Multi-modal vibration energy harvesting using the piezoelectric effect

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**MULTI-MODAL VIBRATION ENERGY HARVESTING
USING THE PIEZOELECTRIC EFFECT**

WU HAO
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
2015

**MULTI-MODAL VIBRATION ENERGY HARVESTING
USING THE PIEZOELECTRIC EFFECT**

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School of Civil and Environmental Engineering

**A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirements for the degree of
Doctor of Philosophy**

2015

This thesis is dedicated

To:

My parents, my wife, and my baby.

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SUMMARY

Over the past decade, the use of remote wireless sensing electronics has grown steadily. One main concern for the development of these kinds of devices is the power supply module. Rather than using the traditional batteries which require periodic maintenance as well as produce chemical waste, harvesting energy from the ambient environment provides a promising solution for implementing self-powered systems. Many kinds of energy sources existing in the environment can be used for energy harvesting, such as solar, wind, thermal gradient, and vibration. Among them, vibration is the most ubiquitous energy source that can be found everywhere in our daily life. There are various mechanisms to convert vibration energy into electrical energy, such as electromagnetic, electrostatic and piezoelectric transduction. Due to the property of high power density and ease of application, vibration energy harvesting using piezoelectric materials has attracted intense research interest in recent years.

A conventional piezoelectric energy harvester (PEH) works as a linear resonator, whose performance greatly relies on its resonant frequency. The working bandwidth of a conventional PEH is quite narrow, while the practical vibration sources in the environment are usually frequency-variant or randomly distributed over a wide frequency range. In this thesis, a novel two-degree-of-freedom (2-DOF) PEH is developed to broaden the working bandwidth by using its first two vibration modes. This novel design can achieve wider bandwidth with two close resonant frequencies, and with no increase of the volume. Besides, such design is more compact and utilizes the material more efficiently. An experimental prototype is fabricated and tested, to investigate the behavior of this harvester. Mathematical model and FEA simulation have been developed to model this 2-DOF energy harvester.

Other than using the linear multi-modal configuration, nonlinear vibration is another promising solution to broaden the bandwidth of a vibration energy harvesting system. Based on the previous linear 2-DOF PEH design, a nonlinear 2-DOF PEH is then developed by incorporating the magnetic nonlinearity. Experimental results show significant improvement of the working bandwidth as well as the powering efficiency. In the meantime, an analytical model is derived, providing good validation compared to the experiment results.

Considering the real environmental vibration are always presented with varying or multiple orientations in a three-dimensional (3-D) or two-dimensional (2-D) domain, it is also important to design a harvester adaptive with different excitation orientations. A multi-modal 2-D PEH with a frame configuration is also studied in this work, which can consistently generate significant power output with excitations from any direction within a 2-D domain. Experimental study is carried out, and numerical simulation is conducted by using the combination of finite element analysis (FEA) and equivalent circuit model (ECM) methods. The results indicate its promising potential for practical vibration energy harvesting.

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CHAPTER 1 INTRODUCTION

1.1 Background

Over the past few decades, the use of wireless sensors and remote electronics has grown steadily. One main concern for the development of such devices is the power supply module. A conventional choice is the use of chemical batteries, which requires periodic maintenance as well as produces chemical waste. To save maintenance cost and also to reduce pollution, environmental energy harvesting technology has captivated both the academics and industrialists. On the other hand, with the recent advancement in integrated circuits techniques, energy consumption of micro-scale electronics has been greatly reduced, making it possible to use the harvested energy from ambient environment for powering those electronics. The ultimate goal for the energy harvesting research is to achieve endless self-power devices for distributed or remote electronic systems, such as wireless sensing network. Many kinds of energy sources existing in the environment can be used for energy harvesting, such as solar, wind, thermal gradient, and vibration. Among them, vibration is the most ubiquitous energy source and can be found everywhere in our daily life. Thus, harvesting energy from mechanical vibration has attracted intense research interest in recent years

The idea for vibration to electricity conversion first appeared in a journal article by Williams and Yates (1996). There are many vibration-to-electric energy conversion mechanisms, such as electromagnetic, electrostatic, and piezoelectric transductions. Among these, piezoelectric transduction is the most popular way because of its high energy density and ease for integration, thus is chosen for the focus of the research in the thesis.

Piezoelectricity is a coupling effect between the mechanical and electrical behaviors for certain materials. In simple terms, when the mechanical strain is applied on a piezoelectric material, the deformation of the material will lead to the electric charge collected at the electrodes located on its surface. This is called direct piezoelectric effect. On the contrary, if the material is subjected to an electric change at its electrodes, it will deform mechanically, which is the converse piezoelectric effect. Both effects usually co-exist in a piezoelectric material. In case of the application for energy harvesting, the direct piezoelectric effect is of particular interest.

Typically, a simple piezoelectric energy harvester (PEH) is designed as a cantilever beam attached with one or two layers of piezoelectric materials (unimorph or bimorph). In order to increase the power output as well as to adjust the working frequency, a proof mass is usually added at the free end of the cantilever beam. The PEH is installed on a vibrating host structure. The vibration motion of the host structure serve as external excitation to the PEH and an alternating current (AC) output will be generated from the piezoelectric layers proportional to the induced dynamic strain. In theoretical analysis as well as experimental research, it is common practice to connect a resistor to the harvester, to evaluate its power generation performance. However, in real application, it is often required to convert the generated AC output into a constant direct current (DC) output using a rectifier (AC-DC converter). Sophisticated interface circuit is required to improve and manage the power generation.

A conventional PEH consisting of a cantilever beam with a tip mass mostly works at its first resonant frequency, while its high-order vibration modes are usually neglected as the frequencies are far away from the fundamental one and can only provide much lower response as compared to the first mode. Thus, only the first mode of the PEH is exploited for energy harvesting, and such kind of PEH is usually

regarded as a single-degree-of-freedom (SDOF) energy harvester. Its performance is greatly relied on the match of the resonant frequency to the excitation source. Only narrow frequency range is effective for energy harvesting, and slight shift from the resonant frequency will result in great reduction of the power output. However, in real applications, the practical vibration source in the environment is usually frequency-variant or random with energy distributed over a wide frequency range, which means, a conventional SDOF PEH with its narrow bandwidth is inefficient for real applications. Therefore, broadening the operation bandwidth is a very important issue for the enhancement of the performance of energy harvesting system. Many researchers have attempted to develop various systems with the capability of broadband energy harvesting. Many approaches have been proposed for broadband vibration energy harvesting in literature, such as frequency tuning, multi-modal and nonlinear techniques. All these approaches have their own advantages and limitations. How to effectively enlarge the bandwidth for vibration energy harvesting still remains a challenge.

Another aspect for the enhancement of piezoelectric energy harvesting system is to improve the efficiency of the generated power output. Advanced structures are proposed by researchers, which can improve the output efficiency by optimizing the structural parameters. Rather than that, many researchers from electrical and electronic engineering disciplines are working on the development of advanced interface circuit to further improve the harvesting efficiency. Many reported works claimed significantly improvement of power output by using sophisticated circuits, however in most of those works, certain assumptions were adopted which greatly reduced the structural complexity. The use of such sophisticated interface circuits combining with complex energy harvesting structure still requires further investigation.

Moreover, for applicable environmental energy harvesting, another important issue is that the real environmental vibration source may include multiple components from different orientations, or the orientation of the excitation may vary with time. Therefore, an adaptive energy harvesting system should be developed to work with any orientation in the three-dimensional (3-D) or two-dimensional (2-D) domain. So far, there are very few attempts reported regarding multi-directional energy harvesting.

1.2 Research Objectives

This research concentrates on enhancing the performance of a vibration piezoelectric energy harvesting system by using multi-modal technique. Firstly, a novel 2-DOF PEH is proposed by the author, which is validated to be more compact and with broader bandwidth. By incorporating the magnetic nonlinearity into the linear 2-DOF system, a nonlinear 2-DOF energy harvester is then studied to further broaden the bandwidth. Moreover, efforts are also devoted for developing a multi-modal multi-directional PEH which can harvest vibration energy in 2-D domain.

To predict the performance of an energy harvester, various modelling methods have been developed by other researchers, such as mathematic modelling, finite element modelling and equivalent circuit modeling. It is already a standard practice when modeling a simple energy harvesting system (i.e. uniform configuration) in simple condition (i.e. sinusoidal excitation, simple circuit load). It still remains challenging to model an energy harvesting system with complex structure and circuit. A distributed parameter mathematical model is derived for the cut-out 2-DOF PEH with segmented configuration, and a lumped parameter model with consideration of magnetic nonlinearity is developed to validate the nonlinear 2-DOF PEH. Additionally, a simulation model with a combination of Finite Element Analysis (FEA) and Equivalent Circuit Model (ECM) methods is developed to provide a robust

tool to simulate and design energy harvesting system with both structural and electrical complexity.

1.3 Original Contributions

The original contributions of this research can be summarized as follows:

- (1) As the narrow bandwidth of a linear SDOF energy harvester cannot fulfill the requirement for real application of energy harvesting, wider operation bandwidth is highly desirable. Hence, a novel 2-DOF PEH has been developed by the author. To provide a larger operation bandwidth as compared to the conventional SDOF PEH, by achieving two close response peaks in the frequency domain. Such 2-DOF harvester is also more compact by utilizing the cantilever materials more efficiently. An experimental prototype is fabricated and tested to investigate the behavior of this harvester. Mathematical model and FEA simulation have also been developed to model this energy harvester.
- (2) Although the linear 2-DOF PEH has already been validated for improving the bandwidth, there still exist a response valley in-between the two resonant response peaks which greatly deteriorates the performance of the harvester, especially when the anti-resonance point is located in-between. To further broaden the working bandwidth, a nonlinear 2-DOF PEH is proposed by incorporating magnetic nonlinearity into the linear 2-DOF PEH design. The experimental parametric study shows that, with a properly chosen structural configuration, much wider operation bandwidth is achieved and more power output can be generated. A lumped parameter model of the nonlinear 2-DOF PEH is also developed with consideration of the dipole-dipole magnetic force.
- (3) As the real environmental vibration sources are mostly presented with multiple components in different orientations, the conventional PEH with its fixed

orientation is inefficient for practical operation. A novel 2-D multi-modal PEH with a frame configuration is developed, which can harvest multi-directional vibration energy in a 2-D domain by utilizing its first two vibration modes. The obtained experimental results suggest promising potential for implementing such 2-D PEH into the practical solution. Furthermore, a general modeling procedure is developed, by using the combination of FEA and ECM simulations. Such modeling method is concluded more suitable for an energy harvesting system with both structural and electrical complexity.

1.4 Organization of the Thesis

This thesis consists of six chapters, including this Introduction Chapter. And Chapter 2 reviews the state-of-the-art technologies for energy harvesting through various mechanisms. Vibration energy harvesting using piezoelectric material is given more attention in this work. Different modeling methods for piezoelectric energy harvesting are discussed. Various techniques for the enhancement of energy harvesting are reviewed in detail too.

Chapter 3 presents a novel 2-DOF PEH proposed by the author. This novel 2-DOF PEH can achieve broader bandwidth with its two resonances tuned close to each other, and both can generate significant output. Moreover, it is more compact than the conventional design, by utilizing materials more efficiently. An experimental prototype is fabricated and tested to investigate the behavior of this harvester. Mathematical model and FEA simulation have also been developed to validate the experiment results. In Chapter 4, the linear 2-DOF PEH design is extended into a nonlinear 2-DOF PEH, by incorporating the nonlinearity using magnetic interaction. As studied in the experiment, with properly chosen structural parameters, significant enhancement for broadband energy harvesting is achieved. An analytical model is also derived, which shows good validation for the experimental finding.

Chapter 5 studies a 2-D multi-modal PEH with a frame configuration for multi-directional energy harvesting, by utilizing its first two vibration modes. Experiment study shows that, with properly chosen structural parameters, this harvester can consistently generate significant power output with excitations from any direction in the 2-D domain. A modeling procedure is also developed by using combination of the FEA and ECM simulation methods. Such method is robust for the piezoelectric energy harvesting system with both structural and electrical complexity.

Finally, Chapter 6 summarizes all the work accomplished in this PHD research, and suggests some recommendations for future development regarding to vibration energy harvesting.

CHAPTER 2 LITERATURE REVIEW

2.1 Overview of Vibration-based Piezoelectric Energy Harvesting

“*Energy harvesting*” is to generate energy by capturing the ambient environmental energy surrounding the system and converting it into electrical energy to directly power small electronic devices or to be stored for future use. In the last decade, great research interest for energy harvesting has grown towards the possibility of self-powered system for distributed or portable electronics systems, such as wireless sensing network.

There are various energy sources existing in the environment, such as solar, wind, thermal gradient, and vibration. For every kind of energy source, different methods have been developed for harvesting the energy. Among all these energy sources, mechanical vibration is the most ubiquitous in our daily life, which has recently attracted intense research interest. High-level mechanical vibrations often occur on machinery and vehicles, while low-level mechanical vibrations occur at every moment and can be found everywhere in the environment. The data listed in Table 2.1 shows the level and frequency region of some ambient vibration sources (Roundy et al., 2003, Butz, C. et al., Reilly et al, 2009, Rahman and Leong 2011, Peigney & Siegert 2013,). Since mechanical vibration is the pervasive energy source, vibration-based energy harvesting is the focus of this research.

Table 2.1 Survey of ambient vibration sources

Vibration Sources	Acceleration (m/s^2)	Frequency (Hz)
Statasys 3D printer	0.6	28
Delta Drill Press	4	41
Motorcycle (Honda wave 125)	3	17

Vehicle on moving	0.2-2	15-25
Grinding machine	4	49
Air handling unit	1	33
Treadmill	0.1	26
Washing machine	0.3	39
Pedestrian bridge	0.2	1.5-4
Girder on a traffic bridge	0.7	5-20
Door frame just after door closes	3	125
Small microwave oven	2.5	121
HVAC vents in office building	0.2-1.5	60
Windows next to busy road	0.7	100
CD on notebook computer	0.6	75
Second storey floor of busy office	0.2	100

2.1.1 Mechanisms for Converting Vibration Energy into Electrical Energy

There are many choices of mechanical-to-electrical conversion mechanisms can be adopted for harvesting environmental vibration energy:

Electrostatic conversion mechanism

The basic concept for electrostatic energy conversion is to use variable capacitor. If the electric charge in the variable capacitor is constrained when the capacitance decreases, the voltage across the capacitor will increase. On the contrary, if the voltage on the capacitor is fixed while the capacitance decreases, electric charge will be collected from the variable capacitor. The typical variable capacitor used for energy harvesting usually consists of two conductors separated by a dielectric material. When the conductors are moved relative to each other, the capacitance is varied, and then the mechanical energy is converted into electrical energy. There are three basic types of electrostatic generators reported in the literature, referring to the different direction of the relative movement of the conductors, as shown in Figure 2.1

(Roundy et al., 2003). The advantage of this conversion mechanism is that such kind of configuration is very easy to be scaled down to very small size, so that it is convenient to be integrated into the micro-electro-mechanical systems (MEMS). However, the drawbacks of this conversion mechanism are that an external power source is required to start the harvesting process, and the power output efficiency for energy harvesting using electrostatic conversion is relatively low compared to the other transduction mechanisms.

Figure 2.1 Three different types of electrostatic generators: (a) in-plane overlap converter, (b) in-plane gap closing converter and (c) out-of-plane gap closing converter (Roundy et al., 2003)

Using dielectric elastomer

Dielectric elastomer material is a sandwich structure where a piece of polymer of high dielectric property is attached with two compliant electrode. It can also be regarded as a variable capacitor. However, different from the electrostatic conversion mentioned above, polymer is presented as dielectric material which is quite soft and is very suitable for large deformation application. With the deformation of the dielectric elastomer, the capacitance of the material changes, then converse the

vibration energy into electrical energy. Such dielectric elastomer can work in both direct (actuator) and converse (harvesting) ways. As an actuator, it is widely studied in robot engineering to design the moveable components (Pelrine *et al.*, 2002; Kovas *et al.*, 2007). In recent years, there are works reported for using dielectric elastomer as energy harvester (Brochu *et al.*, 2010; Koh *et al.*, 2011). However, due to its material characteristics, the application of dielectric material is limited, with problems like: material rupture, loss of tension, electrical breakdown and electromechanical instability (Koh *et al.*, 2011). Also, due to the low stiffness of the polymer, such material is more likely to be used in large amplitude low-frequency application, like human motion or ocean waves.

Electromagnetic conversion

Energy harvesting using electromagnetic conversion is based on the Faraday's law of electromagnetic induction (Arnold 2007). It is similar to the large-scale traditional generator but in micro size, where current is generated when a coil moved within a magnetic field. For this kind of conversion mechanism, a permanent magnetic field is required. However, permanent magnets are usually bulky and difficult to be scaled into MEMS size. The recent attempts to miniaturize the electromagnetic generator using micro-engineering technology were found to have reduced the efficiency considerably. Furthermore, the output voltage from electromagnetic conversion is normally very small ($<1V$), thus it is required to be transformed into usable voltage levels before practical use (Cook Chennault *et al.*, 2008)

Using magnetostrictive materials

Magnetostrictive materials change their susceptibility when subjected with vibratory force, thus inducing alternating current in the pick-up coils. Advantages of using magnetostrictive materials include their high electromechanical coupling coefficient, high flexibility and suitability for high-frequency vibrations (Wang and Yuan, 2008).

However, similar to the electromagnetic generators, the bulky dimension due to the pick-up coils limits their applicability in MEMS devices and the low voltage output requires voltage transformation in the post-processing circuit.

Piezoelectric transduction

Another alternative conversion mechanism adopted for vibration energy harvesting is to use piezoelectric materials, which has attracted great research interest in recent years (Anton and Sodano, 2007). Generally, the conventional PEH is a cantilever configuration consisting of a cantilever substrate and one or two layers of piezoelectric materials attached on the substrate, as shown in Figure 2.2. Such configuration is easy to scale down for MEMS fabrication.

Figure 2.2 A cantilevered PEH configuration

Figure 2.3 MEMS piezoelectric cantilever beam (Jeon et al., 2005)

Figure 2.3 shows a thin film MEMS piezoelectric energy harvesting device fabricated by Jeon et al. (2005). Usually, a proof mass is placed at the free end of the cantilever beam to increase the power output as well as to adjust the resonant frequency. Various techniques were developed to enhance the performance of PEHs, which will be discussed in later sections. A power density versus voltage comparison for different kinds of conversion mechanisms was given by Cook-Chennault et al. (2008), as shown in Figure 2.4. It shows that piezoelectric energy harvesting covers the largest area in this graph, and the power density is comparable to others like electromagnetic conversion, thermoelectric generators and lithium-ion batteries. As apparent from Figure 2.4, voltage output for energy harvesting using electromagnetic conversion is typically much lower than using piezoelectric materials.

Figure 2.4 Comparison of power density and voltage level for various solutions
(Cook-Chennault et al., 2008)

In summary, there are several available solutions for vibration-to-electrical energy conversion. However, due to the high power density and high voltage output of the piezoelectric conversion, as well as the ease of fabrication in both macro and micro scales, vibration energy harvesting using piezoelectric materials has received great attention in recent years, and is chosen for further exploration in this research.

2.1.2 Introduction of Piezoelectricity

The term “piezoelectricity” is used to describe the coupling behavior of the piezoelectric materials, between mechanical and electrical domain, where “piezo” is the Greek word for pressure. This phenomenon of piezoelectricity was first discovered in 1880 by Pierre and Paul-Jacques Curie, in certain crystalline minerals that such as quartz, tourmaline, and Rochelle salt. They found such crystals can develop electric charge on their surface when mechanically deformed, which is called *direct piezoelectric effect*. Conversely, when the piezoelectric materials are subjected to an electric field, the materials will be mechanically deformed in proportion to the strength of the electric field, which is named as *converse piezoelectric effect*.

At that time, due to the low piezoelectric property of those crystals, the development of such material was limited. This situation last until the major breakthrough of the discovery of piezoelectric-ceramics, like Barium Titanate in 1940s and Lead Zirconate Titanate (PZT) in 1950s.

Figure 2.5 Electric dipoles (a) before (b) during and (c) after the poling process of the piezoelectric ceramics. (figure from PI Ceramic)

The most important process in the fabrication of the piezoelectric-ceramics is “*polarizing*” or “*poling*” process. Prior to polarization, the microscopic dipoles are randomly orientated, thus no overall piezoelectric behavior is observable, as shown in Figure 2.5a. When the piezoelectric ceramics are exposed to a strong direct current electric field, usually at a temperature slightly below the Curie point (Crawley and

Anderson, 1990), the dipoles will be oriented according to the direction of the electric field (Figure 2.5b). Upon switching off the electric field, most dipoles will not return to their original orientation as a result of the pinning effect, making numerous microscopic dipoles roughly oriented in the same direction, which is known as “*poling direction*” (Figure 2.5c). Therefore, the materials now present strong permanent polarization, and thus a strong piezoelectric coupling. It is noteworthy that the material can be de-poled if it is subjected to a very high electric field oriented opposite to the poling direction or is exposed to a temperature higher than the Curie temperature of the material.

Piezoelectric constitutive relations

The general constitutive equations for a piezoelectric material under small field considerations can be written as (Ikeda, 1990)

$$\begin{cases} S = s^E T + dE \\ D = dT + \varepsilon^T E \end{cases} \quad (2.1a)$$

$$\begin{cases} T = c^E S - eE \\ D = eS + \varepsilon^S E \end{cases} \quad (2.1b)$$

where S is the strain tensor; T (N/m²) is the stress tensor; D (C/m²) is the electric displacement tensor; E (V/m²) is the applied electric field tensor; s (m²/N) is the elastic compliance tensor; ε (F/m) is the dielectric constant tensor; c (N/m²) is the elastic constant tensor; d (m/V) and e (C/m²) are two different form of piezoelectric coefficients and the superscripts $()^T$, $()^S$ and $()^E$ indicate the coefficient is measured at constant stress, constant strain and constant electric field, respectively.

The coupling term in Equation (2.1b) indicates the electrical output resulted from mechanical domain, corresponding to the *direct piezoelectric effect*, and is named as *forward electromechanical coupling*. On the contrary, the coupling term in Equation (2.1a) depicts the backward effect in mechanical domain from electrical domain,

corresponding to the *converse piezoelectric effect*, and is termed as *backward electromechanical coupling*.

Piezoelectric materials

Piezoelectric materials are widely available in many different forms. The most commonly used piezoelectric material is lead zirconate titanate ($\text{Pb}[\text{Zr}_x\text{Ti}_{1-x}]\text{O}_3$, shorted as PZT), as shown in Figure 2.6(a). PZT transducers exhibit similar characteristics as ceramics, such as high elastic modulus, low tensile strength, and brittleness (Sirohi and Chopra, 2000b). The highly brittle feature of PZT transducers makes it hard to bond onto host structures, especially on curved surfaces.

Figure 2.6 (a) Conventional PZT ceramics (PI Ceramic Co.) and (b) Macro-fiber composites (Smart Material Corp.)

These limitations had motivated researchers to develop alternative materials. One solution to overcome the brittleness of PZT is using a composite material consisting of piezoelectric ceramic fibers embedded in polymeric matrix (Sodano et al., 2004a). The polymeric matrix provides a protective layer for the piezoelectric material thus increases the flexibility of piezoelectric ceramic fibers and makes it conformable to curved surfaces. Several technologies for piezoelectric ceramic fibers are now

commercially available, such as active fiber composite (AFC) actuators developed by MIT (Bent and Hagood, 1993), and macro fiber composite (MFC) actuators by NASA Langley Research Center (Wilkie et al., 2000). The polymer layer in AFC or MFC provides the piezoelectric ceramic fibers excellent flexibility to withstand large deformation, as shown in Figure 2.6(b).

Two working modes for piezoelectric materials

Piezoelectric materials are anisotropic materials. Thus, the properties of the material depend on the orientation of the polarization and direction of forces applied. There are two common operating modes in which piezoelectric materials may be used. These two modes are distinguished by the piezoelectric strain constant tensor d_{ij} , indicating electric field is generated in the i -axis and under stressed in the j -axis. Thus, piezoelectric transducers that rely on a compressive strain applied perpendicular to the electrodes utilize the d_{33} mode; while those rely on a transverse strain parallel to the electrodes utilize the d_{31} mode (Roundy et al. 2003). These two working modes are illustrated in Figure 2.7.

Figure 2.7 Illustration of 33 mode and 31 mode of operation for piezoelectric material

Piezoelectric transducers with the d_{33} mode of operation are mostly used in bulky structure that suffer deformation directly, while the d_{31} mode of operation are used on thin bending structures like cantilever beams.

Other applications for piezoelectric materials

Besides application for vibration energy harvesting, piezoelectric materials have been applied in other research areas. Piezoelectric materials have been playing an important role for structure health monitoring (SHM) in recent decades (Annamdas and Soh, 2010), to detect and evaluate the damage of a structure. Piezoelectric materials are also widely used for vibration control in flexible structures, especially for applications in space structures like sun plate and satellite antenna, to suppress the vibration of such structures (Moheimani 2003).

2.1.3 Piezoelectric Energy Harvesting System Diagram

Usually, the vibration energy harvesting devices can be divided into two groups, i.e., resonant and non-resonant (displacement depended) energy harvesters (duToit, 2005). The resonant energy harvester is mostly used where the input vibrations are regular, frequencies are high, and the input vibration amplitude is smaller than the device critical dimensions, like vibration on machinery or vehicle. On the other hand, non-resonant energy harvester is more efficient where the input vibration motion is irregular or of low frequency, but with amplitudes much larger than the device critical dimensions, such as human motions.

Figure 2.8 shows two different installation schemes to achieve energy harvesting from environmental vibration (Liang and Liao, 2010). One is to directly bond the piezoelectric elements onto the host structures, whose performance is only related to the deformation of the host structure (non-resonant). With a given vibration source, little can be done to further improve the harvester's structural performance, unless

modifying the host structure. The other scheme is to install an add-on system comprises of cantilever beam bonded with piezoelectric elements onto the host structure undergoing a base excitation. In this scheme, since the piezoelectric energy harvester is installed as an add-on system onto the host structure, one can further enhance by modifying the add-on structure. The add-on system of the piezoelectric energy harvester is normally a cantilever beam configuration, whose response greatly rely to its resonances. With a base excitation source matching with its resonances, the harvester will perform high dynamic response. Hence, this kind of add-on system with cantilevered harvester is the focus of most research interest for piezoelectric energy harvesting. A typical PEH is just like the one shown in Figure 2.8(b), which comprise of a cantilever substrate and tip mass, with piezoelectric layers bonded onto the substrate.

Figure 2.8 Different piezoelectric energy harvesting schemes (a) surface bonded (b) add-on system (Liang and Liao, 2010)

Base on the scheme shown in Figure 2.8(b), due to the base vibration motion been applied to the system, an AC output is obtained from the harvester. In the mechanical research for energy harvesting to estimate the performance of AC power generation by the harvester, one common practice is to connect a resistive load to the harvester to represent the electronic load. However, in practice, the electronic components usually are supplied with DC and regulated power. Hence, the AC output should be converted to a stable rectified voltage through a rectifier bridge (AC-DC converter), and usually a secondary stage regulator (DC-DC converter) is employed to regulate

the voltage output, as shown in Figure 2.9. Furthermore, to enhance the power, various circuit techniques were developed by researchers, and will be reviewed in the later sections.

Figure 2.9 Practical energy harvesting circuit of piezoelectric energy harvester

Figure 2.10 System block diagram for energy harvesting wireless sensing node with data logging and RF communications capabilities (Arms et al., 2005)

The final goal for energy harvesting is to achieve a self-powered system. Figure 2.10 shows the layout for a self-powered wireless sensing system (Arms et al. 2005). Connected with energy harvesting and storage, the system has three main parts of components to be powered: sensor nodes, micro-controller and components for RF communication. The power management strategy is also a big challenge to design such systems. A practical application of integrated system with harvester and sensor

node was developed by (Arms et al. 2005) too, as shown in Figure 2.11. It was found that the piezoelectric generator was capable of supplying enough energy to perpetually operate the sensor with *low duty cycle* wireless transmissions. “A *low duty cycle*” shows that the piezoelectric harvester did not generate enough power to continually operate the system. In other words, further enhancement for the piezoelectric energy harvesting is highly desired.

Figure 2.11 Integrated piezoelectric vibration energy harvester and wireless temperature and humidity sensing node (Arms et al., 2005)

2.2 Modeling Method for Vibration PEH

During the design stage, it is important to establish certain efficient models to estimate the performance of the energy harvesting system. Several mathematical models have been established in the past few years, including lumped parameter models, distributed parameter models, as well as approximate models using Rayleigh-Ritz approach. Rather than the analytical models, Finite Element Analysis (FEA) is found reasonable for modeling the harvester's dynamic performance,

especially when the structure is more complicated. Moreover, another method for modeling energy harvesters is from the view of electric domain, by using equivalent circuit modeling (ECM) simulation.

2.2.1 Mathematical modeling method

2.2.1.1 Lumped parameter models

Simplified uncoupled SDOF model

Considering the conventional PEH shown in Figure 2.8(b), its system can be simplified into a lumped mass single-degree-of-freedom (SDOF) system under a base excitation. SDOF model (a mass-spring-damper system) is one common modeling approach to obtain a fundamental understanding of the dynamics of an energy harvesting system. The term “*uncouple*” means, the coupling effect between mechanical and electrical domains is neglected, or simplified into viscous damping. Such method was first used for modeling energy harvester by Willams and Yates (1996), in their work of an electromagnetic generators. The governing equation for this model can be expressed as

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} \quad (2.2)$$

where m is the seismic mass; k is the spring constant; the electrical damping induced by electromechanical coupling of the harvester is treated as a viscous damping c_e , which is included in the total damping coefficient c , together with the structural damping; y is the base excitation amplitude; z is the displacement of the seismic mass relative to the base excitation; and a dot above a variable represents differentiation respect to time.

For this model, one can work out the power generation as

$$\left| \frac{P}{\ddot{y}^2} \right| = \frac{m}{\omega_n} \frac{\zeta_e \Omega^2}{[1 - \Omega^2]^2 + [2\zeta\Omega]^2} \quad (2.3)$$

where ζ_e is the electrically induced damping ratio; ζ is the total damping ratio; ω_n is the natural frequency; and $\Omega=\omega/\omega_n$ is the dimensionless frequency. This model is fairly accurate, as it did not just simply neglect the backward coupling, but assumed the backward coupling effect is proportional to the harvester's velocity response only. (Erturk and Inman, 2008b).

Coupled SDOF model

Considering the electromechanical coupling behavior of the piezoelectric materials, the response in electric domain will definitely feedback to the mechanical domain as the backward coupling effect. duToit et al. (2005) established a SDOF model including the backward coupling effect for an energy harvester using d_{33} effect. The governing equations of their model are

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\ddot{w}_B \quad (2.4a)$$

$$R_l C_p \dot{v} + v + m_{eff} R_l d_{33} \omega_n^2 \dot{w} = 0 \quad (2.4b)$$

where w_B is the base displacement; w is the displacement of the proof mass relative to the base; R_l is the load resistor; v is the voltage output across R_l ; m_{eff} is the effective mass; ζ_m is the mechanical damping ratio; ω_n is the un-damped natural frequency; and C_p is the capacitance of the piezoelectric material.

In this model, they used two separate equations, one to describe the dynamics of the harvester in the mechanical domain, while another describes the electrical response of the harvester in the electrical domain. The backward coupling effect in the mechanical domain caused by the electrical output is described as $\omega_n^2 d_{33} v$ in Equation (2.4b), while $m_{eff} R_l d_{33} \omega_n^2 \dot{w}$ in Equation (2.4a) is presenting the forward coupling effect in electrical domain. Thus, this model is named as “coupled model” to

differentiate it from the “*uncoupled model*” as mentioned above. The power response of this coupled SDOF model can be expressed as,

$$\left| \frac{P}{\ddot{w}_b^2} \right| = \frac{m_{eff}}{\omega_n} \frac{\alpha k_e^2 \Omega^2}{\left[1 - (1 + 2\zeta_m \alpha) \Omega^2 \right]^2 + \left[(2\zeta_m + (1 + k_e^2) \alpha) \Omega - \alpha \Omega^3 \right]^2} \quad (2.5)$$

where k_e is the alternative electromechanical coupling coefficient ($k_e^2 = d_{33}^2 \cdot c_{33}^E / \epsilon_{33}^S$) and $\alpha = \omega_n R_1 C^S$ is the dimensionless electric load of the system. Comparing Equations (2.5) and (2.3), the backward piezoelectric coupling effect in power generation obviously acts in a more complicated way than viscous damping. Moreover, using the coupled model, dutToit *et al.* (2005) observed the shift of short circuit and open circuit resonances, which the uncoupled SDOF model failed to predict.

SDOF correction factors

Although the coupled SDOF model (dutToit *et al.*, 2005) improved the prediction of system performance with proper handling of electromechanical coupling effect, Erturk and Inman (2008a) pointed out that this model failed to predict the system performance accurately if the proof mass to the distributed mass ratio was not very high. The inaccuracy was due to the simplification for SDOF system by ignoring the contribution of the distributed beam mass. Although the SDOF model predicts the resonance accurately by using the effective mass which normalize the distributed mass to the point of tip mass. There still has certain underestimation of performance due to contribution of the distributed mass in the excitation amplitude. It is suggested that correction factors should be added to the SDOF equation to make the result more accurate (Erturk and Inman, 2008a). The equation for uncoupled SDOF system was expressed as

$$m\ddot{z} + c\dot{z} + kz = -\mu_1 m\ddot{y} \quad (2.6a)$$

$$m\ddot{z} + c\dot{z} + kz = -\kappa_1 m\ddot{y} \quad (2.6b)$$

where μ_1 and κ_1 are the forcing amplitude correction factors for transverse and longitudinal vibrations, respectively. The correction factors were derived based on distributed parameter modeling

$$\mu_1 = \frac{(M_t/mL)^2 + 0.603(M_t/mL) + 0.08955}{(M_t/mL)^2 + 0.4637(M_t/mL) + 0.05718} \quad (2.7a)$$

$$\kappa_1 = \frac{(M_t/mL)^2 + 0.7664(M_t/mL) + 0.2049}{(M_t/mL)^2 + 0.6005(M_t/mL) + 0.161} \quad (2.7b)$$

The correction factor is also applied to the equation of the coupled SDOF model as

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\kappa_1 \ddot{w}_B \quad (2.8)$$

These corrected equations are consistent with the former SDOF equations (Equations (2.2), (2.3) and (2.4)) when M_t/mL becomes very large, thus making factors μ_1 and κ_1 tend to unity. This is reasonable because a very large M_t/mL value indicates that the large proof mass dominates the inertia for the vibration while the contribution from distributed beam mass becomes negligible. In the numerical case study presented by duToit *et al.* (2005), the mass ratio of proof mass to bar mass is $M_t/mL=1.33$, which is not large enough to ignore the correction factor. As reported by Erturk and Inman (2008a), with a mass ratio of 1.33, their modified equations considering the correct factors avoid underestimation of the tip motion and voltage output with an error of 8.83% and the power output with an error of 16.9%.

2.2.1.2 Distributed parameter models

Uncoupled distributed parameter model

The lumped parameter SDOF model is a simplified modeling method for energy harvesting system, and it can only present the first vibration mode of the harvester. If higher order vibration modes are required, another model should be developed by considering the distributed parameters. The uncoupled distributed parameter model was developed by neglecting the backward coupling effect of piezoelectric elements

(Chen et al., 2006; Lin et al., 2007). Similar to the uncoupled SDOF model, this model results in error for estimating the maximum performance and failed to predict the resonance shifting for short circuit and open circuit conditions, especially when the electromechanical coupling is strong. However, such model still can provide good solution for weakly coupled systems.

Coupled distributed parameter model

To get more accurate estimation of performance for piezoelectric energy harvesters, the backward coupling effect should be properly considered in the model. A coupled distributed parameter model based on Euler-Bernoulli beam assumption was developed by Erturk and Inman (2008b), considering a uniform cantilevered piezoelectric energy harvester connected with a resistive load. In the absence of a proof mass, the partial differential equation of the motion of a cantilevered beam with proof mass can be written as

$$\begin{aligned}
 YI \frac{\partial^4 w_{rel}(x,t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x,t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x,t)}{\partial t} + m \frac{\partial^2 w_{rel}(x,t)}{\partial t^2} \\
 + \theta V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] = -m \frac{\partial^2 w_b(x,t)}{\partial t^2} - c_a \frac{\partial w_b(x,t)}{\partial t}
 \end{aligned} \quad (2.9)$$

where YI is the average bending stiffness; I is the equivalent area moment of inertia of the composite cross section; $w_b(x,t)$ and $w_{rel}(x,t)$ are the base excitation and the deflection relative to the base motion, respectively; c_s and c_a are the strain rate damping coefficient and viscous air damping coefficient, respectively; L is the length of the beam; m is the mass per unit length; θ is the electromechanical coupling coefficient; $V(t)$ is the output voltage of the energy harvester; $\delta(x)$ is the Dirac delta function; x is the longitudinal coordinate; and t is time. If a tip mass M_t is added, m in the equation should be replaced by $m + M_t \delta(x-L)$.

The vibration motion of the beam can be represented by an absolutely and uniformly convergent series of eigenfunctions as

$$w_{rel}(x, t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (2.10)$$

where $\phi_r(x)$ is the r -th mode shape function, and $\eta_r(t)$ is the modal coordinate. By substituting Equation (2.10) into Equation (2.9) and applying the orthogonality conditions of the eigenfunctions, the response in the modal coordinate of the harvester is obtained as,

$$\frac{d^2 \eta_r(t)}{dt^2} + 2\zeta_r \omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2 \eta_r(t) + \chi_r V(t) = -\frac{\partial^2 w_b(x, t)}{\partial t^2} \left(m \int_0^L \phi_r(x) dx + M_t \phi_r(L) \right) \quad (2.11)$$

where ω_r and ζ_r are the natural frequency and damping ratio of the r -th mode; and χ_r is the modal electromechanical coupling coefficient. The coupled equation in electrical domain is

$$\frac{V(t)}{R_l} + C^s \frac{dV(t)}{dt} - \sum_{r=1}^{\infty} \chi_r \frac{d\eta_r(t)}{dt} = 0 \quad (2.12)$$

The modal electromechanical coupling coefficient can be expressed as

$$\chi_r = \int_0^L \theta \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] \phi_r(x) dx = \int_0^L \theta \frac{d^2 \phi_r(x)}{dx^2} dx = \theta \frac{d\phi_r(x)}{dx} \Big|_{x=L} \quad (2.13)$$

When a harmonic base excitation $u_g(t) = A e^{j\omega t}$ is applied (j is the unit imaginary number and ω is the excitation frequency), the steady state voltage response across the resistive load can be worked out using Equations (2.11) and (2.12), as

$$V(t) = \frac{\sum_{r=1}^{\infty} \frac{j\omega \chi_r f_r}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega}}{\sum_{r=1}^{\infty} \frac{j\omega \chi_r^2}{\omega_r^2 - \omega^2 + j2\zeta_r \omega_r \omega} + j\omega C^s + \frac{1}{R_l}} A \omega^2 e^{j\omega t} \quad (2.14)$$

With this model, more accurate estimations for the system performance can be obtained, which fit well with the experiment results. Moreover, by using this model, the resonance shifting phenomenon caused by the backward coupling effect can also be predicted properly.

This work by Erturk and Inman (2008b) is regarded as a benchmark of coupled distributed parameter model for piezoelectric energy harvesters. However, this modal is only applied to simple structure like a simple rectangular cantilever beam with uniform properties. It is much more complicated or even impossible to derive an analytical modal for a complicated structure with varying or irregular configuration. This method will be later adopted in the author's work and further developed for more complicated application.

2.2.1.3 Approximate model by Rayleigh-Ritz approach

Other than the accurate analytical distributed parameter model, another way to model the energy harvesting system to use the Rayleigh-Ritz approach (Sodano et al., 2004b; Elvin and Elvin, 2009a; Elvin and Elvin, 2009b). The equation of motions using Rayleigh-Ritz approach is written as,

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} - \mathbf{\Theta}\mathbf{v} = -\ddot{w}_b \left(\int_0^L m\boldsymbol{\varphi}(x)dx + M_t\boldsymbol{\varphi}(L) \right) \quad (2.15)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{\Theta}$ are the mass, damping, stiffness and piezoelectric coupling matrices after the Rayleigh-Ritz formulation; $\mathbf{r}$ denotes the displacement vector; m is the mass distribution per unit length (can be a function of x); M_t is the proof mass; w_b is the base excitation; $\boldsymbol{\varphi}$ is the vector of assumed mode shape functions; and $\mathbf{v}$ is the voltage coordinate vector.

By using Rayleigh-Ritz approach, the accuracy is depending on the mode shape functions and the number of assumed modes. If more assumed modes are considered in the model, more accurate estimation can be achieved. These mode shape functions are admissible functions which can be chosen as a set of any functions satisfying the geometric boundary conditions. Thus, it is quite easy to increase the accuracy of this modeling method simply by including more assumed mode shape functions. The most advantage compared to the analytical modeling method is that, the Rayleigh-

Ritz approximate modeling is applicable for modeling complex structures which may not be able to be achieved with analytical solution.

2.2.2 Finite element analysis

Other than the mathematical modelling methods, numerical simulation is another way to model the energy harvesting behaviors, and it is much easier to be implemented for complex structures. FEA is an advanced method for solving difficult problems across many field of physics and also has the capability of solving coupled-field problem. It is one of the most popular methods for simulating the energy harvester.

De Marqui Junior et al. (2009) derived an electromechanical finite element model for piezoelectric energy harvesting plates, with results consistent with the mathematical modeling and experiment data. Rather than deriving the finite element model theoretically, there are several robust commercial FEA software suitable for modeling piezoelectric energy harvesting, such as ANSYS and ABAQUS. Zhu et al., (2009) first presented a piezoelectric-circuit coupled model to analyze the power output of a PEH, using ANSYS software.

With such FEA software, the steady state response for a linear energy harvesting system can be easily worked out through harmonic analysis. They are also capable of solving nonlinear dynamics problems through transient analysis, however the computation time cost may be quite high. Complex harvester structures can be modeled in FEA easily, while only a few linear electric components can be modeled, such as resistor. It is quite hard to simulate the energy harvesting system with complex circuit interface including nonlinear electric components like rectifier by using FEA modeling.

2.2.3 Equivalent circuit model

Another modeling method was proposed to model the system's behavior in the electrical domain using ECM method, which can include both mechanical and circuit complexity (Elvin and Elvin, 2009a; Yang and Tang, 2009).

The development of equivalent circuit modeling method is based on the analogies between mechanical and electrical domains. For an electromechanical system such as piezoelectric energy harvesting system, parameters in mechanical domain can be transferred into electrical domain (Yang and Tang, 2009). Then, the circuit simulation can be carried out by using those parameters in a circuit simulator (i.e. SPICE). An example of such circuit simulation is shown in Figure 2.12, which represent the model for a multiple-modal energy harvester connected with a load resistor (first three modes are modeled). The results obtained from ECM simulation are consistent with the experimental results and analytical results. It is also applicable for complex structure configuration as long as the parameters can be determined through theoretical analysis or FEA. Once the parameters are obtained, a complicated energy harvesting system with both structural and circuit complexity can be simulated in the same network.

Figure 2.12 Multi-mode equivalent circuit model of piezoelectric energy harvester (Yang and Tang, 2009)

2.3 Enhancing Performance of Energy Harvesting Systems

Although power consumption decreases dramatically with the advancement of circuit technologies, it is still necessary to improve the harvester's performance to match the power requirements of most current electronics. In the literature, various ways have been proposed by researchers from various engineering communities in order to enhance the performance of energy harvesting systems. The enhancement can be considered from different aspects:

1) Firstly, power output efficiency is required to be further improved to match with the requirement of electronics. Typically, the efficiency improvement can be further achieved from two different ways: (a) improve efficiency by optimizing the structural configuration and (b) improve efficiency with adaptive energy harvesting circuit. These approaches usually focus on how to improve the maximum power harvesting at the resonant frequency of the harvester.

2) Most energy harvesters work as linear resonators with limited bandwidth. However, environmental excitations may not always be located at a fixed frequency. A slight shift of the working frequency from the harvester's resonance will result in significant power decrease. Therefore, rather than focusing on how to improve harvesting efficiency at the resonance, researchers are attempting to develop different schemes to broaden the operation bandwidth for energy harvesting.

3) Moreover, a real environmental vibration source may include multiple components from different directions in a 3-D or 2-D domain, or the excitation orientation may vary with different status. Thus, an adaptive energy harvester should be able to work in such condition with consistent power output. The conventional PEH which can only work with single excitation direction is inefficient for a real 3-D or 2-D

environment. It is an important concern to design an applicable energy harvester with ability to harvest energy in a 3-D or 2-D domain.

2.3.1 Efficiency Enhancement and Optimizing

2.3.1.1 Enhancement of efficiency by structural optimization

The structural configuration is a major factor that influences the efficiency of piezoelectric energy harvesting. Many works have been reported in literature, to optimize the energy harvester's structure, thus to improve the power output efficiency.

As most PEHs are designed as a cantilever beam, structural parameters are mostly related to the geometry of the cantilever and the distribution of mass. Anderson and Sexton (2006) investigated the relationship between the performance of a bimorph cantilevered harvester with a proof mass and its structural parameters. By varying the proof mass, length and width, they found that changes of the proof mass has the largest effect on the energy harvesting performance. Rather than using a uniform cantilever beam, Roundy et al. (2005) suggested that, with a trapezoidal shaped cantilever, the strain can be more evenly distributed throughout the structure compared to a rectangular beam, as shown in Figure 2.13. It was concluded that the output energy density (per volume of PZT) of a trapezoidal cantilever is improved more than twice of the rectangular beam. Xu et al. (2010) also proposed another design of a right-angle cantilever piezoelectric harvester, which was announced that can make the strain distribution more uniform and produce two times larger energy output compared to the conventional cantilever PEH under the same strain limitation.

Figure 2.13 Strain distributions for different cantilever beam configurations
(Roundy et al., 2005)

Figure 2.14 Segmented multi-modal piezoelectric energy harvester (Lee and Youn, 2011)

Besides modifying the structure of the cantilever substrate to achieve higher power output, one can also optimize the segmentation strategy of the PZT materials to improve the efficiency, especially when multiple modes are considered. For example, Lee and Youn (2011) developed an optimized topology segmented multi-modal PEH, by removing the PZT material along the inflection lines from multiple modal shapes to minimize the cancellation effect, as shown in Figure 2.14. Similar optimization work was reported by Carrara *et al.*, (2014), to design a spatially distributed

piezoelectric energy harvester on a cantilever plate, for generation of electrical energy from propagating electroacoustic waves.

2.3.1.2 Enhancement of efficiency with advanced circuit interface

Other than optimizing the structural configurations of the piezoelectric energy harvesters, many researchers from electrical engineering paid more attention to developing advanced energy harvesting circuit. Usually, a *standard energy harvesting circuit* comprises a rectifying component followed by energy storage like a rechargeable battery or directly connected with an electric load, as shown in Figure 2.15. However, standard circuit is simple but not efficient. To extract more energy from the system, many adaptive energy circuit techniques have been developed in recent years.

Figure 2.15 Standard circuit with (a) a rechargeable battery or (b) a resistive load

Impedance adaptation technique

A PEH can be regarded as a voltage source with very high internal impedance. The harvester will generate its maximum power output when the external impedance is matched with its internal impedance. Assuming the simple condition as an uncoupled model, where its optimal matching impedance is equal to $R_l^{\text{opt}} = 1/\omega C^S$. The optimal voltage output is equal to half of the open circuit voltage when the impedance is matched, and maximum power output is achieved at that time. Therefore, in order to get the maximum power output, the load impedance should be optimized to match with the internal impedance. However, for the standard circuit, the impedance of

charging a battery or capacitor usually does not match with the internal impedance of the PEH. To achieve the optimal power output, some researchers proposed using impedance adaptation circuit interface to control and tune the load impedance, by adding DC-DC converters.

Figure 2.16 Comparison of (a) the available power, (b) the power to charge battery by impedance adaptation circuit and (c) the power of directly charging the battery (Ottman et al., 2002)

Ottman et al. (2002) proposed an adaptive circuit using a buck DC-DC converter to maximize the power flow from energy harvester to rechargeable battery. An adaptive control algorithm was developed to continuously implement optimal power transfer, by controlling the voltage on the rectifier to nearby half of the open-circuit voltage. It was reported that 400% increase of the energy flow to the battery compared to the standard circuit was achieved by using such circuit, as shown in Figure 2.16. As the converter only works when the input voltage is higher than the output voltage, significant power was lost at low external excitation levels.

SCE technique

The *synchronous charge extraction* (SCE) interface circuit was first proposed by Lefeuvre *et al.* (2005), in which a flyback switch-mode DC-DC converter is used, and an additional control circuit is required to sense the voltage across the rectifier.

When the voltage reaches a maximum or minimum, the flyback converter is activated and transfers the charge to the battery or directly to the electric load. The energy on the piezoelectric element is extracted and the voltage drops to zero at these instances.

Figure 2.17 compares the waveforms for the SCE circuit and standard circuit.

Figure 2.17 Waveform of (a) standard circuit, (b) SCE circuit

With this circuit, Lefeuvre et al. (2005) claimed that they achieved nearly 400% power output compared to the standard circuit, which means saving 70~75% of piezoelectric material to reach the same maximum power. However, Tang and Yang (2011) found out that such conclusion is only valid when the electromechanical coupling is very weak. When the coupling is medium or strong, the output from SCE circuit would be similar or even lower compared to the standard circuit, at the resonant frequency. But the SCE circuit can still improve the performance for the off-resonance frequency range, which is useful to broaden the bandwidth of the energy harvesting system.

SSHI technique

Another effective circuit for improving the performance of energy harvesting was developed by Guyomar et al. (2005), named as *synchronized switch harvesting on inductor* (SSHI). As reported in literature, by using this technique, the harvested power can be increased (compared to the *standard energy harvesting circuit*) by as

much as 250% to 900% depending on the electromechanical coupling of the system (Badel et al., 2005; Shu et al., 2007; Lefeuvre et al., 2010).

Figure 2.18 (a) Parallel SSHI technique and (b) Series SSHI technique (Liang and Liao, 2012)

There are two different configurations for SSHI technique, namely Parallel-SSHI and Series SSHI depending on whether the switch is placed in parallel or series with the piezoelectric elements, as shown in Figure 2.18. During the operation of this circuit, the switch remains open, except when the maxima or minima of voltage is reached. At the instant when this occurs, the switch is closed for an extremely short time until the voltage on the piezoelectric elements is reversed. Typical waveform for the parallel SSHI and the series SSHI are shown in Figure 2.19, note that, the voltage inversion is not perfect because a part of the energy stored on the piezoelectric element's capacitance is lost in the switching network.

Figure 2.19 Typical waveforms of two SSHI schemes (a) parallel-SSHI (b) series-SSHI (c) Inversion of voltage at the instant of extreme displacements technique (Liang and Liao, 2012)

Although all the above techniques were announced to be efficient for harvesting more energy, the researchers evaluated the performance without considering the power consumption of those adaptive components like the switches in SSHI technique. To achieve the autonomous system, a self-switching circuit should be implemented and many researchers are now focusing on this. In the report of Liang and Liao (2012), they found the self-powered SSHI can outperform the standard circuit only when the excitation is above certain level.

2.3.2 Broadband Energy Harvesting

The conventional energy harvester is usually designed as a linear resonator with a very narrow operation bandwidth around its resonant frequency. However, most environmental vibration sources are frequency-variant or randomly distributed in a wide frequency range. Hence, rather than improving the maximum power density at the resonances of the energy harvesting system, broadening the operation bandwidth is even more important. This section reviews the different techniques reported by various researchers focusing on broadening the bandwidth for energy harvesting systems, including resonance frequency tuning, frequency up-conversion, multi-modal and nonlinear techniques.

2.3.2.1 Resonant frequency tuning technique

As the environmental excitation frequency is variable in certain frequency range for different operation conditions, an energy harvester with fixed resonance cannot achieve its optimal power output if the excitation frequency is not matched with its resonance. Therefore, the energy harvester is expected to be tunable within certain frequency range. The resonance frequency tuning techniques can be divided into two different modes: active and passive (Roundy and Zhang 2005). In the active mode, continuous power input is required for tuning the resonance. While in the passive

mode, power input will only be required when the tuning is processing, and no more input power is required after the frequency is matched, until the excitation frequency varies again.

According to the basic theory of vibration mechanics, the natural frequency will change if the stiffness or seismic mass is changed. The structural stiffness can be changed by simply adding a pre-load to the structure. The pre-load can be added via different ways.

Eichhorn et al. (2008) and Hu et al. (2007) proposed a tunable energy harvesting devices by apply axial preload to alter the stiffness, as shown in Figure 2.20. However, such proposed device only worked in the active mode, where the devices should be adjusted manually.

Figure 2.20 (a) Generator with arms (upper and bottom sides) and (b) schematic of the entire setup (Eichhorn et al. 2008)

Piezoelectric materials can work in either direct or reverse conversion. Thus, the structural stiffness can also be adjusted by the piezoelectric materials driven by electric input. Roundy and Zhang (2005) presented a piezoelectric generator with an active tuning actuator, as shown in Figure 21. The electrode of this bimorph beam was etched to a harvesting and a tuning part. However, according to their study, they

suggested that an active tuning scheme *never* resulted in a net increase for power output as an external power supply is required for maintaining the resonance tuning.

Figure 2.21 An active tuning piezoelectric generator (the surface electrode is divided into a harvesting and a tuning part, Roundy and Zhang, 2005)

Magnetic force is another option to apply a pre-load to the structure and to tune the resonance. Challa et al. (2008) developed a tunable cantilever harvester in which two magnets were fixed at the free end of a beam, while the other two magnets were fixed at the top and bottom of the enclosure of the device. The stiffness of the harvester can be tuned by adjusting the distance of the two groups of magnets. However, when the magnetic force is strong enough, nonlinear vibration will occur.

Figure 2.22 Resonance tunable harvester using magnets (Challa et al., 2008)

Instead of changing the stiffness of an energy harvesting system, another option for tuning the resonant frequency is changing of the seismic mass. As it seems impossible to change the mass value if the system is already installed, adjusting the gravity center of the seismic mass is more reasonable. Wu et al. (2008) proposed a tunable cantilever energy harvester by using a movable tip mass. Gu and Livermore (2012) proposed a self-tuning energy harvester used in rotation application such as tire pressure monitoring system, in which the tip mass location is passively tuned with centrifugal force. The schematic of that device is shown in Figure 2.23.

Figure 2.23 Self-tuning harvester in rotation application (Gu and Livermore, 2012)

2.3.2.2 Frequency up-conversion technique

Different from the resonant frequency tuning technique which tunes the harvester's resonance to match with the environment vibration frequency, the frequency up-conversion technique focuses on converting the environmental vibration source to the resonant frequency of the harvester. Usually, these techniques work in applications to pump the ultra-low frequency excitation (like human motion or ocean wave) into higher resonant frequency of the harvester.

Rastegar et al. (2006) proposed a design of energy harvesting system with frequency up-conversion as shown in Figure 2.24. When in operation, the base excitation will

lead the teeth to impact the piezoelectric energy harvesters, making them vibrating at their own natural frequencies. Thus, the low frequency base excitation can be transferred to the high frequency vibration of the harvesters. To avoid energy loss when the teeth impact the beams, magnets can be added to such system. Such kind of frequency up-conversion technique can be further developed for harvesting energy from human motion, low speed machinery or buoy-type ocean wave energy harvester (Rastegar and Murray 2009).

Figure 2.24 Schematic of the two-stage vibration energy harvesting design for frequency up-conversion (Rastegar et al., 2006)

2.3.2.3 Multi-modal energy harvesting

A SDOF harvesting system only present narrow operation bandwidth around its single frequency response peak. It is more advantageous if multiple response peaks can be utilized. Multi-modal energy harvesting is a promising solution to broaden the bandwidth. There are two schemes to achieve multi-modal energy harvesting, one is using multiple harvester components in an array, and another is utilizing the higher order vibration modes based on one single harvester.

Shahruz (2006) and Ferrari et al. (2008) proposed similar systems comprising an array of cantilevers with various lengths and tip masses. These cantilevers with different working frequencies can be carefully designed to cover certain range of

frequency to achieve a broader bandwidth, as shown in Figure 2.25. However, when the excitation frequency is matched to the resonance of one cantilever, the other cantilevers will not work efficiently as the excitation frequency is not at their resonances. Such design significantly increases the volume and weight of the system, which not only sacrifices the power density but also limits its applicability.

Figure 2.25 Schematic of the array of PEH cantilevers and its frequency response (Shahruz, 2006)

Rather than using the cantilever array configuration, some researchers have developed multiple-degree-of-freedom (MDOF) energy harvesters based on one single cantilever beam. Tadesse et al. (2009) presented a design of multi-modal energy harvesting beam employing both electromagnetic and piezoelectric transduction mechanisms, each of which was efficient for a specific mode. Ou et al. (2010) presented a 2-DOF system by using a two-mass cantilever beam. Although two useful modes were obtained, they were separated quite far away (at 25 Hz and 150 Hz). Arafa et al. (2011) proposed a 2-DOF harvester in which a dynamic magnifier was adopted. It can magnify the power output from the harvester, and also present multi-modal response. However, this design could not achieve two close working frequencies unless an impractical huge magnifier was employed. Erturk et al. (2009b) developed a PEH with L-shaped beam configuration where the second natural frequency approximately doubled the first. Generally, the purpose for designing a broadband multi-modal energy harvester is to achieve several close

resonances with significant magnitudes for effective energy conversion, but most of the previous designs can only achieve resonances far away from each other with second peak much smaller than the first. Kim et al. (2011) developed a 2-DOF system that could achieve two close resonances by using both translational and rotational degrees of freedom of a vibration body, but this design required an additional vibration body to be attached to two cantilevers, which increased the volume as well as the complexity of the harvester as shown in Figure 2.26. In later chapter, a novel 2-DOF PEH which is much space efficient will be proposed and studied to achieve broadband multi-modal energy harvesting with close resonance.

Figure 2.26 (a)Simplified mechanical model of proposed device (b)schematic view of device. (Kim et al. 2011)

2.3.2.4 Nonlinear techniques

As mentioned in section 2.3.2.1, one option for resonant frequency tuning is adding magnetic field to adjust the structural stiffness of the harvesters. However, other than the linear stiffness change caused by the magnetic interaction, nonlinear stiffness is also introduced into the system too, especially when the magnetic field is strong. The nonlinear behavior is also beneficial for broadening the bandwidth for energy harvesting.

According to the different equilibrium conditions, nonlinear energy harvesters are usually divided into mono-stable and bi-stable configurations. Multi-stable nonlinear

energy harvesting also appears in literature in recent years (Avvari et al., 2013, Trigona et al., 2013). A typical potential energy function of the seismic mass is as presented by Cottone et al. (2009) in Figure 2.27. As shown in Figure 2.27, a mono-stable configuration (only one equilibrium position at the center) is achieved when the distance of two magnets is larger ($>10\text{mm}$). As the distance of the two magnets tuned closer, the system then changed into a bi-stable configuration, two new equilibrium positions were located besides the central position while the former equilibrium point became a potential barrier.

Figure 2.27 Potential function $U(x)$ for inverted pendulum with different distance of magnets (Cottone et al., 2009)

Mono-stable nonlinear configuration for energy harvesting

In the mono-stable configuration of nonlinear energy harvesting, it can be further divided into two different configurations according to the sign of nonlinear stiffness. They are hardening configuration and softening configuration, depending on whether the stiffness will increase or decrease. Their typical frequency responses are shown in Figure 2.28. The frequency response curves will be bent to the right direction (higher frequency range) or left direction (lower frequency range) respectively. This behavior will be more obvious when the excitation level is higher (higher response curves in Figure 2.28). Their frequency responses are almost similar to the linear vibration, if the excitation level is low (lower response curves in Figure 2.28).

Figure 2.28 Response amplitudes of output voltage for softening and hardening configuration with different excitation levels (Stanton et al., 2009)

As shown in Figure 2.28, within certain frequency range, there are two stable oscillation states (solid line) at one frequency point, as well as one unstable state (dot line). The two stable oscillation states co-exist at the same frequency point with same excitation level, depending on the different initial conditions which lead to different attractors. If the harvester is vibrating in the higher energy attractor, its bandwidth is greatly improved. But if the harvester is located in the lower energy attractor, its bandwidth is reduced significantly. An initial condition with large vibration motion is helpful to capture the higher energy attractor, however, the higher amplitude vibration cannot be guaranteed in a practical application. Besides, even if a harvester is located in the lower energy attractor, a trigger such as a sudden impact can make it jump to the higher energy attractor. Thus, a promising solution is to design certain trigger mechanism, to provide an impact and excite the harvester into the higher energy attractor when it is located in the lower energy attractor. For example, Masuda *et al.*, (2013) developed a nonlinear energy harvesting system, in which a switch circuit is used to give the system a self-excitation capability.

Bi-stable nonlinear configuration for energy harvesting

If the harvester is tuned into a bi-stable configuration, various types of oscillation statuses will occur, such as, chaotic oscillation, large-amplitude periodic oscillation and large-amplitude quasi-periodic oscillation (Moehlis *et al.*, 2009).

Erturk et al. (2009a) developed a broadband bi-stable energy harvester consisted of a ferromagnetic cantilever beam attached with two piezoelectric layers and two magnets near the free end of the beam, as shown in Figure 2.29. Under harmonic base excitation, the response of this system can be chaotic motion or large-amplitude periodic oscillation (limited cycle oscillation). The large-amplitude periodic oscillation could also be obtained under small base excitation level by simply applying a perturbation or an initial velocity condition to the beam. Therefore, large-amplitude periodic oscillation can be obtained at off-resonance frequency range, which means the bandwidth can be broadened as compared to the linear counterpart.

Figure 2.29 Bi-stable energy harvester (Erturk et al., 2009a).

Cottone et al. (2009) proposed the inverted piezoelectric pendulum with two polar opposing magnets. They studied the response of bi-stable configuration with two magnets close enough; and under random excitation, the oscillation of the pendulum was confined in one potential well or swung from one to the other. The maximum power output reached 4-6 times larger than the linear one. This is because: for closer distance of two magnets, two potential wells were separated farther away making the oscillation amplitude increased significantly. However, increase of the height for potential barrier with closer magnets will make the jump probability decreases as well.

Other than achieving nonlinear vibration using magnetic interaction which requires an additional magnetic field, bi-stable structures (i.e. buckled beam) provide an alternative solution for bi-stable nonlinear energy harvesting. Arrieta et al., (2010) investigated the dynamics of a piezoelectric bi-stable plate for nonlinear broadband energy harvesting. Cottone et al., (2012) developed a bi-stable piezoelectric energy harvester using a buckled beam, which can produce up to an order of magnitude more power than unbuckled one. Such bi-stable structures are more desirable in MEMS applications as no additional magnets are required.

Researchers have verified that bi-stable harvesters show enhanced performance for improving the power output as well as broadening the operation bandwidth, as long as the seismic mass is able to vibrate between the two stable positions. The vibration motion jumping cross the barrier between the two potential wells is termed as “*snap-through*” mechanism. This is one important concern when designing a bi-stable energy harvester because the bi-stable energy harvester can outperform only if the snap-through can be achieved and global oscillation is guaranteed. Ramlan *et al.* (2010) studied the behavior of the *snap-through*, and concluded that this mechanism could provide much better performance than the linear mechanism. Normally, higher excitation will help the system to snap-through. Thus the recent researches of bi-stable energy harvesting are mostly based on high excitation levels. To design an effective mechanism which can easier trigger snap-through will improve the performance of a bi-stable energy harvester, as well as expand its application in the lower excitation environment.

2.3.3 Multi-directional Energy Harvesting

As reviewed in the previous section, most reported energy harvesters focus on harvesting more energy from wider frequency range but in only one single excitation direction (normally perpendicular to the cantilever). However, a practical

environmental vibration source may include multiple components from different directions, as concluded in a survey done by Reilly *et al.*, (2009). For example, a Statasys 3-D printer produces three frequency response peaks at 28, 28.3 and 44.1 Hz along three perpendicular directions, and a washing machine undergoes resonance at 85.0 Hz in two perpendicular directions. Hence, it is important to design a vibration energy harvester that can harvest vibration energy in three-dimension (3-D) domain or two-dimension 2-D domain,

So far, only a few attempts are reported in the literature, regarding multi-directional energy harvesting. Bartsch *et al.*, (2009) and Liu *et al.*, (2012) developed similar electromagnetic energy harvesting device using a disk shape mass connecting with concentric circular springs which can work in the 3-D domain (Figure 2.30). Zhu *et al* (2011) developed a 2-D ultrasonic electrostatic harvesting device, with a central seismic mass suspended by 16 small beams attached to four corner anchors.

Figure 2.30 Three-dimensional electromagnetic energy harvester (Liu *et al.*, 2012).

Yang *et al.*, (2014) studied a 2-D vibration energy harvester with magnetostrictive transducers, supported by a cantilever rod with circular cross-section instead of rectangular cantilever beam, as shown in Figure 2.31.

Figure 2.31 Two-dimensional harvester with rod cantilever (Yang *et al.*, 2014).

These designs employed similar scheme in which a seismic mass is connected with support springs such that multi-directional displacements are achieved in 2-D or 3-D space. Such a scheme is suitable for the conversion mechanisms such as electromagnetic conversion, as only displacement is concerned. However, in piezoelectric energy harvesting, the induced strains in piezoelectric layers are the essential concern, rather than the tip displacement. Rectangular cross-sectioned cantilevers which can develop high strain at its top or bottom layer are employed in most piezoelectric energy harvesting systems.

A work of bi-axial bi-stable 2-D PEH is reported by Andò *et al* (2012), in which two separate conventional cantilever PEHs in different directions were coupled with magnetic interaction. 2-D energy harvesting was achieved as the magnetic force between the two harvesters could excite each other when the orientation of vibration changed in the 2-D domain. Su and Zu (2014) investigated a similar system with

beam-spring and beam-beam piezoelectric energy harvesters to working in two orthogonal directions with the magnetic coupling mechanism. They also applied the similar system for a tri-directional vibration application (Su and Zu, 2013). In these designs, permanent magnets were required in this design, which led to certain complexity in the system.

Rather than using magnets, Xu and Tang (2015) proposed a cantilevered piezoelectric energy harvester attached with a pendulum at its free end (as shown in Figure 2.32), that the pendulum's large amplitude sway motion induce resonance of beam bending motion when the excitation is from other two directions.

Figure 2.32 Tri-directional cantilever-pendulum harvester (Xu and Tang 2015)

2.4 Chapter Summary

This chapter reviews the basic principle of energy harvesting technologies as well as state-of-the-art techniques developed by other researchers. Among various energy harvesting approaches, vibration energy harvesting using piezoelectric material is given more attention. The whole system for piezoelectric energy harvesting is briefly introduced in this chapter. To evaluate the performance of energy harvesting systems, different modeling methods are reviewed. As the basic principles for piezoelectric energy harvesting are already well investigated in the past years, enhancing the performance is now the focus of current research. The enhancement can be achieved

from different aspects: optimization to achieve higher power density (structural or electrical), broadening bandwidth for practical operation, and multiple-directional energy harvesting for adaptive application in real environment. Various approaches for enhancing the performance of piezoelectric energy harvesting are also reviewed in this chapter.

CHAPTER 3 A COMPACT 2-DOF PIEZOELECTRIC ENERGY HARVESTER WITH CUT-OUT BEAM

3.1 Introduction

A conventional vibration piezoelectric energy harvester is usually designed as a SDOF system with cantilever configuration, which only works when the excitation frequency is near its resonance. Such kind of harvesting system is insufficient for real application in which the energy source may be frequency variant or random distributed. Thus, a critical issue for the vibration energy harvesting research is to broaden the operation bandwidth of the harvesters.

As reviewed in Section 2.3.2, broadband energy harvesting techniques are usually divided as: resonant frequency tuning techniques, frequency conversion techniques, multi-modal techniques and nonlinear techniques. Among which, the multi-modal energy harvesting is easy to implement, various research works have been proposed by other researchers, such as using cantilever array or exploiting high order vibration modes for one cantilever beam or vibration body. However, each of them has its own advantages and limitations. For piezoelectric energy harvester array configuration, the main drawback is the increase of volume and mass which scarifies the efficiency for bandwidth. For most prototypes of multi-modal harvesting by exploiting higher order vibration modes, their higher order resonant frequencies are usually located far away from its first one, but only present much lower outputs compared to the first resonant response. Which means, such harvester is not really broadband, as the higher vibration modes contribute little. One device proposed by Kim et al. (2011) has achieved two close resonance peaks. However, their device required an additional vibration body to be attached to two cantilevers, which increased the volume as well as the system complexity.

Thus, to design an applicable multi-modal piezoelectric energy harvester, three aspects should be considered properly: (1) the multiple vibration response peaks are close enough to each other, (2) every vibration mode can provide significant power output, (3) the improvement of bandwidth is achieved with none or only slight increase of the volume of the system.

In this chapter, a novel compact 2-DOF PEH is developed towards these three challenges mentioned above. This novel 2-DOF PEH comprises one main cantilever and an inner secondary cantilever. It can be easily fabricated from a conventional SDOF PEH by cutting out the inner beam inside and attaching an additional proof mass on that. This configuration is referred to as “*cut-out*” beam hereafter. By using this design, without increase of volume, two close resonant peaks with significant magnitudes can be obtained, thus wider bandwidth is achieved.

3.2 Comparison of 2-DOF Cantilever PEHs

Conventionally, a 2-DOF cantilever PEH comprises a continuous beam and two proof masses, as shown in Figure 3.1a. Although such design produces two resonances, the second resonant frequency is about 5 times larger than the first one, assuming L_1 is equal to L_2 and the weights of two masses are the same, as shown in Figure 3.1b. Even if the two resonant frequencies can be tuned by adjusting the length and weight of tip masses, they are not able to be tuned very close to each other, unless extraordinary mass or length ratio.

Figure 3.1 (a) A conventional 2-DOF cantilever PEH (b) Typical frequency response for this 2-DOF DOF cantilever PEH

Figure 3.2 Comparison of (a) SDOF cantilever, (b) conventional continuous cantilever, (c) equivalent continuous cantilever, (d) simplified cut-out cantilever and (e) actual cut-out cantilever studied in experiment

The fundamental difference between this proposed PEH and the conventional 2-DOF PEH is that the secondary beam (L_2) is cut inside the main beam (L_1) rather than extended outwards from the main beam. This geometric discrepancy results in major difference in the stiffness matrix of the proposed PEH and conventional 2-DOF PEH. To illustrate this point, the cut-out cantilever beam is simplified to compare with the continuous cantilever beam model (other's design), as shown in Figure 3.2.

For simplicity, the primary cantilever beam (length L_1) in the cut-out configuration (Figure 3.2d) is assumed to have the same elastic modulus, thickness and overall width as the secondary beam (length L_2). Thus, the flexural rigidity EI is uniform throughout. This assumption also applies to the conventional continuous 2-DOF configuration in Figure 3.2c. Although there is slight difference between the simplified model and the actual model in the experiment due to the different width at the root of main beam, it can be neglected as the simplified model is only used to illustrate the difference of natural frequencies between cut-out configuration and continuous configuration. Both configurations can be modeled as the lumped parameter system by neglecting the distributed mass of the cantilever beam. The mass matrices are the same for both configuration

$$[M] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = M_1 \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix} \quad (3.1)$$

The difference in stiffness matrices is the key that two configurations generate different frequency responses. The stiffness matrix for the cut-out beam configuration is

$$[K]_a = \frac{6EI}{(3\beta^2 + 4\beta^3)L_1^3} \begin{bmatrix} 2 - 6\beta + 6\beta^2 + 2\beta^3 & 3\beta - 2 \\ 3\beta - 2 & 2 \end{bmatrix} \quad (3.2)$$

While the stiffness matrix of the conventional continuous beam configuration is

$$[K]_b = \frac{6EI}{(3\beta^2 + 4\beta^3)L_1^3} \begin{bmatrix} 2(1 + \beta)^3 & -3\beta - 2 \\ -3\beta - 2 & 2 \end{bmatrix} \quad (3.3)$$

where the non-dimensional parameter α denotes the proof mass ratio M_2/M_1 , and β denotes the ratio L_2/L_1 . Solving the eigenvalue problem of the two configurations, it gives out two roots of ω , where ω is the natural frequency. The non-dimensional difference of the two roots of the eigenvalue problem can be given as,

$$\frac{\Delta(\omega^2)^a}{\omega_s^2} = \frac{4\sqrt{[\alpha(1-3\beta+3\beta^2+\beta^3)+1]^2 - \alpha(3\beta^2+4\beta^3)}}{\alpha(3\beta^2+4\beta^3)} \quad (3.4)$$

$$\frac{\Delta(\omega^2)^b}{\omega_s^2} = \frac{4\sqrt{[\alpha(1+3\beta+3\beta^2+\beta^3)+1]^2 - \alpha(3\beta^2+4\beta^3)}}{\alpha(3\beta^2+4\beta^3)} \quad (3.5)$$

where ω_s denotes the natural frequency of the SDOF cantilever beam with length L_1 and proof mass M_1 . Note that the only difference in the above equations is that the term “ 3β ” has opposite sign, however this small discrepancy is the key for the differences of their resonances and frequency responses. For both configurations, when α approaches zero, two resonant frequencies will approach each other. But this means that the mass for the secondary beam decreases to zero, making the system degrade to a SDOF system, which is of no interest. Other than that, by taking derivative of Equation (3.4), it is found that, the cut-out cantilever beam can also achieve two equal resonant frequencies when $\beta \rightarrow 2/3$ and $\alpha \rightarrow 27/17$. However, from Equation. (3.5) of the continuous beam configuration, it is not possible to obtain two close resonances, with any non-zero α value. This unique property of cut-out cantilever configuration provides a practical parametric option to easily implement a 2-DOF PEH with two close resonances.

3.3 Experimental Study

Based on the cut-out cantilever concept discussed above, the author devised a 2-DOF cut-out PEH, as well as a conventional SDOF PEH for comparison. Experiment is performed to compare the two harvesters and to show the advantage of this novel design. As stated in Section 3.2, a simplified model is used to illustrate the concept of 2-DOF cut-out beam and that simplified model neglects the non-uniform stiffness

and the distributed mass of the beam. Thus it cannot be directly used as a guide to design the parameter for the 2-DOF cut-out configuration. A preliminary FEA simulation for determining the parameters and the natural frequencies of the 2-DOF beam is carried out before the experiment. After the experiment, more works of the FEA simulation are conducted to validate the experimental results, which will be discussed in Section 3.4.

3.3.1 Experiment Setup

Figure 3.3 shows the fabricated prototypes installed on a vertical seismic shaker. The detailed dimensions of the two harvesters are shown in Figure 3.4.

Figure 3.3 Conventional SDOF and proposed 2-DOF cut-out PEHs installed on seismic shaker

The SDOF cantilever beam and the 2-DOF cut-out cantilever beam are both made from pieces of aluminum plates (110mm*40mm*0.6mm). Specially, the cut-out 2-DOF cantilever beam is fabricated by cutting inside of the SDOF one. Pieces of small

steel plates are screwed at the free end of the beams in the experiment, such that the weight of the proof masses can be adjusted conveniently. Besides, Macro-Fiber-Composite (MFC, model: M-2814-P2, Smart Material Corp.) with d_{31} piezoelectric effect are used for the vibration-to-electricity transduction. Two pieces of MFC sheets are bonded at the root of the main beam, while another one piece bonded at the root of the secondary beam. For comparison, the conventional SDOF harvester also has 2 pieces of MFC sheets at its beam root.

Figure 3.4 Geometry of conventional SDOF and proposed 2-DOF PEHs, (All dimensions in mm)

The schematic of the whole experiment setup is shown in Figure 3.5. The harmonic excitation source is generated by the function generator, adjusted by the power amplifier and finally fed to the seismic shaker. In the experiment, the excitation frequency is manually swept from 10 Hz to 30 Hz. During this sweeping procedure, the excitation acceleration is monitored by an acceleration data logger as feedback loop and always controlled at same level of 1m/s^2 . The voltage outputs generated by the MFC sheets are logged by the digital multimeter.

Figure 3.5 Schematic of experiment setup

3.3.2 Open Circuit Voltage Response

The open circuit voltage responses are recorded from both the main beam and the secondary beam. As explained in Equations 3.1 to 3.5, the harvester's response pattern will be dominated by the length ratio and mass ratio. It is not necessary to change both the outer mass and inner mass in the experiment study. Therefore, the weight of the inner mass M_2 is varied in the experiment while the outer mass M_1 keeping unchanged. The mass values are determined by trail tests in the experimental study. In the experiment, various mass values for both M_1 and M_2 were tested, for better presentation of the harvester's response pattern, a proper value was chosen as $M_1=7.2$ g. To obtain the open circuit voltage response, it requires an extremely high impedance value for the measuring equipment. However, the impedance value of the measuring equipment (NI9221) used in the experiment is only 1 M Ω . As a result, the measured open circuit voltage response is slightly lower than that in "ideal open circuit" condition. Thus, "measured open circuit voltage" response is used in following sections to present the experiment results.

As shown in Figure 3.6, different frequency responses are obtained when M_2 varies from 8.8 grams to 16.8 grams while keeping M_1 at 7.2 grams. When $M_2=8.8$ grams, the two resonant peaks obtained are at 17.5 Hz and 21.6 Hz. The voltage output from the main beam is higher at the first resonance than that at the second while the voltage output from the secondary beam voltage is just on the contrary. When M_2 increased to 11.2 grams, the two resonant peaks are quite close to each other (17.4 Hz and 19.6 Hz), and the amplitudes of the two peaks of main beam voltage output are almost equal. Although the amplitudes are slightly smaller than the first resonance peak in Figure 3.6(a), the two resonant peaks are much closer, which will benefit energy harvesting from a given continuous working frequency range. However, in this case, the secondary beam does not provide two equal peaks in voltage response.

Figure 3.6 Measured open circuit voltage output with different second mass when $M_1=7.2$ grams. (a) $M_2=8.8$ grams, (b) $M_2=11.2$ grams, (c) $M_2=14.2$ grams and (d) $M_2=16.8$ grams

When $M_2=14.2$ grams, the two resonant frequencies are 16.8 Hz and 18.0 Hz, and the two equal peaks appear in the voltage response of the secondary beam while the two peaks in the response from main beam is not equal. As M_2 further increase, different from Figure 3.6(a), a reverse trend of voltage response is observed in Figure 3.6(d). Meanwhile, the open circuit voltage response of SDOF PEH for different tip mass values are also obtained in the experiment, as shown in Figure 3.7.

Figure 3.7 Measured open circuit voltage output for SDOF PEH

Figure 3.8 Comparison of open circuit voltage responses

Figure 3.8 compares the open circuit voltage responses from the main beam of the cut-out 2-DOF PEH and the conventional SDOF PEH. The tip masses on the main beam of the cut-out 2-DOF cantilever and on the SDOF beam are both 7.2 grams. The tip mass on the secondary beam of the cut-out cantilever is $M_2=11.2$ grams. It is obvious that the 2-DOF configuration has two close peaks with same magnitude. The amplitudes of these peaks are almost the same as that of the SDOF configuration (about 15 V). But the cut-out 2-DOF configuration has significantly wider bandwidth than the conventional SDOF PEH. As shown in Figure 3.8, the bandwidth in the open circuit voltage spectrum at voltage level of 3 V for the cut-out 2-DOF PEH is about 3.0 Hz (by adding up the two segments near the two resonances), which is much more advantageous over the 2.1 Hz of the SDOF PEH.

Other than broader bandwidth achieved by the cut-out 2-DOF configuration, the proposed cut-out design can also fully utilize the cantilever beam for harvesting energy, which means such 2-DOF PEH is more compact with higher efficiency. Conventionally, the area of the secondary beam is not used or used inefficiently because of the low voltage output (due to low strain level) in the SDOF cantilever beam configuration. But in this cut-out configuration, by adjusting the tip masses, the response level of the secondary beam can also be tuned to be comparable with that of the main beam, as shown in Figure 3.6. Thus the secondary beam can also have significant contribution to energy harvesting. As refer to the three points listed in previous section, this prototype is more applicable for broadband energy harvesting with its two resonant response peaks close enough with each other and both provide significant output, with no increase of the volume, and it is more compact and efficient.

It is noted that the SDOF PEH has lower mass values as compared to the 2-DOF PEH, in Figure 3.6. By attaching a heavier mass to the harvester, its peak amplitude will definitely increase, and the resonant frequency will change as well. But SDOF harvester only can produce one response peak, which limits its application to a practical environment where vibration energy distributed within a wide frequency range. Thus, broadband piezoelectric energy harvester with multi-modal technique is required. The purpose for proposing such novel 2-DOF in this work is to achieve wider operation bandwidth with two response peaks can be designed close to enough to each other. Moreover, to design a piezoelectric energy harvester, the size of the device is more critical than the weight, especially for MEMS application. This proposed 2-DOF design is more efficient for the using of the space as the secondary beam could also contribute significantly while a conventional SDOF piezoelectric energy harvester waste the space due to low strain distribution near the free end of the cantilever

3.3.3 Power Output Response

Other than the open circuit voltage response, power output response is also concerned to evaluate the performance of a piezoelectric energy harvester. Usually, a resistor is connected to the harvester served as an electrical load to evaluate its power output performance. To obtain the maximum power output from a harvester, the optimal resistor value should be determined according to impedance matching technique. In the experiment, a variable resistor ranging from $1\text{k}\Omega$ to $999\text{k}\Omega$ is connected to the SDOF and the 2-DOF cut-out PEHs respectively, to study their performance with different resistances. The exact optimal resistor values actually vary slightly at each frequency point. However, the peak values around the resonance frequencies are the most interest for the research of power output from a piezoelectric energy harvester. Thus, for simplicity, rather than finding out every optimal resistor value at each frequency, it is focused on the optimal resistor at the resonances because the

responses at off-resonance frequencies are much lower. An approximate value of optimal resistor should be chosen for further study. Figure 3.9 shows the frequency response of the power output with different resistor values for the main beam of the 2-DOF cut-out PEH when $M_1=7.2$ grams and $M_2=8.8$ grams. It can be observed that when the resistor value is around 120 k Ω , the PEH had a maximum response for power output, for the both two peaks, as shown in Figure 3.9

Figure 3.9 Frequency response of the power output for the main beam of the 2-DOF cut-out PEH when $M_1=7.2$ grams and $M_2=8.8$ grams.

Figure 3.10 Power output versus resistor value for the main beam of the 2-DOF cut-out PEH when $M_1=7.2$ grams and $M_2=8.8$ grams at (a) first resonant frequency of 17.4 Hz (b) second resonant frequency of 19.6 Hz

Figure 3.10 illustrates more clearly the power output from the main beam of the 2-DOF PEH versus the resistor value at 17.4 Hz and 19.6 Hz (two resonant frequencies obtained in the open circuit condition). It is noted that the optimal resistor value is located in the range from about 120 to 160k Ω for both resonances. In such a range, the maximum power output does not vary much. Therefore, by choosing a resistor value in the range, the frequency response for optimal power output can be obtained. Although this value is not the exact value for optimization at each frequency, the error by using this is quite small especially when the frequency range is quite near the resonance.

More results of the optimal power and its corresponding optimal resistor value for both the main beam and the secondary beam with different configuration of the 2-DOF cut-out PEH as well as the SDOF PEH are given in Figure 3.11. The power output responses versus resistor values for these configurations with different proof masses are studied at their resonant frequencies (obtained from the open circuit condition). From these results, even with different configurations, the optimal resistor value for the main beam of the 2-DOF cut-out PEH all located in the similar range, thus the value $R_1=130\text{ k}\Omega$ is chosen for later use. For the secondary beam, the optimal resistor value of $R_2=250\text{ k}\Omega$ is chosen. For the SDOF PEH, it is almost the same as the main beam of the 2-DOF PEH, $R=130\text{ k}\Omega$. By using these optimal resistor values, the maximum power output from these harvesters connected with simple resistor is obtained as, $P=U^2/R_{\text{opt}}$. The results of the maximum power output for different configurations are shown in Figure 3.12.

Figure 3.11 Optimal power output of (a) main beam and (b) secondary beam of cut-out PEH at first resonance; (c) main beam and (d) secondary beam of cut-out PEH at second resonance; and (e) SDOF PEH at its resonance

Figure 3.12 Experiment results of power output for 2-DOF cut-out PEH of $R_1=130$ k Ω and $R_2=250$ k Ω when $M_1=7.2$ grams and (a) $M_2=8.8$ grams, (b) $M_2=11.2$ grams, (c) $M_2=14.2$ grams, (d) $M_2=16.8$ grams, and for (e) SDOF PEH ($R=130$ k Ω)

Figure 3.12 shows that both the MFC transducers on the main and secondary beams can generate significant power output when tip masses are properly selected. It is

noted that the best overall power output (power outputs from both the main and secondary beams) may not occur when the two resonances are very close (Figure 3.12(c)). In such case, only one peak in the response of the main beam can significantly contribute to energy harvesting and the contribution from the secondary beam is negligible, which cannot be regarded as an efficient design in terms of bandwidth. Instead, when $M_2=8.8$ grams or 16.8 grams, although the two resonances are separated slightly away, both the main and secondary beams have one significant peak. Thus the cut-out harvester has significant overall power output at both resonant frequencies and broadband energy harvesting is achieved. To achieve this, the detailed parameters of the 2-DOF cut-out configuration should be carefully designed. Compared to the SDOF PEH with $M=7.2$ grams, the peaks of the cut-out PEH can have larger magnitudes (e.g., Figures 3.12(c) and 3.12(d)). For an increased tip mass $M=13.2$ grams for SDOF harvester, the peak magnitude of the SDOF harvester is comparable with that of the cut-out harvester. However, the cut-out harvester is still advantageous in terms of bandwidth.

In conclusion, this proposed cut-out 2-DOF PEH can achieve not only broader bandwidth, but also greater power outputs as compared to the SDOF PEH by fully utilizing the cantilever beam.

3.4 Mathematical Modelling for The 2-DOF PEH

As reviewed in Chapter 2, various mathematical models were developed to model the piezoelectric energy harvesters. A coupled distributed parameter modelling method based on the Euler-Bernoulli beam assumption was developed by Erturk and Inman (2008b) for a uniform cantilever configuration, and was expanded to model an L-shape broadband piezoelectric energy harvester (Erturk et al. 2009b). In this section, such method is then further expanded to develop a distributed parameter model for the proposed 2-DOF PEH.

3.4.1 Distributed parameter model and modal analysis

In the experiment study, MFC transducers are attached to the cantilever substrate, thus the cross-sections are not uniform along the whole structure. To build the distributed parameter model, the structure is divided into five uniform segments equipped with local coordinates, as shown in Figure 3-13. The two tip masses are simplified as two point masses located at the end of segments 3 and 5.

Figure 3.13 (a) Segments of the cut-out 2-DOF PEH , (b) The local coordinate system for each segment

The equation of motion for undamped free vibrations of each segment in its lateral direction can be written as

$$\begin{aligned} \frac{\partial^2 M_k(x_k, t)}{\partial x_k^2} + m_k \frac{\partial^2 \omega_k(x_k, t)}{\partial t^2} + \delta_{3k} M t_1 g \frac{\partial^2 M_k(x_k, t)}{\partial x_k^2} \\ + \delta_{5k} M t_2 g \frac{\partial^2 M_k(x_k, t)}{\partial x_k^2} = 0, \quad k = 1, 2, 3, 4, 5 \end{aligned} \quad (3.6)$$

where $M_k(x_k, t)$ is the bending moment, m_k is the distributed mass per length and $\omega_k(x_k, t)$ is the lateral displacement of the k -th segment, Mt_1 and Mt_2 are the two tip masses, g is the gravitational acceleration, and δ_{rs} is Kronecker delta (equal to unit when $r=s$ and equal to zero when $r \neq s$). For the segment without MFC attached ($k=2, 3, 5$), its bending moment is

$$M_k(x, t) = YI_k \frac{\partial^2 \omega_k(x, t)}{\partial x^2}, \quad k = 2, 3, 5 \quad (3.7)$$

While for the segment with MFC attached ($k=1, 4$), the piezoelectric coupling effect should be included, as

$$M_k(x, t) = YI_k \frac{\partial^2 \omega_k(x, t)}{\partial x^2} + \theta_i v_i(t), \quad k = 1, 4 \quad (3.8)$$

$$\theta_i = \int_{h_{pb}}^{h_{pt}} e_{31} \frac{b_p}{h_p} z dz \quad (3.9)$$

θ_i is the piezoelectric coupling term for the i -th piezoelectric transducer, YI_k is the bending stiffness of the cross-section for the composite beam, expressed as,

$$YI_k = \begin{cases} \frac{1}{12} E_s b_s h_s^3 & k = 2, 3, 5 \\ \frac{1}{3} [E_s b_s (h_{st}^3 - h_{sb}^3) + E_p b_p (h_{pt}^3 - h_{pb}^3)] & k = 1, 4 \end{cases} \quad (3.10)$$

and E is the elastic module, b is the width of the cross-section, the subscripts of 's' and 'p' refer to the substrate layer and piezoelectric material layer; h_{sb} , h_{st} , h_{pb} and h_{pt} are the positions of the bottom and top of the substrate and piezoelectric layers from the neutral axis.

The vibration response can be expressed as a convergent series of the eigenfunctions:

$$\omega_k(x, t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (3.11)$$

where $\phi_r(x)$ is the r -th mode shape function, and $\eta_r(t)$ is modal coordinate. The eigenfunctions $\phi_r(x)$ can be expressed as

$$\phi_r(x) = \begin{cases} A_1 \sin(\beta_1 x_1) + B_1 \cos(\beta_1 x_1) + C_1 \sinh(\beta_1 x_1) + D_1 \cosh(\beta_1 x_1), & 0 < x_1 < L_1 \\ A_2 \sin(\beta_2 x_2) + B_2 \cos(\beta_2 x_2) + C_2 \sinh(\beta_2 x_2) + D_2 \cosh(\beta_2 x_2), & 0 < x_2 < L_2 \\ A_3 \sin(\beta_3 x_3) + B_3 \cos(\beta_3 x_3) + C_3 \sinh(\beta_3 x_3) + D_3 \cosh(\beta_3 x_3), & 0 < x_3 < L_3 \\ A_4 \sin(\beta_4 x_4) + B_4 \cos(\beta_4 x_4) + C_4 \sinh(\beta_4 x_4) + D_4 \cosh(\beta_4 x_4), & 0 < x_4 < L_4 \\ A_5 \sin(\beta_5 x_5) + B_5 \cos(\beta_5 x_5) + C_5 \sinh(\beta_5 x_5) + D_5 \cosh(\beta_5 x_5), & 0 < x_5 < L_5 \end{cases} \quad (3.12)$$

$$\beta_k^4 = \frac{\omega_r^2 m_k}{Y I_k} \quad (3.13)$$

By applying the boundary conditions to the modal shape function, the coefficients of A_k , B_k , C_k , and D_k can be worked out. The boundary conditions and the continuity conditions are stated as follow:

At the clamped end ($x_1=0$),

$$\phi_r(x_1)|_{x_1=0} = 0, \quad (3.14a)$$

$$\left. \frac{d\phi_r(x_1)}{dx_1} \right|_{x_1=0} = 0, \quad (3.14b)$$

For the connections of the sections 1-2, 2-3 and 4-5,

$$\phi_r(x_k)|_{x_k=L_k} = \phi_r(x_{k+1})|_{x_{k+1}=0}, \quad k = 1, 2, 4 \quad (3.14c)$$

$$\left. \frac{d\phi_r(x_k)}{dx_k} \right|_{x_k=L_k} = \left. \frac{d\phi_r(x_{k+1})}{dx_{k+1}} \right|_{x_{k+1}=0}, \quad k = 1, 2, 4 \quad (3.14d)$$

$$Y I_k \left. \frac{d^2 \phi_r(x_k)}{dx_k^2} \right|_{x_k=L_k} = Y I_{k+1} \left. \frac{d^2 \phi_r(x_{k+1})}{dx_{k+1}^2} \right|_{x_{k+1}=0}, \quad k = 1, 2, 4 \quad (3.14e)$$

$$Y I_k \left. \frac{d^3 \phi_r(x_k)}{dx_k^3} \right|_{x_k=L_k} = Y I_{k+1} \left. \frac{d^3 \phi_r(x_{k+1})}{dx_{k+1}^3} \right|_{x_{k+1}=0}, \quad k = 1, 2, 4 \quad (3.14f)$$

While for the connection of the section 3-4, the local coordinate has been changed to opposite direction, thus the continuity condition should be modified as

$$\phi_r(x_3)|_{x_3=L_3} = -\phi_r(x_4)|_{x_4=0} \quad (3.14g)$$

$$\left. \frac{d\phi_r(x_3)}{dx_3} \right|_{x_3=L_3} = \left. \frac{d\phi_r(x_4)}{dx_4} \right|_{x_4=0} \quad (3.14h)$$

$$YI_3 \left. \frac{d^2\phi_r(x_3)}{dx_3^2} \right|_{x_3=L_3} = YI_4 \left. \frac{d^2\phi_r(x_4)}{dx_4^2} \right|_{x_4=0} \quad (3.14i)$$

$$YI_3 \left. \frac{d^3\phi_r(x_3)}{dx_3^3} \right|_{x_3=L_3} - \omega_r^2 M_{t1} \phi_r(x_3)|_{x_3=L_3} = -YI_4 \left. \frac{d^3\phi_r(x_4)}{dx_4^3} \right|_{x_4=0} \quad (3.14j)$$

At the free end location of the inner mass, the boundary conditions are

$$EI_5 \left. \frac{d^2\phi_r(x_5)}{dx_5^2} \right|_{x_5=L_5} = 0 \quad (3.14k)$$

$$EI_5 \left. \frac{d^3\phi_r(x_5)}{dx_5^3} \right|_{x_5=L_5} - \omega_r^2 M_{t2} \phi_r(x_5)|_{x_5=L_5} = 0 \quad (3.14l)$$

Moreover, all the eigenfunctions obtained from above equations should satisfy the following orthogonality conditions,

$$\int_0^L \phi_s(x) m \phi_r(x) dx + \phi_s(L) M_t \phi_r(L) = \delta_{sr} \quad (3.15)$$

With above equations, one can obtain the harvester's modal shapes and related resonant frequencies.

3.4.2 Coupled voltage frequency response for harmonic base excitation

In this study, the first two vibration modes are particularly interested, and two separate voltage outputs are required from two piezoelectric segments. Thus, similar to the Equation 2.11 and 2.12, the forced equation of the motion in the modal coordinate can be written as,

$$\begin{cases} \frac{d^2\eta_1(t)}{dt^2} + 2\zeta_1\omega_1 \frac{d\eta_1(t)}{dt} + \omega_1^2\eta_1(t) + \chi_{11}V_1(t) + \chi_{12}V_2(t) = -f_1u_g(t) \\ \frac{d^2\eta_2(t)}{dt^2} + 2\zeta_2\omega_2 \frac{d\eta_2(t)}{dt} + \omega_2^2\eta_2(t) + \chi_{21}V_1(t) + \chi_{22}V_2(t) = -f_2u_g(t) \\ \frac{V_1(t)}{R_1} + C_{p1} \frac{dV_1(t)}{dt} - \chi_{11} \frac{d\eta_1(t)}{dt} - \chi_{21} \frac{d\eta_2(t)}{dt} = 0 \\ \frac{V_2(t)}{R_2} + C_{p2} \frac{dV_2(t)}{dt} - \chi_{12} \frac{d\eta_1(t)}{dt} - \chi_{22} \frac{d\eta_2(t)}{dt} = 0 \end{cases} \quad (3.16)$$

$$\chi_{ij} = \int_{x_a}^{x_b} \theta_{ij} \frac{d^2\phi_i(x)}{dx^2} dx \quad (3.17)$$

$$f_i = \int_0^L m(x)\phi_i(x) dx + M_{t1}\phi_i(L_3) + M_{t2}\phi_i(L_5) \quad (3.18)$$

where ζ_1 and ζ_2 are the damping ratios of related vibration mode, $V_1(t)$ and $V_2(t)$ are the voltage responses from the piezoelectric transducers in segments 1 and 4, C_{p1} and C_{p2} are the capacitances of the two piezoelectric transducers, R_1 and R_2 are the load resistors connected to the respective piezoelectric transducer, and $u_g(t)$ is the external harmonic excitation. χ_{ij} is the electromechanical coupling coefficient for the j -th piezoelectric transducer in the i -th vibration mode.

By solving Equation 3.15, the amplitude of the voltage frequency responses at the two different piezoelectric segments can be obtained as,

$$\begin{cases} V_1 = \left| \frac{\vartheta_1 \nabla_2 - \vartheta_2 \nabla_3}{\nabla_1 \nabla_2 - \nabla_3^2} \right| \\ V_2 = \left| \frac{\vartheta_1 \nabla_3 - \vartheta_2 \nabla_1}{\nabla_1 \nabla_2 - \nabla_3^2} \right| \end{cases} \quad (3.19a)$$

$$\vartheta_1 = \frac{j\omega\chi_{11}f_1}{\omega_1^2 - \omega^2 + 2j\zeta_1\omega\omega_1} + \frac{j\omega\chi_{21}f_2}{\omega_2^2 - \omega^2 + 2j\zeta_2\omega\omega_2} \quad (3.19b)$$

$$\vartheta_2 = \frac{j\omega\chi_{12}f_1}{\omega_1^2 - \omega^2 + 2j\zeta_1\omega\omega_1} + \frac{j\omega\chi_{22}f_2}{\omega_2^2 - \omega^2 + 2j\zeta_2\omega\omega_2} \quad (3.19c)$$

$$\nabla_1 = \frac{1}{R_1} + j\omega C_{p1} + \frac{j\omega\chi_{11}^2}{\omega_1^2 - \omega^2 + 2j\zeta_1\omega\omega_1} + \frac{j\omega\chi_{21}^2}{\omega_2^2 - \omega^2 + 2j\zeta_2\omega\omega_2} \quad (3.19d)$$

$$\nabla_2 = \frac{1}{R_2} + j\omega C_{p2} + \frac{j\omega\chi_{12}^2}{\omega_1^2 - \omega^2 + 2j\zeta_1\omega\omega_1} + \frac{j\omega\chi_{22}^2}{\omega_2^2 - \omega^2 + 2j\zeta_2\omega\omega_2} \quad (3.19e)$$

$$\nabla_3 = \frac{j\omega\chi_{11}\chi_{12}}{\omega_1^2 - \omega^2 + 2j\zeta_1\omega\omega_1} + \frac{j\omega\chi_{21}\chi_{22}}{\omega_2^2 - \omega^2 + 2j\zeta_2\omega\omega_2} \quad (3.19f)$$

where j is the unit imaginary number and ω is the excitation frequency.

3.4.3 Results from the distributed parameter model

This distributed parameter model is then applied to the proposed 2-DOF PEH to validate the experiment results. By using Equations 3.12-15 the PEH's modal shapes and related resonant frequencies can be worked out. Figure 3.14 shows the example of the modal shapes for the configuration with $M_1=7.2$ grams and $M_2=8.8$ grams. The resonant frequencies for these two modes are 18.01 Hz and 21.74 Hz, which is slightly higher than the experiment results (17.5Hz and 21.6Hz). In other configurations with different tip masses, the resonant frequencies obtained from the distributed model are also very close to the experiment results.

Figure 3.14 First two vibration modal shapes for $M_1=7.2$ grams and $M_2=8.8$ grams

To obtain the open circuit voltage frequency responses for the PEH by using Equation 3.19, the two resistor values are set extremely large (i.e. $R_1 = R_2 = 10^9 \Omega$). And the damping ratios (ζ_1 and ζ_2) are set as 0.7%, which is obtained from the experimental attenuation test. The capacitance for each piece of MFC is $25.7 \mu\text{F}$, thus $C_{p1}=51.4 \mu\text{F}$

and $C_{p2}=25.7 \mu\text{F}$. For the different configurations, their open circuit voltage frequency responses are obtained as in Figure 3.15.

Figure 3.15 Open circuit voltage response from the distributed parameter model, with $M_1=7.2$ grams while (a) $M_2=8.8$ grams, (b) $M_2=11.2$ grams, (c) $M_2=14.2$ grams and (d) $M_2=16.8$ grams

There are certain discrepancies can be observed in Figure 3.15 and Figure 3.6, for the resonant frequencies and the amplitude of the peaks, which are mainly due to the fabrication defects of the experiment prototype. A more precise fabrication process is expected to reduce such discrepancy. However, the trend of the how the two response peaks be changed with different masses are both demonstrated well for both figure. Despite of some discrepancies as compared to the experiment outcomes, the results obtained from the distributed model predict well for resonant frequencies and the voltage frequency response with the change of the tip masses.

3.5 Model Validation Using Finite Element Analysis (FEA)

Rather than the mathematical modelling method, numerical simulation using Finite Element Analysis (FEA) is another option to model such electromechanical coupled system. In this section, a FEA model is developed to validate the experimental findings.

3.5.1 FEA model of The 2-DOF Cut-Out PEH

The finite element model of the cut-out beam is developed in the commercial FEA software ANSYS. ANSYS provides a unique element (SOLID 226 element) for coupled-field analysis which can be used to model the piezoelectric transducers. Conventional SOLID 186 element is used to model the aluminum substrate of beams and tip masses. The load resistor connected to the piezoelectric transducers is modeled using CIRCU 94 element. Between the piezoelectric transducer and the substrate beam, an adhesive bond layer (epoxy) of 0.2 mm thickness is also simulated with Solid 186 element, with an elastic modulus of $1e8 \text{ N/mm}^2$ (Yang et, al. 2010). As assuming no de-bonding will happen during the experiment, different layers are connected to each other with same nodes which present the same displacement at the interface. The degree of freedoms of electrical potential of the nodes on the top and bottom surfaces of the piezoelectric transducers are coupled respectively to implement the uniform electrical potential on electrodes, and then connected to the two terminals of the load resistor. Two resistors are connected to the two transducers on the main and secondary beams separately. In the FEA model, all the geometry parameters are just followed to the experiment setup, as shown in Figure 3.4. Finally, the FEA model of the 2-DOF cut-out PEH with connection of resistors is established. Modal analysis is conducted first, to determine the first two vibration modes and the steady state analysis is performed to obtain the voltage responses from the harvester.

The first two vibration modes are predicted by modal analysis, as shown in Figure 3.16. In this case of FEA simulation, the values of two tip masses are: $M_1=7.2$ grams,

and $M_2=8.8$ grams. The resonant frequencies obtained from simulation are almost consistent with the experimental results (17.5Hz and 21.6Hz). In other cases of different tip mass values, the predictions of natural frequencies are also consistent with the experimental results.

First mode, resonant frequency=17.5Hz

Second mode, resonant frequency=21.7Hz

Figure 3.16 First and second modal shapes of 2-DOF cut-out PEH

3.5.2 Steady-State Analysis for Open Circuit Voltage Output

Other than the modal analysis to work out the vibration modes and resonances, the steady-state open circuit voltage frequency response of the proposed harvester can also be obtained by harmonic analysis in ANSYS. Same as the mathematical model in the previous section, the load resistors connected to the piezoelectric transducers are set extremely large ($10^9 \Omega$), to obtain the open circuit voltage response. Also, a constant damping ratio of 0.7% is adopted in the harmonic analysis.

Figure 3.17 Comparison of simulation and experiment results for open circuit response with different second mass when $M_1=7.2$ grams. (a) $M_2=8.8$ grams, (b) $M_2=11.2$ grams, (c) $M_2=14.2$ grams and (d) $M_2=16.8$ grams

Figure 3.17 shows the predicted open circuit voltage responses obtained from the FEA simulation, together with the results obtained from the mathematical model and experiment. Although there are certain discrepancies for resonances and magnitudes from these three groups of data, in general, both mathematical model and FEA

simulation predict well for the voltage frequency response of the 2-DOF PEH. Both of these two modelling methods are efficient to model this 2-DOF PEH. However, FEA simulation is regarded as a more robust method which can be conveniently used to model more complicated structures (i.e. with varying cross-section).

3.5.3 Steady-State Analysis for Power Output

By setting different values of the resistor, different power responses at various frequencies can be obtained, which can be compared with the experimental results. The simulation results shown in Figure 3.18 are for the same configurations as for Figure 3.9, i.e., $M_1=7.2$ grams and $M_2=8.8$ grams. Comparing Figures 3.9 and 3.18 it is apparent that the simulation results are similar to the experimental ones except the slight shift of resonance frequencies and different magnitudes of the power output peaks. However, the results for impedance match study are very close, they both indicate the optimal resistance value is around $120\text{ K}\Omega$. Here the value of $R_1=120\text{ k}\Omega$ for the resistor connected to the main beam is also used to calculate the maximum power output in FEA simulation. For the secondary beam, similar results are obtained and $R_2=230\text{ k}\Omega$ is adopted as the optimal resistor value.

Figure 3.18 Simulation results of power output response versus frequency for the main beam of the 2-DOF cut-out beam when $M_1=7.2$ grams and $M_2=8.8$ grams

In Figure 3.19, the maximum power output responses versus frequency are also worked out by using these optimal resistor values. Again, these results are similar to the experimental ones shown in Figure 3.12, with minor discrepancy in resonant frequencies and magnitudes.

Figure 3.19 Simulation results of power output for 2-DOF cut-out PEH for $R_1=120$ k Ω and $R_2=230$ k Ω when $M_1=7.2$ grams (a) $M_2=8.8$ grams, (b) $M_2=11.2$ grams, (c) $M_2=14.2$ grams and (d) $M_2=16.8$ grams

Overall, these simulation results suggest that numerical simulation can be employed as a useful tool to provide guidelines for the design of 2-DOF cut-out harvester.

3.6 Comparison Study of the Proposed 2-DOF Cut-out PEH and Conventional SDOF PEH

To evaluate the overall performance of the proposed 2-DOF cut-out PEH, a study is conducted in this section to compare with a conventional SDOF PEH.

As shown in Figure 3.20, the SDOF PEH is designed with same size of the 2-DOF PEH (Figure 3.4). As mentioned in previous sections, to design an energy harvester, the size is a most critical concern as the space of the device is limited, especially for MEMS. Thus, the two harvesters are designed with same space requirement. And the thickness of the substrate is also assumed as 0.6 mm for comparison, which is same as experiment prototype. There are two patches of MFC attached on the SDOF PEH, with the same size and location as the 2-DOF design. From the above experiment study (Figure 3.6), the working frequency is located within 15-22 Hz for different configuration. Based on the above assumption, a theoretical distributed mass model is built followed by Erturk's modeling method for a conventional SDOF PEH, which is reviewed in Section 2.2.1.2 (Erturk and Inman (2008b)). From the model, the tip mass value is determined as 9 grams, and the resonant frequency is 19.07 Hz. The 2-DOF PEH is modeled with the proposed mathematical model as presented in Section 3.4, which is basically developed from the same method by Erturk and Inman (2008b).

Figure 3.20 Layout of the conventional SDOF PEH

Based on the two mathematical models, the power output results obtained after impedance matching, are shown in the Figure 3.21. It can be seen that, the SDOF PEH present higher power output for MFC-1 patch (about 0.8 mw), while MFC-2 can only contribute very little (about 0.003 mw) due to the very low strain distribution

at the free end of the conventional cantilever structure. In 2-DOF design, both the MFC-1 and MFC-2 patches can generate significant power output (about 0.55 mw), as both parts can generate high strain distribution. Moreover, there is another response peak can also generate significant power output, which is benefit for broadband energy harvesting.

Figure 3.21 Power output obtained from the mathematic models for (a) 2-DOF PEH and (b) SDOF PEH

Through this comparison study, it is demonstrated that the proposed 2-DOF PEH has larger working bandwidth. And it is more space efficient, as it utilizes the material of cantilever beam by generating significant power output from both the main and secondary beams.

3.7 Frequency Response Patterns for The 2-DOF Cut-out Harvesters

The previous experiment results and modelling results show the advantage by employing such cut-out configuration for piezoelectric energy harvesting. Rather than doing parametric study every time, it is more important to conclude the regular patterns for such harvester with various configuration.

For simplicity, uncoupled model is used here, in which only the displacements of the two tip masses are concerned, as the power generation is related to the strain level which is proportional to the tip displacement. A cut-out cantilever beam with uniform property is model, as shown in Figure 3.22(a), which is same as in Figure 3.2(d). Either through the mathematical modelling or FEA simulation, the frequency response for displacement of the two tip masses can be obtained.

Figure 3.22 (a) A typical cut-out PEH (b) its first two vibration model shapes

The frequency response curves can be classified into different groups according to the values of L_1/L_2 and two natural frequencies f_1 and f_2 , as listed in Table 3.1. Here the two natural frequencies f_1 and f_2 are not defined by the order of the absolute value of the frequency. Instead, f_1 denotes the natural mode in which the movement of the outer mass dominates (upper one in Figure 3.20(b)), while f_2 relates to the inner mass (lower one in Figure 3.20(b)).

Table 3.1 Frequency response patterns for different configurations of cut-out PEHs

L_2/L_1	f_2/f_1	Displacement frequency response for outer mass M_1	Displacement frequency response for inner mass M_2	Remark
$=2/3$				Group A
$>2/3$	>1			Group B
	<1			Group C
$<2/3$	>1			Group D
	<1			Group E

First of all, this table shows that, the frequency responses of the 2-DOF beam greatly rely on the length ratio of the inner beam to outer beam, while the value of $2/3$ is a critical point that divides the pattern into three categories. This is agreed with previous discussion in section 3.2, that the 2-DOF system with this length ratio is fully decomposed to be two SDOF systems. This can be observed from Group A in Table 3.1, the response curves are the same as the two independent SDOF systems.

It is worth mention that, as observed from the experiment results (Figure 3.6), in-between the two frequency response peaks, there exist a response valley. Actually, there are two different types for this response valley, one is a very deep valley whose response almost approaching to zero, named as anti-resonance; another is just normal frequency response as the frequency shifted off resonance.

If the length ratio L_2/L_1 is larger than $2/3$, the system will follow the response pattern in Group B or C. As shown in group B, when $f_2 > f_1$, there is an anti-resonance point (highlighted with a red circle) in-between the two response peaks for the outer beam. This anti-resonance separates the two peaks and forms a deep valley in the response curve. The outer beam will not be able to generate sufficient power output around this valley due to the ultra-low response. The existence of anti-resonance between the two peaks will greatly deteriorate the performance of the harvester. There is an anti-resonance point for the inner beam as well, with its position in front of the two response peaks. In this condition, the valley in-between the two response peaks is not deep, and the inner beam can still generate significant power output throughout this frequency range.

By adjusting the values of the moment of inertia and the tip masses, natural frequency f_2 for the vibration mode dominated by the inner mass can be tuned lower than f_1 , as

represented by Group C in Table 3.1. By comparing Groups B and C, it can be observed that the response patterns for the outer beam and inner beam is just swapped.

In Groups D and E with the length ratio smaller than $2/3$, for $f_2 > f_1$, the anti-resonance point is located at the right of the two response peaks for the outer beam, while the inner beam has the anti-resonance response in-between the two peaks. Moreover, it also observed from Groups D and E, when the order of the natural frequencies exchanges, the patterns of the response curve swaps.

As conclude from Table 3.1, to design such a linear “cut-out” 2-DOF broadband energy harvesting system, one may prefer to design its frequency response with two close significant peaks, but without the anti-resonance in-between them (i.e. the response from the inner beam in Group B in Table 3.1).

There is a study for harvesting energy from a vehicle bridge, and the recorded acceleration spectrum data is shown in the following Figure 3.22. It can be seen that there several peaks for different frequencies, thus multi-modal broadband PEH is definitely required. As compared to the results in this Chapter, the proposed 2-DOF PEH would be a good match to work in such environment.

Figure 3.23 Recorded acceleration spectrum for a vehicle bridge with different locations (Peigney and Siegert, 2013)

3.8 Chapter Summary

In this chapter, a novel design of 2-DOF cut-out cantilever PEH is proposed and studied. This novel 2-DOF PEH meets the requirements for multi-modal broadband energy harvesting. It provides larger bandwidth compared to the conventional SDOF and 2-DOF PEHs. Meanwhile the proposed harvester is more compact than the conventional 2-DOF PEH, as it efficiently utilizes the material of cantilever beam by generating significant power output from both the main and secondary beams. With different proof masses, the open circuit voltage and the power output responses with a resistor as the electrical load connected to the harvester have been studied in experiment. Subsequently, a mathematical distributed parameter model as well as a FEA model have been developed to validate the experiment results. The development of this novel 2-DOF cut-out PEH provides a new idea for designing a broadband harvester using the multi-modal technique, which would be useful in the practical design of PEHs, especially when space constraint is imposed in the design such as for micro-electro-mechanical systems (MEMS).

However, one obvious drawback of this design is that a response valley (anti-resonance point) always presents in-between these two resonant peaks, either for main beam or the secondary beam. Thus, the frequency response patterns for different configurations of this 2-DOF PEH are concluded, to provide a guideline for design and chosen structural parameters. With that, one can easily decide to design a harvester with required response pattern, to avoid the anti-resonance point presenting in-between. Both the mathematical model and the FEM model can be used for a tool for general design purpose, for the linear 2-DOF piezoelectric energy harvester. As compared to the experimental results, it is proved that the models can predict the harvester's response reasonably. There are certain discrepancies remain, mainly due

to the fabrication defects of the experiment prototype. A more precise fabrication process is expected to implement the models into real application.

In the next chapter, a nonlinear 2-DOF harvester is studied, in which this design strategy is utilized. With the introducing of the magnetic interaction, its working bandwidth is further broadened.

CHAPTER 4 DEVELOPMENT OF A BROADBAND NONLINEAR TWO-DEGREE-OF-FREEDOM PIEZOELECTRIC ENERGY HARVESTER

4.1 Introduction

A novel compact 2-DOF PEH has been proposed for broadband energy harvesting, as discussed in Chapter 3. This harvester has the three advantages, wider bandwidth achieved with two close resonant peaks, both with significant power output and compact configuration for efficient use of materials without any increase of volume. However, as show in Figure 3.6, between the two resonant peaks, there is always a response valley, for responses from both the main beam and the secondary beam. The presence of this valley will definitely deteriorate the performance of this harvester. However, as catalogued into different patterns in Table 3.1, it can be found that the response valleys are in two different types. An anti-resonance point always presents in the frequency response curve, which cannot be eliminated. The best choice is to locate the anti-resonance point outside the two response peaks range, with properly chosen configuration. The other type of the frequency response valley is just normal behavior of the off-resonance frequency response, which has great potential to be further improved.

On the other hand, as reviewed in Section 2.3.2.4, nonlinear technique is another option to for broadband energy harvesting. A typical frequency response with nonlinear vibration is illustrated in Figure 2.30; the bent curve can cover larger frequency range making the bandwidth much wider than that of a linear harvester. Thus, it is promising to improve the harvesters' performance, by combing the nonlinear vibration technique together with the multi-modal energy harvesting,

especially for the proposed 2-DOF configuration. Moreover, most researchers have focused the application of nonlinear vibration in SDOF energy harvesting configuration. Very few attempts have been reported in the literature for the nonlinear multi-DOF energy harvesting system.

Based on the design of the novel 2-DOF harvester, nonlinearity is introduced to the system by adding a pair of polar opposite magnets, making it as a nonlinear 2-DOF PEH. The objective of this study is to raise the response valley existing in the response of the linear 2-DOF harvester, to achieve broader operation bandwidth. In this chapter, experimental parametric study will be presented to illustrate the characteristics of this nonlinear 2-DOF PEH. Through the study, an optimal configuration is determined, which provides significantly wider bandwidth. The response valley in the linear 2-DOF system is also raised in this nonlinear 2-DOF system. Furthermore, an analytical model is developed for the nonlinear 2-DOF system by considering the dipole-dipole magnetic interaction. Results are obtained by solving the model numerically, which provide good validation compared to the experiment finding.

4.2 Experimental Study of The Nonlinear 2-DOF Harvester

4.2.1 Design of Nonlinear 2-DOF Harvester

Based on the design of the linear 2-DOF “cut-out” PEH, the nonlinear 2-DOF PEH is developed by introducing magnetic interaction. It comprises a main beam (outer beam) and a secondary beam (inner beam), both with tip masses, which is same as the linear 2-DOF PEH. Two polar opposite magnets are installed at the tip of the inner beam and the clamped base, respectively. Figure 4.1 shows the prototype installed on a vertical seismic shaker.

Figure 4.1 Nonlinear 2-DOF piezoelectric energy harvester installed on the verital shaker

As discussed in Chapter 2, except for Group A in Table 3.1, an anti-resonance valley always exists in the frequency responses of either the outer beam or inner beams, but at different locations. Thus, only one beam (either outer beam or inner beam) can avoid the anti-resonance present in-between the two response peaks for further broadening the bandwidth. In this experimental study, the inner beam is selected and optimized for further study of broadband energy harvesting, by avoiding the anti-resonance in-between two peaks. With magnetic nonlinearity introduced, nonlinear vibration behaviors are studied. Two repulsive NdFeB permanent magnets with diameter of 10 mm, thickness of 5 mm and surface flux of 3500 gauss are embedded in two plastic holders, separated at the distance of D . One holder with the magnet serves as the tip mass of the inner beam ($M_2=7.4$ grams), while the other one is clamped to the shaker with a short support beam. The length of the short support beam is adjustable, making it convenient to adjust the distance between the two magnets while keeping other structural parameters unchanged. The outer and inner beams have the thickness of 1 mm and 0.6 mm, respectively, both of which are fabricated from same aluminum plate. For the convenience of fabrication, these two

parts are fabricated separately and assembled with screws. The screws and several pieces of steel plate at the free end of the outer beam serve as its tip mass (M_1), and M_1 is adjustable by adding or removing small steel plates. Each piece of small plate weighs about 1.9 grams, while the minimum value of M_1 including the screws and nuts is 3.6 grams. One d_{31} piezoelectric sheet, Macro-Fiber-Composite (MFC, model: M-2814-P2, Smart Material Corp.), is attached on the inner beam for energy generation. Detailed dimensions of the proposed nonlinear 2-DOF PEH are shown in Figure 4.2.

Figure 4.2 The illustration of nonlinear 2-DOF harvester (all demension in mm)

In the experimental parametric study, three parameters are adjusted to study the behavior of the system. They are: the distance of the two magnets (D), the tip mass of the outer beam (M_1), and the base excitation level (A). The distance of two magnets which affects the nonlinear stiffness of the system and the base excitation level are two key parameters which determining the nonlinear dynamics of the PEH. Also, adjusting M_1 affects the nonlinear response patterns of various configurations. Before the experiment, a preliminary study was carried out by using FEM model as described

in the Chapter 3, to predict the vibration frequencies in linear condition. The operation frequency is considered within 10-30 Hz which is regarded as a low frequency vibration environment (e.g. vehicle vibration, bridge or building environment, etc.). Therefore, the mass values are roughly determined, but the exact mass values are limited to the fabrication of the experiment prototype. Different values of these three parameters used in the experiment are listed in Table 4.1.

Table 4.1 Structural parameters used in the experiment study

Parameters	Values
Distance of two magnets, D (mm)	14, 12, 11, 10, 9
Tip mass at the end of outer beam, M_1 (g)	13.1, 11.2, 9.3, 7.4, 5.5, 3.6
Harmonic base excitation level, A (m/s^2 , RMS)	0.5, 1, 2

If the distance between two magnets is smaller than certain value, due to the strong repulsive magnetic force, the structure will have two stable equilibrium positions (thus two potential wells). This is tuned into bi-stable configuration, as illustrated in Figure 4.3. The central position is the stable equilibrium position in the mono-stable configuration, which however becomes the potential barrier in the bi-stable configuration. When the distance between two magnets decreases further, the two equilibrium positions will be separated further away from each other, and the central potential barrier will increase, making it harder to jump across. The dynamics of the bi-stable configuration will be more complicated than that of mono-stable one. In this study, the focus is the performance of the PEH in mono-stable configuration, while the bi-stable configuration will be only briefly illustrated to show the difference. For the prototype studied in the experiment, the critical distance D for transition from mono-stable configuration to bi-stable one is observed as between 10 and 9 mm.

The nonlinear 2-DOF PEH is firstly tested with sinusoidal sweep to obtain the frequency response curves for different configurations, from which an optimal

configuration is determined. Subsequently, the optimal nonlinear 2-DOF PEH configuration is tested under random excitation to compare its performance with that of the linear counterpart.

Figure 4.3 Illustration of equilibrium position for mono-stable and bi-stable vibrations

4.2.2 Frequency Response for Sinusoidal Sweep

The acceleration for the base excitation is kept constant for every sinusoidal sweep. The frequency responses of open circuit voltage from the nonlinear 2-DOF PEH are recorded in terms of the root mean square (RMS) values as the PEH is vibrating in the steady state. At some unique frequencies, the transient responses are recorded as well. With various distances between the two magnets, linear response, quasi-linear response, mono-stable nonlinear response and bi-stable nonlinear response are observed, and an optimal configuration is concluded which can achieve significantly wider bandwidth.

Frequency response of linear 2-DOF PEH (without magnetic force)

By removing the magnet which clamped at the base, the system thus becomes a linear 2-DOF PEH. Its frequency responses, including three groups of experimental results with different mass values, are shown in Figure 4.4. It is observed that the first peak slightly changes due to different M_1 while the second peak is almost not affected.

Here, the two natural frequencies (about 15 Hz and 28 Hz for case (a)) are still a bit far away from each other, and the magnitude of the first peak is relatively small compared to the second peak. Thus, the linear PEH is not optimized to achieve two close response peaks with adequate magnitudes.

Figure 4.4 Frequency response of 2-DOF PEH without magnets, (a) $M_1=11.2$ grams, (b) $M_1=9.3$ grams, and (c) $M_1=7.4$ grams.

Quasi-linear response of nonlinear configuration with lower excitation level

When the 2-DOF PEH with nonlinear configuration is tested under low excitation level (0.5 m/s^2), it exhibits a quasi-linear behavior which is similar to the linear harvester (Figure 4.5). Under low excitations, the vibration amplitude of the structure is not significant, thus the linear component of the magnetic force dominates and changes the linear stiffness of the system (mostly affect the inner beam), and frequency tuning effect is observed. It can be seen from Figure 4.4a and Figure 4.5, with different distances between the two magnets D , the resonant frequency for the second peak is tuned (from 28 Hz to 17.5 Hz) while the first resonant peak remains at the same position (around 15 Hz). Meanwhile, the magnitude of the first peak increases and the second peak decreases gradually with the decrease of the magnets distance. Moreover, an anti-resonance point can be clearly observed in front of the

first peak (highlighted with red circle), which is very similar to the pattern of the inner beam in linear 2-DOF case (Group B in Table 3.1).

Figure 4.5 Quasi-linear frequency response for nonlinear 2-DOF PEH under base excitation of 0.5 m/s^2 with $M_1=11.2$ grams and (a) $D=14$ mm, (b) $D=12$ mm, (c) $D=11$ mm (d) $D=10$ mm

It is important to note that the configuration in case (d) of Figure 4.5 meets the requirements for the design of a broadband multi-modal piezoelectric energy harvester (as discussed in Section 3.1), which presents two close response peaks of adequate magnitudes with anti-resonance point outside the two peaks range. Under higher excitations, the bandwidth for such system can be further increased, which will be detailed in the following section. Besides, it should be mentioned that further decrease of the magnets distance (D) tunes the harvester into bi-stable configuration. As observed from Figure 4.5, when the distance between the magnets decreases, the amplitude of the second response peak slightly drops. Meanwhile, the amplitude of first peak, as well as the response for frequency 15-17 Hz increases significantly. This indicates energy redistribution phenomenon in the frequency domain when the

parameters are varied and the broader bandwidth is achieved at the minor cost of peak amplitude.

Mono-stable nonlinear response with higher excitation level

With two magnets placed closer but still maintain mono-stable configuration ($D=10$ mm), the response curve presents very minor hardening nonlinear behavior under low excitation, that is, the second peak is slightly bent to the higher frequency direction, as shown in Figure 4.6(a). However, the upward and downward sweeps do not have obvious difference, thus it still can be regarded as a quasi-linear response.

Figure 4.6 Frequency responses for nonlinear 2-DOF harvester with $M_1=11.2g$ and $D=10mm$ under excitation of (a) $0.5m/s^2$ (b) $1m/s^2$ and (c) $2m/s^2$.

When the excitation level increases, the response curve, especially the second peak, is bent further to the higher frequency direction, providing enlargement in bandwidth,

as shown in Figure 4.6(b) and (c). Also, the typical jump phenomenon and multi-valued response of mono-stable configurations are observed (i.e. from 18.5Hz to 20Hz in Figure 4.6(c), large-amplitude and small-amplitude oscillation orbits co-exist). If 10V is regarded as a useful working voltage level, the upward sweep ensures the harvester to capture the higher energy orbit and cover a bandwidth of about 5 Hz (15 Hz to 20 Hz). However, the higher voltage response obtained from the upward sweep cannot be always guaranteed in the practical application. By considering the downward sweep response curve, which is robust for any initial condition, the bandwidth is still quite large of about 3.5 Hz (15 to 18.5 Hz). The frequency response of the nonlinear 2-DOF PEH is a perfect match for the recorded environmental vibration energy source on a vehicle bridge, which also presents two significant peaks (Peigney and Siegert, 2013, Figure 7).

Figure 4.7 Transient voltage responses of nonlinear 2-DOF PEH at (a) 16.4Hz, (b) 16.9Hz, (c) 17.4Hz and (d) 17.8Hz.

Moreover, within certain frequency range (as indicated in Figure 4.6(c)), the response is not harmonic though the system is under harmonic excitation. Transient voltage responses for several frequency points are shown in Figure 4.7. It is clear in Figure 4.7 that any of these waveforms can be viewed as a combination of several harmonic waveforms with different frequency. As observed in the experiment, the vibration motion of the nonlinear 2-DOF PEH was constantly swapping between its two vibration modes, which can be regarded as the internal resonance for the nonlinear system. In Figure 4.6(c), an additional peak presents within such frequency range. Actually, the presence of this peak is due to the variation of the RMS value for those non-harmonic waveforms. As observed in the experiment, this phenomenon is more obvious when the excitation level is higher.

It is worth mentioning that the broadband performance of the nonlinear 2-DOF PEH is achieved by properly selecting the structural parameters ($M_1=11.2\text{g}$) and the distance between magnets ($D=10\text{mm}$). Similar to the linear 2-DOF PEH, two close response peaks are achieved, both with adequate amplitudes, and the negative effect of anti-resonance for broadband performance is mitigated by avoiding its appearance in-between the peaks. Moreover, the nonlinearity introduced into the system further widens the bandwidth by bending the second peak and raising the valley between the peaks. But such broadband performance may be deteriorated with improperly selected parameters, as shown in the following cases studied.

Figure 4.8 provides more cases of the linear 2-DOF PEH with different M_1 . For case (a) in Figure 4.8, the anti-resonance point is located in-between the two response peaks, which is similar to the pattern of inner beam for Group C in Table 3.1. The presence of the anti-resonance point greatly deteriorates the performance. For cases (b) and (c), the two resonant frequencies are tuned much closer to each other, and the bandwidth is reduced as compared to Figure 4.6. On the contrary, for case (d), the

two resonant frequencies are tuned too far away, resulting lower output for the range between the two response peaks. Thus, considering the case in Figure 4.6 and the four cases in Figure 4.8, it is concluded that the configuration of nonlinear 2-DOF PEH with $M_1=11.2$ grams and $D=10$ mm is the optimal configuration for this mono-stable nonlinear 2-DOF PEH. Here, the optimal configuration refers to the best one which produces largest bandwidth with respectable amplitude, when the tip masses vary from 7.4 to 13.1 grams and the distance between magnets changes from 14 to 10mm, through the experimental parametric study.

Figure 4.8 Frequency response for nonlinear 2-DOF harvester with $D=10$ mm, $A=2\text{m/s}^2$ and (a) $M_1=5.5$ grams, (b) $M_1=7.4$ grams, (c) $M_1=9.3$ grams and (d) $M_1=13.1$ grams

When the distance between two magnets is further decreased ($D \leq 9\text{mm}$), the PEH will change into bi-stable configuration, which is more complicated than the mono-stable vibration. According to the experimental observation, the harvester was easily confined in one potential well due to the gravity and fabrication defect. The dynamics

and frequency response will be much different from the mono-stable configuration. The investigation of the bi-stable configuration is beyond the scope of this work.

In this work, the electromechanical coupling is quite low as $Ke^2/\zeta=0.2\sim0.3$, (Ke is electromechanical coupling coefficient and ζ is mechanical damping ratio), which can be regarded as weak coupling condition (Shu and Lien, 2006). For weak electromechanical coupling, the frequency responses of the optimal power and open circuit voltage have very similar trends in spite of the slight resonant frequency shift. Thus, the frequency response of open circuit voltage can be used to study the bandwidth of the system and the conclusions in terms of open circuit voltage response apply to the harvested power as well.

4.2.3 Test Under Random Excitation

In real applications, the majority of vibration energy sources present in random patterns. In this section, the nonlinear 2-DOF PEH is tested under random excitation with a uniformly distributed acceleration spectrum from 8 Hz to 35 Hz, which covers all the frequency range for both linear and nonlinear configurations developed in this work. Such profile is close to the vibration spectrum for application of energy harvesting in building or bridge environment (Roundy et al., 2003; Peigney and Siegert, 2013). In the experiment, a shaker controller (VR9500) is used to control the random vibrations of the shaker. Figure 4.9(a) shows an example of the controlled excitation spectrum for the experiment test, in which the demanded spectrum is a uniform distribution with RMS acceleration of 0.1G (gravitational acceleration). The time history of the acceleration of the base excitation from the shaker is also recorded, as shown in Figure 4.9(b). Three random excitation levels, 0.1 G, 0.15 G and 0.2 G, are considered for random tests.

(a)

(b)

Figure 4.9 (a) Power density of demanded spectrum and controlled value for RMS acceleration=0.1 G, (b) Time history of base excitation

The optimal 2-DOF nonlinear configuration ($M_1=11.2$ grams and $D=10$ mm) obtained from the previous section is tested and evaluated under random excitation. Its performance will be compared with its linear counterpart (simply remove the magnet at clamped base).

Figure 4.10 provides the examples of the waveforms of the open circuit voltage response for both the nonlinear and linear configurations, under the same level of

random excitation. Obviously, the magnitude of the response for the nonlinear 2-DOF PEH is much larger than its linear counterpart.

Figure 4.10 Recorded waveforms under random excitation of RMS acceleration=0.1 G, (a) Linear, (b) Nonlinear

Figure 4.11 FFT result for recorded waveform, (a) Linear, (b) Nonlinear

Power spectrums in the frequency domain are obtained by taken Fast Fourier Transformation (FFT), for both linear and nonlinear configurations, as shown in Figure 4.11. It can be observed that both the magnitude and the bandwidth are greatly improved for the nonlinear configuration compared to its linear counterpart

To further evaluate the performance of the system, the harvester is connected with an energy storage circuit composed of an AC/DC full-wave rectifier and a storage capacitor ($330\mu F$). In the experiment, the charging procedure is carried out for same time period (2 minutes) each time, and the voltage at the capacitor is monitored to calculate how much energy can be accumulated, as shown in Figure 4.12. For each

random excitation level, the same procedure is repeated for 4-5 times to ensure the reliability of the results.

Figure 4.12 Charging record for nonlinear and linear 2-DOF harvester with different excitation levels

From the comparison shown in Figure 4.12, the performance of the nonlinear 2-DOF PEH is much better than that of its linear counterpart. For all the tested cases, the accumulated energy in the capacitor ($E = 1/2CU^2$) by the nonlinear 2-DOF PEH is about 2.5 times larger than its linear counterpart. For example, the average energy stored for the nonlinear 2-DOF PEH with excitation of 0.2G is about 5.5 mJ, while the linear one only achieves about 2.2mJ. Moreover, other than this standard charging circuit, many other circuit techniques proposed by researchers, i.e. SCE circuit or SSHI techniques, can be considered in future to further improve the power output of the developed nonlinear harvester.

4.3 Modeling of Nonlinear 2-DOF Harvester and Validation

4.3.1 Lumped-mass Modeling of Linear 2-DOF Harvester

In this section, a lumped parameter model for the nonlinear 2-DOF PEH is presented. The modeling results are obtained using numerical integration in Matlab and validated against the experiment outcome.

The vibration motion of the linear 2-DOF system subjected to base vibration can be described by the governing equation:

$$\begin{cases} M_1\ddot{x} + C_{11}\dot{x} + K_{11}x + C_{12}\dot{y} + K_{12}y = -M_1\ddot{u}_0 \\ M_2\ddot{y} + C_{12}\dot{x} + K_{21}x + C_{22}\dot{y} + K_{22}y = -M_2\ddot{u}_0 \end{cases} \quad (4.1)$$

where x and y are the displacements for outer mass and inner mass, respectively; $\ddot{u}_0$ is the base acceleration; and M_i , K_{ij} and C_{ij} denote the related components in the mass, stiffness and damping matrixes, respectively. With assumption of uniform cross-section for the whole system, mass and stiffness matrixes are following the same equation of Equation (3.1) and (3.2). When simplifying the harvester using the lumped-mass model, the mass values used in the equations should be modified with a correction factor, as the distributed mass of the cantilever will also contribute to the vibration motion (Erturk and Inman 2008a). However, when the tip mass is much larger than the distributed mass of cantilever, the correction factor is very close to unity. For example, by considering the optimal configuration of the experiment, tip mass $M_1=11.2$ grams and the distributed mass of outer beam is about 2.7 grams. The correction factor can be calculated as 1.03, by using the equations in Erturk and Inman (2008a). Thus, for qualitative analysis using the lumped-mass model, the correction factor is not applied in this work.

The damping matrix is assumed to be proportional to the mass and stiffness matrices as:

$$[C] = \mu[K] + \lambda[M] \quad (4.2)$$

where μ and λ are two coefficients. To determine these coefficients, an attenuation test for the experiment prototype is carried out to measure the damping ratio (ζ) at the first and second resonances. The measured damping ratio does not vary too much for different conditions, and the value is around 0.8% for both resonances. Thus a value of 0.8% is used for a fixed damping ratio in later calculation. With known damping ratios at the resonant frequencies, the two coefficients as well as the damping matrix are then obtained.

With the piezoelectric transducer attached on the inner beam, an electrical-mechanical coupling equation is required to relate the vibration motion with the electrical output. However, in this 2-DOF cantilever beam, the strain is not simply related to the displacements 'x' and 'y'. The angle of rotation at the tip mass is also contributed to the strain in the beam. Figure 4.13 illustrated the displacements and angle of rotation in stationary condition, where the dashed line of the inner beam indicates the “free position” that no strain occurs in it.

Figure 4.13 Stationary displacement and angle rotation relation

The strain distribution in the inner beam should be proportional to the overall displacement from the “free position” to the forced position, which is indicated as “ Δ ” in Figure 4.13. To obtain Δ , the angle of rotation at the outer mass should be obtained first, which is expressed as

$$\theta_1 = \varphi_{11}(K_{11}x + K_{21}y) + \varphi_{21}(K_{12}x + K_{22}y) \quad (4.3)$$

where φ_{ij} denotes the angle of rotation at position j when a unit force is applied at position i ,

$$\varphi_{11} = \frac{L_1^2}{2EI_1} \quad (4.4)$$

$$\varphi_{21} = \frac{(L_1 - L_2)^2 - L_2^2}{2EI_1}$$

Finally the overall displacement Δ is

$$\Delta = y - x + \theta_1 L_2 = \alpha y - \beta x \quad (4.5)$$

$$\alpha = 1 + (\varphi_{11}K_{21} + \varphi_{21}K_{22})L_2$$

$$\beta = 1 - (\varphi_{11}K_{11} + \varphi_{21}K_{12})L_2$$

Therefore, the coupled governing equation of the 2-DOF system should be written as,

$$\begin{cases} M_1\ddot{x} + C_{11}\dot{x} + K_{11}x + C_{12}\dot{y} + K_{12}y - \psi V = -M_1\ddot{u}_0 \\ M_2\ddot{y} + C_{12}\dot{x} + K_{21}x + K_{22}y + C_{22}\dot{y} + \psi V = -M_2\ddot{u}_0 \\ -\psi(\dot{\Delta}) + C_s\dot{V} + V/R = 0 \end{cases} \quad (4.6)$$

where ψ is an electrical-mechanical coupling coefficient related to the property of piezoelectric material and the vibration modal shape; C_s is the capacitance of the piezoelectric element; R is the electric load connected to the harvester; and V is the voltage cross the load. In this model, ψ is equal to $9.2\text{e-}5 \text{ NV}^{-1}$ and C_s is 25 nF . To emulate the open circuit condition, R is set to be $1000\text{M}\Omega$.

4.3.2 Dipole-dipole Magnetic Interaction

In the proposed nonlinear 2-DOF PEH, the magnetic force is simplified as a dipole-dipole magnetic interaction. In vector form, its general expression is (Levitt and Malcolm, 2001),

$$\vec{F}_{mag} = \frac{3\mu_0 m_a m_b}{4\pi |r|^4} [\hat{r} \times \hat{m}_a \times \hat{m}_b + \hat{r} \times \hat{m}_b \times \hat{m}_a - 2\hat{r}(\hat{m}_a \cdot \hat{m}_b) + 5\hat{r}(\hat{r} \times \hat{m}_a) \cdot (\hat{r} \times \hat{m}_b)] \quad (4.7)$$

where μ_0 is the permeability of space ($4\pi \times 10^{-7}$ Tm/A); m_a and m_b are the magnetic moment for the two magnets (in the experiment, $m_a=m_b=0.218\text{Am}^2$); r is the distance of two magnetic dipoles; $\hat{r}$, $\hat{m}_a$, $\hat{m}_b$, $\hat{j}$ and $\hat{k}$ are the units vector with directions shown in Figure 4.14. In this nonlinear 2-DOF PEH, one magnet is fixed at the shaker thus its position and orientation does not change, while the other is attached at the tip of the inner beam with displacement and angle of rotation during vibration.

By applying these restrictions into Equation (4.7), the magnetic force can be expressed as,

$$\vec{F}_{mag} = \frac{3\mu_0 m_a m_b}{4\pi |r|^4} [\hat{k} \sin(a) + \hat{j} \sin(a + b) + 2\hat{r} \cos(b) - 5\hat{r} \sin(a) \sin(a + b)] \quad (4.8)$$

The angles a and b in Figure 4.14 are also related to the displacements of the outer mass (x) and inner mass (y),

$$a = \arctan\left(\frac{y}{D'}\right) \quad (4.9)$$

$$b = \theta_2 = \varphi_{12}(K_{11}x + K_{21}y) + \varphi_{22}(K_{12}x + K_{22}y) \quad (4.10)$$

Finally the magnetic force in the vertical direction is obtained as

$$F_{\text{mag}_v} = \vec{F}_{\text{mag}} \cdot \hat{j} = \frac{3\mu_0 m_a m_b}{4\pi |r|^4} [\sin(a) \cos(b) + \sin(a+b) + 2 \cos(b) \sin(a) - 5\hat{r} \sin(a) \sin(a+b)] \sin(a) \quad (4.11)$$

Figure 4.14 Relative position of the magnets

It should be noted that, in the experiment, the distance D between the magnets is measured from the facing surfaces of the two magnets. While, in the modeling with the assumption of dipole-dipole magnetic interaction, the horizontal distance D' is calculated from center to center of the magnets, i.e., $D' = D + 5$, as the thickness of the two cylinder magnets are both 5 mm.

By adding this magnetic force into Equation (4.6), the governing equation of the nonlinear 2-DOF system is again modified as

$$\begin{cases} M_1 \ddot{x} + C_{11} \dot{x} + K_{11} x + C_{12} \dot{y} + K_{12} y - \psi V = -M_1 \ddot{u}_0 \\ M_2 \ddot{y} + C_{12} \dot{x} + K_{21} x + K_{22} y + C_{22} \dot{y} + \psi V + F_{\text{mag}_v} = -M_2 \ddot{u}_0 \\ -\psi(\alpha \dot{y} - \beta \dot{x}) + C_s \dot{V} + V/R = 0 \end{cases} \quad (4.12)$$

For simplicity, only the vertical component of the magnetic force is considered in Equation (4.11), while the horizontal component is neglected. Note that, the magnetic force is only applied to the inner mass. This is why in the experiment the nonlinearity

mostly affects the second response peak which is dominated by the inner beam, leaving another peak in linear behavior.

4.3.3 Numerical Computations and Results

Since it is very difficult to solve Equation (4.12) including nonlinear term analytically, numerical integration technique is resorted by using Runge-Kutta method (which are readily available in Matlab) to obtain the numerical results. By numerically solving the ordinary differential equation with given initial conditions, the steady-state vibration waveform can be obtained for every frequency point. To simulate the same procedure of the continuous frequency sweep, the initial conditions used for the numerical integration at a frequency point are obtained from the previous results before frequency shifted (either upwards or downwards). Thus the response curve for the frequency sweep can be plotted. All structural parameters are set according to the experiment setup, as shown in Figure 4.2, and the optimal configuration is chosen to be validated. In lumped mass modeling, an effective mass value is normally adopted with consideration of the contribution of the distributed mass along the beam (i.e. the contribution coefficient would be 33/140 for a simple cantilever beam (Hibbeler, 2011)). Therefore, the effective mass values are slightly higher than the tip mass values used in the experiment. Other parameters used for numerical computation are listed in the following Table 4.2

Figure 4.15 shows simulation results under low excitation level of 0.5m/s^2 , to illustrate the trend of the resonance tuning by adjusting the value of the distance between two magnets, which is similar to the experiment results in Figure 6 except for slight differences in the peak locations and amplitudes. The value of the distance between two magnets does not exactly match with the experiment to achieve the optimal configuration. In the experiment, the critical point that the harvester changed from mono-stable to bi-stable vibration is with a distance value between 9-10 mm.

While in the model, this critical point is slightly lower than that, is between 8-9 mm. To observe the vibration response with the minimum distance value for the mono-stable vibration, slightly different values of the distance are used here.

Table 4.2 Parameters used for numerical computation

Parameters	Values
Outer effective mass, M_1 (grams)	11.8
Inner effective mass, M_2 (grams)	7.9
Damping ratio, ζ	0.8%
Stiffness matrix, K (N/m)	$\begin{bmatrix} 138.5 & 10.4 \\ 10.4 & 207.4 \end{bmatrix}$
Damping matrix, C (10^{-3} N·sec/m)	$\begin{bmatrix} 27.31 & 1.22 \\ 1.22 & 31.46 \end{bmatrix}$
Magnetic moment, m_a & m_b (Am^2)	0.218
Permeability of space, μ_0 (Tm/A)	$4\pi\epsilon^{-7}$
Electromechanical coupling coefficient, ψ (N/V)	$9.2\text{e-}5$
Capacitance of the piezoelectric element, C_s (nF)	25
Electric load resistor, R (Ω)	$1\text{e}9$

Figure 4.15 Voltage response for optimal configuration under low excitation level of 0.5 m/s^2 and with (a) $D'=18 \text{ mm}$, (b) $D'=16 \text{ mm}$, (c) $D'=15 \text{ mm}$ (d) $D'=14 \text{ mm}$, with experiment data

Figure 4.16 shows the theoretical results for different base excitation levels, which is similar to the experiment results shown in Figure 4.6, however the peak locations and amplitudes does not exactly match. It is also observed the existence of an anti-resonance point in front of the first response peak. Moreover, as shown in Figure 4.16(c), there is a fluctuation of the frequency response curve around 18 Hz, which indicates the occurrence of the internal resonance. The transient voltage waveform for the frequency of 18.3 Hz is recorded in Figure 4.17, which shows the similar phenomenon observed in the experiment.

Figure 4.16 Voltage response for optimal configuration with $D'=14\text{mm}$ under (a) 0.5 m/s^2 (b) 1 m/s^2 and (c) 2 m/s^2 as compared with experiment data (dots)

Note : distance of magnets is slightly different with experiment result ($D+5=15\text{mm}$)

Figure 4.17 Waveform of the voltage response at 18.3 Hz

In summary, the results from the lumped parameter model indicate similar trend in the frequency response of the nonlinear harvester. The internal resonance behavior is captured as well in the modeling results. The results from the lumped parameter modeling validates that this nonlinear 2-DOF PEH can achieve broader operation bandwidth with proper chosen parameters. Although discrepancies exist between the

theoretical modeling and the experimental results, the model predicts similar trend regarding the peak change and nonlinear vibration phenomenon to the experiment. Therefore, it can be used as a tool for parametric study of resonances and bandwidth tuning, which would provide initial estimate for the parameters in the optimal configuration. When doing so, the parameters should be chosen such that (1) no anti-resonance exists in the desired frequency range (e.g. choose the parameters that produce the inner beam response in Group B of Table 3.1); and (2) two response peaks are close with significant output.

4.4 Optimization Study of the Proposed Nonlinear 2-DOF PEH

As the analytical model is already developed in the above section, an optimization study is conducted to investigate the effect of each structural parameters. The parameters used for the optimization study are the length, mass, and the distance of the two magnets.

Figure 4.18 Power output spectrum for intergration

Based on the different structural parameters, the frequency response spectrum can be obtained by solving the analytical model. The power output response will be obtained through impedance matching. To assess the performance of the nonlinear 2-DOF PEH for the desired frequency range (8-35 Hz, same as the experiment test), the

power output response spectrum is simply integrated within such range, presenting the overall power output. An example is given in the following Figure 4.18, and the integral read as 0.24 mw*Hz for forward sweep and 0.099 mw*Hz for backward sweep.

Firstly, the effect of the distance of the two magnets are studied, and the results are shown in Figure 4.19. As known from the literature, larger magnetic force (closer distance) will increase the nonlinear vibration amplitude. However, when the two magnets are too close enough to form a bi-stable configuration, and the excitation force is not large enough to help the oscillator to snap-through, its vibration oscillation will be confined within one potential well only, resulting even worse performance.

Figure 4.19 Overall power output for different magnet distances

As seen from Figure 4.19, with the decrease of the distance, the power output increase significantly for the forward sweep as higher energy oscillation state captured, while it almost remain same for the backward sweep. But, when the distance is further reduced to 13 mm which tuned the harvester into bi-stable configuration. The power outputs for both forward and backward sweep are both reduced as the large amplitude oscillation to snap through cannot be guaranteed. Therefore, it can be conclude that

the maximum power output will be achieved around the critical point that the harvester transferred from mono-stable configuration into bi-stable configuration. Such conclusion is also consistent as reported in Tang et al., (2012). In the following optimization, the distance of the magnets is tuned to close to the critical point as discussed above, to achieve maximum power output for different configurations.

As discussed in Section 3.7, the frequency response pattern is mainly determined by the length ratio and mass ratio. In this study, the response patterns in Group B and C are preferred, with the length ratio larger than $2/3$. The configurations with length ratio lesser than $2/3$ are not desired. One example is given in the following Figure 4.20, for the length ratio equal to 0.6, the mass ratio is $M_1/M_2=1.5$ (as indicated in Table 4.2), with its power output index equal to 0.03 mw*Hz which is much smaller than the cases in Figure 4.18. Similar cases can also be found when length ratio is larger than $2/3$, but with a low value of mass ratio as indicated in Figure 4.21.

Figure 4.20 Power output spectrum for length ratio of 0.6

Figure 4.21 Power output spectrum for length ratio of 0.7 and mass ratio of 0.6

Figure 4.22 Overall power output for different mass ratio with length ratio of 0.7

Figure 4.22 show a comparison of overall power output for those configuration could produce the desired frequency response pattern as show in Table 3.1. There are two peaks can be observed, one is for mass ratio of 1.0, and another is for mass ratio of 2.0. For the case mass ratio equal to 2.0, a stronger magnetic forced is required to achieve the maximum output. However, if the distance of the two magnets are tuned two close, it will be tuned into bi-stable configuration and be confined in a very strong potential well that will produce quite low output. For the case of mass ratio of 1.0, the high power output is achieved because the two response peaks are adjusted very close to each other, thus the harvester is benefited from the broader bandwidth even though the magnetic interaction is smaller.

According to this optimization study, it can be concluded with few key points:

1. The maximum output is achieved when the distance of the two magnets is tuned close to the transfer point between mono-stable and bi-stable configuration.
2. The structural parameters should be carefully chosen that the desired frequency response pattern could be produced.

3. The maximum output can be achieved either by a configuration with higher magnetic interaction, or by a configuration with two close resonances. Thus, this analytical model could provide a tool for designing such nonlinear 2-DOF harvester with specific condition.

4.5 Chapter Summary

In this chapter, a nonlinear 2-DOF PEH is proposed and studied experimentally. This nonlinear 2-DOF PEH is extended from the linear 2-DOF PEH presented in Chapter 3 by introducing a magnetic field using two polar repulsive magnets. By changing the parameters like mass and distance of magnets, different configurations are studied in the experiment.

With mono-stable vibration condition, this PEH exhibits a typical hardening nonlinear behavior, of which the frequency response curve is bent towards higher frequency range with jump phenomenon observed. With the decrease of distance between the magnets and the increase of the base excitation level, the nonlinear behavior are strengthened. By carefully adjusting the structural parameters ($D=10\text{mm}$, $M_1=11.2\text{g}$), a significant large bandwidth with adequate magnitude is achieved at the larger excitation level ($A=2\text{ m/s}^2$). The response valley presented in two resonant peaks of previous linear 2-DOF system is raised with significant output. The experiment results show that, this nonlinear 2-DOF PEH can significantly broaden the bandwidth for energy harvesting, and is validated more efficient when charging a same storage capacitor, which is more advantageous than its linear counterpart. .

Moreover, an analytical lumped parameter model is developed to evaluate this nonlinear 2-DOF system, in which dipole-dipole magnetic interaction is considered. This model successfully predicts the similar broadband response trend of the

proposed harvester as experiment with slight discrepancy, providing good validation for the experiment finding.

CHAPTER 5 A TWO-DIMENSIONAL VIBRATION PIEZOELECTRIC ENERGY HARVESTER WITH A FRAME CONFIGURATION

5.1 Introduction

As discussed in Section 2.3, most reported multi-modal piezoelectric energy harvesters are focused on harvesting energy from certain frequency range, but only single excitation direction is concerned (normally perpendicular to the cantilever). However, a practical environmental vibration source may include multiple components from different directions. For example, in (Reilly et al, 2009), a Statasys 3D printer produces three frequency response peaks at 28, 28.3 and 44.1 Hz along three perpendicular directions, and a washing machine undergoes resonance at 85.0 Hz in two perpendicular directions. To harvest energy from such environment, it is an important issue to design a vibration energy harvester that can work with multiple excitation directions.

Several designs of 3-D or 2-D energy harvesters are reviewed in Section 2.3.3, with similar scheme that multi-direction displacement is achieved by utilizing seismic mass connected with space support springs. Such a scheme is suitable for the conversion mechanisms such as electromagnetic conversion, as only displacement is concerned. However, in piezoelectric energy harvesting, the induced strains in piezoelectric layers are the essential concern, rather than the tip displacement. Rectangular cross-sectioned cantilevers which can develop high strain at its root were employed in most piezoelectric energy harvesting systems.

This chapter presents a novel 2-D multi-modal PEH with a frame configuration, which utilizes its first two vibration modes to work with various vibration

orientations. Firstly, a frame-type PEH is prototyped and studied in experiment. With properly chosen structural parameters, this harvester can consistently provide significant power output with excitations from any direction with the same operation frequency, which satisfies the requirement of a practical energy harvester working in the real environment. In addition, it can also be tuned to harvest vibration energy from multiple directions with different working frequencies or to harvest broadband vibration energy in specific orientations.

Moreover, to validate the experimental results of the proposed 2-D energy harvester, a simulation model with combination of FEA and ECM methods is developed, to provide a robust tool to simulate and design such system with both structural and electrical complexity.

5.2 Design and Preliminary Analysis of the 2-D Piezoelectric Energy Harvester

The proposed 2-D multi-modal vibration PEH is designed as a frame structures with several segmented piezoelectric transducers attached. Figure 5.1 shows a schematic drawing of the proposed PEH. Generally, the frame structure can be regarded as a beam (horizontal plate) supported by two columns (vertical plate). From simple structural analysis, it is known that: if the two columns are strong enough compared to the beam, the structure can be regarded as a clamped-clamped beam that only vibrates along the vertical direction, or if the beam is stiff enough compared to the columns, the structure turns into a sway frame which only deforms along the horizontal direction. Thus, by adjusting the stiffness of the beam and columns properly, the harvester can work with the combination of both the horizontal and vertical modes. And the two resonances for the respective modes can be easily tuned by adjusting the structural parameters (mass, length,

and thickness). By using this design, it is possible to develop a robust energy harvesting system to harvest vibration energy from different directions. Figure 5.2 shows the strain distribution patterns for the two different vibration modes, which are obtained from a FEA model with uniform structural configuration. As shown, the strain distributions along the outer (or inner) surface of frame structure are not always of the same sign. To avoid the cancellation, the piezoelectric transducers on the frame are segmented (8 segments as shown in Figure 5.1). In this study, all 8 transducers are placed on the outer surface in this study; similar 8 transducers can be bonded to the inner surface if needed.

Figure 5. 1 Schematic of the proposed 2-D vibration piezoelectric energy harvester

Figure 5. 2 Illustration of the strain distributions for two different vibration modes

Generally, this 2-D harvester can be regarded of two single-degree-of-freedom (SDOF) harvesters in two perpendicular directions, while the response in other orientation is a combination from the two harvesters. Which means, its response is similar to a system with two separate vibration harvesters placed in two perpendicular directions. However, this proposed harvester is a more integrated system that only one single structure is required. Moreover, different from the conventional cantilever with triangle strain distribution which is only efficient at the root area, the proposed harvester utilizes the material more efficiently as both the root and tip areas are useful for converting vibration energy.

It is convenient to work out the two natural frequencies with the lumped mass modeling method for the two SDOF systems in the two different directions. For the horizontal vibration mode, its equivalent stiffness can be calculated by using the standard stiffness influence coefficient method (Meirovitch, 2003), as

$$K_h = \frac{\frac{L_1 L_2^2}{EI_1} + \frac{L_2^3}{6EI_2}}{\frac{L_1^3}{6EI_1} \left(\frac{L_1 L_2^2}{EI_1} + \frac{L_2^3}{6EI_2} \right) - \frac{1}{8} \left(\frac{L_1^2 L_2}{EI_1} \right)^2} \quad (5.1)$$

K_v

$$= \frac{\frac{L_1^2}{2EI_1} + \frac{L_1 L_2}{EI_2}}{\left(\frac{L_2^3}{48EI_2} + \frac{L_1^2 L_2}{4EI_1} - \frac{3L_1 L_2^2}{32EI_1} \right) \left(\frac{L_1^2}{2EI_1} + \frac{L_1 L_2}{EI_2} \right) - \left(\frac{L_1 L_2}{16EI_1} + \frac{L_2^2}{16EI_2} \right) \left(\frac{L_1^2 L_2}{4EI_1} + \frac{L_1 L_2^2}{4EI_2} \right)} \quad (5.2)$$

where E is the elastic module of the structure, L_1 and L_2 are the length for wall and beam respectively, and I_1 and I_2 are the moment of inertia for the wall and beam respectively (for simplicity, the moment of inertia is assumed uniform for

the wall and beam). Here and hereafter, the subscripts of ‘ h ’ and ‘ v ’ refer to the vibration mode in the horizontal and vertical direction, respectively.

As the lumped mass is not very large compared to the distributed mass, the distributed mass of the beam and column cannot be ignored when calculating the value of effective mass. For this 2-D harvester, the effective mass (with consideration of the contribution of distributed mass) for the horizontal and vertical modes are different. For the vertical vibration mode, its effective mass comprises of the central mass and partial contribution of the mass from two columns and beam. While when vibrating in the horizontal direction, the whole horizontal beam actually are moving together with the central mass of the same amplitude. Thus, its effective mass should include one more term for the total mass of the horizontal beam. The two effective masses can be written as

$$\begin{aligned} M_{ev} &= M_c + \alpha_v M_1 + \beta_v M_2 \\ M_{eh} &= M_c + M_2 + \alpha_h M_1 + \beta_h M_2 \end{aligned} \quad (5.3)$$

where M_c is the central mass value; M_1 and M_2 are the mass values for the columns and beam, respectively; α and β are the participation coefficients for the columns and beams, respectively. The participation coefficients normalize the vibration motion of distributed mass with the displacement of the central mass, and can be calculated through the integration of the kinetic energy based on the vibration mode shapes, which satisfy the equation of

$$T = \frac{1}{2} \int_0^L \rho(\xi) \phi(\xi) \dot{x}^2 d\xi = \frac{1}{2} \kappa \dot{x}^2 \quad (5.4)$$

where T is the total kinetic energy based on the mode shapes; $\rho(\xi)$ is the distributed mass function; $\phi(\xi)$ is the modal shape function; and α is the participation coefficient which can be calculated from the equation. It is well

known that the effective mass coefficient is $33/140$ for a simple cantilever beam, which can be obtained using the above equation. For our proposed structure with uniform cross section, by setting the lengths of the beam and columns the same, the coefficients are obtained as: $\alpha_h \approx 0.29$, $\beta_h \approx 0.014$, $\alpha_v \approx 0.006$, $\beta_v \approx 0.43$. These four coefficients slightly vary with different configurations of length, thickness, width, or mass. Except for extreme cases (e.g., the length ratio of beam over column is very large or very small), predictions of the effective mass using these four values are reasonable with an acceptable error ($<5\%$). Thus, these values can be used for a preliminary analysis to obtain the two natural frequencies of the frame structure.

By using the two groups of equivalent stiffness and effective mass, two natural frequencies for the horizontal and vertical vibration modes can be worked out as two separate SDOF systems, given as

$$\omega_{h,v} = \sqrt{\frac{K_{h,v}}{M_{eh,ev}}} \quad (5.5)$$

From the above analysis, the two natural frequencies can be easily tuned to any values to match with the vibration source by adjusting the structural parameters. Therefore, the harvester can work in any orientations with any required operation frequencies to harvest multi-directional vibration energy in a 2-D domain.

Due to the difference of the effective mass values for the two vibration modes, the two natural frequencies can be tuned closer or separate by adjusting the center mass value, even when the harvester structure is already fabricated. More

discussions will be presented in the following sections in experiment study and FEA simulation.

This section only presents a preliminary analysis for this harvester using the lumped mass model with uniform structural configuration, to show how the natural frequencies can be tuned. However, the prototype used in the experiment study is more complicated to be modeled with mathematical model, as several piezoelectric transducers are attached at different locations. A simulation model with FEA and ECM will be developed to study the behavior of the harvester.

5.3 Experiment Study of the 2-D PEH

5.3.1 Experiment setup

Based on the schematic in Figure 5.1, an experiment prototype for the proposed 2-D harvester is fabricated and installed on a vibration shaker, as shown in Figure 5.3. The frame substrate is fabricated from aluminum plate with thickness of 0.6mm. Macro-fiber-composites (MFC) patches (M-2814-P2, Smart Material Corp.) are served as piezoelectric transducers attached on the outer surface of the aluminum frame substrate. In this study, totally 8 pieces of MFC are attached to avoid the cancellation, and they are numbered as MFC-1 to MFC-8, as shown in Figure 5.3a. The dimensions of the harvester are listed in Table 5.1.

As the shaker used in the experiment is not convenient to change its angle, an additional circular plate is designed for the purpose of tuning the orientation of the 2-D harvester. The circular plate comprises two separate parts. One part is fixed on the shaker, while the other can be rotated around the center, with an interval of 15 degrees, as illustrated in Figure 5.3b. Thus, the orientation of the

2-D harvester can be tuned by rotating the circular plate, while the base excitation from the shaker is always provided in the vertical direction. Then, the vertical excitation from the shaker can be easily converted into the vibration with arbitrary direction in the 2-D plane for the harvester.

Figure 5. 3 (a) Experiment setup, (b) Rotatable circular plate

Table 5. 1 Dimensions of the experiment prototype

<u>Aluminum frame substrate</u>		<u>MFC patches</u>	
Thickness	0.6 mm	Active length	28 mm
Width	20 mm	Active width	14 mm
Length (wall)	72 mm		
Length (beam)	180 mm		

From the preliminary analysis in the previous section, it is known that the difference between the effective mass for two vibration modes can be utilized to tune the two natural frequencies. Once the frame structure is fabricated and installed on the shaker, it is not possible to change its thickness and length, which

means its equivalent stiffness for two modes has already been fixed. However, the two effective masses for the two modes are variable. By changing the value of the central mass, the ratio of two effective masses thus the two natural frequencies can be adjusted. For example, when the central mass is chosen as 9 grams, the two natural frequencies are 40.5 Hz (horizontal mode) and 43.7 Hz (vertical mode); while they can be adjusted to 36.8 Hz (horizontal mode) and 37.2 Hz (vertical mode) for the central mass of 14 grams. As the central mass is increased to 21 grams, the natural frequency of vertical mode (32.5 Hz) is tuned lower than that of the horizontal mode (34.4 Hz). In the experiment, it is difficult to tune the two natural frequencies to be exactly the same, as slightly change of the boundary condition will affect the natural frequencies. After a number of adjustments of the central mass, the configuration with central mass of 14 grams is chosen for later study, as it can harvest the vibration energy in different orientations with a nearly constant operation frequency close to 37 Hz.

In the experiment, a harmonic vibration signal is generated by a function generator and magnified by an amplifier, driving the shaker to vibrate with required harmonic motion. An accelerometer is attached to the shaker to monitor and control the base acceleration to be maintained at the same level during the sweeping of the testing frequency. The base acceleration is kept at 2 m/s^2 during the whole experiment. A data acquisition system (NI 9229) is used to read and record the output response.

5.3.2 Frequency response of open circuit voltage

The frequency responses of open circuit voltage output with different orientations are firstly studied and recorded in the experiment. To obtain the response under

the excitations from different orientations, the harvester is rotated in the 2-D plane with an interval of 15 degree. Due to symmetry, the whole 2-D plane can be divided into 4 quarters which have similar pattern of response. Thus, only one quarter of the 2-D plane is studied in the experiment, i.e., the harvester is orientated from 0 to 90 degree with 15 degree intervals. Totally 7 groups of frequency responses for different directions are plotted in Figure 5.4. As shown in these graphs, the two natural frequencies of the harvester in two directions are almost the same. The resonance frequency for the vertical vibration mode (0 degree) is about 36.8 Hz, while for the horizontal mode (90 degree) it is 37.2 Hz. For other orientations in-between 0-90 degree, only one significant response peak is observed for most cases, locating around 37 Hz. This indicates the harvester can always generate significant output from the excitations in any direction in the 2-D plane, but with same operation frequency.

It is observed from Figure 5.4 that every piece of MFC can generate significant voltage output. At the orientation of 90 degree, the harvester is vibrating in the horizontal mode. According to the strain distribution from Figure 5.2, the two pieces of MFC near the root of the columns (MFC-1 and MFC-8) should have the largest output, while the two pieces of MFC near the central mass (MFC-4 and MFC-5) generate the smallest output. On the contrary, at the orientation of 0 degree, two pieces of MFC near the root (MFC-1 and MFC-8) produce the smallest output, while MFC-4 and MFC-5 generate significant output. It is also noted that MFC-6 and MFC-7 generate even larger output, mainly because of the fabrication and installation defects. According to the strain distribution, in an ideal condition, MFC-6 and MFC-7 should generate slightly lower output than MFC-4 and MFC-5.

For other orientations in-between 0-90 degree, the harvester's vibration motion can be regarded as a combination of the two modes. Cancellation or enhancement of voltage output may happen in MFCs due to the superposition of the strain generated from the two vibration modes. For example, at the orientations of 45 and 60 degree, cancellation is observed in MFC-6 and MFC-7 as their voltage outputs are reduced, while MFC-3 and MFC-2 have enhanced voltage responses. The voltage outputs of all MFCs at excitation frequency of 37 Hz are plotted in Figure 5.5, from which the variations of voltage with different orientations can be easily observed.

Figure 5. 4 Frequency response for different MFC with various orientations

Figure 5. 5 Open circuit voltage versus orientation (37.0 Hz)

If the system is perfectly symmetric, the responses of MFCs for the orientations from 90 to 180 degree can be easily mapped with those for the orientations from 0 to 90 degree. For example, the response of MFC-2 at 165 degree should be the same as that of MFC-7 at 15 degree, and the response of MFC-7 at 165 degree should be the same as that of MFC-2 at 15 degree. Furthermore, for the orientations between 180 and 360 degree, the responses are exactly the same as those between 0 and 180 degree. Thus, the responses in Figures 5.4 and 5.5 from 0 to 90 degree can be used to represent the overall response pattern in the entire 2-D plane.

5.3.3 Power output evaluation

Power output is an important criterion to evaluate the performance of the harvester. In this study, variable resistors and rectifiers are used in the experiment to evaluate the power output of the 2-D harvester.

For every single piece of MFC, its power output is evaluated by direct connection to a variable resistor, while keeping other MFCs in open circuit condition. For different orientations, the optimal resistor values for the MFCs are slightly different. However, as observed from all the experiment data, the optimal resistor values are always around the range of 100 to 120 k Ω . Many cases have been studied to evaluate the power output from individual MFCs, while Figure 5.6 only shows one typical case with MFC-6 at the 60 degree orientation.

Figure 5. 6 Individual power output evaluation

As indicated from the strain distribution pattern in Figure 5.2, it is apparent that there are different phase angles for different MFCs at different vibration modes. Thus, the MFCs cannot be simply connected to each other to get the overall power output. Therefore, in the experiment, 8 rectifiers are used to rectify the outputs of 8 MFCs. The rectified outputs are then connected in series and in parallel for overall power output evaluation, and the results are shown in Figure 5.7 and Figure 5.8, respectively. However, there is certain voltage drop when alternating current (AC) is converted into direct current (DC) across the rectifier. The value of drop slightly varies for different voltage level, which is about 0.8-1.2 V according to the experiment observation. Thus, substantial amount of energy is lost during rectifying.

Figure 5.7 shows the results of the overall power output evaluation with series connection, for different orientations. It can be seen that, this 2-D harvester can always generate significant power output with a base excitation from any orientation in the 2-D plane. The maximum of the overall power output is about 2.9 mw at 60 degree, while the minimum is about 1.8 mw at 15 degree.

Figure 5. 7 Overall power evaluation with series connection after rectification

Similar results are obtained for the power evaluation with parallel connection. The overall optimal resistor value is around 700 k Ω for series connection and about 20 k Ω for parallel condition. The maximum power achieved with parallel connection is about 2.5 mw at 90 degree, while the minimum is about 1.6 mw at 0 degree, which is slightly lower than series connection. The phase angles of the outputs from different MFCs are different. Although cancellation is avoid after rectification, different connection conditions will still present slightly different results. This may able be improved by developing certain regulation circuit. For all the frequency-power responses, slight resonance shifts due to the electromechanical coupling can be observed.

However, it is worth mentioning that, in the experiment, there is an inevitable voltage drop about 1 V or more in each rectifier, which scarifies certain portion of the harvested power. Especially for those MFCs having low voltage outputs, the loss in power is significant. Overall, considerable loss of the power output is observed due to rectifying. Therefore, more efficient rectifying interfaces for such frame harvester are highly desirable, which deserve further investigation.

5.3.4 Other results with different mass

In previous sections, the 2-D harvester working in various orientations but with the same operation frequency (two natural frequencies are very close given the central mass of 14 grams) are studied experimentally. As discussed above, by changing the central mass, the two natural frequencies can be tuned to be separate from each other. Open circuit voltage frequency responses for the configuration of the central mass of 9 grams are shown in Figure 5.8.

Figure 5. 8 Frequency response with central mass of 9 grams

Unlike the previous configuration that provides only one significant response peak for different orientations, this configuration provides different frequency responses as the orientation changes. As shown in Figure 5.8, the natural frequency for the vertical vibration mode (0 degree) is 43.7 Hz, while it is 40.5 Hz for the horizontal vibration mode (90 degree). For these two modes, the harvester responds with single peak at the resonances. However, at other orientations in-between 0-90 degree, the frequency responses exhibit two peaks corresponding to the two modes. For example, at 45 degree, there are two response peaks located at the resonant frequencies of the vertical and horizontal modes. Such kind of configuration is valuable in the scenarios where the environmental vibration sources have not only two directional components but also a broad bandwidth. The frame harvester in this case can serve as a broadband harvester for multi-directional vibrations, which definitely deserves further investigation.

5.4 Validation by Numerical Simulation with Finite Element Analysis and Equivalent Circuit Modelling

It is easy to use FEA to simulate a harvester with complicated structural configuration, but FEA can only work out electrical responses in certain simply condition (i.e. single voltage response with resistor). On the contrary, ECM can be used for the system with complicated interface circuits while the electrical parameters are determined from theoretical analysis or FEA. By combining these two methods, one can model coupled systems like piezoelectric energy harvesters with both structural and electrical complexity. In this section, numerical simulations are carried out by using the combination of FEA and ECM to validate the experimental results.

5.4.1 FEA simulation of 2-D piezoelectric energy harvester

FEA model of the 2-D harvester

The finite element model of the 2-D piezoelectric energy harvester is built in the common FEA software ANSYS. Different types of modelling elements are used for different components: SOLID 226 for the piezoelectric transducers, SOLID 186 for the aluminum substrate and central mass, and CIRCU 94 for the load resistors. The same configuration in the experiment is presented in the FEA model, with 8 pieces of piezoelectric segments attached on the substrate, as shown in Figure 5.9, where different colors represent different types of elements. The electrical displacement of the nodes on the top and bottom surfaces of the piezoelectric transducers are coupled and connected to the load resistor. The parameters are listed in Table 5.2, while the geometric parameters are exactly the same as the experiment setup.

Figure 5. 9 FEA model of 2-D piezoelectric energy harvester

Table 5. 2 Parameters used in the FEA

Items	Value
Tension modulus of MFC (rod direction)	30.336 GPa
Tension modulus of MFC (electrode direction)	15.857 GPa
Modulus of aluminum	69 GPa
Density of MFC	5440 Kg/m ³
Density of aluminum	2700 Kg/m ³
Piezoelectric constant (d_{31})	-170 pC/N
Piezoelectric constant (e_{31})	5.157 C/m ²
Damping ratio	1.1%
Capacitance (single piece of MFC-2814-p2)	25.7 nF

It is convenient to obtain the harvester's vibration modes and related natural frequencies through modal analysis in FEA. As the higher order vibration modes

presented at higher frequencies are of no interest in this study, only the first two vibration modes are considered. The resonance frequencies for the first two vibration modes are listed in Table 5.3, with comparison to the experimental results. Only minor differences are observed, indicating that the FEA model gives good prediction of the harvester's vibration motions. The configuration with central mass of 14 grams is mainly studied in the following simulation, with two open circuit resonances of 37.205 Hz (vertical) and 37.443 Hz (horizontal), which are close to the experimental results of 36.8 Hz and 37.2 Hz.

Table 5. 3 Comparison of the resonance frequencies from experiment and FEA
(unit: Hz)

Central mass value	Horizontal mode		Vertical mode	
	Experiment	FEA	Experiment	FEA
9 grams	40.5	41.480	43.7	43.938
14 grams	37.2	37.443	36.8	37.205
21 grams	34.4	33.783	32.5	31.809

Frequency response the 2-D harvester from FEA

With the steady-state harmonic analysis, the frequency responses of voltage of each piezoelectric transducer can be obtained. To simulate the open circuit condition, all the load resistors are set with an extremely high value ($10^9 \Omega$). While the structural damping ratio is set as 1.1%, which is obtained through the attenuation test with the experiment prototype. In the harmonic analysis, the acceleration of the base excitation is set as 2m/s^2 , while its orientation can be changed in the 2-D plane (X-Z plane, as indicated in Figure 5.9). Thus, with the same orientations tested in the experiment, similar open circuit voltage frequency responses are worked out.

Figure 5. 10 Open circuit voltage frequency response obtained from FEA
(central mass of 14 grams)

Figure 5.10 shows 4 groups of results of the open circuit voltage response with different orientations (0, 15, 45 and 90 degrees), which can be compared with the experimental results in Figure 5.4. Because of the ideal symmetric condition of the FEA model, the voltage responses of MFC-1,2,3,4 are exactly the same as those of MFC-8,7,6,5 at both 0-degree and 90-degree orientations. Similar pattern of the frequency responses to the experiment results in Figure 5.4 can be observed. For example, MFC-1 and MFC-8 generate the lowest output at 0 degree, while the highest output at 90 degree. It can be observed that for MFC-4 and MFC-5 present large difference as compared to the experiment results. Which may because of the imperfect fabrication. And this imperfection is observed more serious when the harvester is located in 0 degree, as the two columns may not located in the purely vertical direction in the experiment. The frequency responses at 15 and 45 degree clearly show the trend of superposition and cancellation effects due to the orientation change, similar to the experiment. There are some discrepancies between the experiment and FEA results, as the experiment prototype is not perfectly fabricated and fixed onto the shaker well

enough for symmetric condition while the FEA model is built in ideal condition. The rest three groups at 30, 60 and 75 degree are not shown here as they are also similar to the experiment.

5.4.2 Identification of parameters to be used in the ECM

As reviewed in Chapter 2, the development of ECM method is based on the analogies exist in mechanical and electrical domain. For an electromechanical system such as a piezoelectric energy harvesting system, those parameters in the mechanical domain can be transferred into the parameters in electrical domain through theoretical analysis or FEA. With the identified electrical parameters, the response of the harvester can be evaluated with a SPICE (*Simulation Program with Integrated Circuit Emphasis*) simulator.

Through the vibration analysis of a coupled piezoelectric energy harvesting system (Erturk and Inman 2008b), the governing equations of a piezoelectric energy harvester can be expressed in the modal coordinate as,

$$\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2\eta_r(t) + \chi_r V(t) = -f_r \ddot{u}_g(t) \quad (5.6)$$

$$\frac{V(t)}{R} + C_p \frac{dV(t)}{dt} - \sum_{r=1}^{\infty} \chi_r \frac{d\eta_r(t)}{dt} = 0 \quad (5.7)$$

$$\chi_r = \int_{x_1}^{x_2} \theta(x) \frac{d^2\phi_r(x)}{dx^2} dx \quad (5.8)$$

$$f_r = \int_0^L m(x)\phi_r(x) dx + M_t\phi_r(L) \quad (5.9)$$

where $\phi_r(x)$ and $\eta_r(t)$ are the mass-normalized eigenfunction and modal coordinate, respectively; $V(t)$ is the voltage response of the harvester; ζ_r is the damping ratio; ω_r is the natural frequency at the short circuit condition; χ_r is the

modal coupling coefficient while $\theta(x)$ is the electromechanical coupling coefficient determined by the property of the cross section; f_r is the normalized modal force amplitude and $\ddot{u}_g(t)$ is the external excitation function; $m(x)$ and M_t are the distributed mass and tip mass, respectively; R is the load resistance and C_p is the capacitance of the piezoelectric transducer. The subscript r refers to the r -th vibration mode. Note that, the integral boundary for the modal coupling coefficient (χ_r) is determined by the position of the piezoelectric transducer.

The analogy between the parameters in the mechanical and electrical domains is listed in Table 5.4. With such analogy, the coupled energy harvesting system can be represented with electrical components only, and modeled in SPICE software. For the harvesters with complex structural configuration that the mechanical parameters (e.g. mode shapes) are difficult to obtain from theoretical analysis, FEA can be used. The electrical parameters can then be calculated through analogy. The detailed procedure is illustrated in (Yang and Tang 2009).

Table 5. 4 Parameter analogy between mechanical and electrical domain

Electrical parameters	Mechanical parameters
Charge: $q_r(t)$	$\eta_r(t)$
Inductance: L_r	1
Resistor: R_r	$2\zeta_r\omega_r$
Capacitance: C_r	$1/\omega_r^2$
Voltage source: $V_r(t)$	$-f_r\ddot{u}_g(t)$
Transformer ratio: N_r	χ_r

For the 2-D proposed harvester, Equations (5.6) and (5.7) should be modified to take into account the multiple voltage outputs, as there are 8 pieces of MFC transducer bonded to the frame. To do so, the governing equations for the vertical vibration mode should be re-written as:

$$\begin{cases} \frac{d^2\eta_v(t)}{dt^2} + 2\zeta_v\omega_v \frac{d\eta_v(t)}{dt} + \omega_v^2\eta_v(t) + \sum_{i=1}^8 \chi_{v-i}V_i(t) = -f_v\ddot{u}_g(t) \\ \frac{V_1(t)}{R_1} + C_p \frac{dV_1(t)}{dt} - \chi_{v-1} \frac{d\eta_v(t)}{dt} = 0 \\ \dots \dots \dots \\ \frac{V_8(t)}{R_8} + C_p \frac{dV_8(t)}{dt} - \chi_{v-8} \frac{d\eta_v(t)}{dt} = 0 \end{cases} \quad (5.10)$$

Similar equations apply to the horizontal vibration mode by changing the subscript v to h in Equation (5.10). It needs to be mentioned again that the natural frequencies ω_v and ω_h are the short circuit ones, that is, all MFCs are short circuited. It is convenient to work out these two values in FEA by setting the fixed boundary conditions for all resistor terminals. For the central mass of 14 grams, the two short circuit resonances are obtained through FEA as, $\omega_v = 36.832$ Hz and $\omega_h = 37.119$ Hz. Subsequently, the values of L_v , C_v and R_v are readily derived as: $L_v=1$ H, $C_v=1/\omega_v^2=1/(2\pi*36.832)^2=1.8672e^{-5}$ F, and $R_v=2\zeta_v\omega_v=2*0.011*2\pi*36.832=5.0913$ Ω . Similar calculation can be carried out for the horizontal vibration mode.

In Equation (5.10), there are totally 8 modal coupling coefficients χ_{v-i} ($i = 1, 2 \dots 8$) to be determined for each vibration mode. To determine these coefficients as well as the modal force amplitude, the normalized modal shape functions are extracted from the nodal displacements obtained by FEA. The two modal shapes for the vertical and horizontal vibration modes are shown in Figure 5.11. With those modal shape functions, the 8 coupling coefficients can be calculated using Equation (5.8), in which $\theta(x)$ is expressed as

$$\theta(x) = e_{31}h_p b \quad (5.11)$$

where h_p is the distance from the center of piezoelectric layer to the neutral axis of the cross section, and b is the width of the piezoelectric patch.

Figure 5. 11 Modal shapes of 2-D harvester from FEA

The modal force amplitudes f_v and f_h can be calculated with the integral of modal shape function using Equation (5.9). For the horizontal vibration mode, the term M_t in Equation (5.9) need be slightly modified, as the tip mass for the horizontal vibration mode not only consists of the central mass, but also includes the total mass of the horizontal beam. As the external excitation is 2m/s^2 , which is twice of the unit harmonic function, the corresponding voltage magnitude should also be twice of f_v or f_h .

With above calculations for both vibration modes, all the electrical parameters are worked out and listed in Table 5.5. It is worth mentioning that the sign of the

transformer ratios (N_{v-i} and N_{h-i}) are very important which will be used in later ECM simulation.

Table 5. 5 Parameters indentified from FEA

Vertical mode		Horizontal mode	
Inductance: L_v (H)	1	Inductance: L_h (H)	1
Resistor: R_v (Ω)	5.0913	Resistor: R_h (Ω)	5.1310
Capacitance: C_v (μF)	18.672	Capacitance: C_h (μF)	18.384
Voltage source amplitude	0.2868	Voltage source amplitude	0.3284
Transformer ratio: N_{v-1}	0.5585e-3	Transformer ratio: N_{h-1}	2.461e-3
N_{v-2}	-1.583e-3	N_{h-2}	-0.793e-3
N_{v-3}	-1.526e-3	N_{h-3}	-1.794e-3
N_{v-4}	2.441e-3	N_{h-4}	-0.214e-3
N_{v-5}	2.441e-3	N_{h-5}	0.214e-3
N_{v-6}	-1.526e-3	N_{h-6}	1.794e-3
N_{v-7}	-1.583e-3	N_{h-7}	0.793e-3
N_{v-8}	0.5585e-3	N_{h-8}	-2.461e-3

5.4.3 ECM simulation and comparison of results

With the parameters identified in the previous section, it is easy to build an ECM to represent the coupling system working in the vertical or horizontal vibration mode. For example, the ECM for the vertical vibration mode is shown in Figure

5.12, by adopting the parameters from Table 5.5. Similar to FEA, each piezoelectric transducer is connected with a resistor with extremely high resistance ($10^9 \Omega$).

Figure 5. 12 ECM of 2-D harvester for vertical vibration mode

To model the harvester working in the other orientations with combined vibration modes, the governing equation should be further modified as

$$\left\{ \begin{array}{l} \frac{d^2 \eta_v(t)}{dt^2} + 2\zeta_v \omega_v \frac{d\eta_v(t)}{dt} + \omega_v^2 \eta_v(t) + \sum_{i=1}^8 \chi_{v-i} V_i(t) = -f_v \ddot{u}_g(t) \cos(\psi) \\ \frac{d^2 \eta_h(t)}{dt^2} + 2\zeta_h \omega_h \frac{d\eta_h(t)}{dt} + \omega_h^2 \eta_h(t) + \sum_{i=1}^8 \chi_{h-i} V_i(t) = -f_h \ddot{u}_g(t) \sin(\psi) \\ \frac{V_1(t)}{R_1} + C_p \frac{dV_1(t)}{dt} - \chi_{v-1} \frac{d\eta_v(t)}{dt} - \chi_{h-1} \frac{d\eta_h(t)}{dt} = 0 \\ \dots \dots \dots \\ \frac{V_8(t)}{R_8} + C_p \frac{dV_8(t)}{dt} - \chi_{v-8} \frac{d\eta_v(t)}{dt} - \chi_{h-8} \frac{d\eta_h(t)}{dt} = 0 \end{array} \right. \quad (5.12)$$

where ψ stands for the angle of the harvester's orientation. With this modification, the influence of different orientations is taken into account by the angular coefficients $\sin(\psi)$ and $\cos(\psi)$. With this governing equation, the ECM is modified accordingly, as shown in Figure 5.13. The two vibration modes are coupled with each other, with the angular coefficients applied to the source amplitudes. In Figure 5.13, it is important to point out that, the output of MFC-4, 6, 7 and 8 for the two vibration modes should be connected with opposite terminals, due to the opposite sign of the coupling coefficients (as listed in Table 5.5). Subsequently, the voltage frequency response can now be worked out with any orientation using this ECM.

Figure 5. 13 ECM of 2-D harvester with combination of two vibration modes

The open circuit voltage frequency responses from ECM simulation and experiment results at the orientation of 45 degree are compared in Figure 5.14.

The comparison indicates good match between the experimental outcome and the simulation results. Similar results can be obtained for other orientations, which are not shown here. It is concluded that FEA and ECM methods are capable of modeling the 2-D harvester connected with a simple resistive load. However, FEA can only work with simple circuit condition; it cannot work with nonlinear electric components, such as rectifiers. If a complex interface circuit is employed in the energy harvesting system, only ECM can be adopted for systemic simulation.

Figure 5.14 Comparison of ECM and experiment results for 45 degree orientation

To simulate the overall power output in the same configuration as the experiment, rectifiers are needed in the interface circuit for series and parallel connections of the MFCs. The ECM for series connection is shown in Figure 5.15, in which a wattmeter is also connected with the resistor to read the overall power output.

Figure 5. 15 ECM for series connection after rectification

One case study with the orientation of 75 degree is shown in Figure 5.16(a), indicating similar trend as compared with the experimental results shown in Figure 5.7, in spite of certain discrepancies for the peaks' amplitudes and frequencies. Moreover, the overall power output for the 2-D harvester working at various orientations but with the same operation frequency (37.1 Hz) is presented in Figure 5.16(b). These simulation results together with the experiment outcomes indicate that the proposed 2-D piezoelectric energy harvester is able to consistently provide significant power output with any orientations in the 2-D domain. Moreover, as the power output obtained in this study is not optimized, there are good potentials to further improve the output

efficiency, either by optimizing the structural parameters or by employing some advanced interface circuits.

(a) Overall power output with series connection at 75 degree

(b) Overall power output versus orientation at 37.1 Hz

Figure 5. 16 Overall Power evaluation by ECM

The study shown in this section demonstrate a robust modeling method by combining FEA and ECM simulations. The results are validated as compared to the experiment. Such method is powerful to model a piezoelectric energy harvesting system with both structural and electrical complexity.

5.5 Chapter Summary

In this chapter, a PEH with a frame configuration is proposed for multiple-directional vibration energy harvesting in 2-D plane. Experimental work is first carried out to study the behavior of such 2-D harvester when subjected to base excitations from various directions. When the structural parameters are well-tuned, this harvester can consistently provide significant power output with excitations of the same frequency from any orientation in the 2-D domain. In addition, it can be designed to harvest vibration energy in two different directions with different working frequencies or to harvest broadband vibration energy in specific orientations.

Moreover, a simulation procedure with combination of FEA and ECM methods is presented, to provide a robust tool to model and design such system with both structural and electrical complexity.

In summary, the proposed 2-D frame-type PEH possesses a promising potential in practical vibration energy harvesting

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

This thesis presents the research work conducted by the author, including the theoretical analysis, numerical simulation and experimental studies on vibration energy harvesting using piezoelectric materials. This work is focused on the enhancement of performance for vibration piezoelectric energy harvesting by using multi-modal technique. The work carried out can be summarized as follows:

- (1) To develop an applicable broadband piezoelectric energy harvesting system, a novel 2-DOF configuration has been proposed, prototyped and experimentally tested. In this novel design, the secondary beam is fabricated by cutting the inside of the main beam, instead of extending it. With this unique design, this novel 2-DOF PEH provides a larger bandwidth by achieving two close effective resonant peaks in the frequency response, where both can generate significant output. Additionally, such 2-DOF PEH is more compact than the conventional SDOF PEH by utilizing the cantilever beam more efficiently. Such PEH is more applicable than the other 2-DOF PEH designs by utilizing its first two resonant peaks which are close and adequate, with no increase of volume. Subsequently, a mathematical distributed parameter model as well as a FEA model have been developed to validate the experiment finding.
- (2) Although the novel linear 2-DOF PEH has already been validated for improving the bandwidth by using its first two resonant peaks, there always exist a response valley in-between the two resonant peaks which greatly deteriorates the performance of the harvester. To further improve the bandwidth, a nonlinear 2-DOF PEH is proposed, by incorporating magnetic

nonlinearity into the linear 2-DOF PEH design. The experimental parametric study shows that, with a properly chosen structural configuration, much wider bandwidth can be achieved. Compared to its linear 2-DOF counterpart, the nonlinear 2-DOF PEH can provide much broader bandwidth and with higher efficiency when charging a storage capacitor. A lumped parameter model of the nonlinear 2-DOF PEH is also developed considering the dipole-dipole magnetic force. This model successfully predicts the similar broadband response of the proposed harvester.

- (3) To ensure the harvester work in applicable environment, in which the energy source may come from different orientations, a 2-D multi-modal PEH is developed to harvest energy in 2-D domain. By utilizing its first two vibration modes of the frame configuration, the PEH can consistently generate significant power output for any orientation in the 2-D domain. Experimental results suggest promising potential for implementing such 2-D PEH in practical application. Furthermore, a general modeling procedure is also presented, by using a combination of FEA and ECM simulation. Such modeling procedure is more robust and suitable for the energy harvesting system with both structural and electrical complexity.

6.2 Recommendations for Future Work

The ultimate goal for energy harvesting technology is to achieve self-powered systems for small electronics (i.e. sensors), so that there is no requirement for batteries. Although numerous research attempts have been made in the past few years, including the contributions reported in this thesis, there are still many challenges ahead before the vibration energy harvesting techniques can be widely deployed in actual practices. Based on the experiences accumulated throughout

this study, the author believes that the research on vibration energy harvesting can be further extended as follows:

(1) Study of bi-stable configuration for the proposed nonlinear 2-DOF PEH

In Chapter 2, due to the limitations in the fabrication and test of the nonlinear 2-DOF PEH, only mono-stable nonlinear vibration is studied. When tuned into bi-stable configuration by changing the magnets distance, it is very hard to maintain a symmetric condition for the two potential wells. Vibration motion is always confined in the lower potential well, due to the gravity and fabrication defect. The prototype may need to be modified to further study the bi-stable behavior.

(2) Snap-through of bi-stable nonlinear vibration.

Although there are many solutions for broadband energy harvesting, among them, nonlinear technique has attracted the most interest in recent years, especially for bi-stable nonlinear vibration. Bi-stable nonlinear energy harvesting has shown its potential to greatly improve the bandwidth as well as the efficiency, provided the external excitation is large enough to make it snap-through. However, if the bi-stable harvester cannot snap-through its potential barrier between its two potential wells, its output will be greatly reduced. It is important to develop certain mechanism to make a bi-stable energy harvesting system easier to snap-through. The author has started to investigate a bi-stable harvester with un-balance configuration, which is believed to be helpful for achieving snap-through.

(3) Multi-directional energy harvesting and optimization

As presented in Chapter 5, the 2-D frame-type multi-modal PEH is validated to achieve multi-directional energy harvesting in 2-D domain. However, the harvester presented is not working efficiently, which requires further optimization. The enhancement can be achieved in two aspects: optimization of the structural parameters and segmentation, and improving efficiency with advanced regulation circuit interface.

(4) Ultra-low frequency energy harvesting (human motion)

In recent years, development of wearable electronics has attracted more and more attention in both academics and industry. There is great potential to integrate harvesters into those wearable electronics to harvesting energy from human motion. The biggest problem is that, the human motion is irregular with very low frequency, a frequency up-conversion mechanism is required. To find an efficient method for the conversion remains a great challenge.

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APPENDIX: AUTHOR'S PUBLICATIONS

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