Properties of soils using laboratory seismic methods and numerical modelling

Cheng, Zhuoyuan

2016

Cheng, Z. (2016). Properties of soils using laboratory seismic methods and numerical modelling. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/68845>

<https://doi.org/10.32657/10356/68845>

NANYANG
TECHNOLOGICAL
UNIVERSITY

**PROPERTIES OF SOILS USING LABORATORY
SEISMIC METHODS AND NUMERICAL
MODELLING**

CHENG ZHUOYUAN

**SCHOOL OF CIVIL AND ENVIRONMENTAL
ENGINEERING**

2015

**PROPERTIES OF SOILS USING LABORATORY
SEISMIC METHODS AND NUMERICAL
MODELLING**

CHENG ZHUOYUAN

SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of Doctor of
Philosophy

2015

Acknowledgments

"Never regret. If it's good, it's wonderful, if it's bad, it's an experience."

--Victoria Holt

Well, I never did regret embarking on this journey despite the various challenges I have encountered along the way. The most important part of this PhD journey is not the knowledge that I have attained but the attitude, approach and wisdom I was inculcated with when handling challenges be it in research and daily life. No amount of words can express my heartfelt gratitude to A/P Leong Eng Choon. He was more than a mentor to me and it is his encouragement, support and advice that has made all these possible and less painful.

Next in line, I would like to thank my seniors Aung Aung Myo Win and Than Than Nyunt for sharing with me their knowledge and experiences. May Natalie grow up to be a kind, intelligent and healthy girl and all the best to your next baby.

I am also thankful to the laboratory executives Mr. Vincent Heng Hiang Kim, Mdm Lim-Ding Susie, Mr Andy Koh Sun Weng and Mr. Eugene Tan Hiap Guan for sharing with me their experiences and helping me with some of the experiments. I would also like to extend my appreciation to Mr. Phua Kok Soon for his help in equipment fabrication.

Next, I would like to thank the many FYP students that I have worked with. They are Wong Kah Chou, Benjamin Lim, Hardy Kek, Yam Chau Min, Sheranne Eng, Ashraff, Terrence and Clive. I have learnt much from my interactions with you all.

The PhD journey would have no doubt been an arduous one without friends walking by your side. For this, I would like to thank the following people: Chew Jiahan, Yeung Jiansheng, Namyo Salim, Li Zhihuan, Winston Huang, Martin Wijaya and Richard Kizza. Thanks for sharing with me the joy and tribulations through this journey.

I am also grateful to Nanyang Technological University for providing me the Nanyang President Graduate Scholarship. The financial support for daily expenses and overseas conference allowed me to widen my horizons and see the world. Last but most important of all, I would like to thank my family members for the support and understanding they have given me.

Table of Contents

Abstract	v
List of Figures	viii
List of Tables	xiii
List of Symbols	xiv
List of Abbreviations	xviii

CHAPTER 1: Introduction	1
1.1 Background	1
1.2 Objectives and Scope of Research	3
1.3 Scope of Research	3
1.4 Organization of Thesis	4

CHAPTER 2: Literature Review	7
2.1 Wave Propagation in Soil	7
2.1.1 Non-linearity of soil properties	10
2.1.2 Effect of confining pressure and degree of saturation on wave velocities	11
2.1.3 Effect of confining pressure and degree of saturation on damping ratio	15
2.2 Advanced Geotechnical Testing	19
2.2.1 Resonant Column Test	19
2.2.2 Cyclic Triaxial Test	20
2.2.3 Wave Propagation Test	21
2.2.3.1 Bender-Extender Element System	
2.2.3.2 Ultrasonic system	
2.3 Determination of Damping Ratio	29
2.3.1 Logarithmic Decrement Method (LDM)	30
2.3.2 Half-Power Bandwidth Method (HPBM)	31
2.3.3 Spectral Ratio Method (SRM)	32
2.3.4 Hilbert Transform Method (HTM)	35
2.4 Residual Soils	37
2.4.1 Saturated and unsaturated soils	39
2.4.2 Dynamic properties of residual soil	44
2.5 Summary and Gaps in Current Research	46

CHAPTER 3: Methodology, Test Procedures and Experimental Set-up	47
3.1 Outline and Purpose of Test Series	47
3.2 Test Series I: The Wave Propagation Transducer	48
3.2.1 Material Utilised	48
3.2.2 Fabrication of Bender/extender elements	51
3.2.3 Experimental Set-up and Methodology	53

3.3	Test Series II: Development of testing framework, effect of saturation and confining pressures on small strain soil properties	57
3.3.1	Material Utilised	57
3.3.2	Experimental Set-up and Methodology	61
3.3.3	Digital Signal Processing	66
3.4	Test Series III: Dynamic properties of soil rubber mixtures	77
3.4.1	Material Utilised	77
3.4.2	Experimental Set-up and Methodology	78
3.4.3	Signal Processing	80
3.5	Test Series IV: Cyclic triaxial and wave propagation test on unsaturated residual soil	82
3.5.1	Material Utilised	82
3.5.2	The modified cyclic triaxial set-up	84
3.5.2.1	<i>Calibration of set-up</i>	
3.5.2.2	<i>Fabrication of local displacement transducer (LDT)</i>	
3.5.3	Experimental Set-up and Methodology	91
3.5.3.1	<i>Saturated consolidated undrained (CU) multi-stage test</i>	
3.5.3.2	<i>Unsaturated constant water content (CW) test</i>	
3.5.4	Digital Signal Processing	94
3.6	Summary	95
CHAPTER 4:	Finite Element Modelling	96
4.1	Outline and Purpose	96
4.2	Introduction to LS-DYNA	98
4.3	Calibration for stiffness response	98
4.3.1	The Timoshenko beam effect	99
4.3.2	Verification of the finite element model	102
4.3.3	Finite element simulation program	104
4.3.4	Correction factor for the Timoshenko beam effect	105
4.3.5	Validation of correction factor χ_m	108
4.4	Calibration for damping response	114
4.4.1	The Viscoelastic material model	114
4.4.2	Input Parameters for Viscoelastic Model	118
4.4.3	Finite element simulation program	120
4.4.4	Calibration of viscoelastic model using simulated resonant column test	122
4.4.5	Wave Propagation in a semi-infinite medium	125
4.5	Simulations for the Verification of the Hilbert Transform Method	128
4.6	Summary	129
CHAPTER 5:	Results and Discussion	131
5.1	Outline	131
5.2	Test Series I: The Wave Propagation Transducer	131

5.2.1	Wave arrival by ultrasonic system	132
5.2.2	Wave arrival by bender/extender element system	138
5.2.3	Wave arrival by hybrid bender element – ultrasonic system	141
5.2.4	Summary and remarks	147
5.3	Test Series II: Development of testing framework, effect of saturation and confining pressures on small-strain soil properties	150
5.3.1	Soil Water Characteristics Curve	150
5.3.2	Techniques to determine the damping ratio	152
5.3.2.1	<i>Damping ratio via SRM</i>	
5.3.2.2	<i>Damping ratio via LDM</i>	
5.3.2.3	<i>Damping ratio via HTM</i>	
5.3.3	Verification of the Hilbert transform method (HTM)	160
5.3.3.1	<i>Verification with Ottawa sand</i>	
5.3.3.2	<i>Verification with finite element simulation</i>	
5.3.4	Effects of degree of saturation and confining pressure on V_p	163
5.3.5	Effects of degree of saturation and confining pressure on V_s	175
5.3.6	Effects of degree of saturation and confining pressure on ξ	178
5.3.7	Summary and Remarks	183
5.4	Test Series III: Dynamic properties of soil-rubber mixtures	184
5.4.1	Comparison of wave signals between the Ottawa sand, river sand and soil- rubber mixtures	185
5.4.2	Wave velocities and stiffnesses of soil rubber mixtures	188
5.4.3	Damping ratios of soil-rubber mixtures using Hilbert transform method	194
5.4.4	Summary and Remarks	198
5.5	Test Series IV: Cyclic triaxial and wave propagation test on unsaturated residual soil	200
5.5.1	Dynamic Properties of Residual Soil	200
5.5.2	Mechanical Properties of Residual Soil	215
5.5.3	Summary and Remarks	221
CHAPTER 6:	Conclusion and Recommendations	223
6.1	Summary	223
6.2	Conclusion	224
6.2.1	Test Series I	224
6.2.2	Test Series II and III	225
6.2.3	Test Series IV	227
6.3	Recommendations and applications	228
6.3.1	Test Series I	229
6.3.2	Test Series II and III	229
6.3.3	Test Series IV	230
6.3.4	Applications	231
References		233
Appendix A: Borehole log of undisturbed specimens		267

Appendix B: Derivation of Cascante et al. (1998)'s method	268
Appendix C: Procedures for Kumar and Madhusudhan (2010)	272
Appendix D: Collation of Results for FIG. 4.3, 4.4 and 4.5	273
Appendix E: Collation of Results for Test Series II, III and IV	278

Abstract

Due to increasing complexity of the infrastructure in urban living environment, the determination of geotechnical parameters such as soil stiffnesses (shear and Young's moduli) and damping ratio at different strains are becoming increasingly important in geotechnical engineering. However, derivation of these parameters requires additional resources and time to conduct advanced geotechnical testing. One such advanced geotechnical test is the wave propagation test which can be incorporated into various tests to determine the parameters simultaneously. It is the intention of this study to further develop the capabilities of the wave propagation test in order to realise its full potential of determining both the soil stiffness and damping ratio. The versatility of the wave propagation test will be demonstrated through its incorporation into a cyclic triaxial test to determine both the dynamic and mechanical properties of geo-materials.

Despite recent popularity, the wave propagation test has yet to be standardised. Furthermore damping ratio was seldom determined using the wave propagation test. Hence, concerted effort was expended to develop proper procedures for the determination of damping ratio. This study was separated into four test series supplemented by numerical modelling.

Test Series I investigates three different configurations of transducers used for wave propagation test: ultrasonic, bender/extender element and the hybrid bender element – ultrasonic system. Despite the non-invasive nature of the ultrasonic system, it was unable to provide accurate and reliable S-wave velocity, hence shear stiffness, due to poor transducer design. Development of the hybrid bender element – ultrasonic system provided better understanding of the ultrasonic system and its limitations. The bender element test system was eventually selected due to its stronger signal. Prior to subsequent tests, it was further improved with better waterproofing.

Test Series II can be separated into two parts. The first part evaluates the different techniques used to determine the material damping ratio from the P- and S-wave signals. The Hilbert transform method (HTM) was shown to be the most reliable

and robust method to determine damping ratio. As the HTM has never been applied in the field of geotechnical test, it is verified experimentally by comparing the results derived for Ottawa 20-30 sand with those reported in the literature. Numerical simulations were further conducted using a simplified viscoelastic material model to verify the HTM. Meanwhile, the study of the numerical model revealed the discrepancy in the analytical solution which is used to obtain Young's modulus E from the flexure natural frequency. A correction factor χ_m was introduced to account for the discrepancy caused by the Timoshenko beam effect and to reconcile the difference in actual and calculated E . The study allowed better understanding of the input parameters of the viscoelastic model.

The establishment of the testing framework allows the investigation of the effects of changing net confining pressures and degrees of saturation (S_r) on P- and S-wave velocities and damping ratios (V_p , V_s , ξ_p and ξ_s , respectively). The relationship of V_p with Skempton pore pressure parameter B , matric suction and S_r were examined. The limitation of using B -value to determine full saturation was highlighted and V_p was proposed as a better proxy for S_r as it was found to increase sharply when S_r is greater than about 80 - 90% to about 1500 m/s at full saturation. The experiments also showed that the effect of matric suction on V_s is not isotropic compared with isotropic effective confining stress. Trends of ξ_p and ξ_s with S_r were shown to exhibit significant variability. However, values of ξ_p and ξ_s were generally close to each other. The damping ratios were also shown to decrease with increasing confining pressure and to increase with increasing S_r .

Test Series III demonstrated the robustness of the testing framework through tests on soil-rubber mixtures – an engineered material where changes in stiffness and damping characteristics can be controlled and varied significantly. Results of V_s and ξ obtained were compared with those obtained from the resonant column test reported in the literature. The rubber can also be treated as a third phase to observe the response of wave propagating through a material with different phases.

In Test Series IV, undisturbed residual soil specimens were tested in a cyclic triaxial set-up equipped with bender elements, local displacement transducers (LDT) and

proximity sensors. Both dynamic and mechanical properties were determined. Dynamic parameters (E and Poisson's ratio ν) determined using the LDT was found to be inaccurate due to slippage between the rubber membrane and the soil specimens. Results of the dynamic soil properties ($G/G_{\max}$ and ξ) with shear strain were shown to agree with the literature. The residual soil specimens were sheared under consolidated undrained condition and constant water content condition for the saturated and unsaturated specimens, respectively. The effective shear strength parameters (ϕ' and c') were observed to fall within the range shown in the literature. The effective cohesion intercept was shown to increase with depth as the deeper specimens have a higher density. The total cohesion was also shown to increase with both the matric suction and depth.

List of Figures

- FIG. 1.1 Shear strain range of dynamic laboratory test (modified from Das, 1993)
- FIG. 1.2 Scope of research
- FIG. 2.1 Propagation of P and S waves (Modified from: <http://www.astro.uwo.ca/~jlandstr/planets/webfigs/earth/images/waves.gif>)
- FIG. 2.2 Variation of soil stiffness and damping ratio with strain (modified from Wang and Santamarina 2007)
- FIG. 2.3 Variation of S_r with B-value (data from Table 2.1)
- FIG. 2.4 Effect of saturation on compressional (P-wave) and shear (S-wave) damping
- FIG. 2.5 Hysteresis loop from cyclic triaxial
- FIG. 2.6 Characteristic signal displaying prospective features for arrival time (A) First deflection, (B) First bump, (C) Zero after first bump and (D) First major peak (modified from Lee and Santamarina, 2005)
- FIG. 2.7 Ultrasonic transducer configuration
- FIG. 2.8 Logarithmic decrement approach
- FIG. 2.9 Forced vibration response plotted with frequency (f) normalized with natural frequency (f_n) at different damping ratio
- FIG. 2.10 Half-power bandwidth approach
- FIG. 2.11 Simplified soil profiles in sedimentary and residual soils (from Wesley, 2010)
- FIG. 2.12 Typical tropical residual soil profile (from Little, 1969)
- FIG. 2.13 Typical SWCC (from Fredlund and Xing, 1994)
- FIG. 2.14 Extended Mohr-Coulomb failure enveloped for unsaturated soil (from Fredlund and Rahardjo, 1993)
- FIG. 2.15 Line of intercept along the failure plane on the τ versus $(u_a - u_w)$ plane (from Fredlund and Rahardjo, 1993)
- FIG. 2.16 Relationship between G/G_{max} , ξ and PI for normally consolidated and over-consolidated soil (from Vucetic and Dobry, 1991)
- FIG. 3.1 Grain size distribution of reconstituted specimen
- FIG. 3.2 Insulation of bender/extender elements and its (a) series and (b) parallel wiring configuration (adapted from Leong et al., 2009)
- FIG. 3.3 Fabrication of bender/extender element (a) after soldering (b) after water-proofing
- FIG. 3.4 Frequency spectrum for face-to-face (a) P- and (b) S-wave signals from ULT-2012
- FIG. 3.5 Arrival times of shear and compression waves for mild steel specimens of different lengths using ULT-2012 transducers
- FIG. 3.6 (a) Schematic and (b) actual representation of fabricated mould used for casting of silicon rubber
- FIG. 3.7 Triaxial set-up
- FIG. 3.8 Typical S-wave signal from the bender element test

- FIG. 3.9 Frequency domain of P- and S-wave at different excitation frequencies and effective confining pressures
- FIG. 3.10 Time and frequency domain of raw and windowed S-wave signals from RS-US_100H, $(u_a - u_w) = 30$ kPa
- FIG. 3.11 Time and frequency domain of raw and windowed P-wave signals from RS-US_100H, $(u_a - u_w) = 0$ kPa
- FIG. 3.12 Time and frequency domain of raw and windowed P-wave signals from RS-US_100H, $(u_a - u_w) = 30$ kPa
- FIG. 3.13 (a) Raw and filtered S-wave signals excited at 5 kHz and its corresponding (b) magnitude of the analytical signal, specimen RS-US_100H with $(u_a - u_w) = 30$ kPa
- FIG. 3.14 Rubber chips used in the sand-rubber mixture
- FIG. 3.15 Sketch of the segregation specimens
- FIG. 3.16 Comparison of P- and S- wave signals from sand rubber mixtures
- FIG. 3.17 Effects of varying (a) excitation frequency, (b) confining pressure and (c) rubber content on the frequency composition of the S wave signal
- FIG. 3.18 Geological map of Singapore with sampling location marked with a star (modified from http://www.srmeg.org.sg/docs/N13072012_2.pdf)
- FIG. 3.19 Grain size distribution curves of the residual soil
- FIG. 3.20 Modified cyclic triaxial set-up (modified from Leong et al. 2006)
- FIG. 3.21 Criteria for acceptable and unacceptable sine curves (adopted from ASTM D3999-11, 2011)
- FIG. 3.22 Schematic diagram of (a) LDT (from Leong et al. 2011)
- FIG. 3.23 Schematic diagram of (a) original pseudo-hinged attachment (from Goto et al. 1991) and (b) modified pseudo-hinged attachment.
- FIG. 3.24 Schematic diagram of (a) Wheatstone bridge circuit and (b) Anderson loop (from Leong et al. 2011)
- FIG. 3.25 Calibration of LDT using micrometer
- FIG. 3.26 Volume of water outflow with time
- FIG. 4.1 Ratio of frequency corrected for Timoshenko beam effect to those from Euler-Bernoulli solution (F) with dimensionless parameter (Λ) for a uniform cantilever beam of $E/\kappa G = 3.20$.
- FIG. 4.2 Finite element model of the resonant column test
- FIG. 4.3 Variation of χ_m with lumped mass and v
- FIG. 4.4 Variation of χ_m with height of steel attachment
- FIG. 4.5 Variation of χ_m with 2 kg lumped mass of different heights ($L/D=2$)
- FIG. 4.6 Variation of J_o with measured resonant frequency (modified from Priest et al. 2005)
- FIG. 4.7 Graphical Representation of (a) Kelvin – Voigt model, (b) Maxwell model, (c) Biot Hysteretic model, and (d) Standard linear solid or Zener model
- FIG. 4.8 Finite element model with nodes examined for amplitude decay
- FIG. 4.9 (a) Time domain signal when input damping ratio is 10% and (b) the corresponding logarithmic decrement plot
- FIG. 4.10 (a) Comparison of output and input damping ratios for flexure, torsional and longitudinal modes of resonant column test
- FIG. 4.11 Damping ratio obtained from LDM when excited in the (a) flexural (b) torsional and (c) longitudinal mode

- FIG. 4.12 Variation of damping ratio with β/ω for $GR = 0$
- FIG. 4.13 Variation of damping ratio with β/ω at different GR
- FIG. 4.14 Model used to simulate vibration in the free field
- FIG. 4.15 Amplitude decrease with distance from excitation source
- FIG. 4.16 (a) Top and (b) front view of finite element model
- FIG. 5.1 Time and frequency domain of (a,c) compression and (b,c) shear wave of Kaolin specimen (KL-100) using ULT-2012 transducers
- FIG. 5.2 Simultaneous monitoring of both P and S wave transducers upon excitation of S wave transducer (KL-140)
- FIG. 5.3 (a) Compression and (b) shear wave signals for KL-100 tested in bender element system at various frequencies. All input pulses occur at $500\mu s$.
- FIG. 5.4 Comparison of P wave signal from (a) Changi sand (CSD-100) and (b) Kaolin (KL-100) using three different set-ups
- FIG. 5.5 (a) Compression and shear wave signals for KL-100 tested in hybrid system at various frequencies. All input pulses occur at $500\mu s$.
- FIG. 5.6 (a) Compression and (b) shear wave signals for CSD-100 tested in hybrid system at various frequencies. All input pulses occur at $500\mu s$.
- FIG. 5.7 Frequency spectrum of CSD-100 tested in hybrid system
- FIG. 5.8 Frequency spectrum of KL-100 tested in hybrid system
- FIG. 5.9 Frequency spectrum of KL-100 tested in bender element system
- FIG. 5.10 Schematic diagram of a typical ultrasonic transducer (adopted from Panametrics-NDT, 2006)
- FIG. 5.11 SWCC of the specimen expressed in terms of (a) volumetric water content and (b) degree of saturation
- FIG. 5.12 Hysteresis of SWCCs for specimens (a,b) RS-US_100H and (c,d) RS-US_400H
- FIG. 5.13 Variation of P wave arrival time with specimens of different heights
- FIG. 5.14 Variation of S wave arrival time with specimens of different heights
- FIG. 5.15 Damping ratios of 100 mm silicon rubber specimen using cyclic triaxial set-up
- FIG. 5.16 Frequency spectra of sand and silicon rubber specimen
- FIG. 5.17 Damping ratios of river sand specimen at different net confining stress using SRM
- FIG. 5.18 Damping ratios of river sand specimen at different net confining stresses using LDM
- FIG. 5.19 S-wave signals of RS-US_200H at excitation frequencies of (a) 3 and (b) 10 kHz
- FIG. 5.20 Damping ratios of river sand specimen at different net confining stress using HTM
- FIG. 5.21 Damping ratios of sand specimen at different net confining stresses using HTM
- FIG. 5.22 (a, b) Time domain, (b) frequency domain and (c) Hilbert transform of the simulated signal (Input damping ratio = 7%)
- FIG. 5.23 Variation of Skempton pore-pressure parameter B with V_p during the saturation phase
- FIG. 5.24 P wave signals propagating through different specimens close to saturation and water (50 kHz excitation frequency)

- FIG. 5.25 Variation of B-value and V_p with increasing with increasing effective stress
- FIG. 5.26 Variation of V_p with void ratio
- FIG. 5.27 Variation of V_p with matric suction
- FIG. 5.28 Variation of V_p with degree of saturation
- FIG. 5.29 Frequency domains of P wave signals (a) before and (b) after the air-entry value of RS-US_400H
- FIG. 5.30 (a) V_p vs degree of saturation (b) V_p/V_{p0} vs degree of saturation (data sources in Table 2.1)
- FIG. 5.31 Variation of V_p with Skempton's Pore-water pressure parameter B
- FIG. 5.32 Effect of subjected stress on V_s for coarse grained specimens
- FIG. 5.33 Effect of subjected stress on V_s for Metrano silty sand compacted (a) at optimum and (b) wet of optimum water content (Alramahi et al. 2007)
- FIG. 5.34 Effect of subjected stress on V_s for SP soil (Hoyos et al. 2008)
- FIG. 5.35 Comparison of P- and S- wave damping for specimens at different height
- FIG. 5.36 Comparison of (a) P- and (b) S- wave damping for specimens subjected to different net isotropic stresses
- FIG. 5.37 Comparison of P- and S- wave damping for (a) RS-US_100H, (b) RS-US_200H and (c) RS-US_400H
- FIG. 5.38 Comparison of P- and S- wave damping subjected to hysteretic saturation
- FIG. 5.39 Collation of S-wave signals from different geo-materials
- FIG. 5.40 (a) S- and (b) P-wave velocities of specimens with confining pressure
- FIG. 5.41 (a) S- and (b) P-wave velocity of specimens with rubber content
- FIG. 5.42 Variations of (a) $V_{s,0}$ and (b) B_R with confining pressure
- FIG. 5.43 Variation of Poisson's ratio with rubber content
- FIG. 5.44 Effects of segregation on V_s for a specimen with 15% rubber content
- FIG. 5.45 Comparison of experimental V_s with the Voigt and Reuss models
- FIG. 5.46 Variation of mean damping ratio with rubber content and confining pressure
- FIG. 5.47 Combined results of damping ratio and normalised V_s with rubber content at effective confining pressures of 100, 200 and 400 kPa
- FIG. 5.48 (a) Young's modulus and (b) damping ratio of AMK2-US_50 subjected to 40 cycles of sinusoidal loading at 0.5 Hz with double amplitude strain (ϵ_{DA}) of 0.2%
- FIG. 5.49 Young's modulus of dense gravel obtained from both LDT and proximeter at different void ratios and effective confining pressures (data from Tatsuoka et al. 1994)
- FIG. 5.50 Difference in the level of strain in LVDT and LDT expressed in the (a) time domain and (b) hysteresis stress-strain plot (Excitation frequency of 0.5 Hz, ϵ_{DA} of 0.2%)
- FIG. 5.51 Variation of $\epsilon_{DA,LVDT}/\epsilon_{DA,LDT}$ with $\epsilon_{DA,LVDT}$ at different excitation frequencies.
- FIG. 5.52 Calibration set-up (a) without and (b) with rubber membrane (adopted from Hoque et al., 1997)
- FIG. 5.53 Variation of G/G_{max} with shear strain for specimens (a) AMK1 and (b) AMK2

- FIG. 5.54 Variation of v with axial strain for specimens (a) AMK1 and (b) AMK2
- FIG. 5.55 Variation of normalised Poisson's ratio with strain for specimens from the literature
- FIG. 5.56 Variation of ξ with shear strain for specimens (a) AMK1 and (b) AMK2
- FIG. 5.57 Comparison of G from specimens (a) AMK1 and (b) AMK2 with Tou (2003)
- FIG. 5.58 FIG. 5.58: Comparison of ξ from Tou (2003) with specimens (a) AMK1 and (b) AMK2
- FIG. 5.59 Variation of (a, b) deviator stress, (c, d) pore water pressure and (e, f) matric suction.
- FIG. 5.60 (a) Mohr circle and (b) effective stress path of AMK1-S
- FIG. 5.61 (a) Mohr circle and (b) effective stress path of AMK2-S
- FIG. 5.62 Extended Mohr circle plot for both saturated and unsaturated specimens of AMK1
- FIG. 5.63 Extended Mohr circle plot for both saturated and unsaturated specimens of AMK2
- FIG. 5.64 Cohesion intercept with matric suction from Test Series IV plotted with results from Rahardjo et al. (2004)

List of Tables

Table 2.1	Summary of data shown in FIG. 2.3
Table 2.2	Summary of bender/extender element tests since 2001
Table 2.3	Summary of ultrasonic soil tests in the literature
Table 3.1	Specimens tested for Test Series I
Table 3.2	Basic soil properties of reconstituted specimens
Table 3.3	Bender element connection configuration
Table 3.4	Specimens tested for Test Series II
Table 3.5	Summary of literatures which conducted resonant column tests on Ottawa sand
Table 3.6	Composition of sand and rubber chips in specimens of different rubber content
Table 3.7	Basic soil properties and test conditions of the residual soil
Table 3.8	PID values for rubber specimen subjected to effective confining pressure of 100, 300 and 600 kPa
Table 4.1	Range of χ_m from FIGs. 4.3 and 4.4
Table 4.2	Properties of Methyl Hydrate specimen H1
Table 4.3	Back-calculation of torsional natural frequency and properties of attached peripherals
Table 4.4	Comparison of Young's modulus calculated from different methods
Table 4.5	Soil parameters used for simulation
Table 5.1	V_p and V_s using ultrasonic transducers - ULT 2012
Table 5.2	Ultrasonic transducers' transmission energy (T) for P and S waves
Table 5.3	V_p and V_s using the hybrid system and bender element system
Table 5.4	L_{tt}/λ ratio for FIGs. 5.3b, 5.5b and 5.6b
Table 5.5	Damping ratios from the SRM for silicon rubber specimen
Table 5.6	Comparison between input damping ratio and those derived from HTM
Table 5.7	Summary of soils used for comparison with experiment results
Table 5.8	Effects of segregation on V_s for a specimen with 15% rubber content
Table 5.9	Details of resonant column test conditions for the damping ratios shown in FIG. 5.47
Table 5.10	Summary of results from bender element tests
Table 5.11	Summary of results from cyclic triaxial tests
Table 5.12	Comparison of G obtained via different methods
Table 5.13	Basic properties of BT and JF soils (Tou, 2003)
Table 5.14	Effective shear strength parameters of Bukit Timah granite (adopted from Leong et al. 2002)

List of Symbols

$\hat{\eta}$	Viscosity coefficient of the dashpot
a	Inflexion point of the SWCC (Equation 2.21)
A	Cross-sectional area
a'	Ordinate intercept of the failure envelope in $p' - q$ plot
a_0	Coefficient for Equation 4.4
A_1	Amplitude at distance x_1
a_1	Coefficient for Equation 4.4
$\bar{a}_1$	Coefficient of equation shown in FIG. 5.42a
a_2	Coefficient for Equation 4.4
$\bar{a}_2$	Coefficient of equation shown in FIG. 5.42a
a_3	Coefficient for Equation 4.4
$\bar{a}_3$	Coefficient of equation shown in FIG. 5.42a
A_i	Peak amplitude at the i^{th} cycle
A_{i+N}	Peak amplitude at the $(i+N)^{\text{th}}$ cycle
A_n	Amplitude at distance x_n
B	Skempton's pore-water pressure parameter
b_1	Coefficient of equation shown in FIG. 5.42b
b_2	Coefficient of equation shown in FIG. 5.42b
B_R	Curve fitting coefficient of Equation 5.3
c	Total cohesion intercept
$C(\psi)$	Correction function of SWCC
c'	Effective cohesion
C_1	Coefficient for Equation 4.5
C_2	Coefficient for Equation 4.5
C_3	Coefficient for Equation 4.5
C_4	Coefficient for Equation 4.5
C_c	Coefficient of gradation
C_u	Coefficient of uniformity
D	Diameter of the specimen
D_{50}	Average grain diameter
e	Euler's number (Equation 2.21)
e	Void ratio
E	Young's modulus
E_c	Young's modulus calculated using Cascante et al. (1998) method
e_{max}	Maximum void ratio
e_{min}	Minimum void ratio
f	Frequency of the wave
f_e	Frequency corresponding to the end of the frequency range for calculating the representative dominant frequency
f_{in}	Input excitation frequency
f_n	Natural frequency
f_{tor}	Torsional natural frequency of the specimen
G	Shear modulus
$G(t)$	Shear modulus at time t

$G(x)$	Frequency independent geometrical factor
G^*	Complex shear modulus
G'	Elastic or storage shear modulus
G''	Loss or viscous shear modulus
G_∞	Long term or infinite shear modulus
G_i	Shear modulus of the i^{th} Maxwell model placed in parallel connection
G_{max}	Small-strain shear modulus
G_o	Instantaneous or short term shear modulus
GR	G_∞/G_o
G_s	Specific gravity of the soil grains
h_o	Height of the bottom of the attached mass measured from the top of specimen
h_l	Height of the top of the attached mass measured from the top of specimen
i	Imaginary number
I	Area moment of inertia
J	Mass polar moment of inertia of the specimen
J_o	Mass polar moment of inertia of the drive plate
K	Bulk modulus
k	Constant for the fundamental mode of flexure vibration (3.5156)
K_b	Bulk modulus of the soil
K_f	Bulk modulus of the fluid
K_{max}	Small-strain bulk modulus
K_s	Bulk modulus of the soil grains
K_w	Bulk modulus of the pore-water
L	Length of the specimen
L_{DA}	Double amplitude load
LL	Liquid limit
L_{tt}	Tip to tip distance of the bender/extender element
m	Gradient
m	Parameter that is related to the residual water content (Equation 2.21)
m_i	Mass of the i^{th} attached mass
M_R	Mass of rubber in the sand-rubber mixture
M_{ru}	Rubber chips content by mass in percent
m_s	Mass of the specimen
M_s	Mass of sand in the sand-rubber mixture
m_x	Mass of the sample per unit length
n	Porosity
n	Parameter that controls the slope at the inflection point in the SWCC (Equation 2.21)
$P(f)$	Ordinate of the power spectrum
p'	$(\sigma'_1 + \sigma'_3)/2$
p_{atm}	Atmospheric pressure (100 kPa)
PI	Plasticity index
PL	Plastic limit
Q	Quality factor
q	$(\sigma_1 - \sigma_3)/2$

R	Reflection coefficient
r	Radius of the specimen
S_{DA}	Double amplitude axial deformation
S_r	Degree of saturation
t	Time
T	Transmission coefficient
t_{100}	Time for 100% primary consolidation
t_{50}	Time for 50% primary consolidation
t_{90}	Time for 90% primary consolidation
T_c	Retardation time of the Kelvin-Voigt model
Tr	Relaxation time of the Maxwell model
T_{ri}	Relaxation time of the i^{th} Maxwell model placed in parallel connection
u_a	Pore-air pressure
u_w	Pore-water pressure
u_{wf}	Pore-water pressure at failure
V	Wave velocity
V_{lf}	Longitudinal wave velocity
V_p	P-wave velocity
V_s	S-wave velocity
$V_{s,0}$	V_s of the specimens with 0% rubber chips
$V_{s,R}$	V_s of rubber in the sand-rubber mixture
$V_{s,Reuss}$	V_s of sand-rubber mixture obtained using the Reuss model
$V_{s,S}$	V_s of sand in the sand-rubber mixture
$V_{s,Voigt}$	V_s of sand-rubber mixture obtained using the Voigt model
w	Gravimetric water content
x	Distance
$x(t)$	Signal in the time domain
$x_a(t)$	Analytic signal
$x^H(t)$	Hilbert transform of the time domain signal
y	Horizontal displacement of the specimen
Z	Acoustic impedance
α	Attenuation coefficient due to material damping
β	Decay constant corresponding to the inverse of the relaxation time
γ	Shear strain
γ_{DA}	Double amplitude shear strain
γ_o	Amplitude of the sinusoidal shear strain
γ_{td}	Threshold shear strain
γ_t	Shear strain at time t
δ	Logarithmic decrement
Δt	Wave travel time
Δu	Change in pore-water pressure
$\Delta \sigma$	Change in confining pressure
ε_{DA}	Double amplitude axial strain
η	Loss factor
θ_a	Residual water content
θ_s	Saturated volumetric water content
θ_w	Volumetric water content
κ	Timoshenko shear coefficient

λ	Wavelength
Λ	Ratio of radius of gyration of cross section and length of beam
ν	Poisson's ratio
ξ	Damping ratio
$\xi_{\min}$	Small-strain damping ratio
ξ_p	P-wave damping ratio
ξ_s	S-wave damping ratio
π	Mathematical constant (3.14159)
ρ	Bulk density
$\rho_{d,\max}$	Maximum dry density
$\rho_{d,\min}$	Minimum dry density
ρ_R	Density of rubber in the sand-rubber mixture
ρ_S	Density of sand in the sand-rubber mixture
σ	Normal stress
σ_1'	Major principal stress
σ_3	Confining pressure or minor principal stress
σ_3'	Effective confining pressure
τ	Shear stress
τ_o	Amplitude of the sinusoidal shear stress
τ_t	Shear stress at time t
ϕ	Phase difference between shear stress and shear strain
ϕ'	Effective friction angle
ϕ^b	Angle describing the rate of change of shear strength with matric suction
χ	Scaling factor for geometric damping
χ_m	Correction factor of Cascante et al. (1998) equation for Timoshenko effect
ψ	Matric suction (Equation 2.21)
ψ'	Angle of the failure envelope in $p' - q$ plot
ψ_r	Matric suction corresponding to the residual water content
ω	Circular radian frequency
ω_n	Natural radian frequency

List of Abbreviations

GCTS	Geotechnical Consulting & Testing Systems
P-wave	Primary or Compression wave
S-wave	Secondary or Shear wave
LDM	Logarithmic Decrement Method
HPBM	Half-Power Bandwidth Method
SRM	Spectral Ratio Method
HTM	Hilbert Transform Method
SWCC	Soil-Water Characteristics Curve
HAE	High Air-Entry
CW	Constant Water Content Test
CD	Consolidated Drained Test
CU	Consolidated Undrained Test
PI	Plasticity Index
ULT-2012	Ultrasonic transducer used in Leong et al. (2004)
ULT-2014	Ultrasonic transducer used in this thesis
PVC	Polyvinyl Chloride
RTV	Room Temperature Vulcanisation
DPVC	Digital Pressure-volume Controller
GDS	Global Digital Systems
EPDM	Ethylene-Propylene-diene-Monomer
UTM	Universal Testing Machine
IPC	Industrial Process Controls
PMMA	Polymethyl methacrylate
LDT	Local Deformation Transducer
LVDT	Linear Variable Displacement Transducer
PID	Proportional, Integral and Derivative
HPS	Homogeneous Partial Saturation
NHPS	Non-homogeneous Partial Saturation

CHAPTER 1: Introduction

1.1 Background

Today, geotechnical designs are facing more stringent and challenging criteria. Increasing population density in built-up cities mean that taller mega-structures would have to be constructed in close proximity to house the population. Areas which were previously shunned like slopes would have to be included for development. In order to maximize the use of land space, infrastructures like railways and subways have to be constructed underground, usually beneath foundation level. In view of this, traditional geotechnical parameters are no longer sufficient for design of these constructions. Geotechnical parameters (Shear modulus G , Young's modulus E) at different strains would have to be used to optimize the design. Dynamic or small-strain parameters (stiffness and damping ratio ξ) would be needed to study the effect of ground vibration from sources ranging from natural disasters (e.g. earthquakes) to machinery (e.g. trains).

Generally, the dynamic parameters needed are the stiffness and damping ratio at various strains with emphasis at small-strain. The derivation of these dynamic parameters requires advanced geotechnical testing apparatuses such as the resonant column, cyclic triaxial and bender element. These tests differ in the shear strain range as seen in FIG 1.1. Among these apparatuses, both the resonant column and the cyclic triaxial apparatuses can be used to determine both stiffness and damping ratio, and have been included in testing standards ASTM D4015-07 (2007) and ASTM D3999-11 (2011), respectively. On the other hand, while wave propagation techniques have been gaining popularity in recent years, it has yet to be included in testing standards.

FIG. 1.1: Shear strain range of dynamic laboratory test (modified from Das, 1993)

Wave propagation techniques use compression (P) and shear (S) waves to determine the Young's modulus and the shear stiffness of the geo-material and hence the Poisson's ratio ν . Moreover, the transducers used to conduct the tests are small and can be incorporated into conventional testing apparatuses like the triaxial and simple shear apparatuses. This will bring about great savings in time and resources as the small-strain modulus can be determined concurrently during the test without the need for additional samples. This time saving aspect will be exceptionally significant in the area of unsaturated soil mechanics.

With the exception of deep foundation where saturated soil is encountered, most geotechnical problems like landslides and seepage problems deal with soils closer to the ground surface which are essentially unsaturated. The seismic response of the soil at different stress-state and degree of saturation was also seldom investigated despite the fact that seismic tests have been used in geotechnical testing and oil exploration (Clayton et al. 2005, Priest et al. 2005). This is mainly due to the large amount of time needed to bring the specimen to its unsaturated condition in the laboratory. By combining wave propagation tests with conventional testing apparatuses, the capability of laboratory tests can be greatly improved and thereby promote the implementation of unsaturated soil mechanics into engineering practice.

However, wave propagation tests still suffers from various inadequacies. One of which is material damping ratio is still not determined in wave propagation test. Therefore, if material damping ratio can be determined using wave propagation test, it will increase the use of wave propagation test in routine geotechnical laboratory tests.

1.2 Objectives of Research

The objective of this study is to further develop the capabilities of the wave propagation test in order to realise its full potential of determining both the wave velocity and damping ratio. The versatility of the wave propagation test is demonstrated through its incorporation into a cyclic triaxial test to determine both the dynamic and mechanical properties of geo-materials.

1.3 Scope of Research

Wave propagation tests in the laboratory are typically performed using ultrasonic transducers or bender elements. In this study, both of the above mentioned transducers are evaluated to determine their suitability for wave propagation test. Despite the popularity of the wave propagation test in small strain testing, it is still incapable of determining the material damping ratio. Therefore the study will first focus on the development of the methodology used to obtain small-strain stiffness and damping ratio. Verification of the methodology was conducted by testing standard materials and finite element simulations using LS-DYNA. The numerical model was calibrated by simulating a simplified resonant column test to ensure that the model gives the correct response in terms of stiffness and damping. The input parameters of the viscoelastic material model used to simulate material damping was studied so that the relationship between input parameters and simulation outcome can be better understood.

As soil exists in various degrees of saturation, the effect of degree of saturation and net confining pressure on the wave propagation tests would have to be investigated.

This is to correctly attribute the results obtained to the stress-state conditions of the soil. Reconstituted soil was used in this case due to the ease of reproducing homogenous specimens of similar properties. Wave propagation tests were also conducted on soil-rubber mixtures to ensure that the methodology developed was able to reflect the stiffness and damping ratio for a wide range of soil materials. Results were compared with those obtained using resonant column tests reported in the literature. Finally, tests were conducted on saturated and unsaturated residual soils using a cyclic triaxial apparatus which was modified to incorporate bender-extender elements. Both dynamic and mechanical soil properties were obtained using this test set-up to demonstrate the applicability of the techniques developed in this study. The scope of the research is summarized in FIG. 1.2.

FIG. 1.2: Scope of research

1.4 Organization of Thesis

The thesis is organized into six chapters:

Chapter 1 provides the background to the problem. The objective and scope of the research are also specified.

Chapter 2 provides a brief introduction to wave propagation in geo-material. The variation of wave velocities, stiffness and damping ratio with strain, degree of saturation and confining pressure are reviewed. Some of the advanced laboratory

tests are reviewed and the need to further develop the wave propagation test is explained. Finally, the residual soil used in this study is briefly introduced together with the basic principles of unsaturated soil mechanics. The correlation between soil properties and its dynamic parameters are also concisely explained.

Chapter 3 focuses on the experimental aspects of this study. The experiments are divided into four different series each with a different scope which will contribute to the determination of the shear modulus and damping ratio of unsaturated residual soils. The approach and methodology of each series are explained prior to the experimental procedures and set-up. The first series consisted of wave propagation test conducted using different transducers. This series was performed to develop the equipment used for subsequent tests. The second series investigate the effect of confining pressure and degree of saturation on the dynamic properties of an unsaturated sand. Different techniques were employed to determine the material damping ratio. Verification using experimental and numerical means was conducted to ascertain the reliability of the method described. In the third series, techniques developed are applied to soil-rubber mixture to ascertain their ability to detect soils of varying stiffness and damping ratio. Finally the framework developed for the wave propagation test was incorporated into the cyclic triaxial set-up to test undisturbed residual soil specimens in both the saturated and unsaturated states. After the cyclic triaxial test, the specimen was sheared to failure.

Chapter 4 illustrates the calibration of the viscoelastic material model in finite element program LS-DYNA. Calibration was done to ensure that the finite element model was able to produce the correct stiffness and damping response. The input material stiffness was compared with those obtained from analytical solution using natural frequencies obtained from finite element simulation. The damping ratio was obtained using the free-vibration decay part of the simulated receiver signal. The numerical model was verified by simulating wave propagation in a semi-infinite medium. Subsequent numerical simulations to verify the Hilbert transform method in obtaining the damping ratio are presented in the last portion of the chapter.

Chapter 5 discusses the results of the four test series. In the first series of test, the reasons for using bender-extender elements for subsequent tests are clarified. After which, techniques used to derive the damping ratio are reviewed and the Hilbert transform method was selected. The Hilbert transform method is justified with verifications from experiments and finite element simulations. Subsequently, the effects of confining pressure and degree of saturation on unsaturated reconstituted soil are discussed. Results of wave propagation test on soil-rubber mixtures are presented and compared with existing literatures. Finally, the dynamic and mechanical properties of unsaturated residual soils derived from the modified cyclic triaxial setup are presented. Results are compared with those obtained from the literature.

Chapter 6 concludes this study. The significance and contribution of the study are highlighted. In addition, recommendations for future research are also presented.

CHAPTER 2: Literature Review

2.1 Wave Propagation in Soil

Perturbation in soil can either occur from natural sources such as earthquakes or from man-made sources like machine vibration. These vibrations lead to the generation of seismic waves which can be broadly classified into two categories – body and surface waves. Body waves consist of compression (P) and shear (S) waves. Body waves propagate radially outward from the source along a hemispherical wave front. Surface waves consist of Rayleigh, Love and Lamb waves. Surface waves propagate radially outward along a cylindrical wave front, concentrating along the earth surface with amplitude decreasing with depth (Richart et al., 1970). Out of the three wave types, P-wave velocity is the fastest followed by S-wave then surface wave. Surface waves only occur in the interface of two different media while body waves propagate within the geo-material. This study focuses only on the propagation of body waves (P and S waves).

Compression wave (FIG. 2.1a) causes particles to oscillate in the direction of wave propagation and is able to travel through all types of medium thus reflecting the medium's bulk modulus (K). On the other hand, S wave (FIG. 2.1b) causes the particles to oscillate perpendicular to the direction of wave propagation and can only travel through solids with shear stiffness i.e. it does not travel through fluids. It thus reflects the medium's shear stiffness (G). Small-strain bulk (K) and shear (G) moduli can be obtained from P- and S-wave velocities using the relationship derived from elastic theory as shown in Equation 2.1.

$$V_p = \sqrt{\frac{K + \frac{4}{3}G}{\rho}} \quad (2.1a)$$

$$V_s = \sqrt{\frac{G}{\rho}} \quad (2.1b)$$

where V_p is the P-wave velocity, V_s is the S-wave velocity, K is the bulk modulus, G is the shear modulus and ρ is the bulk density of the soil.

FIG. 2.1: Propagation of P and S waves

(Modified from:

<http://www.astro.uwo.ca/~jlandstr/planets/webfigs/earth/images/waves.gif>)

Propagation of wave involves the transfer of energy through a medium. During this process, the wave will experience attenuation which can be broadly classified as geometric damping and material damping as shown in Equation 2.2.

$$A_n = A_1 \left(\frac{r_1}{r_n} \right)^\chi e^{[-\alpha(x_n - x_1)]} \quad (2.2)$$

where A_n and A_1 are amplitudes at distances x_n and x_1 , respectively, α is the attenuation coefficient due to material damping and χ is the scaling factor for geometric damping.

Geometric damping is the decrease in wave amplitude with the increasing size of the wave front during wave propagation i.e. energy being redistributed to a larger area. Udías (1999) suggested that the geometric damping scaling factor χ can range from 0.5 to 2 depending on the type and location of the propagated wave (Das,

1993). Geometric damping is however not crucial in laboratory experiments as the cylindrical shape and the short specimen height would limit geometric spreading and scattering leaving only material damping (Ishihara, 1996). Cha et al. (2009) also observed the absence of geometric attenuation when wave propagates in a cylindrical rock column. The magnitude of geometrical damping can be brought into perspective using Equation 2.2. Assuming that x_1 represents a small distance of 0.01 mm at the source of perturbation and x_n represents the height of the specimen. For a 140 mm high cylindrical specimen without material damping, reduction in amplitude due to geometric damping ranges from $8.45 \times 10^{-1} - 5.10 \times 10^{-7}\%$ as χ increases from 0.5 to 2.

This study will focus on material damping which occurs due to the energy dissipation of real earth material. Energy dissipation due to material damping can arise due to various mechanisms. For the case of geo-materials, energy loss can arise from friction at the point of inter-granular contact, anelasticity of bonds and the relative motion between fluid and skeletal frame (Stoll, 1979). However, the loss mechanism is not within the scope of this study and damping is measured regardless of the mechanism involved. In the literature, damping can be measured by a variety of parameters like the attenuation coefficient (α), quality factor (Q), logarithmic decrement (δ), damping ratio (ξ) and loss factor (η). The correlation between the different parameters is given in Equation 2.3.

$$\frac{1}{Q} = \frac{V\alpha}{\pi f} = 2\xi = \eta = \frac{\delta}{\pi} \quad (2.3)$$

where V is the velocity and f is the frequency of the wave.

The attenuation coefficient (α) carries a unit of dB/cm and is shown to vary with frequency. The rest of the parameters are dimensionless and are typically assumed to be frequency independent. For standardization, material damping in this study will be reported in terms of damping ratio (ξ).

2.1.1 Non-linearity of soil properties

Stress-strain behaviour of soil is known to be highly non-linear (Atkinson, 2000) as depicted in FIG. 2.2 where shear stiffness and damping ratio are shown to vary with increasing strain. Below the threshold strain (γ_{td}), soil properties are independent of strain. The threshold strain (γ_{td}) is defined as the strain where the soil microstructure and engineering properties are irreversibly altered by cyclic shearing (Vucetic, 1994). Vucetic (1994) shows that the value of γ_{td} increases with decreasing size of soil particles and can be correlated to the plasticity index (PI).

Soil properties obtained below γ_{td} are termed small-strain soil properties. Due to their linear elastic behaviour, the values at small strain (G_{max} and ξ_{min}) are used to normalise shear modulus and damping ratio at higher strains (Macari and Hoyos 1996, Zhang et al. 2005). Shear modulus (G) is usually normalised with G_{max} to give G/G_{max} . However, for damping ratio ξ , there is no fixed expression for the normalization since ξ increases with increasing strain. Macari and Hoyos (1996) used ξ/ξ_{min} while Zhang et al. (2005) used $(\xi-\xi_{min})$. Often, the damping ratio is not normalised as shown in FIG. 2.2. In this study, small-strain stiffness is expressed in terms of wave velocities to facilitate the discussion especially in terms of the relationship of V_p and S_r which will be mentioned in later sections. The wave velocities also take into account of the effect of density on the soil thereby enabling better visualisation of the soil structure. Damping ratio, on the other hand, will not be normalised.

FIG. 2.2: Variation of soil stiffness and damping ratio with strain (modified from Wang and Santamarina 2007)

2.1.2 Effect of confining pressure and degree of saturation on wave velocities

Wave propagation through geo-material is a complicated phenomenon as different types of waves are known to travel through different paths in a multi-phase medium thereby leading to vastly different response. As such, confining pressure and degree of saturation are known to affect P- and S-wave velocities of the soil differently.

P-wave is able to travel through both solid and fluid phases and therefore is greatly influenced by the degree of saturation of the soil. When approaching full saturation, V_p tends towards or exceeds those of V_p in water (=1500 m/s). However, effect of confining pressure and degree of saturation (S_r) on V_p were seldom investigated in detail. To date, although V_p was touted to be a proxy to Skempton's pore-water pressure parameter B (Skempton, 1954) for S_r (Yoshimi et al. 1989, Tamura et al. 2002, Hatanaka and Masuda 2008), little has been done to characterize the trend of V_p with degree of saturation for different confining pressures and geo-materials. The characteristics of P-wave signals at different degree of saturation using bender/extender elements were also rarely explored. While correlation of S_r with the B -value is convenient in the laboratory setting, it cannot be determined in-situ. Even

for the determination of B-value in a laboratory, there are limitations using B-value as a proxy for degree of saturation.

The Skempton's pore-water pressure parameter B was introduced as a proxy for degree of saturation and is commonly used in laboratory tests to ascertain full saturation. The B-value has the following expression:

$$B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1 + n \left(\frac{K_b}{K_f} \right)} \quad (2.4)$$

where Δu is the change in pore-water pressure, $\Delta \sigma$ is the change in confining pressure, n is the porosity of the soil specimen, K_b is the bulk modulus of the soil, and K_f is the bulk modulus of the fluid.

The B-value increases from 0 to 1 as a dry soil reaches full saturation. Bishop (1973) later extended Equation 2.4 to give Equation 2.5 to account for the compressibility of the soil skeleton:

$$B = \frac{\Delta u}{\Delta \sigma} = \frac{1}{1 + n \left(\frac{1}{K_w} - \frac{1}{K_s} \right) / \left(\frac{1}{K_b} - \frac{1}{K_s} \right)} \quad (2.5)$$

where K_w is the bulk modulus of the pore-water and K_s is the bulk modulus of the soil grains.

Equation 2.5 bridges the discontinuity between experimental results and theoretical derivations especially for the case of stiff soil specimens whereby saturated specimens have a B-value less than 0.95 – the value representing full saturation (ASTM D7181-11, 2011). The above limitation was first shown in Black and Lee (1973) where soft to medium clay was able to reach a B-value of 0.96 at 99.5 % saturation but stiff clay and sands can only reach a B-value of 0.69 at a similar degree of saturation. Similar observation was obtained by Kokusho (2000) for Masa soil at different relative densities i.e., stiffnesses. In more extreme cases like rock specimens, Mesri et al. (1976) found that the B-value decreased from values close

to 1 to values in the range of 0.33 – 0.69 as effective stress increases from 0 to 11 MPa. Yang (2005) plotted B-value with S_r for different K_b/K_s ratios and showed that as K_b/K_s approaches unity, the B-value at similar degree of saturation decreased significantly. As the B-value at or near full saturation depends on parameters like porosity and stiffnesses of the constituent phases, it becomes difficult to ascertain the B-value at full saturation. Furthermore, K_b and K_s in Equation 2.5 cannot be measured independently.

The variation of the B-value with degree of saturation for different soils is shown in FIG. 2.3. Details of the soil specimens shown in FIG. 2.3 are tabulated in Table 2.1. FIG. 2.3 shows that there is no unique relationship between S_r and B. Furthermore, the parameter B at full saturation varies from 0.79 to 0.95. As such, it will be useful to use another parameter as an indicator of full saturation for stiff geo-materials like silty clays and sands.

FIG. 2.3: Variation of S_r with B-value (data from Table 2.1)

Table 2.1: Summary of data shown in FIG. 2.3

Author	Soil Tested	Specimen Tag	Testing Condition
Hatanaka and Masuda (2008)	Toyoura sand		<ul style="list-style-type: none"> • Confining Pressure = 50 kPa
	Futtsu sand	H-TA-RXX	<ul style="list-style-type: none"> • $D_r = 45, 55$ and 70%
	Keisa Sand No. 2	H-FU-RXX	<ul style="list-style-type: none"> • Saturating fluid – Tap water, De-aired water and ground water
	Keisa sand No. 3	H-KS2-RXX H-KS3-RXX	<ul style="list-style-type: none"> • Diameter = 50 mm, Height = 100 mm • Outside diameter = 100mm, Inside diameter = 60 mm and Height = 100 mm • $D_r = 60\%$
Yoshimi et al. (1989)	Toyoura sand	Y-TA-RXX	
Huang et al. (1999)	Toyoura sand	HG-TA	
George et al. (2009)	Bonny Dam Silt		
	Heterogeneous Soil (85% sand, 13% silt and 2% clay)	G-BDS G-HS	<ul style="list-style-type: none"> • Diameter = 100 mm, Height = 125 mm
Byun et al. (2013)	Uniform-sized sand of $D_{50} = 0.45$		
	Sand silt mixture of 20% silt	BY-SA BY-SS	
Barrière et al. (2012)	Sand from Landes, France	BA-SA	<ul style="list-style-type: none"> • Confining Pressure = 0 kPa • Rectangular container $(1.07 \times 0.4 \times 0.35) \text{m}^3$
Kumar and Madhusudhan (2012)	Poorly graded sand, $D_{50} = 0.37$ mm	KM-S-P100-RXX	<ul style="list-style-type: none"> • Confining Pressure = 100 and 300 kPa • $D_r = 40, 50, 60, 70$ and 80%
		KM-S-P300-RXX	<ul style="list-style-type: none"> • Diameter = 50 mm, Height = 100 mm • Confining Pressure = 49 kPa
Tamura et al. (2002)	Toyoura sand	TA-TA-RXX	<ul style="list-style-type: none"> • $D_r = 64$ and 84% • Diameter = 300 mm, Height = 600 mm

(to be continued on the next page)

Tsukamoto et al. (2002)	Toyoura sand	TO-TA-RXX	<ul style="list-style-type: none"> • Confining Pressure = 98 kPa • $D_r = 30, 40, 50, 60$ and 70% • Diameter = 60 mm, Height = 120 mm
Valle-Molina (2006)	Washed Mortar Sand	VM-MS	<ul style="list-style-type: none"> • Used both bender elements and

Note: The specimen tag is labelled in three parts. The first part represents the author's name, followed by the soil tested and then the corresponding relative density. For example, TM-TA-R40 refers to the Toyoura sand tested by Tamura et al. (2002) at relative density of 40%. For Hatanaka and Masuda (2008) where different saturating fluid was used, a further extension of the specimen tag represent the saturating fluid. TW is for Tap water, DAW is for de-aired water and GW is for ground water. For Barrière et al. (2012), the soil specimen went through 2 drying and wetting process. The extension W and D stands for the wetting and drying, respectively. For Kumar and Madhusudhan (2012), P stands for the confining pressure in kPa

As S-wave only travels through the solid phase, S-wave velocity (V_s) is rather insensitive to changes in degree of saturation. Most investigations of V_s with confining pressure involve either dry or full-saturated specimens (Hardin and Richart 1963, Oztoprak and Bolton 2013). For the case of unsaturated soil, the degree of saturation will indirectly alter the stress-state variables in unsaturated soils i.e. matric suction and net normal stress. Reduction in saturation can lead to an increase in matric suction resulting in a stiffer soil which will give rise to higher V_s . The effect of matric suction on the soil skeleton may not be akin to positive pore-water pressure which is isotropic. Difference in trends with changing confining pressure between saturated and unsaturated soil were seldom compared in the literature and will be investigated in this study.

2.1.3 Effect of confining pressure and degree of saturation on damping ratio

Energy loss in geo-material is typically represented by the damping ratio. Stoll (1985) suggested that energy loss in a dry soil specimen is a result of slippage between the particles. When the soil specimen is saturated, the observed damping is larger due to the additional viscous loss from the fluid. The following is congruent

to that of Bolton and Wilson (1990) who suggested that the derived damping ratio is a summation of hysteretic (due to aggregate) and viscous damping (due to viscosity of fluid).

The literature (Hardin 1965, Murphy 1982, Bolton and Wilson 1989; 1990, Zavoral and Campanella 1994) shows theoretically and experimentally that hysteretic damping which occurs in dry specimens is solely dependent on strain and independent of frequency. Lin et al. (1996) disagree with the above and suggested that hysteretic damping is dependent on the excitation frequency. However, the approach undertaken by Lin et al. (1996) remains questionable as they had used values (amplitude of applied torque, amplitude of angle of twist and phase angle) derived from cyclic torsional shear test as input into their numerical model to obtain the shear modulus and damping ratio instead of from the test.

Viscous damping is a result of the relative motion between the fluid field and the skeletal frame and is hence a function of frequency, fluid viscosity and permeability. Stoll (1979, 1980, 1985) and Bolton and Wilson (1990) showed that the damping ratio changes as the excitation frequency transits from low frequency range to high frequency range. In the low frequency range, water saturated specimens behave in a hysteretic manner. However, as the frequency increases, effects of viscous damping become more dominant and the apparent damping ratio is observed to increase over a narrow frequency range. For geo-materials of high permeability such as coarse sand, fluid movement is relatively easier thereby allowing effects of viscous damping to dominate at lower frequency compared to a fine-grained geo-material of low permeability. While the effect of frequency is not within the scope of this study, it is noted that excitation frequency of wave propagation test in a laboratory setting would most likely fall under the high frequency range (Stoll, 1985).

In most cases, researchers do not differentiate between the different modes of excitation i.e. compression and shear. This indirectly leads to some complication as seen in Kokusho (1980) where the damping ratio of sand with shear strain obtained from the cyclic triaxial does not compare well with those from resonant column,

torsional shear and cyclic simple shear. This is because unlike the cyclic triaxial which excites the specimen in a compressional mode, the other apparatuses excite the specimen in shear mode. Literature on the effect of saturation on damping ratio for different excitation modes was seldom encountered in the field of soil dynamics but was discussed in geophysics. The fluctuation of damping ratio with degree of saturation can be explained by the interplay of fluid flow in the soil/rock skeleton and the excitation mode.

It has been widely established that damping of a geo-material at full and partial saturation is higher than that of a dry specimen (Wyllie et al. 1962, Gardner et al. 1964, Winkler and Nur 1982, Madhusudhan and Kumar 2013). In the saturated state, the S-wave damping is reported to be higher than the P-wave damping (Toksöz et al. 1979, Spencer 1979, Winkler and Nur 1982). This is because when saturated, compressional wave causes the crack to be compressed by the same amount leading to no substantial fluid flow. On the other hand, shear wave causes dilation and compression zones in different parts of the specimen resulting in a pressure gradient that encourages water flow which leads to greater viscous loss. At partial saturation, attenuation in compressional wave is stronger than shear wave. This is because as the P-wave passes through the specimen, it gives rise to compression and rarefaction zones. As the voids become compressed in the compression zones, water will flow out leading to viscous loss. Conversely, under shear motion, some voids which are favourably oriented will not experience compression thus the amount of water flow is reduced (Winkler and Nur 1982).

Murphy (1982, 1984), Winkler and Nur (1982) presented results of P- and S- wave damping with saturation for Massillon sandstone, Fort Union sandstone and Vycor porous glass. Results obtained using the resonant bar technique (Gardner et al., 1964) were expressed in $1000/Q$ and had been converted to damping ratio using Equation 2.3 and presented in FIG. 2.4 to better illustrate the general trend of damping ratio with degree of saturation S_r . While both sets of data show similar trend with degree of saturation, data from Murphy (1982) show more variation compared to those from Winkler and Nur (1982). Winkler and Nur (1982)

suggested that it is due to the nature of the Vycor pores which responded at different frequencies and were less conducive for fluid flow. Thus, well-defined trends for P- and S- wave damping with degree of saturation are absent. Moreover, a fixed degree of saturation where S-wave damping becomes higher than P-wave damping does not exist. In this study, attempts were made to obtain both P- and S-wave damping using techniques developed for the wave propagation test.

FIG. 2.4: Effect of saturation on compressional (P-wave) and shear (S-wave) damping

Trends of damping ratio with confining pressure will be more intuitive and stable compared to that with degree of saturation. As confining pressure increases, the contact force between the soil grains increases leading to diminished frictional loss and hence lower damping ratio (Hardin and Drnevich, 1972a and b). The relationship is further illustrated in the later part of this study where damping ratios of Ottawa sand at various confining pressures were compared with those obtained from the wave propagation test.

2.2 Advanced Geotechnical Testing

Advanced geotechnical testing apparatuses are required to obtain stiffness and damping ratio at varying strains. In this section, developments of the various apparatuses used in this study are reviewed.

2.2.1 Resonant Column Test

The resonant column is one of the earliest apparatuses designed to measure dynamic soil properties such as soil stiffness and damping ratio (Richart et al. 1970, Hardin and Drnevich 1972a and b). The test has been standardized in ASTM D4015-07 (2013). The apparatus makes use of an electromagnetic driving mechanism to excite a cylindrical soil specimen in either longitudinal or torsional mode. In each mode, the excitation frequency is varied until resonance is achieved. Resonant frequencies corresponding to longitudinal and torsional modes of excitation are used to derive the Young's modulus (E) and shear modulus (G), respectively. Material damping (ξ) is obtained from the amplitude decay of the vibration after excitation stopped at the resonant frequency. This method also known as the logarithmic decrement method (ASTM D4015-07, 2013) will be elaborated later.

Most resonant column tests reported were conducted using the torsional mode of vibration (Hardin and Drnevich 1972a and b, Saxena et al. 1988, Shibuya and Tanaka 1996, Stokoe et al. 1999, Kumar and Clayton 2007, Clayton et al. 2009, Pineda et al. 2010, Madhusudhan and Kumar 2013, Schaeffer et al. 2013). There is a lack of literature on resonant column tests for the longitudinal mode of vibration.

Recently, Cascante et al. (1998) modified a torsional mode resonant column apparatus to enable vibration in the flexural mode thereby allowing the determination of E . The modification to the driving mechanism from the torsional to flexural mode of excitation is minor. As such, a number of commercial resonant column apparatuses now have the flexural mode of excitation available. Cascante et al. (1998) described how the Young's modulus of the specimen can be determined

from the resonant frequency for the flexural mode of excitation. This is the method often used for resonant column tests in flexural mode of vibration (Karl et al. 2003, Kumar and Madhusudhan 2010) due to its simplicity. Kumar and Madhusudhan (2010) derived another method to account for the insufficiency of the method presented by Cascante et al. (1998) but the method requires iteration of the Young's modulus of the specimen. Neither method was included in ASTM D4015-07 (2013).

2.2.2 Cyclic Triaxial Test

The cyclic triaxial test, standardized in ASTM D3999-11 (2011), measures the response of a cyclic axial deviator stress or a cyclic axial deformation of fixed magnitude imposed on a cylindrical soil specimen. Tatsuoka et al. (1994) advocated the use of localised axial and radial strain measurement to improve the accuracy of cyclic testing. The usage of local deformation transducer (LDT) was then standardised in the international round-robin test conducted by TC-29 (Yamashita et al., 2001). Axial stress and strain recorded are used to plot the hysteresis loop as shown in FIG. 2.5. Calculations are performed on each individual loop. The secant modulus can be obtained from the gradient of the line joining both ends of the hysteresis loop. The damping ratio is obtained using Equation 2.6.

FIG. 2.5: Hysteresis loop from cyclic triaxial

$$\xi = \frac{1}{2\pi} \times \frac{\text{Area of Hysteresis Loop}}{\text{Area of } \Delta A1 \text{ and } \Delta A2} \quad (2.6)$$

As seen from FIG. 1.1, the strain range of the cyclic triaxial tests is larger than that of both the resonant column and the wave travel time methods. Several attempts (Tatsuoka et al. 1994, Hoque and Tatsuoka 2004, Kokusho 1980) have been made to improve the small strain measurement in triaxial tests using high sensitivity internal load and displacement measuring transducer such as the local displacement transducer (LDT) by Goto et al. (1991) and the gap or proximity sensors. Some of these transducers will be used in the cyclic triaxial set-up in this study and will be further elaborated in Chapter 3. Due to the excitation mode of the cyclic triaxial test, only the Young's modulus E can be derived unless the radial displacement is measured so that the Poisson's ratio can be obtained to convert E to G .

Due to the limitations of the cyclic triaxial test, it will be useful to incorporate wave propagation test into the set-up to obtain the small-strain Young's, shear, bulk modulus and Poisson's ratio from the P- and S-wave velocities (Equation 2.1). The wave propagation test is reviewed in the next section.

2.2.3 Wave Propagation Test

In the wave propagation test, compression (P) and shear (S) waves are transmitted and received with the help of piezoelectric materials. Piezoelectric elements are materials that are able to convert mechanical energy into electrical signals and vice versa. Piezoelectric elements were first discovered and announced by the brothers Pierre and Jacques Curies in 1880 but their application only became widespread during World War 1 to transmit ultrasonic waves underwater to detect submarines (Gautchi, 2002). To date, piezoelectric elements have been widely used in the geotechnical field as passive sensors for the measurement of displacement, pressure and acceleration. Despite the widespread popularity, the wave propagation test has only been standardised in ASTM D2845-08 (2008) for laboratory determination of

pulse velocities and ultrasonic elastic constants of rock. A standard for bender element test on geo-materials has yet to be developed.

Piezoelectric transducers used for wave propagation test is generally classified into bender-extender elements (made of piezo-ceramics) and ultrasonic transducers (made of piezo-crystals). The subsequent subsections will elaborate on the development of these transducers and their testing methods.

2.2.3.1 Bender-Extender Element System

When bender elements were first used in geotechnical testing, it was excited in its flexure mode to produce shear (S) waves (Shirley and Hampton, 1978). Since then, the bender element has been used in various laboratory apparatuses: triaxial apparatus (Bates 1989, Viggiani and Atkinson 1995, Brignoli et al. 1996, Pennington et al. 2001, Leong et al. 2005), oedometer (Dyvik and Madshus 1985, Kawaguchi et al. 2001), resonant column (Dyvik and Madshus 1985, Souto et al. 1994), direct simple shear apparatus (Dyvik and Madshus 1985, Kuwano et al. 1999) and triaxial cubical box device (Agarwal and Ishibashi 1991, Jenkins et al. 1992). The height of specimen used ranges from about 20 mm to 200 mm, the diameter of specimen used ranges from 38 mm to 200mm and the excitation frequency used ranges from 5 Hz to 60 kHz (Dyvik and Madshus 1985, Yamashita et al. 2009). Lings and Greening (2001) extended the capability of the bender element through a simple modification which excites the bender element in its extender mode thus enabling the production of compression (P) waves.

Despite the widespread usage, the bender/extender element is still plagued with various problems. The most obvious drawback is the ease of setting up varies with soil type. Literature review has shown that experiments involving bender/extender elements are usually conducted on clay and sand where the transducers can be pushed into the soil specimen with relative ease. Arroyo et al. (2010) attempted to install use bender elements to measure V_s in very stiff argillaceous rocks and had reported difficulties in transducer insertion and in-plane alignment between

transmitter and receiver. A milling machine was eventually used to mill a slot for the transducer installation. This invasive approach can introduce undesirable sample disturbances thus affecting the dynamic properties of the rock. Insertion into stiff soil can also damage the waterproofing layers of the bender/extender elements which were usually made of epoxy (Brignoli et al. 1996, Leong et al. 2009) or polyurethane (Deniz 2008, Styler and Howie 2014). Furthermore, it is almost impossible to achieve a good contact condition in stiff specimen thus leading to weak signal strength and poor data.

While the bender/extender elements can transmit and receive both P and S waves, most researchers only choose to use S wave only. Table 2.2 summarises the tests conducted using bender/extender elements since Lings and Greening (2001) modified the bender element to produce P wave. Table 2.2 shows that P wave with respect to soil conditions like degree of saturation has not been thoroughly investigated.

Other researchers (Ferreira and Viana da Fonseca 2005, Alramahi et al. 2007, Valle-Molina and Stokoe II 2012, Eseller-Bayat et al. 2013) instead chose to make use of bender elements and piezoelectric discs for shear and compression wave velocity measurements, respectively. This shift was made as P wave signal is significantly weaker than that of S wave signal resulting in difficulties in detecting P wave signals in loose granular soil at low confining pressure. Deniz (2008) attributed this to the smaller contact area between transducer and soil in extender mode. Lings and Greenings (2001) used a laser velocimeter to characterise the deformation of bender/extender elements in both flexure and extender modes to show that the deformation in the longitudinal mode is one order of magnitude smaller than that in the flexure mode. The usage of both bender element and piezoelectric disc essentially results in an off centre configuration with unknown repercussions.

Table 2.2: Summary of bender/extender element tests since 2001

Reference	Specimen Properties	Specimen Dimension (mm)	Wave Procured	Excitation Frequency (kHz)	S _r (%)
Arroyo et al. (2003a and b)	Gault Clay	Height: 180 Diameter: 90	S wave	2.7-10 kHz	≈100%
Dano et al. (2003)	Loire River Sand	Height: 200 Diameter: 100	P and S wave	5 kHz	≈100%
Greening and Nash (2004)	Gault Clay	Not mentioned	S wave	Not mentioned	≈100%
Greening et al. (2007)	Polyurethane rubber	Height: 6-76 Diameter: 38-75	S wave	0.25-8.4 kHz	-
Viana da Fonseca et al. (2009)	Residual Soil	Height: 140 Diameter: 70	P and S wave	S wave: 4-20 kHz P wave: 25 kHz	≈100%
Leong et al. (2009)	Residual Soil Sand	Height: 100 Diameter: 50	P and S wave	1-30 kHz	90% 0%
Airey and Mohsin (2013)	Sand Kaolin	Height: 50-200 Diameter: 50-100	S wave	Not mentioned	0% ≈100%
Heitor et al. (2013)	Compacted silty sand	Height: 100 Diameter: 50	S wave	0.5-15 kHz	Wide range of S _r
Marjanovic and Germaine (2013)	Ticino sand Boston blue clay	Height: 90 Diameter: 75-200	S wave	4-8 kHz	0% ≈100%

Bender elements are also highly susceptible to electrical drift, noise and crosstalk as reported by many researchers. Electrical drift and noise are usually due to background electromagnetic signals in the environment and can be eliminated by making sure that signal cables run parallel to each another and do not cross.

Crosstalk is the phenomenon whereby the transmitter signals leaks into the receiver signals through electromagnetic coupling (Lee and Santamarina, 2005). Lee and Santamarina (2005) recommended the removal of undesirable interferences mentioned above by shielding the bender/extender elements with conductive paint

and then grounding of either source or the receiver. However, experiments conducted by Rio (2006) showed difficulties in removing them despite many trials. Rio (2006) attributed the crosstalk to be due to the oscilloscope after rigorous testing. This was further confirmed by a tutorial from National Instruments (2011) which stated that the close proximity of the digital input/output signals on the data acquisition board can be a possible source of crosstalk. However the tutorial also pointed out another probable source which is inadequate magnetic shielding due to the inherent magnetic permeability of the shielding materials.

Last but not least, the topic most debatable when utilising bender/extender elements is the determination of arrival time. Numerous methods have been employed and they can be generally classified into time and frequency domain methods.

The time domain methods vary according to the identification of the “correct” feature which indicates the arrival wave. They vary from first deflection, first maximum, zero after first bump and first major peak (FIG. 2.5). Usage of this method is further complicated by near field effects whereby the faster P wave interferes with the propagating S wave making the determination of S wave arrival difficult (Sanchez-Salinero et al. 1986, Arulnathan et al. 1998). Leong et al. (2009) suggested keeping the L_{tt}/λ ratio above 3.33 (Equation 2.7) for clearer identification of S-wave arrival time.

$$\frac{L_{tt}}{\lambda} = \left(\frac{L_{tt}}{V} \right) f_{in} = f_{in} \Delta t \quad (2.7)$$

where L_{tt} is the tip to tip distance of the bender/extender element, λ is the wavelength, V is the wave velocity, f_{in} is the input excitation frequency and Δt is the wave travel time.

However, the wavelength (λ) to be used in the equation remains an uncertainty since the excitation frequency may differ from the actual excitation frequency due to the coupling between the bender element and soil specimen. While Arulnathan et al. (1998) and Leong et al. (2009) suggested using the input excitation frequency;

Brignoli et al. (1996) calculated the L_{tt}/λ ratio using the predominant output frequency of the receiver. Regardless of the values used, the literatures cited above suggested the importance of exciting at a wide range of frequency such that the region affected by the near-field effect can be identified and the wave arrival be detected. Determination of the P-wave arrival time also helps to determine the possible region which affected by near field effect. It can be observed that as L_{tt}/λ increases, the near field effect (usually represented by region ABC of FIG. 2.6) decreases and the S-wave arrival time starts at point C of FIG. 2.6.

FIG. 2.6: Characteristic signal displaying prospective features for arrival time (A) First deflection, (B) First bump, (C) Zero after first bump and (D) First major peak (modified from Lee and Santamarina, 2005)

Arulnathan et al. (1998) suggested using the reflection of the signal from the transmitter to determine the arrival time. However in most circumstances, the reflected signal is either too weak to be detected due to attenuation or shows interference of reflected waves from the boundary.

The frequency domain methods consist of the cross-correlation and cross spectrum techniques. The former calculates the cross correlation or similarities between the transmitter and receiver signals (Santamarina and Fratta, 2005) and the time corresponding to maximum amplitude is taken as the arrival time. The latter is an extension of the cross-correlation technique where phase analysis or “unwrapping”

is conducted on the cross-power spectrum derived from the frequency spectra (Viggiani and Atkinson, 1995). The arrival time is then calculated from the slope of the relationships of amplitude and phase angle with frequency. Greening and Nash (2004) extended the cross-spectrum technique by using a broadband sinusoidal sweep to obtain the slope of the unwrapped phase angle of the cross-power spectrum prior to deriving the wave travel time. Usage of the cross correlation method carries the implicit assumption that the transmitted wave is able to retain its shape or frequency (Jovicic et al. 1996, Santamarina and Fam, 1997). This phenomenon only occurs when the excitation frequency exceeds the natural frequency of the bender/extender element (Rio 2006, Marjanovic and Germaine 2013). Yet even in this case, the transmitted signals are frequently subjected to distortion due to reflected waves.

More often than not, the maximum amplitude of the receiver signal also does not appear at the first peak denoted by D in FIG. 2.6. Therefore, although the cross correlation method is deemed the most accurate by Viggiani and Atkinson (1995), Jovicic et al. (1996) abandoned the method on grounds of complexity (Arroyo et al., 2003a and b). Camacho-tauta et al. (2013) compared the velocities obtained from bender element tests using time- and frequency-domain methods with those from resonant column tests and found that V_s determined using the time-domain methods are in better agreement with those obtained from the resonant column test. This is important as it ensures consistency between the different test apparatuses.

The above review shows that time-domain methods are not only reliable but also simple and popular (Yamashita et al., 2009). As such, the time domain method will be used in this study for the determination of arrival time. The zero after first bump (point C in FIG. 2.5) is selected to be the S wave arrival as convergence of arrival time was observed when $L_{tt}/\lambda > 3.33$. The first deflection and first bump was shown to be due to P wave influence in Leong et al. (2009). In addition, selection of the first major peak would over-estimate the S wave arrival time.

2.2.3.2 Ultrasonic system

The ultrasonic system offers a way to obtain dynamic soil properties without the invasive nature of the bender elements. A typical configuration of the ultrasonic transducer is shown in FIG. 2.7. Ultrasonic transducers use frequencies above 20 kHz. Unlike bender/extender element which needs extensive water proofing, the piezoelectric crystals are protected within a metallic housing which also acts as a good electrical shield against background noises. Despite the numerous advantages, there have been studies showing difficulties in detecting and identifying the shear wave arrival time.

FIG. 2.7: Ultrasonic transducer configuration

Weidinger (2008) reported that despite proper contact conditions and usage of various acoustic couplants, shear wave signals could not be obtained for silt specimens due to large attenuation. The specimen height was eventually reduced from 114 mm to 30 mm before the signal could be detected. Nonetheless, the shear wave arrival still could not be identified due to strong interference of compression wave which masked the weaker shear wave signal. Slavova et al. (2010) also tested silt specimens ranging from 13 mm to 52 mm in height and chose the third peak to represent shear wave arrival since the first peak is contaminated by P wave. Both Weidinger (2008) and Slavova et al. (2010) estimated the shear wave arrival time based on an assumed Poisson's ratio (ν) and compression wave velocity. The inability to obtain a proper signal from loose granular material due to high attenuation was also mentioned in Nakagawa et al. (1996) and Shi et al. (2007) who

conducted tests on sandy silts using Geotechnical Consulting & Testing Systems (GCTS) ultrasonic transducers. They reported difficulty in obtaining S wave signals at confining pressures less than 50 kPa. Experimental details from various researches that have used ultrasonic transducers are summarised in Table 2.3.

Table 2.3: Summary of ultrasonic soil tests in the literature

Reference	Specimen Properties	Confining Pressure (kPa)	Specimen Dimension (mm)	Dominant Frequency	Remarks
Nakagawa et al. (1996)	Saturated Kaolin and Nearly Saturated Fine Sand	98 - 490	Diameter: Not mentioned Height: 101	Transducer only P: 500 kHz S: 50 kHz Platen P: 200 kHz S: 3-5 kHz	Self-made platen
Leong et al. (2004)	Kaolin	800	Diameter: 70 Height: 60	Platen P: 90 kHz S: 76 kHz	GCTS platen
Shi et al. (2007)	Sandy Silt	40 - 70	Not mentioned	Not mentioned	GCTS platen
Mulmi et al. (2008)	Dry Toyoura Sand	50, 100, 200, 400	Diameter: 50 Height: 100	Applied input excitation of 3.9 – 15.6 kHz	Self-made Platen
Weidinger (2008)	Silt	None	Diameter: Not mentioned Height: 30	Platen P: 46 kHz S: 39 kHz	GCTS ULT-100 Ultrasonic Velocity Testing System
Slavova et al. (2010)	Silt	Not mentioned	Diameter: 71.1 Height: 12.7 – 50.8	Platen P: 46 kHz S: 39 kHz	GCTS ULT-100 Ultrasonic Velocity Testing System

2.3 Determination of Damping Ratio

This section examines the common methods that were used to determine the damping ratio from wave propagation tests. Out of the four methods considered,

only the Hilbert transform method was not employed for the determination of damping ratio in the geotechnical field.

2.3.1 Logarithmic Decrement Method (LDM)

The Logarithmic Decrement Method (LDM) is a widely popular method based on the decay of free oscillation in the time domain (FIG 2.8). Upon initiation of free vibration, only the relative displacement amplitudes need to be measured. The peak amplitude is then plotted on a log scale versus the cycle number. The gradient of the plot gives the logarithmic decrement (δ) and can be correlated to damping ratio (ξ) using Equation 2.8.

$$\delta = \ln\left(\frac{A_i}{A_{i+N}}\right) \frac{1}{N} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad (2.8)$$

where A_i is the peak amplitude at the i^{th} cycle and A_{i+N} is the amplitude at $(i+N)^{\text{th}}$ cycle. (FIG 2.8)

FIG. 2.8: Logarithmic decrement approach

Despite the convenience, the LDM is only applicable when a clear signal of amplitude with time is obtained. In most cases, the signal is affected by other reflected waves (i.e. signals containing more than one frequency peak) making

interpretation of the logarithmic decrement difficult. Another shortcoming is that the damping ratio derived is found to be amplitude dependent (Clough and Penzien, 2003). Damping ratio obtained from the earlier part of the free vibration response defers from that obtained from the later stage of the response.

2.3.2 Half-Power Bandwidth Method (HPBM)

The Half-Power Bandwidth Method (HPBM) works on the principle that the frequency response curve of steady forced vibration has a shape controlled by the damping ratio of the system (FIG. 2.9). The frequency response curve (FIG. 2.10) can be obtained by measuring the steady-state amplitudes of relative displacement response produced by harmonic loading at discrete excitation frequency and fixed amplitude. The range of excitation frequency should include the natural frequency of the system. The frequencies corresponding to $1/\sqrt{2}$ of the peak amplitude, f_1 and f_2 , are then substituted into Equation 2.9 to obtain the damping ratio.

$$\xi = \frac{f_2 - f_1}{f_2 + f_1} \quad (2.9)$$

FIG. 2.9: Forced vibration response plotted with frequency (f) normalised with natural frequency (f_n) at different damping ratio

FIG. 2.10: Half-power bandwidth approach

While the HPBM appears straightforward, the natural frequency of the system is seldom known and tests over a wide range of frequency need to be conducted to determine the natural frequency. Accuracy will hence be dependent on the frequency increment used. For the case of low damping, a narrow peak is expected and the frequency increment will have to be greatly reduced in order to obtain the correct frequency response curve. Damping ratio would not be investigated using the HPBM in this study due to its inherent unreliability.

2.3.3 Spectral Ratio Method (SRM)

The theory for the development of the spectral ratio method (SRM) first appeared in the field of geophysics (Bath, 1974). In this method, the amplitude of signals obtained from two general cases is compared. The aim of this method is to eliminate all the similarities between the two cases and attribute the difference in signal amplitude to the difference in condition of the two cases (Toksöz et al., 1979). This difference can be the distance travelled by the wave or the material the specimen is made up of. The following will be elaborated in detail below starting with the expression for the Fourier amplitude:

$$A_n(f) = G_n(x)e^{-\alpha_n(f)x_n} \quad (2.10)$$

where subscript n represents both Cases 1 and 2, respectively, A is the amplitude of the wave, f is the frequency, x is the distance, G(x) is a frequency independent geometrical factor which includes spreading, reflections etc., and $\alpha(g)$ is the frequency dependent attenuation coefficient.

Using the correlation between α and Q from Equation 2.3 and comparing the two cases, the general form is derived as shown in Equation 2.11. If Q_1 is known, the value of Q_2 can be obtained by plotting $\ln\left(\frac{A_1}{A_2}\right)$ versus f whereby the term $\left(\frac{\pi x_2}{Q_2 V_2} - \frac{\pi x_1}{Q_1 V_1}\right)$ is the gradient and $\ln\left(\frac{G_1}{G_2}\right)$ is the y-intercept. This will mean that geometric damping can be eliminated in the calculation of damping.

$$\frac{A_1}{A_2} = \frac{G_1}{G_2} e^{-\left(\frac{\pi x_1}{Q_1 V_1} - \frac{\pi x_2}{Q_2 V_2}\right)f} \quad (2.11)$$

Applying natural logarithm on both sides of Equation 2.11,

$$\ln\left(\frac{A_1}{A_2}\right) = \ln\left(\frac{G_1}{G_2}\right) + \left(\frac{\pi x_2}{Q_2 V_2} - \frac{\pi x_1}{Q_1 V_1}\right)f \quad (2.12)$$

Toksöz et al. (1979), Johnston and Toksöz (1980) and Frempong et al. (2005, 2007) make use of a reference specimen of similar geometry and known quality factor (Q). When the geometry is similar, $x_1 = x_2 = x$, therefore Equation 2.12 is reduced to

$$\ln\left(\frac{A_1}{A_2}\right) = \ln\left(\frac{G_1}{G_2}\right) + \left(\frac{\pi}{Q_2 V_2} - \frac{\pi}{Q_1 V_1}\right)fx \quad (2.13)$$

Toksöz et al. (1979), Johnston and Toksöz (1980) and Frempong et al. (2005, 2007) used an aluminium reference specimen. The quality factor (Q) of aluminium is about 150,000 (Zamanek and Rudnick, 1961) as opposed to soil and rock where $Q <$

1,000. When Q of the reference material is high, $\frac{\pi}{QV} \rightarrow 0$ and the specimen's Q can be directly determined from the slope.

Alternatively, similar materials of identical cross section but of varying height/distance can be used. Wang et al. (2006) made use of bender elements placed at different distances apart in a box filled with Toyoura sand. Other researchers (Karl et al. 2008 and Khan et al. 2011) “lengthen the specimen” by comparing the main arrival wave with the echoes. In all the above cases, both Q and V remain the same except for the distance travelled. Equation 2.12 is hence reduced to

$$\ln\left(\frac{A_1}{A_2}\right) = \ln\left(\frac{G_1}{G_2}\right) + \left(\frac{\pi}{QV}\right)(x_2 - x_1)f \quad (2.14)$$

Despite the SRM's versatile application, Johnston (1978) found that the SRM is only applicable for specimens with $Q < 100$ due to the difficulty in obtaining a good correlation coefficient for a linear regression analysis of data of low slope (high Q).

The usage of reflected wave signals/echoes to determine attenuation carries the implicit assumption that no energy is lost during reflection at the boundaries. The degree of reflection at boundaries can be measured using the reflection coefficient (R) shown in Equation 2.15.

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 \quad (2.15)$$

where Z is the acoustic impedance given by the product of density (ρ) and wave velocity (V), the subscripts 1 and 2 represent the media at the boundary.

Equation 2.15 shows that the greater the impedance mismatch, the greater the percentage of energy that will be reflected at the interface between the media. For a technique like SRM where wave amplitude is of crucial importance, discrepancies like impedance mismatch should be reduced if not eliminated. Therefore,

comparison of signals using echoes and reference material will be deemed less reliable compared to measuring at various distances where wave reflection can be eliminated in both cases.

Literature review shows that there is no common consensus of the frequency range to be selected for use in the SRM. Toksöz et al. (1979) took into consideration of the region where spectral ratios are linear with frequency where signals-to-noise ratio is high. Other researchers like Frempong et al. (2007), Wang et al. (2006), Karl et al. (2008) and Khan et al. (2011) obtained slopes around the dominant frequency but different ranges of frequencies were used.

2.3.4 Hilbert Transform Method (HTM)

The Hilbert transform is an operator which convolutes a signal by $1/\pi x$ (Agneni and Balis-Crema, 1986). In other words, it is a filter which transforms the signal by shifting their phases by $\pm \pi/2$ while maintaining the magnitudes of their respective spectral components. It was first used by Agneni and Balis-Crema (1986) to derive damping ratios of composite materials using free vibration decay data. However, measurement of most engineering structures involved vibration responses due to wind and background noises. In order to overcome this, Huang et al. (1998) incorporated the ‘empirical mode decomposition’ method which helps to decompose complicated signals into their respective modal components to yield well-behaved Hilbert transforms. This method was also employed by Salvino (2000), Zhang and Tamura (2003), Yang et al. (2004) and Le et al. (2010) to investigate damping in structures.

The signal processing mentioned above are however not required in bender element tests because the transmitting bender element is excited using a single sinusoidal pulse and the receiver bender element records the free vibration decay. For completeness, the derivation of material damping using the Hilbert transform (Salvino 2000, Iglesias 2000) is reproduced below.

If $x(t)$ is the signal in the time domain (Equation 2.16a) and $x^H(t)$ is the Hilbert transform of the time domain signal (Equation 2.16b), the combination will give the analytic signal $x_a(t)$ as shown below

$$x(t) = Ae^{-\xi\omega_n t} \sin(\omega_n t \sqrt{1-\xi^2}) \quad (2.16a)$$

$$x^H(t) = Ae^{-\xi\omega_n t} \cos(\omega_n t \sqrt{1-\xi^2}) \quad (2.16b)$$

$$x_a(t) = x(t) - ix^H(t) \quad (2.16c)$$

where A is the amplitude of the signal, ξ is the damping ratio, ω_n is the natural radian frequency in radian, t is the time in seconds and i is the imaginary number

The magnitude of the analytic signal gives the envelope of the time signal $x_a(t)$ as shown in Equation 2.17:

$$|x_a(t)| = Ae^{-\xi\omega_n t} \quad (2.17)$$

By taking natural logarithm on both sides of Equation 2.17, the damping ratio ξ can be separated as shown in Equation 2.18.

$$\ln(|x_a(t)|) = \ln(A) - (\xi\omega_n)t \quad (2.18)$$

Equation 2.18 shows that the gradient used to derive the damping ratio from the plot of $\ln(|x_a(t)|)$ with time has to be negative. The damping ratio can be derived from the gradient as shown in Equation 2.19. When considering the time window to derive the gradient, it is important to note that the tail end of the analytic signal can be distorted by the Hilbert transform (Luo et al., 2009). Therefore, it is suggested that the time window of the signal extracted for best-fitting does not extend to the end of the signal.

$$\xi = \frac{m}{\omega_n} = \frac{m}{2\pi f_n} \quad (2.19)$$

where m is the gradient and f_n is the natural frequency of the signal.

Equation 2.19 shows that in order to obtain the damping ratio, the natural frequency of the signal will be necessary. The situation is straight-forward in the case where the signal only consists of a single dominant frequency. However, none of the literature explains alternatives when a range of closely spaced frequency peaks were detected due to superposition of the reflected waves. In most cases, the closeness of the frequency peaks meant that they could not be isolated using signal filtering. As such, a representative dominant frequency ω_n would be needed. The representative dominant frequency ω_n can be calculated from the second moment of the power spectrum (Barnes, 1993) after filtering as shown below.

$$\omega_n = 2\pi \sqrt{\frac{\int_0^{f_e} f^2 P(f) df}{\int_0^{f_e} P(f) df}} \quad (2.20)$$

where f_e are the frequencies corresponding to the end of the frequency range, $P(f)$ is the ordinate of the power spectrum.

2.4 Residual Soils

Residual soils are the remnants of the physical and chemical weathering of rocks. Unlike sedimentary soil, they remain at the place of origin and are not transported thereby exhibiting a certain degree of heterogeneity (Wesley, 2010). Differences of sedimentary and residual soil are further elaborated in FIG. 2.11.

FIG. 2.11: Simplified soil profiles in sedimentary and residual soils (from Wesley, 2010)

Contrary to popular belief, presence of residual soils is not limited to the tropics. However, residual soils are more prevalent in the tropics as the warm and humid climatic condition subject the bed rock to a greater degree of chemical weathering. Residual soils are characterised by their degree of weathering as shown in FIG. 2.12. However, the classification system is only intended for the residue resulting from the weathering of igneous rocks in the humid topics. The profile is subjected to variation depending on the parent rocks. At times, the transition from soil to rock can be abrupt. Residual soils weathered from igneous rocks have better engineering properties compared to the residual soils weathered from sedimentary rocks which tend to produce clay minerals. The residual soils tested in this study were derived from weathered Bukit Timah Granite (Leong et al. 2002).

FIG. 2.12: Typical tropical residual soil profile (from Little, 1969)

According to FIG. 2.12, residual soils are usually found close to the ground surface. As such, residual soils are likely to be above the ground water table and hence exist in the unsaturated state. As such, the effective stress principle introduced by Terzaghi (1943) is not suitable for analysis. The following sections review the fundamental concepts of unsaturated soil mechanics together with the triaxial tests for unsaturated soils. The dynamic properties of residual soils will also be reviewed.

2.4.1 Saturated and unsaturated soils

In reality, most soils, especially those near the ground surface are unsaturated. Unsaturated soil consists of three phases namely, soil solids, water and air. Fredlund and Morgenstern (1977) suggested the fourth phase which is the air-water interface or contractile skin. Negative pore-water pressure also known as matric suction when referenced to atmospheric pressure can arise in nature due to desiccation of soil by evaporation and evapotranspiration. Application of unsaturated soil mechanics is especially important in the area of slope stability where the high negative pore-water pressure is responsible for increasing soil shear strength hence keeping the slope stable.

Unlike saturated soil, the mechanical behaviour of unsaturated soil is governed by two stress-state variables which are net normal stress ($\sigma - u_a$) and matric suction ($u_a - u_w$) where σ is the normal stress, u_a is the pore-air pressure and u_w is the pore-water pressure. As the matric suction changes, water will move either in or out of the soil until equilibrium is reached. The relationship between moisture content and matric suction can be described using the Soil-Water Characteristics Curve (SWCC) as shown in FIG 2.13.

FIG. 2.13: Typical SWCC (from Fredlund and Xing, 1994)

The air-entry value is the matric suction that must be exceeded before air can enter the soil pores. The residual water content is defined as the water content at which an increase in matric suctions does not bring about a significant change in water content. FIG. 2.13 also shows different drying and wetting paths of the soil otherwise known as hysteresis. This is due to the non-uniform pore size distribution in the soil. The SWCC can be curve fitted using the equation by Fredlund and Xing (1994):

$$\theta_w = C(\psi) \frac{\theta_s}{\left\{ \ln \left[e + \left(\frac{\psi}{a} \right)^n \right] \right\}^m} \quad (2.21a)$$

$$C(\psi) = 1 - \frac{\ln \left(1 + \frac{\psi}{\psi_r} \right)}{\ln \left(1 + \frac{1000000}{\psi_r} \right)} \quad (2.21b)$$

where θ_w is the volumetric water content, θ_s is the saturated volumetric water content, a is the inflexion point of the curve, n is the parameter that controls the slope at the inflection point in the soil-water characteristic curve, m is the parameter that is related to the residual water content, ψ is the matric suction, e is the Euler's number, $C(\psi)$ is the correction function and ψ_r is the matric suction corresponding to the residual water content. According to Leong and Rahardjo (1997), $C(\psi)$ can be assumed to be unity.

Shear strength (τ) of a saturated soil can be expressed using the Mohr-Coulomb failure criterion as shown in Equation 2.22. The shear strength is expressed as a function of the stress-state variable of saturated soil, $(\sigma - u_w)$, widely known as effective normal stress.

$$\tau = c' + (\sigma - u_w) \tan \phi' \quad (2.22)$$

where c' is the effective cohesion, σ is the normal stress, ϕ' is the effective friction angle.

In the case of unsaturated soil mechanics, an extended Mohr-Coulomb failure criterion (Equation 2.23) which consists of both stress-state variables is suggested by Fredlund et al. (1978). Unlike saturated soils, shear strength in unsaturated soils is depicted in a three-dimensional plot as seen in FIG. 2.14. The Mohr-Coulomb failure envelope is expressed as a failure plane tangent to the Mohr circles at failure. Though depicted as a planar surface, the failure envelope can be curved. The cohesion intercept (c') and the slope angles (ϕ' and ϕ^b) were used to correlate the stress-state variables to the shear strength. As the friction angle (ϕ') is an inherent soil property which is dependent on the frictional properties of the soil particles, its

value does not change regardless of the degree of saturation. The angle ϕ^b , on the other hand, is used to describe the increase in shear strength due to matric suction. Values of ϕ^b can be less than or similar to that of ϕ' .

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi^b \quad (2.23)$$

where ϕ^b is the angle describing the rate of change of shear strength with matric suction

Equation 2.23a can be rewritten to resemble Equation 2.22 by combining the effective cohesion together with the matric suction term as shown in Equation 2.24a. The new cohesion intercept is termed the total cohesion intercept (Equation 2.24b). The matric suction is grouped with the cohesion intercept to collapse the 3D plots (FIG. 2.15) into a 2D plot. It does not imply that the matric suction is a cohesion component of shear strength.

$$\tau = c + (\sigma - u_a) \tan(\phi') \quad (2.24a)$$

$$c = c' + (u_a - u_w) \tan(\phi^b) \quad (2.24b)$$

FIG. 2.14: Extended Mohr-Coulomb failure enveloped for unsaturated soil (from Fredlund and Rahardjo, 1993)

FIG. 2.15: Line of intercept along the failure plane on the τ versus $(u_a - u_w)$ plane
(from Fredlund and Rahardjo, 1993)

The shear strength of unsaturated soil can be determined using the triaxial set-up similar to that for saturated soil. A summary of the various triaxial testing configurations is given in Fredlund and Rahardjo (1993). However, these experiments require the incorporation of high air-entry (HAE) disk which is used to separate the air and water phases. Low permeability of the HAE disk results in extended period of time required for the soil specimen to reach equilibrium hence making unsaturated soil testing time-consuming.

One of the unsaturated triaxial tests utilised in this study is the constant water content (CW) test. In the CW test, the specimen is first consolidated and then sheared with the pore-air drained and the pore-water undrained. Since the air pressure valve remains open during shearing, the net confining pressure ($\sigma_3 - u_a$) remains unchanged. Matric suction ($u_a - u_w$) is expected to change as the u_w changes with degree of saturation. Change in degree of saturation arises as the void space changes while water content remains constant.

As the CW test is not included in any test standards, different guidelines are adopted in the literature especially in terms of shearing rate. Shearing rate of the unsaturated soil is governed by the low permeability of the HAE disk and the physical properties of the unsaturated soil (Ho and Fredlund, 1982). Satiija and

Gulhati (1979) used changes in matric suction and water content to determine an appropriate strain rate for Dhanauri Clay before concluding that the strain rate of CW test should be five times higher than the consolidated drained (CD) tests. Rahardjo et al. (2004) conducted CW test on residual soil from the sedimentary Jurong Formation and used a strain rate of 0.009 mm/min which is ten times higher than the corresponding CD test.

2.4.2 Dynamic properties of residual soil

Residual soils have always been a subject of study due to their frequent occurrence in geotechnical problems. However, studies on the dynamic behaviour of residual soils are generally scarce. Field and laboratory studies on residual soils are highly localised take for example Piedmont residual soils from Piedmont Province (Borden et al. 1996, Macari and Hoyos 1996, Hoyos and Macari 1999, Schneider et al. 1999), tropical residual soil from Brazil (Correia et al. 2001) and residual soils from Columbia (Villarraga 2001). In Singapore, studies on dynamic behaviour of residual soils are mostly limited to geophysical seismic surveys (Tan 1997, Veijayaratnam et al. 1993 and Anand et al. 2000). Laboratory testing using cyclic triaxial tests has been conducted on marine clay (Leong and Pan 1998), Bukit Timah and Jurong Formation residual soils (Tou 2003, Leong et al. 2003). The cited literature depicted large variability in the dynamic behaviour of residual soils and further highlighted the need to classify these soils such that a general trend can be obtained. It will also allow practitioners to estimate the dynamic properties of residual soils in the event where tests to determine dynamic properties are unavailable.

Dobry and Vucetic (1987) evaluated various soil parameters on both normally consolidated and over-consolidated residual soils and concluded that the plasticity index (PI) correlates well with the dynamic parameters, $G/G_{\max}$ and ξ . Later, Vucetic and Dobry (1991) focus on the effect of soil plasticity and published the plots illustrated in FIG. 2.16. The plots combine results from various literature and are consistent with those from Seed and Idriss (1970) and Seed et al. (1986) who

worked on cohesionless soils. It can be observed that cohesionless non-plastic soils are the most non-linear. In other words, fine-grained soils like clay with high plasticity (larger PI) are more flexible and exhibit non-linearity and inelastic response at larger strain i.e. larger γ_{td} . The opposite is true for coarse-grained soil like sand which is less flexible and will have a smaller γ_{td} (Vucetic 1994). Vucetic (1994) also suggested that unsaturated soils will display a larger γ_{td} compared to their saturated and dry counterparts. FIG. 2.16 will be useful when analysing the results obtained from the residual soil.

FIG. 2.16: Relationship between $G/G_{\max}$, ξ and PI for normally consolidated and over-consolidated soil (from Vucetic and Dobry, 1991)

2.5 Summary and Gaps in Current Research

The wave propagation test is a simple and convenient test to determine the small-strain soil properties such as stiffness and damping ratio. Despite its versatility to be incorporated into conventional testing apparatus such as the triaxial apparatus, the test technique has many variants and has not been standardised. First and foremost, the variety of transducers consisting mainly of ultrasonic platens and bender/extender elements was seldom verified to ensure reliable and accurate results. Furthermore, while various literatures mentioned methods to obtain V_p and V_s , the determination of material damping ratio ξ was seldom discussed much less a methodology to determine ξ . As such, one of the focuses of this study centres on the verification of equipment and the development of a robust methodology to ascertain dynamic soil properties (V_p , V_s and ξ). In addition, understanding wave propagation in such tests through finite element simulations is lacking.

While the relationship between confining pressure and V_s has been investigated in saturated geo-material, the relationships of both confining pressure and degree of saturation on the V_s of geo-materials have been less well studied. The complex interplay between degree of saturation and the corresponding stress state variables as well as the long test duration required to bring the specimen to a stress equilibrium condition makes such study difficult. Experimental work will hence be devised to investigate the effect of these variables (confining pressure and saturation) on the dynamic soil properties. Reconstituted soil specimens will first be tested to eliminate variability due to heterogeneity of the soil specimens before progressing to testing undisturbed samples of residual soil.

While residual soils are widely encountered, studies of their dynamic behaviour are scarce and generally localised. Furthermore, advanced geotechnical tests to determine dynamic soil properties were rarely prescribed in civil engineering projects. As such, it would be useful to have a generalised trend where practitioners can estimate the values of the soil's dynamic parameters.

CHAPTER 3: Methodology, Test Procedures and Experimental Set-up

3.1 Outline and Purpose of Test Series

It is the intention of this study to incorporate the wave propagation technique into other advanced geotechnical testing apparatus so that a more rigorous solution can be developed to determine the normalised stiffness and damping ratio over a large strain range. However, before testing can be conducted on undisturbed residual soil specimens, it is imperative to validate the equipment used and further develop the capabilities of the equipment to determine material damping which is rarely determined in the wave propagation method. As geo-materials are subjected to various in-situ saturation and stress-state condition, it is important to study the effect of these conditions on the dynamic properties of geo-materials. During the developmental phase of this study, experiments will be conducted on specimens that are reproducible (i.e. reconstituted soil and engineered mixtures) so that variability between the specimens can be minimized while the influences of other effects are investigated. Eventually, the wave propagation test will be incorporated in the cyclic triaxial apparatus to perform tests on the unsaturated residual soil. In view of this, the study is separated into four test series as described below:

Test Series I was designed to characterize the behaviour of different transducers used to conduct the wave propagation test. They are the bender-extender elements, ultrasonic transducer and a hybrid bender element–ultrasonic system.

The intention of Test Series II was to investigate the various methods (logarithmic decrement, spectral ratio and Hilbert transform method) available for the determination of damping ratio in geo-materials via the wave propagation test. The method selected would be further evaluated using Ottawa sand since damping ratio of Ottawa sand using the resonant column is available in the literature. Test Series

II aims to develop a framework that can be used to determine the small-strain stiffness and damping ratio. The effects of degree of saturation and confining pressure on small-strain soil properties (V_p , V_s and ξ) were also investigated. With this framework, variations in the test results can be correctly quantified to reduce the uncertainty when testing is conducted on highly heterogeneous specimens.

The trend of ξ with shear strain in FIG. 2.16b shows that at small strains, variation of ξ with PI is small. In order for the framework to be considered robust and reliable, it should be able to detect small changes in wave velocities and ξ . As such, Test Series III applies the framework developed in Test Series II to soil-rubber mixtures – an engineered material where changes in stiffness and damping characteristics can be controlled and varied significantly. Results of V_s and ξ obtained from Test Series III were compared with those obtained from the resonant column test reported in the literature. The rubber can also be treated as a third phase to observe the response of wave propagating through material with different phases.

Finally, Test Series IV involved tests using wave propagation tests in a cyclic triaxial apparatus. The tests were conducted on saturated and unsaturated residual soils in the undisturbed state. Both dynamic and static properties were obtained. Soil stiffness and damping ratio were determined at various strain levels. Multi-stage consolidated-undrained (CU) test were conducted on fully saturated specimens while constant water content (CW) test were conducted on unsaturated specimens.

3.2 Test Series I: The Wave Propagation Transducer

3.2.1 Material Utilised

Four different materials, acrylic, nylon 66, kaolin and Changi sand were tested and their properties are summarized in Table 3.1. For kaolin and Changi sand, the basic soil properties are tabulated in Table 3.2 while grain size distribution curves are shown in FIG 3.1. Acrylic and nylon 66 served as standard materials for calibration purposes. All specimens have a diameter of 70 mm and no confining pressure was

applied. The zero confining pressure condition will provide the worst possible test condition for bender elements as increase in confining pressure will improve contact condition between bender elements and soil. The specimen tag is in the format X-N where X is the material type and N is the specimen height in mm, e.g. AC-100 is for an acrylic specimen of height 100 mm and NY-100 is for a nylon 66 specimen of 100 mm height.

Table 3.1: Specimens tested for Test Series I

Material	Specimen Tag	Density (kg/m ³)	Height (mm)	Void Ratio	Degree of Saturation (%)
Acrylic	AC-100	1182	100	-	-
Nylon 66	NY-100	1153	100	-	-
Kaolin	KL-60, KL-100, KL-140	1621	60, 100, 140	1.66	~100
Changi Sand	CSD-60	1706	60	0.73	40.1
	CSD-100	1782	100	0.67	46.2
	CSD-140	1851	140	0.60	50.4

The kaolin, from Malaysia, was prepared by consolidating kaolin slurry at an initial water content of 230% in a consolidation tank 30 cm in internal diameter under a consolidation pressure of 100 kPa. The kaolin specimen was initially trimmed to a height of 140 mm for the tests. It was then subsequently trimmed to heights of 100 and 60 mm, respectively, so that the same specimen was used in the subsequent tests. The kaolin specimen was fully saturated and had an initial void ratio of 1.624.

The Changi sand, dredged from seabeds in the region, consists mainly of uniform medium grains with minute amount of sea shells. Due to the presence of a tiny amount of large sea shells, the sand was sieved through sieve No. 8 (2.36 mm). Predetermined amounts of sand and water were mixed and compacted to achieve a relative density of 70% ($e \approx 0.63$) and a degree of saturation (S_r) of about 50%. The undercompaction procedures used by Ladd (1978) were considered and modified. This method works on the principle that the compaction of the top layer will densify the subsequent layers of sand below it and therefore the lower layers should be compacted less than the top layers so as to produce uniform and homogeneous

specimens. The wet pouring (pluvial) method was not chosen due to the inherent difficulty in producing a homogeneous specimen as segregation occurs between the fine and coarser sand particles. Compaction was performed using a 70-mm-diameter split mould lined with a transparency sheet to facilitate removal of the sand specimen from the split mould. The moist sand was placed in 10 mm lifts, and each lift was tamped about 50 times using a metal rod of 50 mm diameter. The top surface of each sand layer was lightly scratched to ensure good bonding between lifts. Sand specimens of various heights (60mm, 100mm, and 140mm) were prepared.

Table 3.2: Basic soil properties of reconstituted specimens

Soil Type	Changi Sand	River Sand (5% fines)	Ottawa Sand 20-30	Kaolin
Specimen Tag	CSD	RS	OTT	KL
G_s	2.66	2.65	2.65	2.65
e_{max}	0.842	0.89	0.69	
e_{min}	0.539	0.5	0.48	
$\rho_{d,max}$ (g/cm³)	1.728	1.767	1.791	
$\rho_{d,min}$ (g/cm³)	1.444	1.402	1.568	
Liquid Limit, LL (%)				60
Plastic Limit, PL (%)				40
Plasticity Index, PI (%)				20
Unified Soil Classification System	SP	SP	SP	MH

FIG 3.1: Grain size distribution of reconstituted specimen

3.2.2 Fabrication of Bender/extender elements

Bender elements from Piezo Systems Inc. USA were used. The top and bottom platens were fitted with a Y (T220-A4-303Y) and X (T220-A4-303X) poled bender elements, respectively. Each bender element protrudes about 5 mm from the face of the platen. The bender elements were fabricated as follows:

Bender element of length 12 mm and width 9.5 mm was cut from the original piece (12.7 mm x 31.5 mm). A small segment of about 4 mm lengthwise was then marked out using a cutter. The piezo-ceramic material within this small segment was filed off cleanly to expose the metal shim at the centre. Resistance and capacitance of the bender element were checked with a multi-meter. The multi-meter should reflect an open circuit at all times. Capacitance of the ceramic to metal shim at each side is approximately twice that of the ceramic to ceramic surface on both sides.

Three wires were soldered onto the bender elements, one on each side of the ceramic sheet and one on the metal shim as shown in FIG. 3.2. During the process of soldering, it was imperative to check the resistance and capacitance as the conductive soldering flux might short the bender element. If the bender element was accidentally shorted, the bender element can be cleaned using alcohol or filed slightly. The completed bender element is shown in FIG 3.3a. The bender element was then wrapped with a thin layer of Teflon tape followed by aluminium tape. The aluminium tape provides shielding from undesirable electrical interferences such as crosstalk and was connected to the grounding wire shield. The whole assembly is then coated in epoxy. Normal epoxy tends to absorb water, expand and crack in a humid/wet environment. The process would be accelerated under the influence of high pressure in the triaxial cell. As such, the epoxy layer tends to peel off after prolonged usage. Metal epoxy was used to overcome this problem but the resulting bender elements were so stiff that they were not able to transmit and receive waves properly. In order to overcome this, an acrylic casing is fabricated to house the bender element as shown in FIG. 3.3b.

In order for the bender element to transmit and receive different waves, they must be connected according to Table 3.3 and FIG. 3.2 as described by Leong et al. (2009).

FIG. 3.2: Insulation of bender/extender elements and its (a) series and (b) parallel wiring configuration (adapted from Leong et al., 2009)

FIG. 3.3: Fabrication of bender/extender element (a) after soldering (b) after waterproofing

Table 3.3: Bender element connection configuration

Wave Type	X-Poled	Y-Poled
P wave	Parallel	Series
S wave	Series	Parallel

3.2.3 Experimental Set-up and Methodology

Three experimental set-ups were used in this study: (1) ultrasonic transducers, (2) bender/extender elements, and (3) a hybrid bender element–ultrasonic system. These are described in detail below. The acrylic and nylon 66 specimens were tested using only ultrasonic transducers and hybrid bender element-ultrasonic system. For the hybrid bender element-ultrasonic system, a slot needs to be cut into one end of the acrylic and nylon 66 specimens to receive the bender element. These specimens were not tested using bender/extender elements because it was difficult to obtain good contact between the bender element and specimen at both ends of the specimen.

A function generator (Hewlett Packard Model 33120 A) was used to generate a sinusoidal pulse which is then amplified using a power amplifier (Piezo EPA-104)

before feeding it to the transmitter. The receiver and the function generator were connected to a digital oscilloscope (Yokogawa Model DL750P) so that both transmitted and received signals can be recorded. The maximum excitation frequency used was 200 kHz, and thus the sampling rate of the oscilloscope was set at 2MHz. This will mean that the oscilloscope should be able to record signals up to 200 kHz accurately as a single sine curve would have at least ten data points.

The ultrasonic transducers used were similar to those described by Leong et al. (2004). However, instead of six shear wave piezoelectric crystals, there were only four shear wave piezoelectric crystals surrounding the central compression wave piezoelectric crystal (FIG. 2.7). The former ultrasonic transducer is referred to as ULT-2004 while the latter ultrasonic transducer is referred to as ULT-2012 in this report. The natural frequencies of both the compression and shear wave piezoelectric crystals transducers are about 100 to 200 kHz as reported by GCTS. However after being epoxied to the aluminium housing, the dominant frequencies of the transducers are expected to be different.

Face-to-face tests of the ultrasonic transducer ULT-2012 were conducted at excitation frequencies of 1, 3, 5, 10, 20, 30, 50, 100 and 200 kHz and the frequency spectrum plots are shown in FIG. 3.4. FIG. 3.4 shows that the dominant frequencies of the compression and shear wave transducers occur at about 49 kHz and 47 kHz, respectively.

Compression wave signals are generally stronger than shear wave signals despite the transducers having more shear wave crystals. Signal strength increases with excitation frequency but the dominant frequency of the receiver signal was not shown to shift accordingly. The excitation frequency leading to the greatest output occurs at about 100 kHz. This shows that the strongest signal was generated when the crystals are excited near their natural frequencies. Therefore, the excitation frequency should be adjusted to provide the maximum output so that the strongest signals can be generated.

FIG. 3.4: Frequency spectrum for face-to-face (a) P- and (b) S-wave signals from ULT-2012

The delay time caused by transmission of the waves through the aluminium housing was obtained using four mild steel specimens of different heights: 80, 100, 120 and 140 mm. The travel times of the waves through the mild steel specimens are plotted with specimen height as shown in FIG. 3.5. Delay time was approximately $3.6 \mu\text{s}$ for both types of wave as shown by the y-intercept of the best-fit line. The delay time was further confirmed by measuring the travel time in the face to face tests. Signals at different excitation frequencies were lined up next to one another to identify the wave arrival time.

FIG. 3.5: Arrival times of shear and compression waves for mild steel specimens of different heights using ULT-2012 transducers

The bender-extender elements were aligned in the same plane to ensure maximum transmission of the P- and S-waves. Wave velocities were derived from the time taken to cover the distance bounded by the tips of the transmitting and receiving transmitter. Excitation frequencies of the bender-extender element follow those of the ultrasonic transducer. Signals at different excitation frequencies were lined up in the time domain to determine the arrival time. This is especially important for S-wave signals so that the region affected by the near-field effect can be identified. Unlike the ultrasonic transducer, wave produced by the bender-extender elements does not have to pass through the metal housing and hence does not have a delay time.

The hybrid bender element-ultrasonic system consists of the ultrasonic transducer as a receiver and the bender/extender element as a transmitter. The ultrasonic transducer is at the bottom of the soil specimen and the platen containing the bender/extender element is placed on top of the soil specimen. This arrangement simplifies the placement of the soil specimen, requiring only a slot on the top of the soil specimen to be made to receive the transmitter bender/extender element. The delay time of the receiver ultrasonic transducer was assumed as $1.8 \mu\text{s}$ (half of that for a pair of ultrasonic transducers) and the travel distance of the wave was taken as from the tip of the bender/extender element to the face of the ultrasonic transducer.

Otherwise, the procedures to obtain wave signals and arrival time are similar to those two set-ups mentioned.

Wave signals and velocities from the different transducers were compared to characterize the different transducers and also to develop a guideline for the determination of wave arrival time. An appropriate transducer will be selected to conduct the subsequent tests in this study.

3.3 Test Series II: Development of testing framework, effect of saturation and confining pressures on small-strain soil properties

Results from Test Series I showed that the bender-extender element is a more suitable transducer for subsequent tests. Details will be further elaborated in Chapter 5. The subsequent sections will be devoted to the development of the testing framework and the effect of saturation and confining pressures on small-strain soil properties

3.3.1 Material Utilised

In Test Series II, experiments were conducted using river sand, Ottawa 20-30 sand and silicon rubber. The river sand initially consists of 95% sand and 5% of non-plastic fines. It has a coefficient of uniformity (C_u) and gradation (C_c) of 3.96 and 1.05, respectively. The maximum grain size of the sand is 1.0 mm and its D_{50} is 0.39 mm. However, in Test Series II, the fines content was proportionately increased to 30% so that the air-entry value of the specimen can be made higher. This is done so that the small-strain soil properties can be observed before and after the air-entry value. The C_u and C_c of the modified mix are 4.67 and 0.21, respectively. Under the Unified Soil Classification System, the classification of the soil changes from SP to SM with the increase in fine content from 5% to 30%. The Ottawa 20-30 sand is a standard sand which adheres to ASTM C778-13 (2013). The grain size distribution curves and properties of the soils used in Test Series II were summarised in FIG. 3.1 and Table 3.2, respectively.

All specimens had a diameter of 50 mm. Soil specimens of 50 and 100 mm height were used in the tests. A predetermined amount of sand and water ($w = 15\%$) were mixed and compacted to a void ratio ranging from 0.45 – 0.50. The sand specimens were prepared using the moist tamping procedures similar to that used for Changi sand in Test Series I. The resulting specimen was then frozen so that slots on both sides of the specimen could be etched out to house the bender elements. This procedure was necessary to ensure good specimen-bender element contact and minimise sample disturbance during set up. Upon setting up, confining pressure and back pressure were applied to create an effective confining pressure of 30 kPa. The specimen was then left to sit in the triaxial cell for one day to allow for thawing.

Combination of different specimen heights allow for the application of the spectral ratio method. The shorter specimen (50 mm) was used to reduce test duration during matric suction equalisation while maintaining reasonable accuracy in wave velocity measurement. This is especially useful when investigating changes in small-strain properties during repeated drying and wetting of the specimen.

The net normal stress and effective stress applied on the specimen were isotropic. The net normal stress (unsaturated test) and the effective stress (saturated test) applied on the specimens are indicated by the specimen tag. The specimen tag is in the format S-P_N(H) where S is the specimen tag representing the material being tested, P for the type of test (S for saturated test, US for unsaturated test and D for dry test) and N for the applied pressure (effective stress or net normal stress). The H at the end of the specimen tag was used to represent specimens of 50 mm height. For example, RS-US_400H refers to an unsaturated river sand specimen of 50 mm height being tested at a net normal stress of 400 kPa. Testing conditions are summarized in Table 3.4.

The river sand was desaturated from full saturation and tested at different degree of saturation. Specimen RS-US_400H and RS-US_100H were subjected to an additional wetting and drying cycle. This was to confirm the effect of degree of

saturation on V_p , V_s and B-value and to ascertain if the V_p at full saturation remained constant when the specimen re-saturates. The Ottawa sand was tested dry at different effective confining pressures. This acts as a verification test set so that the damping ratio obtained from the chosen method can be compared with those from the literature which had determined the damping ratio of dry Ottawa sand using the resonant column test (Table 3.5). A mean void ratio from Table 3.5 was selected for the Ottawa sand specimen used in this study so that the damping ratio can be compared with those obtained from the literature. Details of the specimens tested in the literature are summarized in Table 3.5.

Table 3.4: Specimens tested for Test Series II

Soil Type	Specimen Tag	Initial Void Ratio	Height (mm)	Stress-State Variables (kPa)		
				$(u_a - u_w)$	$(\sigma_3 - u_a)$	$(\sigma_3 - u_w)$
River Sand	RS-US_100H	0.512	50	0 – 38	100	-
	RS-US_100	0.509	100	0 – 20	100	-
	RS-US_200H	0.469	50	0 – 38	200	-
	RS-US_200	0.506	100	0 – 38	200	-
	RS-US_400H	0.505	50	0 – 38	400	-
	RS-S	0.499	50	-	-	30 – 430
Ottawa Sand	OTT-D	0.55	100	-	-	50 - 800

Table 3.5: Summary of literatures which conducted resonant column tests on Ottawa sand

Author	Soil Tested	Specimen Tag	e_{min}	e_{max}	e
Hall (1962)	Ottawa 20-30	HL	0.55	0.77	0.67
Hardin (1965)	Ottawa 20-30	HN	-	-	0.50
Ray and Woods (1988)	Ottawa 20-30	RW	0.48	0.73	0.48 – 0.58
Shen et al. (1985)	Ottawa C109	SN	-	-	0.51
Maher et al. (1994)	Ottawa 20-30	MR	0.5	0.78	-
Hardin et al. (1994)	Ottawa	HN2	-	-	-
Zhang and Aggour (1996)	Ottawa	ZA	-	-	0.56

Silicon rubber specimens (denoted by specimen tag SR) were used as a standard material to calibrate wave arrival time and damping ratio. Signals obtained from the

silicon rubber specimen also act as reference signals to determine damping ratio in the spectral ratio method (SRM). Silicon rubber was chosen as it can be moulded into a homogenous specimen that is of similar dimensions as the soil specimen. Dow Corning® 3120 RTV Rubber was used. The density of the silicone rubber is 1450 kg/m^3 and the shore hardness A is 56. A total of three silicon rubber specimens of different heights (25, 50 and 100 mm) and 50 mm diameter were made.

A PVC mould was fabricated as shown in FIG 3.6. The PVC mould was secured to two acrylic plates with bolts and nuts. The acrylic plate was made such that there is a protrusion of slightly smaller size than the bender element. This was to ensure that the slot created in the silicon rubber specimen provides a tight fit condition for the bender element.

FIG. 3.6: (a) Schematic and (b) actual representation of fabricated mould used for casting of silicon rubber

A non-silicon based releasing agent (Meguiar's Mirror Gaze Maximum Mold Release Wax) was used to ease removal of the silicon rubber specimen. It is important to use a releasing agent without silicon derivatives as silicon derivatives will hinder the setting of the silicon rubber. A piece of transparency coated with

releasing agent was used to line the PVC tube to facilitate extrusion of the SR specimen from the PVC tube.

The base and catalyst of the Dow Corning® 3120 RTV Rubber were mixed in the ratio of 10:1. This will provide an approximate working time of two hours whereby the mixture can be de-aired and then poured into the mould slowly. The mould was then screwed tight between the acrylic plates so that no air bubbles were trapped. The silicon rubber specimen was de-moulded on the second day. The finished silicon rubber specimens are shown in FIG 3.6b.

3.3.2 Experimental Set-up and Methodology

In Test Series II, the river sand specimens were first tested at various degrees of saturation. Damping ratios of the specimens were obtained using the various methods proposed in Section 2.3. Details of the tests are elaborated below:

The specimens were saturated in a triaxial cell as shown in FIG 3.7. The top cap was fitted with a porous disk while the bottom platen was fitted with a ceramic disk. The air-entry value of the ceramic disk was 0.5 bar for the sand specimens. Saturation of soil specimens is a time consuming process as it may take several weeks for the air bubbles to dissolve in solution (Lee and Black 1972). To hasten the saturation process, a low effective confining pressure of 10 kPa was maintained. Water was allowed to enter the specimen from the bottom platen using a digital pressure-volume controller (DPVC). As the rate of water entry slowed down and stopped, the top drainage line connected to a vacuum of approximately 80 kPa was opened to allow any entrapped air bubbles to escape from the system. The top drainage line was shut when no further air bubbles were observed in the top drainage line. The routine described above was repeated to remove as much air as possible.

FIG. 3.7: Triaxial set-up

A B-value check was conducted each morning after leaving the specimen to saturate overnight. During the B-value check, the confining pressure was increased by 100 kPa and the maximum pore-water pressure response within 10 minutes was recorded. This was to ensure that there was no further change in the B-value (Black and Lee, 1973). These procedures served to monitor the degree of saturation of the specimen through the B-value as recommended by ASTM D7181-11 (2011). Both P and S-wave signals were recorded before and after the B-value check. The P-wave signals were recorded so that the characteristics of the wave during saturation can be observed and compared with the B-value. The S-wave signals were used to detect changes in the soil structure during saturation and during the B-value check.

The bender/extender elements were excited using a sinusoidal pulse generated by the function generator. Signals were recorded at a sampling frequency of 2 MHz and recorded for a duration of 5 ms. Signals were obtained at different excitation frequencies so that accurate arrival time could be obtained by comparing the recorded signals. The excitation frequencies for P-wave signal were 10, 20 and 50 kHz. The excitation frequencies for S-wave signal were 3, 5, 10 and 20 kHz. The excitation frequencies of the S-wave signal were lower than those for the P-wave signals because the resonant frequency of the bender extender element in flexure

mode is lower than its resonant frequency in extension mode (Cheng and Leong, 2014).

Saturation continued until the following conditions were met: (1) V_p remained constant; (2) No air bubbles were observed in the top drainage line; and (3) Skempton's pore-pressure parameter, $B > 0.95$ (Head, 1998). If air bubbles have ceased to emerge from the top drainage line but $B < 0.95$, the back pressure and confining pressure were increased in equal amount to dissolve the remaining air trapped within the soil specimen.

Upon saturation, the river sand specimens were consolidated to the various stress conditions (Table 3.4). Matric suction ($u_a - u_w$) was imposed using the axis-translation technique (Hilf, 1956). Water outflow from the back pressure and top drainage line were monitored via the GDS Digital Pressure/Volume controller (DPVC). Upon equilibrium, the top drainage line was connected to the air pressure which was adjusted to be at the same value as the back pressure.

Matric suction was imposed by applying air pressure (u_a) through the top drainage line via the top platen. The air pressure (u_a) was held constant while the water pressure (u_w) at the bottom platen was reduced to establish the desired matric suctions ($u_a - u_w$) while maintaining the net normal stress ($\sigma_3 - u_a$). Changes in the volume of water moving in and out of the specimen were monitored by the DPVC so that water content of the soil specimen at each stage can be calculated and the soil-water characteristics curve established. At equilibrium, the P- and S-wave signals were obtained. This was followed by a B-value check after which the wave signals were recorded again. The above testing procedure will allow the relationship between S_r , B-value and V_p to be studied. Testing under saturated and unsaturated conditions using similar pressure range enabled the wave velocities to be compared when the soil specimen was subjected to an isotropic effective stress (saturated) and a combination of isotropic net normal stress and matric suction.

The river sand specimens were tested at matric suctions ranging from 0 to 38 kPa. Two of the river sand specimens (RS-US_400H and RS-US_100H) of 50 mm height were further made to go through another cycle of wetting and drying to investigate the effect of hysteretic saturation on small-strain soil properties (Table 3.5). Upon completion of the test, the specimen was removed and dried in the oven at 105°C for water content determination (ASTM D2216-10, 2010). Equilibrium water content of the specimen at each matric suction was obtained by back calculation from the final water content. Receiver signals were processed with a MATLAB script to determine the wave velocity and damping ratio via the methods previously described in Section 2.3 (LDM, SRM and HTM).

By testing specimens at different confining pressures, degrees of saturation and of various soil types, their effects on V_p , V_s and ξ can be better understood. This will further reduce the amount of uncertainties when wave propagation test is conducted on unsaturated residual soil which is highly heterogeneous.

As stated in Chapter 2, determination of damping ratio using the SRM can be carried out using two cases. The first case uses a reference specimen of similar geometry and known damping ratio. Material damping ratio of the soil specimen is determined from the change in Fourier amplitude due to difference in material type. The second case involves using similar materials of identical cross section but varying the distance of wave propagation. Material damping ratio is obtained from the reduction of Fourier amplitude with increasing distance of wave propagation.

In this study, the first case is carried out by using the silicon rubber specimen as a reference specimen. Damping ratio of silicon rubber was determined by applying the spectral ratio method on signals obtained from identical specimens of different height (Case 2 of SRM). When comparing with the soil specimens, the silicone rubber should be subjected to the same effective confining pressure to ensure that the only variation between the two signals is the medium of wave propagation. Sampling frequency and recording time were identical to those obtained for the soil specimens. The damping ratio of the silicon rubber was also obtained by testing the

100 mm height silicon rubber specimen in a cyclic triaxial set-up. The specimen was subjected to 40 sinusoidal excitations with axial strains of 0.05, 0.1 and 0.2%. At each strain, the specimen was tested at excitation frequencies of 0.2 and 05 Hz. The Case 2 of SRM was carried out by comparing the Fourier amplitudes of river sand specimens of different heights.

The most reliable method for determining the damping ratio will undergo further verifications. The first verification involves the determination of damping ratio for dry Ottawa sand at effective confining pressures of 50, 100, 200, 400 and 800 kPa. Ottawa sand was chosen as dynamic properties such as stiffness and damping ratio at different strains are widely available in the literature (Table 3.5). A predetermined mass of dry Ottawa sand corresponding to a void ratio of 0.55 for a specimen of 50 mm diameter and 100 mm height was measured. The sand was poured into a rubber membrane stretched by a membrane stretcher and attached to the bottom platen of the triaxial cell with O-rings. The rubber membrane was supported on the outside by a transparency film rolled to a cylinder of 50 mm diameter and 100 mm high and fixed with adhesive to provide some support of the sand specimen when the membrane stretcher was removed. The top platen was then lowered down so that the top portion of the rubber membrane can be sealed with O-ring. The specimen was then lightly tapped at the sides to densify the Ottawa sand while the top platen was slowly lowered until a height of 100 mm was attained. Care was taken to ensure that the load cell does not register an increase in load as the top platen was lowered. Prior to the removal of the supporting transparency film, a suction of 50 kPa was applied using a handheld vacuum pump at the top drainage valve to ensure that the specimen was able to maintain its cylindrical shape without collapsing. The suction was only removed after the triaxial cell was filled and confining pressure applied. During testing, both top and bottom drainage line was opened to attain a drained condition. Wave propagation test was conducted using procedures similar to those for the river sand specimens.

Wave propagation test was also conducted on the 100 mm height silicon rubber specimen. Using procedures previously mentioned, the receiver signals were

analysed. Damping ratios obtained were compared with those obtained from the cyclic triaxial test.

The final verification was conducted using finite element simulation and will be elaborated further in Chapter 4. Wave propagation in a soil specimen was modelled as a 3D solid cylinder. Sinusoidal pulses of 3, 5 and 10 kHz were used to excite the model separately. Using a viscoelastic model for the soil, damping ratios of 0.03 and 0.07 were assigned to the model for each of the excitation frequency. Damping ratio obtained by processing the receiver signal was compared with the input damping ratio.

3.3.3 Digital Signal Processing

Signals obtained from the bender/extender elements are highly affected by electronic drift and noise. This is especially so for the weaker P-wave. In view of this, the signals were filtered through a fourth order, band pass Butterworth filter with a lower and upper bound of 0.5 and 100 kHz, respectively. The lower bound of 0.5 kHz serves to remove low frequency electrical drift which can affect the damping results. A wide frequency band was selected so that the receiver wave will not be heavily distorted by the filtering. The filtered and raw signals were both used to determine the arrival times of the P- and S-waves.

However, to obtain the damping ratio, the digital filter would have to be designed to retain the natural frequencies of the P- and S-wave signals. In the resonant column test, the material damping ratio is obtained from the free vibration decay after the excitation stops at the natural frequency which correspond to the resonant frequency at the particular vibration mode. However, signals from the bender element test were different as they were excited with a single pulse which most likely does not correspond to the natural frequency of the specimen. As such, the receiver signal will contain two parts (1) transient part which is caused by the initial excitation pulse, and (2) steady-state free vibration decay which is prevalent when the transient effect from the excitation pulse dies off (Rossing and Fletcher, 2004) (FIG.

3.8). Oscillations at the steady-state free vibration decay correspond to the natural frequency of the specimen.

FIG. 3.8: Typical S-wave signal from the bender element test

S-wave signals at various excitation frequencies and matric suctions were studied to determine the natural frequency of the S-wave. It was found that regardless of the net confining pressure and matric suction, the S-wave signal remained relatively unchanged. S-wave signals of river sand specimens at a matric suction of 30 kPa and different net confining pressure were expressed in their frequency domain in FIGs. 3.9c, f and i. The dominant frequency of the S-wave signals was observed to be around 2 kHz as highlighted in FIGs. 3.9c, f and i. This intensity at the dominant frequency was the strongest when the specimen was excited at 3 kHz. Moreover, the plot in the frequency domain shows only one sharp peak which means that the amount of interference from reflected waves was negligible. The signal in the time domain is similar to that observed in a resonant column apparatus during free vibration decay (FIG. 2.8). At higher excitation frequencies, the dominant frequency was observed to be higher and can range from 10 to 20 kHz. This is due to the initial excitation pulse and interference from the reflected and refracted waves from the specimen's boundaries and when the wave travel from one phase to another. However, the peak at about 2 kHz remains visible albeit at a smaller intensity. At higher confining pressure, the peaks in the range of 10 to 20 kHz

become more prominent as the larger contact force between the soil grains reduces the frictional loss of the higher frequency waves. Nevertheless, the peaks at 2 kHz are still visible.

The natural frequency at 2 kHz can be further justified by windowing. Receiver signals from specimen RS-US_100H at matric suction of 30 kPa were passed through the Hanning window twice to remove the transient response before conducting an FFT. The time and frequency domains of the receiver signals before and after windowing are shown in FIG. 3.10. Ordinates in the frequency domain plots are normalised using the highest intensity of each series. The higher peaks in the range of 10 to 20 kHz were seen to decrease with windowing. The peaks at around 2 kHz were present even after windowing regardless of the excitation frequency thereby confirming it to be the natural frequency of the specimen.

The natural frequency of the bender element system at 2 kHz was further validated with the help of the finite element program, LS-DYNA. A specimen (50 mm diameter and height) was modelled using implicit eigenvalue analysis (see Chapter 4) to derive the natural frequency at the various modes of vibration. Bulk and shear stiffnesses derived from the P- and S-wave velocities (V_p and V_s) together with the bulk density were input into the finite element model. The natural frequencies in the bending mode for the fixed-free and fixed-fixed mode of a cylindrical soil specimen were found to be 1 and 2.6 kHz, respectively. Since the boundary condition of the specimen lies between those of fixed-free and fixed-fixed conditions, it was expected to fall between 1 and 2.6 kHz and thus, the experimental results gave good agreement. As such, a Butterworth band-pass filter with lower and upper bounds of 0.5 and 5 kHz was applied to the S-wave signal from the bender element test to retain the natural frequency.

P-wave signal was observed to be different before and after the air-entry value. As such, P-wave signals at matric suctions of 0 and 30 kPa were analysed. FIGs. 3.9a, d and g shows that full saturation give rise to peaks above 30 kHz. The intensity of these peaks increases with excitation frequencies. As the specimen desaturates, the

intensity of the peaks above 30 kHz reduces drastically (FIGs. 3.9b, e and h). This phenomenon is consistent regardless of the net confining pressure. Hence, it can be deduced that these high frequency components is a result of P-wave propagating through the water phase. The above observation is further verified in Section 5.3.4 whereby the frequency components of P-wave traversing through water coincided with the peaks above 30 kHz observed in FIG 3.9.

The P-wave signals from specimen RS-US_100H at matric suctions of 0 and 30 kPa were also passed through the Hanning window twice. The time and frequency domains of the P-wave signals at matric suctions of 0 and 30 kPa were shown in FIGs. 3.11 and 3.12, respectively. The figures showed a reduction in intensity of the peaks above 10 kHz upon windowing. Intensity of the peaks below 10 kHz remain strong thereby implying that the natural frequency of the specimen in the P-wave excitation mode lies between 0 to 10 kHz. As such, a Butterworth band-pass filter with lower and upper bounds of 0.5 and 15 kHz was applied to the P-wave signal to retain the natural frequency.

The P- and S-wave signals were processed using the filtering procedures stated above before they were subjected to the LDM and HTM. The SRM were processed separately and will be elaborated later in this section.

FIG. 3.9: Frequency domain of P- and S-wave at different excitation frequencies and effective confining pressures

FIG. 3.10: Time and frequency domains of raw and windowed S-wave signals from RS-US_100H, $(u_a - u_w) = 30$ kPa

FIG. 3.11: Time and frequency domains of raw and windowed P-wave signals from RS-US_100H, $(u_a - u_w) = 0$ kPa

FIG. 3.12: Time and frequency domains of raw and windowed P-wave signals from RS-US_100H, $(u_a - u_w) = 30$ kPa

For the logarithmic decrement method (LDM), a MATLAB script was written to pick the peaks and troughs in the filtered signal (FIG. 2.8). Absolute values of negative troughs were used as half cycle to increase the number of data points available for linear regression. Procedures of the LDM can be seen in Section 2.3.1.

For the Hilbert transform method (HTM), a MATLAB Hilbert transform function was used to convert the filtered signal (FIG 3.13a) to the analytic signal. The natural logarithm of the magnitude of the analytic signal was plotted with time (FIG. 3.13b). The time window of the signal which depicts free vibration decay was then isolated (demarcated by the dotted lines in FIG. 3.13b) so that a best-fit line can be constructed through the data points in order to obtain the gradient used for calculating the damping ratio (Equation 2.19). However, the presence of destructive interference will give rise to the troughs as seen in FIG. 3.13b. Inclusion of these irregularities led to problematic determination of the best fit line and hence peaks in the decaying portion of the FIG. 3.13b was used to construct the best fit line as shown in FIG. 3.13b. The representative natural frequency can be calculated using Equation 2.20. The frequency range for Equation 2.20 was set to be 0 – 10 kHz for S-wave and 0 – 30 kHz for P-wave. Using the gradient and the natural frequency, the damping ratio can be obtained using the HTM.

FIG. 3.13: (a) Raw and filtered S-wave signals excited at 5 kHz and its corresponding (b) magnitude of the analytical signal, specimen RS-US_100H with $(u_a - u_w) = 30$ kPa

The SRM focuses on the first incident sine wave unlike the LDM and HTM which works on the steady-state free vibration decay. According to the derivations from Section 2.3.3, the SRM compares the signal amplitude of waves from either different specimens or different travel lengths. Waves from the free decay portion can be sometimes subjected to interferences from reflected waves leading to multiple peaks in the frequency domain. Furthermore, when comparing between different specimens or specimens of different heights, it is very likely that the natural frequency reflected in the steady-state free vibration portion can be different. This difference can lead to misalignment of the peaks in the frequency domain

thereby affecting the process of determining the gradient on the $\ln\left(\frac{A_1}{A_2}\right)$ versus f plot.

The above will be further elaborated in Chapter 5. In view of the above consideration, a time window was used to extract the first sine wave from the receiver signal. This is because the amplitude of the first sine wave is a result of the excitation pulse travelling through the specimen and is least affected by reflection and frequency mismatch between the receiver signals. The extracted signal was then zero padded to 2^{16} points before performing a FFT. The frequency spectra for both the reference and subject specimens were hence compared.

From Equation 2.14, it can be observed that when comparing between specimens of the same material but with different heights (i.e. same Q), the resulting slope obtained should be positive as the shorter specimen (subscript 1) produces a stronger signal. In this case, $A_1 > A_2$ and $x_1 < x_2$, the gradient selected around the dominant frequency should have a positive slope. Hence, the selection of frequency range to determine gradient will be based on:

- Positive gradient of $\ln\left(\frac{A_1}{A_2}\right)$ versus frequency (f) graph
- Frequency range corresponding to peak amplitude and $1/\sqrt{2}$ of the peak amplitude.
- Frequency range selected should be maximised while ensuring a minimal R^2 of 0.99

When comparing two materials of similar geometry, subscript 1 will refer to the standard material (silicon rubber). The value of the slope obtained should be similar throughout the test series since damping ratio was not expected to vary significantly.

3.4 Test Series III: Dynamic properties of soil rubber mixtures

Results from Test Series II showed that the Hilbert transform method (HTM) is the most reliable method to determine damping using bender-extender elements. As such, the HTM was used to obtain the damping ratio of the soil-rubber mixtures.

3.4.1 Material Utilised

The soil-rubber mixture consists of the river sand (95% sand, 5% non-plastic fines) and commercially available rubber chips. The non-spherical rubber chips (FIG. 3.14) were made of Ethylene-Propylene-diene-Monomer (EPDM) which was manufactured from recycled rubber tyres with non-toxic elements. For triaxial tests, ASTM D7181-11 (2011) specifies that the maximum grain size diameter should be less than $1/6$ of the specimen's diameter whereas BS1377-8:1990 (1990) specifies that the maximum grain size diameter should be less than $1/10$ of the specimen's diameter. Therefore, the rubber chips were sieved to pass through a sieve size of 4.75 mm to satisfy both the ASTM and BS standards for a 50 mm diameter specimen. To reduce the effects of powdered rubber chips, only rubber chips retained on 2.36 mm sieve were used. The specific gravity of the rubber chips is 1.83.

FIG. 3.14: Rubber chips used in the sand-rubber mixture

3.4.2 Experimental Set-up and Methodology

The sand-rubber mixtures were mixed by weight ratio. The void ratio (e) of the sand rubber mixtures specimens were maintained at 0.60 throughout the test program. The compositions of the sand-rubber mixtures are tabulated in Table 3.6.

To prepare a sand-rubber mixture specimen, the sand and rubber were first divided into six equal parts and then placed into a PVC tube of 250 mm in height and 48 mm in diameter, in alternating layers with the sand as the first layer. The tube was then overturned five times to ensure that the sand and rubber were evenly mixed. The specimens were tested dry in a triaxial cell with bender elements embedded in the top and bottom platens. The bender elements protruded about 5 mm from the face of the platen. A split mould (100 mm in height and 50 mm in diameter) lined with rubber membrane was attached to the bottom platen. The soil-rubber mixture from the PVC tube was poured into the split mould in five lifts. Each lift was tamped for twenty times with a tamping rod of 50 mm diameter to obtain a specimen with void ratio of 0.60.

Table 3.6: Composition of sand and rubber chips in specimens of different rubber content

Weighted rubber content (%)	Mass of sand (kg)	Mass of rubber chips (kg)	Bulk density (kg/m³)	Volumetric rubber content (%)	Volumetric sand content (%)
0	0.3252	0	1656.25	0	100
15	0.2590	0.0457	1551.94	20.35	79.65
30	0.2007	0.0860	1459.99	38.29	61.71
60	0.1025	0.1538	1305.31	68.48	31.51
100	0	0.2246	1143.75	100	0

The percentages of rubber chips by weight tested were 0, 15, 30, 60 and 100%. For 100% rubber content, the specimen's height was reduced to 50 mm before a clear receiver signal could be recorded. After assembling the triaxial cell, the specimen was subjected to effective confining pressures of 0, 100, 200 and 400 kPa. The top

and bottom drainage valves were left open to ensure that the pore air was fully drained.

For ease of discussion, the specimens are referred using the notation of Cxxx-P/SWyy-Rzz where C represents the effective confining pressure, P/SW represents the P- or S-wave excitation frequency and R represents the rubber content. The alphabets x, y and z represent the effective confining pressure in kPa, excitation frequency in Hz and rubber chip content in percent by weight, respectively.

Segregation is an effect which cannot be avoided both in-situ and during laboratory testing of the soil-rubber mixture. While all possible measures were taken to ensure homogeneity during preparation of the specimen, the extent of inhomogeneity needs to be quantified. As such, the worst case scenario of segregation was used to investigate its effects on dynamic parameters. Using 15% rubber content, the rubber chips and sand were placed as separate layers as shown in FIG. 3.15. A specimen height of 80 mm was used.

FIG. 3.15: Sketch of the segregation specimens

While both P- and S-wave signals were recorded, only the S wave signals were analysed for the material damping ratio. The P-wave signal was weak because of the smaller displacement of the bender element in the extension mode (Lings and Greening 2001) rendering the P wave signal too noisy for the Hilbert transform

analysis. This is illustrated in FIG. 3.16 where P-wave signals need to be significantly amplified to have comparable amplitude as the S-wave signals.

FIG. 3.16: Comparison of P- and S- wave signals from sand rubber mixtures

3.4.3 Signal Processing

Signal processing to obtain arrival times of the P- and S-wave is similar to that elaborated in Section 3.3.3. However, the signal processing of the S-wave signals would have to be re-examined to ensure that the Butterworth filter previously established applies to the soil rubber mixtures. Frequency domain of the S-wave signals is shown in FIG. 3.17. FIG. 3.17 shows that the dominant frequency range does not change with the excitation frequency (FIG. 3.17a), increases marginally with effective confining pressures (FIG. 3.17b), and decreases marginally as the rubber content of the specimen increases (FIG. 3.17c). Regardless of the condition, the dominant frequency ranges between 0.5 and 5 kHz. Hence, the same fourth order Butterworth filter was used to retain frequencies within this band.

FIG. 3.17: Effects of varying (a) excitation frequency, (b) confining pressure and (c) rubber content on the frequency composition of the S wave signal

3.5 Test Series IV: Cyclic triaxial and wave propagation test on unsaturated residual soil

3.5.1 Material Utilised

Undisturbed residual soil obtained from Ang Mo Kio Avenue 4 (FIG. 3.18) using a Mazier sampler was used in this test. The borehole log (Appendix A) shows that the soil was weathered from the Bukit Timah granite. Sampling depth ranges from 3 to 11 m below the surface of the ground. Specimens sampled from depth close to one another were grouped together as shown in Table 3.7.

The undisturbed samples were extruded from PVC tubes of 1 m length. After which, 0.1 m at each end was trimmed and the remaining sample was divided into four samples of 0.2 m each before wrapping up in cling wraps, aluminium foil and then sealed with wax. At times, an entire sample would have to be discarded due to the presence of a layer of sand or gravel causing the specimen to break upon extrusion. The four samples were trimmed to specimens of diameter 50 mm and height of 100 mm. Out of the four specimens; three were tested. One of the specimens was subjected to a saturated consolidated undrained (CU) multi-stage test at effective 100, 200 and 350 kPa. The other two specimens were subjected to unsaturated constant water content test at net confining pressure of 200 kPa and matric suction of 50 and 100 kPa each. The fourth specimen was a reserve specimen. The grain size distribution curves of the three tested specimens are shown in FIG. 3.19. According to USCS, the residual soil is classified as MH. The liquid and plastic limits of the residual soil are 62 and 33, respectively. The plasticity index of the soil is 29. Basic soil properties and test conditions of the three tested specimens are tabulated in Table 3.7.

FIG. 3.18: Geological map of Singapore with sampling location marked with a star
(modified from http://www.srmeg.org.sg/docs/N13072012_2.pdf)

Table 3.7: Basic soil properties and test conditions of the residual soil

Depth (m)	$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	Specimen Tag	Void Ratio	G_s	ρ (kg/m ³)	Test Type
4.6	Multi-stage	0	AMK1-S	0.85	2.44	1717	CU
3.6	200	50	AMK1-US_50	0.86	2.44	1737	CW
3.4	200	100	AMK1-US_100	0.85	2.44	1722	CW
8.6	Multi-stage	0	AMK2-S	1.26	2.68	1721	CU
8.2	200	50	AMK2-US_50	1.26	2.68	1656	CW
9.6	200	100	AMK2-US_100	1.07	2.68	1620	CW

FIG. 3.19: Grain size distribution curves of the residual soil

3.5.2 The modified cyclic triaxial set-up

3.5.2.1 Calibration of set-up

The cyclic triaxial apparatus used in this study was the UTM-14 from Industrial Process Controls (IPC) Global. The apparatus was modified to house a pair of bender-extender elements at the top and bottom platens. The bottom platen was fitted with a high air-entry ceramic disk while the top platen was fitted with a porous stone. The air-entry value of the ceramic disk was 500 kPa. The schematic diagram of the modified cyclic triaxial set-up is shown in FIG. 3.20. The apparatus is fitted with the following transducers:

- A 5 kN submersible load cell to measure axial load.
- A Pore-water pressure transducer attached to the bottom platen for monitoring changes in pore-water pressure in the soil specimen.

- A submersible linear variable displacement transducer (LVDT) with a range of 5 mm. The spindle of the LVDT was attached to the top of the load cell by adhesive. The submersible LVDT not only records the displacement of the load cell, it is also used as a feedback for the pneumatic servo actuator for cyclic loading.
- An external LDVT to record the large strain especially during shearing
- A pair of proximity transducers to measure radial strain. The transducer (6U KDM-8200 from KAMAN, USA) has a measurement range up to 6 mm and is mounted diametrically opposite of the specimen. Displacement is measured by recording the changes in current induced by the interaction of the conductive target with the electromagnetic field generated by the transducer. A piece of aluminium sheet (25 x 25 mm) was pasted onto the specimen using silicone grease and served as a target for the proximity transducer.
- A pair of local displacement transducers (LDT) to measure local axial strain at the mid-height of the specimen. Details of the LDT can be found in Leong et al. (2011).

FIG. 3.20: Modified cyclic triaxial set-up (modified from Leong et al. 2006)

All the transducers were connected to a control and data acquisition system (CDAS) and a computer. Both the transducers and the pneumatic servo actuator have to be properly calibrated before use. The transducers were calibrated using a loading and unloading path to ensure that voltage change in the transducer follow an identical path, i.e., no hysteretic effect.

The pneumatic servo actuator located at the top of the triaxial cell controls the movement of the loading piston via the proportional, integral and derivative (PID) controller. The PID controller is essentially a feedback control compensator whereby the difference between the loading waveform measured by the transducer (feedback) and the desired waveform is used to drive the servo valve to regulate the flow of air pressure in order to minimize the difference between the waveforms. The proportional loop gain (proportional) gives an output that is directly proportional to the difference between the feedback and input waveform. The error time constant (integral) ensures that the servo system reaches a zero error condition within a given time. The mass compensation (derivative) is used to compensate for the weight of any add-ons on the loading piston (IPC, 2001). In this study, the feedback is provided by the submersible LVDT which rest on top of the submersible load cell.

The cyclic triaxial cell is calibrated with the help of the silicone rubber specimen of 50 mm diameter and 100 mm height and the built-in performance tuning test. Upon initiation of the test program, the actuator attempts to undergo displacement according to the input waveform. The PID values were adjusted until a sine curve with the desired frequency is obtained. Criteria for acceptable sine curve are shown in FIG. 3.21. The silicone rubber specimen was used as a dummy specimen for the calibration tests. A sinusoidal displacement with time was used. The double amplitude cyclic axial displacements were 0.05, 0.1, 0.2, 0.5 and 1 mm. For a 100 mm high specimen, the value will translate to percent strain. Each displacement was programmed to cycle at excitation frequencies of 0.2 and 0.5 Hz. In order to overcome compliance issues due to fluid medium and the confining pressure, the dummy specimen was calibrated while being subjected to effective confining

pressures of 100, 300 and 600 kPa. The calibrated PID values are tabulated in Table 3.8.

FIG. 3.21: Criteria for acceptable and unacceptable sine curves (adopted from ASTM D3999-11, 2011)

Table 3.8: PID values for rubber specimen subjected to effective confining pressure of 100, 300 and 600 kPa

Displacement (mm)	Frequency (Hz)		
	1	0.5	0.2
1	650-70-13000*	650-27-13000	650-11-13000
0.5	650-55-13000	650-22-13000	650-11-13000
0.2	1100-43-13000	1100-14-13000	1100-9-13000
0.1	1900-38-13000	1900-14-13000	1900-9-13000
0.05	1900-38-13000	1900-14-13000	1900-9-13000

*values are listed in the order of P-I-D

The PID values were found to be unaffected by changes in the confining pressure. While the PID values are specific to the equipment used, Table 3.8 shows the trend of the PID values with changes in displacement and driving frequency and is useful

as a guide to other researchers using similar pneumatic servo actuator. The value of the derivative term (D) remains unchanged regardless of changes in frequency and displacement. The value of the integral term (I) was shown to decrease with decreasing frequency and displacement. The value of the proportional term (P) was only shown to change with displacement.

3.5.2.2 Fabrication of local displacement transducer (LDT)

Conventionally, measurement of stress-strain behaviour in a triaxial set-up is usually carried out using externally mounted transducer. However, deformation derived from these conventional transducers can have inherent errors due to the seating, alignment, bedding and compliance issues (Baldi et al. 1988). These errors can however be minimized by measuring strains locally or internally (Lo Presti et al., 1994). Rowe and Barden (1964), Jardine et al. (1984) and Jamiolkowski et al. (1994) further emphasized the importance of local strain measurement when conducting triaxial tests in small to intermediate strain range. Local strain measurements come in many forms like the self-releasing cantilever type local displacement transducer (Yimsiri et al. 2005), inclinometer level (Burland and Symes 1982, Jardine et al. 1984, Ackerley et al. 1987), Hall effect gauges (Clayton and Khatrush 1986) and the local displacement transducer (Goto et al. 1991). In this study, the local displacement transducer (LDT) by Goto et al. (1991) was used due to its simplicity in fabrication and low cost.

The LDT is fabricated by attaching four 350 Ω strain gauges (TML FLA-2-350-11) on a thin rectangular strip of phosphor bronze. The strain gauge has a gauge length of 2 mm and a gauge factor of 2.1. Two of such gauges were attached to the center of each side of the strip of phosphor bronze (FIG. 3.22) using cyanoacrylate. According to Hoque et al. (1997), the dimension of the rectangular strip should be 3.5 mm wide, 80 mm long and 0.20 mm thick for a specimen with 50 mm diameter and 100 mm height. The LDT is positioned in the middle of the specimen covering about 80% of the specimen height (Yamashita et al. 2001) using a pseudo-hinged attachment made of phosphor bronze (FIG. 3.23a). Nyunt (2012) mentioned that

slippage at the pseudo hinges can lead to “apparent” inaccuracy of the LDT especially when it is subjected to cyclic loading. To overcome the limitation in this study, a modified pseudo-hinged attachment was 3D printed using hard plastic (Polymethyl methacrylate also known as PMMA) to minimize the effect of slippage. The modified hinge has a 0.2 mm wide slot which is equivalent to the thickness of the phosphor bronze strip so that it is able to fit snugly onto it and rotate within the slot without slippage (FIG. 3.23b). The LDT was calibrated using the modified hinge.

FIG. 3.22: Schematic diagram of (a) LDT (from Leong et al. 2011)

FIG. 3.23: Schematic diagram of (a) original pseudo-hinged attachment (from Goto et al. 1991) and (b) modified pseudo-hinged attachment.

The LDT proposed by Goto et al. (1991) was wired such that the four strain gauges formed a Wheatstone bridge as shown in FIG. 3.24a. The Wheatstone bridge is an electrical bridge which is powered by a voltage source to measure resistance,

inductance and capacitance. In this study, the Anderson loop (FIG. 3.24b) which is powered by a current source was used instead due to advantages it has over the Wheatstone bridge (Anderson, 1995; 1998) when operating in off-balance bridge operation like strain gauges. Additional circuitries elaborated in Leong et al. (2011) were used together with the Anderson loop in this study. The combination was found to be twice as sensitive as the Wheatstone bridge circuit and offer better performance in terms of lower deviation, hysteresis and repeatability errors.

FIG. 3.24: Schematic diagram of (a) Wheatstone bridge circuit and (b) Anderson loop (from Leong et al. 2011)

The strain gauges were first coated with three layers of polyurethane. Upon drying, the centre portion of the LDT was wrapped in 3M vinyl/mastic tape. The exposed wires and the tape were further coated with three layers of 1521B Synthetic Rubber Adhesive by Three Bond. The waterproofed LDT was then calibrated using a digital micrometer (Mitutoyo) which has a resolution of $1\mu\text{m}$ using the modified pseudo-hinged attachment (FIG. 3.25).

FIG. 3.25: Calibration of LDT using micrometer

3.5.3 Experimental Set-up and Methodology

Upon setting up, all the specimens were saturated and tested for saturation using procedures described in Test Series II. The saturated specimens were then consolidated to the test conditions stated in Table 3.7. Changes in water content were monitored using the GDS DPVC. The degree of consolidation was monitored from the volume of water outflow from the specimen with time. The log time method (FIG. 3.26) was used to calculate the time for 50% primary consolidation (t_{50}) which was used to determine the strain rate for CU test according to ASTM D4767-11 (2011). Displacement controlled cyclic triaxial test was conducted under undrained condition, i.e., all drainage lines were closed. A total of 40 loading cycles were imposed for each test at each specified frequency and amplitude.

FIG. 3.26: Volume of water outflow with time

3.5.3.1 Saturated consolidated undrained (CU) multi-stage test

Saturated specimens were made to undergo a consolidated undrained multi-stage test. After consolidation to an effective stress of 100 kPa, a wave propagation test was conducted according to the procedures mentioned in Test Series II. P-wave signals were derived by exciting at 10, 20 and 50 kHz while S-wave signals were derived by exciting at 3, 5, 10 and 20 kHz. After which, the cyclic triaxial test was conducted on the specimen at double amplitude axial strains of 0.05, 0.1 and 0.2% starting with the smallest strains. Each axial strain was excited at 0.2 and 0.5 Hz. Pore-water pressure was seen to rise slightly during the course of the undrained cyclic loading. In order to keep the soil structure consistent in between different strain of cyclic loading, the water drainage valve was kept closed. The pore-water pressure was allowed to reach a steady-state before the next cyclic loading begins. It was observed to take approximately 30 – 60 minutes for the pore-water pressure to stabilise after each cyclic loading strain.

Upon completion of the cyclic triaxial tests, the specimen was sheared to failure. However, to determine the failure envelope, at least three identical specimens are required. This is not advisable for undisturbed residual soil due to heterogeneity of the soil. As such, the multi-stage test (Taylor 1950, Schoenemann and Pyles 1988)

was prescribed. The difficulties in the multi-stage test lie in the premature failure of the specimen during shearing and interpretation of the shear strength parameters. Furthermore, the multi-stage test was not part of the ASTM standards and only briefly mentioned in BS1377-7 (1990). As such, researchers have formulated different criteria to interpret the test results. Ho and Fredlund (1982), Krahn et al. (1989) and Rahardjo et al. (1995) stopped shearing when the stress-strain curve becomes horizontal ($d\sigma/d\varepsilon = 0$) and removed the applied load before consolidating to the next higher effective confining pressure. While this method was able to produce effective stress parameters (c' and ϕ') close to those from single stage triaxial test, it can also lead to irreversible changes in the soil structure for weakly cemented, structured and dilatant soil (Ravi Sharma et al. 2011). In order to overcome this, Sridharan and Narasimha Rao (1972) and Nambiar et al. (1985) limited the strain to about 2 - 4% then use a hyperbolic model to extend the deviator stress-strain and pore-water pressure-strain relationships to failure. Nambiar et al. (1985) further proposed that the effective confining pressure of the subsequent stages to be twice the previous stage to reduce the effect of stress anisotropy induced during shear.

Based on the above, the specimen were sheared until the maximum stress is imminent but before the point where $d\sigma/d\varepsilon = 0$ for intermediate stages of the multi-stage test. The shearing was conducted in the strain-controlled mode and the strain rate was determined in accordance to ASTM D4767-11 (2011). After which, the hyperbolic method (Chin 1970; 1971, Chin and Vail 1973) was used to obtain the stress and pore-water pressure at failure. After shearing was halted, the deviator stress was rapidly removed and the specimen was consolidated to the next higher effective confining stress of 200 kPa. The removal of the deviator stress prevents 'creep' of the soil structure between loading stages and allow a small elastic recovery of the strain (Head, 1998).

The above procedures were then repeated again before consolidating the specimen to the final effective confining pressure of 350 kPa. After the wave propagation test, the cyclic triaxial test was conducted on the specimen at double amplitude axial

strains of 0.05, 0.1, 0.2, 0.5 and 1%. Each axial strain was applied at 0.2 and 0.5 Hz for 40 cycles. The specimen was then sheared to failure.

3.5.3.2 Unsaturated constant water content (CW) test

For the unsaturated specimen, wave propagation test was conducted when the specimen reached equilibrium at the stipulated stress-state condition in Table 3.7. The cyclic triaxial test was conducted on the specimen at double amplitude axial strains of 0.05, 0.1, 0.2, 0.5 and 1%. Each axial strain was applied at 0.2 and 0.5 Hz for 40 cycles. The specimen was then sheared to failure under constant water content condition, i.e., the air pressure valve remained open while the water pressure valve was closed during shearing. The strain rate for CW test was five and ten times higher than the corresponding consolidated drained (CD) test in Satiija and Gulhati (1979) and Rahardjo et al. (2004), respectively. Using both literature as reference, the axial strain rate from the consolidated drained (CD) test was determined from ASTM D7181-11 (2011) using the time to 90% consolidation (t_{90}) via the log time method (FIG.3.26). At five times of the calculated strain rate of CD tests, the strain rate was found to fall in the range from 0.008 – 0.0095 mm/min. By selecting a value of 0.008 mm/min for all the CW tests in this study, the value proposed by Rahardjo et al. (2004) was not exceeded and deemed to be sufficient for any pore-water pressure build up to equalize.

3.5.4 Digital Signal Processing

The sampling rate of the data acquisition program provided by UTM-14 was only sufficient for the sinusoidal response curve of the transducer to be plotted. As such, the time step would have been too coarse to perform numerical integration (trapezoidal rule) on the hysteresis loop. Transducers' response was resampled using a MATLAB script. The resampling rate was increased until it did not give rise to a significant change in the area of the hysteresis loop.

3.6 Summary

In this chapter, four series of experiments were described so that a testing framework which allows the determination of dynamic properties (small-strain stiffness and damping ratio) using wave propagation test could be progressively developed. While determination of the small-strain stiffness from wave velocities are established, damping ratio is seldom obtained from wave propagation test. As such, various techniques to obtain the damping ratio were evaluated using unsaturated reconstituted soil specimens and soil-rubber specimens so that the effects of confining pressure and degree of saturation can be studied. Testing on homogeneous specimens allow the changes in wave signal and dynamic properties to be correctly attributed to the imposed conditions and reduce the uncertainties when testing heterogeneous specimens such as unsaturated residual soil. Further experiments to verify the reliability of the selected technique were also described. Numerical modelling to validate the experiments will be described in the next chapter. As the strain range of the cyclic triaxial test was unable to go below the threshold strain, the wave propagation test was incorporated into the cyclic triaxial test so that the normalised shear stiffness and damping ratio of residual soil specimens can be determined at both saturated and unsaturated states.

CHAPTER 4: Finite Element Modelling

4.1 Outline and Purpose

In order to perform numerical modelling of wave propagation in soil, it is important to ensure that the material model would be able to reflect the soil's stiffness and material damping ratio. Most literature on wave propagation in soil does not describe the modelling methodology in detail and there is little or no verification on the accuracy of the model. As such, this chapter aims to validate that the finite element simulation and the material model employed give the correct response in terms of stiffness and damping.

To date modelling efforts on dynamic laboratory tests include modelling of resonant column tests by Karl et al. (2003, 2008) using ABAQUS, modelling of bender element tests by Arroyo et al. (2002, 2003a and b, 2006), Rio (2006) using FLAC3D, and Hardy et al. (2002) using Fourier Series Aided Finite Element Method. Karl et al. (2003, 2008) determined numerically the natural frequency of the soil specimen for various modes of vibration in the resonant column tests. The numerical study, however, did not address the issue of material damping and did not describe the type of material model used. Arroyo et al. (2002, 2003a and b, 2006) and Rio (2006) employed a mass proportional damping model in FLAC3D to model wave propagation in soil for bender element tests but did not provide any validation. The incorporation of damping in the model utilized by Hardy et al. (2002) was not described.

Carcione et al. (1988a, b), Bardet (1992), Carcione (1993) and Dwivedi et al. (2011) employed the viscoelastic model to investigate dynamic response and wave propagation in soil. While the formulation of the viscoelastic model has been well documented in the literature, they contain lengthy and complicated differential equations with parameters which prove challenging in terms of application and parameter input.

Hayner (1994), Spanos and Tsavachidis (2001) and Lee et al. (2012) employed complicated configurations of rheological elements (springs and dashpots) to simulate wave propagation in soil. However, usage of such elements meant that numerous input parameters corresponding to the rheological elements have to be obtained. Values of these input parameters are obtained by calibrating with results from specialized apparatuses and techniques (e.g. Dynamic Mechanical Analysis or DMA) via Monte Carlo simulation. The DMA records the strain response of the soil to an applied sinusoidal loading and requires specialist who are not commonly found in commercial geotechnical testing laboratory. Moreover, the calibration of the various input parameters needs considerable effort.

Chapter 4 aims to use a simplified viscoelastic model from finite element program LS-DYNA to model wave propagation in soil. A simplified model of the resonant column test was used for the calibration work. In terms of the stiffness response, the input small-strain stiffness was compared with those calculated from well-established analytical solutions which are functions of the natural frequency and dimensions. The natural frequencies for different vibration modes (torsional, flexural and longitudinal) were obtained from LS-DYNA using implicit eigenvalue analysis. Natural frequency in the flexural mode was found to deviate from the analytical solution due to the Timoshenko beam effect. In view of this, the extent of the Timoshenko beam effect on the flexural natural frequency of the specimen was investigated in the first half of Chapter 4.

In terms of the damping response, material damping ratio derived from the application of the logarithmic decrement method (ASTM D4015-07 2013) on the simulated free decay vibration was compared with the input parameters of the simplified viscoelastic model. This was followed by calibration of the viscoelastic model and the methodology for the verification tests. The logarithmic decrement method was used as it is the prescribed method of resonant column test (ASTM D4015-07 2013). The simplified viscoelastic model enables parameters derived from laboratory geotechnical tests to be used in the numerical simulations. The

study hopes to provide better understanding of the relationships between the input parameters and the simulation outcome instead of treating the material model as a “black box”.

4.2 Introduction to LS-DYNA

The finite element program LS-DYNA (LSTC, 2013) was used to model the resonant column test on a soil specimen under different excitation modes. LS-DYNA was chosen as it is able to perform highly non-linear, transient dynamic analysis using explicit and implicit time integration. The explicit approach uses the central difference method to march forward in time to solve transient dynamic problem. The implicit approach, on the other hand, is used to solve static, low strain rate problems and to extract the corresponding eigenvalue (resonant frequency) and eigenvector (mode shape). The implicit approach, however, uses much more memory compared to the explicit approach. The simulations were run on a 64 bit CPU equipped with Intel Processor of i7-3610QM and 8GB of RAM. The implicit and explicit approaches take about 15 and 60 minutes, respectively.

4.3 Calibration for stiffness response

The resonant column tests could be conducted in both the torsional and longitudinal modes at various strains to obtain the shear and Young’s modulus, respectively. In recent years, Cascante et al. (1998) introduced a minor modification to the torsional driving mechanism of the resonant column apparatus to allow for testing in the flexural mode. Similar to the longitudinal mode, the natural frequency obtained from the flexural mode can be correlated to the Young’s modulus. Due to the simplicity of the modification, a number of commercial resonant column apparatuses now have the flexural mode of excitation. However, the flexural mode of testing in the resonant column apparatus has yet to be standardised.

The Young’s modulus obtained from the flexural mode in Cascante et al. (1998) method deviates from the true Young’s modulus due to Timoshenko beam effect (Timoshenko 1937). This is especially so for short and stocky specimens. The

difference between the actual Young's modulus and those derived from Cascante et al. (1998) method is reconciled using finite element modelling in this section.

4.3.1 The Timoshenko beam effect

The flexural resonant column test is analogous to deflection of a cantilever beam. The natural frequency of cantilever beam in flexural mode can be described using the Euler-Bernoulli's method (Equation 4.1) which is derived from the equation of motion shown as Equation 4.2.

$$f = \frac{1}{2\pi} \left(\frac{k}{L^2} \right) \sqrt{\frac{EI}{\rho A}} \quad (4.1)$$

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} = 0 \quad (4.2)$$

where f is the natural frequency, $k = 3.5156$ for the fundamental mode of flexure vibration, L is the length, E is the Young's modulus, I is the area moment of inertia, ρ is the bulk density, A is the cross-sectional area, x is the distance measured along the height of the specimen and y is the horizontal displacement of the specimen.

However, Equation 4.1 is valid only for homogeneous, isotropic beam of uniform cross section whose length is greater than its cross-section's dimensions (Stokey 2009). Furthermore, the Euler-Bernoulli's method becomes complicated for a cantilever beam with a lumped mass which simulates the driving mechanism in a flexural resonant column test. As such, Cascante et al. (1998) made use of the Rayleigh's energy method to relate flexural resonant frequency to Young's modulus. However, this method only provides an approximation to the fundamental natural frequency. The true natural frequency can only be obtained if the deflection of the specimen is known (Stokey 2009). Accuracy of the Rayleigh's energy method varies with the assumed mode shape function used to describe the deflection of the specimen.

Cascante et al. (1998) modelled the resonant column test as a cantilever beam with an attached lumped mass (driving mechanism) using the Rayleigh's energy method. The solution for the flexural mode natural frequency (f_n) is presented in Equation 4.3 and was derived using the assumed mode shape function shown in Equation 4.4. The attached lump mass was assumed to have uniform cross section and density. Derivation of the equation is explained in detail in Appendix B.

$$f_n^2 = \frac{3EI}{4\pi^2 L^3 \left[\frac{33}{140} m_s + \sum_{i=1}^N m_i h(h_{0i}, h_{1i}) \right]} \quad (4.3a)$$

$$h(h_{0i}, h_{1i}) = 1 + 3 \frac{(h_{0i} + h_{1i})}{2L} + \frac{9}{16L^2} (h_{0i}^2 + 2h_{0i}h_{1i} + h_{1i}^2) \quad (4.3b)$$

where m_s is the mass of the specimen, m_i is the total mass of the i^{th} attached mass and h_0 and h_1 are the height of the bottom and top of the attached mass measured from the top of specimen.

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad (4.4)$$

where a_0 , a_1 , a_2 and a_3 are coefficients for the equation.

Kumar and Madhusudhan (2010) proposed a different mode shape function from Cascante et al. (1998) for the flexural resonant column test as shown in Equation 4.5.

$$y(x) = C_1 \sin \beta_K x + C_2 \cos \beta_K x + C_3 \sinh \beta_K x + C_4 \cosh \beta_K x \quad (4.5a)$$

$$\beta_K = \left(\frac{m_x \omega^2}{EI} \right)^{\frac{1}{4}} \quad (4.5b)$$

where C_1 , C_2 , C_3 and C_4 are coefficients for the equation and m_x is the mass of the sample per unit length.

Due to the more complex mode shape function, an iterative method is needed to solve Equation 4.2 to obtain the Young's modulus. Kumar and Madhusudhan (2010)

procedures are summarised in Appendix C. Both Cascante et al. (1998), and Kumar and Madhusudhan (2010) methods did not account for the Timoshenko beam effect (combination of rotary motion and shearing forces) since Equation 4.2 was employed in both methods to solve for the Young's modulus.

The Timoshenko beam effect is a complication that arises in flexural resonant column test when specimens are stocky. Although ASTM D4015-07 (2013) limits the length-to-diameter (L/D) ratio of the specimen to be between 2 and 7, and between 2 and 3 when the specimen is subjected to “an ambient axial stress greater than the ambient lateral stress”, resonant column tests on specimens with L/D ratio less than 2 have also been reported in the literature (Karl et al. 2003). Such specimens, described as stocky specimens, are subjected to shear and rotary inertia otherwise known as the Timoshenko beam effect (van Rensburg and van der Merwe 2006) during flexural vibration. Effects of shear and rotary inertia occur when curvature of the beam is large relative to its thickness and during higher modes of vibration where the plane section, which is initially normal to the beam axis, ceases to remain normal and rotates out of planes causing points on the beam to approach one another (Stokey 2009). To account for the shear and rotary inertia, Timoshenko (1937) extended Equation 4.2 to give the differential equation of motion as:

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} - \left(I_p + \frac{EI\rho}{\kappa G} \right) \frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{I_p^2}{\kappa G} \frac{\partial^4 y}{\partial t^4} = 0 \quad (4.6)$$

To date, there is no simple solution of Timoshenko beam theory for a cantilever beam (van Rensburg and van der Merwe 2006). Some researchers (Tefft 1960, Leissa and So 1995) published tables of correction factors consisting of dependent variables (L/D ratio and Poisson's ratio) to correct the frequencies obtained from the Euler-Bernoulli's equation. Sutherland and Goodman (1951) showed the ratio of frequency (F) corrected for Timoshenko beam effect to that derived from the Euler-Bernoulli's equation in FIG. 4.1. FIG. 4.1 plotted with dimensionless parameter Λ (Equation 4.7) is only valid for uniform cantilever beam of $E/\kappa G = 3.20$ where κ is the Timoshenko shear coefficient. Sutherland and Goodman (1951) proposed κ to

be 5/6 and 9/10 for rectangular and circular cross sections, respectively. The dimensionless parameter Λ is defined as the ratio of radius of gyration of cross section and length of beam. For a circular cross section beam, Λ is given by:

$$\Lambda = \frac{D}{4L} \quad (4.7)$$

FIG. 4.1: Ratio of frequency corrected for Timoshenko beam effect to those from Euler-Bernoulli solution (F) with dimensionless parameter (Λ) for a uniform cantilever beam of $E/\kappa G = 3.20$.

4.3.2 Verification of the finite element model

Since the curve reproduced from Sutherland and Goodman (1951) shown in FIG. 4.1 did not cover the full range of Λ indicated in ASTM D4015-07 (2013), finite element simulations were conducted using LS-DYNA (LSTC 2013) to extend the range. These simulations also served to ascertain the ability of LS-DYNA to model the Timoshenko beam effect for a cylindrical specimen.

Three-dimensional soil specimens of 50 mm diameter and various heights were modelled to produce the points shown in FIG. 4.1. The model was fixed from translational and rotational motion at the bottom to simulate the degree of freedom

in a resonant column. Finite element models in this study consist of 8-node solid elements (ELFORM = -2) with full integrations i.e. eight integration points. This element formulation was chosen to eliminate possible hourglassing which is zero energy deformation arising from single point integration (LSTC 2013). For each specimen height, the finite element mesh was gradually refined until the changes in the flexural resonant frequency become insignificant. Similar approach was applied to subsequent finite element models unless otherwise stated.

In this study, the flexural resonant frequencies were obtained using an implicit eigenvalue analysis. The elastic material model (MAT_001) with input parameters density (ρ), Young's modulus (E) and Poisson's ratio (ν) was used. In order to have typical input parameters, shear modulus of sand was estimated from the equation by Hardin and Black (1968) given as Equation 4.8.

$$G_{\max} = \frac{3230(2.97 - e)^2}{1 + e} \sigma_3'^{0.5} \quad (4.8)$$

where e is the void ratio, σ_3' is the effective confining pressure in kPa and $G_{\max}$ is the small-strain shear modulus in kPa.

The $G_{\max}$ was obtained for effective confining pressures σ_3' of 50, 200 and 1000 kPa using Equation 4.8. The corresponding void ratios were 0.6, 0.55 and 0.5. Densities of the specimen were 1700, 2000 and 2200 kg/m³. The Poisson's ratio to satisfy the condition of $E/\kappa G = 3.20$ in FIG. 4.1 was 0.44. The $G_{\max}$ was converted to E and input into the material model using Equation 4.9.

$$E = 2G_{\max}(\nu + 1) \quad (4.9)$$

The frequency ratio F was shown to coincide with the curve from Sutherland and Goodman (1951) in FIG. 4.1. FIG. 4.1 shows that when the Timoshenko beam effect was considered, reduction of Euler-Bernoulli's frequency can be as large as 10% for $L/D = 2$. Despite variation in Young's modulus, FIG. 4.1 shows that the frequency ratio F remains unchanged when ν is constant. This shows that F is

independent of E and ρ , provided that ν is constant. Variation of F with ν was investigated in Tefft (1960), and Leissa and So (1995). This implies that F is only affected by the change in ν and not E or G . As such, the discrepancy in E due to the Timoshenko effect will be investigated by varying L/D and ν .

4.3.3 Finite element simulation program

Deviation of the Young's modulus from the original input into LS-DYNA was investigated using the equation of Cascante et al. (1998). This is due to its ease in application. The discrepancy was studied in terms of Young's modulus instead of the flexural natural frequency as it is deemed more theoretically appropriate to correct the Young's modulus which has deviated due to the application of Equation 4.3. The correction factor χ_m is calculated using the equation below:

$$\text{Correction factor, } \chi_m = \frac{E \text{ from Cascante et al. (1998) (Eq. 4.3)}}{E \text{ input into LS-DYNA}} \times 100\% \quad (4.10)$$

Specimens with L/D ratios of 1, 2 and 3 were modelled with properties of aluminium ($\rho = 2690 \text{ kg/m}^3$, $E = 68.93 \text{ GPa}$) since the material has been widely used as calibration bars in the resonant column apparatus. The difference in Young's modulus will not affect the results obtained as mentioned in Section 4.3.2. Poisson's ratio (ν) was varied over the typical range for soil and rock i.e., from $\nu = 0.1$ to 0.49 to study the effect of Poisson's ratio on the flexural natural frequency.

Specimens of diameters 38, 50 and 70 mm were modelled for each L/D ratio. The peripherals were assumed as a lumped mass of 10 mm height. The lumped mass was modelled as a rigid body (MAT_020) with input parameters E , ρ and ν . The Young's modulus E and Poisson's ratio ν of the lumped mass were shown to not affect the outcome of the finite element simulation. Changes in the weight of the lumped mass were reflected using the density ρ . Since the weight of the attached peripherals varies with the make of the resonant column apparatus, the density of the lumped mass was varied such that the weight of the attached peripherals covers

the expected range in resonant column apparatuses. The weight of the lumped mass was varied from 100 to 8000g. The bottom of the specimen was fixed from rotational and translation motion. The finite element model is shown in FIG. 4.2.

FIG. 4.2: Finite element model of the resonant column test

Since the flexural natural frequency is also a function of the centroid of the lumped mass from the top of the specimen, two additional series of LS-DYNA runs were included to study this effect. The first series was performed on a specimen of 50 mm diameter and L/D of 2. Using a fixed lumped mass of 2 kg, its height was varied from 5 to 70 mm by changing its density. The second series was performed using a constant density lumped mass equal to that of steel (7850 kg/m^3) and varying its height from 20 to 70 mm.

4.3.4 Correction factor for the Timoshenko beam effect

Variations of the correction factor χ_m with weight of lumped mass for various ν and L/D ratios of 1, 2 and 3 are shown in FIG. 4.3. For each ν and L/D ratio, the

variation of χ_m with weight of lumped mass is less than 5% for the range of weight of lumped mass investigated, i.e., from 0.1 to 8 kg. The variation of χ_m with weight of lumped mass is less than 1% when the weight of the lumped mass exceeds 0.4 kg. The minimum and maximum values of χ_m for each L/D ratio are at $v = 0.1$ and 0.49 , respectively.

As flexural natural frequency is also a function of the height of the lumped mass, it is expected that χ_m will also change accordingly. FIG. 4.4 shows the effect of increasing the height of the lumped mass on χ_m while maintaining the density of the lumped mass to be constant. The effect of increasing the height of the lumped mass from 20 to 70 mm on χ_m is the same effect as increasing the weight of the lumped mass from 0.308 to 1.079 kg on χ_m . The ranges of χ_m in FIG. 4.4 compared to those in FIG. 4.3 are summarized in Table 4.1. Except for $L/D = 1$, Table 4.1 shows that the Young's modulus obtained using Equation 4.3 deviates by similar extent from the actual Young's modulus when the height of the lumped mass was varied from 20 to 70 mm. If the flexural resonant column tests are performed for $L/D = 2$ with a 2 kg lumped mass, χ_m ranges from 86.8% to 92.9% as shown in FIG. 4.5. Data used to plot FIGs. 4.3, 4.4 and 4.5 are tabulated in Appendix D.

FIG. 4.3: Variation of χ_m with lumped mass and v

FIG. 4.4: Variation of χ_m with height of steel attachment

FIG. 4.5: Variation of χ_m with 2 kg lumped mass of different heights (L/D = 2)

Table 4.1: Range of χ_m from FIGs. 4.3 and 4.4

	L/D = 1		L/D = 2		L/D = 3	
	FIG. 4.3	FIG. 4.4	FIG. 4.3	FIG. 4.4	FIG. 4.3	FIG. 4.4
Minimum (%)	64.4	70.1	86.7	86.9	93.1	92.3
Maximum (%)	66.9	81.8	90.6	92.4	96.8	97.0
Mean (%)	65.7	76.0	88.7	89.7	95.0	94.7

4.3.5 Validation of correction factor χ_m

Clayton et al. (2005), Priest et al. (2005) and Rees (2009) tested reconstituted Methyl Hydrate soil using the resonant column apparatus in both flexural and torsional modes. A number of specimens were tested. For brevity, only tests on specimen, H1, as shown in Table 4.2 are discussed. Specimen H1 was subjected to different values of confining pressures and the resonant frequencies for the torsional and flexural modes were obtained.

Table 4.2: Properties of Methyl Hydrate specimen H1

Specimen H1	Reported Values			Calculated Values		
	Confining Pressure	V_{lf}	V_s	G	E	ν
	(kPa)	(m/s)	(m/s)	(Pa)	(Pa)	
Diameter = 70 mm	250	599.4	411.5	2.70E+08	5.72E+08	0.06
Height = 140mm	500	658.0	471.4	3.54E+08	6.90E+08	-0.03
	750	716.6	503.1	4.03E+08	8.18E+08	0.02
Dry Density = 1589.4 kg/m ³	1000	735.9	520.7	4.32E+08	8.63E+08	-0.00
	1250	768.1	541.9	4.68E+08	9.40E+08	0.01
Density = 1593.3 kg/m ³	1500	787.2	559.5	4.99E+08	9.87E+08	-0.01
	1750	813.0	573.6	5.24E+08	1.05E+09	0.01
w = 0.244%	2000	832.1	591.2	5.57E+08	1.10E+09	-0.01

However, only the shear (V_s) and longitudinal (V_{lf}) wave velocities were reported as shown in Table 4.2. While the details pertaining to the soil specimens were reported, details of the resonant column test such as the weight and dimensions of the attached peripherals (top platen and drive plate) are not available. Relationship

between V_s and V_{lf} are shown in Equations 2.1b and 4.11, respectively. The Poisson's ratio (ν) can be calculated using Equation 4.9.

$$V_{lf} = \sqrt{\frac{E_{flex}}{\rho}} \quad (4.11)$$

Using Equation 4.9, the calculated ν values for the various confining pressures are shown in Table 4.2. Some of the calculated ν values are negative and are unreasonable. The highest calculated ν value is 0.06. As such, ν for the numerical modelling was assumed to be 0.1. Input values for the Young's modulus were then calculated using $\nu = 0.1$, G from Table 4.2 and Equation 4.9.

To simplify the numerical model, the attached peripherals were assumed to have the same diameter as the specimen. The torsional resonant frequencies from the tests were obtained using Equation 4.12 which is a function of the mass polar moment of the drive plate or attached peripherals (Equation 4.13):

$$\frac{J}{J_o} = \frac{2\pi fL}{V_s} \tan \frac{2\pi fL}{V_s} \quad (4.12)$$

where J_o is the mass polar moment of inertia of the drive plate and J is the mass polar moment of inertia for the specimen given as follows:

$$J = \frac{1}{2} m_s r^2 \quad (4.13)$$

where m_s is the mass of the specimen and r is the corresponding radius.

FIG. 4.6 shows the relationship of J_o obtained from the calibration using aluminium calibration bars with the measured resonant frequency as reported by Priest et al. (2005). The relationship of J_o with resonant frequency f can be best-fitted with the following quadratic polynomial function:

$$J_o = 8.347 \times 10^{-9} f^2 - 1.716 \times 10^{-7} f + 3.398 \times 10^{-3} \quad (4.14)$$

FIG. 4.6: Variation of J_0 with measured resonant frequency (modified from Priest et al. 2005)

Using Equation 4.14, Equation 4.12 can be made a function of the torsional natural frequency, f , and f can be determined. The value of J for the specimen calculated using Equation 4.13 was $5.29 \times 10^{-4} \text{ kgm}^2$. The torsional natural frequency f and the corresponding J_0 are tabulated in Table 4.3. Values of J_0 are observed to increase by 6.26% from an initial value of $3.62 \times 10^{-3} \text{ kgm}^2$. If equivalent diameter of the attached peripherals was similar to that of the specimen's diameter (70 mm), the minimum mass of the attached peripherals calculated using Equation 4.13 is 5.91 kg (Table 4.3). On the other hand, if the mass of the attached peripherals was 1.5 kg, the minimum equivalent diameter of the attached peripherals is 0.139 m. For simplicity, the former assumption was used in the numerical model. The density of the attached peripherals was changed to reflect the weight of equivalent attached peripherals. The numerical model was then used to model the resonant column tests at other confining pressures. Table 4.4 compares the numerical and experimental resonant frequencies. The numerical torsional resonant frequencies agree well with the experimental resonant frequencies, with a maximum difference of less than 3%.

Table 4.3: Back-calculation of torsional natural frequency and properties of attached peripherals

Confining Pressure (kPa)	f_{tor} (Hz)	J_0 (kgm ²)	Attached Peripherals	
			Mass (D = 0.07 m) (kg)	Diameter (Mass =1.5 kg) (m)
250	173.53	3.62E-03	5.91	0.139
500	197.02	3.69E-03	6.02	0.140
750	209.22	3.73E-03	6.09	0.141
1000	215.92	3.75E-03	6.12	0.141
1250	223.9	3.78E-03	6.17	0.142
1500	230.49	3.80E-03	6.21	0.142
1750	235.72	3.82E-03	6.24	0.143
2000	242.21	3.85E-03	6.28	0.143

Using the same numerical model, the flexural resonant frequencies were obtained for the H1 specimen test series. From the numerical flexural resonant frequency, Young's modulus E was calculated using Cascante et al. (1998) and, Kumar and Madhusudhan (2010) methods and presented in Table 4.4. Table 4.4 shows that the differences between the input E and E calculated using Cascante et al. (1998) and, Kumar and Madhusudhan (2010) methods are 12% and 9.9%, respectively. The E calculated using Cascante et al. (1998) method can be corrected as follows:

$$\text{Corrected } E = \frac{1}{\chi_m} E_c \quad (4.15)$$

where $\chi_m = 0.887$ for $L/D = 2$ (Table 4.1) and E_c is the E calculated using Cascante et al. (1998) method. The last column of Table 4.4 shows that the correction factor χ_m successfully reduced the difference between the derived and input E to 0.8%.

This section showed that the finite element program LS-DYNA was able to reproduce the Timoshenko beam effect which affects short and stocky specimens in flexural vibration and gave the correct stiffness response for the input value. The discrepancy lies in the methods proposed by Cascante et al. (1998) and Kumar and Madhusudhan (2010) which under-estimated the Young's modulus for a given flexural resonant frequency. In view of this, a correction factor was introduced to

correct the Young's modulus derived from the method proposed by Cascante et al. (1998).

Table 4.4: Comparison of Young's modulus calculated from different methods

Confining Pressure	Input Values		Torsional Mode				Flexural Mode			Difference		
	Input E		LS-		Difference	LS-	E _{calculated}			$\frac{(a)-(1)}{(a)}$	$\frac{(a)-(2)}{(a)}$	$\frac{(a)-(3)}{(a)}$
	G	(a)	Expt f _n	DYNA		DYNA	(1)	(2)	(3)			
			f _n	f _n		f _n						
	(kPa)	(Pa)	(Pa)	(Hz)	(Hz)	(%)	(Hz)	(Pa)	(Pa)	(Pa)	(%)	(%)
250	2.70E+08	5.93E+08	173.53	173.44	0.0505	38.66	5.22E+08	5.34E+08	5.89E+08	12	9.9	0.8
500	3.54E+08	7.79E+08	197.02	198.69	0.8483	44.29	6.86E+08	7.01E+08	7.73E+08	12	9.9	0.8
750	4.03E+08	8.87E+08	209.22	212.06	1.359	47.27	7.81E+08	7.99E+08	8.80E+08	12	9.9	0.8
1000	4.32E+08	9.50E+08	215.92	219.49	1.6522	48.93	8.37E+08	8.56E+08	9.43E+08	12	9.9	0.8
1250	4.68E+08	1.03E+09	223.9	228.4	2.0125	50.91	9.06E+08	9.27E+08	1.02E+09	12	9.9	0.8
1500	4.99E+08	1.10E+09	230.49	235.83	2.3195	52.57	9.66E+08	9.88E+08	1.09E+09	12	9.9	0.8
1750	5.24E+08	1.15E+09	235.72	241.77	2.5693	53.89	1.02E+09	1.04E+09	1.14E+09	12	9.9	0.8
2000	5.57E+08	1.23E+09	242.21	249.2	2.8865	55.55	1.08E+09	1.10E+09	1.22E+09	12	9.9	0.8

Note:

- (1) Young's modulus back-calculated from Cascante et al. (1998)
- (2) Young's modulus back-calculated from Kumar and Madhusudhan (2010)
- (3) Young's modulus after application of correction factor χ_m

4.4 Calibration for damping response

In this section, input parameters of the simplified viscoelastic material model were calibrated with the damping ratio obtained from the simulated free decay vibration. The damping ratio was obtained using the LDM as prescribed in ASTM D4015-07 (2013). Verification was obtained by simulating wave propagation in a semi-infinite medium.

4.4.1 The Viscoelastic material model

The difference between elastic and viscoelastic materials centres on their behaviour when subjected to vibratory motion. Assuming that a sinusoidal deformation is applied to a specimen, the corresponding strain (γ) can be described using Equation 4.16a. If the material is elastic, the strain and stress will be in phase. However, a viscoelastic material will experience a phase lag in its resulting stress (τ) as described in Equation 4.16b.

$$\gamma = \gamma_o e^{i\omega t} \quad (4.16a)$$

$$\tau = \tau_o e^{i(\omega t + \phi)} \quad (4.16b)$$

where γ_o is the amplitude of the sinusoidal shear strain, τ_o is the amplitude of the sinusoidal shear stress, ϕ is the phase difference between them, ω is the circular frequency and t is the time.

The corresponding shear stiffness can be expressed in terms of the dynamic complex shear modulus as shown in Equation 4.17:

$$G^* = \frac{\tau_o}{\gamma_o} (\cos \phi + i \sin \phi) = G' + iG'' \quad (4.17)$$

where G^* is the complex shear modulus, G' is the elastic (or storage) modulus which measures the ability of the material to store energy and G'' is the loss (or viscous) modulus which measures the ability of the material to dissipate energy and i is the imaginary number ($=\sqrt{-1}$).

Ratio of the loss modulus and elastic modulus gives the loss factor η which is related to the damping ratio ξ as given below:

$$\eta = \frac{G''}{G'} = \tan \varphi = 2\xi \quad (4.18)$$

Viscoelastic material exhibits volume change due to loading (elasticity) and time (viscosity) i.e., deformation is time dependent. Phenomenon of viscoelastic materials includes creep (or retardation) and relaxation, which is described mathematically with the help of constitutive models. These models are described using different combinations of springs and dashpots. Examples of these models include the Kelvin-Voigt, Maxwell and Standard Linear Solid models which will be illustrated in greater details below.

The Kelvin-Voigt model shown in FIG. 4.7a is used to describe the creep effect. The dashpot which is connected in parallel with the spring prevents the spring from reacting immediately to the applied force (Tschoegl 1989). Simply explained, the dashpot provides the delay in deformation upon application of stress (Lakes 2009). The deformation in the Kelvin-Voigt model is represented by (Vyalov 1986):

$$\gamma_t = \frac{\tau_t}{G} (1 - e^{-t/T_c}) \quad (4.19)$$

where τ_t and γ_t are the shear stress and strain at the time t , G is the shear modulus and T_c is the retardation time.

The Maxwell model shown in FIG. 4.7b is used to describe the relaxation effect. Relaxation is characterized by the gradual decrease in stress when the material is subjected to deformation (Lakes 2009). In this case, the dashpot can be seen to be gradually relaxed upon the application of a sudden deformation. The stress invoked in the spring will therefore decrease slowly, hence the term relaxation. The change in stress with time in the Maxwell model is represented by (Vyalov 1986):

$$\tau_t = G\gamma_0 \left(e^{-t/T_r} \right) \quad (4.20)$$

where γ_0 is the initial shear strain and T_r is the relaxation time.

Expression for retardation and relaxation time is shown in Equation 4.21. While both T_r and T_c is represented by the same equation, they differ in physical meaning and value (Lakes 2009) through the viscosity coefficient $\hat{\eta}$:

$$T = \frac{\hat{\eta}}{G} \quad (4.21)$$

where T can be either retardation or relaxation time and $\hat{\eta}$ is the viscosity coefficient of the dashpot.

Compared to the Kelvin-Voigt model, the Maxwell model is more suited to model damping in soil (Stucky and Lord 2001). In most cases, the Biot hysteretic model (FIG. 4.7c) is employed to handle more complex dynamic soil behaviour (Muscolino et al. 2005). The Biot hysteretic model consists of a spring element in parallel with an infinite number of Maxwell models in parallel connection. The model can be expressed mathematically using Equation 4.22.

$$G(t) = G_\infty + \sum_{i=1}^N G_i e^{-t/T_{ri}} \quad (4.22)$$

where $G(t)$ is the shear modulus at time t , G_∞ is the long term or infinite shear modulus, G_i and T_{ri} are the shear modulus and the relaxation time of the i^{th} Maxwell model placed in parallel connection.

The second term of Equation 4.22 is known as the Prony series (Tschoegl 1989, Makris and Zhang 2000) and shares a similar expression as the Maxwell model shown by Equation 4.20. The Prony series dictates the decrease in shear modulus with time. Upon total relaxation, the Biot hysteretic model will be left with the long term shear modulus (G_∞). However, the large number of rheological elements present will mean that considerable effort have to be undertaken to calibrate the

model. This can be done experimentally using a Dynamic Mechanical Analyser (DMA) or by calibrating with known data.

The DMA is used to characterize viscoelastic behaviour in material by recording the response of the material subjected to sinusoidal stress so that the complex modulus can be determined (Menard, 2008). However, the apparatus is not readily available in most geotechnical testing laboratories. Calibration of the parameters with Monte Carlo simulation and other optimization algorithm will also take significant effort and time (Hayner 1994, Spanos and Tsavachidis 2001 and Lee et al. 2012). Hence, there is a need for a simplified model where parameters can be obtained from accessible laboratory tests.

Biesen (2006), Hu et al. (2007) and Smith and Duris (2009) used a model simplified from the Biot hysteretic model as shown in FIG. 4.7d otherwise known as the Standard Linear Solid or Zener model. This model is available in the finite element program LS-DYNA. The time dependent shear modulus (LSTC 2013) is given by:

$$G(t) = G_{\infty} + (G_0 - G_{\infty})e^{-\beta t} \quad (4.23)$$

where G_0 is the instantaneous or short term shear modulus and β is the decay constant corresponding to the inverse of the relaxation time.

The input parameters for the viscoelastic material model are bulk modulus (K), bulk density (ρ), G_0 , G_{∞} and β . Small-strain bulk and shear moduli ($K_{\max}$ and $G_{\max}$) can be obtained from advanced geotechnical laboratory tests like the bender element, cyclic triaxial and resonant column tests. However, neither the combination of G_0 and G_{∞} with $G_{\max}$ nor the correlation of β with damping ratio is mentioned in the literature (Biesen 2006, Hu et al. 2007 and Smith and Duris 2009). The present study therefore aims to fill this knowledge gap by investigating the relationship between the above-mentioned parameters and provides correlations that can be used for the viscoelastic model.

FIG. 4.7: Graphical Representation of (a) Kelvin – Voigt model, (b) Maxwell model, (c) Biot Hysteretic model, and (d) Standard linear solid or Zener model

4.4.2 Input Parameters for Viscoelastic Model

Implicit eigenvalue analysis was used to obtain the fundamental frequencies for the various vibration modes (flexural, torsional and longitudinal). The fundamental frequency for each vibration mode was then used to derive the decay constant (β) for use in the explicit time integration. Explicit time integration was used to obtain the amplitude decay in the time domain upon reaching steady-state so that material damping ratio can be derived from the logarithmic decrement method.

Before damping can be studied using the Zener model, the input parameters (K , ρ , G_0 , G_∞ and β) need to be correlated to the actual parameters ($K_{\max}$, ρ , $G_{\max}$ and ξ). Due to the mismatch in the number of parameters available, $G_{\max}$ is assumed to be G_0 and $G_\infty = 0$, thereby reducing the Zener model to a Maxwell model. From which, the correlation between parameters can be obtained by reconciling the constitutive model to resemble Equation 4.17:

In the Maxwell model, the stress in both the spring and dashpot is similar. Total deformation is therefore the summation of strains which is given as:

$$\frac{d\gamma}{dt} = \frac{1}{G} \frac{d\tau}{dt} + \frac{\tau}{\hat{\eta}} \quad (4.24)$$

Substituting Equations 4.16a and 4.16b into Equation 4.24,

$$(i\omega)\gamma_0 e^{i(\omega t)} = \frac{1}{G} (i\omega)\tau_0 e^{i(\omega t + \phi)} + \frac{1}{\hat{\eta}} \tau_0 e^{i(\omega t + \phi)} \quad (4.24)$$

Dividing Equation 4.24 by $\gamma_0 e^{i(\omega t)}$ gives

$$i\omega = \frac{i\omega}{G} G^* + \frac{1}{\hat{\eta}} G^* = G^* \left[\frac{i\omega\hat{\eta} + G}{G\hat{\eta}} \right] \quad (4.25)$$

$$G^* = \left[\frac{i\omega G\hat{\eta}}{i\omega\hat{\eta} + G} \right]$$

Multiplying $\left[\frac{i\omega\hat{\eta} - G}{i\omega\hat{\eta} + G} \right]$ on the right hand side of Equation 4.25 gives

$$G^* = G' + iG'' = \frac{\omega^2 G\hat{\eta}^2}{\omega^2 \hat{\eta}^2 + G^2} + i \frac{\omega G^2 \hat{\eta}}{\omega^2 \hat{\eta}^2 + G^2} \quad (4.26)$$

By substituting Equation 4.26 into Equation 4.18 and simplifying using Equation 4.21, the correlation between β and ξ was derived and is shown in Equation 4.27:

$$\beta = \frac{G}{\hat{\eta}} = 2\xi\omega \quad (4.27)$$

The Zener model will be investigated subsequently by allowing G_∞ to be a fixed proportion of $G_{\max}$ while keeping the other parameters (G_0 , K , ρ and β) constant. Its effect on the damping ratio was monitored so as to develop proper correlations.

4.4.3 Finite element simulation program

A shorter cylindrical specimen (30 mm height $\times$ 50 mm diameter) consisting of 14400 solid elements was modelled using the viscoelastic material model (MAT_006) for flexural, torsional and longitudinal vibration. The base of the specimen was fixed from translational and rotational movements. A short specimen was modelled to increase the fundamental frequency for the various vibration modes mentioned thereby reducing the time required for explicit time integration. Parameters for the numerical simulations are summarized in Table 4.5. The flexural, torsional and longitudinal fundamental frequencies were first obtained using implicit time integration. The model was then excited using ten sinusoidal pulses at the respective fundamental frequency of each vibration mode such that steady-state was reached. When excitation ceases, the amplitude decay was monitored at three different nodes (5608, 5599 and 5593) on the specimen (FIG. 4.8). Displacements along the y-axis, xy-plane and z-axis were monitored for the flexural, torsional and longitudinal modes of vibration, respectively. Decay constant β corresponding to damping ratios of 2, 3, 5, 7, 10, 15 and 20% were input into the viscoelastic model using Equation 4.27. The Maxwell material model was employed as described in the previous section. The damping ratio ξ was computed from the decaying amplitudes using the logarithmic decrement method (LDM) described in ASTM 4015-07 (2013) using Equation 2.8.

Table 4.5: Soil parameters used for simulation

Density, ρ	1783 kg/m ³
Bulk Modulus, K	80.88 MPa
Short-time Shear Modulus, G_0	39.50 MPa
Infinite Shear Modulus, G_∞	0 MPa

FIG. 4.8: Finite element model with nodes examined for amplitude decay

The effects of input parameters (G_∞ and G_0) on the model response are studied below. Long term shear modulus G_∞ was expressed as a shear modulus ratio, GR, of the instantaneous shear modulus G_0 (i.e. $GR = G_\infty/G_0$) and used for further analysis. This serves to expand the analysis from the Maxwell model to the Zener model. Using GR of 0, 0.2 and 0.4, G_∞ was input into the viscoelastic model. Two cases were considered in order to study the effects on the amplitude decay and stiffness by changing GR. The first case involves changing the value of G_0 such that the summation of G_0 and G_∞ equals to $G_{\max}$. The second case involves keeping G_0 constant and changing G_∞ according to the GR i.e. $GR = G_\infty/G_{\max}$. In order to ascertain that the results obtained using the viscoelastic material model was identical to the elastic material model (MAT_001) mentioned in Section 4.3, simulations for the second case were repeated using MAT_001. The K and $G_{\max}$ were converted to E and ν using Equations 4.11 and 4.28.

$$K = \frac{E}{3(1-2\nu)} \quad (4.28)$$

The simulations showed that the natural frequency is only a function of the stiffness parameters, G_0 and K , and is independent of the decay constant (β) and G_∞ . The

long term shear modulus (G_∞) only changes the damping ratio of the material model. Relationship of G_∞ with ξ is elaborated in greater details later. The simulations imply that G_0 should be $G_{\max}$. The natural frequencies in the various modes were also shown to be the same regardless of the material model as long as the stiffness parameters were properly correlated using Equations 4.11 and 4.28.

4.4.4 Calibration of viscoelastic model using simulated resonant column test

Material damping ratio is obtained from the amplitude decay of the three nodes seen in FIG. 4.8. A typical nodal displacement and logarithmic decrement plot is shown in FIG. 4.9. A comparison of the output and input damping ratios for the three modes of vibration is shown in FIG. 4.10. Both the flexural and longitudinal modes gave lower output damping ratios compared to the input damping ratios. The torsional mode was shown to best reproduce the input damping ratio. This could be due to the inadequacy of the Prony series in estimating the relationship between G' and G'' . Furthermore, the lack of a relaxation time parameter relating to the bulk modulus (K) may also be a contributing factor. In order to correct for such a limitation, the simulation was re-ran at 1.1 and 1.2 times of the β value derived from Equation 4.27 and the results are shown in FIG. 4.11. All results shown in FIG. 4.11 were for $GR = 0$. The increases in β were shown to bring the output damping ratio closer to input damping ratio for the flexural and longitudinal modes. The output damping ratio in the torsional mode was shown to be minimally affected by the increased decay constant β .

FIG. 4.9: (a) Time domain signal when input damping ratio is 10% and (b) the corresponding logarithmic decrement plot

FIG. 4.10: (a) Comparison of output and input damping ratios for flexure, torsional and longitudinal modes of resonant column test

FIG. 4.11: Damping ratio obtained from LDM when excited in the (a) flexural (b) torsional and (c) longitudinal mode

Equation 4.27 shows that the damping ratio is a function of β/ω . Results from FIG. 4.11 were henceforth re-plotted together to give FIG. 4.12. FIG. 4.12 shows that the damping ratio is dependent on β/ω and independent of the vibrational mode. Results show a scatter of $\pm 20\%$ about the best-fit line (Equation 4.29) thereby

suggesting that the following equation can be used to relate the damping ratios to β/ω regardless of the vibration mode.

$$\xi = 0.4249 \frac{\beta}{\omega} \text{ for GR} = 0 \quad (4.29)$$

FIG. 4.12: Variation of damping ratio with β/ω for $\text{GR} = 0$

4.4.5 Wave Propagation in a semi-infinite medium

Equation 4.29 is only applicable to the Maxwell model. To investigate the applicability of the viscoelastic model for modelling wave propagation in a semi-infinite medium, the analysis was extended to the Zener model. As mentioned previously, G_0 is equated to $G_{\max}$ in order for the material model to display the correct stiffness. As such, G_{∞} is expressed as a ratio of G_0 , i.e., GR as stated previously. The value of β was taken to be 1.2 times of that derived from Equation 4.27.

It is observed that increasing G_{∞} while keeping G_0 constant, i.e., increasing GR leads to a decrease in the derived damping ratio as shown in FIG. 4.13. The damping ratio is obtained using 1.2β . This indirectly implies that the response of the viscoelastic model is dependent on GR. Therefore the simpler Maxwell model

where $GR = 0$ is sufficient to account for the material properties (stiffness and damping) associated with wave propagation.

FIG. 4.13: Variation of damping ratio with β/ω at different GR

The simplified viscoelastic model ($GR = 0$) was used to simulate wave propagation in a semi-infinite medium. A two-dimensional finite element model, 300×300 mm, (FIG. 4.14) was created with 5625 2D axisymmetric shell elements. The node at the top was excited using three sinusoidal pulses at 5 kHz. The axis of symmetry boundary was restrained against motion in the x-direction. The bottom and right boundaries of the finite element model were assigned non-reflecting boundary condition. Details of non-reflecting boundary condition in LS-DYNA are given in Aung and Leong (2010).

FIG. 4.14: Model used to simulate vibration in the free field

The finite element model was first modelled using an elastic material model (MAT_ELASTIC). The displacements of the nodes at the top of the model at 20, 60, 100, 140, 180, 220 and 260 mm from the excitation node were monitored for the first S-wave arrival and curve fitted using Equation 2.2.

Since the elastic material model does not account for material damping, there will be no attenuation coefficient (α) and hence the geometric damping factor (χ) could be determined. The value of χ was determined to be 0.904. After which, the elastic material model was replaced with the simplified viscoelastic model. A β decay constant corresponding to 5 and 10% material damping ratio was calculated using Equation 4.29. Input frequency for Equation 4.29 should correspond to the frequency of the propagating wave. Density of the material model was obtained from Table 4.5. Material model's stiffness (K , G_∞ and G_0) from Table 4.5 was increased by 100 times in order to increase the dominant frequency of the system thereby reducing the computation time. Numerical results were curve-fitted using Equation 2.2 and χ derived from the results of the elastic material model to obtain the attenuation coefficient (α). The material damping ratio can then be obtained using the relationship shown in Equation 2.3.

FIG. 4.15: Amplitude decrease with distance from excitation source

The back-calculated damping ratio for 5 and 10% damping ratios were 4.81 and 10.02%, respectively. The results show that the viscoelastic model with $GR = 0$ and β as determined using Equation 4.29 is able to model wave propagation in a semi-infinite medium.

4.5 Simulations for the Verification of the Hilbert Transform Method

The main objective of this simulation is to determine if the input damping ratio can be recovered from the wave signal using the HTM for S-wave. Wave propagation in a cylindrical soil specimen was modelled using a 3D cylindrical FE model with 35000 elements (FIG. 4.16). An explicit time integration mode was used. A row of five nodes, corresponding to 5.3 mm, situated in the middle of the cross-section and at the top surface of the numerical model were excited transversely. The responses at a node in the middle of the cross-section and, 4 mm from the bottom end were recorded. The bottom end of the model was fixed from translational and rotational movement. Excitation frequencies of 3, 5 and 10 kHz were used. Soil properties were referenced from the results of the wave propagation test of a fully saturated specimen RS-US_400H. The density, small-strain bulk and shear modulus are 2096 kg/m³, 6274 MPa and 168 MPa, respectively. Using the viscoelastic model with GR

$= 0$, damping ratios of 0.03 and 0.07 were assigned to the model for each of the excitation frequency using Equation 4.29. Since the natural frequency in the S-wave excitation mode is to be damped, the decay constant (β) from Equation 4.29 was calculated using the natural frequency.

The model was also subjected to an implicit eigenvalue analysis whereby the flexural and torsional natural frequencies were identified at 0.983 and 1.413 kHz, respectively. The receiver signals from the explicit time integration were then filtered to retain peaks around frequency range of the flexural and torsional natural frequencies. The filtered signal was then subjected to the HTM as described in Section 3.3.3.

FIG. 4.16: (a) Top and (b) front view of finite element model

4.6 Summary

While some researchers have performed finite element modelling of resonant column tests and wave propagation in soil, most do not describe the modelling in sufficient details to be replicated. Furthermore, there was little or no verification of the accuracy of the model. This chapter described the use of a simplified viscoelastic model with easily obtainable parameters to simulate propagation of wave in soil medium taking material damping into consideration. The viscoelastic

material model was first verified by simulating torsional, longitudinal and flexural modes resonant column tests. The material damping ratio was obtained through the application of the logarithmic decrement method on the free vibration signals from the simulated resonant column test. Effects of the input parameters on material damping ratio were investigated. A relationship between the material damping ratio and the decay constant used in the viscoelastic model was proposed. Results indicated that the viscoelastic model with parameters as proposed in this chapter is able to model wave propagation in soils. The simplified viscoelastic model will henceforth be employed to verify the technique selected for the determination of damping ratio in Test Series II.

CHAPTER 5: Results and Discussion

5.1 Outline

In this chapter, the results from the four test series are discussed. Test Series I discusses the response obtained from the three configurations – the bender-extender elements, ultrasonic transducer and a hybrid bender element-ultrasonic system. The bender element system was proven to be more reliable than the rest and hence used for the following test series. Test Series II evaluates the different techniques to obtain damping ratio from the bender elements receiver signals. The Hilbert transform method (HTM) was shown to be the most robust and trustworthy. Further verifications were conducted on the HTM via experimental (Section 3.3.2) and numerical (Section 4.5) means. The effects of varying confining pressures and degrees of saturation were studied. Test Series III sees the framework developed in Test Series I and II applied on soil-rubber mixtures. The stiffness and damping ratio of the soil-rubber mixtures were altered by changing the rubber content of the specimen. The response of wave propagating through different phases was studied. Test Series IV sees the bender elements incorporated into the cyclic triaxial set-up. The dynamic and mechanical properties of unsaturated undisturbed residual soils were determined in the saturated and unsaturated state.

5.2 Test Series I: The Wave Propagation Transducer

As mentioned previously, one of the focuses of this thesis centres on the determination of small-strain soil-stiffness and damping ratio so that the effect of saturation and confining pressure on these parameters can be investigated. Of the many advanced geotechnical laboratory test equipment, the wave propagation test is probably the most flexible as it can be incorporated in apparatuses such as the triaxial, oedometer and resonant column apparatuses. The incorporation aids to improve testing efficiency since both mechanical and dynamic properties can be

simultaneously determined. In a laboratory setting, P- and S- waves are generally transmitted with the help of bender/extender elements and ultrasonic transducers.

Bender/extender elements have several disadvantages when applied to soil testing. The most apparent limitation of the bender/extender element is the need for it to be sufficiently robust such that it can survive the insertion into stiff specimens. This may also lead to some specimen disturbances. Bender/extender elements are also found to be more susceptible to crosstalk and background noise as well as malfunction due to water ingress and hence there is a need for proper shielding and waterproofing. Finally, signal quality is greatly dependent on the contact condition between soil and the bender/extender elements. The greatest advantage the ultrasonic transducers have over bender/extender elements is its non-invasive nature. In most cases, ultrasonic transducers come in metallic housing which provides shielding against crosstalk and background noises. Despite this, the ultrasonic transducer has its own set of problems which will be further elaborated.

This section investigates the signal quality of ultrasonic transducer, bender/extender elements and a hybrid system consisting of an ultrasonic transducer and a bender/extender element. Wave arrival from three configurations – ultrasonic, bender/extender element and hybrid configurations will be evaluated using both standard and soil specimens so that the best configuration can be used for subsequent tests. The nature of wave propagation and detection by the transducers will be analysed.

5.2.1 Wave arrival by ultrasonic system

As shown in Table 5.1, there are generally no issues pertaining to the arrival times of both compression and shear waves when using the ultrasonic transducers for standard specimens such as acrylic and nylon 66. Problems are, however, encountered when testing Changi sand and kaolin.

Table 5.1: V_p and V_s using ultrasonic transducers - ULT 2012

Material	Specimen Tag	V_p (m/s)	V_s (m/s)	Reported Values
Acrylic	AC-100	2769	1634	$^1V_p = 2667 - 2769$ m/s $^1V_s = 1118 - 1448$ m/s
Nylon 66	NY-100	2786	1045	$^2V_p = 2663$ m/s $^2V_s = 1141$ m/s
Kaolin	KL-60	1441	1167(unreliable)	$^3V_s = 69 - 146$ m/s $^4V_s = 98 - 185$ m/s
	KL-100	1499	1157(unreliable)	
	KL-140	1508	1183(unreliable)	
Changi Sand	CSD-60	260	-	-
	CSD-100	298		
	CSD-140	274		

¹Reported values for acrylic from NDT Systems (2012)

²Reported values for nylon 66 from Leong et al. (2004)

³Reported values for kaolin at $\sigma_3' = 24 - 104$ kPa from Oh et al. (2008)

⁴Reported values for kaolin at $\sigma_3' = 100$ kPa from Black et al. (2009)

For Changi sand, compression wave signals were detected for various specimen heights (Table 5.1) but shear wave signals could not be detected at all for specimen of heights ranging from 40 mm to 140 mm despite numerous attempts to improve contact condition between the specimen and the transducer. This is congruent to the findings by Nakagawa et al. (1996) and Shi et al. (2007). The energy transmitted (T) through the aluminium housing to the sand specimen can be computed using the following equation:

$$T = 1 - \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2 \quad (5.1)$$

where Z is the impedance ($=\rho V$) and subscripts t and s represents transducer and soil, respectively, ρ is the material density and V is the wave velocity.

The density of the aluminium is taken to be 2700 kg/m^3 . For aluminium, compression wave velocity, V_p , and shear wave velocity, V_s , are 6390 m/s and 3180 m/s , respectively (Lee and Waite 2010).

Table 5.2 tabulates the transmission energy (T) of both compression (P) and shear (S) waves for kaolin and Changi sand. The V_p and V_s used for kaolin specimen KL-

100 and Changi sand specimen CSD-100 were obtained from bender element tests given in Table 5.3. For the Changi sand specimen CSD-100, the estimated amount of energy transmitted were about 11% for both P- and S-waves. From FIG. 3.4, the maximum intensities for P- and S-waves from the ultrasonic transducers in the face-to-face test were 200 and 70, respectively. Since the power spectrum can be obtained by squaring the output from the frequency spectrum, the power delivered at maximum intensity was approximately eight times higher for the P-wave compared to the S-wave. Therefore, even when the transmission energy was similar, the S-wave transmitted was still weaker compared to the P wave.

Table 5.2: Ultrasonic transducers' transmission energy (T) for P- and S-waves

Material	Specimen Tag	Density (kg/m ³)	T (%)	
			P wave	S wave
Kaolin	KL-100	1621	44.2	7.5
Changi Sand	CSD-100	1782	11.1	11.7

Table 5.3: V_p and V_s using the hybrid system and bender element system

Material	Specimen Tag	Wave velocity from Hybrid System (m/s)		Wave velocity from Bender Elements		Difference (%)	
		V_p	V_s	V_p	V_s	V_p	V_s
Acrylic	AC-100	2668	1492	NP	NP	3.6 ^a	8.7 ^a
Nylon 66	NY-100	2945	883	NP	NP	5.7 ^a	15.5 ^a
	KL-60*	1594	102	1542	95	3.4 ^b	7.4 ^b
Kaolin	KL-100*	1569	107	1540	103	1.9 ^b	3.9 ^b
	KL-140	1596	112	1467	104	8.8 ^b	7.7 ^b
Changi Sand	CSD-60	271	93	289	93	6.2 ^b	0.0 ^b
	CSD-100	277	157	285	150	2.8 ^b	4.7 ^b
	CSD-140	264	151	249	156	6.0 ^b	3.2 ^b

* Trimmed from KL-140 specimen

NP - Not performed

^a Referenced to ultrasonic transducers, Table 5.1, Columns 3 and 4

^b Referenced to bender elements, Table 5.3, Columns 5 and 6

If energy loss due to attenuation was accounted for, the S-wave signal could be even weaker. The effect of attenuation on the S-wave signal could be qualitatively evaluated following ASTM D2845-08 (2008). ASTM D2845-08 (2008) specified that the ultrasonic transmitter should be designed to generate waves with

wavelengths of at least three times the average grain size of rock specimen to reduce scattering and poorly defined first arrivals at the receiver. The average grain diameter of Changi sand is about 0.4 mm (D_{50}). Using the wave velocities from bender elements for CSD-100 presented in Table 5.3, the wavelengths for both P- and S-waves at excitation frequency of 100 kHz were about 2.85 mm and 1.5 mm, respectively. The wavelength of P-wave was 7.1 times whereas the wavelength of the S-wave was about 3.8 times the mean grain size diameter of Changi sand. Therefore, it was more difficult to detect S-wave as compared to P-wave. Both of these reasons explained the failure to detect the S wave signals in the ultrasonic transducers.

For kaolin specimen KL-140, the V_s of 1183 m/s obtained using ultrasonic transducers was much higher than the expected V_s of about 100 m/s (Table 5.1). The filtered S-wave and corresponding frequency spectrum are shown in FIGs.-5.1b and 5.1c, respectively. A low pass fourth-order Butterworth filter with a cut-off frequency of 60 kHz (10 kHz above the dominant frequency) was used. Arrival time of the S-wave was determined at the first deflection as shown in FIG 5.1b. Poisson's ratio (ν) back-calculated from V_p and V_s using Equation 5.2, gives a value of -0.30 which is clearly unreasonable.

$$\nu = \frac{0.5 \left(\frac{V_p}{V_s} \right)^2 - 1}{\left(\frac{V_p}{V_s} \right)^2 - 1} \quad (5.2)$$

FIG. 5.1: Time and frequency domain of (a,c) compression and (b,c) shear wave of Kaolin specimen (KL-100) using ULT-2012 transducers

The faster and stronger P-wave has masked the arrival of the slower and weaker S-wave making it hard to detect the point of S-wave arrival. This complication was made worse by the closeness in dominant frequencies of the P- and S-wave transducers (FIG. 5.1c) making digital signal filtering ineffective. The same problem was reported by Weidinger (2008) when using similar ultrasonic transducers. He had proposed to use v and V_p to estimate possible S-wave arrival time. However, Nakagawa et al. (1996) and Leong et al. (2004) did not report similar problem for kaolin. Nakagawa et al. (1996) tested kaolin specimens of about 100 mm height under an increasing confining pressure from 49 kPa to 627.2 kPa while Leong et al. (2004) tested kaolin specimen of 60 mm height under a confining pressure of 800 kPa. The ULT-2004 used by Leong et al. (2004) has different characteristics from ULT-2012 as mentioned in Section 3.2.3.

The problem mentioned above was investigated further by capturing the receiver signals of both the P-wave and S-wave crystals of the ultrasonic transducer on the digital oscilloscope when an S-wave was transmitted through specimen KL-100. The P-wave crystal showed an output signal of higher intensity compared to the S-wave crystals despite the fact that only the S-wave crystals were excited at the transmitting end, implying that the transmitted wave has high P-wave content (FIG. 5.2). The frequency spectra of the P- and S-waves show that the dominant frequencies are similar at the lower end of the spectrum (FIG 5.1c). Comparing FIG 5.1c to the frequency spectrum reported in Nakagawa et al. (1996) and Leong et al. (2004), there is a much smaller separation of dominant frequencies of both compression and shear waves using ULT-2012 and hence making it difficult to separate the stronger P-wave from the S-wave via digital signal processing (stacking, filtering, windowing).

FIG. 5.2: Simultaneous monitoring of both P and S wave transducers upon excitation of S wave transducer (KL-140)

5.2.2 Wave arrival by bender/extender element system

The S-wave receiver signal obtained from the bender element was usually stronger than that of the P-wave (FIG 5.3). As the bender element's deformations in shear and extension are different, S-wave signal is expected to be less affected by P-wave. However, signals from bender elements are more susceptible to crosstalk/electromagnetic induction. The phenomenon was persistent despite numerous attempts being made to ensure proper grounding and shielding of the bender elements following Lee and Santamarina (2005) suggestions and ensuring that the cables do not cross each another. It was also observed that a series connection was more susceptible to crosstalk/ electromagnetic induction as compared to parallel connection. Moreover, the receiver signal was also subjected to a weak electronic drift.

FIG. 5.3: (a) Compression and (b) shear wave signals for KL-100 tested in bender element system at various frequencies. All input pulses occur at 500μs.

Results from the bender/extender elements are tabulated in Table 5.3. The P-wave signals from the three different configurations are vertically offset for comparison in FIG. 5.4. For Changi sand, visual inspection of FIG. 5.4a shows that P-wave signal strength from bender element was comparable to those from the ultrasonic system but it was heavily affected by crosstalk and electronic noises. These noises

were present even after digital signal processing. A cleaner signal is always preferred since the arrival time can be more clearly determined. The S-wave signals were less affected by crosstalk and electronic noises as shown in FIG. 5.4b.

For the kaolin specimen, the P-wave signal from bender element was comparatively weaker than both the ultrasonic and hybrid configurations (FIG 5.4b). This is to be expected as the kaolin specimen was significantly softer as compared to the Changi sand and was more severely affected by specimen disturbance as the bender/extender intruded into the soil specimen.

(to be continued on the next page)

FIG. 5.4: Comparison of P wave signal from (a) Changi sand (CSD-100) and (b) Kaolin (KL-100) using three different set-ups

5.2.3 Wave arrival by hybrid bender element – ultrasonic system

Signals from the hybrid system are subjected to smaller electronic noise as compared to those from the bender/extender element system. Response of the hybrid system (FIG. 5.5), like those from the bender/extender element system (FIG. 5.3) varied with excitation frequency. This is in contrast to the ultrasonic transducers whose transmitter signal was independent of excitation frequency (FIG. 5.1c). Wave velocities measured using the hybrid system are also tabulated in Table 5.3.

For acrylic and nylon 66, differences in determining wave velocities were calculated via comparison with wave velocities measured using the ultrasonic system. For kaolin and Changi sand specimens, differences in determining wave velocities were compared with those measured using the bender/extender element

system. The comparison is shown in Table 5.3. Table 5.3 shows that V_s and V_p obtained from the hybrid system agreed well with errors of less than 10% except for V_s of nylon 66. The higher error was attributed to the poor contact condition between the bender element and the nylon 66 specimen, where a slot was made to receive the bender element transmitter.

FIG. 5.6 shows the P- and S-wave signals for Changi sand specimen CSD-100 using the hybrid system. Signals are generally distinct and the arrival times can be identified even without digital signal processing due to smaller degree of interferences from P-wave. The frequency spectrum in FIG. 5.7 shows that the signal strength of the S-wave is higher than that of the P-wave. The S-wave signal also has a lower dominant frequency which increases in intensity as the resonant frequency is approached. Resonant frequency is about 1 kHz for S-wave but is greater than 10 kHz for P-wave.

Similar signal trends can be observed for the kaolin specimen KL-100 using the hybrid system (FIG. 5.5). At higher excitation frequencies, P-wave gradually becomes stronger as the initial flat portion of the S-wave signal becomes increasingly obscured by high frequency P-waves (FIG. 5.5). At an excitation frequency of 20 kHz, the stronger P-wave signal masked the S-wave signal significantly. The frequency spectrum (FIG. 5.8) also shows that the signal intensities from 20 to 50 kHz (due to P-wave) are higher than those around 1 kHz (due to S-wave). The interference from P-wave is attributed to the high water content of the kaolin specimen which facilitates the propagation of P-wave. On the contrary, the bender element system is less susceptible to the interference of the P-wave signals as shown in FIG. 5.3 and its frequency spectrum in FIG. 5.9. However, this should not be a source of concern because a low pass filter can be used to easily separate the waves in the hybrid system as they have widely differing resonant frequencies.

FIG. 5.5: (a) Compression and shear wave signals for KL-100 tested in hybrid system at various frequencies. All input pulses occur at 500 μs .

FIG. 5.6: (a) Compression and (b) shear wave signals for CSD-100 tested in hybrid system at various frequencies. All input pulses occur at 500 μ s.

FIG. 5.7: Frequency spectrum of CSD-100 tested in hybrid system

FIG. 5.8: Frequency spectrum of KL-100 tested in hybrid system

FIG. 5.9: Frequency spectrum of KL-100 tested in bender element system

The results above further showed that even though it is possible to transmit a “pure” P-wave, S-wave can be accompanied by stronger P-wave depending on the frequency of excitation and degree of saturation. This is also the case for bender elements (Lee and Santamarina 2005). In view of this, it is strongly advisable that ultrasonic transducers be made with P- and S-wave transducers of widely differing resonant frequencies. This will enable easier identification of the wave arrival. Moreover, S-wave being the weaker wave should be generated with a transducer with a stronger output and lower resonant frequency. This is because a wave of lower frequency will go through fewer cycles before reaching the receiver and hence will experience less attenuation.

Leong et al. (2009) suggest that near-field effects on S-wave can be minimized by ensuring that L_{tt}/λ ratio is more than 3.33. Table 5.4 shows the L_{tt}/λ ratio of FIGs. 5.3b, 5.5b and 5.6b. As observed in Table 5.4, when L_{tt}/λ ratio is low (excitation at 1 kHz), the gradual rise in signal makes it difficult to determine the exact arrival time. However as excitation frequency increases, a sharper rise can be observed and the arrival time can be determined with less difficulty.

It is suggested that at least three different excitation frequencies should be used for determination of S-wave arrival time in the hybrid system. The first excitation frequency should be at the frequency where the receiver S-wave has the highest

intensity in the frequency spectrum. The second and third excitation frequencies should be at $L_{tt}/\lambda \approx 3.33$ and 10. Receiver wave signals at these frequencies should then have a more distinct arrival time to enable accurate determination of wave travel time.

Table 5.4: L_{tt}/λ ratio for FIGs. 5.3b, 5.5b and 5.6b

Material	Specimen Tag	L_{tt}/λ ratio		
		1 kHz	5 kHz	20 kHz
Kaolin	KL-100 Hybrid System	0.848	4.24	16.96
	KL-100 Bender Element System	0.748	3.74	14.96
Changi Sand	CSD-100 Hybrid System	0.578	2.89	11.56

5.2.4 Summary and remarks

Three system configurations, ultrasonic, bender/extender element and hybrid, were investigated with standard and soil specimens. Apart from the system configuration, there is a need to look at the signals generated in totality. This enables better understanding of the workings of the transducers and provides guidelines to obtain signals of optimum quality for determining the dynamic soil properties.

The ultrasonic transducer despite having various advantages over the bender-extender element and the hybrid system was unable to produce reliable S-wave results due to the interference from P-wave that cannot be separated with digital signal processing. This problem is not only attributed to the close proximity of the P- and S-wave resonant frequencies but also the low transmissivity of the S-wave signal. By using the bender element as a transmitter and the ultrasonic transducer as the receiver in the hybrid configuration, the working of the ultrasonic transducer was further understood. Response of the hybrid system, unlike the ultrasonic transducer, was shown to vary with excitation frequency of the bender element. The received signal typically consists of a transient response due to the excitation pulse followed by the steady-state free vibration response as explained in Section 3.3.3. This implicitly means that the ultrasonic receiver was able to act as passive receiver

to register perturbation from the excitation similar to the bender/extender elements and that the long ringing shown in FIG. 5.2 and 5.4 can be attributed to the transmitting ultrasonic transducer in the ultrasonic system.

The schematic diagram of a typical ultrasonic transducer is shown in FIG. 5.10. The presence of the backing layer and matching layer are known to alter the behaviour of the ultrasonic transducer. When the ultrasonic piezoceramic is sandwiched between two media with large difference in impedances, the mismatch will cause most of the wave energy to be reflected away from the test specimen. In order to maximise transmission, the matching layer should usually be made of materials of intermediate impedance to improve transmission. On the other hand, the backing layer is usually made of high damping material in order to absorb the energy that is radiated backwards from the piezoceramics (Baun 2004, Ramadas et al. 2009, Zhou et al. 2011). The features described above were however not found in the ultrasonic transducer (ULT-2012) used. As such, upon excitation the piezoceramic will begin to ring at its natural frequency. The impedance mismatch will cause the waves generated to be reflected back into the ultrasonic transducer. In the absence of the backing layer, the reflected wave is not absorbed leading to after-ringing which lengthens the duration of the acoustic pulse and giving the response observed in FIGs. 5.2 and 5.4.

FIG. 5.10: Schematic diagram of a typical ultrasonic transducer (adopted from Panametrics-NDT, 2006)

Designing of the matching and backing layers requires extensive modelling effort and is out of the scope of this study. As such, the ultrasonic transducers were not considered. Between the hybrid system and the bender elements, the bender elements are shown to have consistently stronger signals regardless of the specimen type and experience lesser interference from P-wave (FIG. 5.5b). Thus, the bender element system was selected to be used for subsequent testing.

Test Series I also provided the basic guidelines for the excitation frequencies for P- and S-waves. In order to obtain signals of high signal to noise ratio (SNR), the excitation frequency should ideally be around the natural frequency of the excitation mode pertaining to the specimen. For a cylindrical specimen, the natural frequency increases with the excitation mode as follows: flexural, torsional and longitudinal excitation. Frequency domain plots (FIG. 5.7 and 5.8) and Section 3.3.3 also show that signals from P-wave excitation (longitudinal) has a higher frequency compared to signals from S-wave excitation (torsional). Hence, P-wave excitation frequency for subsequent tests would be fixed at 10, 20 and 50 kHz.

For S-wave signals, excitation at the natural frequency (1 – 3 kHz as shown in FIG. 5.7, 5.8 and 5.9) is insufficient due to near-field effect. At low excitation frequency, there is a lack of distinct feature that can be identified as the arrival time (FIG. 5.3). As such, there is a need to satisfy the L_{tt}/λ ratio. The L_{tt}/λ ratio illustrates the changes in the initial transient portion of the received wave which is dependent on the excitation frequency. Therefore, it should be calculated based on the excitation frequency instead of the dominant frequency as suggested by Brignoli et al. (1996). Nevertheless, excitation close to the dominant frequency is required as it will translate to signals of high SNR during the steady-state decay thereby allowing the damping ratio to be determined. Hence, S-wave excitation frequency for subsequent tests would be fixed at 3, 5, 10 and 20 kHz.

5.3 Test Series II: Development of testing framework, effect of saturation and confining pressures on small-strain soil properties

While Test Series I set out the framework for the transducer used and the excitation frequency of P- and S-waves for determination of wave velocities, Test Series II first focuses on evaluating the technique used to obtain damping ratio from the signals. With a complete guideline to determine the wave velocities and damping ratio, the effects of degree of saturation and confining pressure on small-strain soil properties were studied.

5.3.1 Soil Water Characteristics Curve

The soil water characteristics curves (SWCC) of the 50 mm specimens at various net confining stresses ($\sigma_3 - u_a$) are expressed in terms of volumetric water content and degree of saturation and shown in FIG. 5.11. FIG. 5.11a shows that the air-entry value of the sand specimen ranges from 6.7 to 9.6 kPa as ($\sigma_3 - u_a$) increases from 100 kPa to 400 kPa. Degree of saturation decreases from 100% to a range of 15 – 30% when the specimens were subjected to a maximum matric suction ($u_a - u_w$) of 38 kPa.

The SWCC of RS-US_100 and RS-US_400 which underwent a further cycle of wetting and drying is shown in FIG. 5.12. FIG. 5.12 shows that both RS-US_100 and RS-US_400 did not reach full saturation upon wetting on both occasions and have residual water contents (θ_a) of 0.099 and 0.077, respectively.

FIG. 5.11: SWCC of the specimen expressed in terms of (a) volumetric water content and (b) degree of saturation

FIG. 5.12: Hysteresis of SWCCs for specimens (a,b) RS-US_100H and (c,d) RS-US_400H

5.3.2 Techniques to determine the damping ratio

In this section, the spectral ratio method (SRM), logarithmic decrement method (LDM) and Hilbert transform method (HTM) were evaluated for their suitability to derive damping ratio from the bender element receiver signals. Only S-wave signals were used in this section as they were comparatively more consistent than P-wave signals which were observed to exhibit greater complexity in the frequency domain (FIG. 3.9) since it passes through both solid and liquid phases. Signals from RS-US_100H and RS-US_200H were used for analysis via LDM and HTM. Signals from RS-US_100, RS-US_100H, RS-US_200 and RS-US_200H were used for analysis via SRM. Signals of excitation frequencies of 3, 5 and 10 kHz were analyzed unless otherwise stated.

5.3.2.1 Damping ratio via SRM

As mentioned previously, damping ratio can be obtained via SRM using two cases: (1) different specimens (with known damping ratio) of the same dimensions, and (2)

similar specimens but of different heights. In this thesis, silicon rubber is used as the reference specimen for the first case as it can be moulded for a snub fit with the bender elements. The damping ratio of silicon rubber was first determined using specimens of different heights (25, 50 and 100 mm) with Case 2 of SRM. The damping ratio was checked against those obtained from cyclic triaxial testing.

The V_p and V_s of the silicon rubber specimen were obtained from the gradient of the best fit lines as shown in FIGs. 5.13 and 5.14. The P- and S-wave velocities of the silicon rubber are 1026.1 and 28.5 m/s, respectively.

FIG. 5.13: Variation of P-wave arrival time with specimens of different heights

FIG. 5.14: Variation of S-wave arrival time with specimens of different heights

Spectral ratio method (SRM) was applied by comparing specimens of different heights. When using this method, only signals which conform to the conditions in Section 3.3.3 are deemed reliable. Damping ratio obtained using the spectral ratio method is tabulated in Table 5.5. As the spectral ratio method is heavily dependent on the signal amplitude and quality, results obtained from poor signal will tend to show abnormality. This can be shown by the wider range of damping ratio in P-wave damping ratios compared to those from S-wave. Signals from the 25 mm height specimen were unreliable as it fell within the near field region where variations in amplitude occurred.

Cyclic triaxial test was conducted on the 100 mm silicon rubber specimen. The specimen was excited for 15 sinusoidal cycles at double amplitude axial strain of 0.2, 0.5 and 1% with excitation frequencies of 0.2 and 0.5 Hz for each strain. The damping ratio was plotted with respect to cycle number in FIG. 5.15. Damping ratio at the smallest strain of 0.2% (C3F02 and C3F05) was shown to have a larger scatter compared to those at larger strains of 0.5 and 1%. This is because the more compressible silicon rubber specimen needs a larger displacement to trigger a clear response in the load cell. Results from FIG. 5.15 also show that the excitation amplitude is well below the threshold strain since the damping ratio is not shown to increase with increasing strain. Damping ratio derived from the SRM was shown to be slightly higher than those determined from the cyclic triaxial test.

Table 5.5: Damping ratios from the SRM for silicon rubber specimen

Wave Type	Specimen Height (mm)		Excitation Frequency (kHz)	Height Difference (mm)	Damping ratio (%)	Mean (%)
	1	2				
S-wave	25	100	3	75	2.18	2.31
			5		2.19	
			10		2.05	
	50	100	3	50	1.63	
			5		2.82	
			10		1.81	
	25	50	3	25	2.99	
			5		1.6	
			10		3.53	
P-wave	25	100	20	75	0.87	3.32
			50		2.46	
	50	100	20	50	5.45	
			50		1.73	
	25	50	20	25	-66.3*	
			50		6.08	

* due to poor signal

FIG. 5.15: Damping ratios of 100 mm silicon rubber specimen using cyclic triaxial set-up

When the silicon rubber specimen was used as a reference specimen against a soil specimen of similar dimensions (Case 1), it was found that the dominant frequency of the silicon rubber specimen is significantly lower than that of the sand specimen

(FIG. 5.16). An implicit assumption in using a reference specimen is that the frequency spectrum of the reference specimen's signal must be within the same frequency range as the specimen to be tested. Attempts to increase the dominant frequency of the silicon rubber specimen by increasing confining pressure were ineffective. The mismatch in frequency spectra leads to a large scatter in the damping ratio and some unreasonable negative values confirming that Case 1 of the SRM is unsuitable for determining damping ratio.

FIG. 5.16: Frequency spectra of sand and silicon rubber specimen

Case 2 of SRM was further tested using combinations of the 50 and 100 mm specimens subjected to confining stresses ($\sigma_3 - u_a$) of 100 and 200 kPa. The damping ratios obtained using excitation frequency of 10 kHz is plotted with matric suction in FIG. 5.17. Results from excitation frequencies of 3 and 5 kHz showed significantly higher scatter and were hence not included. Selection of the frequency range to determine the gradient was challenging. A time window is first applied on both signals to select the first sine waves starting from the arrival time. The later part of the wave signals was not used due to interferences from the reflected waves. The frequency range used in SRM was selected based on the following criteria:

- Dominant frequencies of both specimens were close to each other

- Signal from the shorter specimen should be stronger than the longer specimen
- Frequency range was maximized while maintaining a minimum coefficient of determination R^2 of at least 0.99
- Frequency range contains the peak amplitude

The damping ratios obtained via Case 2 of SRM did not decrease with increasing matric suction as expected. A large scatter of about $\pm 40\%$ was observed. This is due to the difficulty in ensuring similar quality of receiver signals and contact conditions between soil and bender elements for both specimens. The unreliability and difficulties in obtaining the damping characteristics of geo-material using the SRM was also experienced by Sams and Goldberg (1990) who conducted their analyses on synthetic and real data. Sams and Goldberg (1990) pointed out the various uncertainties in the method ranging from the large scatter of the results, windowing of the signal and the optimum frequency range to fit a straight line.

FIG. 5.17: Damping ratios of river sand specimen at different net confining stress using SRM

5.3.2.2 Damping ratio via LDM

The LDM uses the free vibration decay to determine the damping ratio of the specimen. Damping ratio obtained using the LDM was plotted together with matric suction and shown in FIG. 5.18. The damping ratios were not observed to vary with net confining stress and decrease slightly with increasing matric suction (decreasing degree of saturation). Scatter of the damping ratio were mainly within $\pm 20\%$ from the mean line. Out of the three excitation frequencies of 3, 5 and 10 kHz, only the 3 and 5 kHz S-wave signals were analyzed for damping ratio. The lower excitation frequencies being closer to the natural frequency of the specimen were able to produce well-behaved free vibration decay signals (FIG. 5.19a). With increasing excitation frequencies, interference from frequency peaks in close proximity of the natural frequency lead to irregularities (FIG 5.19b) which cannot be removed via signal filtering. Despite the signal showing free vibration decay with time, the selection of peaks for LDM is challenging and prone to error due to the interferences shown in FIG. 5.19b.

FIG. 5.18: Damping ratios of river sand specimen at different net confining stresses using LDM

FIG. 5.19: S-wave signals of RS-US_200H at excitation frequencies of (a) 3 and (b) 10 kHz

5.3.2.3 Damping ratio via HTM

The HTM is conceptually similar to the LDM but it is able to overcome the limitation of the LDM since a regression line is fitted through the decaying portion of the signal. In this way, fluctuations due to interferences and the actual decrease in amplitude can be determined. Damping ratios obtained using the HTM are plotted together with matric suction and shown in FIG. 5.20. Damping ratios were averaged from the values determined from 3, 5 and 10 kHz and plotted with error bars to show the maximum and minimum deviations. This corresponds to a scatter

of about $\pm 10\%$. Similar to FIG. 5.18, FIG. 5.20 shows a decrease in damping ratio with increasing matric suction. FIG. 5.20 shows a marginal decrease in damping ratio when the specimen is subjected to a higher net confining stress. The following trend was not observed in FIG. 5.18 as the large scatter in values would have masked the slight decrease in damping ratio due to the stiffer specimen. The following observations were in accordance to those described in literatures. Damping ratio obtained using the HTM was observed to be slightly lower than those obtained using the LDM.

FIG. 5.20: Damping ratios of river sand specimen at different net confining stress using HTM

5.3.3 Verification of the Hilbert transform method (HTM)

Three techniques were evaluated for their suitability in determining the damping ratio of geo-materials using S-wave signals from bender-extender elements. Determination of damping ratio using the SRM using a reference specimen of different material is not feasible due to the mismatch in the frequency spectra of the incident pulse. While the damping ratio can be obtained using similar specimens of different heights, implementation of the method is almost impractical due to the large scatter in results. The procedures are not only inefficient and require additional resources; obtaining identical specimens is almost impossible due to the heterogeneity in undisturbed specimens.

While LDM is similar to HTM, the HTM was shown to be able to overcome the limitations of the LDM. Expected trends with increasing confining pressure and matric suction can be observed with the smaller scatter of the results obtained using the HTM. In this section, the HTM is further verified using experimental results from the literature and the finite element simulations.

5.3.3.1 Verification with Ottawa sand

Damping ratios of dry Ottawa sand determined using the resonant column tests from the literature were compared with those obtained using the bender element tests by HTM. Ottawa sand specimens of 50 mm diameter and 100 mm height were subjected to effective confining pressures ranging from 50 to 800 kPa. Sinusoidal pulses of excitation frequencies of 3, 5, 10 kHz were used. The damping ratios determined at these frequencies were averaged and plotted with effective confining pressure in FIG. 5.21. Damping ratios obtained from the HTM and resonant column tests show a decrease in value as the specimen becomes stiffer at higher effective confining pressure. The range of values of the damping ratio agrees with those in the literature thereby providing support on the use of HTM in determining damping ratio from wave propagation signals.

FIG. 5.21: Damping ratios of sand specimen at different net confining stresses using HTM

5.3.3.2 Verification with finite element simulation

Finite element modelling using LS-DYNA was further used to verify the HTM. Details of the finite element modeling are given in Section 4.5. The damping ratio was recovered from the simulated received waveform (FIGs. 5.22a and b) using HTM and compared with the input damping ratio. The model was first subjected to an implicit eigenvalue analysis whereby the flexural and torsional natural frequencies were identified at 0.983 and 1.413 kHz, respectively. The peaks at approximately the lowest frequency of 1 kHz (FIG 5.22c) which lies within the range of the flexural and torsional natural frequencies were retained. The time and frequency domain plots (FIG. 5.22a, b and c) show peaks at higher frequencies due to the absence of the non-reflecting boundary condition. This will more realistically simulate the actual situation where the reflected waves interfere with the incident wave. While this high frequency content persisted beyond the initial transient region (FIG. 3.8), it will not affect the result after signal filtering. The magnitude of the analytic signal with time together with the best-fit line for the identified peaks is shown are FIG. 5.22d. The input damping ratios are compared with those derived from the HTM in Table 5.6.

FIG. 5.22: (a, b) Time domain, (b) frequency domain and (c) Hilbert transform of the simulated signal (Input damping ratio = 7%)

Differences between the input damping ratio and those derived from HTM are within $\pm 10\%$. FIG. 5.22 and Table 5.6 further show that exciting the model close to the natural frequency will lead to a stronger received signal at the natural frequency. This in turn reduces the discrepancy between the input and derived damping ratio. The following is congruent to experimental observation whereby the weak natural frequency component of some S-wave signals excited at 20 kHz gave rise to negative values for damping ratio. The experimental and numerical verifications of the HTM provided further support that HTM is sufficiently robust and reliable in the determination of damping ratio.

Table 5.6: Comparison between input damping ratio and those derived from HTM

Excitation Frequency (kHz)	Input ξ (%)	ξ from HTM	Difference (%)
3	3	3.1316	4.39
	7	7.2329	3.33
5	3	3.2524	8.41
	7	7.1722	2.46
10	3	3.3249	10.83
	7	7.5874	8.39

With the testing framework properly established, the effects of degree of saturation and confining pressure on the dynamic parameters V_p , V_s and ξ were investigated. The dynamic parameters will be evaluated separately for both effects in the subsequent sections.

5.3.4 Effects of degree of saturation and confining pressure on V_p

The behaviour of V_p is studied in conjunction with the B-value as they are both known to vary with degree of saturation and can be used as proxies for saturation. An effective proxy for degree of saturation should only be a function of the degree of saturation and independent of the material type and stiffness. As such, the B-

value and V_p were evaluated for their effectiveness in reflecting saturation using the river sand specimens at different effective confining pressures (i.e. stiffnesses). The B-value was obtained followed by the V_p during the saturation process where effective confining pressure was maintained at 30 kPa (FIG. 5.23). The P- and S-wave velocities obtained before and after the B-value check remained unchanged therefore implying that the sand specimen was not affected by the measurement of B-values. The B-value was shown to increase as the degree of saturation increases. The V_p of the specimen also increases from an initial value of approximately 200 m/s to 1600 m/s at full saturation.

FIG. 5.23: Variation of Skempton pore-pressure parameter B with V_p during the saturation phase

While it is known that at full saturation, V_p and B-value tend towards 1500 m/s and 0.95, respectively, the P-wave signal is seldom analysed with respect to the time and frequency domains. FIG. 5.24a shows the P-wave signals of a typical sand specimen and water in the time domain. Specimen RS-US_400H was subjected to an isotropic confining pressure and pore-water pressure of 330 and 300 kPa, respectively. The water medium was pressurized to 300 kPa. The initial bump at the 500 μ s was due to the crosstalk as a result of the excitation pulse (Cheng and Leong, 2014). The arrival time of the wave is indicated by the vertical dotted lines. It can

be observed that as B-value tends towards 1, the dominant frequency of the P-wave signal increases.

The dominant frequency range shifts from a range of 1 – 10 kHz to a range of 20 – 80 kHz as B-value increases (i.e., increasing S_r) as shown in FIG. 5.24b. This higher dominant frequency in the initial part of the P-wave signal is consistent and coincident with the P-wave signals through water (see FIG. 5.24c). The amplitude of the dominant frequency increases with degree of saturation as well. This indicates that the high frequency P-wave component was due to P-wave travelling through water. Emerson and Foray (2006) also observed that the initial high frequency component in a P-wave signal for a near saturated specimen was absent in a dry specimen. Hence, full saturation is not only indicated by the values of V_p and B but can also be observed from the frequency and time domains.

(a) Time-domain of P-wave signals

(b) Frequency-domain P-wave signal for sand specimens

(c) Frequency-domain P-wave signal through water

FIG.5.24: P-wave signals propagating through different specimens close to saturation and water (50 kHz excitation frequency)

Upon reaching full saturation, specimen RS-S was consolidated to different effective confining pressures. The B and V_p values in response to increasing isotropic effective stress for a fully saturated specimen are shown in FIG. 5.25. The B -value decreases as the soil stiffness increases at higher effective stresses. The effect of confining pressure on V_p is small. Thus, V_p is a better proxy compared to the B -value as it remains unaffected by the changes in the soil's stiffness.

FIG. 5.25: Variation of B -value and V_p with increasing with increasing effective stress

However, V_p did not display a constant value at full saturation. Since it has been previously established that P-wave travels through the water phase at full saturation, it was intuitive to deduce that V_p is given by V_p of water (≈ 1500 m/s). To investigate if V_p is affected by the volume of water in the specimen, V_p was plotted against void ratio in FIG. 5.26. Data from the literature (Table 2.1) are also included in FIG. 5.26. FIG. 5.26 shows that V_p does not vary with void ratio and has a mean value of 1684 m/s. All the V_p values are within $\pm 10\%$ of the mean value.

FIG. 5.26: Variation of V_p with void ratio

The trend of V_p , B -value and S_r were investigated in greater details using the measurements on the specimens undergoing desaturation from full saturation at various isotropic net confining pressures. The variations in V_p are plotted with matric suction in FIG. 5.27.

For the saturated sand specimens, V_p was about 1600 m/s and remained constant during the initial increase in matric suction and started to decrease only after the air-entry value (range demarcated by the dotted lines in FIG. 5.27). After the residual matric suction, the changes in V_p were minimal and lies within $\pm 10\%$ of 419 m/s as the isotropic net normal stress increased from 100 to 400 kPa. FIGs. 5.27b and c show V_p during drying and wetting of specimen RS-US_100H and RS-US_400H, respectively. On wetting, V_p was not seen to increase as matric suction decreases. This is because both specimens only reached a S_r of 89% upon wetting. The V_p of RS-US_100H (FIG. 5.27b) was observed to decrease slightly during the wetting phase. This decrease is less noticeable on the wetting V_p of RS-US_400H as reduction in matric suction is small compared to the high isotropic net normal stress the specimen was already subjected to.

FIG. 5.27 can be correlated to FIG.5.28 after taking into consideration the volume of water entering or leaving the specimen. Initially it might appear to be difficult to ascertain full saturation by observing V_p since V_p does not change significantly after the abrupt increase to about 1500 m/s at S_r of 80 – 92%. However, the issue can be resolved by examining the P-wave signals in the frequency domain as shown in FIG. 5.29.

(a) Effect of isotropic net normal stress

(b) Effect of drying and wetting on specimen RS-US_100H

(c) Effect of drying and wetting on specimen RS-US_400H

FIG. 5.27: Variation of V_p with matric suction

FIG. 5.28: Variation of V_p with degree of saturation

FIG. 5.29: Frequency domains of P-wave signals (a) before and (b) after the air-entry value of RS-US_400H

Prior to the air-entry value, signals with high frequency component (range of 25 – 35 kHz) were identified (FIG. 5.29a). Upon drying, the amplitude of the high frequency components was reduced significantly. Further drying sees a decrease in the dominant frequency range to a range of 5 – 10 kHz. When matric suction increases beyond the air-entry value, the high frequency component ceases to exist and the amplitude of the low frequency component intensifies as seen in FIG. 5.29b. If V_p is obtained during the saturation phase, the reduction in high frequency component can be easily identified from the P-wave signals. It is also noted that the intensity of the high frequency component in FIG. 5.29a corresponds to the intensity of the P-wave travelling through water (FIG. 5.24a) at amplitude of about 0.63.

FIG. 5.28 shows V_p decreasing rapidly from 1600 m/s when S_r drops below 80 – 92%. This degree of saturation corresponds to the air-entry value as shown in FIG. 5.27a. The unique relationship between V_p and S_r shown in FIG. 5.28 was also found by others as shown in FIG. 5.30a. FIG. 5.30b shows V_p normalised with the lowest V_p for the same series so that the trend can be more distinctly seen. FIG. 5.30b shows V_p remains relatively constant until S_r exceeds 95%.

FIG. 5.30: (a) V_p vs degree of saturation (b) V_p/V_{p0} vs degree of saturation (data sources in Table 2.1)

The range from $S_r = 90\%$ to 100% appears to correspond to the zone of ‘partial saturation’ proposed by Kamata et al. (2009) where the soil is not fully saturated and occluded air bubbles are present yet matric suction is negligible. In this zone,

the P-wave was able to traverse through the stiffer water phase thereby giving a velocity close to 1500 m/s despite giving a low B-value upon drying as seen in FIG. 5.31. The arrows show the drying and wetting paths undertaken by the specimens. Since θ_w of subsequent wetting does not increase beyond that corresponding to the air-entry value, both the B-value and V_p remain low.

FIG. 5.31: Variation of V_p with Skempton's Pore-water pressure parameter B

The above observations concur with the case of non-homogeneous partial saturation as defined in Naesgaard et al. (2007). Naesgaard et al. (2007) proposed two types of saturation conditions known as homogeneous (HPS) and non-homogeneous (NHPS) partial saturation. The HPS condition is represented by the situation when air bubbles are small and scattered throughout the void spaces. On the other hand, the NHPS condition is represented by the occurrence of a few large air bubbles in localised area. Numerical simulation performed by Naesgaard et al. (2007) shows that during HPS, air bubbles dispersed within the void spaces give rise to a homogenous fluid stiffness resulting in V_p to be a function of B-value. However, HPS only exists in a theoretical situation. In most cases, drainage will first occur at the large pores as soon as the air-entry value is reached creating discontinuity in the water phase. As such, the P-wave will route through the stiffer saturated zones where the water phase is continuous. Therefore, V_p remains relatively unchanged at around 1600 m/s but the B-value is significantly reduced. The above illustrated the

relationship between B-value, S_r and V_p . The limitations of both B-value and V_p were highlighted to provide better understanding when using them for analysis. The study suggests that V_p , being independent of the stiffness and soil type, is a better alternative to the B-value as a proxy for full saturation. The effective confining pressure has negligible effects on the V_p at various degrees of saturation.

5.3.5 Effects of degree of saturation and confining pressure on V_s

The effect of confining pressure on the V_s has been investigated by Hardin and Richart (1963), and Oztoprak and Bolton (2013). In this study, the interaction between both saturated and unsaturated stress-state variables on the V_s of soils were investigated. The matric suction can again be correlated to the degree of saturation as seen in FIG. 5.11. If matric suction ($u_a - u_w$) is assumed to act isotropically then the sum of net normal stress ($\sigma_3 - u_a$) and matric suction can be compared with the isotropic effective stress. FIG. 5.32 shows V_s plotted against σ_3' or $[(\sigma_3 - u_a) + (u_a - u_w)]$. FIG. 5.32 shows V_s increases linearly with σ_3' and with $[(\sigma_3 - u_a) + (u_a - u_w)]$. However the slopes of the lines are different. Similar data from Vinale et al. (1999) and Hoyos et al. (2008) are also plotted in FIGs. 5.33 and 5.34, respectively. Details of the soils tested by Vinale et al. (1999) and Hoyos et al. (2008) are given in Table 5.7.

FIG. 5.32: Effect of subjected stress on V_s for coarse grained specimens

FIG. 5.33: Effect of subjected stress on V_s for Metrano silty sand compacted (a) at optimum and (b) wet of optimum water content (Alramahi et al. 2007)

FIG.5.34: Effect of subjected stress on V_s for SP soil (Hoyos et al. 2008)

Table 5.7: Summary of soils used for comparison with experiment results

Description	Composition	USCS Classification	G_s	C_u	C_c
Metrano Silty Sand, Vinale et al. (1999)	63% Sand 21% silt 16% Clay	SC	2.64	400	-
Loose sand, Hoyos et al. (2008)	98% Sand 2% Fines	SP	2.65	1.78	0.819

Results of Metrano Silty Sand obtained from Alramahi et al. (2007) and found in Vinale et al. (1999). Properties of soil tested for Hoyos et al. (2008) was found in Takkabutr (2006).

Vinale et al. (1999) conducted resonant column test on compacted Metrano silty sand at net normal stresses of 100, 200 and 400 kPa. The soil was de-saturated from full saturation by subjecting the soil to matric suctions ranging from 20 to 400 kPa. Results were presented in term of small-strain shear stiffness with matric suction at different net normal stress. However, Alramahi et al. (2007) quoted the paper and presented them in term of V_s with matric suction for specimens which were compacted at optimum water content and wet of optimum water content. Plots of V_s with σ_3' or $[(\sigma_3 - u_a) + (u_a - u_w)]$ show different trends where the effect of matric suction can show lower or greater influence than that of isotropic effective stress. Matric suction of specimen compacted at optimum water content (FIG. 5.33a) exerts greater influence compared to the isotropic effective stress. The reverse is true for the specimen compacted at wet of optimum. This divergent effect created by the matric suction reduces at higher matric suctions.

Hoyos et al. (2008) conducted resonant column tests on SP soil at net normal stresses of 6.9, 17.25 and 34.5 kPa. The specimens were subjected to matric suctions of 6.9 to 110 kPa. Results were presented in term of small-strain shear stiffness with matric suction at different net normal stresses. A constant bulk

density of 1700 kg/m^3 was assumed based on Takkabutr (2006) to convert the shear modulus to V_s . In this case, effect of matric suction is lesser than σ_3' .

Trend of V_s with matric suction at different net normal stresses was shown to vary with different soil fabric and soil type. Nevertheless, it can be generally seen that the effect of matric suction on V_s is not isotropic and can be greater or lesser than the isotropic effective stress σ_3' as seen from FIGs. 5.32, 5.33 and 5.34.

5.3.6 Effects of degree of saturation and confining pressure on ξ

While it has been recognised that the damping ratio will decrease when the geo-material stiffens at increasing confining pressure and decreasing saturation (increasing matric suction), changes in damping ratio is generally not well-defined like V_p versus S_r . This is further illustrated in FIG. 2.4 where the difference between P-wave damping (ξ_p) and S-wave damping (ξ_s) can be subtle depending on the material tested and the testing frequency. However, it can be observed that while ξ_s increases with saturation, ξ_p will reach an inflection point and decrease such that it is lower than ξ_s . The S_r corresponding to the inflection point appears to vary with the specimens tested. In this section, the P- and S-wave damping ratios of the river sand specimens were obtained using the HTM. Figures were presented to illustrate the effects of confining pressure, degree of saturation and the difference between ξ_s and ξ_p at different matric suction. The matric suction can be correlated to the S_r using FIG. 5.11. The damping ratios plotted in this section are mean values. Mean ξ_s are derived by averaging ξ_s obtained from excitation frequencies of 3, 5 and 10 kHz. Mean ξ_p are derived by averaging ξ_p obtained from excitation frequencies of 10, 20 and 50 kHz.

As specimens of different height were tested, the ξ_p and ξ_s were first compared in FIG. 5.35 to ensure that the results obtained were independent of the amplitude of the incident wave caused by differences in height. The following also serves to verify the repeatability of the HTM. FIG. 5.35 shows that the deviation in damping ratio of the 50 and 100 mm height specimens were small especially for ξ_s (FIG.

5.35a, b). Both sets of data show similar trend hence illustrating the robustness of the HTM. Subsequent studies will henceforth be conducted using the 50 mm specimens.

FIG.5.35: Comparison of P- and S- wave damping for specimens at different height

Damping ratios of the specimens subjected to different isotropic net normal stresses are plotted with matric suction in FIG. 5.36. For the case of S-wave damping (FIG. 5.36a), decrease of damping ratio with matric suction is evident in specimen RS-US_100H. This trend is less pronounced at higher isotropic net normal stresses of 200 and 400 kPa. This is because at high confining pressure, the contribution from the matric suction in stiffening the soil skeleton is small. The ξ_s at different isotropic net normal stresses were seen to converge to a single value after a matric suction of about 10 kPa which correspond to the air-entry value of the specimen. The ξ_p was also seen to exhibit significant scatter prior to the air-entry value (10 kPa). As the specimens desaturate, ξ_p converges to a single value.

FIG.5.36: Comparison of (a) P- and (b) S- wave damping for specimens subjected to different net isotropic stresses

FIG. 5.36 was replotted by placing ξ_p and ξ_s obtained from similar isotropic net normal stress together in FIG 5.37. While ξ_p was observed to drop when the specimens neared saturation as per FIG. 2.4, some of the results before the air-entry value did exhibit significant fluctuations due to interference from the water phase. Generally ξ_s is higher than ξ_p unlike FIG. 2.4 where ξ_s is higher than ξ_p only at high degrees of saturation. This difference in behaviour can be attributed to the ease of flow of the pore fluid. The sandstone specimens (Murphy 1982, 1984) and the Vycour porous glass specimen in FIG. 2.4 have a porosity of 22 and 28%, respectively. It can be seen that as the porosity increases, the difference between ξ_p

and ξ_s reduces. Since the porosity of the river sand specimen is higher at 33%, it becomes highly possible that a different trend will be observed. The error bars in FIG. 5.37 show the maximum and minimum deviation of the respective damping ratios at different excitation frequencies from the mean value. Deviations from the mean value are about $\pm 10\%$ for ξ_p and $\pm 20\%$ for ξ_s .

Signals obtained from specimens RS-US_100H and RS-US_400H were analysed to investigate the effect of hysteretic saturation on ξ_p and ξ_s . FIGs. 5.38a and 5.38c show that for a specimen subjected to an isotropic net normal stress of 100 kPa, the damping ratio was shown to change with degree of saturation in the same manner as the hysteretic saturation curve (FIG. 2.13). This change is more evident for ξ_s compared to ξ_p due to the fluctuations. However, at higher isotropic net normal stresses (FIGs. 5.38b and 5.38d), the variation of damping ratio with degree of saturation is less obvious. Mean values of ξ_s become closer to that of ξ_p .

FIG.5.37: Comparison of P- and S-wave damping for (a) RS-US_100H, (b) RS-US_200H and (c) RS-US_400H

FIG.5.38: Comparison of P- and S-wave damping subjected to hysteretic saturation

5.3.7 Summary and Remarks

In this section, three techniques to ascertain their suitability in determining damping ratios from bender element signals were evaluated. The Hilbert transform method (HTM) was selected due to its robustness. The HTM was further verified by experimental and numerical means. For the experimental aspect, damping ratios of standard Ottawa 20-30 sand from resonant column tests in the literature were compared with those derived using HTM at different effective confining pressures. For the numerical aspect, damping ratios calculated from the simulated receiver waveform were compared with the input damping ratios for the viscoelastic model. The damping ratio was shown to agree well in both cases.

The effects of confining pressure and degree of saturation on dynamic parameters V_p , V_s , ξ_p and ξ_s were investigated. There is a unique relationship between V_p and degree of saturation S_r . Characteristics of P-wave travelling through a soil near full saturation were confirmed when comparing the time and frequency domains of the P-waves recorded with those of P-wave travelling through water. It was also shown

that V_p decreases when matric suction exceeds the air-entry value. Changes in effective stress (σ_3') or net normal stress ($\sigma_3 - u_a$) were not shown to affect the values of V_p significantly.

Based on V_s , matric suction was observed to act anisotropically. Results from experiments conducted and data obtained by other researchers show that matric suction can exert either a greater or lesser influence compared to isotropic effective stress. This effect is likely to be caused by differences in soil structure (extent of flocculation or dispersion during compaction of specimens) and soil type causing inhomogeneous saturation and should be investigated in future research.

Finally, ξ_p and ξ_s were determined from the wave signals obtained. It was observed that while damping ratio was shown to decrease with increasing confining pressure, the change is not significant especially at high isotropic net normal stresses. Even though ξ_p and ξ_s were shown to increase with degree of saturation, a fixed trend was not observed. Generally, ξ_p is higher than ξ_s except when close to full saturation. Furthermore, values of ξ_p and ξ_s are not close to each another and differ from that shown in FIG. 2.4. Winkler and Nur (1982) attributed this discrepancy to the dependency of the attenuation mechanism on both saturation and permeability. Winkler and Nur (1982) suggested that each medium of wave propagation will have a corresponding frequency which is more conducive for fluid flow and enhance the damping mechanism. As such, similar material tested at different frequencies can lead to different trends in damping ratio.

5.4 Test Series III: Dynamic properties of soil-rubber mixtures

Test Series II sees testing being conducted on river sand specimens where the damping ratio ranges between 1 and 4% for sand specimens. Hence, there is a need to show that the HTM is able to detect more varied changes in the damping ratio. In view of this, soil-rubber mixtures were used. Being a highly customised material, the damping ratio and stiffness of this engineered material can be varied by changing the rubber content. The response of wave propagating through materials

of different phases can be further studied by comparing the wave characteristics of the soil-rubber mixtures with the reconstituted specimens previously tested. The wave velocities and damping ratio obtained via the HTM were compared with those in the literature obtained using resonant column test.

5.4.1 Comparison of wave signals between the Ottawa sand, river sand and soil- rubber mixtures

The characteristics of the P-wave were seen to vary greatly before and after the air-entry value but remained relatively consistent regardless of the excitation frequencies of 10, 20 and 50 kHz. However, S-wave signals show significant variations which is worthy of further study. The S-wave signals at excitation frequencies of 3, 5 and 10 kHz were presented in both time and frequency domain in FIG. 5.39. Signals were obtained from Ottawa sand, river sand and soil-rubber mixture specimens of 50 mm diameter and 100 mm height. The specimens were subjected to either an effective confining stress or isotropic net normal stress of 200 kPa.

The time domain plots show that each signal can be broadly classified into two portions: The transient portion is the result of the forced excitation from the sinusoidal pulse and the free vibration portion which is more evident when the transient portion dies off.

The frequency content of the transient portion is largely dependent on the frequency of the excitation pulse. Take for example, at an excitation of 10 kHz, the dominant frequency of the receiver signals would be between 10 and 30 kHz (FIGs. 5.39b, d and f). This high frequency content can also be due to interference from P wave especially in a wet specimen as explained for Test Series I. The frequency content in the free vibration portion will correspond to the natural frequency of the specimen from which the damping ratio is calculated. Thus, it is important to excite the specimen close to its natural frequency to obtain a strong free vibration response.

Study of the frequency domain plots shows that natural frequency of the specimens generally lies below 5 kHz. However, when observing signals from higher excitation frequencies (10 kHz), it is easy to be mistaken that the free vibration frequency has shifted to 10 kHz and beyond (FIGs. 5.39a, c and e). This is due to the strong transient portion (which persisted and masked the weaker lower frequency components. Nevertheless, the existence of the lower natural frequency components can still be observed in the frequency domain (FIGs. 5.39b, d and f) and verified using the soil-rubber mixtures. FIGs. 5.39g and 5.39h show the time and frequency domains of the S-wave passing through pure dry river sand used for the soil- rubber mixtures. The higher frequency transient portion can be observed from approximately 1000 to 2000 μ s after which the free vibration portion reigns. This higher frequency portion is comparatively shorter compared to wet river sand and Ottawa sand S-wave signals since it is compacted to a void ratio of 0.60 rather than 0.5. The looser dry river sand damps out the high frequency wave more quickly. When rubber inclusion is increased to 60% by weight, the signal appears to enter the free vibration state immediately (FIGs. 5.39i and j). The higher frequency component from the excitation pulse cannot be found as the highly dissipative rubber chips have damped out the high frequency waves. The above analysis allows the response of the wave signal to be better understood and further justify the need to retain S-wave signals below 5 kHz during filtering for the determination of the damping ratio.

Time Domain**Frequency Domain****Dry Ottawa 20-30 sand** **River sand, $(u_a - u_w) = 0$ kPa or fully saturated** **River sand, $(u_a - u_w) = 38$ kPa** **Dry river sand for soil rubber mixture**

(to be continued on the next page)

FIG. 5.39: Collation of S-wave signals from different geo-materials

5.4.2 Wave velocities and stiffnesses of soil rubber mixtures

P and S wave velocities (V_p and V_s) were obtained by dividing the tip-to-tip distance of the bender elements by the travel time and are shown in FIG. 5.40 for effective confining pressures of 0 to 400 kPa. Arrival time was determined from signals at different excitation frequencies. At low confining pressure, poor contact condition between the specimen and the bender elements gave rise to signals of low signal to noise ratio (SNR). As such, the arrival time could not be determined and hence the wave velocities are not plotted in FIG. 5.40. The situation was worse for P-wave due to the smaller perturbation as mentioned in Lings and Greening (2001).

FIG. 5.40: (a) S- and (b) P-wave velocities of specimens with confining pressure

FIG. 5.40 was re-plotted as FIG. 5.41 to give the relationship of V_s and V_p with rubber content at effective confining pressures of 0, 100, 200 and 400 kPa. The V_s relationship with rubber content, M_{ru} , could be described using an equation of the following form:

$$V_s = V_{s,0} e^{-B_R M_{ru}} \quad (5.3)$$

where $V_{s,0}$ corresponds to the V_s of the specimens with 0% rubber chips, coefficient B_R is a curve fitting constant and M_{ru} is the rubber chips content by mass in percent.

FIG. 5.41: (a) S- and (b) P-wave velocity of specimens with rubber content

Values of $V_{s,0}$ shown were plotted with confining pressure and curve fitted with a logarithmic equation shown in FIG. 5.42a. Values of B were plotted with confining pressure and curve fitted with a linear equation in FIG. 5.42b. The coefficients $\bar{a}_1$, $\bar{a}_2$ and $\bar{a}_3$ have values of 62.03 m/s, 0.1846 and -184.8 m/s, respectively. The atmospheric pressure (p_{atm}) is taken as 100 kPa and is introduced to normalise σ_o . The coefficients b_1 and b_2 have values of 2.429×10^{-4} and 1.049×10^{-2} , respectively.

FIG. 5.42: Variations of (a) $V_{s,0}$ and (b) B_R with confining pressure

Poisson's ratio (ν) of the specimen can be calculated from both V_p and V_s using Equation 5.2. At an effective confining pressure of 400 kPa, ν increases from 0.381 to 0.426 as the rubber content changes from 0 to 100% (FIG. 5.43). The Poisson's ratios are reasonable when compared to the Poisson's ratio range of 0.47 – 0.49 obtained by O'Hara (1983) and Beatty (1981) for solid rubber specimens.

FIG. 5.43: Variation of Poisson's ratio with rubber content

Tests were performed on specimens at 15% rubber content to investigate the effect of segregation. The V_s of segregated and homogenous specimens versus confining pressure are shown in FIG. 5.44. FIG. 5.44 shows that V_s does not vary significantly with changes in the position of the segregated layer of rubber chips. Segregation of rubber chip gave rise to a maximum discrepancy in V_s of 19% compared to a homogeneous sand-rubber mixture. This shows that while a perfectly homogeneous mixture might be impossible, an entirely segregated specimen does not bring about drastic changes to V_s .

FIG. 5.44: Effects of segregation on V_s for a specimen with 15% rubber content

Propagation of S-wave through a homogenous and segregated specimen was analysed in greater detail in Table 5.8. The measured travel time of the segregated specimen was calculated using the average velocity of the segregated specimens (bottom, center and top) shown in FIG. 5.44. The height of the rubber and sand layer were 19 and 61 mm, respectively. Due to random errors in measurements, the height of the layers may show measurement errors of ± 1 mm. Range of calculated travel time and difference between the calculated and measured travel time are tabulated in Table 5.8. The difference between the calculated and the actual travel time was around 10% thereby confirming that the perfectly segregated specimens behaved as two separate materials: pure sand and pure rubber.

Table 5.8: Effects of segregation on V_s for a specimen with 15% rubber content

Confining Pressure (kPa)	100	200	400
V_s in sand (m/s)	189.5	225.0	268.7
V_s in rubber (m/s)	68.8	86.1	103.2
Calculated travel time (μs)	589 - 607	485 - 499	405 - 417
Measured travel time in specimen (μs)	533	447	397
Percentage Difference (%)	10.47 – 13.94	8.41 – 11.62	2.06 – 5.06

According to the theory of mixtures, properties of a composite material like soil-rubber mixtures can be predicted with a theoretical upper bound (Voigt model) and a theoretical lower bound (Reuss model). The upper and lower bounds are modified accordingly in the context of this study and presented in Equations 5.4a and b, respectively. The V_s calculated using Equation 5.4 is plotted with experimental values in FIG. 5.45. The V_s obtained experimentally were observed to trend towards the upper bound or Voigt model regardless of the segregation configuration.

$$V_{s,voigt} = \left(\frac{1}{M_s \rho_R + M_R \rho_S} \right) (M_s \rho_R V_{s,S} + M_R \rho_S V_{s,R}) \quad (5.4a)$$

$$V_{s,reuss} = (M_s \rho_R + M_R \rho_S) \left[\frac{V_{s,S} V_{s,R}}{M_R \rho_S V_{s,S} + M_s \rho_R V_{s,R}} \right] \quad (5.4b)$$

where M and ρ is the mass and density of the respective components of the mixture, the subscript S and R represents the sand and rubber chips in the sand-rubber mixtures.

FIG. 5.45: Comparison of experimental V_s with the Voigt and Reuss models

5.4.3 Damping ratios of soil-rubber mixtures using Hilbert transform method

S-wave signals were analysed to obtain the corresponding damping ratio as P wave signals were considerably weaker as seen in FIG. 3.11. The scatter in damping ratio obtained from different excitation frequencies was small at about 2 to 5% from the mean value. Maximum percentage deviation of damping ratio from the mean is approximately 10%. This suggests that the Hilbert transform method is capable of producing repeatable results from the S wave signals. For ease of presentation, the mean damping ratio is presented in FIG. 5.46 and fitted with a polynomial best-fit line. FIG. 5.46 shows that the effect of confining pressure on damping ratio is negligible.

FIG. 5.46: Variation of mean damping ratio with rubber content and confining pressure

Mean values of damping ratio obtained from the Hilbert transform method are plotted together with those collated from the literatures in FIG. 5.47. The data from the literatures are summarised in Table 5.9. Damping ratios from the literatures were obtained from resonant column tests. These damping ratios were extracted from the lowest shear strain as it is expected that the strain level associated with bender element tests will not exceed $10^{-4}\%$ (Leong et al. 2005). Plots in FIG. 5.47 are arranged according to the effective confining pressure applied to the specimens. The corresponding V_s was normalised with the square root of specific volume ($1+e$), and presented beside the damping ratio plots to compare the stiffness and soil structure of the different specimens. The following normalisation was used after considering the relationship put forth by Hardin and Richart (1963) and Rahman et al. (2012) where small-strain shear stiffness ($G_{\max}$) was shown to be a function of specific volume ($1+e$). Since V_s is proportional to the square root of $G_{\max}$, it becomes intuitive to use $V_s/\sqrt{1+e}$ to represent the stiffness of the specimen.

FIG. 5.47: Combined results of damping ratio and normalised V_s with rubber content at effective confining pressures of 100, 200 and 400 kPa

Data from the literature was labelled such that the first part stands for the author's designation. For literatures which have tested more than one type of soil rubber mixture, the alphabets were presided by a number linked to the specific mixture of sand and rubber. The second part was usually labelled with a "D", "PS" or "S" which stands for dry, partially saturated and saturated, respectively. The final part represents the effective confining pressure in kPa of the specimens tested.

Damping ratio is plotted with an error bar to illustrate its variation with frequency. The small error bar implies that the damping ratio is independent of the frequency. Trends of damping ratio obtained using Hilbert transform method was seen to agree

well with those extracted from the literature in FIG. 5.47a, c and e. Damping ratio increases as the stiffness decreases when sand is gradually replaced by the softer rubber chips. Results from Pamukcu and Akbulut (2006) appear to deviate from the trend described above. Despite being stiffer (FIG. 5.47d), it has a higher damping ratio (FIG. 5.47c). This could be because the specimen was prepared by adding a small amount of water to the soil-rubber mixture to reduce segregation between the rubber chips and sand. The introduction of moisture could have given rise to viscous damping which exists due to presence of pore fluid. Viscous damping adds on to the hysteretic damping from the aggregate to give a higher damping value (Bolton and Wilson, 1990). Damping ratio from Rocco (2012) was observed to be significantly higher than the rest. This is because unlike the rest, the soil-rubber mixture is made of softer kaolin and expanded polystyrene (EPS) which are of lower density. The overall softer specimen with higher void ratio is reflected by the lower normalised V_s values. The partially saturated specimens also have a minimum S_r of about 50% giving rise to a greater amount of viscous damping which contributes to the overall damping ratio.

The Hilbert transform method was hence proven to be a useful technique to determine the damping ratio from signals obtained from bender element. Nevertheless, it is important to note that determination of damping ratio requires signals of good quality. This is only possible if good contact condition between the medium and the transducer is achieved. Limitations pertaining to the medium e.g. large grain size leading to reduce contact area with bender elements might be difficult to overcome. However, operators could improve the contact conditions by increasing the confining pressure. Excitation frequency of the bender elements could tend towards the dominant frequency of the specimen to increase the signal to noise ratio. Alternatively, the wave travel path can be shortened to increase the amplitude of the waves when geo-materials of high damping ratios are tested.

5.4.4 Summary and Remarks

Test Series III provides a greater insight into the effects of excitation frequency and geo-material type on the response of S-wave signals. The need to retain S-wave signals below 5 kHz during filtering for the determination of damping ratio was further justified.

Both P and S-wave velocities were shown to decrease as the soil was being replaced by rubber chips. Wave velocities also increase with increasing confining pressure. The relationship between V_s , rubber content and effective confining pressure are expressed using an empirical relationship. While there is no available mean to quantify the degree of segregation in a composite material like the soil-rubber mixture, the effect of V_s on a perfectly segregated specimen was studied. Perfect segregation was shown to reduce V_s by a maximum of 19% thereby suggesting that intermediate degree of segregation will give rise to a much lower discrepancy. However, regardless of degree of segregation, the V_s of a composite material was found to tend towards the upper bound (Voigt model) according to the theory of mixture.

Damping ratio was shown to increase with rubber content. Trends of the damping ratio with rubber content and confining pressure agreed with the results from the literature. Increasing confining pressure was not seen to cause significant changes in the damping ratio of the dry specimen. The Hilbert transform method was hence shown to be a suitable method for the determination of material damping from signals derived from bender elements.

Table 5.9: Details of resonant column test conditions for the damping ratios shown in FIG. 5.47.

Author Designation	Soil Type	D_{50,soil}/D_{50,rubber} (mm), D50 ratio	G_{s,soil} G_{s,rubber}	Initial Void Ratio Range	Dimension (Dia×Ht in mm)
Senetakis et al. (2012) SS	Quarry Sandy Gravel	7.8/1.5, 5.2	2.67 1.10	-	71.1×142.2
Anastasadis et al. (2012a, b) AS	Sand of different origins	AS1: 0.27/2.8, 0.0964 AS2: 0.27/1.5, 0.18 AS3: 0.56/0.4, 1.4 AS4: 0.56/2.8, 0.2	2.67 1.10	0.43 - 0.65 0.57 - 0.67 0.56 - 0.62 0.42 - 0.59	71.1×142.2 35.7×82.2
Feng and Sutter (2000) FS	Ottawa Sand	0.725/3.375, 0.215	2.67 1.11	0.35 - 0.46	70.0×150.0
Pamukcu and Akbulut (2006) PA	Ottawa sand with 11% kaolinite	0.725/0.76, 0.954	2.65 1.125	0.37 – 0.40	71.0×140.0
Rocco (2012) RO	Kaolin	4×10 ⁻⁴ /300, 1.33×10 ⁻⁶	2.64 0.03	0.948 – 1.915	70.0×140.0

Note: Rocco (2012) mixes kaolin with expanded polystyrene (EPS).

5.5 Test Series IV: Cyclic triaxial and wave propagation test on unsaturated residual soil

In this test series, the dynamic properties of the residual soil were determined using a cyclic triaxial apparatus with bender elements incorporated. The residual soil specimens were sheared monotonically after the bender element and cyclic triaxial tests. Multi-stage consolidated undrained shearing tests were conducted on saturated specimens and constant water content shearing tests were conducted on unsaturated specimens.

5.5.1 Dynamic Properties of Residual Soil

Small-strain dynamic properties (stiffness and damping ratio) were obtained from wave propagation tests using bender elements embedded in the top and bottom platens. The P- and S-wave velocities obtained were used to calculate the shear, bulk, Young's modulus and Poisson's ratio. The damping ratio was obtained by processing the signals using the HTM. The strain level of the dynamic properties obtained from the bender element test is not expected to exceed $10^{-4}\%$ according to Leong et al. (2005). Dynamic properties at higher strain ranges (0.05 – 1%) were obtained using the submersible load cell, internal submersible LVDT, a pair of LDT and a pair of proximeters. The Young's modulus, damping ratio and Poisson's ratio were calculated based on the displacement measured by the internal submersible LVDT and the LDT at excitation frequencies of 0.2 and 0.5 Hz. The details are given in Appendix E.

Even though the dynamic properties from both internal LVDT and LDT were expected to be similar, the results in FIG. 5.48 show that the Young's modulus obtained from the LDT was 2 – 4 times higher than those obtained from the internal LVDT. For the damping ratio, results from the LDT were generally 1 – 2 times higher. This phenomenon was also observed by Tatsuoka et al. (1994) who conducted cyclic triaxial test on dense gravel. Instead of an internal LVDT, Tatsuoka et al. (1994) used a proximeter to measure the axial strain at the top cap.

The Young's modulus from LDT was generally shown to be two times higher than that of the proximeter. Some of the data are replicated in FIG. 5.49 for ease of reference. Tatsuoka et al. (1994) did not comment further about the discrepancy in the Young's modulus.

FIG. 5.48: (a) Young's modulus and (b) damping ratio of AMK2-US_50 subjected to 40 cycles of sinusoidal loading at 0.5 Hz with double amplitude axial strain (ϵ_{DA}) of 0.2%

FIG. 5.49: Young's modulus of dense gravel obtained from both LDT and proximeter at different void ratios and effective confining pressures (data from Tatsuoka et al. 1994)

Discrepancies in the Young's moduli E from LDT and LVDT can be investigated using Equation 5.5 provided by ASTM D3999-11 (2011). From the equation, it can be seen that the variables which cause the discrepancy between the transducers used

were the height of the specimen L and the double amplitude axial deformation S_{DA} . The strain range obtained from the displacement transducers are plotted with time in FIG. 5.50a. Double amplitude axial strain obtained from the internal LVDT corresponded to that programmed in the strain-controlled cyclic triaxial test at 0.2%. Double amplitude axial strain obtained from the LDT was 3.4 times smaller. The smaller strain at the same double amplitude load will translate to a steeper gradient of the hysteresis loop (FIG. 5.50b) giving rise to a larger E .

$$E = \left(\frac{L_{DA}}{S_{DA}} \right) \left(\frac{L}{A} \right) \quad (5.5)$$

where L_{DA} is the double amplitude load and A is the cross-sectional area of the specimen.

FIG. 5.50: Difference in the level of strain in LVDT and LDT expressed in the (a) time domain and (b) hysteresis stress-strain plot (Excitation frequency of 0.5 Hz, ϵ_{DA} of 0.2%)

The smaller strain registered by the LDT was largely attributed to the slippage between the rubber membrane and the soil specimen during cyclic loading. During cyclic loading, the rubber membrane did not follow the deformation of the specimen leading to significantly lower strain. This is further corroborated by FIG. 5.51 which shows the ratio of the double amplitude axial strain (ϵ_{DA}) of the internal LVDT to the LDT at different axial strains and excitation frequencies. The slippage is more evident at higher excitation frequencies where difference between the ϵ_{DA} of the internal LVDT and LDT increases.

FIG. 5.51: Variation of $\epsilon_{DA,LVDT}/\epsilon_{DA,LDT}$ with $\epsilon_{DA,LVDT}$ at different excitation frequencies.

This error was briefly mentioned in Hoque et al. (1997) who found that the loading and unloading of LDT during calibration using a micrometer calibrator (FIG. 3.25) does not equate to that experienced by the LDT during cyclic loading using a small strain amplitude. However, the calibration set-up was not representative of the condition experienced in a cyclic triaxial set-up. The test consists of two configurations: the first consists of attaching the pseudo hinges directly on the steel attachment (FIG. 5.52a) while the second consists of a rubber membrane being glued onto steel attachment followed by the pseudo hinges (FIG. 5.52b). While there were some discrepancies during the single load/unload test, it does not mimic the slippage between the rubber membrane and the soil specimen since the rubber membrane was glued onto the steel attachment. Furthermore, the LDT was also not subjected to repeated load/unload cycles.

FIG. 5.52: Calibration set-up (a) without and (b) with rubber membrane (adopted from Hoque et al., 1997)

Hoque and Tatsuoka (2004) further highlighted the need for the load and displacement measuring transducers to be of the same nature to prevent any time lag which would affect the determination of dynamic parameters. In other words, if the LDT (a strain gauge type device) is used to measure axial displacement, the load cell should also be a strain gauge type device. Since it has been shown that LDT measurements were not accurate due to slippage between the rubber membrane and the specimen, subsequent dynamic parameters were derived using measurements from the internal submersible LVDT. Hence, the LDT is only suitable for monotonic loading and not for cyclic loading due to slippage between the rubber membrane and the soil specimen.

In this section, small-strain stiffness and damping ratio obtained from the bender element tests were compared with those obtained at various strains using the cyclic triaxial test. Dynamic soil properties from the cyclic triaxial tests were averaged from those obtained at excitation frequencies of 0.2 and 0.5 Hz as there were negligible differences. The V_p and V_s obtained from bender element tests can be converted into K , G , E and ν using Equations 2.1, 4.9 and 5.2, respectively. However, E is normally derived from the cyclic triaxial test. Yet, in soil mechanics and geotechnical engineering, G is more useful. Hence, E from the cyclic triaxial test was converted to G using ν calculated from the radial and axial deformations. The shear modulus obtained from both bender element and cyclic triaxial tests are normalised as per FIG. 2.16a and plotted together in FIG. 5.53. Axial strain was converted to shear strain using Equation 5.6:

$$\gamma_{DA} = \left(\frac{\epsilon_{DA}}{1 + \nu} \right) \quad (5.6)$$

where γ_{DA} is the double amplitude shear strain in percentage, respectively.

FIG. 5.53: Variation of $G/G_{\max}$ with shear strain for specimens (a) AMK1 and (b) AMK2

The trend of $G/G_{\max}$ from AMK1 and AMK2 in FIG 5.53 were shown to agree well with those generalised from Vucetic and Dobry (1991) in FIG. 2.16a. The parameters derived from the bender element tests and cyclic triaxial tests were also tabulated in Table 5.10 and 5.11, respectively. Table 5.11 shows that G can be derived from either the v obtained from the V_p and V_s in the bender element test or the v obtained from the radial and axial deformation of the cyclic triaxial tests.

Table 5.10: Summary of results from bender element tests

Specimen Tag	V_p	V_s	G_{max}	K_{max}	E_{max}	v	ξ - P-wave	ξ - S-wave
	(m/s)	(m/s)	(MPa)	(MPa)	(MPa)		(%)	(%)
AMK1-S_100	1607.0	155.2	41	4378	124	0.495	1.36	4.62
AMK1-S_200	1551.3	192.2	63	4047	189	0.492	2.07	4.99
AMK1-S_350	1506.7	232.1	92	3774	275	0.488	0.85	4.12
AMK1-US_50	1578.9	250.0	109	4186	323	0.487	0.67	4.34
AMK1-US_100	1591.2	266.8	123	4198	364	0.486	0.67	4.52
AMK1-S_100	1544.1	159.8	44	4044	131	0.495	1.04	4.70
AMK1-S_200	1490.3	177.6	54	3750	162	0.493	1.19	4.92
AMK1-S_350	1498.0	210.8	76	3760	228	0.490	0.52	4.01
AMK1-US_50	488.4	194.3	63	312	176	0.406	1.14	4.49
AMK1-US_100	481.3	228.6	85	262	229	0.354	1.06	4.49

Table 5.11: Summary of results from cyclic triaxial tests

Specimen Tag	Strain (%)	γ (%)	E (MPa)	ν	1 - G (MPa)	2 - G (MPa)	ξ (%)
AMK1-S_100	0.05	0.0334	42	0.289	16	14	12.16
	0.1	0.0669	33	0.298	13	11	13.38
	0.2	0.1338	24	0.305	9	8	14.68
AMK1-S_200	0.05	0.0335	68	0.485	23	23	10.88
	0.1	0.0670	50	0.620	16	17	12.90
	0.2	0.1340	35	0.627	11	12	14.51
AMK1-S_350	0.05	0.0336	114	0.564	36	38	9.75
	0.1	0.0672	85	0.227	35	29	11.78
	0.2	0.1344	63	0.338	24	21	13.32
	0.5	0.3360	38	0.329	14	13	15.33
	1	0.6720	24	0.709	7	8	16.83
AMK1-US_50	0.05	0.0336	100	0.477	34	34	9.84
	0.1	0.0672	80	0.159	34	27	12.21
	0.2	0.1345	62	0.160	27	21	14.34
	0.5	0.3362	38	0.548	12	13	16.61
	1	0.6724	23	0.572	7	8	17.09
AMK1-US_100	0.05	0.0337	114	0.415	40	38	17.27
	0.1	0.0673	79	0.463	27	27	19.21
	0.2	0.1346	63	0.444	22	21	26.81
	0.5	0.3366	31	0.531	10	10	29.10
	1	0.6732	16	0.592	5	5	29.70
AMK2-S_100	0.05	0.0335	54	0.469	18	18	8.47
	0.1	0.0669	42	0.509	14	14	9.38
	0.2	0.1338	32	0.505	11	11	10.36
AMK2-S_200	0.05	0.0335	70	0.404	25	24	8.65
	0.1	0.0670	54	0.441	19	18	10.26
	0.2	0.1340	40	0.449	14	13	11.38
AMK2-S_350	0.05	0.0336	120	0.287	47	40	8.45
	0.1	0.0671	95	0.364	35	32	9.56
	0.2	0.1342	71	0.312	27	24	10.89
	0.5	0.3356	40	0.366	15	14	13.13
	1	0.6712	24	0.409	9	8	14.31
AMK2-US_50	0.05	0.0356	93	0.332	35	33	6.81
	0.1	0.0711	77	0.290	30	27	7.96
	0.2	0.1423	63	0.370	23	22	9.39
	0.5	0.3556	41	0.356	15	14	11.60
	1	0.7113	24	0.405	9	9	13.45
AMK2-US_100	0.05	0.0369	132	0.234	54	49	10.04
	0.1	0.0738	117	0.596	37	43	8.48
	0.2	0.1477	90	0.332	34	33	8.83
	0.5	0.3692	63	0.272	25	23	10.26
	1	0.7384	38	0.291	15	14	11.41

Note to Table 5.11:

- 1 - G Shear modulus obtained using v derived from the internal LVDT and proximeter of the cyclic triaxial test (column 5 of Table 5.11)
- 2 - G Shear modulus obtained using v from the P- and S-wave of bender element tests (Table 5.10).

While the deviation arising from the difference in v is small as observed in Table 5.11, it is still important to investigate the extent of this variation. The Poisson's ratios derived from both bender elements and cyclic triaxial tests are plotted in FIG. 5.54. Poisson's ratios corresponding to a strain of $1 \times 10^{-4}\%$ were derived from bender element tests. These values were found to approach 0.49 for saturated specimen and decreases to about 0.35 upon desaturation (FIG. 5.54b). The Poisson's ratios obtained from proximeter and internal LVDT were shown to exhibit some scatter even though most of the results were generally seen to increase slightly with axial strain. This observation agrees with the findings of Yokota and Konno (1980), Kokusho (1980) and Tatsuoka et al. (1994). Generally, v is not expected to exceed 0.5 (Yokota and Konno 1980, Kokusho 1980 and Tatsuoka et al. 1994) but FIG. 5.54 showed some anomalies. This is likely to be due to the dilatant nature of these stiff soils as suggested by Tatsuoka et al. (1994). The scatter in values could also be due to the degradation of the aluminium target sheet after prolonged testing which affects the sensitivity of the proximity sensors. Nevertheless, it should not be a cause of concern as discrepancy in v will not lead to a large change in G as demonstrated in Table 5.12.

FIG. 5.54: Variation of v with axial strain for specimens (a) AMK1 and (b) AMK2

Results derived from the bender element test were assigned a strain of $10^{-4}\%$ following the suggestion of Leong et al. (2005) and underlined in Table 5.12.

Results which are not underlined were derived from the cyclic triaxial test. The G in Column 4 was calculated from the E of Column 2 and ν of Column 3 using Equation 4.9. On the other hand, G from Column 5 was calculated using ν derived from bender element test which is underlined in Table 5.12. Most of the discrepancies in G lie within 10%. Furthermore, a 68% change in ν , will only lead to a 15% change in the G value. Variation of ν with shear strain was shown in FIG. 5.55. The Poisson's ratio was normalised with the minimum Poisson's ratio ($\nu_{\min}$) of each series. The specimen tag in FIG. 5.55 is labelled in two parts. The first part represents the author's name, TA for Tatsuoka et al. (1994) and K for Kokusho (1980). The second part represents the effective confining pressure applied to the specimen.

FIG. 5.55 shows that the increase in ν with shear strain is generally within 1.4 times (40%) of $\nu_{\min}$. Given that G is insensitive to changes in ν (Table 5.12), the proportionate change in G from the 40% increase in ν would be about 8.8%. This resulting change in G would be insignificant and thus the small-strain Poisson's ratio from V_p and V_s can also be used as an alternative to those derived from the cyclic triaxial test.

Table 5.12: Comparison of G obtained via different methods

ϵ_{DA} (%)	E (MPa)	ν	1 - G (MPa)	2 - G (MPa)	ν - Diff (%)	G Diff (%)
<u>AMK2-S 350</u>						
<u>0.0001</u>	<u>227.920</u>	<u>0.490</u>	=	<u>76.489</u>	<u>0.000</u>	<u>0.000</u>
0.05	120.145	0.287	46.693	40.320	-41.510	15.807
0.1	94.520	0.364	34.642	31.720	-25.647	9.210
0.2	70.929	0.312	27.032	23.803	-36.321	13.563
0.5	40.404	0.366	14.788	13.559	-25.267	9.061
1	24.402	0.409	8.662	8.189	-16.589	5.769
<u>AMK2-US 50</u>						
<u>0.0001</u>	<u>175.774</u>	<u>0.406</u>	=	<u>62.509</u>	<u>0.000</u>	<u>0.000</u>
0.05	92.850	0.332	34.843	33.020	-18.120	5.521
0.1	76.501	0.290	29.661	27.206	-28.671	9.026
0.2	62.717	0.370	22.884	22.304	-8.786	2.603
0.5	40.608	0.356	14.973	14.441	-12.296	3.681
1	24.465	0.405	8.704	8.700	-0.146	0.042
<u>AMK2-US 100</u>						
<u>0.0001</u>	<u>229.427</u>	<u>0.354</u>	=	<u>84.702</u>	<u>0.000</u>	<u>0.000</u>
0.05	132.465	0.234	53.657	48.905	-33.851	9.717
0.1	117.405	0.596	36.771	43.345	68.339	-15.167
0.2	89.592	0.332	33.622	33.076	-6.203	1.650
0.5	63.368	0.272	24.906	23.395	-23.187	6.458
1	37.657	0.291	14.590	13.903	-17.994	4.940

Note:

- 1 - G Shear modulus obtained using ν derived from the internal LVDT and proximeter of the cyclic triaxial
- 2 - G Shear modulus obtained using ν from the P- and S-wave of bender element tests.

FIG. 5.55: Variation of normalised Poisson's ratio with strain for specimens from the literature

The damping ratios are plotted against shear strain in FIG. 5.56. Normalised shear modulus (FIG. 5.53) and damping ratio (FIG. 5.56) of the specimens were shown to lie between the curves corresponding to $PI = 0$ and $PI = 30$ in Vucetic and Dobry (1991) thereby showing the robustness of these curves. Damping ratios from AMK2 (FIG. 5.56b) are shown to span between the curves of $PI = 0$ and $PI = 50$. The curves in FIG. 5.56 were not derived from bender element tests. Hence, some discrepancies in the damping ratios may be attributed to the different excitation modes. Even though the curves in FIG. 5.56 were derived from saturated residual soil specimens, it was found to be applicable to the unsaturated specimens as well. The S-wave damping ratio was again shown to be higher than the P-wave damping ratio. Damping ratios derived from P- and S-wave are also shown to form the lower and upper boundary of the curves from Vucetic and Dobry (1991) extrapolated to low strains of 0.0001%.

FIG. 5.56: Variation of ξ with shear strain for specimens (a) AMK1 and (b) AMK2

The shear moduli of the residual soils from Table 5.11 were also compared with those obtained from Tou (2003) who conducted cyclic triaxial tests on a specimen of residual soil from the Bukit Timah Granite and two specimens from the Jurong Formation. The residual soils were denoted BT, JF1 and JF2, respectively. Dimensions of the specimens are similar to those used in this study. The basic properties of the specimens from Tou (2003) were tabulated in Table 5.13. The specimens BT, JF1 and JF2 were tested at effective confining pressures of 28, 32 and 47 kPa, respectively. Shear moduli and damping ratios from Tou (2003) were plotted with those from specimens AMK1 and AMK2 in separate figures as shown in FIG. 5.57 and 5.58, respectively.

Shear moduli and damping ratios of specimens AMK1 and AMK2 exhibit similar trends with increasing shear strain as those from Tou (2003). However, the shear moduli were slightly higher than those from Tou (2003) as specimens AMK1 and AMK2 were tested at higher effective confining pressure leading to stiffer specimens. The reverse can be said for the damping ratio in FIG. 5.58 where the stiffer specimens gave rise to a slightly lower damping ratio.

Table 5.13: Basic properties of BT and JF soils (Tou, 2003)

Parameters	Jurong Formation		Bukit Timah Granite
	JF1	JF2	BT
Sand (%)	32		63
Silt and Clay (%)	68		37
PL	30		32
LL	55		55
G_s	2.71		2.62
USCS Symbol	CH		SC
Bulk Density (kg/m^3)	1926	1917	2434
Void Ratio	0.752	0.756	0.357

(to be continued on the next page)

FIG. 5.57: Comparison of G from specimens (a) AMK1 and (b) AMK2 with Tou (2003)

FIG. 5.58: Comparison of ξ from Tou (2003) with specimens (a) AMK1 and (b) AMK2

5.5.2 Mechanical Properties of Residual Soil

Upon completion of both the bender element and the cyclic triaxial tests, the residual soil specimens were sheared to failure. Saturated specimens were subjected to a multi-stage consolidated-drained test at effective stresses of 100, 200 and 350 kPa. Chin's method (Chin 1970; 1971, Chin and Vail 1973) was employed to determine the failure stress for the first two stages. The specimen was sheared to failure for the final stage. Unsaturated specimens were sheared to failure using the constant water content condition. The change in deviator stress, pore-water pressure and matric suction with axial strain for both AMK1 and AMK2 specimens are plotted in FIG. 5.59.

FIG. 5.59: Variation of (a, b) deviator stress, (c, d) pore water pressure and (e, f) matric suction.

Pore-water pressure at failure (u_{wf}) was observed to be higher than the initially imposed pore-water pressure of the specimen due to the development of excess u_w with cyclic loading in the undrained condition. Cyclic loading was carried out in the undrained water condition to prevent the specimen from consolidating with the applied vibration. However, it can be seen from FIGs. 5.59c and d that u_w quickly reached a steady value with shearing. Pore-water pressure of AMK2-S_350 shows slight fluctuation during shearing. This is probably due to the presence of soil particles of different sizes and alignments along the failure plane getting realigned during shearing. It is however, interesting to note that the build-up of pore-water pressure for the unsaturated specimen can be higher than that of the imposed pore-air pressure (u_a) during shearing. This indirectly leads to the development of a negative matric suction or excess pore-water pressure in the specimen as shown in FIGs. 5.59e and f.

Matric suctions of AMK1-US_50, AMK1-US_100 and AMK-US_50 were found to drop to -53.76, -43.86 and -6.56 kPa, respectively, during shearing. While matric suction has been observed to decrease by about 100 kPa in constant water content test (Rahardjo et al. 2004), it was not expect to become negative (i.e., positive pore-water pressure) as shown in FIG. 5.59e and f. This is especially so when the above mentioned specimens were partially saturated at S_r of 91.9, 88.2 and 84.7%, respectively. One probable reason might be the accumulation of a thin layer of water at the bottom of the specimen that behaved in the undrained pore-water condition. Excess pore-water pressure builds up in the trapped water and the pore-water pressure transducer connected to the bottom platen will hence registers the positive excess pore-water pressure which is not representative of that in the specimen. It is expected that the change in pore-water pressure is negligible as pore-air pressure is drained. Hence, the pore-water pressure measurements were ignored in the analyses. For more reliable measurement of the pore-water pressure and hence matric suction in a constant water content test, it is imperative to measure the pore-water pressure along the height of the specimen using high suction tensiometers such as those described in He et al. (2006).

The stress-state variables used for construction of the Mohr circle were derived from the plot of deviator stress vs axial strain (FIGs. 5.59a and b). The stress-state variables corresponding to the peak deviator stress for specimen AMK1-US_50 were used to construct the Mohr circle. With the exception of specimen AMK1-US_50, the other specimens did not showed a peak deviator stress. In order to obtain a consistent estimation of the failure deviator stress, Chin's method was applied on these specimens as well.

The Mohr circles and the effective stress path of the saturated consolidated-undrained multi-stage tests were presented in FIGs. 5.60 and 5.61 for AMK1-S and AMK2-S, respectively. The cohesion intercept (c') and friction angle (ϕ') shown in FIGs. 5.60a and 5.61a were used to determine the intercept (a') and angle of the failure line (ψ') in the p' - q plot (FIG. 5.58b and 5.59b). Equation 5.7 was used to correlate the coefficients of the failure lines in both plots.

FIG. 5.60: (a) Mohr circle and (b) effective stress path of AMK1-S

FIG. 5.61: (a) Mohr circle and (b) effective stress path of AMK2-S

$$\sin\phi' = \tan\psi' \quad (5.7a)$$

$$c' = \frac{a}{\cos\phi'} \quad (5.7b)$$

The multi-stage test enables the shear strength parameters to be determined for the same specimen thus reducing errors due to specimen inhomogeneity. The effective friction angle ϕ' and effective cohesion c' was found to lie within the range reported in the literature shown in Table 5.14. Effective friction angle ϕ' was not shown to vary significantly with depth since the grain size distributions (FIG. 3.19) of the specimens at different depths are almost similar. On the other hand, c' was observed to increase with depth. Since ϕ' was not shown to change significantly with depth, the same ϕ' was used for the failure envelope for the unsaturated AMK1 and AMK2 soil specimens. For the constant water content tests, it was assumed that the change

in matric suction was negligible as pore-air pressure was drained. Hence, the extended Mohr-Coulomb plots for AMK1 and AMK2 soil specimens are plotted using the imposed matric suctions of 50 and 100 kPa in FIGs. 5.62 and 5.63, respectively.

Table 5.14: Effective shear strength parameters of Bukit Timah granite (adopted from Leong et al. 2002)

References	Range of c' (kPa)	Average c' (kPa)	Range of ϕ' (°)	Range of ϕ' (°)
Dames and Moore (1983)	0 – 125	0	13 – 36	30
Poh et al. (1985)	0 – 42	-	20 – 35	-
Yang and Tang (1997)	5 – 10	-	35 – 40	-
Tan et al. (1987)	0 – 40	15	30 – 35	32
Rahardjo (2000)	12 – 50	26	29 – 33	30
	0 – 14	9	27 – 31	29
Winn et al. (2001)	-	-	20 – 40	-
Zhou (2001)	-	7	-	32

FIG. 5.62: Extended Mohr circle plot for both saturated and unsaturated specimens of AMK1

FIG. 5.63: Extended Mohr circle plot for both saturated and unsaturated specimens of AMK2

The total cohesion intercept for AMK1-S, AMK1-US_50 and AMK1-US_100 (FIG. 5.62) are 16, 32 and 0 kPa, respectively. The total cohesion intercept for AMK2-S, AMK2-US_50 and AMK2-US_100 (FIG. 5.63) were 32, 44 and 68 kPa, respectively. While total cohesion intercepts were generally shown to increase with matric suction (FIG. 5.64), AMK1-US_100 showed otherwise. The anomaly is likely due to the heterogeneity in the specimen causing it to differ from other tests. The difference could be further observed in FIG. 5.59a where the deviator stress at failure for AMK1-US_100 was smaller compared to that of AMK1-US_50. Thus, AMK1 demonstrated the difficulties associated with testing undisturbed residual soils which may be heterogeneous. FIG. 5.59b, on the other hand, shows that AMK2 exhibited the expected trend whereby the deviator stress at failure increases with increasing matric suction.

The cohesion intercepts from FIGs. 5.62 and 5.63 are plotted against matric suction together with data from Rahardjo et al. (2004) in FIG. 5.64. Rahardjo et al. (2004)

conducted both consolidated-drained (CD) and constant water (CW) test on compacted residual soil from Jurong sedimentary formation. The cohesion intercept of the failure envelope was derived from a constant friction angle of 31.5° . This value was obtained from the CD triaxial test results and assumed to be similar for all the CW tests (Rahardjo et al., 2004). The numbers following the “CW” label in FIG. 5.64 represents the net confining stress in kPa of the specimen. The plot shows an increase in total cohesion intercept with matric suction. The test results concur with the observations in this study except for the anomaly present in AMK1-US_100. The specimens from larger depths (AMK2) were shown to exhibit higher values of total cohesion compared to specimens from shallower depths (AMK1).

FIG. 5.64: Cohesion intercept with matric suction from Test Series IV plotted with results from Rahardjo et al. (2004)

5.5.3 Summary and Remarks

Test Series IV demonstrated how bender elements can be incorporated into the cyclic triaxial test to obtain both the dynamic properties at various strains as well as the mechanical properties of the specimen. The Poisson's ratio obtained from the bender element test can be used to check the Poisson's ratio obtained from axial and lateral deformations in the cyclic triaxial test. The LDT was found to have limited use during cyclic testing as it was unable to deform with the specimens due to

slippage between the rubber membrane and the specimen. Discrepancies between LDT and internal submersible LVDT was observed to increase with increasing frequency of cyclic loading. Results of $G/G_{\max}$ and ξ with shear strain were shown to agree with the literature.

Pore-water pressure observed during shearing at constant water content condition was deduced to be unreliable as it was not expected to become positive during cyclic loading. It is suggested that for future studies, pore-water pressure measurement should be conducted along the height of the specimen so that the change in the matric suction can be more accurately measured. The improvements made to the modified cyclic triaxial set-up will allow the effects of cyclic loading followed by shearing on pore-water pressure response of the soil specimen to be more effectively studied and analysed. Despite the slight discrepancies, the effective shear strength parameters (ϕ' and c') of the undisturbed residual soils fall within the range shown in the literature. The total cohesion is also shown to increase with matric suction.

CHAPTER 6: Conclusion and Recommendations

6.1 Summary

The objective of this study is to make use of laboratory seismic methods to determine soil properties. More focus was given to the development of a testing framework to determine the small-strain soil stiffness and damping ratio from wave propagation test using bender/extender elements. Verifications were performed using both experiments and numerical modelling. The bender/extender elements were incorporated into a cyclic triaxial set-up and undisturbed residual soil specimens were tested

Four test series were performed. Test Series I studies the response of the bender/extender elements and ultrasonic transducers used for wave propagation test. The bender/extender elements were selected to be used for subsequent tests. Test Series II evaluates three different techniques for the determination of damping ratio of reconstituted sand specimens using bender elements test signals. The Hilbert transform method (HTM) was evaluated to be the most robust and reliable. The HTM was further verified by comparing the damping ratios ξ_s obtained with those from resonant column test and also via numerical simulation using LS-DYNA. The V_p , V_s , ξ_p and ξ_s at various net confining pressures and matric suctions were studied. In Test Series III, bender element tests were conducted on soil-rubber mixtures. This test series served to investigate the effect of the different phases in a geo-material on the wave signature. It also tests the robustness of the HTM by comparing the wave velocities and damping ratio obtained by HTM with those from the literature using resonant column tests. Finally, in Test Series IV, undisturbed residual soils were tested in a cyclic triaxial set-up modified to include a pair of bender/extender elements. The properties of the residual soil specimens over a large strain range were investigated together with the capabilities of the various transducers used in the set-up.

6.2 Conclusion

The significant findings of this research are arranged according to the respective test series and presented below.

6.2.1 Test Series I

In Test Series I, response from three different configurations were studied: ultrasonic system, bender/extender element system and the hybrid bender element – ultrasonic system. Investigation of this nature was seldom conducted especially in the context of geotechnical laboratory testing. The findings from this section were published in Cheng and Leong (2014).

Commercially available ultrasonic platens were found to be unable to obtain reliable S-wave signals due to poor transducer design. The lack of a proper backing and matching layer promotes ringing after excitation and reduces the transmissivity of the S-wave signal. Furthermore, resonant frequencies of P- and S-wave piezo-crystals were too close and cannot be separated using digital signal processing. The hybrid system acts as an alternative to the ultrasonic system and its response provided better understanding of the working principles of the ultrasonic platens. More importantly, this test series highlighted the importance of not only to understand the equipment used but also to conduct verification tests using different materials in order to determine the limitation of the equipment. In this way, reliable and consistent results can be derived.

Between the hybrid system and the bender elements, the bender elements were shown to have consistently stronger S-wave signals regardless of the specimen type and experience lesser interference from P-wave. Hence, bender elements were selected. Despite popularity of bender elements in geotechnical laboratory tests, no standard for bender element tests is available to guide testing. A framework was formulated upon reviewing numerous literatures in order to produce a set of

procedures that can be used to obtain good quality signals for the determination of wave velocities and material damping ratio. It is recommended that S-wave signals be obtained by exciting the bender element at lower frequencies of 3, 5, 10 and 20 kHz while P-wave signals be obtained by exciting the extender element at higher frequencies of 10, 20 and 50 kHz. The following are derived from the fact that the natural frequency of the excitation mode increases in the order of flexural, torsional and longitudinal. Although low frequencies S-wave can lead to a reduced L_{tt}/λ ratio and hence significant near-field effect, it will give rise to a receiver signal of higher signal-to-noise ratio (SNR) and hence allows the damping ratio to be determined. Furthermore, S-wave signals at different excitation frequencies also allow the region in the receiver signals affected by the near-field effect to be identified so that the arrival time can be more accurately determined. It is hope that that this study will serve as a reference for developing a standard for bender element testing so that users can employ the tests with greater confidence.

6.2.2 Test Series II and III

In Test Series II, three techniques used to determine damping ratio from bender element receiver signals were evaluated. The Hilbert transform method (HTM) which has not been applied in the field of geotechnical laboratory testing was introduced. The HTM was deemed the most robust and reliable out of the three techniques. Procedures for the HTM were developed and elaborated. The HTM was applied on standard Ottawa 20-30 sand in Test Series II and soil-rubber mixtures in Test Series III for verification purposes. Damping ratio from HTM was found to agree well with those determined from the resonant column test reported in the literature. The test results were reported in Cheng and Leong (2015a). Numerical simulation was also conducted to verify the HTM. The introduction of the HTM allows the bender element tests to overcome its limitation of not being able to determine the damping ratio.

Variations of V_p , V_s , ξ_p and ξ_s with changing net confining pressure and S_r were investigated in Test Series II. The relationship of V_p with Skempton B-value,

matric suction and S_r were examined. The limitation of the B-value was highlighted and V_p was proposed as a better proxy for S_r compared to Skempton B-value. The V_p was found to change significantly at S_r greater than about 80 - 90% and reached about 1500 m/s when close to full saturation. Changes in effective stress or net normal stress were not shown to affect the V_p value at matric suction less than the air-entry value. Even though numerous studies have been conducted to discuss the contribution of matric suction to the shear strength, its contribution to the soil structure which is reflected in its stiffness and hence V_s has seldom been discussed. The experiments demonstrated that the effect of matric suction on V_s is not isotropic compared with isotropic effective confining stress. The extent of influence may vary with soil type. The findings have been reported in Cheng and Leong (2015b). The contribution of matric suction to soil stiffness will be valuable to engineers conducting numerical simulations on soils subjected to repeatedly drying and wetting cycles.

In geotechnical engineering, damping ratio is commonly determined using the resonant column test. However, it is seldom analysed according to the vibration mode i.e. ξ_p and ξ_s . Damping ratios ξ_p and ξ_s were found generally to decrease with increasing confining pressure and increase with increasing S_r . Trends of ξ_p and ξ_s with S_r are not fixed but ξ_p is generally higher than ξ_s except when close to full saturation. This was however not observed but ξ_p and ξ_s are close to each another. The discrepancy can be attributed to the permeability of the specimen but requires further research for verification. The closeness of ξ_p and ξ_s and the lack of a typical trend could also suggest that the determination of their respective values for practical purposes may not be advisable in the face of greater uncertainties. For engineering practice, it is more practical to consider a range of probable values.

Signals from both Test Series II and III were also reanalysed in the time and frequency domains. By comparing the wave characteristics found in different specimens subjected to various conditions, different aspects of the wave characteristics in the time and frequency domains can be attributed to variables such as the degree of saturation and the excitation frequency. This allows better

understanding of wave propagation in soil and serves to connect the theoretical framework with the experimental observation.

Further investigation of the bender element and resonant column tests using the finite element method provided deeper understanding of the tests and interpretations. In finite element simulation of a simplified resonant column test, it was found that there will always be some discrepancies between the input Young's modulus and those derived from the Cascante et al. (1998) equation. This was attributed to the Timoshenko's beam effect present in short and stocky specimens. A correction factor χ_m was introduced to correct the Young's modulus obtained from Cascante et al. (1998) equation. Damping was simulated in the finite element model using a viscoelastic material model. The basic concept of the viscoelastic model was reviewed. Literature review shows that the input parameters of a viscoelastic model were generally calibrated with experimental results hence there is a lack of understanding of the effects of the various input parameters. In order to achieve a better understanding of the viscoelastic model (Zener model) used in this study, the various input parameters (G_0 , G_∞ and β) were studied. Relationships of G_0 , G_∞ and β with the simulated stiffness and damping ratio were studied. An equation was proposed to correlate the decay constant β with the damping ratio.

6.2.3 Test Series IV

In Test Series IV, a modified cyclic triaxial set-up consisting of bender elements was used to perform dynamic and mechanical tests on undisturbed residual soil specimens. The cyclic triaxial tests reviewed the limitations of the local displacement transducer (LDT) in obtaining axial strains. The LDT mounted on the rubber membrane was unable to follow the deformation of the specimen. Discrepancies between the readings from the LDT and LVDT were shown to increase at higher excitation frequencies. This discrepancy was attributed to slippage between the membrane and the soil specimen and has not been identified previously by researchers as the LDT was usually used during monotonic shearing.

Normalised shear modulus and damping ratio obtained from the modified cyclic triaxial setup was shown to agree with the curves of Vucetic and Dobry (1991).

Multi-stage shearing was conducted on saturated undisturbed residual soil specimens in the undrained condition. Unsaturated undisturbed residual soil specimens were sheared at constant water content conditions i.e. under drained pore-air but undrained pore-water condition. During shearing, the pore-water pressure was observed to increase beyond that of the imposed pore-air pressure resulting in positive pore-water pressure. This is likely due to the pore-water pressure transducer registering the pore-water pressure build-up in a thin water layer that accumulated at the bottom of the specimen. The effective shear strength parameters were found to fall within the range reported in the literature. The effective friction angle did not show large changes since the grain size distribution of the specimens did not vary significantly with depth. As such, the friction angle obtained from the saturated specimens was applied to the unsaturated specimens of the same group. The total cohesion was observed to increase with sample depth and matric suction. The study discovered some equipment limitations especially when they are extended beyond their testing norm. Other than the LDT, the testing of unsaturated soil in constant water conditions would have to be further investigated in order to address the issues mentioned above. A proper testing standard for unsaturated soil should be eventually developed so that constant water content test and multi-stage test can be better accepted by the engineering community and employed in the analysis of engineering problems.

6.3 Recommendations and applications

Due to constraint of time and resources, some aspects of the research that deserve further study were not investigated. These aspects are recommended for future research. The recommendations for future research are arranged according to the respective test series and presented below.

6.3.1 Test Series I

While bender elements are reliable, its intrusive nature can be overcome by ultrasonic transducers. However, current design of ultrasonic transducers needs to be improved to overcome the problems highlighted in the current study to make them more suitable for testing geo-materials.

6.3.2 Test Series II and III

Testing unsaturated soils involve long testing time due to the need to allow the specimen to reach matric suction equilibrium at the stipulated stress-state conditions. The sand used in Test Series II took close to a month to complete one drying cycle. Hence, one would expect the testing time of clayey soils which have lower permeability to be longer. Furthermore, clayey soils desaturate at significantly higher matric suction. Future research should be conducted on clayey soils following the same procedures as those used for sand in this study to investigate the behaviour of V_s , ξ_p and ξ_s with S_r to establish more general relationships. The extent of influence from matric suction was also found to vary with soil type and structure while determining V_s at different matric suction. Further research will be carried out to confirm the effect of soil type and structure on the matric suction through V_s . As testing of unsaturated soil is known to be time consuming, it will be convenient if subsequent works can determine the changes in SWCC due to factors such as the effective confining pressure and soil type. In this way, the stresses imposed on the soil can be more easily determined from the in-situ conditions.

In the finite element modelling, the viscoelastic model used in this study is the Zener model. A more comprehensive viscoelastic model is the Biot hysteretic model which is an extension of the Zener model with multiple Maxwell models arranged in parallel configuration. The Biot hysteretic model should be investigated to see if it offers any advantages compared to the Zener model in particular to quantify the frequency range whereby the decay constant β can reflect the input damping ratio.

The soil-rubber mixture tested in Test Series III holds significant potential as a seismic barrier. Future studies should involve the quantification of viscous damping in unsaturated and saturated soils thereby enabling the determination of the range of the damping ratio for the geo-material studied. This will serve to further enhance the chart from Vucetic and Dobry (1991) and allow better estimation of the damping ratio. Accurate and reliable determination of both the dynamic and mechanical properties of the soil-rubber mixtures at different strains, confining pressures and degrees of saturation will also further promote their application in practice.

6.3.3 Test Series IV

During the course of the cyclic triaxial tests, some equipment limitations were discovered. One of which is the inability of the LDT to follow the deformation of the specimen during cyclic loading. It is suggested that the LDT should be tested at different strain rates (from monotonic to dynamic loading) in order to quantify the severity of slippage between the specimen and the LDT and to find a solution to avoid the slippage problem.

It was also observed that the Poisson's ratio obtained from axial and lateral deformations in the cyclic triaxial test showed scatter. This was likely attributed to the weak flux-cutting ability of the aluminium target in the electromagnetic field generated by the proximity sensor. The aluminium target was observed to disintegrate with prolonged testing duration which might have further degraded the quality of the data. In view of this, it is proposed that other materials be used as the target to improve the performance of the proximity sensor.

Finally, the matric suction from the constant water content test was observed to become positive pore-water pressure during shearing. This was attributed to the position of the pore-water pressure transducer (bottom of the specimen). It was suspected that a thin layer of trapped water at the bottom of the specimen generated

excess positive pore-water pressure which was not reflective of the pore-water pressure in the specimen. In view of this, it is recommended that miniature tensiometers be placed at different heights of the specimen so that the variation in the matric suction during the cyclic triaxial tests can be measured more accurately.

6.3.4 Applications

This thesis first introduced the wave propagation test and provided a thorough review of both the ultrasonic transducer and the bender elements. Potential complications related to the usage of the ultrasonic transducers were highlighted for future improvement in design to increase their reliability and accuracy.

The framework developed in this thesis for the determination of P- and S-wave velocities and damping ratio using the bender element test is useful for standardisation of the bender element test. The Hilbert transform method has been verified to be robust and reliable in the determination of the material damping ratio. With this methodology, the contribution of different fluids to viscous damping in geo-materials can be more efficiently investigated.

Finite element simulation of the resonant column for calibration purpose also identified the short coming of the currently available equations to determine E from flexural mode vibrations. Young's modulus calculated from these equations (Cascante et al. 1998, Kumar and Madhusudhan 2010) was found to be unable to correct for the effect due to short and stocky specimens otherwise known as Timoshenko's beam effect. It is hoped that by highlighting this shortcoming, more care are taken when preparing resonant column test samples. The review of the viscoelastic model and its corresponding parameters gave a better insight of the various parameters that are used as input.

The soil-rubber mixture mentioned in Test series III has shown great potential to be used as a seismic barrier material. Although suggestions have been made to use them to shield building foundation (Tsang et al. 2007, Tsang 2008) and even

underground structures like tunnels (Tsang 2009) from vibrations and shockwave, more research is needed before field implementation can be made. Furthermore, it can be used as a replacement for soft soil to minimize amplification of ground vibration (Dobry and Iai 2000).

Test Series IV illustrated the versatility of the cyclic triaxial set-up. Through the incorporation of the bender element, the cyclic triaxial set-up was able to obtain dynamic soil properties over a wider strain range. The dynamic soil properties obtained were shown to agree with the trends proposed by Vucetic and Dobry (1991).

Unlike the resonant column, the cyclic triaxial set-up was also able to determine the mechanical properties of the soil specimens. While this provides an edge over the resonant column test, equipment limitation results in a gap in the testing strain range (1×10^{-3} - 1×10^{-2}) as shown in FIG 1.1. With more extensive research and improvement in sensor technology, it is hoped that this gap can be closed in future geotechnical testing equipment.

References

Ackerley, S.K., Hellings, J.E. R.J. Jardine, R.J. (1987), “Discussion: A new device for measuring local axial strain strains on triaxial specimens” (paper by Clayton and Khatrush, 1986), Géotechnique, Vol. 37, No. 3, pp. 413–417.

Agneni, A. and Balis-Crema, L. (1986). “Damping measurements by Hilbert transform on composite materials”, Damping Proceedings Volume 2, Las Vegas, Nevada, United States, FI1–FI13.

Airey, D. and Moshin, A.K.M. (2013), “Evaluation of shear wave velocity from bender elements using cross-correlation”, Geotechnical Testing Journal, Vol. 36, No. 4, pp. 1-9.

Alramahi, B., Alshibli, K.A., Fratta, D. and Trautwein, S. (2007), “A suction-control apparatus for the measurement of P and S-wave velocity in soils”, Geotechnical Testing Journal, Vol. 31, No. 1, pp. 1-12.

Anand, S., Leong, E.C. and Cheong, H.K. (2000), “Measurement of shear modulus profile using a continuous surface wave measurement system”, Proceedings of the 4th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, San Diego, USA, Paper No. 1.53, pp. 1-6

Anastasiadis, A., Senetakis, K. and Pitilakis, K. (2012a), “Small-strain shear modulus and damping ratio of sand-rubber and gravel-rubber mixtures”, Geotechnical and Geological Engineering, Vol. 30, No. 2, pp. 363-382.

Anastasiadis, A., Senetakis, K., Pitilakis, K., Gargala, C. and Karakasi I. (2012b), “Dynamic behaviour of sand/rubber mixtures, Part I: Effect of rubber content and duration of confinement on small-strain shear modulus and damping ratio”, Journal of ASTM International, Vol. 9, No. 2, pp. 221-247.

Anderson, K.F. (1995), A Conversion of Wheatstone Bridge to Current-Loop Signal Conditioning for Strain Gages, National Aeronautics and Space Administration, Dryden Flight Research Facility, Edwards, California.

Anderson, K.F. (1998). "Your successor to the Wheatstone Bridge? NASA's Anderson Loop." IEEE Instrumentation & Measurement Magazine, Vol. 1, No. 1, pp. 5–15.

Argawal, T.K., and Ishibashi, I. (1991), "Multi-directional wave velocity by Piezoelectric Crystals," Proceedings of the Recent Advances in Instrumentation, Data Acquisition and Testing in Soil Dynamics, Orlando, Florida, United States, ASCE, pp. 102-117.

Arroyo, M., Medina, L. and Wood, D.M. (2002), "Numerical modelling of scale effects in bender-based pulse tests", Proceedings of the 8th International Symposium Numerical Models in Geomechanics, Rome, Italy, pp. 589-594.

Arroyo, M., Wood, D.M. and Greening, P.D. (2003a), "Source near-field effects and pulse tests in soil samples", Géotechnique, Vol. 53, No. 3, pp. 337-345.

Arroyo, M., Greening, P.D. and Wood, D.M. (2003b), "An estimate of uncertainty in current laboratory pulse test practice", Rivista Italiana di Geotecnica, Vol. 37, No. 1, pp. 33-51.

Arroyo, M., Wood, D.M., Greening, P.D., Medina, L. and Rio, J. (2006), "Effects of sample size on bender-based axial $G(0)$ measurements", Géotechnique, Vol. 56, No. 1, pp 39-52.

Arroyo, M., Pineda, J.A. and Romero, E. (2010), "Shear Wave Measurements Using Bender Elements in Argillaceous Rocks", Geotechnical Testing Journal, Vol. 33, No. 6, pp. 1-11.

Arulnathan, R., Boulanger, R.W., Riemer, M.F. (1998), “Analysis of Bender Element Tests,” Geotechnical Testing Journal, Vol. 21, No. 2, pp. 120–131.

ASTM (2007), D4015-07 Standard test methods for modulus and damping of soils by resonant-column method, ASTM International, West Conshohocken, PA, USA.

ASTM (2008), D2845–08 Standard test method for laboratory determination of pulse velocities and ultrasonic elastic constants of rock, ASTM International, West Conshohocken, PA, USA.

ASTM (2010), D2216-10 Standard Test Methods for Laboratory Determination of Water (Moisture) Content of Soil and Rock by Mass, ASTM International, West Conshohocken, PA, USA.

ASTM (2011), D3999-11 Standard test methods for the determination of the modulus and damping properties of soils using the cyclic triaxial apparatus, ASTM International, West Conshohocken, PA, USA.

ASTM (2011), D4767-11 Standard test method for consolidated undrained triaxial compression test for cohesive soils, ASTM International, West Conshohocken, PA, USA.

ASTM (2011), D7181-11 Method for consolidated drained triaxial compression test for soils, ASTM International, West Conshohocken, PA, USA.

ASTM (2013), C778-13 Standard Specification for Standard Sand, ASTM International, West Conshohocken, PA, USA.

Atkinson, J.H. (2000), “Non-linear soil stiffness in routine design”, Géotechnique, Vol. 50, No. 5, pp. 487–508.

Aung, A.M.W., and Leong, E.C. (2010), “Discussion: Near-field effects on array-based surface wave methods with active sources” (Paper by Yoon and Rix 2009), Journal of Geotechnical and Geoenvironmental Engineering, Vol. 136, No. 5, pp. 773–775.

Baldi, G., Hight, D.W. and Thomas, G.E. (1988), “A re-evaluation of conventional triaxial test methods”, ASTM STP977 Advanced Triaxial Testing of Soil and Rock, pp. 219-263.

Bardet, J.P. (1992), “A viscoelastic model for the dynamic behaviour of saturated poroelastic soils”, Journal of Applied Mechanics, Vol. 59, No. 1, pp. 128-135.

Barnes, A.E. (1993), “Instantaneous spectral bandwidth and dominant frequency with applications to seismic reflection data”, Geophysics, Vol. 58, No. 3, pp. 419-428.

Barrière, J., Bordes, C., Brito, D., Sénéchal, P. and Perroud, H. (2012), “Laboratory monitoring of P waves in partially saturated sand”, Geophysical Journal International, Vol. 191, No. 3, pp. 1152-1170.

Bates, C.R. (1989), “Dynamic soil property measurements during triaxial testing”, Geotechnique, Vol. 39, No. 4, pp. 721–726.

Bath, M. (1974), Spectral analysis in geophysics- Developments in solid earth geophysics, Elsevier Scientific Publishing Company, Amsterdam, The Netherlands.

Baun, J. (2004), Principles of General & Vascular Sonography, San Francisco, California, United States.

Beatty, J.R. (1981), “Physical properties of rubber compounds”, Mechanics of Pneumatic Tires, University of Michigan, U.S Department of Transportation, National Highway Traffic Safety Administration, Washington, D.C., pp. 871-885.

Biesen, E.D. (2006), “Prediction of plastic deformation in aluminium softball bats using finite element analysis”, Master Thesis, Washington State University, United States.

Bishop, A.W. (1973), “The influence of an undrained change in stress on the pore pressure in porous media of low compressibility”, Géotechnique, Vol. 23, No. 3, pp. 435-442.

Black, D.K. and Lee, K.L. (1973), “Saturating laboratory samples by back pressure”, Journal of the Soil Mechanics and Foundations Division, Vol. 99, No. 1, pp. 75-93.

Black, J.A., Stanier, S.A., and Clarke, S.D. (2009), “Shear wave velocity measurement of kaolin during undrained unconsolidated triaxial compression”, Proceedings of the 62nd Canadian Geotechnical Conference, Halifax, Canada.

Bolton, M.D. and Wilson, J.M.R. (1989), “An experimental and theoretical comparison between static and dynamic torsional soil tests”, Géotechnique, Vol. 39, No. 4, pp. 585-599.

Bolton, M.D. and Wilson, J.M.R. (1990), “Soil stiffness and damping”, Cambridge University Engineering Department, United Kingdom, pp. 209-216.

Borden, R.H., Shao, L. and Gupta, A. (1996), “Dynamic properties of Piedmont residual soils”, Journal of Geotechnical Engineering, Vol. 122, No. 10, pp. 813-821

Brignoli, E.G.M., Gotti, M. and Stokoe, K.H., II (1996), “Measurement of shear waves in laboratory specimens by means of Piezoelectric transducers”, Geotechnical Testing Journal, Vol. 19, No. 4, pp. 384–397.

BS (1990), BS 1377-7:1990 Methods of test for soils for civil engineering purposes. Shear strength tests (total stress), British Standards Institution, London.

BS (1990), BS 1377-8:1990 Methods of test for soils for civil engineering purposes. Shear strength tests (effective stress), British Standards Institution, London.

Burland, J.B., & Symes, M. (1982), “A simple axial displacement gauge for use in the triaxial apparatus”, Géotechnique, Vol. 32, No. 1, pp. 62-65.

Byun, Y.H., Lee, J.S., Cho, S.H. and Yoon, H.K. (2013), “Evaluation of void ratio and elastic modulus of unsaturated soil using elastic waves”, Proceedings of the 18th International Conference on Soil Mechanics and Geotechnical Engineering, Paris, France, pp. 1089-1092

Camacho-Tauta, J.F., Reyes-Ortiz, O.J. and Jimenez Alvarez, J.D. (2013), “Comparison between resonant –column and bender element tests on three types of soils”, Dyna, Vol. 80, No. 182, pp. 163-172.

Carcione, J.M., Kosloff, D. and Kosloff, R. (1988a), “Wave propagation simulation in a linear viscoacoustic medium”, Geophysical Journal, Vol. 93, No. 2, pp. 393-401.

Carcione, J.M., Kosloff, D. and Kosloff, R. (1988b), “Wave propagation simulation in a linear viscoelastic medium”, Geophysical Journal, Vol. 95, No. 3, pp. 597–611.

Carcione, J.M. (1993), “Seismic modelling in viscoelastic media”, Geophysics, Vol. 58, No. 1. pp. 110-120.

Cascante, G., Santamarina, C. and Yassir, N. (1998), “Flexural excitation in a standard torsional-resonant column device”, Canadian Geotechnical Journal, Vol. 35, No. 3, pp. 478-490.

Cha, M., Cho, G.C. and Santamarina, J.C. (2009), “Long-wavelength P-wave and S-wave propagation in jointed rock masses”, Geophysics, Vol. 74, No. 5, pp. E205-E214.

Cheng, Z. and Leong, E.C. (2014), “A hybrid bender element–ultrasonic system for measurement of wave velocity in soils”, Geotechnical Testing Journal, Vol. 37, No. 3, pp. 1-12

Cheng, Z. and Leong, E.C. (2015a), “Damping characteristics of sand-rubber chip mixtures as a seismic barrier”, Proceedings of the 5th International Conference on Design and Analysis of Protective Structures (DAPS2015), Singapore, pp. 1-8.

Cheng, Z.Y. and Leong, E.C. (2015b), “Effects of confining pressure and degree of saturation on wave velocities of soils”, ASCE International Journal of Geomechanics, Accepted with revision.

Chin, F.K. (1970), "Estimation of the Ultimate Load of Piles Not Carried to Failure", Proceedings of the 2nd Southeast Asian Conference on Soil Engineering, Singapore, pp. 81-90.

Chin, F.K. (1971). "Discussion: Pile Tests—Arkansas River Project" (Paper by Mansur and Hunter, 1970), Journal of the Soil Mechanics and Foundations Division, Vol. 97, No. SM6, pp. 930-932.

Chin, F.K. and Vail, A.J. (1973), "Behaviour of Piles in Alluvium", Proceedings of the Eighth International Conference on Soil Mechanics and Foundation Engineering, Moscow, Russia, pp. 47-52.

Clayton, C.R.I. and Khatrush, S.A. (1986), “A new device for measuring local axial strains on triaxial specimens”, Géotechnique, Vol. 36, No. 4, pp. 593-597.

Clayton, C.R.I., Priest, J.A. and Best, A.I. (2005), “The effects of disseminated methane hydrate on the dynamic stiffness and damping of sand”, Géotechnique, Vol. 55, No. 6, pp. 423–434.

Clayton, C.R.I., Priest, J.A., Bui, M., Zervos, A. and Kim, S.G. (2009), “The Stokoe resonant column apparatus: effects of stiffness, mass and specimen fixity”, Géotechnique, Vol. 59, No. 5, pp. 429-437.

Clough, R.W. and Penzien, J. (2003), Dynamics of structures, Computers & Structures Inc., United States of America.

Correia, A.G., Santos, J.A., Barros, J.M.C. and Niyama, S. (2001), “An Approach to Predict Shear Modulus of Soils in the Range of 10^{-6} to 10^{-2} Strain Levels”, Proceedings of the 4th International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics and Symposium in Honor of Professor W.D. Liam Finn, San Diego, California, United States, Paper No. 1.22

Dames and Moore (1983), Mass rapid transit system, Singapore: Detailed geotechnical study, Interpretative Report, Provisional Mass Rapid Transit Authority, Singapore.

Dano, C., Hareb, H., and Hicher, P.Y. (2003), “Characterisation of loire river sand in the small strain domain using new bender-extender elements,” Proceedings of the 16th ASCE Engineering Mechanics Conference, University of Washington, Seattle, pp. 1-9.

Das, B.M. (1993), Principles of Soil Dynamics, PWS – KENT Publishing Company, USA.

Deniz, R.O. (2008), “Bender elements and bending disks for measurement of shear and compressional wave velocities in large sand specimens”, Master's Thesis, Northeastern University, United States.

Dobry, R. and Vucetic, M. (1987), “State-of-the-Art Report: Dynamic properties and seismic response of soft clay deposits”, Proceedings of the International Symposium on Geotechnical Engineering of Soft Soils, Mexico City, pp. 51-87

Dobry, R., and Iai, S. (2000), “Recent developments in the understanding of earthquake site response and associated seismic code implementation”, Proceedings of GeoEng2000: An International Conference on Geotechnical & Geological Engineering, Melbourne, Australia, pp. 186-219

Dwivedi, J.P., Singh, V.P. and Lal, R.K. (2011), “Dynamic response of partially sealed circular tunnel in viscoelastic soil condition”, Vol. 7, No. 26, pp. 37-45.

Dyvik, R. and Madshus, C. (1985), Lab measurements of $G_{\max}$ using bender elements, Proceedings of the ASCE Annual Convention on Advances in the Art of Testing Soils under Cyclic Conditions, Detroit, Michigan, pp. 186-196.

Emerson. M. and Foray P. (2006), “Laboratory P-wave measurements in dry and saturated sand”, Acta Geotechnica, Vol. 1, No. 3, pp. 167-177.

Eseller-Bayat, E., Gokyer, S., Yegian, M.K., Deniz, R.O., and Alshawabkeh, A. (2013), “Bender elements and bending disks for measurement of shear and compression wave velocities in large fully and partially saturated sand specimens”, Geotechnical Testing Journal, Vol. 36, No. 2, pp. 1-8

Feng, Z.Y. and Sutter, K.G. (2000), “Dynamic properties of granulated rubber/sand mixtures”, Geotechnical Testing Journal, Vol. 23, No. 3, pp. 338-344.

Ferreira, C. and Viana da Fonseca, A. (2005), International Parallel Tests on Bender Elements at the University of Porto, Report, Faculdade de Engenharia da Universidade do Porto, Portugal.

Fredlund, D.G. and Morgenstern, N.R. (1977), "Stress state variables for unsaturated soil," Journal of Geotechnical and Geoenvironmental Engineering, Vol. 103, No. GT5, pp. 447-466

Fredlund, D.G., Morgenstern, N.R. and Widger, R.A. (1978), "The shear strength of unsaturated soils", Canadian Geotechnical Journal, Vol. 15, No. 3, pp. 313-321.

Fredlund, D.G. and Xing, A. (1994), "Equations for the soil-water characteristic curve", Canadian Geotechnical Journal, Vol. 31, No. 4, pp. 521-532.

Fredlund, D.G. and Rahardjo, H. (1993), Soil Mechanics for Unsaturated Soils, John Wiley and Sons, USA.

Frempong, P., Butt, S.D., Donald A. (2005), "Frequency dependent spectral ratio technique for Q estimate", Proceedings of Rainbow in the Earth – 2nd International Workshop, Lawrence Berkeley National Laboratory, Berkeley, California, pp. 1–4.

Frempong, P., Donald, A. and Butt, S.D. (2007), "The effect of pore pressure depletion and injection cycles on ultrasonic velocity and quality factor in a quartz sandstone", Geophysics, Vol. 72, No. 2, pp. E43 – E51

Gardner, G.H.F., Wyllie, M.R.J. and Droschak, D. (1964), "Effects of pressure and fluid saturation on the attenuation of elastic waves in sands", Journal of Petroleum Technology, Vol. 16, No. 2, pp. 189-198.

Gautchi, G. (2002), Piezoelectric Sensorics: Force, Strain, Pressure, Acceleration and Acoustic Emission Sensors, Materials and Amplifiers, Springer-Verlag Berlin Heidelberg, Switzerland.

George, L.A., Dewoolkar, M.M. and Znidarcic, D. (2009). Simultaneous Laboratory Measurement of Acoustic and Hydraulic Properties of Unsaturated Soils, Vadose Zone Journal, Vol. 8(3): 633-642

Goto, S., Tatsuoka, F., Shibuya, S., Kim, Y.S. and Sato, T. (1991), “A simple gauge for local small strain measurements in the laboratory”, Soils and Foundations, Vol. 31, No. 1, pp. 169-180.

Greening, P., Rio, J. and Arroyo, M. (2007), “Some observations from a parametric study of the behaviour of benders in a polyurethane rubber”, Rivista Italiana Di Geotecnica, Vol. 41, pp. 48-55.

Greening, P.D. and Nash, D.F.T. (2004), “Frequency domain determination of G_0 using bender elements”, Geotechnical Testing Journal, Vol. 27, No. 3, pp. 288-294

Hall, J.R. Jr. (1962), “Effect of amplitude on damping and wave propagation in granular materials”, PhD thesis, University of Florida, Gainesville, Florida

Hardin, B.O. (1965), “The nature of damping in sands”, Journal of Soil Mechanics & Foundations Division, Vol. 91, No. SM1, pp. 63-97.

Hardin, B.O. and Drnevich, V.P. (1972a), “Shear modulus and damping in soils: measurement and parameter effects”, Journal of Soil Mechanics & Foundations Division, Vol. 98, No. 6, pp. 603-624.

Hardin, B.O. and Drnevich, V.P. (1972b), “Modulus and damping of soils I: Measurement and parameters effects”, Journal of the Soil Mechanics and Foundations Division, Vol. 98, No. 7, pp. 667-692.

Hardin, B.O. and Black, W.L. (1968), “Vibration Modulus of Normally Consolidated Clays”, Journal of the Soil Mechanics and Foundations Division, Vol. 94, No. SM2, pp. 353-369.

Hardin, B.O. and Richart, F.E. Jr. (1963), “Elastic Wave Velocity in Granular Soils,” Journal of the Soil Mechanics and Foundations Division, Vol. 89, No. SM1, pp. 33-65.

Hardin, K.O., Drnevich, V.P., Wang, J. and Sams, C.E. (1994), “Resonant column testing at pressures up to 3.5 MPa (500 psi)”, Dynamic Geotechnical Testing II STP 1213, pp. 222-233.

Hardy, S., Zdravkovi'c, L. and Potts, D.M. (2002), “Numerical interpretation of continuously cycled bender element tests”, Proceedings of the 8th International Symposium Numerical Models in Geomechanics, Rome, Italy, pp. 595–600.

Hatanaka, M. and Masuda, T. (2008), “Experimental Study on the relationship between degree of saturation and P wave velocity in sandy soils”, Proceedings of the 2nd International Conference GEDMAR08 - Geotechnical Engineering for Disaster Mitigation and Rehabilitation, Nanjing, China, pp. 346-351.

Hayner, M.A. (1994), “Optimised Finite Difference Schemes for Wave Propagation in High Loss Viscoelastic Material”, PhD Thesis, Massachusetts Institute of Technology, Cambridge, United States.

He, L., Leong, E.C. and Algamal, A. (2006), “A miniature tensiometer for measurement of high matric suction”, Proceedings of the 4th International Conference on Unsaturated Soils, Carefree, Arizona, pp. 1897-1907.

Head, K.H. (1998), Manual of Soil Laboratory Testing, Volume 3: Effective stress tests, John Wiley and Sons. Ltd, England.

Heitor, A., Indraratna, B. and Rujikiatkamjorn, C. (2013), “Laboratory study of small-strain behavior of a compacted silty sand”, Canadian Geotechnical Journal, Vol. 50, No. 2, pp. 179-188.

Hilf, J.W. (1956), "An investigation of pore-water pressure in compacted cohesive soils", PhD Thesis, University of Colorado, Denver, Colorado, United States.

Ho, D.Y.F. and Fredlund, D.G. (1982), "Multi-stage triaxial tests for unsaturated soils", Geotechnical Testing Journal, Vol. 5, No. 1, pp. 18–25.

Hoque, E., & Tatsuoka, F. (2004), "Triaxial testing system for measuring loading-rate effects during cyclic tests of sand", Geotechnical Testing Journal, Vol. 27, No. 5, pp. 483-495.

Hoque, E., Sato, T., and Tatsuoka, F. (1997). "Performance Evaluation of LDTs for Use in Triaxial Tests", Geotechnical Testing Journal, Vol. 20, No. 2, pp. 149-167.

Hoyos, L.R., Takkabutr, P., Puppala, A.J. and Hossain, Md.S. (2008), "Dynamic response of unsaturated soils using resonant column and bender element testing techniques", Proceedings of the Geotechnical Earthquake Engineering and Soil Dynamics IV, Sacramento, California, pp. 1-8.

Hoyos, L. and Macari, E. J. (1999), "Influence of in situ factors on dynamic response of Piedmont residual soils", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 125, No. 4, pp. 271-279

Hu, J., Jin, X., Lee, J.B., Zhang, L., Chaudhary, V., Guthikonda, M., Yang, K.H. and King A.I. (2007), "Intraoperative brain shift prediction using a 3D inhomogeneous patient-specific finite element model", Journal of Neurosurgery, Vol. 106, No. 1, pp. 164–169.

Huang, N.E., Shen, Z., Long, S.R., Wu, M.C., Shih, H.H., Zheng, Q., Yen, N.C., Tung C.C. and Liu, H.H. (1998), "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis", Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences, Vol. 454, No. 1971, pp. 903-995.

Huang, Y., Tsuchiya, H., and Ishihara, K. (1999), "Estimation of partial saturation effect on liquefaction resistance of sand using P-wave velocity", Proceedings of Japanese Geotechnical Society, Vol. 113, pp. 431-434.

Iglesias, A.M. (2000), "Investigating various modal analysis extraction techniques to estimate damping ratio", Masters Thesis, Virginia Polytechnic Institute and State University, United States

IPC (Industrial Process Controls Limited) (2001), UTM-5P/14P Universal Testing Machine Reference Manual, Melbourne, Australia.

Ishihara, K. (1996), Soil Behaviour in Earthquake Geotechnics, Oxford University Press, New York.

Jamiolkowski, M., Lancellotta, R., Lo Presti, D.C.F. and Pallara, O. (1994), "Stiffness of Toyoura sand at small and intermediate strain", Proceeding of the 13th International conference of Soil Mechanics and Foundation Engineering, New Delhi, India, pp. 169-172

Jardine, R.J., Symes, M.J., & Burland, J.B. (1984), "The measurement of soil stiffness in the triaxial apparatus", Géotechnique, Vol. 34, No. 3, pp. 323-340.

Jenkins, J.T., Cundall, J.T. and Ishibashi, I. (1992), "Experiments, theory, and simulation on the evolution of fabric in granular materials", Research Report, Cornell University, Ithaca, New York.

Johnston, D.H. and Toksöz, M.N., 1980, "Ultrasonic P and S wave attenuation in dry and saturated rocks under pressure," Journal of Geophysical Research, Vol. 85, No. B2, pp. 925 – 936

Johnston, D.H., 1978, “The attenuation of seismic waves in dry and saturated rocks”, PhD Thesis, Massachusetts Institute of Technology, Cambridge, United States.

Jovicic, M., Coop, R., Simic, M. (1996), “Objective criteria for determining G_{max} from bender element tests”, Géotechnique, Vol. 46, No. 2, pp. 357–362.

Kamata, T., Tsukamoto, Y. and Ishihara, K. (2009). Undrained shear strength of partially saturated sand in triaxial test, Bulletin of the New Zealand Society for Earthquake Engineering, Vol. 42 (1):57-62

Karl, L., Haegeman, W., Pyl, L. and Degrande, G. (2003), “Measurement of material damping with bender elements in triaxial cell”, In Proceedings of the 3rd International Symposium on Deformation Characteristics of Geomaterials, Lyon, France, Vol. 1, pp. 3-11.

Karl, L., Haegeman, W., Degrande, G. and Doms, D. (2008), “Determination of the Material Damping Ratio with the Bender Element Test,” Journal of Geotechnical and Geoenvironmental Engineering, Vol. 134, No. 12, pp. 1743-1756

Kawaguchi, T., Mitachi, T. Shibuya, S. (2001), “Evaluation of shear wave travel time in laboratory bender element test,” Proceedings of the 15th International Conference on Soil Mechanics and Geotechnical Engineering, Istanbul, Balkema, pp. 155 – 158

Khan, Z. Cascante, G. and Naggar, M.H.E. (2011), “Measurement of dynamic properties of stiff specimens using ultrasonic waves”, Canadian Geotechnical Journal, Vol. 48, No. 1, pp. 1-15

Kokusho, T. (1980), “Cyclic triaxial test of dynamic soil properties for wide strain range”, Soils and Foundations, Vol. 20, No. 2, pp. 45-60.

Kokusho, T. (2000), "Correlation of pore-pressure B-value with P-wave velocity and Poisson's ratio for imperfectly saturated sand or gravel", Soils and Foundations, Vol. 40, No. 4, pp. 95-102.

Krahn, J., Fredlund, D.G., and Klassen, M.J. (1989). "Effect of soil suction on slope stability at Notch Hill", Canadian Geotechnical Journal, 26, No. 2, pp. 269–278.

Kumar, J. and Clayton, C.R.I. (2007), "Effect of sample torsional stiffness on resonant column test results", Canadian Geotechnical Journal, Vol. 44, No. 2, pp. 221-230.

Kumar, J. and Madhusudhan, B.N. (2010), "On determining the elastic modulus of a cylindrical sample subjected to flexural excitation in a resonant column apparatus", Canadian Geotechnical Journal, Vol. 47, No. 11, pp. 1288-1298.

Kumar, J. and Madhusudhan, B.N. (2012), "Dynamic properties of sand from dry to fully saturated states", Géotechnique, Vol. 62, No. 1, pp. 45-54.

Kuwano, R., Connolly, T.M. and Kuwano, J. (1999), "Shear stiffness anisotropy measured by multi-directional bender element transducers", Proceedings of the 2nd International Symposium on Pre-failure Deformation Characteristics of Geomaterials, Torino, Italy, Vol. 1, pp. 205–212.

Ladd, R.S. (1978), "Preparing Test Specimens Using Undercompaction", Geotechnical Testing Journal, Vol. 1, No. 1, pp. 16–23

Lakes, R. (2009), Viscoelastic Materials, Cambridge University Press, New York, United States.

Le, T.H., Tamura, Y. and Yoshida, A. (2010), "Modal parameter estimation from ambient data using time-frequency analysis", Proceeding of the 24th International Conference of Noise and Vibration, Leuven, Belgium, pp. 1-15.

Lee, J.S. and Santamarina J.C. (2005), “Bender elements: Performance and signal interpretation”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 131, No. 9, pp. 1063 – 1070

Lee, K.H., Szajewski, B.A., Hah, Z., Parker, K.J. and Maniatty, A.M. (2012), “Modelling shear waves through a viscoelastic medium induced by acoustic radiation force”, International Journal for Numerical Methods in Biomedical Engineering, Vol. 28, No. 6-7, pp. 678-696

Lee, K.L. and Black, D.K. (1972), “Time to dissolve air bubble in drain line”, Journal of the Soil Mechanics and Foundations Division, Vol. 98, No. 2, pp. 181-194.

Leissa, A.W. and So, J. (1995), “Comparisons of vibration frequencies for rods and beams from one-dimensional and three-dimensional analyses”, Journal of the Acoustical Society of America, Vol. 98, No. 4, pp. 2122-2135.

Leong, E.C. and Rahardjo, H., (1997), “Review of Soil-Water Characteristic Curve Equations”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 123, No. 12, pp. 1106–1117.

Leong, E.C. and Pan, T.C. (1998), “Geophysical survey methods for soft soil”, Geotechnical Site Characterisation, Proceedings of the 1st International Conference on Site, Georgia, pp. 485-490

Leong, E.C., Rahardjo, H., Tang, S.K. (2002), “Characterization and engineering properties of Singapore residual soils”, Proceedings of the International Workshop on Characterisation and Engineering Properties of Natural Soils, Singapore, pp. 1–10.

Leong, E.C., Rahardjo, H. and Cheong, H.K. (2003), "Stiffness-strain relationship of Singapore residual soils", Proceedings of the 7th Pacific Conference on Earthquake Engineering, Christchurch, New Zealand, Paper Number 160, pp. 1-9

Leong, E.C., Yeo, S.H. and Rahardjo, H. (2004), "Measurement of wave velocities and attenuation using an ultrasonic test system", Canadian Geotechnical Journal, Vol. 41, No. 5, pp. 844–860.

Leong, E.C., Yeo, S.H. and Rahardjo, H. (2005), "Measuring shear wave velocity using bender elements", Geotechnical Testing Journal, Vol. 28, No. 5, pp. 488–498.

Leong, E.C., Cahyadi, J., and Rahardjo, H. (2006), "Stiffness of a Compacted Residual Soil", Proceedings of the 4th International Conference on Unsaturated Soils, Carefree, Arizona, United States, pp. 1169-1180.

Leong, E.C., Cahyadi, J. and Rahardjo, H. (2009), "Measuring shear and compression velocities of soil using bender-extender elements", Geotechnical Testing Journal, Vol. 28, No. 5, pp. 792–812.

Leong, E.C., Nyunt, T.T. and Low, K.S. (2011), "Local displacement transducer with Anderson loop", Geotechnical Testing Journal, Vol. 34, No. 6 , pp. 676-684.

Lin, M.L., Huang, T.H., and You, J.C. (1996), "The effects of frequency on damping properties of sand", Soil Dynamics and Earthquake Engineering, Vol. 15, No. 4, pp. 269-278.

Lings, M.L. and Greening, P.D. (2001), "A novel bender/extender element for soil testing," Géotechnique, Vol. 51, No. 8, pp. 713-717.

Little, A.L. (1969), "The engineering classification of residual tropical soils," Proceedings of the Seventh International Conference on Soil Mechanics and Foundation Engineering Vol. 1, Mexico, pp. 1–10

Lo Presti, D.C.F., Pallara, O., Rainò, M., & Maniscalco, R. (1994), "A computer-controlled triaxial apparatus: preliminary results", Rivista Italiana di Geotecnica, Vol. 1, No. 94, pp. 43-60.

LSTC (2013), LS-DYNA User Manual R7.0, Livermore Software Technology Corporation, California, United States.

Luo, H., Fang, X. and Ertas, B. (2009), "Hilbert transform and its engineering applications", American Institute of Aeronautics and Astronautics Journal, Vol. 47, No. 4, pp. 923-932.

Macari, E.J. and Hoyos Jr., L. (1996), "Effect of degree of weathering on dynamic properties of residual soils", Journal of Geotechnical Engineering, Vol. 122, No. 12, pp. 988-997.

Madhusudhan, B.N. and Kumar, J. (2013), "Damping of Sands for Varying Saturation", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 139, No. 9, pp. 1625-1630.

Maher, M.H., Ro, K.S. and Welsh, J.P. (1994), "High strain dynamic modulus and damping of chemically grouted sand", Soil Dynamics and Earthquake Engineering, Vol. 13, No. 2, pp. 131-138.

Makris, N. and Zhang, J. (2000), "Time-domain viscoelastic analysis of earth structures", Earthquake Engineering and Structural Dynamics, Vol. 29, No. 6, pp. 745–768.

Mansur, C.I. and Hunter, A.H. (1970), "Pile Tests—Arkansas River Project", Journal of the Soil Mechanics and Foundations Division, Vol. 96, No.5, pp. 1545-1582.

Marjanovic, J. and Germaine, J.T. (2013), "Experimental Study Investigating the Effects of Setup Conditions on Bender Element Velocity Results," Geotechnical Testing Journal, Vol. 36, No. 2, pp. 1-11.

Sams, M. and Goldberg, D. (1990), "The validity of Q estimates from borehole data using spectral ratios", Geophysics, Vol. 55, No. 1, pp. 97-101.

Menard, K.P. (2008), Dynamic mechanical analysis: a practical introduction, CRC Press, Florida, USA.

Mesri, G., Adachi, K. and Ullrich, C.R. (1976), "Pore-pressure response in rock to undrained change in all-round stress", Géotechnique, Vol. 26, No. 2, pp. 317-330.

Mulmi, S., Sato, T. and Kuwano, R. (2008), "Performance of plate type piezo-ceramic transducer for elastic wave measurements in laboratory soil specimens," Seisan Kenkyu, Vol. 60, No. 6, pp. 565–569

Murphy, W.F. (1982), "Effects of partial water saturation on attenuation in Massillon sandstone and Vycor porous glass", The Journal of the Acoustical Society of America, Vol. 71, No. 6, pp. 1458-1468.

Murphy, W.F. (1984), "Acoustic measures of partial gas saturation in tight sandstones", Journal of Geophysical Research: Solid Earth, Vol. 89, No. B13, pp. 11549-11559.

Muscolino, G., Palmeri, A. and Ricciardelli, F. (2005), "Time-domain response of linear hysteretic systems to deterministic and random excitations", Earthquake Engineering and Structural Dynamics, Vol. 34, No. 9, pp. 1129–1147

Naesgaard, E., Byrne, P.M. and Wijewickreme, D. (2007), “Is P wave velocity an indicator of saturation in sand with viscous pore fluid?”, International Journal of Geomechanics, Vol. 7, No. 6, pp. 437-443.

Nakagawa, K., Soga, K., and Mitchell, J.K. (1996), “Pulse transmission system for measuring wave propagation in soils”, Journal of Geotechnical Engineering, Vol. 122, No. 4, pp. 302–308.

Nambiar, M.R.M., Rao, G.V. and Gulhati, K.S. (1985), “Multistage triaxial testing: A rational procedure”, ASTM STP 883 Strength Testing of Marine Sediments: Laboratory and In-Situ Measurements, pp. 274–293.

National Instruments, Citing online sources: Field Wiring and Noise Considerations for Analog Signals [online]. Available from <http://www.ni.com/white-paper/3344/en> [cited 31 March 2013]

NDT Systems (2012), Citing online sources: Velocity Table [online], Available from: http://ndtsystems.com/Reference/Velocity_Table/velocity_table.html

Nyunt, T.T. (2012), “Strength and stiffness characteristics of unsaturated soils”, PhD Thesis, Nanyang Technological University, Singapore.

O’Hara, G.P. (1983), “Mechanical properties of silicone rubber in a closed volume”, Technical Report ARLCB-83045, US Army Armament Research and Development Center, Large Caliber Systems Laboratory, Watervliet, New York..

Oh, S.H., Park, D.S., Kim, B.J., Kim, E.J., and Mok, Y.J. (2008), “Laboratory measurements of stiffness of soft clay using bender elements”, Proceedings of the 14th World Conference on Earthquake Engineering (CD-ROM), Beijing, China.

Oztoprak, S. and Bolton, M. D. (2013), “Stiffness of sands through a laboratory test database”, Géotechnique, Vol. 63, No. 1, pp. 54-70.

Pamukcu, S. and Akbulut, S. (2006), “Thermoelastic enhancement of damping of sand using synthetic ground rubber”, Journal of Geotechnical and Geoenvironmental Engineering, Vol. 132, No. 4, pp. 501–510.

Panametrics-NDT (2006), Citing online sources: Ultrasonic transducers and technical notes [online], Available from: <https://www.olympus-ims.com/data/File/panametrics/UT-technotes.en.pdf>.

Pennington, D.S., Nash, D.F.T., and Lings, M.L. (2001), “Horizontally mounted bender elements for measuring anisotropic shear moduli in triaxial clay specimens,” Geotechnical Testing Journal, Vol. 24, No. 2, pp. 133–144

Pineda, J., Hoyos, L. and Colmenares, J. (2010), “Stiffness response of residual and saprolitic soils using resonant column and bender element testing techniques”, In Proceedings of GeoFlorida 2010: Advances in Analysis, Modeling & Design, Florida, USA, pp. 783-792.

Poh, K.B., Chua, N.L. and Tan, S.B. (1985), “Residual granite soil of Singapore”, Proceedings of 8th Southeast Asian Geotechnical Conference, Kuala Lumpur, pp. 3-1 – 3-9.

Priest, J.A., Best, A.I. and Clayton, C.R.I. (2005), “A laboratory investigation into the seismic velocities of methane gas hydrate-bearing sand”, Journal of Geotechnical Research: Solid Earth, Vol. 110, No. B4, pp. 1-13.

Rahardjo, H., Lim, T.T., Chang, M.F. and Fredlund, D.G. (1995), “Shear-strength characteristics of a residual soil”, Canadian Geotechnical Journal, Vol. 32, No. 1, pp. 60–77.

Rahardjo, H. (2000), "Rainfall-Induced Slope Failures", NSTB 17/6/16 Main Report, Nanyang Technological University, Singapore.

Rahardjo, H., Ong, B.H. and Leong, E.C. (2004), "Shear strength of a compacted residual soil from consolidated drained and constant water content triaxial tests", Canadian Geotechnical Journal, Vol. 41, No. 3, pp. 421-436.

Rahman, M.M., Cubrinovski, M. and Lo, S.R. (2012), "Initial shear modulus of sandy soils and equivalent granular void ratio", Geomechanics and Geoengineering, Vol. 7, No. 3, pp. 219-226.

Ramadas, S.N., O'Leary, R.L. and Gachagan, A. (2009), "Ultrasonic Sensor Design for NDE Application: Design Challenges & Considerations", Proceedings of the National Seminar & Exhibition on Non-Destructive Evaluation, Tiruchirapalli, India, pp. 88-91.

Ravi Sharma, M.S., Baxter, C.D.P., Moran, K., Vaziri, H. and Narayanasamy, R. (2011), "Strength of weakly cemented sands from drained multistage triaxial tests", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 137, No. 12, pp. 1202-1210.

Ray, R.P. and Woods, R.D. (1988), "Modulus and damping due to uniform and variable cyclic loading", Journal of Geotechnical Engineering, Vol. 114, No. 8, pp. 861-876.

Rees, E.V.L. (2009), "Methane gas hydrate morphology and its effect on the stiffness and damping of some sediments", PhD Thesis, University of Southampton, England, UK.

Richart, F.E., Hall Jr., J.R. and Woods, R.D. (1970), Vibrations of Soils and Foundations, Prentice-Hall, Eaglewood Cliffs, New Jersey.

Rio, J.F.M.E. (2006), “Advances in laboratory geophysics using bender elements”, PhD thesis, University College London, England, United Kingdom.

Rocco, N.T. (2012), “Characterization of expanded polystyrene (EPS) and cohesive soil mixtures”, PhD thesis, Missouri University of Science and Technology, Missouri, United States.

Rossing, T.D. and Fletcher, N.H. (2004), Principles of Vibration and Sound, 2nd Edition, Springer Science+Business Media, New York, United States.

Rowe, P.W. and Barden, L. (1964), “Importance of free ends in triaxial testing”, Journal of the Soil Mechanics and Foundations Division, Vol. 90, No. 1, pp. 1-28.

Salvino, L.W. (2000), “Empirical mode analysis of structural response and damping”, Proceedings of the 18th International Modal Analysis Conference, San Antonio, Texas, United States, pp. 503-509.

Sanchez-Salinero, I., Roesset, J.M. and Stokoe, K.H. (1986), “Analytical Studies of Body Wave Propagation and Attenuation,” Report GR 86-15, Civil Engineering Department, University of Texas, Austin, United States.

Santamarina, J.C. and Fam, M.A. (1997), "Discussion on interpretation of bender element test" (paper by Viggiani and Atkinson, 1995), Géotechnique, Vol. 47, No. 4, pp. 873–877

Santamarina, J.C. and Fratta, D. (2005), Discrete Signals and Inverse Problems: An introduction for engineers and scientists, John Wiley and Sons Ltd, West Sussex, England.

Satija, B.S. and Gulhati, S.K. (1979), “Strain rate for shearing testing of unsaturated soil”, Proceedings of the 6th Asian Regional Conference on Soil Mechanics and Foundations Engineering, Singapore, pp. 83-86.

Saxena, S.K., Avramidis, A.S. and Reddy, K.R. (1988), "Dynamic moduli and damping ratios for cemented sands at low strains", Canadian Geotechnical Journal, Vol. 25, No. 2, pp. 353–368.

Schaeffer, K., Bearce, R. and Wang, J. (2013), "Dynamic modulus and damping ratio measurements from free-free resonance and fixed-free resonant column procedures", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 139, No. 12, pp. 2145–2155.

Schneider, J.A., Hoyos, L., Mayne, P.W., Macari, E.J. and Rix, G.J. (1999), "Field and laboratory measurements of dynamic shear modulus of piedmont residual soils", Behaviour Characteristics of Residual Soils, Proceedings of Sessions of Geo-Congress, Charlotte, North Carolina, United States, pp. 12-25

Schoenemann, M.R., and Pyles, M.R. (1988), "Stress path considerations in multistage triaxial testing", ASTM STP977 Advanced Triaxial Testing of Soil and Rock, pp. 732–739.

Seed, H.B., and Idriss, I.M. (1970), "Soil moduli and damping factors for dynamic response analyses", Report No. EERC 70-10, University of California, Berkeley, California.

Seed, H.B., Wong, R.T., Idriss, I.M., and Tokimatsu, K. (1986), "Moduli and damping factors for dynamic analyses of cohesionless soils", Journal of Geotechnical Engineering, Vol. 112, No. 11, pp. 1016-1032.

Senetakis, K., Anastasiadis, A. and Pitilakis, K. (2012), "Dynamic properties of dry sand/rubber (SRM) and gravel/rubber (GRM) mixtures in a wide range of shearing strain amplitudes", Soil Dynamics and Earthquake Engineering, Vol. 33, No. 1, pp. 38-53.

Shen, C.K., Li, X.S. and Gu, Y.Z. (1985), "Microcomputer based free torsional vibration test", Journal of Geotechnical Engineering, Vol. 111, No. 8, pp. 971-986.

Shi, Q., Padilla, M. and Houston, W.N. (2007), "Comparison of field wave velocity and velocity of ultrasound through a soil specimen under laboratory condition", Proceedings of the Sri Lankan Geotechnical Society's First International Conference on Soil & Rock Engineering, Vol. 1

Shibuya, S. and Tanaka, H. (1996), "Estimate of elastic shear modulus in Holocene soil deposits", Soils and Foundations, Vol. 36, No. 4, pp. 45–55.

Shirley, D.J. and Hampton L.D. (1978), "Shear wave measurements in laboratory sediments", Journal of the Acoustical Society of America, Vol. 63, No.2, pp. 607–613.

Skempton, A.W. (1954), "The pore-pressure coefficients A and B", Géotechnique, Vol. 4, No. 4, pp. 143-147.

Slavova, D.Z., Weidinger, D.M., Sevi, A.F. and Ge, L. (2010), "Evaluation of compacted silt characteristics by ultrasonic pulse velocity testing", Proceedings of GeoFlorida 2010: Advances in Analysis Modeling & Design, ASCE Geotechnical Special Publication No. 199, pp. 1284 – 1293

Smith, L.V. and Duris, J.G. (2009), "Progress and challenges in numerically modelling solid sports ball with application to soft balls", Journal of Sports Science, Vol. 27, No. 4, pp. 353–360

Souto, A., Hartikainen, J. and Ozudogru, K. (1994), "Measurement of dynamic parameters of road pavement materials by the bender element and resonant column tests," Géotechnique, Vol. 44, No. 3, pp. 519–526.

Spanos, P.D. and Tsavachidis, S. (2001), “Deterministic and stochastic analyses of a nonlinear system with a Biot visco-elastic element”, Earthquake Engineering and Structural Dynamics, Vol. 30, No. 4, pp. 595-612.

Spencer, J.W. (1979), “Bulk and shear attenuation in Berea sandstone: the effects of pore fluids”, Journal of Geophysical Research: Solid Earth, Vol. 84, No. B13, pp. 7521-7523.

Sridharan, A., and Narasimha Rao, S. (1972). “A new approach to multistage triaxial test”, Journal of the Soil Mechanics and Foundations Division, Vol. 98, No. 11, pp. 1279–1286.

Stokey, W.F. (2009), Harris' Shock and Vibration Handbook, McGraw Hill, USA.

Stokoe, K.H. II, Darendeli, M.B., Andrus, R.D. and Brown, L.T. (1999), “Dynamic soil properties: Laboratory, field and correlation studies”, In Proceedings of the 2nd International Conference on Earthquake Geotechnical Engineering. Balkema, Lisbon, Portugal, pp. 811-845.

Stoll, R.D. (1979), “Experimental studies of attenuation in sediments”, The Journal of the Acoustical Society of America, Vol. 66, No. 4, pp. 1152-1160.

Stoll, R.D. (1980), “Theoretical aspects of sound transmission in sediments”, The Journal of the Acoustical Society of America, Vol. 68, No. 5, pp. 1341-1350.

Stoll, R.D. (1985), “Marine sediment acoustics”, The Journal of the Acoustical Society of America, Vol. 77, No. 5, pp. 1789-1799.

Stucky, P. and Lord, W. (2001), “Finite element modeling of transient ultrasonic waves in linear viscoelastic media”, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control, Vol. 48, No. 1, pp. 6-16.

Styler, M.A. and Howie, J.A. (2014), “Continuous Monitoring of Bender Element Shear Wave Velocities During Triaxial Testing”, Geotechnical Testing Journal, Vol. 37, No. 2, pp. 1–12.

Sutherland, J.G. and Goodman, L.E. (1951), “Vibrations of Prismatic Bars Including Rotatory Inertia and Shear Corrections”, Technical Report of a Cooperative Investigation with the Office of Naval Research, University of Illinois, Urbana, USA.

Takkabutr P. (2006), “Experimental investigations on small strain stiffness properties of partially saturated soils via resonant column and bender element testing”, PhD thesis, University of Texas, Arlington.

Tamura, S., Tokimatsu, K., Abe, A. and Sato, M. (2002), “Effects of air bubbles on B-value and P-wave velocity of a partly saturated sand”, Soils and Foundations, Vol. 42, No. 1, pp. 121-129

Tan, S.B., Tan, S.L., Lim, T.L. and Yang, K.S. (1987), “Landslide problems and their control in Singapore”, Proceedings of 9th Southeast Asian Geotechnical Conference, Bangkok, Thailand, Vol. 1, pp. 25-36.

Tan, S.B. (1997), “Thirty years of geotechnical developments in Singapore public works”, Geotechnical Engineering in Asia: 2000 and Beyond, Proceedings of the 3rd Asian Young Geotechnical Engineers Conference, Singapore, pp. 1-46

Tatsuoka, F., Teachavorasinskun, S., Dong, J., Kohata, Y. and Sato, T. (1994), “Importance of measuring local strains in cyclic triaxial tests on granular materials”, Dynamic Geotechnical Testing II STP 1213, pp. 288-288.

Taylor, D.W. (1950), “A triaxial shear investigation on a partially saturated soil”, STP106 Triaxial Testing of Soils and Bituminous Mixtures, pp. 180–191.

Tefft, W.E. (1960), “Numerical solution of the frequency equations for the flexural vibration of cylindrical rods”, Journal of Research of the National Bureau of Standards, Vol. 64B, pp. 237-250

Terzaghi, K. (1943), Theoretical Soil Mechanics, John Wiley and Sons, United States of America.

Timoshenko, S. (1937), Vibration Problems in Engineering, D van Nostrand Company, New York, USA.

Toksöz, M.N., Johnston, D.H. and Timur, A. (1979), “Attenuation of seismic waves in dry and saturated rocks: I. Laboratory measurements”, Geophysics, Vol. 44, No. 4, pp. 681–690.

Tsang, H.H., Sheikh, M.N. and Lam, N. (2007), “Rubber-soil cushion for earthquake protection”, Proceedings of the Australian Earthquake Engineering Society Conference, Wollongong, Australia, pp. 1-8

Tsang, H.H. (2008), “Seismic isolation by rubber-soil mixtures for developing countries”, Earthquake engineering & structural dynamics, Vol. 37, No. 2, pp. 283-303

Tsang, H.H., Lam, J.Y.K., Yaghmaei-Sabegh, S. and Lo, S.H. (2009), “Protecting underground tunnel by rubber-soil mixtures”, Proceedings of the Seventh International Conference on Lifeline Earthquake Engineering, ASCE, California, United States, pp. 401-411

Tou, J.H. (2003), “Cyclic triaxial test apparatus for evaluating dynamic properties of residual soils”, M.Eng thesis, Nanyang Technological University, Singapore.

Tschoegl, N.W. (1989), The Phenomenological Theory of Linear Viscoelastic Behaviour, Springer-Verlag, Berlin Heidelberg, Germany

Tsukamoto, Y., Ishihara, K. Nakazawa, H., Kamada, K. and Huang, Y. (2002), “Resistance of partly saturated sand to liquefaction with reference to longitudinal and shear wave velocities”, Soils and Foundations, Vol. 42, No. 6, pp. 93-104.

Udías, A. (1999), Principles of Seismology, Cambridge University Press, Cambridge, United Kingdom.

Valle-Molina, C. (2006), “Measurements of V_p and V_s in dry unsaturated and saturated sand specimens with piezoelectric transducers”, PhD Thesis, The University of Texas, Austin, United States of America.

Valle-Molina, C. and Stokoe II, K.H. (2012), “Seismic measurements in sand specimens with varying degrees of saturation using piezoelectric transducers”, Canadian Geotechnical Journal, Vol. 49, No. 6, pp. 671–685.

van Rensburg, N.F.J. and van der Merwe, A.J. (2006), “Natural frequencies and modes of a Timoshenko beam”, Wave Motion, Vol. 44, No. 1, pp. 58-69.

Veijayaratnam, M., Poh, K.B. and Tan, S.L. (1993), “Seismic velocities on Singapore soils and some geotechnical applications”, Proceedings of the 11th South East Asian Geotechnical Conference, Singapore pp. 813-818

Viana da Fonseca, A., Ferreira, C. and Fahey, M. (2009), “A Framework Interpreting Bender Element Tests, Combining Time-Domain and Frequency-Domain Methods”, Geotechnical Testing Journal, Vol. 32, No. 2, pp. 1-17.

Viggiani, G. and Atkinson, J.H. (1995), “Interpretation of Bender Element Tests,” Géotechnique, Vol. 45, No. 1, pp. 149–154.

Villarraga, M.R. (2000), “Dynamic Behavior of Tropical Soils from Medellín, Colombia”, GeoEng 2000 Proceedings of the International Conferences on Recent

Advances in Geotechnical Earthquake Engineering and Soil Dynamics, Melbourne, Australia, Paper No. 1.32

Vinale, F., d'Onofrio, A., Mancuso, C., Santucci de Magistris, F., and Tatsuoka, F., (1999), "The pre-failure behavior of soils as construction materials", Proceedings of the 2nd International Symposium on Pre-failure Deformation Characteristics of Geomaterials, IS Torino, Italy, pp. 955–1007.

Vucetic, M. (1994), "Cyclic threshold shear strains in soils", Journal of Geotechnical Engineering, Vol. 120, No. 12, pp. 2208-2228.

Vucetic, M. and Dobry, R. (1991), "Effect of soil plasticity on cyclic response", Journal of Geotechnical Engineering, Vol. 117, No. 1, pp. 89-107.

Vyalov, S.S. (1986), Developments in Geotechnical Engineering Volume 36: Rheological fundamentals of soil mechanics, Elsevier, New York, United States.

Wang, Y.H. and Santamarina, J.C. (2007), "Attenuation in sand: an exploratory study on the small strain behaviour and the influence of moisture condensation", Granular Matter, Vol. 9, No. 6, pp. 365-376.

Wang, Y.H., Yan, W.M., Lo, K.F. (2006), "Damping-ratio measurements by the spectral ratio method", Canadian Geotechnical Journal, Vol. 43, No. 11, pp. 1180-1194.

Weidinger, D.M. (2008), "Laboratory analysis of small strain moduli in compacted silts", Master's Thesis, Missouri University of Science and Technology, Rolla, United States.

Wesley, L.D. (2010), Geotechnical engineering in residual soils, John Wiley and Sons, Hoboken, New Jersey.

Winkler, K.W. and Nur, A. (1982), "Seismic attenuation: Effects of pore fluids and frictional-sliding", Geophysics, Vol. 47, No. 1, pp. 1-15.

Winn, K., Rahardjo, H. and Seh, C.P. (2001), "Characterisation of Residual Soils in Singapore", Geotechnical Engineering, Vol. 23, No. 1, pp. 1-14.

Wyllie, M.R.J., Gardner, G.H.F. and Gregory, A.R. (1962), "Studies of elastic wave attenuation in porous media", Geophysics, Vol. 27, No. 5, pp. 569-589.

Yamashita, S. Kawaguchi, T. Nakata, Y., Mikami, T., Fujiwara, T. and Shibuya, S. (2009), "Interpretation of international parallel test on the measurement of $G_{\max}$ using bender elements", Soils and Foundations, Vol. 49, No. 4, pp. 631-650.

Yamashita, S., Kohata, Y., Kawaguchi, T. and Shibuya, S. (2001), "International round-robin test organized by TC-29", Advanced Laboratory Stress-Strain Testing of Geomaterials, pp. 65-110.

Yang, K.S. and Tang, S.K. (1997), "Stabilising the slope of Bukit Gombak", Proceedings of 3YGEC, Singapore, pp. 589-605.

Yang, J.N., Lei, Y., Lin, S. and Huang, N. (2004), "Identification of natural frequencies and dampings of in situ tall buildings using ambient wind vibration data", Journal of Engineering Mechanics, Vol. 130, No. 5, pp. 570-577.

Yang, J. (2005), "Pore pressure coefficient for soil and rock and its relation to compressional wave velocity", Géotechnique, Vol. 55, No. 3, pp. 251-256

Yimsiri, S., Soga, K. and Chandler, S.G. (2005), "Cantilever-type local deformation transducer for local axial strain measurement in triaxial test", Geotechnical Testing Journal, Vol. 28, No. 5, pp. 1-7.

Yokota, K. and Konno, M. (1980), "Dynamic Poisson's ratio of soil", Proceeding of the 7th World Conference Earthquake Engineering Vol. 3, Istanbul, pp. 475-478.

Yoon, S. and Rix, G.J. (2009), "Near-field effects on array-based surface wave methods with active sources", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 135, No. 3, pp. 399-406.

Yoshimi, Y. and Tanaka, K. and Tokimatsu, K. (1989), "Liquefaction resistance of a partially saturated sand", Soils and Foundations, Vol. 29, No. 3, pp. 157-162.

Zamanek, J. Jr., and Rudnick, I. (1961), "Attenuation and dispersion of elastic waves in a cylindrical bar", Journal of the Acoustical Society of America, Vol. 33, No. 10, pp. 1283-1288.

Zavoral, D.Z. and Campanella, R.G. (1994), "Frequency Effects of Damping/Modulus of Cohesive Soil", Dynamic Geotechnical Testing II STP 1213, pp. 191-191.

Zhang, J., Andrus, R.D., Juang, C.H. (2005), "Normalized shear modulus and material damping ratio relationships", Journal of Geotechnical and Geoenvironmental Engineering, Vol. 131, Vol. 4, pp. 453-464.

Zhang, L. and Tamura, Y. (2003), "Damping estimation of engineering structures with ambient response measurements", Proceeding of the 21st International Modal Analysis Conference, Kissimmee, Florida, United States.

Zhang, X.J. and Aggour, M.S. (1996), "Damping determination of sands under different loadings", Proceedings of the 11th World Conference on Earthquake Engineering, Elsevier Science.

Zhou, Y. (2001), “Engineering geology and rock mass properties of the Bukit Timah Granite”, Proceedings of Underground Singapore 2001, Singapore, pp. 308-314.

Zhou, Q., Lau, S., Wu, D. and Shung, K.K. (2011), “Piezoelectric films for high frequency ultrasonic transducers in biomedical applications”, Progress in Materials Science, Vol. 56, No. 2, pp. 139-174.

Appendix A: Borehole log of undisturbed specimens

BOREHOLE LOG													
PROJECT: HDB					NTU CFJ								
DATE: 13.04.09					BOREHOLE NO: 901-1 SHEET: 1 of								
LOCATION: Ang Mo Kio Ave-4					DRILLER: shakil.								
COORDINATES: N: E:					DRILL TYPE: ROTARY ELEVATION: (m)								
GROUND WATER LEVEL- (m)													
Date	VISUAL DESCRIPTION	Depth (m)	Legend	Sampling		Standard Penetration Test				Undisturbed sample & Core			
				Sample No.	Depth (m)	1 st	2 nd	3 rd	N-value	REC	TCR%	SCR%	RQD%
14/04/09	Firm, yellowish brown, slightly sandy CLAY. (FILL)			MZ 1	1.0 ~ 2.0						0		
	Firm, purple, yellowish brown, slightly sandy CLAY. (Residual GVI)			MZ 2	2.0 ~ 3.0						97		
				MZ 3	3.0 ~ 4.0						98		
	Firm, reddish brown with yellow silty CLAY (GVI)			MZ 4	4.0 ~ 5.0						95		
	Stiff, yellowish brown, fine to coarse grained sandy CLAY.			MZ 5	5.0 ~ 6.0						60		
	Stiff, reddish brown, mixed with yellow. silty CLAY trace fine sand.			MZ 6	6.0 ~ 7.0						98		
				MZ 7	7.0 ~ 8.0						95		
	Stiff, reddish brown with purple sandy CLAY. (completely weathered granite)			MZ 8	8.0 ~ 9.0						80		
				MZ 9	9.0 ~ 10.0						90		
				MZ 10	10.0 ~ 11.0						90		
				MZ 11	11.0 ~ 12.0						0		
MZ-11 ~ MZ-14 were missed. Borehole was terminated at 15.0 m													

Ground Water Observation During Operation	Date	Time	BH Depth	Casing depth	Water level
FOSTA® FOSTA Pte Ltd BBR Building 50 Changi South Street 1 Singapore 486126 Tel: +65 65619885 Fax: +65 65613766 Email: info@fos-ta.com: www.fos-ta.com					

☒ Standard Penetration Test(SPT)	☐ Core Run (CR)
☒ Undisturbed Sample(UD)	☒ Vane Shear Test (V)
☒ Mazier Sample(MZ)	☒ Pressuremeter Test (PMT)
☒ Piston Sample(PS)	☒ Permeability Test (PBT)

Appendix B: Derivation of Cascante et al. (1998)'s method

For a vertical specimen, assume that the horizontal displacement y can be approximated with a third order polynomial

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 \quad (A1)$$

Consider the boundary conditions:

1. Fixed at the bottom $\rightarrow$ When $x=0$, $y=0$, $y'=0$
2. Moment is zero when $x=L$, $EIy''(L)=0$

Solve for the unknown coefficients

When $x=0$ and $y=0$,

$$a_0 = 0 \quad (A2)$$

When $x=0$ and $y'=0$,

$$\begin{aligned} 0 &= a_1 + 2a_2x + 3a_3x^2 \\ \therefore a_1 &= 0 \end{aligned} \quad (A3)$$

When $x=L$ and $y''=0$,

$$\begin{aligned} 0 &= 2a_2 + 6a_3x \\ a_2 &= -3a_3L \end{aligned} \quad (A4)$$

Substitute into Equation (A1),

$$y(x) = a_2x^2 - \frac{a_2}{3L}x^3 = \frac{a_2}{3L}x^2(3L-x) = \alpha x^2(3L-x) \quad (A5)$$

$$\text{Where } \alpha = \frac{a_2}{3L}$$

Assume that the horizontal displacement of the rigid mass follow that at $x=L$,

$$\begin{aligned} y(x) &= 3L\alpha x^2 - \alpha x^3 \\ y'(x) &= 6L\alpha x - 3\alpha x^2 \\ y''(x) &= 6L\alpha - 6\alpha x \end{aligned} \quad (A6)$$

When $x = L$,

$$\begin{aligned} y(x) &= 3\alpha L^3 - \alpha L^3 = 2\alpha L^3 \\ y'(x) &= 6\alpha L^2 - 3\alpha L^2 = 3\alpha L^2 \end{aligned} \quad (\text{A7})$$

Displacement of the rigid mass

= Distance moved by cantilever + further displacement due to the inherent slope

$$= 2\alpha L^3 + (x - L)(3\alpha L^2) = \alpha L^2 [2L + 3(x - L)] \quad (\text{A8})$$

Potential energy is the work done in deforming the beam, disregarding the work done due to shear forces, it can be expressed as $\frac{1}{2} \int_0^L M d\theta$ where M is the bending moment.

$$\text{Using, } M = EI \frac{d^2 y}{dx^2}, \frac{d\theta}{dx} = \frac{d^2 y}{dx^2} \text{ and} \quad (\text{A9})$$

Potential energy, U

$$\begin{aligned} &= \frac{1}{2} \int_0^L EI \frac{d^2 y}{dx^2} \frac{d^2 y}{dx^2} dx = \frac{1}{2} \int_0^L EI \left(\frac{d^2 y}{dx^2} \right)^2 dx = \frac{1}{2} \int_0^L EI \alpha^2 (6L - 6x)^2 dx = \frac{1}{2} EI \alpha^2 (12L^3) \\ &= 6EI \alpha^2 L^3 \end{aligned} \quad (\text{A10})$$

Kinetic energy consists of both the energy due to the vibrating specimen (K_1) and the attached mass at the end of the specimen (K_2). For the vibrating specimen, the kinetic energy can be expressed as

$$K_1 = \frac{1}{2} \int_0^L \dot{y}^2 dm = \frac{1}{2} \int_0^L [\omega y(x)]^2 \rho A(x) dx = \frac{1}{2} \rho A \omega^2 \int_0^L [y(x)]^2 dx \quad (\text{A11})$$

$$\text{Using } dm = \rho A(x) dx \text{ and assuming harmonic variation, } y(x, t) = y(x) \cos(\omega t) \quad (\text{A12})$$

$$\text{Velocity, } \dot{y} = \omega y(x) \sin(\omega t). \text{ For kinetic energy to be maximum, } \sin(\omega t) = 1 \quad (\text{A13})$$

K_1

$$\begin{aligned}
 &= \frac{1}{2} \rho A \alpha^2 \omega^2 \int_0^L [3Lx^2 - x^3]^2 dx = \frac{1}{2} \rho A \alpha^2 \omega^2 \int_0^L [9L^2 x^4 - 6Lx^5 + x^6] dx \\
 &= \frac{1}{2} \rho A \alpha^2 \omega^2 \left(\frac{33}{35} L^7 \right) = \frac{33}{70} \rho A \alpha^2 \omega^2 L^7 = \frac{33}{70} m \alpha^2 \omega^2 L^6
 \end{aligned} \tag{A14}$$

For the attached lumped mass (m_L), the kinetic energy can be expressed as $K_2 =$

$$\frac{1}{2} m_L v^2 \tag{A15}$$

K_2

$$\begin{aligned}
 &= \frac{1}{2} m_L [\omega y(x)]^2 = \frac{1}{2} \omega^2 m_L \alpha^2 L^4 [2L + 3(x - L)]^2 \\
 &= \frac{1}{2} \omega^2 m_L \alpha^2 L^4 [2L + 3h]^2 = \frac{1}{2} \omega^2 m_L \alpha^2 L^4 (4L^2 + 12Lh + 9h^2) \\
 &= \omega^2 m_L \alpha^2 L^6 \left(2 + 6 \frac{h}{L} + \frac{9h^2}{2L^2} \right)
 \end{aligned} \tag{A16}$$

where $h = x - L$ and is measured from the top of the specimen to the centroid of the lumped mass

For conservation of energy, maximum internal potential energy = maximum kinetic energy

$$U = K_1 + K_2$$

$$\begin{aligned}
 6EI \alpha^2 L^3 &= \frac{33}{70} m \alpha^2 \omega^2 L^6 + \omega^2 m_L \alpha^2 L^6 \left(2 + 6 \frac{h}{L} + \frac{9h^2}{2L^2} \right) \\
 6EI &= \frac{33}{70} m \omega^2 L^3 + \omega^2 m_L L^3 \left(2 + 6 \frac{h}{L} + \frac{9h^2}{2L^2} \right) = \omega^2 \left[\frac{33}{70} m L^3 + m_L L^3 \left(2 + 6 \frac{h}{L} + \frac{9h^2}{2L^2} \right) \right] \\
 \omega^2 &= \frac{3EI}{L^3 \left[\frac{33}{140} m + m_L \left(1 + 3 \frac{h}{L} + \frac{9h^2}{4L^2} \right) \right]}
 \end{aligned} \tag{A17}$$

The term $m_L(1 + 3\frac{h}{L} + \frac{9h^2}{4L^2})$ is the result of a single lumped mass and can be expanded into

$$\sum_{i=1}^N m_{iL} h(h_{0i}, h_{1i}) \text{ where}$$

$$h(h_{0i}, h_{1i}) = 1 + 3\frac{(h_{0i} + h_{1i})}{2L} + \frac{9}{4L^2} \left[\frac{(h_{0i} + h_{1i})}{2} \right]^2 = 1 + 3\frac{(h_{0i} + h_{1i})}{2L} + \frac{9}{16L^2} (h_{0i}^2 + 2h_{0i}h_{1i} + h_{1i}^2) \quad (\text{A18})$$

Appendix C: Procedures for Kumar and Madhusudhan (2010)

1. Assume a value for the Young's modulus of the soil, E_s

2. Calculate:

i. Summation of attached masses $\sum_{i=1}^N m_i$

ii. Summation of the first moment of all the masses about the top of the specimen

$$\sum_{i=1}^N J_{i1} = \sum_{i=1}^N m_i \frac{(h_{i1} + h_{i2})}{2}$$

iii. Summation of the second moment of all the masses about the top of the specimen

$$\sum_{i=1}^N J_{i2} = \sum_{i=1}^N m_i \left(\frac{h_{i1} + h_{i2}}{2} \right)^2$$

iv. $a_{11} = -E_s I_s \beta^2 \sin(\beta L) - E_s I_s \beta^2 \sinh(\beta L) - \omega^2 \sin(\beta L) \sum_{i=1}^N J_{i1} + \omega^2 \sinh(\beta L) \sum_{i=1}^N J_{i1} - \omega^2 \beta \cos(\beta L) \sum_{i=1}^N J_{i2} + \omega^2 \beta \cosh(\beta L) \sum_{i=1}^N J_{i2}$

v. $a_{12} = -E_s I_s \beta^2 \cos(\beta L) - E_s I_s \beta^2 \cosh(\beta L) - \omega^2 \cos(\beta L) \sum_{i=1}^N J_{i1} + \omega^2 \cosh(\beta L) \sum_{i=1}^N J_{i1} + \omega^2 \beta \sin(\beta L) \sum_{i=1}^N J_{i2} + \omega^2 \beta \sinh(\beta L) \sum_{i=1}^N J_{i2}$

vi. $a_{21} = -E_s I_s \beta^3 \cos(\beta L) - E_s I_s \beta^3 \cosh(\beta L) + \omega^2 \sin(\beta L) \sum_{i=1}^N m_i - \omega^2 \sinh(\beta L) \sum_{i=1}^N m_i + \omega^2 \beta \cos(\beta L) \sum_{i=1}^N J_{i1} - \omega^2 \beta \cosh(\beta L) \sum_{i=1}^N J_{i1}$

vii. $a_{22} = E_s I_s \beta^3 \sin(\beta L) - E_s I_s \beta^3 \sinh(\beta L) + \omega^2 \cos(\beta L) \sum_{i=1}^N m_i - \omega^2 \cosh(\beta L) \sum_{i=1}^N m_i - \omega^2 \beta \sin(\beta L) \sum_{i=1}^N J_{i1} - \omega^2 \beta \sinh(\beta L) \sum_{i=1}^N J_{i1}$

3. Iterate value of E_s such that $a_{11}a_{22} - a_{12}a_{21} = 0$

Appendix D: Collation of Results for FIG. 4.3, 4.4 and 4.5

Input Young's Modulus $E = 68.93 \text{ GPa}$

FIG. 4.3 - Variation of γ_m with lumped mass and ν

L/D=1												
Attached Mass (kg)	$\nu=0.1$		$\nu=0.2$		$\nu=0.3$		$\nu=0.35$		$\nu=0.4$		$\nu=0.49$	
	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)
0.1	4.11E+10	59.629	4.07E+10	58.983	4.06E+10	58.925	4.08E+10	59.147	4.11E+10	59.566	4.20E+10	60.963
0.2	4.26E+10	61.858	4.22E+10	61.255	4.22E+10	61.253	4.24E+10	61.507	4.27E+10	61.965	4.37E+10	63.450
0.3	4.33E+10	62.782	4.29E+10	62.198	4.29E+10	62.218	4.31E+10	62.486	4.34E+10	62.959	4.44E+10	64.476
0.4	4.36E+10	63.301	4.32E+10	62.727	4.33E+10	62.759	4.35E+10	63.034	4.38E+10	63.514	4.48E+10	65.049
0.5	4.39E+10	63.629	4.35E+10	63.060	4.35E+10	63.100	4.37E+10	63.379	4.40E+10	63.864	4.51E+10	65.409
0.6	4.40E+10	63.854	4.36E+10	63.290	4.37E+10	63.334	4.39E+10	63.616	4.42E+10	64.104	4.53E+10	65.656
0.7	4.41E+10	64.018	4.37E+10	63.457	4.38E+10	63.505	4.40E+10	63.789	4.43E+10	64.280	4.54E+10	65.836
0.8	4.42E+10	64.137	4.38E+10	63.578	4.39E+10	63.629	4.41E+10	63.914	4.44E+10	64.407	4.55E+10	65.966
0.9	4.43E+10	64.242	4.39E+10	63.685	4.39E+10	63.738	4.41E+10	64.025	4.45E+10	64.518	4.56E+10	66.081
1	4.43E+10	64.327	4.40E+10	63.771	4.40E+10	63.826	4.42E+10	64.114	4.45E+10	64.608	4.56E+10	66.174
2	4.46E+10	64.690	4.42E+10	64.141	4.43E+10	64.204	4.45E+10	64.496	4.48E+10	64.995	4.59E+10	66.570
4	4.47E+10	64.863	4.43E+10	64.317	4.44E+10	64.383	4.46E+10	64.678	4.49E+10	65.179	4.60E+10	66.759
8	4.48E+10	64.957	4.44E+10	64.414	4.44E+10	64.482	4.47E+10	64.777	4.50E+10	65.280	4.61E+10	66.862

Note:

Back Calculated (denoted as Back Cal. E) is derived from the equation by Cascante et al. (1998) using the flexural frequency obtained from the finite element simulation

FIG. 4.3 - Variation of γ_m with lumped mass and v

L/D=2												
Attached Mass (kg)	v=0.1		v=0.2		v=0.3		v=0.35		v=0.4		v=0.49	
	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)
0.1	5.68E+10	82.344	5.67E+10	82.321	5.71E+10	82.896	5.75E+10	83.446	5.80E+10	84.206	5.95E+10	86.264
0.2	5.79E+10	84.001	5.79E+10	84.010	5.83E+10	84.607	5.87E+10	85.163	5.92E+10	85.925	6.06E+10	87.970
0.3	5.84E+10	84.777	5.85E+10	84.801	5.89E+10	85.405	5.93E+10	85.961	5.98E+10	86.721	6.12E+10	88.752
0.4	5.87E+10	85.222	5.88E+10	85.254	5.92E+10	85.860	5.96E+10	86.415	6.01E+10	87.172	6.15E+10	89.192
0.5	5.89E+10	85.507	5.90E+10	85.544	5.94E+10	86.153	5.98E+10	86.707	6.03E+10	87.461	6.17E+10	89.472
0.6	5.91E+10	85.695	5.91E+10	85.736	5.95E+10	86.344	5.99E+10	86.898	6.04E+10	87.650	6.18E+10	89.654
0.7	5.92E+10	85.842	5.92E+10	85.885	5.96E+10	86.494	6.00E+10	87.047	6.05E+10	87.798	6.19E+10	89.796
0.8	5.93E+10	85.955	5.93E+10	86.000	5.97E+10	86.609	6.01E+10	87.161	6.06E+10	87.911	6.20E+10	89.904
0.9	5.93E+10	86.044	5.93E+10	86.090	5.98E+10	86.700	6.01E+10	87.251	6.07E+10	88.000	6.20E+10	89.989
1	5.94E+10	86.123	5.94E+10	86.170	5.98E+10	86.780	6.02E+10	87.331	6.07E+10	88.079	6.21E+10	90.065
2	5.96E+10	86.452	5.96E+10	86.504	6.00E+10	87.113	6.04E+10	87.661	6.09E+10	88.404	6.23E+10	90.375
4	5.97E+10	86.620	5.97E+10	86.675	6.02E+10	87.284	6.05E+10	87.830	6.11E+10	88.570	6.24E+10	90.531
8	5.98E+10	86.696	5.98E+10	86.752	6.02E+10	87.360	6.06E+10	87.905	6.11E+10	88.643	6.25E+10	90.599

FIG. 4.3 - Variation of γ_m with lumped mass and v

L/D=3												
Attached Mass (kg)	v=0.1		v=0.2		v=0.3		v=0.35		v=0.4		v=0.49	
	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)	Back Cal E (Pa)	Ratio (%)
0.1	6.19E+10	89.773	6.20E+10	89.959	6.25E+10	90.639	6.29E+10	91.197	6.34E+10	91.928	6.47E+10	93.804
0.2	6.27E+10	90.958	6.28E+10	91.156	6.33E+10	91.835	6.37E+10	92.385	6.42E+10	93.103	6.54E+10	94.938
0.3	6.31E+10	91.573	6.33E+10	91.776	6.37E+10	92.451	6.41E+10	92.995	6.46E+10	93.703	6.58E+10	95.509
0.4	6.34E+10	91.922	6.35E+10	92.128	6.40E+10	92.799	6.43E+10	93.338	6.48E+10	94.039	6.61E+10	95.822
0.5	6.35E+10	92.162	6.37E+10	92.369	6.41E+10	93.037	6.45E+10	93.572	6.50E+10	94.267	6.62E+10	96.035
0.6	6.36E+10	92.319	6.38E+10	92.527	6.42E+10	93.192	6.46E+10	93.724	6.51E+10	94.415	6.63E+10	96.169
0.7	6.37E+10	92.444	6.39E+10	92.652	6.43E+10	93.315	6.47E+10	93.844	6.52E+10	94.532	6.64E+10	96.276
0.8	6.38E+10	92.530	6.39E+10	92.739	6.44E+10	93.400	6.47E+10	93.927	6.52E+10	94.612	6.64E+10	96.348
0.9	6.38E+10	92.607	6.40E+10	92.816	6.44E+10	93.475	6.48E+10	94.000	6.53E+10	94.682	6.65E+10	96.412
1	6.39E+10	92.698	6.40E+10	92.907	6.45E+10	93.565	6.49E+10	94.089	6.53E+10	94.769	6.65E+10	96.494
2	6.41E+10	92.925	6.42E+10	93.135	6.46E+10	93.784	6.50E+10	94.301	6.55E+10	94.970	6.66E+10	96.665
4	6.41E+10	93.057	6.43E+10	93.267	6.47E+10	93.911	6.51E+10	94.423	6.55E+10	95.087	6.67E+10	96.764
8	6.42E+10	93.122	6.43E+10	93.332	6.48E+10	93.974	6.51E+10	94.483	6.56E+10	95.143	6.67E+10	96.811

FIG. 4.4 - Variation of χ_m with height of steel attachment

Input Young's Modulus E = 68.93 GPa

L/D=1

Attached Mass		v=0.1		v=0.2		v=0.3		v=0.4		v=0.49	
Height (m)	Weight (kg)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)
0.02	0.3082688	4.83E+10	70.141	4.83E+10	70.141	4.84E+10	70.189	4.92E+10	71.349	5.06E+10	73.349
0.05	0.7706719	5.16E+10	74.857	5.16E+10	74.857	5.25E+10	76.146	5.38E+10	78.082	5.58E+10	80.964
0.07	1.0789407	5.15E+10	74.665	5.15E+10	74.665	5.27E+10	76.399	5.42E+10	78.596	5.64E+10	81.763

L/D=2

Attached Mass		v=0.1		v=0.2		v=0.3		v=0.4		v=0.49	
Height (m)	Weight (kg)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)
0.02	0.3082688	6.01E+10	87.161	6.01E+10	87.250	6.06E+10	87.897	6.15E+10	89.225	6.29E+10	91.221
0.05	0.7706719	6.07E+10	88.014	6.08E+10	88.272	6.14E+10	89.044	6.23E+10	90.446	6.37E+10	92.448
0.07	1.0789407	5.99E+10	86.861	6.01E+10	87.188	6.07E+10	88.005	6.16E+10	89.425	6.30E+10	91.412

L/D=3

Attached Mass		v=0.1		v=0.2		v=0.3		v=0.4		v=0.49	
Height (m)	Weight (kg)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)	Back Cal. E (Pa)	Ratio (%)
0.02	0.3082688	6.40E+10	92.874	6.42E+10	93.090	6.46E+10	93.756	6.55E+10	94.967	6.66E+10	96.688
0.05	0.7706719	6.43E+10	93.278	6.45E+10	93.544	6.49E+10	94.213	6.57E+10	95.370	6.68E+10	96.975
0.07	1.0789407	6.36E+10	92.328	6.38E+10	92.616	6.43E+10	93.283	6.51E+10	94.412	6.61E+10	95.960

FIG. 4.5 - χ_m with 2 kg lumped mass of different heights (L/D=2)

Input Young's Modulus E = 68.93 GPa

Attached Mass		v=0.1		v=0.2		v=0.3		v=0.4		v=0.49	
Height	Density	Back	Ratio	Back	Ratio	Back	Ratio	Back	Ratio	Back	Ratio
(m)	(kg/m ³)	Cal. E	(%)	Cal. E	(%)	Cal. E	(%)	Cal. E	(%)	Cal. E	(%)
		(Pa)		(Pa)		(Pa)		(Pa)		(Pa)	
0.005	203718.3	6.02E+10	87.318	6.02E+10	87.358	6.06E+10	87.942	6.15E+10	89.188	6.28E+10	91.082
0.03	33953.05	6.11E+10	88.636	6.12E+10	88.823	6.17E+10	89.536	6.27E+10	90.889	6.4E+10	92.859
0.05	20371.83	6.07E+10	88.064	6.09E+10	88.337	6.14E+10	89.116	6.24E+10	90.513	6.38E+10	92.498
0.07	14551.31	5.98E+10	86.794	6.01E+10	87.127	6.06E+10	87.947	6.16E+10	89.364	6.3E+10	91.341

Appendix E: Collation of Results for Test Series II, III and IV

Test Series II

Specimen Tag: RS-US_100H

(u _a -u _w) (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
				Excitation Frequency (kHz)							
				10	20	50	AVG	3	5	10	AVG
0	100.00	1600.00	205.13	2.37	2.35	2.91	2.54	3.70	3.34	2.91	3.32
2	99.86	1600.00	195.12	2.11	2.34	2.22	2.22	3.86	3.31	3.11	3.42
4	84.32	1600.00	195.12	2.47	2.21	2.77	2.48	3.87	3.36	2.82	3.35
6	80.50	1600.00	195.12	1.82	1.65	2.19	1.89	3.31	2.93	2.69	2.98
8	79.40	1600.00	195.12	1.84	1.44	1.45	1.58	3.33	2.89	2.79	3.00
10	76.46	666.67	200.00	2.79	2.39	2.63	2.60	3.58	2.72	1.90	2.73
12	73.24	333.33	202.02	2.45	1.95	1.91	2.10	2.72	1.98	1.53	2.08
14	68.84	347.83	208.33	2.05	1.67	1.65	1.79	2.04	1.90	1.50	1.81
16	62.27	363.64	219.78	1.38	1.41	1.47	1.42	2.00	2.24	1.76	2.00
18	56.74	370.37	219.78	1.29	1.34	1.50	1.38	2.51	2.08	1.62	2.07
20	52.40	370.37	228.57	0.91	1.21	1.21	1.11	2.38	2.04	1.81	2.08
22	49.38	370.37	231.21	0.97	1.12	1.10	1.06	2.39	2.00	1.68	2.03
24	45.87	370.37	232.56	0.81	0.84	0.94	0.86	2.75	2.16	1.42	2.11
26	42.88	377.36	235.29	0.75	0.80	0.83	0.79	2.19	1.71	1.43	1.78
30	37.69	360.36	235.29	0.80	0.82	0.87	0.83	2.16	1.58	1.50	1.75
34	33.54	370.37	235.29	0.73	0.76	1.17	0.89	2.42	1.94	1.61	1.99
38	29.18	370.37	242.42	0.82	0.85	0.94	0.87	2.51	1.91	1.44	1.96
34	30.10	360.36	235.29	0.68	0.69	0.83	0.73	2.25	1.90	1.77	1.97
30	31.20	360.36	235.29	0.58	0.60	0.71	0.63	2.09	1.75	1.53	1.79
25	33.30	366.97	235.29	0.56	0.74	0.81	0.70	2.24	1.81	1.70	1.91
20	35.93	366.97	235.29	1.07	1.13	1.13	1.11	2.06	1.59	1.41	1.69
15	40.60	363.64	232.56	1.19	1.27	1.28	1.25	2.29	1.65	1.44	1.79
10	49.30	338.98	215.05	0.77	1.09	1.12	0.99	2.54	2.16	2.04	2.24
5	58.27	338.98	210.53	0.92	0.94	0.94	0.93	3.19	2.17	1.84	2.40
0	89.17	307.69	186.92	1.16	1.68	1.51	1.45	3.48	3.35	2.84	3.22
2	88.87	320.00	190.48	1.74	1.65	1.23	1.54	3.58	3.01	2.69	3.09
4	86.99	320.00	192.31	1.70	1.76	1.41	1.62	3.51	2.84	2.71	3.02
6	83.93	333.33	198.02	2.05	1.93	2.08	2.02	3.27	1.82	2.02	2.37
8	76.61	333.33	203.05	2.43	2.35	2.52	2.43	3.19	1.71	1.46	2.12
10	69.71	347.83	208.33	1.25	2.20	1.24	1.56	2.98	1.63	1.57	2.06
12	66.02	333.33	208.33	1.43	1.35	1.51	1.43	2.90	1.46	1.35	1.90
14	61.65	347.83	217.39	1.28	1.24	1.22	1.24	2.50	2.12	2.02	2.21
16	58.27	341.88	217.39	0.78	0.90	0.82	0.83	2.59	1.97	1.98	2.18
18	55.42	347.83	222.22	0.94	0.96	0.95	0.95	2.93	2.36	1.92	2.40
20	52.20	353.98	225.99	1.02	1.09	1.11	1.07	2.81	2.27	1.92	2.33
22	49.54	357.14	228.57	1.11	1.12	1.14	1.12	2.70	2.14	2.02	2.29
24	47.42	357.14	228.57	0.99	1.01	0.99	1.00	2.60	2.06	1.92	2.19
26	44.27	347.83	228.57	1.23	1.31	1.24	1.26	2.47	2.00	1.70	2.06
30	32.15	392.16	251.57	1.38	1.16	1.07	1.20	2.70	2.10	2.12	2.30
34	28.53	380.95	248.45	1.36	1.37	1.30	1.34	2.82	2.29	2.09	2.40
38	25.74	384.62	248.45	1.10	1.14	1.14	1.13	2.80	2.24	1.96	2.33

Specimen Tag: RS-US_100

(u _a -u _w) (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
				Excitation Frequency (kHz)							
				10	20	50	AVG	3	5	10	AVG
0	99.59	1634.62	197.67	1.31	1.40	1.36	1.36	3.38	3.79	3.50	3.55
2	85.17	1634.62	202.38	1.71	1.80	1.54	1.68	4.42	3.35	2.86	3.54
4	83.84	1634.62	202.38	2.09	1.92	2.06	2.02	3.05	2.96	2.77	2.93
6	81.64	1545.45	202.38	1.41	1.58	1.53	1.51	2.75	2.54	2.08	2.46
8	77.25	425.00	223.68	0.79	1.48	1.47	1.24	2.37	2.58	2.31	2.42
10	74.73	425.00	223.68	1.19	1.39	1.44	1.34	2.61	2.17	2.63	2.47
12	68.00	425.00	229.73	1.19	0.82	1.45	1.15	3.01	2.54	2.72	2.76
14	66.55	425.00	229.73	0.77	0.99	1.43	1.07	3.43	2.32	2.30	2.69
16	64.70	425.00	236.11	0.67	0.67	0.89	0.74	2.77	2.61	2.33	2.57
18	61.41	425.00	239.44	0.71	0.70	0.67	0.69	2.72	1.94	2.04	2.23
20	56.92	425.00	242.86	0.71	0.69	0.75	0.72	2.48	2.44	2.26	2.39

Specimen Tag: RS-S

(u _a -u _w) (kPa)	V _p (m/s)	V _s (m/s)
10	1600.00	105.26
60	1600.00	166.67
100	1600.00	181.82
110	1600.00	190.48
120	1600.00	195.12
130	1600.00	195.12
138	1600.00	195.12
200	1600.00	205.13
210	1600.00	210.53
220	1600.00	216.22
230	1600.00	210.53
238	1600.00	210.53
400	1600.00	266.67
410	1600.00	266.67
420	1600.00	272.11
430	1600.00	273.97
438	1600.00	273.97

Specimen Tag: OTT-D

σ' (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ_s (%)			
				Excitation Frequency (kHz)			
				3	5	10	AVG
50	0	500.00	242.86	1.62	1.47	1.22	1.44
100	0	505.95	274.19	0.81	1.13	1.02	0.99
200	0	586.21	329.46	1.53	1.31	0.86	1.23
400	0	680.00	386.36	3.03	1.51	1.50	2.01
800	0	739.13	445.03	1.19	0.69	0.64	0.84

Specimen Tag: RS-US_200H

(u _a -u _w) (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
				Excitation Frequency (kHz)							
				10	20	50	AVG	3	5	10	AVG
0	100.00	1500.00	220.00	0.51	0.65	0.82	0.66	4.06	2.18	1.70	2.65
2	89.61	1500.00	220.00	0.98	0.59	0.78	0.78	3.09	2.13	1.86	2.36
4	89.22	1500.00	220.00	0.90	1.46	1.45	1.27	2.99	2.28	1.52	2.26
6	86.70	600.00	220.00	1.51	1.54	1.44	1.49	2.76	1.77	1.28	1.94
8	77.68	507.69	235.71	1.45	1.95	1.60	1.67	2.48	1.70	1.43	1.87
10	68.37	440.00	235.71	2.12	1.60	1.19	1.64	2.35	1.56	1.65	1.85
12	62.21	440.00	235.71	1.86	1.81	1.26	1.64	2.06	1.83	2.31	2.07
14	54.87	440.00	235.71	1.66	1.41	1.52	1.53	2.46	1.78	1.49	1.91
16	50.13	440.00	235.71	1.53	1.30	0.98	1.27	2.45	1.64	2.63	2.24
18	45.36	440.00	244.44	1.55	0.95	0.97	1.16	2.30	2.07	1.67	2.01
20	40.73	440.00	244.44	1.05	1.19	1.24	1.16	1.96	1.95	0.99	1.63
22	35.06	440.00	253.85	1.15	1.14	1.09	1.12	1.91	1.69	1.60	1.73
24	31.76	440.00	253.85	1.20	1.10	1.10	1.13	2.08	1.72	1.29	1.70
26	28.71	440.00	253.85	1.31	1.27	1.24	1.27	2.06	1.05	1.11	1.41
30	24.22	471.43	264.00	1.35	1.11	1.11	1.19	2.20	1.90	1.32	1.81
34	18.05	471.43	264.00	1.19	1.09	1.01	1.10	2.00	1.92	1.53	1.82
38	15.13	471.43	275.00	1.34	1.26	1.25	1.28	2.05	1.60	1.67	1.77

Specimen Tag: RS-US_200

(u _a -u _w) (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
				Excitation Frequency (kHz)							
				10	20	50	AVG	3	5	10	AVG
0	100.00	1607.14	225.00	0.41	0.63	0.66	0.57	3.26	2.91	2.43	2.87
2	91.50	1636.36	230.77	0.64	0.49	1.06	0.73	3.34	3.00	2.82	3.06
4	91.03	1636.36	225.00	1.36	0.68	1.48	1.17	3.23	2.03	2.40	2.56
6	89.44	400.00	225.00	1.02	0.91	0.49	0.81	2.02	2.38	1.30	1.90
8	85.69	450.00	216.87	0.70	0.77	0.61	0.69	2.04	1.48	0.86	1.46
10	78.39	382.98	219.51	0.76	0.93	0.51	0.74	1.85	2.00	1.26	1.71
12	65.47	400.00	225.00	0.78	1.13	1.52	1.14	2.56	1.14	0.92	1.54
14	60.25	428.57	225.00	0.54	1.30	1.02	0.95	1.71	1.93	1.26	1.63
16	56.43	450.00	230.77	0.58	0.68	1.59	0.95	2.29	1.45	1.32	1.69
18	50.42	418.60	233.77	0.62	0.85	0.50	0.66	1.79	1.37	1.43	1.53
20	46.93	450.00	246.58	0.61	0.57	0.49	0.56	2.00	1.54	0.97	1.51
22	43.72	450.00	253.52	0.72	0.59	0.60	0.63	2.03	1.23	1.61	1.63
24	40.06	450.00	253.52	0.66	0.58	0.64	0.62	1.86	1.42	1.70	1.66
26	36.81	450.00	257.14	0.43	0.54	0.52	0.50	2.48	1.37	1.27	1.71
30	30.24	384.62	260.87	0.51	0.99	0.33	0.61	2.47	1.23	1.37	1.69
34	26.05	384.62	264.71	0.54	0.87	0.50	0.64	1.62	2.18	1.17	1.66
38	22.79	384.62	290.32	0.42	0.24	0.58	0.41	1.12	1.73	1.43	1.43

Specimen Tag: RS-US_400H

(u _a -u _w) (kPa)	S _r (%)	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
				Excitation Frequency (kHz)							
				10	20	50	AVG	3	5	10	AVG
0	100.00	1760.87	283.22	1.50	1.58	1.93	1.67	3.00	2.68	3.31	3.00
2	94.96	1687.50	279.31	1.69	2.10	2.09	1.96	2.32	2.34	1.57	2.08
4	94.09	1687.50	279.31	1.63	1.85	1.86	1.78	1.88	1.84	1.38	1.70
6	93.44	1687.50	279.31	2.64	1.67	1.65	1.99	1.83	1.69	1.71	1.75
8	92.54	1687.50	279.31	1.47	1.52	1.33	1.44	1.83	1.56	1.60	1.67
10	89.31	1446.43	285.21	1.90	1.91	2.00	1.94	1.70	1.70	1.68	1.69
12	80.41	405.00	285.21	2.01	1.86	1.65	1.84	1.63	1.42	1.32	1.46
14	72.95	506.25	285.21	1.90	1.68	1.77	1.78	1.54	0.96	1.13	1.21
16	65.45	460.23	287.23	1.54	1.47	1.47	1.49	1.81	1.22	0.98	1.34
18	58.92	445.05	287.23	1.56	1.54	1.58	1.56	1.80	0.76	1.53	1.36
20	53.72	445.05	287.23	1.51	1.48	1.51	1.50	2.25	0.97	1.35	1.52
22	47.23	440.22	289.29	1.44	1.35	1.43	1.40	2.50	1.57	1.36	1.81
24	42.51	440.22	291.37	1.28	1.24	1.21	1.24	2.14	1.98	1.87	2.00
26	38.17	435.48	291.37	1.25	1.19	1.11	1.18	1.95	1.68	1.52	1.72
30	31.32	440.22	300.00	1.29	1.25	1.22	1.25	1.98	2.04	1.66	1.89
34	26.46	455.06	302.24	1.27	1.28	1.25	1.27	2.07	1.91	1.77	1.92
38	22.80	455.06	304.51	1.29	1.32	1.36	1.32	2.56	1.90	1.45	1.97
34	23.55	455.06	302.24	1.31	1.29	1.23	1.27	2.90	2.05	2.16	2.37
30	24.81	455.06	304.51	1.32	1.12	1.26	1.23	2.94	2.77	2.35	2.69
26	26.54	455.06	304.51	1.31	1.12	1.09	1.17	2.52	2.70	2.19	2.47
22	29.74	455.06	302.24	1.18	1.07	1.15	1.13	2.25	2.13	1.61	2.00
18	33.79	455.06	295.62	1.27	1.18	1.16	1.20	2.04	1.78	1.17	1.66
14	40.93	440.22	291.37	1.28	1.25	1.20	1.25	1.75	1.72	1.13	1.53
12	44.27	440.22	287.23	1.29	1.28	1.23	1.27	1.93	1.54	1.55	1.67
10	49.56	426.32	285.21	1.28	1.31	1.29	1.29	2.08	1.73	1.38	1.73
8	53.22	426.32	283.22	1.21	1.21	1.23	1.22	1.86	1.92	2.13	1.97
6	56.97	426.32	281.25	1.36	1.29	1.33	1.33	2.25	1.95	2.11	2.10
4	62.40	435.48	277.40	1.39	1.41	1.41	1.40	1.99	1.67	1.83	1.83
2	75.52	435.48	277.40	1.85	1.80	1.81	1.82	2.19	2.26	1.88	2.11
0	89.05	435.48	261.29	2.06	1.85	1.92	1.94	2.28	1.54	1.57	1.80
2	89.04	455.06	261.29	1.81	1.76	1.88	1.82	2.20	1.81	1.76	1.92
4	88.78	455.06	270.00	1.92	1.75	2.01	1.89	2.17	1.60	1.39	1.72
6	88.26	455.06	270.00	1.94	1.78	1.99	1.90	2.13	1.49	1.60	1.74
8	87.61	455.06	270.00	1.59	1.65	1.61	1.62	2.08	1.55	1.51	1.71
10	87.18	455.06	270.00	1.58	1.67	1.44	1.57	2.29	1.93	1.41	1.87
12	80.91	455.06	273.65	1.54	1.56	1.51	1.53	1.76	1.52	1.52	1.60
14	72.72	455.06	285.21	1.52	1.58	1.49	1.53	1.43	1.15	0.99	1.19
18	59.22	413.27	279.31	1.36	1.24	1.41	1.34	1.69	1.69	1.54	1.64
22	49.81	413.27	279.31	1.17	1.19	1.22	1.19	1.95	2.03	1.56	1.85
26	39.79	413.27	283.22	1.08	1.23	1.23	1.18	2.51	1.63	1.59	1.91
30	31.86	421.88	289.29	1.13	1.10	1.12	1.12	2.50	2.01	1.94	2.15
34	25.69	421.88	289.29	1.04	1.02	1.03	1.03	2.67	1.10	1.63	1.80
38	20.95	430.85	293.48	1.02	1.01	1.16	1.06	2.61	1.67	2.06	2.12

Test Series III

Rubber Content by Mass (%)	V _s (m/s)				V _p (m/s)			
	Effective Confining Pressure (kPa)							
	0	100	200	400	0	100	200	400
0	82.60	189.50	225.00	268.70		400.00	473.70	529.40
15	68.20	173.10	204.50	243.20		375.00	450.00	500.00
30		144.00	173.10	202.20		321.40	383.00	450.00
60		104.70	126.80	158.50				409.10
100		68.80	86.10	103.20				295.50

Effective Confining Pressure (kPa)	Rubber Content by Mass (%)	ξ_s (%)			
		Excitation Frequency (kHz)			
		3	5	10	AVG
100	0	2.11	2.10	1.95	2.06
100	15	2.83	2.55	2.50	2.63
100	30	2.93	2.71	2.63	2.75
100	60	5.31	5.63	4.59	5.18
100	100	6.63	6.58	6.48	6.56
200	0	1.97	1.92	1.84	1.91
200	15	2.77	2.68	2.82	2.76
200	30	2.86	3.03	3.13	3.01
200	60	4.17	5.00	4.89	4.69
200	100	6.59	6.72	5.89	6.40
400	0	1.57	1.54	1.73	1.61
400	15	2.48	2.40	2.36	2.41
400	30	2.80	3.49	3.34	3.21
400	60	5.01	5.27	5.34	5.21
400	100	6.06	6.05	5.54	5.88

Test Series IV

Results from bender element test

Specimen Tag	V _p (m/s)	V _s (m/s)	ξ _p (%)				ξ _s (%)			
			Excitation Frequency (kHz)							
			10	20	50	AVG	3	5	10	AVG
AMK1-S_100	1606.96	155.25	1.48	1.52	1.08	1.36	5.98	4.25	3.63	4.62
AMK1-S_200	1551.33	192.23	2.25	2.21	1.74	2.07	5.07	5.16	4.74	4.99
AMK1-S_350	1506.70	232.11	0.43	0.66	1.47	0.85	4.89	4.53	2.93	4.12
AMK1-US_50	1578.95	250.00	0.78	0.69	0.54	0.67	4.57	4.75	3.69	4.34
AMK1-US_100	1591.23	266.76	0.68	0.83	0.50	0.67	5.24	4.93	3.40	4.52
AMK2-S_100	1544.07	159.82	1.09	0.99		1.04	5.40	4.99	3.70	4.70
AMK2-S_200	1490.32	177.63	1.37	1.00	1.20	1.19	5.60	5.26	3.89	4.92
AMK2-S_350	1497.99	210.83	0.65	0.28	0.63	0.52	5.19	4.67	2.17	4.01
AMK2-US_50	488.41	194.32	0.94	1.30	1.18	1.14	4.84	4.99	3.65	4.49
AMK2-US_100	481.33	228.63	1.17	0.97	1.04	1.06	4.84	4.99	3.65	4.49

Results from cyclic triaxial test

Specimen Tag: AMK1

Specimen Tag	Damping Ratio, % (Freq=0.2)					Damping Ratio, % (Freq=0.5)				
	Strain (%)					Strain (%)				
	0.05	0.1	0.2	0.5	1	0.05	0.1	0.2	0.5	1
AMK1-S_100	11.74	13.31	14.44			12.57	13.45	14.91		
AMK1-S_200	11.01	12.58	14.57			10.76	13.21	14.44		
AMK1-S_350	9.56	11.55	13.74	15.73	17.00	9.95	12.00	12.90	14.93	16.66
AMK1-US_50	9.48	12.05	13.92	17.20	17.81	10.20	12.36	14.75	16.02	16.36
AMK1-US_100	11.23	13.80	16.12	18.75	18.74	11.51	14.27	15.96	17.53	19.54

Specimen Tag	E, MPa (Freq=0.2)					E, MPa (Freq=0.5)				
	Strain (%)					Strain (%)				
	0.05	0.1	0.2	0.5	1	0.05	0.1	0.2	0.5	1
AMK1-S_100	43.73	33.52	25.51			41.11	32.13	22.80		
AMK1-S_200	67.91	51.44	36.64			67.41	49.38	34.33		
AMK1-S_350	115.72	86.10	64.18	38.02	24.00	111.44	84.61	61.91	37.28	23.11
AMK1-US_50	104.55	79.64	59.60	36.85	23.40	96.15	79.98	64.31	39.07	23.01
AMK1-US_100	120.13	81.75	69.32	30.88	15.45	107.15	76.53	56.92	31.04	16.99

Specimen Tag: AMK2

Specimen Tag	Damping Ratio, % (Freq=0.2)					Damping Ratio, % (Freq=0.5)				
	Strain (%)					Strain (%)				
	0.05	0.1	0.2	0.5	1	0.05	0.1	0.2	0.5	1
AMK2-S_100	8.34	9.36	10.64			8.60	9.39	10.08		
AMK2-S_200	8.94	10.15	11.47			8.36	10.37	11.28		
AMK2-S_350	8.63	9.82	11.05	13.40	14.70	8.28	9.30	10.73	12.87	13.91
AMK2-US_50	6.78	8.06	9.19	12.01	13.77	6.84	7.86	9.58	11.18	13.14
AMK2-US_100	11.07	8.72	8.73	10.68	11.32	9.01	8.23	8.92	9.84	11.49

Specimen Tag	E, MPa (Freq=0.2)					E, MPa (Freq=0.5)				
	Strain (%)					Strain (%)				
	0.05	0.1	0.2	0.5	1	0.05	0.1	0.2	0.5	1
AMK2-S_100	54.95	43.17	32.92			53.02	41.62	30.74		
AMK2-S_200	69.09	54.05	40.61			71.68	53.12	39.85		
AMK2-S_350	123.01	95.56	72.12	43.94	24.58	117.28	93.48	69.74	36.87	24.22
AMK2-US_50	94.08	77.39	63.57	39.56	24.22	91.62	75.61	61.87	41.65	24.71
AMK2-US_100	132.97	119.11	100.21	63.04	36.05	131.96	115.70	78.97	63.69	39.27

Specimen Name: AMK1-S

1- Results from bender elements tests

σ_3' (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
100	1607.0	155.2	41	4378	124	0.495	1.36	4.62
200	1551.3	192.2	63	4047	189	0.492	2.07	4.99
350	1506.7	232.1	92	3774	275	0.488	0.85	4.12

2 - Results from cyclic triaxial tests

Specimen Name: AMK1-S_100

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	42	0.289	16	14	12.16
0.1	33	0.298	13	11	13.38
0.2	24	0.305	9	8	14.68

Specimen Name: AMK1-S_200

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	68	0.485	23	23	10.88
0.1	50	0.620	16	17	12.90
0.2	35	0.627	11	12	14.51

Specimen Name: AMK1-S_350

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	114	0.564	36	38	9.75
0.1	85	0.227	35	29	11.78
0.2	63	0.338	24	21	13.32
0.5	38	0.329	14	13	15.33
1	24	0.709	7	8	16.83

E: Derived from LVDT results

G - 1: Converted shear modulus from cyclic triaxial test derived Poisson's ratio ν

G - 2: Converted shear modulus from bender element test derived Poisson's ratio ν

Specimen Name: AMK1-US_50

1- Results from bender elements tests

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
200	50	1578.9	250.0	109	4186	323	0.487	0.67	4.34

2 - Results from cyclic triaxial tests

Specimen Name: AMK1-US_50

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	100	0.477	34	34	9.84
0.1	80	0.159	34	27	12.21
0.2	62	0.160	27	21	14.34
0.5	38	0.548	12	13	16.61
1	23	0.572	7	8	17.09

Specimen Name: AMK1-US_100

1- Results from bender elements tests

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
200	100	1591.2	266.8	123	4198	364	0.486	0.67	4.52

2 - Results from cyclic triaxial tests

Specimen Name: AMK1-US_100

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	114	0.415	40	38	17.27
0.1	79	0.463	27	27	19.21
0.2	63	0.444	22	21	26.81
0.5	31	0.531	10	10	29.10
1	16	0.592	5	5	29.70

E: Derived from LVDT results

G - 1: Converted shear modulus from cyclic triaxial test derived Poisson's ratio ν

G - 2: Converted shear modulus from bender element test derived Poisson's ratio ν

Specimen Name: AMK2-S

1- Results from bender elements tests

σ_3' (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
100	1544.1	159.8	44	4044	131	0.495	1.04	4.70
200	1490.3	177.6	54	3750	162	0.493	1.19	4.92
350	1498.0	210.8	76	3760	228	0.490	0.52	4.01

2 - Results from cyclic triaxial tests

Specimen Name: AMK2-S_100

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	54	0.469	18	18	8.47
0.1	42	0.509	14	14	9.38
0.2	32	0.505	11	11	10.36

Specimen Name: AMK2-S_200

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	70	0.404	25	24	8.65
0.1	54	0.441	19	18	10.26
0.2	40	0.449	14	13	11.38

Specimen Name: AMK2-S_350

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	120	0.287	47	40	8.45
0.1	95	0.364	35	32	9.56
0.2	71	0.312	27	24	10.89
0.5	40	0.366	15	14	13.13
1	24	0.409	9	8	14.31

E: Derived from LVDT results

G - 1: Converted shear modulus from cyclic triaxial test derived Poisson's ratio ν

G - 2: Converted shear modulus from bender element test derived Poisson's ratio ν

Specimen Name: AMK2-US_50

1- Results from bender elements tests

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
200	50	488.4	194.3	63	312	176	0.406	1.14	4.49

2 - Results from cyclic triaxial tests

Specimen Name: AMK2-US_50

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	93	0.332	35	33	6.81
0.1	77	0.290	30	27	7.96
0.2	63	0.370	23	22	9.39
0.5	41	0.356	15	14	11.60
1	24	0.405	9	9	13.45

Specimen Name: AMK2-US_100

1- Results from bender elements tests

$(\sigma_3 - u_a)$ (kPa)	$(u_a - u_w)$ (kPa)	V_p (m/s)	V_s (m/s)	G_{max} (MPa)	K_{max} (MPa)	E_{max} (MPa)	ν	ξ - P-wave (%)	ξ - S-wave (%)
200	100	481.3	228.6	85	262	229	0.354	1.06	4.49

2 - Results from cyclic triaxial tests

Specimen Name: AMK2-US_100

Strain (%)	E (MPa)	ν	G - 1 (MPa)	G - 2 (MPa)	ξ (%)
0.05	132	0.234	54	49	10.04
0.1	117	0.596	37	43	8.48
0.2	90	0.332	34	33	8.83
0.5	63	0.272	25	23	10.26
1	38	0.291	15	14	11.41

E: Derived from LVDT results

G - 1: Converted shear modulus from cyclic triaxial test derived Poisson's ratio ν

G - 2: Converted shear modulus from bender element test derived Poisson's ratio ν