Study of LQR weight matrix on Cp-Max

Yan, Feifei

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Garching bei München, July 2015

Masterarbeit

Study of LQR weight matrix on Cp-Max

Yan Feifei
WE-MT-01-03

Author:	Yan Feifei
Registration number:	03656259
Professor:	Carlo L. Bottasso
Supervisor:	Pietro Bortolotti

Garching bei München, July 2015

STATEMENT OF AUTHORSHIP

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ABSTRACT

In this study the effect of the weights of an LQR controller into the design of a conceptual 10 MW offshore wind turbine is analyzed. First, a load analysis is performed in the multi-body based aeroelastic code Cp-Lambda at varying sets of controller weights. Then, different wind turbine rotors are designed at different set of LQR weights. A design improvement in terms of blade mass and blade cost is found, while changes in the cost of energy are limited. Two weight parameters, Q_3 and R_{31} have large influence on wind turbine design. When Q_3 is enlarged by 3 times, blade mass and cost can be reduced by 2.1% and 1.45% respectively with -0.01% change of CoE. When R_{31} is enlarged by 3 times, blade mass and cost can be reduced by 2.86% and 1.37% respectively with 0.009% change of CoE.

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CHAPTER 1

Introduction

1.1 Background

A wind turbine is a machine that converts the power in the wind into electricity. The initial idea of a wind turbine comes from wind mill, which

Figure 1.1: Worldwide growth of wind energy usage

transfers wind power into mechanical power. And the use of wind for electricity generation begins at the end of 19th century when electrical generators appeared. Since early 1990s, there has been a re-emergence in wind energy industry. The averaged size of new invented wind turbines, as illustrated in Figure.1.1a has increased from 30m in diameter, 20m in height to 120m in diameter, 95m in height. Accompanying the trend to bigger wind turbines is a shift to megawatt-sized wind turbines. The averaged nominal power of new turbines reached 2.6 MW in 2013 [19] and the largest commercial wind turbines nowadays is 178m in diameter 116 m in height, which is capable to produce 10 MW nominal power [18]. A new 20 MW wind turbine is under design and expected to be in use in 2020 as

well.

Thanks to the development of material science, computer science, aerodynamics, analysis methods, testing and monitoring and power electronics hasten over the last 4 decades, wind turbine is becoming more reliable, cost effective, and quieter. Different from other technologies that converts fossil fuels to useful energy, wind turbine can get power from wind repeatedly, which makes it a potential energy supplier in future. However, to enlarge the commercial usage of wind turbines, there will be continuing pressure for designers to improve the cost effectiveness of turbines.

The design of wind turbine should not only satisfy aerodynamic optimisation as described in Reference. [17] [14], but also takes structural analysis into account [15] [3]. To realize the multi-disciplinary wind turbine optimization, a control law is required to optimize structure size under the effect of loads within design constrains. Two main codes that are implemented in such area are RotorOpt [2] and FOCUS5 [11]. The integrated multi-disciplinary wind turbine design tool used in this project is C_P -Max, which takes multi-stage process that first alternates between an aerodynamic shape optimisation step and a structural blade optimisation one, and then combines the two to yield the final optimum solution [6]. In this approach, the design parameters are optimised by maximising or minimising merit functions under various design constrains. And a high fidelity aero-servo-elastic multi body simulator, C_P -Lambda is installed to ensure a high level of fidelity of the design.

1.2 Motivation

The integrated multidisciplinary design of wind turbine depends on a controller, which can optimise blade design within acceptable control power. In C_P -Max design loop, a multi input multi output (MIMO) linear quadratic regulator (LQR) is used. This controller works by minimizing a merit function of state and control vectors within design constrains. Thus, the performance of wind turbine designed by C_P -Max can be influenced by the design of this merit function.

Considering that the merit function in LQR control is based on a weight matrix, which decides the distribution of control power over state and control parameters. It is thus suspected that the cost of energy or blades can be reduced by appropriate setting of LQR weight matrix.

Therefore, the target of this project is

- to study the effect of LQR weight matrix on wind turbines in C_P -Max
- to reduce the cost of energy of wind turbine designed by C_P -Max by changing LQR weight matrix.

1.3 Approach

The reduction of cost of energy is challenging because it is influenced by numerous design parameters. Thus, the effect of LQR weight matrix on CoE is studied by performing simulations. In order to reduce computational cost, a reduced set of simulation is performed in C_P -Lambda controller loop with the adjusted controller weight. The simulation environment is changed by applying three different turbulent seeds into the 11 m/s mean wind. These simulations can indicate the effect of LQR weight matrix on maximum load, energy and actuator duty cycle of wind turbine.

Based on simulations in C_P -Lambda loop, weight parameters that have comparatively significant effect on wind turbine performance are changed in weight matrix. A reduced set of simulations are then performed in C_P -Max design loop with these new weight matrices. By observing the response of CoE and blade cost to the change of weight matrix, new LQR designs are expected to reduce the cost the energy.

1.4 Outline

This report contains five chapters, which are Introduction, Theory, Methodology, Simulations and Results and Conclusions and Recommendations for Future Work.

In Chapter 1, the background, motivation, approach, and outline of this project are introduced, which gives brief idea about the whole project.

In Chapter 2, principles and control strategies of wind turbine machine are explained. A brief description of Linear Quadratic Regular(LQR) used in the wind turbine system is also given in this chapter.

In Chapter 3, the wind turbine design tool, C_P -Max is introduced. And the aero-servo-elastic multi-body simulator, C_P -Lambda and the LQR controller implied in the design system are also described.

In chapter 4, the static and dynamic simulations with LQR controller

weight adjustment in Cp-Lambda and Cp-Max are described. The discussion of simulation result can also be found in this chapter.

Chapter 5 gives conclusion of this project and a discussion about the limitation of this project. It is also followed by recommendations for future work on this topic.

CHAPTER 2

Theory

2.1 Wind turbine design strategy

The wind turbine to be designed in this project is a horizontal axial wind turbine (HAWT). The main subsystems of this wind turbine includes the rotor, the drive train, the nacelle and main frame, the tower and the foundation, the machine controls and the electrical system(Fig.2.1).

Modern wind turbines use basic aerodynamic force of lift to produce a new

Figure 2.1: Major components of a horizontal axis wind turbine (HAWT)

positive torque on a rotating shaft, resulting in the production of mechanical power and then in its transformation to electricity in a generator. Thus, wind turbine blades must not only be designed to convert the kinetic energy in the wind into torque, but at the same time have structural properties that ensure the required static and fatigue strength for a long operation

life. In addition, they must have material and manufacturing costs that is low enough makes for the system to be acceptable in the market place. For a HAWT, a family of airfoil shapes are distributed along its blades. The choice of airfoils, chord length and twist along the blades can be updated with certain design strategies in the design loop. A traditional design strategy is to maximize power coefficient of the rotor given airfoil shapes. Rotors designed under this strategy have optimal aerodynamic efficiency under certain condition, but there will be much loss of aerodynamic strategy under off-design condition. Recently, more concerns are drawn into the financial efficiency of wind turbines. Thus wind turbine design strategy has transferred into the minimum of cost of energy (CoE). And the development of computing power also enables design loop to simultaneously optimize aero and structural constrains.

2.1.1 1-D Momentum theory and the Betz Limit

To predict the performance of a HAWT, a one dimensional simple model is built by Betz (1926) based on a linear momentum theory, where the power of an ideal turbine, the thrust of the wind on the ideal rotor and the effect of the rotor operation on local wind field can be determined.

In this theory, a wind turbine is simplified as a uniform 'actuator disk', around which an imaginary boundary, steam tube is built. Analysis is then performed on a control volume inside the steam tube. Assume that homogeneous, incompressible, steady state fluid flow comes into this control volume through crosssection 1, and leaves through cross-section 2. after passing actuator disk without friction drag (see Figure.2.2). And there is a flow discontinuity is induced by the turbine blades. Apply the conservation of linear momentum to the control volume of the whole system, the thrust can be expressed by

$$F_x = \dot{m}(U_1 - U_4) \quad (2.1)$$

where $\dot{m}$ is the mass flow rate, and $\rho AU_1 = \rho AU_4$ for steady state flow. Then apply Bernoulli function on either side of the actuator disc

$$p_1 + \frac{1}{2}\rho U_1^2 = p_2 + \frac{1}{2}\rho U_2^2 \quad (2.2)$$

$$p_3 + \frac{1}{2}\rho U_3^2 = p_4 + \frac{1}{2}\rho U_4^2 \quad (2.3)$$

Figure 2.2: Actuator disc model of a wind turbine

Assume that far upstream and far downstream pressures are equal ($p_1 = p_4$), and the velocity remains the same when passing the actuator disk ($U_2 = U_3$). The thrust becomes

$$F_x = \frac{1}{2}\rho A(U_1^2 - U_4^2) \quad (2.4)$$

$$= \frac{1}{2}\rho A U^3 4a(1-a)^2 \quad (2.5)$$

where A is actuator disk area, U_1 is replaced by U , and a is the axial induction factor

$$a = \frac{U_1 - U_2}{U_1} \quad (2.6)$$

The power of the rotor can then be calculated by multiplying thrust by income velocity on disk.

$$P = \frac{1}{2}\rho A U^3 4a(1-a)^2 \quad (2.7)$$

from which the expression of pressure coefficient, C_p is defined as

$$C_P = \frac{P}{\frac{1}{2}\rho U^3 A} = 4a(1-a)^2 \quad (2.8)$$

From Equation.2.9 , it can be calculated that the C_P of a HAWT has a maximum value when $a = 1/3$, which is the ideal states of wind turbine.

$$C_{P,max} = 16/27 = 0.5926 \quad (2.9)$$

And the thrust of the rotor is similarly defined by

$$C_{F_x} = \frac{F_x}{\frac{1}{2}\rho U^2 A} = 4a(1 - a) \quad (2.10)$$

C_{F_x} has a maximum of 1 at $a = 0.5$. At maximum power output ($a = 1/3$), C_{F_x} has a value of $8/9$. It should be noticed that this one-dimensional simplified model is only valid when $a < 0.5$, which is so called Betz limit.

2.1.2 Rotor aerodynamics and blade element theory

The aerodynamic performance of a rotor is largely determined by blade geometry. A HAWT is with a family of airfoils along the blades, which generate lifts and drags during the work of wind turbine. And it is important to have a reasonable choice of these airfoils, and distribute their chord length and twist along the blade to satisfy aerodynamic design target, which can be evaluated by annual energy production (AEP) [9].

$$AEP = \int_{V_{in}}^{V_{out}} P(V) f_W dV \quad (2.11)$$

Where $P(V)$ is the designed power distribution over incoming wind speed, f_W is the probability distribution function (PDF) of Weibull distribution, V_{in} and V_{out} are cut in and cut out velocity of wind turbine machine separately (Fig.2.3). To analyze wind turbine rotor behaviour in complex oper-

Figure 2.3: Wind turbine power curve and Weibull distribution

ating environment, it is necessary to use computer codes for blade design. The aeroelastic codes, Cp-Lambda used in this project is based on the

blade element moment (BEM) theory, which is a combination of momentum theory and blade element theory. From momentum theory, thrust and torque on an annular section of the rotor are defined as a function of axial and angular induction factors. While from the element theory, analysis is based on the N blade sections. Forces on the blade is a function of lift, drag coefficients and the angle of attack. The basic parameter definition from blade element theory is shown in Figure.2.4. where

Figure 2.4: Blade geometry for analysis of a HAWT

$U(1 - a)$ = Wind velocity at blades

U_{rel} = Relative wind velocity

β = Section pitch angle

α = Angle of attack

$\phi = \beta + \alpha$ = Angle of relative wind

β_{tip} = Blade pitch angle at tip

$\beta_{twist} = \beta - \beta_{tip}$ = Section twist angle

dF_L = Incremental lift force

dF_D = Incremental drag force

dF_N = incremental force normal to the plan of rotation

dF_T = incremental force tangential to the circles sweep by the rotor

Note that the lift and drag are parallel and perpendicular to the relative wind speed. Thus, for a three rotors HAWT, the normal force on the section at a distance, r , from the centre is

$$dF_N = \frac{3}{2} \rho U_{rel}^2 (C_l \cos \phi + C_d \sin \phi) c dr \quad (2.12)$$

The differential torque due to the tangential force operating at a distance, r from centre can be calculated by

$$dT = 3r dF_T = \frac{3}{2} \rho U_{rel}^2 (C_l \sin \phi + C_d \cos \phi) c r dr \quad (2.13)$$

From blade elementary theory, thrust and torque can be defined as a function of the flow angles of the blade and airfoil characteristics. Thus the shape and performance of blade can be related by combining moment theory with elementary theory. For example, the optimal design of blade shape is to approximate one that maximizes the power coefficient within Betz limit.

2.1.3 Blade loads and structure

The interplay of forces from the external environment and the motions of various components of the wind turbine results not only in the desired

energy production of the blade, but also in stresses in the constituent materials. Thus loads withstand should also be taken into consideration in wind turbine design [12].

The most important rotor loads on a wind turbine are those associated with thrust on the blades and torque to drive rotor. Blade bending moments are designated as either flapwise or edgewise. Flapwise bending moments cause the blade to bend upwind and downwind. Edgewise bending moments are parallel to the rotor axis and give rise to the power-producing torque, which is also called 'lead-lag'.

Assume the wind turbine to be an ideal rigid body, the rotor flapwise bending moment, M_β , on a single blade, when working at designed state is

$$M_\beta = \frac{1}{3} \int_0^R r \left(\frac{1}{2} \rho \pi \frac{8}{9} U^2 2r \right) dr = \frac{4}{9} F_x R \quad (2.14)$$

and the maximum flapwise stress, $\sigma_{\beta,max}$ is

$$\sigma_{\beta,max} = M_\beta c / I_b \quad (2.15)$$

where c is the distance from the flapwise neutral axis and I_b is the area moment of inertia of blade cross-section root. Variation in wind flow can deviate the blades from this ideal working state and then induce blade flapping.

The edgewise moments, which gives rise to the power-producing torque are less significant than flapwise moments in terms of blade strength. The mean torque is the power divided by the rotational speed, thus the bending moment in edgewise direction at the root of a single blade is

$$M_\zeta = \frac{P}{3\Omega} = \frac{1}{6} \frac{C_P}{\lambda} \rho \pi R^2 U^2 \quad (2.16)$$

For an ideal rotor, the rotational speed varies with the wind speed, the edgewise moment varies as the square of the wind speed. And rotors designed for higher tip speed ratio experience lower torque. It should be noted that for large sized wind turbine, blades rotating in gravitational field have large effect on lead-lag motion. And this moment can be counter acted by moment induced by centrifugal force.

The interior architecture of blade are designed to withstand extreme loads and to survive many fatigue cycles. A box type three-cell configuration for blade structure is taken in this project, which is made up of 9 main components (shell, 2 spar caps, 3 shear webs, TE, LE and root reinforcement).

As shown in Figure.2.5, webs provides interior strength to the blade. Loads on outer skin are transferred to the shear webs by spar caps. There are also reinforcement components that strengthens blade at its leading edge, trailing edge and root. evaluation of structural design is based on the cost, which is strongly combined with the weight of the system. The design of blade structure is parameterized by defining thickness at multiple stations along the blade span. The component thickness distribution is shown in Figure.2.6. And the structure mass or cost is used to evaluate this design under certain aerodynamic constrains.

Figure 2.5: Typical wind turbine blade structure architecture

Figure 2.6: Typical wind turbine blade structure architecture

2.1.4 Aero-structural combined design

The design of rotor requires a trade off between aero and structural merit [6] [8]. In order to parameterize the combined aero-structural merit, a global parameter, the cost of energy (CoE) is introduced [16]. It is the unit

cost to produce energy (in \$/kWh) from the wind energy system, which is expressed in Equ.2.17.

$$CoE = \frac{TotalCost}{EnergyProduction} = \frac{ICC(P) \times FCR + C_{O\&M}}{AEP(p)} \quad (2.17)$$

Where ICC is the installed capital cost, FCR is the fixed charge rate, $C_{O\&M}$ is the average annual operation and maintenance cost, p denotes the design parameters.

With this design strategy, the aerodynamic shape and structural sizing are strongly combined together in design process, which is performed in a multi-disciplinary optimisation tool, Cp-Max. The detailed introduction of Cp-Max can be found in the following chapter.

2.2 Regulation strategy

2.2.1 $C_P - \lambda$ Curve

The power generated by wind turbine can be expressed as

$$P = \frac{1}{2} \rho A V^3 C_P \quad (2.18)$$

Where ρ is air density, A is the front area of wind turbine, V is incoming wind velocity, C_P is power coefficient.

The power of wind turbine can be influenced by many factors, including the incoming wind velocity, rotor velocity, blade pitch angle and so on. For the convenience of understanding, a new non-dimensional parameter, tip speed ratio (TSR), is defined. It is the ratio between rotor tip speed and incoming wind speed, which is denoted by λ (Equ.2.19).

$$\lambda = \frac{\Omega R}{V_w} \quad (2.19)$$

where Ω is rotation rate, R is rotor radius, V_w is incoming wind speed.

$C_P - \lambda$ curves are used to determine the rotor power of any combination of wind and rotor speed (Fig.2.7). They provide immediate information on the maximum rotor power coefficient and optimum speed ratio [?]. It can be observed that when blade pitch angle, β is fixed, C_P initially increases

Figure 2.7: C_P - λ Curves with various β values

and then decreases with λ . For each pitch angle, there exists a pitch angle which maximizes C_P . It can also be observed that when λ is small, a small β generates high C_P , when the λ is big, a large β generates high C_P .

2.2.2 Regulation policy

To maximize the efficiency of wind turbine, rotation rate and pitch angle are adjusted to reach a power coefficient as high as possible. However, this may lead to an increase of torque oscillation. Therefore, a trade off between efficiency and safety is required in designing of regulation policy. The control strategy of wind turbine is based initially on its rated wind velocity, V_r and maximum power coefficient, C_{Pmax} . And then a rated tip speed ratio can be set accordingly. Depending on the cut-in wind speed, V_{in} , cut-out wind speed, V_{out} , and the rated wind speed, V_r , the work of wind turbine can be divided into several regions. And the control strategy varies with these regions accordingly (Fig.2.8).

In regionII, the incoming wind speed is larger than V_{in} but smaller than V_r , which is also called 'low wind region'. In this region, the blade control strategy is to keep blade tip speed ratio at certain magnitude which maximizes power coefficient. In this region, TSR and blade pitch angle are kept constant, while the rotor rotation rate, magnitude of power and torque increase with wind speed, which can be expressed in the following equations.

Figure 2.8: Wind turbine control strategy with variable V_W

$$P(V) = \frac{1}{2} \rho A V^3 C_P^* \quad (2.20)$$

$$\beta(V) = \beta^* \quad (2.21)$$

$$\Omega(V) = V \lambda^* / R \quad (2.22)$$

$$T(V) = \frac{1}{2} \frac{C_P}{\lambda^*} \rho A V^2 R C_P^* \quad (2.23)$$

In wind region III, relative velocity is larger than V_r but smaller than cut-out velocity V_{out} . This region is also called 'high wind region'. The regulation strategy in high wind region is to keep power generated by wind turbine at a rated value, P_r to avoid wind turbine overload, which can leads to an increase of torque oscillation. Thus, when the incoming wind speed increases from V_r , the rotor rotation rate stops increasing and be constant at Ω^* , and the thrust on rotor becomes constant as well. And β and λ decreases to a certain magnitude that can induce desired C_P corresponding to the wind speed. The change of P , β , Ω and T with wind speed V in

region III is listed below.

$$P(V) = \frac{1}{2}\rho AV^3 C_P(\lambda, \beta) = \text{const} \quad (2.24)$$

$$\beta(V) = \beta(\lambda, C_P^*) \quad (2.25)$$

$$\Omega(V) = \Omega^* \quad (2.26)$$

$$T(V) = \frac{P^*}{\Omega^*} \quad (2.27)$$

2.3 LQR controller

A linear quadratic controller works by minimising a quadratic performance function (cost function) subject to the linear dynamic constraints of the system. Compared to other control methods, LQR controller deals with multivariable systems in a relatively simple way and can be used more universally. It is applicable to both time-varying and time-invariant systems over finite or infinite time intervals.

In order to perform control of wind, a Multi-Input Multi-Output (MIMO) Linear Quadratic Regulator (LQR) is implemented into the wind turbine design loop, where pitch-torque control is performed. A brief introduction about the principles and functions of LQR is provided in this section. And the specific LQR used in the C_P -Max design tool will be described in Chapter 3.

2.3.1 LQR problem statement

The basic optimisation of a LQR control problem is to find a state-feedback control law of the form $u = -Kx$ that minimizes a cost function 2.28, which functions as a quadratic performance measure of the system.

$$J = \int_0^T [x^T(t)Qx(t) + u^T(t)Ru(t)]dt + x^T(T)Mx(T) \quad (2.28)$$

where the system has linear dynamics constraints given by

$$\dot{x} = Ax + Bu \quad (2.29)$$

and Q and M are typically positive-semidefinite matrices, R is a positive-definite matrix, x is an n -dimensional state vector, and u is an m -dimensional

input vector. The final time T is considered fixed while the final state $x(T)$ is free. The state x here represents an incremental state. Thus, the control goal is to keep the state x as close to the state $x_r = 0$ as possible, which is so called a regular problem. And the problem above is referred to as the optimal linear-quadratic-regulator problem(LQR). The term $x^T Q x$ in Equation.3.47 measures control accuracy while the term $u^T R u$ measures control effort, and the term $x^T(T) M x(T)$ is a measure of terminal control accuracy. Thus, the design of Q , R and M matrix can be understood as trade off between the control accuracy, control effort and final state optimisation [21].

2.3.2 Hamilton-Jacobi Equation

An optimal state feed-back control law $u = -Kx$ is defined as the one that minimise cost function over time T given system dynamics. According to the optimality principle, if $u^*(\tau)$ is optimal over the interval $[t, T]$, starting at state $x(t)$, then $u^*(\tau)$ is necessarily optimal over the subinterval $[t + \Delta t, T]$ for any Δt such that $T - t \geq \Delta t > 0$. If the minimal value of cost function is denoted by $J^*(x, t)$ when initial time is t and the initial state is $x(t) = x$ under the feed-back control law $u[t, T]$ over time interval $[t, T]$, then

$$J^*(x, t) = \min_{u[t, T]} \left\{ \int_t^T [x^T Q x + u^T(\tau) R u(\tau)] d\tau + m[x(T)] \right\} \quad (2.30)$$

$$J^*(x, t) = \min_{u[t, t + \Delta t]} \left\{ \int_t^{t + \Delta t} [x^T Q x + u^T(\tau) R u(\tau)] d\tau + J^*[x(t + \Delta t), t + \Delta t] \right\} \quad (2.31)$$

If the term $J^*[x(t + \Delta t), t + \Delta t]$ in Equation.2.31 is written in Taylor-series expansion about the point $(x(t), t)$ with $x(t + \Delta t) - x(t) = [x(t)^T Q x(t) + u^T(t) R u(t)] \Delta t$, then

$$J^*(x, t) = \min_{u(t)} \left\{ (x^T Q x + u^T R u) \Delta t + J^*(x, t) + \frac{\partial J^*(x, t)}{\partial t} \Delta t + \left[\frac{\partial J^*}{\partial x} \right]' \dot{x} \Delta t + o(t) \right\} \quad (2.32)$$

where $o(t)$ denotes higher-order terms in Δt . If the Δt goes to zero and $J^*(x, T) = m(x)$ for all x , the optimization equation can be expressed in

Hamilton-Jacobi equation below

$$-\frac{\partial J^*(x, t)}{\partial t} = \min_{u(t)} \{ (x^T Q x + u^T R u) + \left[\frac{\partial J^*}{\partial x} \right] (Ax + Bu) \} \quad (2.33)$$

$$J^*(x, T) = m(x), \text{ for all } x \quad (2.34)$$

The solution to this optimisation function is obtained by two steps. The first one is to perform the indicated minimization which leads to a control law of the form

$$u^* = \psi\left(\frac{\partial J^*}{\partial x}, x, t\right) \quad (2.35)$$

And the second is to substitute Equation.2.35 back into Equation.2.33 and solve the non-linear, partial-differential Equation.2.37

$$-\frac{\partial J^*(x, t)}{\partial t} = \{ (x^T Q x + \psi^T R \psi) + \left[\frac{\partial J^*}{\partial x} \right]' (Ax + B\psi) \} \quad (2.36)$$

for $J^*(x, t)$ subjected to boundary condition

$$J^*(x, T) = m(x) \quad (2.37)$$

Then the gradient of $J^*(x, t)$ with respect to x can be computed. And hence the optimal state-feedback control law can be obtained in Equation.2.38

$$u^* = \psi\left(\frac{\partial J^*}{\partial x}, x, t\right) = \phi(x, t) \quad (2.38)$$

2.3.3 LQR solution: Matrix Riccati equation

The matrices A , B , Q , R , and M in general may all be time varying. The first step to obtain a solution of LQR problem is to minimise

$$x^T Q x + u^T R u + \left[\frac{\partial J^*}{\partial x} \right]' (Ax + Bu) \quad (2.39)$$

with respect to u . This minimisation can be calculated by setting the gradient of (2.39) to be zero and the u^* can be expressed in Equation.2.40.

$$u^* = -\frac{1}{2} R^{-1} B' \frac{\partial J^*}{\partial x} \quad (2.40)$$

where R to be positive definite is a necessary condition. Since u is a liner-feedback equation of initial state x , the cost function with optimal feedback

control can be expressed in a quadratic form in the initial state of the system. The optimal cost function is assumed to be

$$J^*(x, t) = x^T P(t)x \quad (2.41)$$

where $P(t)$ is symmetric. And the gradient of J^* becomes $2P(t)x$. After substituting Equation.2.41 back into Equation.2.37, it becomes

$$-x^T \dot{P}x = x^T [A^T P + PA + Q - PBR^{-1}B^T P]x \quad (2.42)$$

with the boundary condition

$$J^*(x, T) = x^T P(T)x = x^T Mx \quad (2.43)$$

Since Equation.2.42 and Equation.2.43 are always true for all x , they can be further simplified into matrix Riccati equation with the final boundary value of $P(t)$ [13].

$$\dot{P} = A^T P + PA + Q - PBR^{-1}B^T P \quad (2.44)$$

$$P(T) = M \quad (2.45)$$

Thus the optimal state-feedback control is given by

$$u^* = -K(t)x \quad (2.46)$$

$$K(t) = R^{-1}B^T P(t) \quad (2.47)$$

2.3.4 Steady-State regulated LQR solution

Consider the LQR problem to be steady-state. The input A , B , R , Q and M are assumed time-invariant and its optimization interval is infinite. The optimal state-feedback can be derived from matrix Riccati equation (Equation.2.44).

$$u^*(t) = -R^{-1}B^T \bar{P}x(t) \quad (2.48)$$

where $\bar{P}$ is a time-invariant and satisfies the algebraic Riccati equation (ARE) (Equation.2.49)

$$0 = A^T \bar{P} + \bar{P}A + Q - \bar{P}BR^{-1}B^T \bar{P} \quad (2.49)$$

Note that the solution of a steady-state LQR problem is independent of the matrix M , and $x(T) = 0$, the cost function is simplified into

$$J^* = \int_0^\infty [x(t)^T Qx(t) + u(t)^T Ru(t)]dt \quad (2.50)$$

For a LQR problem with $M = 0$, $R > 0$, $Q = D^T D$, where the pair (A, D) is observable and the pair (A, B) is controllable, there exists a unique positive-definite solution to the ARE (2.49), $\bar{P}$, and the optimal closed-loop system, $\dot{x} = (A - BK)x$, where $K = -R^{-1}B^T \bar{P}$, is asymptotically stable. The linear system $\dot{x} = Ax + Bu$, $y = Cx$ is controllable if and only if

$$\text{rank}[B|AB|\dots|A^{n-1}B] = n \quad (2.51)$$

and observable if and only if

$$\text{rank} \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix} = n \quad (2.52)$$

2.3.5 Selection of Q and R matrices

For a steady-state LQR problem, the optimal state-feedback is independent of M , thus only the selection of Q and R matrices is discussed here. The weight matrices Q and R are normally diagonal, which has the expression

$$Q = \begin{pmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_n \end{pmatrix} \quad R = \begin{pmatrix} r_1 & & & \\ & r_2 & & \\ & & \ddots & \\ & & & r_m \end{pmatrix} \quad (2.53)$$

where $q_1, q_1 \dots q_n$ and $r_1, r_1 \dots r_m$ are weight parameters which are positive. One advantage of LQR control is that the system preserves asymptotic stability with any adjusting of Q within its positive-semidefiniteness constraint and any adjusting of R within its positive-definiteness constraint [1].

Since the term $x^T Q x$ in cost function (Equation.2.50) measures control accuracy while the term $u^T R u$ measures control effort, the design of Q , R matrices can be understood as trade off between the control accuracy and control effort. A large magnitude of a weight parameter therefore can intensify the control of its corresponding variables to the rated value [13].

For example, for a "cheap control" system where the control effort is viewed to be inexpensive the system performance is measured by cost function in the form

$$J^* = \int_0^\infty [x(t)^T Q x(t) + \rho u(t)^T R u(t)] dt \quad (2.54)$$

---

where ρ is a positive scalar that approaches zero.
The design of weight matrices Q and R can be adjusted according to other system performance requirements.

CHAPTER 3

Methodology

In this project, the study of LQR weight matrix design on wind turbine design is based on the multi-disciplinary wind turbine design tool, Cp-Max. In Cp-Max design loop, design parameters are optimised by fulfilling merit functions within constrains [7] [8]. And the evaluation of merit functions and constrains are performed by the simulations in an aero-servo-elastic multi-body simulator, Cp-Lambda.

3.1 C_P -Max

Figure 3.1: C_P -Max design loop

3.1.1 Aerodynamic Optimization

The aerodynamic optimisation is performed in a sequential quadratic process (SQP) based on a fixed structural design [10]. In this process, blade chords, twist and thickness distribution along blade span is designed to maximize AEP under constrains.

Figure 3.2: Aerodynamic parameters optimization

$$\begin{aligned}
 c(\eta) &= s_c(\eta)c_{bl}(\eta) \\
 \theta(\eta) &= s_\theta(\eta)\theta_{bl}(\eta) \\
 t(\eta) &= s_t(\eta)t_{bl}(\eta)
 \end{aligned} \tag{3.1}$$

where s_c , s_θ and s_t are deformation functions calculated by Equ.3.2.

$$\begin{aligned}
 s_c(\eta) &= n_c(\eta)c \\
 s_\theta(\eta) &= n_\theta(\eta)\theta \\
 s_t(\eta) &= n_t(\eta)t
 \end{aligned} \tag{3.2}$$

where n_c , n_θ and n_t are spline shape functions. η , c , θ , and t are non-dimensional blade length, chord, associate nodal twist and thickness. Design parameters, p is then updated into $p_a = \{c(\eta), \theta(\eta), t(\eta)\}$ after aerodynamic optimisation.

It should be noted that at each design loop, the optimisation are constrained by maximum solidity, σ_c and tapering, τ_c . Equ.3.3 and Equ.3.4 are the expression of these two constrains.

$$\sigma_c = \frac{\int_0^1 t \cdot d\eta}{100\%} \tag{3.3}$$

$$\tau_c = \frac{\int_0^1 \eta \cdot t \cdot d\eta}{\int_0^1 t \cdot d\eta} \tag{3.4}$$

The spanwise distribution of airfoil chord, twist angle and blade thickness is shown in Figure.3.2a ,3.2b ,3.2c.

3.1.2 Structural optimization

The structural optimization is performed in an aero-servo-elastic multi-body model, C_P - λ . Structural sizing is performed on both blades and tower,

which is evaluated by the mass or cost of the whole structure. Structural parameters are then updated into $p_s = \{t_b, D_t, t_t\}$, where t_b is blade thickness distributions of structural components, D_t is tower thickness distribution and t_t is wall thickness distribution. Considering that the constraints in this design process are much complex than that in aerodynamic optimisation, a MIMO LQR controller is implemented in the structural design loop. Generally, the loop converges after 2-3 iterations. [5]. Structural optimization is initially performed on blades and tower respectively. Blade structural optimization starts with the definition of aerodynamic parameters, which is the p_a from aerodynamics optimisation, and the definition of structural parameters, which carries information about component thickness distribution, composite materials and structural constraints. A blade model can then be build based on the definition(Figure.3.3). After

Figure 3.3: Blade core component thickness

that, blade cross sectional performance is taken into concern. Anisotropic Beam Analysis(ANBA) is performed on a 2-D sectional model to calculate 6 by 6 stiffness matrices based on the input stresses and strains. After that, a geometrically exact 3-D blade model is built though span-wise interpolation.

Tower optimisation starts with the definition of structural parameters describing diameter and wall thickness distributions. Based on this definition,

tower stiffness matrices on cross sections can be directly calculated as below

$$K_{axial} = A \cdot E \quad (3.5)$$

$$K_{tors} = J_P \cdot G \quad (3.6)$$

$$K_{bending} = 0.5 \cdot J_P \cdot E \quad (3.7)$$

$$K_{shear} = 0.5 \cdot A \cdot G \quad (3.8)$$

where K_{axial} , K_{tors} , $K_{bending}$, and K_{shear} are axial, torsional, bending and shear stiffness, area moment inertia J_P is calculated by $J_P = \frac{\pi}{32} \cdot (D_{out}^4 - D_{in}^4)$, Sectional area $A = \frac{\pi}{4} \cdot (D_{out}^2 - D_{in}^2)$, mass distribution $m = A \cdot \rho$. After that, the geometrically 3-D exact tower model can be built by height-wise interpolation.

The well defined 3-D blade and tower model is then combined to generate

Figure 3.4: Structural parameters of Tower

a complete HAWT C_p -Lambda multi body model. Loads and performance analysis are performed on this complete model by statics or dynamic simulation, and the main results are shown in diagrams. For example, a campbell diagram shows eigenfrequencies and their correspondent rotation rate. C_p -Lambda curves can be used to evaluate the wind turbine aerodynamic performance, to compute the regulation policy and AEP as well.

3.1.3 Global optimisation

Global optimization is a loop which consists of aerodynamic optimization, structural optimization and global sizing. In this loop, global parameters

Figure 3.5: Merit magnitud of iterations

p_g is updated to certain combination that minimizes the merit function. In C_P -Max, cost of energy is used to evaluate the global design. The global parameter $p_g = \{R, \sigma, \tau, H, \gamma, \xi\}$, where R is blade radius, σ denotes blade solidity, τ is blade tapering, H is tower height, γ is rotor cone angle, ξ is nacelle uptilt angle. The reduction of cost after global iterations is shown in Figure.3.5.

3.2 Cp-Lambda

C_P -Lambda (Code for Performance, Loads, Aero-elasticity, by multi-Body Dynamic Analysis) is an aero-servo-elastic multi-body model. This model is built on Cartesian coordinates, where wind turbine are modelled with beams, rigid bodies, joints, actuators with boundary conditions. C_P -Lambda can not only model aerodynamics of the rotor, but also the motions of the elastically deforming blades and the interactions between these blade motions and the flow.

3.2.1 Cartesian coordinate

The multibody systems are built on Cartesian coordinates, which specifies each point uniquely in a plane by a pair of numerical coordinates. The strains to assemble a 3-beam system is listed below

Figure 3.6: 3-beam system in Cartesian coordinate

$$x_1 - \frac{l_1}{2} \cos \theta_1 = 0 \quad y_1 - \frac{l_1}{2} \sin \theta_1 = 0 \quad (3.9)$$

$$x_1 + \frac{l_1}{2} \cos \theta_1 = x_2 - \frac{l_2}{2} \cos \theta_2 \quad y_1 + \frac{l_1}{2} \sin \theta_1 = y_2 - \frac{l_2}{2} \sin \theta_2 \quad (3.10)$$

$$x_2 + \frac{l_2}{2} \cos \theta_2 = x_3 - \frac{l_3}{2} \cos \theta_3 \quad y_2 + \frac{l_2}{2} \sin \theta_2 = y_3 - \frac{l_3}{2} \sin \theta_3 \quad (3.11)$$

$$x_3 + \frac{l_3}{2} \cos \theta_3 = 0 \quad y_2 + \frac{l_2}{2} \sin \theta_2 = 0 \quad (3.12)$$

In Cartesian coordinates, strain energy is unaffected by arbitrarily large rigid motions, which simplifies the expression of the strain energy. And the system is easy to extend in Cartesian coordinate.

3.2.2 Component model

Geometrically exact beam model is built in C_P -Lambda. Considering the asymmetric cross section of blades, a reference line is defined besides its mass centre line. And its sectional frames is assumed to remain planar after deformation. Its stiffness matrix can be obtained by solving 2D FEM sectional problem. And the beams are constrained using augmented Lagrangian method.

Joins are components that connect two bodies with relative motion allowed. In C_P -Lambda, 6 simplest surface contact elements, which is so called lower pairs are defined. Other point or line contact between elements can be expressed as combinations of these lower pairs. Models for flexible joints and

unilateral joints are also built by setting constraints on lower pairs. Models for pitch, torque and yaw actuators combining low frequent structural response with high frequent hydraulic response are built in *C_p-Lambda*. Pitch and yaw actuator are designed 2nd ordered while the torque actuator is 1st ordered. Take pitch controller as an example, the work of this actuator begins with computing bearing reaction from blade inertial and aerodynamic loads. After that, bearing friction and torque can be calculated. And the torque about blade pitch axis within limit can then be applied.

3.2.3 Aerodynamic model

Aerodynamic models in *C_p-Lambda* consists of lifting lines, inflow models and generic interface. Lift, drag and moment coefficient at local stations are got from experimental tables, based on which air-load for components can be calculated using thin airfoil theory. Inflow models are built by Iterating between thrust and torque change from lifting line and that calculated using blade moment theory. Inflow models consists of dynamic Pitt-Peters & Peters-He models and free wake models. To transport air-loads to beam reference line, CFD coupling of fluid dynamic mesh and geometric configuration is required.

3.2.4 Wind model

To simulate the environment the wind turbine works in, pre-calculated time history of wind models are applied. Wind models of deterministic gusts, 3D stochastic turbulence and shear can be built according to IEC 61400-1. Wind speeds distribution over an extend of time is described by Weibull distribution functions (Equation.3.14,3.15).

$$F_W = 1 - \exp\left(-\left(\frac{V_0}{C}\right)^k\right) \quad (3.13)$$

$$f_W = -k \frac{V_0^{k-1}}{C^k} \exp\left(-\left(\frac{V_0}{C}\right)^k\right) \quad (3.14)$$

$$(3.15)$$

where

F_W = cumulative probability function

f_W = probability density function

V_0 = wind speed

C = scale parameter

k = shape parameter

The change of wind velocity with altitude is modelled using Logarithmic or power law profiles. According to power law

$$V(z) = V(z_r) \left(\frac{z}{z_r} \right)^2 \quad (3.16)$$

where

$V(z)$ = Wind speed at height z

z = Height above ground

z_r = Reference height above ground for fitting the profile

z_0 = Roughness length

The expression of wind profile taking hub height as reference is

$$V(z) = V_{hub} \left(\frac{z}{z_{hub}} \right)^2 \quad (3.17)$$

The extreme wind conditions include wind shear events, as well as peak wind speeds due to storms and rapid changes in wind speed and direction are modelled using explicit expressions (Equation.3.19)

$$V_{e50}(z) = 1.4V_{ref} \left(\frac{z}{z_{hub}} \right)^{0.11} \quad (3.18)$$

$$V_{e1}(z) = 0.8V_{e50}(z) \quad (3.19)$$

where V_{ref} = reference wind speed used for defining wind turbine classes. Other special extreme conditions such as extreme operation gust (EOG), extreme direction change (EDC), extreme coherent gust with direction change (ECD) and extreme wind shear (EWS) can also be modelled using similar approach.

Turbulent flow is described using Kaimal spectral and exponential coherent

model or mann uniform shear model. The expression of normal turbulence is

$$\sigma_1 = I_{ref}(0.75V_{hub} + b) \tag{3.20}$$

$$b = 5.6m/s \tag{3.21}$$

where

- σ_1 = Turbulent standard deviation
- I_{ref} = Expected value of turbulence intensity at 15m/s
- V_{ref} = Reference wind speed averaged over ten minutes
- V_{hub} = Wind speed at hub height averaged over ten minutes

And the wind turbulence can be classified by standard table.3.1 The tower

Wind turbulence class		I	II	III	S
V_{ref}	m/s	50	42.5	37.5	2*Values specified by designer
A	$I_{ref}[-]$		0.16		
B	$I_{ref}[-]$		0.14		
C	$I_{ref}[-]$		0.12		

Table 3.1: Wind turbulence class

shadow can be modelled by potential models for a conical tower, downwind empirical tower model or an interpolation of these two.

3.3 LQR controller in Cp-Lambda

3.3.1 Dynamic model

To perform effective control of a wind turbine, the wind turbine system needs to be described by a model with acceptable cost. In this project, a 6 state- 2 input collective-pitch-only model is used (Fig. 3.7).

6-dimensional state vector: $x = (d, \dot{d}, \Omega, \beta_e, \dot{\beta}_e, T_{ele})^T$

2-dimensional control vector: $u = (\beta_c, T_{elc})^T$

Based on this reduced model, dynamics of the wind turbine system is simplified into 4 equations of motion, which describes the drive-train shaft dynamics (Equ.3.22), eleastic tower fore-aft motion(Equ.3.23), blade pitch actuator dynamics (Equ.3.24), and electrical generator dynamics (Equ.3.25).

Figure 3.7: 6-state 2-control input wind turbine model

$$(J_R + J_G)\dot{\Omega} + T_l(\Omega) + T_{ele} - T_a(\Omega, \beta_e, V_W - \dot{d}, V_m) = 0 \quad (3.22)$$

$$M_T\ddot{d} + C_T\dot{d} + K_Td - F_a(\Omega, \beta_e, V_W - \dot{d}, V_m) = 0 \quad (3.23)$$

$$\ddot{\beta}_e + 2\xi\omega\dot{\beta}_e + \omega^2(\beta_e - \beta_c) = 0 \quad (3.24)$$

$$\dot{T}_{ele} = 0 + \frac{1}{\tau}(T_{ele} - T_{elc}) = 0 \quad (3.25)$$

where

J_R = Rotating inertia of rotor

J_G = Rotating of inertia of generator

Ω = Angular velocity of rotor

β_e = Pitch angle of blade

β_c = Teerred blade pitch angle

V_W = Wind speed

V_m = Mean wind speed

d = Tower displacement

T_l = Initial torque

T_{ele} = Torque from electrical load

T_{elc} = Teerred torque from electrical load

T_a = Torque from actuator

M_T = Mass of tower

C_T = Damping coefficient of tower

K_T = Spring constant in tower flapping direction

F_a = Actuator generated force in tower flapping direction

ξ = Blade pitch damping ratio

ω = Blade pitch frequency

τ = Control constant of electric generator

The wind speed is composed of mean wind speed and turbulent wind speed as shown in Equ.3.26

$$V_W = V_m + V_t \quad (3.26)$$

Rotor force coefficient, C_{F_e} and moment coefficient, C_{P_e} can be calculated from Equ.3.27 and Equ. 3.28.

$$F_a = \frac{1}{2} \rho \pi R^2 \frac{C_{F_e}(\lambda, \beta_e, V_m)}{\lambda} (V_W - \dot{d})^2 \quad (3.27)$$

$$T_a = \frac{1}{2} \rho \pi R^3 \frac{C_{P_e}(\lambda, \beta_e, V_m)}{\lambda} (V_W - \dot{d})^2 \quad (3.28)$$

And the calculation of C_{F_e} and C_{P_e} are performed off-line with Cp-Lambda by averaging periodic response over one rotor revolution.

3.3.2 Model linearization

Steady states parameters of wind turbine and can be calculated by performing static simulation, which is expressed in x^* , u^* and V_m^* . Assume that wind turbine is working near this steady state, around which the high ordered states and control deviation can be ignored. Thus, the wind turbine incremental state has a linear behaviour whose dynamics can be given by Equ.3.29.

$$\Delta \dot{x} = A\Delta x + B\Delta u \quad (3.29)$$

where

$$\Delta x = x - x^*(V_M)$$

$$\Delta u = u - u^*(V_M)$$

A and B are state vector and control vector of the increment state respectively.

$$A(V_M) = A(x^*, u^*, V - M)$$

$$B(V_M) = B(x^*, u^*, V - M)$$

3.3.3 LQR controller

Figure 3.8: MIMO LQR loop

The wind turbine works in an time variant environment, which requires an on-time all-state control. To fullfil this requirement, LQR control method was taken in the control system (Fig.3.8).

When the wind turbine works, its states are either directly measured by sensors or calculated from other parameters. Both of these methods may

lead to errors. In order to have a more reliable data of wind turbine states, an extra state is included in the 7-state vector. This new 7-state vector is defined as augment state vector denoted by Equ.3.30.

$$x_{aug} = (d, \dot{d}, \Omega, \beta_e, \dot{\beta}_e, T_{ele}, \int \Omega dt)^T \quad (3.30)$$

Thus the augmented dynamic system can be expressed in

$$\Delta \dot{x}_{aug} = A_{aug} \Delta x_{aug} + B_{aug} \Delta u \quad (3.31)$$

where augmented state matrix, A_{aug} and augmented control matrix, B_{aug} are expressed

$$A_{aug} = \begin{bmatrix} A(x^*, u^*, V - M) & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix} & 0 \end{bmatrix} \quad (3.32)$$

$$B_{aug} = \begin{bmatrix} B(x^*, u^*, V - M) \\ \begin{bmatrix} 0 & 0 \end{bmatrix} \end{bmatrix} \quad (3.33)$$

The state-feedback control law of the form $u = -Kx$ is defined by minimizing the cost function (Equ.3.47).

$$J = \int_0^\infty [x_{aug}^T(t) Q x_{aug}(t) + u^T(t) R u(t)] dt \quad (3.34)$$

Here, weight matrix Q is a 7-dimensional positive-semidefinite matrix and R is a 2-dimensional positive-definite matrix. Since the control goal is to keep the state Δx as close to zero as possible, it is a so called regular problem. The term $x_{aug}^T(t) Q x_{aug}(t)$ is a measure of control accuracy while the term $u^T(t) R u(t)$ is a measure of control effort. Thus, the design of Q and R matrix affects trade of between control accuracy and control effort.

In this project, Q and R matrix are initially evaluated based on incoming wind speed vector, $W = [3, 9, 11, 15, 25]^T$. According to these five base wind velocities, five sets of Q , R and Q_W weight parameters has been set,

which makes up of three base matrices listed below.

$$W.Q = \begin{bmatrix} 0 & 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 & 0 \end{bmatrix} \quad (3.35)$$

$$W.R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \\ R_{31} & R_{32} \\ R_{41} & R_{42} \\ R_{51} & R_{52} \end{bmatrix} \quad (3.36)$$

$$W.Q_W = \begin{bmatrix} Q_{W1} \\ Q_{W2} \\ Q_{W3} \\ Q_{W4} \\ Q_{W5} \end{bmatrix} \quad (3.37)$$

When faced with a incoming wind velocity in between of $W(i)$ and $W(i+1)$, the weight parameter $w.Q$, $w.R$ and $w.Q_W$ is calculated by Equ.3.38,3.39 and 3.40.

$$\frac{V_m - W(i)}{W(i+1) - W(i)} = \frac{w.Q - W.Q(i)}{W.Q(i+1) - W.Q(i)} \quad (3.38)$$

$$\frac{V_m - W(i)}{W(i+1) - W(i)} = \frac{w.R - W.R(i)}{W.R(i+1) - W.R(i)} \quad (3.39)$$

$$\frac{V_m - W(i)}{W(i+1) - W(i)} = \frac{w.Q_W - W.Q_W(i)}{W.Q_W(i+1) - W.Q_W(i)} \quad (3.40)$$

Since the maximum loads that the wind turbine sustained often exists at rated wind speed 11m/s, the simulations are performed in this mean wind speed environment. Thus, the base weight parameters are selected as

$$w.Q = [0 \ 0 \ Q_3 \ 0 \ 0 \ 0] \quad (3.41)$$

$$w.R = [R_{31} \ R_{32}] \quad (3.42)$$

$$w.Q_W = [Q_{W3}] \quad (3.43)$$

$$(3.44)$$

Thus, weight matrix Q and R is built as below

$$Q = \begin{bmatrix} \text{diag}(w.Q) & 0 \\ 0 & \text{diag}(w.Q_w) \end{bmatrix} = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & Q_3 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \\ & & & & & & Q_{w3} \end{bmatrix} \quad (3.45)$$

$$R = \text{diag}(w.R) = \begin{bmatrix} R_{31} \\ R_{32} \end{bmatrix} \quad (3.46)$$

Input Q, R and Q_w into

$$J = \int_0^\infty [\Delta\Omega Q_3 \Delta\Omega + (\Delta \int \Omega dt) Q_{w3} (\Delta \int \Omega dt) + \Delta\beta R_{31} \Delta\beta + \Delta T R_{32} \Delta T] dt \quad (3.47)$$

Therefore, the magnitude of Q_3 , Q_{w3} , R_{31} and R_{32} can influence the controller performance of wind turbine system. And the effect of these four weight parameters is evaluated from simulations in Cp-Max.

CHAPTER 4

Simulations and results

4.1 Description of 10 MW machine

The baseline of the wind turbine design is a 10 MW 3-blade HAWT model. This wind turbine is initially set to be 178.3 m in diameter, 119 m in height with rated power production capability of 10 MW. Blade spanwise distribution of chord and twist are set at 8 base locations with 7 initial airfoils applied in between. More detailed input data for the wind turbine design is listed in Table.4.1.

where d_{zN} is the distance between tower top and nacelle coordinate system,

Rotor diameter, D_R	178.3	[m]
Hub height, H	119	[m]
Blade root distance from hub, d_{zN}	2.8	[m]
Over hang, d_{xH}	7.07	[m]
Rotor tilt angle, α_t	5	[deg]
Cone angle, α_c	4.65	[deg]

Table 4.1: Global inputs for wind turbine optimization

whose position is determined by the intersection of the tower centre line with rotor axis. The rotor overhang d_{xH} is the distance between the centre of the hub coordinate system (xyz_H) and the intersection of the rotor axis with the tower centre line. Rotor tilt angle, α_t is the angle between the horizontal axis and the rotor shaft, and a positive angle increases hub height and tower clearance. Cone angle, α_c is the angle between the rotor plane and the blade axis, the blades are directed downwind for positive conning. The additional gross properties describing the rated working state of the

wind turbine is listed in Table.4.2.

Rated rotation speed, Ω^*	9.64	[rpm]
Rated torque, T^*	9.9	[MNm]
Cut-in wind speed, V_{in}^*	3	[m/s]
Rated wind speed, V_r^*	11.3	[m/s]
Cut-out wind speed, V_{out}^*	25	[m/s]
Maximum power coefficient, C_{Pmax}^*	0.45	[-]
Rated tip speed ratio, λ^*	8.1	[-]

Table 4.2: Wind turbine rated state

4.2 Time history of loads and performances

Figure 4.1: Main reference systems of wind turbine [20]

The sensitivity of wind turbine to the change of Q and R are studied by observing response of loads and performances by simulations in C_P -Lambda. Time histories of blade combined moment, M_{xyb} , blade tip displacement, D_{tip} , thrust, F_x , torque, T , tower fore-aft moment, M_{yt} , blade

pitch angle, , rotation rate, Ω and power, P are measured by sensors shown in Figure.4.2. Considering that the C_P -Lambda is a multi-body model, these loads are measured on a series of Cartesian, right-handed reference systems. M_{xyb} is calculated from lead-lag moment M_{xb} and flapping moment M_{yb} with reference to blade rotating frame, which can be sensed at blade root. The expression of this combined moment is

$$M_{xyb}(t) = \sqrt{M_{xb}^2(t) + M_{yb}^2(t)} \quad (4.1)$$

$F_{xh}(t)$ and $T(t)$ are the thrust and torque exerted on hub. $M_{fore-aft}(t)$ is the fore-aft moment at tower root, which is also M_y with reference to $(xyz)_T$. $D_{tip}(t)$ denotes blade tip displacement with reference to $(xyz)_B$, which is calculated from D_{xB} and D_{yB} .

$$D_{tip}(t) = \sqrt{D_{xB}^2(t) + D_{yB}^2(t)} \quad (4.2)$$

Maximum values of M_{xyb} , D_{tip} , F_x , T and M_{yt} of the the 610-second dynamic simulation in C_P -Lambda. Actuator duty cycle is the calculated by integrating blade pitch rate over time with the expression $\int \dot{\beta}(t)dt$.

$$\int \dot{\beta}(t)dt = \sum_{i=1}^{n-1} (\beta_{i+1} - \beta_i) \quad (4.3)$$

. To evaluate the oscillation of rotation rate, the integration of rotation rate over time,

$$\int \Omega(t) = \sum_{i=1}^{n-1} \frac{\Omega(t_i) + \Omega(t_i + t_{i+1})}{2} (t_{i+1} - t_i) + \frac{\Omega(t_i)}{2} (t_i - t_{i-1}) \quad (4.4)$$

is also taken into concern. The total energy produced by the wind turbine throughout the 610 seconds is calculated by

$$E = \sum_{i=1}^{n-1} \frac{P(t_i) + P(t_i + t_{i+1})}{2} (t_{i+1} - t_i) + \frac{P(t_i)}{2} (t_i - t_{i-1}) \quad (4.5)$$

Figure 4.2: Time history of wind turbine typical performances in 610 second, with 11 m/s mean wind, turbulent seed 1, LQR base control matrix

4.3 Dynamic simulations in C_P -Lambda with varying LQR controllers

4.3.1 Base line performance

The baseline of LQR weight matrix, Q^* and R^* can be expressed into

$$Q^* = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & Q_3^* & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 0 \\ & & & & & & Q_{w3}^* \end{bmatrix} \quad R^* = \begin{bmatrix} R_{31}^* & \\ & R_{32}^* \end{bmatrix} \quad (4.6)$$

Thus, the cost function of the 10 MW wind turbine system is $J = \int_0^\infty [\Delta\Omega Q_3^* \Delta\Omega + (\Delta \int \Omega dt) Q_{w3}^* (\Delta \int \Omega dt) + \Delta\beta R_{31}^* \Delta\beta + \Delta T R_{32}^* \Delta T] dt$, where weight parameters are set to be

$$Q_3^* = 35 \quad (4.7)$$

$$Q_{w3}^* = 0.01 \quad (4.8)$$

$$R_{31}^* = 2 \quad (4.9)$$

$$R_{32}^* = 0.05 \quad (4.10)$$

Thus, the wind turbine loads and performance under the control of baseline LQR weight matrices are listed in Table.4.3.

$\max M_{xyb}^*$ [MNm]	$\max F_{xh}^*$ [MN]	$\max T^*$ [MNm]	$\max M_{fore-aft}^*$ [MNm]	$\max D_{tip}^*$ [m]	$\int \dot{\beta}^* dt$ [rad]	$\int \dot{\Omega}^* dt$ [rad]	$\max P^*$ [MW]	E^* [MWh]
44.284	1.946	10.374	215.350	12.191	3.139	6.410	11.270	6.436

Table 4.3: Max loads and control efforts with base LQR controller

4.3.2 Effect of Q_3

Simulations with varying Q_3 are performed in 610-second, 11 m/s incoming wind model with three different turbulent seeds. Some regular effect of Q_3

Figure 4.3: Non-dimensionalized effect of Q_3 in 11m/s V_m wind with turbulent seed 1,2,3

Figure 4.4: Safety margin check of Q_3

on maximum torque, maximum thrust, actuator duty cycle and maximum rotation rate can be observed in Fig. 4.3.

It can be observed from Fig.4.3a that maximum thrust decreases with Q_3 with turbulent seed 1 and 2, which indicates a positive influence of Q_3 on thrust control. However, there's almost no change of maximum thrust with Q_3 in seed3 case.

Figure.4.3b, 4.13, 4.3d show that the increase of Q_3 can decrease the maximum rotation rate and actuator duty cycle with an increase of maximum torque. This can be understood from the definition of cost function $J = \int_0^\infty [\Delta\Omega Q_3 \Delta\Omega + (\Delta \int \Omega dt) Q_{w3} (\Delta \int \Omega dt) + \Delta\beta R_{31} \Delta\beta + \Delta T R_{32} \Delta T] dt$, where the increase of Q_3 can directly increase the control accuracy of rotation rate with the loss of control accuracy on torque and larger actuator duty cycle required.

Figure. 4.4a shows that the change of Q_3 has almost no effect on energy generated. However, it should be noted that with turbulent seed1, there is a large energy loss when $\Delta Q_3 < -30\%Q_3^*$. The similar energy drop can also be observed in Figure.4.3a and 4.13, where the maximum torque and actuator duty cycle has a sudden increase when $\Delta Q_3 < -30\%Q_3^*$. This is caused by system shut down because of the lack of control on rotation rate. Such shut down is also induced by particular turbulence, which only exists in seed1. Thus a safety margin of the minimum Q_3 is set to be $0.7Q_3^*$.

4.3.3 Effect of Q_{w3}

Simulation with varying Q_3 shows that the state response of the change of Q_{w3} is much smaller than that of Q_3 , some of the response are mixed with noises. Regular response can only be observed in maximum thrust and actuator duty cycle in Fig. 4.5, which is also very small in magnitude. Maximum thrust and actuator duty cycle is increased with Q_{w3} . From the definition of cost function, Q_{w3} directly affect the control accuracy of $\int \Omega dt$, which is an augmented states used for states tracing accuracy improvement. Thus, the magnitude of Q_{w3} has little affect on the system.

Figure. figures/Qw3Energy.eps shows that the energy generated is not much affected to the change of Q_{w3} as well. And there's no system shutdown occurred within Q_{w3} simulation range from $10\%Q_{w3}^*$ to $500\%Q_{w3}^*$.

Figure 4.5: Non-dimensionalized effect of Q_{w3} in 11m/s V_m wind with turbulent seed 1,2,3

Figure 4.6: Non-dimensionalized effect of Q_{w3} on Energy in 11m/s V_m wind with turbulent seed 1,2,3

4.3.4 Effect of R_{31}

Figure 4.7: Non-dimensionalized effect of R_{31} in 11m/s V_m wind with turbulent seed 1,2,3

Simulations with varying R_{31} are performed in 610-second, 11 m/s incoming wind model with three different turbulent seeds. Figure 4.7 show the response of maximum thrust, torque and rotation rate on the change of R_{31} and the corresponding actuator duty cycle change. A large effect of R_{31} on the wind turbine performance can be observed.

According to the cost function, $J = \int_0^\infty [\Delta \Omega Q_3 \Delta \Omega + (\Delta \int \Omega dt) Q_{W3} (\Delta \int \Omega dt) + \Delta \beta R_{31} \Delta \beta + \Delta T R_{32} \Delta T] dt$, R_{31} directly affect the pitch control effort. The increase of R_{31} can intensify the control over pitch angle. Thus, the oscillation of pitch angle decreases with R_{31} (Fig. 4.18), and the maximum torque decreases with R_{31} as is shown in Fig. 4.3d. Note that the $\Delta R_{31} - \Delta \max(T)$ curves in seed 1, 2, 3 do not coincide with each other, the

Figure 4.8: Non-dimensionalized effect of R_{31} on Energy in 11m/s V_m wind with turbulent seed 1,2,3

sensitivity of torque control with R_{31} may be influenced by other parameters.

In Figure.4.7a, big decrease of maximum thrust decreases with R_{31} can be observed in all three seeds, which shows positive control effect of R_{31} on thrust.

Figure.4.7d shows the insensitivity of Ω on R_{31} change. According to the Energy- R_{31} curves in Figure.4.8, the energy keeps almost constant with the change of R_{31} , and no design margin that leads to system shut down is found from 50% R_{31}^* to 400% R_{31}^*

4.3.5 Effect of R_{32}

Similar simulations are also performed with varying R_{32} in 610-second, 11 m/s incoming wind model with turbulent seeds 1, 2 and 3.

The simulation results show that R_{32} can have big influence on the wind turbine loads and actuator performance, however, these response don't have regularly related with the change of R_{31} and changes with seeds. Only a regular decrease of maximum torque value can be observed in all seeds with R_{32} increase. And the energy doesn't show large response to R_{32} change as well. Look back to the cost function

According to the definition of cost function,

$$J = \int_0^\infty [\Delta\Omega Q_3 \Delta\Omega + (\Delta \int \Omega dt) Q_{W3} (\Delta \int \Omega dt) + \Delta\beta R_{31} \Delta\beta + \Delta T R_{32} \Delta T] dt$$

Figure 4.9: Non-dimensionalized effect of R_{32} on $\max(T)$ in 11m/s V_m wind with turbulent seed 1,2,3

(a) $\Delta R_{32} - \Delta Energy$

(b) Time – Power

Figure 4.10: Safety margin check for R_{32}

, R_{32} directly affects the approximation of torque to its reference magnitude. An increase of R_{32} therefore enlarges the control power on torque, and at the same time destabilises other states. It should be noted that in seed 1, a break point can be observed when $\Delta R_{32} = 50\%R_{32}^*$ in both Figure.4.9 and Figure.4.10a. When R_{32} is larger than $150\% R_{32}^*$, there is not enough control on other states, which induces system shutdown. Since the turbulent seed 2 and 3 is not as severe as seed 1, such shut down can only be observed in seed1 with the simulation range from $50\%R_{31}^*$ to $400\%R_{31}^*$ (Figure.4.10b). It is highly suspected that the system will also shut down if R_{32} is further enlarged. Based on current simulation result, a safety margin for maximum R_{32} is set to be $1.5R_{32}^*$.

4.3.6 Comparison of system response on Q_3, Q_{w3}, R_{31} and R_{32}

Based on the dynamic simulation results in seed 1, seed 2 and seed 3, the regulation of the effect of LQR controller weight Q_3, Q_{w3}, R_{31} and R_{32} can be observed. The simulation results of LQR controller with twice magnitude of $Q_3^*, Q_{w3}^*, R_{31}^*$ and R_{32}^* are listed in Table.4.4. It should be noted that system shut down exists in the simulation with turbulent seed 1, which indicates a lower design margin of Q_3 at $0.7Q_3^*$ and a upper design margin of R_{32} at $1.5R_{32}^*$. Thus, the design of $2R_{32}^*$ LQR controller is not acceptable. However, for the convenience of comparison, simulation results with $R_{32} = 2R_{32}^*$, turbulent seed 2 was listed in Table.4.4. It can be indicated

	Q_3	Q_{w3}	R_{31}	R_{32}
	$2Q_3^*$	$2Q_{w3}^*$	$2R_{31}^*$	$2R_{32}^*$
$\Delta max(F_X)/max(F_x^*)$	-3.2%	1.2%	-2.5%	5.5%
$\Delta max(T)/max(T^*)$	1.6%	0.0%	5.6%	-6%
$\Delta \int \beta dt / \int \beta^* dt$	25.7%	-0.4%	-11.2%	-3.2%
$\Delta max(\Omega)/max(\Omega^*)$	-2.6%	0.0%	-0.7%	1.6%

Table 4.4: Turbulent seed 2, effect of double Q_3, Q_{w3}, R_{31} and R_{32}

from Table.4.4 that

- compared to other weight parameters, the effect of Q_{w3} magnitude on wind turbine performance is much weaker, which can be ignored.
- Increase of Q_3 can induce large increase of actuator duty cycle, more control accuracy of thrust and rotation rate with loss of torque control accuracy.
- System response of R_{31} increase is lower but similar as that of Q_3 . However, the actuator duty cycle decreases with R_{31} increase.
- Unlike Q_3 and R_{31} , the increase of R_{32} doesn't induce much change of actuator duty cycle. It only has regular trend of increasing the control accuracy of torque. And its effect on thrust and rotation speed control show large uncertainty depending on the incoming wind.

4.4 Wind turbine designs in C_P -Max

4.4.1 WT design with Q_3 selection

Figure 4.11: Effect of Q_3 on CoE and blade mass & cost

In order to study the effect of Q_3 on blade design, dynamics simulation are performed in C_P -Max design loops, where Q_3 is set to from 70% to 160% of its base value, Q_3^* .

Figure.4.11b shows the response of CoE to Q_3 change, where a minimum CoE can be observed when Q_3 is set to $4Q_3^*$. Considering that the -0.01%

$\frac{Q_3}{Q_{3r}} [-]$	$\frac{\Delta CoE}{CoE_r} [\%]$	$\frac{m_b}{m_{br}} [\%]$	$\frac{C_b}{C_{br}} [\%]$
0.7	0.032	1.15	0.36
2	0.005	-1.13	-0.75
4	-0.010	-2.10	-1.45
8	0.014	-1.64	-1.18
16	0.044	0.15	-0.19

Table 4.5: Effect of Q3 on CoE and blade mass & cost

reduction is small, the response of balde mass and cost are also shown in Figure.

with the increase of Q_3 .

This response corresponds the definition of LQR cost function(Equation.3.47)

Figure 4.12: Time history signal of thrust and torque

$J = \int_0^\infty [\Delta\Omega Q_3 \Delta\Omega + (\Delta \int \Omega dt) Q_{W3} (\Delta \int \Omega dt) + \Delta\beta R_{31} \Delta\beta + \Delta T R_{32} \Delta T] dt.$
The comparative magnitude of Q_3 can have direct affect on the control accuracy of rotation rate, Ω , thus the increase of Q_3 can intensify the approximation of Ω to its rated magnitude, which can also stabilizes thrust to some extent. While the same time, blade pitch angle and torque more likely to be adjusted to generate Ω_r than to approach their own rated magnitude. Thus, the increase of actuator duty cycle with Q_3 increase can be observed in Figure.4.13, and so does the increased torque oscillation in Figure.4.12b. It should be noted that since the oscillation of thrust is a mutual result of rotation rate, wind speed and pitch angle (Equation.2.12), thus the oscilation of thrust may initially be reduced with the stabilization of rotation

Figure 4.13: Effect of Q_3 on Actuator duty cycle

(a) Spars

(b) Webs

(c) Leading edge reinforcement

(d) Trailing edge reinforcement

Figure 4.14: Spanwise thickness distributions of blade component with different Q_3

rate, and start to grow with destabilization of blade pitch angle. Note that the oscillation of thrust and torque can affect the blade structure design within strain and stress constrains. For example, recall the expression of blade root flapwise bending moment, $M_\beta = \frac{4}{9}F_xR$ (Equation.2.14) and the maximum flapwise stress at blade root, $\sigma_{\beta,max} = M_\beta c/I_b$ (Equation.2.15). For a blade with aerodynamically fixed design, flapwise moment is almost proportional to thrust. Thus, the thickness of webs can be reduced with the reduction of maximum thrust, as is shown in Figure.4.14b. That is because webs provides strengths to and keep blade cross section from deformation. Also, the thickness of spars and LE, TE reinforcements with $4Q_{3r}$ is also reduced from base line.

4.4.2 WT design with R_{31} selection

Similar to the study of Q_3 , dynamics simulation are also performed in a series C_P -Max design loops, with R_{31} set from 100% to 1600% of its base value, R_{31}^* .

Figure.4.15b shows the response of CoE to R_{31} change. CoE can be reduced by -0.014% when R_{31} is set to $4R_{31}^*$, which is also not significant compared to the effect on blade mass and cost. The response of blade mass and cost to R_{31} are shown in Figure.4.16, where blade mass and cost can be mostly reduced by -2.86% and -1.37% at $R_{31} = 4R_{31}^*$.

Since blade component thickness is designed with structural constrains,

Figure 4.15: Effect of R_{31} on CoE and blade mass & cost

the maximum thrust, maximum torque and energy provided during the simulations in C_P -Max with R_{31}^* , $2R_{31}^*$, $4R_{31}^*$, $8R_{31}^*$ and $16R_{31r}$ is shown

$\frac{R_{31}}{R_{31}^*} [-]$	$\frac{\Delta CoE}{CoE^*} [\%]$	$\frac{m_b}{m_b^*} [\%]$	$\frac{C_b}{C_b^*} [\%]$
2	0.014	-1.89	-1.26
4	-0.009	-2.86	-1.37
8	0.027	0.15	-0.19
16	0.016	0.91	0.29

Table 4.6: Effect of R_{31} on CoE and blade mass & cost

in Figure.4.16. The energy produced by wind turbine has little reduction with R_{31} increase. Since this reduction is small, the system is still seen as under effective control within this R_{31} range. Similar to that with Q_3 , the maximum thrust reduces with R_{31} while the maximum torque has a small increase with R_{31} .

The time history of thrust and torque signals are also shown in Fig-

Figure 4.16: Effect of R_{31} on maximum thrust, torque and energy

ure.4.17a and Figure.4.17b. A significant reduction of thrust with R_{31} reduction can be seen in Figure.4.17a. Although the maximum torque is increased by $8R_{31}^*$, this torque oscillation with is not intensified as seriously as that with $8Q_3^*$. Thus, the effect of R_{31} is more significant on thrust than that on torque.

According to the definition of LQR cost function (Equation.3.47), the magnitude of R_{31} directly affect the pitch control effort. The increase of

Figure 4.17: Time history signal of thrust and torque

R_{31} thus turns control strategy into the reduction of actuator duty cycle, which is shown in Figure.4.18. Unlike that with Q_3 change, wind turbine thrust is reduced with R_{31} from R_{31}^* to $4R_{31}^*$ with small loss of energy and negligible torque change. Thus, larger reduction of blade mass and cost can be achieved with $4R_{31}$ and than that with $4Q_3$. However, the reduction of thrust with R_{31} turns to be very small after $4R_{31}^*$ and almost keeps constant after $8R_{31}^*$, while torque oscillation keeps increasing. And at the same time, the control accuracy of rotation speed keeps reducing as well. Thus, the blade cost and mass starts to increase after $4R_{31}^*$ as a result of loads increase.

Similar effects can also be seen in Figure.4.19, where the thickness of spars, leading edge and trailing edge reinforcement are all minimized with $4Q_{31}^*$ controller weight, while the width of webs does not have much deviation from baseline. Thus, it can be suspected that the reduction of blade mass and cost with R_{31} adjustment mostly comes from thickness reduction of spars and reinforcement.

4.4.3 Comparison of blade design optimization by Q_3 and R_{31} adjustment

- The total cost of energy, blade mass and cost can be minimized by appropriate selection of Q_3 and R_{31} . When Q_3 is adjusted to $4Q_3^*$, CoE, blade mass and cost can be reduced by -0.01%, -2.1% and -0.75% respectively. When R_{31} is adjusted to R_{31}^* , CoE, blade mass and cost can be reduced by -0.01%, -2.86% and -1.37% respectively.

Figure 4.18: Effect of R_{31} on Actuator duty cycle

(a) Spars

(b) Webs

(c) Leading edge reinforcement

(d) Trailing edge reinforcement

Figure 4.19: Spanwise thickness distributions of blade component with different R_{31}

- Although the reduction of CoE is not significant, the change of CoE can indicate how to select Q_3 and R_{31} to reduce blade structural cost, which is a combined result of thrust, torque and blade pitch angle control.
- An appropriate setting of Q_3 can reduce maximum thrust, while at the same time induce large torque and pitch oscillation. As a result of combined loads change, the blades are designed with thinner webs, spars and reinforcement, among which the reduction of webs thickness is the main factor reducing blade cost.
- An appropriate setting of Q_3 can reduce maximum thrust with no lost of pitch angle control and only small intensification of torque oscillation. Thus, blade cost has steeper reduction with Q_3 adjustment. Blade components, webs, spars and reinforcement are all reduced. Among them, the reduction of spars and reinforcement thickness contribute more to blade costs reduction than that of webs.

CHAPTER 5

Conclusions and Recommendations

In this project, the effect of LQR weight matrix on wind turbine performance is studied by performing simulations in the multi-disciplinary wind turbine design tool, *C_P-Max*. The design of wind turbine in *C_P-Max* contains multi stage iteration of aerodynamic optimization, structural optimization and global optimization. At each stage, wind turbine design parameters are updated within constraints. The LQR controller is installed in the structural optimization loop. And the aero-servo-elastic multi-body model, *C_P-Lambda* is installed to model aerodynamics of rotors, motions of the blade elastically deformation, and the interactions between blades and flow.

The dynamic model of wind turbine is linearised into an augmented system, $\dot{x}_{aug} = A_{aug}x_{aug} + B_{aug}u$ where x_{aug} is a 7-state vector and u is the pitch-torque control vector. According to LQR control theory, the whole system is stabilised by minimizing the cost function, $J = \int_0^\infty [\Delta\Omega Q_3 \Delta\Omega + (\Delta \int \Omega dt) Q_{w3} (\Delta \int \Omega dt) + \Delta\beta R_{31} \Delta\beta + \Delta T R_{32} \Delta T] dt$. The sensitivity of wind turbine to Q_3 , Q_{w3} , R_{31} and R_{32} is studied by simulations in *C_P-Lambda* in 610s, 11m/s wind combined with 3 different turbulent seeds.

- Q_3 has a lower margin at $0.7Q_3^*$, R_{32} has an upper margin at $1.5R_{32}^*$
- Wind turbine energy is nearly constant with the change of weight matrices. Compared to Q_{w3} and R_{32} , wind turbine performance is more sensitive to Q_3 , R_{31} .
- With the increase of Q_3 , maximum thrust decreases, maximum torque increases, actuator duty cycle increases.

- With the increase of R_{31} , maximum thrust decreases, maximum torque increases, actuator duty cycle decreases.

Two weight matrices installed in C_P -Max that can reduce CoE of designed WT is found. The matrices is selected by altering Q_3 or R_{31} .

- When $Q_3 = 4Q_3^*$, an -0.01% reduction of CoE, -2.1% reduction of blade mass, -1.45% reduction of blade cost can be achieved. This comes along with lower maximum thrust, larger torque oscillation and an increase of actuator duty cycle. Compared to baseline design, the blade is designed with thinner spars and webs.
- When $R_{31} = 4R_{31}^*$, an -0.009% reduction of CoE, -2.86% reduction of blade mass, -1.37% reduction of blade cost can be achieved. This comes along with smaller thrust oscillation, smaller increase of maximum torque, and a decrease of actuator duty cycle. Compared to designs with other R_{31} , the blade is designed with thinner spars, webs, LE and TE reinforcement.

Suggestions for future work

- Joint effect of Q_3 and R_{31} on the system can be studied by performing further simulations in C_P -Max systems. A further reduction of CoE may be found by a combined design of Q_3 and R_{31} .
- The effect of LQR weight matrices on wind turbine can also be studied in other incoming wind speed by performing further simulations. Better selections of other weight parameters are expected.

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