Scheduling and resource optimization in rail operations : models and algorithms

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**SCHEDULING AND RESOURCE OPTIMIZATION IN
RAIL OPERATIONS: MODELS AND ALGORITHMS**

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2016

Scheduling and Resource Optimization in Rail Operations: Models and Algorithms

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In loving memory of my parents.....

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Abstract

In this thesis, we address important optimization issues in railway operations planning, namely *train scheduling* and *resource optimization*. Railway resources such as tracks, overhead equipment, locomotives, passenger and freight cars require huge capital investments and long lead time for procurement and installation. Large railway systems such as those in US, China, Europe and India need investment of billions of dollars on an on-going basis. Typically, installation of new tracks needs investment of USD 2 million per kilometre for normal tracks and more than USD 10 million for high speed tracks. Countries such as China and India have the railway network span over 120,000 and 64,000 track kilometres respectively, and are rapidly expanding their rail network. Indian Railways have more than 10,000 locomotives in inventory, and this represents an investment of about the USD 30 billion. Given these large investments and growth in traffic, even marginal improvements in resource utilization can result in significant cost savings. The specific problems identified and studied in this thesis are in the context of Indian Railways where the author has been working for more than 17 years. However, the problems studied and contributions made are fundamental and can be applied across all railway systems. We present these selected problems as three independent essays in this thesis.

The first essay entitled *Integrated Train Timetabling and Platforming Problem* comprehensively addresses the problem of train scheduling in heterogeneous, high density double track corridors on Indian Railways (IR). As these corridors are intensively utilized, efficient scheduling of trains with different priorities, speeds and halt pattern is critical for optimization of track capacity. The key challenge for train planners is the planning of overtaking of trains having different priorities such as *non-stop*, *suburban*, *express*, *commuter*, *container* and *heavy haul freight* trains. While freight trains generate significantly higher

revenue as compared to passenger trains, non-stop and express trains are given higher priority in IR due to the political sensitivity of delay in passenger services. Therefore, planning for overtaking of multiple trains without negative impact on throughput time is crucial for efficient scheduling of freight trains on these corridors dominated by passenger trains.

The existing approaches divide train scheduling into two distinct problems the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP) and solve them sequentially. In this essay, we develop a novel Integer Programming (IP) model that integrates the TTP and TPP into a single problem called the Integrated Train Timetabling and Platforming Problem (ITTPP). Our model explicitly handles station capacity constraints, assigns trains to platforms in a conflict free-manner with a possibility of overtaking of multiple trains at a station, and directly generates feasible timetable for the entire corridor that can be implemented without any further adjustments. We test our model with real data obtained from IR. With the proposed new model and the solution algorithm, we are able to schedule on average 26% additional trains as compared to existing timetable without much impact on the average throughput time of the trains.

The second essay entitled *Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization* (STMPMR) addresses the issue of energy efficient timetabling for Mass Rapid Rail Systems (MRRS) in urban areas. Energy cost accounts for a significant percentage of the cost of rail operations. With the current focus on cost rationalization and carbon emission in rail systems, this area provides a rich set of contemporary research questions to implement sustainable rail operations. The energy cost, power generation capacities and related CO₂ emissions are impacted by both energy consumption as well as peak power demand.

In this essay, we present a comprehensive Integer Programming (IP) model that seeks to minimize both the peak power demand as well as the total power consumption while keeping

average trip time of the trains within acceptable limits. We incorporate all phases of train movement such as acceleration, coasting/constant speed, deceleration and halt in the formulation. Our proposed model minimizes both the peak power demand and total energy consumption by reducing the number of trains accelerating simultaneously and synchronizing acceleration phase of trains with deceleration phase of other trains thus maximizing the usage of regenerative braking power. Computational results show that on average our model is able to cut the peak power demand by 28% and increase the regenerative braking power usage by 7.5 % while limiting the trip time increase to 2%.

The third essay entitled *Integrated Locomotive Scheduling and Routing Problem* addresses the issue of efficient scheduling of locomotives and their day to day routing and maintenance decisions. The Locomotive Scheduling Problem (LSP) and the Locomotive Routing Problem (LRP) are considered one of the most important resource scheduling problems in rail operations planning. While the existing approaches treat the LSP and LRP as sequential problems, in this essay, we develop an Integer Programming (IP) model that integrates LSP and LRP into a single unified problem called Integrated Locomotive Scheduling and Routing Problem (ILSRP). Our model incorporates deadheading, light heading and maintenance constraints of locomotives and generates a locomotive schedule for predefined timetable and route for each locomotive that can be implemented without further adjustment.

The key contribution of this thesis is in the newness of the proposed models and the comprehensive nature of the issues they address. They contribute to the railway operations planning literature both in terms of innovative problem formulations and solution algorithms. Our models have the potential to make significant improvements in rail operations practice and performance.

Chapter 1

Introduction

1.1 Overview

Rail systems have traditionally occupied an important position in both passenger and freight transportation due to their significantly lower operating cost and sustainability profile as compared to other modes of land transport. The growing concern about the reduction in carbon emissions has led to an increased global focus on rail systems as a preferred mode of transport (Alessandrini et al. 2012, Thompson 2010 and Bitzan and Keeler 2011). However, expanding rail capacity to meet rising demand requires long-term planning as rail resources such as tracks, overhead equipment, locomotives, passenger and freight cars need huge capital investments and long lead time for procurement and installations. Large railway systems such as those in US, China, Europe and India have invested billions of dollars in rail resources and also incur significant costs in rail operations. For example, track length of Chinese Railway network is 120,000 kilometres and that of Indian Railways network is 64,400 kilometres, which represent the investment of more than a few hundred billion USD. Indian Railways hold inventory of more than 10,000 locomotives for operating passenger and freight trains that represent an investment of about the USD 30 billion. These investments are likely to continue in the future. Given these large investments, even marginal improvements in resource utilization can result in significant cost savings. This potential for huge cost savings in railway operations arising from improved resource utilization has provided motivation for a rich set of research problems in rail operations.

1.2 Motivation

The primary motivation for this research has come from the complex, real-life train scheduling and resource optimization problems the author has encountered during his seventeen years of experience as an officer in the Indian Railways (IR). IR is the fourth largest network in the world, which is owned, regulated and operated by the government. IR operates 13,000 passenger trains and 8,000 freight trains daily, covering 64,400 route kilometres across the country. It transports more than 8.5 billion passengers and more than 1 billion tonnes freight every year and generates revenue more than USD 18 billion annually (IRSP 13-14). Sixty-five percent of the total freight traffic and fifty-five percent of the total passenger traffic is carried on only about fifteen percent of the total network called Golden Quadrilateral corridors connecting four Indian metro cities namely New Delhi, Mumbai, Chennai and Kolkata. The projected future growth of IR is also centred on the Golden Quadrilateral corridors. Scheduling of new trains on these intensively utilized corridors results in traffic congestion and increase in average throughput time of trains. Therefore, efficient scheduling of trains to improve utilization of track capacity and efficiency of resources is one of the foremost challenges faced by rail planners in India and this provides a very relevant backdrop for our research.

The rail corridors in IR carry both freight and passenger trains. In general, passenger trains have priority over freight trains. Among the passenger trains the priority is *Non-stop, Suburban, Express and Commuter Trains* in that order. For freight trains, *Container* trains are given priority over *Heavy haul freight* trains. Passenger trains run as per published time-table and freight trains are planned strictly in the intervals between two passenger trains. As both freight trains and passenger trains share the same track of the rail network, addition of every passenger train directly affects running performance of freight trains. Even though freight trains generate significantly higher revenue as compared to passenger trains, non-stop and fast express trains are given much higher priority in IR due to the political sensitivity of delay in passenger

services. Efficient scheduling of freight trains is therefore, crucial for improving overall operational performance of IR.

IR holds an inventory of more than 10,000 locomotives to haul passenger and freight trains. Locomotives are classified as diesel locomotives and electric locomotives. Electric locomotives can work only in an electrified territory while diesel locomotives have no such limitations. Under each traction type, they are further categorized as passenger train locomotives and freight train locomotives. Passenger train locomotives are designed to achieve higher speeds while freight train locomotives are aimed to generate higher hauling power rather than higher speeds. Passenger train locomotives are not suitable to haul heavy freight trains and freight train locomotives cannot reach the maximum scheduled speeds of passenger trains. Therefore, freight and passenger locomotives are not used interchangeably, except in case of emergency such as locomotive failure, signal failure and accident. Within each locomotive category, they are further divided into various types based on features such as horsepower rating and number of axles. Due to these complexities, allocation of right type of locomotives to appropriate train is critical for efficient management of rail operations in IR. Presently, locomotive allocation is done manually, and IR is continuously striving to improve the operating performance by close monitoring. However, this approach has resulted in only marginal improvement in the operating performance. Given the complexity and massive nature of operations in IR, substantial improvements in operating performance can be achieved only by integrating optimization techniques into train scheduling and rail operations planning. In the next section, we describe the relevance and importance of train scheduling and planning in rail operations as well as briefly introduce three essays that we present in this thesis.

1.3 Scheduling and Rail Operations Planning

The key to efficient and cost effective management of rail operations lies in scheduling of trains optimally and optimization of resources at strategic and tactical level of rail operations planning. At strategic level, issues having the long-term impact on operating performance and investment decisions are addressed. A prospective infrastructure and resource plan is prepared to meet the expected passenger and freight traffic demand of next 10 to 20 years. While forecasting the demand of passenger and freight traffic, factors such as GDP growth rate, location of prospective industries, development plans for ports and other infrastructure and growth pattern of cities are considered.

In the first phase of strategic planning, issues such as capacity expansion of rail network, addition of corridors, conversion of single track to double track corridor, electrification of corridors, first mile and last mile connectivity, construction of new stations or yards, development of new freight stations and additions of locomotive maintenance depots are addressed. In the second phase, the number of passenger and freight trains serving different pairs of stations is derived from forecast of passenger and freight traffic for next five to ten years. Tentative departure and arrival times from originating and at destination stations are finalized taking into consideration factors such as customer feedback, operational and business constraints. Based on these details, requirement of additional locomotives, passenger and freight cars and crew is worked out and process of procurement of resources and selection of the crew is undertaken. In the third phase, with a timeframe of one to two years, the number of passenger and freight trains scheduled to be run between different pairs of stations is worked out along with tentative departure and arrival times of trains. This information is fed as one of the inputs for generation of train scheduling at tactical planning level.

At tactical level, issues such as preparation of train schedule, time schedule for periodical maintenance of tracks and other assets, scheduling and routing of locomotives, scheduling of passenger cars, freight cars and crew, and repositioning of empty freight cars are addressed. Train scheduling is performed in two stages namely, train timetabling and train platforming. In the train timetabling stage arrival and departure times of trains at all stations are determined for the entire corridor. In train platforming problem stage, trains are assigned to platforms in a conflict-free manner at each station and train timings are adjusted subject to station capacity (number of platforms/lines at the station) constraints. For reliable train operations, it is important to carry out regular preventive maintenance of tracks and other assets. Therefore, time slots are earmarked for maintenance of assets in a train schedule. Train schedule or a train timetable is implemented from few months to a year and becomes the basis or input for other tactical level problems.

Locomotive scheduling and routing are other important issues that are addressed at a operational, tactical and strategic level. Locomotives play a vital role in efficient management of rail operations as punctuality and reliability of trains depend on the availability of the right type of locomotive at the right time and location. Procurement of locomotives needs huge capital investment and cost of locomotive accounts for significant percentage of total operating costs. Therefore, even a small improvement in the utilization of locomotive can result in significant cost savings. Large rail systems hold inventory of thousands of different types of locomotives for train operations. Therefore, efficient scheduling of locomotives is crucial for improving operational performance. The Locomotive Scheduling Problem (LSP) ensures assignment of right type of locomotives to right train and the Locomotive Routing Problem (LRP) determines the route for each locomotive subject to fuelling and maintenance constraints.

Like the LSP, Crew Scheduling Problem (CSP) is an important problem as reliability and punctuality of train operations also depend on the availability of a crew at right location. The CSP aims to find a set of rosters covering every train trip in a given time horizon while satisfying duty hours, business and operational constraints. The objective of CSP is to minimize cost of the crew. Finally the Freight Car Scheduling Problem (FCSP) aims at distribution of empty cars dynamically to improve the railway's ability to meet the demand for empty cars promptly in the rail network while minimizing transportation costs of the cars.

Optimization problems formulated to address train scheduling and resource optimization are challenging in nature. The challenge in solving these problems is on two fronts. First, we need to convert the complex set of operational, structural and safety requirements of the rail system into a mathematical model. Second we need to solve these complex problems optimally using commercial software. Most of these formulations are NP-hard Mixed Integer Programming (MIP) problems (Caprara, Fischetti, and Toth, 2002) and therefore, solving realistic sized problems in reasonable time is a difficult task.

In this thesis, we address two crucial issues in rail operations planning namely *train scheduling* and *resource optimization*. We organize these problems as three independent essays. All the three essays we propose in this thesis address tactical level problems in rail operations planning. The first essay addresses the issue of the train timetabling and platforming for high density double track corridors carrying heterogeneous traffic. We propose a new formulation that integrates the Train Timetabling Problem and the Train Platforming Problem into a single model. Our model is able to maximize the track capacity utilization by scheduling additional new trains while keeping the throughput time within prescribed limits. The second essay addresses the issue of minimization of operating cost of Mass Rapid Rail System in urban areas using energy-efficient train scheduling. Energy efficient timetabling problem aims to minimize both the peak and total power consumption of trains simultaneously while keeping

the average trip times within allowable limits. In the third essay, we address the issue of efficient scheduling and routing of locomotives, which is one of the most important and complex problems in rail operations planning.

The first essay entitled *Integrated Train Timetabling and Platforming Problem* addresses the problem of train scheduling in high density double track corridors. The current approach used for train scheduling is to divide the problem into two distinct problems, namely, the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP). The TTP is solved first for the entire corridor, and thereafter the TPP is solved for each station on the corridor. The TTP seeks to determine the arrival and departure times of trains at all stations while the TPP's objective is to find a conflict-free train schedule at each station subject to station capacity constraints. In this essay, we integrate the TTP and TPP into a single problem called the Integrated Train Timetabling and Platforming Problem (ITTPP). Our model is able to explicitly handle station capacity constraints (i.e. the number of platforms/lines in a station), and generate conflict-free platform assignments with the possibility of overtaking of multiple trains at a station. The solution to our model directly generates feasible timetable for the entire corridor without any further adjustments. We develop a heuristic algorithm as well as an optimal Branch and bound method for solving the model. Computational results on real-life problem sets obtained from Indian Railways for corridors with heterogeneous traffic show that our heuristic algorithm yields close to optimal solutions, and on average takes 7 minutes to solve problems of realistic size. With the proposed new model and the solution algorithm, we are able to schedule on average 26% additional trains without much impact on the average throughput time of the trains. The proposed model and the solution methodology developed in this essay will potentially lead to significant improvements and cost savings for Indian Railways.

The second essay entitled *Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization* (STMPMR) addresses the issue of energy-efficient timetabling for Mass Rapid Rail Systems (MRRS) in urban areas. MRRS is considered cheaper, energy-efficient and more environmentally friendly as compared to other modes of passenger transport. Due to increased focus on carbon emission and rising energy costs, rail companies are motivated to seek newer ways of further minimizing both traction energy consumption and peak power demand. Traction energy cost, power generation capacities and related CO₂ emissions are impacted by both total energy consumption as well as peak power demand. In this essay, we present a comprehensive Integer Programming (IP) model that seeks to minimize both the peak power demand as well as total power consumption while keeping the trip times within acceptable limits. The peak power demand and the total energy consumption is minimized by reducing the number of trains accelerating simultaneously and synchronizing acceleration phase of trains with deceleration phase of other trains for maximizing the utilization of regenerative braking power. Our model integrates the speed-time and the power-time train characteristics with features of rail network to incorporate acceleration time, constant speed/coasting time and their corresponding energy consumption, as well as deceleration time and corresponding regeneration of braking energy and is able to minimize both peak power and energy consumption by considering weighted sum of peak power and power consumption in objective function. We test our model on real-life problem sets obtained from the Mumbai Suburban System (MSS) of Indian Railways. MSS operates more than 2800 trains that carry over eight million commuters daily on six corridors. Test results show that on average our model is able to reduce the peak power demand by 28% and increase the regenerative braking energy utilization by 7.5 % while limiting the average trip time increases to only 2%.

The third essay entitled *Integrated Locomotive Scheduling and Routing Problem* (ILSRP) addresses the issue of efficient scheduling of locomotives and their day to day routing.

Locomotives represent billions of dollars of investment and, also account for a significant percentage of operating cost of trains. In a large system, even a small improvement in locomotive utilization can result in significant savings. Therefore, the Locomotive Scheduling Problem (LSP) and the Locomotive Routing Problem (LRP) is considered one of the most important resource scheduling problems in rail transportation. The LSP aims to assign locomotives to each train in a pre-planned schedule so that adequate hauling power is provided to each train from its origin station to destination stations while satisfying all operational and business constraints. The objective of LSP is to minimize the operating cost of locomotives. The LRP aims to determine the day to day routing of a locomotive while satisfying the maintenance constraints. The existing approaches treat the LSP and LRP as distinct problems and solve them sequentially. In this essay, we develop an Integer Programming (IP) model that integrates the LSP and LRP into a single unified problem called Integrated Locomotive Scheduling and Routing Problem (ILSRP). The model we develop in this essay is new as well as comprehensive. It incorporates deadheading, light heading and maintenance constraints of locomotives and generates a locomotive schedule for a predefined timetable and route for each locomotive that can be implemented without further adjustment.

All the models that we propose in this thesis are new and contribute to the railway operations planning literature both in terms of innovative problem formulations and solution algorithms. They also have the potential to make significant improvements in rail operations practice and performance. The key contribution of this thesis is in the richness of the proposed models arising from the comprehensive nature of the issues they address.

The rest of the thesis is organised as follows. In chapter 2, we describe the first essay Integrated Train Timetabling and Platforming Problem. In chapter 3, we elucidate the second essay Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization. In chapter 4, we present the third essay Integrated Locomotive Scheduling and

Routing Problem. Each essay includes self-contained literature review. Finally, in chapter 5 we summarize the thesis with conclusion and directions for future research.

Chapter 2

Integrated Train Timetabling and Platforming Problem

2.1 Introduction

In this paper we address the problem of train scheduling in double track corridors in which each track along with its associated platforms/lines at the stations is dedicated for trains running in one direction. Our paper is motivated by the real life scenario in Indian Railways (IR), where the author works as part of the senior management team. Trains having different priorities, speeds, and halt patterns such as *non-stop*, *suburban*, *express*, *commuter*, *container* and *heavy haul freight trains*¹ share the same track in IR. Scheduling these trains efficiently to minimize the total throughput time and maximize the number of trains that can be run in a corridor is one of the core issues of managing the operations in IR. The planning of overtaking of trains is a complex decision process due to the heterogeneous nature of traffic. Even though freight trains generate significantly higher revenue in IR, non-stop and fast express trains are given much higher priority due to the political sensitivity of delay in passenger services. Efficient scheduling of freight trains in these dense, passenger dominated rail corridors therefore poses a challenge for train planners. Freight trains need to be scheduled from one station to another station that has multiple platforms/lines. This will facilitate its overtaking by higher priority trains. Given the large number of freight trains in the system, it is necessary to plan for

¹ In Indian Railways both *Non-stop* and *express* trains are long distance passenger trains while *commuter* trains are regional trains travelling over short distances. *Non-stop* trains do not have any commercial halt while *express* trains have commercial halts at a few important stations between the origin and destination station. *Commuter* trains halt at almost all the stations on their route. *Container* and *heavy haul freight* are freight trains which carry containers and bulk commodities respectively. Average speed of non-stop trains is highest followed by express, suburban, commuter, container and heavy freight trains respectively. Non-stop trains are given the highest priority followed by suburban, express, commuter, container and heavy freight in that order.

overtaking of multiple trains at a station. An example of how overtaking of multiple trains can substantially improve the throughput time and effectively schedule more freight trains is presented in the Appendix 1.

The current approach used for train scheduling is to divide the problem into two distinct problems, namely, the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP) which are solved sequentially. First the TTP is solved for the entire corridor and next the TPP is solved for each station on the corridor. The TTP determines the arrival and departure times of the trains at all stations on the corridor. The objective is to minimize the total throughput time (the time taken by a train from the departure at the originating station to the arrival at the destination station) of all trains while satisfying the operational and safety constraints. The schedule generated by the TTP is not directly implementable as it ignores the station capacity and assumes that there are infinite platforms/lines available at all stations. The TPP adjusts the train schedule generated by the TTP to eliminate conflicts. The cost of deviation from the ideal schedule is minimized while satisfying station capacity, safety and operational constraints. The adjusted schedule generated by the TTP might necessitate cancellation of some trains to ensure its feasibility. The only attempt to incorporate station capacity constraint into the TTP is in the paper by Caprara et al. (2006). However they restricted station capacity to two while solving the problem heuristically.

The key contribution of this paper is the integration of TTP and TPP into a single problem that we call the *Integrated Train Timetabling and Platforming Problem* (ITTPP). To our knowledge, no previous paper has addressed the ITTPP. The ITTPP is able to explicitly handle station capacity constraints at each station, and generate conflict-free schedule with the possibility of multiple trains at a station. The solution to the ITTPP directly generates a feasible timetable that does not require any further adjustment. We develop a heuristic method as well as an optimal Branch and Bound algorithm for solving the ITTPP. Computational results on

real life problem sets obtained from IR show that our heuristic yields close to optimal solutions, and on average takes 7 minutes to solve problems of realistic size. With the proposed new ITTPP formulation, we are able to schedule on average 26% additional trains in the corridor without much impact on the average throughput time of the trains. We test our model using existing timetables that are currently implemented on each of the six Indian Railways corridors. For each corridor we solve two set of problem instances. One instance corresponds to current practice (existing timetable) while the other improved version has additional trains included while keeping the same train mix. On average we are able to add 26% more trains with only 0.36% increase in throughput time. This increase in throughput time as compared to the increase in number trains is insignificant. The ITTPP formulation and the solution methodology developed in this paper has the potential to significantly improve the operational performance of IR.

The rest of the paper is organised as follows. Section 2 is devoted to a review of relevant literature. The model notation and formulation is described in Section 3. Efficient methods for solving the ITTPP are discussed in Section 4. In Section 5, we develop an efficient heuristic as well as an improved branch and bound method for obtaining the exact optimum solution to the problem. Section 6 presents computational results tested on real life data obtained from Indian Railways. Finally, section 7 concludes with a summary and directions for future research.

2.2 Literature Review

Train Timetabling models can be categorized into double track and single track corridor models depending on whether the traffic flow is bi-directional on the same track (as in the single track models) or not. Single track models require additional constraints to address train crossing while they do not consider multiple overtaking of trains as this may result in inordinate delay to trains running in both directions. In each category, timetabling models address either tactical

or operational train scheduling issues. A tactical schedule is a daily/weekly timetable that is kept fixed for a period ranging from few months to a year. In tactical scheduling, the quality of the solution is more important than the computational time required to obtain it, as it is solved only once or twice in a year. An operational train schedule is prepared for a period ranging from a few hours to a day on a real time basis. Operational scheduling uses the tactical schedule as an input and determines the actual scheduling during a day dynamically depending on the traffic conditions. In operational scheduling, getting quick solutions assumes greater importance and therefore heuristic approaches are used. The approach used to address tactical train scheduling in the literature is to divide the problem into two distinct problems, namely, the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP). The TTP is solved for the entire corridor first and thereafter the TPP is solved for each station on the corridor. The TTP and TPP have both received considerable attention in the literature.

The earliest TTP models addressed train scheduling on a single track corridor, as most of the corridors had single track at that time. Szpigel (1973), Jovanovic and Harker (1991), Kraay et al. (1991), Brannlund et al. (1998) presented tactical train scheduling models for single track corridor while Cai and Goh (1994), Caray (1994), Kraay and Harker (1995), Higgins et al. (1997), Cai, Goh and Mees (1998) and Sahin (1999) presented real time operational models.

In a double track corridor, only train overtaking is important since train crossing does not take place. Carey and Lockwood (1995) presented an Integrated Programming (IP) formulation for the TTP in a double track corridor. They developed a heuristic which scheduled one train at a time and reported reasonable results for small instances involving 10 trains and 10 stations. Caprara, Fischetti, and Toth (2002) developed a Mixed Integer Programming (MIP) model to address the TTP. As the resulting TTP formulation was NP-hard, the effectiveness of the heuristics was evaluated against upper and lower bounds obtained by Lagrange Relaxation combined with sub gradient optimization and a heuristic procedure. The authors tested their

model using real-life data from Italian Railways and reported that for less congested instances, the optimality gap was within a few percent; however with increase in congestion this gap went up to 20%. Their model was able to schedule up to 193 trains in a corridor with 17 stations. Cacchiani, Caprara and Toth (2008) presented a column generation approach to solve the TTP model presented in Caprara, Fischetti, and Toth (2002). They used an LP relaxation approach to derive heuristics and exact branch and bound algorithms and reported that this approach yielded better upper bounds and shorter computational times as compared to the Lagrangean Relaxation approach used in Caprara et al. (2006). Zhou and Zhong (2005) presented an MIP formulation with multiple objective functions. The objective of this model was to minimize both the expected waiting time for high speed trains and total throughput time for high speed and medium speed trains. The authors proposed a heuristic solution which decomposed the problem into a multi-mode resource constraint project for each train which was solved using a branch and bound procedure. The model was developed specifically for the Beijing-Shanghai high speed corridor in China and was able to generate a feasible schedule for 24 high speed and 12 medium speed trains on the corridor having 17 stations.

The output of the TTP, is an ideal timetable that serves as an input to the TPP. TPP models do not have any specific objective function and therefore most of the approaches attempt to achieve feasibility rather than to optimize the train schedule. Cardillo and Mione (1998) proposed a graph colouring approach to solve the TPP, in which trains are assigned to platforms in a conflict free manner. They presented a heuristic algorithm and reduction techniques to solve the TPP efficiently. Billionnet (2003) proposed two integer programming formulations based on the graph colouring model presented in Cardillo and Mione (1998). The author used a commercial IP solver to find the exact solution to the TPP efficiently and solved the problem involving 200 trains and 14 platforms. Carey and Carville (2003) considered the TPP for large, busy and complex stations. Corlensen and Stefano (2007) also presented a model

for TPP based on graph colouring approach. The interested reader is also referred to Lusby et al. (2011) for a detailed review of the TPP.

Almost all TTP formulations assume infinite station capacity. When the ideal timetable generated by the TTP is used as an input to the TPP, it might necessitate cancellation of some trains and/or interruption of train schedules generated by the TTP, resulting in a solution that is significantly different from the TTP solution in terms of its objective function value. The only attempt to incorporate station capacity constraints into the TTP is the paper by Caprara et al. (2006). They solve problems using real-life data from the Italian Railways with congested traffic. Imposition of station capacity constraint results in cancellation of more than 15% of trains and more than 9% reduction in the value of objective function (profits) as compared to the case when station capacity is ignored. Caprara et al. (2006) assume station capacity equal to one for minor stations and two for major stations while solving the problem heuristically. The assumption of less than the actual number of platforms not only makes the model unnecessarily restrictive but also eliminates the possibility of overtaking of multiple trains. As discussed, in IR, where there is heterogeneous traffic of different types of passenger and freight trains, multiple overtaking of trains becomes essential for efficient scheduling. An example of how overtaking of multiple trains can substantially improve the throughput time and effectively schedule more freight trains is presented in the Appendix 1.

The ITTPP model that we propose and develop in this paper, integrates the TTP and TPP into a single problem, explicitly considers actual station capacity and allows for the possibility of overtaking multiple trains at a time. Our model also assigns trains to platforms/lines in a conflict free manner and generates feasible train schedules for the entire corridor without any further adjustment. To our knowledge, this is the first paper in literature that addresses all these issues in a single model.

2.3 Model Formulation

In this paper, we consider a double track rail corridor. Without loss of generality, we assume that there is a mutually exclusive segregation of the corridor into tracks and associated platforms/lines to carry dedicated traffic in each direction. We also assume that all platforms at any station are long enough to accommodate any of the trains. Using the proposed problem formulation, the train schedule can be developed independently for each direction of the double track corridor. The Integrated Train Timetabling and Platforming Problem (ITTPP) is formulated as an Integer Programming (IP) problem. The notations used in this paper are summarised in Table 2.1. There are S stations sequentially numbered from the first to the last. Each station $s \in (1, 2, 3 \dots S)$ has a finite *station capacity* (number of platforms/lines) represented by C_s . A total of T trains must be scheduled and each train $t \in (1, 2, 3 \dots T)$ starts from its origin station S_t^o and terminates at its destination station S_t^d , $1 \leq S_t^o < S_t^d \leq S$. The *earliest* and the *latest departure times* of train t at the origin station are denoted by d_t^e and d_t^l respectively. The *running time* for train t between station $s-1$ and s is denoted by r_t^s and defined for $\forall t, S_t^o < s \leq S_t^d$ while the minimum *halt time* for train t at station s is represented by w_t^s and defined for $\forall t, S_t^o \leq s \leq S_t^d$. At originating station S_t^o , each train halts at the platform for boarding of the passengers before its departure. Similarly, at the destination station S_t^d , each train t halts at the platform for passengers to alight before it is moved to the yard. It is also possible that certain (long distance) trains run through multiple corridors. In such cases, S_t^o (S_t^d) is the station at which train t starts (ends) its run in the corridor of interest. The *departure (arrival) headway* h_d (h_a) is the minimum time interval between departures (arrivals) of two consecutive trains at any station. The definition of departure and arrival headways is used in line with previous train scheduling models such as Caprara, Fischetti, and Toth (2002).

Table 2.1 Problem Parameters and Decision Variables

Problem Parameters/Input	
$s \in (1, 2, 3 \dots S)$	Station index
S	Number of stations in the corridor
$C_s, s \in (1, 2, 3 \dots S)$	Station Capacity (number of platforms/lines) at station s
$t, q \in (1, 2, 3 \dots T)$	Train indices
$S_t^o, (1 \leq S_t^o < S_t^d \leq S)$	Originating station for train t
$S_t^d, (1 \leq S_t^o < S_t^d \leq S)$	Destination station for train t
$w_t^s, (\forall t, S_t^o \leq s \leq S_t^d)$	Minimum halt time (wait time) for train t at station s
$r_t^s, (\forall t, S_t^o < s \leq S_t^d)$	Running time for train t between stations $s-1$ and s
h_d, h_a, h_p	Departure, Arrival and Platform headways in minutes
P_t	Priority cost for train t in dollars/minute
d_t^e, d_t^l	Earliest and latest departure time for train t from the originating station
Decision Variables	
$D_t^s, \forall t, S_t^o \leq s \leq S_t^d$	Departure time of train t from station s
$A_t^s, \forall t, S_t^o \leq s \leq S_t^d$	Arrival time of train t at station s
Auxiliary Variable	
$X_{qt}^s, \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\}$	$X_{qt}^s = \begin{cases} 1, & \text{if train } t \text{ departs after train } q \text{ at station } s \\ 0, & \text{otherwise} \end{cases}$
$Y_{qt}^s, \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\}$	$Y_{qt}^s = \begin{cases} 1, & \text{implies train } t \text{ arrives at } s \text{ only after train } q \text{ has} \\ & \text{departed from station } s \\ 0, & \text{otherwise} \end{cases}$
$f_t^s, S_t^o \leq s \leq S_t^d$	Number of trains departing from station s before train t
M	Large integer value

We introduce a new parameter called “*platform headway*” h_p , which we define as the minimum time interval between departure of a train from a platform and the arrival of the next train on the same platform. The departure headway (governed by the signalling system) ensures that a track-segment outside the station is used by only one train at any instant. The arrival headway (governed by the station layout) ensures that a track from entry point of the station to the platform is used by only one train at any instant. The platform headway ensures that each platform at the station is used by only one train at a time, thereby eliminating any platform conflict at a particular station. The three headways together ensure safety in train operations. Previous papers in the literature had not considered platform headway, as they ignored station

capacity in their formulation. The *Departure (Arrival) time* of train t at station s is denoted by D_t^s (A_t^s) and defined for $\forall t, S_t^o \leq s \leq S_t^d$. Both departure and arrival times are discretized continuous variables and expressed as integers. W represents the length of the planning horizon which we assume equal to one day. The *train sequencing* at station s is captured by the binary variable X_{qt}^s .

$$X_{qt}^s = \begin{cases} 1, & \text{if train } t \text{ departs after train } q \text{ at station } s \\ 0, & \text{otherwise} \end{cases} \quad \text{where } t, q \in (1, 2 \dots T) \text{ which is defined}$$

for $\text{Max} \{S_q^o, S_t^o\} \leq s \leq \text{Min} \{S_q^d, S_t^d\}$. The binary variable Y_{qt}^s captures the sequencing of arrival of train t and departure of train q at station s ,

$$Y_{qt}^s = \begin{cases} 1, & \text{implies train } t \text{ arrives at } s \text{ only after train } q \text{ has departed from the station } s \\ 0, & \text{otherwise} \end{cases} \quad \text{and is}$$

defined for $\text{Max} \{S_q^o, S_t^o\} \leq s \leq \text{Min} \{S_q^d, S_t^d\}$. The number of trains departing before train t from station s in the planning period W is represented by the integer variable f_t^s . The *priority cost* of train t in dollars/ minute is denoted by P_t . Finally, M is a very large integer number which is used for operationalizing the train sequencing using binary variables. The Integrated Train Timetabling and Platforming Problem (ITTPP) is given below.

Problem ITTPP

$\text{Min } \sum_{t=1}^T P_t (D_t^{S^d} - A_t^{S^o}) \quad (1)$	(1)
Subject to:	
$D_t^{S^o} \geq d_t^e, \forall t \quad (2)$	(2)
$D_t^{S^o} \leq d_t^l, \forall t \quad (3)$	(3)
$A_t^s = D_t^{s-1} + r_t^s, \forall t, S_t^o < s \leq S_t^d \quad (4)$	(4)
$(D_t^s - A_t^s) \geq w_t^s, \forall t, S_t^o \leq s \leq S_t^d \quad (5)$	(5)
$D_t^s \geq (D_q^s + h_d) - M(1 - X_{qt}^s), \forall q, t, q < t \text{ and } \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\} \quad (6a)$	(6a)
$D_q^s \geq (D_t^s + h_d) - M X_{qt}^s, \forall q, t, q < t \text{ and } \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\} \quad (6b)$	(6b)
$A_t^{s+1} \geq (A_q^{s+1} + h_a) - M(1 - X_{qt}^s), \forall q, t, q < t \text{ and } \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\} \quad (7a)$	(7a)
$A_q^{s+1} \geq (A_t^{s+1} + h_a) - M X_{qt}^s, \forall q, t, q < t \text{ and } \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\} \quad (7b)$	(7b)
$f_t^s = \sum_{q=1}^{t-1} X_{qt}^s + \sum_{q=t+1}^T (1 - X_{tq}^s), \forall t, S_t^o \leq s \leq S_t^d \quad (8)$	(8)
$A_t^s \geq (D_q^s + h_p) - M(1 - Y_{qt}^s), \forall q, t, \text{Max}\{S_q^o, S_t^o\} \leq s \leq \text{Min}\{S_q^d, S_t^d\} \quad (9)$	(9)
$\sum_{q \neq t} Y_{qt}^s \geq f_t^{s-1} - C_s + 1, \forall t, S_t^o \leq s \leq S_t^d \quad (10)$	(10)
$f_t^{S_t^o-1} = f_t^{S_t^o}, \forall t \quad (11)$	(11)
$X_{qt}^s, Y_{qt}^s = 0 \text{ or } 1, \forall q, t, s, \quad (12)$	(12)
$D_t^s, A_t^s \text{ and } f_t^s \geq 0 \text{ and integer}, \forall t, s \quad (13)$	(13)

The objective function defined by equation (1) minimizes the total throughput time (weighted by the priority cost P_t) of all the trains. The priority cost P_t (\$/minute) is specified by the rail company based on the relative importance of different trains. It is higher for a train with higher priority than for one with a lower priority. The theoretical minimum throughput time of a train is the sum of the minimum running and halt times required by it to travel from the origin to the destination station. The actual throughput time achieved would be higher because of headway restrictions, overtaking constraints and station capacity restrictions. Constraints (2) and (3) ensure that each train t departs from the originating station S_t^o within a specified time window. Constraint (4) ensures that the running time between stations is accounted for in the departure and arrival time of a train at contiguous stations while (5) ensures that the minimum halt time

required at each station is enforced. Constraints (6a) and (6b) enforce the minimum departure headway between two consecutive trains at each station while (7a) and (7b) enforce the minimum arrival headway at each station. The constraints (6a), (6b), (7a) and (7b) are defined for the train pair (q, t) only for the stations that are visited by both trains. They also ensure that overtaking takes place only at a station. Constraint (8) is an equation which determines the number of trains departing before train t from station s . The number of trains that arrive before train t at station “ s ” is same as the number of trains that depart before train t from station “ $s-1$ ” as overtaking is possible only at a station. However, not all the trains that preceded train t at station $s-1$, need to have departed from station s when train t arrives at station s . Clearly, the number of trains running ahead of train t at station $s-1$, that have not departed from station s cannot be more than $C_s - 1$ (as at least one vacant platform/line is required for the arrival of train t). Constraints (8), (9) and (10) ensure that the number of trains simultaneously present at any station s cannot be greater than the station capacity C_s . Constraint (9) is implemented at station level as well as at platform level. If a platform/ line is available at station s for the next train t to be received from the previous station $s-1$ then constraint (9) is not enforced. It is enforced only when no platform/line is available at s to receive the next train t . Constraint (9) ensures that the next train t arrives on the first available platform at s vacated by the departed train q after lapse of platform headway h_p . Therefore, constraint (9) is implemented both at the station as well as platform level.

Constraints (8), (9) and (10) are to our best knowledge, new and a surprisingly simple and novel way to incorporate actual station capacity into the train scheduling model. This enables us to integrate timetabling and platforming problem into a single problem and generate the train schedule for the whole corridor. Constraint (11) is a logical constraint to ensure the feasibility of station capacity at the originating station of a train. Constraint (12) ensures variables X_{qt}^s

and Y_{qt}^s take only binary values either 0 or 1. Finally, constraint (13) ensures that variables D_t^s , A_t^s and f_t^s takes only non-negative integer values.

2.3.1 Model Analysis

The number of variables (both integer and binary) and constraints in the ITTPP formulation increase exponentially for instances of realistic size required in practice. For example, a problem involving 201 trains and 18 stations, has 1,458,054 integer variables (of which 1,447,200 are binary variables) and 2,510,491 constraints. Problems of such magnitude cannot be solved optimally even with commercial grade optimization software within a reasonable time. Note that we have already eliminated a large number of binary variables in our formulation above by using only the lower triangular part of the $\mathbf{X}^s$ matrix for each station s .

We therefore further revise the model to reduce the feasibility region and consequently the solution time. This is similar to the use of cutting planes to improve the efficiency of the integer programming problems. Intuitively, one would expect only a higher priority train to overtake a lower priority train at a station. In the optimal train schedule, trains of lower priority cannot overtake trains of higher priority and similarly, trains of same priority cannot overtake each other at any station s . We first state a few assumptions and one lemma before we formally prove this in a theorem. Note that P_q and P_t are the priority costs of train q and train t respectively and $P_t < P_q$ implies that train q has higher priority than train t .

Assumptions:

For any pair of trains q and ℓ ,

- (i) $P_\ell < P_q \Rightarrow r_\ell^s \geq r_q^s$ and $w_\ell^s \geq w_q^s, \forall s$ with $\text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$
- (ii) $P_\ell = P_q \Rightarrow r_\ell^s = r_q^s$ and $w_\ell^s = w_q^s, \forall s$ with $\text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$
- (iii) $P_\ell > P_q \Rightarrow r_\ell^s \leq r_q^s$ and $w_\ell^s \leq w_q^s, \forall s$ with $\text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$

These assumptions are very reasonable, logical and are usually satisfied in practice.

Lemma 1:

Let train ℓ and q be such that $P_\ell \leq P_q$ and $A_q^s < A_\ell^s$ at station s . That is train ℓ has the same or lower priority than train q and train ℓ arrived at station s after train q . Consider two possible scenario

- (i) Train ℓ departs from station s immediately after train q

Let the corresponding departure times for train ℓ and q be D_ℓ^s and D_q^s and let the corresponding delay times at station s be δ_ℓ^s and δ_q^s respectively.

- (ii) Train ℓ overtakes train q and departs immediately ahead of train q at station s .

Let the corresponding departure times for train ℓ and train q be $\bar{D}_\ell^s$ and $\bar{D}_q^s$ and let the corresponding delay times at station s be $\bar{\delta}_\ell^s$ and $\bar{\delta}_q^s$ respectively.

$$\text{Then } (P_\ell \bar{\delta}_\ell^s + P_q \bar{\delta}_q^s) \geq (P_\ell \delta_\ell^s + P_q \delta_q^s) \quad (14)$$

Let ∇_ℓ^s be the reduction in delay of train ℓ at station s by overtaking train q .

$$\nabla_\ell^s = (\delta_\ell^s - \bar{\delta}_\ell^s) \quad (15)$$

Let ∇_q^s be the increase in delay of train q at station s by overtaking of q by train ℓ .

$$\nabla_q^s = (\bar{\delta}_q^s - \delta_q^s) \quad (16)$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s \quad (17)$$

Further when $P_\ell < P_q$

$$(P_\ell \bar{\delta}_\ell^s + P_q \bar{\delta}_q^s) > (P_\ell \delta_\ell^s + P_q \delta_q^s) \quad (18)$$

$$P_q \nabla_q^s > P_\ell \nabla_\ell^s \quad (19)$$

The proof of lemma 1 is presented in the Appendix 2.

Theorem 1:

In the optimal solution to the problem (ITTPP), a train ℓ cannot overtake train q at any station s if $P_\ell \leq P_q$.

Proof:

Let's assume that Ω is an optimal schedule for a given problem (ITTPP). On the contrary to theorem 1, there exists trains of lower priority which overtake trains of higher priority at some stations.

Let station s be the last station in the direction of the movement of trains at which such an overtaking takes place with train ℓ overtakes train q and $P_\ell \leq P_q$. Then, let us generate a new but modified schedule Ω' for problem ITTPP which otherwise is Ω except at station s train ℓ does not overtake q (if necessary plans after station s may have to be modified accordingly)

From Lemma 1, we have already proved that, if lower priority train ℓ overtakes higher priority train q at station s then the total cost of delay of train ℓ and q is higher than or equal to the cost of delay of train ℓ and q , at station s , if ℓ does not overtake train q . i.e. $P_\ell \bar{\delta}_\ell^s + P_q \bar{\delta}_q^s \geq P_\ell \delta_\ell^s + P_q \delta_q^s$.

Further if schedule of train ℓ and train q is required to be changed after station s then delay of higher priority train q will not increase if lower priority train ℓ immediately leading train q as $\sum_s^{s_q^d} r_q^s + w_q^s \leq \sum_s^{s_\ell^d} r_\ell^s + w_\ell^s$ for $s \leq \text{Min} \{s_q^d, s_\ell^d\}$.

Therefore, value of objective function of modified schedule Ω' is as good as original schedule Ω as schedule of trains ℓ and q before overtaking at station s remains same.

By induction, it proves that in the optimal solution to the problem (ITTPP), a train ℓ cannot overtake train q at any station s if $P_\ell \leq P_q$.

Thus theorem is proved. $\square$.

The result of theorem 1 can be formally incorporated in the problem (ITTPP) as additional constraints and these are outlined below.

Additional Constraints

$$X_{qt}^s \geq X_{qt}^{s-1} \quad \forall s, q < t \text{ for } P_t < P_q \quad (21)$$

$$X_{qt}^s \leq X_{qt}^{s-1} \quad \forall s, q < t \text{ for } P_t > P_q \quad (22)$$

$$X_{qt}^s = X_{qt}^{s-1} \quad \forall s, q < t \text{ for } P_t = P_q \quad (23)$$

$$1) \quad X_{qt}^s \geq X_{qt}^{s-1} \quad \forall s, q < t, P_t < P_q.$$

When $P_t < P_q$, train t cannot overtake q at station s , but the vice versa is possible. So if train t departed after q at station $s-1$ (i.e. $X_{qt}^{s-1}=1$), then train t must follow train q at station s (i.e. $X_{qt}^s=1$). On the other hand, if q departed after train t at station $s-1$ (i.e. $X_{qt}^{s-1} = 0$), it is possible for train q to overtake and depart earlier than t at station s

(i.e. $X_{qt}^s = \begin{cases} 1, & \text{if } q \text{ overtakes } t \text{ at station } s \\ 0, & \text{otherwise} \end{cases}$). Comparing the two cases, we have

$$X_{qt}^s \geq X_{qt}^{s-1} \quad \forall s, q < t, P_t < P_q.$$

$$2) \quad X_{qt}^s \leq X_{qt}^{s-1} \quad \forall s, q < t, P_t > P_q.$$

The logic for the constraint (22) is similar to that of (21). Using the same logic, if $P_t = P_q$, the sequence of train q and t cannot change at station s and this leads to the constraint (23).

2.4 Revised Model Formulation

Problem RITTPP

$\text{Min } \sum_{t=1}^T P_t \left(D_t^{S_t^d} - A_t^{S_t^o} \right) \quad (24)$ <p>Subject to</p> <p>Constraints (1) to (13) and (21) to (23).</p>

The additional constraints (21) - (23) reduce the feasible region and the search space for the variables in the Branch and Bound algorithm. They are similar in spirit to adding cutting planes for solving integer programming models efficiently. With these additional constraints,

commercial optimization software LINGO is able to obtain solution to the original problem much faster. For example, the computational time reduced from 22507 seconds to 4010 seconds for a problem with 40 trains and 30 stations using Lingo 14.0 on a PC using an i7- 4700HQ CPU, running at 2.4 GHZ. Though the computational time reduced significantly after the inclusion of constraints (21) to (23), it still remained relatively high for solving realistic size problem instances. Our test bed (described in detail in Section 2.6) consists of problems with the number of trains ranging from 45 to 201 and the number of stations ranging from 17 to 64 stations. We therefore propose a heuristic method in the next section to solve large instances of RITTPP within reasonable time. We further use the solution obtained by the heuristic as an upper bound in an improved branch and bound algorithm to get the exact solution.

2.5 Heuristic and Improved Branch and Bound Method

The two heuristics we propose are referred to as *train block heuristic* and *station block heuristic*, respectively. The key idea behind the train block (station block) heuristic is to divide the main problem into several smaller sub-problems based on fewer trains (stations) and solving one sub-problem at a time. The sub-problems are easier to solve as they have significantly fewer variables and constraints. Both heuristics perform equally well for problems involving up to 15 trains. However, the efficiency of station block heuristic deteriorates significantly as the number of trains to be scheduled increases. The train block heuristic turns out to be computationally efficient and provides near optimal solutions in reasonable times even for very large problems because it needs to solve sub-problems involving only a few trains at a time. We therefore only discuss the train block heuristic in this paper, however report computational results obtained using train block as well as station block heuristics in computational section.

2.5.1 Train Block Heuristic

In the train block heuristic, we divide T trains to be scheduled into several train blocks (sub problems). In each block, let γ be the number of trains. The trains in the contiguous train blocks are represented by $(1, 2 \dots \gamma)$, $(\gamma + 1, \gamma + 2 \dots 2 \gamma)$, $(2\gamma + 1, 2\gamma + 2 \dots 3 \gamma) \dots$ etc. respectively. All the blocks except the last one will have exactly γ trains. Therefore the number of sub problems to be solved is given by $\lceil T/\gamma \rceil$. We solve the optimization problem sequentially starting with the first block. All the subsequent blocks except the first will have additional constraints (26) and (27) along with constraints (1) to (13) and (21) to (23) to ensure that (a) the departure times of all the trains in the block g at all the stations is greater or equal to the maximum of the departure times of all the trains in preceding blocks plus the departure headway, (b) the arrival times of all the trains in the block g at all the stations is greater than or equal to the maximum of the arrival times of all the trains in the preceding block plus the arrival headway. In other words, trains in the subsequent blocks are not permitted to overtake trains in the earlier block. Let sub problem TRITTPP^g refer to the revised ITTPP problem for the trains in the block g .

Train Block heuristic for Problem RITTPP

T trains running on a corridor are divided into U blocks. $U = \lceil T/\gamma \rceil$

For $g=1, 2 \dots U$, Do

{

$$j_1^g = \lceil (g-1) \gamma \rceil + 1$$

$$j_e^g = \text{Min} \{ (g\gamma), T \}$$

(Note that j_1^g and j_e^g are first and last trains of sub problem g).

Solve TRITTPP^g

}

Sub problem TRITTPP^g

$$\text{Min } \sum_{t=t_1^g}^{t_e^g} P_t \left(D_t^{S_t^d} - A_t^{S_t^o} \right) \quad (25)$$

Subject to:

Constraints (1) to (13) and (21) to (23).

Additional Constraints (applicable only for $g > 1$)

Departure time constraints

$$\{ D_{j_1^g+1}^s \dots D_{j_e^g+1}^s \} \geq \text{Max} \{ D_{j_1^g}^s \dots D_{j_e^g}^s \} + h_d, \forall s, S_t^o \leq s \leq S_t^d \quad (26)$$

Arrival time constraints

$$\{ A_{t_1^g+1}^s \dots A_{t_e^g+1}^s \} \geq \text{Max} \{ A_{t_1^g}^s \dots A_{t_e^g}^s \} + h_a \forall s, S_t^o \leq s \leq S_t^d \quad (27)$$

(Sub problem TRITTPP^g is solved for trains from j_1^g to j_e^g)

2.5.2 An Improved Optimal Branch and Bound Method

The heuristics described above yield near optimal solutions to the complete problem. We used this near optimal objective function value as an upper bound or a hurdle tolerance in the branch-and-bound algorithm to narrow the search for the optimum value. The branch and bound

algorithm searches only for integer solutions with objective function values lower than the upper bound. As our heuristic provides near optimal solutions and a tight upper bound, the additional computational time required to obtain the optimal solution is reduced dramatically.

2.6 Computational Results

The algorithm we develop for the RITTPP is tested extensively on the data obtained from Indian Railways (IR). Both the heuristic and the optimal branch and bound algorithm is implemented using LINGO 14.0. We select six dense corridors of IR namely Mumbai-Kalyan (MK), Igatpuri-Bhusawal (IB), Ballarshah-Itarsi (BI), Bhusawal-Khandwa (BK), Badnera-Bhusawal (BB), and Lonavala-Daund (LD) for obtaining the problem data. All these corridors have a traffic intensity (i.e. average number of trains per hour) that is significantly higher than the average traffic intensity across the entire IR network and are therefore defined as *dense* corridors. Mumbai-Kalyan (18 stations) is a 53 kilometre long *suburban* corridor with nearly 55% suburban trains, 40% long distance passenger trains and 5% freight trains. In suburban corridors, suburban trains are accorded priority on par with express trains. Igatpuri-Bhusawal (30 stations) is a 307 kilometre corridor with nearly 80% long distance passenger trains and 20% freight trains. Ballarshah-Itarsi (64 stations) is a 510 kilometre corridor with 70% long distance passenger trains and 30% freight trains. Bhusawal-Khandwa (17 stations) is a 124 kilometre corridor with nearly 78% long distance passenger trains and 22% freight trains. Badnera-Bhusawal (25 stations) is a 203 kilometres corridor with nearly 68% long distance passenger trains and 32% freight trains and Lonavala-Daund (28 stations) is a 140 kilometre semi-suburban corridor with nearly 53% long distance passenger trains, 32 % suburban trains and 15% freight trains. Taken together, these corridors exemplify the wide variety of traffic conditions encountered in the IR. For each corridor, we obtained data for two problem sets. (a) One problem set represents the current train traffic and for the second problem set, we added

extra trains in the corridor while maintaining the same train mix. On the Mumbai-Kalyan (MK) corridor, for example, the problem data set represents two different scenarios one with 169 trains (current level) and another with 201 trains (32 additional trains). In line with normal practice, the time horizon is assumed to be one day (i.e. $W=1440$ minutes). The timetable that is generated by our algorithm can then be cyclically repeated on a daily basis. To keep the analysis simple, S_t^o is set to 1 and S_t^d is set to S which is the last station in the corridor. In line with the current practice in IR, the departure, arrival and platform headways are taken as 3, 2 and 3 minutes respectively for the suburban corridor, and 4, 3 and 4 minutes respectively for other corridors. In our model, P_t represents the transit cost of train t in dollars per minute. Let $TT_t = (D_t^{S_t^d} - A_t^{S_t^o})$ represent the throughput time of train t from the origin to its destination. The total cost of train t travelling from origin to destination stations is therefore $(TT_t P_t)$. Table 2.2 shows the cost P_t used in the objective function for trains with different priorities. Non-stop trains are accorded the highest priority followed by suburban, express, commuter, container and freight trains in that order. The relative values of P_t used in the problem sets are reflective of the policy used by IR of giving non-stop trains (suburban trains in suburban corridor) an uninterrupted schedule while keeping container and freight trains waiting at intermediate stations for overtaking.

Table 2.2: Priority Cost for Different Trains

Train Type	Priority Cost (P_t) in \$/min
Non-stop Trains	40
Suburban Trains	20
Express Trains	20
Commuter Trains	10
Container Cargo Trains	2
Heavy Haul Freight Trains	1

The main characteristics of the problem sets viz. number of stations, number of trains per day and, train mix, length of the corridor are outlined in table 2.3. The train mix gives the number of heavy haul freight, container, commuter, express, suburban and non-stop trains per day in that order. The last column in Table 2.3 gives the total number of trains departing in each 4 hour period (0.00 am -3.59 am, 4.00 am- 7.59 am, 8.00 am-11.59 am, 12.00 am-3.59 pm, 4.00 pm – 7.59 pm, and 8.00 pm- 11.59 pm.).

Table 2.3 Train Mix Data in the Problem Sets

Problem Set	Corridor	# of Stations	# of Trains per day	Train Mix	# of Trains in 4 hour period
1	MK	18	169	(4,4,0,63,93,5)	(9,22,46,22,43,27)
2	MK	18	201	(5,5,0,70,114,7)	(8,25,55,29,49,35)
3	IB	30	69	(7,6,2,49,0,5)	(13,7,13,9,14,13)
4	IB	30	82	(8,8,2,57,0,7)	(15,09,15,10,17,16)
5	BK	17	49	(4,5,4,30,0,6)	(8,11,2,9,10,9)
6	BK	17	65	(7,7,4,39,0,8)	(12,14,2,12,14,11)
7	BI	64	46	(9,5,2,25,0,5)	(9,10,9,5,7,6)
8	BI	64	59	(11,7,2,33,0,6)	(9,13,9,7,12,9)
9	BB	25	45	(9,4,4,25,0,3)	(7,4,8,9,10,7)
10	BB	25	61	(13,7,4,32,0,5)	(9,8,11,11,12,10)
11	LD	28	61	(5,4,2,29,19,2)	(12,10,12,6,11,10)
12	LD	28	104	(8,5,2,41,44,4)	(17,12,15,15,20,25)

Table 2.4 shows number of stations, length of corridor in kilometres and the minimum throughput time of different trains used in the problem set. The minimum throughput time is the sum of running times for all segments in the corridor and the sum of the minimum halt times at all the stations in the corridor.

Table 2.4 Length of Corridors and Travel Time in the Problem Sets

Corridor Name	# of Stations	Distance (KM)	Minimum Train Throughput Time (minutes)					
			Non-stop	Express	Suburban	Commuter	Container	Heavy haul Freight
MK	18	53	71	81	62	0	84	88
IB	30	307	204	245	0	289	273	300
BK	17	124	115	151	0	184	183	194
BI	64	510	395	474	0	540	547	561
BB	25	221	160	191	0	211	207	228
LD	28	139.42	140	173	154	202	191	215

The results of our computational tests using station block heuristics for the 12 problem sets are reported in Table 2.5. The results reported include the cost of heuristic solution, its corresponding computational time, the optimal cost, the corresponding computational time required for the improved Branch and Bound algorithm, and finally the percentage gap between the heuristic cost and the optimal cost.

Table 2.5 Computational Results for the Problem Sets Using Station Block Heuristic

Problem Set	Corridor	# of Trains , # of Stations	Heuristic solution Station Block		Optimal solution		% Optimality Gap Station block Heuristic and Optimal Solution
			Cost	Time (seconds)	Cost	Time (seconds)	
1	MK	169,18	237124	204	235804	8047	0.56
2	MK	201,18	284415	315	282207	132425	0.78
3	IB	69,30	297190	288	293092	128687	1.4
4	IB	82,30	363951	694	350722	154800	3.8
5	BK	49,17	128810	48	128680	284	0.1
6	BK	65,17	165639	89	163526	7219	1.3
7	BI	46,64	344277	286	339888	82800	1.2
8	BI	59,64	449539	549	435167	170400	3.3
9	BB	45,25	128466	31	127375	444	0.85
10	BB	61,25	172529	230	169332	2930	1.9
11	LD	61,28	189863	203	178414	2932	6.4
12	LD	89,28	270154	61	262327	2719	3.0

Similarly Table 2.6 show results of train block heuristic with respective optimal solution.

Table 2.6 Computational Results for the Problem Sets Using Train Block Heuristic

Problem Set	Corridor	# of Trains , # of Stations	Heuristic solution		Optimal solution		% Optimality Gap (Between Train Block Heuristic and optimal solution)
			Cost	Time (seconds)	Cost	Time (seconds)	
1	MK	169,18	236764	221	235804	8047	0.41%
2	MK	201,18	282602	317	282207	132425	0.14%
3	IB	69,30	293726	367	293092	128687	0.22%
4	IB	82,30	352250	505	350722	154800	0.4%
5	BK	49,17	128810	76	128680	284	0.1%
6	BK	65,17	163764	145	163526	7219	0.14%
7	BI	46,64	341455	758	339888	82800	0.48%
8	BI	59,64	435725	725	435167	170400	0.1%
9	BB	45,25	127395	113	127375	444	0.1%
10	BB	61,25	170002	116	169332	2930	0.39%
11	LD	61,28	178697	108	178414	2932	0.16%
12	LD	89,28	263450	105	262327	2719	0.43%

It can be seen from the comparison that overall train block heuristic performs better than station block heuristic. Minimum optimality gap in case of train block heuristic is 0.1 percent, maximum optimality gap is 0.48 percent and average gap is 0.36%. In case of station block heuristic minimum optimality gap is 0.1 percent while maximum optimality gap is 6.4 percent and average gap is 2.05%. Station block heuristic gives faster results when number of stations on the corridor are relatively less i.e. less than or equal to 25. In all other cases train block heuristic is faster. Therefore, we choose train block heuristic over station block heuristic and discuss the results obtained using train block heuristic as shown in Table 2.6 in detail.

The computational results of train block heuristic show that our proposed heuristic obtains close to optimal solution with a maximum optimality gap (percentage gap between the cost of heuristic solution and the optimal cost) of 0.48 %. The average optimality gap across

all the problem instances is 0.26 %. The average computational time taken by the heuristic solution is 7 minutes. The computational time reported for the improved Branch and Bound algorithm is the additional computational time required to obtain the optimal solution by using the heuristic solution as the hurdle tolerance for the branch and bound. The time taken by the improved Branch and Bound algorithm for obtaining the optimal solution ranged from a few minutes to 48 hours (average time was 15.82 hours).

Table 2.7 Average Delay at Present level of Traffic and with Additional Trains

Problem Set	Corridor	# of Stations	Present Level of traffic		Increased Traffic level			
			# of Trains	Average Throughput time (minutes)	Additional # of Trains	% increase in trains	Average Throughput time (minutes)	% Increase in throughput time
(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(H -E)/E
1	MK	18	169	73	32	18.9	73.2	0.41
2	IB	30	69	253	13	18.8	253.2	0.2
3	BK	17	49	157	16	34.7	157.3	0.24
4	BI	64	46	502	13	28.2	503.5	0.29
5	BB	25	45	200	16	35.6	200.5	0.17
6	LD	28	61	172	28	45.9	173.7	0.87

Even though some of the corridors had high traffic intensity in terms of the number of trains per hour, we are able to schedule an average of 26% additional trains as compared to the trains in the existing timetable with the proposed timetabling algorithm without much impact on the average throughput time of the trains. As shown in Table 2.6, the average throughput time increased only by 0.36%. The proposed RITTPP formulation and the solution methodology therefore have the potential to substantially improve the existing scheduling practices and thus the operational performance of Indian Railways.

2.7 Summary and Conclusions

Train scheduling is one of the core issues in managing rail operations efficiently. The current approach used to address train scheduling is to divide the problem into two distinct problems, namely, the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP). In this paper we present an analytical model which integrates TTP and TPP into a unified Integrated Train Timetabling and Platforming Problem (ITTPP), whose output yields a feasible timetable that can be implemented without further modification. Our model explicitly considers actual station capacity (in terms of number of platforms) and the possibility of multiple overtaking. Through the use of novel constraints and binary variables, the ITTPP is formulated as a compact Integer Programming (IP) problem. We solve very large problems of realistic sizes by adding a few cutting plane like constraints in the model to reduce the feasibility region without which the problem could not be solved optimally in reasonable time. We generate these additional constraints by using technical results on the sequencing and overtaking of trains with different priorities. While the solution speed increased after the addition of these constraints, the computational time is still prohibitive for large problems. We therefore develop an efficient train block heuristic by partitioning the ITTPP formulation into problems involving lesser number of trains. The smaller sub problems are then solved optimally to obtain a heuristic solution for the larger master problem. The heuristic solution is used as hurdle tolerance in an improved branch-and-bound procedure to obtain the optimal solution. As per current practice, timetabling is done manually in IR. In a manual timetabling exercise, generating efficient schedules is difficult if the freight trains are also included in the exercise. Therefore freight trains are scheduled dynamically on a daily basis after the manual timetable is generated for the other trains. Through the use of our timetabling methodology, freight trains can also be efficiently scheduled in advance. Our methodology is computationally tested on real data obtained from six corridors of IR with a heterogeneous mix of train priorities and high traffic

intensity. The data used in the computational study exemplify the wide variety of traffic conditions encountered in the IR. We solved two sets of problems for each corridor, one representing the current train mix and number of trains and another which included additional trains. Computational results on real problem sets obtained from IR show that our proposed heuristics obtained close to optimal solutions, with a maximum gap of 0.48 % between the heuristic solution and the optimal cost. The average computational time taken by the heuristic solution is 7 minutes. Even though the corridors are dense, we are able to schedule an average of 26% additional trains, without much impact on the average throughput time. Given the high capital intensity needed to build railway infrastructure, the proposed methodology can result in superior utilization of resources and has the potential to dramatically transform the operational performance of Indian Railways.

Further research could investigate the possibility of extending the ITTPP formulation to single track corridors and to rail networks involving multiple intersecting corridors. Operational real-time scheduling of trains using the ITTPP formulation as the basis is another research avenue.

Chapter 3

Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization

3.1 Introduction

The cost of traction energy is a significant component of the total cost of operating Mass Rapid Rail Systems (MRRS) in urban areas and it is crucially impacted by two factors - total energy consumption and peak power demand (Albrecht, 2004). Additionally, power generation capacities and related CO₂ emissions are also determined by both peak power demand and total energy consumption. Reduction of peak power demand, concurrent with lower average energy consumption is critical from the twin perspective of cost rationalization and environmental protection. Rail companies are therefore seeking newer ways of further improving both traction energy consumption and peak power demand. Prior research has highlighted that the traction energy consumption can be potentially reduced by 25-35% through strategies such as implementing energy efficient timetables and maximizing utilization of regenerative braking energy (Gonzalez-Gil et al., 2014). However, very few previous papers have considered the need to simultaneously reduce total energy consumption and peak power demand. The model presented in this chapter seeks to fill this important gap in literature by minimizing both peak power demand as well as total energy consumption while keeping trip times within acceptable limits.

Our research is motivated by the real life problem encountered by the Mumbai Suburban System (MSS) of Indian Railways (IR), where the author works as a member of the senior management team. MSS operates more than 2800 trains that carry over eight million

commuters daily on six corridors. Out of these six corridors, four are dedicated to run suburban trains while the remaining two corridors run heterogeneous traffic consisting of passenger, freight and fast suburban trains. Almost 80% of the suburban trains are scheduled to run on the four dedicated corridors and these trains are required to stop at every station. Suburban Trains operating on MSS are called Electrical Multiple Units (EMU). There are more than 200 EMUs comprising 9, 12 or 15 car combinations. EMUs have self-propelled carriages and therefore do not require a separate locomotive to haul them. They are designed to have quick acceleration (0.54 m/s^2) and deceleration ($0.74 - 0.84 \text{ m/s}^2$) as the distance between contiguous stations is only about 2 to 3 kilometres. EMUs accelerate quickly and can reach speeds in the range of 70 to 100 kilometres per hour before coasting and subsequently applying regenerative braking in preparation for the halt at the next station. The peak power demand and energy consumption of the EMUs are determined by both the speed-time and the power-time train characteristics. Figure 3.1 displays the typical speed-time and power-time characteristics of a train running between two stations. *Acceleration* phase is the time required for a train to reach peak or near peak speed from a halt position and *Braking* phase is the time required for a train to stop after application of braking. Train consumes very high power during acceleration phase while during deceleration phase it regenerates braking energy which is exported back to the traction system. Between the acceleration and braking phases the train will either be in *coasting* or in *constant speed* phase depending on the distance between successive stations. During the coasting phase the train consumes zero power as it continues to run due to its own momentum, while in the constant speed phase, the train consumes minimal power to maintain its constant speed.

Figure 3.1 Speed Time and Power Time Characteristics of EMU in MSS

In the regenerative braking phase, traction motors function as generators and convert kinetic energy into electrical energy altering the direction of power flow. Regenerative braking energy which is fed back to the traction system has to be used instantaneously by other accelerating/constant speed trains; otherwise, this power is wasted as heat. The peak power demand is primarily determined by the amount of power consumed by the number of trains accelerating simultaneously in a corridor and the amount of regenerative braking power that is exported back into the system. The total energy consumption is determined by the amount of regenerated power that is instantaneously utilized by the accelerating and constant speed trains. As stated earlier, trains on the four dedicated suburban corridors of MSS are required to stop at every station which provides potential for regenerating up to 30% of the energy requirement, thus reducing CO₂ emissions by 380,000 tonnes and saving 443 million units of energy (Singh, 2013). For achieving the full potential of these savings, we can either use storage devices or alternatively implement energy efficient timetables. Since the efficiency of storage devices is

very low, energy efficient timetables provide a promising alternative for achieving the goals of cost rationalization and environmental protection.

The existing approaches to generate energy efficient timetables assume a starting reference timetable and then seek to modify this timetable to either minimize the peak power demand or the total energy consumption subject to feasibility of the modified timetable. These approaches use approximate optimization methods such as genetic algorithms or simulation and are therefore not guaranteed to provide optimal solutions. Besides, most of these papers only attempt to adjust the arrival/departure time at one station at a time rather than considering the entire corridor. Our paper makes three distinct contributions to existing literature in energy efficient train timetabling. First, we develop a new and comprehensive Integer Programming (IP) formulation that integrates the speed-time and the power-time train characteristics with other features of the rail corridor. Our model incorporates *acceleration* and *constant speed/coasting* phases along with their corresponding power consumptions, as well as the *braking* phase along with its corresponding regenerative braking power, minimizes the number of trains accelerating simultaneously and synchronizes acceleration of trains with deceleration of other trains running in the corridor. Second, we generate a train timetable that minimizes both the peak power demand as well as the total energy consumption while keeping the average trip times of trains within acceptable limits. We also incorporate maximum trip times in our model so that the scheduler/decision maker can modify its value until a satisfactory solution in terms of peak power demand, total energy consumption and trip time is generated by the model. Third, since we simultaneously consider the impact of implementing the timetable on the entire, double track corridor, our approach leads to significantly improved results.

We test our model on real life problem sets obtained from four dedicated suburban rail corridors of MSS to generate energy efficient timetables. We solve the problem sets using Lingo 15.0, commercial optimization software. Our results indicate that, on average, our model

is able to reduce the peak power demand by 28 % and increase the utilization of regenerative braking power by 7.5 % while limiting the increase in trip times to 50 seconds (2%) in trips involving on average 42 minutes.

The rest of the chapter is organised as follows. Section 3.2 is devoted to a review of the relevant literature. In section 3.3, we present the model notation and formulation. In section 3.4, we develop an algorithm for solving the model. In Section 3.5, we present computational results on problem sets based on real life data obtained from the Mumbai Suburban System followed by discussion. Finally, in section 3.6, we conclude with a summary and directions for future research.

3.2 Literature Review

In this chapter, we study energy efficient Train Timetabling Problem (TTP) for Mass Rapid Rail Systems (MRRS). Energy efficient timetabling is a train scheduling problem which focuses on minimization of peak power and total power consumption. The Train Timetabling Problem (TTP) has received considerable attention in the operations research literature. TTP determines the arrival and departure times of the trains at all stations subject to various safety, business and operational constraints to optimize the objective function. The focus and objectives of TTP for long distance trains and Mass Rapid Rail Systems are entirely different. MRRS trains have identical priorities therefore, track capacity utilization of the corridor is determined by arrival and departure headways. On MRRS corridor, therefore the objective is to minimize either waiting time for passengers or cost of operations. Corridors used for running long distance trains are heterogeneous. Train having different priorities, speed characteristics and halt pattern run on these corridors. The focus of TTP for long distance trains therefore is to maximize capacity utilization by scheduling as many trains as possible.

The models reported in prior literature for long distance trains have focused on different objective functions. Kraay, Harker and Chen (1991) propose a model to minimize fuel consumption by optimally pacing freight trains, Jovanovic and Harker (1991) suggest an approach for scheduling trains without using any explicit objective function, Brannlund et al. (1998) present a model to maximize operating profit, Cacchiani, Caprara, and Fischetti (2012) propose a model to improve the robustness of the timetable and Salicrúa, Fleurentb and Armengolc (2011) develop a model to optimize the reliability of the train schedule.

Trains used in MRRS are operationally identical and it is critical to minimize passenger wait times, total energy consumption and peak power demand. Wong et al. (2008) present a MIP model to generate synchronised timetables that minimizes total transfer waiting time (at interchange stations) for passengers while Kroon and Peeters (2003) propose a model to reduce wait times at connecting stations by including variable trip times. Several papers addressing the timetable optimization problem for MRRS seek to minimize the peak power demand or the total energy consumption. This stream of literature is referred to as energy efficient timetabling and we review it in a greater detail below.

Sanso and Girard (1997) propose an optimization model to minimize the instantaneous peak power demand of a subway system during rush hour by attempting to desynchronize train departure times. To solve the model the authors develop a heuristic algorithm that divides the rush hour problem into a series of smaller desynchronization problems of shorter intervals. They report average reduction of 16% in the peak power demand while limiting the increase in accumulated throughput time to 28 seconds for 22% trains and zero delay for 78% trains. However, this model does not take regenerative braking into consideration.

Maite et al. (2011) develop an optimization model that synchronizes acceleration of one train with braking of another train at the same station or at the adjoining station to reduce the total traction energy consumption by maximizing the utilization of regenerative energy. They

adjust the halting time and the running time of trains without affecting the feasibility of the existing timetable. The authors use CPLEX 12.1 software to solve the model and test this model on the data obtained from the Madrid underground system and report energy savings of 7% without affecting the quality of customer service. This model considers trains running in only a single direction and does not address the issue of minimizing peak power demand.

Yang et al. (2013) too present a timetable optimization model to improve utilization of regenerative braking energy. They propose a cooperative scheduling approach to maximize the overlapping period of acceleration phase of one train with deceleration phase of another train running in the same section and powered by the same substation. The authors propose a Genetic Algorithm to solve the problem and report computational results using data from the Beijing Yizhuang subway line in China. They too address only the minimization of total power rather than peak power demand. Li and Yang (2013) present a stochastic cooperative scheduling model for single direction trains to optimize the utilization of regenerative braking energy to minimize total power consumption.

Yang et al. (2014) propose an Integer Programming model with the twin objective of maximizing the utilization of regenerative energy and the reduction of the passenger waiting time by using headway and halt time control. They design a Genetic Algorithm with binary encoding to compute the optimal solution. The interested reader is also referred to Albrecht. (2004), Chen et al. (2005), Kim, Oh and Han (2010) Kim and Kim (2010) Nasri et al. (2010) for a review of related models. It can be seen from the above discussion that most of these papers start from an existing timetable and modify this timetable using approximate optimization methods such as genetic algorithm. They do not attempt to develop a comprehensive timetable that minimizes both peak power demand as well as total energy consumption for trains running in both directions. This is a significant gap from the perspective of both academic research as well as practice.

In this paper we formulate a new comprehensive Integer Programming (IP) model that concurrently considers minimization of total number of trains accelerating simultaneously and synchronization of acceleration of trains with deceleration of other trains producing regenerative braking power and improves both peak power demand as well as total energy consumption on the entire corridor.

3.3 Model Formulation

Our model, Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization (STMPMR) considers a double track suburban rail system, in which one track is dedicated for trains carrying traffic in a particular (up) direction while the other track is dedicated to traffic in the opposite (down) direction to generate an integrated timetable for both up and down trains simultaneously. We make the following assumptions (i) regenerated energy from braking has to be consumed instantaneously by other accelerating and constant speed trains otherwise it is wasted as heat as there are no storage devices on the train or in the system (ii) the traction system supplying power to both “up” and “down” trains is obtained from the same substation; therefore regenerated braking energy produced by “up” or “down” trains can be consumed by other accelerating or constant speed trains running in either direction (iii) trains consume no power when halted. In reality, for all the phases (halt, acceleration, constant speed or coasting and deceleration) constant power is consumed by the railcars for running auxiliary equipment such as lights, fans etc. However, as this power requirement is significantly lower than the traction power, we ignore the power consumption of auxiliary equipment in our model.

Table 3.1 and Table 3.2 below list the notations (parameters, decision variables and auxiliary variables) used in the STMPMR model. All the notations for trains running in up direction are same as that for trains running in the “down” direction except they are indicated

Table 3.1 Notations-Parameters and Decision Variables

Problem Parameters/Input	
S	Number of stations on the corridor
$s = 1, \dots, S$	Station index for down direction trains
$s = S \dots 1$	Station index for up direction trains
$\tau = 1, \dots, T$	Time index
$t = 1, \dots, N$	Train index for down direction trains
$u = 1 \dots U$	Train index for up direction trains
$w^s(\bar{w}^s)$	Minimum halt time for down (up) train $t(u)$ at station s
$r^s(\bar{r}^s)$	Running time for train $t(u)$ between stations $s-1(s)$ and $s(s-1)$
$w_{max}^s(\bar{w}_{max}^s)$	Maximum halt time for train $t(u)$ at any station s
h_d, h_a	Departure (Arrival) headway in minutes
a	Acceleration time before train t reaches constant speed or starts coasting
b	Deceleration time (regenerative braking phase)
π	Power consumption per unit time by an accelerating train
θ	Power consumption per unit time by a train to keep constant speed
λ	Regenerative power per unit time regenerated through braking by a decelerating train
f	Frequency of trains in terms of time interval between departure at station
α	Assigned value from 0 to 1 depending on relative weight for reduction of peak power and total power
M	A large positive real number
Decision Variables	
D_t^s	Departure time of train t at station s , $1 \leq s < S$
$\bar{D}_u^s$	Departure time of train u at station s , $1 < s \leq S$
A_t^s	Arrival time of train t at station s , $1 < s \leq S$
$\bar{A}_u^s$	Arrival time of train u at station s , $1 \leq s < S$
e^τ	Power consumption at any time τ , $\forall \tau = 1, \dots, T$ for $Y^\tau > 0$
E	Total energy consumption during period T
Z	Peak power requirement at any time τ , $\forall \tau$
Y^τ	Net power requirement at time τ , $\tau = 1, \dots, T$

Table 3.2 Notations-Auxiliary Variables

$E_{st}^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is accelerating between station } s \text{ and } s + 1 \\ s = 1, \dots, S - 1, \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{E}_{su}^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is accelerating between station } s \text{ and } s - 1 \\ s = 2, \dots, S, \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$F_{st}^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is at constant speed/coasting between station } s \text{ and } s + 1 \\ s = 1, \dots, S - 1, \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{F}_{su}^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is at constant speed or coasting between } s \text{ and } s - 1 \\ s = 2, \dots, S, \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$H_{st}^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is decelerating between station } s \text{ and } s + 1 \text{ (regenerative braking)} \\ s = 1, \dots, S - 1, \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{H}_{su}^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is decelerating between } s \text{ and } s - 1 \text{ (regenerative braking)} \\ s = 2, \dots, S, \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$G_{st}^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is halting at station } s \\ s = 2, \dots, S - 1, t = 1, \dots, N, \tau = 1, \dots, T \\ 0, \text{ otherwise} \end{cases}$ Note: $G_{1t}^\tau = 1$ implies that train t has not departed from station 1 yet.
$\bar{G}_{su}^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is halting at station } s \\ s = 2, \dots, S, \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$X_t^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is accelerating and consuming power at time } \tau \\ \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{X}_u^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is accelerating and consuming power at time } \tau \\ \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$Q_t^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is in constant speed/coasting and consuming power at time } \tau \\ \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{Q}_u^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is in constant speed or coasting mode and consuming power at time } \tau \\ \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$
$V_t^\tau =$	$\begin{cases} 1, \text{ if train } t \text{ is in regenerative braking mode and regenerating power at time } \tau \\ \tau = 1, \dots, T, t = 1, \dots, N \\ 0, \text{ otherwise} \end{cases}$
$\bar{V}_u^\tau =$	$\begin{cases} 1, \text{ if train } u \text{ is decelerating and regenerating power at time } \tau \\ \tau = 1, \dots, T, u = 1, \dots, U \\ 0, \text{ otherwise} \end{cases}$

with the “bar” subscript and the train index t is replaced by train index u . There are S stations, sequentially numbered from 1 to S in the down direction and from S to 1 in up direction. The

station index is represented by s . In the down (up) direction N (U) trains depart sequentially $1 \dots N$ ($1 \dots U$) over the planning horizon T . It is assumed that trains are operationally identical and each train starts from the first station and terminates at the last station. The planning horizon T could either be 24 hours or the duration of peak traffic period (i.e. 3 hours). The *running time* between stations $s-1(s)$ and s ($s-1$) is represented by $r^s(\bar{r}^s)$ while the *halt time* of train t (u) at station s is represented by $w^s(\bar{w}^s)$. Each train halts at every station for boarding and alighting of passengers. The maximum halt time for train t (u) at station s is represented by $w_{max}^s(\bar{w}_{max}^s)$. The *departure (arrival) headway* h_d (h_a) is the minimum time interval between the departures (arrivals) of two consecutive trains at a station to ensure consecutive trains do not share the same track at the same time. The *Departure time* (the *arrival time*) of down train t at station s is represented by $D_t^s(A_t^s)$ and the *Departure time* (the *arrival time*) of up train u at station s is represented by $\bar{D}_u^s(\bar{A}_u^s)$. Both departure and arrival times are discretized continuous variables and expressed as integers. Parameter “ a ” is the acceleration time and “ π ” is the power consumption per unit time during this period. Parameter “ b ” is the deceleration time during which regenerative braking energy “ λ ” per unit time is regenerated. In between acceleration and deceleration phase train is in either constant speed or coasting phase before beginning of regenerative braking. During constant speed phase a small amount of power “ θ ” per unit time is consumed by the train to keep its speed constant while during coasting phase “ θ ” becomes zero. The time duration for constant speed (or coasting phase) is the difference between the running time for a section and time for acceleration plus deceleration. Note that in reality, acceleration and deceleration times will be different depending on the inter-station distance. Therefore the values of “ a ” and “ b ” will also depend on the specific station-pairs s and $s+1$. To keep the presentation simple, in this chapter we assume that “ a ” and “ b ” are constant across all the stations. This is a reasonable and valid assumption if inter-station distances are similar across the entire corridor as is the case in the Mumbai Suburban System.

Our formulation can easily accommodate station-pair dependent values for “ a ” and “ b ”. Train $t(u)$ would be accelerating, coasting/constant speed or decelerating between stations $s(s)$ and $s+1(s-1)$ or halting at station s at time τ . Status of train $t(u)$ whether accelerating, or at constant speed/coasting, or decelerating, or halting is captured by the binary variables $E_{st}^\tau (\bar{E}_{su}^\tau)$, $F_{st}^\tau (\bar{F}_{su}^\tau)$, $H_{st}^\tau (\bar{H}_{su}^\tau)$ and $G_{st}^\tau (\bar{G}_{su}^\tau)$ respectively. Power consumption status of a train $t(u)$ whether in accelerating or constant speed/coasting phase at any time τ is captured by binary variables $X_t^\tau (\bar{X}_u^\tau)$ and $Q_t^\tau (\bar{Q}_u^\tau)$ respectively while regenerative braking status of train $t(u)$ in deceleration phase is captured by binary variable $V_t^\tau (\bar{V}_u^\tau)$. Variables Y^τ , e^τ and E capture the amount of net energy consumption, positive net energy consumption and the total energy consumption by train $t(u)$ at time τ over the planning horizon T . Variable Z captures peak power demand at time τ over the planning horizon T . Parameter α is assigned value between 0 and 1 depending on the relative weights given for reduction of peak power versus total energy consumption. The problem formulation for Scheduling Trains to Minimize Peak power and Maximize Regenerative braking power utilization (STMPMR) is presented below.

STMPMR

Min $\alpha Z + (1-\alpha) E$	(1)
Subject to:	
$D_t^1 - D_{t-1}^1 = f, 1 < t \leq N$	(2d)
$\bar{D}_u^1 - \bar{D}_{u-1}^1 = f, 1 < u \leq U$	(2u)
$D_1^1 \leq f$	(3d)
$\bar{D}_1^1 \leq f$	(3u)
$A_t^s = D_t^{s-1} + r^s, 1 < s \leq S, \forall t$	(4d)
$\bar{A}_u^s = \bar{D}_u^{s-1} + \bar{r}^s, 1 \leq s < S, \forall u$	(4u)
$(D_t^s - A_t^s) \geq w^s, 1 \leq s \leq S, \forall t$	(5d)
$(\bar{D}_u^s - \bar{A}_u^s) \geq \bar{w}^s, 1 \leq s \leq S, \forall u$	(5u)
$(D_t^s - A_t^s) \leq w_{max}^s, 1 \leq s \leq S, \forall t$	(6d)
$(\bar{D}_u^s - \bar{A}_u^s) \leq \bar{w}_{max}^s, 1 \leq s \leq S, \forall u$	(6u)
$D_t^s \geq (D_{t-1}^s + h_d), 1 \leq s < S, 1 < t \leq N$	(7d)
$\bar{D}_u^s \geq (\bar{D}_{u-1}^s + h_d), 1 < s \leq S, 1 < u \leq U$	(7u)
$M(1 - E_{st}^r) + E_{st}^r \tau \geq D_t^s, s=1, \dots, S-1, \forall \tau, t$	(8d)
$M(1 - \bar{E}_{su}^r) + \bar{E}_{su}^r \tau \geq \bar{D}_u^s, s=2, \dots, S, \forall \tau, u$	(8u)
$E_{st}^r \tau \leq (D_t^s + a - 1), s=1, \dots, S-1, \forall \tau, t$	(9d)
$\bar{E}_{su}^r \tau \leq (\bar{D}_u^s + a - 1), s=2, \dots, S, \forall \tau, u$	(9u)
$M(1 - F_{st}^r) + F_{st}^r \tau \geq D_t^s + a, s=1, \dots, S-1, \forall \tau, t$	(10d)
$M(1 - \bar{F}_{su}^r) + \bar{F}_{su}^r \tau \geq \bar{D}_u^s + a, s=2, \dots, S, \forall \tau, u$	(10u)
$F_{st}^r \tau \leq (A_t^{s+1} - (b + 1)), s=1, \dots, S-1, \forall \tau, t$	(11d)
$\bar{F}_{su}^r \tau \leq (\bar{A}_u^{s+1} - (b + 1)), s=2, \dots, S, \forall \tau, u$	(11u)
$M(1 - H_{st}^r) + H_{st}^r \tau \geq (A_t^{s+1} - b), s=1, \dots, S-1, \forall \tau, t$	(12d)
$M(1 - \bar{H}_{su}^r) + \bar{H}_{su}^r \tau \geq (\bar{A}_u^{s+1} - b), s=2, \dots, S, \forall \tau, u$	(12u)
$H_{st}^r \tau \leq (A_t^{s+1} - 1), s=1, \dots, S-1, \forall \tau, t$	(13d)
$\bar{H}_{su}^r \tau \leq (\bar{A}_u^{s+1} - 1), s=2, \dots, S, \forall \tau, u$	(13u)
$M(1 - G_{st}^r) + G_{st}^r \tau \geq A_t^s, s=1, \dots, S-1, \forall \tau, t$	(14d)
$M(1 - \bar{G}_{su}^r) + \bar{G}_{su}^r \tau \geq \bar{A}_u^s, s=2, \dots, S, \forall \tau, u$	(14u)
$G_{st}^r \tau \leq (D_t^s - 1), s=1, \dots, S-1, \forall \tau, t$	(15d)
$\bar{G}_{su}^r \tau \leq (\bar{D}_u^s - 1), s=2, \dots, S, \forall \tau, u$	(15u)
$\sum_{s=1}^{S-1} (F_{st}^r + G_{st}^r + E_{st}^r + H_{st}^r) = 1, \forall \tau, t$	(16d)
$\sum_{s=2}^S (\bar{F}_{su}^r + \bar{G}_{su}^r + \bar{E}_{su}^r + \bar{H}_{su}^r) = 1, \forall \tau, u$	(16u)
$X_t^r = \sum_{s=1}^{S-1} E_{st}^r, \forall \tau, t$	(17d)
$\bar{X}_u^r = \sum_{s=2}^S \bar{E}_{su}^r, \forall \tau, u$	(17u)
$Q_t^r = \sum_{s=1}^{S-1} F_{st}^r, \forall \tau, t$	(18d)
$\bar{Q}_u^r = \sum_{s=2}^S \bar{F}_{su}^r, \forall \tau, u$	(18u)
$V_t^r = \sum_{s=1}^{S-1} H_{st}^r, \forall \tau, t$	(19d)
$\bar{V}_u^r = \sum_{s=2}^S \bar{H}_{su}^r, \forall \tau, u$	(19u)
$Y^r = \sum_{t=1}^N [\pi X_t^r + \theta Q_t^r - \lambda V_t^r] + \sum_{u=1}^U [\pi \bar{X}_u^r + \theta \bar{Q}_u^r - \lambda \bar{V}_u^r], \forall \tau$	(20)
$Z \geq Y^r, \forall \tau$	(21)
$e^r \geq Y^r, \forall \tau$	(22)
$E = \sum_{\tau=1}^T e^r, \forall \tau$	(23)
$E_{st}^r, F_{st}^r, G_{st}^r, H_{st}^r \in \{0,1\}, s=1, \dots, S-1, \forall \tau, t$	(24d)
$\bar{E}_{su}^r, \bar{F}_{su}^r, \bar{G}_{su}^r, \bar{H}_{su}^r \in \{0,1\}, s=2, \dots, S, \forall \tau, u$	(24u)
$X_t^r, Q_t^r, V_t^r, \bar{X}_u^r, \bar{Q}_u^r, \bar{V}_u^r \in \{0,1\}, \forall \tau, t, u$	(25)
$e^r, E \geq 0, \forall \tau$	(26)

As discussed earlier, we consider double track corridors and simultaneously generate integrated timetable for both down and up trains. Constraints 2 to 19 and 24 to 26 are marked as “d” for the “down” direction trains and as “u” for the “up” direction trains. Since the logic of formulation is same, to avoid repetition, we provide explanation only for down direction trains and for remaining common constraints. The *Objective function* minimizes the weighted value of the peak energy and the total energy consumption. Constraints (2d) ensures that time interval between departure of two consecutive trains at the originating station is equal to the frequency of the trains while (3d) ensures that the first train from the first station departs not later than the value of train frequency. Constraint (4d) ensures that the running time between stations is accounted for in the departure and the arrival time of a train at contiguous stations, while (5d) enforces the actual halt time at the station not to be less than the minimum halt time. Constraint (6d) limits the actual halt time at any station $1 \leq s \leq S$ to predefined maximum halt time. The acceptable level of average train trip time can be obtained by manoeuvring w_{max}^s (maximum halt time) allowed at any station. Constraint (7d) enforces the departure headway between two consecutive trains at any station. Logical constraints (8d) and (9d) ensure binary variable E_{st}^τ ($t=1...N, s=1...S-1, \tau=1...T$) takes a correct value at time τ . $E_{st}^\tau=1$ implies that the train t is accelerating after starting from the halt i.e. $(D_t^s \leq \tau < D_t^s + a)$. The term involving M is incorporated into (8d) to ensure that no unwanted constraint is imposed on D_t^s when $E_{st}^\tau=0$. Logical constraints (10d) and (11d) ensure binary variable F_{st}^τ ($t=1...N, s=1...S-1, \tau=1...T$) takes the correct value at time τ . $F_{st}^\tau=1$ implies that the train t is in constant speed or coasting phase, i.e. $(D_t^s + a \leq \tau < A_t^{s+1} - b)$. Logical constraints (12d) and (13d) ensure that binary variable H_{st}^τ ($t=1...N, s=1...S-1, \tau=1...T$) takes the correct value at time τ . $H_{st}^\tau=1$ implies that the train t is decelerating before the halt at station $s+1$ i.e. $(A_t^{s+1} - b \leq \tau < A_t^{s+1})$. Logical constraints (14d) and (15d) ensure that the binary variable G_{st}^τ takes the correct value at time τ . Logical constraints (16d) ensure that at any time τ , a train has to be halting at one of the

stations $s = 2, \dots, S$, or accelerating, or running at constant speed/coasting, or decelerating between two stations or not yet departed from the first station. Logical constraints (17d), (18d), (19d) ensure the correct values are assigned to X_t^r , Q_t^r , V_t^r . Constraints (20) and (21) ensure that correct values of power at any point of time and peak power requirement taking both up and down trains simultaneously are assigned to Y^τ and Z ($t=1 \dots N$, $\tau=1 \dots T$). Similarly, constraints (22) and (23) capture the positive energy consumption and sum it over a time period T respectively taking both up and down trains simultaneously. Constraints (24d), (24u), (25), (26) are non-negativity, integer and binary constraints.

3.4 Heuristic Algorithm

We test our model using commercial optimization software LINGO 15.0 for different sizes of problem instances ranging from 10 trains to 120 trains and 8 stations to 19 stations. Size of the problem is determined both by the number of trains to be scheduled and number of stations on the corridor. We found that LINGO can solve problem instances of small size up to 20 to 25 trains and 10 to 12 stations optimally in a reasonable time. However, as the problem size increases, the solution time increases exponentially. Therefore, we propose a train partition heuristic to solve the larger real-life problem instances.

Train partition heuristic divides a master problem into smaller problems called partitions. Accordingly, N trains which are required to be scheduled are divided into smaller groups and assigned to each partition. Each partition will have exactly κ number of trains except the last partition. We solve $\lceil N/\kappa \rceil$ partitions sequentially to obtain solution for the master problem. LINGO is able to solve each partition optimally. The first partition has exactly same constraints as in STMPMR i.e. form (2) to (26), however subsequent partitions ($STMPMR^p$) have four additional constraints (28d), (28u), (29d) and (29u) to ensure that departure times of all the down (up) direction trains in the p^{th} ($\bar{p}^{\text{th}}$) block are not less than the

maximum of the departure times of all the down (up) trains in the preceding blocks plus the departure headway at all the stations and the arrival times of all the down (up) trains in the p^{th} ($\bar{p}^{\text{th}}$) block are not less than the maximum of the arrival times of all the down (up) trains in the preceding blocks plus the arrival headway at all intermediate and destination stations. As trains running on the dedicated suburban corridor are operationally identical, they do not overtake each other. Therefore, it is possible to construct a full timetable by sequentially solving $\lceil N/\kappa \rceil$ partitions without compromising the integrity of the solution. Determining the value of $\{ \kappa \}$ that will not compromise the quality of the solution is an important aspect of the train partitioning heuristic. The peak power demand depends on the maximum number of trains running simultaneously in the corridor. The maximum number of train running simultaneously at any time will be X (where, $X = \lceil \text{average trip time} / \text{frequency of trains} \rceil$). Therefore the value of $\{ \kappa \}$ should not be less than X . For example, Mumbai-Vashi (MV) corridor has 16 stations and trains are scheduled at frequency of 4 minutes with average trip time of 44 minutes and 20 seconds. Therefore, the value of X for this corridor is 11 and as a result value of κ cannot be less than 11.

Train Partition Heuristic for “STMPMR”

The number of trains N running on the given corridor are divided into P partitions.

$$P = \lceil N/\kappa \rceil$$

For $p = 1, 2 \dots P$,

Do

{

$$j_1^p = \{(p-1) \kappa\} + 1$$

$$j_e^p = \text{Min} \{p \kappa, T\}$$

(Note j_1^p and j_e^p are the first and the last train of the p^{th} partition respectively)

Solve STMPMR^p

}

Sub problem STMPMR^p

$$\text{Min} \quad \alpha Z + (1-\alpha) E \quad (27)$$

Subject to,

Constraints 2 to 26,

Additional constraints (only for $p > 1$)

Departure time constraint,

$$\{D_{j_1^{p+1}}^s \dots D_{j_e^{p+1}}^s\} \geq \text{Max} \{D_{j_e^p}^s\} + h_d, \forall s, 1 \leq s < S \quad (28d)$$

$$\{\bar{D}_{j_1^{\bar{p}+1}}^s \dots \bar{D}_{j_e^{\bar{p}+1}}^s\} \geq \text{Max} \{\bar{D}_{j_e^{\bar{p}}}^s\} + h_d, \forall s, 1 < s \leq S \quad (28u)$$

Arrival time constraint,

$$\{A_{t_1^{p+1}}^s \dots A_{t_e^{p+1}}^s\} \geq \text{Max} \{A_p^s\} + h_a \forall s, 1 < s \leq S \quad (29d)$$

$$\{\bar{A}_{t_1^{\bar{p}+1}}^s \dots \bar{A}_{t_e^{\bar{p}+1}}^s\} \geq \text{Max} \{\bar{A}_{\bar{p}}^s\} + h_a \forall s, 1 \leq s < S \quad (29u)$$

(Partition p is solved for down direction trains from j_1^p to j_e^p and $\bar{p}$ is solved for up direction trains from $j_1^{\bar{p}}$ to $j_e^{\bar{p}}$)

3.5 Computational Results and Discussion

We test our model extensively on the data obtained from Mumbai Suburban System of Indian Railways using LINGO 15.0. We select Mumbai-Vashi (MV), Mumbai-Thane (MT), Thane-Vashi (TV), Churchgate-Andheri (CA) dedicated suburban corridors for testing as they provide a huge potential for reduction of peak power demand and total power consumption. Details of these suburban rail corridors are presented in Table 3.3 below which includes corridor name, number of stations on the corridor, length of corridor in kilometres, average trip time of the trains and the number of trains to be scheduled.

Table 3.3 Features of suburban Rail Corridors used in the Computational Results

Corridor Name	# of Stations	Distance (KM)	Average Trip Time (Seconds)	# of trains to be scheduled
Mumbai -Vashi (MV)	16	28.9	2660	90
Mumbai -Thane (MT)	19	33	3110	90
Thane- Vashi (TV)	8	18.6	1680	120
Churchgate- Andheri (CA)	16	28	2540	120

In line with the current practice in Mumbai Suburban System, the departure and arrival headways are taken as 3 and 2 minutes respectively. Train t consumes power “ π ” during acceleration and power “ θ ” while running at constant speed and regenerates power “ λ ” during deceleration phase. We assume the values of accelerations and deceleration period as 40 seconds and 60 seconds respectively. The values of power consumption during acceleration and constant speed phases of train are 500 KW and 50 KW respectively and value of regenerative braking power is 100 KW. Our problem instances consider timetabling during peak period of a day for and take time horizon of 6, 3, 3 and 4 hours for TV, MV, MT and CA respectively. We divide the time horizon into slots of 10 seconds and each slot is expressed as 1 discrete integer unit.

3.5.1 Comparison of trains in single direction versus both directions

We generate train timetables using STMPMR for trains running in a single direction and later for trains running in both up and down direction simultaneously. We test the model on the data obtained from Mumbai-Vashi corridor and present the comparative performance in Table 3.4. This table shows the sum of maximum halt times across all the stations, total energy consumption (E) in KWH, percentage utilization of regenerative braking energy, peak power demand (Z) in KW and percentage reduction in peak power demand. To keep the problem size

equal, we schedule 24 trains in a single direction and 12 trains in each direction to test STMPMR.

Table 3.4 Comparison of STMPMR for Single Direction and Both Direction Trains

$\sum_{s=1}^S w_{max}^s$ (seconds)	Trip time (Seconds)		Total Energy Consumption (E) KWH		% Utilization of Regenerative Energy		Peak power consumption (Z) (KW) (% Reduction)	
	single	Both	single	Both	single	both	single	both
340	2660	2660	1570	1438	66.16	88.16	2300 (0)	4100 (0)
360	2690	2670	1481	1429	81	90.16	2200 (4.3)	3300 (19.5)
400	2700	2700	1477	1405	82.5	93.5	2100 (8.7)	3000 (26.8)

Table 3.4 indicates that timetable generated by STMPMR for trains running in both the direction requires peak power demand of 3000 KW (maximum 24 trains running at any time), utilizes 93.5 % of regenerated energy and restricts the trip time increase to 40 seconds while for trains running in a single direction requires peak power demand of 2100 KW (maximum 12 trains running at any time), utilizes 82.5% of the regenerated energy and restricts the trip time increase to 40 seconds. The integrated timetable generated by STMPMR with trains running in both directions simultaneously produces superior results as compared to the case with trains running in only one direction. On MV corridor trains run at 4 minute frequency and have average trip time of 44.3 minute therefore maximum 12 trains can run in each of the double track corridors. In case of trains running in single direction, model considers maximum 12 trains running at a time while in case of both direction trains, model considers maximum 24 trains running at any time. Therefore, in case of both direction trains, potential for synchronization of acceleration of trains with deceleration of other trains is much higher due to pooling effect of trains running in both the direction than that in single direction trains. STMPMR with both direction trains exploits the pooling effect of trains running in both directions.

3.5.2 Optimality of Heuristic Solution

We test our model using LINGO 15.0 for different size of problem instances ranging from 10 to 120 trains and 8 to 19 stations. Size of the problem is determined both by the number of trains to be scheduled and number of stations on the corridor. We find that LINGO can solve problem instances of small size up to 20 to 25 trains and 10 to 12 stations optimally in a reasonable time. However, as we gradually increase the size of the problem, the solution time increases exponentially. Therefore, we adopt heuristic approach to solve the larger real-life problem instances. The smallest size of the problem instance we test our model is 120 trains and 8 stations while the largest size of the problem instance is 120 trains and 19 stations.

We develop a train partition heuristic to solve real-life problem instances in a reasonable time. To test the optimality of our heuristic solution we solve three instances of small size involving 40 trains and 8 stations optimally using Lingo and compare the results with heuristic solution.

Table 3.5 Optimality of Heuristic Solution

$\sum_{s=1}^S w_{max}^s$ (seconds)	Trip time (Seconds)		Total Energy Consumption (E) KWH		% Utilization of Regenerative Energy		Peak power consumption (Z) (KW) (% Reduction)		Solution Time (Seconds)	
	Heuri stic	Opti mal	Heuri stic	Optimal	Heuri stic	Optimal	Heuri stic	Optimal	Heurist ic	Optimal
120	1680	1680	752	726	59	70	0	0	110	91
180	1720	1710	699	686	81	86	6.67	6.67	2506	28030
210	1740	1710	698	686	81.5	86	6.67	6.67	54963	>180000

First, we solve the full problem using LINGO to obtain the optimal solution. Later, we use train partition heuristic to solve the same problem instances. Computational results of the problem instances are given in Table 3.5. The test results tabulated in Table 3.5 indicate that our formulation is able to improve the regenerative braking by 22.8% and peak power usage by 6.67% while it restricts the average trip time increase to only 2%. Comparatively, heuristic improves the regenerative braking utilization by 38% and peak power usage by 6.67% while it restricts the average trip time increase to 4%. It can be seen that as maximum halt time increases from 120 to 210 seconds, the optimality gap for regenerative braking utilization reduces from 11% to 4.5% and for peak power usage to 0% while trip time optimality gap increases from 0% to 2%. We further test our heuristic for one more instance with $\sum_{s=1}^S w_{max}^s = 240$ seconds. In this instance, our heuristic is able to improve the regenerative braking power utilization by 48.3% and peak power usage by 40% while restricting the trip time increase to 4%. In the problem instant with maximum halt time of 210 seconds the heuristic takes 12 hours and 22 minutes to solve this problem while optimal solution of full problem is not possible to obtain for commercial optimization software LINGO even in 48 hours.

3.5.3 Impact on Peak and Total Power Consumption

In this section, we present the computational results of problem instances obtained from dedicated suburban corridors of Mumbai Suburban Network. We use the train partition heuristic to solve the model and tabulate the results. Table 3.6 which shows, name of the corridor, the number of trains to be scheduled, sum of the maximum halt time across all the stations, average train trip time, total energy consumption (E) in KWH, percentage improvement in utilization of regenerative braking energy and peak power along with percentage peak power reduction.

Table 3.6 Computational Results of Dedicated Suburban Corridor Problem Sets

Corridor	# of trains	$\sum_{s=1}^S w_{max}^s$ (seconds)	Trip time (seconds)	Total Energy Consumption (KWH)	% Utilization of Regenerative Energy	Peak Power (KW) (% Reduction)
Thane -Vashi (TV) Corridor						
TV	120	120	1680	101.5	65	1500 (0)
		180	1720	97	84	1400 (6.7)
		240	1750	95.1	89	900 (40)
Mumbai -Vashi (MV) Corridor						
MV	90	340	2660	153.8	90	4100 (0)
		360	2670	152.6	93	3300 (19.5)
		380	2680	151.9	95	2800 (31.7)
Mumbai -Thane (MT) Corridor						
MT	90	320	3110	181.4	97	3900 (0)
		350	3130	181.1	97.5	3100 (20.5)
		360	3140	180.9	98	3000 (23)
Churchgate -Andheri (CA) Corridor						
CA	120	290	2540	202	96	3400 (0)
		330	2560	201.8	96.5	2900 (14.7)
		340	2570	202.5	95	2800(17.6)

We solve three problem instances for each corridor and generate corresponding timetables. The first problem instance represents the existing timetable i.e. maximum halt time at each station is equal to the actual halt time in the existing timetable. Solution to the first problem instance gives reference values of peak power requirement and total power consumption and current average throughput time of trains. Two additional problem instances are designed by increasing the maximum halt times of trains. Solution of additional problem instances provide the comparison of peak power, total power consumption and average throughput times with respect to reference timetable. We generate three timetables for three problem instances and compare the train trip time, peak power demand and total power consumption. Examination of computational results of the TV corridor in Table 3.6 shows that our model is able to reduce the peak power demand by 40 percent and increase the utilization of regenerative power by 24 percent while restricting the trip time increase to only 70 seconds as compared to the current

practice (existing timetable). On average the proposed model is able to reduce the peak power demand by 28 percent and improve the utilization of regenerative braking energy by 7.5 percent while restricting the increase in trip time to 50 seconds. It is interesting to note that the utilization of regenerative braking energy is 97% and 96% on MT and CA corridors respectively with the actual halt times. On the other hand the utilization of regenerative braking energy is only 65% on TV corridor. The reasons behind this huge gap in regenerative braking utilization are higher train density and almost uniform distribution of stations across MT and CA corridors as compared to that in TV corridor. Stations are located at short distances and train frequency is 4 minute on MT and CA corridors while distance between stations is comparatively longer and train frequency is 6 minute on TV corridor. While on TV corridor the utilization of regenerative braking energy improved significantly by 37%, on MT corridor it improved marginally by 1%. Our model is able to improve the peak power demand significantly across all the corridors from 17.6% on the CA corridor to 40% on the TV corridor and in between this range on other corridors. The increase in the trip time is within allowable limits. The increase is from 20 seconds on the MV corridor to 70 seconds on the TV corridor and in between this range on other corridors. It can be seen that on TV corridor our model is able to significantly improve the peak power demand by 40% and regenerative braking utilization by 37% while limiting the increase in trip time to 70 seconds which is approximately 4% increase. Out of various timetable options generated by STMPMR, train timetable planners can choose any one option depending upon the trade-off between peak power and regenerative braking energy utilization on one hand and trip time increase on the other.

3.5.4 STMPMR as a Tool to Reduce Peak Power Demand

Our model can be used as a tool to reduce peak power demand by controlling the increase in average trip time of trains. This feature of the model has very important application in meeting increased peak power demand due to increase in frequency of trains without actually increasing

the capacity of the traction substation. A new rail corridor is generally operated at a lower train frequency due to lower passenger demand for the initial years of commissioning of the corridor. However, as the passenger demand grows, the number of trains running per hour is correspondingly increased to match the passenger demand. Increase in train frequency results in increased number of trains running on the corridor drawing more peak power from the substation. Normally, increase in peak power demand is met by augmenting the capacity of substation and traction equipment which requires additional investment. Alternatively, it is possible to reduce the peak power demand by generating an energy efficient timetable using the STMPMR model just by adjusting the halt times and relatively smaller increase in the trip time.

Table 3.7 Comparison of Trains with 5 and 4 Minutes Frequency on MV Corridor

$\sum_{s=1}^S w_{max}^s$ (seconds)	Trip time (Seconds)		Total Energy Consumption (E) KWH		% Utilization of Regenerative Energy		Peak Power (Z) (KW) (% Reduction)	
	Train Frequency (Minutes)		Train Frequency (Minutes)		Train Frequency (Minutes)		Train Frequency (Minutes)	
	5	4	5	4	5	4	5	4
340	2660	2660	154.8	153.8	87.2	90	3600 (0)	4100 (0)
360	2680	2670	151.5	152.6	96	93	3100 (13.9)	3300 (19.5)
380	2690	2680	151.7	151.9	95.5	95	2700 (25)	2800 (31.7)

In Table 3.7, we present a comparison of set of problem instances for trains running on MV corridor. One problem set considers a train frequency of 5 minute and another 4 minute respectively. In each set there are three problem instances one with existing halt time and other two with higher halt times. The peak power demand for actual halt time corresponding to existing timetable and at train frequency of 5 minute is 3600 KW. Peak power demand increases to 4100 KW when trains run at 4 minute frequency. In the first set our model is able to reduce the peak from 3600 KW to 2700 KW and in second set from 4100 KW to 2800 KW by adjusting the halt times without any negative impact on average trip time.

3.5.5 Generating Timetable for 24 hour period.

A published timetable for a suburban system such as Mumbai's has a time horizon of 24 hours and is repeated daily. 24 hour period of a day is divided into 5 sub-periods namely pre-morning peak (4 am to 8.29 am), morning peak (8.30 am to 11 am), post-morning peak (11.01 am to 4.59 pm), evening peak (5 pm to 8 pm), post-evening peak (8.01 am to 1 am) and maintenance period (1.01 am to 3.59 am). On MT, MV and CA corridors, during peak hours the trains run at 4 minute frequency and during the off peak period at 6 minute frequency. On the TV corridor, during peak period train frequency is 6 minutes which reduces to 8-10 minutes during off-peak periods. It is possible to build a full timetable by independently solving these 5 sub-period problems using the train partition heuristic.

3.6. Summary and Conclusion

This paper addresses the problem of minimizing the energy cost and CO₂ emission of Mass Rapid Rail Systems (MRRS) by minimizing both peak power demand and total energy consumption. The traction energy cost is a major component of the cost of operating a MRRS. The energy cost is determined by two factors first total energy consumption and second peak power demand. Moreover, power generation capacities and related CO₂ emissions are also determined by both peak power demand as well as total energy consumption. Therefore, reduction of peak power demand along with total energy consumption is crucial. However few previous papers have considered the need to simultaneously reduce total energy consumption and peak power demand. In this paper we endeavour to fill this important gap in literature by minimizing both peak power demand as well as total power consumption while keeping trip times within acceptable limits. We develop a comprehensive Integer Programming (IP) model that minimizes the peak power demand by reducing the number of trains accelerating simultaneously and total energy consumption by maximizing utilization of regenerative

braking power while keeping trip times within acceptable limits. Our model integrates the speed-time and the power-time train characteristics with features of rail network to incorporate acceleration time, constant speed/coasting time and their corresponding power consumptions, as well as deceleration time and corresponding regeneration of braking power. We tested our model on real life problem sets obtained from the Mumbai Suburban System (MSS) of Indian Railways which operates more than 2800 trains that carry over eight million commuters daily on six corridors. On MSS there exists a potential for reduction of CO₂ emission by 380,000 tonnes and saving of 443 million units of energy. For our study, we select four dedicated suburban rail corridors which have maximum potential for energy saving. Computational results show that on Thane-Vashi corridor our model is able to reduce the peak power demand by 40 percent and increase the utilization of regenerative energy by 24 percent while restricting the trip time increase to only 70 seconds. On average our model is able to reduce the peak power demand by 28% and increase the regenerative braking energy utilization by 7.5 % while limiting the trip time increase to only 50 seconds. Our model can also be used as a tool to keep the peak power demand within range when the train frequency is increased. The methodologies we suggest in this paper have the potential to significantly reduce the CO₂ emission and energy cost of Mumbai Suburban System.

Chapter 4

Integrated Locomotive Scheduling and Routing Problem

4.1 Introduction

Locomotives play an important role in cost-effective and efficient management of train operations as on-time performance of trains and quality of service depend on the availability of the right type of locomotive at the right time and at the right place. In large railway systems such as those in United States, Europe, China and India where thousands of trains are run daily, a huge fleet of locomotives having different types and pulling powers is required to manage the rail operations. Locomotives are a capital intensive resource. They represent billions of dollars of investment and also account for a significant percentage of operating cost. Therefore, even a small improvement in the efficient utilization of locomotives can result in substantial cost savings.

A locomotive schedule is generated to implement a given train timetable such that each train is assigned a suitable type of locomotive/consist (consist is a prime-mover comprised of two or more locomotives linked together as a single unit) from a given pool of locomotives/consists. Train originate from one station and terminate at a destination station. It is pulled by a locomotive/consist, therefore for efficient train operations availability of right type of locomotive/consist at right time is a goal of locomotive/scheduling problem. Implementing a feasible locomotive schedule might require idling of locomotives at a particular station, dead-heading of locomotive (when it is attached to another train and not powered) between two stations or light-heading between stations (when it is not pulling any train). Idling, dead-heading and light-heading of locomotives have to be minimized for efficient use of the locomotives. The Locomotive Scheduling Problem (LSP) and the Locomotive Routing Problem (LRP) considered one of the most important problems in rail operations as it ensures

optimum utilization of locomotives. In the prior literature, the LSP and LRP are treated as distinct problems and solved sequentially.

The LSP aims at assigning a set of locomotives/consists to each train optimally in a predefined train schedule so that sufficient pulling power is provided to each train from its origin to destination station while satisfying several operational constraints. The objective of the LSP is to minimize the operating cost of locomotives. The solution to the LSP assigns locomotives to trains and determines the optimal *fleet mix* (locomotive types) for a pre-defined train schedule. LSP helps locomotive planners in tactical decision-making process. The schedule generated by LSP is not directly implementable as it provides only type and corresponding number of locomotives required for implementation of a given train schedule and does not include fuelling and maintenance constraints of locomotives.

The LRP aims to determine the day to day routing of a locomotive/ consist in the given planning horizon including, hauling, dead-heading, light-heading, fuelling and maintenance of the locomotive. The LRP receives schedule generated by LSP as an input and determines the route for each locomotive/consist. The LRP helps train planners to take tactical as well as operational decisions. The schedule generated by LRP is feasible and implementable as it contains the route for each individual locomotive/consist including maintenance and fuelling constraints.

The motivation for this paper comes from the Locomotive Scheduling and Routing Problem faced by Indian Railways (IR) where the author works as a member of the senior management team. IR runs more than 13000 passenger trains and 8000 freight trains daily and holds inventory of more than 10000 different types of locomotives. Each locomotive costs 2-3 million USD, therefore, even a small improvement in locomotive utilization can result in significant economic saving. In this chapter, we develop an Integer Programming (IP) formulation that integrates LSP and LRP into a single unified problem called *Integrated*

Locomotive Scheduling and Routing Problem (ILSRP) for a rail network. The ILSRP generates the locomotive assignment for a pre-defined train schedule as well as routing plan for each locomotive/consist while minimizing the operating cost of locomotives. Our model considers the light-heading and dead-heading of locomotives/consist and repositions them from locomotive surplus to locomotive deficit station. Locomotives/consist move from one station (surplus) to another station (deficit) by actively pulling the train or by light-heading or by dead-heading. In light heading mode, consist travels to a new location without actively pulling the train and requires a separate crew to operate it. In a deadheading, separate crew is not required to operate the locomotives as locomotives are attached to another train as a set of inactive locomotives. Dead-heading cost less than light heading. However, its feasibility depends on the availability of a scheduled train at an appropriate time. Light-heading does not face any timing or scheduling constraint and can be scheduled independently. Our model also incorporates maintenance of locomotives. It considers maintenance depots along with their maintenance capacity and ensures that each locomotive/consist is dispatched for servicing periodically to the depot as per the existing practices of IR.

As mentioned earlier, previous approaches treat LSP and LRP as distinct problems and solve them sequentially resulting in a suboptimal solution. There exists a crucial link between tactical planning and operational planning. Therefore, integrating scheduling and routing phases and solving them concurrently leads to an optimal solution. The key contribution of this paper is to develop a new formulation that integrates LSP and LRP into a single unified model called *Integrated Locomotive Scheduling and Routing Problem (ILSRP)*. The ILSRP is a comprehensive formulation. It considers deadheading, light heading and maintenance constraints of locomotives and generates a locomotive schedule including routing of individual locomotive for a pre-defined train schedule that can be implemented without further

adjustment. We tested our model with problem instances of smaller, medium as well as large size.

The rest of the chapter is organised as follows. Section 4.2 is devoted to a review of the relevant literature. In section 4.3, we develop a problem statement that we use for developing the formulation. In section 4.4, we present the model notation and formulation. In Section 4.5, we present computational results followed by discussion. Finally, in section 4.6, we conclude with a summary and directions for future research.

4.2 Literature Review

A significant amount of research has been devoted to locomotive scheduling in railway operations planning literature. Earliest work on locomotive scheduling was aimed at developing decision support systems to assist planners in locomotive assignment using simulation methods and decision rules based on experience. However, 1980 onwards, the focus of research changed from simulation to optimization. Locomotive scheduling optimization models are classified into single locomotive and multiple locomotive models based on the number of locomotives used for hauling a train. In each of the above categories, models are further classified based on the purpose they are expected to serve e.g. strategic, tactical and operational planning. Strategic planning models aim to estimate the number and types of locomotives required to haul each train at a maximum permissible speed and optimal fleet mix (locomotive types) for a pre-defined train schedule while minimizing the capital investment and locomotive maintenance cost. Generally, a planning horizon for strategic models ranges from two to ten years. Tactical planning models aim to assign locomotives to each train in a given train schedule subject to all operational constraints. The objective of tactical models is to minimize operating cost of locomotives. The planning horizon of tactical models ranges from one year to a few months. The aim of operational planning model is to determine day to

day routing of individual locomotive that includes fuelling and servicing constraints. Planning horizon for operational model is usually from a day to a week. On the basis of number of locomotives assigned to a train, models are classified as single locomotive scheduling and multiple locomotive scheduling models.

Single locomotive scheduling models consider assignment of a single locomotive to a train. Most of the papers, formulate them as multi commodity network flow problems and consider locomotive type as commodity. Booler (1980) develops a linear programming model that assigns locomotives to a given set of trains at the minimum operating cost. The model is useful for tactical planning, and it could solve the problems involving 50 trains. Wright (1989) examines the application of a stochastic algorithm to solve single locomotive assignment problems. Test results indicate that stochastic algorithms performed better than deterministic methods, and the simulated annealing algorithm performed the best. The model aims at addressing strategic planning issues, and is able to solve problems up to 200 trains involving five types of locomotives. Forbes, Holt and Watts (1991) present an exact algorithm based on an approach used to solve multiple-depot bus scheduling problems. Their model considers fleet size which is an improvement over Write (1989). This model aims at addressing tactical planning issues and solve problem instances involving up to 200 trains, 5 types of locomotive in 21,000 seconds. Booler (1995) revises his own work and suggests relaxing the linking constraint using Lagrangean Relaxation to solve the problem more efficiently. He incorporates both fixed locomotive costs and variable costs of light engines into the analysis. Tests were conducted on small instances involving 14 trains and 3 locomotive types.

Fügenschuh et al. (2006) develop a linear integer programming formulation with objective to minimize the cost of locomotive. They propose two versions of the problems - fixed and flexible starting times of trains. In a flexible starting time model, they further consider two sub-versions, one with the constant running time and another with flexible running time.

Their models aim at supporting strategic simulation tool used by Deutsche Bahn for working out the future network load in freight transportation. The fixed starting time model can solve problem instances up to 1537 freight trains and 4 locomotive types while the flexible starting time model can solve up to 120 freight trains and 4 locomotive types with time windows ranging from ± 10 to ± 120 minutes intervals around the prescheduled starting time. While the fixed starting time model solves the problem optimally, the flexible time model solves the problem with the optimality gap ranging from 0% to 79%. Therefore, the authors develop a heuristic to solve a flexible time model. Fügenschuh et al. (2008) develop a randomized parametric greedy heuristic approach. They consider the same instances used in Fügenschuh et al. (2006) for fixed starting time and more complex instances for flexible time model. Comparison of CPLEX solution and heuristic approach indicate that heuristic can solve bigger and complex instances with improved optimality gaps. Ghoseiri and Ghannadpour (2010) present a hybrid genetic algorithm to assign homogeneous locomotives located at multiple depots to set of trains starting in pre-specified time windows. The model aims to address operational planning issues. Computational results indicate that algorithm developed by them is efficient and can solve the problems in a polynomial time.

Multiple Locomotive Scheduling Models assign sufficient number of locomotives of different types (heterogeneous consist) to trains in a pre-defined train schedule. Florian et al. (1976) present a mixed integer programming model to address the strategic level planning. They provide an exact solution to heterogeneous consist assignment problem using Bender's decomposition. They consider a weekly time horizon. Their model cannot converge even for small problems. Ziarati et al. (1997) present a time-space network approach for locomotive assignment problem involving heterogeneous consists. They formulate a non-linear integer programming (NLIP) model for a large scale rail network and consider loco inspection schedule and shop movement along with the deadheading. The NLIP problem is solved using

Dantzig-Wolfe decomposition method. They test a large scale problem having 1300 locomotives and 2000 trains, a real data of CN North American railway. They report an optimality gap of around 5.5%. Ziarati et al. (1999) propose branch first cut second method to solve the model presented in Ziarati et al. (1997). The initial formulation is strengthened by problem specific cutting planes and then solved using Dantzig-Wolfe decomposition. This method can reduce the optimality gap by 52% for two types of locomotives and by 34% for all other types.

Ahuja et al. (2005) present a Mixed Integer Programming formulation for locomotive assignment problem. Objective function of the model is to minimize the cost of active, dead-heading and light-heading of locomotives. It also includes penalty for consist bursting, inconsistency and assignment of single locomotive to a train. They consider a time horizon of one week. They propose a sequence of heuristics using large scale neighbourhood search. Their model can solve the problem instance involving 3324 trains and 3316 locomotives. They report potential annual savings of more than 400 locomotives. However, the output of the model is not directly implementable as they do not consider many practical issues such as late running of trains, fuelling and maintenance of locomotives. Vaidyanathan et al. (2007) propose a new LSP formulation based on consist instead of locomotives. Consist based formulation reduces the solution space and improves the speed of solution from few hours to less than 10 minutes. The solution obtained is as good as locomotive based solution. They add few practical constraints such as the cab signalling, accounting of foreign power and incremental locomotive planning. Interested reader is also referred to Rouillon, Desaulniers and Soumis (2005) and Ching-Chung Kuo and Gillian M. Nicholls (2007) for additional models on multiple locomotive scheduling problems.

Vaidyanathan et al. (2008) present a Locomotive Routing problem (LRP) that considers fuelling and servicing of locomotives. They adopt a two-phase sequential approach to address

the problem. In the first phase, they solve the Locomotive Scheduling Problem (LSP) to determine assignment of locomotive types to trains while satisfying operational and business constraints at minimum cost. In the second phase, they solve the Locomotive Routing Problem to determine routing of each locomotive, including fuelling and servicing of locomotives. They develop a fast aggregation-disaggregation heuristic, which could solve the LRP within a few minutes and obtain near optimal solution. They test their model on real data obtained from a major Class I U.S. railroad. This is the only paper which addresses the issue of routing of locomotives comprehensively.

In the literature Locomotive Scheduling and Routing Problems are treated as distinct problems and are solved sequentially. Due to the crucial link between various planning phases, these problems ideally should be solved concurrently. This paper attempts to fill this significant gap in the locomotive scheduling literature by integrating tactical and operational phases of planning in a single formulation.

4.3 Problem Statement

In this paper, we address the issue of locomotive scheduling and routing of passenger as well as freight trains in a rail network. Rail network consists of several stations/yards connected by tracks. Stations/yards are activity centres where most of the business and the operational activities such as boarding and alighting of passengers, loading and unloading of freight, fuelling and servicing of locomotives are carried out in stations/yards. In our problem, we consider only those stations/yards which perform at least one of the activities mentioned above.

While passenger trains generally need a single locomotive to haul, freight trains need multiple locomotives to haul as they carry heavy load. Coupling of two or more locomotives together to increase a hauling power is called *consist*. Consist formation or consist breaking is a technical task which can be carried out only in the maintenance shop. Consist formation or

bursting incurs a cost due to the requirement of the crew for movement of consist to and from the shop and idling of consist during this period. Therefore, once consist is formed generally it is not broken. Therefore, we treat consist as a single unit and consider types of consist instead of types of locomotives. This is in line with existing practice in Indian Railways. The Locomotive Scheduling / Routing problem gets transformed into a Consist Scheduling/Routing Problem.

Assignment of consist should ensure that adequate horsepower is provided to haul the train at the maximum permissible speed. A train with a given tonnage may require higher hauling power, in the sections with a rising gradient as compared to the section with a flat profile. Trains may get stalled in the section if horse power provided by consist is inadequate while overpowering of a train will result in wastage of resources and increase the cost of operations. Further, every train needs to be hauled only by consist of suitable traction (Diesel/Electric) and of appropriate horse power. While electric locomotives can operate only on electrified sections, diesel locomotives have no such limitation. A combination of diesel and electric locomotives is prohibited.

Consists are categorized as Preferred (Appropriate), Less Preferred (Over powered) and Prohibited (unsuitable) consists. When hauling power along with locomotive type exactly matches with the train requirement, the locomotive is called appropriate. One of the objectives of Consist Scheduling Problem (CSP) problem is to find the assignment of consist to trains at minimum cost. Every consist, after working for a certain number of days or running for a certain kilometres has to be serviced in the maintenance shop otherwise it is declared unfit to haul the train. Diesel locomotive needs fuelling after completion of certain kilometres of working, otherwise, it is declared unfit to haul the train. Locomotives might have to be detached from the train in the middle of its journey, if consist gets due for either servicing or fuelling.

Our model considers servicing and fuelling constraints (for diesel locomotives), light-heading and dead-heading of locomotives. We consider a fortnightly planning horizon in which trains can have different frequencies such as daily, thrice a week or weekly. Trains require to be scheduled in a planning horizon have attributes such as starting station, departure time, destination station, arrival time, train frequency and permissible set of consists that can be assigned to the train. A set of permissible consists is determined based on whether a train is a passenger or freight train, the train route, number of cars, train load and nature of commodity to be carried. Each consist has got attributes such as consist types, fleet size and capital and operation cost. We assume fixed train schedule for both passenger and freight trains. However, in a passenger dominated heterogeneous corridor, freight trains are scheduled in the gaps between two passenger trains.

4.4 Model Formulation

Table 4.1 lists the notations (parameters, decision variables and auxiliary variables) used in the ILSRP model that we develop in this section. G is the total number of real trains, and D is the total number of dummy trains. The time horizon is represented by T . The train index is represented by j, k, m and d . The departure and arrival times of the train j from its originating and at destination stations are represented by a_j and d_j respectively. N is the total number of consists available for allocation and N_l is total number of consist of l type. The route index is represented by i and the maximum number of routes cannot be more than N (total number of consists available for allocation). Time required for moving consist from the train j (at its destination-node) to train k at (its origin-node), including time for light heading if any, is represented by t_{jk} . Cost of consist, which includes light-heading from the destination-node of the train j to the origin-node of train k is represented by C_{jk} ($C_{jk}=0$ if the destination node of train j is same as the origin node of train k).

Table 4.1 Notations –Parameters and Decision Variables

Problem Parameters and Inputs	
k, j, d, m	Train index
$1, 2, 3, \dots, G$	Real Trains
$G + 1 \dots G + D$	Dummy trains
$i = \{1, 2, 3 \dots N\}$	Route index. A route is a set of trains served by the same consist in the time horizon of the problem
$d_j, j = 1, 2, 3, \dots, G + D$	Departure time of train j from its origin-node
$a_j, j = 1, 2, 3, \dots, G + D$	Arrival time of train j at its destination-node
N	Total number of consists
$N_l, l = 1 \dots R$	Total number of l type of consists
t_{jk}	Time required for moving consist from train j (at its destination-node) to train k at (its origin-node), including time for light heading if any
C_{jk}	Cost of consist which includes light-heading from the destination-node of train j to the origin-node of train k . $C_{jk} = 0$ if the destination node of train j is same as the origin node of train k .
D_m	Cost of dead-heading a consist in train m .
H_m	Maximum number of consists that can be dead-headed with train m
I_j	set of permissible consists for train j
$\pi_l, l = 1, 2, \dots, N$	cost of consist l
δ	Number of maintenance depots
$\lambda_{t\alpha}$	Set of dummy trains corresponding to a time slot t , at depot α . $t=1 \dots T$ and $\alpha=1 \dots \delta$.
$\theta_{t\alpha}$	Capacity at maintenance depot α in a slot t
T	Time Horizon of the problem (typically one week or one day measured in hours)
Decision Variables	
$X_{li} =$	$\begin{cases} 1, \text{ if consist } l \text{ is assigned to route } i \\ l = 1 \dots N \\ 0, \text{ otherwise} \end{cases}$
Auxiliary Variable	
$W_i =$	$\begin{cases} 1, \text{ if route } i \text{ is used (at least one train is assigned)} \\ 0, \text{ otherwise} \end{cases}$
$r_{ij} =$	$\begin{cases} 1, \text{ if train } j \text{ is assigned to route } i \\ 0, \text{ otherwise} \end{cases}$
$Z_{jkm} =$	$\begin{cases} 1, \text{ if consist of train } j \text{ is assigned to train } k \text{ after dead heading train } m \\ 0, \text{ otherwise} \end{cases}$
$Y_{jk} =$	$\begin{cases} 1, \text{ if consist of train } j \text{ is assigned to train } k \text{ without any deadheading} \\ 0, \text{ otherwise} \end{cases}$
$Z_{jkk} = Z_{jkj} = 0, Y_{jj} = 0$	By definition
$F_{ij} =$	$\begin{cases} 1, \text{ if } j \text{ is the first train in a route } i \\ 0, \text{ otherwise} \end{cases}$
$\bar{F}_j =$	$\begin{cases} 1, \text{ if } j \text{ is the first train in a route} \\ 0, \text{ otherwise} \end{cases}$

Cost of dead-heading with train m is represented by D_m , while the maximum number of consists that can be dead-headed with train m is represented by H_m . Set of permissible consist that can haul the train j is represented by I_j . The cost of consist l is represented by π_l . The number of maintenance depots is represented by δ and $\theta_{t\alpha}$ is the capacity of maintenance depot α in a slot t . X_{li} is the binary variable that takes one of the appropriate values,

$$X_{li} = \begin{cases} 1, & \text{if consist } l \text{ is assigned to route } i \\ & l=1 \dots N \\ 0, & \text{otherwise} \end{cases}.$$

The binary variable $W_i = \begin{cases} 1, & \text{if route } i \text{ is used (at least one train is assigned)} \\ 0, & \text{otherwise} \end{cases}$ indicates

whether the route is active or otherwise, The binary variable r_{ij} captures whether train j is

assign to the route i or otherwise, $r_{ij} = \begin{cases} 1, & \text{if train } j \text{ is assigned to route } i \\ 0, & \text{otherwise} \end{cases}$. The binary variable

Y_{jk} takes one of the values, and ensures correct assignment of consist from one train to other.

$Y_{jk} = \begin{cases} 1, & \text{if consist of train } j \text{ is assigned to train } k \text{ without any deadheading} \\ 0, & \text{otherwise} \end{cases}$. The binary

variable Z_{jkm} becomes active when dead-heading of locomotive is involved

$Z_{jkm} = \begin{cases} 1, & \text{if consist of train } j \text{ is assigned to train } k \text{ after dead heading train } m \\ 0, & \text{otherwise} \end{cases}$.

By definition, $Y_{jj} = 0$ as consist after hauling train j cannot haul the same train. Similarly, Z_{jkk}

$= Z_{j kj} = 0$ as consist of the train j cannot be connected to train k by dead-heading train j ,

similarly, consist of train j cannot be connected to train k by dead-heading train k . The binary

variable F_{ij} captures whether train j is the first train on the route i , and accordingly takes correct

values $F_{ij} = \begin{cases} 1, & \text{if } j \text{ is the first train in a route } i \\ 0, & \text{otherwise} \end{cases}$. The binary variable $\bar{F}_j$ is an indicator

variable which, indicate whether train j is the first train on the route or otherwise,

$$\bar{F}_j = \sum_{i=1}^N F_{ij} = \begin{cases} 1, & \text{if } j \text{ is the first train in a route} \\ 0, & \text{otherwise} \end{cases}.$$

The formulation ILSRP is described below. First, we describe the formulation ILSRP without maintenance constraints and later, we describe ILSRP formulation with periodical maintenance constraints.

4.4.1 Model ILSRP without Maintenance Constraints

Min $\sum_{l=1}^R \sum_{i=1}^N \pi_l X_{li} + \sum_{j=1}^G \sum_{k=1}^G Y_{jk} C_{jk} + \sum_{j=1}^G \sum_{k=1}^G \sum_{m=1}^G Z_{jkm} (C_{jm} + C_{mk} + D_m)$	(1)
Subject to	
$\sum_{i=1}^N X_{li} \leq N_l, l = 1 \dots N.$	(2)
$\sum_{i=1}^N r_{ij} = 1, j = 1 \dots G.$	(3)
$(r_{ij} - r_{ik}) \leq (1 - Y_{jk} - (\sum_{m=1}^G Z_{jkm})), I = 1 \dots N \text{ and } j, k = 1 \dots G.$	(4)
$(r_{ik} - r_{ij}) \leq (1 - Y_{jk} - (\sum_{m=1}^G Z_{jkm})), (i = 1 \dots N \text{ and } j, k = 1 \dots G).$	(5)
$\sum_{k=1}^G (Y_{jk} + (\sum_{m=1}^G Z_{jkm})) = 1, j = 1 \dots G.$	(6)
$\sum_{j=1}^G (Y_{jk} + (\sum_{m=1}^G Z_{jkm})) = 1, k = 1 \dots G.$	(7)
$Y_{jj} = 0, j = 1, \dots, G$	(8a)
$Z_{jkk} = 0, j = 1, \dots, G \text{ and } k = 1, \dots, G$	(8b)
$Z_{jkj} = 0, j = 1, \dots, G \text{ and } k = 1, \dots, G$	(8c)
$\sum_{l=1}^N X_{li} = W_i, i = 1 \dots N.$	(9)
$\sum_{j=1}^G F_{ij} = W_i, i = 1 \dots N.$	(10)
$F_{ij} \leq r_{ij}, j = 1 \dots G \text{ and } i = 1 \dots N.$	(11)
$\bar{F}_j \leq \sum_{i=1}^N F_{ij}, j = 1 \dots G.$	(12)
$(Y_{jk} (d_k - a_j - t_{jk}) + \bar{F}_k T) \geq 0, j = 1 \dots G \text{ and } k = 1 \dots G.$	(13)
$(Z_{jkm} (d_m - a_j - t_{jm}) + \bar{F}_m T) \geq 0, \forall m = 1 \dots G \text{ and } \forall j, k = 1 \dots G.$	(14)
$(Z_{jkm} (d_k - a_m - t_{mk}) + \bar{F}_k T) \geq 0, \forall m = 1 \dots G \text{ and } \forall j, k = 1 \dots G.$	(15)
$\sum_{l \in I_j} X_{li} \leq r_{ij}, j = 1 \dots G \text{ and } i = 1 \dots N.$	(16)
$\sum_{j=1}^G r_{ij} \leq GW_i, i = 1 \dots N \text{ and } j = 1 \dots G.$	(17)
$\sum_{j=1}^G \sum_{k=1}^G Z_{jkm} \leq H_m, m = 1, \dots, G$	(18)
$X_{li}, W_i, r_{ij}, Y_{jk}, F_{ij}, \bar{F}_j \text{ and } Z_{jkm} \in \{0-1\}$	(19)

The objective function (1) minimizes the cost of locomotive assignment that includes cost of consists, light heading and deadheading of locomotives. Constraints (2) and (3) ensure that only one consist is assigned to a route, and a train is assigned exactly to one route. Constraints (4), (5), (6) and (7) together ensure that train k can be a successor to train j and vice versa only if, both trains are in the same route and consist arrives at the destination either by directly

hauling a train j or by deadheading with some other train m before getting attached to train k and vice versa. Constraints (8a), (8b) and (8c) ensure that a train j cannot be train j 's successor or train k cannot be train k 's successor. Constraint (9) ensures that the exactly one consist is assigned to an active route and zero consist to an inactive route. Constraint (10) ensures that each route has exactly one first train, while constraints (11) and (12) ensure the logic that train j is first train only in a route it is assigned and indicator variable $\bar{F}_j$ takes correct values. Constraint (13) ensures that a particular consist attached with train j can be used by train k only after satisfying the time feasibility constraint. Similarly constraints (14) and (15) ensure that consist after hauling train j and deadheading with train m can be used by train k after only after satisfying time feasibility constraints. Constraint (16) ensures that the consist l assigned to route i must belong to a set of permissible consist I_j for all trains j in route i . Constraint (17) ensures that no train is assigned to an inactive route. Constraint (18) imposes the maximum bound on the number of consists that can be deadheaded with train m .

4.4.2 Model ILSRP with Maintenance Constraints

We add constraints (8), (9), (10d), (10e), (10f), (21) and (22) to the ILSRP model (without maintenance constraints) and introduce a concept of dummy trains to incorporate maintenance of locomotives in the formulation. Let's assume there are δ depots in the rail network, each depot operates in 3 shifts per day and periodicity of maintenance is J days ($t = 1 \dots J$), then $3 \delta J$ dummy trains are required for each consist. Constraints (8) and (9) ensure that when a dummy train is assigned to a route then this dummy train must have a predecessor or successor real train. Constraints (10d) ensures that a locomotive/consist of train j cannot be assigned to a train k by dead-heading with a dummy train while (10e) ensures that consist of a dummy train

Model ILSRP with Maintenance Constraints

Min $\sum_{l=1}^R \sum_{i=1}^N \pi_l X_{li} + \sum_{j=1}^{G+D} \sum_{k=1}^{G+D} Y_{jk} C_{jk} + \sum_{j=1}^{G+D} \sum_{k=1}^{G+D} \sum_{m=1}^G Z_{jkm} (C_{jm} + C_{mk} + D_m)$	(1)
Subject to	
$\sum_{i=1}^N X_{li} \leq N_l, l = 1 \dots N.$	(2)
$\sum_{i=1}^N r_{ij} = 1, j = 1 \dots G.$	(3)
$(r_{ij} - r_{ik}) \leq (1 - Y_{jk} - (\sum_{m=1}^G Z_{jkm})), i = 1 \dots N \text{ and } j, k = 1 \dots G+D.$	(4)
$(r_{ik} - r_{ij}) \leq (1 - Y_{jk} - (\sum_{m=1}^G Z_{jkm})), (i = 1 \dots N \text{ and } j, k = 1 \dots G+D).$	(5)
$\sum_{k=1}^G (Y_{jk} + (\sum_{m=1}^G Z_{jkm})) = 1, j = 1 \dots G.$	(6)
$\sum_{j=1}^G (Y_{jk} + (\sum_{m=1}^G Z_{jkm})) = 1, k = 1 \dots G.$	(7)
$\sum_{k=1}^G (Y_{dk} + (\sum_{m=1}^G Z_{dkm})) \geq \sum_{i=1}^N r_{id}, d = G+1 \dots G+D.$	(8)
$\sum_{j=1}^G (Y_{jd} + (\sum_{m=1}^G Z_{jdm})) \geq \sum_{i=1}^N r_{id}, d = G+1 \dots G+D.$	(9)
$Y_{jj} = 0, j = 1, \dots, G+D$	(10a)
$Z_{jkk} = 0, j = 1, \dots, G \text{ and } k = 1, \dots, G$	(10b)
$Z_{jkj} = 0, j = 1, \dots, G \text{ and } k = 1, \dots, G+D$	(10c)
$Z_{jkd} = 0, j = 1, \dots, G \text{ and } k = 1, \dots, G, d = G+1, \dots, G+D$	(10d)
$Z_{xdm} = 0, x = G+1, \dots, G+D, d = G+1, \dots, G+D, m = 1, \dots, G+D$	(10e)
$Y_{xd} = 0, x = G+1, \dots, G+D, d = G+1, \dots, G+D$	(10f)
$\sum_{l=1}^N X_{li} = W_i, i = 1 \dots N.$	(11)
$\sum_{j=1}^{G+D} F_{ij} = W_i, i = 1 \dots N.$	(12)
$F_{ij} \leq r_{ij}, j = 1 \dots G+D \text{ and } i = 1 \dots N.$	(13)
$\bar{F}_j \leq \sum_{i=1}^N F_{ij}, j = 1 \dots G+D.$	(14)
$(Y_{jk} (d_k - a_j - t_{jk}) + \bar{F}_k T) \geq 0, j = 1 \dots G+D \text{ and } k = 1 \dots G+D.$	(15)
$(Z_{jkm} (d_m - a_j - t_{jm}) + \bar{F}_m T) \geq 0, \forall m = 1 \dots G \text{ and } \forall j, k = 1 \dots G+D.$	(16)
$(Z_{jkm} (d_k - a_m - t_{mk}) + \bar{F}_k T) \geq 0, \forall m = 1 \dots G \text{ and } \forall j, k = 1 \dots G+D.$	(17)
$\sum_{l \in I_j} X_{li} \leq r_{ij}, j = 1 \dots G \text{ and } i = 1 \dots N.$	(18)
$\sum_{j=1}^{G+D} r_{ij} \leq (G+1)W_i, i = 1 \dots N \text{ and } j = 1 \dots G+D.$	(19)
$\sum_{j=1}^G \sum_{k=1}^G Z_{jkm} \leq H_m, m = 1, \dots, G$	(20)
$\sum_{j=G+(i-1)3\delta T+1}^{G+3i\delta T} r_{ij} \leq W_i, \forall i$	(21)
$\sum_{i=1}^N r_{i(i-1)\alpha T+\alpha t} \leq \theta_t, t = 1 \dots T, \alpha = 1 \dots \delta$	(22)
$X_{li}, W_i, r_{ij}, Y_{jk}, F_{ij}, \bar{F}_j \text{ and } Z_{jkm} \in \{0-1\}$	(23)

cannot be assigned to a dummy train by dead-heading with a real train and (10f) ensures that consist of a dummy train cannot be assigned to another dummy train. Constraint (21) ensures exactly one route i can be assigned to a maintenance slot corresponding to dummy trains j in a maintenance depot. Constraint (22) imposes upper bound ($\theta_{t\alpha}$) on the number of consist that can be maintained at depot α in slot t .

4.5 Computational Results

We use LINGO 15.0, commercial optimization software to solve different problem instances. Our problem instances cover a triangular network. We test various features of the model such as light-heading, dead-heading and maximum number locomotives to be dead-headed with a train. We assume (i) cost of light-heading of consist is the same irrespective of number of locomotives in consist as only one locomotive is active at a time, (ii) cost of dead-heading is much lower than light-heading and can be different for different trains but in this problem we assume it to be the same for all the trains.

4.5.1 Locomotive Schedule for 9 Daily Trains with 3 Deadheading

We test our model using commercial optimization software LINGO 15.0. The input parameters and values are shown table 4.2 and 4.3. Figure 4.1 indicate a rail network having 3 stations.

Table 4.2 Input Parameters for Locomotive Schedule for 9 Daily Trains

Sr. No.	Train Input/Parameters	Value of Input/parameter	
1	Number of Trains	9	
2	Number of stations	3	
3	Cost of light-heading (\$/hour)	20	
4	Cost of dead-heading	5	
5	Consist Type	1	2
6	Total Number of Consist	2	2
7	Cost of consist (\$)	3000	5000

Figure 4.1. Station Network for problem instances

Table 4.3 Locomotive Schedule for 9 Daily 9 trains with 3 Dead-heading

Trains	d_j Departure time	Origin Station	a_j Arrival time	Destination Station	Set Permissible consists	Maximum # of locomotives dead-headed (H_m)
T1	1	1	5	2	1, 2	3
T2	2	1	6	2	1, 2	3
T3	3	1	7	2	1, 2	3
T4	4	1	8	2	1, 2	3
T5	9	2	13	1	1, 2	3
T6	16	1	20	3	1, 2	3
T7	17	1	21	3	1, 2	3
T8	18	1	22	3	1, 2	3
T9	24	2	27	3	1, 2	3

Optimal solution of this problem instance is presented in Table 4.4.

Table 4.4 Solution for Locomotive Schedule for 9 Daily Trains with 3 Dead-heading

Consist	Hauling of Trains	Light-heading cost in \$	Dead-heading cost in \$	Total Cost in \$
1	T1- T6	80	T5 (25)	3105
2	T2-T5-T7	80	0	3080
3	T3-T9	80	0	5080
4	T4-T8	80	Train 5 (25)	5105
Grand Total		320	50	16370

4.5.2 Locomotive Schedule for 9 Daily Trains with 1 Dead-heading

We test problem instance 2 by changing the value of number of locomotives that can be dead-headed with a train. We change the value of H_m from 3 to 1 and record the results in table 4.5. It is seen from the table that the number of locomotives dead-headed is reduced from two to one, while the number of locomotives light-headed was increased from four to five.

Table 4.5 Solution for Locomotive Schedule for 9 Daily Trains with 1 Dead-heading

Consist	Hauling of Trains	Light-heading cost in \$	Dead-heading cost in \$	Total Cost in \$
1	T4- T6	80	T5 (25)	3105
2	T1T-8	160	0	3160
3	T3-T5-T7	80	0	5080
4	T2-T9	80	0	5080
Grand Total		400	25	16425

4.5.3 Effect of Change in permissible consist

In this problem instance we change the permissible set of consist for trains to test the model. Set of permissible consist is as shown in table 4.6. Due to change in the locomotive type requirements, with the same number locomotive and type as shown in table 4.2, the solution was infeasible. Therefore, we increase the number of locomotives of type 2 from two to three and simultaneously reduce the number of consist of type 1 from two to one.

Table 4.6 Changed Set of Permissible Consist

Trains	d_j Departure time	Origin Station	a_j Arrival time	Destination Station	Set of Permissible consists	Maximum # of locomotives dead-headed (H_m)
T1	1	1	5	2	2	3
T2	2	1	6	2	2	3
T3	3	1	7	2	1, 2	3
T4	4	1	8	2	2	3
T5	9	2	13	1	1, 2	3
T6	16	1	20	3	1, 2	3
T7	17	1	21	3	1, 2	3
T8	18	1	22	3	2	3
T9	24	2	27	3	2	3

Our model is able to generate optimal solution to problem as shown in table 4.7 below.

Table 4.7 Solution for Changed Set of Permissible Consists

Consist	Hauling Trains	of	Light-heading cost in \$	Dead-heading cost in \$	Total Cost in \$
1	T3- T5-T6		80	0	3080
2	T1-T7		80	25	5105
3	T2-T9		80	0	5080
4	T4-T8		80	25	5105
Grand Total			320	50	18370

It can be seen from the Table 4.7 that restriction on permissible consist impact the cost of operation adversely.

4.5.4 Locomotive Schedule for 27 Trains

In this section we solve a medium size problem involving 27 trains including 21 passenger trains and 6 freight train. There are 3 types of locomotives used to implement the timetable as shown in Table 4.9. The input parameters are given in Table 4.8 below.

Table 4.8 Input Parameters for Locomotive Schedule for 27 Trains

Sr. No.	Train Input/Parameters	Value of Input/parameter		
1	Number of Trains	27		
2	Number of Origin/ Destination Stations	6		
3	Cost of light-heading (\$/hour)	100		
4	Cost of dead-heading	25		
5	Consist Type	1	2	3
6	Total Number of Consist	4	4	3
7	Cost of consist (\$)	5000	6000	9000

Table 4.9 Timetable for 27 Trains

Trains	Type of Train	Origin Station	Destination Station	d_j Departure time	a_j Arrival time	Permissible Consists
1	Passenger	1	2	1	5	1,2
T2	Passenger	1	2	2	6	1,2
T3	Passenger	1	2	3	7	1,2
T4	Freight	1	2	4	8	3
T5	Passenger	2	1	9	13	1,2
T6	Passenger	1	3	16	20	1,2
T7	Passenger	1	3	17	21	1,2
T8	Freight	1	3	18	22	3
T9	Passenger	2	3	24	27	1,2
T10	Passenger	1	2	25	29	1,2
T11	Passenger	1	2	26	30	1,2
T12	Passenger	1	2	27	31	1,2
T13	Freight	1	2	28	32	3
T14	Passenger	2	1	33	37	1,2
T15	Passenger	1	3	40	44	1,2
T16	Passenger	1	3	41	45	1,2
T17	Freight	1	3	42	46	3
T18	Passenger	2	3	48	51	1,2
T19	Passenger	1	2	49	53	1,2
T20	Passenger	1	2	50	54	1,2
T21	Passenger	1	2	51	55	1,2
T22	Passenger	1	2	52	56	3
T23	Passenger	2	1	57	61	1,2
T24	Freight	1	3	64	68	1,2
T25	Passenger	1	3	65	69	1,2
T26	Passenger	1	3	66	70	3
T27	Passenger	2	3	72	75	1,2

Table 4.10 Solution for Locomotive Schedule for 27 Trains

Consist	Hauling of Trains
C11	T2-T5-T7
C21	T11-T14-T16
C22	T20-T23-T25
C31	T1-T9
C22	T10-T18
C23	T19-T27
C24	T3-T6
C25	T12-T15
C26	T21-T24
C31	T4-T8
C32	T13-T17
C33	T22-T26

Light engine running cost of 4500 and deadheading cost of 1125 and total operating cost of locomotive operation is 83571.

4.5.5 Locomotive Schedule for 129 Trains

In this section we solve a fairly large size problem involving 150 trains including 106 passenger trains and 42 freight train. There are 5 types of locomotives used to implement the timetable as shown in Table 4.12. The input parameters are given in Table 4.11 below.

Table 4.11 Input Parameters for Locomotive Schedule for 129 Trains

Sr. No.	Train Input/Parameters	Value of Input/parameter				
1	Number of Trains	129				
2	Number of Origin/ Destination Stations	6				
3	Cost of light-heading (\$/hour)	100				
4	Cost of dead-heading	25				
5	Consist Type	1	2	3	4	5
6	Total Number of Consist	3	3	3	1	0
7	Cost of consist (\$)	9000	11000	6000	8000	13000

Table 4.12 Timetable for 129 Trains

Sr. No.	Type of Train	Train Frequency	Origin Station	Destination Station	d_j Departure time	a_j Arrival time	Permissible Consists
1	Passenger	1,2,3,4,5,6,7	1	2	7	11	3
2	Passenger	1,2,3,4,5,6,7	1	2	10	14	3
3	Freight	1,2,3,4,5,6,7	1	2	13	20	5
4	Passenger	1,2,3,4,5,6,7	1	2	15	19	3
5	Passenger	2,5	1	2	18	22	3
6	Freight	1,2,3,4,5,6,7	1	3	8	12	5
7	Passenger	1,2,3,4,5,6,7	1	3	11	14	1,2
8	Passenger	1,2,3,4,5,6,7	1	3	15	18	1,2
9	Freight	1,3,5	2	3	2	10	4,5
10	Passenger	1,2,3,4,5,6,7	2	3	12	17	1,2
11	Passenger	1,2,3,4,5,6,7	2	3	14	19	1,2
12	Passenger	1,2,3,4,5,6,7	2	1	9	13	3
13	Passenger	2,5	2	1	14	18	3
14	Freight	1,2,3,4,5,6,7	2	1	1	8	5
15	Passenger	1,2,3,4,5,6,7	2	1	18	23	3
16	Freight	1,2,3,4,5,6,7	3	1	14	19	4,5
17	Passenger	1,3,5	3	1	17	20	4,5
18	Passenger	1,2,3,4,5,6,7	3	1	20	23	1,2
19	Freight	1,2,3,4,5,6,7	3	2	19	24	1,2
20	Passenger	1,2,3,4,5,6,7	3	2	18	23	1,2
21	Passenger	1,2,3,4,5,6,7	3	2	19	24	1,2

Table 4.13 Solution for Locomotive Schedule for 129 Trains

Consist	Hauling of Trains	Light-heading cost in \$	Dead-heading cost in \$	Total Cost in \$
C11	T3 (1,2,3,4,5,6,7)-T4(1,2,3,4,5,6,7)	0	0	9000
C12	T5 (1,2,3,4,5,6,7)-T2(1,2,3,4,5,6,7)	0	25	9000
C13	T1(1,2,3,4,5,6,7) –T5 (2,5)-T13(2,5)	0	0	9000
C21	T14 (1,2,3,4,5,6,7)- T3(1,2,3,4,5,6,7)	0	0	11000
C22	T6 (1,2,3,4,5,6,7)- T16 (1,2,3,4,5,6,7)-	0	0	11000
C23	T9 (1,3,5)- T17 (1,3,5)	1200	0	11000
C31	T10 (1,2,3,4,5,6,7)-T20 (1,2,3,4,5,6,7)-	0	0	6000
C32	T11 (1,2,3,4,5,6,7)-T19 (1,2,3,4,5,6,7)	0	0	6000
C33	T8 (1,2,3,4,5,6,7)-T18 (1,2,3,4,5,6,7)	0	0	6000
C41	T7 (1,2,3,4,5,6,7)-T21 (1,2,3,4,5,6,7)	0	0	8000
Grand Total		1200	25	86000

4.6. Summary and Conclusion

Locomotives are considered as the most important resource in train operations, as they represent billions of dollars of investment and, also account for a significant percentage of operating cost of trains. In a large system, even a small improvement in locomotive utilization can result in significant savings. Therefore, Locomotive Scheduling Problem (LSP) and Locomotive Routing Problem (LRP) are considered as the most important resource scheduling problems in rail operations planning. The LSP aims to assign locomotives to each train in a pre-planned schedule so that adequate hauling power is provided to each train from its origin station to destination station while satisfying all operations constraints. The objective of LSP is to minimize the operating cost of locomotives. The LRP aims to determine the day to day routing of a locomotive while satisfying the fuelling and maintenance constraints. The existing approaches treat the LSP and LRP as distinct problems and solve them sequentially. In this essay, we develop an Integer Programming (IP) model that integrates the LSP and LRP into a single unified problem called Integrated Locomotive Scheduling and Routing Problem (ILSRP). The model we develop in this essay is new as well as comprehensive. It incorporates

deadheading, light heading and maintenance constraints of locomotives and generates a locomotive schedule for a predefined timetable and route for each locomotive that can be implemented without further adjustment. We test our model on various problem instance sizes. The model is working and able to obtain optimal solution for small to large problem instances.

Chapter 5

Summary and Future Directions

5.1 Summary of main contribution

Rail transportation is a preferred mode of transport as it is cheaper and more environmentally friendly as compared to other modes of land transport. In the recent years, the growing concerns about the reduction in carbon emissions, has led to an increased demand for rail transportation. Expansion of rail capacity to meet the growing demand is a challenging task as resources such as rail track, overhead equipment, locomotives, passenger and freight cars need huge capital investment and require long lead times for procurement and installation. Given these large investments, even marginal improvements in resource utilization can result in significant cost savings. It is therefore, critical to optimize the utilization of available track capacity and resources for efficient and cost effective rail operations. In this thesis, we address two important optimization issues in railway operations planning namely *train scheduling* and *resource optimization*. The specific problems identified and studied in this thesis are in the context of Indian Railways where the author has been working for more than 17 years. However, the problems studied and contributions made are fundamental and can be applied across all railway systems. The author while working in Indian Railways encountered various problems such as scheduling of new trains in the existing timetable, scheduling of freight trains in the heterogeneous corridors, issue of peak power minimization during rush hours, assignment of locomotives to trains in the efficient manner and most importantly planning of overtaking of

multiple trains at a station for trains with different priorities. In this thesis, we have addressed three different problems and presented them as three independent essays.

In the first essay, entitled *Integrated Train Timetabling and Platforming Problem*, we address the problem of train scheduling in high density double track corridors on Indian Railways (IR). High density corridors are intensively utilized. Therefore, efficient scheduling of trains with different priorities, speeds, and halt pattern is critical for maximizing the utilization of existing track capacity. The key challenge is planning of overtaking of trains having different priorities such as *Non-stop*, *Suburban*, *Express*, *Commuter*, *Container* and *Heavy haul freight* trains due to the complexity of the decision process. Even though freight trains generate significantly higher revenue as compared to passenger trains, non-stop and fast express trains are given much higher priority due to the political sensitivity of delay in passenger services. As freight trains are accorded lower priority, they get excessively delayed in heterogeneous corridors dominated by passenger trains. Given the large number of freight trains, planning for overtaking of multiple trains at a station without negative impact on throughput time is crucial for efficient scheduling of trains and particularly of freight trains.

The existing approaches divide train scheduling into two distinct problems the Train Timetabling Problem (TTP) and the Train Platforming Problem (TPP) and solve them sequentially. The TTP determines arrival and departure times of trains at all stations on the corridor. The TPP assigns trains to a platform in a conflict-free manner. First, the TTP is solved for the entire corridor, and then the TPP is solved for each station subject to station capacity constraints (number of platforms/lines). We develop a novel Integer Programming (IP) model that integrates the TTP and TPP into a single problem called the Integrated Train Timetabling and Platforming Problem (ITTPP). The significant contribution of our model is integrating two distinct problems into a single unified problem, handling of station capacity constraints explicitly and assigning trains to platforms in a conflict-free manner with a possibility of

overtaking of multiple trains at a station. Our model is able to directly generate feasible timetables for the entire corridor that can be implemented without any further adjustments. We develop a heuristic method as well as an optimal Branch and Bound method for solving the model. Computational results on real-life problem sets obtained from Indian Railways for corridors with heterogeneous traffic show that our heuristic algorithm yields close to optimal solutions, and on average takes 7 minutes to solve problems of realistic size. With the proposed new model and the solution algorithm, we are able to schedule on average 26% additional trains without much impact on the average throughput time of the trains.

In Indian Railways, sixty-five percent of the total freight traffic and fifty-five percent of the total passenger traffic is carried on only about fifteen percent of the total network called golden quadrilateral corridors connecting four Indian metro cities namely New Delhi, Mumbai, Chennai and Kolkata. The projected future growth of Indian Railways is also centred on the golden quadrilateral corridors. Scheduling of additional new trains on these corridors results in congestion and increased throughput time as these corridors are intensively utilized. However, by using our model that we develop in this chapter, it is possible to schedule about 26% additional trains on dense corridors without much negative impact on the throughput times of trains. In Indian Railways, particularly on the golden quadrilateral corridors a huge demand for additional passenger and freight trains remains unfulfilled. Using our model and solution methods it is possible to schedule additional passenger and freight trains, which will improve the service quality and lead to earning surplus revenue. Annual revenue of Indian Railways in 2013-14 was about USD 18 billion. Therefore, potential for additional earning by running additional trains is huge. The proposed model and the solution methodology developed in this paper have the potential to improve operational performance of Indian Railways significantly.

The second essay entitled *Scheduling Trains to Minimize Peak Power and Maximize Regenerative braking power utilization* (STMPMR) addresses the issue of energy-efficient

timetabling for Mass Rapid Rail Systems (MRRS) in the urban areas. Traction energy cost accounts for a significant percentage of train operating cost. With the current focus on cost rationalization and environment protection in rail operations, this area provides a rich set of contemporary research questions. The energy cost, power generation capacities and related CO₂ emissions are impacted by both total energy consumption as well as peak power demand. In this essay, we present an Integer Programming (IP) model that seeks to minimize both the peak power demand as well as the total power consumption while keeping average trip time of trains within acceptable limits. Our model incorporates all phases of train running such as acceleration, coasting/constant speed, deceleration and halt and minimizes the peak power demand by reducing the number of trains accelerating simultaneously and synchronizing it with regenerative braking, and reduces the total energy consumption by maximizing the utilization of regenerative braking energy. The key contributions of our model are it is new and comprehensive and its ability to minimize peak power usage and maximize utilization of regenerative braking power concurrently. We propose an efficient train partition heuristic to solve large problem instances. We obtained real-life data of four dedicated corridors of Mumbai Suburban System (MSS) to test our model. Computational results indicate that on average our model is able to reduce the peak power demand by 28% and increase the regenerative braking energy utilization by 7.5 % while limiting the trip time increases to 2%.

Trains running on dedicated suburban corridors of MSS have the potential to regenerate up to 30% of the energy consumption as they halt at every station. Therefore, on MSS, it is possible to reduce CO₂ emissions by 380,000 tonnes and save 443 million units of energy by using full potential of regenerative braking power. To achieve full potential of regenerative braking, it is necessary to install storage devices on the train to avoid wastage of power. However, efficiency of storage devices is very low. Therefore, storage devices cannot provide cost-effective solution. Alternatively, our model has the potential to generate energy-efficient timetable that

can maximize the utilization of generative braking. The model that we develop in this chapter also can be used as a tool to minimize peak power demand by adjusting the halt time while restricting the trip time increases within allowable limits. The number of trains running at any point of time on the corridor increases as the frequency of trains increases. Increased number of trains draws higher peak power from the substation, which necessitate enhancement of the substation capacity and more power from the grid. Alternately, it is possible to minimize the peak power by implementing energy-efficient train schedule generated by our model.

The third essay entitled *Integrated Locomotive Scheduling and Routing Problem* addresses the issue of efficient scheduling of locomotives and their day to day routing. Large rail systems hold thousands of different types of locomotives for hauling passenger and freight trains at maximum permissible speeds. Locomotives are considered as the most important resource in train operations, as they represent billions of dollars of investment and, also account for a significant percentage of operating cost of trains. In a large system, even a small improvement in locomotive utilization can result in significant savings. Therefore, Locomotive Scheduling Problem (LSP) and Locomotive Routing Problem (LRP) are considered the most important resource scheduling problems in rail operations planning. While the existing approaches treat the LSP and the LRP as sequential problems, in this paper, we develop an Integer Programming (IP) formulation that integrates the LSP and LRP into a single unified problem called Integrated Locomotive Scheduling and Routing Problem (ILSRP). Our model is new as well as comprehensive. It incorporates deadheading, light heading and maintenance constraints of locomotives and generates a locomotive schedule for predefined timetable and route for each locomotive that can be implemented without further adjustment.

All the models that we propose in this thesis are new and contribute to the academic literature on rail operation planning both in terms of innovative problem formulations and solution algorithms. They also have the potential to make significant improvements in rail

operations practice. The key contribution of this thesis is in the richness of the proposed models arising from the comprehensive nature of the issues they address.

5.2 Limitations of Models

ITTPP model that we develop in second chapter, assumes that tracks and platforms can be assigned to up and down trains exclusively. This assumption is realistic for high density corridors. However, in corridors, which are less intensively utilized, to optimize platform/line utilization, both up and down trains might be required to share platforms/lines at few stations at different times. Our model does not consider sharing of platform/line by trains running in opposite directions. Therefore, application of ITTPP to these corridors might require some manual adjustment to the timetable.

Secondly, while our model can directly generate a timetable for an entire corridor, it cannot directly generate timetable for entire rail network. In order to solve a network problem, we need to first deconstruct the network into a sequence of corridors and then generate a timetable for each corridor sequentially. Departure times at destination stations of first corridor are taken as departure times at the originating station of the contiguous corridor.

The STMPMR model that we develop in third chapter is able to reduce both the peak power and total power consumption by adjusting departure times at originating station and halt times at all the stations. This model assumes that trains perfectly follow the pre-defined timetable. However, in practice, sometimes, trains might get delayed due to breakdown of assets such as signals, track and locomotives. This results in bunching of trains in some sections and increases the peak power usage during that period. As our model does not consider train disruptions in day to day train operations, further research is necessary to address this issue.

5.3 Future Direction

The ITTPP model that we develop in the second chapter considers double track corridor and able to generate a timetable for trains running in single direction. Further research could investigate the possibility of extending the ITTPP model to single track corridors for trains running in both up and down directions on a single corridor. Extension of ITTPP to single corridor would necessitate incorporation of a few additional constraints such as train crossing and platform sharing by both up and down trains at a station.

This model could further be extended to generate an integrated timetable for both directions trains on a double track corridor. Similarly, it can also be extended to a rail network involving multiple intersecting corridors. In case of rail network, handling of trains at junction stations where one corridor intersects the other would be critical. Using the ITTPP formulation for real-time scheduling of trains can be a basis for another research avenue.

The STMPMR model that we develop in third chapter addresses only energy-efficient timetabling. This model can further be extended to include scheduling of passenger car and crew in a unified model along with the energy-efficient timetabling. The integrated model can address the issue of optimization of track capacity, crew and passenger car and energy efficiency jointly. One of the possible extensions of this model can be to maximize the regenerative braking power utilization in long distance corridors.

The ILSRP model that we develop in the fourth chapter considers fixed departure and arrival times of trains. The possible extension could include variable departure and arrival times of trains. The ILSRP model can be extended to include real-time locomotive scheduling. The extended model will comprehensively address all issues in locomotive planning such as light-heading, dead-heading, fuelling, maintenance, train delays and train disruptions due to asset and equipment failures.

In rail operations planning, repositioning of empty freight cars is an important issue as it directly impacts the loading performance of the rail system. Effective repositioning of empty cars also depends on the availability of locomotives. Therefore, integrating repositioning of empty cars and locomotive scheduling and routing problem in a unified problem has the potential to improve the overall efficiency of the rail operations.

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Appendix 1

The following example describes how incorporating multiple overtaking results in efficient scheduling as compared to simple overtaking. Figure 2.1 depicts a part of a corridor on Indian Railways.

Figure 2.1 A portion of a Corridor and Location of Different Priority Trains

CSTM is the originating station (12 platforms), Dadar (3 platforms/lines), CLA (3 platforms/lines), GC (1 platform), MLND (1 platform) and TNA (2 platforms) are intermediate stations. Trains with different priorities namely Freight (F), commuter (C), express (E) and non-stop (N) are running on the corridor in that order. The non-stop train has the highest priority and it does not have a scheduled halt at any intermediate stations. The Express train is next in priority and has a 2 minute scheduled halt each at Dadar and at TNA. The Commuter train is third in priority and has a 1 minute scheduled halt each at Dadar, CLA, GC, MLND and TNA while the freight train is the lowest in priority and has a 10 minute halt at TNA. High priority trains can overtake trains with low priority to attain the desired objective of minimizing delays.

With simple overtaking, a train can overtake only one train at a time resulting in a time distance graph (of the movement of the trains over this corridor) as depicted in Figure 2.2.

Figure 2.2 Time-Distance Graph with simple overtaking

The non-stop train overtakes three lower priority trains at *three* different stations namely Dadar, CLA and TNA due to pairwise overtaking. As a result it gets delayed by 12 minutes, 3 minutes at CLA, 5 minute at GC and 4 minutes at MLND. This delay to the non-stop train may discourage train planners to avoid scheduling of the freight train in the gap between high priority trains. Multiple overtaking as shown in Figure 2.3 can avoid delay to non-stop train. The non-stop train overtakes express train at Dadar and freight and commuter trains (multiple trains) simultaneously at CLA. As a result the non-stop train gets an uninterrupted schedule. The uninterrupted schedule of non-stop train will encourage train planners to schedule the freight train up to intermediate station (in this case Dadar). In a similar manner other gaps can be used to schedule the freight train up to its destination. Note that the total throughput time in case of simple overtaking was 222 minutes which reduced to 211 minutes by incorporating multiple overtaking.

Figure 2.3 Example of multiple overtaking

Appendix 2

Assumptions:

For any pair of trains q and ℓ ,

$$(i) P_\ell < P_q \Rightarrow r_\ell^s \geq r_q^s \text{ and } w_\ell^s \geq w_q^s, \forall s \text{ with } \text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$$

$$(ii) P_\ell = P_q \Rightarrow r_\ell^s = r_q^s \text{ and } w_\ell^s = w_q^s, \forall s \text{ with } \text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$$

$$(iii) P_\ell > P_q \Rightarrow r_\ell^s \leq r_q^s \text{ and } w_\ell^s \leq w_q^s, \forall s \text{ with } \text{Max} \{S_q^o, S_\ell^o\} \leq s \leq \text{Min} \{S_q^d, S_\ell^d\}$$

These assumptions are very reasonable, logical and are usually satisfied in practice.

Lemma 1:

Let train ℓ and q be such that $P_\ell \leq P_q$ and $A_q^s < A_\ell^s$ at station s . That is train ℓ has the same or lower priority than train q and train ℓ arrived at station s after train q . Consider two possible scenario

(iii) Train ℓ departs from station s immediately after train q

Let the corresponding departure times for train ℓ and q be D_ℓ^s and D_q^s and let the corresponding delay times at station s be δ_ℓ^s and δ_q^s respectively.

(iv) Train ℓ overtakes train q and departs immediately ahead of train q at station s .

Let the corresponding departure times for train ℓ and train q be $\bar{D}_\ell^s$ and $\bar{D}_q^s$ and let the corresponding delay times at station s be $\bar{\delta}_\ell^s$ and $\bar{\delta}_q^s$ respectively.

$$\text{Then } (P_\ell \bar{\delta}_\ell^s + P_q \bar{\delta}_q^s) \geq (P_\ell \delta_\ell^s + P_q \delta_q^s) \quad (14)$$

Let ∇_ℓ^s be the reduction in delay of train ℓ at station s by overtaking train q .

$$\nabla_\ell^s = (\delta_\ell^s - \bar{\delta}_\ell^s) \quad (15)$$

Let ∇_q^s be the increase in delay of train q at station s by overtaking of q by train ℓ .

$$\nabla_q^s = (\bar{\delta}_q^s - \delta_q^s) \quad (16)$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s \quad (17)$$

Further when $P_\ell < P_q$

$$(P_\ell \bar{\delta}_\ell^s + P_q \bar{\delta}_q^s) > (P_\ell \delta_\ell^s + P_q \delta_q^s) \quad (18)$$

$$P_q \nabla_q^s > P_\ell \nabla_\ell^s \quad (19)$$

Proof:

To prove the lemma we need to show that

$$P_q \nabla_q^s > P_\ell \nabla_\ell^s \text{ when } P_\ell < P_q \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s \text{ when } P_\ell \leq P_q.$$

Let $j_1 = \underset{j \in \theta}{\operatorname{argmin}} (D_j^{s+1})$, j_1 is the first train among the set of trains θ that depart station $s+1$.

Let $j_2 = \underset{j \in (\theta - j_1)}{\operatorname{argmin}} (D_j^{s+1})$, j_2 is the second train among the set of trains θ that depart station $s+1$.

[Note that if $|\theta|=1$, then $j_2 = q$ if there is no overtaking and $j_2 = \ell$ if ℓ overtakes q .]

At station s , if ℓ does not overtake train q , the departure and arrival times of train q at s and $s+1$ respectively are determined by the departure, running and arrival times of trains in the set θ at stations s and $s+1$. The departure of train q at station s is governed by four constraints (5), (6a/6b), (7a/7b) and (9) namely the departure headway at station s , arrival headway at $s+1$, platform headway at $s+1$, arrival time plus minimum halt time at station s . This leads the requirement that,

$$D_q^s = \operatorname{Max} \left\{ \begin{array}{l} \operatorname{Max}_{j \in \theta} \{D_j^s + h_d\} \\ \operatorname{Max}_{j \in \theta} \{A_j^{s+1} + h_a\} - r_q^{s+1} \\ \operatorname{Min}_{j \in \theta} \{D_j^{s+1} + h_p\} - r_q^{s+1} \\ A_q^s + w_q^s \end{array} \right. \quad (20)$$

Let $j^* = \underset{j \in \theta}{\operatorname{argmax}} (D_j^s)$ therefore,

$$D_q^s = \operatorname{Max} \left\{ \begin{array}{l} D_{j^*}^s + h_d \\ D_{j^*}^s + h_a + r_{j^*}^{s+1} - r_q^{s+1} \\ D_{j_1}^{s+1} + h_p - r_q^{s+1} \\ A_q^s + w_q^s \end{array} \right. \quad \begin{array}{l} \text{(A)} \\ \text{(B)} \\ \text{(C)} \\ \text{(D)} \end{array}$$

Similarly, the value of departure time of train ℓ that immediately follows train q is determined by following equation.

$$D_{\ell}^s = \begin{cases} D_q^s + h_d & (L) \\ D_q^s + h_a + r_q^{s+1} - r_{\ell}^{s+1} & (O) \\ D_{j2}^{s+1} + h_p - r_{\ell}^{s+1} & (M) \\ A_{\ell}^s + w_{\ell}^s & (N) \end{cases}$$

At station s , if ℓ overtakes train q , the departure and arrival times of train ℓ at s and $s+1$ are determined by following equations.

$$\bar{D}_{\ell}^s = \begin{cases} (D_{j^*}^s + h_d) & (P) \\ D_{j^*}^s + h_a + r_{j^*}^{s+1} - r_{\ell}^{s+1} & (Q) \\ (D_{j1}^{s+1} + h_p) - r_{\ell}^{s+1} & (R) \\ (A_{\ell}^s + w_{\ell}^s) & (S) \end{cases}$$

At station s , if ℓ overtakes train q , the departure and arrival times of train q at s and $s+1$ are determined by following equations.

$$\bar{D}_q^s = \begin{cases} (\bar{D}_{\ell}^s + h_d) & (W) \\ (\bar{D}_{\ell}^s + h_a + (r_{\ell}^{s+1} - r_q^{s+1})) & (X) \\ (D_{j2}^{s+1} + h_p) - r_q^{s+1} & (Y) \\ (A_q^s + w_q^s) & (Z) \end{cases}$$

There are total $4 \times 4 \times 4 \times 4 = 256$ possible combinations of train departure times. However, certain combinations are not possible and some of the combinations can be eliminated due to redundancies.

- a) D_{ℓ}^s can take maximum value of A,B,C or D. In this case $D_q^s + h_d < D_q^s + h_a + r_q^{s+1} - r_{\ell}^{s+1}$ as $r_q^{s+1} \leq r_{\ell}^{s+1}$ therefore (L > O) D_{ℓ}^s can never take value of O.
- b) As $\bar{D}_{\ell}^s + h_a + r_{\ell}^{s+1} - r_q^{s+1} \geq \bar{D}_{\ell}^s + h_d$ and train ℓ overtakes train q at station s equation W becomes redundant and can be eliminated.

- c) Equation $\bar{D}_q^s = Z$ is not possible because if train q cannot depart at $A_q^s + w_q^s$ after train ℓ overtakes it. Therefore combination involving Z can be eliminated.
- d) Let $D_q^s = D_{j^*}^s + h_d$ i.e. equation A is applicable $A \geq \begin{cases} B \\ C \\ D \end{cases}$ therefore $P = A$, $Q \leq B$, $R \leq C$ and $S \geq D$ as $r_q^{s+1} \leq r_\ell^{s+1}$, $A_\ell^s \geq A_q^s + h_d$ and $w_\ell^s \geq w_q^s$. Therefore AQ and AR combinations are not possible.
- e) Let $D_q^s = A_q^s + w_q^s$ i.e. equation D is applicable only DS combination is possible and DP, DQ and DR combinations are not possible.
- f) BPNX, BPNX, BQNX, BQNY, BQNY, BRNX, BRNY, CPNX, CPNY, CQNX, CQNY, CRNX and CRNY combinations are not possible as $D_\ell^s = A_\ell^s + w_\ell^s$ therefore $\bar{D}_\ell^s$ cannot be departed earlier than $A_\ell^s + w_\ell^s$.
- g) BSNX, BSNY, CSNX, CSNY, DSNX and DSNY combinations can be eliminated as $D_\ell^s = \bar{D}_\ell^s = A_\ell^s + w_\ell^s$ condition is not possible.
- h) BRLX and BRLY can be eliminated as $D_q^s \leq \bar{D}_q^s$ is not possible.
- i) BRMX and BRMX can be eliminated. Priority of train q is higher than priority of train ℓ $P_q \geq P_\ell$ therefore, at station $s+1$ if a platform is available for train q then it will certainly be available for train ℓ .

Therefore we need to prove only following combinations.

D_q^s	$\bar{D}_\ell^s$	D_ℓ^s	$\bar{D}_q^s$
A	P	L	X
A	S		
B	P		
B	Q		
B	R	M	Y
B	S		
C	P		
C	Q		
C	R	N	
C	S		
D	S		

(i) APLX

$$\delta_q^s = D_\ell^s - (A_q^s + w_q^s)$$

From A

$$\delta_q^s = D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = \bar{D}_q^s - (A_q^s + w_q^s)$$

From X

$$\bar{\delta}_q^s = (\bar{D}_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1})$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + 2 h_d - (A_q^s + w_q^s) \text{ (as } \bar{D}_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) \geq (\bar{D}_\ell^s + h_d))$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = D_\ell^s - (A_\ell^s + w_\ell^s)$$

From L

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

From P

$$\bar{\delta}_\ell^s = \bar{D}_\ell^s - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(ii) APLY

$$\delta_q^s = D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = (D_{j_2}^{s+1} + h_p - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + 2 h_d - (A_q^s + w_q^s) \text{ (as } D_{j_2}^{s+1} + h_p - r_q^{s+1} \geq \bar{D}_\ell^s + h_d)$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(iii) APMX

$$\delta_q^s = D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = (\bar{D}_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_a + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + W_{j^*}^{s+1} + h_p + (r_{j^*}^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = W_{j^*}^{s+1} + (h_p - h_d) + (r_{j^*}^{s+1} - r_\ell^{s+1})$$

The condition is true for the problem instances we solve in this chapter,

$$P_q (h_a + r_\ell^{s+1} - r_q^{s+1}) \geq P_\ell (W_{j^*}^{s+1} + h_p - h_d + r_{j^*}^{s+1} - r_\ell^{s+1})$$

$$\text{Therefore, for } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(iv) APMY

$$\delta_q^s = D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = (D_{j_2}^{s+1} + h_p) - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\nabla_q^s = (D_{j2}^{s+1} - D_{j^*}^s) + (h_p - h_d) - r_q^{s+1}$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = (D_{j2}^{s+1} - D_{j^*}^s) + (h_p - h_d) - r_\ell^{s+1}$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(v) BPLX

$$\delta_q^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_q^{s+1})) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_d + (r_\ell^{s+1} - r_j^{s+1})$$

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(vi) BPLY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_q^{s+1})) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = (D_{j2}^{s+1} + h_p) - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + 2 h_d - (A_q^s + w_q^s) \text{ (as } (D_{j2}^{s+1} + h_p) - r_q^{s+1} \geq \bar{D}_\ell^s + h_d)$$

$$\nabla_q^s \geq (2 h_d - h_a) - (r_j^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(vii) BPMX

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_d + (r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + W_{j^*}^{s+1} + h_p + (r_{j^*}^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = W_{j^*}^{s+1} + (h_p - h_d) + (r_{j^*}^{s+1} - r_\ell^{s+1})$$

The below condition is true for the problem instances we solve in this chapter,

$$P_q (2h_d + r_\ell^{s+1} - r_{j^*}^{s+1}) \geq P_\ell (W_{j^*}^{s+1} + h_p + r_{j^*}^{s+1} - r_\ell^{s+1})$$

Therefore, for $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(viii) BPMY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = (D_{j_2}^{s+1} + h_p - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = (D_{j_2}^{s+1} - D_{j^*}^s) + (h_p - h_a) - r_{j^*}^{s+1}$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = (D_{j_2}^{s+1} - D_{j^*}^s) + (h_p - h_a) - r_\ell^{s+1}$$

The relationship of running times of trains j^* , ℓ and q is given by

$r_j^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1}$ for the problem instances we solve in this chapter,

Therefore, for $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(ix) BQLX

$$\delta_q^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_a + (r_j^{s+1} - r_\ell^{s+1}) + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + 2h_a + (r_j^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_a$$

$$\delta_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s))$$

$$\nabla_\ell^s = h_d - h_a + (r_\ell^{s+1} - r_j^{s+1}) \text{ as } r_j^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1}$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(x) BQLY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = (D_{j_2}^{s+1} + h_p - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + h_a + (r_j^{s+1} - r_\ell^{s+1}) + h_d - (A_q^s + w_q^s)$$

$$(\text{as } D_{j_2}^{s+1} + h_p - r_q^{s+1} \geq \bar{D}_\ell^s + h_d)$$

$$\nabla_q^s \geq h_d + (r_q^{s+1} - r_\ell^{s+1})$$

$$\delta_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_j^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s))$$

$$\nabla_\ell^s = h_d - h_a + (r_\ell^{s+1} - r_j^{s+1})$$

$$h_d + (r_q^{s+1} - r_\ell^{s+1}) \geq h_d - h_a + (r_\ell^{s+1} - r_j^{s+1})$$

Above condition is true for the problem instances we solved,

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xi) BQMX

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_a$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + 2h_d - (A_\ell^s + w_\ell^s) \text{ (as } (D_{j_2}^{s+1} + h_p) - r_q^{s+1} \geq D_q^s + h_d)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s))$$

$$\nabla_\ell^s = 2h_d - h_a + (r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\text{As } P_\ell \leq P_q \text{ (} 2P_\ell \leq P_q \text{) and } r_\ell^{s+1} \geq r_{j^*}^{s+1}$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$$

(xii) BQMY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = (D_{j_2}^{s+1} + h_p - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) + h_d - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d + (r_q^{s+1} - r_\ell^{s+1})$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s))$$

$$\nabla_\ell^s \geq h_d + r_\ell^{s+1} - r_q^{s+1}$$

As $P_\ell \leq P_q$ ($P_\ell : P_q, 1:2$)

Therefore, $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$

(xiii) BSLY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s \geq (D_{j2}^{s+1} + h_p) - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + 2h_d - (A_q^s + w_q^s) \text{ (as } (D_{j2}^{s+1} + h_p) - r_q^{s+1} \geq \bar{D}_\ell^s + h_d)$$

$$\nabla_q^s \geq (2h_d - h_a) + (r_{j^*}^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = (D_{j^*}^s + 2h_d - (A_\ell^s + w_\ell^s))$$

$$\bar{\delta}_\ell^s \geq D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \leq h_d$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xiv) BSMX

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq (h_d + r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) + h_d - (D_{j^*}^s + h_d)$$

$$\bar{\delta}_\ell^s \geq A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_a + (r_{j^*}^{s+1} - r_q^{s+1})$$

The problem instances we solve in this chapter satisfy condition $4P_{j^*} \leq 2P_\ell \leq P_q$.

$$P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xv) BSMY

$$\delta_q^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s))$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + 2h_d - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq (2h_d + r_q^{s+1} - r_{j^*}^{s+1})$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_q^{s+1}) + h_d - (D_{j^*}^s + h_d)$$

$$\bar{\delta}_\ell^s \geq A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_a + (r_{j^*}^{s+1} - r_q^{s+1})$$

The problem instances we solve in this chapter satisfy condition $4P_{j^*} \leq 2P_\ell \leq P_q$.

$$P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xvi) CPLX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq h_a + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j^*}^s + 2h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

Therefore, as $P_\ell \leq P_q$ ($P_\ell : P_q, 1:2$)

$$P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xvii) CPLY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d + h_a - h_p + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s = D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = h_d$$

Therefore, as $P_\ell \leq P_q$ ($P_\ell : P_q, 1:2$)

$$P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xviii) CPMX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq h_a + r_\ell^{s+1} - r_q^{s+1}$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s \geq h_d$$

Therefore, for $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xix) CPMY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq D_{j^*}^s + h_d + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d + h_a - h_p + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j^*}^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s \geq h_d$$

Therefore, for $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xx) CQLX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq 2 h_a - h_d + (r_{j^*}^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1})) - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s = 2 h_d - h_a + (r_\ell^{s+1} - r_{j^*}^{s+1}) \text{ as } r_{j^*}^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1}$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxi) CQLY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq 2 h_a - h_d + (r_{j^*}^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1})) - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s \geq 2 h_d - h_a + (r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\text{as } r_{j^*}^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1} \text{ and } P_{j^*} \leq P_\ell \leq P_q \text{ } (P_{j^*} : P_\ell : P_q, 1:2:4)$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxii) CQMX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1}) + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq 2 h_a - h_d + r_{j^*}^{s+1} - r_q^{s+1}$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + 2 h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1})) - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s \geq 2h_d - h_a + (r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\text{as } r_{j^*}^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1} \text{ and } P_{j^*} \leq P_\ell \leq P_q \text{ } (P_{j^*} : P_\ell : P_q, 1:2:4)$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxiii) CQMY

$$\delta_q^s = D_{j_1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j_2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1})) + h_d - (A_q^s + w_q^s)$$

$$\nabla_q^s \leq 2h_a - h_d + r_{j^*}^{s+1} - r_q^{s+1}$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq D_{j^*}^s + 2h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = (D_{j^*}^s + h_a + (r_{j^*}^{s+1} - r_\ell^{s+1})) - (A_\ell^s + w_\ell^s)$$

$$\nabla_\ell^s \geq 2h_d - h_a + (r_\ell^{s+1} - r_{j^*}^{s+1})$$

$$\text{as } r_{j^*}^{s+1} \geq r_\ell^{s+1} \geq r_q^{s+1} \text{ and } P_{j^*} \leq P_\ell \leq P_q \text{ } (P_{j^*} : P_\ell : P_q, 1:2:4)$$

$$\text{Therefore, } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxiv) CRLX

$$\delta_q^s = D_{j_1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j_1}^{s+1} + h_p - r_\ell^{s+1} + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s = h_a$$

$$\delta_\ell^s = (D_{j_1}^{s+1} + h_p - r_q^{s+1} + h_d) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j_1}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = h_d + (r_\ell^{s+1} - r_q^{s+1})$$

$$r_\ell^{s+1} \geq r_q^{s+1} \text{ and } 2 P_\ell \leq P_q$$

Therefore, for the problem instances we solve in this chapter $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxv) CRLY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = (D_{j1}^{s+1} + h_p - r_q^{s+1} + h_d) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j1}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = h_d + (r_\ell^{s+1} - r_q^{s+1})$$

$$r_\ell^{s+1} \geq r_q^{s+1} \text{ and } 2 P_\ell \leq P_q$$

Therefore, for the problem instances we solve in this chapter $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxvi) CRMY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = (D_{j2}^{s+1} + h_p - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = D_{j1}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_d$$

$$\text{As } 2 P_\ell \leq P_q$$

Therefore, for the problem instances we solve in this chapter $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxvii) CSLX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = A_\ell^s + w_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_a + h_d + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = (D_{j1}^{s+1} + h_p - r_\ell^{s+1} + h_d) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_d$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxviii) CSLY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = (D_{j1}^{s+1} + h_p - r_\ell^{s+1} + h_d) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \leq h_d$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxix) CSMX

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = A_\ell^s + w_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_a + h_d + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = (D_{j2}^{s+1} + h_p - r_\ell^{s+1} + h_d) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_d$$

$$\text{As } 2 P_\ell \leq P_q$$

Therefore, for the problem instances we solve in this chapter $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxx) CSMY

$$\delta_q^s = D_{j1}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\delta_q^s \geq D_{j^*}^s + h_d - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\nabla_q^s \geq h_d$$

$$\delta_\ell^s = (D_{j2}^{s+1} + h_p - r_\ell^{s+1}) - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s \geq h_d$$

$$\text{As } 2 P_\ell \leq P_q$$

Therefore, for the problem instances we solve in this chapter $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxxi) DSLX

$$\delta_q^s = A_q^s + w_q^s - (A_q^s + w_q^s) = 0$$

$$\bar{\delta}_q^s = A_\ell^s + w_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$A_\ell^s + w_\ell^s \geq A_q^s + w_q^s + h_d$$

$$\nabla_q^s \geq h_d + h_a + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = A_q^s + w_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = 0$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxxii) DSLY

$$\delta_q^s = A_q^s + w_q^s - (A_q^s + w_q^s) = 0$$

$$\bar{\delta}_q^s = D_{j2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq A_\ell^s + w_\ell^s + h_d - (A_q^s + w_q^s) \text{ (as } A_\ell^s + w_\ell^s \geq A_q^s + w_q^s + h_d)$$

$$\nabla_q^s \geq 2h_d$$

$$\delta_\ell^s = A_q^s + w_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = 0$$

$$\text{Therefore, } \nabla_q^s \geq \nabla_\ell^s \text{ and } P_q \nabla_q^s \geq P_\ell \nabla_\ell^s.$$

(xxxiii) DSMX

$$\delta_q^s = A_q^s + w_q^s - (A_q^s + w_q^s) = 0$$

$$\bar{\delta}_q^s = A_\ell^s + w_\ell^s + h_a + (r_\ell^{s+1} - r_q^{s+1}) - (A_q^s + w_q^s)$$

$$A_\ell^s + w_\ell^s \geq A_q^s + w_q^s + h_d$$

$$\nabla_q^s = h_d + h_a + (r_\ell^{s+1} - r_q^{s+1})$$

$$\delta_\ell^s = D_{j2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq A_q^s + w_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = 0$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

(xxxiv)DSMY

$$\delta_q^s = A_q^s + w_q^s - (A_q^s + w_q^s) = 0$$

$$\bar{\delta}_q^s = D_{j_2}^{s+1} + h_p - r_q^{s+1} - (A_q^s + w_q^s)$$

$$\bar{\delta}_q^s \geq A_\ell^s + w_\ell^s + h_d - (A_q^s + w_q^s) \text{ (as } A_\ell^s + w_\ell^s \geq A_q^s + w_q^s + h_d)$$

$$\nabla_q^s \geq 2h_d$$

$$\delta_\ell^s = D_{j_2}^{s+1} + h_p - r_\ell^{s+1} - (A_\ell^s + w_\ell^s)$$

$$\delta_\ell^s \geq A_q^s + w_q^s + h_d - (A_\ell^s + w_\ell^s)$$

$$\bar{\delta}_\ell^s = A_\ell^s + w_\ell^s - (A_\ell^s + w_\ell^s) = 0$$

$$\nabla_\ell^s = 0$$

Therefore, $\nabla_q^s \geq \nabla_\ell^s$ and $P_q \nabla_q^s \geq P_\ell \nabla_\ell^s$.

Appendix 3

Running time and Halt time data of six corridors used in Chapter 2

Running time and halt Time details of IGP-BSL Corridor

[illegible]

Running time and halt Time details of LNL-DD Corridor

[illegible]

Running time and halt Time details of BSL-KND Corridor

	Running Time																	
Non Stop	11	6	6	6	6	5	5	7	5	5	5	5	5	3	4	11	0	95
Intercity	12	7	7	7	6	7	7	9	6	6	7	6	7	4	5	13	0	116
Passenger	12	8	8	8	8	8	8	11	7	7	7	7	8	5	6	16	0	134
Container	12	9	8	9	8	10	10	12	7	8	9	7	9	6	7	17	0	148
Freight	13	10	9	10	8	11	11	12	8	9	9	8	9	6	8	18	0	159
	Halt Time																	
Non Stop	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Intercity	0	0	0	0	5	0	3	0	0	2	0	0	0	0	0	0	5	15
Passenger	0	1	2	2	2	2	2	2	1	2	1	1	1	1	1	1	8	30
Container	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	15
Freight	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	15	15

Running time and halt Time details of BD-BSL Corridor

	Running Time																									
Stations	BD	TKI	KUM	MHAD	MANA	MZR	KTP	BGN	YAD	AK	GAO	PS	NGZ	SEG	JM	NN	KJL	BIS	WOD	MKU	KMKD	BDWD	AGC	VNA	BSL	
Non Stop	7	4	4	4	3	7	5	6	5	7	4	6	5	7	7	5	3	4	5	6	7	8	4	14	0	137
Intercity	9	5	4	4	7	6	6	7	6	9	5	6	7	7	7	5	3	4	5	6	7	8	4	16	0	153
Passenger	10	4	4	4	8	7	5	8	7	9	5	6	6	7	7	6	4	4	6	7	8	9	5	17	0	163
Container	10	4	4	5	9	8	6	8	9	8	6	6	7	7	8	6	4	4	5	8	9	10	6	20	0	177
Freight	11	5	4	5	9	10	7	9	9	9	8	7	8	8	8	7	6	5	6	9	9	11	7	21	0	198
					Halt time																		0			
Non Stop	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	5
Intercity	0	0	0	0	0	0	0	0	0	5	0	0	0	2	0	0	0	0	0	1	0	0	0	0	10	18
Passenger	0	0	0	0	0	2	0	0	0	5	0	0	0	2	0	2	2	0	0	0	2	2	0	1	10	28
Container	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Freight	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Running time and halt Time details of CSTM-KYN Corridor

		Running Time																		
	CSTM	MSD	SNRD	BY	PR	DR	MTN	SIN	CLA	VK	GC	BND	NHR	MLND	TNA	DW	DI	KYN		
Suburban	4	2	3	3	3	3	2	2	2	2	3	3	2	3	6	4	6	0	53	
Intercity	5	2	2	3	3	3	2	2	2	1	3	3	3	4	6	4	8	0	56	
Non stop	4	2	1	3	1	1	2	1	2	1	3	3	3	3	5	4	7	0	46	
Container	5	3	3	3	3	4	3	3	7	3	2	2	2	2	8	5	8	0	66	
Freight	6	3	3	3	3	4	3	3	8	3	3	2	2	3	8	6	7	0	70	
		Halt time																		
Suburban	0	0	0	1	0	1	0	0	1	0	1	0	0	1	1	0	1	1	8	
Intercity	0	0	0	0	0	2	0	0	0	0	0	0	0	0	2	0	0	2	6	
Non stop	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Container	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Freight	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	

Running time and halt Time details of BPQ-IT Corridor

[illegible]