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INVESTIGATION ON ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING WITH INDEX MODULATION

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Abstract

This thesis is devoted to investigating orthogonal frequency division multiplexing (OFDM) with index modulation (OFDM-IM) to improve its bit error rate (BER) performance and spectral efficiency.

In OFDM-IM, by selecting a number of subcarriers as active subcarriers to carry constellation symbols, the indices of these active subcarriers may carry additional bits of information. Compared to classical OFDM, OFDM-IM offers spectral efficiency boost and BER performance improvement in certain circumstances. To be specific, in low SNR region, OFDM-IM's BER performance is not as good as classical OFDM's while in high SNR region, OFDM-IM outperforms classical OFDM in terms of BER performance. In addition, OFDM-IM's spectral efficiency superiority over classical OFDM is evident only when Binary Phase Shift Keying (BPSK) is adopted. For constellation sizes higher than BPSK, such superiority is only observed for very few carefully chosen system configurations. Motivated by the above analysis, this thesis focuses on improving OFDM-IM in terms of BER performance and spectral efficiency when BPSK, Quadrature Phase Shift Keying (QPSK) and constellation sizes higher than QPSK are adopted, respectively.

In OFDM-IM, for a given subblock size, only a fixed number of active subcarriers are selected to carry constellation symbols, which does not make full use of other possible numbers of active subcarriers. Hence, OFDM with type-1 generalized index modulation (OFDM-GIM1) is proposed, where the number of active subcarriers in an OFDM subblock is no longer fixed. Dependent on the input binary string, different number of active subcarriers are assigned to carry constellation symbols. With the developed modulation and demodulation of indices in the OFDM-GIM1, the spectral efficiency of the scheme is further improved when BPSK is adopted.

To tackle the problem of the spectral efficiency of OFDM-IM for QPSK, a

novel scheme named as OFDM with type-2 generalized index modulation (OFDM-GIM2) is proposed. In this scheme, independent index modulation is performed on the in-phase and quadrature component of each subcarrier. Through such a way, a higher spectral efficiency than that of OFDM-IM may be achieved. OFDM-GIM2 may be further extended, such that the input bit string jointly decides the active indices for in-phase and quadrature components. As a result, more bits can be transmitted per OFDM frame.

The symbol constellations of Phase Shift Keying (PSK) or amplitude modulation are identical when the constellation sizes are of 2 or 4. When the constellation sizes are larger, in OFDM, it is well known that Quadrature Amplitude Modulation (QAM) outperforms PSK in terms of BER performance. In this thesis, the 8PSK and 8QAM performances in OFDM-GIM1 are investigated, and the results show that OFDM-GIM1 adopting 8PSK provides a higher index detection accuracy than 8QAM, leading to a better BER performance in the low SNR region.

Throughout this thesis, the implementation complexities for the proposed schemes are analyzed in respective chapters. The theoretical analysis for the index modulation systems proposed in this thesis is conducted. The mathematical demonstration of the 8PSK's superiority over 8QAM in indices detection is also detailed.

Abbreviations

AWGN	Additive White Gaussian Noise.
BER	Bit Error Ratio.
BLAST	Bell-Labs Layered Space-Time Architecture.
BPSK	Binary Phase Shift Keying.
CDMA	Code Division Multiple Access.
CPEP	Conditional Pairwise Error Probability.
DFT	Discrete Fourier Transform.
CIR	Channel Impulse Response.
CP	Cyclic Prefix.
DSL	Digital Subscriber Line.
ESIM	Enhanced Subcarrier-Index Modulation.
FDMA	Frequency Division Multiple Access.
FFT	Fast Fourier Transform.
GSM	Global System for Mobile Communications.
ICI	Interchannel interference.
IDFT	Inverse Discrete Fourier Transform.
IFFT	Inverse Fast Fourier Transform.
IGCH	Information Guided Channel Hopping.
ISI	Intersymbol Interference.
LTE	Long Term Evolution.
LLR	Log Likelihood Ratio.
MC-CDMA	Multicarrier Code Division Multiple Access.

ML	Maximum Likelihood
MIMO	Multiple-Input Multiple-Output.
MMSE	Minimum Mean Square Error.
OFDM	Orthogonal Frequency Division Multiplexing.
OFDM-IM	Orthogonal Frequency Division Multiplexing with Index Modulation.
OFDM-IIM	Orthogonal Frequency Division Multiplexing with Interleaved Index Modulation.
OFDM-GIM1	Orthogonal Frequency Division Multiplexing with type-1 Generalized Index Modulation.
OFDM-IGIM1	Orthogonal Frequency Division Multiplexing with type-1 Interleaved Generalized Index Modulation.
OFDM-GIM2	Orthogonal Frequency Division Multiplexing with type-2 Generalized Index Modulation.
OFDM-IGIM2	Orthogonal Frequency Division Multiplexing with type-2 Interleaved Generalized Index Modulation.
OFDM-GIM3	Orthogonal Frequency Division Multiplexing with type-3 Generalized Index Modulation.
OFDM-IGIM3	Orthogonal Frequency Division Multiplexing with type-3 Interleaved Generalized Index Modulation.
OOK	On-Off Keying.
PAPR	Peak to Average Power Ratio.
PEP	Pairwise Error Probability.
PSK	Phase Shift Keying.
P/S	Parallel to Serial.
QAM	Quadrature Amplitude Modulation.
QPSK	Quadrature Phase Shift Keying.
RF	Radio Frequency.
SM	Spatial Modulation.

SIM	Subcarrier-Index Modulation.
SNR	Signal-to-Noise Ratio.
SSK	Space Shift Keying.
S/P	Serial to Parallel.
SUI	Stanford University Interim.
TA	Transmit Antenna.
TDMA	Time Division Multiple Access.
UPEP	Unconditional Pairwise Error Probability.
WLAN	Wireless Local Area Networks.
WMAN	Wireless Metropolitan Area Networks.
ZF	Zero Forcing.

Contents

Acknowledgements	ii
Abstract	iii
Abbreviations	v
List of Figures	x
List of Tables	xii
1 Introduction	1
1.1 Background	1
1.2 Motivation and Objectives	6
1.3 Major Contribution of the Thesis	7
1.3.1 Generalization of OFDM-IM	7
1.3.2 Joint I/Q Index Modulation	8
1.3.3 OFDM-GIM1 using PSK	9
1.4 Organization of the Thesis	9
2 Literature Review	11
2.1 OFDM	11
2.2 Pioneering Works on OFDM-IM	16
2.2.1 MIMO Transmission	16
2.2.2 Subcarrier Index Modulation OFDM	20
2.2.3 Enhanced Subcarrier Index Modulation OFDM	21
2.3 OFDM with Index Modulation	24
2.3.1 System Model	24
2.3.2 Index Selection and Demapping using Look-Up Table Method	28
2.3.3 Index Selection and Demapping using Combinatorial Method	28
2.3.4 Detection using ML detector	32
2.3.5 Detection using LLR detector	33
2.4 Chapter Summary	34
3 Type 1 Generalization of OFDM-IM	36
3.1 Main Idea	36
3.2 Problem Formulation	38

3.3	Generalized Index Modulation Block	40
3.4	Upgraded LLR detector	46
3.5	Implementation Complexity	48
3.6	Numerical Examples	50
3.7	Simulation Results	54
3.8	OFDM with type-1 Interleaved GIM	56
3.9	Chapter Summary	59
4	Type 2 Generalization Scheme of OFDM-IM	61
4.1	Proposed Generalized Scheme For QPSK	62
4.1.1	Numerical Examples	66
4.1.2	Simulation Results	73
4.1.3	OFDM with type-2 Interleaved GIM	74
4.2	Joint I/Q Index Modulation for OFDM-IGIM2	77
4.2.1	Motivation	77
4.2.2	Joint Index Modulation	80
4.2.3	The Corresponding Detection Rule	83
4.2.4	Simulation Results	84
4.3	OFDM with type-3 Interleaved Generalized Index Modulation (OFDM-IGIM3)	85
4.4	Implementation Complexity	88
4.5	Chapter Summary	89
5	OFDM with Generalized Index Modulation Using PSK	90
5.1	PSK and QAM	91
5.2	Mathematical Model	92
5.2.1	An Active Subcarrier Detected as Inactive	93
5.2.2	An Inactive Subcarrier Detected as Active	102
5.3	Simulation Results	104
5.4	Diversity Order Analysis	106
5.5	Chapter Summary	113
6	Conclusions and Future Work	114
6.1	Conclusions	114
6.2	Future Work	115
6.2.1	Further Generalizations	115
6.2.2	Optimal Selection Strategy for OFDM-GIM1	116
6.2.3	Applications	117
	Bibliography	118

List of Figures

1.1	Block diagram of a General Communication System.	2
1.2	Single carrier and Multicarrier Communications.	5
2.1	Block diagram of classical OFDM transmitter.	13
2.2	BER performances of classical OFDM using BPSK, QPSK and 8QAM.	15
2.3	Basic Principle of MIMO.	16
2.4	Basic Principle of spatial multiplexing	17
2.5	Basic Principle of space-time coding.	17
2.6	Basic Principle of beamforming.	18
2.7	Basic Principle of Spatial Modulation.	19
2.8	Block diagram of SIM-OFDM transmitter, 1 is the majority bit in B_{OOK}	20
2.9	Block diagram of ESIM-OFDM transmitter.	23
2.10	Block diagram of the OFDM-IM transmitter	24
2.11	The logic computational diagram of Combinatorial Method	30
3.1	Subcarrier combinations of OFDM-IM.	37
3.2	Subcarrier combinations of OFDM-GIM1, $n = 4$, $K = 1, 2$	38
3.3	The generalized index modulation block of the g -th subblock for OFDM-GIM1.	45
3.4	BER performance of OFDM-GIM1 when original and upgraded LLR decoding, $n = 16$, BPSK.	50
3.5	BER performance of the classical OFDM and OFDM-IM, $n = 4$, ML and LLR based decoding, BPSK.	53
3.6	BER performance of OFDM-IM with fixed n and different K , LLR based decoding, BPSK.	54
3.7	BER performance of OFDM-GIM1 and OFDM-IM, $n = 8$, BPSK.	56
3.8	BER performance of OFDM-GIM1 and OFDM-IM, $n = 16$, BPSK.	56
3.9	The process of Interleaving.	57
3.10	BER performance of OFDM-IIM and OFDM-IGIM1, $n = 8$, BPSK.	58
3.11	BER performance of OFDM-IIM and OFDM-IGIM1, $n = 16$, BPSK.	58
4.1	Block diagram of the OFDM-GIM2 transmitter.	63
4.2	BER performance of OFDM-IM and OFDM-GIM2, $n = 8$, QPSK.	72
4.3	BER performance of OFDM-IM and OFDM-GIM2, $n = 16$, QPSK.	72
4.4	BER performance of OFDM-IIM and OFDM-IGIM2, $n = 8$, QPSK.	74

4.5	BER performance of OFDM-IIM and OFDM-IGIM2, $n = 16$, QPSK.	75
4.6	BER performance of OFDM-IM (BPSK), OFDM-GIM1 (BPSK) and OFDM-GIM2 (QPSK), $n = 16$.	76
4.7	BER performance of OFDM-IM (BPSK), OFDM-GIM1 (BPSK) and OFDM-GIM2 (QPSK) under SUI-5 channel, $n = 16$.	77
4.8	Spectral efficiency comparison of classical OFDM, OFDM-GIM and the proposed scheme for (a) $n = 4$, and (b) $n = 8$, QPSK.	79
4.9	The joint index modulation block of g -th subblock for OFDM-GIM	81
4.10	BER performance of classical OFDM, OFDM-GIM and the proposed scheme, $n = 4$, $K = 2$, QPSK	86
4.11	BER performance of classical OFDM, OFDM-GIM and proposed scheme, $n = 8$, $K = 3$, QPSK	86
4.12	BER performance of OFDM-IIM and OFDM-IGIM3, $n = 8$, QPSK.	87
4.13	BER performance of OFDM-IIM and OFDM-IGIM3, $n = 16$, QPSK.	88
5.1	Constellation Diagrams of 8PSK and 8QAM.	92
5.2	Constellation Diagrams of Circular 8QAM.	93
5.3	Calculation of distances for active index detection for 8QAM.	94
5.4	Calculation of distances for active index detection for circular 8QAM.	95
5.5	Calculation of distances for active index detection for 8PSK.	96
5.6	BER performance of OFDM-GIM using 8PSK, circular 8QAM and 8QAM, $n = 8$, $\mathbf{K} = \{4, 5\}$.	105
5.7	BER performance of OFDM-GIM using 8PSK, circular 8QAM and 8QAM, $n = 16$, $\mathbf{K} = \{10, 11\}$.	106
5.8	BER performance of OFDM-GIM, $n = 8$, $\mathbf{K} = \{4, 5\}$ using 8QAM, circular 8QAM and 8PSK when convolutional code is adopted.	107
5.9	BER performance of OFDM-GIM, $n = 16$, $\mathbf{K} = \{10, 11\}$ using 8QAM, circular 8QAM and 8PSK when convolutional code is adopted with rate $\frac{1}{8}$.	108

List of Tables

2.1	Look-Up Table for $n = 4, K = 2$	28
2.2	Combinadic Table for $n = 8, K = 4$	31
3.1	The 16 combinations represented by an active subcarrier out of eight subcarriers	41
3.2	The 448 combinations represented by three active subcarriers out of eight subcarriers	43
3.3	The 1584 combinations represented by five active subcarriers out of eight subcarriers	44
3.4	LLR calculation example for $n = 8, K_2 = 3$	53
4.1	LLR calculation example for $n = 16, K = 10$, in-phase component .	70
4.2	LLR calculation example for $n = 16, K = 10$, quadrature component	71

Chapter 1

Introduction

1.1 Background

Nowadays, communication comes into our daily lives in so many different ways that it is very easy to overlook the multitude of its facets. The mobile phones and portable radios in our hands, cable TV systems in our homes, the laptops and tablets enabling us to surf the Internet, and the sensors keeping us informed of weather conditions, are all capable of providing different kinds of communications we need from almost every corner of this planet.

Indeed, the list of applications involving the use of communication in one way or another is almost endless. Irrespective of the form of communication process being considered, a communication system deals with information or data transmission from one point to another. There are three basic elements in every communication system, namely, transmitter, channel, and receiver, as shown in Fig. 1.1 where the block diagram of a general communication system is given. The transmitter is located at one point in space, the receiver is located at some other point separate from the transmitter, and the channel is the physical medium that connects them.

The transmitter is a collection of electronic components and circuits, such as oscillators, amplifiers, tuned circuits and filters, modulators, frequency mixers,

FIGURE 1.1: Block diagram of a General Communication System.

frequency synthesizers, and other circuits. The task of the transmitter is to convert the message signal produced by the source of information into a form suitable for transmission over the channel. The message signal can either be analog such as speech, audio, image, and video, or digital such as text or multimedia.

The data stream produced by the transmitter is then modulated to generate waveforms for transmission over a channel, which is a physical link such as a telephone line, a high frequency radio link, or a storage medium. However, as the transmitted signal propagates through the channel, it is distorted due to channel imperfections. Moreover, noise and interfering signals (originating from other sources) are added to the channel output, with the consequence that the received signal is a corrupted version of the transmitted signal.

A receiver is a collection of electronic components and circuits that accepts the transmitted message from the channel and converts it back into a form understandable by a user. Receivers contain amplifiers, oscillators, mixers, tuned circuits and filters, and a demodulator or detector that recovers the original message signal from the modulated carrier.

Dependent on the channel types, there are two types of common communication systems: wireline communication systems and wireless communication systems. Coaxial cable line [1] was once a major wireline channel adopted in

wireline communication systems. The long-distance telephone network once used coaxial cable lines, which has now been replaced by optical fiber [2]. The wireless communication systems use the atmosphere or free space as the channel, which is susceptible to various types of noise.

The spectacular and booming growth of voice, video and data communication promises great expectations for mobile multimedia. In harmony with this growth, there have been increasing attempts to extend such communication services available in wireline communication systems to wireless communication systems [3].

Generally speaking, there are four major periods in the history of wireless communications [4]. In the first period, a single wireless communication system usually occupies the whole frequency band in a certain area. Such a wireless communication system has severe problems with congestion and call completion. In other words, a customer using a particular frequency in a geographic area excludes the other customers from using the wireless communication service on that same frequency. In addition, the number of frequencies allocated was small, limiting the number of simultaneous calls.

In the second period, the first generation wireless communication systems developed around the world adopting distinct and largely incompatible techniques. For example, in February 1983, The U.K. government chose the Total Access Communications System [5] as the first UK national cellular system. While eight months later, Bell Labs officially introduced the Advanced Mobile Phone System in the United States [6]. Two years later, DeTeMobil (currently Deutsche Telekom) developed C-Netz in Germany [7]. Such incompatible and different systems around the world make the international roaming impossible.

In the third period, Second generation (2G) wireless communication used digital technology. Unfortunately, the adoption of second generation technology still differed substantially between the United States and Europe. Specifically, in

July 1991, GSM (Global System for Mobile Communications), a standard developed by the European Telecommunications Standards Institute, was deployed in Europe. Although now GSM has become the default global standard for mobile communications, at that time, a couple of different techniques were developed in the United States, for example, Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA). Such difference undermines the earlier mobile experience.

Then the development of wireless communication moves to third generation (3G) and fourth generation (4G) technology. These systems promise significant transmission speed boosts and are very suitable for customers demanding data services. For example, mobile phones using 3G or 4G provide customers with services like email, music downloading and video conferencing. Such services used to be provided only by computers.

The major concerns of wireless communication systems lie in the efficient use of the radio spectrum and the resistance to frequency selective channels. The radio spectrum is a scarce and limited natural resource. What's worse, only part of the electromagnetic spectrum is suitable for wireless communications. Therefore, wireless communication systems with high spectral efficiencies are preferred to make full use of the scarce radio spectrum. The wireless channels, on the other hand, offer very harsh transmission condition, such as large attenuation, noise, interference, time variation and non-linearities. This makes reliable data transmission very difficult.

In recent years, multicarrier communications have become an attractive technique in many wireless communication systems to meet the increasing demand for high data rate communication systems in frequency selective channels. The basic idea of multicarrier communications is to divide the transmitted bit stream into different sub-streams and send them over different subchannels. The subchannels are generally orthogonal under ideal propagation conditions.

FIGURE 1.2: Single carrier and Multicarrier Communications.

As shown in Fig. 1.2, in a conventional single carrier communications system, symbols are transmitted sequentially such that the frequency spectrum of each data symbol is allowed to occupy the entire available bandwidth. The equalizer of frequency selective channels in single carrier communication systems, therefore, is often sophisticated with many taps of equalization coefficients. However, in a multicarrier communications system, several symbols are transmitted at the same time through subcarriers with different frequencies, but over a longer time period. Such a system offers possibilities for alleviating many of the problems encountered by single carrier systems.

The major benefit of multicarrier communications is that it allows one to

achieve high data rates in a frequency selective radio propagation environments. One of the most popular multicarrier communications techniques, orthogonal frequency division multiplexing (OFDM), has developed into a widely-used scheme for wideband digital communication. This is credited to OFDM's high spectral efficiency, robustness to channel fading, excellent implementation advantages and ability to support several access schemes such as TDMA, frequency division multiple access (FDMA), multicarrier code division multiple access (MC-CDMA) [8–10] as well as various modulation schemes. The major advantage of OFDM over single-carrier schemes is its ability to cope with frequency-selective fading channel with only one-tap equalizer.

1.2 Motivation and Objectives

Driven by the same requirements, i.e., the efficient use of spectrum and the resistance to frequency selective channels, OFDM is improved in various aspects. One of the most recent improvements is OFDM with Index Modulation (OFDM-IM) proposed in [11, 12]. The major contribution of this scheme is the utilization of subcarrier indices as a source of information which results in error performance significantly better than that of classical OFDM under frequency selective channels. In addition, a better spectral efficiency is achieved when Binary Phase Shift Keying (BPSK) is adopted.

OFDM-IM provides novel encoding/decoding and mapping/demapping algorithms to transmit information bits by selecting different sets of active indices. However, there is plenty of room to improve OFDM-IM, as listed below.

- In OFDM-IM, for a given subblock size, only a fixed number of active subcarriers are selected to carry constellation symbols, which does not make full use of the other possible numbers of active subcarriers. In other words,

OFDM-IM's spectral efficiency can be improved if flexibility is offered for the selection of active subcarriers.

- Compared to classical OFDM, OFDM-IM offers inconsistent bit error rate (BER) performance improvements. To be specific, in the high signal-to-noise ratio (SNR) region, OFDM-IM outperforms the classical OFDM in terms of BER performance while in the low SNR region, OFDM-IM's BER performance is not as good as the classical OFDM's.
- OFDM-IM's spectral efficiency superiority over the classical OFDM is only evident when BPSK is adopted. For constellation sizes higher than BPSK, such superiority is only observed for a few carefully chosen system configurations.

Noting the above points, the objective of this thesis is to further improve OFDM-IM's spectral efficiency and BER performance by proposing several generalizations including the use of higher order constellation symbols in OFDM-IM and the introduction of M-PSK with $M > 4$.

1.3 Major Contribution of the Thesis

The main research contributions of this thesis are summarized as follows.

1.3.1 Generalization of OFDM-IM

Based on OFDM-IM, OFDM with generalized index modulation schemes are proposed. The generalization is proposed in two aspects. First, a more flexible selection of active subcarriers, which is named as OFDM with type-1 generalized index modulation (OFDM-GIM1), is proposed to further improve the spectral efficiency. However, the generalization in this aspect cannot fundamentally overcome OFDM-IM's difficulty in adopting Quadrature Phase Shift Keying (QPSK)

and other higher constellation symbols. In the second aspect of generalization, named as OFDM with type-2 generalized index modulation (OFDM-GIM2), the in-phase component and quadrature component of QPSK symbols are split into two independent components so that index modulation is independently applied to them.

The main results related to the generalization of OFDM-IM have been presented in a conference paper and a journal paper listed in the following:

- R. Fan, Y. J. Yu, and Y. L. Guan, “Orthogonal frequency division multiplexing with generalized index modulation,” in *Proc. IEEE Global Communications Conference (GLOBECOM’14)*, Dec. 2014, pp. 3880-3885.
- R. Fan, Y. J. Yu, and Y. L. Guan, “Generalization of orthogonal frequency division multiplexing with index modulation,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 10, pp. 5350-5359, Oct. 2015.

1.3.2 Joint I/Q Index Modulation

In OFDM-GIM2, index modulation is performed independently on the in-phase and quadrature components of the QPSK symbol on each subcarrier. In this way, a higher spectral efficiency than that offered by OFDM-IM may be achieved. By introducing joint I/Q index modulation where the two index modulations on the in-phase and quadrature components are no longer independent, OFDM-GIM2’s spectral efficiency is further improved. In other words, in the proposed scheme, the input bit string jointly decides the active indices for in-phase and quadrature components. As a result, more bits can be transmitted per OFDM frame.

The main result for this part has been accepted for publication as a journal given in the following:

- R. Fan, Y. J. Yu, and Y. L. Guan, “Improved Orthogonal Frequency Division Multiplexing with Index Modulation,” *IET Communications*, vol. 8, iss. 10, pp. 969-974, 2016.

1.3.3 OFDM-GIM1 using PSK

For M -ary Quadrature Amplitude Modulation (QAM) constellation symbols, if $M \geq 8$, the constellation symbols have different distances from origin, which degrades the accurate detection of active indices. This is one of the factors that hinders the OFDM-GIM1 from successfully employing higher order ($M \geq 8$) QAM constellation symbols. Therefore, PSK is introduced to replace QAM to tackle this problem. For an M -ary PSK constellation, all the constellation symbols have the same distance from origin, which has a potential of achieving higher index detection accuracy. As a result, a better BER performance in the low SNR region can be achieved. However, compared to QAM, introduction of PSK degrades the BER performance in high SNR region since PSK's minimum Euclidean distance between constellation symbols are smaller than QAM's. Noting that PSK and QAM offer the best tradeoff, an adaptive scheme switching between QAM and PSK may be adopted to make full use of their advantages in different SNR regions.

The main result has been included in the following journal paper:

- R. Fan, Y. J. Yu, and Y. L. Guan, “Orthogonal Frequency Division Multiplexing with Generalized Index Modulation Using Phase Shift Keying,” submitted to *IET Communications*.

1.4 Organization of the Thesis

The rest of the thesis is organized as follows.

Chapter 2 first reviews the literature and techniques related to OFDM-IM as a background for the subsequent chapters and then introduces OFDM-IM in detail, on which all the results of this thesis are fundamentally based.

Chapter 3 presents OFDM-GIM1, where a more flexible selection of active subcarriers is proposed to improve the spectral efficiency of OFDM-IM when BPSK is adopted. Then the BER performance loss that may be incurred in OFDM-GIM1 in the low SNR region is mitigated by introducing an interleaver.

Chapter 4 derives OFDM-GIM2 by splitting the in-phase component and quadrature component of QPSK symbols into two independent components so that index modulation is independently applied to these two components. Then OFDM-GIM2 is extended, where the input bit string jointly decides the active indices for in-phase and quadrature components. As a result, more bits can be transmitted per OFDM frame. The reasons for the improved BER performance are analyzed in depth. Noting that OFDM-GIM1 and OFDM-GIM2 are compatible with each other, to further improve the spectral efficiency, a combination of these two schemes, named as OFDM-GIM3 is also investigated.

Chapter 5 presents OFDM-GIM1 using 8PSK, which provides a higher accuracy in index detection compared to 8QAM. As a result, the BER performance of OFDM-GIM1 in the low SNR region is improved. The reason behind 8PSK's superiority in the index detection accuracy is also mathematically demonstrated. In addition, to avoid the BER performance loss in the high SNR region due to 8PSK's lower minimum Euclidean distance between constellation symbols, an adaptive switching scheme between 8PSK and 8QAM is introduced. The diversity order analysis for the index modulation systems proposed in this thesis is also conducted.

Chapter 6 concludes the thesis and points out possible future research directions.

Chapter 2

Literature Review

Orthogonal Frequency Division Multiplexing (OFDM) and Multiple-Input Multiple-Output (MIMO) systems are two key technologies in wireless communications. These two technologies' great potential for commercial deployment is a consequence of decades of research work by researchers. As a matter of fact, OFDM-IM is based on OFDM and inspired by one of the major techniques proposed in MIMO. A careful review of the two key technologies' main breakthroughs as well as the pioneering works of OFDM-IM is essential to the understanding of OFDM-IM, which is then described in detail in the rest of this chapter.

2.1 OFDM

The idea of data transmission by frequency division using orthogonal signals was put forward in the 1960s [13–16], with a couple of United States patents [17, 18]. By using parallel data and frequency division multiplexing with overlapping subchannels, a better bandwidth efficiency may be achieved.

The initial applications were military communications. At that time, all the implementations were based on parallel banks of modulators, filters and correlators. It was not until 1971 that OFDM was born in its modern form, when Weinstein

proposed the use of the Discrete Fourier Transform (DFT) as a means for generating multitone baseband signals [19]. With the introduction of DFT, which has a set of harmonically related sinusoidal and co-sinusoidal basis functions, the overall implementation complexity of OFDM is reduced. The modulation of data into a complex waveform occurs at the Inverse Discrete Fourier Transform (IDFT) stage of the transmitter and these harmonically related frequencies can hence be used as the set of carriers required by the OFDM system. The demodulation of data into a complex waveform occurs at the DFT stage of the receiver. DFT multiplies the signal successively with complex exponentials over a range of frequencies, sums each product and displays the results as a coefficient of that frequency.

After the discovery of fast Fourier transform (FFT) by Cooley and Tuckey in 1965 [20], Weinstein, who came up with the idea of guard intervals to reduce ISI, suggested the use of a completely digital modem built around a special-purpose computer performing the fast Fourier transform for practical implementation.

The use of a cyclic prefix (CP) as a guard interval was introduced in 1980 [21]. A cyclic prefix is in fact just a repetition of the last section of a frame that is appended to the beginning of the frame. The cyclic prefix is usually about 10% to 25% of the symbol time. Short CP length is used for channels with low time delay, for example, Stanford University Interim (SUI)-1, SUI-2 and SUI-3 channel models [22]. Long CP length is used for channels with high time delay, for example, SUI-5 and SUI-6 channel models. Although cyclic prefix can help to combat ISI and mitigates the effects of multipath fading, the tradeoff is that it increases the bandwidth requirements or reduces data rate.

The introduction of CP lays the foundation for the current, widely adopted, classical OFDM, whose block diagram is given in Fig. 2.1. This scheme can be described as follows. The incoming serial data is first converted from serial to parallel (S/P) and grouped into $\log_2 M$ bits each to be mapped to an M -ary constellation symbol by the Mapper. The constellation symbols then go through

FIGURE 2.1: Block diagram of classical OFDM transmitter.

the N -point IFFT block, where N is the total number of subcarriers. A length L CP is added to the output of IFFT block to avoid ISI caused by multipath distortion. After that, parallel to serial (P/S) conversion is performed. The receiver performs the inverse process of the transmitter. A one-tap equalizer is used to correct channel distortion. The tap coefficients of the filter are calculated based on channel information. Note that, in classical OFDM, all subcarriers are active and carry M -ary constellation symbols.

Later, in 1985, the topic of classical OFDM modulation for transmission over multipath fading channels was revisited, and the use of pilot symbols for determining the phase and magnitude correction of an OFDM signal transmitted over a Rayleigh fading channel was introduced [23]. In 1989, Kalet showed that the optimum power allocation in a QAM modulated multitone signal is ruled by the water-pouring solution of information theory [24], and thus established the maximum bit rate that can be transmitted using multitone QAM in white Gaussian noise over a frequency-selective channel [25].

By 1990, the advances in Very Large Scale Integration technology in digital signal processing made high speed large size FFT chips commercially affordable [26], which provided a good opportunity for OFDM to become the modulation technique of choice in several major communication standards worldwide. OFDM was adopted in Europe for digital audio and video broadcasting [27–29], for Digital Subscriber Line (DSL) applications, and for the last mile in IEEE 802.16a standard for Wireless Metropolitan Area Networks (WMAN) [30]. In the early 2000s, OFDM reached its crowning moment when the popular IEEE 802.11a standard for packet-switched Wireless Local Area Networks (WLAN) [31] was successfully deployed.

Researchers then focused on solving the key issues in OFDM. For example, inter-carrier interference (ICI), which is the cross-talk between different subcarriers destroying the orthogonality between OFDM subcarriers [32–35], can result in a substantially higher BER. The sources of ICI can be carrier frequency offset and time variation in the channel, with solutions proposed in [36–38] and [39–42], respectively.

Another major issue in OFDM is the high peak-to-average power ratio (PAPR) due to the use of large number of subcarriers [43]. These high signal peaks can lead to in-band distortion and spectral spreading in the presence of non-linear devices such as power amplifiers. A number of approaches have been proposed to deal with the PAPR problem. These techniques include amplitude clipping [44], clipping and filtering [45–47], coding [48–54], tone reservation [55], tone injection [56], active constellation extension [56], and multiple signal representation techniques such as partial transmit sequence [57–63], selected mapping [64–66], and interleaving [67–69]. These techniques achieve PAPR reduction at the expense of transmit signal power increase, BER increase, data rate loss and computational complexity increase.

Fig. 2.2 shows the BER performances of classical OFDM for BPSK, QPSK

FIGURE 2.2: BER performances of classical OFDM using BPSK, QPSK and 8QAM.

and 8QAM over a frequency selective channel. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (2.1)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

As seen from Fig. 2.2, one of the attractiveness in classical OFDM is that from BPSK to QPSK, the spectral efficiency is increased, but the BER performance is maintained. Compared to classical OFDM using BPSK and QPSK, classical OFDM using 8QAM offers better spectral efficiency at the cost of BER performance degradation.

FIGURE 2.3: Basic Principle of MIMO.

2.2 Pioneering Works on OFDM-IM

In this section, the techniques related to OFDM-IM and the pioneering works on OFDM-IM are reviewed.

2.2.1 MIMO Transmission

In conventional wireless communication systems, a single antenna is used at the transmitter, and another antenna is used at the receiver. With the rapid development of wireless services, the antenna part of the wireless communication systems can also be utilized to meet the growing demand. As a matter of fact, when utilizing multiple antennas, the previously unused spatial domain can be exploited. As shown in Fig. 2.3, in MIMO systems [70–77], both the transmitter and the receiver adopt multiple antennas, which are used to reduce the BER and increase the transmission capacity. Several books [78–81] on MIMO systems have been published. Three types of fundamental gains, i.e., a multiplexing gain, a diversity gain, and an antenna gain, can be obtained by using MIMO in a wireless communication system.

FIGURE 2.4: Basic Principle of spatial multiplexing

FIGURE 2.5: Basic Principle of space-time coding.

The multiplexing gain can be obtained by spatial multiplexing [82–90]. As shown in Fig. 2.4, the basic principle of spatial multiplexing is that, at the transmitter, the information bit sequence is split into several sub-sequences (demultiplexing), which are modulated and transmitted simultaneously over the transmit antennas using the same frequency band. At the receiver, the transmitted sequences are separated by employing an interference cancellation type of algorithm.

In contrast to spatial multiplexing, whose main objective is to provide higher

FIGURE 2.6: Basic Principle of beamforming.

bit rate compared to a single-antenna system, spatial diversity [91–98] predominantly aims at an improved error rate performance. As shown in Fig. 2.5, by transmitting and receiving redundant signals representing the same information sequence, multiple antennas can also be used to reduce the error rate of a system, akin to channel coding.

In addition to higher bit rates and lower error rates, multiple-antenna techniques can also be utilized to improve the SNR at the receiver and to suppress cochannel interference in a multiuser scenario. This is achieved by means of adaptive antenna arrays [70, 99–102], which is named as beamforming, as shown in Fig. 2.6.

Besides spatial multiplexing, spatial diversity and beamforming, the recently proposed spatial modulation (SM) [103–105] is particularly promising. In SM, the indices of the transmit antennas (TAs) is regarded as an additional dimension for transmitting information [106]. As shown in Fig. 2.7, only one out of all the antennas is selected to transmit and different antenna selections carry different information. At the receiver, such information is decoded by determining which antenna is selected at the transmitter.

The concept of space modulation was introduced for the first time in [107]. The scheme exploits the distinct multipath fading characteristics for antenna index

FIGURE 2.7: Basic Principle of Spatial Modulation.

detection to discriminate the transmitted information messages. The extensive research of SM was then fueled by several follow-up works [108, 109]. In 2004, in [108], a modulation scheme, named as channel hopping technique, was put forward. This scheme is exactly what is known today as SM. In 2005, the same modulation scheme as in [108] was independently proposed [109].

In 2006, the same authors further investigated the scheme proposed in [109], and they used the terminology of spatial modulation to name this encoding mechanism for the first time [110–113]. Two years later in 2008, various papers were published to further improve and investigate the SM concepts. In [114], the channel capacity was further studied based on [108]. In this paper, the parlance of Information-Guided Channel Hopping (IGCH) was proposed to achieve a high throughput.

In [103], the bit-to-symbol mapping rule of SM was explicitly made. The authors in [115] then developed the Maximum Likelihood (ML) optimum demodulator for SM, and they show that some performance improvements can be expected compared to the suboptimal demodulator introduced in [103]. Later, by only utilizing on the TA indices to transmit information between the transmitter and receiver, Space Shift Keying (SSK) [116] modulation was put forward.

Inspired by the concept of SM/SSK, the subcarrier orthogonality can also be exploited and the indices of active subcarriers of OFDM symbols can be employed

FIGURE 2.8: Block diagram of SIM-OFDM transmitter, 1 is the majority bit in B_{OOK} .

to convey additional information. By treating the subcarrier indices of an OFDM system as the antenna indices in a MIMO system, SM is successfully applied to OFDM.

The pioneering works of such application are Subcarrier-Index Modulation OFDM (SIM-OFDM) [117] and Enhanced Subcarrier Index Modulation OFDM (ESIM-OFDM) [118], which will be introduced in the following two subsections.

2.2.2 Subcarrier Index Modulation OFDM

In classical OFDM, modulation techniques, such as BPSK and QAM, map a fixed number of information bits into constellation symbols. Each constellation symbol represents a point in the 2-D baseband signal space. SIM-OFDM exploits the subcarrier orthogonality in an on-off keying (OOK) fashion. In that way, a new dimension, which also conveys information, is added to the complex 2-D signal plan.

The block diagram of SIM-OFDM is given in Fig. 2.8. Compared to the classical OFDM, SIM-OFDM has an additional module named as subcarrier-index modulator. This module first splits the input bit strings into two equal length bit

strings. The first bit string, named as B_{OOK} , decides the majority bit value and the locations of active subcarriers. The type of the majority bit value (one or zero) is determined by counting the number of ones and zeros in this bit string. If the number of ones is larger or equal to the number of zeros, the majority bit value is one. Otherwise, the majority bit value is zero. The locations of the majority bits serve as the indices of the active subcarriers. Then the second bit string, named as B_{QAM} , are mapped to QAM symbols and modulated onto those active subcarriers accordingly.

In SIM-OFDM, the group of subcarriers associated with the subset of the majority bit-value are regarded as active and are selected to be modulated by the second bit string. Meanwhile, the remaining subcarriers are treated as inactive and are turned-off (suppressed before the signal modulation), hence the term on-off keying (OOK).

At the receiver, the signal is transformed using a Fast Fourier Transform as in classical OFDM. Then every subcarrier is inspected to see whether it is active. The subcarrier whose power is above a certain threshold is marked as active, otherwise it is marked as inactive. After inspecting all the subcarriers, B_{OOK} is reconstructed from the detected activeness of the subcarriers as well as the known majority bit type. Afterwards, B_{QAM} is reconstructed by demodulating active subcarriers according to the respective QAM scheme.

The SIM-OFDM scheme exploits the orthogonality of the multicarrier dimension in a radically different approach for information transmission. Meanwhile, it reduces the ICI without any degradation in spectral efficiency.

2.2.3 Enhanced Subcarrier Index Modulation OFDM

Despite of the advantages offered by SIM-OFDM, there are three main issues which limit the performance of SIM-OFDM.

- First, the SIM-OFDM scheme has a potential bit error propagation which could lead to significant burst errors. To be specific, an incorrect detection of a subcarrier's activeness leads not only to incorrect demodulation of the M -ary constellation symbols it encodes, but also to incorrect demodulation of all subsequent QAM symbols. As the total number of subcarriers increases, the issue becomes worse.
- Second, it is difficult to find an appropriate demodulation threshold. Even though an average significant power is allocated to subcarriers for M -ary constellation symbols, the threshold needs to be below the smallest symbol power. Hence, the system's ability to correctly distinguish subcarrier's activeness does not improve significantly.
- Third, a perfect feedforward from the transmitter to the receiver needs to be assumed in SIM-OFDM. Only in that way will the receiver know the majority bit value for the subcarrier index selecting bits. Though the majority bit type can be signaled either through secure communication channels, or by reserving one particular frequency carrier and transmitting the desired value with sufficiently high SNR, this only leads to a slight improvement in the SIM-OFDM performance.

The ESIM-OFDM is a solution to these three problems. Different from SIM-OFDM, ESIM-OFDM uses one bit to control two adjacent subcarriers such that only one subcarrier is activated at a time.

The block diagram of ESIM-OFDM is given in Fig. 2.9. Instead of each information bit being encoded in a single subcarrier state, it can be encoded in the states of two consecutive subcarriers, which are regarded as a subcarrier pair. Whenever a zero is encountered in B_{OOK} , the first subcarrier of the pair is set as passive and the second one as active. Whenever a one is encountered in B_{OOK} , the first subcarrier of the pair is set as active and the second one as passive. The benefit, however, is that for each pair it is certain that exactly one of the subcarriers is

FIGURE 2.9: Block diagram of ESIM-OFDM transmitter.

active. This means that bits from B_{QAM} can no longer be misplaced due to wrong detection of previous subcarrier states. The error that can be made is limited within each pair of subcarriers. In addition, there is no longer a need to define a bit-majority in B_{OOK} , and the total number of active carriers is always the same. The number of active subcarriers within each pair is known and is always one, so there is no need to use a threshold for demodulation. Instead, the subcarrier with higher power in a subcarrier pair can be recognized as active, which would lead to better performance in the subcarrier activeness detection.

The disadvantage of the modified scheme, compared to SIM-OFDM, is the reduced spectral efficiency. To achieve the same spectral efficiency as that of classical OFDM, this scheme needs to adopt higher size of constellation symbols.

FIGURE 2.10: Block diagram of the OFDM-IM transmitter

2.3 OFDM with Index Modulation

Based on the same principle, but following a different approach from that of SIM-OFDM and ESIM-OFDM, a novel transmission scheme termed as OFDM with Index Modulation (OFDM-IM) was proposed in [11] for frequency selective fading channels. Besides the utilization of subcarrier indices as a source of information, the major contribution of this scheme is that the spectral efficiency of this scheme under BPSK can exceed that of classical OFDM without increasing the size of the signal constellation.

2.3.1 System Model

The block diagram of the OFDM-IM transmitter is given in Fig. 2.10. This scheme, transmitting B bits per OFDM frame over a frequency selective Rayleigh fading channel, can be described as follows. First, these B bits are divided into G groups and every group has p bits, i.e., $B = pG$. Every group of p bits is then assigned to one of G OFDM subblocks. Let the length of each subblock be n . Thus, the total

number of OFDM subcarriers N , is nG . In classical OFDM, all n subcarriers in an OFDM subblock are active and may carry in total n M -ary signal constellation symbols, while in OFDM-IM, not all subcarriers are active.

A subcarrier is defined as active if the subcarrier carries an M -ary constellation symbol and a subcarrier is defined as inactive if the subcarrier actually carries nothing but zero. In such a way, in OFDM-IM, not only active subcarriers but also the indices of active subcarriers carry information.

Assume that there are K active subcarriers for $K \leq n$. Thus, the incoming p bits are divided into two parts. The first part has p_1 bits and the second part has p_2 bits, and $p = p_1 + p_2$, as shown in Fig. 2.10. The dashed block in Fig. 2.10 is named as the index modulation block. The p_1 bits are fed to the Index Selector, mapping the incoming p_1 bits to a combination of K active subcarriers. The K indices of the active subcarriers of the g -th OFDM subblock, for $1 \leq g \leq G$, are represented as a set $\mathbf{I}^g$ as shown in Fig. 2.10, given by

$$\mathbf{I}^g = \{i_1^g, \dots, i_K^g\}, \quad (2.2)$$

where $i_k^g \in [1, \dots, n]$ for $g = 1, \dots, G$, $k = 1, \dots, K$ and $i_{k_1}^g \neq i_{k_2}^g$ if $k_1 \neq k_2$.

On the other hand, the p_2 bits of the g -th OFDM subblock go through the M -ary Mapper (modulator) to be mapped to K constellation symbols. The output of the Mapper are a set of constellation symbols, carried by the active subcarriers specified in the corresponding $\mathbf{I}^g$ and given by

$$\mathbf{s}^g = \{s_1^g, \dots, s_K^g\}, \quad (2.3)$$

where $s_k^g \in \mathcal{S}$ for $g = 1, \dots, G$, $k = 1, \dots, K$ and $\mathcal{S}$ is the set of M -ary constellation symbols.

Hence, the maximum number of bits that can be carried by the selection of K active subcarriers is given by,

$$B_1 = p_1 G = \lfloor \log_2 C_n^K \rfloor G, \quad (2.4)$$

where C_n^K denotes the number of K combinations from a given set of n elements.

The maximum number of bits carried by the K M -ary constellation symbols is,

$$B_2 = p_2 G = K(\log_2 M)G. \quad (2.5)$$

As a result, the maximum number of bits that can be transmitted by a single block of OFDM-IM scheme is

$$\begin{aligned} B = B_1 + B_2 &= \lfloor \log_2 C_n^K \rfloor G + K(\log_2 M)G \\ &= \lfloor \log_2 (M^K C_n^K) \rfloor G. \end{aligned} \quad (2.6)$$

For the K active subcarriers in an OFDM subblock, the constellation symbols are normalized to unit average power by setting,

$$E \{ \mathbf{s}^g \mathbf{s}^{gH} \} = K, \quad (2.7)$$

where $(\cdot)^H$ stands for Hermitian transposition.

Then the output of OFDM Block Creator is given by,

$$\mathbf{x} = \{ \mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^G \} = \{ x(1), x(2), \dots, x(N) \}, \quad (2.8)$$

where $x(\xi) \in \{0, \mathcal{S}\}$ for $\xi = 1, \dots, N$, i.e., each subcarrier is either active carrying a signal constellation, or inactive carrying zero. Here a normalization factor $N/\sqrt{K_{\text{tot}}}$ (or $\sqrt{K_{\text{tot}}}/N$) is applied to ensure that the input to the IFFT (or the output of FFT) $\mathbf{x}$ satisfies $E \{ \mathbf{x}^H \mathbf{x} \} = N$, where $K_{\text{tot}} = GK$ is the total number of active subcarriers within the OFDM block.

Next, as shown in Fig. 2.10, the IFFT algorithm is implemented. After adding CP, P/S conversion and digital-to-analog conversion, the signal is sent through the frequency-selective Rayleigh fading channel defined in Sec. 2.1. The output of the FFT algorithm at the receiver, $Y(\xi)$, is given by

$$Y(\xi) = x(\xi)H(\xi) + W(\xi), \quad \xi = 1, \dots, N \quad (2.9)$$

where $H(\xi)$ are the channel fading coefficients, $W(\xi)$ denote the frequency domain noise samples with the distribution of $\mathcal{CN}(0, N_{0,F})$ and $N_{0,F}$ is the noise variance in the frequency domain.

The relationship between the noise variance in time domain, denoted as $N_{0,T}$, and that in frequency domain is given by

$$N_{0,F} = \left(\frac{K_{\text{tot}}}{N} \right) N_{0,T}. \quad (2.10)$$

The SNR for OFDM-IM is defined as $\rho = E_b/N_{0,T}$, where $E_b = (N + L)/m$ is the average transmitted energy per bit. The spectral efficiency of OFDM-IM is given by $B/(N + L)$ [bits/s/Hz].

Compared to classical OFDM, the major implementation difference of OFDM-IM lies in the index selection at transmitter and the index demapping at the receiver. The Index Selector maps the incoming bits to a combination of active indices out of all the available subcarriers. The Index Demapper, which is the inverse of Index Selector, provides an estimate of these bits by processing the active indices. The active indices are obtained by the ML or Log-likelihood Ratio (LLR) detector before the processing of Index Demapper. Two different index selection and demapping schemes, which were proposed in [11] to facilitate different implementation scenarios, will be reviewed in the following two subsections.

TABLE 2.1: Look-Up Table for $n = 4, K = 2$

Incoming Bits	Selected Indices	Subblock Realization
00	$\{1, 2\}$	$[s_\chi \ s_\zeta \ 0 \ 0]$
01	$\{2, 3\}$	$[0 \ s_\chi \ s_\zeta \ 0]$
10	$\{3, 4\}$	$[0 \ 0 \ s_\chi \ s_\zeta]$
11	$\{1, 4\}$	$[s_\chi \ 0 \ 0 \ s_\zeta]$

2.3.2 Index Selection and Demapping using Look-Up Table Method

In this mapping method, a look-up table of size 2^{p_1} is created to be used at both the transmitter and the receiver sides. At the transmitter, the look-up table provides the corresponding indices for the incoming bits for each subblock, and it performs the inverse operation at the receiver.

A look-up table example is presented in Table 2.1 for $n = 4, K = 2$. Although selecting 2 active subcarriers from 4 subcarriers has $C_4^2 = 6$ possible combinations, it maximally carries 2 bit signals. Therefore, only 4 out of the total 6 combinations are used, and the others are discarded. The column of the Selected Indices in Table 2.1 shows a possible selection of active subcarriers. Although this is a very efficient and simple method for smaller C_n^K values, the mapping method is not feasible for higher values of C_n^K due to the extraordinary size of the required table.

2.3.3 Index Selection and Demapping using Combinatorial Method

In this section, the Combinatorial Method [119, 120], an elegant algorithm which generates a unique mathematical combination element from an arbitrary decimal number, is introduced. This method provides a one-to-one mapping between decimal numbers (from 0 to $C_n^K - 1$) and K -combinations, for all n and K . In other words, this method accepts a decimal number as well as n, K as input, and returns an appropriate index selection.

To better understand this method, we first define the combinadic of an integer as an alternative representation of this integer based on combinations. For instance, for the integer 50, if we fix $n = 8$ and $K = 4$, it turns out that the combinadic of 50 is $(7, 5, 3, 2)$. This means that,

$$C_7^4 + C_5^3 + C_3^2 + C_2^1 = 50. \quad (2.11)$$

With $n = 8$ and $K = 4$, any decimal number Z_{p_1} between 0 and $2^{p_1} - 1$ can be uniquely represented as,

$$Z_{p_1} = C_{c_K}^K + C_{c_{K-1}}^{K-1} + C_{c_{K-2}}^{K-2} + \cdots + C_{c_1}^1. \quad (2.12)$$

In this representation, $n > c_K > c_{K-1} > c_{K-2} > \cdots > c_1$. Just like all single digits in ordinary base 10 (i.e., decimal number) are between 0 and 9, all combinadic digits for a given n are between 0 and $n - 1$. The K value determines the number of terms in the combinadic.

For any given p_1 , n and K , the combinadic of Z_{p_1} , $c_K, c_{K-1}, \dots, c_1$, may be computed by the Combinatorial Method illustrated in Fig. 2.11. The Combinatorial Method starts by choosing the maximal c_K that satisfies $C_{c_K}^K \leq Z_{p_1}$, and then choose the maximal c_{K-1} that satisfies $C_{c_{K-1}}^{K-1} \leq Z_{p_1} - C_{c_K}^K$ and so on.

Let us take the decimal number $Z_{p_1} = 27$ as an example, with n and K set to 8 and 4, respectively. The calculation of the combinadic of a number follows a strictly decreasing order. Since $K = 4$, the calculation starts by c_4 , then c_3 , c_2 and finally c_1 . To determine the value of c_4 , the largest number less than $n = 8$, i.e., 7 is first attempted. In the light of $C_7^4 = 35$, which is larger than 27, 7 is not the right choice for c_4 . Next, we try 6 and get $C_6^4 = 15$, which is less than 27. As a result, $c_4 = 6$ is one of the combinadic of 27.

Since 15 out of the original decimal number 27 have already been used up, 12 is left to obtain $c_3 = 5$, and then $c_2 = 2$, $c_1 = 1$ in a similar way.

FIGURE 2.11: The logic computational diagram of Combinatorial Method

By the same token, the combinadic of Z_{p_1} , for $Z_{p_1} \in [0, 63]$, can be calculated. The combinadic for $n = 8$ and $K = 4$ are listed in Table 2.2.

In OFDM-IM, for each subblock, the bits entering the Index Selector are first converted to a decimal number, and this decimal number is then fed to the

TABLE 2.2: Combinadic Table for $n = 8, K = 4$

Incoming Bits	Decimal Number Z_{p_1}	Combinadic
1 1 1 1 1 1	63	(7,6,4,2)
0 1 1 1 1 1	62	(7,6,4,1)
1 0 1 1 1 1	61	(7,6,4,0)
0 0 1 1 1 1	60	(7,6,3,2)
1 1 0 1 1 1	59	(7,6,3,1)
0 1 0 1 1 1	58	(7,6,3,0)
$\vdots$	$\vdots$	$\vdots$
0 1 1 1 1 0	30	(6,5,3,2)
1 0 1 1 1 0	29	(6,5,3,1)
0 0 1 1 1 0	28	(6,5,3,0)
1 1 0 1 1 0	27	(6,5,2,1)
0 1 0 1 1 0	26	(6,5,2,0)
1 0 0 1 1 0	25	(6,5,1,0)
$\vdots$	$\vdots$	$\vdots$
1 0 1 0 0 0	5	(5,2,1,0)
0 0 1 0 0 0	4	(4,3,2,1)
1 1 0 0 0 0	3	(4,3,2,0)
0 1 0 0 0 0	2	(4,3,1,0)
1 0 0 0 0 0	1	(4,2,1,0)
0 0 0 0 0 0	0	(3,2,1,0)

Combinatorial Method to select the active indices. At the receiver, after the indices of active subcarriers are determined (which is described in the next subsection), the decimal number can be easily reverted back using (2.12). This number is then recovered to a bit string.

Unlike classical OFDM, in the OFDM-IM scheme, the receiver not only needs to detect the information bits on the active subcarriers, but also needs to detect the indices of the active subcarriers. For any subcarrier, the metric of determining whether this subcarrier is active is to investigate what kind of frequency domain symbols it is carrying. Specifically, if it is carrying a non-zero symbol, then it is active. Otherwise, it is inactive. Enlightened by the above statement, in [11], two different types of detectors, i.e., ML detector and LLR detector have been proposed, which will be reviewed in the following two subsections.

2.3.4 Detection using ML detector

The ML detector takes all possible subblock realizations into consideration. For the g -th subblock, consider all possible sets of active subcarrier indices ($\mathbf{I}^g$) and all possible sets of M -ary constellation symbols ($\mathbf{s}^g$) carried by each possible $\mathbf{I}^g$; the sets, $\hat{\mathbf{I}}^g$ and $\hat{\mathbf{s}}^g$, that correspond to the minimum distance between the assumed transmitted signal (with channel conditions considered) and the received signal, are regarded as the set of active subcarrier indices and the M -ary constellation symbols carried by these subcarriers.

Mathematically, $(\hat{\mathbf{I}}^g, \hat{\mathbf{s}}^g)$ is given by

$$(\hat{\mathbf{I}}^g, \hat{\mathbf{s}}^g) = \arg \min_{\substack{\mathbf{I}^g, \mathbf{s}^g \\ \xi \notin \mathbf{I}^g}} \left(\sum_{\xi=1}^n |Y^g(\xi)|^2 + \sum_{k=1}^K |Y^g(\mathbf{I}^g(k)) - H^g(\mathbf{I}^g(k)) \mathbf{s}^g(k)|^2 \right), \quad (2.13)$$

where $Y^g(\xi)$ and $H^g(\xi)$ are the received signals and the corresponding fading coefficients for the g -th subblock, respectively, given by

$$Y^g(\xi) = Y(n(g-1) + \xi), \quad (2.14)$$

$$H^g(\xi) = H(n(g-1) + \xi), \quad (2.15)$$

for $1 \leq \xi \leq n$.

The ML detector offers the best detection precision since all possible subblock realizations are processed and compared. However, the total computational complexity of the ML detector is very high and grows exponentially. To be specific, the number of complex multiplications in ML detector is $\sim \mathcal{O}(C_n^K M^K)$. Therefore, ML detector may only be applied to the cases where $M^K C_n^K$ is small. Large $M^K C_n^K$ values produce a large number of possible subcarrier index combinations, making it very time-consuming and impractical.

2.3.5 Detection using LLR detector

Compared to the ML detector, the LLR Detector has a much lower decoding complexity and is more suitable for higher $M^K C_n^K$ values. The LLR detector evaluates the logarithm of the ratio of *a posteriori* probabilities of non-zero symbols to that of zero for every subcarrier. This ratio, providing the information about the active status of the corresponding subcarrier, is formulated as

$$\lambda(\xi) = \ln \frac{\sum_{m=1}^M P(x(\xi) = s_m | Y(\xi))}{P(x(\xi) = 0 | Y(\xi))} \quad (2.16)$$

According to Bayes' formula [121], we have

$$\lambda(\xi) = \ln \frac{\sum_{m=1}^M P(Y(\xi) | x(\xi) = s_m) P(x(\xi) = s_m)}{P(Y(\xi) | x(\xi) = 0) P(x(\xi) = 0)}. \quad (2.17)$$

Since $x(\xi) = s_m$, for $m = 1, \dots, M$ is equally distributed and note the fact that

$$\sum_{m=1}^M P(x(\xi) = s_m) = \frac{K}{n}, \quad (2.18)$$

and

$$P(x(\xi) = 0) = \frac{n - K}{n}, \quad (2.19)$$

(2.17) is further simplified to

$$\lambda(\xi) = \ln(K) - \ln(n - K) + \ln \frac{\sum_{m=1}^M P(Y(\xi) | x(\xi) = s_m)}{P(Y(\xi) | x(\xi) = 0)}. \quad (2.20)$$

Given that $Y(\xi) = X(\xi)H(\xi) + W(\xi)$, for $\xi = 1, \dots, n$ and $W(\xi)$ follows the distribution of $\mathcal{CN}(0, N_{0,F})$, we have

$$\lambda(\xi) = \ln(K) - \ln(n - K) + \frac{|Y(\xi)|^2}{N_{0,F}} + \ln \left(\sum_{m=1}^M \exp \left(-\frac{1}{N_{0,F}} |Y(\xi) - H(\xi)s_m|^2 \right) \right), \quad (2.21)$$

where $\xi = 1, \dots, n$, $s_m \in \mathcal{S}$.

The larger the value of $\lambda(\xi)$, the higher the probability that the subcarrier transmitting $Y(\xi)$ is active.

The computational complexity of LLR detector in (2.21), in terms of complex multiplications, is $\sim \mathcal{O}(M)$ per subcarrier, which is the same as that of classical OFDM.

After calculation of the n LLR values, the subcarriers with the highest K probabilities are assumed to be active. The set of active indices are then passed to the Index Demapper at the receiver, which performs the inverse action of the Index Selector block given in Fig. 2.10. Demodulation of the constellation symbols on the active indices is then straightforward.

However, if the output of Index Demapper, Z_{p_1} , is larger than C_n^K , it can result in a catastrophic error. A catastrophic error occurs when the LLR detector decides on an unused combination of active subcarriers. For example, in Table 2.2 where $n = 8$ and $K = 4$, Z_{p_1} ranges from 0 to 63 while $C_n^K = 70$. This means that $70 - 64 = 6$ combinations of active subcarriers are unused at the transmitter. The LLR detector at the receiver may decide on these 6 combinations of active subcarriers, which may not only result in the wrong detection of active subcarriers but also the wrong decoding of constellation symbols since the decoding of constellation symbols is based on the result of active subcarrier detection.

2.4 Chapter Summary

The development of classical OFDM, SM and the pioneering works of applying SM to classical OFDM, i.e., SIM-OFDM and ESIM-OFDM, are reviewed in this chapter. Then a novel multicarrier scheme named as OFDM with index modulation, which also uses the indices of the active subcarriers to transmit data, is described in detail. In this scheme, the incoming information bits are transmitted in a unique fashion to improve BER performance as well as to increase spectral

efficiency. OFDM-IM achieves significantly better BER performance than classical OFDM when BPSK is adopted.

The originality of OFDM-IM compared to ESIM-OFDM is the following:

- the number of active subcarriers and the spectral efficiency of the system can be adjusted as desired,
- novel methods have been implemented to select active subcarrier indices according to the information bits,
- novel low complexity detection techniques have been proposed,
- fading channels have been considered, while ESIM-OFDM has been investigated only for channels with Additive White Gaussian Noise (AWGN).

Chapter 3

Type 1 Generalization of OFDM-IM

In the OFDM-IM scheme, the number of active subcarriers in each OFDM subblock is fixed to a value of K and the best performance in terms of BER and spectral efficiency is achieved when $K = n/2$ if BPSK is used. For the case of $n = 8$ and $n = 32$, the maximum spectral efficiency achieved are 1.1111 bits/s/Hz and 1.25 bits/s/Hz, respectively, and the BER performance outperforms that of classical OFDM in high SNR region.

To achieve higher spectral efficiency and better BER performance, in this chapter, the OFDM-IM is generalized such that the number of active subcarriers in an OFDM subblock may be different. This generalization is named as OFDM with type-1 generalized index modulation, denoted as OFDM-GIM1 [122].

3.1 Main Idea

Fig. 3.1 and Fig. 3.2 show a simple example of subcarrier combinations of OFDM-IM and OFDM-GIM1, respectively. For an OFDM subblock with a group of

FIGURE 3.1: Subcarrier combinations of OFDM-IM.

subcarriers, let n be the total number of subcarriers of the subblock. In the OFDM-IM scheme, the number of active subcarriers in each OFDM subblock is fixed. Fig. 3.1 shows all the possible subcarrier combinations for $n = 4$ when the number of active subcarriers K is 2. However, the number of subcarriers that are active to carry constellation symbols is not necessary to be fixed; it may vary dependent on the input signal to be transmitted.

Fig. 3.2 shows the possible active subcarrier combinations for the same case of $n = 4$, when the number of active subcarriers may be 1 or 2. In such a way, intuitively, the number of bits that may be carried by the indices of the active subcarriers and by the active subcarriers themselves may be increased. The main problem to solve is how to map the input signal to different numbers of active subcarriers, and how they are subsequently detected. The following sections of this chapter will answer these questions.

FIGURE 3.2: Subcarrier combinations of OFDM-GIM1, $n = 4$, $\mathbf{K} = 1, 2$.

3.2 Problem Formulation

Considering a subblock of OFDM-IM, say the g -th subblock, let $\mathbf{K}$ be the set of all allowed numbers of active subcarriers, and R is defined as the size of set $\mathbf{K}$. For example, if 1 or 3 out of n subcarriers are allowed to carry signal constellation symbols, $\mathbf{K} = \{1, 3\}$ and R is equal to 2. Thus, for the OFDM subblock, $\mathbf{K}$ is given by,

$$\mathbf{K} = \{K_1, K_2, \dots, K_R\}. \quad (3.1)$$

In the case of K_r subcarriers that are active for $K_r \in \mathbf{K}$ and $r = 1, \dots, R$, the indices of the selected K_r active subcarriers of the g -th subblock are denoted as $\mathbf{I}_r^g$, given by,

$$\mathbf{I}_r^g = \{i_{r,1}^g, \dots, i_{r,K_r}^g\}, \quad (3.2)$$

where $i_{r,k}^g \in \{1, \dots, n\}$ for $g = 1, \dots, G$, $k = 1, \dots, K_r$ and $i_{r,k_1}^g \neq i_{r,k_2}^g$ if $k_1 \neq k_2$. Similarly, the signal constellation symbols at the output of the mapper to be put onto the subcarriers with indices in $\mathbf{I}_r^g$ are denoted as $\mathbf{s}_r^g$, given by

$$\mathbf{s}_r^g = \{s_{r,1}^g, \dots, s_{r,K_r}^g\}, \quad (3.3)$$

where $s_{r,k}^g \in \mathcal{S}$ and $\mathcal{S}$ is the set of M -ary constellation symbols.

In the OFDM-IM scheme, once n is given, $\mathbf{K}$ is fixed and only has a single element for all the subblocks. As a result, the OFDM-IM scheme is a special case of our OFDM-GIM1 scheme. For a certain $K_r \in \mathbf{K}$, the total number of bits that can be transmitted by an OFDM subblock is given by

$$B_r^g = \lfloor \log_2 (M^{K_r} C_n^{K_r}) \rfloor, \quad (3.4)$$

and the total number of bits that can be transmitted by all $K_r \in \mathbf{K}$ of an OFDM subblock is given by,

$$\sum_{K_r \in \mathbf{K}} B_r^g = \left\lfloor \log_2 \left(\sum_{K_r \in \mathbf{K}} M^{K_r} C_n^{K_r} \right) \right\rfloor. \quad (3.5)$$

Considering an extreme case where $\mathbf{K} = \{0, 1, \dots, n\}$, we have,

$$\begin{aligned} \sum_{r=1}^{n+1} B_r^g &= \left\lfloor \log_2 \left(\sum_{r=1}^{n+1} M^{K_r} C_n^{K_r} \right) \right\rfloor \\ &= \log_2 (M+1)^n. \end{aligned} \quad (3.6)$$

Clearly, $\log_2 (M+1)^n$ is much larger than B_r^g , the number of bits that can be transmitted by a fixed K_r . This extreme case indicates that by allowing multiple values of K , much more possible ways of selecting active subcarriers can be obtained for a given subblock size n . Hence, compared to the OFDM-IM scheme, more information bits per subblock can be transmitted in the OFDM-GIM1 scheme. For example, the spectral efficiency of the OFDM-GIM1 scheme will be up to 20%

higher than that of OFDM-IM when $n = 8$.

To implement the proposed OFDM-GIM1 scheme, several changes are necessary.

1. The normalization factors in OFDM-IM needs to be adjusted. To ensure $E\{\mathbf{x}^H \mathbf{x}\}$ equal to N , $E\{\mathbf{x}^g \mathbf{x}^{gH}\}$ is set to p , where p is the number of incoming bits per subblock as defined in Chapter 2. And the normalization factor of IFFT is changed to $\sqrt{N}$ and accordingly the normalization factor of FFT is changed to $1/\sqrt{N}$.
2. The index modulation block in Fig. 2.10 works for $\mathbf{K}$ containing only a single element, where the p bit input binary string is split to fixed p_1 bits and p_2 bits for a given K value, regardless of the information represented by the string. Generalizing this block is necessary so that p bit input string with different values may be split to different p_1 bits and p_2 bits, and thus to use different number of active subcarriers to carry M -ary constellation symbols for a given set $\mathbf{K}$ with multiple elements. In other words, for OFDM-GIM1, p remains constant for all transmission subblocks while p_1 and p_2 may vary according to different incoming input strings.
3. The LLR detector at the receiver to detect the active subcarriers and the M -ary constellation symbols carried by the active subcarriers are upgraded to adapt to a given set $\mathbf{K}$ with multiple elements.

Compared to OFDM-IM, 1) is a minor change and 2) and 3) are major changes. The details of these two major changes are addressed in the following 2 subsections.

3.3 Generalized Index Modulation Block

In this subsection, we first take $n = 8$, $\mathbf{K} = \{1, 3, 5\}$ and $M = 2$ (i.e., BPSK) as an example to demonstrate how the generalized index modulation block works.

TABLE 3.1: The 16 combinations represented by an active subcarrier out of eight subcarriers

Z_p	p_1 binary bits determining the active subcarrier	p_2 binary bit carried by the active subcarrier
<i>0</i>	000000000*	<u>0</u>
<i>1</i>	000000000	<u>1</u>
<i>2</i>	0000000001	<u>0</u>
$\vdots$	$\vdots$	$\vdots$
<i>14</i>	0000000111	<u>0</u>
<i>15</i>	0000000111	<u>1</u>

* The offset value to satisfy the requirement of Combinatorial Method, $Z_{p_1}^1$, i.e., the p_1 bits of the input binary string carried by the first combination of K_1 active subcarrier.

The elements of the set $\mathbf{K}$ are assumed to be in strict ascending order. These parameters mean that for the g -th subblock consisting of 8 subcarriers, either a subcarrier, 3 subcarriers or 5 subcarriers out of 8 subcarriers may carry BPSK symbols. Hence, according to (3.5), 11 bits, i.e., $p = 11$, can be transmitted by the g -th subblock.

To map the 11 bits to the subcarrier indices and BPSK symbols, let's first consider the case that $K_1 = 1$, i.e., a subcarrier out of 8 subcarriers is chosen and a BPSK symbol is put onto the selected subcarrier. From $M^{K_1}C_n^{K_1} = 2^1C_8^1 = 16$, it is known that in such a case 16 combinations out of $2^p = 2^{11} = 2048$ combinations can be expressed. The first 16 combinations, listed in Table 3.1, are designated to map with $K_1 = 1$, i.e., if the corresponding decimal value of the input string is in between 0 and 15 inclusive, the signal will be carried by a single active subcarrier.

The mapping is illustrated in Table 3.1, consisting of three columns. The first column (in italic style) contains the sequence numbers of the combinations, Z_p , which are also the decimal expressions of the binary strings to be transmitted by the subblock. The second column (in bold style) and the third column (underlined) together are the binary strings to be transmitted (arranged in ascending order). Since only one subcarrier is active when $K_1 = 1$, the number of the information

bits which can be carried by the active subcarrier in BPSK symbol, denoted as p_2 , is one, i.e., $p_2 = 1$ in this case. Taking the p_2 least significant bit (in column three) to be the bit carried by the active subcarrier, the remaining p_1 more significant bits (in column two) are mapped to determine the subcarrier index that is active.

The Combinatorial Method introduced in Chapter 2 may be used for the mapping of the p_1 bits. The Combinatorial Method suggests that the range of input decimal numbers to be mapped should be a contiguous integer set starting from zero. For OFDM-IM, since there is only one element in $\mathbf{K}$, i.e., all p bits decimal values from 0 to $2^p - 1$ are mapped based on a fixed number of active subcarriers, the above requirement is automatically satisfied. However, for the proposed OFDM-GIM1, only when the input numbers (the p_1 bits of binary string) are mapped based on the first element in $\mathbf{K}$, i.e., mapped to K_1 active subcarriers, the requirement for the range of input numbers is satisfied for sure. The range of input numbers mapped based on the other elements, i.e., $K_r \in \mathbf{K}, r = 2, \dots, R$, is not necessarily starting from zero. To circumvent this problem, an offset is introduced for each K_r to ensure that the range of the input decimal numbers satisfies the requirement of the Combinatorial Method.

Let Z_{p_1} represent the decimal number of the p_1 bit binary string, and denote $Z_{p_1}^r$ as the decimal number of the first combination of the p_1 bits carried by K_r active subcarriers (see $Z_{p_1}^1$ in Table 3.1). This Z_{p_1} is the offset to make the range of input numbers satisfy the requirement of the Combinatorial Method. Offsetting Z_{p_1} by a value $Z_{p_1}^r$, the resultant value, denoted as Z'_{p_1} , is $Z'_{p_1} = Z_{p_1} - Z_{p_1}^r$. For instance, for $Z_p = 2$, we have $Z_{p_1} = 1, Z_{p_1}^1 = 0$ and $Z'_{p_1} = Z_{p_1} - Z_{p_1}^1 = 1$. This Z'_{p_1} , together with $n = 8, K_1 = 1$ are fed to the Combinatorial Method. The output of the Combinatorial Method for this example is $\mathbf{I}_1^g = \{2\}$, indicating that the second subcarrier in this subblock is the selected active subcarrier. Meanwhile, the p_2 bit underlined, i.e., 0, is mapped to BPSK symbol $\mathbf{s}_1^g = \{-1\}$ and to be put onto the second subcarrier.

TABLE 3.2: The 448 combinations represented by three active subcarriers out of eight subcarriers

Z_p	p_1 binary bits determining the active subcarriers	p_2 binary bits carried by the the active subcarriers
16	00000010 *	<u>000</u>
17	00000010	<u>001</u>
18	00000010	<u>010</u>
$\vdots$	$\vdots$	$\vdots$
462	00111001	<u>110</u>
463	00111001	<u>111</u>

* The offset value to satisfy the requirement of Combinatorial Method, $Z_{p_1}^2$, i.e., the p_1 bits of the input binary string carried by the first combination of K_2 active subcarriers.

Now let's proceed to consider the case that $K_2 = 3$, i.e., 3 subcarriers out of 8 subcarriers are chosen and 3 BPSK symbols are put onto those selected subcarriers. In this case, the total number of combinations can be represented is given by:

$$M^{K_2} C_n^{K_2} = 2^3 C_8^3 = 448. \quad (3.7)$$

The 448 combinations that are designated to $K_2 = 3$ are given in Table 3.2. The procedure of the combination mappings for $K_2 = 3$ is similar to that of $K_1 = 1$. For example when Z_p is 462, $p_2 = 3$, $p_1 = 8$, $Z_{p_1}^2 = 2$ and the offset decimal number Z_{p_1}' , given by $Z_{p_1} - Z_{p_1}^2$, thus is $57 - 2 = 55$. This number, together with $n = 8$, $K_2 = 3$ are fed to the Combinatorial Method. The output of the Combinatorial Method will be $\mathbf{I}_2^g = \{6, 7, 8\}$, i.e., the sixth, seventh and eighth subcarriers are selected in this subblock as the active subcarriers. Meanwhile, the p_2 bits in underlined type, i.e., 110, are mapped to BPSK symbols $\mathbf{s}_2^g = \{1, 1, -1\}$ and to be put onto those selected subcarriers respectively.

According to the above description, with $K_1 = 1$ and $K_2 = 3$, 464 out of 2048 combinations have already been mapped. The remaining 1584 combinations are to be mapped by using $K_3 = 5$. Theoretically, the total number of combinations

TABLE 3.3: The 1584 combinations represented by five active subcarriers out of eight subcarriers

Z_p	p_1 binary bits determining the active subcarriers	p_2 binary bits carried by the the active subcarriers
464	001110 *	<u>10000</u>
465	001110	<u>10001</u>
466	001110	<u>10010</u>
$\vdots$	$\vdots$	$\vdots$
2046	111111	<u>11110</u>
2047	111111	<u>11111</u>

* The offset value to satisfy the requirement of Combinatorial Method, $Z_{p_1}^3$, i.e., the p_1 bits of the input binary string carried by the first combination of K_3 active subcarriers.

that $K_3 = 5$ can represent is given by:

$$M^{K_3} C_n^{K_3} = 2^5 C_8^5 = 1792. \quad (3.8)$$

Therefore, only 1584 out of the 1792 combinations will be used. By noting this, the procedure of the combination mappings for $K_3 = 5$ is the same as that of $K_1 = 1$ and $K_2 = 3$, which is listed in Table 3.3.

Through this example, it can be seen that the proposed index modulation block has one more level of operation than the original one. In the original technique, the number of active subcarriers is fixed, such that the splitting of p bits to p_1 and p_2 bits is also fixed. However, in the proposed technique, the p bit signal is split into different p_1 and p_2 bits for different input binary strings for a given set $\mathbf{K}$.

For a certain Z_p , the number of bits p_2 entering the mapper is given by,

$$p_2 = \begin{cases} K_1 \log_2 M, & Z_p \in [0, M^{K_1} C_n^{K_1} - 1], \\ K_2 \log_2 M, & Z_p \in [M^{K_1} C_n^{K_1}, M^{K_2} C_n^{K_2} - 1], \\ \vdots \\ K_R \log_2 M, & Z_p \in [M^{K_{R-1}} C_n^{K_{R-1}}, M^{K_R} C_n^{K_R} - 1]. \end{cases} \quad (3.9)$$

FIGURE 3.3: The generalized index modulation block of the g -th subblock for OFDM-GIM1.

In the example mentioned above, we have

$$p_2 = \begin{cases} 1, & Z_p \in [0, 15], \\ 3, & Z_p \in [16, 463], \\ 5, & Z_p \in [464, 2047]. \end{cases} \quad (3.10)$$

Thus, the index modulation block in Fig. 2.10 is modified to the generalized structure in Fig. 3.3, where the subblock splitter takes the p bit signal, the set $\mathbf{K}$ and M as input to determine the value of p_2 for this particular p bit input (with decimal value Z_p) according to (3.9). Once p_2 is determined, the number of bits entering the index selector is given by $p_1 = p - p_2$. Thereafter, the Index Selector takes the p_1 most significant bits of the input binary string and generates the indices of p_2 active subcarriers $\mathbf{I}_r^g$ using the Combinatorial Method with the offset $Z_{p_1}^r$ considered, where r satisfies $p_2 = K_r \log_2 M$ and $r \in \{1, \dots, R\}$. Meanwhile, the p_2 least significant bits are sent to the Mapper to be mapped to M -ary constellation symbols $\mathbf{s}_r^g$. The constellation symbols $\mathbf{s}_r^g$ are then put onto the active subcarriers

with indices in $\mathbf{I}_r^g$ respectively.

3.4 Upgraded LLR detector

As mentioned in Chapter 2, the ML detector is optimum in the detection of received symbols in OFDM-IM and may only be applied to the cases where $M^K C_n^K$ is small. Similarly, OFDM-GIM1 may use ML detector when the number of possible subcarrier index combinations is small. However, in general, OFDM-GIM1 has many more combinations than OFDM-IM, making ML detector generally impractical in most cases where $\sum_{r=1}^R M^{K_r} C_n^{K_r}$ is large. Hence, LLR detector is a practical choice to trade off between the detection precision and detection complexity. In this subsection, the upgraded LLR detector for OFDM-GIM1 is proposed. The original LLR detector is only suitable for a given set $\mathbf{K}$ with single element while the upgraded LLR detector aims at adapting to a given set $\mathbf{K}$ with multiple elements.

In the original OFDM-IM, for the g -th subblock, there is only a single element in the set $\mathbf{K}$, i.e., $\mathbf{K} = \{K_1\}$. In the receiver, once the K_1 indices of the subcarriers which have the maximum K_1 LLR values out of the n LLR values, computed according to (2.21), are obtained, the demodulation of M -ary constellation symbols is straightforward from those chosen subcarriers. Different from OFDM-IM, the generalized scheme has a flexible $K_r \in \mathbf{K}$ for p bit input signal with different values in an OFDM subblock. Though the set $\mathbf{K}$ is known by the receiver in advance, for each received symbol, the receiver actually does not know what K_r is. To detect the information, every possible K_r must be considered.

The detection procedure is done subblock by subblock. Take the g -th subblock as an example. The detection procedure starts by calculating the LLR values of all the n subcarriers in the g -th subblock. According to (2.21), for every $K_r \in \mathbf{K}$,

we have

$$\begin{aligned} \lambda_r^g(\xi) = & \ln \left(\sum_{m=1}^M \exp \left(-\frac{1}{N_{0,F}} |Y^g(\xi) - \sqrt{\frac{n}{K_r}} H^g(\xi) s_m|^2 \right) \right) \\ & + \frac{|Y^g(\xi)|^2}{N_{0,F}} + \ln(K_r) - \ln(n - K_r), \end{aligned} \quad (3.11)$$

where

$$Y^g(\xi) = Y(n(g-1) + \xi), \quad (3.12)$$

$$H^g(\xi) = H(n(g-1) + \xi), \quad (3.13)$$

$$\xi = 1, \dots, n, \quad (3.14)$$

and

$$s_m \in \mathcal{S}. \quad (3.15)$$

Here, $Y^g(\xi)$ is the ξ -th received signal in the g -th subblock, and $H^g(\xi)$ is the ξ -th channel fading coefficient in the g -th subblock. In (3.11), a factor $\sqrt{n/K_r}$ is introduced to normalize the received signal according to the assumed number of active subcarriers. In OFDM-IM, no such factor is used in the LLR detector because K is fixed in OFDM-IM, and the normalization by a factor of $\sqrt{N/K_{tot}} = \sqrt{n/K}$ has been done collectively before the LLR detector.

For BPSK modulation, by using the Jacobian logarithm [123] to prevent numerical overflow, (3.11) can be further simplified to

$$\begin{aligned} \lambda_r^g(\xi) = & \max(a, b) + \ln(1 + \exp(-|b - a|)) + \frac{|Y^g(\xi)|^2}{N_{0,F}} \\ & + \ln(K_r) - \ln(n - K_r), \end{aligned} \quad (3.16)$$

where

$$a = -|Y^g(\xi) - \sqrt{\frac{n}{K_r}} H^g(\xi)|^2 / N_{0,F}, \quad (3.17)$$

$$b = -|Y^g(\xi) + \sqrt{\frac{n}{K_r}} H^g(\xi)|^2 / N_{0,F}. \quad (3.18)$$

For every $K_r \in \mathbf{K}$, the maximum K_r LLR values in $\lambda_r^g(\xi)$, for $\xi = 1, \dots, n$, are selected and their corresponding indices ξ , are grouped into $\mathbf{I}_r^g$ (given in (3.2)) to form the K_r active subcarriers. The M -ary constellation symbols $\mathbf{s}_r^g$, as given in (3.3), are obtained from the subcarriers with indices in $\mathbf{I}_r^g$.

Based on the obtained $\mathbf{I}_r^g$ and $\mathbf{s}_r^g$ for all r , the set $\hat{\mathbf{I}}_{\hat{r}}^g$ and $\hat{\mathbf{s}}_{\hat{r}}^g$, that minimize the distance between the assumed transmitted signal (with channel conditions considered) and the received signal, are regarded as the set of active subcarrier indices and the M -ary constellation symbols carried by these subcarriers, i.e.,

$$(\hat{\mathbf{I}}_{\hat{r}}^g, \hat{\mathbf{s}}_{\hat{r}}^g) = \arg \min_{r \in [1, \dots, R]} \left(\sum_{\substack{\xi=1 \\ \xi \notin \mathbf{I}_r^g}}^n |Y^g(\xi)|^2 + \sum_{k=1}^{K_r} \left| Y^g(\mathbf{I}_r^g(k)) - \sqrt{\frac{n}{K_r}} H^g(\mathbf{I}_r^g(k)) \mathbf{s}_r^g(k) \right|^2 \right). \quad (3.19)$$

The Upgraded LLR detector is an LLR-ML detector, where the detection of active indices is based on LLR and the detection of K_r value is ML. Thereafter, the obtained $\hat{\mathbf{I}}_{\hat{r}}^g$ and $\hat{\mathbf{s}}_{\hat{r}}^g$ are passed to the index demodulation block at the receiver which performs the opposite action of the index modulation block in Fig. 3.3, to provide an estimate of the p bit input binary string.

3.5 Implementation Complexity

For the proposed generalization scheme, the major difference of implementation complexity compared to that of OFDM-IM lies in the computational complexity of the upgraded LLR detectors. For OFDM-IM, the complex multiplications is $\sim \mathcal{O}(M)$ per subcarrier when M -ary constellation symbols are carried.

For OFDM-GIM1, the complex multiplications is $\sim \mathcal{O}(RMn)$ per subcarrier, where R is the size of the set $\mathbf{K}$ (i.e., the set of all allowed numbers of active subcarriers) and n is the length of subblock. The parameter R here indicates

that R LLR detections are needed since the number of active subcarriers varies within subblocks. The parameter n here represents the computational complexity to compute the distance between the assumed transmitted signal (with channel conditions considered) and the received signal, for every LLR detection out of the total R LLR detections within each subcarrier.

Next, the actual running time of the upgraded LLR, original LLR and ML detectors as well as their corresponding performance impacts are studied by simulation. The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (3.20)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16, V = 10$ and the number of OFDM subcarriers N is set to be 128. The SNR is defined as $E_b/N_{0,T}$, where $E_b = (N + L)/B$ is the average transmitted energy per bit. The BER performance of these schemes was evaluated via Monte Carlo simulations. The simulation is done in a computer with Intel Core i5 processor and 4GB RAM. The simulation software is Matlab 2013b.

To ensure the accuracy of simulation results, every BER value for a certain SNR is obtained by averaging the results of 10,000 transmissions. In OFDM-GIM1, $n = 16, \mathbf{K} = \{9, 12\}$ is chosen.

For the upgraded LLR detector, it takes 91,728.018 seconds to complete the whole simulation for every 5dB of SNR from 0dB to 40dB.

However, the original LLR detector is only suitable for a given set $\mathbf{K}$ with single element. As shown in Fig. 3.4, when used with OFDM-GIM1, the BER performance shows an error floor at the BER of about 0.3.

For ML detector, the simulation of 1 transmission costs 138,564.572 seconds, which is approximately 38.5 hours. It will take about 395.5 years to complete the whole simulation, which is not a possible option.

FIGURE 3.4: BER performance of OFDM-GIM1 when original and upgraded LLR decoding, $n = 16$, BPSK.

3.6 Numerical Examples

This numerical example shows how a bit string is modulated and demodulated under the proposed scheme. In this example, the subblock size n is set to 8 and $\mathbf{K} = \{1, 3, 5\}$. According to (3.5), it is known that 11 bits can be transmitted per subblock. As one subblock of OFDM-GIM1 is suffice to present the behavior of a whole OFDM-GIM1 frame, for simplicity, the first subblock, i.e., $g = 1$, is chosen. This examples adopts BPSK constellation symbols.

The incoming $p = 11$ bits are given by 10101100110, which needs to be divided into two parts. One part has p_1 bits and the other part has p_2 bits. Unlike OFDM-IM, where p_1 and p_2 are fixed, for OFDM-GIM1, p_1 and p_2 are first determined according to (3.9).

The decimal number Z_p , derived from the input $p = 11$ bits, is equal to 1382. According to (3.10), which is derived from (3.9) when $n = 8$, $\mathbf{K} = \{1, 3, 5\}$ and

$M = 2$, p_2 for this particular p bit input is set to 5. In other words, $r = 3$ and $K_r = K_3 = 5$ subcarriers out of $n = 8$ subcarriers in this subblock are active to carry constellation symbols.

Once p_2 is determined, the number of bits entering the Index Selector is given by $p_1 = p - p_2 = 11 - 5 = 6$. Thereafter, the Index Selector takes the $p_1 = 6$ bits of the input binary string, i.e., 101011, and generates the indices of $p_2 = 5$ active subcarriers $\mathbf{I}_r^g$ using the Combinatorial Method with the offset $Z_{p_1}^3$ considered.

Specifically, according to Table 3.3, the offset value $Z_{p_1}^3 = 14$ is the decimal number of the p_1 bits of the input binary string carried by the first combination of K_3 active subcarriers, i.e., 001110. The decimal number $Z_{p_1} = 43$ is derived from $p_1 = 6$ bits 101011. Then the offset decimal number Z'_{p_1} is given by

$$Z'_{p_1} = Z_{p_1} - Z_{p_1}^3 = 29. \quad (3.21)$$

This Z'_{p_1} , together with $n = 8$, $K_3 = 5$ are fed to the Combinatorial Method. The combinadic of 29 is (7, 5, 3, 2, 1). Hence, the second, third, fourth, sixth and eighth subcarriers are selected in this subblock as the active subcarriers.

Meanwhile, the $p_2 = 5$ least significant bits, i.e., 00110, are sent to the Mapper and are mapped to BPSK constellation symbols $\mathbf{s}_3^1$, which is put onto the active subcarriers with indices in $\mathbf{I}_3^1$.

After passing through OFDM Block Creator, N-point IFFT, adding CP, P/S and digital-to-analog conversion, the signal is sent through a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (3.22)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than

V . Here, $L = 16, V = 10$ and the number of OFDM subcarriers N is set to be 128.

The channel fading coefficients used in this numerical example are a set of randomly produced coefficients according to the channel model. The channel fading coefficients for the first subblock are given by

$$\begin{aligned} H(1) &= 0.1708 + 0.0832i, & H(2) &= 0.3742 + 0.0721i, \\ H(3) &= 0.5965 - 0.0389i, & H(4) &= 0.7748 - 0.2814i, \\ H(5) &= 0.8167 - 0.6326i, & H(6) &= 0.6515 - 0.9982i, \\ H(7) &= 0.2841 - 1.2385i, & H(8) &= -0.1807 - 1.2330i. \end{aligned} \tag{3.23}$$

And the noise samples used are a set of randomly produced coefficients according to the distribution of $\mathcal{CN}(0, N_{0,F})$, given by

$$\begin{aligned} W(1) &= 0.2537 - 1.0311i, & W(2) &= -0.7293 + 0.5798i, \\ W(3) &= 0.8733 - 0.4746i, & W(4) &= 0.4879 - 0.0066i, \\ W(5) &= 0.6432 - 0.9090i, & W(6) &= -0.1959 + 1.2248i, \\ W(7) &= 0.5236 + 0.2477i, & W(8) &= -0.0967 + 0.8635i. \end{aligned} \tag{3.24}$$

As $\mathbf{K} = \{1, 3, 5\}$, for K_1, K_2 and K_3 , the detection procedure will calculate three sets of n LLR values according to (3.11) and three distances between the assumed transmitted signals (with channel conditions considered) and the received signal, respectively. The calculated LLR values when $K_2 = 3$ are shown in Table 3.4.

Based on Table 3.4, the subcarriers with the highest $K_2 = 3$ probabilities are assumed to be active, i.e., the second, third, fourth subcarriers in this subblock are regarded as the active subcarriers. Then the inverse of Combinatorial Method is applied and the input bits carried by such selection of three subcarriers are given by 10100000.

Demodulation of the constellation symbol on the active subcarrier is then

TABLE 3.4: LLR calculation example for $n = 8, K_2 = 3$

Subcarrier Index	LLR value	Largest Order
1	-13905	8
2	5622	1
3	3672	2
4	2461	3
5	-2272	7
6	672	4
7	-797	6
8	608	5

FIGURE 3.5: BER performance of the classical OFDM and OFDM-IM, $n = 4$, ML and LLR based decoding, BPSK.

straightforward. Specifically, the BPSK symbols carried by the three subcarriers are -1 , -1 and 1 . Then the bits carried by the active subcarriers are 110. The assumed transmitted signal is $0 \ -1.9149 \ -1.9149 \ 1.9149 \ 0 \ 0 \ 0 \ 0$.

In the light of (3.19), the distance d_2 is 2.5572.

By the same token, the distance d_1 when $K_1 = 1$ is 14.5657 and the distance d_3 when $K_3 = 5$ is 0.0086. Compared to d_1 and d_2 , d_3 is the minimal. Hence, for this subblock, $K_3 = 5$ is regarded as the number of active subcarriers that was implemented at the transmitter. Combining the bits carried by such selection of $K_3 = 5$ subcarriers, the input bits 10101100110 for the first subblock is finally recovered.

FIGURE 3.6: BER performance of OFDM-IM with fixed n and different K , LLR based decoding, BPSK.

3.7 Simulation Results

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (3.25)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

The simulation results of classical OFDM and OFDM-IM with ML and LLR decoding are shown in Fig. 3.5. The OFDM-IM exhibits inferior BER performance than the classical OFDM in the low SNR region because the bits transmitted by the active indices are affected more in the low SNR region than the data transmitted in the frequency-domain. As the SNR increases, the error performance of these bits improve substantially and the OFDM-IM scheme begins to achieve better error

performance.

Fig. 3.6 shows the tradeoff between BER performance and spectral efficiency when n is fixed to 4 and K varies. It can be concluded that when K is small, OFDM-IM offers better BER performance and lower spectral efficiency while when K is large, OFDM-IM offers worse BER performance and higher spectral efficiency. The best performance trade-off in terms of BER and spectral efficiency is achieved when $K = n/2$.

Fig. 3.7 shows the BER performances of the OFDM-GIM1 schemes with two different $\mathbf{K}$ sets, classical OFDM and OFDM-IM for BPSK. The subblock size n is set to 8. When $\mathbf{K} = \{1, 3, 5\}$, according to (3.5), it is known that 11 bits can be transmitted per subblock. However, for OFDM-IM with $n = 8$ and $K = 4$, only 10 bits can be transmitted per subblock.

In Fig. 3.7, the proposed OFDM-GIM1 scheme achieves 10% higher spectral efficiency at the cost of a BER performance loss lower than 0.5 dB. When $\mathbf{K} = \{1, 2, 3, 4, 5, 6\}$, according to (3.5), 12 bits can be transmitted per subblock, and thus a 20% higher spectral efficiency is achieved at the cost of a BER performance loss up to 2.5 dB. The 2.5 dB BER performance loss was observed at the BER probability of 10^{-3} .

Fig. 3.8 shows the BER performances of the OFDM-GIM1 schemes with two different $\mathbf{K}$ sets, classical OFDM and OFDM-IM for BPSK. The subblock size n is set to 16. When $\mathbf{K} = \{9, 12\}$, according to (3.5), it is known that 23 bits can be transmitted per subblock. However, for OFDM-IM with $n = 16$ and $K = 8$, only 21 bits can be transmitted per subblock.

In Fig. 3.8, the proposed OFDM-GIM1 scheme achieves 9.52% higher spectral efficiency at the cost of a BER performance loss lower than 0.3 dB. When $\mathbf{K} = \{10, 11\}$, according to (3.5), 24 bits can be transmitted per subblock, and thus a 14.29% higher spectral efficiency is achieved at the cost of a BER performance loss up to 3 dB.

FIGURE 3.7: BER performance of OFDM-GIM1 and OFDM-IM, $n = 8$, BPSK. FIGURE 3.8: BER performance of OFDM-GIM1 and OFDM-IM, $n = 16$, BPSK.

3.8 OFDM with type-1 Interleaved GIM

As shown in Fig. 3.7 and Fig. 3.8, the proposed OFDM-GIM1 suffers from BER performance loss at low SNR. An interleaved subcarrier-index modulation technique was proposed in [124] to mitigate the BER performance loss at low SNR for OFDM-IM. This interleaving technique may also be applied to the proposed OFDM-GIM1 since OFDM-GIM1 is the generalized versions of OFDM-IM.

FIGURE 3.9: The process of Interleaving.

Unlike what is illustrated in Fig. 2.10, where the output of OFDM Block Creator is directly processed by an N -point IFFT, the output of OFDM Block Creator is first interleaved and then processed by N -point IFFT in the proposed OFDM with type-1 interleaved generalized index modulation, denoted as OFDM-IGIM1. To maximize the frequency separation, the depth of the interleaving is set to the length of subblock, i.e., n . The process of interleaving is demonstrated in Fig. 3.9. The effect of this interleaving is to convert the correlated channel fading within different subcarriers into independent channel fading so that OFDM-IGIM1 may harvest on the channel diversity.

At the receiver, the received sequence is first processed by N -point FFT and then deinterleaved. The resultant sequence is then fed to a revised LLR detector proposed for OFDM-IGIM1. Specifically, since the channel effect is applied to the permuted signal sequences, the LLR detector first deinterleaves the channel fading coefficients $H(\xi)$ to get the deinterleaved channel fading coefficients $H'(\xi)$. Thereafter, the detection procedures are the same with that of OFDM-GIM1's, except that $H(\xi)$ is replaced with $H'(\xi)$ in (3.16)-(3.19).

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its

FIGURE 3.10: BER performance of OFDM-IIM and OFDM-IGIM1, $n = 8$, BPSK.

FIGURE 3.11: BER performance of OFDM-IIM and OFDM-IGIM1, $n = 16$, BPSK.

channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (3.26)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant

during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

Fig. 3.10 shows the BER performances of classical OFDM, OFDM-IM, OFDM-IIM (OFDM with interleaved index modulation), OFDM-GIM1 and OFDM-IGIM1 for BPSK. For OFDM-IM and OFDM-IIM, the subblock size n is set to 8 and the number of active subcarriers in each subblock K is set to 4. For OFDM-GIM1 and OFDM-IGIM1, the subblock size n is also set to 8 and two different $\mathbf{K}$ sets, i.e., $\mathbf{K} = \{1, 3, 5\}$ and $\mathbf{K} = \{1, 2, 3, 4, 5, 6\}$ are used.

Fig. 3.11 shows the BER performances of classical OFDM, OFDM-IM, OFDM-IIM, OFDM-GIM1 and OFDM-IGIM1 for BPSK. For OFDM-IM and OFDM-IIM, the subblock size n is set to 16 and the number of active subcarriers in each subblock K is set to 8. For OFDM-GIM1 and OFDM-IGIM1, the subblock size n is also set to 26 and two different $\mathbf{K}$ sets, i.e., $\mathbf{K} = \{9, 12\}$ and $\mathbf{K} = \{10, 11\}$ are used.

Fig. 3.10 and Fig. 3.11 indicate that compared to OFDM-IM, the proposed OFDM-IGIM1 shows significant BER performance improvement and at the same time, achieves higher spectral efficiency. Compared to OFDM-IIM [124], the proposed OFDM-IGIM1 shows spectral efficiency improvements dependent on different selections of $\mathbf{K}$ with no BER performance improvements. Compared to OFDM-GIM1, the proposed OFDM-IGIM1 also show significant BER performance improvement at a cost of negligible interleaving and deinterleaving overhead. These comparisons clearly demonstrate the superiority of the proposed scheme.

3.9 Chapter Summary

In this chapter, OFDM-GIM1 is proposed by varying the number of active subcarriers, which carry constellation symbols, in an OFDM subblock. The generalized index modulation block and the corresponding detector at the receiver for the variable number of active subcarriers are presented. With these techniques, more

bits can be transmitted for a given subblock length n . To mitigate the BER performance loss that may be incurred in OFDM-GIM1 in the low SNR region, an interleaver is introduced. Simulation results indicate that a much higher spectral efficiency is achieved by the proposed OFDM-GIM1 scheme at a price of a marginal BER performance loss.

Chapter 4

Type 2 Generalization Scheme of OFDM-IM

It is well known that classical OFDM using QPSK doubles spectral efficiency of that using BPSK with no BER performance loss. In OFDM-IM, the generalization scheme of OFDM-IM proposed in Chapter 3, i.e., OFDM-GIM1, successfully increases the transmitting spectral efficiency and meanwhile improves the BER performance for signals with high SNR under BPSK symbols. However, OFDM-GIM1 becomes ineffective when higher constellation symbols other than BPSK are implemented.

For example, when M -ary constellation symbols are implemented, for a certain n and a set $\mathbf{K}$, the total number of bit combinations that OFDM-IM can represent is given by $\sum_{r=1}^R M^{K_r} C_n^{K_r} G$. However, to achieve the same spectral efficiency as classical OFDM using QPSK, the total number of combinations required is $M^n G$. Noting that

$$\frac{M^n G}{\sum_{r=1}^R M^{K_r} C_n^{K_r} G} = \frac{M^n}{\sum_{r=1}^R M^{K_r} C_n^{K_r}} > 1 \quad (4.1)$$

for most choices of n and $\mathbf{K}$ when $M \geq 4$, it is important to find a scheme to remedy this shortcoming of OFDM-GIM1.

Even though OFDM-GIM1 can adopt a $\mathbf{K}$ which produces higher spectral

efficiency, such spectral efficiency is less than that of classical OFDM using QPSK and the BER performance is poor. To solve these problems, the type-2 generalization of OFDM-IM is introduced in this chapter. This scheme is based on the original OFDM-IM and named as OFDM with type-2 generalized index modulation (OFDM-GIM2) [125]. An interleaver is also introduced to reduce the BER performance loss. Noting that OFDM-GIM1 and OFDM-GIM2 are compatible with each other in further improving the spectral efficiency, the combination of these two schemes, named as OFDM-GIM3 is also investigated. Then OFDM-GIM2 is further improved by introducing joint I/Q index modulation [126].

4.1 Proposed Generalized Scheme For QPSK

In wireless communications, a QPSK constellation symbol consists of an in-phase component and a quadrature component. In OFDM-IM, the in-phase and quadrature components are regarded as inseparable and index modulation is applied coherently to the complex constellation symbol as a whole. In other words, in the original OFDM-IM, if a subcarrier is inactive, both the in-phase and quadrature components carried are 0, and if a subcarrier is active, both the in-phase and quadrature components carried are non-zero. The basic idea of OFDM-GIM2 is to split the in-phase component and quadrature component into two independent components so that index modulation is independently applied to these two components, i.e., a subcarrier is not necessary to be active or inactive simultaneously for the in-phase and quadrature components.

The block diagram of the OFDM-GIM2 transmitter is given in Fig. 4.1. This scheme, transmitting B bits per OFDM frame over a frequency selective Rayleigh fading channel, can be described as follows.

First, these B bits are divided into G groups and every group has p bits, i.e., $B = pG$. Every group of p bits is then assigned to one of G OFDM subblocks.

FIGURE 4.1: Block diagram of the OFDM-GIM2 transmitter.

For QPSK, the in-phase and quadrature components can be regarded as two independent BPSK streams. If two independent index modulations are applied to these two independent BPSK streams, the total number of combinations that can be represented is given by

$$2^K C_n^K 2^K C_n^K G = 4^K C_n^K C_n^K G. \quad (4.2)$$

On the other hand, the total combination that OFDM-IM using QPSK represents is $4^K C_n^K G$. As a result, we have

$$4^K C_n^K C_n^K G > 4^K C_n^K G. \quad (4.3)$$

For example, when $n = 16$, $K = 10$, the total number of bits that the proposed scheme can transmit is given by

$$(\lfloor \log_2 2^{10} C_{16}^{10} \rfloor + \lfloor \log_2 2^{10} C_{16}^{10} \rfloor) G = 44G. \quad (4.4)$$

While the total numbers of bits that OFDM-IM and OFDM can transmit are both

$$(\lfloor \log_2 4^{10} C_{16}^{10} \rfloor) G = (\lfloor \log_2 4^n \rfloor) G = 32G. \quad (4.5)$$

In other words, when $n = 16, K = 10$, the proposed OFDM-GIM2 offers a 37.5% higher spectral efficiency compared to OFDM-IM.

Different from OFDM-IM, at the transmitter, the input bit strings allocated to each subblock are equally split into two parts, one for the in-phase components' index modulation and the other for quadrature components' index modulation. The outputs of these two index modulation are then combined and constructed into a complex constellation symbol.

Assume that there are K active subcarriers for $K \leq n$. In OFDM-GIM2, the p input bit strings allocated to each subblock are first equally split into two parts, $p/2$ bits for in-phase components' index modulation and the other $p/2$ bits for quadrature components' index modulation.

For the in-phase component, the incoming $p/2$ bits are further divided into $p_1/2$ and $p_2/2$ bits, as shown in Fig. 4.1, where $p/2 = p_1/2 + p_2/2$.

The dashed block in Fig. 4.1 is named as type-2 generalized index modulation block. The $p_1/2$ bits are fed to the index selector, mapping the incoming $p_1/2$ bits to a combination of K active subcarriers, i.e., $\mathbf{I}_{\text{Real}}^g$ for $0 < g \leq G$ in Fig. 4.1, where $\mathbf{I}_{\text{Real}}^g$ is the set of K indices of the active subcarriers for the in-phase component of the g -th OFDM subblock, given by

$$\mathbf{I}_{\text{Real}}^g = \{i_{\text{Real},1}^g, \dots, i_{\text{Real},K}^g\}. \quad (4.6)$$

Here $i_{\text{Real},k}^g \in [1, \dots, n]$ for $g = 1, \dots, G$, $k = 1, \dots, K$ and $i_{\text{Real},k_1}^g \neq i_{\text{Real},k_2}^g$ if $k_1 \neq k_2$.

The remaining $p_2/2$ of the in-phase component bits go through the Mapper

(modulator) to be mapped to K BPSK constellation symbols. The output of the Mapper are given by

$$\mathbf{s}_{\text{Real}}^g = \{s_{\text{Real},1}^g, \dots, s_{\text{Real},K}^g\}, \quad (4.7)$$

where $s_{\text{Real},k}^g \in \mathcal{S}$ for $g = 1, \dots, G$, $k = 1, \dots, K$ and $\mathcal{S}$ is the set of BPSK constellation symbols.

Similarly, for the quadrature component of the g -th OFDM subblock, $\mathbf{I}_{\text{Imag}}^g$ and $\mathbf{s}_{\text{Imag}}^g$ are obtained according to the other $p/2$ bits of the input bit string.

Based on the active subcarriers in $\mathbf{I}_{\text{Real}}^g$ and $\mathbf{I}_{\text{Imag}}^g$, the constellation symbols in $\mathbf{s}_{\text{Real}}^g$ and $\mathbf{s}_{\text{Imag}}^g$ are then combined and constructed into a complex symbol vector $\mathbf{x}$ in the OFDM Block Creator, where $x \in \sqrt{\frac{n}{K}} \times \{\pm 1 \pm i, \pm 1, \pm i\}$.

Hence, for either in-phase or quadrature components, the total number of bits that can be carried by a single block of OFDM-GIM2 is given by,

$$B_{\text{Real}} = B_{\text{Imag}} = \lfloor \log_2 (2^K C_n^K) \rfloor G, \quad (4.8)$$

As a result, the total number of bits that can be transmitted by a single block of OFDM-GIM2 scheme is,

$$B = B_{\text{Real}} + B_{\text{Imag}} = 2 \lfloor \log_2 (2^K C_n^K) \rfloor G. \quad (4.9)$$

Next, as shown in Fig. 4.1, after adding CP, P/S conversion and digital-to-analog conversion, the signal is sent through a frequency-selective Rayleigh fading channel.

To implement the proposed OFDM-GIM2 at the receiver, the LLR detector in OFDM-IM is revised accordingly.

The revised LLR detector for OFDM-GIM2 performs zero-forcing equalization first. For the ξ -th frequency domain received signal $Y(\xi)$, for $\xi = 1, \dots, n$, let

$$Y'(\xi) = \frac{Y(\xi)}{H(\xi)}, \quad (4.10)$$

where $Y'(\xi)$ is the received signal after zero-forcing equalization.

Note that the zero-forcing equalization amplifies the noise power so the noise power in (2.21) should be changed accordingly by introducing a factor of $H^2(\xi)$. After equalization, what the detectors will detect are symbols of BPSK, denoted as s_m for $m = 1, 2$. Therefore, the revised LLR detector for the in-phase components is given by

$$\begin{aligned} \lambda_I(\xi) = & \ln(K) - \ln(n - K) + \frac{H^2(\xi)|\text{Re}(Y'(\xi))|^2}{N_{0,F}} \\ & + \ln \left(\sum_{m=1}^M \exp \left(-\frac{H^2(\xi)}{N_{0,F}} |\text{Re}(Y'(\xi)) - s_m|^2 \right) \right), \end{aligned} \quad (4.11)$$

where $\text{Re}(Y'(\xi))$ returns the real part of $Y'(\xi)$.

And the revised LLR detector for the quadrature components is given by

$$\begin{aligned} \lambda_Q(\xi) = & \ln(K) - \ln(n - K) + \frac{H^2(\xi)|\text{Imag}(Y'(\xi))|^2}{N_{0,F}} \\ & + \ln \left(\sum_{m=1}^M \exp \left(-\frac{H^2(\xi)}{N_{0,F}} |\text{Imag}(Y'(\xi)) - s_m|^2 \right) \right), \end{aligned} \quad (4.12)$$

where $\text{Imag}(Y'(\xi))$ returns the imaginary part of $Y'(\xi)$.

After that, the two sets of LLR values are independently fed to the inverse index modulation block to get an estimate of the input bit string.

4.1.1 Numerical Examples

This subsection demonstrates how bit strings are modulated and demodulated. In this example, the subblock size n is set to 16 and the number of active subcarriers

per subblock K is set to 10. According to (4.4), it is known that 44 bits can be transmitted per subblock. As one subblock of OFDM-GIM2 is sufficient to represent the behavior of a whole OFDM-GIM2 frame, for simplicity, the first subblock, i.e., $g = 1$, is chosen.

The incoming $p = 44$ bits are given by

$$10101100110101100110100110011010101010110011, \quad (4.13)$$

which is equally split into two parts, one for in-phase component's index modulation and the other for quadrature component's index modulation.

For the in-phase component, the incoming $p/2 = 22$ bits,

$$1010110011010110011010, \quad (4.14)$$

are further divided into $p_1/2 = 12$ bits and $p_2/2 = 10$ bits.

The $p_1/2$ bits, 101011001101, are fed to the Index Selector, mapping the incoming $p_1/2$ bits to a combination of K active subcarriers. By applying the Combinatorial Method, the set of K indices of the active subcarriers for the in-phase component of the first OFDM subblock is given by

$$\mathbf{I}_{\text{Real}}^1 = \{2, 5, 7, 8, 9, 11, 12, 13, 14, 15\}. \quad (4.15)$$

The remaining $p_2/2 = 10$ of the in-phase component bits, 0110011010, go through the Mapper to be mapped to K BPSK constellation symbols. The output of the Mapper are given by

$$\mathbf{s}_{\text{Real}}^1 = \{-1, 1, 1, -1, -1, 1, 1, -1, 1, -1\}. \quad (4.16)$$

Based on the active subcarriers in $\mathbf{I}_{\text{Real}}^1$ and the constellation symbols in $\mathbf{s}_{\text{Real}}^1$, we have

$$\begin{aligned} \mathbf{x}_{\text{Real}}^1 = \{ & 0, -1.0488, 0, 0, 1.0488, 0, 1.0488, -1.0488, -1.0488, \\ & 0, 1.0488, 1.0488, -1.0488, 1.0488, -1.0488, 0 \}. \end{aligned} \quad (4.17)$$

Similarly, according to the other $p/2$ bits of the input bit string,

$$0110011010101010110011, \quad (4.18)$$

for the quadrature component of the first OFDM subblock, $\mathbf{I}_{\text{Imag}}^1$ and $\mathbf{s}_{\text{Imag}}^1$ are obtained, given by

$$\mathbf{I}_{\text{Imag}}^1 = \{1, 5, 6, 7, 8, 9, 10, 11, 13, 15\}, \quad (4.19)$$

and

$$\mathbf{s}_{\text{Imag}}^1 = \{1, -1, 1, -1, 1, 1, -1, -1, 1, 1\}. \quad (4.20)$$

Based on the active subcarriers in $\mathbf{I}_{\text{Imag}}^1$ and the constellation symbols in $\mathbf{s}_{\text{Imag}}^1$, we have

$$\begin{aligned} \mathbf{x}_{\text{Imag}}^1 = \{ & 1.0488, 0, 0, 0, -1.0488, 1.0488, -1.0488, 1.0488, \\ & 1.0488, -1.0488, -1.0488, 0, 1.0488, 0, 1.0488, 0 \}. \end{aligned} \quad (4.21)$$

$\mathbf{x}_{\text{Real}}^1$ and $\mathbf{x}_{\text{Imag}}^1$ are then combined and constructed into a complex symbol vector $\mathbf{x}$ in the OFDM Block Creator, given by

$$\begin{aligned} \mathbf{x}^1 = \{ & 1.0488i, -1.0488, 0, 0, 1.0488 - 1.0488i, 0 + 1.0488i, \\ & 1.0488 - 1.0488i, -1.0488 + 1.0488i, -1.0488 + 1.0488i, \\ & 1.0488i, 1.0488 - 1.0488i, 1.0488, -1.0488 + 1.0488i, \\ & 1.0488, -1.0488 + 1.0488i, 0 \}. \end{aligned} \quad (4.22)$$

After passing through OFDM Block Creator, N-point IFFT, adding CP, P/S and digital-to-analog conversion, the signal is sent through a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (4.23)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16, V = 10$ and the number of OFDM subcarriers N is set to be 128.

The channel fading coefficients used in this numerical example are a set of randomly produced coefficients according to the channel model. The channel fading coefficients for the first subblock are given by

$$\begin{aligned} H(1) &= 0.8511 + 1.7563i, & H(2) &= 1.0905 + 1.2810i, \\ H(3) &= 1.1570 + 0.7664i, & H(4) &= 1.0461 + 0.2770i, \\ H(5) &= 0.7784 - 0.1235i, & H(6) &= 0.3977 - 0.3829i, \\ H(7) &= -0.0344 - 0.4685i, & H(8) &= -0.4470 - 0.3732i, \\ H(9) &= -0.7709 - 0.1181i, & H(10) &= -0.9488 + 0.2501i, \\ H(11) &= -0.9456 + 0.6652i, & H(12) &= -0.7540 + 1.0509i, \\ H(13) &= -0.3969 + 1.3330i, & H(14) &= 0.0747 + 1.4506i, \\ H(15) &= 0.5897 + 1.3665i, & H(16) &= 1.0664 + 1.0730i. \end{aligned} \quad (4.24)$$

And the noise samples used are a set of randomly produced coefficients according to the distribution of $\mathcal{CN}(0, N_{0,F})$, given by

TABLE 4.1: LLR calculation example for $n = 16, K = 10$, in-phase component

Subcarrier Index	LLR value	Largest Order
1	-8332.3	16
2	6166.9	1
3	-4077.0	14
4	-2496.73	13
5	1362.4	8
6	-625.5	11
7	476.0	10
8	688.0	9
9	1369.1	7
10	-1987.6	12
11	2841.1	6
12	3764.9	5
13	4083.4	4
14	4696.1	3
15	4873.3	2
16	-4866.9	15

$$\begin{aligned}
W(1) &= 0.2537 - 1.0311i, & W(2) &= -0.7293 + 0.5798i, \\
W(3) &= 0.8733 - 0.4746i, & W(4) &= 0.4879 - 0.0066i, \\
W(5) &= 0.6432 - 0.9090i, & W(6) &= -0.1959 + 1.2248i, \\
W(7) &= 0.5236 + 0.2477i, & W(8) &= -0.0967 + 0.8635i, \\
W(9) &= -0.8459 - 0.4423i, & W(10) &= 0.3397 - 0.3949i, \\
W(11) &= 0.6707 + 0.1175i, & W(12) &= -0.5536 + 0.4405i, \\
W(13) &= -0.2709 + 0.8429i, & W(14) &= 0.3933 - 0.5382i, \\
W(15) &= 0.5366 + 0.1447i, & W(16) &= 0.7877 - 0.4524i.
\end{aligned} \tag{4.25}$$

Next, at receiver, the LLR values of the in-phase component and the quadrature component are calculated according to (4.11) and (4.12), respectively. The calculation results are listed in Table 4.1 and Table 4.2, respectively.

Based on Table 4.1, the subcarriers with the highest $K = 10$ probability are assumed to be active. Then the inverse of Combinatorial Method is applied and

TABLE 4.2: LLR calculation example for $n = 16, K = 10$, quadrature component

Subcarrier Index	LLR value	Largest Order
1	8468.4	1
2	-6011.0	16
3	-4121.9	13
4	-2564.8	11
5	1337.3	7
6	639.9	9
7	470.4	10
8	796.5	8
9	1391.8	6
10	2171.0	5
11	2960.0	4
12	-3609.7	12
13	4376.3	3
14	-4512.2	14
15	4756.1	2
16	-4942.3	15

the input bits carried by such selection of $K = 10$ subcarriers are given by

$$101011001101. \quad (4.26)$$

Demodulation of the constellation symbols on the active subcarrier is then straightforward, give by

$$0110011010 \quad (4.27)$$

Hence the bits transmitted by the in-phase component are reconstructed as

$$1010110011010110011010. \quad (4.28)$$

By the same token, based on Table 4.2, the bits transmitted by the quadrature component are reconstructed as

$$0110011010101010110011. \quad (4.29)$$

FIGURE 4.2: BER performance of OFDM-IM and OFDM-GIM2, $n = 8$, QPSK. FIGURE 4.3: BER performance of OFDM-IM and OFDM-GIM2, $n = 16$, QPSK.

Combining the bits carried by in-phase and quadrature component, the input bits for the first subblock are finally recovered as

$$10101100110101100110100110011010101010110011. \quad (4.30)$$

4.1.2 Simulation Results

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (4.31)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16, V = 10$ and the number of OFDM subcarriers N is set to be 128.

Fig. 4.2 shows the BER performances of classical OFDM, OFDM-IM, and OFDM-GIM2 for QPSK. The subblock size n is set to 8 and the number of active subcarriers in each subblock K is set to 3. According to (4.4) and (4.5), the proposed OFDM-GIM2 scheme achieves 45.5% higher spectral efficiency at the cost of up to 1.2 dB BER performance loss in the low SNR region.

Fig. 4.3 shows the BER performances of classical OFDM, OFDM-IM, and OFDM-GIM2 for QPSK. The subblock size n is set to 16 and the number of active subcarriers in each subblock K is set to 10. According to (4.4) and (4.5), the proposed OFDM-GIM2 scheme achieves 37.5% higher spectral efficiency at the cost of up to 1 dB BER performance loss in the low SNR region.

Both Fig. 4.2 and Fig. 4.3 indicate that the proposed scheme outperforms OFDM-IM not only in spectral efficiency but also in BER performance when the BER is lower than 10^{-3} , which clearly demonstrate the superiority of OFDM-GIM2.

FIGURE 4.4: BER performance of OFDM-IIM and OFDM-IGIM2, $n = 8$, QPSK.

4.1.3 OFDM with type-2 Interleaved GIM

As OFDM-GIM2 also suffers from BER performance loss at low SNR, the interleaving technique introduced in Chapter 3 may also be applied to the proposed OFDM-GIM2. The only difference of this application lies at the receiver, where the received sequence is first processed by N -point FFT and then deinterleaved. The resultant sequence is then fed to a revised LLR detector proposed for OFDM-IGIM2. Specifically, since the channel effect is applied to the permuted signal sequences, the LLR detector first deinterleaves the channel fading coefficients $H(\xi)$ to get the deinterleaved channel fading coefficients $H'(\xi)$. Thereafter, the detection procedures are the same with that of OFDM-GIM2's, except that $H(\xi)$ is replaced with (4.11)-(4.12).

Fig. 4.4 and Fig. 4.5 show the BER performances of classical OFDM, OFDM-IM, OFDM-IIM, OFDM-GIM2 and OFDM-IGIM2 for QPSK, with $n = 8, K = 3$ and $n = 16, K = 10$, respectively. They indicate that compared to OFDM-IM, the proposed OFDM-IGIM2 show significant BER performance improvement and at the same time, achieve higher spectral efficiency. Compared to OFDM-GIM2, the proposed OFDM-IGIM2 also shows significant BER performance improvement

FIGURE 4.5: BER performance of OFDM-IIM and OFDM-IGIM2, $n = 16$, QPSK.

at a cost of negligible interleaving and deinterleaving overhead. Compared to OFDM-IIM [124], the proposed OFDM-IGIM2 shows significant spectral efficiency improvement at a cost of acceptable BER performance loss in the low SNR region.

To show the advantages of the proposed schemes more clearly, a setting was found where OFDM-IM and OFDM-GIM1 have exactly the same spectral efficiency, but OFDM-GIM2 has slightly higher spectral efficiency than both OFDM-IM and OFDM-GIM1 as it is difficult to have a setting where all three schemes have exactly the same spectral efficiency. As seen in Fig. 4.6, despite of OFDM-GIM2's slightly higher spectral efficiency, OFDM-GIM1 and OFDM-GIM2 both achieve better BER performance than OFDM-IM. To achieve the same spectral efficiency, the number of active subcarriers in each OFDM-GIM2 subblock is much lower than that of OFDM-GIM1 and OFDM-IM. In other words, for a fixed transmission power, the lower the number of active subcarriers, the more power on those active subcarriers. This is the reason why OFDM-GIM2 shows outstanding BER performance compared to OFDM-GIM1 and OFDM-IM. For the proposed schemes, a tradeoff exists between the BER performance and spectral efficiency.

Under the same spectral efficiency setting as that of Fig. 4.6, a different channel model, named as SUI-5 [22], is chosen to give deeper insight into the proposed index

FIGURE 4.6: BER performance of OFDM-IM (BPSK), OFDM-GIM1 (BPSK) and OFDM-GIM2 (QPSK), $n = 16$.

modulation schemes. As defined in [22], SUI-5 is one of the channels which were selected to represent typical terrain type of the continental US. The delay profile of SUI-5 is given by

$$[0 \quad 4 \quad 10] \mu s, \quad (4.32)$$

and the corresponding power profile is given by

$$[0 \quad -5 \quad -10] \text{ dB}. \quad (4.33)$$

As seen in Fig. 4.7, under SUI-5 channel, OFDM-GIM2 still shows outstanding BER performance compared to OFDM-GIM1 and OFDM-IM. Compared to Fig. 4.6 where the channels are 10 taps with equal power, there are slight differences in BER performances in Fig. 4.7. Such comparison demonstrates that the proposed index modulation schemes support channels with not only equal power taps, but also other multipath delay/power profiles.

FIGURE 4.7: BER performance of OFDM-IM (BPSK), OFDM-GIM1 (BPSK) and OFDM-GIM2 (QPSK) under SUI-5 channel, $n = 16$.

4.2 Joint I/Q Index Modulation for OFDM-IGIM2

OFDM-GIM2 successfully increases the transmitting spectral efficiency and meanwhile improves the BER performance for signals with high SNR under QPSK symbols. However, it is found that OFDM-GIM2 can be further improved to reach its full potential in terms of spectral efficiency, which will be discussed in this section. As this section only focuses on OFDM-IGIM2, for the sake of simplicity, we denote OFDM-IGIM2 as OFDM with generalized index modulation (OFDM-GIM) in the remainder of this section.

4.2.1 Motivation

From (4.9), note that

$$0 \leq \left(\log_2 (2^K C_n^K) \right)^2 G - 2 \lfloor \log_2 (2^K C_n^K) \rfloor G < 2G, \quad (4.34)$$

where the term $(2^K C_n^K)^2$ is the total number of combinations that can be represented by the K active subcarriers of in-phase components and K active subcarriers

of quadrature components. This indicates that OFDM-GIM proposed in Section 4.1 does not always achieve its full capability in improving spectral efficiency.

When

$$\left(\log_2 (2^K C_n^K)^2 \right) - 2 \lfloor \log_2 (2^K C_n^K) \rfloor \geq 1, \quad (4.35)$$

for a subblock, if the indices of the active subcarriers of both in-phase and quadrature components are jointly modulated according to the input bit string, one more bit may be transmitted. As a result, for the whole OFDM block, a total of G bits may be additionally transmitted.

For example, when $n = 4, K = 2$, for every subblock in OFDM-GIM, the number of bits that can be transmitted is given by

$$2 \lfloor \log_2 (2^K C_n^K) \rfloor = 2 \lfloor \log_2 (2^2 C_4^2) \rfloor = 8. \quad (4.36)$$

In the same case, if joint I/Q index modulation are implemented in OFDM-GIM, the number of bits that can be transmitted is given by

$$\lfloor 2 (\log_2 (2^K C_n^K)) \rfloor = \lfloor 2 (\log_2 (2^2 C_4^2)) \rfloor = 9. \quad (4.37)$$

The increase in the number of bits that can be transmitted results in better spectral efficiency. As defined in [11], the spectral efficiency of classical OFDM when QPSK is adopted is given by

$$\frac{2N}{N + L}. \quad (4.38)$$

The spectral efficiency of OFDM-GIM when QPSK is adopted is given by

$$\frac{2 \lfloor \log_2 (2^K C_n^K) \rfloor G}{N + L}. \quad (4.39)$$

FIGURE 4.8: Spectral efficiency comparison of classical OFDM, OFDM-GIM and the proposed scheme for (a) $n = 4$, and (b) $n = 8$, QPSK.

The spectral efficiency of the proposed scheme when QPSK is adopted is given by

$$\frac{\lfloor 2 (\log_2 (2^K C_n^K)) \rfloor G}{N + L}. \quad (4.40)$$

Figs. 4.8 (a) and (b) show the spectral efficiency comparison of classical OFDM, OFDM-GIM and the proposed scheme for $n = 4$ and $n = 8$, respectively. They indicate that, for commonly used n and K combinations in OFDM-GIM which demonstrate better BER performance and/or higher spectral efficiency than classical OFDM, such as $n = 4, K = 2$, or $n = 8, K = 3$, or $n = 8, K = 5$, the spectral efficiency of the joint index modulation is improved.

On the other hand, joint I/Q index modulation may achieve better BER performance. This is because the total number of combinations that can be represented by OFDM-GIM is not an integer power of 2, while only integer power of

combinations can be transmitted in OFDM-GIM. Thus, some combinations are not used for transmission. However, at the receiver, the received signal may be detected to be one of the unused combinations, leading to the so called catastrophic error [11]. Obviously, if more combinations are in use, the probability of catastrophic errors is reduced. With the help of joint I/Q index modulation, the number of unused combinations out of all possible index combinations is reduced. In the same example, for OFDM-GIM, the percentage of the number of unused combinations out of all possible index combinations is given by

$$1 - \frac{2^{\lfloor \log_2(2^K C_n^K) \rfloor}}{2^K C_n^K} = 1 - \frac{2^{\lfloor \log_2(2^2 C_4^2) \rfloor}}{2^2 C_4^2} = 1 - \frac{16}{24} = 33.33\%. \quad (4.41)$$

With the introduction of joint I/Q index modulation, the percentage of unused combinations out of all possible index combinations is reduced to

$$1 - \frac{2^{\lfloor 2(\log_2(2^K C_n^K)) \rfloor}}{2^K C_n^K \cdot 2^K C_n^K} = 1 - \frac{2^{\lfloor 2(\log_2(2^2 C_4^2)) \rfloor}}{2^2 C_4^2 \cdot 2^2 C_4^2} = 1 - \frac{512}{576} = 11.11\%. \quad (4.42)$$

In this scenario, every subblock transmits 12.5% more bits compared to OFDM-GIM and meanwhile the BER performance may potentially be improved due to the reduced probability of catastrophic errors.

Based on these two motivations, the joint I/Q index modulation and its detection technique are proposed in the following two subsections.

4.2.2 Joint Index Modulation

Fig. 4.9 shows the overall structure of the proposed joint I/Q index modulation block for the g -th subblock. In the generalized index modulation block proposed in Section 4.1, the input p bits are equally split into two parts and two independent index modulations are proceeded separately based on the two $p/2$ bits. In the proposed joint index modulation block, the input p bits are first split into three parts. The first part consisting of $p-2K$ bits are fed to the joint I/Q index selector

FIGURE 4.9: The joint index modulation block of g -th subblock for OFDM-GIM

to jointly decide the active subcarrier sets $\mathbf{I}_{\text{Real}}^g$ and $\mathbf{I}_{\text{Imag}}^g$, and the second and third parts both consisting of K bits go through the mapper to be mapped to $\mathbf{s}_{\text{Real}}^g$ and $\mathbf{s}_{\text{Imag}}^g$, respectively.

The Combinatorial Method introduced in Chapter 2 provides a one-to-one mapping between natural decimal numbers and K indices selected out of n subcarriers, for all n and K . In order to obtain two sets of K active subcarrier indices, the $p - 2K$ bits need to be transformed to two decimal numbers. The transform between the $p - 2K$ bits and the two decimal numbers requires a one-to-one mapping, and the resultant decimal numbers are constrained to the range of $[0, 2^K - 1]$. Such transformation may not be unique and the straightforward way is to use division-and-modulo operation.

Denote Z^g as the decimal number represented by the $p - 2K$ bit string, Z_I^g as the decimal number representing the active indices information for the in-phase component and Z_Q^g as the decimal number for the quadrature component. Hence, we have

$$Z_I^g = \lfloor Z^g / C_n^K \rfloor, \quad (4.43)$$

and

$$Z_Q^g = Z^g - Z_I^g \cdot C_n^K. \quad (4.44)$$

Thus, Z_I^g and Z_Q^g uniquely respond to the input $p - 2K$ bit string. Together with n and K , Z_I^g and Z_Q^g are fed to the Combinatorial Method respectively to get $\mathbf{I}_{\text{Real}}^g$ and $\mathbf{I}_{\text{Imag}}^g$.

Take $n = 4, K = 2, p = 9, g = 1$ and the incoming bit string 101110111 as an example. First of all, the incoming bit string is split into three parts, given by

$$\underline{10111} \quad \mathbf{01} \quad \mathbf{11}. \quad (4.45)$$

The first part of $p - 2K = 5$ bits, i.e., 10111, is first converted to a decimal number $Z^g = 29$. Then Z_I^g , the decimal number representing the active indices information for the in-phase component, is

$$Z_I^g = \lfloor Z^g / C_n^K \rfloor = \lfloor 29/6 \rfloor = 4. \quad (4.46)$$

And Z_Q^g , the decimal number representing the active indices information for the quadrature component, is

$$Z_Q^g = Z^g - Z_I^g \cdot C_n^K = 29 - 4 \cdot 6 = 5. \quad (4.47)$$

Thus $n = 4, K = 2$ and $Z_I^g = 4$ are fed to the Combinatorial Method and the output of the Combinatorial Method is $\mathbf{I}_{\text{Real}}^1 = \{2, 4\}$, indicating that the second and the fourth subcarriers in the in-phase component of this subblock are the selected active subcarriers. By the same token, with $n = 4, K = 2$ and $Z_Q^g = 5$ as input, the output of the Combinatorial Method is $\mathbf{I}_{\text{Imag}}^1 = \{3, 4\}$, indicating that the third and the fourth subcarriers in the quadrature component of this subblock are the selected active subcarriers.

On the other hand, both the second part and third part consist of K bits, i.e.,

01 and **11** go through the mapper to be mapped to $\mathbf{s}_{\text{Real}}^g = [-1, 1]$ and $\mathbf{s}_{\text{Imag}}^g = [1, 1]$, respectively. Overall, the joint I/Q index modulation converts the incoming bit string 101110111 to active subcarriers 2 and 4 in the in-phase component, carrying BPSK symbols -1 and 1 respectively, and active subcarriers 3 and 4 in the quadrature component, carrying BPSK symbols 1 and 1 , respectively.

4.2.3 The Corresponding Detection Rule

At the receiver, the reverse operations of generalized index modulation are carried out. The received sequence is first processed by N -point FFT and then deinterleaved. Unlike classical OFDM, in the OFDM-GIM scheme, for either in-phase or quadrature components, the receiver not only need to detect the information bits on the active subcarriers, but also need to detect the indices of the active subcarriers. For any subcarrier, the metric of determining whether this subcarrier is active is to investigate what kind of frequency domain symbols it is carrying. Specifically, if it is carrying a non-zero symbol, then it is active. Otherwise, it is inactive. In Section 4.1, the LLR detector for OFDM-GIM has been proposed, which evaluates the logarithm of the ratio of *a posteriori* probabilities of non-zero to that of zero for every subcarrier.

For the proposed scheme, since the I/Q components are jointly index modulated at the transmitter, the joint I/Q detection is adopted accordingly at the receiver. Take the g -th subblock as an example. For the in-phase component, the set of active indices $\mathbf{I}_{\text{Real}}^g$ is converted back to the decimal number Z_I^g using reverse Combinatorial Method. Meanwhile, the constellation symbol carried by the active subcarrier which are indexed by $\mathbf{I}_{\text{Real}}^g$, is demodulated and the output of all K active subcarriers is formed into a K bit string. By the same token, Z_Q^g and the other K bit string are obtained for the quadrature component.

Thus, Z^g is uniquely recovered by

$$Z^g = Z_I^g \cdot C_n^K + Z_Q^g. \quad (4.48)$$

Z^g is then converted to a $p - 2K$ bit string, which, together with the two K bit strings, are reconstructed into the input p bit string.

4.2.4 Simulation Results

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (4.49)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

Fig. 4.10 shows the BER performances of classical OFDM, OFDM-GIM, and the proposed scheme for QPSK when $n = 4$, $K = 2$. Compared to OFDM-GIM, the proposed scheme achieves 12.5% higher spectral efficiency. Meanwhile, the proposed scheme shows the same BER performance in the low SNR region and exhibits up to 1 dB improvement at high SNR region.

Fig. 4.11 shows the BER performances of classical OFDM, OFDM-GIM, and the proposed scheme for QPSK when $n = 8$, $K = 3$, the proposed scheme shows the same BER performance as OFDM-GIM and at the same time, achieves 6.25% higher spectral efficiency.

The BER performance improvement in the high SNR region is attributed to two

factors. One is because of the reduction of the percentage of unused combinations as discussed in Section 4.2.1. The other is because of the difference of average transmitted energy per bit, i.e., E_b , where $E_b = (N + L)/B$. For both OFDM-GIM and the proposed scheme, N and L are the same. However, the proposed scheme transmits more bits per OFDM frame than OFDM-GIM. Hence there is a difference in B for OFDM-GIM and the proposed scheme. Therefore, E_b for OFDM-GIM is higher than that for the proposed scheme. The simulation is done under the scenario where the same $E_b/N_{0,T}$ (x-axis) is guaranteed and the bit error rate (y-axis) is compared under the same $E_b/N_{0,T}$. In order to make the comparison fair, compared to $N_{0,T}$ for OFDM-GIM, the noise variance for the proposed scheme needs to be reduced.

The benefit of the reduced percentage of unused combinations and noise variance in the low SNR region is not evident since the large noise variance due to low SNR greatly affects the detection of the indices of the active subcarriers and overwhelms such reductions. This is the reason why OFDM-GIM and the proposed scheme show almost the same BER performance in the low SNR region. As the SNR increases, the benefit of the reduced percentage of unused combinations and noise variance surfaces. As seen from Fig. 4.10 and Fig. 4.11, the proposed scheme shows better BER performance progressively with the increase of SNR.

4.3 OFDM with type-3 Interleaved Generalized Index Modulation (OFDM-IGIM3)

The advantage of the two generalization schemes proposed in Chapter 3 and Chapter 4 lies in their compatibility with each other. The combination of OFDM-GIM1 and OFDM-GIM2, is named as OFDM-GIM3. As a result, OFDM-IGIM3 represents OFDM with type-3 Interleaved GIM.

FIGURE 4.10: BER performance of classical OFDM, OFDM-GIM and the proposed scheme, $n = 4$, $K = 2$, QPSK

FIGURE 4.11: BER performance of classical OFDM, OFDM-GIM and proposed scheme, $n = 8$, $K = 3$, QPSK

In the generalized index modulation block employing OFDM-IGIM3, the input bit strings allocated to different subblocks are equally split into two parts, one for in-phase components' generalized index modulation and the other for quadrature components' generalized index modulation. Both index modulations use the block in Fig. 3.3. The outputs of these two generalized index modulation block are then combined and constructed into one complex constellation symbol. At the receiver, the index demodulation block will simply do the inverse procedures of the

FIGURE 4.12: BER performance of OFDM-IIM and OFDM-IGIM3, $n = 8$, QPSK.

generalized index modulation block.

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (4.50)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

Fig. 4.12 and Fig. 4.13 show the BER performances of classical OFDM, OFDM-IIM, and OFDM-IGIM3 for $n = 8$, $\mathbf{K} = \{1, 3\}$ and $n = 16$, $\mathbf{K} = \{9, 12\}$, respectively. Compared to the other 2 schemes, the proposed OFDM-IGIM3 shows significant BER performance improvement as well as spectral efficiency boost.

FIGURE 4.13: BER performance of OFDM-IIM and OFDM-IGIM3, $n = 16$, QPSK.

4.4 Implementation Complexity

For OFDM-GIM2, the complex multiplications is $\sim \mathcal{O}(2M)$ per subcarrier. The value 2 here indicates the complexity of the independent detections for in-phase and quadrature components.

For OFDM-IGIM3, the complex multiplications is $\sim \mathcal{O}(2RMn)$ per subcarrier. The complexity incurred due to interleaving is negligible.

The implementation complexities of the examples in Fig. 4.6 are analyzed as follows. For OFDM-IM, the complex multiplications is $\sim \mathcal{O}(2)$ per subcarrier. For OFDM-GIM1, the complex multiplications is $\sim \mathcal{O}(64)$ per subcarrier. For OFDM-GIM2, the complex multiplications is $\sim \mathcal{O}(4)$ per subcarrier.

Compared to OFDM-IGIM2, the major complexity difference of the proposed joint I/Q index modulation scheme lies in the division-and-modulo operation at the transmitter, as well as the corresponding multiplication operation at the receiver. Apparently, this complexity difference is negligible and the proposed scheme has no complexity overhead compared to OFDM-IGIM2.

Overall, for OFDM-GIM1, OFDM-GIM2 and OFDM-IGIM3, the implementation complexity is linear with respect to R , M and n . When the total number of subcarrier is high in practice, the large number of subcarriers can always be divided into more subblocks to maintain n as low as 16. The number of subblocks does not affect the overall BER performance while lower n keeps the low computational complexity of the system.

4.5 Chapter Summary

In this chapter, OFDM-GIM2 is presented. To implement these schemes, type-2 generalized index modulation block and upgraded LLR detector are proposed. Like OFDM-GIM1, interleaving is introduced to improve the BER performance of the proposed schemes in the low SNR region. OFDM-GIM2 achieves higher spectral efficiency than OFDM-IM. For OFDM-GIM1 and OFDM-GIM2, when the same spectral efficiencies are considered, both generalization schemes show consistent BER performance gain in all SNR regions. The joint I/Q index modulation and its detection scheme for OFDM-IGIM2 are also presented. To implement this scheme, joint I/Q index modulation and the corresponding detection block are proposed for QPSK.

The two generalization schemes, i.e., OFDM-GIM1 and OFDM-GIM2 are compatible with each other and their combined scheme greatly outperforms existing works in spectral efficiency and BER performance, at the cost of a little higher complexity.

Simulation results indicate that the proposed scheme outperforms OFDM-IGIM2 in spectral efficiency and at the same time achieves a BER performance which is at least not worse than OFDM-IGIM2.

Chapter 5

OFDM with Generalized Index Modulation Using PSK

In all the index modulation schemes, the dominating factor of the BER performance loss in the low SNR region is the incorrect detection of the activeness of subcarriers, which is catastrophic and is very likely to result in the incorrect detection of constellation symbols as well. For M -ary QAM constellation symbols, if $M \geq 8$, the constellation symbols have different distances from 0, which degrades the accuracy of active indices detection. This is one of the factors that hinders the index modulation systems from successfully employing higher order ($M \geq 8$) QAM constellation symbols, especially for OFDM-GIM1. OFDM-GIM1 has a flexible choice of the number of active subcarriers and its receiver actually does not know which number is chosen at the transmitter. To detect the information, every possible situation must be considered, requiring a very high accuracy of active index detection to help distinguish different numbers of active subcarriers.

In this chapter, PSK is introduced to replace QAM to tackle this problem for OFDM-GIM1. For any M of an M -ary PSK constellation, all the constellation symbols have the same distance from 0, which has a potential of achieving higher index detection accuracy. As a result, a better BER performance in the low

SNR region can be achieved. However, compared to QAM, the introduction of PSK degrades BER performance in the high SNR region since PSK's minimum Euclidean distance between constellation symbols is smaller than QAM's. An adaptive scheme switching between QAM and PSK may be adopted to make full use of their advantages in different SNR regions. The theoretical analysis for the index modulation systems proposed in this thesis is also conducted.

As this chapter mainly focuses on OFDM-GIM1, for the sake of simplicity, we denote OFDM-GIM1 as OFDM with generalized index modulation (OFDM-GIM) in the remainder of this chapter.

5.1 PSK and QAM

As discussed in the previous two chapters, OFDM-IM and its generalizations have been successfully developed for adopting constellation symbols up to QPSK. In this section, the commonly used modulation schemes, i.e., PSK and QAM (both rectangular QAM and circular QAM, also known as Amplitude-Phase Shift Keying) are briefly reviewed. PSK conveys data by changing the phase of the carrier. On the other hand, QAM relies the change of amplitude and/or phase of the constellation symbols to carry the modulated message.

The normalized constellation diagrams of 8PSK and conventional rectangular 8QAM are given in Fig. 5.1, and their normalized constellation symbols are given by

$$S_R = \frac{1}{\sqrt{6}} [\pm 3 \pm i, \pm 1 \pm i], \quad (5.1)$$

and

$$S_P = \left[\pm 1, \pm i, \pm \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}i \right], \quad (5.2)$$

respectively.

FIGURE 5.1: Constellation Diagrams of 8PSK and 8QAM.

However, although the rectangular 8QAM is the most commonly used 8QAM scheme, it is not optimal in the sense of requiring the minimum mean power for a given minimum Euclidean distance. This may degrade the accuracy of the active subcarrier detection of 8QAM. While we consider to use 8PSK replacing 8QAM, the optimal circular 8QAM is also discussed for a fair comparison. The normalized constellation diagram of the circular 8QAM is given in Fig. 5.2, with symbols given by

$$S_C = \frac{1}{\sqrt{3 + \sqrt{3}}} \left[\pm 1 \pm i, \pm(1 + \sqrt{3}), \pm(1 + \sqrt{3})i \right]. \quad (5.3)$$

As can be seen from Fig. 5.1 and Fig. 5.2, all the PSK constellation symbols have the same distance from 0 while QAM constellation symbols may have different distances from 0.

5.2 Mathematical Model

The 8PSK and 8QAM constellations introduced in the previous section can be directly applied to OFDM-GIM. In this section, under the assumption that AWGN and frequency-selective Rayleigh fading channel exist, the mathematical analysis

FIGURE 5.2: Constellation Diagrams of Circular 8QAM.

of 8PSK's superiority over that of QAMs in the detection accuracy of active indices is derived.

Assume that channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \quad (5.4)$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths.

The index detection errors consist of two scenarios. One is that an active subcarrier is detected as inactive and the other vice versa.

5.2.1 An Active Subcarrier Detected as Inactive

When an active subcarrier, which is carrying a constellation symbol, is wrongly detected as inactive and regarded as carrying 0, incorrect detection occurs.

FIGURE 5.3: Calculation of distances for active index detection for 8QAM.

Consider a constellation symbol represented by a complex number $x(\xi)$, for $x(\xi) \in S_R, S_P$ or S_C and $\xi = 1, \dots, n$. In OFDM-GIM1, $x(\xi)$ is first processed by IFFT, after adding CP, P/S conversion and digital-to-analog conversion, the signal is sent through the channel, represented by the channel fading coefficient $h(\xi)$. The AWGN sample $n(\xi)$ is also added. After FFT processing at the receiver, the corresponding received complex signal is represented by $y(\xi)$.

Hence we have

$$\begin{aligned}
 y(\xi) &= \text{FFT} \{ \text{conv} (\text{IFFT} \{x(\xi)\}, h(\xi)) + n(\xi) \}, \\
 &= \text{FFT} \{ \text{IFFT} \{x(\xi)\} \} \cdot \text{FFT} \{h(\xi)\} + \text{FFT} \{n(\xi)\}, \\
 &= x(\xi) \cdot \text{FFT} \{h(\xi)\} + \text{FFT} \{n(\xi)\}.
 \end{aligned} \tag{5.5}$$

The FFT of $h(\xi)$ is a linear addition of weighted $h(\xi)$, for $\xi = 1, \dots, n$. And $h(\xi)$ follows complex Gaussian distribution. As a result, $\text{FFT} \{h(\xi)\}$ also follows complex Gaussian distribution. By the same token, $\text{FFT} \{n(\xi)\}$ also follows complex

FIGURE 5.4: Calculation of distances for active index detection for circular 8QAM.

Gaussian distribution.

For simplicity, (5.5) is rewritten as

$$y = hx + n. \quad (5.6)$$

Consider the following two hypotheses:

$$\begin{cases} H_0 : y \text{ represents an inactive subcarrier} \Leftrightarrow H_0 : y = n, \\ H_1 : y \text{ represents an active subcarrier} \Leftrightarrow H_1 : y = hx + n. \end{cases} \quad (5.7)$$

We have

$$P(y | H_0) = \frac{1}{\sqrt{2\pi N_{0,F}}} e^{-\frac{|y|^2}{2N_{0,F}}}, \quad (5.8)$$

and

$$P(y | H_1) = \max \left\{ \frac{1}{\sqrt{2\pi N_{0,F}}} e^{-\frac{|y-x_\Theta|^2}{2N_{0,F}}} \right\}, \Theta = 1, \dots, M. \quad (5.9)$$

FIGURE 5.5: Calculation of distances for active index detection for 8PSK.

The decision criteria of deciding whether this y represents an active or inactive subcarrier is given as

$$\begin{cases} y \text{ represents an inactive subcarrier : } P(y | H_0) \geq P(y | H_1), \\ y \text{ represents an active subcarrier : } P(y | H_0) < P(y | H_1). \end{cases} \quad (5.10)$$

In other words, the criteria of deciding whether this y represents an active or inactive subcarrier is to evaluate its distance from any constellation symbol x_Θ ($\Theta = 1, \dots, M$) of a given constellation and from 0. If any distance from the constellation symbols is larger than its distance from 0, y is wrongly detected as inactive.

When 8QAM is taken into consideration, in (5.9), compared to $x \in \{x_1, x_2, x_3, x_4\}$, the probability that $x \in \{x_5, x_6, x_7, x_8\}$ is wrongly detected as inactive is much less. As a result, the error probability of $x \in \{x_1, x_2, x_3, x_4\}$ wrongly detected as inactive will dominate. Hence, for any constellation symbol x , if the corresponding received

complex signal y falls into the shadowed area in Fig. 5.3, the subcarrier carrying y is wrongly detected as inactive.

To be specific, when $x \in \{x_1, x_2, x_3, x_4\}$, detection error occurs when

$$|y - x_\Theta|^2 > |y - 0|^2, \quad (5.11)$$

where x_Θ is the constellation symbol that is closest to y , for $\Theta = 1, \dots, 4$. Note that when (5.11) is held for $\Theta = 5, \dots, 8$, wrong detection does not occur necessarily.

Expanding (5.11), we have

$$\begin{aligned} \Rightarrow |y|^2 + |x_\Theta|^2 - (x_\Theta^* y)^* - x_\Theta^* y &> |y|^2, \\ \Rightarrow |x_\Theta|^2 - 2\text{Re}(x_\Theta^* y) &> 0. \end{aligned} \quad (5.12)$$

In the following analysis, we assume that x is x_1 in Fig. 5.3; the analysis for x equal to other symbols is the same. Under such assumption, there are two cases:

Case 1: When $x_\Theta = x$

Let's first consider the probability that y falls into the second quadrant. Let $x = a + bi$, $n = c + di$. Since $x = x_1$, we have $a = -\frac{1}{\sqrt{6}}$, $b = \frac{1}{\sqrt{6}}$. The condition of $y = hx + n = ha + c + (hb + d)i$ falls into the second quadrant is given by

$$\begin{aligned} ha + c &< 0, \text{ and } hb + d > 0, \\ \Rightarrow c &< -ha, \text{ and } d > -hb, \\ \Rightarrow c &< \frac{1}{\sqrt{6}}h, \text{ and } d > -\frac{1}{\sqrt{6}}h. \end{aligned} \quad (5.13)$$

As n follows complex Gaussian distribution $\mathcal{CN}(0, 1)$, c and d also follow Gaussian distribution. The probability of y falls into the second quadrant is given by $(1 - Q(\frac{1}{\sqrt{6}}h))Q(-\frac{1}{\sqrt{6}}h)$, where $Q(\cdot)$ denotes the tail probability of the standard Gaussian distribution.

According to [127], $Q(z)$ can be approximated by

$$Q(z) \cong \frac{1}{12}e^{-z^2/2} + \frac{1}{4}e^{-2z^2/3}. \quad (5.14)$$

Let $t = h^2$. The probability density function of chi-squared distribution is given by

$$\frac{1}{2^{k/2}\Gamma(k/2)}z^{k/2-1}e^{-z/2}, z > 0 \quad (5.15)$$

where $\Gamma(n) = (n-1)!$ is the Gamma Function and k is the degree of freedom.

Since t has a chi-squared distribution with two degrees of freedom, the distribution of t is simplified to

$$\frac{1}{2}e^{-z/2}, z > 0. \quad (5.16)$$

Therefore, we have

$$\begin{aligned} & Q\left(\frac{1}{\sqrt{6}}h\right) \\ &= \int_0^{+\infty} \left\{ \frac{1}{12}e^{-t/12} + \frac{1}{4}e^{-t/9} \right\} \frac{1}{2}e^{-t/2} dt \\ &= 0.2760. \end{aligned} \quad (5.17)$$

Hence, the probability of y falls into the second quadrant is given by $(1 - Q(\frac{1}{\sqrt{6}}h))Q(-\frac{1}{\sqrt{6}}h) = (1 - Q(\frac{1}{\sqrt{6}}h))(1 - Q(\frac{1}{\sqrt{6}}h)) = 0.5242$. Under the case that y falls in the second quadrant, the probability that y falls in the shadowed area in Fig. 5.3 is computed as follows. Substitute (5.6) into (5.12), we have

$$\begin{aligned} & |x|^2 - 2\text{Re}(x^*(hx + n)) > 0, \\ \Rightarrow & |x|^2 - 2h|x|^2 - 2\text{Re}(x^*n) > 0, \\ \Rightarrow & 2\text{Re}(x^*n) < (1 - 2h)|x|^2. \end{aligned} \quad (5.18)$$

Since $x = a + bi$, $n = c + di$, (5.18) is written as

$$\begin{aligned} 2\operatorname{Re}((a + bi)^*(c + di)) &< -(1 - 2h) |a + bi|^2, \\ \Rightarrow 2(ac + bd) &< -(1 - 2h)(a^2 + b^2). \end{aligned} \quad (5.19)$$

As n is circularly symmetric noise and follows complex Gaussian distribution $\mathcal{CN}(0, 1)$, c and d also follow Gaussian distribution. Therefore, let $n' = ac + bd$, n' also follows Gaussian distribution $\mathcal{N}(0, \sqrt{a^2 + b^2})$. In addition, let $n' = (\sqrt{a^2 + b^2}) n''$, where n'' follows Gaussian distribution $\mathcal{N}(0, 1)$.

As a result, (5.19) is further written as

$$\begin{aligned} 2n' &< -(1 - 2h)(a^2 + b^2), \\ \Rightarrow 2(\sqrt{a^2 + b^2}) n'' &< -(1 - 2h)(a^2 + b^2), \\ \Rightarrow n'' &< -\frac{1}{2}(1 - 2h)\sqrt{a^2 + b^2}, \\ \Rightarrow n'' &< -\frac{1}{2}(1 - 2h)|x|. \end{aligned} \quad (5.20)$$

As n'' follows Gaussian distribution $\mathcal{N}(0, 1)$, the probability of incorrect detection of active index when $x_\Theta = x$ is given by $Q(\frac{1}{2}(1 - 2h)|x|)$.

Therefore, we have

$$\begin{aligned} &Q\left(\frac{1}{2}(1 - 2h) \times \frac{\sqrt{2}}{\sqrt{6}}\right) \\ &= \int_0^{+\infty} \left\{ \frac{1}{12} e^{-\frac{2}{6}(1-2\sqrt{t})^2/8} + \frac{1}{4} e^{-2\frac{2}{6}(1-2\sqrt{t})^2/12} \right\} \frac{1}{2} e^{-t/2} dt \\ &= 0.2801. \end{aligned} \quad (5.21)$$

Hence, the probability of an active subcarrier detected as inactive when $x_\Theta = x$ is given by $0.2801 \times 0.5242 = 0.1468$.

Case 2: When $x_\Theta \neq x$

Use the same principle in Case 1, the probabilities of y falls into the first, third

and fourth quadrant are given by 0.1998, 0.1998, and 0.0762 respectively. Only the situation where y falls into the first and third quadrant will be considered since the other situation's probability is low and ignored. Thanks to the symmetry of the first and third quadrant, only the probability that y falls in the shadowed area when y is in the first quadrant is derived.

Since $x = a + bi$, $n = c + di$, (5.12) is written as

$$\begin{aligned} c &< \frac{1}{2} (|x_{\Theta}| - 2ha), \\ \Rightarrow c &< \frac{1}{2} \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{6}}h \right). \end{aligned} \quad (5.22)$$

As n follows complex Gaussian distribution $\mathcal{CN}(0, 1)$, c also follow Gaussian distribution. Thus, the probability of incorrect detection of active index when $x_{\Theta} \neq x$ is given by $Q\left(\frac{1}{2} \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{6}}h \right)\right)$.

Therefore, we have

$$\begin{aligned} &Q\left(\frac{1}{2} \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{6}}h \right)\right) \\ &= \int_0^{+\infty} \left\{ \frac{1}{12} e^{-\left(\frac{1}{2} \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{6}}\sqrt{t} \right)\right)^2 / 8} + \frac{1}{4} e^{-2 \left(\frac{1}{2} \left(\frac{\sqrt{2}}{\sqrt{6}} + \frac{2}{\sqrt{6}}\sqrt{t} \right)\right)^2 / 12} \right\} \frac{1}{2} e^{-t/2} dt \quad (5.23) \\ &= 0.2226. \end{aligned}$$

Hence, the probability of an active subcarrier detected as inactive when $x_{\Theta} \neq x$ is given by $0.2226 \times 0.1998 = 0.0445$. As a result, the probability of incorrect detection of active index for 8QAM is $0.1468 + 0.0445 \times 2 = 0.2358$.

By the same token, for circular 8QAM, as demonstrated in Fig. 5.4, the inner ring wrongly detected as inactive will dominate the error probability. Hence we

have

$$\begin{aligned}
& Q\left(\frac{1}{2}(1-2h) \times \frac{\sqrt{2}}{\sqrt{3+\sqrt{3}}}\right) \\
&= \int_0^{+\infty} \left\{ \frac{1}{12} e^{-\frac{\sqrt{2}}{\sqrt{3+\sqrt{3}}}(1-2\sqrt{t})^2/8} + \frac{1}{4} e^{-2\frac{\sqrt{2}}{\sqrt{3+\sqrt{3}}}(1-2\sqrt{t})^2/12} \right\} \frac{1}{2} e^{-t/2} dt \\
&= 0.2697.
\end{aligned} \tag{5.24}$$

The probability of an active subcarrier detected as inactive when $x_\Theta \neq x$ is given by

$$Q\left(\frac{1}{2}\left(\frac{\sqrt{2}}{\sqrt{3+\sqrt{3}}} + \frac{2}{\sqrt{3+\sqrt{3}}}\right)h\right) = 0.1583. \tag{5.25}$$

The probabilities of y falls into the first, second, third quadrant are given by 0.5420, 0.1942 and 0.1942, respectively. Therefore, the probability of incorrect detection of active index for circular 8QAM is $0.2697 \times 0.5420 + 0.1583 \times 0.1942 \times 2 = 0.2077$.

For 8PSK, when $x_\Theta = x$, an active subcarrier detected as inactive only if y falls into the shadowed area constructed by line L1, L2 and L3 in Fig. 5.5.

L1 is given by

$$y_l + (1 + \sqrt{2})x_l = 0, \tag{5.26}$$

L2 is given by

$$y_l - (1 + \sqrt{2})x_l = 0, \tag{5.27}$$

and L3 is given by

$$y_l - \frac{1}{2} = 0. \tag{5.28}$$

As $y = hx + n = ha + c + (\sqrt{t}b + d)i = c + (\sqrt{t}b + d)i$, to make sure y falls into the shadowed area constructed by line L1, L2 and L3, we have

$$\left\{ \begin{array}{l} (\sqrt{t} + d) + (1 + \sqrt{2})c > 0 \\ (\sqrt{t} + d) - (1 + \sqrt{2})c > 0 \\ (\sqrt{t} + d) - 0 > 0 \\ (\sqrt{t} + d) - \frac{1}{2} < 0 \end{array} \right. \quad (5.29)$$

Simplifying (5.29), we have

$$\left\{ \begin{array}{l} c > -\frac{\sqrt{t}+d}{1+\sqrt{2}} \\ c < \frac{\sqrt{t}+d}{1+\sqrt{2}} \\ d > -\sqrt{t} \\ d < -\sqrt{t} + \frac{1}{2} \end{array} \right. \quad (5.30)$$

Hence the probability that y falls into the shadowed area constructed by line L1, L2 and L3 is given by

$$\begin{aligned} & \int_0^{+\infty} \int_{-\sqrt{t}}^{-\sqrt{t}+\frac{1}{2}} \int_{-\frac{\sqrt{t}+d}{1+\sqrt{2}}}^{\frac{\sqrt{t}+d}{1+\sqrt{2}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{c^2}{2}} dc \frac{1}{\sqrt{2\pi}} e^{-\frac{d^2}{2}} dd \frac{1}{2} e^{-t/2} dt \\ &= 0.1410. \end{aligned} \quad (5.31)$$

Using the same principle of (5.31), the probability of incorrect detection of active index when $x_{\Theta} \neq x$ is 0.0410. Therefore, the probability of incorrect detection of active index for 8PSK is $0.1410 + 0.0410 = 0.1820$.

For detection of an active subcarrier, 8PSK demonstrates the best accuracy while 8QAM shows the worst accuracy.

5.2.2 An Inactive Subcarrier Detected as Active

When an inactive subcarrier, which is carrying 0, is wrongly detected as active and regarded as carrying a constellation symbol, incorrect detection occurs as well. For

an inactive subcarrier, the received complex signal y is given by

$$y = h \times 0 + n = n. \quad (5.32)$$

If the distance of y from the closest constellation symbols is smaller than its distance from 0, y is wrongly detected as active.

To be specific, we have

$$|y - x_\Theta|^2 < |y - 0|^2, \quad (5.33)$$

where $\Theta = 1, \dots, 8$.

Expanding (5.33), we have

$$\Rightarrow |n|^2 > |n|^2 - 2\text{Re}(nx_\Theta^*) + |x_\Theta|^2 \quad (5.34)$$

Use the same principle of (5.19) and (5.20), we have

$$n'' > \frac{|x_\Theta|}{2}. \quad (5.35)$$

Use the same analysis principle of Section 5.2.1, for 8PSK, we have

$$\begin{aligned} & Q\left(\frac{1}{2}\right) \\ &= 0.3085 \end{aligned} \quad (5.36)$$

For 8QAM, we have

$$\begin{aligned} & Q\left(\frac{1}{2\sqrt{3}}\right) \\ &= 0.4096. \end{aligned} \quad (5.37)$$

For circular 8QAM, we have

$$\begin{aligned} Q\left(\frac{1}{\sqrt{6+2\sqrt{3}}}\right) \\ = 0.3726. \end{aligned} \tag{5.38}$$

For the detection of an inactive subcarrier, 8PSK also demonstrates the best accuracy while 8QAM also shows the worst accuracy. As a result, it can be concluded that 8PSK has the best index detection accuracy among all the three schemes, in the detection of both active and inactive subcarriers. Compared to 8QAM, circular 8QAM has a better index detection accuracy.

However, the minimum Euclidean distance between circular 8QAM constellation symbols is 0.9194, that between PSK constellation symbols is 0.7654 and that between QAM constellation symbols is 0.8165. Hence it can be expected that in the high SNR region, where the dominating factor is the minimum Euclidean distance between constellation symbols, index modulation systems using circular 8QAM achieves better BER performance than index modulation systems using 8PSK and 8QAM.

5.3 Simulation Results

The following assumptions for the channels are repeated for easy references. We assume that the channel is a frequency-selective Rayleigh fading channel. Its channel impulse response (CIR) coefficients are given by,

$$\mathbf{h} = [h(1), h(2), \dots, h(V)]^T, \tag{5.39}$$

where $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$ and V is the number of paths. Assuming further that the channel remains constant

FIGURE 5.6: BER performance of OFDM-GIM using 8PSK, circular 8QAM and 8QAM, $n = 8$, $\mathbf{K} = \{4, 5\}$.

during the transmission of an OFDM block and the length of CP, L , is larger than V . Here, $L = 16$, $V = 10$ and the number of OFDM subcarriers N is set to be 128.

Fig. 5.6 and Fig. 5.7 show the BER performances of the OFDM-GIM using 8PSK, circular 8QAM and 8QAM. The subblock sizes n are set to 8 and 16, respectively. The set of the number of active subcarriers in each subblock $\mathbf{K}$ is set to $\{4, 5\}$ and $\{10, 11\}$, respectively. Compared to OFDM-GIM using 8QAM, the proposed OFDM-GIM using 8PSK scheme achieves better BER performance in the low SNR region. In the high SNR region, OFDM-GIM using circular 8QAM and OFDM-GIM using 8QAM outperform OFDM-GIM using 8PSK.

Based on the mathematical analysis in Section 5.2 and the above simulation results, to make full use of PSK's and circular QAM's advantage in different SNR regions, an adaptive switching scheme between PSK and circular QAM may be adopted for all index modulation schemes. In low SNR region, PSK is preferred since it offers higher indices detection accuracy. In high SNR region, circular QAM is implemented as it has a higher minimum Euclidean distance between constellation symbols, which is the dominating factor in this region.

To show the benefit of introduction of PSK in a more practical scenario where

FIGURE 5.7: BER performance of OFDM-GIM using 8PSK, circular 8QAM and 8QAM, $n = 16$, $\mathbf{K} = \{10, 11\}$.

channel coding is adopted, Fig. 5.8 and Fig. 5.9 plot the BER performances of OFDM-GIM using 8QAM, circular 8QAM and 8PSK when convolutional code with rate $\frac{1}{8}$ is adopted for $n = 8$ and $n = 16$, respectively. As seen from Fig. 5.8 and Fig. 5.9, OFDM-GIM using 8PSK efficiently brings the BER down to below 10^{-3} when SNR larger than 10 dB, and it greatly outperforms OFDM-GIM using 8QAM and circular 8QAM. Thus, under channel coding, OFDM-GIM using 8PSK may be used by itself.

5.4 Diversity Order Analysis

In this section, the diversity order of the index modulation schemes proposed in this thesis are analytically evaluated by studying their pairwise error probability (PEP) events. PEP refers to the error probability of transmitting a signal and receiving its corresponding distorted signal. To simplify the analyses, we first prove that PEP events within different subblocks are identical, and it is sufficient to investigate the PEP events within a single subblock to determine the overall system performance.

Let $\mathbf{I}_N$ and $\mathbf{0}_{N_1 \times N_2}$ denote identity and zero matrices with dimensions $N \times N$ and $N_1 \times N_2$, respectively. The channel coefficients in the frequency domain,

FIGURE 5.8: BER performance of OFDM-GIM, $n = 8$, $\mathbf{K} = \{4, 5\}$ using 8QAM, circular 8QAM and 8PSK when convolutional code is adopted.

denoted as $\mathbf{H}$, are related to the coefficients in the time domain by

$$\mathbf{H} = \mathbf{W}_N \mathbf{h}_0, \quad (5.40)$$

where $\mathbf{W}_N$ is the DFT matrix with $\mathbf{W}_N^H \mathbf{W}_N = N \mathbf{I}_N$ and $(\cdot)^H$ stands for Hermitian transposition. $\mathbf{h}_0$ is a length N and zero-padded version of the vector of the time domain channel coefficients $\mathbf{h}$, i.e.,

$$\mathbf{h}_0 = [h(1), \dots, h(V), 0, \dots, 0]^T. \quad (5.41)$$

Assume that $h(v)$, for $v = 1, \dots, V$, follows the complex Gaussian distribution $\mathcal{CN}(0, \frac{1}{V})$. Thus $\mathbf{H}(\xi)$, for $\xi = 1, \dots, N$, also follows the complex Gaussian distribution since taking the Fourier transform of a Gaussian vector gives another Gaussian vector. However, the elements of $\mathbf{H}$ are no longer uncorrelated. The

FIGURE 5.9: BER performance of OFDM-GIM, $n = 16$, $\mathbf{K} = \{10, 11\}$ using 8QAM, circular 8QAM and 8PSK when convolutional code is adopted with rate $\frac{1}{8}$.

correlation matrix of $\mathbf{H}$ is given as

$$\mathbf{C} = E \{ \mathbf{H} \mathbf{H}^H \} = \mathbf{W}_N \tilde{\mathbf{I}} \mathbf{W}_N^H, \quad (5.42)$$

where

$$\tilde{\mathbf{I}} = \begin{bmatrix} \frac{1}{v} \mathbf{I}_{v \times v} & \mathbf{0}_{v \times (N-v)} \\ \mathbf{0}_{(N-v) \times v} & \mathbf{0}_{(N-v) \times (N-v)} \end{bmatrix} \quad (5.43)$$

is an all-zero matrix except for its first diagonal elements which are all equal to $\frac{1}{v}$. As $\mathbf{C}$ is the correlation matrix of $\mathbf{H}$, it is concluded that PEP events within different subblocks are identical.

Without loss of generality, the first subblock with length n is chosen to simplify the theoretical analysis. The following matrix notation for the input-output

relationship in the frequency domain is introduced,

$$\mathbf{Y}^1 = \mathbf{X}^1 \mathbf{H}^1 + \mathbf{W}^1. \quad (5.44)$$

In (5.44), $\mathbf{Y}^1$ and $\mathbf{X}^1$ represent the transmitted signal and the received signal for the first subblock, respectively. $\mathbf{Y}^1 = [Y(1), \dots, Y(n)]^T$ and $\mathbf{X}^1$ is an all-zero matrix except for its main diagonal elements denoted by $X(1), \dots, X(n)$. $\mathbf{H}^1 = [H(1), \dots, H(n)]^T$ and $\mathbf{W}^1 = [W(1), \dots, W(n)]^T$ denote the channel fading coefficients and the frequency domain noise samples for the first subblock, respectively.

Let us define $\mathbf{C}_n = E \left\{ \mathbf{H}^1 \mathbf{H}^{1H} \right\}$. In fact, $\mathbf{C}_n$ is an $n \times n$ submatrix centered along the main diagonal of the matrix $\mathbf{C}$ because $n < N$. If $\mathbf{X}^1$ is transmitted and it is erroneously detected as $\hat{\mathbf{X}}^1$, the receiver can make decision errors on active indices and/or constellation symbols. The well-known conditional pairwise error probability (CPEP) expression for the model in (5.44) is given as [128]

$$P \left(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1 | \mathbf{H}^1 \right) = Q \left(\sqrt{\frac{\|(\mathbf{X}^1 - \hat{\mathbf{X}}^1) \mathbf{H}^1\|^2}{2N_{0,F}}} \right), \quad (5.45)$$

where $Q(\cdot)$ denotes the tail probability of the standard Gaussian distribution and $\|\cdot\|$ denotes the Frobenius norm.

Let $\mathbf{A} = (\mathbf{X}^1 - \hat{\mathbf{X}}^1)^H (\mathbf{X}^1 - \hat{\mathbf{X}}^1)$, $\psi = \|(\mathbf{X}^1 - \hat{\mathbf{X}}^1) \mathbf{H}^1\|^2 = \mathbf{H}^{1H} \mathbf{A} \mathbf{H}^1$. Thus (5.45) is rewritten as,

$$P \left(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1 | \mathbf{H}^1 \right) = Q \left(\sqrt{\frac{\psi}{2N_{0,F}}} \right). \quad (5.46)$$

According to [127], $Q(x)$ can be approximated by

$$Q(x) \cong \frac{1}{12} e^{-x^2/2} + \frac{1}{4} e^{-2x^2/3}. \quad (5.47)$$

Hence, (5.46) is further simplified as,

$$P\left(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1 | \mathbf{H}^1\right) \cong \frac{1}{12}e^{-q_1\psi} + \frac{1}{4}e^{-q_2\psi}, \quad (5.48)$$

where $q_1 = \frac{1}{4N_{0,F}}$ and $q_2 = \frac{1}{3N_{0,F}}$.

Thus, the unconditional PEP (UPEP) of the index modulation schemes can be obtained by

$$P\left(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1\right) \cong E_{\mathbf{H}^1} \left\{ \frac{1}{12}e^{-q_1\psi} + \frac{1}{4}e^{-q_2\psi} \right\}, \quad (5.49)$$

where $E_{\mathbf{H}^1} \{\cdot\}$ represents the expectation with respect to $\mathbf{H}^1$.

Let $r_1 = \text{rank}(\mathbf{C}_n)$ denote the rank of $\mathbf{C}_n$. Since $r_1 < n$ for all the index modulation schemes, the spectral theorem [129] is used to calculate the expectation above. To be specific, $\mathbf{C}_n$ can be decomposed as $\mathbf{Q}\mathbf{D}\mathbf{Q}^H$ where $\mathbf{D} = E\{\mathbf{u}\mathbf{u}^H\}$ is an $r_1 \times r_1$ diagonal matrix, $\mathbf{u}$ is an $r_1 \times 1$ vector and $\mathbf{Q}$ is an $n \times r_1$ matrix. Hence we have

$$\mathbf{C}_n = E\left\{\mathbf{H}^1\mathbf{H}^{1H}\right\} = \mathbf{Q}\mathbf{D}\mathbf{Q}^H = \mathbf{Q}E\{\mathbf{u}\mathbf{u}^H\}\mathbf{Q}^H = E\left\{\mathbf{Q}\mathbf{u}(\mathbf{Q}\mathbf{u})^H\right\}. \quad (5.50)$$

From (5.50), $\mathbf{H}^1 = \mathbf{Q}\mathbf{u}$. Thus, $\psi = \mathbf{H}^{1H}\mathbf{A}\mathbf{H}^1 = \mathbf{u}^H\mathbf{Q}^H\mathbf{A}\mathbf{Q}\mathbf{u}$. The power density function of $\mathbf{u}$ is given by

$$f(\mathbf{u}) = \frac{\pi^{-r_1}}{\det(\mathbf{D})} e^{-\mathbf{u}^H\mathbf{D}^{-1}\mathbf{u}}, \quad (5.51)$$

where $\det(\mathbf{D})$ denote the determinant of $\mathbf{D}$.

The UPEP in (5.49) can be calculated as

$$\begin{aligned} P\left(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1\right) &\cong \frac{\pi^{-r_1}}{12\det(\mathbf{D})} \int_{\mathbf{u}} e^{-\mathbf{u}^H(\mathbf{D}^{-1}+q_1\mathbf{Q}^H\mathbf{A}\mathbf{Q})\mathbf{u}} d\mathbf{u} \\ &+ \frac{\pi^{-r_1}}{4\det(\mathbf{D})} \int_{\mathbf{u}} e^{-\mathbf{u}^H(\mathbf{D}^{-1}+q_2\mathbf{Q}^H\mathbf{A}\mathbf{Q})\mathbf{u}} d\mathbf{u}. \end{aligned} \quad (5.52)$$

From (5.51), we have

$$e^{-\mathbf{u}^H \mathbf{D}^{-1} \mathbf{u}} = \frac{f(\mathbf{u}) \det(\mathbf{D})}{\pi^{-r_1}} = \frac{f(\mathbf{u})}{\pi^{-r_1} \det(\mathbf{D}^{-1})}. \quad (5.53)$$

Integrating (5.53) with respect to $\mathbf{u}$,

$$\begin{aligned} & \int_{\mathbf{u}} e^{-\mathbf{u}^H \mathbf{D}^{-1} \mathbf{u}} d\mathbf{u} \\ &= \int_{\mathbf{u}} \frac{f(\mathbf{u})}{\pi^{-r_1} \det(\mathbf{D}^{-1})} d\mathbf{u} \\ &= \frac{1}{\pi^{-r_1} \det(\mathbf{D}^{-1})}. \end{aligned} \quad (5.54)$$

Hence, we have

$$\begin{aligned} & \int_{\mathbf{u}} e^{-\mathbf{u}^H (\mathbf{D}^{-1} + q_1 \mathbf{Q}^H \mathbf{A} \mathbf{Q}) \mathbf{u}} d\mathbf{u} \\ &= \frac{1}{\pi^{-r_1} \det(\mathbf{D}^{-1} + q_1 \mathbf{Q}^H \mathbf{A} \mathbf{Q})}, \end{aligned} \quad (5.55)$$

and

$$\begin{aligned} & \int_{\mathbf{u}} e^{-\mathbf{u}^H (\mathbf{D}^{-1} + q_2 \mathbf{Q}^H \mathbf{A} \mathbf{Q}) \mathbf{u}} d\mathbf{u} \\ &= \frac{1}{\pi^{-r_1} \det(\mathbf{D}^{-1} + q_2 \mathbf{Q}^H \mathbf{A} \mathbf{Q})}. \end{aligned} \quad (5.56)$$

According (5.55) and (5.56), (5.52) is written as

$$P(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1) \cong \frac{1}{12 \det(\mathbf{I}_{r_1} + q_1 \mathbf{D} \mathbf{Q}^H \mathbf{A} \mathbf{Q})} + \frac{1}{4 \det(\mathbf{I}_{r_1} + q_2 \mathbf{D} \mathbf{Q}^H \mathbf{A} \mathbf{Q})}. \quad (5.57)$$

If $\mathbf{M}$ and $\mathbf{N}$ are two matrices with dimensions of $r_1 \times n$ and $n \times r_1$, respectively, we have

$$\det(\mathbf{I}_{r_1} + \mathbf{M} \mathbf{N}) = \det(\mathbf{I}_n + \mathbf{N} \mathbf{M}). \quad (5.58)$$

As a result,

$$\begin{aligned}
 P(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1) &\cong \frac{1}{12\det(\mathbf{I}_{r_1} + q_1 \mathbf{DQ}^H \mathbf{AQ})} + \frac{1}{4\det(\mathbf{I}_{r_1} + q_2 \mathbf{DQ}^H \mathbf{AQ})} \\
 &= \frac{1}{12\det(\mathbf{I}_n + q_1 \mathbf{QDQ}^H \mathbf{A})} + \frac{1}{4\det(\mathbf{I}_n + q_2 \mathbf{QDQ}^H \mathbf{A})} \quad (5.59) \\
 &= \frac{1}{12\det(\mathbf{I}_n + q_1 \mathbf{C}_n \mathbf{A})} + \frac{1}{4\det(\mathbf{I}_n + q_2 \mathbf{C}_n \mathbf{A})}.
 \end{aligned}$$

Let us define $\mathbf{A}_i = \mathbf{I}_n + q_i \mathbf{C}_n \mathbf{A} = \mathbf{I}_n + q_i \mathbf{B}$ for $i = 1, 2$. We have

$$\det(\mathbf{A}_i) = \prod_{\alpha=1}^n \lambda_{\alpha}(\mathbf{A}_i) = \prod_{\alpha=1}^r (1 + q_i \lambda_{\alpha}(\mathbf{B})), \quad (5.60)$$

where $\mathbf{B} = \mathbf{C}_n \mathbf{A}$, $r = \text{rank}(\mathbf{B})$ and $\lambda_{\alpha}(\mathbf{B})$ is the α -th eigenvalue of $\mathbf{B}$.

For high SNR values, $q_i \gg 1$, (5.59) can be rewritten as

$$P(\mathbf{X}^1 \rightarrow \hat{\mathbf{X}}^1) = \left(12q_1^r \prod_{\alpha=1}^r \lambda_{\alpha}(\mathbf{B})\right)^{-1} + \left(4q_2^r \prod_{\alpha=1}^r \lambda_{\alpha}(\mathbf{B})\right)^{-1}. \quad (5.61)$$

As seen from (5.61), the diversity order of the system is determined by r , which is upper bounded according to the rank inequality [129] by $r \leq \min\{r_1, r_2\}$, where $r_2 = \text{rank}(\mathbf{A})$. As $\mathbf{A} = (\mathbf{X}^1 - \hat{\mathbf{X}}^1)^H (\mathbf{X}^1 - \hat{\mathbf{X}}^1)$, $(\mathbf{X}^1 - \hat{\mathbf{X}}^1)$ indicates the proposed index modulation schemes' error patterns between the transmitted signal and the received signal. For all the index modulation schemes throughout the thesis, although $(\mathbf{X}^1 - \hat{\mathbf{X}}^1)$ may be different because of implementing different generalizations or different constellation symbols, all the index modulation schemes' error patterns can always be divided into three categories.

1. The receiver wrongly detects at least one of the active indices and at least one of the M -ary symbols carried by the active subcarriers is also mistaken.
2. The receiver wrongly detects at least one of the active indices but none of the M -ary symbols carried by the active subcarriers is mistaken.

3. The receiver correctly detects all the active indices but at least one of the M -ary symbols carried by the active subcarriers is mistaken.

For the first and second category of error patterns, $(\mathbf{X}^1 - \hat{\mathbf{X}}^1)$ is an all-zero matrix except for at least two main diagonal elements related to the wrong detection of active indices. Hence the rank of $\mathbf{A} = (\mathbf{X}^1 - \hat{\mathbf{X}}^1)^H (\mathbf{X}^1 - \hat{\mathbf{X}}^1)$ is at least 2. However, when the third category of error patterns is taken into consideration, $(\mathbf{X}^1 - \hat{\mathbf{X}}^1)$ can be an all-zero matrix except for only one main diagonal element. Such case happens when the receiver correctly detects all of the active indices and makes a single decision error out of the M -ary symbols carried by the active subcarriers. Under such scenario, $\text{rank}(\mathbf{A}) = 1$. Hence we have $\min\{r_2\} = \min\{\text{rank}(\mathbf{A})\} = 1$.

Through the above analysis, the diversity orders of all the proposed index modulation schemes, which are all determined by the worst case PEP scenario when $r_2 = \text{rank}(\mathbf{A}) = 1$, are the same. This means all the index modulation schemes throughout the thesis will demonstrate the same BER slope as classical OFDM in multipath Rayleigh fading channel, as long as the SNR values are significantly high.

5.5 Chapter Summary

In this chapter, OFDM-GIM using PSK are presented. Mathematical analysis has shown that PSK provides a higher indices detection accuracy than QAM, and therefore the BER performance of OFDM-GIM in the low SNR region are improved. To avoid the BER performance loss in the high SNR region due to PSK's lower minimum Euclidean distance between constellation symbols, an adaptive switching scheme between PSK and QAM is introduced. The theoretical analysis for the index modulation systems proposed in this thesis has also been conducted.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, OFDM-IM's BER performance and spectral efficiency have been improved for the implementation of different sizes of constellation symbols. Two generalized schemes, OFDM-GIM1 and OFDM-GIM2, are proposed to achieve higher spectral efficiency and better BER performance for BPSK and QPSK, respectively. Interleaving is introduced to improve the BER performance of the proposed schemes in the low SNR region. Noting that OFDM-GIM2 has not reached its full potential, the joint I/Q index modulation for OFDM-GIM2 is also presented.

The OFDM-GIM1 and OFDM-GIM2 are later found to be compatible with each other and their combination greatly outperforms the existing works in spectral efficiency and BER performance, at the cost of a slightly increased complexity.

With the introduction of PSK, in which all the constellation symbols have the same distance from 0 for constellation sizes larger than QPSK, a higher index detection accuracy can be achieved. The theoretical analysis for the index modulation systems proposed in this thesis is conducted. The mathematical demonstration of the 8PSK's superiority over 8QAM in indices detection is also detailed.

Computer simulation results clearly show the proposed schemes' superiority in both spectral efficiency and BER performance compared to the existing works on OFDM-IM in the literature.

6.2 Future Work

Some future research directions that may extend the results presented in this thesis are given in the following subsections.

6.2.1 Further Generalizations

The following further generalizations are promising research directions.

- In [130], the authors proposed an upgraded version of OFDM-GIM1 where each subblock can use different constellation symbols as well as different numbers of active subcarriers to transmit information. The corresponding transmitter and receiver structures have also been proposed. However, [130] failed to incorporate OFDM-GIM2 to further generalize the proposed scheme. The design of the receiver to incorporate OFDM-GIM2 may be challenging. However, OFDM-GIM2 can significantly boost the spectral efficiency of the subblocks using QPSK.
- In OFDM-GIM1, flexibility is offered to the number of active subcarriers which results in the improvement of spectral efficiency. However, for all the index modulation schemes proposed throughout the thesis, the subblock size n is always assumed to be fixed. If flexibility is offered to n , the spectral efficiency can be further improved.
- In [131], Multiple-Input Multiple-Output OFDM with index modulation is proposed by combining MIMO and OFDM-IM transmission techniques as a possible alternative to classical MIMO-OFDM. In this scheme, each transmit

antenna transmits its own OFDM-IM frame to boost data rate and at the receiver, these frames are separated and demodulated. The introduction of MIMO transmission offers a new degree of flexibility, i.e., the selection of active antennas. Combining this flexibility with the flexibilities offered by OFDM-GIM1 and OFDM-GIM2 raises an interesting problem of the tradeoff between complexity, spectral efficiency and BER performance. In addition, the design of the corresponding detectors remains an open problem.

6.2.2 Optimal Selection Strategy for OFDM-GIM1

In [132], the optimal selection strategy of n and K for OFDM-IM is proposed to maximize the energy efficiency. For OFDM-GIM1, as the generalization offers more freedom of n and $\mathbf{K}$ selections, the optimal selection strategy to maximize energy efficiency, as in [132], is a promising research direction. Moreover, the target of the optimal selection strategy can be further extended to BER performance, spectral efficiency and the tradeoff between them.

In OFDM-GIM1, in terms of $\mathbf{K} = \{K_1, K_2, \dots, K_R\}$, the following factors should be taken into consideration,

- The largest number of subcarrier combinations $C_n^{K_r}$, where $r \in [1, \dots, R]$, is usually achieved when K_r is around $n/2$. The larger the number of subcarrier combinations $C_n^{K_r}$ produces, the higher it contributes to the overall spectral efficiency.
- The best BER performance is achieved when $K_r = 1$. The smaller K_r is, the more it helps to improve the overall BER performance.
- The distance between neighbouring elements in $\mathbf{K}$, given by $|K_r - K_{r+1}|$ where $r \in [1, \dots, R-1]$, also has an impact on the BER performance of the overall system. The larger the distance is, the better BER performance is.

6.2.3 Applications

Index modulation schemes have many unique features which are of great value to certain communication scenarios.

- As discussed in [133], Machine-to-machine communications require low power consumption, especially for devices that have no or limited access to power sources, wake only on demand or experience infrequent human interaction. In [132], it is revealed that OFDM-IM is much more energy efficient than classical OFDM.
- Visible light communications is a promising new technology for next generation wireless communications systems due to its advantages over radio frequency based systems. In [134], optical orthogonal frequency division multiplexing with index modulation for visible light communications systems have been proposed. Computer simulations show that a better BER is achieved compared to classical optical OFDM schemes.
- Vehicle-to-Vehicle and Vehicle-to-Infrastructure communication bring about stringent demand for robust and efficient communication techniques under high mobility. As shown in [11], OFDM-IM shows superiority over classical OFDM under high mobility.

The performance of OFDM-GIM1 and OFDM-GIM2 under such scenarios may also be studied as they offer more design flexibility compared to OFDM-IM.

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