Nanostructures with quantized angular momentum in the strong light-matter coupling regime

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NANYANG
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**NANOSTRUCTURES WITH QUANTIZED
ANGULAR MOMENTUM IN THE STRONG
LIGHT-MATTER COUPLING REGIME**

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PhD Scientiarum degree in Theoretical Physics

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by

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ABSTRACT

A great deal of both theoretically and experimental investigation is currently being devoted into the regime of strong light-matter coupling in optically confining systems. In this strong coupling regime, bare matter particle states are heavily influenced by photon modes trapped within the system. The matter particles are said to become "dressed" in the optical field, picking up the properties of the photons therein. A large portion of this thesis is devoted to a type of such phenomena, the *exciton-polariton*, a quasiparticle which arises due to strong coupling between quantum well excitons and microcavity photons.

Exciton-polaritons are exciting candidates for a number of practical optoelectronic applications. Being spin ± 1 quasiparticles with high natural nonlinearities inherited from their excitonic part, and fast scattering dynamics from their photonic part, they open the possibility of a new era in spin-dependent devices with great speed and efficient signal processing. In terms of waveguide geometries, they can propagate coherently over hundreds of microns with small losses. This coherence can be sustained indefinitely as exciton-polaritons can form an analog of a driven-dissipative Bose-Einstein condensate, a macroscopic quantum fluid so to speak.

In this thesis we explore novel angular momenta effects, arising in such systems, through both numerical and analytical methods. In the case of exciton- and exciton-polariton Bose-Einstein condensates, unique types of quantum vortices appear due to the particle spin structure. These vortex states have quantized angular momentum and offer new possibilities in topologically robust elements in future applications. Here, the advantage of using exciton-polaritons comes from the fact that they can be easily controlled and monitored through the application of an optical field.

Angular phenomenon arising in quantum rings are also studied in the regime of light-matter coupling. Both electron- and exciton states become "field-dressed" in a strong, external, circularly polarized electromagnetic field. In quantum ring structures, the field-dressed particle states reveal the onset of an artificial $U(1)$ gauge associated with breaking of time-reversal symmetry, analogous to the well known Aharonov-Bohm effect.

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CHAPTER 1

INTRODUCTION

The study of light-matter interactions has both grown and spread into nearly every field of physics ever since the birth of quantum mechanics. It has had enormous effect on our daily lives and plays a role in many modern devices relying on processing and transfer of information. Specifically, in the field of quantum optics and condensed matter physics, scientist have striven to understand and explain the intricate action of the light quanta, better known as *photons*, on both single matter particles such as individual atoms, and also in the framework of particle ensembles such as the free electron gas. The most commonly known example of a light-matter device is the LASER (Light Amplification by Stimulated Emission of Radiation). A device which, when powered, produces a coherent source of light by utilizing the interaction of natural cavity-electromagnetic modes with the device optical media, a process better known as *stimulated emission*. Today, a field dedicated to the development and integration of optical and electronic devices, known as *optoelectronics*, is under intense research as such devices offer a path towards more efficient communication methods (optical fibre cables), signal processing, energy harvesting devices (solar cell industry), and many other future applications.

The introduction to this thesis will focus on a very special regime of light-matter interaction, namely the *strong light-matter coupling regime*. By ‘strong’ I do not mean that the photons in the system are necessarily high in energy or intensity. But rather that they are kept ‘alive’ long enough to interact with the same matter particle multiple times. It is in this regime that a new type of elementary excitation arises, a quasiparticle dubbed the *exciton-polariton* (henceforth *polaritons*), a coupled state between photons and quantum well excitons. A large part of this thesis is dedicated to this new light-matter particle (Chap. 3-4) in planar microcavity systems but the exciton by itself will also be separately addressed in Chap. 2. The latter part of the thesis is, on the other hand, dedicated to a specific light-matter coupling phenomenon where charged particles in ring-like structures exhibit non-equivalent behavior between clockwise and anti-clockwise propagation when strongly coupled to an external circularly polarized electromagnetic field (Chap. 5).

The introduction is organized as such to inform the reader on the most general aspects of light-matter coupling and the systems associated with it. The most

commonly used systems utilize two-dimensional optical confinement, i.e. planar microcavity systems. It is thus worth explaining how such systems of reduced dimensionality can affect the physical properties of the particles living in them. Most importantly, the arise of band structures in periodic lattices will be addressed with primary focus on semiconductor materials in Sec. 1.1.1. We then introduce the difference between fermionic and bosonic gases in Sec. 1.1.2-1.1.3 where the bosonic particle nature can lead to exciting quantum collective phenomenon such a superfluids, Bose-Einstein condensates, and superconductors. Due to the strong nonlinear nature of these systems, a plethora of phase transitions and topologically distinct solutions become possible. In Sec. 1.1.4 we will address such a type of topological solution, the *quantum vortex*. An irrotational quantity characterized by discrete integer values of angular momentum.

Moving onto Sec. 1.2.1, an elementary excitation arising in semiconductor materials (usually localized to quantum wells) classified as an *exciton* is introduced (also known as the *Wannier-Mott exciton*). The exciton is a bound pair of an electron and a hole in the conduction band and in the valance band respectively. Being of opposite charges, the attractive Coulomb interaction causes the electron and hole to form a bound state, where the electron-hole wavefunction overlap is associated with the exciton lifetime, which can be in the range of dozens of microseconds. We will focus our interest specifically to indirect-excitons, a long-living type of an exciton corresponding to electron and hole coupling across spatially different quantum wells. Sec. 1.2.2 will briefly address the current interest and challenges in achieving excitonic condensates. In Sec. 1.2.3 we will introduce the role of an external electromagnetic field to a system of optically receptive particles and address the nature of strong coupling between light and matter. The strong coupling regime is a playground of quantum electrodynamics (QED) giving rise to a vast variety of physical phenomenon picking up the properties of the photons. The full QED formalism is out of the scope of this thesis as we will only pay mind to the mean-field theory associated with the exciton- and polariton Bose-Einstein condensates. It will be the goal of Sec. 1.2.5 to introduce the coupling between excitons and photons giving rise to a renormalized spectrum associated with these new light-matter quasiparticles, followed by their spin formalism in Sec. 1.2.6. Furthermore, in Sec. 1.2.7 we will address the non-equilibrium nature of the polariton Bose-Einstein condensate when supported by an external driving field, allowing macroscopic coherent phenomenon to take place over hundreds of microns.

The last part of the introduction (Sec. 1.3) is dedicated to electron transport phenomenon in quantum rings and their fabrication techniques. A great deal of interest in fabricating smaller (nanoscale) quantum rings has lead to a clearer picture on phenomenon related to their non-single-connected nature (i.e., topological nature). The most well known such effect is the Aharonov-Bohm (AB) effect where

electrons traveling the quantum ring feel an incursion of phase when magnetic flux penetrates the ring, but the field itself is zero in the vicinity of the electrons. In this thesis, we will show that in the strong light-matter coupling regime one can call forward the same effect by using not a magnetic field but a circularly polarized electromagnetic field. It should be noted that this introductory part will only detail the magnetic AB theory in Sec. 1.3.1, with the new light-matter theory detailed in Chap. 5.

The organization of the thesis chapters is as follows: In Chap. 2 we investigate the possible vortex solutions arising in an planar equilibrium condensate of spinor indirect-excitons with spin projections $s_X = \pm 1, \pm 2$ along the system growth axis. The unique four-component condensate structure allows for several interesting vortex solutions to take place and even more so under the presence of spin-orbit coupling of Rashba- or Dresselhaus type. Chap. 3 is further devoted to vortex phenomenon but this time in a incoherently driven non-equilibrium system of exciton-polaritons where we neglect the spin degree of freedom. Using a well accepted method to model the generation and decay of polaritons through a reservoir of active excitons, one can utilize the self-trapping of polaritons with ring-shaped pump spots which will naturally exhibit the vortex state as a steady state solution. We further demonstrate the these vortex states can be manipulated via pump positioning in patterned potential landscape, allowing for information transfer and inversion. Chap. 4 presents recent experimental results demonstrating a unique whirl-shaped polarization pattern in the polarization emission of a condensate of polaritons. The patterns are directly linked to an effect known as the optical spin Hall effect which arises naturally in planar microcavities due to splitting of longitudinal and transverse optical modes in the cavity plane. The experimental results are then reproduced numerically using a set of coupled mean-field equations mimicking the dynamics of the polariton condensate. In the final chapter of the thesis, Chap. 5, results are presented on a peculiar strong-coupling effect between light and matter in quantum rings where the new light-dressed angular momentum states of the ring give rise to an analogue of the Aharonov-Bohm effect.

1.1 TWO-DIMENSIONAL SYSTEMS

Advances in modern nanotechnology and fabrication of mesoscopic systems of reduced dimensionality have proven to be an exciting playground of great physical interest. Today the most commonly known low dimensional systems are zero-dimensional (0D) quantum dots, one-dimensional (1D) quantum nanowires, and two-dimensional (2D) planar quantum wells. These systems are important building blocks in construction of future optoelectronic devices where the interactions of light and matter play an important role. Indeed, the optical properties of materials go hand-in-hand with their *density of states* which depends on the dimensionality of the structure. Considering a system of free non-interactive electrons with energy E ; for 1D systems one can write the density of states as $D(E) \propto E^{-1/2}$, in 2D materials one has $D(E) = \text{const.}$, and in 3D materials $D(E) \propto E^{1/2}$. The fact that one has finite density of states at the bottom of the band structure makes low dimensionality systems preferential for low power optoelectrical devices.

Starting with planar 2D systems, the transport property of particles living in the structure is modified by the sudden absence of the third axis (let's call it the z -axis whereas the plane itself will be characterized by the x and y coordinates). The motion of a free particle with mass m is now restricted only to the xy -plane and consequently its Hamiltonian is described by the 2D Laplacian:

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla_{\perp}^2 + V(x, y), \quad (1.1)$$

where $\hbar$ is the Planck's constant, $\nabla_{\perp}^2 = \partial_x^2 + \partial_y^2$ is the 2D Laplacian, and $V(x, y)$ is some static potential. In periodic lattices this potential is as well periodic due to the ordering of the atoms corresponding to the lattice unit cell.¹ Making use of the symmetry in lattice structures is highly advantageous in understanding the underlying physics. As an example, semiconductor materials (one of the most important sandboxes in condensed matter physics) are characterized by two main crystal structures, both possessing several such symmetries. The *diamond*, and *zinc-blende* lattice structures. As an example, the former is the structure of Si whereas the latter is for GaAs. One can often show what sort of physical processes (such as optical absorptions) are forbidden and allowed by looking at the symmetry alone in the lattice structure.

In metallic materials the density of states, and several other physical properties, can be understood through the free electron model (also known as the *Drude-Sommerfeld model*). Devised in principle by Paul Drude in 1900 and extended to

¹The lattice unit cell is the minimum unit volume which allows one to construct the lattice by a translational operation.

atomic theory by Arnold Sommerfeld in 1933, it can be used to describe the behavior of electrons in the *valence band* of metals. *Band theory*, in short, describes the allowed energies and wavevectors of a wavefunction in a solid (not necessarily a metal). The whole range of these energies and wavevectors is called a *band structure* and explains how insulators are different from conductors using the formalism of the free electron model. However, a more complete picture can be obtained by taking into account the periodicity of the atomic lattice which gives rise to so-called *band gaps*. These forbidden regions are vitally important in the band structure of semiconductor physics as they are responsible for the unique conducting and optical properties of semiconductor materials. In order to understand how these band structures arise in materials, we will write our potential as a periodic function:

$$V(\mathbf{r}) = \sum_{m,n,w=-\infty}^{\infty} V_{mnw} \exp\left(2\pi i \frac{mx}{a} + 2\pi i \frac{ny}{b} + 2\pi i \frac{wz}{c}\right) = \sum_{m,n,w} V_{mnw} e^{i\mathbf{G}_{mnw} \cdot \mathbf{r}}. \quad (1.2)$$

We have returned here to a more general 3D case, but the following formalism can be easily applied to 2D systems. Here $\mathbf{G}_{mnw}$ is the reciprocal lattice vector, V_{mnw} is an element of the lattice cell (e.g. the cell can have different atoms in its vertices), and $\{m, n, w\} \in \mathbb{Z}$. One can apply *Bloch's theorem* to write the solution for noninteracting particles in the form,

$$\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r}) e^{i\mathbf{k} \cdot \mathbf{r}}, \quad (1.3)$$

where $u_{\mathbf{k}}(\mathbf{r})$ is a periodic function with the same period as the potential $V(\mathbf{r})$, and $\exp(i\mathbf{k} \cdot \mathbf{r})$ are plane wave solutions with wavevector $\mathbf{k}$. If one has a very complicated periodic potential then the particle waves will scatter around in the lattice in a very complicated manner. A condition exists however known as *Bragg reflection*, which in the case of a material with periodic crystal planes, can be written neatly as,

$$\frac{2\pi n}{k} = 2a \sin(\theta), \quad (1.4)$$

where θ is the wave's angle of incidence on the plane, $k = |\mathbf{k}|$, and a is the period of the lattice planes (see Fig. 1.1[a]). Waves which satisfy this condition are reflected perfectly back and form standing waves. So there are points in k -space (reciprocal space) where the particle cannot possibly propagate through the lattice, an equivalent way of wording this is to say that the wave function group velocity becomes zero. Instead of a parabolic dispersion one will have at points $\mathbf{G}_{mnw}$ in k -space "splits" in the spectrum. These splits form forbidden regions in the band structure of free particles in a periodic potential (see Fig. 1.1[b]). In the case of semiconductor lattices, the symmetry planes are somewhat complicated but nonetheless result in such forbidden regions which are named *band gaps*. The band gap can be charac-

Figure 1.1: (a) Schematic showing Bragg reflection between periodic crystal planes for a wave (red arrow) at an angle θ of incidence. (b) Band structure of Si, plotted as a function of $\mathbf{k}$ within the first Brillouin zone (the ticks correspond to common labels used for the Brillouin zone critical points), showing the bandgap separating the Valence and conduction band.

terized as an energy gap between the valence band maximum and the conduction band minimum with the electron Fermi level caught in between.²

The notion of band gaps is extremely important in modern condensed matter physics. It gives one a degree of freedom to manipulate electrons in semiconductor materials by exciting them from the valence band to the conduction band by either an optical- or electrical excitation. Conversely, electrons in the conduction band can recombine with their positive "empty-spot" known as a *hole* left behind in the valence band to emit light. This forms the very fundamentals of semiconductor coherent light sources with the most famous example being the laser-diode which is found now in numerous everyday appliances.

1.1.1 HETEROSTRUCTURES

Nearly every modern electronic device is based on semiconductor physics. A system of different (usually layered) semiconducting materials forms the building blocks of these devices, such as the ones used in telecommunication systems, high-mobility transistors, and low-threshold lasing. Realized in the mid 20th century [1], the first *p-n* homojunction transistors were patented by W. Shockley in 1951 (e.g., *pnpn*-diode or equivalently *the thyristor*) which was then followed by work done by H. Kroemer who paved the way to more efficient heterostructured transistors [2]. Here *p* and *n* stand for negative and positive charge carrier *doping* respectively in a

²In wave optics, *stop-bands* are analogous to these forbidden regions, where light is reflected nearly perfectly from a structure with periodic layers of different refractive indices.

Figure 1.2: Heterojunction based laser device with the active region (orange) being driven by an external current through the contacts (black).

semiconductor material. Doping being a term used for adding impurities into the pure semiconductor material with additional electrons or electron vacancies.

A system of reduced dimensions, such as a layered system, can be realized with heterojunctions which mark the interface of two different semiconductor materials. The two materials can be of different crystalline properties such that free particles cannot pass from into another through diffusion. That is, a heterojunction can work as an effective barrier against propagating particles. Multiple such junctions can then be used to create a heterostructure in order to achieve a system of quantum confinement. As an example, one can sandwich a GaAs layer between two other materials with a wider band gap (such as AlAs) to effectively create a quantum well which confines the electrons living inside the GaAs.

The process of matching different semiconductor lattices is called *band engineering*. This has allowed researchers to control the band gap of the heterostructure material via the different compositions (*lattice constants*) of semiconductor materials, creating scenarios where electrons and/or holes are trapped in a optically active region (see Fig. 1.2). This is used for example in laser diodes such as double heterostructures lasers, quantum well lasers, vertical-cavity surface-emitting lasers, distributed Bragg reflector lasers, etc. All of which rely on confining the electrons and holes into the optically active region (e.g., GaAs or InGaAs) in order to increase the emission amplitude. Band engineering also allows one to tune the band-gap of the alloys from indirect- to direct gaps by changing the alloy fraction x . For aluminum-gallium-arsenide it can be written as $\text{Al}_x\text{Ga}_{1-x}\text{As}$.

Doped semiconductor heterojunctions serve a purpose in a device known as the *field-effect transistor* (FET). In short, it's a device where the conductivity between a source and drain terminals is controlled via a gate terminal using high mobility

electrons forming at the interfaces of different semiconductor materials. At the interface of a heterojunction (or *homojunction* of two differently doped semiconductor materials) there is a region of trapped electrons due to dissimilar band gaps of the materials. As more electrons travel towards the lower energy band a Coulomb potential is formed due to the increasing concentration of electrons moving away from one material to the other. The Coulomb potential tries to pull the electrons back towards their original structure but the different conduction band energies create a strong barrier forbidding them to enter. The result is a trapped two-dimensional electron gas (2DEG) at the heterojunction interface, the nature of 2DEG will be discussed in Sec. 1.1.2. This 2DEG forms the basis of FETs in general (other FET variations include JFETs, MOSFETs, MODFETs). Heterostructures thus prove to be excellent ground of localizing electrons and holes to planar systems and modifying the density of states. Quantum wires and quantum dots are also possible systems through controlled growth techniques and self-organizing behavior of atoms. However, we will keep our focus mostly on planar systems.

The next challenge of heterostructures is to show that they cannot only confine charge carriers, but also optical modes. The narrow geometry of the heterostructure is necessary in order to effectively create a quantum well confinement for charge carriers, this ranges in the tens of nanometers. These length scales are however far too small for optical modes which have wavelengths in the hundreds of nanometers. To overcome this problem, a larger periodic structure is imposed, usually called a superlattice since it imposes an additional periodic nature to the system. This structure composes of alternating layers of different *refractive indices*.³ The idea is to confine charge carriers and the photons separately, giving the optical mode in question its needed space to interact with the system. Another confinement method is to use added semiconductor layers on the initial heterostructure of lower refractive index. This is commonly known as *separate confinement heterostructure* (SCH). In Sec. 1.2.4 we will discuss optical confinement in more detail for the most general optical cavity systems where interactions of light and matter become important.

Fabrication of high quality heterostructures can be done using *metal-organic chemical vapour deposition* (MOCVD or MOVPE) or *molecular-beam epitaxy* (MBE). The former relies on the surface reaction of organic or metalorganic gases which are injected in a controlled manner into a system containing a semiconductor substrate at moderate pressures. The reaction induces crystalline growth, creating a compound semiconductor. The high accuracy of this method goes hand-in-hand with the fast control of the different gases and is commonly used for creating optoelectronic devices. MBE uses near vacuum conditions where the substrate is rotated as atomic beams are fired upon it. The flux of the atomic beams can be controlled

³A superlattice of alternating semiconductor materials will give rise to electron *minibands* which affects their transport properties.

by heating the chamber (so called Knudsen cells), the process can be realized as a subliming and then condensing onto a substrate. Though an accurate method, the process is time consuming as opposed to the MOCVD.

1.1.2 TWO-DIMENSIONAL ELECTRON GAS

We have mentioned that two type of charge carriers can arise various solids. Electrons and holes. What these two particles have in common is that they are both classified as *fermions*. As we shall soon see, there arise two fundamental groups of particles in nature; *fermions* and *bosons*. This section serves to address the fundamental difference between these two different types of particles and elaborate on the statistics which describes an ensemble of fermions leading to an accurate picture of the electron (hole) gas. In the next section we will discuss the statistics of an ensemble of bosons.

Let us imagine a system of N identical (indistinguishable) particles described by the state vector $|\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)\rangle$ corresponding to some Hamiltonian $\hat{H}$. Here $\mathbf{r}_i$ is the position coordinate of the i -th particle. We now define an exchange operator, $\hat{P}$, which interchanges two particles (for the sake of brevity we will let them be $\mathbf{r}_1$ and $\mathbf{r}_2$),

$$\hat{P}|\psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)\rangle = |\psi(\mathbf{r}_2, \mathbf{r}_1, \dots, \mathbf{r}_N)\rangle. \quad (1.5)$$

It is clear that applying the operator twice returns us to the original state, i.e. $\hat{P}^2 = \mathbb{1}$, and that its eigenvalues are $\lambda = \pm 1$. Since all the particles are identical the exchange operator commutes with the Hamiltonian,

$$[\hat{P}, \hat{H}] = 0. \quad (1.6)$$

Thus $\hat{P}$ and $\hat{H}$ share the same complete set of eigenstates which we can classify as either symmetric states ($\lambda = 1$) corresponding to bosons or antisymmetric ($\lambda = -1$) corresponding to fermions. An interesting property of the antisymmetric states is the requirement that no two fermions can sit in the same state. Indeed, writing out the antisymmetric wave function composed of single particle states $\{\psi_a(\mathbf{r}_1), \psi_b(\mathbf{r}_2), \psi_c(\mathbf{r}_3), \dots\}$ will reveal that if two particles are in the same state (e.g. $a = b$) then the full state vector becomes zero. This is famously known as the *Pauli exclusion principle* and gives rise to Fermi-Dirac statistics where the i -th state occupation number is written,

$$N_i = \frac{g_i}{e^{(\varepsilon_i - \mu)/k_B T} + 1}, \quad (1.7)$$

Here, ε_i is the energy of the single particle state, g_i is the degeneracy of the i -th state, μ is the chemical potential of the ensemble, k_B is the Boltzmann constant, and

T is the temperature. At zero temperature, the chemical potential of the highest occupied state in a Fermi system corresponds to the *Fermi energy* of the system. A highly important feature in condensed matter physics.

Another important characteristic of the Pauli exclusion principle is that all bosons possess integer spins and all fermions possess half integer spins. Thus, electrons are classified as fermions since they possess half-integer spin, namely $s_e = 1/2$. The connection between the spin structure and particle statistics can be proven in relativistic quantum mechanics but here we will take it as an axiom.

As mentioned in Sec. 1.1.1, two-dimensional electron gas can be realized at the junction of two differently doped semiconductor materials where the different band structures help trap the electrons at the junction (experimentally, the MODFET has become very popular due to the high electron mobility attained). Quantum wells can also serve as a 2D confinement for metallic layers where the electrons are free to move in the plane of the metallic sheet but have quantized motion in the perpendicular direction, these quantized levels are also known as subbands and, as an example, can give rise to inter-subband polaritons.⁴ Topological insulators can also provide 2D surface electronic states. Though the 2DEG is not in the focus of this thesis, it's worth mentioning that multiple exciting phenomenon can arise related to the 2D electron transport. Most famous is the quantum Hall effect where the conductance of the 2DEG becomes quantized in the presence of a magnetic field, or the extreme fast electron mobility in the 2D honeycomb lattices of graphene.

1.1.3 TWO-DIMENSIONAL BOSE GAS

Bosonic particles are no less commonplace than fermions in nature. A good example of a boson is the photon (the elementary excitation of the electromagnetic field) which carries spin $s = 1$ with spin projections $m_s = \pm 1$ which are associated with the two circular polarization degrees of the electromagnetic wave (usually written σ_+ and σ_- for right and left hand circular polarizations).⁵ In the standard model the fundamental force carriers are so-called *gauge bosons* and then there is the recently experimentally confirmed *Higgs boson* classified as a *scalar boson*. Helium is probably the most famous boson in physics, alongside other *cold-atoms*.⁶ A type of bosons arising in semiconductor systems are excitons, a charge neutral elementary excitation corresponding to a bound pair of conduction band electron and valance

⁴Polaritons will be discussed in Sec. 1.2.5.

⁵Note that $m_s = 0$ doesn't exist due to the massless nature of the photon, i.e., there doesn't exist a rest frame corresponding to an eigenfunction of zero spin projection for the photon, the spin can only be along the direction of propagation.

⁶The classification *cold-atoms* applies to atoms which can be sustained at extremely low temperatures. Such atoms are bosonic since fermionic systems are limited by their *Fermi temperature*.

band hole, and phonons which correspond to lattice waves. Suffice to say, bosons arise everywhere in nature and obey their own statistics known as *bosonic statistics*.

Let us first stick to the case of a homogeneous system with no requirements set on its dimensionality. A system of N non-interacting fermions in thermal equilibrium can be described by Eq. 1.7 from statistical mechanics, in an analogous manner a system of N non-interacting bosons can be described with the occupation number of the i -th particle state,

$$N_i = \frac{g_i}{e^{(\varepsilon_i - \mu)/k_B T} - 1}, \quad (1.8)$$

where the total number of particles is,

$$N = \sum_i \frac{g_i}{e^{(\varepsilon_i - \mu)/k_B T} - 1}. \quad (1.9)$$

Note that opposed to Eq. 1.7 the bosonic occupation number can take any positive value (not only between 0 and 1). This is a consequence of the symmetric bosonic wavefunctions which don't impose any restriction on how many particles can sit in a given state.

Looking at Eq. 1.8, it is obvious that in order for it to make sense then $\varepsilon_i > \mu$ since otherwise $N_i < 0$. Luckily this is always satisfied since the definition of the chemical potential in statistical mechanics for a system with N particles, described by the total energy $E(N)$, can roughly be written:⁷

$$\mu = E(N) - E(N - 1). \quad (1.10)$$

This states that it's equivalent to the energy released when removing one particle from the system. It becomes then obvious that the maximum amount of energy the chemical potential can take is to remove a particle from its lowest energy state, thus $\varepsilon_0 > \mu$. Another important feature of the chemical potential is that in a system with well defined energy levels ε_i and temperature T it is uniquely determined by the total number of particles N according to Eq. 1.9.

We come now to an interesting result due to the degeneracy of the bosonic states. When taking the limit $\mu \rightarrow \varepsilon_0$ it can be seen that it results in the population of the lowest energy state to diverge to infinity, an obvious nonphysical effect but gives an insight into theory of Bose-Einstein condensation. Originally, Bose statistics were developed for massless particles (photons) by S. N. Bose [3] in the 1920s and then extended by A. Einstein to massive particles whom then predicted the possibility of a peculiar phase of matter called a *Bose-Einstein condensate* (BEC) [4, 5]. The

⁷The chemical potential is sometimes referred to as *partial molar free energy* in chemistry and corresponds to the amount of energy released or obtained during a chemical reaction, particles escaping/entering, and phase transitions. In terms of the i -th particle state at constant pressure and temperature, it can be written $\mu_i = \left(\frac{\partial G_i}{\partial N_i} \right)_{T,P}$ where G_i is the *Gibbs free energy*.

onset of a BEC is a critical result of this thesis, playing a major role in Chapters 2-4. Thus, the rest of this section will be devoted to explaining the physics behind a BEC.

Let's assume $g_i = 1$ and consider the total number of particles in our system written as,

$$N = N_0 + N_{i \neq 0} = \frac{1}{e^{(\varepsilon_0 - \mu)/k_B T} - 1} + \sum_{i \neq 0} \frac{1}{e^{(\varepsilon_i - \mu)/k_B T} - 1}, \quad (1.11)$$

where N_0 is the number of particles in the ground state, which we will also call the condensed state, and $N_{i \neq 0}$ are non-condensed particles. For a fixed value of ε_0 and T the population $N_{i \neq 0}$ reaches a maximum value N_c when $\mu \rightarrow \varepsilon_0$.

Let's now imagine a system with $N = N_1$ particles (see Fig. 1.3) at some T . We take the limit $\mu \rightarrow \varepsilon_0$ and get N_c . If $N_c > N_1$ then the system population N stays normalized with N_0 relatively small and no extreme behavior taking place. That is, for typical values of μ the fraction of condensed and non-condensed particles behaves. Since N_c is an increasing function of T (more particles are thermally excited to higher states) then we can say that $N_c > N$ for some temperature $T > T_c$. However, if $N_c < N$, or equivalently $T < T_c$, then in order for the system to stay normalized according to Eq. 1.11 the condensate portion of the system shows extreme behavior in the thermodynamic limit $N \rightarrow \infty$ where N_0 starts to greatly exceed $N_{i \neq 0}$ (point $N = N_2$ in Fig. 1.3). This phenomenon is known as Bose-Einstein condensation for an ideal non-interacting gas of bosons.

Another way to look at this phenomenon is that there exists a statistical pressure towards particles populating the ground state. This pressure shows an extreme exponential behavior below T_c for a given system of N particles.

As an example of deriving this BEC critical temperature, one can look at the case of an non-interacting bose gas enclosed in a box of volume V , described by the Hamiltonian,

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2, \quad (1.12)$$

where m is the mass of individual bosons, and ∇^2 is the 3D Laplacian. For periodic boundary conditions one has plane waves, $\psi_{\mathbf{k}}(\mathbf{r}) = e^{-i\mathbf{k} \cdot \mathbf{r}}/\sqrt{V}$ as eigensolutions with energy $\varepsilon_k = \hbar^2 k^2/2m$. Replacing the sum of $N_{i \neq 0}$ with an integral over the momentum states $\mathbf{k}$ and applying the condition of the BEC critical temperature ($N(T_c, \varepsilon_0 = \mu) = N_T$) one gets the critical temperature,

$$T_c = \frac{2\pi\hbar^2}{k_B m} \left(\frac{n}{2.612} \right)^{2/3}, \quad (1.13)$$

where $n = N/V$ is the density of the bose gas. This underlines the importance of the gas density which characterizes the critical temperature. An important result

Figure 1.3: Number of particles in the condensed (N_0) and the non-condensed ($N_{i \neq 0}$) state as a function of the chemical potential for a given temperature T .

which we will revisit again in Sec. 1.2.1 when we discuss the condensation threshold of a gas of indirect excitons.

Bose and Einstein's predictions were somewhat disregarded since they could only be applied to ideal non-interacting systems within the framework of statistical mechanics. However, in 1947, N. N. Bogoliubov devised a quantum BEC theory applicable to the interacting bose gas [6]. The generalization of the Bogoliubov theory (*Hartree-Fock approximation*) allows one to study the dynamics of BECs through a mean-field equation commonly known as the *Gross-Pitaevskii equation* which is used extensively today to understand and analyze the macroscopic coherence phenomenon in atomic systems and, relevant to this thesis, systems of exciton-polaritons.⁸ In order to derive it properly, we must introduce the notion of *field operators* $\hat{\Psi}$ in a nonuniform system of interacting particles,

$$\hat{\Psi}(\mathbf{r}) = \sum_i \varphi_i(\mathbf{r}) \hat{a}_i, \quad (1.14)$$

where

$$[\hat{\Psi}(\mathbf{r}), \hat{\Psi}^\dagger(\mathbf{r}')] = \delta(\mathbf{r} - \mathbf{r}'). \quad (1.15)$$

Here $\hat{a}_i$ and its hermitian conjugate $\hat{a}_i^\dagger$ are the annihilation and creation operators of a particle in the state φ_i respectively. They follow the standard bosonic commutation rules where $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$ and $[\hat{a}_i, \hat{a}_j] = 0$, where δ_{ij} is the Kronicker-Delta function. The single particle states φ_i would evolve individually according to the

⁸The Gross-Pitaevskii equation is similar to the Ginzburg-Landau equation where the latter was designed to describe type-I superconductors. It is also sometimes referred as a nonlinear Schrödinger equation, an analogy in the field of optics.

standard Schrödinger equation if the particles were noninteracting. However, since the particles can ‘bounce’ and interact with each other, we must take into account the standard formalism of many-particle quantum mechanics which starts out with the field operator. The expectation value of the state operators is now given by $\langle \hat{a}_i^\dagger \hat{a}_i \rangle = N_i$, where N_i is the number of particles in state i . Writing the most general type of Hamiltonian describing a system of interacting particles (binary interactions), we can write the dynamical equation in the Heisenberg representation as,

$$i\hbar \frac{d\hat{\Psi}(\mathbf{r}, t)}{dt} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) + \int \hat{\Psi}^\dagger(\mathbf{r}', t) V_{\text{int}}(\mathbf{r} - \mathbf{r}') \hat{\Psi}(\mathbf{r}', t) d^3r' \right) \hat{\Psi}(\mathbf{r}, t). \quad (1.16)$$

Here, $V_{\text{int}}(\mathbf{r} - \mathbf{r}')$ is the two body potential between the system particles.

Let us now write our field operator in two parts, one for particles belonging to the condensate ($i = 0$) and second for any higher energy states ($i \neq 0$),

$$\hat{\Psi}(\mathbf{r}) = \varphi_0(\mathbf{r}) \hat{a}_0 + \sum_{i \neq 0} \varphi_i(\mathbf{r}) \hat{a}_i. \quad (1.17)$$

Up to this point the field operator is still perfectly general and no unnecessary adjustments have been made to the model. We now come to the most important step of our BEC theory named the *Bogoliubov approximation*. It states that when a large fraction of the particles in the system occupy the same state (namely the ground state), one can safely neglect the noncommutativity between $\hat{a}_0$ and $\hat{a}_0^\dagger$ by replacing them with a complex number with the amplitude of the ground state population, i.e. $\hat{a}_0 = \sqrt{N_0} e^{i\phi}$. This is equivalent to treating the ground state component of the field operator as a classical field or by saying the the physical system is not changed by adding a particle to the ground state or removing a particle from the ground state since $N_0 \gg 1$. The field operator can then be written,

$$\hat{\Psi}(\mathbf{r}) = \sqrt{N_0} \varphi_0(\mathbf{r}) + \delta\hat{\Psi}(\mathbf{r}), \quad (1.18)$$

where I have chosen $\phi = 0$ for brevity and the latter term accounts for non-condensed particles (e.g., thermal fluctuations). In dilute bose gases one can neglect the non-condensed part and the field operator can be written as a classical field $\hat{\Psi}(\mathbf{r}) = \Psi_0(\mathbf{r}) = \sqrt{N_0} \varphi_0(\mathbf{r})$. This is also known as the *mean field treatment* as it accounts for an average of all the condensed particles in the system, reducing the many body problem into a simpler one body problem. In the case of photons, this treatment is analogous to reverse quantization of the quantum electrodynamic picture to arrive at the classical description of the electromagnetic field. That is, having a large number of photons in the same coherent quantum state creating a classical electromagnetic wave. The complex function $\Psi_0(\mathbf{r})$ is known as the *order parameter* of the condensate

and in the case of a uniform condensate it evolves with the time average of the stationary states, i.e., the chemical potential $\mu = \partial E_0 / \partial N$,

$$\Psi_0(\mathbf{r}, t) = \Psi_0(\mathbf{r}) e^{-i\mu t/\hbar}. \quad (1.19)$$

Let us now look into the dynamics of a interacting bose gas system which will need to be described by the Hamiltonian operator from Eq. 1.16. In order to simplify the integral term we can work in the *Born approximation* where we assume that the field operator varies very slowly compared to some effective interaction potential $V_{\text{eff}}(\mathbf{r}')$. This is equivalent to saying that the spatial form of the initial field operator doesn't differ considerably from the scattered field operator. Note that our new effective potential should produce the same low energy scattering processes as given by $V_{\text{int}}(\mathbf{r} - \mathbf{r}')$. We can then replace $\mathbf{r}'$ for $\mathbf{r}$ in the arguments of $\hat{\Psi}$ and proceed by substituting our field operators $\hat{\Psi}(\mathbf{r}, t)$ with the condensate order parameter $\Psi_0(\mathbf{r}, t)$ to arrive at,

$$i\hbar \frac{d\Psi_0(\mathbf{r}, t)}{dt} = \left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) + \alpha |\Psi_0(\mathbf{r}, t)|^2 \right) \Psi_0(\mathbf{r}, t), \quad (1.20)$$

where

$$\alpha = \int V_{\text{eff}}(\mathbf{r}) \, d^3r, \quad (1.21)$$

and $V(\mathbf{r}, t)$ is an effective potential producing the scattering energy of the condensate.

Eq. 1.20 is the Gross-Pitaevskii (GP) equation derived separately by E. P. Gross and L. P. Pitaevskii in 1961. A great deal of this thesis is based on complex types of this very equation describing systems of indirect-exciton- and polariton BECs. The parameter α is denoted as the interaction constant of the condensate. For repulsive interaction one has $\alpha > 0$ and for attractive $\alpha < 0$. Thus condensate experiences an continuous energy shift depending nonlinearly on the order parameter or, to word it differently, the condensate density,

$$n_0(\mathbf{r}, t) = |\Psi_0(\mathbf{r}, t)|^2, \quad (1.22)$$

where the total number of particles in the condensate satisfies

$$N_0 = \int n_0(\mathbf{r}, t) \, d^3r. \quad (1.23)$$

The inclusion of interactions removes certain unphysical aspects such as the infinite compressibility of the non-interacting gas since the particles couldn't "see" each other up until now. By including two-body interactions the pressure of the BEC obeys,

$$p = \frac{\alpha n^2}{2}, \quad (1.24)$$

where $n = N/V$. The interactive picture also leads to the renormalization of the condensate spectrum. Setting $V(\mathbf{r}, t) = 0$ and applying the standard approach of elementary excitations in the form of plane waves where the solution of Eq. 1.20 is expanded as,

$$\Psi(\mathbf{r}, t) = \Psi_0(\mathbf{r})e^{-i\mu t/\hbar} \left[1 + \sum_{\mathbf{k}} A_{\mathbf{k}}e^{i(\mathbf{k}\mathbf{r}-\omega t)} + B_{\mathbf{k}}e^{-i(\mathbf{k}\mathbf{r}-\omega t)} \right]. \quad (1.25)$$

Here the chemical potential follows $\mu = \hbar\alpha|\Psi_0(\mathbf{r})|^2$. Solving the obtained system equations leads to the new spectrum of the GP-equation,

$$\varepsilon = \pm \sqrt{\frac{\hbar^2 k^2}{2m} \left(\frac{\hbar^2 k^2}{2m} + 2\mu \right)}. \quad (1.26)$$

This is known as the *Bogoliubov dispersion law* [6] and can also be achieved from the microscopic approach of second quantization (i.e. applying the operators $\hat{a}$ and $\hat{a}^\dagger$). For small wave vectors the dispersion is approximately linear corresponding to a phonon-like dispersion. This linear excitation can be regarded as Nambu-Goldstone modes of the spontaneously broken gauge symmetry due to the condensation, just like for normal fluids where the longitudinal phonon modes come from spontaneously broken Galilean symmetry. For large wave vectors it approaches the free particle form (parabolic curve).

The first ever experimental observation of such quantum collective phenomenon was made with superfluid Helium-4 in 1938 by Kapitsa, Allen, and Misener [7, 8]. A superfluid is a phase of matter, sometimes mistaken for a condensation, which takes place below a critical temperature called the *Lambda point*. Pioneering work made by L. D. Landau and R. Feynman showed that the viscosity of the superfluid goes to zero below a certain critical velocity known as the Landau critical velocity. In fact, the linear part of the Bogoliubov dispersion (Eq. 1.26) corresponds to the onset of superfluidity with the critical velocity defined as,

$$v_c = \frac{1}{\hbar} \left(\frac{\partial \varepsilon}{\partial k} \right)_{k=0} = \frac{1}{\hbar} \sqrt{\frac{\alpha n}{m}}, \quad (1.27)$$

where n is the density of the superfluid.

On the other hand, Bose-Einstein condensation wasn't experimentally confirmed until 1995 first in a vapor of Rubidium-87 atoms [9] cooled to the range of hundreds of nanokelvins, and four months later in Sodium gas [10], by using interference techniques to confirm the long range order of the quantum fluid. Today, state of the art cryogenic experiments can achieve extremely low temperatures well below the critical condensation threshold for various systems.

The nonlinear nature of the GP-equation is an important analogy between BECs and nonlinear optics, the former devoted to a system of massive particles and the latter to photonic systems, and gives rise to a plethora of topologically distinct solutions. The appearance of such solution is directly associated with BEC phase transitions and symmetry breaking. Most famously of such is the quantization of angular momentum, i.e. quantum vortex, which was experimentally created and observed in a two-component Rubidium-87 condensate [11]. Vortex solutions will be discussed in more detail in Sec. 1.1.4.

We now move onto the titled topic, the two-dimensional Bose gas. We already showed how statistical mechanics can predict the existence of BEC in ideal non-interacting Bose gas where quantities such as the critical temperature can be derived, such as Eq. 1.13 for a 3D bose gas confined in a box. In fact, looking at the free particle gas which obeys the dispersion

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m} \quad (1.28)$$

one has different behaviour of the density of states depending on dimensionality (as mentioned in Sec. 1.1).

$$\rho(\varepsilon) = \begin{cases} \left(\frac{L}{2\pi}\right)^3 \left(\frac{2m}{\hbar^2}\right)^{3/2} 2\pi\sqrt{\varepsilon} & 3\text{D}, \\ \left(\frac{L}{2\pi}\right)^2 \left(\frac{2m}{\hbar^2}\right) \pi & 2\text{D}, \\ \frac{L}{2\pi} \left(\frac{m}{2\hbar^2}\right)^{1/2} \sqrt{\varepsilon}^{-1} & 1\text{D}. \end{cases} \quad (1.29)$$

In 3D the density of states approaches zero when $\varepsilon \rightarrow 0$ whereas it is constant in 2D and infinite in 1D. This radically different behavior causes a divergent result when determining the critical temperature of the condensate. The fraction of particles out of the condensate, N_T , does not approach a finite value for any nonzero temperature in infinite 2D or 1D systems [12]. This is also commonly known as the *no-go* theorem [13] which was proved by Mermin and Wagner in 1966 [14]. The reason being that long-wavelength thermal fluctuations quench the long range order of the bose gas making BEC impossible to achieve.

In order to show that BECs can exist in 1D and 2D systems, one needs to introduce a trapping potential that adjusts the density of states, allowing a BEC transition at $T \neq 0$. For 2D systems, let us imagine that motion along the axial coordinate (z -axis) is frozen and only planar motion contributes to the dynamics of the problem. In the case of parabolic confinement along the z -axis, the chemical potential reads,

$$\mu' = \mu - \frac{\hbar\omega_z}{2}, \quad (1.30)$$

where μ is the old non-confined chemical potential, and ω_z is the trap frequency. This is completely valid for confinement tighter than the healing length where the axial extent of the system wave function will be $a_z = \sqrt{\hbar/m\omega_z}$. We can then approximately treat the Bose gas as 2D on a surface S where the density $n(x, y)$ obeys,

$$N = \int n(x, y) dS. \quad (1.31)$$

Let us now imagine a parabolic planar trapping potential written,

$$V(r) = \frac{1}{2}m\omega^2 r^2, \quad (1.32)$$

where $r = \sqrt{x^2 + y^2}$. For an ideal non-interacting Bose gas, the total number of particles can be written,

$$N = N_0 + \int_0^\infty \frac{\rho(\varepsilon) d\varepsilon}{e^{(\varepsilon - \mu)/k_B T} - 1}, \quad (1.33)$$

where N_0 corresponds to particles in the condensed state ($\varepsilon_0 = 0$) and the integral covers all particles with energy $\varepsilon > 0$. For our choice of a trapping potential, the density of states becomes,

$$\rho(\varepsilon) = \frac{\varepsilon}{(\hbar\omega)^2}. \quad (1.34)$$

This allows the integral to converge as opposed to the case of $V(r) = 0$ and $\rho(\varepsilon)$ being a constant valued as according to Eq. 1.29. Setting $N_0 = 0$ and $\mu = 0$,⁹ we can find the critical number of the non-condensed particles.

$$N_c \simeq \frac{1}{6} \left(\frac{\pi k_B T_c}{\hbar\omega} \right)^2, \quad (1.35)$$

which defines the critical temperature T_c . The density of the trapped gas can then be approximately found by using the effective trap size $k_B T_c = m\omega^2 r_{\text{eff}}^2/2$ which gives,

$$n_c = \frac{N_c}{\pi r_{\text{eff}}^2} = \frac{\pi k_B T_c m}{12 \hbar^2}. \quad (1.36)$$

This result does not conflict with the Hohenberg theorem [12] which only applies to uniform systems. Here the planar parabolic trapping decreases the density of states and quenches phase fluctuations which would normally make it impossible to realize a BEC in 2D (and 1D) systems.

⁹Note that our initial analysis determined the critical temperature by finding $N_{i \neq 0}$ for $\mu = \varepsilon_0$. This is absolutely equivalent to our current case where the smallest energy of the free gas is $\varepsilon_0 = 0$.

The above formalism shows that the problem of BEC transitions for $T \neq 0$ in low dimensional systems is solved for the case of non-interacting bose gases. When interactions are included, the derivation becomes more complicated and relies on the formalism of coherence functions and accounting for *long-range order*. The meaning of long range order is simply the degree of correlation between two spatially separate particles in the system. If all particles are in the condensate and occupy thus a single state, then the system is said to be fully coherent (ordered). We will simply take it as an axiom that long-range order can exist in interacting bose gases systems at finite temperatures T .

Another important consequence of including interactions is the *Berezinskii Kosterlitz Thouless transition* (BKT transition) [15, 16]. It defines a second critical temperature between the onset of superfluidity and condensation. The BKT critical temperature corresponds to a transition where one can no longer thermally excite single vortices and any existing vortices in the superfluid system form vortex-antivortex pairs. In terms of statistical mechanics, the correlation in the gas goes from an exponential spatial decay to a power-law decay, such that the superfluid density is extended. In 3D systems this is not a problem since it costs a macroscopic amount of energy in order to generate a vortex state (it will be proportional to the vortex line length). Thus thermal generation of 3D vortices can be safely neglected. In 2D systems, this transition very well exists and can pose problems since it's not very well understood how the presence of bound vortex pairs affects the BEC transition in trapped gases.

1.1.4 QUANTUM VORTICES

The onset of topological phases and excitations can be regarded as an embodiment of unique and universal laws of physics. In this section we will give a special attention to such a topologically excitation of the Gross-Pitaevskii equation called the *quantum vortex*. Such topological excitations, which are widely studied in various condensed matter systems, were first attained for Bose-Einstein condensates of ultracold atoms [9, 10], where the quantized angular momenta was experimentally observed in a two-level Rubidium-87 condensate [11].

Quantum vortices can exist in BECs, superconductors, and superfluids and are characterized by a vortex core where the condensate density becomes zero and phase of the order parameter becomes singular. The superfluid nature of the system evolves the vortex into an irrotational state¹⁰ with a circulating superfluid flow around with

¹⁰A consequence of the zero-viscosity of superfluids. A normal rotating fluid enclosed by a cylinder (e.g., water in a bucket) feels a force gradient from the surrounding cylinder, which sets the flow into a *rotational state*. This force gradient is absent for a superfluid.

a phase winding being an integer number of 2π [5, 13] (known also as *vorticity* or *topological charge*). So to speak, one can regard them as quantized excitations of angular momenta. They were first predicted by Lars Onsager in 1949 in his work on superfluids [17] which was then further developed by Richard Feynman in 1955 [18]. We will not address the detailed nature of *superfluids* which can be considered as more thermally excited type of a BEC which makes it easier to approach experimentally. Indeed, the ideal BEC has its origin from the non-interacting Bose gas. However, within the framework of this thesis, we will consider the interacting Bose gas which permits solutions such as quantum vortices. Thus, much of the theoretical work done on superfluids applies to interacting BECs.

The quantum vortex state can be understood nicely in terms of the GP-equation which describes a system where interacting bosons have formed a BEC. We will make use of the fact that the order parameter of the BEC can be written as,

$$\Psi_0(\mathbf{r}, t) = \sqrt{n_0(\mathbf{r}, t)} e^{iS(\mathbf{r}, t)}, \quad (1.37)$$

where $n_0(\mathbf{r}, t) > 0$ is the local density of the BEC. Since $n_0(\mathbf{r}, t)$ is a purely real function, it doesn't carry any net propagating velocity just like standing wave solutions on a string. Looking at the order parameter current density (analogous to the probability current in single particle QM) and using Eq. 1.37 we find that,

$$j(\mathbf{r}, t) = \frac{i\hbar}{2m} (\Psi_0 \nabla \Psi_0^* - \Psi_0^* \nabla \Psi_0) = \frac{\hbar}{m} n_0(\mathbf{r}, t) \nabla S(\mathbf{r}, t). \quad (1.38)$$

The velocity component of the condensate can then be written,

$$\mathbf{v}(\mathbf{r}, t) = \frac{\hbar}{m} \nabla S(\mathbf{r}, t). \quad (1.39)$$

In mathematics this is known as a *conservative vector field* for any scalar function $S(\mathbf{r}, t)$ (scalar potential). Integration along a path in such fields only depends on the chosen end points but not on the path taken. In the special case of a closed path which begins and ends in some point $\mathbf{r}'$ one has,

$$\oint \mathbf{v}(\mathbf{r}, t) \cdot d\mathbf{r} = \frac{\hbar}{m} [S(\mathbf{r}', t) - S(\mathbf{r}', t)] = 0. \quad (1.40)$$

In physics the velocity field is said to be *irrotational* since $\nabla \times \mathbf{v}(\mathbf{r}, t) = 0$ and is analogous to a conservative field provided that the region, where the field is defined, is simply connected.¹¹ The question now remains of determining $n_0(\mathbf{r}, t)$ and $S(\mathbf{r}, t)$.

¹¹It can be stated that every conservative vector field is also an irrotational vector field, and that the converse is also true if the region S is simply connected. This can be seen from the fact that a conservative vector field is defined as the gradient of some scalar function (in our case ∇S) and using the well known identity; $\nabla \times \nabla S = 0$. In case of vortices, the gradient ∇S is singular at its core and the region is no longer simply connected.

Figure 1.4: (a) Irrotational vector field in a non-simply connected region. (b) Solutions of Eq. 1.46 for $n = 1$ (whole line) and $n = 2$ (dashed line), reproduced from Ref. [5].

We will focus our attention to 2D systems,¹² where a stationary solution to the GP-equations can be written as $\Psi_0(\mathbf{r}, t) = \sqrt{n_0(r)}e^{in\varphi}e^{-i\mu t/\hbar}$ where φ is the system polar angle, r is the radial coordinate, μ is the condensate chemical potential, and n is some integer to assure that the order parameter stays single valued. This function is an eigenfunction of the 2D angular momentum operator $\hat{L}_z$ with eigenvalues $\hat{L}_z\Psi_0(\mathbf{r}, t) = \hbar n\Psi_0(\mathbf{r}, t)$ where the total angular momentum of the condensate will be $N_0 n \hbar$. Inserting this ansatz into Eq. 1.39 we get,

$$\mathbf{v}(r) = \frac{\hbar n}{m r} \hat{\mathbf{r}}. \quad (1.41)$$

Note the singular behavior of the velocity at $r = 0$. This is a consequence of our function $S(\mathbf{r}, t)$ not associating a scalar value to the z -axis of our system, or in other words, the chosen ansatz makes the field values on the z -axis meaningless. So our region is not simply connected and thus the field is not conservative as can also be seen from integrating over a closed path around the origin,

$$\oint \mathbf{v}(\mathbf{r}) \cdot d\mathbf{r} = \frac{\hbar}{m} 2\pi n. \quad (1.42)$$

This is a fundamental result since it confirms that all the rotation (vorticity) is quantized in integers of n and concentrated at the center of system. In fact, integration over any closed path which does not involve the origin is still zero (see Fig. 1.4[a]). Thus our field is irrotational everywhere except when including the origin where it becomes,

$$\nabla \times \mathbf{v}(r) = \frac{\hbar}{m} 2\pi n \delta(r) \hat{\mathbf{z}}, \quad (1.43)$$

¹²In the case of 3D systems, one has more complicated solutions such as vortex rings. Here the vortex line can form various patterns including connecting in a ring shape with the flow somewhat similar to a solenoid like velocity field.

where $\delta(r)$ is the radial Dirac-Delta function.

The solution of the density function $n_0(r)$ is not possible to obtain in a closed form due to the nonlinearity of the GP-equation. However, we will arrive at a nice differential equation which is possible to solve numerically. Let us plug in our ansatz into the GP-equation to get,

$$-\frac{\hbar^2}{2m} \frac{1}{r} \frac{d}{dr} \left(r \frac{d}{dr} |\Psi_0| \right) + \frac{\hbar^2 n^2}{2mr^2} |\Psi_0| + \alpha |\Psi_0|^3 - \mu |\Psi_0| = 0. \quad (1.44)$$

We will assume that the solution can be written as $|\Psi_0| = \sqrt{n_0} f(\eta)$ where $\eta = r/\xi(r)$ and,

$$\xi(r) = \frac{\hbar}{\sqrt{2m\alpha n_0}}, \quad (1.45)$$

is the *healing length* of the vortex. We then arrive at,

$$\frac{1}{\eta} \frac{d}{d\eta} \left(\eta \frac{df}{d\eta} \right) + \left(1 - \frac{n^2}{\eta^2} \right) f - f^3 = 0, \quad (1.46)$$

where $\lim_{\eta \rightarrow \infty} f(\eta) = 1$ since the condensate must become uniform when we move away from the vortex core. Solutions to Eq. 1.46 are plotted in Fig. 1.4[b]. For small η the solution f decreases to zero roughly as $\eta^{|s|}$, an expected result since a faster rotation increases the size of the vortex core.

Introducing a spin degree of freedom leads to other examples of vortex type solutions including half vortices [19, 20], warped vortices [21], merons [22], skyrmions [23, 24], and fractional vortices which can appear in multicomponent [25] or spinor condensate systems [26]. Deriving such vortex solutions is beyond the scope of this thesis but we will comment on some of the characteristics of these solutions in the following chapters.

Though the focus is set on BECs, it's worth mentioning vortices arising in superconductor systems. Specifically, in type-II superconductors one can have circulating persistent currents which exist on a length scale corresponding to the *London penetration depth* (usually denoted as λ). These currents circulate around a density minimum with a magnetic flux corresponding to the *fundamental flux quantum* $\Phi_0 = h/2|e|$ (the quantized nature of the flux is directly linked with the quantized rotation of the vortex state). These vortices are commonly known as *Abrikosov vortices* (magnetic vortices). Another type of such circulating persistent currents can be found in Josephson junctions giving rise to the *Josephson vortex* where the vortex core is no longer characterized by a healing length ξ from Ginzburg-Landau theory but the parameters of the Josephson barrier.

1.2 LIGHT AND MATTER SYSTEMS

This section of the introduction addresses two particles which are fundamental to the results of this thesis. Firstly; an elementary excitation arising in matter systems labeled as an *exciton* (Sec. 1.2.1-1.2.2). Secondly; the polariton quasiparticle (Sec. 1.2.5-1.2.7) which arises as a result of strong coupling between light and matter, and possesses unique optical properties. In fact, as will later be made clear, the polariton is composed of an exciton state strongly coupled to an optically confined photonic mode. In this fashion, the two particles are closely linked.

The theory of strong-light matter coupling is introduced for both the case of a classical system, and a quantum system (Sec. 1.2.3). Systems where such strong coupling between light and matter occurs are also presented with a special highlight on the planar microcavity (Sec. 1.2.4), which has become a very popular system for experimental research on polaritonic properties in the past decade.

1.2.1 DIRECT AND INDIRECT SEMICONDUCTOR EXCITONS

Solid state systems contain a very high number of atoms which are usually organized in a very orderly fashion making up crystalline structure of the solid. Instead of describing every single atom and its electron orbitals, one can regard the ground state of such a system as a new quasivacuum where elementary excitations play the role of new weakly interacting quasiparticles in this vacuum. A type of such quasiparticles is the exciton.

An exciton state is a bound pair of a conduction band electron and valance band hole through an electrostatic Coulomb force. It can be thought of as the solid state analogue of the hydrogen atom. In materials of small dielectric constant such as organic crystals and alkali metals one can find the *Frenkel exciton*. A type of exciton with a high binding energy (0.1-1.0 eV) such that the Bohr radius is of the order of the lattice unit cell. Another type of exciton arising in semiconductor systems is the *Wannier-Mott exciton*. An exciton with a large Bohr radius due to the large dielectric constant (screened interactions) and low effective mass of the electrons and holes. In contrast to the Frenkel excitons, the Wannier-Mott excitons have small binding energies measured around 0.01 eV [27].

Since the exciton is essentially a hydrogen atom system, the Hamiltonian can be simply written as

$$\hat{H} = -\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 - \frac{e^2}{4\pi\epsilon\epsilon_0|\mathbf{r}_e - \mathbf{r}_h|}, \quad (1.47)$$

where the first two terms are the kinetic energies of the electron and hole respectively (with effective masses m_e and m_h), and the last term corresponds to the Coulomb

attraction between them. Here ϵ_0 is the vacuum permittivity and ϵ is the relative permittivity of the material (e.g., $\epsilon = 12.9$ for GaAs). This Hamiltonian can be simplified by moving into the center-of-mass frame where it can be written,

$$\hat{H} = -\frac{\hbar^2}{2m_X} \nabla_{\mathbf{R}}^2 - \frac{\hbar^2}{2\mu} \nabla_{\mathbf{r}}^2 - \frac{e^2}{4\pi\epsilon\epsilon_0|\mathbf{r}|}, \quad (1.48)$$

where $m_X = m_e + m_h$ is the exciton mass, $\mu = m_e m_h / m_X$ is the reduced mass and,

$$\mathbf{R} = \frac{m_e \mathbf{r}_e + m_h \mathbf{r}_h}{m_e + m_h} \quad (1.49)$$

$$\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h. \quad (1.50)$$

The first term on the right hand side (R.H.S.) of Eq. 1.48 governs the free motion of the exciton as a whole, and the last two terms determine the wave function of the bound state and its corresponding binding energies [28]. We will specifically focus on 2D systems where the first three excited states of the exciton wave function can be written,

$$\psi_{1s}(r) = \sqrt{\frac{2}{\pi}} \frac{1}{a_B} e^{-r/a_B}, \quad (1.51)$$

$$\psi_{2s}(r) = \sqrt{\frac{4}{3\pi}} \frac{1}{a_B} \left(1 - \frac{2r}{3a_B}\right) e^{-r/3a_B}, \quad (1.52)$$

$$\psi_{2p}(r) = \sqrt{\frac{4}{3\pi}} \frac{r}{(3a_B)^2} e^{-r/3a_B} e^{\pm i\varphi}. \quad (1.53)$$

Here (r, φ) are the polar coordinates and a_B is the 2D exciton Bohr radius which can be derived as,

$$a_B = \frac{2\pi\hbar^2\epsilon\epsilon_0}{\mu e^2}. \quad (1.54)$$

The corresponding 2D binding energy of the ground state is,

$$\varepsilon_b = \frac{e^4\mu}{8\pi^2\hbar^2\epsilon^2\epsilon_0^2}, \quad (1.55)$$

and is usually in the range meV in typical semiconductor materials. Photon selection rules now state that the 2s states cannot be optically excited whereas the 2p states are optically active (two photon absorption is although possible but not considered here).

The exciton effective mass (arising through the periodic nature of the semiconductor structure) can easily be evaluated through a well known relation which

utilizes the curvature of the electron and hole dispersions and their free particle rest-mass $m_{e,h}^{(0)}$,

$$m_{e,h}(k) = \frac{\hbar^2}{m_{e,h}^{(0)}} \left(\frac{d^2 \varepsilon_{e,h}}{dk^2} \right)^{-1}. \quad (1.56)$$

Here ε is the dispersion of the particle in question. At the band gap in typical semiconductor systems the dispersion is roughly parabolic and the effective mass can be regarded as a constant.

In conventional bulk semiconductor systems, there is an emergence of two different bands which converge at the valance band maximum which are termed *light hole* (*lh*) and *heavy hole* (*hh*) bands (see Fig. 1.5[a]). As the name suggests, these bands have different parabolic shapes corresponding to two different hole effective masses in the growth direction of the lattice, namely $m_{lh} = 0.062m_e^{(0)}$ and $m_{hh} = 0.45m_e^{(0)}$ in GaAs systems.

The reason for these two different bands lies in the orbital structure of the valance band holes. The holes at the valance band edge are *p*-orbitals corresponding to orbital angular momentum $l = 1$ and spin $s = 1/2$. In the absence of spin-orbit interaction (SOI), these bands correspond to the projection of the orbital angular momentum on the helicity of the hole. Thus heavy holes correspond to $m_l = \pm 1$ whereas light holes have $m_l = 0$. Including SOI, we need to work with the total angular momentum $\mathbf{j} = \mathbf{s} + \mathbf{l}$ which now splits off the bands with $j = 1/2$ from the $j = 3/2$ bands such that we can safely disregard the former. This is known as the *spin-orbit gap* which GaAs is around 0.3 eV. The origin of the spin-orbit gap is beyond the scope of this introduction but it can be derived using $\mathbf{k} \cdot \mathbf{p}$ perturbation theory for the band-structure of spin-orbit coupled particles. As a consequence, we are left only with heavy holes corresponding to $m_j = \pm 3/2$ and light holes to $m_j = \pm 1/2$. The heavy hole and light hole dispersions are approximately parabolic and degenerate at $k = 0$, and due to the bigger effective mass of the heavy holes their density of states tends to dominate at the Γ -point (indeed, in what follows we will disregard light holes altogether).

For this reason, the dominating exciton type has a mass corresponding to the effective electron and heavy-hole masses, $m_X = m_e + m_{hh} = 0.517m_e^{(0)}$ in GaAs, and a spin structure composed of electron spin $s_e = \pm 1/2$ and heavy-hole spin $s_{hh} = \pm 3/2$ (the total angular momentum projection of the heavy hole is simply taken as its new spin structure). The total exciton spin thus reads as $s_X = \pm 1, \pm 2$ where the ± 1 exciton are labeled as *bright excitons* and the ± 2 ones as *dark excitons* (see Fig. 1.5[b]) [29, 30]. An important difference between the bright and dark excitons lies in their optical properties. The bright excitons can be generated via optical absorption and can undergo radiative decay since the optical selection rules are satisfied. Dark excitons on the other hand cannot absorb or emit single photon quanta. Also, radiative transitions between ± 2 and ± 1 spin states are forbidden

Figure 1.5: (a) GaAs band structure at the Γ point showing the light-hole band (lh), heavy-hole band (hh), and the spin-orbit split off light-hole band (SOI). (b) The exciton spin structure formed by a superposition of the electron spin (e) and heavy hole spin (hh).

since they have the same parity. Hence the name “dark” excitons, since they cannot be detected by optical means.

The exciton state possesses narrow absorption peaks lying below the interband continuum with energy separation characterized by its binding energy. At low temperatures, it provides an important absorption mechanism due to its large exciton transition oscillator strength since thermal fluctuations are quick to dissociate weakly bound excitons. In narrow and medium band gap semiconductors they can survive up to 100 K whereas in large band gap material such as GaN or ZnO they can stay bound up to room temperatures, an important result if the optical properties of excitons are to be implemented in optoelectronic devices.

The concluding words of this subsection will be devoted to two different realizations of excitons. Namely, *direct excitons* and *indirect excitons*. Very simply put, direct excitons arise in single quantum wells where the electron and the hole wave functions overlap in the same quantum well. Indirect excitons on the other hand arise from overlap of spatially separate electron hole wave functions in different quantum wells (see Fig. 1.6) [31, 32]. The small wave function overlap gives rise to an increased exciton lifetime, and their large dipole moment in the normal of the QW plane results in stronger exciton-exciton interactions.

1.2.2 EXCITON CONDENSATION

Since excitons possess integer spin structure they can be regarded as bosonic quasi-particles which can undergo BEC phase transition. The promise of exciton BEC and

Figure 1.6: (a) A double quantum well schematic showing an electron from one quantum well coupled with a heavy hole in the other, forming an indirect exciton. (b) The energy structure of an electron-hole bilayer showing the separation of the wave functions under an external bias.

superfluids can result in a plethora of exciting effects including persistent currents and Josephson related phenomenon [33]. However, the condition for their existence is a low electron and hole density regime. One needs to stay below a so called *Mott transition*, associated with material going from being an insulator into a conductor. At a high enough excitation intensity one enters into a regime of electron-hole plasma where exciton formation is no longer observable due to dissociation through the Auger recombination process. In order to stay within the validity of a dilute 2D bosonic exciton gas one must satisfy,

$$na_B^2 \ll 1, \quad (1.57)$$

where n is the exciton density and a_B is the exciton Bohr radius. Another problem of acquiring exciton BEC is the exciton localization by lattice defects causing a large inhomogeneous broadening. Thus ruining the bosonic nature of the exciton gas.

Bose-Einstein condensation of excitons was theoretically proposed more than 50 years ago [34, 35] and has since then been a challenging task for solid states physicists around the world. The light effective mass of the exciton shifts the critical temperature from the regime of nano-Kelvins to Kelvins, a step forward from the usual difficulty of achieving of cold atom systems at nanoscale temperatures (< 1 nK) where the condensation of atoms can take place in magnetic traps. However, the short exciton lifetime which is usually less than a nanosecond proves to be insufficient for excitons to achieve lattice temperature and consequently reach thermal equilibrium. Indirect excitons have proven to be the best bet in order to achieve BEC since they can be cooled down to the lattice temperature within their lifetime which can extend to hundreds of nanoseconds [36–38]. Theoretical works [39–41] and measurement started in earnest in 1990 where pulsed excitations were used [42] but with still not clear enough evidence of exciton BEC. In 2004 condensation of excitons was

proposed in parallel layers of conduction band electrons [43]. Measurements then revealed the onset of spontaneous coherence of in regions of macroscopically ordered indirect exciton states in coupled quantum well structures [44–47]. These recent result are still somewhat under debate since the true BEC will need to satisfy the equilibrium requirement which remains dubious for excitons.

1.2.3 STRONG COUPLING

Coupling of light and matter can be described by writing an appropriate Hamiltonian for a system possessing separate energy levels where photons can excite electrons from the valance band to the conduction band leaving behind a hole (creating thus an electron-hole pair). These optical transitions are however not the only consequence of light-matter interaction as the photons can also influence particles such to pick up some of their properties. In this case the particle is said to be *dressed* in the electromagnetic (EM) field. A common example (though not related to an EM field) is the correction to the electron mass, also known as effective mass, in various materials due to the periodic lattice potential dressing.

The physics of light-matter interaction are usually characterized by a so called light-matter interaction constant. The derivation of this constant depends on the susceptibility of the matter particle in question and the polarization of the external EM field. The efficiency of an optical transition due to the incoming field is determined by this interaction constant which needs to be large in order to achieve *strong light-matter coupling*. Physically, we are after a system where the optical transitions are taking place at a much higher rate then any other natural transitions which characterize the lifetime of the particle in question. With the photon trapped in such a system, it will interact again and again with the material that shares its confinement, giving rise to a high interaction constant which leads to strong coupling. Such strong coupling is difficult to achieve experimentally but was achieved in 1992 in a monolithic Fabry-Perot cavity [48]. Today, using state of the art technology to confine optical modes, one can have various system geometries which allow efficient light confinement. These systems have all sorts of names depending on the method of trapping the EM field, but in the next section we will specifically consider types of microcavities (see Sec. 1.2.4).

The fundamental idea of strong coupling can be visualized with a classical system of two masses on a frictionless surface [49], each connected by an ideal spring to opposite facing walls and also connected between themselves by another spring with a different spring constant (see Fig. 1.7). According to Hooke’s law, the force needed to displace the spring from equilibrium by distance x is equal to $F = -kx$. If the two masses were uncoupled we would have a noninteracting system where each mass is

Figure 1.7: A classical system simple harmonic oscillators (masses m connected to a background via spring constants k) coupled through a third spring with spring constant γ .

follows harmonic motion $\cos(\omega t)$ along the x -axis where $\omega = \sqrt{k/m}$. If we introduce now a spring connecting the two masses which has a spring constant γ we arrive at the following Lagrangian:

$$\mathcal{L} = \frac{m}{2} \left(\frac{dx_1}{dt} \right)^2 + \frac{m}{2} \left(\frac{dx_2}{dt} \right)^2 - \frac{kx_1^2}{2} - \frac{kx_2^2}{2} - \frac{\gamma}{2} (x_1 - x_2)^2, \quad (1.58)$$

where x_1 and x_2 are the coordinates of each mass. The evolution of a physical system is described by the solutions of the Euler-Lagrange equations,

$$m \frac{d^2 x_1}{dt^2} + kx_1 + \gamma(x_1 - x_2) = 0, \quad (1.59)$$

$$m \frac{d^2 x_2}{dt^2} + kx_2 - \gamma(x_1 - x_2) = 0. \quad (1.60)$$

The second order differential equations leads to two linearly independent solutions, namely $x_i = A_i e^{-i\omega_{\pm} t}$, where the new frequencies of the system $\omega_{\pm}$ are derived from solving the determinant corresponding to the system of equations,

$$\begin{vmatrix} \omega_{\pm}^2 - \omega^2 & \gamma/m \\ \gamma/m & \omega_{\pm}^2 - \omega^2 \end{vmatrix} = 0, \quad (1.61)$$

where $\omega = \sqrt{(k + \gamma)/m}$. The case of equal wall-mass spring constants k is equivalent to zero detuning between the coupled modes. The new frequencies can be written,

$$\omega_{\pm}^2 = \omega^2 \pm \frac{\gamma}{m} = \begin{cases} (k + 2\gamma)/m, \\ k/m. \end{cases} \quad (1.62)$$

By coupling the two springs together we have arrived at two new eigenfrequencies. These frequencies correspond to the cases where the masses are moving in 'antiphase'

Figure 1.8: The effects of classical strong coupling demonstrated for the case of $\gamma = 0$ (left) and $\gamma \neq 0$ (right) between spring-oscillating masses with spring constants k_1 and k_2 (see Fig. 1.7).

causing the middle spring to pull/push them together/apart or when the masses are moving ‘in phase’ and not displacing the middle spring at all.

The new spectrum now possesses *anticrossing* behaviour with a frequency splitting $\omega_+ - \omega_- = \Delta\omega$ which is demonstrated in Fig. 1.8[b] at k_2/k_1 . This anticrossing behavior is strongly associated with strong-coupling phenomenon. Furthermore, in order for the system to display strong-coupling in the presence of damping Γ (for each spring k), one must have damping linewidth that does not exceed the splitting of the modes,

$$\frac{\Delta\omega}{2\Gamma/m} > 1. \quad (1.63)$$

We now move to a quantum two-level system interacting with an EM field. The levels are separated by an energy $\hbar\omega_0$ and the frequency of the EM radiation is ω . The Hamiltonian of the considered system can be written using a standard notation of the quantized EM field where the photon energy is $\hbar\omega$:

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{\hbar\omega_0}{2}\hat{\sigma}_z + \hbar g (\hat{a} + \hat{a}^\dagger) (\hat{\sigma}^+ + \hat{\sigma}^-). \quad (1.64)$$

This model is sometimes called the *quantum Rabi model*. Here, $\hat{a}$ and $\hat{a}^\dagger$ are the photon creation and annihilation operators, $\hat{\sigma}_z$ is the third Pauli matrix characterizing the energy of the two levels, g is a coupling parameter, and $\hat{\sigma}^\pm$ are the raising and lowering operators of the two level system. In a non-interacting system, the bare energies of the photon field and the two levels would be given exactly by the first two

terms with the eigenstates $|N, \psi_i\rangle$ where N is the photon occupancy number and ψ_i is the i -th level. The third term in this Hamiltonian is analogous to the coupling in our classical model. This Hamiltonian is commonly known as the *Jaynes-Cummings Hamiltonian* in quantum optics [50]. In order to simplify it a little, we can define the detuning of the system as $\delta = \omega_0 - \omega$ and work in the *rotating wave approximation* to arrive at:

$$\hat{H} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{\hat{\sigma}_z}{2} \right) + \frac{\hbar\delta}{2} \hat{\sigma}_z + \hbar g \left(\hat{a}^\dagger \hat{\sigma}^- + \hat{a} \hat{\sigma}^+ \right). \quad (1.65)$$

Using a coherent superposition of the the bare Hamiltonian eigenstates $|N-1, \psi_1\rangle$ and $|N, \psi_2\rangle$, where ψ_1 denotes the lower energy state, we can diagonalize Eq. 1.65 to arrive at a new set of eigenenergies,

$$\hbar\omega_{N,\pm} = \hbar\omega N \pm \frac{\hbar}{2} \sqrt{\delta^2 + 4g^2 N}, \quad (1.66)$$

which correspond to new dressed states $|N, \phi_\pm\rangle$ of our original two-level system which can be written,

$$|N, \phi_+\rangle = \cos\left(\frac{\alpha(N)}{2}\right) |N, \psi_1\rangle + \sin\left(\frac{\alpha(N)}{2}\right) |N, \psi_2\rangle, \quad (1.67)$$

$$|N, \phi_-\rangle = -\sin\left(\frac{\alpha(N)}{2}\right) |N, \psi_1\rangle + \cos\left(\frac{\alpha(N)}{2}\right) |N, \psi_2\rangle, \quad (1.68)$$

where $\alpha(N) = \tan^{-1}(g\sqrt{N+1}/\delta)$.

Eq. 1.65 shows that the system is governed by three parameters, the resonance and driving frequencies, ω_0 and ω respectively, and the coupling g . However, just like in our classical system, one must take account of decay processes, namely the cavity decay rate κ and the two-level decay rate γ . In the weak coupling regime one has $g \ll \gamma, \kappa, \omega, \omega_0$ and Eq. 1.65 holds fast. In the strong coupling regime one has $\gamma, \kappa \ll g \ll \omega, \omega_0$ where Eq. 1.65 is still valid (i.e., the rotating wave approximation has not broken down). There are also two more regimes commonly classified as the ultrastrong ($g \lesssim \omega_0$) and deep strong ($g \gtrsim \omega_0$) coupling regimes. The former is associated with photon blockades, superradiant phase transitions and ultraefficient light emissions [51]; the latter one has yet to be realized experimentally but some theoretical works have been addressed [52]. However, in order to stay within the scope of this thesis, only the strong coupling regime will be considered.

1.2.4 MICROCAVITIES

Microcavities are micrometer sized EM field traps [53, 54], which allow the confined mode to survive long enough to interact with the cavity material. An example of

Figure 1.9: (a) Schematic showing the first three standing wave solutions inside a cavity. Red, green, and blue correspond to λ_1 , λ_2 and λ_3 as given by Eq. 1.69. (b) The Fabry-Perot resonator. Interference pattern can be obtained by controlling the distance d inducing a phase difference $2kd \cos(\theta) = \Delta\phi$. Constructive interference will occur between two parallel beams when $\Delta\phi = 2\pi n$ where $n \in \mathbb{Z}$.

the usefulness of microcavities is their low-threshold for lasing (as opposed to bulk lasers). This low-threshold comes from the fact that a microcavity has a small effective volume, which enhances its *Purcell factor*,¹³ and that only a small number of optical modes can be present in the cavity which increases the chances of a an emitted photon to stimulate the active material to emit another photon into the same mode (the spontaneous emission coupling factor in conventional laser is usually around 10^{-5} whereas in microcavities it is around 0.1) [53].

Essentially, microcavities are electromagnetic resonators such as the well known *Fabry-Perot resonator* (also known as a *Fabry-Perot etalon*¹⁴) which allows only integer values of the half-wavelength to form between the cavity walls, satisfying the boundary condition that the wave must be zero at the cavity interface. Let us imagine a vacuum cavity of width d where $\lambda\nu = c$. The condition needed to be satisfied is then,

$$\lambda_n = \frac{2d}{n}, \quad (1.69)$$

where $n \in \mathbb{N}$. The allowed frequencies can then be written,

$$\nu_n = \frac{nc}{2d}, \quad (1.70)$$

with the frequency spacing (free spectral range) equal to $\Delta\nu = \nu_1 = c/2d$.

The quality of a cavity is characterized by its *Q-factor*, a dimensionless parameter that describes the average amount of energy escaping the system per radian of

¹³In the weak coupling regime the Purcell factor is a characteristic of optical resonators which describes the enhancement of spontaneous emission. In the strong coupling regime the situation is more complicated and is a subject of QED.

¹⁴*Etalon* comes from the French *étalon*, meaning *measuring gauge* or *standard*.

oscillation, or equivalently the average number of round trips before a photon escapes the system (this is analogous to RLC circuits). A high Q -factor corresponds to high quality cavity where the photon remains trapped for a relatively long time. The photon lifetime τ scales with the Q -factor according to,

$$Q = 2\pi\tau\nu_n, \quad (1.71)$$

where ν_n is given by Eq. 1.70.¹⁵ The Q -factor is naturally related to the linewidth Γ of the cavity mode. In a perfect cavity a delta peak would appear at the resonant cavity frequency but due to radiative losses and cavity absorption the resonant frequency is ‘smeared’ accross a range frequencies. The cavity photon lifetime is defined as $\tau = (2\pi\Gamma)^{-1}$ and thus the Q -factor can be neatly written,

$$Q = \frac{\nu_n}{\Gamma}. \quad (1.72)$$

Another important quantity to keep in mind when dealing with cavities is the *cavity finesse* which can be written,

$$F = \frac{\Delta\nu}{\Gamma}. \quad (1.73)$$

Thus in order to have high cavity resolution, $F \gg 1$, the frequency separation $\Delta\nu$ must considerably exceed the linewidth. One can see now that one of the obstacles of microcavity fabrication is to have a high finesse and high Q -factor at the same time. While the former can be achieved by reducing the size of the cavity d and increase frequency separation, while a high quality factor can be quite difficult to achieve.

Another physical parameter which needs to be addressed is the effective mass of the cavity photon. Let’s stick to the case of 2D microcavity systems where the cavity is sandwiched between two mirrors and can be regarded as a defect layer with a refractive index n_{ref} . Here, $\mathbf{k} = (k_x, k_y)$ is the in-plane momentum and ω is the frequency of light trapped inside a cavity. One can then write:

$$\hbar\omega = \sqrt{\left(\frac{\hbar\pi nc}{n_{\text{ref}}d}\right)^2 + (\hbar ck)^2}. \quad (1.74)$$

Here we have used the fact that in the growth direction (z -axis) the energy is quantized as $h\nu_n$ where ν_n is given by Eq. 1.70. The in plane momenta follows the classical photon dispersion as can be seen from the last term. When the quantized

¹⁵Eq. 1.71 is the perfect scenario inside a vacuum resonator whereas in more realistic situations the dielectric constant ϵ at the resonator boundary, characterizing reflectivity and absorption, needs to be accounted for.

Figure 1.10: (a) Etching of GaAs/AlGaAs distributed Bragg reflector by chlorine chemistry. Figure taken from Ref. [55]. (b) Schematic showing two DBRs (blue and purple layers) sandwiching a 2D quantum well (green) in the center with a gold alloy contact (yellow).

mode perpendicular to the quantum well plane is much higher in energy than the in-plane dispersion (i.e., $\pi n/d \gg k$) one can approximate the cavity photon dispersion as:

$$\hbar\omega \simeq \frac{\hbar\pi nc}{n_{\text{ref}}d} + \frac{c\hbar k^2}{2\pi n} = \varepsilon_0 + \frac{\hbar^2 k^2}{2m_C}, \quad (1.75)$$

where

$$m_C = \frac{\hbar n_{\text{ref}}}{c} k_n, \quad (1.76)$$

is the effective mass of the cavity photon for the n -th mode with momentum k_n . Later we will see that instead of using FPR cavities one can design cavity structures where only one frequency resonates with the QW, making the index n unnecessary.

Microcavities can roughly be categorized into three groups: Standing-wave (or linear) microcavities where the light is trapped between two reflective surfaces, ring cavities where the light goes in a circular loop via total internal reflection, and photonic crystals. It is also convenient to categorize cavities into groups based on the photon propagation: 2D cavities are confined only along the z -axis but are free to move in the xy -plane, 1D cavities are confined except along the x -axis, and 0D cavities are confined in all directions and allow only standing modes to form in the system. Here we will skim over the most commonly known types of microcavities and finalizing this section with a more detailed discussion on the planar microcavity.

The Fabry-Perot resonator (FPR) gives rise to a discrete set of allowed wavelengths and frequencies according to Eq. 1.70. It consists of two opposite reflective surfaces characterized by some reflection and transmission coefficients, and its main advantages are its high interferometry resolution, and in laser devices. The small spacing d between the two reflective surfaces allowed one to control very accurately the phase difference between parallel light beams and collect via lenses to form a

Figure 1.11: (a) A schematic showing the basic structure of a VCSEL. The blue wavy line indicates escaping light. Power is supplied with a current from the contacts (yellow). (b) A scanning electron microscope (SEM) image of a VCSEL mesa from Ref. [56]

strong interference pattern. This was a great improvement to the Michelson Interferometer which utilized only a beam splitter for interference. The advantage in lasers comes from the fact that only a discrete set of frequencies are allowed inside the resonator. The laser output is never truly monochromatic since it will be affected by Doppler broadening due to the atoms having a finite velocity in the laser media. This broadening however is quenched since only resonance frequencies will survive inside the resonator.

The problem with the FPR lies in its reflection, and transmission coefficients which tells us how much of the light is reflected and transmitted at the interfaces of the FPR. As mentioned earlier the quality of a cavity is defined by its Q -factor, so in order to have high quality cavities one must have a high reflective coefficient which increases the photon confinement and reduces the mode linewidth. A huge improvement came with the implementation of the *Distributed-Bragg Reflector* (DBR)¹⁶. It consists of alternating semiconductor layers of different refractive indexes. Each layer is designed such that its optical thickness is a quarter of the wavelength of the confined light in order to achieve constructive interference of reflected waves thus creating a high-quality reflector (see Fig. 1.10)). Analogous to our derivation (Eq. 1.70) where we had an electromagnetic mode confined between two reflective surfaces with linewidth Γ we have for the DBR,

$$\Gamma = \frac{4\nu_{\text{DBR}}}{\pi} \sin^{-1} \left(\frac{n_2 - n_1}{n_2 + n_1} \right), \quad (1.77)$$

¹⁶Also known as a *dielectric Bragg mirror*.

where ν_{DBR} is the central frequency of the mode in question and n_1 and n_2 are the refractive indexes of the alternating DBR layers.

For periodic structures, designed to confine light, there exist intervals of k -vectors of the incident light called *stop-bands* where the k -vector of the propagating wave becomes purely imaginary. In this case the wave is perfectly reflected from the DBR¹⁷. The frequency of the light at the center of the stop-band is usually written $\bar{\omega}$. If the frequency of the trapped cavity mode is the same as $\bar{\omega}$, that is $\Delta = \omega - \bar{\omega} = 0$, then it can be shown that the cavity photon spectrum will correspond to Eq. 1.74. When $\Delta \neq 0$ one has splitting between the TE- and TM- polarized cavity modes which gives rise to an effect called the *optical spin Hall effect*. This will be discussed further in Chap. 4.

Using *molecular beam epitaxy* high quality DBRs can be fabricated easily although the process is more demanding as opposed to more economical *chemical vapor deposition* method which results in DBRs of lesser quality. Most planar microcavities today are designed using DBRs to confine the electromagnetic wave within. For example, pillar microcavities utilize total internal refraction to confine light laterally and a DBR mesa to reflect light vertically. The most commonly known type is the *vertical-cavity surface emitting laser* (VCSEL), a type of a laser diode which emits a laser beam perpendicular to its structure axis (see Fig. 1.11) greatly reducing absorption losses as opposed to the edge emitting laser diodes. This design, although initially designed weak coupling regime (regime of laser diodes) it also opens the way towards strong coupling systems since it can achieve a Q -factor in the thousands. Another type of microcavities are *spherical mirror cavities* where instead of planar reflective surfaces, one has a curved surface, allowing one to reach a finesse in the orders of hundreds [57].

Ring shaped resonators based on total internal reflection can achieve extremely high Q -factors (see Fig. 1.12). Here the mode favored by the system is called *whispering gallery mode* and have experimentally demonstrated strong coupling of light and matter [58]. We can roughly categorize such circular resonators as of high and ultrahigh quality. The former includes the *microdisk* [59] with a Q -factor in the thousands and can be constructed either from semiconductor or polymer. The latter includes the *microsphere* [60] and *microtoroid* [61] which have a Q -factor in the order of $10^8 - 10^9$. The downside to the whispering gallery mode resonators is the complicated spatial profile of the trapped electromagnetic mode, as opposed to the simple planar resonators. Indeed, because of the 2D degree of freedom particles (e.g., excitons and polaritons) have in the quantum well of planar microcavity, one can expect interesting transport phenomena such as spin currents and polarization patterns to take place (Chap. 2 and 4).

¹⁷This is analogous to the electronic band-gaps in semiconductor materials where the Bragg condition arises due to the periodicity of the lattice

Figure 1.12: (a) SEM image of a microdisk mesa. (b) Schematic showing a silica microsphere resonator. (c) A SEM image of a silica microtoroidal resonator.

1.2.5 EXCITON POLARITONS

Previous sections have addressed the existence and properties of the exciton state arising in semiconductors, strong coupling of matter and light, and techniques in designing a system favoring strong light-matter interaction. We come now to the part where a new type of a quasiparticle which arises in the regime of strong coupling is introduced. This particle is known as the *cavity exciton-polariton* (or simply polariton). Though several types of polaritons can be realized such as the Tamm-Plasmon polaritons, intersubband polaritons, phonon polaritons, and Bragg polaritons, we will focus exclusively on the exciton-polariton arising in semiconductor planar microcavity systems [53, 62, 63]. In the strong light-matter regime (see Sec. 1.2.3) interactions between excitons and cavity photons give rise to a new quasiparticle named the exciton-polariton (henceforth, polariton). It's characterized by a very small effective mass (down to 10^{-5} of the free electron mass) and short lifetimes

Figure 1.13: Schematic showing the excitonic wavefunction χ inside the QW coupling with the photonic field of the cavity, ϕ . In the regime of strong coupling this leads to the formation of the polariton quasiparticle.

(around picoseconds depending on the cavity Q -factor). Due to its light effective mass, the polariton is extremely versatile with high velocities, allowing it to travel coherently across hundreds of microns before decaying. It also possesses a natural nonlinearity from its interactive excitonic part, making it a possible candidate for various optoelectronic devices [64].

The polariton was theorized long before its experimental observation due to technical equipment difficulties. The initial theory was introduced first by S. I. Pekar [65], V. M. Agranovich [66], and J. J. Hopfield [67]. It wasn't until 1992 by Weisbuch *et al.* that polaritons confined within a planar microcavity were first observed [48].

We will derive the formation of the cavity polaritons starting from the Hamiltonian of bare excitons (here 'bare' simply means that the particle is not dressed, yet) and bare photons coupled together through some interaction potential $V(\mathbf{k})$.

$$\hat{H} = \sum_{\mathbf{k}} \varepsilon_X(\mathbf{k}) \hat{b}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \sum_{\mathbf{k}} \varepsilon_C(\mathbf{k}) \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + \sum_{\mathbf{k}} \frac{V(\mathbf{k})}{2} (\hat{a}_{\mathbf{k}}^\dagger \hat{b}_{\mathbf{k}} + \hat{a}_{\mathbf{k}} \hat{b}_{\mathbf{k}}^\dagger) \quad (1.78)$$

where $\hat{b}_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}$ are creation operators for the excitons and photons with in plane momentum $\mathbf{k}$ respectively and $\hat{b}_{\mathbf{k}}^\dagger$ and $\hat{a}_{\mathbf{k}}^\dagger$ are their annihilation operators. All together these satisfy the standard commutation rules of bosonic particles,

$$[\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}] = 0, \quad [\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}') \quad (1.79)$$

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}] = 0, \quad [\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}') \quad (1.80)$$

where $\delta(\mathbf{k} - \mathbf{k}')$ is the Dirac-Delta function. Working within the parabolic approximation we have for the kinetic terms,

$$\varepsilon_{X,C}(k) = \frac{\hbar^2 k^2}{2m_{X,C}} \quad (1.81)$$

where $m_{X,C}$ are the effective masses of the exciton and polariton respectively. The exciton effective mass is estimated as according to Eq. 1.56 and the cavity photon to Eq. 1.76. In Fig. 1.14 we have plotted Eq. 1.81 (dashed lines). Due to different effective masses the spectrum of the excitons seems nearly constant compared to the spectrum of the cavity photons. We will define the detuning parameter $\Delta = \varepsilon_X(0) - \varepsilon_C(0)$. In the case of negative detuning the bare spectra will crossover at a point,

$$k_0 = \frac{2\Delta(m_X - m_C)}{\hbar^2 m_X m_C}. \quad (1.82)$$

Naturally, if one wants to account for gain or decay terms in the spectrum then they would have to be rewritten,

$$\varepsilon_{X,C}(k) = \frac{\hbar^2 k^2}{2m_{X,C}} - i \frac{\hbar \gamma_{X,C}}{2}, \quad (1.83)$$

where γ_X and γ_C are the decay rate of excitons and cavity photons respectively. The decay rate can be understood in terms of the particle lifetime τ through $\gamma = 1/\tau$. It should be noted that the factor $\hbar/2$ is purely for convenience when looking at the probability density current of the particles in question. For the time being, we will neglect the decay rates of the particles in order to keep the formalism clearer.

The third term of our Hamiltonian is the interaction term, much so similar to the one introduced in Eq. 1.65 in the rotating wave approximation. We will refrain from deriving the form of this interaction $V(\mathbf{k})$ which was derived in Ref. [68]:

$$V(\mathbf{k}) = \frac{\varepsilon_X(k)\mu_{cv}}{\hbar c} \sqrt{\frac{2\pi\hbar c}{n_{\text{ref}}d(k^2 + k_n^2)}} F_k(0)I. \quad (1.84)$$

Here $\mu_{cv}(\mathbf{k}, k_n) = e \langle v | \epsilon_{\mathbf{k}, k_n} \cdot \mathbf{x} | c \rangle$ is the dipolar matrix element of the exciton transition between the valance ($|v\rangle$) and the conduction band ($|c\rangle$), k_n is the momentum of the n -th quantized mode between the DBRs, n_{ref} is the refractive index of the cavity, $F_k(\boldsymbol{\rho})$ is the exciton envelope function with in-plane displacement vector $\boldsymbol{\rho}$, and $I < 1$ is determined by the geometry of the QW with exciton resonance [68].

In the following analysis, the exciton envelope function and dipolar matrix element are taken to be constant with $\mathbf{k}$, and the envelope function of the photon mode k_n approximated as a step function with value $1/d$ inside the QW and zero

Figure 1.14: Bare exciton and polariton spectrum plotted with blue and red dashed lines respectively. The whole green and black lines show the upper and lower branches respectively of the renormalized spectrum of the exciton cavity-polariton (Eq. 1.88) for different detunings Δ . Here, $m_X = 0.6m_0$ and $m_C = 5 \times 10^{-5}m_0$ where m_0 is the free electron rest mass. **(From left to right)** $\Delta = \{-10, 0, 10\}$ meV, and the interactions are set to a constant value $V = 15$ meV.

outside (reasonable for k_n close to exciton resonance). We can then define an effective interaction constant V which does not depend on the in-plane momentum of the excitons or photons.

Our Hamiltonian can be diagonalized using the following linear transformation,

$$\hat{\varphi}_{L,\mathbf{k}} = C_{L,\mathbf{k}}\hat{a}_{\mathbf{k}} + X_{L,\mathbf{k}}\hat{b}_{\mathbf{k}}, \quad (1.85)$$

$$\hat{\varphi}_{U,\mathbf{k}} = C_{U,\mathbf{k}}\hat{a}_{\mathbf{k}} + X_{U,\mathbf{k}}\hat{b}_{\mathbf{k}}, \quad (1.86)$$

where the indices U and L stand for the ‘upper’ and ‘lower’ polariton branches (see Fig. 1.14, green and black whole lines respectively). Here $C_{L(U),\mathbf{k}}$ and $X_{L(U),\mathbf{k}}$ are the *Hopfield coefficients* whose amplitude squared corresponds to the photonic and excitonic fraction of the polaritons. Our new Hamiltonian now reads,

$$\hat{H} = \sum_{\mathbf{k}} \varepsilon_L(\mathbf{k}) \hat{\varphi}_{L,\mathbf{k}}^\dagger \hat{\varphi}_{L,\mathbf{k}} + \sum_{\mathbf{k}} \varepsilon_U(\mathbf{k}) \hat{\varphi}_{U,\mathbf{k}}^\dagger \hat{\varphi}_{U,\mathbf{k}}, \quad (1.87)$$

where the spectrum of the lower and the upper branch can be written,

$$\varepsilon_{U,L}(\mathbf{k}) = \frac{\varepsilon_C(k) + \varepsilon_X(k)}{2} \pm \frac{1}{2} \sqrt{(\varepsilon_C(k) - \varepsilon_X(k))^2 + V^2}. \quad (1.88)$$

The Hopfield coefficients can be determined as,

$$C_{U,\mathbf{k}} = X_{L,\mathbf{k}} = \frac{\varepsilon_U(\mathbf{k}) - \varepsilon_X(\mathbf{k})}{\sqrt{V^2/4 + (\varepsilon_U(\mathbf{k}) - \varepsilon_X(\mathbf{k}))^2}}, \quad (1.89)$$

$$C_{L,\mathbf{k}} = -X_{U,\mathbf{k}} = \frac{-V^2/2}{\sqrt{V^2/4 + (\varepsilon_U(\mathbf{k}) - \varepsilon_X(\mathbf{k}))^2}}. \quad (1.90)$$

One can see that for zero detuning at the in plane momentum the photon and exciton fractions are equal, $|C_{U(L)}|^2 = |X_{U(L)}|^2 = 1/2$. Analogous to our classical strong-coupling model, the interaction between the excitons and photons results in an anticrossing behavior in the renormalized dispersion. The point of intersection between the bare photon and exciton dispersions (Eq. 1.82) is where the LP and UP dispersions are exactly split in the energy V . This splitting is usually called the *Rabi splitting*¹⁸ denoted $\hbar\Omega$ where Ω is the Rabi frequency.

Let us now again take account of the decay rates of the cavity photons and QW excitons (γ_C , γ_X) as given by Eq. 1.83. The resulting polariton spectrum will understandably then become complex where the imaginary branches correspond to the polariton decay rate and the real branches correspond to the polariton dispersion. In this case the Rabi splitting can be written as,

$$\hbar\Omega = \begin{cases} \sqrt{V^2 - (\gamma_X - \gamma_C)^2}, & V^2 > (\gamma_X - \gamma_C)^2, \\ 0, & V^2 \leq (\gamma_X - \gamma_C)^2, \end{cases} \quad (1.91)$$

So the splitting and anticrossing behavior vanish if the decay rates are too strong (strong damping). This is analogous to our statement in Sec. 1.2.3 that the regime of strong coupling is determined by damping parameters. In the case of the polariton system, one must satisfy $V^2 > (\gamma_X - \gamma_C)^2$ in order to have strong coupling.

In the Schrödinger picture, the dynamical equations describing a coherent gas of exciton-polaritons can be achieved by coupling the exciton and cavity-photon order parameters (ϕ , χ) through their Rabi splitting,

$$i\hbar \frac{d}{dt} \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \begin{pmatrix} \hat{H}_\phi & \hbar\Omega/2 \\ \hbar\Omega/2 & \hat{H}_\chi \end{pmatrix} \begin{pmatrix} \phi \\ \chi \end{pmatrix}. \quad (1.92)$$

Here, $\hat{H}_\phi$ accounts for the dispersion of the cavity-photons (including their lifetime) and $\hat{H}_\chi$ accounts for the exciton-exciton interactions which give rise to the nonlinear nature of the polaritons.

There are several things worth discussing from Fig. 1.14. One can see that for small k the effective mass of polaritons is very similar to the one of cavity

¹⁸Also known as *normal mode splitting* in analogy to single atom cavity systems.

photons. An inflection point in the anti-crossing region turns the effective mass from positive to negative since the effective mass can be determined from the second k -space derivative of the dispersion (see Eq. 1.56). This will play an important role in a type of Bose-Einstein condensation of polaritons, and quantum collective phenomenon in general for light-matter systems which will be discussed in Sec. 1.2.7.

According to Eq. 1.84 the Rabi splitting depends on the bare exciton spectrum. This has given researchers the opportunity to explore different coupling strengths depending on the material being used. For example, measurement on GaAs cavities have revealed several meV splittings up to $T = 40$ K [69], CdTe cavities were measured with splitting of 26 meV up to $T = 100$ K [70], GaN cavities were measured with a splitting of 56 meV up to room temperatures [71], and ZnO cavities with a splitting to astounding 300 meV at room temperatures [72]. These findings have opened the way towards realizing room temperature polaritonic devices [64].

1.2.6 POLARITON SPIN FORMALISM

Polaritons possess an integer spin with two possible projections of the angular momentum ($s_z = \pm 1$) on the structural growth axis (z) of the microcavity and can be regarded as a two level system just like a spin 1/2 electron system.

It is well known that any two level system can be represented by a pseudospin as it accounts for all possible linear arrangements of the two levels [73]. A good example is the polarization of light which can be described in the basis of clockwise (σ_+) and anticlockwise (σ_-) circular polarization, which then can be superposed to form linearly and diagonally polarized photon states. This is analogous for the polariton. In fact, the polariton spin state $s_z = \pm 1$ is directly related to the photon spin (circular polarization) $\sigma_{\pm}$ such that it can be directly accessed by optical measurements, where the polarization of the emitted light corresponds to the spin dynamics of the polaritons [74].

The pseudospin formalism is as follows: A polariton state corresponding to some wave vector $\mathbf{k}$ can be described by the 2×2 density matrix which is structured using the Pauli spin matrices

$$\rho_{\mathbf{k}} = \frac{N_{\mathbf{k}}}{2} \sigma_0 + S_{\mathbf{k}} \bullet \boldsymbol{\sigma}_{\mathbf{k}}. \quad (1.93)$$

Here $S_{\mathbf{k}}$ is the pseudospin, and $\boldsymbol{\sigma}_{\mathbf{k}} = (\sigma_z, \sigma_y, \sigma_x)_{\mathbf{k}}$ where the Pauli matrices are written,

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (1.94)$$

The general convention is to associate the states with pseudospin $S_z = \pm 1$ with right- or left-circular polarizations, the states $S_x = \pm 1$ with X- and Y-linearly

Figure 1.15: (a) The polarization of the coherent polariton state can be fully characterized with the pseudospin which lies on the surface of the Poincaré sphere. For example, a fully circularly polarized state correspond to the north and south poles whereas fully linearly and diagonally polarized states lie somewhere on the equator. (b) Circularly polarized light is realized through the superposition X (green) and Y (blue) linearly polarized beams whom are exactly $\pm\pi/2$ out of phase. The signs correspond to clockwise σ_+ and anticlockwise σ_- circular polarization respectively.

polarized light, and the states $S_y = \pm 1$ with linear diagonal polarizations. These are also known as the *Stokes vector* components.¹⁹ It's common to use the direct relation between the pseudospin and the Stokes vector since the polarization of the emitted light is directly related to the polariton spin state. Furthermore, if the polariton state is described by a coherent spinor wavefunction $\Psi = (\Psi_+, \Psi_-)$, then information of its polarization can be directly accessed through the following relations,

$$S_z = \frac{|\Psi_+|^2 - |\Psi_-|^2}{n_0}, \quad S_x = 2 \frac{\Re(\Psi_- \Psi_+^*)}{n_0}, \quad S_y = 2 \frac{\Im(\Psi_- \Psi_+^*)}{n_0}, \quad (1.95)$$

where $n_0 = |\Psi_+|^2 + |\Psi_-|^2$ normalizes the pseudospin to unity. Graphically, the pseudospin can be projected on the Poincaré sphere (see Fig. 1.15) which fully characterizes the spin state of the coherent polaritons ensemble.

¹⁹Proposed by George Gabriel Stokes (1819 - 1903), a mathematician and a physicist from Ireland, the Stokes parameters trace their origin to the formalism of classical electromagnetic propagation described by a plane wave $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$. In the case of z -axis propagation we can write the plane wave as $\mathbf{E} = (E_{0x} \cos(kz - \omega t) ; E_{0y} \cos(kz - \omega t + \phi))$ where E_{0x} and E_{0y} are its transverse amplitudes and ϕ their phase difference. The unnormalized Stokes parameters describing the polarization of the wave are then written: $S_0 = E_{0x}^2 + E_{0y}^2$, $S_x = E_{0x}^2 - E_{0y}^2$, $S_y = 2E_{0x}E_{0y} \cos(\phi)$, and $S_z = 2E_{0x}E_{0y} \sin(\phi)$.

The spin structure of polaritons (or rather the excitons) introduces spin-dependent interactions (singlet vs triplet). Fundamentally, exciton interactions can be classified into two: Coulomb interactions and direct interactions. The former depends on the exciton (polariton) spin whereas the latter does not and can be safely neglected [75, 76]. Triplet interactions (α_1), estimated as repulsive, do not change the spin state of the excitons through an exchange of an electron or hole. However, singlet interactions (α_2), estimated as attractive, can lead to bright excitons transforming into dark ones and vice versa. It has been shown that the dominant interaction comes from the triplet case, namely, the ratio of the two contributions has been estimated as $|\alpha_2/\alpha_1| = 0.1$ [77, 78]. The relative strength of these interaction terms depends on the detuning and can in general vary over a broad range [79], however for the work presented in this thesis the singlet interaction is usually regarded as negligible ($\alpha_2 = 0$).

The polariton spin is of great significance to the field of spinoptronics since spin related phenomenon and devices (such as spin Hall effect and spin transistors) have now an optical analog. Currently a great deal of research effort is being expended on investigating various spin related dynamics in hopes to bridge the gap between state of the art electronic and photonic devices.

1.2.7 CONDENSATION OF POLARITONS

The strong dissipative and nonlinear nature of polaritons allows them to reach a metastable steady state very different from their equilibrium case. This steady state is often classified as "out-of-equilibrium" Bose-Einstein condensate and only exists where external driving fields, which match the polariton decay, allow polaritons to undergo stimulated scattering into a macroscopically coherent state [80]. The long extend of the polariton wavefunction allows them to maintain their coherence over long distances which in turn suppresses the effects of disorder and system defects which would otherwise quench any quantum phase transitions.

Evidence of polariton phase transitions has been experimentally established in several open microcavity systems of GaAs, CdTe, and ZnO where long range coherence was measured [81, 82], and in systems with harmonic trapping in the cavity plane [83]. The onset of macroscopic occupation of the $k = 0$ state from Ref. [81] is shown for different intensities in Fig. 1.16. The realization of the polariton BEC at room temperature in both planar microcavity [84] and microwire [85] structures has a fundamental practical importance in optoelectronic technologies. Indeed, purely electronic devices require the Coulomb interaction of electrons in order to switch currents by an external voltage, whereas optical systems get their nonlinear

Figure 1.16: Experimental figure taken from Ref. [81]. Exciton-polariton far-field emissions showing (a) real space and (b) k -space occupation, measured at 5 K, for three different excitation intensities. The increased intensity of the incoherent optical excitation shows the onset of a macroscopically occupied coherent state at $k = 0$.

coefficients through the material in question²⁰ which unfortunately are usually very small, making purely optical devices only realizable at high excitation powers.

Since the polariton BEC is a macroscopic coherent state it can be regarded as a coherent source of radiation through spontaneous emission of light [86] as opposed to the conventional stimulated emission in today's lasers. These types of lasers realized in microcavity systems are termed *polariton lasers* [87] and possess extremely low threshold since the only requirement is to create an ensemble of electron and holes via incoherent pumping. The consequent onset of exciton formation, followed by polariton formation, allows the particles to relax into the ground state via polariton-phonon and polariton-polariton interactions creating a coherent source of light (see Fig. 1.17[a]). These polariton lasers have today been observed in various semiconductor systems such as GaN, GaAs, and CdTe thus opening a wide variety of materials for creation of low-power opto-electronic laser devices [88, 89] where

²⁰The *Kerr effect* is an example of such optical nonlinear behavior where the material refractive index changes with the square of the electric field propagating through it.

even the polarization properties of the emitted light can be controlled [90, 91].

Another important phenomenon arising in polariton BEC systems is *bistability*. Simply put, the condensate is said to be bistable when two different densities (intensities) can be supported in the same interval of pumping power (see Fig. 1.17[b]). In semiconductor microcavities, bistability arises from an interplay of their density dependent blueshift (caused by their repulsive binary interactions) and their stimulated coherent absorption when the polariton BEC energy matches the energy of an external resonant pump [92]. Bistability can also be realized with nonresonant pumping schemes by using polarization gratings, density dependent lifetimes of the electron-hole reservoir, and specific pump profiles, although this is out of the scope of the thesis.

Though exciton-polaritons undergo fast scattering processes which allows them quickly to decay into the system ground state, their short lifetimes make thermal equilibrium difficult to achieve. The spontaneous onset of phase coherence is assisted by driving the system continuously at high enough intensities in order to induce stimulated scattering (not unlike photon lasers and amplifiers) into a low energy state²¹. This can be regarded as a BEC threshold density, which quickly decays if the external driving is not kept present. Due to the polariton's small effective mass their spontaneous coherence can be observed up to room temperatures [84]. This promising result has galvanized the solid state physics community to find ways for implementing polariton BECs into future light-matter devices operating at room temperature conditions.

As stated earlier, the excitonic part of polaritons gives them an ability to interact. The two possible interactions which need to be considered are *phonon-polariton interaction* and *polariton-polariton interaction*. The former arises from interaction between excitons and longitudinal-acoustic phonons and allows UPB polaritons to relax down to the LPB at high k values. LPB polaritons can then continue to scatter via phonons down along their dispersion but will face a slight conundrum, a so-called *bottleneck effect* takes place at the anticrossing region of the LPB [93], preventing polaritons from relaxing into the $k = 0$ state through phonon scattering. It is here that polariton-polariton interactions come to rescue. By increasing the density of LPB polaritons, the rate of polariton-polariton scattering is increased such that stimulated scattering takes place into the $k = 0$ state of the LPB, effectively forming a macroscopic single state occupation [94]. This stimulated scattering process is associated with *polariton lasing* which analogous to photonic lasers but in the absence of population inversion. Indeed, the spontaneous emission coupling

²¹Stimulated scattering is a characteristic of a system of identical bosons. The presence of N particles in a final state enhances scattering into that state by a factor of $N + 1$ and thus the particles are more inclined to populate the state. The small LPB density of states allows one to easily obtain a quantum degenerate seed of polaritons in the low energy state which kickstarts the scattering [80].

Figure 1.17: (a) A schematic showing the generation of a polariton BEC through nonresonant excitation (orange area). A hot exciton reservoir is created which generates polaritons in the lower branch. Scattering processes then allow polaritons to decay fast enough into the ground state forming a type of BEC. (b) Bistable phenomenon demonstrated for varying pump power P (for both resonant or not). The nonlinear nature of the polariton BEC allows the support two intensity separated states at the same pump value.

constant of a laser setup allows the lasing regime to be determined by the population inversion.

The polariton system can also be regarded as a type of an optical amplifier where bottleneck polaritons in the LPB are induced to coherently scatter into the $k = 0$ state by selectively exciting the bottleneck region [95]. Another method is generating LPB population at a *magic angle* corresponding to a wave number where $k = k_{MA}$ polaritons can coherently scatter into the $k = 0$ and $k = 2k_{MA}$ states and conserve energy [94]. This is also known as a coherent optical four-wave mixing.

As we mentioned in the case of excitons, establishing thermal equilibrium is quite challenging and would seem even more so for exciton-polaritons due to their extremely short lifetimes were it not for their fast scattering. Classifying the exciton-polariton condensate as a true BEC is rather misleading as it can only exist when the decay is matched by some external driving fields. However, it has been shown that the polariton distribution in reciprocal space reveals a close match to the Bose distribution function [70] such that today it's univocally agreed that the macroscopic coherence of polaritons can be regarded as a type of BEC phenomenon as long as the exciton density stays within the dilute gas limit (see Eq. 1.57). Such systems are termed non-equilibrium systems but can none the less be described by a Gross-Pitaevskii type equation as long as all the relevant dynamical factors are taken account of [96].

Another important feature of a polariton BEC is the existence of superfluid flow close to the ground state where the spectrum should show Bogoliubov-like renor-

malization (see Eq. 1.26) due to the interactive nature of the polaritons. This was experimentally confirmed in 2008 by Utsunomiya et. al. [97]. Furthermore, superfluidity was also confirmed in experiments where backscattering effects from defects were absent with the formation of Cherenkov patterns [98–100]. In general, the polariton system size is small enough that a finite polariton BEC can be realized before a BKT transition [101].

1.3 QUANTUM RINGS

Progress in modern nanotechnologies [102] has resulted in rapid developments in the fabrication of mesoscopic objects, including non-single connected nanostructures such as quantum rings. The fundamental interest attracted by these systems is caused by a wide variety of purely quantum-mechanical topological effects which can be observed in ring-like mesoscopic structures. One such effect, which will be the highlight of Sec. 1.3.1 and Chap. 5, is the *Aharonov-Bohm effect* (AB effect) where measurable interference in the ring conductance arises due to seemingly non-physical influence of electromagnetic potentials on the charged particle phase. Commonly this effect is derived for the case of a magnetic field $\mathbf{B}$ threading a conducting ring with vanishing magnetic field lines at the ring circumference, i.e. the electrons do not 'feel' $\mathbf{B}$ but still acquire a phase related to the fields vector potential (this will be detailed in the next section). Another type of such phases associated with QRs is the phase acquired by spin-orbit coupling and is known as *Aharonov-Casher effect*. When taking into account a more detailed model including spin, an exact Coulomb interaction potential of the electrons, and structure impurities one finds that oscillations with even smaller periods can exist [103]. The effect was accordingly dubbed the *fractional Aharonov-Bohm effect*.

Another famous phenomenon predicted by quantum mechanics is the existence of persistent circulating electric currents in the coherent equilibrium state of a quantum ring threaded by a magnetic flux [104]. This is not unlike vortex states arising in bosonic systems since they can form stable low-energy solutions associated with some finite quantized angular momenta. All of these effects have been possible to observe due to more and more accurate nanoscale fabrication methods and low temperature experimental techniques in order to maximize the phase coherence nanostructures.

There are various techniques in fabricating quantum rings. A partial capping process has allowed one to transform self-assembled quantum dots, grown by molecular beam epitaxy (MBE), into self-assembled QRs. The process relies in In atoms to diffuse radially away from the QDs and leave behind a center of Ga surrounded by In atoms, effectively forming a InGaAs QRs. Another method relies on a *dewet-*

Figure 1.18: (i) Filled states topography image of a cleaved QR in the $(1\bar{1}0)$ plane and (110) plane, corresponding to the short and long axis of the ring-shaped islands. The inset shows the modeled volcano shaped QR. Taken from Ref. [109]. (ii) Lateral ordering of quantum rings achieved by using InAs quantum dots layered on GaAs which are then capped and transformed into QRs. Taken from Ref. [108]

ting process (also known as *wetting droplet instability*) which arises when InAs QDs are partially capped with a layer of GaAs, the dynamical instability results in the deformation of the QDs into QRs (shown in Fig. 1.18[b]). Self-assembled QRs show great promise to work in the purely quantum regime where one can have long dephasing-lengths due to reduced rate of scattering processes. Fabrication of such mass QR structures is highly feasible for various modern nanodevices which rely on quantum effects to tune their electronic and magnetic properties. Such devices range from broad-area lasers [105], spintronic gates [106], solar cells [107], etc. Lately there has been a great deal of research involved in making such stacks of QRs in an ordered fashion in hopes to gain further control of the structure properties [108]. Another interesting consequence of the self-assembled method is that the formation of the QRs does not guarantee a doubly-connected structure (i.e. a proper ring link geometry) but rather a singly-connected volcano shaped structures [109] (See Fig. 1.18[a]). Surprisingly these QR structures still revealed strong AB oscillations, the reason being that the electrons decayed rapidly towards the “volcano” center such that the wave functions were approximately topologically identical to electron wave functions in doubly-connected QRs.

Various *lithographic* techniques have proven to be extremely useful to create nanostructures but face size-restriction (< 20 nm). Regardless, optical lithographic techniques have become extremely advanced in controlling shape and sizes within

their working regime, and can cover large areas cost-effectively. *Partial overgrowth* and *Droplet-Epitaxy* allows the formation of strain free QR complexes. It has demonstrated great fabrication control in design of structures such as single QRs, double-QRs, and concentric higher-order multiple QRs.

1.3.1 THE AHARONOV-BOHM EFFECT

The Aharonov-Bohm effect was first predicted by Werner Ehrenberg and Raymond E. Siday in 1949 [110] and then revisited again by Yakir Aharonov and David Bohm in 1959 [111]. The effect describes how a charged particle in negligible electric and magnetic field can still be affected by the electromagnetic potentials spanning those fields. The effect was experimentally confirmed in 1982 in quantum rings threaded by a magnetic field where a current of electrons divided clockwise and anti-clockwise along the ring would interfere destructively or constructively as a function of magnetic flux [112–114]. Even though the magnetic field itself was confined within the ring and not touching the electrons there would be nonzero vector potential $\mathbf{A}$ which would affect them. This stunning result underlined the importance of the electromagnetic potentials which had before been disregarded as purely mathematical identities in vector calculus with no real effect in the physical world save for the electric- and magnetic field.

In order to derive the effect, we will need to address an important identity named *gauge transformation*. We know that in classical physics the electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) fields are directly measurable physical quantities and are described by Maxwell's equations,²²

$$\mathbf{E} = -\nabla\varphi - \frac{\partial\mathbf{A}}{\partial t}, \quad (1.96)$$

$$\mathbf{B} = \nabla \times \mathbf{A}. \quad (1.97)$$

Here $\mathbf{A}$ and φ are some vector- and scalar potential which define the electric and magnetic field in question. However, $\mathbf{A}$ and φ are not uniquely defined for some

²²Maxwell's equations are a set of equations put together by the Scottish physicist James Clerk Maxwell (1831 - 1879) which completely describe the classical laws of electricity and magnetism. They can be neatly written as:

$$\begin{aligned} \nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0}, & \nabla \times \mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}, \\ \nabla \cdot \mathbf{B} &= 0, & \nabla \times \mathbf{B} &= \mu_0 \left(\mathbf{J} + \epsilon_0 \frac{\partial\mathbf{E}}{\partial t} \right). \end{aligned}$$

Here $\mathbf{J}$ is the material current density, ρ is charge per unit volume, $\epsilon_0 = 8.854 \times 10^{-12}$ F/m is the electric permittivity of free space, and $\mu_0 = 4\pi \times 10^{-7}$ N/A² is the magnetic permeability of free space.

$\mathbf{E}$ and $\mathbf{B}$. In fact, for any scalar potential Λ one can apply the following Gauge transformation,

$$\mathbf{A}' = \mathbf{A} + \nabla\Lambda, \quad \varphi' = \varphi - \frac{\partial\Lambda}{\partial t}, \quad (1.98)$$

to arrive at exactly the same results given by Eqs. 1.96-1.97. This is commonly known as *gauge invariance*. The question of whether a physical system is gauge invariant arises when $\mathbf{A}$ and φ must be involved in the dynamical equations describing the system. For example, Newton's equations depend explicitly on $\mathbf{E}$ and $\mathbf{B}$ where a particle with charge q is placed in an electromagnetic field and is acted on by the Lorentz force,

$$\mathbf{F} = q [\mathbf{E} + \mathbf{v} \times \mathbf{B}]. \quad (1.99)$$

From Newton's first law the particle position $\mathbf{r}(t)$ and momentum $\mathbf{p}(t)$ will depend only on $\mathbf{E}$ and $\mathbf{B}$ and are therefore gauge invariant quantities. In the Hamiltonian formalism, one can derive a proper general result for a particle subjected to the Lorentz force which states that,

$$\mathbf{p} = \nabla_{\mathbf{v}}\mathcal{L} = m\mathbf{v} + q\mathbf{A}, \quad (1.100)$$

where $\mathcal{L}$ is the system Lagrangian, m is the particle mass, and $\nabla_{\mathbf{v}}$ is the velocity gradient. Eq. 1.100 is called the canonical momentum. We can now write our Hamiltonian of a particle in an electromagnetic field as,

$$\hat{H} = \frac{1}{2m} (-i\hbar\nabla - q\mathbf{A})^2 + q\varphi. \quad (1.101)$$

The Hamiltonian depends on the vector- and scalar potential $\mathbf{A}$ and φ its solutions are not necessarily gauge invariant. In fact, an eigensolution ψ to $\hat{H}$ will undergo a phase change when one introduces a scalar potential $\Lambda(\mathbf{r}, t)$ to our gauge (Eq. 1.98),

$$\psi'(\mathbf{r}, t) = e^{iq\Lambda(\mathbf{r}, t)/\hbar} \psi(\mathbf{r}, t). \quad (1.102)$$

This result is critical to the origin of the Aharonov-Bohm effect. Let us now consider the same thing but with $\Lambda = 0$ and assume that a solution ψ' exists to Eq. 1.101 and can be written as,

$$\psi'(\mathbf{r}, t) = e^{ig(\mathbf{r})} \psi(\mathbf{r}, t), \quad (1.103)$$

where using the fact that $\nabla \times \mathbf{A} = 0$ we have,

$$g(\mathbf{r}) = \frac{q}{\hbar} \int_{\mathcal{O}}^{\mathbf{r}} \mathbf{A}(\mathbf{r}') \cdot d\mathbf{r}'. \quad (1.104)$$

Here $\psi(\mathbf{r}, t)$ is some solution to the problem where $\mathbf{A} = 0$. So in the presence of a vector potential $\mathbf{A}$ we only pick up a phase factor $g(\mathbf{r})$. This phase factor is of no significance since we can choose the gauge in such manner that the additional

Figure 1.19: (a) A quantum ring threaded by a magnetic field $\mathbf{B}$ corresponding to a vector potential $\mathbf{A}$. (b) Results from Ref. [113] showing magnetoresistance oscillations of period $h/|e|$. The Fourier spectrum shows two peaks where the smaller one corresponds to $h/2|e|$ oscillations associated with electron weak localization. The inset shows the QR of $R = 784$ nm drawn with a scanning transmission electron microscope (STEM) on a polycrystalline gold film.

phase $g(\mathbf{r})$ cancels against $q\Lambda(\mathbf{r})/\hbar$. But we run up against a wall when the path of the integral in Eq. 1.104 closes around the magnetic field $\mathbf{B}$. In this case, there is no gauge which can remove the additional phase for clockwise integration and anticlockwise integration. The particle will pick up a phase factor no matter what Λ is chosen. Let us now choose a convenient gauge, namely the Coulomb gauge, where $\nabla \cdot \mathbf{A} = 0$ and integrate anticlockwise along the half circle threaded by a magnetic flux we have,

$$g = \frac{q}{\hbar} \int_{\mathcal{O}} \mathbf{A}(\mathbf{r}) \cdot d\mathbf{r} = \frac{q\Phi}{2\pi\hbar} \int_0^\pi \left(\frac{\hat{\varphi}}{R} \right) \cdot (R\hat{\varphi}d\varphi) = \frac{q\Phi}{2\hbar}. \quad (1.105)$$

Here $\hat{\varphi}$ is the polar unit vector, Φ is the magnetic flux, and R is the radius of the ring. If we travel clockwise then the limits of the integral switch and we get $g = -q\Phi/2\hbar$. It is now obvious that a gauge transformation Λ cannot remove the phase difference from both paths at the same time. This is the famous Aharonov-Bohm effect and is sometime referred to as a *geometric phase* or *Berry phase*. The spectrum of electron states on the ring can be written,

$$\varepsilon_n = \frac{\hbar^2}{2m_e R^2} \left(m - \frac{q\Phi}{2\pi\hbar} \right), \quad (1.106)$$

where m is the angular momentum quantum number of the electron and $q = -e$. A common notation for electrons is to use the fundamental flux quantum $\Phi_0 = h/e$,

this allows us to write the spectrum as,

$$\varepsilon_n = \frac{\hbar^2}{2m_e R^2} \left(m + \frac{\Phi}{\Phi_0} \right). \quad (1.107)$$

From another fundamental viewpoint, the AB effect arises from the broken time-reversal symmetry in the electron. Namely, the magnetic flux breaks the equivalence of clockwise and counterclockwise electron rotations inside the ring, which results in the flux-controlled interference of electron waves corresponding to these rotations. This time reversal breaking is evident from the Lorentz force (Eq. 1.99) since the path of a charged particle in free space, subjected to this force, will not be the same if time is reversed.

The AB effect has been experimentally confirmed in numerous experiments [115] which have also revealed that it is not exclusive to charged particles such as electrons. The bound electron-hole pair, better known as the exciton state (see Sec. 1.2.1), is also subject to AB oscillations due to different magnetic fluxes penetrating the different QR paths of the electron and hole. Since the exciton is approximately a charge neutral particle, the effect is not as pronounced as for electronic QR states. However, techniques in increasing the polarization of the particle make it a possible candidate in optical devices such as photon sources and detectors. Other uses involve capturing single photon states and releasing them at later times by control of external electric and magnetic fields. Emissions between exciton and biexciton states have demonstrated photon antibunching, an important property for single photon emitters. Such devices would have huge impact on photon computing devices, quantum computers, and communications technology.

CHAPTER 2

VORTICES IN SPIN-ORBIT COUPLED INDIRECT-EXCITON CONDENSATES

Ever since atomic condensates were experimentally realized in the late 20th century there has been a great deal of work expended in understanding their unique macroscopic coherent nature and superfluid properties. Systems which can interact with light, such as excitons and polaritons, in a controlled manner are especially exciting due to their place in modern devices. For example, controlling the spin statistics of excitons is important in spin-based devices (the field being classified as spintronics [116]) and has already shown great promise in organic semiconductors [117]. Here the idea of spin-orbit interaction (SOI) is of great interest since it forms a basis for many spin-based devices such as the *spin field effect transistor* (spin FET).

Control of atomic condensates can be performed with implementation of an artificial SOI between its spin components [118]. An example of a typical scheme for vortex nucleation in atomic physics is based on the effective Lorentz force appearing corresponding to the BEC rotation [119, 120]. A novel alternative approach to create vortices with an optically-induced artificial gauge field generation was also proposed recently [121]. Followed by numerous theoretical proposals [122–124], such a system was shown to be rich in diverse spin-related topological phases and excitations [125], including single plane wave and striped phases [126], hexagonal phase pattern [127], skyrmion grid [128], square vortex lattice,[129], and even a quasicrystalline phase for cold dipolar bosons [130].

Today, solid-state physics offers a variety of systems, where bosonic quasiparticles with a light effective mass can form a BEC at higher temperatures than conventional atomic BECs. They include Quantum Hall bilayers [43], magnons [131], indirect excitons [45, 132, 133] (see Sec. 1.2.1), and cavity exciton-polaritons [81, 83, 134, 135] (see Sec. 1.2.5). In the case of indirect-excitons, high spin degeneracy can be achieved such that one can expect a great variety of distinct vortex solutions. Two-spin-component and three-spin-component condensates have already undergone some investigation but never has a four-component spinor condensate been investigated thoroughly theoretically or experimentally. It is thus highly interesting to extend the spin structure of the exciton condensate and account for both bright and dark

excitons to see what unique topological solutions can arise in such a system where the presence of SOI can play a major role.

The spin structure of the exciton, $S_z = \pm 1, \pm 2$, leads to an ambiguous choice of a BEC ground state [136, 137]. This results in non-trivial condensate topology and the possibility for generation of various topological defects [138]. Moreover, complex spin textures around fragmented beads of cold exciton condensates were observed [139]. They were explained with an influence of SOI of various types, which affects the center-of-mass exciton motion [137, 140, 141]. This assures that physics similar to atomic spin-orbit coupled condensates, even artificial magnetic field generation [142], can be studied with condensed indirect excitons.

In the current chapter we investigate theoretically the ground states of various topological defects in a 2D spinor condensate of cold indirect excitons. The work presented here is directly related to the author's previously published results in Ref. [A6]. Using the imaginary-time propagation on a four-component Gross-Pitaevskii (GP) equations for the spinor macroscopic wave function we numerically analyze these states for different stable scenarios of vortices and half-vortices. Including a SOI mechanism of either *Rashba* or *Dresselhaus* nature for electrons only (see Sec. 2.1 for more details) can result in stable exciton BEC states of cylindrically symmetric half-vortex or half vortex-antivortex pairs, or a non-trivial pattern with warped vortices. However, when both of Rashba and Dresselhaus SOI are present at the same time in the exciton system, the ground state of a condensate represents a stripe phase where vortex type solutions are absent.

2.1 SPINOR INDIRECT EXCITON MODEL

As detailed in Sec. 1.2.1, an indirect exciton is a composite boson consisting of a spatially separated electron and hole [Fig. 1.5]. Its spin is defined by the electron and the heavy hole spins projections on the structure growth axis resulting in the four possible exciton spin projections $S_z = \pm 1, \pm 2$ labeled bright excitons and dark exciton respectively. Bright excitons can be manipulated by an external optical driving field whereas the dark excitons are optically inactive due to angular momentum conservation selection rules. However, they can appear due to exchange interaction between bright states or as a result of SOI mechanisms. As mentioned earlier, our system is chosen as a 2D system.

It can be shown that interband Coulomb processes of the exciton states result in bright excitons being higher in energy than dark excitons which has been confirmed by photoluminescence measurements where the dark excitons are split-off by about 1 μeV [143]. Thus, in the absence of mechanisms which mix the dark and bright

excitons together, one can have the onset of a dark or “gray” condensation in the corresponding systems, which prevents direct observation of macroscopic coherence in the photoluminescence measurements [33, 76]. Moreover, the effects of spin-orbit interactions were shown to interplay with a bright-dark splitting, leading to unconventional pairing effects in the dense BCS-like direct exciton condensates [144]. In the case of indirect excitons the small overlap between electron and hole wave functions leads to approximately equal energies of all four indirect exciton states. The dark states still play an important role and cannot be excluded from our current consideration [145, 146]. Thus our model will fully account for both bright and dark states and regard them as fully degenerate in the absence of any mixing.

As introduced in Sec. 1.1.3, we describe a fully coherent state of indirect excitons using the mean-field treatment where the GP-equation controls the dynamics of the four-component order parameter $\Psi = (\Psi_{+2}, \Psi_{+1}, \Psi_{-1}, \Psi_{-2})^T$ where T stands for the vector transpose. In the general form it can be derived varying the Hamiltonian density over the macroscopic order parameter,

$$i\hbar \frac{d\Psi_\sigma}{dt} = \frac{\partial \mathcal{H}}{\partial \Psi_\sigma^*}, \quad (2.1)$$

where Ψ_σ^* is the complex conjugate of the order parameter σ spin component. The Hamilton density can be written as a sum of a linear single particle operators and nonlinear interaction terms, $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{int}}$.

The single particle operator of the Hamiltonian density is composed of the kinetic energy operator and SOI coupling operator. Electrons with some finite wavevector in the reciprocal plane moving in an electric field experience a magnetic field in their rest frame according to,

$$\mathbf{B}_{\text{eff}} = -\frac{\mathbf{v} \times \mathbf{E}}{c^2}, \quad (2.2)$$

where $\mathbf{v}$ is the velocity of the electron, $\mathbf{E}$ is the electric field in question, and c is the speed of light. This effective magnetic field couples to the spins of the electrons through the operator,

$$\hat{H}_{\text{SOI}} = \mu_B g (\boldsymbol{\sigma} \bullet \mathbf{B}_{\text{eff}}(\mathbf{k})), \quad (2.3)$$

where μ_B is the Bohr magneton, $\mathbf{k} = (k_x, k_y)$ is the exciton in-plane wave vector (taken to be the same as the electrons), and g is the effective exciton Zeeman factor. The effective magnetic field causes splitting in the spin-bands that otherwise would be degenerate in k -space. In what follows we will account only for SOI acting on the spin of the electron, whose eigenstates are usually denoted $\{|\uparrow\rangle, |\downarrow\rangle\}$, and not the heavy hole. It consists of two terms, namely the Dresselhaus term and the Rashba term.

The Dresselhaus term arises from crystal bulk inversion asymmetry [147] where the crystal gives rise to an intrinsic electric field and for a [001] quantum well is

Figure 2.1: (a) The modified electron dispersion in the presence of either Rashba or Dresselhaus SOI can be realized as a revolution of two shifted parabolas. The inner cone-shaped surface corresponds to $|\uparrow\rangle$ states and the outer surface for $|\downarrow\rangle$ states. Schematic representation of the orientation in reciprocal space of the Rashba (a) and Dresselhaus (b) effective magnetic fields (shown with red arrows) with strain applied in the $[001]$ direction.

described by the Hamiltonian $\hat{H}_D = \beta(\hat{\sigma}_x k_x - \hat{\sigma}_y k_y)$ (see Fig. 2.1[b]), where $k_{x,y}$ are Cartesian components of the electron wave vector, $\sigma_{x,y}$ are Pauli matrices, and β denotes the strength of the Dresselhaus interaction. Here we have neglected cubic contribution of the Dresselhaus SOI since they are usually very weak in GaAs.

The Rashba term appears due to structure inversion asymmetry, corresponding to an external electric field skewing the potential wells in a direction perpendicular to the system plane [148], and is described by the Hamiltonian $\hat{H}_R = \alpha(\hat{\sigma}_x k_y - \hat{\sigma}_y k_x)$ (see Fig. 2.1[a]), with α being the strength of the Rashba interaction. It has also been shown that both types of SOI can also occur in certain centrosymmetric crystals [149].

The single particle term in the Hamiltonian density thus reads [137]:

$$\mathcal{H}_0 = \Psi^* \hat{\mathbf{T}} \Psi, \quad (2.4)$$

where,

$$\hat{\mathbf{T}} = \begin{pmatrix} -\frac{\hbar^2 \nabla^2}{2m_X} + V(\mathbf{r}) & \hat{S} & 0 & 0 \\ \hat{S}^\dagger & -\frac{\hbar^2 \nabla^2}{2m_X} + V(\mathbf{r}) & 0 & 0 \\ 0 & 0 & -\frac{\hbar^2 \nabla^2}{2m_X} + V(\mathbf{r}) & \hat{S} \\ 0 & 0 & \hat{S}^\dagger & -\frac{\hbar^2 \nabla^2}{2m_X} + V(\mathbf{r}) \end{pmatrix}. \quad (2.5)$$

Here m_X is the indirect-exciton mass, $V(\mathbf{r})$ is a harmonic trapping potential, and $\hat{S}$ is the SOI operator accounting for both Rashba (α) and Dresselhaus (β) contributions.

It is necessary to introduce a trap to the Hamiltonian in order to keep the condensate localized within the system and to set realistic conditions where 2D BEC can be realized (see Sec. 1.1.3 for the low dimensionality criteria).

$$\hat{S} = \chi \left[\beta \left(\frac{\partial}{\partial y} - i \frac{\partial}{\partial x} \right) + \alpha \left(\frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right) \right], \quad (2.6)$$

which can equivalently be written in k -space as,

$$\hat{S}_k = \chi [\beta(k_x + ik_y) + \alpha(k_y + ik_x)]. \quad (2.7)$$

Here, $\chi = m_e/m_X$ is the electron-to-exciton mass ratio and $\mathbf{k} = -i\nabla$ denotes the center of mass wave vector of the indirect exciton in a 2D planar system. Note that in the described Hamiltonian the bright-dark splitting of the indirect exciton states is neglected. This however can be straightforwardly introduced for the systems, where such a splitting was observed [33, 141].

The nonlinear part of the Hamiltonian density $\mathcal{H}_{\text{int}}$ describes the interactive nature between the indirect excitons. Since excitons are composite bosons, there are four possible types of interactions corresponding to the exchange of electrons (V_e), exchange of holes (V_h), simultaneous exchange of electron and hole (or exciton exchange, V_X), and direct Coulomb repulsion (V_{dir}). Defining the interaction constants $V_0 \equiv V_e + V_h + V_{\text{dir}} + V_X$ and $W \equiv V_e + V_h$, the interaction (nonlinear) part of the Hamiltonian density can be written:

$$\begin{aligned} \mathcal{H}_{\text{int}} = & \frac{V_0}{2} \left(|\Psi_{+2}|^2 + |\Psi_{+1}|^2 + |\Psi_{-1}|^2 + |\Psi_{-2}|^2 \right)^2 \\ & + W \left(\Psi_{+1}^* \Psi_{-1}^* \Psi_{+2} \Psi_{-2} + \Psi_{+2}^* \Psi_{-2}^* \Psi_{+1} \Psi_{-1} \right) \\ & - W \left(|\Psi_{+2}|^2 |\Psi_{-2}|^2 + |\Psi_{+1}|^2 |\Psi_{-1}|^2 \right). \end{aligned} \quad (2.8)$$

We mainly focus on a dilute BECs of indirect excitons where the most important interaction contribution comes from vanishing transferred momentum q , thus working in the long wavelength limit ($q \rightarrow 0$) the interaction parameters satisfy $V_{\text{dir}} = V_X$ and $V_e = V_h$ (s -wave approximation). The interaction parameters can be further estimated using a narrow QW approximation [137].

For our numerical modeling; we consider the indirect exciton system investigated in Ref. [45] where a high quality GaAs/AlGaAs structure consisting of two coupled 8nm GaAs QWs separated by a 4nm $\text{Al}_{0.33}\text{Ga}_{0.67}\text{As}$ layer was used to detect macroscopic coherence of indirect exciton gas. The observation of nontrivial spin structures in the same sample presumes an importance of spin-orbit interaction in the described system [139].

Straightforward calculation from Eq. 2.1 gives us that the exciton dynamics of the system can be modeled through a set of four spin-coupled GP-equations:

$$i\hbar \frac{d\Psi_{+2}}{dt} = \hat{E}\Psi_{+2} + \hat{S}\Psi_{+1} + V_0 \sum_{\sigma} |\Psi_{\sigma}|^2 \Psi_{+2} - W|\Psi_{-2}|^2 \Psi_{+2} + W\Psi_{-2}^* \Psi_{+1} \Psi_{-1}, \quad (2.9)$$

$$i\hbar \frac{d\Psi_{+1}}{dt} = \hat{E}\Psi_{+1} - \hat{S}^\dagger \Psi_{+2} + V_0 \sum_{\sigma} |\Psi_{\sigma}|^2 \Psi_{+1} - W|\Psi_{-1}|^2 \Psi_{+1} + W\Psi_{-1}^* \Psi_{+2} \Psi_{-2}, \quad (2.10)$$

$$i\hbar \frac{d\Psi_{-1}}{dt} = \hat{E}\Psi_{-1} + \hat{S}\Psi_{-2} + V_0 \sum_{\sigma} |\Psi_{\sigma}|^2 \Psi_{-1} - W|\Psi_{+1}|^2 \Psi_{-1} + W\Psi_{+1}^* \Psi_{+2} \Psi_{-2}, \quad (2.11)$$

$$i\hbar \frac{d\Psi_{-2}}{dt} = \hat{E}\Psi_{-2} - \hat{S}^\dagger \Psi_{-1} + V_0 \sum_{\sigma} |\Psi_{\sigma}|^2 \Psi_{-2} - W|\Psi_{+2}|^2 \Psi_{-2} + W\Psi_{+2}^* \Psi_{+1} \Psi_{-1}, \quad (2.12)$$

where we have defined $\hat{E} \equiv -\hbar^2 \nabla^2 / 2m_X$ for brevity.

2.2 NUMERICAL IMAGINARY TIME PROPAGATION

We apply an imaginary time method to find a state corresponding to the local energy minima of the Hamiltonian of the interacting exciton system described by Eqs. 2.9-2.12. The method relies on solving the dynamical equations where time is substituted as $t \rightarrow -i\tau$. This imaginary transform changes the role of energy into the one of decay where high energy solutions will decay faster than low energy ones. I.e.,

$$\Psi(\mathbf{r})e^{-i\omega t} \rightarrow \Psi(\mathbf{r})e^{-\omega\tau}. \quad (2.13)$$

This allows one to start with some initial condition in the given energy landscape and find local energy minimum by propagating the solution forward in imaginary time, τ . Fast-Fourier-Transform methods are used to evaluate the spatial dependence, and a variable order Adams-Bashforth-Moulton method in time to achieve accurate discrete gradient flow towards a possible low energy solution. Being a set of nonlinear equations, the energy landscape can have multiple minima, and the one that is reached in the numerical procedure strongly depends on the initial condition.

As an example; the true ground state of the system in absence of SOI corresponds to a homogeneous condensate with a spatial profile corresponding to the ground state of the trapping potential, $V(\mathbf{r})$, which for a harmonic trap will be a Gaussian. Now, if one imprints angular momentum onto the exciton condensate the low-energy solution of the system will correspond to an energy minima of non-zero vorticity (if it's a possible solution). The imaginary time method does not necessarily force the order parameter Ψ into a non-vortex state. If the system doesn't possess any stable solutions in the form of vortices, state with no angular momentum will be recovered independently of the initial condition.

A choice of an initial condition is not always trivial when dealing with a nonlinear set of equations controlled by many parameters. In our case the typical initial condition corresponds to the vortex solution in a polar coordinate system (r, θ) :

$$\Psi_\sigma(r, \theta) = G^{(0)}(r) \frac{r/\xi_\sigma}{\sqrt{(r/\xi_\sigma)^2 + 1}} e^{i(m_\sigma\theta + \phi_\sigma)}. \quad (2.14)$$

Here $G^{(0)}(r)$ is a Gaussian function localizing the trapped exciton gas, σ is the spin index, ξ_σ is the vortex healing length¹ analogous to Eq. 1.45 [150], m_σ is the vortex angular momentum, and ϕ_σ is its phase. The effective mass of the exciton is taken to be $m_X = 0.21m_e$, where m_e is the free electron mass [37]. We assume that the healing length of a vortex in a four component BEC is comparable with one component BEC case. In the following results the exciton density in the harmonic trap is kept around $n \propto 10^8 \text{ cm}^{-2}$, corresponding to realistic experimental values. The lateral size of the system of $20 \text{ } \mu\text{m}$ was chosen corresponding to localized bright spots observed in past experiments on exciton condensates [139]. The 2D harmonic trap profile is given by $V_{\mathbf{r}} = m_X \omega^2 r^2 / 2$, where we choose our trap strength to satisfy $m_X \omega^2 / 2 = 1 \text{ } \mu\text{eV } \mu\text{m}^{-2}$.

It should be stressed that the initial condition is used here only to set different topologies in the system. The final result of imaginary time propagation obtains the minimum energy state for a given topology (if such a state exists), that is, the ground state of a given topological defect characterized by winding numbers m_σ . If we decide to set $m_\sigma = 0$ for some spin-components then correspondingly we only use a uniform initial condition for the spin (if there is no rotation there is no density dip).

Note that the relative phases between the components in the initial condition (set by ϕ_σ in Eq. 2.14) can affect the solution. Where this is so, we minimize over different values of ϕ_σ to find the minimum energy state. Finally, the observed solutions are propagated in real time numerically to confirm that they are indeed stationary and not some metastable states in the potential landscape.

2.3 TRIVIAL VORTEX STATES AND VORTEX PAIRS

In order to get an idea of what sort of solutions are possible in the four-component exciton BEC, it is ideal to consider the cylindrically symmetric stationary order

¹For our case the healing length is simply written with the appropriate parameters as $\xi_\sigma = \hbar / \sqrt{2m_X V_0 n_\sigma}$, where V_0 is the nonlinear interaction parameter defined before and n_σ is the 2D density of the exciton gas.

parameter of our coupled GP-equations in the absence of trapping ($V(\mathbf{r}) = 0$) as a possible minimal energy state for a rotating BEC around the z -axis [150],

$$\Psi_\sigma(r, \theta, t) = R_\sigma(r) e^{i(m_\sigma \theta + \phi_\sigma)} e^{-i\mu t/\hbar}, \quad (2.15)$$

where μ is the chemical potential of the condensate analogous to Eq. 1.19 and $R_\sigma(r)$ is some radial amplitude. Note that Eq. 2.15 is purely for analytical investigation whereas Eq. 2.14 is for numerics. Now, the circulation of the tangential velocity over a closed contour for quantum vortices is quantized in units of $2\pi\hbar/m_X$ controlled by the winding number m_σ (see Sec. 1.1.4). Recent works on spinor exciton condensates have concluded that one of the simplest vortex solutions is of opposite vorticity in the $\Psi_{\pm 1}$ components (half vortex-antivortex pair) and zero vorticity in the dark components (or vice versa) [138, 151]. This will later be shown to be indeed a possible low energy solution amongst other interesting vortex solutions for different m_σ and ϕ_σ .

The radial part is taken to be purely real and is related to the total density n of the condensate as

$$n_0 = |R_{+1}|^2 + |R_{-1}|^2 + |R_{+2}|^2 + |R_{-2}|^2, \quad (2.16)$$

where

$$N = \int n_0 d^2r, \quad (2.17)$$

is the total number of excitons in the system.

The phase ϕ_σ plays an important role in whether a vortex solution is present in the condensate or not. For example, setting $\phi_\sigma = n\pi$ switches the sign of the wave function (where $n \in \mathbb{Z}$) and thus switches the sign of the second line term in the nonlinear part of the Hamiltonian density (Eq. 2.8) corresponding to bright to dark exciton conversion.

Plugging Eq. 2.15 into the dynamical equations (Eqs. 2.9-2.12) and setting $\hat{S} = 0$, one arrives at the following bound of the winding numbers and phases:

$$m_{+1} + m_{-1} = m_{+2} + m_{-2}, \quad (2.18)$$

$$\phi_{+1} + \phi_{-1} = \phi_{+2} + \phi_{-2} + n\pi, \quad (2.19)$$

where $n = 0, 1$ and plays an important role in stabilizing certain solutions. Note that $e^{i(m_\sigma \theta + \phi_\sigma)}$ are eigensolutions of the $\hat{E}$ operator. This bound must necessarily be satisfied if a solution on the form of Eq. 2.15 is to exist in the BEC.

Let us rewrite Eqs. 2.9-2.12 (in the absence of SOI):

$$i\hbar \frac{d\Psi_{+2}}{dt} = \hat{E}\Psi_{+2} + V_0 n_0 \Psi_{+2} + W \Psi_{-2}^* \Psi_\Delta^2, \quad (2.20)$$

$$i\hbar \frac{d\Psi_{+1}}{dt} = \hat{E}\Psi_{+1} + V_0 n_0 \Psi_{+1} - W\Psi_{-1}^* \Psi_{\Delta}^2, \quad (2.21)$$

$$i\hbar \frac{d\Psi_{-1}}{dt} = \hat{E}\Psi_{-1} + V_0 n_0 \Psi_{-1} - W\Psi_{+1}^* \Psi_{\Delta}^2, \quad (2.22)$$

$$i\hbar \frac{d\Psi_{-2}}{dt} = \hat{E}\Psi_{-2} + V_0 n_0 \Psi_{-2} + W\Psi_{+2}^* \Psi_{\Delta}^2, \quad (2.23)$$

where we define $\Psi_{\Delta}^2 \equiv \Psi_{+1}\Psi_{-1} - \Psi_{+2}\Psi_{-2}$. Eqs. 2.20-2.23 show that the only difference between the bright and dark exciton equations is the sign of the W term describing bright to dark exciton conversion. This symmetry between bright and dark components means that if topologically distinct solutions exist for the bright excitons then the same defects can exist for the dark excitons.

Of main interest are winding number $\mathbf{m} = (m_{+2}, m_{+1}, m_{-1}, m_{-2})$ configurations such as:

$$\{(0, 1, -1, 0), (1, 1, 1, 1), (1, 0, 1, 0)\}, \quad (2.24)$$

which all satisfy Eq. 2.18. If Eq. 2.18 is not satisfied, then there is no observation of an energy minimum for a trapped state of the considered topological defect, cylindrically symmetric or not. Real time propagation revealed that if for example a stable solution of $\mathbf{m} = (0, 1, -1, 0)$ was suddenly switched to $\mathbf{m} = (0, 1, 1, 0)$ by conjugating the Ψ_{-1} component then the solution became immediately non-stationary and the vortex state was destroyed.

The vortices with high topological charges, $|m_{\sigma}| > 1$, were shown to be unstable in single component BECs depending on interaction strength [152]. This holds as well in our case: stable vortex states are no longer observed for $|m_{\sigma}| > 1$. It is however possible to obtain an energy minimum containing multiple single-charged vortices in the system if SOI is taken into account as it will be discussed in the next section.

In Fig. 2.2-2.3 we show four cases of low energy solutions for vortex topological defects in the four-component trapped exciton condensate. Fig. 2.2[a-b] correspond to a vortex-antivortex pair in $\Psi_{\pm 1}$. Fig. 2.2[c-d] corresponds to a trivial vortex composed of two vortex-antivortex pairs in both bright and dark components. Fig. 2.3[a-b] corresponds to a trivial vortex composed of two bright-dark vortex pairs in both bright and dark components both with a π phase difference. Fig. 2.3[c-d] correspond to a bright-dark vortex pair in Ψ_{-1} and Ψ_{+2} components. One can see that for a vortex pair the core stabilizes at a greater healing length due to the nonrotating components trying to fill in the density dips. The densities of bright and dark excitons try to complement each other, staying close to the Thomas-Fermi profile.

The existence of a low energy solution with vortices is determined by a competition between kinetic operator $\hat{E}$ and the nonlinear mixing of the spins. The following analysis determines what types of vortex solutions can form a stable en-

Figure 2.2: Density (left) and phase (right) profiles of the trapped exciton condensate with different initial configurations of vortex solutions. **(a-b):** $\mathbf{m} = (0, 1, -1, 0)$ and $\phi = (0, 0, 0, 0)$. **(c-d):** $\mathbf{m} = (1, -1, 1, -1)$ and $\phi = (0, 0, 0, 0)$. In all pictures: $V_0 = 1 \mu\text{eV} \mu\text{m}^{-2}$ and $W/V_0 = -0.1$.

ergy minima by introducing appropriate ansatz into the dynamical equations. A summary of stable solutions is gathered in Fig. 2.4.

2.3.1 TRIVIAL VORTEX STATE

Let's start with the trivial vortex state where all spin-components are rotating. The ansatz in question can be written,

$$\begin{aligned} \Psi_{+2} &= v e^{i(m_{+2}\theta + \phi_{+2})}, & \Psi_{+1} &= v e^{i(m_{+1}\theta + \phi_{+1})}, \\ \Psi_{-1} &= v e^{i(m_{-1}\theta + \phi_{-1})}, & \Psi_{-2} &= v e^{i(m_{-2}\theta + \phi_{-2})}. \end{aligned}$$

Figure 2.3: Density (left) and phase (right) profiles of the trapped exciton condensate with different configurations of vortex solutions. **(a-b):** $\mathbf{m} = (1, 1, 1, 1)$ and $\phi = (\pi, -\pi, 0, 0)$. **(c-d):** $\mathbf{m} = (1, 0, 1, 0)$ and $\phi = (\pi, 0, 0, 0)$. In top picture: $V_0 = 1 \mu\text{eV} \mu\text{m}^{-2}$ and $W/V_0 = -0.1$. In bottom picture: $W/V_0 = 0.1$.

Here v is some radial density profile (i.e., cylindrically symmetric) which satisfies the coupled dynamical equations. The Hamiltonian density (Eq. 2.8) becomes,

$$\mathcal{H}_{\text{int}} = v^4(8V_0 + 2W(\cos(\Delta\phi) - 1)), \quad (2.25)$$

where $\Delta\phi = \phi_{+2} + \phi_{-2} - \phi_{+1} - \phi_{-1}$. Using Eq. 2.19 we can rewrite this as,

$$\mathcal{H}_{\text{int}} = v^4(8V_0 + 2W(\cos(n\pi) - 1)). \quad (2.26)$$

Thus, when the whole condensate is rotating (note that different components still can have opposite vorticity) there exists two distinct solutions; in-phase ($n = 0$) and anti-phase ($n = 1$). The in-phase solution causes the W term to vanish, making the sign choice of W irrelevant (see Fig. 2.2[c-d] and Fig. 2.3[a-b]). The anti-phase solution however becomes $\mathcal{H}_{\text{int}} = v^4(8V_0 - 2W)$. The sign choice of W then only serves as a shift on the interaction strength of the excitons (i.e., blueshifts or redshifts the solution) but doesn't remove the energy minima of the trivial vortex solution.

2.3.2 TWO-VORTEX STATES

Here we are concerned with vortex states which exists as pairs. It is obvious from Eq. 2.18 that a single vortex or three vortices in the order parameter cannot exist. The following analysis creates a distinction between two different solutions. Vortex-antivortex pair, and Bright-dark vortex pair.

2.3.2.1 Vortex-antivortex pair

Here we look at states of the type $\mathbf{m} = \{0, 1, -1, 0\}$. The ansatz in question can be written,

$$\begin{aligned}\Psi_{+2} &= ue^{i\phi_{+2}}, & \Psi_{+1} &= ve^{i(m_{+1}+\phi_{+1})}, \\ \Psi_{-1} &= ve^{i(m_{-1}+\phi_{-1})}, & \Psi_{-2} &= ue^{i\phi_{-2}}\end{aligned}$$

The Hamiltonian density can then be written as,

$$\mathcal{H}_{\text{int}} = 2V_0(v^2 + u^2)^2 + W \left(2u^2v^2 \cos(n\pi) - u^4 - v^4 \right). \quad (2.27)$$

If $n = 0, 1$ then we get respectively,

$$\mathcal{H}_{\text{int}} = 2V_0(v^2 + u^2)^2 \pm W(u^2 \mp v^2)^2. \quad (2.28)$$

Two different scenarios exist here. If the anti-phase solution is chosen then $\mathcal{H}_{\text{int}} = (2V_0 - W)(v^2 + u^2)^2$ which is similar to the trivial solution. Except, here there exist different kinetic energies between the components with no possible way of being compensated through the nonlinearity of the equations. Thus, the anti-phase solution is always rejected.

The in-phase solution however possesses a balancing effect where W term matches the vortex rotational energy in order to form a stable stationary state. Indeed, the term $(u^2 - v^2)^2$ is maximum at the center of the vortex core and serves to distribute the BEC energy such that it stays the same everywhere. This naturally depends on the sign of W where it can be seen from plugging in the above ansatz into Eqs. 2.20-2.23 that the ± 2 spins gain energy from the W term whereas ± 1 spins lose energy for $W < 0$. This serves as a mechanism to bring all the spins to the same level of energy, thus enabling a steady vortex state (see Fig. 2.2[a-b]). The contrary takes place when $W > 0$ which further separates the dark and bright excitons in energy, with no steady state found.

Figure 2.4: Observed vortex solutions of the four-component exciton condensate which possess a stable energy minimum (in the absence of SOI). Solutions are verified numerically via the imaginary-time-method and by propagation in real-time using the coupled GP-equations (Eqs. 2.9-2.9).

2.3.2.2 Bright-dark vortex pair

Here we look at states of the type $\mathbf{m} = \{1, 0, 1, 0\}$ (or $\{1, 1, 0, 0\}$ for that matter). The ansatz in question can be written,

$$\begin{aligned}\Psi_{+2} &= ue^{i(m_{+2}+\phi_{+2})}, & \Psi_{+1} &= ve^{i\phi_{+1}}, \\ \Psi_{-1} &= ve^{i(m_{-1}+\phi_{-1})}, & \Psi_{-2} &= ue^{i\phi_{-2}}\end{aligned}$$

The Hamiltonian density can then be written as,

$$\mathcal{H}_{\text{int}} = 2V_0(v^2 + u^2)^2 + 2Wu^2v^2 (\cos(n\pi) - 1). \quad (2.29)$$

If the in-phase solution is chosen ($n = 0$) then we retrieve the same density function as for the trivial solution. The same argument then applies as for the vortex-antivortex, that is, the kinetic energy of the vortices cannot be compensated through the nonlinear terms. The anti-phase solution ($n = 1$) gives us $-4Wu^2v^2$ which may stabilize the state, since it provides a reduction of the energy when all components are populated (see Fig. 2.3[c-d]); if one component is depleted then this term can no longer contribute to minimization of the energy.

2.4 CYLINDRICALLY SYMMETRIC GROUND STATE SOLUTIONS UNDER SPIN-ORBIT INTERACTION

When SOI of Rashba or/and Dresselhaus type is included in the Hamiltonian, the analysis of low energy state solutions becomes more tricky. Prior studies in the field

of atomic condensates revealed a plethora of phenomena emerging due to spin-orbit interaction [118, 125]. Indirect-exciton condensates can be expected to show also a great variety in possible low energy solutions with phase separation and density modulations between different spin-components.

Just like at the start of Sec. 2.3, a simple analysis of the dynamical equations reveals the possible setups of winding numbers satisfying cylindrically symmetric stationary solutions (Eq. 2.15). Again, we assume that the angular momentum of the ansatz is an eigensolution of the kinetic operator $\hat{E}$ (i.e., $\hat{E}e^{-im_\sigma\theta} \propto e^{-im_\sigma\theta}$). For only Dresselhaus SOI, the winding numbers result in the following bound (in addition to those given by Eq. 2.18):

$$\begin{aligned} m_{+2} &= 1 + n, & m_{+1} &= n, \\ m_{-1} &= 1 + m, & m_{-2} &= m. \end{aligned} \quad (2.30)$$

where $n, m \in \mathbb{Z}$. On the other hand, if only Rashba SOI is present the bound is:

$$\begin{aligned} m_{+2} &= n, & m_{+1} &= 1 + n, \\ m_{-1} &= m, & m_{-2} &= 1 + m. \end{aligned} \quad (2.31)$$

We limit our consideration in this section to three distinct types of cylindrical vortex configurations for SOI of either Dresselhaus or Rashba type: $\mathbf{m} = (0, 1, -1, 0)$, $(0, -1, 1, 0)$, and $(1, 0, 1, 0)$. Numerics show that in the case where the bounds Eq. 2.30 or Eq. 2.31 are not satisfied the initial topological charge configuration from Eq. 2.14 is not preserved, but instead a steady state configuration of *warped vortices* with a mix of winding numbers distributed amongst the spin components with a complicated density profile appears [21]. Unsurprisingly varying the initial condition allows one to also retrieve a density modulated stripe phase with no vorticity (not shown here). This underlines the sensitivity of the found solution on the initial condition.

For Dresselhaus SOI only the configurations $\mathbf{m} = (0, -1, 1, 0)$ and $\mathbf{m} = (1, 0, 1, 0)$ satisfy Eq. 2.30, and we observe formation of the cylindrically symmetric vortices (see Fig. 2.5[a-b] and Fig. 2.9), whereas $\mathbf{m} = (0, 1, -1, 0)$ does not satisfy the bound, the cylindrical symmetry is no longer present, and configuration with higher winding numbers $\mathbf{m} = (+2, +3, -3, -2)$ is formed (see Fig. 2.5[c-d]). The similar behavior can be observed for the case of the Rashba SOI (see Fig. 2.6) but this time the cylindrical symmetry is manifested for $\mathbf{m} = (0, 1, -1, 0)$ and $(0, 1, 0, 1)$ configurations. This unsurprising symmetry between the Dresselhaus and Rashba condensate solutions comes naturally from the symmetry of the SOI operator $\hat{S}_k$ (Eq. 2.7).

A noteworthy feature seen in Figs. 2.5-2.6 is the sudden radial phase boundary where the phase of the components suddenly undergoes a π transition. This domain

Figure 2.5: Density (left) and phase (right) profiles of the trapped condensate components with different vortex defects as initial condition in the presence of Dresselhaus SOI ($\beta = 1 \mu\text{eV}\mu$). **(a-b):** Initial ansatz was set to $\mathbf{m} = (0, -1, 1, 0)$. Eq. 2.30 is satisfied and cylindrically symmetric vortex type solution is obtained. **(c-d):** Initial ansatz was set to $\mathbf{m} = (0, 1, -1, 0)$. Eq. 2.30 is not satisfied, and as a result warped vortex corresponding to $\mathbf{m} = (+2, +3, -3, -2)$ is formed in the stationary regime. In all pictures: $\phi = (0, 0, 0, 0)$, $V_0 = 22 \mu\text{eV}\mu\text{m}^2$, and $W/V_0 = 0.1$.

wall is associated with the density profile of bright excitons giving way to dark excitons. Currently there is no clear understanding on why this sudden jump in phase takes place under SOI, and whether it can be associated with solitonic phenomenon.

So far, in Figs. 2.5-2.6, we have neglected the phases of each spin component. It is clear that cylindrically symmetric solutions are possible when the global phase of all the components is kept the same, i.e. $\phi = (0, 0, 0, 0)$. However, if one introduces phase difference between the condensate components chosen as an initial condition, another type of the vortex solution corresponding to the spiral phase pattern is obtained (see Fig. 2.7[a-b]). One can see that the discontinuous radial phase boundary is no longer present, making it a unique feature of the case where the spin

Figure 2.6: Density (left) and phase (right) profiles of the trapped condensate components with different vortex defects as initial condition in the presence of Rashba SOI ($\alpha = 1 \mu\text{eV}\mu$). **(a-b):** Initial ansatz was set to $\mathbf{m} = (0, 1, -1, 0)$. Eq. 2.31 is satisfied and cylindrically symmetric vortex type solution is obtained. **(c-d):** Initial ansatz was set to $\mathbf{m} = (0, -1, 1, 0)$. Eq. 2.31 is not satisfied, and as a result warped vortex corresponding to $\mathbf{m} = (-2, -3, +3, +2)$ is formed in the stationary regime. In all pictures: $\phi = (0, 0, 0, 0)$, $V_0 = 22 \mu\text{eV}\mu\text{m}^2$, $W/V_0 = 0.1$.

components share the same phase.

The topological charges of Fig. 2.5 and Fig. 2.7 are the same, and to distinguish between them one can analyze their polarization patterns shown in Fig. 2.7[c]. Since the bright exciton states ($S_z = \pm 1$) are optically active; polarization resolved imaging gives an important insight into the spin-dynamics of the system. For global phase symmetry, we observe a four leaf S_x pattern (Fig. 2.7[c]) whereas if the phase symmetry between the components is broken, we observe a spiral S_x pattern (Fig. 2.7[d]). Such spiral patterns were also analyzed for exciton spin currents in localized bright spots (see Fig. 2[a-b] in Ref. [139]). The complete analysis of these different configurations has yet to be addressed and remains a subject for future investigation.

Figure 2.7: Density (a) and phase (b) profiles of the trapped exciton condensate components for only Dresselhaus SOI. $\mathbf{m} = (0, -1, 1, 0)$, $\phi = (0, \pi, 0, 0)$ and $W = 2 \mu\text{eV}\mu\text{m}^2$, $V_0 = 28 \mu\text{eV}\mu\text{m}^2$, and $\beta = 1 \mu\text{eV}\mu\text{m}$. (c) Linear and diagonal polarization patterns of the cylindrically symmetric cases for Dresselhaus SOI only with $\mathbf{m} = (0, -1, 1, 0)$ and $\phi = (0, 0, 0, 0)$ initial condition corresponding to Fig. 2.5[a-b], the colorscale runs from $[-1 \rightarrow 1]$. (d) Same as frame (c) but now with $\phi = (0, \pi, 0, 0)$, corresponding to the plots (a-b) in this figure.

Also, it should be noted that the sign of exchange interaction W affects the possible states of stable topological defects. To illustrate its role, we focus on a configuration $\mathbf{m} = (0, -1, 1, 0)$, the same as in Fig. 2.5[a-b], but now use a negative mixing interaction parameter: $W/V_0 = -0.1$. We observe a half vortex in a condensate half depleted with a spiral phase pattern resulting from negative W (see Fig. 2.8). The results are clearly different from those shown in Fig. 2.5[a-b] corresponding to opposite sign of the exchange interaction, $W/V_0 = 0.1$. In Fig. 2.9 a half vortex pair solution between the bright and dark excitons is shown with $\mathbf{m} = (1, 0, 1, 0)$ for Dresselhaus SOI (in case of only Rashba it would be $\mathbf{m} = (0, 1, 0, 1)$). The solution remained the same for both signs of the mixing parameter W and was lost when the components phase symmetry was broken.

The case of weak nonlinearities was also investigated where the impact of the SOI terms becomes dominant ($V_0, W \ll \beta, \alpha$). Fig. 2.10 illustrates the case when

Figure 2.8: Density (a) and phase (b) profiles of the exciton condensate components for only Dresselhaus SOI. $m_\sigma = (0, -1, 1, 0)$, $\phi_\sigma = (0, 0, 0, 0)$, $V_0 = 28 \mu\text{eV}\mu\text{m}^2$, $W/V_0 = -0.07$, and $\beta = 1 \mu\text{eV}\mu\text{m}$

Figure 2.9: Density (a) and phase (b) profiles of the exciton condensate half vortex pair for only Dresselhaus SOI. $\mathbf{m}_\sigma = (1, 0, 1, 0)$, $\phi_\sigma = (0, 0, 0, 0)$, $V_0 = 28 \mu\text{eV}\mu\text{m}^2$, $W/V_0 = \pm 0.07$, and $\beta = 1 \mu\text{eV}\mu\text{m}$

Figure 2.10: Density (left) and phase (right) profiles of the trapped exciton condensate components for only Dresselhaus SOI and small nonlinear parameters. **(a-b):** $\mathbf{m} = (1, 0, 1, 0)$. **(c-d):** $\mathbf{m} = (0, -1, 1, 0)$. **(e-f):** $\mathbf{m} = (0, 1, -1, 0)$. In all pictures $\phi = (0, 0, 0, 0)$ and parameters were $\beta = 1 \mu\text{eV}\mu\text{m}$, $V_0 = 2.8 \text{ neV}\mu\text{m}^2$, and $W/V_0 = \pm 0.07$.

only Dresselhaus SOI is present. If the bound (Eq. 2.30) is satisfied the solutions still retain their cylindrical symmetry (Fig. 2.10[a-d]) as one would expect. For the case $\mathbf{m} = (0, 1, -1, 0)$, where a system of warped vortices appeared for the nonlinear regime, now the solution still contains its original vortex configuration but resembles a dipole density profile (see Fig. 2.10[e-f]). In this weakly nonlinear limit the sign of W becomes irrelevant.

2.5 PRESENCE OF BOTH DRESSELHAUS AND RASHBA SPIN-ORBIT INTERACTION

This section will conclude the chapter with a brief numerical analysis on minimum energy solutions when both Dresselhaus and Rashba SOI are present in the system. First thing to be noted is that with $\alpha \neq 0$ and $\beta \neq 0$ the exciton dispersion becomes anisotropic. This is different from the cases of $\alpha = 0$ or $\beta = 0$ where the energy minima of the non-interacting particle system corresponds to a circle in the k -space, but now it corresponds to two fixed points situated along the k -space diagonal [153],

$$\mathbf{k}_0 = \pm \frac{m_e(\alpha + \beta)}{\hbar^2} \frac{(\mathbf{e}_x + \mathbf{e}_y)}{\sqrt{2}}. \quad (2.32)$$

Here $\mathbf{e}_x$ and $\mathbf{e}_y$ are the x and y Cartesian coordinate unit vectors. One can thus

Figure 2.11: Density (a) and phase (b) profile of condensate components for $m_\sigma = (0, 0, 0, 0)$ and $\phi_\sigma = (0, \pi, 0, 0)$, the parameters were $\beta = 1 \mu\text{eV}\mu\text{m}$, $\alpha/\beta = 1/2$, $V_0 = 28 \mu\text{eV}\mu\text{m}^2$ and $W/V_0 = -0.07$.

expect formation of a striped ground state corresponding to the spatial modulation of the density $(e^{i\mathbf{k}_0 \cdot \mathbf{r}} + e^{-i\mathbf{k}_0 \cdot \mathbf{r}}) = 2 \cos(\mathbf{k}_0 \cdot \mathbf{r})$. This is indeed the case as can be seen in Fig. 2.11. As the ground state of the condensate reveals spatial anisotropy, no cylindrically symmetric vortex solutions can be expected to appear in this case.

The stability of the vortex-type versus striped phase solutions depends on the ratio α/β . Fixing the parameters describing nonlinearities as $V_0 = 28 \mu\text{eV}\mu\text{m}^2$ and $W = \pm 2 \mu\text{eV}\mu\text{m}^2$, our numerical analysis shows that for $\alpha/\beta \sim 10^{-3}$ the vortex type solutions shown in Figs. 2.5-2.9 still persist. However, already at $\alpha/\beta \sim 10^{-2}$ all vortex solutions disappear and only stripe phase solutions are stable. This is illustrated in Fig. 2.11, where we have set $\beta = 1 \mu\text{eV}\mu\text{m}$ and $\alpha/\beta = 1/2$ for a spatially uniform condensate as a initial condition of the imaginary time method. In fact, any sort of starting vortex configuration immediately gets wiped away and replaced by this striped condensate pattern.

CHAPTER 3

VORTEX MEMORY TRANSFER IN INCOHERENTLY DRIVEN POLARITON CONDENSATES

We now move on from the indirect-exciton quasiparticle into regime of strong light-matter coupling where the subject of next investigation is the exciton-polariton. As detailed in Sec. 1.2.5, the exciton-polariton is a quasiparticle formed by the strong coupling of a microcavity photon mode to electronic excitations in quantum wells embedded in said microcavity. The most notable features of this composite bosonic particle are its light effective mass (about 4 to 5 orders of magnitude smaller than the electron rest mass) which arises from the photonic component, and strong binary interactions from the excitonic one. Polaritons also have a short lifetime (few orders of picoseconds), which inhibits thermalization to the lattice temperature and into an equilibrium state. However, polariton-polariton scattering processes allow polaritons to relax fast enough into a macroscopically occupied ground state, effectively creating an out-of-equilibrium polariton condensate [86].

The quantum vortex will be a subject of heavy investigation in this chapter. It is already one of the most well studied topological defects in atomic BECs (see Sec. 1.1.4) but recently it has also been observed in both polariton parametric oscillators [154–156] and non-resonantly excited polariton BECs [157, 158]. The topological stability of the quantum vortex makes it a prime candidate for a robust binary memory component, and recent works have considered control of the path of moving vortices [159–164].

The primary objective of this chapter will be to show that stable vortex solutions of charge (winding number) $m = \pm 1$ can be supported by an CW incoherent ring-shaped pump. Furthermore, by making two such spatially separate condensates interact, one can deterministically choose what sort of a vortex configuration will arise in the system. Such pump shapes were considered experimentally [165], demonstrating pattern formation [166, 167], evaporative cooling [168], and vortex-antivortex arrays [169]. Unlike in cases of deterministic vortex generation [170, 171], the vortices that one can predict in incoherently supported systems represent multi-

stability and are challenging to observe experimentally due to the random selection of the vortex sign during condensation, which is averaged to zero in multishot experiments. Nevertheless, one can expect that coherent pulses can deterministically select the vortex sign allowing their detection [172]. Alternatively, it was recently shown that very high quality microcavities can be used to make single shot measurements, allowing vortex detection in ring-shaped traps [173]. Vortex states can be read by using interferometry [157], while other methods have been discussed in Ref. [174], such as detection of the wavevector of polaritons around the vortex core.

Introducing simple potential guides to a microcavity system (see Fig. 3.1), it can be shown numerically that single vortex charges can be both copied or inverted to a second spatially separate ring pump even under a large amount of stochastic noise. This satisfies the standards of copying binary memory, with high fidelity, and of a NOT gate. The choice of whether transferring the same charge ($\pm 1 \rightarrow \pm 1$) or the inverse charge ($\pm 1 \rightarrow \mp 1$) can be controlled by either changing the length scales of the said potential guides or the distance between the ring pumps supporting the two condensates. The work presented here is directly related to the author's previously published results in Ref. [A5].

3.1 THEORETICAL NONEQUILIBRIUM APPROACH

Using mean field theory and assuming the spontaneous formation of the exciton-polariton condensate, an open-dissipative Gross-Pitaevskii (GP) model describes our incoherently pumped condensate coupled with an exciton reservoir. The polariton order parameter $\Psi(\mathbf{r}, t)$, where $\mathbf{r}$ is the 2D in-plane spatial coordinate, is described by a GP-type equation and the exciton reservoir density $\mathcal{N}(\mathbf{r}, t)$ by a rate equation [96].

$$i\hbar \frac{d\Psi}{dt} = \left[-\frac{\hbar^2 \nabla^2}{2m} - \frac{i\hbar\gamma}{2} + V(\mathbf{r}) + \alpha|\Psi|^2 + \left(g_R + \frac{i\hbar R}{2} \right) \mathcal{N} \right] \Psi + P_c(\mathbf{r}, t), \quad (3.1)$$

$$\frac{d\mathcal{N}}{dt} = -(\gamma_R + R|\Psi|^2)\mathcal{N} + P_R(\mathbf{r}), \quad (3.2)$$

where ∇^2 is the 2D Laplacian. That is, the dispersion of the polaritons in the condensed state is taken to be parabolic. Parameters chosen for our system correspond to the experimental results of Ref. [175]. The polariton mass is set to $m = 10^{-4}m_e$ where m_e is the free electron mass. The decay rates for the polariton condensate and exciton reservoir respectively are chosen as $\gamma = 0.033 \text{ ps}^{-1}$ and $\gamma_R = 1.5\gamma$. The polariton-polariton and polariton-reservoir interaction strengths are set to $\alpha = 6 \text{ } \mu\text{eV } \mu\text{m}^2$ and $g_R = 2\alpha$ respectively, and condensation rate to $R = 0.01 \text{ ps}^{-1} \mu\text{m}^2$. $V(\mathbf{r})$ represents any potential patterning of the microcavity, which can be achieved

Figure 3.1: A schematic showing the exciton-polariton microcavity composed of $\lambda/2$ AlAs cavity with a single GaAs quantum well (QW) inside (multiple QWs pose no obstacle of course). Distributed Bragg reflectors (DBR) localize the photonic field within the cavity. The ring-shaped incoherent optical pumps create an exciton reservoir in the QW which in turn generates polaritons which form a vortex state. A grid like pattern of metallic plates helps to guide the polaritons from one pump to another.

by a variety of techniques, such as: reactive ion etching [176–178], mirror thickness variation [179], stress application [83], metal surface deposition [82, 180] or optical means [181]. For the most part, results depend on the incoherent (non-resonant) pumping represented by the pump profile $P_R(\mathbf{r})$. However, in order to demonstrate a degree of control in the considered system, a coherent (resonant) pump $P_c(\mathbf{r}, t)$ is also introduced into the order parameter dynamical equation.

3.2 BISTABILITY OF VORTICES USING INCOHERENT RING-SHAPED PUMPS

Let us first consider the case of a ring-shaped incoherent pump for a uniform system in the absence of potential patterning ($V(\mathbf{r}) = 0$). The ring pump profile is chosen

Figure 3.2: (a) The gray curves show the frequency shift (blue shift), $\Omega(r)$, induced by the exciton reservoir created by the ring shaped incoherent pump. Polaritons typically condense inside the effective trap potential represented by $\Omega(r)$, with frequency marked in red. (b) Density profile $|\Psi|^2$ of the vortex with charge $m = 1$ and its phase profile (c) for $P_{R0} = 1 \text{ ps}^{-1} \mu\text{m}^{-2}$. (d) Bifurcation diagrams of different states supported by the ring pump. Maxima of the polariton density $|\Psi|^2$ versus pump amplitude P_{R0} . Dashed lines represent unstable solutions.

to be described by the function type:

$$P_R(r) = P_{R0} \left(\frac{r}{w_1} \right)^{10} e^{-(r/w_2)^2}, \quad (3.3)$$

where $r = \sqrt{x^2 + y^2}$, and w_1 and w_2 are parameters controlling the functions geometric shape, such as the inner and the outer radius of the ring pump, set to $w_1 = 4 \mu\text{m}$ and $w_2 = 3 \mu\text{m}$. A cross section of the pump can be visualized from the grey shaded area in Fig. 3.2[a]. The reason for choosing this pump profile is through simple trial and error of finding a pump that would support vortex states.

Owing to the many particle interaction effects; the incoherent pump induces a blue shift in the frequencies of the polaritons $\Omega(r) = g_R n_R / \hbar = g_R P_R(r) / (\hbar \gamma_R)$, shown in Fig. 3.2[a]. It is found that the ring-shaped pump supports a stable vortex solution of charge $m = \pm 1$, provided that the pump intensity overcomes some threshold value (Figs. 3.2[b-c]). In general, among these fundamental vortices (with charges $m = \pm 1$) the ring-shaped pump also supports the solutions with other charges including a non-vortex state with $m = 0$ (Fig. 3.2[d]). The latter one is characterized by the formation of the polariton condensate in the center of the ring pump. Similar solutions have been observed recently in Refs. [167, 168] and even showing interesting bistable patterns [182]. A standard linear stability analysis can be performed to prove stability of the solutions. It turns out that non-vortex solu-

tions, as well as vortices with the charge $m = \pm 2$, are unstable for the particular set of parameters of the ring pump used in the model of this chapter (this is in contrast to the stability of $m = \pm 2$ vortices injected in polariton parametric oscillators with small momentum [183, 184]). The fundamental vortices with charges $m = \pm 1$ are the only stable solutions and experience destabilization for very high pumping rate (for $P_{R0} \gtrsim 1.7$ in Fig. 3.2[d]). The vortex solutions with $m = +1$ and $m = -1$ are equivalent and can be used for bistability schemes. Note that the ring shaped pumping is essential; Gaussian shaped pumps are known not to support stable vortices in the steady state [185]. The ring shaped pumping also negates the need for an additional hard parabolic trapping, where vortex solutions are also known to exist [186, 187].

3.3 GENERATING SINGLE VORTEX STATES

The results of vortex generation and vortex transfer presented in this chapter are calculated using different realizations of stochastic noise both as an initial condition and as a weak background noise [188] in order to test the robustness of the results.¹

One starts by creating a vortex with either charge 1 or -1 set deterministically using a coherent pulse $P_c(\mathbf{r}, t)$. Indeed, using only an incoherent pump the vortex charge is chosen spontaneously corresponding to the degeneracy of such rotational solutions in 2D systems. Writing the vortex states with a definite sign can be accomplished with the coherent Gaussian laser pulse applied within the ring pump area, incident at an angle $\mathbf{k}_c$ to the quantum well plane in order to induce a polariton rotation,

$$P_c(\mathbf{r}, \mathbf{k}_c, t, E_c) = P_{c0} \exp \left[- \left(\frac{\mathbf{r} - \mathbf{r}_c}{w_c} \right)^2 - i \left(\frac{E_c t}{\hbar} - \mathbf{k}_c \cdot \mathbf{r} \right) \right]. \quad (3.4)$$

For the results displayed in Fig. 3.3, the pulse energy is set to $E_c = 0.18$ meV (on resonance), width $w_c = 3 \mu\text{m}$, momentum $|\mathbf{k}_c| = 0.46 \mu\text{m}^{-1}$, and amplitude $P_{c0} = 0.3$ meV μm^{-1} . In order to start the circulating flow of polaritons, the coherent pulse is placed a small distance $\mathbf{r}_c$ away from the center of the ring CW pump (yet staying inside the ring) and a clockwise or anticlockwise circulation is created by setting $\mathbf{k}_c$ perpendicular to $\mathbf{r}_c$. This creates an initial current of coherent polaritons which determine the vorticity of the to-be-formed vortex state. The results of creating a stationary vortex state of charge $m = -1$ can be seen stepwise

¹At the time of this work, it was deemed unnecessary to introduce static disorder to the potential landscape due the continuous pumping of the system. However, the author acknowledges that the present results (although shown to be robust) should also be done with disorder present.

in Fig. 3.3. Starting both the coherent- and incoherent pump at the same time, we observe a quick injection of polaritons into the system that starts to rotate. After 50 ps the coherent pump is shut off and the system allowed to reach a stationary state through the balance of incoherent gain and decay. After a few hundred picoseconds the vortex becomes cylindrically symmetric and stable, maintaining its form for as long as the CW ring-shaped pump is applied. Arguably, the activation time of the coherent pump can be reduced for different setups in order to optimize operational time.

Fig. 3.3 furthermore shows the edges of our patterned potential (yellow dashed lines) depicted by $V(\mathbf{r})$. The presence of this potential pattern is of course not necessary in order to generate and support the vortex state. It does however show that the stationary vortex state is not skewed by the corners of this potential but rather settles into a nice cylindrically symmetric shape. In Sec. 3.3.2 we will show that for more restricted potential patterning the vortex state starts to lose its cylindrical symmetry.

The next sub-section will demonstrate that the vortex states can be initialized with this procedure using a range of different coherent pump wavevectors, powers, positions and energies; making the scheme of vortex generation in the exciton-polariton condensate rather promising given the ring-shaped pump profile. It should also be noted that it is quite possible to generate vortex states with similar shaped pumps (i.e., w_1 , w_2 , or even the power factor r^{10} can be varied). But in order to keep things simplistic, the pump profile given by Eq. 3.3 will remain unchanged and only the parameters of the coherent pump will be checked.

3.3.1 DEPENDANCE ON COHERENT PUMP PARAMETERS

To characterize the process of deterministic vortex formation and study the dependence on the parameters $\{\mathbf{r}_c, \mathbf{k}_c, E_c, P_{c0}\}$, let us introduce the fidelity $F(\Psi, \Psi_0)$ of the vortex state created, $\Psi(\mathbf{r}, t)$:

$$F(\Psi, \Psi_0) = \left\langle \frac{|\int \Psi_0^*(\mathbf{r}) \Psi(\mathbf{r}, t) d\mathbf{r}|}{\sqrt{\int |\Psi_0(\mathbf{r})|^2 d\mathbf{r} \int |\Psi(\mathbf{r}, t)|^2 d\mathbf{r}}} \right\rangle, \quad (3.5)$$

where $\Psi_0(\mathbf{r})$ is the target stationary vortex state with charge $m = -1$. When $\Psi(\mathbf{r}, t)$ reaches the same state as $\Psi_0(\mathbf{r})$ then $F(\Psi, \Psi_0) = 1$. Take note that all the results displayed in this sub-section are in a potential free system ($V(\mathbf{r}) = 0$) and all calculations are averaged over with different realizations of stochastic noise.

In Fig. 3.4[a] the results of shifting the coherent pump center $\mathbf{r}_c$ along the x -axis of the system (the incoherent ring pump being centered at the origin) are shown. Different shifts reveal that the vortex stabilizes at the same rate. However, as

Figure 3.3: Polariton density and phase profiles respectively showing the generation of a vortex with charge $m = -1$ by coherent pumping at $\mathbf{r}_c = -3.8\hat{\mathbf{e}}_x \mu\text{m}$ in a potential grid node with $\mathbf{k}_c = 0.46\hat{\mathbf{y}} \mu\text{m}^{-1}$. Edges of the potential guides are outlined by yellow dashed lines. Both the coherent pump and the incoherent ring pump are activated at the same time and after 50 ps the coherent pump is shut off. At 500 ps the polaritons have formed a stable vortex state. Polariton streamlines are plotted along with the phase profiles (blue arrows). Trails of polaritons can be clearly seen as they diffuse away along the guides. The energy of the potential grid is set to 1 meV, and $P_{R0} = 1 \text{ ps}^{-1} \mu\text{m}^{-2}$.

Figure 3.4: Using the same method as in Fig. 3.3 (excluding the potential grid) we calculate the fidelity $F(\Psi, \Psi_0)$ (Eq. 3.5) of the state created $\Psi(\mathbf{r}, t)$ against a stable vortex state $\Psi_0(\mathbf{r})$ with charge $m = -1$ over 20 realizations of stochastic noise. (a) $k_c = 0.46 \mu\text{m}^{-1}$, $P_{c0} = 0.3 \text{ meV } \mu\text{m}^{-1}$ and $E_c = 0.18 \text{ meV}$. (b) $r_c = 3.8 \mu\text{m}$, $P_{c0} = 0.3 \text{ meV } \mu\text{m}^{-1}$ and $E_c = 0.18 \text{ meV}$. (c) $r_c = 3.8 \mu\text{m}$, $k_c = 0.46 \mu\text{m}^{-1}$ and $E_c = 0.18 \text{ meV}$. (d) $r_c = 3.8 \mu\text{m}$, $k_c = 0.46 \mu\text{m}^{-1}$ and $P_{c0} = 0.3 \text{ meV } \mu\text{m}^{-1}$.

one shifts closer to the center of the ring pump ($\mathbf{r}_c \rightarrow 0$) the circulation induced is no longer definite since the tangential momentum of the polaritons injected becomes random (blue line).

Different wavevector magnitudes, where $\mathbf{k}_c$ is oriented along the positive y -axis, have a small effect on the vortex created (see Fig. 3.4[b]). A notable difference is for smaller wavevectors (e.g., $|\mathbf{k}_c| = 0.3 \mu\text{m}^{-1}$) and large wavevectors (e.g., $|\mathbf{k}_c| = 1.2 \mu\text{m}^{-1}$) where longer times, about 100-200 ps more, are needed to reach a steady vortex state. If the wavevector is set to zero, then the charge created is random (blue line), corresponding to a fidelity of 0.5.

We also investigated the dependence on the coherent pumping strength, P_{c0} , in Fig. 3.4[c]. When the pumping strength is weak (blue line) then polaritons generated incoherently from the exciton reservoir will overcome the coherent injection, leading to a slower formation of a random vortex state. When the pump is too strong (orange and black line), the polaritons will overcome the potential set by the incoherent ring pump and diffuse away, making the process unreliable.

Finally, in Fig. 3.4[d] the coherent pump energy, E_c , reveals that the stability of the vortex created is challenged if one deviates too far away from the polariton resonance energy and a vortex of a random charge is created (black line). Near the regime of resonance, it's possible to have vortices with a definite charge but not necessarily the one aimed for (orange line). When close to resonance (blue, purple

and red lines) the choice of a vortex charge becomes definite.

Let us note that when the fidelity is shown as being unity in Fig. 3.4, this value is obtained from averaging over a finite number of noise realizations. We can not rule out the possibility of very rare events that may reduce slightly the fidelity in the limit of a very large set of repetitions. Still, in any case, we can conclude that the fidelity is very close to unity from finite numbers of calculations.

3.3.2 $2\pi/3$ AND π ROTATIONAL SYMMETRIC GUIDE SETUPS

It was shown in Fig. 3.3 that a vortex created in a $\pi/2$ rotationally symmetric guide scheme (4 guides leading away from the vortex) becomes stable and cylindrically symmetric after a few hundred picoseconds. However, if the confinement of the vortex is increased, there is an expected increase in polaritons scattering off the guide walls which affects the stability of the vortex. In Fig. 3.5 we show that a $2\pi/3$ rotationally symmetric guide setup [a,b] one can support a stationary vortex state with only slight deformation, whereas in a π rotationally symmetric guide setup [c,d] their density profile is non-stationary, indicating an unstable vortex state.

Though the vortex density dip and phase singularity can still be observed in Fig. 3.5(c,d), the overall density profile of the state has become severely deformed and any attempts at using it for transferring charge information results in a random charge transferred. Thus π symmetric setups are not favorable for vortex control.

3.4 OPERATIONS WITH VORTEX STATES

In this section, we will see that by using simple potential guides of different length scales one can manipulate the polariton flow pattern, making it possible to copy the same (or inverted) vortex state by activating a second spatially separate incoherent ring pump. Furthermore, if the blue shift of the potential grid is substantially changed then so does the polariton flow pattern in the guides, opening the possibility of controlling the vortex information transfer by having different types of metallic layers on the microcavity. The distance between the two pumps must be chosen such that the vortices can interfere accordingly with each other. If the distance is too small, then polaritons from each pump interfere strongly and the vortex states are lost. If the distance is too great, random noise will overcome the polaritons traveling in the guide.

Fixing the width of the guide at $15\text{ }\mu\text{m}$, the two relevant length scales are the lateral length of the guide between the two ring pumps, D_g , and the distance between

Figure 3.5: (a,b) Density and phase profiles of a stable $m = -1$ vortex created in a $2\pi/3$ rotationally symmetric guide setup. (c,d) Density and phase profiles of an unstable $m = -1$ vortex created in a π rotationally symmetric guide setup. Blue arrows in (b,d) show the polariton streamlines and yellow dotted lines in (a,c) outline the potential guide edge.

Figure 3.6: Schematic of the vortex information transfer setup showing the outline of the potential guide edges (dashed lines) and ring pumps (orange circles). The guide length is defined by D_g and the distance between the ring pump centers is D_p . In the results presented in this chapter; the vertical width of the guide is fixed at $15\ \mu\text{m}$.

the two ring pump centers, D_p as shown in Fig. 3.6. Changing these length scales dramatically affects the diffracted flow pattern of polaritons which can then favor one process above the other even under a large amount of stochastic noise. Of course, the width of the guide could also be used as a variable in that sense. But in order to keep the potential patterning as simple as possible, we stick with a $15\ \mu\text{m}$ guide width.

The method of copying the same or inverted vortex state from one pump to another is as follows: The potential guide is set to the desired length scales (D_p, D_g) and on one side a deterministic vortex state is prepared (this can be achieved by using the coherent pulse as shown in Fig. 3.3). After the first vortex has settled to a steady state, a second ring pump is activated at the other end of the guide with strong random noise in its center as an initial condition. By itself, the second ring pump would normally develop into a vortex state with sign chosen spontaneously as polaritons condense. However, polaritons traveling from the first preset vortex state arrive at the second pump with a definite momentum, which depends on the sign of the first vortex state. As in the case of writing the vortex state with a coherent pulse, these polaritons introduce a preferential direction of flow at the position of the second ring pump through interference, which overcomes the strong polariton noise introduced to the system. This allows the second vortex state to form in a way logically dependent on the state of the first.

Results are presented in Fig. 3.7 where the transfer of charge is shown stepwise in time for two different cases. The yellow dashed lines outline the edges of the guides. Setting $D_g = 30\ \mu\text{m}$ and $D_p = 40\ \mu\text{m}$, a formation of a vortex in the second pump with an inverted charge with respect to the charge of the initial vortex is observed (see Fig. 3.7, right column). This process is named *inverter*, $(m_1 = \pm 1) \rightarrow (m_2 =$

Figure 3.7: **Left column:** Density plots of the copier process taking place at different times. Yellow dashed lines show the edges of the guide. At $t = 300$ ps the transfer is complete and at $t = 1000$ ps the state is nearly stationary. **Right column:** The inverter process taking place at different times. At $t = 400$ ps the transfer is complete and at $t = 1000$ ps the state is nearly stationary. Bottom panels show the phase profiles at $t = 1000$ ps.

∓ 1). In the left column of Fig. 3.7 for $D_g = 35 \mu\text{m}$ and $D_p = 40 \mu\text{m}$, the formation of a vortex in the second pump with the same charge with respect to the charge of the initial vortex is observed. This process is named *copier*, $(m_1 = \pm 1) \rightarrow (m_2 = \pm 1)$. After 1000 ps the system in both cases has become stationary. One can see that in both cases the vortex density lobes become slightly deformed in a dipole manner with a periodic density pattern between them corresponding to the flow pattern of the polaritons between the pumps forming a standing wave solution.

Faster transfer times can be expected in microcavities with shorter polariton lifetime or lighter polariton effective mass. Using these results, we can start with any vortex state in one node (pump spot) and transfer either the inverted or same state to any other node in the grid by controlling the distance between the pump centers at each guide (see supplemental material to Ref. [A5]). Other grid symmetries, e.g., hexagonal, are also feasible setups (see Fig. 3.5[a,b]).

A total of four robust copier and inverter processes were uncovered by varying the length scales D_p and D_g (see Fig. 3.9). For simplicity, the current chapter is restricted to these four cases, while one can expect more processes over a wider range of guide parameters. The copier and inverter from Fig. 3.7 are shown again in Fig. 3.9[c,a] respectively. Fig. 3.9[b] shows a copier with $D_p = 35 \mu\text{m}$ and $D_g = 30 \mu\text{m}$, and Fig. 3.9[d] an inverter with $D_p = 35 \mu\text{m}$ and $D_g = 22.5 \mu\text{m}$. In order to confirm that these processes are robust, the fidelity (Eq. 3.5) of each case is checked and plotted in Fig. 3.8. Each line is calculated for twenty different realizations of stochastic noise. The results show the lines for each process converging to unity indicating that the transfer of charge aimed for takes place with 100% fidelity within

Figure 3.8: Fidelity shown for each process in Fig. 3.9 calculated over twenty realizations of stochastic noise. The results show excellent convergence at unity, confirming the robustness of the vortex transfer.

Figure 3.9: Four different setups which show a completed transfer of charge information after 2.5 ns for the copier (b,c) and the inverter process (a,d). Yellow dashed lines outline the edges of the potential grid. (a) $D_p = 40 \mu\text{m}$ and $D_g = 30 \mu\text{m}$. (b) $D_p = 35 \mu\text{m}$ and $D_g = 30 \mu\text{m}$. (c) $D_p = 40 \mu\text{m}$ and $D_g = 35 \mu\text{m}$. (d) $D_p = 35 \mu\text{m}$ and $D_g = 22.5 \mu\text{m}$.

our simulations.

As a concluding remark for this chapter: The four different processes shown in Fig. 3.9, along with $\pi/2$ and $2\pi/3$ rotationally symmetric setups shown in Fig. 3.3 and Fig. 3.5[a,b] respectively, can offer many different possibilities in creating potential guide systems in order to efficiently manipulate the transfer of polariton vortex charges. Here, an analog of the NOT gate (inverter) has been realized using such guides with vortex states.

However, a big question still remains unanswered, can either a AND and/or OR gates be created using this scheme of guided vortical polariton flow. A complete logic gate architecture cannot be realized except with either NOT and AND (OR) gates, so this becomes a crucial next step to investigate. The AND and OR gates can possibly be realized using the interference of three vortices, two serving as signals and the third being anchored to the system (i.e., ancilla bit). This creates a type of majority vote scenario. Let us imagine that m and n are the signal winding numbers and j is the anchored winding number, then we are looking at an interference which gives the following,

$$(n, m, j) = (1, 1, 1) \rightarrow 1 ; (1, 0, 1) \rightarrow 1 ; (0, 1, 1) \rightarrow 1 ; (0, 0, 1) \rightarrow 0.$$

Such setups will have to await future investigation.

It is also worth to mention again that for simplicity sake, the spin degree of polaritons has been neglected throughout this work. Half-vortices which can form in spinor polariton condensates [19, 158, 189] can possibly offer a wider alphabet for topologically protected spin based logic. Future work should focus on the adaptation of vortex bits for use in cascable logical circuits [190, 191].

CHAPTER 4

ROTATING SPIN TEXTURES IN SPINOR POLARITON CONDENSATES

In this section we will address a recent observation of so called “polariton spin whirls” in a radially expanding polariton condensate formed under non-resonant optical excitation. The work, which can be found in Ref. [A1], was done in collaboration with the research group of Prof. Pavlos Lagoudakis, lead by Dr. Pasquale Cilibrizzi, at the University of Southampton.

As mentioned before, polariton condensates are strictly non-equilibrium systems which can only be realized over a certain threshold excitation intensity where rapid interactions allow polaritons to scatter into the ground state of the lower polariton branch (see Fig. 1.14) effectively creating a BEC. In this experiment, a nonresonant laser pulse excites a small spot in a typical microcavity system (details in Sec. 4.3) creating a reservoir of excitons in the microcavity quantum wells [192]. These excitons then start populating the lower polariton branch through exciton-phonon and exciton-exciton scattering. In fact, the theory applied in this chapter is analogous to the one in Chap. 3 where mean field equations describe the polariton condensate and the exciton reservoir. But here we include the spin degree of the polaritons.

The spin degree of polaritons (see Sec. 1.2.6) is directly related to the polarization of the emitted cavity light where $s_p = \pm 1$ corresponds to the right and left circular polarization of the photons (usually denoted $\sigma_{\pm}$). Information about the spin state of the system can thus be directly accessed by measuring the polarization of the emitted light. In this experiment, a dynamical spin texture in a polariton microcavity is studied for the first time. The appearance of the spin texture can be traced to the *optical spin Hall effect* (OSHE) originally predicted by Kavokin and co-workers in 2005 [193], and has now been observed in both polaritonic [194] and photonic [195] microcavities (see Sec. 4.1 for details).

Real space imaging of polarization- and time-resolved photoluminescence reveal a spiraling polarization pattern in the plane of the microcavity. In order to understand the observed phenomenon, simulations are performed on the spatiotemporal dynamics of a spinor polariton condensate. Numerical results reveal the crucial role of the polariton interactions with the corresponding spinor exciton reservoir. The

importance of these results is highlighted by the fact that one can harness the spin dependent interactions between the exciton reservoir and polariton condensates, allowing for manipulation of spin currents and the realization of dynamic collective spin effects in solid state systems.

It is worth mentioning that spontaneous rotation of the spin textures and breaking of chiral symmetry (associated with BEC phase transitions) has been reported in a spinor BEC with ferromagnetic interactions [196]. Skyrmions and other nontrivial spin structures have also been observed in 2D superfluid Fermi gas [197], topological insulators [198] and magnetic thin film materials [199]. This tremendous interest in exploring the physics of spin textures is motivated by their strong relation with fundamental phenomena, such as the spin Hall effect in semiconductors [200, 201] and spontaneous symmetry breaking in BECs [202], but also by their potential in future applications, such as low-power magnetic data storage [203] and logic devices [204].

4.1 THE OPTICAL SPIN HALL EFFECT

The OSHE is an important mechanism affecting the spin dynamics of polaritons. In short, the effect describes a type of precession of the polariton pseudospin in the plane of microcavities analogous to the presence of a magnetic field, and has received a great deal of attention ever since its prediction [193].

The OSHE is enabled by the energy splitting between transverse-electric and transverse-magnetic (TE-TM) light polarizations [205] and the longitudinal-transverse (LT) splitting of the exciton states inside the microcavity [143]. It should be noted that in a semiconductor microcavity the longitudinal-transverse splitting for polaritons is dramatically increased as compared to that for bare excitons. This is due to the coupling with the cavity mode. The longitudinal exciton is coupled with the TM polarized cavity mode, whilst the transverse exciton is coupled with the TE polarized cavity mode.

The TE-TM splitting arises from the fact that different polarized optical modes will have different phase and penetration depths into the Bragg mirrors sandwiching the quantum wells. The LT splitting of the excitonic states, on the other hand, is mainly due to the long-range exciton exchange interaction and arises from the different alignment of the dipole moments (i.e., exciton states having dipole moments in different directions will have different energies [206]).

For polariton microcavities, the OSHE can be realized as a momentum dependent in-plane effective magnetic field (see orange arrows in Fig. 4.1) analogous to the case of SOI acting on the electron spin in Chap. 2, causing a pseudospin precession (red, green, blue, yellow arrows) [193]. This momentum (wave vector) dependence results

in spin currents propagating over hundreds of microns in both resonant [207] and non-resonant configurations [208]. Taking account of the long range coherence [209] and fast spin dynamics [210] of polaritons, they have been proposed as a potential candidate for the realization of a new generation of spinoptronic devices [64].

In order to visualize the pseudospin precession, the effective magnetic field can be written as follows:

$$\mathbf{B}_{\text{eff}} = \frac{\hbar}{\mu_B g} \boldsymbol{\Omega}_{\text{LT}}(\mathbf{k}), \quad (4.1)$$

where μ_B is the Bohr magneton, $\mathbf{k} = (k_x, k_y)$ is the polariton in-plane wave vector, and g is the effective exciton Zeeman factor (the field $\boldsymbol{\Omega}_{\text{LT}}$ serves simply for brevity). The corresponding component of the polariton Hamiltonian can be written:

$$\hat{H}_{\text{LT}} = \mu_B g (\boldsymbol{\sigma} \bullet \mathbf{B}_{\text{eff}}), \quad (4.2)$$

where m is the polariton effective mass, and $\boldsymbol{\sigma}$ is the Pauli matrix corresponding to the two spin projections. The components of the effective magnetic field can be written [193],

$$\Omega_x = \frac{\Delta_{\text{LT}}}{\hbar k^2} (k_x^2 - k_y^2), \quad \Omega_y = \frac{\Delta_{\text{LT}}}{\hbar k^2} 2k_x k_y, \quad \Omega_z = 0. \quad (4.3)$$

Here, Δ_{LT} is the TE-TM and LT splitting combined. Using polar coordinates in the k -plane, where $k_x = k \cos(\varphi)$ and $k_y = k \sin(\varphi)$, one can rewrite the above into,

$$\Omega_x = \frac{\Delta_{\text{LT}}}{\hbar} \cos(2\varphi), \quad \Omega_y = \frac{\Delta_{\text{LT}}}{\hbar} \sin(2\varphi), \quad \Omega_z = 0. \quad (4.4)$$

Equations 4.4 determine the orientation of the effective magnetic field in the plane of the microcavity as a function of the k -vector orientation.

Using the fact that $\mathbf{k} = -i\nabla$, and assuming that the polariton BEC dispersion is parabolic, and adopting the effective mass approximation, Eq. 4.2 then takes a very simple form:

$$\hat{H}_{\text{LT}} = \frac{\Delta_{\text{LT}}}{k_{\text{LT}}^2} \left(i \frac{\partial}{\partial x} \pm \frac{\partial}{\partial y} \right)^2, \quad (4.5)$$

with Δ_{LT} being TE-TM splitting at the wavevector magnitude k_{LT} .

Using the pseudospin formalism (see Sec. 1.2.5), it is easy to see that the pseudospin $\mathbf{S}_{\mathbf{k}}$ precesses around $\boldsymbol{\Omega}_{\mathbf{k}}$ (see Fig. 4.1) according to,

$$\hbar \frac{d\mathbf{S}_{\mathbf{k}}}{dt} = \mathbf{S}_{\mathbf{k}} \times \boldsymbol{\Omega}_{\text{LT}}(\mathbf{k}). \quad (4.6)$$

This is of course analogous to rotation of the electron spin under Rashba (or Dresselhaus) spin-orbit interaction. The key difference here is that for one full cycle in k -space the pseudospin rotates twice, whereas for SOI the electron spin rotates only once.

Figure 4.1: (a) The TE-TM operator (Eq. 4.5) splits the dispersion of spin-up and spin-down polaritons into two paraboloids of different weight. (b) Sketch of the optical spin Hall effect in k -space. The orange arrows show the effective magnetic field, the other arrows show the gradual rotation of the polariton pseudospin vector along the circle in k -space. The colorscaled arrows corresponds to the rotated polariton Stokes vector ($\mathbf{S}$) starting from a linearly polarized state. The inset shows the projections in the xy plane. (c) Surface plots showing the three Stokes vector components, S_x , S_y , S_z corresponding to X -polarized initial condition being driven by a horizontally polarized nonresonant pump at the system center. The patterns then appear when X -polarized polaritons move from the center with some finite k -vector (colorscale $[-1 \rightarrow 1]$).

4.2 THE RESERVOIR MEAN FIELD MODEL

To accurately model the spin dynamics in the exciton-polariton system, an open-dissipative Gross-Pitaevskii equation (Eq. 4.7) describes the polariton spinor order parameter ($\Psi_{\pm}$), which is then coupled with the exciton reservoir density ($\mathcal{N}_{\pm}$) [96]:

$$i\hbar \frac{d\Psi_{\pm}}{dt} = \left[-\frac{\hbar^2 \nabla^2}{2m} - \frac{i\hbar\gamma}{2} + \alpha |\Psi_{\pm}|^2 + GP_{\pm}(\mathbf{r}, t) + \left(g_R + \frac{i\hbar R}{2} \right) \mathcal{N}_{\pm} \right] \Psi_{\pm} + \hat{H}_{\text{LT}} \Psi_{\mp}, \quad (4.7)$$

$$\frac{d\mathcal{N}_{\pm}}{dt} = -\left(\gamma_R + R|\Psi_{\pm}|^2\right)\mathcal{N}_{\pm} + P_{\pm}(\mathbf{r}, t). \quad (4.8)$$

Here, $\hat{H}_{LT}$ is the same as Eq. 4.5 and causes a mixing between the polariton spins. These equations can model the process of polaritons being generated from a hot exciton reservoir and then scattered into the ground state of the condensate. The blueshift of the condensate due to interactions with excitons is characterized by the parameter g_R . The mass of the polariton is m , polariton and exciton reservoir lifetimes are $\tau = 1/\gamma$ and $\tau_R = 1/\gamma_R$ respectively. It has been shown that the dominant component of interactions between polaritons comes from the Coulomb exchange interaction [75]. In the model, the same-spin polariton interactions strength is characterized by the parameter α whereas interactions between polaritons with opposite spins are neglected since they are typically smaller in magnitude [79] at energies far from the biexciton resonance [211] (as detailed in Sec. 1.2.6). The exciton reservoir is driven by a Gaussian pump, $P(\mathbf{r}, t)$, as in the experiment, and feeds the polariton condensate with a condensation rate (R). An additional pump-induced shift is described by the interaction constant G to take into account other excitonic contribution to the blueshift [96]. The excitation polarization is controlled by the two nonresonant pump terms $P_{\pm}$ (e.g., a linearly polarized pump would correspond to $P_+ = P_-$).

In all the theoretical calculations the following parameters were set to values corresponding to experimentally estimated values: $m = 5 \times 10^{-5}m_0$, $\alpha = 2.4 \mu\text{eV} \mu\text{m}^2$, $g_R = 1.5\alpha$, $G = 4\alpha$, $R = 0.01 \mu\text{m}^2 \text{ps}^{-1}$, $\Delta_{LT}/k_{LT}^2 = 11.9 \mu\text{eV} \mu\text{m}^2$, $\tau = 9 \text{ps}$, $\tau_R = 10 \text{ps}$. Some parameters are verified experimentally whereas some have an ambiguous choice such as R and τ_R which are critical for the exciton reservoir. These values were chosen to give the highest accuracy between experimentally observed and numerically calculated patterns.

4.3 EXPERIMENTAL AND NUMERICAL RESULTS

4.3.1 ELLIPTICALLY POLARIZED EXCITATION

The sample used is a $5\lambda/2$ AlGaAs/GaAs microcavity, composed by 32 (35) top (bottom) distributed Bragg reflectors (DBRs) and 4 triplets of 10 nm thick GaAs QWs. The cavity quality factor is measured to exceed $Q \gtrsim 8000$, with transfer matrix simulations giving $Q = 20000$, corresponding to a cavity photon lifetime $\sim 9 \text{ps}$. The Rabi splitting is 9 meV. This is the the same sample as used in Ref. [208]. All the data presented here are recorded at negative detuning $\Delta = -4 \text{meV}$. Since we

Figure 4.2: Snapshots of the spatio-temporal dynamics of the degree of circular polarization S_z under non-resonant linearly polarised excitation at: (a) 38 ps, (b) 41 ps and (c) 46 ps showing the clockwise rotation of the spin texture within the microcavity plane. (d-f) Theoretical simulations showing the circular Stokes vector of the spin whirls at: (d) 30 ps, (e) 45 ps and (f) 60 ps. Note that the colorscales are different.

work directly with the lower polariton branch in the parabolic regime, this detuning does not play directly into Eq. 4.7.

Under nonresonant elliptically polarized excitation, time and polarization resolved measurements reveal a clockwise rotation of the entire spin texture in the plane of the microcavity at an angular velocity of about 0.11 rad/ps. This is shown in Fig. 4.2[a-c] for the circular polarization $S_z = (I_+ - I_-)/I_{\text{tot}}$, with I_+ and I_- being the measured intensity of the two circular polarization components and $I_{\text{tot}} = I_+ + I_-$. The intensity emitted by the microcavity is time-resolved by using a tomography scanning technique.

At the pump spot position, due to the repulsive interactions between polaritons and the exciton reservoir, the condensate is blueshifted in energy. Outside the pump spot, this potential energy is converted to kinetic energy with an in-plane wavevector (here $k \leq 2.8 \mu\text{m}^{-1}$) determined by the cavity lifetime and the gradient of the potential [188]. Thus, highly focused Gaussian excitation ($\sim 2 \mu\text{m}$ FWHM) at the center of the system, produces a cylindrically symmetric potential that leads to the radial expansion of polaritons in the plane of the microcavity.

In Figure 4.3, the theoretical circular Stokes polarization patterns are plotted for different polarized pump ratios. early circularly $P_-/P_+ = 0.1$ (Fig. 4.3[a]), linearly $P_-/P_+ = 1$ (Fig. 4.3[b]), and elliptically $P_-/P_+ = 0.9$ (Fig. 4.3[c]) polarized pump are shown. Technically, $P_-/P_+ = 0.1$ should also be classified as “elliptical” but in order to distinguish the three cases we name it “nearly circular”. The different patterns obtained highlight their radical dependence on the ratio of generated spin populations. The nearly circularly polarized pump allows for injection of a one spin dominant condensate that due to OSHE evolves to concentric rings of alternating

Figure 4.3: (a-c) Spin textures showing the evolution of the degree of circular polarization S_z after 50 ps in a system excited with (a) nearly circular ($P_-/P_+ = 0.1$), (b) linear ($P_-/P_+ = 1$) and (c) elliptical ($P_-/P_+ = 0.9$) pump polarization. (d-f) Simulated density, energy splitting, and polarization vs time of the polariton condensate and exciton reservoir at the pump center, corresponding to frame (c). Dashed line marks the onset of polariton condensation.

spin [208], as shown in Fig. 4.3[a]. The cylindrically symmetric patterns observed here are due to the fact that the polariton pseudospin, directed along the z -axis in the Poincaré sphere, is here always perpendicular to the effective magnetic field, lying very close on the xy plane. Under linearly polarized pump (Fig. 4.3 [b]) there is no spin imbalance in the exciton reservoir, and the fermionic component of excitons produces strong exchange coupling between bright and dark states that force the condensate to be linearly polarized [76]. In this case, the typical OSHE pattern is

retrieved due to the Stokes vector precessing at 45° to the x, y axis [193]. It has been predicted that under linear excitation the condensate forms a skyrmion pattern [212] and more recently confirmed that this skyrmion pattern is in fact composed of half-skyrmions of spin topological index $n = \pm 0.5$ [213]. Combining the presence of topologically stable half-skyrmions with the rotating spin textures, one can move the half-skyrmions by some angle using an elliptically polarized source beam or an external magnetic field.

For the creation of polarization symmetry breaking textures such as the spin whirls observed here, a spin imbalance is necessary. Although the excitation beam is highly linearly polarized, (extinction ratio higher than $1 : 10^3$), an ellipticity is created due to the high numerical aperture (NA) of the focusing lens. Indeed, the electric field of a linearly polarized beam, when focused by a high-NA objective, acquires non-zero components in the two directions perpendicular to the polarization of the incident field (i.e., at the focal plane the electric field vector sweeps an ellipse) [214, 215]. Thus, the tight focus of a linearly polarized excitation beam, breaks the rotational symmetry of the σ_+ and σ_- polarizations and introduces an ellipticity in the pump spot. The measurements reveal an ellipticity of 10% for the excitation conditions used in the experiment.

In the simulations, a 10% ellipticity is introduced in the linearly polarized pump, i.e., elliptical pulse with ($P_-/P_+ = 0.9$), and one can observe that the circular polarization patterns rotate, as shown in Fig. 4.3[c]. To understand this behavior, we must first consider that polaritons can only be generated in the vicinity of the localized pump spot, which serves as the source for the entire spatial spin pattern. The time-dependent spatial whirl observed in the system is, in fact, a manifestation of varying polarization at the pump spot.

The varying polarization at the pump spot is generated by the ellipticity of the Gaussian pump, which populates one circular component of the reservoir faster than the other. This leads to a splitting $g_R(\mathcal{N}_+ - \mathcal{N}_-)$ of polaritons (Fig. 4.3[e]), which can be thought of as an effective Zeeman splitting at the pump spot. Here, the imbalance between the two populations (Fig. 4.3[d]) induces an effective magnetic field along the z -direction (Ω_z) [77], which causes the precession of the Stokes vector in the Poincaré sphere, as shown schematically in Fig. 4.4[a]. Due to its excitonic nature, Ω_z exists only at the pump spot position where the exciton reservoir is localized. Away from the excitation spot, the polariton pseudospin dynamics is essentially driven by the TE-TM splitting of the polariton mode, represented by an in-plane effective magnetic field, Ω_{LT} [193] (see Fig. 4.4[b]). The combination of these two rotations is at the origin of the polariton spin whirls. The rotating polarization at the source results in the appearance of rotating spiral arms in the spatial distribution of the circular polarization degree, in analogy to the water jets created by a rotating sprinkler head (Figs. 4.2[d-f]). The energy splitting between Ψ_+ and Ψ_- states at

Figure 4.4: The pseudospin vector $S(t)$ (blue arrows) in the Poincaré sphere at: (a) the pump spot and (b) outside the pump. At the pump spot position, (a), $S(t)$ precesses around the z -direction since $|\Omega_z| > |\Omega_{LT}|$. Outside the pump spot, (b), $S(t)$ precesses around Ω_{LT} since $|\Omega_{LT}| > |\Omega_z|$. (c) Time-resolved, spatially integrated measurements of the two circular polarization components (Ψ_+ , red and Ψ_- , blue) PL intensity, normalized and integrated over the area imaged in Figs. 4.2[a-c], i.e., $(460 \times 340) \mu\text{m}^2$. In green we show the time resolved degree of circular polarization S_z averaged over an area $(1.78 \times 1.78) \mu\text{m}^2$, centered at origin. The blue solid circles annotated with (A), (B), (C) refer to the three snapshots of Figs. 4.2[a-c].

the pump spot can also be generated by interactions between polaritons, $\alpha(|\Psi_+|^2 - |\Psi_-|^2)$, where the corresponding precession in linear polarization was previously described [216]. However; due to the high exciton reservoir density measurements reveal that the dominant contribution to the splitting is caused by the exciton reservoir splitting, $g_R(\mathcal{N}_+ - \mathcal{N}_-)$ as is also evident from Fig. 4.3[e].

This spin imbalance, induced by the ellipticity of the pump polarization, results in picosecond scale oscillation in the circular emission (red and blue profile in Fig. 4.4[c]) indicated in literature as features of bosonic stimulation [217, 218]. Experimentally, the rotation of the polarization at the pump spot is confirmed by the area-average of the degree of circular polarization at the pump spot position,

Figure 4.5: (a-c) Theoretical simulations showing the circular Stokes vector S_z of the spin whirls in presence of disorders at (a) 30 ps, (b) 45 ps and (c) 60 ps.

which oscillates between ± 0.1 , as shown in Fig. 4.4[c] (green profile) and coincides with the rotation of the spin textures (Figs. 4.2[a-c]).

In Fig. 4.5, the formation of polariton spin whirls is additionally calculated in presence of disorders, resembling the experimental results shown in Figs. 4.2[a-c]. The disorder potential was generated with 0.05 meV root mean squared amplitude and 1.5 μm correlation length. The theoretical calculations show that, although disorder introduces some additional fine structure, it does not affect the basic spin textures.

4.3.2 CIRCULARLY POLARIZED EXCITATION

The experiment was redone at the same conditions of detuning, power and excitation spot-size but now exciting with a nearly circularly polarized beam. In this case, polariton condensation results in highly imbalanced population and the small ellipticity induced by the tightly focused spot will not play a relevant role as in the case of linearly polarized pump. As a consequence, the imbalance between the two polariton populations is set by the pump and preserved throughout the entire process so that no noticeable oscillations of the polarization appear at the pump spot (Fig. 4.6[d]) and the spin texture does not result in a whirl pattern as shown in Fig. 4.2. Of course, if the beam were perfectly circularly polarized then the $S_x = 0$ at the excitation spot, but from Fig. 4.6[d] one can see the the S_x component fluctuates at the pump spot.

This type of circularly polarized excitation corresponds to setting $P_-/P_+ = 0.1$,

Figure 4.6: (a-c) Real space experimental S_x Stokes parameters at different times for circularly polarized excitation. After the hot excitons relax down on the lower polariton dispersion, polaritons are formed with $k \leq 2.8 \mu\text{m}^{-1}$. (d) Linear degree of polarization versus time calculated by averaging the experimental circular Stokes parameters over the pump area comparable with FWHM of the excitation. The letters (A), (B) and (C) in the graph refer to frames (a-c).

Figure 4.7: Simulation showing the linear Stokes parameters (S_x) at (a) 40 ps, (b) 55 ps and (c) 70 ps for a nearly circular excitation, i.e., $P_-/P_+ = 0.1$. The author would like to point out that the y -axis here is flipped as opposed to Fig. 4.6[a-c].

an imbalance which naturally results in a spin +1 dominant condensate. This imbalance of the pump results in same spin lobes staying connected until the imbalance between the spin populations vanishes (in that case, one retrieves the typical OSHE pattern already observed in Ref. [208]). A good agreement is achieved between numerical and experimental results (see Fig. 4.6 and 4.7) for circular excitation where no whirl dynamics take place in the polarization pattern.¹

4.4 EXCITON RESERVOIR DYNAMICS

This section of the chapter will make an attempt at explaining the differences in the time dynamics observed in experiment and theory from Sec. 4.3.1. It can be seen from theory in Fig. 4.3 that the dynamics of the polariton condensate and exciton reservoir under elliptically polarized nonresonant excitation ($P_-/P_+ = 0.9$) are qualitatively different from experiment (see Fig. 4.4[c]). Starting from a linearly polarized condensate, the evolution of the polarization follows hand in hand with the splitting. When Ψ_+ polaritons are generated faster they deplete the $\mathcal{N}_+$ excitons, causing the density to suddenly drop faster than $\mathcal{N}_-$ and thus changing the sign of the splitting. Numerically, the spin +1 polaritons condense first since they are being pumped at a higher rate due to the ellipticity of the pump. This is displayed as a concentric polarization ring which expands outward. Then, as $S_z = -1$ polaritons condense and the reservoir densities deplete (i.e., the splitting switches from weak positive to strong negative) the spin whirl appears. This is marked by the grey dashed line in Fig. 4.3, corresponding to the Stokes vector reversing its precession around the magnetic field Ω_z .

However, experimentally, there is an equal and steady formation of the spin components (under elliptical pumping) which at their maximum intensity suddenly form this spin whirl (Fig. 4.4[c], green points). The experiment builds up to a certain moment where picosecond polarization oscillations take place at the pump spot, whereas the theory immediately picks up a singular oscillation from the imbalance between the spin components $\Psi_{\pm}$.

In general, these differences are due to reservoir dynamics occurring at the pump spot position which is a weakness of the single reservoir mean-field model [188] at such short time scales. Indeed, parameters such as the saturation rate R and exciton dissipation rate γ_R are taken as purely phenomenological and it is impossible to say accurately what those parameter values are exactly at the onset of condensation.

¹It should be stressed that there still exists a mismatch between theory and experiments in the dynamics of the condensate polarization pattern for the case of a linearly (elliptically) pump. This is addressed in Sec. 4.4

The single reservoir (Eq. 4.8) approach is known to result in an exaggerated depletion of the reservoir, which is emptied once condensation is stimulated. In fact, the model is found to describe more accurately polariton fields in the steady state under continuous excitation where dynamics play little role. An accurate descriptions of polariton condensate dynamics would require the multi-level structure of the reservoir to be accounted for [219, 220]. However, the single reservoir model is able to predict the spatial pattern of the OSHE spin whirl, which is the main focus of this chapter, and its qualitative rotation in time. To avoid using an overly complicated model to describe this effect it is preferable to use the single reservoir model, while sacrificing an exact match to the timescales observed experimentally.

CHAPTER 5

OPTICALLY INDUCED AHARONOV-BOHM EFFECT FOR ELECTRONS AND EXCITONS

The current chapter is devoted to theoretical analysis on a type of strong electron coupling to circularly polarized photons in non-single connected nanostructures. It results in the appearance of an artificial $U(1)$ gauge field changing the electron phase as it traverses the ring structure. The effect arises from the breaking of time-reversal symmetry and is analogous to the well-known Aharonov-Bohm (AB) phase effect (see Sec. 1.3.1). Just like in the case of a normal magnetic field threading the ring, the circularly polarized light can manifest itself in the oscillations of the ring conductance as a function of the intensity and frequency of the illumination. In Sec. 5.2 this effect will also be demonstrated (though less pronounced) for excitonic states of a quantum ring.

Progression in modern nanotechnologies has resulted in rapid developments in the fabrication of mesoscopic objects, including non-single connected nanostructures such as quantum rings. The fundamental interest attracted by these systems is caused by a wide variety of purely quantum-mechanical topological effects which can be observed in ring-like mesoscopic structures. The most notable phenomenon amongst them is the Aharonov-Bohm effect arisen from the direct influence of a vector potential ($\mathbf{A}$) on the electron phase [111, 221]. In the ballistic regime, where the electron phase coherence length is longer than the path of the ring, this effect results in magnetic-flux-dependent oscillations of the conductance in ring-like structures with a period equal to the fundamental magnetic flux quantum $\Phi_0 = h/e$ [113, 114, 222–224]. In the diffusive regime, a second type of conductance oscillations with the period $\Phi_0/2$ can be observed. They are known as the Altshuler-Aronov-Spivak (AAS) oscillations and are associated with the weak localization of electrons [225, 226].

As mentioned in Sec. 1.3.1, one can think of the AB-AAS oscillations arising from the broken time-reversal symmetry in the electron system (for our consideration, a conducting mesoscopic ring) subjected to a magnetic flux through the ring. The similar breaking equivalence of electron motion for mutually opposite directions caused by a magnetic field takes place in various nanostructures, including quantum

wells [227], carbon nanotubes [228], and hybrid semiconductor/ferromagnet nanostructures [229]. However, the time-reversal symmetry can be broken not only by a magnetic flux but also by a circularly polarized electromagnetic field as mentioned above. Indeed, the field breaks the symmetry since time reversal turns clockwise polarized photons into counterclockwise polarized ones and vice versa. As a result, the electron coupling to circularly polarized photons can change electron energy spectrum of single quantum rings [230] and even induces band-gap openings in periodic arrays of quantum rings [231]. Therefore, phenomena similar to the AB effect can take place in ring-like electronic systems interacting with a circularly polarized electromagnetic field. The theory can be regarded as lying at the border between condensed-matter physics and quantum optics.

The chapter is organized in two main sections; firstly to electrons in the QR, and secondly to excitons in the QR. The former demonstrates that the conductance of these electron-photon systems can exhibit oscillations which are formally equivalent to the AB-AAS oscillations induced by a magnetic flux (Sec. 5.1). The phenomenon can be described in terms of an artificial $U(1)$ gauge field generated by the strong coupling between electron and circularly polarized photons. The peer-reviewed published work of the thesis author can be found in Refs. [A2,A4]. In the second part of the chapter (Sec. 5.2), a QR exciton Hamiltonian is introduced to the model where both electron and hole become strongly coupled with the external electromagnetic field. However, due to the neutrality of the exciton, the effect will be less pronounced but none the less observable in the state of the art experiments (see Ref. [A3]).

5.1 FIELD DRESSED ELECTRONS

5.1.1 THE ELECTROMAGNETIC AHARONOV-BOHM FORMALISM

Let us consider the conventional model of an electron interference device (see, e.g., Refs. [232, 233]) consisting of an one-dimensional (1D) mesoscopic ring with radius R and two one-dimensional leads which are connected at the quantum point contacts (see Fig. 5.1[a]). Generally, the phase shift between the clockwise and counterclockwise traveling electron waves,

$$\Delta\phi = \phi_+ - \phi_-, \quad (5.1)$$

can be nonzero: The shift can be caused by the application of an external magnetic field (conventional AB effect) or result from spin-orbit interaction [234–237]. Experimentally, it can be detected by measuring the field-dependent oscillations of the conductance of the device.

Figure 5.1: (a) The scheme of the electron interference device consisting of an 1D mesoscopic ring which is connected with two 1D leads at the quantum point contacts (QPCs). An electron wave, which enters into the device with the amplitude $A = 1$, is split between the two different paths with the transmission amplitudes λ and exits the device with the amplitude C . The phase shift of the electron waves traveling clockwise and counterclockwise inside the ring, $\Delta\phi = \phi_+ - \phi_-$, arises from the coupling to an external electromagnetic field. (b) The scheme of the electron energy spectrum $\varepsilon(k)$ in a mesoscopic ring subjected to a circularly polarized electromagnetic field or a stationary magnetic field. The spectrum is shifted along the k axis by the wave vector k_0 which depends on the parameters of the field.

In order to write the phase shift (Eq. 5.1) as a function of the field parameters, one must consider the electron energy spectrum of an isolated ring subjected to an electromagnetic field with the vector potential $\mathbf{A}$. If the field is time-independent, then the electron energy spectrum can be found from the stationary Schrödinger equation with the Hamiltonian

$$\hat{H}_0 = \frac{1}{2m_e} (\hat{p}_\varphi + eA_\varphi)^2, \quad (5.2)$$

where φ is the electron angular coordinate in the ring, e is the electron charge, and m_e is the effective electron mass in the ring. The operator of electron momentum in the 1D ring is written,

$$\hat{p}_\varphi = -i\frac{\hbar}{R}\frac{\partial}{\partial\varphi}. \quad (5.3)$$

Furthermore, considering the problem within the conventional quantum-field approach [238–240], the classical vector potential in the Hamiltonian (Eq. 5.2), A_φ , should be replaced with the operator, $\hat{A}_\varphi$.

For now, let us revisit the well-known case of a stationary magnetic field, $\mathbf{B}$, threading the quantum ring. Then the electron energy spectrum of the ring has the form,

$$\varepsilon(m) = \frac{\hbar^2}{2m_e R^2} \left(m + \frac{\Phi}{\Phi_0} \right)^2, \quad m = 0, \pm 1, \pm 2, \pm 3 \dots, \quad (5.4)$$

where m is the electron angular momentum along the ring axis, and $\Phi = B\pi R^2$ is the magnetic flux through the ring. In the considered case of a mesoscopic ring, it is convenient to rewrite this spectrum as

$$\varepsilon(k) = \frac{\hbar^2}{2m_e} \left(k + \frac{\Phi}{R\Phi_0} \right)^2, \quad (5.5)$$

where $k = m/R$ is the electron wave vector along the ring. Graphically, the energy spectrum (5.5) can be pictured as a parabola shifted along the k axis by the wave vector,

$$k_0 = -\frac{\Phi}{R\Phi_0} \quad (5.6)$$

(see Fig. 5.1[b]). Formally, just the wave vector (5.6) defines the nonzero phase shift (5.1) since $\Delta\phi = 2\pi Rk_0$.

Any physical phenomenon, which results to such a shifted electron energy spectrum with $k_0 \neq 0$, can generate the oscillations of conductance of the considered electron interference device. However, in the case of a time-dependent electromagnetic field with the vector potential $\hat{A}_\varphi(t)$, the Schrödinger equation with the Hamiltonian (5.52) is non-stationary and cannot be used to find the electron energy spectrum. The regular approach to solve this quantum-mechanical problem should be based on the conventional methodology of quantum optics [239, 240]. Namely, we have to consider the system “electrons in the ring + electromagnetic field” as a whole and to write the Hamiltonian of this electron-photon system. If the field frequency lies far from the resonant frequencies of the electron subsystem (i.e. the field is purely “dressing”), then the energy spectrum of the electron-photon system can be written as a sum of field energy and energy of the electrons strongly coupled to the field (dressed electrons). This energy spectrum of dressed electrons will be responsible for all electron characteristics of the ring subjected to the strong high-frequency electromagnetic field.

The Hamiltonian (5.52) is written as a function of the vector potential $\hat{A}_\varphi(t)$ which depends on the gauge. In order to rewrite the Hamiltonian in gauge invariant form, let us apply the unitary transformation [239],

$$\hat{U}(t) = \exp \left[-\frac{ieR}{\hbar} \int \hat{A}_\varphi(t) d\varphi \right]. \quad (5.7)$$

This is of course completely analogous to Eq. 1.104. Then the transformed electron Hamiltonian (Eq. 5.2) is written,

$$\hat{H}'_0 = \hat{U}^\dagger \hat{H}_0 \hat{U} + i\hbar \hat{U}^\dagger \frac{\partial \hat{U}}{\partial t}, \quad (5.8)$$

and takes the form,

$$\hat{H}'_0(\hat{E}_\varphi) = \frac{\hat{p}_\varphi^2}{2m_e} - eR \int \hat{E}_\varphi d\varphi, \quad (5.9)$$

where, according to Maxwell's equations, $\hat{E}_\varphi = -\partial\hat{A}_\varphi(t)/\partial t$ is the angular component of the electric field which does not depend on the field gauge (see Eq. 1.98). The 1D structure of the ring allows us to neglect any influence from the magnetic component of the EM field (the electric field is polarized along the ring perimeter). Though the influence of stationary electric field on the ring-like structures has been studied (see, e.g., Ref. [241]), phase-shift phenomena caused by a high-frequency field escaped attention before. Then the complete electron-photon Hamiltonian reads

$$\hat{H} = \sum_{\mathbf{q}} \hbar\omega_{\mathbf{q}} \hat{a}_{\mathbf{q}}^\dagger \hat{a}_{\mathbf{q}} + \hat{H}'_0(\hat{E}_\varphi), \quad (5.10)$$

where the first term describes the field energy, $\mathbf{q}$ is the photon wave vector, $\omega_{\mathbf{q}}$ is the photon frequency, $\hat{a}_{\mathbf{q}}^\dagger$ and $\hat{a}_{\mathbf{q}}$ are the photon operators of creation and annihilation respectively, and the summation is assumed to be performed over all photon modes of the electromagnetic field.

5.1.2 THE CIRCULAR ELECTROMAGNETIC DRESSING FIELD

In what follows the theory of ring-electrons coupled to a strong external electromagnetic field is laid out. It should be stressed that here the classification "strong-coupling" is in the sense that the external field is necessarily intense and high in frequency. This is different from the polariton states, discussed in Chap. 3 and Chap. 4, where the strong-coupling happens between long-lived cavity-photons and real transitions of electrons from the valance band. In this chapter, the interaction of the EM field is through the charge of the electron as is evident from Eq. 5.7.

Let us say that the ring is subjected to a monochromatic circularly polarized electromagnetic wave propagating perpendicularly to the ring, and assuming only single photon absorption-reemission, then the Hamiltonian (5.10) takes a simple form,

$$\hat{H} = \hbar\omega \hat{a}^\dagger \hat{a} + \frac{\hat{p}_\varphi^2}{2m_e} + ieR \sqrt{\frac{\hbar\omega}{4\epsilon_0 V_0}} (e^{i\varphi} \hat{a} - e^{-i\varphi} \hat{a}^\dagger), \quad (5.11)$$

where ω is the field frequency, V_0 is the quantization volume, ϵ_0 is the vacuum permittivity, and the fields classical amplitude can be written $E_0 = \sqrt{4\pi N_0 \hbar\omega/V_0}$ with the mean photon occupancy number N_0 . In contrast to the case of a ring interacting with a weak photon mode inside a cavity, [242] an amplitude of the strong field does not depend on the electron-photon interaction. Considering the last terms in the Hamiltonian (5.11) as a perturbation, we can apply the approach

developed in Ref. [230] to solve the electron-photon Schrödinger equation with this Hamiltonian.

To describe the electron-photon coupling in the considered system, we will use the joined electron-photon space $|m, N\rangle = |\psi_m(\varphi)\rangle \otimes |N\rangle$. This corresponds to the electromagnetic field being in the state with the photon occupation number $N = 1, 2, 3, \dots$, and the electron being in the state with the wave function

$$\psi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}, \quad (5.12)$$

where $m = 0, \pm 1, \pm 2, \dots$ is the electron angular momentum along the ring axis analogous to Eq. 5.4. The electron-photon states $|m, N\rangle$ are the true eigenstates of the Hamiltonian of the noninteracting electron-photon system,

$$\hat{H}^{(0)} = \hbar\omega \hat{a}^\dagger \hat{a} + \frac{\hat{p}_\varphi^2}{2m_e}, \quad (5.13)$$

and their energy spectrum is

$$\varepsilon_{m,N}^{(0)} = N\hbar\omega + \frac{\hbar^2 m^2}{2m_e R^2}. \quad (5.14)$$

Considering the last term in the Hamiltonian (Eq. 5.11) as a perturbation and performing trivial calculations within perturbation theory, the matrix elements of $\hat{U}$ will read,

$$\langle m', N' | \hat{U} | m, N \rangle = ieR \sqrt{\frac{\pi \hbar \omega}{V_0}} \left[\sqrt{N} \delta_{m,m'-1} \delta_{N,N'+1} - \sqrt{N+1} \delta_{m,m'+1} \delta_{N,N'-1} \right]. \quad (5.15)$$

The corrected spectrum within the second order of perturbation theory then takes the form,

$$\varepsilon_{m,N} = \varepsilon_{m,N}^{(0)} + \frac{|\langle m+1, N-1 | \hat{U} | m, N \rangle|^2}{\varepsilon_{m,N}^{(0)} - \varepsilon_{m+1,N-1}^{(0)}} + \frac{|\langle m-1, N+1 | \hat{U} | m, N \rangle|^2}{\varepsilon_{m,N}^{(0)} - \varepsilon_{m-1,N+1}^{(0)}}. \quad (5.16)$$

The photon occupancy is set to be macroscopic such that,

$$\sqrt{N_0} = \sqrt{N} \simeq \sqrt{N+1} \gg 1, \quad (5.17)$$

can be safely assumed (strong electromagnetic field). It should be noted that the Hamiltonian (Eq. 5.11) describes electrons in an isolated ring, where the electron lifetime is $\tau \rightarrow \infty$. In the interference device pictured in Fig. 5.1[a], this lifetime is the traveling time of an electron from one QPC to the other one, i.e. $\tau \sim \pi R/v_F$,

where v_F is the Fermi velocity of an electron in the ring. Therefore, the developed theory is consistent if the field frequency, ω , is large enough to satisfy the condition,

$$\frac{\omega\tau}{2\pi} \gg 1 \quad (5.18)$$

which allows one to consider the incident electromagnetic field as a dressing field. The corrected electron spectrum can then be written (Eq. 5, from Ref. [230]),

$$\varepsilon(m) = m^2\varepsilon_R + eE_0R \left[\frac{eE_0R/(2\varepsilon_R)}{(2m - \hbar\omega/\varepsilon_R)^2 - 1} \right], \quad (5.19)$$

where $\varepsilon_R = \hbar^2/2m_eR^2$ is the characteristic energy of the ring. The form of Eq. 5.19 does not offer a clear image of the physical properties of the field dressed electrons. Specifically, the equation breaks down close to the singularities $\hbar\omega = (2m \pm 1)\varepsilon_R = \varepsilon^{(0)}(m+1) - \varepsilon^{(0)}(m)$. In what follows, some useful approximations are applied in order to get the simplest possible form of Eq. 5.19, analogous to Eq. 5.5.

To begin with, the ring is assumed to be mesoscopic such that we can work directly with the electron wavevector along the ring $k = m/R$. The ring can be regarded as large enough to satisfy $\hbar\omega/\varepsilon_R \gg 1$, which means that as long as we stay in the regime of low momenta we will avoid any singular behavior corresponding to the optical absorption of single photons. The corrected spectrum (Eq. 5.19) can then be expanded into Taylor series around small wave vectors $k = 0$,

$$\varepsilon(k) = \varepsilon^{(0)}(k) + \frac{m_e^*R^4e^2E_0^2}{\hbar^2} \left[\frac{1}{a^2 - 1} + a \frac{4kR}{(a^2 - 1)^2} + a^2 \frac{(4kR)^2}{(a^2 - 1)^3} + \dots \right], \quad (5.20)$$

where we have written for brevity $a = \hbar\omega/\varepsilon_R$. Since $a \gg 1$ one can write,

$$\varepsilon(k) = \varepsilon^{(0)}(k) + \frac{m_e^*R^4e^2E_0^2}{\hbar^2} \left[\frac{1}{a^2} + \frac{4kR}{a^3} + \frac{(4kR)^2}{a^4} + \dots \right]. \quad (5.21)$$

The inequality $eE_0/m_eR\omega^2 \ll 1$ must be satisfied in order to stay within the applicability of perturbation theory. Accounting only for the first two terms in the expansion, the corrected spectrum of dressed electrons in the ring can be written as,

$$\varepsilon(k) \simeq \frac{\hbar^2k^2}{2m_e} + \frac{\hbar e^2E_0^2}{2m_e^2R\omega^3}k + \frac{e^2E_0^2}{2m_e\omega^2}, \quad (5.22)$$

Setting the zero energy such that the last term can be discarded, the energy spectrum (Eq. 5.22) has the form plotted in Fig. 5.1[b] with a minimum located at,

$$k_0 = -\frac{e^2E_0^2}{2m_e\hbar R\omega^3}. \quad (5.23)$$

Having reduced the dressed electron spectrum to the parabolic form of Eq. 5.22, one can find that states with the same Fermi energy will obey,

$$\Delta\phi = \pi R \Delta k = \pi \frac{e^2 E_0^2}{\hbar m_e \omega^3}, \quad (5.24)$$

and states with mutually opposite angular momenta,

$$\Delta\varepsilon = |\varepsilon(k) - \varepsilon(-k)| = \frac{\hbar e^2 E_0^2}{m_e^2 R \omega^3} k. \quad (5.25)$$

As one can see, $\Delta\varepsilon$ vanishes as $\sim 1/R$, whereas the phase accumulated, $\Delta\phi$, does not due to the length of the electron path in the ring (πR).

It follows from the comparison of Eqs. 5.6 and 5.23 that the high-frequency electromagnetic field results in the same phase shift (Eq. 5.1) as an effective magnetic flux

$$\Phi_{\text{eff}} = \frac{e\pi E_0^2}{m_e \omega^3}. \quad (5.26)$$

The effective magnetic flux (5.26) can be described in terms of an artificial $U(1)$ gauge field with the vector potential,

$$A_{\varphi}^{\text{eff}} = \frac{\Phi_{\text{eff}}}{2\pi R} = \frac{eE_0^2}{2Rm_e\omega^3}, \quad (5.27)$$

which is produced by the strong electron-photon coupling.

As a concluding remark to this analysis, the asymmetric spectrum corresponding to the field-dressed electron Hamiltonian (Eq. 5.11) has been simplified from Eq. 5.19 to Eq. 5.22. One can of course solve Eq. 5.19 straightforwardly but in the end, there is no loss of generality in Eq. 5.22 which provides a very clear and simple picture of the effect which quantum ring electrons undergo in the presence of strong-light matter coupling. A comparison of Eq. 5.19 and Eq. 5.22 is shown in Fig. 5.2; one can see that as long as the radius R of the ring is kept appreciably large (as approximated in the Taylor expansion of Eqs. 5.20-5.21) the dispersions become comparable.

5.1.3 THE ARTIFICIAL GAUGE FIELD

Eq. 5.27 can also be derived within the adiabatic theorem where the onset of geometric phases takes place for slowly varying Hamiltonians. Of course, one can argue that the high-frequency electromagnetic field is out of the scope of the adiabatic theorem since it should depend rapidly on time. However, as elaborated earlier, the

Figure 5.2: Comparison of Eq. 5.19 and Eq. 5.22 for two different ring radii. In (a,c,e) $R = 1 \mu\text{m}$, and in (b,d,f) $R = 10 \mu\text{m}$. (a,b) The corrected spectrums are compared against the bare electron spectrum. (c,d) The splitting of electrons traveling in the ring with mutually opposite angular momenta. Note that the axes scales are different, indeed, the splitting reduces when the ring size is increased. (e,f) Phase difference between electrons traveling along opposite paths of the ring (here they travel the distance πR) for different electron Fermi energies. Here $m_e = 0.1m_0$, $I_0 = |E_0|^2 \epsilon_0 c = 0.5 \text{ W/cm}^2$, and $\omega = 100 \text{ GHz}$.

high-frequency field is taken as a purely dressing field in which its presence can be regarded as stationary with only the appearance of broken time reversal symmetry.

In order to derive the effective vector potential as a geometric phase, one must derive corrected eigenstates of the Hamiltonian (Eq. 5.11). Within the second order of perturbation theory, one arrives at

$$\begin{aligned} |\Psi_{m,N}\rangle = & |m, N\rangle + \frac{\langle m+1, N-1|\hat{U}|m, N\rangle}{\varepsilon_{m,N}^{(0)} - \varepsilon_{m+1,N-1}^{(0)}} |m+1, N-1\rangle \\ & + \frac{\langle m-1, N+1|\hat{U}|m, N\rangle}{\varepsilon_{m,N}^{(0)} - \varepsilon_{m-1,N+1}^{(0)}} |m-1, N+1\rangle. \end{aligned} \quad (5.28)$$

Substituting Eqs. 5.14–5.15 into Eq. 5.28 and assuming the electromagnetic field to be strong ($N \gg 1$), we arrive at the expression

$$|\Psi_{m,N}\rangle = |m, N\rangle + \frac{ieRE_0}{2} \left[\frac{|m+1, N-1\rangle}{\hbar\omega - \varepsilon_R(1+2m)} + \frac{|m-1, N+1\rangle}{\hbar\omega + \varepsilon_R(1-2m)} \right]. \quad (5.29)$$

Taking into account Eq. 5.12, we can rewrite the basis electron-photon states as,

$$|m \pm 1, N\rangle = e^{\pm i\varphi} |m, N\rangle. \quad (5.30)$$

Then the new corrected eigenstates take the form,

$$|\Psi_{m,N}\rangle = |m, N\rangle + \frac{ieRE_0}{2} \left[\frac{e^{i\varphi}|m, N-1\rangle}{\hbar\omega - \varepsilon_R(1+2m)} + \frac{e^{-i\varphi}|m, N+1\rangle}{\hbar\omega + \varepsilon_R(1-2m)} \right]. \quad (5.31)$$

In the basis of the three electron-photon states,

$$\begin{pmatrix} |m, N+1\rangle \\ |m, N\rangle \\ |m, N-1\rangle \end{pmatrix} \quad (5.32)$$

the eigenstate (Eq. 5.31) can be written formally as a vector

$$|\Psi_{m,N}\rangle = \begin{pmatrix} \frac{ieRE_0/2}{\hbar\omega + \varepsilon_R(1-2m)} e^{-i\varphi} \\ 1 \\ \frac{ieRE_0/2}{\hbar\omega - \varepsilon_R(1+2m)} e^{i\varphi} \end{pmatrix}. \quad (5.33)$$

It should be noted that each of the basis states corresponds to the same electron angular momentum m . Therefore, the influence of the electromagnetic field on the

electron results only in the phase incursion describing by the exponential factors $e^{\pm i\varphi}$ in the state vector (see Eq. 5.33).

According to the adiabatic theorem, the geometric phase which arises when varying a Hamiltonian slowly can be written as a path integral through parameter space,

$$g = i \oint \langle \psi | \nabla_{\mathbf{R}} | \psi \rangle d\mathbf{R}. \quad (5.34)$$

Here, $\mathbf{R}$ is a slowly varying parameter of the Hamiltonian in question and ψ are the eigenstates of the initial (unvaried) Hamiltonian. In the case of the Aharonov-Bohm effect, the adiabatically varying parameter is the magnetic field enclosed by the two interference paths of the quantum ring (i.e. the variation is directly related to the coordinate vector of the ring path). The eigenstates of the Hamiltonian, Eq. 1.101, can be then directly associated with the real vector potential of the AB problem through Eq. 5.34,

$$-\frac{ie}{\hbar} \mathbf{A} = \langle \psi_m | \nabla_{\mathbf{r}} | \psi_m \rangle, \quad (5.35)$$

where $\mathbf{r}$ is the coordinate vector of the ring path taken by the ring electrons. In the case of the circularly polarized light strongly coupled to the ring electrons, the appearance of an effective vector potential can be realized using the corrected states $|\Psi_{m,N}\rangle$ of Eq. 5.11,

$$-\frac{ie}{\hbar} \mathbf{A}^{\text{eff}} = \langle \Psi_{m,N} | \nabla_{\mathbf{r}} | \Psi_{m,N} \rangle. \quad (5.36)$$

For a 1D quantum ring, the effective vector potential has the form $\mathbf{A}^{\text{eff}} = (0, 0, A_{\varphi}^{\text{eff}})$, where

$$-\frac{ie}{\hbar} A_{\varphi}^{\text{eff}} = \frac{1}{R} \left\langle \Psi_{m,N} \left| \frac{\partial}{\partial \varphi} \right| \Psi_{m,N} \right\rangle. \quad (5.37)$$

Substituting Eq. 5.33 into Eq. 5.37, we arrive at the expression

$$A_{\varphi}^{\text{eff}} = \frac{\hbar e R E_0^2}{4} \left[\frac{1}{(\hbar\omega + \varepsilon_R(1 - 2m))^2} - \frac{1}{(\hbar\omega - \varepsilon_R(1 + 2m))^2} \right]. \quad (5.38)$$

Let's keep in mind that $\hbar\omega \gg \varepsilon_R$. Taylor expanding Eq. 5.38 around $\varepsilon_R = 0$ (large ring approximation) gives us,

$$\begin{aligned} A_{\varphi}^{\text{eff}} &= \frac{\hbar e R E_0^2}{4} \left[\left(\frac{1}{(\hbar\omega - 2m\varepsilon_R)^2} + \frac{2\varepsilon_R}{(\hbar\omega - 2m\varepsilon_R)^3} + \dots \right) \right. \\ &\quad \left. - \left(\frac{1}{(\hbar\omega - 2m\varepsilon_R)^2} - \frac{2\varepsilon_R}{(\hbar\omega - 2m\varepsilon_R)^3} + \dots \right) \right] \\ &\simeq \frac{\hbar e R E_0^2}{4} \frac{4\varepsilon_R}{(\hbar\omega - 2m\varepsilon_R)^3}. \end{aligned} \quad (5.39)$$

Keeping in mind that we want to work with small wave vectors (corresponding to the Taylor expansion in Eq. 5.20) so we can easily justify that $\hbar\omega - 2m\varepsilon_R \approx \hbar\omega$. We then have,

$$A_\varphi^{\text{eff}} = \frac{eE_0^2}{2Rm_e\omega^3}, \quad (5.40)$$

which is exactly the same as the artificial vector potential from Eq. 5.27.

5.1.4 CONDUCTANCE OSCILLATIONS IN BALLISTIC AND DIFFUSIVE REGIMES

In order to investigate the oscillations of conductance in the quantum ring, one can simply replace the magnetic flux Φ with the pseudo-flux (Eq. 5.26) in known expressions which describe these oscillations of the considered interference device. In what follows, I will explain the standard formalism from which these equations are derived from.

First of all, let us consider the ballistic regime. In this case, the conductance is described by the Landauer formula,

$$G = \frac{2e^2}{h} |C|^2, \quad (5.41)$$

where the transmission amplitude of the interference device, C (see Fig. 5.1[a]), depends on the coupling between the leads and the ring. Generally, this coupling can be described by lead-to-ring and ring-to-lead transmission amplitudes, λ , within the scattering matrix formalism [232, 233]. If the reflection from one lead to itself is absent (i.e., there is no electron backscattering from QPCs), the transmission amplitude is $\lambda = \pm 1/\sqrt{2}$. This corresponds to the incoming electron wave being divided equally in the ring along the clockwise (ϕ_+) and counterclockwise (ϕ_-) paths (see Fig. 5.1[a]). In this simplest case, the replacement $\Phi \rightarrow \Phi_{\text{eff}}$ in the expression describing the AB-oscillations [233] yields

$$G = \frac{2e^2}{h} \left[1 - \left| \frac{\sin^2(\Phi_{\text{eff}}/2\Phi_0)}{1 - \exp(i2\pi Rk_F) \cos^2(\Phi_{\text{eff}}/2\Phi_0)} \right|^2 \right], \quad (5.42)$$

where k_F is the Fermi electron wave vector in the ring. In Fig. 5.1 results are displayed for different values of the transmission amplitudes λ . Unlike nice result given by Eq. 5.42, here one must resort to numerical methods. The results are displayed as a function of field intensity $I_0 = |E_0|^2 \epsilon_0 c$ instead of field amplitude for greater clarity.

For absolutely transparent QPCs ($\lambda = 0.707$), the regular AB-like oscillations take place (see Fig. 5.3[a]). Decreasing the transparency (decreasing λ) changes

Figure 5.3: Conductance of a mesoscopic ring, G , under a circularly polarized electromagnetic wave as a function of wave intensity I_0 and wave frequency ω . Plots (a), (b) and (c) correspond to different transmission amplitudes λ between the current leads and the ring. Frames in the left column are fixed at the wave frequency $\omega = 100$ GHz. In all plots, the ring parameters are assumed to be $R = 10 \mu\text{m}$, $\varepsilon_F = 10 \text{ meV}$, and $m_e = 0.1 m_{e0}$, where m_{e0} is the mass of free electron.

Figure 5.4: Weak-localization correction to the conductance of a mesoscopic ring, ΔG , under a circularly polarized electromagnetic field: **(a)** the correction is plotted as a function of field intensity I_0 for different values of L_φ with $\omega = 100$ GHz and $R = 10 \mu\text{m}$; **(b)** the correction is plotted as a function of field intensity I_0 and of field frequency ω for $L_\varphi = 3R$ and $R = 10 \mu\text{m}$.

the shape of the oscillation pattern (see Figs. 5.3[b-c]). In the Fourier spectrum of the conductance, the role of the higher harmonics increases, and eventually these harmonics with a half period become dominant (see Fig. 5.3[c]). Physically, this reduction of the period arises from an increased confinement of electrons inside the ring, caused by the decrease of transparency of the QPCs. This leads to an increase of the role of round trips of an electron inside the ring, which results in the increment of the effective electron path and, as a consequence, decrease of the period of the oscillations.

In the diffusive regime, the conductivity of a disordered ring-shaped conductor with the dephasing length L_φ can be described by the expression,

$$\Delta\sigma = -\frac{e^2 L_\varphi}{\pi^2 h} \frac{\sinh(2\pi R/L_\varphi)}{\cosh(2\pi R/L_\varphi) + \cos(4\pi\Phi_{\text{eff}}/\Phi_0)}, \quad (5.43)$$

which is derived from the conventional theory of AAS-oscillations [225] by the replacement $\Phi \rightarrow \Phi_{\text{eff}}$.

The weak-localization correction to the conductance, $\Delta G = \Delta\sigma/\pi R$, is plotted in Fig. 5.4 for different values of the dephasing length L_φ . As expected, the correction oscillates with a period which is less than the period of AB-like oscillations (Fig. 5.3[a]) by a factor of 2. As for the amplitude of the oscillations, it decays exponentially when the dephasing length L_φ is much smaller than the distance between the QPCs, πR . Physically, this decay is caused by the electron waves losing their coherence quickly. It should be noted that an electromagnetic field can cause additional decoherence of electrons in conducting systems [243–245] and, therefore, influences on the dephasing length L_φ . Plotting the correction to the conductance, ΔG , in Fig. 5.4, we assumed the field to be high-frequency enough to neglect this effect.

5.2 FIELD DRESSED EXCITONS

Since the AB effect takes place for both a single electron and many-particle quantum states [246], it can be observed for elementary excitations in semiconductor nanostructures as well. The simplest of them is an exciton which is a bound quantum state of a negative charged electron in the conduction band and a positive charged hole in the valence band. Manifestations of various excitonic effects in semiconductor ring-like structures, including the AB effect induced by a magnetic field, have attracted great attention of both theorists [247–255] and experimentalists [256–259].

This section is devoted to the same theory as detailed in Sec. 5.1, however, the particle of interest here will be the exciton state which arises in semiconductor

materials where the attractive Coulomb forces between the electron- and the hole wavefunctions create a bound pair. Different from Sec. 5.1 where the electron conductance was calculated, here we restrict our attention only to the energy difference of the lowest angular momenta eigenstates of the exciton in the QR. As we will soon see, the correction to the exciton dispersion cannot be reduced to the same simplistic form as Eq. 5.5. Thus, we limit the analysis to only the induced splitting between excitons traveling along opposite paths in the QR.

5.2.1 THE EXCITON RING MODEL

An electron-hole pair in a one-dimensional quantum ring (see Fig. 5.5) can be described by the following Hamiltonian,

$$\hat{H}_0 = -\frac{\hbar^2}{2m_h R^2} \frac{\partial^2}{\partial \varphi_h^2} - \frac{\hbar^2}{2m_e R^2} \frac{\partial^2}{\partial \varphi_e^2} + V(\varphi_e - \varphi_h), \quad (5.44)$$

where R is the radius of the ring, $m_{e,h}$ are the effective masses of an electron (hole) in the ring, $V(\varphi_e - \varphi_h)$ is the potential energy of hole-electron interaction, and $\varphi_{e,h}$ are the azimuthal angles of the electron (hole) in the ring. Introducing the new center of mass variables,

$$\varphi = \frac{m_e \varphi_e + m_h \varphi_h}{m_e + m_h}, \quad \theta = \varphi_e - \varphi_h, \quad (5.45)$$

the derivatives take the following form,

$$\frac{\partial}{\partial \varphi_e} = \frac{\partial}{\partial \varphi} \frac{\partial \varphi}{\partial \varphi_e} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial \varphi_e} = \frac{m_e}{M} \frac{\partial}{\partial \varphi} + \frac{\partial}{\partial \theta} \quad (5.46)$$

$$\frac{\partial}{\partial \varphi_h} = \frac{\partial}{\partial \varphi} \frac{\partial \varphi}{\partial \varphi_h} + \frac{\partial}{\partial \theta} \frac{\partial \theta}{\partial \varphi_h} = \frac{m_h}{M} \frac{\partial}{\partial \varphi} - \frac{\partial}{\partial \theta}. \quad (5.47)$$

where $M = m_e + m_h$ is the exciton mass, and $\mu = m_e m_h / M$ is the reduced exciton mass. The Hamiltonian (Eq. 5.44) can then be rewritten as,

$$\hat{H}_0 = -\frac{\hbar^2}{2MR^2} \frac{\partial^2}{\partial \varphi^2} - \frac{\hbar^2}{2\mu R^2} \frac{\partial^2}{\partial \theta^2} + V(\theta). \quad (5.48)$$

Using separation of variables, the eigenfunctions of the stationary Schrödinger equation with the Hamiltonian (Eq. 5.48) have the form,

$$\psi_{n,m}(\varphi, \theta) = \chi_n(\theta) \frac{e^{im\varphi}}{\sqrt{2\pi}}, \quad (5.49)$$

Figure 5.5: Sketch of an exciton-field system in a quantum ring under consideration. The exciton coupling to the circularly polarized electromagnetic field results in physical nonequivalence of exciton states corresponding to clockwise and counterclockwise rotations of the exciton as a whole along the ring (shown by the black arrows). These exciton states are described by mutually opposite angular momenta m and $-m$ along the ring axis.

where the function $\chi(\theta)$ meets the Schrödinger equation

$$-\frac{\hbar^2}{2\mu R^2} \frac{\partial^2 \chi_n(\theta)}{\partial \theta^2} + V(\theta) \chi_n(\theta) = \varepsilon_n \chi_n(\theta), \quad (5.50)$$

$m = 0, \pm 1, \pm 2, \dots$ is the exciton angular momentum along the ring axis, $n = 0, 1, 2, \dots$ is the principal quantum number of the exciton bound state, and ε_n is the exciton binding energy. Correspondingly, the full energy of exciton reads as

$$\varepsilon_{n,m} = \varepsilon_n + \frac{\hbar^2 m^2}{2MR^2}, \quad (5.51)$$

where the second term is the kinetic energy of rotational motion of exciton in the ring.

If now the ring is subjected to a circularly polarized electromagnetic wave with the frequency ω , which propagates along the ring axis (see Fig. 5.5). Then the full Hamiltonian of the exciton-photon system, including both the field energy, $\hbar\omega\hat{a}^\dagger\hat{a}$, and the exciton Hamiltonian, $\hat{H}_0$, is

$$\hat{H} = \hbar\omega\hat{a}^\dagger\hat{a} + \hat{H}_0 + \hat{U}, \quad (5.52)$$

where $\hat{a}$ and $\hat{a}^\dagger$ are the operators of photon annihilation and creation, respectively, written in the Schrödinger representation (the representation of occupation num-

bers), and $\hat{U}$ is the operator of exciton-photon interaction. Generalizing the operator of electron-photon interaction in a quantum ring [230] for the considered case of electron-hole pair, we can write this operator as

$$\hat{U} = \frac{ieR}{2} \sqrt{\frac{\hbar\omega}{\epsilon_0 V_0}} \left[(e^{-i\varphi_h} - e^{-i\varphi_e}) \hat{a}^\dagger + (e^{i\varphi_e} - e^{i\varphi_h}) \hat{a} \right], \quad (5.53)$$

where e is the electron charge, V_0 is the quantization volume, and ϵ_0 is the vacuum permittivity. To describe the exciton-photon system, let us use the notation $|n, m, N\rangle$ which indicates that the electromagnetic field is in a quantum state with the photon occupation number $N = 1, 2, 3, \dots$, and the exciton is in a quantum state with the wave function described by Eq. 5.49. The electron-photon states $|n, m, N\rangle$ are then the true eigenstates of the Hamiltonian,

$$\hat{H}_R^{(0)} = \hbar\omega \hat{a}^\dagger \hat{a} + \hat{H}_0, \quad (5.54)$$

which describes the non-interacting exciton-photon system. Correspondingly, their energy spectrum is

$$\varepsilon_{n,m,N}^{(0)} = N\hbar\omega + \varepsilon_{n,m}. \quad (5.55)$$

Just as detailed in Sec. 5.1.2, we can find the corrected energy spectrum of the full electron-photon Hamiltonian (Eq. 5.52), by utilizing conventional perturbation theory, considering the term $\hat{U}$ as a perturbation with the matrix elements $\langle n', m', N' | \hat{U} | n, m, N \rangle$. Taking into account in Eq. 5.53 that $\varphi_e = \varphi + m_h \theta / M$ and $\varphi_h = \varphi - m_e \theta / M$. The matrix elements can then be written as,

$$\langle n', m', N' | \hat{U} | n, m, N \rangle = eR \sqrt{\frac{\hbar\omega}{\epsilon_0 V_0}} \left[I_{n'n}^* \sqrt{N} \delta_{m,m'-1} \delta_{N,N'+1} - I_{n'n} \sqrt{N+1} \delta_{m,m'+1} \delta_{N,N'-1} \right], \quad (5.56)$$

where the charge neutral nature of the exciton can be described with the following matrix element,

$$I_{n'n} = \int_{-\pi}^{\pi} \chi_{n'}^*(\theta) \chi_n(\theta) \exp\left(-i\theta \frac{m_h - m_e}{2M}\right) \sin\left(\frac{\theta}{2}\right) d\theta. \quad (5.57)$$

Eq. 5.57 is in general a small quantity since it evaluates the ‘spread’ of the wavefunction (i.e., the exciton is not truly a neutral particle but a dipole). Performing trivial calculations within the second order of the perturbation theory, we can derive eigenenergies of the exciton-photon Hamiltonian (Eq. 5.52),

$$\begin{aligned} \varepsilon_{n,m,N} = \varepsilon_{n,m,N}^{(0)} + \sum_{n'} & \left[\frac{|\langle n', m+1, N-1 | \hat{U} | n, m, N \rangle|^2}{\varepsilon_{n,m,N}^{(0)} - \varepsilon_{n',m+1,N-1}^{(0)}} \right. \\ & \left. + \frac{|\langle n', m-1, N+1 | \hat{U} | n, m, N \rangle|^2}{\varepsilon_{n,m,N}^{(0)} - \varepsilon_{n',m-1,N+1}^{(0)}} \right]. \end{aligned} \quad (5.58)$$

Since Eq. 5.58 is derived within the second order of the perturbation theory, it describes the problem correctly if the energy differences in denominators of all terms lie far from zero. In what follows, we have to keep in mind that all parameters of the problem must lie far from these resonant points (i.e., we want neglect any chance of optical absorption).

The energy spectrum of exciton-photon system (Eq. 5.58) can be written formally as $\varepsilon_{n,m,N} = N\hbar\omega + \tilde{\varepsilon}_{n,m,N}$, where the first term is the field energy. Following the conventional terminology of quantum optics [239, 240], the second term, $\tilde{\varepsilon}_{n,m,N}$, is the energy spectrum of the exciton dressed by the circularly polarized field (dressing field). As before, we restrict our analysis to the most interesting case of classically strong dressing field ($N \gg 1$), where we arrive from Eq. 5.58 to the sought energy spectrum of the dressed exciton state,

$$\tilde{\varepsilon}_{n,m} = \sum_{n'} \left[\frac{(eE_0R)^2 |I_{nn'}|^2}{\varepsilon_{n,m} - \varepsilon_{n',m+1} + \hbar\omega} + \frac{(eE_0R)^2 |I_{nn'}|^2}{\varepsilon_{n,m} - \varepsilon_{n',m-1} - \hbar\omega} \right], \quad (5.59)$$

where $E_0 = \sqrt{N\hbar\omega/\epsilon_0 V_0}$ is the classical amplitude of electric field of the electromagnetic wave. It is apparent that dressed exciton states with mutually opposite angular momenta, m and $-m$, have different energies. Physically, this should be treated as a field-induced nonequivalence of clockwise and counterclockwise exciton rotations in the ring. As a consequence, the excitonic Aharonov-Bohm effect induced by the circularly polarized field appears. In order to simplify the calculation of the field-induced splitting,

$$\Delta\tilde{\varepsilon}_{n,m} = \tilde{\varepsilon}_{n,m} - \tilde{\varepsilon}_{n,-m}, \quad (5.60)$$

we will restrict our consideration to the case of the ground exciton state with $n = 0$. Let us assume that the binding energy of exciton, $e^2/4\pi\epsilon_0 R$, is much more than both the characteristic energy of rotational exciton motion, $\hbar^2|m|/2MR^2$, and the photon energy $\hbar\omega$. Then we can neglect the field-induced mixing of exciton states with $n' \neq 0$ in Eq. 5.59. As a result, we arrive from Eq. 5.59 to the field-induced splitting of exciton states with mutually opposite angular momenta,

$$\begin{aligned} \Delta\tilde{\varepsilon}_{0,m} &= \left| \int_{-\pi}^{\pi} |\chi_0(\theta)|^2 \sin\left(\frac{m_h - m_e}{2M} \theta\right) \sin\left(\frac{\theta}{2}\right) d\theta \right|^2 \\ &\times \left[\frac{2\hbar\omega(q_e E_0 R)^2}{\varepsilon_R^2(1 - 2m)^2 - (\hbar\omega)^2} - \frac{2\hbar\omega(q_e E_0 R)^2}{\varepsilon_R^2(1 + 2m)^2 - (\hbar\omega)^2} \right], \end{aligned} \quad (5.61)$$

where $\varepsilon_R = \hbar^2/2MR^2$ is the characteristic energy of exciton rotation. In order to calculate the integral in Eq. 5.61, we have to solve the Schrödinger equation (Eq. 5.50)

and find the wave function $\chi_0(\theta)$. Approximating the electron-hole interaction potential in Eq. 5.50 by the delta-function,

$$V(\theta) = -U\delta(\theta), \quad (5.62)$$

where U is the Coulomb energy, and assuming the characteristic exciton size, $a_X = \hbar/\sqrt{8\mu\varepsilon_0}$, to be much less than the ring length $2\pi R$ (i.e., the bound wavefunction periodicity is neglected), we can write the splitting in the simple form,

$$\Delta\tilde{\varepsilon}_{0,m} = \frac{\hbar\omega}{2} \left(\frac{m_h - m_e}{M} \right)^2 \left[\frac{(eE_0a_X)^2}{\varepsilon_R^2(1 - 2m)^2 - (\hbar\omega)^2} - \frac{(eE_0a_X)^2}{\varepsilon_R^2(1 + 2m)^2 - (\hbar\omega)^2} \right]. \quad (5.63)$$

It should be stressed that the simplest delta-function model leads to reasonable results. This follows formally from the fact that the final expression (Eq. 5.63) does not depend on model parameters: It depends only on the exciton binding energy ε_0 which should be treated as a phenomenological parameter.

Though Eq. 5.63 offers a very clean form of the exciton splitting in the case of a ring radius much larger than the exciton radius ($2\pi R \gg a_X$); it can be further extended by taking into account the periodicity of the ring which becomes important when $2\pi R \sim a_X$. The exact solution is derived in Ref. [253] by using Green's function procedure. The true ground state solution to the Delta-function potential in a QR can be written,

$$\chi_0(\theta) = \frac{U^2 A^2}{2B\Delta^2 \sinh^2(\pi\sqrt{B})} \cosh^2(\sqrt{B}(\pi - \theta)). \quad (5.64)$$

where the parameters Δ and B are defined as,

$$\Delta = \frac{\hbar^2}{2\mu R^2}, \quad B = \left(\frac{\mu R \sqrt{U}}{\hbar} \right)^4, \quad (5.65)$$

and A is a normalization constant. However, the correction to the exciton splitting by using Eq. 5.64, as opposed to neglecting the periodic boundary, is very small as long as one assumes that the spread of the wavefunction is small compared to the ring size.¹ In this case, Eq. 5.63 holds true for varying exciton radius a_X .

Numerical calculation using the 1D quantum ring Coulomb potential were also investigated. Here the potential is described with,

$$V(\theta) = \frac{e^2}{4\varepsilon_0\epsilon\sqrt{a^2 + 2R^2(1 - \cos(\theta))}}, \quad (5.66)$$

¹Indeed, the exponential behavior of $\chi_0(\theta)$ makes it vanish quickly as one goes along the ring, i.e., $\chi_0(\pm\pi) \sim 0$.

where the parameter a takes account of the finite width of the QR in order to avoid singular behavior. However, numerical solutions revealed very similar results to those obtained analytically for the case of the Delta-function potential if the exciton binding energy ε_0 is kept the same. In the light of this, we will strictly stick to results using the Delta potential.

Let us estimate the main limitation of the model one-dimensional exciton Hamiltonian (Eq. 5.44) which neglects the exciton motion in the radial direction. It can be important since the radial motion weakens the AB effect in wide rings [260]. If one imagines a ring with radius R and width ΔR , it follows from the numerical calculations that amplitudes of the AB oscillations for the case of $R/\Delta R > 5$ and for the case of ideal one-dimensional ring ($\Delta R \rightarrow 0$) are almost identical [260]. Therefore, the one-dimensional exciton Hamiltonian (Eq. 5.44) correctly describes the solved AB problem for typical semiconductor rings with radius R in the tens of nanometers and width ΔR in the nanometer range.

5.2.2 ENERGY SPLITTING OF OPTICALLY DRESSED EXCITONS

Unlike Sec. 5.1, where a mesoscopic ring with a radius of 10 μm was used, we will stick to a quantum ring with a radius in the nanometer range. As shown by Eq. 5.25 in previous section, the effect of splitting between electrons vanishes as $1/R$. However, since the phase accumulated increases also with R (Eq. 5.24) the size of the ring could be chosen as large as one wants as long as the derived equations stay within the applicability of perturbation theory (where the perturbation scales with eE_0R). For the case of excitons, we are only interested in the splitting $\Delta\tilde{\varepsilon}_{0,m}$ which also vanishes fast for large ring radii and small exciton Bohr radii a_X . For this reason, a choice of a small quantum ring (yet larger than a_X) is ideal in order to produce a maximum amount of splitting.

The field-induced splitting (Eqs. 5.61 and 5.63) vanishes if the electron mass is equal to the hole mass, $m_e = m_h$. Physically, this can be explained in terms of an artificial $U(1)$ gauge field produced by the coupling of a charged particle to circularly polarized photons [A4]. Since the artificial field [A4] depends on a particle mass, it interacts differently with an electron and a hole in the case of $m_e \neq m_h$. As a consequence, the splitting (Eqs. 5.61 and 5.63) is nonzero in the case of $m_e \neq m_h$, though an exciton is electrically neutral as a whole. In the case of $m_e = m_h$, the artificial gauge field interacts equally with both electron and hole. However, signs of the interaction are different for the electron and the hole since electrical charges of electron and hole are opposite. Therefore, the interaction of the artificial gauge field with an exciton is zero in the case of $m_e = m_h$.

The splitting (Eq. 5.63) for exciton states with the angular momenta $m = 1$ and $m = -1$ in a GaAs quantum ring is presented graphically in Figs. 5.6–5.7 for

Figure 5.6: The energy splitting of the exciton states with angular momenta $m = 1$ and $m = -1$ in a GaAs ring with the radius $R = 9.6$ nm as a function of the exciton binding energy ε_0 for a circularly polarized dressing field with the frequency $\omega = 1050$ GHz and different intensities I_0 .

various intensities of the dressing field, $I_0 = \epsilon_0 E_0^2 c$. The effective masses of electron and holes in GaAs were set to $m_e/m_0 = 0.063$ and $m_h/m_0 = 0.51$ where m_0 is the mass of electron in vacuum. The values are taken from Ref. [261]. In Fig. 5.6, the splitting $\Delta\varepsilon = \tilde{\varepsilon}_{0,1} - \tilde{\varepsilon}_{0,-1}$ is plotted as a function of the exciton binding energy, ε_0 , which depends on the type of confinement potential associated with the quantum ring [249] such as parabolic vs hard wall potentials. It is apparent that the splitting decreases with increasing the binding energy. Physically, this is a consequence of decreasing the exciton size, a_X . Indeed, an exciton with a very small size looks like an electrically neutral particle from viewpoint of the dressing electromagnetic field. As a consequence, the splitting (Eq. 5.63) is small for small excitons.

It follows from Figs. 5.6–5.7 that the typical splitting is of μeV scale for stationary irradiation intensities of tens W/cm^2 . This splitting is comparable to the Lamb shift in atoms and can be detected experimentally by optical methods. In order to increase the splitting, the irradiation intensity I_0 should also be increased. However, the increasing of stationary irradiation can fluidize a semiconductor ring. To avoid the fluidizing, it is reasonable to use narrow pulses of a strong dressing field which splits exciton states and narrow pulses of a weak probing field which detects the splitting. This well-known pump-and-probe methodology was elaborated long ago and is commonly used to observe quantum optics effects, particularly, modifications of energy spectrum of dressed electrons arisen from the optical Stark effect in

Figure 5.7: The energy splitting of exciton states with angular momenta $m = 1$ and $m = -1$ in a GaAs ring with the radius $R = 9.6$ nm as a function of the field intensity I_0 and the field frequency ω for different binding energies of the exciton: **(a)** $\epsilon_0 = 2$ meV; **(b)** $\epsilon_0 = 4$ meV; **(c)** $\epsilon_0 = 6$ meV; **(d)** $\epsilon_0 = 8$ meV. The physically relevant areas of the field parameters, which correspond to applicability of the basic expressions derived within the perturbation theory, lie below of the dashed white lines.

semiconductor structures (see, e.g., Refs. [262, 263]). Within this approach, giant dressing fields (up to GW/cm^2) can be applied to semiconductor structures. As a consequence, the splitting can be of meV scale in state-of-the-art optical experiments.

In Sec. 5.1.4 we showed conductance oscillations of the electron as a function of the external dressing field frequencies (ω) and intensities (I_0). Here we find that interference of exciton currents in the QR are practically vanishing for our typical parameters. This is understandable from the both the neutral nature of the exciton and the small size of the QR (short traveling length). The effect can possibly be detected when taking account of the radial degree of the wavefunction spread. The effect would then ideally scale with an area corresponding to the difference in the hole path and the electron path. From an experimental point of view, exciton currents can be created with a specific in-plane k -vector using optical means and even controlled using acoustic waves [264]. This is however out of the scope of this chapter and will wait further investigation.

It should be stressed that the discussed effect is qualitatively different to those that arise from absorption of circularly polarized light in quantum rings (see, e.g., Refs. [265–267]). In those works the absorption of photons with non-zero angular momentum by electrons leads to the transfer of angular momentum from light to electrons in a ring. Correspondingly, photoinduced currents in the ring appear. Since this effect is caused by light absorption, it can be described within the classical electrodynamics of ring-shaped conductors. In contrast, we consider the Aharonov-Bohm effect induced by light in the regime of electromagnetic dressing, when absorption of real photons is absent. To be more specific, the discussed AB effect arises from light-induced changing phase of electron wave function, which results in the appearance of the artificial gauge field [A4] and shifts exciton energy levels in the ring. Evidently, this purely quantum phenomenon cannot be described within classical physics.

CHAPTER 6

CONCLUSIONS

Research devoted to the regime of strong light-matter coupling reveals an abundance of novel effects in both the purely quantum (Chap. 5) and mean-field system (Chaps. 2-4). Specific to this thesis, we have studied effects closely associated with solutions of angular momenta in 2D systems.

In Chap. 2 we studied the stationary solutions describing various topological defects in a planar coupled-QW system of a spinor indirect exciton condensate. Numerically, the solutions were found by applying the imaginary time method to a set of coupled Gross-Pitaevskii equations describing the exciton spinor condensate in the mean-field picture. The role of SOI of Rashba and Dresselhaus types was analyzed and connected to the formation of single vortices, half vortices and half vortex-antivortex pairs. The transition between warped vortex states and striped phase solutions was described in the presence of both Rashba and Dresselhaus SOI.

In Chap. 3 it was shown that it's possible to sustain a stable vortex state of charge $m = \pm 1$ in an open-dissipative planar microcavity system of exciton polaritons using an incoherent ring shaped pump to self-trap the polariton condensate. The charge of the vortex state can be deterministically be set by using a coherent Gaussian pump (within a wide range of pump parameters). These vortex states can furthermore be copied to a different ring pump using simple rectangular potential guide geometry. The choice of copying the same charge or the inverted charge can be controlled by either adjusting the lateral length of the guides or the distance between the pumps spots (see Fig. 3.6). Calculating the fidelity of these processes over many realizations of stochastic noise confirmed that they are robust. With this in mind, a new area of future information mechanisms can possibly be started using these basic *inverter* and *copier* schemes as building blocks.

In Chap. 4 the dynamics of sudden spin whirls of a spinor polariton condensate in a planar microcavity system where observed and studied. It is well known that static optical spin Hall effect patterns appear in the polarization of the polariton condensate emission due the TE-TM splitting of different propagating optical modes. Here, the dynamic spin whirl (as opposed to the static OSHE pattern) originates from a self-induced Zeeman splitting at the pump spot due to the ellipticity of the pump (i.e., imbalanced spin populations are created in the background exciton

reservoir). The strong nonlinear interactions between polaritons and the exciton reservoir then induce a collective rotation of the 2D textures in the plane of the microcavity. It should be emphasized that the dynamic induction of an effective magnetic field on the picosecond scale and the resulting dynamic control of spin currents is an additional step toward the realization of spinoptronic devices.

Finally, in Chap. 5 it was shown that the interference of electron waves traveling through a mesoscopic ring exposed to a circularly polarized electromagnetic field is formally the same as in a ring subjected to a magnetic flux. As a consequence, the *optically-induced* Aharonov-Bohm effect appears. This effect manifests itself in the oscillating dependence of the ring conductance on the field intensity and field frequency. The effect can be described formally in terms of the artificial $U(1)$ gauge field arisen from the strong electron-photon coupling. Furthermore, the effect can also manifest itself in excitonic states since it also applies to many-particle phenomenon. The charge neutrality of the exciton however results in a less pronounced effect as is demonstrated by the induced μeV splitting of excitonic states with mutually opposite angular momenta in the QR.

LIST OF PUBLICATIONS

RELEVANT TO CURRENT THESIS

- [A1] P. Cilibrizzi, H. Sigurðsson, T. C. H. Liew, H. Ohadi, S. Wilkinson, A. Askitopoulos, I. A. Shelykh, and P. G. Lagoudakis, “Polariton Spin Whirls”, *Physical Review B*, vol. **92**, no. 155308, (2015).
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