Production-inventory-distribution coordination and performance optimization for integrated multi-stage supply chains

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**PRODUCTION-INVENTORY-DISTRIBUTION
COORDINATION AND PERFORMANCE
OPTIMIZATION FOR INTEGRATED MULTI-STAGE
SUPPLY CHAINS**

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2016

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SUPPLY CHAINS**

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Doctor of Philosophy

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DEDICATION

To my loving wife, Li who has been the strong foundation for my life.

*To my wonderful son, Yu Tai, and newly born daughter, Doreen Yu Cheng, the source
of my great happiness.*

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LIST OF ABBREVIATIONS

ACM	Analytical coordination model
API	Active pharmaceutical ingredient
CPFR	Collaborative planning, forecasting, replenishment
DC	Distribution centre
EOQ	Economic order quantity
EPQ	Economical production quantity
FPW	Finished product warehouse
IIC	Integer-ratio inventory coordination
JCIR	Joint consideration of inventory replenishment
JIT	Just-in-time
MINLP	Mixed-integer nonlinear programming
MNC	Multinational corporation
MUP	Manufacturing plant
OCT	Optimal cycle time
PP	Production plant
R	retailer
RMW	Raw material warehouse
SCC	Supply chain coordination
SCM	Supply chain management
SCOR	Supply chain operations reference
TOC	Total operational cost
WDAG	Weighted directed acyclic graph

ZIO

Zero-inventory ordering

ABSTRACT

The objective of this dissertation is to study the production-inventory-distribution coordination and performance optimization problems for integrated multi-stage supply chains by adopting the coordination mechanism and framework, primarily the joint consideration of inventory replenishment and the Supply Chain Operations Reference (SCOR) model. The supply chains under study include raw materials, production, transportation and distribution. There are four stages in the supply chain process: the raw material warehouse, the manufacturing/production plant, the finished product warehouse and the distribution centre/retailer.

To support coordination among the supply chain parties, both the joint consideration of inventory replenishment and an SCOR model are adopted as the coordination mechanism and the framework of the research. The scope of the work is to study three distinct but inseparable problems by using rigorous analytical approaches: an integrated multi-stage supply chain with constant demand, a SCOR-based analytical coordination model for an integrated supply chain with constant demand and an integrated multi-stage supply chain with time-varying demand. The fundamental goal of this research is to study the effects of minimizing the total operational cost of multi-stage supply chains by adopting different coordination mechanisms and frameworks.

The first part of the study considers an integrated production-inventory-distribution planning problem that is faced by a multinational corporation (MNC) that manages a multi-stage supply chain over an infinite time horizon. Based on the supply chain management practices of this company, the joint consideration of inventory replenishment is adopted as the coordination mechanism at the tactical and operational

levels. We devise an optimal integer-ratio coordination policy for inventory replenishment across its supply chain. Under the proposed optimal integer-ratio inventory coordination policy, the total operational cost of the supply chain is demonstrated to reach its global minimum after the integrality constraints are relaxed. Numerical examples are presented with a sensitivity analysis. The computational results demonstrate that the difference in the optimal total operational costs between integer and real-number solutions is not significant.

In the second part of the research, both the joint consideration of inventory replenishment and an SCOR model are adopted as the coordination mechanism and the framework in an integrated supply chain with constant demand. In the existing literature, it remains a challenge to quantify the coordination effects on supply chain performance after the implementation of such models as Collaborative Planning, Forecasting, Replenishment and SCOR. An analytical coordination model for a supply chain of an MNC is presented. To improve supply chain performance, we study the coordination among the supply chain parties from the strategic to the operational levels. *Supply chain management cost* which is one of level 1 metrics from the SCOR model is selected to support the MNC to select and refine the SCM strategies. An optimal integer-ratio inventory coordination policy is devised to coordinate the inventory replenishment at the tactical and operational levels. By combining the SCOR model and the integer-ratio inventory coordination policy, a systematic approach is proposed. We focus on the derivation and analysis of the total operational cost of the supply chain based on cost performance metrics across three levels of the SCOR model version 10. The total operational cost is demonstrated to reach its global minimum after the integer constraint is relaxed. The findings reinforce the proposition that the adoption of an analytical coordination model based on the metrics of the SCOR model is promising in terms of

its capacity to assist decision makers in improving supply chain performance. Numerical experiments are conducted to demonstrate how to compute the optimal total operational cost in practice. The computational results demonstrate that the total operational cost savings through the SCOR-based analytical coordination model are significant.

By extending the results from constant and continuous demand, an integrated multi-stage supply chain with time-varying demand over a finite planning horizon is considered in the last part of the study. With the joint consideration of inventory replenishment coordination mechanism, an optimal production-inventory-distribution policy is devised to minimize the total operational cost. The model is formulated as a mixed-integer nonlinear programming optimization problem. The problem is represented as a weighted directed acyclic graph. The global minimum total operational cost is computed in polynomial time by the developed algorithm. Two numerical examples of a seasonal product and a product over its life cycle are studied to illustrate the results. A sensitivity analysis of the system parameters is conducted to help elucidate the supply chain decision making process.

CHAPTER 1

INTRODUCTION

1.1 Overview

As per the definition of the Council of Supply Chain Management Professionals, Supply Chain Management (SCM) encompasses the planning and management of all activities that are involved in sourcing and procurement, conversion, and logistics management (Council of Supply Chain Management Professionals). A typical supply chain consists of all of the involved parties, which include not only the manufacturers and suppliers but also the transporters and warehouses, and the customers themselves. The primary purpose of a supply chain is to satisfy customers' needs and generate profits for the supply chain owners (Chopra & Meindl, 2013). A basic serial supply chain is shown in Figure 1.1.

Figure 1.1 A serial supply chain.

In today's rapidly changing world, multinational corporations' (MNCs') profits have decreased due to fierce competition. Companies cannot simply pursue market share and cut operational costs from an individual enterprise to ensure profitability. They must

also constantly evaluate and redesign their supply chains to deliver the desired customer service at the lowest cost (Asgari *et al.*, 2016).

In the modern era, supply chains are becoming more complex due to rising fuel prices, dynamic business environments, and demanding customers. One of the most important drivers for decision making in running of a business lies in understanding the total operational cost (TOC) of ownership across an entire supply chain. By understanding the drivers of TOC, decision makers are able to deploy the correct strategies to reduce such costs (Simchi-Levi *et al.*, 2008).

1.2 Inventory control and supply chain management

Inventory spreads throughout a supply chain. It acts as a buffer between suppliers, manufacturers, retailers, and customers. Inventories at different stages are different in name, but all cost money to maintain. The holding cost varies between industries and contributes significantly to the total supply chain's operational cost. Thus, effective inventory management can optimize supply chain performance and reduce the possibility of shortages that are caused by volatile demands (Christopher, 2011).

In 1925, Harris investigated the classic economic order quantity (EOQ) model. Thousands of articles have been published on production and distribution models. Significant research progress in this area was made in the 1980s. The EOQ model is used mainly for a single stage of product-inventory problems. The solution is optimal for all feasible policies. A finite production rate and allowing for backlogging are simple extensions of the EOQ model (Zipkin, 2000).

In practice, there are multi-echelon structures in supply chains. It is a completely different case in which optimal inventory policies are difficult to achieve. Sometimes,

we may even doubt that an optimal inventory policy for a supply chain is possible (Axsäter, 2006). The difficulties arise from the fact that the different stages of a supply chain may have conflicting objectives if each stage belongs to a different owner. As a result, each stage tries to maximize its own profits. However, this often leads to a degradation of responsiveness and reduces the total supply chain profits. Many MNCs face this type of supply chain coordination (SCC) challenge today (Bushuev *et al.*, 2015; Glock, 2012; Silver *et al.*, 1998; Wang *et al.*, 2015). For example, the Pampers diapers maker Procter & Gamble found that the raw material ordering quantity fluctuated significantly over time. However, sales fluctuations at the upstream retail stores were small. This “bullwhip” effect (Lee *et al.*, 1997; Wang & Disney, 2016) in the Procter & Gamble supply chain increased the operational cost and made it difficult to match supply with demand.

1.3 Supply chain coordination

In the last two decades, SCM and its coordination have received a great deal of attention from researchers and practitioners (Bushuev *et al.*, 2015). The coordination of decentralized production planning is challenging because supply chain parties are often independent of and are guided by individual and conflicting objectives. They are not willing to share certain pieces of private information, *e.g.*, cost parameters and customer demand (Glock, 2012).

Consider a typical example of a supply chain (Figure 1.2) in which the vendors supply materials or products and buyers order from vendors. When each party makes decisions independently, buyers determine an inventory replenishment that minimizes their own operational costs. Because the buyers’ decisions about shipping schedules and

lot size neglect the vendors' operational costs, such decisions may not be preferred by the vendors. As a result, the total profits of the supply chain reduce as each stage maximizes its own profits. The lack of coordination has a negative impact on supply chain performance by increasing TOC and reducing responsiveness. The consequences of the lack of coordination are: inaccurate forecasts, low capacity utilization, excessive inventory, inadequate customer service, high inventory cost, slow order fulfilment response, bad product quality, and low customer satisfaction (Chopra & Meindl, 2013). The impact of the lack of coordination on supply chain performance measures is summarized in Table 1.1.

Figure 1.2 An independent decision making (uncoordinated) supply chain.

Table 1.1 The impact of lack of coordination on supply chain performance.

Supply Chain Performance Measures	Impact
Manufacturing Cost	Increase
Inventory Cost	Increase
Transportation Cost and Lead Time	Increase
Profitability	Decrease
Responsiveness	Decrease
Customer Service Level	Decrease

Alternately, a coordinated supply chain in Figure 1.3 is able to foster potential benefits for the individual organization by using different coordination mechanisms. It can minimize inventories and lower the shipping costs or improve the resource utilization of the vendors. Thus, it improves the entire supply chain performance.

Figure 1.3 A coordinated supply chain.

Current supply chains face complexity from multi-stage hierarchies and uncertainties in supply and demand. To meet these challenges, supply chain parties must work toward an integrated system and coordinate with each other (Arshinder *et al.*, 2008).

1.4 Coordination mechanism and framework adopted in supply chains

When supply chain parties act independently, efficient coordination mechanisms are needed to handle the conflicting issues and the demand and supply uncertainties. The supply chain performance can be improved with the right coordination mechanism and framework (Egri, 2008). Both the joint consideration of inventory replenishment (JCIR) and the Supply Chain Operations Reference (SCOR) model are adopted in this research as discussed below.

1.4.1 Joint consideration of inventory replenishment

The JCIR decision making mechanism is well-known in the literature of SCC. It is able to resolve conflicts among supply chain parties. The JCIR model focuses on coordinated inventory replenishment decisions among supply chain parties and the impact on supply chain performance. The model determines the lot sizes of order, production and shipment to minimize TOC. It is especially useful as a planning tool in situations where companies have established long-term relationships with their suppliers and customers (Arshinder *et al.*, 2008; Glock, 2012).

Starting with Goyal (1976), a stream of research has emerged in recent years that targets the JCIR decisions among individual parties to benefit an entire supply chain. All of the JCIR models in the existing literature attempt to find an optimal operational solution and entice supply chain parties to coordinate (Glock, 2012).

There are other supply chain coordination mechanisms in addition to JCIR. For instance, decision makers may design schemes to share profits and risk among the supply chain parties within a planning horizon. Optimal performance can be achieved if

the supply chain parties coordinate with each other such that each party's effort is aligned with the overall objective of the supply chain (Sarmah *et al.*, 2006).

1.4.2 Supply chain operations reference model

The SCOR model was introduced by the Supply-Chain Council. It is designed to benchmark operational measurements to improve supply chain performance and profitability. It facilitates collaboration among supply chain parties. It identifies performance measurements and the supporting tools that are suitable for each activity along a supply chain. This process reference model enables all of the departments and businesses that are involved in developing and managing an integrated supply chain to collaborate effectively (Supply Chain Council, 2010). Figure 1.4 is a schematic framework that illustrates the interrelationship among supply chain parties.

Figure 1.4 Supply chain decision categories mapped to the SCOR model version 10

(source: Supply Chain Council 2010)

The SCOR model is widely adopted and used in many industries (Ntabe *et al.*, 2015). It has been applied to evaluate the efficiency and effectiveness of integrated

supply chains in the manufacturing industry. Some examples were provided in Hwang *et al.* (2008). Intel is one of the SCOR model beneficiaries worldwide that were forced to improve their complex and dynamic virtual supply chain networks (Intel, 2002). Many other organizations, such as Avon, LEGO, and Siemens Medicals, pursued the SCOR methodology and road-map to achieve significant improvements in their supply chains (Huang *et al.*, 2005). The application of the SCOR model has been also reported in the construction industry (Cheng *et al.*, 2010; Pan *et al.*, 2011), the marine industry (Zangouinezhad *et al.*, 2011), the professional services industry (Lisa M. Ellram *et al.*, 2004) and the tourism industry (Yildirim & Umit, 2006).

The SCOR model is adopted in Chapter 4 of this thesis to evaluate the supply chain's TOC, which is one of five supply chain performance metrics. The framework is used to support the coordination of production-inventory-distribution at the strategic levels among supply chain parties.

1.5 Research problems and motivation

1.5.1 Research background

This research is motivated by SCC problems in a pharmaceutical MNC. The MNC is one of the world's leading research-based healthcare organizations with approximately \$27 billion in sales, which accounts for seven percent of global pharmaceutical sales in 2013. Headquartered in the UK, the extensive manufacturing organization and supply chain makes and distributes its products to over 150 countries around the world.

This MNC addresses many different products, which are produced by three primary business units: pharmaceuticals, vaccines and consumer healthcare. Some vaccine products are in nearly constant demand annually. Most pharmaceutical products undergo primary and secondary production with raw materials. The primary production

process involves making the basic molecules, which are called active pharmaceutical ingredients (APIs) through multi-stage chemical synthesis or bioprocesses. The secondary production involves formulating these materials into the final drug products (Shah, 2004). The final drug products are stored at the MNC's distribution centre or delivered to its in-country distribution centre. The clinic centre dispensary receives the drugs from the MNC's in-country distribution centre and sells the drugs to patients (Figure 1.5).

Figure 1.5 The original MNC's clinical trial supply chain.

1.5.2 Need for supply chain coordination

During early-phase development, APIs are commonly over-produced to buffer the uncertainties those arise due to the absence of information, which is inherent in the current clinical trial supply chain. The excessive APIs drive TOC of the supply chain much higher. Due to the long clinical trial supply chain (Figure 1.5), it commonly includes many decision-making parties with different objectives. The supply chain parties have different priorities in their strategies. For example, the external raw material suppliers put responsiveness first due to the intense competition for supplies. Cost is most important metric in production processes. However, distribution centres are always looking at the reliability of the finished product supply. If the MNC cannot align all of its supply chain parties at the strategic, tactical and operational levels, it will be difficult to manage the supply chain successfully. This uncoordinated condition among the supply chain parties could lead to supply chain performance degradation (Arshinder *et*

al., 2008; Glock, 2012). Therefore, collaborative planning, information sharing and JCIR are critical in the reduction of TOC of the MNC's clinical trial supply chain (GlaxoSmithKline, 2013, 2014).

The existing research (Arshinder *et al.*, 2008; Glock, 2012) has shown that SCC with the right mechanisms and frameworks can improve overall supply chain performance and reduce TOC. However, most of analytical models in the existing literature that discussed coordination mechanisms have addressed no more than three-stage supply chains, and they did not consider transportation (Axsater, 1997; Glock, 2012; Goyal & Gupta, 1989; Lu, 1995; Sarker & Parija, 1996; Wang, 2013).

1.5.3 Research problems, general assumptions and justifications

To reduce the complexity of the pharmaceutical supply chain, some models (Chen *et al.*, 2012; Chen *et al.*, 2013) in the existing literature have combined the stages with similar process functions. To achieve the research goals of this preliminary study, the original MNC's clinical trial supply chain in Figure 1.5 is reduced from eight stages to four stages. In the re-designed MNC's multi-stage supply chain in Figure 1.6, the primary and secondary production are combined into a single stage that is called the manufacturing/production plant. Both of the MNC's in-country distribution centre and the clinical centre dispensary are combined as the distribution centre because they are usually located near each other.

decision maker (Baboli *et al.*, 2011; Chiadamrong & Prasertwattana, 2006; Fu *et al.*, 2014).

- 2) The demand is deterministic and is known in advance. Suppliers have a perfect understanding of the price-demand relationship.
- 3) The multi-stage supply chain operates under constant (Chapters 3 and 4) and time-varying demand (Chapter 5) because some vaccine products are in nearly constant in demand year-round.
- 4) The production rate is finite. It is not lower than the demand rate to avoid shortage.
- 5) The ordering/setup and holding cost rates are constant over time.
- 6) This dissertation does not consider the lost-sale models, *i.e.*, customer demand will be fulfilled, and no shortage is allowed. This approach reflects a research stream (Almeder *et al.*, 2015; Archetti *et al.*, 2014; Axsäter, 2006; Hoque, 2011; Massonnet *et al.*, 2014; Rau & OuYang, 2008; Ventura *et al.*, 2013; Wagner & Whitin, 1958; Zhang *et al.*, 2014; Zipkin, 2000) in the existing literature.
- 7) For the purpose of this dissertation, the focus is on a single type of product because the research intention is to offer a simple overview of the possible outcomes after the implementation of the SCC mechanism and framework in integrated multi-stage supply chain with different demand patterns.
- 8) The product defective rate is assumed to be zero.
- 9) An ordering cost is incurred whenever an order is placed at a supply chain stage.

1.5.4 Research motivation

The models that are studied in this dissertation can be used as tools for strategic and tactical planning to analyse the effects of cost components in various practical situations

for performance evaluation in integrated multi-stage supply chains with different demand patterns. In general, they are closer to practical applications and are motivated by the following four aspects:

- A study on multi-stage supply chains that considers transportation is needed because most of the analytical models in the existing literature (Bushuev *et al.*, 2015; Glock, 2012) are analysed in no more than three-stage.
- Some papers (Boissière *et al.*, 2008; Kim & Glock, 2013) have studied integrated supply chains and included more than three stages with the following conventional assumptions:
 - ❖ infinite production rate
 - ❖ fixed and variable costs at each supply chain stage are not necessarily considered
 - ❖ holding cost rates increase as the material/product flows down the supply chain

These assumptions are not true in many industries. We do not know enough about the impact of system parameters on TOC of multi-stage supply chains based on these assumptions. The associated costs in multi-stage supply chains that include inventory holding cost, setup/ordering cost and transportation cost are considered in this thesis (The cost components are outlined in Table 1.2). It will further supply chain research to develop a new and more practical model to examine supply chain performance.

- Since the 1980s, many researchers (Arshinder *et al.*, 2008; Glock, 2012; Li & Wang, 2007) have focused on SCC with different coordination mechanisms. However, they studied the coordination at either the strategic level or the tactical and operational levels. To enhance supply chain performance, an effective

coordination mechanism and a framework, which are able to coordinate from the strategic to the tactical and operational levels, are needed. We could benefit from knowing more about this particular policy and framework in integrated multi-stage supply chains, including the integer-ratio inventory coordination (IIC) policy and the SCOR model in SCC for performance optimization.

- There has been limited research that addressed integrated multi-stage supply chains with time-varying demands in the existing literature (Arshinder *et al.*, 2008; Glock, 2012; Hwang *et al.*, 2013; Kaminsky & Simchi-levi, 2003; Pahl & Voss, 2014; Zhao *et al.*, 2016b). In this dissertation, the customer demand in the integrated multi-stage supply chain is extended from constant to time-varying demand. The time-varying demand is generalized to multiple phases up to an arbitrary integer.

Table 1.2 The definition of cost components.

S/N	Cost Component	Definition
1	Material / Finished product ordering cost	It is the fixed cost of placing an order for the material / finished product, which includes the cost of administration associated with an order. It is independent of the size of an order.
2	Material / Finished product receiving cost	It is the fixed cost of receiving a batch of the material / finished product, which includes the cost of administration associated with receiving an order. It is independent of the size of the batch of the material/finished product.
3	Production setup cost	It is the cost incurred to get production line ready to process the products, which includes the cost of labour, material and idle time associated with setting up and shutting down the equipment.
4	Inventory holding cost	It is the variable cost of holding a single unit of the material/finished product on stock in a unit time. It includes variable opportunity cost, and other cost due to storage, insurance, possible theft, obsolescence and spoilage.
5	Fixed transportation cost	It is the fixed cost of the transport incurred regardless of length of journey. It includes the costs are from insurance, depreciation of the car quota license, maintenance.

1.6 Research goals and objectives

1.6.1 Research goals

TOC is one of the important performance metrics in SCM. Continuous cost reduction in MNCs is a good practice because it enhances supply chain efficiency (Chopra & Meindl, 2013; Simchi-Levi *et al.*, 2008). The explicit goals of this research are:

- to minimize the TOC of integrated multi-stage supply chains;
- to study the system parameters' impacts on minimizing the TOC by adopting different coordination mechanisms and frameworks.

By adopting the JCIR and SCOR as the coordination mechanism and the framework to support the coordination among the supply chain parties, the effects of minimizing the TOC can be expected to have a greater impact on raw materials and finished goods order sizes, manufacturing batch sizes, and shipment delivery scheduling in multi-stage supply chains. This research is motivated the study of these effects. The obtained critical managerial insights will assist decision makers in improving supply chain performance in real-life cases.

1.6.2 Research objectives

The objective is to study the production-inventory-distribution coordination and performance optimization problems for integrated multi-stage supply chains with constant and time-varying demand by adopting the JCIR coordination mechanism and the SCOR framework. The optimal production-inventory-distribution policies are devised to minimize the TOCs of integrated multi-stage supply chains. By obtaining the global minimum TOCs in this research, the following critical questions can be answered:

- What are the optimal ordering/production batch sizes at each stage by adopting the proposed coordination mechanism and the framework in the multi-stage supply chain?
- What is the optimal production and shipping schedule over the planning horizon by adopting the proposed coordination mechanism and the framework in the multi-stage supply chain?

1.7 Scope of the research

This research focuses on production-inventory-distribution coordination and performance optimization problems for integrated multi-stage supply chains. SCOR and JCIR are adopted as the framework and the coordination mechanism and to support the coordination from the strategic to the operational levels among the supply chain parties. *Supply chain management cost* which is one of level 1 metrics from the SCOR model is selected to align the supply chain parties' strategic objectives in Chapter 4. The JCIR coordination mechanism, IIC policies for inventory replenishment operations across integrated supply chains are devised in all chapters. Both constant and time-varying demand patterns are considered. By using rigorous analytical approaches, three distinct but inseparable problems are studied in this dissertation: an integrated multi-stage supply chain with constant demand, a SCOR-based analytical coordination model (ACM) for an integrated supply chain with constant demand and an integrated multi-stage supply chain with time-varying demand. Theoretical models of the problems are formulated and analytical solutions are proposed. Three models are studied in this thesis are illustrated by numerical examples.

1.8 Dissertation overview

1.8.1 Dissertation outline

The organization of this dissertation is presented in Figure 1.7. The next chapter reviews the literature that is related to the research on these three types of problem. By adopting the JCIR coordination mechanism, an IIC policy for inventory replenishment operations across an integrated multi-stage supply chain with constant demand is devised in Chapter 3. We aim to find the optimal IIC policy to minimize the TOC.

In Chapter 4, the JCIR coordination mechanism and the SCOR framework are adopted into an integrated supply chain with constant demand. The SCOR model supports the MNC in choosing and refining the SCM strategies. An IIC policy is devised as a coordination mechanism for the inventory replenishment operations among the supply chain parties. A systematic approach of combining the SCOR model with the IIC policy to build and analyse an ACM is proposed. By using this approach, *supply chain management cost*, which is one of the SCOR level 1 metrics, is selected to build an ACM for supply chain performance evaluation. Both quantitative and qualitative methodologies are employed in this chapter.

By extending the results of constant demand in Chapters 3 and 4, the integrated multi-stage supply chain with time-varying demand is studied in Chapter 5. It is formulated as a mixed-integer nonlinear programming (MINLP) optimization problem. The model is represented as a weighted directed acyclic graph (WDAG). The last chapter is the conclusion with remarks.

Figure 1.7 The outline of this dissertation

1.8.2 Rationale for generating data in numerical examples

Data in numerical examples are randomly generated or derived from the real world, are two general approaches in the operations research literature for the computational testing of algorithms or procedures (Hall & Posner, 2001). However, the availability of real data for proposed research models is limited (Hall & Posner, 2001; Morales, 2000). This is mainly due to consideration for protecting company confidential information. Numerical examples are presented for illustrative purposes throughout the thesis, although analytical solutions for three proposed models in Chapters 3, 4 and 5 are obtained. In Chapters 3 and 5, random numbers are assigned to the data for the parameters in numerical examples, which gives more insights into the general properties

of the analytical solutions. Numerical examples in Chapter 4 that were partially derived from a case study in the pharmaceutical industry (Kannan *et al.*, 2013). This is to test how the proposed systematic approach could perform in practical situations. Comparing with Chapters 3 and 5, the numerical examples with the actual data in Chapter 4 are better to show the feasibility of the proposed decision support system in the pharmaceutical industry. However, the actual data in Chapter 4 are insufficient for performing numerical experiments in Chapters 3 and 5. Because both models in Chapters 3 and 5 are four-stage supply chain with transportation and the time-varying demand problem is considered at Chapter 5. All of these settings in Chapters 3 and 5 need more information on the systems parameters than the model at Chapter 4.

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CHAPTER 2

LITERATURE REVIEW

This chapter reviews works that are related to the research of this dissertation. An overview of the production-inventory-distribution coordination and performance optimization for integrated supply chains is discussed in Section 2.1. In Section 2.2, works that pertain to the integrated supply chain with constant demand are discussed. In Section 2.3, works that are related to the SCOR models and the ACM of an integrated supply chain are reviewed. The works that are related to the integrated supply chain with time-varying demand are reviewed in Section 2.4. Sections 2.2, 2.3 and 2.4 provide technical background and a literature survey for Chapters 3, 4 and 5, respectively. In the last section, the research gap in the existing literature is identified.

2.1 Overview of production-inventory-distribution coordination and performance optimization

Since the 1980s, many researchers have focused on SCC and its performance optimization due to global competition and the constant changing business environment (Arshinder *et al.*, 2008; Chan & Chan, 2010; Li & Wang, 2007). Efficient coordination mechanisms and frameworks are needed to handle conflicting issues when supply chain parties act independently. Various coordination mechanisms and frameworks are adopted in supply chains to study the effects on TOC in both centralized and decentralized settings in the existing literature (Abdul-Jalbar *et al.*, 2016; Abdul-Jalbar *et al.*, 2003; Chen *et al.*, 2001; Chen *et al.*, 2016; Duan & Liao, 2013; Egri, 2008; Fattahi

et al., 2015; Geunes *et al.*, 2016; Giri *et al.*, 2016; Glock, 2012; Lee & Billington, 1993; Salcedo *et al.*, 2013).

Figure 2.1 Production-inventory-distribution problems in a taxonomy diagram

This dissertation presents general insights into the importance of the right coordination mechanism and framework for integrated multi-stage supply chains in centralized settings. The relevant studies are classified into three categories based on

coordination levels and type of customer demand: coordination at the tactical and operational levels in integrated supply chains with constant demand (Section 2.2), coordination from the strategic to the operational levels in integrated supply chains with constant demand (Section 2.3), and coordination at the tactical and operational levels in integrated supply chains with time-varying demand (Section 2.5). The research focuses of this thesis are described in Figure 2.1. They are reviewed in the following sections.

2.2 Integrated supply chain with constant demand

The IIC policy which is the JCIR coordination mechanism is devised in the first part of this research. This policy has been studied in production-inventory-distribution systems in a supply chain with up to three stages (Boissière *et al.*, 2008; Omar *et al.*, 2013) and in multi-echelon inventory systems (Abdul-Jalbar *et al.*, 2005; Abdul-Jalbar *et al.*, 2007; Axsäter, 2006; Wang, 2013; Zipkin, 2000).

In production-inventory-distribution systems, the existing literature mainly focused on the analysis of two-stage supply chains without considering transportation (Abdul-Jalbar *et al.*, 2010; Chang, 2014; Gorji *et al.*, 2014; Huang & Ye, 2010; Jha & Shanker, 2013; Liu & Lian, 2009; Rahdar & Nookabadi, 2014). Goyal (1976) analyzed a two-stage supply chain consisting a single-vendor single-buyer with an infinite production rate. Banerjee (1986) and Hill (1999) relaxed the assumption of the infinite production rate in the model of Goyal (1976). Lu (1995) studied a one-vendor multi-buyer integrated inventory model with an integer-ratio policy. The objective is to minimize the vendor's total cost subject to the maximum costs that buyers will incur. Sarker and Parija (1996) developed an optimal multi-order policy in a two-stage supply chain, which the raw material procurement is for a single manufacturing batch. Although

the infinite production rate was relaxed, transportation was not considered in this model. Hill (1997) and Hill and Omar (2006) considered a single-vendor single-buyer integrated production-inventory model. In their study, the production batches were assumed to increase in size by a fixed geometric growth factor $\lambda \in [1, P/D]$, P denotes the production rate and D the demand rate. In addition, zero transportation time was assumed. Sarkar (2013) developed a production-inventory model with a deteriorating item in a two-stage supply chain. Through a simpler model of a single manufacturer-buyer with a finite production rate, deterministic demand and a single type of product, Eben-Chaime (2004) proposed an integer-ratio policy of inventory replenishment, and proved its optimality.

Some models in the existing literature also discussed coordinated inventory replenishment decisions in supply chains with more than two stages. Boissière *et al.* (2008) studied a three-stage production-inventory-distribution system without the consideration of transportation lead time, the production setup cost and the material ordering cost. Jaber and Goyal (2008) and Jaber and Goyal (2009) studied the coordination of order quantities in three-stage and four-stage supply chains with infinite production rates, respectively. However, the transportation was not considered. Lee (2005) considered a single-manufacturer single-buyer supply chain to determine the joint economic lot size of raw materials and production batch to minimize the TOC per unit time. Different from the model in Chapter 3, the raw material ordering batch was only fulfilled for one cycle of the production in Lee (2005). Seliaman and Ahmad (2009) studied a non-serial multi-stage supply chain with multiple customers. They used an integer multiplier of the cycle time policy of the inventory replenishment intervals. By using the same methodology of Hill (Hill, 1997; Hill & Omar, 2006), Kim and Glock

(2013) studied a multi-stage supply chain where equal- and unequal-sized batches were transported between the stages and penalty costs were imposed on long lead times.

In multi-echelon inventory systems, Roundy (1985) studied a lot-sizing rule in a single-warehouse multi-retailer system, and introduced the integer-ratio policy of inventory replenishment intervals. The infinite production rate was assumed in Roundy (1985). Chen (1998) proposed a stationary integer inventory policy in a multi-echelon inventory system with the assumptions of an infinite production rate and no transportation. A summary of the existing literature of the multi-stage supply chain with constant demand is in Table 2.1

2.3 SCOR-based analytical coordination model for an integrated supply chain

In addition to JCIR, SCOR is adopted as a framework to an integrated supply chain in the second part of the research. The proposed model is to support the coordination among the supply chain parties from the strategic to the operational levels. Numerous studies (Bushuev *et al.*, 2015; Glock, 2012; Li & Wang, 2007) have been conducted on the SCC at either the strategic level or the tactical and operational levels. The literature review of this section is presented in two subsections. First, the SCOR models are applied at the strategic levels. Second, ACMs are applied at the tactical and operational levels.

Table 2.1 The summary of the literature of integrated supply chains with constant demand.

Authors	Demand	Supply Chain Stages	Coordination Mechanisms	Production Rate	Transportation	Solution Method
Goyal (1976)	Constant	Two-stage	Integer-ratio of inventory replenishment interval	Infinite	No	Optimal
Roundy (1985)	Constant	Two-stage	Integer-ratio of inventory replenishment interval	Infinite	No	Optimal
Lu (1995)	Constant	Two-stage	Integer-ratio of inventory replenishment interval	Finite	No	Optimal and Heuristic
Sarker and Parija (1996)	Constant	Two-stage	Integer-ratio of inventory replenishment quantity	Finite	No	Optimal
Hill (1997)	Constant	Two-stage	Integer-ratio of inventory replenishment interval	Finite	No	Optimal
Chen (1998)	Constant	N -stage	Stationary Policy	Infinite	No	Optimal
Banerjee (1986), Hill (1999)	Constant	Two-stage	Integer-ratio of inventory replenishment interval	Infinite	No	Optimal
Eben-Chaïme (2004)	Constant	Two-stage	Integer-ratio of inventory replenishment quantity	Finite	No	Optimal
Lee (2005)	Constant	Three-stage	Integer-ratio of inventory replenishment interval	Finite	No	Optimal
Jaber and Goyal (2008)	Constant	Three-stage	Integer-ratio of inventory replenishment interval	Infinite	No	Heuristic
Boissière <i>et al.</i> (2008)	Constant	Three-stage	Stationary Policy	Finite	Yes (Constant cost)	Optimal
Jaber and Goyal (2009)	Constant	Four-stage	Integer-ratio of inventory replenishment interval	Infinite	No	Heuristic
Seliaman and Ahmad (2009)	Constant	N -stage	Integer-ratio of inventory replenishment interval	Finite	No	Heuristic
Kim <i>et al.</i> (2013)	Constant	N -stage	Integer-ratio of inventory replenishment interval	Finite	No	Optimal for four-stage, simulation for N -stage
Sarker (2013)	Constant	Two-stage	Integer-ratio of inventory replenishment quantity	Finite	Yes (Constant cost)	Optimal
First part of our research	Constant	Four-stage	Integer-ratio of inventory replenishment quantity	Finite	Yes (Linear cost function)	Optimal

2.3.1 SCOR models are applied at strategic levels

SCOR models are applied through qualitative models for the strategic decision making in the existing literature. The SCOR model helps suppliers, manufacturers, downstream retailers/distributors and customers to improve the efficiency of SCM by communicating effectively (Kevan, 2005). Companies using the SCOR model to drive supply chain improvements spend less time putting on firefighting and more time in planning (Bolstorff, 2003). Anecdotal evidence and trade journals have also reported significant improvements after firms have adopted the SCOR model. Empirical validation of the SCOR model for organizations was conducted by Zhou *et al.* (2011). Wang *et al.* (2004a) related product characteristics to supply chain strategy and adopted SCOR model level 1 performance metrics as the decision criteria. An integrated analytic hierarchy process and pre-emptive goal programming based on multi-criteria decision making methodology was developed to take into account both qualitative and quantitative factors in supplier selection. Yuan *et al.* (2010) extended the SCOR model to the air cargo industry and proposed air cargo supply chain operations reference model. Furthermore, Li *et al.* (2011) extended the five decision areas of the SCOR model by integrating quality assurance measures in supply chain process. Sellitto *et al.* (2015) presented a SCOR-based model for supply chain performance measurement and applied it in the context of Brazilian footwear industry.

By applying the SCOR model, there is a group of researchers who worked on simulation models to test various configurations of supply chains and what-if scenarios (Persson, 2011; Persson & Araldi, 2009; Rabelo *et al.*, 2007). Gumus *et al.* (2010) proposed a methodology for effective multi-echelon inventory management. With a neural network simulation model, the supply chain performance was calculated using performance metrics selected from the SCOR model. Pan *et al.* (2011) provided a

systematic approach for analysis and design of construction supply chain operation models. They integrated the SCOR model into the Dynamic Simulation software to assist in establishing a hierarchical model to explore the behavior of construction supply chain processes. Devos Ganga and Ribeiro Carpinetti (2011) proposed a supply chain performance model based on fuzzy logic to predict performance based on causal relationships between metrics of the SCOR model.

There are some researchers using the SCOR model to build analytical model for supply chain performance optimization. Zhang and Reimann (2014) aimed at multi-objective performance assessment and optimization of a two-echelon supply chain by using SCOR metrics. Kurien and Qureshi (2015) adopted the SCOR model to propose a framework and a methodology for flexibility performance measurement of supply chains. However, they (Kurien & Qureshi, 2015; Zhang & Reimann, 2014) did not study SCC in their models.

2.3.2 ACMs are applied at tactical and operational levels

Most analytical coordination models (Banerjee, 1986; Boissière *et al.*, 2008; Bushuev *et al.*, 2015; Glock, 2012; Goyal, 1976; Hill, 1997, 1999; Kim & Glock, 2013) in the existing literature only proposed coordination mechanisms at the tactical and operational levels for supply chain parties. In addition, the devised policy was on integer-ratio of inventory replenishment interval, which is different from the IIC on replenishment quantity in this study. They are reviewed in Section 2.2. A summary of the literature on SCOR models and ACMs with inventory coordination mechanisms is presented in Table 2.2.

Table 2.2 The summary of the literature of SCOR models and analytical coordination models with inventory coordination mechanisms.

Authors	Production Rate	Transportation	Solution Method	Coordination Mechanisms	Coordination Level	Model
Goyal (1976)	Infinite	No	Optimal	Integer-ratio of inventory replenishment interval	Tactical and operational levels	Analytical coordination models
Banerjee (1986)	Finite	No	Optimal	Integer-ratio of inventory replenishment Quantity		
Hill (1999)	Finite	Yes (Constant Cost)	Optimal	Integer-ratio of inventory replenishment interval		
Boissière, et al. (2008)	Finite	Yes (Constant Cost)	Optimal	Stationary Policy		
Hill (1997)	Finite	No	Optimal	Integer-ratio of inventory replenishment interval		
Kim, et al. (2013)	Finite	No	Optimal solution for 4-stage supply chain. Simulation on N -stage supply chain	Integer-ratio of inventory replenishment interval		
Bolstorff (2003); Kevan (2005); Li et al. (2011); Wang, et al (2004a); Yuan, et al (2010) and Zhou. et al (2011).				–	Strategic level	The SCOR-based qualitative models
Devos Ganga and Ribeiro Carpinetti (2011); Gumus, et al (2010); Pan, et al (2011); Persson (2011), Persson, et al (2009) and Rabelo, et al (2007)				–	Strategic level	The SCOR-based simulation models
Zhang and Reiman (2014)	–	–	Optimal	–	Coordination effects were not studied from strategic to operational levels	The SCOR-based quantitative models
Kurien and Qureshi (2015)			–			
The second part of our research	Finite	Yes (Linear cost function)	Optimal	Integer-ratio of inventory replenishment quantity and SCOR model	From strategic to operational levels	Chapter Four selects <i>supply chain management cost</i> , one of level 1 metrics from the SCOR model to build an analytical coordination model

2.4 Integrated supply chain with time-varying demand

Based on the nature of demand, production-inventory-distribution problems can be classified as deterministic or stochastic (Axsäter, 2006; Cachon & Fisher, 2000; Cachon & Lariviere, 2005; Liu & Lian, 2009; Sana & Goyal, 2014; Song *et al.*, 2014; Zhao *et al.*, 2016a, 2016b; Zipkin, 2000). In the deterministic class, it can be further classified into constant and time-varying demand. There have been several studies on time-varying demand with the assumption of an infinite production rate or an instantaneous inventory replenishment for integrated supply chains (Almeder *et al.*, 2015; Bylka, 1999; Hwang *et al.*, 2013; Kaminsky & Simchi-levi, 2003; Van Hoesel *et al.*, 2005; Ventura *et al.*, 2013; Zangwill, 1969) since the classical article by Wagner and Whitin (1958).

Inspired by the practice in the production-inventory-distribution problems, many researchers focused on supply chains with a linear trend in demand and a finite production rate. However, the models did not consider supply chains with more than two stages. For instance, Hariga (1996) studied an optimal inventory model of deteriorating items with a linear trend in demand in a single-stage supply chain. Lo *et al.* (2002) presented the exact optimal solution of inventory replenishment for both linear increasing and decreasing trends in demand in a single-stage supply chain. Rau and OuYang (2008) presented an optimal production–inventory policy for a two-stage supply chain with linear increasing and decreasing trends in demand. Although the single-phase linear trend in demand is an approximation of the practical situation (Chang & Chou, 2013; Chen & Chang, 2007), the demand for most products has a typical cycle with the multi-phase patterns due to the life cycle or boom-and-bust seasons in a year. Hill (1995) considered a product subject to a period of increasing demand, followed by a period of constant demand for a single-stage system. Diponegoro and Sarker (2007)

studied a production-supply problem for a two-stage supply chain with a fixed interval delivery to buyers. The product demand is time-dependent following its life cycle. However, the multi-phase demand over the planning horizon is represented by a piece-wise linear function up to three phases. Goyal and Giri (2003) considered a production-inventory model with time-varying demand to devise the optimal replenishment policies. Sana (2010) also developed a production-inventory model to determine the optimal product reliability and optimal production rate that achieves the maximal total profit under an imperfect manufacturing process. The two-stage supply chain with time-varying demand were considered in both Goyal and Giri (2003) and Sana (2010).

In the existing literature, some researchers studied more than two stages supply chain with time-varying demand. The linear trend in demand is just up to two phases. For example, Sarker and Balan (1999) studied a multi-stage kanban system with a linear trend in demand. Omar *et al.* (2013) considered a three-stage supply chain with the single-phase linearly decreasing demand under the just-in-time (JIT) manufacturing environment. A summary of the literature on time-varying demand is presented in Table 2.3.

Table 2.3 The summary of the literature on time-varying demand.

Authors	Time-varying Demand	Supply Chain Stages	Production Rate	Transportation Cost	Assumption on Holding Cost Rates
Wagner <i>et al</i> (1958)	Discrete-time lot sizing	One-stage	Infinite	No	Not applicable
Bylka (1999)	Discrete-time lot sizing	Two-stage	Infinite	No	
Kaminsky and Simchi-levi (2003)	Discrete-time lot sizing	Two-stage	Infinite	Yes (Non-linear)	Yes
Hwang <i>et al</i> (2013); Van Hoesel <i>et al</i> (2005); Zangwill (1969)	Discrete-time lot sizing	N -stage	Infinite	No	No
Ventura <i>et al</i> (2013)	Discrete-time lot sizing	N -stage	Infinite	Yes (Fixed)	No
Almeder <i>et al</i> (2015)	Discrete-time lot sizing	N -stage	Finite	No	No
Chang and Chou (2013)	Discrete-time lot sizing	Two-stage	Finite	Yes	Yes
Hariga (1996)	Linear trend in two phases	One-stage	Finite	No	Not applicable
Lo <i>et al</i> (2002)	Linear trend in two phases	One-stage	Finite	No	Not applicable
Chen and Chang (2007)	Linear trend in single phase	One-stage	Finite	No	Not applicable
Hill (1995)	Linear trend in two phases	One-stage	Finite	No	Not applicable
Rau and Ouyang (2008)	Linear trend in two phases	Two-stage	Finite	No	No
Goyal and Giri (2003); Sana (2010)	Linear trend in single phase	Two-stage	Finite	No	No
Diponegoro and Sarker (2007)	Linear trend in three phases	Two-stage	Finite	No	No
Omar <i>et al</i> (2013)	Linear trend in single phase	Three-stage	Finite	No	No
Sarker and Balan (1999)	Linear trend in single phase	N -stage	Finite	No	No
The third part of our research	Linear trend in multi-phase	Four-stage	Finite	Yes (Linear)	No

2.5 Research gap

It is evident that SCC and its performance optimization have received a great deal of attention from researchers and practitioners in many aspects. By adopting the JCIR coordination mechanism and the SCOR framework, three distinct but inseparable problems are considered in this dissertation: integrated multi-stage supply chain with constant demand, SCOR-based ACM for an integrated supply chain and integrated multi-stage supply chain with time-varying demand.

For an integrated multi-stage supply chain with constant demand, most of the models in the existing literature have no more than three stages, or the supply chain's operational costs are not fully considered if the models have more than three stages. They cannot reflect the practical situations of the multi-stage hierarchies with high transportation costs. To address this limitation, Chapter 3 extends the models in the existing literature from three-stage to four-stage supply chains with consideration of transportation. With these more general and practical settings, the proposed model can assist decision makers in managing their supply chains in real-life cases. The raw material warehouse and transportation are considered into the three-stage supply chain to study the feasibility of the supply chain integration and the cost effectiveness in different manufacturing settings. Different from prior integer-ratio coordination policy of the replenishment intervals approach, the study focuses on the IIC policy of replenishment quantity in a multi-stage supply chain. The supply chain's operational costs: the production setup cost, the product ordering cost, the inventory holding cost and the transportation cost are fully considered in our model. We aim to find the optimal IIC policy to minimize TOC. In both three- and four-stage supply chains, we analyze the impact of various system parameters on the variables of the optimal relaxed IIC

policy and the global minimum TOC, respectively. This more general and practical study is able to offer a simple overview of the possible outcomes after the implementation of the IIC policy in a multi-stage supply chain.

In addition to the JCIR coordination mechanism in the proposed model in Chapter 3, SCOR is adopted in an MNC's integrated supply chain in Chapter 4. Based on Table 2.2, it is evident that more studies of using the SCOR models to build ACMs are needed. Most analytical models (Glock, 2012; Li & Wang, 2007) only proposed inventory coordination mechanisms at the tactical and operational levels for supply chain parties. In a practical situation, this is not sufficient when supply chain parties do not align their objectives at the strategic level (Ntabe *et al.*, 2015). Many researchers also made use of SCOR models to design both descriptive and analytical models for supply chain performance evaluation (Lai *et al.*, 2002; Lockamy & McCormack, 2004; Lu *et al.*, 2013; Medini & Bourey, 2012; Wang *et al.*, 2010) . However, their models did not adopt the SCOR model to establish critical metrics to build an ACM for supply chain cost optimization. To address this limitation, a systematic approach of combining the SCOR model with the IIC policy to build and analyze an ACM is proposed in Chapter 4. It supports SCC from the strategic to the operational levels. *Supply chain management cost* which is one of level 1 metrics from the SCOR model is selected to align the supply chain parties' strategic objectives. The production setup cost, the product ordering cost, the inventory holding cost, and the transportation cost are considered based on *supply chain management cost*. At the tactical and operational levels, the IIC policy is devised to coordinate the inventory replenishment to formulate TOC. Different from the models in the existing literature, the IIC policy in this research is for replenishment quantities instead of replenishment intervals (Hill, 1997; Jaber & Goyal, 2008; Kim & Glock, 2013).

For the integrated multi-stage supply chain with time-varying demand in Chapter 5, the research is inspired by a two-stage supply chain analyzed by Diponegoro and Sarker (2007) in the high-tech industry. The models in Table 2.3 considered the supply chains with a linear trend in demand up to three phases. There is a need to study multi-stage supply chains with a linear trend in demand to multiple phases. A new model of four-stage supply chain which includes raw material warehouse, manufacturing plant, finished product warehouse, transportation and retailer is presented. This model generalizes the customer time-varying demand to multiple phases up to an arbitrary integer. Several properties are proven for this integrated multi-stage supply chain. The conventional assumption that holding cost rates increase as the material / product flows down the supply chain is relaxed in this model.

This chapter reviews the research works that are related to the research problems proposed in Section 1.4. The supply chain coordination and performance optimization problems with different demand patterns are analysed in the following chapters. Referring to Scope of the research in Section 1.7, two problems with constant demand patterns are studied in Chapters 3 and 4. Then the time-varying problem is studied in Chapter 5.

CHAPTER 3

OPTIMAL INTEGER-RATIO INVENTORY

COORDINATION POLICY FOR AN INTEGRATED

MULTI-STAGE SUPPLY CHAIN

An integrated multi-stage supply chain with constant demand is studied in this chapter. The research problem we consider is described in Section 3.2. In Section 3.3, the IIC policy is devised, and the TOC function is formulated. We derive the continuous relaxation of the optimal TOC and prove that it is the global minimum in Section 3.4. In addition, an integer approximation scheme is developed to refine the optimal IIC policy in this section. In Section 3.5, the impact of various system parameters on the variables of the relaxed optimal IIC policy and the global minimum TOC is analysed for both three- and four-stage supply chains. Computational results and analysis are presented in Section 3.6. The summary of this chapter are in Section 3.7.

3.1 Introduction

This chapter considers a multi-stage supply chain over an infinite horizon, which is motivated by the practice of a pharmaceutical MNC. The MNC is introduced in Section 1.4. Most pharmaceutical products undergo the primary and secondary production. Both of the production stages are characterized by low production rates due to quality assurance activities at each stage. This makes the overall supply chain cycle time long (Chen *et al.*, 2012; Chen *et al.*, 2013; Dario Pacciarelli *et al.*, 2011; Shah, 2004). The

multi-stage supply chain consists of suppliers, manufacturers and distributors. Each supply chain party optimizes its own performance due to the different goals. The uncoordinated supply chain with long cycle time degrades its performance and consequently, weakens the competitive position of the MNC. In this environment, the coordination of inventory replenishment decisions among the pharmaceutical supply chain parties is crucial for reducing the bullwhip effect and saving TOC (Chen *et al.*, 2013).

To achieve the desired coordination of inventory replenishment decisions among the supply chain parties, we devise an optimal IIC policy: inventory replenishment is made based on the optimal integer-ratio quantities with a constant time interval. The IIC policy is able to capture the interest from both academia and industry because it is an abstract of the operations and product flows at different supply chain stages. It is also a channel-wide coordination mechanism to entice all supply chain parties to work towards a common goal (Celebi, 2015; Glock, 2012). A mathematical model is formulated to represent the integrated multi-stage supply chain. The objective of this part of the research is to determine the optimal IIC policy to minimize TOC. Both three- and four-stage supply chains are studied to draw the managerial insights. In this preliminary study, a single type of product with constant demand, like the vaccine, is selected to focus on because the research here is to offer a simple overview of the possible outcomes after the IIC policy implementation in the multi-stage supply chain.

3.2 Problem description, notations and model assumptions

There are four stages in the supply chain: raw material warehouse (RMW) (the upstream stage), manufacturing plant (MUP), finished product warehouse (FPW), and distribution centre (DC) (the downstream stage). The supply chain is illustrated in Figure 3.1.

Figure 3.1 The multi-stage supply chain with constant demand.

Pharmaceutical production processes are strictly and highly regulated to ensure the quality of the semi-finished or finished products. Process constraints are dictated by production departments to avoid the risk of cross-contamination between different products or between different lots of the same product. Cross-contamination issues require processing of one product at a time and long labour hours of cleaning production

lines when switching from one product to another. To avoid these, the materials or products must be shifted from one stage to another in batches and the changeover at production lines is kept to a minimum (Kempf *et al.*, 2011; Shah, 2004).

Materials are stored in RMW for a certain period of time and are shifted to a buffer area in batches before MUP converts them into finished products. The finished products are not kept in MUP once they are manufactured. They are sent to FPW as soon as possible for two reasons. First, this is to prevent cross-contamination at production lines. Second, the finished products have limited shelf life (Kempf *et al.*, 2011). So, the inventory holding cost is not considered at MUP. FPW and MUP are within the same premises. The finished products are sent to FPW directly without much administrative work. The ordering cost is not considered at FPW.

Transferring raw materials from RMW to MUP in batches to the buffer area makes the raw material inventory dynamics different from the earlier works (Lee, 2005; Sarker & Parija, 1996). The finished products may be stored in FPW before they are transported to DC for fulfilling customer demand.

To coordinate the inventory replenishment operations among RMW, FPW and DC, we devise the IIC policy: inventory replenishment is made according to the optimal integer-ratio quantities with a constant time interval. For the inventory ordering at each stage, we adopt the classical zero-inventory ordering (ZIO) policy (Wagner & Whitin, 1958), *i.e.* an order is placed when the inventory position reaches zero. However, the reorder point is used to manage the inventory at DC because the positive transportation lead time from FPW to DC is considered. The notations used in this chapter are summarized in Table 3.1.

Table 3.1 Notations for the multi-stage supply chain with constant demand

Parameters Definition		Parameters Definition	
A_1	ordering cost rate at Raw Material Warehouse (\$/order)	Q_1	batch quantity ordered at Raw Material Warehouse (units)
A_2	production setup cost (\$/setup)	Q_3	batch quantity ordered at Finished Product Warehouse (units)
A_4	finished goods ordering cost rate at Distribution Centre (\$/batch)	Q_4	batch quantity ordered at Distributed Centre (units)
A_s	fixed transportation cost rate (\$/trip)	Q_{avg}	average inventory at Finished Product Warehouse (units)
A_T	system setup cost rate	$Q_{avg'}$	average inventory at Raw Material Warehouse (units)
d	demand rate at time t (units/unit time)	Q_s	shipping quantity (units)
h_1	inventory holding cost rate at Raw Material Warehouse (\$/unit/unit time)	r	conversion factor of raw materials to finished goods
h_1'	normalised inventory holding cost rate at Raw Material Warehouse (\$/unit/unit time)	R_4	inventory reorder point at Distribution Centre (units)
h_3	inventory holding cost rate at Finished Product Warehouse (\$/unit/unit time)	T	ordering cycle time at Distribution Centre
h_4	inventory holding cost rate at Distribution Centre (\$/unit/unit time)	T_1	ordering cycle time at Raw Material Warehouse (unit time)
h_s	transient inventory holding cost rate (\$/unit/unit time)	T_2	production cycle period (unit time)
h_T	system holding cost rate	T_3	production time (unit time)
L_s	transportation lead time from Finished Product Warehouse to Distribution Centre	TC	total operational cost of the supply chain (\$)
p	production rate (units/unit time)		

Decision Variables	Definition
M	positive integer, $M=Q_1/Q_3$
N	positive integer, $N=Q_3/Q_4$
Q	basic replenishment quantity at Distribution Centre (units), $Q=Q_4$
T	ordering cycle time at Distribution Centre

3.2.1 Distribution centre (Stage 4)

The inventory ordering policy (R_4, Q_4) is applied at DC. When the inventory position reaches reorder point $R_4 (>0)$, a batch quantity of size Q_4 is ordered. To simplify the notation, $Q_4 = Q$ is used, where Q is the basic replenishment quantity. Customer demand rate is d units per unit time. The ordering cycle time at stage 4 is $T = Q_4/d = Q/d$. The lead time L_s is the time from an order placed to the order received, which models the transportation time from FPW to DC. The inventory holding cost rate is h_4 per unit per unit time. The ordering cost of finished products is A_4 per order. The operational cost incurred at DC is the sum of the inventory holding cost and the finished product ordering cost.

3.2.2 Transportation from FPW to DC

The transit inventory holding cost rate from FPW to DC is h_s per unit per unit time. The fixed cost per shipment is A_s per trip. The operational cost incurred for transportation is the sum of the transit inventory holding cost and the fixed cost per shipment.

3.2.3 Finished product warehouse (Stage 3)

The ZIO policy is applied at FPW, that is $(0, Q_3)$. When the inventory position is zero, a batch quantity of size Q_3 is ordered and MUP starts to produce finished products. The inventory holding cost rate is h_3 per unit per unit time. The operational cost incurred at FPW is the inventory holding cost.

3.2.4 Manufacturing plant (Stage 2)

The constant production rate of MUP is p units per unit time which is no smaller than the customer demand rate, *i.e.*, $p \geq d$. The inventory keeps increasing during the

production period, T_3 On-hand finished products deplete at a regular time interval of T time units till the end of the FPW cycle time T_2 . The quantity of products manufactured during the production time Q_3/p must exactly match the demand over cycle T_2 . The production period is not greater than the cycle time, *i.e.*, $Q_3/p = T_3 \leq T_2$. The closer the two values are, the greater is the chance that the manufacturing facility performs to its full capacity. The inventory holding cost is not considered at MUP. So, the operational cost incurred at MUP is the production setup cost A_2 per setup. And there is a setup cost for switching on the production.

3.2.5 Raw material warehouse (Stage 1)

The inventory ordering policy at RMW is also ZIO that is $(0, Q_1)$. When the inventory position is zero, a batch quantity size of Q_1 is ordered. The raw materials are supplied to RMW in batched immediately. We consider r units of raw materials which can be converted to one unit of finished products. The inventory holding cost rate is h'_1 per unit per unit time. To normalise the inventory holding cost rate, h'_1 is introduced, where $h'_1 = r \times h_1$. The ordering cost of raw material is A_1 per order and the ordering cycle time is T_1 . The operational cost incurred at RMW is the sum of the inventory holding cost and the raw material ordering cost.

3.2.6 Model assumptions

In addition to the general assumptions in Section 1.4.3, the following assumptions are necessary to model this problem:

- 1) All of the inventories at RAW, FPW and DC are under continuous review.
The inventory level is constantly monitored. When the reorder point is reached, an order is placed.

- 2) The raw materials are supplied to RMW in batches immediately after the manufacturer places an order.
- 3) The shipping quantity Q_s is equal to the basic replenishment quantity at DC, *i.e.*, $Q_s=Q_4=Q$.
- 4) It is more expensive to hold the inventory at a downstream stage than at an upstream stage, *i.e.*, $h_4 \geq h_3 \geq h_1$ (and $\geq h_1'$) due to the value-added processes.

3.3 Optimal integer-ratio inventory coordination policy

To coordinate the replenishment operations among four stages in the supply chain, we devise the IIC policy in which the inventory replenishment is made based on the optimal integer-ratio quantities with a constant time interval. In each cycle, the ratio of the replenishment quantities at the upstream stage to adjacent downstream stage inventories is an integer. Some researchers (Atkins & Sun, 1995; Boissière *et al.*, 2008) proposed the best integer frequency policy in deterministic serial inventory systems. It is proven that the best integer frequency policy provides a solution which is within two percent of the optimal result. The IIC policy in the research is important for three reasons (Axsäter, 2006; Chen, 1998):

- 1) The integer-ratio constraint simplifies material handling (e.g., packaging and bulk breaking) by restricting the shipment to each stage to be multiples of a fixed quantity which may represent a truckload in reality. So, it is attractive for small and medium size companies that generally prefer to schedule their operations by using simple replenishment rules;

- 2) It is easy to implement and manage. It leads to very smooth operations and product flows at different stages in the integrated production-inventory-distribution planning model;
- 3) It offers the advantage of giving freedom in determining the order and production quantities at different supply chain stages.

In our devised policy, it is assumed that $Q_1=MQ_3$, $Q_3=NQ_4$. So, $Q_1=MNQ_4=MNQ$, where M , N and Q are positive integers. Under the optimal IIC policy ($M=M^*$, $N=N^*$, $Q=Q^*$), TOC reaches its global minimum after the integrality constraints are relaxed. The inventory dynamics and synchronization across the integrated multi-stage supply chain are illustrated in Figure 3.2.

We now derive the operational costs at each stage for the integrated multi-stage supply chain: the operational cost incurred at DC is

$$\frac{Q_4}{2}h_4 + \frac{d}{Q_4}A_4 = \frac{d}{2}h_4T + \frac{A_4}{T}. \quad (3.1)$$

The operational cost incurred for transportation is

$$\frac{L_s h_s Q_s + A_s}{T} = \frac{L_s h_s dT + A_s}{T} = L_s h_s d + \frac{A_s}{T}. \quad (3.2)$$

The operational cost incurred at FPW is

$$h_3 Q_{avg} = \frac{h_3 dT}{2} \left[N \left(1 - \frac{d}{p} \right) + 1 \right]. \quad (3.3)$$

The operational cost incurred at MUP is

$$\frac{A_2}{NT}. \quad (3.4)$$

Figure 3.2 The inventory dynamics and synchronization across the integrated supply chain with constant demand.

The operational cost incurred at RMW is

$$h_1' \mathcal{Q}_{avg}' + \frac{A_1}{TMN} = \frac{h_1' dN(M-1)T}{2} + \frac{A_1}{TMN}, \quad (3.5)$$

where $\mathcal{Q}_{avg}$ and $\mathcal{Q}_{avg}'$ are obtained from Appendix A and Appendix B respectively.

The TOC function of the integrated multi-stage supply chain, TC is the sum of TOCs incurred at RMW, MUP, FPW, transportation and DC. By summing up all the costs in Eqs. (3.1), (3.2), (3.3), (3.4) and (3.5), we get

$$TC = \frac{dT}{2} \left\{ h_4 + h_3 \left[N \left(1 - \frac{d}{p} \right) + 1 \right] + h_1' (M-1)N \right\} + \frac{1}{T} \left(A_4 + A_s + \frac{A_2}{N} + \frac{A_1}{MN} \right) + L_s h_s d. \quad (3.6)$$

3.4 Global minimum total operational cost

The objective is to minimize the long-run average TOC per unit time for the integrated multi-stage supply chain. Combining Eq. (3.6) with constraints yields the following:

$$\text{Min } TC = \frac{dT}{2} \left\{ h_4 + h_3 \left[N \left(1 - \frac{d}{p} \right) + 1 \right] + h_1' (M-1)N \right\} + \frac{1}{T} \left(A_4 + A_s + \frac{A_2}{N} + \frac{A_1}{MN} \right) + L_s h_s d.$$

$$\text{s.t. } M, N, T > 0; M, N \text{ integer}$$

$$L_s \leq T.$$

When integrality constraints are relaxed, M and N are treated as continuous variables. Proposition 3.1 shows that the TOC function TC is convex.

Proposition 3.1. The TOC function TC of the integrated four-stage supply chain is convex in the continuous variables M , N , and T , where the stationary point (M_0, N_0, T_0) is given by

$$M_0 = \sqrt{\frac{A_1}{A_2} \left[\frac{h_3}{h_1'} \left(1 - \frac{d}{p}\right) - 1 \right]} \quad (3.7)$$

$$N_0 = \sqrt{\frac{A_2 \left(\frac{h_4}{h_3} + 1 \right)}{(A_4 + A_s) \left[\left(1 - \frac{d}{p}\right) - \frac{h_1'}{h_3} \right]}} \quad (3.8)$$

$$T_0 = \sqrt{\frac{2(A_4 + A_s)}{d(h_4 + h_3)}}. \quad (3.9)$$

And the global minimum TOC is given by

$$TC(M_0, N_0, T_0) = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1 h_1' d} + \sqrt{2A_2 d \left[h_3 \left(1 - \frac{d}{p}\right) - h_1' \right]} + L_s h_s d. \quad (3.10)$$

The proof of Proposition 3.1 is in Appendix C. Before the optimal IIC policy is obtained, we can develop some properties of the model from Eqs. (3.7), (3.8) and (3.9), as shown in the following Corollary 3.1.

Definition 3.1. The system setup cost rate is $A_T = (A_1, A_2, A_4)$ and the system holding cost rates is $h_T = (h_1', h_3, h_4)$.

Based on Proposition 3.1 and Definition 3.1, several properties of the integrated multi-stage supply chain are developed, which are shown in Corollary 3.1.

Corollary 3.1. In the optimal relaxed IIC policy, the following properties hold.

- (i) M_0 is not correlated with both A_T and h_T ;
- (ii) N_0 is not correlated with h_T , but it is a decreasing function of A_s on the region $(0, +\infty)$; and
- (iii) T_0 is an increasing function of A_s on the region $(0, +\infty)$.

The proof of Corollary 3.1 is in Appendix D. It shows that the shopping cost A_s highly affects the variables, N_0 and T_0 . Based on Eqs. (3.7), (3.8) and (3.9), L_s does not affect the values of M_0 , N_0 and T_0 . It indicates that the constant lead time case is essentially the same as the zero lead time case (Chen, 1998) for variables of the IIC policy.

After the global minimum point (M_0, N_0, Q_0) is obtained, we can do a simple approximation to get the integer solution if M_0 , N_0 and Q_0 ($Q_0 = dT_0$) are not integers. The optimal integers (M^*, N^*, Q^*) of the IIC policy can be selected among the integer sets (Eq. (3.11)), where TOC is at the minimum.

$$(M^*, N^*, Q^*) = \arg \min \left\{ \begin{array}{l} TC(\lfloor M_0 \rfloor, \lfloor N_0 \rfloor, \lfloor Q_0 \rfloor); TC(\lceil M_0 \rceil, \lfloor N_0 \rfloor, \lfloor Q_0 \rfloor); \\ TC(\lceil M_0 \rceil, \lceil N_0 \rceil, \lfloor Q_0 \rfloor); TC(\lceil M_0 \rceil, \lceil N_0 \rceil, \lceil Q_0 \rceil); \\ TC(\lfloor M_0 \rfloor, \lceil N_0 \rceil, \lfloor Q_0 \rfloor); TC(\lfloor M_0 \rfloor, \lceil N_0 \rceil, \lceil Q_0 \rceil); \\ TC(\lfloor M_0 \rfloor, \lfloor N_0 \rfloor, \lceil Q_0 \rceil); TC(\lceil M_0 \rceil, \lfloor N_0 \rfloor, \lceil Q_0 \rceil). \end{array} \right. \quad (3.11)$$

3.5 Discussion

The properties of both the three- and four-stage supply chains are discussed in this section. The impact of various system parameters on the variables of the optimal relaxed IIC policy and the global minimum TOC for is analysed. We set

$$C = \frac{h_1'}{h_3(1 - \frac{d}{p}) - h_1'} \text{ and } W = \frac{A_1}{A_2}. \text{ Therefore, } M_0 = \sqrt{\frac{W}{C}} \text{ and } N_0 = \sqrt{\frac{A_2(h_4 + h_3)C}{(A_4 + A_s)h_1'}}.$$

The three-stage supply chains have been studied by many researchers (Banerjee & Kim, 1995; Boissière *et al.*, 2008; Lee, 2005; Omar *et al.*, 2013). Before the optimal

IIC policy for the four-stage supply chain is discussed, let us focus on the three-stage supply chain first.

3.5.1 Supply chain without raw material warehouse

Some MNCs, like Dell and Walmart, strategically allocate RMW of their vendors to implement the Just-in-time (JIT) purchasing (Myerson, 2012). Because the operational cost incurred at RMW is not considered, Eq. (3.10) becomes

$$TC'_{N_0, T_0} = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_2d(h_3 - \frac{h_3d}{p})} + L_s h_s d. \quad (3.12)$$

Corollary 3.2. In the supply chain without RMW, the global minimum TOC is bounded and is given by

$$\sqrt{2d(A_4 + A_s)(h_4 + h_3)} + L_s h_s d \leq TC'_{N_0, T_0} < \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_2dh_3} + L_s h_s d.$$

Proof. The proof follows in a straightforward way from Eq. (3.12). When $p \approx d$, the production is the JIT manufacturing, it is the lower bound. When $p \gg d$, it is an instantaneous replenishment of finished products at FPW, so, $d/p \approx 0$. It is the upper bound.

The JIT manufacturing strategy minimizes storage place and there is no inventory holding cost in FPW. The production never stops running and the setup cost is also at the minimum due to $N_0 \rightarrow \infty$ (Refer to Figure 3.3.). This optimal case was proven by Boissière *et al.* (2008) who studied a three-stage supply chain with constant demand. They proposed a stationary inventory policy to compute lot-sizes to minimize the total cost for the JIT manufacturing. Their model is a special case in our discussion.

To implement the JIT manufacturing, MNCs need to ensure the raw material supply to be always ready for the production plant. This can be achieved through tight relationship with raw material suppliers (Lee & Chu, 2005). In the late 1980s, Walmart was one of first retailers to collaborate with suppliers. Dell also develops very strong partnerships, such as, profit and risk sharing scheme with a few of key suppliers. This cuts down the raw material inventory holding costs for both Walmart and Dell (Myerson, 2012).

Figure 3.3 JIT production inventory dynamics in the three-stage supply chain.

The instantaneous replenishment of finished products has been studied in multi-echelon inventory systems (Axsäter, 2006; Zipkin, 2000) since the 1960s. Despite that it could be unrealistic, it does offer the benefits that there is no interruption in finished product supply to customers. The global minimum TOC is the upper bound. The variable of the relaxed IIC policy is given by

$$N_0 \approx \sqrt{\frac{A_2(h_4 + h_3)}{(A_4 + A_s)h_3}}.$$

3.5.2 Supply chain integrated with raw material warehouse

The supply chain becomes four stages after RMW is integrated. And the bound of the global minimum TOC is shown in Corollary 3.3.

Corollary 3.3. In the supply chain with raw material warehouse, the global minimum TOC is bounded and is given by

$$\begin{aligned} \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1h_1'd} + L_s h_s d &\leq TC_0 \\ &< \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1h_1'd} + \sqrt{2A_2d(h_3 - h_1')} + L_s h_s d. \end{aligned}$$

Proof. The proof follows in a straightforward way from Eq. (3.10) and it is similar to that of Corollary 2.

In the supply chain integrated with RMW, the global minimum TOC is the lower bound when $p \approx d$. In order to be valid for the variables M_0 and N_0 , h_1' must be equal to zero and this is another extreme case: M_0 is undefined and $N_0 \rightarrow \infty$. This affects the raw material lot size. So, it is not easy to integrate RMW into the supply chain when the

company would like to implement the JIT manufacturing. The raw material supply needs to be always ready for production. That is one of the reasons why manufacturers want the raw material suppliers to have warehouses near their production plants (Lubben, 1988).

In the instantaneous replenishment of finished products case, the global minimum TOC is the upper bound. The variables of the optimal relaxed IIC policy are given by

$$M_0 = \sqrt{\frac{A_1(h_3 - h'_1)}{h'_1 A_2}}, \text{ and}$$

$$N_0 = \sqrt{\frac{A_2(h_4 + h_3)}{(A_4 + A_s)(h_3 - h'_1)}}.$$

The inventory dynamics of the integrated four-stage supply chain with instantaneous inventory replenishment is illustrated in Figure 3.4. Now, let us discuss some special cases.

Case 1. When $h'_1 \rightarrow 0$, $C \rightarrow 0$

The variables of the optimal relaxed IIC policy are $M_0 \rightarrow \infty$ and $N_0 \rightarrow 0$. The integrated supply chain tends to have an infinitely large amount of raw material because the holding cost approaches to zero. And the global minimum TOC is given by

$$TC_0 = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_2d \left[h_3 \left(1 - \frac{d}{p} \right) \right]} + L_s h_s d.$$

Figure 3.4 Instantaneous replenishment inventory dynamics in the four-stage supply chain.

Case 2. When $h_3(1-d/p)=h_1'$, $C \rightarrow \infty$

The variables of the optimal relaxed IIC policy are $M_0 \rightarrow 0$ and $N_0 \rightarrow \infty$. The integrated supply chain tends to have an infinitely large production batch size of finished products.

And the global minimum TOC is given by

$$TC_0 = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1 h_1' d} + L_s h_s d.$$

Case 3. When $h_3 = 0$

To have a valid value for C , $h_3(1-d/p) - h_1' = 0 - h_1' \Rightarrow h_1' = 0$. The variables of the optimal relaxed IIC policy M_0 cannot be defined and $N_0 \rightarrow \infty$. This is similar to the JIT manufacturing. It is difficult to integrate RMW into the supply chain. The zero finished product inventory holding cost splits the supply chain into two parts, MUP and DC with the transportation. It is straightforward that we can optimize the two individual parts to get the minimum TOC by applying the EOQ formula. And the minimum production setup cost (The production never stops.) is incurred at MUP. Combining Eqs. (3.1) and (3.2), TOC is given by

$$\frac{d}{2} h_4 T + \frac{A_4 + A_s}{T} + L_s h_s d. \quad (3.13)$$

Because the transportation lead time is fixed, the transit inventory holding cost might not be considered. We can further simplify Eq. (3.13) into

$$\frac{d}{2} h_4 T + \frac{A_4 + A_s}{T}. \quad (3.14)$$

Hence, the stationary point is $T_0 = \sqrt{2(A_4 + A_s)/dh_4}$ and optimal cost is $\sqrt{2d(A_4 + A_s)h_4}$. It is a typical EOQ model.

Case 4. When $A_1 \rightarrow 0$

The variables of the optimal relaxed IIC policy $M_0 \rightarrow 0$ and N_0 remains unchanged. The raw material supply becomes continuous (They are not in batches.) because the raw material ordering cost approaches zero. And the global minimum TOC is given by

$$TC_0 = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_2d \left[h_3 \left(1 - \frac{d}{p} \right) - h_1' \right]} + L_s h_3 d .$$

Case 5. Analysis of production setup cost rate

When the production setup cost rate is extremely high, $A_2 \rightarrow \infty$, the variables of the optimal relaxed IIC policy are $M_0 \rightarrow 0$ and $N_0 \rightarrow \infty$. It is good to have the continuous raw material supply to minimize product changeover in the production line. When the production setup cost is reduced significantly, $A_2 \rightarrow 0$, the variables of the optimal relaxed IIC policy are $M_0 \rightarrow \infty$ and $N_0 \rightarrow 0$. It is optimal to transfer finished products to DC continuously at a rate d (There is no batching.). The raw material supply must be infinitely large as finished products are continuously supplied to DC. The global minimum TOC is given by

$$TC_0 = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1 h_1' d} + L_s h_3 d .$$

Case 6. Analysis of Basic Replenishment Quantity Q

The basic replenishment quantity Q or the cycle time T is a part of the IIC policy. Based on Eq. (3.9), $Q = Q_0 = T_0 d = \sqrt{2d(A_4 + A_s) / (h_4 + h_3)}$, the base replenishment quantity of this integrated supply chain is only affected by the parameters of downstream stages. And these stages are DC, transportation and FPW. Therefore, decision makers can determine the basic replenishment quantity or the cycle time without considering the parameters of MUP and RMW.

3.6 Computational results and analysis

Numerical examples with randomly generated values of the parameters are conducted in this section to study the difference in the optimal TOC between integer and real-number solutions of the optimal inventory coordination policy. A sensitivity analysis is performed to assist supply chain decision makers in gaining useful insights on the optimal IIC policy and TOC of the proposed model in this chapter.

3.6.1 Numerical example

Example 3.1: If $d=5$ units/day, $p=7$ units/day, $L_s=2$ days, $h_4=\$1/\text{unit/day}$, $h_s=\$0.1/\text{unit/day}$, $h_3=\$0.6/\text{unit/day}$, $h_1=\$0.002/\text{unit/day}$, $r=2$, $A_4=\$100/\text{order}$, $A_s=\$60/\text{order}$, $A_2=\$1200/\text{setup}$, $A_1=\$80/\text{order}$.

$h'_1 = h_1 r = \$0.004/\text{unit/day}$. From Eqs. (3.10), (3.11), (3.12) and (3.13), the analytical solution can be obtained:

$M_0=1.6705$, $N_0=8.4660$, $T_0=6.3246(\text{days})$ ($Q_0 = dT_0=31.6228$ (units)), and $TC_0=\$98.2088$. The convex nature of the TOC function can be observed from its graph shown in Figures 3.5, 3.6 and 3.7.

Figure 3.5 Graph of the TOC function of M and Q when $N=8$.

Figure 3.6 Graph of the TOC function of M and N when $Q=32$.

Figure 3.7 Graph of the TOC function of N and Q when $M=2$.

Based on Eq. (3.11), the eight integer solutions are summarized in Table 3 1. TOCs of eight integer solutions are higher than the real-number solution. It is clear that when $M^*=2$, $N^*=8$, and $Q^*=32$, TOC \$98.27 is the lowest. However, it is higher than TC_0 by 0.0660 percent. The solution procedure is given in Appendix E.

Table 3.2 Results of numerical example 3.1.

	M	N	Quantity Q	Cycle Time T	Difference in Cycle Time	Total Operational Cost TC	Difference in Cost	
Real Numbers	1.6705	8.4660	31.6228	6.3246		98.2088		
Integer cases	1	1	8	31	6.2000	-1.9694%	98.67	0.4697%
	2	1	8	32	6.4000	1.1929%	98.54	0.3402%
	3	1	9	31	6.2000	-1.9694%	98.46	0.2556%
	4	1	9	32	6.4000	1.1929%	98.51	0.3046%
	5	2	8	31	6.2000	-1.9694%	98.36	0.1536%
	<u>6</u>	<u>2</u>	<u>8</u>	<u>32</u>	<u>6.4000</u>	<u>1.1929%</u>	<u>98.27</u>	<u>0.0660%</u>
	7	2	9	31	6.2000	-1.9694%	98.30	0.0939%
	8	2	9	32	6.4000	1.1929%	98.39	0.1840%

3.6.2 Sensitivity analysis

3.6.2.1 Effects of the system cost rate

MNCs need to constantly reduce the system setup/ordering cost rates due to advancements in technology. Another challenge is the increase in the system holding cost rates due to inflation (Asgari *et al.*, 2016). The sensitivity analysis is performed on how the system cost rates affect the relaxed optimal IIC policy and the global minimum TOC, respectively. We first decrease the current values of the system setup/ordering cost rates by 25 percent, 50 percent and finally 75 percent. The results are presented in Table 3.2, Figures 3.8 and 3.9. We then increase the system holding cost rates by 25 percent, 50 percent, 75 percent and 100 percent of the original values. The results are presented in Table 3.3 and Figures 3.10 and 3.11.

Table 3.3 Effects of system setup/ordering cost rates in numerical example 3.1.

Parameters	System setup/ordering cost rates decrease by (due to advancements in technology):			
	0%	25%	50%	75%
M_0	1.6705	1.6705	1.6705	1.6705
N_0	8.4660	7.9818	7.2198	5.8076
T_0	6.3246	5.8095	5.2440	4.6098
TC_0	98.21	87.84	75.91	61.18

Figure 3.8 Effects of the system setup/ordering cost rates on the policy

Figure 3.9 Effects of the system setup/ordering cost rates on TOC

Table 3.4 Effects of system holding cost rates in numerical example 3.1.

Parameters	System holding cost rates increase by (due to inflation):				
	0%	25%	50%	75%	100%
M_0	1.6705	1.6705	1.6705	1.6705	1.6705
N_0	8.4660	8.4660	8.4660	8.4660	8.4660
T_0	6.3246	5.6569	5.1640	4.7809	4.4721
TC_0	98.21	109.68	120.06	129.60	138.47

Figure 3.10 Effects of the system holding cost rates on the policy

Figure 3.11 Effects of the system holding cost rates on TOC

In this numerical example, the above results demonstrate M_0 does not change with both A_T and h_T . N_0 decreases as A_T increases. However, it remains constant in h_T . However, T_0 decreases in both A_T and h_T . The global minimum TOC TC_0 decreases as A_T decreases and it increases as h_T increases. These phenomena are in line with Proposition 3.1 and Corollary 3.1. The sensitivity analysis provides the useful managerial insights for decision makers when they deploy the IIC policy in the supply chain.

3.6.2.2 Effects of the production rate

Production is one of the most important stages need to be monitored in the supply chain. The production rate cannot be smaller than the demand rate to avoid stock shortage. However, the excess production causes a gradual inventory build-up to increase the inventory holding costs (Chopra & Meindl, 2013). By fixing the value of demand rate, we increase d/p from 0.01 to 0.95. This is to analyze on how p affects the variables of the relaxed optimal IIC policy and the global minimum TOC, respectively. The results are presented in Table 3.4 and Figures 3.12 and 3.13.

Table 3.5 Effects of d/p in numerical example 3.1

Parameters	Demand Rate to Production Rate d/p										
	0.01	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.9	0.95
M_0	3.1358	2.9889	2.8166	2.6331	2.4358	2.2211	1.9833	1.7127	1.3904	0.9661	0.6583
N_0	4.5099	4.7316	5.0210	5.3709	5.8058	6.3671	7.1307	8.2572	10.1710	14.6385	21.4834
T_0	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246	6.3246
TC_0	137.53	133.59	128.96	124.04	118.75	112.98	106.60	99.34	90.69	79.31	71.05

The results show that M_0 decreases, N_0 increases as d/p increases. From Figure 3.12, N_0 is more sensitive than M_0 . When d/p is approaching to 1, N_0 becomes infinitely large (Eq. (3.8)) and M_0 is approaching to 0. The JIT manufacturing strategy causes non-stop production in MUP to avoid the stock shortage. And the global minimum TOC is the lower bound. Many MNCs, like Toyota, Dell practice this strategy to reduce their operational costs. However, To have an effective JIT manufacturing, the integrated supply chain requires a level of consistent raw material availability and quality (Lubben, 1988).

Figure 3.12 Effects of p on the policy

Figure 3.13 Effects of p on TOC

3.6.3 Computational results of ten numerical examples

Ten more numerical examples with random input data listed in Table 3.5 are tested. The optimal relaxed (real numbers) solutions and the modified solutions (integers) are given in Table 3.6. TOC with the integer solution is always higher than that with the real-number case. However, the difference is not significant and this comparison proves the accuracy of the integer approximation developed in this chapter.

Table 3.6 Input data for computational results.

Input Information													
Example	Stage 4			Transportation			Stage 3	Stage 2		Stage 1			
	d	A_4	h_4	A_s	h_s	L_s	h_3	A_2	p	A_1	h_1	r	h'_1
1	4	300	0.6	20	0.1	2	0.4	150	8	50	0.04	2	0.08
2	5	400	1	30	0.05	3	0.6	200	6	100	0.01	8	0.04
3	8	500	1.2	30	0.06	1	0.8	300	12	400	0.03	4	0.12
4	8	400	1.1	30	0.08	2	0.7	200	20	300	0.05	2	0.10
5	3	100	0.9	10	0.04	1.5	0.5	300	24	500	0.1	1	0.10
6	7	300	0.7	15	0.02	2.5	0.4	200	9	400	0.02	3	0.06
7	3	400	0.5	40	0.2	2	0.2	300	5	300	0	5	0.01
8	9	100	1.3	5	0.09	1	0.9	100	18	500	0.05	4	0.20
9	3	200	1.1	30	0.03	3.5	0.6	50	5	100	0.02	4	0.08
10	10	100	0.4	30	0.01	2.5	0.2	120	12	250	0.01	2	0.02

Table 3.7 Computational results of the optimization and integer approximation.

Example	Optimal Results (Real numbers)					Modified Optimal Results (Integers)					Difference	
	M_0	N_0	T_0	Q_0	TC_0	M^*	N^*	T^*	Q^*	TC^*	$\Delta T \%$	$\Delta TC \%$
1	0.7071	1.976	12.6491	50.5964	69.0533	1	2	12.5000	50	69.40	-1.1787%	0.4996%
2	0.8660	3.5218	10.3682	51.8410	100.9748	1	3	10.4000	52	101.11	0.3067%	0.1352%
3	1.2766	2.7783	8.1394	65.1152	184.9564	1	3	8.2500	66	185.41	1.3588%	0.2421%
4	2.1909	1.6175	7.7280	61.8240	166.4723	2	2	7.6250	61	167.24	-1.3328%	0.4616%
5	2.3717	3.3635	7.2375	21.7125	72.5454	2	4	7.0000	21	72.81	-3.2815%	0.3663%
6	0.9813	4.9169	9.0453	63.3171	97.3232	1	5	9.0000	63	97.33	-0.5008%	0.0104%
7	2.6458	2.6112	20.4707	61.4121	59.6560	2	3	20.3333	61	59.80	-0.6710%	0.2431%
8	2.5000	2.8950	3.2567	29.3103	128.9322	2	3	3.3333	30	129.56	2.3531%	0.4846%
9	2.0000	1.5198	9.4972	28.4916	62.6069	2	2	9.3333	28	63.08	-1.7254%	0.7421%
10	1.1785	6.4450	6.5828	65.8280	55.4037	1	7	6.6000	66	55.46	0.2613%	0.0936%

3.7 Summary

Facing the fierce competition in the market, MNCs must integrate their supply chains to improve the production-inventory-distribution coordination. Based on the scope of the research in Section 1.7, a supply, manufacturing, transportation and distribution problem faced by an MNC is considered in this chapter. By adopting the JCIR coordination mechanism, we formulate the TOC function, and derive the global minimum TOC to improve the multi-stage supply chain profitability.

In addition to the JCIR coordination mechanism, the industrial reference model, SCOR is used as a framework to support an MNC in choosing and refining the SCM strategies(Ntabe *et al.*, 2015; Supply Chain Council, 2010). The SCOR-based ACM for an integrated supply chain with constant demand is presented in the next chapter.

CHAPTER 4

SCOR-BASED ANALYTICAL COORDINATION MODEL FOR AN INTEGRATED SUPPLY CHAIN

An SCOR-based ACM for an integrated supply chain is investigated in this chapter. A systematic approach of combining the SCOR model with IIC is proposed to achieve the analytical solution. The problem description is in Section 4.2. The justifications for proposing the systematic approach are presented in Section 4.3. Section 4.4 outlines the research methodology, devises an IIC policy, and formulates TOC based on the SCOR model. In Section 4.5, the global minimum TOC is derived and two special cases of this SCOR-based ACM are discussed. The uncoordinated supply chain performance is analyzed in this section. Numerical experiments are conducted in Section 4.6 to show how to compute the optimal TOC of our proposed model and evaluate supply chain performance in practice. The last section presents this chapter's summary.

4.1 Introduction

It is mentioned in Section 1.4 that the supply chain parties in the MNC studied here have different priorities in their strategic objectives. Each party optimizes its own performance. Different from Chapter 3, we study the coordination among supply chain parties from the strategic to the operational levels to improve supply chain performance. The SCOR model is adopted to support the supply chain parties in choosing and refining the SCM strategies. By providing the clear purposes from the SCOR model, the supply

chain owner keeps sight of the TOC reduction strategy and is able to devise tactical steps to achieve its goals. Moreover, the SCOR model is able to model the linkage of the strategic objectives and operational metrics in a hierarchical way for supply chains (Sakka *et al.*, 2011). The SCOR model was introduced by the Supply Chain Council to benchmark operational measurement of an organization to improve supply chain performance and profitability. Because it provides a common language and framework which makes communication easier, all departments and business units involved in developing and managing the supply chain can collaborate effectively (Supply Chain Council, 2010). It is discussed in Section 1.3.2 that the SCOR model is widely adopted and used in many industries.

In this chapter, the SCOR model is used to align the strategic objectives among the supply chain parties. At the operational level, the inventory replenishment coordination among supply chain parties is the MNC's general concern because the inventories account for as high as fifty percent of the total supply chain operating cost (Jaber & Goyal, 2009; Lancioni, 2000). The challenge is from the efficiency of allocating the right amount of inventory to flow through the entire supply chain. Therefore, the inventory coordination among supply chain parties is essential (Alamri *et al.*, 2016). The IIC policy is devised as a coordination mechanism for the inventory replenishment operations among the supply chain parties.

4.2 Problem description

The MNC is introduced in Section 1.4. A typical vaccine product with constant annual demand is considered in this chapter. The integrated supply chain includes raw material supply, production, transportation and distribution (Figure 4.1). The raw materials are always ready for production according to the JIT purchasing agreement between the MNC and multiple suppliers. This is to ensure the continuity of raw material supply and to cut down the raw material supply cost (Friedli *et al.*, 2010; Mukhopadhyay *et al.*, 1998; Strohhecker *et al.*, 2014). The JIT partnership needs the open communication between the MNC and its suppliers to achieve a common goal of driving down TOC (Gunasekaran, 1999). This could also help the MNC cut raw material storage space for other business purposes. Batches of raw materials are shifted to the buffer area which is a temporary storage place before production.

After the conversion from raw materials to finished products at the production plant (PP), the manufacturer keeps finished products in the finished product warehouse (FPW) for transportation. Eventually, finished products reach the distribution centre (DC) to meet the multiple retailers demand. Customer demand is determined by contract. This supply chain and its SCOR thread diagram are illustrated in Figure 4.1. The arrows in the SCOR thread diagram, Figure 4.2 represent the direction of the material flow.

Figure 4.1 The integrated supply chain of an MNC.

Figure 4.2 The SCOR thread diagram of the supply chain.

The supply chain parties have different priorities in their strategies. The raw material suppliers put responsiveness first due to the intense supply competition. Cost is the most important metric to PP and FPW. However, DC is always looking at the reliability of the finished product supply. If the MNC cannot align all of its supply chain parties at the strategic, tactical and operational levels, it will be difficult to manage the supply chain successfully. And this uncoordinated condition among the supply chain

parties could lead to supply chain performance degradation (Arshinder *et al.*, 2008; Glock, 2012).

One of main objectives of the MNC set by the global manufacturing and supply chain division is to reduce production overage at upper stream stages to minimize the TOC (GlaxoSmithKline, 2013, 2014). It can be achieved through the supply chain strategic objectives alignment and inventory replenishment coordination among the supply chain parties. The SCOR model and the IIC policy have been identified as the potential solutions to achieve the expected outcome.

To achieve the desire production-inventory-distribution coordination from the strategic to the operational levels, the SCOR model is adopted to support the MNC in the choosing and refining of the SCM strategies for all the supply chain parties. By providing the clear purposes from the SCOR model, the MNC keeps sight of the TOC reduction strategy and is able to devise tactical steps to achieve its goals. An IIC policy is devised to coordinate inventory replenishment operations between FPW and DC. The ZIO policy is adopted for inventory ordering at FPW. A reorder point is used to manage the inventory at DC because the transportation lead time from FPW to DC is considered.

4.3 Rationale for the proposed methodology

It is discussed in Section 1.4.1 that this research is motivated by SCC problems in an MNC's clinical trial supply chain with seven stages. The supply chain parties have different priorities in their strategies. Each party makes decisions independently at the tactical and operational levels. The uncoordinated condition from the strategic to the operational levels could lead to the entire supply chain performance degradation (Arshinder *et al.*, 2008; Glock, 2012)..

The SCOR model has been recognized as a framework is to support the supply chain coordination at the strategic level in different industries (De Souza *et al.*, 2011; Huan *et al.*, 2004; Huang *et al.*, 2005; Hvolby & Trienekens, 2010; Hwang *et al.*, 2008; Knackstedt *et al.*, 2009; Kocaoglu *et al.*, 2013; Li *et al.*, 2011; Lima & Carpinetti, 2016; Liu *et al.*, 2014; Lockamy & McCormack, 2004; Long, 2014; Medini & Bourey, 2012; Ntabe *et al.*, 2015; Okongwu *et al.*, 2016; Sellitto *et al.*, 2015; Supply Chain Council, 2010; Thunberg & Persson, 2014; Wang *et al.*, 2010; Yuan *et al.*, 2010; Zhou *et al.*, 2011). The SCOR performance section consists of performance attributes and metrics. A performance attribute is a group of metrics used to set the strategic direction and cannot be measured. However, metrics measure the ability of a supply chain to achieve the attributes (Supply Chain Council, 2010). The coordination of strategies could be achieved by sharing the common performance attributes and metrics from the SCOR model among the supply chain parties. In other words, the SCOR provides a common language and framework, which makes it easier for supply chain parties to coordinate. In addition, it is theoretically plausible and understandable to practitioners (Huang *et al.*, 2005; Hwang *et al.*, 2008; Liu *et al.*, 2014; Persson *et al.*, 2010; Sellitto *et al.*, 2015; Thunberg & Persson, 2014). Therefore, the SCOR framework is chosen to support the coordination at the strategic level among the supply chain parties in this part of the research.

At the tactical and operational levels, the IIC policy has been identified as one of the effective mechanisms in most analytical models (Banerjee, 1986; Boissière *et al.*, 2008; Bushuev *et al.*, 2015; Glock, 2012; Goyal, 1976; Hill, 1997, 1999; Kim & Glock, 2013; Zhao *et al.*, 2016a) in the existing literature. The benefits of implementing the IIC policy to coordinate the supply chain parties are discussed in Section 3.3.

To fully realize the benefits of value creation from supply chain integration, it is necessary to formulate strategies and implement coordination mechanisms at the tactical and operational levels to guarantee that all of the supply chain parties work towards a common goal (Celebi, 2015; Glock, 2012). A systematic approach of combining the SCOR model with the IIC policy is proposed in this chapter to achieve this objective. The SCOR model is adopted to choose and refine the SCM strategies. The supply chain processes' performance is evaluated based on the SCOR model standard performance metrics. Then the IIC policy is used as a mechanism to coordinate the inventory replenishment operations among the supply chain parties at the tactical and operational levels.

In summary, An ACM is built to analyze supply chain performance by using the proposed systematic approach. It serves as a better decision support system for supply chain performance management.

4.4 Research methodology

Having justified for proposing the systematic approach of combining the SCOR model with IIC policy, this section is to present the research methodology. Enterprises fully recognize the impact from their supply chain performance. However, it is difficult for them to design an efficient and effective supply chain without a systematic methodology (Hwang *et al.*, 2008). At the strategic level, typical barriers include ambiguous definition of supply chain operations model, incorrect or incomplete performance metrics, and the inability to locate problems affecting supply chain performance. To overcome these obstacles, The SCOR model has often been recognized as a systematic approach to identify, evaluate and monitor supply chain performance at the strategic level (Cho *et*

al., 2012; Govindu & Chinnam, 2007; Ming *et al.*, 2014). In order to fulfil the integration of the ACM with the SCOR model to improve supply chain performance, a systematic approach is proposed in this chapter. This approach is able to provide guidelines for industrial players to design and build their own ACMs to address the SCC problems from the strategic to the operational levels. It is divided into six steps which are illustrated in Figure 4.3.

Figure 4.3 The six steps of the proposed approach.

Step 1 of the proposed approach is discussed in section 4.4.1 to define the performance attributes based on the strategic objectives of this MNC. Section 4.4.2 defines the performance metrics from the first level to the last level based on the selected performance attributes, which is **Step 2**. In **Step 3**, each of metrics is identified and modeled by mathematical equations in section 4.4.3. The IIC policy is devised in section

4.4.4 to formulate the TOC function based on the SCOR model level 1 metrics. The SCOR-based ACM is formulated in section 4.4.5, which is **Step 4**. **Step 5** of the proposed approach (in section 4.5) is to optimize supply chain performance. The last step of this proposed approach in section 4.6 is to conduct the numerical examples to validate the SCOR-based ACM and compare TOC with the uncoordinated supply chain. The SCOR model version 10 (Supply Chain Council, 2010) is used throughout this study.

Table 4.1 The SCOR model (version 10) level 1 performance metrics in performance attributes (*Supply Chain Management cost* is selected in the study).

S/N	Leve 1 Strategic Metrics	Performance Attributes			
		Customer Facing			Internal Facing
		Reliability	Responsiveness	Agility	Costs Assets
1	Perfect Order Fulfillment	√			
2	Order Fulfillment Cycle Time		√		
3	Upside Supply Chain Flexibility			√	
4	Upside Supply Chain Adaptability			√	
5	Downside Supply Chain Adaptability			√	
6	Overall Value at Risk(VAR)			√	
7	Supply Chain Management Cost				√
8	Cost of Goods Sold				√
9	Cash-to-Cash Cycle Time				√
10	Return on Supply Chain Fixed Assets				√
11	Return on Working Capital				√

4.4.1 SCOR model performance measures

In this section, the first step of the proposed approach is discussed, which is to define performance attributes from the SCOR model. SCOR identifies five core supply chain performance attributes into two categories: internally focused attributes include *Costs* and *Assets*, customer-focused attributes include *Reliability*, *Responsiveness*, and *Agility*

(Table 4.1). A performance attribute is a group of metrics used to express a strategy of an organization. It is mentioned in Section 4.2, the TOC reduction is the main objective of the MNC. Therefore, *Costs* is chosen from the five attributes of the SCOR model.

4.4.2 Supply chain cost performance metrics

After an attribute is defined, **Step 2** is to define the SCOR model performance metrics from the first level to the last level. The definition of performance metrics is a critical task in the development of a performance measurement system for the supply chain. The adopted SCOR model consists of three levels which include all customer interactions (from order entry through paid invoice), all product (physical material and service) transactions, and all market interactions (from understanding aggregate demand to the fulfilment of each order). Level 1 is the top level that deals with process types. Level 2 is the configuration level which deals with process categories. Level 3 is the process element level which is the lowest level in our model.

The SCOR model version 10 endorses 11 performance metrics and five distinct processes: *Plan*, *Source*, *Make*, *Deliver*, and *Return* at level 1. The research focuses on *supply chain management cost*, and three processes, *source*, *make* and *deliver* (Table 4.2). These three processes are further divided into process elements, tasks and activities.

Table 4.2 The SCOR level 1 process definitions (SCOR model, version 10.).

SCOR	
Processes	Definitions
Plan	The Plan processes describe the planning activities associated with operating a supply chain. This includes gathering customer requirements, collecting information on available resources, and balancing requirements and resources to determine planned capabilities and resource gaps.
Source	The Source processes describe the ordering (or scheduling) and receipt of goods and services. It includes issuing purchase orders, scheduling deliveries, receiving, shipment validation and storage, and accepting supplier invoices.
Make	The Make processes describe the activities associated with the conversion of materials or creation of the content for services. It focuses on conversion of materials rather than production or manufacturing.
Deliver	The Deliver processes describe the activities associated with the creation, maintenance, and fulfillment of customer orders. It includes the receipt, validation, and creation of customer orders; scheduling order delivery; pick, pack, and shipment; and invoicing the customer.
Return	The Return processes describe the activities associated with the reverse flow of goods back from the customer. It includes the identification of the need for a return, the disposition decision making, the scheduling of the return, and the shipment and receipt of the returned goods.

The first level is defined to span across three activities which are *Source*, *Make* and *Deliver* in the supply chain. The MNC implements the JIT strategy for the raw material purchasing, sourcing activities are done by suppliers. Making refers to the manufacturing process that converts raw materials into finished products. It covers production setup, material handling and finished product storage. Delivering refers to the processes that distribute finished products to DC to fulfill the demand. It involves the receipt, validation, creation of customer orders, scheduling order delivery, products handling, shipment and invoicing (Supply Chain Council, 2010).

Table 4.3 The supply chain's operational cost metrics for an MNC (excluding *Plan* and *Return*).

	Level-1 Metric	Level-2 Metric	Level-3 Metric	Summary	Metric Code	Stage	Remarks
Supply Chain's Total Operational Cost	Supply Chain's Total Operational Cost			Supply Chain's Total Operational Cost	TC	All of the Stages	
				Cost to Source	Cost to Source	-	
				Raw material ordering cost	Raw material ordering cost		Costs borne by suppliers due to JIT purchasing strategy
				Raw material transportation cost	Raw material transportation cost		
				Cost to Make	Cost to Make	-	
				Production setup cost	Production setup cost	S	Production Plant
				Finished product inventory holding cost at finished product warehouse	Finished product inventory holding cost at finished product warehouse	h_s	Finished Product Warehouse
				Cost to Deliver	Cost to Deliver	-	-
				Finished product transportation cost	Finished product transportation cost	A_s & h_s	Transportation
				Finished product inventory holding cost at distribution centre	Finished product inventory holding cost at distribution centre	h_3	Distribution Centre
				Ordering cost at distribution centre	Distribution centre ordering cost	A_3	

At the second level, TOC of the supply chain can be described by two types of cost: *cost to make* and *cost to deliver*. The detailed cost elements are further considered at the third level of the SCOR model. In *cost to make*, the production setup cost and the finished product inventory holding cost are considered. Three detailed cost elements in *cost to deliver* are taken into account. They are finished product inventory holding cost in DC warehouse, transportation cost, and ordering cost at DC. The supply chain's operational cost performance metrics of three levels are categorized in Table 4.3.

The objective of this part of the study is to derive an optimal IIC policy that minimizes TOC of the supply chain. TOC is the sum of the production setup cost, the product ordering cost, the inventory holding cost, and the transportation cost based on the SCOR model cost metrics.

4.4.3 Notations and assumptions for the SCOR-based analytical coordination model

At **Step 3**, we identify each of cost metrics at three levels for different stages in the integrated supply chain. The detailed notations for this SCOR-based ACM are as follows.

Distribution Centre: the inventory ordering policy (R_3 , Q_3) is applied, *i.e.*, when the inventory position decreases to reorder point R_3 (>0), a batch with size Q_3 is ordered. To simplify the notation, $Q_3=Q$ is denoted. Customer demand rate is d units per unit time. Thus, the ordering cycle time is $T=Q/d$. The lead time L is the time from the time when an order is placed to the time when the order is received, which models the transportation time from FPW to DC. The inventory holding cost rate at DC is h_3 per unit per unit time. The finished product ordering cost rate is A_3 per order. From Table

4.3, the operational cost incurred at DC is the sum of the inventory holding cost and the ordering cost for finished products.

Transportation from FPW to DC: the transit inventory holding cost rate from FPW to DC is h_s per unit per unit time. The fixed transportation cost rate is A_s per trip. Thus, the operational cost incurred for transportation is the sum of the fixed transportation cost and the transit inventory holding cost (refer to Table 4.3).

Finished Product Warehouse: ZIO policy is used, *i.e.*, $(0, Q_2)$. When the inventory position is zero, a batch of size Q_2 is ordered and PP starts to produce finished products. The inventory holding cost rate is h_2 per unit per unit time. From Table 4.3, the operational cost incurred at FPW is the inventory holding cost.

Production Plant: The production rate is p units per unit time. It is constant and not smaller than the customer demand rate, *i.e.*, $p \geq d$. The quantity of products manufactured during the production time, $Q_2/p = T_1$ must exactly match the demand over the cycle time, T_2 . And the production time is at most equal to the cycle time, *i.e.*, $T_1 \leq T_2$. When the closer the two values are, the closer will the production facility perform to its full capacity. From Table 4.3, the operational cost incurred at PP is the production setup cost S . And there is a setup cost for switching on production.

Table 4.4 Notations for SCOR-based analytical coordination model

Parameters Definition		Parameters Definition	
A_3	finished goods ordering cost rate at Distribution Centre (\$/batch)	Q_3	batch quantity ordered at Distributed Centre (units)
A_s	fixed transportation cost rate (\$/trip)	Q_{avg}	average inventory at Finished Product Warehouse (units)
d	demand rate at time t (units/unit time)	Q_s	shipping quantity (units)
h_2	inventory holding cost rate at Finished Product Warehouse (\$/unit/unit time)	R_3	inventory reorder point at Distribution Centre (units)
h_3	inventory holding cost rate at Distribution Centre (\$/unit/unit time)	S	production setup cost (\$/setup)
h_s	transient inventory holding cost rate (\$/unit/unit time)	T_1	production time (unit time)
L	transportation lead time from Finished Product Warehouse to Distribution Centre	T_2	production cycle period (unit time)
p	production rate (units/unit time)	TC	total operational cost of the supply chain (\$)
Q_2	batch quantity ordered at Finished Product Warehouse (units)		

Decision Variables	Definition
N	positive integer, $N=Q_2/Q_3$
Q	basic replenishment quantity at Distribution Centre (units), $Q=Q_3$
T	ordering cycle time at Distribution Centre

Assumptions: In addition to the general assumptions in Section 1.4.3, the following assumptions are necessary to the SCOR-based ACM developed in this Chapter:

- 1) Both inventories at FPW and DC are under continuous review, where inventory levels are constantly monitored. When the reorder point is reached, an order is placed.

2) The shipping quantity is assumed to be the replenishment quantity at DC,

$$i.e., Q_s=Q_3=Q.$$

4.4.4 Integer-ratio inventory coordination policy of the SCOR-based analytical coordination model

Lacking of coordination causes the bullwhip effect (Lee *et al.*, 1997; Wang & Disney, 2016) which increases supply chain's TOC tremendously. An IIC policy (Abdul-Jalbar *et al.*, 2005; Atkins *et al.*, 1992) is proposed to coordinate inventory replenishment operations between FPW and DC. In each finished product replenishment cycle, inventory replenishment is made based on the optimal integer-ratio quantities with a constant time interval. The IIC policy is able to simplify material handling by restricting the production quantity at FPW to be multiples of a fixed shipment size to DC. It leads to smooth operations and product flow between FPW and DC.

In the devised inventory coordination policy, it is assumed that $Q_2=NQ_3$, N is a positive integer. The best positive integer N^* in the SCOR-based ACM needs to be computed so that TOC is at the minimum. The inventory dynamics and synchronization across the integrated supply chain are illustrated in Figure 4.4.

Figure 4.4 The inventory dynamics and synchronization across the integrated supply chain.

4.4.5 Total operational cost of the SCOR-based analytical coordination model

We are now ready to derive the operational cost at each stage in the integrated supply chain (**Step 4**): the operational cost incurred at DC is

$$\frac{Q_3}{2} h_3 + \frac{d}{Q_3} A_3 = \frac{d h_3}{2} T + \frac{A_3}{T}, \quad (4.1)$$

the operational cost incurred for transportation is

$$\frac{L h_s Q_s + A_s}{T} = L h_s d + \frac{A_s}{T}, \quad (4.2)$$

the operational cost incurred at FPW is

$$h_2 Q_{avg} = \frac{h_2 dT}{2} \left\{ N \left(1 - \frac{d}{p} \right) + 1 \right\}, \quad (4.3)$$

the operational cost incurred at PP is

$$\frac{S}{NT}, \quad (4.4)$$

where Q_{avg} is obtained from Appendix F.

The level 1 metric code, TOC of the integrated supply chain, TC is the sum of the operational costs incurred at DC, transportation, FPW and PP. By summing up the costs in Eqs. (4.1), (4.2) (4.3) and (4.4), the TOC function TC is

$$TC = \frac{dT}{2} \left(h_3 + h_2 \left(N \left(1 - \frac{d}{p} \right) + 1 \right) \right) + \frac{1}{T} \left(A_3 + A_s + \frac{S}{N} \right) + L_s h d \quad (4.5)$$

4.5 Supply chain performance optimization

4.5.1 SCOR-based analytical coordination supply chain

The supply chain performance optimization is **Step 5** in the proposed approach. In this study, the objective is to minimize the long-run average TOC per unit time for the integrated supply chain based on the SCOR model metrics. Combining Eq. (4.5) with the constraints yields the following:

$$\begin{aligned} \text{Min TC} &= \frac{dT}{2} \left(h_3 + h_2 \left(N \left(1 - \frac{d}{p} \right) + 1 \right) \right) + \frac{1}{T} \left(A_3 + A_s + \frac{S}{N} \right) + L_s h d. \\ \text{s.t. } N, T &> 0; N \text{ integer} \end{aligned}$$

It is an MINLP optimization problem. When the integer constraint is relaxed, N is treated as a continuous variable. Proposition 4.1 shows that the TOC function TC is convex.

Proposition 4.1. (i) For the SCOR-based ACM, the TOC function TC is convex in both continuous variables N and T , TC reaches its global minimum at (N_0, T_0) , where

$$N_0 = \sqrt{\frac{S(1 + \frac{h_3}{h_2})}{(A_3 + A_s)(1 - \frac{d}{p})}}, \quad (4.6)$$

$$T_0 = \sqrt{\frac{2(A_3 + A_s)}{d(h_2 + h_3)}}. \quad (4.7)$$

(ii) the global minimum TOC is given by

$$TC_0 = TC(N_0, T_0) = \sqrt{2d(A_3 + A_s)(h_2 + h_3)} + \sqrt{2dSh_2(1 - \frac{d}{p})} + Lh_s d. \quad (4.8)$$

The proof of Proposition 4.1 is provided in Appendix G. (N_0, Q_0) is the optimal relaxed IIC policy ($Q_0 = dT_0$). From Eqs. (4.6) and (4.7), they are highly dependent on the SCOR metric codes, as well as the ratio of production rate to demand rate. Based on Proposition 4.1, two special cases are derived in the following Corollary 4.1.

Corollary 4.1. For the SCOR-based ACM, we have the following results:

- (i) In the JIT production, the global minimum TOC and the variable of the optimal relaxed IIC policy (N_0, Q_0) are given by

$$\sqrt{2d(A_3 + A_s)(h_2 + h_3)} + Lh_s d,$$

$$N_0 \rightarrow \infty, \text{ respectively;}$$

- (ii) In the case of instantaneous replenishment of finished products at DC, the global minimum TOC and the variable of the optimal relaxed IIC policy (N_0 , Q_0) are given by

$$\sqrt{2d(A_3 + A_s)(h_2 + h_3)} + \sqrt{2Sh_2d} + Lh_sd ,$$

$$N_0 \approx \sqrt{\frac{S(h_2 + h_3)}{h_2(A_3 + A_s)}} , \text{ respectively.}$$

The proof of Corollary 4.1 is referred to Appendix H. JIT production minimizes the storage place and there is no inventory holding cost occurred in FPW for the MNC. Moreover, the production never stops running and the setup cost for production is also at the minimum (Bonney & Jaber, 2011; Jaber *et al.*, 2004) (Refer to Figure 4.5 for JIT production inventory dynamics).

For the instantaneous replenishment of finished products at FPW, the global minimum TOC is higher than the first case. However, it does offer benefits such as no interruption in finished product supply to DC (Axsäter, 2006; Zipkin, 2000) (Refer to Figure 4.6).

Figure 4.5 JIT production inventory dynamics.

Figure 4.6 Instantaneous replenishment inventory dynamics.

After the global minimum point (N_0, T_0) is obtained, a simple approximation can be done to get workable solution if N_0 and Q_0 ($Q_0 = dT_0$) are not integers. The optimal integers (N^*, Q^*) of the IIC policy can be selected among the integer solution sets, where TOC is at the minimum.

$$(N^*, Q^*) = \arg \min \left\{ \begin{array}{l} \text{TC}(\lfloor N_0 \rfloor, \lfloor Q_0 \rfloor); \text{TC}(\lfloor N_0 \rfloor, \lceil Q_0 \rceil) \\ \text{TC}(\lceil N_0 \rceil, \lfloor Q_0 \rfloor); \text{TC}(\lceil N_0 \rceil, \lceil Q_0 \rceil) \end{array} \right\} \quad (4.9)$$

4.5.2 Supply chain without coordination

Based on the current practice of the MNC, both DC and FPW make decision independently to minimize operational cost at their own stages. They determine their optimal cycle time (OCT), EOQ and economical production quantity (EPQ), as shown in the following lemma.

Lemma 4.1. At the production plant in the uncoordinated supply chain, EPQ is given by

$$Q_2' = \sqrt{\frac{2Sd}{h_2(1-\frac{d}{p})}}.$$

And the optimal operation cost is given by

$$\sqrt{2Sdh_2(1-\frac{d}{p})} + h_2\sqrt{\frac{A_3d}{2h_3}}.$$

The proof of Lemma 4.1 is referred to Appendix I.

Proposition 4.2. In the supply chain without coordination, the minimum TOC function TC_t in this problem is given by

$$TC_t = \sqrt{2A_3dh_3} + Lh_s d + A_s\sqrt{\frac{h_3d}{2A_3}} + \sqrt{2Sdh_2(1-\frac{d}{p})} + h_2\sqrt{\frac{A_3d}{2h_3}}. \quad (4.10)$$

Comparing with TC_0 for the proposed SCOR-based ACM, TC_t is not smaller than TC_0 , *i.e.*, $TC_t \geq TC_0$. And the difference is given by

$$\Delta TC = TC_t - TC_0 = \sqrt{\frac{A_3 dh_3}{2}} \left(\sqrt{1 + \frac{A_s}{A_3}} - \sqrt{1 + \frac{h_2}{h_3}} \right)^2 \geq 0. \quad (4.11)$$

The proof of Proposition 4.2 is in Appendix J. Based on Eq. (4.11), function ΔTC has a critical point, which $A_s/A_3 = h_2/h_3$ in the region $[0, \infty)$, the global minimum of ΔTC is 0. In other words, comparing the proposed SCOR- based ACM with the supply chain without coordination, there are no savings on TOC when $A_s/A_3 = h_2/h_3$.

4.6 Computational results and analysis

Numerical examples are conducted to validate the SCOR-based ACM in this section. To demonstrate the utility of the proposed approach, the data for the parameters in our numerical examples are partially derived from a case study in the pharmaceutical industry (Kannan *et al.*, 2013). The high transportation cost is considered in this chapter because a typical product, like vaccines must be stored at the right temperature from the time they are manufactured until they are used (Shah, 2004). A sensitivity analysis of the inventory holding cost and the transportation cost is performed to assist decision makers. The supply chain performance evaluation on TOC between our proposed model and the uncoordinated supply chain is conducted as well. This is the last step of the proposed systematic approach. The following numerical example is to illustrate how to get the optimal TOC in practice.

Example 4.1: $d=6200$ units/year, $p=7000$ units/year, $L=0.003$ year,
 $h_3=\$12400$ /unit/year, $h_s=\$120$ /unit/year, $h_2=\$1240$ /unit/year, $A_3=\$650$ /order,
 $A_s=\$1050$ /order, $S=\$2500$ /setup.

From Eqs. (4.6), (4.7), (4.8) and (4.10), the real-number solution can be computed: $N_0=11.8972$, $T_0=0.0063(\text{year})=2.30(\text{days})$ ($Q_0=dT_0=39.3123(\text{units})$), $TC_0=\$604732.1416$ and $TC_t=\$655802.17$. Figure 4.7 shows the convexity of the TOC function, TC.

Figure 4.7 Graph of the TOC function TC in N and Q .

Based on Eq. (4.9), the four integer cases are summarized in Table 4.4. TOC of four integer cases is higher than the real-number solution. It is clear that when $N^*=12$ and $Q^*=39$, the optimal TOC \$604749.21 is the lowest among the four integer cases. However, it is higher than TC_0 by 0.0028 percent. The solution procedure is given in Appendix K. However, comparing with the uncoordinated supply chain, the TOC-saving is $\Delta TC / TC_t = 7.79\%$.

Table 4.5 Results for numerical example 4.1.

		N	Quantity Q	Cycle Time T	Difference in Cycle Time	Total Operational Cost TC	Difference in Cost
Real Numbers		11.8972	39.3123	0.0063		604732.1461	
Integer Case	1	11	39	0.0063	-0.7943%	604996.66	0.0437%
	<u>2</u>	<u>12</u>	<u>39</u>	<u>0.0063</u>	<u>-0.7943%</u>	<u>604749.21</u>	<u>0.0028%</u>
	3	11	40	0.0065	1.7494%	604936.42	0.0338%
	4	12	40	0.0065	1.7494%	604835.10	0.0170%

From Proposition 4.2, the TOC-saving highly depends on the SCOR metric codes, A_s / A_3 and h_2 / h_3 . The sensitivity analysis of this typical example is done to obtain managerial insights to assist the supply chain decision makers. Due to the market inflation, the rising fuel price is one of the major concerns for this MNC. By fixing the value of A_3 , we increase the value of A_s from 0 to \$2000 with interval of \$20 to analyze the effect of the transportation cost on the TOC-saving for this proposed the SCOR-based ACM. To study the holding cost effects on the TOC-saving, we fix the value of h_3 and increase the value of h_2 from 0 to \$40000 with interval \$200. The results are presented in Figure 4.8.

It is observed from Figure 4.8 that the TOC-saving function is convex in both the SCOR-based metric codes, A_s and h_2 . There are no savings on TOC when $A_s / A_3 = h_2 / h_3$ in this example. And these are in line with Proposition 4.2. The following insights can be drawn:

Observation 4.1: Due to the convexity of the total operational saving-cost function on A_s and h_2 , the SCOR-based ACM is effective when $A_s / A_3 \neq h_2 / h_3$.

Figure 4.8 Effects of A_s , h_2 on the TOC-saving in numerical example 4.1.

From Observation 4.1, the SCOR-based ACM is more effective with the high transportation and finished products holding costs. This could be one of the promising factors for decision makers from some industries to consider our recommendation. And those industries could be from retailing, such as Walmart and Carrefour who depend on transportation and storage heavily in their supply chains (Myerson, 2012).

In the current literature, $h_2 < h_3$ is assumed in most papers (Abdul-Jalbar *et al.*, 2007; Glock, 2012; Kim & Glock, 2013; Lee, 2005). However, the recent research (Hill & Omar, 2006) shows that there are some industrial cases where the condition of $h_2 > h_3$ is also hold, particularly for the automotive supply chain with the consignment stock

policy (Braglia & Zavanella, 2003; Valentini & Zavanella, 2003). Many researchers (Bylka, 2013; Corbett, 2001; Wang *et al.*, 2004b) discussed the benefits of using the consignment stock policy for supply chains. In this study, we do not make the above assumption. The results demonstrate the proposed SCOR-based ACM is effective for both cases.

To compare the SCOR-based ACM with the uncoordinated supply chain, another 30 sets of data are developed based on the industrial motivated problem (Kannan *et al.*, 2013) to study supply chain performance. The optimal relaxed (real numbers) solutions, the modified solutions (integers) and TOC without SCOR are given in Table 4.5.

In the SCOR-based ACM, it is observed that TOC at the integer solution is always higher than the real-number case. However, the difference is not significant. The computational results show the overall average difference is 0.0207 percent. Comparing with the uncoordinated supply chain, the average saving on TOC is 4.89 percent. Having analyzed 30 sets of data, the insights can be summarized from Table 4.5 as follows.

Observation 4.2: If $A_s / A_3 \neq h_2 / h_3$, the SCOR-based ACM supports more effective coordination among the supply chain members. Comparing with the supply chain without coordination, the savings on TOC are significant.

Observation 4.3: The IIC policy in the SCOR-based ACM offers the benefits such as simple material handling, smooth operation and production flow. The difference on TOC between the integer solution and real-number solution is insignificant.

Table 4.6 Computational results of supply chain performance

S/N	Input Information									Computational Results										
	Distribution Centre			Transportation			Final Product Warehouse	Production Plant		Optimal Results(Real Numbers)				Modified Optimal Results(Integers)				Difference in TC*	Total Cost without SCOR	Total Cost Saving
d	A_3	h_3	A_s	h_s	L	h_2	S	p	N_0	T_0	Q_0	TC_0	N^*	T^*	Q^*	TC^*	ΔTC^*	TC_t	ΔTC	
1	6500	600	15000	1000	380	0.003	1500	2000	7000	13.8744	0.0055	35.5050	646022.6069	14	0.0054	35	646083.47	0.0094%	704388.98	8.28%
2	6300	650	15100	1100	250	0.0045	1680	2500	6800	13.9303	0.0058	36.2500	677742.2262	14	0.0057	36	677756.91	0.0022%	738259.28	8.20%
3	5400	550	14400	1050	360	0.003	1550	3000	6000	13.8904	0.0061	32.9148	601689.4994	14	0.0061	33	601695.12	0.0009%	664075.28	9.39%
4	5800	660	14000	1150	440	0.006	1500	3500	6300	15.8672	0.0063	36.8046	655305.9133	16	0.0064	37	655320.37	0.0022%	714988.44	8.35%
5	5900	700	13790	1050	230	0.0045	1450	2500	6300	15.3780	0.0062	36.8101	619207.1694	15	0.0063	37	619224.75	0.0028%	666587.10	7.11%
6	5700	580	14170	950	280	0.006	1420	2000	6100	14.7939	0.0059	33.4484	577112.4402	15	0.0058	33	577159.93	0.0082%	627759.17	8.06%
7	6330	500	3550	900	135	0.006	900	1000	6800	7.1483	0.0100	63.1104	314031.4383	7	0.0100	63	314039.11	0.0024%	337011.68	6.82%
8	6250	450	3200	880	145	0.0045	800	1200	6600	9.2233	0.0103	64.4690	287180.3131	9	0.0104	65	287192.34	0.0042%	311421.62	7.78%
9	6350	460	3240	900	200	0.012	850	1500	6600	11.8367	0.0102	64.9845	305792.4808	12	0.0102	65	305794.90	0.0008%	330220.49	7.40%
10	6250	510	3200	850	270	0.003	900	1800	6500	12.5206	0.0103	64.3921	296977.8909	13	0.0102	64	296996.63	0.0063%	314907.96	5.69%
11	6150	440	3150	800	220	0.0045	920	1300	6400	10.8964	0.0100	61.2162	279209.9974	10	0.0099	61	279211.98	0.0007%	298391.82	6.43%
12	6350	400	3370	700	210	0.006	810	900	6800	7.9876	0.0091	57.8109	274403.1528	8	0.0091	58	274404.73	0.0006%	293806.27	6.60%
13	6010	420	3280	720	135	0.003	940	1400	6500	8.5519	0.0095	56.9834	277436.0245	9	0.0095	57	277481.59	0.0164%	294383.71	5.74%
14	5850	400	3200	720	180	0.006	840	1600	6300	9.8077	0.0097	56.9523	269919.4639	10	0.0097	57	269926.42	0.0026%	288409.38	6.41%
15	5900	480	3250	850	300	0.012	880	1200	6400	7.3621	0.0104	61.6441	307031.2598	7	0.0105	62	307066.63	0.0115%	326615.37	5.99%
16	42000	400	350	800	180	0.0045	195	600	47000	3.6243	0.0102	430.0629	300738.9858	4	0.0102	430	300895.89	0.0522%	313451.02	4.01%
17	42500	420	290	820	225	0.006	200	700	52000	2.7514	0.0109	463.7909	331259.1485	3	0.0109	463	331427.14	0.0507%	340164.46	2.57%
18	42500	410	275	780	270	0.003	210	700	50300	2.9599	0.0107	456.6801	299932.9566	3	0.0107	456	299936.36	0.0011%	306839.64	2.25%
19	43500	370	230	750	370	0.003	180	750	52000	3.0547	0.0112	487.5023	291977.1203	3	0.0112	488	291983.59	0.0022%	299023.12	2.35%
20	41300	350	250	650	100	0.0045	120	650	51300	3.2065	0.0114	472.4862	228843.6775	3	0.0115	473	228919.75	0.0332%	238384.23	3.97%
21	41500	360	185	650	270	0.003	100	500	50000	2.8809	0.0131	542.3471	214745.1666	3	0.0131	542	214766.33	0.0099%	221740.58	3.15%
22	40500	320	220	600	160	0.0045	110	700	45900	4.4048	0.0117	475.2033	213065.1825	4	0.0118	476	213187.02	0.0572%	221435.49	3.72%
23	39150	380	195	700	130	0.006	150	800	45000	3.6201	0.0126	495.0889	236292.3572	4	0.0126	495	236465.86	0.0734%	241111.85	1.93%
24	6660	300	1010	500	100	0.003	650	600	8000	3.3816	0.0120	80.1204	164495.7987	3	0.0122	81	164678.53	0.1111%	168408.73	2.21%
25	41950	310	210	500	320	0.003	130	720	47000	4.6515	0.0107	447.0788	221326.7380	5	0.0107	447	221402.21	0.0341%	225700.28	1.90%
26	42450	330	220	520	150	0.0045	100	540	50400	3.5900	0.0112	474.8849	207508.6507	4	0.0112	474	207660.93	0.0734%	213754.01	2.85%
27	6050	340	950	520	100	0.003	730	800	6480	5.6799	0.013	78.7023	155689.2010	6	0.0129	78	155717.29	0.0180%	157811.90	1.33%
28	5200	300	760	500	100	0.003	650	950	6350	3.7714	0.0148	76.8161	143974.0273	4	0.0146	76	144019.76	0.0318%	145760.98	1.19%
29	6300	330	580	600	105	0.006	400	750	6500	8.0133	0.0174	109.3487	121915.3513	8	0.0173	109	121916.03	0.0006%	125440.05	2.81%
30	5400	360	450	660	95	0.0045	350	1000	6300	3.9606	0.0217	117.3456	119422.9147	4	0.0217	117	119423.88	0.0008%	121983.69	2.10%

In summary, the SCOR-based ACM with IIC policy offers an effective solution to address the SCC problems. The computational results validate the feasibility and potential benefits of adopting the proposed approach for the MNC's supply chain.

4.7 Summary

An ACM for an integrated supply chain is presented in this chapter. The SCOR model is adopted to support the MNC in choosing and refining of the level 1 metric, *supply chain management cost* at the strategic level for the supply chain parties. To address the inventory replenishment coordination issue at the tactical and operational levels, we propose and find an optimal IIC policy to minimize the TOC. The finding reinforces the proposition that the adoption of an ACM based on the metrics of the SCOR model is promising in terms of its capacity to assist decision makers in improving supply chain performance.

The integrated multi-stage supply chain with constant and continuous demand under the JCIR coordination mechanism and the SCOR model are studied in Chapters 3 and 4. However, based on the literature review in Chapter 2, the supply chain with constant and continuous demand cannot reflect the practical situations as the demand for most products is changing with time due to the seasonal variation, business cycle, irregular fluctuation, and product life-cycle (Axsäter, 2006; Glock, 2012; Teng *et al.*, 2012). There is a limited number of research addressed integrated multi-stage supply chains with time-varying demand (Arshinder *et al.*, 2008; Glock, 2012; Hwang *et al.*, 2013; Kaminsky & Simchi-levi, 2003; Pahl & Voss, 2014; Zhao *et al.*, 2016b). To gain deeper understanding of supply chain coordination and performance optimization problems, this thesis considers time-varying multi-phase demand which generalizes

continuous and constant demand in the next chapter. It caters for more dynamic business environments.

CHAPTER 5

OPTIMAL PRODUCTION-INVENTORY-DISTRIBUTION COORDINATION POLICY FOR AN INTEGRATED MULTI-STAGE SUPPLY CHAIN WITH TIME-VARYING DEMAND

In this chapter, an integrated multi-stage supply chain with time-varying demand is considered. Section 5.2 describes the problem. TOC of the integrated multi-stage supply chain is formulated in Section 5.3. In Section 5.4, the model is represented as the WDAG. The global minimum TOC is computed by the developed algorithm. In addition, some properties of the proposed model are discussed in this section. Two industrial applications of time-varying demand, a seasonal product and a product over its life cycle are illustrated in Section 5.5. Section 5.6 presents this chapter's summary.

5.1 Introduction

Integrating various processes in a supply chain to devise the optimal production-inventory-distribution policy is essential in a competitive industrial environment. Numerous studies have been conducted on integrated supply chains to minimize the TOCs by finding the optimal lot sizes (Ben-Daya *et al.*, 2008; Brahimi *et al.*, 2006; Glock, 2012) since the 1990s. However, most of them considered the constant demand. They cannot reflect a practical situation because the demand rates are changing with time due to seasonal variation, business cycle, irregular fluctuation, and product life

cycle. Seasonal variations result from both natural randomness and human decisions. For example, umbrellas are in high demand during the monsoon period in Southeast Asia. Customers may change their behaviours during an umbrella sales promotion. So, the demand for umbrellas is subject to seasonal variations. In a typical business cycle, the economy slowly moves through stages of expansion, slowdown, recession, and recovery (Wang *et al.*, 2004a). The demand for most products is positively correlated with the business cycle. Unpredictable events such as accidents and natural disasters may lead to irregular fluctuations in demand. The product life cycle can be divided into four discrete stages: introduction, growth, maturity and decline (Aitken *et al.*, 2003). The product demand is dynamically correlated with its life cycle, particularly for high-tech industries (Marquez & Blanchar, 2006; Teng *et al.*, 2012; Vonderembse *et al.*, 2006). For time-varying demand, linearization techniques can be applied to approximate the actual demand curve over the planning horizon, which consists of multiple phases. The approximation is accurate if the interval is sufficiently small (Andriolo *et al.*, 2014; Hill, 1995; Silver, 1979; Silver *et al.*, 1998). Therefore, the multi-phase function shown in Fig. 1 can represent the demand for most products. It is composed of piece-wise linear demand patterns over a planning horizon T_f , where f is the index of phases, $f = 1, 2, 3, \dots, F$.

To adapt to demand fluctuations, supply chain decision makers should carefully manage their inventories and constantly monitor the production schedule to minimize TOC. Indeed, effectively managing the supply chain with time-varying demand is an important issue for current MNCs.

Figure 5.1 The timer-varying multi-phase demand.

In this chapter, a new model for an integrated multi-stage supply chain with time-varying demand over a finite planning horizon is developed. The supply chain includes supply, production, transportation and distribution. The objective of this part of the study is to devise an optimal production-inventory-distribution policy to minimize TOC which includes production setup cost, product ordering cost, inventory holding cost, and transportation cost. The integrated multi-stage supply chain is formulated as an MINLP problem. It is represented as the WDAG. The global minimum TOC is computed in polynomial time. Furthermore, a conventional assumption for many models in previous studies is that the holding cost rates increase as the material/product flows down the supply chain (Glock, 2012; Kaminsky & Simchi-levi, 2003; Kim & Glock, 2013; Lee, 2005; Liu & Lian, 2009) because the product increases in value as it moves down the supply chain. However, some studies show that this is not true in several industries, such as the aircraft, automotive, personal computer and retailer supply chains with consignment stock policies (Braglia & Zavanella, 2003; Chen *et al.*, 2010; Diabat, 2014;

Valentini & Zavanella, 2003; Verma *et al.*, 2014; Yi & Sarker, 2014). Our model is more generic than the existing models in previous studies as this assumption is relaxed in this chapter.

5.2 Problem description, notations and model assumptions

An integrated multi-stage supply chain in Figure 5.2, which includes supply, production, transportation and distribution, is discussed in this chapter. There are four stages: raw material warehouse (RMW), manufacturing plant (MUP), finished product warehouse (FPW), and retailer (R). Customer demand is time-varying and can be modelled as a piece-wise linear function over a finite planning horizon consisting of multiple phases.

The inventory dynamics and synchronization across the integrated multi-stage supply chain are illustrated in Figure 5.5. The raw materials are supplied to RMW in batches immediately after the manufacturer places an order. One time purchase of raw materials provides an exact amount of materials for a production cycle. Materials are sent to MUP and are converted into finished products with a finite and constant rate. Finished products are transported to R in batches of unequal size periodically with a fixed interval L . At the final stage, R fulfils the customer time-varying demand. The notations used in this chapter are summarized in Table 5.1.

Figure 5.2 The integrated multi-stage supply chain with time-varying demand.

Table 5.1 Notations of the time-varying demand.

Decision Variables	Definition
l_j	number of shipment periods in cycle $j - 1$
M	total number of production cycles or subplans over the planning horizon
m_j	number of shipments in cycle j
Q_{pj}	quantity of finished goods manufactured in cycle j , $Q_{pj} = Q_{3j}$ (units)
T_{0j}	interval between T_{bj} and the time to start production in cycle j (production setup time)

Parameters Definition		Parameters Definition	
A_1	ordering cost rate at Raw Material Warehouse (\$/order)	Q_{4f}	Quantity demanded at R in phase f (units)
A_s	fixed transportation cost rate (\$/trip)	Q_{1j}	quantity of raw materials ordered in cycle j (units)
A_4	finished goods receiving cost rate at Retailer (\$/batch)	Q_{3j}	quantity of finished good ordered in cycle j (units)
b	number of the last shipment in the increasing demand phase	Q_{pj}	production quantity in cycle j (units)
C_j	operational cost of the supply chain in subplan j (\$)	r	conversion factor of raw materials to finished goods
C	total operational cost of the supply chain (\$)	S	production setup cost (\$/setup)
d_{of}	initial demand rate in phase f (units/unit time)	t	index of the planning horizon, $t = 1, \dots, T$
$d(t)$	demand rate at time t (units/unit time)	T	length of the planning horizon
E	set of directed edges in WDAG	T_f	period of phase f
f	index of phases over the planning horizon, $f = 1, 2, \dots, F$	T_{bj}	beginning time of cycle j
F	total number of phases over the planning horizon	T_{ej}	ending time of cycle j
h_1	inventory holding cost rate at Raw Material Warehouse (\$/unit/unit time)	T_{pj}	production time for cycle j
h_s	transient inventory holding cost rate (\$/unit/unit time)	D_0	initial demand in single-phase demand case
h_3	inventory holding cost rate at Finished Goods Warehouse (\$/unit/unit time)	D_{of}	initial demand in phase f (units)
h_4	inventory holding cost rate at Retailer (\$/unit/unit time)	$D(\gamma_f)$	aggregate demand during an interval $[\gamma_f L, (\gamma_f + 1) L]$ at Retailer (units)
$I_j(t)$	finished goods inventory on hand at time t in cycle j	x_k^j	k th shipment size from Finished Goods Warehouse to Reatiler in cycle j (units)
j	index of production cycles over the planning horizon	v	vertex in WDAG
k	index of shipments from Finished Goods Warehouse to Retailer	V	set of vertices in WDAG
L	interval between successive shipments at Retailer	w	weight of edges in WDAG
L_s	transportation lead time from Finished Goods Warehouse to Retailer	W	set of weights of edges in WDAG
n_f	total number of shipment intervals in phase f	α, β	indexes of the shipment period in WDAG, $\alpha, \beta = 0, 1, \dots, N$
N	total number of demand periods over the planning horizon at Retailer	λ_f	rate of increment/decrement of the demand rate in phase f (units/unit time ²)
p	production rate (units/unit time)	θ_f	increment / decrement of demand in phase f (units)
$q_{Fj}(t)$	finished goods produced by time t in cycle j (units)	γ_f	index of shipment intervals at Retailer in phase f , $\gamma_f = 1, \dots, n_f$
$q_{sj}(t)$	total quantity of finished goods shipped by time t in cycle j (units)	δ	path in WDAG
Q	total quantity demanded of the supply chain over F phases (units)	δ^*	the shortest path in WDAG

5.2.1 Retailer

The customer demand at R varies over the planning horizon T , which consists of F phases. So, the ordering and production batches and optimal lengths of the production cycles at upstream stages are not necessarily equal. The review periods at R are not fixed and are computed in this chapter. It is assumed that T and all T_f are multiples of L (Figure 5.3). The piece-wise linear demand rate function is given by

$$d(t) = \begin{cases} d_{01} + \lambda_1 t & t \in [0, T_1), T_1 = n_1 L \\ d_{02} + \lambda_2 t & t \in [T_1, T_2), T_2 = (n_1 + n_2) L \\ \dots & \dots \\ d_{0f} + \lambda_f t & t \in [T_{f-1}, T_f), T_f = (n_1 + n_2 + \dots + n_f) L \\ \dots & \dots \\ d_{0F} + \lambda_F t & t \in [T_{F-1}, T_F), T_F = (n_1 + n_2 + \dots + n_f + \dots + n_F) L \end{cases} \quad (5.1)$$

where

$$d_{0f} \geq 0, f=1, 2, \dots, F \quad (5.1a)$$

$$\sum_{f=1}^F n_f = N, n_f \text{ positive integer}, \quad (5.1b)$$

$$T = T_F = NL. \quad (5.1c)$$

Eq. (5.1) is the function of the demand rate at each phase. $[T_{f-1}, T_f)$ is the interval of phase f . $d(t)$ is the customer demand rate at time t . d_{0f} is the initial demand rate and λ_f is the rate of increment/decrement of the demand rate in phase f . Constraint (5.1a) assures the initial demand rate at each phase is positive. Constraint. (5.1b) assures the total number of shipment intervals from phase 1 to F is equal to N . Constraint. (5.1c) assures the length of the planning period is equal to NL .

Based on Eq. (5.1), the aggregate demand during an interval $[\gamma_f L, (\gamma_f + 1) L] \in [T_{f-1}, T_f)$ at R is $D(\gamma_f)$, where $\gamma_f = 1, 2, \dots, n_f - 1$.

$$D(\gamma_f) = \int_{\gamma_f L}^{(\gamma_f+1)L} d(t)dt = \left[d_{0f}t + \frac{1}{2}\lambda_f t^2 \right]_{\gamma_f L}^{(\gamma_f+1)L} = (d_{0f} + \frac{1}{2}\lambda_f L)L + \gamma_f \lambda_f L^2 = D_{0f} + \gamma_f \theta_f, \quad (5.2)$$

where $D_{0f} = (d_{0f} + 1/2\lambda_f L)L$, $\theta_f = \lambda_f L^2$ are the initial demand and increment/decrement of demand in phase f , respectively. Based on Eq. (5.2), the total quantity demanded at R over F phases is shown in Proposition 5.1.

Proposition 5.1. The total demanded quantity at R over F phases is given by

$$Q = \sum_{f=1}^F Q_{4f} = L \left[\sum_{f=1}^F d_{0f} n_f + \frac{1}{2} \sum_{f=1}^F n_f \lambda_f (n_f + 2 \sum_{f=1}^{f-1} n_f) \right]. \quad (5.3)$$

The proof of Proposition 5.1 is in Appendix L. A batch with size $D(\gamma_f)$ is received in the interval $[\gamma_f L, (\gamma_f + 1)L]$ at R. L_s is the transportation lead time from FPW to R. The inventory holding cost rate is h_4 per unit per unit time. The finished products receiving cost rate is A_4 per batch. The operational cost incurred at R is the sum of the finished product inventory holding cost and the material receiving cost.

Figure 5.3 The time-varying demand over the planning horizon T .

5.2.2 Transportation from FPW to R

The transit inventory holding cost rate is h_s per unit per unit time. The fixed transportation cost rate is A_s per trip. The operational cost incurred from transportation is the sum of the fixed transportation cost and the transit inventory holding cost.

5.2.3 Manufacturing plant

Because shipment sizes are not constant over the planning horizon, production batch sizes vary from cycle to cycle. The planning horizon T consists of M production cycles (Figure 5.4), $1 \leq M \leq N$. Each production cycle j ($j = 1, 2, \dots, M$) begins at time T_{bj} and ends at time T_{ej} , so, $T_{b1} = 0$ and $T_{ej} = T_{b(j+1)}$. Both T_{bj} and T_{ej} are multiples of L , i.e., $T_{bj} = l_j L$, $T_{ej} = l_{j+1} L$, where l_j is the number of shipment periods in the production cycle $j-1$, $0 \leq l_j \leq N$, $T_{ej} - T_{bj} = (l_{j+1} - l_j) L = m_j L$. There are m_j shipments in cycle j . Both l_{j+1} and m_j are positive integers, $l_1=0$. The k th shipment size in production cycle j is x_k^j , $k=1, 2, \dots$

m_j . And $N = \sum_{j=1}^M m_j$. The production quantity in cycle j is given by

$$Q_{pj} = \sum_{k=1}^{m_j} x_k^j. \quad (5.4)$$

The production rate is p units per unit time. The production time in the cycle j is $T_{pj} = Q_{pj} / p$. The production start time T_{0j} is defined as the interval between T_{bj} and the time to actually start the production in cycle j . It must be coordinated to ensure in-time supply within the supply chain. To have a lean supply chain, the value of T_{0j} should be determined that for an exact amount of the first shipment in cycle j , x_1^j are manufactured just-in-time to be shipped to R (Fig. 5.5). Therefore, $x_1^j = D_{0f} + \theta_f = p(L - T_{0j})$. The production setup time of cycle j is given by

$$T_{0j} = L - \frac{D_{0f} + \theta_f}{p}. \quad (5.5)$$

Figure 5.4 The production schedule over planning horizon T .

A generic time-varying demand may be composed of increasing, constant and decreasing phases (Diponegoro & Sarker, 2007). To avoid finished product supply shortages, the production rate cannot be lower than the demand rate over the planning horizon. The following proposition gives the minimum production capacity of the integrated multi-stage supply chain with time-varying demand.

Proposition 5.2.

- (i) In the increasing demand phase, *i.e.*, $\lambda_f > 0$, the production rate is $p \geq d_{0f} + (b+0.5)\lambda_f L$, b is the number of the last shipment in the increasing demand phase;
- (ii) In the constant demand phase, *i.e.*, $\lambda_f = 0$, the production rate is $p \geq d_{0f}$;
- (iii) In the decreasing demand phase, *i.e.*, $\lambda_f < 0$, the production rate is $p \geq d_{0f} + 1.5\lambda_f L$.

The proof of Proposition 5.2 is in Appendix M. The holding cost at MUP is not considered. The operational cost incurred at MUP is the production setup cost S .

5.2.4 Finished product warehouse

A batch with size Q_{3j} is ordered in cycle j . The inventory holding cost rate is h_3 per unit per unit time. The operational cost incurred at FPW is the inventory holding cost.

Figure 5.5 The inventory dynamics and synchronization diagram for the integrated supply chain with time-varying demand.

5.2.5 Raw material warehouse

A batch with size Q_{1j} is ordered in cycle j . The raw materials are supplied to RMW immediately. It is considered that r units of raw materials which can be converted to one unit of finished product. The inventory holding cost rate is h_1 per unit per unit time. The ordering cost of raw materials is A_1 per order. The operational cost incurred at RMW is the sum of the inventory holding cost and the raw material ordering cost.

5.2.6 Model assumptions

In addition to the general assumptions in Section 1.4.3, the following assumptions are necessary to model this time-varying problem in Chapter 5:

- 1) The initial and final inventories at both RMW and FPW are zero without loss of generality.
- 2) The raw materials are supplied to RMW immediately after the manufacturer places an order.
- 3) There is no shipment at the beginning of the planning horizon because R has the residual stocks to satisfy the customer demand.
- 4) The shipping quantity is assumed to be the quantity demanded in the interval L at R (Figure 5.5).

The objective of this part of the study is to determine the optimal production-inventory-distribution policy to minimize the TOC of the integrated multi-stage supply chain with time-varying demand. By obtaining the global minimum TOC, the following questions can be answered for the proposed model in this chapter:

- What is the optimal batch size of finished products at FPW in each production cycle?

- What is the optimal number of production cycles at MUP over the finite planning horizon?
- What is the optimal production setup time at MUP in each production cycle?
- What is the raw material ordering time at RMW?
- What are the batch sizes of the raw materials at RMW?

5.3 Total operational cost function

Because the customer demand varies over time, the optimal lengths of production cycles are not necessarily equal. Before the TOC function is formulated to solve the problem optimally, a subplan is defined as follows:

Definition 5.1: Subplan j , $(l_j L + L_s, l_{j+1} L + L_s, T_{bj}, T_{ej})$ means the quantity of finished products produced in a period starting at time T_{bj} and ending at time T_{ej} and satisfies the demand in consecutive periods, starting at time $l_j L + L_s$ and ending at time $l_{j+1} L + L_s$ at the retailer.

Based on Definition 5.1, it is clear that the finished products produced in a production cycle must always cover the demand in an integer of consecutive demand periods at the retailer in a subplan. Therefore, any feasible operations policy for this integrated supply chain can be divided into a sequence of consecutive subplans. The number of subplans is equal to the number of the production cycle M . The production cycle starts at time zero in the first subplan. It ends at time T in the last subplan M . We are now ready to derive the operational cost at each stage of the integrated supply chain. In the subplan j , the operational cost incurred at R is

$$m_j A_4 + \frac{\sum_{k=1}^{l_{j+1}-l_j} x_k^j}{2} h_4 L = (l_{j+1} - l_j) A_4 + \frac{Q_{pj}}{2} h_4 L, \quad (5.6)$$

the operational cost incurred for transportation is

$$m_j A_s + \sum_{k=1}^{l_{j+1}-l_j} x_k^j h_s L_s = (l_{j+1} - l_j) A_s + Q_{pj} h_s L_s, \quad (5.7)$$

the operational cost incurred at FPW is

$$h_3 \int_{T_{bj}}^{T_{ej}} I_j(t) dt = h_3 \left\{ Q_{pj} \{ l_{j+1} L - l_j L - L + \frac{D_{0f} + (l_j + 1 - n_{f-1}) \theta_f}{p} - \frac{Q_{pj}}{2p} \} \right. \\ \left. - L \sum_{k=1}^{l_{j+1}-l_j-1} x_k^j (l_{j+1} - l_j - k) \right\}, \quad (5.8)$$

where $\int_{T_{bj}}^{T_{ej}} I_j(t) dt$ is obtained from Appendix N. The operational cost incurred at MUP is

S. The operational cost incurred at RMW is

$$A_1 + \frac{r \sum_{k=1}^{l_{j+1}-l_j} x_k^j}{2} T_{pj} h_1 = A_1 + \frac{r Q_{pj}^2}{2p} h_1. \quad (5.9)$$

The operational cost function for the integrated supply chain in subplan j , C_j is the sum of the operational cost incurred at R, transportation, FPW, MUP, and RMW.

By summing up the costs from Eqs. (5.6) to (5.9), we get

$$C_j(l_j, l_{j+1}) = A_1 + \frac{r Q_{pj}^2}{2p} h_1 + S + h_3 \int_{T_{bj}}^{T_{ej}} I_j(t) dt + (l_{j+1} - l_j) A_s + Q_{pj} h_s L_s + (l_{j+1} - l_j) A_4 + \frac{Q_{pj}}{2} h_4 L. \quad (5.10)$$

The objective function: TOC of the integrated supply chain over the planning horizon T , C is the aggregate cycle cost with M subplans:

$$\min C = \sum_{j=1}^M C_j(l_j, l_{j+1}),$$

$$\sum_{j=1}^M C_j(l_j, l_{j+1}) = MA_1 + \frac{rh_1}{2p} \sum_{j=1}^M \left(\sum_{k=1}^{m_j} x_k^j \right)^2 + MS + h_3 \sum_{j=1}^M \int_{T_{bj}}^{T_{ej}} I_j(t) dt + A_s \sum_{j=1}^M m_j$$

$$+ h_s L_s \sum_{j=1}^M \sum_{k=1}^{m_j} x_k^j + A_4 \sum_{j=1}^M m_j + \frac{h_4 L}{2} \sum_{j=1}^M \sum_{k=1}^{m_j} x_k^j, \quad (5.11)$$

Subject to

$$0 = l_1 < l_2 < \dots < l_M \leq N, \quad (5.11a)$$

$$0 < M \leq N, \quad (5.11b)$$

$$L_s \leq L, \quad (5.11c)$$

$$M, N, l_j, m_j \text{ positive integers.} \quad (5.11d)$$

Decision Variables: $l_j, l_{j+1}, Q_{pj}, M, T_{0j}$.

Constraint (5.11a) assures that the cycle j beginning must be earlier than the cycle $j+1$ and the first cycle's beginning is at the starting time of the planning horizon. Constraint (5.11b) assures the total number of production cycles is at least one and at most equal to the total number of demand periods over the planning horizon at Retailer/shipments. Constraint (5.11c) assures the transportation lead time is not greater than the fixed demand period at Retailer. Constraint (5.11d) ensures M, N, l_j, m_j are positive integers.

5.4 Solution methodology

5.4.1 Solution algorithm

By including the objective function with constraints, an MINLP optimization problem is postulated. The operational decision in solving the time-varying demand problem in a period depends on the action in the preceding periods. This problem is represented as a WDAG with $(N + 1)$ vertices and $O(N^2)$ edges. The shortest path through the network represents the optimal operations policy for this integrated multi-stage supply chain. It can be found in $O(N^2)$ time. The following definitions are given to represent the problem as a network model.

Definition 5.2. A WDAG $G(V, E)$, with weighted function $w: E \rightarrow \mathbf{R}$ mapping edges to real-valued weights has the following structure:

- (1) V is the set of vertices, $V = \{v_\alpha: \alpha=0, \dots, N\}$ Council of Supply Chain Management Professionals Council of Supply Chain Management Professionals.
A vertex v_α represents the α th shipment period when $\alpha > 0$. v_0 and v_N are the beginning and ending of the planning horizon T , respectively;
- (2) E is the set of directed edges, $E = \{e(v_\alpha, v_\beta): v_\alpha, v_\beta \in V\}$. An edge $e(v_\alpha, v_\beta)$ represents the subplan j for which the production cycle starts at time T_{bj} and ends at time T_{ej} ;
- (3) W is the set of weights, $W = \{w(v_\alpha, v_\beta): e(v_\alpha, v_\beta) \in E\}$. The weight $w(v_\alpha, v_\beta)$ of the edge $e(v_\alpha, v_\beta)$ is the operational cost of the subplan j . So, $w(v_\alpha, v_\beta) = C_j$;

where both α and β are indexes of vertices, which are in sequence with the shipment schedule at R over the planning horizon T , $\beta - \alpha = m_j$, $0 \leq \alpha < \beta \leq N$.

Definition 5.3. In subplan j , $(l_j L + L_s, l_{j+1} L + L_s, T_{bj}, T_{ej})$, the production cycle starts at the α th shipment period and ends at the β th shipment period in the WDAG $G(V, E)$.

The α th shipment period is always in front of the β th shipment period. Therefore, for an edge $e(v_\alpha, v_\beta)$, v_α always appears before v_β in ordering. Based on Definition 2 and 3, V has $(N+1)$ vertices. The number of edges is $N(N+1)/2$.

Definition 5.4. A path δ through the WDAG $G(V, E)$ is a sequence of $\langle v(1), \dots, v(M+1) \rangle$ from vertex v_0 to v_N , where $v(0) = v_0$, $v(M+1) = v_N$, $e(v(j), v(j+1)) \in E$ for $j = 1, \dots, M$. The weight of path δ is the sum of the weights of the edges from subplans 1 to M ,

$$w(\delta) = \sum_{j=1}^M w(v(j), v(j+1)). \quad (5.12)$$

From Definition 5.4 and Eq. (5.12), the sum of the weights of the edges in the path $w(\delta)$ is equivalent to the sum of the subplans' costs over the planning horizon

$\sum_{j=1}^M w(v(j), v(j+1))$. For example, there are five shipments over the planning horizon as

shown in Figure 5.4. The path $\delta = \langle v_1, v_3, v_5, v_6 \rangle$ represents a three-cycle production or three subplans with length of $m_1=2$, $m_2=2$ and $m_3=1$ shipments, respectively. TOC associated with the feasible operations policy is

$$w(\delta) = w(v_1, v_3) + w(v_3, v_5) + w(v_5, v_6).$$

Figure 5.6 A WDAG representation for the integrate supply chain with five shipments.

For a WDAG with $(N+1)$ vertices, there are many paths from v_0 to v_N . The objective is to find a path with minimum weight which is the shortest path. The shortest path and the weight of the shortest path are defined by δ^* and $w(\delta^*) = \min \{w(\delta)\}$, respectively. Therefore, the shortest path weight $w(\delta^*)$ is the global minimum value of the objective function. The optimal number of subplans and production cycles and the length of each production cycle can be obtained as well.

Theorem 5.1: If $\delta^* = \langle v^*(1), \dots, v^*(j), \dots, v^*(M^*+1) \rangle$ is the shortest path from v_0 to v_N of the WDAG $G(V, E)$ with the weight $w(\delta^*)$, then M^*, l_j^* for $j=1, 2, \dots, M^*$ is an optimal solution for the corresponding the time-varying demand problem. The weight $w(\delta^*) = TC$ is the global minimum TOC of the integrated multi-stage supply chain.

Proof: From Definition 5.3, $w(v(j), v(j+1))$ is the operational cost for the subplan j . From Eq. (13) $w(\delta^*)$ is TOC of the integrated supply chain. By Definition 5.4, $0 = l_1^* < l_2^* < l_3^* \dots < l_{M^*}^* < N, l_j$ is a positive integer for $j=1, 2, \dots, M^*$. So, l_j^* is feasible. By contradiction, suppose $l_j' \neq l_j^*$ for some $j \in \{1, 2, \dots, M^*\}$, the associated TOCS $TC' <$

TC. Then, $v(j) = \iota_j^*$, $v(j) \neq v^*(j)$ that creates another path $\delta' \neq \delta^*$ which has TOC $TC = w(\delta') < w(\delta^*)$. This implies that δ^* is not the shortest path. Therefore, ι_j^* for $j=1, 2, \dots, M^*$ is optimal. The proof is completed.

The above procedure can be summarized in an algorithm as follows.

Solution Algorithm (polynomial) Finding the optimal solution to the time-varying demand problem

Step 1:

(1.1) To construct the graph $G(V, E)$ by using Definitions 5.2 and 5.3;

(1.2) To compute the edge weights:

- a. To define each v_α , where $\alpha = 0, 1, \dots, N-3$;
- b. To define each v_β , where $\beta = \alpha+2, \alpha+3, \dots, N-1$;
- c. To create an edge $e(v_\alpha, v_\beta)$ and compute $w(v_\alpha, v_\beta)$;

Step 2: To find the shortest path and compute the weight of the shortest path by setting v_0 as the source vertex of the graph $G(V, E)$. The weight of the shortest path is the global minimum TOC of the integrated multi-stage supply chain.

We apply the algorithm, Single-source Shortest Path in directed acyclic graphs (Cormen *et al.*, 2009) to find the shortest path in the graph representation of this problem. The graph, $G(V, E)$, is acyclic. For an edge $e(v_\alpha, v_\beta)$, v_α always appears before v_β in ordering. Therefore, $G(V, E)$ is already in topological order and it is not required to do the topological order sorting for the problem. The computation of weights for the edges is implemented on an EXCEL spread sheet. The sparse matrix is constructed after all of the feasible edges and corresponding weights are determined. Then, we use the function “graphshortestpath” in Matlab to search the shortest path. This shortest path

algorithm stores the output into an array $\pi = \{v_1, \dots, v_t\}$ of vertices, and any two vertices in this array indicate the beginning and ending times of the production cycle in the subplan j . Here, v_j for $j > 1$ is the follower of the vertex v_{j-1} in the shortest path. v_0 is the source and does not have any predecessor. The shortest path computational time is $O(V+E)$ when the graph $G(V, E)$ is constructed.

5.4.2 Special cases

Based on Eq. (5.1), the multi-phase demand becomes the single-phase demand or linear trend in demand when $f=1$. We set $d_{01} = d_{02} = \dots = d_{0F} = d_0$, $\lambda_1 = \lambda_2 = \dots = \lambda_F = \lambda$, $\theta_1 = \theta_2 = \dots = \theta_F = \theta$, $\gamma = 1, 2, \dots, N$. The single-phase linear demand function is $d(t) = d_0 + \lambda t$, $t \in [0, T_F)$. Several properties of the multi-stage supply chain with single-phase demand are developed in this section.

Lemma 5.1. For the single-phase demand, the the shipment size from FPW to R in the subplan j , x_k^j is the aggregate demand during an interval $[kL, (k+1)L]$. It is given by

$$x_k^j = D_0 + (l_j + k)\theta, \quad (5.13)$$

Where $D_0 = (d_0 + 1/2\lambda L)L$, $\theta = \lambda L^2$, $0 \leq l_j + k \leq \sum_{j=1}^M m_j$, k positive integer.

The proof of Lemma 5.1 is in Appendix O. The planning horizon T consists of M subplans or production cycles. From Eq. (5.5) and Definition 5.1, the finished products produced in the subplan j may cover more than one demand phase. θ_f is not constant in the multi-phase demand model. So, Q_{pj} in Eq (5.5) is not a closed-form solution. However, it becomes a closed-form in the single-phase demand case, which is shown in Lemma 5.2.

Lemma 5.2: For the single-phase demand, the production quantity produced in subplan j has a closed-form solution and is given by

$$Q_{pj} = (l_{j+1} - l_j)[x_0 + \theta \frac{(l_{j+1} + l_j + 1)}{2}]. \quad (5.14)$$

The proof of Lemma 5.2 is in Appendix P. Based on Eq. (5.14), the first shipment size $x_1^j = x_0 + (l_j + 1)\theta = p(L - T_{0j})$. Therefore, the production setup time for the production cycle j for the single-phase demand is given by

$$T_{0j} = L - \frac{D_0 + (l_j + 1)\theta}{p}. \quad (5.15)$$

Based on Eq. (5.15), some properties of the production set time can be developed shown in Proposition 5.3.

Proposition 5.3: For the single-phase demand, the following properties of the production setup time of the subplan j hold:

- (i) In the increasing demand case, *i.e.*, $\lambda > 0$, the production cycle setup time is $T_{0j} > T_{0(j+1)}$.
- (ii) In the constant demand case, *i.e.*, $\lambda = 0$, production cycle setup time is $T_{0j} = T_{0(j+1)}$.
- (iii) In the decreasing demand case, *i.e.*, $\lambda < 0$, the production cycle setup time is $T_{0j} < T_{0(j+1)}$.

The proof of Proposition 5.3 is in Appendix Q. Many researchers (Hoque, 2011; Huang & Ye, 2010; Huang *et al.*, 2010; Kim & Glock, 2013; Lee, 2005; Lu, 1995; Pal *et al.*, 2012; Sana *et al.*, 2014; Sarker & Diponegoro, 2009) studied the multi-stage supply chain with the single-phase constant demand, where it is a special case of ours.

Lemma 5.3: For the single-phase constant demand, *i.e.*, $\lambda_f = 0$, the shipment size from FPW to R, $x_k^j = d_0 L$ is constant over the planning horizon. The lengths of the production cycles are equal, *i.e.*, $m_1 = m_2 = \dots = m_j$.

The proof of Lemma 5.3 is a straightforward. Let $m_j = m$. We have the following results.

Theorem 5.2. For the single-phase constant demand, the TOC function of the integrated multi-stage supply chain, C is strictly convex in the production cycle length m , the positive integer.

The proof of theorem 5.2 is in Appendix R.

5.5 Computational results and analysis

Numerical examples are conducted in this chapter to illustrate how to devise the optimal production-inventory-distribution policy to minimize TOC in practice. Two types of industrial applications, a seasonal product and a product over its life cycle, are illustrated. The sensitivity analysis is conducted to evaluate how system parameters affect the production-inventory-distribution policy and the global minimum TOC, respectively. Random numbers are assigned to all of the parameters in the following examples.

5.5.1 Demonstrative example of seasonal products

Example 5.1: The planning horizon is two years (or 24 months) for a seasonal product,

$D_f(t) = D_{0f} + \lambda_f t$ in demand phases $f = \{1, 2, 3, 4, 5, 6, 7, 8\}$, demand and increment /

decrement rates in different phases are listed in Table 5.2.

Table 5.2 Demand rates of the seasonal product.

f	1	2	3	4	5	6	7	8
d_{0f} (units/month)	160	280	280	220	220	340	340	280
λ_f (units/month ²)	40	0	-20	0	40	0	-20	0

The interval between successive shipments is $L=1$ month. The sets of shipment time points for demand phases (Figure 5.5) 1, 2, 3, 4, 5, 6, 7, 8 are $\omega_1 = \{1, 2\}$, $\omega_2 = \{3, 4, 5\}$, $\omega_3 = \{6, 7, 8\}$, $\omega_4 = \{9, 10, 11\}$, $\omega_5 = \{12, 13, 14\}$, $\omega_6 = \{15, 16, 17\}$, $\omega_7 = \{18, 19, 20\}$, and $\omega_8 = \{21, 22, 23\}$ respectively. Let $p = 350$ units/month, $r = 3$, $h_1 = \$0.02$ /unit/month, $h_3 = \$0.08$ /unit/month, $h_s = \$0.01$ /unit/month, $h_4 = \$0.1$ /unit/month. $A_1 = \$40$ /order, $A_4 = \$60$ /order, $S = \$200$ /setup, $A_s = \$80$ /trip, and $L_s = 0.05$ month.

Figure 5.7 The seasonal product demand over the planning horizon.

By using Eqs. (5.2) and (5.3), the initial shipment sizes and increment / decrement of shipment size in eight phases can be determined and they are listed in Table 5.3. The ordering quantities at Retailer are: $D(\gamma_f) = \{180, 220, 260, 280, 280, 280, 270, 250, 230, 220, 220, 220, 240, 280, 320, 340, 340, 340, 330, 310, 290, 280, 280, 280\}$.

Table 5.3 Initial shipment size and increment/decrement rates of the seasonal product.

f	1	2	3	4	5	6	7	8
D_{0f} (units)	180	280	270	220	240	340	330	280
θ_{0f} (units)	40	0	-20	0	40	0	-20	0

Based on Definition 5.1, we randomly consider in subplan j that the production cycle starts at $l_j L = 10$ and ends at $l_{j+1} L = 16$. Because the shipment time point 11 belongs to ω_4 , 12 to 14 belongs to ω_5 and 15 to 16 belongs to ω_6 , the shipping quantities in the subplan are

$$x_k^j = \begin{cases} 220 + (10 + k - 9) \times 0 & k \in \{1\} \\ 240 + (10 + k - 12) \times 40 & k \in \{2, 3, 4\} \\ 340 + (10 + k - 15) \times 0 & \text{for } k \in \{5, 6\}. \end{cases}$$

Based on Definition 5.2 and 5.3, we can model this problem into a WDAG. There are 23 shipments within the planning horizon of 24 months. It has 24 vertices $V = \{v_0, \dots, v_{23}\}$ and 276 edges.

The production setup time is $T_{0j} = L - x_1^j / p = 0.371$ (month) = 11.14 (days).

The production quantity of cycle j is $Q_{pj}(10, 16) = 1740$ (units).

The time-weighted aggregate production

$$\int_{10}^{16} q_{pj}(t) dt = Q_{pj}(m_j L - T_{0j} - \frac{1}{2} T_{pj}) = 5468 \text{ (units} \cdot \text{month)}.$$

The time-weighted aggregate shipment

$$\int_{10}^{16} q_{sj}(t)dt = L \sum_{k=1}^{l_{j+1}-l_j-1} x_k^j (l_{j+1} - l_j - k) = 3880 (\text{units} \cdot \text{month}).$$

The time-weighted finished product inventory

$$\int_{10}^{16} I_j(t)dt = 1588 (\text{units} \cdot \text{month}).$$

Based on Eqs (5.6) to (5.9), the operational costs of the multi-stage supply chain can be computed and they are listed in Table 5.4.

Table 5.4 The cycle operational cost of the seasonal product.

	Retailer	Transportation	Finished Goods Warehouse	Manufacturing Plant	Raw Material Warehouse
Operational Cost	\$447.00	\$480.87	\$127.09	\$200.00	\$299.51
			Cycle Operational Cost		\$1,554.47

The operational cost for subplan j where the production cycle starts at $l_j = 10$ and ends at $l_{j+1} = 16$ is \$1554.47. By using the aforementioned method, we can calculate the operational costs for the rest of the feasible subplans and form the sparse matrix in Table 5.5. Based on Definitions 5.2 and 5.3, this problem can be represented as a WDAG. There are 23 shipments within the planning horizon in 24 months. It has 24 vertices $V = \{v_0, \dots, v_{23}\}$ and 276 edges. Then, we use the function “graphshortestpath” in Matlab to find the shortest path.

Table 5.5 The optimal production-inventory-distribution policy of the seasonal product.

Cycle j	l_j^*	l_{j+1}^*	T_{0j} (Days)	Q_{pj}	Q_{1j}	Raw Material Ordering Time (Months)	Cycle Operational Cost
1	0	7	11.14	1590	4770	0.37	\$1,498.42
2	7	13	10.29	1380	4140	6.34	\$1,446.94
3	13	18	2.57	1620	4860	5.09	\$1,320.91
4	18	24	3.43	1770	3540	6.11	\$1,553.38
Global Minimum Total Operational Cost							\$5,819.64

The computation time of this problem is rather short due to the low order of the complexity $O(n^2)$. The result of the predecessor field of the shortest path is: $\pi = (0, \text{NaN}, 1, 1, 1, 1, \underline{1}, 1, 5, 5, 6, 6, \underline{7}, 8, 11, 11, 12, \underline{13}, 13, 15, 16, 17, 18, \underline{18})$, where NaN represents Not-a-Number. Therefore, the shortest path is an array of vertices: $\{1, 7, 13, 18, 24\}$. Based on Definition 5.4 and theorem 5.1, the global minimum TOC of the integrated supply chain is \$ 5819.64. The optimal value of l_j^* , l_{j+1}^* , and the respective values of T_{0j} , Q_{pj} and Q_{1j} , are summarized in Table 5.6.

Table 5.6 Sparse matrix of the seasonal product numerical example

Vertex	V_0	V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}	V_{21}	V_{22}	V_{23}
V_0	0	-	582.59	785.69	1006.714	1245.65	1498.42	1759.11	2025.40	2300.97	2591.37	2896.61	3232.908	3620.17	4062.87	4545.27	5048.27	5571.86	6104.25	6632.00	7152.95	7678.61	8222.19	8783.69
V_1	0	0	-	593.43	798.13	1020.75	1257.78	1503.90	1756.78	2019.53	2297.11	2589.52	2911.83	3282.77	3706.82	4169.41	4652.59	5156.36	5669.52	6179.20	6683.25	7192.59	7719.85	8265.03
V_2	0	0	0	-	597.56	803.22	1023.89	1254.87	1493.82	1743.24	2007.50	2286.58	2594.36	2948.34	3353.00	3795.00	4257.58	4740.76	5233.93	5703.12	6211.32	6703.70	7214.00	7742.22
V_3	0	0	0	0	-	597.56	800.95	1015.93	1240.16	1475.50	1725.68	1990.68	2283.10	2619.16	3003.34	3423.58	3864.40	4325.82	4590.77	5268.93	5736.86	6211.32	6703.70	7214.00
V_4	0	0	0	0	0	-	595.93	794.91	1004.42	1225.68	1461.78	1712.70	1989.76	2307.90	2671.60	3070.08	3489.14	3928.80	4379.73	4830.95	5280.32	5736.86	6211.32	6703.70
V_5	0	0	0	0	0	0	-	590.63	784.89	991.57	1213.08	1449.42	1710.57	2010.15	2352.64	2728.58	3125.11	3542.23	3971.29	4410.89	4832.10	5270.08	5725.98	6199.80
V_6	0	0	0	0	0	0	0	-	583.49	774.95	981.25	1202.38	1446.94	1727.16	2047.52	2399.95	2772.97	3166.59	3572.83	3998.70	4392.15	4810.77	5247.31	5701.77
V_7	0	0	0	0	0	0	0	0	-	578.20	769.03	974.70	1202.39	1462.93	1760.80	2089.33	2438.46	2808.18	3191.22	3594.16	3968.38	4367.32	4784.18	5218.96
V_8	0	0	0	0	0	0	0	0	0	-	576.41	766.87	977.96	1219.14	1494.89	1799.91	2125.53	2471.74	2831.97	3205.20	3567.64	3947.22	4344.72	4760.14
V_9	0	0	0	0	0	0	0	0	0	0	-	576.41	771.32	993.62	1247.79	1529.89	1832.58	2155.86	2493.84	2838.95	3189.06	3549.76	3928.38	4324.92
V_{10}	0	0	0	0	0	0	0	0	0	0	0	-	579.52	782.94	1015.53	1274.70	1554.46	1854.82	2170.55	2494.75	2825.31	3167.13	3526.87	3904.53
V_{11}	0	0	0	0	0	0	0	0	0	0	0	0	-	589.46	801.94	1039.74	1298.13	1577.12	1872.10	2165.39	2489.14	2813.36	3155.50	3515.56
V_{12}	0	0	0	0	0	0	0	0	0	0	0	0	0	-	604.01	822.38	1061.35	1320.91	1597.03	1863.38	2179.79	2488.01	2814.15	3158.21
V_{13}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	616.36	836.67	1077.58	1335.60	1587.50	1885.44	2178.30	2489.08	2817.78
V_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	621.17	842.65	1081.82	1326.52	1597.38	1874.24	2169.02	2481.72
V_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	621.17	840.35	1073.86	1319.57	1579.47	1857.29	2153.03
V_{16}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	619.48	834.21	1062.35	1305.29	1566.15	1844.93
V_{17}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	615.15	823.60	1048.94	1292.20	1553.38
V_{18}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	605.73	812.67	1037.53	1280.31
V_{19}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	599.69	805.83	1029.89
V_{20}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	597.56	803.22
V_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-	597.56
V_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-
V_{23}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

“_” signs mean the infeasible subplans.

To study the effects of the holding costs and setup/ordering costs on the optimal number of production cycles or subplans M^* and the global minimum TOC C of the numerical example, multiple factor ρ is introduced. By setting the value of ρ from 0 to 10, the shortest paths and the global minimum TOCs are computed. The results are provided in Tables 5.7 and 5.8. Figures 5.8 to 5.9 are plotted for better visual effects. The following insights can be drawn:

Observation 5.1. The optimal number of production cycles or subplans over the planning horizon M^* is a piece-wise increasing function of both h_1 and h_3 . It is a piece-wise decreasing function of both A_1 and S . Compared with A_1 , S has a higher impact on the value of M^*

From Table 5.7 and Figure 5.8, the optimal number of subplans or production cycles M^* tends to increase with higher holding cost rates at FPW and RMW. In contrast, M^* tends to decrease with a higher production setup cost rate and raw material ordering cost rate (Table 5.8 and Figure 5.9).

Table 5.7 Effects of holding cost rates on M^* .

Effects of Holding Cost Rates				
$h_3=0.08$			$h_1=0.02$	
ρ	The Shortest Path	M^*	The Shortest Path	M^*
0	{1, 7, 14, 19, 24}	4	{1, 12, 24}	2
0.25	{1, 7, 13, 18, 24}	4	{1, 8, 14, 24}	3
0.5	{1, 7, 13, 18, 24}	4	{1, 7, 13, 19, 24}	4
0.75	{1, 7, 13, 18, 24}	4	{1, 7, 13, 18, 24}	4
1	{1, 7, 13, 18, 24}	4	{1, 7, 13, 18, 24}	4
2	{1, 5, 10, 14, 19, 24}	5	{1, 5, 9, 13, 17, 20, 24}	6
3	{1, 5, 10, 14, 19, 24}	5	{1, 5, 8, 12, 15, 18, 21, 24}	7
4	{1, 5, 9, 13, 17, 21, 24}	6	{1, 4, 7, 10, 13, 16, 18, 21, 24}	8
5	{1, 5, 9, 12, 16, 21, 24}	6	{1, 4, 7, 10, 13, 15, 17, 19, 21, 24}	9
6	{1, 4, 7, 10, 13, 17, 21, 24}	7	{1, 4, 6, 9, 12, 14, 16, 18, 20, 22, 24}	10
7	{1, 4, 7, 10, 13, 17, 21, 24}	7	{1, 3, 5, 7, 9, 12, 14, 16, 18, 20, 22, 24}	11
8	{1, 4, 7, 10, 13, 17, 21, 24}	7	{1, 3, 5, 7, 9, 12, 14, 16, 18, 20, 22, 24}	11
9	{1, 4, 7, 10, 13, 17, 21, 24}	7	{1, 3, 5, 7, 9, 12, 14, 16, 18, 20, 22, 24}	11
10	{1, 4, 7, 10, 13, 17, 21, 24}	7	{1, 3, 5, 7, 9, 12, 14, 16, 18, 20, 22, 24}	11

Table 5.8 Effects of setup/ordering cost rates on M^* .

Effects of Setup/ordering Cost Rates				
$A_1=40$			$S=200$	
ρ	The Shortest Path	M^*	The Shortest Path	M^*
0	{1, 6, 11, 15, 19, 24}	5	{1, 3, 5, 7, 9, 12, 14, 16, 18, 20, 22, 24}	11
0.25	{1, 6, 11, 15, 19, 24}	5	{1, 4, 8, 12, 15, 18, 21, 24}	7
0.5	{1, 6, 11, 15, 19, 24}	5	{1, 5, 9, 13, 17, 20, 24}	6
0.75	{1, 7, 13, 18, 24}	4	{1, 6, 11, 15, 19, 24}	5
1	{1, 7, 13, 18, 24}	4	{1, 7, 13, 18, 24}	4
2	{1, 7, 13, 18, 24}	4	{1, 9, 16, 24}	3
3	{1, 7, 13, 18, 24}	4	{1, 9, 16, 24}	3
4	{1, 7, 13, 18, 24}	4	{1, 13, 24}	2
5	{1, 9, 16, 24}	3	{1, 13, 24}	2
6	{1, 9, 16, 24}	3	{1, 13, 24}	2
7	{1, 9, 16, 24}	3	{1, 13, 24}	2
8	{1, 9, 16, 24}	3	{1, 13, 24}	2
9	{1, 9, 16, 24}	3	{1, 13, 24}	2
10	{1, 9, 16, 24}	3	{1, 13, 24}	2

Figure 5.8 Effects of holding cost rates on M^* .

Figure 5.9 Effects of Setup/ordering cost rates on M^* .

Effects of System Parameters

Figure 5.10 Effects of systems parameters on the global minimum TOC.

Observation 5.2. The global minimum TOC increases in the system parameters of the multi-stage supply chain.

From Figure 5.10, the fixed transportation cost rate A_s affects the global minimum TOC the most, followed by A_4 , S , h_1 , h_3 , h_4 , A_1 , and h_s . Therefore, changes in holding cost and ordering/setup cost rates affect both the optimal number of subplans or production cycles M^* and the global minimum TOC of the integrated supply chain.

In practice, the production rate p can be regulated according to the overall sales of the finished products. To study the effects of p on the optimal number of subplans or production cycles M^* and the global minimum TOC in the numerical example, we set the value of p from 341 to 4900. The computational results are provided in Figures 5.9 and 5.10. The following insights can be drawn:

Observation 5.3: M^* is a constant value five when $p \in [700, 4900]$. The global minimum TOC decreases as the value of p increases. However, the difference in the global minimum TOC is only 2.41 percent, which is not significant.

Figure 5.11 Effects of production rate on M^* .

Figure 5.12 Effects of production rate on the global minimum TOC.

From Figures 5.11 and 5.12, the production rate p does not affect both the value of M^* and the global minimum TOC considerably. However, the higher production capacity leads to a significant cost increase to the manufacturer due to new equipment purchases and more labour inputs (Chopra & Meindl, 2016).

The sensitivity analysis shows how the system parameters affect the number of the optimal subplans or production cycles M^* and the global minimum TOC C . Decision makers should consider these effects on the optimal production-inventory-distribution policy for the integrated multi-stage supply chain.

5.5.2 Demonstrative example of products over their life cycles

Another practical application of time-varying demand is on a product over its life cycle. An understanding of a product life cycle is important in current MNCs for launching a

new product (Seifert & Langenberg, 2011). A typical product life cycle can be divided into four discrete stages: introduction, growth, maturity and decline (Aitken *et al.*, 2003). The customer demand starts to pick up during the introduction and growth stages due to the innovative technology of the new product. On maturity, the customer demand remains constant for a period of time until a new innovative technology launches in the market to replace the existing product. The customer demand for this existing product then starts to decline slowly. In a competitive market, MNCs need to constantly evaluate their supply chains to accommodate the impact of the product life cycle to minimize TOCs. The following numerical example shows the application on a product over its life cycle.

Example 5.2: The planning horizon is three years (or 36 months) for a product. The product demand over its life cycle is shown in Figure 5.11. $D_f(t) = D_{0f} + \lambda_f t$ is the demand rate function in phases f ($f = 1$: Introduction and Growth, 2: Maturity, 3: Decline.), where $D_{01}=120$ units/month, $D_{02}= D_{03}=360$ units/month and $\lambda_f = \{20, 0, -10\}$ units/month². The interval of the shipments $L = 1$ month. The sets of shipment time points for demand phases 1, 2, 3 are $\omega_1 = \{1, 2, \dots, 11\}$, $\omega_2 = \{12, 14, \dots, 23\}$, and $\omega_3 = \{24, 26, \dots, 35\}$, respectively. Let $p = 400$ units/month, $r = 2$, $h_1 = \$0.004/\text{unit/month}$, $h_3 = \$0.05/\text{unit/month}$, $h_s = \$0.02/\text{unit/month}$, $h_4 = \$0.08/\text{unit/month}$. $A_1 = \$50/\text{order}$, $A_4 = \$75/\text{order}$, $S = \$300/\text{setup}$, $A_s = \$100/\text{trip}$, and $L_s = 0.1$ month.

Table 5.9 The optimal production-inventory-distribution policy of the product over its life cycle.

The result of the predecessor field of the shortest path is: $\pi=(0, \text{NaN}, 1, 1, 1, 1,$
 $1, 1, 1, \underline{1}, 1, 1, 1, 7, 8, 8, 8, 9, 9, 9, 9, \underline{10}, 10, 10, 10, 10, 11, 18, 20, 20, 20, 20, 20, 20,$

20, 22}. So, the shortest path is {1, 10, 22, 36}. The computational results are provided in Table 5.10.

From the above two numerical examples, the computational results validate the feasibility and show potential benefits of the proposed model. The sensitivity analysis is conducted to obtain managerial insights to assist decision makers in improving supply chain performance through a joint decision making process.

Table 5.10 Sparse matrix of the product over its life cycle.

Vertex	V ₀	V ₁	V ₂	V ₃	V ₄	V ₅	V ₆	V ₇	V ₈	V ₉	V ₁₀	V ₁₁	V ₁₂	V ₁₃	V ₁₄	V ₁₅	V ₁₆	V ₁₇	V ₁₈	V ₁₉	V ₂₀	V ₂₁	V ₂₂	V ₂₃	V ₂₄	V ₂₅	V ₂₆	V ₂₇	V ₂₈	V ₂₉	V ₃₀	V ₃₁	V ₃₂	V ₃₃	V ₃₄	V ₃₅
V ₀	0	-	722.56	919.83	1125.52	1340.33	1564.8	1799.36	2044.3	2299.80	2565.9	2842.53	3126.65	3415.16	3708.06	4005.35	4307.03	4613.11	4923.58	5238.44	5557.69	5881.33	6209.37	6541.80	6876.46	7211.20	7546.10	7881.22	8216.57	8552.16	8887.93	9223.84	9559.75	9895.56	10231.09	10566.14
V ₁	0	0	-	725.47	924.50	1132	1348.53	1574.51	1810.25	2055.90	2311.53	2577.05	2849.74	3126.81	3408.28	3694.15	3984.4	4279.05	4578.08	4881.51	5189.34	5501.55	5818.16	6139.15	6462.55	6786.33	7110.60	7435.39	7760.75	8086.65	8413.06	8739.91	9067.10	9394.49	9721.92	10049.19
V ₂	0	0	0	-	728.4	929.09	1138.2	1356.18	1583.32	1819.78	2065.62	2320.77	2582.78	2849.18	3119.98	3395.17	3674.75	3958.72	4247.08	4539.84	4836.99	5138.53	5444.46	5754.78	6067.65	6381.21	6695.54	7010.70	7326.71	7643.57	7961.24	8279.64	8598.67	8918.21	9238.07	9558.08
V ₃	0	0	0	0	-	731.37	933.60	1144.14	1363.28	1591.21	1827.96	2073.46	2325.55	2582.04	2842.91	3108.18	3377.85	3651.90	3901.98	4213.18	4500.41	4792.04	5088.05	5388.46	5691.54	5995.60	6300.70	6606.90	6914.24	7222.70	7532.23	7842.78	8154.24	8466.48	8779.32	9092.58
V ₄	0	0	0	0	0	-	734.36	938.03	1149.8	1369.83	1598.18	1834.78	2077.71	2325.03	2576.75	2832.86	3093.36	3358.25	3627.53	3901.21	4179.28	4461.74	4748.59	5039.83	5333.88	5629.16	5925.73	6223.67	6522.99	6823.68	7125.71	7429.01	7733.47	8038.96	8345.31	8652.33
V ₅	0	0	0	0	0	0	-	737.39	942.39	1155.18	1375.83	1604.25	1838.78	2077.69	2321.00	2568.71	2820.8	3077.29	3338.16	3603.43	3873.10	4147.15	4425.60	4708.43	4994.20	5281.41	5570.17	5860.51	6152.48	6446.05	6741.19	7037.83	7335.87	7635.17	7935.57	8236.87
V ₆	0	0	0	0	0	0	0	-	740.46	946.67	1160.3	1381.28	1608.16	1839.43	2075.09	2315.14	2559.58	2808.42	3061.65	3319.27	3581.28	3847.68	4118.48	4393.67	4671.89	4951.77	5233.40	5516.85	5802.11	6089.21	6378.08	6668.67	6960.86	7254.53	7549.51	7845.61
V ₇	0	0	0	0	0	0	0	0	-	743.55	950.87	1165.14	1385.13	1609.5	1838.27	2071.42	2308.97	2550.92	2797.25	3047.98	3303.09	3562.60	3826.51	4094.80	4366.22	4639.50	4914.72	5191.94	5471.17	5752.42	6035.65	6320.78	6607.70	6896.30	7186.40	7477.80
V ₈	0	0	0	0	0	0	0	0	0	-	746.68	954.99	1168.83	1387.07	1609.70	1836.72	2068.13	2303.93	2544.13	2788.72	3037.70	3291.07	3548.83	3810.99	4076.36	4343.75	4613.26	4884.94	5158.80	5434.85	5713.05	5993.32	6275.55	6559.63	6845.38	7132.61

5.6 Summary

A new production-inventory-distribution model for an integrated multi-stage supply chain with time-varying demand is studied in this chapter. The objective of the part of the thesis is to devise the optimal production-inventory-distribution policy to minimize the TOC incurred by the supply chain parties. The model generalizes the customer time-varying demand to multiple phases up to an arbitrary integer. It is formulated as an MINLP optimization problem. According to the scope of the research in Section 1.7, three supply chain coordination and performance optimization problems with various demand patterns are studied in Chapters 3, 4 and 5. The following chapter presents the conclusions of this dissertation with some remarks.

CHAPTER 6

CONCLUSIONS

The objective of this dissertation is to characterise production-inventory-distribution coordination and performance optimization by studying integrated multi-stage supply chains with various customer demand patterns. JCIR and SCOR are adopted as the coordination mechanism and the framework to obtain the optimal operational policy to minimize the TOC. By using the using rigorous analytical approaches, three types of problem are studied: an integrated multi-stage supply chain with constant demand, a SCOR-based ACM for an integrated supply chain with constant demand and an integrated multi-stage supply chain with time-varying demand. This chapter presents the conclusions of the dissertation with some remarks.

6.1 Contributions

The contributions of this dissertation are summarized as follows:

1. Most of the models in the existing literature (Bushuev *et al.*, 2015; Glock, 2012) have studied production-inventory-distribution coordination and performance optimization problems with only two-stage supply chains. This dissertation considers integrated multi-stage supply chains that include raw material supply, production, transportation and distribution. It represents a more practical and expanded supply chain with full consideration of the supply chain's operational costs.

2. This thesis provides a theoretical contribution to supply chain coordination by proposing a systematic approach to build and analyse an ACM. This approach combines the SCOR model with the IIC policy and improves the coordination from the strategic to the operational levels among supply chain parties. The SCOR-based ACM overcomes the challenge that is represented by the difficulty in quantifying the coordination effects on supply chain performance after the implementation of such models like SCOR.
3. This thesis advances the theory on the SCOR models by linking the operational costs up to the tactical and strategic levels. The focus of the research is the total supply chain's operational cost. The costs addressed in the analytical models of the thesis are for the operational level. They are a very important key performance indicators in the SCOR models. The cost addressed in the SCOR models are not only for the operational level, but also for the tactical and strategic levels. This demonstrates the importance of the SCOR model in the thesis. It has lifted the thesis up to another height.
4. Only limited research has addressed integrated multi-stage supply chains with time-varying demand in the existing literature (Arshinder *et al.*, 2008; Glock, 2012; Hwang *et al.*, 2013; Kaminsky & Simchi-levi, 2003; Pahl & Voss, 2014; Zhao *et al.*, 2016b). The customer demand of integrated multi-stage supply chains is extended from constant to time-varying demand in this thesis. The time-varying demand is generalized to multiple phases up to an arbitrary integer.
5. By adopting the JCIR and SCOR as the coordination mechanism and the framework in the research, three analytical SCC models are built to obtain the optimal operational policies by minimizing the TOCs. In Chapters 3 and 4, the TOC reaches its global minimum after the integrality constraints are relaxed.

The impact of various system parameters on both of the variables of the optimal relaxed IIC policy and the global minimum TOC is analysed in multi-stage supply chains.

6. The conventional assumptions that infinite production rates and holding cost rates increase as materials/products flow down a supply chain, are among the SCC and performance optimization problems in the existing literature (Glock, 2012; Kim & Glock, 2013). The research relaxes the first assumption in the three models and relaxes the second assumption in models in Chapters 4 and 5.
7. This thesis provides an insightful analysis of the production-inventory-distribution coordination and performance optimization problems for integrated multi-stage supply chains in centralized settings. The obtained insights of the impact of the JCIR and SCOR model can assist decision makers in improving supply chain performance in real-life cases.
8. This dissertation includes an extensive literature review of studies that address the production-inventory-distribution coordination for integrated supply chains with constant and time-varying demand. Thus, a research gap is identified in the existing literature.

6.2 Managerial perspectives

Three new and more practical models are developed to examine supply chain performance. Having presented the contributions of the thesis, the usefulness of the developed methods are discussed from managerial perspectives in the following sections.

6.2.1 Integrated multi-stage supply chain with constant demand

We begin with a study of the production-inventory-distribution coordination of an integrated multi-stage supply chain with constant demand. The results have important managerial implications for practitioners. First, the research shows that the devised optimal IIC policy is able to assist decision makers in addressing inventory coordination problems among the supply chain parties at the tactical and operational levels. The TOC function of the integrated four-stage supply chain is convex in the continuous variables of the relaxed optimal IIC policy. Under the optimal IIC policy, the integer ratios of the inventory replenishment quantities are highly dependent on the holding, setup/ordering and fixed shipping cost rates that are associated with supply chain stages, as well as the ratio of the production rate to the demand rate. Thus, TOC is strongly related to the supply chain system parameters and decision variables of an IIC policy. Second, the study shows that TOC is the lower bound of the global minimum if a company implements the JIT manufacturing. This is in line with the JIT philosophy, which generally seeks to eliminate waste in all aspects of a supply chain (Myerson, 2012). JIT also promotes smaller lot sizes in a supply chain. To do so, the setup/ordering cost at each supply chain stage must be reduced. However, this study shows that it is difficult to integrate RMW into a multi-stage supply chain in a JIT manufacturing environment. In a practical situation, this finding implies that a supply chain owner needs to ensure that the raw material supply is always ready for production when the JIT manufacturing is implemented (Jaber *et al.*, 2004). Third, the basic replenishment quantity is only affected by the parameters of the downstream stages, *i.e.*, DC, transportation and FPW. This finding implies that the supply chain owner can determine the optimal basic replenishment quantity without considering the parameters of the upstream stages. Finally, the numerical examples show how the system parameters affect the variables of

the relaxed optimal IIC policy and the global minimum TOC. The integer approximation scheme of the IIC policy is developed to refine the optimal solution to a workable industrial solution without much deviation from the global minimum TOC. This also shows the feasibility of implementing the IIC policy in practice for an integrated multi-stage supply chain.

6.2.2 SCOR-based analytical coordination model for an integrated supply chain

To make our model practical, the SCOR model is adopted to support the MNC in the choosing and refining of the level 1 metrics of *supply chain management cost* at the strategic level. To address inventory replenishment coordination problems at the tactical and operational levels, we propose and find an optimal IIC policy to minimize the TOC.

The results of the second part of the study have important managerial implications for practitioners. First, this proposed systematic approach of combining the SCOR model and IIC policy is able to provide guidelines for industrial players to design and build their own ACMs (Huang *et al.*, 2005). It addresses the SCC problems from the strategic to the operational levels (Ntabe *et al.*, 2015). It can also assist decision makers in designing and re-configuring their supply chains to achieve the desired performance. It should be noted that this research methodology is not designed with these metrics in mind and that its performance could be measured with others. In other words, the same methodology can be used if a company decides to either remove or add metrics at the strategic level. Second, the optimal TOC is obtained and is demonstrated to reach its global minimum after the integer constraint is relaxed. An integer approximation scheme is developed to refine the real-number solution to a workable industrial solution without much deviation from the global minimum TOC. Third, the integers of the optimal IIC policy are highly dependent on the cost metrics of the SCOR

model, as well as the ratio of the production rate to the demand rate. Finally, the computational results on the TOC savings show the potential benefits of the proposed systematic approach.

6.2.3 Integrated multi-stage supply chain with time-varying demand

In the last part of the research, time-varying demand, which can reflect the demand for most products in practical situations, is considered. The customer time-varying demand is generalized for multiple phases up to an arbitrary integer. The model is formulated as an MINLP optimization problem, which can be represented as a WDAG. A polynomial algorithm is proposed to devise the optimal production-inventory-distribution policy and compute the global minimum TOC. Furthermore, it is demonstrated that TOC is strictly convex in the number of production cycles where the customer demand is constant.

The proposed algorithm performs well in the numerical examples with the sensitivity analysis. The computational results demonstrate that our model is applicable to both seasonal products and products over their life cycles. However, the industrial applications are not only limited to these two types.

6.3 Limitation of the research

The research is a preliminary study on the production-inventory-distribution coordination and performance optimization for integrated multi-stage supply chains in centralized settings. It extended the past research in several ways with contributions to the theory and practice of SCM. However, there are inevitably some limitations which are discussed as follows:

1. The three multi-stage integrated supply chain problems studied in this dissertation are in centralized settings. The theoretical solutions to these problems are to appoint a central decision maker, whom each supplier chain party has to share all relevant information. The resulted planning task is rather complex because the information about the future is uncertain. In addition, different conflicting objectives, *e.g.*, strategic objectives and service level should be considered at the same time. Thus this centralised coordination approach is practically very difficult to achieve (Egri, 2008; Kovacs *et al.*, 2013; Pibernik & Sucky, 2007).
2. The deterministic models that are studied in this dissertation can be used as tools for strategic and tactical planning in various practical situations for supply chain performance evaluation. In practice, information about some system parameters is available based on forecast (Wang & Disney, 2016). For example, the inventory holding cost rates may not be constant over time due to the market inflation. The changes in the rates result difference in supply chain performance.
3. This research is motivated by SCC problems in a pharmaceutical MNC. To achieve the research goals of this preliminary study, the original clinical trial supply chain is reduced from seven stages to four stages by combining the supply chain stages with similar process functions. A real life supply chain is complex because it commonly deals with multi-player with multi-product setting (Glock, 2012; Pahl & Voss, 2014).
4. SCOR is used to synchronize the supply chain and we adopt the SCOR model to synchronize the supply chain from the cost perspective only in Chapter 4. Since the thesis only focuses on minimizing the cost and all resources are arranged from the cost perspective, the performance of the other four dimensions may

deteriorate. Therefore, the other four dimensions *Agility*, *Assets*, *Reliability* and *Responsiveness* in the SCOR model are not covered in this research.

5. The benefits of implementing the proposed systematic approach by combining the SCOR model and the IIC policy are studied in Chapter 4. It is arguable that the IIC policy alone may have the same impact on supply chain performance if only *supply chain management cost* is considered. The desired performance can easily be achieved without the SCOR model. In a practical situation, there is considerable conflict between the top-down strategy decomposition and the bottom-up implementation among the supply chain parties without the application of the SCOR model (Kocaoglu *et al.*, 2013; Ntabe *et al.*, 2015). If the supply chain parties have different priorities in their strategies, the coordination at the tactical and operational levels is even worse. Supply chain integration is almost impossible, and thus the performance is degraded. According to the annual report (GlaxoSmithKline, 2013, 2014), the MNC fully recognized the importance of SCC from the strategic to the operational levels. There are several restructuring programmes that are able to improve coordination to achieve an integrated multi-stage supply chain. These programmes helped the MNC deliver incremental savings of £400 million in 2013. The decision makers aimed to deliver a total annual savings of £3.9 billion by 2016. We believe that they can deliver more incremental savings through our proposed systematic approach because the SCOR-based ACM with the IIC policy is user-friendly, and this can lead to more efficient and effective SCM. However, to fully realize the benefits of our proposed systematic approach, the MNC must take steps to successfully implement the system (Ntabe *et al.*, 2015).

6. The fixed shipping cost and the transient inventory cost are considered in three proposed models that are studied in this thesis. . In a real world, there are constraints to transport, such as a limited number of available vehicles and limited transport vehicle capacities (Beck & Glock, 2016; Ben-Daya *et al.*, 2008; Ertogral, 2008).
7. Random numbers are assigned to all of the system parameters in numerical examples in Chapters 3 and 5. The data for the parameters in numerical examples in Chapter 4 are based on a case study in a pharmaceutical industry (Kannan *et al.*, 2013). It might be good to conduct the numerical experiments with actual industrial data in Chapters 3 and 5, although the analytical solutions are obtained in three proposed models. The insights drawn from the actual industrial data could better assist decision makers in improving supply chain performance. However, the actual data from the MNC for conducting numerical experiments are not used because of considerations for protecting confidentiality.
8. The research assumes no stock backlogging, zero product defective rate and no restricted service levels at different stages of supply chains. The models with these assumptions have been extensively studied in the existing literature (Glock, 2012). However, these might not be true in practice.

6.4 Future research

This research is motivated by SCC problems in a pharmaceutical MNC. By adopting the JCIR coordination mechanism and the SCOR framework, the integrated multi-stage supply chains with various demand patterns are studied in this dissertation. Possible extensions of the research can be summarized as follows.

1. Stochastic demand

The results of the research are dependent on the assumptions that demand is deterministic and suppliers have perfect information of the price-demand relationship. In most practical situations, demand is stochastic at the downstream stages of a supply chain. By considering the stochastic demand, we are able to study how supply chains respond quickly to unanticipated demand changes from customers (Nosoohi & Nookabadi, 2016; Schildbach & Morari, 2016).

2. Stochastic system parameters

In practice, uncertainty is the most common factor that needs to be considered in supply chains. Useful results and managerial insights might be drawn when we evaluate the impact of uncertainties from various supply chain system parameters (Chen *et al.*, 2000; Kim *et al.*, 2015).

3. Multi-player and multi-product supply chain

The research focuses on serial supply chains with a single product, which does not fully reflect the reality due to the complex nature of production-inventory-distribution systems in many practical situations. It is reasonable to develop a supply chain with multiple players and a multi-product setting (Bozorgi, 2016; Sarker & Diponegoro, 2009). This approach can provide more practical and insightful decision support for current MNCs.

4. SCOR model adoption

The SCOR model is adopted to synchronize the supply chain from the cost perspective only and all resources are arranged from the cost perspective in this thesis. However, the performance of the other four dimensions may deteriorate when our problems are considered in stochastic environments. These are left for future study. In addition, it is more comprehensive to incorporate *Plan*, *Return*

processes in level 1 and other functionality at lower levels, such as *Engineer-to-order* to ACM (Lima & Carpinetti, 2016; Ntabe *et al.*, 2015) in the research.

5. Different coordination mechanisms and frameworks

The JCIR coordination mechanism and the SCOR framework are adopted in this dissertation. Other types of mechanisms, such as revenue sharing and quantity flexibility can be used in the system to study supply chain performance (Biswas *et al.*, 2016; Lee & Whang, 1999).

6. Different constraints

Under real-word conditions, there are many constraints for supply chains. It would be interesting and useful to devise operations policies that include stock backlogging, a limited number of available vehicles, limited transport vehicle capacities, production and inventory, and restricted service levels at different stages of supply chains (Protopappa-Sieke *et al.*, 2016; Qu *et al.*, 2015).

REFERENCES

- Abdul-Jalbar, B., Colebrook, M., Dorta-Guerra, R., & Gutierrez, J. M. (2016). Centralized and decentralized inventory policies for a single-vendor two-buyer system with permissible delay in payments. *Computers & Operations Research*, 74, 187-195.
- Abdul-Jalbar, B., Gutierrez, J., Puerto, J., & Sicilia, J. (2003). Policies for inventory/distribution systems: The effect of centralization vs. decentralization. *International Journal of Production Economics*, 81-2, 281-293.
- Abdul-Jalbar, B., Gutierrez, J., & Sicilia, J. (2005). Integer-ratio policies for distribution/inventory systems. *International Journal of Production Economics*, 93-94, 407-415.
- Abdul-Jalbar, B., Gutierrez, J. M., & Sicilia, J. (2007). An integrated inventory model for the single-vendor two-buyer problem. *International Journal of Production Economics*, 108(1-2), 246-258.
- Abdul-Jalbar, B., Segerstedt, A., Sicilia, J., & Nilsson, A. (2010). A new heuristic to solve the one-warehouse N-retailer problem. *Computers & Operations Research*, 37(2), 265-272.
- Aitken, J., Childerhouse, P., & Towill, D. (2003). The impact of product life cycle on supply chain strategy. *International Journal of Production Economics*, 85(2), 127-140.
- Alamri, A. A., Harris, I., & Syntetos, A. A. (2016). Efficient inventory control for imperfect quality items. *European Journal of Operational Research*, 254(1), 92-104.

- Almeder, C., Klabjan, D., Traxler, R., & Almada-Lobo, B. (2015). Lead time considerations for the multi-level capacitated lot-sizing problem. *European Journal of Operational Research*, 241(3), 727-738.
- Andriolo, A., Battini, D., Grubbstrom, R. W., Persona, A., & Sgarbossa, F. (2014). A century of evolution from Harris's basic lot size model: Survey and research agenda. *International Journal of Production Economics*, 155, 16-38.
- Archetti, C., Bertazzi, L., & Speranza, M. G. (2014). Polynomial cases of the economic lot sizing problem with cost discounts. *European Journal of Operational Research*, 237(2), 519-527.
- Arshinder, S., Kanda, A., & Deshmukh, S. G. (2008). Supply chain coordination: Perspectives, empirical studies and research directions. *International Journal of Production Economics*, 115(2), 316-335.
- Asgari, N., Nikbakhsh, E., Hill, A., & Farahani, R. Z. (2016). Supply chain management 1982-2015: a review. *Ima Journal of Management Mathematics*, 27(3), 353-379.
- Atkins, D., Queyranne, M., & Sun, D. N. (1992). Lot sizing policies for finite production-rate assembly systems. *Operations Research*, 40(1), 126-141.
- Atkins, D., & Sun, D. (1995). 98-percent-effective lot-sizing for series inventory systems with backlogging. *Operations Research*, 43(2), 335-345.
- Axsater, S. (1997). Simple evaluation of echelon stock (R,Q) policies for two-level inventory systems. *Iie Transactions*, 29(8), 661-669.
- Axsäter, S. (2006). *Inventory control* New York : Springer.
- Baboli, A., Fondrevelle, J., Tavakkoli-Moghaddam, R., & Mehrabi, A. (2011). A replenishment policy based on joint optimization in a downstream pharmaceutical supply chain: centralized vs. decentralized replenishment.

- International Journal of Advanced Manufacturing Technology*, 57(1-4), 367-378.
- Banerjee, A. (1986). A joint economic-lot-size model for purchaser and vendor. *Decision Sciences*, 17(3), 292-311.
- Banerjee, A., & Kim, S. L. (1995). An integrated JIT inventory model. *International Journal of Operations & Production Management*, 15(9), 237-&.
- Beck, F. G., & Glock, C. H. (2016). The impact of batch shipments on the Economic Lot Scheduling Problem. *Computers & Industrial Engineering*, 96, 126-139.
- Ben-Daya, M., Darwish, M., & Ertogral, K. (2008). The joint economic lot sizing problem: Review and extensions. *European Journal of Operational Research*, 185(2), 726-742.
- Biswas, I., Avittathur, B., & Chatterjee, A. K. (2016). Impact of structure, market share and information asymmetry on supply contracts for a single supplier multiple buyer network. *European Journal of Operational Research*, 253(3), 593-601.
- Boissière, J., Frein, Y., & Rapine, C. (2008). Optimal stationary policies in a 3-stage serial production-distribution logistic chain facing constant and continuous demand. *European Journal of Operational Research*, 186(2), 608-619.
- Bolstorff, P. (2003). Everything's according to plan. *Transportation & Distribution*, 44(6), 28-29.
- Bonney, M., & Jaber, M. Y. (2011). Environmentally responsible inventory models: Non-classical models for a non-classical era. *International Journal of Production Economics*, 133(1), 43-53.
- Bozorgi, A. (2016). Multi-product inventory model for cold items with cost and emission consideration. *International Journal of Production Economics*, 176, 123-142.

- Braglia, M., & Zavanella, L. (2003). Modelling an industrial strategy for inventory management in supply chains: the 'Consignment Stock' case. *International Journal of Production Research*, 41(16), 3793-3808.
- Brahimi, N., Dauzere-Peres, S., Najid, N. M., & Nordli, A. (2006). Single item lot sizing problems. *European Journal of Operational Research*, 168(1), 1-16.
- Bushuev, M. A., Guiffrida, A., Jaber, M. Y., & Khan, M. (2015). A review of inventory lot sizing review papers. *Management Research Review*, 38(3), 283-298.
- Bylka, S. (1999). A dynamic model for the single-vendor, multi-buyer problem. *International Journal of Production Economics*, 59(1-3), 297-304.
- Bylka, S. (2013). Non-cooperative consignment stock strategies for management in supply chain. *International Journal of Production Economics*, 143(2), 424-433.
- Cachon, G. P., & Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management Science*, 46(8), 1032-1048.
- Cachon, G. P., & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Management Science*, 51(1), 30-44.
- Celebi, D. (2015). Inventory control in a centralized distribution network using genetic algorithms: A case study. *Computers & Industrial Engineering*, 87, 532-539.
- Chan, H. K., & Chan, F. T. S. (2010). A review of coordination studies in the context of supply chain dynamics. *International Journal of Production Research*, 48(10), 2793-2819.
- Chang, C.-T., & Chou, H.-C. (2013). A coordination system for seasonal demand problems in the supply chain. *Applied Mathematical Modelling*, 37(6), 3674-3686.

- Chang, H.-C. (2014). An analysis of production-inventory models with deteriorating items in a two-echelon supply chain. *Applied Mathematical Modelling*, 38(3), 1187-1191.
- Chen, F. (1998). Stationary policies in multi-echelon inventory systems with deterministic demand and backlogging. *Operations Research*, 46, S26-S34.
- Chen, F., Drezner, Z., Ryan, J. K., & Simchi-Levi, D. (2000). Quantifying the bullwhip effect in a simple supply chain: The impact of forecasting, lead times, and information. *Management Science*, 46(3), 436-443.
- Chen, F. R., Federgruen, A., & Zheng, Y. S. (2001). Coordination mechanisms for a distribution system with one supplier and multiple retailers. *Management Science*, 47(5), 693-708.
- Chen, J.-M., Lin, I. C., & Cheng, H.-L. (2010). Channel coordination under consignment and vendor-managed inventory in a distribution system. *Transportation Research Part E-Logistics and Transportation Review*, 46(6), 831-843.
- Chen, K. K., & Chang, C.-T. (2007). A seasonal demand inventory model with variable lead time and resource constraints. *Applied Mathematical Modelling*, 31(11), 2433-2445.
- Chen, X., Wang, X. J., & Chan, H. K. (2016). Channel coordination through subsidy contract design in the mobile phone industry. *International Journal of Production Economics*, 171, 97-104.
- Chen, Y., Mockus, L., Orcun, S., & Reklaitis, G. V. (2012). Simulation-optimization approach to clinical trial supply chain management with demand scenario forecast. *Computers & Chemical Engineering*, 40, 82-96.

- Chen, Y., Pekny, J. F., & Reklaitis, G. V. (2013). Integrated Planning and Optimization of Clinical Trial Supply Chain System with Risk Pooling. *Industrial & Engineering Chemistry Research*, 52(1), 152-165.
- Cheng, J. C. P., Law, K. H., Bjornsson, H., Jones, A., & Sriram, R. D. (2010). Modeling and monitoring of construction supply chains. *Advanced Engineering Informatics*, 24(4), 435-455.
- Chiadamrong, N., & Prasertwattana, K. (2006). A comparative study of supply chain models under the traditional centralized and coordinating policies with incentive schemes. *Computers & Industrial Engineering*, 50(4), 367-384.
- Cho, D. W., Lee, Y. H., Ahn, S. H., & Hwang, M. K. (2012). A framework for measuring the performance of service supply chain management. *Computers & Industrial Engineering*, 62(3), 801-818.
- Chopra, S., & Meindl, P. (2013). *Supply chain management : strategy, planning, and operation* (5th ed.): Upper Saddle River, N.J. : Pearson.
- Chopra, S., & Meindl, P. (2016). *Supply chain management : strategy, planning, and operation* (Sixth ed.): Boston : Pearson, 2016.
- Christopher, M. (2011). *Logistics & supply chain management* (4th ed.): Harlow, England ; New York : Financial Times Prentice Hall.
- Corbett, C. J. (2001). Stochastic inventory systems in a supply chain with asymmetric information: Cycle stocks, safety stocks, and consignment stock. *Operations Research*, 49(4), 487-500.
- Cormen, T. H., Leiserson, C. E., Rivest, R. L., & Stein, C. (2009). *Introduction to algorithms* (3rd ed.): The MIT Press.
- Council of Supply Chain Management Professionals. CSCMP Supply Chain Management, from <https://cscmp.org/>

- Dario Pacciarelli, Carlo Meloni, & Pranzo, M. (2011). Models and Methods for Production Scheduling in the Pharmaceutical Industry. In Kempf Karl G, Keskinocak Pinar & U. Reha (Eds.), *Planning Production and Inventories in the Extended Enterprise* (Vol. 2, pp. Pages 429-459). New York: Springer-Verlag
- De Souza, R., Goh, M., Sundarakani, B., Wai, W. T., Toh, K., & Yong, W. (2011). Return on investment calculator for RFID ecosystem of high tech company. *Computers in Industry*, 62(8-9), 820-829.
- Devos Ganga, G. M., & Ribeiro Carpinetti, L. C. (2011). A fuzzy logic approach to supply chain performance management. *International Journal of Production Economics*, 134(1), 177-187.
- Diabat, A. (2014). Hybrid algorithm for a vendor managed inventory system in a two-echelon supply chain. *European Journal of Operational Research*, 238(1), 114-121.
- Diponegoro, A., & Sarker, B. R. (2007). Operations policy for a supply chain system with fixed-interval delivery and linear demand. *Journal of the Operational Research Society*, 58(7), 901-910.
- Duan, Q. L., & Liao, T. W. (2013). Optimization of replenishment policies for decentralized and centralized capacitated supply chains under various demands. *International Journal of Production Economics*, 142(1), 194-204.
- Eben-Chaïme, M. (2004). The effect of discreteness in vendor-buyer relationships. *IIE Transactions*, 36(6), 583-589.
- Egri, P. e. (2008). *Coordination in Production Networks*. Ph.D, Eötvös Loránd University.

- Ertogral, K. (2008). Multi-item single source ordering problem with transportation cost: A Lagrangian decomposition approach. *European Journal of Operational Research*, 191(1), 156-165.
- Fattahi, M., Mahootchi, M., Husseini, S. M. M., Keyvanshokoo, E., & Alborzi, F. (2015). Investigating replenishment policies for centralised and decentralised supply chains using stochastic programming approach. *International Journal of Production Research*, 53(1), 41-69.
- Friedli, T., Goetzfried, M., & Basu, P. (2010). Analysis of the Implementation of Total Productive Maintenance, Total Quality Management, and Just-In-Time in Pharmaceutical Manufacturing. *Journal of Pharmaceutical Innovation*, 5(4), 181-192.
- Fu, D. F., Ionescu, C. M., Aghezzaf, E. H., & De Keyser, R. (2014). Decentralized and centralized model predictive control to reduce the bullwhip effect in supply chain management. *Computers & Industrial Engineering*, 73, 21-31.
- Geunes, J., Romeijn, H. E., & van den Heuvel, W. (2016). Improving the efficiency of decentralized supply chains with fixed ordering costs. *European Journal of Operational Research*, 252(3), 815-828.
- Giri, B. C., Bardhan, S., & Maiti, T. (2016). Coordinating a three-layer supply chain with uncertain demand and random yield. *International Journal of Production Research*, 54(8), 2499-2518.
- GlaxoSmithKline. (2013). GSK Annual Report 2013. United Kingdom.
- GlaxoSmithKline. (2014). GSK Annual Report 2014. United Kingdom.
- Glock, C. H. (2012). The joint economic lot size problem: A review. *International Journal of Production Economics*, 135(2), 671-686.

- Gorji, M. H., Setak, M., & Karimi, H. (2014). Optimizing inventory decisions in a two-level supply chain with order quantity constraints. *Applied Mathematical Modelling*, 38(3), 814-827.
- Govindu, R., & Chinnam, R. B. (2007). MASCF: A generic process-centered methodological framework for analysis and design of multi-agent supply chain systems. *Computers & Industrial Engineering*, 53(4), 584-609.
- Goyal, S. K. (1976). An integrated inventory model for a single supplier-single customer problem. *International Journal of Production Research*, 14, 107-111.
- Goyal, S. K., & Giri, B. C. (2003). The production-inventory problem of a product with time varying demand, production and deterioration rates. *European Journal of Operational Research*, 147(3), 549-557.
- Goyal, S. K., & Gupta, Y. P. (1989). Integrated inventory models - the buyer vendor coordination. *European Journal of Operational Research*, 41(3), 261-269.
- Gumus, A. T., Guneri, A. F., & Ulengin, F. (2010). A new methodology for multi-echelon inventory management in stochastic and neuro-fuzzy environments. *International Journal of Production Economics*, 128(1), 248-260.
- Gunasekaran, A. (1999). Just-in-time purchasing: An investigation for research and applications. *International Journal of Production Economics*, 59(1-3), 77-84.
- Hall, N. G., & Posner, M. E. (2001). Generating experimental data for computational testing with machine scheduling applications. *Operations Research*, 49(6), 854-865.
- Hariga, M. (1996). Optimal EOQ models for deteriorating items with time-varying demand. *Journal of the Operational Research Society*, 47(10), 1228-1246.
- Heath, M. T. (2002). *Scientific computing : an introductory survey* (2nd ed.): New York : McGraw-Hill.

- Hill, R. M. (1995). Inventory models for increasing demand followed by level demand. *Journal of the Operational Research Society*, 46(10), 1250-1259.
- Hill, R. M. (1997). The single-vendor single-buyer integrated production-inventory model with a generalised policy. *European Journal of Operational Research*, 97, 493-499.
- Hill, R. M. (1999). The optimal production and shipment policy for the single-vendor singlebuyer integrated production-inventory problem. *International Journal of Production Research*, 37(11), 2463-2475.
- Hill, R. M., & Omar, M. (2006). Another look at the single-vendor single-buyer integrated production-inventory problem. *International Journal of Production Research*, 44(4), 791-800.
- Hoque, M. A. (2011). An optimal solution technique to the single-vendor multi-buyer integrated inventory supply chain by incorporating some realistic factors. *European Journal of Operational Research*, 215(1), 80-88.
- Huan, S. H., Sheoran, S. K., & Wang, G. (2004). A review and analysis of supply chain operations reference (SCOR) model. *Supply Chain Management-an International Journal*, 9(1), 23-29.
- Huang, B., & Ye, Z. (2010). The effects of lumpy demand and shipment size constraint: A response to "Revisit the note on supply chain integration in vendor-managed inventory". *Decision Support Systems*, 48(2), 421-425.
- Huang, C.-K., Tsai, D.-M., Wu, J.-C., & Chung, K.-J. (2010). An integrated vendor-buyer inventory model with order-processing cost reduction and permissible delay in payments. *European Journal of Operational Research*, 202(2), 473-478.

- Huang, S. H., Sheoran, S. K., & Keskar, H. (2005). Computer-assisted supply chain configuration based on supply chain operations reference (SCOR) model. *Computers & Industrial Engineering*, 48(2), 377-394.
- Hvolby, H.-H., & Trienekens, J. H. (2010). Challenges in business systems integration. *Computers in Industry*, 61(9), 808-812.
- Hwang, H.-C., Ahn, H.-S., & Kaminsky, P. (2013). Basis paths and a polynomial algorithm for the multistage production-capacitated lot-sizing problem. *Operations Research*, 61(2), 469-482.
- Hwang, Y.-D., Lin, Y.-C., & Jr, J. L. (2008). The performance evaluation of SCOR sourcing process—The case study of Taiwan's TFT-LCD industry. *International Journal of Production Economics*, 115, 411-423.
- Intel. (2002). SCOR experience at Intel: Methods and tools for supply chain management.
- Jaber, M. Y., & Goyal, S. K. (2008). Coordinating a three-level supply chain with multiple suppliers, a vendor and multiple buyers. *International Journal of Production Economics*, 116(1), 95-103.
- Jaber, M. Y., & Goyal, S. K. (2009). A basic model for co-ordinating a four-level supply chain of a product with a vendor, multiple buyers and tier-1 and tier-2 suppliers. *International Journal of Production Research*, 47(13), 3691-3704.
- Jaber, M. Y., Nuwayhid, R. Y., & Rosen, M. A. (2004). Price-driven economic order systems from a thermodynamic point of view. *International Journal of Production Research*, 42(24), 5167-5184.
- Jha, J. K., & Shanker, K. (2013). Single-vendor multi-buyer integrated production-inventory model with controllable lead time and service level constraints. *Applied Mathematical Modelling*, 37(4), 1753-1767.

- Kaminsky, P., & Simchi-levi, D. (2003). Production and distribution lot sizing in a two stage supply chain. *IIE Transactions*, 35, 1065-1075.
- Kannan, G., Grigore, M. C., Devika, K., & Senthilkumar, A. (2013). An analysis of the general benefits of a centralised VMI system based on the EOQ model. *International Journal of Production Research*, 51(1), 172-188.
- Kempf, K. G., Keskinocak, P., & Uzsoy, R. (2011). *Planning Production and Inventories in the Extended Enterprise. [electronic resource] : A State of the Art Handbook, Volume 1 Chapter. 17*: Boston, MA : Springer Science+Business Media, LLC, 2011.
- Kevan, T. (2005). Modeling the future. *Frontline Solutions*, 6(1), 22-24.
- Kim, G., Wu, K., & Huang, E. (2015). Optimal inventory control in a multi-period newsvendor problem with non-stationary demand. *Advanced Engineering Informatics*, 29(1), 139-145.
- Kim, T., & Glock, C. H. (2013). A multi-stage joint economic lot size model with lead time penalty costs. *Computers & Industrial Engineering*, 66(1), 133-146.
- Knackstedt, R., Stein, A., & Becker, J. (2009). *Modeling of integrated production and services with the SCOR model - Existing approaches and development perspectives*. Paper presented at the 9th International Conference on economy computer science, Vienna.
- Kocaoglu, B., Gulsun, B., & Tanyas, M. (2013). A SCOR based approach for measuring a benchmarkable supply chain performance. *Journal of Intelligent Manufacturing*, 24(1), 113-132.
- Kovacs, A., Egri, P., Kis, T., & Vancza, J. (2013). Inventory control in supply chains: Alternative approaches to a two-stage lot-sizing problem. *International Journal of Production Economics*, 143(2), 385-394.

- Kurien, G. P., & Qureshi, M. N. (2015). Analysis and measurement of supply chain flexibility. *International Journal of Logistics Systems and Management*, 21(1), 70-91.
- Lai, K. H., Ngai, E. W. T., & Cheng, T. C. E. (2002). Measures for evaluating supply chain performance in transport logistics. *Transportation Research Part E-Logistics and Transportation Review*, 38(6), 439-456.
- Lancioni, R. A. (2000). New developments in supply chain management for the millennium. *Industrial Marketing Management*, 29(1), 1-6.
- Lee, C. C., & Chu, W. H. J. (2005). Who should control inventory in a supply chain? *European Journal of Operational Research*, 164(1), 158-172.
- Lee, H., & Whang, S. J. (1999). Decentralized multi-echelon supply chains: Incentives and information. *Management Science*, 45(5), 633-640.
- Lee, H. L., & Billington, C. (1993). Material management in decentralized supply chains. *Operations Research*, 41(5), 835-847.
- Lee, H. L., Padmanabhan, V., & Whang, S. J. (1997). Information distortion in a supply chain: The bullwhip effect. *Management Science*, 43(4), 546-558.
- Lee, W. (2005). A joint economic lot size model for raw material ordering, manufacturing setup, and finished goods delivering. *Omega*, 33(2), 163-174.
- Li, L., Su, Q., & Chen, X. (2011). Ensuring supply chain quality performance through applying the SCOR model. *International Journal of Production Research*, 49(1), 33-57.
- Li, X., & Wang, Q. (2007). Coordination mechanisms of supply chain systems. *European Journal of Operational Research*, 179(1), 1-16.

- Lima, F. R., & Carpinetti, L. C. R. (2016). Combining SCOR (R) model and fuzzy TOPSIS for supplier evaluation and management. *International Journal of Production Economics*, 174, 128-141.
- Lisa M. Ellram, Wendy L. Tate, & Billington, C. (2004). Understanding and managing the services supply chain. *Journal of Supply Chain Management*, 40(4), 17-32.
- Liu, P., Huang, S. H., Mokasdar, A., Zhou, H., & Hou, L. (2014). The impact of additive manufacturing in the aircraft spare parts supply chain: supply chain operation reference (scor) model based analysis. *Production Planning & Control*, 25(13-14), 1169-1181.
- Liu, X., & Lian, Z. (2009). Cost-effective inventory control in a value-added manufacturing system. *European Journal of Operational Research*, 196(2), 534-543.
- Lo, W. Y., Tsai, C. H., & Li, R. K. (2002). Exact solution of inventory replenishment policy for a linear trend in demand - two-equation model. *International Journal of Production Economics*, 76(2), 111-120.
- Lockamy, A., & McCormack, K. (2004). Linking SCOR planning practices to supply chain performance - An exploratory study. *International Journal of Operations & Production Management*, 24(11-12), 1192-1218.
- Long, Q. Q. (2014). Distributed supply chain network modelling and simulation: integration of agent-based distributed simulation and improved SCOR model. *International Journal of Production Research*, 52(23), 6899-6917.
- Lu, L. (1995). A one-vendor multi-buyer integrated inventory model. *European Journal of Operational Research*, 81(2), 312-323.

- Lu, Y., Panetto, H., Ni, Y., & Gu, X. (2013). Ontology alignment for networked enterprise information system interoperability in supply chain environment. *International Journal of Computer Integrated Manufacturing*, 26(1-2), 140-151.
- Lubben, R. T. (1988). *Just-in-time manufacturing : an aggressive manufacturing strategy* New York : McGraw-Hill.
- Marquez, A. C., & Blanchar, C. (2006). A Decision Support System for evaluating operations investments in high-technology business. *Decision Support Systems*, 41(2), 472-487.
- Massonnet, G., Gayon, J. P., & Rapine, C. (2014). Approximation algorithms for deterministic continuous-review inventory lot-sizing problems with time-varying demand. *European Journal of Operational Research*, 234(3), 641-649.
- Medini, K., & Bourey, J. P. (2012). SCOR-based enterprise architecture methodology. *International Journal of Computer Integrated Manufacturing*, 25(7), 594-607.
- Ming, Y., Grabot, B., & Houé, R. (2014). A typology of the situations of cooperation in supply chains. *Computers & Industrial Engineering*, 67, 56-71.
- Morales, M. 1. D. R. (2000). *Optimization Problems in Supply Chain Management*. Ph.D, erasmus university.
- Mukhopadhyay, S. K., Dwivedy, J., & Kumar, A. (1998). Design and implementation of an integrated production planning system for a pharmaceutical manufacturing concern in India. *Production Planning & Control*, 9(4), 391-402.
- Myerson, P. (2012). *Lean supply chain and logistics management* New York : McGraw-Hill, .
- Nosoohi, I., & Nookabadi, A. S. (2016). Outsource planning through option contracts with demand and cost uncertainty. *European Journal of Operational Research*, 250(1), 131-142.

- Ntabe, E. N., LeBel, L., Munson, A. D., & Santa-Eulalia, L. A. (2015). A systematic literature review of the supply chain operations reference (SCOR) model application with special attention to environmental issues. *International Journal of Production Economics*, 169, 310-332.
- Okongwu, U., Lauras, M., Francois, J., & Deschamps, J. C. (2016). Impact of the integration of tactical supply chain planning determinants on performance. *Journal of Manufacturing Systems*, 38, 181-194.
- Omar, M., Sarker, R., & Othman, W. A. M. (2013). A just-in-time three-level integrated manufacturing system for linearly time-varying demand process. *Applied Mathematical Modelling*, 37(3), 1275-1281.
- Pahl, J., & Voss, S. (2014). Integrating deterioration and lifetime constraints in production and supply chain planning: A survey. *European Journal of Operational Research*, 238(3), 654-674.
- Pal, B., Sana, S. S., & Chaudhuri, K. (2012). A three layer multi-item production-inventory model for multiple suppliers and retailers. *Economic Modelling*, 29(6), 2704-2710.
- Pan, N.-H., Lee, M.-L., & Chen, S.-Q. (2011). Construction material supply chain process analysis and optimization. *Journal of Civil Engineering and Management*, 17(3), 357-370.
- Persson, F. (2011). SCOR template-A simulation based dynamic supply chain analysis tool. *International Journal of Production Economics*, 131(1), 288-294.
- Persson, F., & Araldi, M. (2009). The development of a dynamic supply chain analysis tool-Integration of SCOR and discrete event simulation. *International Journal of Production Economics*, 121(2), 574-583.

- Persson, F., Bengtsson, J., & Gustad, O. (2010). Construction Logistics Improvements Using the SCOR Model - Tornet Case. In B. Vallespir & T. Alix (Eds.), *Advances in Production Management Systems: New Challenges, New Approaches* (Vol. 338, pp. 211-218).
- Pibernik, R., & Sucky, E. (2007). An approach to inter-domain master planning in supply chains. *International Journal of Production Economics*, 108(1-2), 200-212.
- Protopappa-Sieke, M., Sieke, M. A., & Thonemann, U. W. (2016). Optimal two-period inventory allocation under multiple service level contracts. *European Journal of Operational Research*, 252(1), 145-155.
- Qu, T., Nie, D. X., Chen, X., Chen, X. D., Dai, Q. Y., & Huang, G. Q. (2015). Optimal configuration of cluster supply chains with augmented Lagrange coordination. *Computers & Industrial Engineering*, 84, 43-55.
- Rabelo, L., Eskandari, H., Shaalan, T., & Helal, M. (2007). Value chain analysis using hybrid simulation and AHP. *International Journal of Production Economics*, 105(2), 536-547.
- Rahdar, M., & Nookabadi, A. S. (2014). Coordination mechanism for a deteriorating item in a two-level supply chain system. *Applied Mathematical Modelling*, 38(11-12), 2884-2900.
- Rau, H., & OuYang, B. C. (2008). An optimal batch size for integrated production-inventory policy in a supply chain. *European Journal of Operational Research*, 185(2), 619-634.
- Roundy, R. (1985). 98%-effective integer-ratio lot-sizing for one-warehouse multi-retailer systems. *Management Science*, 31(11), 1416-1430.

- Sakka, O., Millet, P.-A., & Botta-Genoulaz, V. (2011). An ontological approach for strategic alignment: a supply chain operations reference case study. *International Journal of Computer Integrated Manufacturing*, 24(11), 1022-1037.
- Salcedo, C. A. G., Hernandez, A. I., Vilanova, R., & Cuartas, J. H. (2013). Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized Internal Model Control approaches. *European Journal of Operational Research*, 224(2), 261-272.
- Sana, S. S. (2010). A production-inventory model in an imperfect production process. *European Journal of Operational Research*, 200(2), 451-464.
- Sana, S. S., Acevedo Chedid, J., & Salas Navarro, K. (2014). A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items. *Applied Mathematics and Computation*, 229, 139-150.
- Sana, S. S., & Goyal, S. K. (2014). (Q, r, L) model for stochastic demand with lead-time dependent partial backlogging. *Annals of Operations Research*.
- Sarkar, B. (2013). A production-inventory model with probabilistic deterioration in two-echelon supply chain management. *Applied Mathematical Modelling*, 37(5), 3138-3151.
- Sarker, B. R., & Balan, C. V. (1999). Operations planning for a multi-stage kanban system. *European Journal of Operational Research*, 112(2), 284-303.
- Sarker, B. R., & Diponegoro, A. (2009). Optimal production plans and shipment schedules in a supply-chain system with multiple suppliers and multiple buyers. *European Journal of Operational Research*, 194(3), 753-773.

- Sarker, B. R., & Parija, G. R. (1996). Optimal batch size and raw material ordering policy for a production system with a fixed-interval, lumpy demand delivery system. *European Journal of Operational Research*, 89(3), 593-608.
- Sarmah, S. P., Acharya, D., & Goyal, S. K. (2006). Buyer vendor coordination models in supply chain management. *European Journal of Operational Research*, 175(1), 1-15.
- Schildbach, G., & Morari, M. (2016). Scenario-based model predictive control for multi-echelon supply chain management. *European Journal of Operational Research*, 252(2), 540-549.
- Seifert, R. W., & Langenberg, K. U. (2011). Managing business dynamics with adaptive supply chain portfolios. *European Journal of Operational Research*, 215(3), 551-562.
- Seliaman, M. E., & Ahmad, A. R. (2009). A generalized algebraic model for optimizing inventory decisions in a multi-stage complex supply chain. *Transportation Research Part E-Logistics and Transportation Review*, 45(3), 409-418.
- Sellitto, M. A., Pereira, G. M., Borchardt, M., da Silva, R. I., & Viegas, C. V. (2015). A SCOR-based model for supply chain performance measurement: application in the footwear industry. *International Journal of Production Research*, 53(16), 4917-4926.
- Shah, N. (2004). Pharmaceutical supply chains: key issues and strategies for optimisation. *Computers & Chemical Engineering*, 28(6-7), 929-941.
- Silver, E. A. (1979). Simple inventory replenishment decision rule for a linear trend in demand. *Journal of the Operational Research Society*, 30(1), 71-75.
- Silver, E. A., Pyke, D. F., & Peterson, R. (1998). *Inventory management and production planning and scheduling* (3rd ed.): New York : Wiley.

- Simchi-Levi, D., Kaminsky, P., & Simchi-Levi, E. (2008). *Designing and managing the supply chain : concepts, strategies, and case studies / David Simchi-Levi, Philip Kaminsky, Edith Simchi-Levi*: Boston : McGraw-Hill/Irwin, c2008. 3rd ed.
- Song, D.-P., Dong, J.-X., & Xu, J. (2014). Integrated inventory management and supplier base reduction in a supply chain with multiple uncertainties. *European Journal of Operational Research*, 232(3), 522-536.
- Strohhecker, J., Sibbel, R., & Dick, M. (2014). Integrating Kanban principles in a pharmaceutical campaign production system. *Production Planning & Control*, 25(15), 1247-1263.
- Supply Chain Council. (2010). Supply Chain Operations Reference (SCOR) model Overview - Version 10.0
- Teng, J.-T., Min, J., & Pan, Q. (2012). Economic order quantity model with trade credit financing for non-decreasing demand. *Omega-International Journal of Management Science*, 40(3), 328-335.
- Thunberg, M., & Persson, F. (2014). Using the SCOR model's performance measurements to improve construction logistics. *Production Planning & Control*, 25(13-14), 1065-1078.
- Valentini, G., & Zavanella, L. (2003). The consignment stock of inventories: industrial case and performance analysis. *International Journal of Production Economics*, 81-82, 215-224.
- Van Hoesel, S., Romeijn, H. E., Morales, D. R., & Wagelmans, A. P. M. (2005). Integrated lot sizing in serial supply chains with production capacities. *Management Science*, 51(11), 1706-1719.

- Ventura, J. A., Valdebenito, V. A., & Golany, B. (2013). A dynamic inventory model with supplier selection in a serial supply chain structure. *European Journal of Operational Research*, 230(2), 258-271.
- Verma, N. K., Chakraborty, A., & Chatterjee, A. K. (2014). Joint replenishment of multi retailer with variable replenishment cycle under VMI. *European Journal of Operational Research*, 233(3), 787-789.
- Vonderembse, M. A., Uppal, M., Huang, S. H., & Dismukes, J. P. (2006). Designing supply chains: Towards theory development. *International Journal of Production Economics*, 100(2), 223-238.
- Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. *Management Science*, 50, 1770-1774.
- Wang, G., Huang, S. H., & Dismukes, J. P. (2004a). Product-driven supply chain selection using integrated multi-criteria decision-making methodology. *International Journal of Production Economics*, 91(1), 1-15.
- Wang, Q. (2013). A periodic-review inventory control policy for a two-level supply chain with multiple retailers and stochastic demand. *European Journal of Operational Research*, 230(1), 53-62.
- Wang, W. Y. C., Chan, H. K., & Pauleen, D. J. (2010). Aligning business process reengineering in implementing global supply chain systems by the SCOR model. *International Journal of Production Research*, 48(19), 5647-5669.
- Wang, X., & Disney, S. M. (2016). The bullwhip effect: Progress, trends and directions. *European Journal of Operational Research*, 250(3), 691-701.
- Wang, Y., Wallace, S. W., Shen, B., & Choi, T.-M. (2015). Service supply chain management: A review of operational models. *European Journal of Operational Research*, 247(3), 685-698.

- Wang, Y. Z., Jiang, L., & Shen, Z. J. (2004b). Channel performance under consignment contract with revenue sharing. *Management Science*, 50(1), 34-47.
- Yi, H. Z., & Sarker, B. R. (2014). An operational consignment stock policy under normally distributed demand with controllable lead time and buyer's space limitation. *International Journal of Production Research*, 52(16), 4853-4875.
- Yildirim, Y., & Umit, B. (2006). Performance measurement in the value chain: manufacturing v. tourism. *International Journal of Productivity and Performance Management*, 55(5), 371-389.
- Yuan, X.-M., Low, J. M. W., & Tang, C. L. (2010). Roles of the airport and logistics services on the economic outcomes of an air cargo supply chain. *International Journal of Production Economics*, 127(2), 215-225.
- Zangoueinezhad, A., Azar, A., & Kazazi, A. (2011). Using SCOR model with fuzzy MCDM approach to assess competitiveness positioning of supply chains: focus on shipbuilding supply chains. *Maritime Policy & Management*, 38(1), 93-109.
- Zangwill, W. I. (1969). A backlogging model and a mutli-echelon model of a dynamic economic lot size production system-a network approach. *Management Science*, 15(9), 506-527.
- Zhang, M., Kucukyavuz, S., & Goel, S. (2014). A Branch-and-Cut Method for Dynamic Decision Making Under Joint Chance Constraints. *Management Science*, 60(5), 1317-1333.
- Zhang, W., & Reimann, M. (2014). Towards a multi-objective performance assessment and optimization model of a two-echelon supply chain using SCOR metrics. *Central European Journal of Operations Research*, 22(4), 591-622.

- Zhao, S. T., Wu, K., & Yuan, X.-M. (2016a). Optimal integer-ratio inventory coordination policy for an integrated multi-stage supply chain. *Applied Mathematical Modelling*, 40(5-6), 3876-3894.
- Zhao, S. T., Wu, K., & Yuan, X.-M. (2016b). Optimal production-inventory policy for an integrated multi-stage supply chain with time-varying demand. *European Journal of Operational Research*, 255(2), 364-379.
- Zhou, H., Benton, W. C., Jr., Schilling, D. A., & Milligan, G. W. (2011). Supply Chain Integration and the SCOR Model. *Journal of Business Logistics*, 32(4), 332-344.
- Zipkin, P. (2000). *Foundations of inventory management* (1st ed.). New York: McGraw-Hill.

APPENDICES

Appendix A

Derivation of the average inventory at FPW

From Figure 3.2, we define

$Q_F(t)$: Finished product inventory on hand at time t ,

$Q_m(t)$: Finished products manufactured by time t ,

$Q_s(t)$: Total quantity of finished products shipped by time t . So, $Q_F(t) = Q_m(t) - Q_s(t)$.

And the average inventory per cycle

$$\begin{aligned} Q_{avg} &= \frac{1}{T_2} \int_0^{T_2} Q_F(t) dt \\ &= \frac{1}{T_2} \left(\int_0^{T_2} Q_m(t) dt - \int_0^{T_2} Q_s(t) dt \right) \end{aligned}$$

$$Q_m(t) = \begin{cases} pt & 0 \leq t < T_3 \\ pT_3 & T_3 \leq t < T_2 \end{cases},$$

$$\begin{aligned} \frac{1}{T_2} \int_0^{T_2} Q_m(t) dt &= \frac{1}{T_2} \left(\int_0^{T_3} pt dt + \int_{T_3}^{T_2} pT_3 dt \right) \\ &= pT_3 \left(1 - \frac{T_3}{2T_2} \right) \end{aligned}$$

$$pT_3 = dT_2, \text{ so, } \frac{T_3}{T_2} = \frac{d}{p}.$$

$$Q_s(t) = kQ_4 = kQ,$$

Where $k = 1, 2, 3, \dots, N-1$, $kT \leq t \leq (k+1)T = T_2$.

$$\begin{aligned} \frac{1}{T_2} \int_0^{T_2} Q_s(t) dt &= \frac{1}{T_2} \int_0^{NT} Q_s(t) dt \\ &= \frac{1}{T_2} \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} kQ dt \\ &= \frac{(N-1)dT}{2} \end{aligned}$$

$$\begin{aligned}
Q_{avg} &= \frac{1}{T_2} \left(\int_0^{T_2} Q_m(t) dt - \int_0^{T_2} Q_s(t) dt \right) \\
&= pT_3 \left(1 - \frac{T_3}{2T_2} \right) - \frac{(N-1)dT}{2} \\
&= \frac{NdT}{2} \left(1 - \frac{T_3}{NT} \right) + \frac{dT}{2} \\
&= \frac{NdT}{2} \left(1 - \frac{T_3}{T_2} \right) + \frac{dT}{2} \\
&= \frac{dT}{2} \left[N \left(1 - \frac{d}{p} \right) + 1 \right].
\end{aligned} \tag{A.1}$$

Appendix B

Derivation of the average inventory at RMW

From Figure 3.2, we can derive the average raw material inventory

$$\begin{aligned}
Q'_{avg} &= \frac{1}{MTN} [NT((M-1)Q_3 + (M-2)Q_3 + \dots + Q_3)] \\
&= \frac{NdT}{2} (M-1)
\end{aligned} \tag{B.1}$$

Appendix C

Proof of Proposition 3.1.

By relaxing the integer variables to continuous variables in Eq. (3.6), we can attain the local minimum point (Heath, 2002). By taking the first derivative of TC in Eq. (3.6) with respect to T , we get

$$\frac{\partial TC}{\partial T} = \frac{d}{2} \left(h_4 + h_3 N - \frac{h_3 N d}{p} + h_3 + h_1' M N - h_1' N \right) - \frac{1}{T^2} \left(A_4 + A_s + \frac{A_2}{N} + \frac{A_1}{M N} \right) = 0.$$

$$T = \sqrt{\frac{(A_4 + A_s + \frac{A_2}{N} + \frac{A_1}{M N})}{\frac{d}{2} \left(h_4 + h_3 \left(N \left(1 - \frac{d}{p} \right) + 1 \right) + h_1' N (M - 1) \right)}}. \quad (C.1)$$

By taking the second derivative of TC with respect to T , we get

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2}{T^3} \left(A_4 + A_s + \frac{A_2}{N} + \frac{A_1}{M N} \right) > 0. \quad (C.2)$$

By taking the derivative of $\partial TC / \partial T$ with respect to M and N , we get

$$\frac{\partial^2 TC}{\partial T \partial M} = \frac{h_1' N d}{2} + \frac{A_1}{T^2 M^2 N}, \quad (C.3)$$

$$\frac{\partial^2 TC}{\partial T \partial N} = \frac{1}{2} \left(h_3 - \frac{h_3 d}{p} + h_1' M - h_1' \right) d + \frac{1}{T^2 N^2} \left(A_2 + \frac{A_1}{M} \right). \quad (C.4)$$

By taking the first derivative of TC with respect to N , we get

$$\frac{\partial TC}{\partial N} = \frac{dT}{2} \left(h_3 - \frac{h_3 d}{p} + h_1' M - h_1' \right) - \frac{1}{T N^2} \left(A_2 + \frac{A_1}{M} \right) = 0,$$

$$N^2 d T^2 = \frac{(A_2 + \frac{A_1}{M})}{\frac{1}{2} \left(h_3 - \frac{h_3 d}{p} + h_1' M - h_1' \right)}. \quad (C.5)$$

By taking the second derivative of TC with respect to N , we get

$$\frac{\partial^2 TC}{\partial N^2} = \frac{2}{T N^3} \left(A_2 + \frac{A_1}{M} \right) > 0. \quad (C.6)$$

By taking the first derivative of TC with respect to M , we get

$$\frac{\partial TC}{\partial M} = \frac{h_1' NdT}{2} - \frac{A_1}{TM^2 N} = 0. \quad (C.7)$$

By taking the second derivative of TC with respect to M , we get

$$\frac{\partial^2 TC}{\partial M^2} = \frac{2A_1}{TM^3 N} > 0. \quad (C.8)$$

By taking the derivative of $\partial TC / \partial M$ with respect to N , we get

$$\frac{\partial^2 TC}{\partial M \partial N} = \frac{h_1' dT}{2} + \frac{A_1}{TM^2 N^2}. \quad (C.9)$$

From Eq. (C.7), we get

$$M^2 = \frac{2A_1}{h_1' N^2 dT^2}. \quad (C.10)$$

By inserting Eq. (C.5) into Eq. (C.10), we get

$$M_0 = \sqrt{\frac{A_1}{A_2 h_1'} \left[h_3 \left(1 - \frac{d}{p} \right) - h_1' \right]}.$$

By inserting Eq. (C.1) into Eq. (C.10), we get

$$N_0 = \sqrt{\frac{A_2 (h_4 + h_3)}{(A_4 + A_s) \left[h_3 \left(1 - \frac{d}{p} \right) - h_1' \right]}}.$$

By inserting M_0 and N_0 , into Eq. (C.10), we get

$$T_0 = \sqrt{\frac{2(A_4 + A_s)}{d(h_4 + h_3)}}.$$

The Hessian Matrix: $H(TC) = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial M} & \frac{\partial^2 TC}{\partial T \partial N} \\ \frac{\partial^2 TC}{\partial M \partial T} & \frac{\partial^2 TC}{\partial M^2} & \frac{\partial^2 TC}{\partial M \partial N} \\ \frac{\partial^2 TC}{\partial N \partial T} & \frac{\partial^2 TC}{\partial N \partial M} & \frac{\partial^2 TC}{\partial N^2} \end{bmatrix}.$

And $|H(TC)_1| = \frac{\partial^2 TC}{\partial T^2} > 0$. For $|H(TC)_2| = \frac{\partial^2 TC}{\partial T^2} \frac{\partial^2 TC}{\partial M^2} - (\frac{\partial^2 TC}{\partial T \partial M})^2$, insert Eqs.

(C.2), (C.3), and (C.8) into $|H(TC)_2|$, we get

$$|H(TC)_2| = \frac{4A_1}{T^4 M^3 N} (A_4 + A_s) + \frac{4A_2 A_1}{T^4 M^3 N^2} - \frac{h_1'^2 N^2 d^2}{4} - \frac{h_1' d A_1}{T^2 M^2} + \frac{3A_1^2}{T^4 M^4 N^2}.$$

By using M_0, N_0, T_0 to replace M, N, T , respectively in $|H(TC)_2|$, we have

$$|H(TC)_2|_{M_0, N_0, T_0} = \frac{h_1' A_2 d^2 (h_4 + h_3) \sqrt{h_1' (h_4 + h_3)}}{(h_3 - \frac{h_3 d}{p} - h_1') \sqrt{A_1 (A_4 + A_s)}} + \frac{h_1' A_2 d^2 (h_4 + h_3) \sqrt{h_1' A_2}}{(A_4 + A_s) \sqrt{A_1 (h_3 - \frac{h_3 d}{p} - h_1')}} > 0.$$

$$|H(TC)_3| = \frac{d^2 TC}{dT^2} \left(\frac{d^2 TC}{dM^2} \frac{d^2 TC}{dN^2} - \left(\frac{d^2 TC}{dNdM} \right)^2 \right) + \frac{d^2 TC}{dMdT} \left(\frac{d^2 TC}{dNdM} \frac{d^2 TC}{dTdN} - \frac{d^2 TC}{dTdM} \frac{d^2 TC}{dN^2} \right) \\ + \frac{d^2 TC}{dNdT} \left(\frac{d^2 TC}{dTdM} \frac{d^2 TC}{dMdN} - \frac{d^2 TC}{dM^2} \frac{d^2 TC}{dTdN} \right).$$

Inserting Eqs. (C.2), (C.3), (C.4), (C.6), (C.8), and (C.9) into $|H(TC)_3|$, we get

$$|H(TC)_3| = \frac{6A_1 A_2^2}{T^5 M^3 N^5} + \frac{10A_1^2 A_2}{T^5 M^4 N^5} - \frac{h_1'^2 d^2 A_2}{2TN} - \frac{4h_1' d A_1 A_2}{T^3 M^2 N^3} + \frac{4A_1^3}{T^5 M^5 N^5} - \frac{3h_1' d A_1^2}{T^3 M^3 N^3} +$$

$$\frac{2}{T^3} (A_4 + A_s) \left(\frac{4A_1 A_2}{T^2 M^3 N^4} + \frac{3A_1^2}{T^2 M^4 N^4} - \frac{h_1'^2 d^2 T^2}{4} - \frac{h_1' d A_1}{M^2 N^2} \right) + \frac{h_1'^2 h_3 d^3 NT}{4} -$$

$$\frac{h_1'^2 h_3 d^4 NT}{4p} + \frac{h_1'^3 d^3 MNT}{4} - \frac{h_1'^3 d^3 NT}{4} - \frac{h_3^2 d^2 A_1}{2TM^3 N} + \frac{h_3^2 d^3 A_1}{pTM^3 N} + \frac{h_1' h_3 d^2 A_1}{TM^3 N} -$$

$$\frac{2h_3 d A_1 A_2}{T^3 M^3 N^3} - \frac{h_3 d A_1^2}{T^3 M^4 N^3} - \frac{h_3^2 d^4 A_1}{2p^2 TM^3 N} - \frac{h_1' h_3 d^3 A_1}{pTM^3 N} + \frac{2h_3 d^2 A_1 A_2}{pT^3 M^3 N^3} + \frac{h_3 d^2 A_1^2}{pT^3 M^4 N^3} -$$

$$\frac{h_1'^2 d^2 A_1}{2TM^3 N} + \frac{2h_1' d A_1 A_2}{T^3 M^3 N^3} + \frac{h_1' d A_1^2}{T^3 M^4 N^3}.$$

By using M_0, N_0, T_0 to replace M, N, T in $|H(TC)_3|$, we have

$$|H(TC)_3|_{M_0, N_0, T_0} = \frac{h_1' d^2 A_2 \sqrt{2dh_1'(A_4 + A_s)}}{\sqrt{\frac{A_1 A_2}{(h_3 - \frac{h_3 d}{p} - h_1')(h_4 + h_3)}}} > 0.$$

We verify $|H(TC)_1|_{M_0, N_0, T_0} > 0$, $|H(TC)_2|_{M_0, N_0, T_0} > 0$ and $|H(TC)_3|_{M_0, N_0, T_0} > 0$.

Hence, TOC is optimal at (M_0, N_0, T_0) .

Next, we need to verify M_0, N_0, T_0 is the global minimum point. Based on the aforementioned derivation, we obtained only one stationary point which is (M_0, N_0, T_0) . It is the global minimum point if Y is convex in M, N and T . For any given M and N , we have Eq. (C.2): $\partial^2 TC / \partial T^2 = 2(A_4 + A_s + A_2 / N + A_1 / MN) / T^3 > 0$. Hence, TC is convex in T . For any given M and N , TC decreases in T when $T \in (0, T_0]$ and decreases in T when $T \in [T_0, \infty]$. Therefore, TC reaches its global minimum at T_0 for any given M and N . Similarly, for any given M and T , TC reaches its global minimums at N_0 (Eq. (C.6)); and for any given N and T , TC reaches its global minimum at M_0 (Eq. (C.8)). Thus, we readily know that TC reaches its global minimum at (M_0, N_0, T_0) .

Using M_0, N_0, T_0 to substitute M, N and T in Eq. (3.6), we have

$$TC(M_0, N_0, T_0) = \sqrt{2d(A_4 + A_s)(h_4 + h_3)} + \sqrt{2A_1 h_1' d} + \sqrt{2A_2 d(h_3 - \frac{h_3 d}{p} - h_1')} + L_s h_s d.$$

The proof is completed.

Appendix D

The proof of Corollary 3.1

When the system setup cost rates change by α , so, $\alpha A_T = (\alpha A_1, \alpha A_2, \alpha A_4)$. And the system holding cost rates change by β , so, $\beta h_T = (\beta h_1', \beta h_3, \beta h_4)$.

(i) From Eq. (3.7),

$$M_0' = \sqrt{\frac{\alpha A_1}{\alpha A_2} \left[\frac{\beta h_3}{\beta h_1'} \left(1 - \frac{d}{p}\right) - 1 \right]} = \sqrt{\frac{A_1}{A_2} \left[\frac{h_3}{h_1'} \left(1 - \frac{d}{p}\right) - 1 \right]} = M_0.$$

(ii) From Eq. (3.8),

$$N_0' = \sqrt{\frac{A_2 \left(\frac{\beta h_4}{\beta h_3} + 1 \right)}{(A_4 + A_s) \left[\left(1 - \frac{d}{p}\right) - \frac{\beta h_1'}{\beta h_3} \right]}} = \sqrt{\frac{A_2 \left(\frac{h_4}{h_3} + 1 \right)}{(A_4 + A_s) \left[\left(1 - \frac{d}{p}\right) - \frac{h_1'}{h_3} \right]}} = N_0.$$

$\partial N_0 / \partial A_s < 0$, for $A_s > 0$, it is a decreasing function of A_s .

(iii) $\partial T_0 / \partial A_s > 0$, for $A_s > 0$, it is an increasing function of A_s .

The proof is completed.

Appendix E

Solution of numerical example 3.1

After the values of (M_0, N_0, T_0) are obtained, we use Eq. (3.11) to compute the optimal inventory coordination policy:

$$(M^*, N^*, Q^*) = \arg \min \left\{ \begin{array}{l} TC(\lfloor 1.6705 \rfloor, \lfloor 8.4660 \rfloor, \lfloor 31.6228 \rfloor); TC(\lceil 1.6705 \rceil, \lfloor 8.4660 \rfloor, \lfloor 31.6228 \rfloor); \\ TC(\lceil 1.6705 \rceil, \lceil 8.4660 \rceil, \lfloor 31.6228 \rfloor); TC(\lceil 1.6705 \rceil, \lceil 8.4660 \rceil, \lceil 31.6228 \rceil); \\ TC(\lfloor 1.6705 \rfloor, \lceil 8.4660 \rceil, \lfloor 31.6228 \rfloor); TC(\lfloor 1.6705 \rfloor, \lceil 8.4660 \rceil, \lceil 31.6228 \rceil); \\ TC(\lfloor 1.6705 \rfloor, \lfloor 8.4660 \rfloor, \lceil 31.6228 \rceil); TC(\lceil 1.6705 \rceil, \lfloor 8.4660 \rfloor, \lceil 31.6228 \rceil). \end{array} \right.$$

Then, we use the Eq. (3.6) to compute TOC, TC , for the eight integer cases.

Step 1: When $M = 1$, $N = 8$, and $Q = 31$

$TC = 98.67$ (0.4697% higher), $T = Q / d = 31 / 5 = 6.2$ (Days) (1.9694% shorter).

Step 2: When $M = 1$, $N = 8$, and $Q = 32$

$TC = 98.54$ (0.3402% higher), $T = Q / d = 32 / 5 = 6.4$ (Days) (1.1929% longer).

Step 3: When $M = 1$, $N = 9$, and $Q = 31$

$TC = 98.46$ (0.2556% higher), $T = Q / d = 31 / 5 = 6.2$ (Days) (1.9694% shorter).

Step 4: When $M = 1$, $N = 9$, and $Q = 32$

$TC = 98.51$ (0.3046% higher), $T = Q / d = 32 / 5 = 6.4$ (Days) (1.1929% longer).

Step 5: When $M = 2$, $N = 8$, and $Q = 31$

$TC = 98.36$ (0.1536% higher), $T = Q / d = 31 / 5 = 6.2$ (Days) (1.9694% shorter).

Step 6: When $M = 2$, $N = 8$, and $Q = 32$

$TC = 98.27$ (0.0660% higher), $T = Q / d = 32 / 5 = 6.4$ (Days) (1.1929% longer).

Step 7: When $M = 2$, $N = 9$, and $Q = 31$

$TC = 98.30$ (0.0939% higher), $T = Q / d = 31 / 5 = 6.2$ (Days) (1.9694% shorter).

Step 8: When $M = 2$, $N = 9$, and $Q = 32$

$TC = 98.39$ (0.1840% higher), $T = Q/d = 32/4 = 6.4$ (Days) (1.1929% longer).

Therefore, the variables of the optimal inventory coordination policy of this numerical example are $M^*=2$, $N^*=8$ and $Q^*=32$. The optimal TOC, $TC^*=\$98.27$, which is higher than the real-number solution by 0.0660%.

Appendix F

Derivation of the average inventory at FPW

According to the inventory and synchronization diagram, Figure 4.4, the average inventory at FPW can be calculated. It is defined as follows:

$Q_f(t)$: Finished product inventory on hand at time t ,

$Q_m(t)$: Finished products produced by time t ,

$Q_s(t)$:: Total quantity of finished products shipped by time t . So, $Q_f(t)=Q_m(t)-Q_s(t)$.

And the average inventory per cycle at FPW is

$$Q_{avg} = \frac{1}{T_2} \int_0^{T_2} Q_f(t) dt = \frac{1}{T_2} \left(\int_0^{T_2} Q_m(t) dt - \int_0^{T_2} Q_s(t) dt \right).$$

$$Q_m(t) = \begin{cases} pt & 0 \leq t < T_1 \\ pT_1 & T_1 \leq t < T_2 \end{cases},$$

$$\frac{1}{T_2} \int_0^{T_2} Q_m(t) dt = \frac{1}{T_2} \left(\int_0^{T_1} ptdt + \int_{T_1}^{T_2} pT_1 dt \right) = pT_1 \left(1 - \frac{T_1}{2T_2} \right).$$

$$pT_1 = dT_2, \text{ so, } \frac{T_1}{T_2} = \frac{d}{p}.$$

$$Q_s(t) = kQ_4 = kQ,$$

Where $k = 1, 2, 3, \dots, N-1; kT \leq t \leq (k+1)T = T_2$

$$\begin{aligned}
\frac{1}{T_2} \int_0^{T_2} Q_s(t) dt &= \frac{1}{T_2} \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} kQ dt \\
&= \frac{(N-1)dT}{2} . \\
Q_{avg} &= \frac{1}{T_2} \left(\int_0^{T_2} Q_m(t) dt - \int_0^{T_2} Q_s(t) dt \right) \\
&= \frac{dT}{2} \left[N \left(1 - \frac{d}{p} \right) + 1 \right] .
\end{aligned} \tag{F.1}$$

Appendix G

Proof of Proposition 4.1

- (i) By relaxing N to continuous variable in Eq. (4.5), the local minimum point can be obtained (Heath, 2002). The first derivative of TC in Eq. (4.5) with respect to T is given by

$$\begin{aligned}
\frac{\partial TC}{\partial T} &= \frac{d}{2} (h_3 + h_2 (N(1 - \frac{d}{p}) + 1)) - \frac{1}{T^2} (A_3 + A_s + \frac{S}{N}) = 0, \\
T &= \sqrt{\frac{(A_3 + A_s + \frac{S}{N})}{\frac{d}{2} (1 + h_2 (N(1 - \frac{d}{p}) + h_3))}} .
\end{aligned} \tag{G.1}$$

The second derivative of TC with respect to T is given by

$$\frac{\partial^2 TC}{\partial T^2} = \frac{2}{T^3} (A_3 + A_s + \frac{S}{N}) > 0 . \tag{G.2}$$

The derivative of $\partial TC / \partial T$ with respect to N is given by

$$\frac{\partial^2 TC}{\partial T \partial N} = \frac{dh_2}{2} (1 - \frac{d}{p}) + \frac{S}{T^2 N^2} . \tag{G.3}$$

The first derivative of TC with respect to N is given by

$$\frac{\partial \text{TC}}{\partial N} = \frac{dh_2 T}{2} \left(1 - \frac{d}{p}\right) - \frac{S}{TN^2}.$$

The second derivative of TC with respect to N is given by

$$\frac{\partial^2 \text{TC}}{\partial N^2} = \frac{2S}{TN^3} > 0, \quad (\text{G.4})$$

$$N_0 = \sqrt{\frac{S(1 + \frac{h_3}{h_2})}{(A_3 + A_s)(1 - \frac{d}{p})}}.$$

Inserting Eq. (4.6) into Eq. (G.1), we get

$$T_0 = \sqrt{\frac{2(A_3 + A_s)}{d(h_2 + h_3)}}.$$

The hessian matrix: $H(\text{TC}) = \begin{bmatrix} \frac{\partial^2 \text{TC}}{\partial T^2} & \frac{\partial^2 \text{TC}}{\partial T \partial N} \\ \frac{\partial^2 \text{TC}}{\partial N \partial T} & \frac{\partial^2 \text{TC}}{\partial N^2} \end{bmatrix}$, and

$$|H(\text{TC})_1| = \frac{\partial^2 \text{TC}}{\partial T^2} > 0, \quad |H(\text{TC})_2| = \frac{\partial^2 \text{TC}}{\partial T^2} \frac{\partial^2 \text{TC}}{\partial N^2} - \left(\frac{\partial^2 \text{TC}}{\partial T \partial N}\right)^2.$$

Insert Eqs. (G.2), (G.3), and (G.4) into $|H(\text{TC})_2|$, we get

$$|H(\text{TC})_2| = \frac{4SA_3}{T^4 N^3} + \frac{4SA_s}{T^4 N^3} + \frac{3A_1^2}{T^4 N^4} - \frac{h_2^2 d^2}{4} + \frac{h_2^2 d^3}{2p} - \frac{h_2 d S}{T^2 N^2} - \frac{h_2^2 d^4}{4p^2} + \frac{S}{T^2 N^2} \frac{h_2 d^2}{p}.$$

By using N_0 and T_0 to replace N and T , respectively in $|H(\text{TC})_2|$, we get

$$|H(\text{TC})_2| = \sqrt{\frac{(A_3 + A_s)(h_2 + h_3)}{S}} (h_2 (1 - \frac{d}{p}))^{1.5} \geq 0.$$

It is verified that $|H(\text{TC})_1|_{N_0, T_0} > 0$ and $|H(\text{TC})_2|_{N_0, T_0} > 0$. Hence, TOC is optimal at (N_0, T_0) . Next, we need to verify (N_0, T_0) is the global minimum point. Based on the aforementioned derivation, (N_0, T_0) is the one stationary point. It is the global minimum point if TC is convex in N and T . For any given N , we have Eq. (G.2): $\partial^2 \text{TC} / \partial T^2 > 0$. So, TC is convex in T . For any given N , TC decreases in T when $T \in (0, T_0]$ and increases in T when $T \in [T_0, \infty)$. Therefore, TC reaches its global minimum at T_0 for any given N . Similarly, for any given T , TC reaches its global minimums at N_0 . Thus, it is readily known that TC reaches its global minimum at (N_0, T_0) .

- (ii) By using N_0 and T_0 to replace N and T , respectively in TC, the global minimum TOC is given by

$$\text{TC}_0 = \text{TC}(N_0, T_0) = \sqrt{2d(A_3 + A_s)(h_2 + h_3)} + \sqrt{2Sh_2d(1 - \frac{d}{p})} + Lh_s d.$$

The proof is completed.

Appendix H

Proof of Corollary 4.1

- (i) In JIT production, $p \approx d$, the global minimum TOC is given by

$$TC_0' = \sqrt{2d(A_3 + A_s)(h_2 + h_3)} + Lh_s d ,$$

and the variable of the IIC policy is given by

$$N_0 \rightarrow \infty ;$$

- (ii) In the case of instantaneous replenishment of finished products at DC, $p \gg d$, $d/p \approx 0$. The global minimum TOC is given by

$$TC_0'' = \sqrt{2d(A_3 + A_s)(h_2 + h_3)} + \sqrt{2Sh_2 d} + Lh_s d ,$$

and the variable of the IIC policy is given by

$$N_0 \approx \sqrt{\frac{S(h_2 + h_3)}{h_2(A_3 + A_s)}} .$$

The proof is completed.

Appendix I

Proof of Lemma 4.1.

At DC, OCT $T' = \sqrt{2A_3d/h_3}$, EOQ $Q_3' = \sqrt{2A_3d/h_3}$, the optimal operational cost is given by

$$C_1 = \sqrt{2A_3dh_3}. \quad (\text{I.1})$$

For transportation, the optimal operational cost is given by

$$C_2 = Lh_s d + A_s \sqrt{\frac{h_3 d}{2A_3}}. \quad (\text{I.2})$$

To avoid the stock out, we assume EPQ at production $Q_2' = mQ_3'$, m is integer.

So, the operational cost is given by

$$h_2 Q_{avg}' + \frac{S}{mT'} = \frac{h_2 Q_3'}{2} \left\{ m \left(1 - \frac{d}{p} \right) + 1 \right\} + \frac{Sd}{mQ_3'}.$$

The derivation of Q_{avg}' is same as Q_{avg} at Appendix B. It is obvious that the operational cost function of production is convex in m in the interval $(0, \infty)$. So, when

$$m = \frac{1}{Q_3'} \sqrt{\frac{2Sd}{h_2(1-\frac{d}{p})}} = \sqrt{\frac{Sh_3}{A_3h_2(1-\frac{d}{p})}},$$

the operational cost is optimal and given by

$$C_3 = \sqrt{2Sdh_2(1-\frac{d}{p})} + h_2 \sqrt{\frac{A_3d}{2h_3}}. \quad (\text{I.3})$$

$$\text{EPQ } Q_2' = \sqrt{\frac{2Sd}{h_2(1-\frac{d}{p})}}.$$

The proof is completed.

Appendix J

Proof of Proposition 4.2

From Appendix I, we sum Eqs. (I.1), (I.2) and (I.3), the TOC function of the supply chain without coordination is given by

$$TC_t = \sqrt{2A_3dh_3} + Lh_s d + A_s \sqrt{\frac{h_3 d}{2A_3}} + \sqrt{2Sdh_2(1 - \frac{d}{p})} + h_2 \sqrt{\frac{A_3 d}{2h_3}}.$$

Comparing with the global minimum TOC TC_0 of the SCOR-based ACM model, we have

$$TC_t - TC_0 = \sqrt{2A_3dh_3} + A_s \sqrt{\frac{h_3 d}{2A_3}} + h_2 \sqrt{\frac{A_3 d}{2h_3}} - \sqrt{2d(A_3 + A_s)(h_2 + h_3)},$$

the difference is given by

$$TC_t - TC_0 = \sqrt{\frac{A_3 dh_3}{2}} \left(\sqrt{1 + \frac{A_s}{A_3}} - \sqrt{1 + \frac{h_2}{h_3}} \right)^2 \geq 0.$$

The proof is completed.

Appendix K

Solution of numerical example 4.1

After the real-number solution of (N_0, T_0) are obtained, the optimal inventory coordination policy is computed by using Eq. (4.9).

$$(N^*, Q^*) = \arg \min \left\{ \begin{array}{l} \text{TC}(\lfloor 11.8972 \rfloor, \lfloor 39.3123 \rfloor); \text{TC}(\lfloor 11.8972 \rfloor, \lceil 39.3123 \rceil); \\ \text{TC}(\lceil 11.8972 \rceil, \lfloor 39.3123 \rfloor); \text{TC}(\lceil 11.8972 \rceil, \lceil 39.3123 \rceil). \end{array} \right\}.$$

Then, Eq. (4.5) is used to compute TOC, TC, for four integer cases.

Step 1: When $N = 11$ and $Q = 39$

TC = \$604996.66 (0.0437% higher), $T = Q / d = 0.00629$ (year) (0.7943% shorter).

Step 2: When $N = 12$ and $Q = 39$

TC = \$604749.21 (0.0028% higher), $T = Q / d = 0.00629$ (year) (0.7943% shorter).

Step 3: When $N = 11$ and $Q = 40$

TC = \$604936.42 (0.0388% higher), $T = Q / d = 0.0065$ (year) (1.7494% longer).

Step 4: When $N = 12$ and $Q = 40$

TC = \$604835.10 (1.0752% higher), $T = Q / d = 0.0065$ (year) (1.7494% longer).

Therefore, the optimal inventory coordination policy of this numerical example is $N^*=12$ and $Q^*=39$. The optimal TOC, $TC^*=\$604749.21$, which is higher than the real-number solution by 0.0028 percent.

Appendix L

Proof of Proposition 5.1

Based on Eq (5.1), the quantity demanded at R in phase f is given by:

$$\begin{aligned} D_f &= \int_{T_{f-1}}^{T_f} d(t)dt = \int_{T_{f-1}}^{T_f} (d_{0f} + \lambda_f t)dt \\ &= \left[d_{0f}t + \frac{1}{2} \lambda_f t^2 \right]_{T_{f-1}}^{T_f} \\ &= d_{0f}(T_f - T_{f-1}) + \frac{1}{2} \lambda_f (T_f^2 - T_{f-1}^2) \\ &= d_{0f} n_f L + \frac{1}{2} \lambda_f (T_f + T_{f-1}) n_f L \\ &= n_f L [d_{0f} + \frac{1}{2} \lambda_f (n_f + 2 \sum_{i=1}^{f-1} n_i)]. \end{aligned}$$

Therefore, the total quantity demanded at R over F phases is given by

$$Q = \sum_{f=1}^F D_f = L \left[\sum_{f=1}^F d_{0f} n_f + \frac{1}{2} \sum_{f=1}^F n_f \lambda_f (n_f + 2 \sum_{i=1}^{f-1} n_i) \right].$$

The proof is completed.

Appendix M

Proof of Proposition 5.2

(i)

The demand at the end of the increasing demand phase is the highest. The finished products produced during interval L , pL must not smaller than the last shipment size over the demand phase, *i.e.*,

$$pL \geq x_{0f} + b\theta_f.$$

$$\text{And } x_{0f} + b\theta_f = (d_{0f} + 0.5\lambda_f L) L + b (\lambda_f L^2).$$

$$\text{So, } p \geq d_{0f} + (b + 0.5) \lambda_f L,$$

where b is the number of the last shipment in the increasing demand phase.

(ii)

In the constant demand phase, it is straightforward that the production rate must not smaller than demand rate. So, $p \geq d_{0f}$.

(iii)

The demand at the beginning of the decreasing demand phase is the highest. The finished products produced during interval L , pL must not smaller than the first shipment size over the demand phase, *i.e.*,

$$pL \geq x_{0f} + \theta_f.$$

$$\text{And } x_{0f} + \theta_f = (d_{0f} + 0.5\lambda_f L) L + (\lambda_f L^2).$$

$$\text{So, } p \geq d_{0f} + 1.5\lambda_f L.$$

The proof is completed.

Appendix N

Derivation of the time-weighted inventory at FPW

We define:

$I_j(t)$: Finished product inventory on hand at time t in production cycle j ,

$q_{Fj}(t)$: Finished products manufactured by time t in production cycle j ,

$q_{sj}(t)$: Total quantity of finished products shipped by time t in production cycle j , so,

$$I_j(t) = q_{Fj}(t) - q_{sj}(t).$$

The time-weighted aggregate finished products produced in production cycle j

$$\begin{aligned} \int_{T_{bj}}^{T_{ej}} q_{Fj}(t) dt &= Q_{pj} (m_j L - T_{0j} - \frac{1}{2} T_{pj}) \\ &= Q_{pj} \{ l_{j+1} L - l_j L - L + \frac{D_{0f} + (l_j + 1 - n_{f-1}) \theta_f}{p} - \frac{Q_{pj}}{2p} \}, \end{aligned}$$

The time-weighted aggregate finished products shipped in production cycle j

$$\begin{aligned} \int_{T_{bj}}^{T_{ej}} q_{sj}(t) dt &= L \left[x_1^j (m_j - 1) + x_2^j (m_j - 2) + \dots + x_{(m_j-1)}^j \right] \\ &= L \sum_{k=1}^{l_{j+1}-l_j-1} x_k^j (l_{j+1} - l_j - i), \end{aligned}$$

The time-weighted inventory in production cycle j

$$\int_{T_{bj}}^{T_{ej}} I_j(t) dt = Q_{pj} \{ l_{j+1} L - l_j L - L + \frac{D_{0f} + (l_j + 1 - n_{f-1}) \theta_f}{p} - \frac{Q_{pj}}{2p} \} - L \sum_{k=1}^{l_{j+1}-l_j-1} x_k^j (l_{j+1} - l_j - i).$$

Appendix O

Proof of Lemma 5.1

From Eqs. (5.1) and (5.2), the k th shipment over the planning horizon x_k is given by

$$\begin{aligned} x_k &= \int_{kL}^{(k+1)L} d(t)dt = \left[d_0 t + \frac{1}{2} \lambda t^2 \right]_{kL}^{(k+1)L} \\ &= (d_0 + \frac{1}{2} \lambda_f L) L + k \lambda L^2 \\ &= D_0 + k \theta \end{aligned}$$

The k th shipment size in cycle j from the beginning is given by

$$x_k^j = D_0 + (l_j + k) \theta,$$

where $D_0 = (d_0 + \frac{1}{2} \lambda L) L$, $\theta = \lambda L^2$, $0 \leq l_j + k \leq \sum_{j=1}^M m_j$. k positive integer.

The proof is completed.

Appendix P

Proof of Lemma 5.2.

From Eq. (5.14), the production quantity in the subplan j is given by

$$\begin{aligned} Q_{pj} &= D_0 + (l_j + 1) \theta + D_0 + (l_j + 2) \theta + \dots + D_0 + (l_j + m_j) \theta \\ &= (l_{j+1} - l_j) [D_0 + \theta \frac{(l_{j+1} + l_j + 1)}{2}] \end{aligned}$$

The proof is completed.

Appendix Q

Proof of Proposition 5.3

Rearranging Eq. (5.16), we can obtain

$$\begin{aligned}\frac{T_{0(j+1)}}{T_{0j}} &= \frac{L - \frac{D_0 + (l_{j+1} + 1)\theta}{p}}{L - \frac{D_0 + (l_j + 1)\theta}{p}} \\ &= 1 - \frac{m_j \lambda L^2}{L - \frac{D_0 + (l_j + 1)\theta}{p}}.\end{aligned}$$

(i)

When $\lambda > 0$, $\frac{\frac{m_j \lambda L^2}{p}}{L - \frac{D_0 + (l_j + 1)\theta}{p}} > 0$, it is proven that $\frac{T_{0(j+1)}}{T_{0j}} < 1$, so, $T_{0j} > T_{0(j+1)}$.

(ii)

Similarly, when $\lambda = 0$, $\frac{\frac{m_j \lambda L^2}{p}}{L - \frac{D_0 + (l_j + 1)\theta}{p}} = 0$, it is proven that $\frac{T_{0(j+1)}}{T_{0j}} = 1$, so,

$$T_{0j} = T_{0(j+1)}.$$

(iii)

Similarly, when $\lambda < 0$, $\frac{\frac{m_j \lambda L^2}{p}}{L - \frac{D_0 + (l_j + 1)\theta}{p}} < 0$, it is proven that $\frac{T_{0(j+1)}}{T_{0j}} > 1$, so,

$$T_{0j} < T_{0(j+1)}.$$

The proof is completed.

Appendix R

Proof of Theorem 5.2

In subplan j , the operational cost incurred at R is

$$m_j A_4 + \frac{1}{2} h_4 L \sum_{k=1}^{l_{j+1}-l_j} x_k^j = m(A_4 + \frac{d_0}{2} h_4 L^2), \quad (R.1)$$

the operational cost incurred for transportation is

$$m_j A_s + \sum_{k=1}^{l_{j+1}-l_j} x_k^j h_s L_s = m(A_s + d_0 h_s L L_s), \quad (R.2)$$

the operational cost incurred at FPW is

$$h_3 \int_{T_{bj}}^{T_{ej}} I_j(t) dt = m d_0 L^2 h_3 \left\{ \frac{d_0}{p} \left(1 - \frac{m}{2}\right) + \frac{1}{2} (m-1) \right\}, \quad (R.3)$$

the operational cost incurred at MUP is S and the operational cost incurred at RMW is

$$A_1 + \frac{r \sum_{k=1}^{l_{j+1}-l_j} x_k^j}{2} T_{pj} h_1 = A_1 + \frac{r(m d_0 L)^2}{2p} h_1. \quad (R.4)$$

By summing up Eqs. (R.1) to (R.4), TOC in subplan j

$$C_j = m \left\{ (A_4 + \frac{d_0}{2} h_4 L^2) + A_s + d_0 h_s L L_s + d_0 L^2 h_3 \left[\frac{d_0}{p} \left(1 - \frac{m}{2}\right) + \frac{1}{2} (m-1) \right] \right\} + S + A_1 + \frac{r(m d_0 L)^2}{2p} h_1.$$

By taking the second derivative of C_j with respect to m , we get

$$\frac{\partial^2 C_j}{\partial m^2} = d_0 L^2 \left[h_3 \left(1 - \frac{d_0}{p}\right) + \frac{r d_0}{p} h_1 \right].$$

To prevent stock out, $p \geq d_0$, $\partial^2 C_j / \partial m^2 > 0$. Then, TOC function C is strict convex in the production cycle length, the positive integer m in the region $(0, +\infty)$.

The proof is completed.

VITA

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PUBLICATIONS

1. Zhao, S. T., K. Wu and X. M. Yuan. (2016). Optimal production-inventory policy for an integrated multi-stage supply chain with time-varying demand, *European Journal of Operational Research* (Impact Factor 2.911), DOI: 10.1016/j.ejor.2016.04.027.
2. Zhao, S. T., Wu, K., & Yuan, X.-M. (2016). Optimal integer-ratio inventory coordination policy for an integrated multi-stage supply chain. *Applied Mathematical Modelling* (Impact Factor 2.326), 40(5-6), 3876-3894.
3. Zhao, S. T., Yuan, X. M., & Ling, S. F. (2011). Production and Distribution Planning Model for Hinterland Supply Chain. *2011 Ieee International Conference on Industrial Engineering and Engineering Management (Ieem)*, 247-251.