State estimation in smart power grids

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State Estimation in Smart Power Grids

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Abstract

State Estimation (SE), first proposed by F. C. Schweppe in 1970s [1], is an essential module in Energy Management System (EMS). The phasor voltage of each bus in the system is estimated based on the measurements, such as power flows, power injection, voltage magnitudes and etc. The estimated results are later utilized to identify bad measurements/inconsistent topology and evaluate the system original condition. It can be claimed that power system State Estimator constitutes the core module for the on-line security analysis.

Smart power grids refer to the modern power systems with the advanced technologies in communication, system control and operation. In contrast to the traditional electric power systems, the smart power grids are advantageous by equipping with computer-based remote control and automation, intelligent devices supported by two-way communication technologies, active consumers and renewable energy resources. High efficiency of energy utilization can be expected in smart power grids.

Traditional monitoring and control techniques over power systems will not be suitable for the smart power grids, both in the transmission and the distribution levels. In this thesis, conventional SE methods are improved to cater for the needs of smart power grids. Phasor Measurement Units (PMUs) are firstly introduced and their application in SE is reviewed. Prior to utilizing PMU measurements or synchrophasor measurements in SE, the placement strategy on PMUs is studied over

various test systems. Case study results verify that the measurement redundancy can be improved via the proposed techniques.

Next, SE in smart transmission systems is conducted. In this work, the adaptive underfrequency load shedding (UFLS) procedure is considered as the pseudo measurements to improve the SE results. By correcting the power injection measurements at the unobservable buses where load shedding takes place, it enables the State Estimator to track the system states more accurately. Case studies on the IEEE test systems are conducted to verify the results.

For SE in smart distribution systems, the study has been carried out on the newly emerging hybrid AC/DC microgrids. A decentralized technique is proposed for the estimation by modelling the AC and DC microgrids separately and utilizing the power flow measurements obtained from the connecting line of the two networks as an equality constraint. By virtue of PMUs, SE in the AC network is formulated in a three-phase linear manner and the unbalanced problem can be investigated. SE in the DC network is built in a linear manner as well, based on the voltage-current relationship. The decentralized SE can perform comparatively with the centralized technique regarding the accuracy, computational speed and bad data detection, while the shared measurement information is reduced and data privacy can thus be improved.

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Chapter 1

Introduction

Traditional monitoring, operation and control techniques have reigned over the electric power system for more than a century. Towards the beginning of the 21st century, traditional operation techniques of electrical power systems were found increasingly inferior to satisfy the needs of both utilities and consumers [2]. For instance, to meet the peak demands, many peaking generators are needed to be stand-by with a low rate of utilization, resulting in a high cost to be borne by the utilities. Besides, due to the integration of different systems, the increasingly complicated networks have brought in many reliability issues. It is imperative to develop a more smart technique of system monitoring, operation and control for both the transmission and distribution systems. In [3], the concept of smart grids is introduced. By general definition, smart grids refer to the modern power systems equipped with advanced technologies in communication, system control and operation. The following key features of the smart grid are summarized, in contrast to the traditional electric power systems.

- Using computer-based remote control and automation.

- Owning devices supported by two-way communication technologies.
- Facilitating more active consumers.
- Offering many benefits to improve the energy efficiency.
- Involving renewable energy resources.

In this section, background knowledge related to smart grids, both in transmission and distribution systems, will be presented. As a core module in Energy Management System for smart grids, SE with its challenges will be discussed as well. Four relevant topics will be illustrated from Section 1.1 to Section 1.4.

1.1 Energy Management System and State Estimation

Energy Management System (EMS) is essential for power system real-time monitoring and steady-state security analysis. Many functions can be realized in EMS, such as Contingency Analysis (CA) [4], Automatic Generation Control (AGC) [5], Load Forecasting (LF) [6] and etc. For power systems, the operating condition or the static states can be reflected by the bus phasor voltages. The operating condition can be classified into three categories: Normal, Emergency and Restorative. The relationship among these three conditions is shown in Figure 1.1. For system operators, the main task is to maintain the system under the normal condition by continuously monitoring the system states. Preventive and other effective methods need to be determined and implemented after the security evaluation.

Figure 1.1 Three operation conditions of power system [7].

EMS is built up for a better monitoring and assessment of the system conditions by equipping with the state-of-the-art technologies [7]. Before any EMS functions are executed, a necessary step is to gather the measurements across the entire power system. Traditional measurements are collected by the Supervisory Control and Data Acquisition (SCADA) system with the Remote Terminal Units (RTUs) installed throughout the system. RTUs are responsible for sending back the analog and digital data to the control center and receiving the control commands from the controller. The typical EMS/SCADA configuration can be found in Figure 1.2.

Figure 1.2 EMS/SCADA system [7].

Since power system states cannot be measured directly from the traditional power

meters and the raw measurements may be erroneous, SE is developed to estimate the states accurately and efficiently from the available raw measurements. A comprehensive description of State Estimator from the EMS is presented as follows [7] [8]:

- **Observability analysis:** This module is to check whether the SE can be solved with a unique solution. If the measurements obtained are insufficient, the system will be unobservable and some states cannot be estimated. The usual remedy for the unobservability is to identify the unobservable parts first and then add relevant measurements, i.e. the pseudo measurements, to make the system observable and SE solvable.
- **Bad data processing:** Measurements directly obtained from the meters are referred to as the “raw data” which means that they are potentially incorrect. Evidently, if these data items are utilized directly, the SE results will be biased. Therefore the bad data processing is a necessary step and it is employed to detect and eliminate the suspicious data.
- **Parameter and topology error processing:** This module is to preprocess the measurements and topology parameters in order to guarantee that the system is correctly modeled. Measurements/parameters in this process include the transmission line model parameters, tap changing transformers parameters, shunt capacitors parameters and the circuit breakers status.
- **SE module:** This is the main module of State Estimator which is to determine the bus phasor voltages from the measurements in the system. The Weighted Least Square (WLS) algorithm is commonly adopted for the SE. Branch power flows and bus power injections in the system can be estimated subsequently.

In summary, SE plays a crucial role in EMS and it contributes significantly to a better security assessment for power systems. SE behaves as a data filter between the SCADA system and EMS functions by providing reliable data for the calculation in

EMS. The importance of SE and its relationship with other EMS functions for the on-line static security assessment can be found from the functional diagram below.

Figure 1.3 On-line static security assessment: functional diagram [7].

1.2 Synchrophasor Measurements

Synchrophasor measurements or the equally-addressed PMU measurements will be introduced in this section. Due to the advanced communication technologies, synchrophasor measurements of voltage and currents can be obtained much more accurately than before, with time stamped and phase angle information.

With the improvement on Global Positioning System (GPS) technology [9] [10], an accurate clock signal can be achieved as the time reference. The first prototype of PMU was invented in Virginia Polytechnic Institute and State University [11], as shown in Figure 1.4. Currently, PMUs have been widely produced in industry. In the Clean Energy Research Laboratory (CERL) at Nanyang Technological University (NTU), Singapore, PMUs modeled as ION 7550/7650 by Schneider Electric have

been installed [12].

(a)

(b)

Figure 1.4 (a) First PMU invented in Virginia Tech. (b) Schneider ION 7550/7650 PMU.

Traditional measurements include voltage magnitudes V , active/reactive power flows P_f/Q_f , active/reactive power injections P_{inj}/Q_{inj} and current magnitudes I . They constitute the online measurements which are collected from the SCADA

system. Voltage and current measurements are only calibrated in magnitudes without the phase angle information, since the global clock signal is missing in the traditional measurement units. Moreover, if the clock signal is not accurate enough, the relevant phase angle measurements will be biased which cannot be used for the phasor calculation in EMS. As reported in [11], by adopting the LORAN-C and GOES satellite with the proper communication technique to set the time reference, the accuracy of the first direct phase angle measurement can be $40\mu\text{s}$ in 1980s'. However the calculation technique involved in these methods is not suitable for the wide-area phasor measurement systems as it focused on the phase angle differences of the single phase voltages.

Despite the different appearances and hardware configurations, the basic and major components of PMUs are similar as shown in Figure 1.5 [11] [13].

Figure 1.5 Hardware configuration of PMU.

By installing PMUs in the system and utilizing the uniform clock signal transmitted from GPS as the time reference, phase angle of the bus voltages can be measured accurately. It can be claimed that the accuracy of the phase angle measurements depends on the accuracy of the GPS. If the error of the GPS can be achieved within $1\mu\text{s}$, the incurring error in the calculated value of phase angle for a 50Hz system will be around 0.02° .

1.3 Smart Transmission Systems

Transmission systems are responsible to transmit electrical energy from power plants to substations across a geographical distance. Power plants are usually built in remote areas for easy access to resources, such as coal and fossil, while substations are near the cities/towns. In this way, long distances can be expected between power plants and substations. Figure 1.6 shows a typical power system with its subsystems. Traditional monitoring and control over the transmission system can be identified by the following features [14]:

- SCADA systems based EMS/SE.
- Direct control over the switching devices, shunt injections and other series- and shunt-adjustable devices.
- Frequency regulation by maintaining the Area Control Error (ACE) within the required limits.
- Load shedding strategies under contingencies.

Figure 1.6 Typical power system with its subsystems.

Nowadays, it is urged to manage and control the transmission systems in a wiser manner to deal with the conflicting objectives [15]. Due to the increasing demands

and environmental protection issues, it is believed that renewable and clean energy resources are the best replacement to the traditional ones. However the higher penetration of renewable energy sources is associated with an unpredictable uncertainty, which brings in many difficulties in the system planning and operation.

For smart transmission systems, one of the key features is the implementation of advanced metering units and robust communication infrastructures [16]. With this, the aforementioned control strategies will be realized in a more smart and robust manner. Another feature of the smart transmission systems is the Wide Area Measurement System (WAMS) which is initiated by virtue of the synchrophasor measurements [16] - [17]. As a result, monitoring and control strategies over the transmission systems will be revolutionized. For instance, State Estimator incorporating the WAMS information will have more timely information than that of traditional SCADA measurements. Besides, situational awareness regarding the reliability and robustness of the power grid or contingency assessment can be achieved through measurement-oriented techniques and such features can be only achieved via the dynamic model-based techniques before [18] [19]. Figure 1.7 suggests a typical control center for the smart transmission systems.

Figure 1.7 A typical smart transmission system control [15].

1.4 Smart Distribution Systems

Distribution systems are the final station in the delivery of electric power from the power plant to the individual consumers. Connecting to the transmission system via step-down transformers, primary distribution lines will carry a medium voltage level ranging between 2kV and 35kV. A downstream distribution station will further lower the voltage level for the use of household electrical appliances [20]. In the past, distribution systems were not equally emphasized by electrical engineers who involved largely with the transmission system [21] and the monitoring/control strategies over the distribution system were not as robust as those of the transmission level.

Until 1990s when distributed generation (DG) emerged, distribution systems could participate in the entire systems more proactively and the reliability issue of the system has become an important topic. DG modeling requires that the analysis tools for the distribution system are able to cover a full three-phase formulation for both urban and rural systems. On top of this, harmonics analysis, full three-phase power flow and short-circuit analysis became necessary [21]. Currently, studies on the demand side management (DSM) in distribution systems have attracted the attention of researchers and system operators [22] - [23]. Through the DSM agents, load demand at peak hours can be effectively reduced and benefits can be earned by both the utilities and consumers. Loads of a single household may be impossible to affect the entire system substantially. However through the load aggregator [23], changes of the residential demand in the distribution system will be influential to the operation of the entire system.

Another closely related concept to the smart distribution systems is microgrids [24]. Microgrids are the localized grids of the small scale low/medium voltage (LV/MV)

networks. Comprised of clusters of loads, energy storage systems (ESSs), renewable and traditional distributed energy resources (DERs), microgrids can be operated either in grid-connected or islanded mode. A microgrid is able to assess its health, predict its behavior, adapt to new environments, handle the uncertainties of the distributed resources and demands, and provide optimal response to smart devices in real-time. Figure 1.8 suggests a typical microgrid [25].

Figure 1.8 A typical microgrid [25]

With the increasing importance of distribution systems in the entire power grid, development of an accurate, reliable and robust Distribution SE (DSE) is suggested. The objective of DSE is to estimate the system states using online measured data for the distribution systems. The characteristics of the SE in a smart distribution system are presented in Figure 1.9 [26].

Figure 1.9 The characteristics of a smart distribution system state estimation [26].

1.5 Thesis Objectives and Technical Contributions

In order to maintain the optimal operation of power systems, the following items should be well initiated: real-time functionalities, fast restoration after outages, reliability evaluation, real-time power quality improvements, demand side management and distributed resources control [27] [28]. To achieve this, smart grids should be equipped with the advanced monitoring/control systems and the robust integrated data exchange architectures among different components [29] [30]. Collected data in the control centers should be processed wisely to determine the system states and the responsive activities [31]. Therefore, it is necessary to build a robust EMS/SE to support the aforementioned functions of smart grid. This thesis focuses on the study of synchrophasor measurements and its applications on power system SE, considering the necessities of smart grids.

The first objective of this thesis is to optimize PMU placement strategy. The optimal PMU placement problem (OPP) in power system SE is mainly for the observability and economic concerns [32] - [33]. Locations for PMU installation are determined

to minimize the cost and guarantee the system observability. Observability-oriented OPP problems arise due to the high cost of the PMU installation in power systems. From the report [34], the average cost for installing one PMU can be found in Figure 1.10, ranging from \$40K to \$180K among different utilities. The high cost of the installation is not necessarily caused by the device itself, but other infrastructure expenditure such as the communication networks, as shown in Figure 1.11.

Figure 1.10 Average overall cost of a PMU

Figure 1.11 Average cost of PMU device compared to average installation cost of PMU

On top of the economic and observability concerns, measurement redundancy is another objective when determining the optimal PMU placement in this thesis. Redundant measurements can detect and identify the bad data with a higher

probability and thus guarantee a more accurate SE result.

Considering that synchrophasor measurements would be possibly utilized in a way to cooperate with SCADA measurements, the second objective is to combine the two types of measurements in hybrid SE. Although PMUs can provide the phase angle information at a much higher reporting rate than the SCADA measurements, most studies deal with these two types of measurements at the same pace, by approximately treating the SCADA measurements as unchanged during the calculation. In this way, the inadequate PMU measurements can be “compensated” by the SCADA measurements which are slower, time-skewed and inaccurate. Such unsynchronized issue of measurements will arise and will become more severe in the real-time analysis, especially when the SCADA measurements are extremely inaccurate. To solve this problem, an effective technique is needed to update/correct SCADA measurements for a robust real-time analysis of smart grids. In this thesis, attention is paid to the hybrid SE in an outage situation that is notified by the fast frequency measurements from PMUs. Under such contingencies, results from the traditional hybrid SE will be inaccurate, misleading and problematic if the biased SCADA data is directly used to compensate the inadequate synchrophasor measurements.

The third objective of this thesis is to better monitor the hybrid AC/DC microgrid with the decentralized SE. Motivation of this work arises from the challenges faced in the smart distribution systems. When distribution systems become increasingly independent and influential in the entire electric power grid, it becomes necessary to design the distribution EMS/SE. For interconnected microgrids, a decentralized EMS/SE will be preferred as measurement information can be processed locally. The information exchanged in the decentralized SE among the interconnected multi-areas could be much reduced and beneficial to data privacy, compared with what is

needed for the centralized one. In this thesis, a decentralized SE for the hybrid AC/DC microgrid is carried out with the Dual Decomposition technique. The decentralized SE proposed for the hybrid AC/DC microgrids is found more appropriate to cater for different formulations of each microgrid. Case study results verify the proposed decentralized SE.

As presented above, this thesis deals with the important issues of the monitoring, control and operation for the smart grids. The innovation and contribution of this thesis are: 1) Measurement redundancy is incorporated into the OPP problem, especially for buses with the poorest redundancy level; 2) Frequency information is utilized in the SE during contingencies and the time-skewed SCADA measurements are corrected before incorporated into the hybrid SE; and 3) A decentralized SE is proposed on the newly emerging hybrid AC/DC microgrids via the Dual Decomposition technique.

1.6 Thesis Organization

This thesis is presented in six chapters. The current chapter provides an introduction to the Energy Management System and smart power grids. The concepts of EMS/SE, synchrophasor measurements, smart transmission and distribution systems are briefly introduced. Technical challenges for an optimal operation of smart grids are also reviewed. The research motivation, objectives, contributions and organization of this thesis are discussed as well.

Chapter 2 presents a literature review of synchrophasor measurements based SE. Classical formulations of the hybrid state estimation (HSE) are introduced. With the SCADA and PMU measurements utilized together in the calculation, different hybrid SE techniques are introduced as the post-processing method [35] and

equality-constrained method [36]. Processing techniques on measurement covariance are also discussed in this chapter.

Chapter 3 reviews the OPP strategies with different objectives and scenarios. Improvements on the existing PMU placement strategies are presented to improve the measurement redundancies for SE and the relevant case studies are conducted.

Chapter 4 proposes an enhanced SE when outages occur in the system. PMU measurements can reflect the real-time situation due to the fast sampling rate whereas the SCADA measurements are time-skewed. When contingencies occur, the mismatch between the two sets of measurements will be substantial. Utilizing them simultaneously will lead to deteriorated SE results. Simulation studies in this chapter demonstrate the necessities to correct the delayed SCADA measurements before including them into the measurement set of SE. On top of this, the adaptive underfrequency load shedding strategies are discussed when outages occur. Through the case studies, it can be demonstrated that the proposed enhanced SE is capable of tracking the system states more accurately than the traditional hybrid SE.

Chapter 5 discusses a decentralized SE technique for the newly emerging network configuration in distribution systems, called the hybrid AC/DC microgrids. The modelling of SE for both AC and DC microgrids is presented respectively. The formulation of the centralized SE for the hybrid AC/DC microgrids is used as a comparison with the proposed decentralized SE. A Dual Decomposition technique is utilized for the decentralized SE and case studies on the simple hybrid AC/DC microgrids are conducted to verify the proposed decentralized SE.

Chapter 6 provides conclusion and suggestion for future studies with regards to the study of SE in smart power grids.

Chapter 2

Synchrophasor Measurements Assisted State Estimation Techniques

Traditional SE utilizes the measurements from the Supervisory Control and Data Acquisition (SCADA) system to estimate the system states, namely the voltage magnitude and phase angle of each bus. Normally, power system stays in steady state without evident changes. Therefore the unsynchronized and slowly reported SCADA measurements can be used in the calculation since it is believed that the delayed measurements will not adversely affect the estimation results under the steady state condition.

In contrast with the SCADA measurements, synchrophasor/PMU measurements are fast and synchronized with time stamp which could be utilized for SE under contingencies or the real-time analysis. Considering that the number of PMUs may be inadequate, synchrophasor measurements and SCADA measurements are expected to work together for the calculation of SE, named as hybrid SE. Methodologies to combine these two measurements in the hybrid SE will be discussed in this chapter.

2.1 Traditional Static State Estimation

2.1.1 Background

State Estimation is an essential module in Energy Management System and it is first proposed by F. C. Schweppe in 1970s [1]. SE results are mainly utilized to detect the bad measurement/topology data and estimate the phasor voltage of each bus. Static SE refers to the procedure of obtaining the phasor voltages for all buses in the system at a given time instant. It is practically impossible to collect the measurements of different buses across the system simultaneously. For static SE, since it is assumed that the power system stays in a static steady condition, a certain amount of time-skew of the SCADA measurements can be tolerated.

2.1.2 System Modelling

Power systems are monitored by examining if the system states, i.e. the phasor voltage, can stay in the normal condition. Traditionally, the measurements captured from the RTUs constitute the SCADA measurement set. They are power flow measurements ($\mathbf{P}_{fl}, \mathbf{Q}_{fl}$), power injection measurements ($\mathbf{P}_{inj}, \mathbf{Q}_{inj}$) and voltage magnitudes ($|\mathbf{V}|$). Most SCADA measurements can only be formulated by the phasor voltages in a non-linear manner. Denote $\mathbf{z}_S$, $\mathbf{x}_{|V|}$ and $\boldsymbol{\theta}_V$ as the measurement set from SCADA, state variables of voltage magnitudes and angles representatively, the traditional SE can be formulated as [7]:

$$\mathbf{z}_S = \begin{bmatrix} |\mathbf{V}| \\ \mathbf{P}_{fl} \\ \mathbf{Q}_{fl} \\ \mathbf{P}_{inj} \\ \mathbf{Q}_{inj} \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \\ h_2(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \\ h_3(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \\ h_4(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \\ h_5(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \end{bmatrix} + \mathbf{e}_S = \mathbf{h}_S(\mathbf{x}) + \mathbf{e}_S \quad (2.1)$$

where

$$\mathbf{x} = [\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}]^T$$

$h_1 \sim h_5$ are the measurement functions related with the different measurements and

they can be formulated as follows:

For the voltage magnitude measurement at bus k :

$$\mathbf{h}_{1(k)} = \mathbf{x}_{|\mathbf{V}|(k)} \quad (2.2)$$

For the power flow measurement from bus i to bus j :

$$\mathbf{h}_{2(i,j)} = \mathbf{x}_{|\mathbf{V}|(i)}^2 g_{(ij)} - \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} (g_{(ij)} \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} + b_{(ij)} \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)}) \quad (2.3)$$

$$\mathbf{h}_{3(i,j)} = -\mathbf{x}_{|\mathbf{V}|(i)}^2 (b_{s(i)} + b_{(ij)}) - \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} (g_{(ij)} \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} - b_{(ij)} \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)}) \quad (2.4)$$

where the parameters of the transmission line can be found in Figure 2.1. A transmission line connecting bus i and j is depicted in Figure 2.1 and its positive sequence series impedance is denoted as $g_{(ij)} + jb_{(ij)}$. The total charging susceptance of $j2b_s$ is modelled as well.

Figure 2.1 Two-port π model of a transmission line

For injection measurement at bus i :

$$\mathbf{h}_4(i) = \mathbf{x}_{|\mathbf{V}|(i)} \sum_{j=1}^{N_{bus}} \mathbf{x}_{|\mathbf{V}|(j)} (\mathbf{G}_{(ij)} \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} + \mathbf{B}_{(ij)} \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)}) \quad (2.5)$$

$$\mathbf{h}_5(i) = \mathbf{x}_{|\mathbf{V}|(i)} \sum_{j=1}^{N_{bus}} \mathbf{x}_{|\mathbf{V}|(j)} (\mathbf{G}_{(ij)} \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} - \mathbf{B}_{(ij)} \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)}) \quad (2.6)$$

where the $\mathbf{G}$ and $\mathbf{B}$ are the real and imaginary part of the admittance matrix.

2.1.3 Weighted Least Square Algorithm

The Weighted Least Square (WLS) technique is commonly adopted for the traditional SE. Results from WLS technique are expected to fit the measurement quantities, namely to minimize the square of the difference between the estimated values and measurements. The objective function of the WLS-based SE can be written as:

$$\begin{aligned} & \text{minimize } \sum_{i=1}^{N_{meas}} \mathbf{W}_{(i,i)} \mathbf{r}_{(i)}^2 \\ & \text{subject to } \mathbf{z}_{S(i)} = \mathbf{h}_{S(i)} + \mathbf{r}_{(i)} \end{aligned} \quad (2.7)$$

where N_{meas} is the number of measurements and $\mathbf{W}$ is the weight matrix which equals the inverse of the covariance matrix of $\mathbf{R}$. For the SCADA measurements, the uncertainties of different types of measurements can be found in Table 2.1 [37]:

TABLE 2. 1 UNCERTAINTIES OF SCADA MEASUREMENTS

$\sigma_{ V }$	$\sigma_{P_i}, \sigma_{Q_i}$	$\sigma_{P_{fl}}, \sigma_{Q_{fl}}$
0.4%	1%	0.8%

The objective function of (2.7) can be transformed into (2.8) as shown below:

$$\begin{aligned} \text{minimize } J &= \sum_{i=1}^{N_{meas}} \frac{(\mathbf{z}_{S(i)} - \mathbf{h}_{S(i)})^2}{\mathbf{R}_{(i,i)}} \\ &= (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x})) \end{aligned} \quad (2.8)$$

To obtain the optimum value of $\mathbf{x}$, the first-order optimality condition should be satisfied, namely:

$$\mathbf{f}(\mathbf{x}) = \frac{\partial J}{\partial \mathbf{x}} = -\mathbf{H}_S^T(\mathbf{x}) \mathbf{R}^{-1} (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x})) = 0 \quad (2.9)$$

where $\mathbf{H}_s$ is the Jacobian matrix and $\mathbf{H}_s = \frac{\partial \mathbf{h}_s}{\partial \mathbf{x}} =$

$$\begin{bmatrix} \frac{\partial |\mathbf{V}|}{\partial \mathbf{x}_{\theta_V}} & \frac{\partial |\mathbf{V}|}{\partial \mathbf{x}_{|\mathbf{V}|}} \\ \frac{\partial \mathbf{P}_{fl}}{\partial \mathbf{x}_{\theta_V}} & \frac{\partial \mathbf{P}_{fl}}{\partial \mathbf{x}_{|\mathbf{V}|}} \\ \frac{\partial \mathbf{Q}_{fl}}{\partial \mathbf{x}_{\theta_V}} & \frac{\partial \mathbf{Q}_{fl}}{\partial \mathbf{x}_{|\mathbf{V}|}} \\ \frac{\partial \mathbf{P}_{inj}}{\partial \mathbf{x}_{\theta_V}} & \frac{\partial \mathbf{P}_{inj}}{\partial \mathbf{x}_{|\mathbf{V}|}} \\ \frac{\partial \mathbf{Q}_{inj}}{\partial \mathbf{x}_{\theta_V}} & \frac{\partial \mathbf{Q}_{inj}}{\partial \mathbf{x}_{|\mathbf{V}|}} \end{bmatrix}.$$

The Jacobian elements can be found in details as shown below:

- **For power flow measurements**

$$\begin{aligned} \frac{\partial \mathbf{P}_{fl}(i,j)}{\partial \mathbf{x}_{\theta_V(i)}} &= \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} - b_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] \\ \frac{\partial \mathbf{P}_{fl}(i,j)}{\partial \mathbf{x}_{\theta_V(j)}} &= -\mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} - b_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] \\ \frac{\partial \mathbf{P}_{fl}(i,j)}{\partial \mathbf{x}_{|\mathbf{V}|(i)}} &= -\mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} + b_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} \right] + 2\mathbf{x}_{|\mathbf{V}|(i)} g_{(ij)} \\ \frac{\partial \mathbf{P}_{fl}(i,j)}{\partial \mathbf{x}_{|\mathbf{V}|(j)}} &= -\mathbf{x}_{|\mathbf{V}|(i)} \left[g_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} + b_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} \right] \end{aligned} \quad (2.10)$$

$$\begin{aligned} \frac{\partial \mathbf{Q}_{fl}(i,j)}{\partial \mathbf{x}_{\theta_V(i)}} &= -\mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} + b_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} \right] \\ \frac{\partial \mathbf{Q}_{fl}(i,j)}{\partial \mathbf{x}_{\theta_V(j)}} &= \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} + b_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} \right] \\ \frac{\partial \mathbf{Q}_{fl}(i,j)}{\partial \mathbf{x}_{|\mathbf{V}|(i)}} &= -\mathbf{x}_{|\mathbf{V}|(j)} \left[g_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} - b_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] - 2\mathbf{x}_{|\mathbf{V}|(i)} (b_{s(i,j)} + b_{(ij)}) \\ \frac{\partial \mathbf{Q}_{fl}(i,j)}{\partial \mathbf{x}_{|\mathbf{V}|(j)}} &= -\mathbf{x}_{|\mathbf{V}|(i)} \left[g_{(ij)} \sin \mathbf{x}_{\theta_{V(i,j)}} - b_{(ij)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] \end{aligned} \quad (2.11)$$

- **For power injection measurements**

$$\begin{aligned} \frac{\partial \mathbf{P}_{inj}(i)}{\partial \mathbf{x}_{\theta_V(i)}} &= \sum \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[-\mathbf{G}_{(i,j)} \sin \mathbf{x}_{\theta_{V(i,j)}} + \mathbf{B}_{(i,j)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] - \mathbf{x}_{|\mathbf{V}|(i)}^2 \mathbf{B}_{(i,i)} \\ \frac{\partial \mathbf{P}_{inj}(i)}{\partial \mathbf{x}_{\theta_V(j)}} &= \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[\mathbf{G}_{(i,j)} \sin \mathbf{x}_{\theta_{V(i,j)}} - \mathbf{B}_{(i,j)} \cos \mathbf{x}_{\theta_{V(i,j)}} \right] \\ \frac{\partial \mathbf{P}_{inj}(i)}{\partial \mathbf{x}_{|\mathbf{V}|(i)}} &= \sum \mathbf{x}_{|\mathbf{V}|(j)} \left[\mathbf{G}_{(i,j)} \cos \mathbf{x}_{\theta_{V(i,j)}} + \mathbf{B}_{(i,j)} \sin \mathbf{x}_{\theta_{V(i,j)}} \right] + \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{G}_{i,i} \end{aligned}$$

$$\frac{\partial \mathbf{P}_{\text{inj}}(i)}{\partial \mathbf{x}_{|\mathbf{V}|(j)}} = \mathbf{x}_{|\mathbf{V}|(i)} \left[\mathbf{G}(i,j) \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} + \mathbf{B}(i,j) \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} \right] \quad (2.12)$$

$$\frac{\partial \mathbf{Q}_{\text{inj}}(i)}{\partial \mathbf{x}_{\theta_{\mathbf{V}}(i)}} = \sum \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[\mathbf{G}(i,j) \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} + \mathbf{B}(i,j) \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} \right] - \mathbf{x}_{|\mathbf{V}|(i)}^2 \mathbf{G}(i,i)$$

$$\frac{\partial \mathbf{Q}_{\text{inj}}(i)}{\partial \mathbf{x}_{\theta_{\mathbf{V}}(j)}} = \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{x}_{|\mathbf{V}|(j)} \left[-\mathbf{G}(i,j) \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} - \mathbf{B}(i,j) \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} \right]$$

$$\frac{\partial \mathbf{Q}_{\text{inj}}(i)}{\partial \mathbf{x}_{|\mathbf{V}|(i)}} = \sum \mathbf{x}_{|\mathbf{V}|(j)} \left[\mathbf{G}(i,j) \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} - \mathbf{B}(i,j) \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} \right] - \mathbf{x}_{|\mathbf{V}|(i)} \mathbf{B}_{i,i}$$

$$\frac{\partial \mathbf{Q}_{\text{inj}}(i)}{\partial \mathbf{x}_{|\mathbf{V}|(j)}} = \mathbf{x}_{|\mathbf{V}|(i)} \left[\mathbf{G}(i,j) \sin \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} - \mathbf{B}(i,j) \cos \mathbf{x}_{\theta_{\mathbf{V}}(i,j)} \right] \quad (2.13)$$

- **For voltage magnitude measurements**

$$\frac{\partial |\mathbf{V}|(i)}{\partial \mathbf{x}_{\theta_{\mathbf{V}}(i)}} = 0 \quad \frac{\partial |\mathbf{V}|(i)}{\partial \mathbf{x}_{\theta_{\mathbf{V}}(j)}} = 0 \quad \frac{\partial |\mathbf{V}|(i)}{\partial \mathbf{x}_{|\mathbf{V}|(i)}} = 1 \quad \frac{\partial |\mathbf{V}|(i)}{\partial \mathbf{x}_{|\mathbf{V}|(j)}} = 0 \quad (2.14)$$

Expanding this non-linear function of $\mathbf{f}(\mathbf{x})$ as indicated in (2.9) into its Taylor series at the state value of $\mathbf{x} = \mathbf{x}_{(k)}$, we have:

$$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}_{(k)}) + \mathbf{F}_{\mathbf{x}_{(k)}} (\mathbf{x} - \mathbf{x}_{(k)}) + \dots = 0 \quad (2.15)$$

where $\mathbf{F}_{\mathbf{x}_{(k)}} = \left. \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x}=\mathbf{x}_{(k)}} = \mathbf{H}_S^T(\mathbf{x}) \mathbf{R}^{-1} \mathbf{H}_S(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_{(k)}}$, which is usually referred as the *Gain* matrix.

Neglecting the higher order of the Tylor series in (2.15), we can obtain the state variables in an iterative way as:

$$\mathbf{x}_{(k+1)} = \mathbf{x}_{(k)} - \mathbf{F}_{\mathbf{x}_{(k)}}^{-1} \mathbf{f}(\mathbf{x}_{(k)}) \quad (2.16)$$

namely

$$\Delta \mathbf{x}_{(k)} = -\mathbf{F}_{\mathbf{x}_{(k)}}^{-1} \mathbf{f}(\mathbf{x}_{(k)})$$

Usually, the *Gain* matrix will be decomposed into triangular factors and the resulting sparse linear sets of equations can be solved via *Forward/Back* substitutions at each iteration. The WLS algorithm for the static SE can be summarized in **Algorithm**

WLS as shown below:

Algorithm WLS

1. Initialize the state variables with a flat start, i.e. $\mathbf{x}_{(k)} = [\mathbf{1}, \mathbf{0}]^T$. Vector $\mathbf{1}$ for $\mathbf{x}_{|V|}$ and vector $\mathbf{0}$ for $\mathbf{x}_{\theta_V}$. Initialize the iteration index, $k = 1$ and tolerance ε .
 2. Calculate the Gain matrix of $\mathbf{F}_{\mathbf{x}_{(k)}} = \mathbf{H}_S^T(\mathbf{x})\mathbf{R}^{-1}\mathbf{H}_S(\mathbf{x})|_{\mathbf{x}=\mathbf{x}_{(k)}}$.
 3. Calculate $f(\mathbf{x}_{(k)}) = -\mathbf{H}_S^T(\mathbf{x}_{(k)})\mathbf{R}^{-1}(\mathbf{z}_S - \mathbf{h}_S(\mathbf{x}_{(k)}))$.
 4. Decompose $\mathbf{F}_{\mathbf{x}_{(k)}}$ and solve $\Delta\mathbf{x}_{(k)} = -\mathbf{F}_{\mathbf{x}_{(k)}}^{-1}f(\mathbf{x}_{(k)})$.
 5. If $|\Delta\mathbf{x}_{(k)}| \geq \varepsilon$
 6. $\mathbf{x}_{(k+1)} = \mathbf{x}_{(k)} + \Delta\mathbf{x}_{(k)}$
 7. $k = k + 1$ and go to step 1.
 8. Else If $|\Delta\mathbf{x}_{(k)}| < \varepsilon$
 9. Stop
 10. End if
-

2.1.4 Bad Data Processing

SE results are closely related with measurement information, therefore any erroneous measurements should be removed or corrected with proper techniques. Many sources can lead to the erroneous measurements, such as the failure of communication, improper wiring and meter breakdown. In the following section, commonly adopted techniques on bad data detection and identification will be introduced and the techniques greatly rely on measurement redundancy. Therefore for measurements with low redundancy, such as the critical measurements¹ or conforming multiple bad data², successful detection/identification of the bad data may not be guaranteed.

¹ Critical measurements refer to the measurements that if removed, the system will be unobservable.

² Conforming multiple bad data refer to two or more measurements that have errors in agreement.

2.1.4.1 Measurement Residuals

In **Algorithm WLS**, consider the linearized measurement equation:

$$\Delta \mathbf{z} = \mathbf{H}\Delta \mathbf{x} + \mathbf{e} \quad (2.17)$$

where $\mathbf{H}$ is the linearized measurement function, namely the Jacobian matrix as illustrated in (2.10) - (2.14). The measurement error $\mathbf{e}$ is assumed to follow a normalized distribution with $E(\mathbf{e}) = 0$ and $Cov(\mathbf{e}) = \mathbf{R}$. The estimated state vector can be obtained by:

$$\begin{aligned} \Delta \hat{\mathbf{x}} &= (\mathbf{H}^T \mathbf{R}^{-1} \mathbf{H})^{-1} \mathbf{H}^T \mathbf{R}^{-1} \Delta \mathbf{z} \\ &= \mathbf{F}^{-1} \mathbf{H}^T \mathbf{R}^{-1} \Delta \mathbf{z} \end{aligned} \quad (2.18)$$

and the estimated value of the measurement set of $\Delta \mathbf{z}$ is:

$$\begin{aligned} \Delta \hat{\mathbf{z}} &= \mathbf{H} \Delta \hat{\mathbf{x}} \\ &= \mathbf{H} \mathbf{F}^{-1} \mathbf{H}^T \mathbf{R}^{-1} \Delta \mathbf{z} \\ &= \mathbf{K} \Delta \mathbf{z} \end{aligned} \quad (2.19)$$

where $\mathbf{F}$ is the Gain matrix as suggested in (2.16) previously. The measurement residual can therefore be deduced as shown in (2.20):

$$\begin{aligned} \mathbf{r}_z &= \Delta \mathbf{z} - \Delta \hat{\mathbf{z}} \\ &= (\mathbf{I} - \mathbf{K}) \Delta \mathbf{z} \\ &= (\mathbf{I} - \mathbf{K})(\mathbf{H} \Delta \mathbf{x} + \mathbf{e}) \quad \text{<Since } \Delta \mathbf{z} = \mathbf{H} \Delta \mathbf{x} + \mathbf{e}\text{>} \\ &= (\mathbf{I} - \mathbf{K}) \mathbf{e} \quad \text{<Since } (\mathbf{I} - \mathbf{K}) \mathbf{H} = \mathbf{0}\text{>} \\ &= \mathbf{S} \mathbf{e} \end{aligned} \quad (2.20)$$

The matrix $\mathbf{S}$, named as the residual sensitivity matrix, reflects the relationship between the measurement residuals and measurement errors. Based on this, we can prove that the measurement residual possesses the following qualities:

$$E(\mathbf{r}_z) = E(\mathbf{S} \mathbf{e}) = \mathbf{S} E(\mathbf{e}) = 0 \quad (2.21)$$

$$Cov(\mathbf{r}_z) = Cov(\mathbf{S} \mathbf{e}) = \mathbf{S} Cov(\mathbf{e}) \mathbf{S}^T = \mathbf{S} \mathbf{R} \mathbf{S}^T \quad (2.22)$$

$$\mathbf{S} \mathbf{R} \mathbf{S}^T = \mathbf{S} \mathbf{R} (\mathbf{I} - \mathbf{K})^T = \mathbf{S} \mathbf{R} (\mathbf{I} - \mathbf{H} \mathbf{F}^{-1} \mathbf{H}^T \mathbf{R}^{-1})^T$$

$$\begin{aligned}
 &= \mathbf{SR} - \mathbf{SR}(\mathbf{H}\mathbf{F}^{-1}\mathbf{H}^T\mathbf{R}^{-1})^T \\
 &= \mathbf{SR} - (\mathbf{I} - \mathbf{K})\mathbf{R}(\mathbf{R}^{-1})^T\mathbf{H}(\mathbf{F}^{-1})^T\mathbf{H}^T \\
 &= \mathbf{SR} - (\mathbf{I} - \mathbf{K})\mathbf{H}(\mathbf{F}^{-1})^T\mathbf{H}^T \quad \text{<Since } \mathbf{R} \text{ is diagonal>} \\
 &= \mathbf{SR} = \mathbf{\Omega} \quad \text{<Since } (\mathbf{I} - \mathbf{K})\mathbf{H} = \mathbf{0}> \quad (2.23)
 \end{aligned}$$

2.1.4.2 Bad Data Detection

The most commonly adopted bad data detection technique is based on the Chi-square χ^2 distribution. Assume that a set of independent variables, $x_1, x_2, \dots, x_N$, follow the normal distribution as:

$$x_i \sim N(0,1), \quad i = 1, 2, \dots, N.$$

Define a new variable of y as:

$$y = \sum x_i^2, \quad i = 1, 2, \dots, N. \quad (2.24)$$

and y will follow the Chi-square χ^2 distribution with N degrees of freedom.

Recall the objective function of WLS based SE in (2.8):

$$J = \sum_{i=1}^{N_{meas}} \frac{(\mathbf{z}_{s(i)} - \mathbf{h}_{s(i)})^2}{\mathbf{R}_{(i,i)}} \quad (2.25)$$

If $\frac{(\mathbf{z}_{s(i)} - \mathbf{h}_{s(i)})^2}{\mathbf{R}_{(i,i)}}$ can be approximately treated as the standard normal distributed variables, the objective J will follow the Chi-square χ^2 distribution with the freedom degree of $(N_{meas} - N_{basic})$ where N_{basic} is the least number of measurements needed to satisfy the system observability.

Using the Chi-square χ^2 distribution to detect bad data, we should first understand the probability density function (p.d.f) of χ^2 distribution as shown in Figure 2.2. The MATLAB function of CHI2PDF(X, DF) is used to compute the value of p.d.f at each value of X using the corresponding degrees of freedom, namely, DF. In Figure 2.2, the degrees of freedom is set as 10, namely DF = 10. The area under the curve where $X \geq X_{th}$ represents the probability of $X \geq X_{th}$. Evidently, a lower

probability is found if the threshold value, X_{th} , increases. For instance, as shown in the figure, the probability is only around 0.05 when X is larger than $X_{th} = 18$.

Figure 2.2 χ^2 Probability density function.

Therefore, a simple judgment on whether the bad data exist is to compare the value of J and the threshold value based on the selected probability in the Chi-square distribution, say, 0.95. If J is found smaller than the threshold value, it is believed that no bad data exists, otherwise, the current measurement set may be tainted with the bad data.

2.1.4.3 Bad Data Identification

If bad data is detected in the measurement set, one would need a technique to identify the erroneous data. In this thesis, an approach based on the *Largest Normalized Residual* method will be introduced and utilized for the following studies. As shown above, the measurement residual, $\mathbf{r}_z$, follows a normal distribution with $E(\mathbf{r}_z) = 0$ and $Cov(\mathbf{r}_z) = \mathbf{\Omega}$. The *Largest Normalized Measurement Residual* test can be conducted in the following steps:

Step 1: Calculate the measurement residual:

$$\mathbf{r}_{z(i)} = \mathbf{z}_{(i)} - \mathbf{h}_{(i)}(\hat{\mathbf{x}}), \quad i = 1, 2, \dots, N_{meas}. \quad (2.26)$$

Step 2: Obtain the normalized residuals:

$$\mathbf{r}_{z(i)}^N = |\mathbf{r}_{z(i)}| / \sqrt{\boldsymbol{\Omega}_{(i,i)}}, \quad i = 1, 2, \dots, N_{meas}. \quad (2.27)$$

Step 3: Find k such that $\mathbf{r}_{z(k)}^N$ is the largest among all $\mathbf{r}_{z(i)}^N$, $i =$

$$1, 2, \dots, N_{meas}.$$

Step 4: Choose an identification threshold value of c . If $\mathbf{r}_{z(k)}^N > c$, then the

k^{th} measurement will be identified as the bad data.

Step 5: Remove the k^{th} measurement from the measurement set. Conduct the SE calculation again and go to Step 1.

The identification process will terminate until the largest normalized residual will not be larger than the threshold value of c anymore. As an alternative method to remove the identified bad data from the measurement set as shown in Step 5, the estimated errors can be subtracted from the bad data and the corrected values can be utilized for the SE processing again. Assume that the bad measurement is:

$$\mathbf{z}_{(i)} + \mathbf{e}_{(i)} = \mathbf{z}_{(i)}^{\text{bad}} \quad (2.28)$$

where $\mathbf{z}_{(i)}^{\text{bad}}$ is the identified bad data in the measurement set, $\mathbf{z}_{(i)}$ is the true value and $\mathbf{e}_{(i)}$ is the associated errors. Recall (2.20) and the relationship between the measurement residual and measurement error (in a linearized formulation) can be used to obtain the error of $\mathbf{e}_{(i)}$:

$$\mathbf{r}_{(i)}^{\text{bad}} \approx \mathbf{S}_{(ii)} \mathbf{e}_{(i)} \quad (2.29)$$

$$\mathbf{e}_{(i)} \approx \frac{\mathbf{r}_{(i)}^{\text{bad}}}{\mathbf{s}_{(ii)}} = \frac{\mathbf{r}_{(i)}^{\text{bad}}}{\boldsymbol{\Omega}_{(ii)}} \mathbf{R}_{(ii)} \quad (2.30)$$

Therefore the corrected measurement can be obtained as:

$$\mathbf{z}_{(i)} = \mathbf{z}_{(i)}^{\text{bad}} - \frac{\mathbf{r}_{(i)}^{\text{bad}}}{\boldsymbol{\Omega}_{(ii)}} \mathbf{R}_{(ii)} \quad (2.31)$$

SE will be repeated with the corrected measurements. Both techniques dealing with the identified bad measurement can be utilized for the SE and their performances are similar.

2.2 Hybrid State Estimation Combining SCADA and Synchrophasor Measurements

In contrast to what can be provided from the SCADA measurements, the PMU measurements, $\mathbf{z}_p$, comprise phasor voltage measurements ($|\mathbf{V}|$, θ_v) and phasor line current measurements ($|\mathbf{I}|$, θ_i), which can be further formulated by the state variables in a linear way if rectangular coordinates are adopted. Since the synchrophasor measurements are usually insufficient, hybrid SE is proposed to process the combined SCADA and PMU measurements in the calculation [35] [36]. Two typical hybrid State Estimators will be introduced in this chapter: post-processing method and equality-constrained method.

2.2.1 Post-processing Method [35]

The basic idea of this method is to keep the traditional SE unchanged and utilize its results as pseudo-measurements in a post-processing procedure together with PMU measurements. In this way, the existing SE in EMS does not need any changes to cater for the newly added synchrophasor measurements. In the post-processing procedure, it is suggested that PMU measurements in the rectangular coordinates are utilized to achieve a linear formulation of SE. For the post-processing method, it is assumed that SE is able to function well with the sufficient SCADA measurements and proper PMU placement.

For the traditional SCADA measurements, the measurement functions of (2.2) – (2.6) can be utilized for the SE. For the PMU measurements. Since phase angle

information becomes available, the measurement function will be different from those of the SCADA measurements. PMU measurements can be listed as:

$$\mathbf{z}_p = \begin{bmatrix} |\mathbf{V}| \\ \boldsymbol{\theta}_v \\ |\mathbf{I}| \\ \boldsymbol{\theta}_i \end{bmatrix} = \begin{bmatrix} \mathbf{V}_r \\ \mathbf{V}_i \\ \mathbf{I}_r \\ \mathbf{I}_i \end{bmatrix} \quad (2.32)$$

where $|\mathbf{V}|$, $\boldsymbol{\theta}_v$, $|\mathbf{I}|$ and $\boldsymbol{\theta}_i$ are the measurements of phasor voltage and currents from PMUs, in polar form. $\mathbf{V}_r$, $\mathbf{V}_i$, $\mathbf{I}_r$ and $\mathbf{I}_i$ are the measurements transformed into rectangular coordinates where the subscripts of $\mathbf{r}$ and $\mathbf{i}$ indicate the real and imaginary part respectively.

The measurement function associated with PMU measurements can be formulated in terms of the state variables as:

$$\mathbf{h}_{V_r} = \mathbf{x}_{V_r} \quad (2.33)$$

$$\mathbf{h}_{V_i} = \mathbf{x}_{V_i} \quad (2.34)$$

$$\mathbf{h}_{I_r} = \mathbf{G}\mathbf{x}_{V_r} - \mathbf{B}\mathbf{x}_{V_i} \quad (2.35)$$

$$\mathbf{h}_{I_i} = \mathbf{B}\mathbf{x}_{V_r} + \mathbf{G}\mathbf{x}_{V_i} \quad (2.36)$$

where $\mathbf{h}_{V_r}$, $\mathbf{h}_{V_i}$, $\mathbf{h}_{I_r}$ and $\mathbf{h}_{I_i}$ are the measurement functions of synchrophasor measurements from PMUs. $\mathbf{x} = [\mathbf{x}_{V_r}, \mathbf{x}_{V_i}]^T$ is the state variables vector in rectangular coordinates.

As discussed above, SE results from the existing EMS will be utilized as the pseudo measurements in the post-processing method and they are denoted as $\mathbf{x}^{(1)} = [\mathbf{x}_{V_r}^{(1)}, \mathbf{x}_{V_i}^{(1)}]^T$. Such pseudo measurements can be formulated by the state variables in the rectangular coordinates as:

$$\mathbf{x}^{(1)} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{V_r} \\ \mathbf{x}_{V_i} \end{bmatrix} \quad (2.37)$$

where $\mathbf{1}$ is the unity matrix.

With the combined pseudo measurements and PMU measurements together, the post-processing method for the hybrid SE can be finally formulated as shown below:

$$\begin{bmatrix} \mathbf{x}_{V_r}^{(1)} \\ \mathbf{x}_{V_i}^{(1)} \\ \mathbf{V}_r \\ \mathbf{V}_i \\ \mathbf{I}_r \\ \mathbf{I}_i \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \\ \mathbf{1}' & \mathbf{0} \\ \mathbf{0} & \mathbf{1}' \\ \mathbf{C}_1 & \mathbf{C}_2 \\ \mathbf{C}_3 & \mathbf{C}_4 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{V_r} \\ \mathbf{x}_{V_i} \end{bmatrix} + \mathbf{e}_p \quad (2.38)$$

where $\mathbf{1}$ is the unity matrix. $\mathbf{1}'$ stands for the matrix with only 1 and 0 as its elements. If the phasor voltage is measured at a bus, the relevant element of that bus in the matrix of $\mathbf{1}'$ will be 1, otherwise it will be 0. $\mathbf{C}_1 \sim \mathbf{C}_4$ are the constant coefficients describing the relationship between the synchrophasor measurements and the state variables. Values of $\mathbf{C}_1 \sim \mathbf{C}_4$ can be obtained from the measurement function of (2.33) – (2.36).

It can be proved that the post-processing method is of the same accuracy as a direct combination of SCADA and PMU measurements for the SE. However, the post-processing method is advantageous since the existing EMS/SE does not need any modification and a linear (non-iterative) State Estimator can be achieved.

Calculation of Covariance Matrix

The post-processing method utilizes the traditional SE results as the pseudo measurements and transforms the synchrophasor measurements from the original polar coordinates to the rectangular ones. Therefore the covariance matrix of the measurements needs to be recalculated before conducting the hybrid SE [38].

- **Pseudo measurements from traditional State Estimation results**

The covariance and weight matrices of SCADA measurements in State Estimator have been discussed above, which are denoted as: $\mathbf{R}$ and $\mathbf{W}$. For the pseudo

measurements of the traditional SE results, $\mathbf{x}^{(1)}$, the covariance matrix can be obtained as $\mathbf{R}_{\mathbf{x}^{(1)}}$:

$$\begin{aligned}\mathbf{R}_{\mathbf{x}^{(1)}} &= \text{cov}(\mathbf{x}^{(1)}) \\ &= \text{cov}\left((\mathbf{H}_S^T \mathbf{R}^{-1} \mathbf{H}_S)^{-1} \mathbf{H}_S^T \mathbf{R}^{-1} [\mathbf{z}_S - \mathbf{h}_S(\mathbf{x})]^T\right) \\ &= (\mathbf{H}_S^T \mathbf{R}^{-1} \mathbf{H}_S)^{-1} \mathbf{H}_S^T \mathbf{R}^{-1} \text{cov}([\mathbf{z}_S - \mathbf{h}_S(\mathbf{x})]^T) [(\mathbf{H}_S^T \mathbf{R}^{-1} \mathbf{H}_S)^{-1} \mathbf{H}_S^T \mathbf{R}^{-1}]^T\end{aligned}\quad (2.39)$$

Considering that $\mathbf{z}_S - \mathbf{h}_S(\mathbf{x}) = \mathbf{e}$ as suggested in (2.1) and $\mathbf{R}$ is a diagonal matrix, we can have the covariance matrix for the pseudo measurements as:

$$\mathbf{R}_{\mathbf{x}^{(1)}} = (\mathbf{H}_S^T \mathbf{R}^{-1} \mathbf{H}_S)^{-1} \quad (2.40)$$

- **Synchrophasor measurements from PMUs**

In the post-processing method, a linear formulation of SE is obtained by utilizing the rectangular coordinates for the synchrophasor measurements which are originally in polar coordinates. The original synchrophasor measurements are $|\mathbf{V}|$, θ_V , $|\mathbf{I}|$ and θ_I . After changing the original polar coordinates of the synchrophasor measurements into the rectangular coordinates, we can have the transformed values of $\mathbf{V}_r$, $\mathbf{V}_i$, $\mathbf{I}_r$ and $\mathbf{I}_i$. Assume the covariance matrix of the original synchrophasor measurements is $\mathbf{R}_P$ as:

$$\mathbf{R}_P = \text{diag}(\sigma_{|V|}^2, \sigma_{\theta_V}^2, \sigma_{|I|}^2, \sigma_{\theta_I}^2) \quad (2.41)$$

These four original measurement uncertainties can be obtained from the technical manual of PMUs as suggested in Table 2.2 [37]:

TABLE 2.2 UNCERTAINTIES OF PMU MEASUREMENTS

$\sigma_{ V }$	σ_{θ_V}	$\sigma_{ I }$	σ_{θ_I}
0.02%	0.02%	0.02%	0.01 °

For any phasor $\vec{a}$:

$$\vec{a} = |a|\angle\theta_a = a_r + a_i \quad (2.42)$$

Assume that there is a small deviation, Δa and $\Delta\theta_a$, in the magnitude and phase angle respectively. Consequently the respective deviations in the real and imaginary parts of the phasor Δa_r and Δa_i , can be determined by (2.43) and (2.44) as:

$$\Delta a_r = \cos\theta_a \cdot \Delta a - a \sin\theta_a \cdot \Delta\theta_a \quad (2.43)$$

$$\Delta a_i = \sin\theta_a \cdot \Delta a + a \cos\theta_a \cdot \Delta\theta_a \quad (2.44)$$

i.e.

$$\begin{bmatrix} \Delta a_r \\ \Delta a_i \end{bmatrix} = \begin{bmatrix} \cos\theta_a & -a \sin\theta_a \\ \sin\theta_a & a \cos\theta_a \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta\theta_a \end{bmatrix} \quad (2.45)$$

The coordinate transformation matrix, $\begin{bmatrix} \cos\theta_a & -a \sin\theta_a \\ \sin\theta_a & a \cos\theta_a \end{bmatrix}$, can be used to calculate the measurement uncertainty from polar to rectangular coordinates. Specifically, for synchrophasor measurements, the following relationship can be referred:

$$\begin{bmatrix} \sigma_{V_r} \\ \sigma_{V_i} \\ \sigma_{I_r} \\ \sigma_{I_i} \end{bmatrix} = \mathbf{T}_c \begin{bmatrix} \sigma_{|V|} \\ \sigma_{\angle V} \\ \sigma_{|I|} \\ \sigma_{\angle I} \end{bmatrix} \quad (2.46)$$

where $\mathbf{T}_c$ can be expanded as shown in (2.47):

$$\mathbf{T}_c = \begin{bmatrix} \text{diag}(\cos\theta_V) & \text{diag}(-|V|\sin\theta_V) & \mathbf{0} & \mathbf{0} \\ \text{diag}(\sin\theta_V) & \text{diag}(|V|\cos\theta_V) & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \text{diag}(\cos\theta_I) & \text{diag}(-|I|\sin\theta_I) \\ \mathbf{0} & \mathbf{0} & \text{diag}(\sin\theta_I) & \text{diag}(|I|\cos\theta_I) \end{bmatrix} \quad (2.47)$$

Denote the covariance matrix of the PMU measurements after coordinate transformation as $\mathbf{R}'_p$. It can be pointed out that:

$$\mathbf{R}'_p = \mathbf{T}_c \mathbf{R}_p \mathbf{T}_c^T \quad (2.48)$$

where $\mathbf{T}_c$ is the coordinate transformation matrix.

2.2.2 Equality-constrained Method [36]

Unlike the post-processing method introduced above, another hybrid SE technique via the equality-constrained formulation is introduced in this section [8]. The synchrophasor measurements will be formulated as an equality constraint, considering the way how zero-injection measurements are incorporated into the traditional SE. Zero-injection measurements are the system information and not necessarily obtained via measuring, therefore they can be treated as absolutely accurate measurements. Instead of assigning high weighted values to the zero-injection measurements, it is suggested to constrain them as a series of equalities. For the zero-injection measurements, we have:

$$\mathbf{P}_{\text{inj}}(k) = \mathbf{Q}_{\text{inj}}(k) = 0, \quad k \in \mathcal{S}\{\text{Zero_injection buses}\} \quad (2.49)$$

Equation (2.49) can be combined into an equality constraint as:

$$\mathbf{c}(\mathbf{x}) = \begin{bmatrix} \mathbf{P}_{\text{inj}} \\ \mathbf{Q}_{\text{inj}} \end{bmatrix} = \begin{bmatrix} \mathbf{h}_4(\mathbf{x}) \\ \mathbf{h}_5(\mathbf{x}) \end{bmatrix} = \mathbf{0} \quad (2.50)$$

With these equality constraints of zero-injection measurements included into SE, it is guaranteed that their estimated values will be zero. By introducing the Lagrange multiplier, λ , to the equality constraint, the objective function of SE can be obtained as:

$$L(\mathbf{x}, \lambda) = (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x}))^T \mathbf{R}^{-1} (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x})) + \lambda^T \mathbf{c}(\mathbf{x}) \quad (2.51)$$

where $\mathbf{c}(\mathbf{x})$ is the equality constraint of zero power injections. The solution of $L(\mathbf{x}, \lambda)$ can be obtained by (2.52) in an iterative way:

$$\begin{bmatrix} \Delta \mathbf{x}_k \\ \lambda_k \end{bmatrix} = \begin{bmatrix} \mathbf{H}_S^T \mathbf{R}^{-1} \mathbf{H}_S & -\mathbf{C} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} (\mathbf{H}_S^T \mathbf{R}^{-1}) (\mathbf{z}_S - \mathbf{h}_S(\mathbf{x}_k)) \\ \mathbf{c}(\mathbf{x}) \end{bmatrix} \quad (2.52)$$

where $\mathbf{C} = \frac{\partial \mathbf{c}(\mathbf{x})}{\partial \mathbf{x}}$ and $\Delta \mathbf{x}_k = \mathbf{x}_{k+1} - \mathbf{x}_k$.

To formulate the equality-constrained SE using the synchrophasor measurements, a new set of state variables is first introduced as:

$$\mathbf{x}^{new} = [\mathbf{x}, \mathbf{x}^{aux}]^T \quad (2.53)$$

where $\mathbf{x}^{aux} = [\mathbf{x}_{\theta_I}, \mathbf{x}_{|I|}]^T$ is the auxiliary set of state variables to consider the phasor line currents. $\mathbf{x}_{\theta_I}$ and $\mathbf{x}_{|I|}$ represent the phase angle and magnitude of the phasor line currents. The purpose of adding the auxiliary set of state variables is for a direct use of phasor measurements in polar form [36]. The measurement set for the hybrid SE, a combination of SCADA measurements and phasor measurements from PMUs, can be found as below:

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_S \\ \mathbf{z}_P \end{bmatrix} = \begin{bmatrix} \mathbf{z}_S \\ |\mathbf{V}| \\ \boldsymbol{\theta}_V \\ \mathbf{I} \\ \boldsymbol{\theta}_I \end{bmatrix} \quad (2.54)$$

and the measurement function can be obtained accordingly as:

$$h(\mathbf{x}^{new}) = \begin{bmatrix} \mathbf{h}_S \\ \mathbf{h}_P \end{bmatrix} = \begin{bmatrix} \mathbf{h}_S(\mathbf{x}_{|V|}, \mathbf{x}_{\theta_V}) \\ \mathbf{x}_{|V|} \\ \mathbf{x}_{\theta_V} \\ \mathbf{x}_{\theta_I} \\ \mathbf{x}_{|I|} \end{bmatrix} \quad (2.55)$$

where $\mathbf{z}_S$ and $\mathbf{h}_S$ can be referred to (2.1) – (2.6). $\mathbf{h}_P$ is the measurement function for PMU measurements. As shown in Figure 2.3, if a PMU is installed at bus i , the phasor voltage of bus k can be expressed via (2.56) and (2.57), i.e. in the basic voltage/current relationship.

Figure 2.3 PMU location in power network.

$$\mathbf{x}_{|V|(k)} = f_V(\mathbf{x}_{|V|(i)}, \mathbf{x}_{\theta_V(i)}, \mathbf{x}_{\theta_I(i)}, \mathbf{x}_{|I|(i)}) \quad (2.56)$$

$$\mathbf{x}_{\theta_V(k)} = f_\theta(\mathbf{x}_{|V|(i)}, \mathbf{x}_{\theta_V(i)}, \mathbf{x}_{\theta_I(i)}, \mathbf{x}_{|I|(i)}) \quad (2.57)$$

Based on this, the equality constraints can therefore be written as:

$$\mathbf{x}_{|V|(k)} - f_V(\mathbf{x}_{|V|(i)}, \mathbf{x}_{\theta_V(i)}, \mathbf{x}_{\theta_I(i)}, \mathbf{x}_{|I|(i)}) = 0 \quad (2.58)$$

$$\mathbf{x}_{\theta_V(k)} - f_V(\mathbf{x}_{|V|(i)}, \mathbf{x}_{\theta_V(i)}, \mathbf{x}_{\theta_I(i)}, \mathbf{x}_{|I|(i)}) = 0 \quad (2.59)$$

where $\mathbf{x}_{|V|(k)}$ and $\mathbf{x}_{\theta_V(k)}$ are the state variables of bus k . In a compact form, (2.58) and (2.59) can be grouped into the equality constraint, $\mathbf{c}_p$, as shown:

$$\mathbf{c}_p(\mathbf{x}^{new}) = 0 \quad (2.60)$$

Compared with the post-processing method, the equality-constrained method can utilize the entire measurement set directly without changing the coordinates from the original polar coordinates to rectangular ones.

2.2.3 Other Methods

On top of the aforementioned hybrid SE techniques, some other related methods can be found in [39] [40] [41] [42] [43]. The differences among the methods lie in the type of coordinate systems utilized. For the post-processing method in Section 2.2.1, a rectangular coordinate is used for both the state variables and synchrophasor measurements, whereas for the equality-constrained formulation in Section 2.2.2, a polar coordinate is adopted by both of them. In [39], the authors propose a rectangular coordinate system for the phasor current measurements while the phasor voltage measurements and the state variables are kept in the polar form. The relationship between the current measurements and state variables will be non-linear and the derivatives need to be calculated and updated in each iteration:

$$\frac{\partial \mathbf{I}_r}{\partial \mathbf{x}_{|V|}}, \frac{\partial \mathbf{I}_r}{\partial \mathbf{x}_{\theta_V}}, \frac{\partial \mathbf{I}_i}{\partial \mathbf{x}_{|V|}}, \frac{\partial \mathbf{I}_i}{\partial \mathbf{x}_{\theta_V}}.$$

In [40], a comparison is conducted among different ways to deal with phasor current measurements. Three methods are discussed as follows:

- Method 1: polar form of the current measurements.

- Method 2: rectangular form of the current measurements.
- Method3: pseudo voltage measurements calculated by the phasor current measurements.

The comparison is conducted from the angle of convergence and accuracy. The convergence is investigated according to the value of the first derivatives of current measurements with regard to the voltage magnitude and phase angle. Method 1 may be found problematic in the convergence since the value of the first derivative is non-continuous, while the other two methods do not have this problem. Another comparison is to examine the accuracy by using the following equation:

$$\frac{1}{N} \sum_{j=1}^{Nbus} \left[\frac{1}{M} \sum_{i=1}^M (\hat{\mathbf{x}} - \mathbf{x}_{true}) \right] \quad (2.61)$$

where $Nbus$ is the number of state variables and M is the number of Monte Carlo simulations. The calculated value from (2.61) is the average error of the estimation values and they will be used as an index to evaluate the SE performance of the three methods. It is pointed out that Method 3 is less accurate than Method 1 and 2. After evaluating both the convergence and the accuracy, Method 2, i.e. including the phasor currents in rectangular form, is believed to be the most advantageous technique among the three of them.

TABLE 2. 3 COMPARISON OF DIFFERENT OPP STRATEGIES

		Advantages	Disadvantages
Post-processing [35]	Method	Don't need to change the existing EMS	PMU measurements need to be transformed into rectangular coordinates
	Equality-constrained Method [36]	PMU measurements can be utilized directly	Need to change the existing EMS
Other Methods [40]	Method 1	Accurate	Problematic in the convergence
	Method 2	Accurate and no convergence problem	PMU measurements need to be transformed into rectangular coordinates
	Method 3	No convergence problem	Less accurate

2.3 Discussion on the Limited Number of Synchrophasor Measurements

In Section 2.2, the hybrid SE is introduced in the context that the synchrophasor measurements may be inadequate to make the system observable. The formulation of hybrid SE introduced above is valid only when the system is said to be in steady state. When tracking the dynamic states, it is problematic to combine the delayed SCADA measurements and the real-time synchrophasor measurements directly since the SE results will be biased. To deal with the inadequate synchrophasor measurements together with the time-skew SCADA measurements, several techniques will be introduced in this section.

2.3.1 SCADA Measurements as Reference

In [44], the authors propose to use the SCADA measurements as the reference measurements. Two types of SCADA measurements are analyzed and selected to compensate the insufficient synchrophasor measurements. They are the voltages of generator buses under automatic voltage regulator (AVR) control and active/reactive powers consumed by loads.

For the generator buses under automatic voltage regulator (AVR) control, the voltage measurement does not vary much as long as the reactive power limit is not violated. Therefore such voltage measurements can be utilized together with synchrophasor measurements for state tracking. The active/reactive powers consumed by loads are sensitive to voltage and frequency. If the disturbances are not serious, changes in the active/reactive power of loads will be small. In this context, the load powers can be used to compensate the inadequate synchrophasor measurements. The relevant reference measurements can be found as:

$$\mathbf{V}^{\text{ref}}, \mathbf{P}^{\text{ref}}, \mathbf{Q}^{\text{ref}}$$

The measurements will be included in the WLS based SE as suggested in (2.5) and (2.6).

In [44], it is also pointed out that the power produced by generator buses will change significantly after disturbances. Therefore it is suggested that PMUs are placed at such buses.

2.3.2 Regularized Method

Consider the measurement model:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{e} \quad (2.62)$$

where $\mathbf{z}$ is the measurement set, $\mathbf{x}$ is the state variable, $\mathbf{h}(\cdot)$ is the measurement function and $\mathbf{e}$ is the error associated with the measurement set. The corresponding weight matrix of the measurement will be $\mathbf{W}$ and $\mathbf{W}_{(i,i)} = \mathbf{R}_{(i,i)}^{-1}$. Obviously, the estimates of the state variables, $\hat{\mathbf{x}}$, can only be obtained when measurements are sufficient to make the system observable. Suppose that another set of measurements, $\mathbf{u}$, which is composed of the voltage magnitudes and angles of all buses, no matter real or pseudo, will be considered. The new measurement set will be obtained as:

$$\bar{\mathbf{z}} = [\mathbf{z}, \mathbf{u}]^T \quad (2.63)$$

The measurement function and the weight matrix will be $\bar{\mathbf{h}}(\mathbf{x}) = [\mathbf{h}(\mathbf{x}), \mathbf{x}]^T$, $\bar{\mathbf{W}} = \begin{bmatrix} \mathbf{W} & \mathbf{0} \\ \mathbf{0} & \mathbf{S} \end{bmatrix}$, where $\mathbf{S}$ is the diagonal weight matrix associated with measurement set of $\bar{\mathbf{u}}$.

The WLS based estimation will be utilized to minimize the following objective function:

$$\text{minimize } \mathbf{W} \|\mathbf{z} - \mathbf{h}(\mathbf{x})\|^2 + \mathbf{S} \|\mathbf{z} - \mathbf{h}(\mathbf{x})\|^2 \quad (2.64)$$

Equation (2.64) can be considered as *Tikhonov* regularization [45] [46] or simply regularized least squares formulation, which can be utilized to solve the underdetermined problems.

The weighted matrix of $\mathbf{S}$ should be set properly in order to reach a reasonable result of $\mathbf{x}$. For instance, the recommendation is to set the weighting factor of the real measurements with typical values and the weighting factors of the pseudo-measurements with smaller values implying that such pseudo-measurements are assigned with large variances.

2.3.3 Machine Learning Techniques

In [47], the lack of measurements of SE in the distribution system is noticed and solved by using a closed-loop information feedback technique via Machine Learning. The estimation results serve as the inputs to the Machine Learning module together with the historical load data. In this way, the reliable nodal load estimates (both the active and reactive power for each node) are expected and thus the SE results can be improved. Figure 2.4 suggests the design of the State Estimator with the Machine Learning module.

Figure 2.4 A closed-loop information feedback technique in SE via Machine Learning

A parallel distributed processing (PDP) method is proposed to implement the Machine Learning function [48]. The five layers of the learning structure can be

referred as: Input, Mapping, Storage, Mapping and Output, as shown in Figure 2.5.

Figure 2.5 The parallel distributed processing (PDP) structure

The first layer receives the input as Index and Load Time Series. Index is used to classify the day into workday, weekend and holiday. Load Time Series of $L_{t-N}, L_{t-N+1}, \dots, L_{t-1}$ are regarded as the inputs and L_t is the expected load value which is used for the Output Layer. The second layer converts the Load Time Series, $L_i, i = t - N, t - N + 1, \dots, t$, into a three-dimensional point $E(x_i, y_i, z_i)$ through the Gaussian function of $\mu(x)$:

$$\mu(x) = e^{\frac{(x-L)^2}{0.5}}, L = \begin{cases} 0 \\ 0.5 \\ 1 \end{cases} \quad (2.65)$$

Therefore the mapping will be $L(L_i) \rightarrow E(x_i, y_i, z_i), i = t - N, t - N + 1, \dots, t$. The third layer, Storage Layer, is for the learning process. The fourth layer is also a Mapping Layer for managing the load/output information and the last layer implements the mapping to reach the expected load value.

2.3.4 Weighted Least Absolute Value (WLAV) Based State Estimator

In [49], it is proposed to utilize a WLAV-based robust alternative estimator to handle the PMU and SCADA measurements which are at different refresh rates. A

conventional WLS estimator is utilized when both PMU and SCADA measurements are refreshed at the same rate. For the subsequent instances when only PMU measurements are refreshed, a WLAV estimator is proposed to be utilized. WLAV estimator is modified on the Least Absolute Value (LAV) estimator which minimizes the L_1 norm of the residuals and the robustness of this method can be proved. The following estimation problem will be solved in the WLAV method:

$$\begin{aligned} \min c^T |r| \\ \text{s. t. } \Delta z - H\Delta x = r \end{aligned} \quad (2.66)$$

where c is the measurement cost vector which is related with the measurement accuracy and r is the measurement residual vector. A flowchart of the WLAV based hybrid SE in [49] can be found as:

Figure 2. 6 Flowchart of the WLAV based SE in [49].

In this flowchart, it can be pointed out that if SCADA, z_{SCADA} and PMU, z_{PMU} measurements are both available at time instant k , the WLS estimator will be used to obtain the estimation results, $x_{estimated}^k$. If new SCADA measurements are not available at next time instant $k + 1$, the WLAV estimator will be used to obtain the

estimation results.

It can be pointed out that the present-day power systems are generally not observable with mere PMUs. State estimators have to employ inaccurate SCADA measurements to solve the state estimation problem between two SCADA updates. The proposed method is testified as a useful diagnosis tool to monitor the system voltage in the case of slow moving voltage problems. The effectiveness and robustness of this approach is highly related to the number and location of PMUs placed in the system, namely if more PMUs are placed at strategic locations in the system, more accurate estimation results will be obtained.

2.4 Summary

In this section, techniques on hybrid SE are introduced. The advantage of the post-processing method is that phasor measurements from PMUs can be incorporated without changing the existing SE software in EMS. A linear formulation is proposed by transforming the original polar coordinates of the synchrophasor measurements into the rectangular ones, as well as the state variables. Due to the change of coordinates, the covariance matrix needs to be recalculated. The equality-constrained method, however, is proposed to use the phasor measurements as the equality constraint. The Lagrange multiplier is utilized to solve the equality-constrained problem. Another reason to incorporate synchrophasor measurements in the equality constraints is that the accuracy of synchrophasor measurements is much higher than that of SCADA measurements.

Besides this, a method to calculate the covariance matrix is discussed in this chapter, namely the transformed matrix method. The transformed matrix method is based on the derivative relationship between the polar and rectangular coordinates.

Discussion on the inadequate synchrophasor measurements for SE is also covered in this chapter. Three typical techniques are introduced to solve the problem.

The work reported later in this thesis adopts the SE formulation with equality constraints and the solution is obtained by using the Lagrange multiplier technique. However, the synchrophasor measurements from PMUs are included in the rectangular coordinates as introduced in [35] and the relevant covariance matrix is calculated via the transformed matrix method.

Chapter 3

Optimal Placement Strategies of Phasor Measurement Units

Before conducting the measurement-based power system analysis, such as SE, the placement problem of measurement units should be well solved. When PMUs are available, a practical problem arises as how to install the measurement units throughout the system [50] [51]. The Optimal PMU Placement (OPP) problem has always been a heated research topic. Some mature techniques to consider single or multiple objectives in the problem formulation have been proposed in [52] [53] [54] [55]. Different PMU placement strategies will affect the subsequent analyses and therefore the OPP problem is an interesting topic to explore. A taxonomy paper on the OPP studies from 1993 to 2011 is available in [56]. It can be found that the OPP techniques are classified into mathematical and heuristic algorithms. For the mathematical algorithms, the two typical methods can be concluded as integer programming and exhaustive search. For the heuristic algorithms, some representative methods can be found as genetic algorithm, tabu search, simulated annealing, particle swarm optimization and so on.

Measurement redundancy is a primary concern for the optimal PMU strategy, since redundancy is useful to eliminate the biased measurements and thus guarantee a more accurate SE solution. In this chapter, the state-of-art strategies on the OPP are introduced and optimizing ideas on the measurement redundancy are proposed.

3.1 Literature Review

3.1.1 Binary Integer Linear Programming (BILP)

Work reported in [52] pioneers the study of OPP strategies by applying the Binary Integer Linear Programming (BILP) to obtain the solution. It is the first time to formulate the OPP problem via the numerical method, instead of the search strategy or graph theory [53]. In the BILP based OPP problem, state variables of each site to install a PMU will be treated as a binary viable as y_i . If a PMU should be placed at bus i , $y_i = 1$; otherwise $y_i = 0$. An observable system means that every bus in the system should be “reached” at least once by PMU and therefore a bus can be made observable either by placing a PMU on it or on any buses connected to it. Based on this placement, the constraint of the OPP problem to guarantee the observability can be obtained as:

$$\mathbf{r} = \mathbf{A}\mathbf{y} \geq \mathbf{1} \quad (3.1)$$

where $\mathbf{r}$ is the redundancy level, $\mathbf{r} = [r_1, r_2, \dots, r_{N_{\text{bus}}}]^T$. $\mathbf{A}$ is the bus-bus connectivity matrix. $\mathbf{y} = [y_1, y_2, \dots, y_{N_{\text{bus}}}]^T$ is the binary vector that indicates the placement solution. The objective of this optimization is to find the minimum number of PMUs which can be formulated as:

$$N_{PMU} = \text{minimize } \sum y_i \quad (3.2)$$

The OPP problem formulated via the BILP will be illustrated using the 5-bus system, as shown in Figure 3.1.

Figure 3.1 An illustrative 5-bus system

According to the system connectivity, the redundancy level of each bus can be written as:

$$\begin{cases} r_1 = y_1 + y_2 + y_4 \\ r_2 = y_1 + y_2 + y_3 + y_5 \\ r_3 = y_2 + y_3 \\ r_4 = y_1 + y_4 \\ r_5 = y_2 + y_5 \end{cases}$$

To guarantee the observability of the system, the optimal placement strategy should make the redundancy level of each bus larger than or equal to 1. Besides, the number of PMUs needed should be minimized. Multiple solutions may be expected, for instance, in this illustrative case study, PMUs can be placed at bus 1 and bus 2 ($y_1 = 1, y_2 = 1$), or bus 2 and bus 4 ($y_2 = 1, y_4 = 1$) to make the system observable with a minimum number of PMUs.

Many commercial solvers can be employed to deal with the BILP based PMU placement problem. In this work, *bintprog()* from MATLAB 2010 is utilized to achieve the OPP solution.

3.1.2 Binary Search Algorithm

The Binary Search Algorithm (BSA) proposed in [57] is based on an exhaustive searching principle. An optimal PMU placement can be achieved through three steps:

Step 1 Determine the minimum number of PMUs satisfying the observability

The minimum number of PMUs needed to satisfy the system observability can be

obtained via binary search. Assume that the minimum number of PMUs needed is N_{pmu} , the number of candidate locations for placing PMUs is P . To examine if N_{pmu} is the minimal, all possible PMU placement strategies with N_{pmu} sets of PMUs will be checked to see whether the system is solvable. The number of combinations of the placement strategies can be determined as:

$$N_{com} = \frac{P!}{N_{pmu}!(P-N_{pmu})!} \quad (3.3)$$

i.e. to choose the number of N_{pmu} locations out of the total number of P candidate positions.

Each possible placement will be examined sequentially to see if the observability can be satisfied. If all placements cannot make the system observable, then $N_{pmu} = N_{pmu} + 1$ and the possible placement strategies will be examined all over again. The procedure will be terminated until the observable placement strategy is found and thus the minimum number of PMUs needed, N_{pmu} , is found. The number of PMU placement strategies satisfying the observability with N_{pmu} sets of PMUs can be counted as N_{sol} .

Step 2 Take the single branch outage into consideration

The aforementioned method to determine the minimum number of PMU is under normal condition. When single-line outages occur, more PMUs are needed to make the system observable since the previous connection between two buses will be interrupted. The second step of BSA is conducted to find the optimal placement with a minimum number of PMUs which can make the system observable under contingencies. Similarly, a search strategy will be used to examine the observability of the possible strategies.

Step 3 Maximize the measurement redundancy

After the minimum number of PMUs is determined, the optimal location for PMUs

can be found using the following procedure:

- 1) Set the minimum redundancy level as $r_{min} = 2$.
- 2) For each candidate PMU placement, let n_i be the number of times when the criteria of $r_k^i > r_{min}$ can be satisfied, where $k = 1, 2, \dots, N_{sol}$, $i = 1, 2, \dots, N_{bus}$, r_k^i is the redundancy level of bus i under solution k . N_{sol} is the number of candidate PMU placements.
- 3) Find the m_{th} placement such that

$$n_m = \max(n_1, n_2, \dots, n_{N_{sol}})$$

- 4) If $r_{min} \leq r_{max}$ and multiple solutions are found for the condition above, then $r_{min} = r_{min} + 1$ and go to 2), otherwise go to 5), where r_{max} is the maximum redundancy level of each buses of all candidate placements.
- 5) Output the optimal solution when the optimization is terminated.

A PMU placement with the best redundancy *distribution level* can be obtained by BSA. Global optimal solution can be found through BSA, as it is technically a searching method. However BSA requires a long execution time due to the heavy computational burden which limits its application to larger power systems [58].

3.1.3 Considering Conventional Measurements

On top of the observability and measurement redundancy, another concern of the OPP problem is the conventional measurements. References [32] and [33] formulate the PMU placement strategy by considering the existing conventional measurements which contribute to the system observability. By considering the existing conventional measurements, the number of PMUs needed will be less than that of the placement strategy without considering the conventional measurements. Augmented Bus Merging (ABM) method is proposed in [32] to deal with the conventional measurements in the OPP problem. This method utilizes matrix

permutation to reorder the bus number according to the location of conventional measurements and thus the observability model is reconstructed.

3.1.4 Considering Condition Number

In numerical stability analysis, condition number, $cond(.)$, is used to evaluate the reliability of the solution in a given calculation. For SE calculation, a high condition number manifests that the solutions will be very sensitive to the measurement error, thus the numerical stability deteriorates. For the following linear equation in (3.4):

$$\mathbf{Ax} = \mathbf{b} \quad (3.4)$$

Assume that the disturbance in $\mathbf{b}$, $\xi_{\mathbf{b}}$, will cause a consequent change in the solution vector, $\mathbf{x}$, as $\xi_{\mathbf{x}}$. The relationship between the two relative errors can be written as [59]:

$$\frac{\|\xi_{\mathbf{x}}\|}{\|\mathbf{x}\|} \leq \left(\|\mathbf{A}\| \cdot \|\mathbf{A}^{-1}\| \right) \frac{\|\xi_{\mathbf{b}}\|}{\|\mathbf{b}\|} = cond(\mathbf{A}) \frac{\|\xi_{\mathbf{b}}\|}{\|\mathbf{b}\|} \quad (3.5)$$

The condition number of the coefficient matrix $\mathbf{A}$, $cond(\mathbf{A})$, reflects the range of error amplification and it can be written as:

$$cond(\mathbf{A}) = \|\mathbf{A}\| \cdot \|\mathbf{A}^{-1}\| \quad (3.6)$$

In the calculation of (3.6), any norm (1-norm, 2-norm and etc.) can be used. For computational efficiency, 1-norm is adopted in this paper and it is defined as the maximum value of the absolute column sum of the matrix:

$$\|\mathbf{A}\| = \max_{i=1}^n \sum_{j=1}^n |\mathbf{A}_{ij}| \quad (3.7)$$

Reference [60] considers the condition number as a criteria when formulating the OPP problem and the condition number of the normalized measurement matrix is minimized by selecting the optimal PMU placement.

3.1.5 Multi-stage Placement Strategy

On top of the OPP topics as introduced above, another research focus on PMU

placement is about multistage placement strategies [61] [62]. The objective is to fulfill the placement task in a given period, for example several years. Reference [61] utilizes Integer Linear Programming (ILP) to obtain the optimal PMU placement location and a multi-criteria decision-making method is utilized to determine the locations for PMU placement in each step. The multi-criteria considered in [61] include: observability with/without line outages and voltage control area. Reference [62] considers the availability of measurements as a probability problem and it is aimed at achieving the maximum average observability at each stage. In summary, the studies on the multi-stage placement strategy mainly focus on how to assign weights to each bus according to its contribution to the observability of the system and thus to decide its priority in the placement strategy.

3.2 Proposed Methodology

3.2.1 Improvement on Total Measurement Redundancy

The proposed methodology to determine an optimal PMU placement is based on a multiple-objective optimization. The first Objective (*Obj 1*) is to minimize the number of PMUs while making the system observable and the second objective (*Obj 2*) is to maximize the measurement redundancy level. *Obj 2* is to be implemented after *Obj 1* is satisfied. The relationship between *Obj 1* and *Obj 2* can be drawn as follows:

Figure 3.2 The relationship between two objectives involved in the optimization.

The first step can be named as *Observability BILP* and is the traditional observability BILP as introduced in Section 3.1.1. In this step, our aim is to determine the

minimum number of PMUs, N_{PMU} , which can satisfy the network's observability. N_{PMU} can be calculated according to (3.2).

The second step can be defined as *Redundancy BILP*. In this step, the total measurement redundancy will be maximized. To ensure that *Obj 2* is implemented after *Obj 1* is achieved, the value of N_{PMU} will be utilized as a constraint in the second step together with the *observability* constraint. Therefore step 1 can be treated as a preparation and step 2 is the final process to optimize the OPP solution. The proposed algorithm can be described in the following flow chart:

Figure 3.3 Proposed algorithm of the optimal PMU placement.

3.2.2 Improvement on Measurement Redundancy Distribution Level

3.2.2.1 Considering Conventional Measurements

Research studies have been done on the OPP problems with conventional measurements when such measurements are considered inadequate to make the system observable. However, for any existing power systems, conventional measurements are sufficient enough to make the system observable. In this context,

observability will not be a decisive factor for the OPP problem, especially when the number of PMUs is inadequate in the initial stage. To build a robust SE, local redundancy of the existing conventional measurements is considered in this work by placing PMUs in the system to improve the local redundancy of the existing conventional measurements.

Recall (2.19) which illustrates the relationship between the estimated measurements and raw measurements.

$$\Delta \hat{\mathbf{z}} = \mathbf{K} \Delta \mathbf{z}$$

For the i^{th} measurement, the entry in the matrix $\mathbf{K}$, $\mathbf{K}(i, j)$, indicates how the estimated i^{th} measurement is related with the j^{th} measurement. It can be claimed that if the diagonal element, $\mathbf{K}(i, i)$, is much larger than all the off-diagonal elements, $\mathbf{K}(i, j)$, then the estimated i^{th} measurement is heavily dependent on itself. This means that the local redundancy of this measurement is poor [7] [63]. Therefore for the i^{th} measurement, the number of times that the value of $\mathbf{K}(i, i)/\mathbf{K}(i, j)$, $i \neq j$, which is larger than a predefined value $\mathbf{d}$ can be used as an index to judge whether the measurement is of a poor local redundancy. And the local redundancy of each measurement, $\mathbf{Lr}$, can be calculated as:

$$\mathbf{Lr}(i) = N_{\text{meas}} - \sum_{j=1}^{N_{\text{meas}}} r(i, j), \quad 1 \leq i, j \leq N_{\text{meas}} \quad (3.8)$$

where $\begin{cases} r(i, j) = 1, & \text{if } \mathbf{K}(i, i)/\mathbf{K}(i, j) \geq \mathbf{d} \\ r(i, j) = 0, & \text{else} \end{cases}$ and N_{meas} is the number of measurements.

Obviously, $1 \leq \mathbf{Lr}(i) \leq N_{\text{meas}}$. After $\mathbf{Lr}(i), i = 1, 2, \dots, N_{\text{meas}}$, is calculated and the candidate sites for PMU placement can be chosen as Table 3.1 suggests if $\mathbf{Lr}(i) = 1$:

TABLE 3. 1 RELATIONSHIP BETWEEN CANDIDATE PMU PLACEMENT SITES AND MEASUREMENT WITH POOR LOCAL REDUNDANCY

Measurement	Type	Location	Candidate Placement Site/Sites
i	Voltage	bus k	k
	Power Flow	line $m-n$	m or n

3.2.2.2 Without Considering the Conventional Measurements

In [57], the OPP strategy is determined by selecting the placement with most buses satisfying $r_k^i > r_{min}$, where $k = 1, 2, \dots, N_{sol}$, $i = 1, 2, \dots, N_{bus}$, and r_{min} is the redundancy level and set as 2 initially, as suggested in Section 3.1.2. In this way, an evenly distributed redundancy level can be achieved. To better illustrate this point, define the red- p buses as the buses which can be reached p times by PMUs and the red-1 buses will be referred as those can be reached only once by the PMUs.

For the 7-bus system shown in Figure 3.4, if the observability and the maximum redundancy are considered as the objectives for the OPP problem, the minimum number of PMUs needed can be calculated as 3 by using the proposed method in Section 3.1.2. Buses 3, 4 and 6 are needed to be placed with PMUs as shown in Figure 3.4 (a). For each bus in the system, the redundancy level can be calculated as: $\mathbf{r} = [1 \ 1 \ 3 \ 3 \ 1 \ 3 \ 1]^T$ and the number of red-1 buses is 4. However, if another placement is adopted, i.e. to place PMUs at buses 1, 4 and 6 as shown in Figure 3.4 (b), the bus redundancy level becomes: $\mathbf{r} = [1 \ 2 \ 3 \ 3 \ 1 \ 2 \ 1]^T$ with only 3 red-1 buses.

Figure 3. 4 Two cases of PMU placement for a 7-bus case study.

The difference between these two placement strategies lies in: under the placement of Figure 3.4 (a), bus 2 is the red-1 bus since it can only be reached by the PMU at bus 4. Therefore the corruption/loss of any measurements of $|\bar{V}_4|$, $\theta_{\bar{V}_4}$, $|\bar{I}_{42}|$ and $\theta_{\bar{I}_{42}}$ will lead to bus 2 to be poorly estimated/unobservable. However for the placement in Figure 3.4 (b), the same measurement problem will not affect the estimation of bus 2 since its redundancy level has increased to 2, either by the PMU placed on bus 2 or bus 4. Therefore the placement in Figure 3.4 (b) is more robust than the placement suggested in Figure 3.4 (a) regarding the reliability of the execution of SE. The optimization on the redundancy distribution level to reduce the number of red-1 buses is proposed as:

- 1) Obtain the solution with maximum measurement redundancy as suggested in Figure 3.3.
- 2) Calculate the redundancy level of each bus by using (3.1), $\mathbf{r} = \mathbf{A}\mathbf{y} \geq 1$.
- 3) For bus i whose redundancy level is equal to or larger than 2, i.e. $\mathbf{r}_i \geq 2$, if bus j and bus i are associated and bus j is installed with PMU, find the bus/buses connected with j . Shift the PMU installed at bus j to its associated buses in an attempt to reduce the number of red-1 buses while satisfying the network observability.
- 4) Optimization is terminated when the number of red-1 buses cannot be reduced.

Figure 3.5 Flow chart of redundancy distribution level optimization

The flow chart of the redundancy distribution level optimization is drawn in Figure 3.5. The redundancy distribution level optimization is efficient and the calculation time is also computed for comparison in the following case studies. Measurement redundancy is also considered for the optimal PMU placement in [64] and [65]. However the focus of [64] and [65] is on the robust SE, i.e. the Least Absolute Value (LAV) SE. Besides, the channel limit of PMUs is considered in the optimization problem in [64] and [65].

3.2.3 Case Studies

The IEEE 14-bus system will be used for the case studies, as shown in Figure 3.6. Assume that the system is fully observable using conventional measurements of 5 voltage measurements and 18 power flow measurements, i.e. 90% of the branches are metered. The five voltage measurements are obtained from generation buses at buses 1, 2, 3, 6, and 8. The flow measurements are assigned using the rules below:

- 1) Lines from important buses, such as generation buses.
- 2) Other buses to ensure the system's observability.

Figure 3.6 IEEE 14-bus system configuration

The conventional measurements with their true values are shown in Table 3.2. With the convergence tolerance set as 10^{-5} , the WLS based SE converges after 5 iterations. The simulated data is based on the principle of “ 3σ ”. This means that measurements within the range of $True\ Values \pm 3\sigma$ are treated as normal data and used for the simulation. The $\mathbf{K}$ matrix in WLS can be obtained by using (2.19) and the local redundancy of each measurement can be calculated according to (3.8). The results shown in Table 3.3 are calculated as the average values from 100 times of simulations.

TABLE 3.2 MEASUREMENTS OBTAINED USING IEEE 14-BUS SYSTEM

Meas.			True Value	Meas.			True Value
	Type				Type		
1	V_1	Voltage	1.0600	21	P_{109}	Power Flow	-0.0526
2	V_2	Voltage	1.0450	22	P_{1011}	Power Flow	-0.0383
3	V_3	Voltage	1.0100	23	P_{1314}	Power Flow	0.0567
4	V_6	Voltage	1.0700	24	Q_{12}	Power Flow	-0.2039
5	V_8	Voltage	1.0900	25	Q_{15}	Power Flow	0.0363
6	P_{12}	Power Flow	1.5680	26	Q_{21}	Power Flow	0.2764
7	P_{15}	Power Flow	0.7556	27	Q_{23}	Power Flow	0.0357
8	P_{21}	Power Flow	-1.5251	28	Q_{25}	Power Flow	0.0087
9	P_{23}	Power Flow	0.7312	29	Q_{32}	Power Flow	0.0156
10	P_{25}	Power Flow	0.4155	30	Q_{34}	Power Flow	0.0422
11	P_{32}	Power Flow	-0.7081	31	Q_{47}	Power Flow	-0.0976
12	P_{34}	Power Flow	-0.2316	32	Q_{49}	Power Flow	-0.0038
13	P_{47}	Power Flow	0.2803	33	Q_{56}	Power Flow	0.1270
14	P_{49}	Power Flow	0.1607	34	Q_{611}	Power Flow	0.0352
15	P_{56}	Power Flow	0.4403	35	Q_{612}	Power Flow	0.0258
16	P_{611}	Power Flow	0.0734	36	Q_{613}	Power Flow	0.0750
17	P_{612}	Power Flow	0.0779	37	Q_{87}	Power Flow	0.1733
18	P_{613}	Power Flow	0.1781	38	Q_{914}	Power Flow	0.0382
19	P_{87}	Power Flow	-0.0000	39	Q_{109}	Power Flow	-0.0423
20	P_{914}	Power Flow	0.0948	40	Q_{1011}	Power Flow	-0.0164
				41	Q_{1314}	Power Flow	0.0178

From Table 3.3, it can be pointed out that the local redundancy of measurements 17, 19 and 35 is 1. According to the relationship between the candidate sites and the local redundancy of measurements suggested in Table 3.2 and Table 3.3, the candidate placement sites in this case study can be determined as shown in Table 3.4. The candidate placement sites are at buses 6, 7, 8, 12, i.e. $C_{pmu} = \{6, 7, 8, 12\}$.

If PMUs are placed at bus 6 and bus 7, the updated local redundancy of the measurements can be found in Table 3.5. From Table 3.5, it can be found that no conventional measurements have a local redundancy of 1 anymore.

TABLE 3.3 LOCAL REDUNDANCY OF THE MEASUREMENTS IN THE CASE STUDY

Measurement	<i>Lr</i>	Measurement	<i>Lr</i>
1	21	21	33
2	24.85	22	34.91
3	28	23	36
4	26	24	26.82
5	15	25	37
6	35	26	28
7	37	27	37.63
8	35	28	36.4
9	36.86	29	37.53
10	36	30	37
11	36.87	31	38
12	38	32	38
13	15	33	34
14	35.9	34	36.85
15	35	35	1
16	35	36	30.34
17	1	37	38
18	30	38	33
19	1	39	32.84
20	34.93	40	36
		41	36.33

TABLE 3.4 CANDIDATE PMU PLACEMENT SITES IN THE CASE STUDY

Measurement	Type	Location	Candidate Placing Site/Sites
17	Power Flow	line 6-12	bus 6 or 12
19	Power Flow	line 8-7	bus 8 or 7
35	Power Flow	line 6-12	bus 6 or 12

TABLE 3.5 LOCAL REDUNDANCY (UPDATED) OF THE MEASUREMENTS IN THE CASE STUDY

Measurement	Lr	Measurement	Lr
1	27.01	21	38.22
2	26.9	22	37
3	25	23	39
4	33	24	41
5	35	25	39.96
6	36.71	26	2
7	39	27	36
8	39	28	34
9	38.05	29	36.53
10	40	30	36.42
11	41.87	31	39
12	43.74	32	38.98
13	40.94	33	25
14	39.4	34	38.97
15	2	35	31.6
16	36	36	38.95
17	2	37	40
18	36.86	38	40
19	37.65	39	40
20	38.71	40	44
		41	45

Various test systems are used to demonstrate the reduction of red-1 buses and the results from BSA are utilized as a benchmark to compare. Based on the flow chart of the process shown in Figure 3.5, the PMU placement with the maximum measurement redundancy will be optimized in terms of the redundancy distribution level. The results of various test systems are presented as follows:

- IEEE 14-bus system

The results are the same as the solution with the maximum redundancy.

- IEEE 30-bus system [66]

The PMU placement with maximum measurement redundancy is obtained as:

$$y = [0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0]$$

Namely, buses 2, 4, 6, 9, 10, 12, 15, 18, 25 and 27 are needed to be placed with PMUs. The redundancy level of each bus is:

$$\mathbf{r} = [1 \ 3 \ 1 \ 4 \ 1 \ 5 \ 1 \ 1 \ 3 \ 3 \ 1 \ 3 \ 1 \ 2 \ 3 \ 1 \ 1 \ 2 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 2 \ 1 \ 2 \ 2 \ 1 \ 1]$$

which amounts to a total of 18 red-1 buses.

The first successful attempt to reduce the number of red-1 buses occurs on bus 6 whose redundancy level is 5 and bus 4 is found to be an associated PMU bus to bus 6. When shifting the PMU located at bus 4 to its associated bus 3, the placement solution $\mathbf{y}$ now becomes:

$$\mathbf{y} = [0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0]$$

This solution is valid since it can satisfy the observability. Now the number of red-1 buses is reduced to 17. The same procedure can be repeated sequentially and another successful optimization can be achieved at bus 15. The PMU placed at bus 18 which is associated to bus 15, is transferred to bus 19. After checking, the observability can be still satisfied and the number of red-1 buses further reduces to 16. When the optimization process completes, the improved placement solution of $\mathbf{y}$ with 16 red-1 buses can be obtained as:

$$\mathbf{y} = [0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1 \ 0 \ 0 \ 0].$$

Buses 2, 3, 6, 9, 10, 12, 15, 19, 25 and 27 are needed to be placed with PMUs.

- Other test systems

A similar process is conducted on IEEE 57-, 118- and 300-bus systems [66] to examine the improvements on redundancy distribution. The optimization on the measurement redundancy level in these systems cannot be executed with BSA due to heavy calculation burden. However, by using the redundancy distribution level optimization, the number of red-1 buses in these systems can be reduced as shown in Table 3.6. Another 72-bus system constructed from the IEEE test system is carried out as shown in Figure 3.7 and a reduction of 8 red-1 buses is

achieved by using the proposed optimization on the redundancy distribution level.

Figure 3.7 An illustrative 72-bus system

The reduction percentage of red-1 buses from the IEEE 30-, 57-, 118-, 300-bus system and the illustrative 72-bus system can be calculated as 11.1%, 2.3%, 1.3%, 1.6% and 17.4% as shown in Table 3.6.

TABLE 3.6 PMU PLACEMENT WITH RED-1 BUSES REDUCED AND THEIR COMPARISON WITH BSA

Test Systems	Before	After	Reduction Percentage on Red-1 Buses	BSA
	Optimization Process	Optimization Process		
	No. of red-1 buses	No. of red-1 buses		No. of red-1 buses
IEEE-14	10	10	-	10
IEEE-30	18	16	11.1%	16
IEEE-57	43	42	2.3%	Time Out
IEEE-118	80	79	1.3%	Time Out
IEEE-300	193	190	1.6%	Time Out
Illustrative 72	46	38	17.4%	Time Out

Figure 3.8 shows a comparison of the computation time between the redundancy

distribution level optimization and the BSA for different cases. For the IEEE 57-bus system and beyond, dotted lines indicate that BSA has timed out.

Figure 3.8 Computation time between the redundancy distribution level optimization and BSA

3.3 Summary

In this chapter, typical methodologies to solve OPP problem are presented. As explained previously, the OPP problem arises mainly due to the high cost of PMUs, including the cost of communication infrastructure, deployment and the devices. To improve the existing OPP strategies, optimization on the measurement redundancy has been carried out on two aspects in this chapter: total measurement redundancy and measurement redundancy distribution level with/without considering the conventional measurements. By using the two-step method indicated in Figure 3.3, the total measurement redundancy can be maximized while the number of PMUs needed is kept as the minimum. As for the measurement redundancy distribution level, more evenly distributed redundancy level can be obtained after the optimization. The number of buses with poor measurement redundancy level can be effectively reduced and in this way a more robust SE can be reached. When conventional measurements are considered, priority is given to the placement which can improve the redundancy level of the conventional measurements. When only

PMU measurements are considered, the optimization method on the redundancy distribution level indicated in Figure 3.5 is adopted. Case study results demonstrate that the number of buses with the worst measurement redundancy can be reduced and a more evenly distributed redundancy level can be achieved for future SE calculations.

Chapter 4

Enhanced State Estimation Incorporating Frequency Information from Synchrophasor Measurements

Traditional SE is conducted for pseudo steady operation when the system is said to be in normal state and the solution is based on the slowly-sampled SCADA measurements [7]. It can be pointed out that the reporting rate of SCADA is around 3-5 seconds, or sometimes even of the order of minutes, taking the latency into consideration [67]. When contingencies occur, the SCADA measurements will temporarily become unusable [44] and the traditional SE cannot be directly implemented during the contingency period.

Unlike SCADA measurements, synchrophasor measurements from PMUs can provide phase angle information and the reporting rate can be as fast as 10-120 samples/second [68]. However, since PMU measurements may be inadequate, it is not possible to track the states of the entire system due to the unobservability.

In this chapter, the adaptive underfrequency load shedding (UFLS) procedure is used as the pseudo measurements to improve the SE results. The improvement is based

on correcting the power injection measurements at the unobservable buses where load shedding takes place and thus enables the State Estimator to track the system states more accurately. Case studies on the IEEE test systems are conducted to verify the results.

4.1 Building State Estimation from a Limited Number of Synchrophasor Measurements

Studies on power system dynamic SE during contingencies can be found in [69]-[72]. Two bifurcations of the studies can be generalized. References [69] and [70] adopt the *static states* (magnitude and phase angle of bus voltage: $|V|, \theta_V$) with a Kalman Filter established between time $k + 1$ and k . By *prediction*, the dynamic behavior of power systems can be estimated. The problem encountered in this method, however, is the choice of the coefficients in Kalman Filter equations. Another research focuses on *dynamic states* (rotor angle and speed) [71] [72]. This dynamic model is more accurate than that of the aforementioned Kalman Filter using the static states, but the formulation is complicated and thus a popular application is hard to realize [71] [73]. When the fast-sampled PMU measurements become available, traditional State Estimator will not be limited to *static* situations. Instead it can be used for states tracking during the contingencies if the problem of insufficient synchrophasor measurements can be solved well [74].

SE and UFLS strategies are independent modules and employed for different purposes. They are however related in a way that the UFLS strategy changes the load powers of selected buses and SE processes such loading information by taking the corresponding active and reactive power measurements. Usually, load powers changed by the UFLS strategies cannot be immediately measured either by SCADA or PMUs. Using inaccurate load power measurements, the SE results will be biased.

Such problem can be concluded as *unobservability* or *limited measurements*. A regularized method is proposed in [75] to solve this underdetermined problem, but the accuracy is not satisfactory enough. In [44], historical power measurements are adopted as pseudo measurements. Again, the accuracy of SE results is not acceptable. Some closed-loop methods are introduced to deal with the unobservable problem. A machine learning technique is proposed in [47]. In [76] the authors suggest to integrate load forecasting with SE to solve the problem of limited measurements. The forecasted load data is utilized by SE and updated by the estimation results.

Different from the works reported in [44] [47] [75] [76], the real-time load shedding information is utilized and incorporated into the traditional SE in this thesis, due to the fast frequency measurements from PMUs. The real-time information of the load measurements are unknown, however, the load shedding information can be immediately obtained according to the system frequency information from PMUs. In this way, we can utilize the limited frequency measurements to obtain all load power measurements and the unobservability problem can be solved. By doing so, a more accurate measurements set can be obtained, compared with the techniques introduced in [44] [47] [75] [76]. Another contribution in this work is the introduction of auxiliary binary variables that reflect the load shedding status. Instead of solving the problem as a mixed integer nonlinear problem (MINLP) which leads to huge computational expenses, we adopt the alternating optimization method (AOM) [77] [78] to obtain an approximate solution which converges in real time. Case studies are conducted subsequently and the advantages of the proposed technique are verified.

4.2 Adaptive Underfrequency Load Shedding Strategy

As stated above, load powers usually change abruptly due to the underfrequency

load shedding events for maintaining system security. Power imbalances between the load and generation will cause frequency fluctuation and affect the normal operation of the system [79]. The UFLS is therefore needed to curtail the excessive load powers and prevent the frequency collapse. It is a common technique to maintain power system stability by removing the overload in some part of the system. This control strategy is essential in maintaining the system reliability. Intensive studies on the adaptive UFLS strategies can be found from literature review and utilized as the technical background for the work reported in this thesis. Generally three aspects are considered in the load shedding strategy:

- 1) Triggering time,
- 2) Required load shedding amount, and
- 3) Distribution of the load shedding amount in each step and at each participating load bus.

As introduced, the adaptive UFLS strategy can shed load powers more responsively and dynamically compared with the traditional and semi-adaptive one [80] [81] [82]. Currently, traditional UFLS strategies are still adopted for most power systems since they do not rely on complicated communication infrastructure and can shed loads locally. However, it must be understood that traditional UFLS strategies are conducted in a comparatively loose manner. The shedding amount and triggering time instants are determined based on the predefined values, regardless of the actual disturbance size, voltage drop and the rate at which frequency drops [80]. The traditional UFLS task is conducted locally and thus there is no cooperation among different locations of load shedding. To monitor the system in a more intelligent way, the *adaptive* UFLS strategies are adopted in this thesis and the relevant issues are discussed regarding the incorporation of the adaptive UFLS information into the SE. Compared with the traditional and semi-adaptive UFLS strategies, the adaptive UFLS strategy offers many benefits as it can shed loads according to the estimated

power deficit and coordinate the load shedding events globally among different load buses. Although a high dependence on the communication infrastructure currently hinders the popularity of the adaptive UFLS in power systems, its promising future can be expected [83] [84].

4.2.1 On-line Frequency Monitoring

As introduced above, a control center is needed for the adaptive UFLS. Frequency data acquisition, load shedding calculation and signal transmission are realized in the control center [85]. Figure 4.1 shows an on-line monitoring of the frequency disturbance and recovery in Clean Energy Research Laboratory (CERL), Nanyang Technological University (NTU), Singapore. Data acquisition is realized via ModBus-TCP/IP I/O Server embedded in the LabVIEW [86]. The sampling rate is 1s and the results shown in Figure. 4.1 are based on 380 samples.

Figure 4.1 An on-line frequency monitoring in CERL, NTU, Singapore.

4.2.2 Calculation of Shedding Amount

Discussion on the adaptive UFLS strategies can be found in [87] [88] [89]. In this work, load shedding will be triggered when the system frequency drops to 59.5 Hz (the nominal frequency is 60 Hz). To calculate the power deficit in the system, the

“first-swing equation” [90] needs to be utilized as:

$$\begin{aligned} P_{def} &= \sum_{m=1}^{N_G} \frac{2H(m)}{f_n} \frac{df_G(m)}{dt} S(m) \\ &= \frac{2 \sum_{m=1}^{N_G} H(m)S(m)}{f_n} \frac{df_{COI}}{dt} \end{aligned} \quad (4.1)$$

where

P_{def} : Power deficit,

$H(m)$: Inertia constant of the m^{th} generator,

$S(m)$: Rated apparent power of the m^{th} generator,

f_n : Rated system frequency,

$f_G(m)$: Frequency of the m^{th} generator,

N_G : Number of generators in the system, and

f_{COI} : Frequency of Center of Inertia. $\frac{df_{COI}}{dt} = \frac{\sum_{m=1}^{N_G} H(m) \frac{df_G(m)}{dt} S(m)}{\sum_{m=1}^{N_G} H(m)S(m)}$.

The power deficit, P_{def} , calculated from (4.1) cannot reflect the load shedding amount accurately, since it ignores the changes of load powers caused by the voltage oscillation. Assume that the load model is represented:

$$P_{L,k} = P_{L0,k} \left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} \quad (4.2)$$

where $P_{L0,k}$ and $P_{L,k}$ are the load power of bus k before and after the disturbances. $U_{0,k}$ and U_k are the voltage magnitude of bus k before and after the disturbances. The parameter α_k determines the type of the load model and it ranges from 0 to 2. When α_k equals 0, the load power is assumed not to vary with the voltage changes and the load model is therefore named as the constant power (CP) model; when α_k equals 1, the load power is assumed to vary with the voltage proportionally and this load model is named as the constant current (CI) model; when α_k equals to 2, the load power is assumed to vary with the voltage quadratically and the load model is named as the constant impedance (CZ) model. The real load model can be treated as a combination of these three models and an

experimental method to determine the load model coefficients can be found in [91].

The difference between the original and current load powers can be calculated as:

$$\begin{aligned}\Delta P_{L,k} &= P_{L,k} - P_{L0,k} \\ &= P_{L0,k} \left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - P_{L0,k} \\ &= P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right]\end{aligned}\quad (4.3)$$

And the power deficit for the system is:

$$P_{def} = P_{tur} - \sum_{k=1}^{N_L} P_{L,k} = P_{tur} - \sum_{k=1}^{N_L} (P_{L0,k} + \Delta P_{L,k}) \quad (4.4)$$

where P_{tur} is the mechanical power on the turbine, N_L is the number of loads and $\sum_{k=1}^{N_L} P_{L,k}$ indicates the total load power consumption.

Equation (4.4) can be further expanded as:

$$\begin{aligned}P_{def} &= P_{tur} - \sum_{k=1}^{N_L} (P_{L0,k} + \Delta P_{L,k}) \\ &= P_{tur} - \sum_{k=1}^{N_L} P_{L0,k} - \sum_{k=1}^{N_L} \Delta P_{L,k} \quad < \text{Substitute (4.3) into (4.4)} > \\ &= P_{tur} - \sum_{k=1}^{N_L} P_{L0,k} - \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right]\end{aligned}\quad (4.5)$$

Rearranging,

$$P_{tur} - \sum_{k=1}^{N_L} P_{L0,k} = P_{def} + \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right] \quad (4.6)$$

Replace P_{def} of (4.6) by (4.1) will yield the following:

$$P_{tur} - \sum_{k=1}^{N_L} P_{L0,k} = \frac{2 \sum_{m=1}^{N_G} H(m) S(m)}{f_n} \frac{df_{COI}}{dt} + \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right] \quad (4.7)$$

Based on (4.7), the load shedding amount could be calculated as shown in (4.8).

$$P_{LS} = P_{tur} - \sum_{k=1}^{N_L} P_{L0,k}$$

$$= \frac{2 \sum_{m=1}^{N_G} H(m)S(m)}{f_n} \frac{df_{COI}}{dt} + \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right] \quad (4.8)$$

In (4.8), the second term represents the influence of voltage drop on the load power during contingencies, as a modification to the traditional system frequency response (SFR) model shown in (4.9) [92]:

$$P_{LS}^{SFR} = \frac{2 \sum_{m=1}^{N_G} H(m)S(m)}{f_n} \frac{df_{COI}}{dt} \quad (4.9)$$

The benefits of considering the influence of voltage drop in the UFLS strategies are evident. If the load shedding amount is calculated without considering the influences of voltage drop as shown in (4.9), the amount calculated will be less than the required to balance the generation and consumption. Detailed discussion can be found in [90]. In the work presented below, the constant impedance load model will be used in the calculation and the simulation.

4.2.3 Adaptive UFLS Schedule

Load shedding will be conducted in multiple steps to avoid the over shedding problem [65]. The total shedding amount will be distributed into multiple steps when the frequency drops to the respective threshold values. In this thesis, a three-step load shedding strategy is adopted as shown in Figure. 4.2. Specifically, when frequency drops to 59.5 Hz, the first load shedding will be triggered and the shedding amount is 50% of the maximum amount. If the frequency continues to drop from 59.5 Hz to 59.2 Hz, an additional load shedding of 30% of the maximum shedding amount will be conducted. This leads to a total 80% of the maximum load shedding amount in the second step of load shedding. Further, if the frequency drops to 58.9 Hz, an extra 20% of the maximum load shedding is implemented which leads to a full load shedding. It should be realized that the actual load shedding depends on the reliability of the communication system and the relevant equipment such as the protective relays and circuit breakers.

Figure 4.2 Multi-step adaptive load shedding design

Although the shedding events can be well scheduled in the control center, the actual load shedding depends on the reliability of the communication systems and the relevant equipment such as the protective relays and circuit breakers. Assume that there are n selected load shedding sites and the probability of the successful load shedding event at *one* site is r . Based on the binomial distribution, the probability of successful load shedding events at all k sites can be calculated as:

$$\text{Pr} = \binom{n}{k} r^k (1 - r)^{n-k} \quad (4.10)$$

where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

4.3 Proposed Enhanced State Estimator

4.3.1 Formulation

The adaptive UFLS is utilized to obtain the accurate pseudo measurements of active/reactive powers which cannot be captured by SCADA instantaneously. Due to a limited number of PMU measurements, the equation of State Estimator will be underdetermined. As mentioned above, the problem can be solved by adding the available but outdated load power measurements from SCADA as they are assumed

to change slowly. Information of the zero-injection buses and voltage buses with automatic voltage regulator (AVR) can be utilized to supplement the measurements during contingencies since such voltage measurements can be treated as constants. Considering that the UFLS events can possibly occur during contingencies, we propose to conduct the SE with load power measurements reduced by the shedding amount whenever the UFLS events are triggered. Figure 4.3 shows the design of the proposed State Estimator.

Figure 4.3 The designed state estimator with adaptive UFLS strategies.

Load shedding information is useful, however the actual load shedding status, namely whether the load shedding is conducted at the selected buses, is still difficult to know. To formulate the proposed enhanced State Estimator, we first identify two sets of buses belonging to PMU and SCADA measurements: $\mathcal{S}\{PMUBus\}$ and $\mathcal{S}\{SCADABus\}$. For buses which are installed with PMUs, they are included into the set of $\mathcal{S}\{PMUBus\}$. For buses which are installed with traditional measurements, they are included into $\mathcal{S}\{SCADABus\}$ which can be further classified into $\mathcal{S}\{LoadShedBus\}$ for buses selected for load shedding and $\mathcal{S}\{OtherSCADABus\}$ for normal SCADA buses. This means that:

$$\mathcal{S}\{SCADABus\} = \mathcal{S}\{LoadShedBus\} \cup \mathcal{S}\{OtherSCADABus\} \quad (4.11)$$

The load shedding amount of bus i will be indicated as δ_{pi} and δ_{qi} for the active and reactive power respectively. The status variable of the load shedding event at bus i , will be indicated β_i . If the load shedding event is not conducted, $\beta_i = 0$

and if the event is conducted as expected, $\beta_i = 1$, $i \in \mathcal{S}\{LoadShedBus\}$.

The weighted least square (WLS) based SE is to find the best estimates of the state variables, $\mathbf{x}$, which can minimize the sum of the square of the differences between the actual and estimated values of the measurements. Compared with the traditional WLS, the proposed formulation of the enhanced SE in this work will include $\delta_{pi}\beta_i$ and $\delta_{qi}\beta_i$ into the active/reactive powers of p_i and q_i from the SCADA measurements, shown as:

minimize $f(\mathbf{x}, \boldsymbol{\beta})$

$$\begin{aligned} &= \sum_{i \in \mathcal{S}\{LoadShedBus\}} \mathbf{W}_{(i,i)} (p_i - h_{pi}(\mathbf{x}) + \delta_{pi}\beta_i)^2 \\ &\quad + \sum_{i \in \mathcal{S}\{LoadShedBus\}} \mathbf{W}_{(i,i)} (q_i - h_{qi}(\mathbf{x}) + \delta_{qi}\beta_i)^2 \\ &\quad + \sum_{i \in \mathcal{S}\{OtherSCADABus\}} \mathbf{W}_{(i,i)} (z_i - h_i(\mathbf{x}))^2 \\ &\quad + \sum_{i \in \mathcal{S}\{PMUBus\}} \mathbf{W}_{(i,i)} (z_i - h_i(\mathbf{x}))^2 \end{aligned}$$

subject to

$$\begin{aligned} \mathbf{g}(\mathbf{x}) &= \mathbf{0} \\ \boldsymbol{\kappa}(\mathbf{x}) &\leq \mathbf{0} \\ \boldsymbol{\beta}_i &= \{0,1\} \end{aligned} \tag{4.12}$$

where $\mathbf{W}_{(i,i)}$ is the weight for measurement i . z_i represents the measurement and $h_i(\cdot)$ is the measurement function. $\mathbf{x}$ is the state variables, comprising the real and imaginary part of the phasor voltage of each bus, namely, $\mathbf{x} = [\mathbf{x}_{V_r}, \mathbf{x}_{V_i}]^T$. δ_{pi} and δ_{qi} are the load shedding amount of the active and reactive power dictated by the adaptive UFLS control center. The binary variable of β_i indicates the on-site load shedding status. $\mathbf{g}(\cdot)$ and $\boldsymbol{\kappa}(\cdot)$ indicate the equality and inequality constraints.

The failure in executing the adaptive UFLS strategies and the time delay in communication will cause a less effective frequency recovery and even a frequency collapse in some cases. However, the issues will not affect the proposed SE

technique and the results, since the status of the load shedding events is fully considered in the proposed enhanced SE.

4.3.2 The Alternating Optimization Method (AOM)

4.3.2.1 Introduction of AOM

Considering the function of $P_0 = P_0(\mathbf{z})$, $\mathbf{z}$ is the set of variables, i.e. $\mathbf{z} \in \mathcal{R}^s$ and $P_0(\mathbf{z}): \mathcal{R}^s \rightarrow \mathcal{R}$. $\mathbf{z}$ can be partitioned into a set of non-overlapping T subsets as $\mathbf{z} = \{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_T\}$ where $\mathbf{Z}_t \in \mathcal{R}^{s_t}, t = 1, 2, \dots, T$ and $\sum_{t=1}^T s_t = s$. Alternating optimization method (AOM) is an iterative procedure to minimize/maximize the function of P_0 jointly over all the variables by alternately minimize/maximize the individual subsets of $\mathcal{S}\{\mathbf{Z}_1, \mathbf{Z}_2, \dots, \mathbf{Z}_T\}$. A detailed illustration of the AOM can be found in [77] [78]:

step 1: Initialize the variables as: $\mathbf{z} = \mathbf{z}^0 = \{\mathbf{Z}_1^0, \mathbf{Z}_2^0, \dots, \mathbf{Z}_T^0\}$. Set the iteration time $k = 1$, the termination criteria of a threshold value ε and the iteration limit, $\mathbf{L}$.

step 2: Find the minimizer of each subset of variables at the k^{th} iteration as:

$$\mathbf{Z}_t^{k+1} = \operatorname{argmin} P_0(\tilde{\mathbf{Z}}_1^{k+1}, \tilde{\mathbf{Z}}_2^{k+1}, \dots, \mathbf{Z}_t, \dots, \tilde{\mathbf{Z}}_T^k)$$

namely find the minimizer of $\mathbf{Z}_t^{k+1}$ with other variables fixed to the latest update. Noted that “ $\sim$ ” indicates a constant.

step 3: Terminate if $\max ||\mathbf{Z}^{k+1} - \mathbf{Z}^k|| \leq \varepsilon$ or $k \geq \mathbf{L}$. Otherwise, let $k = k + 1$ and go to **step 2**.

We prove the convergence of the AOM here. For the original problem formulated as $f(\mathbf{x}, \boldsymbol{\beta})$ in (4.12), we fix $\boldsymbol{\beta}$ at $\boldsymbol{\beta} = \boldsymbol{\beta}_0$ in the first iteration and minimize the problem of $f(\mathbf{x}, \boldsymbol{\beta})|_{\boldsymbol{\beta}=\boldsymbol{\beta}_0}$ with the initial value of $\mathbf{x} = \mathbf{x}_0$. If we can find the optimal value $\mathbf{x} = \mathbf{x}_1$ as the minimizer, it can be proven that:

$$f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_0, \boldsymbol{\beta}=\boldsymbol{\beta}_0} \geq f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_1, \boldsymbol{\beta}=\boldsymbol{\beta}_0}$$

Next, $\mathbf{x} = \mathbf{x}_1$ is fixed to minimize the problem of $f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_1}$ over $\boldsymbol{\beta}$. If $\boldsymbol{\beta} = \boldsymbol{\beta}_1$ is found as the minimizer, it can be proven that:

$$f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_1, \boldsymbol{\beta}=\boldsymbol{\beta}_0} \geq f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_1, \boldsymbol{\beta}=\boldsymbol{\beta}_1}$$

and further:

$$f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_0, \boldsymbol{\beta}=\boldsymbol{\beta}_0} \geq f(\mathbf{x}, \boldsymbol{\beta})|_{\mathbf{x}=\mathbf{x}_1, \boldsymbol{\beta}=\boldsymbol{\beta}_1}$$

In a similar fashion, we can prove that the problem is solved in a *non-increasing* manner starting from the initial values and the convergence of the AOM is therefore proved.

4.3.2.2 Solving the Enhanced State Estimator via AOM

The enhanced State Estimator formulated in (4.12) can be solved as a mixed integer nonlinear problem (MINLP) and many commercial solvers can be utilized to deal with the problem. However, the computational expense of utilizing the MINLP solver is huge, especially when we apply the enhanced SE to large scale systems or for the purpose of fast states tracking. To this end, AOM is investigated to solve the proposed enhanced SE. Considering that $\mathbf{x}$ and $\boldsymbol{\beta}$ constitute the entire state variables and affect the estimation results of each other, we optimize them in an alternating manner as introduced by **step 1** to **step 3** in Section 4.3.2.1.

The first term of $f(\mathbf{x}, \boldsymbol{\beta})$ in (4.12) will be expanded as suggested in (4.13), considering that the binary variable of $\boldsymbol{\beta}_i$ satisfies $\boldsymbol{\beta}_i^2 = \boldsymbol{\beta}_i$.

$$\begin{aligned} & \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} (p_i - h_{pi}(\mathbf{x}) + \delta_{pi} \boldsymbol{\beta}_i)^2 \\ &= \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left[(p_i - h_{pi}(\mathbf{x}))^2 + (\delta_{pi} \boldsymbol{\beta}_i)^2 + 2(p_i - h_{pi}(\mathbf{x})) \delta_{pi} \boldsymbol{\beta}_i \right] \\ &= \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left\{ (p_i - h_{pi}(\mathbf{x}))^2 + [\delta_{pi}^2 + 2(p_i - h_{pi}(\mathbf{x})) \delta_{pi}] \boldsymbol{\beta}_i \right\} \\ &= \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left\{ (p_i - h_{pi}(\mathbf{x}))^2 + [\delta_{pi} + 2(p_i - h_{pi}(\mathbf{x}))] \delta_{pi} \boldsymbol{\beta}_i \right\} \end{aligned}$$

(4.13)

Similarly, the second item in (4.12) can be expanded as:

$$\begin{aligned} & \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} (q_i - h_{qi}(\mathbf{x}) + \delta_{qi} \boldsymbol{\beta}_i)^2 \\ &= \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left\{ (q_i - h_{qi}(\mathbf{x}))^2 + [\delta_{qi} + 2(q_i - h_{qi}(\mathbf{x}))] \delta_{qi} \boldsymbol{\beta}_i \right\} \end{aligned} \quad (4.14)$$

In this way, (4.12) can be rewritten as:

$$\begin{aligned} & f(\mathbf{x}, \boldsymbol{\beta}) \\ &= \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left[(p_i - h_{pi}(\mathbf{x}))^2 + (q_i - h_{qi}(\mathbf{x}))^2 \right] \\ & \quad + \sum_{i \in \mathcal{S}\{\text{OtherSCADABus}\}} \mathbf{W}_{(i,i)} (z_i - h_i(\mathbf{x}))^2 \\ & \quad + \sum_{i \in \mathcal{S}\{\text{PMUBus}\}} \mathbf{W}_{(i,i)} (z_i - h_i(\mathbf{x}))^2 \\ & \quad + \sum_{i \in \mathcal{S}\{\text{LoadShedBus}\}} \mathbf{W}_{(i,i)} \left\{ [\delta_{pi} + 2(p_i - h_{pi}(\mathbf{x}))] \delta_{pi} + [\delta_{qi} + 2(q_i - h_{qi}(\mathbf{x}))] \delta_{qi} \right\} \boldsymbol{\beta}_i \end{aligned} \quad (4.15)$$

In (4.15), compared with (4.12), the terms of $\boldsymbol{\beta}$ are expanded. As suggested in **step 2**, the two sets of variables of $\mathbf{x}$ and $\boldsymbol{\beta}$ will be optimized alternately and it can be obtained in the $(k + 1)^{th}$ iteration:

$$\mathbf{x}^{k+1} = \underset{\mathbf{x}}{\text{argmin}} f(\mathbf{x}, \tilde{\boldsymbol{\beta}}^k) \quad (4.16)$$

$$\boldsymbol{\beta}^{k+1} = \underset{\boldsymbol{\beta}}{\text{argmin}} f(\tilde{\mathbf{x}}^{k+1}, \boldsymbol{\beta}) \quad (4.17)$$

It is not difficult to solve (4.16) with the help of Quadratic Optimization solver. The *quadprog()* in MATLAB is adopted to solve (4.16). To minimize (4.12), $\boldsymbol{\beta}$ in (4.17) can be solved via (4.18) which says that if the coefficient of $\boldsymbol{\beta}_i$, $\mathbf{W}_{(i,i)} \left\{ [\delta_{pi} + 2(p_i - h_{pi}(\mathbf{x}))] \delta_{pi} + [\delta_{qi} + 2(q_i - h_{qi}(\mathbf{x}))] \delta_{qi} \right\}$, is larger than 0, $\boldsymbol{\beta}_i$ will be 0, otherwise it will be 1.

$$\beta_i^{k+1} = \begin{cases} 0, & \mathbf{W}_{(i,i)} \left\{ \left[\delta_{pi} + 2(p_i - h_{pi}(\mathbf{x})) \right] \delta_{pi} + \left[\delta_{qi} + 2(q_i - h_{qi}(\mathbf{x})) \right] \delta_{qi} \right\} > 0 \\ 1, & \mathbf{W}_{(i,i)} \left\{ \left[\delta_{pi} + 2(p_i - h_{pi}(\mathbf{x})) \right] \delta_{pi} + \left[\delta_{qi} + 2(q_i - h_{qi}(\mathbf{x})) \right] \delta_{qi} \right\} \leq 0 \end{cases} \quad (4.18)$$

To avoid that the AOM falls into a local optimum, we design a multiple start-up strategy within the computation limit, as shown in Figure 4.4. First we initialize the β with an all-one vector $\mathbf{1}$ and optimize the problem formulated in (4.12) via AOM based on (4.16) and (4.17). The solution of $f(\mathbf{x}, \beta)$, $\mathbf{x}$ and β will be stored as $\mathbf{Sol}_1$. Next, we initialize the β with all-zero vector $\mathbf{0}$ and the optimization result will be stored as $\mathbf{Sol}_2$. Afterwards, β will be kept initializing with a random binary vector of **rand** and the solution will be stored as $\mathbf{Sol}_k$, $k \geq 3$. After all the solutions obtained within the time limit are stored, the solution with the minimum value of $f(\mathbf{x}, \beta)$ will be chosen as the final result.

Figure 4.4 Flow chart of the multiple start-up AOM

4.3.3 Case Studies

4.3.3.1 A Small Test System of IEEE 30-Bus System

The proposed enhanced SE during contingencies is tested on the IEEE 30-Bus system as shown in Figure 4.5. A simultaneous tripping of generator at bus 2 and line 2-6 at time $t = 1s$ [88] is assumed for the following study. PowerWorld V.17 Transient Analysis Simulator is used to obtain the real-time information of the voltage magnitude, bus angle and frequency of each bus during the contingency. Load information of the testing system can be found in Table 4.1.

Figure 4.5 IEEE 30-Bus test system.

Due to contingencies, the bus voltages will drop if the effective actions are not taken in time. As shown in Figure 4.6, the voltage magnitudes of buses 2 and 30 drop to unacceptably low values when the UFLS is not utilized. Frequency will also drop to a risky level due to the loss of power generation.

TABLE 4. 1 LOAD INFORMATION OF IEEE 30-BUS SYSTEM

Bus No.	P (MW)	Q (Mvar)	Bus No.	P (MW)	Q (Mvar)
2	21.7	12.7	17	9	5.8
3	2.4	1.2	18	3.2	0.9
4	7.6	1.6	19	9.5	3.4
5	94.2	19	20	2.2	0.7
7	22.8	10.9	21	17.5	11.2
8	30	30	23	3.2	1.6
10	5.8	2	24	8.7	6.7
12	11.2	7.5	26	3.5	2.3
14	6.2	1.6	29	2.4	0.9
15	8.2	2.5	30	10.6	1.9
16	3.5	1.8

Figure 4.6 Voltage measurement of bus 2 and 30 during contingencies without taking the UFLS strategies.

The adaptive UFLS strategy will be triggered when the frequency drops to 59.5Hz. In the adaptive UFLS control center, the total load shedding amount can be determined according to (4.8) and the load shedding strategy will be scheduled in multiple steps as planned in Figure 4.2. In the simulation, f_{COI} drops to 59.5Hz at time $t = 2.1s$ and the adaptive UFLS is initiated accordingly. Considering the

delay in the processing and the communication, the time to shed loads will be within the range of 2.2s~2.3s. A uniform inertia constant of $H = 3s$ is adopted in the simulation and the first derivative value of f_{COI} can be calculated as -0.4Hz/s . The voltage magnitudes before/after the disturbances and the initial load powers can be found in Table A.1. Other parameters needed in (4.8) to calculate the total generation deficit are:

$$N_G = 6; S = 100 \text{ MVA}; f_n = 60 \text{ Hz}; N_L = 21; \alpha_k = 2$$

Based on the above, the generation deficit can be calculated as:

$$\begin{aligned} \Delta P_T &= \frac{2 \sum_{m=1}^{N_G} H(m)S(m)}{f_n} \frac{df_{COI}}{dt} + \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right] \\ &= -24 - 16.4 = -40.4 \approx -40\text{MW} \end{aligned}$$

For the IEEE 30-Bus system, Table 4.2 presents the load shedding schedule based on the total load shedding amount and the multiple shedding strategy indicated in Figure 4.2.

TABLE 4.2 UFLS STRATEGIES WITH SELECTIVE LOADS (LARGEST SEVEN LOAD BUSES)

BUS No.	TOTAL SHEDDING AMOUNT (MW)	SHEDDING AMOUNT IN EACH STEP (MW)	
		STEP 1	STEP 2
2	1.6	1.0	0.6
5	6.7	4.2	2.5
7	1.6	1.0	0.6
8	2.1	1.3	0.8
12	0.8	0.5	0.3
21	12.0	7.5	4.5
30	7.3	4.5	2.7

In this case study, only two steps of the load shedding can recover the frequency back to the normal state as shown in Figure 4.7.

Figure 4.7 Frequency recover curve after the load shedding events: (a) the first UFLS event. (b) the second UFLS event.

Topology processing is very important for SE and any topology changes should be updated immediately. For the topology changes occurring in the IEEE 30-Bus system, the State Estimator responds in the following way:

- i Power injection from generator 2 will be zero.
- ii Line admittance between bus 2 and bus 6 will be zero.
- iii The relevant Jacobian matrix in the SE equation will be altered due to ii.

Next, the influence of the adaptive UFLS on SE under contingencies is considered. In this case study, it is assumed that PMUs are installed at generation buses as shown in Figure 4.5 and $\mathcal{S}\{PMUBus\} = \mathcal{S}\{1, 2, 5, 8, 11, 13\}$. Phasor voltage and line current measurements can be obtained for buses in $\mathcal{S}\{PMUBus\}$ and the measurements are assumed to be updated at every 1s. As suggested in Table 4.2, seven loads are selected for load shedding and four of them are SCADA buses. We can have $\mathcal{S}\{LoadShedBus\} = \mathcal{S}\{7, 12, 21, 30\}$ and other buses will be included into $\mathcal{S}\{OtherSCADABus\}$ as normal SCADA buses. Active/reactive power measurements can be obtained for $\mathcal{S}\{LoadShedBus\}$ and $\mathcal{S}\{OtherSCADABus\}$ which are assumed to be unchanged during the states tracking in the traditional SE. The shedding amount of δ_{pi} can be found in Table 4.2 for each step and each bus belonging to $\mathcal{S}\{LoadShedBus\}$. As explained above, the probability of successful execution of the adaptive UFLS depends on the reliability of the communication

system and the associated equipment such as the protective relays and circuit breakers. Therefore for $i \in \mathcal{S}\{LoadShedBus\}$, β_i is used to indicate that the shedding is successfully conducted on site when $\beta_i = 1$ and not conducted when $\beta_i = 0$.

In these case studies, SE is conducted under two schemes: traditional SE and the proposed AOM based SE. For both schemes, only measurements of the buses belonging to $\mathcal{S}\{PMUBus\}$ are updated during the tracking period. Other measurements are unchanged and treated as historical data. However, load shedding information of δ_{pi} and δ_{qi} and the shedding status variables β_i , $i \in \mathcal{S}\{LoadShedBus\}$, are considered in the proposed AOM based SE. Both schemes are conducted every 1s and the whole tracking period is from 0s – 40s. The estimated errors of each bus at each time instant are used to evaluate the performances of the two schemes as suggested by (4.19).

$$Error_{i,k} = |\hat{V}_{i,k} - V_{i,k}^{true}| \quad (4.19)$$

where $\hat{V}_{i,k}$ and $V_{i,k}^{true}$ are the estimated and true value of the voltage magnitude of bus i at time instant k , $i = 1, 2, \dots, 30$, $k = 0, 1, \dots, 40$.

Figure. 4.8 shows the statistical results of the estimated errors from the two schemes. In the figure, the middle line in the box indicates the median value of the estimated errors for each bus during the time period of 0s – 40s. The edges of the box are the 25th and 75th percentiles. The whiskers extend to the extreme data points which are not considered outliers and the detected outliers are plotted individually.

From Figure. 4.8, it can be observed that the estimation error of the traditional SE is generally more than 10 times larger than the proposed AOM based estimation results. It can be pointed out that the proposed enhanced SE performs better in terms of the

accuracy. Figure 4.8 also demonstrates that the load shedding events affect the SE results during contingencies and it is not accurate enough to utilize the historical SCADA data for the states tracking.

(a)

(b)

Figure 4.8 Estimated errors of the two schemes: (a) traditional SE, (b) proposed AOM based SE.

The apparent power on line 22-24 is reconstructed based on the SE results from the two schemes. Since power flow calculation plays a crucial role in evaluating system stability, its accuracy should be guaranteed. From Figure 4.9, it can be found that the reconstructed results from the *traditional SE* are underestimated compared with the exact values and this may lead to an undetected over-loaded line. However, the reconstructed values from the *proposed enhanced SE* are much more accurate.

Figure 4.9 Reconstructed load power measurements

Table 4.3 shows the results of the estimated voltage magnitudes of the test system at $t = 40s$. For *PMU-observable* buses (white rows), the estimation results are of the same accuracy for both the traditional and the proposed SE. However, for the *non-PMU observable* buses (grey rows), the results from the traditional SE results deviate more from the true values as compared to those of the proposed enhanced SE. This means that the proposed SE is more accurate.

The impact of the time delays of PMU database operations on the SE has not been addressed specifically in this work. However, it is proposed to set the State Estimator to be executed every 1s. Theoretically speaking, if the communication time delay plus the SE computational time is less than 1s, the SE results can be obtained for the real-time analysis. If the time delay of the PMU database operations is unacceptably large, the real-time analysis will be hard to realize.

TABLE 4.3 RESULTS OF THE VOLTAGE MAGNITUDES OF THE IEEE 30-BUS SYSTEM AT $t = 40s$

Bus No.	True Values	Traditional SE	Proposed SE
1	1.0207	1.0207	1.0207
2	0.9948	0.9948	0.9948
3	0.9932	0.9932	0.9932
4	0.9875	0.9875	0.9875
5	0.9894	0.9894	0.9894
6	0.9879	0.9879	0.9879
7	0.9813	0.9813	0.9813
8	0.9908	0.9908	0.9908
9	1.0132	1.0132	1.0132
10	0.9937	0.9920	0.9938
11	1.0767	1.0767	1.0767
12	1.0132	1.0132	1.0132
13	1.0633	1.0633	1.0633
14	0.9968	0.9976	0.9966
15	0.9913	0.9926	0.9913
16	0.9974	0.9975	0.9975
17	0.9896	0.9891	0.9897
18	0.9797	0.9828	0.9796
19	0.9760	0.9791	0.9758
20	0.9796	0.9821	0.9796
21	0.9867	0.9805	0.9868
22	0.9859	0.9811	0.9860
23	0.9781	0.9794	0.9782
24	0.9690	0.9689	0.9692
25	0.9675	0.9700	0.9676
26	0.9492	0.9566	0.9493
27	0.9755	0.9748	0.9754
28	0.9869	0.9869	0.9869
29	0.9644	0.9593	0.9641
30	0.9625	0.9480	0.9623

4.3.3.2 The IEEE 118-Bus System

The IEEE 118-Bus system [93] is studied in this section and the load power information can be found in Table 4.4. A tripping of the generator at bus 80 is simulated at $t = 1s$. The power imbalance between the supply and the loads will cause the frequency to drop and the UFLS strategies will be implemented

subsequently. The generation deficit can be calculated according to (4.8) with $\frac{df_{COI}}{dt} = -0.83\text{Hz/s}$. For the calculation of the second term in (4.8), the voltage magnitudes before/after the disturbances can be found in Table A-2 and the initial load power for each load bus can be found in Table 4.4. The generation deficit can be obtained as:

$$\begin{aligned}\Delta P_T &= \frac{2 \sum_{m=1}^{N_G} H(m)S(m)}{f_n} \frac{df_{COI}}{dt} + \sum_{k=1}^{N_L} P_{L0,k} \left[\left(\frac{U_k}{U_{0,k}} \right)^{\alpha_k} - 1 \right] \\ &= -415 - 53.8 \approx -470\text{MW}\end{aligned}$$

Thirty load buses are chosen in the load shedding strategies. The first load shedding is conducted when frequency drops to 59.5Hz at time $t = 1.6\text{s}$. Another two load shedding events are conducted when the frequency drops to 59.2Hz at $t = 2.3\text{s}$ and 58.9Hz at $t = 6\text{s}$. The load shedding buses and the shedding amount in each step can be found in Table 4.5.

TABLE 4.4 LOAD INFORMATION OF IEEE 118-BUS SYSTEM (MW)

Bus No.	P	Q	Bus No.	P	Q	Bus No.	P	Q
2	20	9	43	18	7	84	11	7
3	39	10	44	16	8	86	21	10
7	19	2	45	53	22	88	48	10
11	70	23	47	34	0	93	12	7
13	34	16	48	20	11	94	30	16
14	14	1	50	17	4	95	42	31
16	25	10	51	17	8	96	38	15
17	11	3	52	18	5	97	15	9
20	18	3	53	23	11	98	34	8
21	14	8	57	12	3	101	22	15
22	10	5	58	12	3	102	5	3
23	7	3	60	78	3	106	43	16
28	17	7	67	28	7	108	2	1
29	24	4	75	47	11	109	8	3
33	23	9	78	71	26	114	8	3
35	33	9	79	39	32	115	22	7
39	27	11	82	54	27	117	20	8
41	37	10	83	20	10	118	33	15

TABLE 4.5 UFLS STRATEGIES WITH SELECTIVE LOADS FOR IEEE 118-BUS SYSTEM

Bus	Total Shedding	Shedding Amount in Each Step (MW)			Bus	Total Shedding	Shedding Amount in Each Step (MW)		
No.	Amount (MW)	Step 1	Step 2	Step 3	No.	Amount (MW)	Step 1	Step 2	Step 3
2	9	4	3	2	75	21	10	6	4
3	18	8	5	3	78	32	15	9	6
11	31	15	9	6	79	18	8	5	3
13	15	7	4	3	82	24	11	7	5
16	11	5	3	2	86	9	4	3	2
29	11	5	3	2	88	22	10	6	4
33	10	5	3	2	94	13	6	4	3
35	15	7	4	3	95	19	9	5	4
39	12	6	3	2	96	17	8	5	3
41	17	8	5	3	98	15	7	4	3
45	24	11	7	4	101	10	5	3	2
47	15	7	4	3	106	19	9	5	4
48	9	4	3	2	115	10	5	3	2
53	10	5	3	2	118	15	7	4	3
60	35	16	10	7	Total	470	235	141	94
67	13	6	4	2					

In this case study, different situations are simulated to compare the traditional SE and the proposed one. Load shedding is scheduled in the control center once the adaptive UFLS is triggered and the shedding event at each selected bus is independent from each other. As suggested in (4.10), the selected buses for shedding in the IEEE 118-Bus system will follow the binomial distribution as shown in Figure 4.10, given the reliability of the load shedding event at each bus is 0.95. To simulate this stochastic procedure, we randomly generate ten sets of β and each set of β contains thirty binary integers, indicating if the load shedding is conducted on the selected buses. The ten sets of β represent ten different cases of load shedding. According to the values of β , the different load shedding events are implemented in the PowerWorld simulator. If the load shedding event is not conducted on the one or more selected buses as expected, the frequency recover will be affected. This means that the load shedding timings will be triggered earlier and the frequency can only be recovered to a lower value than the nominal.

Figure 4.10 Binomial distribution with $n = 30, p = 0.95$. Probability = $C_k^n 0.95^k 0.05^{n-k}$

For the adaptive UFLS strategies, the thirty load buses will be involved as shown in Table 4.5 and the set of load shedding buses can be obtained as:

$\mathcal{S}\{LoadShedBus\} = \mathcal{S}\{2, 3, 11, 13, 16 \dots 118\}$. The shedding amount of δ_{pi} of each bus at each step can be referred to Table 4.5. The load shedding status of β_i for the thirty buses will be used to indicate if the load shedding is conducted as stated above.

Different PMU placements will be investigated with the aforementioned ten randomly generated load shedding strategies. As indicated in [94], a minimum number of 32 PMUs are needed to make the IEEE 118-Bus system observable without the help of SCADA measurements. We choose 26 PMUs for the testing which is 20% less than the necessary 32 PMUs. Different PMU placements satisfying the observability with 26 units are randomly generated and tested for the two SE schemes.

Figure 4.11 shows the comparison between the two SE schemes regarding the estimation errors. In this comparison, we focus on the time instant of $t = 20s$ after the load shedding events are implemented according to the UFLS schedule. It can be pointed out that the proposed AOM based SE behaves better than the traditional SE as the estimation errors are much smaller. The errors are calculated as the average value of all the simulations. The error of the AOM based estimation results is within 0.5%. It is noted that such results are from the PMU placement only with 80% of the number of PMUs that can make the system fully observable.

Figure 4.11 Estimation errors of the two schemes with a number of 26 PMUs.

4.4 Summary

In this section, the adaptive underfrequency load shedding strategy and frequency information are incorporated into the SE. The work conducted in this chapter falls under the Wide Area Monitoring, Protection and Control (WAMPAC) area for a more smart transmission system. By utilizing the fast-sampled but limited synchronized frequency measurements, the dynamic states during contingencies can be accurately tracked. These frequency measurements are used as the pseudo measurements which are subsequently utilized for the SE purpose. As the resulting SE problem is a mixed-integer linear program which is NP-hard in general, we propose an approximate solution which can be obtained in real time with the alternating optimization method. The results of case studies on both the IEEE 30-Bus and IEEE 118-Bus test systems show that the achieved SE results using the proposed technique are always more accurate than those obtained with a traditional

SE method that is unable to incorporate the load shedding information. In particular, we observe that the improvements are especially noteworthy in the reconstructed voltage magnitudes and power flows selected from the non-PMU observable zones. The operating condition of the power system can thus be better evaluated by adopting the proposed enhanced SE.

Moreover, using the centralized adaptive UFLS strategy to evaluate *all* load powers in the system is only dependent on the frequency value of COI. Therefore the problem of insufficient measurements can be solved. Since the adaptive UFLS strategy has not been widely applied, future work will be focused on the studies of real power systems implemented with the adaptive UFLS strategy and the influence of its reliability on the SE results.

As to the drawbacks of the frequency based load shedding strategies on other parts of the power system, two aspects need to be considered. The first aspect is the loss of loads. Since load shedding is the last resort to recover the frequency back to normal operating conditions during generation deficit, load power has to be curtailed to balance the power difference between the load and the generation. The second aspect is that the accurate calculation of the rate of frequency change and the proper setting of the relays may be hard to achieve [95]. These drawbacks are not the focuses of the work since they are not aimed at designing or improving the current load shedding schemes. Instead the author proposes to utilize the current available load shedding scheme to enhance the performance of SE.

Chapter 5

Decentralized State Estimation for Hybrid AC/DC Microgrid

SE studies on the transmission systems have been extensively conducted. However, for the distribution system, especially for the newly emerging hybrid AC/DC microgrids, the relevant discussion is lacking and/or insufficiently covered. In this chapter, a decentralized SE on the newly emerging hybrid AC/DC microgrid is proposed. The power flow measurements on the connecting line between the two networks are utilized as an equality constraint. SE is formulated for the AC and DC microgrids separately, each considering its own characteristics. By virtue of the phasor measurement units (PMUs), SE in the AC network is formulated in a three-phase linear manner and the unbalanced problem can be investigated. SE in the DC network is also built in a linear manner based on the voltage-current relationship.

In a large scale system, a decentralized technique will certainly reduce the computational time if a good convergence and fast communication mechanism can be guaranteed. For a small scale system, other benefits can be claimed such as better data privacy and a more robust solution.

Case Studies have been conducted to demonstrate the satisfactory performances of

the proposed decentralized SE for the hybrid AC/DC microgrids on accuracy, convergence, bad data detection with the adequate or inadequate measurements.

5.1 Decentralized State Estimation Techniques

Decentralized SE or Multi-area SE (MASE) studies were firstly conducted in the late 1970s, shortly after SE was implemented for power system analysis [96]. Such studies were motivated to reduce the computation time, given the unfavorable computational resources at that time. Another motivation is to process the measurements in each control center, instead of gathering them from various centers across the system for the centralized calculation. Different decentralized SE can be classified based on the following items:

- Area overlapping level
- Computing architecture
- Solution methodology

Area overlapping level of the decentralized SE refers to whether the bordering buses will be overlapped in different areas. References [97] and [98] discuss the decentralized SE with non-overlapping areas and [99] deals with the decentralized SE with overlapping of bordering buses. Computation architecture refers to whether a central coordinator is needed for the decentralized SE. A central coordinator can be used as a platform for information exchange and to tune the SE results obtained from different areas [100]. Decentralized SE can be conducted without a central coordinator as well and the information exchange should be realized among the connecting areas [97] [98] [101]. Solution methodology for the decentralized SE is mainly based on the traditional WLS objective function, using the approach of Decentralized Lagrange, Alternating Direction of Multiplier Method or Dual Decomposition and Penalty Technique.

The basic idea of the decentralized technique is to decompose the original problem into several solvable *subproblems* which then exchange the relevant information through communication links. Most of the existing decomposition techniques can be classified as the primal or the dual decomposition method. The former is based on decomposing the original primal problem, whereas the latter is based on decomposing a Lagrangian dual problem [102]. In this thesis, the author intends to obtain the decentralized solution to the SE problem and the algorithm should enable the AC and DC networks to perform the calculation separately with the minimum information exchanged in between the two networks.

The dual decomposition method can be applied to a generalized problem formulated as:

$$\begin{aligned} & \text{minimize } \sum_i f_i(x_i) \\ & \text{subject to } \sum_i h_i(x_i) \leq c \end{aligned} \quad (5.1)$$

where i denotes the subproblems decomposed from the original objective function; $x_i \in \mathbf{R}^n$ is the optimization variable, f_i is the objective function; h_i is the inequality constraint function and c is a constant.

In a decentralized manner, each subproblem can be solved in the k^{th} iteration by:

$$\text{minimize } f_i(x_i^k) + \lambda_i^k h_i(x_i^k) \quad (5.2)$$

The dual variable of the Lagrange multiplier λ_i^k in the k^{th} iteration is updated based on the *Gradient Method*, as suggested by (5.3).

$$\lambda^{k+1} = \lambda^k + \rho[\sum_i h_i(x_i^k) - c] \quad (5.3)$$

where ρ is a positive step-size. Iteration counts and execution time of the *Dual Decomposition* will be determined when the termination criteria are satisfied.

5.2 Modelling the Hybrid AC/DC Microgrids

Distribution networks become increasingly attractive nowadays as their participation in the entire power system is much more active than before [103]. Compared with the traditional distribution systems, the contemporary systems are regarded as “smart distribution systems” since they are endowed with the following features:

- Installation of the distributed generation [104],
- Implementation of demand response strategies [105] [106], and
- Placement of smart meters [107].

One of the promising network structures in the distribution level is the hybrid AC/DC microgrid [108] and it provides many benefits to both the electrical power suppliers and the consumers [109]. In the hybrid system, microgrids of AC and DC will undertake their respective responsibilities independently. For instance, the growing number of DC loads can be supplied by the DC microgrid and the unnecessary conversion losses from AC to DC will be avoided. Since most home appliances and office utilities are internally driven by DC, substantial losses could be avoided if the DC microgrid is built for them. Besides, the electrical power consumed by the single-phase loads is better supplied by DC than the traditional AC since the unbalanced problem will not occur. For the three-phase loads and AC sources, such as the wind turbines, an AC microgrid will be tasked to sustain the operation. Therefore it is forecasted that such a hybrid AC/DC network with its coordinated control strategies will be a trend in future power systems [108].

SE is an essential module in Energy Management System (EMS) of power systems when it was first proposed by F. C. Schweppe in 1970s [1]. The SE results are mainly utilized for bad measurement/topology data detection and phasor voltage estimation

of each bus in the system. In [110] [111] [112], SE is dealt with multiple DC and AC terminals when the HVDC transmission lines are included in the system. However, the SE in these works is limited to the centralized techniques and no synchrophasor measurements are considered.

5.2.1 DC Microgrid

SE in the DC network is different from that of the traditional AC network. The state variables are merely composed of the bus voltage magnitudes and the formulation of SE can be built in a simple linear current-voltage relationship.

Denote the state variable set for the DC microgrid as $\mathbf{x}_{DC}$ and it can be written as:

$$\mathbf{x}_{DC} = \mathbf{x}_{V_{DC}} \quad (5.4)$$

where $\mathbf{x}_{V_{DC}}$ is the voltage magnitude of each bus in the DC network. The measurement set for the DC microgrid can be denoted as $\mathbf{z}_{DC}$ and it includes the measurement of voltage magnitudes, line currents and current injections:

$$\mathbf{z}_{DC} = [\mathbf{V}_{DC}; \mathbf{I}_{DC}^{fl}; \mathbf{I}_{DC}^{inj}] \quad (5.5)$$

With a linear formulation, the measurement equation can be obtained as:

$$\mathbf{z}_{DC} = \mathbf{H}_{DC}\mathbf{x}_{DC} + \mathbf{e}_{DC} \quad (5.6)$$

For the voltage measurement of bus p , the measurement function can be written as:

$$\mathbf{V}_{DC}(p) = \mathbf{x}_{DC}(p) + e \quad (5.7)$$

For the line current measurement on line p, q and the current injection of bus p , the measurement function can be written as:

$$\mathbf{I}_{DC}^{fl}(p, q) = -\mathbf{G}(p, q)[\mathbf{x}_{DC}(p) - \mathbf{x}_{DC}(q)] + e \quad (5.8)$$

$$\mathbf{I}_{DC}^{inj}(p) = \sum \mathbf{G}(p, q)\mathbf{x}_{DC}(q) + e \quad (5.9)$$

where $\mathbf{G}$ is the conductance matrix.

5.2.2 AC Microgrid

For the AC microgrid, three-phase State Estimator will be built to consider the unbalanced situation. The measurements obtained for each phase are denoted as:

$\mathbf{z}_{AC}^A$, $\mathbf{z}_{AC}^B$ and $\mathbf{z}_{AC}^C$ which can be expanded as:

$$[\mathbf{z}_{AC}^A; \mathbf{z}_{AC}^B; \mathbf{z}_{AC}^C] = [\mathbf{V}_{AC}^A; \mathbf{I}_{AC}^A; \mathbf{V}_{AC}^B; \mathbf{I}_{AC}^B; \mathbf{V}_{AC}^C; \mathbf{I}_{AC}^C] \quad (5.10)$$

For each phase, the measurement set is composed of phasor voltages, $\mathbf{V}_{AC}$, and phasor current injection and current flow, $\mathbf{I}_{AC}^{inj}$, $\mathbf{I}_{AC}^{fl}$. Preceding to the state estimation, the phase domain components (A, B, C) should be decoupled to the modal domain (Positive, Negative, Zero) via the following transformation:

$$\begin{bmatrix} \mathbf{z}_{AC}^+ \\ \mathbf{z}_{AC}^- \\ \mathbf{z}_{AC}^0 \end{bmatrix} = \mathbf{T} \begin{bmatrix} \mathbf{z}_{AC}^A \\ \mathbf{z}_{AC}^B \\ \mathbf{z}_{AC}^C \end{bmatrix} \quad (5.11)$$

where $\mathbf{T}$ is the transformation matrix and it can be written as:

$$\mathbf{T} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \quad (5.12)$$

where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$.

After the measurements are transformed into the modal domain, SE will be conducted in the decoupled positive, negative and zero network respectively. The state variable in each network can be denoted as:

$$\mathbf{x}_{AC} = [\mathbf{x}_{AC}^+; \mathbf{x}_{AC}^-; \mathbf{x}_{AC}^0] \quad (5.13)$$

where

$$\begin{aligned} \mathbf{x}_{AC}^+ &= [\mathbf{x}_{V_{real}^+}; \mathbf{x}_{V_{imag}^+}]^T \\ \mathbf{x}_{AC}^- &= [\mathbf{x}_{V_{real}^-}; \mathbf{x}_{V_{imag}^-}]^T \\ \mathbf{x}_{AC}^0 &= [\mathbf{x}_{V_{real}^0}; \mathbf{x}_{V_{imag}^0}]^T \end{aligned} \quad (5.14)$$

For the positive network, the measurement set could be formed as:

$$\mathbf{z}_{AC}^+ = [\mathbf{V}^+; \mathbf{I}_{AC}^{inj+}; \mathbf{I}_{AC}^{fl+}] = \mathbf{H}_{AC} \mathbf{x}_{AC} + \mathbf{e}_{AC}$$

and the measurement function can be experienced in the rectangular form as:

$$\mathbf{V}^+ = \mathbf{x}_{V_{real}^+} + j\mathbf{x}_{V_{imag}^+} + \mathbf{e} \quad (5.15)$$

$$\begin{aligned} \mathbf{I}_{AC}^{inj+}(p) = & \sum [\mathbf{G}^+(p, q)\mathbf{x}_{V_{real}^+}(q) - \mathbf{B}^+(p, q)\mathbf{x}_{V_{imag}^+}(q)] \\ & + j \sum [\mathbf{G}^+(p, q)\mathbf{x}_{V_{imag}^+}(q) + \mathbf{B}^+(p, q)\mathbf{x}_{V_{real}^+}(q)] + \mathbf{e} \end{aligned} \quad (5.16)$$

$$\begin{aligned} \mathbf{I}_{AC}^{fl+}(p, q) = & -\mathbf{G}^+(p, q) [\mathbf{x}_{V_{real}^+}(p) - \mathbf{x}_{V_{real}^+}(q)] + \mathbf{B}^+(p, q) [\mathbf{x}_{V_{imag}^+}(p) - \mathbf{x}_{V_{imag}^+}(q)] \\ & -j \{ \mathbf{B}^+(p, q) [\mathbf{x}_{V_{real}^+}(p) - \mathbf{x}_{V_{real}^+}(q)] + \mathbf{G}^+(p, q) [\mathbf{x}_{V_{imag}^+}(p) - \mathbf{x}_{V_{imag}^+}(q)] \} + \mathbf{e} \end{aligned} \quad (5.17)$$

where $\mathbf{G}^+ + j\mathbf{B}^+$ is the admittance matrix of the positive network.

5.2.3 Interlinking Converter

The interlinking converter (IC) plays an important role in the operation of the AC/DC microgrid as power transmission is achieved through it [113]. Moreover a suitable control strategy to coordinate the two microgrids is crucial. Regardless of the control strategies adopted, a basic modelling the IC with regards to the two microgrids is based on equal active power, namely:

$$P_{AC} = P_{DC} \quad (5.18)$$

5.2.4 Problem Formulation

SE for the hybrid AC/DC microgrids can be formulated as:

$$\begin{aligned} & \text{minimize } J_{DC}(\mathbf{x}_{DC}) + J_{AC}(\mathbf{x}_{AC}) \\ & \text{subject to } P_{DC} + \eta P_{AC} = 0 \end{aligned} \quad (5.19)$$

where $\mathbf{x}_{DC}$ and $\mathbf{x}_{AC}$ are the state variables of the DC and AC microgrids respectively. J_{DC} and J_{AC} are the objective function of each microgrid and can be

expanded as:

$$J_{DC}(\mathbf{x}_{DC}) = (\mathbf{z}_{DC} - \mathbf{H}_{DC}\mathbf{x}_{DC})^T \mathbf{W}_{DC} (\mathbf{z}_{DC} - \mathbf{H}_{DC}\mathbf{x}_{DC}) \quad (5.20)$$

$$J_{AC}(\mathbf{x}_{AC}) = \sum_{m=+, -, 0} (\mathbf{z}_{AC}^m - \mathbf{H}_{AC}^m \mathbf{x}_{AC}^m)^T \mathbf{W}_{AC} (\mathbf{z}_{AC}^m - \mathbf{H}_{AC}^m \mathbf{x}_{AC}^m) \quad (5.21)$$

where $\mathbf{z}_{DC}$ and $\mathbf{z}_{AC}^m$ are the measurement vectors for DC and AC microgrid. m denotes the three independent sequence networks of the AC microgrid, namely the positive-, negative- and zero-sequence networks. $\mathbf{W}_{DC}$ and $\mathbf{W}_{AC}$ are the weight matrices of the measurements from the DC and AC microgrids which are the inverse of the measurement covariance matrices. $\mathbf{H}_{DC}$ and $\mathbf{H}_{AC}^m$ are the Jacobian matrices derived from the measurement function. With phasor measurements, a linear measurement function can be obtained for both the DC and AC microgrids and it will be unnecessary to differentiate the measurement function to linearize $\mathbf{H}_{DC}$ and $\mathbf{H}_{AC}^m$ accordingly [103]. To achieve a linear formulation, phasor voltage and current measurements obtained in polar coordinates should be transferred into rectangular ones and the relevant measurement covariance should be calculated via the transformation technique reported in [35] or [36]. In (5.19), P_{DC} represents the active power transmitted from DC to AC and P_{AC} represents the reverse. The transferring active powers of P_{DC} and P_{AC} will be constrained as suggested in (5.19). Assume that the transmission efficiency is η and

$$\eta = \begin{cases} \leq 1 & \text{the active power is transferred from AC to DC} \\ \geq 1 & \text{otherwise} \end{cases}$$

Power measurements in the original three-phase AC microgrid and three-sequence networks can be related as shown in (5.22) and illustration of the equation can be found in Appendix B:

$$P_{AC} = 3 \sum_{m=+, -, 0} P_{AC}^m \quad (5.22)$$

Assume that bus p in the DC microgrid and bus q in the AC microgrid are connected. A detailed description of the two power injection measurements can be obtained by:

$$P_{DC}(p) = \mathbf{x}_{DC}(p)\mathbf{G}(p, :)\mathbf{x}_{DC} - \mathbf{P}_{Gen_{DC}}(p) + e \quad (5.23.1)$$

$$P_{AC}^+(q) = [\mathbf{x}_{V_{real}^+}(q)\mathbf{G}^+(q, :) - \mathbf{x}_{V_{imag}^+}(q)\mathbf{B}^+(q, :), \\ -\mathbf{x}_{V_{real}^+}(q)\mathbf{B}^+(q, :) - \mathbf{x}_{V_{imag}^+}(q)\mathbf{G}^+(q, :)] \mathbf{x}_{AC}^+ - \mathbf{P}_{Gen_{AC}}(q) + e \quad (5.23.2)$$

where $\mathbf{P}_{Gen_{DC}}$ and $\mathbf{P}_{Gen_{AC}}$ are the active generation power in the DC and AC microgrids, if any. The active powers in negative- and zero-sequence network, $P_{AC}^-(q)$ and $P_{AC}^0(q)$, can be obtained in a similar way. e denotes the measurement error.

5.3 Solution Techniques

5.3.1 Relax the Quadratically Constrained Quadratic Problem (QCQP)

The equality constraint in (5.23.1) and (5.23.2) of the presented SE problem is quadratic in terms of the state variables and they can be written in a general form as:

$$P_{DC}(p) = \mathbf{x}_{DC}^T \mathbf{A}_{\lambda D} \mathbf{x}_{DC} - c_D \quad (5.24)$$

$$P_{AC}^+(q) = \mathbf{x}_{AC}^{+T} \mathbf{A}_{\lambda A} \mathbf{x}_{AC}^+ - c_A \quad (5.25)$$

where $\mathbf{A}_{\lambda D}$ and $\mathbf{A}_{\lambda A}$ are constant matrices for the quadratic term, and c_D and c_A are the constants which are given as $c_D = \mathbf{P}_{Gen_{DC}}(p)$ and $c_A = \mathbf{P}_{Gen_{AC}}(q)$. The decentralized SE for the hybrid AC/DC microgrids therefore can be induced as a QCQP. Reference [114] introduces several techniques to relax the QCQP as such a problem is hard to solve. By using the *Lagrangian Relaxation* method, the objective function of the hybrid SE problem formulated in (5.19) can be transformed into:

$$\text{minimize } L_p = J_{DC}(\mathbf{x}_{DC}) + J_{AC}(\mathbf{x}_{AC}) + \lambda(P_{DC} + \eta P_{AC}) \quad (5.26)$$

The formulation of the *Dual* function of the relaxed QCQP and the discussion of the duality have also been presented in [114] for interested readers.

5.3.2 Distributed Algorithm

As discussed above, the proposed SE for the hybrid AC/DC microgrids is a QCQP and will be relaxed into a Lagrangian form with the help of dual variables. The *Dual Decomposition* method can then be applied to find the optimal values of the primal and dual variables. The proposed algorithm of the decentralized state estimation via *Dual Decomposition* for the hybrid AC/DC microgrids can be summarized in **Distributed Algorithm**.

Distributed Algorithm

1. Relaxed the original problem:

$$L_\rho = J_{\text{DC}}(\mathbf{x}_{\text{DC}}) + J_{\text{AC}}(\mathbf{x}_{\text{AC}}) + \lambda(P_{\text{DC}} + \eta P_{\text{AC}})$$
 2. Initialize $\mathbf{x}_{\text{DC}}$, $\mathbf{x}_{\text{AC}}$, λ and ρ .
 3. Calculate residual: $r = P_{\text{DC}} + \eta P_{\text{AC}}$.
 4. $k \rightarrow 0$
 5. $\mathbf{x}_{\text{DC}}^{k+1} := \underset{\mathbf{x}_{\text{DC}}}{\operatorname{argmin}} L_\rho(\mathbf{x}_{\text{DC}}, \mathbf{x}_{\text{AC}}^k, \lambda^k)$
 6. $\mathbf{x}_{\text{AC}}^{k+1} := \underset{\mathbf{x}_{\text{AC}}}{\operatorname{argmin}} L_\rho(\mathbf{x}_{\text{DC}}^{k+1}, \mathbf{x}_{\text{AC}}, \lambda^k)$
 7. $\lambda^{k+1} := \lambda^k + \rho(P_{\text{DC}} + \eta P_{\text{AC}})$
 8. $k + 1 \rightarrow k$ and go to step 5 $\rightarrow$ 7 (until satisfying the termination criteria).
-

where $\operatorname{argmin} L_\rho$ denotes any minimizer of the function of L_ρ .

A few comments on **Distributed Algorithm** can be made here. Firstly, problems formulated in Step 6 and Step 7 are classical quadratic optimization problems and their solutions can be obtained by many existing solvers that are commercially available. In this paper, the *quadprog()* from MATLAB is utilized. The quadratic formulation of L_ρ can be found in details in Appendix C. Secondly, the optimal solutions of the hybrid microgrids are achieved in an iterative way and the iteration counter is denoted as k . Thirdly, the roles of $\mathbf{x}_{\text{DC}}$ and $\mathbf{x}_{\text{AC}}$ are almost symmetric [115], namely, Step 5 and Step 6 in **Distributed Algorithm** can be reversed via the *sequential* process as shown in Figure 5.1. Lastly, information exchange in the decentralized technique will include the following: 1) Power injection estimates of

the two bordering buses; 2) Lagrangian multiplier of λ ; and 3) Power measurements transmitted on the connecting line.

Figure 5.1 Sequential process for decentralized technique via *Dual Decomposition*.

5.3.3 Termination and Convergence

It can be proved that the dual variable of λ will converge to the dual optimal λ^* and the state variables of $\mathbf{x}_{DC}$ and $\mathbf{x}_{AC}$ will converge to the optimal $\mathbf{x}_{DC}^*$ and $\mathbf{x}_{AC}^*$ when the iteration time $k \rightarrow \infty$ [102]. Conventionally, the termination criteria can be made on the residual, r , of the consensus constraint as:

$$r = P_{DC} + \eta P_{AC} \leq r_{th} \quad (5.27)$$

where r_{th} is a predefined value.

Besides, since the original problem is non-convex quadratic constrained and it can only be solved in a decentralized manner via the Lagrangian relaxation method. The duality of the Lagrangian relaxation determines the solution gap from the original problem and the discussion can be found in [114].

5.3.4 AC Microgrid: Transforming to Phase Values

For the AC microgrid side, a three-phase state estimator will be built to consider the unbalanced situation [116]. After the state variables from the three sequence networks are obtained, the values of the original three phases of A, B and C can be obtained as:

$$\begin{bmatrix} \mathbf{x}_{AC}^A \\ \mathbf{x}_{AC}^B \\ \mathbf{x}_{AC}^C \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} \mathbf{x}_{AC}^+ \\ \mathbf{x}_{AC}^- \\ \mathbf{x}_{AC}^0 \end{bmatrix} \quad (5.28)$$

where $\mathbf{T}$ is the transformation matrix and it can be written as:

$$\mathbf{T} = \frac{1}{3} \begin{bmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{bmatrix} \quad (5.29)$$

where $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$. The above transformation is only valid for fully transposed transmission lines in AC system. For phasor current measurements on the untransposed lines, it is advised to use the technique introduced in [116].

5.4 Centralized Hybrid AC/DC State Estimation

The centralized formulation will be introduced in this section and it will be used as a comparison with the performances of the decentralized technique as presented above. Based on the equal active power constraint in (5.19), a pseudo measurement can be introduced in the centralized calculation [110]. Assume that the connecting buses in DC and AC microgrids are bus p and bus q respectively. The following pseudo measurement, $\mathbf{z}_{pseu}$, can be constructed:

$$\mathbf{z}_{pseu} = P_{DC}(p) + \eta P_{AC}(q) = 0 \quad (5.30)$$

The centralized formulation will be obtained as:

$$\min J = (\mathbf{z}_{cen} - \mathbf{h}_{cen}(\mathbf{x}_{cen}))^T \mathbf{W} (\mathbf{z}_{cen} - \mathbf{h}_{cen}(\mathbf{x}_{cen})) \quad (5.31)$$

where the measurement set is $\mathbf{z}_{cen}$ and $\mathbf{z}_{cen} = [\mathbf{z}_{DC}, \mathbf{z}_{AC}^+, \mathbf{z}_{pseu}]^T$; the state variables is $\mathbf{x}_{cen}$ and $\mathbf{x}_{cen} = [\mathbf{x}_{DC}, \mathbf{x}_{AC}^+]^T$; $\mathbf{h}_{cen}$ is the measurement function and $\mathbf{W}$ is the weight matrix, inverse to the measurement covariance.

The solution of (5.31) can be achieved by using many available quadratic

optimization solvers including the standard methods via KKT conditions [110]. A linearized Jacobian matrix, $\mathbf{H}_{\text{cen}}$, will be needed in each iteration and it can be obtained by taking the first derivative of the measurement function as

$$\mathbf{H}_{\text{cen}} = \begin{bmatrix} \mathbf{H}_{\text{DC}} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{\text{AC}}^+ \\ \frac{\partial \mathbf{z}_{\text{pseu}}}{\partial \mathbf{x}_{\text{DC}}} & \frac{\partial \mathbf{z}_{\text{pseu}}}{\partial \mathbf{x}_{\text{AC}}} \end{bmatrix} \quad (5.32)$$

5.5 Case Studies

5.5.1 System Description

A simple hybrid AC/DC microgrid is designed and used for the case studies in this work. As shown in Figure 5.2, there are five buses in each microgrid. Bus 1 from the DC microgrid is connected to bus 5 in the AC microgrid through an interlinking converter, represented by a switch in the figure. Line resistance and reactance values of the two microgrids are set according to [117] and they can be found in Table 5.1 and Table 5.2. Due to the short line length in the distribution systems, line shunts are ignored in this work [118].

Figure 5.2 A simple hybrid AC/DC microgrid.

TABLE 5.1 LINE PARAMETER IN DC MICROGRID

From Bus	To Bus	R (p.u.)
1	3	0.45
1	2	0.15
3	5	0.25
2	4	0.25

TABLE 5.2 LINE PARAMETER IN AC MICROGRID

From Bus	To Bus	R (p.u.)	X (p.u.)
3	5	0.138	0.140
2	4	0.138	0.140
1	2	0.277	0.280
1	3	0.276	0.280
2	3	0.554	0.552

Load and generation information in the hybrid microgrids can be found in Table 5.3. In the DC microgrid, bus 1 is the bordering bus to be connected with the AC microgrid. It is assumed that 3MW is transferred from AC to DC. If the transmission efficiency is assumed as 1, i.e. $\eta = 1$, the generation power will be 3MW for bus 1 in the DC microgrid. In the AC microgrid, the loading information (single phase), both active and reactive power, can be found as listed next to the DC microgrid in Table 5.3. Bus 5 in the AC microgrid is the bordering bus to supply the active power to the DC microgrid. Therefore for bus 5 in the AC microgrid, the load power can be found as $1\text{MW} + j0\text{MVar}$.

TABLE 5.3 LOAD CONDITION IN DC AND AC MICROGRID

Bus No.	DC		AC	
	Load	Gen.	Load	Gen.
	(MW)	(MW)	(MW+jMvar)	(MW+jMvar)
1	0	3	$6 + j1$	0
2	2	5	$7 + j2$	0
3	1	4	$5 + j2$	0
4	4	0	$5 + j1$	0
5	4	1	$1 + j0$	$26 + j8$

True values of the bus voltage magnitudes/phase angles in the hybrid AC/DC microgrid are achieved using PowerWorld Simulator V17 and MATLAB together.

Figure 5.3 AC microgrid constructed via PowerWorld simulator

5.5.2 Measurements

Bus voltage magnitudes and phase angles in the hybrid AC/DC microgrid are obtained using PowerWorld Simulator V17 and MATLAB which are solved simultaneously. All measurements are built using the true values plus the random errors of three standard deviations ($\pm 3\sigma$). Enough PMUs are assumed to be installed in the AC microgrid to guarantee the observability and sufficient measurements are assigned to the DC microgrid as well. For the following case studies, the measurement information will be utilized:

DC Microgrid: **Current flows** on line 1-3, line 1-2, line 3-5, line 2-4; **voltage magnitudes** on bus 1-5; **current injections** on bus 1.

AC Microgrid: **Phasor Current flows** on line 1-2, line 1-3, line 2-4, line 2-3, line 2-1 and line 5-3; **phasor voltages** on bus 1, bus 2 and bus 5; **phasor current injection** on bus 5.

On top of the *local measurements* in each microgrid, relevant information from the other microgrid will be considered as *exchange measurements*. As stated above, the exchange information between the two systems is the power transmitted on the connecting line. For the DC microgrid, the measurement *sent out* is the power

injection at bus 1 and the exchange measurement *received* is the power injection from bus 5 of the AC microgrid, both in active and reactive. The reverse situation is for the AC microgrid. All the measurements will be transferred into currents before included into the SE calculation for a linear formulation. For the centralized SE, one more measurement of the *pseudo measurement* will be included as shown in (5.30). A summary of the number of measurements can be found in Table 5.4.

TABLE 5.4 SUMMARY OF THE NUMBER OF MEASUREMENTS

		Dec. SE	Cen. SE
Local Measurements	DC	10	10
	AC	20	20
Exchange Measurements		3	-
Pseudo Measurements		-	1

5.5.3 Simulation and Results

In this section, case studies will be conducted to demonstrate the validity of the decentralized SE on the hybrid AC/DC microgrids. The accuracy, convergence and the gap with the centralized solution will be evaluated when the measurements are sufficient. Bad data detection is conducted in this section as well. When measurements are insufficient for one of the two microgrids, the centralized SE is only solvable when enough pseudo measurements are added. For the decentralized SE, however, it can be found that the SE solution of the observable island will not be affected by the other unobservable island and can be obtained via the decentralized technique directly.

5.5.3.1 On Accuracy

As indicated above, measurements utilized in this case study are sufficient and the errors within $\pm 3\sigma$ are added. Using **Distributed Algorithm** in Section 5.2.3.2, the decentralized SE results can be shown in Table 5.5. The results are based on the

average values from 100 times of Monte Carlo simulations. From Table 5.5, the maximum estimation error can be calculated around 0.4% in voltage magnitude and 0.9% in phase angle when compared with the true values, as shown in Table 5.5. The good accuracy of the decentralized SE can be therefore demonstrated.

TABLE 5.5 SE RESULTS FROM THE DECENTRALIZED FORMULATION

Bus No.	$ V _{true}$	$ V $	$\theta_{true}(^\circ)$	$\theta(^\circ)$
DC				
1	1.0500	1.0496	-	-
2	1.0501	1.0498	-	-
3	1.0370	1.0368	-	-
4	1.0404	1.0401	-	-
5	1.0295	1.0293	-	-
AC Phase-A				
1	0.9700	0.9692	-2.67	-2.57
2	0.9500	0.9516	-3.24	-3.06
3	1.0100	1.0074	-1.30	-1.31
4	0.9400	0.9422	-3.60	-3.37
5	1.0500	1.0506	0.00	0.00
AC Phase-B				
1	0.9700	0.9692	-122.67	-122.50
2	0.9500	0.9515	-123.24	-123.13
3	1.0100	1.0073	-121.30	-121.04
4	0.9400	0.9424	-123.60	-123.56
5	1.0500	1.0504	-120.00	-119.71
AC Phase-C				
1	0.9700	0.9691	117.33	117.45
2	0.9500	0.9513	116.76	116.88
3	1.0100	1.0074	118.70	118.74
4	0.9400	0.9420	116.40	116.51
5	1.0500	1.0507	120.00	120.05

Next, the decentralized SE results are compared with those from the centralized one. The gap between the two SE solutions can be calculated by using the indices in (5.33) – (5.35). The comparison results are shown in Table 5.6 and it can be observed that the gap is very small, which demonstrates that the comparability of the decentralized

SE to the centralized one.

$$\mathbf{gap}_{|V_{DC}|} = \sqrt{\sum_{n=1}^{100} \sum_{m=1}^{N_{DC}} [|V_{DC}|_{dec}(m, n) - |V_{DC}|_{cen}(m, n)]^2 \frac{1}{100 \times N_{DC}}} \quad (5.33)$$

$$\mathbf{gap}_{|V_{AC}|} = \sqrt{\sum_{n=1}^{100} \sum_{m=1}^{N_{AC}} [|V_{AC}|_{dec}(m, n) - |V_{AC}|_{cen}(m, n)]^2 \frac{1}{100 \times N_{AC}}} \quad (5.34)$$

$$\mathbf{gap}_{\theta} = \sqrt{\sum_{n=1}^{100} \sum_{m=1}^{N_{AC}} [\theta_{dec}(m, n) - \theta_{cen}(m, n)]^2 \frac{1}{100 \times N_{AC}}} \quad (5.35)$$

where N_{DC} and N_{AC} are the number of state variables in the DC and AC microgrids respectively.

TABLE 5.6 COMPARISON BETWEEN CENTRALIZED AND DECENTRALIZED FORMULATION

Results	
$\mathbf{gap}_{ V_{DC} }$	4.9e-4
$\mathbf{gap}_{ V_{AC} }$	6.6e-4
$\mathbf{gap}_{\theta}$	0.001 rad

5.5.3.2 On Convergence

Convergence of the decentralized SE is evaluated in this section. An approximate 5 iterations can make the residual of the equality constraint of (5.27) less than 0.1%. Here, one iteration denotes one information exchange between the two decentralized SE modules in the AC and DC microgrids.

5.5.3.3 On Bad Data Detection

Bad data detection is conducted in this section. Firstly, we consider a situation when *interacting and conforming* measurement errors [7] are found in the DC microgrid. We choose the measurement of I_{13} , I_{12} and V_1 in the DC microgrid as an example. The relationships between these measurements are:

$$V_1 - V_3 = I_{13} \times R_{13} \quad (5.36)$$

$$V_1 - V_2 = I_{12} \times R_{12} \quad (5.37)$$

where R_{13} and R_{12} are the resistances on line 1-3 and line 1-2, which can be found in Table 5.1.

The correct measurement values of (5.36) and (5.37) can be found in the second column in Table 5.7. An interacting and conforming measurement error may apply in the way as shown in the third column in the same table. Since the biased measurements in the DC microgrid appear in a conforming way, it is impossible to identify the bad data merely using the measurements from the DC microgrid. However, in the proposed decentralized SE, bad data detection is conducted successfully by exchanging the information with the neighboring AC microgrid. Table 5.7 shows the detection results by the *Largest Normalized Residual* (LNR) method [7]. The details are suggested from the fourth column to the seventh column in Table 5.7. After three times of elimination of the largest normalized residuals, no more measurements are identified as the bad data as shown in the last column.

TABLE 5.7 NORMALIZED RESIDUAL RESULTS FOR BAD DATA DETECTION IN THE DECENTRALIZED SE WITH INTERACTIVE AND CONFORMING ERRORS

	Cor. Meas.	Act. Meas.	Normalized Residuals			
			1 st	2 nd	3 rd	4 th
$I_{13}^{(*)}$	0.0289	1.1918	1306.6	259.2	-	-
$I_{12}^{(*)}$	-0.0007	3.4767	1623.8	-	-	-
I_{35}	0.0500	0.0502	208.8	107.9	35.0	1.03
I_{24}	0.0388	0.0390	73.6	194.4	94.5	0.6
$V_1^{(*)}$	1.0500	1.5798	262.1	176.8	228.4	-
V_2	1.0501	1.0553	75.4	206.5	98.9	0.6
V_3	1.0370	1.0384	210.4	111.2	34.7	1.04
V_4	1.0404	1.0399	73.6	194.4	94.5	0.6
V_5	1.0295	1.0239	208.8	107.9	35.0	1.03
I_1	0.0282	0.0283	398.5	19.3	8.0	0.42
I_{AC-DC}	0.0293	0.0272	398.8	19.6	6.2	0.27

5.5.3.4 On Measurement Deficiency

In this section, the author intends to investigate a situation when measurements of one microgrid are lost or unavailable. Under such circumstances, we expect that the SE solutions of the microgrid with sufficient measurements will not be affected and they can be obtained normally. As suggested in Table 5.8, measurements of the AC

microgrid are assumed unavailable and the numerical analysis of the centralized SE can be found in the table.

TABLE 5.8 NUMERICAL PROOF OF CENTRALIZED STATE ESTIMATOR (AC MICROGRID

MEASUREMENTS LOSS)	
	Value
No. of state variables	14
No. of measurements	$10 + 1 = 11$
Rank of H_{cen}	6
Inverse of Gain Matrix of $(H_{cen}^T W H_{cen})$	N.A.

From Table 5.8, it can be found that when AC measurements are missing, the total measurements of the hybrid system will be reduced to 11, out of which 10 measurements are from the DC microgrid and 1 measurement is the pseudo measurement in the centralized SE as indicated by (5.30). The number of total state variables will be 14 where 5 states variables are DC bus voltage magnitudes, 5 are AC bus voltage magnitudes and the other 4 are phase angles of AC bus voltages considering the phase angle of the slack bus in the AC microgrid as zero. The rank of the Jacobian matrix, H_{cen} , will reduce to 6 and it will cause problems when taking the inverse of the Gain matrix of $H_{cen}^T W H_{cen}$ due to singularity. This means that SE results of the DC microgrid can be obtained *only* when enough pseudo measurements are added in the AC microgrid.

In the decentralized SE, since the measurements in the DC microgrid are still available, SE results of the DC microgrid can be directly obtained under such circumstances although SE of the AC microgrid cannot be fully observed. By using the **Distributed Algorithm**, the results from the decentralized SE can be achieved in Table 5.9 with the true values listed next to them.

TABLE 5.9 DECENTRALIZED SE RESULTS OF THE DC MICROGRID WITH LOSS OF AC MICROGRID

MEASUREMENTS		
Bus No.	$ V _{true}$	$ V _{Dec}$
1	1.0500	1.0506
2	1.0501	1.0501
3	1.0370	1.0363
4	1.0404	1.0401
5	1.0295	1.0289

A similar case study can be conducted when the measurements of the DC microgrid are lost. Likewise, for the centralized SE, the following numerical analysis can be found in Table 5.10. The total number of state variables is still 14 as stated above. After the measurements from the DC microgrid are lost, the total number of available measurements becomes 21, where 20 measurements are from the AC microgrid and 1 measurement is the pseudo measurement. The rank of the Jacobian matrix increases to 11, compared with that from Table 5.8. However it still causes problems when taking the inverse of the Gain matrix due to singularity. As a consequence, the centralized SE results can only be obtained when pseudo measurements are added to make the system observable.

TABLE 5.10 NUMERICAL PROOF OF CENTRALIZED STATE ESTIMATOR (DC MICROGRID

MEASUREMENTS LOSS)	
	Value
No. of state variables	14
No. of measurements	$20 + 1 = 21$
Rank of H_{cen}	11
Inverse of Gain Matrix of $(H_{cen}^T W H_{cen})$	N.A.

In contrast, results of the AC microgrid from the decentralized SE can be directly obtained as shown in Table 5.11 with the true values listed next to them.

TABLE 5.11 DECENTRALIZED SE RESULTS OF THE AC MICROGRID WITH LOSS OF DC MICROGRID

MEASUREMENTS				
Bus No.	$ V _{true}$	$ V _{cen}$	$\theta_{true}(^\circ)$	$\theta_{cen}(^\circ)$
<i>Phase A</i>				
1	0.9700	0.9647	-2.67	-2.65
2	0.9500	0.9466	-3.24	-3.20
3	1.0100	1.0041	-1.30	-1.27
4	0.9400	0.9371	-3.60	-3.61
5	1.0500	1.0460	0.00	0.00
<i>Phase B</i>				
1	0.9700	0.9663	-122.67	-122.71
2	0.9500	0.9483	-123.24	-123.30
3	1.0100	1.0043	-121.30	-121.31
4	0.9400	0.9398	-123.60	-123.65
5	1.0500	1.0495	-120.00	-120.07
<i>Phase C</i>				
1	0.9700	0.9668	117.33	117.31
2	0.9500	0.9497	116.76	116.74
3	1.0100	1.0048	118.70	118.67
4	0.9400	0.9398	116.40	116.41
5	1.0500	1.0484	120.00	120.10

5.6 Summary

In this chapter, a decentralized SE is proposed for the newly emerging network of the hybrid AC/DC microgrids. A *Dual Decomposition* technique based on *Lagrangian Relaxation* and *Gradient Method* is applied to obtain the decentralized solution. SE of the AC and DC microgrids are formulated separately based on their own characteristics. The proposed decentralized SE is fully distributed without any central coordinator and the exchange information is limited which alleviates the communication burden. Moreover, compared with the centralized SE, the decentralized SE method has the advantages of avoiding the use of the entire measurement information of the microgrids and being scalable in computation. Case studies have been carried out on a simple hybrid AC/DC microgrid and the following

features can be achieved from the simulation results:

- 1) The SE is fully decentralized with no central coordinator. Information exchange will be reduced to the power injection estimates of the two bordering buses, Lagrange multiplier of λ and the active power measurements transmitted.
- 2) A good convergence and fast computation speed can be proven from the case studies.
- 3) By calculating the gap between the estimated values, it can be found that the decentralized SE results approximate the centralized SE. When the measurements of one out of the two microgrids are lost, the decentralized SE can still be directly utilized to obtain the partial results.
- 4) Phasor measurements are given a full consideration when formulating the SE problem in this thesis.

Chapter 6

Conclusion and Recommendations

6.1 Conclusion

In this section, the significance, objectives and outcomes of the thesis are summarized. This thesis focuses on power system SE for smart power grids by incorporating the PMUs into the calculation.

Synchrophasor measurements from PMUs are firstly studied. As introduced, SE is a measurement-related module which is to determine the system states based on the available measurements. Since synchrophasor measurements are different from the traditional measurements in many aspects, SE based on the synchrophasor measurements will be distinct from the conventional one. The opportunities and challenges of using the synchrophasor measurements in power system SE are reviewed in Chapter 2.

Next, studies on the optimal PMU placement are conducted. The motivation to conduct this study is the relatively high cost of PMUs nowadays. It is expected to utilize the minimum number of units to achieve the system observability. In Chapter 3, the background and the state-of-art techniques of the OPP studies are introduced. Improvements on the existing OPP strategies in terms of the total measurement

redundancy and the redundancy distribution level among all buses are proposed. Case studies are conducted on IEEE 14-, 30-, 57- and 118-Bus test systems. The results are as accurate as those of classical techniques, however the proposed technique is more advantageous as it can be applied to solve large scale systems while the existing techniques are invalid due to time-out.

Work reported in Chapter 4 is intended to study the SE of the smart transmission systems which is the continuation of PMU application on smart power grids. The works done in this chapter fall under the research topic of Wide Area Monitoring, Protection and Control (WAMPC). Different from the previous studies on the SE with inadequate synchrophasor measurements from PMUs as introduced in Section 2.3, the adaptive Underfrequency Load Shedding (UFLS) is considered in the scope of smart transmission systems. During contingencies, it is proposed to utilize the adaptive UFLS information, if any, to update the SCADA measurements and thus to improve the SE results.

Considering that it is hard to know whether the adaptive UFLS events are successfully conducted, the auxiliary binary variable set is introduced into the SE algorithm. The load shedding status indicated by the auxiliary binary variables of β are estimated as well as the traditional state variables of phasor voltages. If the load shedding status is estimated as “successfully conducted”, the load shedding amount will be reduced from the relevant SCADA measurements accordingly. Otherwise, the historical SCADA measurements will remain as unchanged. The formulation of the proposed SE becomes a Mixed Integer Non-linear Problem (MINLP). In Chapter 4, a real-time algorithm of Alternating Optimization Method (AOM) is applied to obtain the SE results. Case study results demonstrate that the load shedding amount should be considered in order to obtain the accurate estimation results in both the small and large scale test systems when the UFLS events are conducted. Besides,

the proposed AOM is capable of estimating the auxiliary state variables of β and the traditional state variables of phasor voltages accurately under different scenarios. Results from the case studies also suggest that the proposed enhanced SE with inadequate PMU measurements can perform comparably with the adequate synchrophasor measurements in terms of accuracy.

In Chapter 5, SE studies in the scope of smart distribution system are conducted. A decentralized solution is obtained for a newly emerging distribution system of hybrid AC/DC microgrid. The case study is conducted on a simulated system based on a real distribution system in Singapore. The AC and the DC microgrid is modelled respectively and the power flow on the connecting line between the two microgrids is then considered as a constraint. Dual decomposition is adopted to obtain the decentralized solution of the SE. The proposed decentralized SE is fully distributed without any central coordinators. Information exchange is limited to the power injection estimates of the two bordering buses in the AC and DC microgrid respectively, Lagrange multiplier of λ and the active power measurements transmitted on the connecting line.

Case studies on the decentralized SE are conducted in terms of accuracy, computational speed, bad data detection and measurement deficiency. Comparison between the decentralized and centralized SE is conducted as well. From the results, it can be observed that the decentralized SE performs comparatively with the centralized one in many aspects. Meanwhile, the decentralized SE deals with the data locally which greatly reduces the shared information and thus improves the data privacy.

6.2 Recommendations for Future Works

The following recommendations discuss some future research and development works which may provide additional insights into the SE for smart power grids.

For the SE in the scope of smart transmission systems, the adaptive Underfrequency Load Shedding (UFLS) strategies are fully considered. However, for other load shedding schemes such as the voltage-based shedding schemes [119] [120], they will be considered as an interesting topic for the future work. If the under-voltage load shedding (UVLS) scheme is adopted, the low voltage of the buses in the system will trigger the load shedding events, similar to the UFLS events triggered by the low frequency as discussed. The shedding information can be incorporated into the state estimation, i.e. the load power measurements from SCADA will be corrected with the load shedding amount. Improvement on the SE accuracy can be expected and needs to be further examined.

It can be demonstrated that the decentralized technique proposed for the hybrid AC/DC microgrids can perform comparatively as the traditional centralized one and is more advantageous for data privacy. One future work would be focused on the practical applications of the distributed algorithm. Communication issues will be examined and its influences on the decentralized technique for the SE will be investigated as well. A microgrid constructed in Clean Energy Research Laboratory (CERL) at Nanyang Technological University, Singapore, can be utilized as a test bed. In the current stage, the EMS/SCADA system of the microgrid is built in a centralized manner via the Modbus I/O Server embedded in LabVIEW. To apply the decentralized SE on the microgrid in CERL, a decentralized SCADA system should be developed first and this will be considered as one future work as well.

Appendix A

TABLE A. 1 VOLTAGE MAGNITUDES OF LOAD BUSES BEFORE/AFTER DISTURBANCES IN IEEE 30-

BUS SYSTEM							
Bus No.	2	3	4	5	7	8	10
$V_{0,k}(\text{p.u.})$	1.045	1.025	1.017	1.010	1.003	1.010	1.001
$V_k(\text{p.u.})$	0.986	0.985	0.979	0.981	0.972	0.983	0.977
$P_{L0,k}(\text{MW})$	21.7	2.4	7.6	94.2	22.8	30	5.8
Bus No.	12	14	15	16	17	18	19
$V_{0,k}(\text{p.u.})$	1.026	1.009	1.002	1.008	0.998	0.989	0.985
$V_k(\text{p.u.})$	1.005	0.988	0.980	0.985	0.975	0.967	0.962
$P_{L0,k}(\text{MW})$	11.2	6.2	8.2	3.5	9	3.2	9.5
Bus No.	20	21	23	24	26	29	30
$V_{0,k}(\text{p.u.})$	0.988	0.988	0.987	0.974	0.956	0.963	0.951
$V_k(\text{p.u.})$	0.965	0.964	0.964	0.950	0.930	0.934	0.923
$P_{L0,k}(\text{MW})$	2.2	17.5	3.2	8.7	3.5	2.4	10.6

TABLE A. 2 VOLTAGE MAGNITUDES OF LOAD BUSES BEFORE/AFTER DISTURBANCES IN IEEE-118 BUS SYSTEM

Bus No.	1	2	3	4	6	7	11	12	13	14	15	16	17	18
$V_{0,k}(\mathbf{p.u})$	0.9572	0.9722	0.9690	0.9980	0.9900	0.9893	0.9851	0.9900	0.9682	0.9836	0.9700	0.9839	0.9952	0.9730
$V_k(\mathbf{p.u})$	0.9566	0.9717	0.9684	0.9974	0.9895	0.9889	0.9845	0.9895	0.9677	0.9831	0.9694	0.9834	0.9944	0.9725
Bus No.	19	20	21	22	23	27	28	29	31	32	33	34	35	36
$V_{0,k}(\mathbf{p.u})$	0.9634	0.9581	0.9587	0.9698	0.9997	0.9680	0.9616	0.9632	0.9670	0.9637	0.9716	0.9859	0.9807	0.9800
$V_k(\mathbf{p.u})$	0.9629	0.9576	0.9582	0.9692	0.9990	0.9678	0.9613	0.9629	0.9667	0.9634	0.9704	0.9842	0.9791	0.9785
Bus No.	39	40	41	42	43	44	45	46	47	48	49	50	51	52
$V_{0,k}(\mathbf{p.u})$	0.9708	0.9700	0.9667	0.9850	0.9787	0.9853	0.9869	1.0050	1.0173	1.0206	1.0250	1.0015	0.9676	0.9575
$V_k(\mathbf{p.u})$	0.9696	0.9691	0.9656	0.9833	0.9763	0.9817	0.9828	1.0012	1.0104	1.0147	1.0184	0.9957	0.9628	0.9531
Bus No.	53	54	55	56	57	58	59	60	62	66	67	70	74	75
$V_{0,k}(\mathbf{p.u})$	0.9463	0.9550	0.9520	0.9546	0.9712	0.9597	0.9850	0.9932	0.9980	1.0500	1.0198	0.9840	0.9586	0.9682
$V_k(\mathbf{p.u})$	0.9424	0.9513	0.9486	0.9510	0.9668	0.9555	0.9785	0.9851	0.9904	1.0390	1.0106	0.9774	0.9498	0.9549
Bus No.	76	77	78	79	80	82	83	84	85	86	88	90	92	93
$V_{0,k}(\mathbf{p.u})$	0.9430	1.0117	1.0084	1.0131	1.0400	1.0000	0.9939	0.9852	0.9883	0.9888	0.9891	0.9850	0.9978	0.9912
$V_k(\mathbf{p.u})$	0.9292	0.9731	0.9679	0.9689	0.9846	0.9752	0.9720	0.9696	0.9761	0.9807	0.9758	0.9761	0.9824	0.9714
Bus No.	94	95	96	97	98	100	101	102	103	104	105	106	107	108
$V_{0,k}(\mathbf{p.u})$	0.9941	0.9853	0.9982	1.0142	1.0235	1.0170	0.9947	0.9954	1.0064	0.9882	0.9824	0.9726	0.9520	0.9767
$V_k(\mathbf{p.u})$	0.9711	0.9585	0.9661	0.9709	0.9827	1.0015	0.9793	0.9799	0.9993	0.9841	0.9791	0.9682	0.9507	0.9741
Bus No.	109	110	112	114	115	117	118							
$V_{0,k}(\mathbf{p.u})$	0.9747	0.9730	0.9750	0.9605	0.9604	0.9824	0.9499							
$V_k(\mathbf{p.u})$	0.9723	0.9712	0.9748	0.9603	0.9601	0.9819	0.9364							

Appendix B

POWER IN THREE PHASE AND SEQUENCE NETWORKS

In the three-phase network, the total active power injection at a bus can be obtained as a summation of the active power calculated from each phase.

$$P_{AC} = \sum_{m=A,B,C} P^m = \sum_{m=A,B,C} V^m I^m \quad (C-1)$$

where P^m , V^m and I^m are the relevant phase power, phase voltage and phase current. Utility factor is assumed as 1.

The relationship between the voltage values of the three phase networks and those of the three sequence networks can be found as:

$$\begin{aligned} V^A &= \mathbf{T}^A [V^+ \ V^- \ V^0]^T \\ V^B &= \mathbf{T}^B [V^+ \ V^- \ V^0]^T \\ V^C &= \mathbf{T}^C [V^+ \ V^- \ V^0]^T \end{aligned} \quad (C-2)$$

where $\mathbf{T}^A$, $\mathbf{T}^B$ and $\mathbf{T}^C$ are the first, second and third row of the matrix $\mathbf{T}^{-1}$ in (5.12) respectively and can be calculated as:

$$\begin{aligned} \mathbf{T}^A &= [1 \ 1 \ 1] \\ \mathbf{T}^B &= [a^2 \ a \ 1] \\ \mathbf{T}^C &= [a \ a^2 \ 1] \end{aligned} \quad (C-3)$$

Similar relationship can be obtained for currents. Substitute (C-1) by (C-2) and

repeat the substitution for I^m , we can obtain:

$$\begin{aligned}
 P_{AC} &= \sum_{m=A,B,C} [V^+ \ V^- \ V^0] T^{mT} T^m [I^+ \ I^- \ I^0]^T \\
 &= 3 \sum_{m=+,-,0} V^m I^m
 \end{aligned} \tag{C-4}$$

Appendix C

MATRICES USED IN *DUAL DECOMPOSITION*

To obtain the optimal SE results of the DC microgrid from step 5 in **Distributed Algorithm** in Section 5.2.3.2 which is based on the decentralized technique of *Dual Decomposition*, the following quadratic formulation with the relevant coefficients can be obtained:

$$L_\rho = \mathbf{x}_{DC}^T (\mathbf{A}_{DC} + \mathbf{A}_{\lambda D}) \mathbf{x}_{DC} + \mathbf{b}_{DC}^T \mathbf{x}_{DC} + \mathcal{C}(\mathbf{x}_{DC}) \quad (\text{B-1})$$

where

$$\mathbf{A}_{DC} = \mathbf{H}_{DC}^T \mathbf{W}_{DC} \mathbf{H}_{DC}$$

$$\mathbf{A}_{\lambda D} = \lambda \mathbf{I}_D \mathbf{G}(\mathbf{p}, :)$$

$$\mathbf{b}_{DC}^T = -2\mathbf{z}_{DC}^T \mathbf{W}_{DC} \mathbf{H}_{DC}$$

$$\mathbf{I}_D = \begin{matrix} & \overbrace{[0 \ 0 \ 1 \ 0 \ 0]}^{N_{DC}} \\ \uparrow \\ \mathbf{p} \end{matrix}^T$$

$\mathcal{C}(\mathbf{x}_{DC})$ is a constant term w.r.t. $\mathbf{x}_{DC}$.

Similarly, to obtain the optimal SE of the AC microgrid from step 6 in **Distributed Algorithm**, the following quadratic formulation can be found as:

$$L_\rho = \mathbf{x}_{AC}^{+T} (\mathbf{A}_{AC} + \mathbf{A}_{\lambda A}) \mathbf{x}_{AC}^+ + \mathbf{b}_{AC}^T \mathbf{x}_{AC}^+ + \mathcal{C}(\mathbf{x}_{AC}^+) \quad (\text{B-2})$$

where

$$\mathbf{A}_{AC} = \mathbf{H}_{AC}^{+T} \mathbf{W}_{AC} \mathbf{H}_{AC}^+$$

$$\mathbf{A}_{\lambda A} = 3\lambda \times \begin{bmatrix} \mathbf{I}_A \mathbf{G}^+(q, :) & -\mathbf{I}_A \mathbf{B}^+(q, :) \\ -\mathbf{I}_A \mathbf{B}^+(q, :) & -\mathbf{I}_A \mathbf{G}^+(q, :) \end{bmatrix}$$

$$\mathbf{x}_{AC}^+ = [\mathbf{x}_{AC_{\text{real}}}^+; \mathbf{x}_{AC_{\text{imag}}}^+]^T$$

$$\mathbf{I}_A = \overbrace{[0 \ 0 \ 1 \ 0 \ 0]^T}^{N_{AC}}$$

$\uparrow$
 q

$$\mathbf{b}_{AC}^T = -2\mathbf{z}_{AC}^{+T} \mathbf{W}_{AC} \mathbf{H}_{AC}^+$$

$\mathcal{C}(\mathbf{x}_{AC}^+)$ is a constant term w.r.t . $\mathbf{x}_{AC}^+$.

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