Strategy change and wealth accumulation in financial markets

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**STRATEGY CHANGE AND WEALTH
ACCUMULATION IN FINANCIAL MARKETS**

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SCHOOL OF HUMANITIES AND SOCIAL SCIENCES

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**STRATEGY CHANGE AND WEALTH
ACCUMULATION IN FINANCIAL MARKETS**

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Contents

1	Introduction	1
1.1	Literature review	1
1.2	Motivation and organization	7
2	Strategy Change and Wealth Accumulation: S&P 500 Data	10
2.1	Introduction	10
2.2	Strategies and strategy change	11
2.2.1	Fundamental analysis	12
2.2.2	Technical analysis	12
2.2.3	Fitness of strategies	13
2.3	Simulations with S&P 500 historical prices	15
2.3.1	The whole history: 1950~2014	18
2.3.2	Bulls, bears and others	20
2.4	Path dependent wealth accumulation	24
2.5	Conclusion	30
3	Strategy Change and Wealth Accumulation: A Heterogeneous Agent Model	31
3.1	Introduction	31
3.2	A discrete dynamic HAM model	36
3.2.1	A baseline model	36
3.2.2	Wealth accumulation	40
3.3	Profitability of strategies	40

3.4	Simulations and discussions	43
3.4.1	Numerical simulations	43
3.4.2	Main findings	50
3.4.3	Robustness	55
3.4.4	Noise term	56
3.5	Conclusion	59
4	Endogenous Fundamental, Financial Bubbles and Stock Cycles	60
4.1	Introduction	60
4.2	The model	63
4.3	Dynamic equilibrium	70
4.3.1	Real economy	72
4.3.2	Stock bubbles	75
4.4	Numerical analysis	78
4.4.1	Constraints on liquidity, investment and consumption	78
4.4.2	Parameters	81
4.4.3	Four phases of a stock cycle	82
4.4.4	Discussion of other issues	91
4.5	Conclusions	93
5	Summary	95

List of Figures

1-1	Total household financial assets (US dollars/capita) in 2012. Source: National Accounts at a Glance.	8
1-2	Households' shares and other equity (% of total financial assets) in 2012. Source: National Accounts at a Glance.	9
2-1	Daily close prices of SP500 from 1/3/1950 to 5/21/2015, 16452 days in total.	16
2-2	Two fitnesses of the fundamental strategy through the history of SP500. Top panel: Daily profit. Bottom panel: Modified fitness based on returns.	17
2-3	Wealth accumulations by four strategies. Top panel: Linear scale of wealth. Bottom panel: Logarithmic scale of wealth.	19
2-4	Agents' frequency of strategy switching (top) and their final wealth (bottom). Note that agents are ranked by their strategy-switching thresholds.	21
2-5	Agents' average frequency of strategy switching (top) and their average final wealth of 1000 simulations (bottom). Note that agents are ranked by their strategy-switching thresholds.	22
2-6	Analysis of bull markets. Left panels: From 12/8/1994 to 3/24/2000. Right panels: From 3/9/2009 to 11/24/2014.	25
2-7	Analysis of bear markets. Left panels: From 11/9/2000 to 11/11/2002. Right panels: From 10/30/2007 to 3/23/2009.	26
2-8	Analysis of markets with major trend reversals. Left panels: From 9/10/1997 to 8/13/2009. Right panels: From 8/30/2000 to 1/14/2013.	27

3-1	Absolute profit of the fundamental strategy (top) and the trend-following strategy (bottom). In the first quadrant, the profit of a strategy is positive in the grey shaded region, zero along two lines and negative in the other area.	42
3-2	Relative profit. Top panel: The value of relative profit in a 3D space. Bottom panel: Details on signs and comparisons of strategies' absolute profits. The first quadrant is divided into ten areas by four solid lines. The shaded regions indicate areas where the relative profit is positive. .	44
3-3	A typical simulation. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy over 200 periods. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high. . .	46
3-4	Time series within a typical simulation. (a) Stock price and fundamentalists' expectation of the fundamental value. (b) Fundamentalists' and chartists' demand weighted by their market fraction. (c) Relative profit of fundamental strategy comparing with chartists' strategy. (d) Average market fraction of fundamental analysis within the whole population. . .	48
3-5	Comparison of the group with maximum strategy-switching propensity and the group with minimum strategy-switching propensity. (a) Fraction of fundamental analysis within a group. (b) Wealth accumulation of a group.	49
3-6	Average of 1000 simulations. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high.	51
3-7	Decomposition of the wealth change of the slowest group. (a) The same simulation shown in Fig. 3. (b) A new simulation with zero initial endowment.	54

3-8	Robustness check. Investors with higher propensity of strategy switching have higher average fraction of fundamental strategy (left column) and lower final wealth (right column) under different risk-free interest rates (a, b), market maker's price-adjustment speeds (c, d) and chartists' trend-following extents (e, f).	57
3-9	Effect of the standard deviation of the noise term	58
4-1	The two-way interaction between the production and the financial market.	63
4-2	Agents' roles in the production process and in the financial market. . . .	65
4-3	A detailed timeline	71
4-4	A sketch of Equation 4.35 when two real roots exist.	74
4-5	A saddle-node bifurcation happens when the total factor productivity parameter A increases.	75
4-6	Agents' consumption. The thin straight line is their initial consumption. The thick polyline shows their actual consumption under constraints. . .	80
4-7	A typical simulation of stock price series and its fundamental.	82
4-8	Four phases of a stock cycle: Accumulation, Boom, Crash and Recovery. Shaded and unshaded areas indicate different phases.	87
4-9	Deviation of stock price from its fundamental.	88
4-10	Fundamentalists' and chartists' intended demands for stock shares and their actual demands limited by constraints.	88
4-11	Agents' cash saving and their purchase of bond.	89
4-12	More samples of stock cycles.	92
4-13	The fraction of stock controlled by technical analysis changes, even though no strategy switching in this model.	93

List of Tables

4.1 Parameter values 81

SUMMARY

This thesis studies investors' strategy change behavior and how such behavior affects investors' wealth accumulation by financial investments. Other issues relate to financial markets, such as the performance of strategies, fundamental value, the formation and burst of bubbles, stock cycles, are also investigated. Heterogeneous agent modeling is used as the main methodology.

Chapter 1 introduces the history and latest development of heterogeneous agent models and describes the motivation of this thesis.

Chapter 2 studies investors' strategy change frequency and their wealth accumulation by financial investments. Artificial investors are put into a stock market. They trade S&P 500 index following common strategies in practice. Driven by past performance of strategies, investors change strategy at different frequencies. There are special regions where investors who change strategy more often end up with less final wealth under diverse market trends. A detailed decomposition of wealth accumulation via financial investment shows the dependence of wealth on investors' past transactions.

Chapter 3 introduces a new heterogeneity, that is, agents' propensity of strategy switching, to the heterogeneous agent model. Numerical analysis shows that agents with higher propensity adopt the better strategy more often but end up with less final wealth. As this parameter, investors' propensity of strategy switching, can be interpreted as the speed of adaptive learning, the economic meaning that fast adaptive learning can hurt wealth accumulation seems counter-intuitive. Further investigation reveals the inconsistency between investors' strategy switching and their wealth accumulation, which causes the counter-intuitive phenomenon. Wealth

accumulation relies heavily on the position of risky asset an investor holds and its corresponding market value, rather than on the profit earned in the latest trading.

Chapter 4 improves the heterogeneous agent model built in Chapter 3. In Chapter 3, the fundamental value of the stock is exogenous. In Chapter 4, the fundamental value is endogenously determined by a production process. This distinguishes my work from other models with endogenous fundamental, as almost all of them take the New-Keynesian approach which relates the fundamental to the demand side of the real economy. More realistic issues like constraints on budget, short-sale and liquidity are covered. Under a nonlinear price dynamics and all constraints faced by investors, the prices of risky asset show cyclical motions. Stock bubbles form and burst along stock cycles. The formation of bubbles hurt the real economy by drawing resource from future production. Though in general chartists are less wealthy than fundamentalists, they have a significant impact on the stock price.

Chapter 5 summarizes this thesis and points out future research directions.

Chapter 1

Introduction

1.1 Literature review

This thesis investigates investors' wealth accumulation by financial investment and the interaction of financial market and real production. The method of heterogeneous agent model is used through the thesis. Since this methodology is relatively new, first appeared in the 1970s and started flourishing from the 1990s, a broad review of this field is necessary. This section gives a general introduction of heterogeneous agent models, unfolding the history and development of this field. Later, at the beginning of each chapter, more detailed literature review can be found.

Heterogeneous agent model (hereafter HAM) is a study approach of dynamic economic systems with heterogeneous interacting agents. Other synonymous names are agent-based computational economics (ACE), agent-based models (ABM), and agent-based computational models (ABC). As shown by its name, the most significant feature of such models is the existence of multiple heterogeneous agents, representing economic subjects such as households, investors, firms, banks, and so on. Agents' behavior in HAMs is designed to mimic real economic subjects. This provides HAMs a good micro-foundation. At the macro level, it is easy to acquire macro data as long as all economic activities among agents are well described in models and programs. HAMs provides a natural description of the economic system. The bottom-up structure of HAMs bridges microeconomics and macroeconomics. In HAMs, macro data are generated by agents'

behavior at the micro level. However, the macro data may not be directly decomposed to a simple summation of microdata. The interaction between agents and macro conditions brings complexity into the economy. In reality, people are involved in the economy as economic subjects rather than spectators. In financial markets, investors make investment decisions according to macro data like price, volume, gross domestic product, money supply, and so on. Their decisions feed back to the system and further affect the formation of macro data in future. Such interactions among agents and between agents and macro conditions distinguish Economics from natural sciences. These nonlinear interactions make the economy an evolving complex system where anomalous phenomena emerge, and no universal law exists. Too little attention was paid to these interactions before the 1990s. They surely deserve more investigation.

During the 1970s and 1980s, the efficient market hypothesis was one of the mainstream paradigms dominating economics and finance. Later, HAMs and behavioral economics emerged from the critics against the efficient market hypothesis on representative agents and rational expectations (Malkiel, 2003; Hommes, 2006). Zeeman (1974) is one of the first HAMs for financial markets. By using catastrophe theory, it provides a qualitative explanation for the alternation of bull and bear markets. The description of fundamentalists and chartists and their basic behavior rules is still widely adopted today. Beja and Goldman (1980) is the first dynamic HAM with a market maker to adjust asset price according to agents' aggregate demand.

The 1990s is the period during which HAMs started to prosper. In contrast to the continuous model of Beja and Goldman (1980), Day and Huang (1990) is in discrete time. More importantly, Day and Huang show the nonlinear deterministic price dynamics under bull and bear markets and the random switch of these two markets. They identify that the market maker's price adjustment speed plays a critical role in the bifurcation and chaotic behavior of price. There is a beauty of concision in the way how their model produces various market scenarios from the balance of heterogeneous agents' demand. Besides bull and bear markets, agents' herding behavior is another central research topic. Lux (1995) focuses on the contagion of market sentiment among agents. The well designed and justified nonlinear contagion dynamics presents

stationary or cyclical price bubbles under different conditions, as well as the switching between bear and bull markets. From another viewpoint, agents' herding behavior is a form of adaptive learning under limited information. Brock and Hommes (1997, 1998) dig deeper into agents' adaptive learning. In their model, heterogeneous agents do myopic mean-variance maximization of wealth. The main nonlinearity comes from the discrete choice assumption for predictor selection. Fundamentalists and chartists are defined by their predictors of future price. The proportion of a predictor being adopted in the whole population correlates with the past performance of this predictor. The idea of learning from experience is essential in adaptive learning. Moreover, the parameter representing the speed of adaptive learning is critical in the nonlinear price dynamics. When the intensity of agents' adaptive learning increases, the system evolves from steady states to bifurcations to chaos. Brock and Hommes' work may not be the first HAM introducing the idea of adjusting the fraction of heterogeneous groups based on experience, but it must be the most influential one. The discrete choice function of strategy switching used in their model is widely adopted in HAMs even nowadays. From the viewpoint of finance, Chiarella and He (2001) study asset price and wealth dynamics. They assume that agents' wealth follows a continuous-time stochastic differential equation and agents' goal is to maximize the expected utility of wealth. Besides nonlinear price dynamics, particular attention is paid to the equilibrium and stability in the homogeneous model, the mixture of different agent types in the heterogeneous agent model, and the reproduction of stylized facts.

In the 2000s, empirical works on stylized facts in financial markets (Cont, 2001; Taylor, 2005; Pacurar, 2006) drew researchers' attention. Lots of HAMs are developed to explain the formation of stylized facts, such as Hommes (2002), Chiarella et al. (2002), LeBaron (2006), He and Li (2008), *etc.* Gradually, the ability to reproduce stylized facts becomes a basic criterion of justifying HAMs for being reasonable and useful. Chen et al. (2012) summarize empirical works on stylized facts and stylized facts explained by theoretical HAMs under various model designs. But still, more is required for model justification. The topic of calibration and estimation of HAMs gains popularity. Following Lux's work on herding, Alfarano et al. (2005, 2006, 2007) estimate the herding tendency

by using maximum likelihood. In HAMs with strategy switching, the intensity of choice is the most critical parameter. It is natural that this parameter gains more attention than any other parameters. Westerhoff and Reitz (2003), Reitz and Westerhoff (2007), Boswijk et al. (2007), de Jong et al. (2006, 2009), Amilon (2008) estimate the intensity of choice by using diverse methods. Chen et al. (2012) provide detailed statistics on literature relating this issue. Apart from econometrics, the human subject experiment is another support for justifying HAMs. Duffy (2006) sheds light on a promising route of combining HAMs and human subject experiments by taking Gode and Sunder (1993, 1997a, 1997b, 2004) as an example. Essentially, both HAMs and human subject experiments are research methods created by pioneers upon the critics against strong assumptions like perfect rationality and representative agents. Both approaches focus on bounded rationality of agents or subjects. Both follow a bottom-up structure to bridge the micro and macro level. Their collaboration is natural and reasonable. Some representative studies are Boswijk et al. (2007), Hommes et al. (2008), Anufriev and Hommes (2008), Heemeijer et al. (2009), *etc.* Recently, Hommes (2013) summarizes his years of research on the collaboration of HAMs and behavioral economics. This book shows how the interactions of heterogeneous expectations lead to instability and further result in chaotic dynamics in the asset price. He presents formal analysis and empirical and experimental validation. Several laboratory experiments are conducted to study people's non-rational expectations, especially in positive feedback systems. These experiments justify underlying assumptions in HAMs, such as the existence of heterogeneous expectations and adaptive learning behavior.

A look at the history of HAMs reveals how the research trend is led by the development of relative disciplines, computational ability, financial crises and other factors. Driven by the development of nonlinear dynamics, most HAMs built in 1990s focus on the pure mathematical analysis of nonlinear dynamics, strange attractors, bifurcations and chaotic behavior in the asset price. Entering 2000s, thanks to the improvement of computer capacity, more and more HAMs rely heavily on numerical simulations. Numerical simulations reproduce a broad range of stylized facts. A mixture of mathematical analysis and numerical analysis gains popularity. Meanwhile, scholars adopt

econometric methods to justify HAMs. Later, the 2008 financial crisis causes a small rush of HAMs investigating financial crises (Huang et al., 2010; Gallegatia et al., 2011; Huang and Zheng, 2012).

After a rapid development of more than 20 years, there are some handbooks and collections of works reviewing HAMs comprehensively. Tesfatsion and Judd (2006) show how agent-based computational economics can contribute to economics, finance, human subject experiments, politics, organizations, market design, social-ecological systems, *etc.* This handbook provides a broad coverage and useful guidance to other literature. However, it comes with the side effect that the depth of discussion varies, depending on how far the contributors of each chapter want to go. Among all fields mentioned, the financial economics is one of the most active research areas. Hens and Schenk-Hoppé (2009) focus on HAMs in financial economics. They relax the central paradigm in finance: optimization and rational expectations, which is the theoretical foundation of the efficient market hypothesis. Instead, contributors of chapters present models of portfolio selection and asset price dynamics based on investors' heterogeneity and the rule of thumb strategies. These models are successful as a descriptive approach because they can explain stylized facts, which are anomalies or puzzles in the traditional finance world, such as fat tails in the return distribution, clustered volatility, bubbles, and crashes. The highlight of their work is the importance of dynamics and heterogeneity in financial markets. Besides, there are other books on heterogeneous agent models, nonlinear price dynamics and financial economics, *e.g.* Lux et al. (2005), Bischi et al. (2010), Gaffard and Napoletano (2012) and Dieci et al. (2014).

Now, in the 2010s, cutting-edge research works of HAMs show new features. HAMs merge with other methods, theorems, and data to investigate untouched fields. The popularity of behavior economics gains the collaboration of HAMs and human subject experiments more adherents (Bao et al., 2012, 2013). Network structures are embedded in HAMs to study the contagion of crises in the money market. A core-periphery network structure of bank system (in't Veld and van Lelyveld, 2014; Fricke and Lux, 2015; Lux, 2015) is discovered from empirical evidence (Allen and Gale, 2000; Gale and Kariv, 2007). An interesting finding is that the bank system is unstable when banks are

homogeneous; but when banks are heterogeneous, and big banks stay in the core, the bank system becomes stable (in't Veld and van Lelyveld, 2014). Other issues, *e.g.* the formation of the core-periphery structure, the stability of the system, the contagion of crises through the network and so on, are currently under intense study. As a perfect tool bridging microeconomics and macroeconomics, the application of HAMs to macroeconomics can be promising. Leijonhufvud (2006) gives a tentative introduction to this route. De Grauwe (2012) introduces essential elements of HAMs into macroeconomics. De Grauwe criticizes the common assumption of rational expectations and dependence on exogenous shocks taken by the mainstream business cycle models in the New Keynesian Macroeconomics. Alternatively, to give an endogenous explanation of business cycles, he proposes a behavioral model where agents adopt heterogeneous forecasting heuristics: fundamentalists rule or extrapolative rule, and the population share of each rule adjust according to past performance. The emphasis is on macroeconomic issues like the aggregate demand, Phillips curve, Taylor rule, inflation, interest rate, business cycle, and so on. Inspired by De Grauwe (2012), some works introduce macroeconomic factors into HAMs to gain endogenous fundamental value in artificial financial markets (Westerhoff, 2012; Lengnick, 2013; Lengnick and Wohltmann, 2013). Moreover, large-scale agent-based models are created to study the impact of monetary policy and fiscal policy on the real economy and financial markets. The EURACE project (Deisenberg et al., 2008; Cincotti et al., 2010, 2011, 2012; Dawid et al., 2012) simulates the European economy filled with households, firms, banks and a government. Empirical data are imported into the simulation system to reproduce statistical regularities and to experiment with various macroeconomic policy scenarios. The EURACE project works as a tool for policy making, and it is carried on as another project named Symphony to build a large-scale multi-country agent-based macroeconomic model. Other rising research branches in HAMs include, but not limited to, high-frequency financial data, artificial intelligence, simulation of the double auction, nonlinear price dynamics under continuous time, *etc.* With a high-level computational capacity and the advent of the big data era, a large number of new research directions emerge.

1.2 Motivation and organization

Nowadays, almost every household is participating in financial markets directly or indirectly. According to OECD (2015), every household in the US held around 189,000 USD financial assets in 2012, out of which 30.3% were shares and other equity. See Fig. 1-1 and Fig. 1-2 for data of other OECD countries¹. Besides households' intentional investment in stock markets, more households are involved in financial markets via insurance reserves and pension funds. Financial investments show large influence on households' wealth. But exactly how? This thesis focuses on how wealth is accumulated via financial investments, rather than general wealth accumulation of households including ordinary income, precautionary saving, and so on.

Investors' trading behaviors affect their capital gain and wealth in various ways. Inspired by Barber and Odean's (2000) work on how active trading hurts investors' wealth, I start to wonder whether a similar thing could happen to investors' strategy change behavior. However, there is a difficulty in getting empirical data. Unlike Barber and Odean's study where investors' transaction record is available for investigation, there is no easy access to the data of investors' strategy change behavior. Moreover, individual investors are influenced by others, *e.g.* their friends who claim in possession of inside information, stock experts showed up on television, other investors who brag about their big wins, and so on. Individual investors may take others' suggestions and make trading decisions without being fully aware of the trading strategies behind. Such examples could weaken the validity of the data derived from survey or questionnaire. Therefore, the method of heterogeneous agent model is used.

In Chapter 2 and Chapter 3, artificial investors mimic real investors' behavior, trading assets according to popular investment strategies in practice. Particular emphasis is put on investors' strategy-switching frequency and their wealth accumulation via financial investments. In Chapter 2, artificial investors trade in a real stock market which is represented by the historical prices of S&P 500.

¹Household financial assets include currency and deposits, securities other than shares, loans, shares and other equity, net equity of households in life insurance reserves, net equity of households in pension funds, prepayments of premiums and reserves against outstanding claims, and other accounts receivable.

Figure 1-1: Total household financial assets (US dollars/capita) in 2012. Source: National Accounts at a Glance.

In Chapter 3, a heterogeneous agent model is built to support the empirical study in Chapter 2. An endogenous price formation completes the model structure so that deeper mathematical and numerical analysis is feasible. The main findings of this chapter are interesting, robust and consistent with empirical results in Chapter 2.

After the study of individual's behavior, my curiosity on the performance of the population at the macro level is aroused. Among all those gaps exposed after the previous literature review, the interaction of financial market and real economy grabs my attention. Besides, this topic can work as an extension of the model in previous chapters. Therefore, Chapter 4 replaces the exogenous fundamental value used in Chapter 3 by an endogenous one. The interaction of financial market and the real economy is revealed during the evolvement of stock cycles.

Chapter 5 summarizes this thesis.

Figure 1-2: Households' shares and other equity (% of total financial assets) in 2012.
Source: National Accounts at a Glance.

Chapter 2

Strategy Change and Wealth Accumulation: S&P 500 Data

2.1 Introduction

The World today is full of changes and choices. Life today moves faster than ever before. To seize opportunities, people often need to make quick decisions. When lack of information, one tends to imitate the behavior of people who are believed having more information. Since most people only have access to limited information, this method is frequently adopted by the majority. Therefore, we see all kinds of trends and fashions in our daily lives. In financial markets, the imitation among investors causes Shiller's irrational exuberance (Shiller, 2000) which may further lead to disastrous financial crises.

In stock markets, investors' adjustment of strategies according to experience is usually called adaptive learning (Brock and Hommes, 1997, 1998). Investors observe others' payoff and adopt others' strategy if such strategy produces a payoff higher than their old strategy does. The motivation of such behavior is to pursue a higher payoff. Meanwhile, the implicit belief behind such behavior is that the past performance of strategies will continue at least in the short run. Since no strategy dominates other strategies, otherwise all investors only need to use this dominant strategy and doing this would eventually invalid this strategy, investors naturally have the incentive to change their

strategies. But finding the right moment of changing strategies is no less difficult than forecasting the beginning of a financial crisis. Even with the same information of the historical performance of strategies, different investors may have different frequency of adjusting their strategies. Does a faster adjustment of strategies lead to a higher final wealth? I devote this chapter to these questions.

In this chapter, artificial investors are put into a real stock market to see how their strategy change behavior affects their wealth accumulation. I take Standard & Poor's 500 index (hereafter S&P 500) as the risky asset. S&P 500 is an American stock market index based on the market capitalizations of 500 large companies having common stock listed on the NYSE or NASDAQ. It is one of the best representations of the US stock market. Investors trade the index while adopting one of many strategies. Some popular trading strategies in practice are formulized for investors to adopt. When the performance of other strategies is better than an investor's current strategy, this investor has an incentive to change his strategy. However, an investor does not abandon his present strategy whenever it has been surpassed. He changes his strategy only when the performance of another strategy is higher than the performance of his old strategy by an amount over a threshold. Different investors have different thresholds. A higher threshold means that the investor changes his strategy less often. I calculate investors' final wealth under various market situations and find that in general investors with higher threshold end up with higher final wealth.

2.2 Strategies and strategy change

The four strategies are fundamental analysis, one-period trend following, 10/20 moving average and 15/30 moving average. Agents only adopt one strategy at a time. These strategies, fundamental or technical, represent typical strategies used in practice. As the daily close prices of S&P 500 is the only information used here, practical strategies which need information beyond prices are not tested here, like price-volume analysis. Strategies involving subjective judgments, for example, chart patterns, are not covered too.

2.2.1 Fundamental analysis

In practice, the fundamental value of an individual stock is usually derived from its price-to-earnings ratio or price-to-dividends ratio. Since the S&P 500 index, rather than a single stock, is taken as the risky asset here, the search for the true fundamental becomes more complicated. To simplify the procedure, the annual moving average of prices is taken as the fundamental value F_t

$$F_t = \frac{P_{t-119} + P_{t-118} + \cdots + P_{t+120}}{240}. \quad (2.1)$$

I assume a year consists of 240 working days. Similarly, 5 days are treated as a week, 20 days as a month and 60 days as a quarter. The fundamental value is calculated according to Eq. (2.1). The result of calculation is fed to investors with fundamental analysis. As for those investors, they receive the exogenous information of fundamental value without knowing how it is derived. Eq. (2.1) is beyond their knowledge, and it is impossible for them to infer accurate future prices from the time series of fundamental. The fact that future prices are used to calculate the current fundamental makes the fundamental analysis a forward-looking strategy, though without investors' awareness.

Investors adopting fundamental analysis believe that the price will converge to the fundamental eventually. So they buy shares when the stock is undervalued compared with its fundamental and sell when it is overvalued. Investors' market orders at time t D_t^f are formulized as

$$D_t^f = F_t - P_t R, \quad (2.2)$$

where P_t is the current price, and R is the total risk-free return of one unit capital.

2.2.2 Technical analysis

Investors with technical analysis derive their anticipation and trading from past information. They read charts, count waves, look for patterns, *etc.* They believe that what has happened before will reoccur. Here I focus on one-period trend following and moving averages.

One-period trend following

Investors adopting one-period trend following strategy believe that the latest price change will continue in short future. They submit buy(sell) orders when the latest price change is positive (negative). The market buy order submitted by a trend follower at period t is

$$D_t^{tf} = P_t - P_{t-1}. \quad (2.3)$$

Moving average

Moving average is widely used in technical analysis to filter out random price fluctuations. It usually takes two moving averages to form a concrete strategy: one with a shorter time frame and the other with a relatively longer time frame. The shorter moving average crossing above the longer moving average, known as a “golden cross”, indicates the shifting up of a trend. It sends a buy signal. The shorter moving average crossing below the longer moving average, known as a “dead/death cross”, sends a sell signal. When the shorter moving average is above or below the longer moving average, it still sends a buy or sell signal, though may not be a chance as good as these crossovers. Essentially, the moving average strategy is following a short-term trend against a relatively long-term trend. Two popular combinations, 10/20 moving averages and 15/30 moving averages, are tested. Investors who adopt these two strategies submit market orders $D_t^{10/20}$ and $D_t^{15/30}$ respectively in period t

$$D_t^{10/20} = \frac{P_{t-9} + P_{t-8} + \cdots + P_t}{10} - \frac{P_{t-19} + P_{t-18} + \cdots + P_t}{20}, \quad (2.4)$$

$$D_t^{15/30} = \frac{P_{t-14} + P_{t-13} + \cdots + P_t}{15} - \frac{P_{t-29} + P_{t-28} + \cdots + P_t}{30}. \quad (2.5)$$

2.2.3 Fitness of strategies

An agent can only pick one strategy at a time. When other strategies work better than his current strategy, naturally this agent has an incentive to change his strategy. Agents' strategy change behavior is driven by comparing strategies' fitness, that is, the past performance of strategies.

Different strategies share one feature in common, that is, to buy shares when expecting higher future price and to sell when expecting lower future price. The difference among strategies is more about the different price expectations. Eq. (2.6) shows a linear relation between the current expectation on future price $E_t^h(P_{t+1})$ and the market order D_t^h determined by strategy h . Eq. (2.7) gives the daily profit π_t^h earned by strategy h .

$$D_t^h = E_t^h(P_{t+1}) - P_t R \quad (2.6)$$

$$\begin{aligned} \pi_t^h &= D_t^h \cdot (P_{t+1} - P_t R) \\ &= (E_t^h(P_{t+1}) - P_t R) \cdot (P_{t+1} - P_t R) \end{aligned} \quad (2.7)$$

The profit earned by the market order determined by a strategy is widely used as the quantitative measure of a strategy's fitness. It is reasonable when the price fluctuates in a moderate range. But when the price changes a lot, proper adjustment is required. For example, a profit of 200 means differently when the price is 20.16 on 11/22/1950 and when the price is 2074.78 on 11/28/2014. As the daily close price of S&P 500 increased over 100 times after more than 60 years, the long-run increase of S&P 500 is taken into consideration. Therefore, the fitness of strategy h is modified as follows

$$\begin{aligned} \pi_t^h &= E_t^h(\rho_t) \cdot \rho_t \\ &= \frac{E_t^h(P_{t+1}) - P_t R}{P_t} \cdot \frac{P_{t+1} - P_t R}{P_t}, \end{aligned} \quad (2.8)$$

where ρ_t is the excess return rate of stock in period t . The modified fitness is determined together by the real return and the expected return of a strategy. Assume $R = 1$, because the risk-free interest rate is trivial on a daily basis.

At a daily trading interval, the modified fitness in Eq. (2.8) is small. But it does not make agents' strategy switching sensitive to parameter choices. Because, agents' strategy switching is driven by the comparison of the fitness of different strategies, not by the absolute fitness of strategies. From the old fitness, *i.e.* the realized profit, to the new modified fitness, the fitness of every strategy is divided by the same value of P_t^2 .

The relative relation of different strategies' fitness does not change. Therefore, under either the realized profit or the modified fitness, agents switch their strategies in the exact same way.

Fig. 2-1 shows the linear and logarithmic daily close prices of S&P 500. Apparently, an exponential growth fits the S&P 500 data better than a linear growth does. Therefore, in the contrast of the daily profit, the modified fitness is more suitable to evaluate the performance of strategies.

Fig. 2-2 shows the daily profit (Eq. (2.7)) and the modified fitness (Eq. (2.8)) of the fundamental strategy. Because of the long-run increase of price, the daily profits in the last twenty years overwhelm profits earned before 1990s. The figure of the modified fitness shows more details of the performance of strategies. But still, the fitness of fundamental strategy becomes more volatile as time passes.

As mentioned before, an investor does not abandon his present strategy whenever it has been surpassed by other strategies. He changes his strategy only when the fitness of another strategy is higher than the fitness of his old strategy by an amount higher than a threshold. When multiple strategies meet this criterion, this investor replaces his old strategy by the strategy with the highest fitness.

2.3 Simulations with S&P 500 historical prices

In the artificial financial market, agents can invest either in the risky S&P 500 index or in a risk-free asset. The exogenous S&P 500 index indicates that all agents discussed here are trivial in the stock market. Their aggregate demand for shares has no impact on the index. After agents submit market orders according to their current strategies, their market orders are satisfied under the current price, by investors uncovered in this model.

As the daily interest rate of risk-free asset is negligible in practice, the interest rate of risk-free asset is assumed as zero, *i.e.* $R = 1$. Agents' investment of risk-free asset can be interpreted as holding cash. Agents' wealth is updated

$$W_{i,t+1} = S_{i,t+1} + P_{t+1}N_{i,t+1}, \quad (2.9)$$

(a) Linear scale

(b) Logarithmic scale

Figure 2-1: Daily close prices of SP500 from 1/3/1950 to 5/21/2015, 16452 days in total.

Figure 2-2: Two fitnesses of the fundamental strategy through the history of SP500. Top panel: Daily profit. Bottom panel: Modified fitness based on returns.

$$S_{i,t+1} = (S_{i,t} - D_{i,t}P_t)R, \quad (2.10)$$

$$N_{i,t+1} = N_{i,t} + D_{i,t}. \quad (2.11)$$

At the beginning of period t , agent i holds $S_{i,t}$ risk-free asset and $N_{i,t}$ shares of risky asset, and his total wealth is $W_{i,t}$. After he places a market order $D_{i,t}$ and the order is later executed by the market maker under the current price P_t , the amount of his risk-free asset, risky asset and total wealth are updated to $S_{i,t+1}$, $N_{i,t+1}$ and $W_{i,t+1}$ respectively.

All agents start with the same zero initial condition, *i.e.* $\forall i$, $S_{i,0} = 0$ and $N_{i,0} = 0$. No budget constraint or short-sale constraint is imposed on agents. So, investors can borrow risk-free asset to buy index, or sell short index to gain risk-free asset.

2.3.1 The whole history: 1950~2014

The daily close prices of S&P 500 from 1/3/1950 to 5/21/2015 are shown in Fig. 2-1.

Fig. 2-3 compares the four strategies mentioned before. In general, the fundamental strategy accumulates more final wealth than other strategies do. But this does not mean fundamental analysis is always the best. During the formation of bubbles, two moving average strategies accumulate wealth faster than the fundamental strategy does. During crises, fundamental strategy is the only one that can make a large increase of wealth, when all technical strategies suffer a huge drop of wealth.

The reason behind is straightforward. When bubbles emerge, short-term trends are strong, and the index price continuously deviates from its fundamental. So, trend following strategies benefit more from the strong short-term price trend than the fundamental strategy does. During crises, price trends reverse and trend following strategies fail. As the price drops, stock price converges to its fundamental. As a result, fundamentalists who are holding right expectations benefit from selling short the index.

Now, put 100 artificial agents into the simulation. The 100 agents' strategy-switching thresholds are evenly distributed from 0 to a maximum threshold. The maximum strategy-switching threshold is properly chosen according to the fitness of strategies, as shown in the bottom panel of Fig. 2-2. A maximum threshold that is too big

Figure 2-3: Wealth accumulations by four strategies. Top panel: Linear scale of wealth. Bottom panel: Logarithmic scale of wealth.

makes some agents never be able to change strategies. A maximum threshold that is too small only considers agents with fast strategy switching and leaves out agents with slow strategy switching. Here, the maximum threshold is 0.025. The actual minimum strategy change threshold is 0.00025, instead of 0, which makes no sense. Every agent's initial strategy is randomly picked from these four strategies. Agents have zero initial endowments. No budget constraint or short-sale constraint is imposed on agents. After ranking agents according to their strategy-switching thresholds, the frequency of their strategy-switching and their final wealth are shown in Fig. 2-4. Agents with higher thresholds change their strategies less often. In the bottom panel of Fig. 2-4, there seems an upward trend, blurred by fluctuations. These fluctuations are caused by agents' random initial strategies. This randomness affects agents with higher thresholds more, as such agents change their strategies less frequently.

I run the simulation 1000 times and average the results to cancel out the randomness of initial strategies. Fig. 2-5 shows the results. Again, agents with higher threshold change their strategies less frequently. Compare with Fig. 2-4(b), Fig. 2-5(b) is less volatile. As agents' threshold increases, agents' final wealth drops at first and then increases. At last, agents' final wealth stays at the level around the maximum.

The lowest point in Fig. 2-5(b) represents the agent with threshold 0.00125 who on average has switched strategies 372.7 times throughout the whole history of S&P 500. For agents with thresholds lower than 0.00125, a higher threshold relates to a lower final wealth. For agents with thresholds higher than 0.00125, a higher threshold leads to a higher final wealth. It seems that these two parts are contradictory. This may be caused by the long history of data, which is over 60 years. Besides, if the cost of strategy switching is taken into consideration, a frequent strategy switching may cost a lot.

2.3.2 Bulls, bears and others

This subsection focuses on special time periods of S&P 500 to investigate how agents' strategy switching behavior affects their wealth accumulation under bulls and bears. The maximum strategy-switching threshold in each segment is adjusted accordingly.

Figure 2-4: Agents' frequency of strategy switching (top) and their final wealth (bottom). Note that agents are ranked by their strategy-switching thresholds.

Figure 2-5: Agents' average frequency of strategy switching (top) and their average final wealth of 1000 simulations (bottom). Note that agents are ranked by their strategy-switching thresholds.

Fig. 2-6 shows two bull markets: one from 12/8/1994 to 3/24/2000 (left panels) and the other from 3/9/2009 to 11/24/2014 (right panels). The former covers the dot-com bubble. The latter shows the current bull market starting from the recovery of 2007/08 financial crisis. The top six panels show the daily close prices and fundamental values, wealth accumulation of four strategies, and the fitness of four strategies. In the second row, four stubborn agents, each sticks to one strategy, start with zero initial endowments. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy-switching thresholds from 0.00005 to 0.005. In the fourth row, agents with higher thresholds change their strategies less frequently. In the last row, as agents' strategy-switching thresholds increase, their final wealth drops at first and then increases. This result is consistent with the finding in the last subsection. It seems that agents' final wealth drops at first and then increases is a feature of bull markets, as the whole history of S&P 500 can be taken as a long-lasting bull market.

Fig. 2-7 demonstrates two bear markets: one from 11/9/2000 to 11/11/2002 (left panels) and the other from 10/30/2007 to 3/23/2009 (right panels). The first one presents the burst of the dot-com bubble. The last one shows the 2007/08 financial crisis. The top six panels show the daily close prices and fundamental values, wealth accumulation of four strategies, and the fitness of four strategies during these two bear markets. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy change thresholds from 0.00005 to 0.005. In the fourth row, again, agents with higher thresholds switch less frequently. In the last row, as agents' strategy-switching thresholds increase, their average final wealth increases. In the bottom right panel, agents' final wealth drops when their thresholds are higher than 0.00385. This is because when agents seldom change strategies, their initial strategies have large impact on their wealth. Agents with better initial strategies end up with being one of the richest. Agents with relatively poorer initial strategies have much less final wealth. As a result of averaging agents with better and poorer initial strategies, the final wealth of agents with thresholds higher than 0.00385 drops with their thresholds.

Fig. 2-8 shows the analysis of markets with major trend reversals: one from 9/10/1997 to 8/13/2009 (left panels) and the other from 8/30/2000 to 1/14/2013 (right panels). The former presents two rounds of boom and bust, covering the dot-com bubble, its burst, the US housing bubble, and the 07/08 financial crisis. The latter shows the burst of the dot-com bubble, its recovery, US housing bubble, 07/08 financial crisis, and the recovery to the peak of the housing bubble. The top six panels show the daily close prices and fundamental values, wealth accumulated by four strategies, and the fitness of four strategies. The bottom four panels show the average results of 1000 simulations. 100 artificial agents are ranked by their strategy-switching thresholds from 0.0001 to 0.01. In the fourth row, agents with higher thresholds switch less frequently. In the last row, agents' average final wealth increases with their strategy-switching thresholds. In the bottom left panel, agents become less wealthy when their thresholds are higher than 0.0094 out of the same reason discussed in the last paragraph.

It is natural that agents with higher strategy-switching thresholds change their strategies less often. From the study of the whole history and typical segments of S&P 500, obviously, no universal relation of agents' final wealth and their strategy-switching frequency exists. In bull markets, fundamental analysis and technical analysis are comparable. The fundamental analysis seems better than technical analysis when sharp price drops happen. This study can provide suggestions to investors under specific price trends. In bull markets, both fast and slow strategy switching work better than a moderate strategy switching. Investors either change their strategies frequently to beat the market, or just stick to one strategy, for example, the buy-and-hold strategy. In bear markets and markets with major trend reversals, active strategy switching hurts investors' wealth. When large price drops happen, fundamental analysis performs much better than technical strategies do. So there is no need for an active strategy switching.

2.4 Path dependent wealth accumulation

Investors' wealth accumulation is path dependent on their past transactions.

Assume investor i 's portfolio at the beginning of period t is $(S_{i,t}, N_{i,t})$, which means

Figure 2-6: Analysis of bull markets. Left panels: From 12/8/1994 to 3/24/2000. Right panels: From 3/9/2009 to 11/24/2014.

Figure 2-7: Analysis of bear markets. Left panels: From 11/9/2000 to 11/11/2002. Right panels: From 10/30/2007 to 3/23/2009.

Figure 2-8: Analysis of markets with major trend reversals. Left panels: From 9/10/1997 to 8/13/2009. Right panels: From 8/30/2000 to 1/14/2013.

he holds $S_{i,t}$ amount of risk-free asset and $N_{i,t}$ shares of risky asset. Under the present price¹ of risky asset P_t , this investor's wealth is $W_{i,t}$. Every period, this investor places a market order $D_{i,t}$, which is executed under the present price P_t . Then the rest of his risk-free asset grow at the rate R . At the beginning of next period, the value of his portfolio is $W_{i,t+1}$.

$$\begin{aligned}\Delta W_{i,t+1} &= W_{i,t+1} - W_{i,t} \\ &= S_{i,t}r + (P_{t+1} - P_t)N_{i,t} + (P_{t+1} - P_tR)D_{i,t}\end{aligned}\quad (2.12)$$

After iterating Eq.s (2.9)-(2.11) twice, agent i 's wealth at period $t+2$ and the change of wealth ($W_{i,t+2} - W_{i,t}$) can be derived

$$W_{i,t+2} = S_{i,t}R^2 + N_{i,t}P_{t+2} + D_{i,t} \cdot (P_{t+2} - R^2P_t) + D_{i,t+1} \cdot (P_{t+2} - RP_{t+1}), \quad (2.13)$$

$$W_{i,t+2} - W_{i,t} = S_{i,t}(R^2 - 1) + N_{i,t}(P_{t+2} - P_t) + D_{i,t} \cdot (P_{t+2} - R^2P_t) + D_{i,t+1} \cdot (P_{t+2} - RP_{t+1}). \quad (2.14)$$

After iteration of Eq.s (2.9)-(2.11) m times, the value of investor i 's portfolio after m period is

$$\begin{aligned}W_{i,t+m} &= S_{i,t}R^m + N_{i,t}P_{t+m} + D_{i,t} \cdot (P_{t+m} - R^mP_t) \\ &\quad + D_{i,t+1} \cdot (P_{t+m} - R^{m-1}P_{t+1}) + \dots + D_{i,t+m-1} \cdot (P_{t+m} - RP_{t+m-1}).\end{aligned}\quad (2.15)$$

The first two parts on the right hand side of Eq. (2.15) are determined by the investor's portfolio at time t ($S_{i,t}, N_{i,t}$), the risk-free return R and the latest price of risky asset P_{t+m} . All rest parts are determined by investor's historical transactions ($D_{i,t}, D_{i,t+1}, \dots, D_{i,t+m-1}$). They show the impact of investor's past transactions on his future wealth. However, such impact is out of the investor's control, because the weight

¹Note that the formation of price is not mentioned here. This means the discussion does not depend on the generation process of price. This discussion can be apply to empirical data, theoretical analysis or numerical simulation.

of every past transaction contains future information P_{t+m} which is unavailable to him when he made his decision $D_{i,t+\tau}$ ($0 \leq \tau \leq m-1$).

Rewrite Eq. (2.15) as follows

$$W_{i,t+m} = F(S_{i,t}, N_{i,t}, R, P_t, P_{t+1}, \dots, P_{t+m}, D_{i,t}, D_{i,t+1}, \dots, D_{i,t+m-1}).$$

The value of investor i 's portfolio at time $t+m$ is a function of his initial portfolio $(S_{i,t}, N_{i,t})$, the risk-free return R , the price series of risk asset $(P_t, P_{t+1}, \dots, P_{t+m})$ and his past transactions $(D_{i,t}, D_{i,t+1}, \dots, D_{i,t+m-1})$. A close look at the right hand side of Eq. (2.15) gives more details. If investor i does not execute any transactions, all the increase of his wealth comes from the interest of his risk-free asset. If investor i places a market order at any time $t+x$, the transaction $D_{i,t+x}$ changes the value of his portfolio at time $t+y$ by $D_{i,t+x} \cdot (P_{t+y} - R^{y-x}P_{t+x})$. The impact of a past transaction on the investor's current wealth is weighted by $(P_{t+y} - R^{y-x}P_{t+x})$, *i.e.* current benefit minus opportunity cost. Since the opportunity cost of every transaction is different, the weight of every transaction in the calculation of current wealth is different. That is why I argue that investors' wealth is path dependent on his past transactions. For example, two investors with identical initial portfolio $(S_{i,t}, N_{i,t}) = (S_{j,t}, N_{j,t})$ must act exactly the same in every period to get identical value of portfolio at any time. Another example: two investors with identical initial portfolio $(S_{i,t}, N_{i,t}) = (S_{j,t}, N_{j,t})$ may get identical value of portfolio at a time $W_{i,t+m} = W_{j,t+m}$; but if they do not share the same transaction history, the values of their portfolio will diverge later.

Previously, the investor adjust his portfolio first and then get the interest from his holdings of risk-free asset. If the order of actions is changed, *i.e.* get risk-free interest first and then buy or sell risky asset, the argument on path dependence still stands.

$$\begin{aligned} W_{i,t+m} = & S_{i,t}R^m + N_{i,t}P_{t+m} + D_{i,t} \cdot (P_{t+m} - R^{m-1}P_t) \\ & + D_{i,t+1} \cdot (P_{t+m} - R^{m-2}P_{t+1}) + \dots + D_{i,t+m-1} \cdot (P_{t+m} - R^0P_{t+m-1}) \end{aligned} \quad (2.16)$$

2.5 Conclusion

Whether a fast strategy switching helps to accumulate more wealth? Out of the curiosity, I put artificial investors into a real stock market to see how their strategy-switching frequency affects their wealth accumulation. Fundamental analysis, 15/30 moving average, 10/20 moving average and one-period trend following form a strategy pool from which agents pick their strategies. Agents either trade the S&P 500 index or hold risk-free cash. Agents change their current strategies only when other strategies surpass their old ones by an amount over their strategy-switching thresholds. Agents with higher thresholds change strategy less often. As for the relation of their final wealth and their threshold, no simple relation exists. In general, there is a threshold range within which agents with higher threshold gain higher final wealth. That is, agents who switch strategy less often accumulate more wealth. The threshold range starts from the minimum threshold in bear markets and markets with reversal trends. In bull markets, a piece of negative relation of threshold and wealth happens before the range where agents' threshold and final wealth show positive relation. Despite market trends, when agents' thresholds are so high that they barely change their strategies, agents stick to their initial strategies. After averaging the final wealth of agents with better or worse initial strategies, the average final wealth drops when agents' threshold further increases.

In this chapter, artificial investors are assumed trivial in the sense that they have no influence on the stock price. Their trading does not affect the formation of future price. Some may wonder when investors are nontrivial, that is, when investors' aggregate demand drives the future price, whether the negative relation between their strategy-switching frequency and their final wealth remains. In the next chapter, a heterogeneous agent model is built to show that the answer is positive.

Chapter 3

Strategy Change and Wealth Accumulation: A Heterogeneous Agent Model

3.1 Introduction

This is a world full of changes and choices. Life today moves faster than ever before. To seize opportunities, people often need to make quick decisions. With limited ability and information, people tend to imitate the behavior of individuals who are more intelligent, more influential or who are believed to possess more information. In stock markets, the imitation of strategies among investors is usually referred to as adaptive learning (Brock and Hommes, 1997, 1998) or herding (Lux, 1995). Investors observe different strategies' performance and adopt a new strategy if such strategy produces a profit higher than their old strategies do. Since no strategy dominates other strategies, otherwise all investors only need to use the dominant strategy and doing this would eventually invalid this strategy, investors naturally have the incentive to change their strategies from time to time.

However, at what pace should investors change their strategies and how this will affect their wealth accumulation? This chapter does not investigate the timing of strat-

egy change in practice, because that requires specific information and the perfect timing is likely to vary case by case in a real stock market. Instead, it studies how investors' strategy change propensity affects their wealth accumulation in an artificial stock market when all investors share the same set of information. The method of heterogeneous agent model is used to facilitate my study.

Over the past two decades, heterogeneous agent models (HAMs) have been widely used to study unsolved problems in financial markets, such as stylized facts, nonlinear price dynamics, investors' bounded rationality, financial crises and so on¹. The most intrinsic feature that distinguishes HAMs from other models is the heterogeneity among agents. It endows HAMs the capability to reproduce various phenomena beyond those that are explained by the conventional economic theorems. In most HAMs on stock markets, agents' heterogeneities are embedded in their strategies. Fundamental analysis and technical analysis are two main types of strategies widely used in practice. HAMs reflect this feature accordingly. Both fundamental and technical strategies are formulated. The whole population is divided into two groups: fundamentalists and chartists. When agents change their strategies, for example, switching from fundamentalists to chartists or the other way around, they have the ability of adaptive learning.

The intensity of choice parameter is a crucial parameter in HAMs with adaptive learning. It measures the speed or the extent of agents' strategy switching. It has been intensively studied, especially on its effect on the nonlinear price dynamics and its estimation. Brock and Hommes (1997) identify the intensity of choice as the critical parameter for the production of complex dynamics ranging from steady states to high order cycles and even to chaos. Brock and Hommes (1998) make a comprehensive investigation of bifurcations related to the intensity of choice. Chiarella et al. (2006) get similar patterns when they examine agents' strategy switching between fundamental strategy and moving average strategy. They exhibit the evolvement of the shape of strange attractors in a phase map of price and market fraction when the intensity of choice increases. After this parameter's influence on nonlinear price dynamics has been

¹See Chiarella and He (2005), Hommes (2006), LeBaron (2006), Chiarella et al. (2009), Hommes and Wagener (2009) and Lux (2009) for a comprehensive review.

explored extensively, researchers shift their attention to its estimation. Boswijk et al. (2007) estimate the intensity of choice by the method of nonlinear least square on yearly S&P500 data from 1871 to 2003, but the estimated value is not significantly different from zero. De Jong et al. (2009) estimates investors' strategy switching between fundamentalists, chartists and internationalists on the interlinked stock markets of Hong Kong and Thailand during a period surrounding the Asian crisis, 1980-2007. The value of intensity of choice estimated is 1.031 in Hong Kong and 2.869 in Thailand, both significant at the 5% level. Amilon (2008) uses daily data of S&P500 from 1980 to 2000 to estimate the intensity of choice and other parameters under different model structures. The maximum likelihood estimation gives a value of 1.91, statistically significant. However, the estimate by the efficient method of moments in a model with fewer free parameters and a more complex disturbance structure is no longer significant. Chen et al. (2012) provide a comprehensive literature review on the estimation of parameters in HAMs with various market structures. Recently, Anufriev et al. (2013, 2015) run laboratory experiments to study how people switch between mutual funds. They find that the estimated intensity of choice increases with correlation between past and future returns.

Besides the intensity of choice, wealth dynamics is another issue which has attracted scholars' attention. Most study on wealth dynamics focuses on the wealth proportion among fundamentalists and chartists. This is because proxy fundamentalist and chartist are used to represent groups of fundamentalists and chartists to make the model analytically tractable. Hommes and Wagener (2009) track wealth accumulated by wealth accumulated by fundamental strategy and technical strategy. However, some subtle problems hide underneath the study of wealth when strategy switching is allowed, for example, how to justify the idea of identifying agents by their strategies when their strategies can change, and how to reallocate wealth when agents change their strategies and, therefore, their identity. Chiarella and He (2005) and Brianzoni et al. (2010, 2012) assume that all agents belonging to the same group agree to share their wealth and when an agent switches from one group to another he brings his share of wealth to the second group. This assumption shows one way to clear up the problems mentioned

earlier.

A new way to resolve agents' ambiguous identity is to adopt new criteria rather than changeable strategies to identify agents. Anufriev and Dindo (2010) endow agents with diverse degrees of risk aversion. They find that when the general growth rate of the risky asset is larger than the risk-free rate, those agents with the lowest risk aversion seize most wealth and dominate the market. This model focuses on agents' propensity of strategy switching, which is embodied by the intensity of choice². Agents' propensity of strategy switching is taken as a behavioral parameter, as suggested in Weisbuch et al. (2000). Then, it is further assumed that agents have heterogeneous propensity of strategy switching. Agents are grouped according to their propensity of strategy switching. Though the so-called fundamentalists and chartists still refer to agents adopting fundamental strategy and technical strategy respectively, they become agents' temporary roles rather than agents' identification criterion. There are both fundamentalists and chartists within every group. When an agent changes his strategy his temporary role as a fundamentalist or a chartist changes, but he still stays in the same group. No agent jumps between groups while strategy switching, and a group's wealth is closely attached to this group. In this case, a group can represent an institutional investor, a household, or even an individual investor who integrates fundamental and technical analyses in his financial investment.

To the best of my knowledge, no study has ever introduced heterogeneity into agents' propensity of strategy switching. Except for the innovation of heterogeneous intensity of choice, all building blocks of this model are borrowed from classical HAMs. The model is an integration of agents with heuristic rule and adaptive learning and the market-maker price mechanism. Following Day and Huang (1990), agents take heuristic rule to form their demand for the stock, and the stock price is adjusted by a market maker according to the excess demand. As agents' propensity of strategy switching is the main focus of this study, naturally, agents show adaptive learning. Within every agent group, agents' strategy change is described by the discrete choice function as in Brock and

²The intensity of choice and the propensity of strategy switching are identical. The first one is a term researchers are more familiar with, and the second one shows its economic intuition better. We choose either one depending on the content.

Hommes (1997, 1998).

In the majority of HAM literature, a basic model structure consists of agents' strategy, market-clearing mechanism, and adaptive learning. Each part has some generally acknowledged designs³. Agents are usually assumed to maximize myopic mean-variance (Brock and Hommes, 1997, 1998) or expected utility of wealth (Chiarella and He, 2001). The market clearing is either done by a Walrasian auctioneer or by a market maker. Chiarella et al. (2009) review the role played by these two market-clearing mechanisms in HAMs. As for the adaptive learning, it gradually becomes a standard feature of HAMs on financial markets after its first appearance in Brock and Hommes (1997, 1998). The popularity of the adaptive learning is caused by two reasons. On one hand, it is a principal source of nonlinearity. Models with adaptive learning can reproduce lots of nonlinear price dynamics. On the other hand, the existence of investors' adaptive learning in practice has been verified by Boswijk et al. (2007).

Different from the majority of HAMs, agents in this model do not maximize myopic mean-variance or expected utility of wealth. I intentionally avoid those designs, because problems exist when either maximization combines with the market-maker price adjustment. For example, the stock price fails to converge to the fundamental value when only fundamentalists exist in the market. Franke (2008) discusses problems caused by the confusion between excess demand and desired holding of risky asset. He suggests that when the price adjustment is done under a market-maker framework, the story of the maximization of myopic mean-variance or expected utility should be dropped. That is exactly what I do in this model. Franke (2008) negates the model structure with a market maker and myopic mean-variance maximization from different aspects. Similar problems exist in models with a market maker and agents who maximize expected utility of wealth. To give this model a better structure, the combination of agents with heuristic rule and the market-maker price adjustment is adopted. Besides, to some extent, the heuristic rule that agents' excess demand for the risky asset is proportional to their expected excess return on risky asset is intuitive.

³Only papers in which these designs first appeared are mentioned. Loads of publications following such designs are ignored.

Our study focuses on how investors' strategy change propensity affects their wealth accumulation. Do agents who quickly react to strategies' latest performance become wealthier than agents who are reluctant to the same information? By mathematical and numerical analyses, my study gives a negative answer. All simulations show a robust result, that is, the higher propensity of strategy switching a group of agents have the less wealthy they are in the end. After decomposing the wealth change in a period, it is found that this result is caused by the inconsistency between short-run profit and long-run wealth accumulation. When agents intensively adjust their strategies to chase after short-run profit, they fail to accumulate a large position in the risky asset, the market value of which wealth accumulation relies heavily on. Of course, all findings in this paper are subject to the model structure and assumptions embedded.

The rest of this chapter is organized as follows. In Section 3.2, an artificial financial market with heterogeneous agents is built. Section 3.3 analyzes strategies' profitability under a single intensity of choice. Section 3.4 presents the main findings, robustness check and further discussion. Section 3.5 concludes this paper.

3.2 A discrete dynamic HAM model

3.2.1 A baseline model

Assume that a financial market consists of heterogeneous investors who invest in both risk-free asset and risky asset. Assume the return rate of the risk-free asset is r , then the gross return is $R = 1 + r$. Let P_t denote the price per share of the risky asset at time t , which includes all kinds of return from investing in this asset⁴.

There are two kinds of strategies: fundamental strategy and technical strategy. Investors adopting fundamental strategy are named as fundamentalists⁵ and denoted by superscript f , while investors adopting technical strategy are named as chartists and denoted by superscript c . At period t , type h investors' expectations about the price of

⁴Our model shows no explicit dividend. Stock price P_t is a cum-dividend price.

⁵In our model, fundamentalists or chartists are only investors' temporary roles. After we introduce investors' propensity for strategy switching as another heterogeneity, we identify investors by their propensities rather than their strategies.

risky asset in the next period $t + 1$ are $E_t^h(P_{t+1})$, where $h = f$ or c .

Fundamentalists believe that there is a fundamental value underneath the fluctuating price series, to which the current price will eventually converge. P_0^f is the initial fundamental value. Later the fundamental value grows at the same rate of the risk-free asset.

$$E_t^f(P_{t+1}) = P_0^f R^t \quad (3.1)$$

Chartists form their expectations of future price by observing historical prices. Lots of technical strategies are developed in practice. Some are naive, and some are sophisticated. Assume that chartists are *trend followers* in the sense that they believe that the current price trend will continue in the short future. Their expectation of future price follows one period trend extrapolation, as shown in Eq. (3.2), in which the parameter a displays the extent of chartists' trend following

$$E_t^c(P_{t+1}) = P_t + a \cdot (P_t - P_{t-1}). \quad (3.2)$$

Investors in the model do not maximize myopic mean-variance (Brock and Hommes, 1997, 1998) or expected utility of wealth (Chiarella and He, 2001). Instead, investors derive their demands for risky asset z_t^h from their expectations on future price $E_t^h(P_{t+1})$. They buy shares when they anticipate an excess profit from investing risky asset and sell when they anticipate a loss. Their excess demand for the risky asset is of the same amount as the expected excess return on the risky asset

$$z_t^h = E_t^h(P_{t+1}) - R \cdot P_t. \quad (3.3)$$

To keep the model neat, I only consider the first moment of investors' expectations and leave second and higher moments untouched. Similar to the majority of HAMS, the heterogeneity in strategies comes from different forecasting rules. Given any price expectation, investors derive their demands following the same Eq. (3.3).

Following Day and Huang (1990), the price of the risky asset is decided by a market maker according to the aggregate demand for the risky asset D_t . The aggregate demand

is the sum of fundamentalists' and chartists' demand weighted by their market fraction n_t^h

$$D_t = n_t^f z_t^f + n_t^c z_t^c. \quad (3.4)$$

A deterministic price dynamics and a stochastic price dynamics are respectively defined as

$$P_{t+1} = P_t + \gamma D_t, \quad (3.5)$$

$$P_{t+1} = P_t + \gamma D_t + \varepsilon_t, \quad (3.6)$$

where γ is the market maker's price-adjustment speed, and ε_t is a white noise that represents the perturbation caused by noise traders or the arrival of trivial new information.

After the asset price has been updated, the realized profit of each strategy can be calculated

$$\pi_t^h = (P_{t+1} - P_t R) \cdot z_t^h = (P_{t+1} - P_t R) \left[E_t^h(P_{t+1}) - P_t R \right]. \quad (3.7)$$

The realized profit assesses the performance of strategies. The comparison of the realized profits of strategies drives investors' strategy switching. Statistically, the fact that a strategy produced a higher profit does not guarantee this strategy will continue doing so in future. However, from investors' viewpoint, they try to infer something from the information available to them. They may think that the recent performance of strategies reveals the latest market condition and the strategy which performed better is more likely to maintain its advantage in the short future. This kind of thought gives investors the incentive to adjust their strategies.

When investors adjust their strategies, the market fraction of fundamentalists and chartists is updated according to the discrete choice model with multinomial logit prob-

abilities in Brock and Hommes (1998):

$$n_{t+1}^h = \frac{e^{\beta\pi_t^h}}{\sum_{k=f,c} e^{\beta\pi_t^k}}, \quad (3.8)$$

where β is the intensity of choice measuring how fast agents switch between different strategies. In most HAMS, agents share the same intensity of choice. But here, β is treated as a behavior parameter (Weisbuch et al., 2000) that its value varies with agents. It is formally referred to as the *propensity of strategy switching*.

In reality, there are flexible people and stubborn people. Even under the same discrepancy between the realized profits of strategies, investors behave differently. Flexible investors abandon current strategy as long as the other strategy works better while stubborn investors do not easily change their strategy unless a significant disadvantage unfolds.

Assume there are N levels of β so that whole the population is divided into N groups, which will be distinguished by the subscript i , $i = 1, 2, \dots, N$.

Eq. (3.4) and Eq. (3.8) are then modified respectively

$$D_t = \sum_{i=1}^N z_{i,t}, \quad (3.9)$$

$$z_{i,t} = n_{i,t}^f z_t^f + n_{i,t}^c z_t^c, \quad (3.10)$$

$$n_{i,t+1}^h = \frac{e^{\beta_i \pi_t^h}}{\sum_{k=f,c} e^{\beta_i \pi_t^k}}. \quad (3.11)$$

Now the aggregate demand for the risky asset is the sum of N groups' demand. Inside any group i , there are fundamentalists and chartists. A group's demand for the risky asset is a weighted sum of fundamentalists' and chartists' demand. The fraction of fundamentalists in a group follows the discrete choice function (3.11). Every group observes the same information on the performance of strategies, but the heterogeneous β values lead dynamically to different fractions of fundamentalist and chartists in different groups. A group with a higher (lower) β react faster (slower) to the past strategies performance, and will be referred to as a fast (slow) group accordingly.

3.2.2 Wealth accumulation

The wealth of a group of investors is the market value of the portfolio held by the group. Numerical simulation makes it possible to track a group's portfolio, trading, and wealth. At the beginning of period t , the value of group i 's investment in the risk-free asset is $S_{i,t}$. Group i holds $Z_{i,t}$ shares of risky asset in total⁶. Group i 's total wealth is $W_{i,t}$. In period t , investors trade stock shares at the current price P_t and then the market maker adjusts the stock price to P_{t+1} . Before the end of period t , every unit of risk-free asset earns a risk-free return R . Group i 's wealth is updated to $W_{i,t+1}$ at the beginning of period $t + 1$ as follows:

$$W_{i,t+1} = S_{i,t+1} + P_{t+1}Z_{i,t+1}, \quad (3.12)$$

$$S_{i,t+1} = R \cdot (S_{i,t} - z_{i,t}P_t), \quad (3.13)$$

$$Z_{i,t+1} = Z_{i,t} + z_{i,t}. \quad (3.14)$$

Substituting Eq. (3.13) and Eq. (3.14) into Eq. (3.12) gives

$$W_{i,t+1} = R \cdot W_{i,t} + (P_{t+1} - R \cdot P_t)Z_{i,t+1}, \quad (3.15)$$

that is, Group i 's wealth at any period is the risk-free return earned by its latest wealth plus the excess return earned by its total holding of the risky asset.

3.3 Profitability of strategies

This section justifies investors' strategy switching behavior. In the situation of homogenous propensity of strategy switching and no noise term, the relative advantage of one strategy is determined by the price expectations of all strategies. No strategy can dominate the other. This provides a motivation for investors to adjust their strategies. Huang (2011) provides a dynamic analysis on profitability similar to ours in a

⁶Please note that in this chapter, the notation of investors' stock position $Z_{i,t}$ is different from the notation $N_{i,t}$ used in Chapter 2 and Chapter 4. As there are more variables in this chapter, both $z_{i,t}$ and $n_{i,t}$ appear. I choose the notation $Z_{i,t}$ over $N_{i,t}$ to relate it to $z_{i,t}$ rather than $n_{i,t}$.

quantity-competition oligopoly case.

For any strategy h , $h = f$ or c , its *absolute profit* π_t^h is the excess profit over the opportunity cost of investing in risk-free asset. It can be derived from substituting (3.3,3.4,3.5) into (3.7)

$$\pi_t^h = [\gamma \sum_{k=f,c} n_t^k E_t^k(P_{t+1}) - (r + \gamma R)P_t][E_t^h(P_{t+1}) - P_t R]. \quad (3.16)$$

When $E_t^h(P_{t+1}) = P_t R$, the absolute profit of strategy h is zero as there is no trading at all. In all other cases, the absolute profit of a strategy is determined by the balance of the price expectations of both strategies, as summarized in Proposition 1.

Proposition 1 *The absolute profit of a strategy can be positive, negative or zero.*

Case 1: $\pi_t^h > 0$ if and only if $E_t^h(P_{t+1}) > \max(P_t R, \tilde{P}_{t+1}^h)$ or $E_t^h(P_{t+1}) < \min(P_t R, \tilde{P}_{t+1}^h)$;

Case 2: $\pi_t^h < 0$ if and only if $\min(P_t R, \tilde{P}_{t+1}^h) < E_t^h(P_{t+1}) < \max(P_t R, \tilde{P}_{t+1}^h)$;

Case 3: When $E_t^h(P_{t+1}) = \tilde{P}_{t+1}^h$, fundamentalists' expectation and chartists' expectation are perfectly balanced so that $P_{t+1} = P_t R$ and $\pi_t^h = 0$;

where $\tilde{P}_{t+1}^h \triangleq [(r/\gamma + R)P_t - n_t^{\bar{h}} E_t^{\bar{h}}(P_{t+1})]/n_t^h$ and $\bar{h}$ denotes the opposite strategy of strategy h .

Based on Proposition 1 and more calculation, the signs of the absolute profits of strategies are shown in Fig. 3-1. To facilitate the demonstration, coordinate axes are divided by the current price. Fig. 3-1 shows that the profitability of a strategy is determined by both strategies simultaneously. The balance of expectations affects future price via the market maker's adjustment, and further influences the profitability of strategies.

The *relative profit* $\Delta\pi_t$ is defined as the difference of fundamentalists' and chartists'

Figure 3-1: Absolute profit of the fundamental strategy (top) and the trend-following strategy (bottom). In the first quadrant, the profit of a strategy is positive in the grey shaded region, zero along two lines and negative in the other area.

absolute profits

$$\begin{aligned}
\Delta\pi_t &\triangleq \pi_t^f - \pi_t^c \\
&= [\gamma \sum_{k=f,c} n_t^k E_t^k(P_{t+1}) - (r + \gamma R)P_t][E_t^f(P_{t+1}) - E_t^c(P_{t+1})] \\
&= \gamma n_t^h [E_t^h(P_{t+1}) - \tilde{P}_{t+1}^h][E_t^f(P_{t+1}) - E_t^c(P_{t+1})].
\end{aligned} \tag{3.17}$$

Proposition 2 *The relative profit of the fundamental strategy can be positive, negative or zero.*

Case 1: $\Delta\pi_t > 0$ if and only if $E_t^f(P_{t+1}) > \max(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$ or $E_t^f(P_{t+1}) < \min(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$.

Case 2: $\Delta\pi_t < 0$ if and only if $\min(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f) < E_t^f(P_{t+1}) < \max(E_t^c(P_{t+1}), \tilde{P}_{t+1}^f)$.

Case 3: $\Delta\pi_t = 0$ when both strategies share the same expectation, or when the expectations of strategies are perfectly balanced, i.e. $E_t^h(P_{t+1}) = \tilde{P}_{t+1}^h$.

Fig. 3-2 shows the relative profitability of two strategies. Depending on the combination of fundamentalists' and chartists' expectations, any strategy can be more profitable than the other. That is to say, no strategy can definitely dominate the other. Under some expectation combination, one strategy is better; while under some other combinations, the other strategy is better. To pursue higher profit, it is reasonable for investors to change their strategies. This provides a motivation for investors' strategy switching behavior.

3.4 Simulations and discussions

3.4.1 Numerical simulations

A typical simulation

With the discrete HAM model given by Eq.s (3.1, 3.2, 3.6, 3.7 and 3.9-3.14), assume there are 100 levels of propensity of strategy switching, following an arithmetic sequence from 0.02 to 2, so that propensity of strategy switching of the i th group is $0.02 * i$. Simulations are conducted under a default set of parameters and initial conditions:

Figure 3-2: Relative profit. Top panel: The value of relative profit in a 3D space. Bottom panel: Details on signs and comparisons of strategies' absolute profits. The first quadrant is divided into ten areas by four solid lines. The shaded regions indicate areas where the relative profit is positive.

$r = 0.0001$, $\gamma = 0.001$, $a = 2$, $\varepsilon_t \sim N(0, 1)$ and $S_{i,0} = 5000$, $Z_{i,0} = 50$, $P_0 = 100$, $E_0^h(P_1) = 100$, $n_{i,1}^f = 0.5$. Every simulation lasts 200 periods.

Fig. 3-3 shows simulation results for the price series, cumulative relative profit, groups' historical average fraction of fundamental strategy and groups' final wealth in a typical simulation.

Investors adjust their strategies according to the relative profit of fundamental strategy. The relative profit of fundamental strategy varies a lot. So daily relative profits are summed up to get a better view on the comparison of strategies. The cumulative relative profit of fundamental strategy is shown in Fig. 3-3(b). Generally speaking, the fundamental strategy performs better than the trend-following strategy, but its advantage becomes faint after 140 periods.

In Fig. 3-3(c) and Fig. 3-3(d), groups of investors are sorted by their propensity of strategy switching from small to large. Fig. 3-3(c) shows that groups with higher propensity have larger historical average fractions of fundamental strategy. As for the wealth held by each group by the end of the simulation, in Fig. 3-3(d), groups with higher propensity of strategy switching have less final wealth. Fig. 3-3(c) and Fig. 3-3(d) show that groups with higher propensity of strategy switching adopt the better strategy more often, but end up with less final wealth.

Because of the white noise in price formation, the trajectory of price varies every simulation. Fig. 3-4 shows the stock price series, fundamentalists' and chartists' demand, relative profit and average fraction of fundamental strategy in a typical simulation. In Fig. 3-4(a), the stock price deviates from its fundamental from time to time. Fig. 3-4(b) shows fundamentalists' and chartists' demand of stock shares weighted by their market fraction. In this model, the market maker satisfies all investors' demand, so investors' demand equals their trading volume. The trading volume of fundamentalists is comparable to that of chartists in absolute value. Every time when the price deviates far from its fundamental, fundamentalists' trading volume increases, dragging the price back to its fundamental. After the price moves close to its fundamental, fundamentalists reduce their trading. Sometimes their trading can be lower than chartists', as a result the price starts to deviate from its fundamental again. The competition between

Figure 3-3: A typical simulation. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy over 200 periods. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high.

fundamental strategy and trend-following strategy continues. Any strategy can gain temporary advantage, but no one can dominate the other for long.

Investors' strategy switching is driven by the relative profit. In Fig. 3-4(c), when the relative profit of fundamental strategy is positive, some investors who have adopted the trend-following strategy switch to fundamental strategy in the next period, and the market fraction of fundamental strategy increases in the next period. When the relative profit of fundamental strategy is negative, some investors initially adopting the fundamental strategy switch to the trend-following strategy, and the market fraction of fundamental strategy decreases in the next period. Because of this cause-effect relation between relative profit and market fraction of fundamental strategy, Fig. 3-4(c) and Fig. 3-4(d) show similar sequences of ups and downs, but with different fluctuation ranges. The market fraction of fundamental strategy lags one period behind the relative profit time series.

Within a group, there are fundamentalists and chartists. The behavior of a group is a convex combination of fundamentalists' and chartists' behavior. Moreover, because the fraction of fundamental analysis in a group is a strictly increasing function of groups' strategy-switching propensity, the behavior of any group is a convex combination of the behavior of the two groups with minimum and maximum strategy-switching propensity⁷. Therefore, to understand the fraction of fundamental strategy in every group, only details of the two groups with minimum and maximum strategy-switching propensity are necessary, and all other groups are located in between. As for the evolution of groups' wealth, a demonstration of these two groups gives us a sketch of other groups, though strictly speaking the wealth of a group in between may go beyond the band defined by these two groups with extreme propensity. Fig. 3-5 shows the fraction of fundamental analysis and the process of wealth accumulation of these two groups. In Fig. 3-5(a), the group with minimum strategy-switching propensity holds a stable combination of fundamental strategy and technical strategy, and the detailed proportion fluctuates slightly under the influence of the previous profitability of strategies.

⁷To be clear, this statement is limited to variables determined by the information one or two periods ago, e.g. price prediction, demand, and fractions of strategies. It cannot be extended to variables involving distant historical information like historical average fraction of strategies and wealth accumulation.

Figure 3-4: Time series within a typical simulation. (a) Stock price and fundamentalists' expectation of the fundamental value. (b) Fundamentalists' and chartists' demand weighted by their market fraction. (c) Relative profit of fundamental strategy comparing with chartists' strategy. (d) Average market fraction of fundamental analysis within the whole population.

Figure 3-5: Comparison of the group with maximum strategy-switching propensity and the group with minimum strategy-switching propensity. (a) Fraction of fundamental analysis within a group. (b) Wealth accumulation of a group.

The group with maximum strategy-switching propensity adjusts its proportion of fundamental and technical strategies intensively, showing a stronger and faster strategy adjustment. Fig. 3-5(b) shows the evolvement of wealth of these two groups. Most of the time, the group with minimum strategy-switching propensity enjoys a higher wealth. But, there are also periods when the group with maximum strategy-switching propensity is wealthier, like around the 122^{ed} period. But as time passes by, the group with minimum propensity becomes more and more wealthy, and the gap of wealth between these two groups gradually widens.

Average 1000 simulations

Because of the noise term in the stochastic price dynamics, the trajectory of stock price varies from simulation to simulation. To eliminate the influence of the noise term, I average the results of 1000 simulations.

Fig. 3-6 shows the average result. In Fig. 3-6(a), the average stock price is below the fundamental expectation because the risk-free interest rate is positive. With a negative risk-free interest rate, the average stock price will be higher than the fundamental

expectation.

Fig. 3-6(b) shows that, in general, the fundamental strategy is better than the trend-following strategy in the sense of producing a higher cumulative relative profit⁸. The advantage of the trend-following strategy only exists in the first few periods and it depends on the initial condition of the simulation. If the initial stock price deviates from the fundamental expectation, the early advantage of the trend-following strategy will become weak or even disappear.

In Fig. 3-6(c) and Fig. 3-6(d), agent groups with higher propensity of strategy switching adopt the fundamental strategy more often but end up with less final wealth. That is to say, frequently adopting a more profitable strategy does not guarantee a higher final wealth.

3.4.2 Main findings

Why adopting a better strategy more often does not guarantee a higher final wealth? In this subsection, I summarize the main findings and show that the key to the question lies in the inconsistency between short-run profit and long-run wealth accumulation.

Fraction of fundamental strategy

Analytically, based on Eq. (3.11) and Eq. (3.17), each group's fraction of fundamental strategy in period $t + 1$ is

$$n_{i,t+1}^f = \frac{1}{1 + e^{-\beta_i \Delta \pi_t}}. \quad (3.18)$$

Proposition 3 *Under any positive relative profit, i.e. $\forall \Delta \pi_t > 0$, groups' fraction of fundamental strategy is a strictly concave increasing function of groups' propensity of strategy switching, because*

$$\frac{\partial n_{i,t+1}^f}{\partial \beta_i} > 0 \quad \text{and} \quad \frac{\partial^2 n_{i,t+1}^f}{\partial \beta_i^2} < 0.$$

⁸This is an observation, not a result. The explanation of this observation is *ex post* and with a taste of model selection. The fundamental strategy performs better as long as the stock price does not diverge. In a model where chartists surpass fundamentalists, the stock price is likely to diverge, producing stock price time series with little economic meaning.

Figure 3-6: Average of 1000 simulations. (a) Time series of stock price and fundamental expectation. (b) Cumulative relative profit of fundamental strategy. (c) 100 groups' historical average fraction of fundamental strategy. (d) 100 groups' final wealth. In (c) and (d), groups of agents are sorted by their propensity of strategy switching from low to high.

Eq. (3.18) and Proposition 3 depict how a group's fraction of fundamental strategy changes. For given propensity of strategy switching β_i , the corresponding group's fraction of fundamental strategy $n_{i,t+1}^f$ is higher (lower) than 0.5 when the relative profit $\Delta\pi_t$ is positive (negative), and it converges to 1 (0) as the absolute value of the relative profit increases. For given positive (negative) relative profit $\Delta\pi_t$, different groups' fractions of fundamental strategy $n_{i,t+1}^f$ start from 0.5 and converge to 1 (0) when groups' propensity of strategy switching increases.

Fig. 3-3(c) and Fig. 3-6(c) show groups' historical average fraction of fundamental strategy over 200 periods in one simulation and in 1000 simulations. As the historical average fraction is the focus here, the time subscript is ignored⁹. Similar to Proposition 3, $n_i^f = 1/(1 + e^{-\beta_i\Delta\pi})$ is a strictly concave increasing function of β_i when $\Delta\pi > 0$. However, curves in Fig. 3-3(c) and Fig. 3-6(c) do not converge to 1 because the fractions $n_{i,t+1}^f$ is averaged over all values of $\Delta\pi_t$. Therefore, the shapes of these curves simply reflect the fact that the fundamental strategy performs better, in the sense that it generated positive relative profits more often than negative relative profits.

This fact echoes the cumulative relative profit shown in Fig. 3-3(b) and Fig. 3-6(b). Intuitively, when the fundamental strategy is generally more profitable, investors adopt this strategy more often. So the average fraction of fundamental strategy in every group is always larger than 0.5. Comparing different groups, groups with higher propensity of strategy switching have faster adaptive learning and adopt the fundamental strategy more often. Therefore, groups with higher propensity have larger average fraction of fundamental strategy.

Final wealth

Fig. 3-3(d) and Fig. 3-6(d) present the most important finding. That is, investors with higher propensity of strategy switching end up with less final wealth.

When the fundamental strategy is generally more profitable, the coexistence of a

⁹Please note this is not a strict proof. A group's historical average fraction of fundamental strategy is actually $n_i^f = \sum_{t=1}^T n_{i,t}^f / T$, and the average relative profit is $\Delta\pi = \sum_{t=1}^T \Delta\pi_t / T$. Here, I ignore the time subscript to show a neat, but not rigorous, proof.

positive relation between groups' propensity of strategy switching and their average fraction of fundamental strategy and a negative relation between groups' propensity and their final wealth seems counterintuitive. That is to say, a group with faster adaptive learning adopts the better strategy more often but ends up with less final wealth. How could this happen? To answer this question, I decompose investors' wealth change in every period. By manipulating Eqs (3.3, 3.7, 3.10, 3.12, 3.13, 3.14), the wealth change in one period is

$$\begin{aligned}
\Delta W_{i,t+1} &= W_{i,t+1} - W_{i,t} \\
&= S_{i,t+1} + P_{t+1}Z_{i,t+1} - S_{i,t} - P_tZ_{i,t} \\
&= S_{i,t}R - z_{i,t}P_tR + P_{t+1}Z_{i,t} + P_{t+1}z_{i,t} - S_{i,t} - P_tZ_{i,t} \\
&= S_{i,t}r + (P_{t+1} - P_t)Z_{i,t} + (P_{t+1} - P_tR)z_{i,t} \\
&= S_{i,t}r + (P_{t+1} - P_t)Z_{i,t} + (n_{i,t}^f\pi_t^f + n_{i,t}^c\pi_t^c).
\end{aligned} \tag{3.19}$$

There are three parts in the fourth line of the right-hand side of Eq. (3.19). The first part is the interest from investing in risk-free asset; the second part is the change of market value of all risky asset a group has already held; and the third part is the excess profit from the latest purchase of risky asset. The excess profit per share equals the change of market value of a share minus the opportunity cost of buying a share. It is a sum of the profits of strategies weighted by the fractions of strategies, *i.e.* $n_{i,t}^f\pi_t^f + n_{i,t}^c\pi_t^c$, as shown in the last line of Eq. (3.19).

Fig. 3-7 shows the detailed decomposition of wealth change of the slowest group, *i.e.* $\beta_1 = 0.02$, in two simulations. Fig. 3-7(a) is the exact same simulation shown in Fig. 3-3.

Fig. 3-7(b) is another simulation under the same parameter set but with zero initial endowments $S_{i,0} = 0$ and $Z_{i,0} = 0$. In both cases, it is obvious that the change of market value of all risky asset this group has already held dominates the wealth change in almost all periods. Though only the slowest group is demonstrated, the analysis of other groups look similar. On average, *i.e.* $\frac{1}{NT} \sum_{i=1}^N \sum_{t=1}^T (P_{t+1} - P_t)Z_{i,t} / \Delta W_{i,t+1}$, the change of market value of risky asset a group has already held contributes to more than

Figure 3-7: Decomposition of the wealth change of the slowest group. (a) The same simulation shown in Fig. 3. (b) A new simulation with zero initial endowment.

90% of a group's wealth change in a period.

The explanation of the main findings becomes clear now. Investors' strategy switching behavior is driven by the comparison of the short-run profitability of strategies. But the sum of strategies' profit weighted by strategies' fraction only contributes to a small part of the wealth change in every period. If investors' ultimate goal is to accumulate wealth as much as possible, then their strategy switching behavior is inconsistent with their ultimate goal. This inconsistency is an inconsistency between short-run profit and long-run wealth accumulation.

When investors adjust their strategies, they may neglect the lasting effect of their strategy switching on the total number of shares they possess in future. A large position

in the risky asset¹⁰ is achieved by continuously buying or selling the risky asset. Agents who stick to the fundamental strategy are more likely to accumulate a large position. Agents with fast strategy switching become fundamentalists from time to time, but they do not stick to the fundamental strategy. The process of risky asset accumulation is interrupted when they switch to chartists. Therefore, they do not hold a large position in the risky asset.

The change of the market value of investors' risky asset position contributes to the majority of their wealth change. When an investor adjusts his strategy or executes a purchase, the immediate effect on one-period wealth change is visible. But there is no way to tell what will happen in the long run because that requires information on future price, which is obviously unavailable to him. This is how the inconsistency between short-run profit and long-run wealth accumulation emerges. This inconsistency eventually causes the interesting phenomenon that investors with faster adaptive learning adopt the better strategy more often but end up with less final wealth.

3.4.3 Robustness

The main findings are based on simulations under the default parameter set. I carry out robustness checks to confirm that the main findings are stable when parameters vary within a reasonable range. In this section, the risk-free interest rate r , the market maker's price-adjustment speed γ and chartists' trend-following extent a are tested. I check one parameter at a time and keep others fixed at their default values. Under every parameter setting, I run 1000 simulations and average the results. Each simulation lasts 1000 periods.

Fig. 3-8 shows the robustness check results. In the left column of Fig. 3-8, groups with higher propensity of strategy switching have higher fraction of fundamental strategy. In the right column, groups with higher propensity have less final wealth. The main findings are robust when the risk-free interest rate varies from -0.001 to 0.001 (top), or when the market maker's price-adjustment speed varies from 0.00005 to 0.002

¹⁰ A large position in the risky asset is either a positive position in a bull market or a negative one in a bear market, from an *ex post* viewpoint.

(middle), or when chartists' trend-following extent varies from 0.05 to 3 (bottom).

Besides, Fig. 3-8 reveals more information on parameters' influence over the main findings. As shown in Fig. 3-8(a) and Fig. 3-8(b), the risk-free interest rate barely affects the results. Even when the interest rate is negative, the main findings are stable, except that the general levels of groups' final wealth decrease with the risk-free interest rate.

When the market maker increases the price-adjustment speed, the positive relation between groups' propensity of strategy switching and their fraction of fundamental strategy becomes more pronounced at first and then less marked, as shown in Fig. 3-8(c). The general levels of groups' final wealth decrease with the price-adjustment speed in Fig. 3-8(d). I do not have an explanation for this so far.

Fig. 3-8(e) and Fig. 3-8(f) show that chartists' trend-following extent has a large impact on the results. When chartists follow the trend more intensively, the positive relation between the propensity and the fraction of fundamental strategy weakens, but the negative relation between the propensity and the final wealth are enhanced. Based on the discussion of the profitability of strategies in Section 3, the strategy with larger impact is more likely to drive the price moving toward the direction benefit to that strategy. As chartists' trend following becomes stronger, chartists have larger influence on the price formation, and the relative advantage of the fundamental strategy is weakened. So the positive relation between the propensity and the fraction of fundamental strategy becomes less pronounced. Meanwhile, the negative relation between strategy switching propensity and final wealth becomes more remarkable. This hints us that the negative relation of investors' propensity of strategy switching and their final wealth does not depend on a strong relative profit of the fundamental strategy.

3.4.4 Noise term

There are a few works focusing on the role that noise term plays in HAMs. He and Li (2007) study how the simulated price, especially the autocorrelation of returns, are affected by different noise processes. Amilon (2008) finds that the way noise added to models shows a large influence on models' ability to generate some stylized facts.

Figure 3-8: Robustness check. Investors with higher propensity of strategy switching have higher average fraction of fundamental strategy (left column) and lower final wealth (right column) under different risk-free interest rates (a, b), market maker's price-adjustment speeds (c, d) and chartists' trend-following extents (e, f).

Figure 3-9: Effect of the standard deviation of the noise term

A robustness check on the standard deviation of noise term shows that the noise term plays an important part in the model dynamics. In Fig. 3-9, the main findings on the fraction of fundamental strategy and the final wealth both become more pronounced when the standard deviation of the noise term in price formation increases. The noise term in the price formation represents unexpected factors, like the uncertainty in short future. Without the uncertainty, *i.e.* the standard deviation of noise term is 0, the main findings disappear. This shows that the noise term plays a critical role in this study. In other words, the main findings depend on the existence of uncertainty. Therefore, to be precise, the main findings should be refined as: when uncertainty exists, frequently adopting a more profitable strategy does not guarantee a higher final wealth.

From another viewpoint, Fig. 3-6 averages the results of 1000 simulations, which to some extent helps to cancel out the influence of the noise term. Inferring from Fig. 3-6(a), without the noise term, the simulated stock price does not converge to its fundamental, but stays at a level slightly lower than its fundamental and grows at the same rate as its fundamental does, *i.e.* at the risk-free interest rate. As long as the stock price grows at the risk-free interest rate, according to Eq. (3.7), both fundamental strategy and trend-following strategy have zero absolute profit. Therefore, all groups' fractions of fundamental strategy are 0.5, and this is shown as a straight line parallel to the axis of propensity of strategy switching in the left panel of Fig. 3-9. Moreover, since all groups behave in the same way, all groups' wealth at any period is equal, and this is shown as a straight line parallel to the axis of propensity of strategy switching

in the right panel of Fig. 3-9.

3.5 Conclusion

In this chapter, a heterogeneous agent model is built to study whether a faster adaptive learning helps investors accumulate more wealth in the end. It is found that, when the fundamental strategy is in general more profitable than the trend-following strategy, investors with higher propensity of strategy switching adopt the fundamental strategy more often but end up with less final wealth. This counterintuitive phenomenon is robust under various parameter settings. After decomposing the wealth change in every period, I attribute the phenomenon to the inconsistency between short-run profit and long-run wealth accumulation. The comparison of the short-run profits of strategies drives investors' strategy switching. But the short-run profit only takes a small proportion of the wealth change in every period. Investors' wealth accumulation relies heavily on the total amount of risky asset possessed by investors and its corresponding market value rather than on the profit from latest trading. Therefore, investors' strategy switching behavior is inconsistent with their wealth accumulation.

This study raises some interesting questions for future study. Because of the inconsistency between short-run profit and long-run wealth, we do not expect that a series of investment actions driven by short-run profit eventually lead to a long-run success, but why to a long-run failure instead of a mediocre ending? The key to this problem may lie in the timing of investors' strategy switching. The model has shown how often investors chose the strategy which was superior *ex ante*. But how often investors chose the strategy which was superior *ex post*? Even the fundamental strategy is superior on average, investors still have to distinguish its well-performing time from its ill-performing time. The timing of investors' strategy switching is significant in theory and practice. Last but not the least, to what extent the conclusions of the model are subject to the model structure? Major revisions on model structure are likely to make a difference. Further investigation is required to get a better understanding of these questions.

Chapter 4

Endogenous Fundamental, Financial Bubbles and Stock Cycles

4.1 Introduction

Financial bubbles are usually defined as the asset prices rising far above their fundamental values. Numerous publications on financial bubbles exist. Among them, heterogeneous agent models show special power on reproducing stock bubbles (Lux, 1995, Huang et al., 2013) and anomalous stylized facts (Hommes, 2002, Lebaron, 2006, Lux 2009). This power gains HAMs popularity among researchers. However, most HAMs either assume exogenous fundamental values (Day and Huang, 1990) or derive the fundamental from an exogenously predetermined dividend process (Brock and Hommes, 1998). That is, there is no connection between the true fundamental value and the real economy. Since the difference between stock price and its fundamental is used to define stock bubbles, it is necessary to make the fundamental value endogenous to gain a further understanding of the dynamics behind stock bubbles.

To the best of my knowledge, only a few papers build heterogeneous agent models on financial economics with endogenous fundamental. Most relevant to my work is Wester-

hoff (2012), in which a Keynesian-type goods market represents the real economy. The author compares situations of isolated markets with connected markets. He studies the steady states of goods market and stock market, in both isolated and connected cases. Special attention is paid to the complex behavior of national income and the stock price under various parameter values. Naimzada and Pireddu (2014) build a model with a real subsystem and a financial subsystem. In the real subsystem, the output is adjusted according to the difference of previous aggregate demand and output. In the financial subsystem, fundamentalists' expectations on fundamental are based on a weighted mean of an exogenous true fundamental and current output, plus a bias. They design an interaction parameter to show the degree of connection between two subsystems. Their work focuses on the transmission of instability between real and financial subsystems under a various degree of connection. Lengnick and Wohltmann (2013) combine agent-based financial market theory with New Keynesian macroeconomics. The fundamental value of the stock is proportional to the output gap. Moreover, with a dynamic stochastic general equilibrium model, they do impulse response analysis on the output gap, inflation, and transitory cost. Special attention is paid to the design of transaction cost, to stabilize markets and to produce tax revenue. Less relevant, Gori and Ricchiuti (2014) investigate an exchange rate model in which the fundamental exchange rate is endogenously determined by the balance of payment between two goods markets. They focus on the mathematical analysis of steady states and how such steady states can be achieved under what conditions. Last but not the least, EURACE project (Deissenberg et al., 2008; Cincotti et al., 2010, 2011, 2012; Dawid et al., 2012) built a large-scale agent-based model of the European economy designed for policy advice. Filled with tons of households, firms, banks and a government, the EURACE model considers encompassing types of real and financial markets. With all kinds of explicitly detailed empirical data, information and behavioral rules feeding into the model, the EURACE model mimics the reality to the greatest extent. It is the largest and most complicated agent-based model ever developed so far. It exhibits possible outcomes of various monetary and fiscal policies by numerical simulation, but at the cost of analytic tractability.

In works mentioned before and others, except the EURACE model, when the fundamental is related to the real economy, a Keynesian-type goods market is usually adopted. That is, the total output is described in an income-expenditure form at the macro-level. While in my model, the output is determined by the Cobb-Douglas production function, at the micro-level. Besides, I try to keep the model simple to gain a clear logic via both mathematical and numerical analysis. I focus on the production process of a representative company in the goods market. This feature distinguishes my model from most HAMs with endogenous fundamental. Moreover, investors in financial markets are all involved in the production, as either firm owners or workers. Agents not only get paid for the production but also acquire some information of the true fundamental which substantially affect their strategies in the financial market.

Our model is a combination of heterogeneous agent model and basic microeconomics. The real economy, which determines the true fundamental of the financial market, is described by a Cobb-Douglas production function. The financial sector of the model follows the design in Day and Huang (1990) and Westerhoff (2012). Constraints of the budget, short-sale and funding liquidity are imposed on investors to make the numerical analysis more realistic. The model shows that the appearance of bubbles is closely related to the real production and people's financial investment. Stock cycles occur repeatedly. After referring to empirical division of stock cycles, I propose to divide a stock cycle into four phases: accumulation, boom, crash and recovery. A closer investigation of each phase reveals specific reasons behind the beginning and ending of each phase. It shows that a prosperous stock market may accelerate the formation of bubbles by drawing resource from future production. The burst of bubbles is usually triggered by the constraint on investors' liquidity. Though in general, chartists are less wealthy and holding smaller stock positions than fundamentalists, they are capable of showing a large impact on the stock market.

There are three highlights distinguishing this model from others. 1) A Cobb-Douglas production process is embedded into an HAM on the financial market so that the true fundamental of stock becomes endogenous. To balance the complexity and economic intuition, I simplify the model, while keeping the production sector and the financial

Figure 4-1: The two-way interaction between the production and the financial market.

market functional. 2) A two-way interaction exists between the production sector and the financial sector (Fig. 4-1). The output of production decides the fundamental of stock. While people's investment in risk-free asset transits to newly increased capital for future production. 3) Agents' roles in both markets are fixed. There is no strategy switching in the financial market. Instead of the implicit assumption of perfect information which is embedded in most HAMs with strategy switching, information is asymmetric in the model. That is, agents with different roles have different accesses to the information of true fundamental.

The rest of this chapter is organized as follows. Section 4.2 presents the model details. Section 4.3 is devoted to the mathematical analysis of steady states. The existence of bubbles under those steady states are also discussed. Section 4.4 shows the numerical simulation of stock cycles under financial constraints. A typical stock cycle is divided into four phases. I elaborate the cause-and-effect relationship in each phase. Section 4.5 concludes this chapter.

4.2 The model

I build a hypothetical economy with a conglomerate consumer goods firm¹. The firm employs the whole population to produce various goods which people consume. In the

¹Some examples are P&G, Nestle, Unilever, LOTTE and so on.

financial market, the firm issues a corporate bond to raise financing for production and a stock to involve people into the economic progress. The firm constantly issues new bonds before every round of production. Each bond quickly matures², and the principal is repaid with an interest by the firm. All discussion of the bond is limited in a primary market. All discussion of stock is limited in a secondary market without new issues. Every period, there are two stages: a production stage and then a financial investment stage. All agents are involved in both stages.

At the production stage, the firm's output follows the Cobb-Douglas production function with capital and labor as inputs. Firm owners provide the capital and workers provide the labor. An agent is either a firm owner or a worker. Before a round of production, firm owners raise money from people by issuing a corporate bond which matures after the production stage. After the payment of the corporate bond, the remaining output is distributed among firm owners and workers according to the output elasticity of capital and labor as owners' business income and workers' wage.

At the financial investment stage, the latest output of production indicates the true fundamental of the firm's stock. In the stock market, based on the information available to them, firm owner use fundamental analysis and workers use technical analysis. Firm owners are fundamentalists as they are insiders. They have access to the latest true fundamental, and they believe that the stock price will converge to it. Workers are chartists because the information of fundamental is unavailable to them. They can only observe past fundamental with a time lag. A market maker adjusts the stock price according to the excess aggregate demand. After receiving their capital gain from the stock investment, people consume a part and keep the rest. Besides the risky stock, agents can hold their money as cash or invest in a risk-free corporate bond. Here the corporate bond is risk-free because its return rate is fixed and its payment is secured even before the distribution of agents' ordinary income. As the corporate bond is risk-free, agents buy the bond with the money left after their consumption. The money raised by

²Later in the simulation, the representative bond matures every period. It seems that the short-term bond is questionable. But if we consider a bond market full of long-term bonds which act in an asynchronous manner, then the representative short-term bond can be created by holding multiple bonds of different maturities.

Figure 4-2: Agents' roles in the production process and in the financial market.

the bond will be reinvested in the production of next period as newly produced capital. Fig. 4-2 shows how agents are involved in the production process and the financial market.

There are three assets in the model: cash, a risk-free corporate bond with interest and a risky stock. Workers' wealth is the sum of them. As for firm owners, besides these three assets, they have physical capital. Since the heterogeneity of agents only exists among groups, *i.e.* between firm owners and workers, a proxy firm owner is used to represent all owners and a proxy worker to represent all workers. There is no difference between agents in the same group. Superscript h denotes agents' identity. $h = f$ represents fundamentalists in stock market, *i.e.* firm owners in production. $h = c$ denotes chartists in stock market, *i.e.* workers in production.

At the beginning of period t , the wealth W_t^h of an agent h is

$$W_t^f = N_t^f \cdot P_t + S_t^f + K_t \quad (4.1)$$

$$W_t^c = N_t^c \cdot P_t + S_t^c, \quad (4.2)$$

where N_t^h is the total number of stock shares held by agent h , P_t is the current stock price and K_t is the physical capital left over from the production in the last period. S_t^h is the total value of agent h 's holding of cash and corporate bond B_t^h , which he bought at the end of the last period.

At the production stage, the physical capital of the last period depreciates at a rate of δ . Firm owners invest the fund raised from the corporate bond in the production process. Note that firm owners buy corporate bond too. As the firm's stock is traded in a secondary stock market, these firm owners are obviously not private owners of this firm, but major shareholders. They have no motivation to invest their saving into future production without any interest. Besides, as the payment of the bond is prior to the distribution of ordinary income, the purchase of corporate bond gives them a higher total income.

The total capital before a new round of production is K_{t+1}

$$K_{t+1} = (1 - \delta) \cdot K_t + B_t, \quad (4.3)$$

in which B_t is the total amount of bond raised from the whole population

$$B_t = B_t^f + B_t^c, \quad (4.4)$$

and δ is the depreciation rate of physical capital.

The production follows a Cobb-Douglas production function with capital and labor as inputs. Assume the amount of labor provided by workers is 1. The total output is Y_{t+1}

$$Y_{t+1} = A \cdot K_{t+1}^\alpha, \quad (4.5)$$

where A is the total factor productivity, K is the capital input, and α denotes the output elasticity of capital. The parameter α is positive and less than 1.

After production, firm owners secure the payment to bond at first and then distributes the rest of output among firm owners and workers according to the output elasticity of capital and labor.

At the financial investment stage, agents decide their demands for stock shares based on their beliefs. The true fundamental of stock in the next period F_{t+1} is determined by the latest output of production Y_{t+1}

$$F_{t+1} = j \cdot Y_{t+1}, \quad (4.6)$$

here j is a parameter to adjust the scale of production output and the scale of fundamental value, so that the fundamental value and the production output are proportional. The definition of fundamental value is different from the usual definition used in investment which is the present value of future cash flow like price-dividend ratio. In my model, the representative stock does not produce any cash flow. Therefore, the common calculation of fundamental value does not apply here. Besides, Eq. (4.6) seems as one of the simplest ways to link the fundamental value with the production.

Firm owners are fundamentalists in the stock market. They know the true fundamental, as they manage the whole production process. Moreover, they believe the stock price will converge to the fundamental. They buy stock shares when the stock is undervalued and sell when the stock is overvalued. Let D_t^f denote fundamentalists' demand for stock shares

$$D_t^f = a \cdot (E_t^f(F_{t+1}) - P_t)^3, \quad (4.7)$$

$$E_t^f(F_{t+1}) = F_{t+1}. \quad (4.8)$$

Here, a is a positive parameter showing fundamentalists' market power in the stock market. The cubic function of fundamentalists' demand is borrowed from Westerhoff (2012). With this cubic function, fundamentalists act more intensively under large price deviation, comparing with the case of small price deviation. The intuition behind the nonlinear demand is that fundamentalists believe the convergence to the fundamental is more likely to happen when the price deviation is large. Fundamentalists can evaluate the probability of the fulfillment of their expectations.

Workers behave as chartists in the stock market. They believe that the price deviation from its fundamental or the current trend of price will continue in the short future.

However, they have no access to the latest true fundamental. They can only observe past fundamental with a time lag τ . They take the observation as their estimate of future fundamental $E_t^c(F_{t+1})$

$$E_t^c(F_{t+1}) = F_{t+1-\tau}. \quad (4.9)$$

Chartists' demand for stock shares is D_t^c

$$D_t^c = b \cdot (P_t - E_t^c(F_{t+1})), \quad (4.10)$$

where b is a positive parameter showing chartists' market power. Chartists' linear demand shows that they do not assess the probability of the fulfillment of their expectations.

In the stock market, there is a market maker who adjusts the price according to the aggregate demand D_t . Parameter γ is the speed of price adjustment. The price dynamics can be deterministic or stochastic, *i.e.* without an error term as in Eq. (4.12) or with an error term in Eq. (4.13). A deterministic dynamics makes the model analytically tractable; while a stochastic price dynamics is more realistic, as in practice, new information constantly pours into the stock market, adding perturbation to the price dynamics. In the model, the deterministic price dynamics is used in the discussion of long-run equilibrium and steady states, and the stochastic price dynamics is adopted in the numerical analysis of stock cycles.

$$D_t = D_t^f + D_t^c \quad (4.11)$$

$$P_{t+1} = P_t + \gamma \cdot D_t \quad (4.12)$$

$$P_{t+1} = P_t + \gamma \cdot D_t + \varepsilon_t \quad (4.13)$$

The market maker provides liquidity to the stock market and satisfies agents' excess demand for stock shares. Agents' market orders D_t^h are executed at the present price. Then, agents' stock holdings are updated to N_{t+1}^h

$$N_{t+1}^h = N_t^h + D_t^h. \quad (4.14)$$

After the market maker adjusts the stock price to P_{t+1} , agents check their total income in this period and consume accordingly. Agents' consumptions consist of autonomous consumption C^h and induced consumption with the marginal propensity to consume mpc^h . Agents' induced consumption is proportional to their total income in a period

$$C_t^f = C_{auto}^f + mpc^f \cdot income_t^f, \quad (4.15)$$

$$C_t^c = C_{auto}^c + mpc^c \cdot income_t^c. \quad (4.16)$$

After consumption, agents' wealth is updated

$$W_{t+1}^f = W_t^f + income_t^f - C_t^f = N_{t+1}^f \cdot P_{t+1} + S_{t+1}^f + K_{t+1}, \quad (4.17)$$

$$W_{t+1}^c = W_t^c + income_t^c - C_t^c = N_{t+1}^c \cdot P_{t+1} + S_{t+1}^c. \quad (4.18)$$

Before the end of period t , agents use their cash holdings S_{t+1}^h to buy corporate bond B_{t+1}^h which will fund the production in the next period.

Within every period, agents' wealth is first updated to $W_{t+1/3}^h$ after the production stage and then to $W_{t+2/3}^h$ after the stock investment stage. Postpone the payment of corporate bond to the update of $W_{t+2/3}^h$ so that $(W_{t+1/3}^h - W_t^h)$ denotes agent's ordinary income, *i.e.* firm owners' business income and workers' wage, and $(W_{t+2/3}^h - W_{t+1/3}^h)$ denotes agent's capital gain from the investment in bond and stock. The sum of agents' ordinary income and capital gain forms their total income in this period, *i.e.* $(W_{t+2/3}^h - W_t^h)$. A detailed timeline of agents' action and the update of variables in period t is shown in Fig. 4-3.

$$\begin{aligned} W_{t+1/3}^f &= N_t^f P_t + S_t^f + \alpha(Y_{t+1} - RB_t) + K_{t+1} \\ &= W_t^f + \alpha(Y_{t+1} - RB_t) + K_{t+1} - K_t \end{aligned} \quad (4.19)$$

$$\begin{aligned} W_{t+1/3}^c &= N_t^c P_t + S_t^c + (1 - \alpha)(Y_{t+1} - RB_t) \\ &= W_t^c + (1 - \alpha)(Y_{t+1} - RB_t) \end{aligned} \quad (4.20)$$

$$\begin{aligned}
W_{t+2/3}^f &= N_{t+1}^f P_{t+1} + S_t^f + \alpha(Y_{t+1} - RB_t) + K_{t+1} + rB_t^f - D_t^f P_t \quad (4.21) \\
&= W_{t+1/3}^f + rB_t^f + N_{t+1}^f (P_{t+1} - P_t)
\end{aligned}$$

$$\begin{aligned}
W_{t+2/3}^c &= N_{t+1}^c P_{t+1} + S_t^c + (1 - \alpha)(Y_{t+1} - RB_t) + rB_t^c - D_t^c P_t \quad (4.22) \\
&= W_{t+1/3}^c + rB_t^c + N_{t+1}^c (P_{t+1} - P_t)
\end{aligned}$$

$$ordinary\ income_t^f = W_{t+1/3}^f - W_t^f = \alpha(Y_{t+1} - RB_t) + K_{t+1} - K_t \quad (4.23)$$

$$ordinary\ income_t^c = W_{t+1/3}^c - W_t^c = (1 - \alpha)(Y_{t+1} - RB_t) \quad (4.24)$$

$$capital\ gain_t^f = W_{t+2/3}^f - W_{t+1/3}^f = rB_t^f + N_{t+1}^f (P_{t+1} - P_t) \quad (4.25)$$

$$capital\ gain_t^c = W_{t+2/3}^c - W_{t+1/3}^c = rB_t^c + N_{t+1}^c (P_{t+1} - P_t) \quad (4.26)$$

$$income_t^f = ordinary\ income_t^f + capital\ gain_t^f = W_{t+2/3}^f - W_t^f \quad (4.27)$$

$$income_t^c = ordinary\ income_t^c + capital\ gain_t^c = W_{t+2/3}^c - W_t^c \quad (4.28)$$

4.3 Dynamic equilibrium

In this subsection, I analyze the dynamic equilibrium in the production sector and the stock market under a deterministic price dynamics. Here, the dynamic equilibrium means that the values of variables stay unchanged while the production and the trading in the stock market continue. One or two long-run equilibria may exist. Stock bubbles can exist at dynamic equilibria.

Figure 4-3: A detailed timeline

4.3.1 Real economy

To study the model from a macro-level, at first, aggregate variables as follows

$$\begin{aligned}
W_t &= W_t^f + W_t^c, \\
N_t &= N_t^f + N_t^c, \\
S_t &= S_t^f + S_t^c, \\
B_t &= B_t^f + B_t^c, \\
D_t &= D_t^f + D_t^c, \\
C_t &= C_t^f + C_t^c, \\
C_{auto} &= C_{auto}^f + C_{auto}^c.
\end{aligned} \tag{4.29}$$

The total wealth of the population at period t is $W_t = N_t \cdot P_t + S_t + K_t$. Under the assumption $B_t^h = S_t^h$ and $mpc^f = mpc^c = mpc$, the model can be described by a five-dimensional system $(D_t, N_t, P_t, S_t, K_t)$. The development of this system follows

$$\begin{aligned}
D_t &= a \left(E_t^f(F_{t+1}) - P_t \right)^3 + b(P_t - E_t^c(F_{t+1})), \\
N_t &= N_{t-1} + D_{t-1}, \\
P_t &= P_{t-1} + \gamma D_{t-1}, \\
S_t &= Y_t - C_{auto} - D_{t-1}P_{t-1} - mpc(Y_t - S_{t-1} + K_t - K_{t-1} + N_tP_t - N_tP_{t-1}), \\
K_t &= (1 - \delta)K_{t-1} + S_{t-1}.
\end{aligned} \tag{4.30}$$

There may exist some steady states $(D^*, N^*, P^*, S^*, K^*)$ under certain parameter setting. Once the system reaches a steady state, the time subscripts of variables can be left out. At the steady state, $D_t = D^* = 0$, the five-dimensional system (D, N, P, S, K) degenerates to a two-dimensional system (S, K)

$$S = Y - C_{auto} - mpc(Y - S), \tag{4.31}$$

$$K = (1 - \delta)K + S, \tag{4.32}$$

while $D^* = 0$ and N^* and P^* are determined by the trajectory along which the system reaches the steady state.

Eq. (4.31) and Eq. (4.32) are further rewritten as follows

$$Y = \frac{C_{auto}}{1 - mpc} + S, \quad (4.33)$$

$$S = \delta \cdot K. \quad (4.34)$$

At the long-run equilibrium, saving equals investment. The investment, as the newly produced capital, compensates for the depreciation of physical capital. Substitute $Y = A \cdot K^\alpha$ and Eq. (4.34) into Eq. (4.33), the following equation with fractional order is derived

$$AK^\alpha = \frac{C_{auto}}{1 - mpc} + \delta \cdot K. \quad (4.35)$$

The real economy in this model is depicted by the production process. That is to say, my model focus more on the supply side of an economy rather than on the demand side as in Westerhoff (2012) and Lengnick and Wohltmann (2013). But still, the income-expenditure relation, *i.e.* $Y = C + I + G + NX$, which says the national income Y is the sum of consumption C , investment I , government expenditure G and net export NX , is maintained when the system is in equilibrium. This model presents a closed economy without government. The income-expenditure relation is simplified to $Y = C + I$, which perfectly corresponds to Eq. (4.35). That is to say, at the long-run equilibrium, the aggregate supply is fully utilized to satisfy people's consumption and to compensate the depreciation of physical capital.

Let $f(K)$ denote the right-hand side of Eq. (4.35) and let $g(K)$ denote the left-hand side of Eq. (4.35). As shown in Fig. 4-4, $g(K)$ is a strictly concave increasing function of K passing through $(0, 0)$ and $(1, A)$ in the $(K, g(K))$ plane. Depending on parameters, there may be zero or one or two real roots³. A saddle-node bifurcation happens when two steady states emerge. The steady state with a higher capital level is stable, and the other is unstable. The analysis of the stability of steady states is straight forward after

³To keep our discussion economically meaningful, we focus on real solutions and ignore complex ones. Mathematically, there are two conjugate complex roots when no real root exists.

Figure 4-4: A sketch of Equation 4.35 when two real roots exist.

rewriting Eq. (4.35) to $K = \frac{1}{\delta} \left(AK^\alpha - \frac{C_{auto}}{1-mpc} \right)$.

Proposition 4 *Steady states of the two-dimensional system (S, K) are determined by parameter A , α , δ , mpc and C_{auto} together. The number of steady states changes from zero to one to two via a saddle-node bifurcation when A or α increases or when δ or mpc or C_{auto} decreases. For given α , δ , mpc and C_{auto} , no steady state exists if $A < (\frac{\delta}{\alpha})^\alpha (\frac{C_{auto}}{(1-mpc)(1-\alpha)})^{1-\alpha}$, unique steady state exists if $A = (\frac{\delta}{\alpha})^\alpha (\frac{C_{auto}}{(1-mpc)(1-\alpha)})^{1-\alpha}$, and two exists when $A > (\frac{\delta}{\alpha})^\alpha (\frac{C_{auto}}{(1-mpc)(1-\alpha)})^{1-\alpha}$.*

Fig. 4-5 shows the saddle-node bifurcation while parameter A increases. In the (A, K) plane, the bifurcation happens at the point $((\frac{\delta}{\alpha})^\alpha (\frac{C_{auto}}{(1-mpc)(1-\alpha)})^{1-\alpha}, \frac{C_{auto}\alpha}{\delta(1-mpc)(1-\alpha)})$.

In Eq. (4.35), though multiple steady states may exist, only one is stable. Let $\tilde{K}$ denote the value of capital at the stable steady state. The system converges to the stable steady state

$$\begin{aligned} D &= 0, N = N^*, P = P^*, S = S^* = \delta \tilde{K}, K = K^* = \tilde{K}, \\ Y &= A \tilde{K}^\alpha, F = j A \tilde{K}^\alpha, B = \delta \tilde{K}, C = A \tilde{K}^\alpha - \delta \tilde{K}, W = N^* P^* + (1 + \delta) \tilde{K}. \end{aligned}$$

The total number of stock shares in the dynamic equilibrium N^* is path dependent to the convergence process. The equilibrium stock price P^* is determined by the balance of fundamentalists' and chartists' demands for stock shares, and it is not necessarily

Figure 4-5: A saddle-node bifurcation happens when the total factor productivity parameter A increases.

equal to the fundamental. As $D = 0$, P^* is the real solution of $P^3 - 3FP^2 + (3F^2 - b/a)P + (bF/a - F^3) = 0$, where $F = jA\tilde{K}^\alpha$.

4.3.2 Stock bubbles

At the stable steady state, the stock price does not change, because fundamentalists' and chartists' market orders perfectly offset the other's, leading to a zero aggregate demand. However, the stock price can constantly deviate from its fundamental. Let X_t denote the deviation

$$X_t = P_t - F_t. \quad (4.36)$$

A positive value shows a bubble, and a negative value means the stock is undervalued.

As mentioned before, $E_t^f(F_{t+1})$ denotes fundamentalists' current estimate of the fundamental in the next period. A simple linear demand follows $D_t^f = a \cdot (E_t^f(F_{t+1}) - P_t)$. A nonlinear demand is given by $D_t^f = a \cdot (E_t^f(F_{t+1}) - P_t) \cdot f(P_t)$ while $P_t \in [m, M]$ in Day and Huang (1990) which is bounded in an interval with f as a bimodal function; or by $D_t^f = a(F_t - P_t)^3$ as in Westerhoff (2012) and in my model. $E_t^c(F_{t+1})$ denotes chartists' estimate or observation of the fundamental. A liner demand is given by $D_t^c = b(P_t - E_t^c(F_{t+1}))$. Limited by the information available to them, chartists' estimate

of fundamental is heavily based on the history. In practice, technical analysts usually take the moving average of past prices as their estimate. This makes their behavior similar to trend following. In my model, though fundamentalists have access to the true fundamental, the stock price can constantly deviate from the fundamental because of chartists' opposite belief and their biased estimate of the fundamental.

Example 1 (Linear demand) *When both fundamentalists' and chartists' demands are linear, i.e. $D_t^f = a \cdot (E_t^f(F_{t+1}) - P_t)$ and $D_t^c = b \cdot (P_t - E_t^c(F_{t+1}))$, the aggregate demand is $D_t = D_t^f + D_t^c = (a \cdot E_t^f(F_{t+1}) - b \cdot E_t^c(F_{t+1})) + (b - a) \cdot P_t$.*

1) *If $E_t^f(F_{t+1})$ is a biased estimate of the true fundamental and $E_t^c(F_{t+1})$ is a biased observation of past fundamental, i.e. $E_t^f(F_{t+1}) = F_{t+1} + m$ and $E_t^c(F_{t+1}) = F_{t-l} + n$, at steady states, $F_{t+1} = F_{t-l} = F, P_t = P, D_t = 0$, and the price deviation $X = P - F = \frac{am-bn}{a-b}$. If $a = b$, then $m = n$ under equilibria and X is arbitrary.*

2) *If $E_t^f(F_{t+1})$ is a biased estimate of the true fundamental and $E_t^c(F_{t+1})$ is a biased observation of past price, i.e. $E_t^f(F_{t+1}) = F_{t+1} + m$ and $E_t^c(F_{t+1}) = P_{t-l} + n$, at steady states, $F_{t+1} = F, P_t = P_{t-l} = P, D_t = 0$, and the price deviation $X = P - F = m - \frac{b}{a}n$.*

Parameters a and b show the comparison of fundamentalists' and chartists' market power and trading willingness. Note that a and b are positive by definition. Parameters m and n show the biases of fundamentalists' and chartists' estimates. When $E_t^f(F_{t+1})$ is unbiased, i.e. $m = 0$, the price deviation is $X = \frac{-bn}{a-b}$ in case 1) and $-\frac{b}{a}n$ in case 2). When $E_t^c(F_{t+1})$ is unbiased, i.e. $n = 0$, chartists' choice over past fundamental or past price do not change the convergence of stock price to its fundamental. But when chartists' estimate is biased, price deviation exists and there may be a bubble.

When chartists take the past fundamental as their estimate of the fundamental as in the case 1), the comparison of fundamentalists' and chartists' market power matters. If fundamentalists are more powerful than chartists, i.e. $a > b$, fundamentalists can draw the price close to the fundamental, around which chartists with pessimistically (optimistically) biased estimate, i.e. $n < 0$ ($n > 0$), tend to buy (sell) more shares. As a result, there is a positive (negative) price deviation under steady states. Alternatively, if chartists are more powerful than fundamentalists, i.e. $b > a$, fundamentalists lose

control over the price. Chartists' biased estimate of the fundamental will be partially fulfilled as $\lim_{a/b \rightarrow 0} X = n$.

When chartists take the past price as their estimate of the fundamental as shown in the case 2), their behavior is trend following. They always tend to buy (sell) more shares when they have pessimistically (optimistically) biased estimates, leading to a positive (negative) price deviation. The comparison of fundamentalists' and chartists' market power, *i.e.* a and b , affects the direction and size of price deviation.

As discussed above, the stock price can constantly deviate from its fundamental because of agents' opposite beliefs and their biased estimates of the fundamental. If fundamentalists understand the underlying mechanism, they can intentionally bias their estimates to eliminate such price deviation, for example by setting $m = \frac{b}{a}n$ in both cases.

Example 2 (Nonlinear demand) *When fundamentalists have nonlinear demand*

$$D_t^f = a(E_t^f(F_{t+1}) - P_t)^3$$

and chartists' demands are linear

$$D_t^c = b(P_t - E_t^c(F_{t+1})),$$

the aggregate demand is

$$D_t = D_t^f + D_t^c = a(E_t^f(F_{t+1}) - P_t)^3 - b(E_t^f(F_{t+1}) - P_t) + b(E_t^f(F_{t+1}) - E_t^c(F_{t+1})).$$

1) If $E_t^f(F_{t+1})$ is a biased estimate of the true fundamental and $E_t^c(F_{t+1})$ is a biased observation of past fundamental, *i.e.* $E_t^f(F_{t+1}) = F_{t+1} + m$ and $E_t^c(F_{t+1}) = F_{t-l} + n$, at steady states, $F_{t+1} = F_{t-l} = F$, $P_t = P$, $D_t = 0$, and the price deviation X is the real root of $a(X - m)^3 - b(X - m) - b(m - n) = 0$. If $m = 0$, the price deviation X is the real root of $aX^3 - bX + bn = 0$.

2) If $E_t^f(F_{t+1})$ is a biased estimate of the true fundamental and $E_t^c(F_{t+1})$ is a biased observation of past price, *i.e.* $E_t^f(F_{t+1}) = F_{t+1} + m$ and $E_t^c(F_{t+1}) = P_{t-l} + n$, at steady states, $F_{t+1} = F$, $P_t = P_{t-l} = P$, $D_t = 0$, and the price deviation is $X = m - (\frac{b}{a}n)^{1/3}$.

The calculation of price deviation under nonlinear demands can be tedious because of cubic equations. But the economic intuition of price deviation is similar to the previous analysis with linear demands. So I leave the discussion here.

4.4 Numerical analysis

4.4.1 Constraints on liquidity, investment and consumption

Built upon simple equations, this model is incapable of preventing extremely unrealistic situations like the divergence of stock price or negative output from happening, unless some constraints are imposed. In practice, investors' behavior is curbed by a limited resource. To make the model more realistic, restrictions are put on agents' funding liquidity, their purchase of bond and stock, and their consumption. In short, there is no short sale of bond or stock. Agents cannot borrow money to make financial investments. Minimum autonomous consumption must be satisfied. Agents' cash saving S_t^h shows their funding liquidity, the shortage of which ($S_t^h < 0$) can force agents to clear their stock positions.

In this section, I relax the assumption of $S_t^h = B_t^h$ used in the last section. Instead, I assume agents' demand for bond is inertial. It changes slower than the available cash saving changes. In practice, such inertia may be caused by the limited supply of bond or by investors' preference for funding liquidity. In this model, a moving average of past cash saving $MA(S_t^h, \tau)$ with a time frame τ is used to denote agents' initial demand for bond when no constraint is imposed

$$MA(S_t^h, \tau) = \frac{1}{\tau}(S_t^h + S_{t-1}^h + \cdots + S_{t-\tau+1}^h). \quad (4.37)$$

Constraints on bond

- 1) *There is no short sale of bond.*
- 2) *Agents' initial demand for bond is constrained by their budget, i.e. their current liquidity. Agents cannot borrow money to buy bond.*

Agents' final demand for bond under constraints is as follows

$$B_t^h = \begin{cases} 0 & (S_t^h \leq 0) \cup (MA(S_t^h, \tau) \leq 0) \\ S_t^h & 0 < S_t^h < MA(S_t^h, \tau) \\ MA(S_t^h, \tau) & S_t^h \geq MA(S_t^h, \tau) > 0 \end{cases}.$$

In practice, investors' initial demand for stock shares may not be fully satisfied because of the bid-ask spread, limited opposite orders and so on. As a result, the actual demand which is eventually satisfied may be different from investors' intended demand. Since the market maker provides supply or demand to clear the market, I focus on internal constraints which are caused by the limited resource available to agents. Let ID_t^h denote agents' initial demand for shares, which is discussed in the last section. Let D_t^h denote agents' final demand after constraints. Though there is no short sale of stock, agents can sell shares if they initially hold some.

Constraints on stock

- 1) *There is no short sale of stock.*
- 2) *Agents' purchase of stock is constrained by their budget, i.e. their current liquidity. Agents cannot borrow money to buy stock shares.*
- 3) *When lack of liquidity, agents have to sell stock shares to close the liquidity shortage without selling short.*

Agents' final demand for stock under constraints is as follows

$$D_t^h = \begin{cases} -N_t^h & (S_t^h < -N_t^h P_t) \cup (ID_t^h < -N_t^h) \\ S_t^h / P_t & -N_t^h P_t \leq S_t^h \leq 0 \\ ID_t^h & (S_t^h > 0) \cap (-N_t^h \leq ID_t^h \leq W_{t+1/3}^h / P_t) \\ W_{t+1/3}^h / P_t & (S_t^h > 0) \cap (ID_t^h > W_{t+1/3}^h / P_t) \end{cases}.$$

Agents' consumption consists of autonomous consumption and induced consumption. Agents' autonomous consumption must be satisfied under any conditions so that they will not starve to death. Agents' income, from which agents derive their induced consumption, is the total revenue from production and financial investment, including bond and stock, i.e. $(W_{t+2/3}^h - W_t^h)$. Agents' consumption under constraints is shown

Figure 4-6: Agents' consumption. The thin straight line is their initial consumption. The thick polyline shows their actual consumption under constraints.

in Fig. 4-6.

Constraints on consumption

- 1) *Autonomous consumption must be satisfied for agents to survive.*
- 2) *Agents' induced consumption is constrained by their income. This means, at some level of income, agents live from paycheck to paycheck.*

Agents' consumption under constraints is as follows

$$C_t^h = \begin{cases} C_{auto}^h & (W_{t+2/3}^h - W_t^h) < C_{auto}^h \\ (W_{t+2/3}^h - W_t^h) & C_{auto}^h \leq (W_{t+2/3}^h - W_t^h) \leq \frac{C_{auto}^h}{1 - mpc^h} \\ C_{auto}^h + mpc^h(W_{t+2/3}^h - W_t^h) & (W_{t+2/3}^h - W_t^h) > \frac{C_{auto}^h}{1 - mpc^h} \end{cases} .$$

Besides explicit constraints shown above, there is one implicit constraint on funding liquidity, which is embedded in previous constraints on bond and stock. It has the highest priority among agents' financial constraints. That is, when facing a liquidity shortage, an agent cannot participate in the bond market, and he has to sell stock to regain liquidity if possible.

These constraints are all initiated with strong economic intuition. They curb the model in a realistic way. So then, plenty of interesting patterns emerge. Moreover, these

Table 4.1: Parameter values

Notation	Description	Value
γ	Price adjustment speed	0.2
A	Total factor productivity	5
α	Output elasticity of capital	0.7
r	Risk-free interest rate	0.0001
C_{auto}^f	Fundamentalists' autonomous consumption	2
C_{auto}^c	Chartists' autonomous consumption	1
a	Fundamentalists' market power	0.01
b	Chartists' market power	0.2
j	Rescaling production output to stock fundamental	0.01
δ	Depreciation rate of physical capital	0.2
mpc^f	Fundamentalists' marginal propensity to consume	0.5
mpc^c	Chartists' marginal propensity to consume	0.5
τ	Agents' inertia of bond purchase and the time lag of chartists' observation of past fundamental	60

constraints become a part of the model dynamics and even trigger critical events in the formation and burst of stock bubbles.

4.4.2 Parameters

Parameter settings in the simulation are shown in Table 4.1.

According to the discussion on steady states in the last section and the parameter values in Table 4.1, a long-run equilibrium can be found in a deterministic price dynamics. There are two steady states in the production sector, one stable and the other unstable. At the stable steady state, *i.e.* the long-run equilibrium, the total capital before production is 45587.7, 9123.54 units of output are produced, and the true fundamental of the stock price is 91.2354.

Run the simulation with a stochastic price dynamics where the error term follows a normal distribution $N(0, 0.1)$. Fundamentalists are initially endowed with 400 cash saving, 100 capital stock, and 100 shares of stock. Chartists are endowed with 500 cash saving and 100 shares of stock. The initial stock price is 5 per share. Fundamentalists and chartists are equally wealthy at the beginning. Fig. 4-7 shows how the stock price and its fundamental evolve. If the price dynamics is deterministic, both the price and its fundamental will converge to the long-run equilibrium level, *i.e.* 91.2354. While, as

Figure 4-7: A typical simulation of stock price series and its fundamental.

the simulation is under a stochastic price dynamics, the price and its fundamental move towards the long-run equilibrium level at first, and then fluctuate around it. Moreover, the stock price and the fundamental show special patterns after the steady state is approximately achieved. There are stock cycles of bubbles and crashes. Though the shapes of cycles vary in details, these cycles reappear for sure.

4.4.3 Four phases of a stock cycle

To examine stock cycles closely, zoom in the Fig. 4-7. The stock price series from period 2000 to period 3000 in Fig. 4-7 shows a cycle of the formation and crash of a bubble. I divide this cycle into four phases: accumulation, boom, crash and recovery, as shown in Fig. 4-8. Such division focuses on the motion of stock price and its comparison with the fundamental⁴.

⁴It is similar to the accumulation, markup, distribution and markdown phases widely used in technical analysis. The later one ignores the fundamental as it is out of their concern.

Fig. 4-9 shows the deviation of stock price from its fundamental. Positive values denote the existence of bubbles. Negative values mean the stock being undervalued. Fig. 4-10 presents fundamentalists' and chartists' initial demand for stock shares and their final demand under constraints. Fundamentalists are barely constrained, so the curves of their intended demand and their demand under constraints overlap. Chartists are frequently constrained during the accumulation phase and the crash phase. Therefore, chartists' demand under constraints deviates from their intended demand in these two phases. Fig. 4-11 shows agents' cash saving and their purchase of the bond. Because of the inertia and constraints on bond purchase, agents hold large liquid cash when they sell stock shares. This causes two segments of large excess cash saving, which is a waste of liquidity.

Now, I elaborate what happens during each phase of the stock cycle in Fig. 4-8. Fig. 4-10 and Fig. 4-11 to better understand the development and changes of four phases.

Accumulation phase The true fundamental and chartists' observation of past fundamental fluctuate at the same level. The stock price fluctuates at a level slightly lower than the fundamental does. As the price is close to its fundamental, fundamentalists' demand for stock is small but persistent. Throughout the accumulation phase, fundamentalists continuously buy stock shares and gradually accumulate a large position. Meanwhile, their bond holdings decline. As for chartists, they want to sell out stock shares, but their intended supply of stock is invalid because they have no stock shares to sell. Unable to participate into the stock market, chartists accumulate cash and bond while the production of goods proceeds. The increase of chartists' bond offsets the decline of fundamentalists' bond. A stable and sustainable production is maintained thanks to this balance. At this stage, the stochastic factor in the price dynamics has a large impact on the price change. In technical analysis, the fundamental value works as a resistance line. The stock price randomly strikes the resistance line, again and again, driven by the stochastic factor. But beneath the randomness, the ending of the current accumulation phase and the advent of a boom phase are supported by the sustainable production and chartists' wealth accumulation.

Boom phase After the stock price breaks through a resistance line, the old resistance line becomes a new support line. Fundamentalists start to sell stock shares as they feel that the stock is overvalued. Chartists start to buy stock as they believe the stock price will further increase. Chartists buy more shares than fundamentalists sell. Together, there is a large excess aggregate demand, which drives the stock price up. A stock bubble forms in the boom phase.

Fundamentalists' cash saving increases while they sell out shares. Their purchase of bond increases too, but with a slower speed because of the inertia of bond. Chartists' cash saving drops as they use the money to buy shares. Their purchase of bond drops at the same speed as their saving drops under the budget constraint. Consider the changes of the population's holding of cash and bond, the increase of fundamentalists' cash saving surpasses the decrease of chartists' cash saving, but the drop of chartists' bond purchase outpaces the rise of fundamentalists' bond purchase (Fig. 4-11), because of the inertia of bond and the budget constraint on purchasing bond. As a result, though the total amount of liquid cash increases in the boom phase, the investment in bond drops with fluctuations. Since the bond is used to raise fund for further production, a lower amount of bond leads to a lower amount of output, and further a lower level of stock fundamental. Detailed fluctuations of the fundamental are caused by the subtle comparison of fundamentalists' and chartists' holdings of bond.

In short, a stock bubble forms during the boom phase. Chartists invest a large amount of their wealth in the stock market. Such behavior draws resource from the production, leading to lower fundamental. The boom phase shows that a prosperous speculative stock market can harm the real economy by drawing resource from future production.

Crash phase In the crash phase, the stock price drops dramatically and the bubble bursts. A crash phase is further divided into three sub-phases: initial rapid drop sub-phase, fluctuating drop sub-phase and further rapid drop sub-phase.

Initial rapid drop The stock price drops sharply in the initial rapid drop sub-

phase, showing the burst of the bubble. The advent of crash phase is triggered by chartists' liquidity shortage. At the end of the previous boom phase, chartists use up their cash saving and expose themselves to the liquidity shortage. Under the liquidity constraint, agents have to sell stock shares to regain liquidity, despite their intended demand. Though chartists' initial intention is to buy more shares, they are forced to sell stock shares, as fundamentalists do. The huge aggregate supply of stock presses down the stock price heavily. The drop of stock price further lowers agents' total wealth and increases the probability that chartists may be exposed to liquidity shortage again in the next period. Throughout the initial rapid drop sub-phase, chartists are constantly limited by the liquidity constraint. Their total position of stock reduces in size and their wealth shrinks dramatically. In the previous boom phase, chartists' bond holding continuously drops. Now, chartists' bond cannot drop any further after it hits zero. Besides, chartists cannot buy the bond as they face liquidity shortage. Fundamentalists totally determine the aggregate bond. For fundamentalists, because of the inertia of bond, though their wealth and cash saving decrease in the current sub-phase, their investment in bond still increases for a while before it drops. As a result, the fundamental increases suddenly at first and then drops back to the long-run equilibrium level in this sub-phases. Compare the stock price and its fundamental; the price is at first higher and then lower than its fundamental. So, fundamentalists at first sell and then buy shares. At the end of the current sub-phase, fundamentalists' demand for shares slows the price drop. Meanwhile, chartists gradually ease their liquidity shortage after selling out stock shares.

Fluctuating drop In the fluctuating drop sub-phase, the price drop slows down with many fluctuations. This sub-phase seems like a market correction located in the middle of a bear market, between two rapid price drops. The stock price fluctuates around the long-term equilibrium of the fundamental. At the first half part of this sub-phase, chartists constantly face liquidity con-

straint. They sell shares, but only sell small amount as the liquidity shortage is low. At the second half part of this sub-phase, chartists are limited by the liquidity constraint from time to time. When the constraint is invalid, chartists' stock purchase leave them a negative saving. Therefore, in the next period, they have to sell roughly the same amount of shares to regain liquidity, after which they can buy shares again, and then sell out of constraint, and so on and so forth. The liquidity constraint and chartists' initial demand for stock shares produce chartists' fluctuating demand, shown in Fig. 4-10. As chartists' bond holding is zero, fundamentalists' bond holding determines the real production and further the fundamental. After the previous rapid drop of stock price, fundamentalists' wealth shrinks, so does their inertial bond holding. As the total bond drops, the fundamental drops dramatically in the current sub-phase. Fundamentalists' demand for the stock is still positive, but its size becomes small. Fundamentalists' little demand and chartists' fluctuating demand lead to fluctuating stock price in the current sub-phase. Usually, the fluctuating drop sub-phase lasts about the same length as the time lag of chartists' observation of past fundamental.

Further rapid drop The stock price declines sharply again in the further rapid drop sub-phase. This is caused by chartists selling out stock. As chartists' observation of past fundamental is lagged, at the beginning of this sub-phase, they start to notice the increase of recent fundamental which happened during the initial rapid drop sub-phase. Once their observation of past fundamental is higher than the price, they start to sell shares. Meanwhile, fundamentalists' demand for shares is ignorable at the beginning as in their opinion the price deviation is small. Together, the stock price drops sharply. Later, when the stock price is significantly below its true fundamental, fundamentalists buy more shares. This decelerates the price drop. At the end of the further rapid drop sub-phase, chartists have cleared out their stock position. There is no more stock shares for them to sell. The stock price reaches its bottom. By the end of this sub-phase, chartists' observation of past fundamental falls

Figure 4-8: Four phases of a stock cycle: Accumulation, Boom, Crash and Recovery. Shaded and unshaded areas indicate different phases.

back to the level around the long-run equilibrium of the fundamental.

Recovery phase In the recovery phase, the stock price rises from the bottom of a cycle to a level lower than the fundamental. Since chartists have exited from the stock market, the change of stock price is driven by fundamentalists in this phase. When the price is far below the true fundamental, fundamentalists have a large demand for stock shares. This demand pushes the stock price up. However, the price does not completely converge to the fundamental. As the price moves towards its fundamental, fundamentalists' demand becomes smaller and smaller. At some point, the stochastic factor surpasses fundamentalists' demand and dominates the price change. Then, the recovery phase ends. A stock cycle of four phases completes.

The four phases of a stock cycle, *i.e.* accumulation, boom, crash, and recovery, always happen successively. It is uncertain how long an accumulation phase lasts. A boom phase may start whenever the stock price surpasses the fundamental. During the boom phase, chartists continuously buy stock and gradually exhaust their liquid

Figure 4-9: Deviation of stock price from its fundamental.

Figure 4-10: Fundamentalmists' and chartists' intended demands for stock shares and their actual demands limited by constraints.

Figure 4-11: Agents' cash saving and their purchase of bond.

cash saving. Chartists facing a liquidity shortage triggers a crash phase. Later, after chartists clear their position of stock, the crash phase ends, and a recovery phase starts. Finally, the recovery phase ends after fundamentalists bring the stock price close to its fundamental, and a stock cycle is completed.

Stock bubbles are usually defined as the stock price far beyond its fundamental. As shown in Fig. 4-9, a bubble forms and grows during the boom phase and bursts in the crash phase. During the crash phase, the bubble disappears almost instantly. After a rebound, the stock price further drops to a level of severe undervaluation. Even after the recovery phase, the stock is undervalued to some extent. Such undervaluation continues in the next accumulation phase. No bubble forms until the next boom phase.

Analyzing agents' demand for shares, especially chartists' demand, is critical to understand the price move better. Constraints are imposed on agents' financial investments, but these constraints barely limit fundamentalists' action. Because they have abundant resources, like plenty of liquid cash saving, large positions of bond and stock shares. In contrast with fundamentalists, chartists have fewer resources. These constraints substantially limit them. Under the short-sale constraint on the stock, chartists participate in the stock market only in the boom phase and in the crash phase. Chartists buy shares in the boom phase and sell, willingly or forced, in the crash phase. That

is to say, the formation of bubbles and their burst are driven by chartists' demand or supply of stock shares under constraints. Though fundamentalists are involved in the whole process too, they stabilize the price by placing orders opposite to chartists'. In general, fundamentalists are much wealthier and holding larger stock position, compare with chartists. However, chartists are capable of showing large impact on the stock market via bubbles and crashes.

Because of the inertia of bond purchase and the budget constraint, there are two periods during which the liquid cash saving is higher than the bond purchase. That is, the liquid cash is not fully utilized. The first period lasts from the boom phase to the first sub-phase of the crash phase. It is caused by fundamentalists' inertial bond purchase. The second period lasts from the last sub-phase of the crash phase to the recovery phase, and it is caused by chartists' inertial bond purchase.

Stock cycles recur in the simulation. Fig. 4-12 shows detailed price move of other stock cycles in Fig. 4-7. The accumulation, boom, crash and recovery phases can be identified in each cycle, while variations in details exist. For example, the stock prices fluctuate a lot through the boom phase in the middle panel. It seems that the subtle balance between fundamentalists' and chartists' market power produces an ascending channel, along which the price climbs to the peak.

The nonlinearity in the model comes from the Cobb-Douglas production function and financial constraints imposed on agents. Though fundamentalists' demand for stock is nonlinear, it is not essential. Similar patterns of stock cycles can be reproduced under a linear demand of fundamentalists. Only the nonlinearity from the Cobb-Douglas production function is critical to the robustness of the model. Based on Eq. (4.35), the key parameters in the model are the depreciation rate of physical capital δ , the total factor productivity A , and the output elasticity of capital α . They strongly affect the level of long-run production and the numerical simulation. Empirical estimates of the output elasticity of capital are around 0.3 for the US, 0.4 for the EU, and 0.6 for China (Zheng et al., 2009). So far, a high output elasticity of capital is necessary to produce the main findings. So is a high depreciation rate of physical capital. At this stage, I interpret the model as an alternative explanation of stock cycles in the economy with

extensive growth (high output elasticity of capital) and fast depreciation of capital.

4.4.4 Discussion of other issues

No strategy switching

Most heterogeneous agent models on financial markets allow agents' strategy switching. The implicit assumption behind strategy switching is perfect information. For agents to evaluate different strategies, they need to know the past profit earned by each strategy. This requires all agents have free access to all historical public and private information. Such assumption is strong, and I exclude it from my model.

However, no strategy switching among agents does not mean a fixed proportion of fundamental analysis and technical analysis. Boswijk et al. (2007) show that the fraction of fundamental and technical analysis varies. But the reason of such variation has not been identified. The variation can be either caused by agents' strategy switching or by the change of the amount of money invested in the market by stubborn fundamentalists and chartists. Theoretic models with strategy switching are built upon the former one. Empirical works does not distinguish these two reasons. The model shows the variation of investment size of fundamental and technical analyses (Fig. 4-13), though there is no strategy switching among agents. While chartists participate into the stock market at the boom and crash phases and exit from the stock market at the recovery and accumulation phases, the proportion of stock shares held by them varies from 0 to 8.5%.

Why fundamentalists do not clear their stock holdings

At the boom phase, fundamentalists sell out stock, but they do not clear their positions. This phenomenon is a natural product of the model design. But still, here I try to support it from the viewpoint of company ownership. In practice, some firm owners, especially magnates, care more about the ownership of a company attached to stock shares, rather than capital gains from stock market. For example, Bill Gates will not clear his stock position of Microsoft when the stock price drops. A majority holding

Figure 4-12: More samples of stock cycles.

Figure 4-13: The fraction of stock controlled by technical analysis changes, even though no strategy switching in this model.

of the stock guarantees his control over the company. To such investors, the target of stock investment is more than capital gain. The ownership of a company and business income should also be considered.

In this model, firm owners are assumed as fundamentalists. To improve, one may distinguish fundamentalists as firm owners and fundamentalists as professional investors, *e.g.* institutional investors. They have different accesses to true fundamental, ultimate goals, degrees of activity, trading behaviors and so on. In the current model, they are put into the same group. Being aware of such segmentation of fundamentalists helps to support the phenomenon that fundamentalists adjust, but do not clear, their positions and still holding large positions even when they are selling out.

4.5 Conclusions

This chapter builds a heterogeneous agent model on a stock market with endogenous fundamental values. There is a two-way interaction between the production and the stock price dynamics: the production output determines the true fundamental of the stock; and agents' investment in risk-free asset transits to newly produced capital in future production. The endogenous fundamental and the two-way interaction between production and stock price distinguish this model from others. A long-run equilibrium

of production is discovered under the deterministic price dynamics. Stock bubbles can exist under equilibria when chartists' observation of the fundamental is biased. Stock cycles reappear in the numerical simulation. There are four phases in a stock cycle: accumulation, boom, crash and recovery. The formation of bubbles draws resource from future production in the boom phase. Later, the crash phase is triggered by chartists' liquidity shortage. After chartists clear their position, the crash phase ends and the recovery phase starts. Realistic constraints of funding liquidity, budget, consumption and short-sale of financial assets are imposed on agents. Chartists buy shares in the boom phase and sell, willingly or forced, in the crash phase. Though chartists are less wealthy than fundamentalists, they have a large impact on the stock price. The formation of bubbles and their burst are closely related to chartists' market orders under constraints.

When a real production is embedded into the heterogeneous agent model, I simplify the process so that capital is the only factor of production considered. This keeps the model simple, clear, but sensitive to some parameters. So far, the real production in the model can only be interpreted as in an economy with extensive growth and strong capital depreciation. To extend the explanatory power of the model, I should release the assumption of constant labor in future work. Besides, agents' utility from consumption and leisure should be considered in order to strength the goods market modeling.

Chapter 5

Summary

This thesis studies investors' strategy change frequency, wealth accumulation and the formation of stock cycles in financial markets. Common sense says that quick reaction to the environment is good. In equity markets, investors may frequently change their strategy to pursue higher profit. With limited information, such behavior is rational. However, my study reveals an inconsistency between short-term profit and long-term wealth accumulation which leads to a counter-intuitive phenomenon that investors with faster strategy change behavior end up with less final wealth.

Chapter 2 and 3 investigate the same problem from different viewpoints. Heterogeneous agent models are built in both chapters. Chapter 2 uses historical data of S&P 500 to represent a risky asset. Fundamental analysis generally surpasses technical analysis in all market situations except boom periods. Though investors' strategy change behavior, which is driven by the past performance of strategies, seems reasonable, a faster strategy change does not guarantee a higher final wealth. Same result is reproduced in Chapter 3. Chapter 3 builds a typical heterogeneous agent model. Agents' aggregate demand for risky asset drives the price change of the risky asset. Agents' strategy change shows their ability of adaptive learning. Again, it is found that agents with faster adaptive learning enjoy less wealth in the end. A detailed decomposition of wealth accumulation identifies the inconsistency between short-term profit and long-term wealth accumulation as the main reason. Investors' strategy change driven by short-term profit may not serve the goal of accumulating more wealth well.

The heterogeneous agent model in Chapter 3 falls into a branch of computational economics which is newly established and prospering within the last few decades. Though similar models are widely used to study financial economics, there is room for improvement. Chapter 4 suggests multiple modifications, all aiming to make the model more realistic. The strong assumption of the exogenous fundamental value of the risky asset is replaced by an endogenous fundamental value relating to a production process. The production sector interacts with the financial sector. The assumption of perfect information is relaxed. Economic constraints on budget, short-sale and liquidity are imposed on investors. Investors' behavior at the micro level affects the development of financial sector and production sector at the macro level. Macro data of production and financial asset, in turn, feed back into investors. As a result, stock bubbles form and burst in stock cycles.

As mentioned in Chapter 1, nowadays households are widely involved in financial markets. The update of their investment portfolio may show considerable influence on their general wealth accumulation. This thesis has studied how wealth is accumulated via financial investments. However, it has not yet linked to households' general wealth accumulation which covers aspects like income, saving, bequest, and so on. To extend the current work to households' general wealth accumulation, I need a model integrating a production sector, a financial sector, a goods market and a labor market. The model in Chapter 4 embeds a real production process into a heterogeneous agent model. Though it is still far from what I plan for future research, at least it shows a beginning. Besides, as Chapter 4 tracks agents' income and wealth, in the future research on households' general wealth accumulation, issues like economic inequality and the linkage from income inequality to wealth inequality can be investigated.

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