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RESONATORS**

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STUDY ON DESIGNER SURFACE PLASMON RESONATORS

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For my family.

List of Acronyms

SPP.....	Surface plasmon polaritons
LSPs	Localized surface plasmons
EM.....	Electromagnetic
PEC	Perfect electric conductor
DSP	Designer surface plasmon
SCS	Scattering cross section
TM.....	Transverse magnetic
2D.....	Two dimensional
3D.....	Three dimensional
FWHM	Full width of half maximum
VNA.....	Vector network analyzer
CMT	Coupled mode theory
DLSP.....	Designer localized surface plasmon
1D.....	One-dimensional
TI.....	Topological insulator
TE.....	Transverse electric

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ABSTRACT

The thesis presents the functional and structural design of designer surface plasmon resonators, and studies novel physics induced by coupling between multiple resonators.

Chapter 1 introduce the basic properties of surface plasmon polaritons which is supported on the interface between dielectrics and metal in optical range, and how to mimic them with artificial plasmonic metamaterials in low frequencies.

Chapter 2 introduces a dispersion tuning method on designer surface plasmon resonators. As an example, high-order multipolar modes are enhanced, which are absent in previous studies. Experimental results successfully verify the existence of high-order modes.

Chapter 3 studies two horizontally coupled two-dimensional resonators. Our study reveals that, selective excitation of two modes of opposite parity can flip the sign of coupling strength between the pair of resonators. The two modes are degenerate multipolar modes with opposite parity. Near-field experiments verify this sign reversal phenomenon.

Chapter 4 studies two vertically stacked two-dimensional resonators. This vertical coupling can be strong enough to induce interference between multipolar modes of successive orders. Spectral minimums associated with asymmetrical line-shapes are observed in near-field transmission spectra. We then construct a vertical chain of these resonators, and find that

vertical coupling enables vertical transport of subwavelength surface electromagnetic modes.

Chapter 5 implement a two-dimensional electromagnetic topological structure with two-dimensional modified designer surface plasmonic resonators. With the help of structural flexibility of this artificial structure, various time-reversal-invariant defects are implemented to probe the limits of robustness of electromagnetic topological edge states. Experimental results show that although all defect are time-reversal-invariant, some of them can still break the topological protection, which are consistent with simulation results.

Chapter 6 extends a two-dimensional cylindrical resonator to a three-dimensional spherical resonator. Incorporating with effective media model, Mie theory is developed to predict the electromagnetic response of the designed spherical structure. Scattering experiments are conducted and verify the above prediction.

CHAPTER 1

INTRODUCTION

1.1 General properties of surface plasmon

Since Ritchie discovered surface plasmon polaritons (SPPs) when studying the energy loss of fast electrons passing through thin metal films [1], SPPs have attracted extensive research efforts [2-3]. It is recognized that electron density oscillations contribute to the existence of SPPs in optical range (shown in Fig. 1-1) [4]. This type of electromagnetic excitations is confined in the subwavelength region around the flat interface between metal and dielectric. Due to its excellent electromagnetic property, significant amount of investigations on SPPs are dedicated to applications for surface enhanced Raman spectroscopy (SERS) [5], nonlinear light generation [6] and subwavelength optical devices enabling the miniaturization of optical components [7-10].

Figure 1-1. schematic of propagating SPP. Bottom grey region represents metal, and its upper surface is air, red colour indicates the decay length of SPP. (Adopted from Ref. [4], © Wikipedia)

In parallel to the propagating SPPs on a metal surface, localized surface plasmons (LSPs) on a metal nanoparticle also attracts much attention [11-12]. The earliest application of LSPs can date back to the Lycurgus cup in 4th century A.D (as shown in Fig. 1-2), which appears red colour in transmission, while showing green in reflection [13]. This peculiar appearance is due to the presence of Au nanoparticles in the glass which show resonances around green wavelength region of the visible spectrum. Nowadays, metal nanoparticles have also been widely utilized in biosensors [14-15], nano antennas [16-17], active [18] or nonlinear processes [6], SERS [5] and subwavelength optical devices [7-10].

Figure 1-2. (a) Lycurgus cup with red transmitted light. (b) scanning electron microscopy (SEM) image of a metal nanocrystal in glass. (c) calculated absorption spectrum of a thin gold film (blue dots), and of 30-nm nanocrystals in water (red dots), measured spectrum (black dots). (Adopted from Ref. [13], © 2005 AIP, by S.A. Maier *et.al*)

1.2 Development of designer surface plasmon polariton

1.2.1 Designer surface plasmon on groove structured metal surface

From far-infrared to radio frequencies, metal surfaces often approximate as perfect electric conductors (PEC) can also support surface electromagnetic (EM) modes which are usually called Sommerfeld or Zenneck waves [19]. However, these low-frequency surface EM modes do not show as strong confinement in comparison to SPPs in optical range. To efficiently improve confinement of the low-frequency surface EM modes, Goubau [20], Harvey [21], and Mills and Maradudin [22] proposed implementation of metal surface structures, such as arrays of grooves.

Figure 1-3. (a) Schematic of perfect electric conductor (PEC) perforated with grooves. (b) equivalent two-layer metamaterial model. (Adopted from Ref. [26], © 2005 IOP, by Garcia-Vidal *et.al.*)

A simpler plasmonic metamaterials is a metal surface perforated with grooves, as shown in Fig. 1-4(a). A two-layer media model in Fig. 1-4(b) can be utilized to describe this material

[26]. The top layer with grooves is equivalent to a media with $\epsilon = \begin{bmatrix} \frac{d}{a} & 0 & 0 \\ 0 & \infty & 0 \\ 0 & 0 & \infty \end{bmatrix}$, $\mu = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{a}{d} & 0 \\ 0 & 0 & \frac{a}{d} \end{bmatrix}$ in Cartesian coordinate; and the bottom layer without grooves is still PEC.

Dispersion properties of surface EM mode is governed by $\frac{\sqrt{k_x^2 - k_0^2}}{k_0} = \frac{a}{d} \tan(k_0 h)$, where k_0 and k_x are wavevector in free space and wavevector of the surface mode respectively. By comparing this dispersion with the one of SPPs on real metal, a similar point is worth noticing. In SPP dispersion, ω tends to $\omega_p/\sqrt{2}$, while in designer SPP dispersion shown in Fig. 1-5, ω tends to the cavity resonance frequency $\pi c_0/2h$. With this groove structure, some applications have been proposed, such as terahertz focusing on periodically corrugated metal wires [27], and collimation of terahertz laser beams [28].

Figure 1-4. Dispersion relation of surface EM mode on a groove structure (Adopted from Ref. [26], © 2005 IOP, by Garcia-Vidal *et.al.*)

Systematic study shows that lateral width of the groove shows negligible effect on the dispersion of designer SPP, which means that impedance mismatch is negligible for two designer SPP waveguides with different lateral width [29]. Based on this property, power divider and concentrator of designer SPP have been proposed and experimentally demonstrated [30]. Further progress pushed the lateral width of waveguide to several micrometers shown in Fig. 1-6, and realized with standard printing circuit board technology [31]. This ultrathin designer SPP waveguide is conformal and flexible to curved surfaces, and can mold the flow of designer SPP.

Figure 1-5. Dispersion of ultrathin designer SPP waveguide with different thickness t . (Adopted from Ref. [31], by Shen *et.al.*)

1.2.2 Designer surface plasmon on surface perforated with holes

Figure 1-6. Schematic of perfect electric conductor (PEC) perforated with subwavelength holes

(Adopted from Ref. [23], © 2004 AAAS, by Pendry *et.al.*)

From viewpoint of metamaterials, Pendry [23] investigated the holey structured metal surface in Fig. 1-3 with effective media theory, and termed the low-frequency surface EM mode as designer (or spoof) surface plasmons (DSP). Based on the assumption that the lowest order (TE₁₀) waveguide mode dominates the subwavelength holes, the effective dielectric functions of this artificial media can be described with local and frequency-dependent permittivity and permeability which are of plasma forms. Later, a more general analysis has been given by de Abajo [24], who generalized Pendry's prediction so that effective medium theory is not limited to holes filled with high index dielectrics. In 2005, Hibbins *et.al* [2005] demonstrated the plasmon-like dispersion of subwavelength square

grid in microwave range. Three years later, Garcia-Vidal .et.al. [19] verified the guiding property of SPPs on holy structured metal surface in terahertz frequency.

1.3 Current state of research on designer localized surface plasmon

Besides the propagating property of designer SPP on a flat groove pattern, also the localization property of designer LSP has been studied on closed surfaces with groove patterns shown in Fig. 1-7(a) [32]. With effective media theory, it can be equivalent to

metamaterial in Fig. 1-7(b) described with constitutive parameter $\epsilon = \begin{bmatrix} \infty & 0 & 0 \\ 0 & \frac{d}{a} & 0 \\ 0 & 0 & \infty \end{bmatrix}$, $\mu = \begin{bmatrix} \frac{a}{d} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{a}{d} \end{bmatrix}$ in cylindrical coordinate (r, θ, z) . Furthermore, its scattering cross section (SCS)

spectrum is the same with a Drude metal cylinder with radius R , and transvers magnetic (TM) multipolar resonance modes are verified [32]. Therefore, this textured PEC cylinder can be utilized to emulate LSP. Field enhancement property has also been investigated.

Figure 1-7. (a) schematic of closed PEC surface textured with grooves. (b) equivalent core-shell metamaterial model. (Adopted from Ref. [32], © 2012 APS, by Pors *et.al.*)

Figure 1-8. Leftmost is the structure fabricated with printed circuit board technology. The middle figure is simulated scattering cross section. The rightmost figure is the simulated E_z field pattern of dipole mode (Adopted from Ref. [33], © 2013 Wiley, by Shen *et.al.*)

To experimentally demonstrate the proposal above, the thickness in z direction has been reduced to several micrometers shown in Fig. 1-8 [33]. Due to the finite thickness, pure TM modes are not supported on this DSP resonator. For the purpose of characterizing the multipolar modes, z components of electric fields are measured. Electric dipole mode is observed as the lowest order mode. As an alternative method to make the resonator significantly smaller than the working wavelength, spiral arms are used in the disk instead of straight arms, as shown in the inset of Fig. 1-9, and magnetic dipole comes out [34].

Figure 1-9. Extinction cross section of the spiral structure shown in the inset (Adopted from Ref. [34],
© 2014 APS, by Huidobro *et.al.*)

1.4 Organization of the thesis

Although a single designer LSP has been well studied in both theory and experiment, there are still many open questions deserving investigation. For example, from the functional viewpoint, tuning the EM response of DSP resonator in real time is very useful for future device design. Reversible tuning methods for DSP resonators have not been realized yet. In addition, the physics of coupling between two neighboring resonators need to be established before building more complex arrays. Only after these basic issues are addressed, can we further understand the applicability of designer LSPs. From the structural

viewpoint, DSP resonators studied currently are all in two dimension. Although the practical ultrathin DSP resonators are three dimensional structures, modes profiles still follow cylindrical symmetries. Then, we extend DSP resonators to spherical cases, which can support spherical mode profiles. However, due to structural design and fabrication difficulties, spherical DSP resonators have not been realized.

The thesis are organized as three parts.

Part 1 includes Chapter 1 which provides a general introduction of the background of this thesis work. Previous efforts on DSP resonators are introduced. Current development progress is summarized.

Part 2 discusses two-dimensional (2D) DSP resonators from Chapter 2 to Chapter 5. Chapter 2, from the functional viewpoint, discusses the tunability of a single DSP resonator. An additional ground plane is introduced beneath the original DSP resonator. Higher-order modes are enhanced and experimentally observed. In Chapter 3, horizontal coupling between two DSP resonators are studied. The two DSP resonators are placed in the same plane. For two degenerate modes of the same order, parities of them determines the signs of the coupling strength, which is validated in near-field experiment by changing excitation position. In Chapter 4, vertical coupling between two resonators are investigated. The two resonators are stacked in a vertical configuration. Strong-coupling-induced interference between two successive multipolar modes is observed. With near-field detection,

transmission dips associated with Fano-like line-shapes are observed. Chapter 5 discusses topological properties of a square DSP ring resonators array. Various time-reversal-invariant defect rings are implemented in the edge of this array. Results show that photonic topological protection can be broken with time-reversal-invariant defects, unlike fermion topological systems.

Part 3 includes Chapter 6, which develops DSP resonators from the structural viewpoint. Dimensionality of DSP are extended from cylindrical (2D) to spherical (three-dimensional: 3D). Scattering cross section (SCS) of the spherical DSP resonator are calculated with Mie theory by incorporating with effective media theory. Resonance peaks in SCS spectrum are verified with far-field experiments.

CHAPTER 2

Dispersion-controllable designer surface plasmonic resonator

Recent investigations disclosed that a structured metallic surface could be considered as a designer-surface-plasmonic (DSP) resonator supporting designer localized surface plasmons [32-34], which is very sensitive to its environmental dielectric properties as a plasmonic sensor [33]. Further experimental and numerical evidences [33] show that a DSP resonator can support multipolar modes and higher-order modes show higher sensitivity. However, DSP resonators studied previously have fixed dispersive property without tunability, which leads to the difficulty in further increasing the number of observable high-order multipolar resonance modes in experiment. When the same resonator is needed to be used in different requirements, both reversibility and flexibility [35-40] are indispensable. An external non-destructive stimulus is desirable to control the EM properties of a DSP resonator with fabricated geometrical structure.

In this chapter, a tunable ultrathin DSP resonator are proposed by uniting a planar DSP resonator with a movable ground plane. By closely laying an external ground metallic plane underneath, we can tune the dispersion of an ultrathin single-period DSP structure with approximately zero thickness. By decreasing the gap between the DSP resonator and the metal ground plane, the dispersion curve tends to approach a lower frequency, and becomes more flat than that of the original bare DSP resonator without the metal ground plane, indicating a better confinement. Based on the analysis above, we can propose an ultrathin

dispersion-tunable DSP resonator by tuning the distance between the DSP resonator and the metal ground plane. Experiments are performed in microwave frequency range to verify the property of the ultrathin dispersion-tunable DSP resonator.

2.1 Dispersion tuning principle

We first investigate the way that the dispersion of an ultrathin single-period DSP structure changes while placing a metal ground plane beneath. Fig. 2-1(a) shows schematic of a unit cell, where periodic subwavelength grooves are fabricated on an ultrathin metallic strip. The groove width and periodicity refer to a and d , respectively. The groove depth is represented with r , and the whole structure height is R . The separation between the planar DSP structure and the metal ground plane refer to h , and dielectric material with relative permittivity 2.2 fulfills the region in between them. The thickness of the metal parts for both the DSP resonator and the metal ground plane is $18\text{ }\mu\text{m}$, which are not specifically denoted. The parameters a , d , R , and r are set to be 0.628 mm , 1.256 mm , 12 mm , and 9.0 mm , respectively. The distance h between the planar DSP structure and the metal ground plane are tunable, and they are selected as 0.125 mm , 0.254 mm , 0.508 mm , 0.762 mm , and 1.016 mm , respectively. The thickness of dielectric layer in DSP structure without a metal ground plane is fixed to be 0.254 mm . Fig. 2-1(b) presents the dispersion curves, in which the black dashed line represents the light line in free space, while the grey, blue, green, red, and purple lines correspond to the planar DSP structure with the metal ground plane below when $h = 1.016\text{ mm}$, 0.762 mm , 0.508 mm , 0.254 mm , and 0.125 mm , respectively. It is

worth noting that the black solid curve, which is from the planar DSP structure without the metal ground plane, is above those from all samples with metal ground plane, implying a weaker confinement.

It can be observed that several changes in dispersion curves are induced after the metal ground plane is introduced below the planar DSP structure, through comparing the red and black lines (the dielectric materials still keep the same thickness for both samples). The most striking change is that ω_p (the asymptotic frequency) downshifts after the metal ground plane is placed below. Secondly, the dispersion curve deviates from the light line further than that without the metal ground plane, which implies less radiative DSP modes. Thirdly, near the asymptotic frequency, the dispersion curve of the DSP structure with metal ground plane below slowly goes flat in a large wavevector range, so its group velocity remains finite, rather than suddenly becoming flat as in the structure of planar DSP structure without the metal ground plane. Otherwise we can explain it like that, if we define a cutoff wavevector $k_{\max}$ where the dispersion curve has reached its ω_p , the $k_{\max}$ in the DSP structure with the metal ground plane is larger than its counterpart without the metal ground plane. Fourthly, another change is shown in their eigen mode patterns in Fig. 2-1(c) and (d). In the planar DSP structure with the metal ground plane, the most of electric field is confined in the dielectric layer between upper planar DSP structure and the lower metal ground plane, while in the structure of a planar DSP structure on a dielectric substrate without the metal ground plane, most part of electric field is distributed in both lower and upper ambient space. This tighter field confinement by the help of the metal ground plane indicates a less

radiative DSP mode, which is consistent with the analysis on dispersion curves in Fig. 2-1(b). Furthermore, we can further tune the dispersion of the planar DSP structure by gradually changing the distance h from the planar DSP structure to the metal ground plane. In Fig. 2-1(b), it can be observed that the asymptotic frequency is downshifted with decreasing h .

Figure 2-1. (a) Illustration of a unit cell for a planar DSP structure with a metal ground plane below. $R = 12$ mm, $r = 9.0$ mm, $d = 1.256$ mm, and $a = 0.628$ mm. The separation between the planar DSP structure and the metal ground plane is full of dielectric material with $\epsilon_r = 2.2$. (b) Numerically obtained dispersion relation of the planar DSP structure with the metal ground plane as h varies. The dispersion curve of a DSP structure without the metal ground plane is also shown for $h = 0.254$ mm. (c-d) Eigen mode profiles of the planar DSP structure (c) without and (d) with the metal ground plane. The part enclosed with dashed black lines is the dielectric layer.

After understanding the variations of DSP dispersion by introducing an extra metal ground plane, we continue to design and experimentally demonstrate a DSP resonator with tunable dispersion and enhanced high-order resonances modes. In the following, we show the design as well as measurement principles. The asymptotic frequency of a planar DSP waveguide can be kept in a DSP resonator. To form a resonance mode in a circular resonator, a basic requirement to be met is $kL = 2n\pi$, where n is a positive integer, and L is the perimeter of the resonator. The field confinement, as revealed in Fig. 2-1(b) by comparing the deviations of each dispersion curve from the light line, can be quantified by quality factor (Q) of each resonance peak or its full width of half maximum (FWHM). As metallic loss is ignorable at microwave frequencies, radiation dominates the dissipation of a resonator. Close to the asymptotic frequency, the cutoff wavenumber $k_{\max}$ determined by the slope change of dispersion curves can determine how many resonance modes are supported in a DSP resonator, since resonances can only be formed for modes with wavevectors k smaller than $k_{\max}$.

2.2 Resonance frequency shift of designer surface plasmon resonator

Figure 2-2. Photo of near-field Experiment setup

Next, we experimentally verify this dispersion-tunable DSP resonator with a metal ground plane below. Fig. 2-2 shows the photo of the experiment setup. Near-field source monopole is below the sample, and connected to the Vector Network Analyzer (VNA). Probe monopole is also connected with VNA, and mounted on the translational scanner. Fig. 2-3(a) shows the photo of a fabricated resonator, where $a = 0.628$ mm, $d = 1.256$ mm, the depth of groove $r = 9.0$ mm, and the outer radius is $R = 12$ mm. The dielectric support is Rogers RT 5880 with relative permittivity 2.2 and loss tangent 0.0009. A metal plate is attached on the back side of the dielectric substrate. In the measurement, a monopole antenna is utilized as a near-field excitation source, and the other as a detector [33]. Fig. 2-3(a) shows their relative positions. The measured transmission (S21) spectra of resonators

with $h = 1.016$ mm, 0.762 mm, 0.508 mm, 0.254 mm are given in Fig. 2(b). From the measured transmission spectra, several conclusions can be made. Firstly, the black dashed line in Fig. 2-3(b) denotes variation of the cutoff frequency, or the asymptotic frequency for the resonances, which increases with dielectric thickness h increases. Secondly, all resonance peaks below cutoff frequency blue shift and their FWHMs becomes larger with h increases. Fig. 2-3(c) and (d) show the variation of frequencies and Q factors of two typical resonance peaks P1 and P8 as h increases. These results experimentally prove the analysis upon dispersion in Fig. 2-1(b).

Figure 2-3. (a) Top view of an ultrathin DSP resonator with a metal ground plane at the bottom side. (b) Detected transmission spectrum at various separations from the resonator to the metal ground plane for $h = 1.016$ mm, 0.762 mm, 0.508 mm, and 0.254 mm. (c) and (d) Measured frequency shift and Q factor variation of the two selected resonances, P1 and P8, respectively, as h increases.

Figure 2-4. Field patterns of several resonance modes from the lowest order (dipole mode; P1) to the thirteenth order (P13) at (a) 3.43 GHz, (b) 5.61 GHz, (c) 5.83 GHz, (d) 5.92 GHz, (e) 5.99 GHz, and (f) 6.011 GHz, respectively.

Now we discuss the number of detected resonance modes. 13 ambiguous resonance peaks are observed in the measured transmission spectra. Especially, the high-order resonance modes P10, P11, P12 and P13 which were missing in previously reported planar DSP resonator [33] now are clearly observed. To clearly exhibit these resonance modes, we employ a microwave near-field scanning system to grab the spatial profile of E_z field with near-field excitation. The source monopole keeps at rest while the probe monopole moves horizontally in the XY plane which is 1.0 mm above the DSP resonator. Field profiles at

3.43 GHz, 5.61 GHz, 5.83 GHz, 5.92 GHz, 5.99 GHz, and 6.01 GHz are given in Figs. 2-4(a-f), respectively. Fig. 2-4(a) and Fig. 2-4(b) show the profiles of fundamental mode (P1) and the eighth-order mode (P8), which correspond to P1 and P8 in Fig. 2-3(b). The other field profiles of P10, P11, P12, and P13 (Fig. 2-4(c-f)) which were absent in previous reports [33], but observed clearly here, verify the high-order mode numbers. Blue and red colors represent negative and positive field respectively. The excitation source induces the brightest spot on the rightmost side in each mode field profile. Wavelength of the pattern far away from the source is slightly smaller than that near the source. This asymmetry is due to the tilted ground plane caused by the source monopole.

2.3 DSP resonator without ground plane

For the purpose of completeness, we also study the effect of varying the thickness of dielectric layer of a planar DSP resonator without a metal ground plane, shown in Fig. 2-5. When we gradually increase the substrate thickness from 0.254 mm to 1.016 mm, cutoff frequencies of dispersion curves shown in Fig 2-5(a) are downshifted, which exhibits opposite trend with respect to that in Fig. 2-1(b) and Fig. 2-3(b). In addition, dispersion curves further deviate away from the light line as h increases, which implies less radiative loss. In the DSP resonator measurement (Fig. 2-5(b)), spectrum of measured transmission show less resonance peaks than the resonator of the identical structure but with a metal ground plane. The reason is that the cutoff wavenumber in the dispersion curve is much smaller than that of the resonator with a metal ground plane.

Figure 2-5. (a) Simulated dispersion curves of planar DSP structure without the metal ground plane when the substrate thickness changes. $h = 1.016$ mm, 0.762 mm, 0.508 mm, and 0.254 mm. (b) Detected transmission spectrum of planar DSP structure without the metal ground plane with various substrate thickness. The resonator geometry is the same as that shown in Fig. 2-3(a) without the metal ground plane.

2.4 Conclusion

In conclusion, we have designed and experimentally verified a tuning method of planar DSP structures with a movable ground plane. The function of this metal ground plane can give rise to more resonance modes that are not observed in previous similar DSP resonators without a metal ground plane. By tuning the separation from the planar resonator to the ground plane, frequencies and Q factors of all resonance modes can be tuned efficiently. This approach could be applicable in future tunable DSP devices.

CHAPTER 3

Sign reversal of coupling strength in horizontal coupling

Since about two decades ago coupled resonators optical waveguides (CROW) have been firstly proposed by Yariv [44], extensive experimental investigation based on photonic crystals and dielectric resonators were conducted, due to their potential application in optical delay lines, reflectionless bends, slow waves or second harmonic generations [46-58]. Inherent dispersion relationship of this novel waveguide can be simply characterized by a coupling coefficient κ , [44] which is determined by near-field coupling between adjacent resonators.

This near-field coupling scheme has also been introduced into plasmonic systems. Coupling between plasmonic particles can induce rich physics, such as mode hybridizations [12], Fano resonances [41], electromagnetically-induced-transparency (EIT) [42], superscattering [43] .et.al. Among all of them, coupling strength κ plays an important role. This physical quantity is determined by several factors, such as geometries of resonators, relative positions between resonators, and excited modes inside resonators. It has been numerically shown that parity of excited multipolar modes can determine the sign of coupling strength between two dielectric cylindrical resonators [44-45]. Here, we will employ DSP resonators to demonstrate this phenomenon with directly imaging mode patterns. For simplification, cylindrical resonators, which are usually placed parallel in the same plane, allow us to only consider the effects of modes.

3.1 Theoretical analysis

A CROW is an optical waveguide which consists of a chain of coupled high- Q factor resonators. Here, we focus on a CROW assembled with dielectric resonators supporting multipolar modes, among which two resonators are shown in Fig. 3-1. In the tight-binding approximation, eigenmode $\mathbf{E}_K(\mathbf{r}, t)$ of a CROW is taken as a linear combination of the high- Q modes of $\mathbf{E}_\Omega(\mathbf{r})$ of the individual resonators along a straight line parallel to the x axis (see Fig. 3-1). Denoting the coordinate of the centre of the n th resonator as $x = na$, where a represents the centre-to-centre distance of two adjacent resonators, we have $\mathbf{E}_K(\mathbf{r}, t) = E_0 \exp(i\omega_K t) \sum_n \exp(-inKa) \times \mathbf{E}_\Omega(\mathbf{r} - na\hat{x})$. $\mathbf{E}_K(\mathbf{r}, t)$ satisfies the Maxwell equations, which leads to $\nabla \times (\nabla \times \mathbf{E}_K) = \varepsilon(\mathbf{r}) \frac{\omega_K^2}{c^2} \mathbf{E}_K$, where $\varepsilon(\mathbf{r})$ is the dielectric constant of the system, and ω_K is eigenfrequency of the waveguide mode. Similarly, $\mathbf{E}_\Omega(\mathbf{r})$ satisfies Maxwell equations but with $\varepsilon(\mathbf{r})$ replaced with $\varepsilon_0(\mathbf{r})$, the dielectric constant of the single resonator, and ω_K replaced with Ω . The normalization process should follow $\int dr^2 \varepsilon_0(\mathbf{r}) E_\Omega^*(\mathbf{r}) \cdot E_\Omega(\mathbf{r}) = 1$. However, in the tight binding approximation, where evanescent coupling is dominant and radiation coupling is negligible, Yariv approximately took $E_\Omega(\mathbf{r})$ to be real, and reduced normalization process to be $\int dr^2 \varepsilon_0(\mathbf{r}) E_\Omega(\mathbf{r}) \cdot E_\Omega(\mathbf{r}) = 1$. After mathematic manipulation, Yariv *et al.* in Ref. 44 has obtained the dispersion relation of the CROW $\omega_K = \Omega[1 + \kappa \cos(Ka)]$. The coupling factor κ between adjacent dielectric resonators can be expressed as follow,

$$\kappa = \int dr^2 [\varepsilon_0(\mathbf{r} - a\hat{x}) - \varepsilon(\mathbf{r} - a\hat{x})] \mathbf{E}_\Omega(\mathbf{r}) \cdot \mathbf{E}_\Omega(\mathbf{r} - a\hat{x}) \quad (3.1)$$

where $\varepsilon_0(\mathbf{r} - a\hat{x}) = \begin{cases} \varepsilon_m; & (|\mathbf{r} - a\hat{x}| < R) \\ \varepsilon_0; & (|\mathbf{r} - a\hat{x}| > R) \end{cases}$ is the relative dielectric constant function of a single resonator, $\varepsilon(\mathbf{r} - a\hat{x}) = \begin{cases} \varepsilon_m; & (|\mathbf{r} - na\hat{x}| < R) \\ \varepsilon_0; & \text{otherwise} \end{cases}; n \in \mathbb{Z}$, represents the relative dielectric constant function of the CROW. In the following we will show the sign reversal of coupling factors for two different coupling configurations with tight binding approximation.

3.1.1 “pole-pole” coupling

Defining the x -axis as in Fig. 3-1A and setting the radius of two identical resonators as R , the multipole resonance mode profile of each resonator can be expressed as $E = B_m(kr)\cos(m\varphi)$, where m is the order of multipole resonance mode, and (r, φ) are the polar coordinates with the center of each resonator as the origin. $B_m(kr) = \begin{cases} J_m(k_d r), & r < R \\ \frac{J_m(k_d R)}{H_m^1(ik_0 R)} H_m^{(1)}(ik_0 r), & r > R \end{cases}$, where $J_m(\cdot)$ is Bessel function, $H_m^1(\cdot)$ is Hankel function of the first-kind, and k_d and k_0 are wavevectors in resonators and the environment, respectively. The coupling factor κ is a mathematical expression that depends on the integration of overlapped resonance wave fields of the two resonators [44]. This kind of “pole-pole” coupling is understandable, because the two intensity maxima facing each other must induce significant coupling between the two resonators.

Figure 3-1. “Pole-pole” coupling configuration with overlapped intensity maxima between resonators.

For the “pole-pole” coupling configuration as shown in Fig. 3-1, the resonance field profile in the left resonator, taking its center as the origin, can be written as,

$$E_1(\vec{r}) = \begin{cases} AJ_m(k_d r_1) \cdot \cos(m\phi_1) & (r_1 < R) \\ AC_m^1 \cdot H_m^{(1)}(ik_0 r_1) \cdot \cos(m\phi_1) & (r_1 > R) \end{cases} ; \quad (3.2)$$

where $C_m^1 = \frac{J_m(k_d R)}{H_m^{(1)}(ik_0 R)}$, and A is a constant for normalization. Similarly, the resonance field profile in the right resonator that is translated by a distance of a in the x direction can be written as:

$$E_2(\vec{r} - a\hat{x}) = \begin{cases} AJ_m(k_d r_2) \cdot \cos(m\phi_2) & (r_2 < R) \\ AC_m^1 \cdot H_m^{(1)}(ik_0 r_2) \cdot \cos(m\phi_2) & (r_2 > R) \end{cases} ; \quad (3.3)$$

We then substitute Eqs. (3.2-3.3) into Eq. (3.1). We can consider only the overlap field from the nearest resonator, because of the tight-binding nature of localized resonance modes. As a result,

$$\kappa = (\varepsilon_0 - \varepsilon_m)A^2 \int_{r_1 < R} dr^2 C_m^1 H_m^{(1)}(ik_0 r_2) \cos(m\phi_2) J_m(k_d r_1) \cos(m\phi_1) \quad (3.4)$$

Since r_1 and r_2 are with different sets of polar coordinates, we apply the addition theorem [59] to unify them with one set of polar coordinates. This gives,

$$\begin{aligned} \kappa = (\varepsilon_0 - \varepsilon_m)A^2 \int_0^R dr_1 C_m^1 \sum_{n=-\infty}^{+\infty} H_{n-m}^{(1)}(ik_0 a) J_n(ik_0 r_1) J_m(k_d r_1) \cdot \\ \int_0^{2\pi} \cos(n\phi_1) \cos(m\phi_1) d\phi_1 \end{aligned} \quad (3.5)$$

Only when $n = \pm m$ can the integral be nonzero. Thus,

$$\begin{aligned} \kappa = (\varepsilon_0 - \varepsilon_m)\pi A^2 \int_0^R r_1 dr_1 \left\{ C_m^1 [H_0^{(1)}(ik_0 a) \cdot J_m(ik_0 r_1) \cdot J_m(k_d r_1) + H_{-2m}^{(1)}(ik_0 a) \right. \\ \left. \cdot J_{-m}(ik_0 r_1) \cdot J_m(k_d r_1)] \right\} \\ = (\varepsilon_0 - \varepsilon_m)\pi A^2 \int_0^R r_1 dr_1 \left\{ \frac{J_m(k_d R)}{K_m(k_0 R)} [i^{2m} K_0(k_0 a) + i^{2m} K_{2m}(k_0 a)] I_m(k_0 r_1) J_m(k_d r_1) \right\} \end{aligned} \quad (3.6)$$

Only when $0 < k_0 a < \tau_c$, where $\frac{K_{2m}(\tau_c)}{K_0(\tau_c)} = 10$, it satisfies $K_{2m}(k_0 a) \gg K_0(k_0 a)$ for tightly localized resonance modes, we can approximately obtain

$$\kappa \approx (\varepsilon_0 - \varepsilon_m) \pi A^2 \int_0^R r_1 dr_1 \cdot (i)^{2m} \cdot \frac{J_m(k_d R)}{K_m(k_0 R)} \cdot K_{2m}(k_0 a) \cdot I_m(k_0 r_1) \cdot J_m(k_d r_1) \quad (3.7)$$

And, when the mode order m is larger, τ_c becomes larger. For the tight binding model, κ should approximately take the real part of above expression.

3.1.2 “Node-node” coupling

Figure 3-2. “Node-node” coupling configuration with overlapped intensity maxima between resonators.

For the “node-node” coupling configuration as shown in Fig. 3-2, the field of the left resonator can be written as:

$$E_1(\vec{r}) = \begin{cases} AJ_m(k_d r_1) \cdot \sin(m\phi_1) & (r_1 < R) \\ AC_m^1 \cdot H_m^{(1)}(ik_0 r_1) \cdot \sin(m\phi_1) & (r_1 > R) \end{cases}; \quad (3.8)$$

The field of the right resonator can be written as:

$$E_2(\vec{r} - a\hat{x}) = \begin{cases} AJ_m(k_d r_2) \cdot \sin(m\phi_2) & (r_2 < R) \\ AC_m^1 \cdot H_m^{(1)}(ik_0 r_2) \cdot \sin(m\phi_2) & (r_2 > R) \end{cases}; \quad (3.9)$$

Substituting Eq. (3.8) and Eq. (3.9) into Eq. (3.1) and performing derivations similar to last session, we can get

$$\begin{aligned} \kappa &= (\varepsilon_0 - \varepsilon_m) A^2 \int_0^R dr_1 C_m^1 \sum_{n=-\infty}^{+\infty} H_{n-m}^{(1)}(ik_0 a) J_n(ik_0 r_1) J_m(k_d r_1) \cdot \\ &\int_0^{2\pi} \sin(n\phi_1) \sin(m\phi_1) d\phi_1 \end{aligned} \quad (3.10)$$

Only when $n = \pm m$ can the integral be nonzero. Thus,

$$\begin{aligned} \kappa &= (\varepsilon_0 - \varepsilon_m) \pi A^2 \int_0^R r_1 dr_1 \left\{ C_m^1 [H_0^{(1)}(ik_0 a) \cdot J_m(ik_0 r_1) \cdot J_m(k_d r_1) - H_{-2m}^{(1)}(ik_0 a) \cdot \right. \\ &\left. J_{-m}(ik_0 r_1) \cdot J_m(k_d r_1)] \right\} \\ &= (\varepsilon_0 - \varepsilon_m) \pi A^2 \int_0^R r_1 dr_1 \left\{ \frac{J_m(k_d R)}{K_m(k_0 R)} [i^{2m} K_0(k_0 a) - i^{2m} K_{2m}(k_0 a)] I_m(k_0 r_1) J_m(k_d r_1) \right\} \end{aligned} \quad (3.11)$$

Since, $K_{2m}(k_0 a) \gg K_0(k_0 a)$ for tightly localized resonance modes, we can approximately obtain

$$\kappa \approx -(\varepsilon_0 - \varepsilon_m)\pi A^2 \int_0^R r_1 dr_1 (i)^{2m} \frac{J_m(k_d R)}{K_m(k_0 R)} K_{2m}(k_0 a) I_m(k_0 r_1) J_m(k_d r_1) \quad (3.12)$$

Similar to the arguments in the pole-pole coupling, for the tight binding model, κ should approximately take the real part of above expression. Note that κ in Eq. (3.12) has the same magnitude but opposite sign compared to that in Eq. (3.7). This shows the sign reversal of coupling factors in the two different coupling configurations.

3.2 Retrieval process of κ

Before demonstration of the theoretical results above, we develop a set of method to extract κ from measured results. By employing coupled mode theory (CMT) [60-61], the two coupled resonator system with tight binding approximation can be described as following

$$\begin{cases} -\frac{da_1}{dt} = i\omega_0 a_1 + i\kappa\omega_0 a_2 \\ -\frac{da_2}{dt} = i\omega_0 a_2 + i\kappa\omega_0 a_1 \end{cases} \quad (3.13)$$

where a_1 and a_2 represent resonance wave fields in the two resonators, and ω_0 denotes the intrinsic resonance frequency of a multipole resonance mode in a single resonator. By solving this eigenvalue problem in Eq. (3.13), we obtain two orthogonal eigen solutions of $[a_1 \ a_2]^T$ as $[1 \ -1]^T$ and $[1 \ 1]^T$, in which the former corresponds to the out-of-phase (a_1 and a_2 differ by a phase of π) coupled mode with eigen frequency $\omega_{1,-1} = \omega_0 - \kappa\omega_0$, and the latter to the in-phase (a_1 and a_2 have the same phase) coupled mode with eigen

frequency $\omega_{1,1} = \omega_0 + \kappa\omega_0$. It can be clearly seen that the magnitude of coupling factor κ determines the frequency difference in mode splitting. As a result, if we can experimentally obtain ω_0 , $\omega_{1,-1}$, and $\omega_{1,1}$, coupling factor can be retrieved with

$$\kappa = (\omega_{1,1} - \omega_{1,-1}) / (2\omega_0) \quad (3.14)$$

3.3 Demonstration with designer surface plasmon resonators

We then characterized intrinsic resonance frequencies of a single resonator which is the same as that used in chapter 2. The transmission spectrum for a single resonator is measured by placing the source and probe at opposite sides of the resonator. Three resonance modes in a single resonator at 5.34 GHz, 6.07 GHz, and 6.44 GHz can be seen in Fig. 3-3A. They correspond to the quadrupole (labeled as “Q”), hexapole (labeled as “H”), and octopole (labeled as “O”) modes.

Figure 3-3. Mode splitting of two coupled resonators. (A) The measured near-field transmission spectra through a coupled-resonator dimer with end and side excitations. Inset: photo of the two designer-surface-plasmon resonators with inter-resonator distance $\Delta = 30$ mm. (B) The measured resonance wave patterns. The position of source is indicated by a red arrow.

Then we study two horizontally coupled DSP resonators. As mentioned above, signs of coupling strength κ depend on parities of multipolar modes. To control the parity of multipolar modes, we utilize point excitations. During studying a single resonator, it is found that poles always come out near the point source. It can be explained that EM waves emit from the monopole source can split into two parts which travels clockwise and anti-clockwise respectively. The two different circulating waves interfere in the DSP resonator, and form stable multipolar modes. Firstly, to implement the “pole-pole” coupling, we locate the source and the probe in symmetric positions at two ends of the dimer, as indicated by a pair of red dots in the inset of Fig. 3-3A. We call this configuration as “end excitation”. When the coupled-resonator dimer is excited with end excitation, each of the three

resonance modes in a single resonator splits into two supermodes: from one mode at $Q = 5.34$ GHz to two modes at $Q_\pi = 5.19$ GHz and $Q_0 = 5.49$ GHz, from one at $H = 6.07$ GHz to two at $H_0 = 6.03$ GHz and $H_\pi = 6.11$ GHz, and from one at $O = 6.44$ GHz to two at $O_\pi = 6.42$ GHz and $O_0 = 6.47$ GHz. Here the subscript “ π ” or “0” denotes out-of-phase or in-phase relation for the two resonators after mode splitting. To explicitly show the phase relation, we measure the resonance wave pattern for each resonance mode by scanning the probe in a transverse plane 1 mm above the coupled-resonator dimer, as shown in Fig. 3-3B. The source position is indicated by a red arrow. We observe that for the coupled quadrupole and octopole resonance modes, the two resonators are out-of-phase at lower resonance frequencies (Q_π and O_π), and in phase at higher resonance frequencies (Q_0 and O_0). However, the situation is reversed for the hexapole mode: the two resonators are in-phase at the lower resonance frequency (H_0), and out-of-phase at the higher resonance frequency (H_π). We obtain coupling strengths with end excitation, $\kappa_Q = 0.0281$, $\kappa_H = -0.0066$, and $\kappa_O = 0.0039$ for the coupled quadrupole, hexapole, and octopole resonance modes, respectively. The different phase relation of two coupled resonators for the coupled hexapole resonance mode is simply because the hexapole mode is an odd mode (mode order $m = 3$) while both the quadrupole (mode order $m = 2$) and the octopole mode (mode order $m = 4$) are even modes.

Secondly, the “node-node” coupling is implemented by placing the source and the probe on one side of the two resonators, as indicated by a pair of blue dots in the inset of Fig. 3-3A. We call this configuration as “side excitation.” For the quadrupole and octopole modes,

because of their four-fold rotational symmetry, the resonance wave patterns for their split modes are almost the same with those in the end excitation. Here we only show the result of hexapole mode. As shown in Fig. 3-3b, we measure the resonance wave patterns for the two split hexapole modes ($H_{\pi}^* = 6.03$ GHz and $H_0^* = 6.11$ GHz) in the coupled-resonator dimer, where the source position is indicated by a red arrow. The retrieved coupling strength is $\kappa_H^* = 0.0066$, which shows reversed sign with κ_H .

3.4 Conclusions

In this chapter, we have presented an analysis of sign reversal of coupling strength κ by switching two degenerate multipolar modes, and then demonstrated with designer surface plasmonic resonators. The two degenerate multipolar modes can be selectively excited with different excitation positions.

CHAPTER 4

Near-field energy transport induced by vertical coupling

The strong electromagnetic coupling between two plasmonic nanoparticles has recently been revisited with transformation optics from an alternative viewpoint adopted in a transformed space [62]. Some novel phenomena have been predicted, such as the minimal cross section area (“invisibility dip”) in scattering spectrum with asymmetric line shapes, being similar to Fano resonance [62]. However, this is fundamentally different from Fano resonance, since Fano resonance requires coupling between bright and dark modes [41, 63-64], while the spectral minimal in scattering spectrum as predicted by transformation optics arises from destructive interference between successive modes [62]. This novel mechanism arising from mode interference but different from Fano resonance deserves a deeper exploration and experimental verification.

The original transformation-optics prediction requires an extremely narrow gap to facilitate the strong coupling between the two nanoparticles [62]. However, in experiments, fabrication imperfections of tiny geometrical features and nonlocal/quantum effects [65-68] at nanometric separations can fail the experimental observation of this effect. To tackle these challenges, it is possible to demonstrate this “invisibility dip” prediction with designer-localized-surface-plasmon (DLSP) resonators [32-34, 69] at a macroscopic scale in microwave regime. Previous experiments generally focused on the interaction between DLSP particles in the transverse arrangement [70]. In this case, the strong confinement of

DLSP modes at extremely small gaps can invalidate the effective medium description of these metamaterial structures, severely weakening the coupling between the two DLSP resonators [71]. Therefore, the predicted spectral minimal with Fano-like asymmetric line-shape (which requires strongly interaction between the two plasmonic particles) has not been observed.

In this chapter, we investigate the strong coupling between two DLSP resonators stacked vertically. Instead of observing in the far field, we probe the near-field energy transport [72] through two resonators. The vertical coupling configuration can reserve a sufficient gap between the two resonators, while the coupling between them can still be strong enough because of their large area overlap. Particularly, the strong coupling contains significant radiation coupling that induces lifetime-contrast mode splitting. The “invisibility dips” predicted by transformation optics for the far-field scattering, is observed in the transmission spectrum for the near-field energy transport through the two resonators. We further demonstrate the underlying mode interference mechanism by directly imaging the field map of interfered waves that are tightly localized around the resonators.

4.1 Comparison between horizontal and vertical coupling

To illustrate that the larger area overlap gives rise to the stronger coupling in vertical configuration, we quantitatively compare the transmission spectrum of horizontal and vertical coupling configurations with simulation. In Fig. 4-1a and 4-1b, both the hexapole

and octopole modes are split in the horizontal and vertical coupling configurations. The edge-edge distance between resonators in both configurations are $D = 6.1$ mm as shown in the insets. To be consistent with later experiment, the horizontally coupled dimer (inset in Fig. 4-1a) is placed on a Teflon plate with thickness $t = 6.1$ mm, and the vertically coupled dimer (inset in Fig. 4-1b) are spaced by the same Teflon plate. Mode splitting in the vertical coupling configuration (shown in Fig. 4-1b) exhibits larger spectral separations (i.e. hexapole mode splitting with $\delta\omega_{\text{Hv}} = 0.412$ GHz, and octopole mode splitting with $\delta\omega_{\text{Ov}} = 0.284$ GHz, where the subscripts ‘H’ and ‘O’ represent hexapole and octopole modes respectively, ‘v’ denotes vertical coupling) than those in the horizontal coupling configuration shown in Fig. 4-1a (i.e. hexapole mode splitting with $\delta\omega_{\text{Hh}} = 0.072$ GHz, and octopole mode splitting with $\delta\omega_{\text{Oh}} = 0.030$ GHz, where the subscript ‘h’ denotes horizontal coupling). The larger mode splitting separation implies a stronger coupling in the vertical coupling configuration. In later analysis we will use a complex number κ to quantify the coupling strength. It is worth mentioning that while linewidths (or lifetimes) of split modes are almost the same in the horizontal coupling configuration, they are different in the vertical coupling configuration. For instance, in Fig. 4-1b the linewidths of split hexapole modes differ by $\delta\gamma_{\text{H}} = 0.0104$ GHz, and those of split octopole modes differ by $\delta\gamma_{\text{O}} = 0.0002$ GHz. Similar to the formation of superradiant states in coherent spontaneous radiation [73], this lifetime-contrast splitting can be attributed to the enhanced radiation coupling, as a result of the large area overlap between the resonators.

Figure 4-1. (a) Mode splitting in horizontally coupled DLSP resonators placed on a Teflon plated with thickness $t = 6.1$ mm. (b) Lifetime-contrast splitting and mode-interference-induced “invisibility dip” in vertically coupled resonators spaced with the same Teflon plate in (a).

4.2 Lifetime-contrast splitting

We then attach a second resonator on the backside of the Teflon plate to fulfil the vertical coupling configuration as shown in Fig. 4-2. Two thickness of the Teflon plates of $t = 9.9$ mm and $t = 8.2$ mm are adopted to tune the coupling between the resonators. First, simulation results in Fig. 4-3a show that both hexapole (resonance frequency $\omega_H = 5.305$ GHz, linewidth $\gamma_H = 0.008$ GHz) and octopole (resonance frequency $\omega_o = 5.725$ GHz, linewidth $\gamma_o = 0.0027$ GHz) modes of an individual resonator are split into two modes with different linewidths. The lower and higher peaks are named as binding and anti-binding modes indicated with subscript ‘b’ and ‘a’ respectively. This lifetime-contrast splitting can be understood with coupled mode theory (CMT) [60-61]. Equations of coupled modes are set up as follows:

$$\begin{cases} i\omega \begin{bmatrix} H_1 \\ O_1 \end{bmatrix} = \begin{bmatrix} i\omega_H - \gamma_H & 0 \\ 0 & i\omega_O - \gamma_O \end{bmatrix} \begin{bmatrix} H_1 \\ O_1 \end{bmatrix} + \begin{bmatrix} i\kappa_H & 0 \\ 0 & i\kappa_O \end{bmatrix} \begin{bmatrix} H_2 \\ O_2 \end{bmatrix} + \begin{bmatrix} \tau_H \\ \tau_O \end{bmatrix} S_{in} \\ i\omega \begin{bmatrix} H_2 \\ O_2 \end{bmatrix} = \begin{bmatrix} i\omega_H - \gamma_H & 0 \\ 0 & i\omega_O - \gamma_O \end{bmatrix} \begin{bmatrix} H_2 \\ O_2 \end{bmatrix} + \begin{bmatrix} i\kappa_H & 0 \\ 0 & i\kappa_O \end{bmatrix} \begin{bmatrix} H_1 \\ O_1 \end{bmatrix} \\ S_{out} = \cos(m_H \varphi) \eta_H H_2 + \cos(m_O \varphi) \eta_O O_2 \end{cases} \quad (4.1)$$

where ‘ H ’ and ‘ O ’ refer to hexapole and octopole modes respectively, subscript ‘1’ and ‘2’ denote different resonators, and symbols ‘ ω ’, ‘ γ ’, ‘ τ ’ and ‘ η ’ represent resonance frequency, dissipation loss, input coupling strength and output coupling strength respectively. Since the monopole probe is placed at the opposite position of the source, phase differences is $\varphi = \pi$. Mode orders are $\varphi_H = 3$ for hexapole mode, and $\varphi_O = 4$ for octopole mode, respectively. The measured transmission spectra are defined as $T = |\frac{S_{out}}{S_{in}}|$. Through CMT fitting, the complex coupling strength can be retrieved as $\kappa_H = 0.0762 + 0.005i$ for the hexapole mode and $\kappa_O = 0.036 + 0.0001i$ for the octopole mode. Other parameters can be obtained as $\tau_H^* \eta_H = 0.7093$, and $\tau_O^* \eta_O = 0.9822$ for the input and output couplings. Real parts of κ are contributed by both evanescent field and radiation coupling, while imaginary parts of κ are only induced by radiation coupling [74-76], which can lead to split modes with contrast lifetimes similar to the formation of superradiant modes in coherent spontaneous radiation [73]. Here the radiation coupling is nontrivial, as a result of large area overlap between the two resonators with relatively small separation. One resonator can thus receive almost half of the radiation from the other. Experimentally detected transmission in Fig. 4-3b verify the lifetime-contrast splitting, in which binding modes show narrower line width than antibinding modes.

Figure 4-2. Photo of the experimental set up. The Teflon plate is long enough to make the two-resonator system free standing.

Figure 4-3. Lifetime-contrast splitting. (a) Simulated trasmission spectra of a single resonator (black circles), vertically coupled resonators with 9.9 mm-thick teflon spacer (blue circlces), and with 8.2 mm-thick teflon spacer (red circles). Black solid curves are fitting results with CMT. (b) Detected transmission spectra of a single resonator (black dashed lines), 9.9 mm-thick teflon spacer (blue lines), and 8.2 mm-thick teflon spacer (red lines). (c)-(d) Experimentally recorded E_z field patterns on X-Y plane at 5.58 GHz, and 5.62 GHz respectively, which correspond to octopole binding mode (O_b), and anti-binding mode (O_a) respectively. (e)-(f) Simulated E_z field pattern on X-Z plane at 5.58 GHz, and 5.62 GHz respectively, which correspond to (c) and (d) respectively.

In order to gain further insight into the mode splitting, field profiles of the binding mode O_b at 5.58 GHz and the anti-binding mode O_a at 5.62 GHz in the X-Y plane are measured by the near-field scanning system with a spacer thickness $t = 9.9$ mm. The experiment setup is shown in Fig. 4-2. In order to avoid colliding with the source monopole when the probe monopole is moving, the source is fixed in the same plane and parallel with the bottom resonator. The probe is second monopole antenna mounted on a translational scanner to scan the mode patterns. Observed field patterns 0.5mm above/below the resonator dimer are shown in Fig. 4-3c and Fig. 4-3d. For further illustration, we show simulated field patterns of O_b (Fig. 4-3e) and O_a (Fig. 4-3f) in the cross section of the resonator dimer in the X-Z plane. Inside the Teflon layer in Fig. 4-3e, the E_z field points from the top resonator to the bottom one, indicating that the two resonators are bound by opposite charge distributions. However inside the spacer in Fig. 4-3f, the E_z field points from both top and bottom resonators to the middle region, demonstrating that the two resonators are anti-bound and repulsed by the same charge distributions.

Next, as we decrease the Teflon thickness to $t = 8.2$ mm, binding and anti-binding modes of both hexapole and octopole are spectrally separated further in Fig. 4-3a. Fitting with Eq. (4.1), retrieved coupling strength are $\kappa_H = 0.1212 + 0.0052i$ for the hexapole mode and $\kappa_o = 0.0702 + 0.0001i$ for the octopole mode, in which the real parts are much larger than their counterparts when $t = 9.9$ mm, while imaginary parts are almost unchanged. The reason is that a closer separation between resonators can enhance their near-field coupling, as reflected in the real part of coupling strength. Yet as one resonator has already covered

almost half space from the viewpoint of the other resonator, the radiation coupling between them is almost unaffected by the further separation reduction. The experimentally measured transmission spectrum is shown in Fig. 4-3b, which show good agreements with simulations and CMT results. In addition, it can be found that decreasing the spacer thickness shifts the anti-binding hexapole mode H'_a and the binding octopole mode O'_b close to each other. This shows the possibility for these two modes to spectrally overlap and interfere with each other. This spectral overlap requires extremely strong coupling between the two DLSP resonators, which is difficult to attain using the transverse arrangement in the horizontal coupling configuration in previous experiments [70]. As we will show later, this spectral overlap enables us to observe the effect of invisibility dip predicted by transformation optics [62]

4.3 Invisibility dips induced by destructive mode interference

To achieve the mode interference, we further decrease thickness of Teflon layer to $t = 6.1$ mm. The transmission spectrum is shown in Fig. 4-4a. H_a and O_b are closer than in Fig. 4-3a. They are no longer in symmetric Lorentz lineshape. An invisibility dip forms between the resonant frequencies of the two modes which is consistent with the theoretical prediction [62]. By fitting the simulation curve with Eq. (4.1), the coupling strength are retrieved as $\kappa_H = 0.2080 + 0.0048i$, $\kappa_o = 0.1420 + 0.0001i$. The experimentally detected asymmetric lineshape in Fig. 4-4c verifies the simulation and CMT results.

Figure 4-4. Mode-interference-induced “invisibility dip”. (a) Simulated transmission spectrum (blue circle line) with spacer thickness $t = 6.1$ mm, and fitting results by CMT (black solid line). (b) Simulated field patterns on top, middle (inside spacer), and bottom X-Y planes at three different frequencies (H_a , Dip, O_b) (c) Experimentally detected transmission spectrum (d) Experimentally recorded E_z field patterns on top and bottom XY planes at peaks H_a , Dip and O_b respectively.

To further illustrate how anti-binding (H_a) and binding (O_b) modes interact with each other, simulated mode profiles at three frequencies (i.e. H_a , Dip, and O_b in Fig. 4-4a) are shown in Fig. 4-4b. The phase difference between top and bottom mode profiles of H_a , and O_b are similar with those in Fig. 4-3c and 4-3d, and are verified by experimental results shown in Fig. 4-4d. In addition, we also show the simulated patterns on the middle plane inside the Teflon spacer as in Fig. 4-4b. As discussed before, the field pattern on the middle plane of anti-binding mode H_a is almost not discernable, but that of the binding mode O_b is comparable with those on top and bottom planes. The interference of H_a , and O_b is depicted at the dip frequency in the middle panel of Fig. 4-4b. Two features can be noticed. First, at

the probe position, anti-binding H_a and binding O_b modes destructively interfere with each other, leading to the null field. Second, in the middle plane, octopole mode dominates while the hexapole mode is relatively weak. This shows that the null field is indeed caused by the mode interference between H_a and O_b . Experiment results in Fig. 4-4(d) confirm the appearance of mode interference on the top and bottom surface of the resonator dimer.

Figure 4-5. “Invisibility dip” and modes crossing. (a) Measured transmission spectrum with spacer thickness $t = 4.9$ mm. (b) Experimentally recorded E_z field patterns on top and bottom XY planes at peaks H_a , and O_b respectively.

Finally, we further demonstrate that transmission dips with asymmetry line shapes arise from mode interference rather than inter-mode coupling between hexapole and octopole modes. The asymmetry property of the lineshape can be flipped by further decreasing the spacer thickness to $t = 4.9$ mm. The underlying mechanism is mode crossing that causes the frequency of the hexapole anti-binding mode to be higher than that of the octopole binding mode as shown in Fig. 4-5a. The mode crossing shows that these two modes are independent and unaffected by each other. This phenomenon has been further verified with

measured near-field patterns shown in Fig. 4-5b. Following the above fitting process, the retrieved coupling strength are $\kappa_H = 0.265 + 0.0049i$, $\kappa_o = 0.2 + 0.0001i$. Note that anomalous modes, arising from mode coupling are not observed here [77].

4.4 Vertical transport of subwavelength designer localized surface plasmon

4.4.1 Motivation

Transporting subwavelength localized surface electromagnetic (EM) modes has been studied in extensive applications such as topological modes [78], Dirac dispersions [79], colourful super-resolution imaging [80], and photonic energy transport [81-82]. In optical frequencies, localized surface EM modes, also called as localized surface plasmons (LSP) [11], are usually supported by metal nanostructures [83-85] usually on a two-dimensional (2D) dielectric substrate. Essence in these systems is the near field coupling between adjacent nanostructures. The theory of coupled resonator optical waveguide (CROW) [86] can account for a lot of in-plane coupling of these localized EM modes, without considering the vertical coupling [87-88]. However, thanks to the development of modern craft, it is possible to fabricate multilayer photonic circuits [89-90], therefore it is feasible to experimentally investigate the vertical-coupling-induced transport of subwavelength localized surface EM modes.

Here, we employ designer surface plasmon (DSP) resonators to experimentally and theoretically demonstrate the vertical transport of subwavelength localized surface EM modes. Along the direction vertical to planes of individual DSP resonators, these resonators are stacked to realize subwavelength vertical transport of these localized modes. First we theoretically study an infinitely long chain consisting of DSP resonators stacked in the vertical direction, and it is shown that the resonance frequency of a single DSP resonator can extend to a finite bandwidth and form a transport band. The dispersion relation of this new DSP resonator chain is verified by measuring spectra and mapping field profiles on a sample of five stacked DSP resonators.

4.4.2 Demonstration

In order to describe the performance of a one-dimensional (1D) infinite chain of stacked DSP resonators, we employ coupled mode theory as follows:

$$-\frac{da_n}{dt} = i\omega_0 a_n + i\kappa\omega_0 a_{n-1} + i\kappa\omega_0 a_{n+1} \quad (4.2)$$

where a_{n-1} , a_n , and a_{n+1} denote fields in $(n-1)$ -th n -th, and $(n+1)$ -th resonators, respectively, as indicated in the inset of Fig. 4-6(a). Due to the periodicity along Z direction, we can apply Bloch theorem $a_{n+1} = a_n e^{iKh}$, where K indicates the wavevector of vertical transport of a DSP mode, and h represents the distance between neighboring resonators. By solving the eigen problem Eq. (4.2) above, we can obtain the intrinsic dispersion relation ω

$=\omega_0 [1+2\kappa \cos(Kh)]$ of the infinite chain as the curve shown in Fig. 4-6(a) which gives negative group velocity as [91, 92].

Figure 4-6. (a) Dispersion relation of the vertical transport of DSP modes along an infinite chain of DSP resonators. The inset shows three resonators of them. Teflon plates are not shown. (b) Schematic of 5 coupled DSP resonators spaced by 4 Teflon plates with thickness h . (c) Measured near-field transmission spectrum of the sample in (b). A control detection on a sample as given in inset (only top resonator and 4 Teflon layers) is also conducted for comparison. (d) Scanned E_z field profile on the top of the sample shown in (b).

To experimentally observe vertical transport of subwavelength DSP modes in a 1D chain, we stack 5 DSP resonators vertically, and space them with 6-mm-thick Teflon plates in-between, as shown in Fig. 4-6(b). An infinite number of DSP resonators is essential to realize a smooth and continuous transmission band as in Fig. 4-6(a), which is the dispersion

of an unterminated 1D chain with infinite length. However, only a finite system with two terminals can be utilized in reality. It is sufficient to demonstrate this “bulk” property with a chain consisting of 5 resonators, among which the bottom and top resonators serve as the two terminals, and multiple resonators (to be precise, 3 resonators) are still left in-between. The inter-resonator distance is just 6 mm which is much shorter than the operating wavelength (about 56.39 mm) of hexapole mode. The detected near-field transmission spectrum, shown in Fig. 4-6(c), clearly shows a transmission band spanning from 5.17 to 5.57 GHz around the resonance frequency of original hexapole mode. The spectrum is consistent with the bandwidth of derived dispersion curve in Fig. 4-6(a). For the purpose of comparison, the measurement of near-field transmission when the lower four resonators are removed is done, whose result in Fig. 4-6(c) shows that the subwavelength EM energy cannot be efficiently delivered from the bottom resonator to the top one through the whole structure. At 5.39 GHz, we scan the field profile on the top surface of the structure, and the result shows a clear hexapole mode pattern. This confirms that the hexapole mode can be transported through the vertical 5-resonator chain from the bottom to the top one.

Figure 4-7. Simulated field profiles of the 5-resonator chain at 5.42 GHz at time $t = 0$ (a), $t = T/4$ (b), and $t = T/2$ (c) respectively, where T represents the period of oscillation.

It is worthy of noting that the field profile above the top surface is out of phase with respect to the field profile near the bottom surface. To show how the Bloch wave propagates along this 5-resonator chain, by numerical simulation we monitor field patterns at 5.42 GHz on 5 planes, each of which is 0.3 mm above corresponding individual resonator. As shown in Fig. 4-7, along the propagation direction, different resonators shows different phases, and the patterns of bottom and top resonators are out of phase.

4.5 Conclusions

In conclusion, we observed strong-coupling-induced spectral dips in two vertically coupled identical spoof surface plasmon resonators. Mode interferences induced by strong coupling are enhanced by gradually decreasing thickness of Teflon plates between the two DLSP resonators. Finally, mode crossing further verifies mode interference mechanism rather than

intermode coupling, in which low-order modes show higher frequencies than high-order modes. In the meantime, lifetime-contrast splittings are observed, where radiation coupling plays a significant role. The whole process is verified with transmission spectra and near-field imaging. Coupled mode theory has been developed to quantitatively describe the process. These results may extend to surface plasmon systems in optical frequencies and open a new avenue for various subwavelength optical applications.

Furthermore, the vertical transport of subwavelength designer LSP modes is demonstrated by stacking DSP resonators along its normal direction. The field in its normal direction around an individual DSP resonator induces mode splitting in a system consisting of two vertically coupled DSP resonators, and generates a transport band for a chain of coupled DSP resonators. Starting from the coupling strength extracted with coupled mode theory, we calculated the dispersion relation of the subwavelength mode transport along a 1D infinite chain. We also experimentally demonstrated the transport bandwidth and capability with a finite chain of 5 vertically coupled DSP resonators. These results may be useful in photonic circuits, filters, or 3D slow light structures.

CHAPTER 5

Probing topological protection using a designer surface plasmon structure

A topological insulator (TI) is a material that is electrically insulating in the bulk, but conducts along the surface via a family of ‘topological edge states’, whose existence is guaranteed by the topological incompatibility between the TI’s electronic band structure and the vacuum [93-94]. Recently, a novel field of ‘topological photonics’ has emerged, seeking to exploit the phenomenon of topological photonic states using classical electromagnetic (EM) waves [95-96]. These states are not only promising for defect-resistant EM wave-guiding applications, but also provide a unique platform for fundamental studies of the physics of topological phases that are not easily available in the condensed-matter context.

The first demonstration of topological photonic states was in a microwave-scale two-dimensional (2D) photonic crystal containing magneto-optic elements biased by an external magnetic field to break time-reversal symmetry [97-98]. Two time-reversal-invariant designs, operating at optical frequencies, followed. Firstly, Rechtsman *et al.* [99] demonstrated an array of three-dimensional (3D) coupled helical waveguides in which the paraxial propagation along the third dimension mapped formally to a periodically-driven, or ‘Floquet,’ 2D TI [100]. Secondly, Hafezi *et al.* [56-57,101] realized a time-reversal-invariant photonic TI in the form of an on-chip lattice of coupled optical ring resonators,

engineered to simulate a uniform magnetic field in the quantum Hall effect. Many other designs have also been proposed recently. [102-104] However, the intrinsic difference between electrons and photons determines that the ‘topological protection’ in such time-reversal-invariant photonic systems [56-57, 99-105] does not share the same robustness as its counterpart in electronic topological insulators, and the limits require further experimental study. Furthermore, the topological phases in previous photonic systems [56, 99,102] all have existing condensed-matter realizations which can already provide experimental platforms [57,99,103] to study them. The usefulness of topological photonics to understanding fundamental topological physics can be demonstrated with the explicit construction of a novel topological phase that still lacks a condensed-matter realization.

In this chapter, we report on the implementation of a topological designer surface plasmon structure operating in the microwave regime. Designer surface plasmons (also called ‘spoof surface-plasmons’) [23-30] are electromagnetic modes analogous to the familiar plasmon modes which occur in metallic surfaces and resonators at infrared and optical frequencies; these modes, however, appear at much lower frequencies and are supported by the presence of periodic sub-wavelength corrugations in the underlying metal structures. They hold considerable promise in device applications ranging from microwave to infrared frequencies, due to the ease with which their properties can be fine-tuned by altering the underlying structural parameters. This highly-tunable platform allows us to probe the robustness of topological edge states under a wide variety of different defect conditions.

Furthermore, this system is a realization of an anomalous Floquet TI, which is a topological phase that has not yet been realized in a condensed-matter setting.

Using the designer surface plasmon platform, we study the performance of the photonic TI in the presence of specific defects. Firstly, we show that the topological edge states are indeed immune to backscattering from a variety of barriers that do not reverse the propagation of edge states, including removal of entire unit cells. However, the path detour of propagation around defects can deteriorate the transmission of topological edge states, because of the intrinsic propagation loss of designer surface plasmons. Next, we construct two kinds of common photonic defects, without counterparts in electronic TIs, to break the topological protection of these edge states. First of all, for electronic edge states under topological protection in TIs, a default law is particle number conservation [93-94]. In contrast, photons can be easily annihilated or created by loss or gain in photonic systems [58]. Although loss is already present in above defect studies, here we construct an extreme case, a strongly dissipative defect that can completely annihilate photonic topological states without backscattering. This kind of annihilation defect has not been observed in electronic topological insulators [93-94]. Second, it is well-known that a ‘magnetic,’ or time-reversal-breaking, defect in TIs can flip the spin of electrons and cause backscattering for topological edge states. In periodically driven Floquet TIs, a defect cannot flip the time harmonic modulation on the whole structure, and thus cannot reverse the propagation of topological edge states. On the other hand, photonic pseudo spins/modulations effectively realized with wave circulation can be flipped by some common time-reversal-invariant defects. We

demonstrate that the photonic TI is not topologically robust against such defects; in a practical photonic TI device, therefore, such defects need to be explicitly suppressed.

Finally, it is worth noting that even though our study is based on an anomalous Floquet photonic TI²³, other photonic TIs which have been demonstrated experimentally [57,99,103] have the same features and limitations. In particular, both Hafezi *et al.*'s pseudo spin approach and Rechtsman *et al.*'s Floquet modulation approach are effectively realized with wave circulation that can be coupled to the opposite circulation. The coupled ring resonator lattice studied by Hafezi *et al.* [56-57,101] relies on having decoupled photonic pseudo spins. Likewise, the coupled helical waveguide arrays studied by Rechtsman *et al.* [99] rely on propagation down one direction along the waveguide axis which determines an effective circulation direction in the reduced 2D lattice as a time harmonic modulation; backscattering along the axial direction, which would induce the opposite circulation, is ignored. Our systematic experimental tests of topological protection including the possibility of flipping the pseudo spin or Floquet modulation are therefore applicable to these systems as well.

5.1 Implementing topological designer surface plasmon structure

The designer surface plasmon structure is shown in Fig. 5-1a. It consists of closely-spaced sub-wavelength metallic rods, placed on a flat metallic surface in an arrangement similar to the design of Hafezi *et al.* [56-57,101] (but with a significant conceptual difference in the

topological phase, to be discussed in the next section). Large rings, called ‘lattice rings’, are set in a square lattice, and each pair of adjacent lattice rings is connected by a smaller ‘coupling ring’. Designer surface plasmon waves can circulate clockwise or counter-clockwise in the lattice rings, serving as ‘pseudo spins’ in the photonic pseudo spin approach as demonstrated in previous designs [56-57, 101]. For the moment, we consider the typical situation where the two circulations do not couple to each other. Modes of the chosen circulation can be excited via U-shaped input/output waveguides at the corners of the lattice (Fig. 5-1b). The field pattern is recorded by a near-field probe scanning above the metal rods, connected to a microwave network analyzer.

Figure 5-1. Construction of a topological designer surface plasmon structure. **a**, Photo of metallic rods with diameter 2.5 mm and height 5.0 mm distributed with center-center distance 5.0 mm on a flat metallic surface. A lattice ring formed by 56 metallic rods is with radius $R_1 = 44.56$ mm. A coupling ring formed by 48 metallic rods is with radius $R_2 = 38.2$ mm. The ring-ring distance is $g = 5.0$ mm. ϕ denotes the phase delay of electromagnetic waves along a quarter of a lattice ring. **b**, Schematic of a 5×5 lattice in experiment. A network analyzer records the field pattern by scanning a near-field probe above the metallic rods. The red meandering curve above the structure represents edge states.

The 5×5 lattice of lattice rings contains a total of 3,320 metal rods, each having diameter 2.5 mm and height 5.0 mm, standing on a flat aluminum plate which is $1 \text{ m} \times 1 \text{ m}$ in size and 5.0 mm thick. Each lattice ring consists of 56 rods arranged in a circle of radius of $R_1 = 44.56 \text{ mm}$. Adjacent lattice rings are coupled through coupling rings, each consisting of 48 rods arranged in a circle of radius $R_2 = 38.20 \text{ mm}$.

The excitation source is a single-mode cable-to-waveguide adaptor with a rectangular port. Half of the port is covered with aluminum foil to increase the cutoff frequency of the waveguide port. Exciting the U-shape waveguide at different legs (input 1 or 2) can excite different circulations of surface EM wave in lattice rings. The field pattern of E_z component is recorded with a 3 mm-in-length monopole probe which scans in the xy plane, 1 mm above the top of the metal rods.

5.2 Mapping to anomalous Floquet topological insulator phase

A significant departure from the design of Hafezi *et al.*, [56-57,101] where the coupling rings were assigned different geometries for constructing an incommensurate ‘magnetic vector potential’ to simulate the quantum Hall effect, is that the coupling rings in the present lattice have identical geometries. Therefore, the lattice is entirely commensurate, and cannot be mapped to a quantum Hall system. Nonetheless, its band structure is topologically non-trivial. [58,106-107] The lattice is described by a network model [108-109] which can be formally mapped onto a Floquet lattice, with the phase delay ϕ (as

marked in Fig. 5-1a) in each quarter of a lattice ring playing the role of a Floquet quasi-energy [58,106-107]. When the coupling between adjacent lattice rings is increased beyond a critical value ($\theta = 0.25\pi$ in the parameterization of Ref. 58 and Ref. 107), the lattice undergoes a topological transition from a topologically trivial quasi-energy band structure to a topologically non-trivial one with robust topological edge states (Fig. 5-2c;).

Before we elaborate on the theoretical modelling, we emphasize that, as the most unusual feature of this topologically non-trivial phase, the Chern numbers, which relates to Berry phase acquired by a Bloch particle when it is adiabatic pumped across the Brillouin zone, are zero for all bands, for each (decoupled) circulation [107]. Normally, in spin-decoupled condensed-matter and photonic systems, the net number of topological edge states in each band gap is equal to the sum of the Chern numbers in all bands below the gap ('bulk-edge correspondence') [93-97,99,102-103,110]. However, 'anomalous Floquet TI' phases are an exception [111-113] for the following reason. Because Floquet quasi-energies, unlike ordinary energies, are angle variables [111-113], Floquet band structures are thus not bounded below by a 'lowest band'. In other words, the infinite number of bands below the gap makes it impossible to apply the usual Chern-number-based bulk-edge correspondence. Although the corresponding non-vanishing topological invariant of the anomalous Floquet TI phase has recently been experimentally confirmed [114], an extended 2D lattice supporting this exotic topological phase has not been demonstrated. Our designer surface plasmon structure thus serves as the first explicit realization of an anomalous Floquet topological phase.

Figure 5-2. Network model description of the topological structure and its topological transition. **a**, Schematic of a unit cell in a two-dimensional lattice of coupled ring waveguides. **b**, The equivalent periodic network. Within the unit cell, we define a surface (red rectangle) which is penetrated by input amplitudes $|a\rangle$ and output amplitudes $|b\rangle$, related by $|b\rangle = e^{-i\phi}|a\rangle$. These amplitudes also scatter with those of neighbouring cells, with coupling matrices S_x and S_y . **c**, Topological transition as the inter-ring coupling strength θ is tuned from weak to strong. Before and after the transition, all bands have zero Chern number $C=0$. Red and blue lines denote edge states confined to the upper and lower edges of the strip respectively.

We start the theoretical modelling with a 2D illustration of this periodic lattice as depicted in Fig. 5-2a. Each lattice ring acts as a waveguide, constraining EM waves to propagate along the ring. When the coupling through the coupling ring (scaled down in Fig. 5-2a) has negligible internal backward scattering, the mode-hopping from a lattice ring to

neighboring lattice rings conserves the circulation. To determine the bandstructure, we consider one unit cell of the lattice, shown schematically in Fig. 5-2b. Let $n \equiv (x_n, y_n)$ denote the site of unit cell. In each unit cell, input amplitudes can be described by a four-vector $|a_n\rangle = [a_{1n}, a_{2n}, a_{3n}, a_{4n}]$, and output amplitudes by another four-vector $|b_n\rangle = [b_{1n}, b_{2n}, b_{3n}, b_{4n}]$. These input and output amplitudes are related by $|a_n\rangle = e^{-i\phi}|b_n\rangle$, where ϕ is the phase delay along a quarter lattice ring. For Bloch modes, which satisfy $|a_n\rangle = |a_K\rangle e^{iK \cdot r_n}$ and $|b_n\rangle = |b_K\rangle e^{iK \cdot r_n}$, the inter-cell scattering can be described by $S(K)|b_K\rangle = |a_K\rangle$, where $S(K)$ is a unitary scattering matrix derived from the couplings between the lattice and coupling rings; it is periodic in K with the periodicity of the Brillouin zone. More specifically, the coupling between the sites at n and $n+x$ can be described as $\begin{bmatrix} a_{2,n} \\ a_{4,n+x} \end{bmatrix} = S_{nx} \begin{bmatrix} b_{1,n} \\ b_{3,n+x} \end{bmatrix}$ and the coupling between the sites at n and $n+y$ can be described as $\begin{bmatrix} a_{1,n} \\ a_{3,n+y} \end{bmatrix} = S_{ny} \begin{bmatrix} b_{4,n} \\ b_{2,n+y} \end{bmatrix}$, where $S_{nx} = S_{ny} = S_n = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix}$ is a unitary matrix containing four complex numbers r, r', t and t' . Combining together, we can write down the following scattering matrix equation

$$\begin{bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n+y} \\ a_{4,n+x} \end{bmatrix} = \begin{bmatrix} 0 & t' & 0 & r \\ r & 0 & t' & 0 \\ 0 & r' & 0 & t \\ t & 0 & r' & 0 \end{bmatrix} \begin{bmatrix} b_{1,n} \\ b_{2,n+y} \\ b_{3,n+x} \\ b_{4,n} \end{bmatrix} \quad (5.1)$$

This periodic network allows Bloch theorem to apply. We can thus get

$$\begin{bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n+y} \\ a_{4,n+x} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{iK_y} & 0 \\ 0 & 0 & 0 & e^{iK_x} \end{bmatrix} \begin{bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ a_{4,n} \end{bmatrix}$$

$$\begin{bmatrix} b_{1,n} \\ b_{2,n+y} \\ b_{3,n+x} \\ b_{4,n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{iK_y} & 0 & 0 \\ 0 & 0 & e^{iK_x} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_{1,n} \\ b_{2,n} \\ b_{3,n} \\ b_{4,n} \end{bmatrix} \quad (5.2)$$

After substituting Eq. (5.2) into Eq. (5.1), we can get:

$$\begin{bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ a_{4,n} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-iK_y} & 0 \\ 0 & 0 & 0 & e^{-iK_x} \end{bmatrix} \begin{bmatrix} 0 & t' & 0 & r \\ r & 0 & t' & 0 \\ 0 & r' & 0 & t \\ t & 0 & r' & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{iK_y} & 0 & 0 \\ 0 & 0 & e^{iK_x} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} b_{1,n} \\ b_{2,n} \\ b_{3,n} \\ b_{4,n} \end{bmatrix} \quad (5.3)$$

By expressing $\begin{bmatrix} a_{1,n} \\ a_{2,n} \\ a_{3,n} \\ a_{4,n} \end{bmatrix} = e^{-i\phi} \begin{bmatrix} b_{1,n} \\ b_{2,n} \\ b_{3,n} \\ b_{4,n} \end{bmatrix}$ in Eq. (5.3), we rewrite the governing scattering matrix

equation as:

$$S(K)|b_K\rangle = e^{-i\phi}|b_K\rangle \quad (5.4)$$

where $S(K) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{-iK_y} & 0 \\ 0 & 0 & 0 & e^{-iK_x} \end{bmatrix} \begin{bmatrix} 0 & t' & 0 & r \\ r & 0 & t' & 0 \\ 0 & r' & 0 & t \\ t & 0 & r' & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{iK_y} & 0 & 0 \\ 0 & 0 & e^{iK_x} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} |b_K\rangle = \begin{bmatrix} b_{1,n} \\ b_{2,n} \\ b_{3,n} \\ b_{4,n} \end{bmatrix}$. We can

regard the eigenvectors $|b_K\rangle$ in the above equation as Bloch eigenstates; then $\phi(K)$ plays the role of a band energy, except for the fact that it is an angle variable ($\phi \equiv \phi + 2\pi$). We refer to ϕ as the ‘quasienergy.’ Following the same parameterization process in Ref. 58, the unitary scattering matrix $S_n = \begin{bmatrix} r & t' \\ t & r' \end{bmatrix}$ can be rewritten as $\begin{bmatrix} \sin\theta e^{i\chi} & -\cos\theta e^{i(\varphi-\xi)} \\ \cos\theta e^{i\xi} & \sin\theta e^{i(\varphi-\chi)} \end{bmatrix}$, where θ , representing the coupling strength between neighboring rings, is relevant with amplitudes of r and t , and χ, φ, ξ are relevant with phases of r and t . We choose $\chi =$

$-0.24\pi, \varphi = \pi, \xi = 0$, extracted from simulated transmission of a unit cell at 11.3 GHz (shown in Fig. 5-3). The band structure of a semi-infinite strip with 50 lattices in y direction and periodic in x direction is plotted in Fig. 5-2c in the main text, where $\theta = 0.2\pi$ for weak coupling, $\theta = 0.25\pi$ for critical coupling, and $\theta = 0.4\pi$ for strong coupling, respectively.

The Bloch modes of a periodic network are equivalent to the Floquet modes of a periodically driven lattice. Suppose we have a lattice, of the same spatial dimensions as our network, with a Hamiltonian $H_K(t)$ that is periodic in time with period T . Then the Floquet state with state vector $|b_n\rangle$ and Floquet quasienergy $\phi(K)/T$ obeys exactly the governing scattering matrix equation, provided $S(K)$ is the time-evolution operator over one period:

$$S(K) = T \exp \left[-i \int_0^T dt H_K(t) \right] \quad (5.5)$$

where T is the time-ordering operator. This relationship between network models and Floquet lattices was pointed out in Ref. 107. One can regard $S(K)$ as a discrete time-evolution operator acting on a particle which is initially localized at one point in the network (say the midpoint of a quarter lattice ring as the link); over one time period, the particle moves along the link, tunnels instantaneously across a node, and moves midway along a neighbouring link.

Using the above formalism, we have calculated the quasi-energy band structure of a semi-infinite strip with 50 lattices in y direction and periodic in x direction; the results are shown in Fig. 5-2c. By tuning the effective inter-ring coupling strength θ , we can achieve a topological phase transition. For weak couplings $\theta < 0.25\pi$, the band structure is gapped; the gaps close at a critical value $\theta = 0.25\pi$, and for strong couplings $\theta > 0.25\pi$ the gaps re-open with topologically protected edge states, while the Chern numbers of all bands are still zero, as verified numerically. The calculation of Chern numbers follows the formula $C = \frac{1}{2\pi i} \int_{\text{Brillouin Zone}} d^2K \nabla \times \vec{A}(K)$, where the Berry connection $\vec{A}(K)$ can be calculated with the obtained Bloch wavefunction $|b_K\rangle$ as $\vec{A}(K) = \langle b_K | \nabla_K | b_K \rangle$.

5.3 Retrieval of coupling strength θ

Figure 5-3. Simulation of coupling through a unit cell in the topological designer surface plasmon structure. The frequency is at 11.3 GHz.

Parameter θ describes coupling strength between neighboring lattice rings, which depends on both frequency ω , and ring-ring distance g . Topological nontrivial phase occurs above the critical $\theta = 0.25\pi$, while trivial phase occurs below this critical θ value. In real structures that support narrowband designer surface plasmons, we tune g to realize different topological phases. To match with band diagrams, we retrieved θ with $\theta = \arcsin[(I_{\text{out}}/I_{\text{in}})^{1/2}]$ from simulations on a unit cell, where I_{in} is the power delivered into the unit cell through the input plane S_{in} , and I_{out} is the output power through the output plane S_{out} , as marked in Fig. 5-3. Topological edge state at 11.3 GHz has $\theta = 0.41\pi (\pm 0.03\pi)$ within the topological band gap), larger than 0.25π . Note that the coupling strength θ characterizes the inter-ring coupling, not the propagation loss. Therefore, the measured propagation loss of 1.44 dB per lattice constant is irrelevant here. Regarding the topologically trivial phase with $g = 7.5$ mm, the demonstrated bulk state at 11.3 GHz corresponds to $\theta = 0.10\pi$, smaller than 0.25π , and the in-gap excitation at 11.45 GHz corresponds to $\theta = 0.08\pi$, also smaller than 0.25π .

5.4 Simulation of designer surface plasmons

We implement designer surface plasmons by arranging periodic metallic rods on a metal surface. The dispersion of the designer surface plasmons can be tuned by changing the rod heights. Fig. 5-4a shows a 1D periodic array of metallic rods with periodicity $p = 5.0$ mm on a flat metallic surface. Each metallic rod has radius $r = 1.25$ mm and the same height h (h can be varied). The background is free space. We simulate the dispersion relation of the guided designer plasmon waves, for different values of h ; the results are shown in Fig. 5-

4b. The mode profile of the electric field, for $h = 5.0$ mm, is shown in Fig. 5-4c. We observe that the fields are tightly confined around the rods; other choices of h give similar mode profiles. This strong waveguiding makes it feasible to probe the field distribution using a near-field scanning measurement.

Figure 5-4. Dispersion and field profile of designer surface plasmons. **a**, Schematic of an array of metallic rods on a flat metallic surface. **b**, Dispersions of designer surface plasmons with heights of rods $h = 3.5$ mm (star curve), $h = 4.3$ mm (triangle curve), and $h = 5.0$ mm (circle curve), respectively. The black solid line is the light line. **c**, Mode profile of $|E|$ field in the cross section of a metallic rod that is perpendicular to the waveguiding direction ($h = 5.0$ mm).

Only part of the quasi-energy band structure shown in Fig. 5-2c is accessible, because the designer surface plasmons propagate only within a narrow frequency band as in Fig. 5-4b. As it is unfeasible to simulate the band structure of the full 3D structure *ab initio*, we adopt an alternative approach based on repetitive frequency scanning, which shortens the computation time. The commercial software CST Microwave Studio is used for numerical simulation. In band structure simulation, a supercell of the designer surface plasmon

structure that consists of 5 lattice rings in y direction and 1 lattice ring in x direction is adopted. The periodic boundary condition is imposed with a phase shift in x direction. An external dipole source close to the top ring is used to excite the network. By scanning the frequency, each dip of the receiving spectrum (widely termed ‘ S_{11} ’ parameter) of this dipole source should correspond to a mode of this network for a specific phase shift in x direction. By further scanning the phase shift through the whole Brillouin zone, the complete band structure can be obtained. Circulation of the edge states can be monitored with dynamic field distributions.

Fig. 5-5a shows the simulated band structure for a semi-infinite strip which has 5 lattice rings in the y direction, and is infinite in the x direction (choosing the circulation where the modes run clockwise along the lattice rings). These results reveal a gap between 11.1 GHz and 11.7 GHz, spanned by unidirectional states localized to opposite edges of the strip. Note that within such a narrow bandwidth, the inter-ring coupling strength θ is mainly determined by the spacing g between lattice and coupling rings. We estimate that for the current setting with $g=5.0$ mm, the coupling strength is $\theta = 0.41\pi \pm 0.03\pi$ within the band gap of interest. According to calculation from the network model in Fig. 5-2, this band gap is topologically nontrivial.

Figure 5-5. Demonstration of topological edge state. **a**, Simulated band diagram of the narrow-band designer surface plasmon structure. **b**, Observed field pattern when the excitation is inside the bulk at frequency 11.3 GHz. **c**, Observed edge state at frequency 11.3 GHz. **d**, Transmission spectra of configurations in **c**, **e**, and **g**. **e**, The defect (shown in **h**) causes strong reflection for a 1D lattice (**f**). **g**, The edge state circumvents a defect lattice (shown in **i**) consisting of metallic rods with height $s = 3.5$ mm. All other rods have the height $h = 5.0$ mm.

Figure 5-6. Simulated field patterns according to experimental results in Fig. 5-5. **a**, An insulating bulk state, corresponding to Fig. 5-5b. **b**, A topological edge state, corresponding to Fig. 5-5c. **c**, States on a 1D lattice, corresponding to Fig. 5-5e. **d**, Circumvention of the topological edge state around a defect ring with 3.5mm height, corresponding to Fig. 5-5g.

5.5 Demonstration of topological protection and its robustness

We now experimentally and numerically study the topological edge state in a finite 5×5 lattice. First, we apply a monopole source to the bulk, at a mid-gap (11.3 GHz) frequency; this produces a mode localized in the vicinity of the source (Fig. 5-5b), verifying that the

bulk is insulating. Next, we excite the structure via one of the U-shaped input/output waveguides at 11.3 GHz. This produces a mode which propagates along the edge (Fig. 5-5c), including around one corner of the lattice. No obvious reflection is observed in experiment. The transmission reaches -12.94 dB at the output as shown in Fig. 5-5d. High transmission within a frequency range around 11.3 GHz corresponds extremely well with the band gap predicted in Fig. 5-5a. The transmission drop of 12.94 dB arises from the propagation loss over 9 lattice constants. We thus estimate that the propagation loss per lattice constant is about 1.44 dB at 11.3 GHz. Finally, in order to verify that the observed topological edge state is a consequence of the bulk structure in the 2D lattice, we remove all the rings except those along the bottom and left boundaries, which form a one-dimensional (1D) chain (Fig. 5-5f, 5-5h). The defect ring is left in place along this chain. The mode is now strongly reflected by the defect ring (Fig. 5-5e), and the transmission at the output reaches the noise level (Fig. 5-5d).

To probe the robustness of the topological edge state, we introduce a defect by altering one of the lattice rings along the edge, decreasing the height of its rods from 5.0 mm to 3.5 mm (Fig. 5-5i). As can be seen in the dispersions in Fig. 5-4b, this decrease of height forms a sharp momentum mismatch when the edge state intends to go through this defect ring. As a result of topological protection, the resultant edge state circumvents the defect ring, and continues propagating along the modified edge (Fig. 5-5g). However, as can be seen in Fig. 5-5d, the path detour of propagation leads to the transmission drop from -12.94 dB to -19.86 dB at the output. This roughly 7 dB drop is consistent with the propagation

loss over extra 5 lattice constants as a result of the path detour. Their corresponding simulation results are shown in Fig. 5-6 respectively.

The robustness of topological protection can be further demonstrated by varying the height of the defect ring, as analogues of defects with different ‘potential barriers’ in electronic materials. The first defect variation is achieved by fully removing a lattice ring (or equivalently, reducing its height to zero) and its surrounding coupling rings (Fig. 5-7e), being similar to the defect in Ref. 57. Because of the zero probability for the waves to couple to this defect, it corresponds to an ‘infinite potential barrier’. Simulation (Fig. 5-7a) and experiment (Fig. 5-7c) show that the edge mode circumvents the defect of missing rings without touching. The transmission at the output reads -16.04 dB, as summarized in Table 1. Compared to the case with no defect, the path detour leads to a longer propagation length with 2 more lattice constants (shorter than the case of 3.5 mm-tall defect ring), which drops the transmission for about 3 dB.

Figure 5-7. Demonstration of the robustness of topological protection against defect variations. Simulated (a) and observed (c) field pattern when removing a lattice ring and its surrounding coupling rings (shown in e). Simulated (b) and observed (d) field pattern when the height of metallic rods of a lattice ring is decreased to $s = 4.3$ mm (shown in f). All other rods maintain their height $h = 5$ mm. Metal is modelled as a perfect electric conductor in simulation.

The second defect variation consists of a lattice ring of metallic rods with 4.3 mm height (Fig. 5-7f). Since the dispersion in the modified lattice ring is close to a regular lattice ring, this defect can be considered as a low potential barrier (substantially weaker than the similar defect in Fig. 5-5i that can be treated as a medium potential barrier). The low potential

barrier allows part of the mode to directly tunnel through this defect, while the remainder still circumvents the defect, as can be seen clearly in simulation (Fig. 5-7b) and experiment (Fig. 5-7d). Therefore, the transmission at the output reads -17.4 dB, at the level between cases of 3.5 mm-tall defect ring and no defect.

5.6 Demonstration of breaking topological protection

The above defects, similar to previous demonstrations in topological photonics, do not break topological protection. The first defect that we will demonstrate to break the topological protection is a backscattering-immune strongly dissipative defect, which is realized by gradually decreasing heights of the rods in the defect ring (inset of Fig. 5-8a). As simulated in Fig. 5-8a, the scattering occurring at this defect is able to fully dissipate, or ‘annihilate,’ the edge states, while backscattering is suppressed due to the adiabatic momentum change on the rods with gradually decreasing heights. The measured field pattern in Fig. 5-8b confirms this dissipating phenomenon. The stronger radiation in Fig. 5-8b than in Fig. 5-8a is because in experiment the monopole probe of finite length collects fields in a finite range of heights, while in simulation only fields at a single height are captured. The transmission at the output reaches noise level.

Figure 5-8. Demonstration of backscattering-immune strongly dissipative defect. **a**, Simulated field pattern when metallic rods of a lattice ring gradually decrease their heights to zero (shown in the **inset**; from s_1 to s_7 , $s_1 = 4.7$ mm, $s_2 = 4.1$ mm, $s_3 = 3.5$ mm, $s_4 = 2.9$ mm, $s_5 = 2.3$ mm, $s_6 = 1.7$ mm, and $s_7 = 1.0$ mm). Metal is modelled as a perfect electric conductor in simulation. **b**, Observed field pattern corresponding to simulation in **a**.

We then demonstrate the second defect, which breaks the topological protection by reversing the propagation of edge states. This propagation reversal of edge states corresponds to spin flip in the photonic pseudo spin approach [56-57,101] or the modulation reversal in the Floquet modulation approach [99]. Hereafter we adopt the ‘spin’ flip picture to describe this process for convenience. First, we implement a partial ‘spin’ flip defect by approaching a metallic block to the lattice ring without touching (Fig. 5-10e). This defect can mix the two pseudo ‘spins’. Similar defects such as a semi-transparent scatterer [56] have been proposed previously, but have not been demonstrated. For the purpose of imaging the propagation of ‘spin’ flipped modes, we deliberately decrease the height of input waveguide from 5.0 mm to 4.3 mm, such that the coupling

of ‘spin’ flipped mode to the input waveguide is weak. We estimate that the coupling loss from the input waveguide to the structure is about 7 dB.

Figure 5-9. Simulation of coupling between a 5-mm-tall ring to a 4.3-mm-tall ring. The frequency is at 11.3 GHz. **a**, Input from the 5-mm-tall ring. **b**, Input from the 4.3 mm-tall ring.

In experiment, the 4.3-mm-tall ring is used as a defect and later used as the input waveguide. In Fig. 5-9, we simulate the coupling between a 5.0-mm-tall ring and a 4.3-mm-tall ring. We can see in Fig. 5-9a that, the wave coupled from the 5.0-mm-tall ring to the 4.3-mm-tall ring is minimal (~ 7 dB) and the wave maintains its propagation (~ 1 dB) on the 5.0-mm-tall ring without reflection. In Fig. 5-9b, the coupling from the 4.3-mm-tall ring to the 5.0-mm-tall ring is also weak (~ 7 dB). There is moderate radiation on the 4.3-mm-tall ring because of its relatively weak capability of confining the wave energy of designer surface plasmons.

Both the simulated (Fig. 5-10a) and measured (Fig. 5-10c) field patterns reveal partial mixing between the two pseudo ‘spins,’ with a portion of the mode with clockwise circulation continuing to propagate to the left and eventually exiting from the upper leg of U-shape waveguide at the upper left corner, while the remainder is reflected to the right with counter-clockwise circulation, eventually exiting from the lower leg of the U-shape waveguide. Because signals reaching the output waveguide are too weak to observe, we measure the transmission/reflection at the location marked by a white horizontal line in Fig. 5-10c, where the transmitted and reflected waves have the same propagation length. The measured transmission/reflection at -20.25 dB/-18.36 dB shows that the ratio between the transmitted ‘spin’ down state and the reflected ‘spin’ up state is about 1:1.54.

Figure 5-10. Demonstration of time-reversal-invariant ‘spin’ flipping defect. Simulated (a) and observed (c) field pattern when a metallic block (shown in e) is located near a lattice ring without touching. Simulated (b) and observed (d) field pattern when the metallic block (shown in f) is inserted in a lattice ring. Metal is modelled as a perfect electric conductor in simulation.

Second, we construct a defect capable of flipping the ‘spin’ completely by inserting the metallic block between two metallic rods in a lattice ring (Fig. 5-10f). Simulation in Fig. 5-10b shows a complete conversion from the clockwise circulation, or pseudo ‘spin-down,’ to the counter-clockwise circulation, or pseudo ‘spin-up,’ by the defect at the bottom edge

of the structure. The edge state only exits from the lower leg of U-shape waveguide at the upper left corner. The measured field pattern (Fig. 5-10d) matches well with the simulated results. The measured reflection of -16.29 dB at the location on the marked horizontal line in Fig. 5-10d is consistent with the transmission in Fig. 5-5c without defect after considering the coupling loss at the input and the reduced propagation length in experiment.

5.7 Demonstration of a topologically trivial phase

For completeness, we finally demonstrate the behaviour of the designer surface plasmon structure in the topologically trivial phase. As shown in Fig. 5-2c, the network band structure is topologically trivial when the coupling between lattice rings is sufficiently weak. To accomplish this, we increase the inter-ring separation g from 5 mm to 7.5 mm. We first simulate the band diagram as shown in Fig. 5-11a. It can be seen that between the two bulk bands, there is no edge state in the bandgap which spans from 11.35 GHz to 11.5 GHz. When the structure is excited at 11.3 GHz (in a bulk band), an extended field pattern is observed in both simulation (Fig. 5-11b) and experiment (Fig. 5-11d). When the excitation is tuned to 11.45 GHz (in the band gap), the field patterns in both simulation (Fig. 5-11c) and experiment (Fig. 5-11e) show a mode that is localized in the vicinity of excitation and does not propagate. The retrieved coupling strength within this frequency band of interest shows weaker coupling strength than the critical value of topological phase transition.

Figure 5-11. Demonstration of a trivial insulator phase. **a**, Simulated band diagram of the topologically trivial designer surface plasmon structure. **b**, Simulated field pattern of bulk state at 11.3 GHz. **c**, Simulated field pattern at 11.45 GHz in the band gap. **d**, Observed field pattern at 11.3 GHz in the bulk band. **e**, Observed field pattern at 11.45 GHz in the topologically trivial band gap. Metal is modelled as a perfect electric conductor in simulation.

5.8 Discussion

The above results, as summarized in Table 1, demonstrate the topological protection and its robustness against various defects on a time-reversal-invariant photonic platform of designer surface plasmons. When facing the defects analogous to low (defect no. 1), medium (defect no. 2), and infinite (defect no. 3) potential barriers, the topological edge states can circumvent these defects without back-scattering. However, the path detour still deteriorates the transmission because of propagation loss over elongated path.

Table 1 Summary of testing results on topological protection against defects						
Defect No.	Figure	Defect type	Physical meaning	Transmission	Reflection	Comment
0	Fig. 5-5c	No defect	Perfect lattice	-12.94 dB	<-35 dB	Measuring propagation loss
1	Fig. 5-7d	Rod height decreased to 4.3 mm	Low potential barrier	-17.4 dB	<-35 dB	Path detour and tunneling
2	Fig. 5-5g	Rod height decreased to 3.5 mm	Medium potential barrier	-19.86 dB	<-35 dB	Path detour
3	Fig. 5-7c	Rings removed	Infinite potential barrier	-16.04 dB	<-35 dB	Shorter path detour
4	Fig. 5-8b	Rod heights gradually decreased	Dissipation	<-35 dB	<-35 dB	>35 dB dissipation at defect
5	Fig. 5-10c	Metallic block approached	Partial flip of spin/modulation	-20.25 dB	-18.36 dB	~7 dB coupling loss at input
6	Fig. 5-10d	Metallic block inserted	Complete flip of spin/modulation	<-35 dB	-16.29 dB	~7 dB coupling loss at input

Table 1. Summary of testing results on topological protection against defects.

On the other hand, the strongly dissipative defect (defect no. 4) and the metallic block (defects no. 5 and 6) can break the topological protection for completely dissipating the edge states, and inducing reflection, respectively. The reflection corresponds to spin flip in Hafezi *et al.*'s pseudo spin approach [56-57,101] and modulation reversal in Rechtsman *et al.*'s Floquet modulation approach. Furthermore, the anomalous Floquet topological phase constructed in this study cannot be predicted by the usual Chern number topological invariants. Our demonstration makes the first step to explicitly construct this exotic topological phase in a physical photonic structure. In view of the tunability of the designer surface plasmon structure, intriguing avenues to explore in the future include the possibility of topologically protected mode amplification [58], which can be achieved by integrating microwave amplifiers, and topological many-body physics by incorporating non-linear components.

CHAPTER 6

Spherical designer surface plasmonic resonator

Besides attractive applications, LSPs supported in metallic nanostructures also bring rich new physics [62, 71]. Those novel phenomena come from the dramatic spatial variations of electromagnetic excitations which rely on the geometries of the structures [83, 85]. Therefore, modes profiles of cylinder LSPs follow cylindrical symmetries. Although two-dimensional (2D) resonators are usually reduced to finite length as disks in reality, mode profiles can still follow cylinder symmetries. Spatial variations of electromagnetic excitations in a spherical metallic nanoparticle are totally different from those in the cylindrical case. Novel and interesting physics have recently been predicted with spherical LSPs. For example while studying two extremely closely placed spherical metallic nanoparticles, anomalous modes are predicted by transformation optics [77], which cannot exist in cylindrical case no matter how close the two are placed [62, 71].

Designer localized surface plasmons (LSPs) provide a flexible platform to demonstrate these new physics. Two-dimensional (2D) designer LSP has been proposed with groove structured metal cylinders, and realized with ultrathin groove structured metal disks. However, three-dimensional (3D) designer LSP has not been achieved. That's because of complexity of structure design and fabrication difficulties. Here, I design a structure which can support spherical LSP, and realize it with 3D printing technology. Experiment results are consistent with theoretical analysis.

6.1 Design of spherical designer LSP

As introduced in Chap. 1, the 2D designer LSP can be described with a two-layer model consisting of an anisotropic metamaterial shell and a PEC core. By averaging $1/\varepsilon$ according to effective media theory $\frac{1}{\varepsilon_{eff}} = \frac{1-a/d}{\varepsilon_{PEC}} + \frac{a/d}{\varepsilon_{air}}$, The anisotropic metamaterial shell

has permittivity $\varepsilon_{r\theta z} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & \frac{d}{a} & 0 \\ 0 & 0 & \infty \end{bmatrix}$ in cylindrical coordinate [32]. By extending to 3D

case, the spherical designer LSP can be realized with an anisotropic spherical metamaterial

shell and a spherical PEC core, and the anisotropic spherical shell has permittivity $\varepsilon_{r\theta\phi} =$

$\begin{bmatrix} \infty & 0 & 0 \\ 0 & \varepsilon_t & 0 \\ 0 & 0 & \varepsilon_t \end{bmatrix}$ in spherical coordinate. To realize this parameter, we designed structure as

shown Fig. 6-1, in which $\theta_d = \phi_d = d$, $\theta_a = \phi_a = a$. According to the effective media theory, it can

be obtained that the shell consisted of pillars satisfies $\varepsilon_t = 1 + \frac{2p}{1-p}$, where $p = \frac{a^2}{d^2}$ [115].

Note that all the PEC pillars point to the spherical center. As the light propagates with the

velocity c along ρ direction in the metamaterial shell, the effective metamaterial must

satisfy the equations [32] $\sqrt{\varepsilon_\theta \mu_\phi} = \sqrt{\varepsilon_\phi \mu_\theta} = 1$, leading to $\mu_{r\theta\phi} = \begin{bmatrix} \infty & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_t \end{bmatrix}$.

Figure 6-1. Schematic of spherical designer LSP.

6.2 Scattering cross section of spherical designer LSP

To disclose the EM response of the spherical DSP resonator, we simplify the resonator as a core-shell structure shown in Fig. 6-2, and study its scattering cross section. The core is a PEC sphere, and the shell can be equivalent to a homogeneous anisotropic media. The DSP resonator is exposed under a linearly polarized plane wave with unit magnitude $E_i = \hat{x}e^{ik_0z}$ along $\hat{z}$ direction, where k_0 is the wavevector in vacuum. $e^{-i\omega t}$ which represents the time dependence is suppressed. According to the effective media theory discussed above, the shell can be characterized by effective permittivity:

$$\bar{\bar{\epsilon}} = \infty \hat{r}\hat{r} + \epsilon_t \hat{\theta}\hat{\theta} + \epsilon_t \hat{\phi}\hat{\phi} \quad (6.1)$$

$$\bar{\bar{\mu}} = \infty \hat{r}\hat{r} + \mu_t \hat{\theta}\hat{\theta} + \mu_t \hat{\phi}\hat{\phi} \quad (6.2)$$

Figure 6-2. Core-shell model of spherical designer LSP.

where μ_t and ϵ_t are permeability and permittivity along the $\hat{\theta}$ and $\hat{\phi}$ direction. The formal expressions for the EM wave in the resonator are first studied. For the cases of source free, the fields inside the resonator can be decomposed into transverse electric (TE) / transverse magnetic (TM) modes (with respect to $\hat{r}$) by employing the scalar potentials Ψ_{TE} and Ψ_{TM} [116]:

$$\vec{B}_{TM} = \nabla \times (\hat{r}\Psi_{TM}) \quad (6.3)$$

$$\vec{D}_{TM} = \frac{1}{-i\omega} \nabla \times \bar{\mu}^{-1} \cdot \nabla \times (\hat{r} \Psi_{TM}) \quad (6.4)$$

$$\vec{B}_{TE} = \frac{1}{-i\omega} \nabla \times \bar{\epsilon}^{-1} \cdot \nabla \times (\hat{r} \Psi_{TE}) \quad (6.5)$$

$$\vec{D}_{TE} = -\nabla \times (\hat{r} \Psi_{TE}) \quad (6.6)$$

According to Eqs. (6.1-6.6), and after some derivations, we can get the wave equations for Ψ_{TM} and Ψ_{TE} :

$$\left\{ \frac{\partial^2}{\partial r^2} + k_t^2 + \text{SR} \cdot \left[\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right] \right\} \Psi = 0 \quad (6.7)$$

Where $k_t = \omega \sqrt{\epsilon_t \mu_t}$; for TE wave, $\text{SR} = \mu_t / \mu_r$, and for TM wave, $\text{SR} = \epsilon_t / \epsilon_r$. Utilizing the variables separation method and assuming $\Psi = f(r)g(\theta)h(\varphi)$, we get $h(\varphi) = e^{\pm i n \varphi}$ as harmonic functions, $g(\theta) = P_n^m(\cos \theta)$ as associated Legendre polynomials, and $f(r)$ as the solution of the equation below:

$$\left[\frac{\partial^2}{\partial r^2} + k_t^2 + \text{SR} \cdot \frac{n(n+1)}{r^2} \right] f(r) = 0 \quad (6.8)$$

For the infinite media we study here, we can take $\text{SR} = 0$. Therefore, no matter what number n takes, the solution of Eq. (6.8) is:

$$f(r) = k_t r b_0(k_t r) \quad (6.9)$$

where b_0 is the zeroth spherical Bessel function. According to the analysis above, it can be seen that the solutions of Eq. (6.7) in the shell layer consists of a superposition of Bessel functions, associated Legendre polynomials, and harmonic functions.

For the purpose of matching the boundary conditions on both the inner and outer surface, the incident wave are expanded with spherical harmonics. Using the solutions of Eq. (6.7) in the shell layer, the scalar potentials are obtained for the incident fields ($r > R_2$), the scattered fields($r > R_2$), and fields in the shell layer ($R_1 < r < R_2$), respectively, to be of the form:

$$\Psi_{TM}^i = \frac{\cos\varphi}{\omega} \sum_n a_n \psi_n(k_0 r) P_n^1(\cos\theta) \quad (6.10)$$

$$\Psi_{TE}^i = \frac{\sin\varphi}{\omega\eta_0} \sum_n a_n \psi_n(k_0 r) P_n^1(\cos\theta) \quad (6.11)$$

$$\Psi_{TM}^s = \frac{\cos\varphi}{\omega} \sum_n a_n T_n^{(M)} \zeta_n(k_0 r) P_n^1(\cos\theta) \quad (6.12)$$

$$\Psi_{TE}^s = \frac{\sin\varphi}{\omega\eta_0} \sum_n a_n T_n^{(M)} \zeta_n(k_0 r) P_n^1(\cos\theta) \quad (6.13)$$

$$\Psi_{TM}^c = \frac{\cos\varphi}{\omega} \sum_n \{d_n^{(M)} \psi_n(k_t(r - R_1)) + f_n^{(M)} \chi_n(k_t(r - R_1))\} P_n^1(\cos\theta) \quad (6.14)$$

$$\Psi_{TM}^c = \frac{\sin\varphi}{\omega\eta_0} \sum_n \{d_n^{(N)} \psi_n(k_t(r - R_1)) + f_n^{(N)} \chi_n(k_t(r - R_1))\} P_n^1(\cos\theta) \quad (6.15)$$

where $a_n = \frac{(-i)^{-n}(2n+1)}{n(n+1)}$, $n = 1, 2, 3, \dots$, $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. $T_n^{(M)}$, $T_n^{(N)}$, $d_n^{(M)}$, $d_n^{(N)}$, $f_n^{(M)}$, and $f_n^{(N)}$ are unknown expansion coefficients, and $\psi_n(\xi)$, $\chi_n(\xi)$ and $\zeta_n(\xi)$ are the Riccati-Bessel functions of the first, the second, and the third kind respectively. With Eq. (6.10 – 6.15), the electromagnetic fields of the outer and shell regions can be expanded according to the corresponding scalar potentials. By using the boundary conditions at both the inner and outer surfaces, two equations can be obtained at $r = R_1$ and other four equations are obtained at $r = R_2$. Boundary condition for E_ϕ at $r = R_1$ is obtained as:

$$\frac{1}{\mu_\theta \varepsilon_\phi} \left[d_n^{(M)} k_t \psi'_0(k_t R_1) + f_n^{(M)} k_t \chi'_0(k_t R_1) \right] = 0 \quad (6.16)$$

According to the continuous H_θ at the boundary of $r = R_2$, we can obtain the coefficients below:

$$T_n^{(M)} = \frac{\varepsilon_\phi A^{(M)} \psi'_n(k_0 R_2) - B^{(M)} \psi_n(k_0 R_2)}{B^{(M)} \zeta_n(k_0 R_2) - \varepsilon_\phi A^{(M)} \zeta'_n(k_0 R_2)} \quad (6.17)$$

$$T_n^{(N)} = \frac{\mu_\phi A^{(N)} \psi'_n(k_0 R_2) - B^{(N)} \psi_n(k_0 R_2)}{B^{(N)} \zeta_n(k_0 R_2) - \mu_\phi A^{(N)} \zeta'_n(k_0 R_2)} \quad (6.18)$$

where $A^{(M)} = k_t \sin(k_t R_2) \cdot \sin(k_t R_1) + k_t \cos(k_t R_1) \cdot \cos(k_t R_2)$; $B^{(M)} = k_t^2 \cos(k_t R_2) \cdot \sin(k_t R_1) + k_t^2 \cos(k_t R_1) \cdot \sin(k_t R_2)$; $A^{(N)} = -\sin(k_t R_2) \cdot \cos(k_t R_1) + \sin(k_t R_1) \cdot \cos(k_t R_2)$; $B^{(N)} = -k_t \cos(k_t R_2) \cdot \cos(k_t R_1) - k_t \sin(k_t R_2) \cdot \sin(k_t R_1)$. The total scattering cross section can be calculated with:

$$\text{SCS} = \frac{2\pi}{k^2} \sum_n (2n+1) (|T_n^{(M)}|^2 + |T_n^{(N)}|^2). \quad (6.19)$$

According to theoretical analysis above, TM modes are in low-frequency region, whereas TE modes locate in higher-frequency region. To gain more insight into the resonance modes of a spherical DSP resonator, we turn to eigen modes analysis. Eigen modes of this spherical DSP resonator can also be obtained with the governing equation Eq. (6.7) by imposing the boundary conditions at $r = R_1$ and $r = \text{infinity}$. Otherwise we can just simply assume incident wave to be zero, and let SCS $T_n^{(M)}$ or $T_n^{(N)}$ diverge, the diverging condition $B^{(M)}\zeta_n(k_0 R_2) = \varepsilon_\phi A^{(M)}\zeta'_n(k_0 R_2)$ or $B^{(N)}\zeta_n(k_0 R_2) = \mu_\phi A^{(N)}\zeta'_n(k_0 R_2)$ can give the resonance condition. Linearly polarized incident wave can be expanded with spherical harmonic functions only of order $m = 1$, so according to modes matching only resonance modes with $m = 1$ can be excited. Here we take $R_1 = 20$ mm, and $R_2 = 50$ mm. Radial components of electric field are shown in Fig. 6-3 below for the three lowest-order TM modes at $r = 51$ mm, which is 1 mm outside of the sphere. They correspond to modes $(n = 1, m = 1)$, $(n = 2, m = 1)$, and $(n = 3, m = 1)$ respectively.

Figure 6-3. Three lowest-orders TM modes.

6.3 Realization of spherical designer LSP

Figure 6-4. Image of 3D printed sample.

Due to the complexity of the structure, conventional mechanical treatment cannot achieve it. Here, we realize it (as shown in Fig. 6-4) with modern 3D printing technology which fabricates samples layer-by-layer. The material is stainless steel which can behave as PEC in microwave. The inner core radius is $R_1 = 20$ mm, and the outer radius is $R_2 = 50$ mm. The pillars are grooved equally with $\theta_d / \theta_a = \phi_d / \phi_a = 2:1$. In θ direction, pillars are grouped into 18 circles, and each circle consists of 36 pillars except the top and bottom (not shown)

circles. The reason is that pillars in these two circles are too thin to stand steady, so we reduce the pillars number to 12. Although the pillars are less, both the ratios θ_d / θ_a and ϕ_d / ϕ_a are still kept to be 2:1.

Figure 6-5. Experiment setup of transmission measurement.

Then we characterize electromagnetic property of the spherical designer LSP with far-field transmission configuration as shown in Fig. 6-5 above. The whole setup is placed in an anechoic chamber. Two linearly polarized horn antennas are setup face to face, and polarization direction is horizontal. Working frequency of the antennas covers 0.8 ~ 18 GHz. The sphere is placed in the middle of the two antennas. Then we change the polarizations of incident wave, however the observed spectrum have little changes. That proves the uniformity of this spherical DSP resonator.

The measured transmission spectrum is shown in Fig. 6-6 as below, along with the theoretical results calculated with Eq. (6.19), and simulated SCS spectrum. Here, we only measured forward scattering spectrum, not the total; so we do not compare magnitudes of theoretic and experimental results. It can be found that simulation and experiment results are well consistent, but the theoretical result shows a little discrepancy. The measured three lowest-frequency peaks shown in Fig. 6-6 locates at 1.07 GHz, 1.58 GHz, and 1.80 GHz respectively, and we can find corresponding counterparts in the theoretical spectrum at 0.95 GHz, 1.44 GHz, and 1.75 GHz.

Figure 6-6. Comparison between theoretical, simulation and experimental results.

6.4 Discussion

The two higher-frequency peaks in theoretical results are not observed in both simulation and experimental results. It can be attributed to the higher Q factor of the higher-order modes, which cannot be efficiently excited. Compared with simulation and experimental results, the little discrepancy in theoretical result may be due to the nonlocal effect which is not included in effective media theory.

6.5 Conclusions

In this chapter, we designed a spherical metamaterial structure, which can support spherical LSP-like surface EM modes. Incorporating effective media theory into Mie theory, resonance peaks of the spherical metamaterial structure is shown in calculated radar cross section. 3D printing technology is employed to realize this structure. Measurement results are consistent with simulation and theoretical analysis.

SUMMARY

The thesis extensively studied the designer surface plasmonic (DSP) resonator in microwave frequencies which is proposed to mimic properties of metallic nanostructures in optical frequency.

This thesis first study a single DSP resonator from the functional aspect. A metal ground plane is introduced to tune the intrinsic dispersion of resonator, and enhance high-order resonance modes which are not observed in previous experiments.

After a single resonator, we studied the coupling between two 2D resonators, which can induce some novel physics. Firstly, we start from the horizontal coupling configuration, where two resonators are placed on the same planar substrate. For a multipolar mode of some order, two degenerate modes with different parity are supported in a single resonator. Experiments demonstrates that the sign of coupling strength can be reversed by exciting modes with different parity. Except being placed in the same plane, two 2D resonators can also been stacked. Here we call it vertical coupling configuration which is totally different from the horizontal coupling above. In the vertical coupling configuration, coupling can be very strong when we decrease the spacer to a small thickness. Interference between multipolar modes of successive orders are observed. Spectral minimums with asymmetrical line-shapes are observed in near-field transmission spectra.

Then, 2D resonators with modified geometries (called ring resonators) are arranged as a square lattice array in a plane. The quasi-energy band structures are calculated with an abstract network model. Topological edge states are predicted when strong coupling occurs between neighbouring ring resonators. With the help of structural flexibility of this artificial resonator, we implemented various time-reversal-invariant defects to probe the limits of robustness of electromagnetic topological edge states. Experimental results show that although all defect are time-reversal-invariant, some of them can still break the topological protection, which are consistent with simulation results.

For the purpose of enriching physics of DSP resonators, we then extend this artificial cylindrical resonator to a spherical case. Results measured from a scattering experiment are well consistent with theoretical analysis based on effective media theory.

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