Variability of soil-water characteristic curve

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2016

Zhai, Q. (2016). Variability of soil-water characteristic curve. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/66938>

<https://doi.org/10.32657/10356/66938>

ACKNOWLEDGEMENTS

First of all, I would like to express my sincere appreciation to my lovely supervisor, Professor Harianto Rahardjo for his continual encouragement and guidance. Without his help and support, it seems impossible to complete this research.

Secondly, I am grateful to Mr. Vincent Heng Hiang Kim, Ms. Susie Lim-Ding and Mr Eugene Tan Hiap Guan for their laboratory technical helps and sharing their valuable knowledge.

I would also like to thank Dr. Alfrendo Satyanaga Nio, Dr. Sugeng Krisnanto for their experience sharing in conducting unsaturated soil tests.

Finally, I would like to express my gratitude to my parents and my deeply love to my wife Miao Miao, lovely daughter Zhai Xiao Yu and son Zhai Xiao Yi for their encouragement and understanding throughout my life.

ABSTRACT

In recent years, unsaturated soil mechanics as related to soil above ground water table and compacted soil, has received increased attention in geotechnical engineering. The soil-water characteristic curve (SWCC) defines the relationship between the water amount in the soil (i.e. gravimetric water content, w , volumetric water content, θ_w , or the degree of saturation, S) and the suction in the soil. The SWCC contains the fundamental information to solve problems related to unsaturated soil mechanics. Other unsaturated soil properties such as permeability and shear strength can be predicted from SWCC. SWCC can be determined from experimental measurement or predicted from basic soil properties. Experimental measurement provides more reliable SWCC rather than the method of prediction from the basic soil properties. However, there is variability in SWCC regardless of which method is used. This variability can affect the accuracy of predicting unsaturated soil properties. Therefore, it is necessary to understand the variability in SWCC before using it to predict other unsaturated soil properties.

In this research variability of SWCC during experimental measurement and data interpretation was investigated and discussed. Equations for quantification of variability in SWCC, determination of SWCC variables such as air-entry value, slope at the inflection point, residual suction and residual water content, hydraulic conductivity from fitting parameters in Fredlund and Xing's (1994) equation were derived. The equation for estimation of hydraulic conductivity from SWCC using capillary model was proposed. The equations for estimation of wetting SWCC from drying SWCC using capillary model was also proposed. The variation in measured water content for compacted mixture of sand and kaolin and residual soil prepared in this research during drying and wetting process was presented. The variability in SWCC during data interpretation with respect to soil volume change, number of data points adopted and suction range covered for the best fit procedure was presented and discussed. The variation in SWCC variables using both conventional graphical method and equations proposed in this research was also presented. The variability in SWCC and hydraulic conductivity for residual soils in Singapore was quantified using confidence limits proposed in this research. A new frame which incorporated with the confidence limits was proposed for prediction of SWCC for residual soils in Singapore from saturated hydraulic conductivity.

From the study in this research, both water content and soil volume change is suggested to be monitored during SWCC measurement, and more data points to be collected within the estimated transition zone. SWCC variable such as air-entry value instead of SWCC fitting parameters is suggested to be correlated with soil properties.

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List of symbols

a in Gardner's (1958) equation = fitting parameter related to the inverse of air-entry value

a in Brooks and Corey's (1964) equation = fitting parameter related to air-entry value

a in Fredlund and Xing's (1994) equation = fitting parameter related to air-entry value

a in Van Genuchten's (1980) equation = fitting parameter related to the inverse of air-entry value

a_g, b_g, c_g, d_g, e_g in Gupta and Larson's (1979) equation = regression coefficients

a_b in Rawl et al.'s (1992) equation = fitting parameter related to air-entry value (cm)

AEV = air-entry value

a_s in Fredlund et al.'s. (2002) equation = a parameter related to the initial breaking point of the curve

a, b in Gardner's (1958) equation = fitting parameters

a, n, m in Leong and Rahardjo's (1997b) equation = fitting parameter in Fredlund and Xing's (1994) equation with correction factor $C(\psi)=1$

$a_{\max}$ and $a_{\min}$ in Zhai and Rahardjo (2013b) equation = upper limit and lower limit of fitting parameter a from Fredlund and Xing (1994) equation, respectively

a_1, n_1 and m_1 in Equation (3-32) are fitting parameters

A_d in Kunze et al.'s (1968) equation = adjusting constant

b_b in Rawl et al.'s (1992) equation = fitting parameter related to pore-size distribution index

b_s in Fredlund et al.'s. (2002) equation = a parameter related to the steepest slope of the curve

$\tilde{b}$ in Mishra and Parker's (1989) equation = the vector of the estimated parameter

b in Mishra and Parker's (1989) equation = the vector of the true parameter

c' = effective cohesion

C_r in Fredlund and Xing's (1994) equation = parameter related to the residual suction, selected as $C_r=1500$ kPa for most cases. (Zhai and Rahardjo (2012a, b)

c_s in Fredlund et al.'s. (2002) equation = a parameter related to the shape of the fines portion of the curve

$C(d)$ in Fredlund et al.'s. (2002) equation = correction factor to ensure that the function goes through a lower limit particle diameter of 0.00001 mm

C in Tinjum et al. (1997) equation = categorical variable, ($C=1$ for mod. proctor, $C=-1$ for std. proctor)

c' in Fredlund et al. 's (1978) equation = effective cohesion

c' in Vanapalli et al.'s (1996) equation = effective cohesion

d in Fredlund et al.'s. (2002) equation = particle diameter (mm)

d_r in Fredlund et al.'s. (2002) equation = a parameter related to the amount of fines in a soil

d_m in Fredlund et al.'s. (2002) equation = the diameter of the minimum allowable size particle

D_{60} in Zapata's (1999) equation = grain diameter corresponding to 60% passing by weight or mass (mm)

e = void ratio

e_0 , e_{max} = initial void ratio

E in Mishra and Parker's (1989) equation = the statistical expectation

Fine% = percentage of fine content

GSD = grain size distribution

G_s = specific gravity

i in Kunze et al.'s (1968) equation = interval number, which increases as the volumetric water content decreases

j in Kunze et al.'s (1968) equation = a count from “i” to “m”

J in Mishra and Parker's (1989) equation = the parameter sensitivity matrix or Jacobian matrix

k_s = saturated hydraulic conductivity

k_w in Gardner's (1958) equation = relative hydraulic conductivity

k_r in Corey's (1954) equation = relative hydraulic conductivity

k_r in Mualem's (1976) equation = relative hydraulic conductivity

k_r in Leong and Rahardjo's (1997b) equation = relative hydraulic conductivity

$k_w(\theta_w)_i$ in Kunze et al.'s (1968) equation = predicted water coefficient of permeability for the volumetric water content

k_s in Kunze et al.'s (1968) equation = measured saturated coefficient of permeability (m/s)

k_{sc} in Kunze et al.'s (1968) equation = calculated saturated coefficient of permeability (m/s)

k, b = fitting parameters for estimation of hysteresis of SWCC due to contact angle different in drying and wetting process

LL = liquid limit

m in Fredlund and Xing's (1994) equation = fitting parameter

m' in Leong and Rahardjo's (1997b) equation = mp

m in Kunze et al.'s (1968) equation = total number of intervals between the saturated volumetric water content, θ_s , and the lowest volumetric water content, θ_L

M in Mishra and Parker's (1989) equation = the number of experimental data points

m_{max} and m_{min} in Zhai and Rahardjo (2013b) equation = upper limit and lower limit of fitting parameter m from Fredlund and Xing (1994) equation, respectively

n in Gardner's (1958) equation = fitting parameter related to the pore-size distribution index

n in Fredlund and Xing's (1994) equation = fitting parameter

n in Rawl et al.'s (1992) equation = porosity

$\bar{n}$ in Zapata's (1999) equation = average value of fitting parameter n

$n = 0.5$, $m=0$ in Mualem's (1976) equation as suggested by Mualem

n_s = porosity corresponding to the saturated state

n_m = porosity corresponding to suction ψ_m

$n_{\max}$ and $n_{\min}$ in Zhai and Rahardjo (2013b) equation = upper limit and lower limit of fitting parameter n from Fredlund and Xing (1994) equation, respectively

PL= plastic limit

p in Van Genuchten's (1980) equation = fitting parameter related to the pore-size distribution of the soil

P in Fredlund et al's. (2002) equation = percent passing at any particular grain-size

PI in Zapata's (1999) equation = plasticity index (%)

PI in Tinjum et al. (1997) equation = plasticity index

p in Leong and Rahardjo's (1997b) equation = additional fitting parameter

PI = Plastic index = LL-PL

P in Mishra and Parker's (1989) equation = the number of fitting parameters

q in Van Genuchten's (1980) equation = fitting parameter related to the asymmetry of the SWCC curve

r = radius of the meniscus

r_t = equivalent effective radius for water flow

SWCC = Soil-water characteristic curve

S = degree of saturation

SSE = sum of squared error

S_e in Corey's (1954) equation = effective degree of saturation defined as $\frac{S - S_r}{1 - S_r}$

$S_{\text{dscanning}}$ = degree of saturation corresponding to the drying scanning curve

s^2 in Mishra and Parker's (1989) equation = the sum of the squared error

Sand% = percentage of sand content

s^2 = standard deviation

T_s = surface tension

u_a = pore-air pressure

u_w = pore-water pressure

$(u_a - u_w)$ in Fredlund et al. 's (1978) equation = matric suction

$(u_a - u_w)$ in Vanapalli et al.'s (1996) equation = matric suction

$(u_a - u_w)_b$ in Ghosh's (1980) equation = air-entry value

Var(a) in Zhai and Rahardjo (2013b) equation = variance of fitting parameter a from Fredlund and Xing (1994) equation

Var(n) in Zhai and Rahardjo (2013b) equation = variance of fitting parameter n from Fredlund and Xing (1994) equation

Var(m) in Zhai and Rahardjo (2013b) equation = variance of fitting parameter m from Fredlund and Xing (1994) equation

w = gravimetric water content

wPI in Zapata's (1999) equation = passing #200*PI

wPI in Zapata's (1999) equation = passing #200*PI

W in Tinjum et al. (1997) equation = compaction water content, %

W_{opt} in Tinjum et al. (1997) equation = optimum water content, %

y = variability

α = contact angle

β in Ghosh's (1980) equation = empirical constant

ΔS_1 = Hysteresis of SWCC due to the difference in contact angle during the drying and wetting process

ΔS_{ri+k} = reduction in the degree of saturation due to the "ink-bottle" effect

ΔS_{wet} = recovering degree of saturation during the wetting process with decreasing suction from ψ_m to ψ_i

δ in Corey's (1954) equation = a constant, = 4 as suggested by Corey

θ_w in Mualem's (1976) equation = volumetric water content

θ_s in Mualem's (1976) equation = saturated volumetric water content

$(\theta_w)_i$ in Kunze et al.'s (1968) equation = corresponding to the i^{th} interval (m/s)

θ_{upper} and θ_{lower} in Zhai and Rahardjo (2013b) equation = volumetric water content corresponding to upper confidence limit and lower confidence limit of SWCC, respectively

θ_w = volumetric water content

θ_w in Gardner's (1958) equation = volumetric water content

θ_r in Gardner's (1958) equation = residual volumetric water content

θ_s in Gardner's (1958) equation = saturated volumetric water content

θ in Gupta and Larson's (1979) equation = predicted volumetric water content

θ_r in Rawl et al.'s (1992) equation = residual volumetric water content

θ_w in Gardner's (1958) equation = volumetric water content

Θ in Vanapalli et al.'s (1996) equation = normalized volumetric water content defined as

$$\frac{\theta - \theta_r}{\theta_s - \theta_r}$$

θ_i = predicted volumetric water content at i suction level, from the confidence limit of the best fitted SWCC or from experimental data

$\hat{\theta}_i$ = best estimated volumetric water content at i suction level, from the best fitted SWCC

θ_s = saturated volumetric water content

θ_r = residual volumetric water content

λ in Brooks and Corey's (1964) equation = fitting parameter termed the pore-size distribution index

ρ_d in Gupta and Larson's (1979) equation = bulk density (gm/cm^3)

ρ_d = dry density

σ in Kosugi's (1994) equation = fitting parameter

$(\sigma_n - u_a)$ in Fredlund et al.'s (1978) equation = net normal stress

$(\sigma_n - u_a)$ in Vanapalli et al.'s (1996) equation = net normal stress

τ in Fredlund et al.'s (1978) equation = shear strength of unsaturated soil

τ in Vanapalli et al.'s (1996) equation = shear strength of unsaturated soil

ϕ' = effective friction angle

ϕ^b = angle indicating the rate of increase in shear strength relative to matric suction

ϕ' in Fredlund et al. 's (1978) equation = effective internal friction angle of saturated soil

ϕ^b in Fredlund et al. 's (1978) equation = internal friction angle with respect to matric suction

ϕ' in Vanapalli et al.'s (1996) equation = effective internal friction angle of saturated soil

ψ in Gardner's (1958) equation = matric suction

ψ_c in Kosugi's (1994) equation = fitting parameter

ψ_0 in Kosugi's (1994) equation = fitting parameter

ψ = matric suction

ψ' = estimated suction on the drying curve

ψ_m = suction state in the soil

ψ_r = residual suction

%S in Rawl et al.'s (1992) equation = percentage of sand

%C in Rawl et al.'s (1992) equation = percentage of clay

CHAPTER ONE

INTRODUCTION

1.1 Research Background

In recent years, unsaturated soil mechanics, as related to soil above the ground water table and compacted soil, has received increased attention in geotechnical engineering. The soil-water characteristic curve (SWCC), which describes the relationship between the amount of water in soil and matric suction, contains the fundamental information to solve problems relating to unsaturated soil mechanics. There are several measures of the amount of water in soil, including the degree of saturation (S), volumetric water content (θ_w) and gravimetric water content (w). As direct measurement is time-consuming and costly, SWCC is commonly used to predict other unsaturated soil properties, such as the permeability function and shear strength. Fredlund et al. (2012) presented that the estimation of unsaturated soil properties from SWCC was approximate, but was generally satisfactory for analyzing unsaturated soil mechanics problems. Fredlund and Houston (2009) provided recommendations and suggestions for the determination and use of soil-water characteristic curve and consequently, for the computation of unsaturated soil property function (USPFs). SWCC is normally considered analogous to the pore-size distribution function and can be determined either from direct experimental measurement or indirect prediction based on the basic soil properties. Direct experimental measurement provides a more reliable SWCC result, while the indirect method provides a less accurate result. However, there is variability in SWCC regardless of which method is used. This variability can affect the accuracy of predicting other unsaturated soil properties. Therefore, it is necessary to understand the variability in SWCC before using it to predict other unsaturated soil properties.

Measurement error always exists in collected data and is dependent on the method and apparatus used in the experiment. Measurement error in water content and volume can lead to uncertainty in the determined SWCC (i.e., it can affect the calculated degree of saturation (S) and volumetric water content (θ_w)). Using different equipment or devices (e.g., Tempe cells, pressure plates, capillary tubes, tri-axial equipment, etc.) can also result in variability in the determined SWCC. SWCC will follow different paths (normally called scanning curves) if the specimen is saturated/de-saturated with different initial water contents. Variation in

measured water content may result from the hysteresis characteristic of SWCC. Therefore, aspects of experimental measurement, such as measurement error and the hysteresis characteristic, can lead to variability in SWCC.

After direct experimental measurement, the experimental data are fitted with the best fit equation to determine SWCC. The best fit equation is a continuous mathematical model representing SWCC. Various best fit equations have been proposed by different researchers, but the fitting parameters are always obtained by a curve fitting technique. In other words, the best fitted SWCC is a regression resulting from the observed samples (i.e., the experimental data). The different best fit equations perform differently at representing the real SWCC from the experimental data. Compared to other best fit equations, Zapata (1999) concluded that the least variability in SWCC was obtained by using Fredlund and Xing's (1994) equation. In a word, variability in SWCC may result from data interpretation.

Residual soil is developed or weathered from parent material and has the same general chemistry as the parent material. Weathering of the parent material can occur via physical weathering (disintegration), chemical weathering (decomposition) or chemical transformation. Therefore, the properties of residual soil can vary drastically due to their parent material and forms of weathering. The different textures and arrangements of soil particles in residual soil can result in different SWCCs.

Tinjum et al. (1997) and Yaldo (1999) showed that SWCC shape could be affected by compaction water content and compactive effort. Zapata (1999), Zapata et al. (2000) investigated the variability in SWCCs for three types of soil analyzed by different laboratories, and concluded that the highest variability in SWCC occurs near the air-entry value (AEV) and residual suction. Thu et al. (2007) presented different SWCCs for the same soil specimen with different confining pressures. Dye et al. (2011) showed that variability in SWCC had a significant effect on seepage analysis results. Rahardjo et al. (2012) demonstrated the variability in residual soil properties by analyzing experimental data, such as grain size distribution data (GSD), soil-water characteristic curve (SWCC), liquid limit (LL), plastic limit (PL), natural water content (w), void ratio (e), effective cohesion (c'), effective friction angle (ϕ') and angle indicating the rate of increase in shear strength relative to the matric suction (ϕ^b), for residual soils weathered from Bukit Timah granite, Jurong Formation and Old Alluvium. The works of Rahardjo et al. (2012) suggested there was

variability in the SWCCs for residual soil derived from the same formation but different depths relative to ground level.

Zapata (1999) summarized the factors that may result in the variability in SWCC as follows:

- (1) The number of data points used to define the SWCC;
- (2) The method used to acquire the matric suction data;
- (3) The range in suction covered by the actual measurements;
- (4) The best fit equation used to fit the experimental data;

Many factors at different stages can lead to variability in SWCC. This research investigates variability in SWCC during the experimental measurement and data interpretation stages. Statistical analysis and theories developed in this research are applied to SWCCs for residual soils in Singapore, as collected from the published literature. The factors considered in this research are as follows:

A. Experimental measurement stage:

- (1) Measurement error in the laboratory.
- (2) Hysteresis behavior of SWCC and scanning curves.
- (3) Soil volume change associated with SWCC measurement.

B. Data interpretation stage:

- (4) Residual error after regression.
- (5) The range of suction covered for determination of SWCC.
- (6) The number of data points used for determination of SWCC.
- (7) The method adopted for determination of SWCC variables.

The variability in SWCC for residual soils in Singapore with respect to soil formation and soil index properties is also presented in this research.

During the computation of unsaturated soil properties from SWCC, additional uncertainty may be resulted from the limitation of the prediction model. In this research, new equations for the prediction of the wetting SWCC and permeability function from the drying SWCC are derived based on the theory of capillary model. The derived equations are verified with experimental data and used throughout this study. As only one method is adopted for the prediction of wetting SWCC and the permeability function, the variability due to different prediction models can be eliminated in this research.

1.2 Objective

In this research, SWCC variability resulting from the factors mentioned in Section 1.1 is investigated. The main objective of this research is to investigate the variability in SWCC associated with experiment measurement and data interpretation. Additional objectives are as follows: to derive equations for determination of the confidence limits of SWCC from the fitting parameters and residual error; to provide a theoretical method for estimation of variability in SWCC from limited measurement data; to derive equations for determination of SWCC variables, such as air-entry value, slope at inflection point, residual water content and residual suction, from fitting parameters; to provide a theoretical method for estimation of variability in SWCC variables; to derive equations for estimation of wetting SWCC from drying SWCC; to derive equations for prediction of the permeability function from fitting parameters; to provide a theoretical method for estimation of variability in the permeability function; and to illustrate the variability in SWCCs for natural residual soils in Singapore with respect to soil formation and basic soil properties.

1.3 Scope

This thesis can be divided into four main parts: i) theoretical development, ii) research program, iii) the results and iv) discussion with conclusions.

The scope of the theoretical development can be summarized as follows:

- 1) Develop equations for determination of the confidence limits of the SWCC from the fitting parameters and residual error;
- 2) Derive equations for determination of the SWCC variables (i.e., air-entry value, slope at inflection point, residual volumetric water content, residual suction) from the fitting parameters;
- 3) Develop equations for estimation of the wetting curve from the drying SWCC.
- 4) Derive equations for prediction of the permeability function from the SWCC.

The scope of the research program can be summarized as follows:

- 5) Verify the proposed theories using published data from the literature.
- 6) Conduct soil basic properties tests for a compacted mixture of sand and kaolin and a residual soil.
- 7) Conduct SWCC measurement for the compacted mixture of sand and kaolin and the residual soil from Bukit Timah granite.
- 8) Apply the proposed theories to describe the characteristics of the collected experimental data, including the confidence limits, SWCC variables, and wetting curves.
- 9) Correlate the SWCC of residual soils in Singapore with saturated hydraulic conductivity, k_s and propose a framework for prediction of the SWCC from k_s .

The scope of the results can be summarized as follows:

- 10) Present soil basic properties for the compacted mixture of sand and kaolin and the residual soil from Bukit Timah granite.
- 11) Present SWCC data for the compacted mixture of sand and kaolin and residual soil.
- 12) Present fitting parameters, confidence limits of SWCC, SWCC variables and wetting curves using the theories proposed in this research.

- 13) Present the variability in SWCC due to the soil volume change associated with SWCC measurement;
- 14) Present the variability in the SWCC variables resulting from the conventional graphical method.

The scope of the discussion with conclusions can be summarized as follows:

- 15) Discuss the variability in the SWCC determined using confidence limits of the SWCC and standard deviation from direct measurement.
- 16) Discuss the variation in SWCC variables determined from the confidence limits of the SWCC and experimental data for the individual specimen.
- 17) Discuss the variability in SWCC due to the different numbers of data point and different suction ranges covered in the best fit procedure.
- 18) Discuss the variability in SWCC for residual soils in Singapore with respect to the saturated hydraulic conductivity, k_s .

1.4 Methodology

Fredlund et al. (2002) proposed equations for estimation of SWCC from grain size distribution data (GSD), and these equations have been incorporated into the Soilvision software. Therefore, Soilvision 2002 was used in this research for the selection of suitable soil samples (i.e., those with suitable GSD data). To minimize the variability in SWCC due to other factors, only the pressure plate was used as the SWCC measurement apparatus and all measurements were carried out solely by the author. In addition, only one best fit equation (i.e., Fredlund and Xing's (1994) equation) was selected as the best fit equation for the regression procedure. The fitting parameters were determined by minimizing the sum of the squared error ($SSE = \sum w_i(\theta_i - \theta'_i)^2$), which was suggested by Leong and Rahardjo (1997), using Solver in Microsoft Excel.

After the fitting parameters and residual error were obtained from the best fit procedure, the confidence limits of SWCC were computed using Zhai and Rahardjo's (2013b) equations,

which were developed as part of this research. The SWCC variables were also determined from the fitting parameters using Zhai and Rahardjo's (2012a) equations, which were also developed in this research. For comparison, the SWCC variables were also determined using the conventional graphical method. The results of the SWCC variables determined from both methods are presented. The variability in the SWCC variables resulting from the conventional graphical method and the confidence limits are both presented and compared.

Statistical summaries consisting of the minimum value, the first quartile, the median value, the third quartile and the maximum value of the data sets (i.e., measured water content) were calculated from the SWCC data for residual soil in Singapore. The box-plot method was used to illustrate variability in the SWCC data. A comparison of the results of the variability in SWCC, which were determined from the box-plot method and confidence limits from equations developed in this research, is also presented.

The variability in the SWCC and permeability function for the residual soils, namely Jurong Formation, Bukit Timah Granite and Old Alluvium soils, was estimated using both the box-plot method and the confidence limits, as suggested by Zhai and Rahardjo (2013b, 2014).

1.5 Outline of report

This report is organized into seven chapters as follows:

Chapter 1: INTRODUCTION includes the research background, objectives, scope, methodology and an outline of this report.

Chapter 2: LITERATURE REVIEW includes a brief review of unsaturated soil mechanics, the concept of suction and SWCC, the characteristics of SWCC, the methodology for determining the soil-water characteristic curve and the estimation of other unsaturated properties, such as permeability function and shear strength, from SWCC. The factors that may lead to the variability in SWCC and the methods for quantification of the uncertainty in SWCC are reviewed and discussed.

Chapter 3: THEORIES presents the applicable theories and theoretical development of the equations for determination of the confidence limits of SWCC from the fitting parameters

and residual error, the equations for determination of SWCC variables from the fitting parameters and equations for estimation of the wetting SWCC from the drying SWCC. The relationship between SWCC and the pore-size distribution function and the equations for calculating the permeability function from SWCC are presented. The factors associated with laboratory measurement and data interpretation that may lead to uncertainty in SWCC are also explained in this section.

Chapter 4: RESEARCH PROGRAM includes three main parts, namely verification of the proposed theories, the experimental program and data interpretation. Verification of the proposed theories includes collection of published data from the literature and a comparison of results from both proposed theories and experimental measurements. The experimental program comprises selection of soil, selection of measurement equipment, preparation of specimens, procedures for SWCC measurement and computation of the standard deviation for the measured data. Data interpretation consists of analysis of the experimental data using theories proposed in this research, such as the confidence limits of SWCC, SWCC variables, the wetting SWCC and hydraulic conductivity. Variability in the SWCC for residual soils in Singapore, with respect to saturated hydraulic conductivity, k_s , is also analyzed.

Chapter 5: RESULTS presents the experimental findings for the basic soil properties, results of SWCC measurements for the compacted mixture of sand and kaolin and for the residual soil from Bukit Timah granite, the confidence limits of the SWCCs, the SWCC variables for the compacted mixture of sand and kaolin and for the residual soil, the wetting SWCC for the compacted mixture of sand and kaolin, the variability in SWCC for residual soils in Singapore and the correlations between the SWCC for residual soils in Singapore and the saturated hydraulic conductivity, k_s .

Chapter 6: DISCUSSION OF RESULTS presents the variability in the SWCC associated with experimental measurement or data interpretation; the variability in the SWCC variables determined from the confidence limits and direct measurements; the effect of volume change on variability in the SWCC; and the variability in the SWCC for residual soils in Singapore with respect to saturated hydraulic conductivity, k_s .

Chapter 7: CONCLUSIONS AND RECOMMENDATIONS presents the conclusions on the variability in SWCC associated with experimental measurement and data

interpretation. Correlations between SWCCs for residual soil and basic soil properties are presented. Recommendations regarding SWCC measurement and prediction from basic soil properties are also made.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter presents a review of the literature related to this research. A brief review of the basic concept of unsaturated soil mechanics is given first. Subsequently, the research work relating to determination of SWCC from direct measurement or prediction from basic soil properties, estimation of other unsaturated properties such as permeability function and shear strength from SWCC, factors that may lead to uncertainty in SWCC and methods of quantification of uncertainty in SWCC is reviewed.

2.2 Unsaturated soil mechanics

Unsaturated soils form the largest category of materials whose behavior does not adhere to classical saturated soil mechanics. The differentiation between saturated and unsaturated soil becomes necessary due to the basic differences in their nature and engineering behavior (Fredlund and Rahardjo, 1993a).

2.2.1 Unsaturated soil

Climates play an important role in soil condition, especially in tropical countries. Whether a soil is saturated or unsaturated is controlled by periodic changes in climate, as well as variation in the ground water table. Basically, soil below the ground water table is typically in a saturated condition with a positive pore-water pressure, while soil above the ground water table is typically in an unsaturated condition with a negative pore-water pressure (also known as suction). In many geotechnical structures (e.g., dams, sub-grade bases for roads, etc.), soil is normally compacted and has a water content level much lower than the saturated water content. In other words, to achieve optimum compaction results, compacted soil is always in an unsaturated condition. Figure 2-1 shows a schematic diagram of the natural hydrology cycle and the role of the environment on unsaturated soil, as presented by Fredlund (2006).

Figure. 2-1 Illustration of the unsaturated zone in a regional and local basis (after Fredlund, 2006)

Bouwer (1978) named the unsaturated soil zone the “vadose zone”, and this term has been adopted by many researchers. Water is removed from soil either by evaporation from the soil surface, which is affected by environment factors such as sun, wind and temperature, or by evapo-transpiration from vegetation. In contrast, water is supplied to the soil by precipitation or rainfall, as shown in Fig. 2-2. Therefore, the pore-water pressure in the soil is affected by rainwater infiltration during rainfall and evaporation/evapo-transpiration by the surrounding environment.

Figure. 2-2 Illustration of the vadose zone in a soil layer (after Fredlund and Rahardjo, 1993b).

In classic soil mechanics, soil is considered a three-phase (soil, water and air) mixture. In unsaturated soil mechanics, soil is commonly treated as a four phase (soil, water, air and air-water interface or contractile skin) mixture. The fourth phase, namely the contractile skin, has an insignificant weight compared with the water and soil phases. However, the contact skin still plays a significant role in the mechanic behavior of unsaturated soil.

2.2.2 Soil suction

Negative pore-water pressure in soil is commonly called soil suction. In the early 1900s, the concept of soil suction was developed in soil physics (Edlefsen and Anderson, 1943). Soil suction was quantified in terms of relative humidity (Fredlund and Rahardjo, 1993a) and was commonly referred to as total suction. Total suction consists of two components: osmotic suction and matric suction.

Osmotic suction is always associated with dissolved salts in the pore-water in the soil. Changes in salt concentration can result in changes in osmotic suction. However, compared to matric suction, osmotic suction plays a less important role since its changes are less significant and less frequent. The total suction (i.e., matric suction + osmotic suction) and matric suction curves have similar shapes (Fredlund and Rahardjo, 1993a). In other words, a change in total suction is essentially equivalent to a change in matric suction. This can be expressed as $\Delta\psi = \Delta(u_a - u_w)$, where $\Delta\psi$ is the change in total suction and $\Delta(u_a - u_w)$ is the change in matric suction. In practice, osmotic suction can be ignored as long as the soil solution is dilute enough and the contained solutes do not significantly affect the matric suction (Hillel, 1998). Besides, most engineering problems in unsaturated soil are the result of environmental changes that cause matric suction changes (Fredlund and Rahardjo, 1993a)

Aitchison (1965) defined matric suction as "the equivalent suction derived from the measurement of the partial pressure of the water vapor in equilibrium with the soil water, relative to the partial pressure of the water vapor in equilibrium with a solution identical in composition with the soil water." The International Society of Soil Science (ISSS) uses the definition of matric suction proposed by Hillel (1998): "the negative gauge pressure, relative to the external gas pressure on the soil water, to which a solution identical in composition with the soil solution must be subjected in order to be in equilibrium through a porous membrane wall with the water in the soil". Alternatively, matric suction can be thought of as the difference between the pore-air pressure (u_a) and the pore-water pressure (u_w) that act across the contractile skin (Fredlund and Rahardjo, 1993a). The pores in soil are like a bundle of capillary tubes of different sizes. The capillary effects cause the water to rise above the water table in a capillary tube; the smaller the radius of the tube, the higher the water will rise above the water table.

2.2.3 Soil-water characteristic curve

The soil-water characteristic curve (SWCC) is a graphical representation of the relationship between the amount of water in a soil (i.e., the degree of saturation (S), volumetric water content (θ_w) or gravimetric water content (w)) and the soil suction. SWCC contains the fundamental information required for describing the mechanical behavior of unsaturated soil.

Fredlund et al. (2012) showed that the estimation procedures for obtaining unsaturated soil properties are approximate, but are generally satisfactory for analyzing unsaturated soil mechanics problems. As a result, the hydraulic conductivity and shear strength of an unsaturated soil are commonly estimated from SWCC instead of being measured directly because direct measurement is time-consuming and costly.

In soil physics, the relationship between water content and matric suction is also referred to as the water retention curve, moisture retention curve, soil-moisture retention curve, soil-water release curve or soil-moisture characteristic curve. A soil-moisture characteristic curve is a graphical representation of the wetness and matric suction of soil, which are functionally related to each other (Hillel, 1998). In civil engineering related disciplines, the term soil-water characteristic curve is preferred for referring to the relationship between the amount of water and the matric suction in a soil (Fredlund, 2002).

Vanapalli et al. (1998) proposed the graphical method, which is still commonly used by researchers, for the determination of the air-entry value, residual suction and residual saturation. Fredlund (2006) introduced three zones, namely the boundary effect zone, transition zone and residual zone, associated with SWCC, as illustrated in Fig 2-3. These three zones are defined by their air-entry value (AEV) and residual suction. The zone where the matric suction is less than AEV is the boundary effect zone, the zone where the matric suction is greater than the residual suction is the residual zone and the zone where the suction is between AEV and residual suction is defined as the transition zone.

Fredlund (2006) also summarized the advantages and disadvantages of the three measures of the amount of water in a soil. The gravimetric water content, w , is consistent with usage in classical soil mechanics, but does not allow differentiation between change in volume and change in degree of saturation, S , and does not yield the correct air-entry value. The volumetric water content, θ_w , emerges in the derivation of transient seepage in unsaturated soil and requires volume measurement during SWCC measurement. The degree of saturation, S , most clearly defines the air-entry value but also requires volume measurement.

Figure. 2-3 Illustration of the three in situ zones of de-saturation defined in the SWCC (after Fredlund, 2006)

SWCC can be categorized as a drying curve and a wetting curve. The drying curve is obtained by measuring the water content of a soil sample during the desorption process, while the wetting curve is obtained by measuring the water content of a soil sample during the absorption process. Different starting points for drying or wetting processes result in different curves. These curves are normally called scanning curves. Phenomena associated with the different curves of the desorption and adsorption processes are commonly referred to as the hysteretic property of soil. In fact, most SWCCs presented in the literature are drying curves because they are easier to measure (Fredlund, 2006). However, wetting curves represent the true unsaturated soil properties during rainfall because, during rainfall, the soil is in a saturation process rather than a de-saturation process. The hysteretic property of SWCC (i.e., drying curve and wetting curve) is illustrated in Fig 2-4.

Figure. 2-4. Commonly used definitions for hysteretic SWCC (Pham et al., 2005)

2.3 Determination of soil-water characteristic curve

Various researchers have proposed different best fit equations or models to describe SWCCs. Best fit equations are continuous mathematical models that can be used for regression with experimental data. In general, best fit equations are governed by a few fitting parameters, which can be determined from the best fit procedure using a curve fitting technique. Besides direct experimental measurements, SWCC can be predicted from basic soil properties using prediction equations or models. Both best fit equations for regression with experimental data and indirect prediction methods are reviewed and discussed in this section.

2.3.1 Best fit equations for soil-water characteristic curve

As discussed by Fredlund (2006), all of the proposed equations contain one variable related to the air-entry value (AEV) of soil and one variable related to the de-saturation rate of soil. If a third variable is used, it will allow the low matric suction range, which is near the AEV, to have a shape that is independent of the high matric suction range, which is near the residual

matric suction. Zapata et al. (2000) stated that best fit equations were needed because many applications of the SWCC require it to be differentiated or integrated and be continuous.

Gardner (1958) proposed one of the first equations for the representation of a real SWCC. Originally, this continuous function was proposed for modeling the unsaturated coefficient of permeability of soil. Gardner's (1958) equation was adapted for describing the SWCC of soil (Sillers et al., 2001.) The equation adopts two fitting parameters, "a" and "n", and is expressed as follows:

$$\theta_w = \theta_r + \frac{\theta_s - \theta_r}{1 + a\psi^n} \quad \text{----- (2-1)}$$

where,

θ_w = volumetric water content

θ_r = residual volumetric water content

θ_s = saturated volumetric water content

ψ = matric suction

a = fitting parameter related to the inverse of AEV

n = fitting parameter related to the pore-size distribution index.

Brooks and Corey (1964) assumed that water content remains constant for the range of matric suctions less than AEV, and decreases with an exponential function for the range of matric suctions greater than AEV. Their equation adopts two parameters, "a" and " λ ", and is as follows:

$$\begin{aligned} \theta_w &= \theta_s && \text{when } \psi < a \\ \theta_w &= \theta_s \left(\frac{\psi}{a} \right)^{-\lambda} && \text{when } \psi > a \end{aligned} \quad \text{----- (2-2)}$$

where,

a = fitting parameter related to AEV

λ = fitting parameter termed the pore-size distribution index.

Brooks and Corey's (1964) equation is not a continuous function for the entire range of suction. An abrupt change in the curve may result in numerical instability when modeling unsaturated soil behavior (Sillers et al., 2001).

To solve this problem, van Genuchten (1980) proposed a continuous best fit equation with great flexibility in fitting SWCC data for different types of soil. This equation has three fitting parameters of "a", "p" and "q" and is expressed as follows:

$$\theta_w = \theta_s \left[\frac{1}{1 + (a\psi)^p} \right]^q \quad \text{----- (2-3)}$$

where:

a = fitting parameter related to the inverse of AEV

p = fitting parameter related to the pore-size distribution of the soil

q = fitting parameter related to the asymmetry of the SWCC curve.

Adopting the concept of pore-size distribution, Fredlund and Xing (1994) proposed another continuous best fit equation similar to Van Genuchten's (1980) equation. Fredlund and Xing (1994) assumed the form of the pore-size distribution function and derived the best fit equation by integrating the pore-size distribution function. As expressed in Equation (2-4), their equation adopts three fitting parameters, "a", "n" and "m", plus the additional input value C_r .

$$\theta = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad \text{----- (2-4)}$$

where,

a = fitting parameter related to AEV

n = fitting parameter

m = fitting parameter

C_r : parameter related to the residual suction, selected as $C_r=1500$ kPa for most cases. (Fredlund and Xing (1994), Zhai and Rahardjo (2012a, 2012b)).

Leong and Rahardjo (1997a) also suggested the use of $C(\psi) = 1$ in Equation (2-4). This parameter makes the equation less accurate in the high suction range, but can best fit the data more closely in the low suction range, which is commonly obtained from Tempe cell and pressure plate tests in laboratory. Zhai and Rahardjo (2012a) termed the Fredlund and Xing (1994) equation with correction factor $C(\psi)$ as Method A and with correction factor $C(\psi)=1$ as Method B. Zhai and Rahardjo (2012a, 2012b) stated there were only three fitting parameters in the Fredlund and Xing (1994) equation: C_r is an input value and can be selected as 1500 kPa for most cases. As the SWCC in the form of degree of saturation, S , most clearly defines the correct air-entry value, Zhai and Rahardjo (2013b) suggested that the Fredlund and Xing (1994) equation could be slightly modified and expressed in the form of degree of saturation, S , rather than volumetric water content, θ_w , as shown in Equation (2-5).

$$S = C(\psi) \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{----- (2-5)}$$

where,

S is the degree of saturation

a_1 , n_1 and m_1 are fitting parameters.

Zhai and Rahardjo (2013b) presented that fitting parameters a_1 , n_1 , and m_1 in Equation (2-5) are different from a , n and m in Equation (2-4) for the same soil if a soil volume change associated with SWCC measurement is considered.

Kosugi (1994) proposed a model using a three-parameter lognormal distribution function to describe the soil-water characteristic curve. The model is as follows:

$$S_e = 0.5 \operatorname{erfc} \left[\frac{\ln \left(\frac{\psi_c - \psi}{\psi_c - \psi_0} \right) - \sigma^2}{\sqrt{2}\sigma} \right] \quad \psi < \psi_c \quad \text{----- (2-6)}$$

$$S_e = 1 \quad \psi \geq \psi_c$$

where,

ψ_c = fitting parameter

ψ_0 = fitting parameter

σ = fitting parameter.

Pedroso et al. (2009) proposed using the concept of reference curves to model the soil-water characteristic curve. Pedroso and Williams (2010) used the reference curves to model the soil-water characteristic curve with hysteresis for computational analysis.

By comparing the performance of different best fit equations, Leong and Rahardjo (1997) concluded that Fredlund and Xing's (1994) equation performed best and could be used for a wide range of soil types over the entire matric suction range.

2.3.2 Prediction equations for a soil-water characteristic curve from basic soil properties

Zapata (1999) summarized and categorized the prediction methods proposed by previous researchers into three categories: i) based on statistical estimation of water content at selected matric suction values; ii) correlating soil properties with the fitting parameters of the best fit equation; and iii) estimating SWCC using a physics based conceptual model approach. Several selected equations from the three categories are presented below.

Gupta and Larson (1979) proposed a correlation equation of volumetric water content with the percentage of soil content as follows:

$$\theta = a_g * \text{sand}\% + b_g * \text{silt}\% + c_g * \text{clay}\% + d_g * \text{organic matter}\% + e_g * \rho_d \quad \text{----- (2-7)}$$

where,

θ = predicted volumetric water content

a_g, b_g, c_g, d_g, e_g : regression coefficients

ρ_d = bulk density (gm/cm³).

Ghosh (1980) proposed a model to predict SWCC based on physical properties of soil, such as the percentage of sand, silt, and clay and saturated volumetric water content, as follows:

$$h = (u_a - u_w)_b \left(\frac{\theta_w}{\theta_s} \right)^{-\beta} \quad \text{----- (2-8)}$$

where,

β = empirical constant

$(u_a - u_w)_b$ = air-entry value.

Rawl et al. (1992) presented regression equations for prediction of the fitting parameters in Brooks and Corey's (1964) best fit equation from the basic soil properties as follows:

$$a_b = \exp(5.34 + 0.00185C - 2.484n - 0.00002C^2 - 0.00004Sn - 0.00617Cn + 0.00001S^2n^2 - 0.00009C^2n^2 - 0.00000001S^2C + 0.0000009C^2S - 0.000007S^2n + 0.0000005n^2S - 0.005n^2C) \quad \text{----- (2-9)}$$

$$b_b = \exp(-0.784 + 0.00018S - 1.062n - 0.0000005S^2 - 0.00003C^2 + 1.1111n^2 - 0.00031Sn + 0.000003S^2n^2 - 0.0000000002S^2C + 0.00008C^2n - 0.00007n^2C) \quad \text{----- (2-10)}$$

$$\theta_r = -0.018 + 0.000009S + 0.00005C + 0.029n - 0.000002C^2 - 0.00001Sn - 0.000002C^2n^2 + 0.000003C^2n - 0.00002n^2C \quad \text{----- (2-11)}$$

where,

a_b = fitting parameter related to air-entry value (cm)

b_b = fitting parameter related to pore-size distribution index

θ_r = residual volumetric water content

%S = percentage of sand

%C = percentage of clay

n = porosity.

Fredlund et al. (1997) redefined the fitting parameters of Fredlund and Xing's (1994) equation and presented the grain size distribution (GSD) model as given in Equation (2-12). Fredlund et al. (2002) proposed a model for prediction of SWCC from GSD data. A divisional SWCC can be estimated from the packing porosity by dividing GSD function into small divisions of uniform soil particles.

$$P = C(d) \left[\frac{100}{\ln \left[e + \left(\frac{d}{a_s} \right)^{b_s} \right]^{c_s}} \right] = \left[1 - \frac{\ln \left(1 + \frac{d_r}{d} \right)}{\ln \left(1 + \frac{d_r}{d_m} \right)} \right]^7 \left[\frac{100}{\ln \left[e + \left(\frac{d}{a_s} \right)^{b_s} \right]^{c_s}} \right] \quad \text{----- (2-12)}$$

where,

P = percent passing at any particular grain-size

d = particle diameter (mm)

a_s = a parameter related to the initial breaking point of the curve

b_s = a parameter related to the steepest slope of the curve

c_s = a parameter related to the shape of the fines portion of the curve

$C(d)$ = correction factor to ensure that the function goes through a lower limit particle diameter of 0.00001 mm

d_r = a parameter related to the amount of fines in a soil

d_m = the diameter of the minimum allowable size particle.

Zapata (1999) proposed a predictive model with prediction of SWCC fitting parameters for Fredlund and Xing's (1994) equation from basic soil properties as follows:

For plastic soil (PI > 0):

$$a = 0.00364(wPI)^{3.35} + 4(wPI) + 11 \quad \text{-----}(2-13)$$

$$n/m = -2.313(wPI)^{0.14} + 5 \quad \text{-----}(2-14)$$

$$m = 0.0514(wPI)^{0.465} + 0.5 \quad \text{-----}(2-15)$$

$$C_r/a = 32.44e^{0.0186(wPI)} \quad \text{-----}(2-16)$$

$$\theta_s = 0.0143(wPI)^{0.75} + 0.36 \quad \text{-----}(2-17)$$

where,

$wPI = \text{passing \#200} \times PI$

$PI = \text{plasticity index (\%)}$.

For non-plastic soil (PI = 0):

$$a = 0.8627 (D_{60})^{-0.751} \quad \text{-----}(2-18)$$

$$n = \bar{n} = 7.5 \quad \text{-----}(2-19)$$

$$m = 0.1772 \ln(D_{60}) + 0.7734 \quad \text{-----}(2-20)$$

$$C_r/a = \frac{1}{D_{60} + 9.7e^{-4}} \quad \text{-----}(2-21)$$

where,

$D_{60} = \text{grain diameter corresponding to 60\% passing by weight or mass (mm)}$

$\bar{n} = \text{average value of fitting parameter } n$.

For Zapata's (1999) proposed equations based on the statistical analysis of a database, accuracy is dependent on the size of the sample (or database).

Tinjum et al. (1997) proposed a predictive model with prediction of the SWCC by correlating fitting parameters in Van Genuchten's (1980) equation with compacted soil properties as follows:

$$\text{Log } a = -1.127 - 0.017\text{PI} - 0.092(W - W_{\text{opt}}) - 0.263C \quad \text{-----}(2-22)$$

$$n = -1.06 + 0.0005\text{PI} - 0.0005(W - W_{\text{opt}}) \quad \text{-----}(2-23)$$

where,

PI = plasticity index

W = compaction water content, %

C = categorical variable, (C=1 for mod. proctor, C=-1 for std. proctor)

W_{opt} = optimum water content, %.

2.4 Estimation of other unsaturated soil properties using the soil-water characteristic curve

As direct measurement is time-consuming and costly, unsaturated soil properties, such as the shear strength and permeability function, are commonly estimated from SWCC. Equations developed by previous researchers with descriptions of the shear strength and permeability function from the SWCC are presented in this section.

2.4.1 Equations of shear strength

Fredlund et al. (1978) proposed another equation with a description of the shear strength of unsaturated soil in terms of two independent stress state variables as follows:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad \text{-----}(2-25)$$

where,

τ = shear strength of unsaturated soil

c' = effective cohesion

$(\sigma_n - u_a)$ = net normal stress

$(u_a - u_w)$ = matric suction

ϕ' = effective internal friction angle of saturated soil

u_a = pore-air pressure

u_w = pore-water pressure

ϕ^b = internal friction angle with respect to matric suction.

Vanapalli et al. (1996) proposed another equation by modifying Fredlund et al. (1978) equation with a description of the shear strength of unsaturated soil as related to SWCC as follows:

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \Theta^k \tan \phi' \quad \text{-----}(2-26)$$

where,

τ = shear strength of unsaturated soil

c' = effective cohesion

$(\sigma_n - u_a)$ = net normal stress

$(u_a - u_w)$ = matric suction

ϕ' = effective internal friction angle of saturated soil

u_a = pore-air pressure

u_w = pore-water pressure

Θ = normalized volumetric water content defined as $\frac{\theta - \theta_r}{\theta_s - \theta_r}$.

2.4.2 Prediction of permeability function

The relationship between hydraulic conductivity and matric suction is referred to as the permeability function. Leong and Rahardjo (1997b) indicated that determination of the permeability function of unsaturated soil is a tedious and time-consuming process. Thus, the permeability function is commonly predicted from SWCC rather than by direct measurements. Various predictive models have been proposed by different researchers. Mualem (1986) categorized these models into three groups: empirical, macroscopic and statistical. These models are presented in this section.

Gardner (1958) proposed an empirical equation for the permeability function as follows:

$$k_w = a\theta_w^b \quad \text{-----}(2-27)$$

where,

k_w = relative hydraulic conductivity

a, b = fitting parameters

θ_w = volumetric water content.

Corey (1954) presented a macroscopic model for the permeability function as follows:

$$k_r = S_e^\delta \quad \text{-----}(2-28)$$

where,

k_r = relative hydraulic conductivity

S_e = effective degree of saturation defined as $\frac{S - S_r}{1 - S_r}$

δ = a constant, = 4 as suggested by Corey.

Mualem (1976) reviewed the previous statistical models and proposed a revised statistical model as follows:

$$k_r = S_e^n \left(\frac{\int_0^{\theta_w} \frac{d\theta_w}{\psi^{1+m}}}{\int_0^{\theta_s} \frac{d\theta_w}{\psi^{1+m}}} \right)^2 \quad \text{-----}(2-29)$$

where,

k_r = relative hydraulic conductivity

θ_w = volumetric water content

θ_s = saturated volumetric water content

$n = 0.5$, $m=0$ as suggested by Mualem.

Based on the empirical equation correlating the permeability function with the volumetric water content, θ_w , Leong and Rahardjo (1997b) proposed a new predictive model based on Fredlund and Xing's (1994) equation for calculation of the permeability function from SWCC as follows:

$$k_r = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^{m'}} \quad \text{-----}(2-30)$$

where,

k_r = relative hydraulic conductivity

$m' = mp$

a , n , m = fitting parameter in Fredlund and Xing's (1994) equation with correction factor $C(\psi)=1$

p = additional fitting parameter.

Childs and Collis-George (1950) proposed a statistical model for estimation of the coefficient of permeability based on the “cutting and random rejoining” concept, which considers pores in the soil as a series of capillary tubes and that these tubes are randomly connected to each

other in a given section. Childs and Collis-George's (1950) model makes three major assumptions: (i) pores in the soil can be considered a set of randomly distributed interconnected pores characterized by a pore radius r and density $f(r)$, and that the density $f(r)$ is the same for any cross-section; (ii) the Poiseuille equation is applicable; and (iii) SWCC can be considered analogous to the pore-size distribution function. Childs and Collis-George (1950) provided a theoretical basis for the statistical model, as categorized by Mualem (1986), for computation of the permeability function. Marshall (1958) improved the model by Childs and Collis-George (1950) and presented a polynomial equation instead of an equation in an integrated form. Equations in statistical models, such as those by Childs and Collis-George (1950), Marshall (1958) and Kunze et al. (1968), use only a few discrete points rather than a continuous mathematical model to represent the permeability function of unsaturated soil. Thus, the accuracy of the prediction results of a statistical model is greatly dependent on the number and locations of these discrete points. That said, the locations of these discrete points are dependent on the method of dividing the range of suction. Kunze et al. (1968) proposed equally dividing the volumetric water content, θ_w , into certain intervals, $\Delta\theta_w$, and calculating the interval of matric suction, $\Delta\Psi$, accordingly. Equal division of the volumetric water content, θ_w , makes the density, $f(r)$, unique for all pore radii, which means the pore-size distribution function follows a uniform distribution. By improving Childs and Collis-George's (1950) and Marshall's (1958) equations, Kunze et al. (1968) presented a simple equation, as illustrated in Equation (2-31), for calculation of the permeability function.

$$k_w(\theta_w)_i = \frac{k_s}{k_{sc}} A_d \sum_{j=i}^m \left\{ (2j+1-2i)(u_a - u_w)_j^{-2} \right\}, i = 1, 2, \dots, m \quad \text{-----}(2-31)$$

where,

$k_w(\theta_w)_i$ = predicted water coefficient of permeability for the volumetric water content

u_a = air pressure (kPa)

u_w = pore-water pressure (kPa)

$(\theta_w)_i$ corresponding to the i^{th} interval (m/s)

i = interval number, which increases as the volumetric water content decreases

j = a count from “i” to “m”

m = total number of intervals between the saturated volumetric water content, θ_s , and the lowest volumetric water content, θ_L

k_s = measured saturated coefficient of permeability (m/s)

k_{sc} = calculated saturated coefficient of permeability (m/s)

A_d = adjusting constant.

In Kunze et al.’s (1968) equation, the interval $\Delta\psi$ in the higher suction range is much greater than the interval $\Delta\psi$ in the lower suction range. The calculation of $\Delta\psi$ from θ_w is dependent on iteration and cannot always be easily solved using MS Excel. In addition, the soil volume change with respect to suction is not incorporated into the models from Childs and Collis-George (1950), Marshall (1958) or Kunze et al. (1968).

Mualem (1976) showed that Kunze et al.’s (1968) equation could be expressed in an analytical form as follows:

$$k(\theta_w) = \frac{\int_0^{\theta_w} \frac{(\theta_w - \vartheta)}{\psi^2} d\vartheta}{\int_0^{\theta_s} \frac{(\theta_w - \vartheta)}{\psi^2} d\vartheta} \quad \text{-----}(2-32)$$

Leong and Rahardjo (1997b) showed that the statistical model is most rigorous and provides the most accurate results.

Romero et al. (1999) and Romero (2013) presented the experimental results of a study on the macrostructure and microstructure of soil. Both the inter-aggregate pores (i.e., macrovoids between soil aggregates or shielding grains) and intra-aggregate pores (i.e., microvoids inside clay aggregates) were presented. Romero et al. (2011) showed that the intra-aggregate pores displayed non-constricted porosity with no bottleneck effects and restricted the capacity for liquid flow. Meanwhile, the water adsorption storage mechanism was not affected by porosity variation in intra-aggregate pores. In contrast, the inter-aggregate porosity displayed an interconnected porosity and the water storage mechanism was affected by void ratio changes. The work from Romero et al. (1999), Romero et al. (2011) and Romero (2013) suggests that

the statistical model for prediction of the permeability function of unsaturated soil is only applicable within inter-aggregate governing suction. In other words, the statistical model is only applicable for soil in which inter-aggregate pores are dominant.

2.5 Factors affecting uncertainty in the soil-water characteristic curve.

There are various factors that may lead to uncertainty in SWCC, and these factors have been discussed by many researchers. Zapata (1999) examined uncertainty arising due to different best fit equations, different operators, different numbers of data points and different suction ranges measured for the regression process and different methods used to obtain the SWCC data. Yaldo (1999) conducted laboratory testing on four compacted cohesive soils to assess the impact of soil type and compaction condition on SWCC. The results indicated that a water content difference of 2% on either side of the optimum did not have significant effect on SWCC. These results also indicated that the residual volumetric water content, θ_r , is a soil characteristic and not a function of the soil compaction condition, and that θ_r increases with increasing plasticity. In order to minimize uncertainty in SWCC due to the soil specimen itself, Yaldo (1999) suggested using a large-scale specimen for SWCC measurements (i.e., 45 cm diameter and 30 cm high). Gharagheer (2009) also reported that compaction effort and molding water content did not have any significant effects on SWCC for non-plastic soils. Gharagheer's (2009) results are in agreement with Yaldo's (1999) report. Furthermore, Tinjum et al. (1997) found that the air-entry value increased as the molding water content and compaction efforts increased; the slope of the SWCC was steeper for soil compacted at the dry optimum than soil compacted at the wet optimum; the dry unit weight had little effect on the SWCC for compacted clay; and that the shape of the SWCC was a function of the soil type. By performing a statistical study, Gurdal et al. (2003) showed that parameter "a" in van Genuchten's (1980) equation had a strong relationship with the saturated hydraulic conductivity (k_s) and was inversely related to the percentage of clay content and remolding water content; however, parameter "n" was less sensitive to the physical and basic properties than k_s . Dye et al. (2011) presented the effect of uncertainty in SWCC on seepage analysis results and concluded that variability in SWCC had a significant effect on the computed pore-water pressure.

2.6 Method of estimation of uncertainty in the soil-water characteristic curve

Mishra et al. (1989) suggested that the error covariance matrix C associated with van Genuchten's (1980) model could be used to represent uncertainty of the predicted result for the permeability function, where error covariance matrix C is a matrix relating to the variance of the fitting parameters. Because van Genuchten's (1980) equation is only a best fit equation, it cannot accurately represent an actual SWCC. Similarly, Fredlund and Xing's (1994) equation is also a best fit equation and cannot represent an actual SWCC. However, the error covariance matrix C can be used to estimate the uncertainty of the determined SWCC. Kool et al. (1987) presented a procedure for parameter estimation for unsaturated flow including an estimation of the uncertainty in the parameters (i.e., fitting parameter "a" and "n" from van Genuchten's (1980) equation). Beck and Arnold (1977), Mishra et al. (1989) and Mishra and Parker (1989) introduced a procedure for quantifying the uncertainty in the parameters using the first-order error analysis approach as follows:

$$Var(f) = E[(f - E(f))^2] \approx \sum_i \sum_j \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} E[(x_i - \hat{x}_i)(x_j - \hat{x}_j)]; \quad \text{-----}(2-33)$$

$$Var(f) \approx \sum_i \sum_j \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} Cov[x_i x_j] \quad \text{-----}(2-34)$$

$$Cov[Y_1 Y_2] \approx \sum_i \sum_j \frac{\partial Y_1}{\partial x_i} \frac{\partial Y_2}{\partial x_j} Cov[x_i x_j] \quad \text{-----}(2-35)$$

where, x_i, x_j = parameter vector

Y_1, Y_2 = random variables (both are functions of parameter vector x_i and x_j)

$$C = E[(\tilde{b} - b)(\tilde{b} - b)^T] \approx \frac{s^2 (J^T J)^{-1}}{M - P} \quad \text{-----}(2-36)$$

where,

$\tilde{b}$ is the vector of the estimated parameter

b is the vector of the true parameter

E denotes the statistical expectation

s^2 is the sum of the squared error

M is the number of experimental data points

P is the number of fitting parameters

J is the parameter sensitivity matrix or Jacobian matrix

$$J_{ij} = -\frac{\partial q_i}{\partial b_j} \approx \frac{\tilde{q}(b_j + \delta b_j) - \tilde{q}(b_j)}{\delta b_j} \quad \text{-----}(2-37)$$

with the forward difference increment taken to be $\delta b_j = 0.01 b_j$.

Kool and Parker (1988) showed that parameter confidence limits can be approximated from the individual parameter variance using t-statistics. In their work, the error covariance matrix C, which defines the variances of the SWCC fitting parameters, was used to represent the uncertainty of the best fitted SWCC (i.e., they determined fitting parameters a, n and m for Fredlund and Xing's (1994) equation) to accurately represent the true SWCC. Confidence level and t-statistics will be used to determine the confidence limits of the best fitted SWCC.

Fitting parameters in best fit equations are commonly determined from the regression procedure. The determined fitting parameters are always accompanied by residual error. In other words, there is no unique solution for the fitting parameters. In addition, only the fitting parameters are used for representation of SWCC and the residual error is always discarded. Statistical theory suggests that variability in the determined SWCC may exist due to the residual error. Zhai and Rahardjo (2013b) proposed Equations (2-38) and (2-39) for determination of the confidence limits of SWCC from the fitting parameters and residual error after the regression procedure. The confidence limits of SWCC can be used for quantification of the variability in SWCC.

$$\begin{array}{ll} \text{When } 0 < \psi < a_{\max} & \text{When } \psi > a_{\max} \\ \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right] \right\}^{m_{\min}}} & \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right] \right\}^{m_{\min}}} \end{array} \quad \text{-----}(2-38)$$

$$\begin{array}{ll}
\text{When } 0 < \psi < a_{\min} & \text{When } \psi > a_{\min} \\
\theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right] \right\}^{m_{\max}}} & \theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right] \right\}^{m_{\max}}}
\end{array} \quad \text{-----}(2-39)$$

where,

$$a_{\max} = a + t_{a/2} \sqrt{\text{Var}(a)}, \quad a_{\min} = a - t_{a/2} \sqrt{\text{Var}(a)},$$

$$n_{\max} = n + t_{a/2} \sqrt{\text{Var}(n)}, \quad n_{\min} = n - t_{a/2} \sqrt{\text{Var}(n)},$$

$$m_{\max} = m + t_{a/2} \sqrt{\text{Var}(m)}, \quad m_{\min} = m - t_{a/2} \sqrt{\text{Var}(m)}$$

Var(a), Var(n) and Var(m) are the variances of fitting parameters a, n and m, respectively.

$t_{a/2}$ is the confidence level.

2.7 Variability of index properties of residual soil in Singapore.

The Singapore Island is situated around 15m above sea level (PWD, 1976) . The geology of Singapore consists essentially of three formations: (i) igneous rocks of granite (Bukit Timah Granite) in the center and northwest, (ii) sedimentary rocks (Jurong Formation) in the west, and (iii) a semi-hardened alluvium (Old Alluvium) which covers older rocks beneath in the east of Singapore (PWD, 1976). Granite occurs in two separate masses. The larger one is found in the central and northern areas, the smaller one in north eastern parts of Singapore. Granite or igneous rocks underlie the Bukit Timah Nature Reserve and Central Catchment Area in the centre of the island. The granite in Singapore, according to radioactive age determination, is more than 200 million years old. The Sedimentary rocks of Jurong Formation form extensive areas in southern, south western and western parts of Singapore. These variations of conglomerate, sandstone and shale are also observed on the islands to the south to west. The semi-hardened Old Alluvium was deposited by an ancient river system, probably in the Pleistocene epoch, during a low stand of the sea. (PWD, 1976)

Based on the works from Rahardjo et al. (2012), the average value and the coefficient of variation for index properties, such as water content, $w\%$, plastic limit, PL, liquid limit, LL, void ratio, e , effective cohesion, c' (kPa), effective friction angle ϕ' (degree), angle indicating the rate of increase in shear strength relative to matric suction, ϕ^b , air-entry value, (AEV), residual suction, ψ_r , and residual saturation, S_r , of residual soil including Jurong sedimentary formation, Bukit Timah granite and Old Alluvium were computed and illustrated in Table 2.1

Table 2.1 Average value (AV) and coefficient of variation (COV) for index properties of residual soil in Singapore (computed from Rahardjo et al. 2012)

		Index properties				Saturated soil properties		Unsaturated soil properties			
		w% in situ	PL	LL	e In situ	c'(kPa)	ϕ' (degree)	ϕ^b (degree)	AEV (kPa)	Ψ_r (kPa)	S _r
Jurong formation	AV	17.36	20.54	42.54	0.49	11.58	32.33	28.28	41.66	4187.37	0.20
	COV	29.92%	19.63%	18.72%	23.58%	29.68%	9.15%	22.72%	70.19%	49.73%	58.58%
Bukit Timah granite	AV	32.00	33.69	43.38	0.51	8.83	36.83	31.60	16.74	2142	0.22
	COV	26.03%	21.62%	17.52%	18.58%	43.74%	9.12%	23.28%	127.22%	98.32	93.37%
Old Alluvium	AV	33.00	35.82	56.82	0.59	20.17	36.17	31.30	18.90	2754.39	0.22
	COV	28.29%	24.58%	13.35%	16.50%	36.98%	10.79%	28.42%	58.30%	106.39%	76.31%

2.8 Research gap on the variability of SWCC.

Based on current literature review, the quantification of the variability of SWCC has not been studied extensively. The results of the graphical method for the determination of SWCC variables such as air-entry value, slope at the inflection point, residual suction and residual saturation, are subjective to different interpretations. The location of the inflection point and tangent line constructed by manual plot is subjective to the visual judgement. The construction method may cause variability in SWCC variables and it is difficult to quantify this kind of variability. The wetting SWCC and permeability function, which are important unsaturated properties for the analysis of infiltration because the infiltration is a wetting process rather than drying process. Using the drying SWCC for infiltration analysis may get incorrect results. Incorrect permeability function also can lead to wrong solution of the seepage analysis. Therefore, the wetting SWCC and the permeability function are derived from the drying SWCC in this research.

CHAPTER THREE

THEORIES

3.1 Introduction

In this chapter, the applicable theories and theoretical development for the confidence limits of the best fitted SWCC are presented. The method for quantification of variability in SWCC will also be presented. Next, the factors affecting variability in SWCC associated with laboratory measurements and data interpretation are investigated and discussed. The variability in SWCC associated with soil volume change is also investigated and discussed in this section. The equations for determination of SWCC variables, such as air-entry value, slope at inflection point, residual suction and residual water content, are then derived. Next, variability in SWCC variables is presented, and the relationship between the SWCC and pore-size distribution curve is derived and discussed. Lastly, the equation for calculation of the permeability function for unsaturated soil from the fitting parameters is derived using the statistical methods.

The theoretical development in this research is illustrated in Figure 3-1 as follows:

Figure 3-1. Illustration of theoretical development

3.2 Determination of confidence limits of best fitted SWCC

After the best fit procedure, only the fitting parameters are used to express SWCC and the residual error is typically discarded. However, as residual error is a result of the regression process, statistical theory suggests that the magnitude of the residual error represents the goodness of the best fitted SWCC with the experimental data. In other words, a higher residual error represents less goodness of fit, while a lower residual error represents better goodness of fit. The confidence limits of the best fitted SWCC can be estimated using the residual error and fitting parameters.

Beck and Arnold (1977), Mishra et al. (1989) and Mishra and Parker (1989) introduced the procedure for quantifying uncertainty in the parameters using the first-order error analysis approach, as given in Equation (3-1):

$$C = E[(\tilde{b}-b)(\tilde{b}-b)^T] \approx \frac{s^2 (J^T J)^{-1}}{M - P} \quad \text{----- (3-1)}$$

where,

$\tilde{b}$ is the vector of the estimated parameter

b is the vector of the true parameter

E denotes the statistical expectation

s^2 is the sum of the squared error

M is the number of experimental data points

P is the number of fitting parameters

J is the parameter sensitivity matrix or Jacobian matrix

$$J_{ij} = -\frac{\partial q_i}{\partial b_j} \approx \frac{\tilde{q}(b_j + \delta b_j) - \tilde{q}(b_j)}{\delta b_j} \quad \text{----- (3-2)}$$

with the forward difference increment taken to be $\delta b_j = 0.01 b_j$.

Kool et al. (1987) presented the procedure for parameter estimation for unsaturated flow including estimation of the uncertainty in the parameters (i.e., fitting parameters "a" and "n" for van Genuchten's (1980) model). Mishra et al. (1989) suggested that error covariance matrix C associated with the van Genuchten's (1980) model to SWCC data could be used to represent the uncertainty of the predicted results for the permeability function, where error covariance matrix C is related to the variances of the fitting parameters. As both equations are best fit equations, covariance matrix C can be used for quantification of variability in best fitted SWCCs using Fredlund and Xing's (1994) equation.

3.2.1 Variance of fitting parameters

As stated by Leong and Rahardjo (1997a), Fredlund and Xing's (1994) equation performs best for different types of soil. Therefore, Fredlund and Xing's (1994) equation, which has only three best fitting parameters (i.e., a, n and m), was selected as the best fit equation. The error covariance matrix, C, can be defined and the Jacobian matrix can then be rewritten as follows:

$$C = \begin{bmatrix} Var(a) & Cov(an) & Cov(am) \\ Cov(na) & Var(n) & Cov(nm) \\ Cov(ma) & Cov(mn) & Var(m) \end{bmatrix} \quad \text{-----}(3-3)$$

$$J_i = \begin{bmatrix} \frac{\partial \theta_i}{\partial a} & \frac{\partial \theta_i}{\partial n} & \frac{\partial \theta_i}{\partial m} \end{bmatrix} \quad \text{-----}(3-4)$$

By considering the fitting parameters a, n and m in Fredlund and Xing's (1994) equation as unknown variables, the function of θ can be expressed based on Taylor's expansion by neglecting the higher-order terms as follows:

$$\theta \approx \theta(x) = \theta(\hat{x}) + \frac{\partial \theta}{\partial x}(x - \hat{x}) \quad \text{-----} (3-5)$$

where,

x is parameter vector [a, n, m]

$\hat{x}$ is the vector of the best fitted parameter [a, n, m]

By applying the expected value operator on both sides of the equation, it becomes:

$$E[\theta(x)] \approx \theta(\hat{x}) + \frac{\partial \theta}{\partial x} E[(x - \hat{x})] \quad \text{----- (3-6)}$$

Since small parameter perturbations are assumed around the mean values, Equation (3-6) can be simplified as

$$E[\theta(x)] \approx \theta(\hat{x}) \quad \text{----- (3-7)}$$

The variance of θ is defined as:

$$Var[\theta] \approx E[(\theta - E[\theta])^2] \quad \text{----- (3-8)}$$

Substituting Equations (3-5) and (3-7) into Equation (3-8), the following equation is obtained:

$$Var[\theta(x)] \approx E \left[\left(\theta(\hat{x}) + \frac{\partial \theta}{\partial x} (x - \hat{x}) - \theta(\hat{x}) \right)^2 \right] = \sum \left(\frac{\partial \theta}{\partial x} \right)^T \frac{\partial \theta}{\partial x} E[(x - \hat{x})^2] \quad \text{----- (3-9)}$$

The expected value of $(x - \hat{x})$ can be expressed as follows:

$$E[(x - \hat{x})^2] = Var[x - \hat{x}] + \left(E[x - \hat{x}] \right)^2 = Var[x] + 0 \quad \text{----- (3-10)}$$

Substituting Equation (3-10) into Equation (3-9) results in Equation (3-11) as follows:

$$Var[\theta] \approx \sum \left(\frac{\partial \theta}{\partial x} \right)^T \frac{\partial \theta}{\partial x} Var[x] \quad \text{----- (3-11)}$$

Replacing parameter vector x with [a,n,m] in Equation (3-11) results in the following Equation (3-12):

$$Var[x] = C = \begin{bmatrix} Var(a) & Cov(a,n) & Cov(a,m) \\ Cov(n,a) & Var(n) & Cov(n,m) \\ Cov(m,a) & Var(m,n) & Var(m) \end{bmatrix} = \frac{Var[\theta]}{\left(\sum \left(\frac{\partial \theta}{\partial x} \right)^T \frac{\partial \theta}{\partial x} \right)} \quad \text{-----}(3-12)$$

There are M experimental data points and three unknown variables, resulting in M-3 degrees of freedom. Considering that $Var[\theta] = \frac{SSE}{M-3}$, Equation (3-12) can be rearranged as follows:

$$C = \frac{SSE}{(M-3) \left(\sum \left(\frac{\partial \theta}{\partial x} \right)^T \frac{\partial \theta}{\partial x} \right)} \quad \text{-----}(3-13)$$

Fredlund and Xing's (1994) equation based on Taylor's expansion of the best fitted parameter $\bar{a}$ can be expressed as:

$$\theta(a,n,m) \approx \theta(\bar{a}) + \frac{\partial \theta}{\partial a} (a - \bar{a}) \quad \text{-----}(3-14)$$

$$Var(\theta(a,n,m)) \approx Var(\theta(\bar{a}) + \frac{\partial \theta}{\partial a} (a - \bar{a})) = \sum \left(\frac{\partial \theta}{\partial a} \right)^2 Var(a) \quad \text{-----}(3-15)$$

where, $\bar{a}$ is the fitting parameter.

Rearranging Equation (3-15) results in Equation (3-16) as follows:

$$Var(a) \approx \frac{Var(\theta)}{\sum \left(\frac{\partial \theta}{\partial a} \right)^2} = \frac{SSE}{(M-3) \sum \left(\frac{\partial \theta}{\partial a} \right)^2} \quad \text{-----}(3-16)$$

Fredlund and Xing's (1994) equation can also be expressed based on Taylor's expansion of the best fitted parameters $\bar{n}$ and $\bar{m}$. The variances of fitting parameters n and m can be expressed as follows:

$$Var(n) \approx \frac{Var(\theta)}{\sum \left(\frac{\partial \theta}{\partial n} \right)^2} = \frac{SSE}{(M-3) \sum \left(\frac{\partial \theta}{\partial n} \right)^2} \quad \text{-----}(3-17)$$

$$Var(a) \approx \frac{Var(\theta)}{\sum \left(\frac{\partial \theta}{\partial a} \right)^2} = \frac{SSE}{(M-3) \sum \left(\frac{\partial \theta}{\partial a} \right)^2} \quad \text{-----}(3-18)$$

Compared with Equation (3-13), Equations (3-16) to (3-18) have less accuracy, but are much simpler and easier to solve. The variances of these fitting parameters can be determined by Equations (3-16) to (3-18) using Microsoft Excel. Equation (3-11) also indicates that the three fitting parameters [a, n and m] of Fredlund and Xing's (1994) equation are dependent, because their covariances (i.e., Cov(a,n), Cov(a,m) and Cov(n,m)) are not equal to zero. This is consistent with the conclusions of probabilistic analyses of experimental data from databases by Phoon et al. (2010).

3.2.2 Confidence limits of best fitted SWCC

Kool and Parker (1988) indicated that confidence limits of the parameters could be determined from the individual parameter variance by approximately using t-statistics. Elkateb et al. (2003) and Rahardjo et al. (2012) suggested the confidence level of 90% for the estimation of the variation of soil properties. In this research, two-sided confidence limits with $\alpha\%$ significance level and t-statistic tool are adopted for the determination of the confidence limits of the fitting parameters. Therefore, t distribution function with 10% significance level (two tails) is adopted in this research.

The confidence limits of the fitting parameters are defined as follows:

$$\begin{aligned} a_{\max} &= a + t_{\alpha/2} (Var(a))^{0.5}, & n_{\max} &= n + t_{\alpha/2} (Var(n))^{0.5}, & m_{\max} &= m + t_{\alpha/2} (Var(m))^{0.5} \\ a_{\min} &= a - t_{\alpha/2} (Var(a))^{0.5}, & n_{\min} &= n - t_{\alpha/2} (Var(n))^{0.5}, & m_{\min} &= m - t_{\alpha/2} (Var(m))^{0.5} \end{aligned}$$

Different combinations of fitting parameters represent different SWCCs. The upper and lower confidence limits can be obtained from these combinations. The correction factor $C(\psi)$ in Fredlund and Xing's (1994) equation does not contain any fitting parameters, which means that the variances of the fitting parameters do not affect the correction factor $C(\psi)$. Therefore, the correction factor $C(\psi)$ can be considered as a constant in the determination of the confidence limits of the best fitted SWCC.

To observe the combinations of fitting parameters that correspond to the upper and lower confidence limits of the best fitted SWCC, a mathematical deduction is carried out as follows:

It is known that:

if $x_1 > x_2 > 0$, when $a > 1$, then $a^{x_1} > a^{x_2}$

when $0 < a < 1$, then $a^{x_1} < a^{x_2}$;

when $a > 0$, $x_1^a > x_2^a$;

Since all of these fitting parameters are positive, the following relationships can be obtained:

$$\text{when } 0 < \psi < a_{\min}, \text{ then } \left(\frac{\psi}{a_{\max}}\right)^{n_{\max}} < \left(\frac{\psi}{a_{\max}}\right)^{n_{\min}} \text{ or } \left(\frac{\psi}{a_{\min}}\right)^{n_{\max}} < \left(\frac{\psi}{a_{\min}}\right)^{n_{\min}} < 1 \quad \text{-----}(3-19)$$

$$\text{when } a_{\min} < \psi < a_{\max}, \text{ then } \left(\frac{\psi}{a_{\max}}\right)^{n_{\max}} < \left(\frac{\psi}{a_{\max}}\right)^{n_{\min}} < 1 < \left(\frac{\psi}{a_{\min}}\right)^{n_{\min}} < \left(\frac{\psi}{a_{\min}}\right)^{n_{\max}} \quad \text{-----}(3-20)$$

$$\text{when } \psi > a_{\max}, \text{ then } \left(\frac{\psi}{a_{\min}}\right)^{n_{\max}} > \left(\frac{\psi}{a_{\max}}\right)^{n_{\max}} \text{ or } \left(\frac{\psi}{a_{\min}}\right)^{n_{\min}} > \left(\frac{\psi}{a_{\max}}\right)^{n_{\min}} > 1 \quad \text{-----}(3-21)$$

As $\ln\left(e + \left(\frac{\psi}{a}\right)^n\right)$ is always greater than 1,

$$\text{then, } \left\{ \ln\left(e + \left(\frac{\psi}{a}\right)^n\right) \right\}^{m_{\min}} < \left\{ \ln\left(e + \left(\frac{\psi}{a}\right)^n\right) \right\}^{m_{\max}} \quad \text{-----}(3-22)$$

It can be concluded from inequalities (3-19) to (3-21):

When $0 < \psi < a_{\min}$, then

$$\frac{\theta_s}{\left\{ \ln\left(e + \left(\frac{\psi}{a_{\max}}\right)^{n_{\max}}\right) \right\}^{m_{\min}}} > \frac{\theta_s}{\ln\left(e + \left(\frac{\psi}{a_{\max}}\right)^{n_{\min}}\right)^{m_{\min}}} \text{ or } > \frac{\theta_s}{\ln\left(e + \left(\frac{\psi}{a_{\min}}\right)^{n_{\max}}\right)^{m_{\min}}} > \frac{\theta_s}{\ln\left(e + \left(\frac{\psi}{a_{\min}}\right)^{n_{\min}}\right)^{m_{\min}}} \quad \text{-----}(3-23)$$

$$\frac{\theta_s}{\left\{ \ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right) \right\}^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right)^{m_{\max}}} \text{ or } > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right)^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right)^{m_{\max}}} \quad \text{-----}(3-24)$$

when $a_{\min} < \psi < a_{\max}$, then

$$\frac{\theta_s}{\left\{ \ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right) \right\}^{m_{\min}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right)^{m_{\min}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right)^{m_{\min}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right)^{m_{\min}}} \quad \text{-----}(3-25)$$

$$\frac{\theta_s}{\left\{ \ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right) \right\}^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right)^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right)^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right)^{m_{\max}}} \quad \text{-----}(3-26)$$

when $a_{\max} < \psi$, then

$$\frac{\theta_s}{\left\{ \ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right) \right\}^{m_{\min}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right)^{m_{\min}}} \text{ or } > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right)^{m_{\min}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right)^{m_{\min}}} \quad \text{-----}(3-27)$$

$$\frac{\theta_s}{\left\{ \ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right) \right\}^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right)^{m_{\max}}} \text{ or } > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right)^{m_{\max}}} > \frac{\theta_s}{\ln \left(e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right)^{m_{\max}}} \quad \text{-----}(3-28)$$

Therefore, it can be concluded:

the condition of $\psi < a_{\min}$, ($a_{\max}$, $n_{\max}$, $m_{\min}$) gives the upper confidence limit, while ($a_{\min}$, $n_{\min}$, $m_{\max}$) gives the lower confidence limit;

the condition of $a_{\min} < \psi < a_{\max}$, ($a_{\max}$, $n_{\max}$, $m_{\min}$) gives the upper confidence limit, while ($a_{\min}$, $n_{\max}$, $m_{\max}$) gives the lower confidence limit;

the condition of $a_{\max} < \psi$, $(a_{\max}, n_{\min}, m_{\min})$ gives the upper confidence limit, while $(a_{\min}, n_{\max}, m_{\max})$ gives the lower confidence limit.

Equations for the determination of the confidence limits of the best fitted SWCC are presented in Equations (3-29) to (3-30), and an illustration of the confidence limits of the best fitted SWCC is shown in Figure. 3-2.

$$\begin{aligned} \text{When } 0 < \psi < a_{\max} \quad & \text{When } \psi > a_{\max} \\ \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right] \right\}^{m_{\min}}} \quad & \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right] \right\}^{m_{\min}}} \end{aligned} \quad \text{-----(3-29)}$$

$$\begin{aligned} \text{When } 0 < \psi < a_{\min} \quad & \text{When } \psi > a_{\min} \\ \theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right] \right\}^{m_{\max}}} \quad & \theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right] \right\}^{m_{\max}}} \end{aligned} \quad \text{-----(3-30)}$$

Figure.3-2 Illustration of confidence limits of best fitted SWCC (Zhai and Rahardjo, 2013b)

3.3 Determination of SWCC variables from the SWCC fitting parameters

According to Fredlund and Xing (1994), the air-entry value (AEV) is the matric suction where air starts to enter the largest pores in the soil. In other words, the air-entry value is the matric suction that breaks the meniscus formed by surface tension in the largest pores. The residual water content (θ_r) is the water content where a large suction change is required to remove additional water from the soil. These definitions of air-entry value and residual suction are very clear. However, it is difficult to measure them directly due to limitations of the existing equipment. The air-entry value and the residual state (residual water content and residual suction) are typically determined using the following procedure: draw a tangent line through the inflection point, followed by a horizontal line through the initial point and then another tangent line through the point where the curve starts to drop linearly in the high suction range (Figure 3-3). The intersections of these tangent lines indicate the AEV and the residual state. The definitions of the SWCC variables are provided in Figure. 3-3.

Figure. 3-3 Definitions of soil-water characteristic curve variables (after Zhai and Rahardjo, 2012a)

3.3.1 Determination of SWCC variables for Method A

Zhai and Rahardjo (2012a) defined the Fredlund and Xing's (1994) equation with correction factor $C(\psi)$ as Method A and with correction factor $C(\psi)=1$ as Method B. The equations for determination of SWCC variables from the fitting parameters for Fredlund and Xing's (1994) equation, as illustrated in Equation (3-31), are derived in this section.

$$\theta = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} = \left[1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad \text{-----}(3-31)$$

Fredlund (2006) summarized the advantages and disadvantages of the three measurements of the amount of water (i.e., gravimetric water content, volumetric water content and degree of saturation) in a soil. Fredlund (2006) indicated that gravimetric water content (w) is consistent with usage in classical soil mechanics, but does not allow differentiation between change in volume and change in degree of saturation, and also does not yield the correct air-entry value. Alternatively, volumetric water content (θ_w) emerges in the derivation of transient seepage (Richard's (1931) equation) in unsaturated soils and requires a volume measurement and degree of saturation (S) that most clearly defines the air-entry value. Zhai and Rahardjo (2013a) presented Fredlund and Xing's (1994) equation in the form of degree of saturation (S) for the derivation of equations to determine SWCC variables as follows:

$$S = C(\psi) \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{-----}(3-32)$$

where,

S is the degree of saturation

a_1 , n_1 and m_1 are fitting parameters, which are different from a , n and m in Equation (3-31) if the soil volume change is considered during measurement.

3.3.1.1 Determination of the inflection point.

The inflection point is the point where the magnitude of the slope of the curve reaches its maximum value. In other words, the differentiation of the slope with respect to suction at this point should be equal to zero. Therefore, the suction corresponding to the inflection point can be calculated by solving Equation (3-33) as follows:

$$\begin{aligned}
 & - \left[1 - \frac{\ln \left(1 + \frac{\psi}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{m_1 n_1}{a_1} \frac{1}{\left[\ln \left(e + \left(\frac{\psi}{a_1} \right)^{n_1} \right) \right]^{m_1+1}} \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n_1-1} \\
 & \left\{ (-m_1 - 1) \frac{1}{\left[\ln \left(e + \left(\frac{\psi}{a_1} \right)^{n_1} \right) \right]} \frac{n_1}{a_1} \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n_1-1} - \frac{n_1}{a_1} \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n_1-1} + \frac{n_1 - 1}{\psi} \right\} + \\
 & 2 \frac{1}{\ln \left(1 + \frac{10^6}{C_r} \right)} \frac{1}{C_r + \psi} \frac{m_1 n_1}{a_1} \frac{1}{\left[\ln \left(e + \left(\frac{\psi}{a_1} \right)^{n_1} \right) \right]^{m_1+1}} \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n_1-1} + \\
 & \frac{1}{\ln \left(1 + \frac{10^6}{C_r} \right)} \left(\frac{1}{C_r + \psi} \right)^2 \frac{1}{\left[\ln \left(e + \left(\frac{\psi}{a_1} \right)^{n_1} \right) \right]^{m_1}} = 0
 \end{aligned}
 \tag{3-33}$$

3.3.1.2 Determination of slope at inflection point s_1

The slope of the SWCC curve can be obtained by differentiating the curve function (i.e., best fitting equation) as follows:

$$\frac{dS}{d \log(\psi)} = \frac{\partial S}{\partial \psi} \psi \ln 10 =$$

$$\psi \ln(10) \left[\frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right) \left(1 + \frac{\psi}{C_r}\right) C_r \left\{ \ln \left[e + \left(\frac{\psi}{a_1}\right)^{n_1} \right] \right\}^{m_1}} + \frac{m_1 n_1 \left(\frac{\psi}{a_1}\right)^{n_1-1} \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right]}{a_1 \left[e + \left(\frac{\psi}{a_1}\right)^{n_1} \right] \left\{ \ln \left[e + \left(\frac{\psi}{a_1}\right)^{n_1} \right] \right\}^{m_1+1}} \right]$$

----- (3-34)

Substituting $\psi=\psi_i$ into Equation (3-34) gives the slope at the inflection point s_1 . As the slope is typically defined as positive, s_1 can be defined as follows:

$$s_1 = \left| \frac{dS}{d \log(\psi)} \right|_{\psi=\psi_i} =$$

$$\psi_i \ln(10) \left[\frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right) \left(1 + \frac{\psi_i}{C_r}\right) C_r \left\{ \ln \left[e + \left(\frac{\psi_i}{a_1}\right)^{n_1} \right] \right\}^{m_1}} + \frac{m_1 n_1 \left(\frac{\psi_i}{a_1}\right)^{n_1-1} \left[1 - \frac{\ln\left(1 + \frac{\psi_i}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right]}{a_1 \left[e + \left(\frac{\psi_i}{a_1}\right)^{n_1} \right] \left\{ \ln \left[e + \left(\frac{\psi_i}{a_1}\right)^{n_1} \right] \right\}^{m_1+1}} \right]$$

----- (3-35)

3.3.1.3 Determination of the air-entry value (AEV)

From the geometrical relationship in Figure 3-6, it is found that s_1 can be obtained by the following definition:

$$s_1 = \frac{1 - S_i}{\log(\psi_i) - \log(\psi_b)} = \frac{1 - S_i}{\log(\psi_i / \psi_b)} \quad \text{----- (3-36)}$$

where,

$$S_i = C(\psi_i) \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1}} = \left[1 - \frac{\ln \left(1 + \frac{\psi_i}{C_r} \right)}{\ln \left(1 + \frac{10^6}{C_r} \right)} \right] \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{----- (3-37)}$$

The solution of Equation (3-36) should be equal to the solution of Equation (3-35) and, as a result, the following equation can be obtained:

$$\psi_b = \psi_i 0.1^{\frac{1 - S_i}{s_1}} \quad \text{----- (3-38)}$$

3.3.1.4 Determination of the residual suction (ψ_r) and the residual degree of saturation (S_r)

Slope s_2 at point (ψ' , S') where the curve starts to drop linearly in the high suction range can be obtained by inputting ψ' into Equation (3-34) as follows:

$$s_2 = \psi' \ln(10) \left[\frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right) \left(1 + \frac{\psi'}{C_r}\right) C_r \left\{ \ln\left[e + \left(\frac{\psi'}{a_1}\right)^{n_1}\right]\right\}^{m_1}} + \frac{m_1 n_1 \left(\frac{\psi'}{a_1}\right)^{n_1-1} \left[1 - \frac{\ln\left(1 + \frac{\psi'}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right]}{a_1 \left[e + \left(\frac{\psi'}{a_1}\right)^{n_1} \right] \left\{ \ln\left[e + \left(\frac{\psi'}{a_1}\right)^{n_1}\right]\right\}^{m_1+1}} \right] \quad \text{----- (3-39)}$$

The degree of saturation S' at this point (ψ' , S') can be obtained by inputting ψ' into Equation (3-32) as follows:

$$S' = \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right] \frac{1}{\left\{ \ln\left[e + \left(\frac{\psi'}{a_1}\right)^{n_1}\right]\right\}^{m_1}} \quad \text{----- (3-40)}$$

From the geometrical relationship in Figure 3-2, the following equations can be obtained:

$$s_1 = \frac{S_i - S_r}{\log(\psi_r) - \log(\psi_i)} \quad \text{----- (3-41)}$$

$$s_2 = \frac{S_r - S'}{\log(\psi') - \log(\psi_r)} \quad \text{----- (3-42)}$$

By equating Equation (3-41) to Equation (3-36) and Equation (3-42) to Equation (3-39), the residual suction and residual water content can be obtained as follows:

$$\psi_r = 10^{\frac{S_i - S' + s_1 \log(\psi_i) - s_2 \log(\psi')}{s_1 - s_2}} \quad \text{----- (3-43)}$$

$$S_r = S_i - s_1 \log\left(\frac{\psi_r}{\psi_i}\right) \quad \text{----- (3-44)}$$

3.3.2 Determination of SWCC variables for Method B

Zhai and Rahardjo (2013a) presented Fredlund and Xing's (1994) equation with correction factor $C(\psi)=1$ in the form of degree of saturation (S) for deriving the equations for determination of the SWCC variables as follows:

$$S = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{----- (3-45)}$$

3.3.2.1 Determination of the inflection point.

Differentiation of the slope with respect to suction at this point should be equal to zero. Therefore, the suction corresponding to the inflection point can be calculated by solving Equation (3-46) as follows:

$$\begin{aligned}
& n_1 \left(\frac{\psi}{a_1} \right)^n \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \frac{m_1}{\left\{ \ln \left[e + \left(\frac{\psi}{a_1} \right)^{n_1} \right] \right\}^{m+1}} \left\{ \frac{\frac{m_1 + 1}{\ln \left[e + \left(\frac{\psi}{a_1} \right)^{n_1} \right]} \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n-1} \frac{n_1}{a_1} + \right.} \\
& \left. \frac{1}{e + \left(\frac{\psi}{a_1} \right)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n-1} \frac{n_1}{a_1} - \frac{n_1 - 1}{\psi} \right\} \\
& + \frac{(-m_1)}{\left\{ \ln \left[e + \left(\psi / a_1 \right)^{n_1} \right] \right\}^{m_1+1}} * \frac{n_1}{e + (\psi / a_1)^{n_1}} * \left(\frac{\psi}{a_1} \right)^{n_1-1} * \frac{1}{a_1} = 0
\end{aligned}
\tag{3-46}$$

3.3.2.2 Determination of slope at inflection point s_1

The slope of the SWCC curve can be obtained by differentiating the curve function (i.e., best fitting equation) as follows:

$$\begin{aligned}
& \frac{dS}{d \log(\psi)} = \frac{\partial S}{\partial \psi} \psi \ln 10 = \\
& \frac{(-m_1)}{\left\{ \ln \left[e + (\psi / a_1)^{n_1} \right] \right\}^{m_1+1}} \frac{n_1}{e + (\psi / a_1)^{n_1}} \left(\frac{\psi}{a_1} \right)^{n_1-1} \frac{1}{a_1} \psi \ln 10
\end{aligned}
\tag{3-47}$$

Substituting $\psi = \psi_i$ into Equation (3-47) gives the slope at the inflection point s_1 . As the slope is typically defined as positive, s_1 can be defined as follows:

$$s_1 = \left| \frac{dS}{d \log(\psi)} \right|_{\psi=\psi_i} = \frac{(-m_1)}{\left\{ \ln \left[e + (\psi_i / a_1)^{n_1} \right] \right\}^{m_1+1}} \frac{n_1}{e + (\psi_i / a_1)^{n_1}} \left(\frac{\psi_i}{a_1} \right)^{n_1-1} \ln 10
\tag{3-48}$$

3.3.2.3 Determination of the air-entry value (AEV)

From the geometrical relationship in Figure 3-6, it is found that s_1 can be obtained by the following definition:

$$s_1 = \frac{S_s - S_i}{\log(\psi_i) - \log(\psi_b)} = \frac{S_s - S_i}{\log(\psi_i / \psi_b)} \quad \text{----- (3-36)}$$

where,

$$S_i = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1}} = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{----- (3-49)}$$

The solution of Equation (3-48) should be equal to the solution of Equation (3-36) and, as a result, the following equation can be obtained:

$$\psi_b = \psi_i 0.1^{\frac{1-S_i}{s_1}} \quad \text{----- (3-38)}$$

3.3.2.4 Determination of the residual suction (ψ_r) and the residual degree of saturation (S_r)

Slope s_2 at the point (ψ' , S') where the curve starts to drop linearly in the high suction range can be obtained by inputting ψ' into Equation (3-47) as follows:

$$s_2 = \left| \frac{dS}{d \log(\psi)} \right|_{\psi=\psi'} = \frac{(-m_1)}{\left\{ \ln[e + (\psi' / a_1)^{n_1}] \right\}^{m_1+1}} \frac{n_1}{e + (\psi' / a_1)^{n_1}} \left(\frac{\psi'}{a_1} \right)^{n_1} \ln 10 \quad \text{----- (3-50)}$$

The degree of saturation (S') at this point (ψ' , S') can be obtained by inputting ψ' into Equation (3-45) as follows:

$$S' = \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi'}{a_1} \right)^{n_1} \right] \right\}^{m_1}} \quad \text{----- (3-51)}$$

From the geometrical relationship in Figure 3-6, the following equations can be obtained:

$$s_1 = \frac{S_i - S_r}{\log(\psi_r) - \log(\psi_i)} \quad \text{----- (3-41)}$$

$$s_2 = \frac{S_r - S'}{\log(\psi') - \log(\psi_r)} \quad \text{----- (3-42)}$$

By equating Equation (3-41) to Equation (3-36) and Equation (3-42) to Equation (3-50), the residual suction and residual water content can be obtained as follows:

$$\psi_r = 10^{\frac{S_i - S' + s_1 \log(\psi_i) - s_2 \log(\psi')}{s_1 - s_2}} \quad \text{----- (3-43)}$$

$$S_r = S_i - s_1 \log\left(\frac{\psi_r}{\psi_i}\right) \quad \text{----- (3-44)}$$

3.4 Determination of the confidence limits of the SWCC variables

By fixing the value of "m" and "n" and varying the value of "a", relationships between the SWCC variables and the fitting parameter "a" can be plotted. Following the same procedure, the relationships between the SWCC variables and the fitting parameters "m" and "n" can also be plotted. The following relationships are observed from Equations (3-38), (3-35), (3-43) and (3-44) and illustrated in Figures 3-4 to 3-7 by fixing the values of $a=1$, $n=1$ and $m=1$.

Figure. 3-4 Relationships between the air-entry values and the fitting parameters (a , n and m)

Figure. 3-5 Relationships between the slope at inflection point s_1 and the fitting parameters (a , n and m)

Figure. 3-6 Relationships between the residual suction (ψ_r) and the fitting parameters (a , n and m)

Figure. 3-7 Relationships between the residual volumetric water content (θ_r) and the fitting parameters (a , n and m).

Figures 3-4 indicates that the air-entry value increases with an increase in fitting parameters " a " and " n ", and decreases with an increase in the fitting parameter " m ". Figure 3-6 indicates that the residual suction (ψ_r) increases with an increase in the fitting parameter " a ", and decreases with an increase in fitting parameters " n " and " m ". Therefore, the combination of ($a_{\max}$, $n_{\max}$ and $m_{\min}$), which defines the upper confidence limit, gives the maximum air-entry value, while the combination of ($a_{\min}$, $n_{\min}$ and $m_{\max}$), which defines the lower confidence limit, gives the minimum air-entry value. The combination of ($a_{\max}$, $n_{\min}$, and $m_{\min}$), which defines the upper confidence limit, gives the maximum value of the residual suction (ψ_r), while the combination of ($a_{\min}$, $n_{\max}$ and $m_{\max}$), which defines the lower confidence limit, gives the minimum value of the residual suction ψ_r . Therefore, the confidence limits of the air-entry value and the residual suction can be determined by the following equations:

$$AEV_{\max} = \psi_b(a_{\max}, n_{\max}, m_{\min}) \text{ and } AEV_{\min} = \psi_b(a_{\min}, n_{\min}, m_{\max}); \quad \text{----- (3-52)}$$

$$\psi_{r\max} = \psi_r(a_{\max}, n_{\min}, m_{\min}) \text{ and } \psi_{r\min} = \psi_r(a_{\min}, n_{\max}, m_{\max}); \quad \text{----- (3-53)}$$

where,

$\psi_b(a, n, m)$ and $\psi_r(a, n, m)$ are the functions for the determination of the air-entry value and the residual suction (Zhai and Rahardjo, 2013a), respectively.

3.5 Estimation of the wetting SWCC from the primary drying SWCC

The model representing hysteresis of SWCC due to the difference in contact angles during the drying and wetting processes and the “ink-bottle” effect is presented in this section.

3.5.1 Capillary model

The capillary model considers the pores in the soil as a series of capillary tubes. As shown in Figure 3-8, pores with different sizes can be simplified as a series of capillary tubes with different radii (e.g., $r_1, r_2, r_3, \dots, r_N$, where $r_1 > r_2 > r_3 > \dots > r_N$). When the soil is fully saturated, all the tubes are filled with water. where there is an increase in suction, water will drain from the largest tube first, then the second largest tube and so on until an equilibrium state is reached (as illustrated in Figure 3-9), based on Kelvin’s capillary law (Fredlund and Rahardjo, 1993 Fredlund et al., 2012). The tubes are randomly connected to each other and the probability of a connection between two capillary tubes is dependent on their statistical distribution (i.e., pore-size distribution function). Zhai and Rahardjo (2015) demonstrated that the pore-size distribution function is a function of matric suction if there is a volume change with a change in matric suction. Zhai and Rahardjo (2015) also suggested that the SWCC in the form of degree of saturation can be simplified as an integrated form of the pore-size distribution function. Therefore, the SWCC in the form of degree of saturation was used for the estimation of the hysteresis of SWCC throughout this paper. As matric suction is commonly referred to in unsaturated soil mechanics, Zhai and Rahardjo (2015) suggested the use of the term "pore-suction distribution function" to replace "pore-size distribution function" by converting the radius into suction using Kelvin's capillary law.

Figure. 3-8 Illustration of the capillary model incorporated into the SWCC.

(a) Water phase in the capillary tubes when the soil is fully saturated.

(b) Water phase in the capillary tubes under suction of ψ_1 .

(c) Water phase in the capillary tubes under suction of ψ_2 ($\psi_2 > \psi_1$).

Figure. 3-9 Water phase in the capillary tubes under different suctions.

3.5.2 Hysteresis of SWCC due to the difference in contact angles during the drying and wetting processes

Bear (1979) demonstrated that the contact angle at an advancing interface during the wetting process is different from that at a receding interface during the drying process. Goebel et al. (2004), Ramirez-Flores et al. (2008) and Bachmann et al. (2013) presented that the contact angle is dependent on water repellency, where higher water repellency represents a greater contact angle. Dekker and Ritsema (1994) proposed that the water repellency of a soil depends on its moisture content and that air-dry soil repels water the most. Experimental data from Bachmann et al. (2002) showed that the contact angle for hydrophobic soil decreases with an increase in water content. In addition, the experimental results from Bachmann and Ploeg (2002) indicated that contact angle in the wetting process was higher than that in the drying process. Therefore, it is reasonable to assume that the contact angle during the wetting process is greater than that during the drying process.

When the soil is dried from A to B, water will drain from tubes between A and B, as illustrated in Figure 3-10 (a) and (b). However, when the soil is wetted from B to A, water may not fill all of the tubes between A and B because the contact angle is greater in the wetting process than in the drying process. Only the tubes between B and C can fill with water and the water content corresponding to point C can be calculated from the drying SWCC as illustrated in Figure 3-10 (c). As illustrated in Figure 3-10 (c), if ψ' can be estimated then the hysteresis of SWCC resulting from the difference in contact angles during the drying and wetting processes can be calculated.

(a) Illustration of the water phase in the capillary tubes at suction of ψ before drying.

(b) Illustration of the water phase in the capillary tubes at suction of ψ_m after the drying process from A to B.

(c) Illustration of the water phase in the capillary tubes at suction of ψ after the wetting process from B to C.

Figure. 3-10 Illustration of water movement in the tubes during the drying and wetting processes.

The final equilibrium state (i.e., water filling the capillary tubes or water draining out from the capillary tubes) is governed by the capillary law, as illustrated in Equation (3-54), as follows:

$$r = \frac{2T_s \cos \alpha}{\psi} \quad \text{----- (3-54)}$$

where,

r = radius of capillary tube,

ψ = matric suction,

T_s = surface tension and

α = contact angle.

Defining the contact angle as α_{dry} at point A and as α_{wet} at point C (where $\alpha_{\text{wet}} > \alpha_{\text{dry}}$), at point A water can fill capillary tubes with a maximum radius of $r_{\text{dry}} = 2T_s \cos \alpha_{\text{dry}} / \psi$, while at point C water can only fill capillary tubes with a maximum radius of $r_{\text{wet}} = 2T_s \cos \alpha_{\text{wet}} / \psi$. It is observed that r_{dry} is greater than r_{wet} , which means, under the same suction, water can fill more tubes during the drying process than during the wetting process. As illustrated in Figure 3-9 (c), r_{wet} can be estimated by considering a higher suction of ψ' instead of ψ (i.e., assuming $\psi' = k\psi + b$, where k, b are constants, $k > 1$ and $b > 0$).

In other words, the hysteresis of SWCC due to the difference in contact angles during the drying and wetting processes can be estimated using Equation (3-55) as follows:

$$\Delta S_1 = S(\psi) - S(\psi') \quad \text{----- (3-55)}$$

where:

ΔS_1 = hysteresis of degree of saturation due to difference in contact angles during the drying and wetting processes,

$S(\psi)$ is the degree of saturation corresponding to ψ on the drying curve,

$S(\psi')$ is the degree of saturation corresponding to ψ' on the drying curve,

$\psi' = k\psi + b$ and

k, b = constant, $k > 1, b > 0$.

3.5.3 Hysteresis of SWCC due to the “ink-bottle” effect

During the wetting process, according to the capillary law, water fills the capillary tubes from those with a smaller radius to those with a bigger radius, as illustrated in Figure 3-9 (c). In reality, these capillary tubes are randomly connected to each other and dry tubes may cause the “ink-bottle” effect and block the water flow. Reinson et al. (2005) presented that water only can flow through the water phase (i.e. wet pores or pendular rings) and cannot flow through the air phase in an unsaturated medium. Therefore, the water filling procedure may be affected by these dry tubes, as illustrated in Figure 3-11 (b). Once equilibrium is

reached under suction of ψ , the capillary tubes with radii less than $2T_s \cos \alpha / \psi$ should be filled with water based on capillary law. On the other hand, the tubes may remain dry because of the larger dry tubes. If the tube is connected with smaller tubes, as illustrated in Figure 3-11 (a), the water will flow through the smaller tubes and fill the tube. However, if the tube is connected with a larger dry tube, as illustrated in Figure 3-11 (b), then the water cannot enter the larger tube, according to the capillary law, and cannot fill the tube. The scenario illustrated in Figure 3-11 (b) is commonly called the “ink-bottle” effect.

Figure. 3-11 Illustration of water filling a pore, pending the conditions of the connected pores.

Figure 3-12 Illustration of the hysteresis of SWCC due to the “ink-bottle” effect.

Due to the “ink-bottle” effect, during the wetting process, the water content may only achieve point D rather than point C, as illustrated in Figure 3-12. Because capillary tubes between zero suction and suction of ψ' are dry, as illustrated in Figure 3-12, if these tubes are connected to smaller tubes and they will cause the "ink-bottle" effect, as illustrated in Figure

3-11 (b). The hysteresis of the SWCC due to the “ink-bottle” effect can be quantified using the concept of pore-size distribution function and random connection. The probability of a tube with a radius of r_1 being connected to a tube with a radius of r_i is: $f(r_1)*f(r_i)$, (where, $f(r_1)$ and $f(r_i)$ are the pore size density for a tube with a radius of r_1 and r_i , respectively). A reduction in the degree of saturation resulted from the "ink-bottle" effect, such as the tube with a radius of r_1 blocking the tube with a radius of r_i , is: $f(r_1)f(r_i)\Delta S_i$, where ΔS_i is the increase in the degree of saturation due to only capillary tubes with a radius of r_i being fully filled.

Similarly, a tube with a radius of r_2 will cause a reduction in the degree of saturation as $f(r_2)f(r_i)\Delta S_i$, then the reduction in the degree of saturation due to the "ink-bottle" effect ΔS_2 can be obtained by summarizing all the scenarios as follows:

$$\Delta S_2 = (1 - S(\psi'))(S(\psi') - S(\psi_m)) \quad \text{----- (3-56)}$$

Therefore, the degree of saturation at point D in Figure 6 can be calculated as follows:

$$S = S(\psi_m) + S(\psi')[S(\psi') - S(\psi_m)] \quad \text{----- (3-57)}$$

where:

S = degree of saturation at Point D,

$S(\psi')$ = degree of saturation at Point C,

$S(\psi_m)$ = degree of saturation at the point where starts the wetting process,

ψ' = suction corresponding to $S(\psi')$ on the drying curve, $\psi' = k\psi + b$ and

ψ_m = the suction where the soil starts to be saturated on the drying curve.

3.5.4 Scanning curve for the drying process

After saturation is completed in the wetting process, the soil is dried again. The drying scanning curve is then generated. During the drying process, no water drains out from the soil

when the suction increases from ψ_i to ψ' because the pores with radii of r_i to r_{i+k} are all in a dry condition. If the suction increases beyond the suction of ψ' , water will gradually drain out from the pores with a size of r_{i+k+1} to the smaller pores. When the water is fully drained out from pores r_{i+k+1} , the reduction in degree of saturation is $S(\psi')f(r_{i+k+1})\Delta r$ instead of $f(r_{i+k+1})\Delta r$ because not all the pores with a size of r_{i+k+1} are fully saturated due to the “ink-bottle” effect.

Therefore, the drying scanning curve can be estimated using Equation (3-58) as follows:

$$S_{dscanning} = \begin{cases} S(\psi_i) & \psi < \psi' \\ S(\psi_i) - \sum_{j=1} S(\psi) f(r_{i+k+j}) \Delta r & \psi > \psi' \end{cases} \quad \text{----- (3-58)}$$

where:

$S_{dscanning}$ = degree of saturation corresponding to the drying scanning curve;

$S(\psi_i)$ = degree of saturation corresponding to the wetting curve when saturation is completed;

$S(\psi)$ = degree of saturation corresponding to the drying curve;

ψ' = the suction on the drying curve.

If the degree of saturation calculated from Equation (3-58) is higher than the one calculated from the drying curve, then the drying scanning curve will follow the drying curve.

3.6 Calculation of permeability function for unsaturated soil from SWCC fitting parameters

The concepts and derived equations for prediction of permeability function from SWCC are presented in this section.

3.6.1 Relationship between SWCC and pore-size distribution function

Several statistical models (e.g., Childs and Collis-George 1950; Marshall 1958; Kunze et al. 1968; and Zhai and Rahardjo 2015) have considered that SWCC is analogous to the pore-size distribution function. As the radius of a pore can be calculated from suction using capillary law as introduced by Fredlund and Rahardjo (1993), and suction is commonly used as the variable for solving problems related to unsaturated soil, the pore-size distribution function is redefined as the pore-suction distribution function, which defines the relationship between pore size density and matric suction. The SWCC and pore-suction distribution functions are illustrated in Figure. 3-13. The degree of saturation (S) defines the ratio of volume of water in the soil to the total volume of voids (or pores) in the soil. Fredlund and Rahardjo (1993) presented that suction (ψ) can be related to the radius of the meniscus using Kelvin's capillary law: a higher suction result in a smaller radius of the meniscus. If the radius of the meniscus is smaller than the radius of the pore, air will break through the meniscus and water in the pore will drain out. In other words, the area under the pore-suction distribution function defines the degree of saturation, as illustrated in Figure. 3-14. The overall area under the pore-suction distribution function should be equal to 1 (i.e., $\int_{0.01}^{10^6} f(\psi) d\psi = 1$). Therefore, the SWCC in the form of degree of saturation is considered analogous to the pore-suction distribution function.

Figure. 3-13 Illustration of the SWCC and pore-suction distribution function.

Figure. 3-14 Illustration of the relationship between the degree of saturation and pore-suction distribution function.

Work from Fredlund and Pham (2006) and Cuisinier et al. (2014) indicated that the pore-suction distribution function is a function of suction. In other words, the pore-suction distribution function changes with changes in suction because of the soil volume change that occurs during the drying or wetting process (Fredlund and Pham, 2006; Cuisinier et al., 2014). The left-hand side of Figure. 3-15 shows the pore-suction distribution function at suction ψ_1 . If suction increases to ψ_2 , the pore-suction distribution function may change to the curve on the right-hand side of Figure 3-15. Although the pore-suction distribution function may change with a change in suction, the definition of the degree of saturation using the pore-suction distribution function is still applicable, as illustrated in Figure. 3-15.

Figure. 3-15 Illustration of degree of saturation using pore-suction distribution functions in states with different suctions.

It must be highlighted that only the SWCC in the form of degree of saturation corresponding to the drying process can be considered analogous to the pore-suction distribution function. The SWCC corresponding to the wetting process cannot be considered analogous to the pore-suction distribution function because during the wetting process, at a given suction of ψ_i , not all pores in the soil with radii less than r_i (i.e., $r_i = 2T/\psi_i$ using Kelvin's law) are fully filled with water. The definition in Figure 3-18 may no longer be applicable.

3.6.2 Probability function for random connections

The coefficient of permeability is derived from Childs and Collis-George's (1950) assumption. Poiseuille's law (Sutera and Skalak 1993) for stream-line flow and the theory of probability of random connection (Benjamin and Cornell 1970) are adopted in the derivation. The SWCC in the form of degree of saturation (S) is referred to as the probability function of the random connection.

The entire range of pores in the soil can be divided into groups. Each group can be treated as a capillary tube with a certain diameter, as illustrated in Figure. 3-16. The diameter of the capillary tube represents the pore radius of the individual group of pores. The minimum value, the maximum value or the mean value of the pore radius can be adopted as the diameter of the capillary tube. Normally, the mean value gives the most accurate results. However, if the interval is very small, either the minimum or the maximum value can give satisfactory results. The normalized volume (i.e., the ratio of the volume of pores from an individual group to the volume of all pores in the soil) of an individual capillary tube can be calculated from the area under the pore-suction distribution function, which is equal to ΔS as illustrated in Figure. 3-16. Therefore, the probability of a capillary tube with a larger diameter being connected to a capillary tube with a smaller diameter is $\Delta S_1 * \Delta S_2$, as illustrated in Figure. 3-16.

Figure. 3-16 Illustration of the equivalent capillary tube associated with the pore-suction distribution function from Zhai and Rahardjo (2015)

3.6.3 Interval of suction ($d\psi$)

The diameter and normalized volume of a capillary cube are dependent on the interval of suction, $d\psi$. There are three approaches to dividing the entire suction range into groups. The first is to create individual groups with the same normalized volume, as illustrated in Figure. 3-17(a). The second is to create equal interval $d\psi$ or $d(\log\psi)$ groups, as illustrated in Figure. 3-17(b). The third is to randomly divide the entire suction range into a number of intervals.

The first approach makes each group of pores to have the same pore size density. Kunze et al. (1968) adopted this approach, but used SWCC in the form of volumetric water content instead of degree of saturation as the probability function for random connection. In Kunze et al.'s (1968) equation, the interval of suction ($d\psi$) needs to be calculated from the interval of volumetric water content ($d\theta_w$). In addition, the intervals in Kunze et al.'s (1968) equation can be very wide at certain ranges, which make the calculated results to be less accurate, as shown in Figure. 3-18. No results were obtained between suctions of 0.01 kPa to 0.3 kPa, and

only two data points were obtained beyond suction of 5 kPa. The calculated results mostly cluster between suctions of 0.3 kPa and 5 kPa. It seems that the results calculated using Kunze et al.'s (1968) equation are less accurate in both the low and high suction ranges as the calculated results mostly cluster in the medium suction range. In this research, an equal interval $d\psi$ is adopted and the calculated results are evenly distributed across the entire suction range.

Figure. 3-17 Approaches for dividing the entire suction range into different groups Zhai and Rahardjo (2015)

Figure. 3-18 Calculated relative coefficient of permeability k_r (i.e. $k_r=k(\psi)/k_s$) using Kunze et al.'s (1968) equation

3.6.4 Equivalent effective radius for water flow

Consider a soil element cut into two pieces and define the two sections on the joint as section A and section B. The groups of pores in the two sections are considered as capillary tubes with different diameters (i.e., $r_1, r_2, \dots, r_i, \dots, r_n$, where $r_1 > r_2 > \dots > r_i > \dots > r_n$). The pores with different radii are distributed following the pore-suction distribution function in both sections, as illustrated in Figure. 3-19.

Figure. 3-19 Illustration of the distribution of pores in both sections.

The soil particles and pores with different radii in section A can be randomly connected with the soil particles and pores in section B. There are six scenarios for the possible connections, as illustrated in Figure. 3-20.

Figure. 3-20 Illustration of the possible scenarios for the connection of two sections.

A circle and rectangle are used to represent soil particles and pores, respectively. There are a total of six possible scenarios of connections, as illustrated in Fig. 3-20. Scenario (a) illustrates a soil particle connected to a soil particle; (b) illustrates a dry pore connected to a soil particle; (c) illustrates a saturated pore connected to a soil particle; (d) illustrates a dry pore connected to a dry pore; (e) illustrates a dry pore connected to a saturated pore; and (f) illustrates a saturated pore connected to a saturated pore.

Scenarios (a) to (e) are observed to be impervious, as the water phase is discontinuous and only scenario (f) allows water to flow through both capillary tubes. The permeability of a section is dependent on the smaller diameter of the capillary tube, as illustrated in scenario (f). Therefore, the permeability of a section is dependent on the probability of scenario (f) occurring and the diameter of the smaller capillary tube.

If the soil is fully saturated, there will be no dry pores and scenarios (b), (d) and (e) will not occur. The equivalent effective radius for water flow is dependent on the probability of the connection of pores. The probability of a pore with a radius of r_1 in section A being connected to a pore with a radius of r_1 in section B is $n \cdot f(r_1) \cdot n \cdot f(r_1) = n^2 \cdot f(r_1)^2$, where n is the porosity of the soil and $f(r_1)$ is the pore size density corresponding to r_1 . If unlimited numbers of pores in the soil are represented by a limited number of capillary tubes, $f(r_1)$ can be calculated by the area under the pore-suction distribution function. Similarly, the probability of a pore with a radius of r_i in section A being connected to a pore with a radius of r_j in section B is $n \cdot f(r_i) \cdot n \cdot f(r_j) = n^2 \cdot f(r_i) \cdot f(r_j)$, where, $r_i > r_j$. As the permeability of the connected section is dependent on the smaller radius of the pore (i.e., πr_j^2), the effective area for water flow is equal to $n^2 \cdot f(r_i) \cdot f(r_j) \pi r_j^2$.

If the soil is fully saturated, a pore with a radius of r_1 in section A connected to pores with radii of r_1 to r_n in section B will make all the sections permeable. The effective area for water flow can be obtained using Equation (3-59):

$$A(r_1) = n^2 \pi \left(f(r_1) \cdot f(r_1) r_1^2 + f(r_1) \cdot f(r_2) r_2^2 + \dots + f(r_1) \cdot f(r_n) r_n^2 \right) \quad \text{----- (3-59)}$$

Similarly, a pore with a radius of r_2 in section A connected to pores with radii of r_1 to r_n in section B will result in the effective area for water flow given by Equation (3-60):

$$A(r_2) = n^2 \pi \left(f(r_2) \cdot f(r_1) r_1^2 + f(r_2) \cdot f(r_2) r_2^2 + \dots + f(r_2) \cdot f(r_n) r_n^2 \right) \quad \text{----- (3-60)}$$

A pore with a radius of r_i in section A connected to pores with radii of r_1 to r_n in section B will result in the effective area for water flow given by Equation (3-61):

$$A(r_i) = n^2 \pi \left(f(r_i) \cdot f(r_1) r_1^2 + f(r_i) \cdot f(r_2) r_2^2 + \dots + f(r_i) \cdot f(r_{i+1}) r_{i+1}^2 + \dots + f(r_i) \cdot f(r_n) r_n^2 \right) \quad \text{----- (3-61)}$$

Summation of all these effective areas for water flow yields Equation (3-62):

$$\begin{aligned}
A &= \sum_{i=1}^n A(r_i) = n^2 \pi \left(f^2(r_1)r_1^2 + [2f(r_1)f(r_2) + f^2(r_2)]r_2^2 + \right. \\
&\quad \left. [2f(r_1)f(r_3) + 2f(r_2)f(r_3) + f^2(r_3)]r_3^2 + \dots \right) \\
&= n^2 \pi \left(f^2(r_1)r_1^2 + \sum_{i=2}^N \left[\left(\sum_{j=1}^i f(r_j) \right)^2 - \left(\sum_{j=1}^{i-1} f(r_j) \right)^2 \right] r_i^2 \right) \quad \text{-----(3-62)}
\end{aligned}$$

Therefore, the equivalent effective radius for water flow in the saturated soil can be expressed as follows:

$$r_t^2 = n^2 \left(f^2(r_1)r_1^2 + \sum_{i=2}^N \left[\left(\sum_{j=1}^i f(r_j) \right)^2 - \left(\sum_{j=1}^{i-1} f(r_j) \right)^2 \right] r_i^2 \right) \quad \text{-----(3-63)}$$

Substituting $\sum_{j=1}^i f(r_j) = 1 - S(\psi_i)$ into Equation (3-63) gives

$$r_t^2 = n^2 \left(\sum_{i=1}^N [1 - S(\psi_i)]^2 - [1 - S(\psi_{i-1})]^2 \right) r_i^2, \text{ where } S(\psi_0) = 1 \quad \text{----- (3-64)}$$

Therefore, the effective radius for the permeability of saturated soil can be obtained using Equation (3-64).

If the soil is unsaturated, dry pores will exist and scenarios (b), (d) and (e) can occur. The number of dry pores is dependent on the suction state; a higher suction results in more dry pores in the soil. Any dry pores or soil particles in a section will make the section impervious. If the suction state in the soil is ψ_m , all pores with radii greater than r_m (i.e. $r_m = 2T/\psi_m$) are dry pores, while all pores with radii less than r_m are saturated pores. The radii of dry pores are termed r_1, r_2 to r_m , and the radii of saturated pores are termed r_{m+1}, r_{m+2} to r_n . The probability of connection of saturated pores is illustrated in Figure. 3-21.

Figure.3-21 Illustration of probability of connection of saturated pores with suction of ψ_m from Zhai and Rahardjo (2015).

Summarizing all the effective radii corresponding to different radii yields the effective radius for water flow in the unsaturated soil with a suction state of ψ_m as follows:

$$r_t^2 = n^2 \left([S(\psi_m) - S(\psi_{m+1})]^2 r_m^2 + \sum_{i=m+1}^N [[S(\psi_m) - S(\psi_i)]^2 - [S(\psi_m) - S(\psi_{i-1})]^2] r_i^2 \right) \text{----- (3-65)}$$

3.6.5 Computation of hydraulic conductivity for the saturated and unsaturated soil

By substituting Equation (3-64) into Poiseuille's equation, the permeability for saturated soil can be obtained as follows:

$$k_s = \frac{1}{8} n^2 \left\{ \sum_{i=1}^N [[1 - S(\psi_i)]^2 - [1 - S(\psi_{i-1})]^2] r_i^2 \right\}, \text{----- (3-66)}$$

where, $S(\psi_0) = 1$.

Substituting Equation (3-65) into Poiseuille's equation results in the relative permeability function k_r for unsaturated soil as follows:

$$k_r = \frac{n_m^2}{n_s^2} \frac{(S(\psi_m) - S(\psi_{m+1}))^2 \psi_m^{-2} + \sum_{i=m+1}^N [(S(\psi_m) - S(\psi_i))^2 - (S(\psi_m) - S(\psi_{i-1}))^2] \psi_i^{-2}}{\sum_{i=1}^N [1 - S(\psi_i)]^2 - [1 - S(\psi_{i-1})]^2] \psi_i^{-2}} \quad \text{----- (3-67)}$$

where, k_s = the saturated coefficient of permeability

ψ_m = suction state in the soil

n_s = porosity corresponding to the saturated state

n_m = porosity corresponding to suction ψ_m .

If the coefficient of permeability $k(\psi_m)$ with respect to suction of ψ_m is known, then the coefficient of permeability $k(\psi_{m+i})$ with respect to suction of ψ_{m+i} can be calculated using Equation (3-68).

$$k(\psi_{m+i}) = k(\psi_m) \frac{n_{m+i}^2 \left\{ (S(\psi_{m+i}) - S(\psi_{m+i+1}))^2 r_{m+i}^2 + \sum_{j=m+i+1}^N [(S(\psi_{m+i}) - S(\psi_j))^2 - (S(\psi_{m+i}) - S(\psi_{j-1}))^2] r_j^2 \right\}}{n_m^2 \left\{ (S(\psi_m) - S(\psi_{m+1}))^2 r_m^2 + \sum_{i=m+1}^N [(S(\psi_m) - S(\psi_i))^2 - (S(\psi_m) - S(\psi_{i-1}))^2] r_i^2 \right\}} \quad \text{----- (3-68)}$$

3.6.6 Relationship between the proposed equation and Marshall's (1958) and Kunze et al.'s (1968) equations

If the pore-suction distribution function is considered to be uniformly distributed, the pore size density $f(\psi_i)$ will be equal to $1/N$. Substituting $\sum_{j=1}^i f(\psi_j) = \frac{i}{N}$ into Equation (3-66), the coefficient of permeability can be obtained from Equation (3-69), which is the same as Marshall's (1958) equation:

$$k = \frac{n^2}{N^2} \sum_{i=1}^N [(i)^2 - (i-1)^2] r_i^2 = \frac{n^2}{N^2} [r_1^2 + 3r_2^2 + 5r_3^2 + \dots (2n-1)r_n^2] / 8 \quad \text{----- (3-69)}$$

By substituting $S(\psi_m) - S(\psi_i) = (i-m)/N$ into Equation (3-67) and ignoring the soil volume change, the coefficient of permeability of unsaturated soil can be obtained as follows:

$$\begin{aligned}
k_r = \frac{k(\psi_m)}{k_s} &= \frac{\sum_{i=m}^N \left[\left(1 - \frac{m}{N} - \left(1 - \frac{i}{N} \right) \right)^2 - \left(1 - \frac{m}{N} - 1 - \frac{i-1}{N} \right)^2 \right] \psi_i^{-2}}{\sum_{i=1}^N \left[\left(1 - \left(1 - \frac{i}{N} \right) \right)^2 - \left(1 - \left(1 - \frac{i-1}{N} \right) \right)^2 \right] \psi_i^{-2}} \\
&= \frac{\sum_{i=m}^N \left[(i-m)^2 - (i-m-1)^2 \right] \psi_i^{-2}}{\sum_{i=1}^N \left[(i)^2 - (i-1)^2 \right] \psi_i^{-2}}
\end{aligned}
\tag{3-70}$$

Equation (3-70) is the same as Kunze et al.'s (1968) equation, as expressed in Equation (2-31).

Therefore, the proposed equation is proven to be a general form of Marshall's (1958) and Kunze et al.'s (1968) equations.

3.7 Variability in relative hydraulic conductivity due to uncertainty in SWCC

Different relative hydraulic conductivities will be obtained if different SWCCs are used for the prediction. Therefore, the upper/lower confidence limits of relative hydraulic conductivity can be calculated from the upper/lower confidence limits of the SWCC using Equation (3-68). In other words, the variability in the relative hydraulic conductivity of unsaturated soil can be estimated using Equations (3-29), (3-30) and (3-68).

3.8 Quantification of variability in SWCC

The variability y in volumetric water content θ_w , which is defined in the following equation, is adopted for representation of the variability in SWCC:

$$\text{variability } y = \frac{\theta_i - \hat{\theta}_i}{\theta_s} 100\%
\tag{3-71}$$

where,

θ_i : predicted volumetric water content at i suction level, from the confidence limit of the best fitted SWCC or from experimental data

$\hat{\theta}_i$: best estimated volumetric water content at i suction level, from the best fitted SWCC

θ_s : saturated volumetric water content.

Equation (3-70) is similar to the equation presented by Zapata (1999). If the volumetric water content determined from the best fitted SWCC is taken as the mean value, then the upper bound and lower bound of the volumetric water content can be obtained from y by comparing the water content obtained from the confidence limits to that determined from the best fitted SWCC, as illustrated in Figure. 3-22. As both ends of the SWCC are well defined by Fredlund and Xing's (1994) equation (i.e., $\theta_w = \theta_s$ at suction of $\psi = 0.01$ kPa and $\theta_w = 0$ at suction of $\psi = 10^6$ kPa), the variability of SWCC at these two ends is defined to be zero as illustrated in Figure 3-22.

Figure 3-22 Illustration of upper bound and lower bound of the determined volumetric water content.

3.9 Variability in SWCC associated with laboratory measurement

Many factors can result in variability in SWCC associated with laboratory measurement. Variability may result from inherent uncertainty in the natural soil, measurement error, the equipment and methods used to acquire SWCC data, soil volume change associated with measurement, the suction range covered in the measurement, the skill of the operator, etc. In this research, all measurements were carried out by the author himself; therefore variability in SWCC due to measurement error and skill of operator was considered insignificant and ignored. A pressure plate was the only apparatus used in this research for determination of SWCC; therefore variability in SWCC due to the equipment and methods used was also considered insignificant and ignored. The causes of variability that were investigated in this research are hysteresis character of SWCC, soil volume change associated with measurement and the suction range covered in the measurement.

3.9.1 Variability in SWCC resulting from the hysteresis character of SWCC

It is known that the water content of soil at a given suction obtained from the wetting path is less than that obtained from the drying process. This phenomenon is known as the hysteresis character of SWCC. Sometimes the drying curve and wetting curve cannot form a closed loop due to entrapped air. SWCC curves may also follow different paths if the soil is dried (or wetted) from the starting point with different water contents. These curves are normally called "scanning curves" and are illustrated in Figure 3.23. Different scanning curves may be obtained due to hysteresis if the soil specimen is dried or wetted at a different water content.

Figure. 3-23 Illustration of variability in SWCC with respect to different initial water contents (Pham et al., 2005).

3.9.2 Variability in SWCC resulting from soil volume change associated with measurement

The relationships between the slopes of the three forms of SWCCs (i.e., gravimetric water content, w , volumetric water content, θ_w and degree of saturation, S) are derived and discussed in this section. It is observed that the fitting parameters of the SWCCs for these three forms are different for the same soil if the soil volume change is considered.

3.9.2.1 Slope of SWCCs in different forms

The relationships between gravimetric water content (w), volumetric water content (θ_w) and degree of saturation (S) are expressed as follows:

$$\theta_w = \frac{V_w}{V} = \frac{Se}{1+e} = \frac{G_s}{1+e} w \quad \text{----- (3-72)}$$

$$S = \frac{G_s}{e} w \quad \text{----- (3-73)}$$

In Equations (3-72) and (3-73), both the gravimetric water content (w) and void ratio (e) are directly measured, while the volumetric water content (θ_w) and degree of saturation (S) are calculated from the gravimetric water content and void ratio. Therefore, the right-hand side of Equations (3-72) and (3-73) can be considered as directly measured values, and the left-hand side can be considered as calculated values. In other words, the accuracy of θ_w and S relies on the accuracy of w and e . Differentiating Equations (3-72) and (3-73) with respect to matric suction results in the following slope functions of SWCC:

$$\frac{\partial \theta_w}{\partial \psi} = \frac{\partial \left(\frac{G_s}{1+e} w \right)}{\partial \psi} = \frac{G_s}{1+e} \frac{\partial w}{\partial \psi} - \frac{w G_s}{(1+e)^2} \frac{\partial e}{\partial \psi} \quad \text{----- (3-74)}$$

$$\frac{\partial S}{\partial \psi} = \frac{\partial \left(\frac{G_s}{e} w \right)}{\partial \psi} = \frac{G_s}{e} \frac{\partial w}{\partial \psi} - \frac{G_s w}{e^2} \frac{\partial e}{\partial \psi} \quad \text{----- (3-75)}$$

Equations (3-74) and (3-75) indicate that both slopes $\frac{\partial \theta_w}{\partial \psi}$ and $\frac{\partial S}{\partial \psi}$ are dependent on two variables, namely the gravimetric water content (w) and void ratio (e). In some cases, volumetric water content (θ_w) and degree of saturation (S) are calculated with the assumption of a constant void ratio (i.e., $e=e_0$, where e_0 is the initial void ratio). The slope of the SWCC can then be calculated from Equations (3-74) and (3-75) considering $e=e_0$ and $\frac{\partial e}{\partial \psi} = 0$ as follows:

$$\frac{\partial \theta_w}{\partial \psi} = \frac{\partial \left(\frac{G_s}{1+e_0} w \right)}{\partial \psi} = \frac{G_s}{1+e_0} \frac{\partial w}{\partial \psi} \quad \text{----- (3-76)}$$

$$\frac{\partial S}{\partial \psi} = \frac{\partial \left(\frac{G_s}{e_0} w \right)}{\partial \psi} = \frac{G_s}{e_0} \frac{\partial w}{\partial \psi} \quad \text{----- (3-77)}$$

Equations (3-76) and (3-77) indicate there are constant linear relationships between the slopes of SWCCs in different forms (i.e., $\frac{\partial \theta_w}{\partial \psi}$, $\frac{\partial S}{\partial \psi}$ and $\frac{\partial w}{\partial \psi}$) if the soil volume change is ignored.

However, in most cases, the soil volume change cannot be ignored especially for soils that are associated with a large volume change. When the suction in the soil is low, the soil remains in a saturated state (i.e., S remains as 1), and Equations (3-74) and (3-75) can be simplified as follows:

$$\frac{\partial \theta_w}{\partial \psi} = \frac{\partial \left(\frac{G_s}{1+e} w \right)}{\partial \psi} = \frac{1}{(1+G_s w)^2} \frac{\partial w}{\partial \psi} = \frac{1}{(1+e)^2} \frac{\partial w}{\partial \psi} \quad \text{----- (3-78)}$$

$$\frac{\partial S}{\partial \psi} = 0 \quad \text{----- (3-79)}$$

Since $\frac{1}{1+e_0} > \frac{1}{(1+e)^2} > 0$, Equations (3-78) and (3-79) indicate that, at a low suction range,

the soil is saturated and the SWCC in the form of gravimetric water content (w) has a steeper slope than the SWCC in the form of volumetric water content (θ_w), and the SWCC in the form of volumetric water content (θ_w) has a steeper slope than the SWCC in the form of degree of saturation (S).

In contrast, at a high suction range, the void change becomes insignificant and the void ratio can be considered constant (i.e., $\frac{\partial e}{\partial \psi} \approx 0$). Equations (3-74) and (3-75) can be simplified as follows:

$$\frac{\partial \theta_w}{\partial \psi} = \frac{G_s}{1+e_i} \frac{\partial w}{\partial \psi} \quad \text{----- (3-80)}$$

$$\frac{\partial S}{\partial \psi} = \frac{G_s}{e_i} \frac{\partial w}{\partial \psi} \quad \text{----- (3-81)}$$

where,

e_i is the final void ratio.

Since $\frac{1}{1+e_i} > \frac{1}{1+e_0}$, $\frac{1}{e_i} > \frac{1}{e_0}$, the slopes calculated from Equations (3-80) and (3-81) are greater than those calculated from Equations (3-76) and (3-77), respectively. In other words, at a high suction range, the SWCC in the form of gravimetric water content (w) has a gentler slope than the SWCC in the form of volumetric water content (θ_w), and the SWCC in the form of volumetric water content (θ_w) has a gentler slope than the SWCC in the form of degree of saturation (S).

In conclusion, ignoring soil volume change can lead to steeper slopes at a low suction range and gentler slopes at a high suction range. The difference between SWCCs that consider or ignore the soil volume change is dependent on the significance of the soil volume change during measurement.

The difference between the slope of SWCCs ignoring and considering soil volume change is illustrated using SWCC data for Oil Sands tailing from Fredlund and Houston (2013) in Figure 3.24.

Figure. 3-24 SWCCs of Oil Sands tailing (after Fredlund and Houston, 2013).

3.9.2.2 Best fit equation for the soil volume change

Equation (3-72) can be made a differential equation and re-arranged as follows:

$$S \frac{\partial e}{\partial \psi} + e \frac{\partial S}{\partial \psi} = G_s \frac{\partial w}{\partial \psi} \quad \text{-----}(3-82)$$

If both the gravimetric water content (w) and degree of saturation (S) are expressed using Fredlund and Xing's (1994) equation, Equation (3-83) can be expressed as follows:

$$\begin{aligned} & \frac{1}{\left\{ \ln \left[e + \left(\frac{\psi}{a_s} \right)^{n_s} \right] \right\}^{m_s}} \frac{\partial e}{\partial \psi} - e \frac{\frac{m_s n_s \left(\frac{\psi}{a_s} \right)^{n_s - 1}}{a_s}}{\left\{ \ln \left[e + \left(\frac{\psi}{a_s} \right)^{n_s} \right] \right\}^{m_s + 1}} \\ &= -G_s \frac{\frac{m_w n_w \left(\frac{\psi}{a_s} \right)^{n_w - 1}}{a_s}}{\left\{ \ln \left[e + \left(\frac{\psi}{a_w} \right)^{n_w} \right] \right\}^{m_w + 1}} \quad \text{-----}(3-83) \end{aligned}$$

The void ratio expressed using Fredlund and Xing's (1994) equation, as given in Equation (3-84), is one possible solution of the differential Equation (3-83):

$$e = \frac{e_{\max}}{\left\{ \ln \left[e + \left(\frac{\psi}{a_e} \right)^{n_e} \right] \right\}^{m_e}} \quad \text{-----}(3-84)$$

where,

a_e, n_e and m_e are fitting parameters

$e_{\max}$ is the initial void ratio.

The fitting parameters in Equation (3-84) can be determined using the curve fitting technique. They are illustrated in Figure 3-25 using data for Regina clay from Fredlund and Houston (2013).

Figure. 3-25 Illustration of Fredlund and Xing's (1994) equation for expressing soil volume change using data from Fredlund and Houston (2013).

3.9.3 Variability in SWCC resulting from different suction ranges covered in the laboratory measurements

Zhai and Rahardjo (2013b) showed that different suction ranges used in SWCC measurements lead to variability in SWCCs. Residual suction is very important for the determination of SWCC, and if an inadequate suction range is adopted in the measurements it can result in an inaccurate SWCC. Leong and Rahardjo (1997a) suggested that SWCC measurements should cover a suction range beyond the residual suction. Zhai and Rahardjo (2015) demonstrated that residual suction is also important for determination of the

permeability function for unsaturated soil. Therefore, variability in SWCC may occur if the maximum measurement suction is less than the residual suction of the soil.

3.10 Variability in SWCC associated with data interpretation

After SWCC data are obtained from laboratory measurements, best fit equations are commonly used for best fitting the experimental data and as a result, the fitting parameters are determined. Various best fit equations have been proposed by different researchers, and variability in SWCC may result from the adoption of different best fit equations. However, only Fredlund and Xing's (1994) equation was used in this research; therefore variability in SWCC due to different best fit equations can be eliminated. Variability in SWCC may also occur if different numbers of data points are selected for determination of the best fitted SWCC.

CHAPTER FOUR

RESEARCH PROGRAM

4.1 Introduction

In this chapter, flow chart of this research, collected published data from literature, experiment measurement including the criteria for soil selection, set up of the experimental equipment, specimen preparation methods and SWCC measurement procedure are described. The procedure for investigating of variability in SWCC associated with hysteresis and soil volume change during measurement is presented. In addition, the variability in SWCC resulting from data interpretation, such as the maximum matric suction and the number of data points used for the best fit procedure, is presented. The variability in the determined SWCC variables using the conventional graphical method and the equations proposed in this research is also discussed in this chapter.

4.2 Flow chart of the research

The flow chart, as illustrated in Figure 4-1, explains the procedure of the research work in this study. There are four main parts in the flow chart: i) theories development, ii) verification of the proposed theories, iii) application of the proposed theories to describe characteristics of experimental data and iv) discussion and conclusions.

Figure. 4-1 Flow chart of the research

4.3 Verification of the proposed theories

SWCC data from residual soils in Singapore, including Bukit Timah granite, Jurong formation and Old Alluvium, were collected from the published literature (such as Agus et al. 2001, 2005; Rahardjo et al. 2005, 2010, 2011, 2012; and Tami et al. 2007) and used for verification of Equations (3-29) and (3-30). SWCC data for sandy materials, including gravelly sand, medium sand, fine sand, clayey sand I and clayey sand II (from Yang et al. 2004) were used for verification of Equations (3-38), (3-35), (3-43) and (3-44). SWCC data for sandy materials from Yang et al. (2004), coarse sand from Viaene et al.(1994) and compacted kaolin from Meilani (2005) were used for verification of Equations (3-57) and (3-58). SWCC and permeability data for volcanic sand, glass beads, fine sand and touch silt loam from Brooks and Corey (1964) were used for verification of Equations (3-67) and (3-68).

The properties of residual soils in Singapore are summarized in Table 4-1 and the properties of sandy materials from Yang et al. (2004) and Viaene et al. (1994) are summarized in Table 4- 2.

Table 4-1: Properties of residual soils in Singapore

S/N	Soil	USCS	k_s (m/s)	G_s	LL(%)	PL(%)	PI(%)	Sand(%)	Fine(%)	Formation	Reference
1	NTU-	CL	1.24E-08	2.77	39	29	10	14	86	Jurong Formation	Agus et al. 2001,2005
2	NTU-	CL	1.00E-08	2.63	32	5	27	25	75	Jurong Formation	Agus et al. 2001,2005
3	NTU-	SM	3.65E-08	2.73	28	18	10	63	37	Jurong Formation	Agus et al. 2001,2005
4	NTU-	CL	4.53E-08	2.63	39	23	16	28	72	Jurong Formation	Agus et al. 2001,2005
5	NTU-	CL	3.59E-09	2.67	47	26	21	23	77	Jurong Formation	Agus et al. 2001,2005
6	NTU-	CL	5.73E-10	2.72	34	21	13	16	84	Jurong Formation	Agus et al. 2001,2005
7	NTU-	MH	3.05E-09	2.52	55	33	22	27	73	Jurong Formation	Agus et al. 2001,2005
8	NTU-	ML	1.18E-08	2.56	36	26	10	50	50	Jurong Formation	Agus et al. 2001,2005
9	NTU-	CL	4.96E-09	2.72	44	30	14	8	92	Jurong Formation	Agus et al. 2001,2005
10	NTU-	ML	2.33E-09	2.67	42	27	15	44	56	Jurong Formation	Agus et al. 2001,2005
11	NTU-	ML	1.04E-09	2.76	43	28	15	44	56	Jurong Formation	Agus et al. 2001,2005
12	NTU-	ML	2.89E-09	2.63	35	26	9	79	21	Jurong Formation	Agus et al. 2001,2005
13	JK-1	SC	8.21E-06	2.53	47	33	14	69	27	Jurong Formation	Rahardjo et al. 2011
14	JK-2	SC	8.21E-06	2.64	30	13	17	73	27	Jurong Formation	Rahardjo et al. 2011
15	OSC-1	CL	5.18E-06	2.67	47	34	13	--	--	Jurong Formation	Rahardjo et al. 2005
16	OSC-2	CL	5.18E-06	2.67	47	34	13	--	--	Jurong Formation	Rahardjo et al. 2005
17	PCS-1	CL	1.67E-07	2.72	34	13.5	20.5	--	--	Jurong Formation	Rahardjo et al. 2005
18	PCS-2	CL	1.67E-07	2.72	34	13.5	20.5	--	--	Jurong Formation	Rahardjo et al. 2005
19	BM1	CL	1.30E-06	2.63	49	24	25	0	100	Jurong Formation	Rahardjo et al. 2010
20	BM2	CL	2.39E-06	2.67	45	25	20	5	95	Jurong Formation	Rahardjo et al. 2010
21	BM3	CL	2.39E-06	2.68	39	19	20	3	97	Jurong Formation	Rahardjo et al. 2010
22	HR	MH	9.20E-07	2.62	62	36	26	37	63	Jurong Formation	Rahardjo et al. 2010
23	CC	CL	4.58E-06	2.72	29	18	11	14	86	Jurong Formation	Rahardjo et al. 2010
24	DR	MH	7.00E-07	2.69	45	24	21	20	80	Jurong Formation	Rahardjo et al. 2010
25	TB	CL	9.00E-06	2.67	41	21	20	1	99	Jurong Formation	Rahardjo et al. 2010
26	BP	CL	5.60E-06	2.7	--	--	--	25	75	Jurong Formation	Rahardjo et al. 2012

27	JKL	CL	1.40E-05	2.69	38	18	20	2	98	Jurong Formation	Rahardjo et al. 2012
28	JBM	CL	3.30E-05	2.68	39	19	20	3	97	Jurong Formation	Rahardjo et al. 2012
29	SLR-1a	SM	2.50E-08	2.6	108	47	61	61	39	Bukit Timah Granite	Agus et al. 2001,2005
30	SLR-1b	SM	7.36E-08	2.61	64	34	30	72	28	Bukit Timah Granite	Agus et al. 2001,2005
31	SLR-1c	SM	2.88E-07	2.55	60	35	25	75	25	Bukit Timah Granite	Agus et al. 2001,2005
32	SLR-2a	MH	2.78E-08	2.6	105	47	58	38	62	Bukit Timah Granite	Agus et al. 2001,2005
33	SLR-2b	MH	3.00E-08	2.61	116	48	68	38	62	Bukit Timah Granite	Agus et al. 2001,2005
34	SLR-2c	CH	1.36E-08	2.55	61	36	25	43	57	Bukit Timah Granite	Agus et al. 2001,2005
35	YS-a	MH	7.27E-08	2.61	54	33	21	44	56	Bukit Timah Granite	Agus et al. 2001,2005
36	YS-b	MH	4.80E-07	2.53	68	49	19	32	68	Bukit Timah Granite	Agus et al. 2001,2005
37	LA-a	SC	9.51E-07	2.53	48	37	11	66	34	Bukit Timah Granite	Agus et al. 2001,2005
38	LA-b	SM	5.37E-08	2.6	97	38	59	72	28	Bukit Timah Granite	Agus et al. 2001,2005
39	LA-c	SM	1.86E-08	2.54	61	4	57	62	38	Bukit Timah Granite	Agus et al. 2001,2005
40	LA-d	SM	3.57E-08	2.6	97	38	59	75	25	Bukit Timah Granite	Agus et al. 2001,2005
41	MR1	SM	6.00E-06	2.66	52	31	21	53	46	Bukit Timah Granite	Rahardjo et al. 2011
42	MR2	MH	3.30E-05	2.58	54	39	15	40	60	Bukit Timah Granite	Rahardjo et al. 2011
43	SS	SM	2.20E-06	2.59	48	27	21	61	38.9	Bukit Timah Granite	Tami et al. 2007
44	TR1	CH	3.19E-08	2.59	71	34	37	27	73	Bukit Timah Granite	Rahardjo et al. 2012
45	TR2	MH	5.65E-05	2.66	51	29	22	30	70	Bukit Timah Granite	Rahardjo et al. 2012
46	BBA1	CL	5.27E-08	2.82	31	20	11	43	54	Bukit Timah Granite	Rahardjo et al. 2012
47	BBA2	CL	8.28E-08	2.64	34	20	14	20	80	Bukit Timah Granite	Rahardjo et al. 2012
48	BB1	MH	1.20E-08	2.65	55	32	23	3	97	Bukit Timah Granite	Rahardjo et al. 2012
49	BB2	SM	2.15E-08	2.71	49	30	19	51	49	Bukit Timah Granite	Rahardjo et al. 2012
50	AMK1	SM	8.52E-06	2.54	56	38	18	58	42	Bukit Timah Granite	Rahardjo et al. 2012
51	AMK2	SM	3.96E-06	2.58	58	36	22	53	47	Bukit Timah Granite	Rahardjo et al. 2012
52	AMA	MH	2.52E-07	2.7	54	29	25	29	70	Bukit Timah Granite	Rahardjo et al. 2012
53	TWL	MH	5.60E-06	2.66	72	45	27	5	95	Old Alluvium	Rahardjo et al. 2012
54	JS1	SM	1.61E-06	2.66	72	28	44	62	6	Old Alluvium	Rahardjo et al. 2012
55	JS2	SM	4.73E-06	2.61	54	28	26	62	11	Old Alluvium	Rahardjo et al. 2012

56	BN1	CL	1.40E-08	2.72	49	27	22	36	64	Old Alluvium	Rahardjo et al. 2012
57	BN2	CL	1.40E-08	2.71	57	27	30	7	93	Old Alluvium	Rahardjo et al. 2012
58	BN3	SC	5.00E-05	2.67	34	18	16	61	20	Old Alluvium	Rahardjo et al. 2012
59	Tam	CL	9.00E-05	2.68	60	36	24	73	25	Old Alluvium	Rahardjo et al. 2012

Table 4-2. Basic soil properties of the sandy materials

	Gravelly sand from Yang et al. (2004)	Medium sand from Yang et al. (2004)	Fine Sand from Yang et al. (2004)	Clayey sand I from Yang et al. (2004)	Clayey sand II from Yang et al. (2004)	Coarse sand from Vianene et al. (1994)
Property						
Specific gravity, G_s	2.62	2.60	2.65	2.64	2.59	--
Grain-size data						
D60 (mm)	5.15	1.25	0.35	0.06	0.56	0.59
D30 (mm)	3.68	0.62	0.23	0.05	0.02	0.41
D10 (mm)	2.73	0.29	0.17	0.003	0.005	0.054
Gravel content	49.9%	0.8%	0	0	0.2%	0
Sand content	50.1%	98.4%	99.2%	68.5%	60.9%	81.4%
Fines content	0	0.8%	0.8%	31.5%	38.9%	18.6%
Dry density ρ_d (Mg/m^3)	1.62	1.69	1.56	1.72	1.47	--
Void ratio, e	0.62	0.54	0.7	0.54	0.76	--

The SWCCs for residual soils are illustrated in Figure 4-2. The SWCCs for sandy materials from Yang et al. (2004) are shown in Figure 4-3, while the drying and wetting SWCCs of sandy materials from Yang et al. (2004), coarse sand from Vianene et al. (1994) and compacted kaolin from Meilani et al. (2005) are illustrated in Figure 4-4. Both experimental SWCCs and relative hydraulic conductivity for volcanic sand, glass beads, fine sand and touch silt loam from Brooks and Corey (1964) are illustrated in Figure 4-5. The air-entry values and residual suction, as presented by Yang et al. (2004), are illustrated in Table 4-3.

Table 4-3. SWCC variables for the sandy materials from Yang et al. (2004)

SWCC variables	Gravelly sand	Medium sand	Fine Sand	Clayey sand I	Clayey sand II
Air-entry value (kPa)	0.11	0.85	1.40	1.80	4.05
Residual suction (kPa)	0.40	3.75	3.94	11.40	14.90
Residual volumetric water content, θ_r	0.021	0.063	0.035	0.237	0.296

Figure. 4-2(a) SWCC data for residual soil of Jurong Formation

Figure. 4-2 (b) SWCC data for residual soil of Bukit Timah granite

Figure 4-2 (c) SWCC data for residual soil of Old Alluvium

Figure. 4-2 SWCC data for residual soils in Singapore

Figure. 4-3 SWCC data for sandy material from Yang et al. (2004)

Figure. 4-4 (a) Drying and wetting SWCCs for sandy material from Yang et al. (2004)

Figure. 4-4 (b). Drying and wetting SWCCs for coarse sand from Vianene et al. (1994) and compacted kaolin from Meilani et al. (2005)

Figure. 4-4 Drying and wetting SWCC data from the literature

Figure. 4-5 (a). SWCC data for four types of soil from Brooks and Corey (1964)

Figure. 4-5 (b) Relative hydraulic conductivity data for four types of soil from Brooks and Corey (1964)

Figure. 4-5 SWCC and relative hydraulic conductivity data from Brooks and Corey (1964)

A comparison of the confidence limits of SWCCs for residual soils estimated using Equations (3-29) and (3-30) and variations in the SWCC expressed using the box-plot method is

illustrated in Figure 4-6. Figure 4-6 indicates that the confidence limits estimated using Equations (3-29) and (3-30) agree with the results of the box-plot method. In other words, defining the confidence limits using Equations (3-29) and (3-30) is a reliable method for estimating variability in SWCCs.

Figure. 4-6 (a) Comparison of SWCC confidence limits and its variability obtained from the box plot method for residual soil of Jurong formation

Figure. 4-6 (b) Comparison of SWCC confidence limits and its variability obtained from the box plot method for residual soil of Bukit Timah granite

Figure. 4-6 (c) Comparison of SWCC confidence limits and its variability obtained from the box plot method for residual soil of Old Alluvium

Figure. 4-6 Comparison of SWCC confidence limits and its variability obtained from the box plot method for residual soil of residual soil in Singapore

The fitting parameters of SWCC and the determined SWCC variables, such as air-entry value and residual suction, for sandy materials from Yang et al. (2004) are illustrated in Table 4-4. The air-entry values and residual suctions calculated using Equations (3-38), (3-43) and (3-44) are consistent with the values presented by Yang et al. (2004). In other words, Equations (3-38), (3-35), (3-43) and (3-44) are capable of providing reasonable values of SWCC variables.

Table 4-4. Fitting parameters for sandy materials from Yang et al. (2004)

	Gravelly sand	Medium sand	Fine Sand	Clayey sand I	Clayey sand II
Fitting parameters					
a (kPa)	0.17	1.27	1.97	2.37	5.72
n	4.05	4.10	5.47	2.38	4.28
m	1.21	0.74	0.99	0.27	0.23
SWCC variables					
Air-entry value (kPa)	0.12	0.87	1.50	1.65	4.70
Residual suction (kPa)	0.42	4.23	4.43	12.49	16.26
Residual volumetric water content, θ_r	0.012	0.054	0.023	0.23	0.29

As the inter-aggregate pores are assumed to be dominant in the derived equation, only sandy soils and compacted coarse kaolin were selected for verification of the proposed equation. The gravelly sand, medium sand, fine sand, clayey sand I and clayey sand II from Yang et al. (2004), coarse sand from Viaene et al. (1994) and compacted coarse kaolin from Meilani et al. (2005) and compacted mixture of sand and kaolin were used to verify Equations (3-57) and (3-58). The index properties of the sandy soils are illustrated in Table 1. As no index properties were reported by Meilani et al. (2005) for compacted kaolin, it is not included in Table 4-5.

Table 4-5. The index soil properties of four sandy materials.

	Gravelly sand from Yang et al. (2004)	Medium sand from Yang et al. (2004)	Fine Sand from Yang et al. (2004)	Clayey sand I from Yang et al. (2004)	Clayey sand II from Yang et al. (2004)	Coarse sand from Viaene et al. (1994)	Compacted mixture of sand and kaolin
Property							
Specific gravity, G_s	2.62	2.60	2.65	2.64	2.59	--	2.66
Grain-size data							
D60 (mm)	5.15	1.25	0.35	0.06	0.56	0.59	0.30
D30 (mm)	3.68	0.62	0.23	0.05	0.02	0.41	0.15
D10 (mm)	2.73	0.29	0.17	0.003	0.005	0.054	0.01
Gravel content	49.9%	0.8%	0	0	0.2%	0	0
Sand content	50.1%	98.4%	99.2%	68.5%	60.9%	81.4%	73%
Fines content	0	0.8%	0.8%	31.5%	38.9%	18.6%	27%
Dry density ρ_d (Mg/m ³)	1.62	1.69	1.56	1.72	1.47	--	1.89
Void ratio, e	0.62	0.54	0.7	0.54	0.76	--	0.41

The fitting parameters of the drying SWCC and wetting SWCC of Fredlund and Xing's (1994) equations were obtained using the curve fitting technique, as suggested by Zhai and Rahardjo (2012a, 2012b), and are summarized in Table 4-6.

Table 4-6. Fitting parameters of SWCCs of the soils used for verification.

Soil name	$a_{dry}(kPa)$	n_{dry}	m_{dry}	$C_r(kPa)$	$a_{wet}(kPa)$	n_{wet}	m_{wet}
Gravelly sand	0.18	4.26	1.23	1500	0.07	1.59	2.39
Medium sand	1.25	3.81	0.78	1500	0.71	1.55	1.89
Fine sand	1.92	5.27	0.99	1500	2.04	2.82	4.83
Clayey sand I	2.40	2.23	0.28	1500	0.47	1.21	0.56
Clayey sand II	5.15	3.83	0.24	1500	1.77	0.51	0.84
Coarse sand	3.54	7.69	0.92	1500	2.19	2.26	1.64
Compacted kaolin	75.41	3.34	0.82	1500	39.73	1.90	1.06
Compacted mixture of sand and kaolin	72.66	0.92	2.08	1500	17.13	0.73	2.01

The wetting curves expressed using Equations (3-57) and (3-58) are illustrated in Figure 4-7. The compacted mixture of sand and kaolin was prepared in air-dry condition before starting the wetting process. As indicated, the main wetting curves and scanning wetting curves expressed using Equations (3-57) and (3-58) fit with the experimental data very well, which means that Equations (3-57) and (3-58) work very well. The values of constants k and b , accompanied by the sum of squared-error (SSE) and coefficient of determination (R^2) best obtained from curve fitting technique, are summarized in Table 4-7.

(a) Estimated main wetting curve and scanning wetting curve for gravelly sand

(b) Estimated main wetting curve and scanning wetting curve for medium sand

(c) Estimated main wetting curve and scanning wetting curve for fine sand

(d) Estimated main wetting curve and scanning wetting curve for clayey sand I

(e) Estimated main wetting curve and scanning wetting curve for clayey sand II

(f) Estimated main wetting curve and scanning wetting curve for coarse sand

(g) Estimated main wetting curve and scanning wetting curve for compacted kaolin

(h) Estimated main wetting curve and scanning wetting curve for compacted mixture of sand and kaolin

Figure. 4-7 Estimated main wetting curves and scanning wetting curves for the tested soils.

Table 4-7. Values of the constant of k and b obtained from the best fit procedure.

Parameters	Gravelly sand	Medium sand	Fine sand	Clayey sand I	Clayey sand II	Coarse sand	Compacted kaolin	Compacted mixture of sand and kaolin
k	1	1	1	1	1	1	1	1
b	0.10	0.58	0.56	1.85	4.56	0.67	14.28	5.13
SSE	0.009	0.02	0.003	0.04	0.007	0.133	0.094	0.05
R ²	0.91	0.98	0.99	0.81	0.71	0.84	0.86	0.80

Figure 4-7 indicates that the main wetting curves estimated using Equation (4) agree with experimental data from Yang et al. (2004). In addition, the scanning wetting curves obtained using Equation (4) agree with the experimental data for coarse sand, compacted kaolin and compacted mixture of sand and kaolin. Viaene et al. (1994) and Meilani et al. (2005) did not specify if the soil samples were wetted at oven-dry conditions or immediately after completion of the drying process. As illustrated in Figure 4-7 (f) and (g), it seems that the coarse sand and compacted kaolin were wetted immediately after completion of the drying process. Table 3 indicates that the value of "k" is constant (i.e., k=1), as obtained from the best fit procedure. Therefore, ψ' can be simplified as $\psi'=\psi+b$. Table 4-7 also indicates that the value of "b" increases with an increase in fine content.

The relative hydraulic conductivities predicted using Equations (3-67) and (3-68) for four types of soil from Brooks and Corey (1964) are illustrated in Figure 4-8. Figure 4-8 indicates that Equations (3-67) and (3-68) can predict the relative hydraulic conductivity well from the SWCC.

Figure. 4-8 Comparison between calculated and measured permeability functions for four types of soil from Brooks and Corey (1964)

4.4 Outline of experimental program and data interpretation.

To verify the theory proposed in Section 3, experimental measurements are carried out in this research. The experimental program includes seven parts:

1. Soil selection.

Works by previous researchers, such as Tinjum et al. (1997), Yaldo (1999) and Gharagheer (2009), suggest that molding water content and compacted density have an insignificant effect on variability in SWCCs for non-plastic soils (i.e., soils with a very low plastic limit). Sand mixed with kaolin, which is classified as silty sand (i.e., SM (USCS)), a non-plastic soil, was prepared as the soil sample in this research. Soilvision (2002) was used to preliminary select the GSD data for the soil sample by simulating SWCCs using the equations developed by Fredlund et al. (2002). As coarse kaolin was selected as the fine-grained material in this research, the input data for the portion of the fine-grained material was the same as the GSD data for the coarse kaolin. Different

SWCCs can be obtained by inputting different GSD data for the coarse grained material. The SWCC with a distinct air-entry value and low residual suction was selected, and the corresponding GSD data was recorded and used for soil sample preparation. Reconstituted soil was chosen as it is homogeneous and its identical properties can be well controlled. In addition, reconstituted soil is reproducible.

2. Basic index properties test

Basic property tests and compaction tests, such as sieve-analysis, hydrometer test, specific gravity test, standard compaction test, modified compaction test and static compaction, were conducted on the selected soil. Since a non-plastic soil was used, Atterberg limit tests were not conducted. After the compaction tests were completed, compaction curves associated with the different compaction efforts were plotted.

3. Specimen preparation

The molding water content and compacted density used for preparation of the soil specimen were selected from the compaction curves. Soil specimens were then compacted using the static compaction shown in Figure 4-16.

4. SWCC measurement for compacted mixture of sand and kaolin

Initially, both Tempe cell and pressure plate were used for SWCC measurements. However, tests using the Tempe cell showed high variability in initial water content associated with a low suction range. It was thought that water might not be fully draining out of the Tempe cell under the low air pressure. In other words, the calculated initial water content might not represent the water content of the real soil specimen in the Tempe cell. Therefore, only the pressure plate was used as the SWCC measurement apparatus in the experimental program.

Several soil specimens with the same soil properties (i.e., same molding water content, same compacted density, same diameter and same height) were prepared and placed on the pressure plate. After the air pressure was applied, the specimens were weighed frequently until equilibrium was reached (i.e., insignificant mass loss over time). The air pressure was increased for the next step of the measurement. The above procedure was repeated until the last step (i.e., the step with the maximum air pressure). Lastly, the

specimens were moved into an oven for 24 hours. The dry specimens were weighed again for determination of the soil mass. The water contents of the specimens associated with different matric suctions were then calculated from the difference between the respective specimen weights and the solid mass. After carrying out the soil volume measurement for the compacted kaolin with sand, no significant volume change was observed. Thus, soil volume change during SWCC measurement is considered negligible for this type of soil.

5. SWCC measurement for residual soil of Bukit Timah granite

Although there was no significant soil volume change associated with the compacted mixture of sand and kaolin, for other types of soil investigation must be carried out first to observe the effect of soil volume change on the measured SWCC. The residual soil of Bukit Timah granite, which is suspected to have a significant soil volume change with increasing matric suctions, taken from Marsiling Ave. 6 was selected for SWCC measurement. The procedure of SWCC measurement was similar to that described in part 4. Additional volume measurement was carried out at each step for calculation of the degree of saturation (S) and volumetric water content (θ_w) from the gravimetric water content (w). Finally, both SWCCs considering and ignoring the volume change for residual soil of Bukit Timah granite were established.

6. Primary analysis of collected data

After completion of the SWCC measurements, experimental data were collected and water contents corresponding to different matric suctions were calculated. The void ratios corresponding to different matric suctions were also calculated.

7. Standard deviation of measured water content and void ratio

The standard deviation of the measured water content for the compacted mixture of sand and kaolin was calculated from the collected data. The standard deviations of both measured water content and void ratio for the residual soil were also calculated.

Data interpretation includes six components:

1. Determination of the fitting parameters

After calculation of the degree of saturation, the fitting parameters from Fredlund and Xing's (1994) equation were determined using a curve fitting technique by minimizing the sum of squared-error (i.e., SSE as in Equation 4-1), as suggested by Leong and Rahardjo (1997).

$$SSE = \sum w_i (S_i - S'_i)^2 \quad \text{----- (4-1)}$$

where,

w_i = weighting factor

S_i = degree of saturation from experimental measurement

S'_i = degree of saturation from best fitted SWCC

SSE = sum of squared-error.

2. Determination of the confidence limits of the best fitted SWCC for residual soil in Singapore

The confidence limits of SWCC for both the compacted mixture of sand and kaolin and the residual soil were estimated from the fitting parameters and residual error. A confidence level of 90%, as suggested by Rahardjo et al. (2012), was adopted for calculation of the confidence limits.

3. Determination of SWCC variables

The SWCC variables, such as air-entry value, slope at the inflection point, residual degree of saturation and residual suction for both the compacted mixture of sand and kaolin and the residual soil, were calculated from the fitting parameters.

4. Determination of confidence limits of SWCC variables

The confidence limits of the SWCC variables for both the compacted mixture of sand and kaolin and the residual soil were calculated using the equations (i.e. Equations 3-29 and 3-30) proposed by Zhai and Rahardjo (2013b).

5. Investigation of the effect of number of data points and suction range covered on SWCC

Different numbers of data points and different suction range of suction measurements were used in the best fit procedure for the investigation of variability in SWCCs.

6. Determination of relative hydraulic conductivity

The relative hydraulic conductivity for both the compacted mixture of sand and kaolin and the residual soil was computed from SWCC using the statistical model proposed in this research.

7. Investigation of the effect of basic soil properties on SWCC

The collected SWCC data of the residual soils were compared with the framework proposed by other researchers. Next, the basic soil properties were set as variables for investigation of the effect of these properties on the variability in SWCC.

4.5 Specimen preparation and equipment set up

To investigate the variability in SWCC associated with the hysteresis of SWCC and soil volume change during measurement, identical specimens were prepared. The specimen preparation method and equipment set up are presented in this section.

4.5.1 Criteria for selection of soil sample

Reconstituted specimens were used in this research program because the soil properties can be carefully controlled and all the experimental results from different specimens will be comparable. Reconstituted soil specimens were chosen since they can be prepared as identical specimens and are also reproducible. During SWCC measurement for the residual soil, both the water content and soil volume were measured and recorded. To investigate variability in SWCC due to the volume change during measurement, the residual soil of Bukit Timah Granite taken from Marsiling Ave. 6 in Singapore was prepared.

The criteria for the selection of soil samples are as follows:

1. The soil should be classified as a non-plastic soil (i.e., with a very low plastic limit, PL).

2. The soil specimen should be uniform and homogeneous in its properties (i.e., molding water content, compacted density, specimen diameter and specimen height) so that the results from the specimens are comparable.
3. The soil specimen should be easy to trim to ensure a certain shape and size.
4. The SWCC for the soil specimen should have a distinct air-entry value and a low residual suction that can be achieved using the pressure plate.
5. A volume change of the reconstituted soil, that was found to be small, can be ignored for the entire measurement period. However, a significant volume change associated with the residual soil cannot be ignored and must be measured and recorded.

Two soils (i.e., graded sand and kaolin) were selected for the preparation of identical soil specimens (i.e., compacted mixture of sand and kaolin). ASTM C778 graded sand, which was produced by Societe Nouvelle du Littoral (France), was selected as the sand material, while coarse Kaolin (L2), which contained 86% silt (75~2 μm) and 14% clay (finer than 2 μm) and was produced by Kaolin Malaysia SDN BHD (Malaysia), was used in this research. The graded sand had a low air-entry value and no cohesion, which made specimen preparation difficult. The coarse kaolin had a relatively high air-entry value and good cohesion, which made specimen preparation easier.

4.5.2 Sand and coarse kaolin ratio selection for the mixture

GSD data, as illustrated in Figure. 4-9, was selected from Soilvision (2002) for the preparation of the soil samples. SWCCs simulated by the uni-modal and bi-modal methods are shown in Figure 4-10 and 4-11, respectively. Graded sand was sieved into four groups of different particle sizes: 600-1250 μm , 300-600 μm , 150-300 μm and 75-150 μm . These particles were weighed with the percentages of the GSD data from Figure 4-9 for the preparation of the soil samples.

Figure. 4-9 Grain size distribution data for the compacted mixture of sand and kaolin

Figure. 4-10 Simulated SWCC using uni-modal model

Figure. 4-11 Simulated SWCC using bi-modal model

4.5.3 Basic soil properties tests

A series of basic soil properties tests, including grain size analysis (sieving analysis and hydrometer tests) and specific gravity tests, were conducted to determine the index properties of the soil. All the basic soil properties tests were based on ASTM soil testing standards, such as ASTM C136-06 for sieve analyses, ASTM E100-95 for hydrometer analyses and ASTM D854-10 for specific gravity test.

4.5.3.1 Grain size distribution data

The results of the grain size distribution data for ASTM C778 graded sand, coarse Kaolin L2, compacted mixture of sand and kaolin and residual soil of Bukit Timah Granite are summarized in Table 4-8 and Figures 4-12 to 4-13.

Table 4-8 Soil properties of graded sand, coarse Kaolin, compacted mixture of sand and kaolin and residual soil of Bukit Timah Granite

Soil Properties	Graded Sand	Coarse Kaolin	Compacted mixture of sand and Kaolin	Residual soil of Bukit Timah Granite
Specific Gravity, G_s	2.66	2.67	2.66	2.68
D_{60} (mm)	0.42	0.0095	0.3	0.099
D_{30} (mm)	0.32	0.0055	0.15	0.0013
D_{10} (mm)	0.2	0.001	0.007	0.0005
Dry density, ρ_d (Mg/m^3)	-	-	1.90	-
Percentage of Sand, %	100	0	73	48
Percentage of Fine, %	0	100	27	52
Void ratio, e	-	-	0.40	0.72
Liquid Limit	-	-	-	54%
Plastic Limit	-	-	-	30%
Plasticity Index	-	-	-	24%

4.5.3.2 Standard (dynamic) compaction test

A standard Proctor compaction test, as given in ASTM D698 (1997), was conducted to determine the compaction curves for the soil. Different water contents were used to obtain different dry densities and to investigate the relationship between water content and dry density of the soil. The compactive effort in the standard Proctor test can be calculated using the following equation:

$$\text{Compactive effort} = \frac{2.495kg * 9.81m/s^2 * 0.3048m * 3layer * 25blows/layer}{0.944 * 10^{-3} m^3} = 592.7kJ .$$

The compactive effort of the standard Proctor test was modified to 60% and 140% in this testing program, and defined as 1/2 standard compaction and modified compaction, respectively. The molding water content and dry density were determined from the compaction curves.

Usually, soil at the dry optimum has more potential for volume change upon wetting and a higher saturated permeability conductivity. On the other hand, soil at the wet optimum is easy

to compact and has a much lower saturated permeability conductivity. Therefore, the point on the compaction curve closest to the optimum water content was selected for preparation of the soil specimens. In this research, 2% less than the optimum water content (i.e., 7.8%) (i.e., dry of optimum) on the 1/2 standard compaction curve was selected, as illustrated in Figure 4-14.

Figure. 4-14 Compaction curves with different compaction energies

4.5.3.3 Static compaction

The properties of the compacted soil specimens could not be well controlled using the dynamic compaction method. Static compaction, which was suggested by Ong (1999), was adopted to obtain identical specimens. Three sets of static compaction molds with different inner diameters (i.e., 50mm, 71mm and 100mm) were used for specimen preparation. A compaction mold with a diameter of 50mm was used throughout the research program. The static compaction apparatus consisted of a stainless steel mold and two stainless steel plugs. Every compaction mold includes two symmetrical parts that can be connected and screwed together, as shown in Figures 4-15 and 4-16.

Figure. 4-15 Compaction molds and plugs

Figure. 4-16 Elements of compaction molds (after Ong, 1999)

The stainless steel plugs consist of a number of removable 10mm thick disks that can be attached to each other through the threaded screw. The screw protrudes from the center of the

face of the disk with the threaded hole drilled into the center of the face of another disk. The screw connects to the hole in the adjacent disk, as illustrated in Figure 4-8. Therefore, the length of the plugs can be changed by adding or removing disks accordingly to prepare a specimen layer by layer. All disks were designed in a T-shape with two different diameters, where the top part has a slightly larger diameter with a 5mm thickness while the bottom part has a slightly smaller diameter with a 5mm thickness. The purpose of the T-shaped disk design is to reduce friction between the trapped soil particles in the gaps and along the wall of the mold. Information about the compaction molds of different sizes is given in Table 4-9.

The specimen was statically compacted between two layers each with a thickness of 10mm. The mass of soil needed for each layer was calculated and weighed before being placed into the mold for compaction. As suggested by Ong (1999), after one layer was compacted successfully, the mold was turned over so that the top plug was at the bottom and the bottom plug was at the top. To ensure all specimens were compacted under the same compaction condition, a gauge, as shown in Fig. 4-17, was used to monitor the load applied to each specimen. The loading was stopped when the reading on the gauge reached 1.5 round (which is equal to 3.82 kN). To ensure the same bonding between layers, the compacted soil surface was scratched roughly before the mixture was placed inside. After the soil was placed in the mold, the top plug was inserted and an axial load was applied at a constant rate until the reading on the gauge reached 1.5 round (which is equal to 3.82 kN). A fixed displacement rate of 1 mm/min was applied using the electrical/manual compression machine (as shown in Figure 4-16) to ensure a uniform density throughout the whole specimen and to avoid excess pore-water pressure building up in certain portions of the specimen. In the end, the specimen was extruded using the same machine. The procedure was repeated until a sufficient number of specimens was obtained.

Table 4-9: Mold information for specimen preparation

	50 mm diameter Compaction mold	71 mm diameter Compaction mold	63.5 mm diameter Compaction mold
Mold inner diameter (mm)	50	71	100
Mold height (mm)	200	200	200
Disk thickness (mm)	10	10	10
Upper disk thickness (mm)	5	5	5
Lower disk thickness (mm)	5	5	5
Upper disk diameter (mm)	50	71	100
Lower disk diameter (mm)	40	61	90
Use of specimen	SWCC test by Tempe cell Pressure Plate	SWCC test by Tempe cell Pressure Plate	SWCC test by Tempe cell Pressure Plate

Figure. 4-17 Electrical/Manual compaction machine for static compaction

4.6 SWCC Test

Identical specimens of the compacted mixture of sand and kaolin were prepared with a diameter of 50mm and a height of 20mm. SWCC measurement was based on the ASTM D6836-02 standard.

4.6.1 SWCC measurement using a Tempe cell

A Tempe cell can be used for SWCC measurement with matric suction up to 100 kPa. The Tempe cell consisted of a one bar (i.e., 100 kPa) air-entry value ceramic disk, three O-rings, a cell cap, cell base, brass cylinder and screws. The Tempe cell was weighed before placement of the soil specimen. After placement of the soil specimen, the cell cap was tightened and the Tempe cell was weighed again so the initial mass of the soil specimen could be calculated. The soil specimen was then saturated from the bottom through the pre-saturated ceramic disk. Before installation in the cell base, the ceramic disk was saturated in a suction tank for 24 hours. It was necessary to flush the air out from between the ceramic disk and the cell base.

After the soil specimen was saturated, the entire Tempe cell was weighed to determine the mass of the saturated soil specimen. The axis-translation technique proposed by Hilf (1956) was adopted to achieve the target matric suction ($u_a - u_w$) in the soil specimen. The air supply was then connected to the inlet point on the top of the Tempe cell. At the same time, a burette was connected to the base for specimen saturation. The water level (which was the same level as the mid-height of the specimen) was open to the atmosphere so that the equilibrated pore-water pressure inside the Tempe cell was considered to be zero. As a result, the matric suction was equal to the air pressure applied: an increase in air pressure would imply a drying process, while a decrease in air pressure would imply a wetting process.

The air pressure was applied at the lowest pressure of 0.5 kPa, and subsequently increased to 1 kPa, 3 kPa, 7 kPa, 10 kPa and 30 kPa. While air pressure was applied, water from the soil specimen started to drain out through the saturated ceramic disk. The Tempe cell with the soil specimen was weighed and the mass was recorded at a designated time schedule. Once the Tempe cell weight changed little with time, which is normally considered the equilibrium condition, the air pressure was increased for the next targeted matric suction measurement. The Tempe cell was weighed frequently to monitor the equilibrium condition of the specimen. The same procedure was repeated until the measurement with an air pressure of 30 kPa (i.e., the maximum air pressure used) was completed. High variability in the measured water content at a low suction range using the Tempe cell was observed, as illustrated in Figure 4-18.

Figure. 4-18 Measured SWCCs for the compacted mixture of sand and kaolin using a Tempe cell

As illustrated in Figure 4-18, high variability in saturated volumetric water content was observed using the Tempe cell. It is expected that at the suction equilibrium condition, the water may remain in the Tempe cell (i.e., in the gap between the specimen and the brass ring) instead of in the soil under the very low suction. Variability in volumetric water content is also observed at suction of 50 kPa in Figure 4-18. It can be attributed to the fact that some of the soil in the specimen collapsed during the saturation stage and this collapse may affect the calculated volumetric water content.

4.6.2 SWCC measurement using a pressure plate

A pressure plate can be used for SWCC measurement up to a maximum matric suction of 500 kPa. The pressure plate consisted of a pressure chamber, a 5 bar (i.e., 500 kPa) air-entry value ceramic disk and a rubber membrane beneath the ceramic disk. The space between the ceramic disk and the rubber membrane serves as a water compartment, which was connected to a burette line that was open to the atmosphere. The layout of the pressure plate apparatus is shown in Figure 4-19.

Figure. 4-19 Set up of the pressure plate (from Agus, 2001)

The ceramic disk is normally saturated by immersing it in distilled water in a vacuum tank for 24 hours. However, sometimes this saturation procedure is insufficient. In this research, after saturation for 24 hours, the ceramic disk was further saturated before starting the test by pouring de-aired distilled water on its surface and applying a slight air pressure. The valve of the burette was opened and exposed to the atmosphere. The de-aired distilled water infiltrated the ceramic disk and flowed out from the pressure chamber due to the pressure gradient. The ceramic disk was considered saturated after repeating these procedures a few times and no air bubbles were observed in the burette. After saturation of the ceramic disk, the soil specimen was placed on it. Next, the pressure chamber was closed and different target air pressures were applied. The equilibrated pore-water pressure of the specimen was determined by the height difference between the specimen and the water level in the burette, which was open to the atmosphere. The weight and volume of the specimens were measured daily to monitor the equilibrium condition. A small amount of de-aired water was sprayed on the surface of the ceramic disk to maintain good contact between the saturated ceramic disk and the soil specimen. The ceramic disk was flushed after each reading was taken. Once the mass of the soil specimen reached the equilibrium condition, the air pressure was increased to the next

targeted pressure and the above procedure was repeated until the measurement with an air pressure of 480 kPa was completed. Saturation of the specimens on the pressure plate is illustrated in Figure 4-20.

Figure. 4-20 Illustration of saturation of soil specimens on the pressure plate

4.6.3 Primary analysis of the collected data

The water content (w) corresponding to each matric suction (i.e., $u_a - u_w$) was calculated from the collected data, which include the mass of the specimens under different suctions. The degree of saturation (S) was calculated from the water content (w) with a constant void ratio for the compacted mixture of sand and kaolin. As the void ratio for the residual soil varied during the SWCC measurements, the degree of saturation (S) was calculated using a varying void ratio (i.e., the measured void ratio). The standard deviations of the water content (w), void ratio (e) and degree of saturation (S), with respect to matric suction, were calculated and plotted. The standard deviation was calculated using Equation (4-2) as follows:

$$s^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N} \quad \text{----- (4-2)}$$

where,

s = standard deviation

x_i = observed value

$\bar{x}$ = mean of observed values

N = size of the sample.

The sample variance (s^2) is the second sample central moment of estimator for variance of a sample. Normally, Bessel's correction factor (i.e., (N-1)/N) is applied to the estimator for the variance of an unbiased sample, as shown in Equation (4-3). The author observed from the box-plot method that the sample of water contents from the residual soils in Singapore seemed to be a biased sample. Equation (4-2) is more precise than Equation (4-3) as an estimator for biased sample variance. Therefore, the standard deviation (s) was calculated using Equation (4-2) throughout this research.

$$s^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N - 1} \quad \text{----- (4-3)}$$

4.7 Data interpretation

The discrete data points of the degree of saturation (S) at different matric suctions were used for the regression procedure (i.e., best fit procedure) by fitting the data with the best fit equation (i.e., Fredlund and Xing's (1994) equation) using the curve fitting technique. Both the fitting parameters and residual error were obtained from the best fit procedure. The confidence limits of SWCC for both the compacted mixture of sand and kaolin and the residual soil were calculated from the fitting parameters and residual error using Equations (3-29) and (3-30) proposed in this research. The SWCC variables for the compacted mixtures of sand and kaolin were determined from the fitting parameters using Equations (3-38), (3-43) and (3-44). The confidence limits of SWCC variables for the compacted mixture of sand and kaolin were also computed using Equations (3-52) and (3-53). The results of the SWCC variables determined using the conventional graphical method are presented to illustrate the

variability associated with the conventional graphical method. Finally, the relative hydraulic conductivity of both the compacted mixture of sand and kaolin and the residual soil were computed from the fitting parameters using the equations proposed in this research.

4.7.1 SWCCs obtained from experimental data with different maximum suctions and different numbers of data points

The full experimental data set for the compacted mixture of sand and kaolin was adopted for the best fit procedure and to obtain the best fitted SWCC. After the best fitted SWCC was obtained from the full set of experimental data (i.e., using all the measured data points during experiment), the experimental data with less data points were used for the best fit procedure again.

Experimental data up to 50 kPa were used for the best fit procedure to obtain best fitted SWCC1. Experimental data up to 100 kPa (i.e., the limited capacity of the Tempe cell) were used for the best fit procedure to obtain best fitted SWCC2. Five values from the experimental data with a maximum suction of 500 kPa were used for the best fit procedure to obtain best fitted SWCC3. Seventy two (72) values from the experimental data with a maximum suction of 100 kPa were used for the best fit procedure to obtain SWCC4. The best fitted results and fitting parameters are presented in Chapter Five.

4.7.2 SWCC variables determined from the conventional graphical method and equations proposed in this research

The best fitted SWCC determined from the full experimental dataset for the compacted mixture of sand and kaolin was used for determination of the SWCC variables. Three copies of the best fitted SWCC were passed to three different operators for determination of the SWCC variables using the conventional graphical method. In addition, the SWCC variables were computed from the fitting parameters using Equations (3-38), (3-43) and (3-44). The results from both the conventional graphical method and the equations proposed in this

research are summarized in Chapter Five. The confidence limits of the SWCC variables for the compacted mixture of sand and kaolin are also given in Chapter Five.

4.7.3 Investigation of the effect of soil properties on the variability in SWCC

The collected SWCC data were fitted using the framework proposed by other researchers (i.e., Zapata 1999 and Perera et al. 2005). In the framework by Zapata (1999) and Perera et al. (2005), SWCC was correlated with wPI. Equation (3-66) in this research indicated that SWCC can be correlated with saturated hydraulic conductivity, k_s . To investigate the correlation between SWCC and soil properties, the air-entry values were correlated with the individual soil properties (including engineering property: saturated hydraulic conductivity, k_s) or the multiple soil properties. The individual soil property was termed independent variable x_i , while the air-entry value (AEV) was termed dependent variable y . The regression between y and x_i was carried out to investigate the significance of the effect of the soil properties on the SWCC of the residual soils in Singapore. The variables used for the regression analysis are summarized in Table 4-10.

Table 4-10: Variables used for regression analysis

Variables	Y	X1	X2	X3	X4	X5	X6	X7	X8
	AEV	Liquid Limit, LL	Plastic Limit, PL	Plastic Index, PI	Sand content, Sand%	Fine content,	Log of k_s , Log(k_s)	Sand%*PI	Fine%*PI
Variables	Y	X9	X10	X11	X12	X13			
	AEV	Log(k_s)*PI	Sand%*PI	Fine%*PI	PI ²	(Log(k_s)) ²			

CHAPTER FIVE

RESULTS

5.1 Introduction

This chapter presents the results of the experimental measurements, including the basic soil index tests and SWCC measurements. The standard deviations of the calculated degree of saturation for both the compacted mixture of sand and kaolin and the residual soil are also presented. Next, the results of data interpretation (i.e., fitting parameters, confidence limits of SWCC, SWCC variables and confidence limits of SWCC variables) are presented. The effects of suction range and number of data points adopted for the best fit procedure on variability in SWCC are also investigated and presented. The confidence limits of both the SWCC and permeability function for residual soils (i.e., Jurong Formation, Bukit Timah granite and Old Alluvium) in Singapore are computed. Lastly, a regression between the air-entry value and basic soil properties is carried out to investigate the correlation between SWCC and basic soil properties of residual soils in Singapore.

5.2 Results of experimental measurements

The basic soil properties determined from the experimental measurement and SWCC results for compacted sand and kaolin and residual soil, are presented in this section.

5.2.1 Soil basic properties

The properties of the compacted mixture of sand and kaolin, which consists of 73% sand and 27% fines, and the residual soil, which consists of 48% sand and 52% fines, are summarized in Table 5-1. The void ratios of the compacted mixture of sand and kaolin and the residual soil were measured two times, once before SWCC measurement, which is termed the initial void ratio, e_i , and again after SWCC measurement, which is termed the final void ratio, e_f .

Table 5-1 Soil properties of a compacted mixture of sand and kaolin and a residual soil of Bukit Timah Granite

Property	Compacted mixture of fine sand with kaolin	Residual soil
(USCS)	SM	CH
Specific gravity, G_s	2.66	2.68
Grain-size data		
D_{60} (mm)	0.3	0.10
D_{30} (mm)	0.15	0.01
D_{10} (mm)	0.01	0.001
Sand (%)	73%	48%
Fines (%)	27%	52%
Dry density, ρ_d (Mg/m^3)	1.89	1.56
Initial void ratio, e_i	0.41	0.72
Final void ratio, e_f	0.39	0.57
Initial water content w_i (%)	7.8%	28.5%
Plastic Limit	--	30%
Plastic Index	--	24%

5.2.2 Experimental SWCC data for compacted mixture of sand and kaolin

SWCC test results from 12 specimens of the compacted mixture of sand and kaolin are illustrated in Figure. 5-1. A comparison of the SWCCs, including the drying curves and wetting curves from the 12 specimens, is illustrated in Figure 5-2. Table 5-1 indicates that the void ratio of the compacted mixture of sand and kaolin decreased by 0.02 after completion of the SWCC tests. Compared with the initial void ratio of 0.41, 0.02 can be considered insignificant. Therefore, the soil volume change during SWCC measurement for the compacted mixture of sand and kaolin is considered negligible.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

(i)

(j)

(k)

(l)

Figure. 5-1 SWCC results (including drying and wetting curves) for the compacted mixture of sand and kaolin

Figure. 5-2 Comparison of SWCC results from 12 specimens of the compacted mixture of sand and kaolin

5.2.3 Experimental SWCC data for residual soil

Table 5-1 indicates that the void ratio of the residual soil decreased by 0.15 after completion of the SWCC. Compared with the initial void ratio of 0.72, 0.15 represents around 21% of the initial void ratio and cannot be ignored. Therefore, the soil volume change during the SWCC measurement for the residual soil should be considered and both the water content and void ratio were monitored during the measurement.

The measured gravimetric water contents for six specimens of the residual soil are illustrated in Figure 5-3. A comparison of the water contents of the six specimens of residual soil is illustrated in Figure 5-4.

(e)

(f)

Figure 5-3 SWCC in the form of gravimetric water content for the residual soil

Figure. 5-4 Comparison of SWCCs in the form of gravimetric water content for six specimens of residual soil

Because the soil volume change could not be ignored, the void ratio was monitored and recorded throughout the SWCC measurement. The changes in the void ratio for the six specimens are illustrated in Figure 5-5. A comparison of changes in the void ratios of the six specimens is illustrated in Figure 5-6.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 5-5 Soil volume change during SWCC measurement for residual soil

Figure 5-6 Comparison of soil volume changes in six specimens of residual soil

The calculated degrees of saturation for the six specimens are illustrated in Figure 5-7. A comparison of the degrees of saturation of the six specimens is illustrated in Figure 5-8.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 5-7 SWCC in the form of degree of saturation for residual soil

Figure 5-8 Comparison of SWCCs in the form of degree of saturation for six specimens of residual soil

5.3 Results of data interpretation

The fitting parameters of SWCC for the compacted mixture of sand and kaolin and the residual soil are presented. The confidence limits of SWCC, SWCC variables, confidence limits of SWCC variables and relative hydraulic conductivity are computed from the fitting parameters.

5.3.1 Fitting parameters after the best fit procedure

The fitting parameters of SWCCs (drying curves and wetting curves) for the compacted mixture of sand and kaolin are summarized in Tables 5-2 and 5-3, while the fitting parameters of SWCCs in the form of gravimetric water content are summarized in Table 5-4. The fitting parameters of SWCCs in the form of degree of saturation, which considers the soil volume change associated with SWCC measurement, are summarized in Table 5-5. Both the standard deviation (s) and coefficient of variance (c_v) were adopted for evaluation of the

variability in the determined results, where $c_v = s/\mu$ (s = standard deviation as defined in Chapter 4, μ = mean value of the determined results).

Tables 5-2 and 5-3 indicate that the variability of fitting parameter “a” was higher than the variability of the other two fitting parameters, such as “n” and “m”. Tables 5-4 and 5-5 indicate that the fitting parameters of SWCC obtained by considering the soil volume change are different from those obtained by ignoring the soil volume change. As illustrated in Table 5-5, high variability of these fitting parameters is observed for natural residual soil. The high variability is believed to be resulted from natural variability of soil context and disturbing during the specimen preparation stage.

Table 5-2 Fitting parameters of SWCCs (drying curves) for the compacted mixture of sand and kaolin

Fitting parameters	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Specimen 7	Specimen 8	Specimen 9	Specimen 10	Specimen 11	Specimen 12	Standard deviation	Coefficient of variation
$a_{(dry)}$ (kPa)	102.44	74.03	72.73	80.95	75.71	69.69	75.81	83.93	68.54	59.20	61.54	64.73	11.57	0.16
$n_{(dry)}$	0.88	0.93	0.92	0.87	0.90	0.90	0.87	0.86	0.91	1.06	1.04	0.97	0.07	0.07
$m_{(dry)}$	2.52	2.09	2.09	2.27	2.16	2.16	2.19	2.31	2.04	1.73	1.77	1.91	0.22	0.11

Table 5-3 Fitting parameters of SWCCs (wetting curves) for the compacted mixture of sand and kaolin

Fitting parameters	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Specimen 7	Specimen 8	Specimen 9	Specimen 10	Specimen 11	Specimen 12	Standard deviation	Coefficient of variation
$a_{(wet)}$ (kPa)	14.50	14.20	14.55	18.27	15.25	17.13	17.92	14.98	28.24	15.70	17.90	25.20	4.45	0.25
$n_{(wet)}$	0.84	0.81	0.79	0.67	0.78	0.73	0.71	0.81	0.63	0.76	0.76	0.69	0.06	0.08
$m_{(wet)}$	1.69	1.71	1.75	2.11	1.78	1.94	2.01	1.71	2.47	1.83	1.89	2.24	0.24	0.13

Table 5-4 Fitting parameters of SWCCs ignoring the soil volume change for the residual soil

Fitting parameters	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Standard deviation	Coefficient of variation
a (kPa)	83.51	105.41	93.46	89.08	83.06	71.56	11.39	0.13
N	1.63	1.43	1.42	1.56	1.61	1.73	0.12	0.08
M	0.19	0.26	0.25	0.20	0.19	0.20	0.03	0.15

Table 5-5 Fitting parameters of SWCC considering the soil volume change for the residual soil

Fitting parameters	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Standard deviation	Coefficient of variation
a_1 (kPa)	100.01	425.53	100.07	688.56	367.28	100.02	241.32	0.81
n_1	1.58	1.59	1.47	1.05	3.73	6.76	2.20	0.82
m_1	0.03	0.16	0.09	0.20	0.04	0.05	0.07	0.74

5.3.2 Confidence limits of SWCC for the compacted mixture of sand and kaolin and residual soil

The confidence limits of the best fitted SWCC for the compacted mixture of sand and kaolin and the residual soil were determined using Equations (3-29) and (3-30) and are illustrated in Figures 5-9 and 5-10. As the properties of the mixture of sand and kaolin are very uniform, the variability in the SWCC of this type of soil is very small. On the other hand, the variability in the SWCC of the residual soil is much higher.

Figure. 5-9 Best fitted SWCC and its confidence limits for the compacted mixture of sand and kaolin determined from the best fit procedure

Figure. 5-10 Best fitted SWCC and its confidence limit for the residual soil determined from the best fit procedure

Figure 5-10 indicates that a suction of 1500 kPa is insufficient for the determination of an accurate SWCC for this type of residual soil. This is because the residual suction is greater than 1500 kPa, which is the maximum capacity of the 15 bar pressure plate from the determined SWCC.

5.3.3 SWCC variables for compacted mixture of sand and kaolin and residual soil

The SWCC variables for both the compacted mixture of sand and kaolin and the residual soil were computed from the fitting parameters and are illustrated in Tables 5-6 to 5-8.

Table 5-6. SWCC variables corresponding to the drying curve for the compacted mixture of sand and kaolin

SWCC variables	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Specimen 7	Specimen 8	Specimen 9	Specimen 10	Specimen 11	Specimen 12	Standard deviation	Coefficient of variation
Air-entry value, AEV (kPa)	11.59	10.92	10.51	9.86	10.26	10.14	9.61	9.71	9.88	12.5	12.49	11.06	1.02	0.10
Slope at inflection point s_1	0.53	0.52	0.52	0.51	0.52	0.52	0.50	0.50	0.51	0.56	0.56	0.53	0.02	0.04
Residual degree of saturation, S_r	0.03	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.06	0.06	0.05	0.01	0.20
Residual suction, ψ_r (kPa)	795.2	723.02	725.30	771.80	749.40	724.40	771.10	800.70	734.10	610.20	628.20	678.2	60.59	0.08

Table 5-7. SWCC variables corresponding to the SWCC ignoring soil volume change for the residual soil

SWCC variables	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Standard deviation	Coefficient of variation
Air-entry value, AEV (kPa)	53.84	62.51	54.08	56.70	53.23	43.4	6.21	0.12
Slope at inflection point s_1	0.23	0.27	0.26	0.24	0.24	0.24	0.02	0.06
Residual degree of saturation, S_r	0.72	0.46	0.43	0.42	0.74	0.70	0.16	0.27
Residual suction, ψ_r (kPa)	845.60	6029	8867	8800	800	787.7	4014.92	0.92

Table 5-8. SWCC variables corresponding to the SWCC considering soil volume change for the residual soil

SWCC variables	Specimen 1	Specimen 2	Specimen 3	Specimen 4	Specimen 5	Specimen 6	Standard deviation	Coefficient of variation
Air-entry value, AEV (kPa)	650.60	294.30	327.10	430.90	626.20	388.90	151.44	0.33
Slope at inflection point s_1	0.30	0.31	0.29	0.33	0.31	0.29	0.02	0.05
Residual degree of saturation, S_r	0.56	0.41	0.62	0.39	0.48	0.45	0.09	0.18
Residual suction, ψ_r (kPa)	19794	24548	6969	31223	30708	31356	9584.00	0.40

Tables 5-2, 5-6, 5-5 and 5-8 indicate that the variability in the SWCC variables is less than the variability in the fitting parameters. Phoon et al. (2010) and Zhai and Rahardjo (2013b) suggested that fitting parameters are dependent on each other. In other words, an individual fitting parameter has no physical meaning and only a combination of fitting parameters can define the SWCC. Therefore, the fitting parameters should not be correlated with the basic

soil properties. It is suggested that the air-entry value and residual suction to be correlated with the basic soil properties.

5.3.4 Wetting curve of compacted mixture of sand and kaolin

The wetting curves expressed using Equation (3-56) and the experimental data are illustrated in Figure 5-11.

(a) Specimen 1, with $k=1$, $b=10.79$ kPa

(b) Specimen 2, with $k=1$, $b=10.93$ kPa

(c) Specimen 3, with $k=1$, $b=10.40$ kPa

(d) Specimen 4, with $k=1$, $b=11.19$ kPa

(e) Specimen 5, with $k=1$, $b=9.59$ kPa

(f) Specimen 6, with $k=1$, $b=7.68$ kPa

(g) Specimen 7, with $k=1$, $b=8.70$ kPa

(h) Specimen 8, with $k=1$, $b=8.20$ kPa

(i) Specimen 9, with $k=1$, $b=9.13$ kPa

(j) Specimen 10, with $k=1$, $b=14.26$ kPa

(k) Specimen 11, with $k=1$, $b=13.11$ kPa

(l) Specimen 12, with $k=1$, $b=8.62$ kPa

Figure. 5-11 Expression of the wetting curves for the compacted mixture of sand and kaolin using Equation (3-57).

Figure 5-11 indicates that Equation (3-57) can express the wetting curve of the compacted mixture of sand and kaolin very well.

5.3.5 Comparison of best fitted SWCC and SWCC1, SWCC2, SWCC3 and SWCC4

Figure 5-12 shows a comparison of the best fitted SWCC, which was determined from the full experimental dataset, with SWCC1, which was determined from experimental data with a maximum suction of 50 kPa, SWCC2, which was determined from experimental data with a maximum suction of 100 kPa, SWCC3, which was determined from experimental data with a maximum suction of 500 kPa and only five data points and SWCC4, which was determined from experimental data with a maximum suction of 50 kPa and 48 data points.

(a) Comparison of best fitted SWCC and SWCC1

(b) Comparison of best fitted SWCC and SWCC2

(c) Comparison of best fitted SWCC and SWCC3

(d) Comparison of best fitted SWCC and SWCC4

Figure. 5-12 Comparison of the best fitted SWCC and SWCC1, SWCC2, SWCC3 and SWCC4

Figure 5-12 indicates that SWCC3, which was determined from experimental data with a maximum suction of 500 kPa and only five data points, is closest to the best fitted SWCC. As there are three fitting parameters (i.e., a , n , and m) in Fredlund and Xing's (1994) equation, theoretically, it needs a minimum of three numbers of data points for the determination of these three fitting parameters. However, Fredlund and Xing's (1994) equation is not in a linear form and therefore, more data points are required in order to obtain the solution for a , n and m . In fact, there are no exact solution for a , n and m for most of the soils and these fitting parameters can only be obtained using the curve fitting technique or the regression procedure. Therefore, the more data used for the regression, the more accurate solution for the fitting parameters can be obtained. Figure 5-12 illustrates that the maximum suction range has a more significant effect than the number of data points on the accuracy of the determined SWCC.

5.3.6 Variability in SWCC variables obtained using the conventional graphical method

The best fitted SWCC for the compacted mixture of sand and kaolin was selected for determination of SWCC variables, such as the air-entry value, slope at inflection point, residual suction and residual water content. The best fitted SWCC was determined by three people using the conventional graphical methods and the results are illustrated in Figure 5-13.

(a) SWCC variables determined by person A

(b) SWCC variables determined by person B

(c) SWCC variables determined by person C

Figure 5-13 SWCC variables for the compacted mixture of sand and kaolin determined by three people using the conventional graphical method

Figure 5-15 indicates that variability in SWCC variables (i.e., the determined results were not consistently determined by three persons, as illustrated in Figure 5-15) could result from different interpretations in using the conventional method for determination of SWCC variables. On the other hand, the SWCC variables can also be obtained using Equations (3-38), (3-43) and (3-44) as follows: $AEV=10.62$ kPa, slope at inflection point $s=0.522$ kPa⁻¹, residual suction $\psi_r=699.6$ kPa and residual degree of saturation $S_r=0.05$. It is concluded that Equations (3-38), (3-43) and (3-44) provide consistent results for the SWCC variables and reduce the variability in the determined results compared with the conventional graphical method.

5.3.7 Variability in SWCCs associated with soil volume change

A comparison of SWCCs obtained by ignoring the soil volume change and SWCCs obtained by considering the soil volume change of six specimens of the residual soil is presented in Figure 5-14.

(a)

(b)

(c)

(d)

(e)

(f)

Figure 5-14 Comparison of SWCCs considering and ignoring the soil volume change for six specimens of residual soil

Figure 5-14 and Tables 5-7 and 5-8 indicate that the air-entry values determined from SWCCs of the residual soil as obtained by considering the soil volume change are much higher than those determined from SWCCs as obtained by ignoring the soil volume change. As suggested by Fredlund (2006), the results in Table 5-9 give the correct values for the SWCC variables. Therefore, ignoring the soil volume change associated with SWCC measurement results in underestimation of the air-entry value and residual suction.

5.4 Statistical analysis of SWCC data for residual soils in Singapore

The confidence limits of the permeability function for the residual soils (i.e., Jurong Formation, Bukit Timah granite and Old Alluvium) are presented. The frameworks proposed by Zapata (1999) and Perera et al. (2005) were tested using SWCC data for residual soils in Singapore. A regression analysis was then carried out for correlation of SWCCs and basic soil properties.

5.4.1 Variability of unsaturated hydraulic properties of residual soil and effects on the slope stability

The permeability function ($k(\psi)$ function) was calculated from the SWCC using the statistical method (i.e., Equation (3-71) proposed in this research). The variability in the calculated $k(\psi)$ function for the three types of residual soil is illustrated in Figures 5-15 to 5-17. Based on the theory of vapor flow (i.e., vapor permeability function), Ebrahimi-B et al. (2004) suggested the value of 10^{-14} m/s as the minimum value of the water hydraulic conductivity. However, the vapor permeability function is dependent on the values of parameters α and β . Different researchers such as Lai et al. (1976), Millington and Quirk (1961), Millington (1959), Marshall (1959), Penman (1940) and Abu-El-Sha'r and Abriola (1997) adopted different values of $\alpha\beta$ and obtained the maximum vapor permeability around 10^{-13} m/s to 10^{-14} m/s. Therefore, the minimum hydraulic conductivity of 10^{-13} m/s is adopted as the lower limit of the permeability function as illustrated in Figures 5-15 to 5-17.

Figure. 5-15 Variability in the permeability function of the residual soil from Jurong Formation

Figure. 5-16 Variability in the permeability function of the residual soil from Bukit Timah granite

Figure. 5-17 Variability in the permeability function of the residual soil from Old Alluvium

Figures 5-15 to 5-17 indicate that the permeability function of residual soil can be affected by both SWCC and saturated hydraulic conductivity k_s . The confidence limits computed using Equations (3-29) and (3-30) agree with the results using box-plot method.

The comparison of the average values and coefficient of variations (COV) of saturated soil properties such as water content, $w\%$, plastic limit, PL, liquid limit, LL, void ratio, e , effective cohesion, c' , effective friction angle, ϕ' , saturated hydraulic conductivity, k_s , and unsaturated soil properties such as air-entry value, AEV, residual suction, ψ_r , and residual saturation, S_r , for the residual soils are summarized in Table 5-9. As illustrated in Table 5-9, the COV of unsaturated soil properties is much higher than that of saturated soil properties.

Table 5-9: Comparison of average value and COV of soil properties for residual soil in Singapore.

		Index properties				Saturated soil properties			Unsaturated soil properties			
		w% in situ	PL	LL	e In situ	c' (kPa)	ϕ' (degree)	Log(k_s)	ϕ^b (degree)	Air-entry value, AEV (kPa)	Residual suction , ψ_r (kPa)	Residual saturation, S_r
Jurong formation	AV	17.36	20.54	42.54	0.49	11.58	32.33	-6.71	28.28	34.18	3238.25	0.30
	COV	29.92%	19.63%	18.72%	23.58%	29.68%	9.15%	22.46%	22.72%	96.95%	72.37%	77.12%
Bukit Timah granite	AV	32.00	33.69	43.38	0.51	8.83	36.83	-6.62	31.60	16.58	2189.76	0.24
	COV	26.03%	21.62%	17.52%	18.58%	43.74%	9.12%	17.05%	23.28%	126.73%	100.49%	83.74%
Old Alluvium	AV	33.00	35.82	56.82	0.59	20.17	36.17	-5.86	31.30	18.90	2754.39	0.22
	COV	28.29%	24.58%	13.35%	16.50%	36.98%	10.79%	28.51%	28.42%	58.30%	106.39%	76.31%

The average values and coefficients of variation of the air-entry value, AEV, residual suction, ψ_r , and residual saturation, S_r , for Jurong Formation and Bukit Timah granite are different from that presented by Rahardjo et al. (2012) as illustrated in Table 2-1. There were total 18 datasets for Jurong Formation and 19 datasets for Bukit Timah granite analyzed by Rahardjo et al. (2012) while 28 datasets for Jurong Formation and 25 datasets for Bukit Timah granite were analyzed in this research.

Zhai et al. (2015) demonstrated the effect of variability of SWCC on the analyses of rainfall-induced slope failure using the numerical analyses. Two-dimensional seepage analyses were carried out using a model as illustrated in Figure 5-18. The distance between the slope and the side of the slope model was set to three times the height of the slope and Q , equal to zero with potential seepage review was assigned to the bottom and along the sides of the slope model above the groundwater table. The constant total head, h_w , on each side was applied as the boundary along the sides of the slope model below the groundwater table. A rainfall with intensity of 22mm/ hour and duration of 24 hours was applied to the slope surface as a flux boundary, q . Ponding was not allowed to occur at the slope surface which made the excess rainfall at the slope surface to be removed as run-off. Three types of soils (i.e. soils with the SWCC as best fitted SWCC, upper limit and lower limit) were assigned to the single layer of soil. To make the analyses results to be comparable, $k_s = 6 \times 10^{-6}$ m/s, which was within the range of saturated coefficient of permeability for the residual soil from Bukit Timah Granite as illustrated in Figures 5-16, was assigned to the three soil types. The permeability functions of the three types of soil were computed from SWCC using Zhai and Rahardjo (2015) equation. The SWCCs and permeability functions adopted in the simulation model are illustrated in Figures 4-6(b) and 5-19.

Figure 5-18. Slope geometry and boundary condition of residual soil (Zhai et al. 2015).

Figure 5-19. Computed permeability functions for residual soil from Bukit Timah Granite (Zhai et al. 2015).

An effective cohesion $c'=12$ kPa and an effective friction angle $\phi'=33^\circ$ as presented by Rahardjo et al. (2012) for Bukit Timah Granite, were assigned to the three types of soil for slope stability analyses. The angle indicating the rate of increase in shear strength relative to the matric suction $\phi^b=0.5 \phi'$ was used in this study.

Pore-water pressures (PWP) on the three sections as shown in Figure 5-20 were computed using Seep/W and illustrated in Figures 5-21 to 5-23.

Figure 5-20. Sections for monitoring of PWP (Zhai et al. 2015).

Figure 5-21. Pore-water pressure profiles on section 1 after 24 hours rainfall (Zhai et al. 2015).

Figure 5-22. Pore-water pressure profiles on section 2 after 24 hours rainfall (Zhai et al. 2015).

Figure 5-23. Pore-water pressure profiles on section 3 after 24 hours rainfall (Zhai et al. 2015).

Figures 5-21 to 5-23 indicate that pore-water pressure is highest in the soil with the upper limit of SWCC and lowest in the soil with the lower limit of SWCC. In other words, the slope in the soil with the upper limit of SWCC can be most easily infiltrated by rainwater and the effect of rainfall on the slope stability is most significant compared with the other two types of soil. On the contrary, the slope in the soil with the lower limit of SWCC can be most difficult to be infiltrated by rainwater and the effect of rainfall on the slope stability is least significant compared with the other two types of soil.

Stability analyses in this study were performed by incorporating the pore-water pressure from seepage analyses using Slope/W. The variation in factor of safety obtained from the slope

stability analyses of the three cases were calculated using Bishop's simplified method of slices. The computed factors of safety (FOS) are illustrated in Figure 5-24.

Figure 5-24. Computed FOS for the slope with residual soil from Bukit Timah Granite (Zhai et al. 2015).

Figure 5-24 indicates that the effect of variability in SWCC and k_w function on slope stability is significant. As shown in Figure 5-24, the FOS can drop by 30% if the upper limit of SWCC instead of the best fitted SWCC was adopted in the analyses. Figure 5-24 also indicates that rebound of FOS after the end of rainfall is also dependent on the unsaturated hydraulic properties of the soil. Therefore, the upper limit of SWCC is suggested to be used the stability analyses of slope under rainfalls.

5.4.2 Application of frameworks by Zapata (1999) and Perera et al. (2005)

Ten datasets were observed with a wPI greater than 20, and eight were selected for fitting using the SWCC framework proposed by Zapata (1999) and Perera et al. (2005), where w is the fines content, which is defined as the percentage of fines passing through a #200 sieve, and PI is the plastic index. Comparisons of the observed SWCCs of three types of residual soil and the frameworks of SWCCs by Zapata (1999), Zapata et al. (2000) and Perera et al. (2005) are provided in Figures 5-25 and 5-26.

Figure. 5-25 Comparison of observed SWCCs of residual soil with wPI greater than 20 and the framework of SWCCs by Zapata (1999)

Figure. 5-26 Comparison of observed SWCCs of residual soil with wPI greater than 20 and the framework of SWCCs by Perera et al. (2005)

Figures 5-25 and 5-26 indicate that the frameworks proposed by Zapata (1999), Zapata et al. (2000) and Perera et al. (2005) may not work well for the residual soils in Singapore. The proposed frameworks were developed based on the statistical analyses of database which consisted of experimental data that were mostly collected from USA. The accuracy of the results from the framework is dependent on the collected soil database. Therefore, the framework from Zapata (1999), Zapata et al. (2000) and Perera et al. (2005) may not necessarily be applicable to the residual soils in Singapore. High variability of SWCC is observed for the residual soils in Singapore if the SWCCs are correlated with wPI as illustrated in Figures 5-27 and 5-28.

The SWCCs for residual soil were divided into three groups based on the wPI of the soil (i.e., $1 < wPI < 10$, $10 < wPI < 20$ and $wPI > 20$) and the confidence limits of the SWCC for each group were computed using Equations (3-29) and (3-30). The SWCC data and confidence limits for each group are illustrated in Figure 5-27 to 5-29. A comparison of the confidence limits of the SWCCs for the three groups is given in Figure 5-30.

Figure. 5-27 SWCCs of residual soil with wPI from 1 to 10

Figure 5-28. SWCCs of residual soil with wPI from 10 to 20

Figure 5-29. SWCCs of residual soil with wPI greater than 20

Figure 5-30. Comparison of confidence limits of SWCCs for soils with different wPIs

Figure 5-30 indicates that the confidence limits were narrowest in SWCCs for soils with a wPI greater than 20. Narrower confidence limits mean lower variability in SWCCs. In other words, variability in SWCC decreases with an increase in wPI.

5.4.3 Correlation of SWCC of residual soil with basic soil properties

Thirteen sets of variables (x_1 to x_{13}), as illustrated in Table 4-2, were defined by their basic soil properties. The regression analyses between the air-entry value of individual SWCCs and variables were carried out and the results from the regression analyses are summarized in Table 5-10.

Table 5-10 indicates that the coefficients of determination R^2 for the regression between the air-entry value and x_6 ($\log(k_s)$) and x_{13} ($\log(k_s)^2$) were 0.140 and 0.155, respectively. On the other hand, the coefficient of determination R^2 for the regression between the air-entry value and x_8 (fine%PI) was only 0.018. In other words, the air-entry value of these three types of residual soils in Singapore was more related to k_s than fine%PI or wPI, as defined by Zapata (1999) and Perera et al. (2005). Table 5-10 indicates that the air-entry value of SWCC for residual soils in Singapore was more related to k_s than other soil properties.

Table 5-10. Results of regression analyses between the air-entry value of residual soils and basic soil properties

Regression	Degree of freedom for Regression df_R	Degree of freedom for residual df_E	R^2
$y=cx_1+b$ ($x_1=LL$)	1	53	7.40e-5
$y=cx_2+b$ ($x_2=PL$)	1	53	0.019
$y=cx_3+b$ ($x_3=PI$)	1	53	0.007
$y=cx_4+b$ ($x_4=sand\%$)	1	53	0.005
$y=cx_5+b$ ($x_5=fine\%$)	1	53	0.002
$y=cx_6+b$ ($x_6=\log(k_s)$)	1	53	0.140
$y=cx_7+b$ ($x_7=sand\%*PI$)	1	53	3.66e-05
$y=cx_8+b$ ($x_8=fine\%*PI$)	1	53	0.018
$y=cx_9+b$ ($x_9=\log(k_s)*PI$)	1	53	0.029
$y=cx_{10}+b$ ($x_{10}=sand\%*\log(k_s)$)	1	53	0.001
$y=cx_{11}+b$ ($x_{11}=fine\%*\log(k_s)$)	1	53	0.068
$y=cx_{12}+b$ ($x_{12}=PI^2$)	1	53	0.012
$y=cx_{13}+b$ ($x_{13}=(\log(k_s))^2$)	1	53	0.155

The SWCCs of residual soils in Singapore are correlated with k_s , as illustrated in Figure 5-31.

(a) SWCCs of residual soil with k_s less than 10^{-8} m/s

(b) SWCCs of residual soil with k_s between 10^{-8} m/s and $5 \cdot 10^{-8}$ m/s

(c) SWCCs of residual soil with k_s between $5 \cdot 10^{-8}$ m/s and 10^{-7} m/s

(d) SWCCs of residual soil with k_s between 10^{-7} m/s and 10^{-6} m/s

(e) SWCCs of residual soil with k_s between 10^{-6} m/s and 10^{-5} m/s

(f) SWCCs of residual soil with k_s between 10^{-5} m/s and 10^{-4} m/s

Figure 5-31. SWCCs of residual soil with respect to saturated hydraulic conductivity, k_s

CHAPTER SIX

DISCUSSION OF RESULTS

6.1 Introduction

This chapter presents a discussion of the results of SWCC measurements and their data interpretation. The variability in SWCC due to experimental error, hysteresis and soil volume change associated with SWCC measurement is discussed. The variability in SWCC due to the different numbers of data points adopted, different matric suction ranges covered for the best fit procedure and different methods for determination of SWCC variables is also discussed. The variability in SWCC for natural residual soils in Singapore, with respect to soil formation and saturated hydraulic conductivity, is also discussed.

6.2 Variability in SWCC associated with SWCC measurement

The compacted mixture of sand and kaolin is a sandy material with an air-entry value of 10.62 kPa, which is determined using equation derived in this research, while the residual soil is a silty material with an air-entry value of around 500 kPa. The sandy material has a low air-entry value and the degree of saturation is assumed to be 1 at suction of 0.01 kPa. The variability of SWCC for the compacted mixture of sand and kaolin is set to zero at suction of 0.01 kPa as illustrated in Figure 6-1. On the other hand, the residual soil has a higher air-entry value of 380kPa, which is determined using equation derived in this research, and the degree of saturation is assumed to be 1 at suction of 20 kPa. The variability of SWCC for the residual soil is set to zero at 20 kPa as illustrated in Figure 6-2.

By using Equation (3-71), the variability in SWCC for the compacted mixture of sand and kaolin and the residual soil can be illustrated as depicted in Figures 6-1 and 6-2, respectively. As shown in these figures, the highest variability in SWCC occurred in the transition zone (i.e., the suction zone between the air-entry value and residual suction). The highest variability in SWCC for the compacted mixture of sand and kaolin was around 3%, while the highest variability in SWCC for the residual soil was around 6.5%.

Figure 6-1. Variability in SWCC for the compacted mixture of sand and kaolin

Figure 6-2. Variability in SWCC for the residual soil

The standard deviation of the measured water contents during the drying and wetting processes for the compacted mixture of sand and kaolin is illustrated in Figure 6-3.

Figure 6-3. Standard deviation of the measured water contents of the compacted mixture of sand and kaolin with respect to different suctions

Figure 6-3 indicates that high variability in the measured water content occurred within the transition zone, which agrees with the results from the confidence limits, as illustrated in Figure 6-1. In addition, Figure 6-3 indicates that the standard deviation of the measured water content during the drying process was consistent with that during the wetting process. The variation in the measured water content may result from measurement errors such as error in reading and error due to measurement equipment. Furthermore, if the soil specimen is de-saturated or saturated from different initial water contents, the SWCC will follow different paths and different scanning curves will be obtained. Consequently, there may be variation in the measured water content. Variation in the measured water content, as illustrated in Figure 6-3, may result from measurement errors or the hysteresis characteristic of SWCC. Figure 6-3 suggests, in order to obtain a more accurate SWCC, that more experimental data should be measured in the transition zone.

The standard deviation and coefficient of variation of the measured water content, void ratio and degree of saturation for residual soils in Singapore are illustrated in Figures 6-4 and 6-5.

(a)

(b)

(c)

Figure 6-4. Standard deviation (s) for the gravimetric water content, void ratio and degree of saturation obtained from six specimens of residual soil

(a)

(b)

(c)

Figure 6-5. Coefficient of variation (c_v) for the gravimetric water content, void ratio and degree of saturation obtained from six specimens of residual soil

Figures 6-4 and 6-5 indicate that high variability in the measured water content, void ratio, and degree of saturation occurred within the transition zone, which agrees with the results from the confidence limits, as illustrated in Figure 6-2. Figures 6-3 and 6-4 indicate that, compared with variability in the measured gravimetric water content, the variability in the measured void ratio is much higher (i.e., the standard deviation of the measured void ratio is almost ten times greater than the standard deviation of the measured water content). The void ratio was determined by measuring the diameter and height of the specimens, while the water content was determined by weighing the specimens. The error associated with volume measurement is much larger than the error associated with mass measurement. To be more precise in the determination of degree of saturation, a more accurate method or technique is needed for measuring the soil volume change during SWCC measurement.

As illustrated in Figures 6-3 and 6-4(a), the standard deviation of the water content for the compacted mixture of sand and kaolin is much lower than that for the natural residual soil. This result agrees with the value determined from the confidence limits, as illustrated in Figure 6-1 and 6-2. The compacted mixture of sand and kaolin is a reconstituted soil and all the compaction conditions are well controlled, while the residual soil is a natural soil that has natural variability. Variability in water content results from factors such as sample disturbance, natural arrangement of soil particles, uniformity of the soil, etc., and therefore, greater variability is expected from natural materials.

The SWCCs that consider and ignore the soil volume change for residual soils are illustrated in Figure 6-6. Figure 6-6 indicates that the slopes of SWCCs obtained by considering the soil volume change are gentler than those of SWCCs obtained by ignoring the soil volume change when suction is low. In contrast, the slopes of SWCCs with the soil volume change measurements are steeper than those of SWCCs without the soil volume change measurements when suction is high. In addition, Figure 6-6 indicates that high variability in SWCC can result from the variability in the soil volume change measurement itself.

Figure 6-6. Comparison of SWCCs obtained by considering and ignoring the soil volume change for six specimens of residual soil

6.3 Variation of SWCC variables

Comparisons of the SWCC variables determined from SWCC data for individual specimens with those determined from the confidence limits of SWCCs (Equations (3-52) and (3-53)) for the compacted mixture of sand and kaolin are illustrated in Figures 6-7 to 6-10.

Figure. 6-7 Comparison between air-entry values determined from the confidence limit and those obtained from individual SWCCs

Figure. 6-8 Comparison between slope at the inflection point determined from the confidence limit and those obtained from individual SWCCs

Figure 6-9. Comparison of residual suctions determined from the confidence limit and individual SWCC

Figure 6-10. Comparison of residual degrees of saturation determined from the confidence limit and individual SWCC

Figures 6-7 to 6-10 indicate that the ranges of SWCC variables estimated from the confidence limits of SWCCs agree with those determined from the individual SWCCs. Therefore, the confidence limits of SWCCs can be used as an indirect method for estimating the variation in SWCC variables.

6.4 Wetting SWCC for the compacted mixture of sand and kaolin

A comparison between the wetting curve expressed using Equation (3-57) and the experimental data for the compacted mixture of sand and kaolin is illustrated in Figure 6-9. The figure indicates that the wetting curve determined from Equation (3-57) agrees with the experimental data. Therefore, Equation (3-57) is recommended for the prediction of the wetting curve from the primary drying curve.

Figure 6-11. Comparison between the wetting curve determined from Equation (3-57) and the experimental data from the compacted mixture of sand and kaolin

6.5 Variability in SWCC associated with data interpretation

The variability in SWCC variables determined using different methods and the variability in SWCC determined using different numbers of data point and suction ranges are discussed in this section.

6.5.1 Variability in SWCC variables

The air-entry value and residual suction are commonly determined using the graphical method, as it is difficult to measure these SWCC variables directly using an experimental apparatus. Conventionally, SWCC variables are determined by manual construction. In this research, equations for determination of the SWCC variables from the fitting parameters were

derived. Comparisons of the air-entry values and residual suctions for the compacted mixture of sand and kaolin are illustrated in Table 6-1. As shown, Equations (3-38), (3-35), (3-43) and (3-44) provide consistent results. The results suggest that the conventional, manual and graphical method should be replaced with the equations developed in this research for determination of the SWCC variables.

Table 6-1. Results of SWCC variables determined using the conventionally graphical method and the equations proposed in this research

SWCC variables	Conventional manual graphical method				Equations proposed in this research
	Person A	Person B	Person C	Coefficient of variation	Equations (3-38), (3-35), (3-43) and (3-44)
Air-entry value (kPa)	11.85	7.61	13.36	27.2%	10.62
Slope at the inflection point	0.54	0.42	0.58	16.2%	0.52
Residual suction (kPa)	634.96	1313.27	563.17	49.4%	699.6
Residual saturation	0.06	0.05	0.06	10.2%	0.05

Table 6-1 indicates that there are high variations in the determined air-entry value, the slope at the inflection point, the residual suction and the residual saturation using the conventional graphical method. Therefore, it is a bit risky to use the results from the conventional graphical method to predict other unsaturated soil properties because the variability of the results would reduce the accuracy of the predicted function (e.g, the permeability function, the shear strength function, etc).

6.5.2 Variability in SWCC due to different numbers of data point adopted and different suction ranges covered for the best fit procedure

Figure 6-12 presents a comparison of the best fitted SWCC with SWCC1, which was determined from experimental data with the maximum suction of 50 kPa, SWCC2, which

was determined from experimental data with the maximum suction of 100 kPa, SWCC3, which was determined from experimental data with the maximum suction of 500 kPa and only five data points and SWCC4, which was determined from experimental data with the maximum suction of 50 kPa and 48 data points. The figure indicates that SWCC3 is the closest to the best fitted SWCC. Figure 6-12 also indicates that the maximum suction range has more significant effect on the accuracy of the determined SWCC than the number of data points.

Figure 6-12. Comparison of best fitted SWCC and SWCC1, SWCC2, SWCC3 and SWCC4.

Leong and Rahardjo (1997a) suggested that the maximum suction adopted for SWCC measurement should be greater than the residual suction. Figure 6-12 illustrates that the accuracy of the determined SWCC can be improved if a wider suction range is covered during measurement (i.e., SWCC1 improving towards SWCC3). In addition, the accuracy of the SWCC can be improved if more experimental data are obtained (i.e., SWCC2 improving towards SWCC4). However, the accuracy of SWCC4 is still less than SWCC3. Therefore, it can be concluded that both the number of data points adopted and the suction range covered for the best fit procedure can affect accuracy or variability in the SWCC. The suction range covered for the best fit procedure has a more significant effect on the variability in SWCC than the number of data points adopted for the best fit procedure.

6.6 Estimation of SWCC from physical soil properties of residual soil in Singapore.

Many factors can affect variability of SWCC and the confidence of the determined results is dependent on relationship between SWCC and these properties and the number of soil properties which are used for the correlation. As illustrated in Figures 5-18 and 5-19, the experimental SWCC data did not fit very well with the framework proposed by Zapata (1999) and Perera et al. (2005). In their framework, only one variable (i.e., wPI) was considered for the correlation with SWCC. Equation (3-66), proposed in this research, illustrates that SWCC has direct correlation with the saturated hydraulic conductivity, k_s . In addition, the regression results in Table 5-9 indicate that the air-entry value of SWCC for residual soil is most related to the saturated hydraulic conductivity, k_s , compared with other soil properties. The bands of SWCCs predicted from different values of k_s are illustrated in Figure 6-13.

Figure. 6-13 Framework for the prediction of SWCC from saturated hydraulic conductivity, k_s , for residual soil in Singapore

Figure 6-13 indicates that the bands of SWCC for k_s between 10^{-5} and 10^{-4} m/s and k_s less than 10^{-8} m/s are significantly less than those for k_s in other ranges. Soil with k_s greater than 10^{-5} m/s represents a coarse-grained material, such as sand, while k_s , less than 10^{-8} m/s represents a fine-grained material, such as clay. Soil with k_s between 10^{-5} m/s and 10^{-8} m/s represents mixed soil, such as sandy clay. Therefore, Figure 6-13 suggests that variability of SWCC is dependent on the mixture of the soil. For more constituent mixture of soil, higher variability in SWCC is expected. On the contrary, when there is less constituent mixture of soil – regardless if it is a coarse-grained or a fine-grained mixture, less variability in SWCC is expected.

CHAPTER SEVEN

CONCLUSIONS AND RECOMMENDATIONS

7.1 Introduction

In this chapter, the concluding remarks on the results of this research are presented. Several recommendations regarding SWCC measurement and prediction from basic soil properties are given.

7.2 Conclusions

The conclusions can be summarized as follows:

1. Equations (i.e., Equations 3-29 and 3-30) for determination of the confidence limits of the best fitted SWCC were proposed, and the confidence limits were correlated with the fitting parameters of SWCC and residual error obtained in the best fit procedure. In addition, it was proven that confidence limits can be used as an indirect method for estimation of variability in SWCC.
2. Equations (i.e., Equations 3-38, 3-35, 3-43, 3-44) for determination of SWCC variables, such as air-entry value, slope at the inflection point, residual suction and residual water content, were derived. These SWCC variables were directly correlated with the fitting parameters of SWCC. The proposed equations provided consistent results and can be used to replace the conventional graphical method.
3. Equations (i.e., Equations 3-57 and 3-58) for estimation of the wetting SWCC from the drying SWCC are derived and verified with experimental data and published data from the literature.

4. A new equation (i.e., Equation 3-68), which can be considered as a statistical method, was derived for prediction of the relative hydraulic conductivity from the fitting parameters of SWCC. In the proposed equation, relative hydraulic conductivity is correlated with the fitting parameters of SWCC and the relative hydraulic conductivity can be computed using Microsoft Excel and the proposed equation.
5. Equations (i.e., Equations 3-52 and 3-53) for estimation of variation in SWCC variables were proposed. These equations provide an alternative method for estimation of SWCC variables.
6. Experimental results indicated that high variability in both the water content and degree of saturation in soil occurred within the transition zone. In addition, the variability in the measured void ratio was much higher than the variability in the measured water content due to measurement error.
7. The soil volume change associated with SWCC measurement should be monitored and recorded, as the SWCC in the form of degree of saturation provides the most accurate air-entry value.
8. Both the number of data points and suction range covered for the best fit procedure can affect the variability in SWCC. However, compared to the number of data points, the suction range has a more significant effect on the variability in SWCC.
9. The SWCC in the form of degree of saturation can be considered analogous to the pore-size distribution function because the area under the pore-size distribution function defines the degree of saturation.
10. Variability in SWCC for residual soil in Singapore is dependent on the mixture of the soil. If there is more constituents mixture of the soil (i.e., coarse-grained soil mixed with fine-grained soil), higher variability in SWCC is expected. On the contrary, if there is less constituents mixture of the soil (i.e., either all coarse-grained or fine-grained soil), lower variability in SWCC is expected.
11. It was observed that SWCC cannot be correlated with a single variable (i.e., a single soil property); many factors affect the shape of SWCC. To increase the accuracy of prediction, it was suggested that SWCC should be correlated with multiple variables (i.e., more soil

properties) instead of a single variable. Alternatively, a range of SWCCs (i.e., confidence limits) instead of a single curve could be correlated with a single variable.

12. Regression analyses indicated that SWCCs of residual soils in Singapore are more related to saturated hydraulic conductivity (k_s) than other soil properties and agrees with Equation (3-66) proposed in this research. A framework of SWCCs that is correlated with k_s was proposed.

The derived equations are summarized in Table 7-1.

Table 7-1. Summary of derived equations in this research.

Equation No.	Derived Equation	Purpose
3-29	$\begin{array}{ll} \text{When } 0 < \psi < a_{\max} & \text{When } \psi > a_{\max} \\ \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\max}} \right] \right\}^{m_{\min}}} & \theta_{\text{upper}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\max}} \right)^{n_{\min}} \right] \right\}^{m_{\min}}} \end{array}$	To determine the upper confidence limits of SWCC
3-30	$\begin{array}{ll} \text{When } 0 < \psi < a_{\min} & \text{When } \psi > a_{\min} \\ \theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\min}} \right] \right\}^{m_{\max}}} & \theta_{\text{lower}} = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a_{\min}} \right)^{n_{\max}} \right] \right\}^{m_{\max}}} \end{array}$	To determine the lower confidence limits of SWCC

3-33	$ \begin{aligned} & - \left[1 - \frac{\ln\left(1 + \frac{\psi}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right] \frac{m_1 n_1}{a_1} \frac{1}{\left[\ln\left(e + \left(\frac{\psi}{a_1}\right)^{n_1}\right) \right]^{m_1+1}} \frac{1}{e + \left(\frac{\psi}{a_1}\right)^{n_1}} \left(\frac{\psi}{a_1}\right)^{n_1-1} \\ & \left\{ (-m_1 - 1) \frac{1}{\left[\ln\left(e + \left(\frac{\psi}{a_1}\right)^{n_1}\right) \right]} \frac{n_1}{a_1} \frac{1}{e + \left(\frac{\psi}{a_1}\right)^{n_1}} \left(\frac{\psi}{a_1}\right)^{n_1-1} - \frac{n_1}{a_1} \frac{1}{e + \left(\frac{\psi}{a_1}\right)^{n_1}} \left(\frac{\psi}{a_1}\right)^{n_1-1} + \frac{n_1 - 1}{\psi} \right\} + \\ & 2 \frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right)} \frac{1}{C_r + \psi} \frac{m_1 n_1}{a_1} \frac{1}{\left[\ln\left(e + \left(\frac{\psi}{a_1}\right)^{n_1}\right) \right]^{m_1+1}} \frac{1}{e + \left(\frac{\psi}{a_1}\right)^{n_1}} \left(\frac{\psi}{a_1}\right)^{n_1-1} + \\ & \frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right)} \left(\frac{1}{C_r + \psi}\right)^2 \frac{1}{\left[\ln\left(e + \left(\frac{\psi}{a_1}\right)^{n_1}\right) \right]^{m_1}} = 0 \end{aligned} $	To locate the inflection point
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3-35	$s_1 = \left \frac{dS}{d \log(\psi)} \right _{\psi=\psi_i} =$ $\psi_i \ln(10) \left[\frac{1}{\ln\left(1 + \frac{10^6}{C_r}\right) \left(1 + \frac{\psi_i}{C_r}\right) C_r \left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1}} + \frac{m_1 n_1 \left(\frac{\psi_i}{a_1} \right)^{n_1-1} \left[1 - \frac{\ln\left(1 + \frac{\psi_i}{C_r}\right)}{\ln\left(1 + \frac{10^6}{C_r}\right)} \right]}{a_1 \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \left\{ \ln \left[e + \left(\frac{\psi_i}{a_1} \right)^{n_1} \right] \right\}^{m_1+1}} \right]$	To determine the slope at the inflection point
3-38	$\psi_b = \psi_i 0.1^{\frac{1-S_i}{s_1}}$	To determine the air-entry value
3-43	$\psi_r = 10^{\frac{S_i - S' + s_1 \log(\psi_i) - s_2 \log(\psi')}{s_1 - s_2}}$	To determine the residual suction

3-44	$S_r = S_i - s_1 \log(\psi_r / \psi_i)$	To determine the residual saturation
3-57	$S = S(\psi_m) + S(\psi') [S(\psi') - S(\psi_m)]$	To estimate the main wetting SWCC
3-58	$S_{dscanning} = \begin{cases} S(\psi_i) & \psi < \psi' \\ S(\psi_i) - \sum_{j=1} S(\psi) f(r_{i+k+j}) \Delta r & \psi > \psi' \end{cases}$	To estimate the wetting scanning SWCC
3-67	$k_r = \frac{n_m^2}{n_s^2} \frac{(S(\psi_m) - S(\psi_{m+1}))^2 \psi_m^{-2} + \sum_{i=m+1}^N [(S(\psi_m) - S(\psi_i))^2 - (S(\psi_m) - S(\psi_{i-1}))^2] \psi_i^{-2}}{\sum_{i=1}^N [1 - S(\psi_i)]^2 - [1 - S(\psi_{i-1})]^2] \psi_i^{-2}}$	To predict the permeability function

3-68	$k(\psi_{m+i}) = k(\psi_m)$ $\frac{n_{m+i}^2 \left\{ (S(\psi_{m+i}) - S(\psi_{m+i+1}))^2 r_{m+i}^2 + \sum_{j=m+i+1}^N [(S(\psi_{m+i}) - S(\psi_j))^2 - (S(\psi_{m+i}) - S(\psi_{j-1}))^2] r_j^2 \right\}}{n_m^2 \left\{ (S(\psi_m) - S(\psi_{m+1}))^2 r_m^2 + \sum_{i=m+1}^N [(S(\psi_m) - S(\psi_i))^2 - (S(\psi_m) - S(\psi_{i-1}))^2] r_i^2 \right\}}$	To predict the permeability function
3-52	$AEV_{\max} = \psi_b(a_{\max}, n_{\max}, m_{\min})$ and $AEV_{\min} = \psi_b(a_{\min}, n_{\min}, m_{\max})$;	To estimate the variation of AEV
3-53	$\psi_{r\max} = \psi_r(a_{\max}, n_{\min}, m_{\min})$ and $\psi_{r\min} = \psi_r(a_{\min}, n_{\max}, m_{\max})$;	To estimate the variation of residual suction

7.3 Recommendations

The recommendations for SWCC measurement and prediction can be summarized as follows:

- (1) Both water content and volume change should be monitored and recorded during SWCC measurement. Volume change monitoring is not only important for determination of a correct SWCC, but also for prediction of a reasonable permeability function.
- (2) The error associated with volume measurement is much larger than that associated with water content measurement. Thus, a more accurate measurement method for volume monitoring is suggested.
- (3) As the fitting parameters in the best fit equations are dependent on each other, any single fitting parameter is sensitive to change during the best fit procedure. Therefore, it is risky to correlate the fitting parameters with the basic soil properties. Instead, the air-entry value and residual suction are suggested to be correlated with the basic soil properties.
- (4) As high variability in SWCC occurs in the transition zone, more experimental data within the transition zone are required in order to obtain a more accurate SWCC.
- (5) Upper confidence limit of SWCC is recommended to be used for the analyses of rainfall-induced slope failures as the upper limit of SWCC will result in the most conservative result.
- (6) Lower confidence limit of SWCC is recommended to be used for the estimation of the shear strength as the lower limit of SWCC will result in the most conservative estimation.
- (7) The equations developed in this research for prediction of the wetting SWCC and the permeability function can be used for capillary barrier design (Rahardjo et al. 2015)
- (8) Suction effect can be considered as an additional cohesion of soil that increases the shear strength of soil to be used for geotechnical analyses such as slope stability, lateral earth pressure, foundation bearing capacity, ground deformation. The reliability of the suction in the soil is dependent on the unsaturated hydraulic properties of the

soil such as SWCC and permeability function. Therefore, results of the variability in SWCC and the permeability function from this research can be extended to quantify the variability in the geotechnical analyses involving unsaturated soil.

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