = r_{out}$, $r_r = 1$, and when $r_{in} = 0$, $r_r = 0.5$. Thus, for $0.5 \leq r_r \leq 1$. Equation (13) can then be rewritten as:

$$l = 2 \pi \sqrt{\frac{L}{\mu}} \sqrt{r_{out}} \sqrt{\frac{r_r}{f_f}} \quad (14)$$

Equation (14) shows that, for any given r_{out} , l is proportional to $\sqrt{r_r/f_f}$. Thus r_r is plotted against $\sqrt{r_r/f_f}$ to show the relationship between the wire length and the average radius. Figure 24 shows that increasing r_r will decrease $\sqrt{r_r/f_f}$, which will decrease l . So the minimum wire length is obtained at $r_{in} = r_{out}$, when $r_r = 1$.

As established in Chapter 4.1, however, the maximum coupling occurs when r_{in} is at most $0.5 r_{out}$, and increasing r_{in} will reduce coupling. In the case where the coupling takes priority due to capacitor stress constraints, $r_{in} = 0.5 r_{out}$ is the most optimum dimension for a circular spiral coil.

Figure 24: Relationship between the wire length and the average radius

4.4 AC resistance of Litz wire in a winding

Litz wire is commonly used in a winding to minimise skin effect. In a winding, proximity effect also plays a part to increase the AC resistance of Litz wire. After determining the number of turns in a geometrically optimised spiral coil, the AC resistance can be determined.

4.4.1 DC resistance

DC resistance formula for Litz wire is given by the manufacturer as

$$R_{dc} = N_t \rho \frac{2\pi r}{A_w} 1.015^{NB} 1.025^{NC} \quad (15)$$

where NB is the number of bunching operations and NC is the number of cabling operations. These numbers denote the physical construction of the Litz wire, for example, a Type 1 Litz wire has one bunching operation, and a Type 2 Litz wire may have one or more cabling operations of several Type 1 Litz wires. These two are the most common types and are illustrated in Figure 25.

Figure 25: Litz wire construction types [28]

The DC resistance values that are calculated from the formula given by the manufacturer, however, are found to have significant differences when compared to the values listed in the catalogue [28]. By using the listed DC resistance values for $NB=1$ and $NC=1$, new factors for NB and NC are derived and the formula is modified into

$$R_{dc} = N_t \rho \frac{2\pi r}{A_w} 1.06^{NB} 1.03^{NC} \quad (16)$$

where conduction area of each wire with strand diameter D_s ,

$$A_w = N_s \pi \left(D_s / 2 \right)^2 \quad (17)$$

4.4.2 Skin effect and proximity effect

According to [30], skin effect can be neglected when strand diameter is less than 1.5 times skin depth. Figure 26 is given in the paper as an experimental result showing the error between the DC resistance and the exact resistance as a function of strand diameter and skin depth ratio. Therefore following this, ratio that would give minimal error should be less than 0.7. Thus,

$$D_s \leq 0.7 \delta \quad (18)$$

where δ is the skin depth at the maximum operating frequency of a Litz wire with strand diameter D_s . So the maximum operating frequency that will allow the skin effect in the AC resistance to be ignored, can be approximated as

$$\omega_0 = \frac{1}{\rho \mu D_s^2} \quad (19)$$

where ρ and μ are the resistivity and permeability of the conductor respectively.

Figure 26: Error between the DC resistance and the exact resistance as a function of strand diameter and skin depth ratio [30]

In [8], a formula is derived to predict AC resistance of a winding due to proximity effect. This formula is valid for frequencies less than 500 kHz and ignores skin effect. This formula has been simplified into

$$R_{ac} = R_{dc} \left(1 + \frac{\pi \mu \sigma}{8 \sqrt{2}} N_t N_s D_s^4 \eta \beta f^2 \right) \quad (20)$$

where N_t is the number of turns in the winding, N_s is the number of strands in the wire, β is the copper-to-winding area ratio,

$$\beta = \frac{N_t A_w}{b t} \quad (21)$$

and η is a function of coil width-to-thickness ratio by the following formulas,

$$\eta = -0.08998 x^6 - 0.005291 x^5 + 0.5015 x^4 + 0.01544 x^3 - 0.9772 x^2 - 0.006905 x + 0.9423$$

$$x = \log_{10} \frac{b}{t} \quad (22)$$

which according to Figure 27, is only defined up to the lower limit $\eta=0.25$, which will be used when it goes below that value.

Figure 27: Numeric values of η for coils with different cross sections. b and t are the coil's width and thickness, as shown in Figure 20 [8]

The coil width

$$b = N_l D_w \quad (23)$$

is found using the number of layers in the winding,

$$N_l = \text{ROUNDUP} \left(\frac{N_t D_w}{t} \right) \quad (24)$$

together with wire diameter D_w data found in the Litz catalogue.

Assuming that for a copper conductor, $\mu = \mu_0 = 4\pi E - 7$ and $\sigma = 5.8E7$, then Equation (20) can be simplified further into

$$R_{ac} = R_{dc} \left(1 + 409.6 N_t N_s D_s^4 \eta \beta f^2 \right) \quad (25)$$

4.5 Conclusion

This chapter has presented optimisation in geometry and coupling, which is used in the estimation of the number of turns required for the inductance. The maximum

operating frequency is dictated by the strand diameter to ignore skin effect. Equation (25), the resistance formula due to proximity effect, is then completed with Equation (16) through Equation (24) together with data from the Litz wire catalogue. The overall flow of the equations is presented in Figure 28. This flow can be input into a spreadsheet along with the given coil design scenario and the list of Litz wire data in the catalogue. It will result in a list of AC resistances that corresponds to the list of Litz wire in the catalogue. As an example, Figure 29 shows the spreadsheet used for designing SS-topology coils used in the experiment, which will be explained in the next chapter.

Figure 28: to find AC resistance

Chapter 4 Coil Resistance

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
1	V1=V2	I1=I2	V_C1	V_C2	I_L1	I_L2	q	max Rac	RL	rho	mu	k	F1	r	t				
2	400	25	1351.35	1351	25	25	81.568	0.065254	16	1.72E-8	1.26E-6	0.296	2.02	0.225	0.15				
3	w.AWG	Ns	Dw(m)	NB /	NC x	Ds(m)	Aw(m^2)	f(Hz)	L2(H)	Nt	NI	b	beta	eta	Rac(Ω)	l(m)	(kg/m)	m(kg)	C2(F)
4	12	150	0.00307	1	1	0.0001803	3.83E-6	67137.23	1.28E-4	14.975	0.3068	0.0031	0.12	0.25	0.1182	21.2	0.0366	0.775	4.39E-8
5	10	210	0.00363	1	1	0.0001803	5.36E-6	67137.23	1.28E-4	14.975	0.3626	0.0036	0.15	0.25	0.0911	21.2	0.0511	1.083	4.39E-8
6	8	329	0.00445	1	1	0.0001803	8.40E-6	67137.23	1.28E-4	14.975	0.4438	0.0044	0.19	0.25	0.06894	21.2	0.0821	1.739	4.39E-8
7	6	525	0.00602	1	2	0.0001803	1.34E-5	67137.23	1.28E-4	14.975	0.601	0.006	0.22	0.272	0.05905	21.2	0.137	2.899	4.39E-8
8	4	850	0.00767	1	2	0.0001803	2.17E-5	67137.23	1.28E-4	14.975	0.7658	0.0077	0.28	0.285	0.05671	21.2	0.219	4.636	4.39E-8
9	2	1320	0.01229	1	1	0.0001803	3.37E-5	67137.23	1.28E-4	14.975	1.2273	0.0246	0.14	0.509	0.04359	21.2	0.3631	7.687	4.39E-8
10	12	265	0.00295	1	1	0.000127	3.36E-6	135376	6.35E-5	10.546	0.2071	0.0029	0.08	0.25	0.09273	14.9	0.0325	0.484	2.17E-8
11	10	420	0.00401	1	2	0.000127	5.32E-6	135376	6.35E-5	10.546	0.2821	0.004	0.09	0.25	0.06527	14.9	0.053	0.791	2.17E-8
12	8	660	0.005	1	2	0.000127	8.36E-6	135376	6.35E-5	10.546	0.3518	0.005	0.12	0.261	0.04896	14.9	0.0878	1.309	2.17E-8
13	6	1050	0.00627	1	2	0.000127	1.33E-5	135376	6.35E-5	10.546	0.4411	0.0063	0.15	0.274	0.04094	14.9	0.1376	2.051	2.17E-8
14	4	1800	0.00818	1	2	0.000127	2.28E-5	135376	6.35E-5	10.546	0.575	0.0082	0.2	0.289	0.03929	14.9	0.2328	3.47	2.17E-8
15	2	2660	0.00947	1	2	0.000127	3.37E-5	135376	6.35E-5	10.546	0.6661	0.0095	0.25	0.301	0.04397	14.9	0.3403	5.073	2.17E-8
16	12	420	0.00297	2	1	0.0001016	3.41E-6	211524	4.07E-5	8.4365	0.1671	0.003	0.06	0.25	0.07768	11.9	0.0323	0.385	1.39E-8
17	10	660	0.00378	2	1	0.0001016	5.35E-6	211524	4.07E-5	8.4365	0.2129	0.0038	0.08	0.25	0.05409	11.9	0.0506	0.603	1.39E-8
18	8	1050	0.00508	1	2	0.0001016	8.51E-6	211524	4.07E-5	8.4365	0.2857	0.0051	0.09	0.262	0.03876	11.9	0.0877	1.046	1.39E-8
19	6	1650	0.00632	1	2	0.0001016	1.34E-5	211524	4.07E-5	8.4365	0.3557	0.0063	0.12	0.275	0.03264	11.9	0.1357	1.619	1.39E-8
20	4	2625	0.00813	2	2	0.0001016	2.13E-5	211524	4.07E-5	8.4365	0.4571	0.0081	0.15	0.288	0.03268	11.9	0.2135	2.546	1.39E-8
21	2	4140	0.01255	2	1	0.0001016	3.36E-5	211524	4.07E-5	8.4365	0.7057	0.0125	0.15	0.339	0.03173	11.9	0.3676	4.384	1.39E-8
22	12	700	0.0032	1	2	7.874E-005	3.41E-6	352174	2.44E-5	6.5383	0.1395	0.0032	0.05	0.25	0.05803	9.24	0.0348	0.321	8.36E-9
23	10	1100	0.00399	1	2	7.874E-005	5.36E-6	352174	2.44E-5	6.5383	0.1738	0.004	0.06	0.25	0.04033	9.24	0.0545	0.504	8.36E-9
24	8	1800	0.00599	2	1	7.874E-005	8.77E-6	352174	2.44E-5	6.5383	0.2613	0.006	0.06	0.272	0.02942	9.24	0.0991	0.916	8.36E-9
25	6	2790	0.00744	2	1	7.874E-005	1.36E-5	352174	2.44E-5	6.5383	0.3244	0.0074	0.08	0.283	0.02446	9.24	0.1533	1.417	8.36E-9
26	4	4455	0.01095	2	1	7.874E-005	2.17E-5	352174	2.44E-5	6.5383	0.4772	0.0109	0.09	0.318	0.02178	9.24	0.2619	2.421	8.36E-9
27	2	7200	0.01453	2	1	7.874E-005	3.51E-5	352174	2.44E-5	6.5383	0.6333	0.0145	0.11	0.367	0.02391	9.24	0.4316	3.989	8.36E-9
28	12	1050	0.00323	1	2	0.0000635	3.33E-6	541502	1.59E-5	5.2728	0.1134	0.0032	0.04	0.25	0.04772	7.45	0.0325	0.242	5.44E-9
29	10	1710	0.0047	2	1	0.0000635	5.42E-6	541502	1.59E-5	5.2728	0.1652	0.0047	0.04	0.255	0.03246	7.45	0.06	0.447	5.44E-9
30	8	2700	0.00587	2	1	0.0000635	8.55E-6	541502	1.59E-5	5.2728	0.2063	0.0059	0.05	0.271	0.0241	7.45	0.0938	0.699	5.44E-9
31	6	4200	0.00729	2	1	0.0000635	1.33E-5	541502	1.59E-5	5.2728	0.2563	0.0073	0.06	0.282	0.01992	7.45	0.1445	1.077	5.44E-9
32	4	6750	0.01102	2	1	0.0000635	2.14E-5	541502	1.59E-5	5.2728	0.3875	0.011	0.07	0.319	0.01747	7.45	0.2515	1.875	5.44E-9
33	2	10800	0.01425	2	1	0.0000635	3.42E-5	541502	1.59E-5	5.2728	0.5009	0.0142	0.08	0.363	0.01914	7.45	0.4048	3.017	5.44E-9

Figure 29: Spreadsheet used for designing SS-topology coils used in the experiment

Chapter 5

Experimental Validation

An experiment had been set up to verify the minimum stress condition and Litz wire resistance calculations. This experiment also served as a design example.

SS topology had been chosen for this experiment because it's the topology that the team has more practical experience in and it works well with the full-bridge square-wave inverter.

5.1 Initial design

Several parameters had to be first chosen as initial design input. The desired coil radius and gap were chosen according to the space constraints on the EV, keeping in mind that larger radius and smaller gap will increase coupling and reduce stress. Coupling factor could then be estimated accordingly. Load resistance was also chosen according to the desired output voltage and power.

For this experiment, these parameters had been chosen: $r_{out}=0.3$ m and for a 15 cm gap, $k=0.296$, input-output voltage ratio $a=1$, and $R_L=16 \Omega$. Substituting these r , k , a , R_L and Equation (19) into L_2 from Table 3 and then into Equation (10) yielded the number of turns and wire length needed by a coil for each Litz wire with a certain strand diameter. For Litz wires with 38 AWG strands, 212 kHz is the maximum frequency that will allow the skin effect in the AC resistance to be ignored. At this frequency, the skin depth is 145 μm . And the results are shown in Table 5.

Table 5: Initial design results for 38AWG-strand wires

f_0 (Hz)	L_2 (H)	N_t	length (m)	C_2 (F)
212000	4.07E-05	8.44	11.9	1.39E-08

Following the flow of the equations presented in Figure 28, Table 6 shows the

result of substituting Equation (10), (16), (19) into Equation (25) for some wires with 38AWG strands, which is a part of the spreadsheet shown in Figure 29. In Table 6, w.AWG is the equivalent wire AWG and LMD is the linear mass density of the wire.

Table 6: AC resistance of coils with 38AWG strands

w.AWG	N_s	R_{ac} (Ω)	LMD (kg/m)	mass (kg)
12	420	0.0777	0.0323	0.385
10	660	0.0541	0.0506	0.604
8	1050	0.0390	0.0877	1.05
6	1650	0.0332	0.136	1.62
4	2625	0.0335	0.213	2.55
2	4140	0.0324	0.368	4.39

It is observed that larger wire tends to decrease resistance but will increase mass proportionally. (The increase in resistance seen is due to going from 1-layer to 2-layer, increasing η .) Using wires with smaller strand diameter will result in coils with lower mass and lower resistance due to higher frequency requiring less inductance thus shorter length of wire. Therefore there is a compromise where decreasing the absolute resistance may mean heavier/costlier coil or more losses in power devices that have to switch at a higher frequency.

5.2 AC resistance from power dissipation

Using this method of calculating the AC resistance, even though it has resulted in very low values of resistance, there is not a single lowest value to choose from. Instead, choosing an appropriate wire is a practical matter depending on real world constraints.

One common constraint is the maximum thermal dissipation of the packaging. For example, power dissipation from natural convection can be used to estimate the maximum permissible AC resistance. The method of “Natural Convection on Isothermal Heated Horizontal Plate Facing Up” [31] is used and adapted into a MATLAB code (Appendix B).

Given the coil structure as shown in Figure 20, it has the convection area of

$$A = \frac{4}{3} \pi r^2 \quad (26)$$

and convection perimeter of

$$p = 2 \pi \frac{4}{3} r \quad (27)$$

With ambient temperature T_a and coil temperature T_p , the power dissipated from natural convection, q , could be found using the code. Then, using maximum coil current found from circuit equations I , the maximum permissible AC resistance could be found as

$$R_{ac} = \frac{q}{I^2} \quad (28)$$

5.3 AC resistance of capacitors

For a more accurate prediction, AC resistance of the compensating capacitors had also been taken into account and estimated as

$$R_{cap} = \frac{1}{\omega_0 C Q} \quad (29)$$

where $Q=1000$ for a 470 pF 3 kV NP0 Multi-Layer Ceramic Capacitor (MLCC) [32]. 30 of these MLCCs were connected in parallel to achieve the required capacitance in Table 5. This capacitor had been chosen because of its temperature and voltage stability and the ability to handle current that is only limited by its operating temperature [33].

Thus the resulting parasitic resistances were now sums of the resistances of the coils and capacitors,

$$R_1 = R_2 = R_{ac} + R_{cap} \quad (30)$$

5.4 Results

For validation, the coil with 10 AWG wire (660 strands 38 AWG) had been chosen as it has reasonably low mass and resistance. Its thermal dissipation would support up to 30 A if calculated according to Chapter 5.2.

The system was set up as shown in Figure 30, wherein the primary compensation capacitor is visible in the bottom left corner. The measured input and output voltages and currents were captured in a screenshot as seen in Figure 31, where C1 (yellow) is V_1 , C2 (green) is I_1 , C3 (magenta) is V_2 and C4 (cyan) is I_2 . M1 (red), mean input power $V_1 I_1$, and M2 (orange), mean output power $V_2 I_2$, were automatically calculated by the oscilloscope. In the screenshot, the voltage are square-shaped and the current waveforms are sinusoidal. The power waveforms, which are the product of the voltage and current waveforms, are mostly positive. They are shaped similar to a rectified sine-wave for the primary, and a sine-squared wave for the secondary.

Figure 30: Primary and secondary coils used for testing

Figure 31: Input and output voltages, currents and power of the 1.7 kW test

The resulting measurements are compared with the calculations in Table 7. The coil inductance and coupling factor of the test setup was lower than predicted, causing the slightly unequal input-output voltage ratio and capacitor voltage. Nevertheless, the predicted efficiency and capacitor voltage stresses closely matched the test result.

Table 7: Comparison between design and experimental setup

	Calculated	Measured	Error %
N_t	8.44	8.5	0.71
f_0 (Hz)	212000	215000	1.4
C_2 (F)	1.39E-08	1.386E-08	-0.29
L_2 (H)	4.07E-05	3.95E-05	-2.9
k	0.296	0.279	-5.7
V_1 (V)	165	156 (FHA)	-5.5
I_1 (A)	10.3	11.1	7.8
V_2 (V)	165	166	0.61
I_2 (A)	10.3	10.3	0
$V_1 I_1$ (W)	1700	1720	1.18
$V_2 I_2$ (W)	1700	1700	0
R_L (Ω)	16	16.0	0
R_{ac} (Ω)	0.0541	-	-

	Calculated	Measured	Error %
$R_{cap} (\Omega)$	0.0541	-	-
Efficiency	0.9865	0.9863	-0.020
$V_{C1} (V)$	558	595	6.6
$V_{C2} (V)$	558	552	-1.1

Using SS-topology, this experiment has put into practice a design procedure for a WPT system that uses the minimum stress condition for the compensation capacitors, finds the AC resistance of the coils and comparing across the range of available Litz wires based on weight and thermal constraints. The measured values have also verified the design calculations.

5.5 A 10 kW test

The coils were tested again at 10 kW with 400 VDC input and output voltage. The setup was modified accordingly.

10 kW with 400 VDC output means 16 ohms DC load resistance. Through FHA, this translates to 360 Vrms with 13 ohms load before the rectifier. Thus the load changed while the inductance and frequency remained the same. The required coupling was then calculated according to the L_2 equation for SS in Table 3. Using the load resistance $R_L=13 \Omega$, the same inductance $L_2=40.7 \mu H$ and the same frequency $f_0=212 \text{ kHz}$, the coupling factor became $k=0.239$. This coupling value was achieved by increasing the air gap to 18 cm.

According to the V_{C1} and V_{C2} equations for SS in Table 3, the capacitor voltage will increase to 1500 Vrms. For an MLCC, the complete peak-to-peak voltage must not exceed the rated DC voltage [34], i.e. there is a $1/2\sqrt{2}$ times de-rating for a sinusoidal AC voltage applied to the capacitor. Since the MLCC used was rated at 3 kV, the AC voltage rating became 1000 Vrms. To handle the new voltage stress of 1500 Vrms, two capacitors in series will be used. Therefore, the capacitor bank have been reconstructed to have 120 of 470 pF capacitors, which was 60 parallel capacitors in series with another 60 parallel capacitors. It was also clamped to a heat-sink via a Laird Technologies T-Flex 3200 thermal pad. This construction is shown in Figure 32.

Figure 32: 60 parallel MLCCs in series with another 60 parallel MLCCs

To provide power, two of Chroma 62100H-600S Programmable DC Power Supply were connected in parallel to feed the full-bridge inverter that drove the primary coil. As the load, three of Chroma Programmable AC/DC Electronic Load Model 63804 were connected in parallel to the output of the full-bridge rectifier whose input was connected to the secondary coil.

The AC input and output voltages and currents were measured using the oscilloscope and were captured in a screenshot as seen in Figure 33, where C1 (yellow) is V_1 , C2 (green) is I_1 , C3 (magenta) is V_2 and C4 (cyan) is I_2 . M1 (red), mean input power $V_1 I_1$, and M2 (orange), mean output power $V_2 I_2$, were automatically calculated by the oscilloscope. In the screenshot, the voltage are square-shaped and the current waveforms are sinusoidal. The power waveforms, which are the product of the voltage and current waveforms, are mostly positive and shaped similar to rectified sine-waves.

Figure 33: Input and output voltages, currents and power of the 10 kW test

For the coils, the resulting measurements are compared with the calculations in Table 8. Similar to the previous results in Table 7, the predicted efficiency and capacitor voltage stresses closely matched the test result.

Table 8: AC results comparison for 10 kW test

	Calculated	Measured	Error %
k	0.2391	0.2314	-3.2
V_1 (V)	366.8 (FHA)	348 (FHA)	-5.1
I_1 (A)	28.3	30.1	6.4
V_2 (V)	366.8 (FHA)	364 (FHA)	-0.76
I_2 (A)	28.3	28.3	0
$V_1 I_1$ (W)	10380	10520	1.3
$V_2 I_2$ (W)	10380	10380	0
R_L (Ω)	12.96	12.87	-0.69
R_{ac} (Ω)	0.0541	-	-
R_{cap} (Ω)	0.0541	-	-
Efficiency	0.9835	0.9867	0.33
V_{C1} (V)	1528	1626	6.4
V_{C2} (V)	1528	1526	-0.13

The DC input and output voltages and currents were read from the display panels of the DC power supplies and the electronic loads. The resulting measurements are compared with the calculations in Table 9. The fairly small error shows that the FHA method is quite reliable in predicting the conversion between AC and DC for

the square-wave inverter and the rectifier. Owing to the very efficient inverter and rectifier, the final DC-to-DC efficiency of 94.8% was achieved.

Table 9: DC results comparison for 10 kW test

	Calculated	Measured	Error %
V_{dc1} (V)	406.5	410	0.86
I_{dc1} (A)	25.38	26.56	4.6
V_{dc2} (V)	406.5	406	-0.12
I_{dc2} (A)	25.41	25.41	0
P_{dc1} (W)	10330	10890	5.7
P_{dc2} (W)	10330	10330	0
R_{Ldc} (Ω)	16.0	16.0	0
Efficiency	-	0.948	-

Chapter 6

Displacement Sensitivity

The design procedure has been established for a WPT system with fixed coupling factor, where the secondary coil is positioned directly above the primary coil at a certain distance. However, during real-world usage, it may be difficult to accurately control the relative positioning of the primary and secondary coils. This in turn will affect the mutual inductance and the coupling factor between the two coils. The change in the value of the coupling factor in turn will affect the values of all the circuit parameters.

Besides the circuit parameters, the input and output voltage relation, i.e. the transfer function, also changes when the coupling changes. This means the control objective of the system has to be first considered, as the circuit parameters change differently for different control situations. To provide an insight into the behaviour of the system with changing coupling, three control objectives are compared in the sensitivity analysis:

- (1) Constant input voltage,
- (2) Constant input current,
- (3) Constant output voltage.

Sensitivity analysis in a constant input voltage and a constant input current situations are useful to show the behaviour of a WPT system that only controls its input voltage or current and does not use any feedback from the secondary side. On the other hand, sensitivity analysis in a constant output voltage situation is useful to show the behaviour of a WPT system that controls its output voltage using feedback from the secondary side. Analysing the differences in the behaviours of the circuit parameters for different topologies will help in choosing the appropriate topology for each control situation.

The sensitivity analysis was done using the 60 cm coils used in the experiment in Chapter 5. It consisted of a few steps:

1. The coupling and positioning data were measured for the 60 cm coils. A surface-fitting was done to correlate the relative positions to their corresponding coupling factors. The correlation equation between coupling and positioning was then obtained.
2. For each control situation, the circuit equations were re-arranged to solve for the parameters of interest.
3. The correlation equation was inserted into the circuit equations to obtain and plot the changes in parameters due to the changes in relative positioning.

6.1 Measurements and surface-fitting

The coupling and positioning data were measured for the 60 cm coils. The positioning was recorded according to the coordinate system shown in Figure 34, where x is the horizontal displacement and y is the vertical displacement. The mutual inductance value was measured using Agilent U1733C LCR meter.

Figure 34: Coordinate system for measurement of coupling and positioning

By trial and error, many functions were used in the attempt to fit the measured data. For each y there was a set of k and x data that was found to be related

according to the following polynomial function,

$$k(x) = p_1 x^3 + p_2 x^2 + p_3 \quad (31)$$

The polynomial coefficients p_1 , p_2 and p_3 were then found to be related to y according to the following exponential function,

$$p_n(y) = e_{1n} \exp(e_{2n} y) + e_{3n} \quad (32)$$

Equation (31) and Equation (32) were combined to obtain the correlation equation between coupling and positioning:

$$k(x, y) = (e_{11} \exp(e_{21} y) + e_{31}) x^3 + (e_{12} \exp(e_{22} y) + e_{32}) x^2 + (e_{13} \exp(e_{23} y) + e_{33}) \quad (33)$$

In Figure 35, this equation is plotted as a surface with the measured data and they were found to agree very well. MATLAB was used to fit the measured data and plot the surface (Appendix C).

Figure 35: Coupling map with estimated surface and measured data

6.2 Constant input voltage V_1

With a constant input voltage V_1 , the circuit equations were re-arranged to solve for the parameters of interest, which are V_2 , I_2 , V_{L2} , I_{L2} , V_{C2} , I_{C2} , V_{L1} , I_{L1} , V_{C1} , I_{C1} and I_1 . These equations were expressed in terms of V_1 , L_2 , C_2 , L_1 , C_1 , ω , R_L , which are considered fixed, and M , which is the variable for this sensitivity analysis. A MATLAB code was written for this purpose (Appendix D).

The correlation equation was inserted into the circuit equations to obtain and plot the changes in parameters due to the changes in relative positioning (Appendix E).

6.2.1 Effect of horizontal displacement

Inserting the correlation equation into the circuit equations, the changes in parameters due to the changes in horizontal displacement were obtained and plotted in Figure 36. The x-axis is the change in horizontal displacement from the original value of 0 cm. The y-axis is the change in parameter values expressed as ratios of original values.

Figure 36: Parameter change due to horizontal displacement with constant input voltage

For SS and SP, Figure 36(a) and Figure 36(b) respectively, all parameters increase as horizontal displacement increases, with parameters on the primary side increasing more than secondary side. Primary stresses increase in proportion to input current, while secondary stresses increase with the same proportion as output current and voltage. Transferred power increases as coupling decreases.

For PS and PP, Figure 36(c) and Figure 36(d) respectively, most of the parameters decrease as horizontal displacement increases, with V_{C1} and I_{C1} remaining the same throughout the displacement range. V_{L1} and I_{L1} increase slightly for PS and decrease slightly for PP. Secondary stresses decrease with the same proportion as output current and voltage. Transferred power decreases as coupling decreases.

6.2.2 Effect of vertical displacement

Similarly, the changes in parameters due to the changes in vertical displacement were obtained and plotted in Figure 37. The x-axis is the change in vertical displacement from the original value of 15 cm. Again, the y-axis is the change in parameter values expressed as ratios of original values.

Figure 37: Parameter change due to vertical displacement with constant input voltage

For SS and SP, Figure 37(a) and Figure 37(b) respectively, all parameters increase as vertical displacement increases, with parameters on the primary side increasing more than secondary side. Primary stresses increase in proportion to input current, while secondary stresses increase with the same proportion as output current and voltage. Transferred power increases as coupling decreases.

For PS and PP, Figure 37(c) and Figure 37(d) respectively, most of the parameters decrease as vertical displacement increases, with V_{C1} and I_{C1} remaining the same throughout the displacement range. V_{L1} and I_{L1} increase slightly for PS and decrease slightly for PP. Secondary stresses decrease with the same proportion as output current and voltage. Transferred power decreases as coupling decreases.

The opposite happens with decreasing vertical displacement. Except V_{L1} and I_{L1} for PP, where they decrease slightly with displacement in both directions.

6.3 Constant input current I_1

With a fixed input current I_1 , the circuit equations were re-arranged to solve for the parameters of interest, which are V_2 , I_2 , V_{L2} , I_{L2} , V_{C2} , I_{C2} , V_{L1} , I_{L1} , V_{C1} , I_{C1} and V_1 . These equations were expressed in terms of I_1 , L_2 , C_2 , L_1 , C_1 , ω , R_L , which are considered fixed, and M , which is the variable for this sensitivity analysis. A MATLAB code was written for this purpose.

The correlation equation was inserted into the circuit equations to obtain and plot the changes in parameters due to the changes in relative positioning.

6.3.1 Effect of horizontal displacement

Inserting the correlation equation into the circuit equations, the changes in parameters due to the changes in horizontal displacement were obtained and plotted in Figure 38. The x-axis is the change in horizontal displacement from the original value of 0 cm. The y-axis is the change in parameter values expressed as ratios of original values.

Figure 38: Parameter change due to horizontal displacement with constant input current

For SS and SP, Figure 38(a) and Figure 38(b) respectively, all secondary side parameters decrease as horizontal displacement increases, with primary side parameters remaining the same throughout the displacement range, except V_1 , which decreases more than the secondary side parameters. Secondary stresses decrease with the same proportion as output current and voltage. Transferred power decreases as coupling decreases.

For PS and PP, Figure 38(c) and Figure 38(d) respectively, all parameters increase as horizontal displacement increases, with parameters on the primary side increasing more than secondary side. Primary stresses increase in proportion to input voltage, while secondary stresses increase with the same proportion as output current and voltage. Transferred power increases as coupling decreases.

6.3.2 Effect of vertical displacement

Similarly, the changes in parameters due to the changes in vertical displacement were obtained and plotted in Figure 39. The x-axis is the change in vertical displacement from the original value of 15 cm. Again, the y-axis is the change in parameter values expressed as ratios of original values.

Figure 39: Parameter change due to vertical displacement with constant input current

For SS and SP, Figure 39(a) and Figure 39(b) respectively, all secondary side parameters decrease as vertical displacement increases, with primary side parameters remaining the same throughout the displacement range, except V_1 , which decreases more than the secondary side parameters. Secondary stresses decrease with the same proportion as output current and voltage. Transferred power decreases as coupling decreases.

For PS and PP, Figure 39(c) and Figure 39(d) respectively, all parameters increase as vertical displacement increases, with parameters on the primary side increasing more than secondary side. Primary stresses increase in proportion to input voltage, while secondary stresses increase with the same proportion as output current and voltage. Transferred power increases as coupling decreases.

The opposite happens with decreasing vertical displacement for all topologies.

6.4 Constant output voltage V_2

With a fixed output voltage V_2 , the circuit equations were re-arranged to solve for the parameters of interest, which are V_{L2} , I_{L2} , V_{C2} , I_{C2} , V_{L1} , I_{L1} , V_{C1} , I_{C1} , V_1 and I_1 . These equations were expressed in terms of V_2 , L_2 , C_2 , L_1 , C_1 , ω , R_L , which are considered fixed, and M , which is the variable for this

sensitivity analysis. A MATLAB code was written for this purpose.

The correlation equation was inserted into the circuit equations to obtain and plot the changes in parameters due to the changes in relative positioning.

6.4.1 Effect of horizontal displacement

Inserting the correlation equation into the circuit equations, the changes in parameters due to the changes in horizontal displacement were obtained and plotted in Figure 40. The x-axis is the change in horizontal displacement from the original value of 0 cm. The y-axis is the change in parameter values expressed as ratios of original values.

(a)

(b)

Figure 40: Parameter change due to horizontal displacement with constant output voltage

For all topologies in Figure 40, all secondary side parameters remains the same throughout the displacement range, with most of the primary side parameters increasing as horizontal displacement increases.

For SS and SP, Figure 40(a) and Figure 40(b) respectively, V_1 decreases and I_1 increases as horizontal displacement increases, with primary stresses increase in proportion to I_1 . Conversely for PS and PP, Figure 40(c) and Figure 40(d) respectively, I_1 decreases and V_1 increases as horizontal displacement increases, with primary stresses increase in proportion to V_1 .

6.4.2 Effect of vertical displacement

Similarly, the changes in parameters due to the changes in vertical displacement were obtained and plotted in Figure 41. The x-axis is the change in vertical displacement from the original value of 15 cm. Again, the y-axis is the change in parameter values expressed as ratios of original values.

Figure 41: Parameter change due to vertical displacement with constant output voltage

For all topologies in Figure 41, all secondary side parameters remain the same throughout the displacement range, with most of the primary side parameters increasing as vertical displacement increases.

For SS and SP, Figure 41(a) and Figure 41(b) respectively, V_1 decreases and I_1 increases as vertical displacement increases, with primary stresses increase in proportion to I_1 . Conversely for PS and PP, Figure 41(c) and Figure 41(d) respectively, I_1 decreases and V_1 increases as vertical displacement increases, with primary stresses increase in proportion to V_1 .

The opposite happens with decreasing vertical displacement for all topologies.

6.5 Effect of angular displacement

Figure 42: Angle due to ride height difference

A scenario as depicted in Figure 42 was imagined where an imbalance in the ride height of a vehicle causes an angle between the primary and secondary coil. On a vehicle with a wheelbase of 200 cm and height difference of 25 cm, an angle of 7° will result.

Figure 43: Coordinate system for measurement of coupling and angle

The same coils were then set up with such an angle and the mutual inductance values were measured with varying vertical displacement as shown in Figure 43.

The resulting coupling values of the coils with 7° angle are compared with the values of the coils with 0° angle in the Figure 44.

Figure 44: Coupling comparison for the two angles

It is clear that the angle has a negligible effect on the coupling, at least up to the small angle of 7° . A larger concern with the height difference of the vehicle is the change in vertical displacement, which the WPT system is more sensitive to, that varies more significantly from the front to the back of the car, depending on where the coil is positioned on the car.

6.6 Conclusion

The WPT system with 60 coils at 15 cm air-gap is much more tolerant in the x direction than y, i.e. there is less change in the parameters for the same magnitude of displacement in the x direction than y direction. This may be due to the shape of the magnetic field of this type of coil. The magnetic field is shaped such that displacement in the y-direction would result in greater change in flux linkage, thus coupling factor, than in the x-direction.

It can be observed that the behaviour of the circuits is largely dependent on the primary compensation type. Generally, SS and SP behave similarly, and PS and PP behave similarly.

In a constant input voltage situation, the transferred power of SS and SP topologies increases when coupling decreases. This is accompanied with an increase in both primary and secondary stresses. The transferred power, primary and secondary stresses of PS and PP topologies increase similarly when coupling decreases in a constant input current situation.

Conversely, in a constant input voltage situation, the transferred power of PS and PP topologies decreases when coupling decreases. This is accompanied with a decrease in secondary stresses and no significant change in the primary stresses. The transferred power and secondary stresses of SS and SP topologies decrease similarly with no significant change in the primary stresses when coupling decreases in a constant input current situation.

To maintain a constant output voltage as the coupling decreases, input voltage decreases and input current increases for SS and SP topologies, while input voltage increases and input current decreases for PS and PP topologies. Secondary stresses experience no change in all topologies due to the fixed output voltage, while primary stresses in all topologies are increased due to the decreasing coupling.

Different topologies behave differently under different control conditions. Thus depending on the application, some circuit behaviours may be more desirable than the others.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

This report has presented a design procedure that intends to address two important problems which have not been studied extensively in the design of a WPT system, first is to minimise the capacitor stress by quantifying the voltage and current stresses in relation to the operating conditions of the WPT system and second to minimise the absolute resistance of a WPT coil.

Many existing coil designs have been reviewed. Most of them resulted in high enough efficiencies, but none designed to reduce AC resistance. Furthermore, most of the time, the reasons behind the inductance and capacitance values chosen were not explained at all.

The study is based on the application of the near-field resonant inductive coupling technique for high power EV charging. The conductor material used for the winding is Litz wire. The coil diameter is 60 cm and air gap is 15 cm which are realistic for this application. And the power is in the range of kilowatts. The circuits analysed are those commonly used for this application: the 2-coil system using the four basic resonant circuit topologies, i.e. SS, SP, PS and PP.

The four topologies are explained and their circuit equations are presented. The electrical parameters of circuit components are derived to establish the minimum stress condition. From this perspective of minimum stress condition, the magnitudes of the stresses don't depend on the values of the components, nor the frequency, instead they depend on the values of input and output voltage and current and the coupling factor. A simulation was then set up to verify the derived equations and the results are found to match the equations well.

Next is geometrical optimisation of coupling, linked to estimation of the number of turns from the required inductance, minimum wire length, maximum operating

frequency for each strand diameter, and resistance formula due to proximity effect. Using data from the Litz catalogue, a given coil design scenario will result in a list of AC resistances that corresponds to the list of Litz wires in the catalogue.

An experiment is set up to verify the design calculations. Using SS-topology, this experiment has put into practice a design procedure for a WPT system that considers the voltage stress of the compensation capacitors, AC resistance of the coils and the capacitors. This design procedure has resulted in the lightest coil designs that can be chosen from commercially available Litz wires. This experimental setup has achieved 98.6 % coil-to-coil efficiency with 15 cm air-gap and 0.6 kg of winding. Using the same coils, a second experimental setup has achieved 10 kW DC-to-DC power transfer with 18 cm air-gap, maintaining 98.6 % coil-to-coil efficiency and achieving 94.8 % DC-to-DC efficiency. The measured values have agreed well with the design calculations.

Sensitivity analysis has been performed to assess the changes in the circuit parameters due to the changes in coupling, using the 60 cm coils used in the experiment. The relationship between coupling and positioning has been mapped. The extent of the circuit parameters sensitivity to changes in horizontal displacement, vertical displacement and angle has been shown.

In conclusion, the design procedure for a WPT system has been devised where capacitor stresses due to resonance and AC resistance are both minimised. This becomes increasingly important as the power level increases, as component ratings and thermal dissipation limits become a larger constraint in the design of WPT systems. This procedure has been successfully put into practice for SS-topology with a unity input-output ratio.

7.2 Future Work

Even though this work was initially motivated by the application of WPT in the wireless charging of EV, the resulting design procedure has only addressed the problems of capacitor stress and AC resistance. While these problems are quite important to the near-field resonant inductive coupling technique, this procedure has neglected to include the factors unique to the EV environment. Consequently,

the tests that were conducted to verify the procedure, even though conducted at the power level of EV charging, might not be not very representative of an actual EV charging scenario.

To further explore WPT using the near-field resonant inductive coupling technique, there are a number of ways to expand this work:

- An experiment may be conducted to validate the simulation in Chapter 3.4. That simulation was set up for a system that draws power from a 3-phase 230VAC mains supply that is rectified into 560VDC, and outputs 400VDC to charge an EV battery at 10 kW. Therefore, the next experiment may follow the simulation setup that uses a 1.4 step-down voltage ratio, and is done for each topology. The experiment may use a higher frequency of 350 kHz and aim for a power of 10 kW, depending on the capabilities of the available inverter.
- An output filter for the inverter has to be designed for topologies that use a parallel compensated primary, i.e. PS and PP. This is because when the square-wave voltage from the inverter imposes too large a dV/dt across the compensation capacitor, it will draw large surges of current.
- Since the WPT system has been analysed at a fixed load operation, circuit behaviour when operated above or below rated load may be analysed. This would be a sensitivity analysis with respect to the load and should be very similar to the sensitivity analysis with respect to the coupling factor that has been presented in Chapter 6.

The minimum stress condition is a fundamental part to the operation of WPT systems. Thus this project can be further expanded to include other applications that have more varied requirements such as smaller power level down to milliwatt level, or larger transfer distance with coupling factor less than 0.1. These applications might include WPT for implantable devices, wireless sensors, mobile devices and isolated power converters.

Publications

Journals

1. Rotaru, M.D.; Tanzania, R.; Ayoob, R.; Tan Y.K.; Sykulski, J.K., "Numerical and experimental study of the effects of load and distance variation on wireless power transfer systems using magnetically coupled resonators," *Science, Measurement & Technology, IET*, vol.9, no.2, pp.160,171, 3 2015
doi: 10.1049/iet-smt.2014.0175

Conferences

1. Cai, A.; Pereira, A.; Tanzania, R.; Tan, Y.K.; Siek, L., "A High Frequency, High Efficiency GaN HFET based Inductive Power Transfer System," *Applied Power Electronics Conference and Exposition (APEC), 2015 Thirtieth Annual IEEE*, 17-19 March 2015
doi: 10.1109/APEC.2015.7104793
2. Tanzania, Robin; Choo, Fook Hoong; Siek, Liter, "Design of WPT coils to minimize AC resistance and capacitor stress applied to SS-topology," in *Industrial Electronics Society, IECON 2015 - 41st Annual Conference of the IEEE*, vol., no., pp.000118-000122, 9-12 Nov. 2015
doi: 10.1109/IECON.2015.7392085

Bibliography

- [1] Wu, H.H.; Gilchrist, A.; Sealy, K.; Israelsen, P.; Muhs, J., "A review on inductive charging for electric vehicles," Electric Machines & Drives Conference (IEMDC), 2011 IEEE International , pp.143-147, 15-18 May 2011
doi: 10.1109/IEMDC.2011.5994820
- [2] Covic, G.A.; Boys, J.T., "Modern Trends in Inductive Power Transfer for Transportation Applications," Emerging and Selected Topics in Power Electronics, IEEE Journal of , vol.1, no.1, pp.28-41, March 2013
doi: 10.1109/JESTPE.2013.2264473
- [3] Rui Chen; Cong Zheng; Zahid, Z.U.; Faraci, E.; Wengsong Yu; Jih-Sheng Lai; Senesky, M.; Anderson, D.; Lisi, G., "Analysis and parameters optimization of a contactless IPT system for EV charger," Applied Power Electronics Conference and Exposition (APEC), 2014 Twenty-Ninth Annual IEEE , pp.1654-1661, 16-20 March 2014
doi: 10.1109/APEC.2014.6803528
- [4] Zaheer, A.; Covic, G.A.; Kacprzak, D., "A Bipolar Pad in a 10-kHz 300-W Distributed IPT System for AGV Applications," Industrial Electronics, IEEE Transactions on , vol.61, no.7, pp.3288-3301, July 2014
doi: 10.1109/TIE.2013.2281167
- [5] Takanashi, H.; Sato, Y.; Kaneko, Y.; Abe, S.; Yasuda, T., "A large air gap 3 kW wireless power transfer system for electric vehicles," Energy Conversion Congress and Exposition (ECCE), 2012 IEEE , pp.269-274, 15-20 Sept. 2012
doi: 10.1109/ECCE.2012.6342813
- [6] Sallan, J.; Villa, J.L.; Llombart, A.; Sanz, J.F., "Optimal Design of ICPT Systems Applied to Electric Vehicle Battery Charge," Industrial Electronics, IEEE Transactions on , vol.56, no.6, pp.2140-2149, June 2009
doi: 10.1109/TIE.2009.2015359
- [7] RamRakhyani, A.K.; Mirabbasi, S.; Mu Chiao, "Design and Optimization of Resonance-Based Efficient Wireless Power Delivery Systems for Biomedical Implants," Biomedical Circuits and Systems, IEEE Transactions on , vol.5, no.1, pp.48-63, Feb. 2011
doi: 10.1109/TBCAS.2010.2072782
- [8] Yang, Zhi; Wentai Liu; Basham, E., "Inductor Modeling in Wireless Links for Implantable Electronics," Magnetics, IEEE Transactions on , vol.43, no.10, pp.3851-3860, Oct. 2007
doi: 10.1109/TMAG.2007.904189
- [9] Hongchang Li; Xu Yang; Kangping Wang; Xiaoshuai Dong, "Study on

- efficiency maximization design principles for Wireless Power Transfer system using magnetic resonant coupling," ECCE Asia Downunder (ECCE Asia), 2013 IEEE , pp.888-892, 3-6 June 2013
doi: 10.1109/ECCE-Asia.2013.6579210
- [10] Olivares-Galvan, J.C.; Campero-Littlewood, E.; Magdaleno-Adame, S.; Maximov, S.; Xu, W., "Wireless Power Transfer: Literature survey," Power, Electronics and Computing (ROPEC), 2013 IEEE International Autumn Meeting on , pp.1,7, 13-15 Nov. 2013
doi: 10.1109/ROPEC.2013.6702725
- [11] "History of electrical resonance," Electrical Engineering , vol.61, no.7, pp.353,353, July 1942
doi: 10.1109/EE.1942.6436401
- [12] Hui, S.Y.R.; Wenxing Zhong; Lee, C.K., "A Critical Review of Recent Progress in Mid-Range Wireless Power Transfer," Power Electronics, IEEE Transactions on , vol.29, no.9, pp.4500,4511, Sept. 2014
doi: 10.1109/TPEL.2013.2249670
- [13] Choi, S.Y.; Gu, B.W.; Jeong, S.Y.; Rim, C.T., "Trends of Wireless Power Transfer Systems for Roadway Powered Electric Vehicles," Vehicular Technology Conference (VTC Spring), 2014 IEEE 79th , vol., no., pp.1,5, 18-21 May 2014
doi: 10.1109/VTCSpring.2014.7023175
- [14] Seungyong Shin; Jaegue Shin; Boyune Song; Seokhwan Lee; Yangsu Kim; Guho Jung; Seongjeub Jeon, "Wireless power transfer system for high power application and a method of segmentation," Wireless Power Transfer (WPT), 2013 IEEE , vol., no., pp.76,78, 15-16 May 2013
doi: 10.1109/WPT.2013.6556886
- [15] Covic, G.A.; Boys, J.T., "Inductive Power Transfer," Proceedings of the IEEE , vol.101, no.6, pp.1276,1289, June 2013
doi: 10.1109/JPROC.2013.2244536
- [16] Fujita, I.; Yamanaka, T.; Kaneko, Y.; Abe, S.; Yasuda, T., "A 10kW transformer with a novel cooling structure of a contactless power transfer system for electric vehicles," Energy Conversion Congress and Exposition (ECCE), 2013 IEEE , vol., no., pp.3643,3650, 15-19 Sept. 2013
doi: 10.1109/ECCE.2013.6647181
- [17] Villa, J.L.; Sallan, J.; Sanz Osorio, J.F.; Llombart, A., "High-Misalignment Tolerant Compensation Topology For ICPT Systems," Industrial Electronics, IEEE Transactions on , vol.59, no.2, pp.945,951, Feb. 2012
doi: 10.1109/TIE.2011.2161055
- [18] Onar, Omer C.; Miller, John M.; Campbell, Steven L.; Coomer, Chester; White, Cliff.P.; Seiber, Larry E., "A novel wireless power transfer for in-motion EV/PHEV charging," Applied Power Electronics Conference and Exposition (APEC), 2013 Twenty-Eighth Annual IEEE , vol., no., pp.3073,3080, 17-21 March 2013
doi: 10.1109/APEC.2013.6520738

- [19] Bosshard, R.; Kolar, J.W.; Wunsch, B., "Accurate finite-element modeling and experimental verification of inductive power transfer coil design," Applied Power Electronics Conference and Exposition (APEC), 2014 Twenty-Ninth Annual IEEE , vol., no., pp.1648,1653, 16-20 March 2014
doi: 10.1109/APEC.2014.680352
- [20] 10 years of electric buses with IPT® Charge. Conductix-Wampfler, 31 May 2012 [Online]. Available: <http://www.conductix.com/en/news/2012-05-31/10-years-electric-buses-iptr-charge>
- [21] Qualcomm Becomes Founding Technology Partner of FIA Formula E Championship. Qualcomm, 9 September 2013 [Online]. Available: <https://www.qualcomm.com/news/releases/2013/09/09/qualcomm-becomes-founding-technology-partner-fia-formula-e-championship>
- [22] Plugless Power [Online]. Available: <https://www.pluglesspower.com>
- [23] Carlson, R. and Normann, B., "Test Results of the PLUGLESS™ Inductive Charging System from Evatran Group, Inc.," SAE Int. J. Alt. Power. 3(1):64-71, 2014, doi:10.4271/2014-01-1824.
- [24] Chwei-Sen Wang; Covic, G.A.; Stielau, O.H., "Power transfer capability and bifurcation phenomena of loosely coupled inductive power transfer systems," Industrial Electronics, IEEE Transactions on , vol.51, no.1, pp.148,157, Feb. 2004
doi: 10.1109/TIE.2003.822038
- [25] Zierhofer, C.M.; Hochmair, E.S., "Geometric approach for coupling enhancement of magnetically coupled coils," Biomedical Engineering, IEEE Transactions on , vol.43, no.7, pp.708-714, July 1996
doi: 10.1109/10.503178
- [26] Mohan, S.S.; del Mar Hershenson, M.; Boyd, S.P.; Lee, T.H., "Simple accurate expressions for planar spiral inductances," Solid-State Circuits, IEEE Journal of , vol.34, no.10, pp.1419-1424, Oct 1999
doi: 10.1109/4.792620
- [27] Petersen, M.; Fuchs F.W., "Comparative Study on Optimal Core Design for Maximizing the Coupling Coefficient in Electric Vehicle Inductive Power Transfer System," PCIM Europe 2013 , pp.14-16, 2013
- [28] Product Selection Guide. New England Wire Technologies [Online]. Available: <http://www.newenglandwire.com/newt-catalog.pdf>, pp.127-136
- [29] Antonio Carlos M. de Queiroz, "Mutual Inductance and Inductance Calculations by Maxwell's Method" [Online]. Available: <http://www.coe.ufrj.br/~acmq/tesla/maxwell.pdf>
- [30] Acero, J.; Hernandez, P.J.; Burdio, J.M.; Alonso, R.; Barragdan, L.A., "Simple resistance calculation in litz-wire planar windings for induction cooking appliances," Magnetics, IEEE Transactions on , vol.41, no.4, pp.1280-1288, April 2005
doi: 10.1109/TMAG.2005.844844
- [31] Natural Convection on Isothermal Heated Horizontal Plate Facing Up.

Bibliography

- MAYA Thermal Wizard [Online]. Available: <http://www.thermal-wizard.com/tmwiz/convect/natural/hup-isot/hup-isot.htm>
- [32] Middle and High Voltage Multilayer Ceramic Capacitors, 200V to 3kV (NP0, X7R & Y5V Dielectrics). Multicomp [Online]. Available: <http://www.farnell.com/datasheets/1876948.pdf>
- [33] Ripple Current for MLCCs. TDK [Online]. Available: <http://product.tdk.com/capacitor/mlcc/en/faq/faq00012.html>
- [34] MLCC Voltage Strength. TDK [Online]. Available: http://product.tdk.com/capacitor/mlcc/en/faq/pdf/20_mlcc_voltage_strength.pdf

Appendices

Appendix A: MATLAB Code for Observing Capacitor Stress Behaviour

In Chapter 3.3, the behaviour of the capacitor stresses is presented as the results of the MATLAB code below. The parameters in Table 1 were used to calculate capacitor stresses while varying the secondary inductance L_2 , resonant frequency f_0 , and coupling factor k . The input and output V_1, I_1, V_2, I_2 were normalised at 1.

```
clear

V2 = 1 ;
I2 = 1 ;
RL = V2/I2 ;
V1 = V2 ;
I1 = I2 ;

k = 0.05:0.05:0.5 ;
k_size = size(k,2) ;

logf = 3.69897:0.1:5.69897 ;
f = 10.^logf ; %5-500 kHz
w = 2*pi()*f ;
w_size = size(w,2) ;

logL2 = -6:.1:-3 ;
L2 = 10.^logL2 ; %1-1000 uH
L2_size = size(L2,2) ;

%struct(L2_size,w_size,k_size)
for ki=1:k_size
    for wi=1:w_size
        for L2i=1:L2_size

%SS.V1 = -j*k*sqrt(L1/C2)*I2;
SS.C2(L2i,wi,ki) = abs(1/w(wi)^2/L2(L2i)) ;
SS.L1(L2i,wi,ki) = abs(V1^2/I2^2/k(ki)^2*SS.C2(L2i,wi,ki)) ;
SS.M(L2i,wi,ki) = abs(k(ki)*sqrt(SS.L1(L2i,wi,ki)*L2(L2i))) ;
SS.C1(L2i,wi,ki) = abs(1/(w(wi)^2*SS.L1(L2i,wi,ki))) ;
%SS.I1 = -j/(w*M)*V2;
SS.VL2(L2i,wi,ki) = abs(j*w(wi)*L2(L2i)*I2) ;
SS.IL2(L2i,wi,ki) = abs(I2) ;
SS.VC2(L2i,wi,ki) = abs(-j*w(wi)*L2(L2i)*I2) ;
SS.IC2(L2i,wi,ki) = abs(I2) ;
SS.VL1(L2i,wi,ki) = abs(SS.L1(L2i,wi,ki)/SS.M(L2i,wi,ki)*V2) ;
```

Appendices

```

SS.IL1(L2i,wi,ki) = abs(-j/w(wi)*SS.M(L2i,wi,ki)*V2) ;
SS.VC1(L2i,wi,ki) = abs(-SS.L1(L2i,wi,ki)/SS.M(L2i,wi,ki)*V2) ;
SS.IC1(L2i,wi,ki) = abs(-j/w(wi)*SS.M(L2i,wi,ki)*V2) ;

%SP.V1 = k*sqrt(L1/L2)*V2;
SP.C2(L2i,wi,ki) = abs(1/w(wi)^2/L2(L2i)) ;
SP.L1(L2i,wi,ki) = abs(V1^2/V2^2/k(ki)^2*L2(L2i)) ;
SP.M(L2i,wi,ki) = abs(k(ki)*sqrt(SP.L1(L2i,wi,ki)*L2(L2i))) ;
SP.C1(L2i,wi,ki) = abs(1/(w(wi)^2*SP.L1(L2i,wi,ki)*(1-k(ki)^2))) ;
%SP.I1 = L2/M*I2;
SP.VL2(L2i,wi,ki) = abs((-RL+j*w(wi)*L2(L2i))*I2) ;
SP.IL2(L2i,wi,ki) = abs((1+j*RL/(w(wi)*L2(L2i)))*I2) ;
SP.VC2(L2i,wi,ki) = abs(V2) ;
SP.IC2(L2i,wi,ki) = abs(j/(w(wi)*L2(L2i))*V2) ;
SP.VL1(L2i,wi,ki) =
abs(j*w(wi)*SP.L1(L2i,wi,ki)*L2(L2i)/SP.M(L2i,wi,ki)*I2) ;
SP.IL1(L2i,wi,ki) = abs(L2(L2i)/SP.M(L2i,wi,ki)*I2) ;
SP.VC1(L2i,wi,ki) = abs(-
j*w(wi)*SP.L1(L2i,wi,ki)*L2(L2i)/SP.M(L2i,wi,ki)*(1-k(ki)^2)*I2) ;
SP.IC1(L2i,wi,ki) = abs(L2(L2i)/SP.M(L2i,wi,ki)*I2) ;

%PS.V1 = (1/k*sqrt(L1/L2)-j*k*sqrt(L1/C2)/RL)*V2;
PS.C2(L2i,wi,ki) = abs(1/w(wi)^2/L2(L2i)) ;
PS.L1(L2i,wi,ki) = abs(V1^2/V2^2/(1/k(ki)^2/L2(L2i)
+k(ki)^2/PS.C2(L2i,wi,ki)/RL^2)) ;
PS.M(L2i,wi,ki) = abs(k(ki)*sqrt(PS.L1(L2i,wi,ki)*L2(L2i))) ;
PS.C1(L2i,wi,ki) = abs(1/(w(wi)^2*PS.L1(L2i,wi,ki)+
(w(wi)^4*PS.M(L2i,wi,ki)^4)/(RL^2*PS.L1(L2i,wi,ki)))) ;
%PS.I1 = 1/(L1/M+j*w*M/RL)*I2;
PS.VL2(L2i,wi,ki) = abs(j*w(wi)*L2(L2i)*I2) ;
PS.IL2(L2i,wi,ki) = abs(I2) ;
PS.VC2(L2i,wi,ki) = abs(-j*w(wi)*L2(L2i)*I2) ;
PS.IC2(L2i,wi,ki) = abs(I2) ;
PS.VL1(L2i,wi,ki) = abs(PS.L1(L2i,wi,ki)/PS.M(L2i,wi,ki)*V2) ;
PS.IL1(L2i,wi,ki) = abs(-j/(w(wi)*PS.M(L2i,wi,ki))*V2) ;
PS.VC1(L2i,wi,ki) = abs((PS.L1(L2i,wi,ki)/PS.M(L2i,wi,ki)-
j*w(wi)*PS.M(L2i,wi,ki)/RL)*V2) ;
PS.IC1(L2i,wi,ki) = abs((w(wi)*PS.M(L2i,wi,ki)
+j*PS.L1(L2i,wi,ki)/PS.M(L2i,wi,ki)*RL)*w(wi)*PS.C1(L2i,wi,ki)*I2)
;

%PP.V1 = (k*sqrt(L1/L2)+j/k*sqrt(L1/C2)/RL*(1-k^2))*V2;
PP.C2(L2i,wi,ki) = abs(1/w(wi)^2/L2(L2i)) ;
PP.L1(L2i,wi,ki) = abs(V1^2/V2^2/(k(ki)^2/L2(L2i)
+1/k(ki)^2/PP.C2(L2i,wi,ki)/RL^2*(1-k(ki)^2)^2)) ;
PP.M(L2i,wi,ki) = abs(k(ki)*sqrt(PP.L1(L2i,wi,ki)*L2(L2i))) ;
PP.C1(L2i,wi,ki) = abs(1/(w(wi)^2*PP.L1(L2i,wi,ki)*(1-k(ki)^2)+
(k(ki)^4*PP.L1(L2i,wi,ki)*RL^2)/((1-k(ki)^2)*L2(L2i)^2))) ;
%PP.I1 = 1/(M/L2-j*w*L1*L2/(M*RL))*(1-k^2)*I2;
PP.VL2(L2i,wi,ki) = abs((-RL+j*w(wi)*L2(L2i))*I2) ;
PP.IL2(L2i,wi,ki) = abs((1+j*RL/(w(wi)*L2(L2i)))*I2) ;
PP.VC2(L2i,wi,ki) = abs(V2) ;
PP.IC2(L2i,wi,ki) = abs(j/(w(wi)*L2(L2i))*V2) ;
PP.VL1(L2i,wi,ki) =
abs(j*w(wi)*PP.L1(L2i,wi,ki)*L2(L2i)/PP.M(L2i,wi,ki)*I2) ;
PP.IL1(L2i,wi,ki) = abs(L2(L2i)/PP.M(L2i,wi,ki)*I2) ;
PP.VC1(L2i,wi,ki) = abs((PP.M(L2i,wi,ki)/L2(L2i)
+j*w(wi)*PP.L1(L2i,wi,ki)*L2(L2i)/(PP.M(L2i,wi,ki)*RL)*(1-
k(ki)^2))*V2) ;
PP.IC1(L2i,wi,ki) = abs((-
w(wi)*PP.L1(L2i,wi,ki)*L2(L2i)/PP.M(L2i,wi,ki)*(1-
k(ki)^2)+j*PP.M(L2i,wi,ki)/L2(L2i)*RL)*w(wi)*PP.C1(L2i,wi,ki)*I2)

```

```
;
    end
end
end

figure;
hold on; surf(f,L2,SS.VC1(:,:,2),'FaceColor','y');
surf(f,L2,SS.VC2(:,:,2),'FaceColor','b');
title('SS topology, k = 0.1','FontSize',14); xlabel('f_0 (Hz)','FontSize',14); ylabel('L_2 (H)','FontSize',14);
zlabel('V_C_1 (V) and V_C_2 (V)','FontSize',14);
set(gca,'XScale','log','YScale','log','ZScale','log','XGrid','on','XMinorGrid','off','YGrid','on','YMinorGrid','off','ZGrid','on','ZMinorGrid','off'); view(-30,10);
figure;
hold on; surf(f,L2,PP.IC1(:,:,2),'FaceColor','y');
surf(f,L2,PP.IC2(:,:,2),'FaceColor','b');
title('PP topology, k = 0.1','FontSize',14); xlabel('f_0 (Hz)','FontSize',14); ylabel('L_2 (H)','FontSize',14);
zlabel('I_C_1 (A) and I_C_2 (A)','FontSize',14);
set(gca,'XScale','log','YScale','log','ZScale','log','XGrid','on','XMinorGrid','off','YGrid','on','YMinorGrid','off','ZGrid','on','ZMinorGrid','off'); view(-30,10);
figure;
hold on; surf(f,L2,SP.VC1(:,:,2),'FaceColor','y');
surf(f,L2,SP.IC2(:,:,2),'FaceColor','b');
title('SP topology, k = 0.1','FontSize',14); xlabel('f_0 (Hz)','FontSize',14); ylabel('L_2 (H)','FontSize',14);
zlabel('V_C_1 (V) and I_C_2 (A)','FontSize',14);
set(gca,'XScale','log','YScale','log','ZScale','log','XGrid','on','XMinorGrid','off','YGrid','on','YMinorGrid','off','ZGrid','on','ZMinorGrid','off'); view(-30,10);
figure;
hold on; surf(f,L2,PS.IC1(:,:,2),'FaceColor','y');
surf(f,L2,PS.VC2(:,:,2),'FaceColor','b');
title('PS topology, k = 0.1','FontSize',14); xlabel('f_0 (Hz)','FontSize',14); ylabel('L_2 (H)','FontSize',14);
zlabel('I_C_1 (A) and V_C_2 (V)','FontSize',14);
set(gca,'XScale','log','YScale','log','ZScale','log','XGrid','on','XMinorGrid','off','YGrid','on','YMinorGrid','off','ZGrid','on','ZMinorGrid','off'); view(-30,10);
```

Appendix B: MATLAB Code for Calculating Power Dissipation by Natural Convection

In Chapter 5.2, one way to find the maximum permissible AC resistance is from the maximum thermal dissipation of the packaging. Here, the power dissipation from natural convection is used to estimate the maximum thermal dissipation. The method of “Natural Convection on Isothermal Heated Horizontal Plate Facing Up”

[17] is used and adapted into the MATLAB code below.

```
function q = convection(Ta,Tp,r) % ambient temp, plate temp,
average radius

A = pi*16/9*r^2; % plate area
p = pi*8/3*r ; % plate perimeter

Tref = 20; % referenced to air at 20 degC
rho_ref = 1.201; % density at reference
Cp = 1007; % specific heat capacity
k = 0.0261; % thermal conductivity
CTE = 0.0034; % coefficient of thermal expansion
mu = 0.0000185; % dynamic viscosity
g = 9.81; % gravity

Tf = (Tp+Ta)/2; % film temperature
rho = rho_ref*(Tref+273)/(Tf+273); % film density
cl = A/p; % characteristic dimension
a = k/rho/Cp; % thermal diffusivity
v = mu/rho; % kinematic viscosity
Pr = v/a; % Prandtl number
Gr = g*CTE*(Tp-Ta)*cl^3/v^2; % Grashof number
Ra = Gr*Pr; % Rayleigh number

if Ra>=2E4 && Ra<=1E6
    Nu = 0.54*(Gr*Pr)^0.25;
elseif Ra>1E6 && Ra<=1E11
    Nu = 0.15*(Gr*Pr)^0.33; % Nusselt Number
else
    error('Rayleigh number is out of range.');
```

```
end

h = Nu*k/cl; % average heat transfer coefficient
q = h*A*(Tp-Ta); % heat flow from the plate

end
```

Appendix C: MATLAB Code for Surface Fitting

In Chapter 6.1, the surface-fitting and the correlation equation between coupling and positioning was obtained using the MATLAB code below.

```
clear

% varying x at y = [0.04;0.12;0.15;0.18;0.36;0.48]
y = [0.04;0.12;0.15;0.18;0.36;0.48];
x = 0:0.04:0.28;

k = zeros(6,8);
k(1,:) = [0.652364357 0.632136005 0.573979492 0.495594628
0.400774227 0.294575378 0.206076338 0.146655553];
```

Appendices

```
k(2,:) = [0.367903155 0.355260435 0.32744645 0.28951829
0.24526877 0.195962161 0.145391281 0.099877489];
k(3,:) = [0.28951829 0.284461202 0.262968578 0.236418866
0.20101925 0.166883905 0.128955745 0.092291857];
k(4,:) = [0.250325858 0.24526877 0.228833234 0.208604882
0.179526625 0.150448369 0.113784481 0.088499041];
k(5,:) = [0.093556129 0.091027585 0.087234769 0.082177681
0.074592049 0.065742145 0.05689224 0.048042336];
k(6,:) = [0.051835152 0.051835152 0.049306608 0.046778064
0.042985248 0.039192432 0.035399616 0.030342528];

poly3fit = 'a*x^3+b*x^2+c';
poly3start = [1 -1 .1];
fx = zeros(3,5);
fx(:,1) = coeffvalues(fit(x',k(1,:)','poly3fit','Start',
poly3start));
fx(:,2) = coeffvalues(fit(x',k(2,:)','poly3fit','Start',
poly3start));
fx(:,3) = coeffvalues(fit(x',k(3,:)','poly3fit','Start',
poly3start));
fx(:,4) = coeffvalues(fit(x',k(4,:)','poly3fit','Start',
poly3start));
fx(:,5) = coeffvalues(fit(x',k(5,:)','poly3fit','Start',
poly3start));
fx(:,6) = coeffvalues(fit(x',k(6,:)','poly3fit','Start',
poly3start));

expfit = 'a*exp(b*x)+c';
expstart = [0 0 0];
fc = zeros(3,3);
fc(:,1) = coeffvalues(fit(y,fx(1,:)','expfit','Start', expstart));
fc(:,2) = coeffvalues(fit(y,fx(2,:)','expfit','Start', expstart));
fc(:,3) = coeffvalues(fit(y,fx(3,:)','expfit','Start', expstart));

y1 = 0.00:0.01:0.50 ;
y1_size = size(y1,2) ;
x1 = 0:0.01:0.30 ;
x1_size = size(x1,2) ;
fxy = zeros(y1_size,x1_size);
for xli=1:x1_size
    for yli=1:y1_size
        fxy(yli,xli) = (fc(1,1)*exp(fc(2,1)*y1(yli))+fc(3,1))*x1(xli)^3 +
        (fc(1,2)*exp(fc(2,2)*y1(yli))+fc(3,2))*x1(xli)^2 +
        (fc(1,3)*exp(fc(2,3)*y1(yli))+fc(3,3));
    end
end
end

o =
[0,0.04,0.651100085;0,0.08,0.485480452;0,0.12,0.369167427;0,0.16,0.
.280668386;0,0.2,0.217454786;0,0.24,0.174469537;0,0.28,0.139069921
;0,0.32,0.112520209;0,0.36,0.091027585;0,0.4,0.074592049;0,0.44,0.
061949328;0,0.48,0.051835152;0,0.04,0.652364357;0.04,0.04,0.632136
005;0.08,0.04,0.573979492;0.12,0.04,0.495594628;0.16,0.04,0.400774
227;0.2,0.04,0.294575378;0.24,0.04,0.206076338;0.28,0.04,0.1466555
53;0,0.12,0.367903155;0.04,0.12,0.355260435;0.08,0.12,0.32744645;0
.12,0.12,0.28951829;0.16,0.12,0.24526877;0.2,0.12,0.195962161;0.24
,0.12,0.145391281;0.28,0.12,0.099877489;0,0.15,0.28951829;0.04,0.1
5,0.284461202;0.08,0.15,0.262968578;0.12,0.15,0.236418866;0.16,0.1
5,0.20101925;0.2,0.15,0.166883905;0.24,0.15,0.128955745;0.28,0.15,
0.092291857;0,0.18,0.250325858;0.04,0.18,0.24526877;0.08,0.18,0.22
8833234;0.12,0.18,0.208604882;0.16,0.18,0.179526625;0.2,0.18,0.150
448369;0.24,0.18,0.113784481;0.28,0.18,0.088499041;0,0.36,0.093556
```

```
129;0.04,0.36,0.091027585;0.08,0.36,0.087234769;0.12,0.36,0.082177
681;0.16,0.36,0.074592049;0.2,0.36,0.065742145;0.24,0.36,0.0568922
4;0.28,0.36,0.048042336;0,0.48,0.051835152;0.04,0.48,0.051835152;0
.08,0.48,0.049306608;0.12,0.48,0.046778064;0.16,0.48,0.042985248;0
.2,0.48,0.039192432;0.24,0.48,0.035399616;0.28,0.48,0.030342528;];
```

```
figure;
hold on; scatter3(o(:,1),o(:,2),o(:,3));
mesh(x1,y1,fxy,'EdgeColor','k','FaceAlpha',0); hold off;
set(legend('measured data','estimated surface'),'FontSize',16);
xlabel('x (m)','FontSize',16); ylabel('y (m)','FontSize',16);
zlabel('k','FontSize',16); view(135,30); xlim([0 .3]);
set(gca, 'XGrid','on', 'XMinorGrid','off', 'YGrid','on',
'YMinorGrid','off', 'ZGrid','on', 'ZMinorGrid','off');
```

Appendix D: MATLAB Code for Solving Circuit Equations

In Chapter 6.2, the MATLAB code below was written to evaluate the effect of changing k for a constant input voltage condition. The circuit equations were re-arranged to solve for the parameters of interest, which are V_2 , I_2 , V_{L2} , I_{L2} , V_{C2} , I_{C2} , V_{L1} , I_{L1} , V_{C1} , I_{C1} and I_1 . The equations were expressed in terms of V_1 , L_2 , C_2 , L_1 , C_1 , ω , R_L , which are considered fixed, and M , which is the variable for this sensitivity analysis.

In Chapter 6.3 and Chapter 6.4, the same code was slightly modified to re-arrange the circuit equations to solve for the parameters of interest for a constant input current condition and a constant output voltage condition, respectively.

```
clear

symSS = solve( 'V1-(j*w*L1+1/(j*w*C1))*IL1+j*w*M*IL2=0',...
'j*w*M*IL1-(j*w*L2+1/(j*w*C2))*IL2-V2=0',...
'V2=RL*I2',...
'IL2=I2',...
'IC2=I2',...
'IL1=I1',...
'IC1=I1',...
'VL2=(j*w*L2)*IL2',...
'VC2=1/(j*w*C2)*IC2',...
'VL1=(j*w*L1)*IL1',...
'VC1=1/(j*w*C1)*IC1',...
'V2','I2','VL2','IL2','VC2','IC2','VL1','IL1','VC1','IC1','I1'
);
symSS = [symSS.V2; symSS.I2; symSS.VL2; symSS.IL2; symSS.VC2;
symSS.IC2; symSS.VL1; symSS.IL1; symSS.VC1; symSS.IC1; symSS.I1]

symSP = solve( 'V1-(j*w*L1+1/(j*w*C1))*IL1+j*w*M*IL2=0',...
```



```

'j*w*M*IL1-(j*w*L2)*IL2-1/(j*w*C2)*(IL2-IL2)=0',...
'-1/(j*w*C2)*(IL2-IL2)-V2=0',...
'V2=RL*I2',...
'IC2=IL2-IL2',...
'IL1=I1',...
'IC1=I1',...
'VL2=(j*w*L2)*IL2',...
'VC2=1/(j*w*C2)*IC2',...
'VL1=(j*w*L1)*IL1',...
'VC1=1/(j*w*C1)*IC1',...
'V2','I2','VL2','IL2','VC2','IC2','VL1','IL1','VC1','IC1','I1'
);
symSP = [symSP.V2; symSP.I2; symSP.VL2; symSP.IL2; symSP.VC2;
symSP.IC2; symSP.VL1; symSP.IL1; symSP.VC1; symSP.IC1; symSP.I1]

symPS = solve('V1-1/(j*w*C1)*(I1-IL1)=0',...
'-1/(j*w*C1)*(IL1-I1)-(j*w*L1)*IL1+j*w*M*IL2=0',...
'j*w*M*IL1-(j*w*L2+1/(j*w*C2))*IL2-V2=0',...
'V2=RL*I2',...
'IC2=I2',...
'IC1=I1-IL1',...
'IL2=I2',...
'VL2=(j*w*L2)*IL2',...
'VC2=1/(j*w*C2)*IC2',...
'VL1=(j*w*L1)*IL1',...
'VC1=1/(j*w*C1)*IC1',...
'V2','I2','VL2','IL2','VC2','IC2','VL1','IL1','VC1','IC1','I1'
);
symPS = [symPS.V2; symPS.I2; symPS.VL2; symPS.IL2; symPS.VC2;
symPS.IC2; symPS.VL1; symPS.IL1; symPS.VC1; symPS.IC1; symPS.I1]

symPP = solve('V1-1/(j*w*C1)*(I1-IL1)=0',...
'-1/(j*w*C1)*(IL1-I1)-(j*w*L1)*IL1+j*w*M*IL2=0',...
'j*w*M*IL1-(j*w*L2)*IL2-1/(j*w*C2)*(IL2-IL2)=0',...
'-1/(j*w*C2)*(IL2-IL2)-V2=0',...
'V2=RL*I2',...
'IC2=IL2-IL2',...
'IC1=I1-IL1',...
'VL2=(j*w*L2)*IL2',...
'VC2=1/(j*w*C2)*IC2',...
'VL1=(j*w*L1)*IL1',...
'VC1=1/(j*w*C1)*IC1',...
'V2','I2','VL2','IL2','VC2','IC2','VL1','IL1','VC1','IC1','I1'
);
symPP = [symPP.V2; symPP.I2; symPP.VL2; symPP.IL2; symPP.VC2;
symPP.IC2; symPP.VL1; symPP.IL1; symPP.VC1; symPP.IC1; symPP.I1]

```

Appendix E: MATLAB Code for Calculating Parameter Change due to Displacement

In Chapter 6.2, the correlation equation was inserted into the circuit equations to obtain and plot the changes in parameters due to the changes in relative positioning for a constant input voltage condition using the MATLAB code below.

In Chapter 6.3 and Chapter 6.4, the same code was again slightly modified to obtain and plot the changes in parameters for a constant input current condition and a constant output voltage condition, respectively.

```
clear
% k and E varying
fc = [ 48.8221775355106    -22.3508838674483    0.842511650725441;
      -12.8617068399440   -10.8798428557297   -7.82840776257341;
        0.653184050206234   -0.402388081618429    0.0372860169017289];
x0 = 0 ;
y0 = .15 ;

k = (fc(1,1)*exp(fc(2,1)*y0)+fc(3,1))*x0^3 +
    (fc(1,2)*exp(fc(2,2)*y0)+fc(3,2))*x0^2 +
    (fc(1,3)*exp(fc(2,3)*y0)+fc(3,3));
k_size = size(k,2) ;

x1 = 0:.02:.2 ;
x1_size = size(x1,2) ;
Ex = zeros(k_size,x1_size);

y1 = .1:.01:.2 ;
y1_size = size(y1,2) ;
Ey = zeros(k_size,y1_size);
y = y1-y0;

RL = 16 ;
a = 1 ;
w = 1350000 ;

for ki=1:k_size

SS.L2(ki) = sqrt(a)*RL/(w*k(ki));
SS.L1(ki) = a*SS.L2(ki);
SS.C2(ki) = 1/w^2/SS.L2(ki);
SS.C1(ki) = 1/w^2/SS.L1(ki);
SS.M(ki) = k(ki)*sqrt(SS.L1(ki)*SS.L2(ki));
SS.V2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.I2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.VL2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.IL2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.VC2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
```

Appendices

```

SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.IC2(ki) = abs(SS.M(ki)/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.VL1(ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.IL1(ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.VC1(ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.IC1(ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));
SS.I1(ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.M(ki)^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1));

SP.L2(ki) = RL/(sqrt(a)*w*sqrt(1/k(ki)^2-1));
SP.L1(ki) = a^2*SP.L2(ki)/k(ki)^2;
SP.C2(ki) = 1/w^2/SP.L2(ki);
SP.C1(ki) = 1/w^2/SP.L1(ki)/(1-k(ki)^2);
SP.M(ki) = k(ki)*sqrt(SP.L1(ki)*SP.L2(ki));
SP.V2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.I2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.VL2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.IL2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +

```

Appendices

```

SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.VC2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.IC2(ki) = abs(SP.M(ki)/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.VL1(ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.IL1(ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.VC1(ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.IC1(ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));
SP.II1(ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.M(ki)^2*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3 +
SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2 -
SP.C1(ki)*SP.C2(ki)*SP.M(ki)^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4));

PS.L2(ki) = RL/(sqrt(a)*w*k(ki));
PS.L1(ki) = a^3*k(ki)^2*PS.L2(ki)/(a+k(ki)^2);
PS.C2(ki) = 1/w^2/PS.L2(ki);
PS.C1(ki) = 1/w^2/PS.L1(ki)/(1+w^2*k(ki)^4*PS.L2(ki)^2/RL^2);
PS.M(ki) = k(ki)*sqrt(PS.L1(ki)*PS.L2(ki));
PS.V2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.I2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.VL2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.IL2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
SP.VC2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +

```

Appendices

```

PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki));
PS.IC2(ki) = abs(PS.M(ki)/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.VL1(ki) = abs(1/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.IL1(ki) = abs(1/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));
PS.VC1(ki) = abs(1);
PS.IC1(ki) = abs(1);
PS.I1(ki) = abs((PS.C1(ki)*PS.L1(ki)*j^2*w^2 +
PS.C2(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*RL*j*w -
PS.C1(ki)*PS.C2(ki)*PS.M(ki)^2*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*RL*j^3*w^3 + 1)...
/(- PS.C2(ki)*PS.M(ki)^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)));

PP.L2(ki) = sqrt(a)*RL/w*k(ki)/sqrt(1-k(ki)^2);
PP.L1(ki) = a^2*PP.L2(ki)/(a+k(ki)^2*(1-a));
PP.C2(ki) = 1/w^2/PP.L2(ki);
PP.C1(ki) = 1/(w^2*PP.L1(ki)*(1-k(ki)^2)+k(ki)^4*PP.L1(ki)*RL^2/
(1-k(ki)^2)/PP.L2(ki)^2);
PP.M(ki) = k(ki)*sqrt(PP.L1(ki)*PP.L2(ki));
PP.V2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.I2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.VL2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.IL2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.VC2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.IC2(ki) = abs(PP.M(ki)/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.VL1(ki) = abs(1/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.IL1(ki) = abs(1/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));
PP.VC1(ki) = abs(1);
PP.IC1(ki) = abs(1);
PP.I1(ki) = abs((RL + PP.L2(ki)*j*w -
PP.C1(ki)*PP.M(ki)^2*j^3*w^3 +
PP.C1(ki)*PP.L1(ki)*PP.L2(ki)*j^3*w^3 +
PP.C1(ki)*PP.L1(ki)*RL*j^2*w^2 + PP.C2(ki)*PP.L2(ki)*RL*j^2*w^2 -
PP.C1(ki)*PP.C2(ki)*PP.M(ki)^2*RL*j^4*w^4 +
PP.C1(ki)*PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^4*w^4)...
/(- PP.C2(ki)*RL*PP.M(ki)^2*j^2*w^2 -
PP.M(ki)^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +

```

Appendices

```

PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL));

for xli=1:x1_size
Ex(xli) = ((fc(1,1)*exp(fc(2,1)*y0)+fc(3,1))*x1(xli)^3 +
(fc(1,2)*exp(fc(2,2)*y0)+fc(3,2))*x1(xli)^2 +
(fc(1,3)*exp(fc(2,3)*y0)+fc(3,3)))/k(ki);

SS.ME = Ex(xli)*SS.M(ki);

SS.V2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.V2(ki);
SS.I2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.I2(ki);
SS.VL2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VL2(ki);
SS.IL2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IL2(ki);
SS.VC2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VC2(ki);
SS.IC2Ex(xli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IC2(ki);
SS.VL1Ex(xli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VL1(ki);
SS.IL1Ex(xli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IL1(ki);
SS.VC1Ex(xli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VC1(ki);
SS.IC1Ex(xli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IC1(ki);
SS.I1Ex(xli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +

```

$$\frac{SS.C1(k_i) * SS.C2(k_i) * SS.L1(k_i) * SS.L2(k_i) * j^4 * w^4 + SS.C1(k_i) * SS.C2(k_i) * SS.L1(k_i) * RL * j^3 * w^3 + 1)}{SS.I1(k_i)};$$
$$SP.ME = E_X(x_{1i}) * SP.M(k_i);$$
[illegible]

```
PS.ME = Ex(x1i)*PS.M(ki);
```

Appendices

```

PS.V2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.V2(ki);
PS.I2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.I2(ki);
PS.VL2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VL2(ki);
PS.IL2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IL2(ki);
PS.VC2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VC2(ki);
PS.IC2Ex(xli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IC2(ki);
PS.VL1Ex(xli,ki) = abs(1/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VL1(ki);
PS.IL1Ex(xli,ki) = abs(1/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IL1(ki);
PS.VC1Ex(xli,ki) = abs(1)/PS.VC1(ki);
PS.IC1Ex(xli,ki) = abs(1)/PS.IC1(ki);
PS.I1Ex(xli,ki) = abs((PS.C1(ki)*PS.L1(ki)*j^2*w^2 +
PS.C2(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*RL*j*w -
PS.C1(ki)*PS.C2(ki)*PS.ME^2*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*RL*j^3*w^3 + 1)...
/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.I1(ki);

```

```

PP.ME = Ex(xli)*PP.M(ki);

```

```

PP.V2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.V2(ki);
PP.I2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.I2(ki);
PP.VL2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.VL2(ki);
PP.IL2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IL2(ki);
PP.VC2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.VC2(ki);
PP.IC2Ex(xli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IC2(ki);
PP.VL1Ex(xli,ki) = abs(1/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.VL1(ki);
PP.IL1Ex(xli,ki) = abs(1/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IL1(ki);

```


Appendices

```

PP.VC1Ex(xli,ki) = abs(1)/PP.VC1(ki);
PP.IC1Ex(xli,ki) = abs(1)/PP.IC1(ki);
PP.I1Ex(xli,ki) = abs((RL + PP.L2(ki)*j*w -
PP.C1(ki)*PP.ME^2*j^3*w^3 + PP.C1(ki)*PP.L1(ki)*PP.L2(ki)*j^3*w^3
+ PP.C1(ki)*PP.L1(ki)*RL*j^2*w^2 + PP.C2(ki)*PP.L2(ki)*RL*j^2*w^2
- PP.C1(ki)*PP.C2(ki)*PP.ME^2*RL*j^4*w^4 +
PP.C1(ki)*PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^4*w^4)...
/((- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.I1(ki);

end

for yli=1:y1_size
Ey(yli) = ((fc(1,1)*exp(fc(2,1)*y1(yli))+fc(3,1))*x0^3 +
(fc(1,2)*exp(fc(2,2)*y1(yli))+fc(3,2))*x0^2 +
(fc(1,3)*exp(fc(2,3)*y1(yli))+fc(3,3)))/k(ki);

SS.ME = Ey(yli)*SS.M(ki);

SS.V2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.V2(ki);
SS.I2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.I2(ki);
SS.VL2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VL2(ki);
SS.IL2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IL2(ki);
SS.VC2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VC2(ki);
SS.IC2Ey(yli,ki) = abs(SS.ME/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IC2(ki);
SS.VL1Ey(yli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.VL1(ki);
SS.IL1Ey(yli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -
SS.C1(ki)*SS.C2(ki)*SS.ME^2*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*SS.L2(ki)*j^4*w^4 +
SS.C1(ki)*SS.C2(ki)*SS.L1(ki)*RL*j^3*w^3 + 1))/SS.IL1(ki);
SS.VC1Ey(yli,ki) = abs(1/(SS.C1(ki)*SS.L1(ki)*j^2*w^2 +
SS.C2(ki)*SS.L2(ki)*j^2*w^2 + SS.C2(ki)*RL*j*w -

```

[illegible]

Appendices

```

SP.C1(ki)*SP.ME^2*j^3*w^3 + SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3
+ SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2
- SP.C1(ki)*SP.C2(ki)*SP.ME^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4)/SP.IC1(ki);
SP.I1Ey(yli,ki) = abs(1/(RL + SP.L2(ki)*j*w -
SP.C1(ki)*SP.ME^2*j^3*w^3 + SP.C1(ki)*SP.L1(ki)*SP.L2(ki)*j^3*w^3
+ SP.C1(ki)*SP.L1(ki)*RL*j^2*w^2 + SP.C2(ki)*SP.L2(ki)*RL*j^2*w^2
- SP.C1(ki)*SP.C2(ki)*SP.ME^2*RL*j^4*w^4 +
SP.C1(ki)*SP.C2(ki)*SP.L1(ki)*SP.L2(ki)*RL*j^4*w^4)/SP.I1(ki);

PS.ME = Ey(yli)*PS.M(ki);

PS.V2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.V2(ki);
PS.I2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.I2(ki);
PS.VL2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VL2(ki);
PS.IL2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IL2(ki);
PS.VC2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VC2(ki);
PS.IC2Ey(yli,ki) = abs(PS.ME/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IC2(ki);
PS.VL1Ey(yli,ki) = abs(1/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.VL1(ki);
PS.IL1Ey(yli,ki) = abs(1/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.IL1(ki);
PS.VC1Ey(yli,ki) = abs(1)/PS.VC1(ki);
PS.IC1Ey(yli,ki) = abs(1)/PS.IC1(ki);
PS.I1Ey(yli,ki) = abs((PS.C1(ki)*PS.L1(ki)*j^2*w^2 +
PS.C2(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*RL*j*w -
PS.C1(ki)*PS.C2(ki)*PS.ME^2*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^4*w^4 +
PS.C1(ki)*PS.C2(ki)*PS.L1(ki)*RL*j^3*w^3 + 1)...
/(- PS.C2(ki)*PS.ME^2*j^2*w^2 +
PS.C2(ki)*PS.L1(ki)*PS.L2(ki)*j^2*w^2 + PS.C2(ki)*PS.L1(ki)*RL*j*w
+ PS.L1(ki)))/PS.I1(ki);

PP.ME = Ey(yli)*PP.M(ki);

PP.V2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.V2(ki);
PP.I2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.I2(ki);
PP.VL2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.VL2(ki);
PP.IL2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IL2(ki);
PP.VC2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -

```

Appendices

```

PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL)/PP.VC2(ki);
PP.IC2Ey(yli,ki) = abs(PP.ME/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IC2(ki);
PP.VL1Ey(yli,ki) = abs(1/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.VL1(ki);
PP.IL1Ey(yli,ki) = abs(1/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.IL1(ki);
PP.VC1Ey(yli,ki) = abs(1)/PP.VC1(ki);
PP.IC1Ey(yli,ki) = abs(1)/PP.IC1(ki);
PP.I1Ey(yli,ki) = abs((RL + PP.L2(ki)*j*w -
PP.C1(ki)*PP.ME^2*j^3*w^3 + PP.C1(ki)*PP.L1(ki)*PP.L2(ki)*j^3*w^3
+ PP.C1(ki)*PP.L1(ki)*RL*j^2*w^2 + PP.C2(ki)*PP.L2(ki)*RL*j^2*w^2
- PP.C1(ki)*PP.C2(ki)*PP.ME^2*RL*j^4*w^4 +
PP.C1(ki)*PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^4*w^4) ...
/(- PP.C2(ki)*RL*PP.ME^2*j^2*w^2 -
PP.ME^2*j*w + PP.C2(ki)*PP.L1(ki)*PP.L2(ki)*RL*j^2*w^2 +
PP.L1(ki)*PP.L2(ki)*j*w + PP.L1(ki)*RL))/PP.I1(ki);

```

```

end
end

```

```

figure;
subplot(2,2,1); hold on; title('SS','FontSize',14); xlabel('\Delta
x (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); ylim([1 3.2]);
plot(x1,SS.V2Ex(:,1),'r-o');
plot(x1,SS.I1Ex(:,1),'b-s');
legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','I_1 , V_L_1 , I_L_1 , V_C_1 ,
I_C_1','Location','northwest');
hold off;
subplot(2,2,2); hold on; title('SP','FontSize',14); xlabel('\Delta
x (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); ylim([1 3.2]);
plot(x1,SP.V2Ex(:,1),'r-o');
plot(x1,SP.I1Ex(:,1),'b-s');
legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','I_1 , V_L_1 , I_L_1 , V_C_1 ,
I_C_1','Location','northwest');
hold off;
subplot(2,2,3); hold on; title('PS','FontSize',14); xlabel('\Delta
x (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); ylim([.3 1.1]);
plot(x1,PS.V2Ex(:,1),'r-o');
plot(x1,PS.VL1Ex(:,1),'k-*');
plot(x1,PS.VC1Ex(:,1),'g-^');
plot(x1,PS.I1Ex(:,1),'b-s');
legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','V_L_1 , I_L_1','V_C_1 ,
I_C_1','I_1','Location','southwest');
hold off;
subplot(2,2,4); hold on; title('PP','FontSize',14); xlabel('\Delta
x (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); ylim([.3 1.1]);
plot(x1,PP.V2Ex(:,1),'r-o');
plot(x1,PP.VL1Ex(:,1),'k-*');
plot(x1,PP.VC1Ex(:,1),'g-^');

```

Appendices

```

        plot(x1,PP.I1Ex(:,1),'b-s');
        legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','V_L_1 , I_L_1','V_C_1 ,
I_C_1','I_1','Location','southwest');
        hold off;

figure;
subplot(2,2,1); hold on; title('SS','FontSize',14); xlabel('\Delta
y (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); xlim([-0.05 0.05]); ylim([0.4 2]);
        plot(y,SS.V2Ey(:,1),'r-o');
        plot(y,SS.I1Ey(:,1),'b-s');
        legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','I_1 , V_L_1 , I_L_1 , V_C_1 ,
I_C_1','Location','northwest');
        hold off;
subplot(2,2,2); hold on; title('SP','FontSize',14); xlabel('\Delta
y (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); xlim([-0.05 0.05]); ylim([0.4 2]);
        plot(y,SP.V2Ey(:,1),'r-o');
        plot(y,SP.I1Ey(:,1),'b-s');
        legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','I_1 , V_L_1 , I_L_1 , V_C_1 ,
I_C_1','Location','northwest');
        hold off;
subplot(2,2,3); hold on; title('PS','FontSize',14); xlabel('\Delta
y (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); xlim([-0.05 0.05]); ylim([0.4 2]);
        plot(y,PS.V2Ey(:,1),'r-o');
        plot(y,PS.VL1Ey(:,1),'k-*');
        plot(y,PS.VC1Ey(:,1),'g-^');
        plot(y,PS.I1Ey(:,1),'b-s');
        legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','V_L_1 , I_L_1','V_C_1 , I_C_1','I_1');
        hold off;
subplot(2,2,4); hold on; title('PP','FontSize',14); xlabel('\Delta
y (m)','FontSize',14); ylabel('Parameter change
ratio','FontSize',14); xlim([-0.05 0.05]); ylim([0.4 2]);
        plot(y,PP.V2Ey(:,1),'r-o');
        plot(y,PP.VL1Ey(:,1),'k-*');
        plot(y,PP.VC1Ey(:,1),'g-^');
        plot(y,PP.I1Ey(:,1),'b-s');
        legend('V_2 , I_2 , V_L_2 , I_L_2 , V_C_2 ,
I_C_2','V_L_1 , I_L_1','V_C_1 , I_C_1','I_1');
        hold off;

```