Dynamic behavior of reinforced concrete slabs under rapid loading and low velocity impact

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NANYANG
TECHNOLOGICAL
UNIVERSITY

**DYNAMIC BEHAVIOR OF REINFORCED
CONCRETE SLABS UNDER RAPID LOADING
AND LOW VELOCITY IMPACT**

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DYNAMIC BEHAVIOR OF REINFORCED CONCRETE SLABS UNDER RAPID LOADING AND LOW VELOCITY IMPACT

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SUMMARY

In addition to static loading, reinforced concrete structures may experience various kinds of dynamic loadings such as earthquake, impact or blast. These types of dynamic loadings can lead to different loading rates on structure and result in dynamic structural response which is different from the static behavior of structure. Therefore, it is desirable to gain better understanding of the influence of loading rate on structural behavior. Nowadays, impact problems on structures have gained greater attention in structural engineering. The impact loads caused by vehicle impact, crash of aircraft, natural hazard, terrorist attack or accidental impact may cause significant damage to structures. Nevertheless, there is still no satisfactory guideline for structural design and evaluation concerning structures subjected to impact loads. Hence, research work is needed to fill the gap between the increasing engineering demand and the limited knowledge in this field.

The objective of this thesis is to experimentally and analytically evaluate the behavior of reinforced concrete slabs subjected to rapid loading and low velocity impact. In this study, an experimental program was designed and carried out. In this experimental program, thirty-nine reinforced concrete slab specimens were tested in three groups. In the first group, eighteen slab specimens with six types of designs were tested under static (0.0004 m/s), medium (0.4 m/s), and high (2m/s) rate loadings to study the influences of loading rates on the behavior of reinforced concrete slab. In the second group, six slab specimens with the same types of designs as the slab specimens used in the first group were tested under low velocity impact (5.43 m/s). A comparison study between the results from the rapid loading tests in the first group and the impact tests in the second group concerning damage process, failure mode, strain rate, and energy absorption capacity was carried out to illustrate the correlation between these two types of experiments. In the third group, fifteen

reinforced concrete slabs were tested under low velocity impact to study the influences of impact velocity, mass, diameter and nose shape of the impactor on the behavior of slab.

From the experimental results of impact experiment, it is found that the assigned impact energy is strongly related with slab damage. Therefore, a finite element (FE) analysis is followed using LS-DYNA software to numerically study the energy capacity of reinforced concrete slabs under impact. The influences of various parameters on the energy capacity of reinforced concrete slab are evaluated.

Both rapid loading and impact experiments are costly. The FE analysis also requires lots of modeling efforts and time of calculation. Therefore, simplified analytical models were proposed for obtaining the load-displacement relationship of reinforced concrete slabs under dynamic loads and predicting the response of reinforced concrete slabs under low velocity impact. The analytical load-displacement relationship is based on a quadrilinear moment-curvature relationship and the Critical Shear Crack Theory which could consider both the strength softening of slab after punching shear failure and the loading rate effect. The model for assessing the impact response of slab is a 2 degrees of freedom (2-DOF) mass-damper-spring system based on structural dynamics. This 2-DOF model is further utilized to perform a probability analysis of reinforced concrete slab subjected to low velocity impact to investigate how the slab response and damage are affected by the inherent uncertainties existed in the impact problem.

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LIST OF SYMBOLS

CHAPTER 6

DR = diameter of reinforcing bars

d = diameter of impactor

Ei = impact energy

E_c = minimum impact energy to cause punching shear failure (energy capacity of impacted slab)

E_{c-Equ} = minimum impact energy for punching shear failure estimated using Eq. (6-1).

E_{c-FEA} = minimum impact energy for punching shear failure obtained in FE analysis.

f_c = compressive strength of concrete

H = slab thickness

MIF = maximum impact force

MMD = maximum mid-span deflection

m_i = mass of impactor

m_s = mass of slab

RMD = residue mid-span deflection

T_{MMD} = time of reaching the maximum mid-span deflection

Vi = impact velocity

ρ = longitudinal reinforcement ratio

CHAPTER 7

- A_s = the area of shear reinforcement intersected by the critical shear crack
- b_0 = punching shear perimeter
- c_c = contact damping coefficient
- c_s = damping coefficient of slab
- c = depth of compression zone
- DAF = Dynamic amplification factor
- d = effective depth of slab
- d_g = maximum aggregate size of concrete
- d_{g0} = reference aggregate size of concrete (16 mm)
- E_c = elastic modulus of concrete
- E_i = impact energy
- E_{ip} = imparted energy
- E_s = elastic modulus of reinforcement (assumed $e_s=10^4 f_c^{1/3}$)
- EI_0 = flexural stiffness before cracking
- EI_1 = tangential flexural stiffness after cracking
- e = coefficient of restitution
- F_c = impact force
- $F_{c,max}$ = maximum impact force
- f_c = compressive strength of concrete
- f_{cd} = dynamic compressive strength of concrete
- f_{ct} = tensile strength of concrete (assumed $f_{ct}=0.3f_c^{2/3}$)
- f_{ctd} = dynamic tensile strength of concrete
- f_y = yield strength of reinforcement
- f_{yd} = dynamic yield strength of reinforcement

g	=	gravitational acceleration
H	=	slab depth
I	=	impulse
k_c	=	contact stiffness
k_e	=	elastic stiffness of slab
k_s	=	slab stiffness
l	=	clear span of slab
m_{cr}	=	cracking moment per unit width
m_i	=	mass of impactor
m_R	=	moment capacity per unit width
m_r	=	radial moment per unit width
m_s	=	equivalent mass of slab
m_t	=	tangential moment per unit width
r_0	=	radius of the critical shear crack
r_1	=	radius of the zone in which cracking is stabilized
r_c	=	radius of loaded area
r_{cr}	=	radius of cracked zone
r_q	=	radius of support reaction force at the perimeter
r_s	=	radius of slab
r_y	=	radius of yielded zone
T_c	=	duration of impact
u_{max}	=	mid-span displacement of slab correspond to V_{max}
u_i	=	displacement of impactor
$\dot{u}_i$	=	velocity of impactor ($\dot{u}_i(0)$ =initial velocity; $\dot{u}_i(t_r)$ =rebound velocity)
$\ddot{u}_i$	=	acceleration of impactor
u_s	=	mid-span displacement of slab
$u_{s,max}$	=	maximum slab displacement under impact load

$u_{s,res}$ = residue slab displacement under impact load

$\dot{u}_s$ = mid-span velocity of slab

$\ddot{u}_s$ = mid-span acceleration of slab

V = static load capacity of rc slab

V^d = dynamic load capacity of slab

V_{max} = maximum static load capacity of slab

V_R = punching shear capacity of slab

V_s = shear force carried by shear reinforcement

$\Delta\varphi$ = angle of a slab sector

β = efficiency factor

$\dot{\epsilon}$ = strain rate

ζ = damping ratio of slab

ζ_c = damping ratio of contact model

ρ = reinforcement ratio

ρ_s = slab density

$\varphi(x)$ = shape function of slab

χ_1 = curvature in stabilized cracking

χ_{cr} = curvature at cracking

χ_r = curvature in radial direction

χ_t = curvature in tangential direction

CHAPTER 8

COV = coefficient of variation

d = effective depth

d_b = depth of slab

E_c = elastic modulus of concrete

E_s = elastic modulus of steel

f_c = compressive strength of concrete

f_y = yield strength of steel

L = length of slab

m_b = mass of impactor

PDF = probability density functions

r_b = diameter of impactor

V_b = Impact velocity

ρ_0 = density of slab

ρ_s = density of slab

ρ_h = Shear reinforcement ratio

CHAPTER 1

INTRODUCTION

1.1 Research Background and Problem Statement

1.1.1 Reinforced concrete members under rapid loading

Nowadays, the design and analysis of reinforced concrete structures are based mainly on their static behavior, and the understanding of their dynamic behavior is still limited. During their service lives, reinforced concrete structures may experience different kinds of dynamic loads, such as seismic, impact, or blast loads. Impact loads may be caused by vehicle or aircraft crashes, falling rocks, or accidents. Blast loads can result from military bombardment or accidental explosions. These types of dynamic loadings can lead to different loading rates on a structure and result in a dynamic structural response that is different from the static behavior of the structure. For example, a structural member designed to have a flexure failure mode under a static load may experience shear failure mode if the loading rate increases. Reinforced concrete members are also found to be able to sustain higher loads when the corresponding loading rate increases. Inertial effects, propagation of stress waves, and the strain rate effect on the constitutive properties of steel and concrete are the main reasons for these differences. According to Bischoff and Perry [B7], various types of loadings can be related to the induced strain rates. As shown in **Fig. 1.1**, static loads generate low strain rates ($10^{-6}/s$ to $10^{-4}/s$); earthquakes result in medium strain rates ($10^{-3}/s$ to $10^0/s$); and impact and blast loadings create high strain rates ($10^0/s$ to $10^3/s$). In general, the inertial force is significant for impact and explosive problems, and the strain rate effect on the constitutive properties of concrete and steel is significant when the strain rate is above $10^0/s$.

Fig. 1.1: Spectrum of strain rates corresponding to various loading environments [B7].

The most important aspect of the loading rate effect is its influence on the load-displacement relationship of a structural member, which can be measured by performing a rapid loading test. In most studies, hydraulic machines that are capable of generating a wide range of loading rates (usually between 10^{-5} m/s and 10^0 m/s) are used to conduct the rapid loading test (**Fig. 1.2**). Before the test, a support system is required to support the specimen being tested, and a steel loading plate is usually placed between the piston of the rapid loading machine and the specimens. The piston and loading plate generally maintain contact with each other throughout the test to prevent sudden contact or impact from occurring. During the test, the piston, powered by the hydraulic system, can be instantly accelerated to the assigned loading rate. The test concludes when the predetermined loading displacement is reached. For reinforced concrete members, the loading rate effect is usually very small and can be ignored when it is below 10^{-3} m/s. Therefore, in this thesis, the term static test refers to a test with a loading rate of 10^{-4} m/s and below, and rapid loading test refers to a test with a loading rate of 10^{-3} m/s and above.

Fig. 1.2: Typical test setups for rapid loading tests: (a) rapid loading on a reinforced concrete beam [A5]; (b) rapid loading on a reinforced concrete slab [X1].

Many rapid loading tests have been conducted on reinforced concrete beams [M10, K10, F4, A5, A6, and A7]. However, when it comes to reinforced concrete slabs, the available experimental database is limited, and few experimental works can be found [S4 and K3]. The scarcity of information from rapid loading tests hinders further understanding of the influence of loading rate on slab behavior. Therefore, high quality rapid loading experiments on reinforced concrete slabs are urgently needed.

In addition, analytical models for obtaining the load-displacement curves of reinforced concrete slabs under different loading rates are also unsatisfactory. First, current analytical models are merely based on moment-curvature analysis, which cannot consider the softening of the load capacity of reinforced concrete slabs after punching shear failure occurs. Moreover, these models mainly attribute the load capacity enhancement of reinforced concrete slabs under high loading rates to the strain rate effect on the slab materials (i.e., concrete and steel). However, the load capacity enhancement may not be solely attributed to the strain rate effect [C4 and C5]. Additionally, the effectiveness and accuracy of these analytical models is rarely verified because of the insufficient experimental database. New analytical

models that can overcome the aforementioned deficiencies are needed to better predict the dynamic behavior of reinforced concrete slabs.

1.1.2 Reinforced concrete members under impact loading

Impact loading cannot be simply considered as an extreme case of rapid loading due to the involvement of significant inertial force, energy and stress wave transfer mechanism [A8]. The impact problem can be categorized as hard or soft impact according to the stiffness of the impactor. In hard impact, the stiffness of the impactor is very large, and the deformation of the impactor during impact is insignificant. As a result, very little impact energy can be dissipated via the deformation of the impactor. Conversely, in soft impact, the stiffness of the impactor is relatively small, and its deformation cannot be ignored during the impact process. As a result, a portion of the impact energy can be dissipated through the deformation of the impactor. It is generally accepted that hard impact is more damaging to structures than soft impact, and thus most of the research on impact focuses on hard-impact problems. Alternatively, the impact problem can be categorized as low velocity (1~10 m/s), medium velocity (10~100 m/s), and high velocity (100~1000 m/s) impact problems. These velocity ranges are also used to define low, medium, and high velocity experiments. Before the 21st century, most research on impact problems focused on high-velocity impact, and a considerable number of high-velocity impact experiments have been conducted to study the impact response of structural members impacted by missiles or aircraft. Based on these experimental results, different types of empirical equations have been proposed for estimating local damage to reinforced concrete members in terms of penetration depth, scabbing limit, and perforation limit [T2, L1, and R4]. Recently, more attention has been paid to low-velocity impact problems. In addition to the local response, low-velocity impact can also cause significant global response on structures.

The most accurate and reliable approach to investigating structural behavior under impact loading is to conduct impact experiments. In a high-velocity impact test, slabs are usually fixed in steel frames or anchor-bolted to a laboratory wall and subjected to missiles' impacts with a speed of more than 100 m/s. The missiles are accelerated by different kinds of latching equipment. The mass of missiles is highly variable, but the missiles with greater mass require more expensive equipment to provide the large amount of kinetic energy needed to launch the missiles. **Fig. 1.3(a)** shows the test apparatus of the high-velocity impact experiment conducted by Kojima [K8]. In low-velocity impact tests, slabs are usually fixed horizontally and impacted by free-drop impactors with impact velocities of less than 10 m/s. The impactor must be lifted to the height necessary to allow gravity to accelerate the impactor to the desired impact velocity. **Fig. 1.3(b)** shows the test setup of the low-velocity impact experiment conducted by Kishi et al. [K5].

Fig. 1.3: Typical test setups for impact tests: (a) high-velocity impact [K8]; (b) low-velocity impact [K5].

Currently, published results of high-quality low-velocity impact tests are still limited, and few give sufficient attention to the global behavior of impacted targets. Therefore, high-quality low-velocity impact experiments that focus on both local and global structural responses are needed to better understand the performance of

impact on structural members.

Compared to impact experiments, finite element (FE) simulation is a more economical tool for studying structural response under impact, and a verified FE model can be used to conduct parametric studies and predict structural response. However, the FE method also has some drawbacks in that it requires considerable modeling, verification, and calculation efforts. Therefore, a simple, efficient, and accurate analytical method is required to analyze impact problems. Available analytical models are generally based on the energy method or a single or multiple degree-of-freedom dynamic system [B4, D3, F4, M4, and S10]. However, these models have the following types of limitations: experimental data is required for determining model parameters or inputs; the models can only consider flexural failure mode; the models cannot sufficiently consider loading rate effect; they lack verification. Thus, a new analytical model without the aforementioned drawbacks is required.

1.1.3 Relationship between rapid loading and impact tests

While the rapid loading test cannot be used to simulate impact problems, it can be used to build a clear relationship between loading rate and the load displacement of reinforced concrete members. On the other hand, due to the large amount of inertial force involved in impact testing, it is very difficult to determine the relationship between the dynamic load-displacement and the corresponding load capacity of the impacted member. By carefully choosing the loading rate, the rapid loading test can produce a similar deflection rate and strain rate as the low-velocity impact problem. Because the load capacities of slabs can be easily and accurately obtained from the displacement-controlled rapid loading rate test, it can be considered as a supplementary approach to studying the impact problem.

1.2 Objectives and Scope

The goal of the present work is to improve the understanding of the dynamic behavior of reinforced concrete slabs subjected to rapid loading and low-velocity impact. The specific objectives of this research are summarized below:

- Conduct rapid loading tests on reinforced concrete slabs with loading rates ranging from 4×10^{-4} m/s to 2 m/s. Understand the influence of loading rate on the performance of reinforced concrete slabs, which includes the dynamic load-displacement relationship; load carrying and energy absorbing capacities; the induced strain rate; the induced inertial effect; the deflected shape; the failure mode; and the damage process. Understand the relationship between the dynamic increase factor (DAF, defined as the ratio of specimens' maximum load capacity at any loading rate to its maximum static load capacity) and the slab properties (longitudinal reinforcement ratio, shear reinforcement ratio, slab depth) under different loading rates.
- Conduct low-velocity impact tests on reinforced concrete slabs. Study both the local and global behaviors of reinforced concrete slabs. Analyze the induced strain rate and inertial force. Investigate how impact energy, diameter, and nose shape of the impactor affect the extent of damage that occurs to the slab. Compare the test results of the rapid loading test and the low-velocity impact test to understand the correlation between these two types of tests. Give suggestions on how to use the data from the rapid loading test to facilitate the understanding of the impact problem.
- Conduct FE analysis of the low-velocity impact problem on a reinforced concrete slab using LS-DYNA software. The modeling approach should at first be verified with experimental data and then used for a series of parameter

studies concerning the energy capacity of reinforced concrete slabs subjected to impact loads.

- Model the dynamic load-rotation curves of reinforced concrete slabs with a simple analytical method. In contrast to earlier methods, the proposed method should be able to rationally consider the punching shear failure of a reinforced concrete slab and the loading rate effect. Propose a simple but effective analytical model based on structural dynamics for analyzing the damage and response of reinforced concrete slabs under low-velocity impact. Compared with previous models, the proposed method should be able to accurately predict the slab deflection and the impact force. The validity of this model should be verified by comparing the analytical results with experimental data from different impact experiments.
- Perform a probability study on the low-velocity impact problem of reinforced concrete slabs. The influence of the uncertainties in the impact problem on the impact response of the slab should be investigated.

1.3 Structure of Thesis

The introduction of the problems, the objectives, and the scope of this study are described in this chapter. The contents of the remaining chapters are briefly described in the following paragraphs.

Chapter 2 presents a comprehensive review of the literature on reinforced concrete members subjected to rapid loading or impact loading. This literature is reviewed in terms of experimental work, numerical simulation, and analytical methods. This literature review intends to present the current understanding of rapid loading and impact problems. Limitations of the available literature are discussed and

summarized.

Chapter 3 introduces the rapid loading experiment on 18 reinforced concrete slabs. The design of the specimens, the properties of specimen materials, the test setup, and the instrumentation are illustrated. The measured and processed experimental data are provided and discussed.

Chapter 4 introduces the low-velocity impact experiment as performed on six reinforced concrete slabs. The design and material properties of the impacted slab specimens are the same as those tested in the rapid loading experiment. The experimental test setup and instrumentation are introduced. The results of these specimens are presented and compared with the experimental data from the rapid loading experiment to illustrate the correlations between these two types of experiments.

The experimental data of the remaining 15 reinforced concrete slab specimens under low-velocity impact are given in **Chapter 5**. Discussions based on the experimental results are made concerning the influences of the impact energy, the diameter, and nose shape of impactor on the response of reinforced concrete slabs.

In **Chapter 6**, FE simulation of a reinforced concrete slab under low-velocity impact is presented. An discussion of the modeling approach describes how the modeling elements were selected and presents material models and a contact law. The modeling approach is then verified using experimental data from different sources. After the verification work, a parameter study is conducted to assess the influences of different parameters on the energy capacity of impacted reinforced concrete slabs.

Chapter 7 illustrates an analytical method of building the dynamic load-displacement relationships of reinforced concrete slabs under different loading

rates and presents a two-degrees-of-freedom (2-DOF) model for predicting the impact response of reinforced concrete slabs. The theory and formulations of the proposed analytical models are thoroughly introduced. Verification of these models is presented as well.

In **Chapter 8**, a probability study of a reinforced concrete slab under low-velocity impact is conducted using the proposed 2-DOF model. The influences of the uncertainties associated with slab properties and the of impact loading on the failure probability of the impacted slab are studied.

Chapter 9 summarizes the conclusions drawn from the presented study and provides recommendations for future research in this field.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This chapter contains six sections. The first section introduces the strain rate effect on concrete and steel reinforcement. The second section introduces the previous experimental works of reinforced concrete member under rapid loading. A review of the analytical methods used to form the load-displacement relationship of reinforced concrete members is given in the third section. Next, a detailed review of the experimental work, numerical simulation, and analytical methods concerning the impact problem of reinforced concrete members is summarized in the following three sections. Although this research focuses on reinforced concrete slabs, this literature review also includes some work that focuses on reinforced concrete beams under rapid and impact loadings to present a more general research background.

2.2 Strain Rate Effect

Both concrete and steel reinforcement are rate sensitive materials whose properties are strain rate dependent. Therefore, understanding the material properties of concrete and steel reinforcement over a wide range of strain rate is a prerequisite for understanding the dynamic behavior of reinforced concrete members.

2.2.1 Strain rate effect on concrete

In general, both the compressive and tensile strength of concrete increase together with loading rate. There are several explanations for this phenomenon. For example, Mihashi and Wittmann [M5] believe that the crack inertial of concrete is governed by the activation energy. When bond-rupture step is assumed to be constant over time, the bond-rupture caused by long-duration loading is more than that of the short-duration

loading. This suggests that higher rate loading leads to an increase in strength. Reinhardt and Weerheijm [R3] included the inertial effect at the material level to explain the strain rate effect on concrete. By calculating the energy when cracks propagate into the material, the authors found that the energy supply rate is too high to be absorbed during the fracture process and considerable energy is stored at the crack tip. This would lead to a change of stress distribution around the crack tip and the material strength would increase with increasing loading rates.

So far, various types of experiments (e.g., split Hopkinson pressure bar test, split-cylinder tests, direct uniaxial tension test, etc.) have been performed to study the dynamic behavior of concrete under different strain rates. For example, Ross et al. [R6] used a 51-mm-diameter split Hopkinson pressure bar to test the compression and tension strengths of concrete with a strain rate ranging from 1/s to 300/s. From this experiment, the authors found that both tension and compression strengths increase with strain rate and exhibit a critical strain rate beyond which large increases in strength occur. This critical strain rate is approximately 5/s for tension and approximately 60/s for compression. Moreover, the tension strength exhibits a larger percentage increase than the compression strength beyond these critical strain rates. Based on the experimental database of concrete strength under different strain rates, regression analyses were performed by researchers to form the empirical equations that govern concrete strength under different strain rates [M1, F1].

2.2.2 Strain rate effect on steel reinforcement

Many researchers have documented the strain rate effect on steel reinforcement. One representative study was made by Malvar, who presented a detailed review of the strain rate effect on steel reinforcement [M2]. The author found that the strength of steel reinforcement increases with increasing strain rate, and this strength enhancement effect for both yield and ultimate stress is inversely related to the yield stress itself.

Moreover, a formulation was proposed by the author to determine the strength enhancement on steel reinforcement.

2.3 Rapid Loading Experiments

Previous rapid loading experiments are summarized in this section. Rapid loading experiments have long been used to study the influence of loading rate on the behavior of structural members. Although rapid loading rarely happens during real engineering practice, the dynamic load-displacement relationships obtained from rapid loading experiments are fundamental for analyzing various structural dynamic problems such as earthquake, impact, and explosion.

2.3.1 Rapid loading experiments on reinforced concrete beams

In previous rapid loading experiments [M10, K10, F4, A5, A6 and A7], similar rapid loading apparatus were used. The beams were simply supported at both ends and loaded at mid-span. Displacement sensors and load cell were installed to measure the applied load and the movement of the mid-span of the beam. Depending on the quality of the loading machine, the maximum loading rate varied from 10^{-1} m/s to 10^0 m/s. Mutsuyoshi and Machida [M10] tested 27 reinforced concrete beams under different loading rates. The beams were singly or doubly reinforced and had different shear-span-to-effective-depth ratios. The loading rate ranged from 10^{-5} m/s to 10^{-1} m/s. It is found that some beam specimens with flexural failure mode under static loading rate fail in shear at a high loading rate. Fourteen singly reinforced concrete beams were tested by Kulkarni and Shah [K10] under static (7.1×10^{-6} m/s) and high (3.8×10^{-1} m/s) loading rates. During the experiment, some beam specimens with shear failure mode under a static loading rate failed in flexure under a high loading rate. The opposite was expected. Moreover, for a beam with shear failure mode, no obvious yield point and yield plateau can be observed from its load-displacement curve. Fujikake et al. [F4]

conducted rapid loading tests on three pairs of reinforced concrete beams with different longitudinal reinforcement ratios under static (5×10^{-4} m/s) and high loading (2 m/s) rates. All of these beam specimens were under-reinforced and had a sufficient amount of shear reinforcement to ensure global flexural failure. It was found that the load capacities of the tested beams increases with increasing tensile reinforcements under both static and high loading rates. In Kulkarni and Shah's experiment [K10], the high loading rate (3.8×10^{-1} m/s) can be correlated with the seismic induced loading rate. In Fujikake et al.'s experiment [F4], the high loading rate (2 m/s) is of the same order as the loading rate induced by low-velocity impact. By summarizing the early experimental studies on reinforced concrete beams under rapid loading, Adhikary et al. [A5, A6, and A7] realized the importance of systemically investigating the performance of reinforced concrete beams under varying loading rates, and tested 72 reinforced concrete beams under a wide range of loading rates (static: 4×10^{-4} m/s; low: 4×10^{-2} m/s; medium: 0.4 m/s; and high: 2 m/s). The variables of the beam specimens include shear reinforcement ratio, longitudinal reinforcement ratio, and shear span to effective depth ratio. Both flexural and shear failure modes were observed during the tests as shown in **Fig. 2.1(a), (b)**. In their research, Adhikary et al. [A5, A6, and A7] emphasized how different parameters affect the DAF of reinforced concrete beams under varying loading rates. From the experimental results, it is found that the ultimate load capacity, energy absorption capacity, and cracking stiffness of reinforced concrete beams increases together with loading rate. Moreover, the DAF also increases when the loading rate increases. Following the experimental work, Adhikary et al. used LS-DYNA FE software to numerically simulate the dynamic load-displacement curves of reinforced concrete slabs (**Fig. 2.2**). By conducting regression analysis of the numerical results, empirical equations were proposed for predicting the DAF of reinforced concrete beams under different loading rates.

Fig. 2.1: Typical load-displacement curves of reinforced concrete beams: (a) shear failure mode; (b) flexural failure mode [A5].

Fig. 2.2: Finite element model of reinforced concrete beams [A5]

2.3.2 Rapid loading experiments on reinforced concrete slabs

Compared to reinforced concrete beams, even less research has been published on rapid loading of reinforced concrete slabs, and very few studies were found. One well-documented experiment was conducted by Saito et al. [S4] where seven kinds of slab specimens with different configurations were tested under static (3×10^{-5} m/s), low (3×10^{-2} m/s), and high (3 m/s) loading rates. The specimens' shapes and details are shown in **Fig. 2.3**. The flat slab specimens were simply supported on all four sides at the 1,000 mm span position. For the remaining specimens, only two sides were supported. A 150-mm square loading plate is placed between the rapid loading machine and the slab center to enlarge the loading area. The experimental results show that flexural deformation occurred in the flat slabs during the initial loading stage in all the tests, and in the final stage, punching shear failure resulted. The final failure modes were nearly the same for all the loading rates. The mid-span displacement and the applied load were measured during the test. A typical load-displacement curve for a flat slab is shown in **Fig. 2.4**. In all the tests, the maximum load capacities of slabs were found to increase with increasing loading rate. Additionally, the DAF under varying loading rates is affected by slab thickness such that the thinner the slab, the higher the DAF. Saito et al. [S4] attributes the enhanced slab capacity to the strain rate effect on the constitutive properties of concrete and steel. However, the main limitation of this experimental work is that no information of the induced strain rate and inertial effect under medium and high loading rates was provided. Therefore, it is not possible to justify the conclusion that the enhanced capacity of reinforced concrete slabs is solely determined by the strain rate effect on concrete and steel.

Fig. 2.3: Configurations of slab specimens [S4]

Fig. 2.4: Typical load-displacement curves of flat slabs with punching shear failure mode [S4].

Kogure et al. [K3] conducted a rapid loading experiment on 12 types of sandwich and reinforced concrete slabs with a loading rate of 4 m/s. These slabs were also

tested statically for comparison purposes. The obtained results show that the load capacity of slabs increases with loading rate. Punching shear failure mode was found during most tests. However, this research mainly focuses on sandwich slabs, and only one type of conventionally reinforced concrete slab was tested.

2.4 Analytical Study of Load-Displacement Curves of Reinforced Concrete Members under Different Loading Rates

An analytical expression of the load-displacement curve of a reinforced concrete member is required by many analytical models to assess the behavior of the member under various types of loadings, and its accuracy is usually considered the most influential factor on the effectiveness of these models. In general, a good analytical load-displacement curve should reasonably reflect the member's load capacity, energy capacity, ductility, failure mode, etc. On the other hand, the load-displacement curve should consider the influence of loading rate when it is being used for to analyze dynamic problems. The loading rate effect can usually be ignored when analyzing seismic problems because the loading rate induced by the seismic load is relatively small and its influence on the load-displacement curve is usually insignificant. As a result, using a static load-displacement curve can also yield a good prediction of the structural response. However, for high loading rate problems, such as impact and explosion, the loading rate effect and the corresponding DAF are significant, and using a static load-displacement curve may overestimate structural response leading to overly conservative structural design.

2.4.1 Static load-displacement curves of reinforced concrete members

Currently, moment-curvature analysis is widely used to form the load-displacement curves of reinforced concrete members. The moment-curvature relationship can be

obtained by conducting section analysis on the structural member with the assumption that the strains are linearly distributed across the section depth. In many studies [S10 and B4], only the yield state and ultimate state are considered in the section analysis (**Fig. 2.5**). The section analysis yields the curvatures and moments of these two states, which can be used to form a bilinear moment-curvature relationship as shown in **Fig. 2.6**. More complex moment-curvature relationships can be formed by including more states in the section analysis.

Fig. 2.5: Typical section analysis: (a) cross-section of a reinforced concrete member; (b) stress diagram of yield state; (c) strain diagram of yield state; (d) stress diagram of ultimate state; (e) strain diagram of ultimate state [S10].

Fig. 2.6: Typical bilinear moment-curvature relationship (M_y , and χ_y are the moment and curvature at yielding status, M_u , and χ_u are the moment and curvature at ultimate status) [B4]

Various methods can be used to convert the moment-curvature to the load-displacement curve of a reinforced concrete member. A simple method used by Stochino and Carta [S10] assumes elastic and elastoplastic behaviors of reinforced concrete member as shown in **Fig. 2.7**. The plastic component v_{Ep} is evaluated by assuming the formation of a plastic hinge at the mid-span of the beam. Based on these assumptions, the relationships between the load and the displacement at the yield and the ultimate status can be derived with the moment-curvature relationship.

Fig. 2.7: Load-displacement curve assuming elastic and elastoplastic behaviors (p_y , and v_{Ey} are the load and displacement at yielding status, p_u and v_{Eu} are the load and displacement at ultimate status) [S10].

Fujikake et al. [F4] adopted a more accurate method to form the load-displacement curve. In their work, the applied load is used to calculate the moment distribution along the span of the reinforced concrete member, and the corresponding curvature distribution can be obtained from the moment-curvature relationship. The displacement of the mid-span of the member can then be calculated by integrating the curvature distribution over half length of the reinforced concrete beam.

The main deficiency of moment-curvature analysis is that it can only capture the flexural behavior of a reinforced concrete member. The section analysis is essentially a flexural analysis in that its ultimate state corresponds to the final flexural capacity irrespective of possible shear failure. For a member with shear failure mode such as **Fig 2.1(a)** and **Fig. 2.4**, softening of load capacity happens immediately after the shear failure point rather than when it reaches its final flexural capacity. As a result, a moment-curvature analysis would overestimate the load and displacement capacities of a member with shear failure mode.

2.4.2 Incorporating loading rate effect into load-displacement curves

The strain rate of structural material increases with increasing loading rate. In most research, the observed load capacity enhancement of a reinforced concrete member under a high rate loading is assumed to be determined by the strain rate effect on concrete and steel. With this assumption, the dynamic load-displacement curve of a reinforced concrete member can be easily formed by replacing the static material strengths with dynamic ones during moment-curvature analysis.

Based on experimental data from dynamic material tests, various empirical equations have been provided for calculating the dynamic strengths of concrete and steel under varying strain rates. In Fujikake et al.'s [F4] research, the following equations proposed by Fujikake et al. [F4] and the Japanese Society of Civil Engineers [J2 and J3] are used to evaluate the dynamic strengths of concrete and steel:

$$\frac{f_{cd}}{f_c} = \left(\frac{\dot{\epsilon}}{1.2 \times 10^{-5}} \right)^{0.006 \left[\log \frac{\dot{\epsilon}}{1.2 \times 10^{-5}} \right]^{1.05}} \quad (2-1)$$

$$\frac{f_{ctd}}{f_{ct}} = \exp \left[0.00126 \left(\log \frac{\dot{\epsilon}}{1 \times 10^{-7}} \right)^{3.373} \right] \quad (2-2)$$

$$\frac{f_{yd}}{f_y} = 1.202 + 0.04 \log \dot{\epsilon} \quad (2-3)$$

where f_{cd} , f_{ctd} , and f_{yd} are the dynamic compression strength of concrete, dynamic tensile strength of concrete, and dynamic yield strength of steel, respectively. f_c , f_{ct} , and f_y are the corresponding static values, and $\dot{\epsilon}$ is the strain rate.

In the research by Stochino and Carta [S10] and Micallef et al. [M4], more widely accepted empirical equations proposed by the International Federation for Structural Concrete [C1 and F1] are used:

$$\frac{f_{cd}}{f_c} = \left(\frac{\dot{\epsilon}}{30 \times 10^{-6}} \right)^{0.014} \quad \text{for } 30 \times 10^{-6} s^{-1} \leq \dot{\epsilon} \leq 30 s^{-1} \quad (2-4)$$

$$\frac{f_{ctd}}{f_{ct}} = \left(\frac{\dot{\epsilon}}{1 \times 10^{-6}} \right)^{0.018} \quad \text{for } 1 \times 10^{-6} s^{-1} \leq \dot{\epsilon} \leq 10 s^{-1} \quad (2-5)$$

$$\frac{f_{yd}}{f_y} = \left[1 + \frac{6}{f_y} \ln \left(\frac{\dot{\epsilon}}{5 \times 10^{-5}} \right) \right] \quad \text{for } \dot{\epsilon} \leq 10 s^{-1} \quad (2-6)$$

Assuming $\dot{\epsilon} = 10/s$ (which corresponds to the impact problem) and using Eqs. (2-1) – (2-3), the values of f_{cd}/f_c , f_{ctd}/f_{ct} , and f_{yd}/f_y are 1.7, 4.1, and 1.2. These values become 1.2, 1.3 and 1.1 when using Eqs. (2-4) – (2-6). The results indicate big differences in the dynamic strengths of concrete when using different empirical equations. These differences are believed to be caused by the scatted experimental results of concrete dynamic tests, and it is difficult to judge which set of empirical equations is superior.

Fig 2.8 shows the comparison between the analytical and experimental load-displacement curves of a reinforced concrete beam under different loading

rates when Eqs. (2-1) – (2-3) are used to calculate the dynamic strengths of concrete and steel [F4]. It can be found that the loading rate effect is effectively incorporated. However, it can be expected that a lower capacity of the reinforced concrete beam under a high loading rate (i.e., 2 m/s; the corresponding maximum strain rate is on the order of 10^0 /s) would be predicted when using Eqs. (2-4) – (2-6).

Fig. 2.8: Comparison between static and dynamic load-displacement curves: (a) static loading rate (5×10^{-4} m/s); (b) high loading rate (2 m/s) [F4].

Though numerical work, Cotsovos [C5] found that the strain rate of reinforced concrete member under impact loading is very low when comparing with the value required to have a significant effect on the strength of concrete. Alternatively, the enhanced load carrying capacity was found to be related to the length of the portion of the beam along which the stress waves travel up to the moment when the member attains its load-carrying capacity [C4 and C5]. As shown in **Fig. 2.9**, more and more localized behavior was observed when loading rate increases (i.e., in a load-controlled loading scheme). As the loading rate increases, a reinforced concrete member should respond within less time. Because stress waves need time to travel from the loading area to the supported area, less member area can be mobilized to response to the applied load.

Fig. 2.9: Deformation profile and crack patterns of reinforced concrete beams under different loading rates (UL: Ultimate Load) [C5]

With this theoretical background, Cotsovos relates the enhanced load capacity to the shortening of effective length of the reinforced concrete member, and proposes a series of formulas that can be used to evaluate this effective length under different loading rates. Therefore, the loading rate effect can be incorporated by replacing the original member's span with the calculated effective length.

A subsequent analysis by Isaac et al. [I1] illustrates how the effective length of a reinforced concrete member varies and affects its shear and bending demands under a concentrated impact load. As shown in **Fig. 2.10**, when load is applied, the effective length gradually increases due to the travel of stress wave. The shorter the effective length, the higher the shear demand and the less the flexural demand. This analysis explains how the loading rate affects a member's failure mode.

Fig. 2.10: Influence of effective length on the shear and flexural demands of a reinforced concrete member [I1].

2.5 Impact Experiments

A reinforced concrete member may respond to impact load locally or globally. When global response dominates, the energy transferred from the impactor to the impacted member is mainly dissipated through the global deformation of the member. When local response dominates, the transferred energy is mainly dissipated through reinforced concrete member's local damage and deformation around the impacted area. According to Bangash [B1], the damage forms of reinforced concrete members under impact loading can be categorized as shown in **Fig. 2.11**. **Fig. 2.11(a)** indicates that light local damage on reinforced concrete members occurs where penetration and concrete scabbing happens in the front surface. A more severe impact would cause concrete spalling in the rear surface of the member (**Fig. 2.11(b)**). By further increasing the impact load, the member may fail in perforation or punching shear (**Fig. 2.11(c), (d)**). Typical flexural behavior is shown in **Fig. 2.11(e)**. The responses of reinforced concrete structural components under impact loading are affected by many factors such as material properties, contact stiffness, impact energy, reinforcement ratios, contact area, boundary condition, etc. Due to the complex nature of impact problems, many impact experiments were carried out to study how these factors affect structural response.

Fig. 2.11: Forms of impact damage on reinforced concrete members [B1]

2.5.1 Medium- and high-velocity impact experiments

From World War II through the Cold War period, impact research focused on the impact performance of protective structures used in the battlefield, nuclear power plants, and important governmental buildings. These structures might be impacted by medium- or high-velocity missiles such as kinetic munitions, vehicles, aircraft, and fragments generated by military or terrorist bombing. Therefore, many medium- and high-velocity impact experiments were conducted during this period. Because of security considerations, the details of these experiments were rarely released to the public. However, based on the experimental data, various versions of the empirical formulas were proposed to evaluate the damage of the impacted structural members. As an example, the Army Corps of Engineers (ACE) formula [K1 and A1] is shown below:

$$\frac{x}{d} = \frac{3.5 \times 10^{-4}}{\sqrt{f_c}} \left(\frac{M}{d^3} \right) d^{0.215} V_0^{1.5} + 0.5 \quad (2-7)$$

$$\frac{e}{d} = 1.23 + 1.07 \frac{x}{d} \quad (2-8)$$

$$\frac{h_s}{d} = 2.28 + 1.13 \frac{x}{d} \quad (2-9)$$

where x is the penetration depth, e is the perforation limit (the minimum thickness of the target to prevent perforation), h_s is the scabbing limit (the minimum thickness of the target to prevent scabbing), and d , M , and V_0 are the diameter, mass, and velocity, respectively, of the impactor. The above formulas are formed through regression analysis from the impact experimental database from the Ordnance Department of the US Army before 1945. Other well-known formulas include the Ballistic Research Laboratory (BRL) formula [L2], the US National Defense Research Committee (NDRC) formula [N1], and the Chang formula [C2]. All of these formulas focus on the local damage of the target because the global

response is usually insignificant under medium- and high-velocity impacts. In some high-velocity impact experiments [K8 and S11], both rigid and deformable impactors were used to study the difference between hard and soft impacts. Experimental results indicate that rigid impact can cause more severe damage to the impacted reinforced concrete member than soft impact does.

2.5.2 Low-velocity impact experiments

After cold war, the interest in high-velocity impact gradually waned. Conversely, more and more attention has been directed toward low-velocity impact problems, of which understanding is limited.

Sawas et al. [S6] designed a low-velocity slab impact experiment by letting a steel ball repeatedly drop on reinforced concrete slabs from increasing drop heights. As shown in **Fig. 2.12**, these reinforced concrete slabs are simply supported on a rigid square steel frame and have the same dimension of $750 \times 750 \times 50$ mm. The reinforcement ratio and drop height (impact velocity) of steel ball were the two variables studied in this experiment. In total, six slabs with reinforcement ratios ranging from 0.45 to 1.47 were cast. The drop heights of the steel ball on all the slabs ranged from 0.2 m to 1.2 m with a 0.1 m interval. The deflection of the mid-point of the slab's bottom surface is the only measured data. The experimental results are summarized in **Fig. 2.13**. Through observation of the experimental results, the author concluded that the maximum slab deflection increases with increased drop height, and decreases with increased reinforcement ratio. However, when the reinforcement ratio is larger than 1%, its effect of reducing slab deflection becomes less significant.

Fig. 2.12: Impact test setup [S6]

Fig. 2.13: Effect of Height of impact (h) on max dynamic deflection for various steel reinforcement ratios (ρ) [S6]

Ong et al. [O2] conducted low-speed impact tests on reinforced concrete and fiber reinforced concrete slabs to investigate the capability of steel fibers to enhance the performance of slabs under impact load. In total, six kinds of slabs were designed

with different layers of longitudinal reinforcements and different volume ratios of steel fiber. These $1000 \times 1000 \times 50$ -mm-thick slabs were clamped on all four edges with a clear span of 900 mm. These slabs were impacted repeatedly until failure with the same impact energy (a 20-kg hemispherical nose-shaped impactor with a drop height of 1.5 m, or a 20-kg flat nose-shaped projectile with a drop height of 4.5 m). The impact force history, slab acceleration, and steel and concrete strains were recorded. The experimental results show that slabs reinforced with higher ratios of steel fiber can sustain higher numbers of impacts before failure. The hemispherical nose-shaped impactor is much more damaging to singly reinforced slabs than doubly reinforced slabs. However, both doubly and singly reinforced slabs can sustain the same number of impacts when the flat nose-shaped impactor is used.

Zhang et al. [Z1] conducted a slab impact experiment to evaluate the damage and failure mode of hybrid-fiber engineered cementitious composite (ECC) slabs. In total, 10 rectangular slabs of various sizes ($1000 \times 2000 \times 50$ mm, $1000 \times 2000 \times 75$ mm, $1000 \times 2000 \times 100$ mm) and reinforcement ratios (1.0%, 0.8%, 0.6%) were tested. Similar to the study of Ong et al. [O2], the slabs were repeatedly impacted by a 45-kg hemispherical nose-shaped hammer with a 4-m drop height until failure. A quartz force ring of 445 kN capacity was attached to the impactor as a load cell to measure the impact forces. Four potentiometers were attached to the underside of the slabs to measure their deformation. A photodiode system was used to measure the impact and rebound velocities of the hammer. The results indicate that slab performance under impact load can be significantly enhanced by increasing slab depth or the ratio of hybrid-fiber ECC. In this study, the energy absorption capabilities of various slabs were evaluated by summing up the lost kinetic energy of the impactor after each strike. A similar approach was used by Rao et al. [R2] to investigate the energy capacity of impacted slabs. However, the energy capacity obtained using this method is not conservative because a portion of the impact

energy would be dissipated by the slab damping during each impact. This is because real engineering structures are more likely to suffer a single severe impact rather than repeated light impacts. The calculated energy capacity can hardly be used to guide structural design and evaluation.

With the intention of studying the dynamic response and behavior of reinforced concrete slabs under impact loading, Zineddin and Krauthammer [Z2] conducted low-velocity impact experiments on three types of rectangular reinforced concrete slabs with various arrangements of steel reinforcement. The slab specimens have the dimensions of $1524 \times 3353 \times 90$ mm, and were impacted by a 2,600-kg impactor from varying drop heights. The test setup is shown in **Fig. 2.14**. Various types of sensors were installed to measure the impact load, the displacement of reinforced concrete slabs, and the strain of reinforcement. Additionally, high-speed video was used during impact tests to record the damage process of the reinforced concrete slabs.

Fig. 2.14: Precision impact testing system.

In analyzing the experimental results, Zineddin and Krauthammer [Z2] divide the slab behavior into the following stages: (a) instantly after impact, the slab behaves like an elastic plate and experiences a localized tension membrane for a very short period; (b) concrete cracking happens around the perimeter of the impacted region; (c) yielding of steel reinforcement starts in the impacted region and gradually spreads along radial cracks; (d) increased slab deflection in the slab center causes the fast progression of yield line mechanism on the top surface; (f) finally, slab deflection at slab center becomes so severe that concrete is ejected from the impacted area. Typical impact damage on reinforced concrete slabs after impact is shown in **Fig. 2.15**. Punching shear failure mode was found for the impacted slabs although they were designed to have a flexural failure mode.

Fig. 2.15: Typical slab damage [Z2].

Kishi et al. [K6] conducted an impact experiment that focuses on the influence of boundary condition on the behavior of slabs. In this experiment, three $2,000 \times 2,000 \times 180$ mm slab specimens with the same design have the boundary conditions shown in **Fig. 2.16**. These slab specimens were impacted five times by a 300-kg drop weight with impact velocities of 1, 2, 3, 4, and 4.5 m/s. The impact load, support reaction force, and impact displacement were measured. Typical punching shear behavior of slab specimens was observed during the tests. The slab damage

after the final impact is shown in **Fig. 2.17**. From this test, it is found that the crack pattern of reinforced concrete slabs is affected by its boundary condition. However, the impact-resistant capacity of the reinforced concrete slabs may be almost the same in spite of the supporting conditions that the slabs exhibited similar levels of damage and mid-span deflections under the same impact velocity. In another impact experiment, Kishi et al. [K5] investigated the influence of impact area on slab behavior using the same slab design, impact apparatus, and instrumentations. The diameter of the impactor varied from 60 mm to 150 mm. Through observing the recorded experimental data, it is found that when loading area increases, the peak impact force and peak reaction force increases and the peak penetration depth decreases.

Fig. 2.16: Test setup and boundary conditions of slab specimens: (a) four sides supported; (b) two sides supported; (c) one side and two corners supported [K5].

Fig. 2.17: Damage of slab specimens after the final impact: (a) four sides supported; (b) two sides supported; (c) one side and two corners supported [K5].

With the intention of providing high-quality impact test data to facilitate the computational simulation work on slab impact problems, Chen and May [C3] conducted impact experiments on 18 reinforced concrete beams and 6 reinforced concrete slabs using a drop-weight system. During the tests, a load cell is installed to the drop weight to measure the impact load, and high-speed video is used to record the damage of reinforced concrete beams. Because the slab damage cannot be observed from its lateral surfaces, no high-speed video on slab damage is provided. The correlation between impact force and damage process of a reinforced concrete beam specimen is shown in **Fig. 2.18**. As can be seen, the impact load reached its maximum value immediately after impact. Meanwhile, the beam had limited deformation, cracking, and damage. Failure and damage on the specimen was not significant until approximately 10 ms, in which the impact force became very small compared with its maximum value. Therefore, Chen and May concluded that it is difficult to draw any conclusion regarding the relationship between the peak impact load and the load carrying capacity of impacted reinforced concrete member.

Fig. 2.18: correlation between impact force and damage process of reinforced concrete member (p =load; t =time) [C3].

Punching shear failure mode was observed for all the tested reinforced concrete slabs. Additionally, Chen and May used a series of empirical equations to predict the damage of tested reinforced concrete slabs, and found that the empirical equations, based on medium- and high-velocity impact experimental data, sometimes cannot satisfactorily predict the damage to reinforced concrete slabs subjected to low-velocity impact.

Hrynyk [H4] carried out a low-velocity impact experiment on reinforced concrete and steel fiber reinforced concrete slabs. In this experiment, eight slabs with a size of $1,800 \times 1800 \times 130$ mm and a simply supported boundary condition were repeatedly impacted until failure. The impact force was calculated from the measured acceleration history of the impactor. The slab supports were installed with load cells to measure the support reaction force. Moreover, 11 accelerometers were installed to the slab to measure its acceleration during impact. By assuming a parabolic shape acceleration distribution and conducting numerical integration of the acceleration data, the inertial force of slab was calculated. To investigate the interaction of how these forces occurred during impact, the time histories of the impact force, support reaction force, and inertial force were plotted together. A typical time history of these forces during an impact event is shown in **Fig. 2.19**. It can be found from the plot that the inertial force played a very important role in impact problems. During the impact period (the period when impactor and impacted slabs are in contact with each other), nearly all the impact force was counterbalanced by the inertial force of slabs while the reaction force was quite small compared with the impact force. During the natural vibration period (the period after the separation of impactor and impacted slab), the impact force disappears, and the residue inertial force of slab was balanced by the support reaction force. This phenomenon explains why the peak value of impact force cannot be considered as the load capacity of the slab and why the impact force history is out of phase with the support reaction force history.

Fig. 2.19: Dynamic equilibrium of an impact event [H4].

Additionally, potentiometers were attached to the back face of each slab specimen to measure its deformation under impact. A typical slab deformation during impact is shown **Fig. 2.20**. Globalized flexural behavior of each slab was observed shortly after impact. After that, the slab deformation in the impacted region increased much more quickly than the slab fringe which showed a typical localized punching shear behavior.

Fig. 2.20: Typical slab deformation during impact [H4].

Delhomme et al. [D2] conducted a large-scale slab impact experiment to study the performance of a new type rock-shed under the impact of falling rock. In this experiment, different parts of the slab specimen were impacted. The experimental result shows that the slab suffered most severe damage when the impact load was applied to its center.

In addition to the aforementioned experiments, other low-speed impact experiments on slabs have been conducted. Kogure [K2] did a low-speed impact experiment to study the influence of reinforcement configuration on slab performance under impact loading. Later, Kogure [K3] did another low-speed impact test to study the impact behavior of sandwich slabs. Sohel and Liew [S9] also conducted low-velocity impact experiments on sandwich slabs. Bhatti [B5] experimentally studied the impact behavior of reinforced concrete slabs strengthened with fiber reinforced plastic sheet. Beckmann et al. [B3] studied the strain behavior of reinforced concrete slabs under impact load. Sato et al. [S5] carried out a large-scale impact experiment to study the impact behavior of large reinforced concrete slabs. Schellenberg et al. [S7] also conducted large-scale impact tests to study the performance of rock-fall protection galleries.

2.6 Numerical Simulations of the Impact Problem

Numerical simulation is commonly used in civil engineering to predict structural behavior in order to reduce the dependence on and expense of experimental work. Once a numerical model is verified by experimental data, it can be used to conduct parametrical study on the variables that were not considered during experiments. The previously discussed numerical simulation work on slab impact problems are briefly summarized in this section.

Delhomme et al. [D3] used the commercial FE software Ansys (version 7.1) to

simulate slab response under block impact. The FE analysis is based on consistent tangents, and the Newton-Raphson iterative approach is employed to solve the nonlinear problems. The time domain method (Newmark step-by-step integration method) is used to solve the dynamic equations of the FE model. A three-dimensional eight-nodes element called “Solid65” with linear displacement interpolation functions was used to model the reinforced concrete. The constitutive model of concrete used in this element can consider cracking in tension and crushing in compression based on Willam-Warnke failure criterion. This element can incorporate a maximum of three pieces of reinforcing bars, which are modeled as uniaxial sub-elements with arbitrary orientations that are capable of bearing axial forces. Plastic behavior and creep are considered in reinforcing bars as well. The amount of reinforcement is defined by specifying a volume ratio and orientation angles of reinforcing bars. The “Contact49” element was used to model the contact between the impact block and the reinforced concrete slab. However, strain rate effects on both concrete and steel were not considered in this analysis. The maximum mid-span displacement of a slab impacted by a falling rock was nicely estimated by this FE analysis.

Zineddin [Z3] used ABAQUS/Explicit FE code to simulate slab response under impact load. In his model, concrete was modeled with a traditional eight-node three-dimensional solid element based on the Lagrangian interpolation function. Steel was modeled with a beam element and assumed to have a perfect bond with concrete. Due to geometric symmetry, only a quarter of the impact system was modeled as shown in **Fig. 2.21**. A total of nine impact cases were analyzed. By comparing the experimental results with the numerical results, it is found that the simulated maximum slab displacement match well with the experimental data. However, the peak impact loads are on average overestimated by 30%. Zineddin concludes that a better correlation between predicted and measured responses might

be achieved by using more representative material models.

Fig. 2.21: Quarter FE model of impact testing system.

Compared with Ansys and ABAQUS, LS-DYNA is more widely used for simulating impact and explosion problems due to its efficient solver and large material database. For example, Sangi et al. [S3] used LS-DYNA software to simulate the slab impact problem. In this study, a solid element was used to mesh concrete. Reinforcing bar was meshed by beam elements that shared the same nodes with concrete elements. The established model is shown in **Fig. 2.22**. In this study, the concrete is modeled by Mat Concrete Damage Rel.3 (Material Type 72R3) and Mat Winfrith Concrete (Material 84). Mat Plastic Kinematic (Material Type 3) is used to model steel reinforcement. Influence of mesh size was also studied by comparing the results from the model with coarse mesh (size of solid element = 19 mm), and the model with refined meshed (size of solid element = 10 mm).

Fig. 2.22: Finite element model of concrete slab and steel reinforcements

The simulated impact forces were compared with the experimental results. As shown in **Fig. 2.23**, both concrete material models overestimate the peak impact load by approximately the same degree. Compared with the coarse mesh, the refined mesh only slightly enhances the accuracy of simulated results. In addition to this work, Kishi et al. [K7], Tai and Tang [T1], Dorsselaer et al. [D5], and Sadiq et al. [S2] also used LS-DYNA to simulate the response of reinforced concrete slabs under high- or low-velocity impact.

Fig. 2.23: Comparison between experimental data and numerical results using different material models and mesh sizes (P = load; t = time).

Apart from using existing commercial FE software, researchers also try to develop more reasonable material and element models to obtain better simulations of the slab impact problem. For example, Hrynyk [H4] improved the nonlinear analysis of reinforced concrete slab and shell structures by employing a layered thick-shell formulation that can account for out-of-plane (through-thickness) shear forces. Based on this analysis, a FE program was developed to analyze the slab impact problem. It is well known that FE method is based on continuum displacement field and the material models used in FE analysis are also mainly based on continuum mechanics, which theoretically contradicts the phenomenon of concrete cracking and ejection observed in impact problems. Therefore, some researchers also use the discrete element method (DEM) to study impact problems [R7].

2.7 Analytical Solutions for the Impact Problem

In general, the FE method is a case-by-case analysis approach, which means a new FE model must be built to analyze a specific problem. In addition, the effectiveness of a new model cannot be ensured unless it is verified by experimental data. Due to the large amount of effort and calculation time required by FE simulations, simpler and more efficient analytical methods are required to analyze the impact problem.

Abdel-Rohman and Sawan [A4] proposed three simple analytical methods (impact factor method, continuous mass method, and equivalent mass method) for analyzing the impact response of reinforced concrete slabs. In the impact factor method, the behavior of the impacted reinforced concrete slab under impact loading is assumed to be the same as that under static loading. Moreover, the impact is assumed to be plastic, which means the impactor and impacted member do not separate during impact. Based on these assumptions, an impact factor can be derived from the principle of conservation of energy. The maximum impact load and the maximum impact deformation in the reinforced concrete slab can be obtained by multiplying

the impact factor with the weight of the impactor and maximum static slab deformation. In the continuous mass method, the slab is idealized as a continuous elastic system. The equation of motion of the impacted slab can thus be formulated from plate theory, and the impact response of the slab can be obtained by solving this equation. The equivalent mass method is actually a single-degree-of-freedom (SDOF) method that models the impacted slab as a lumped mass. In this study, the slab damping is ignored, and the equation of motion of the slab is given as:

$$m_s \ddot{u}_s + k_s u_s = F_c \quad (2-10)$$

where m_s and k_s are the equivalent mass and equivalent stiffness of the slab, and F_c is the impact force estimated from the Petry formula [D6]. There are several limitations to these proposed analytical methods. First, the slab is assumed to be an elastic system in the continuous mass method. Based on this assumption, the continuous mass method cannot produce reliable analysis if the slab enters the plastic range. Second, the influence of the loading rate is not considered in any of the three methods. Moreover, the Petry formula was originally used to determine the penetration depth of a reinforced concrete member. Although it can be processed to give the expression of the impact force, the accuracy of the estimated impact force should be verified with experimental data, which was not provided in this study.

Stochino and Carta [S10] also used an SDOF model to study the impact response of reinforced concrete beams. In this model, the k_s is based on moment-curvature analysis, and the strain rate effect is also included. In addition, an energy model is proposed in this study. This model is based on the conservation of energy, which requires the external work W on the target to equal the sum of the strain energy U and the kinetic energy K of the beam:

$$W=K+U \quad (2-11)$$

However, no method is provided for evaluating the impact force. Therefore, in both models, knowing the impact force is a prerequisite for conducting the analysis.

Fujikake et al. [F4] proposed a 2-degrees-of-freedom (2-DOF) model for predicting the impact response of reinforced concrete beams (**Fig. 2.24**). In contrast to the SDOF model, 2-DOF has an extra lumped mass that represents the impactor. The 2-DOF model also includes a contact model to simulate the interaction between impactor and impacted beam. Therefore, the impact force can be solved during the analysis which rules out its dependence on impact load for initiating the analysis. Evaluation of the impact force during the impact process is also more theoretically rational than evaluating it in advance of impact because the value of the impact load is always affected by the slab response during impact. Moreover, Fujikake et al. also used the energy method to estimate the maximum beam displacement under impact by assuming a perfectly plastic collision between the impactor and the impacted beam.

Fig. 2.24: 2-DOF dynamic model [F4].

Delhomme et al. [D3] observed a phenomenon of progressive bending of the slab from his own numerical and experimental analyses. Consequently, he concluded that only a small area of the slab was bending during the period that the impactor is in contact with the reinforced concrete slab. This conclusion is similar to the concept of “effective length” introduced in **Section 2.3.2**. Thus, instead of the strain rate effect, Delhomme et al. attribute the enhanced load capacity of the slab and slab stiffness to the reduced “response area” of the slab during the contact period. Based on this idea, Delhomme et al. proposed a two-stage 2-DOF model as shown in **Fig. 2.25**. During the contact period (Contact model), the effective mass and stiffness of the slab are calculated based on a reduced slab size. After contact, the impactor is separated from the slab, and the residue response of the slab is evaluated using an SDOF model (post-impact model). In the post-impact model, the effective mass and stiffness of the slab are calculated based on its full size. A main drawback of this method is that the contact time should be known before analysis. In this study, the contact time is determined based on experimental results and numerical analysis. In addition, numerical analysis is also required to help determine most of the model parameters.

Fig. 2.25: Two stage 2-DOF model [D3].

Micallef et al. [M4] employed the same two-stage 2-DOF model for analyzing the

impact response of reinforced concrete slabs. Different from Delhomme et al.'s work, the model parameters can be obtained without experimental data or numerical simulation. The contact force and time are estimated using the Petry formula. The method for calculating the effective length proposed by Cotsovos [C5] is used to evaluate the response area of the slab during contact period. The contact stiffness is set through an iterative process to ensure the calculated contact time equals the contact time predicted by the Petry formula. However, this process cannot ensure that the calculated impact load agrees with the predicted impact load, and the predicted contact time would be questionable if the calculated and predicted impact loads do not match.

2.8 Summary

Based on the presented literature review, the following conclusions can be drawn considering existing research:

- The material properties of both concrete and steel reinforcement vary under different strain rates. In general, the compression and tension strengths of concrete increase with increasing strain rate. For steel reinforcement, both its yield and ultimate strengths increase with increasing strain rate. However, this enhancing effect decreases when the grade of steel reinforcing bar increases.
- The rapid loading test is very important for understanding the dynamic behavior of structural members. However, the rapid loading database on reinforced concrete slabs is currently very limited. Therefore, experimental work in this field is urgently needed to improve the understanding of this field. In this study, 18 reinforced concrete slabs with various shear reinforcement ratios, longitudinal reinforcement ratios, and slab depths were tested using a rapid loading machine. The assigned loading rates are in the range of 4×10^{-4} m/s to 2

- m/s, which corresponds to seismic and low-velocity impact problems. Unlike early research on reinforced concrete slabs, the induced inertial force and strain rates were studied as well.
- Moment-curvature analysis was used in most studies to obtain the load-displacement curves of reinforced concrete members. However, moment-curvature analysis is not suitable for reinforced concrete members with shear failure mode (e.g., **Fig. 2.1(a)**, **Fig. 2.4**). Thus, a new analytical method for obtaining the load-displacement curves of reinforced concrete slabs that can consider shear failure mode is proposed in this study. Current studies attribute the enhanced capacity of reinforced concrete members under dynamic loading to the strain rate effect or to a reduced response area of the slab. Each of these methods has a sound theoretical background. However, their application has not been sufficiently verified by experimental data.
 - Previous low-velocity impact experiments did not pay enough attention to the globalized flexural behavior of reinforced concrete slabs. Moreover, the energy capacity evaluated by summing up the impact energies of repeated impacts is not conservative. In this study, an impact experimental program was conducted to study the impact response of reinforced concrete slabs. Emphasis was placed on both local and global response of impacted slabs. The induced inertial effect and strain rate were also examined. The slab specimens were impacted only once in all the tests to study the relationship between impact energy and slab damage.
 - Many FE software packages can be used to analyze structural behavior. LS-DYNA is the most widely used software for impact problems due to its efficiency and high-quality material models. Therefore, LS-DYNA is used in

this study to numerically study the impact performance of reinforced concrete slabs. Emphasis is placed on various parameters' influence on the energy capacity of reinforced concrete slabs.

- By reviewing current analytical models on impact analysis. It is found that the assumptions for the impact factor and continuous mass methods are overly idealistic. Therefore, these two methods can produce only very coarse estimates of slab response. In addition, the SDOF model requires the impact force as an input, which can hardly be predicted in advance of impact. Thus, the 2-DOF model is used in this study to analyze the impact response of slabs, and in contrast to early models, this model can accurately predict the impact displacement and the impact force on reinforced concrete slabs that fail in punching shear.

CHAPTER 3

EXPERIMENTAL STUDY ON THE DYNAMIC BEHAVIOR OF REINFORCED CONCRETE SLABS UNDER RAPID LOADING

3.1 Introduction

In addition to static loading, reinforced concrete structures may experience various kinds of dynamic loading such as earthquake, impact, or blast. These types of dynamic loadings can lead to different loading rates on structure and result in a dynamic structural response that is different from the static behavior of structure. For example, the maximum load capacity, cracking patterns, and failure modes of structures may change under different loading rates [M10 and I1]. In general, strain rate effects on concrete and steel reinforcement and participation of inertial forces are the two main reasons for the aforementioned differences. Both concrete and steel reinforcement are strain-rate-sensitive materials. It has been proven that the yield strength of steel reinforcement and the tensile and compressive strengths of concrete increase with increasing strain rate [B7, F3, M1 and M2]. Reinforced concrete structures may also gain acceleration that induces inertial force under dynamic loads. As a result, it is important to understand the effect of the loading rate on the behavior of reinforced concrete structures and its relationship with strain rate and inertial effect.

Early experimental investigations mainly focused on the influence of loading rate on the behavior of reinforced concrete beams. For example, Mutsuyoshi and Machida [M10], Kulkarni & Shah [K9], Fujikake et al. [F4], and Adhikary et al. [A5, A6 and A7] carried out a series of tests on reinforced concrete beams by applying a

concentrated load at the beam's mid-span with the loading rate ranging from 0.0000071 to 2 m/s. Considerably less experimental data is available for reinforced concrete slabs. As discussed in the literature review, one well-documented experiment was conducted by Saito et al. [S4], who tested seven types of reinforced concrete slabs by applying a concentrated load at slab center with static (0.00003 m/s), low (0.03 m/s), and high (3 m/s) loading rates. Saito et al. found that the load carrying capacity of slabs increases together with the loading rate, and they attributed the increase in load carrying capacity to the strain rate effect on concrete and steel. Unfortunately, Saito et al.'s experiment did not provide information on strain rate and inertial force during testing. Therefore, it would be difficult for other researchers to further study and use the experiment data (e.g., to relate the data to earthquake or impact problems or use it to help develop analytical models). In order to gain greater understanding and provide high-quality experimental data in this research field, an experimental program was designed and conducted to investigate the behavior of reinforced concrete slabs under different loading rates. The experimental details such as specimen properties, test setup and procedures, instrumentation, and test results are presented in this thesis. Emphasis of this experiment was placed on the influence of loading rate on slab response. Unlike Saito et al.'s work, this experiment also investigates the strain rate and inertial effect that induced by rapid loading. The load carrying and energy absorption capacities and the failure mechanism of slabs obtained from this research will be useful in understanding the dynamic behavior of reinforced concrete slabs. The evaluated inertial effect and strain rate can also facilitate the development and verification of analytical or numerical models.

3.2 Experimental Overview

3.2.1 Design and preparation of slab specimens

A total of six types of 1,200-mm-square reinforced concrete slabs with different

longitudinal reinforcement ratios (ranging from 0.25% to 0.66%), shear reinforcement ratios (ranging from 0% to 0.56%), and slab depths (100 mm and 150 mm) were designed for the experimental program. For each type of design, three slabs were made for static loading rate, medium loading rate, and high loading rate tests. The specimen details are summarized in **Table 3.1** and the reinforcement layout is shown in **Fig. 3.1**. The process of preparing the slab specimens is shown in **Fig. 3.2**. Twenty-four 20-mm-diameter holes are made with PVC pipes on four sides of each slab, which are used for fixing the slab into the test support with bolts. Each type of slab is named by its thickness and longitudinal and shear reinforcement ratios. For example, 10-0.59 means the slab is 100 mm thick and has a longitudinal reinforcement ratio of 0.59% but no shear reinforcement. 15-0.39-0.28 means the slab is 150 mm thick and its longitudinal reinforcement ratio and shear reinforcement ratio are 0.39 and 0.28, respectively.

Table 3.1: Summary of slab specimens

Name of specimen	H, mm	f_c , MPa	LR	SR	Flexure capacity, kN	Shear capacity, kN	Expected failure mode	Observed failure mode
10-0.59	100	42.9	T10	-	161	140	Shear	Shear
15-0.25	150	42.9	T8	-	134	266	Flexure	Shear
15-0.39	150	45.0	T10	-	268	266	Shear	Shear
15-0.66	150	42.9	T13	-	462	266	Shear	Shear
15-0.39-0.14	150	42.3	T10	R6	268	326	Flexure	Shear
15-0.39-0.28	150	42.3	T10	R6	268	416	Flexure	Shear

Note: H = Slab depth; f_c = Compressive strength of concrete; LR = Longitudinal Reinforcement; SR = Shear Reinforcement

clear cover: 20 for 15-0.25

18 for 15-0.39, 15-0.39-0.28 and 15-0.39-0.56

15 for 15-0.66 and 10-0.59

⊙ : Strain gauges installed on top steel mat

● : Strain gauges installed on bottom steel mat

▲ : Strain gauges installed on shear reinforcement

Fig. 3.1: Reinforcement layout of slab specimens and geometry of shear reinforcements.

(a) manufacturing reinforcement cages

(b) attaching strain gauges

(c) preparing molds

(d) slab casting

Fig. 3.2: Preparation of slab specimens.

Although all the slabs are doubly reinforced, their longitudinal reinforcement ratios are calculated based on one layer of steel. When calculating the shear reinforcement ratio, the marginal slab area without shear reinforcement was excluded and the slab area was taken as 780×780 mm. All the slabs were constructed using normal-strength concrete, and the designed concrete strength is 40 MPa. Due to the large quantity of concrete required, the 18 slab specimens were cast in three batches. For each slab casting, three concrete cylinders 150 mm in diameter and 300 mm in height were cast with the same concrete. These cylinders were then tested at 28 days, and their average compressive strengths were found to be 45.0 MPa, 42.9 MPa, and 42.3 MPa for the

three batches. The maximum concrete aggregate size is 10 mm for all specimens. Because all the slabs have the same kind of longitudinal reinforcement layout. T8, T10, and T13 rebars were used as longitudinal reinforcements to produce different longitudinal reinforcement ratios. R6 rebar was used as shear reinforcement. Three pieces of steel bars were tested using a tensile test machine for each type of rebar. The properties of the steel bars are summarized in **Table 3.2**. In order to predict the failure mode of specimens during the static test, the flexure-resistant capacities of slabs were estimated using yield line theory, and the shear-resistant capacities of slabs were evaluated according to ACI 318-08 [A2]. The estimated slab capacities and failure modes are summarized in **Table 3.1**.

Table 3.2: Properties of reinforcing bars

Rebar	Diameter, mm	Yield strength, MPa	Ultimate strength, MPa
T8	8	443	504
T10	10	576	655
T13	13	605	690
R6	13	282	392

3.2.2 Experimental procedure and test setup

All six types of specimens described in the last part were tested under a static loading rate (0.0004 m/s), a medium loading rate (0.4 m/s), and a high loading rate (2 m/s), and each test was named by adding the loading rate (S/M/H) to the front of the specimen's name. The slabs were loaded by the servo-controlled rapid loading machine shown in **Fig. 3.3(a)** and **Fig. 3.3 (b)**. This loading machine was custom made for the experiment and its maximum loading capacity and loading rate are 980 kN and 4 m/s, respectively. Powered by a hydraulic accumulator, the steel piston of

the rapid loading machine could be instantly accelerated to the assigned loading rate and move downward. A circular steel loading plate 200 mm in diameter was placed in the center of the tested slab to increase the loading area of the rapid loading machine. The piston and loading plate generally maintained contact with each other throughout the test to avoid sudden contact or impact. The maximum loading displacement was fixed at 70 mm for all the tests.

The supporting system shown in **Fig. 3.3(c)** was used for static, medium, and high loading rate tests. A 1,200×1,200 mm steel support was constructed using a series of H steel beams that were fixed to the strong floor with high-tension bolts to provide enough rigidity during testing. Because the clear span between opposite H beams was only 800 mm, four thin steel plates 100 mm wide and 980 mm long were installed above the H beams to increase the clear span to 1,000 mm. The specimen was bolted to the support with 24 high-tension bolts. The diameter of the high-tension bolts is 18 mm, which is 2 mm less than the diameter of the holes left on slab specimen. These dimensions were chosen to minimize possible friction between slab and bolts. On top of the slab, 24 100-mm square steel plates were installed together with the bolts to confine the top surface of the specimen. In summary, the boundary condition of specimens during testing is close to a fully fixed boundary condition.

(a) Rapid loading machine.

(b) Rapid loading machine.

Fig. 3.3: Test setup for rapid loading test (continued).

(c) Supporting system.

Fig. 3.3: Test setup for rapid loading test.

3.2.3 Instrumentation

In order to provide good-quality data, various kinds of sensors were installed to monitor the experiment. In the static/medium/high loading rate test, a load cell with a capacity of 980 kN and a measuring frequency of 5 kHz was installed to the rapid loading machine to record the load. Two laser-type displacement sensors (D1 and D2) with a sampling rate of 50 kHz were installed to capture slab's displacement distribution during test. As the loading rate increases, the inertial force will also increase and cannot be neglected. Therefore, four accelerometers with the capacity of 1,000 times gravity and a measured frequency of 10 kHz were installed in medium and high loading rate tests to capture the acceleration distribution of slabs. The arrangement and designation of displacement sensors and accelerometers are given in **Fig. 3.4**. All of these sensors were mounted to measure the response of the slab's top surface. In order to observe the response and strain rate of reinforcement during the experiment, 12 strain gauges were installed to both top and bottom layers of the reinforcement mats. For slab specimens with shear reinforcement, four additional strain gauges were installed to the shear reinforcement. The arrangement and

designation of strain gauges is shown in **Fig. 3.1**.

Fig. 3.4: Arrangement of displacement sensors and accelerometers.

3.3 Experimental Results and Discussion of Rapid Loading Tests

3.3.1 Load-displacement relationship

The load versus loading point displacement curves of various specimens are shown in **Fig. 3.5**. The cable of the data logger was cut off during the test M-15-0.25, and its corresponding result was not recorded. Therefore, the response of test M-15-0.25 is not reported in this thesis. A higher loading rate would lead to a higher maximum load carrying capacity of specimen. The maximum load capacities and dynamic increase factors (DAF, defined as the ratio of specimens' maximum load capacity at any loading rate to its maximum static load capacity) of tested specimens are summarized in **Fig. 3.6**. The maximum load capacities of specimens are enhanced by

15% to 54% at the medium loading rate and 42% to 73% percent at the high loading rate compared with their static load capacities. The average dynamic increase factor under the medium loading rate and high loading rate were calculated to be 1.3 and 1.52, respectively. Studying the result, it is found that specimens with more longitudinal reinforcement have bigger DAFs than those with less longitudinal reinforcement. For example, Specimen 15-0.66 has the largest DAFs of 1.52 and 1.7 under medium and high rate loadings. By comparison, Specimen 15-0.25 has the smallest DAF of 1.3 for the high loading rate.

During all the tests, the load on the slab increased sharply when the slab was initially loaded and quickly reached its maximum value with limited displacement. Additionally, the stiffness of specimens during this stage was found to be affected by loading rate. It can be found from the load-displacement curves that the specimen's stiffness increases as the loading rate increases, and this phenomenon becomes very obvious during high loading rate tests where the stiffness of the specimen is much higher than the one loaded statically. This kind of stiffness enhancement effect leads to a higher load on specimens at the same displacement point when the loading rate increases. After reaching the maximum load, the responses of slabs can be categorized into two kinds depending on whether they have shear reinforcement or not. For slabs without shear reinforcement, their loads fell suddenly to very low values after a peak point, which implies typical shear failure mode. After that, their residual load capacities began to increase again at a very low rate and finally reached a second peak. For slabs with shear reinforcement, although the shear failure mode was still dominant, a much more ductile behavior can be observed than those without shear reinforcement. Their loads dropped more mildly and maintained relatively high magnitudes of residual load capacities at large displacement compare with specimens without shear reinforcement.

Fig. 3.5: Load-displacement curves, (continued).

Fig. 3.5: Load-displacement curves, (continued).

Fig. 3.5: Load-displacement curves.

Fig. 3.6: Maximum load capacity and dynamic amplification factor (DAF).

3.3.2 Energy absorption capacity

In addition to load carrying capacity, energy absorption capacity is another important specimen property that enables us to assess the performance of slabs from the energy aspect. Energy absorption capacity is more important for dynamic problems because it is easier to use energy than maximum load to quantify the dynamic input. Take the impact problem, for example. The impact energy can be easily calculated once the impact velocity and mass of the impactor are known, but the maximum load can hardly be obtained in advance. However, assessing the energy absorption capacities of structures during impact tests is not easy because the damage caused by the given input may be too light or too severe to the impacted structure. On the contrary, the energy absorption capacity of structures can be easily obtained during the rapid loading test by defining a failure criteria and integrating the structural member's load-displacement curve. Therefore, the energy absorption capacities of the tested specimens were also examined in this study.

Punching shear failure mode was found on all the specimens during the conducted experiment. Specimens are considered as to have failed once their load capacities dropped below 80% of the maximum load carrying capacity. The energy absorption capacities of slabs are calculated by integrating their load-displacement curves until the shear failure points. The energy absorption capacity of slabs and the corresponding dynamic energy amplification factor (DEAF, defined as the ratio of specimens' energy absorption capacity at any loading rate to its static energy absorption capacity) are given in **Fig. 3.7**.

Fig. 3.7: Energy absorption capacity and dynamic energy amplification factor (DEAF).

It can be observed that an increase in the slab's thickness, its longitudinal reinforcement ratio, or its shear reinforcement ratio can improve the energy

absorption capacity of the slab. The shear reinforcement ratio is the most effective approach because in addition to providing extra shear resistance, it also enhances the rigidity of slab through its confining effect. The energy absorption capacity of slabs also increases with increasing loading rate. The dynamic energy increase factor (DEIF) of the tested specimens ranges from 1.23 to 1.94 for the medium loading rate and 1.83 to 2.79 for the high loading rate. On average, slab's DEIFs were calculated to be 1.56 for the medium loading rate and 2.34 for the high loading rate. Additionally, it is found that the DEIF is larger for slabs with more longitudinal or shear reinforcement ratios.

3.3.3 Inertial effect

In order to assess the inertial effect of slabs under different loading rates, the measured acceleration data were used to evaluate the generated inertial force of slabs. A simple integration approach is proposed to fully use the recorded acceleration points to estimate the inertial force of slab. As shown in **Fig. 3.4**, a quarter of the slab is divided into four tributary integration areas according to the positions of accelerometers. For area I1, a uniformly distributed acceleration is assumed with the value of A1 sensor. For the rest of the remaining fan area, the acceleration is assumed to have a linear distribution and the values of its inner or outer boundary is the same as the accelerometer that installed in the boundary. Zero acceleration was assumed in the supported regions and slab corners. By assuming symmetric slab acceleration distribution, the inertial force of the slab can be calculated by summing up the integrated inertial forces of the four tributary areas and multiplying this sum by four. Similar inertial force histories were found for all specimens under the same loading rate. Therefore, the measured acceleration data and calculated inertial forces of test M-15-0.39 and H-15-0.39 are given in **Fig. 3.8** to show the typical inertial force history. The maximum inertial forces (absolute value) from all the medium and high loading rate tests are summarized in **Table 3.3**.

Fig. 3.8: Acceleration histories and integrated inertial force histories of 15-0.39, (continued).

Fig. 3.8: Acceleration histories and integrated inertial force histories of 15-0.39.

For medium loading rate tests, inertial force only becomes significant when the specimen's load capacity is reduced. Mostly, the inertial force is too small to affect the response of the slab and the measured load force. By comparison, large and widespread inertial forces are found during high loading rate tests. Due to the high

velocity of the loading plate, the slab is accelerated downward, and upward (negative) inertial force was generated at very early loading stage. Due to the interruption of generated inertial force, the slab stiffness is exaggerated during this period. In the region where the slab reaches its maximum load capacity, the whole inertial force comprises a small amount of upward inertial force in the loading area (A1) and a large amount of downward inertial force in the remaining area (A2, A3). The downward inertial force is not induced by the loading plate, but by the restoring force in those areas. Because the link between loading area and its surroundings is severely weakened due to the formation process of punching shear cone, the downward inertial force in the surrounding areas is more likely to be balanced by the support reaction force and may not effectively affect the measured load cell force. Therefore, the load cell force at this stage is very close to the true load capacity of the slab specimen.

Table 3.3: Maximum inertial forces

Name of specimen	Medium loading rate		High loading rate	
	Maximum inertial force, kN	Corresponding mid-span displacement, mm	Maximum upward inertial force, kN	Corresponding mid-span displacement, mm
10-0.59	50	13.9	102	1.0
15-0.25	—	—	186	0.67
15-0.39	62	10.3	136	0.85
15-0.66	57	7.9	129	0.69
15-0.39-0.28	33	55.5	136	0.65
15-0.39-0.56	60	64.7	126	0.55

3.3.4 Deflected shapes

As mentioned previously, the displacement histories of selected points were recorded by the installed displacement sensors. In this section, the displacement history measured by sensors D1 and D2 were used to plot the deflected shape along the mid-line of slabs. Uniform displacement distribution with the value measured by D1 is assumed in the loading area, and zero displacement is assumed in the supported region. By assuming symmetric displacement response in the slabs, the deflected shape can be plotted by linking the data points with straight lines. **Fig. 3.9** shows the deflected shapes of specimen 15-0.39 during static, medium, and high loading rate tests. For each loading rate, a new deflected shape was plotted when the slab's middle point moves every 2mm.

Fig. 3.9: Deflected shapes under different loading rates (15-0.39), (continued).

Fig. 3.9: Deflected shapes under different loading rates (15-0.39).

Obvious punching shear failure mode can be observed from the deflected shapes of 15-0.39 irrespective of loading rate. The exactly shear failure points are characterized by the rebound of unloaded slab area which indicate the full formation of punching

shear cone and its separation with nearby concrete. The same failure modes were found for all the remaining specimens by observing their deflected shapes. By summarizing the failure points of all the specimens, it was found the punching shear failure usually happens shortly after reaching the specimen's maximum load capacity, and the corresponding failure loads ranged from 60% to 99% of the specimen's maximum load capacity. This result verified the shear failure criteria (80% of specimen's maximum load capacity) proposed early for assessing the specimen's energy absorption capacity.

3.3.5 Strain, failure mechanism, and corresponding strain rate

As mentioned, strain gauges with the arrangement shown in **Fig. 3.1** were installed to capture strain development of reinforcements. In order to illustrate strain development and the corresponding failure mechanism, the strain histories of selected data points along the mid-line of specimen 15-0.39 under different loading rates are plotted in **Fig. 3.10**. Similar strain histories can be seen in spite of the loading rate.

Rapid tensile strain increase and yielding of bottom reinforcement can be observed at a very early loading stage (about 2 mm mid-point displacement). Yielding points are found in the sequence of B1, B2, and B3, which means a gradually globalized response of bottom reinforcement. In the meantime, the strain of the top reinforcement is still small and no yielding point can be found due to the limited slab deformation. Accompanied by a further increase in the displacement, the load capacity of the specimen continues to increase despite the yielding of the bottom reinforcement, which indicates the extra load capacity is mainly contributed by the concrete. As discussed earlier, the punching shear cone fully forms and punching shear failure occurs just after the specimen reaches its maximum load capacity. A sudden drop in tensile strain for both the bottom and top reinforcements can be found

at the same time. After that, the loading plate continues its movement and the strain of reinforcement increases again. Yielding of the top reinforcement also happens during this period. Because the concrete in the loading area has already failed at this stage, the residual load capacity of specimen is controlled solely by the reinforcement. Similar strain histories are found in the slab's diagonal direction. By analyzing all the collected strain data, it is found that all kinds of specimens share the same type strain behavior and failure mechanism described above. Additionally, by studying strain the histories of shear reinforcements of those slabs with stirrups, it is found that the yielding of shear reinforcements usually occurs when a specimen reaches its maximum load capacity, which proves the effectiveness of the shear stirrups in enhancing the slab's punching shear load capacity. Based on the above discussion, the general failure process of slabs is sketched in **Fig. 3.11**.

The strain rates of reinforcement were calculated by differentiating corresponding strain histories in time domain. The maximum strain rates of the B1/B4 data point of all the tested specimens are summarized in **Table 3.4**. The results show that the maximum strain rate is strongly affected by the loading rate. The average maximum strain rates are 0.75 s^{-1} and 4.35 s^{-1} for the medium and high loading rate tests. For the purpose of providing more detailed information on the relationship between loading rate and strain rate, the distribution of strain rate of T1, T2, B1 and B2 of test S-15-0.39, M-15-0.39 and H-15-0.39 are also plotted in **Fig. 3.10**. It is found that the strain rate increases with increasing loading rate. The strain rates of medium and high loading rate tests are mainly distributed from 10^{-3} s^{-1} to 10^0 s^{-1} . It is generally accepted that the load capacity enhancement effect on both steel and concrete should not been ignored when the strain rate is beyond 10^{-2} s^{-1} . Accordingly, it can be concluded that the stain rate effect contributed to the enhanced load capacities of the tested specimens under medium and high loading rate tests.

Fig.3.10: Strain histories and corresponding strain rate under different loading rates (15-0.39), (continued).

Fig.3.10: Strain histories and corresponding strain rate under different loading rates (15-0.39), (continued).

Fig.3.10: Strain histories and corresponding strain rate under different loading rates (15-0.39).

(a) Yielding of bottom reinforcement and global slab response at very early loading stage.

(b) Shear failure and fully formation of punching shear cone just after reaching slab's maximum load capacity.

(c) Residue load capacity of slab is mainly contributed by steel reinforcement.

Fig. 3.11: General failure process of slabs.

Table 3.4: Maximum strain rate of B1/B4 data point

Name of specimen	Static loading rate, s^{-1}	Medium loading rate, s^{-1}	High loading rate, s^{-1}
10-0.59	0.0015	0.33	6.3
15-0.25	0.0010	—	4
15-0.39	0.0029	0.95	3.06
15-0.66	0.0018	0.85	3.18
15-0.39-0.28	0.0008	0.36	6.5
15-0.39-0.56	0.0026	1.27	3.07

3.3.6 Crack pattern and damage

All the specimens exhibited similar damage characteristics and crack patterns as shown in **Fig. 3.12**.

Fig.3.12: Typical damages on top and bottom surfaces of slab (15-0.66).

On the front surface, except for a hole drilled by the loading plate in the loading area, the remaining slab area is nearly intact, and no crack can be observed. On the bottom surface, radial and circumferential cracks can be found, which are indicative of the slab's flexure and punching shear behaviors, respectively. Obvious concrete scabbing can be seen at the bottom surface, which mainly initiates between the outmost circumferential crack and the loading area. The concrete of the loading area is found to be completely separated from the nearby concrete and forms a punching shear cone. The punching shear cone is severely cracked but still manages to hang on the reinforcements. It is believed that at the final loading stage, the punching shear cone is too weak to transfer a large amount of load to the bottom reinforcement. By observing the exposed steel bars, globalized flexural behavior is found on the bottom reinforcements. Comparatively, the deformation of the top reinforcement is centralized in the loading area. However, the top steel reinforcement in the loading area is more severely bent than the bottom reinforcement, and fracturing of the top rebar is found on M-10-0.59. Therefore, it can be concluded that the residual load

capacity of the specimen is mainly contributed by the top reinforcement.

The final states of damage of various specimens under the high loading rate are presented in **Fig. 3.13**. Not much difference in the final damage status can be seen in comparing specimens with different depth and longitudinal reinforcement ratios. However, shear reinforcement is observed to be effective in reducing concrete scabbing.

Fig. 3.13: Damage of bottom surface of slabs tested under high rate loading,
(continued).

Fig. 3.13: Damage of bottom surface of slabs tested under high rate loading.

3.4 Conclusions

The main purpose of this experimental program is to investigate the behavior of reinforced concrete slabs under various loading rates. The results of this study support the following conclusions:

- Both the load carrying and energy absorption capacities of reinforced concrete slabs are found to increase when the loading rate increases. Higher loading rates can also lead to higher specimen stiffness at initial loading stage. The maximum strain rate increased approximately 100 times from the static loading rate to medium loading rate and another 10 times from the medium loading rate to the high loading rate. Inertial force also rises when the loading rate increases. However, the inertial effect did not become significant until the high loading rate was applied.
- Punching shear failure mode was observed for all the specimens that failed during the experiment regardless of loading rate and loading type. Yielding of the bottom reinforcement happens at a very early loading stage. The punching shear failure of specimens is featured by the full formation of the punching shear cone

and its separation from nearby concrete. The residual load capacity of the slab is mainly contributed by top reinforcements whose yielding usually occurred after punching shear failure.

- Both longitudinal and shear reinforcements are effective in enhancing the maximum load capacity of specimens. However, the benefits to the slab's energy absorption and load carrying capacities made by longitudinal reinforcement are limited comparing with the addition of shear reinforcement. The experimental results show that the damage to slabs under rapid loading can be efficiently reduced by adding shear stirrups.
- One main limitation of this experimental study is that only the punching shear failure mode of slabs was observed even though some specimens are designed to fail in flexure under static loading. Therefore, the influence of the loading rate on the transformation of failure modes of reinforced concrete slabs was not covered in this study. Future work in this area should place more attention on the loading rate's influence on the failure model of reinforced concrete slabs.

CHAPTER 4

CORRELATION BETWEEN LOW-VELOCITY IMPACT TEST AND RAPID-LOADING TEST

4.1 Introduction

By carefully choosing the loading rate, specimens tested under rapid loading can reach similar strain rates as those induced under impact loads, which implies that the displacement-controlled rapid loading test can be used to study impact problems. Compared with the impact test [D4, D2, Z2, C3 and H5], the displacement-controlled rapid loading test has a distinct advantage in obtaining the full load-displacement curve of tested slab specimens, which can be easily used to assess the load carrying and energy absorption capacities of tested structural members. Conversely, using the measured impact force history or the cumulative energy absorption method (the summation of imparted impact energies from repeated impact tests) to evaluate structure's resistant capacity is still unsatisfactory in impact experiments. In order to use the advantage of rapid loading test in the field of impact problems, a low-velocity impact experiment was conducted, and its experimental results were carefully analyzed to compare it with the high loading rate (2 m/s) test results presented in **Chapter 3**. This impact experiment can also provide useful information and data for assessing the performance of reinforced concrete slabs under impact loading.

4.2 Experimental Overview

4.2.1 Specimens

Six slab specimens were manufactured for this low-velocity impact experiment. These specimens have the same naming convention, design, and material properties

as those used in the rapid loading test. Therefore, the names, design and material properties of the specimens can be found in **Table 3.1**, **Table 3.2**, and **Fig. 3.1**.

4.2.2 Test setup and instrumentation

During the impact test, the six slab specimens were impacted using the free drop machine shown in **Fig. 4.1**. The drop height of the drop hammer was fixed at 1.5 m for all the tests, and the corresponding impact velocity was calculated to be 5.43 m/s. The drop weight system has a track to guide the movement of the drop weight in order to ensure it hits the target vertically. For comparison purposes, a 500-kg flat nose drop hammer with a diameter of 200 mm was used to impact the center of slab specimen and the impacted area on slab is the same as the loading area of the rapid loading test. In order to differentiate the impact test from the rapid loading test presented in **Chapter 3**, each impact test was differentiated by adding I in front of the specimen's name. The supporting system used for this impact test is exactly the same as the one used in the rapid loading test (**Fig 3.3**). Hence, the boundary conditions of the slab specimens in this impact experiment are the same as those in the rapid loading experiment.

Fig. 4.1: Impact testing system.

During the impact test, except for the two changes illustrated below, the arrangement of the remaining sensors and strain gauges was exactly the same as for the rapid loading experiment introduced in **Section 3.2.3**. First, because there is no load cell on the impact machine, two accelerometers were attached to the dropping hammer to record the acceleration history of the hammer, which can be used for calculating impact force history. Moreover, due to concerns for the safety of the equipment, the D2 displacement sensor was relocated to the D2' positions (**Fig. 3.4**). The arrangement of the other sensors was exactly the same as the rapid loading test. In order to reduce the noise, all the raw data from the accelerometers attached to the impactor were filtered using a fast Fourier transform (FFT) low-pass filter with a cutoff frequency of 1,250 Hz. The cut-off frequency was determined based on the following principles: (1) In the corresponding power spectrum, the cut-off frequency should include most of the high-amplitude sections; (2) The impulse (applied to the slab by the drop weight) calculated with filtered data should not change much compared with the impulse calculated with raw acceleration data.

4.3 Experimental Results of Impact Test and Its Comparison with Rapid Loading Test

4.3.1 Crack patterns and damage

As shown in **Fig. 4.2**, the specimens tested under impact loading are found to have a very similar crack pattern and damage style compared with those of the high loading rate test. On the front surface, the loading area of the specimen is penetrated by the loading hammer but the remaining area is nearly undamaged. The penetration depths of impacted specimens are measured after testing and are summarized in **Table 4.1**. Taking I-15-0.39 as a reference, the penetration depth is increased by 26% and 71% for I-15-0.25 and I-10-0.59, and reduced by 33%, 78%, and 94% for I-15-0.66, I-15-0.39-0.28, and I-15-0.66. On the bottom surface, radial and circumferential

cracks are found on all of the specimens. For specimens without shear reinforcement, the circumferential cracks are fully developed. Concrete scabbing initiated along these circumferential cracks and steel bars are exposed. For I-15-0.39-0.28, circumferential cracks are formed as well. However, only light concrete scabbing happens in a limited area, and no steel bar is exposed. The damage to I-15-0.39-0.28 is quite limited in that the diagonal flexural crack dominates the crack pattern and no concrete scabbing can be observed.

Table 4.1: Summary of impact experiment

Name of specimen	PD, mm	V_i , m/s	V_r , m/s	E_a , J	E_c , J	E_a/E	Estimated punching shear failure	Observed punching shear failure	$F \cdot t$, N·s	$M \cdot \Delta V$, kg·m/s
10-0.59	46.3	5.43	0.96	7129	3,368	2.12	Fail	Fail	3300	3,190
15-0.25	33.9		0.74	7219	3,937	1.83	Fail	Fail	3134	3,085
15-0.39	27.0		1.22	6988	4,185	1.67	Fail	Fail	3391	3,321
15-0.66	18.0		1.26	6963	4,477	1.56	Fail	Fail	3489	3,341
15-0.39-0.28	6.0		1.19	7006	8,139	0.86	Not fail	Not fail	3354	3,306
15-0.39-0.56	1.7		1.70	6637	12,907	0.51	Not fail	Not fail	3703	3,562

Note: PD = penetration depth; V_i = impact velocity; V_r = rebound velocity; E_a = absorbed Energy; E_c = energy absorption capacity (from high loading rate test); $F \cdot t$ = impulse; $M \cdot \Delta V$ = change of momentum of drop hammer.

Fig. 4.2: Damage on the bottom surface of slabs tested under impact loading.

Using the same method described in the rapid loading experiment, the deflected shapes of specimens during impact test are plotted using the data collected from the displacement sensors. For purposes of discussion, the deflected shapes of I-15-0.66 and I-15-0.39-0.56 are presented in **Fig. 4.3**. The exact punching shear failure point defined previously in the rapid loading test is the full formation of the punching shear cone, which is characterized by the reversed movement between the loading area and its surroundings. The same principle is used for the impact tests to determine whether punching shear failure happens during testing. It can be seen that the unloaded area of I-15-0.66 reaches its maximum deformation and begins to rebound at 2.8 ms. Simultaneously, the loading area continues its downward movement at a relatively high rate. The same behavior is found in I-10-0.59, I-15-0.39, and I-15-0.66. Therefore, these slabs can be considered as failed. For I-15-0.39-0.56, loaded and unloaded areas reach maximum displacement at 5.9 ms and begin to rebound together. Therefore, I-15-0.39-0.56 did not fail during the test. The loaded and unloaded areas of I-15-0.39-0.28 also reached their maximum displacement at nearly the same instant. However, a 2.5 ms lag is found between the rebound times of the loaded and unloaded areas. Although this specimen did not fail, this phenomenon indicates the severely weakened link between the loaded and unloaded regions.

Fig. 4.3: Comparison of deflected shapes between slab without reinforcement (I-15-0.66) and slab with reinforcement (I-15-0.39-0.56).

4.3.2 Absorbed energy

The absorbed energy of specimens during impact testing is calculated according to the impact and rebound velocities of the dropping hammer. The impact velocity is directly calculated from the 1.5-m drop height and the rebound velocity is obtained by differentiating the recorded displacement history of the dropping hammer. The absorbed energy together with other experimental details are also presented in **Table 4.1** and compared with the energy absorption capacities of slabs under the high loading rate test (2 m/s). It is found that the punching shear failure of specimens under impact loading is well predicted using the energy absorption capacity of specimens tested under the high loading rate.

4.3.3 Strain and corresponding strain rate

By examining the recorded strain histories, it is found that the strain behavior and damage process of impacted specimens are nearly the same as the specimens tested under high loading rate test. Yielding of bottom reinforcements happens in all specimens at the very beginning of impact. For specimens without shear stirrups, yielding of top reinforcements is observed after the punching shear failure point is reached. For specimens with shear reinforcements, no yielding of the top reinforcement is found due to the limited deformation and penetration depth.

The strain rate distribution of H-15-0.39 and I-15-0.39 are plotted in time domain in **Fig. 4.4**. Additionally, the maximum strain rates of the impact tests are listed in **Table 4.2** and compared with the experimental data from the high loading rate test (2 m/s). It is found that the maximum strain rates of the impact test and the high loading rate test are on the same order of 10^0 s^{-1} . Moreover, the strain rates of both tests mainly ranged from 10^{-2} s^{-1} to 10^0 s^{-1} . The similarity in strain rate between the impact test and the high loading rate test indicates a similar strain rate effect on the specimen materials.

Fig. 4.4: Comparison of strain rate distribution between high loading rate test (H-15-0.39) and impact test (I-15-0.39).

Table 4.2: Comparison of maximum strain rates (B1/B4 data point) between high loading rate test and low velocity impact test.

Name of specimen	Strain rate, s ⁻¹	
	High loading rate test	Impact test
10-0.59	6.3	9.15
15-0.25	4	5.01
15-0.39	3.06	5.85
15-0.66	3.18	2.65
15-0.39-0.28	6.5	5.4
15-0.39-0.56	3.07	4.8

4.3.4 Impact force and inertial effect

As mentioned earlier, two accelerometers are attached to the drop hammer to measure its acceleration. The inertia force of the drop hammer can be calculated by multiplying the mass of the impactor with the measured acceleration. Because the calculated inertia force equals the difference between impact load and the gravity of the impactor, the impact force can thus be calculated. Additionally, the inertial force of a specimen during the impact test is calculated using the same integration method used in the rapid loading test illustrated in **Section 3.3.3**. The impact force histories and corresponding inertial forces of different impact tests are given in **Fig. 4.5**. The maximum impact forces are listed **Table. 4.3**. For all the specimens, immediately after impact, the impact force and inertial force rise quickly and reach their maximum values. After that, the impact force reduces and forms a long tail. Accompanied with the reduction of impact force, the inertial force also drops and gradually dissipates. Compared with the high loading rate test, the induced inertial force under the impact load is more significant and interruptive, which makes it very difficult to directly relate the measured impact force with the true resistance of the slab.

The impulse of each impact test was evaluated by integrating the corresponding

impact force history. Additionally, the change of momentum of the drop hammer during each test was calculated using the mass, impact, and rebound velocities of the drop hammer. The impulse and change of momentum of the drop hammer are listed in **Table 4.1**. It can be seen that in each test, the impulse is nearly same as the change in the momentum of drop hammer, which proves the conservation of momentum during the impact test.

Fig. 4.5: Impact force and inertial force during impact test.

Table 4.3: Maximum impact forces

Name of specimen	Maximum impact force, kN	
	Test	FE simulation
10-0.59	512	577
15-0.25	798	935
15-0.39	990	994
15-0.66	1071	981
15-0.39-0.28	674	972
15-0.39-0.56	799	961

4.4 Conclusions

In this chapter, an impact experimental program is introduced and compared with the rapid loading experiment to study their correlations. Similar crack patterns, damage, and failure process of specimens were found through the comparison of the results from high loading rate test and the impact test. Additionally, the distributions of strain rate of the slab specimens subjected to low-velocity impact (5.43 m/s) and high rate loading (2 m/s) were found to be very similar to one another. The punching shear failure of impacted slabs was well predicted by their energy absorption capacities obtained from the high loading rate test. In sum, the high loading rate test can produce similar slab behavior and strain rate as the low-velocity impact problem while reducing the interruption from the inertial effect. Because the load and energy capacities of slabs can be easily obtained from the displacement-controlled high loading rate test, the high loading rate test can be considered as a supplementary approach for studying impact problem.

Additionally, it is found that the impact damage on slabs can be effectively reduced by shear reinforcement. By comparison, the capacity of slabs under impact loading is only slightly enhanced by increasing longitudinal reinforcement.

CHAPTER 5

BEHAVIOR OF REINFORCED CONCRETE SLABS UNDER LOW-VELOCITY IMPACT

5.1 Introduction

In **Chapter 4**, six slabs were tested under low-velocity impact to reveal the correlations between the impact and rapid loading experiments. In this chapter, the experimental results of more slab specimens under low-velocity impact are presented and discussed to form a more systematic understanding of the impact behavior of reinforced concrete slabs. The effects of impact energy and of the impactor's diameter and nose shape on slab damage and response were examined through the designed impact experimental program.

Reinforced concrete structures may be subjected to impacts at low velocities during their service lives. Typical scenarios include impact from debris brought by tsunamis or floods, impact from falling objects during earthquakes, collision from vehicles, etc. The impact load, which is characterized by the application of a force for a short duration but at high intensity [F4], can generate high strain rate, significant inertial effect, and stress wave on impacted reinforced concrete structure and thereby lead to a dynamic structural response that is different from the static performance of structure. Therefore, it is important to understand the behavior of reinforced concrete structures subjected to impact loading in order to facilitate the corresponding design and evaluation works.

Most of the previous research on the impact problem of reinforced concrete slabs focused on the localized behavior of impacted slabs. This yielded a number of empirical equations for evaluating localized damage indexes such as penetration

depth, scabbing limits, and perforation limits [S8, L1 and T2]. These empirical equations are mainly based on the experimental database concerning high-velocity impact (impact velocity > 100 m/s) such as ballistic or aircraft collisions, and their application to low-velocity impacts (impact velocity < 10 m/s) is sometimes not satisfactory [C3]. However, under low-velocity impacts, the global behavior of slabs becomes more important [Z2 and H5] than their localized behavior, which dominates the slab response in high-velocity impact scenarios [Å9, D1, S11, and S12]. Therefore, in order to gain a better understanding of the behavior of reinforced concrete slabs subjected to low-velocity impact, an experimental program was designed and conducted. Both the local and global behaviors of slabs are presented in this chapter through the recorded deformation, strain, and acceleration data. The induced strain rate and inertial effect are presented as well. In addition, influence of the diameter and nose shape of the impactor on the slab response is also examined. Lastly, a novel loading protocol was used in this experimental program. Among different tests, the impact energy or impact momentum was fixed by changing both the mass of the impactor and the impact velocity. This loading protocol enables researchers to study impact problems from energy and momentum perspectives and to learn how they are related to the response and damage of slabs.

5.2 Experimental Investigation

5.2.1 Specimens

Fifteen reinforced concrete slabs with the design shown in **Fig. 5.1**, which is the same as the 15-0.39 specimen introduced in **Section 3.2.1**, were made for this experimental program. The compressive strength of the concrete is 42.3 MPa. The diameter, yielding, and ultimate strengths of the reinforcing bar are 10 mm, 576 MPa, and 655 MPa, respectively. The flexural strength of the designed specimen

was calculated to be 268 kN based on the yield line theory. The shear capacities of the designed specimens were assessed according to ACI 318-08 [A2]. The calculated shear capacities are 196 kN (assuming a 100-mm diameter circular loading area in the slab center) and 282 kN (assuming a 200-mm diameter circular loading area in the slab center). Comparing the calculated flexure and shear capacities of specimen, it is found that the specimen is predicted to have a shear failure mode under static loading when the diameter of the loading area is 100 mm, but a flexure failure mode when the diameter of loading area becomes 200 mm.

Fig. 5.1: Size and reinforcement layout of slab specimens.

5.2.2 Test procedure

The details of this experimental program are presented in **Table 5.1**. These 15 slab specimens were tested in 3 groups named 10H, 10F, and 20F, according to the nose shape and diameter of the impactor. As shown in **Fig. 5.2**, 100-mm diameter hemispherical nose (10H), 100-mm diameter flat nose (10F), and 200-mm diameter

flat nose (20F) impactors were used for these three groups. In each group, five slabs were impacted under different drop weights and drop heights, which were carefully chosen to produce the same impact energy or impact momentum. The tests named a, b, and d had the same impact energy, and the tests named a, c, and e shared the same impact momentum. The test setup and instrumentation of this experimental program are exactly the same as the impact tests introduced in **Chapter 4**.

Table 5.1: Details of experimental program.

Designation of test	M, kg	h, m	V_i , m/s	E_i , J	M_i , kg·m/s	d, mm	Nose shape
10H-a	200	3	7.67	5,886	1,534	100	Hemispherical
10H-b	300	2	6.26	5,886	1,878		
10H-c	300	1.33	5.11	3,914	1,534		
10H-d	500	1.2	4.85	5,886	2,425		
10H-e	500	0.48	3.07	2,354	1,534		
10F-a	200	3	7.67	5,886	1,534	100	Flat
10F-b	300	2	6.26	5,886	1,878		
10F-c	300	1.33	5.11	3,914	1,534		
10F-d	500	1.2	4.85	5,886	2,425		
10F-e	500	0.48	3.07	2,354	1,534		
20F-a	200	3	7.67	5,886	1,534	200	Flat
20F-b	300	2	6.26	5,886	1,878		
20F-c	300	1.33	5.11	3,914	1,534		
20F-d	500	1.2	4.85	5,886	2,425		
20F-e	500	0.48	3.07	2,354	1,534		

Note: M = Mass of impactor; h = Drop height; V_i = Impact velocity; E_i = Impact energy; M_i = Impact momentum; d = diameter of impactor.

Fig. 5.2: Different noses installed on the impactor: (a) 100-mm diameter hemispherical nose; (b) 100-mm diameter flat nose; (c) 200-mm diameter flat nose.

5.3 Experimental Results and Discussion

5.3.1 Crack patterns and damage

The damage on the front and bottom surfaces of the tested slab specimens is shown in **Fig. 5.3** and **Fig. 5.4**. On the front surface, apart from a hole drilled by the drop weight in the impacted area, the remaining slab area was nearly intact. For each specimen, the drilled hole had the same diameter and shape as the nose of the impactor, and its corresponding penetration depth was measured after testing and listed in **Table 5.2**. In general, the penetration depth was found to increase when impact energy increases despite the equivalence of impact momentum. Differences in penetration depth were not significant among tests a, b, and d, which have the same impact energy. The average penetration depths are 16.6 mm, 17.8 mm, and 8.8 mm for the 10H, 10F, and 20F groups. This result indicates that penetration depth decreases when the diameter of loading area increases. Considering the influence of impactor's nose shape, it is found that the hemispherical nose can penetrate slabs with relatively small impact energy, while the flat nose causes a larger penetration depth than the hemispherical nose when the impact energy is large.

Fig. 5.3: Typical damage on the front surface of specimens.

(a)

(b)

(c)

Fig. 5.4: Damage on the bottom surface of specimens (a) 10H group; (b) 10F group; (c) 20F group, (continued).

Fig. 5.4: Damage on the bottom surface of specimens (a) 10H group; (b) 10F group; (c) 20F group.

Similar to the front surface, damage on the bottom surface of the slabs also increased with increasing impact energy. For tests 10H-e, 10F-e, and 20F-e, only light radial cracks, which are indicative of the slab's flexural behavior, were found on the bottom surfaces of the tested specimens. For tests 10H-c, 10F-c, and 20F-c, in addition to radial cracks, circumferential cracks that are indicative of slab's punching shear behavior were also observed. Compared with 20F-c, the damage to 10H-c and 10F-c is more severe in that light concrete scabbing can be observed. With the further increase of impact energy (tests a, b, and d), the width of circumferential cracks increased and more severe concrete scabbing were seen, which are representative of typical punching shear failure mode. The concrete scabbing mainly initiated between the outmost circumferential cracks and the loading area. Reinforcing bars in the scabbing region were exposed and bent. In the loading area, the concrete was severely crushed but still hung on the reinforcement. The slab area surrounded by the

exterior circumferential crack is defined as damaged area, and the diameter of the damaged area was measured for each specimen and listed in **Table 5.2**. The average diameters of damaged areas for groups 10H, 10F, and 20F are 695 mm, 735 mm, and 872 mm, respectively. In sum, the damage to slabs under an impact load is strongly related to impact energy.

Table 5.2: Summary of experimental results.

Designation of test	PD, mm	DA, mm	Punching shear failure	V_r , m/s	E_a , J	E_a/E_i	MIF_{test} , kN	MIF_{FEM} , kN
10H-a	17.5	603	Yes	1.28	5,390	0.97	415	395
10H-b	26.5	815	Yes	1.08	5,711	0.97	452	349
10H-c	13.5	707	About to fail	1.04	3,763	0.96	397	326
10H-d	21.4	654	Yes	0.88	5,688	0.97	389	375
10H-e	4.3	—	No	0.93	2,139	0.91	297	316
10F-a	23.7	761	Yes	1.31	5,577	0.97	694	646
10F-b	26.6	703	Yes	1.05	5,721	0.97	651	636
10F-c	9.8	610	Yes	0.72	3,846	0.98	603	584
10F-d	28.1	871	Yes	0.99	5,636	0.96	672	627
10F-e	0.7	—	About to fail	1.07	2,071	0.88	538	556
20F-a	10.3	853	Yes				-	-
20F-b	18.6	755	Yes				-	-
20F-c	3.6	878	About to fail	0.9	3,802	0.97	680	757
20F-d	11.3	901	Yes	0.71	5,757	0.98	794	846
20F-e	none	—	No	1.34	1,908	0.81	705	670

Note: PD = Penetration depth; DA = Diameter of damaged slab area; V_r = Rebound velocity;

E_a = Absorbed energy; MIF_{test} = Maximum impact force in test; MIF_{FEM} = Maximum impact force in FEM.

5.3.2 Deflected shapes

In this section, the measured displacement histories of D1 and D2' data points (shown in **Fig. 3.4**) are used to plot the deflected shape along the midline of the front surface of tested slab. Uniform displacement with the value measured by D1 is assumed in the loading area, and zero displacement is assumed in the supported region. By assuming a symmetric displacement response of the slab, the deflected shape is plotted by linking the measured data points with straight lines. For example, the deflected shapes of slabs during the tests of 20F-e, 20F-c, and 20F-d are plotted at an interval of 1.5 ms and shown in **Fig. 5.5**. For 20F-e, an elastic flexural behavior could be observed that the impact caused a global deformation on the slab. After reaching its maximum displacement at around 6 ms, the slab began to rebound and the residue deformation was very limited. For test 20F-c, the global flexural response could be seen in the early loading stage. Additionally, the deflection of the impacted region increased much more rapidly than the deflection of the unloaded area, which suggests punching shear behavior. This punching shear behavior was accompanied by the development of shear cracks and the formation of the punching shear cone. After 4.5 ms, the unloaded area began to rebound while the loaded area continued its downward movement for another few millimeters before rebounding at 7.5 ms. This phenomenon suggests a severely weakened concrete link between the loaded and unloaded slab areas due to the formation process of the punching shear cone. For test 20F-d, the impact energy was further increased and a typical punching shear failure mode could be observed from the deflected shape. The exact punching shear failure point was characterized by the obviously reversed movement between the loaded and unloaded area, which indicates the full formation of a punching shear cone and its separation from the surrounding concrete. This principle of determining the punching shear failure points is used for other specimens, and their failure situations are summarized in **Table 5.2**. For specimens with deflected shapes that are similar to

20F-c, they were at the brim of punching shear failure because the impacted area still had certain connections with the surrounding concrete. Therefore, these specimens are categorized as “about to fail.” From **Table 5.2**, it can be found that the 100-mm diameter flat nose impactor requires the least energy to cause punching shear failure on the designed specimen. By comparison, slabs that were impacted by the 200-mm diameter flat nose impactor had a longer shear perimeter than those impacted by the 100-mm diameter flat nose impactor and therefore possess a greater capability to resist punching shear failure.

Fig. 5.5: Comparison of deflected shapes of slabs under different impact energies, (continued).

Fig. 5.5: Comparison of deflected shapes of slabs under different impact energies.

5.3.3 Strain histories of steel reinforcements and damage process

In order to illustrate the strain development of reinforcement, the recorded strain histories of tests 20F-d, 10F-d, and 10H-d are plotted in **Fig. 5.6**. In each plot, the times of punching shear failure and rebound of impactor are marked with vertical lines. For 20F-d, tensile strain increased rapidly in the bottom reinforcement at a very early loading stage. Yielding points of the bottom reinforcement were found in the sequence of B1, B2, and B3, which indicates a global flexural response of impacted specimen. In the meantime, no yielding of the top reinforcement could be seen due to the limited slab deformation. During the same period, inclined shear cracks also initiated in the impacted region and began to propagate. Once the shear cracks had fully developed, punching shear failure occurred, and the severely damaged concrete in the loading area was unable to transfer load to the bottom reinforcement, which led to a sudden release of strain in the bottom reinforcement. By contrast, the strain on the top reinforcement continued to increase after the failure point, and the residual impact was mainly resisted by the top reinforcement. In general, the strain behavior and damage process of 10F-d and 10H-d are similar to that of 20F-d, that is, (1) yielding of the bottom reinforcement and flexural behavior in the early stages of impact; (2) failure of the specimen was governed by the failure of the concrete regardless of the reinforcement strain; and (3) strain of the top reinforcement continued to increase after the failure point had been reached.

Despite their similarities, the strain of the T1 data point, which represents mass penetration of the tested specimen, was very different among these three tests. For 20F-d, yielding of T1 happened after the shear failure of specimen. For 10F-d, yielding of T1 occurred at the instant of shear failure. For 10H-d, T1 yielded much earlier, before the flexural behavior and the shear failure of the specimen. Due to stress concentration, the hemispherical nose impactor can easily penetrate the impacted slab with a small amount of impact energy. On the other hand, impact

energy and force are not transferred efficiently from the hemispherical nose impactor because of its limited impact area. This explains why the hemispherical nose impactor requires more impact energy to cause punching shear failure on the impacted slab compared to flat nose impactors. The observed difference among the strain histories of T1 suggest that mass penetration does not correlate well with the development of shear cracks and the formation of a punching shear cone. As such, penetration depth should not be used as the sole index for determining the damage of impacted targets (In many empirical equations used to evaluate the impact performance of reinforced concrete members, the damage of the impacted target is merely correlated with the penetration depth). The strain rate of reinforcement was calculated by differentiating the measured strain histories and the maximum strain rate of point B1 ranging from 1.3 s^{-1} to 3.2 s^{-1} for all the tests.

Fig. 5.6: Strain histories of top and bottom reinforcements of tests 20F-d, 10F-d and 10H-d, (continued).

Fig. 5.6: Strain histories of top and bottom reinforcements of tests 20F-d, 10F-d and 10H-d.

5.3.4 Ratio between absorbed energy and impact energy

As mentioned earlier, accelerometers were attached to the impactor to measure its acceleration history. By integrating the measured acceleration history in the time

domain, the velocity history of the impactor was obtained and the rebound velocity could be determined. Once the rebound velocity of the impactor is known, the energy imparted to slabs can be calculated by subtracting the residual kinetic energy of the impactor from the impact energy. The calculated rebound velocities, absorbed energies, and its ratio over impact energies of different tests are listed in **Table 5.2**. The accelerometers of tests 20F-a and 20F-b were damaged during testing and therefore no result is presented for those two tests.

It is found that the ratio between imparted energy and impact energy is strongly correlated with the damage level of tested specimens. For the tests of 10H-e, 10F-e, and 20F-e, where specimens were slightly damaged, approximately 80% to 90% of the impact energy was imparted to the tested slabs. For the remaining tests, 95% to 98% of the impact energy was imparted to the tested slabs. This phenomenon is expected for the less damaged slabs as more impact energy is stored through the temporary elastic deformation of slab. A portion of the stored energy will return to the impactor when the elastic deformation recovers. On the contrary, severely damaged slabs have already entered the plastic stage, and more impact energy is dissipated in the format of permanent deformation or fracture damage and limited energy can return to the impactor.

5.3.5 Impact force and inertial effect

The acceleration history of impactor is multiplied with its mass during each test to calculate the corresponding impact force history. The calculated impact force histories are shown in **Fig. 5.7**. The impact force histories of tests 20F-a and 20F-b are not presented here because the accelerometers were damaged during these two tests and no acceleration data was recorded. The impact force histories of different tests are similar in shape, which comprises a head part with high magnitude but short duration and a tail part with lower magnitude but longer duration. The maximum

impact forces during different tests are listed in **Table 5.2**. It can be seen that the maximum impact force increases when contact area between impactor and specimen increases such that the average maximum impact forces of 10H, 10F, and 20F groups are 390 kN, 632 kN, and 726 kN, respectively. In each group, the maximum impact force is found to increase when impact energy increases, and a similar maximum impact force was observed for tests a, b, and d, which have the same impact energy. However, it is worth mentioning that the maximum impact force usually occurs around 1 ms after impact, and the damage and deformation of the specimen is very limited. No direct relationship can be observed between the maximum impact load and the resistance capacity of the specimen.

With the measured slab acceleration data, the inertial force of the reinforced concrete slab during impact is evaluated using the integration method introduced in **Section 3.3.3**. When ignoring the damping force, the difference between the impact force and the support reaction force is mainly balanced by the inertial force during impact. Therefore, the support reaction force is roughly estimated via subtracting the inertial force from the impact force. The estimated inertial forces and reaction forces are plotted in **Fig. 5.7** as well. Slabs gained significant downward acceleration immediately after impact and upward (negative) inertial force was induced. At this stage, the impact load is mainly balanced by the generated inertial force. This phenomenon helps explain why the maximum impact load cannot be considered as slab's resistance capacity. Accompanied with the increase of slab deformation, the reaction force increased and reversely accelerated the slab. Once the reaction force surpassed the impact load, the inertial force became downward (positive) to balance the reaction force. Therefore, it can be concluded that the inertial effect is significant and interruptive during impact testing, which makes it difficult to directly relate the measured impact or reaction force with the true impact resistance of the tested specimens.

Fig. 5.7: Time histories of impact, inertial and reaction forces, (continued).

Fig. 5.7: Time histories of impact, inertial and reaction forces.

5.4 Conclusions

The main purpose of this chapter is to experimentally investigate the behavior of reinforced concrete slabs under low-velocity impact. The results presented in this thesis support the following conclusions.

- The damage extent of slabs under low-velocity impact increases with increasing

impact energy. Changing the mass and drop height of the impactor only slightly affects the slab damage when the impact energy is fixed. Therefore, in the domain of low-velocity impact, it is possible to directly relate the impact energy to the slab response and damage without knowing the exact velocity and mass of the impactor. Compared to the flat-nose-shaped impactor, the hemispherical-nose-shaped impactor can cause penetration on reinforced concrete slabs with relatively small impact energy, but it requires more impact energy to cause punching shear failure. In addition, more impact energy is required to cause reinforced concrete slabs to fail when the diameter of the impactor increases.

- Although some specimens were designed to have a flexural failure mode under static loading, punching shear failure mode was observed for all the tested specimens. The corresponding damage process could be summarized as: (1) globalized flexural response at early loading stage; (2) shear cracks initiate and propagate to form the punching shear cone, and punching shear failures occur when the punching shear cone is fully developed and separated from its surrounding concrete; (3) concrete in the loading area was crushed after punching shear failure and the residue impact of the slab was mainly resisted by the top reinforcement.
- The impact force reaches its maximum value immediately after impact and was not found in the same phase of the punching shear failure of the impacted slab. Large inertial forces were found during impact, which were coupled with the both impact and support reaction forces. Therefore, the impact force or support reaction force cannot be directly considered as the true resistance capacity.

CHAPTER 6

FINITE ELEMENT ANALYSIS OF REINFORCED CONCRETE SLABS SUBJECTED TO LOW-VELOCITY IMPACT

6.1 Introduction

As discussed in the literature review, performing impact experiments is costly and time consuming. For this reason, FE analysis is used to acquire information that was not covered or recorded during the experimental work.

The evaluation of the maximum load carrying capacity of reinforced concrete members under impact loading has long been a challenge for engineers. In some research, the maximum impact force or maximum support reaction force were related to the load carrying capacity of the tested specimens [C5 and K4]. However, both impact and support reaction forces obtained from impact testing comprise a large amount of inertial force, and therefore cannot accurately represent the actual load-carrying capacity of slabs. In the experimental part, a close relationship has been observed between the impact energy and the damage extent of slabs. However, determining the energy capacity of a reinforced concrete member under impact has achieved limited success as the impact energy needs to be adjusted gradually through a series of trial tests to determine the exact energy capacity of the designed specimen. In some past studies, the reinforced concrete targets were impacted repeatedly until failure with relatively small impact energies, and the corresponding impact energy capacity was calculated by summing up the impact energies of these repeated impacts (Z1 and R2). However, this method tends to overestimate the energy capacity of reinforced concrete members because during each impact, a portion of impact energy

is dissipated through structural dampers without causing damage to the specimen. Although impact experimental results can produce qualitative explanations of the relationship between the impact input and the slab response, they cannot quantify this relationship. Considering the difficulty of experimentally assessing the energy capacity of tested specimens, a three-dimensional (3D) nonlinear FE analysis was performed using the LS-DYNA software. The proposed modeling method was verified with the experimental results and then used for a series of parametric studies. Emphasis of this FE analysis is placed on various parameters' influence on the energy capacity of impacted reinforced concrete slabs. In order to evaluate the energy capacity of reinforced concrete slabs under low-velocity impact, the impact energy is gradually adjusted during the analysis to find the amount of impact energy that is just enough to cause the modeled reinforced concrete member to fail.

6.2 Details of FE Modeling

In this study, the commercial FE software LS-DYNA is employed because of its proven effectiveness for analyzing impact problems and its high-quality database of material models. Moreover, LS-DYNA also contains an efficient solver that uses an explicit method to solve dynamic FE formulations. In this section, the FE modeling approach and the selected material models are illustrated in detail.

6.2.1 Structural configuration

Fig. 6.1 shows the three-dimensional FE model of the slabs tested in **Chapter 5**. Due to geometric symmetry, only a quarter of the impact system is modeled to reduce the calculation efforts. Translational restrains are assigned to both of the symmetry planes in the normal direction. In this model, the concrete, impactor, and support are discretized by the eight-node hexahedron element with a Lagrange interpolation function (**Fig 6.2(a)**). The mesh size for concrete elements is around 10 mm in each direction, and the corresponding aspect ratio is about one. A three-dimensional beam

element with three nodes (one direction node defining the normal direction of the beam element) is used to model the steel reinforcement (**Fig 6.2(b)**). These beam elements share the same nodes and boundaries with the concrete element and therefore a perfect bond between the concrete and steel reinforcement is assumed in this model. This assumption is reasonable for an impact problem because concrete members usually fail within several or tens of milliseconds after impact, and this amount of time is usually insufficient for developing significant bond slip between the concrete and the reinforcing bars. In order to better represent the experimental boundary condition, the support, which contains two steel strips beneath the slab and six steel plates confining top surface of slab, is modeled explicitly. The steel strips and plates are connected with steel bars that are inserted through the slab. These two steel strips are fully fixed and no restraint is assigned to the slab in the supported region. With the described modeling method, contact occurs between the impactor and the slab or between the support and the slab. The “Contact Automatic Surface to Surface” algorithm in LS-DYNA is used to model the contact during analysis.

Fig. 6.1: Quarter FE model of reinforced concrete slab: (a) mesh of concrete; (b) steel reinforcements.

Fig. 6.2: Model elements: (a) eight-node solid element; (b) three-node beam element.

6.2.2 Constitutive model of concrete

The constitutive relationship, also known as the stress-strain relationship, is the fundamental part of the FE analysis. For three-dimensional solid elements, a three-dimensional constitutive model of concrete is required to transform the strain space into stress space. Compared to many other civil engineering materials, the constitutive relationship of concrete is much more difficult to define due to its heterogeneous composition, highly nonlinearity, and cracking behavior. So far, many constitutive models for concrete, such as linear and nonlinear elasticity models, elasto-plastic models, elasto-plastic damage models, fracture models, and endochronic models, have been developed by researchers. Different models may have distinct advantages when simulating particular problems. For impact problems, a suitable constitutive law for concrete should be strain-rate sensitive and capable of capturing its nonlinear behavior. Additionally, it should be easy to define with a few conventional material parameters and have a rational failure surface. LS-DYNA provides many constitutive models that are capable of representing concrete material, such as Mat Soil and Foam (Material Type 5); Mat Soil and Foam Failure (Material Type 14); Mat Pseudo Tensor (Material Type 16); Mat Geologic Cap Model (Material Type 25); Mat Concrete Damage (Material Type 72); Mat Winfrith Concrete (Material type 84); Mat Brittle Damage (Material Type 96); Mat CSCM

Concrete (Material Type 159); and Mat Concrete Damage Rel3 (Material Type 72R3).

In this study, the Continuous Surface Cap Model (CSCM) constitutive model is chosen for modeling the constitutive behavior of concrete. This concrete model is primarily developed to simulate the deformation and failure of concrete in roadside safety structures impacted by vehicles and hence is quite suitable for impact problems. This concrete model is an isotropic elasto-plastic damage model that can consider both hardening and softening effects. This model is based on isotropic constitutive equations and contains a continuous yield surface that is formulated in terms of three stress invariants of the deviatoric stress tensor as shown in **Fig. 6.3**. This yield surface can account for ductile and brittle damages with a damage parameter ranging from zero (undamaged) to one (fully damaged). Strength softening of concrete is also considered with a softening function. This model also allows for element erosion once the damage parameter reaches one. Therefore, fully damaged concrete elements are deleted from the FE model and the concrete damage can be visualized.

Fig. 6.3: General shape of yield surface of CSCM concrete model in two dimensions [M8].

It is necessary for a concrete model to include the strain rate effect when dealing with high strain rate problems. The experimental results of the strain rate effect on the compressive and tensile strength of concrete are summarized in **Fig. 6.4** and **Fig. 6.5**. It can be found that concrete strength increases with increasing strain rate. When strain rate is in the range from 1/s to 10/s (low-velocity impact), the peak strength of concrete is increased from 20% to 50% in compression, and more than 100% in tension. In the selected model, the strain rate effect is defined by several parameters and can produce a reasonable enhancement rate of the strength of concrete. More detailed information and formulations of this model can be found in [M8].

Fig. 6.4: A variety of data sources indicates that the compressive strength of concrete increases with increasing strain rate [B7].

Fig. 6.5: Strain rate effect of concrete is more pronounced in tension than in compression.

6.2.3 Constitutive model for steel reinforcements

The Piecewise Linear Isotropic Plasticity material model is chosen to model the constitutive relationship of steel reinforcement. This model could consider the strength hardening of steel after its yielding and allows users to define the stress versus strain curves under various strain rates. In this thesis, the strength enhancement ratios of steel under different strain rates proposed by Marval [M2] (**Fig. 6.6**) is chosen to define these curves.

Fig. 6.6: Strain rate effect on steel reinforcements [M2].

6.2.4 Material model for impactor and support

The support and impactor are modeled with the rigid material model. In this model, the support and impactor are idealized as rigid body that their deformation is ignored. Detailed formulation and explanation of all the aforementioned materials models can be found in Ref. [H2 and H3].

6.3 Verification of Finite Element Modeling Method

The effectiveness of a modeling method cannot be ensured unless it can be verified with experimental data. Therefore, the modeling methods introduced above are used to analyze the impact experiment introduced in **Chapter 4** and **Chapter 5**. With the purpose of further proving the generality of this modeling method, the experimental work conducted by Kishi et al. [K6] is also analyzed.

6.3.1 Verification using the author's impact experiment

In order to prove the effectiveness of the modeling method described above, FE analysis was conducted for all 21 impact tests introduced in **Chapter 4** and **Chapter 5**. The simulated maximum impact forces are listed in **Table 4.3** and **Table 5.2** and are compared with the experimental results. The simulated impact force histories, displacement histories of points D1 and D2, and crack patterns of tests 20F-c, 20F-d, and 20F-e are presented in **Fig. 6.7–Fig. 6.9**. It can be observed that the simulated results match well with the experimental data. The simulated crack pattern and slab damage also agree well with the experimental results shown in **Fig 5.4**. In the experimental part introduced in **Chapter 4** and **Chapter 5**, the punching shear failure is defined as the obviously reversed movement between the impacted area and its surroundings. The same principle is used in this FE analysis. More precisely, the punching shear failure is defined as to have occurred if D1 (mid-span of slab) continues moving downward for more than 3 mm after the rebound of D2' (245 mm from mid-span). From **Fig. 6.8**, it is observed that 20F-d failed but 20F-e and 20F-c

did not. In **Fig 6.9**, it can be found that the inclined shear cracks are fully developed in the loading area for 20F-d and lead to punching shear failure. For 20F-c, shear cracks are partially formed. For 20F-e, no obvious shear crack could be observed. By comparing the numerical results with the experimental data of the remaining cases, it is found that the simulated damage extents are in good agreement with the experimental results for the specimens without shear reinforcement.

Fig. 6.7: Comparison of simulated and tested impact forces (20F-e, 20F-c and 20F-d).

Fig. 6.8: Comparison of simulated and tested displacement histories of points D1 and D2 (20F-e, 20F-c, and 20F-d).

Fig. 6.9: Comparison of simulated and tested slab damage (20F-e, 20F-c, and 20F-d).

For specimens with shear reinforcement, the effect of shear reinforcement in enhancing the impact resistant capacity is captured by the FE simulation as shown in **Fig. 6.10**. However, in comparing the test results, this effect is underestimated in the FE simulation. During the test, punching shear failure did not occur on specimen I-15-0.39-0.28 and I-15-0.39-0.56 as discussed in **Section 4.3.1**. In the simulation, inclined shear cracks and punching shear cones are fully formed for I-15-0.39-0.28 and I-15-0.39-0.56, which indicates punching shear failure. The reason for this underestimation is believed to be due to the deficiency of the current method in modeling the confining effect of shear reinforcement. In real specimens, the longitudinal reinforcements are wrapped and confined by the shear

reinforcements. In the modeled specimens, the steel reinforcements are modeled using beam elements that share the same nodes with the solid concrete elements, and the interaction between longitudinal and shear reinforcements cannot be modeled.

(a) I-15-0.39

(b) I-15-0.39-0.28

(c) I-15-0.39-0.56

Fig. 6.10: Comparison of simulated and tested slab damages (I-15-0.39, I-15-0.39-0.28, and I-15-0.39-0.56).

6.3.2 Verification using external experimental data

In this part, the modeling method is further verified using the external experimental data from Kishi, et al. [K6]. This experiment investigated the influence of boundary conditions on the behavior of slabs under impact loading. Three 2000×2000×180mm slabs with the boundary conditions shown in **Fig. 6.11(a)**, **(c)**, and **(e)** were tested. In **Fig. 6.11(a)**, the slab is supported on all four sides; in **Fig. 6.11(c)**, only two sides of the slab specimen are supported; in **Fig. 6.11(e)**, the slab has one side and two corners supported. **Fig. 6.11(g)** shows the arrangement of the reinforcement of the tested slabs. The yield strength of the steel reinforcement is 383 MPa, and the compressive strength of the concrete is 26.6 MPa. A 300-kg steel hammer with a 90-mm diameter circular flat nose is used to impact the slabs. The impact velocity is 3 m/s for all the tested slabs. **Fig. 6.11(b)**, **(d)**, and **(f)** shows the FE models of slabs with different boundary conditions. **Fig. 6.11(h)** shows the modeled steel reinforcement. In the real slab specimens, the reinforcements were welded to four thin steel plates attached to each side of the slab in order to fix these reinforcements. This configuration is not included in the FE model because a perfect bond is assumed between the concrete and the reinforcement.

(a) Slab with four sides supported

(b) Quarter model with four sides supported

(c) Slab with two sides supported

(d) Quarter model with two sides supported

(e) Slab with one side and two corners supported

(f) Half model with one side and two corners supported

(g) Slab Reinforcement

(h) Modeled slab reinforcement

Fig. 6.11: FE method models of impact experiment.

The loading point displacement, impact force, and support reaction force time histories from both experimental and numerical results are plotted and compared in **Fig. 6.12–Fig. 6.14**. **Fig. 6.15** shows the comparison between the experimental slab damage and the simulated slab damage. In these plots, 4S refers to the boundary condition with four sides supported; 2S refers to the boundary condition with two sides supported; and 1S refers to the boundary condition with one side and two corners supported. The simulated results agree well with the experimental data.

Fig. 6.12: Mid-span displacement history.

Fig. 6.13: Impact force history.

Fig. 6.14: Support reaction force history.

(a) 4s (Test)

(b) 4s (FE Model)

(a) 2s (Test)

(b) 2s (FE Model)

(a) 1s (Test)

(b) 1s (FE Model)

Fig. 6.15: Comparison between observed and simulated slab damages.

6.4 Influence of Mass Ratio on Response of Reinforced Concrete Slab

After proving the effectiveness of the FE simulation method, FE analysis was conducted to study the influence of the mass ratio on the response of reinforced concrete slabs. In **Chapter 5**, 200-kg, 300-kg, and 500-kg impactors were used to impact the slab specimens whose weight was 540 kg, and the corresponding ratios between impactor mass (m_i) and slab mass (m_s) were 0.37, 0.56, and 0.93. In this part, a numerical study is conducted to include a wider range of mass ratios. Details of this analysis are summarized in **Table 6.1**. Similar to the experimental part introduced in **Chapter 5**, the impact energy is fixed for different cases. The properties of the reinforced concrete slabs is the same as the specimens used in the experiment (15-0.39).

The influence of the mass ratio on the mid-span deflection of the bottom surface of the reinforced concrete slabs is shown in **Fig. 6.16**. The maximum impact force (MIF), maximum mid-span deflection (MMD), residue mid-span deflection (RMD), and time of reaching the maximum mid-span deflection (T_{MMD}) are summarized in **Table 6.1**. The results show that the maximum impact force and residual mid-span deflection decreases when the ratio between m_i and m_s increases. The maximum mid-span deflection of the slab increases when the mass ratio increases from 0.19 to 0.56, but then decreases with the further increase of mass ratio. The time required for the slab to reach its maximum mid-span deflection increases together with the mass ratio. However, it should be noted that the differences among these results made by varying the mass ratio is in general are very small and insignificant when the impact energy is fixed.

Fig. 6.16: Influence of mass ratio on mid-span deflection (bottom surface) of reinforced concrete slab.

Table 6.1: Variation in mass ratio.

m_i , kg	m_s , kg	m_i/m_s	V_i , m/s	E_i , J	d , mm	Nose shape	MIF, kN	MMD, mm	RMD, mm	T_{MMD} , ms
100	540	0.19	8.66	3750	200	flat	772	6.26	5.3	1.8
300		0.56	5.				750	6.42	5.1	2.5
500		0.93	3.87				735	6.07	4.8	2.8
700		1.3	3.27				679	6.03	4.5	3.4
1000		1.85	2.74				658	5.75	4.3	3.7

Note: d = diameter of impactor; V_i = impact velocity; E_i = impact energy.

6.5 Parametric Study on Energy Capacity of Reinforced Concrete Slab

A parametric investigation is presented in this section to include more variables that were not covered in the previous experimental study and to learn how they affect the energy capacity and angle of shear cracks of reinforced concrete slab. The variables considered in this study are the compressive strength of concrete (f_c : 25 MPa, 35 MPa, and 45 MPa), the longitudinal reinforcement ratio (ρ : 0.21%~0.66%), the slab thickness (H : 100 mm and 150 mm) and the diameter of impactor (d : 50 mm, 100 mm, and 200 mm). The weight of the impactor is fixed at 300 kg, and only flat nose impactors are used. Details of this parametric study program and specimen characteristics are summarized in **Table 6.2**. It should be noted that the steel reinforcement in this parametric study has the same layout as **Fig. 5.1**, and the difference in reinforcement ratio among different cases is made by varying the diameter of reinforcing bars. The yield and ultimate strengths of steel reinforcement are 576 MPa and 655 MPa. A total of 54 cases are analyzed. In each case, a relatively small impact velocity is assigned at first. If the specimen did not fail with the assigned impact velocity, the analysis is conducted again with a new impact velocity that is 0.1 m/s higher than the previous one. The analysis ends when the punching shear failure can be observed from the displacement histories of points D1 and D2' (failure criterion defined in **Section 6.3.1**). The corresponding impact velocity is used to calculate the impact energy that is just enough to cause punching shear failure. As introduced in **Section 5.3.4**, for reinforced concrete slabs suffered from punching shear failure when the ratio between the energy absorbed by the slab and the impact energy is close to 1. Therefore, the impact energy that just enough to cause punching shear failure can be considered as the energy capacity of the impacted slab (E_c).

Table 6.2: Details of parametric study.

Designatio n	Dimension of slab (mm)	f_c , MPa	DR, mm	ρ , %	E_{c-FEA} , J (V_i , m/s)			E_{c-Equ} , J		
					$d = 50$ mm	$d = 100$ mm	$d = 200$ mm	$d = 50$ mm	$d = 100$ mm	$d = 200$ mm
15-25-0.25	1,200×1,200 ×150	25	8	0.25	1,634 (3.3)	2,166 (3.8)	2,774 (4.3)	1,606	1,956	3,066
15-25-0.39		25	10	0.39	1,634 (3.3)	2,166 (3.8)	3,038 (4.5)			
15-25-0.66		25	13	0.66	1,838 (3.5)	2,522 (4.1)	3,314 (4.7)			
15-35-0.25		35	8	0.25	2,166 (3.8)	2,400 (4)	4,374 (5.4)	2,249	2,739	4,292
15-35-0.39		35	10	0.39	2,282 (3.9)	2,646 (4.2)	4,538 (5.5)			
15-35-0.66		35	13	0.66	2,400 (4)	2,774 (4.3)	4,704 (5.6)			
15-45-0.25		45	8	0.25	2,646 (4.2)	3,038 (4.5)	4,704 (5.6)	2,891	3,521	5,518
15-45-0.39		45	10	0.39	2,646 (4.2)	3,174 (4.6)	5,046 (5.8)			
15-45-0.66		45	13	0.66	2,904 (4.4)	3,602 (4.9)	5,400 (6)			
10-25-0.21	1,200×1,200 ×100	25	6	0.21	938 (2.5)	1,442 (3.1)	2,522 (4.1)	503	1,450	2,525
10-25-0.38		25	8	0.38	938 (2.5)	1,442 (3.1)	2,522 (4.1)			
10-25-0.59		25	10	0.59	938 (2.5)	1,634 (3.3)	2,522 (4.1)			
10-35-0.21		35	6	0.21	1,176 (2.8)	1,838 (3.5)	3,456 (4.8)	704	2,030	3,535
10-35-0.38		35	8	0.38	1,176 (2.8)	1,944 (3.6)	3,602 (4.9)			
10-35-0.59		35	10	0.59	1,262 (2.9)	2,053 (3.7)	3,602 (4.9)			
10-45-0.21		45	6	0.21	1,442 (3.1)	2,774 (4.3)	4,214 (5.3)	906	2,610	4,545
10-45-0.38		45	8	0.38	1,536 (3.2)	2,774 (4.3)	4,374 (5.4)			
10-45-0.59		45	10	0.59	1,634 (3.3)	2,646 (4.2)	4,538 (5.5)			

Note: f_c = compressive strength of concrete; DR = Diameter of reinforcing bars; ρ = reinforcement ratio; E_{c-FEA} = Minimum impact energy for punching shear failure obtained in FE analysis; V_i = Corresponding impact velocity (using 300 kg impactor); E_{c-Equ} = Minimum impact energy for punching shear failure estimated using Eq. (6-1).

6.5.1 Effects of longitudinal reinforcement ratio and compressive strength of concrete on E_c

Fig. 6.17 illustrates the effects of the longitudinal reinforcement ratio and compressive strength of concrete on the energy capacity of reinforced concrete slabs in resisting punching shear failure. The enhancement on E_c made by increasing the longitudinal reinforcement ratio is observed to be very limited. For a 100-mm-thick slab, E_c is on average enhanced by 6% when the reinforcement ratio increased from 0.21% to 0.59%. For a 15-cm-thick slab, E_c was on average enhanced by 14% when the reinforcement ratio increased from 0.25% to 0.66%. The enhancement on E_c brought by increased longitudinal reinforcement ratio is expected because flexural behavior and yielding of reinforcement occur at an early stage of impact. However, this flexural behavior and the corresponding global deformation are limited to before the shear failure of slab, and therefore prevent the longitudinal reinforcement in dissipating more impact energy. Only lightly reinforced slabs are considered in this analysis. If the slabs are highly reinforced, they are likely to have a more pronounced flexural response and global deformation, which would enable the reinforcement in consuming more energy under impact loading.

E_c is found to be significantly affected by the compressive strength of concrete in **Fig. 6.17**. On average, E_c is enhanced by 33% when f_c increased from 25 MPa to 35 MPa and another 25% when f_c increased from 35 MPa to 45 MPa. These results suggest an approximately linear relationship between E_c and f_c . As introduced in the experimental study, the punching shear failure of reinforced concrete slabs under impact loading is governed by the failure of concrete in the impacted region. This phenomenon explains why, compared to the longitudinal reinforcement ratio, the strength of concrete is more influential on E_c .

Fig. 6.17: Influence of ρ and f_c on E_c .

6.5.2 Effects of diameter of impactor and slab thickness on E_c

The diameter of the impactor and slab thickness are the two parameters that affect the amount of concrete that participates in carrying the impact-induced shear force. Therefore, The effects of d and H on E_c are shown in **Fig. 6.18**. E_c is on average increased by 67% and 70% when d increased from 50 mm to 100 mm and from 100

mm to 200 mm. When it comes to slab thickness, the average E_c of 150-mm-thick slabs is found to be 46% higher than the average E_c of 100-mm-thick slabs.

Fig. 6.18: Influence of d and H on E_c , (continued).

Fig. 6.18: Influence of d and H on E_c .

6.5.3 Effects of diameter of impactor and slab thickness on the angle of shear cracks

In addition to the energy capacity of reinforced concrete slabs, the angle of shear cracks is another important index that could be used to predict the damaged area of reinforced concrete slabs under impact loading. The angle of shear cracks is defined as the angle between the outmost thoroughly developed inclined shear crack and the horizontal line. The influence of the diameter of the impactor and slab thickness on the angle of shear cracks are plotted in **Fig. 6.19**. The angles of shear cracks are summarized in **Table 6.3**. The average angles of shear cracks for 100 mm thick slabs and 150 mm thick slabs are 41.6° and 42.6° respectively. The angle of shear cracks decreases when the diameter of the impactor increases, and this phenomenon is less

pronounced when the slab thickness increases. The angle of shear cracks can also be related using the ratio between slab depth and the diameter of impactor (H/d), which in general increases with the increase of H/d .

Fig. 6.19: Influence of H and d on the angle of shear cracks.

Table 6.3: Angle of shear cracks.

	$H=100 \text{ mm}$			$H=150 \text{ mm}$		
	$d=50$ mm	$d=100$ mm	$d=200$ mm	$d=50$ mm	$d=100$ mm	$d=200$ mm
H/d	2	1	0.5	3	1.5	0.75
Angle of shear crack	59°	42.3°	23.5°	49.1°	43.2°	35.5°

6.5.4 Proposed empirical equations for estimating the minimum impact energy for punching shear failure of reinforced concrete slabs

The energy capacity (E_c) obtained from the above parametric study are summarized in **Table 6.2**. For clearer visualization, these results are again plotted in **Fig. 6.20**.

Fig. 6.20: Minimum impact energy for punching shear failure (a): slab depth = 100 mm (b) slab depth = 150 mm.

In order to quantify the relationship between the various parameters and E_c , two dimensionless empirical equations are proposed for estimating the energy capacity of lightly reinforced concrete slabs in resisting punching shear failure:

$$\frac{E_c}{d^3 f_c} = [1.65 \frac{H}{d} - 4.53 \left(\frac{H}{d}\right)^2 + 3.47 \left(\frac{H}{d}\right)^3] 10^{-4} \quad \text{for} \quad 0.5 \leq \frac{H}{d} < 1 \quad (6-1a)$$

$$\frac{E_c}{d^3 f_c} = [2.05 - 2.72 \frac{H}{d} + 1.25 \left(\frac{H}{d}\right)^2] 10^{-4} \quad \text{for} \quad 1 \leq \frac{H}{d} \leq 3 \quad (6-1b)$$

where the units for the parameters used in these equations are J for energy, MPa for strength, and mm for length. The above equations are obtained through regression analysis of the FE results and plotted in **Fig. 6.21**. In the proposed equation, the influence of reinforcement ratio was excluded and the E_c was assumed proportional to the compressive strength of concrete. The dimensional parameters E_c , f_c , d , and H were substituted with the dimensionless parameters of $E_c/(d^3 f_c)$ and H/d . The impact energy for punching shear failure estimated using the proposed empirical equations are compared with the FE results in **Table 6.2**. It can be seen that in most cases the estimated results are in good agreement with the FE results. It should be noted that these proposed empirical equations are not suitable for predicting the shear failure energy capacity of highly reinforced concrete slabs in which the influence of longitudinal reinforcement ratio cannot be ignored. Additionally, highly reinforced concrete slabs are more likely to fail in flexural under low-velocity impact than lightly reinforced concrete slabs.

Fig. 6.21: Dimensionless equations for estimating E_c .

6.6 Conclusion

FE simulation was performed in this chapter to assess the performance of reinforced concrete slabs under low-velocity impact. Experimental data from different sources were used to verify the effectiveness of the adopted modeling method. After the verification work, FE analysis was conducted concerning the influence of mass ratio on the impact response of reinforced concrete slabs. The results suggest that in the domain of low-velocity impact the difference in slab responses made by varying the mass ratio is insignificant when the impact energy is fixed.

As both experimental and numerical works suggest a direct relationship between impact energy and slab response, a parametric study was performed that focuses on various parameters' influences on the energy capacity of reinforced concrete slabs. Results from the FE analysis show that an increase in concrete strength, diameter of impacted area, or slab thickness can effectively boost the energy capacity of lightly reinforced concrete slab whereas the effect of the longitudinal reinforcement ratio was limited. Additionally, the angle of shear cracks was on average 42° , which decreases when the diameter of the impactor increases. Based on regression analysis

of the FE results, two dimensionless empirical equations were proposed for assessing the energy capacity of lightly reinforced concrete slabs in resisting punching shear failure, which could be used for initial design or a quick check of a slab's performance under low-velocity impact. However, verification involving more experiments and a wider range of parametric study are required to further update the proposed equations to enhance their accuracy and range of application.

A main limitation of the FE modeling method used in this chapter is that it cannot account for the confining effect of shear reinforcement. As a result, the effect of shear reinforcement in enhancing the impact resistance of RC slabs was underestimated. For this reason, shear reinforcement was not included in the subsequent parametric study. In Chapter 4, the test results showed that shear reinforcement is very useful in preventing punching shear failure. Therefore, further research should try to build a better FE model that can rationally reflect the impact response of RC slabs with shear reinforcement, and to incorporate the shear reinforcement ratio into the empirical equation for predicting the energy capacity of RC slabs under impact loading.

CHAPTER 7

ANALYTICAL SOLUTIONS FOR EVALUATING THE DYNAMIC LOAD-DISPLACEMENT RELATIONSHIP AND THE IMPACT RESPONSE OF REINFORCED CONCRETE SLABS

7.1 Introduction

In previous chapters, experimental and FE studies on the low-velocity impact problem of reinforced concrete slabs were presented. However, conducting impact experiments is complex, costly, time consuming, and relies on high-quality testing and measuring instruments. By contrast, FE simulation of impact problems also requires plenty of modeling and calculation effort. Therefore, a simplified analytical model is needed to quickly predict the impact response and corresponding damage of impacted slabs.

As introduced in the literature review, a rational simplification of the impact problem is the equivalent single degree of freedom (SDOF) model. This method models the structural member as a lumped mass whose stiffness and equivalent mass are determined by assessing the load-displacement relationship and the deflected shape of the member. A thorough introduction of the SDOF model can be found in [B6]. The main deficiency of the SDOF model is that it cannot simulate the interaction between the impactor and the impacted member and knowing the impact load is a prerequisite for analysis. To overcome this deficiency, an extra lumped mass that represents the impactor can be added to the SDOF model, thus forming a two-degrees-of-freedom (2-DOF) model. In addition, a contact model that describes the interaction between the impactor and the impacted reinforced concrete member is

needed to determine the impact load.

The load-displacement curve has been recognized to be of fundamental importance for an SDOF or 2-DOF model to produce reliable results [C6]. Therefore, a good load-displacement curve should rationally reflect the dynamic load-displacement relationship of the impacted member. In many impact experiments, punching shear failure has been found to dominate the failure model of reinforced concrete slabs [D2, Z2, C3, and H5]. Nevertheless, as summarized in the literature review, the load-displacement curves used in most current SDOF and 2-DOF models are essentially based on moment-curvature analysis, which cannot consider the softening of the load capacity of reinforced concrete slabs after punching shear failure. Therefore, these models underestimate the slab response. Additionally, the impact load leads to a dynamic load-displacement relationship of the impacted member whose load capacity is higher than a static one. In current SDOF or 2-DOF models, this load capacity enhancement is usually considered by taking account of the strain rate effects on the concrete and steel. However, the strain rate of reinforced concrete members under low-velocity impact is very small ($10^{-2}/s \sim 10^0/s$) when compared to the values considered to have a significant effect on the mechanical characteristics of concrete and steel [C5]. Therefore, solely attributing the load capacity enhancement to the strain rate effects of material properties may underestimate the load capacity of reinforced concrete members under impact loading and overestimate the slab response.

The objective of this chapter is to develop a 2-DOF model for predicting the response and damage of reinforced concrete slabs under low-velocity hard impact. In contrast to earlier research, the load-displacement curve used in this model reasonably considers softening of the load capacity due to punching shear failure and load capacity enhancement due to loading rate effect. In this 2-DOF model, the

load-displacement curve is formed based on the load-rotation equation and the Critical Shear Crack Theory of reinforced concrete slabs that has been proposed by Muttoni [M11]. The load-rotation equation is used to form the ascending branch of the load-displacement curve, and the Critical Shear Crack Theory is used to determine the punching shear failure point and the softening effect of the load capacity after punching shear failure. After that, this load-displacement curve is further extended to include the loading rate effect. The effectiveness of the modeled load-displacement curves is verified by comparing with the load-displacement curves from the rapid loading tests introduced in Chapter 3. Moreover, the effectiveness of this model in producing slab deflection and impact force should be verified with experimental data.

7.2 Dynamic Load-Displacement Relationship of Reinforced Concrete Slabs

In the presented experimental study, punching shear failure mode was found for all the tested slab specimens. For the experimental works summarized in the literature review, most of the tested reinforced concrete slabs also fail in punching shear. Therefore, a rational analytical model of the load-displacement relationship should include this shear failure mode and be capable of considering the softening of the load capacity of reinforced concrete slabs after shear failure. In this study, the load-displacement curve of reinforced concrete slabs is separated into two parts by its punching shear capacity.

7.2.1 Load-rotation relationship of reinforced concrete slabs before punching shear capacity

The low-velocity impact experiments summarized in the literature review show that globalized flexural behavior of slab was involved at the early loading stage [Z2, H5, and X1]. This flexural behavior begins immediately after the loading and ends at the punching shear failure point. Therefore, the load-rotation relationship of reinforced concrete slab during this stage can be obtained through a moment-curvature analysis.

In the literature review part, the introduced moment-curvature relationships are based on sectional analysis of reinforced concrete beams, which are not suitable for reinforced concrete slabs. In this study, an analytical study made by Muttoni [M11] concerning the load-rotation relationship of axisymmetric reinforced concrete slabs on columns is introduced to describe the load-displacement relationship of reinforced concrete slabs. **Fig. 7.1** shows the assumed behavior of a reinforced concrete slab.

Fig. 7.1: Assumed behavior for axisymmetric slab: (a) geometrical parameters and rotation of slab; (b) forces in concrete and in reinforcement acting on slab sector; (c) internal forces acting on slab sector; (d) distribution of radial curvature; (e) distribution of radial moment; (f) distribution of tangential curvature; and (g) distribution of tangential moments for quadrilinear moment-curvature relationship

[M11].

According to Muttoni, the tangential cracks and the radial curvature of a reinforced concrete slab are concentrated in the vicinity of the column-supported region. Outside r_0 (assumed to be at a distance d , the effective depth of the slab, from the face of the column), the radial moment and the corresponding curvature decrease rapidly as shown in **Fig. 7.1(d)** and **(e)**. Therefore, the deformation of the corresponding slab portion is assumed to have a conical shape and a constant slab rotation ψ (**Fig. 7.1(a)**). Inside r_0 , the slab deformation is assumed to have a spherical shape. With the assumed slab deformation, the radial curvature χ_r and tangent curvature χ_t of slabs can be obtained using the following expression:

$$\chi_t = -\frac{\psi}{r} \quad \chi_r = 0 \quad (r > r_0) \quad (7-1a)$$

$$\chi_t = \chi_r = -\frac{\psi}{r_0} \quad (r < r_0) \quad (7-1b)$$

The radial and tangent moments can then be calculated with the quadrilinear moment-curvature relationship shown in **Fig. 7.2**.

Fig. 7.2: Quadrilinear moment-curvature relationship of reinforced concrete slab

[M11].

This quadrilinear moment-curvature relationship is characterized by the stiffnesses EI_0 before and EI_1 after cracking, the cracking moment m_{cr} , the moment capacity m_R , and the tension stiffening effect χ_{ts} . Ignoring the effect of reinforcement before cracking, the expressions of m_{cr} , EI_0 and corresponding curvature χ_{cr} are given as:

$$m_{cr} = \frac{f_{ct} H^2}{6} \quad (7-2)$$

$$EI_0 = \frac{E_c H^3}{12} \quad (7-3)$$

$$-\chi_{cr} = \frac{m_{cr}}{EI_0} = \frac{2f_{ct}}{HE_c} \quad (7-4)$$

Assuming linear-elastic behavior of steel and concrete after cracking, the expressions of EI_1 and the depth of compression zone c can be derived as:

$$EI_1 = \rho\beta E_s d^3 \left(1 - \frac{c}{d}\right) \left(1 - \frac{c}{3d}\right) \quad (7-5)$$

$$c = \rho\beta \frac{E_s}{E_c} d \left(\sqrt{1 + \frac{2E_c}{\rho\beta E_s}} - 1 \right) \quad (7-6)$$

where β is an efficiency factor that account for the stiffness reduction of the slab after cracking by its orthogonal reinforcement layout. An empirical value of 0.6 is recommended for β .

Assuming perfect plastic behavior of the reinforcement after yielding, a rectangular stress block for concrete in the compression zone and neglecting compression reinforcement, the moment capacity m_R is given by:

$$m_R = \rho f_y d^2 \left(1 - \frac{\rho f_y}{2f_c} \right) \quad (7-7)$$

The decrease in curvature due to tension stiffening can be calculated by:

$$\chi_{TS} = \frac{f_{ct}}{6H\rho\beta E_s} \quad (7-7)$$

The curvatures at the beginning of the stabilized cracked regime χ_1 and at yielding χ_y can be calculated by:

$$-\chi_1 = \frac{m_{cr}}{EI_1} - \chi_{TS} \quad (7-8)$$

$$-\chi_y = \frac{m_R}{EI_1} - \chi_{TS} \quad (7-9)$$

The four segments of the quadrilinear moment-curvature relationship correspond to the four regions of the slab shown in **Fig. 7.1(f)** and **Fig. 7.1(g)**. The radii r_{cr} , r_1 and r_y that divide these four regions can be obtained via substituting Eq. (7-4), Eq. (7-8), and Eq. (7-9) into Eq. (7-1a):

The zone within which the reinforcement is yielding, plastic radius r_y is given by:

$$r_y = \frac{\psi}{\frac{m_R}{EI_1} - \chi_{TS}} \leq r_s \quad (7-10)$$

The zone within which the cracking is stabilized, radius r_1 is given by:

$$r_1 = \frac{\psi}{\frac{m_{cr}}{EI_1} - \chi_{TS}} \leq r_s \quad (7-11)$$

The zone up to which the concrete is cracked, cracking radius r_{cr} is given by:

$$r_{cr} = \frac{\psi EI_0}{m_{cr}} \leq r_s \quad (7-12)$$

Using the radii calculated above, the distribution of tangent moment m_t along the

slab section can thus be evaluated. By contrast, the radial moment is uniformly distributed within the radius r_0 with the value of m_r , which can be obtained with the curvature at r_0 .

Finally, the following expression can be obtained by equating the moments induced by external and internal forces acting on the slab sector in **Fig. 7.1(c)**:

$$V \frac{\Delta\phi}{2\pi} (r_q - r_c) = -m_r \Delta\phi r_0 - \Delta\phi \int_{r_0}^{r_s} m_t dr \quad (7-13)$$

which can be further manipulated to have the load-rotation relationship:

$$V = \frac{2\pi}{r_q - r_c} \left(\frac{-m_r r_0 + m_R \langle r_y - r_0 \rangle + EI_I \psi \langle \ln(r_1) - \ln(r_y) \rangle +}{EI_I \chi_{TS} \langle r_l - r_y \rangle + m_{cr} \langle r_{cr} - r_l \rangle + EI_0 \psi \langle \ln(r_s) - \ln(r_{cr}) \rangle} \right) \quad (7-14)$$

where the operator $\langle x \rangle$ is x for $x \geq 0$ and 0 for $x < 0$.

Although the original analysis was designed to study the behavior of column-supported flat slabs, it can be easily applied to reinforced concrete slabs subjected to any type of concentrated load by considering the column-supported region as the location where the concentrated load is applied (**Fig. 7.3**).

Fig. 7.3: Slab under concentrated loading: (a) geometrical parameters and rotation of slab; (b) internal forces acting on slab sector

7.2.2 Load-rotation relationship of reinforced concrete slabs after punching shear capacity

As mentioned earlier, reinforced concrete slabs usually fail with punching shear failure mode under impact loads. In this thesis, the Critical Shear Crack Theory (CSCT) presented by Muttoni [M11] is used to obtain the punching shear capacity of reinforced concrete slabs. As shown in **Fig. 7.4**, the CSCT is based on the assumption that the shear capacity of a reinforced concrete slab is governed by the roughness and the width of the critical crack w developed in a compression strut. When the slab rotation ψ increases, w increases proportionally and the capacity of the compression strut in transmitting shear stress via aggregate interlock action decreases.

Fig. 7.4: Position of critical shear crack developing through compression strut [M12].

Therefore, the punching shear capacity of a reinforced concrete slab can be expressed as a function of its rotation, given below:

$$\frac{V_R}{b_0 d \sqrt{f_c}} = \frac{3/4}{1 + 15 \frac{\psi d}{d_{g0} + d_g}} \quad (7-15)$$

where V_R is the punching shear capacity, b_0 is the punching shear perimeter, d is the effective depth, f_c is the compressive strength of concrete, d_g is the maximum aggregate size, and d_{g0} is the reference aggregate size, which equals 16 mm.

As shown in **Fig. 7.5**, the punching shear capacity of the slab is reached when the load-rotation curve obtained from Eq. 7-14 intersects (solid line) with the failure criterion given by Eq. 7-15 (dash line). After reaching the punching shear capacity, Eq. 7-14 can no longer describe the load-rotation relationship of reinforced concrete slab because of its inability to consider the corresponding softening behavior of the load capacity. On the contrary, Eq. 15 can explain the relationship between reduced punching shear capacity and increased slab rotation. Therefore, in this thesis, Eq. 15 is used to describe the descending branch of the load-rotation curve of reinforced concrete slabs.

Another advantage of incorporating the punching shear failure criteria as a part of slab's load-rotation curve is that the failure mode and corresponding slab damage can be directly known from the constructed load-rotation curve. As shown in **Fig. 7.5(a)**, a slab is expected to have a punching shear failure mode if the slab reaches its punching shear capacity before its flexural capacity. Conversely, a slab is expected to have a flexural failure mode if the slab reaches its flexural capacity before its punching shear capacity (**Fig. 7.5(b)**).

Fig. 7.5: Typical Load-rotation curves of reinforced concrete slabs: (a) slab with punching shear failure mode; (b) slab with flexural failure mode.

7.2.3 Contribution of shear reinforcement on shear capacity of reinforced concrete slabs

The application of CSCT can be further extended to incorporate the contribution of shear reinforcement on the punching shear capacity of a reinforced concrete slab using the following equation [R8]:

$$V_s = \frac{E_s \psi}{6} A_s \leq f_y A_s \quad (7-16)$$

where V_s is the shear force carried by shear reinforcement, E_s is the elastic modulus of shear reinforcement, f_y is the yield strength of shear reinforcement, and A_s is the area of shear reinforcement intersected by the critical shear crack. For slabs with punching shear failure mode, the shear force carried by shear reinforcement is added to the slabs' load-rotation curve as shown in **Fig. 7.6**. However, for slabs with flexural failure mode, the contribution of shear reinforcement is only added to the failure criterion given by Eq. 7-15 to prevent it from affecting the flexural capacity of slab.

Fig. 7.6: Typical load-rotation curve of reinforced concrete slab with shear reinforcement.

7.2.3 Convert load-rotation curves to load-displacement curves

As illustrated in **Section 7.2.1**, the slab deformation is assumed to have a spherical shape inside r_0 , and a conical shape outside r_0 . Therefore, the displacement of the mid-span of slab can be calculated once the slab rotation is known.

For simplicity, the tangent curvature of the slab inside r_0 is ignored and the following expression is used to relate the slab rotation with the mid-span displacement of slab:

$$u_s = r_s \psi \quad (7-17)$$

where r_s is the radius of slab.

7.2.4 Residue capacity of reinforced concrete slabs with punching shear failure mode

After punching shear failure, the ability of concrete to carry the applied load decreases when the slab rotation/displacement increases. With the further increase of slab displacement, the residual capacity is mainly carried by flexural reinforcements. During this stage, the residual capacity of the reinforced concrete slab becomes stable, and the softening behavior of load capacity disappears. In this thesis, it is assumed that the softening behavior of the load capacity ends when the slab displacement reaches 20% of the slab depth. After this displacement point, the load-displacement curve becomes a horizontal line. The value of 20% is empirically determined by summarizing the experimental load-displacement curves of reinforced concrete slabs with punching shear failure mode.

7.2.5 Influence of loading rate on capacity of reinforced concrete slab

As introduced earlier, the capacities of reinforced concrete members increase with increasing loading rates. In this study, two methods are used to obtain the

load-displacement relationships of reinforced concrete slabs under high loading rates. One method attributes the capacity enhancement to the strain rate effect of structural materials and uses the dynamic material properties instead of static ones to calculate the load-displacement relationship of reinforced concrete members (i.e., the strain rate method). In this study, Eq. (7-18)–Eq. (7-20) are used to describe the strain rate effects on the strength of concrete and steel. The influences of strain rate on the compressive and tensile strengths of concrete can be expressed using Eq. (7-18) and Eq. (7-19), provided by the 2010 Model Codes [F1]:

$$\frac{f_{cd}}{f_c} = \left(\frac{\dot{\epsilon}}{30 \times 10^{-6}} \right)^{0.014} \quad \text{for } 30 \times 10^{-6} s^{-1} \leq \dot{\epsilon} \leq 30 s^{-1} \quad (7-18a)$$

$$\frac{f_{cd}}{f_c} = 0.012 \left(\frac{\dot{\epsilon}}{30 \times 10^{-6}} \right)^{\frac{1}{3}} \quad \text{for } 30 s^{-1} < \dot{\epsilon} \leq 300 s^{-1} \quad (7-18b)$$

$$\frac{f_{ctd}}{f_{ct}} = \left(\frac{\dot{\epsilon}}{1 \times 10^{-6}} \right)^{0.018} \quad \text{for } 1 \times 10^{-6} s^{-1} \leq \dot{\epsilon} \leq 10 s^{-1} \quad (7-19a)$$

$$\frac{f_{ctd}}{f_{ct}} = 0.0062 \left(\frac{\dot{\epsilon}}{1 \times 10^{-6}} \right)^{\frac{1}{3}} \quad \text{for } 10 s^{-1} < \dot{\epsilon} \leq 300 s^{-1} \quad (7-19b)$$

where f_{cd} , f_{ctd} , and $\dot{\epsilon}$ are the dynamic compressive strength, dynamic tensile strength, and corresponding strain rate.

The influence of strain rate on the dynamic strength of steel (both in compression and in tension) is given by Eq. (7-20) [C1]:

$$\frac{f_{yd}}{f_y} = \left[1 + \frac{6}{f_y} \ln \left(\frac{\dot{\epsilon}}{5 \times 10^{-5}} \right) \right] \quad \text{for } \dot{\epsilon} \leq 10 s^{-1} \quad (7-20a)$$

$$\frac{f_{yd}}{f_y} = \left[1 + \frac{6}{f_y} \ln(2 \times 10^5) \right] \quad \text{for } \dot{\varepsilon} > 10 \text{ s}^{-1} \quad (7-20b)$$

where f_{yd} and f_y are the yield strengths of steel under dynamic and static loadings.

The other method is to use the dynamic increase (DAF) obtained from high loading rate tests to empirically form the dynamic load-displacement curve of reinforced concrete members (DAF method).

In this study, it is assumed that the slab's maximum dynamic load capacity equals the product of DAF and its maximum static load capacity V_{max} . Before the maximum static load capacity, the enhanced load capacity due to loading rate effect increases linearly from 0 to $(DAF-1) \times V_{max}$; After the maximum static load capacity, the enhanced load capacity due to loading rate effect decreases linearly from $(DAF-1) \times V_{max}$ to 0. Based on these assumptions, the following empirical expressions are used to account for the load capacity enhancement due to the loading rate effect:

$$V^d(u) = V(u) + u \frac{V_{max} \times (DAF - 1)}{u_{max}} \quad \text{for } u \leq u_{max} \quad (7-21)$$

$$V^d(u_s) = V^d(u_{max}) - (u_s - u_{max}) \frac{V_{max} \times (DAF - 1)}{0.1H - u_{max}} \quad \text{for } A_s = 0, u_{max} < u_s \leq 0.1H \quad (7-21)$$

$$V^d(u_s) = V^d(u_{max}) - (u_s - u_{max}) \frac{V_{max} \times (DAF - 1)}{0.2H - u_{max}} \quad \text{for } A_s \neq 0, u_{max} < u_s \leq 0.2H \quad (7-21)$$

where V^d is the slab's dynamic load capacity, V_{max} and u_{max} are the maximum static load capacity and the corresponding displacement of the slab, and H is the slab depth.

7.3 Comparison with Experimental Data

In this section, the load-displacement curves formed with the described method are compared with the rapid loading experimental data presented in **Chapter 3**. The design of these specimens can be referred to **Fig. 3.1** and **Table 7.1**.

Table 7.1: Summary of slab specimens

Name of specimen	H (mm)	f_c , MPa	Flexural reinforcement			Shear reinforcement		
			ρ (%)	f_y , MPa	f_u , MPa	ρ (%)	f_y , MPa	f_u , MPa
10-0.59	100	42.9	0.59	576	655	-	-	-
15-0.25	150	42.9	0.25	443	504	-	-	-
15-0.39	150	45.0	0.39	576	655	-	-	-
15-0.66	150	42.9	0.66	605	690	-	-	-
15-0.39-0.14	150	42.3	0.39	576	655	0.14	282	392
15-0.39-0.28	150	42.3	0.39	576	655	0.28	282	392

Note: H = slab depth; f_c =compressive strength of concrete; ρ = reinforcement ratio; f_y =yield strength of reinforcement; f_u = ultimate strength of reinforcement.

The load-displacement curves of all six types of reinforced concrete slabs are built with the described method when subjected to static loading rate (0.0004 m/s) and the high loading rate (2 m/s), and compared with the experimental data in **Fig. 7.7**. The analytical results show that all six types of slabs meet their punching shear capacities before their flexural capacities and therefore have punching shear failure mode. This estimated failure mode matches the failure mode observed during the experiments. The analytical maximum load capacities and corresponding displacements of reinforced concrete slabs are also compared with the experimental values in **Table**

7.2. For static cases, the estimated load-displacement curves (black dashed lines) in general matched well with the experimental curves (gray dash lines). The residual load capacity, maximum load capacity, and corresponding displacement are also nicely estimated using the proposed method. However, the analytical curves cannot capture the sudden decrease of slab load capacity after punching shear failure point for those slabs without shear reinforcement. This experimental phenomenon is mainly caused by the full formation of the punching shear cone in the loading area and its separation from the surrounding concrete after punching shear failure occurs. In the meantime, the shear cracks suddenly lose their ability to carry shear force. By contrast, for slabs with shear reinforcement, the sudden separation between the punching shear cone and its surroundings after punching shear failure occurs is prevented by the confining effect of the shear reinforcements, and the concrete can still transfer shear force through shear cracks. This explains why the softening of the load capacity of slabs with shear reinforcement is well estimated by the CSCT.

Fig. 7.7: Comparison between load-displacement curves of reinforced concrete slabs obtained from analytical study and rapid loading experiment, (continued).

Fig. 7.7: Comparison between load-displacement curves of reinforced concrete slabs obtained from analytical study and rapid loading experiment, (continued).

Fig. 7.7: Comparison between load-displacement curves of reinforced concrete slabs obtained from analytical study and rapid loading experiment.

Table 7.2: Summary of maximum load capacities and corresponding displacements of reinforced concrete slabs

Name of specimen	Static loading rate				High loading rate					
	Test		Analytical method		Test		$\dot{\epsilon}=10/s$		DAF=1.5	
	V_{max} , kN	u_{max} , mm	V_{max} , kN	u_{max} , mm	V_{max} , kN	u_{max} , mm	V_{max} , kN	u_{max} , mm	V_{max} , kN	u_{max} , mm
10-0.59	240	7.4	187	9.9	360	9.5	204	9.9	281	9.9
15-0.25	303	8.9	234	9.8	435	7.7	259	9.6	350	9.8
15-0.39	304	10.4	279	8.2	474	8.1	321	7.7	419	8.2
15-0.66	379	6.3	367	4.5	663	8.3	420	4.3	553	4.5
15-0.39- 0.14	387	8.0	343	8.2	569	7.72	371	8.1	514	8.2
15-0.39- 0.28	440	11.8	438	8.2	611	10.1	466	8.1	658	8.2

Note: V_{max} = Maximum load capacity; u_{max} = Displacement when slab reaches its maximum load capacity.

For dynamic cases, the loading rate effect is assessed using both strain rate and DAF methods introduced earlier. In the strain rate method, the dynamic load-displacement curves are formed by assuming a 10/s strain rate. As illustrated in **Chapter 3**, the maximum strain rates calculated by differentiating the measured strain data are in the order of $10^0/s$ for the high loading rate tests. Therefore, assuming a 10/s strain rate should yield the upper bound of the load-displacement curves. The calculated dynamic load capacities are on average 10.6% higher than the calculated static load capacities. Similar load capacity enhancement ratios (4% for 0.1/s strain rate; 10% for 10/s strain rate; 15% for 300/s strain rate) were found by Micallef et al. [M4]

using the same approach. This enhancement ratio is far less than the experimental value of 52%. As a result, the strain rate effect on the material lever cannot sufficiently account for the load capacity enhancement on the structural lever when the loading rate is in the order of 10^0 m/s. In the DAF method, the average DAF 1.5 found from the experimental data is used to calculate the loading rate effect. Compared with strain rate method, the DAF method gives a much better estimation of the dynamic load-displacement curves.

In summary, when the DAF method is used, the proposed method can reasonably estimate the load-displacement curves of reinforced concrete slabs under different loading rates. Therefore, this method is used to form the resistance function in the 2-DOF model. In **Chapter 4**, a low-velocity impact experiment (4.85m/s) on reinforced concrete slabs that have the same design as those used in **Chapter 3** was conducted. The results of impact experiment show that the deflection rate (loading rate) of slab center and strain rate of reinforcements are very close to those found in the high loading rate test (2 m/s). Therefore, it is possible to use the DAF obtained from the high loading rate test to form the dynamic load-displacement curves of slabs subjected to low-velocity impact loads. So far, very limited high loading rate experiments have been conducted on reinforced concrete slabs, in addition to the experiment presented **Chapter 3**. Satio et al. [S4] also tested three reinforced concrete flat slabs with a loading rate of 3 m/s, and the average DAF is also 1.5. Therefore, the DAF is assumed to be 1.5 in the subsequent low-velocity impact analysis.

7.4 2-DOF Model for Impact Analysis

7.4.1 Governing equation

In this study, the 2-DOF mass-spring-damper model shown in **Fig. 7.8** is used to analyze the slab response subjected to low-velocity impact. The dynamic governing

equation of this model is given below:

$$\begin{bmatrix} m_i & \\ & m_s \end{bmatrix} \begin{bmatrix} \ddot{u}_i(t) \\ \ddot{u}_s(t) \end{bmatrix} + \begin{bmatrix} c_c & -c_c \\ -c_c & c_c + c_s \end{bmatrix} \begin{bmatrix} \dot{u}_i(t) \\ \dot{u}_s(t) \end{bmatrix} + \begin{bmatrix} k_c & -k_c \\ -k_c & k_c + k_s \end{bmatrix} \begin{bmatrix} u_i(t) \\ u_s(t) \end{bmatrix} = \begin{bmatrix} m_i g \\ 0 \end{bmatrix} \quad (7-22)$$

where m_i is the mass of the impactor; c_c and k_c are the damping coefficient and stiffness of the contact model that describes the interaction between impactor and reinforced concrete slab; c_s , k_s , and m_s are the damper, stiffness, and equivalent mass of the reinforced concrete slab; g is the gravitational acceleration, which equals 9.81 m/s^2 when the slab is vertically impacted and 0 m/s^2 when the slab is horizontally impacted.

Fig. 7.8: Two degrees of freedom (2-DOF) model.

The model parameters mentioned above depend on the properties of the reinforced concrete slab (size, depth, material capacities, reinforcement ratio, etc.) and the impactor (diameter of contact area and mass). Once these model parameters are determined, the impact velocity $\dot{u}_i(0)$ is known as the initial condition. The governing equation can be solved to generate the accelerations, velocities and

displacements of the impactor and the impacted slab in time domain ($\ddot{u}_i(t)$, $\ddot{u}_s(t)$, $\dot{u}_i(t)$, $\dot{u}_s(t)$, $u_i(t)$, $u_s(t)$). Among these outputs, the maximum displacement of the reinforced concrete slab can be used to assess the damage and residual load capacity of the reinforced concrete slab by retracing its load-displacement curve.

Eq. (7-22) can be solved using different kinds of numerical methods (Runge-kutta, Central-difference, Newmark-beta, etc.). In this study, a solver based on the Newmark-beta method was compiled using Fortran language to perform the analysis. This solver also includes a sub-program that forms the load-displacement curves using the method introduced earlier. If the time-step is set as 10^{-5} s and the calculation time is set as 10^{-1} s, the analysis of a single impact case can be finished in approximately 1 s.

After introducing the 2-DOF model, the methods for determining the model parameters are presented in the following subsections.

7.4.2 Stiffness of reinforced concrete slab, k_s

Before the slab reaches its maximum displacement ($\dot{u}_s(t) \geq 0$), k_s equals the gradient of the load-displacement curve of the reinforced concrete slab. After the slab reaches its maximum displacement ($\dot{u}_s(t) < 0$), it begins to rebound, and k_s should be the unloading stiffness. In this study, the unloading stiffness is assumed to be the secant slope of the load-displacement curve when the abscissa equals 5% of the slab depth.

7.4.3 Equivalent mass, m_s

The equivalent mass of the reinforced concrete slab in the 2-DOF model is calculated using the following expression:

$$m_s = \rho_s H \int_0^{2\pi} \int_0^{\frac{l}{2}} x \varphi^2(x) dx d\theta \quad (7-23)$$

where ρ_s is the slab density, l is the clear span of the slab, $\varphi(x)$ is a function that describes the deflected shape of the slab. In this study, the $\varphi(x)$ is assumed with the following format:

$$\varphi(x) = 1 - \frac{12x^2}{l^2} + \frac{16x^3}{l^3} \quad (7-24)$$

In **Fig. 7.9**, the assumed slab deformation is compared with the recorded slab displacements at different times during the impact test I-15-0.39 introduced in **Chapter 4**. As can be seen, Eq. (7-24) matches well with the experimental data. By substituting Eq. (7-24) into Eq. (7-23), it is found $m_s \approx 0.043\pi l^2 \rho_s h$, which is about 0.135 times of the real slab mass.

Fig. 7.9: Comparison between the assumed slab deformation (**Eq. 7-23**) and the experimental data of I-15-0.39.

7.4.4 Slab Damping, c_s

In this study, slab damping is considered to account for the energy dissipated by the

slab during impact problem. The damping value is calculated using the following equation based on structural dynamics:

$$c_s = 2\zeta \sqrt{k_e m_s} \quad (7-25)$$

where ζ is the damping ratio, k_e is the elastic stiffness of the reinforced concrete slab, which is defined as the secant slope of its load-displacement curve when the abscissa equals 1% of slab depth.

With the purpose of characterizing the damping behaviors exhibited by reinforced concrete slabs under impact loads, Hrynyk [H5] had conducted forced free vibration tests on two reinforced concrete slabs before and after they were impacted. The value of ζ was assessed based on the recorded natural vibration of the reinforced concrete slab. Hrynyk found that the value of ζ increases when the slab damage increases. In the undamaged condition, the average damping ratios were found to be 1.6%. After impact, the slabs sustained appreciable damage and the damping ratios increased to 2.3%. Delhomme et al. [D3] used a similar method to study the damping ratio of a reinforced concrete slab used in their impact test, and the damping ratio was estimated to be 14%. For simplicity, the influence of slab damage on the damping ratio is ignored in this study, and the value of ζ is set as 5% in the subsequent analysis.

7.4.5 Contact model, k_c and c_c

The contact between the impactor and the impacted reinforced concrete slab is modeled with Kelvin-Voigt model, which comprises an elastic spring and a viscous damper. The impact force $F_c(t)$ can therefore be calculated using the following expression:

$$F_c(t) = (u_i(t) - u_s(t))k_c + (\dot{u}_i(t) - \dot{u}_s(t))c_c \quad \text{for } u_i(t) - u_s(t) \geq 0 \quad (7-26a)$$

$$F_c(t)=0 \quad \text{for } u_i(t)-u_s(t) < 0 \quad (7-26b)$$

The use of Hertz contact equations to analytically calculate the contact stiffness k_c is commonly reported in the literature [B4 and F4]. However, the Hertz contact equations are based on the assumption that the impactor has a spherical nose shape and is therefore not suitable for estimating the contact stiffness when the impactor has other nose shapes. By contrast, experimental studies of contact stiffness rarely involve concrete as the material. In the absence of sufficient experimental data concerning the contact stiffness of concrete, the rule of thumb is to set the contact stiffness proportional to the stiffness of the impacted reinforced concrete slab [M9]. Using a trial and error method, it is found that the estimated impact forces matches well with the experimental data when the ratio between the contact stiffness k_c and the elastic stiffness k_e equals 5. Therefore, k_c is calculated by multiplying k_e with 5 in this research.

As a rule, energy would be lost in the form of heat and deformation during impact, and the lost energy can be related to the coefficient of restitution. By equating the lost energy, the following expression of contact damping coefficient can be derived [J1]:

$$c_c = 2\xi_c \sqrt{\frac{k_c m_i m_s}{(m_i + m_s)}}; \quad \xi_c = -\frac{\ln e}{\sqrt{\pi^2 + (\ln e)^2}} \quad (7-27)$$

where ξ_c is the damping ratio of the contact damper and e is the coefficient of restitution. By assuming the reinforced concrete slab is stationary during impact, e can be calculated by:

$$e = \left| \frac{\dot{u}_i(t_r)}{\dot{u}_i(0)} \right| \quad (7-28)$$

where $\dot{u}_i(0)$ and $\dot{u}_i(t_r)$ are the impact and rebound velocities of the impactor.

Because the rebound velocity is unknown before analysis, an iterative process is required to determine e . Available low-velocity impact experimental data shows that e is usually in the range from 0.1 to 0.2. This would lead to a ζ_c in the range between 0.46 and 0.59. Therefore, it is suggested to set the initial trial value of ζ_c as 0.5.

7.5 Validation of 2-DOF Model

In this section, the validity of the proposed 2-DOF model is examined by analyzing the impact experiments illustrated in **Chapter 4** and **Chapter 5** and comparing the analytical results with the experimental data.

In **Chapter 4** and **Chapter 5**, the measured displacement histories of the mid-span (bottom surface) of the tested slabs were not reported. Therefore, this data is reported here. In total, 11 impact tests are analyzed. The masses of drop weights and the impact velocities of these 11 impact tests are summarized in **Table 7.3**. The measured mid-span displacement histories and impact force histories are compared with the analytical results in **Fig. 7.10**. The maximum and residue slab deformations are listed in **Table 7.3**. The characteristics of the impact force histories are given in **Table 7.4**.

Fig. 7.10: Comparison between analytical results and experimental data, (continued).

Fig. 7.10: Comparison between analytical results and experimental data, (continued).

Fig. 7.10: Comparison between analytical results and experimental data.

Table 7.3: Comparison between experimental and analytical results: Displacement histories

Name of test	m_i , kg	$\dot{u}_i(0)$, m/s	$u_{s,max}$, mm			$u_{s,res}$, mm			Loading rate, m/s	
			Test	2-DOF	Ratio	Test	2-DOF	Ratio	Test	2-DOF
I-10-0.59	500	5.43	-	54.1	-	-	51.7	-		2.3
I-10-0.59(2)	300	6.26	43.9	38.1	0.87	35.0	35.8	1.02	2.6	2.4
I-10-0.59(3)	200	7.67	36.0	35.0	0.97	28.6	33.0	1.15	3.0	2.8
I-15-0.25	500	5.43	44.7	46.5	1.04	-	41.7	-	2.1	2.1
I-15-0.39	500	5.43	38.4	28.9	0.75	30.2	26.8	0.89	2.4	1.9
I-15-0.39(2)	500	4.85	20.3	19.7	0.97	14.9	16.8	1.13	1.4	1.9
I-15-0.39(3)	300	5.11	10.7	10.5	0.98	5.9	4.6	0.78	2.3	2.3
I-15-0.39(4)	200	7.67	17.9	15.6	0.87	11.7	12.3	1.05	2.2	2.7
I-15-0.66	500	5.43	26.8	20.7	0.77	20.0	20.2	1.01	2.1	1.9
I-15-0.39- 0.28	500	5.43	16.6	16.8	1.01	10.0	11.6	1.16	2.4	2.4
I-15-0.39- 0.56	500	5.43	15.0	13.0	0.87	7.9	6.5	0.82	2.6	2.5

Note: $u_{s,max}$ = maximum displacement; $u_{s,res}$ = residue deformation; Ratio = ratio between analytical result and experimental value.

Table 7.4: Comparison between experimental and analytical results: Impact force histories

Name of test	m _i , kg	$\dot{u}_i(0)$, m/s	E _i , J	F _{c,max} , kN			T _c , ms			I, N·s			$\dot{u}_i(t_r)$, m/s			E _{ip} , J		
				Test	2-DOF	Ratio	Test	2-DOF	Ratio	Test	2-DOF	Ratio	Test	2-DOF	Ratio	Test	2-DOF	Ratio
I-10-0.59	500	5.43	7,371	515	559	1.09	33.8	30.0	0.89	3,300	3,168	0.96	1.17	0.91	0.78	7,029	7,164	1.02
I-10-0.59(2)	300	6.26	5,886	663	642	0.97	22.4	20.3	0.91	2,276	2,178	0.96	1.33	1.00	0.75	5,613	5,728	1.02
I-10-0.59(3)	200	7.67	5,886	-	767	-	-	16.2	-	-	1,756	-	-	1.11	-	-	5,760	-
I-15-0.25	500	5.43	7,371	785	895	1.14	28.5	29.4	1.03	3,134	3,278	1.05	0.84	1.13	1.35	7,195	7,052	0.98
I-15-0.39	500	5.43	7,371	984	927	0.94	24.2	22.2	0.92	3,391	3,140	0.93	1.35	0.85	0.63	6,915	7,191	1.04
I-15-0.39(2)	500	4.85	5,886	794	829	1.04	22.2	15.9	0.72	2,895	2,951	1.02	0.94	1.05	1.12	5,660	5,605	0.99
I-15-0.39(3)	300	5.11	3,917	679	722	1.06	10.8	8.8	0.81	2,279	2,272	1.00	2.49	2.46	0.99	2,987	3,009	1.01
I-15-0.39(4)	200	7.67	5,886	-	1,227	-	-	9.2	-	-	1,865	-	-	1.66	-	-	5,607	-
I-15-0.66	500	5.43	7,371	1077	1,127	1.05	19.0	17.1	0.90	3,490	3,191	0.91	1.55	0.95	0.61	6,771	7,146	1.06
I-15-0.39-0.28	500	5.43	7,371	687	835	1.22	12.8	11.7	0.91	3,353	3,668	1.09	1.28	1.91	1.49	6,962	6,459	0.93
I-15-0.39-0.56	500	5.43	7,371	794	917	1.15	11.1	9.3	0.84	3,702	3,672	0.99	1.97	1.91	0.97	6,401	6,459	1.01

Note: E_i= impact energy; F_{c,max}= maximum impact force; T_c= Duration of impact; I= impulse; E_{ip}= imparted energy; Ratio= ratio between analytical (2-DOF) and test results.

7.5.1 Mid-span displacement histories, $u_s(t)$

For the mid-span displacement histories, the analytical ones in general matched well with the experimental ones in terms of shape, maximum, and residual values. On average, the maximum mid-span displacement of reinforced concrete slab is slightly underestimated by 9%. The largest difference between the analytical and experimental maximum mid-span displacements is found to be 9.5 mm (underestimated by 25% in test I-15-0.39). By contrast, the residual mid-span displacement of reinforced concrete slabs is on average overestimated by 0.1%, and the largest difference between the analytical and experimental values is found to be 1.3 mm [underestimated by 22% in test I-15-0.39(3)]. The slab failure is assumed happen immediately after the slab reaches its maximum load capacity. The damage of a slab can be known by comparing its maximum displacement under the impact load ($u_{s,max}$ in **Table 7.3**) with the displacement when the slab reaches its maximum load capacity (u_{max} in **Table 7.2**). Using this method, it is found that all 11 slabs failed during the impact tests.

7.5.2 Loading rate

The loading rates of the 11 impact tests are evaluated by dividing the maximum slab displacements with the corresponding times. As shown in **Table 7.3**, the loading rates are found to range from 1.4 m/s to 3 m/s and to increase proportionally with impact velocity. These results suggest that it is reasonable to use the DAF taken from the rapid loading test (loading rate on the order of 10^0 m/s) to evaluate the dynamic load-displacement relationship of reinforced concrete members subjected to low-velocity impact ($1 < \dot{u}_i(t) < 10$ m/s).

7.5.3 Impact force histories, $F_c(t)$

Similar to the displacement histories, the estimated impact force histories also match nicely with the experimental data in terms of shape, maximum value, and

duration. The maximum impact force is on average overestimated by 7%, and the maximum difference is found in test I-15-0.39-0.28, where the analytical value is 22% higher than the experimental one. The duration of impact is on average underestimated by 12%, and the maximum difference is found in test I-15-0.39(2), where the analytical value is 28% less than the experimental one.

7.5.4 Impulse and imparted energy

The impulse I given by the impactor can be obtained by integrating the impact force in the time domain. The calculated impulse can also be used to assess the rebound velocity of the impactor and the amount of energy E_{ip} that imparted to the slab by the impactor using the following expressions:

$$\dot{u}_i(t_r) = \frac{I - m_i \dot{u}_i(0)}{m_i} \quad (7-29)$$

$$E_{ip} = \frac{m_i (\dot{u}_i(0)^2 - \dot{u}_i(t_r)^2)}{2} \quad (7-30)$$

The calculated impulse and imparted energy are given in **Table 7.4**. It can be found that both values accurately estimate that the error between analytical and experimental results is within 10%.

7.5.5 Generality

In order to test the generality of the 2-DOF model, two low-velocity impact tests N-IS4 by Bhatti et al. [B5] and TH2-1 by Hrynyk and Vecchio [H4] are analyzed. For N-IS4, the slab has a dimension of 1,650×1650×150 mm and a longitudinal reinforcement ratio of 0.6%. The compressive strength of concrete and the yield strength of reinforcement are 16.7 MPa and 355 MPa. The impact velocity, mass, and diameter of the impactor are 4 m/s, 300 kg, and 60 mm. For TH2-1, the slab has a dimension of 1,800×1,800×130 mm and longitudinal reinforcement ratio of

0.42%. The compressive strength of concrete and the yield strength of reinforcement are 69.4 MPa and 489 MPa. The impact velocity, mass, and diameter of the impactor are 8 m/s, 180 kg, and 300 mm. The comparisons between analytical and experimental results of these two tests are shown in **Fig. 7.11** and **Fig. 7.12**. In general, the slab responses are found nicely predicted by the 2-DOF model.

Fig. 7.11: Comparison between analytical results and experimental data of N-IS4 [B5].

Fig. 7.12: Comparison between analytical results and experimental data of TH2-1 [H4].

7.6 Distribution of Impact Energy and Fast Estimation of Slab Damage

The results obtained from the 2-DOF model can be further used to assess the distribution of impact energy during the impact process. As an example, the distribution of energy during the impact test I-15-0.39(2) is shown in **Fig. 7.13**.

Fig. 7.13: Typical energy distribution of impact problem (I-15-0.39(2)).

Immediately after impact, the kinetic energy of the impactor reduces quickly, and the reduced energy is converted to the strain energy of the contact spring, the kinetic and strain energies of the reinforced concrete slab. The strain energy of the reinforced concrete slab reaches its maximum when the kinetic energy of the impactor becomes 0. After that, the strain energy of the reinforced concrete slab slightly reduces due to the recovery of its elastic deformation, and the recovered strain energy in turn becomes the kinetic energy of the impactor. When the reinforced concrete slab separates from the impactor, the strain energy of the

contact spring becomes 0. Moreover, the kinetic energy of the reinforced concrete slab also gradually reduces to 0 during the following natural vibration of the reinforced concrete slab. At the end of the impact test I-15-0.39(2), 84% of the impact energy becomes the strain energy of reinforced concrete slab (actually it has been dissipated due to the permanent slab deformation); 7.3% of the impact energy was dissipated by contact damper; 5.3% of the impact energy was dissipated by slab damper; and 3.4% of the impact energy becomes the residue kinetic energy of the impactor.

Noting that the majority of impact energy is absorbed by slab deformation, it is reasonable to simply consider the area under the slab's load-displacement curve before the failure point as the energy capacity of the slab as shown in **Fig. 7.14**.

Fig. 7.14: Energy capacity of reinforced concrete slab (15-0.39).

Whether the slab can sustain a given impact can be quickly estimated by comparing the slab's energy absorption capacity and the assigned impact energy. Because not

all the impact energy was dissipated by slab deformation, this method can yield a conservative estimate of the energy capacity of and damage to the reinforced concrete slab. As an example, the energy capacity of Specimen 15-0.39 is estimated by integrating its dynamic load-displacement curve (**Fig. 7.14**) is 2328 J. By contrast, the actual energy capacity of the specimen is evaluated using the 2-DOF model. As shown in **Table 7.5**, the slab fails when the impact velocity reaches 3.35 m/s and the corresponding impact energy is 2,806 J. In this case, the quickly estimated energy capacity underestimated the real energy capacity by 17%, which is conservative and acceptable. Therefore, this quick estimation approach can be used instead of the 2-DOF model for evaluating slab damage in the situations where detailed and accurate slab response is not required.

Table 7.5: Responses of Specimen 15-0.39 subjected different impact energies

m_i , kg	$\dot{u}_i(0)$, m/s	E_i , J	$u_{s,max}$ (TDOF), mm	u_{max} , mm	Fail
500	3.3	2723	8.1	8.2	No
500	3.35	2806	8.3	8.2	Yes
500	3.4	2890	8.5	8.2	Yes

Note: E_i = Impact energy; $u_{s,max}$ = Maximum slab deformation under impact; u_{max} = Displacement when slab reaches its maximum strength

7.7 Conclusions

The following conclusions are drawn based on the results presented in this chapter:

- A novel approach is proposed for constructing the dynamic load-displacement curves of reinforced concrete slabs. In this approach, the load-displacement curve of the reinforced concrete slab before punching shear capacity is formed through a moment-curvature analysis. A failure criteria based on critical shear crack theory is used to determine the punching shear capacity and the resulting load-displacement curve. In this way, the softening behavior of the load capacity of reinforced concrete slabs after punching shear failure can be considered, and the failure modes and failure points of slabs can be directly known from the constructed load-displacement curves. The effectiveness of this approach is verified by comparing the analytical results with experimental data. For the loading rate effect, it is found that the traditional strain rate method cannot sufficiently consider the load capacity enhancement. By comparison, the load capacity enhancement can be properly incorporated directly by using the DAF obtained from the rapid loading test. One main limitation in this approach is that the DAF is set to 1.5 for reinforced concrete slabs when the loading rates are on the order of 10^0 m/s based on the available experimental data. It worth mentioning that, in addition to the loading rate, the DAF can also be influenced by other parameters such as reinforcement ratio, size of specimen, etc. [A5]. However, the current database of rapid loading tests is still limited in terms of obtaining an analytical expression for DAF. Therefore, future work is required in this area to facilitate a more proper estimation of DAF.
- A 2-DOF model is presented for analyzing the response and damage of reinforced concrete slabs under low-velocity impact. In this model, the slab stiffness is evaluated based on the proposed dynamic load-displacement curve,

and the contact between the impactor and the reinforced concrete slab is modeled with an elastic spring and a viscous damper. The model parameters can be easily determined once the properties of the impact load and slab specimen are known. Using a numerical method, this model can be solved very efficiently. In the subsequent validation work, the slab responses in terms of maximum mid-span displacement, maximum impact force, contact duration, impulse, imparted energy, etc. have been estimated with high accuracy. In summary, this 2-DOF model provides a simple, efficient, but accurate method for analyzing slab impact problems.

- The results from the 2-DOF model are further processed to show slab behavior from an energetic standpoint. By analyzing the intrinsic distribution of impact energy, it is found that the impact energy can be dissipated by slab damper, contact damper, and permanent slab deformation. Among these parameters, the energy dissipated by permanent slab deformation reaches very high percentage of the impact energy. This result proves why a rational load-displacement curve is necessary for a 2-DOF model to produce good results.
- In the situation where detailed and accurate slab response is not required, the energy capacity of slab can be quickly and conservatively estimated by integrating its load-displacement curve until its failure point. Whether the slab fails or not during impact can be known by comparing this energy capacity with the exerted impact energy.

CHAPTER 8

PROBABILITY ANALYSIS OF REINFORCED CONCRETE SLABS UNDER LOW-VELOCITY IMPACT

8.1. Introduction

In the previous chapters, the problem of reinforced concrete slabs under low-velocity impact has been experimentally, numerically, and analytically studied.

However, the above studies are essentially deterministic, and the unavoidable uncertainties that existed in the reinforced concrete slabs and the impact loads have not been considered. In reality, the properties of structural dimension, material strength, and the potential magnitude of impact loadings inevitably fluctuate. This merits the further study of reinforced concrete slabs subjected to low-velocity impact loading in a probabilistic manner. So far, probabilistic assessment of structural damage due to blast loads has been undertaken by several researchers [L3, L4, R5, E1, H1, and A3]. When it comes to low-velocity impacts, the only available information is reported by Bertrand et al. [B4], where fragility curves of a reinforced concrete member impacted by falling rocks are derived for risk analysis. Unfortunately, the SDOF model used in this study is only suitable for reinforced concrete members with flexural failure mode and cannot consider the punching shear failure that dominates the failure mode of RC slabs under impact loadings. This SDOF model also lacks verification to justify its analytical results. Therefore, understanding of the probabilistic performance of reinforced concrete slabs subjected to low-velocity impact is still limited and cannot satisfy the demands of

risk analysis and management.

The objective of the study presented in this chapter is to make up for this deficiency and develop a procedure that can be used to estimate the failure probability of reinforced concrete slabs under low-velocity impact. Unlike the study of Bertrand et al. [B4], the 2-DOF model introduced and verified in **Chapter 7**, which can consider both the global (flexural) and local (punching shear) failure modes, is employed to generate the impact response and damage of reinforced concrete slabs. Instead of deterministic parameters, the Latin Hypercube Sampling method is used to produce random input parameters to reflect the uncertainties that exist in both specimen properties and impact loads. These random parameters are used in this 2-DOF model to generate random slab responses for calculating the failure probabilities of reinforced concrete slabs under different impact velocities.

The principles for determining the failure mode and the failure criteria of this probability analysis are the same as the ones introduced in **Section 7.2.2** and **Section 7.5.1**. As shown in **Fig. 7.5(a)**, a slab is assumed to have a punching shear failure mode if its load-rotation curve reaches the punching shear capacity before the flexural capacity. Conversely, a slab is assumed to have a flexural failure mode when its load-rotation curve reaches the flexural capacity before its punching shear capacity (**Fig. 7.5(b)**). In the impact analysis, the maximum displacement of the slab under the assigned impact load is compared with its displacement capacity (slab displacement corresponds to the maximum load capacity of the slab's load-displacement curve). Slab failure is assumed to happen if the maximum impact displacement of the slab is beyond its displacement capacity.

8.2 Input Parameters

In order to use the 2-DOF model to calculate the failure probability of RC slabs under impact, a set of 13 important input parameters should be determined first. These parameters cover the properties of the reinforced concrete slab (L , d_b , d , ρ_0 , ρ_s , ρ_h , f_c , f_y , E_c and E_s) and the impact loading (m_b , V_b , and r_b). The definitions of these parameters are given in **Table 8.1** and **Table 8.2**. In this study, a 1,200-mm-square reinforced concrete slab with a depth of 100 mm and a longitudinal reinforcement ratio of 0.55% was used as the representative sample. The thickness of the concrete cover is 15 mm. The compressive strength of concrete is 23.4 MPa, and the yield strength of the longitudinal reinforcement is 490 MPa. The Young's modulus of concrete and reinforcement are 2.43×10^4 MPa and 2×10^5 MPa, respectively. The variability of material strengths (f_c and f_y) and density of the slab (ρ_0) is considered to be representative of the inherent uncertainties of reinforced concrete slabs. The uncertainties that exist in impact loadings are considered by the randomness of the mass (m_b) and diameter (r_b) of impactor. The imposed impact velocities range from 0 m/s to 10 m/s. The deterministic input parameters used for the representative reinforced concrete slab sample are summarized in **Table 8.1**. The probabilistic features of the considered random variables in terms of the probability distribution type, the mean values, and the coefficients of variations (COV) are shown in **Table 8.2** and **Fig 8.1**.

Table 8.1: Deterministic input parameters of 2-DOF model.

Parameter	Definition	Value
L	Length of slab	1,200 mm
d_b	Depth of slab	100 mm
d	Effective depth	85 mm
ρ_s	Longitudinal reinforcement ratio	0.55%
ρ_h	Shear reinforcement ratio	0
E_s	Elastic modulus of steel	2×10^5 MPa
E_c	Elastic modulus of concrete	2.43×10^4 MPa
V_b	Impact velocity	0–10 m/s

Table 8.2: Probabilistic features of random variables used in 2-DOF model.

Parameter	Definition	Distribution	Mean	COV	Reference
ρ_0	Density of slab	Normal	$1.05 \times \text{Density of concrete}$	10%	Ellingwood et al. 1980 [E2]
f_c	Compressive strength of concrete	Normal	23.4 MPa	18%	
f_y	Yield strength of steel	Beta	490 MPa	10.7%	Mirza and MacGregor [M6]
r_b	Diameter of impactor	Normal	100 mm	20%	Bertrand et al. [B4]
m_b	Mass of impactor	Normal	300 kg	20%

Fig. 8.1: Probability density functions (PDF) of random variables.

8.3 Uncertainty Analysis

The objective of the uncertainty analysis is to properly model the propagation of the uncertainties from the input parameters to the response of the reinforced concrete slab under impact. To achieve this goal, the Latin Hypercube Sampling technique that includes a reduction of spurious correlation of samples [O1] is used to generate the input samples of the 2-DOF model to consider the uncertainties existed in slab property and impact load.

The influences of the uncertainties that exist in slab properties (f_c , f_y , and ρ) on the load-displacement curves are presented in **Fig. 8.2**. The variation of the mid-span displacement curves of the slab under the impact velocities of 1.5 m/s, 2.5 m/s, and 3.5 m/s are shown in **Fig. 8.3**, and their maximum values are marked as well. The large variations observed from these curves reveal the significant effect of the inherent uncertainties of reinforced concrete slab parameters on the impact capacity and response of slabs. **Fig. 8.4** illustrates the variation of the maximum mid-span displacement when the impact velocities increase from 1 m/s to 10 m/s. It is clear that the variation in slab response increases significantly when the imposed impact velocity increases. The COV of the maximum mid-span displacement can reach more than 0.3 when the impact velocity exceeds 4.0 m/s (**Fig. 8.5**). This observation further consolidates the significant effect of the considered uncertainties on the performance of reinforced concrete slabs to resist low-velocity impact. To deal with these inherent uncertainties, a probabilistic analysis instead of a conventional deterministic one is needed.

Fig. 8.2: Variation of load-displacement relationship of reinforced concrete slab due to uncertainties existed in slab properties.

Fig. 8.3: Possible mid-span displacement histories of slab under impact.

Fig. 8.4: Variation of maximum mid-span displacement of slab along under different impact velocities.

Fig. 8.5: COV of maximum mid-span displacements of impacted slab

The failure probability of the slab with increasing impact velocity is shown in **Fig. 8.6**. The maximum mid-span displacements obtained from random input samples are compared with the displacement capacity of the slab to determine the failure conditions of these analyses. The failure probability is set as the ratio of the number of failure and the total number of samples. As shown **Fig. 8.6**, the failure probability of the impacted slab increases from 0 to 1 when the impact velocity increases from 2 m/s and 5 m/s. For comparison purposes, a deterministic impact analysis is also performed by replacing the random variables with their mean values. Because the uncertainties are not considered, the failure probability curve becomes a vertical line and the minimum impact velocity to cause failure of the slab is 3.2 m/s. This deterministic failure impact velocity is apparently not sufficient to cover the potential damages of the reinforced concrete slab. Accordingly, it is suggested that the design of reinforced concrete slabs under low-velocity impact carefully consider the influence of the uncertainties in slab properties and impact loading.

Fig. 8.6: Failure probability of impacted slab.

8.4 Sensitivity Analysis

Sensitivity analysis is conducted to investigate the significance of the effect of the random variables on the impact performance of reinforced concrete slab. In this part, nonlinear impact analyses are conducted with only one random variable changes while the others are set as their means. **Fig. 8.7** shows the variations of the maximum mid-span displacement of slab caused by different random variables. As can be seen from the figure, the random variables related to impact loading (m_b and r_b) are more influential on the variation of the impact response of slab than the random variables related to the properties of reinforced concrete slab (f_c , f_y , and ρ_0). Compared to the radius of impactor r_b , the mass of impactor m_b (impact energy) has more influence on slab displacement. When considering the uncertainties associated with slab properties, the influences of the random variables on slab displacement from large to small are ranked in the sequence of concrete strength f_c , the slab density ρ_s , and yield strength of reinforcement f_y . This observation can be explained by the smaller contribution to the punching shear capacity of the reinforced concrete slab made by the flexural reinforcement than the contribution made by the concrete.

The failure probabilities of the reinforced concrete slab, based on different random variables, are compared in **Fig. 8.8**. It is clear that a higher influential random variable generally implies a flatter failure probability curve, and vice versa. Due to the negligible influences of ρ_0 and f_y on slab response (**Fig. 8.7(d), (e)**), the failure probability curves corresponding to these two random variables are very close to a step function curve whose value suddenly jumps from 0 to 1 after reaching a certain impact velocity. Based on this observation, these two parameters are taken as deterministic parameters in the following probabilistic studies.

Fig. 8.7: Influence of different random variables on maximum mid-span displacement of impacted slab.

Fig. 8.8: Influence of different random variables on failure probability of impacted slab.

8.5 Parametric Study

The above study considers only a specific design of the slab impacted by a particular impactor with an emphasis on illustrating the effect of uncertainty on slab response. However, it is interesting to know how the performance of a slab under low-velocity impact loading would be affected when the design parameters of the slab change. With this purpose in mind, a comprehensive parametric study is conducted in this section to investigate the effects of slab length L , slab thickness d_b , concrete strength f_c , longitudinal reinforcement ratio ρ_s , shear reinforcement ratio ρ_h , mass of impactor m_b , and diameter of impactor r_b on the failure probability of the reinforced concrete slab subjected to low-velocity impacts. **Table 8.3** summarizes all the cases under consideration. To prevent a possible coupling effect, only the considered parameter is varied in each considered case, while the other parameters are determined according to **Table 8.1** and **Table 8.2**.

Table 8.3: Considered cases of parameteric study.

Name	Descriptions
f_c-1	Normal distribution, mean = 19.0 MPa, COV = 18%
f_c-2	Normal distribution, mean = 23.4 MPa, COV = 18%
f_c-3	Normal distribution, mean = 27.8 MPa, COV = 15%
m_b-1	Normal distribution, mean = 100kg, COV = 20%
m_b-2	Normal distribution, mean = 200kg, COV = 20%
m_b-3	Normal distribution, mean = 300kg, COV = 20%
m_b-4	Normal distribution, mean = 400kg, COV = 20%
m_b-5	Normal distribution, mean = 500kg, COV = 20%
ρ_s-1	$\rho_s = 0.25\%$
ρ_s-2	$\rho_s = 0.40\%$
ρ_s-3	$\rho_s = 0.55\%$
ρ_s-4	$\rho_s = 0.70\%$
ρ_s-5	$\rho_s = 0.85\%$
ρ_s-6	$\rho_s = 1.00\%$
d_b-1	$d_b = 75\text{mm}$
d_b-2	$d_b = 100\text{mm}$
d_b-3	$d_b = 150\text{mm}$
r_b-1	Normal distribution, mean = 100mm, COV = 20%
r_b-2	Normal distribution, mean = 200mm, COV = 20%
r_b-3	Normal distribution, mean = 300mm, COV = 20%
$L-1$	$L = 750\text{mm}$
$L-2$	$L = 1,200\text{mm}$
$L-3$	$L = 1,800\text{mm}$
ρ_h-1	$\rho_h = 0.00\%$
ρ_h-2	$\rho_h = 0.28\%$
ρ_h-3	$\rho_h = 0.56\%$

8.5.1 Effect of length and thickness of slab

The effect of the length and thickness of the slab dimensions on the performance of the slab under low-velocity impact loadings are investigated first. The failure probabilities of the slab with different slab lengths are compared in **Fig. 8.9(a)**. Under a certain impact velocity, it can be observed that the failure probability of the slab is reduced when the length of slab increases. This is mainly because the stiffness of the slab is lower when its length increases, which leads to more pronounced flexural behavior and a higher deformation capacity of the slab before failure. Thus the energy that can be absorbed by the slab would increase as well.

Increasing the thickness of the slab is found to be an effective way to improve the impact performance of the slab (**Fig. 8.9(b)**). For instance, the failure probability at the impact velocity of 3 m/s decreases from 90% to 36% when the slab thickness grows from 75 mm to 100 mm. When the slab thickness is further increased to 150 mm, the failure probability greatly decreases to 7%. The main reason for this is because the area of the reinforced concrete slab that is resisting the shear force increases with increasing slab depth, which leads to a higher slab capacity in preventing punching shear failure.

(a) Influence of slab length L (b) Influence of slab depth d_b **Fig. 8.9:** Influence of slab size on failure probability of impacted slab.

8.5.2 Effect of concrete strength

Three grades of concrete with the compressive strengths of 19.0 MPa, 23.4 MPa, and 27.8 MPa are considered here to study the effect of concrete strength on the failure probability of reinforced concrete slab under low-velocity impact. An important reason for selecting these three grades of concrete is that the statistical

descriptions are available for them (M7 and E2). Using a higher grade of concrete can improve the performance of the slab and reduce its failure probability (**Fig. 8.10**). Nevertheless, this improvement is observed to be insignificant. For a given impact velocity of 3 m/s, the failure probabilities for low, medium, and high grades of concrete are 54%, 36%, and 28%, respectively. This reduction rate is obviously not as significant as that caused by increasing the length or the thickness of the slab. It is noteworthy that this conclusion is limited to the current study using normal concretes, which should not be extended to high-performance concrete. Some available test data show that using high-performance concrete can effectively enhance the impact resistance of the slab [B3]. However, this condition is beyond the scope of this study.

Fig. 8.10: Influence of concrete strength on failure probability of impacted slab.

8.5.3 Effect of longitudinal and shear reinforcements

Slabs with different longitudinal and shear reinforcement ratios are analyzed to investigate the effects of steel reinforcements. The considered longitudinal reinforcement ratios range from 0.25% to 1.00%. The failure probability curves for different ρ_s are close to each other (**Fig. 8.12(a)**). This result reveals that the

longitudinal reinforcement ratio is of limited importance in contributing to the impact resistance of reinforced concrete slab. This conclusion has been confirmed by the available test results [O3 and X1]. The main cause of this result is that the dominant failure mode of the slab under impact is punching shear where the effect of longitudinal reinforcement is limited. It should be noted that there is a probability for the slab to fail in flexural mode when its longitudinal reinforcement ratio is below 0.4% (**Fig. 8.11**). However, this probability of flexural failure is still much smaller compared with the probability of punching shear failure.

Fig. 8.11: Influence of longitudinal reinforcement ratio on flexural failure probability of impacted slab.

Three shear reinforcement ratios as 0%, 0.28%, and 0.56% are selected to examine the effect of the shear reinforcement. Compared to the longitudinal reinforcement, the shear reinforcement exhibits much better ability in enhancing the impact performance of the slab (**Fig. 8.12(b)**). When the impact velocity is 3 m/s, the failure probability of the slab is 36% without adding shear reinforcement, while it is reduced to 5% when the shear reinforcement ratio becomes 0.28% and 0% when

shear reinforcement ratio becomes 0.56%. The importance of the shear reinforcement on the energy capacity of reinforced concrete slab has also been proved by the experimental work presented in **Chapter 3** and **Chapter 4**.

(a) Longitudinal reinforcement ρ_s

(b) Shear reinforcement ρ_h

Fig. 8.12: Influence of reinforcement on failure probability of impacted slab.

8.5.4 Effect of impactor properties

Fig. 8.13 shows the effect of impactor properties on the failure probabilities of the slab. As can be seen from **Fig. 8.13(a)**, the failure probability of the impacted slab increases when the mass (impact energy) of the impactor increases. For an impactor with the same mass (impact energy), the failure probability of the slab increases when the diameter of the impactor decreases (**Fig 8.13(b)**).

(a) Weight of impactor m_b

(b) Impactor diameter r_b

Fig. 8.13: Influence of impactor properties on failure probability of slabs.

8.6 Failure Probability Curve

In the previous section, the failure probability curve of impacted slabs was formed by connecting the scattered failure probability points at different impact velocities. Therefore, it is desired to obtain smooth and continuous failure probability curves of impacted slabs. In this section, a cumulative normal distribution function is employed to fit the failure probability curves of reinforced concrete slabs. Twenty-six failure probability curves corresponding to the cases shown in **Table 8.3** are fitted based on the data from the parametric study (**Section 8.5**). The mean values and the coefficients of variation of these fitted curves are summarized in **Fig. 8.14** and **Fig. 8.15**, respectively. If the increase of a parameter is beneficial to the impact performance of slab, the mean value of the fitted curve would increase together with this parameter and vice versa. As can be seen from **Fig. 8.14**, increasing the mass of the impactor (i.e., the impact energy) would cause more severe damage to the slab. The influence of the longitudinal reinforcement ratio on slab performance is generally insignificant. Increasing the values of other parameters can boost the impact capacity of slab. These observations agree well with the results of the parametric study in **Section 8.5**. The value of COV reflects the extent of uncertainty of the impact damage of slab. It can be found from **Fig. 8.15** that the values of COV in most cases range between 0.15 and 0.20.

Fig. 8.14: Comparison between mean failure impact velocity at mean parameters and that calculated by statistics.

Fig. 8.15: Comparison of COV values for all the study cases.

8.7 Conclusions

This study developed a procedure to assess the failure probabilities of reinforced concrete slabs subjected to low-velocity impacts. The process and results are useful for investigating the effect of the uncertainties that exist in slab properties and impact loading on the impact performance of reinforced concrete slabs. The

2-DOF nonlinear model introduced and verified in **Chapter 7** was used to predict the dynamic performance of the slab under low-velocity impact. This analytical model was then used to calculate the failure probabilities with the input parameter samples generated by the Latin Hypercube Sampling method. A sensitivity analysis was performed to identify influential random variables and a parametric study was conducted to examine the influence of different parameters on the impact resistance of reinforced concrete slab in terms of probability. The following conclusions have been drawn based on the presented analysis:

- The uncertainties inherent in slab properties and impact loading show significant effects on the performance of reinforced concrete slabs against low-velocity impact. In the presented sample analysis, these uncertainties led to a large COV of the maximum slab displacement whose value exceeds 0.3 when the impact velocity is greater than 4.0 m/s. This result confirms the necessity of establishing a probabilistic study on the impact resistance of the reinforced concrete slab.
- The random variables related to impact loading (m_b and r_b) can significantly affect the impact performance of reinforced concrete slabs, while the uncertainties associated with slab properties (f_c , f_y , ρ_s) have less influence on the impact performance of slabs. In detail, the mass of the impactor m_b is more influential than the radius of the impactor r_b on slab response. For the random variables associated with slab properties, their influences on slab response are ranked in the sequence of the compressive strength of concrete f_c , the slab density ρ , and the yield strength of reinforcement strength f_y .
- For a given impact velocity, the failure probability of the slab can be reduced by increasing the length or thickness of slab. The failure probability of slab

can be slightly reduced by using a higher grade of concrete. However, this result is not applicable for high-performance concrete, which is not studied here. Compared with longitudinal reinforcement, the shear reinforcement exhibits much better ability to enhance the impact resistance of the slab and reduce its failure probability. When the impact velocity is fixed, the increase of the mass of the impactor (i.e., the impact energy) can lead to the growth of the failure probability of the impacted slab. Moreover, failure probability increases as the diameter of the impactor decreases.

CHAPTER 9

CONCLUSIONS AND FUTURE WORKS

9.1 Conclusions

The dynamic behavior of reinforced concrete slabs subjected to rapid loading and low velocity impact was experimentally, numerically and analytically investigated in this study.

In the experimental part, thirty-nine reinforced concrete slabs specimens were tested in three groups. Rapid loading tests were performed at first to study the dynamic behavior of reinforced concrete slabs under static, medium and high rate loadings that correspond to static, seismic and low velocity impact problems. Subsequently, six slabs with the same boundary condition, loading area, designs and material properties as those used in the rapid loading tests were tested under low velocity impact to study the correlations between rapid loading experiment and impact experiment. The remaining fifteen slabs were also tested under low velocity impact to have a more systematic understanding of the influences of various parameters on the impact performance and capacity of reinforced concrete slabs.

Following the experimental works, a numerical study is performed with the emphasis placed on the influences of various parameters on the exact energy capacity of reinforced concrete slab.

After that, analytical method was proposed to predict the dynamic load-displacement curves of reinforced concrete slabs under rapid loading, and the response and damage of reinforced concrete slabs subjected to low velocity impact. This analytical method was further utilized for a probabilistic analysis of

reinforced concrete slabs under low velocity impact in order to examine this problem from the point view of probability.

Based on the presented works, the subsequent conclusions are drawn.

9.1.1 Rapid loading test

The main purpose of this experimental program is to investigate the behavior of reinforced concrete slabs under different loading rates. The results of this study support the following conclusions:

- Both load carrying and energy absorption capacities of reinforced concrete slabs are found to increase when the loading rate increases. Higher loading rate can also lead to higher specimen stiffness at initial loading stage. The maximum strain rate approximately increased 100 times from static loading rate to medium loading rate and another 10 times from medium loading rate to high loading rate. Inertial force also rises when loading rate increase. However, the inertial effect did not become significant until a high loading rate.
- Punching shear failure mode was observed for all the specimens that failed during experiment irrespective of loading rate and loading type. Yielding of bottom reinforcement happens at very early loading stage. The punching shear failure of specimens is featured by the fully formation of punching shear cone and its separation with nearby concrete, the residual load capacity of slab is mainly contributed by top reinforcements with yielding usually happened after punching shear failure.
- Both longitudinal and shear reinforcements are effective in enhancing the maximum load capacity of specimens. However, the benefits on slab's energy absorption and load carrying capacities made by longitudinal reinforcement are

limited comparing with shear reinforcement. The experimental results show that the damage of slab under rapid loadings can be efficiently reduced by adding shear stirrups.

- One main limitation of this experimental study is that only punching shear failure mode of slab was observed although some specimens are designed to fail in flexure under static loading. Therefore, the influence of loading rate on the transformation of failure modes of reinforced concrete slab was not covered in this study. Future work in this area should place more attention on the loading rate's influence on the failure mode of reinforced concrete slab.

9.1.2 Correlations between rapid loading and impact experiments

Following the rapid loading test, an impact experiment is introduced and compared with rapid loading test to study their correlations. Similar crack patterns, damage and failure process of specimen were found through the comparison between high loading rate test and impact test. Additionally, similar strain rates were observed not only on maximum value but also on an average lever, which would lead to similar strain rate effects on specimen's materials. The punching shear failure of impacted slabs was very well predicted using the energy absorption capacities from high loading rate test. In sum, the high loading rate test can produce similar slab behavior and strain rate as low velocity impact problem while reducing the interruption from the inertial effect. Since the load and energy capacities of slabs can be easily obtained from the displacement controlled high loading rate test. The high loading rate test can be considered as a supplementary approach for studying impact problem.

Additionally, it is found that the impact damage on slab can be effective reduced by shear reinforcements. Comparatively, the capacity of slabs under impact

loading is only slightly enhanced by increasing longitudinal reinforcement.

9.1.3 Low velocity impact experiment

The main purpose of the third group experiment is to experimentally investigate the behavior of reinforced concrete slabs under low velocity impact. The results presented in this part support the following conclusions.

- The damage extent of slabs under low velocity impact increases with increasing impact energy. The change of mass and drop height of impactor do not affect the slab damage so much as long as the impact energy is fixed. Therefore, in the domain of low velocity impact, it is possible to directly relate the impact energy with the slab response and damage without knowing the exact velocity and mass of impactor. Compared to the flat nose shape impactor, hemispherical nose shape impactor can cause penetration on reinforced concrete slabs with relatively small impact energy but requires more impact energy to cause punching shear failure on them. In addition, more impact energy is required to fail reinforced concrete slabs when the diameter of impactor increases.
- Although some specimens were designed to have a flexural failure mode under static loading, punching shear failure mode was observed for all the tested specimens. The corresponding damaged process could be summarized as: 1) globalized flexural response at early loading stage; 2) shear cracks initiates and propagates to form the punching shear cone, punching shear failures happens when the punching shear cone is fully developed and separated from its surrounding concrete; 3) concrete in the loading area was crushed after punching shear failure and the residue impact of slab was mainly resisted by the top reinforcement.
- The impact force reaches its maximum value immediately after impact and was

not found in the same phase of the punching shear failure of impacted slab. Large inertial forces were found during impact which was coupled with both impact and support reaction forces. Therefore, the impact force or support reaction force cannot be directly considered as the true resistance capacity.

9.1.4 Numerical simulation

FE model was developed in this part to assess the performance of reinforced concrete slabs under low velocity impact. Experimental data from different sources were used to verify the effectiveness of the adopted modeling method. After the verification work, FE analysis was conducted concerning the influence of mass ratio on the impact response of reinforced concrete slab. The results suggest that, in the domain of low velocity impact, the difference in slab responses made by varying the mass ratio is insignificant when the impact energy is fixed.

As both experimental and numerical works suggest a direct relationship between impact energy and slab response, a parametric study was performed which focuses on various parameters' influences on the energy capacity of reinforced concrete slabs. Results from the finite element analysis show that an increase in any of concrete strength, diameter of impacted area and slab thickness can effectively boost the energy capacity of lightly reinforced concrete slab whereas the effect of longitudinal reinforcement ratio was limited. Additionally, the angle of shear cracks was on average 42° , which decreases when the diameter of impactor increases. Based on regression analysis of the FE results, two dimensionless empirical equations were proposed for assessing the energy capacity of lightly reinforced concrete slabs in resisting punching shear failure, which could be used for initial design or quick check of slab's ability under low velocity impact. However, verification involving more experiments and a wider range of parametric study are required to further update the proposed equations to enhance its accuracy

and range of application.

A mainly limitation of the FE modeling method used in this chapter is that it cannot account for the confining effect of shear reinforcement. As a result, the effect of shear reinforcement in enhancing the impact resistance of RC slabs was underestimated. Due to this reason, shear reinforcement was not included in the subsequent parametric study. In Chapter 4, the test results showed that shear reinforcement is very useful in preventing punching shear failure. Therefore, further research should try to build a better FE model that can rationally reflect the impact response of RC slabs with shear reinforcement, and to incorporate shear reinforcement ratio into the empirical equation for predicting the energy capacity of RC slabs under impact loading.

9.1.5 Analytical study

The following conclusions are drawn based on the results from the analytical study:

- A novel approach is proposed for constructing the dynamic load-displacement curves of reinforced concrete slabs. In this approach, the load-displacement curve of reinforced concrete slab before punching shear capacity is formed through a moment-curvature analysis. A failure criteria based on critical shear crack theory is used to determine the punching shear capacity and the followed load-displacement curve. In this way, the softening behavior of load capacity of reinforced concrete slabs after punching shear failure could be considered, and the failure modes and failure points of slabs can be directly known from the constructed load-displacement curves. The effectiveness of this approach is verified by comparing with experimental data. For the loading rate effect, it is found that the traditional strain rate method could not sufficiently consider the

load capacity enhancement. Comparatively, the load capacity enhancement could be properly incorporated directly using the DAF obtained from rapid loading test. One main limitation in this approach is that the DAF is set 1.5 for reinforced concrete slabs when the loading rates are in the order of 10^0 m/s based on available experimental data. It worth mention that, in addition to loading rate, the DAF can also be influenced by other parameters such as reinforcement ratio, size of specimen, etc. (Adhikary *et al.* 2012). However, current database of rapid loading tests is still limited for obtaining an analytical expression of DAF. Therefore, future work is required in this area for a more proper estimation of DAF.

- A 2-DOF model is presented for analyzing the response and damage of reinforced concrete slabs under low velocity impact. In this model, the slab stiffness is evaluated base on the proposed dynamic load-displacement curve, and the contact between impactor and reinforced concrete slab is modeled with an elastic spring and a viscous damper. The model parameters can be easily determined once the properties of impact load and slab specimen are known. Using numerical method, this model can be solved very efficiently. In the followed validation work, the slab responses in terms of maximum mid-span displacement, maximum impact force, contact duration, impulse, imparted energy, etc. have been estimated with high accuracy. In sum, this 2-DOF model provides a simple, efficient, yet accurate access for analyzing slab impact problems.
- The results from the 2-DOF model are further processed to show slab behavior from an energetic standpoint. By analyzing the intrinsic distribution of impact energy, it is found that the impact energy can be dissipated by slab damper, contact damper and permanent slab deformation. Among them, the energy

dissipated by permanent slab deformation reaches very high percentage of the impact energy. This result proves why a rational load-displacement curve is a necessity for a 2-DOF model to produce good results.

- In the situation where detailed and accurate slab response is not required, the energy capacity of slab can be fast and conservatively estimated via integrating its load-displacement curve till its failure point. Whether the slab fails or not during impact can be known by comparing this energy capacity with the exerted impact energy.

9.1.6 Probability analysis

The following conclusions are drawn based on the presented probability analysis:

- The uncertainties inherent in slab property and impact loading showed significant effect on the performance of reinforced concrete slabs against low velocity impact. In the presented sample analysis, these uncertainties led to a large COV of the maximum slab displacement whose value exceeds 0.3 when the impact velocity is larger than 4.0 m/s. This result confirms the necessity of establishing a probabilistic study on the impact resistance of the reinforced concrete slab.
- The random variables related to impact loading (m_b and r_b) can significantly affect the impact performance of reinforced concrete slab, while the uncertainties associated with slab properties (f_c, f_y, ρ_s) have less influences on the impact performance of slab. In detail, the mass of impactor m_b is more influential than the radius of impactor r_b on slab response. For the random variables associated with slab properties, their influences on slab response are ranked in the sequence of the compressive strength of concrete f_c , the slab density ρ_0 , and the yield strength of reinforcement strength f_y .

- For a given impact velocity, the failure probability of the slab can be reduced by increasing the length or thickness of slab. The failure probability of slab can be slightly reduced by using a higher grade of concrete. However, this result is not applicable for high performance concrete which is not studied here. Comparing with longitudinal reinforcement, the shear reinforcement exhibits much better ability in enhancing the impact resistance of the slab and reducing its failure probability. When the impact velocity is fixed, the increase of the mass of impactor (impact energy) can lead to the growth of the failure probability of the impacted slab. Moreover, the failure probability increases as the diameter of impactor decreases.

9.2 Future Works

In the light of experience gained from this research program, recommendations are made for the future works considering the dynamic behavior of reinforced concrete slabs under rapid loading and low velocity impact.

9.2.1 Future works about experimental investigation

As summarized in the literature review, current database of rapid loading tests is very limited. Therefore, more rapid loading experiments on reinforced concrete slabs are urgently needed to fill this deficiency. It is recommended that:

- The future rapid loading experiments should consider more variables such as size of loading area, size of slab, strength of concrete, reinforcement layout, etc.
- The future rapid loading experiments may cover a wider range of loading rate. Some advanced rapid loading machine can produce loading rates in the order of 10^1 m/s or even higher, which corresponds to medium or high velocity

impact problem.

- Inertial effect induced by rapid loading test has found very small comparing with that caused by impact test. However, due to the limited accelerometers installed to the slab specimens, this inertial effects cannot be precisely evaluated and ruled out in this study. As a result, future work should try to solve this problem.
- Once the experimental database becomes sufficient, future works should try to produce empirical expression for assessing the DAF of reinforced concrete slabs under different loading rates, which can be used for design works or analytical models.

When it comes to low velocity experiment, the following recommendations on the future works are made:

- Current impact experiments only focus on localized impact. Hence, uniformly distributed impact loading may be considered in the future impact experiment.
- Reinforced concrete slabs may be impacted by soft objects (vehicle, human, etc.). Therefore, future impact experiments may include soft impact and evaluate how the stiffness of the impactor affects slab response.
- For simply supported reinforced concrete slabs, there is a possibility of punching shear failure to occur on the corners of slab. Future works may investigate this failure mode. Additionally, future works may study the impact event when the impact load is applied at other locations of the slab rather than at its center.

- Similar to rapid loading test, future works should more precisely investigate the induced inertial effect.

9.2.2 Future works about numerical simulation

So far, many numerical works of slab impact problems have been conducted, and the simulated results agree well with the experimental data. On the contrary, limited numerical work about rapid loading has been performed. In Adhikary's work [A5 and A6], the simulated results deviate severely from the experimental data when the loading rate beyond 10^{-1} m/s. Therefore, future works should try to present more precise numerical approach for studying the performance of reinforced concrete slabs under rapid loading.

9.2.3 Future works about analytical method

The subsequent recommendations are made for the future analytical works:

- In the presented analytical models of forming the dynamic load-displacement curves and predicting the impact responses of reinforced concrete slabs, some model parameters are empirically determined based on current experimental database. Therefore, future works may try to give analytical expressions for these parameters, or update these parameters with a larger experimental database.
- In the proposed method of generating the dynamic load-displacement curves of reinforced concrete slabs. The loading rate effect was included empirically using the DAF obtained from experiments. However, analytical method is more versatile because it is independent of experimental data. As a result, future works should try to improve the current analytical methods (strain rate method and effective length method), to make sure they can accurately account for the loading rate effect.

REFERENCES

- [A1] ACE (1946), “Fundamentals of Protective Structures”, Report AT120 AT1207821, Army Corps of Engineers, Office of the Chief of Engineers.
- [A2] ACI Committee 318 (2008), “Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary”, American Concrete Institute, Farmington Hills, MI, pp. 473.
- [A3] Acito, M., Stochino, F., and Tattoni, S. (2011), “Structural Response and Reliability Analysis of RC Beam Subjected to Explosive Loading”, *Applied Mechanics and Materials*, Vol. 82, pp. 434-439.
- [A4] Abde-Rohman, M., and Sawan, J. (1985), “Impact Effect on RC Slabs: Analytical Approach”, *Journal of Structural Engineering*, Vol. 111, No. 7, pp. 1590-1601.
- [A5] Adhikary, S. D., Li, B., and Fujikake, K. (2012), “Dynamic Behavior of Reinforced Concrete Beams under Varying Rates of Concentrated Loading”, *International Journal of Impact Engineering*, Vol. 47, pp. 24-38.
- [A6] Adhikary, S. D., Li, B., and Fujikake, K. (2013), “Strength and Behavior in Shear of Reinforced Concrete Deep Beams under Dynamic Loading Conditions”, *Nuclear Engineering and Design*, 259, pp. 14-28.
- [A7] Adhikary, S. D., Li, B., and Fujikake, K. (2013), “Effects of High Loading Rate on Reinforced Concrete Beams”, *ACI Structural journal*, Vol. 111, No. 3, pp. 651-660.
- [A8] Adhikary, S. D. (2013), “Dynamic Behavior of Reinforced Concrete Beams

under Varying Rates of Concentrated and Impact Loading”, PhD thesis, School of Civil and Environmental Engineering, Nanyang Technological University, Singapore.

[A9] Ågårdh, L., and Laine, L. (1999), “3D FE-Simulation of High-Velocity Fragment Perforation of Reinforced Concrete Slabs”, *International Journal of Impact Engineering*, Vol. 22, No. 9, pp. 911-922.

[B1] Bangash, M. Y. H. (1993), “Impact and Explosion, Analysis and Design”, CRC Printing.

[B2] Banthia, N., Mindess, S., Bentur, A., and Pigeon, M. (1989), “Impact Testing of Concrete Using a Drop-Weight Impact Machine”, *Experimental Mechanics*, Vol. 29, No. 1, pp. 63-69.

[B3] Beckmann, B., Hummeltenberg, A., Weber, T., and Curbach, M. (2012), “Strain Behaviour of Concrete Slabs under Impact Load”, *Structural Engineering International*, Vol. 22, No. 4, pp. 562-568.

[B4] Bertrand, D., Kassem, F., Delhomme, F., and Limam, A. (2015), “Reliability Analysis of an RC Member Impacted by a Rockfall Using a Nonlinear SDOF model”, *Engineering Structures*, Vol. 89, pp. 93-102.

[B5] Bhatti, A. Q., Kishi, N., and Tan, K. H. (2011), “Impact Resistant Behaviour of RC Slab Strengthened with FRP Sheet”, *Materials and structures*, Vol. 44, No. 10, pp. 1855-1864.

[B6] Biggs, J. M., and Testa, B. (1964), “Introduction to Structural Dynamics”, McGraw-Hill Companies, New York.

[B7] Bischoff, P. H., and Perry, S. H. (1991), “Compressive Behaviour of Concrete at High Strain Rates”, *Materials and structures*, Vol. 24, No. 6, pp. 425-450.

[C1] CEB (1988), “Bulletin N. 187”, European Committee for Concrete, Lausanne, Switzerland.

[C2] Chang, W. S. (1981), “Impact of Solid Missiles on Concrete Barriers”, *Journal of Structural Division, ASCE*, Vol. 107, No. 2, pp. 257-271.

[C3] Chen, Y., and May, I. M. (2009), “Reinforced Concrete Members under Drop-weight Impacts”, *Proceedings of the ICE-Structures and Buildings*, Vol. 162, No. 1, pp. 45-56.

[C4] Cotsovos, D. M., Stathopoulos, N. D. and Zeris. C. A. (2008), “Behavior of RC Beams Subjected to High Rates of Concentrated Loading”, *Journal of structural engineering*, Vol. 134, No. 12, pp. 1839-1851.

[C5] Cotsovos, D. M. (2010), “A Simplified Approach for Assessing the Load-carrying Capacity of Reinforced Concrete Beams under Concentrated Load Applied at High Rates”, *International Journal of Impact Engineering*, Vol. 37, No. 8, pp. 907-917.

[C6] Crawford, J. E., Anderson, M., Bogosian, D. D., Hasselman, T. K., and Dunn, B. W. (2000), “Enhanced Single Degree-of-freedom Models for Predicting Slab Responses to Blasts”, *SPIE proceedings series. Society of Photo-Optical Instrumentation Engineers*, Bellingham, Washington, pp. 740-746.

[D1] Dancygier, A. N., Yankelevsky, D.Z., and Jaegermann, C. (2007), “Response of High Performance Concrete Plates to Impact of Non-Deforming Projectiles”, *International Journal of Impact Engineering*, Vol. 34, No. 11, pp. 1768-1779.

- [D2] Delhomme, F., Mommessin, M., Mougin, J-P., and Perrotin, P. (2005) “Behavior of a Structurally Dissipating Rock-shed: Experimental Analysis and Study of Punching Effects”, *International Journal of Solids and Structures*, Vol. 42, pp. 4204–4219.
- [D3] Delhomme, F., Mommessin, M., Mougin, J. P., and Perrotin, P. (2007) “Simulation of a Block Impacting a Reinforced Concrete Slab with a Finite Element Model and a Mass-spring System”, *Engineering Structures*, Vol. 29, No. 11, 2844-2852.
- [D4] Dinic G., and Perry., W. S. (1990), “Shear Plug Formation in Concrete Slabs Subjected to Hard Impact”, *Engineering Fracture Mechanics*, Vol. 35, pp. 343-350.
- [D5] Dorsselaer, N. VAN., Lapoujade, V., Nahas, G., Tarallo, F., and Rambach, J. (2011), “Impact Simulations on Concrete Slabs: LS-OPT Fitting Approach”, 8th European LS-DYNA Users Conference, Strasbourg.
- [D6] Doyle, J. M. (1973), “Design of Missile Resistance Concrete Panels”, Reprints of 2nd International Conference on Structural Mechanics, Vol. 4, pp. 6.
- [E1] Eamon, C. D. (2007), “Reliability of Concrete Masonry Unit Walls Subjected to Explosive Loads”, *Journal of Structural Engineering*, Vol. 133, No. 7, pp. 935-944.
- [E2] Ellingwood, B., Galambos, T. V., MacGregor, J. G., and Cornell, C. A. (1980), “Development of a Probability Based Load Criterion for American National Standard A58 – Building Code Requirement for Minimum Design Loads in Buildings and Other Structures”, National Bureau of Standards Department of Commerce Washington, DC 20234.

- [F1] FIB (2010), “Bulletin N. 66: Model Code 2010 (Volume 2) Final Draft”, International Federation for Structural Concrete, Lausanne, Switzerland.
- [F2] Fujikake, K., Mori, K., Uebayashi, K., Ohno, T., and Mizuno, J. (2001), “Constitutive Model for Concrete Materials with High-Rates of Loading under Tri-axial Compressive Stress States”, Proc., 3rd Int. Conf. on Concrete under Severe Conditions, 1, JSCE, Tokyo, pp. 636–643.
- [F3] Fu, H. C., Erki, M. A., and Seckin, M. (1991), “Review of Effects of Loading Rate on Concrete in Compression.” *Journal of Structural Engineering*, Vol. 117, No. 12, pp. 3645-3659.
- [F4] Fujikake, K., Li, B., and Soeun, S. (2009), “Impact Response of Reinforced Concrete Beam and Its Analytical Evaluation”, *Journal of Structural Engineering*, Vol. 135, No. 8, 938-950.
- [H1] Hao, H., Stewart, M., Li, Z. X., and Shi, Y. (2010), “RC Column Failure Probabilities to Blast Loads.” *International Journal of Protective Structures*, Vol. 1, No. 4, pp. 571-591.
- [H2] Hallquist, J. O. (2006), “LS-DYNA Theory Manual”, Livermore Software Technology Corporation.
- [H3] Hallquist, J. O. (2007), “LS-DYNA Keyword User’s Manual”, Livermore Software Technology Corporation, pp. 970.
- [H4] Hrynyk, T. D. (2013), “Behavior and Modelling of Reinforced Concrete Slabs and Shells under Static and Dynamic Loads”, PhD Thesis, University of Toronto, Canada.

[H5] Hrynyk, T. D., and Vecchio, F.J. (2014), “Behavior of Steel Fiber-Reinforced Concrete Slabs under Impact Load”. *ACI Structural Journal*, Vol. 111, No. 5, 1213-1223.

[I1] Isaac, P., Darby, A., Ibell, T., and Evernden, M. (2013), “Influence of Loading Rate on Failure Mode of Reinforced Concrete Members”, *Proceedings of the ICE-Engineering and Computational Mechanics*, Vol. 166, No. 4, pp. 194-203.

[J1] Jakubowski, E. H. (1964), “Dynamic Formulation of Coefficient of Restitution”, No. RIA-77-U91, SPRINGFIELD ARMORY MA.

[J2] Japan Society of Civil Engineers (JSCE), (1993), “Impact Behavior and Design of Structures”, *Structural Engineering Series 6*, JSCE, Tokyo.

[J3] Japan Society of Civil Engineers (JSCE), (2002), “Standard Specifications for Concrete Structures 2002”, *Structural Performance Verification*, JSCE, Tokyo.

[K1] Kennedy, R. P. “A Review of Procedures for the Analysis and Design of Concrete Structures to Resist Missile Impact Effects”, *Nuclear Engineering and Design*, Vol. 37, pp. 183-203, 1976.

[K2] Kogure, K., Ui, M., Ohno, T., Andou, T., and Fujimoto, Kazuo, (1996), “On Impact Behaviors and Resistance of Reinforced Concrete Slabs with Different Rebar Arrangements under Impact Loadings”, *Structural Engineering, JSCE*, Vol. 42A, pp. 1315-1323.

[K3] Kogure, K., Ui, M., Ohno, T., Andou, T., Yamaguchi, H. (1997), “An Experimental Study on Dynamic Behaviors and Impact Resistance of Steel-concrete Composite Panels under Impact Loadings”, *Structural Engineering, JSCE*, Vol. 43A, No. 1489-1498.

- [K4] Kishi, N., Mikami, H., Matsuoka, K.G., and Ando T. (2002), "Impact Behavior of Shear-Failure-Type RC Beams without Shear Rebar." *International Journal of Impact Engineering*, Vol. 27, No. 9, pp. 955-968.
- [K5] Kishi, N., Mikami, H., and Kurihashi, Y. (2008), "An Influence of Loading-ram Diameter on Impact Response Behavior of Rectangular RC Slabs", *Structural Engineering, JSCE*, Vol. 54A, pp. 1034-1043.
- [K6] Kishi, N., Mikami, H., and Kurihashi, Y. (2010), "Static and Impact Loading Tests of RC Slabs with Various Supporting Conditions", *Structural Engineering, JSCE*, Vol. 56A, pp. 1160-1168.
- [K7] Kishi, N., Kurihashi, Y., Ghadimi Khasraghy, S., and Mikami, H. (2011), "Numerical Simulation of Impact Response Behavior of Rectangular Reinforced Concrete Slabs under Falling-Weight Impact Loading", *Applied Mechanics and Materials*, Vol. 82, pp. 266-271.
- [K8] Kojima, I. (1991), "An Experimental Study on Local Behavior of Reinforced Concrete Slabs to Missile Impact", *Nuclear engineering and design*, Vol. 130, No. 2, pp. 121-132.
- [K9] Kulkarni, S. M. (1997), "Effect of High Strain Rate on Reinforced Concrete Beams", PhD thesis, Field of Civil Engineering, Northwestern University, Evanston, Illinois.
- [K10] Kulkarni, S. M., and Shah, S. P. (1998), "Response of Reinforced Concrete Beams at High Strain Rates", *ACI Structural Journal*, Vol. 95, No. 6, pp. 705-715.
- [L1] Li, Q. M., Reid, S. R., Wen, H. M., and Telford, A. R. (2005), "Local Impact Effects of Hard Missiles on Concrete Targets", *International Journal of Impact*

Engineering, Vol. 32, No. 1, pp. 224-284.

[L2] Linderman, R. B., Fakhari, M. and Rotz, J. V. (1973), “Design of Structures for Missile Impact”, BC-TOP-9, Rev. 1, Bechtel Power Corporation, San Francisco, July.

[L3] Low, H. Y., and Hao, H. (2001), “Reliability Analysis of Reinforced Concrete Slabs under Explosive Loading.” *Structural Safety*, Vol. 23, No. 2, pp. 157-178.

[L4] Low, H. Y., and Hao, H. (2001), “Reliability Analysis of Direct Shear and Flexural Failure Modes of RC Slabs under Explosive Loading”, *Engineering Structures*, Vol. 24, No. 2, pp. 189-198.

[M1] Malvar, L. J., and Rose, C. A. (1998), “Review of Strain Rate Effects for Concrete in Tension”, *ACI Materials Journal*, Vol. 95, No. 6, 735-739.

[M2] Malvar, L. J. (1998), “Review of Static and Dynamic Properties of Steel Reinforcing Bars”, *ACI Material Journal*, Vol. 95, No. 5, 609-616.

[M3] Melchers, R. E. (1999), “Structural Reliability Analysis and Prediction (Second Edition)”, John Wiley & Sons.

[M4] Micallef, K., Sagaseta, J., Ruiz, M. F., and Muttoni, A. (2014), “Assessing Punching Shear Failure in Reinforced Concrete Flat Slabs Subjected to Localized Impact Loading”, *International Journal of Impact Engineering*, Vol. 71, pp. 17-33.

[M5] Mihashi, H., and Wittmann, F. H. (1980), “Stochastic approach to study the influence of the rate loading on strength of concrete.” *HERON*, 25(3).

[M6] Mirza, S. A., and MacGregor, J. G. (1979), “Variability of Mechanical

Properties of Reinforcing Bars”, Journal of Structural Division, ASCE, Vol. 105, No. 5, 921-937.

[M7] Mirza SA, Hatzinikolas M, and MacGregor JG. (1979), “Statistical Descriptions of Strength of Concrete”, Journal of Structural Division, ASCE, Vol, 105, No. 6, pp. 1021-1037.

[M8] Murray, Y. D., Abu-Odeh, A. Y., and Bligh, R. P. (2007), “Evaluation of LS-DYNA Concrete Material Model 159”, No. FHWA-HRT-05-063.

[M9] Muthukumar, S., and DesRoches, R. (2006), “A Hertz Contact Model with Non-Linear Damping for Pounding Simulation.” Earthquake Engineering and Structural Dynamics, Vol. 35, No. 7, pp. 811-828.

[M10] Mutsuyoshi, H., and Machida, A. (1984), “Properties and Failure of Reinforced Concrete Members Subjected to Dynamic Loading”, Transactions of the Japan Concrete Institute, Vol. 6, pp. 521-528.

[M11] Muttoni, A. (2008), “Punching Shear Strength of Reinforced Concrete Slabs without Transverse Reinforcement”, ACI Structural Journal, 105, 440-450, 2008.

[M12] Muttoni, A., and Ruiz, M. F. “The Levels-of-approximation Approach in MC 2010: Application to Punching Shear Provisions”, Structural Concrete, Vol. 13. No. 1, pp. 32-41.

[N1] NDRC (1946), “Effects of Impact and Explosion-Summary Technical Report of Division 2, vol. 1”, National Defence Research Committee, Washington, DC.

[O1] Olsson, A. M. J. and Sandberg, G. E. (2002), “Latin Hypercube Sampling for Stochastic Finite Element Analysis”, Journal of Engineering Mechanics, Vol. 128,

No. 1, pp. 121-125.

[O2] Ong, K. C. G., Paramasivam, P. and Basheerkhan, M. (1999), “Behavior of Fiber-Reinforced Concrete Slabs under Low Velocity Projectile Impact”, ACI special Publication, Vol. 172, pp. 993-1011.

[O3] Othman, H., Marzouk, H. (2016), “An Experimental Investigation on the Effect of Steel Reinforcement on Impact Response of Reinforced Concrete Plates”, International Journal of Impact Engineering, Vol. 88, pp. 12-21.

[R1] Rahman, A., Zaidi, A. M. A., and Latif, Q. B. (2011), “Review on Empirical Studies of Local Impact Effects of Hard Missile on Concrete Structures”, International Journal of Sustainable Construction Engineering and Technology, Vol. 1, No. 1, pp. 73-98.

[R2] Rao, H. S., Ghorpade, V. G., Ramana, N. V., and Gnaneswar, K. (2010), “Response of SIFCON Two-way Slabs Under Impact Loading.” International Journal of Impact Engineering, Vol. 37, No. 4, pp. 452-458.

[R3] Reinhardt, H.W., Weerheijm, J. (1991) “Tensile fracture of concrete at high loading rates taking account of inertia and crack velocity effects.” International Journal of Fracture, 51, pp. 31-42.

[R4] Remennikov, A., and Kaewunruen, S. (2006), “Impact Resistance of Reinforced Concrete Columns: Experimental Studies and Design Considerations”, 19th Australasian Conference on the Mechanics of Structures and Materials, Christchurch, New Zealand, pp. 817-824.

[R5] Rong, H. C., and Li, B. (2007), “Probabilistic Response Evaluation for RC Flexural Members Subjected to Blast Loadings”, Structural Safety, Vol. 29, No. 2,

pp. 146-163.

[R6] Ross, A., Tedesco, J.W. and Kuennen, S.T. (1995) "Effect of strain rate on concrete strength." *ACI Material Journal*, 92(1), pp. 37-47.

[R7] Rousseau, J., Frangin, E., Marin, P., and Daudeville, L. (2009), "Multidomain Finite and Discrete Elements Method for Impact Analysis of a Concrete Structure", *Engineering structures*, Vol. 31, No. 11, pp. 2735-2743.

[R8] Ruiz, M. F., and Muttoni, A. (2009), "Applications of Critical Shear Crack Theory To Punching of Reinforced Concrete Slabs with Transverse Reinforcement", *ACI Structural Journal*, Vol. 106, No. 4, pp 485-494.

[S1] Saatci, S., and Vecchio, F. J. (2009), "Effects of Shear Mechanisms on Impact Behavior of Reinforced Concrete Beams", *ACI structural Journal*, Vol. 106, No. 1, pp. 78-86.

[S2] Sadiq, M., Yun, Z. X., and Rong, P. (2014), "Simulation Analysis of Impact Tests of Steel Plate Reinforced Concrete and Reinforced Concrete Slabs Against Aircraft Impact and Its Validation with Experimental Results", *Nuclear Engineering and Design*, Vol. 273, pp. 653-667.

[S3] Sangi, A. J., and May, I. M. (2009), "High-Mass, Low-velocity Impacts on Reinforced Concrete Slabs", In 7th European LS-DYNA conference, Salzburg, Austria.

[S4] Satio H., Imamura A., Takeuchi M., Okamoto S., Kasai Y., Tsubota H., and Yoshimura M. (1995), "Loading Capacities and Failure Modes of Various Reinforced-concrete Slabs Subjected to High Speed Loading", *Nuclear Engineering and Design*, Vol. 156, No. 1, pp. 277-286.

- [S5] Sato, M., Kishi, N., Mikami, H., and Matsuoka, K. (1997), "Anti-impact Capacity of Prototype RC Slabs", *Structural Engineering, JSCE*, Vol. 43A, pp. 1499-1506.
- [S6] Sawan, J., and Abdel-Rohman, M. (1986), "Impact Effect on RC Slabs: Experimental Approach." *Journal of Structural Engineering*, Vol. 112, No. 9, pp. 2057-2065.
- [S7] Schellenberg, K., Volkwein, A., Roth, A., and Vogel, T. (2007), "Large-scale Impact Tests on Rock Fall Galleries", In *7th Int. Conference on Shock & Impact Loads on Structures*, pp. 17-19.
- [S8] Sliter, G. E. (1980), "Assessment of Empirical Concrete Impact Formulas." *Journal of the Structural Division* 1980, Vol. 106, No. 5, pp. 1023-1045.
- [S9] Sohel, K. M. A., and Liew, J. R. (2014), "Behavior of Steel–concrete–steel Sandwich Slabs Subject to Impact Load", *Journal of Constructional Steel Research*, Vol. 100, pp. 163-175.
- [S10] Stochino, F., and Carta, G. (2014), "SDOF Models for Reinforced Concrete Beams under Impulsive Loads Accounting for Strain Rate Effects", *Nuclear Engineering and Design*, Vol. 276, pp. 74-86.
- [S11] Sugano, T., Tsubota, H., Kasai, Y., Koshika, N., Ohnuma, H., Von, Riesemann, W. A., Bickel, D. C., and Parks, M. B. (1993), "Local Damage to Reinforced Concrete Structures Caused by Impact of Aircraft Engine Missiles Part 1. Test Program, Method and Results", *Nuclear Engineering and Design*, Vol. 140, No. 3, pp. 387-405.
- [S12] Sugano, T., Tsubota, H., Kasai, Y., Koshika, N., Itoh, C., Shirai, K., Von,

Riesemann, W. A., Bickel, D. C., and Parks, M. B. (1993), "Local Damage to Reinforced Concrete Structures Caused by Impact of Aircraft Engine Missiles Part 2. Evaluation of Test Results", *Nuclear Engineering and Design*, Vol. 140, No. 3, pp. 407-423.

[T1] Tai, Y. S., and Tang, C. C. (2006), "Numerical Simulation: the Dynamic Behavior of Reinforced Concrete Plates under Normal Impact", *Theoretical and Applied Fracture Mechanics*, Vol. 45, No. 2, pp. 117-127.

[T2] Teland, J. A. (1998), "A Review of Empirical Equations for Missile Impact Effects on Concrete", Report FFI/RAPPORT 97, 05856.

[X1] Xiao, Y., Li, B., and Fujikake, K. (2016), "Experimental Study of Reinforced Concrete Slabs under Different Loading Rates", *ACI structural Journal*.

[Z1] Zhang, J., Maalei, M. and Quek, S. T. (2007), "Performance of Hybrid-Fiber ECC Blast/Shelter Panels Subjected to Drop Weight Impact", *Journal of Materials in Civil Engineering*, Vol. 19, No. 10, pp. 855-863.

[Z2] Zineddin, M., and Krauthammer, T. (2007), "Dynamic Response and Behavior of Reinforced Concrete Slabs under Impact Loading", *International Journal of Impact Engineering*, Vol. 34, No. 9, pp. 1517-1534.

[Z3] Zineddin, M. (2008), "Simulation of Reinforced Concrete Slab Behavior under Impact Loading", In *Proceedings of the AEI 2008 Conference: Building Integration Solutions*, New York, pp. 1-8.