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Sep 2016

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Abstract

With the fragmentation of mass markets in many industries, organizations are increasingly competing on product lines as a marketing strategy instead of focusing on single product. By offering product lines, companies can cover a broader range of market segments but yet do not incur significant cost disadvantages because of economies of scale in material and production processes. Product lines usually evolve in response to market or technology changes with new products phased in and poor-performing products phased out. The objective of this research is to develop an evolutionary product line design methodology in adapting product lines for improved profit in order to survive and proliferate in a competitive marketplace. Some companies may develop new products and select elements for new product lines in sequence. Some companies may conduct these two activities simultaneously. In this regard, this thesis studies two versions of integrated decisions of product line adaptation: 1) including new product design, product phase out and pricing, and 2)including product phase in, product phase out and pricing.

A mixed logit (ML) discrete choice model and a time-driven activity-based costing (ABC) model are respectively developed to quantify the demand and cost implications of product line adaptation. Quasi-random simulated maximum likelihood estimation is developed to estimate customer preferences for the mixed logit model. The decisions regarding changes in product attributes, mix and prices are modeled as a mixed integer or a mixed integer-discrete-continuous non-linear programming problem with the goal to maximize profit. A bi-level optimization procedure, combining genetic algorithm (GA) and differential evolution (DE), is developed for problem-solving.

The proposed methodology is illustrated with an example of mobile phone product line design. It is indicated that the adaptation of product mix and prices improves a product line's profit and that updating product mix and prices simultaneously outperforms updating either one separately. The case example has

demonstrated the feasibility of the proposed method for product line adaptation. The proposed methodology can help develop profitable product lines in competitive market.

Traditional choice models assume that consumers have well-defined preferences and are not influenced by additional information. However, consumers do not always behave rationally in making a purchase decision. A behavioral choice model is developed to employ reference dependence, diminishing sensitivity and loss aversion in assessing the value of each attribute. The behavioral choice model is illustrated with another survey data of the mobile phone, in which each attribute possesses more levels to be more suitable for parameter estimate. It outperforms logit choice model with a larger log-likelihood value at convergence and a smaller squared error in the case study.

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Chapter 1

Introduction

This chapter starts by introducing the research background and motivation (section 1.1), defining the research objective and scope (section 1.2), then briefly describing the research methodology (section 1.3) and finally outlining the organization of the thesis (section 1.4).

1.1 RESEARCH BACKGROUND AND MOTIVATION

With the increasing diversification of customer needs and the resulting high fragmentation of mass markets in many industries, organizations are driven to compete on product lines instead of focusing on single products. A product line is a string of closely related product models grouped together for marketing or technical reasons (Kotler et al. 2012). Product models are distinct units within the product line that are distinguishable by size, color or some other attributes. Products are considered differentiated if one or more product attributes are different from another. They are considered related because they function in similar manners, are sold to the same customer groups and marketed through the same types of outlets, or fall within given price ranges. In this thesis, a product line is defined as a set of product models that are marketed by the same company to one general market, have similar functions, share most of their characteristics while possessing some unique characteristics. The products in the product line can come in various size, colours, qualities, and prices. For instance, the variety of coffees that are offered at a café is one of its product lines and it could consist of flat white, cappuccinos, short black, lattes, mochas and etc. Alternatively, Product line of juices and pastries can also be found at a café. Product family is a group of products which derives from a similar product platform. These goods or services makes use of the same physical properties and share customer segments, pricing techniques, distribution channels, advertisement campaigns and such other elements of marketing. Various product families comprises as a product portfolio. Other terminologies used in this thesis are defined as follows:

A product attribute is an element or feature that a product possesses (Mowen 1993). Attributes can be categorized as concrete or abstract (Peter and Olson 1994). Concrete attributes "are the most objective, tangible characteristics of a product and can be assessed based on some criteria such as color, or shape"(Aaker 1992). Abstract attributes "represent intangible and subjective characteristics that are not easily measured", e.g. aesthetics, or the operating system of a phone. Concrete attributes are often the basis for product modification or optimization, while more abstract attributes are often the basis for brand positioning. This thesis covers both concrete and abstract attributes to differentiate product models. Product attributes can affect products' appeal or acceptance in the market. Product mix is the set of all products and items that a particular seller offers for sale. A company's product mix might consist of various product lines(Panda 2009). In this thesis, it is defined more narrowly as to refer only to the set of product models in the product line that is of interest to the seller.

There are many cited advantages of offering a product line. For example, by offering a larger variety of differentiated but closely related products, organizations could satisfy customers' needs better, and cover a broader range of market segments which may result in a more significant market share benefit. Samsung, by offering 23 smartphone variants, is able to target the different segments of consumers; reaching 45% of the Singapore market in Oct 2012 (Blackbox 2012: online). BMW offers 32 automobile models ranging series 1 to series 7. It achieved the highest number of new car registrations in 2012 in Singapore (LTA 2012: online). For both Samsung and BMW, their successes could be attributed in part to their strategies of supplying many distinctively different product models. At the same time, high product variety in a product line does not result in significant cost disadvantages because of the economies of scope achieved in procurement, production and distribution processes (Kekre and Srinivasan 1990). This applies particularly to companies employing modularity, platform and standardization of components and sub-assemblies. For example, Swatch achieves low manufacturing cost through a broad product line by basing its watches on few standardized components (Boulding and Christen 2005).

However, an overextended product line could cannibalize sales of more profitable product models, muddle the strategic role of each product, undermine brand loyalty and consequently jeopardize profitability (Quelch and Kenny 1994). It is no coincidence that each time Hewlett-Packard and Canon introduce the latest desktop printers, the older models with fewer features became less desirable and eventually become obsolete. Therefore, product variety is critical for the success of a product line.

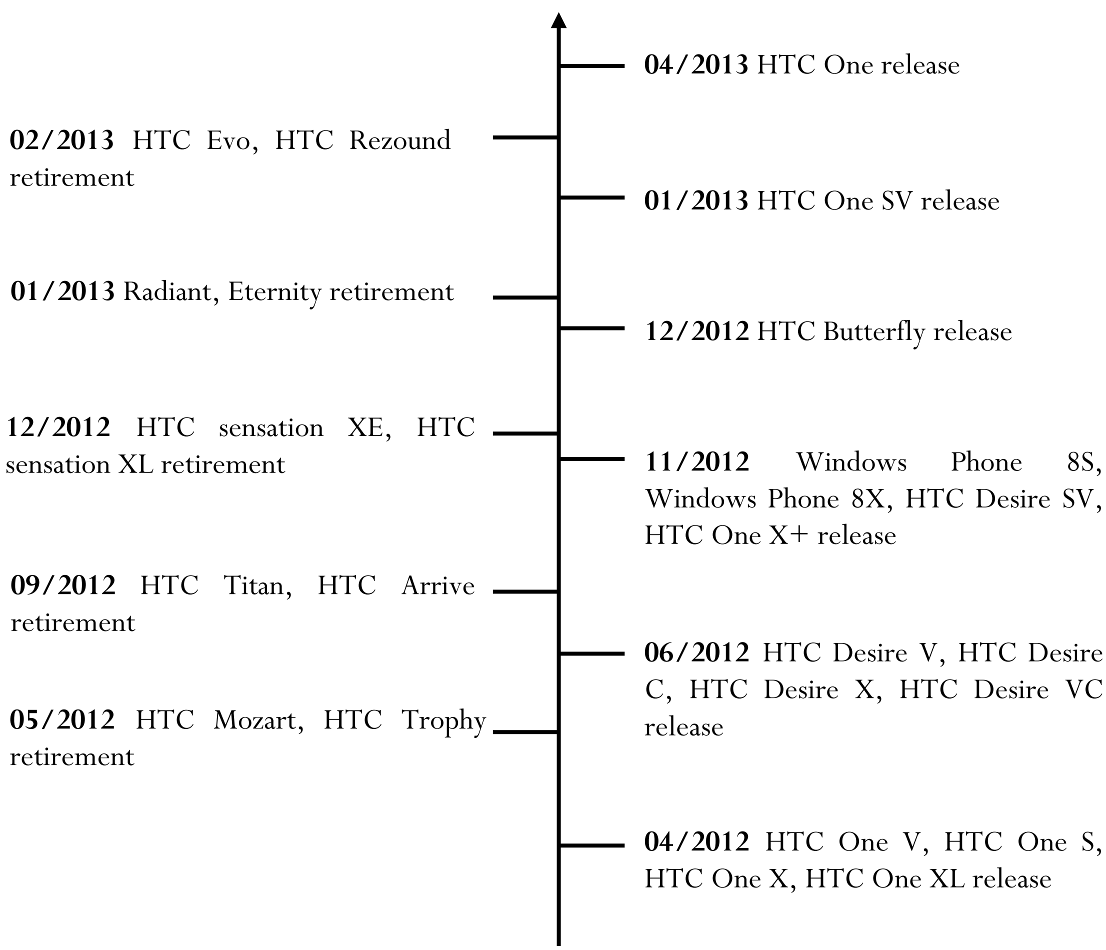


Figure 1-1 Timeline of HTC product line

Product lines cannot stay constant for a long time. Market globalization, shorter product life cycles and rapid technology development put high pressure on firms' profitability. Under such circumstances, new product development becomes one of the most vital success factors for every organization, ranging from the pharmaceutical sector to the high-tech industry (Tsafarakis et al. 2011). In a sense, product retirement is becoming a routine activity as companies constantly replace old product models with improved ones. The addition of new products and the retirement of old products are primary parts of the process of designing a product line. Figure 1-1 is a timeline showing HTC's smartphone models in 2012 and 2013. It is observed that HTC releases newer smartphone models regularly, whilst removing old product models from its product line. HTC launched 78 phone models from January 2009 to April 2013, while only 13 models were available for sale as at June 2013 (HTC 2013: online).

Price directly affects consumers' purchasing decisions and can be used to control cannibalization inter- and intra-product lines and to raise the overall profit. Consumers are willing to pay more for a product that is closer to their specification. However, consumers also make tradeoffs and may buy a product that is not ideal but is offered at a considerably lower price. For instance, as we consider adding new products, prices should be adjusted so that the existing products remain attractive in the midst of changing customer preferences and the profitability can be maintained. Hence it is imperative to include pricing in product line design. Product line design has attracted enormous attention from the perspectives of marketing, manufacturing and engineering design research. Researchers have looked into the various aspects of product line design including product selection (e.g. Green and Krieger 1985, Mcbride and Zufryden 1988), positioning and pricing (e.g. Dobson and Kalish 1988, Thakur et al.2000, Day and Venkataramanan 2006) and manufacturing (e.g. Yano and Dobson 1998, Morgan et al. 2001). It has been generally recognized that an effective product line design requires an intricate balance among marketing, manufacturing and engineering objectives in terms of product variety, component and process commonality and etc. (Michalek et al. 2006, Chen et al. 2009, Kumar et al. 2009). Various optimization-based models and algorithms have been developed to support decision-making in product line design (e.g. Green and Krieger 1985, Kohli and Sukumar 1990, Nair et al. 1995, Li and Azarm 2002, Michalek et al. 2006).

However, in most previous research, new product models are assumed to be in the pipeline and decision makers select a subset from the pool of new product models (Green and Krieger 1985, Alexouda and Paparrizos 2001, Kraus and Yano 2003, Chen et al. 2009, Schön 2010, Luo 2011, Chen et al. 2014). This exclusion of the old product models in the candidate pool would lead to heavy cannibalization among new and old product models due to functional commonality (Desai 2001) and will hurt the overall profitability. Furthermore, selection decisions that are based only on predetermined product models may miss achieving the optimal point in the product attribute spaces at the system level. Product line design with product decisions based on the product attributes of new product models and on a selection of old product models can be more comprehensive.

In addition, the majority of the methodologies and models reported in literature take a path-independent approach by assuming that product lines are optimized based on given market and technological conditions. In practice, however, product lines are rarely designed from scratch. Instead, they usually evolve in response to market or technology changes with new products phased in and poor-performing products phased out (Sorenson 2000). In other words, a product line follows an evolutionary path with its constituent product models undergoing "birth-death" cycles. The new product line depends heavily on the existing one. The evolutionary framework thus provides a more realistic descriptive model to explain product line dynamics and the path-dependent feature of evolution. This calls for new methodologies for product line design.

This research conceptualizes product line design as decisions regarding the adaptation of a product line to improve its fitness (measured in profitability) in order to survive and proliferate in a competitive marketplace. More specifically, this thesis studies the integrated decisions of product line adaptation including new product design, product phase in, product phase out and pricing. It takes into account that in practice, product mix and prices are usually updated simultaneously. Changes in product mix and prices have complicated implications on market demand and manufacturing cost, both of which affect a product line's competitiveness in the marketplace.

Product attributes collectively define products and are key factors that influence customer purchasing decisions. Products design can be viewed as the process of setting product attributes. The thesis considers

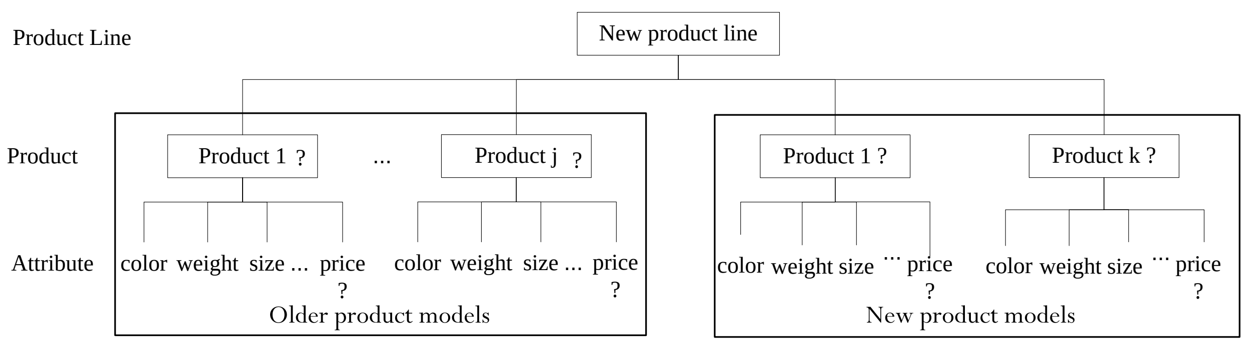


Figure 1-2 Components to be determined in hierarchical structure of product line when new products are ready for selection

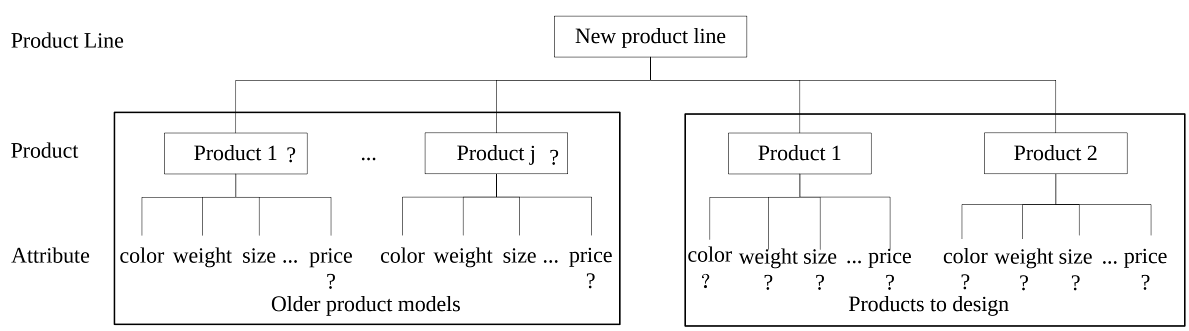


Figure 1-3 Components to be determined in hierarchical structure of product line when new products require being designed

two product line design scenarios with regard to design variables. In the first scenario, new product models have been designed and are ready for selection. The design variables are a subset of new and old product models and prices of each selected product models. As illustrated in Figure 1-2, with regard to product line adaptation in this scenario, questions are raised which include:

- What new candidate product models should be phased in?
- What existing product models should be phased out from the current product line?
- What should be the optimal prices for each of the product models in a product line?

In the second scenario, new product models are partly designed, with some key attributes to be determined but the number of new products is given. The attribute determination, old product selection and pricing are decided in one step. As illustrated in Figure 1-3, the questions raised are:

- What level each attribute of the new product models should be selected?
- What existing product models should be phased out from the current product line? and
- What should be the optimal prices for each of the product models in a product line?

1.2 RESEARCH OBJECTIVE AND SCOPE

The objective of this research is to develop an evolutionary product line design methodology to assist manufacturers in adapting product lines for improved profit in order to survive and proliferate in a competitive marketplace. This research focuses on the consumer product industry which is characterized by diversified consumer lifestyles and customer needs and mass markets fragmentation. The research targets competitive markets where there are multiple competing manufacturers. Manufacturers, thus, have to consider the competitive impact on market share when adapting a product line. However, competitors' reaction to the change of the manufacturer's product line is not included in this research. The decisions to be made are with regard to product mix and prices of a product line based on an old product line and candidate products; and product attributes in the scenario where new candidates have yet been designed.

Market demand, manufacturing cost and prices are primary factors that drive manufacturers' profit and are thus quantified in an integrated model proposed in this thesis. When predicting market demand, the old product line, competitors' offerings and their prices are considered because all these affect customers' purchase choices. Design and development costs are incurred in the process of new product design and prior to the process of product line design. Design and development costs are therefore excluded in the product line cost model if no new products are designed and included otherwise. The formulated profit is gross profit rather than net profit to assess the manufacturer's decision. This research is investigated from the manufacturers' perspective and can be applied to support their decision making. Consumers' purchase behaviors are studied to support manufacturers' decisions while consumers' welfares may not be maximized.

1.3 RESEARCH METHODOLOGY

This thesis conceptualizes product line design as decisions regarding the adaptation of product line to improve its fitness. The integrated decisions of product line adaptation are new product design, product phase in, product phase out, and pricing, taking into account that product mix and prices are usually updated simultaneously in practice. The inter-relationships among decision factors which are old product line, candidate products, competitors' products, demand, price, cost and profit, are delineated in a decision framework.

The implications of product line adaptation on demand and revenues are estimated by mixed logit discrete choice model, which is capable of capturing the interactions among multiple alternatives. This model assumes a probabilistic utility function which contains the customer's preference multiplying the product's attributes and an unobservable random factor. Mixed logit employs the first choice rule which assumes that the consumer seeks to achieve the highest utility among all alternatives. By assuming that the unobservable factor is an identically independent distributed extreme value, the purchase probability is transferred into an integral of conditional purchase probability and probability density function of preferences' distribution over preference, where conditional purchase probability is a function of given preference and product attributes. Maximum likelihood method supports this model by solving parameters of customers' preference distributions with stated-choice data or/and sales data. Quasi-Monte Carlo simulation calculates the purchase probability integral which cannot be solved analytically. Demand is simply formulated as the product of market size and purchase choice probability.

The cost impacts of product line adaptation are modeled in a time-driven activity-based costing (hereafter, TDABC), which has been recognized as an effective costing method, especially in a high product variety environment. TDABC defines the practical capacity of resources, and allocates this capacity to products or customers based on the time that consumed. The cost of a product line is the sum of activity cost and direct component cost, in which the discount of the unit material cost is considered.

Two profit functions are formulated to measure the fitness of proposed product lines: one with new and old product selection and prices as variables and the other with new product attributes and old product selection and prices as variables. The optimization problem is formulated as a mixed integer or a mixed integer-discrete-continuous nonlinear programming problem, respectively, because product mix is an integer vector, attributes are discrete or continuous variables and prices are continuous. A bi-level heuristic method is proposed which combines genetic algorithm (GA) for product mix and differential evolution (DE) for prices and attributes. GA proposes different product mixes and evaluates their fitness based on each product mix's highest profit, because each product mix's profit is a function of all product models' prices and attributes. With each proposed product mix, DE optimizes prices and attributes for all product models and maximizes profit.

1.4 ORGANIZATION OF THE THESIS

The rest of this thesis consists of 6 chapters. Chapter 2 covers the recent relevant literature on product line design from perspectives of marketing, manufacturing and engineering research. The evolutionary design process development which is applied in this thesis is reviewed as well. The research gap is identified as the lack of an integrated methodology for the product line that includes evolutionary design process, demand, manufacturing cost and pricing. Chapter 3 proposes a decision-making framework and develops a computational model for product line design, utilizing mixed logit (ML) and time-driven activity-based costing (TDABC) to quantify the demand and cost impact of product line adaptation respectively. Product line adaptation is then formulated as a mixed integer non-linear programming problem or a mixed integer-discrete-continuous nonlinear programming for profit maximization. In Chapter 4, a bi-level optimization procedure combining genetic algorithm (GA) and differential evolution (DE) is subsequently developed for optimizing product selection, attribute determination and pricing simultaneously. Chapter 5 illustrates the methodology using HTC smartphone as an example and demonstrates the feasibility of proposed methodology. Chapter 6 extends the research by developing a behavioral choice model with reference dependent, nonlinearly and asymmetric multi-attribute utility to address irrational behaviors in consumer purchase decision and illustrates it with smartphone demand prediction. Chapter 7 concludes the research, discusses the limitations in the research and outlines the future work.

Chapter 2

Literature Review

To systematically review the literature, this thesis classifies product line design research from three perspectives: marketing, manufacturing and engineering. The first three sections in this chapter discuss each perspective in turn. Section 2.4 reviews the utilization of the evolutionary methodology in the design process. Section 2.5 summarizes the literature and identifies the research gap.

2.1 PRODUCT LINE DESIGN IN MARKETING RESEARCH

The product line design problem was originally formulated as a product selection and positioning problem in marketing research. Green and Krieger (1985) then extended the single product design research to product line decisions by formulating two versions of the problem according to research objectives and by presenting and evaluating various heuristic approaches. The product line design problem was firstly defined as the selection of a subset of products with substitutability and/or complementarity relationship to maximize the buyers' utility or all sellers' utility. Two versions of formulation were:

The decision maker selected a small subset of product models while each buyer selected one product model from that subset. The objective of the buyer's welfare problem was to maximize all buyers' utility, as follows:

$$\max \quad \sum_{n\in N}\sum_{j\in J} U_{nj}\gamma_{nj} \text{ , subject to} \tag{2.1}$$

$$\sum_{j\in J}\gamma_{nj} = 1 \text{ for } n\in N \text{ and } j\in J; \tag{2.2}$$

$$\sum_{j \in J} \xi_j = NP \; ; \qquad (2.3)$$

$$\gamma_{nj} \le \xi_j \text{ for } n \in N \,,\, j \in J \; ; \qquad (2.4)$$

$$\gamma_{nj}, \xi_j \in \{0,1\} \; ; \qquad (2.5)$$

NP denoted the predetermined quantity of product models in the product line. U_{nj} was the utility of product j for buyer n. $\gamma_{nj} = 1$ if buyer n chose product model j; 0 otherwise. $\xi_j = 1$ if product model j was offered; otherwise $\xi_j = 0$. The problem was translated into finding the optimal value of ξ_j to maximize the sum of all buyers' utilities.

The seller's problem was a two-party game: the seller offered a subset of product models to a group of buyers and each buyer selected one product model from the subset. The seller determined the size of and components of the subset to maximize the seller's welfare while taking the buyers' preference into consideration. It was assumed that the buyer had his own status quo for the product and that the buyer chose the product which maximized utility among those offered but only if the utility exceeded the status quo. The formulation for the seller's welfare was:

$$\max \quad \sum_{j=1}^{J} \sum_{n=1}^{N} \gamma_{nj} V_{nj} \,, \quad \text{subject to} \qquad (2.6)$$

$$\gamma_{nj} \le \xi_j \qquad (2.7)$$

$$\text{and} \quad \sum_{j=0}^{J} \xi_j \le K + 1 \qquad (2.8)$$

V_{nj} was the welfare to the seller if buyer n chose product model j and assumed known. The number of items, NP, could be assumed to be a variable subject to the constraint that $NP < NP'$ (NP' was the size of the choice set). To estimate each buyer's utility (i.e. U_{nj}) for each proposed product alternative, a survey using conjoint analysis was required to be conducted. Green and Krieger (1985) research was a start toward product line design and was frequently cited by research that followed; despite the exclusion of product variety, price, cost and competition issues from the model.

Development in problem formulation

The seller's welfare/profit is a more practical objective than the buyer's welfare because companies are usually profit-oriented. Most of following product line design models and mathematical algorithm methodology were based on this objective (Mcbride and Zufryden 1988, Dobson and Kalish 1988, Yano and Dobson 1998, Day 2006, Kumar 2013).

In practical problems, the number of feasible combinations of products can be extremely large even when given the modest size of product alternative. The formulation of the problem was based on the measurements of individual consumer's preferences from a conjoint analysis. Considering the complexity of the problem, Mcbride and Zufryden (1988) proposed a zero-one integer mathematical programming formulation to solve optimal product line selection. The mathematical programming formulation sought to find the solution vector ξ and matrix γ of zero-one variables whose components were defined as follows:

- $\xi_j = 1$, if the respective product j was included within the product line and $\xi_j = 0$ otherwise.
- $\gamma_{nj} = 1$, if the corresponding consumer n switched to the corresponding product j and $\gamma_{nj} = 1$ otherwise.

The feasible solution region was restricted by the assumption that a consumer would only accept product alternative that offered a higher utility than the status-quo product.

- If $U_{nj} < U_{n0}$ then set $\gamma_{nj} = 0$ (2.10)
- If $U_{nj} < U_{n0}$, then set $\xi_j = 0$ (2.11)

(U_{n0} was the status-quo product's utility and U_{nj} was the utility of product j for consumer n.) The zero-one integer programming model possessed a flexible structure which was adopted in later strategies for solving the product line selection problem.

Dobson and Kalish (1988) formulated the positioning and pricing of a product line problem faced by a monopolist. It was assumed that the market was composed of several market segments of various sizes and that consumers were homogenous in each segment and would choose the same product from the choice set. The company faced both fixed and variable costs for each product. The profit was formulated as:

$$\max \quad \sum_{g=1}^{G}\sum_{j=0}^{J} M_g (p_j - c_j) x_{gj} - \sum_{j=0}^{J} f_j y_j \tag{2.12}$$

$x_{gj} = 1$ if segment g ($g = 1,2,\ldots,G$) was assigned to product j, and $x_{gj} = 0$ otherwise; $\xi_j = 1$ if product was offered j, and $\xi_j = 0$ otherwise. M_g was the market size of segment g; f_j and c_j were the fixed and variable costs of product j; p_j was price of product j.

Aydin and Ryan (2000) employed a consumer choice model in which individuals had stochastic utility for each product and acted to maximize their consumer surplus, instead of assuming that the consumer population consisted of a number of segments populated with identical customers, each with known utility. The research took a retailer's perspective and assumed that the arrival of customers to the store was a Poisson process with mean $\lambda^{poisson}$. The number of customers who chose product model j from a choice set J is a Poisson random variable with mean $\lambda^{poisson}\phi_j^J$, where ϕ_j^J was the choice probability calculated from multinomial logit choice model. Multinomial logit choice model will be discussed in detail in the following sub-section. The expected profit function per unit time could be written as

$$\pi^J(p_1,\ldots,p_J) = \sum_{j=1}^{J} \lambda^{poisson}(p_j - c_j)\phi_j^J \tag{2.13}$$

where c_j was the cost per unit. Based on this model, the authors found that, in a monopolistic market,

- the optimal profit for the product line increased with the number of product models in the product line, given that there was no limitation to shelf space, no inventory cost, no new product introduction cost, and only fixed cost;
- the magnitude of profit increase, however, became smaller as the number of models increased.
- the optimal profit and the optimal profit margin were increasing functions of the average margin; if the number of product models was limited, then the optimal product line consists of a number of models with the highest average margins. The average margin was the difference between the average reservation price of the product and the cost to acquire the product;
- If the retailer replaced one of its current models with a model that had a higher average margin, the retailer should increase the prices of all models in the product line.

Market share generally has a positive effect on business profitability (Szymanski et al. 1993). Once the demand for non-price determinants was optimized, the manufacturer price at the corporate level negotiated, the problems that focused on profit were ill-conditioned because errors in cost estimates impacted profit in a much more discontinuous ways than do errors in the part-worth estimates. (Wang et al. 2009) developed a brand-and-price algorithm based on the objective of the product line's "share-of-choice", i.e. the number of respondents with at least one new product utility exceeding the respondent's reservation utility. Thakur et al.'s (2000) work aims to maximize the number of customers who buy the products.

Utility and consumer decision choice

Capturing consumers' utility accurately and then applying a suitable choice rule to transfer utility to purchasing choice are critical for predicting demand and therefore determining the most profitable product line and prices. Utility is the measure of a consumers' valuation of a product or a service. It determines the consumers' desirability for the product and the willingness-to-pay. Utility is usually transferred to purchasing choice under choice rules to support marketing decision-making. There are two main techniques for analyzing consumer response concerning preferences and intentions to buy based on rational decision assumption: conjoint analysis and discrete choice model.

Conjoint analysis has been applied widely in product line design to quantify consumer evaluations on products (Green et al. 2001, Mcbride and Zufryden 1988, Kohli and Sukumar 1990, Dobson and Kalish 1993, Kuzmanovic and Martic 2012). It was evolved from the seminal research of Luce and Tukey (1964). There are several forms of conjoint analysis, namely, vector model, ideal-point model and part-worth utility model of which part-worth utility model is the most widely used (Green et al. 2001). Part-worth model identifies the products' distinctive attributes and estimates the part-worths at discrete levels of each attribute and interpolate between levels of continuous attributes. In the part-worth model (Green and Krieger 1985, Green et al. 2001), the respondent's preference β_j for product j was formulated as:

$$\beta_j = \sum_{a=1}^{A} f_a(x_{j,a}) \tag{2.14}$$

where $a = 1,2,\ldots,A$ denoted the attributes and $x_{j,a} = 1,2,\ldots,X_{j,a}$ represented attribute levels. For any respondent, f_k was a function denoting the part-worth corresponding to level x_{jk}. Conjoint analysis requires the estimation of relative the part-worths of all attributes at all levels except the one used as the scale. The number of independent parameters of part-worth to be estimated is equal to $\left\{\sum_{a=1}^{A}(X_a - 1)\right\} - 1$, where A is the number of attributes and X_a is the number of levels of attribute a. To apply this model, respondents could be asked to first rate the desirability of each set of attribute levels on a 0 to 100 scale (where 100 means most desired), before going on to rate the importance of the attributes (Green et al. 2001). Another and more rudimentary conjoint analysis form is to require respondents to rate the product models on a 100-point likelihood-of-purchase scale where 100 means most preferred. To reduce the task of survey respondents, conjoint analyst makes extensive use of orthogonal arrays and other types of fractional factorial designs to reduce the number of stimulus descriptions that a respondent sees to a small fraction of the total number of combinations. This technique could approximate each attribute level's preference. However, conjoint analysis is also constrained by its complex design, burdensome data collection process. Survey respondents perform a self-explicated evaluation and rate their "likelihood-of-purchase". Revealed preference data, which reflects the actual choice in the real market environment, cannot be applied with conjoint analysis.

Under a deterministic preference model like conjoint analysis, consumers are assumed to choose the product associated with the highest utility from his/her choice set with certainty. This deterministic model is too restrictive, as the analyst is possibly not able to consider all factors that influence the decision to purchase. Discrete choice model is a newer technique under probabilistic choice rule in predicting product line demand (Hanson and Martin 1996, Aydin and Ryan 2000, Chen and Hausman 2000, Jeremy et.al. 2011, Kuzmanovic and Martic 2012). It overcomes conjoint analysis's disadvantage of complexity by allowing decision makers to choose among two or more discrete options and thus provide a more realistic representation of the consumer decision-making process (Kaul and Rao 1995). It presumes that a consumer has predetermined/well-defined independent preference parameter (in nest logit, standard logit) or parameters (in mixed logit) for each product attribute; preference factors are independent of the attribute levels and the attribute utility is the product of the attribute level and the attributes parameter/weighting. Utility comprised part-worths and an error factor which represents the unobserved factors contributing to utility. Choice probabilities are approached among a finite set of products, given distribution assumption on errors. This technique enables for larger scale of attributes and revealed data.

Aydin and Ryan (2000) employed logit model and reservation prices in solving product line design problems. A reservation price was defined as the highest price that a customer was willing to pay for a corresponding product. It could be viewed as the utility of a product excluding the contribution of price.

Authors assumed that all consumers' reservation prices for each product model in one market segment followed a Weibull distribution. The purchasing choice model was derived under the first choice rule, as follows:

$$\phi_j^J(p_1,\ldots,p_I) = e^{-(\frac{p_j-\alpha_j}{\mu}+\gamma)} \frac{\delta^J(p_1,\ldots,p_J)}{\Psi^J(p_1,\ldots,p_J)} \tag{2.15}$$

$$\text{where } \Psi^J(p_1,\ldots,p_J) = \sum_{j=1}^{J} \exp\left(-\left(\frac{p_j-\alpha_j}{\mu}+\gamma\right)\right) \text{ and} \tag{2.16}$$

$$\delta^J(p_1,\ldots,p_J) = 1-\exp\left(-\Psi^J(p_1,\ldots,p_J)\right) \tag{2.17}$$

μ was the shape parameter of Gumbel distribution, γ=0.5572. α_j and p_j were the mean reservation price and the price of product j, respectively.

Jiao and Zhang (2005) and Steiner and Hruschka (2002) adopted another version of logit model in product line design problem, assuming that the error term in utility was independent and identically distributed extreme value. The purchase probability was derived as:

$$\phi_{nj} = \frac{\exp(\mu V_{nj})}{\sum_{j=1}^{J}\exp(\mu V_{nj})} \tag{2.18}$$

where μ was a scaling parameter, optimization of which could be calibrated on actual market sales data. V_{nj} was the observed deterministic component of the utility of product j. Steiner and Hruschka (2002) Tsafarakis et al. (2008) applied generalized Bradley-Terry-Luce share-of-utility rule (GBTL, α -rule):

$$\phi_{nj} = \frac{V_{nj}^{\ \alpha}}{\sum_{j=1}^{J} V_{nj}^{\ \alpha}} \tag{2.19}$$

The GBTL model could be calibrated based on actual market shares by the optimization of the decision constant α. With α approaching to infinity, GBTL model approximated the first choice rule. With α approaching to 1, GBTL model approximated the share-of-utility rule.

Beyond rational choice models, few researchers have solved the product line design problem by the engaging irrational behaviors of consumers. For example, Orhun (2009) investigated local context effects on consumer preferences in product line design problem. An individual n's valuation of product j, U_{nj}, was formulated as a sum of self-independent absolute valuations $v_{na}(x_{ja})$ on level x_{ja} of attribute $a=\{1,2,\ldots,A\}$ and comparative evaluations $f_a(v_{na}(x_{ja})-v_{na}(x_{ra}))$, which depended on departures from a choice set-specific reference point x_{ra} for attribute a.

$$U_{nj}=\sum_{a=1}^{A}v_{na}(x_{ja})+\sum_{a=1}^{A}f_a(v_{na}(x_{ja})-v_{na}(x_{ra})) \tag{2.20}$$

The author incorporated the reference dependency by using loss aversion. A linear comparative valuation function was applied to capture the effects of loss aversion:

$$f_k(v_{nk}(x_{ik})-v_{nk}(x_{rk}))=\begin{cases}\lambda_k\left[v_{nk}(x_{ik})-v_{nk}(x_{rk})\right] & if \quad v_{na}(x_{ja})<v_{na}(x_{ra})\\ \gamma_k\left[v_{nk}(x_{ik})-v_{nk}(x_{rk})\right] & if \quad v_{na}(x_{ja})\geq v_{na}(x_{ra})\end{cases} \tag{2.21}$$

where $\lambda_a>\gamma_a$. The loss sensitivity parameter λ_a captured the dislike of being in the region of losses in attribute a compared to the reference point. The degree of loss aversion in either attribute can be described as λ_a/γ_a.

Choice rule refers to the assumption under which products' utilities are transferred to purchasing choices. The first choice rule/maximum-utility choice rule has been mostly applied. This rule assumes that consumers would buy the product that has the highest utility value in a choice set. To apply the first choice rule, the purchasing scenario should be characterized by high-involvement and infrequent purchase, such as buying cars, computers and so on. Some papers separated logit choice rule from the first choice rule.

Actually, logit choice rule is derived from the first choice rule under the assumption that error term in utility is independent, identically distributed (IID) extreme value. There is another choice rule applied in product line design problem: share-of-surplus. Yano (2003) was concerned with markets with repetitive purchases such as foods and personal care products and adopted the share-of-surplus choice model in addressing product line selection and pricing. The fraction of a customer segment that selects a product was defined as the ratio of the segment's surplus from a particular product to the segment's total surplus across all products offered with positive surplus for that segment.

The first choice rule with deterministic utility can be too extreme, because it has a tendency to generate market share closer to zero and one. It is also less robust, as small changes in utility values of products can drastically change market shares. On the other hand, the share-of-surplus and logit choice rules are sensitive to the scale range on which utility is measured. The market share prediction under the share-of-surplus will change if each product's utility is added with a constant. The market share prediction under logit choice rule will change if each product's utility is multiplied by a constant.

Development in algorithms

Even for a modest size of proposed or referenced set of product alternatives, the number of feasible product combinations is large. At the time, this could be challenging to solve for an optimal product line by means of standard mathematical programming methods. A stream of follow-up research has been devoted to improving the algorithm efficiency.

Many practical product line design problems have a large number of attributes and levels. In this case, if most attribute level combinations define feasible products, then constructing product lines directly from part-worths data is necessary. Kohli and Sukumar (1990) presented a procedure which structured product lines directly from part-worth preference functions estimated using conjoint or hybrid conjoint analysis.

This was similar to the approaches of Sudharshan et al. (1987) for continuous attributes. The procedure built upon a previously proposed dynamic-programming heuristic which selected a single multi-attribute product model to maximize share of choices (Kohli and Krishnamurti 1987). The problem is NP-Hard and thus they proposed the heuristic solution procedures, which could be implemented in a computationally efficient manner and provided near-optimal solutions.

McBride and Zufryden (1988) formulated Green and Krieger's (1985) seller's problem as a mathematical program and used integer programming code to obtain optimal solutions for a special case corresponding to the share-of-choice problem and for small instances of the general seller's problem. The authors noted that as the problem was NP-Hard; solving the general problem with larger numbers of reference set items might be difficult using a general integer programming code. This is a conclusion that is consistent with the experience of researchers in operations research.

Dobson and Kalish (1988) assumed that each consumer's utility for a product was measured in dollar unit and that each customer chose one from a set of products that maximized the difference between the value of the product and its price. A buyer's problem of maximizing buyers' welfare and a seller's problem of maximizing the seller's profit subject to the choice constraint for each buyer were formulated as mathematical programs. Dobson and Kalish (1988) explicitly considered fixed and variable costs in their formulations. A heuristics was described to solve the NP-Hard problems. Computational tests with small problems suggested that the heuristics closely approximated the optimal solution. Nair et al. (1995) developed improved heuristics based on a beam search approach for solving these problems. The research (in this subsection) has offered simplified models to handle a larger number data and various methodologies to obtain optimal solutions and to improve algorithms efficiency.

Traditional optimization algorithms such as simulated annealing algorithm, greedy and reverse greedy fail to solve large-scale problems especially with nonlinear objective functions. Most of the traditional optimization algorithms require gradient information and hence it is not capable to solve non-differentiable functions. Evolutionary algorithms usually are population-based and proved effective to solve product line design problems, which are NP-Hard. Evolutionary algorithms are less complex and more straightforward compared to traditional optimization algorithms. Moreover, evolutionary algorithms have the capability to generate efficient solution for multiple objective problems. However, evolutionary algorithms don't always come with global optimum all the time. Every time evolutionary algorithm provides different results which only allow situation that tolerate with trial and failure results.

Pricing in product line design

Companies are also concerned with the prices they will charge for products. The more a company charges for a product, the more the marginal profit it obtains from each unit sold. However, higher prices typically lead to reduced sales. As commonly observed in industries, companies often re-price a product line with changes in product mix. Another stream of marketing research has been devoted to addressing the pricing issue in product line design. In the conjoint analysis proposed by Green and Krieger (1985), price was treated on par as the other product attributes and was predetermined. However, from a design perspective, price is a marketing decision that is subjected to economic rather than technical constraints. Three types of pricing product line methods are summarized in this thesis as follows: simplifying and transferring the problem to the shortest path problem, transferring continuous price space to discrete, and applying a probabilistic choice model and deriving the demand functions of prices.

Dobson and Kalish (1988) proposed a model in which prices of products were design variables to be determined. Price was considered as a separate attribute in utility/reservation price analysis and consumer choice was molded as finding a product that maximized the difference between value and price. Prices were determined prior to the other design variables on the basis of products' reservation utilities. The dual pricing problem was decomposed into shortest path problems. Suppose that a monopolistic seller possessed two product models with different qualities of q_h, q_l to price and would position them to two market segments which differed in that the high segment valued the quality higher than the low segment ($v_h > v_l$). All consumers in each segment chose the product which maximized their surplus which is defined as product's utility minus its price. The cost of providing a unit of quality was assumed to increase at an increasing rate as the level of quality increases. The seller could extract all the surpluses from the customers in low segment by setting price for the low-end product at its reservation price $p_l = v_l(q_l)$. The seller extracted as much surplus as possible from consumers in high segment while preventing consumers switching to low segment. The price for the high-end product was set as $p_h = v_h(q_h) - q_l(v_h - v_l)$ to maximize profit. It was propositioned that a product would constrain sellers from charging a higher price for another product that was the son of the first product in the solutions tree; otherwise the segment would switch to the first product. These findings were applied by the following researchers, such as Heese and Swaminathan (2006) and Netessine and Taylor (2007). After the introduction of the price issue to product line design problems, price was involved in most of the product line design models that followed.

Considering that sellers often choose prices for products from a discrete set, Day and Venkataramanan (2006) set the highest customer segment reservation utility value for the product as upper bound and the variable cost of the product as lower bound. A finite number of prices between those two bound were determined ahead of time and were entered as price set. This moved away from continuous pricing variables and turned the profit formulation into a binary linear model with decision variables of product-market segment assignment and product offering through this discrete price reduction.

These two methods assume constant reservation utility, but consumers in the same market segment may not have exactly the same preferences. Kohli and Mahajan (1991) and Aydin and Ryan (2000) also employed random reservation price to describe consumer's utility excluding price. Aydin and Ryan (2000) assumed reservation prices for one product of different customers in one market segment followed Gumbel distribution. Kohli and Mahajan (1991) assumed that each consumer' reservation prices were normally distributed, the average of all consumers' reservation prices could be described by another normal distribution and that the standard deviations of all consumer's reservation prices were the same. The purchase probability of each product was derived as the probability that the product possessed the largest reservation utility in its choice set and it was a highly complicated function of all products' prices in the product line. By applying this model, Aydin and Ryan (2000) found that if the retailer replaced one of its current models with a model that had a higher average margin, the retailer should increase the prices of all models in the product line for customers to retain the new product. This founding was based on the assumption that market was monopolistic and that customers would buy one product with certainty. When authors used probabilistic choice model, prices can be linked to demand, and then formulated as a decision variable in the profit equation (Steiner and Hruschka 2002, Jeremy et al. 2011).

The aforementioned method is the simplest but makes the assumptions that consumers in the same market segment possess the same evaluation of quality and that the market is monopolistic. The assumptions are not realistic. As Day and Venkataramanan (2006) discussed, discrete pricing can limit profitability and the large scale nature of the model made it impossible to solve in a reasonable time. Formulating prices into a probabilistic choice model is complicated but more accurate, because it allows for the heterogeneity in consumers' valuation.

2.2 PRODUCT LINE DESIGN IN MANUFACTURING RESEARCH

The marketing research on product line design has generally focused on product variety and the consequent effects upon customer utility and market demand. The impacts of product line decisions, however, go beyond marketing and affect product fulfillment as well. Cost reduction becomes more important when competition increases and price becomes a differentiator in the market. To maintain the highest level of competitiveness, companies are driven to produce high quality and low-cost products. It is, therefore, essential to allocate and estimate costs to the right products when making decisions with regards to product lines. Researchers have progressively extended product line design research to include various manufacturing processes and costs.

Variable costs, which could provide a basic understanding of the effects of volume on a manufacturer's profit, was the first cost component involved in product line formulation. Green and Krieger's (1985) assumed that variable costs for all varieties of products were identical and constant. Kraus and Yano (2003) also considered only variable costs by assuming that the company had already the capabilities to manufacture a wide variety of products and the costs of introducing minor to moderate variations of products were small in comparison to the revenue and variable costs over the product life cycle. The majority of researchers added fixed cost of production to profit function to make the formulation simple but more realistic (Dobson and Kalish 1993, Schön 2010, Michalek et al. 2011). For instance, Aydin and Ryan (2000) studied the problem from a retailer's perspective when selecting and pricing a product line under cannibalization and assuming profit maximization. Total cost composed of constant wholesale price of manufacturers and the fixed cost associated with the introduction of each new product model.

Variable cost and fixed cost of a product may not remain unchanged in all circumstances over time. Variable costs can be affected by factors such as the discount factor and labor productivity. Fixed costs can be influenced by production processes. To capture these costs accurately, researchers developed complex models for variable and fixed costs, rather than assuming that variable and fixed cost keep permanently the same. Considering the components shared by different product models within the product line and economies of scale, Luo (2011) defined that variable cost c_j was jointly determined by the component cost c_{rwj} of competent r ,a discount factor λ_{rwj} associated with component sharing, the assembly cost c_{aj} , maintenance cost c_{mj} and the salvage cost c_{sj} ,i.e.

$$c_j = \sum_{r=1}^{R} (1-\lambda_{rwj}) \times c_{rwj} + c_{aj} + c_{mj} + c_{sj} \tag{2.22}$$

Heesse and Swaminathan (2006) and Kim and Chhajed (2000) assumed component costs to be convex in quality and per-unit production cost of a component r with quality q was

$$c_r(q) = k_r q^2 \tag{2.23}$$

where cost coefficient $k_r > 0$. Variable costs of a product which was the sum of its components' costs could be a multivariate quadratic function of components' qualities.

The classification of costs into fixed and variable helps inform how manufacturing costs response to the changes of product mix and production volume. However, the classification is not perfect because the underlying assumption that costs are influenced only by volume is not true (Bhattacharyya 2004). Volume is only one of many cost drivers that influence costs. There are also costs related to product specification, product mix, methods of production, technology, and productivity and organization structure. Besides traditional variable costs and fixed costs, researchers introduced other types of costs that are more specific to product processes. Yano and Dobson (1998) highlighted the need to accommodate variable manufacturing costs in modeling product line profit. Fixed costs of the entire product line were the aggregate sum of costs of all manufacturing resources shared by products. Let F_s denote fixed cost of process s. $\xi_s = 1$ if the process s is required for the production of the product line, and 0 otherwise, the profit function was formulated as follows:

$$\max \quad \sum\nolimits_g M_g (p_g - \sum\nolimits_j c_{gj} \gamma_{gj}) - \sum\nolimits_s F_s \xi_s \tag{2.24}$$

Day and Venkataramanan (2006) also include fixed costs of manufacturing class, which were incurred when one or more products assigned to that manufacturing class. Deng et al. (2014) decomposed costs into variable direct cost, fixed direct cost, indirect cost in proposing a methodology for integrated product line design and supplier selection. Fixed direct cost included the cost of product development, software development, prototypes making and licensing and had no relationship with sourcing quantity. Variable direct cost was the sum of all component costs. Indirect cost was referred to the cost associated with suppliers, such as the cost of quality inspection of materials and ordering.

Michalek et al. (2011) insisted that synergy of a product line was strongly correlated with product commonality and that when similar products were offered, as in most product lines; manufacturing could often utilize not only production process but also common resources for the production of several products. Michalek et al. (2011) presumed that product commonalities enable investment cost sharing and that improving economies of scale did not exist. Hence, each new product design required new manufacturing investment. Accordingly, the production cost consists of unit variable cost and investment cost for the product.

Besides manufacturing costs, logistics costs have also been included to make the product line design model more complete. Netessine and Taylor (2007) combined the product line design problem with the classical EOQ (Economic Order Quantity) production cost model, which considered production setup cost, inventory cost. The firm determined the number of products to offer and selected the quality q_j, price p_j and production batch size Q for each product to maximize the profit rate per unit time. Morgan et al. (2001) developed a mathematical programming model for product line design that comprised product variable cost, inventory holding cost, and manufacturing set-up cost. The objective function was formulated as:

$$\max \quad \sum_{j\in J}(p_j - c_j)\sum_{g\in G}\gamma_{gj}M_g - \left(\frac{1}{2T}\right)\sum_{j\in J}\sum_{g\in G}\gamma_{gj}M_g h_j - T\sum_{k\in K}r_k S_k - T\sum_{j\in N}\xi_j s_j \qquad (2.25)$$

With T as the number of production cycles over the given planning horizon; h_j the unit inventory holding cost over the planning horizon; $r_k = 1$ if class k set up occurred and 0 otherwise; S_k the manufacturing class set-up cost incurred for manufacturing class k assigned to the product line; s_j the individual product set-up cost.

Researchers in manufacturing are also devoted to reducing the production cost of a product line. Heese and Swaminathan (2006) formulated the relationship among cost-reduction effort, the cost and quality and determined the component quality levels, the amount of effort to reduce production costs, and whether to use common or different components for products. The production cost per unit of a component j with quality q was $c_j(q) = k_j q^2$, $k_j > 0$ was cost coefficient that reflected the differences in cost of producing quality across different component types. The effort e and unit production cost of a component of quality q was $c_j(q,e) = a_j(e)k_j q^2 - b_j(e)$; a_j and b_j were coefficients.

Greater commonality decreased production cost but made the products more indistinguishable from one another. Kim and Chhajed (2000) developed a model to examine when modular products should be introduced and how much modularity to offer. Cost saving would occur if a common modular design was used for the design of multiple products. The cost of providing a unit of quality increases at an increasing rate as the level of quality increases and was represented by cq^2 where c was a constant. Let $f(q_m)$ denote the cost saving function if modular m was utilized, $f(q_m)$ is non-decreasing in q_m and $0 \le \alpha < 1$ was the cost saving parameter. The total cost function was $c(q^2 - \alpha f(q_m))$.

2.3 PRODUCT LINE DESIGN IN ENGINEERING RESEARCH

Product line design can be categorized into two groups with regards to design variables: determining product mix and assigning levels to product attributes (Schön 2010). As discussed in section 2.1, most research belong to the first stream which views product line design as a selection of product portfolios from a candidate product pools to achieve research objectives (Green and Krieger 1985, Alexouda and Paparrizos

2001, Li and Azarm 2002, Kraus and Yano 2003, Chen et al. 2009, Schön 2010, Luo 2011, Chen et al. 2014). It is also referred as a two-step approach to product line design. The first step is to construct a set of candidate products and the second step to select the final menu of products from these product models and/or price. Research in the first group focuses only on the second step. The other stream can be called the all-in-one method, determining product attribute levels, product variants and/or price simultaneously (Jiao and Zhang 2005, Michalek et al. 2005, Kumar et al. 2006, Michalek et al. 2006, Albritton and McMullen 2007, Luo 2011, Tsafarakis et al. 2013, Deng et al. 2014).

A common approach employed was a decomposition facilitated by the analytical target cascading(ATC) (Michalek et al. 2005, Kumar et al. 2006, Michalek et al. 2011) in which product marketing, design and manufacturing problems were decomposed into a hierarchy of sub-problems. Each sub-problem found feasible solutions to achieve its target as closely as possible. In Michalek et al. (2006), ATC decomposed the product line design problem into

- *a marketing sub-problem* to set price for each product in the line, target for each product's characteristics, production volume, and cost, so that the predicted profit was maximized;
- *one design sub-problem for each product in the product line* to achieve target cost and product characteristics by manipulating the design of each product; and
- *a manufacturing sub-problem* to achieve production volume targets by allocating designs of the product's components to available machines while ensuring that each component could only be made on machines capable of manufacturing the component design.

By coordinating the optimization of each sub-problem iteratively, a consistent optimal design for the overall problem was achieved which reduces practical difficulties associated with problem dimensionality, since the model of each subsystem typically had fewer variables and constraints than the combined full system model.

Jiao and Zhang (2005) integrated customer concern over product offerings with engineering implication and addressed the product portfolio planning as a maximizing shared-surplus (utility per cost) problem. A probabilistic choice model and a platform-based product costing method were employed to predict customers' utilities and to estimate engineering costs, respectively. A pragmatic costing approach which allocated costs to established time standards was adopted in the paper. This method required the developing of mapping relationships between different attribute levels and their expected consumptions of standard times within legacy process capabilities. The expected cycle time was used as a performance indicator of variations in process capabilities. The cycle time demonstrated the distinctions between variables that differed as a result of random error and followed a normal distribution. The one-sided specification limit process capability index, PCI, was formulated as:

$$PCI = \frac{\mu^T - LSL^T}{3\sigma^T} \tag{2.26}$$

LSL^T, μ^T, σ^T were lower specification limit, the mean and the standard deviation of the estimated cycle time, respectively. PCI indicated the expected cost of a product produced within the current capabilities. The cost function C_j of product j was formulated as:

$$C_j = \beta \exp(\frac{1}{PCI_j}) = \beta \exp(\frac{3\sigma_j^T}{\mu_j^T - LSL^T}) \tag{2.27}$$

β is a coefficient that indicating the average dollar cost per variation of process capability. The estimated cycle time of a product was the sum of the standard times of the attribute levels.

Kwong et al. (2011) proposed a one-step multi-objective optimization approach for product line design be applied to products with level-based attributes and attributes that had continuous values. Similar to Jiao and Zhang (2005) and Kwong et al. (2011) applied a linear function with a part-worth structure to estimate product development cost. A product's development cost was formulated as the sum of all attribute levels' costs and the product's fixed cost. Each attribute level has fixed cost and variable cost as well. Due to commonality of product design and manufacturing resources, the development cost of a product line decreased with the increase of the product variety and derived as the sum of all the attribute costs of all products in the product line multiply by a coefficient plus the fixed costs.

Besides determining attribute levels for a product line, engineering design community has researched product family design and product platform development. Product family "is a group related products that are derived from a product platform to satisfy a variety of market niches"(Simpson et al. 2006). Both product lines and product families refer to a group of related products. The difference between the two terms is in terms of perspective. A product line is a market- or customer-driven concept, whereas a product family is a technology- or implementation-dependent concept. A product line need not be built as a product family (i.e. from a common set of core assets). Conversely, a product family need not be a product line (i.e., addressing a particular market niche).

Meyer and Lehnerd (1997) illustrated the power of product platforms which provided a cost-effective foundation to generate a large variety of product variants to cover heterogeneous market segments. Jiao et al. (1998) developed a modeling framework for product family design as a key enabler for mass customization. The main focus of research in this stream has been *how* to structure and design a product family or product platform. A triple-view (functional, technical, structural) scheme was proposed for modeling product families. Functional view depicted product families from the viewpoint of customers and sales department. Technical view consisted of the repository of design building blocks recognized through optimizing reusability among design parameters. Structural view described the synthesis knowledge of configuring end-products from building blocks for specific customer needs. The links between different views embodied various activities of product realization. Research has looked into the architecting issue in product platform design (Gonzalez et al. 2000), product platform design methodologies (Simpson et al. 2001), flexibility of product platforms (Suh et al. 2007), and optimal tradeoff between product variety and commonality in product family design (Huang et al. 2008) etc.

Engineering research focused on physical product design processes and realization of product family while product line design mainly concentrated on the selection of products or attributes. Product family design aims to meet the diverse customers' needs in the shortest time possible and with lowest cost, while the objective of product line design is to maximize the buyers' welfare, the product line's market share or profit, or minimize a product line's cost.

2.4 EVOLUTIONARY DESIGN

Despite the increasing sophistication of the mathematical models, most of the methodologies reported in literature have taken product line design as a static optimization problem. In practice, however, companies usually adapt their product lines by incrementally introducing new products and/or retiring old products, the process of which mimics that of natural evolution (e.g. Tellis and Merle Crawford 1981, Sorenson 2000). The concept of evolution has been recognized as a general design methodology that promises a wide range of applications (Hingston et al. 2008). Its application in the realm of product design has been concentrated on individual products (e.g. Otto and Wood 1998, Maher and Tang 2003). There has been very limited research on evolutionary design in the context of product line design. Among the few papers available, Ramdas and Sawhney (2001) presented a cross-functional approach and utilized incremental revenue and incremental life-cycle costs to evaluate multiple product line extensions. Bryan et al. (2007) focused on the product-process interaction in a product line and proposed a co-evolution model, with the objective of maximizing incremental profit for joint design of product lines and assembly system configurations. Co-evolution was a method for the incorporation of product variants and assembly system changes within a family generation, as well as between product generations, through continuously reconfiguring modular products and manufacturing systems. The inputs to the model were the existing product family, the required design changes and the re-configuration constraints of the product family and assembly system.

Recently, Chen et al. (2009) proposed an evolutionary framework for product line design. By representing product lines as clusters of products with different distances, they demonstrated that there was a need to balance variety, commonality, and continuity in product line evolution, as a multi-objective optimization problem. They derived market share with discrete choice model, defined in-line commonality among the constituent products of a product line as the inverse of the average 1-norm distance and cross-line commonality between the constituent products of two generations of a product line; and then formulated the product line overall fitness as a multi-objective optimization as a weighted sum of market share and two commonalities. These researchers introduced the concept of evolutionary approach for designing a product line, and there are opportunities to investigate this realm from other perspectives.

2.5 RESEARCH GAP

A summary of literature shows that product line design has attracted multidisciplinary research that includes marketing, manufacturing and engineering design with different perspectives and methodologies. Table 2-1 summarizes the coverage of literature reviewed in this thesis.

The focus in marketing research stream of research on product line mainly has concentrated on the optimization of the portfolio of product models in a product line with repose to diversified market segments. They excluded or simplified manufacturing cost by including constant fixed costs or/and constant variable costs. The research from the manufacturing stream expanded the scope to include manufacturing and supply chain costs in deriving the optimal product line with objectives of minimizing total cost or product commonality. In the engineering design community, product family design and platform-based product development were used. The focus mainly concentrated on the design methodology

Table 2-1 The classification of product line research

Research Area	Authors & years	Decision Variables				Objectives			
		Product Selection	Product Design	Price	Maximize Seller's Profit	Maximize Buyers' welfare	Minimizing Cost	Maximizing Demand/ share-of-choice	Other objectives
Marketing	Green and Krieger (1985)	×			×	×			
	Dobson and Kalish (1988)	×		×	×	×			
	McBride and Zufryden (1988)	×			×				
	Kohli and Sukumar (1990)	×		×	×	×			
	Nair et al. (1995)	×						×	
	Aydin and Ryan (2000)	×		×	×				
	Kraus and Yano (2003)	×		×	×				
	Orhun (2009)	×		×	×				
Manufacturing	Day and Venkataramanan (2006)	×		×	×				
	Netessine and Taylor (2007)	×		×	×				
	Huang et al. (2008)		×				×		Minimize commonality degree
	Chen et al. (2009)	×		×				×	Maximize in-line commonality and maximize cross-line continuity
	Kwong et.al (2011)		×				×	×	Min development cycle time
	Deng et al. (2014)	×		×	×		×		Max performance and quality
Engineering	Thakur et al. (2000)		×	×				×	
	Alexouda and Paparrizos (2001)		×	×	×				
	Steiner and Hruschka (2002)		×	×	×				
	Jiao and Zhang (2005)		×	×	×				

Research Area	Authors & years	Decision Variables				Objectives			
		Product Selection	Product Design	Price	Maximize Seller's Profit	Maximize Buyers' welfare	Minimizing Cost	Maximizing Demand/ share-of-choice	Other objectives
	Kumar et al (2006)		×		×				
	Michalek et al. (2006)		×	×	×				
	Albritoon and McMullen (2007)		×					×	
	Kumar (2009)		×		×				
	Wang et al. (2009)		×					×	
	Luo (2011)		×	×	×				
	Michalek et al. (2011)		×	×	×				
	Kuzmanovic and Martic (2012)		×	×	×				
	Tsafarakis et al. (2013)		×	×				×	

Table 2-1 The classification of product line research (continued)

Research Area	Authors & years	Demand model			Costing			Optimization method				Evolution Process
		Conjoint Analysis	Probabilistic choice model	Behavioral choice model	Fixed cost	Variable cost	Other costing methods	Simulated annealing algorithm	Greedy, reverse greedy	Heuristic algorithms	analytical target cascading	
Marketing	Green and Krieger (1985)	×				×						
	Dobson and Kalish (1988)	×			×	×		×	×			
	McBride and Zufryden (1988)	×			×							
	Kohli and Sukumar (1990)	×			×	×			×			
	Nair et al. (1995)	×								×		

Research Area	Authors & years	Demand model			Costing			Optimization method				Evolution Process
		Conjoint Analysis	Probabilistic choice model	Behavioral choice model	Fixed cost	Variable cost	Other costing methods	Simulated annealing algorithm	Greedy, reverse greedy	Heuristic algorithms	analytical target cascading	
	Aydin and Ryan (2000)		×		×	×						
	Kraus and Yano (2003)		×			×		×		×		
	Orhun (2009)			×			Product unit cost is a quadratic function of quality					
Manufacturing	Day and Venkataramanan (2006)	×			×	×	Fixed manufacturing class costs			×		
	Netessine and Taylor (2007)	×					Unit cost is a quadratic function of quality; Economic order quantity model					
	Huang et al. (2008)									×		
	Chen et al. (2009)		×							×		×
	Kwong et.al (2011)		×		×	×	The development cost of a product line decrease with the increase of the number of product profiles			×		
	Deng et al. (2014)		×		×	×				×		
Engineering	Thakur et al. (2000)	×								×		
	Alexouda and Paparrizos (2001)	×				×				×		
	Steiner and Hruschka (2002)		×		×	×				×		

Research Area	Authors & years	Demand model			Costing			Optimization method				Evolution Process
		Conjoint Analysis	Probabilistic choice model	Behavioral choice model	Fixed cost	Variable cost	Other costing methods	Simulated annealing algorithm	Greedy, reverse greedy	Heuristic algorithms	analytical target cascading	
	Jiao and Zhang (2005)		×			×	Production cost is based on time standard and expected cycle time			×		
	Kumar et al (2006)		×				Including material cost , labor cost, repair and warranty costs, design costs, and overhead costs			×	×	
	Michalek et al. (2006)		×				Including investment and operating cost				×	
	Albritoon and McMullen (2007)	×										
	Kumar (2009)		×		×	×	Including material cost, labour cost, repair and warranty costs, design costs, overhead costs					×
	Wang et al. (2009)	×										
	Luo (2011)		×		×	×	Variable product cost is a function of discount factor of components cost, and assembly cost, maintenance cost and salvage cost					
	Michalek et al. (2011)		×		×	×				×	×	
	Kuzmanovic and Martic (2012)		×			×						
	Tsafarakis et al. (2013)		×							×		

Heuristic algorithms in this table include beam search, genetic algorithm, hybrid particle swarm optimization and ant colony algorithm.

and realization a family of products that simultaneously provided variety while maintaining certain commonality. Most research in marketing and manufacturing fields use conjoint analysis, measured consumers' utility in dollar and assumed that consumers in one market segment had the same and definite utility of products and chose the product with the highest surplus (utility minus price) for certain. In this circumstance, prices of all products can be optimized by applying simulated annealing algorithm prior to selecting product models for each market segment. Conjoint analysis requires respondents to rank or rate various alternative combinations and self-explication. In reality, customers have varied preferences and the purchasing process would be one where a consumer chooses one product among multiple alternatives. Moreover, these papers usually refer manufacturing cost as comprising variable and fixed cost but ignoring the cost interaction due to commonalities in components and production processes. Therefore, there is a lack of an integrated computational model that simultaneously takes into consideration demand, manufacturing cost and price issue handling the inter-relationship with various other factors.

Most of the existing research literature assumes that product line design was a static optimization problem. The decision maker selects product models from a big pool of new product models. However, the reality is that most companies have legacy products and they need to adapt their product lines instead of designing products from scratch. Product line design is a path-dependent process and decision makers incrementally phase in new candidates and phase out old product models to adapt a product line. An evolutionary framework thus provides a more realistic model to describe product line dynamics. However, there is a general lack of methodologies and computational models that assists making decisions in product line adaptation. This research is positioned to advance evolutionary product line design research by developing an integrated computational model to support cross-functional decision-making regarding new products' attributes, product mix and prices in product line adaptation.

Chapter 3

Problem Modeling

In this chapter, a decision framework is first developed to illustrate the product line adaptation process and to clarify the interrelationships among the key decision factors, including product mix, competitors' offerings, prices, demand and cost. Section 3.2 shows how mixed logit choice model is adopted to study consumers' intention of purchase and the demand of products on the basis of consumer behavior, given choice sets and prices. Section 3.3 utilizes time-driven activity-based costing model (TDABC) to estimate the costs of products. With demand and cost formulated, section 3.4 subsequently formulates the profit equation with product mix, attributes levels and prices as variables and analyzes its properties.

3.1 PRODUCT LINE DESIGN AND PRICING DECISION FRAMEWORK

This thesis views product line evolution as the result of the periodical adaptations in response to market or technological changes. As illustrated in Figure 3-1, each adaptation starts with an old product line (Λ^{old}) and involves a "birth" process with products phased-in and a "death" process with products phased-out. The phase-in products are either drawn from a set of candidate new products (Λ^{can}), which can be products in the development pipeline, or are products to be designed by decision makers by determining their attribute levels. The phase-out products are a subset of the current product line. The process of adaptation results in a new product line (Λ^{new}), the prices of which can be adjusted. The objective of product line adaptation can be generally described as finding optimal decisions regarding product phase-in

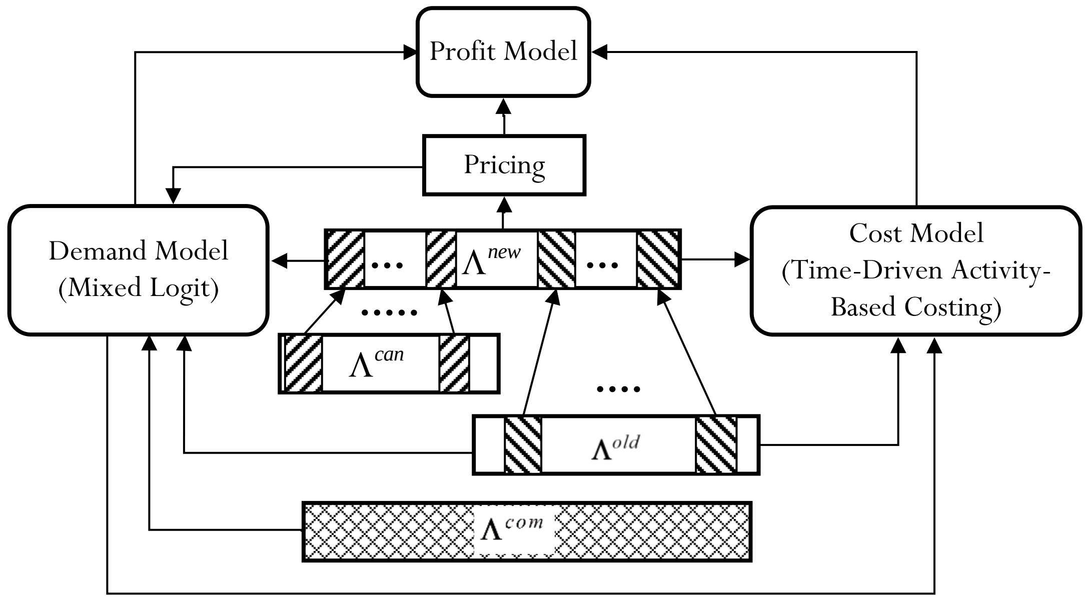


Figure 3-1 A decision framework for product line adaptation and pricing

or new product attribute levels, product phase-out and pricing to maximize the competitiveness of the resulting product line relative to other products in the market (Λ^{com}).

Changes in products or prices affect both the demand quantity and cost of each product model in a product line. This thesis adopts mixed logit (ML) model to predict the demand of a product line, given its ability to capture the interactions among different products (e.g. Hensher and Greene 2003, Train 2003). The cost impact of product line adaptation is modeled based on time-driven activity-based costing (TDABC), which is recognized as a simple and effective costing method, especially in a high product variety environment and in an environment where components are commonly shared. Similar to recent research in product line design (e.g. Michalek et al. 2006, Kumar et al. 2009), this thesis assumes that the goal of product line design is to maximize profit, which serves as an aggregate for fitness evaluation for product line adaptation.

3.2 PRODUCT LINE DEMAND MODELING WITH MIXED LOGIT

Product line adaptation impacts consumers' preferences and product models' market demand, which is an important factor of performance of a product line. Manufacturers generally possess dozens of product models within a product line, likewise their competitors. The demand investigation is, therefore, in the context characterized by very high product variety. It is however not practical for researchers to survey consumers for decisions on all possible combinations of products and prices. A demand model is needed to make the process of choosing easier and for survey responders to investigate only several products. This section presents product line demand modeling with mixed logit discrete choice.

Mixed logit model is regarded as the state of the art discrete choice method (Hensher and Greene 2003). Mixed logit model provides a causal explanation of the behavioral process that leads to consumers' choices (Wassenaar et al. 2005). It allows random taste variation and unrestricted substitution patterns in choice behavior by using distribution preference. It can also approximate any random utility function (Train 2003). Such features are desirable to model the demand of a product line as there are usually complex substitutability and complementarity relationships among different product models (Green and Krieger 1985).

Utility evaluation

Mixed logit model is an example of rational choice theory which assumes that consumers are rational decision makers and have a well-organized and stable system of preferences. Consumers assign evaluations/utilities to products based on their preferences and products attributes. Exact utilities cannot be measured or observed directly; instead, underlying relative utilities are inferred from observed choices. In line with random utility theory (RUT), the utility, $U_{n,j}$, for an alternative varies across consumers as a random variable, considering unobservable or unmeasurable decision factors. The utility that a person n obtains from an alternative j is:

$$U_{nj} = \beta_n' x_{nj} + \varepsilon_{nj} \tag{3.1}$$

where β_n is a column vector of coefficients for each decision maker, n; x_{nj} is a column vector of observed explanatory variables and includes attributes of product j and demographic attributes of decision maker n; ε_{nj} is an unobservable variable that contributes to product utility. For example, smartphones possess attributes such as CPU, storage, screen size and so on; buyers' demographic attributes includes income, age, gender and so on. Explanatory variables of a smartphone include both its attributes and buyers' demographic attributes. Unobservable variable ε_{nj} can be economy, the buyer's emotion and other factors that are not included in x_{nj}. Attribute coefficients, β_n, which reflect decision makers' preference and weighting of explanatory variables in utility evaluation, can be approximated using maximum likelihood method (discussed in the next subsection); while unobservable factors ε_{nj} cannot be captured. As a result, the utility cannot be exactly determined. Thus, rather than calculating for utility, the probability to predict purchase decisions is calculated instead.

Purchase probability

Different types of products require different choice rules. This thesis focuses on the consumer products that consumers do not change frequently, such as computers and smartphones. We consider situations of certainty in which consumers are faced with more than two consumer products with multi-dimensional attributes but choose only one from the choice set. Given this situation, the decision maker chooses the alternative j if and only if it is assigned with the largest utility ($U_{nj} > U_{ni}, \forall i \neq j$) within a choice set. Substituting utility with equation 3.1 and transferring, the probability that customer n chooses alternative j can be expressed as an integral:

$$\Pr ob(U_{nj} > U_{ni}, \forall i \neq j) = \int_{-\infty}^{\infty} \left(\prod_{i \neq j} \int_{-\infty}^{\beta_n' x_{nj} + \varepsilon_{nj} - \beta_n' x_{ni}} f(\varepsilon_{nj}, \varepsilon_{ni}) d\varepsilon_{ni} \right) d\varepsilon_{nj} \tag{3.2}$$

where $f(\varepsilon_{nj}, \varepsilon_{ni})$ represents the joint density function of ε_{ni} and ε_{nj}. Different distribution assumptions of ε_{nj} lead to different forms of discrete choice probability. In mixed logit model, ε_{nj} is assumed to be an independent and identically distributed (i.i.d.) extreme value (Train 2003). The probability density distribution for each unobserved component of utility and the cumulative distribution are:

$$f(\varepsilon_{nj}) = e^{-\varepsilon_{nj}} e^{-e^{-\varepsilon_{nj}}} \tag{3.3}$$

$$F(\varepsilon_{nj}) = e^{-e^{-\varepsilon_{nj}}} \tag{3.4}$$

The probability that decision maker n chooses alternative j conditional on $\beta_n = \beta$ is (Train 2003):

$$\phi_{nj}(\beta) = \text{Prob}(U_{nj} > U_{ni}, \forall i \neq j) = \frac{e^{\beta' x_{nj}}}{\sum_i e^{\beta' x_{ni}}} \tag{3.5}$$

which is the ratio of the natural exponential of alternative j's observed utility and the sum of all natural exponential of all alternatives' observed utilities. Mixed logit model assumes that decision makers' tastes vary in the population of a market segment and over time with a density function $f(\beta|\theta)$, where θ refers collectively to the parameters (i.e. mean and deviation) of the preference distribution. With this specification of preference factors, the unconditional choice probability is:

$$\phi_{nj}(\theta) = \int \phi_{nj}(\beta) f(\beta|\theta) d\beta \tag{3.6}$$

Maximum likelihood method

The parameters of a mixed logit model (i.e. θ) can be estimated using the maximum likelihood method (Train 2003). The method of maximum likelihood selects the set of values of the model parameters that maximize the likelihood function. The likelihood function $L(\theta)$ is defined as the probability of each decision maker in the sample population choosing the alternative that was observed to have been chosen, i.e.:

$$L(\theta) = \prod_{n=1}^{N} \prod_{j} (\phi_{nj})^{\tau_{nj}} \tag{3.7}$$

where ϕ_{nj} refers to equation 3.6, indicator $\tau^{nj} = 1$, if decision maker n selects alternative j, and $\tau^{nj} = 0$ otherwise. $\sum_{j=1}^{J} \tau^{nj} = 1 \quad (j = 1,2,\ldots,J)$ because each consumer is assumed to choose only one product from a choice set. The information on selection is collected from surveys or actual sales data. For example, as in the cases, 5 choice sets, each of which contains 3 alternatives, are given to a survey responder. If the responder chooses the 2nd, 1st, 1st, 3rd, 3rd alternatives for choice set 1 – 5, correspondingly. The likelihood function is $\phi_{1,2}\phi_{2,1}\phi_{3,1}\phi_{4,3}\phi_{5,3}$. It is more convenient to maximize the log-likelihood function since ϕ_{nj} is a positive value and the log function is monotonically increasing. Maximizing $L(\theta)$ is equivalent to maximizing the log-likelihood function:

$$LL(\theta) = \sum_{n=1}^{N} \sum_{j=1}^{J} \tau_{nj} \ln \phi_{nj} \tag{3.8}$$

Quasi-Monte Carlo simulation

Since the integral in equation 3.6 is not in a closed form and cannot be analytically calculated, mixed logit models are usually estimated via Monte Carlo simulation (Train 2003 and Bhat 2001). The method approximates an integral as an average of the function values at a sequence of random points. There are two methods to draw random sequences: pseudo-random points and quasi-random points. Quasi-random (i.e., low discrepancy sequence) is designed to span the domain of unit interval uniformly and efficiently, and therefore provides considerably better accuracy with much fewer draws and shorter computational time than pseudo-random method (Bhat 2001). Quasi-Monte Carlo is employed in this thesis and Halton sequence is hired to generate quasi-random numbers.

Halton sequence is generated by choosing a prime integer number r ($r \geq 2$) and expanding the sequence of integer $g(g = 1,2,\ldots,G)$ in terms of base r (Bhat 2003)

$$g = \sum_{l=0}^{L} b_l r^l \ , \tag{3.9}$$

where $0 \leq b_l \leq r-1$ and $r^L \leq g \leq r^{L+1}$. l is the power index from which the base is raised. The first constraint $0 \leq b_l \leq r-1$ indicates that the digits in the base b_l expansion cannot exceed the base value less one. $r^L \leq g \leq r^{L+1}$ determines the largest value of l (i.e., L) which can be transferred to $\log_r(g) - 1 \leq L \leq \log_r(g)$. Therefore, g ($g = 1,2,\ldots,G$ and $g = b_L r^L + b_{L-1} r^{L-1} + \ldots + b_1 r^1 + b_0 r^0$) can be represented by the r -adic integer string $b_L b_{L-1} \ldots b_1 b_0$. The Halton sequence in the prime base r is obtained by taking the radical inverse of $g(g = 1,2,\ldots,G)$ to the base r by reflecting through the radical point:

$$\varphi_r(g) = 0.b_0 b_1 \ldots b_L (in \quad base \quad r) = \sum_{l=0}^{L} b_l r^{-l-1} \tag{3.10}$$

As an illustration, take the prime number 4 and the integer 7 can be written as $7 = 1 \times 4^1 + 3 \times 4^0$ in base 4. Therefore, the digitized form of the integer 7 in base 4 is 13. The radical inverse function of the integer 7 in base 4 is then obtained by reflecting the digitized number 13 about the "decimal point" as 0.31 and expanding 0.31 in base 4 as $3\times 4^{-1}+1\times 4^{-2}=13/16$. The number 13/16 then forms the 7th number in the sequence. The Halton sequence for other integers can be similarly obtained. As illustrated in Figure 3-2, the first 15 numbers in the sequence corresponding to base 4 are ¼, ½, ¾, 1/16, 5/16, 9/16, 13/16, 1/8, 3/8, 5/8, 7/8, 3/16, 7/16, 11/16, 15/16. New numbers added to the sequence tend to fill in the gaps left by the previous numbers, making for a well-distributed coverage in the interval (0, 1). It is also noted

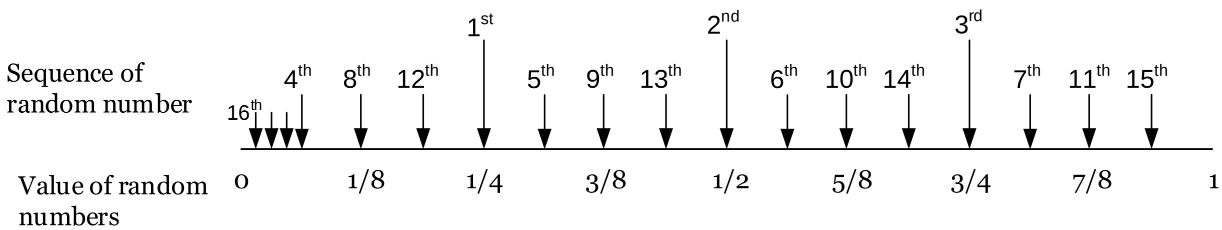


Figure 3-2 Halton sequence in base 4

that the sequence corresponding to the prime number 4 comprises cycles of length 4 of monotonically increasing numbers (after adding a phantom value of zero at the beginning of the sequence). More generally, the sequence corresponding to the prime r comprises cycles of length r of monotonically increasing numbers.

The Halton sequence in K-dimensions is obtained by pairing K one-dimensional sequences based on K pairwise relatively prime integers, $r_1, r_2, \ldots, r_K$ (usually the first K primes). Thus, the gth multi-dimensional point of sequence is as follows:

$$\Psi_g = (\varphi_{r_1}(g), \varphi_{r_2}(g), \ldots, \varphi_{r_s}(g)) . \qquad (3.11)$$

To obtain the corresponding multivariate normal, lognormal or other distributed points over the multi-dimensional domain of the real line, the inverse normal, inverse lognormal or the corresponding inverse distribution transformation of the points are taken. For any given parameter θ, a series of β^d can be simulated by sampling from $f(\beta|\theta)$ with Halton sequence and $\phi_{nj}(\beta^d)$ for each β^d can be calculated using equation 3.5. The average simulated probability with D draws, when D is sufficiently large, can approximate the unconditional choice probability with sufficient accuracy. The expected purchase probability with given θ is as:

$$\hat{\phi}_{nj}(\theta) = \frac{1}{D}\sum_{d=1}^{D}\phi_{nj}(\beta^{d}) \tag{3.12}$$

The simulated log-likelihood function can be approximated as:

$$SLL(\theta) = \sum_{n=1}^{N}\sum_{j=1}^{J}\tau_{nj}\ln\hat{\phi}_{nj} \tag{3.13}$$

The objective in mixed logit model estimation is thus transformed to find $\hat{\theta}$ that maximizes $SLL(\theta)$. This can be approximated with heuristic optimization methods, and genetic algorithm is used for the case study in this thesis. The mixed logit model represented by $\hat{\theta}$ captures customers' preferences and thus can be used to estimate the demand of a product line.

Demand estimation

In the context of product line design, the products offered by a company (Λ^{new} and Λ^{old}) and those offered by the competitors (Λ^{com}) form a choice set, i.e. $\Omega = \Lambda^{new} \cup \Lambda^{old} \cup \Lambda^{com}$. Each product model can be described by a vector of product attributes (A_k) and a scalar of price (p_k), i.e. $Z_k = [A_k, p_k]$. Assuming that there are N market segments and that each segment has M_n homogeneous customers, customers' demographic attributes S_n and product attributes Z_k correspond to the observable variables in the mixed logit model, i.e. $x_{nk} = [S_n, A_k, p_k]$.

A market survey can be conducted to collect customers' choice decisions under different choice situations. The survey can focus on a particular market segment or a number of different segments.

Alternatively, another way to observe customers' choice behavior is to collect actual sales data. However, the sales data needs to include the actual choice set that the customers are faced with when making purchasing decisions. With the choice data, a mixed logit model can be constructed and estimated based on the methods described in equation 3.1 to 3.13. With the mixed logit model constructed, the probability of customers from market segment g choosing product k can be estimated based on equation 3.9.

$$\hat{\phi}_{gk}(\hat{\theta}) = \frac{1}{D}\sum_{d=1}^{D}\phi_{gk}(\beta^{d}) \tag{3.14}$$

The overall demand for product k can thus be estimated as:

$$d_k = \sum_{g=1}^{G} M_g \phi_{gk}(\hat{\theta}) \tag{3.15}$$

Supposing that there are K product models in a product line Λ, the total revenue of the product line can be estimated as:

$$R(\Lambda) = \sum_{k=1}^{K} d_k p_k \tag{3.16}$$

Individuals do not make the same purchase decision all the time for a variety of reasons that analysts cannot fully observe or explain. ML allows customers' preferences among a population to follow any distribution; customer preference heterogeneity is captured by the mean and variance of systematic and random components. The estimated distribution of tastes provides valuable information that is useful for manufacturers in differentiating customers for targeted markets and designing their offers.

3.3 PRODUCT LINE COST MODELING WITH TIME-DRIVEN ACTIVITY-BASED COSTING

Product line evolution can impact costs significantly and it becomes critical to be able to estimate each potential product line's cost for profit maximization. Product lines are characterized by high product variety and commonality of platforms, components and production processes across multiple product models. The cost relationship among product models is complicated and the assigning of accurate costs to products thus becomes more significant for the success of a product line. This section focuses on product line cost modeling with time-driven activity-based costing (TDABC) in the context of product line design.

The traditional costing approach assigns costs and resources directly to products on the basis of single volume measures, such as material consumption or labor hours. However, some overhead costs can be significant and has no direct relation with the volume of products, such as machine setup cost in the semiconductor industry. Therefore, the traditional costing systems may be misleading when assigning costs to products.

Activity-based costing (ABC) traces overhead costs to products via the underlying activities that actually consume the resources. ABC was a more accurate methodology for cost estimation when ABC was introduced. However, ABC has not been widely applied among companies because it usually requires companies to survey employees to determine how they spend time among all the various activities, and uses a large number of cost drivers to handle the heterogeneity in activities (Öker and Adigüzel 2010). When adding new activities or the resource requirement of activities changes, ABC requires a re-estimate of the cost capacity and cost driver rates. It is, therefore, very difficult, time-consuming and costly to install and update an ABC system.

TDABC, on the other hand, is simpler and more accurate compared to ABC. It allows for more heterogeneity in activities without placing burdensome demands for calculating activity and product cost. It also allows cost driver rates to be based on the practical capacity of the resources supplied rather than using theoretical capacity (Kaplan and Anderson 2003). TDABC was originally developed by Steven Anderson in 1997 and improved by Kaplan and Anderson (2003).

Although the product line design is associated with such an early stage of design, most engineering designs involve modifying existing products instead of starting from scratch. Accordingly, patterns of cost estimation in existing products are applicable to a new design. For this type of variant design, where similar work has been done before, there is greater knowledge of costs that can be extrapolated to a new product with a higher degree of confidence. Time-driven ABC in the thesis determines resource consumption according to the estimated processing time for each activity timed by the cost price of time. This intermediate measurement provides a common, consistent metrology to approximate various cost functions for different activities and cost drivers.

Cost Estimate

Similar to Labro (2004), the total cost to produce and deliver a line of products is considered as consisting of two parts: direct component costs and indirect costs that can be traced by activities. The amount of component m that is consumed for the production of a product line which possesses K product models can be derived as:

$$q_m(\Lambda) = \sum_{k=1}^{K} d_k u_{km} \tag{3.17}$$

with $m = 1,\ldots,M$, the index of materials used in the product line; u_{km} is the amount of material m used in one unit of product k; d_k the demand quantity of product k. Component consumption per unit of per product u_{km} can be read directly from the product's bill of material (BOM), and d_k is predicted in demand model. The direct component cost of a product line is the aggregate material cost of its constituent products:

$$C^m(\Lambda) = \sum_{m=1}^{M} q_m(\Lambda) c_m(q_m) \tag{3.18}$$

where c_m is unit material cost; q_m is purchasing volume of component m. It is worth noting that unit material cost c_m is a function of purchasing volume q_m due to quantity discounts. There are many different quantity discount policies used in practice and reported in the literature (Schotanus et al. 2009). Without loss of generality, this thesis uses a continuous quantity discount model, as in the equation below:

$$c_m(q_m) = c_0 + \frac{s}{(q_m)^\eta} \tag{3.19}$$

where c_0 is the lowest price obtainable; s is the price spread; η is the steepness of the quantity discount.

In calculating activity costs, TDABC requires the estimates of two parameters to derive the cost driver rate: the unit cost of supplying capacity and the time required to perform an activity. To get unit cost of capacity, analysts need to go through steps 1 to 3:

(1) Estimate the cost of supplying capacity $C_a^{capacity}$ by identifying and summing up the various groups of resources that perform the activity, a. For example, for the activities of cutting, the analyst identifies the employees who conduct the activities, their supervisors, and the machines and support resources required to perform their functions – blades, electricity, computers, and space.

(2) Estimate the practical capacity of resources supplied $Cap_a^{practical}$. Employees take time for preparation, breaks, arrival, departure, communicating and reading unrelated to actual work performed. Machines have downtime due to maintenance, repair and scheduling fluctuation. The practical capacity is obtained by subtracting these from the full capability.

(3) The unit cost c_a of capacity is calculated by

$$c_a = \frac{C_a^{capacity}}{Cap_a^{practical}} \tag{3.20}$$

Unit time t_a refers to the time required to perform the activity a. TDABC requires the time duration spent on each activity to be derived by observation and/or interviewing managers, which is more accurate than interviewing employees about the percentage of their time spent on all activities. Employees report percentages that sum up to 100, but people do not spend all the time on these activity-related work.

Not all activities of the same sort are the same and require the same amount of time to perform. Companies can generally predict the drivers that cause some activities to be simpler or more complex. Rather than define a separate activity for every possible combination of activity characteristics, or use a duration driver for every activity combination, the TDABC approach estimates the resource demand by a simple time equation: activity time, t_a, is the sum of activity time and the time of specific activity characteristics required if any. The data for special activity characteristics are typically already in the company's ERP system. TDABC operates with fewer equations than the number of activities used in ABC, while permitting more variety and complex activities and products and delivering more accuracy.

The cost driver rate can be calculated by multiplying unit cost of capacity and the unit times that consumed to conduct the activity:

$$r_a = c_a^{capacity} t_a \tag{3.21}$$

The total labor and overhead cost of providing a product line are traced based on underlying activities and can be generally modeled as:

$$C^a(\Lambda) = \sum_{a=1}^{A} r_a t_a(\Lambda) \tag{3.22}$$

By aggregating material, labor, and overhead cost, the total cost impact of product line adaptation can be derived as in the following:

$$C(\Lambda) = C^m(\Lambda) + C^a(\Lambda) \tag{3.23}$$

Besides the cost estimate of a product line, TDABC is also able to reveal individual processes and customer service and expose the magnitude of the source of profit or losses. The insights could support companies' strategic decisions such as the selection of processes, suppliers, channels and so on, and to assist operations management in improving productivity and removing waste.

3.4 PRODUCT LINE PROFIT FORMULATION

There are two product line design scenarios discussed in this thesis: a scenario where new product models are ready for selection and another scenario where the number of new products is determined but the attributes are variables to be designed.

(1) Product line adaptation for the first scenario entails a combinatorial process of retaining some product models from the old product line and selecting some new models from a pool of candidate new products to form a new product line. The decisions regarding changes in the product mix and prices can be formulated as follows:

$$\max \quad \pi(I^{old}, I^{can}, p_k) = R(I^{old}, I^{can}, p_k) - C(I^{old}, I^{can}, d_k) \tag{3.24}$$

$$s.t. \quad I^{old}, I^{can} \in \{0,1\} \tag{3.25}$$

$$p_k > 0 \tag{3.26}$$

There are three variables in the equations: I^{old} and I^{can} are two binary vectors of the same length of the old product line and the pool of candidate new products, respectively. p_k is the price of product model k included in the new product line.

(2) For the second scenario where new products have yet to be designed, the number of new products W is predetermined. Product models are viewed as combinations of attribute levels. Product design and product portfolio management take place simultaneously. The profit function is then formulated with variables of the new product model(s) attributes, the selection of product models from the existing product line and the prices for all product models in the new product line.

$$\max \quad \pi(I^{old}, A^{new}, p_k) = R(I^{old}, A^{new}, p_k) - C(I^{old}, A^{new}, d_k) \tag{3.27}$$

$$s.t. \quad I^{old} \in \{0,1\} \tag{3.28}$$

$$p_k > 0 \tag{3.29}$$

There are three variables in the equations: I^{old}, p_k are the same as in the first scenario; A^{new} are new products' attribute vectors of the product of the number of attributes and the product of new products. Components of A^{new} can be both continuous and discrete.

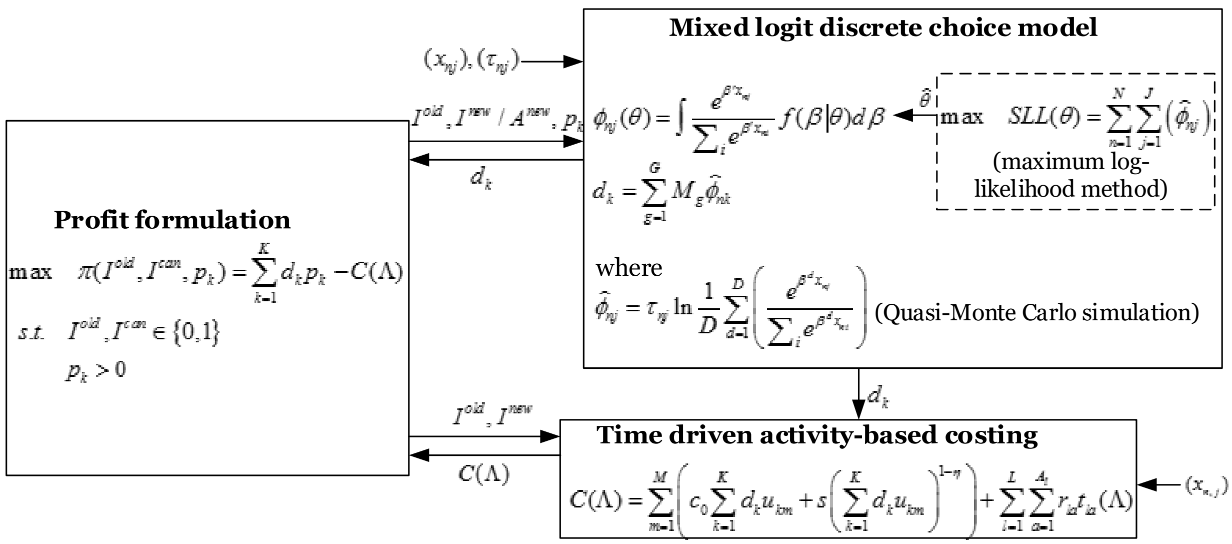


Figure 3-3 The integration of demand model, costing model and profit formulations

The aforementioned demand model, costing model and profit formulation are integrated as illustrated in Figure 3-3. The calculation of profit starts with utilizing maximum likelihood method to extract customers' preferences, as represented by $\hat{\theta}$ in the mixed logit model, from sales data or stated-choice data, i.e. τ_{nj}. Customers' preferences assumed to remain stable during the time horizon of the product line planning. Purchase probabilities and market demand are then predicted with given information of competitors' offerings, the manufacturer's products ($I^{old}, I^{new} / A^{new}$) and prices ($p_k$). Manufacturing cost is calculated with inputs of the quantity value, d_k, and product mix (I^{old}, I^{new}) and product attributes

(A^{new}). The profit of the product line can be approached easily with the demand quantity and the manufacturing cost. Old product attributes (x_k), customer demographic attributes (S_n), market segment size (M_n), each product's BOM (u_{km}), cost driver rate (r_a) and time usage (t_a), as well as the quantity discount policy represented by c_0, s and η, can be assumed as given.

The optimization problem is a mixed integer (Mixed Integer Nonlinear Programming or MINLP refers to mathematical programming with continuous and discrete variables and nonlinearities in the objective function and constraints), or a mixed integer-discrete-continuous nonlinear programming problem because product mix is a integer vector, attribute levels of new products are continuous or discrete, price is continuous, and the objective is nonlinear function. Furthermore, the calculation of choice probability, as indicated in equation 3.6, involves the integral that generally does not have analytical solutions. Simulation-based methods can be used to approximate the choice probability. Product attributes and prices are characteristics of products; a product mix is a property of product line. Each product line with given product line has its optimal product attributes and prices. Thus, there is generally no algorithm that can simultaneously solve the product line design and pricing problem to optimality. This thesis proposes a bi-level heuristic method that combines genetic algorithm (GA) and differential evolution (DE).

Chapter 4

A Bi-Level Optimization Algorithm

This chapter reports the development of a Bi-Level optimization algorithm to solve the problems as formulated in Chapter 3. The overall architecture of the bi-level optimization algorithm which combines genetic algorithm (GA) and differential evolution (DE) is presented in section 4.1 for the scenario where new product models are ready for selection. Section 4.2 and section 4.3 explain the detailed implementations of GA to optimize price and DE to optimize product line, respectively. Lastly, the entire flowchart of the combined algorithm is given. Section 4.4 discusses the scenario where new products' attributes to be determined and the changes to be made when applying the algorithm.

4.1 COMBINATION OF GA AND DE

Both GA and DE are members of evolutionary techniques that are inspired by natural evolution to find approximate solutions to optimization problems. GA and DE are stochastic algorithms (Sivanandam and Deepa 2007, Wang and Zhang 2007), handling (sampling) a population of possible solutions and finding out the fittest solutions to optimization problems. The candidate solutions are evaluated with the objective function of the problem. The ones that fit best are the ones with the highest chances to survive and to generate offspring through operations like crossover, mutation, and selection (Dasgupta and Michalewicz 2001); so that the solutions approach optima. Due to this specific optimization process, GA and DE do not have many particular mathematical requirements about the optimization problems. All that is needed is an evaluation function of the objective. Relatively speaking, GA is more appropriate for discrete optimization, using binary mutation strategy, and is thus adopted for product selection. Despite DE is more appropriate for continuous optimization, incorporating a differential mutation, it is easy to extend DE for discrete optimization, thus DE is adopted for pricing and product attributes, as solution spaces of prices are continuous and solution spaces of product attributes are continuous, discrete or mixed.

Decision makers of the product mix can be regarded as leader and decision makers of price and other product attributes are followers. The upper level optimization of GA has a priority to determine its design vector which is a product mix and announce it to DE. The lower level optimization of DE decides prices based on given product mix and return optimal prices as the feedback to GA.

Figure 4-1 illustrates the overall architecture and procedure of the bi-level optimization algorithm for selecting and pricing old and new product models. The procedure starts by encoding a product line in a discrete space (Step 1). GA is then used for selecting and improving the product mix which is a subset of the group of products from both the old product line and the pool of candidate new products to form a new product line (Step 2). The price of each product model in any proposed product line can be encoded in a continuous space (Step 3) and the price will be optimized by the DE algorithm. Prices will be updated for each product model correspondingly (Step 4). The resulting product lines, including product mix and prices, will be evaluated based on its impact on demand, cost, revenue and eventually profit, which is adopted as the fitness function (Step 5). The outcomes of evaluations will then be used as a basis to determine the direction of the further search (Step 6), with which the process goes back to Step 1 and starts another cycle of evolution. The process continues until it reaches some predetermined stopping criteria, which can be defined based on the convergence of the average fitness value.

In case that new product design is involved in the product line design problem, this bi-level algorithm is capable to support the decision-making by offering optima of new products' attributes, the retention of old product and prices. Price is actually one of product attributes but usually discussed separately from other attributes. This is because price greatly affects demand, contribute to revenue and profit directly and

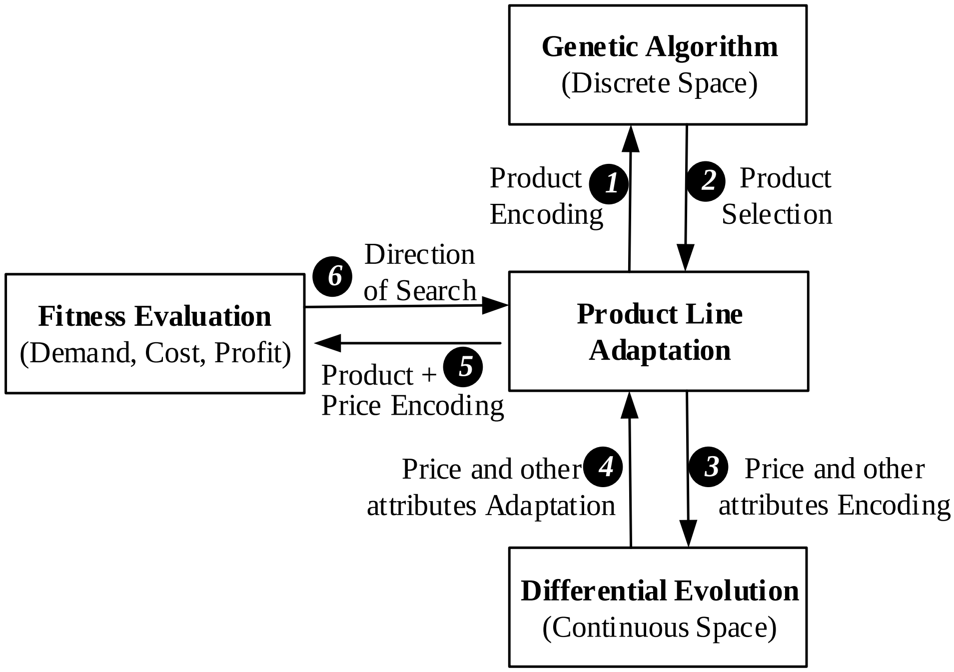


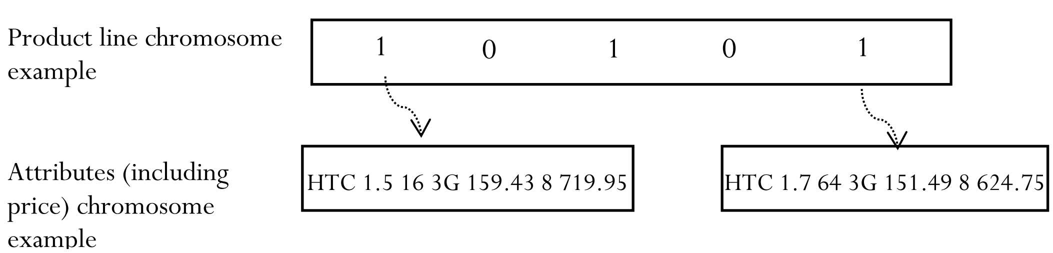
Figure 4-1 Overall architecture and procedure of the algorithm GA and DE

can be changed easily. In the scenario where new products are required to be designed, prices and product attributes are optimized simultaneously in differential evolution. GA and DE will perform different functions in this problem: GA selects product models from only the old product line and DE optimizes product attributes and prices simultaneously with given selection of old products. Product encoding (Step 1) will encode only old products as a binary vector; subsequently GA optimizes product mix of old products (Step 2). In Step 3, prices and new product attributes are encoded to continuous and continuous, discrete or mixed, respectively. DE offers optimal prices and new product attributes for each proposed product mix. The resulting product lines' profits obtained in Step 5 guide future searches of feasible solutions (Step 6). Again, the procedures above iterates until predetermined stopping criteria are satisfied.

4.2 GENETIC ALGORITHM

Genetic algorithm (GA) has been used in solving product line design problems in the literature (Alexouda and Paparrizos 2001). It is able to find approximate solutions to optimization problems with objective functions that do not possess "nice" properties such as continuity, differentiability, satisfaction of the Lipchitz. Therefore, in this research GA is used to search the best product line in a specific set of solutions.

To apply GA, each candidate product model is represented as a binary bit and the entire choice set is encoded as a binary vector; 1indicating the attendance of the corresponding product and 0 the absence in a potential product line (refer to Figure 4-2). The search space covers all binary chromosomes; there are many possible bit strings (or chromosomes) and each represents one feasible solution in the search space. The new product line in Figure 4-2 is one example of possible new product lines. Chromosomes in GA work with one population of solutions rather than a single solution. The initial generation of candidate solutions is specified by using chromosome of the current product line, as this research aims to evolve it. With the initialized solutions, GA can operate reproduction of candidate solution by operators of parent selection, crossover and mutation, as illustrated in figure 4-3.



Attributes presented in chromosome are brand, CPU, memory, network, size and camera and price

Figure 4-2 Representation of product line chromosome in GA

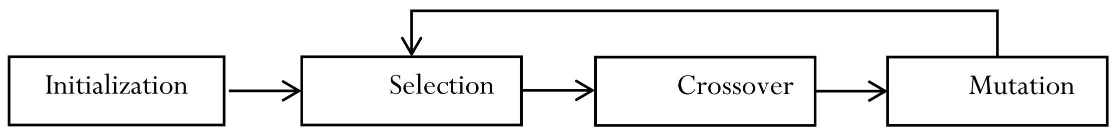


Figure 4-3 Genetic algorithm (GA) procedure

Each individual bit string has an associated value corresponding to the fitness of the solution. In this research, the associated value refers to the maximum profit the product line can achieve with its optimal price, because a product line's profit varies with its price (more details in section 4.3). GA selects parents for crossover from the current candidate solution pool according to their fitness performances. The higher value of the fitness function, the greater the chance that an individual solution is selected. There are multiple selection operators in practice, and a traditional one of "Roulette wheel" (Sivanandam and Deepa 2007) is applied to select parents in this thesis. Each individual's probability of selection is dependent on its fitness value over the sum of all individual's fitness value in that generation. For each and every selected individual solution in a population, a counterpart individual is randomly selected for crossover, which recombines the genetic materials of parent chromosomes to produce offspring. There are different versions of crossover; such as one-point crossover, two-point crossover and uniform crossover. The number of crossover points depends on the length of the binary vector; generally, the number of crossover points is in similar direction proportion to the length of the binary vector. One or more crossover points are randomly chosen and a portion of chromosomes are subsequently exchanged between parents. The crossover rate is typically 0.5 to 1.0 (Srinivas and Patnaik 1994). After the crossover, a bit by bit mutation (flipping a bit, changing 0 to 1 and vice-versa) is executed randomly. The mutation rate for each bit in the offspring is small, typically between 0.005 and 0.05 (Srinivas and Patnaik 1994). Following this, the resulting individual is compared with its parents and the fittest one selected (Alexouda and Paparrizos 2001). GA iterates selection, crossover and mutation operators until its objective function value converges or preset situation achieved.

4.3 DIFFERENTIAL EVOLUTION

Differential evolution algorithm (DE) is another stochastic and population-based method (Wang and Zhang 2007). It was initially developed for global optimization problems over continuous spaces and has been successfully applied for discrete and mixed problems. To utilize DE to optimize prices and product attributes with continuous, each variable (either price or product attribute) is encoded as a real number and all variables together forms a vector. As illustrated in Figure 4-4, prices for product models included in the product line are positive values while prices for product models excluded in the product line are set to zero. The research spaces for prices can be primarily set as positive numbers and updated according to specific cases. The research spaces for product attributes are limited by technology and engineering constraints. Initial candidate population is randomly selected from its research space, and the population with *NP* individuals maintained in each generation.

DE reproduction starts with mutation operator, in which DE employs the weighted differences of two randomly selected parameter vectors as the source of random variations for a third parameter vector (Storn and Price 1997). It is assumed that the individual i in generation k is denoted by p_i^k , $i = 1,2,..,NP$. To generate a donor vector v_i^{k+1} in mutation, three vectors ($p_{r1,i}^k$ $p_{r2,i}^k$ and $p_{r3,i}^k$) are randomly chosen from the current population; the equation is:

Product line chromosome example	1	0	0	1	1	1
Price chromosome example	645.34	0	0	324.83	454.53	341.83

Figure 4-4 Representation of price chromosome in DE

$$v_i^{k+1} = p_{r1,i}^k + F(p_{r2,i}^k - p_{r3,i}^k) \tag{4.1}$$

where F is mutation rate, which is usually a constant from [0, 2] and recommended as 0.5 (Storn and Price 1997). Donor vectors incorporate successful solutions from the previous generation through crossover and thus *trial vector* $u_{j,i}^{k+1}$ is developed with equation:

$$u_{j,i}^{k+1} = \begin{cases} v_{j,i}^{k+1} & rand_{j,i} \leq CR \quad or \quad j = I_{rand} \\ p_{j,i}^k & otherwise \end{cases} \tag{4.2}$$

where CR is the crossover probability, which is between 0 and 1, and recommended as 0.1 (Storn and Price 1997). $rand_{j,i}$ is continuously uniform and distributed between 0 and 1 ($rand_{j,i} \sim$ U[0, 1]), I_{rand} is a random integer from [1, 2,…, G], with G as the maximum generation number. DE selects new individuals from the trial vector and previous generation population according to their profit performance. The equation for selection is:

$$p_i^{k+1} = \begin{cases} u_i^{k+1} & if \quad \pi(u_i^{k+1}) \geq \pi(p_i^k) \\ p_i^k & otherwise \end{cases} \tag{4.3}$$

DE iterates crossover, mutation and selection until the value of the profit function (i.e. $\pi(p_i^k)$) converges or achieves the preset situation.

4.4 DE FOR DISCRETE VARIABLES

Product attributes usually are constrained to certain levels; therefore it is essential for DE which is employed to optimize product attributes to solve discrete optimization problems. Discrete variables in product line design problem refer to discrete product attributes, such as CPU, memory, network and camera pixel of smartphones. Manufacturers usually set these product attributes at certain limited levels. Only a couple of simple modifications are required to be made on DE to optimize discrete variables. Suppose that discrete variables, $x_i (i = 1,\ldots, I)$, have N_i elements $(y_1, y_2,\ldots, y_{n_i},\ldots y_{N_i})$ that can be assigned to them $x_i \in (y_1, y_2,\ldots, y_{n_i},\ldots y_{N_i})$ and $y_{n_i} < y_{n_i+1}$. Instead of randomly generate the first generation of population within the boundaries, DE for discrete variables starts by initializing as follows:

$$x_{i,j}^{(0)} = r_{i,j}(y_{N_i} - y_1 + 1) + y_1 \quad i = 1,\ldots, I \quad j = 1,\ldots, J \tag{4.4}$$

where $r_{i,j}$ denotes a uniformly distributed value within a range of [0, 1]. y_{N_i} and y_1 are the largest and the smallest feasible solutions of $x_{i,j}$ respectively. J denotes the population of solutions. When evaluate the fitness of variables, perform truncation to continuous variable to get integer $n_{i,j}$. $n_{i,j}$ would take the integer part of $x_{i,j}$ and plus 1, if the fraction part of $x_{i,j}$ is bigger than 0.5; and otherwise $n_{i,j}$ take the integer part of $x_{i,j}$.

$$n_{i,j} = \begin{cases} \overline{x}_{i,j} + 1 & if \quad x'_{i,j} \geq 0.5 \\ \overline{x}_{i,j} & else \end{cases} \tag{4.5}$$

$\overline{x}_{i,j}$ and $x'_{i,j}$ are the integer part and fractional part of $x_{i,j}$, respectively. Take the discrete value y_{n_i} that integer $n_{i,j}$ index and use it to calculate fitness, $f_{fitness}(y_{n_i})$. When the reproduction operator of DE extends the search outside the range of search space, the solution $x_{i,j}$ would be replaced by a random number generated using equation (4.4) if it goes beyond research space.

Instead of optimizing the value of discrete variable directly, we optimize the value of index $n_{i,j}$. Only during evaluating the fitness, the indicated discrete values are used. The process of DE for discrete variables iterates until the index converges into an integer. The corresponding numbers labeled by the index numbers are the optima.

Flowchart of the algorithm

Figure 4-5 illustrates the detailed flowchart of associating GA (outer level in grey color) and DE (inner level in while color). GA selects product models from candidates. Even with the same product mix, the profit obtained varies if the price for any product model changes. DE supports GA to find each proposed product mix's best profit with optimal price for each product model in the line and optimal new product attributes if new products are required to be designed. The best profit is used as the corresponding product line's fitness evaluation to guide the search direction of GA. GA outputs results of optimal product line including new products, product selection and prices. DE optimizes the price for each product model of any product line proposed by GA and the attributes of new products and supplies the maximum profit that the product line can achieve. In one run of DE, with one inputted product mix, DE goes through initialization, reproduction and fitness test which has prices and new products' attributes as variables, while the product mix is constant. DE outputs results of optimal prices and maximized profit for each product line that GA proposes. The procedures of this bi-level algorithm are:

Step 1: Initialize product mix with the current product line as one of them in GA.

Step 2: Input one product mix from the corresponding population into DE.

Step 3: Initialize new products' attributes and prices for all product models.

Step 4: Reproduce product attributes and prices through mutation, crossover and selection in DE (refer to section 4.3).

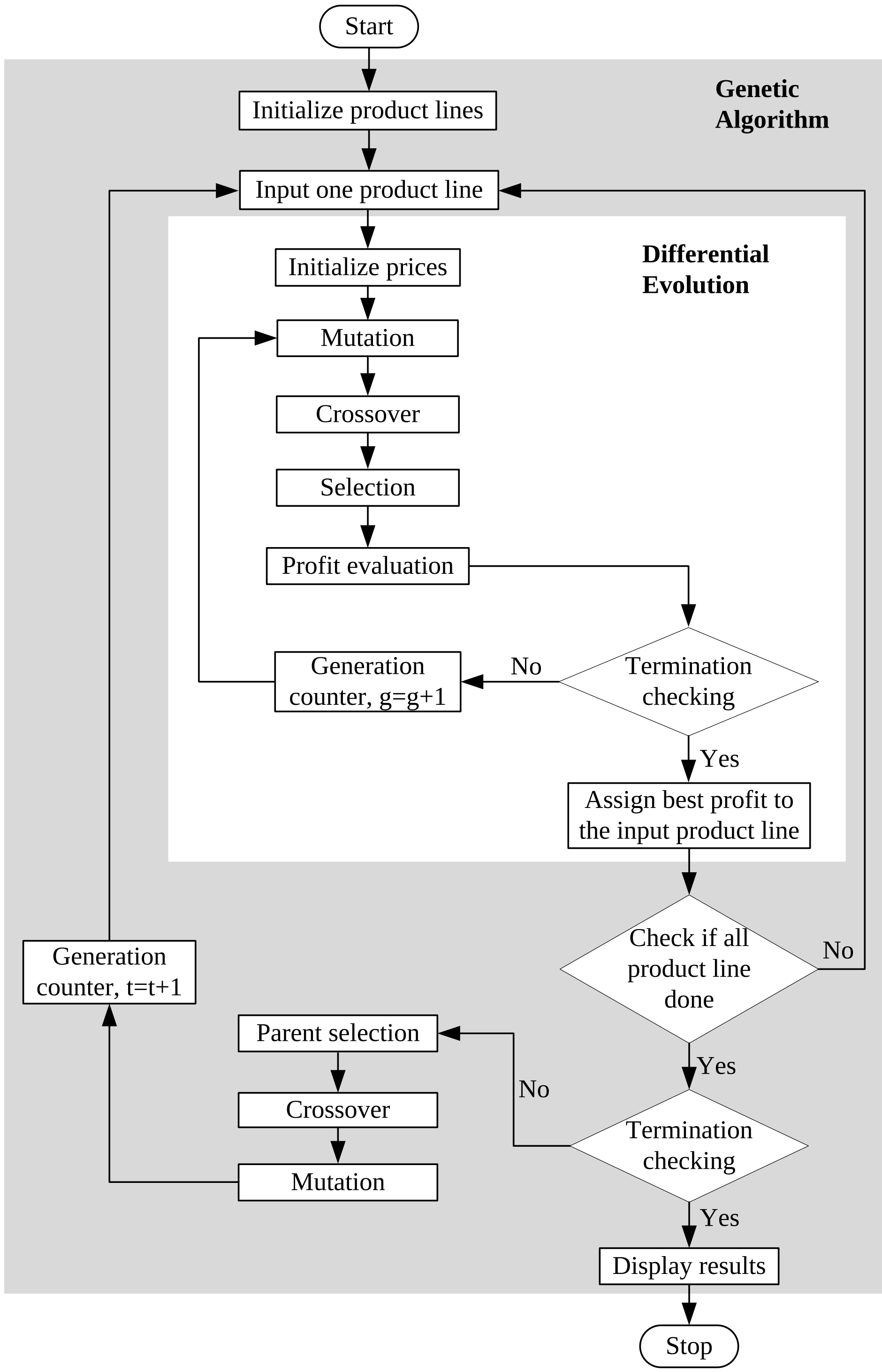


Figure 4-5 The flowchart of a bi-level algorithm

Step 5: Test the fitness of the product attributes and prices by estimating the associated profit (refer to chapter 3).

Step 6: Check the termination criteria, which is profit convergence or other preset situations; if no, go back to step 3; if yes, terminate the run of DE and assign the highest profit and optimal prices to the inputted product line.

Step 7: Iterate step 2 to step 6 with different product mixes until all product mixes in the population are associated with their best profits and optimal prices.

Step 8: Reproduce next generations of product mixes through mutation, crossover and selection in GA (refer to section 4.2), and go through step 2 to step 7 to calculate each product line's best profit.

Step 9: Check the termination criteria for GA, which is the convergence of product lines' profit or other preset situations. If no, go back to step 8; if yes, terminate the algorithm.

The solutions (price and attributes) form GA and DE were considered optimal, because the objective function converged at the same solutions in around 50 runs using MatLab. The value of objective function and candidate solutions of each run were displayed and observed. The objective function was considered converge only when it retained the same value (at six decimal places) in about 100 continuous iterations.

The bi-level optimization algorithm supports the proposed model by providing the most adapted product line, including new product design, product selection and prices. In condition that only either product selection or prices are required to evolved, GA or DE separately can be used, respectively.

Chapter 5

A Case Study

In this chapter, the methodology presented in last two chapters is illustrated with a smartphone product line design problem. Section 5.1 introduces the case and section 5.2 clarifies the data and preparation work needed, applies the demand model and quantifies the customer preferences which are used to predict demand. Section 5.3 identifies the cost data and section 5.4 gives suggested results under different strategies for the case study.

5.1 CASE STUDY INTRODUCTION

The proposed design methodology can be applied to product line design of consumer products, for which there are a large population of consumers with diverse preferences and tastes. However, full-scale implementation of the methodology requires a large amount of data from multiple disciplines including engineering, marketing and manufacturing. Without loss of generality, this thesis uses smartphones sold on a university campus as a case study to illustrate the proposed methodology.

A smartphone is a rapidly growing category of personal digital assistant with advanced computing capability connectivity. The rise of smartphones in recent years has been swift and closely watched by consumer brands who are interested in knowing what their customers prefer. Manufacturers have launched a large variety of smartphones and regularly refresh their smartphone product lines by phasing in new models and/or phasing out old models to cater to changing consumer preferences. This case study aims to support HTC with its product line decision-making for sales of smartphones on the university campus.

It is important to note that smartphones are generally modular in design and that manufacturers can offer almost unlimited variety through product configuration, especially over the internet. Product variants can be derived from a base product by modifying, adding or subtracting some features, e.g. by choosing different storages, RAMs, and/or CPUs etc. In this thesis, we study a traditional sales channel in which manufacturers sell through retail stores. Due to constraints like shelf space and inventory related costs, manufacturers carry only a limited number of product models on campus. The product variants derived from the same base product with minor modifications are assumed to belong to the same product model. HTC's current product line (Λ^{old}) on campus and its major competitors' offerings (Λ^{com}) are summarized in Table 5-1. Smartphone models offered in other market segments by HTC can be taken as the candidate pool of new products (Λ^{new}). Alternatively, HTC can derive new product models by setting some attribute levels.

5.2 DEMAND DATA

The attributes (A_k) of smartphones are collected based on the product descriptions from manufacturers' websites and further refined based on a survey of students that seeks to identify the most important criteria in purchasing smartphones. Some features like colors of cover, applications and accessories, although apparently important, are excluded because different manufacturers offer more or less the same choices and consequently they are not the differentiating features that affect students' purchasing decisions for a smartphone.

Table 5-1 Major Smartphone models for university students

	Product Model		**Brand**	**CPU (GHz)**	**Memory (GB)**	**Network**	**Size (mm)**	**Camera (M pixel)**	**Prices (USD)**
Current product line	Butterfly X920d	$(x_{,1})$	HTC	1.5	16	3G	159.43	8	719.95
	One SV	$(x_{,2})$	HTC	1.2	8	4G	144.43	5	508.30
	8X windows Phone	$(x_{,3})$	HTC	1.5	16	4G	148.03	8	618.80
Candidate new products	8S Windows Phone	$(x_{,4})$	HTC	1	4	3G	135.98	5	338.30
	One X Plus	$(x_{,5})$	HTC	1.7	64	3G	151.49	8	624.75
Competitors' offerings	Galaxy Ace 2	$(x_{,6})$	Samsung	0.8	4	3G	133.65	5	253.30
	Galaxy S 2	$(x_{,7})$	Samsung	1.2	16	3G	141.67	8	428.80
	Galaxy Note 2 N7105	$(x_{,8})$	Samsung	1.6	16	4G	171.21	8	720.80
	Galaxy S 3	$(x_{,9})$	Samsung	1.4	16	3G	153.77	8	551.65
	Galaxy Y S5360	$(x_{,10})$	Samsung	0.8	0.2	3G	119.08	2	135.15
	iPhone 5	$(x_{,11})$	Apple	1.2	16	4G	136.97	8	786.25
	iPhone 4S	$(x_{,12})$	Apple	1	8	3G	129.25	8	654.00
	iPhone 4	$(x_{,13})$	Apple	1	8	3G	129.25	5	488.00
	9790	$(x_{,14})$	Blackberry	1	8	3G	125.30	5	448.80
	9860	$(x_{,15})$	Blackberry	1.2	4	3G	135.07	5	361.25
	9900	$(x_{,16})$	Blackberry	1.2	8	3G	132.59	5	590.75
	Ericsson Xperia Neo V	$(x_{,17})$	Sony	1	0.3	3G	129.25	5	273.70
	Ericsson Xperia Ray	$(x_{,18})$	Sony	1	1	3G	123.00	8	278.80
	Ericsson Xperia Arc S	$(x_{,19})$	Sony	1.4	1	3G	139.98	8	391.00

Mixed logit model is employed to estimate the students' preferences and tastes for smartphones. As this study focuses only on the university student market segment, customer demographic attributes (S_n) are neglected in mixed logit model as the students can be taken as homogeneous from a demographic point of view. The market segment size (M_g) was estimated to be around 10,000 based on the annual enrolment of students and on historical smartphone sales on campus. To collect students' choices among different smartphones, a survey was conducted. All major smartphone models (a total of 17) offered on campus, 2 candidate new smartphones, i.e. $\Lambda^{old} \cup \Lambda^{can} \cup \Lambda^{com}$ and their typical variants were collected and input into a database. Each student was asked to select the smartphone that he/she would buy from a choice situation with 3 different alternatives, which were randomly selected from the choice pool. Each student was presented with 5 different choice situations, and a total number of 1000 students randomly selected on campus participated in the survey.

All the attributes are denoted in real values except '*brand' and 'network'*, which are nonquantifiable and have multiple discrete alternatives. As there is no clear ranking among different brands and networks, it is difficult to encode brand and network as single variables. To circumvent this difficulty, this thesis further assigns '*brand'* and 'network' to vectors that include the specific brand names and specific network names as in the following.

'brand' = [HTC, Samsung, Apple, Blackberry, Sony]

'network' = [*3G, 4G*]

Each brand and network name assumes a binary value that indicates the manufacturer's choice alternative. Thus, there are a total of 12 explanatory variables. For example, the first product model's attributes in table 5-1are represented as:

$$x_{n1} = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1.5 \quad 16 \quad 1 \quad 0 \quad 159.43 \quad 8 \quad 719.95]$$

A vector of the same size is defined correspondingly to indicate students' preferences and tastes over these attributes. As customers have clear preferences over some attributes, e.g. customers always prefer lower price to higher price (other attributes being equal), customers' tastes over these attributes are assumed to follow log-normal distributions, the values in which are always positive (Train 2003). Such attributes include '*CPU*', '*memory*', and '*camera pixel*' besides '*price*'. Preferences over other attributes like brand names and networks are assumed to follow normal distributions because there is no clear preference structure in the attributes. Table 5-2 gives customers' preferences in the mixed model estimated based on the survey.

Table 5-2 Mixed logit model estimation

product attributes (x)			Taste parameters (β)	ML model ($\hat{\theta}$)
Brand	HTC	Binary	Normal	(1.738 5.045)
	Samsung	Binary	Normal	(1.984 1.136)
	Apple	Binary	Normal	(5.933 4.789)
	Blackberry	Binary	Normal	(2.591 7.129)
	Sony	Binary	Normal	(-0.447 4.164)
CPU		Real	Log_normal	(-0.368 1.413)
memory		Real	Log_normal	(-3.167 0.402)
Network	3G	Binary	Normal	(0.658 0.769)
	4G	Binary	Normal	(2.641 0.706)
Size		Real	Normal	(0.260 0.080)
Camera		Real	Log_normal	(0.996 0.206)
Prices		Real	Log_normal	(-2.929 0.044)

5.3 COST DATA

To model the cost of a product line, a practical difficulty in implementing the TDABC framework as presented in equations 3.12 to 3.14 is that production-related costs are based on aggregate production levels of a manufacturer. The university student market accounts for a very small percentage in the overall cost structure for a large manufacturer like HTC. Thus, it would be difficult to model the cost implications of product line adaptation at batch or component level. To circumvent this difficulty without losing the generality of the proposed methodology, this thesis focuses on the product level for cost estimation. More specifically, list cost of a smartphone is estimated by summing the list prices of its key components plus an estimated manufacturing overhead cost rate based on published industry data. The list cost can be taken as the highest cost of a product model as manufacturers usually get quantity discounts from the list prices in material sourcing. The base cost of a smartphone can be estimated by deducting a certain gross profit margin from the wholesale price. The base cost and cost spread are represented as c_0 and s respectively in the quantity discount formula (equation 3.13). The costs of phasing in and phasing out a product model are also estimated based on the logistics costs of a university smartphone shop. The quantity discount rate (η) depends on product models and can be estimated based on the wholesale and retail prices of smartphones. Table 5-3 summarizes the estimated cost data for HTC's smartphone models.

The manufacturer may intend to provide the market segment with newly designed smartphones in order to better satisfy students' needs rather than bringing in smartphones from other market segments. In this illustration, the manufacturer creates one new product by determining its key attributes' levels, in view of that products usually are represented by a bundle of attributes. Components of the new smartphone are divided into two types: fixed components and changeable components. Fixed components are those that are predetermined and excluded in our consideration. Changeable components are those related to attributes

Table 5-3 Cost estimates of HTC's smartphones

Model ID	Lowest cost c_0 (USD)	Cost spread s (USD)	Cost of phase in/out (USD) C^a	Discount rate η
1	525	53	1,200	0.1
2	318	32	1,200	0.1
3	422	45	1,200	0.1
4	151	16	1,200	0.1
5	432	46	1,200	0.1
6	400	70	10,000	0.1

included in the survey and to be determined in this design model. The new smartphone is labeled as the 6th model in Table 5-3. The total cost of all fixed components has the lowest value of $400, a cost spread of $70 and a discount rate of 0.1. The changeable attributes, their feasible levels, and corresponding costs are listed in Table 5-4.

With the data supplied in table 5-1 and table 5-2 product line (with assigned prices) demand and revenue can be estimated. With table 5-3 and demand, the product line's overall cost can be computed as well. Finally, its profit can be computed based on the estimated revenue and cost. The integrated GA and DE algorithm is implemented in Matlab and offers the optimal results.

Table 5-4 New Smartphone's feasible attribute levels and corresponding costs

Attributes	**CPU**		**Memory**		**Network**		**Size**		**Camera**	
Units	*GHz*	USD	*GB*	USD		USD	*mm*	USD	*M pixel*	USD
Lower level	1.7	28	16	24	3G	34	142	52	8	28
Upper level	2.5	46	32	56	4G	54	162	58	13	50

5.4 RESULTS AND DISCUSSION

Based on the proposed model, we investigated different strategies of product line adaptation, which included remain status quo (no change), re-price (optimizing prices of current product line only), re-mix (Optimizing product mix only), re-mix + re-price (Optimizing product mix and prices simultaneously), and design + re-mix + re-price (Designing a new product, optimizing product mix and prices simultaneously). Table 5-5 summarizes the implications of these different strategies of product line adaptation on product mix, price, demand, revenue, cost, and profit. Product mix is the set of all products and items that a particular seller offers for sale. For example, product mix of [1 1 1 0 0] in Table 5-5 means that the product line consists of the first three products of five HTC products in table 5-1, which are Butterfly X920d, HTC One SV and HTC 8X windows Phone.

Assuming that HTC does not change its current product line, i.e. the status quo remains, then the demand for the first two product models (Butterfly X920d and One SV) is small relative to the market size of 10,000 units. The total profit will be $120,850.

If, however, HTC retains the current product models in its product line and re-prices them only, the optimal prices are achieved by dropping them from $719.95, $508.30 and $618.80 to $649.88, $415.53, and $531.96, respectively. Lower prices attract more customers and increase the demand from 18, 21 and 664 to 28, 121 and 2,322, respectively. The increases in production volume decrease manufacturing costs; average costs decline from $564.70, $341.60 and $445.50 to $562.98, $337.81 and $442.73, respectively. Lower prices, larger demand and lower costs influence profit collectively. Profit of the first product model (Butterfly X920d) will decrease from $2,795 to $2,433, while the profits of other two product models' will increase from $3,500 and $115,074 to $9,404 and $207,189, respectively. The profit decline of the first product model is compensated by the profit increments of the other two and the total profit of the entire product line will grow to $219,026.

Table 5-5 Different strategies of product line adaptation and implications

	Status quo	**Re-price**	**Re-mix**	**Re-mix +Re-price**	**New product design + re-mix + re-price**
Product mix	[1 1 1 0 0]	[1 1 1 0 0]	[1 1 1 1 1]	[0 0 11 1]	[1 1 1]
New product's attributes					[1 0 0 1 1]
Price	[719.95; 508.30; 618.80]	[649.88; 415.53; 531.96]	[719.95; 508.30; 618.80; 338.30; 624.75]	[548.90; 260.55; 562.86]	[629.75; 408.32; 548.38; 817.88]
Demand	[18; 21; 664]	[28; 121; 2,322]	[4; 6; 231; 380; 770]	[558; 1,268; 1,306]	[38; 32; 668; 1779]
Total demand	703	2,471	1,391	3,132	2,517
Revenue	[12,959; 10,166; 410,883]	[18,197; 50,279; 1,235,211]	[2,880; 3,050; 142,943; 128,554; 481,058]	[0; 0; 306,286; 330,377; 735,095]	[23931; 13066; 366,320; 1,455,000]
Total revenue	$434,008	$1,303,687	$758,485	$1,371,758	$1,858,317
Cost	[10,060; 7,170; 295,810]	[15,800; 40,900; 1,028,000]	[2,280; 2,070; 103,510; 61,940; 352,060]	[1,200; 1,200; 248,820; 202,600; 594,710]	[21,350; 10,900; 297,600; 1,196,800]
Average cost	[564.70; 341.60; 445.50]	[562.98; 337.81; 442.73]	[571.14; 344.75; 448.11; 162.99; 457.22]	[445.91; 159.78; 455.37]	[561.84; 340.63; 445.48; 672.74]
Profit	[2,795; 3,500; 115,074]	[2,433; 9,404; 207,189]	[595; 981; 39,429; 66,617; 128,995]	[-1,200; -1,200; 57,469; 127,780; 140,387]	[2,581;2,166; 68,720; 258,200];
Total Profit	**$121,369**	**$219,026**	**$236,618**	**$323,236**	**$331,667**

If HTC reselects the product models without updating the prices (8S Windows phone and One X Plus are sold with the same prices as in the other markets), the product mix will be optimized by extending to include both 8S Windows Phone and One X Plus (phasing in). The new smartphones introduced cannibalize the sales of existing ones and the demands for the old three smartphone models decrease significantly, from 18, 21 and 664 to 4, 6 and 231, respectively. Smaller production volumes of the three old smartphone models cause higher average costs, as cost and demand generally have an inverse relationship. Nevertheless, a higher variety of products meet diverse customers' needs better; the overall market demand of the entire product line will increase from 703 to 1,391 units. The total profit will improve to $236,618 because of large sales. Compared with the re-pricing strategy, re-selects strategy creates larger demand, higher revenue, and higher cost but lower profit.

Should HTC chooses to update product mix and prices simultaneously, both 8S Windows Phone and One X Plus will be added to the product line(phase in), but Butterfly X920d and One SV will be obsoleted (phase out); then the prices for the selected product models are $548.90, $260.55 and $562.86, respectively. The demands for all three models will be 558, 1,268, 1,306, respectively which are remarkably larger compared with those in the aforementioned strategies. Because of the phase-out activity cost, each product model phased out costs $1,200. Even though this strategy suggests a lower variety of products and lower prices, the total profit will improve to $323,236, which is higher than either re-pricing or re-selecting strategy alone.

Finally, if HTC determines to design one new smartphone and add it to the product line, reconsiders retaining or obsoleting the old product models and sets prices for each product model in the product line. We select a product mix from the old smartphones, add one new smartphone and optimize the prices simultaneously, the best profit is achieved when all the three old smartphone models are selected and the new smartphone has a technical specification of CPU of 2.5GHz, Memory of 16GB, network of 3G, size of 162mm and Camera of 13M Pixel. The prices of the old product models' are reduced to $629.75, $408.32,

$548.38, respectively; the new product model's price is set at $817.88. Compared to status quo, the optimal prices and the market demand of the old product models decline because of the cannibalization by the new product model. With lower demand quantities, the average unit cost of the old product models increases due to economies of scale and, consequently, the profit from the old product model decreases. Nevertheless, the total profit from the new product exceeds the decrease in the total profit from the old product models and the overall profit of the entire product line improves to $331.667.

We see that when either product mix or prices changes, the demand of all product models changes which in turns results in changes to revenue, cost and profit. In a competitive environment, a product's demand does not necessarily increase when its price decreases; it also depends on other products. When the product mixes changes, each product model's optimal price may change as well. Referring re-price strategy and new product design + re-mix + re-price strategy, When the product line is [1 1 1 0 0], the product line optimal price is [719.95; 508.30; 618.80], but changes to [629.75; 408.32; 548.38] when one new product model is designed and added to the product line. The increase in product variety with same prices decrease each product's demand because of cannibalization within the product line but increase total demand as different customers' needs can be satisfied better. Observing the relationship between demand and cost, it is found that when the demand quantity of a product model increases, the total cost increases but the unit cost of the product increases nonlinearly. In relation to the 5 strategies, higher demand (market shares) does not necessarily mean higher profit. Therefore, demand should be used to evaluate a product line if the company pursues higher profit as an objective. The relative superiority of these 5 strategies of product line adaptation depends on both the impact of market (revenue) and cost (including the cost of re-mix and/or re-price).In this case study, optimal prices are lower than the current prices and demand should increase. However, it is not always the case. It depends on product similarity, customers' preferences and manufacturing costs. The company does not necessarily decrease prices sufficiently low, or offer all product models it has to market to make a good profit.

Chapter 6

A Behavioral Choice Model

Product line design problem have been investigated under the rational choice theory, but rational choice models such as conjoint analysis and discrete choice models neglect customers' irrational behaviors. Consumers do behave irrationally sometimes. For instance, shoppers are more likely to purchase an item on promotion than to one which has the same quantity and price but no promotion. This chapter extends the research by including irrational preferences in utility evaluation and developing a behavioral choice model. We discuss the limitation of traditional choice models in section 6.1, build a framework for utility, in section 6.2 and formulate value function which cover reference dependence, diminishing sensitivity and loss aversion for product attribute in section 6.3. The identification of reference point is then discussed in following section 6.4. A choice probability is then derived from the utility function under the first choice rule with the assumption that unobserved factors are independently identically distributed extreme values. In section 6.6, a case study is conducted and the behavioral choice model is demonstrated to outperform logit choice model with smaller squared error for predicting product demand.

6.1 LIMITATION OF RATIONAL CHOICE THEORY

In mixed logit and other discrete choice models, consumers are usually viewed as rational and self-interested decision makers (Zinkhan 1992, schiffman et.al. 2008). They are assumed to have well-defined preferences and are not influenced by the relative availability of certain information. Each product in the choice set is evaluated and assigned a utility value by consumers and the optimal product is chosen given the information available (Bettman et al. 1998).

Based on the assumptions of well-defined preferences, rational decision theory postulates that the purchase decision depends only on the difference of utilities among alternatives (Train 2009). In discrete choice models, a utility is formulated as a weighted sum of product attributes. Weightings represent consumer preferences on attributes and are independent of the attribute levels. As such, the determining factors of choice decisions among multiple products are based on differences in attributes, rather than final states of products. However, people do not always discard components that are shared all products under consideration; especially when the attribute level significantly exceeds the decision maker's expectations. For example, consumers might pay 100 dollars to upgrade the storage of a handphone from 16GB to 32GB, but balk at paying 200 dollar to upgrade from 32GB to 64GB, even though the capacity/price ratio remains the same.

Another essential feature of discrete choice models for rational decision is that choice alternatives are assumed to be independent from irrelevant alternatives. The relative preference between two options does not depend on the presence or absence of other options (Tversky and Simonson 1993). If A is preferred to B out of the choice set {A, B}, then introducing a third alternative X and expanding the choice set to {A, B, X} do not make B preferable to A. This feature goes against the decoy effect which is a well-known observation in marketing research. Decoy effect describes a phenomenon whereby a product can asymmetrically dominate another two products (Huber et al. 1982).

It's been increasingly recognized that consumer preferences are partly driven by the context that the set of alternatives are presented and by the framing of choices (Orhun 2009, Rooderkerk et al. 2011). The compromise effect, attraction effect and similarity effect are examples of the phenomenon of how context affects relative preferences. In this thesis, a behavioral choice model is developed with reference dependent, nonlinearly and asymmetric multi-attribute utility to address irrational behaviors in consumer purchase decision.

6.2 PRODUCT UTILITY FUNCTION

In line with random utility theory, utility, $U_{n,j}$, is composite of a deterministic systematic component $V_{n,j}$, and a random part $\varepsilon_{n,j}$.

$$U_{n,j} = V_{n,j} + \varepsilon_{n,j} \tag{6.1}$$

where $V_{n,j}$ is the explainable component of utility and $\varepsilon_{n,j}$ is the component that affects utility but is however not included in $V_{n,j}$. $V_{n,j}$ depends on both the number of measurable attributes and consumer preference on each attribute. Conjoint analysis primarily formulates it as the sum of partworths of all corresponding attributes. Discrete choice models formulate $V_{n,j}$ as a weighted sum of all corresponding attributes.

The biggest difference in the behavioral choice model with traditional choice models is how the systematic utility is structured. A lot of experimental and empirical evidence has indicated that consumers' preferences and choices are reference-dependent (Keeney 1976, Hardie et al. 1993, Tversky and Kahneman 1991). A reference could be status quo, expectations; more details are included in section 6.4. Consumers make decisions based on the potential value of losses and gains with respect to reference points rather than absolute outcomes. A change of reference point alters the preference order for prospects.

In this thesis, it is assumed that consumers have a reference point for each attribute of the alternatives. The value of an attribute is not the literal/numerical magnitude of the attribute, but a nonlinear function of its reference point. It is also assumed that the attributes are mutually independent and the utility function for multi-attribute outcomes can be decomposed into separate utility functions over different attributes

(Keeney and Raiffa 1976).The overall representative utility $V_{n,j}$ is formulated as a weighted sum of all corresponding function value of attributes.

$$V_{n,j} = \sum_{k=1}^{K} w_{n,k} v(x_{n,j,k}, x_{n,0,k}) \tag{6.2}$$

with $k = 1,2,\ldots,K$, where K represents the number of observed attributes; $x_{n,j,k}$ is an observed attribute k of product j ; $x_{n,0,k}$ is the reference point of attribute k . $v(x_{n,j,k}, x_{n,0,k})$ is the valuation function of attribute $x_{n,j,k}$ and $x_{n,0,k}$. $w_{n,k}$ is the weighting factor the valuation of $v(x_{n,j,k}, x_{n,0,k})$ contributed to utility.

6.3 ATTRIBUTE VALUE FUNCTION

The derivation of value function is on the basis of observation of consumer behaviors. Three characteristics of consumer behavior are summarized below. First, consumers' choices depend not only on the outcomes of the alternatives but also on the reference point to which the outcomes can be compared. This "reference dependence" was introduced by Kahneman and Tversky (1979) to apply to at most two non-zero outcomes. It was then extended to contain more outcomes (Bleichrodt et al. 2009, Liu et.al. 2011). Utility and disutility, or gain and loss, are derived from the reference points, rather than from the absolute levels of attributes. Changes of reference point often lead to reversals of preference. For instance, if a consumer has used a smartphone with an 8 megapixel camera for years, it is it's more likely that the consumer will evaluate a 4 megapixel smartphone camera against an 8 megapixel as disutility rather than as a utility. In the formulation of the value function, the argument of the value function is $x_{n,j,k} - x_{n,0,k}$ instead of $x_{n,j,k}$.

Second, consumers become less sensitive as the attribute level moves away from the reference point. For example, if the reference point for camera megapixel is 8, then upgrading camera by 2 megapixel from

8 megapixel has significant utility impact, while upgrading camera by 2 megapixel from 18 megapixel has a relatively smaller utility impact. This characteristic makes the value function concave in the region of gains but convex in the region of losses. In this thesis, it is assumed that the value function is a power function of $x_{n,j,k} - x_{n,0,k}$ with a positive fractional exponent when $x_{n,j,k} \geq x_{n,0,k}$ and takes a negative value of power function of $x_{n,0,k} - x_{n,j,k}$ with the same exponent when $x_{n,j,k} < x_{n,0,k}$. Without loss of generality, the exponents for gains and losses from each attribute are assumed to be identical and constant and labeled as α.

Third, consumers are more sensitive to losses than to gains of the same magnitude. If the reference point for camera pixel is 8 megapixel, the value placed on a 1 megapixel v(1) camera, v(1), is smaller in

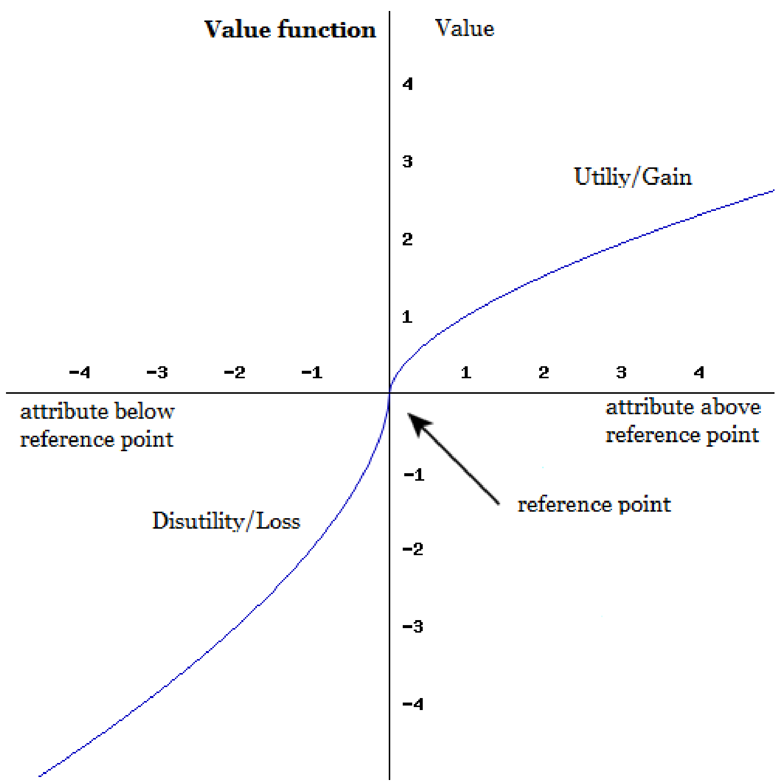


Figure 6-1 Value function of an attribute in behavioral choice model

absolute magnitude than v(-1) the value placed on -1 megapixel camera. Loss aversion makes the value function steeper in the region of loss than in the region of gain. This model is in line with constant loss aversion. A coefficient $\lambda > 1$ is introduced into the disutility function to describe the degree of loss aversion for each attribute. λ is assumed to be identical for all attributes. The shape of value function is illustrated in Figure 6-1 by summarizing its characteristics.

The value function is then formulated as

$$v(x_{n,j,k}, x_{n,0,k}) = \begin{cases} (x_{n,j,k} - x_{n,0,k})^{\alpha} & \text{if} \quad x_{n,j,k} \geq x_{n,0,k} \\ -\lambda(x_{n,0,k} - x_{n,j,k})^{\alpha} & \text{if} \quad x_{n,j,k} \leq x_{n,0,k} \end{cases} \tag{6.3}$$

where $0 < \alpha < 1$ and $\lambda > 1$. α and λ are unknown and therefore estimated statistically.

6.4 REFERENCE POINT IDENTIFICATION

A significant challenge presented in modeling reference dependence preferences is the identification of reference points. The reference points determine whether attributes are characterized as gains or losses. A change in a reference point can alter the comparative value of alternatives and, therefore, alter preference (preference reversal) even if the attributes of alternatives remain unchanged. Therefore, how consumers identify their reference points and hence frame a choice problem is critical.

Reference points, defined as “the stimuli that generate a neutral experience” are viewed as subjective points and their identification is a cognitive process (Thaler 1985, Wakker 2010). Reference points may be different in different contexts among individuals and attributes (Tversky and Kahneman 1981). In the initial phase of prospect theory, reference points are assumed to be single. Status quo, aspiration levels, expectations were generally used as reference points in prospect theory. They are however not exclusive. For instance, information of context can affect consumers’ expectation of their next goods. In later research, multiple reference points are involved in marketing models (Lopes 1987, Neale and Bazerman 1991, Koop and Johnson 2012). Some theories combine multiple reference points into one single composite point. Some other hypotheses suggest that individuals have a separation of reference points and compare each reference point to their outcomes before making decisions (Thaler 1985). For simplicity of analysis, this paper designates the most popular product in the market as the benchmark/reference point.

The utility formulation is embedded in an independent multinomial logit model by assuming independent, identically distributed extreme value of errors, and assuming the maximum utility rule. The probability that a decision maker selects a product j rather than any other product i is

$$\phi_j = \frac{e^{\sum_{k=1}^{K} w_{n,k} v(x_{n,j,k}, x_{n,0,k})}}{\sum_i e^{\sum_{k=1}^{K} w_{n,k} v(x_{n,i,k}, x_{n,0,k})}} \tag{6.4}$$

6.5 BEHAVIORAL CHOICE, STANDARD LOGIT AND MIXED LOGIT MODELS

The multinomial logit discrete choice model can be viewed as a special case of the behavioral choice model with both the exponent and the degree of loss aversion in the value function equal to 1 (i.e. $\alpha = 1$, $\lambda = 1$). In this situation, the value function becomes as $x_{n,j,k} - x_{n,0,k}$; it is a linear function of the attribute. As the reference point is a constant for each attribute, different selections of reference points have the same impact on the value of the attribute as well as the product's overall utility. The same amount of change of utility has no impact on the choice under logit choice rule. In other words, when $\alpha = 1$ and $\lambda = 1$, the selection of reference point does not affect the choice probability among a choice set.

Both mixed logit model and behavioral choice model can be viewed as refined/improved versions of standard logit model but in two different directions. They both admit that consumers' utilities on products are not always in line with attributes within a market segment. Mixed logit model attributes the variation of utility to the dynamic of preferences and allows for random taste variation, unrestricted substitution patterns, and correlation in unobserved factors over time (Train 2003). Behavioral choice model attributes heterogeneity to psychological reasons, conceptualizes the utility variation and explains it by introducing reference points, loss aversion and diminishing sensitivity.

Each of the models covers one behavior of consumers. The selection for application between them depends on the characteristics of the scenarios. When the time horizon of the demand prediction is significantly long and there are a very high variety of product alternative, mixed logit model is recommended because consumers preferences variation are more likely due to time and heterogeneity. When the product attribute levels cover very wide domains, the behavioral factors play more significant roles than heterogeneity; therefore, behavioral choice model should be hired.

6.6 A CASE STUDY

To trace out the curve of utility function, a sufficient variety of product models are required to be presented to customer and analyzed. Another case study was conducted among university students to illustrate the proposed behavioral choice model and evaluate its performance. The multinomial logit model is one of most widely applied discrete choice models and it can be viewed as a special case of the behavioral choice model as discussed earlier. Therefore, the multinomial logit model is employed as a benchmark to assess the performance of the behavioral choice model. The log-likelihood values in model estimation and squared errors in demand estimation by both models are compared. The maximum likelihood method is employed to estimate unknown parameters in models. The split-half method is employed to assess the consistency of the survey. This is done by comparing the prediction based on the estimate of the first half with the data of the second half.

The case study focuses on iPhones. Five essential attributes are used to characterized a smartphone, i.e. CPU speed, storage, diagonal screen size, battery life and price. The remaining attributes remain the same for all the smartphones. The exclusion of other attributes in the survey does not indicate that they are of least significance but that they are designed in narrower ranges, related to or included in the selected attributes. For instance, rear camera of all iPhones is 8 megapixels; the number of independent actual processing units (called "cores") is correlated with the CPU speed; therefore rear camera and the number of cores are excluded in model construction. To extract the parameters (weightings, the exponent of the value function, the degree of loss aversion) from the survey data, a sufficient variety of attribute levels are needed to form alternatives. Instead of limiting the attribute levels to available iPhones, the case study extends the ranges of attribute levels to include an adequate variety, as shown in Table 6-1. A split-half method is employed in the survey: with the first part for parameter estimation and the second part for the consistency assessment. In the first part, each product composition (hypothetical products, called profiles) is formed by randomly selecting a level for each attribute with price as an integer within the interval. Each survey respondent is asked to choose one profile out of three different alternatives that form a choice set. A total of 4,200 choice sets are presented to 350 students for the first part of data collection. The second half survey is designed to statistically check the fitness of behavioral choice model based on squared errors. Due to the large number of possible product compositions (6*5*4*5*801=680,600), 20 sample products are taken, as listed in Table 6- 2, and their selection are observed in the survey. Three of these products are randomly

Table 6-1 Attribute levels and reference points

Attributes	**Levels**	**Reference points**
CPU speed （GHz)	[0.8; 1.0; 1.3; 1.4; 1.8; 2]	1.3
Storage (GB)	[8;16; 32; 64; 128]	16
Diagonal screen size (inch)	[4; 4.7; 5; 5.5]	4
Battery life (hour)	[8;10; 14;20;24]	10
Price (SGD)	[600 – 1400]	848

Table 6-2 Intangible phones for survey

Attributes / Series No.	CPU speed (GHz)	Storage (GB)	Diagonal Screen Size (inch)	Battery life (hour)	Price (SGD)
1	1	16	4	14	608
2	1	8	5	16	698
3	1.3	8	4	8	588
4	1.3	16	4	10	848
5	1.3	32	4	10	918
6	1.4	16	4.7	14	988
7	1.4	64	4.7	14	1148
8	1.4	128	4.7	14	1288
9	1.4	16	5.5	24	1148
10	1.4	64	5.5	24	1288
11	1.4	128	5.5	24	1448
12	1.6	16	5.2	20	1028
13	1.6	32	5	17	968
14	1.6	64	5	20	1088
15	2.3	16	5.2	20	1268
16	2.3	64	5.5	24	1488
17	2.5	32	5	20	1288
18	2.5	64	5.5	20	1468
19	2.7	64	5	20	1368
20	2.7	128	5.5	24	1508

selected to form a choice set in each questionnaire. 1302 choice sets questions are distributed to 110 students for the second part of survey.

The first part of the data is used to compute parameters by applying the maximum likelihood method and the estimates are shown in Table 6-3. Two scenarios are compared –behavioral choice model with iPhone 5s as the reference point and multinomial logit, which does not assume any reference point. iPhone 5s is employed as the reference point because it was the best known exemplar of the iPhone when the survey was conducted. As in shown in Table 6-4, parameters with the same definitions differ in value among different scenarios. Weightings of all attributes, except price, are positive and they make positive contributions to products' utility. Price has a negative weighting. α has the value of 0.8321, revealing that $(x_{n,j,k} - x_{n,0,k})^{0.8321}$ fit the data best among all fractional power functions. λ has the value of 4.0202, suggesting that losses loom 4.0202 times as large as gain. The behavioral choice model gives a larger log-likelihood value than multinomial logit model, thus indicating better fitness to the data. The log-likelihood at convergence represents the maximum agreement of the selected model with the observed data. The bigger the log-likelihood at convergence represents a better fitness. The estimated purchasing probabilities for survey results are less than 1, therefore the log-likelihood at convergence are negative.

Table 6-3 Estimate variables for survey

Models / **Variables**		Behavioral choice model	Multinomial logit
Weighting	CPU	0.5087	0.6621
	Storage	0.0098	0.0037
	Camera	0.1175	0.1012
	Battery	0.0700	0.0602
	Price	-0.0030	-0.0038
α		0.8321	1
λ		4.0202	1
Log-likelihood at convergence		-4253	-4321

Table 6-4 Comparison of squared errors between behavioral choice and multinomial logit models

Series No.	Real selected percentage (%)	Estimated Choice Probability using Behavioral Choice Model (%)	Squared Error using Behavioral Choice Model (10^{-4})	Estimated Choice Probability using Multinomial Logit Model (%)	Squared Error using Multinomial Logit Model (10^{-4})
1	4.53	6.34	4.26	12.30	64.37
2	2.3	5.68	12.12	10.59	70.43
3	2.23	5.32	9.94	10.95	77.12
4	4.45	8.07	15.77	4.74	0.41
5	4.15	5.89	2.55	3.85	0.20
6	6.76	5.68	0.52	4.06	5.48
7	4.69	3.78	1.50	2.64	5.59
8	2.92	2.89	0.03	1.97	1.22
9	5.38	4.8	0.31	4.38	0.96
10	6.22	3.64	5.50	3.07	8.50
11	1.38	2.67	1.98	2.12	0.74
12	6.91	7.24	0.00	6.01	1.46
13	10.6	9	4.57	6.55	21.05
14	8.83	7.1	2.20	5.60	8.90
15	6.76	3.94	7.60	3.84	8.16
16	3.84	2.8	0.68	2.61	1.03
17	6.68	4.32	5.79	4.22	6.29
18	3.3	2.79	0.27	2.52	0.63
19	5.53	4.08	3.24	4.00	3.54
20	2.53	3.96	1.58	3.99	1.66
total	100	100	80.42	100	287.70

We count the times of selection for each variant in the second part of the survey and then calculate the percentage of selection as shown in Table 6-4. Based on the parameters estimated in the first part, we predict each variant's choice probability in the choice set of 20 alternatives and estimate the squared errors to compare the performance of the two scenarios. The sum squared error of the behavioral choice model is smaller than that of the multinomial Logit model. The results are consistent with the first part observation that the behavioral choice model offers a more accurate prediction of product demand.

Chapter 7

Conclusion

This chapter summarizes the research work that has been done (section 7.1), discusses the limitation of this research (section 7.2), and outlines the plan for future research (section 7.3).

7.1 RESEARCH SUMMARY

Manufacturers are increasingly competing on product lines instead of single products to satisfy diverse customer needs and be competitive. By sharing commonalities in components and production processes, product models in a product line can be produced with lower costs. However, products cannibalize in the marketplaces; an overextended product line may hurt the overall profitability. Prices directly affect customers' purchasing decisions and can be used to control product cannibalization. Therefore, pricing is included in product line design problem for the purpose of raising the profit of the overall product line. As product variety continues to increase while product lifecycles decrease, it is becoming an increasingly challenging task to optimize a product line in terms of its composition, features and prices.

Product line design has been studied for three decades. Most previous research assumes that new product models are complete and the decision to make is selecting a subset from new product models. This exclusion of the existing product models in the candidate pool would lead to heavy cannibalization among new and existing product models due to functional commonality. In reality, manufacturers usually adapt their product lines by the regular phasing in new candidates and/or phasing out poor performing product models instead of designing product lines from scratch. The new product line depends heavily on existing ones. Therefore, this thesis proposes an evolutionary approach for product line design and pricing. This proposal considers two scenarios where new products have been completely designed and where new products are partially designed with some key attributes to be determined. If manufacturers have already designed new product models, this model support them with decisions on:

- What new candidate product models should be phased in?
- What existing product models should be phased out from the current product line?
- What should be the optimal prices for each of the product models in a product line?

If manufacturers would like to design new products represented by attributes that are optimal at the system level and make decisions on the product design and selection in one step, this model facilitate the decisions on:

- What level each attribute of the new product models should be set?
- What existing product models should be phased out from the current product line?
- What should be the optimal prices for each of the product models in a product line?

This thesis develops a framework to illustrate the product line adaption process and clarify the interrelationships among the key decision factors, including product mix, competitors, prices, demand and cost. The product line adaptation starts with an old product line, evolves with new products phasing-in and old products phasing-out. The adaptation of a product line has complicated implications on market demand and manufacturing cost due to the coexistence of both competitive and complimentary relationship among different product models. A mixed logit discrete choice model is developed to estimate the purchase probability of a product model based on the weighted average of the logit function evaluated at different values of consumer preferences. Quasi-random simulated maximum likelihood estimation is developed to estimate the parameters for the mixed logit model. A time-driven activity-based costing model is developed to estimate the manufacturing cost of a product line by aggregating the volume of components from different product models using bill of materials, and taking into consideration of volume discounts as well as common overhead activities.

Product line adaptation is then formulated as a mixed integer or mixed integer-discrete-continuous non-linear programming problem, which aims to identify the optimal product line phasing in, phasing out, attribute setting and pricing that maximize the competitiveness (represented by profitability) of the resulting product line. Each product line with fixed product mix has its unique optimal solutions for prices and new products' attributes. As there are discrete variables (product mix and product attributes) and continuous variables (price and product attributes) and product attributes, a bi-level optimization procedure integrating genetic algorithm (GA) and differential evolution (DE) is developed for the optimization problem solving.

The proposed methodology is applied to a case study that involves a smartphone product line design for a university campus. The result has demonstrated the feasibility of the proposed methodology and its potential of being utilized as a theoretical foundation to develop decision support systems to facilitate integrated and cross-functional decision making regarding product line adaptation.

The behavioral choice model presented in this thesis offers a psychologically richer version of the standard utility framework of discrete choice models to study consumer purchasing behaviors, by taking into account of reference dependence, loss aversion and diminishing sensitivity in attribute utility. To better capture consumers' seemingly irrational behaviors, the value function is specified to be concave for attribute levels above references, while convex and with relatively steeper slopes for attribute levels that are below references. A power function with a fractional exponent and a coefficient that is larger than one are adopted to construct the value function. Involving behavioral factors in the value function offers a better performance in terms of data fitting and product demand prediction. The model provides a framework for studying consumers' irrational behaviors.

The analytical model proposed can serve as a decision-making framework to support manufacturers' decisions on product line adaptation including new product design, product phase in, product phase out and pricing. It delineates the intricate relationships between product line composition, features and prices of its constituent product models, and shows the consequent impact on market demand as well as product fulfillment. A key feature of the evolutionary product line design model, the discrete choice model for demand estimation, in particular assumes that customers' preferences and tastes are heterogeneous and stochastic. Thus, the model is ideally suitable for consumer goods industry where there are a large number of consumers who are faced with a large variety of choices when making purchasing decisions. Another key feature of the evolutionary product line design model lies in the focus on the change of a product line, with the search for a better product line starting with an existing product line and its neighborhood in the solution space. Thus, the methodology is suitable for industries where frequent and incremental changes of product lines are required.

The algorithms developed in this research can be implemented in a product line design software module that links to the database in marketing, manufacturing and engineering to extract the relevant information for product line adaptation. Changes in marketing (e.g. consumer demographics and competitive offerings), in manufacturing (e.g. material cost and quantity discounts) or in engineering (e.g. technology and product development) could trigger product line adaptation. The impact in terms of demand, revenue, cost and profit could be quickly evaluated at the system level and fed back to individual functional departments for further assessment and/or to guide implementation. This research makes a contribution towards developing product line design methodologies and systems that facilitate cross-functional decision making. It promotes continuous improvement in product line competitiveness.

7.2 RESEARCH LIMITATIONS

There are, however, some limitations in this research work. This research is premised on selecting product models and product attribute levels from what manufacturers have, rather than on creating new features in the product design and development process. Therefore, the research cannot guide engineers and designers in product innovation. It seeks only to support manufacturers to test and select the desired but already successful product models at product line level.

This research aims to support firms' tactical strategies. Technologies, market, suppliers and production capacity are essential factors to product line design and assumed to be fixed in the time horizon of planning. Technology evolutions' impacts on product line design are hard to capture unless they reflect on functional features. To make the product line more suitable for market, the model should be run more frequently whenever there are big changes in technologies, market and suppliers and have big impacts on demand or cost. Surveys on the new function attributes should be conducted to collect data on customer preference on these attributes. Disruptive innovation's impact on product line design is very difficult to predict and control, therefore it beyond the scope of the research.

It is very difficult to delineate a product line's actual economic consequence; this research uses a case example to demonstrate the feasibility of the proposed method for product line adaptation.

This research's objective is to maximize manufacturers profit from product lines, while other indicators of manufacturers' competitiveness such as market share, manufacturing cost and product sustainability are not optimized. At the same time, consumer purchase decision is based on the choice set supplied by the manufacturers; consumers' welfare may therefore not actually be maximized by product line adaptation.

Factors such as a new product's releasing time, competitors' new launches, new technologies and commercials are important for consumers' preferences and product line sales but are beyond the scope of this research. Consumers' preferences are assumed to remain static during the time horizon of product line planning. In addition, like most stochastic methods, GA and DE do not guarantee the finding of global optimum solutions for product mix, attributes and prices. They are satisfied with finding "acceptably good" solutions to the problem.

The behavioral choice model was developed for the purpose of providing a more accurate demand prediction than mixed logit discrete choice model. Because it requires more work in combining reference points and discovering appropriate unity function, the behavioral choice model hasn't been implemented in case study. The performance comparison between the behavioral choice model and the mixed logit model has not been studied in predicting a product's line profit.

7.3 FUTURE WORK

Based on the current framework, the computational model and the heuristic algorithm, more work can be done to make this research more complete for enhanced performance. Future research is needed to improve this research in two directions:

- Develop a more specific and complete behavioral choice model for product line design;
- Carry out a more comprehensive case study.

The behavioral choice model can be improved in two ways in future work: combining reference points and discovering utility function. In this thesis, only one reference point, i.e. a benchmark product, is used, but consumers' expectations and their past experiences, as well as present contexts could also be stimuli that are related to the reference point. Future study is needed to investigate the possibility of incorporating multiple reference points in the value function. The second way is regarding the value function, which is assumed to be a power function was applied. Future study is needed to study other forms of functions, like logarithm functions and exponential functions, for the possibility of better capturing consumers' behaviors.

Moreover, the current case study only covers a small market segment i.e. a university campus, where customers are more homogenous and with a limited number of product models. Among product attributes, only brand, CPU, memory, network, size, camera pixel, and prices are investigated in the case study. In reality, customer products, such as smartphone, have extremely high variety in the market; customers' purchase considerations are more than those in the case study. Markets are highly segmented due to geographical and other reasons, with heterogeneous preferences on products. In the manufacturing, products share production processes, platform and components, prices of which are related to quantity. As a result, cost calculation is much more complicated. A more comprehensive case study will be needed that takes into consideration multiple market segments and a larger number of product models.

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Appendix A1

Deviation of choice probability

$$prob(U_{nj} > U_{ni}, \forall i \neq j) = prob(A_{nj} + \varepsilon_{nj} > A_{ni} + \varepsilon_{ni}, \forall i \neq j)$$

$$= prob(\varepsilon_{ni} < \varepsilon_{nj} + A_{nj} - A_{ni}, \forall i \neq j)$$

$$= \int \left(\prod_{i \neq j} F(\varepsilon_{nj} + A_{nj} - A_{ni}) \right) f(\varepsilon_{nj}) d\varepsilon_{nj}$$

$$= \int \left(\prod_{i \neq j} e^{-e^{-(\varepsilon_{nj}+A_{nj}-A_{ni})}} \right) e^{-\varepsilon_{nj}} e^{-e^{-\varepsilon_{nj}}} d\varepsilon_{nj}$$

$$= \int \left(\prod_{i \neq j} e^{-e^{-(\varepsilon_{nj}+A_{nj}-A_{ni})}} \right) e^{-\varepsilon_{nj}} e^{-e^{-\varepsilon_{nj}+A_{ni}-A_{ni}}} d\varepsilon_{nj}$$

$$= \int \left(\prod_{i} e^{-e^{-(\varepsilon_{nj}+A_{nj}-A_{ni})}} \right) e^{-\varepsilon_{nj}} d\varepsilon_{nj}$$

$$= \int \exp\left(-\sum_{i} e^{-(\varepsilon_{nj}+A_{nj}-A_{ni})} \right) e^{-\varepsilon_{nj}} d\varepsilon_{nj}$$

$$= \int_{\varepsilon_{nj}=-\infty}^{\infty} \exp\left(-e^{-\varepsilon_{nj}} \sum_{i} e^{-(A_{nj}-A_{ni})} \right) e^{-\varepsilon_{nj}} d\varepsilon_{nj}$$

$$= \int_{\infty}^{0} \exp\left(-t \sum_{i} e^{-(A_{nj}-A_{ni})} \right) (-dt)$$

$$= \int_{0}^{\infty} \exp\left(-t \sum_{i} e^{-(A_{nj}-A_{ni})} \right) dt$$

$$= \left. \frac{\exp\left(-t \sum_{i} e^{-(A_{nj}-A_{ni})} \right)}{-\sum_{i} e^{-(A_{nj}-A_{ni})}} \right|_{0}^{\infty}$$

$$= \frac{1}{\sum_{i} e^{-(A_{nj}-A_{ni})}} = \frac{e^{A_{nj}}}{\sum_{i} e^{A_{ni}}}$$

Appendix B1

Survey of college students' preference on smartphone

Thank you very much for taking time to fill out this survey. This survey is aimed at investigating college students' preference on smartphone. The results will be used in a research case study. Included in this page you will find:

- The first row (brand, CPU and so on) shows smartphone features that are considered in this survey.
- Every **choice set** consists of 3 smartphones. Please compare the 3 smartphones in each choice set and tick the one that you like most.
- The first choice set is a sample. Please start from choice set 1.

Note: the numbers in the left most column is for the investigator's reference only, please ignore them. The ***size*** *refers to smartphone* ***diagonal length*** *and the prices are in* ***US dollar*** *(1USD=1.25SGD)*

	Brand	*CPU (GHz)*	*Memory (GB)*	*Network*	*Size (mm)*	*Camera (M pixel)*	*Price (USD)*	
				Sample Choice Set				
18	Sony	1	1	3G	123.00	8	278.80	☐
13	Apple	1	8	3G	129.25	5	488.00	☐
19	Sony	1.4	1	3G	139.98	8	391.00	✓
				Choice Set 1				
19	Sony	1.4	1	3G	139.98	8	391.00	☐
17	Sony	1	0.3	3G	129.25	5	273.70	☐
10	Samsung	0.8	0.2	3G	119.08	2	135.15	☐
				Choice Set 2				
6	Samsung	0.8	4	3G	133.65	5	253.30	☐
15	Blackberry	1.2	4	3G	135.07	5	361.25	☐
5	HTC	1.7	64	3G	151.49	8	624.75	☐
				Choice Set 3				
4	HTC	1	4	3G	135.98	5	338.30	☐
2	HTC	1.2	8	4G	144.43	5	508.30	☐
5	HTC	1.7	64	3G	151.49	8	624.75	☐
				Choice Set 4				
17	Sony	1	0.3	3G	129.25	5	273.70	☐
18	Sony	1	1	3G	123.00	8	278.80	☐
14	Blackberry	1	8	3G	125.30	5	448.80	☐
				Choice Set 5				
14	Blackberry	1	8	3G	125.30	5	448.80	☐
5	HTC	1.7	64	3G	151.49	8	624.75	☐
11	Apple	1.2	16	4G	136.97	8	786.25	☐

Appendix C1

Matlab code for case study

```
% Product, survey, and respondents' choice data
format short
global PRODUCTS TAU SURVEY CANDIDATES POPULATION THETA

%brand      CPU Hard_drive network   size    camera  price
PRODUCTS = [...
1 0 0 0 0   1.5    16.0    1 0     159.43      8   719.95
1 0 0 0 0   1.2     8.0    0 1     144.43      5   508.30
1 0 0 0 0   1.5    16.0    0 1     148.03      8   618.80
1 0 0 0 0   1.0     4.0    1 0     135.98      5   338.30
1 0 0 0 0   1.7    64.0    1 0     151.49      8   624.75
0 1 0 0 0   0.8     4.0    1 0     133.65      5   253.30
0 1 0 0 0   1.2    16.0    1 0     141.67      8   428.80
0 1 0 0 0   1.6    16.0    1 0     171.21      8   720.80
0 1 0 0 0   1.4    16.0    1 0     153.77      8   551.65
0 1 0 0 0   0.8     0.2    1 0     119.08      2   135.15
0 0 1 0 0   1.2    16.0    0 1     136.97      8   786.25
0 0 1 0 0   1.0     8.0    1 0     129.25      8   654.00
0 0 1 0 0   1.0     8.0    1 0     129.25      5   488.00
0 0 0 1 0   1.0     8.0    1 0     125.3       5   448.80
0 0 0 1 0   1.2     4.0    1 0     135.07      5   361.25
0 0 0 1 0   1.2     8.0    1 0     132.59      5   590.75
0 0 0 0 1   1.0     0.3    1 0     129.25      5   273.70
0 0 0 0 1   1.0     1.0    1 0     123.00      8   278.80
0 0 0 0 1   1.4     1.0    1 0     139.98      8   391.00
];

CANDIDATES = [...% older product model and new canidates without prices
1 0 0 0 0   1.5     2.0     16.0    1 0     159.43      8
1 0 0 0 0   1.2     1.0      8.0    0 1     144.43      5
1 0 0 0 0   1.5     1.0     16.0    0 1     148.03      8
1 0 0 0 0   1.0     0.5      4.0    1 0     135.98      5
1 0 0 0 0   1.7     1.0     64.0    1 0     151.49      8];

POPULATION = 10000; % Market size
THETA = [-3.113125  15.44015625 1.085625    1.7421875   4.54109375  1.8003125
2.645    1.32875  -0.41890625  5.2715625    0.73828125   1.00375  -12.96191406
0.33183655    -2.40890625  0.670314941  -0.409375     11.32546875  0.368564453
3.31234375     -3.218125       0.137694615  -0.45484375  0.147532564  -4.499
0.19421875]; % Estimated theta

SURVEY = [...% Each row is a choice set, 305 choice sets in total
16  19  13;
1   17  18;
8   13  4;
14  1   6;
1   2   16;
14  7   19;
1   9   8;
15  16  4;
…
```

```
10  6   2;
10  12  15;
2   13  10];

TAU=[...% Each row is a choice(1 indicates the selected choice)
    0 1 0;
    0 0 1;
    1 0 0;
    0 0 1;
    0 1 0;
    0 1 0;
      …

    0 1 0;
    0 0 1;
    0 1 0];
```

```
% demand functions; input product line and prices, output demand for each
function demand = demanding (prod_line1,prices1)
global POPULATION
format short
demand = ave_prob(prod_line1,prices1)*POPULATION;
return;
end

function  phi_es = ave_prob (prod_line2,prices2)
global THETA
format short
    beta2 = [];
    for R = 1:500
      beta2                                                              =
      [beta2;THETA(1,1)+randn*THETA(1,2),THETA(1,3)+randn*THETA(1,4),THETA(1,
      5)+randn*THETA(1,6),THETA(1,7)+randn*THETA(1,8),THETA(1,9)+randn*THETA(
      1,10),lognrnd(THETA(1,11),THETA(1,12)),lognrnd(THETA(1,13),THETA(1,14))
      ,lognrnd(THETA(1,15),THETA(1,16)),THETA(1,17)+randn*THETA(1,18),THETA(1
      ,19)+randn*THETA(1,20),lognrnd(THETA(1,21),THETA(1,22)),lognrnd(THETA(1
      ,23),THETA(1,24)),-lognrnd(THETA(1,25),THETA(1,26))];
    end

    sum_phi = 0;
    for r = 1:(size(beta2,1))
        sum_phi = sum_phi + cond_prob(prod_line2, prices2, beta2(r,:));
    end
 phi_es = sum_phi/size(beta2,1);
 return;
end

 function phi_beta = cond_prob (prod_line3, prices3, beta_single)
 global CANDIDATES PRODUCTS
    format short
    candidates = [CANDIDATES,prices3'];
    denominator = 0.0;

   for n = 1:size(prod_line3,2)
```

```
        if prod_line3(1,n) == 1
           denominator = denominator + exp(beta_single*(candidates(n,:))') ;
        else
            denominator = denominator + 0;
        end
   end

   for c = 6:19 % compititior's contribution to denominator
       denominator = denominator + exp(beta_single*(PRODUCTS(c,:))') ;
   end

   phi_beta = [];
    for n = 1:size(prod_line3,2)
        if prod_line3(1,n) == 1
            phi_beta                        =                        [phi_beta,
exp(beta_single*(candidates(n,:))')/denominator];
        else
             phi_beta = [phi_beta,0];
        end
    end

 return;
 end
```

```
% loglikelihood function
function sll = loglikelihood(theta)
inputs_testing;
    global TAU
   sll=-sum(sum(TAU.*log(ave_prob (theta))));
return;
end

function  phi_es = ave_prob (theta1)
    beta1 = draw(theta1);
    sum_phi = 0;
    for r = 1:(size(beta1,1))
        sum_phi = sum_phi+cond_prob(beta1(r,:));
    end
 phi_es = sum_phi/size(beta1,1);
 return;
end

  function phi_beta = cond_prob (beta_single)
       global PRODUCTS SURVEY
    phi_beta = [];
    for n = 1:305
        A = PRODUCTS(SURVEY(n,1),:);
        B = PRODUCTS(SURVEY(n,2),:);
        C = PRODUCTS(SURVEY(n,3),:);
        denominator    =    exp(beta_single*A')    +    exp(beta_single*B')    +
exp(beta_single*C');
     phi_beta                         =                                [phi_beta;
exp(beta_single*A')/denominator,exp(beta_single*B')/denominator,exp(beta_sing
le*C')/denominator];
```

```
    end
    return;
 end

 function beta2 = draw (theta2)
   beta2 = [];
    for R = 1:5000
    beta2                                                                    =
[beta2;theta2(1,1)+randn*theta2(1,2),theta2(1,3)+randn*theta2(1,4),theta2(1,5
)+randn*theta2(1,6),theta2(1,7)+randn*theta2(1,8),theta2(1,9)+randn*theta2(1,
10),lognrnd(theta2(1,11),theta2(1,12)),lognrnd(theta2(1,13),theta2(1,14)),log
nrnd(theta2(1,15),theta2(1,16)),theta2(1,17)+randn*theta2(1,18),theta2(1,19)+
randn*theta2(1,20),lognrnd(theta2(1,21),theta2(1,22)),lognrnd(theta2(1,23),th
eta2(1,24)),-lognrnd(theta2(1,25),theta2(1,26))];
    end
    return;
 end
```

```
%cost data
global  LOWEST_PRICE PRICE_SPREAD STEEPNESS  RE_MIX_COST
LOWEST_PRICE = [525 318 422 151 432];
PRICE_SPREAD = [53 32 45 16 46];
STEEPNESS = [0.1 0.1 0.1 0.1 0.1];
RE_MIX_COST = [1200,1200,1200,1200,1200];
```

```
% Total costing of product line

function total_cost = total_costing(demand1)
format short
total_cost = comp_costing(demand1) + act_costing(demand1);
return;
end

function comp_cost = comp_costing(demand2)
global LOWEST_PRICE PRICE_SPREAD STEEPNESS

comp_cost=0;

for a = 1:size(demand2,2)
    if demand2(1,a)> 0
        comp_cost    =    comp_cost    +    demand2(1,a)*(LOWEST_PRICE(1,a)+
PRICE_SPREAD(1,a)/(demand2(1,a)^STEEPNESS(1,a)));
    else
        comp_cost = comp_cost;
    end
end

return;
end

function act_cost = act_costing (demand5)
```

```
global  RE_MIX_COST
format short
act_cost = 0;

for a = 1:3
    if demand5(1,a)> 0
        act_cost = act_cost;
    else
        act_cost = act_cost + RE_MIX_COST(1,a);
    end
end
for b = 4:5
    if demand5(1,b)> 0
        act_cost = act_cost + RE_MIX_COST(1,b);
    else
        act_cost = act_cost;
    end
end

return;
end
```

```
% Profit function
function profit2 = profiting2(prod_line2,prices2)
demand2 = demanding(prod_line2, prices2);
profit2 = prices2*demand2'- total_costing (demand2);
return;
end
```

```
% Optimize prices with DE
function max_profit = optimize_differential_evolutionary(prod_line)
format short

cost_data;
inputs_testing;

%display(prod_line);

  % set default parameters
 population_size    = 30;
 max_generation     = 100;
 F                  = 0.6;
 CR                 = 0.5;
 upper_bound = [1500 1500 1500 1500 1500];
 lower_bound = [525 318 422 151 432];

 prices_info = [];

  %initialization of soultions
  solutions = [];
  for n = 1:population_size
  solutions = [solutions;lower_bound + rand(1,5).*(upper_bound-lower_bound)];
  end
```

```
  for g = 1:max_generation % each generation
      solutions_new = [];
      for j = 1:population_size % each price

      %mutation
      r1 = floor(population_size*rand(1,5))+1;
      r2 = floor(population_size*rand(1,5))+1;
      r3 = floor(population_size*rand(1,5))+1;
      donor_vector      =      [solutions(r1(1,1),1)+F*(solutions(r2(1,1),1)-
solutions(r3(1,1),1))];
      donor_vector                                                          =
[donor_vector,solutions(r1(1,2),2)+F*(solutions(r2(1,2),2)-
solutions(r3(1,2),2))];
      donor_vector                                                          =
[donor_vector,solutions(r1(1,3),3)+F*(solutions(r2(1,3),3)-
solutions(r3(1,3),3))];
      donor_vector                                                          =
[donor_vector,solutions(r1(1,4),4)+F*(solutions(r2(1,4),4)-
solutions(r3(1,4),4))];
      donor_vector                                                          =
[donor_vector,solutions(r1(1,5),5)+F*(solutions(r2(1,5),5)-
solutions(r3(1,5),5))];

        %ensuring donor_vector in the bound
      for m=1:5
      if
(donor_vector(1,m)<lower_bound(1,m))||(donor_vector(1,m)>upper_bound(1,m))
          donor_vector(1,m)=solutions(j,m);
      else
          continue;
      end
      end

      %recombination
      if rand(1)< CR || j == (floor(population_size*rand(1))+1)
          trial_vector = donor_vector;
      else
          trial_vector = solutions(j,:);
      end

      %selection
      if                                                  profiting2(prod_line,
trial_vector)<profiting2(prod_line,solutions(j,:))
          target_vector = solutions(j,:);
      else
          target_vector = trial_vector;
      end

      solutions_new = [solutions_new;target_vector];
      end
      solutions = solutions_new;
      prices_info = [prices_info; solutions];

    end
```

```
  %selecting the best prices from the last generation
  max_profit = 0;
  for p = 1:population_size
      if max_profit <profiting2(prod_line,solutions(p,:))
      max_profit = profiting2(prod_line, solutions(p,:));
      prices = solutions(p,:);
      else
          continue;
      end
  end
  %product_index = [1 2 3 4 5];
  %plot(product_index,prices_info);
  display(solutions);
  display(prices);
  return;
end
```

```
% product_line_solving

clear all;
clc;
format short
cost_data;
inputs_testing;

% set default parameters
 population_size    = 20;
 max_generation     = 30;
 CR                 = 0.5;
 MU                 = 0.1;

 %initialization of soultions
  solutions = [];
  for n = 1:population_size
  solutions = [solutions;floor(2*rand(1,5))];
  end

  for g = 1:max_generation
  %selecting parents
  %calculateing chances
  fitness = [];
  fitness_sum = 0;
  chance = [];
for m = 1:population_size
    fitness                                                              =
[fitness,optimize_differential_evolutionary(solutions(m,:))];  %1xpopulation_s
ize
    fitness_sum = fitness_sum + fitness(1,m);
end
    chance = [fitness/fitness_sum]; %1xpolution size

%selecting
parents = [];
s = 0;
```

```
while s < population_size
    rand_vector = binornd(1,chance);
    if sum(rand_vector)==1; % ensuring rand_vector only one element is 1
        s = s+1;
        for i=1:population_size %checking each element of rand_vector
            if rand_vector(1,i)==1
                parents = [parents; solutions(i,:)];
            else
                continue;
            end
        end
    else
        continue;
    end

end

%crossover
offspring = [];
for j = 1: (0.5*population_size)
    %rand(1)<
    if rand(1)<CR
    crossover_point = floor(4*rand(1))+1;  %crossover_point= 1,2,3,4
    offspring1= [];
    offspring2 = [];
    for a = 1:crossover_point
        offspring1 = [offspring1,parents((2*j-1),a)];
        offspring2 = [offspring2,parents(2*j,a)];
    end

    for b = crossover_point:4
        offspring1 = [offspring1,parents(2*j,(b+1))];
        offspring2 = [offspring2,parents((2*j-1),(b+1))];
    end
       offspring = [offspring; offspring1;offspring2];
    else
        offspring = [offspring;parents((2*j-1),:); parents(2*j,:)];
    end
end

% mutation
for c = 1:population_size  % each row
    for d = 1:5
        if rand(1)<MU
            offspring(c,d)= not(offspring(c,d));
        else
            continue;
        end
    end
end
solutions = offspring;
display(offspring);
  end

final_profit = 0;
for h = population_size
```

```
    if final_profit < optimize_differential_evolutionary(solutions(h,:))
        final_profit = optimize_differential_evolutionary(solutions(h,:));
        final_productline = solutions(h,:);
    else
        continue;
    end
end

display(final_productline);
display(final_profit);
```