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FAULT TOLERANT CONTROL FOR SPACECRAFT ATTITUDE CONTROL SYSTEMS

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Date

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Abstract

Future spacecraft are expected to achieve highly accurate pointing, fast slewing from large initial condition, and reliable maneuvers in the presence of environmental disturbances, unknown or uncertain inertia parameters, control input saturation, and component failures. Actuators play an important role of generating control torque/force in the desired direction under command from the controller to achieve specific aerospace objectives. However, an abrupt occurrence of an actuator fault/failure could significantly disrupt mission performance or even lead to totally loss of the spacecraft. In order to enhance the reliability and reduce critical failure of spacecraft, fault-tolerant capability should be addressed in the control system of spacecraft. Generally, the available fault-tolerant control (FTC) schemes can be classified into two categories: active FTC and passive FTC. In this thesis, both active and passive FTC approaches are developed for attitude control systems of spacecraft to enhance reliability and safety while achieving high precision attitude maneuver.

Two control allocation (CA) based active FTC schemes are developed for overactuated spacecraft attitude control systems to accommodate actuator faults/failures. The active FTC approach reacts to the system component malfunctions by reconfiguring the controller based on real-time information from a fault detection and diagnosis (FDD) scheme. The proposed two active FTC schemes use the estimated fault information in the on-line CA to redistribute the virtual control efforts into

each individual actuator judiciously without reconfiguring the controller. In the first active FTC scheme, taking the effects of FDD imprecision into account, an inertia-free adaptive fault-tolerant attitude tracking controller is developed to handle multiplicative actuator faults/failures. A novel time-varying dead-zone modification is proposed in the design of parameter adaptation to stop the adaptive gain from increasing, and it also improves the robustness of the adaptive law. In the second active FTC scheme, instead of ensuring robustness with respect to imprecise fault estimation in controller design, a novel robust control allocation (RobCA) is proposed to ensure some robustness in CA design such that design complexity of the controller can be reduced. Consequently, a model reference adaptive high-level controller incorporating RobCA is designed to achieve asymptotic attitude tracking irrespective of actuator faults/failures, imprecise fault estimations, external disturbances, and actuator amplitude and rate constraints.

Contrary to CA-based active FTC method, passive FTC method utilizes an unique robust controller to tackle all expected faults. It has a simple structure as neither FDD scheme nor a controller reconfiguration mechanism is needed. This thesis also presents two passive FTC strategies to solve the fault-tolerant attitude control stabilization problem for a rigid spacecraft. Firstly, based on the establishment of an integral-type sliding manifold and a novel simple saturated proportional-derivative control law, an adaptive FTC strategy is developed such that the resultant closed-loop system is capable of tolerating partial loss of control effectiveness fault and additive fault. Furthermore, with consideration of inertia uncertainties, actuator redundancy, and actuator saturation constraints, another finite-time fault-tolerant attitude stabilization scheme is proposed to guarantee that the attitude and angular velocity converge to a neighbourhood of origin in finite time despite four types of common actuator faults/failures.

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Symbols and Acronyms

$\|\cdot\|$ Euclidean norm of vector or its induced norm of matrix.

$\|\mathbf{x}\|_p$ p -norm of a vector $\mathbf{x}$ for $p \geq 1$.

$\lambda_{min}(\cdot)$ Minimum eigenvalue of a matrix.

$\mathbf{I}_n$ n -by- n identity matrix.

$\mathcal{I}$ Inertial fixed reference frame.

$\mathcal{B}$ Body-fixed reference frame.

$\mathcal{R}$ Desired reference frame.

$\mathbf{Q} \in \mathbb{R}^4$ Unit-quaternion in body frame $\mathcal{B}$ w.r.t inertial frame $\mathcal{I}$; $\mathbf{Q} = [q_0, \mathbf{q}^T]^T$,
 $\mathbf{q} \in \mathbb{R}^3$; $q_0^2 + \mathbf{q}^T \mathbf{q} = 1$.

$\mathbf{Q}_d \in \mathbb{R}^4$ Desired attitude trajectory representing by unit-quaternion in body frame
 $\mathcal{B}$ w.r.t desired reference frame $\mathcal{R}$; $\mathbf{Q}_d = [q_{d0}, \mathbf{q}_d^T]^T$, $\mathbf{q}_d \in \mathbb{R}^3$; $q_{d0}^2 + \mathbf{q}_d^T \mathbf{q}_d = 1$.

$\mathbf{Q}_e \in \mathbb{R}^4$ Error quaternion representing the relative orientation between $\mathbf{Q}$ and $\mathbf{Q}_d$;
 $\mathbf{Q}_e = [q_{e0}, \mathbf{q}_e^T]^T$, $\mathbf{q}_e \in \mathbb{R}^3$; $q_{e0}^2 + \mathbf{q}_e^T \mathbf{q}_e = 1$.

$\otimes$ Quaternion multiplication operator.

$\boldsymbol{\omega} \in \mathbb{R}^3$ Angular velocity expressed in body frame $\mathcal{B}$.

$\boldsymbol{\omega}_d \in \mathbb{R}^3$ Desired angular velocity expressed in the desired reference frame $\mathcal{R}$.

$\boldsymbol{\omega}_e \in \mathbb{R}^3$ Angular velocity tracking error.

$\boldsymbol{C}(\boldsymbol{Q}_e) \in \mathbb{R}^{3 \times 3}$ Directional cosine matrix associated with $\boldsymbol{Q}_e$.

$\boldsymbol{x}^\times$ Cross-product matrix associated with vector $\boldsymbol{x} = [x_1, x_2, x_3]^T$.

$\boldsymbol{J} \in \mathbb{R}^{3 \times 3}$ Symmetric positive-definite inertia matrix.

$\boldsymbol{D} \in \mathbb{R}^{3 \times n}$ Actuator distribution matrix.

$\boldsymbol{\tau} \in \mathbb{R}^3$ Total control torques.

$\boldsymbol{u} \in \mathbb{R}^n$ Applied control torques generated by n actuators.

$\boldsymbol{u}_c \in \mathbb{R}^3$ Commanded control torques from controller.

$\boldsymbol{d} \in \mathbb{R}^3$ External disturbances.

$\boldsymbol{s} \in \mathbb{R}^3$ Sliding manifold or filtered angular velocity error.

$\boldsymbol{u}_{nom}$ Nominal controller.

$\text{Proj}_\theta(\bullet)$ Projection operator for scalar variable θ .

$\boldsymbol{E}(t) \in \mathbb{R}^{n \times n}$ Actuator effective matrix.

$\bar{\boldsymbol{u}}(t) \in \mathbb{R}^n$ Additive actuator fault vector.

$\hat{\boldsymbol{E}}(t) \in \mathbb{R}^{n \times n}$ Estimated actuator effective matrix from fault diagnosis scheme.

$\hat{\bar{\boldsymbol{u}}}(t) \in \mathbb{R}^n$ Estimated additive actuator fault vector from fault diagnosis scheme.

$\boldsymbol{J}_w \in \mathbb{R}^{3 \times 3}$ Diagonal inertia matrix of n reaction wheels.

$\boldsymbol{\Omega} \in \mathbb{R}^n$ Angular velocity of n reaction wheels.

$\boldsymbol{\Delta}_E \in \mathbb{R}^{n \times n}$ Diagonal matrix representing the level of imprecision in the estimation of actuator effectiveness.

$\Delta_{\bar{u}} \in \mathbb{R}^{n \times n}$ Diagonal matrix representing the level of imprecision in the estimation of additive actuator fault.

$\bar{\omega}_d \in \mathbb{R}^3$ Auxiliary desired angular velocity.

$D_\epsilon[\cdot]$ Dead-zone operator.

$D^\dagger$ Pseudo-inverse of matrix D ; $D^\dagger = \hat{E}^3(t)D^T(D\hat{E}^3(t)D^T)^{-1}$.

(ψ, θ, ϕ) Euler angles: yaw, pitch, roll.

FTC Fault-tolerant control.

FDD Fault detection and diagnosis.

SMC Sliding mode control.

MRA Model reference adaptive.

CA Control allocation.

RobCA Robust control allocation.

SCOP Second-order cone programming.

RW Reaction wheel.

ISMC Integral-type sliding mode control.

FTSM Fast terminal sliding mode.

PDC Proportional-derivative controller.

IRA-FTC Indirect robust adaptive FTC.

SIRA-FTC Saturated version of IRA-FTC.

AFT-FTC Adaptive finite-time FTC.

SAFT-FTC Saturated version of AFT-FTC.

Chapter 1

Introduction

In safety-critical systems such as spacecraft and aircraft, reliability is particularly important as a minor fault in such systems can result in significant performance degradation or even instability. The attitude control subsystem of a 3-axis stabilized spacecraft is of great importance in many space missions, and the life of attitude control components essentially decides the operational life of a spacecraft. Its degradation can significantly disrupt mission performance, and its failure is fatal to the spacecraft.

As the heart of spacecraft, the attitude control subsystem basically consists of three parts: sensors, actuators, and control electronics (on-board computer). Actuators play an important role of generating control torque/force in the desired direction under command from the controller to achieve specific aerospace objectives. However, when a fault occurs in the actuator, the influence of the controller on the spacecraft might be interrupted or modified. As a result, if the controller is designed without any fault tolerance capability, an abrupt occurrence of an actuator fault could significantly disrupt mission performance or even lead to totally loss of the spacecraft.

In order to ensure system reliability and safety, the attitude control subsystem of

spacecraft has to be capable of tolerating potential faults and maintaining desirable system performance. A way to ensure reliable operation of systems in the presence of undesirable faults is by means of fault-tolerant control (FTC) strategies [2–4]. An FTC system has the capacity of compensating for the effects of faults automatically and maintaining the overall system stability with acceptable system performance despite the occurrence of component faults. It satisfies the industrial demand for enhanced availability and safety, in contrast to traditional reactions to faults that bring about sudden shutdowns and loss of availability [5]. As is well known, the available FTC schemes can be generally classified into two categories, i.e. active and passive ones. Even though the control objectives of these two approaches are the same, each method can give rise to some unique properties due to its distinctive design mechanisms [6]. Besides safety and reliability requirements, due to increasing mission demands for performance, modern spacecraft is expected to achieve high-precision pointing and fast rotational maneuver in the presence of large environmental disturbances, modeling uncertainties, and actuator constraints [7]. In this thesis, both active FTC and passive FTC approaches are developed for spacecraft attitude control subsystems to handle actuator faults/failures and maintain desired system performance despite environmental disturbances, modeling uncertainties, and actuator constraints.

1.1 Literature Review

1.1.1 Spacecraft Attitude Control

The attitude control subsystem is required to orient and maintain the spacecraft or the application payloads, such as camera, antennae, and radiometer, in a desired direction. For some satellite, a number of primary mission tasks such as earth observation and space monitoring, require attitude maneuver throughout

their lifetime and even in their final mission stage. As a result, the attitude control problem of spacecraft has attracted considerable attention, and various nonlinear control approaches have been proposed in the literature, such as nonlinear feedback control [8], sliding mode control (SMC) [9–12], inverse optimal control [13,14], adaptive control [15–17]. Specially, SMC based scheme is known as an efficient way to withstand matched external disturbances and model uncertainties, and has been widely adopted in the area of spacecraft attitude control. Meanwhile, sliding mode controllers are always designed with the aid of adaptation mechanisms to accommodate large uncertain structural and parametric variation. In [18], an adaptive sliding mode attitude controller was designed to adjust the current attitude of a spacecraft to track the desired attitude despite external disturbances and internal inertia variations. In [19], two sliding mode control laws were presented by combining the adaptive technique and extended state observer to solve the attitude tracking problem of a rigid spacecraft with external disturbances and an unknown inertia matrix. In [20], with consideration measurement errors, guaranteed analytical performance bounds are obtained by using an adaptive sliding mode controller for spacecraft attitude tracking systems.

Although all these approaches can accomplish attitude stabilization or tracking maneuver, most of them only guarantee asymptotic stability, which means that the system states converge to the equilibrium as time tends to infinity. For some spacecraft missions, such as remote sensing or reconnaissance, the spacecraft is required to achieve rapid attitude maneuvering together with high targeting accuracy and stability between the maneuvers [7]. Thus, finite-time convergence becomes applicable, as it ensures fast convergence, high precision control performance, and better disturbance rejection [21,22]. Owing to these advantages, a number of researchers have investigated finite-time spacecraft attitude control recently. In [23], based on terminal sliding mode control approach, two robust control strategies were pro-

posed to achieve finite-time attitude tracking of a rigid spacecraft. As an extension to [23], fast terminal sliding mode (FTSM) control schemes were presented in [24] for spacecraft attitude control. In [25], Chebyshev neural network was introduced to approximate the desired system nonlinearities and bounded external disturbances, and a finite-time attitude tracking control scheme was derived by using modified FTSM. Based on geometric homogeneity technique, two simple PD controllers were proposed for finite-time stabilization of spacecraft in [26] to achieve fast transient and high-precision performance.

Another key issue that should be addressed in spacecraft attitude control system is actuator saturation. From a view point of practical application, the spacecraft actuators are not able to provide any requested torque because the available control torque amplitude is always limited in actual spacecraft. If the controller requires more control efforts than the actuators can supply, the degraded or unpredictable performance and thermal or mechanical failure may emerge [27]. Recognizing these difficulties, a number of spacecraft attitude control schemes which handle actuator saturation have been proposed. In [28], a saturated feedback control law on the basis of a non-standard description of the attitude motion was developed to achieve attitude tracking for an underactuated spacecraft. Boskovic et al. [29] proposed a robust continuous tracking algorithm for a rigid spacecraft in the presence of parametric uncertainty, external disturbances, and control input saturation. The algorithm was based on variable structure control design approach and a single parameter was adapted dynamically to guarantee global stability of the overall system. In [30], a bounded attitude-stabilizing control law by using hyperbolic tangent function was developed. In [31], by combining SMC approach and adaptive technique, the attitude stabilization problem of a rigid spacecraft with control input saturation was solved. In [32], two simple saturated PD control laws were designed to ensure globally asymptotic stabilization of spacecraft subject to actuator saturation in terms

of the Rodrigues parameter attitude representations and the modified Rodrigues parameter attitude representations, respectively. Recently, in [33], a global finite-time controller based on homogeneous system theory was developed to stabilize the attitude and angular velocity of a spacecraft under actuator saturation constraint.

Nevertheless, it should be pointed out that the aforementioned approaches do not consider actuator faults explicitly, and thus the attitude control system may suffer performance degradation or critical failure when actuator faults happen. In order to enhance safety and reduce critical failure, fault tolerance of attitude control system is one of the key issues that needs to be addressed. Moreover, in the event of actuator faults/failures, the remaining actuators are required to generate higher torques to compensate for the faulty actuators to maintain system performance [34–37]. As a result, actuator faults/failures may potentially lead to saturation of the actuators, which will lead to further damage to the rest of the system or even instability of the closed-loop system if there is no appropriate control strategy to dump the saturated actuators [34,37]. Hence, the problem of actuator saturation should be considered in the fault-tolerant control system design as discussed in the survey paper [4,38].

1.1.2 Active Fault Tolerant Control

The active FTC approach reacts to the occurrence of component faults on-line by reconfiguring the controller based on a FDD scheme which provides real-time information about the fault, so that the capability of the closed-loop system to achieve its given objective is preserved despite faults. The typical structure of an active FTC system is shown in Fig. 1.1. Based on this structure, the design objectives of an active FTC system can be stated as follows:

1. design an effective FDD scheme to not only detect the fault but also identify the fault location and fault magnitude with sufficient precision in a timely manner;

2. reconfigure the existing controller to compensate the effect of fault so that the stability and acceptable performance of the overall closed-loop system can be maintained.

A number of active FTC methods have been proposed in literature, such as eigen-structure assignment [39], adaptive control [40], multiple model [41,42], model predictive control [43], SMC based approach [44], and neural networks/fuzzy logic based scheme [45–47].

Figure 1.1: Graphical illustration of the active FTC system.

As shown in Fig. 1.1, active FTC can reconfigure the controller by using the real-time information provided by the FDD scheme. Since fault information can be obtained in active FTC approach, the optimal solution of the reconfigured controller may exist with certain pre-set performance criteria. Another advantage of an active FTC approach is that it is more flexible to deal with different types of faults, including failure scenario beyond designed basis faults. Moreover, the controller used in the active FTC approach is less conservative than that of the controller used in a passive FTC approach. However, the performance of an active FTC approach depends heavily on the accuracy of the FDD scheme. An error or imprecision in FDD can lead to loss of effectiveness in the synthesized reconfigurable controller. In

addition, the amount of available time for FDD and control reconfiguration is very limited. If either FDD scheme or control reconfiguration process takes an unexpectedly long time, system states may become unbounded due to the effect of faults and the performance of the system may deteriorate severely. Lastly, the undesirable transients in switching between the nominal pre-fault controller and reconfigured controller may generate shocks to the systems, and potentially give rise to further damage to the system components.

Although various active FTC schemes have been developed in different application areas, most of them are only feasible for linear systems and unified methods for dealing with nonlinear active FTC are not yet available. Particularly, because the attitude kinematics and dynamics of a spacecraft are nonlinear and strongly coupled, only very few research papers consider the active fault-tolerant controller design for spacecraft attitude control system. In [48], actuator failure detection, identification and adaptive reconfigurable controller for spacecraft were proposed, where fast and accurate detection and diagnosis of actuator failure, and convergence of tracking errors to zero can be achieved despite the constraint of control input saturation. In [49], an iterative learning observer was designed to estimate time-varying actuator faults. Based on the FDD scheme developed in [49], an FTC law was reconfigured in [50] to accomplish attitude stabilization maneuver under partial loss of actuator effectiveness fault and external disturbances. In [51], the problem of nonlinear fault detection, isolation, and recovery for a rigid spacecraft's orbital and attitude control was investigated, where system anomalies caused by faults were viewed as changes of certain parameters in the system and least-squares parameter estimation techniques were used to generate residuals. In [52,53], based on the reconstructed faults and disturbances information from a terminal sliding mode observer, a fault compensation controller was developed for spacecraft to follow desired attitude trajectories after a finite time despite actuator faults.

The availability of actuator redundancies greatly enhances the achievable system performance for FTC systems, and modern spacecraft often use redundant actuators to improve their reliability and survivability. Since more actuators than strictly needed are used to meet the control objectives, the control system with actuator redundancies is over-actuated [54,55]. To make effective use of these redundancies, control allocation (CA) is one promising and effective approach [56], which distributes the virtual control efforts from the high level controller to the individual actuators, especially in the case of actuator faults and failures [57], [58]. The benefits of using CA as a means for FTC are that the virtual control efforts can be reallocated to the remaining available actuators without reconfiguring the controller [59], while allowing flexibility in choice of control technique to compensate the effects of actuator faults/failures [60]. In [61], adaptive CA scheme was developed to handle loss of actuator effectiveness in nonlinear discrete-time systems. Recently, in [59] and [62], SMC techniques are combined with CA to develop fault-tolerant controllers for over-actuated linear systems when actuators were subject to partial loss of effectiveness or complete failures. The results were further extended to the situation where only measured output were available in [63] by using a full order linear unknown input observer. Particularly, in [64], a velocity-free nonlinear proportional-integral controller is designed as high-level controller for spacecraft attitude stabilization problem and a robust least-squares-based CA is employed to distribute the three-axis moments over the available thrusters under redundancy. However, most of the aforementioned control allocation strategies are proposed and implemented in an open-loop manner [65,66], as the stability of such closed-loop system under using control allocation with the baseline controller could not be guaranteed theoretically [67].

1.1.3 Passive Fault-Tolerant Control

Passive FTC is also referred to as reliable control in literature [4]. In a passive FTC approach, a set of component malfunctions (including actuators, sensors, and system itself) is assumed known a priori as designed basis faults, and all faulty modes are considered together with the normal system operating conditions at the design stage. Based on robust control theory, a single fixed fault-tolerant controller is synthesized so that the system is able to achieve its given objectives throughout the healthy situation and the faulty situation. The general structure of a passive FTC system is shown in Fig. 1.2. Since neither FDD capability nor controller re-configuration is required, a passive FTC approach is simpler to implement from a practical point of view. Furthermore, it has the advantage of avoiding time delay between the online FDD and controller reconfiguration in active FTC. In practice, the fault may progress rapidly and the critical system may even become unstable before a desired active fault-tolerant controller can be synthesized [46,68]. Therefore, passive FTC schemes are necessary during the FDD phase, to ensure the boundedness and stability of the faulty systems, before switching to the reconfigured controller. However, the passive approach is often conservative because the achieved system performance is based on worst case fault or failure. Moreover, the fault-tolerant capabilities of passive FTC are limited, especially in the case that the number of fault scenarios increases beyond the considered basis faults. As the number of fault cases increases, the stability of the overall system and acceptable performance cannot be guaranteed and one may not be able to find a passive FTC scheme taking all fault scenarios together into consideration.

Some effective passive FTC approaches have been proposed for both linear systems and nonlinear systems, such as H_∞ theory [69], linear matrix inequalities (LMIs) techniques [70], Lyapunov reconstruction [68,71], and SMC [72]. Regarding actuator faults in spacecraft attitude control systems, there have been a great deal of results

Figure 1.2: Graphical illustration of the passive FTC system.

in the literature. For instance, in [73], sliding mode fault-tolerant controller was proposed for a set of second-order nonlinear systems under actuator faults, where the proposed fault-tolerant controller was shown to result in local asymptotic stability when it was applied to spacecraft attitude stabilization. In [74], an indirect robust adaptive FTC strategy for spacecraft attitude tracking was presented to accommodate actuator failures. In [75], by using dynamic inversion and time-delay control theory, an FTC method for attitude tracking control with four reaction wheels was developed. Using backstepping techniques, an adaptive FTC scheme was developed for spacecraft attitude stabilization in [76], where actuator faults were estimated by three implicit estimation filters and effective attitude stabilization was achieved. In [77], two adaptive fault-tolerant sliding mode control laws were developed for multiple spacecraft formation flying, where the global asymptotic convergence of the position tracking error was achieved in the presence of uncertain system parameters, external disturbances, and actuator faults. In [78], adaptive sliding mode fault-tolerant attitude tracking control scheme was developed for flexible spacecraft with partial loss of actuator effectiveness fault, where a neural network was employed to account for system uncertainties and an on-line updating law was used to estimate the upper bound of actuator fault information. With the use of fuzzy logic and TSMC technique, a decentralized adaptive fuzzy fault-tolerant controller was proposed for satellite attitude synchronization in the presence of external disturbances and actuator faults, where the fuzzy logic system was introduced to approximate un-

known individual satellite attitude dynamics caused by actuator faults [79]. In [80], a velocity-free feedback FTC law was proposed to solve the problem of fault-tolerant attitude stabilization for a rigid spacecraft suffering from partial loss of actuator effectiveness fault and actuator saturation. In [81], a fault-tolerant attitude tracking controller based on variable structure approach was proposed for attitude control of spacecraft, where the transient response of the closed-loop system was controlled by suitable choice of a dedicated parameter.

1.2 Motivation

A recent satellite failure study [82] shows that the attitude control subsystem is a major contributor to spacecraft unreliability and is responsible for approximately 20% of all spacecraft failures. Comparing with other subsystems in a spacecraft, the attitude control subsystem has higher number of anomalies that leads to a premature loss of a spacecraft or loss of its ability to perform its primary mission during its design life [83]. In [84], 156 on-orbit failures that occurred on 129 different civil and military spacecrafts from 1980 to 2005 are studied. Based on the results from [84], the component failures involved in attitude control subsystems are shown in Fig. 1.3. It is observed that 44% of attitude control system failures are attributed to actuators including thruster, reaction wheel, momentum wheel, and control momentum gyro (CMG). Faults/failures associated with actuators may result in performance loss, mission interruption, or even total loss of the spacecraft. For example, in September 1997, NASA's Earth Orbiting Lewis Spacecraft suffered a serious malfunction in the attitude control system, which led to improper vehicle attitude, inability to charge the solar array, discharge of batteries, and loss of command and control. The spacecraft failed on 28 September 1997, and the investigation board report stated that the failure of spacecraft was mainly due to a

technical flaw in the attitude control system design which result in excessive thruster firings and eventual shutdown of all thrusters [85]. Another accident occurred with TOPEX, a satellite developed by U.S. and French to map ocean surface topography. During October 2005, the satellite could not perform attitude maneuvers because of the failure of pitch reaction wheel, and its mission was ended [86]. More examples of actuator faults/failures in spacecraft attitude control subsystems can be found from the satellite digest website [86]. Since actuators of on-board spacecraft are prone to faults/failures, actuator fault tolerance capability needs to be addressed in attitude control subsystems such that the desirable degree of overall stability and performance can be achieved despite the presence of actuator faults/failures.

Figure 1.3: Component failures in spacecraft attitude control systems

In order to enhance reliability and achieve high precision attitude pointing for future spacecraft, several significant challenges must be overcome. The problem statements for attitude control systems of future spacecraft in this thesis can be classified as follows:

- **Prob1:** *Nonlinear spacecraft models.* Since the equations that govern attitude kinematics and dynamics of the spacecraft are highly nonlinear and coupled,

the problem of how to accomplish attitude maneuver with high attitude pointing accuracy is challenging.

- **Prob2:** *External disturbances and uncertain mass properties.* Disturbance signals such as gravity gradient torques, aerodynamics torques, magnetic torques and solar radiation torques, which always exist when knowledge of external environment is incomplete. As a result, the nonlinear kinematics models of spacecraft can be imprecise and high pointing accuracy may not be maintained. In addition, due to fuel consumption, deployable appendage, sensor boom or articulation, the mass properties of the spacecraft may be uncertain or may change, which makes the inertia matrix different from that determined during preflight testing [15,17]. This inertia parameter discrepancy also leads to the degradation of attitude control performance [87].
- **Prob3:** *Actuator faults/failures.* As discussed in [88], actuators such as reaction wheels or thrusters in spacecraft attitude control systems are vulnerable to several types of faults/failures, and they can be classified on the basis of their effects on the output behaviour of the actuators as partial loss of effectiveness fault, bias fault, stuck failure, and outage failure, respectively. When a fault occurs on an actuator, the actuator may be still operational but have a slower response or become less effective. However, when a failure occurs, the actuator is complete breakdown and cannot perform its required function under specified operating conditions.
- **Prob4:** *Imprecise fault estimation.* In active fault-tolerant control design, an FDD scheme is used to detect and estimate fault. However, no matter which FDD scheme is employed, it is inevitable to have some estimation or identification errors [59,71] that may introduce substantial performance degradation. Moreover, if CA are used in active FTC with imprecise knowledge of the con-

trol effectiveness, CA algorithm may not be able to realize the desired control effect, particularly when actuator saturation constraints are present [89].

- **Prob5: *Actuator constraints.*** In practice, due to the physical limitations of the actuator, it may not be able to generate typically commanded control torque. Particularly, in the event of actuator faults/failures, the remaining actuators are required to generate higher torques to compensate for the faulty actuators such that the required system performance can be maintained [34–37]. As a result, actuator faults/failures may potentially bring about saturation of the actuators, which will lead to further damage to the rest of the system or even instability of the closed-loop system if there is no appropriate control strategy to dump the saturated actuators [34,37].

1.3 Contributions

In response to the problems described in previous section, this thesis aims to develop CA-based active FTC and passive FTC to handle actuator faults/failures for attitude control systems of a rigid spacecraft in the presence of bounded external disturbances, inertia uncertainties, actuator faults/failures, and actuator constraints. The main contributions of this thesis are listed as follows:

- **Inertia-free fault-tolerant attitude tracking using CA (Prob1 - Prob4, in Chapter 3)**

A novel inertia-free adaptive FTC design incorporated with CA for spacecraft attitude tracking systems is proposed. The effectiveness level of actuator is used in the on-line CA for redistributing the virtual control efforts into each individual actuator judiciously without reconfiguring the controller, although it is estimated imperfectly from the FDD scheme. Some modifications are made for the existing on-line CA scheme proposed in [59] so that partially effective

actuators can be leveraged for control use. Unlike existing methods that estimate the elements in inertia matrix directly for solving the attitude tracking control problem, indirect adaptive techniques are employed so that inertia matrix information is not required in the FTC design. Based on the structure of the proposed indirect adaptive control technique, a novel time-varying dead-zone modification is proposed in the design of parameter adaptation to stop the adaptive gain from increasing, and which also improves the robustness of the adaptive law to measurement or system noises.

- **Robust CA for attitude tracking under actuator faults (Prob1 - Prob5, in Chapter 4)**

A model reference adaptive (MRA) controller incorporated with robust CA (RobCA) is proposed for an overactuated spacecraft. The external disturbances can be rejected by the MRA high-level controller without considering actuator faults, and the closed-loop performance is defined by a reference model, which creates a desired trajectory for the attitude tracking system to follow. Comparing with existing robust control allocation methods such as [64,90], the optimal index of minimizing the energy consumption is introduced to guarantee that the solution is unique in both unsaturated and saturated cases of RobCA. Furthermore, to reduce computation burden in the unsaturated case of RobCA, the original vector optimization problem is converted to a one-dimensional search problem.

- **Integral-type sliding mode FTC for attitude stabilization (Prob1 - Prob3, in Chapter 5)**

A new adaptive FTC law on the basis of integral-type sliding mode control (ISMC) technique for spacecraft attitude stabilization is designed to compensate for actuator faults without fault estimation and controller redesign. The

proposed adaptive FTC scheme has a simple structure and only one parameter is required to be adapted, thus simplifying the design process and reducing the online computation load significantly. Besides, since the proposed adaptive controller is within the ISMC framework, the initial sliding manifold deviation in the conventional adaptive SMC approach can be removed during the switching gain adaptation process, and thus the obtained control magnitude is less demanding than that of conventional adaptive SMC designs.

- **Finite-time FTC for attitude stabilization with actuator saturation (Prob1 - Prob3, and Prob5, in Chapter 6)**

Finite-time fault-tolerant attitude stabilization control problem is investigated for a rigid spacecraft. In order to achieve finite-time stability, FTSM manifold is introduced and the spacecraft attitude dynamics is transformed into a lagrange-like expression with respect to FTSM. With the aid of adaptation mechanism, the bounds of external disturbances, inertia uncertainties, and actuator faults are estimated and a fault-tolerant attitude control approach is proposed which can ensure finite-time convergence in the reaching phase and the sliding phase despite actuator faults/failures. Furthermore, as an extension to the designed adaptive fault-tolerant controller, control input saturation is considered in the controller design procedure. It is established that attitude and angular velocity converge to a neighbourhood of the origin in finite time without FDD scheme even in the presence of control input constraints and actuator faults/failures.

1.4 Thesis Outline

In this thesis, chapter 2 reviews the mathematic models of rigid-body kinematics and dynamics as well as actuator fault model. Chapters 3-4 propose CA-based

active FTC approaches for spacecraft attitude tracking control. Chapters 5-6 present passive FTC designs for spacecraft attitude stabilization. The outline of this thesis is shown in Fig. 1.4. The details of each chapter are as follows.

Chapter 2 presents some background knowledge on spacecraft attitude control systems and the mathematical model of actuator faults/failures. A detailed derivation of the rigid body attitude dynamics is provided along with a description of quaternion-based attitude parameterizations. The actuator faults/failures in attitude control systems are also analyzed and a mathematical model of actuator faults/failures is developed. In addition, a brief introduction of finite-time control is presented along with pertinent lemmas to allow the reader to have a better understanding of the theoretical and practical developments in this thesis. In addition, a brief introduction of finite-time control and CA is presented along with pertinent lemmas to allow the reader to have a better understanding of the theoretical and practical developments in this thesis.

Chapter 3 presents an CA-based FTC scheme for attitude tracking of an over-actuated spacecraft. Firstly, with consideration of fault estimation errors, a discontinuous FTC law incorporating on-line CA is proposed to handle actuator faults/failures and ensure asymptotic convergence of tracking errors. When information on the inertia and bounds on the disturbances are unknown, a novel inertia-free adaptive continuous FTC law is developed, where a time-varying dead-zone modification technique is employed in parameter adaptations. It is shown that uniform ultimate boundedness of the tracking errors can be ensured despite actuator faults/failures, FDD uncertainties, and external disturbances. To illustrate the efficiency of the CA-based FTC strategy, numerical simulations are carried out for a rigid spacecraft under different actuator fault and failure scenarios.

Chapter 4 develops a MRA controller incorporated with RobCA for a rigid spacecraft to reject disturbances and handle actuator faults without reconfiguring the

controller. First, MRA control technique is used to design a high-level controller to produce the three-axis virtual control torque and reject disturbances. Then, taking fault estimation uncertainties into account, a RobCA strategy is proposed to redistribute virtual control signals to the remaining actuators when an actuator fault occurs. The RobCA is formulated as a min-max optimization problem, which deals with actuator faults directly without reconfiguring the controller and ensures some robustness of system performances. Finally, Simulation results are provided to show the effectiveness of the overall control strategy.

Chapter 5 introduces an ISMC-based fault-tolerant attitude control scheme for a rigid spacecraft in the presence of external disturbances and two kinds of actuator faults. A basic integral-type sliding mode fault-tolerant control scheme is first designed so that sliding manifold can be maintained from the very beginning. Once the system enters the sliding mode, the dynamics of the closed-loop system with actuator fault is identical to that of the nominal healthy system. Then, the integral-type sliding mode fault-tolerant controller is incorporated with adaptive technique to accommodate actuator faults so that the required boundary information can be relaxed. The effectiveness of the proposed schemes against actuator faults is demonstrated in simulation.

Chapter 6 addresses the finite-time fault-tolerant attitude stabilization control problem for a rigid spacecraft under external disturbances, inertia uncertainties, and actuator saturation constraints. With consideration of the actuator redundancy, four types of common actuator faults/failures are taken into account. The proposed control law provides fault-tolerant capability despite control input saturation and also ensures that attitude and angular velocity converge to a neighbourhood of the origin in finite time.

Chapter 7 summarizes the contributions of the thesis and provides suggestions for future works.

Figure 1.4: Thesis outline

Chapter 2

Background

In this chapter, the nonlinear differential equations that govern the kinematics and dynamics of the spacecraft in terms of quaternion are provided. As reaction wheels are commonly-used actuators for three-axis stabilized spacecraft to achieve high performance attitude control, the actuator considered in this thesis is reaction wheel. Firstly, fault/failure types of reaction wheel are investigated. Then, fault models for reaction wheels are proposed and analyzed. In addition, a brief introduction of relevant mathematical concepts is provided.

2.1 Rigid Body Kinematics and Dynamics

2.1.1 Attitude Representation

The formulation of spacecraft attitude control problems involves consideration of attitude kinematics and dynamics. The attitude of the rigid body is represented by a 3×3 rotation matrix that transforms a vector resolved in the body-fixed frame into its representation resolved in the reference frame [91]. The set of all rotation matrices is described by $SO(3)$, which is the special orthogonal group of rigid rotation in $\mathbb{R}^3$ [92]. There exist various typical parameterizations for the representation of the

orientation of a rigid body relative to the inertia frame, such as unit quaternions, Euler angles, Modified Rodrigues parameters, rotation matrices. For details of each attitude representation, please refer to [93]. Three-parameter representations of attitude such as Euler angles or Modified Rodrigues parameters possess singularities and thus the stability results using continuous control laws are only of a local nature [94]. Four-parameter representations of attitude such as unit quaternions can represent all attitudes of a rigid body globally but not uniquely. This is due to the fact that the map from unit quaternions in the space $\mathbb{S}^3$ to the physical rotations in $SO(3)$ is not unique. However, because unit quaternions are well suited for onboard real-time computation (only products and no trigonometric relations in the quaternion kinematic differential equations) [95] and the number of redundant parameters is less than rotation matrices representation, the spacecraft orientation is commonly described in terms of unit quaternions.

2.1.2 Attitude Kinematics and Dynamics

In this thesis, the unit quaternion $\mathbf{Q} = [q_0, q_1, q_2, q_3]^T = [q_0, \mathbf{q}^T]^T \in \mathbb{R} \times \mathbb{R}^3$ is used to represent the orientation of the body frame $\mathcal{B}$ with respect to the inertial frame $\mathcal{I}$ and satisfies the unit-norm constraint $q_0^2 + \mathbf{q}^T \mathbf{q} = 1$. For the sake of notational simplicity, the time argument t is left out except for emphasis in certain places. The scalar part q_0 and vector part $\mathbf{q}$ of unit quaternion are defined as

$$\mathbf{Q} = \begin{bmatrix} q_0 \\ \mathbf{q} \end{bmatrix} = \begin{bmatrix} \cos(\frac{\bar{\phi}}{2}) \\ \mathbf{e} \sin(\frac{\bar{\phi}}{2}) \end{bmatrix}, \quad (2.1)$$

where $\mathbf{e}$ is the Euler axis and $\bar{\phi}$ is the Euler angle.

The nonlinear differential equations that govern the kinematics and dynamics of

the spacecraft in terms of quaternion can be expressed by [95,96]

$$\begin{cases} \dot{\mathbf{Q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\mathbf{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}^T \\ \mathbf{q}^\times + q_0 \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \boldsymbol{\tau} + \mathbf{d} \end{cases} \quad (2.2)$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the inertial angular velocity vector of the spacecraft with respect to an inertial frame $\mathcal{I}$ and expressed in the body frame $\mathcal{B}$, $\mathbf{I}_3 \in \mathbb{R}^{3 \times 3}$ is the 3×3 identity matrix, $\boldsymbol{\tau} \in \mathbb{R}^3$ denotes the combined control torques acting on the spacecraft body axes, and $\mathbf{d} \in \mathbb{R}^3$ is the external disturbances. The notation $\mathbf{a}^\times \in \mathbb{R}^{3 \times 3}$ for a vector $\mathbf{a} = [a_1, a_2, a_3]^T$ is used to represent the skew-symmetric matrix

$$\mathbf{a}^\times = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \quad (2.3)$$

The skew-symmetric matrix satisfies the following important properties

$$\begin{aligned} \mathbf{a}^\times \mathbf{b} &= -\mathbf{b}^\times \mathbf{a}, \quad \mathbf{a}^\times \mathbf{a} = 0, \quad \mathbf{a}^T \mathbf{b}^\times \mathbf{a} = 0 \\ \mathbf{a}^\times \mathbf{b}^\times &= \mathbf{b} \mathbf{a}^T - \mathbf{a}^T \mathbf{b} \mathbf{I}_3, \quad (\mathbf{a}^\times \mathbf{b})^\times = \mathbf{b} \mathbf{a}^T - \mathbf{a} \mathbf{b}^T. \end{aligned} \quad (2.4)$$

Modern spacecraft often uses redundant actuators to enhance the reliability, maneuverability and survivability. If the spacecraft has more than three actuators, the spacecraft is overactuated and the combined control torques $\boldsymbol{\tau}$ is given as

$$\boldsymbol{\tau} = \mathbf{D} \mathbf{u}, \quad (2.5)$$

where $\mathbf{D} \in \mathbb{R}^{3 \times n}$ is the actuator distribution matrix (for a given spacecraft, $\mathbf{D}$ is available and has full-row rank by properly placing the actuator at certain locations

and directions), $\mathbf{u} \in \mathbb{R}^n$ denotes the control torques generated by the n actuators and $n > 3$. Therefore, the attitude kinematics and dynamics of spacecraft can be described as

$$\begin{cases} \dot{\mathbf{Q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\mathbf{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}^T \\ \mathbf{q}^\times + q_0 \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{D}\mathbf{u} + \mathbf{d}. \end{cases} \quad (2.6)$$

If only three actuators (aligned with the body-fixed axes) are equipped for attitude control, the distribution matrix $\mathbf{D}$ in (2.6) is an identity matrix and $\mathbf{D} = \mathbf{I}_3$. If the spacecraft is overactuated, the distribution matrix $\mathbf{D} \in \mathbb{R}^{3 \times n}$ ($n > 3$) satisfies $\text{rank}(\mathbf{D}) = 3$.

2.1.3 Attitude Tracking Error Dynamics

To address the attitude tracking issue, the desired motion of the spacecraft in the desired reference frame $\mathcal{R}$ with respect to inertial frame $\mathcal{I}$ is specified by unit-quaternion $\mathbf{Q}_d = [q_{d0}, \mathbf{q}_d^T]^T$ and satisfies the constraints $q_{d0}^2 + \mathbf{q}_d^2 = 1$. The attitude tracking error $\mathbf{Q}_e = [q_{e0}, \mathbf{q}_e^T]^T$ is defined as the relative orientation between the attitude $\mathbf{Q}$ and the desired attitude $\mathbf{Q}_d$, which is computed as

$$\mathbf{Q}_e = \mathbf{Q}_d^{-1} \otimes \mathbf{Q}, \quad (2.7)$$

where $\mathbf{Q}_d^{-1}$ is the inverse or conjugate of the desired quaternion and is determined by $\mathbf{Q}_d^{-1} = [q_{d0}, -\mathbf{q}_d^T]^T$, and $\otimes$ denotes the quaternion multiplication operator of two unit-quaternion $\mathbf{Q}_i = [q_{i0}, \mathbf{q}_i^T]^T$ and $\mathbf{Q}_j = [q_{j0}, \mathbf{q}_j^T]^T$, which is defined as follows:

$$\mathbf{Q}_i \otimes \mathbf{Q}_j = \begin{bmatrix} q_{i0}q_{j0} - \mathbf{q}_i^T \mathbf{q}_j \\ q_{i0}\mathbf{q}_j + q_{j0}\mathbf{q}_i + \mathbf{q}_i^\times \mathbf{q}_j \end{bmatrix}. \quad (2.8)$$

The mapping from unit quaternion space to the proper orthogonal matrix space $SO(3)$ is given in terms of rotation matrix $\mathbf{C}$. Note that the rotation matrix $\mathbf{C} \in SO(3) = \{\mathbf{C} \in \mathbb{R}^{3 \times 3} : \mathbf{C}^T \mathbf{C} = \mathbf{C} \mathbf{C}^T = \mathbf{I}_3, \det(\mathbf{C}) = 1\}$. The rotation matrix $\mathbf{C}(\mathbf{Q})$ denotes a transformation from inertial frame $\mathcal{I}$ to body-fixed frame $\mathcal{B}$, i.e., $\mathcal{I} \rightarrow \mathcal{B}$ and can be parameterized in terms of $\mathbf{Q}$ as follows [91]:

$$\mathbf{C}(\mathbf{Q}) = (q_0^2 - \mathbf{q}^T \mathbf{q}) \mathbf{I}_3 + 2\mathbf{q} \mathbf{q}^T - 2q_0 \mathbf{q}^\times. \quad (2.9)$$

As a result, the composite rotation from the reference frame $\mathcal{R}$ to the body frame $\mathcal{B}$ can be obtained by combining the two rotation matrices $\mathbf{C}(\mathbf{Q})$ and $\mathbf{C}(\mathbf{Q}_d)$ into a single rotation matrix $\mathbf{C}(\mathbf{Q}_e)$, which is given by

$$\mathbf{C}(\mathbf{Q}_e) = \mathbf{C}(\mathbf{Q})(\mathbf{C}(\mathbf{Q}_d))^T. \quad (2.10)$$

Furthermore, it can be obtained that $\mathbf{C}(\mathbf{Q}_e) = (q_{e0}^2 - \mathbf{q}_e^T \mathbf{q}_e) \mathbf{I}_3 + 2\mathbf{q}_e \mathbf{q}_e^T - 2q_{e0} \mathbf{q}_e^\times$. Then, the angular velocity error $\boldsymbol{\omega}_e \in \mathbb{R}^3$ is expressed as

$$\boldsymbol{\omega}_e = \boldsymbol{\omega} - \mathbf{C}(\mathbf{Q}_e) \boldsymbol{\omega}_d, \quad (2.11)$$

where $\boldsymbol{\omega}_d$ is the desired angular velocity expressed in the reference frame $\mathcal{R}$ with respect to inertial frame $\mathcal{I}$. Then, it is found that $\dot{\mathbf{C}}(\mathbf{Q}_e) = -\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e)$ [8].

Consequently, the equations that govern the attitude tracking error dynamics are given by [97]

$$\begin{cases} \dot{\mathbf{Q}}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}_e^T \\ \mathbf{q}_e^\times + q_{e0} \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{J} \dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} (\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e) \boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e) \dot{\boldsymbol{\omega}}_d) + \mathbf{D} \mathbf{u} + \mathbf{d}. \end{cases} \quad (2.12)$$

From (2.12), it can be found that attitude tracking problem is equivalent to a sta-

bilization problem with respect to ω_e and Q_e .

2.2 Actuator Fault Modeling

Before we proceed to discuss the detailed fault modeling, the definitions of fault and failure are given firstly as follows [98]:

- **Fault:** an unpermitted deviation of at least one characteristic property or parameter of the system from the acceptable/usual/standard condition.
- **Failure:** a permanent interruption of a system's ability to perform a required function under specified operating conditions.

In other words, a failure is a condition which is much more severe than a fault. When a fault occurs on an actuator, the actuator remains operational but may have a slower response or become less effective. But when a failure occurs, the actuator has to be shut off and the redundant actuator is needed to produce the same desired effect.

2.2.1 Fault and Failure Classification

Each actuator is a combination of mechanical and electronic components, and thus a fault or failure may occur in many different ways. According to the effect of a fault or failure, we can classify the fault or failure into several general types. For the actuators, there are mainly three types of failures:

1) Lock in place: In this case as shown in Fig. 2.1(a), the output torque is stuck at a constant value. For example, if lock in place failure occurs in a reaction wheel, then the reaction wheel will keep accelerating or decelerating until the angular velocity reaches maximum.

2) Float: As shown in Fig. 2.1(b), there is no output torque although the command torque is not zero. The actuator output torque will float around zero.

3) Hardover: As depicted in Fig. 2.1(c), this case is similar to the case of lock in place. Once this failure occurs, the output torque of the actuator will reach the maximum limit at its maximum rate, which results in system failure.

Figure 2.1: Type of actuator failures. (t_f is the time instant that actuator failure occurs)

As compared to actuator failure, actuator fault may not break down the system. Once an actuator fault occurs, the system performance may be degraded. Actuator faults can be categorized into two types:

1) Loss of effectiveness: In this case as illustrated in Fig. 2.2(a), the effectiveness

of the actuation is reduced.

2) Bias: As shown in Fig. 2.2(b), there is an additive effect on the output. However, this bias fault may vary with time.

Figure 2.2: Type of actuator fault.

Based on the way faults/failures enter to the system, actuator faults/failures can be classified as *multiplicative* and *additive* faults or failures [5]. For example, loss of effectiveness fault is described as multiplicative fault because the actual actuator outputs depending on the fault size are multiplied with the input of the actuator. While bias fault of actuator is usually described as additive input signal or as input deviation, it belongs to additive fault.

Actuator faults/failures can also be categorized according to their temporal behaviors as abrupt faults/failures and incipient faults. Abrupt faults/failures exhibit a sudden and unexpected change. An example of an abrupt failure is an actuator jam, or a jam with offset and hardover. Incipient faults are subtler and the effect is not so obvious. However, if incipient faults are left unattended for a long period of time, they might degrade the required system performance and finally lead to abrupt and catastrophic failures. Incipient faults can be caused by operational wear and tear as the effect is negligible at the beginning but becomes gradually worse

before it fails abruptly.

2.2.2 Faults and Failures in Reaction Wheels

Reaction wheel consists of a flywheel and an electric motor. When the rotation speed of the flywheel is changed, the spacecraft begins to counter-rotate proportionately through conservation of angular momentum. A fault/failure may occur in several components of the reaction wheel, such as faulty electronics, drive motor, and power supply, etc. As discussed in [88], reaction wheels are sensitive devices that are vulnerable to the following four main sources of faults/failures:

1. *Decreased reaction torque (F1)*: Due to increased friction between stator and rotor, inadequate lubrication and marginal failures in bearings, and decreased motor torque, the rate of change of the wheel speed might be less than the nominal value. As a consequence, the generated reaction wheel output torque could be less than the commanded torque by the controller.
2. *Increased bias torque (F2)*: When the external disturbances are neglected and the demanded reaction torque by the attitude control system is zero, the reaction wheel should hold its speed and generate no torque. However, because of changes in coulomb friction and viscous friction of the bearings caused by aging, time-varying temperature, and lubrication, the wheel may not be capable of holding its speed, which may accelerate or decelerate the wheel gradually. Therefore, a low bias torque is produced even when the commanded attitude control torque is zero.
3. *Failure to respond to control signals (F3)*: If malfunction occurs in drive electronics, drive motor, and power supply, the reaction wheel may decelerate slowly or hold its speed, without any response to control signals.

4. *Continuous generation of reaction torque (F4)*: Faults in the bus voltage and intermittent time-varying faults in the motor current might lead to continuous increase or decrease in wheel speed, thereby generating reaction torque, independent of the commanded torque by the controller.

Based on the definitions of fault and failure, **F1** and **F2** could be categorized as actuator fault, while **F3** and **F4** could be categorized as actuator failure. When a fault occurs in one reaction wheel, the reaction wheel is still usable but may have a slower response or become less effective. But once a failure occurs, the reaction wheel may undergo a catastrophic or complete breakdown, which is much more severe than a fault. If one of the reaction wheel experiences **F3-F4**, actuator redundancy should be used to ensure three-axis control of the attitude control system and maintain performance. On the other hand, the above four types of reaction wheel faults/failures can also be classified on the basis of their effects on the output behaviour of the reaction wheel, and **F1-F4** of the reaction wheel can be described as partial loss of effectiveness fault, bias fault, float failure, and lock in place failure or hardover failure, respectively.

In the fault-free case, the actual output torques $\mathbf{u}$ of n actuators are equal to the desired values $\mathbf{u}_c$ commanded by controller, i.e., $\mathbf{u} = \mathbf{u}_c$. When reaction wheel faults/failures occur, the above four types of reaction wheel faults/failures can be mathematically modelled as

$$u_i = e_i(t)u_{ci} + \bar{u}_i(t), \quad (2.13)$$

where u_{ci} denotes the desired torque signal of the i th reaction wheel generated by the controller with $i = 1, 2, \dots, n$, $0 \leq e_i(t) \leq 1$ represents the effectiveness factor of the i th reaction wheel, $\bar{u}_i$ is the uncertain faulty input entering the spacecraft in an additive way, and τ_i denotes the actual control action exerted on the spacecraft.

In the situation of fault-free, the actual output torque of an actuator is equal to its desired value, i.e., $u_i = u_{ci}$, with $e_i(t) = 1$ and $\bar{u}_i = 0$. The corresponding relations between model parameters and reaction wheel faults/failure are summarized in Table 2.1.

Table 2.1: Relations between model parameters and reaction wheel faults/failures

Fault/Failure	Type	e_i	$\bar{u}_i$
F1	Partial loss of effectiveness fault	$0 < e_i < 1$	0
F2	Bias fault	1	$\bar{u}_i \neq 0$
F3	Float failure	0	0
F4	Lock in place failure or Hardover failure	0	$\bar{u}_i \neq 0$

According to equation (2.13), the actual control torque τ generated by n reaction wheels can be written as follows:

$$\mathbf{u} = \mathbf{E}(t)\mathbf{u}_c + \bar{\mathbf{u}}(t), \quad (2.14)$$

where $\mathbf{u}_c = [u_{c1}, u_{c1}, \dots, u_{cn}]^T \in \mathbb{R}^n$ denotes the input torque vector of the n actuators, $\mathbf{E}(t) = \text{diag}([e_1(t), e_2(t), \dots, e_n(t)]) \in \mathbb{R}^{n \times n}$ represents the actuator control effectiveness matrix, and $\bar{\mathbf{u}}(t) = [\bar{u}_1(t), \bar{u}_2(t), \dots, \bar{u}_n(t)]^T \in \mathbb{R}^n$ is the additive actuator fault vector.

Since the dynamics of the reaction wheel is faster than the satellite attitude dynamics, it is usually ignored in the attitude controller design [96]. Thus, the attitude dynamics with reaction wheel faults/failures **F1-F4** can be described as

$$\begin{cases} \dot{\mathbf{Q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\mathbf{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}^T \\ \mathbf{q}^\times + q_0 \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{D}(\mathbf{E}\mathbf{u}_c + \bar{\mathbf{u}}) + \mathbf{d}. \end{cases} \quad (2.15)$$

Consequently, the faulty attitude tracking error dynamics is given by

$$\begin{cases} \dot{\mathbf{Q}}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}_e^T \\ \mathbf{q}_e^\times + q_{e0}\mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{J}(\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + \mathbf{D}(\mathbf{E}\mathbf{u}_c + \bar{\mathbf{u}}) + \mathbf{d}. \end{cases} \quad (2.16)$$

2.3 Performance Analysis under Actuator Faults and Failures

Since actuator faults and failures are inevitable, it is better to understand the performance degradation of the attitude control system in the presence of actuator faults and failures in advance. In this section, attitude tracking performance under different actuator faults and failures will be analyzed via simulation. A nominal PD controller is used to achieve attitude tracking in the absence of actuator faults and failures. Then, different actuator faults and failures are considered in the simulation. In the simulation, it is assumed that actuator faults or failures are not detected and isolated and there is no action for control reconfiguration, so that the nominal PD controller is used throughout this section. The nominal PD controller is developed as follows [8,95]:

$$\mathbf{u}_c = (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d + \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d - k_p\mathbf{q}_e - k_d\boldsymbol{\omega}_d, \quad (2.17)$$

where k_p and k_d are two positive constants.

In the simulation, the spacecraft is assumed to have three reaction wheels as actuator and external disturbances are ignored. The inertia matrix is $\mathbf{J} = \text{diag}([20, 17, 15])$ $\text{kg} \cdot \text{m}^2$. The initial attitude is $\mathbf{Q}(0) = [-0.2, 0.15, 0.35, 0.9028]^T$ and the initial angular velocity is $\boldsymbol{\omega} = [0, 0, 0]^T$ rad/s . The desired motion expressed in reference frame $\mathcal{R}$ with respect to inertial frame $\mathcal{I}$ is $\boldsymbol{\omega}_d(t) = 0.05[\cos(t/40), \sin(t/60), \cos(t/50)]^T$

rad/s.

Before we proceed to discuss the effect of actuator faults/failures, attitude tracking performance in fault-free situation is shown by using the nominal PD controller. It can be observed from Fig. 2.3 that spacecraft can track the desired attitude and angular velocity asymptotically by using the nominal PD controller.

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

(c) Commanded control torques.

Figure 2.3: Attitude tracking performance in fault-free situation.

2.3.1 Effect of Actuator Faults

Two kinds of the most common reaction wheel faults are considered, i.e. partial loss of effectiveness fault (**F1**) and bias fault (**F2**).

Case I: partial loss of effectiveness fault (**F1**). At time $t = 10$ s, a loss of effective fault occurs in the reaction wheel aligned with x axis and the other two reaction wheels can work normally. The effectiveness gain after occurrence of **F1** is changed

to $e_1 = 0.3$. It is noted from Fig. 2.4 that attitude and angular velocity tracking errors only converge to a neighbourhood of zero eventually. Therefore, the steady-state tracking precision is inferior than that under fault-free condition.

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

(c) Commanded control torques.

Figure 2.4: Attitude tracking performance in the event of partial loss of effectiveness faults.

Case II: bias fault (F2). At time $t = 50$ s, a bias fault occurs in the reaction wheel aligned with y axis and the other two reaction wheels can work normally. The negative additive bias torque is $\bar{u}_2 = -0.1$ N·m. The simulation results are described in Fig. 2.5. It is noted that, spacecraft angular velocity finally converges to the desired one while there exists a constant error in the state-state of attitude tracking response.

These two results verify that a fault does not break down a system but may lead to system performance degradation.

2.3.2 Effect of Actuator Failures

Two kinds of reaction wheel failures are considered in the following simulation, i.e. float failure (**F3**) and lock in place failures (**F4**).

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

(c) Commanded control torques.

Figure 2.5: Attitude tracking performance in the event of bias faults.

Case I: float failure (**F3**). The reaction wheel aligned with y axis experiences a float failure at time $t = 10$ s and the other two reaction wheels can work normally. Simulation results are shown in Figs. 2.6. It is observed from Fig. 2.6(a) and Fig. 2.6(b) that after 10 s, spacecraft attitude and velocity tracking errors begin to increase rapidly and the closed-loop control system becomes unstable finally. That is, if float failure is not properly accommodated, system will fail to execute missions due to the large attitude error.

Case II: lock in place failure (**F4**). The reaction wheel aligned with y axis expe-

periences a lock in place failure at time $t = 10$ s and the other two reaction wheels can work normally. Simulation results are shown in Figs. 2.7. It is observed from Fig. 2.7(a) and Fig. 2.7(b) that after 10 s, the closed-loop attitude control system also becomes unstable. However, as compared to the float failure case, the overall attitude control system is affected more significantly and quickly in case of the lock in place failure. Hence, it can be known that lock in failure has severer influence on the satellite than that of the float failure.

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

(c) Commanded control torques.

Figure 2.6: Attitude tracking performance in the event of float failures.

Based on the above simulation results, it can be concluded that an actuator failure may lead to mission interruption and disordered attitude maneuver. While a fault may degrade the attitude control performance, such as a slow convergence rate and reduced tracking accuracy. Therefore, it is necessary to design a fault tolerant controller for spacecraft attitude control systems to accommodate actuator

faults/failures automatically and maintain system stability with a desired level of overall performance.

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

(c) Commanded control torques.

Figure 2.7: Attitude tracking performance in the event of lock in place failure.

2.4 Mathematical Concepts

The following important mathematical results are used in this thesis.

2.4.1 Finite-Time Convergence Theory

The following results will be used to derive finite-time stability proof for the proposed FTC design.

Lemma 2.1. [24] *Suppose $a_1, a_2, \dots, a_n$ are positive numbers and $0 < p < 2$, then*

the following inequality holds:

$$(a_1^2 + a_2^2 + \cdots + a_n^2)^p \leq (a_1^p + a_2^p + \cdots + a_n^p)^2. \quad (2.18)$$

Lemma 2.2. [21] Suppose $V(x)$ is a continuous positive-definite function (defined on $\mathcal{D} \in \mathbb{R}$) and $\dot{V}(x) + \lambda V^\rho(x)$ is negative semi-definite on $\mathcal{D}$ for $0 < \rho < 1$ and $\lambda > 0$, then there exist an area $\mathcal{D}_0 \in \mathbb{R}$ such that any $V(x)$ which starts from $\mathcal{D}_0 \in \mathbb{R}$ can reach $V(x) \equiv 0$ in finite time. Moreover, if T_{reach} is the time needed to reach $V(x) \equiv 0$, then

$$T_{reach} \leq \frac{V^{1-\rho}(x_0)}{\lambda(1-\rho)}, \quad (2.19)$$

where $V(x_0)$ is the initial value of $V(x)$.

Lemma 2.3. [99] Assume $\rho_1 > 0$, $\rho_2 > 0$, and $0 < \rho < 1$, if a continuous positive definite function V satisfies the following inequality:

$$\dot{V} \leq -\rho_1 V - \rho_2 V^\rho, \quad \forall t > 0 \quad (2.20)$$

then, it can be found that V which starts from $V(x_0)$ can reach $V \equiv 0$ in finite time. Moreover, the reaching time T_{reach} is given by

$$T_{reach} \leq \frac{1}{\rho_1(1-\rho)} \ln \frac{\rho_1 V^{1-\rho}(0) + \rho_2}{\rho_2}. \quad (2.21)$$

2.4.2 Barbalat's Lemma

Barbalat's lemma is frequently used for time-varying systems for stability analysis, which is stated as follows:

Lemma 2.4. (Barbalat's Lemma) [100,101] Let $f(t): \mathbb{R} \rightarrow \mathbb{R}$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t \rightarrow \infty} \int_0^t f(\tau) d\tau$ exists and is finite.

Then, $\lim_{t \rightarrow \infty} f(t) = 0$.

A practical corollary to Barbalat's lemma is also given as:

Corollary 2.1. [102] *Consider the function $f(t): \mathbb{R}_+ \rightarrow \mathbb{R}$. If $f(t), \dot{f}(t) \in \mathcal{L}_\infty$, and $f(t) \in \mathcal{L}_p$ for some $p \in [1, \infty)$, then $\lim_{t \rightarrow \infty} f(t) = 0$.*

Chapter 3

Inertia-Free Fault-Tolerant Attitude Tracking Using CA

The aim of this chapter is to propose a novel inertia-free adaptive FTC design incorporated with CA for spacecraft attitude tracking systems subject to unknown physical inertia information, external disturbances, and actuator faults/failures. The estimated effectiveness level of actuator, although it is assumed to be imprecise, is used in the on-line CA for redistributing the virtual control efforts into each individual actuator judiciously without reconfiguring the controller.

Two CA-based control inputs, one a nonadaptive discontinuous signal and the other an inertia-free adaptive continuous signal, are designed to provide fault-tolerant attitude tracking capability. The main contributions of this chapter are listed as follows. Firstly, the existing works that design CA-based FTC approach for linear systems in [59], [62], [63], are extended to strongly coupled nonlinear attitude tracking systems, and external disturbances are rejected. Secondly, some modifications are made for the on-line CA scheme proposed in [59] so that partially effective actuators can be leveraged for control use. Thirdly, unlike existing methods in [14], [15] that estimate the elements in inertia matrix directly for solving the attitude track-

ing control problem for spacecraft, indirect adaptive techniques are employed to remove information of inertia matrix in the FTC design. Since only one parameter is updated in the proposed virtual control input, the design complexity and on-line computation load can be greatly reduced. Fourthly, a novel time-varying dead-zone modification is proposed in the design of parameter adaptations to stop the adaptive gain from increasing, and which also improves the robustness of the adaptive law with respect to measurement or system noises. The stability of the closed-loop systems under the proposed CA-based adaptive FTC schemes is analyzed by using Lyapunov-like approach, and attitude orientation and angular velocity tracking errors are uniformly ultimately bounded and converge to a small invariant set containing the origin.

The remaining of this chapter is organized as follows. Section 3.1 presents the mathematical models for spacecraft attitude tracking under multiplicative actuator fault/failure, and also formulate the FTC problem. In section 3.2, the existing CA-based FTC schemes are extended to nonlinear spacecraft attitude tracking systems with consideration of external disturbances, and a discontinuous FTC law is presented. Section 3.3 is devoted to the presentation of the main contributions of this chapter, where a novel inertia-free adaptive continuous FTC scheme is proposed. In section 3.4, simulation results are included to show the efficiency of the two proposed schemes for a rigid spacecraft with actuator faults/failures and external disturbances. Finally, conclusions are given in section 3.5.

3.1 Preliminary and Problem Statement

3.1.1 Preliminary

In this chapter, only multiplicative faults/falireus of reaction wheels are taken into account, i.e., loss of effective fault and float failure.

From chapter 2, the kinematics and dynamics for the attitude tracking error systems of a rigid spacecraft with multiplicative actuator faults/faliures can be expressed by the following equations

$$\begin{cases} \dot{\mathbf{Q}}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}_e^T \\ \mathbf{q}_e^\times + q_{e0}\mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{J}(\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + \mathbf{D}\mathbf{E}(t)\mathbf{u}_c + \mathbf{d}, \end{cases} \quad (3.1)$$

where $\mathbf{E} = \text{diag}\{e_1(t), e_2(t), \dots, e_n(t)\} \in \mathbb{R}^{n \times n}$ denotes the effectiveness gain matrix of actuators with scalars $e_i(t)$ satisfying $0 \leq e_i \leq 1$, $i = 1, 2, \dots, n$. Note that the case $e_i(t) = 1$ indicates that the i th actuator works normally, and $0 < e_i(t) < 1$ implies that the i th actuator partially loses its effectiveness, but still has not totally failed. The value $e_i(t) = 0$ means that the i th actuator undergoes a complete failure.

Here, it is assumed that actuator effectiveness can be successfully diagnosed by an FDD mechanism. Similar to [59], [62], the estimate of the actuator effectiveness, $\hat{\mathbf{E}}(t) = \text{diag}\{\hat{e}_1(t), \hat{e}_2(t), \dots, \hat{e}_n(t)\}$, where the scalars $0 \leq \hat{e}_i(t) \leq 1$, are supposed to satisfy

$$\mathbf{E}(t) = (\mathbf{I}_n - \boldsymbol{\Delta}(t))\hat{\mathbf{E}}(t) \quad (3.2)$$

where $\boldsymbol{\Delta}(t) = \text{diag}\{\delta_1(t), \delta_2(t), \dots, \delta_n(t)\}$, and the unknown scalars $\delta_i(t)$ represent the level of imprecision (relative error with respect to the actual actuator efficiency) in the estimation of actuator effectiveness.

To develop the control scheme, it would be useful to define an auxiliary desired angular velocity, given by

$$\bar{\boldsymbol{\omega}}_d = \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - k\mathbf{q}_e, \quad (3.3)$$

where k is a positive constant chosen by the designer. The differentiable signal $\bar{\omega}_d$ satisfies

$$\dot{\bar{\omega}}_d = -\omega_e^\times C(Q_e)\omega_d + C(Q_e)\dot{\omega}_d - \frac{k}{2}(\mathbf{q}_e^\times + q_{e0}\mathbf{I}_3)\omega_e. \quad (3.4)$$

Based on the auxiliary desired angular velocity, a sliding vector $\mathbf{s}$ is given by [14]

$$\mathbf{s} = \boldsymbol{\omega} - \bar{\boldsymbol{\omega}}_d = \boldsymbol{\omega}_e + k\mathbf{q}_e. \quad (3.5)$$

Consequently, from the attitude dynamics in equation (3.1) with actuator efficiency in equation (3.2), the attitude tracking error in terms of the sliding vector can be described as

$$\mathbf{J}\dot{\mathbf{s}} = -\mathbf{s}^\times \mathbf{J}\boldsymbol{\omega} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{J}\boldsymbol{\omega} - \mathbf{J}\dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{D}(\mathbf{I}_n - \boldsymbol{\Delta}(t))\hat{\mathbf{E}}(t)\mathbf{u}_c + \mathbf{d}. \quad (3.6)$$

To facilitate control system design, the following assumptions are used in the subsequent developments.

Assumption 3.1. *The inertia matrix $\mathbf{J}$ is a symmetric, positive-definite and bounded **constant** matrix such that $\|\mathbf{J}\| \leq c_J$, where c_J is a positive constant.*

Assumption 3.2. *The desired angular velocity of spacecraft and its time derivative are bounded, that is $\|\boldsymbol{\omega}_d\| \leq c_\omega$ and $\|\dot{\boldsymbol{\omega}}_d\| \leq c_{d\omega}$, where c_ω and $c_{d\omega}$ are positive constants.*

Assumption 3.3. *The external disturbance $\mathbf{d}$ is bounded such that $\|\mathbf{d}\| \leq d_{max}$, where d_{max} is a positive constant.*

Assumption 3.4. *The spacecraft is so constructed that potentially up to $n-3$ actuators can suffer from total failures simultaneously, and the estimate of the actuator*

effectiveness satisfies $\hat{\mathbf{E}}(t) \in \mathcal{E}$, where the set $\mathcal{E}$ is defined as

$$\mathcal{E} = \left\{ (\hat{e}_1, \hat{e}_2, \dots, \hat{e}_n) \in [0 \ 1]^N : \det(\mathbf{D}\hat{\mathbf{E}}^3(t)\mathbf{D}^T) \neq 0 \right\}$$

with $[0 \ 1]^n = \underbrace{[0 \ 1] \times \dots \times [0 \ 1]}_{n \text{ times}}$.

Remark 3.1. Since the environmental disturbances due to gravitation, solar radiation pressure, magnetic forces or aerodynamic drag are bounded in practice, Assumption 3 is reasonable. Assumption 4 guarantees that the faulty attitude tracking system is controllable and closed-loop stability can be maintained, otherwise the systems shall be under-actuated.

3.1.2 Problem Statement

For overactuated systems, it is possible to divide the controller design into two steps [56]. In the first step, a high-level controller is designed as virtual controller to specify the total desired control efforts to the system. Then, a control allocation algorithm is developed to map the virtual control efforts into individual actuators such that the total actual control signals generated by all actuators amount to the commanded virtual inputs. Overall structure of the CA-based fault-tolerant attitude tracking scheme is shown in Fig. 3.1.

Figure 3.1: Structure of the overall proposed FTC system.

Motivated by [59], [62], the following l_2 -optimal CA problem is considered:

$$\begin{aligned} \min_{\mathbf{u}_c(t)} \quad & \mathbf{u}_c^T \hat{\mathbf{E}}^{-1}(t) \mathbf{u}_c \\ \text{subject to} \quad & \mathbf{D} \hat{\mathbf{E}}(t) \mathbf{u}_c = \mathbf{v}_c \end{aligned} \quad (3.7)$$

where $\mathbf{v}_c$ is the virtual control input generated by the high level controller. Comparing with [59], the difference is that estimate of actuator effectiveness is considered in the constraint of control allocation design. This modification makes the constraint is in accord with the actual situation, and partially effective actuators can be leveraged for control use. If an actuator fault occurs, the weighting matrix $\hat{\mathbf{E}}^{-1}(t)$ will change online and the command control input $\mathbf{u}_c$ is reallocated to minimize the use of the faulty actuator. For example, in the event of the total failure of the i th actuator, the corresponding diagonal element $e_i(t)$ in the weighting matrix becomes to be a very large value. Hence, the command control torque allocated to the i th actuator will be a value near zero, i.e., the actuator that experiences failure will not be used anymore. It should be noted that $\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T$ is invertible since $\hat{\mathbf{E}}(t) \in \mathcal{E}$ from Assumption 3.4. Thus, it is straightforward to get the solution of problem (3.7) given by

$$\mathbf{u}_c = \hat{\mathbf{E}}^2(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c. \quad (3.8)$$

Substituting the CA solution (3.8) into equation (2.16), the attitude tracking error dynamics can be further written as

$$\mathbf{J} \dot{\mathbf{s}} = -\mathbf{s}^\times \mathbf{J} \boldsymbol{\omega} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{v}_c - \mathbf{D} \boldsymbol{\Delta}(t) \mathbf{D}^\dagger \mathbf{v}_c + \mathbf{d}, \quad (3.9)$$

where $\mathbf{D}^\dagger = \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1}$. Since $\mathbf{D} \mathbf{D}^\dagger = \mathbf{I}_3$, $\mathbf{D}^\dagger$ is a pseudo-inverse of matrix $\mathbf{D}$ under the condition that $\hat{\mathbf{E}}(t) \in \mathcal{E}$ in Assumption 3.4. According to the

property of pseudo-inverse in [59], [103], if $\hat{\mathbf{E}}(t) \in \mathcal{E}$, then there exists a finite scalar ξ independent of $\hat{\mathbf{E}}(t)$ such that

$$\|\mathbf{D}^\dagger\| = \|\hat{\mathbf{E}}^3(t)\mathbf{D}^T(\mathbf{D}\hat{\mathbf{E}}^3(t)\mathbf{D}^T)^{-1}\| < \xi. \quad (3.10)$$

To achieve all the above control objectives, another assumption is also applied in addition to Assumptions 3.1-3.4.

Assumption 3.5. *The mismatch between the actual efficiency matrix $\mathbf{E}(t)$ and its estimate $\hat{\mathbf{E}}(t)$ is bounded, and there exists a positive constant Δ_{max} such that $\Delta(t)$ in equation (3.2) satisfies*

$$\|\Delta(t)\| \leq \Delta_{max} < \frac{1}{\xi\|\mathbf{D}\|} \quad (3.11)$$

where ξ is defined in equation (3.10).

Remark 3.2. *Assumption 3.5 means that the mismatch between the actual efficiency matrix $\mathbf{E}(t)$ and the estimate $\hat{\mathbf{E}}(t)$ cannot be arbitrarily large. In practice, since the designed FDD scheme could always provide estimation on faults with a certain degree of precision, this assumption is reasonable.*

3.2 Discontinuous FTC Design

In this section, a discontinuous fault-tolerant controller is first proposed for the spacecraft attitude tracking system, and then the stability of the overall CA-based closed-loop FTC system is proved. The nonadaptive discontinuous virtual control

input for spacecraft is proposed as

$$\mathbf{v}_c = \mathbf{v}_1 + \mathbf{v}_2 \quad (3.12)$$

$$\mathbf{v}_1 = \bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d - k_e \mathbf{q}_e \quad (3.13)$$

$$\mathbf{v}_2 = - \frac{k_v + d_{max} + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \frac{\mathbf{s}}{\|\mathbf{s}\|} \quad (3.14)$$

where k_e and k_v are two positive constants.

To analyze the stability of the closed-loop system, a Lyapunov-like candidate is chosen as

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{J} \mathbf{s} + k_e [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2]. \quad (3.15)$$

Differentiating V with respect to time leads to

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \mathbf{J} \dot{\mathbf{s}} + k_e \mathbf{q}_e^T \dot{\boldsymbol{\omega}}_e \\ &= \mathbf{s}^T \left(-\mathbf{s}^\times \mathbf{J} \boldsymbol{\omega} - \bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} \right. \\ &\quad \left. - \mathbf{D} \boldsymbol{\Delta}(t) \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c + \mathbf{v}_c \right) + k_e \mathbf{q}_e^T \dot{\boldsymbol{\omega}}_e. \end{aligned} \quad (3.16)$$

Here, it should be noted that $\dot{\mathbf{J}} = 0$ because $\mathbf{J}$ is a constant matrix. Since $\mathbf{s}^\times$ is a skew-symmetric matrix, it is clear that $\mathbf{s}^T \mathbf{s}^\times \mathbf{J} \boldsymbol{\omega} = 0$. Then substituting the virtual control input (3.12) into equation (3.16) yields

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \left(\mathbf{d} - \mathbf{D} \boldsymbol{\Delta}(t) \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c + \mathbf{v}_2 \right) - k k_e \|\mathbf{q}_e\|^2 \\ &\leq - \frac{k_v + d_{max} + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \frac{\mathbf{s}^T \mathbf{s}}{\|\mathbf{s}\|} \\ &\quad + (d_{max} + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_c\|) \|\mathbf{s}\| - k k_e \|\mathbf{q}_e\|^2. \end{aligned} \quad (3.17)$$

Notice that

$$\|\mathbf{v}_c\| \leq \|\mathbf{v}_1\| + \frac{k_v + d_{max} + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \leq \frac{k_v + d_{max} + \|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \quad (3.18)$$

under Assumption 3.5, then it can be shown that

$$\begin{aligned} \dot{V} &\leq - \frac{k_v + d_{max} + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \|\mathbf{s}\| \\ &\quad + \frac{k_v\Delta_{max}\xi\|\mathbf{D}\| + d_{max} + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \|\mathbf{s}\| - k k_e \|\mathbf{q}_e\|^2 \\ &\leq -k_v \|\mathbf{s}\| - k k_e \|\mathbf{q}_e\|^2. \end{aligned} \quad (3.19)$$

Finally, the actual commanded control input, which will be sent to the actuator, is given by

$$\begin{aligned} \mathbf{u}_c = & \hat{\mathbf{E}}^2(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \left(\bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d - k_e \mathbf{q}_e \right. \\ & \left. - \frac{k_v + d_{max} + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \frac{\mathbf{s}}{\|\mathbf{s}\|} \right). \end{aligned} \quad (3.20)$$

Now, it is ready to state the following theorem:

Theorem 3.1. *Consider the attitude tracking error system in equation (3.1) under Assumptions 3.1-3.5. The commanded control input $\mathbf{u}_c$ given by equation (3.20) can guarantee that $\lim_{t \rightarrow \infty} \mathbf{q}_e(t) = 0$ and $\lim_{t \rightarrow \infty} \boldsymbol{\omega}_e(t) = 0$.*

Proof. Equation (3.19) implies that V is nonincreasing and bounded for all $t \geq 0$, and it follows that $\mathbf{s}$ and $\mathbf{q}_e$ are all bounded. As $\mathbf{s}$ and $\mathbf{q}_e$ are bounded, it is clear that $\boldsymbol{\omega}_e$ is also bounded from equation (3.5), which further gives rise to the boundedness of $\boldsymbol{\omega}$. Consequently, based on the tracking error kinematics in equation (3.1), we obtain that $\dot{\mathbf{q}}_e$ is bounded for all $t \geq 0$. Therefore, $\mathbf{q}_e$ is uniformly continuous. Invoking Assumption 3.2 and boundedness of $\boldsymbol{\omega}_e$ and $\mathbf{q}_e$, one can show that $\bar{\boldsymbol{\omega}}_d$ and $\dot{\bar{\boldsymbol{\omega}}}_d$ are

bounded. Since $\boldsymbol{\omega}$, $\bar{\boldsymbol{\omega}}_d$ and $\mathbf{q}_e$ are all bounded, it is clear from equation (3.13) that $\mathbf{v}_1$ is bounded, which further implies that $\mathbf{v}_2$ and the nonadaptive discontinuous virtual control input $\mathbf{v}_c$ are bounded. As a results, we obtain that the actual commanded control input $\mathbf{u}_c$ is also bounded from equation (3.20). Since $\mathbf{s}$, $\boldsymbol{\omega}$, $\bar{\boldsymbol{\omega}}_d$, $\dot{\bar{\boldsymbol{\omega}}}_d$, and $\mathbf{u}_c$ are bounded, one can easily verify from equation (3.6) that $\dot{\mathbf{s}}$ is bounded for all $t \geq 0$. Therefore, $\mathbf{s}$ is also uniformly continuous. On the other hand, integrating both sides of equation (3.19) with respect to time, we obtain

$$V(0) - V(\infty) \geq k_v \int_0^\infty \|\mathbf{s}(\varsigma)\| d\varsigma + k_e \int_0^\infty \|\mathbf{q}_e(\varsigma)\|^2 d\varsigma. \quad (3.21)$$

Therefore, $\lim_{t \rightarrow \infty} \int_0^\infty \|\mathbf{s}(\varsigma)\| d\varsigma$ and $\lim_{t \rightarrow \infty} \int_0^\infty \|\mathbf{q}_e(\varsigma)\|^2 d\varsigma$ exist and are finite. Consequently, since $\mathbf{s}$ and $\mathbf{q}_e$ are uniformly continuous in t for all $t \geq 0$, $\|\mathbf{s}(t)\|$ and $\|\mathbf{q}_e(t)\|^2$ are also uniformly continuous in t for all $t \geq 0$. Then, by using Lemma 2.4 (Barbalat's Lemma), it can be seen that $\lim_{t \rightarrow \infty} \|\mathbf{s}(t)\| = 0$ and $\lim_{t \rightarrow \infty} \|\mathbf{q}_e(t)\|^2 = 0$. Therefore, we can conclude that $\lim_{t \rightarrow \infty} \mathbf{s}(t) = 0$ and $\lim_{t \rightarrow \infty} \mathbf{q}_e(t) = 0$, which further leads to $\lim_{t \rightarrow \infty} \boldsymbol{\omega}_e(t) = 0$ because of equation (3.5). $\square$

3.3 Inertia-Free Adaptive FTC Design

In the previous section, a discontinuous controller is proposed for the attitude tracking control problem in the presence of actuator faults/failures and external disturbances. However, since the term $\mathbf{v}_1$ is used in the proposed virtual control input in equation (3.12) to enhance the tracking performance, knowledge of the spacecraft inertia is required. Due to fuel consumption, deployable appendage, sensor boom or articulation, the mass properties of the spacecraft may be uncertain or unknown, which makes the inertia matrix different from that determined during preflight testing [104], [105]. In addition, the constants d_{max} , c_J , c_ω , and $c_{d\omega}$ may also be unknown to the designer. Moreover, because of the discontinuity of control input,

chattering in the vicinity of a discontinuity may be introduced, especially in the presence of sensor noise. In this section, an inertia-free adaptive continuous virtual control input is proposed to achieve attitude tracking under actuator faults/failures without requiring spacecraft inertia knowledge.

To eliminate the requirement of inertia information in virtual control input, indirect adaptive technique is employed. First, using the assumptions 3.1-3.3 and properties that $\|\mathbf{C}(\mathbf{Q}_e)\| = 1$ and $\|\mathbf{q}_e^\times + q_{e0}\mathbf{I}_3\| = 1$, the following inequalities are obtained:

$$\|\bar{\boldsymbol{\omega}}_d\| \leq c_\omega + k, \quad (3.22)$$

$$\|\bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega}\| \leq c_J (c_\omega + k) \|\boldsymbol{\omega}\|, \quad (3.23)$$

$$\|\dot{\bar{\boldsymbol{\omega}}}_d\| \leq \left(c_\omega + \frac{k}{2}\right) \|\boldsymbol{\omega}\| + c_\omega^2 + \frac{k}{2}c_\omega + c_{d\omega}. \quad (3.24)$$

Consequently, the lumped nonlinear terms and external disturbances in (2.16) are upper bounded by

$$\begin{aligned} \|\bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d}\| &\leq c_J \left(2c_\omega + \frac{3}{2}k\right) \|\boldsymbol{\omega}\| \\ &\quad + c_J \left(c_\omega^2 + \frac{k}{2}c_\omega + c_{d\omega}\right) + d_{max} \end{aligned} \quad (3.25)$$

Furthermore, it can conclude that

$$\|\bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d}\| \leq c\Theta \quad (3.26)$$

where the unknown constant c is given by

$$c = \max \left\{ c_J \left(2c_\omega + \frac{3}{2}k\right), c_J \left(c_\omega^2 + \frac{k}{2}c_\omega + c_{d\omega}\right) + d_{max} \right\}, \quad (3.27)$$

and the time-varying variable Θ is defined as

$$\Theta = \|\boldsymbol{\omega}\| + 1. \quad (3.28)$$

Next, the inertia-free adaptive continuous virtual control input for the spacecraft is proposed as

$$\mathbf{v}_c = \mathbf{v}_1 + \mathbf{v}_2 \quad (3.29)$$

$$\mathbf{v}_1 = -k_1 \mathbf{s} - k_2 \mathbf{q}_e \quad (3.30)$$

$$\mathbf{v}_2 = -\frac{k_v + \hat{c}(t)\Theta + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \mathbf{s}_a, \quad (3.31)$$

where the variable $\mathbf{s}_a$ is defined as

$$\mathbf{s}_a = \begin{cases} \frac{\mathbf{s}}{\|\mathbf{s}\|}, & \text{if } \|\mathbf{s}\| > \epsilon \\ \frac{\mathbf{s}}{\epsilon}, & \text{if } \|\mathbf{s}\| \leq \epsilon \end{cases} \quad (3.32)$$

and k_1 and k_2 are two positive constants, ϵ is a positive scalar variable defined as $\epsilon = \frac{\sigma}{\Theta}$, and σ is a small positive constant. Since $\Theta = \|\boldsymbol{\omega}\| + 1 \geq 1$, it is clear that $\epsilon \leq \sigma$. The parameter $\hat{c}(t)$ is the estimate of constant c , which is updated as

$$\dot{\hat{c}}(t) = \alpha_1 \Theta \mathbf{s}_a^T \mathbf{D}_\epsilon[\mathbf{s}] - \frac{\alpha_1 \alpha_2}{\Theta} \hat{c}(t) \quad (3.33)$$

where α_1 and α_2 are two positive constants, and $\mathbf{D}_\epsilon[\cdot]$ is the dead-zone operator defined as

$$\mathbf{D}_\epsilon[\mathbf{s}] = \begin{cases} \mathbf{s}, & \text{if } \|\mathbf{s}\| > \epsilon \\ \mathbf{0}, & \text{if } \|\mathbf{s}\| \leq \epsilon. \end{cases} \quad (3.34)$$

Since the time-varying variable Θ is incorporated into ϵ , the dead-zone width ϵ is

also time-varying.

The properties of $\mathbf{s}_a$ and Θ are described by the following lemmas.

Lemma 3.1. *The function $\mathbf{s}_a$ defined in equation (3.32) has the following properties:*

1. $\|\mathbf{s}_a\| \leq 1$.
2. $\mathbf{s}^T \mathbf{s}_a = \|\mathbf{s}\|$, if $\|\mathbf{s}\| > \epsilon$
 $\mathbf{s}^T \mathbf{s}_a \leq \sigma$, if $\|\mathbf{s}\| \leq \epsilon$.

Proof. From equation (3.32), $\|\tilde{\boldsymbol{\omega}}_a\| = 1$ when $\|\tilde{\boldsymbol{\omega}}\| > \epsilon$ which implies that $\tilde{\boldsymbol{\omega}}^T \tilde{\boldsymbol{\omega}}_a = \tilde{\boldsymbol{\omega}}^T \frac{\tilde{\boldsymbol{\omega}}}{\|\tilde{\boldsymbol{\omega}}\|} = \|\tilde{\boldsymbol{\omega}}\|$ for $\|\tilde{\boldsymbol{\omega}}\| > \epsilon$. If $\|\tilde{\boldsymbol{\omega}}\| \leq \epsilon$, then $\|\tilde{\boldsymbol{\omega}}_a\| = \frac{\|\tilde{\boldsymbol{\omega}}\|}{\epsilon} \leq 1$ and $\tilde{\boldsymbol{\omega}}^T \tilde{\boldsymbol{\omega}}_a = \frac{\|\tilde{\boldsymbol{\omega}}\|^2}{\epsilon} \leq \epsilon$. Notice that $\epsilon = \frac{\sigma}{\Theta}$ and $\Theta \geq 1$, it follows that $\tilde{\boldsymbol{\omega}}^T \tilde{\boldsymbol{\omega}}_a \leq \sigma$ for $\|\tilde{\boldsymbol{\omega}}\| \leq \epsilon$. Therefore, the results in Lemma 1 are established. $\square$

Lemma 3.2. *For $\|\mathbf{s}\| \leq \epsilon$, the variable Θ defined in equation (3.28) is bounded by*

$$1 \leq \Theta \leq \Theta_m \quad (3.35)$$

where the finite constant $\Theta_m = \frac{c_\omega + k + 1 + \sqrt{(c_\omega + k + 1)^2 + 4\sigma}}{2}$.

Proof. When $\|\mathbf{s}\| \leq \epsilon$, it is clear that $\|\mathbf{s}\| \leq \frac{\sigma}{\Theta}$. Since $\|\mathbf{s}\| = \|\boldsymbol{\omega} - \tilde{\boldsymbol{\omega}}_d\|$ from equation (3.5), it follows that $\|\mathbf{s}\| \geq \|\boldsymbol{\omega}\| - \|\tilde{\boldsymbol{\omega}}_d\|$. This gives

$$\Theta \leq \frac{\sigma}{\Theta} + c_\omega + k + 1. \quad (3.36)$$

where the inequality in equation (3.22) is used.

Due to the fact that $\Theta = \|\boldsymbol{\omega}\| + 1 \geq 1$, multiplying both sides of equation (3.36) by Θ leads to

$$\Theta^2 - (c_\omega + k + 1)\Theta - \sigma \leq 0. \quad (3.37)$$

Solving above inequality and combining with $\Theta \geq 1$, it is obtained that

$$1 \leq \Theta \leq \Theta_m \quad (3.38)$$

where $\Theta_m = \frac{c_\omega + k + 1 + \sqrt{(c_\omega + k + 1)^2 + 4\sigma}}{2}$. $\square$

To analyze the stability, a Lyapunov-like candidate V_s is proposed as

$$V_s = \frac{1}{2} \mathbf{s}^T \mathbf{J} \mathbf{s} + k_2 [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2] + \frac{1}{2\alpha_1} \tilde{c}^2(t) \quad (3.39)$$

where $\tilde{c}(t) = c - \hat{c}(t)$, and $\dot{\tilde{c}}(t) = -\dot{\hat{c}}(t)$. The time derivative of V_s is given by

$$\dot{V}_s = \mathbf{s}^T \mathbf{J} \dot{\mathbf{s}} + k_2 \mathbf{q}_e^T \boldsymbol{\omega}_e - \frac{1}{\alpha_1} \tilde{c}(t) \dot{\tilde{c}}(t), \quad (3.40)$$

which leads to

$$\begin{aligned} \dot{V}_s = & \mathbf{s}^T \left(-\mathbf{S}(s) \mathbf{J} \boldsymbol{\omega} - \mathbf{S}(\bar{\boldsymbol{\omega}}_d) \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} \right. \\ & \left. - \mathbf{D} \boldsymbol{\Delta}(t) \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c + \mathbf{v}_c \right) \\ & + k_2 \mathbf{q}_e^T \boldsymbol{\omega}_e - \frac{1}{\alpha_1} \tilde{c}(t) \dot{\tilde{c}}(t). \end{aligned} \quad (3.41)$$

Since $\hat{c}(0) > 0$ is chosen, $\hat{c}(t) \geq 0$ can always be guaranteed in view of the update law of $\hat{c}(t)$ in equation (3.33). Consequently, under Assumption 3.5 and using the first property of Lemma 3.1, it is clear that

$$\begin{aligned} \|\mathbf{v}_c\| & \leq \|\mathbf{v}_1\| + \frac{k_v + \hat{c}(t)\Theta + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \\ & \leq \frac{k_v + \hat{c}(t)\Theta + \|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|}. \end{aligned} \quad (3.42)$$

Next, substituting the inertia-free continuous virtual control input (3.29) and parameter update law (3.33) into equation (3.41) and using equation (3.42), it follows

that

$$\begin{aligned}
\dot{V}_s &= \mathbf{s}^T \left(-\mathbf{S}(\bar{\boldsymbol{\omega}}_d) \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} - k_1 \mathbf{s} - k_2 \mathbf{q}_e \right. \\
&\quad \left. - \mathbf{D} \boldsymbol{\Delta}(t) \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c + \mathbf{v}_2 \right) \\
&\quad + k_2 \mathbf{q}_e^T \boldsymbol{\omega}_e - \tilde{c}(t) \Theta \mathbf{s}_a^T \mathbf{D}_\epsilon [\mathbf{s}] + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t) \\
&\leq -k_1 \mathbf{q}_e^T (\boldsymbol{\omega}_e + k \mathbf{q}_e) - k_2 \|\mathbf{s}\|^2 + c \Theta \|\mathbf{s}\| + \Delta_{max} \xi \|\mathbf{D}\| \frac{k_v + \hat{c} \Theta + \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \\
&\quad - \frac{k_v + \hat{c}(t) \Theta + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \mathbf{s}^T \mathbf{s}_a + k_1 \mathbf{q}_e^T \boldsymbol{\omega}_e + \frac{1}{\alpha_1} \tilde{c} \dot{\tilde{c}} \\
&\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 + c \Theta \|\mathbf{s}\| + \Delta_{max} \xi \|\mathbf{D}\| \frac{k_v + \hat{c}(t) \Theta + \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\
&\quad - \frac{k_v + \hat{c}(t) \Theta + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \mathbf{s}^T \mathbf{s}_a - \tilde{c}(t) \Theta \mathbf{s}_a^T \mathbf{D}_\epsilon [\mathbf{s}] + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t). \quad (3.43)
\end{aligned}$$

For further stability analysis, two cases are considered here.

Case I: If $\|\mathbf{s}\| > \epsilon$, then $\mathbf{s}^T \mathbf{s}_a = \|\mathbf{s}\|$ from the second property of Lemma 3.1 and $\mathbf{D}_\epsilon [\mathbf{s}] = \mathbf{s}$. Therefore, equation (3.43) can be rewritten as

$$\begin{aligned}
\dot{V}_s &\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 - \tilde{c}(t) \Theta \|\mathbf{s}\| + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t) + \frac{k_v \Delta_{max} \xi \|\mathbf{D}\| - k_v}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\
&\quad + \frac{\hat{c} \Theta \Delta_{max} \xi \|\mathbf{D}\| + \tilde{c}(t) \Theta - c \Theta \Delta_{max} \xi \|\mathbf{D}\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\
&\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 - k_v \|\mathbf{s}\| + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t). \quad (3.44)
\end{aligned}$$

By completion of squares $\tilde{c}(t) \hat{c}(t) \leq -\frac{\tilde{c}^2(t)}{2} + \frac{c^2}{2}$ and using $\|\mathbf{s}\| > \frac{\sigma}{\Theta}$, one obtains that

$$\dot{V}_s \leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 - \frac{1}{\Theta} \left(k_v \sigma - \frac{\alpha_2 c^2}{2} \right). \quad (3.45)$$

If the design parameters k_v , σ , and α_2 are chosen so that

$$k_v \sigma \geq \frac{\alpha_2 c^2}{2}, \quad (3.46)$$

then

$$\dot{V}_s \leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 < 0. \quad (3.47)$$

Therefore, when $\|\mathbf{s}\| > \epsilon$, $V_s(t)$ decreases monotonically until $\|\mathbf{s}\| \leq \epsilon$ is achieved. Furthermore, from equation (3.47), it is obtained that $V_s(t)$ is upper bounded by its initial value $V_s(0)$, i.e., $V_s(t) \leq V_s(0)$. Since $\tilde{c}^2(t) \leq 2\alpha_1 V_s(t)$, it follows that

$$\hat{c}(t) \leq c_m \quad (3.48)$$

where $c_m = c + \sqrt{2\alpha_1 V_s(0)}$.

Case II: If $\|\mathbf{s}\| \leq \epsilon$, then $\mathbf{s}^T \mathbf{s}_a = \frac{\|\mathbf{s}\|^2}{\epsilon} \leq \sigma$ from the second property of Lemma 3.1 and $\mathbf{D}_\epsilon[\mathbf{s}] = \mathbf{0}$. Therefore, equation (3.43) can be rewritten as

$$\begin{aligned} \dot{V}_s &\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t) + \frac{k_v \Delta_{max} \xi \|\mathbf{D}\| - k_v}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\ &\quad + \frac{\hat{c} \Theta \Delta_{max} \xi \|\mathbf{D}\| + \tilde{c}(t) \Theta - c \Theta \Delta_{max} \xi \|\mathbf{D}\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\ &\quad - \frac{k_v + \hat{c}(t) \Theta + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \left(\frac{\|\mathbf{s}\|^2}{\epsilon} - \|\mathbf{s}\| \right) \\ &\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 - k_v \|\mathbf{s}\| + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t) \\ &\quad + \tilde{c}(t) \Theta \|\mathbf{s}\| + \frac{k_v + \hat{c}(t) \Theta + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\|, \end{aligned} \quad (3.49)$$

which further leads to

$$\begin{aligned} \dot{V}_s &\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 - \frac{\alpha_2}{2\Theta_m} \tilde{c}^2(t) + \frac{\alpha_2 c^2}{2} + c \Theta \|\mathbf{s}\| \\ &\quad + \frac{\Delta_{max} \xi \|\mathbf{D}\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} (k_v + \hat{c} \Theta + \|\mathbf{v}_1\|) \|\mathbf{s}\|. \end{aligned} \quad (3.50)$$

Assuming that $\|\mathbf{s}\| \leq \epsilon$ is satisfied at $t = t^*$. As soon as $\|\mathbf{s}\| \leq \epsilon$ is fulfilled at $t = t^*$, the adaptive parameter $\hat{c}(t)$ decreases monotonically in accordance with

$\dot{\hat{c}}(t) = -\frac{\alpha_1\alpha_2}{\Theta}\hat{c}(t)$, which implies $\hat{c}(t) \leq \hat{c}(t^*)$. If $t^* = 0$ or $\|\mathbf{s}\|$ increase to ϵ at $t = t^*$, then $\hat{c}(t^*) \leq \hat{c}(0)$. Otherwise, $\|\mathbf{s}\|$ decreases to ϵ at $t = t^*$, and it follows from equation (3.48) that $\hat{c}(t^*) \leq c_m$. Since $\hat{c}(0) \leq V_s(0)$, it is guaranteed that $\hat{c}(t) \leq c_m$. Therefore, from equation (3.50) and using the facts $|q_0| \leq 1$ as well as $\Theta\|\mathbf{s}\| \leq \sigma$ when $\|\mathbf{s}\| \leq \epsilon$, it is clear that

$$\begin{aligned}\dot{V}_s &\leq -k_1\|\mathbf{s}\|^2 - k_2k\|\mathbf{q}_e\|^2 - \frac{\alpha_2}{2\Theta_m}\tilde{c}^2(t) \\ &\quad + \frac{\Delta_{max}\xi\|\mathbf{D}\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} (k_v + c_m + k_2 + k_1\sigma)\sigma + \frac{\alpha_2c^2}{2} + c\sigma \\ &= -k_1\|\mathbf{s}\|^2 - k_2k\|\mathbf{q}_e\|^2 - \frac{\alpha_2}{2\Theta_m}\tilde{c}^2(t) + \eta\end{aligned}\tag{3.51}$$

with

$$\eta = \frac{\Delta_{max}\xi\|\mathbf{D}\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} (k_v + c_m + k_2 + k_1\sigma)\sigma + \frac{\alpha_2c^2}{2} + c\sigma.\tag{3.52}$$

Then, it is clear from equation (3.51) that

$$\dot{V}_s \leq -\kappa V_s + \varrho,\tag{3.53}$$

where the two positive constants κ and ϱ are given by

$$\kappa = \min \left\{ \frac{k_1}{c_J}, k, \frac{\alpha_1\alpha_2}{\Theta_m} \right\},\tag{3.54}$$

$$\varrho = \frac{\Delta_{max}\xi\|\mathbf{D}\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} (k_v + c_m + k_2 + k_1\sigma)\sigma + \frac{\alpha_2c^2}{2} + c\sigma + 4k_2k.\tag{3.55}$$

Next, it can be found that $\dot{V}_s(t) < 0$ if

$$\|\mathbf{s}\| > \sqrt{\frac{\eta}{k_1}}, \text{ or } \|\mathbf{q}_e\| > \sqrt{\frac{\eta}{k_2k}}\tag{3.56}$$

according to equation (3.51), which implies that $V_s(t)$ decreases monotonically.

Moreover, to show that angular velocity tracking error $\boldsymbol{\omega}_e$ is bounded as well, a new Lyapunov-like candidate is given as

$$V_{s1} = \frac{1}{2} \mathbf{s}^T \mathbf{J} \mathbf{s} + (2k_1k + k_2) [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2] + \frac{1}{2\alpha_1} \tilde{c}^2(t). \quad (3.57)$$

Following the same lines as in the proof of *Case II* and using equation (3.5), it is straightforward to show that

$$\begin{aligned} \dot{V}_{s1} &\leq -k_1(\boldsymbol{\omega}_e + k\mathbf{q}_e)^T(\boldsymbol{\omega}_e + k\mathbf{q}_e) - k_2k\|\mathbf{q}_e\|^2 + 2k_1k\mathbf{q}_e^T\boldsymbol{\omega}_e + \eta \\ &\leq -k_1\|\boldsymbol{\omega}_e\|^2 - k(k_1k + k_2)\|\mathbf{q}_e\|^2 + \eta. \end{aligned} \quad (3.58)$$

Clearly, $\dot{V}_{s1}(t) < 0$ if

$$\|\boldsymbol{\omega}_e\| > \sqrt{\frac{\eta}{k_1}}, \text{ or } \|\mathbf{q}_e\| > \sqrt{\frac{\eta}{k(k_1k + k_2)}} \quad (3.59)$$

according to equation (3.58), which implies that $V_{s1}(t)$ decreases monotonically.

Finally, the actual inertia-free adaptive continuous commanded control input, which will be sent to the actuator, is given by

$$\begin{aligned} \mathbf{u}_c = \hat{\mathbf{E}}^2(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} &\left(-k_1 \mathbf{s} - k_2 \mathbf{q}_e \right. \\ &\left. - \frac{k_v + \hat{c}(t)\Theta + \Delta_{max}\xi\|\mathbf{D}\|\|\mathbf{v}_1\|}{1 - \Delta_{max}\xi\|\mathbf{D}\|} \mathbf{s}_a \right). \end{aligned} \quad (3.60)$$

Now, we shall state the following theorem:

Theorem 3.2. *Consider the attitude tracking error system in (3.1) under assumptions 3.1-3.5, where the inertia is now unknown. Suppose that the control parameters k_v , α_2 , and σ are chosen to satisfy the condition (3.46). If the inertia-free adaptive continuous commanded control input $\mathbf{u}_c$ is given by (3.60) and the update law is provided by (3.33), then the attitude orientation and angular velocity*

tracking errors are uniformly ultimately bounded and converge to a small invariant set containing the origin, that is, $\lim_{t \rightarrow \infty} \mathbf{q}_e(t) \in \Omega_{\mathbf{q}_e}$ and $\lim_{t \rightarrow \infty} \boldsymbol{\omega}_e(t) \in \Omega_{\boldsymbol{\omega}_e}$, where $\Omega_{\mathbf{q}_e} = \left\{ \mathbf{q}_e \mid \|\mathbf{q}_e\| \leq \sqrt{\frac{\eta}{k(k_1 k + k_2)}} \right\}$ and $\Omega_{\boldsymbol{\omega}_e} = \left\{ \boldsymbol{\omega}_e \mid \|\boldsymbol{\omega}_e\| \leq \sqrt{\frac{\eta}{k_1}} \right\}$.

Proof. In *Case I*, if the condition (3.46) is satisfied, then $\dot{V}_s < 0$, and thus $\|\mathbf{s}\| \leq \epsilon \leq \sigma$ can always be achieved in finite time. In *Case II*, from (3.53), it can be shown that the closed-loop systems is uniformly ultimately bounded stable [101]. Summarizing Cases *I* and *II*, it can be concluded that the overall closed-loop system is stable and all the internal signals are bounded. Furthermore, if the condition (3.46) and equation (3.56) are satisfied, it follows that $\dot{V}_s < 0$ for both *Case I* and *Case II*. As a result, the overall closed-loop system is globally bounded and the decrease of $\dot{V}_s$ drives $\|\mathbf{s}\|$ and $\|\mathbf{q}_e\|$ into $\|\mathbf{s}\| \leq \min \left\{ \sqrt{\frac{\phi}{k_1}}, \sigma \right\}$ and $\|\mathbf{q}_e\| \leq \sqrt{\frac{\phi}{k_2 k}}$ ultimately. Moreover, if equation (3.59) is satisfied, it follows that $\dot{V}_{s1} < 0$ for *Case II*, and thus $\|\boldsymbol{\omega}_e\|$ and $\|\mathbf{q}_e\|$ converge to $\|\boldsymbol{\omega}_e\| \leq \sqrt{\frac{\phi}{k_1}}$ and $\|\mathbf{q}_e\| \leq \sqrt{\frac{\phi}{k(k_1 k + k_2)}}$ ultimately. Since $\sqrt{\frac{\phi}{k(k_1 k + k_2)}} < \sqrt{\frac{\phi}{k_1}}$, $\|\mathbf{q}_e\|$ shall be confined to $\|\mathbf{q}_e\| \leq \sqrt{\frac{\phi}{k(k_1 k + k_2)}}$ ultimately. Therefore, it is obtained that attitude orientation and angular velocity tracking errors are uniformly ultimately bounded as $\lim_{t \rightarrow \infty} \mathbf{q}_e(t) \in \Omega_{\mathbf{q}_e}$ and $\lim_{t \rightarrow \infty} \boldsymbol{\omega}_e(t) \in \Omega_{\boldsymbol{\omega}_e}$, where $\Omega_{\mathbf{q}_e}$ and $\Omega_{\boldsymbol{\omega}_e}$ are small invariant sets containing the origin defined in Theorem 3.2. This completes the proof. $\square$

In Theorem 3.1 and Theorem 3.2, we assume that the inertia matrix of spacecraft is a symmetric, positive-definite and bounded *constant* matrix. However, the mass property of spacecraft may be time-varying due to fuel consumption, deployable appendage, sensor boom or articulation [15,17,87]. As a consequence, the inertia matrix of spacecraft may be a symmetric, positive-definite and bounded *time-varying* matrix. The adaptive attitude tracking scheme under time-varying inertia matrix and multiplicative actuator faults/failures is given in the following Corollary.

Corollary 3.1. *Consider the attitude tracking error system in (3.1) under as-*

assumptions 3.2-3.5, where the inertia matrix is a symmetric, positive-definite and bounded time-varying matrix satisfying $\|\mathbf{J}(t)\| \leq c_J$ and $\left\|\frac{d\mathbf{J}(t)}{dt}\right\| \leq c_{dJ}$. Suppose that the control parameters k_v , α_2 , and σ are chosen to satisfy the condition (3.46). If the inertia-free adaptive continuous commanded control input $\mathbf{u}_c$ is given by equation (3.60) and the update law is provided by equation (3.33), then the attitude orientation and angular velocity tracking errors are uniformly ultimately bounded and converge to a small invariant set containing the origin, that is, $\lim_{t \rightarrow \infty} \mathbf{q}_e(t) \in \bar{\Omega}_{\mathbf{q}_e}$ and $\lim_{t \rightarrow \infty} \boldsymbol{\omega}_e(t) \in \bar{\Omega}_{\boldsymbol{\omega}_e}$, where $\bar{\Omega}_{\mathbf{q}_e} = \left\{ \mathbf{q}_e \mid \|\mathbf{q}_e\| \leq \sqrt{\frac{\bar{\eta}}{k(k_1 k + k_2)}} \right\}$ and $\bar{\Omega}_{\boldsymbol{\omega}_e} = \left\{ \boldsymbol{\omega}_e \mid \|\boldsymbol{\omega}_e\| \leq \sqrt{\frac{\bar{\eta}}{k_1}} \right\}$, and $\bar{\eta}$ has the same expression as given in equation (3.52) except that the parameter c is replaced by

$$c = \max \left\{ c_J \left(2c_\omega + \frac{3}{2}k \right) + \frac{1}{2}c_{dJ}, c_J \left(c_\omega^2 + \frac{k}{2}c_\omega + c_{d\omega} \right) + \frac{1}{2}c_{dJ}(c_\omega + k) + d_{max} \right\}.$$

Proof. The Lyapunov-like candidate V_s is considered as

$$V_s = \frac{1}{2} \mathbf{s}^T \mathbf{J}(t) \mathbf{s} + k_2 [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2] + \frac{1}{2\alpha_1} \tilde{c}^2(t). \quad (3.61)$$

The time derivative of V_s along the trajectories of the system is given by

$$\begin{aligned} \dot{V}_s &= \mathbf{s}^T \mathbf{J}(t) \dot{\mathbf{s}} + \frac{1}{2} \mathbf{s}^T \frac{d\mathbf{J}(t)}{dt} \mathbf{s} + k_2 \mathbf{q}_e^T \boldsymbol{\omega}_e - \frac{1}{\alpha_1} \tilde{c}(t) \dot{\tilde{c}}(t) \\ &= \mathbf{s}^T \left(-\mathbf{S}(\mathbf{s}) \mathbf{J} \boldsymbol{\omega} - \mathbf{S}(\bar{\boldsymbol{\omega}}_d) \mathbf{J} \boldsymbol{\omega} - \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} + \frac{1}{2} \frac{d\mathbf{J}(t)}{dt} \mathbf{s} \right. \\ &\quad \left. - \mathbf{D} \boldsymbol{\Delta}(t) \hat{\mathbf{E}}^3(t) \mathbf{D}^T (\mathbf{D} \hat{\mathbf{E}}^3(t) \mathbf{D}^T)^{-1} \mathbf{v}_c + \mathbf{v}_c \right) \\ &\quad + k_2 \mathbf{q}_e^T \boldsymbol{\omega}_e - \frac{1}{\alpha_1} \tilde{c}(t) \dot{\tilde{c}}(t). \end{aligned} \quad (3.62)$$

Due to the facts that

$$\|\bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d}\| \leq c_J \left(2c_\omega + \frac{3}{2}k \right) \|\boldsymbol{\omega}\| + c_J \left(c_\omega^2 + \frac{k}{2}c_\omega + c_{d\omega} \right) + d_{max}$$

from equation (3.25) and $\left\| \frac{1}{2} \frac{d\mathbf{J}(t)}{dt} \mathbf{s} \right\| \leq \frac{1}{2} c_{dJ} \|\boldsymbol{\omega} - \mathbf{C}(\mathbf{Q}_e) \boldsymbol{\omega}_d + k \mathbf{q}_e\| \leq \frac{1}{2} c_{dJ} (\|\boldsymbol{\omega}\| + c_\omega + k)$, it can be obtained that

$$\begin{aligned} \left\| \bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} + \frac{1}{2} \frac{d\mathbf{J}(t)}{dt} \mathbf{s} \right\| &\leq \left\{ c_J \left(2c_\omega + \frac{3}{2}k \right) + \frac{1}{2} c_{dJ} \right\} \|\boldsymbol{\omega}\| \\ &+ c_J \left(c_\omega^2 + \frac{k}{2} c_\omega + c_{d\omega} \right) + \frac{1}{2} c_{dJ} (c_\omega + k) + d_{max}. \end{aligned} \quad (3.63)$$

The above equation can be rewritten as

$$\left\| \bar{\boldsymbol{\omega}}_d^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{J} \dot{\bar{\boldsymbol{\omega}}}_d + \mathbf{d} + \frac{1}{2} \frac{d\mathbf{J}(t)}{dt} \mathbf{s} \right\| \leq c\Theta \quad (3.64)$$

where the unknown constant c is redefined as

$$c = \max \left\{ c_J \left(2c_\omega + \frac{3}{2}k \right) + \frac{1}{2} c_{dJ}, \right. \\ \left. c_J \left(c_\omega^2 + \frac{k}{2} c_\omega + c_{d\omega} \right) + \frac{1}{2} c_{dJ} (c_\omega + k) + d_{max} \right\}. \quad (3.65)$$

and the time-varying variable $\Theta = \|\boldsymbol{\omega}\| + 1$. Then, it can be obtained that

$$\begin{aligned} \dot{V}_s &\leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 + c\Theta \|\mathbf{s}\| + \Delta_{max} \xi \|\mathbf{D}\| \frac{k_v + \hat{c}(t)\Theta + \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \|\mathbf{s}\| \\ &- \frac{k_v + \hat{c}(t)\Theta + \Delta_{max} \xi \|\mathbf{D}\| \|\mathbf{v}_1\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} \mathbf{s}^T \mathbf{s}_a - \tilde{c}(t) \Theta \mathbf{s}_a^T \mathbf{D}_\epsilon [\mathbf{s}] + \frac{\alpha_2}{\Theta} \tilde{c}(t) \hat{c}(t), \end{aligned} \quad (3.66)$$

which is the same as the equation (3.43) in the stability analysis of Theorem 3.2. Consequently, following the same lines as in the proof of Theorem 3.2, the results in Corollary 3.1 can be established. $\square$

Remark 3.3. From parameter update law in (3.33), the constant α_1 mainly adjusts the increasing rate of the parameter $\hat{c}(t)$ (the rising phase of $\hat{c}(t)$ if the initial value $\hat{c}(0)$ is small), while α_2 adjusts the decreasing rate (the steady phase of $\hat{c}(t)$). In order to ensure a smooth change of the $\hat{c}(t)$, α_1 and α_2 should be small constants.

On the other hand, due to the stability condition in (3.46), a large value of α_2 should not be select.

Figure 3.2: Different levels of RW1 fault.

Remark 3.4. The stability condition on the control parameters k_v , α_2 , and σ in equation (3.46) of Theorem 3.2 and Corollary 3.1 can be relaxed if some modifications are made to the update law (3.33). The modified update law is proposed as

$$\dot{\hat{c}}(t) = \alpha_1 \Theta \mathbf{s}_a^T \mathbf{D}_\epsilon[\mathbf{s}] - \frac{\alpha_1}{\Theta} \alpha_2^2(t) \hat{c}(t). \quad (3.67)$$

This modified update law is employed with an auxiliary time-varying gain function $\alpha_2(t)$ that satisfies

$$\dot{\alpha}_2(t) = -\frac{k_\alpha}{\Theta} \alpha_2(t) \quad (3.68)$$

Figure 3.3: Simulation results under *Scenario 1*: Actuator fault/failure scenario and tracking errors using nonadaptive FTC design in equation (3.12) under different level of loss of effectiveness fault.

where k_α is a positive constant. To show the stability of the closed-loop system under the virtual control input (3.29) and modified adaptive law (3.67), the candidate Lyapunov function is chosen as $V_s = \frac{1}{2} \mathbf{s}^T \mathbf{J} \mathbf{s} + k_2 [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2] + \frac{1}{2\alpha_1} \tilde{c}^2(t) + \frac{1}{8k_\alpha} c^2 \alpha_2^2(t)$. Differentiating V_s with respect to time and substituting the virtual control input and modified update law into it, $\dot{V}_s \leq -k_1 \|\mathbf{s}\|^2 - k_2 k \|\mathbf{q}_e\|^2 < 0$ can also be obtained in Case I without restrictions on the control parameters. This modification on the adaptive law shall not affect the result in Case II except that the constant η about the convergent regions is changed to $\eta = \frac{\Delta_{max} \xi \|\mathbf{D}\|}{1 - \Delta_{max} \xi \|\mathbf{D}\|} (k_v + c_m + k_2 + k_1 \sigma) \sigma + c\sigma$. Therefore, it is proved similarly that the results stated in Theorem 3.2 and Corollary 3.1 can also be achieved.

3.4 Simulation Results

To study the effectiveness and performance of the CA-based FTC strategies, numerical simulations have been carried out for spacecraft attitude tracking system given in equation (3.1) under multiplicative actuator faults/failures. The spacecraft is assumed to have the inertia matrix of $\mathbf{J} = [20 \ 1.2 \ 0.9; 1.2 \ 17 \ 1.4; 0.9 \ 1.4 \ 15]$ $\text{kg} \cdot \text{m}^2$. Due to the gravitational perturbation, solar radiation pressure, and atmo-

(a) Commanded control input for RW1.

(b) Commanded control input for RW2.

(c) Commanded control input for RW3.

(d) Commanded control input for RW4.

Figure 3.4: Simulation results under *Scenario 1*: commanded control input u_c using nonadaptive FTC design in equation (3.12) under different level of loss of effectiveness fault.

(a) Attitude orientation tracking error.

(b) Angular velocity tracking error.

Figure 3.5: Simulation results under *Scenario 1*: tracking errors using inertia-free adaptive continuous FTC design in equation (3.29) under different level of loss of effectiveness fault.

(a) Commanded control input for RW1.

(b) Commanded control input for RW2.

(c) Commanded control input for RW3.

(d) Commanded control input for RW4.

Figure 3.6: Simulation results under *Scenario 1*: commanded control input $\mathbf{u}_c$ using inertia-free adaptive continuous FTC design in equation (3.29) under different level of loss of effectiveness fault.

spheric drag, etc., external disturbances are also taken into account, and they are assumed as $\mathbf{d} = 5 \times 10^{-2} [\sin(0.8t) \ \cos(0.5t) \ \sin(0.2t)]^T \text{ N} \cdot \text{m}$. In order to achieve three axes control of a spacecraft, at least three actuators are required to force the attitude to follow a commanded attitude trajectory. In the simulation, four reaction wheels (RW) in a pyramid configuration are considered as actuators, which indicates the existence of actuator redundancy in the attitude control system. The distribution matrix of four reaction wheels ($n = 4$) is as follows:

$$\mathbf{D} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Although the unit quaternion is used to represent attitude, simulation results of attitude are expressed in Euler angles (yaw, pitch, and roll are ψ , θ , and ϕ) converted from unit quaternion for a clear interpretation of the results. The initial attitude is set as $[\psi, \theta, \phi]^T = [38.85, 22.33, -16.32]^T \text{ deg}$, corresponding to $\mathbf{Q}(0) = [-0.2, 0.15, 0.35, 0.9028]^T$. The initial angular velocity is $\boldsymbol{\omega}(0) = [0, 0, 0]^T \text{ rad/s}$. The desired motion expressed in the reference frame $\mathcal{R}$ with respect to inertial frame $\mathcal{I}$ is $\boldsymbol{\omega}_d(t) = 0.01[\cos(t/40), \sin(t/60), -\cos(t/50)]^T \text{ rad/s}$. Using $\dot{\mathbf{q}}_d = \frac{1}{2}(\mathbf{q}_d^\times + q_{d0}\mathbf{I}_3)\boldsymbol{\omega}_d$ and $\dot{q}_{d0} = -\frac{1}{2}\mathbf{q}_d^T\boldsymbol{\omega}_d$ subject to $\mathbf{q}_d^T\mathbf{q}_d + q_{d0}^2 = 1$, the desired quaternion $\mathbf{Q}_d$ can be obtained, and then the desired attitude represented by Euler angles (ψ_d , θ_d , and ϕ_d) can also be obtained. Here it is assumed that the upper bound on the relative error in fault estimation is 10%, so the constant $\Delta_{max} = 0.1$. The parameter of the auxiliary desired angular velocity in equation (3.3) is set as $k = 0.3$. The parameter of the discontinuous virtual control input in equation (3.12) is set as $k_v = 0.2$. In the simulation, the discontinuity control action in equation (3.12) is smoothed using a sigmoidal approximation [106], $\tilde{\boldsymbol{\omega}}/(\|\tilde{\boldsymbol{\omega}}\| + \delta)$ where the value of the positive constant δ is chosen to be 0.0001. The parameters of the inertia-

free adaptive continuous virtual control input in equation (3.29) is set as $k_1 = 0.1$, $k_2 = 0.1$, $\alpha_1 = 0.01$, $\alpha_2 = 0.01$, $\sigma = 0.001$, and the value of k_v is the same as in discontinuous virtual control input. The initial value of the $\hat{c}$ in equation (3.33) is chosen as $\hat{c}(0) = 0.1$. The initial value of desired quaternion is set as $\mathbf{Q}_d(0) = [0, 0, 0, 1]^T$. Two scenarios of actuator faults/failures are considered respectively.

3.4.1 Scenario 1: Single Actuator Fault/Failure

In this scenario, the RW1 undergoes different levels of loss of effectiveness fault ranging from 0% to 100% (including healthy, partial loss of effectiveness, and float failures). According to the assumption that $\Delta_{max} = 0.1$ in the simulation, it is found that $\xi = 1.732$ in equation (3.10) by using a numerical search. Consequently, it is verified that $\frac{1}{\xi\|\mathbf{D}\|} = 0.5 > \Delta_{max}$, and therefore the condition (3.11) in Assumption 3.5 is satisfied.

The actuator fault scenario in *Scenario 1* is depicted in Fig. 3.2, where 11 different RW1 fault conditions including healthy and total failure situations are considered. The tracking performance of closed-loop system using the nonadaptive virtual control input (3.12) and CA scheme (3.7) is shown in Figs. 3.3(a) and 3.3(b). It can be seen that the tracking errors converge to zero in about 45 s, and the tracking performance under different levels of fault is almost identical to that of the healthy situation. That is to say, in the event of single actuator fault/failure, there is no degradation in performance as compared with the fault-free condition. Fig. 3.4 shows the commanded reaction wheel control input, and control chattering is avoided because of the sigmoidal approximation. It is observed from Fig. 3.4 that the CA scheme redistributes the control signals to the fault-free RW2-RW4 systematically, and the faulty RW1 is used reasonably. Fig. 3.5 and Fig. 3.6 present tracking errors and commanded control input using the inertia-free adaptive continuous FTC design

(3.29) and CA scheme (3.7), respectively. It can be seen that all signals are bounded and asymptotic tracking can be achieved, which is similar to the result obtained by the nonadaptive virtual control input. However, it should be pointed out that the adaptive continuous FTC design (3.29) can provide a smoother control input signal than the virtual control input (3.12) (see Fig. 3.4 and Figs. 3.6(a)-3.6(d)).

Figure 3.7: Simulation results under *Scenario 2*: tracking errors using nonadaptive FTC design in equation (3.12) under multiple simultaneous actuator fault/failure

Figure 3.8: Simulation results under *Scenario 2*: commanded control input u_c using nonadaptive FTC design in equation (3.12) under multiple simultaneous actuator fault/failure

Figure 3.9: Simulation results under *Scenario 2*: tracking errors using inertia-free adaptive continuous FTC design in equation (3.29) under multiple simultaneous actuator fault/failure.

3.4.2 Scenario 2: Multiple Actuator Faults/Failures

In this case, four reaction wheels experience partial loss of effectiveness fault or total failure simultaneously. RW1-RW3 undergo partial loss of effectiveness fault, while RW4 has totally failed. The detailed scenario is described as

$$\begin{cases} e_1(t) = 0.5 + 0.02 \sin(0.05t) + 0.025 \text{rand}(\cdot) \\ e_2(t) = 0.6 + 0.03 \cos(0.05t) + 0.03 \text{rand}(\cdot) \\ e_3(t) = 0.4 + 0.01 \sin(0.05t) + 0.025 \text{rand}(\cdot) \\ e_4(t) = 0 \end{cases}$$

where the function $\text{rand}(\cdot)$ generates a random value from the normal distribution with mean 0 and standard deviation 1. In *Scenario 2*, it is also found that $\xi = 1.732$ in (3.10). Thus, the condition (3.11) in Assumption 3.5 is also satisfied.

The attitude and angular velocity tracking errors using nonadaptive FTC design (3.12) and inertial-free continuous adaptive FTC design (3.29) under *Scenario 2* are given in Fig. 3.7 and Fig. 3.9, respectively. It can be seen that all signals are bounded and asymptotic tracking can be ensured under both FTC designs despite

Figure 3.10: Simulation results under *Scenario 2*: commanded control input u_c using using inertia-free adaptive continuous FTC design in equation (3.29) under multiple simultaneous actuator fault/failure.

external disturbances, and actuator faults/failures. In particular, it is observed from Figs. 3.8 and 3.10 that no commanded control input is reallocated to RW4 and the virtual control input is totally reallocated to RW1-RW3. This observation is in accordance with the CA scheme in (3.7). In addition, similar to previous scenario, since the adaptive FTC input (3.29) is continuous, it can provide a smoother control input signal than the nonadaptive virtual control input (3.12) (see Figs. 3.8 and 3.10).

3.5 Summary

In this chapter, two CA-based FTC schemes for attitude tracking systems of over-actuated spacecraft in the presence of actuator faults/failures and external disturbances are presented. Due to the availability of actuator redundancies, CA is used to redistribute the control efforts among remaining actuators without reconfiguring the controller, and faulty but not completely failed actuators are fully leveraged. In the first FTC scheme, a discontinuous controller is proposed, which is an extension of existing CA-based FTC scheme from linear systems to nonlinear highly coupled attitude control systems. Then, a novel inertia-free adaptive continuous FTC scheme

is further developed which assumes that no information about the inertia matrix and external disturbances is available. Since the second FTC scheme not only prevents control chattering phenomena but also has the capacity to handle actuator faults/failures, it can be directly and widely used to enhance the reliability and safety in practical applications. The effectiveness of the proposed FTC schemes is verified through simulation of a rigid spacecraft, which shows almost no performance degradation against the fault-free condition despite actuator faults/failures.

Chapter 4

RobCA for Attitude Tracking under Actuator Faults

In chapter 3, the proposed CA-based fault-tolerant attitude controller solves the attitude tracking problem for spacecraft in the presence of actuator faults/failures and FDD uncertainties. Since the robustness issue respect to imprecise fault estimation is considered in high-level controller design and actuator saturation constraints are not considered in CA of chapter 3, an analytical solution to CA is derived, which enables that the overall stability of such closed-loop system using CA with the high-level controller could be guaranteed theoretically. However, this design philosophy leads to a complicated high-level controller design, especially when actuator saturation constraints are present. In addition, the additive actuator faults/faliures are not considered in previous chapter. In order to deal with these issues, robust control allocation (RobCA) method is proposed to ensure robustness of system performances with respect to imprecise fault estimation in control allocation design instead of in high-level controller design. Therefore, design complexity of the high-level controller can be reduced. Moreover, actuator saturation constraints are also taken into account in RobCA design.

In this chapter, a model reference adaptive (MRA) high-level controller incorporating RobCA scheme is proposed for spacecraft attitude tracking in the presence of actuator faults, imprecise FDD estimations, external disturbances, and actuator amplitude and rate constraints. Specifically, to produce the total three-axis control torques for the over-actuated spacecraft attitude control system, a reference model is designed for the attitude tracking error dynamics to follow and an MRA high-level controller is synthesized to guarantee that the attitude and angular velocity tracking error can converge to zero asymptotically despite the external disturbances. Then, based on the fault information provided by an FDD scheme, a novel RobCA method is proposed to distribute the total desired control command generated by the MRA high-level controller when the actuator fault information estimated by FDD is not precise.

The benefits of the proposed MRA high-level controller incorporating RobCA strategy are threefold. Firstly, external disturbances can be rejected by the MRA high-level controller without considering actuator faults, and the closed-loop performance is defined by a reference model, which creates a desired trajectory for the attitude tracking system to follow. Secondly, in contrast to the existing control allocation based FTC schemes such as [59,62], imprecise fault estimation in both actuator effectiveness and additive fault is considered. Moreover, the proposed methods ensure robustness of system performances with respect to imprecise fault estimation in control allocation design instead of in high-level controller design. Therefore, design complexity of high-level controller can be reduced. Thirdly, comparing with aforementioned robust control allocation methods such as [64,90], the optimal index of minimizing the energy consumption is introduced to guarantee that the solution is unique in both unsaturated and saturated cases of RobCA. Furthermore, to reduce computation burden in the unsaturated case of RobCA, the original vector optimization problem is converted to a one-dimensional search problem.

The chapter is organized as follows. In section 4.1, mathematic models for spacecraft attitude tracking systems with actuator faults are presented, and the RobCA-based FTC problem is formulated. In section 4.2, the MRA high-level controller is designed. Section 4.3 presents the solutions for unsaturated RobCA problem and saturated RobCA problem. Section 4.4 illustrates application of the overall attitude control strategy to a rigid spacecraft, followed by conclusions in section 4.5.

4.1 Preliminary and Problem Statement

4.1.1 Preliminary

In this chapter, actuator redundancy is considered to enhance reliability and safety of spacecraft. The attitude tracking error dynamics with actuator faults/failures is given by

$$\begin{cases} \dot{\mathbf{Q}}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}_e^T \\ \mathbf{q}_e^\times + q_{e0}\mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega}_e \\ \mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{J}(\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + \mathbf{D}(\mathbf{E}\mathbf{u}_c(t) + \bar{\mathbf{u}}(t)) + \mathbf{d}, \end{cases} \quad (4.1)$$

where n actuators are used as actuators and $n > 3$.

To facilitate the RobCA-based FTC design, the following assumptions are presented and will be used in the subsequent developments.

Assumption 4.1. *The external disturbances $\mathbf{d}(t) = [d_1(t), d_2(t), d_3(t)]^T \in \mathbb{R}^3$ may come in many forms, and they are considered in this paper as a combination of a constant and a series of sinusoidal functions [17,107], i.e., each component $d_i(t)$, $i \in \{1, 2, 3\}$, is given by*

$$d_i(t) = h_{i0} + \sum_{j=1}^{n_i} h_{ij} \sin(\omega_{ij}t + \Upsilon_{ij}) \quad (4.2)$$

where h_{i0} and h_{ij} are arbitrarily unknown amplitudes, Υ_{ij} are unknown phase angles, and ω_{ij} are known frequencies. Furthermore, the disturbance model (4.2) can be written in a compact form:

$$d_i(t) = \mathbf{H}_i^T \boldsymbol{\Psi}_i(t) \quad (4.3)$$

where $\mathbf{H}_i = [h_{i0}, h_{i1} \cos \Upsilon_{i1}, \dots, h_{in_i} \cos \Upsilon_{in_i}, h_{i1} \sin \Upsilon_{i1}, \dots, h_{in_i} \sin \Upsilon_{in_i}]^T \in \mathbb{R}^{2n_i+1}$ is an unknown vector with constant elements, and $\boldsymbol{\Psi}_i(t) = [1, \sin(\omega_{i1}t), \dots, \sin(\omega_{in_i}t), \cos(\omega_{i1}t), \dots, \cos(\omega_{in_i}t)]^T \in \mathbb{R}^{2n_i+1}$ is known.

Assumption 4.2. The inertia matrix $\mathbf{J}$ is a symmetric, positive definite and bounded constant matrix. There exists a positive constant c_J such that $\mathbf{x}^T \mathbf{J} \mathbf{x} \leq c_J \|\mathbf{x}\|^2$ for any $\mathbf{x} \in \mathbb{R}^3$. The notation $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

Assumption 4.3. The desired angular velocity of spacecraft and its time derivative (i.e., $\boldsymbol{\omega}_d$ and $\dot{\boldsymbol{\omega}}_d$) are bounded.

Remark 4.1. As stated in [17,108], it is reasonable to approximate the disturbances in a form of equation (4.2) stated in Assumption 4.1 with known frequencies. For example, in the case of orbit-dependent disturbances, the frequencies can be estimated from the orbital parameters. If disturbances are originating from orbital devices, its spectral content may be known. In other cases, it may be possible to estimate the spectrum of the disturbances through signal processing.

4.1.2 Problem Statement

The overall structure of the proposed fault-tolerant attitude tracking scheme is shown in Fig. 4.1.

Step 1: High-Level Controller Design

In the first step, the virtual control torque or combined total control torque $\boldsymbol{\tau} \in \mathbb{R}^3$

Figure 4.1: Structure of the overall proposed FTC system.

is introduced as

$$\tau = D\mathbf{u} = D(E\mathbf{u}_c(t) + \bar{\mathbf{u}}(t)). \quad (4.4)$$

Then, the attitude tracking error dynamics in (4.1) can be rewritten as the following virtual equivalent system

$$\begin{cases} \dot{\mathbf{Q}}_e = \begin{bmatrix} \dot{q}_{e0} \\ \dot{\mathbf{q}}_e \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}_e^T \\ \mathbf{q}_e^\times + q_{e0}\mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega}_e \\ J\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times J\boldsymbol{\omega} + J(\boldsymbol{\omega}_e^\times \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + \tau + \mathbf{d}. \end{cases} \quad (4.5)$$

An MRA control approach is designed as high-level controller to specify total attitude control torque and simultaneously reject external disturbances.

Step 2: Control Allocation Design

In the second step, with consideration of the imprecision in FDD, a RobCA algorithm is designed to distribute the desired total control commands determined by the high-level controller equivalently to each individual actuator.

4.2 High-Level Controller Design

In this section, an MRA high-level controller is proposed for the spacecraft attitude tracking system in the presence of external disturbances. Define a filtered error

variable or sliding vector as

$$\mathbf{s} = \boldsymbol{\omega}_e + \beta \mathbf{q}_e, \quad (4.6)$$

where β is a positive constant. Then, from equation (4.5), the dynamic equation for the filtered error variable $\mathbf{s}$ is obtained as

$$\mathbf{J}\dot{\mathbf{s}} = \mathbf{f}(\boldsymbol{\omega}, \boldsymbol{\omega}_d, \dot{\boldsymbol{\omega}}_d, \mathbf{Q}, \mathbf{Q}_d) + \boldsymbol{\tau} + \mathbf{d}, \quad (4.7)$$

where $\mathbf{f}(\boldsymbol{\omega}, \boldsymbol{\omega}_d, \dot{\boldsymbol{\omega}}_d, \mathbf{Q}, \mathbf{Q}_d) \in \mathbb{R}^3$ is the lumped nonlinear term given by

$$\begin{aligned} \mathbf{f} = & -(\boldsymbol{\omega}_e + \mathbf{C}\boldsymbol{\omega}_d)^\times \mathbf{J}(\boldsymbol{\omega}_e + \mathbf{C}\boldsymbol{\omega}_d) \\ & + \frac{\beta}{2}(\mathbf{q}_e^\times + q_{e0}\mathbf{I}_3)\boldsymbol{\omega}_e + \mathbf{J}(\boldsymbol{\omega}_e^\times \mathbf{C}\boldsymbol{\omega}_d - \mathbf{C}\dot{\boldsymbol{\omega}}_d). \end{aligned} \quad (4.8)$$

To obtain the online information on disturbances, an auxiliary system that can be regarded as a reference model to the filtered error dynamics (4.7), is introduced as

$$\mathbf{J}\dot{\hat{\mathbf{s}}} = \mathbf{f} + \boldsymbol{\chi} + \hat{\mathbf{d}}(t), \quad (4.9)$$

where $\hat{\mathbf{s}} = [\hat{s}_1 \ \hat{s}_2 \ \hat{s}_3]^T \in \mathbb{R}^3$ is the estimate of filtered error variable, $\boldsymbol{\chi} \in \mathbb{R}^3$ is the input of the auxiliary system that will be given later. The vector $\hat{\mathbf{d}}(t) = [\widehat{\mathbf{H}}_1^T \boldsymbol{\Psi}_1(t), \widehat{\mathbf{H}}_2^T \boldsymbol{\Psi}_2(t), \widehat{\mathbf{H}}_3^T \boldsymbol{\Psi}_3(t)]^T \in \mathbb{R}^3$ denotes the estimate of the disturbance, where $\widehat{\mathbf{H}}_i \in \mathbb{R}^{2n_i+1}$, $i \in \{1, 2, 3\}$, is the estimate of $\mathbf{H}_i$ in equation (4.2).

Define the reference model tracking error as $\tilde{\mathbf{s}} = [\tilde{s}_1, \tilde{s}_2, \tilde{s}_3]^T \in \mathbb{R}^3$ with $\tilde{s}_i = \hat{s}_i - s_i$, $i \in \{1, 2, 3\}$. Then, dynamics for $\tilde{\mathbf{s}}$ is written as

$$\mathbf{J}\dot{\tilde{\mathbf{s}}} = \boldsymbol{\chi} - \boldsymbol{\tau} + \tilde{\mathbf{d}}(t), \quad (4.10)$$

where $\tilde{\mathbf{d}}(t) = [\widetilde{\mathbf{H}}_1^T \boldsymbol{\Psi}_1(t), \widetilde{\mathbf{H}}_2^T \boldsymbol{\Psi}_2(t), \widetilde{\mathbf{H}}_3^T \boldsymbol{\Psi}_3(t)]^T \in \mathbb{R}^3$ with $\widetilde{\mathbf{H}}_i = \widehat{\mathbf{H}}_i - \mathbf{H}_i$, $i \in$

$\{1, 2, 3\}$, is the disturbance estimate error.

Next, the theorem with regard to the MRA high-level controller is illustrated as follows:

Theorem 4.1. *Consider that the attitude tracking error dynamics is described by equation (4.5) and that Assumptions 4.1-4.3 are satisfied. Define the MRA high-level controller as*

$$\boldsymbol{\tau} = \boldsymbol{\chi} + k_1 \tilde{\mathbf{s}} - k_3 \mathbf{q}_e \quad (4.11)$$

with $\boldsymbol{\chi}$ given by

$$\boldsymbol{\chi} = -\hat{\mathbf{d}}(t) - \mathbf{f} - k_2 \hat{\mathbf{s}}, \quad (4.12)$$

where k_1 , k_2 , and k_3 are positive constant gains, and k_2 and k_3 are chosen so that $4\beta k_2 > k_3$. Let the adaptive law for $\widehat{\mathbf{H}}_i$ in $\hat{\mathbf{d}}(t)$ of equation (4.12) be updated by

$$\dot{\widehat{\mathbf{H}}}_i(t) = -\boldsymbol{\Lambda}_i \boldsymbol{\Psi}_i(t) \tilde{s}_i, \quad (4.13)$$

where $\boldsymbol{\Lambda}_i$ is a diagonal matrix with positive entries. Then, the closed-loop attitude tracking error system will converge to the equilibrium point $\mathbf{Q}_e = [\mathbf{q}_{e0}, \mathbf{q}_e^T]^T = [1, 0]^T$, $\boldsymbol{\omega}_e = 0$ asymptotically.

Proof. To analyze the stability of attitude tracking systems under above controller, consider the following candidate Lyapunov function:

$$V = \frac{1}{2} \tilde{\mathbf{s}}^T \mathbf{J} \tilde{\mathbf{s}} + \frac{1}{2} \hat{\mathbf{s}}^T \mathbf{J} \hat{\mathbf{s}} + k_3 [\mathbf{q}_e^T \mathbf{q}_e + (1 - q_{e0})^2] + \frac{1}{2} \sum_{i=1}^3 \widetilde{\mathbf{H}}_i^T \boldsymbol{\Lambda}_i^{-1} \widetilde{\mathbf{H}}_i. \quad (4.14)$$

Differentiating V along trajectories of equations (4.9) and (4.10) results in

$$\begin{aligned}\dot{V} &= \tilde{\mathbf{s}}^T \mathbf{J} \dot{\tilde{\mathbf{s}}} + \hat{\mathbf{s}}^T \mathbf{J} \dot{\hat{\mathbf{s}}} + k_3 \mathbf{q}_e^T \boldsymbol{\omega}_e + \sum_{i=1}^3 \widetilde{\mathbf{H}}_i^T \Lambda_i^{-1} \dot{\widetilde{\mathbf{H}}}_i \\ &= \tilde{\mathbf{s}}^T (\boldsymbol{\chi} - \boldsymbol{\tau} + \tilde{\mathbf{d}}) + \hat{\mathbf{s}}^T (\mathbf{f} + \boldsymbol{\chi} + \hat{\mathbf{d}}) + k_3 \mathbf{q}_e^T \mathbf{s} - \beta k_3 \|\mathbf{q}_e\|^2 + \sum_{i=1}^3 \widetilde{\mathbf{H}}_i^T \Lambda_i^{-1} \dot{\widetilde{\mathbf{H}}}_i.\end{aligned}$$

Since $\mathbf{H}_i$ is a vector with constant elements, it follows that $\dot{\widetilde{\mathbf{H}}}_i = \dot{\hat{\mathbf{H}}}_i$. Substituting the high-level controller given in equations (4.11) and (4.12) and adaptive law given by equation (4.13) into above yields

$$\begin{aligned}\dot{V} &\leq -k_1 \|\tilde{\mathbf{s}}\|^2 - k_2 \|\hat{\mathbf{s}}\|^2 + k_3 \hat{\mathbf{s}}^T \mathbf{q}_e - \beta k_3 \|\mathbf{q}_e\|^2 + \tilde{\mathbf{s}}^T \tilde{\mathbf{d}} - \sum_{i=1}^3 \widetilde{\mathbf{H}}_i^T \Psi_i(t) \tilde{s}_i \\ &\leq -k_1 \|\tilde{\mathbf{s}}\|^2 - \begin{bmatrix} \hat{\mathbf{s}}^T & \mathbf{q}_e^T \end{bmatrix} \mathbf{P} \begin{bmatrix} \hat{\mathbf{s}}^T & \mathbf{q}_e^T \end{bmatrix}^T,\end{aligned}\quad (4.15)$$

where $\mathbf{P} = \begin{bmatrix} k_2 \mathbf{I}_3 & -\frac{1}{2} k_3 \mathbf{I}_3 \\ -\frac{1}{2} k_3 \mathbf{I}_3 & \beta k_3 \mathbf{I}_3 \end{bmatrix}$.

Since $4\beta k_2 > k_3$, it is clear that $\mathbf{P}$ is positive definite, which implies that V is nonincreasing and bounded for all $t \geq 0$, and it follows that $\tilde{\mathbf{s}}$, $\hat{\mathbf{s}}$, $\mathbf{q}_e$, and $\widetilde{\mathbf{H}}_i$ are all bounded. As $\tilde{\mathbf{s}}$ and $\hat{\mathbf{s}}$ are bounded, it is clear that the filtered error variable $\mathbf{s}$ is bounded for all $t \geq 0$, which implies that $\boldsymbol{\omega}_e$ is bounded due to the boundedness of $\mathbf{q}_e$. Consequently, using assumptions 4.1-4.3 and boundedness of $\boldsymbol{\omega}_e$ and $\widetilde{\mathbf{H}}_i$, one can show that $\mathbf{f}(\boldsymbol{\omega}, \boldsymbol{\omega}_d, \dot{\boldsymbol{\omega}}_d, \mathbf{Q}, \mathbf{Q}_d)$ and $\hat{\mathbf{d}}$ are bounded for all $t \geq 0$. Since $\hat{\mathbf{d}}$, $\mathbf{f}$, and $\hat{\mathbf{s}}$ are bounded, it is clear from (4.12) that $\boldsymbol{\chi}$ is bounded, and hence one can conclude that $\boldsymbol{\tau}$ is also bounded for all $t \geq 0$. Therefore, from equations (4.9), (4.10), and second equation of (4.5), one can easily verify that $\dot{\hat{\mathbf{s}}}$, $\dot{\tilde{\mathbf{s}}}$, and $\dot{\mathbf{q}}_e$ are bounded for all $t \geq 0$, which implies that $\hat{\mathbf{s}}$, $\tilde{\mathbf{s}}$, and $\mathbf{q}_e$ are uniformly continuous functions. Moreover, from equation (4.15), it is easy to verify that $\hat{\mathbf{s}}$, $\tilde{\mathbf{s}}$, $\mathbf{q}_e \in \mathcal{L}_2$. Then, from the Barbalat's Lemma (using the alternative statement of this lemma from *Corollary A.7* of [102]),

it yields $\tilde{\mathbf{s}} \rightarrow 0$, $\hat{\mathbf{s}} \rightarrow 0$, and $\mathbf{q}_e \rightarrow 0$ as $t \rightarrow \infty$. Hence, from $\tilde{\mathbf{s}} \rightarrow 0$ and $\hat{\mathbf{s}} \rightarrow 0$, one concludes that $\mathbf{s} \rightarrow 0$. Since $\mathbf{s} \rightarrow 0$ and $\mathbf{q}_e \rightarrow 0$, it follows from equation (4.6) that $\boldsymbol{\omega}_e \rightarrow 0$ as well. Moreover, since $\mathbf{q}_e \rightarrow 0$ and $\mathbf{q}_e^2 + q_{e0}^2 = 1$, it leads to $q_{e0} \rightarrow \pm 1$. As discussed in [8,23] that $q_{e0} = -1$ is not a stable equilibrium, it is obtained that $q_{e0} \rightarrow 1$ as $t \rightarrow \infty$. Therefore, results in Theorem 4.1 are established. $\square$

Remark 4.2. *The introduction of the auxiliary system (4.9) provides a reference model for the filtered error system (4.7) to follow. In fact, by substituting the auxiliary system input (4.12) in equation (4.9), it is obtained that $\mathbf{J}\dot{\hat{\mathbf{s}}} = -k_2\hat{\mathbf{s}}$. Consequently, one can choose a candidate Lyapunov function $V = \frac{1}{2}\hat{\mathbf{s}}^T\mathbf{J}\hat{\mathbf{s}}$, and by Lyapunov stability analysis, it is proved that $\hat{\mathbf{s}}$ converge to zero with an exponential rate greater than $\frac{2k_2}{c_J}$. This state trajectory of the auxiliary system creates a desired reference trajectory for the state of the filtered error system to follow. Therefore, if the high-level controller is designed such that the filtered error variable s can asymptotically track $\hat{\mathbf{s}}$, then s will converge to zero asymptotically.*

4.3 RobCA Design

Due to physical limitations on actuators, the amplitude and rate constraints of actuators are considered in the control allocation design. For simplicity, it is assumed that the actuators have the same amplitude and rate constraint values, and the command control inputs $\mathbf{u}_c$ are restricted by

$$\delta_{min} \leq u_{ci} \leq \delta_{max}, \quad |\dot{u}_{ci}| \leq \eta_{rate} \quad (4.16)$$

where lower and upper position constraints of the i th actuator are defined by δ_{min} and δ_{max} , respectively, and η_{rate} is the maximum control rate of the i th actuator. Because modern spacecraft attitude control systems are implemented in a digital computer, rate constraints can be converted into position constraints. Then, overall

constraints are further specified as

$$\underline{\mathbf{u}}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c, \quad (4.17)$$

with $\underline{u}_{ci} = \max\{\delta_{min}, u_{ci}(t - T) - T\eta_{rate}\}$ and $\bar{u}_{ci} = \min\{\delta_{max}, u_{ci}(t - T) + T\eta_{rate}\}$, where $\underline{\mathbf{u}}_c = [\underline{u}_{c1}, \underline{u}_{c2}, \dots, \underline{u}_{cn}]^T \in \mathbb{R}^n$ and $\bar{\mathbf{u}}_c = [\bar{u}_{c1}, \bar{u}_{c2}, \dots, \bar{u}_{cn}]^T \in \mathbb{R}^n$ are the combined constraints for n actuators, T is the sampling time, and $u_{ci}(t - T)$ is the i th command control input in the previous sampling instant.

In addition, in order to handle actuator faults, estimated fault information is used in control allocation. Assuming that actuator fault information, $\hat{\mathbf{E}} = \text{diag}\{\hat{e}_1, \hat{e}_2, \dots, \hat{e}_n\} \in \mathbb{R}^{n \times n}$ and $\hat{\mathbf{u}} = [\hat{u}_1, \hat{u}_2, \dots, \hat{u}_n]^T \in \mathbb{R}^n$, is detected and estimated by an FDD scheme. However, no matter which FDD scheme is employed, it is inevitable to have some estimation or identification errors [59], and $\hat{\mathbf{E}}$ and $\hat{\mathbf{u}}$ may not be identical to their actual values. Hence, the level of imprecision of fault estimation is introduced. Relations between actual fault and their estimated values are assumed to satisfy the following conditions:

$$\mathbf{E} = (\mathbf{I}_n - \Delta_{\mathbf{E}})\hat{\mathbf{E}}, \quad \bar{\mathbf{u}} = (\mathbf{I}_n - \Delta_{\bar{\mathbf{u}}})\hat{\mathbf{u}}, \quad (4.18)$$

where $\Delta_{\mathbf{E}} = \text{diag}\{\delta_{e1}, \delta_{e2}, \dots, \delta_{en}\} \in \mathbb{R}^{n \times n}$ and $\Delta_{\bar{\mathbf{u}}} = \text{diag}\{\delta_{\bar{u}1}, \delta_{\bar{u}2}, \dots, \delta_{\bar{u}n}\} \in \mathbb{R}^n$, and unknown scalars δ_{ei} and $\delta_{\bar{u}i}$ represent the level of imprecision in the estimation of actuator effectiveness and additive actuator fault, respectively. These two scalars can be regarded as relative errors between actual fault sizes and their estimated ones.

Thus, during the control allocation, commanded control efforts $\mathbf{u}_c$ in equation (4.4) are found such that

$$\boldsymbol{\tau} = \mathbf{D}[(\mathbf{I}_n - \Delta_{\mathbf{E}})\hat{\mathbf{E}}\mathbf{u}_c + (\mathbf{I}_n - \Delta_{\bar{\mathbf{u}}})\hat{\mathbf{u}}]. \quad (4.19)$$

In light of uncertainties in equation (4.19), RobCA is formulated as

$$\begin{aligned} \mathbf{u}_c = \arg \min_{\mathbf{u}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c} \max_{\substack{\|\Delta_{\mathbf{E}}\| \leq \rho_1, \\ \|\Delta_{\bar{\mathbf{u}}}\| \leq \rho_2}} \left\{ \|\mathbf{u}_c\|_M^2 \right. \\ \left. + h \|D(I_n - \Delta_{\mathbf{E}})\hat{\mathbf{E}}\mathbf{u}_c + D(I_n - \Delta_{\bar{\mathbf{u}}})\hat{\bar{\mathbf{u}}} - \boldsymbol{\tau}\|^2 \right\}, \end{aligned} \quad (4.20)$$

where $\|\mathbf{u}_c\|_M^2$ stands for $\mathbf{u}_c^T \mathbf{M} \mathbf{u}_c$, ρ_1 and ρ_2 are two known positive scalars such that $\|\Delta_{\mathbf{E}}\| \leq \rho_1$ and $\|\Delta_{\bar{\mathbf{u}}}\| \leq \rho_2$, respectively. For the RobCA in equation (4.20), the primary objective in the second term is to find the optimal $\mathbf{u}_c$ by minimizing the worse-case residual. This achieves some robustness given by the worst residual to control allocation with respect to the imprecision in the fault estimation [71,90]. Comparing with results in [64,90], since there is no guarantee that $\boldsymbol{\tau}$ is attainable or that the solution $\mathbf{u}_c$ is unique, a secondary objective in the first term of equation (4.20), which minimizes the commanded power consumption, is also considered.

To reduce the notational burden, let $\mathbf{A} = D\hat{\mathbf{E}}$, $\mathbf{b} = \boldsymbol{\tau} - D\hat{\bar{\mathbf{u}}}$, $\Delta\mathbf{A} = -D\Delta_{\mathbf{E}}\hat{\mathbf{E}}$, and $\Delta\mathbf{b} = D\Delta_{\bar{\mathbf{u}}}\hat{\bar{\mathbf{u}}}$. Then, the proposed RobCA becomes

$$\mathbf{u}_c = \arg \min_{\mathbf{u}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c} \left\{ \|\mathbf{u}_c\|_M^2 + \max_{\substack{\|\Delta\mathbf{A}\| \leq \rho_A, \\ \|\Delta\mathbf{b}\| \leq \rho_b}} h \|(\mathbf{A} + \Delta\mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta\mathbf{b})\|^2 \right\}, \quad (4.21)$$

where $\|\Delta\mathbf{A}\| \leq \rho_A$, $\|\Delta\mathbf{b}\| \leq \rho_b$, $\rho_A = \rho_1 \|D\| \|\hat{\mathbf{E}}\|$, $\rho_b = \rho_2 \|D\| \|\hat{\bar{\mathbf{u}}}\|$. In the following, the proposed RobCA is solved for both unsaturated and saturated actuators.

4.3.1 Unsaturated Actuators

When no actuator saturates, actuator constraints can be disregarded. Then, the RobCA problem in equation (4.21) reduces to

$$\mathbf{u}_{RobCA} = \arg \min_{\mathbf{u}_c} \left\{ \|\mathbf{u}_c\|_M^2 + \max_{\substack{\|\Delta\mathbf{A}\| \leq \rho_A, \\ \|\Delta\mathbf{b}\| \leq \rho_b}} h \|(\mathbf{A} + \Delta\mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta\mathbf{b})\|^2 \right\}. \quad (4.22)$$

Furthermore, it is verified that the unsaturated RobCA problem is equivalent to the following problem [109]

$$\mathbf{u}_c = \arg \min_{\mathbf{u}_c} \left\{ \|\mathbf{u}_c\|_M^2 + \max_{\|\mathbf{z}\| \leq \phi(\mathbf{u}_c)} h \|\mathbf{A}\mathbf{u}_c - \mathbf{b} + \mathbf{z}\|^2 \right\} \quad (4.23)$$

where $\phi(\mathbf{u}_c)$ is defined as $\phi(\mathbf{u}_c) = \rho_A \|\mathbf{u}_c\| + \rho_b$. To solve equation (4.23), the inner maximization problem is solved first, then followed by the outer minimization problem. For the inner maximization problem, the maximum

$$C(\mathbf{u}_c) \triangleq \max_{\|\mathbf{z}\| \leq \phi(\mathbf{u}_c)} h \|\mathbf{A}\mathbf{u}_c - \mathbf{b} + \mathbf{z}\|^2 \quad (4.24)$$

is convex in $\mathbf{u}_c$, and the inequality constraint is also convex in $\mathbf{z}$, so the maximum over $\mathbf{z}$ is achieved at the boundary, i.e., $\|\mathbf{z}\| = \phi(\mathbf{u}_c)$. Introducing a nonnegative Lagrange multiplier λ , the constrained maximization problem in equation (4.24) is transformed into the following unconstrained problem

$$C(\mathbf{u}_c) = \max_{\mathbf{z}, \lambda} [h \|\mathbf{A}\mathbf{u}_c - \mathbf{b} + \mathbf{z}\|^2 - \lambda (\|\mathbf{z}\|^2 - \phi^2(\mathbf{u}_c))] . \quad (4.25)$$

Differentiating equation (4.25) with respect to $\mathbf{z}$ and λ yields

$$(\lambda^* - h)\mathbf{z}^* = h(\mathbf{A}\mathbf{u}_c - \mathbf{b}), \quad \|\mathbf{z}^*\| = \phi(\mathbf{u}_c), \quad (4.26)$$

where $\mathbf{z}^*$ and λ^* denote the optimal solution of the maximization problem in equation (4.25). Computing the Hessian of $C(\mathbf{u}_c)$ with respect to $\mathbf{z}$, and let it be negative semi-definite when $\lambda = \lambda^*$. Then, it can be found that λ^* should satisfy $\lambda^* \geq h$. In view of (4.26), the maximum cost in equation (4.25) is given by

$$C(\mathbf{u}_c) = \frac{h\lambda^*}{\lambda^* - h} \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|^2 + \lambda^* \phi^2(\mathbf{u}_c). \quad (4.27)$$

Substituting equation (4.27) into the original problem (4.23), the unsaturated RobCA problem is equivalent to the following minimization problem:

$$\begin{aligned} \mathbf{u}_c &= \arg \min_{\mathbf{u}_c} \left\{ \|\mathbf{u}_c\|_Q^2 + C(\mathbf{u}_c) \right\} \\ &= \arg \min_{\mathbf{u}_c} \left\{ \|\mathbf{u}_c\|_M^2 + \frac{h\lambda^*}{\lambda^* - h} \times \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|^2 + \lambda^* \phi^2(\mathbf{u}_c) \right\}. \end{aligned} \quad (4.28)$$

Next, the outer minimization problem (4.28) is solved. To reduce the computation burden, the minimization problem, which is a vector optimization problem on $\mathbf{u}_c$, is converted to a one-dimensional search problem. For this purpose, the following function with two independent variables $\mathbf{u}_c$ and λ is introduced

$$R(\mathbf{u}_c, \lambda) = \frac{h\lambda}{\lambda - h} \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|^2 + \lambda \phi^2(\mathbf{u}_c), \quad (4.29)$$

where λ belongs to the interval $[h, +\infty)$. Then, the cost of the inner maximization in equation (4.24) can be equal to the constrained minimization problem over the scalar λ [110],

$$C(\mathbf{u}_c) = \arg \min_{\lambda \geq h} R(\mathbf{u}_c, \lambda). \quad (4.30)$$

As a result, the original problem turns out to be equivalent to

$$\begin{aligned} \mathbf{u}_c &= \arg \min_{\mathbf{u}_c} \left\{ \|\mathbf{u}_c\|_Q^2 + \min_{\lambda \geq h} R(\mathbf{u}_c, \lambda) \right\} \\ &= \arg \min_{\lambda \geq h} \min_{\mathbf{u}_c} J(\mathbf{u}_c, \lambda) \end{aligned} \quad (4.31)$$

where $J(\mathbf{u}_c, \lambda) = \|\mathbf{u}_c\|_M^2 + R(\mathbf{u}_c, \lambda)$.

Note that, because $J(\mathbf{u}_c, \lambda)$ is a quadratic function, it is possible to derive a closed-form expression for the solution of the innermost minimization in equation (4.31) with respect to $\mathbf{u}_c$ for fixed value of the Lagrange multiplier λ . Since $\phi(\mathbf{u}_c) =$

$\rho_A \|\mathbf{u}_c\| + \rho_b$, taking derivative of $J(\mathbf{u}_c, \lambda)$ with respect to $\mathbf{u}_c$, it yields

$$\left[\mathbf{G}(\lambda) + \lambda \rho_A \left(\rho_A + \frac{\rho_b}{\|\mathbf{u}_c(\lambda)\|} \right) \mathbf{I}_n \right] \mathbf{u}_c(\lambda) = \mathbf{N}(\lambda) \quad (4.32)$$

where $\mathbf{G}(\lambda) = \mathbf{M} + W(\lambda) \mathbf{A}^T \mathbf{A}$, $\mathbf{N}(\lambda) = W(\lambda) \mathbf{A}^T \mathbf{b}$, and $W(\lambda) = \frac{h\lambda}{\lambda-h}$. Then, for any nonzero $\mathbf{u}_c$ in equation (4.32), we get

$$\mathbf{u}_c(\lambda) = \left[\mathbf{G}(\lambda) + \lambda \rho_A \left(\rho_A + \frac{\rho_b}{\|\mathbf{u}_c(\lambda)\|} \right) \mathbf{I}_n \right]^{-1} \mathbf{N}(\lambda) \quad (4.33)$$

Because $\mathbf{u}_c$ appears on both sides of the equality, $\mathbf{u}_c$ can not be obtained directly. With a view to tackle the above challenge, two cases are considered.

Case I: If $\rho_b = 0$, the term $\|\mathbf{u}_c(\lambda)\|$ will disappear on the right-hand side of the expression in (4.33). In this case, the expression for $\mathbf{u}_c(\lambda)$ is given by

$$\mathbf{u}_c(\lambda) = [\mathbf{M}(\lambda) + \lambda \rho_A^2 \mathbf{I}]^{-1} \mathbf{D}(\lambda). \quad (4.34)$$

Case II: If $\rho_b \neq 0$, $\mathbf{u}_c$ is included on both sides of equation (4.33). To solve u_c , the scalar $\beta = \|\mathbf{u}_c(\lambda)\|$ is introduced. As a result, the equation (4.33) is converted to the following equation with a scalar variable β :

$$\beta^2 - \mathbf{N}^T(\lambda) \left[\mathbf{M}(\lambda) + \lambda \rho_A \left(\rho_A + \frac{\rho_b}{\beta} \right) \mathbf{I}_n \right]^{-2} \mathbf{N}(\lambda) = 0. \quad (4.35)$$

It can be shown that a unique solution $\beta^* > 0$ exists for above equation if $\lambda \rho_A \rho_b < \|\mathbf{N}(\lambda)\|$, and otherwise, $\beta^* = 0$, which means that $\mathbf{u}_c(\lambda) = 0$ [110].

Since we assume that there exists imprecision in the estimations of both actuator effectiveness and additive bias fault, the condition $\rho_b \neq 0$ is always satisfied. As a

result, it is obtained that

$$\mathbf{u}_c^*(\lambda) = \left[\mathbf{G}(\lambda) + \lambda \rho_A \left(\rho_A + \frac{\rho_b}{\beta^*} \right) \mathbf{I}_n \right]^{-1} \mathbf{N}(\lambda) \quad (4.36)$$

when $\lambda \rho_A \rho_b \leq \|\mathbf{N}(\lambda)\|$, and otherwise, $\mathbf{u}_c^*(\lambda) = 0$.

Now, let $J(\lambda)$ denote the minimum value of $J(\mathbf{u}_c, \lambda)$ over $\mathbf{u}_c$ in equation (4.31), i.e.,

$$\begin{aligned} J(\lambda) &= \min_{\mathbf{u}_c} J(\mathbf{u}_c, \lambda) = J(\mathbf{u}_c^*(\lambda), \lambda) \\ &= \|\mathbf{u}_c^*(\lambda)\|_M^2 + W(\lambda) \|\mathbf{A} \mathbf{u}_c^*(\lambda) - \mathbf{b}\|^2 + \lambda \phi^2(\mathbf{u}_c^*(\lambda)). \end{aligned}$$

Finally, the unsaturated RobCA problem can be solved by determining λ^* from the following scalar-valued optimization problem

$$\lambda^* = \arg \min_{\lambda \geq h} J(\lambda). \quad (4.37)$$

Because the function $J(\lambda)$ is unimodal [110], the minimization problem (4.37) is always well-posed such that an unique minimum on its domain is attainable.

In sum, the unsaturated RobCA algorithm is implemented as Algorithm 4.1.

4.3.2 Saturated Actuators

When actuator amplitude and rate constraints are considered, the RobCA defined in equation (4.21) can be converted into a second-order cone programming (SOCP) problem formulation. Similar to the unsaturated case, the inner maximization problem is solved first to find the worse-case residual, and then the outer minimization problem is solved by an SOCP problem.

Algorithm 4.1 Unsaturated RobCA Algorithm**Inputs:**

- $\hat{\mathbf{E}}$ and $\hat{\mathbf{u}}$ are from the FDD scheme, and the virtual actuator torque $\boldsymbol{\tau}$ is from high-level controller.
- Choose the weighting parameters $\mathbf{M}$ and h .

Steps:

1. Solve the equation (4.35) and obtain the unique positive solution β^* .
2. Solve the innermost minimization problem in equation (4.31) with respect to $\mathbf{u}_c$. The solution $\mathbf{u}_c^*(\lambda)$ is given by

$$\mathbf{u}_c^*(\lambda) = \begin{cases} [\mathbf{G}(\lambda) + \lambda \rho_A (\rho_A + \frac{\rho_b}{\beta^*}) \mathbf{I}_n]^{-1} \mathbf{N}(\lambda), & \text{if } \lambda \rho_A \rho_b < \|\mathbf{N}(\lambda)\| \\ 0, & \text{if } \lambda \rho_A \rho_b \geq \|\mathbf{N}(\lambda)\|. \end{cases} \quad (4.38)$$

3. Compute λ^* by solving the scalar-valued minimization problem defined in equation (4.37).
4. Substitute λ^* into equation (4.38) to get the solution $\mathbf{u}_c = \mathbf{u}_c^*(\lambda^*)$.

For the inner maximization problem in (4.21), using triangle inequality, it yields

$$\begin{aligned} \|(\mathbf{A} + \Delta \mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta \mathbf{b})\| &\leq \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \|\Delta \mathbf{A}\| \|\mathbf{u}_c\| + \|\Delta \mathbf{b}\| \\ &\leq \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \rho_A \|\mathbf{u}_c\| + \rho_b, \end{aligned} \quad (4.39)$$

which provides an upper bound for $\|(\mathbf{A} + \Delta \mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta \mathbf{b})\|$. In fact, this upper bound is achievable if $\Delta \mathbf{A}$ and $\Delta \mathbf{b}$ have forms of

$$\Delta \mathbf{A} = \frac{\mathbf{A}\mathbf{u}_c - \mathbf{b}}{\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|} \frac{\mathbf{u}_c^T}{\|\mathbf{u}_c\|} \rho_A, \quad \Delta \mathbf{b} = -\frac{\mathbf{A}\mathbf{u}_c - \mathbf{b}}{\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|} \rho_b. \quad (4.40)$$

Then, it follows that

$$\|(\mathbf{A} + \Delta \mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta \mathbf{b})\| = \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \rho_A \|\mathbf{u}_c\| + \rho_b$$

which is the desired upper bound from the triangle inequality.

The equation (4.40) is used to show that there exist $\Delta \mathbf{A}$ and $\Delta \mathbf{b}$ such that the upper bound obtained from triangle inequality for inner maximization problem is achievable. If $\Delta \mathbf{A}$ and $\Delta \mathbf{b}$ are in a form as shown in equation (4.40), for $\mathbf{A}\mathbf{u}_c \neq \mathbf{b}$, we have

$$\|\Delta \mathbf{A}\| = \left\| \frac{\mathbf{A}\mathbf{u}_c - \mathbf{b}}{\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|} \frac{\mathbf{u}_c^T}{\|\mathbf{u}_c\|} \rho_A \right\| = \rho_A \quad (4.41)$$

and

$$\|\Delta \mathbf{b}\| = \left\| -\frac{\mathbf{A}\mathbf{u}_c - \mathbf{b}}{\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\|} \rho_b \right\| = \rho_b, \quad (4.42)$$

which are valid perturbations since the constraints $\|\mathbf{A}\| \leq \rho_A$ and $\|\mathbf{b}\| \leq \rho_b$ are satisfied. For the case that $\mathbf{A}\mathbf{u}_c = \mathbf{b}$, we select $\Delta \mathbf{A} = \frac{\rho_A}{\|\mathbf{u}_c\|} \boldsymbol{\varepsilon} \mathbf{u}_c^T$ and $\Delta \mathbf{b} = \boldsymbol{\varepsilon} \rho_b$, where $\boldsymbol{\varepsilon}$ is an arbitrary unit norm vector. The similar analysis about uncertainty can also be found in references [90,111].

Therefore, we may conclude that the maximum cost of the inner maximization problem can be given by

$$\begin{aligned} C(\mathbf{u}_c) &= \max_{\substack{\|\Delta \mathbf{A}\| \leq \rho_A, \\ \|\Delta \mathbf{b}\| \leq \rho_b}} h\|(\mathbf{A} + \Delta \mathbf{A})\mathbf{u}_c - (\mathbf{b} + \Delta \mathbf{b})\|^2 \\ &= h(\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \rho_A\|\mathbf{u}_c\| + \rho_b)^2. \end{aligned} \quad (4.43)$$

As a result, the saturated RobCA problem reduces to

$$\mathbf{u}_c = \arg \min_{\mathbf{u}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c} \left\{ \|\mathbf{u}_c\|_M^2 + h(\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \rho_A\|\mathbf{u}_c\| + \rho_b)^2 \right\}. \quad (4.44)$$

The solution of problem (4.44) can be obtained by solving the following minimiza-

tion problem

$$\begin{aligned} & \min_{\mathbf{u}_c, t_i} t_1 \\ & \text{subject to} \quad \begin{cases} \|col(t_2, t_3)\| \leq t_1 \\ \|\mathbf{Q}^{1/2}\mathbf{u}_c\| \leq t_2 \\ \sqrt{h} (\|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| + \rho_A\|\mathbf{u}_c\| + \rho_b) \leq t_3 \\ \underline{\mathbf{u}}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c \end{cases} \end{aligned} \quad (4.45)$$

which further can be cast as an SOCP:

$$\begin{aligned} & \min_{\mathbf{u}_c, t_i} t_1 \\ & \text{subject to} \quad \begin{cases} \|col(t_2, t_3)\| \leq t_1 \\ \|\mathbf{Q}^{1/2}\mathbf{u}_c\| \leq t_2 \\ \|\mathbf{A}\mathbf{u}_c - \mathbf{b}\| \leq \frac{t_3}{\sqrt{h}} - \rho_b - t_4 \\ \|\mathbf{u}_c\| \leq \frac{t_4}{\rho_A} \\ \underline{\mathbf{u}}_c \leq \mathbf{u}_c \leq \bar{\mathbf{u}}_c, \end{cases} \end{aligned} \quad (4.46)$$

where the variables $t_i \in \mathbb{R}^+$ with $i \in \{1, 2, \dots, 4\}$, and $col(t_2, t_3) \in \mathbb{R}^2$ is a column vector composed of the variables t_2 and t_3 . The above SOCP problem in (4.46) can be solved by using nonlinear optimization softwares, such as YALMIP, CVX, etc.

4.4 Simulation Results

To study the effectiveness and performance of the proposed MRA high-level control scheme and the RobCA strategy, numerical simulations have been carried out for the rigid spacecraft attitude tracking system under actuator faults. The spacecraft

Figure 4.2: External disturbances.

(a) MRA controller with PICA

(b) PD+ controller with PICA

Figure 4.3: Response of attitude tracking error in the absence of FDD imprecisions.

(a) MRA controller with PICA. (b) PD+ controller with PICA.

is assumed to have the inertia matrix of

$$\mathbf{J} = \begin{bmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0.9 & 0 & 15 \end{bmatrix} \text{ kg} \cdot \text{m}^2.$$

Figure 4.4: Response of angular velocity tracking error ω_e in the absence of FDD imprecisions. (a) MRA controller with PICA. (b) PD+ controller with PICA.

Figure 4.5: Response of command control torque u_c in the absence of FDD imprecisions. (a) MRA controller with PICA. (b) PD+ controller with PICA.

In the simulation, external disturbances are assumed to be

$$\mathbf{d}(t) = \begin{bmatrix} 0.01 + 0.01 \cos(0.05t) \\ 0.01 \sin(0.08t) + 0.01 \cos(0.06t) \\ 0.01 + 0.015 \sin(0.06t) \end{bmatrix} \text{ N} \cdot \text{m}.$$

The plot of disturbances is shown in Fig. 4.2.

The initial attitude is $\mathbf{Q}(0) = [0.2, -0.15, -0.25, 0.9354]^T$ with a zero initial an-

gular velocity. The desired reference angular velocity is given as

$$\boldsymbol{\omega}_d(t) = 0.573 \times \left[\cos\left(\frac{t}{40}\right), \sin\left(\frac{t}{30}\right), -\cos\left(\frac{t}{50}\right) \right]^T \text{ deg/s.}$$

For the consideration of practical application, attitude and angular velocity sensor noises ($\boldsymbol{\sigma}_p^2$): $0.0001(1\sigma)$ are modelled as zero-mean Gaussian random variables with variance $\boldsymbol{\sigma}_p^2$. The measurement bias of angular velocity sensor is 1 deg/h on each axis. Moreover, due to the limited manufacturing tolerances, the mounting misalignments of attitude sensor in the simulation is assumed to be 0.005 deg, respectively.

In order to achieve three-axis control, four reaction wheels in a pyramid configuration are considered as actuators, which indicates the existence of actuator redundancy in simulation. The distribution matrix of four reaction wheels is

$$\mathbf{D} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

During the attitude tracking maneuver, both loss of effectiveness faults and additive faults are considered. The scenarios of time-varying actuator faults are described as follows.

1. The first reaction wheel experiences partial loss of control effectiveness with a time-varying effectiveness gain $e_1 = 0.5 + 0.5e^{-t}$.
2. The second reaction wheel undergoes an additive fault with $\bar{u}_2 = -0.03 + 0.03e^{-0.5t} + 0.001\text{rand}(\cdot)$ N·m.
3. The third reaction wheel experiences loss of effectiveness and additive fault simultaneously with $e_3 = 0.6 + 0.4e^{-0.5t}$ and $\bar{u}_3 = -0.02 + 0.02e^{-t} + 0.002\text{rand}(\cdot)$ N·m, respectively.

(a) Estimated actuator effectiveness gain.

(b) Estimated additive bias.

Figure 4.6: Estimated fault information from [1] under MRA high-level controller and unsaturated RobCA.

Figure 4.7: System performance under MRA controller with unsaturated RobCA.

4. The fourth reaction wheel undergoes a complete failure.

Here, the function $\text{rand}(\cdot)$ generates a random value from the normal distribution with mean 0 and standard deviation 1.

4.4.1 Pseudoinverse based CA with MRA High-Level Controller

The first case is to investigate the performance of the proposed high-level controller. In this case, the actuator fault information is assumed to be identified precisely by the FDD scheme. Since there is no estimation uncertainty in FDD, it is noted that $\hat{e}_j(t) = e_j$ and $\hat{\bar{u}}_j(t) = \bar{u}_j$, $j \in \{1, 2, \dots, 4\}$, and the pseudoinverse based

Figure 4.8: System performance under PD controller with unsaturated pseudoinverse based control allocation.

CA (PICA) method is used to distribute the total control torques generated by the high-level controller to each individual actuator. The high-level controller parameters in the first scenario are chosen with $\beta = 0.5$, $k_1 = 5$, $k_2 = 20$, and $\gamma_i = 0.2$, $i \in \{1, 2, 3\}$. To have an easy interpretation, attitude tracking errors represented by unit quaternion are re-expressed in Euler angles.

Applying the MRA high-level controller defined in (4.11) and (4.12) with PICA method, the simulation results are shown in Figs. 4.3(a)-4.5(a). For the purpose of comparison, the PD+ controller [8] with the PICA method is also carried out for the attitude tracking systems, and the results are shown in Figs. 4.3(b)-4.5(b). Figs. 4.3-4.5 depict the comparisons between the proposed MRA controller and

(a) Estimated actuator effectiveness gain.

(b) Estimated additive bias.

Figure 4.9: Estimated fault information from [1] under MRA high-level controller and saturated RobCA.

Figure 4.10: System performance under MRA controller with saturated RobCA.

the PD+ controller without FDD imprecision. It is observed from Figs. 4.3 and 4.4 that the proposed MRA controller achieves a good performance on the attitude tracking with satisfactory tracking error and a rapid convergence, while the PD+ controller shows a degradation of the performance due to the external disturbances. Fig. 4.5 plots the time histories of control efforts given by the MRA controller and PD+ controller. From these simulation results, it is clear that the total control torques provided by the proposed MRA high-level controller not only achieves the asymptotic stability of the attitude tracking error systems but also suppresses the slow-varying disturbances when the FDD is precise?

4.4.2 Unsaturated RobCA with MRA High-Level Controller

The estimate of the actuator fault information can be provided by an FDD scheme as developed in [1], where local observers are used to estimate fault-related parameters online for each of the actuators. The maximum imprecision in fault estimation is assumed to be 20%.

The high-level controller parameters are chosen with $\beta = 0.42$, $k_1 = 7.5$, $k_2 = 50$, $k_3 = 0.5$, and $\Lambda_i = 0.8\mathbf{I}_2$, $i \in \{1, 2, 3\}$. For RobCA, the weighting matrix $\mathbf{M}$ and weighting scalar h in equation (4.23) have been chosen as $\mathbf{M} = \mathbf{I}_4$ and $h = 1 \times 10^4$, respectively. To have an easy interpretation, attitude tracking errors are re-expressed in Euler angles. For the purpose of comparison, PD controller [8] with pseudoinverse based control allocation method is also carried out.

By applying the FDD scheme from [1], the fault estimation information under MRA controller with unsaturated RobCA is shown in Fig. 4.6. Figures 4.7 and 4.8 show the closed-loop system performance under MRA controller with unsaturated RobCA and PD controller with pseudoinverse control allocation, respectively. It is observed from Fig. 4.7 that, although there exists estimate imprecision in FDD and external disturbances, the desired attitude and angular velocity can still be tracked with small tracking errors and a rapid convergence rate when the proposed scheme is used. Whereas for performance of the PD controller with pseudoinverse control allocation method in Fig. 4.8, due to the FDD imprecisions and external disturbances, the steady-state of attitude tracking error is about 1 deg and angular velocity tracking error is about 1.5×10^{-3} deg/s, which are much larger than that under the proposed scheme. This is due to the fact that the proposed scheme possesses some robustness with respect to external disturbances and imprecise FDD estimation. The time histories of the commanded control efforts under two control schemes are depicted in Figs. 4.7(c) and 4.8(c), respectively.

Figure 4.11: System performance under PD controller with saturated pseudoinverse based control allocation.

4.4.3 Saturated BobCA with MRA High-Level Controller

Next, the actuator saturation is taken into account in simulation. In this case, the limitations on the reaction wheel control torque are assumed to be $\underline{u}_{ci} = -0.25$ N·m and $\bar{u}_{ci} = 0.25$ N·m, $i \in \{1, 2, 3, 4\}$. The fault estimation information under MRA controller with saturated RobCA is shown in Figs. 4.9. Referring to Figs. 4.10 and 4.11, it is observed that steady-state tracking errors under the proposed scheme are still superior to that under the PD controller with pseudoinverse based control allocation method even in the presence of actuator saturation.

4.5 Summary

In this chapter, the MRA high-level controller incorporating RobCA strategy has been proposed for spacecraft attitude tracking system in the presence of external disturbances, FDD imprecisions, and actuator saturation. First, an MRA controller is designed as the high-level controller to specify total control efforts such that the desired attitude trajectories can be followed asymptotically. Then, a RobCA scheme is further developed to distribute the total control command determined by the high-level controller to individual actuators. The RobCA problem is formulated as a min-max optimization problem, and further converted to a scalar-valued optimization problem to reduce computation burden. Moreover, actuator amplitude and rate constraints are taken into account in RobCA design. The feasibility and effectiveness of the proposed strategy are illustrated in simulation.

Chapter 5

Integral-Type Sliding Mode FTC for Attitude Stabilization

5.1 Introduction

In contrast to active FTC approach, passive method utilizes a single robust controller to deal with all expected faults. The passive FTC approach has a simple structure, and has the advantage of avoiding time delay introduced by the online FDD and controller reconfiguration in active FTC. In practice, the fault can progress rapidly and the critical system may even become unstable before a desired active fault-tolerant controller can be synthesized [46,68]. Motivated by the above, this chapter focuses on the development of a passive fault-tolerant controller for a spacecraft attitude stabilization to handle actuator faults/failures.

Based on SMC techniques, some passive fault-tolerant strategies have been developed for FTC systems. However, they have some disadvantages. It was reported that the system dynamics might be vulnerable to faults or uncertainties during the reaching phase in which the system states have not yet reached the sliding manifold [112]. With a view to tackling the reaching phase problem, the concept

of ISMC was proposed in [113–115] to design a proper sliding manifold such that the sliding mode starts from the initial time instant. As a result, robustness of the system can be guaranteed from the beginning of the process and the reaching phase is eliminated. Numerous research results of this technique can be found over the last few years. For example, in [116], a robust ISMC design for the uncertain stochastic system with state delay was studied by means of the feasibility of LMIs. In [117], combining ISMC with robust model predictive control, a control scheme was proposed for nonlinear constrained continuous-time uncertain systems. In particular, some research results based on ISMC have been developed for FTC systems. In [118], active FTC issues were studied from the ISMC viewpoint to accommodate actuator faults whenever the fault detection and diagnosis information was available. With consideration of managing actuator redundancy, an integral-type sliding mode fault-tolerant controller incorporated with control allocation was developed to handle the total failure of certain actuators for a class of over-actuated linear systems [119]. In [120], the passive FTC issues were studied from the ISMC viewpoint to accommodate actuator faults for linear time-varying system.

Inspired by the above research, this chapter investigates ISMC-based FTC strategies for rigid spacecraft attitude stabilization in the presence of external disturbances and two kinds of actuator faults. Specifically, for the zero-disturbance fault-free attitude control system, a simple saturated PD control law which is regarded as nominal controller, is proposed to asymptotically stabilize the attitude motion, where a scalar sharpness function is contained to improve dynamic performance. Then, bounded external disturbances and two kinds of actuator faults, including both partial loss of control effectiveness fault and additive fault, are taken into account for attitude dynamics. Assuming that there exists a priori knowledge on the bounds of disturbances and actuator faults, a basic ISMC-based fault-tolerant controller is proposed to maintain close to the nominal closed-loop performance from the beginning of the

process. Moreover, by means of adaptive mechanism, an adaptive ISMC-based FTC scheme is designed such that the resultant closed-loop system is capable of tolerating potential actuator faults without requiring any information on the boundaries of disturbances and faults except for their existence.

The remainder of this chapter is organized as follows. In section 5.2, the FTC problem for spacecraft attitude stabilization in the presence of external disturbances is formulated. In section 5.3, a nominal controller based on a simple saturated PD control is proposed. In section 5.4, basic ISMC-based FTC scheme and adaptive ISMC-based FTC scheme are presented, respectively, with and without requiring the knowledge of the boundary of the external disturbances and faults. The simulation results are given in section 5.5, followed by conclusions in section 5.6.

5.2 Problem Statement

In this chapter, spacecraft attitude stabilization problem is considered with three reaction wheels used as actuator. Since there is no actuator redundancy, only two kinds of actuator faults (**F1** and **F2**) are considered in this chapter. The actuator faults of interests are modeled as follows:

$$\mathbf{u} = [\mathbf{I}_3 - \mathbf{E}(t)] \mathbf{u}_c + \bar{\mathbf{u}}(t), \quad (5.1)$$

where $\mathbf{E}(t) = \text{diag}\{e_1(t), e_2(t), e_3(t)\} \in \mathbb{R}^{3 \times 3}$ denotes the *effectiveness loss factor matrix* for the spacecraft actuators with $0 \leq e_i(t) < 1$ ($i = 1, 2, 3$). The case $e_i(t) = 0$ indicates that the i th actuator works normally, $0 < e_i(t) < 1$ implies that the i th actuator partially loses its effectiveness, but still not totally fail, $\bar{\mathbf{u}}(t) \in \mathbb{R}^3$ represents the bounded time-varying additive actuator fault. Note that the actuator model equation (5.1) is equivalent to the model in equation (2.14). Such an actuator fault formulation can be found in many previous results, such as [68,75,121]. In

view of actuator fault model in equation (5.1), the dynamics model for attitude stabilization incorporating with actuator fault can be written as:

$$\begin{cases} \dot{\mathbf{Q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\mathbf{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}^T \\ \mathbf{q}^\times + q_0 \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega} \\ \mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + [\mathbf{I}_3 - \mathbf{E}(t)] \mathbf{u}_c + \bar{\mathbf{u}} + \mathbf{d}. \end{cases} \quad (5.2)$$

To facilitate the controller development, the actuator faults and the external disturbances are assumed to satisfy the following assumptions.

Assumption 5.1. *The external disturbance $\mathbf{d}$ is bounded such that $\|\mathbf{d}\| \leq d_{max}$, where d_{max} is a positive constant.*

Assumption 5.2. *The actuator effectiveness loss factor satisfies $0 \leq \max\{e_1, e_2, e_3\} \leq e_m < 1$, where e_m is a positive constant.*

Assumption 5.3. *The actuator additive fault may be time-varying and unknown, but it is always bounded such that $\|\bar{\mathbf{u}}\| \leq f_m$, where f_m is a positive constant.*

Remark 5.1. *Since all the environmental disturbances due to solar radiation pressure, gravitation, magnetic forces and aerodynamic drag can be assumed to be bounded, Assumption 5.1 is reasonable. Assumption 5.2 can be viewed as the requirement of controllability of the plant and the existence of a fault-tolerant controller for compensating the actuator faults. The detailed discussions can be found in [74,122,123]. Assumption 5.3 is reasonable since the magnitude of actuator is always limited. A similar assumption was also proposed in [68,121].*

Finally, with consideration of actuator faults, the control objective is now stated. The FTC objective is to stabilize the attitude of the rigid spacecraft for all initial conditions $[\mathbf{q}(0), \boldsymbol{\omega}(0)]$ despite actuator faults. That is, design a control torque $\mathbf{u}_c$ such that the attitude and angular velocity converge, $\lim_{t \rightarrow \infty} \mathbf{q}(t) = 0$ and $\lim_{t \rightarrow \infty} \boldsymbol{\omega}(t) = 0$, while ensuring that all closed-loop signals remain bounded at all times.

5.3 Nominal Attitude Controller Design

For the zero-disturbance fault-free attitude control system, the following nominal controller is developed and will be used in section 5.4 for the proposed fault-tolerant controller design.

Lemma 5.1. *Consider the normal attitude dynamics without any actuator faults and external disturbances*

$$\mathbf{J}\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} + \mathbf{u}_c \quad (5.3)$$

if the nominal controller is designed as the following simple saturated PD controller:

$$\mathbf{u}_{nom} = -k_p \mathbf{q} - k_d \text{Tanh} \left(\frac{\boldsymbol{\omega}}{p^2} \right) \quad (5.4)$$

where k_p and k_d are two positive constants, $\text{Tanh} \left(\frac{\boldsymbol{\omega}}{p^2} \right) = \left[\tanh \left(\frac{\omega_1}{p^2} \right), \tanh \left(\frac{\omega_2}{p^2} \right), \tanh \left(\frac{\omega_3}{p^2} \right) \right]^T$, p^2 is a nonzero scalar sharpness function satisfying $0 < p_{min}^2 \leq p^2 \in \mathcal{L}_\infty$ and $\dot{p} \in \mathcal{L}_\infty$. Then, the zero-disturbance closed-loop system is asymptotically stable, i.e., $\mathbf{q}(t) \rightarrow 0$, $\boldsymbol{\omega}(t) \rightarrow 0$ as $t \rightarrow \infty$.

Proof. Consider the following positive-definite radially unbounded candidate Lyapunov function:

$$V = k_p(1 - q_0)^2 + k_p \mathbf{q}^T \mathbf{q} + \frac{1}{2} \boldsymbol{\omega}^T \mathbf{J} \boldsymbol{\omega}. \quad (5.5)$$

The time derivative along the attitude kinematics equation (5.2) and attitude dynamics equation (5.3) is given by

$$\begin{aligned} \dot{V} &= -2k_p \dot{q}_0 + \boldsymbol{\omega}^T \mathbf{J} \dot{\boldsymbol{\omega}} = k_p \mathbf{q}^T \boldsymbol{\omega} + \boldsymbol{\omega}^T (-\boldsymbol{\omega}^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{u}_{nom}) \\ &= k_p \mathbf{q}^T \boldsymbol{\omega} - k_p \boldsymbol{\omega}^T \mathbf{q} - k_d \boldsymbol{\omega}^T \text{Tanh}(\boldsymbol{\omega}/p^2) \\ &= -k_d \boldsymbol{\omega}^T \text{Tanh}(\boldsymbol{\omega}/p^2). \end{aligned} \quad (5.6)$$

By using the fact that $x \tanh(x/p^2) \geq 0$ for all x , $\dot{V} \leq 0$ can be obtained. Therefore,

one can show that q_0 , $\mathbf{q}_v$, and $\boldsymbol{\omega}$ are globally bounded. In view of the constraints on p^2 , it is clear that $\ddot{V}$ is bounded. Hence, according to the Barbalat's Lemma, one can conclude that $\lim_{t \rightarrow \infty} \boldsymbol{\omega}(t) = 0$. At this point, it remains to be shown that $\lim_{t \rightarrow \infty} \mathbf{q}(t) = 0$. From (5.3) and (5.4), it can be derived that

$$\begin{aligned} \mathbf{J}\ddot{\boldsymbol{\omega}} &= -\dot{\boldsymbol{\omega}}^\times \mathbf{J}\boldsymbol{\omega} - \boldsymbol{\omega}^\times \mathbf{J}\dot{\boldsymbol{\omega}} + \dot{\mathbf{u}}_{nom} \\ &= -\dot{\boldsymbol{\omega}}^\times \mathbf{J}\boldsymbol{\omega} - \boldsymbol{\omega}^\times \mathbf{J}\dot{\boldsymbol{\omega}} - k_p \dot{\mathbf{q}} - k_d \frac{d}{dt} \text{Tanh}(\boldsymbol{\omega}/p^2). \end{aligned} \quad (5.7)$$

Now, consider the last term in (5.7)

$$\frac{d}{dt} \tanh\left(\frac{\omega_i}{p^2}\right) = \text{sech}^2\left(\frac{\omega_i}{p^2}\right) \frac{\dot{\omega}_i p - 2\omega_i \dot{p}}{p^3} \quad (5.8)$$

where ω_i is the i th element of $\boldsymbol{\omega}$ ($i = 1, 2, 3$). Since $\boldsymbol{\omega}$, $\dot{\boldsymbol{\omega}}$, $\dot{\mathbf{q}}$, and $\dot{p}$ are bounded and $0 < p_{min}^2 \leq p^2$, from equation (5.7) and equation (5.8), one can show that $\ddot{\boldsymbol{\omega}}$ is bounded. Consequently, by virtue of the Barbalat's Lemma again, together with convergence of $\boldsymbol{\omega}$ to the origin and uniform continuity of $\dot{\boldsymbol{\omega}}$, it leads to $\lim_{t \rightarrow \infty} \dot{\boldsymbol{\omega}}(t) = 0$. As t goes to infinity, from equation (5.3), we have $\mathbf{u}_{nom} = 0$. Therefore, from equation (5.4), it is clear that $\lim_{t \rightarrow \infty} \left[k_p \mathbf{q} + k_d \text{Tanh}\left(\frac{\boldsymbol{\omega}}{p^2}\right) \right] = 0$, which implies that $\lim_{t \rightarrow \infty} \mathbf{q}(t) = 0$ since $\lim_{t \rightarrow \infty} \boldsymbol{\omega}(t) = 0$. Thus, the results as stated in Lemma 5.1 is established. $\square$

Remark 5.2. Using the property of the unit quaternion and the standard hyperbolic tangent function, the $\mathbf{u}_{nom}$ in equation (5.4) can be upper bounded as

$$|u_{nomi}| \leq |k_p q_i| + |k_d \text{Tanh}(\omega_i/p^2)| \leq k_p + k_d \quad (5.9)$$

where u_{nomi} , q_{vi} , and ω_i represent the i th element of $\mathbf{u}_{nom}$, $\mathbf{q}$, and $\boldsymbol{\omega}$, respectively. Hence, a natural saturation, in terms of the nominal control gains, is achieved and the designer can set the limits of the nominal control effort through gains k_p and k_d .

Remark 5.3. *In comparison with the similar results presented in [32], where Rodrigues parameters and modified Rodrigues parameters are used for the attitude representation, a nonzero sharpness function p^2 is introduced in the proposed nominal controller. It should be noted that p^2 appears in the denominator of the argument of the hyperbolic tangent function, so that its value determines how strongly the control varies with the signal $\boldsymbol{\omega}$. Adjusting p^2 changes the rate of change of nominal control torque, and thus, better dynamic performance could be obtained [30].*

5.4 Fault-Tolerant Attitude Controller Design

In this section, based on the existing results on the integral-type sliding mode manifold, the ISMC-based FTC schemes are developed to solve the attitude stabilization problem for rigid spacecraft under actuator faults and external disturbances.

5.4.1 Integral-Type Sliding Manifold Design

Adopting the integral-type sliding mode manifold design procedure, the sliding manifold is introduced as follows:

$$\mathbf{s} = \boldsymbol{\Lambda} \left\{ \boldsymbol{\omega}(t) - \boldsymbol{\omega}(t_0) - \int_{t_0}^t \mathbf{J}^{-1} [-\boldsymbol{\omega}(\sigma)^\times \mathbf{J} \boldsymbol{\omega}(\sigma) + \mathbf{u}_{nom}(\mathbf{q}, \boldsymbol{\omega})] d\sigma \right\}, \quad (5.10)$$

where $\boldsymbol{\Lambda} \in \mathbb{R}^{3 \times 3}$ is a constant matrix. The matrix $\boldsymbol{\Lambda}$ is chosen such that $\boldsymbol{\Lambda} \mathbf{J}^{-1}$ is invertible. Notice that, at $t = t_0$, the sliding manifold satisfies $\mathbf{s}(\boldsymbol{\omega}(t_0), t_0) = 0$, and hence the reaching phase is eliminated [114,115,124].

In order to analyze the sliding dynamics (motion equations on the sliding manifold) in the presence of actuator faults, the equivalent control method [112] is used. Taking the derivative of the sliding manifold in equation (5.10) with respect to time yields

$$\dot{\mathbf{s}} = \boldsymbol{\Lambda} [\dot{\boldsymbol{\omega}} - \mathbf{J}^{-1} (-\boldsymbol{\omega}^\times \mathbf{J} \boldsymbol{\omega} + \mathbf{u}_{nom})]. \quad (5.11)$$

Substituting the attitude dynamics model with actuator faults model defined in equation (5.2) into equation (5.11), it follows that

$$\dot{\mathbf{s}} = \mathbf{\Lambda} \mathbf{J}^{-1} [(\mathbf{I}_3 - \mathbf{E}(t))\mathbf{u} + \bar{\mathbf{u}} - \mathbf{u}_{nom} + \mathbf{d}]. \quad (5.12)$$

To derive the equivalent control, the algebraic equation $\dot{\mathbf{s}} = 0$ should be solved, which gives

$$\mathbf{u}_{eq} = [\mathbf{I}_3 - \mathbf{E}(t)]^{-1}(\mathbf{u}_{nom} - \bar{\mathbf{u}} - \mathbf{d}) \quad (5.13)$$

where $\mathbf{\Lambda} \mathbf{J}^{-1}$ is invertible. Thus, the sliding dynamics can be obtained by substituting (5.13) into (5.2) as

$$\mathbf{J} \dot{\boldsymbol{\omega}}_h = -\boldsymbol{\omega}_h^\times \mathbf{J} \boldsymbol{\omega}_h + \mathbf{u}_{nom} \quad (5.14)$$

where the subscript h denotes the state vector in the sliding mode. Furthermore, according to the Lemma 5.1, it can be concluded that the sliding dynamics in equation (5.14) is asymptotically stable.

Remark 5.4. *Based on the aforementioned analysis, it is found that actuator faults and external disturbances can be completely rejected when the dynamics of spacecraft remains on the sliding manifold. That is, the closed-loop dynamics of the faulty system in the sliding mode is identical to that of the zero-disturbance healthy system controlled by the nominal controller. Hence, the designer has much more freedom in designing a suitable nominal controller according to the system requirements for the nominal healthy system in the absence of external disturbances.*

5.4.2 Basic Integral-Type Sliding Mode FTC Design

Now, a sliding mode control law must be synthesized such that the reachability of the specified integral-type sliding manifold is ensured under partial loss of actuator effectiveness fault and additive actuator fault.

The proposed basic ISMC-based FTC scheme has a form given by

$$\mathbf{u} = \mathbf{u}_{nom} + \mathbf{u}_N \quad (5.15)$$

where the nominal control $\mathbf{u}_{nom}$ is the same as equation (5.4) which determines the behavior of the nominal system restricted to the sliding manifold, and the second term $\mathbf{u}_N$ is a discontinuous control component that compensates for the possible actuator fault effects on the system and drives the system trajectories toward the sliding manifold.

In this section, the upper bounds of actuator effectiveness fault and additive fault in Assumptions 5.1 and 5.2 are assumed to be known to the designer. Thus, the discontinuous control $\mathbf{u}_N$ is selected as

$$\mathbf{u}_N = \begin{cases} -\rho(t) \frac{(\Lambda \mathbf{J}^{-1})^T \mathbf{s}}{\|(\Lambda \mathbf{J}^{-1})^T \mathbf{s}\|}, & \text{if } \mathbf{s} \neq 0 \\ 0, & \text{otherwise} \end{cases} \quad (5.16)$$

with the switching gain function

$$\rho(t) = \frac{\sqrt{3}e_m \|\mathbf{u}_{nom}\|_\infty + f_m + d_{max} + \varepsilon}{1 - e_m} > 0 \quad (5.17)$$

where ε is a bounded positive constant.

Theorem 5.1. *Consider the attitude control systems described by equation (5.2) in the presence of partial loss of actuator effectiveness fault and additive actuator fault. Suppose that Assumptions 5.1-5.3 are valid with known bounds on actuator faults and external disturbances. Then the reachability of the sliding manifold $\mathbf{s} = 0$ can be maintained by employing the controller in equation (5.15) with $\mathbf{u}_{nom}$ and $\mathbf{u}_N$ given in equations (5.4) and (5.16), respectively.*

Proof. To analyze the reachability, consider the candidate Lyapunov function

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{s}. \quad (5.18)$$

Taking the time derivative of the Lyapunov function for $\mathbf{s} \neq 0$ with substitution of FTC law in equation (5.15), yields

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \mathbf{\Lambda} \mathbf{J}^{-1} \left[-(\mathbf{I}_3 - \mathbf{E}(t))\rho(t) \frac{(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}}{\|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\|} - \mathbf{E}(t) \mathbf{u}_{nom} + \bar{\mathbf{u}} + \mathbf{d} \right] \\ &\leq - \left[(1 - e_m)\rho(t) - \sqrt{3}e_m \|\mathbf{u}_{nom}\|_\infty - f_m - d_{max} \right] \|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\| \end{aligned} \quad (5.19)$$

where the inequality $\mathbf{x}^T \mathbf{y} \leq \sqrt{3} \|\mathbf{x}\| \|\mathbf{y}\|_\infty$ with $\mathbf{x} \in \mathbb{R}^3$ and $\mathbf{y} \in \mathbb{R}^3$ has been used. Furthermore, since $\mathbf{\Lambda} \mathbf{J}^{-1}$ is nonsingular, substituting $\rho(t)$ from equation (5.17) into equation (5.19) gives

$$\dot{V} \leq -\varepsilon \|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\| < 0, \quad \text{for } \mathbf{s} \neq 0. \quad (5.20)$$

This implies that the sliding motion can be maintained despite partial loss of actuator effectiveness fault and additive actuator fault as well as external disturbances. This completes the proof. $\square$

5.4.3 Integral-Type Sliding Mode FTC Design with Adaptive Estimation

It is seen that the upper bounds d_{max} , e_m , and f_m of external disturbances and actuator faults are required to synthesize the basic FTC law in equation (5.15). However, from a practical point of view, the exact knowledge of these bounds is not easily obtained due to the uncertain and unexpected characteristics of disturbances and faults. Therefore, an adaptive integral sliding mode fault-tolerant controller

is further developed by incorporating parameter adaptive mechanism to relax the requirement of a priori knowledge of these bounds.

Notice that the switching gain function $\rho(t)$ in equation (5.17) is upper bounded with the property that

$$\rho(t) \leq \rho_m = \frac{\sqrt{3}e_m(k_p + k_d) + f_m + d_{max} + \varepsilon}{1 - e_m} \quad (5.21)$$

where ρ_m is a positive scalar representing the upper bound of the switching gain function $\rho(t)$.

Remark 5.5. Recalling the switching gain $\rho(t)$ defined in equation (5.17), from Assumption 5.1 and Remark 5.2, it is clear that $\rho(t) \leq \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}+\varepsilon}{1-e_m}$. Since k_p and k_d are bounded positive constant design parameters, and e_m , f_m , d_{max} and ε are bounded positive constants, the positive scalar ρ_m always exists. Therefore, the upper bounded property of the switching gain $\rho(t)$ can be obtained.

Next, we shall introduce an adaptive scheme which is capable of performing an estimation of the upper bound ρ_m and design a suitable adaptive fault-tolerant controller using this estimated upper bound. Based on the aforementioned analysis, the basic fault-tolerant control law in equation (5.15) is modified as

$$\mathbf{u} = \mathbf{u}_{nom} + \mathbf{u}_{aN} \quad (5.22)$$

where the nominal control $\mathbf{u}_{nom}$ is the same as equation (5.4). Due to the discontinuity of $\mathbf{u}_N$ in equation (5.15), the basic ISMC-based FTC is discontinuous across the sliding manifold $\mathbf{s} = 0$, thus leading to control chattering. In order to alleviate undesirable chattering, the discontinuous control in equation (5.16) is smoothed by

$$\mathbf{u}_{aN} = \begin{cases} -\hat{\rho}_m \frac{(\mathbf{\Lambda J}^{-1})^T \mathbf{s}}{\|(\mathbf{\Lambda J}^{-1})^T \mathbf{s}\|}, & \text{if } \hat{\rho}_m \|(\mathbf{\Lambda J}^{-1})^T \mathbf{s}\| \geq \xi \\ -\hat{\rho}_m^2 \frac{(\mathbf{\Lambda J}^{-1})^T \mathbf{s}}{\xi}, & \text{if } \hat{\rho}_m \|(\mathbf{\Lambda J}^{-1})^T \mathbf{s}\| < \xi \end{cases} \quad (5.23)$$

where ξ is a small positive scalar, and $\hat{\rho}_m \geq 0$ is obtained by the following adaptive law:

$$\dot{\hat{\rho}}_m = \beta(\|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| - \mu\hat{\rho}_m), \text{ with } \hat{\rho}_m(0) \geq 0 \quad (5.24)$$

where β and μ are positive scalars, and the second term $-\mu\hat{\rho}_m$ is used to establish robustness with respect to disturbances and unmodeled dynamics [125] and prevent the increase of the adaptive gain [126,127].

Lemma 5.2. *Given the switching gain update law in equation (5.24), the gain $\hat{\rho}_m$ has an upper bound, i.e., there always exists a positive scalar $\bar{\rho}_m$ such that $\hat{\rho}_m \leq \bar{\rho}_m$ and $\rho_m \leq \bar{\rho}_m$ for all $t > 0$.*

Proof. Consider a Lyapunov function $V_s = \frac{1}{2}\mathbf{s}^T \mathbf{s}$. Differentiating V_s with respect to time and substituting the closed-loop equations into it, we have

$$\begin{aligned} \dot{V}_s &\leq -(1 - e_m) \left[\hat{\rho}_m(t) - \frac{\sqrt{3}e_m\|u_{nom}\|_\infty + f_m + d_{max} + \varepsilon}{1 - e_m} \right] \|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| \\ &\leq -(1 - e_m) (\hat{\rho}_m(t) - \rho_m) \|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| \end{aligned} \quad (5.25)$$

where $\rho_m = \frac{\sqrt{3}e_m(k_p+k_d)+f_m+d_{max}+\varepsilon}{1-e_m}$ defined in equation (25) is a finite constant. To show $\hat{\rho}_m$ is upper bounded, the following four cases are considered here.

Case I: If $\hat{\rho}_m(t) > \rho_m$ and $\|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| \geq \mu\rho_m$, it follows that $\dot{V}_s(t) < 0$ and $\dot{\hat{\rho}}_m(t) \geq 0$. Since $\dot{V}_s(t) \leq 0$, the sliding function $\mathbf{s}$ will decrease and it can be obtained that $V_s(t) \leq V_s(0)$, which implies that $\|\mathbf{s}\| \leq \sqrt{2V_s(0)}$. Therefore, although $\dot{\hat{\rho}}_m \geq 0$, the adaptive gain $\hat{\rho}_m$ is upper bounded in view of the updating law $\dot{\hat{\rho}}_m = \beta(\|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| - \mu\hat{\rho}_m)$.

Case II: If $\hat{\rho}_m(t) > \rho_m$ and $\|(\mathbf{A}\mathbf{J}^{-1})^T \mathbf{s}\| < \mu\rho_m$, it is clear that $\dot{V}_s(t) < 0$ and $\dot{\hat{\rho}}_m(t) \leq 0$. As a result, both the Lyapunov function $V_s(t)$ and the adaptive gain $\hat{\rho}_m(t)$ are upper bounded by their initial values $V_s(0)$ and $\hat{\rho}_m(0)$, respectively, i.e.,

Figure 5.1: Relations among four cases of the adaptive gain $\hat{\rho}_m(t)$.

$V_s(t) \leq V_s(0)$ and $\hat{\rho}_m(t) \leq \hat{\rho}_m(0)$.

Case III: If $\hat{\rho}_m(t) \leq \rho_m$ and $\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T \mathbf{s}\| < \mu\rho_m$, it follows that $\dot{V}_s(t) > 0$ and $\dot{\hat{\rho}}_m(t) \leq 0$. In this case, it is obvious that $\hat{\rho}_m$ is upper bounded by the constant ρ_m . Moreover, although $\dot{V}_s > 0$, we shall further prove that V_s is bounded. Since the sliding function $\mathbf{s}$ is under the condition that $\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T \mathbf{s}\| \leq \mu\rho_m$ is satisfied, it could be shown that $\|\mathbf{s}\| \leq \frac{\mu\rho_m}{\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\|}$. Hence, the Lyapunov function $V_s(t)$ is upper bounded by a constant $V_s^* = \frac{1}{2} \left(\frac{\mu\rho_m}{\|(\mathbf{D}\mathbf{J}^{-1})^T\|} \right)^2$ in this case.

Case IV: If $\hat{\rho}_m(t) \leq \rho_m$ and $\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T \mathbf{s}\| \geq \mu\rho_m$, it is clear that $\dot{V}_s(t) > 0$ and $\dot{\hat{\rho}}_m \geq 0$. It is also obvious that $\hat{\rho}_m$ is upper bounded by the constant ρ_m in this case. Since $\mathbf{s}$ is a large value such that $\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T \mathbf{s}\| > \mu\rho_m$, the adaptive law could be approximated as $\dot{\hat{\rho}}_m(t) = \beta(\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T \mathbf{s}\|)$. Comparing the expressions of $\dot{V}_s(t)$ and $\dot{\hat{\rho}}_m(t)$, it is noted that $\dot{V}_s(t) \leq k_m \dot{\hat{\rho}}_m(t)$, where k_m is a constant defined as $k_m = \frac{(1-e_m)\rho_m}{\beta}$. Thus, the Lyapunov function is upper bounded by $V_s(t) \leq V_s(0) + k_m[\hat{\rho}_m(t) + \hat{\rho}_m(0)]$ on the basis of the comparison principle. Furthermore, on the condition that $\hat{\rho}_m(t) \leq \rho_m$, it is found that $V_s(t) \leq V_s(0) + 2k_m\rho_m$.

In fact, the relations among these four cases are shown by Fig. 5.1. Based on the above analysis of these four cases, we prove that the adaptive gain $\hat{\rho}_m(t)$ is always upper bounded. Therefore, there always exists a positive scalar $\bar{\rho}_m$ such that $\hat{\rho}_m \leq \bar{\rho}_m$ and $\rho_m \leq \bar{\rho}_m$ for all $t > 0$. $\square$

Theorem 5.2. Consider the attitude control systems described by equation (5.2)

in the presence of partial loss of actuator effectiveness fault and additive actuator fault. Suppose that Assumptions 5.1-5.3 are valid with unknown upper bounds on the actuator faults (only the boundedness property is known). Then the trajectory of the closed-loop system can be driven into a neighborhood of the sliding manifold $\mathbf{s} = 0$ in finite time by employing the adaptive controller in equation (5.22) and the parameter adaptive law in equation (5.24).

Proof. Consider the following candidate Lyapunov function:

$$V = \frac{1}{2} \mathbf{s}^T \mathbf{s} + \frac{1 - e_m}{2\beta} (\hat{\rho}_m - \bar{\rho}_m)^2 \quad (5.26)$$

where $\bar{\rho}_m$ is defined in Lemma 5.2 representing the upper bound of $\hat{\rho}_m$. In order to get the result in Theorem 5.2, two cases are considered in the following.

Case I: If $\hat{\rho}_m \|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\| \geq \xi$, the time derivative of V along with equations (5.22)-(5.24) results in

$$\begin{aligned} \dot{V} &= \mathbf{s}^T \dot{\mathbf{s}} + \frac{1 - e_m}{\beta} (\hat{\rho}_m - \bar{\rho}_m) \dot{\hat{\rho}}_m \\ &\leq - (1 - e_m) \left[\bar{\rho}_m - \frac{\sqrt{3}e_m(k_p + k_d) + f_m + d_{max}}{1 - e_m} \right] \\ &\quad \times \|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\| - \mu(1 - e_m)(\hat{\rho}_m - \bar{\rho}_m)\hat{\rho}_m. \end{aligned} \quad (5.27)$$

where the inequality $\mathbf{x}^T \mathbf{y} \leq \sqrt{3} \|\mathbf{x}\| \|\mathbf{y}\|_\infty$ with $\mathbf{x} \in \mathbb{R}^3$ and $\mathbf{y} \in \mathbb{R}^3$ has been used. In view of Lemma 5.2, it is clear that $\bar{\rho}_m - \frac{\sqrt{3}e_m(k_p + k_d) + f_m + d_{max}}{1 - e_m} \geq \frac{\varepsilon}{1 - e_m}$ and $|\hat{\rho}_m - \bar{\rho}_m| \leq \bar{\rho}_m$. Now, it is readily obtained from (5.27) that

$$\begin{aligned} \dot{V} &\leq - \mu(1 - e_m)|\hat{\rho}_m - \bar{\rho}_m| - \varepsilon \|(\mathbf{\Lambda} \mathbf{J}^{-1})^T \mathbf{s}\| \\ &\quad + \mu(1 - e_m) \left(|\hat{\rho}_m - \bar{\rho}_m| + \frac{1}{4} \bar{\rho}_m^2 \right) \\ &= - \delta_1 |\hat{\rho}_m - \bar{\rho}_m| - \delta_2 \|\mathbf{s}\| + \eta_1, \end{aligned}$$

where δ_1 , δ_2 , and η_0 are positive scalars defined by $\delta_1 = \mu(1 - e_m)$, $\delta_2 = \varepsilon\|\mathbf{\Lambda}\mathbf{J}^{-1}\|$, and $\eta_1 = \mu(1 - e_m)(\bar{\rho}_m + \frac{1}{4}\bar{\rho}_m^2)$, respectively. Thus, it follows that

$$\begin{aligned}\dot{V} &\leq -\sqrt{\frac{2\beta\delta_1^2}{1-e_m}}\sqrt{\frac{1-e_m}{2\beta}(\hat{\rho}_m - \bar{\rho}_m)^2} - \sqrt{2}\delta_2\sqrt{\frac{1}{2}\mathbf{s}^T\mathbf{s}} + \eta_1 \\ &\leq -\min\left\{\sqrt{\frac{2\beta\delta_1^2}{1-e_m}}, \sqrt{2}\delta_2\right\}\left(\sqrt{\frac{1-e_m}{2\beta}(\hat{\rho}_m - \bar{\rho}_m)^2} + \sqrt{\frac{1}{2}\mathbf{s}^T\mathbf{s}}\right) + \eta_1 \\ &\leq -\delta V^{\frac{1}{2}} + \eta_1,\end{aligned}\tag{5.28}$$

where $\delta = \sqrt{2}\min\left\{\sqrt{\frac{\beta\delta_1^2}{1-e_m}}, \delta_2\right\}$.

Case II: If $\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| < \xi$, the time derivative of the Lyapunov candidate function defined in equation (5.26) is

$$\begin{aligned}\dot{V} &= -\frac{1-e_m}{\xi}\hat{\rho}_m^2\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\|^2 + (1-e_m)\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| \\ &\quad - (1-e_m)\left[\hat{\rho}_m - \frac{\sqrt{3}e_m(k_p + k_d) + f_m + d_{max}}{1-e_m}\right]\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| \\ &\quad + (1-e_m)(\hat{\rho}_m - \bar{\rho}_m)\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| - \mu(1-e_m)(\hat{\rho}_m^2 - \hat{\rho}_m\bar{\rho}_m).\end{aligned}\tag{5.29}$$

Since $\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| < \xi$, it is obtained that the term $-\frac{1-e_m}{\xi}\hat{\rho}_m^2\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\|^2 + (1-e_m)\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\|$ reaches its maximum value of $\frac{(1-e_m)\xi}{4}$ when $\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\| = \frac{\xi}{2}$.

With the help of Lemma 5.2 and follow the same lines as in *case I*, it yields

$$\dot{V} \leq -\delta V^{\frac{1}{2}} + \eta_2.\tag{5.30}$$

where δ_1 and δ_2 have the same definitions as in case I, and η_2 is a positive scalar defined by $\eta_2 = \frac{1-e_m}{4}(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)$.

Therefore, combining above two cases together, for any $\hat{\rho}_m\|(\mathbf{\Lambda}\mathbf{J}^{-1})^T\mathbf{s}\|$, one has

$$\dot{V} \leq -\delta V^{\frac{1}{2}} + \eta,\tag{5.31}$$

where $\eta = \max\{\eta_1, \eta_2\} = \frac{1-e_m}{4}(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)$. Thus, the trajectory of this system is practically finite-time stable [24]. Furthermore, the decrease of V can in finite time drive the trajectories of the closed-loop system into $V^{\frac{1}{2}} \leq \frac{\eta}{(1-\theta_0)\delta}$, where θ_0 is a positive scalar satisfying $0 < \theta_0 < 1$. As a result, the trajectories of the closed-loop system is bounded in a small set containing the origin in finite time as

$$\|\mathbf{s}\| \leq \frac{\sqrt{2}\eta}{(1-\theta_0)\delta} = \frac{(1-e_m)(\mu\bar{\rho}_m^2 + 4\mu\bar{\rho}_m + \xi)}{4(1-\theta_0) \cdot \min\left\{\mu\sqrt{\beta(1-e_m)}, \varepsilon\|\mathbf{\Lambda}\mathbf{J}^{-1}\|\right\}}. \quad (5.32)$$

This completes the proof. $\square$

Remark 5.6. *From inequality (5.32), it can be seen that the smaller the boundary thickness ξ , the smaller the desired $\mathbf{s}$ is obtained. Furthermore, the ultimate convergence set for $\mathbf{s}$ can be small enough around the origin by choosing large enough parameters β and $\mathbf{\Lambda}$.*

Remark 5.7. *If a sufficiently large value for $\hat{\rho}_m(0)$ can be obtained to compensate the potential disturbance and fault in the systems, the sliding function will stay in the small convergence set even after a fault occurs. However, because the magnitude of potential actuator fault in the system is always unknown to the designer, a proper initial value of the adaptive gain $\hat{\rho}_m(0)$ is difficult to select. In this case, although the sliding function may move out of the small convergence set after a fault occurs, it will be attracted back to the small convergence set in finite time.*

5.5 Simulation Results

Simulation results are presented in this section to illustrate the effectiveness of the proposed ISMC-based FTC schemes. Similar to [13], the inertia matrix of a rigid spacecraft is given as $\mathbf{J} = \text{diag}\{10, 15, 20\} \text{ kg} \cdot \text{m}^2$. The external disturbances are of the form $\mathbf{d} = 0.1[\sin(t/10), \cos(t/15), \sin(t/20)]^T + [0.1, 0.1, 0.1]^T \text{ Nm}$. The

Figure 5.2: Time response of quaternion with different sharpness parameters in the absence of actuator faults and external disturbances.

initial attitude and angular velocity are set as $\mathbf{Q}(0) = [-0.5, 0.5, 0.5, -0.5]^T$ and $\boldsymbol{\omega}(0) = [0.5, -0.8, 0.3]^T$ rad/s, respectively.

In the context of simulation, a severe fault scenario is considered. At $t = 10$ s, each actuator suffers from a partial loss of effectiveness fault, while at $t = 50$ s, these actuators also undergo an additive time-varying fault that enters the spacecraft dynamics in an additive way. The details are described as follows:

$$e_i(t) = \begin{cases} 0, & \text{if } t < 10 \\ 0.5, & \text{if } t \geq 10 \end{cases}$$

$$\bar{u}_i(t) = \begin{cases} 0, & \text{if } t < 50 \\ 0.95 + 0.05\sin(t), & \text{if } t \geq 50. \end{cases}$$

5.5.1 Nominal Saturated PD Controller

In order to demonstrate the effectiveness of the nominal saturated PD controller, the nominal situation is simulated in which all actuators are healthy and there is no external disturbance. The gains for the nominal controller are chosen as $k_p = 1$ and $k_d = 1$. For convenience, the sharpness function p^2 is set as a constant. To analyze

(a) $p^2 = 0.2$ (b) $p^2 = 1$

Figure 5.3: Time response of angular velocity with different sharpness parameters in the absence of actuator faults and external disturbances.

(a) $p^2 = 0.2$ (b) $p^2 = 1$

Figure 5.4: Time response of control input with different sharpness parameters in the absence of actuator faults and external disturbances.

the effect of changes in p^2 , the nominal controller is applied twice with different sharpness parameters (constant p^2) in the simulation.

The response of attitude quaternion and angular velocity are shown in Figs. 5.2 and 5.3, from which it is clear that asymptotic attitude stabilization of the spacecraft is achieved by applying the proposed nominal controller. Fig. 5.4 depicts the time history of the nominal control effort u_{nom} given by equation (5.4), which shows that the applied nominal control torque remains bounded ($|u_{nomi}| \leq k_p + k_d = 2$ Nm for all time). In addition, referring to Figs. 5.2-5.4, it is shown that the sharpness parameter affects the transient performance greatly and, more specifically,

decreasing p^2 can improve the closed-loop convergence rate and reduce the overshoot.

5.5.2 Basic FTC Scheme

For the basic controller proposed in equation (5.15), the control parameters are chosen as $k_p = 1$, $k_d = 1$, $p^2 = 0.2$, and $\varepsilon = 1$, while the parameter of the integral-type sliding mode manifold is $\mathbf{\Lambda} = 2\mathbf{I}_3$. In order to eliminate control chattering, the discontinuous control action in equation (5.16) is smoothed by using the boundary modification [128], and the boundary layer width is chosen as 0.0001. For the purpose of comparison, the nominal saturated PD controller, which does not have a mechanism to accommodate the actuator fault, is also applied to the spacecraft attitude control system in the presence of actuator faults with $k_p = 1$, $k_d = 1$, and $p^2 = 0.2$.

Figs. 5.5 and 5.6 show the attitude quaternion and angular velocity trajectory. It can be observed that the basic ISMC-based fault-tolerant controller obtains a good performance in the attitude stabilization even in the presence of partial loss of actuator effectiveness fault and additive actuator fault. As compared with the basic FTC case, the attitude control systems can not be stabilized by the nominal saturated PD controller when actuator faults are introduced in the system, especially after the occurrence of additive actuator fault in 50s. From Fig. 5.7, it is shown that the basic FTC scheme in equation (5.15) manages to produce a control torque to accommodate the effect of actuator faults. On the other hand, for the nominal controller case, it is observed that the effect of actuator faults can not be compensated by the nominal saturated PD controller, which may result in severe control chattering.

(a) Basic FTC scheme

(b) Saturated PD scheme

Figure 5.5: Time response of quaternion under different control schemes in the presence of actuator faults and external disturbances.

(a) Basic FTC scheme

(b) Saturated PD scheme

Figure 5.6: Time response of angular velocity under different control schemes in the presence of actuator faults and external disturbances.

(a) Basic FTC scheme

(b) Saturated PD scheme

Figure 5.7: Time response of control input under different control schemes in the presence of actuator faults and external disturbances.

5.5.3 Adaptive FTC Scheme

To illustrate the effectiveness of the proposed adaptive fault-tolerant controller defined in equation (5.22), the attitude stabilization problem in the absence of a priori knowledge of upper bounds on disturbances and the actuator faults is simulated in this section. In the simulation, the same gains of $k_p = 1$, $k_d = 1$ and $p^2 = 0.2$ as in the case of basic controller are used for the nominal saturated PD controller. The gain of integral-type sliding mode manifold is chosen as $\mathbf{\Lambda} = 5\mathbf{I}_3$, and the boundary layer width is chosen as $\xi = 0.01$. The design parameters of the adaptive law are chosen as $\beta = 10$ and $\mu = 0.00025$ with initial value $\hat{\rho}_m(0) = 1$.

Figure 5.8: Simulation results under adaptive FTC scheme in the presence of actuator faults and external disturbances.

From Figs. 5.8(a) and 5.8(b), it is shown that the overall attitude quaternion and

angular velocity trajectories are stabilized within 110 s, and acceptable performance is also achieved. Figs. 5.8(c) and 5.8(d) depict the time histories of the estimate of the switching gain $\hat{\rho}_m(t)$ and the control torque u given by equation(5.22). Referring to Figs. 5.8(a)-5.8(d), it is clear that the proposed adaptive FTC scheme is capable of accommodating actuator faults even under the condition that the upper bounds of the actuator faults are unknown. However, compared with the basic FTC scheme, it is observed that the convergence of the system states takes a much longer time when adaptive FTC scheme is used. This is due to the fact that the estimated switching gain requires a period of time to become a value large enough to compensate for the efforts of the actuator fault once a fault happens.

5.6 Summary

In this chapter, two ISMC-based FTC schemes for the attitude stabilization problem of a rigid spacecraft subject to two kinds of actuator faults and external disturbances have been proposed. In zero-disturbance fault-free case, a simple saturated PD control law is proposed to achieve asymptotic attitude stabilization. When actuator faults occur in the system, a basic FTC scheme completely compensates the effect of actuator faults from the beginning of the process. In the event that the bounds on the actuator faults and disturbances are unknown, an adaptive fault-tolerant controller is proposed to ensure that the trajectory of the system is practical finite-time stable with a reasonable switching adaptive gain. Numerical simulations are performed to verify the fault-tolerant capability of the proposed FTC laws in the presence of two kinds of actuator faults and external disturbances.

Chapter 6

Finite-Time FTC for Attitude Stabilization

6.1 Introduction

For some spacecraft missions, such as remote sensing or reconnaissance, the spacecraft is required to achieve rapid attitude maneuvering together with high targeting accuracy and stability between the maneuvers [7]. Due to fast convergence, high precision control performance, and better disturbance rejection [21,22], several finite-time control approaches have been presented in the literature for attitude control problems [23,25,26]. However, these controllers are designed without considering actuator faults/failures, so they are unsuitable for FTC of the spacecraft attitude control system. Recently, in [129], a finite-time fault-tolerant controller was proposed for spacecraft attitude tracking system to accommodate reaction wheel misalignment and fault. Since actuator redundancy is not taken into account, the proposed controller in [129] is only able to handle at most two types of reaction wheel actuator faults.

In addition, actuator saturation should be addressed in spacecraft attitude control

systems as well as FTC systems. Due to the physical limitations of the actuator, it may not be able to generate typically commanded control torque. Particularly, in the event of actuator faults/failures, the remaining actuators are required to generate higher torques to compensate for the faulty actuators to maintain system performance [34–37]. As a result, actuator faults/failures may potentially lead to saturation of the actuators, which will lead to further damage to the rest of the system or even instability of the closed-loop system if there is no appropriate control strategy to dump the saturated actuators [34,37]. Hence, the problem of actuator saturation should be addressed in the fault-tolerant control system design as discussed in [4]. A number of spacecraft attitude control schemes which handle actuator saturation can be found in [11,28,31,130] and the reference therein.

In this chapter, four reaction wheels are used as actuators for attitude control, that is, there exists one degree of actuator redundancy. As a result, four types of common reaction wheel faults/failures discussed in chapter 2 (**F1-F4**) can be accommodated by the proposed fault-tolerant controllers. Three finite-time fault-tolerant controllers based on FTSM control schemes are proposed for spacecraft attitude stabilization under external disturbances, inertia matrix uncertainties, actuator faults/failures, and actuator saturation constraints. Since the proposed controller guarantees a priori boundedness of the control input in the final FTC scheme, a natural constraint on the actuator is achieved. It is established that ultimate boundedness of the closed-loop trajectories subject to control input constraints can be guaranteed no matter whether actuator faults/failures occur or not.

The remainder of this chapter is organized as follows. In section 6.2, the fault-tolerant attitude stabilization problem for spacecraft with redundancy actuators in the presence of external disturbances, inertia uncertainties, actuator faults/failures, and actuator saturation, is formulated. In section 6.3, design and analysis of a basic finite-time fault-tolerant controller and two adaptive finite-time FTC schemes are

presented. In section 6.4, simulations are carried out to verify the effectiveness of the proposed finite-time fault-tolerant controllers. The conclusions and future works are given in Section 6.5.

6.2 Problem Formulation and Statement

Although the dynamics of the reaction wheel is usually ignored in the attitude controller design, it may affect the whole performance of the control system when it is not fast enough to ignore or it is highly nonlinear. In this section, in order to get a more precise attitude dynamic model for real applications, reaction wheel dynamics are taken into account.

6.2.1 Attitude Kinetics with Reaction Wheel Dynamics

The attitude kinematics and kinetics with the dynamics of reaction wheels and actuator faults/falures are given by [75]

$$\begin{cases} \dot{\mathbf{Q}} = \begin{bmatrix} \dot{q}_0 \\ \dot{\mathbf{q}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\mathbf{q}^T \\ \mathbf{q}^\times + q_0 \mathbf{I}_3 \end{bmatrix} \boldsymbol{\omega} \\ \mathbf{J}(\cdot) \dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times (\mathbf{J}(\cdot) \boldsymbol{\omega} + \mathbf{D} \mathbf{J}_w \boldsymbol{\Omega}) + \mathbf{D}(\mathbf{E}(t) \mathbf{u}_c + \bar{\mathbf{u}}(t)) + \mathbf{d}. \end{cases} \quad (6.1)$$

where $\mathbf{J}(\cdot) \in \mathbb{R}^{3 \times 3}$ is the inertia matrix of the spacecraft, $\mathbf{J}_w = \text{diag}([J_{w1}, J_{w2}, \dots, J_{wn}]) \in \mathbb{R}^{n \times n}$ is the inertia matrix of reaction wheels, $\boldsymbol{\Omega} \in \mathbb{R}^n$ is the angular velocity of the reaction wheel. The control torques $\mathbf{u}$ generated by the reaction wheel are given by

$$\mathbf{u} = -\mathbf{J}_w \dot{\boldsymbol{\Omega}}. \quad (6.2)$$

Here, it is assumed that there exists uncertainty in the matrix $\mathbf{J}(\cdot)$, i.e. $\mathbf{J}(\cdot) =$

$\mathbf{J}_0 + \Delta\mathbf{J}$, where $\mathbf{J}_0$ and $\Delta\mathbf{J}$ denote the nominal part and uncertain part of $\mathbf{J}(\cdot)$, respectively. Then kinetics equation of (6.1) can be rewritten as

$$\mathbf{J}_0\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega}^\times \mathbf{J}_0\boldsymbol{\omega} - \boldsymbol{\omega}^\times \mathbf{D}\mathbf{J}_w\boldsymbol{\Omega} + \mathbf{D}(\mathbf{E}(t)\mathbf{u}_c + \bar{\mathbf{u}}(t)) + \mathbf{d} - \Delta\mathbf{J}\dot{\boldsymbol{\omega}} - \boldsymbol{\omega}^\times \Delta\mathbf{J}\boldsymbol{\omega}. \quad (6.3)$$

Property 6.1. *The nominal part $\mathbf{J}_0$ of inertia matrix is a symmetric and positive definite matrix and can be lower and upper bounded as follows:*

$$J_1\|\mathbf{x}\|^2 \leq \mathbf{x}^T \mathbf{J}_0 \mathbf{x} \leq J_2\|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \mathbb{R}^3 \quad (6.4)$$

where J_1 and J_2 are some positive constants, and $\|\cdot\|$ denotes Euclidean norm of vectors and largest singular value for matrices.

6.2.2 Transformed Spacecraft Attitude Dynamics

To express the attitude kinetics in a more convenient manner for controller design, the Lagrange-like equation is adopted to describe spacecraft attitude dynamics. Denoting $\mathbf{T} = \frac{1}{2}(\mathbf{q}^\times + q_0\mathbf{I}_3) \in \mathbb{R}^{3 \times 3}$, kinematics equation (6.1) can be expressed as

$$\boldsymbol{\omega} = \mathbf{P}\dot{\mathbf{q}}, \quad (6.5)$$

with

$$\mathbf{P} = \mathbf{T}^{-1} = \left[\frac{1}{2}(\mathbf{q}^\times + q_0\mathbf{I}_3) \right]^{-1}. \quad (6.6)$$

After taking the time derivative of equation (6.5), the following expression for $\dot{\boldsymbol{\omega}}$ is obtained:

$$\dot{\boldsymbol{\omega}} = \dot{\mathbf{P}}\dot{\mathbf{q}} + \mathbf{P}\ddot{\mathbf{q}}. \quad (6.7)$$

Substituting equations (6.5) and (6.7) into equation (6.3) and premultiplying both sides of the resulting expression by $\mathbf{P}^T$ lead to

$$\mathbf{J}^* \ddot{\mathbf{q}} = -\mathbf{\Xi} \dot{\mathbf{q}} + \mathbf{P}^T (\mathbf{D} \mathbf{J}_w \boldsymbol{\Omega})^\times \mathbf{P} \dot{\mathbf{q}} + \mathbf{P}^T \mathbf{D} \mathbf{E}(t) \mathbf{u}_c + \mathbf{P}^T \mathbf{D} \bar{\mathbf{u}}(t) + \mathbf{T}_d, \quad (6.8)$$

where $\mathbf{J}^* = \mathbf{P}^T \mathbf{J}_0 \mathbf{P}$, $\mathbf{\Xi} = \mathbf{P}^T \mathbf{J}_0 \dot{\mathbf{P}} - \mathbf{P}^T (\mathbf{J}_0 \mathbf{P} \dot{\mathbf{q}})^\times \mathbf{P}$, and $\mathbf{T}_d = \mathbf{P}^T (\mathbf{d} - \Delta \mathbf{J} \dot{\boldsymbol{\omega}} - \boldsymbol{\omega}^\times \Delta \mathbf{J} \boldsymbol{\omega})$. Here, $\mathbf{T}_d$ is considered as the lumped disturbances and uncertainties. Regarding the dynamic model given in equation (6.8) and Property 6.1, the following properties are given according to [105].

Property 6.2. *The inertia matrix $\mathbf{J}^*$ is symmetric positive definite, and the matrix $\dot{\mathbf{J}}^* - 2\mathbf{\Xi}$ satisfies the following skew-symmetric relationship:*

$$\mathbf{x}^T (\dot{\mathbf{J}}^* - 2\mathbf{\Xi}) \mathbf{x} = 0, \quad \forall \mathbf{x} \in \mathbb{R}^3 \quad (6.9)$$

Property 6.3. *The inertia matrix $\mathbf{J}^*$ is lower and upper bounded, and satisfies the following bounded condition:*

$$\underline{J} \|\mathbf{x}\|^2 \leq \mathbf{x}^T \mathbf{J}^* \mathbf{x} \leq \bar{J} \|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \mathbb{R}^3 \quad (6.10)$$

where $\underline{J}$ and $\bar{J}$ are positive constants.

For the development of control law, the following assumptions are given.

Assumption 6.1. *To ensure that $\mathbf{P}$ defined in equation (6.6) exists, the following condition should be satisfied [105]:*

$$\det(\mathbf{T}) = \frac{1}{2} q_0 \neq 0, \quad \forall t \in [0, \infty) \quad (6.11)$$

In order to guarantee that (6.11) remains valid, it is required that the initial value of q_0 satisfy this constraint, and that the subsequent controller is designed to guarantee

that $q_0 \neq 0$ for all $t > 0$.

Assumption 6.2. *The external disturbance $\mathbf{d}$ in equation (6.3) is upper bounded by a constant, and the lumped term $\mathbf{T}_d$ of the disturbances and uncertainties satisfies the following condition [131]:*

$$\|\mathbf{T}_d\| \leq \gamma_0 \Phi, \quad (6.12)$$

where $\Phi = 1 + \|\boldsymbol{\omega}\| + \|\boldsymbol{\omega}\|^2$, and γ_0 is a positive constant.

Assumption 6.3. *The additive fault introduced in the fault model (2.14) satisfies*

$$\|\bar{\mathbf{u}}(t)\| \leq f_0, \quad (6.13)$$

where f_0 is a positive constant.

Assumption 6.4. *The positive definite matrix $\mathbf{DED}^T$ satisfies*

$$0 < e_0 \leq \min\{\lambda_{\min}(\mathbf{DED}^T), 1\}, \quad (6.14)$$

where $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue of a matrix and e_0 is a positive constant.

Remark 6.1. *The model of the lumped variable $\mathbf{T}_d$ in Assumption 6.2 can be generalized to $\|\mathbf{T}_d\| \leq \gamma_0 + \gamma_{01}\|\boldsymbol{\omega}\| + \gamma_{02}\|\boldsymbol{\omega}\|^2$ to reduce the conservativeness, where γ_{01} and γ_{02} are two constants. However, two additional design parameters are introduced, which will complicate the design process and increase the computational complexity, especially to the case of adaptive fault-tolerant controller design. The Assumption 6.4 means that, although the n actuators ($n > 3$) may suffer from partial loss of actuator effectiveness or even complete failure, the number of totally failed actuators is no more than $n - 3$ such that $\mathbf{DED}^T$ remains positive definite. If more than $n - 3$*

actuators have totally failed, the matrix $\mathbf{D}\mathbf{E}\mathbf{D}^T$ becomes singular and the system is underactuated. In such a situation, other types of actuator have to be employed [75]. The underactuated system is not considered further in this paper.

6.3 Finite-Time Fault-Tolerant Controller Design

In this section, we shall show the development of an adaptive finite-time fault-tolerant controller for solving the attitude stabilization problem under actuator faults/failures (**F1-F4**) and control input saturation. Three control schemes are proposed. Firstly, a basic finite-time fault tolerant controller is designed assuming a priori knowledge of the bounds on disturbances, uncertainties, and actuator faults/failures is known. Subsequently, an adaptive finite-time fault-tolerant controller is designed without relying on any prior knowledge. Finally, the results are extended to the case in which actuator saturation is considered.

6.3.1 FTSM Manifold Design

To achieve finite-time convergence, the FTSM manifold $\mathbf{s} \in \mathbb{R}^3$ is defined as

$$\mathbf{s} = \dot{\mathbf{q}} + \alpha \mathbf{q} + \beta \text{sig}(\mathbf{q})^r = 0, \quad (6.15)$$

where α and β are positive constants, $r = \frac{r_1}{r_2}$, r_1 and r_2 are positive odd integers and $0 < r < 1$, and the function $\text{sig}(\mathbf{x})^r$ is defined as

$$\text{sig}(\mathbf{x})^r = [|x_1|^r \text{sign}(x_1), |x_2|^r \text{sign}(x_2), |x_3|^r \text{sign}(x_3)]^T$$

where x_j is the j th component of $\mathbf{x} \in \mathbb{R}^3$, $j = 1, 2, 3$.

Differentiating sliding function $\mathbf{s}$ with respect to time yields

$$\dot{\mathbf{s}} = \ddot{\mathbf{q}} + \alpha \dot{\mathbf{q}} + \beta r \text{diag}(|\mathbf{q}|^{r-1}) \dot{\mathbf{q}}. \quad (6.16)$$

Since equation (6.16) contains a negative fractional power $r - 1$, the singularity will occur if $q_j = 0$ and $\dot{q}_j \neq 0$. To avoid singularity, first order derivative of $\mathbf{s}$ is modified as [132]

$$\dot{\mathbf{s}} = \ddot{\mathbf{q}} + \alpha \dot{\mathbf{q}} + \beta \mathbf{q}_r, \quad (6.17)$$

with $\mathbf{q}_r \in \mathbb{R}^3$ defined as

$$\mathbf{q}_{r,j} = \begin{cases} r|q_j|^{r-1}\dot{q}_j, & \text{if } |q_j| \geq \epsilon, \text{ and } \dot{q}_j \neq 0 \\ r|\epsilon|^{r-1}\dot{q}_j, & \text{if } |q_j| < \epsilon, \text{ and } \dot{q}_j \neq 0 \\ 0, & \dot{q}_j = 0 \end{cases} \quad (6.18)$$

where $q_{r,j}$ is the j th component of $\mathbf{q}_r$, ϵ is a small positive constant. Then, considering equations (6.8), (6.15), and (6.17), it can be shown that

$$\begin{aligned} J^* \dot{\mathbf{s}} &= J^* \ddot{\mathbf{q}} + \alpha J^* \dot{\mathbf{q}} + \beta J^* \mathbf{q}_r \\ &= -\Xi \dot{\mathbf{q}} + P^T (DJ_w \Omega)^\times P \dot{\mathbf{q}} + P^T DE(t) \mathbf{u}_c + P^T D\bar{\mathbf{u}}(t) + \mathbf{T}_d + \alpha J^* \dot{\mathbf{q}} + \beta J^* \mathbf{q}_r \\ &= -\Xi \mathbf{s} + P^T DE \mathbf{u}_c + P^T D\bar{\mathbf{u}} + P^T (DJ_w \Omega)^\times P \dot{\mathbf{q}} \\ &\quad + \alpha \Xi \mathbf{q} + \beta \Xi \text{sig}(\mathbf{q})^r + \alpha J^* \dot{\mathbf{q}} + \beta J^* \mathbf{q}_r + \mathbf{T}_d. \end{aligned} \quad (6.19)$$

The above equation can be further simplified as

$$J^* \dot{\mathbf{s}} = -\Xi \mathbf{s} + P^T DE \mathbf{u}_c + P^T D\bar{\mathbf{u}}(t) + \mathbf{F} + \mathbf{T}_d, \quad (6.20)$$

where $\mathbf{F} = P^T (DJ_w \Omega)^\times P \dot{\mathbf{q}} + \alpha \Xi \mathbf{q} + \beta \Xi \text{sig}(\mathbf{q})^r + \alpha J^* \dot{\mathbf{q}} + \beta J^* \mathbf{q}_r$.

The following results will be used to derive finite-time stability proof for the pro-

posed controller design.

Lemma 6.1. *Consider the terminal sliding mode manifold $\mathbf{s}$ defined by (6.15). If the sliding mode manifold satisfies $\mathbf{s} = 0$, then the system states $\mathbf{q}$ and $\dot{\mathbf{q}}$ are finite-time stable, i.e., the system states $\mathbf{q}$ and $\dot{\mathbf{q}}$ which start from $\mathbf{q}(0)$ and $\dot{\mathbf{q}}(0)$ will converge to $\mathbf{q} = 0$ and $\dot{\mathbf{q}} = 0$ in finite time, respectively.*

Proof. In order to prove that q_v can converge to $\mathbf{q} = 0$ in finite time after the sliding mode manifold $\mathbf{s} = 0$ is achieved, a candidate Lyapunov function is constructed as follows:

$$V_0 = \frac{1}{2} \mathbf{q}^T \mathbf{q}. \quad (6.21)$$

Taking the time derivative of the Lyapunov function (6.21) along sliding mode manifold $\mathbf{s} = 0$ yields

$$\begin{aligned} \dot{V}_0 &= -\alpha \mathbf{q}^T \mathbf{q} - \beta \sum_{i=1}^3 |q_i|^{r+1} \\ &\leq -2\alpha V_0 - 2^{\frac{r+1}{2}} \beta V_0^{\frac{r+1}{2}} \\ &\leq -\varrho_1 V_0 - \varrho_2 V_0^\varsigma, \end{aligned} \quad (6.22)$$

where $\varrho_1 = 2\alpha$, $\varrho_2 = 2^{\frac{r+1}{2}} \beta$, $\varsigma = \frac{r+1}{2}$, and Lemma 1 is applied. It is clear that equation (6.22) has a similar structure to equation (2.20), and therefore, using Lemma 2.3, it can be concluded that $\mathbf{q}$ converges to origin in finite time. Furthermore, because the sliding manifold $\mathbf{s} = 0$ is guaranteed, $\dot{\mathbf{q}}$ will also converge to the origin along the sliding manifold in finite time. This completes the proof. $\square$

6.3.2 Basic Finite-Time FTC Design

For the attitude dynamics defined in equation (6.20) with actuator faults/failures, assuming parameters γ_0 , f_0 , and e_0 are known, the basic finite-time FTC law is

designed as

$$\mathbf{u}_c = -\mathbf{D}^T \mathbf{P} \left[\mathbf{u}_{nom} + (\gamma_1 \|\mathbf{D}\| + \gamma_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\|) \frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|} \right], \quad (6.23)$$

with

$$\mathbf{u}_{nom} = \frac{k \|\mathbf{s}_\rho\|_1 + (\|\mathbf{F}\| + \gamma_0 \Phi) \|\mathbf{s}\|}{\|\mathbf{P}\mathbf{s}\|^2} \mathbf{s}, \quad (6.24)$$

where k is a positive scalar, $\mathbf{s}_\rho$ is a column vector defined as $\mathbf{s}_\rho = [s_1^{\rho+1}, s_2^{\rho+1}, s_3^{\rho+1}]^T$, ρ is a positive scalar satisfying $0 < \rho < 1$, γ_1 and γ_2 are positive scalars satisfying $\gamma_1 = \frac{f_0 + \varepsilon}{e_0}$ and $\gamma_2 = \frac{1 - e_0 + \varepsilon}{e_0}$ respectively with ε is a small positive scalar.

Theorem 6.1. *Consider the attitude control systems described by equation (6.1) in the presence of four types of reaction wheel faults/failures **F1-F4**. If Assumptions 6.1-6.4 are satisfied and the controller (6.23) is applied, then the closed-loop system will be locally stabilized in finite time, i.e., $\mathbf{q} \rightarrow 0$, $\boldsymbol{\omega} \rightarrow 0$ in finite time.*

Proof. Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{2} \mathbf{s}^T \mathbf{J}^* \mathbf{s}. \quad (6.25)$$

Using equation (6.20), the time derivative of V_1 is

$$\dot{V}_1 = \frac{1}{2} \mathbf{s}^T \dot{\mathbf{J}}^* \mathbf{s} + \mathbf{s}^T (-\boldsymbol{\Xi} \mathbf{s} + \mathbf{P}^T \mathbf{D} \mathbf{E} \mathbf{u}_c + \mathbf{P}^T \mathbf{D} \bar{\mathbf{u}} + \mathbf{F} + \mathbf{T}_d). \quad (6.26)$$

In view of the control law in equation (6.23) and Property 6.2, it follows that

$$\begin{aligned} \dot{V}_1 \leq & \mathbf{s}^T \mathbf{P}^T (\mathbf{I}_3 - \mathbf{D} \mathbf{E} \mathbf{D}^T) \mathbf{P} \mathbf{u}_{nom} - \mathbf{s}^T \mathbf{P}^T \mathbf{P} \mathbf{u}_{nom} + f_0 \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| + \|\mathbf{F}\| \|\mathbf{s}\| \\ & - (\gamma_1 \|\mathbf{D}\| + \gamma_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\|) \frac{\mathbf{s}^T \mathbf{P}^T \mathbf{D} \mathbf{E} \mathbf{D}^T \mathbf{P} \mathbf{s}}{\|\mathbf{P}\mathbf{s}\|} + \|\mathbf{T}_d\| \|\mathbf{s}\|, \end{aligned} \quad (6.27)$$

which further leads to

$$\begin{aligned}
\dot{V}_1 &\leq (1 - e_0) \|\mathbf{P}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| - e_0 \gamma_1 \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| \\
&\quad - \mathbf{s}^T \mathbf{P}^T \mathbf{P} \frac{k \|\mathbf{s}_\rho\|_1 + (\|\mathbf{F}\| + \gamma_0 \Phi) \|\mathbf{s}\|}{\|\mathbf{P}\mathbf{s}\|^2} \mathbf{s} + \|\mathbf{F}\| \|\mathbf{s}\| \\
&\quad - e_0 \gamma_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| + f_0 \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| + \|\mathbf{T}_d\| \|\mathbf{s}\| \\
&\leq -k \sum_{j=1}^3 |s_j|^{\rho+1} + (1 - e_0 - e_0 \gamma_2) \|\mathbf{p}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| \\
&\quad + (f_0 - e_0 \gamma_1) \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| + (\|\mathbf{T}_d\| - \gamma_0 \Omega) \|\mathbf{s}\|. \tag{6.28}
\end{aligned}$$

With the definition of γ_1 and γ_2 , and by using Assumption 6.2, it can be shown that

$$\begin{aligned}
\dot{V}_1 &\leq -k \sum_{j=1}^3 |s_j|^{\rho+1} - \varepsilon \|\mathbf{p}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| - \varepsilon \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| \\
&\leq -k \sum_{j=1}^3 |s_j|^{\rho+1}, \tag{6.29}
\end{aligned}$$

From Property 6.3 and Lemma 2.1, the following inequalities can be obtained:

$$\begin{aligned}
\dot{V}_1 &\leq -k \left(\frac{2}{\bar{J}} \right)^{\frac{\rho+1}{2}} \left(\sum_{j=1}^3 \frac{\bar{J}}{2} s_j^2 \right)^{\frac{\rho+1}{2}} \\
&\leq -k \left(\frac{2}{\bar{J}} \right)^{\frac{\rho+1}{2}} V_1^{\frac{\rho+1}{2}} \\
&\leq -KV_1^{\frac{\rho+1}{2}}, \tag{6.30}
\end{aligned}$$

where $K = k \left(\frac{2}{\bar{J}} \right)^{\frac{\rho+1}{2}} > 0$. According to Lemma 2.2, because K is a positive scalar, and $0 < \frac{\rho+1}{2} < 1$, the sliding manifold will reach $\mathbf{s} = 0$ in finite time. Furthermore, from Lemma 6.1, $\mathbf{q} = 0$ is achieved in finite time as well as $\dot{\mathbf{q}}$ will converge to origin. Then, according to kinematics of (6.1) and Assumption 6.1, it can be concluded that $\boldsymbol{\omega} = 0$ is also achieved in finite time. This completes the proof. $\square$

Remark 6.2. *Since finite-time stability requires that system states converge to the equilibrium in finite time, finite-time stability is therefore a much stronger requirement than asymptotic stability [133], whose convergence rate is at best exponential. Hence, the closed-loop systems under finite-time controller will have a faster convergence rate [21,22]. Moreover, the convergence time for achieving attitude stabilization can be given by Lemma 2.2 and Lemma 2.3.*

Remark 6.3. *It should be pointed out that $\mathbf{q} \rightarrow 0$, $\boldsymbol{\omega} \rightarrow 0$ in finite time is proved here, and the state q_0 is not considered. When $\mathbf{q} = 0$, it is obtained that $q_0 = \pm 1$ from the constraint condition on unit quaternion. Therefore, there exist two equilibrium points, i.e., $\mathbf{Q} = [1, 0, 0, 0]^T$ and $\mathbf{Q} = [-1, 0, 0, 0]^T$. However, due to the fact that both equilibrium points correspond to a unique physical equilibrium orientation [91], it is only needed to guarantee that $\mathbf{q} \rightarrow 0$ in finite time. In addition, due to the Assumption 6.1 that requires $q_0(t) \neq 0$ for all $t \geq 0$, local finite-time stability is obtained in Theorem 6.1, as well as the following two theorems. In order to remove this assumption and get global result, some switching method may be incorporated into the controller design, such as [33], but this is not within the scope investigated in this work.*

Remark 6.4. *Note that for the fault-free case with all actuators functional, finite-time stability can also be guaranteed by choosing the same control scheme in equation (6.23). This is easily shown by applying the same procedure outlined in the proof of Theorem 6.1, which implies that the proposed controller in equation (6.23) is able to achieve attitude stabilization in finite time irrespective of whether actuator faults occur or not.*

6.3.3 Adaptive Finite-Time FTC Design

In the first basic finite-time FTC law, the bounds on the inertia uncertainties, external disturbances and actuator faults/failures are assumed to be known to the

designer in advance. In practice, these information on the bounds are not always available due to the complicated structure of disturbance, time-varying inertia property, and unexpected characteristics of faults/failures. Therefore, in order to avoid the requirements of a priori knowledge of these bounds, an adaptive mechanism is introduced to estimate these bounds information.

The adaptive finite-time FTC law is designed as

$$\mathbf{u}_c = -\mathbf{D}^T \mathbf{P} \left[\mathbf{u}_{nom} + (\hat{\gamma}_1 \|\mathbf{D}\| + \hat{\gamma}_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\|) \frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|} \right], \quad (6.31)$$

with

$$\mathbf{u}_{nom} = \frac{(k + \|\mathbf{F}\| + \hat{\gamma}_0 \Phi) \|\mathbf{s}\|}{\|\mathbf{P}\mathbf{s}\|^2} \mathbf{s}, \quad (6.32)$$

where k is a positive scalar, $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are the estimated values of γ_0 , $\gamma_1 = \frac{f_0 + \varepsilon}{e_0}$, and $\gamma_2 = \frac{1 - e_0 + \varepsilon}{e_0}$, respectively. The adaptive laws are chosen as

$$\dot{\hat{\gamma}}_0 = c_0(\Phi \|\mathbf{s}\| - \delta_0 \hat{\gamma}_0), \quad (6.33a)$$

$$\dot{\hat{\gamma}}_1 = c_1(\|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| - \delta_1 \hat{\gamma}_1), \quad (6.33b)$$

$$\dot{\hat{\gamma}}_2 = c_2(\|\mathbf{P}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| - \delta_2 \hat{\gamma}_2), \quad (6.33c)$$

where $c_i > 0$ and $\delta_i > 0$ ($i = 0, 1, 2$) are the design parameters, and the initial estimated values satisfy $\hat{\gamma}_i(0) > 0$.

Lemma 6.2. *Given the parameter adaptive law in (6.33a)-(6.33c), the parameters γ_i as well as their estimates $\hat{\gamma}_i$ have upper bounds, i.e., there always exist positive scalars $\bar{\gamma}_i$ such that $\hat{\gamma}_i \leq \bar{\gamma}_i$ and $\gamma_i \leq \bar{\gamma}_i$ for all $t > 0$, respectively.*

Theorem 6.2. *Consider the attitude control systems described by equation (6.1) in the presence of four types of reaction wheel faults/failures **F1-F4**. If the adaptive*

controller (6.31) and update laws in (6.33a-6.33c) are applied under Assumptions 6.1-6.4, the sliding mode manifold will converge to the neighborhood of $\mathbf{s} = 0$ as

$$\|\mathbf{s}\| \leq \Delta_s \quad (6.34)$$

in finite time, where $\Delta_s = \sqrt{\frac{2}{J}\frac{\phi}{\delta}}$, δ and ϕ are positive scalars to be defined later. Furthermore, the spacecraft attitude q_j and angular velocity ω_j ($j = 1, 2, 3$) will locally converge to the regions

$$|q_j| \leq \min \left\{ \frac{|\Delta_s|}{\alpha}, \left(\frac{|\Delta_s|}{\beta} \right)^{\frac{1}{r}} \right\}, \quad |\omega_j| \leq 6\sqrt{3}\Delta_s \quad (6.35)$$

in finite time, respectively.

Proof. Consider the following Lyapunov function candidate V_2 :

$$V_2 = \frac{1}{2} \mathbf{s}^T \mathbf{J}^* \mathbf{s} + \frac{1}{2c_0} (\hat{\gamma}_0 - \bar{\gamma}_0)^2 + \frac{e_0}{2c_1} (\hat{\gamma}_1 - \bar{\gamma}_1)^2 + \frac{e_0}{2c_2} (\hat{\gamma}_2 - \bar{\gamma}_2)^2. \quad (6.36)$$

Taking the time derivative of V_2 and using (6.20), it leads to

$$\begin{aligned} \dot{V}_2 = & \mathbf{s}^T (\mathbf{P}^T \mathbf{D} \mathbf{E} \mathbf{u}_c + \mathbf{P}^T \mathbf{D} \bar{\mathbf{u}} + \mathbf{F} + \mathbf{T}_d) \\ & + \frac{1}{c_0} (\hat{\gamma}_0 - \bar{\gamma}_0) \dot{\gamma}_0 + \frac{e_0}{c_1} (\hat{\gamma}_1 - \bar{\gamma}_1) \dot{\gamma}_1 + \frac{e_0}{c_2} (\hat{\gamma}_2 - \bar{\gamma}_2) \dot{\gamma}_2. \end{aligned} \quad (6.37)$$

By substituting the control law (6.31) and Property 6.2 into equation (6.37), it follows that

$$\begin{aligned} \dot{V}_2 \leq & (1 - e_0) \|\mathbf{P}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| - \mathbf{s}^T \mathbf{P}^T \mathbf{P} \mathbf{u}_{nom} - e_0 \hat{\gamma}_1 \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| \\ & - e_0 \hat{\gamma}_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\| \|\mathbf{P}\mathbf{s}\| + f_0 \|\mathbf{D}\| \|\mathbf{P}\mathbf{s}\| + \|\mathbf{F}\| \|\mathbf{s}\| + \|\mathbf{T}_d\| \|\mathbf{s}\| \\ & + \frac{1}{c_0} (\hat{\gamma}_0 - \bar{\gamma}_0) \dot{\gamma}_0 + \frac{e_0}{c_1} (\hat{\gamma}_1 - \bar{\gamma}_1) \dot{\gamma}_1 + \frac{e_0}{c_2} (\hat{\gamma}_2 - \bar{\gamma}_2) \dot{\gamma}_2. \end{aligned} \quad (6.38)$$

Applying adaptive laws (6.33a)-(6.33c) and u_{nom} defined in equation (6.32), the following inequalities can be obtained:

$$\begin{aligned}
\dot{V}_2 &\leq -k\|\mathbf{s}\| + (\gamma_0 - \hat{\gamma}_0)\Phi\|\mathbf{s}\| + (f_0 - e_0\hat{\gamma}_1)\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| \\
&\quad + (1 - e_0 - e_0\hat{\gamma}_2)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| + (\hat{\gamma}_0 - \bar{\gamma}_0)\Phi\|\mathbf{s}\| \\
&\quad - (\hat{\gamma}_0 - \bar{\gamma}_0)\delta_0\hat{\gamma}_0 + e_0(\hat{\gamma}_1 - \bar{\gamma}_1)\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| - e_0\delta_1(\hat{\gamma}_1 - \bar{\gamma}_1)\hat{\gamma}_1 \\
&\quad + e_0(\hat{\gamma}_2 - \bar{\gamma}_2)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| - e_0\delta_2(\hat{\gamma}_2 - \bar{\gamma}_2)\hat{\gamma}_2 \\
&= -k\|\mathbf{s}\| + (\gamma_0 - \bar{\gamma}_0)\Phi\|\mathbf{s}\| + (f_0 - e_0\bar{\gamma}_1)\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| \\
&\quad + (1 - e_0 - e_0\bar{\gamma}_2)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| \\
&\quad - \delta_0(\hat{\gamma}_0 - \bar{\gamma}_0)\hat{\gamma}_0 - e_0\delta_1(\hat{\gamma}_1 - \bar{\gamma}_1)\hat{\gamma}_1 - e_0\delta_2(\hat{\gamma}_2 - \bar{\gamma}_2)\hat{\gamma}_2. \tag{6.39}
\end{aligned}$$

Furthermore, based on the result from Lemma 6.2, we have

$$\dot{V}_2 \leq -k\|\mathbf{s}\| - \delta_0(\hat{\gamma}_0 - \bar{\gamma}_0)\hat{\gamma}_0 - e_0\delta_1(\hat{\gamma}_1 - \bar{\gamma}_1)\hat{\gamma}_1 - e_0\delta_2(\hat{\gamma}_2 - \bar{\gamma}_2)\hat{\gamma}_2. \tag{6.40}$$

Using the following facts by completion of squares:

$$-(\hat{\gamma}_i - \bar{\gamma}_i)\hat{\gamma}_i \leq -\left(\hat{\gamma}_i - \frac{\bar{\gamma}_i}{2}\right)^2 + \frac{\bar{\gamma}_i^2}{4}, \tag{6.41}$$

where $i = 0, 1, 2$, we have

$$\begin{aligned}
\dot{V}_2 &\leq -k\|\mathbf{s}\| - \delta_0 \left[\left(\hat{\gamma}_0 - \frac{\bar{\gamma}_0}{2} \right)^2 - \frac{\bar{\gamma}_0^2}{4} \right] \\
&\quad - e_0\delta_1 \left[\left(\hat{\gamma}_1 - \frac{\bar{\gamma}_1}{2} \right)^2 - \frac{\bar{\gamma}_1^2}{4} \right] - e_0\delta_2 \left[\left(\hat{\gamma}_2 - \frac{\bar{\gamma}_2}{2} \right)^2 - \frac{\bar{\gamma}_2^2}{4} \right] \\
&\leq -k\|\mathbf{s}\| + \frac{\delta_0}{4}\bar{\gamma}_0^2 + \frac{\delta_1}{4}\bar{\gamma}_1^2 + \frac{\delta_2}{4}\bar{\gamma}_2^2 \\
&= -k\|\mathbf{s}\| - \delta_0|\hat{\gamma}_0 - \bar{\gamma}_0| - e_0\delta_1|\hat{\gamma}_1 - \bar{\gamma}_1| - e_0\delta_2|\hat{\gamma}_2 - \bar{\gamma}_2| \\
&\quad + \delta_0 \left(|\hat{\gamma}_0 - \bar{\gamma}_0| + \frac{\bar{\gamma}_0^2}{4} \right) + e_0\delta_1 \left(|\hat{\gamma}_1 - \bar{\gamma}_1| + \frac{\bar{\gamma}_1^2}{4} \right) + e_0\delta_2 \left(|\hat{\gamma}_2 - \bar{\gamma}_2| + \frac{\bar{\gamma}_2^2}{4} \right).
\end{aligned}$$

With the help of Lemma 6.2 and Property 6.3, it can be found that

$$\begin{aligned}
\dot{V}_2 &\leq -k \left(\frac{2}{\bar{J}} \right)^{\frac{1}{2}} \left(\sum_{j=1}^3 \frac{\bar{J}}{2} s_j^2 \right)^{\frac{1}{2}} - (2c_0 \delta_0^2)^{\frac{1}{2}} \left[\frac{1}{2c_0} (\hat{\gamma}_0 - \bar{\gamma}_0)^2 \right]^{\frac{1}{2}} \\
&\quad - (2c_1 e_0 \delta_1^2)^{\frac{1}{2}} \left[\frac{e_0}{2c_1} (\hat{\gamma}_1 - \bar{\gamma}_1)^2 \right]^{\frac{1}{2}} - (2c_2 e_0 \delta_2^2)^{\frac{1}{2}} \left[\frac{e_0}{2c_2} (\hat{\gamma}_2 - \bar{\gamma}_2)^2 \right]^{\frac{1}{2}} \\
&\quad + \delta_0 \left(\bar{\gamma}_0 + \frac{\bar{\gamma}_0^2}{4} \right) + e_0 \delta_1 \left(\bar{\gamma}_1 + \frac{\bar{\gamma}_1^2}{4} \right) + e_0 \delta_2 \left(\bar{\gamma}_2 + \frac{\bar{\gamma}_2^2}{4} \right) \\
&\leq -\min \left\{ k \left(\frac{2}{\bar{J}} \right)^{\frac{1}{2}}, (2c_0 \delta_0^2)^{\frac{1}{2}}, (2c_1 e_0 \delta_1^2)^{\frac{1}{2}}, (2c_2 e_0 \delta_2^2)^{\frac{1}{2}} \right\} \\
&\quad \times \left\{ \left(\sum_{i=1}^3 \frac{\bar{J}}{2} s_i^2 \right)^{\frac{1}{2}} + \left[\frac{1}{2c_0} (\hat{\gamma}_0 - \bar{\gamma}_0)^2 \right]^{\frac{1}{2}} + \left[\frac{e_0}{2c_1} (\hat{\gamma}_1 - \bar{\gamma}_1)^2 \right]^{\frac{1}{2}} + \left[\frac{e_0}{2c_2} (\hat{\gamma}_2 - \bar{\gamma}_2)^2 \right]^{\frac{1}{2}} \right\} \\
&\quad + \delta_0 \left(\bar{\gamma}_0 + \frac{\bar{\gamma}_0^2}{4} \right) + e_0 \delta_1 \left(\bar{\gamma}_1 + \frac{\bar{\gamma}_1^2}{4} \right) + e_0 \delta_2 \left(\bar{\gamma}_2 + \frac{\bar{\gamma}_2^2}{4} \right).
\end{aligned}$$

Invoking Lemma 1, the time derivative of the candidate Lyapunov function becomes

$$\dot{V}_2 \leq -\delta V_2^{\frac{1}{2}} + \phi, \quad (6.42)$$

where δ and ϕ are positive scalars defined by $\delta = \min \left\{ k \left(\frac{2}{\bar{J}} \right)^{\frac{1}{2}}, (2c_0 \delta_0^2)^{\frac{1}{2}}, (2c_1 e_0 \delta_1^2)^{\frac{1}{2}}, (2c_2 e_0 \delta_2^2)^{\frac{1}{2}} \right\}$ and $\phi = \delta_0 \left(\bar{\gamma}_0 + \frac{\bar{\gamma}_0^2}{4} \right) + e_0 \delta_1 \left(\bar{\gamma}_1 + \frac{\bar{\gamma}_1^2}{4} \right) + e_0 \delta_2 \left(\bar{\gamma}_2 + \frac{\bar{\gamma}_2^2}{4} \right)$, respectively.

Thus, the time derivative of equation (6.42) can be further written as

$$\dot{V}_2 \leq - \left(\delta - \frac{\phi}{V_2^{\frac{1}{2}}} \right) V_2^{\frac{1}{2}}. \quad (6.43)$$

From equation (6.43), if $\delta - \frac{\phi}{(\frac{1}{2} \mathbf{s}^T \mathbf{J}^* \mathbf{s})^{\frac{1}{2}}} > 0$, then $\delta - \frac{\phi}{V_2^{\frac{1}{2}}} > 0$ is obtained such that the finite-time reachability of sliding manifold can be guaranteed by using Lemma 2.2. Hence, the region $\|\mathbf{s}\| \leq \Delta_s$ is reached in finite time, where Δ_s is defined as

$$\Delta_s = \sqrt{\frac{2}{\bar{J}} \frac{\phi}{\delta}}. \quad (6.44)$$

Moreover, for any sliding variable s_j in the region Δ_s , we have $|s_j| \leq \Delta_s$, $j = 1, 2, 3$. Then, the sliding mode manifold defined in equation (6.15) can be written as follows:

$$\dot{q}_j + \alpha q_j + \beta \text{sig}(q_j)^r = \eta_j, \quad |\eta_j| \leq \Delta_s \quad (6.45)$$

Then, equation (6.45) can be written as the following two forms:

$$\dot{q}_j + \left(\alpha - \frac{\eta_j}{q_j} \right) q_j + \beta \text{sig}(q_j)^r = 0 \quad (6.46a)$$

$$\dot{q}_j + \alpha q_j + \left(\beta - \frac{\eta_j}{\text{sig}(q_j)^r} \right) \text{sig}(q_j)^r = 0. \quad (6.46b)$$

From equation (6.46a), if $\alpha - \frac{\eta_j}{q_j} > 0$, a similar structure to the proposed sliding mode manifold is kept. Therefore, the finite-time attitude stabilization is guaranteed by using Lemma 6.1. Furthermore, the attitude q_j will converge to the region

$$|q_j| \leq \frac{|\eta_j|}{\alpha} \leq \frac{\Delta_s}{\alpha} \quad (6.47a)$$

in finite time. By similar analysis for equation (6.46b) and Lemma 6.1, the attitude q_j will also converge to the region

$$|q_j| \leq \left(\frac{|\eta_j|}{\beta} \right)^{\frac{1}{r}} \leq \left(\frac{\Delta_s}{\beta} \right)^{\frac{1}{r}} \quad (6.47b)$$

in finite time. Finally, the attitude q_j will converge to the region

$$|q_j| \leq \min \left\{ \frac{\Delta_s}{\alpha}, \left(\frac{\Delta_s}{\beta} \right)^{\frac{1}{r}} \right\} \quad (6.48)$$

in finite time. Moreover, from equation (6.45), $\dot{q}_j$ will converge to the region

$$|\dot{q}_j| \leq |\eta_j| + \alpha |q_j| + \beta |q_j|^r \leq 3\Delta_s \quad (6.49)$$

in finite time.

It should be noted that $\|\mathbf{q}^\times + q_0 \mathbf{I}_3\| = 1$. From (6.1), it can be obtained that $\|\boldsymbol{\omega}\| = 2\|\dot{\mathbf{q}}\|$, which further indicates that $\|\boldsymbol{\omega}\|_\infty \leq 2\sqrt{3}\|\dot{\mathbf{q}}\|_\infty$. On the other hand, since $\dot{\mathbf{q}}$ will converge to the region $|\dot{q}_j| \leq 3\Delta_s$ ($j = 1, 2, 3$) in finite time, it is clear that $\|\dot{\mathbf{q}}\|_\infty \leq 3\Delta_s$ will be satisfied in finite time. Therefore, it can be shown that the angular velocity will converge the region $|\omega_j| \leq 6\sqrt{3}\Delta_s$ in finite time. This completes the proof. $\square$

Remark 6.5. *It should be noted that, the control scheme in equation (6.31) is independent of the parameters e_0 and f_0 . Thus, there is no need to involve a FDD block to identify actuator faults and the proposed fault-tolerant controller is able to accommodate actuator faults/failures automatically and adaptively whenever actuator faults/failures occur.*

Remark 6.6. *As can be seen from equation (6.34), the accuracy of the sliding manifold is related to the parameters δ and ϕ . Specifically, the larger the parameter δ or the smaller the parameter ϕ is, the smaller Δ_s becomes. Also, it is clear in equation (6.35) that the final accuracy of attitude stabilization is related to the parameters α , β , and r . Thus, it can be concluded that larger α and β and a smaller r lead to better performance.*

6.3.4 Adaptive FTC with Input Saturation

To deal with input saturation problem, the adaptive finite-time fault-tolerant controller from Theorem 6.2 is further enhanced with explicit consideration of actuator saturation. If actuator saturation is taken into consideration, assume that all n actuator control torques are constrained by a common saturation value given by

$$|u_{ci}| \leq u_{\max}, \quad (6.50)$$

where $u_{\max}$ is a positive constant representing the maximum allowable control torque and $i = 1, \dots, n$. The following input saturation function is taken into consideration:

$$\mathbf{u}_c^{\text{sat}} = \mathcal{X}(\mathbf{u}_c) \mathbf{u}_c, \quad (6.51)$$

where $\mathcal{X}(\mathbf{u}_c) = \text{diag}(\mathcal{X}_1(u_{c1}), \mathcal{X}_2(u_{c2}), \dots, \mathcal{X}_n(u_{cn}))$, with

$$\mathcal{X}_i(u_{ci}) = \begin{cases} \frac{u_{\max}}{u_{ci}} \text{sign}(u_{ci}), & \text{if } |u_{ci}| > u_{\max} \\ 1, & \text{if } |u_{ci}| \leq u_{\max} \end{cases}. \quad (6.52)$$

Therefore, the dynamics of spacecraft defined in equation (6.20) can be written as

$$\mathbf{J}^* \dot{\mathbf{s}} = -\mathbf{\Xi} \mathbf{s} + \mathbf{P}^T \mathbf{D} \mathbf{E} \mathbf{u}_c^{\text{sat}} + \mathbf{P}^T \mathbf{D} \bar{\mathbf{u}} + \mathbf{F} + \mathbf{T}_d. \quad (6.53)$$

Since $0 < \min_i (\mathcal{X}_i(u_{ci})) \leq 1$, according to the density property of real number [134,135], there always exists a constant κ such that

$$0 < \kappa \leq \min_i (\mathcal{X}_i(u_{ci})) \leq 1, \quad \forall u_{ci}(t), t \in [0, \infty). \quad (6.54)$$

Now, the adaptive fault-tolerant control law ensuring attitude stabilization under actuator fault and control input saturation is proposed as

$$\mathbf{u}_c = -\mathbf{D}^T \mathbf{P} \left[\mathbf{u}_{\text{nom}} + \zeta \hat{\theta}_3 \left(\hat{\theta}_1 \|\mathbf{D}\| + \hat{\theta}_2 \|\mathbf{P}\| \|\mathbf{u}_{\text{nom}}\| \right) \frac{\mathbf{s}}{\|\mathbf{P} \mathbf{s}\|} \right], \quad (6.55)$$

with

$$\mathbf{u}_{\text{nom}} = \frac{(k + \|\mathbf{F}\| + \hat{\theta}_0 \Phi) \|\mathbf{s}\|}{\|\mathbf{P} \mathbf{s}\|^2} \mathbf{s}. \quad (6.56)$$

where k is a positive scalar, $\hat{\theta}_0$, $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$ denote the estimations of constants

$\theta_0 = \gamma_0$, $\theta_1 = \frac{f_0 + \varepsilon}{e_0}$, and $\theta_2 = \frac{1 - e_0 \kappa + \varepsilon}{e_0}$, and $\theta_3 = \kappa^{-1}$, respectively. The adaptive laws are chosen as

$$\dot{\hat{\theta}}_0 = c_0(\Phi\|\mathbf{s}\| - \delta_0\hat{\theta}_0), \quad (6.57a)$$

$$\dot{\hat{\theta}}_1 = c_1(\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| - \delta_1\hat{\theta}_1), \quad (6.57b)$$

$$\dot{\hat{\theta}}_2 = c_2(\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| - \delta_2\hat{\theta}_2), \quad (6.57c)$$

$$\dot{\hat{\theta}}_3 = \text{Proj}_{\hat{\theta}_3} \left\{ c_3 \zeta \hat{\theta}_3^3 \left[\left(\hat{\theta}_1 \|\mathbf{D}\| + \hat{\theta}_2 \|\mathbf{P}\|\|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| - \delta_3 \hat{\theta}_3 \right] \right\}, \quad (6.57d)$$

where $\zeta > 1$, $c_i > 0$ and $\delta_i > 0$ ($i = 0, 1, \dots, 3$) are the design parameters, and the initial estimated values satisfy $\hat{\theta}_0(0) > 0$, $\hat{\theta}_1(0) > 0$, $\hat{\theta}_2(0) > 0$, and $\hat{\theta}_3(0) \geq 1$, respectively. $\text{Proj}_{\hat{\theta}_3}(\bullet)$ denotes the projection operator defined as

$$\text{Proj}_{\hat{\theta}_3}(\bullet) = \begin{cases} \bullet, & \text{if } \hat{\theta}_3 > 1 \\ \bullet, & \text{if } \hat{\theta}_3 = 1 \text{ and } \bullet \geq 0 \\ 0, & \text{if } \hat{\theta}_3 = 1 \text{ and } \bullet < 0 \end{cases}$$

whose role is to keep the estimate $\hat{\theta}_3$ within the interval $[1, +\infty)$ because of the physical meaning of κ .

Lemma 6.3. *Given the parameter adaptive law in equations (6.57a)-(6.57d), the parameters θ_i as well as their estimates $\hat{\theta}_i$ have upper bounds, i.e., there always exist positive scalars $\bar{\theta}_i$ such that $\hat{\theta}_i \leq \bar{\theta}_i$ and $\theta_i \leq \bar{\theta}_i$, $i = 0, 1, \dots, 3$, for all $t > 0$, respectively.*

Theorem 6.3. *Consider the attitude control systems described by equation (6.1) with control input saturation in the presence of four types of reaction wheel faults/failures **F1-F4**. If the adaptive controller (6.55) and update laws in equations (6.57a)-(6.57d) are applied under Assumptions 6.1-6.4, the sliding mode manifold will locally*

converge to the neighborhood of $\mathbf{s} = 0$ as

$$\|\mathbf{s}\| \leq \Delta_{sat} \quad (6.58)$$

in finite time, where $\Delta_{sat} = \sqrt{\frac{2}{J}\frac{\varphi}{\eta}}$, and φ and η are two positive scalars to be defined later. Furthermore, the spacecraft attitude q_j and angular velocity ω_j ($j = 1, 2, 3$) will converge to the region

$$|q_j| \leq \min \left\{ \frac{\Delta_{sat}}{\alpha}, \left(\frac{\Delta_{sat}}{\beta} \right)^{\frac{1}{r}} \right\}, \quad |\omega_j| \leq 6\sqrt{3}\Delta_{sat} \quad (6.59)$$

in finite time, respectively.

Proof. Consider the following Lyapunov function candidate V_3 :

$$\begin{aligned} V_3 = & \frac{1}{2} \mathbf{s}^T \mathbf{J}^* \mathbf{s} + \frac{1}{2c_0} (\hat{\theta}_0 - \bar{\theta}_0)^2 + \frac{e_0}{2c_1} (\hat{\theta}_1 - \bar{\theta}_1)^2 \\ & + \frac{e_0}{2c_2} (\hat{\theta}_2 - \bar{\theta}_2)^2 + \frac{e_0}{2c_3} (\hat{\theta}_3^{-1} - \bar{\theta}_1^{-1})^2. \end{aligned} \quad (6.60)$$

With the application of controller (6.55) and Property 6.2, the time derivative of V_3 results in

$$\begin{aligned} \dot{V}_3 \leq & -k\|\mathbf{s}\| + (1 - e_0\kappa)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| + f_0\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| + \theta_0\boldsymbol{\Omega}\|\mathbf{s}\| \\ & - e_0\kappa\zeta\hat{\theta}_3 \left(\hat{\theta}_1\|\mathbf{D}\| + \hat{\theta}_2\|\mathbf{P}\|\|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| - \hat{\theta}_0\boldsymbol{\Omega}\|\mathbf{s}\| + \frac{1}{c_0}(\hat{\theta}_0 - \bar{\theta}_0)\dot{\hat{\theta}}_0 \\ & + \frac{e_0}{c_1}(\hat{\theta}_1 - \bar{\theta}_1)\dot{\hat{\theta}}_1 + \frac{e_0}{c_2}(\hat{\theta}_2 - \bar{\theta}_2)\dot{\hat{\theta}}_2 - \frac{e_0}{c_3\hat{\theta}_3^2}(\hat{\theta}_3^{-1} - \bar{\theta}_1^{-1})\dot{\hat{\theta}}_3. \end{aligned} \quad (6.61)$$

To prove the stability of the closed-loop system with saturated adaptive finite-time FTC law, two cases need to be considered based on the defined projection operator.

Case I: $\hat{\theta}_3 > 1$ or $\hat{\theta}_3 = 1$ and $\bullet \geq 0$. In this case, it can be found from equation

(6.57d) that

$$\dot{\hat{\theta}}_3 = c_3 \zeta \hat{\theta}_3^3 \left[\left(\hat{\theta}_1 \|\mathbf{D}\| + \hat{\theta}_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| - \delta_3 \hat{\theta}_3 \right] \quad (6.62)$$

Then, according to the adaptive laws defined in equations (6.57a)-(6.57c) and equation (6.62), it follows from expression (6.61) that

$$\begin{aligned} \dot{V}_3 &\leq -k\|\mathbf{s}\| + (\theta_0 - \bar{\theta}_0)\Phi\|\mathbf{s}\| + f_0\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| + (1 - e_0\kappa)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| \\ &\quad - e_0\kappa\zeta\hat{\theta}_3 \left(\hat{\theta}_1\|\mathbf{D}\| + \hat{\theta}_2\|\mathbf{P}\|\|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| + (\hat{\theta}_0 - \bar{\theta}_0)\Phi\|\mathbf{s}\| \\ &\quad + e_0(\hat{\theta}_1 - \bar{\theta}_1)\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| + e_0(\hat{\theta}_2 - \bar{\theta}_2)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| \\ &\quad - \delta_0(\hat{\theta}_0 - \bar{\theta}_0)\hat{\theta}_0 - e_0\delta_1(\hat{\theta}_1 - \bar{\theta}_1)\hat{\theta}_1 - e_0\delta_2(\hat{\theta}_2 - \bar{\theta}_2)\hat{\theta}_2 \\ &\quad - e_0\zeta(1 - \bar{\theta}_3^{-1}\hat{\theta}_3) \left(\hat{\theta}_1\|\mathbf{D}\| + \hat{\theta}_2\|\mathbf{P}\|\|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| + e_0\zeta\delta_3(1 - \bar{\theta}_3^{-1}\hat{\theta}_3)\hat{\theta}_3 \\ &\leq -k\|\mathbf{s}\| + (\theta_0 - \bar{\theta}_0)\Phi\|\mathbf{s}\| + (f_0 - e_0\bar{\theta}_1)\|\mathbf{D}\|\|\mathbf{P}\mathbf{s}\| \\ &\quad + (1 - e_0\kappa - e_0\bar{\theta}_2)\|\mathbf{P}\|\|\mathbf{u}_{nom}\|\|\mathbf{P}\mathbf{s}\| \\ &\quad + (\bar{\theta}_3^{-1} - \kappa)e_0\zeta\hat{\theta}_3 \left(\hat{\theta}_1\|\mathbf{D}\| + \hat{\theta}_2\|\mathbf{P}\|\|\mathbf{u}_{nom}\| \right) \|\mathbf{P}\mathbf{s}\| \\ &\quad - \delta_0(\hat{\theta}_0 - \bar{\theta}_0)\hat{\theta}_0 - e_0\delta_1(\hat{\theta}_1 - \bar{\theta}_1)\hat{\theta}_1 - e_0\delta_2(\hat{\theta}_2 - \bar{\theta}_2)\hat{\theta}_2 - e_0\zeta\delta_3(\bar{\theta}_3^{-1}\hat{\theta}_3 - 1)\hat{\theta}_3. \end{aligned}$$

Furthermore, based on the result from Lemma 6.3, we have

$$\dot{V}_2 \leq -k\|\mathbf{s}\| - \delta_0(\hat{\theta}_0 - \bar{\theta}_0)\hat{\theta}_0 - e_0\delta_1(\hat{\theta}_1 - \bar{\theta}_1)\hat{\theta}_1 - e_0\delta_2(\hat{\theta}_2 - \bar{\theta}_2)\hat{\theta}_2 - e_0\zeta\delta_3(\bar{\theta}_3^{-1}\hat{\theta}_3 - 1)\hat{\theta}_3$$

Here, it should be noted that $\hat{\theta}_1(t) \geq 0$ and $\hat{\theta}_2(t) \geq 0$. Considering the following inequalities:

$$-(\hat{\theta}_i - \bar{\theta}_i)\hat{\theta}_i \leq -\left(\hat{\theta}_i - \frac{\bar{\theta}_i}{2}\right)^2 + \frac{\bar{\theta}_i^2}{4}, \quad (6.63)$$

$$-(\bar{\theta}_3^{-1}\hat{\theta}_3 - 1)\hat{\theta}_3 \leq -\bar{\theta}_3^{-1} \left[\left(\hat{\theta}_3 - \frac{\bar{\theta}_3}{2}\right)^2 + \frac{\bar{\theta}_3^2}{4} \right], \quad (6.64)$$

the Lyapunov function candidate can be further written as

$$\begin{aligned}\dot{V}_3 \leq & -k\|\mathbf{s}\| - \delta_0 \left| \hat{\theta}_0 - \bar{\theta}_0 \right| - e_0 \delta_1 \left| \hat{\theta}_1 - \bar{\theta}_1 \right| - e_0 \delta_2 \left| \hat{\theta}_2 - \bar{\theta}_2 \right| \\ & - e_0 \zeta \delta_3 \left| \hat{\theta}_3^{-1} - \bar{\theta}_3^{-1} \right| + \delta_0 \left(\left| \hat{\theta}_0 - \bar{\theta}_0 \right| + \frac{\bar{\theta}_0^2}{4} \right) + e_0 \delta_1 \left(\left| \hat{\theta}_1 - \bar{\theta}_1 \right| + \frac{\bar{\theta}_1^2}{4} \right) \\ & + e_0 \delta_2 \left(\left| \hat{\theta}_2 - \bar{\theta}_2 \right| + \frac{\bar{\theta}_2^2}{4} \right) + e_0 \zeta \delta_3 \left(\left| \hat{\theta}_3^{-1} - \bar{\theta}_3^{-1} \right| + \frac{\bar{\theta}_3}{4} \right).\end{aligned}\quad (6.65)$$

Invoking the fact $\hat{\theta}_3 \geq 1$ and Lemma 6.3, the following inequalities are held:

$$\begin{aligned}\dot{V}_3 \leq & -\min \left\{ k \left(\frac{2}{\bar{J}} \right)^{\frac{1}{2}}, (2c_0 \delta_0^2)^{\frac{1}{2}}, (2c_1 e_0 \delta_1^2)^{\frac{1}{2}}, (2c_2 e_0 \delta_2^2)^{\frac{1}{2}}, (2c_3 e_0 \zeta^2 \delta_3^2)^{\frac{1}{2}} \right\} \\ & \times \left\{ \left(\sum_{i=1}^3 \frac{\bar{J}}{2} s_i^2 \right)^{\frac{1}{2}} + \left[\frac{1}{2c_0} (\hat{\theta}_0 - \bar{\theta}_0)^2 \right]^{\frac{1}{2}} + \left[\frac{e_0}{2c_1} (\hat{\theta}_1 - \bar{\theta}_1)^2 \right]^{\frac{1}{2}} + \left[\frac{e_0}{2c_2} (\hat{\theta}_2 - \bar{\theta}_2)^2 \right]^{\frac{1}{2}} \right. \\ & \left. + \left[\frac{e_0}{2c_3} (\hat{\theta}_3^{-1} - \bar{\theta}_3^{-1})^2 \right]^{\frac{1}{2}} \right\} + \delta_0 \left(\bar{\theta}_0 + \frac{\bar{\theta}_0^2}{4} \right) + e_0 \delta_1 \left(\bar{\theta}_1 + \frac{\bar{\theta}_1^2}{4} \right) \\ & + e_0 \delta_2 \left(\bar{\theta}_2 + \frac{\bar{\theta}_2^2}{4} \right) + e_0 \zeta \delta_3 \left(1 + \frac{\bar{\theta}_3}{4} \right).\end{aligned}\quad (6.66)$$

Then, based on Lemma 2.1, the time derivative of the candidate Lyapunov function becomes

$$\dot{V}_3 \leq -\eta_1 V_3^{\frac{1}{2}} + \varphi_1, \quad (6.67)$$

where η_1 and φ_1 are defined as $\eta_1 = \min \left\{ k \left(\frac{2}{\bar{J}} \right)^{\frac{1}{2}}, (2c_0 \delta_0^2)^{\frac{1}{2}}, (2c_1 e_0 \delta_1^2)^{\frac{1}{2}}, (2c_2 e_0 \delta_2^2)^{\frac{1}{2}}, (2c_3 e_0 \zeta^2 \delta_3^2)^{\frac{1}{2}} \right\}$, and $\varphi_1 = \delta_0 \left(\bar{\theta}_0 + \frac{\bar{\theta}_0^2}{4} \right) + e_0 \delta_1 \left(\bar{\theta}_1 + \frac{\bar{\theta}_1^2}{4} \right) + e_0 \delta_2 \left(\bar{\theta}_2 + \frac{\bar{\theta}_2^2}{4} \right) + e_0 \zeta \delta_3 \left(1 + \frac{\bar{\theta}_3}{4} \right)$.

Comparing the inequalities (6.67) with (6.42), it is clear that two inequalities have a similar structure. Following the same lines as in the proof of Theorem 6.2, it can be shown that the sliding mode manifold will converge to the neighborhood of $\mathbf{s} = 0$ as

$$\|\mathbf{s}\| \leq \Delta_{sat1}, \quad (6.68)$$

in finite time, where $\Delta_{sat1} = \sqrt{\frac{2}{J} \frac{\varphi_1}{\eta_1}}$. Furthermore, the spacecraft attitude q_j and angular velocity ω_j will converge to the regions

$$|q_j| \leq \min \left\{ \frac{\Delta_{sat1}}{\alpha}, \left(\frac{\Delta_{sat1}}{\beta} \right)^{\frac{1}{r}} \right\}, \quad (6.69)$$

$$|\omega_j| \leq 6\sqrt{3}\Delta_{sat1}, \quad (6.70)$$

in finite time, respectively.

Case II: $\hat{\theta}_3 = 1$ and $\bullet < 0$. In this case, there does not exist input saturation and $\hat{\theta}_3 = 1$. Substituting $\hat{\theta}_3 = 1$ into the control law (6.55), the saturated adaptive finite-time FTC law is obtained as

$$\mathbf{u}_c = -\mathbf{D}^T \mathbf{P} \left[\mathbf{u}_{nom} + \zeta (\hat{\gamma}_1 \|\mathbf{D}\| + \hat{\gamma}_2 \|\mathbf{P}\| \|\mathbf{u}_{nom}\|) \frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|} \right],$$

which is similar to the unsaturated adaptive finite-time FTC law as given in equation (6.31) except for the gain constant ζ . Following the same lines as in the proof of Theorem 6.2, it can also be shown that the sliding variable s , the spacecraft attitude q_j , and angular velocity ω_j ($j = 1, 2, 3$) will converge to the regions

$$\|\mathbf{s}\| \leq \Delta_{sat2}, \quad (6.71)$$

$$|q_j| \leq \min \left\{ \frac{\Delta_{sat2}}{\alpha}, \left(\frac{\Delta_{sat2}}{\beta} \right)^{\frac{1}{r}} \right\}, \quad (6.72)$$

$$|\omega_j| \leq 6\sqrt{3}\Delta_{sat2}, \quad (6.73)$$

in finite time, where $\Delta_{sat2} = \sqrt{\frac{2}{J} \frac{\varphi_2}{\eta_2}}$ with $\eta_2 = \min \left\{ k \left(\frac{2}{J} \right)^{\frac{1}{2}}, (2c_0\delta_0^2)^{\frac{1}{2}}, (2c_1e_0\delta_1^2)^{\frac{1}{2}}, (2c_2e_0\delta_2^2)^{\frac{1}{2}} \right\}$, and $\varphi_2 = \delta_0 \left(\bar{\theta}_0 + \frac{\bar{\theta}_0^2}{4} \right) + e_0\delta_1 \left(\bar{\theta}_1 + \frac{\bar{\theta}_1^2}{4} \right) + e_0\delta_2 \left(\bar{\theta}_2 + \frac{\bar{\theta}_2^2}{4} \right)$.

Finally, summarizing Cases 1 and 2, it can be concluded that the sliding variable $\mathbf{s}$, the spacecraft attitude q_j , and angular velocity ω_j ($j = 1, 2, 3$) will converge to

the regions

$$\|\mathbf{s}\| \leq \Delta_{sat}, \quad (6.74)$$

$$|q_j| \leq \min \left\{ \frac{\Delta_{sat}}{\alpha}, \left(\frac{\Delta_{sat}}{\beta} \right)^{\frac{1}{r}} \right\}, \quad (6.75)$$

$$|\omega_j| \leq 6\sqrt{3}\Delta_{sat}, \quad (6.76)$$

in finite time, where $\Delta_{sat} = \max\{\Delta_{sat1}, \Delta_{sat2}\} = \sqrt{\frac{2}{J}\frac{\varphi}{\eta}}$ with $\eta = \min\{\eta_1, \eta_2\} = \min \left\{ k \left(\frac{2}{J} \right)^{\frac{1}{2}}, (2c_0\delta_0^2)^{\frac{1}{2}}, (2c_1e_0\delta_1^2)^{\frac{1}{2}}, (2c_2e_0\delta_2^2)^{\frac{1}{2}}, (2c_3e_0\zeta^2\delta_3^2)^{\frac{1}{2}} \right\}$, and $\varphi = \max\{\varphi_1, \varphi_2\} = \delta_0 \left(\bar{\theta}_0 + \frac{\bar{\theta}_0^2}{4} \right) + e_0\delta_1 \left(\bar{\theta}_1 + \frac{\bar{\theta}_1^2}{4} \right) + e_0\delta_2 \left(\bar{\theta}_2 + \frac{\bar{\theta}_2^2}{4} \right) + e_0\zeta\delta_3 \left(1 + \frac{\bar{\theta}_3}{4} \right)$. This completes the proof. $\square$

Remark 6.7. *The saturated adaptive fault-tolerant controller in Theorem 6.3 is also applicable when the actuator is fault-free but with actuator saturation constraint. Since the fault-free case ($e_i(t) = 1$ and $\bar{u}_i(t) = 0$) is also included in the controller design of Theorem 6.3, by following the same procedure outlined in the proof of Theorem 6.3, it is guaranteed that attitude and angular velocity converge to a neighbourhood of the origin in finite time with explicit consideration of the actuator saturation no matter actuator faults/failures occur or not.*

Remark 6.8. *The proposed control law is discontinuous due to the functions $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|}$ and $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|^2}$, which may lead to undesirable phenomenon of chattering in the sliding mode. This problem can be alleviated by replacing the discontinuous functions $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|}$ and $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|^2}$ by continuous functions $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|+\xi}$ and $\frac{\mathbf{s}}{\|\mathbf{P}\mathbf{s}\|^2+\xi}$, respectively, where ξ is a small positive scalar [136].*

6.4 Simulation Results

To demonstrate the effectiveness and performance of the proposed controller, simulation results are presented in this section. Consider the spacecraft model given

in equation (6.1) with four reaction wheels in a pyramid configuration with the actuator distribution matrix $\mathbf{D}$ given as [75]

$$\mathbf{D} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ \sqrt{2} & \sqrt{2} & \sqrt{2} & \sqrt{2} \end{bmatrix},$$

and the moment inertia of each reaction wheel is $\mathbf{J}_w = 0.015\mathbf{I}_4 \text{ kg}\cdot\text{m}^2$.

The nominal inertia matrix of the spacecraft is $\mathbf{J}_0 = \text{diag}([140, 120, 130]) \text{ kg}\cdot\text{m}^2$. Due to energy consumption and onboard payload deployment, the uncertainty in the inertia matrix $\Delta\mathbf{J}$ is depicted in Fig. 6.1(a) [74]. The initial attitude orientation is chosen as $\mathbf{q}(0) = [0.3, -0.3, 0.2]^T$ and $q_0(0) = \sqrt{1 - \mathbf{q}^T \mathbf{q}}$. The initial angular velocity of the spacecraft is $\boldsymbol{\omega}(0) = [0, 0, 0]^T \text{ rad/s}$. The initial angular velocity of reaction wheel is set as $\boldsymbol{\Omega}(0) = [50, 50, 50, 50]^T \text{ rad/s}$. The external disturbance model is in the form of $\mathbf{d}(t) = 0.005 \times [\sin(0.8t), \cos(0.5t), \cos(0.3t)]^T \text{ N}\cdot\text{m}$. The parameters in FTSM manifold defined in equation (6.15) are chosen as $\alpha = 0.05$, $\beta = 0.01$, $r_1 = 3$, and $r_2 = 5$. In order to accomplish critical mission, such as ground station tracking, the spacecraft must provide high attitude pointing accuracy and stability of the pointing such that $|q_j| \leq 1.0 \times 10^{-4}$ and $|\omega_j| \leq 5.0 \times 10^{-5}$, $j = 1, 2, 3$.

6.4.1 Basic Finite-Time FTC Scheme

In this section, the actuator fault scenario is described in Fig. 6.1(b). That is, the first reaction wheel experiences an increased bias torque $\mathbf{F2}$ (bias fault) with a negative additive bias torque $\bar{u}_1 = -(0.06 \sin(0.8t) + 0.04) \text{ N}\cdot\text{m}$ after 50 s and the second reaction wheel decreases 50% of its normal actuation torque $\mathbf{F1}$ (partial loss of effectiveness fault) after 5 s. The third reaction wheel loses 60% of its effectiveness in the time interval from 5 s to 60 s, and further experiences an additive bias fault with a negative bias torque $\bar{u}_3 = -(0.02 \sin(0.3t) + 0.02) \text{ N}\cdot\text{m}$ after 60 s. The fourth

reaction wheel fails to respond to control signals **F3** (float) after 10 s. The model of bias fault can be found in [137].

Figure 6.1: Spacecraft inertia uncertainty and actuator fault scenario.

For the purpose of comparison, the proposed basic finite-time fault-tolerant control (BFT-FTC) scheme in equation (6.23) and the traditional proportional-derivative control (PDC) scheme [138] are compared under the aforementioned reaction wheel faults. For the BFT-FTC scheme, the control parameters are chosen as $k = 0.01$, $\rho = 0.6$, $\gamma_0 = 0.01$, $\varepsilon = 0.1$, and $\xi = 0.0015$. To make a fair comparison of the control performances (e.g., strong robustness and convergence rate), the magnitude of control torques under two control schemes are limited to the same level. The performances of attitude quaternion, angular velocity as well as command control inputs using the BFT-FTC scheme and the PDC method are presented in Figs. 6.2-6.4, respectively. From Figs. 6.2(a) and 6.3(a), the attitude control requirements for the mission are met by using the BFT-FTC scheme despite external disturbances, inertia uncertainties, and actuator faults. In contrast, the attitude control system with PDC scheme cannot accomplish the mission due to the reaction wheel faults/failures. The control performance comparison of the BFT-FTC scheme and the PDC scheme is shown in Table 6.1 in terms of settling time and steady-state precision. As shown in Table 6.1, the BFT-FTC scheme performs significantly bet-

ter than the PDC scheme with higher steady precision and shorter settling time in both attitude and angular velocity response. The commanded reaction wheel control torque under the two schemes are depicted in Fig. 6.4. Fig. 6.5(a) shows the sliding manifold response which indicates that the sliding function s converges to a small neighborhood of origin in finite time, roughly 3.5 seconds. The four reaction wheel speeds under the BFT-FTC scheme are shown in Fig. 6.5(b). These simulation results show that not only good attitude stabilization performance but also fault tolerance capability can be obtained with the proposed BFT-FTC scheme.

Figure 6.2: Spacecraft attitude response under BFT-FTC scheme and PDC scheme.

Figure 6.3: Spacecraft angular velocity response under BFT-FTC scheme and PDC scheme.

Figure 6.4: Commanded control torque under BFT-FTC scheme and PDC scheme.

Figure 6.5: Time response of sliding function and reaction wheel speed under BFT-FTC scheme.

Table 6.1: Comparison of control performance between BFT-FTC scheme and PDC scheme

Controller	Control performance			
	Settling time of $\mathbf{q}$ (s)	Steady preci- sion of $\mathbf{q}$	Settling time of $\boldsymbol{\omega}$ (s)	Steady preci- sion of $\boldsymbol{\omega}$
BFT-FTC	68	5.0×10^{-5}	70	4.5×10^{-5}
PDC	120	1.5×10^{-2}	135	8.0×10^{-4}

6.4.2 Adaptive Finite-Time FTC Scheme

To illustrate the effectiveness of the proposed adaptive finite-time fault-tolerant control (AFT-FTC) scheme defined in (6.31), the attitude stabilization problem under actuator faults/failures in the absence of a priori knowledge of system parameters is simulated in this section. For comparison purpose, the indirect robust adaptive fault-tolerant control (IRA-FTC) method presented in [74] is also simulated. In the simulation, the reaction wheels are assumed to experience the same fault scenario as described in previous section. All the controller parameters of AFT-FTC scheme in equation (6.31) are identical to those used in the BTF-FTC scheme in previous section, and the corresponding parameters of the adaptation law defined in (6.33a-6.33c) are chosen with $c_0 = 0.1$, $c_1 = 1$, $c_2 = 1$, $\delta_0 = 0.02$, $\delta_1 = 0.002$, and $\delta_2 = 0.001$. The initial values of $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are selected as $\hat{\gamma}_0(0) = 0.005$, $\hat{\gamma}_1(0) = 0.1$, and $\hat{\gamma}_2(0) = 0.1$, respectively. To smooth the proposed fault-tolerant controller, the parameter ξ in Remark 6.8 is chosen as 0.0002. Similarly, in order to properly evaluate the performance and allow fair comparison between the AFT-FTC and IRA-FTC scheme, the magnitude of control torques under two controllers are limited to the same level.

The attitude quaternion and spacecraft angular velocity are shown in Figs. 6.6 and 6.7. The results show that both the AFT-FTC scheme and the IRA-FTC scheme can stabilize the attitude quaternion and angular velocity to a neighborhood of zero in spite of inertia uncertainties, time-varying disturbances, and reaction wheel faults. As shown by the steady-state behavior in Figs. 6.6 and 6.7, the attitude pointing accuracy and the stabilization accuracy under the AFT-FTC scheme are 4.0×10^{-5} and 3.5×10^{-5} , respectively. Thus, the mission can be successfully accomplished by the AFT-FTC scheme. In contrast, the IRA-FTC scheme cannot meet the attitude control requirements for the mission when the control torques are at the same level as under the AFT-FTC scheme. The control performance

comparison of the BFT-FTC scheme and the IRA-FTC scheme is shown in Table 6.2. By comparing the performance of the two schemes, it is clear that the AFT-FTC can achieve better attitude pointing accuracy and faster convergence rate than that of the IRA-FTC scheme. The commanded reaction wheel control torques of the two schemes are shown in Fig. 6.8. Fig. 6.9(a) depicts the time history of sliding function under the AFT-FTC scheme, where finite-time convergence to the neighborhood of origin is demonstrated. Referring to Fig. 6.9(b), it is observed that the adaptive parameters $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ are bounded, and thus the efficacy of the proposed adaptation laws in (6.33a)-(6.33c) are verified. The four reaction wheel speeds under AFT-FTC scheme are given in Fig. 6.10. These simulation results show that fairly good control performance is achieved by the proposed AFT-FTC approach even under such severe reaction wheel faults and unknown uncertainty bounds which are required to be known in the BFT-FTC scheme.

Figure 6.6: Spacecraft attitude under AFT-FTC scheme and IRA-FTC scheme.

6.4.3 Adaptive Finite-Time FTC with Actuator Saturation

In this subsection, besides reaction wheel faults/failures, control input saturation is also simulated. For comparison study, the saturated version of IRA-FTC (SIRA-FTC) scheme in [74] and the AFT-FTC scheme with actuator saturation (SAFT-

Figure 6.7: Spacecraft angular velocity response under AFT-FTC scheme and IRA-FTC scheme.

Figure 6.8: Commanded control torque under AFT-FTC scheme and IRA-FTC scheme.

Table 6.2: Comparison of control performance between AFT-FTC scheme and IRA-FTC scheme

Controller	Control performance			
	Settling time of q (s)	Steady precision of q	Settling time of ω (s)	Steady precision of ω
AFT-FTC	73	4.0×10^{-5}	72	3.5×10^{-5}
IRA-FTC	128	8.0×10^{-4}	138	1.5×10^{-4}

Figure 6.9: Time response of sliding function and adaptive parameters $\hat{\gamma}_0(t)$, $\hat{\gamma}_1(t)$, and $\hat{\gamma}_2(t)$ under AFT-FTC scheme.

FTC) designed in equation (6.55) are investigated. The maximum torque of each reaction wheel for both two control schemes is constrained to be 0.2 N·m, i.e., $u_{max} = 0.2$ N·m. The reaction wheel fault scenario is the same as presented in previous section except the bias faults are changed to be $\bar{u}_1 = -(0.012 \sin(0.8t) + 0.008)$ N·m for the first reaction wheel and $\bar{u}_3 = -(0.004 \sin(0.8t) + 0.004)$ N·m for the third one, respectively.

Figure 6.10: Time response of reaction wheel speed under AFT-FTC scheme.

The control gains for the SAFT-FTC scheme are chosen as $\zeta = 1.5$, $c_0 = 0.1$, $c_1 = 1$, $c_2 = 1$, $c_3 = 0.1$, $\delta_0 = 0.02$, $\delta_1 = 0.001$, $\delta_2 = 0.001$, and $\delta_3 = 0.01$. The initial values of $\hat{\theta}_0$, $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$ are chosen to be $\hat{\theta}_0(0) = 0.005$, $\hat{\theta}_1(0) = 0.1$, $\hat{\theta}_2(0) = 0.1$,

and $\hat{\theta}_3(0) = 1$, respectively. In order to smooth the proposed SAFT-FTC scheme, the parameter ξ in Remark 6.8 is chosen as 0.004. The other design parameters remain unchanged from those used in the previous AFT-FTC scheme.

From Figs. 6.11(a) and 6.12(a), it can be observed that the attitude and the angular velocity under the SAFT-FTC scheme converge to the neighborhood of zero in finite time despite actuator faults and control input saturation limits, and that satisfactory stabilization error and a good convergency can be achieved. However, due to the effects of control input saturation, the convergence rate of attitude in Fig. 6.11(a) is slower than the situation without input saturation as shown in Fig. 6.6(a). This phenomena is often referred to as graceful degradation in performance [34]. As shown in Fig. 6.13(a), it is clearly seen that the commanded control input is always within the maximum magnitude of the reaction wheel. Figs. 6.11(b), 6.12(b), and 6.13(b) show attitude quaternion, angular velocity, and command control input when the SIRA-FTC scheme is applied. The control performance comparison of the SAFT-FTC scheme and the SIRA-FTC scheme is shown in Table 6.3. Although the SIRA-FTC scheme can stabilize the spacecraft attitude to a certain level, the attitude pointing accuracy is lower than that of the SAFT-FTC scheme and doesn't meet the mission performance requirements. In addition, similar to the previous simulation case, the proposed SAFT-FTC scheme can provide faster convergence rate than the SIRA-FTC scheme. The bounded adaptive parameters $\hat{\theta}_0$, $\hat{\theta}_1$, $\hat{\theta}_2$, and $\hat{\theta}_3$ are shown in Figs. 6.14(a) and 6.14(b). From Fig. 6.15(a), it can be observed that the sliding manifold s converges to the neighbourhood of $s = 0$ after 56 seconds. The four reaction wheel speeds under SAFT-FTC scheme are shown in Fig. 6.15(b). Moreover, referring to Fig. 6.13, it is interesting to see that actuator saturation constraints can be satisfied by both fault-tolerant controllers. However, the allowed control torque ($u_{max} = 0.2$ N·m) under the SIRA-FTC scheme is not fully used, which can be observed from Fig. 6.13(b), where the actual maximal

control torque is only about 0.15 N·m. Hence, the SIRA-FTC method in [74] is much more conservative than the proposed SAFT-FTC scheme.

Figure 6.11: Spacecraft attitude under SAFT-FTC scheme and SIRA-FTC scheme.

Figure 6.12: Spacecraft angular velocity response under SAFT-FTC scheme and SIRA-FTC scheme.

6.5 Summary

In this chapter, adaptive finite-time FTC scheme has been investigated for rigid spacecraft subject to external disturbances, uncertainties in inertia matrix, four types of actuator faults/failures as well as actuator saturation. Firstly, based on the

Figure 6.13: Commanded control torque under SAFT-FTC scheme and SIRA-FTC scheme.

(a) $\hat{\theta}_0(t)$ and $\hat{\theta}_1(t)$ under SAFT-FTC scheme (b) $\hat{\theta}_2(t)$ and $\hat{\theta}_3(t)$ under SAFT-FTC scheme

Figure 6.14: Time response of adaptive parameters $\hat{\theta}_0(t)$, $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, and $\hat{\theta}_3(t)$ under SAFT-FTC scheme.

Table 6.3: Comparison of control performance between SAFT-FTC scheme and SIRA-FTC scheme

Controller	Control performance			
	Settling time of $\mathbf{q}$ (s)	Steady preci- sion of $\mathbf{q}$	Settling time of $\boldsymbol{\omega}$ (s)	Steady preci- sion of $\boldsymbol{\omega}$
SAFT-FTC	82	3.5×10^{-5}	83	4.0×10^{-5}
SIRA-FTC	183	4.0×10^{-4}	160	4.0×10^{-5}

Figure 6.15: Time response of sliding function and reaction wheel speed under SAFT-FTC scheme.

terminal sliding mode theory, a basic finite-time fault-tolerant controller has been proposed. Then, incorporating the adaptive strategy, an adaptive finite-time fault-tolerant controller is developed. The proposed adaptive controller is effective against actuator faults/failures and also guarantees finite-time convergence without requiring any a priori knowledge of fault and uncertainty information. Furthermore, the adaptive fault-tolerant controller is enhanced to address control input constraints. The Lyapunov direct approach is employed to prove the local stability of the closed-loop system and to show the finite-time convergence of the system states. Finally, simulation studies have been presented to illustrate the effectiveness of the proposed finite-time FTC schemes.

Chapter 7

Conclusion and Recommendations

7.1 Conclusion

In the field of aerospace engineering, high precision control of a spacecrafts orientation is often crucial to mission success, especially for those carrying out earth-based observations, surveillance, or rendezvous applications. However, actuators used for attitude control are potentially subjected to unexpected anomalies and failures during the attitude maneuver mission, which may give rise to severe performance degradation or even totally loss of the spacecraft. Therefore, actuator fault-tolerant capacity should be addressed in attitude control systems to enhance the reliability and safety of the spacecraft.

This thesis is devoted to developing FTC techniques to handle actuator faults/failures whilst maintaining the desirable degree of overall stability and performance despite external disturbances and modeling uncertainties. Towards this objective, both active and passive FTC schemes are presented for spacecraft attitude control systems in this thesis. Some conclusions of this thesis are summarized as follows:

1. CA-based active FTC scheme is proposed for attitude tracking problem. The estimated effectiveness level of actuator is used in the on-line CA for redis-

tributing the virtual control efforts into each individual actuator judiciously without reconfiguring the controller. With consideration of imprecision in fault estimation, indirect robust adaptive control is incorporated with CA to deal with loss of effective faults/failures and ensure robustness with respect to external disturbances, inertia uncertainties, and imprecise fault estimation. It is worth mention that a time-varying dead-zone modification is employed in the design of parameter adaptation to stop the adaptive gain from increasing, which also improves the robustness of the adaptive law to measurement or system noises.

2. The results of the CA-based active FTC is further extended to the situation that imprecise fault estimation in both actuator effectiveness and additive fault are considered. Instead of ensuring robustness with respect to imprecise fault estimation in high-level controller design, a novel RobCA is proposed to ensure some robustness in control allocation design such that design complexity of high-level controller can be reduced. To guarantee the uniqueness of the solution of RobCA, an optimal index of minimizing the energy consumption is introduced. The RobCA problem is formulated as a min-max optimization problem, and further converted to a one-dimensional search problem to reduce computation burden. Moreover, actuator amplitude and rate constraints are also taken into account in RobCA design.
3. Two ISMC-based passive FTC schemes are presented for attitude stabilization problem. Two kinds of actuator faults, including partial loss of effectiveness fault and additive bias fault, are compensated by the proposed schemes without controller reconfiguration. The first basic FTC approach maintains the nominal closed-loop performance from the very beginning of the process. Then, the basic controller is further enhanced by combining with adaptive

mechanism to ensure that the required information on the boundaries of disturbances and faults can be relaxed except for their existence and a reasonable adaptive gain is obtained.

4. Finite-time convergence of fault-tolerant attitude stabilization problem is investigated for a rigid spacecraft in the presence of external disturbances, uncertainties in inertia matrix, four types of actuator faults/failures, and actuator saturation constraints. Based on FTSM technique, the proposed attitude controller provides fault-tolerant capability despite control input saturation, and also ensures that attitude and angular velocity converge to a neighbourhood of the origin in finite time. Since actuator redundancy is considered, not only actuator faults but also actuator failures can be accommodated by the proposed controller.

7.2 Recommendations for Future Research

Figure 7.1: Graphical illustration of the hybrid fault-tolerant attitude control system.

Some future research works and possible extensions are identified as follows:

1. Attitude constraints are another practical issue may be addressed in spacecraft fault-tolerant attitude control. For example, some sensitive payloads, such as optical instruments, infrared telescopes, star tracker, and low energy ion composition analyzers, are required to point to a certain direction or track a desired trajectory while keeping away from direct exposure to the sunlight or other bright objects. For such missions, the attitude control system of the spacecraft has the additional task of applying the necessary control torque on the spacecraft such that the spacecraft attitude remains in particular attitude-constrained zones (defined by the minimum avoidance angles) emanating from these sensitive optical devices exclude the bright or heat generating objects during large angle attitude maneuvers. This problem is similar to obstacle avoidance problem, and potential function can be used to model these attitude constraints. In addition, the fault-tolerant attitude control problem in the presence of attitude-constrained zones should also be examined in the research work.
2. Attitude coordinate control for spacecraft formation flying (SFF) has received considerable attention in recent years. The existing attitude synchronization literature addresses several important issues for attitude control of multiple spacecraft, such as robustness to modeling uncertainties, external disturbances attenuation, time delay in the communication channel, and actuator saturation constraint accommodation. However, the problem of actuator failure/fault tolerance is not considered in these works. Although some results have been proposed in [79,139] recently, these fault-tolerant controllers belong to passive FTC scheme, and they have limited fault-tolerant capability. Active FTC design for spacecraft formation flying is not available as far as author's knowledge in the field.

3. In most of the existing active FTC methods, the stability of the post-fault system cannot be guaranteed during the period that the FDD unit is performing its diagnostic functions. In certain practical situation, there is only limited amount of time available for FDD and for the control systems reconfiguration to react to the faults. If FDD process takes an unexpectedly long time, the system may enter an unrecoverable state or even be unstable. In order to overcome this limitation of active FTC approach, a hybrid FTC scheme may be developed by combining the merits of active and passive FTC approach. The general structure of the hybrid FTC scheme is shown in Fig. 7.1. When a fault is detected, but not yet completely diagnosed, the performance deterioration of the closed-loop system should be slowed down through a passive control system. After the fault diagnosis is completed, the improved system performance can then be achieved through an appropriate reconfigurable controller. In fact, such hybrid FTC control scheme has been proposed in [140]. However, this hybrid scheme is only carried out in the frame work of aircraft flight control and further results for dealing with the nonlinear systems like spacecraft attitude control systems are not yet available. Moreover, there are still some other problems need to be solved, such as switching transients among different controllers and integration design of FDD and reconfigurable control.
4. Existing fault-tolerant schemes for spacecraft attitude control are based on the assumption that the attitude control systems is at least controllable although there exist actuator faults/failures. That is, no more than $n - 3$ (n is the number of actuators and $n > 3$) experience actuator failures. Otherwise, the attitude control systems become underactuated. In this situation, three-axis stabilization of an underactuated spacecraft is a challenging problem and the spacecraft dynamics cannot be asymptotically stabilized to any equilibrium using a time invariant continuous feedback control law [141].

5. Experimental study including designing testbed and testing the proposed fault-tolerant control approach for the spacecraft is also important from an application point of view. However, experimental test and further practical research and development have been restricted due to safety concern and cost constraints [142]. On the other hand, since quadrotor aircrafts provide a cheap and operative experimental testbed and their mathematical model of attitude dynamics is similar to that of the spacecraft, the proposed attitude FTC control algorithms may be tested and implemented on quadrotor aircrafts firstly to illustrate the effectiveness.

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