Spin dynamics of two-dimensional systems in strong-coupling regime

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**SPIN DYNAMICS OF TWO-DIMENSIONAL
SYSTEMS IN STRONG-COUPLING REGIME**

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**SCHOOL OF PHYSICAL AND MATHEMATICAL
SCIENCES**

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Abstract

A possibility to use the spin properties of the particles opens an alternative path for the creation of new devices for information storage and transfer, which in comparison to the currently existing charge-based devices have several manufacturing benefits such as a compact size of the systems and ability of easy-going state control. Additionally apart from strongly applied motives, the study of particle spin dynamics has a significant fundamental potential for the understanding of the nature of spin-induced effects.

Speaking about the difficulties of spin usage, note that initial spin alignment can be lost with time under the multiple scattering events and various interactions in the absence of external influences, which regarding device manufacturing results in the information loss. For that reason, the study of the spin dynamics and the search for alternative particles, which can carry the information for relatively long time are the topics of high priority nowadays.

The current thesis is devoted to the theoretical study of the spin transport effects in two-dimensional systems in strong-coupling regime. Following discussion covers several aspects of the spin dynamics and, in particular, includes:

- investigation of the spin and the transport properties of two-dimensional electrons dressed by laser field;
- investigation of polariton spin dynamics in diffusive regime;
- investigation of the effect of spin control in polariton systems using the effect of polariton Rabi oscillations.

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1. Introduction

Archeologically history of humankind has been classified into three broad categories on the basis of technological development and common material used for tool and weapon manufacturing, so-called the Stone Age, the Bronze Age, and the Iron Age. According to this distinction, modern industrial era can rightfully be called the “Semiconductor Age”, due to the rapid development and widespread application of this type of materials in daily human routine. One can start counting the beginning of this period from the day of the first semiconductor transistor creation in 1947 by Shockley, Bardeen, and Brattain [1, 2]. This invention made a revolution in the human way of life, which eventually made an impact on individual performance in different tasks from work to leisure [3, 4, 5, 6]. From the last half of the XX century until now semiconductor electronic technology has come a long way from the first point-contact transistor to the modern powerful electronic devices. However, nowadays the desire to speed up the device operating processes with a simultaneous decrease in the size of the systems is still relevant booster among commercial semiconductor manufacturers.

Although the first transistor was created on germanium (Ge) material basis, in a short time the electrical properties of silicon (Si) have been found more suitable for electronic engineering purposes [7]. For many years, silicon remained the dominant material in electronics, because of cost and better electric characteristics in comparison to germanium. However in last decades along with improvement of the performance of currently existing devices an extensive study of various semiconductor structures showed a great potential of another class of semiconductor materials, known as III-V compound semiconductors. This group includes GaAs, InP, and other structures, which are formed by a mixture of elements from III and V group in the periodic table, respectively. Development of several semiconductor growth techniques, such as MOCVD and MBE, allowed to manufacture a high-quality III-V

structures of a given thicknesses with high electron mobility, and therefore from applied point of interest, nominated III-V compound semiconductors to become ideal materials in ultra-fast nanoelectronics [8, 9, 10].

From the fundamental point of view technological capability to fabricate a nanoscale semiconductor layers gave a rise to experimental implementation of theoretical models of spatially confined systems. In other words, it allowed creating systems of reduced dimensionality, such as quantum wells, quantum wires and quantum dots, which in return stimulated the study of fundamental quantum effects and engineering of devices, where novel quantum phenomena play the major role. In addition to the use of III-V compound semiconductors as a principal component in high-speed nanoelectronics, they also took over the leading position in optoelectronics and photonics, where their predominance mainly comes from the fundamental property of the solids, known as the band structure. Indeed, recall that in contrast to indirect band gap silicon, most of III-V materials are direct band gap semiconductors, which make them highly efficient for the creation of light emitting devices with a particular wavelength governed by the semiconductor elemental composition.

Back to the recent technological needs, the modern industry also seeks for an alternative way for information storage and transmission, since existing devices, where the charge property of the carrier is exploited, tend to reach their performance limit. Remember that according to the quantum physics every particle can also be characterized by another fundamental quantum property, called spin, which can be described as the self-spinning of the selected particle. Talking about the electron spin, we need to mention that it only performs in two states, so-called up and down states, distinguished by the rotation direction of the electron - clockwise and anticlockwise. Therefore, due to this discrete set of spin states, electron spin can potentially be used as a new kind of binary logic and, as the flow of charges can transfer information along the device. In comparison to the currently existing

charge-based devices, an ability to utilize electron spin has several manufacturing benefits, which started a new promising area of physics known as spintronics, where the central place is given to the study of spin dynamics of charged carriers in semiconductors [11, 12, 13, 14]. One of the advantages of the electron spin in the area of data storage lays in the possibility of easy-going spin manipulation by an external field, which was already implemented in magnetic storage technology, where spin control was realized by applied magnetic field [15]. In addition to the previous spin benefit, a new generation of semiconductor devices based on spin flow might be much smaller and more powerful than current charge-based instruments, which makes spin-based devices very promising within the framework of current market requirements [16, 17, 18, 19, 20, 21]. Talking about difficulties of the spin appliance, one needs to note that electrons might lose their initial spin orientation with time under the multiple electron scatterings and various interactions in the absence of external influence, which regarding device manufacturing results in the information loss. Summing up this paragraph, one can notice that the study of spin dynamics and the search for alternative particles, which can carry the information for relatively long distance are the topics of high priority nowadays, which still hold the difficulties waiting to be solved.

Therefore, the current thesis is a contribution to the field of condensed matter physics focusing on the study of non-equilibrium spin dynamics of alternative promising quasi-particles, called dressed electrons and polaritons, in quantum confined semiconductor systems. The outline of the thesis is organized as follows: Arising from the summarized significance and motivation of the current thesis topic in the first chapter “Introduction”, the second chapter “Theoretical concepts” consists of the brief overview of basic semiconductor features related to the behavior of particles in bulk and low-dimensional systems, which can give the reader primary theoretical perspectives necessary for the understanding of the subsequent chapters. Following

thesis intent, in this chapter, our attention will be mainly centered on the spin behavior in semiconductor materials, which will be discussed in details using relevant theoretical models. Reverting to the problem of a novel spin-based device realization, fundamental mechanisms of spin evolution and loss of initial spin direction in the medium, including the effects of spin-orbit interaction, will be highlighted. In particular, it will be shown that in clean systems, meaning that disorder is absent in the system, spin dynamics of the particle is completely governed by the structure growth direction and presence of built-in or external electromagnetic fields. After that, we will introduce a quasi-particle, called exciton, which could be created under the optical excitation of the electron, and thence later we will define another novel quasi-particle formed under the effect of strong coupling between the exciton and the photon, called exciton-polariton. At the end of the chapter, talking about the experimental observation of exciton-polaritons, we will discuss the particular type of complex systems, called microcavities, where exciton-polaritons can be created, and outline their specific features.

The third chapter “Spin and transport properties of two-dimensional electrons dressed by laser field” is devoted to the study of spin dynamics of a two-dimensional electron gas (2DEG) dressed by light [22]. Therefore, primarily we will introduce the concept of a 2DEG in the presence of a strong high-frequency electromagnetic field (dressing field). Following the motive of the thesis, this chapter will be continued by the calculation of the eigenfunctions and eigenvalues of the dressed electrons in the presence of the Rashba spin-orbit interaction and a brief overview of a powerful approach for the study of spin dynamics, called the density matrix response function formalism. As a result of this formalism implementation, it will be shown that the dressing field leads to the drastic change in the carrier spin properties. Particularly one will be able to see that the strong high-frequency electromagnetic field may give rise to the transition between the governing mechanisms of spin relaxation, known as the Elliott-Yafet and the D’yakonov-Perel’ regimes,

which subsequently, drives to the ability to control the spin properties of a 2DEG by a high-frequency electromagnetic field.

The fourth chapter “Transport of polaritons in diffusive regime” is assigned to the discussion of spin transport of polaritons in two-dimensional systems with impurities, where initial polariton spin alignment originates from the polarization direction of the polariton’s photon component. For the accuracy of the polariton system study, in analogy to electron and hole spin interactions, polariton spin-orbit interaction, so-called TE-TM splitting, will be introduced. Limiting by diffusive, low-density and non-interacting particle behavior, we will discuss the evolution of the polariton spin states under the spatially localized external pump and compare obtained results to well-known solutions for two-dimensional electron and hole systems. As a consequence of analytical and numerical calculations, we will show that selected polariton spin signals have a strong correlation with the initial light polarization. In particular, we will demonstrate that external magnetic field leads to the specific multi-pole spin pattern formation in the plane of the structure. From the fundamental point of view developed approach presented in this chapter allows to identify the dominating impacts on the polariton pseudospin behavior and predict the polariton scattering and relaxation times, which in return, can be found practically useful for the design of the system with significant spin signals.

The fifth chapter “Polaritons spin Rabi oscillations” is attributed to the novel method of light generation, where the polarization of emitted light can be potentially tuned. Physical basis of proposed model will lay in the effect of the Rabi oscillations between two polariton fields excited by two counter-polarized and delayed in time light pulses. Qualitatively above idea can be understood via the precession of the polariton spin, which at long time scales reduces to the specified spin direction, selected by the value of the two-pulse time delay. It should be mentioned that feasibility of this concept has been proved experimentally and, therefore, in continuation of

this chapter, correlation between the theoretical prediction and experimental results will be shown [23]. Regarding manufacturing needs, we will discuss the applicability of proposed idea of spin polarization control and will show that it can be found useful for various engineering problems, for example, for the realization of the multitasking devices, where polarization spanning is required.

2. Theoretical concepts

2.1 Low-dimensional semiconductor structures

2.1.1 Band structure of solids

In the early XXth century, elaborated band theory of solids allowed to introduce the general classification of the crystal structures and explain electronic properties of various materials on its basis [24, 25, 26]. In particular, it demonstrated that in contrast to Bohr postulated discrete energy states in a free atom the energy spectrum of the system drastically changes under transition to the solid macroscopic system. According to the energy band theory, it was obtained that separated discrete atomic electron levels transform in the crystal into new closely placed states with negligibly small energy difference, which consequently can be treated as continuous bands. Recalling that since at equilibrium electron distribution is subject to the Pauli exclusion principle along with the minimum energy requirement, it was concluded that electrons cannot occupy the same energy state in these continuous bands and are forced to take different states and climb along the allowed energy levels. In order to distinguish these bands and their impact on material properties, the following terminology has been introduced. At absolute zero the last completely filled by electrons band was called valence band, and the next empty band where electrons can admittedly be located was called conduction band [27]. Under the above circumstances, solids have been divided into three broad classes, such as metals, semiconductors, and insulators based on the energy band arrangement. Typically, metals are materials with the overlap between valence and conduction bands, which drives to the smooth electron transfer from one band to another even under the small external energy influence. In contrast to metals, valence and conduction bands in semiconductors and insulators are separated by the energy gap,

known as band gap, where the arrangement of electron states is forbidden. Note that the principal difference between semiconductors and insulators only lies in the magnitude of band gap energy. The value of band gap in semiconductors usually reaches several *meV*, and for example, in most known semiconductors, such as silicon (Si), germanium (Ge) and gallium arsenide (GaAs) the band gap is equal to 1.17 eV, 0.74 eV, and 1.51 eV, respectively [28]. Nevertheless, it should be mentioned that categorization of materials to insulator and semiconductor classes is quite rough and mostly depends on the material application and conventional terminology in the particular research field. The band structures of the main semiconductor materials are shown schematically in Fig. 2.1. One can notice that an additional categorization of semiconductor structures can be done depending on the mutual arrangement of valence and conduction bands on semiconductor crystal momentum. Consider the band structure of GaAs (Fig. 2.1(c)), where the maximum energy value of valence band occurs at the same momentum as the minimum energy value of conduction band. Following conventional classification materials with this band arrangement are called direct band gap semiconductors. Otherwise, semiconductors with the band structure where the maximum energy of valence band and the minimum energy of conduction band happen at the different values of momentum, are called indirect semiconductors. The most known examples of this class of materials are Si and Ge (see Fig. 2.1(a,b)) [29]. Note that accurate calculation of the energy dispersion in the bulk material is a many-body problem with a large number of interacting particles, which requires massive computing power. Therefore to simplify the procedure for semiconductor bands accounting, several intuitive and approximate methods were created, which can be found in various fundamental textbooks [30, 31, 32].

As to the main result of theoretical study of semiconductor band structure, it was found that at small momentum parabolic energy dependence of the free electron can serve as a good approximation for the semiconductor

Figure 2.1: Schematic band structure of selected bulk semiconductors: (a) silicon, (b) germanium, and (c) gallium arsenide with the values of band gap $E_g = 1.17$ eV, $E_g = 0.74$ eV and $E_g = 1.51$ eV at low temperature, respectively. According to mutual arrangement of the bands, semiconductors are classified into two groups: indirect band gap semiconductors, such as Si and Ge, and direct bandgap semiconductors, like GaAs [29].

conduction band, measured from the bottom of valence band:

$$E(\mathbf{k}) = E_g + \frac{\hbar^2 \mathbf{k}^2}{2m^*}, \quad (2.1)$$

where E_g is the energy band gap, $\mathbf{k}$ is the electron wave vector, and m^* is the effective mass of the electron, which generally differs from the real carrier mass. The emergence of effective mass, m^* , in Eq. (2.1) has a great practical significance and is related to the aspiration to simplify the many-body problem by vanishing of the influence of the lattice periodic potential, which, in addition, allows to apply the laws of free particle to the problem of the carrier in solids [33, 34]. The shape of the valence band can be calculated within well-known Luttinger model [33, 34, 35, 36], which takes into account interaction of the electrons with the crystal core. It was shown that valence band in the semiconductor is a combination of two degenerate at zero momentum bands, separated by the distance, called the energy of spin-orbit coupling. At the same time, it was demonstrated that one of degenerate bands unfolds into two degenerate energy states at non-zero momentum, known as the light-hole band and the heavy-hole band, distinguished by

the difference in angular momentum, which takes the values equal to $\pm 3/2$ for heavy-hole states and $\pm 1/2$ values for light-hole states. The qualitative picture of the band structure of bulk GaAs is shown in Fig. 2.2.

Figure 2.2: The process of photon absorption in direct band gap bulk semiconductor, accompanied by absorption of the photon with energy $\hbar\omega$, the transition of the electron from the valence band (VB) to the conduction band (CB) and creation of the hole in valence band. Here, LH is the light-hole valence band, HH is the heavy-hole valence band, and SO is the split-off valence band.

2.1.2 Semiconductor systems of reduced dimensionality

Apart an adaptation of natural crystals for technological needs, development of semiconductor growth techniques allowed to create man-made systems with a specified set of parameters and even to grow multi-layered structures, which brought a great success to the modern technology. In particular, its prosperity was gained by systems, called heterostructures, which are formed by contacting semiconductors with different band gap values [37, 38, 39]. Note that crystal properties of semiconductors, such as the value of lattice constant, differ widely from one material to another, which during the process of heterostructure growth results in the unguided formation of defects on the edge between two semiconductor layers [40, 41]. It was shown experimentally that presence of randomly placed defects in heterostructures has a

drastic influence on system properties through the action on the carrier propagation characteristics. For now, there are only a few pairs of semiconductor materials with close values of lattice constant, which meet the requirements of designed theoretical setups and the industrial needs. A heterostructure formed by layers of GaAs and a solid solution of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ for any x -value of aluminum can serve as one of the successful examples of heterostructure realization, which is also the most studied complex nowadays.

Another experimentally implemented composite system is formed by thin layer of narrow band gap semiconductor placed between two layers of semiconductor with larger band gap energy value, which according to the models of quantum physics reproduces a system of the thin quantum well (QW) for electrons and holes where various quantum effects can occur. This analogy comes from the fact that electrons and holes become trapped inside of thin layer and, therefore, their motion becomes confined in one direction, while it still can be treated as free propagation in the others [36, 39]. For clarity of the following discussion let us introduce Cartesian coordinate system, where two (x and y) axes lie in-plane of this thin layer (or, in other words, in-plane of the quantum well) and z axis is oriented perpendicularly to them and coincides with the growth direction of the heterostructure. Then, in terms of carrier properties, two components of its wave vector, k_x and k_y , lying in-plane of a QW remain continuous as in bulk material, while k_z projection of the wave vector is only allowed to take a discrete set of values as $k_z = \pi n/d$, where n is the integer number and d is the thickness of a QW. As to the electron energy in a QW along the z axis of quantum confinement, it also changes and turns into a set of discrete dimensionally quantized energy levels. Therefore under the transformation from quadratic and isotropic energy dispersion of the electron in bulk material (see Eq. (2.1)) to confined 2D system, the energy spectrum of the electron reads [42] :

$$E(\mathbf{k}) = E_g + \frac{\hbar^2}{2m^*} \frac{\pi^2 n^2}{d^2} + \frac{\hbar^2(k_x^2 + k_y^2)}{2m^*}. \quad (2.2)$$

Following conventional notation a system of 2D confined electrons is also called a two-dimensional electron gas (2DEG). Note that for experimental realization quantum confined system should meet two requirements regarding the thickness of a QW and the value of energy quantization. The first limit comes from the particle-wave duality and states that the thickness of the well should be comparable and proportional to the half of de Broglie wavelength of the carrier. The second restriction refers to the value of energy quantization, which should be greater than both electron thermal energy and the value of state broadening arising from collisions. The physical meaning of the last limit lies in an effort to impose a restriction on the particle transfer between quantized levels. For illustrative purposes energy levels of electrons and holes in a QW are shown schematically in Fig. 2.3. It should be mentioned that in comparison to degenerate at zero-momentum valence band in bulk materials, the degeneracy of light-hole and heavy-hole bands lifts in confined semiconductor systems, which under excitation results in experimentally distinguishable formation of the light and heavy-holes.

Figure 2.3: Energy levels in a semiconductor quantum well created by placing a thin layer of the semiconductor with a band gap E_{g2} between two layers of semiconductor with a band gap E_{g1} , where $E_{g1} > E_{g2}$. Note that energy levels and values of band gaps are not shown on a scale.

2.1.3 Optical phenomena in semiconductor structures of reduced dimensionality

Since we have mentioned earlier the effect of electron excitation in bulk and QW structures, let us briefly recall the laws governing optical transitions.

Remember that any system should satisfy the Pauli principle, which states that the only allowed electron transfer occurs under transition from the occupied to the empty energy state. For that reason electron transition in pure semiconductors at ground state and low temperature is connected entirely with electron excitation from valence band to conduction band since valence band is fully occupied. Then electron transfer in semiconductors can only occur in the presence of an additional energy originating from the energy of electron thermal motion or external optical effects such as absorption of photons with the energy value greater than the value of semiconductor band gap. Imagine that the electron obtained the energy required for the transition between bands and transferred to the empty state in the conduction band leaving the vacant state in the valence band, which is shown qualitatively in Fig. 2.2. The empty state in valence band is called a hole, which regarding the interaction with electric and magnetic fields can be treated as a particle with the charge opposite to the electron charge and equal to it in magnitude.

Speaking about restrictions, it is also useful to note that for allowed processes of optical excitation the value of total angular momentum, J , of the electron should satisfy the selection rule, which is valid for both bulk and confined semiconductor systems. Graphically it is shown in Fig. 2.4, where the band structure of the semiconductor is represented by atomic-like energy levels. Note the difference between the degenerate valence band levels in bulk semiconductors (panel Fig. 2.4(a)) and the separated energy levels for the case of a QW, shown in panel Fig. 2.4(b).

In Fig. 2.4 electron transfer from the valence band to the conduction band under the optical excitation is marked by symbols σ^+ , σ^- and π , which refer to irradiation of the system by clockwise and anti-clockwise circularly and linearly polarized light, respectively. One can notice that since in quantum wells light-hole and heavy-hole levels are spatially separated, light with selected polarization allows to control the spin direction of the

Figure 2.4: Optical selection rule in (a) a bulk semiconductor and (b) a semiconductor quantum well, where σ^+ , σ^- and π correspond to the system illumination with specific light polarization: clockwise and anti-clockwise circularly and linearly polarized light, respectively, and labels of the states refer to the value of total angular momentum of the electron, J .

excited electron in the confined system. For example, illumination of the quantum well by σ^+ circularly polarized light with the energy sufficient for heavy-hole - conduction band transfer only leads to the coupling between the heavy-hole band with total momentum value $J = -3/2$ and the conduction band with $J = -1/2$.

In addition, we should mention the restriction on optical excitations between the states in valence and conduction bands related to the momentum conservation law. It says that electron transition accompanied by photon absorption is only allowed when the momentum of the final electron state differs from the initial state by the value of absorbed photon. Since the last is negligibly small in comparison to the electron momentum in crystal, we can assume that optical transitions in semiconductors only occur when quasi-momentum of initial and final states are roughly equal. Regarding the probability of electron transfer, this restriction states that electron transfer is more probable in direct semiconductors rather than in indirect band materials under the optical excitation. Nevertheless, it should be noted that optical transitions are still possible in indirect semiconductors, but they require to involve additional particles, like phonons, to meet the requirement of momentum conservation.

In the following one can see that band arrangement is a crucial ingredient for optoelectronic purposes and recalling the motivation of this thesis, in

the next chapters we will only focus on direct semiconductor structures, and particularly on GaAs material. In continuation of above discussion let us talk about the elementary electronic excitation and its interaction with external light in more details.

2.1.4 Excitons in semiconductor materials

As it was shown previously, the process of electron excitation is based on the emergence of the electron in conduction band and the creation of the hole in valence band. Recalling the fact that they both are charged particles, it is clear that they should experience an attraction arising from Coulomb interaction, which makes their motion mutually correlated and results in a formation of the electron-hole bound state. It was proved experimentally that this electron-hole coupled state is potentially able to move as a whole inside of the material and interact with other electron-hole bound states, which allows to consider it as a novel quasi-particle, called exciton, with the energy value close to band gap in studied semiconductor material.

From the historical point of view, detection of the excitons in the middle of last century was a result of a series of experimentally observed photon absorption peaks at the band gap energy range. Indeed, it was unexpected result that time since intuitively it was reasonable to anticipate that photons with energies smaller than band gap value will not be absorbed by semiconductor. This discovery stimulated theoretical studies of solids, which subsequently proved the existence of excitons in semiconductors and showed their fundamental properties [43, 44].

Nowadays two limiting exciton types can be distinguished depending on the radius of the quasi-particle. The first type mainly manifesting in insulator materials is called Frenkel exciton [45, 46], which is formed by the tightly bounded electron and hole placed on the distance comparable with the magnitude of the lattice constant of the material. The second type prevalent in semiconductors is called Mott-Wannier excitons [47]. In comparison to

Frenkel exciton, it has weaker electron-hole bounding and scales much larger than the lattice constant. Since we will consider only semiconductor materials in this thesis, only Mott-Wannier excitons will be present in the system and therefore, in the coming chapters, we will call them simply excitons keeping another type of excitons in mind.

Mathematically Hamiltonian of the system with interaction between the electron and the hole reads:

$$\hat{H} = E_g + \frac{\hbar^2 \nabla_e^2}{2m_e^*} + \frac{\hbar^2 \nabla_h^2}{2m_h^*} - \frac{e^2}{\epsilon_0 \epsilon |\mathbf{r}_e - \mathbf{r}_h|}, \quad (2.3)$$

where $\mathbf{r}_e$ and $\mathbf{r}_h$ are the coordinate vectors of the electron and the hole, respectively, ∇_e and ∇_h are corresponding Laplace operators, ϵ and ϵ_0 are the vacuum permittivity and dielectric constant of the medium, respectively.

From Eq. (2.3) the energy of the exciton can be easily found by the approach used in the problem of hydrogen atom and enclosed in the coordinate transformation [48]. Introducing the position vectors, such as the vector of center of mass, $\mathbf{R} = (m_e^* \mathbf{r}_e + m_h^* \mathbf{r}_h) / (m_e^* + m_h^*)$, and the vector of reduced mass, $\mathbf{r} = \mathbf{r}_e - \mathbf{r}_h$, the Hamiltonian $\hat{H}$ can be splitted in two independent parts. After that performing the well-known method for the calculation of the eigenvalues, the exciton energy spectrum is given by:

$$E_n(\mathbf{K}) = E_g - \frac{E_b}{n^2} + \frac{\hbar^2 \mathbf{K}^2}{2M}, \quad (2.4)$$

where M is the effective exciton mass, n is an integer, E_b is the binding energy, and $\mathbf{K}$ is the exciton wave vector.

Typically for any hydrogen-like system, the magnitude of binding energy, E_b , is proportional to the ionization energy of the hydrogen atom, which can be found in various fundamental textbooks, for e.g. see Ref. [49].

Speaking about quantum wells, where the carrier motion is confined in one direction, the Hamiltonian (2.4) should be corrected by an additional terms, performing the confining potential. Therefore total Hamiltonian for

the exciton in a 2D system is given by:

$$\hat{H}^{QW} = \frac{\hbar^2 \nabla_e^2}{2m_e^*} + \frac{\hbar^2 \nabla_h^2}{2m_h^*} - \hat{V}_e(z_e) - \hat{V}_h(z_h) - \frac{e^2}{4\pi\epsilon_0\epsilon|\mathbf{r}_e - \mathbf{r}_h|}. \quad (2.5)$$

Following the same routine as in a 3D case, the eigenvalues of the exciton in a QW read:

$$E_n^{QW}(\mathbf{K}) = E_g + E_e + E_h + \frac{\hbar^2 \mathbf{K}^2}{2M} - \frac{E_b}{(n - 1/2)^2}, \quad (2.6)$$

where E_e and E_h are the confining energies for the electron and the hole.

As one can see, 2D exciton spectrum (2.6) is similar to the exciton energy in the bulk material (2.4), meaning that it still has a quadratic dependence on the momentum with the only difference lying in the shift of the energy level by the amount of confining energies E_e and E_h .

In analogy to the electron, the quantum property, called a total angular momentum, has also been introduced for the excitons. By definition, total angular momentum of the exciton is a sum of the electron momentum, which equals to $J = \pm 1/2$, and the momentum of light-hole or heavy-hole with $J = \pm 1/2$ or $\pm 3/2$, respectively. Then the momentum of the heavy-hole exciton can take the values ± 1 and ± 2 while possible momentum values of the light-hole exciton are 0 and ± 1 .

It is interesting to note that apart from mutual interaction, excitons can also interact with light through the coupling with the photons with respect to selection rules. According to this restriction couplings between exciton states with momentum equal to 0 or ± 2 and a light with clockwise or anti-clockwise circular polarization describing the photons with values of angular momentum 1 or -1 , respectively, are forbidden. The group of excitons, which cannot couple with light with certain polarization, are called dark excitons. In contrast to dark excitons, light-hole excitons with momentum $J = \pm 1$ are allowed to interact with circularly polarized light and following conventional terminology are called bright excitons. Discussed process of

coupling between matter and light is nontrivial by itself and particularly, gives an opportunity for a new quasi-particle formation, which will be shown in the following sections.

2.2 Spin related phenomena in semiconductor systems

Initially quantum-mechanical description of the electron state only included the use of spatial coordinates and time with wave functions $\psi(x, y, z, t)$, which were sufficient for characterization of a simple system like a hydrogen atom. However the detailed study of a large group of phenomena and experiments such as Zeeman effect [50], Stern-Gerlach experiment [51] and others, forced to admit that this simple treatment of the electron states was not enough.

The additional quantum degree of freedom, which allowed explaining these detected phenomena, was called spin. By definition, it has a pure quantum nature and describes intrinsic angular momentum of the particle over its orbital angular momentum. Generally speaking, the spin of the particle is a vector quantity, which refers to its rotation around selected direction and can potentially change direction with time under external influences or internal interactions. As to the one of the most known examples, the magnitude of the electron spin is equal to $1/2$.

2.2.1 Spintronics overview

Referring to technological implementation, for many years quantum nature of the carriers was disregarded in macroscopic computing electronics, where the encoding of information data is mainly based on selected two-valued system states, which are usually represented by a distinct voltage levels [52, 53]. However, the aspiration to increase the performance of these electronic devices by miniaturization and simultaneous magnification of data

processing speed both arising from the industrial requirements faced the pure physical limit in structure size of currently existing systems and, therefore, led to a search for qualitatively different methods of information transport [54, 55, 56]. As an alternative to the charge-based electronics, the interest in capability to use spin properties of the carrier for data transfer has started to grow in recent years. In view of the fact that this approach is mainly focused on the use of the particle spin of freedom as basic working element for manufacturing of various kinds of devices, similar to the currently existing transistors, memory devices and logic circuits in charge-based electronics, this novel branch of technology has been called spintronics [11, 12, 13, 57]. Talking about the material suitability, diverse amount of metals and semiconductors were proposed and implemented for spintronics purposes nowadays. However, currently the most successful and well-studied structure in spintronics is a stack of layers composed of an array of alternating ferromagnetic and non-magnetic layers. The physical origin of its prosperity principally comes from the phenomenon, called giant magnetoresistance (GMR), which was independently shown by research teams of A. Fert and P. Grünberg in the end of last century [58, 59, 60]. Generally speaking, the effect of GMR lays in the drastic change in the resistance of the system under the mutual orientation of magnetization vectors of the magnetic layers from the relatively high value of resistivity for anti-parallel magnetization vector configuration and the relatively low value of resistivity for parallel magnetization vector configuration in the structure. This effect was theoretically justified later by J. Barnas and R. E. Campley. Their explanation of this unusual behavior of structure resistance was done under the idea that electron scattering strongly depends on its spin alignment [61, 62]. Following the predicted model, a subsequently enhanced study of GMR effect opened a new way for data storage, which in the short time was commercially implemented in magnetoresistive random-access memory (MRAM) cards, data reading devices and others [63, 64].

At present these metal-based structures widely occupy manufacturing sector of high technological industry, while semiconductor materials remain unused in modern spintronics. Nevertheless from the manufacturing point of view an ability to vary the properties of semiconductor materials along with an extensive knowledge of semiconductor fabrication processes hold a great potential of semiconductor materials in spintronics. Therefore following successful path of metal-based devices, profound study of spin dynamics in semiconductor systems has been started in last decades. Speaking about information transfer, in analogy to binary logic of current charge-based devices, most of the proposed semiconductor spin-based devices use the property of electron spin to perform only in two states, like spin state aligned parallel and anti-parallel along selected direction.

Regarding implementation of a spin degree of freedom in semiconductor technology, it is reasonable to start our discussion from the problem, connected with the process of creation of particular spin state in the system. Several methods, which allow to achieve particular spin alignment in semiconductor structure, have been developed nowadays. The most popular method among them is based on the use of optical effects, where electron spin alignment occurs due to external illumination of the sample. For the first time the effect of interaction between polarized light and electron spin was shown experimentally in silicon by Lampel in 1968 [65], and since then this method became the most common approach for spin alignment in spintronics, due to its convenience and simultaneous feasibility. Another technique used to create a particular spin state lies in the controlled injection of electrons with certain spin alignment from the ferromagnetic sample to the semiconductor. This idea was also proved experimentally [66, 67], however, from the point of view of technological implementation this method has faced difficulties, due to the difference in material conductivities [68].

From the motives mentioned above, one can conclude that realization of spin-based devices and their profitability mainly depend on the time scale

corresponding to the conservation of initial spin state during the carrier propagation through the structure, where we keep in mind that even in the absents of external fields, spin alignment can be changed due to various interactions between the carriers. Therefore, for spin-information transfer, it seems reasonable to choose the suitable semiconductor material with respect to the value of the parameter, called coherence length (or in ballistic regime, mean free path), which defines the scale corresponding to the conservation of the specific spin state. Qualitatively it is easy to understand that spin coherence length requires being relatively long for secure spin state transport along the structure. Therefore recalling the study of Kikkawa and Awschalom, where they found that spin coherence length in n-type GaAs is greater than in Si or Ge materials and, moreover, can reach hundreds of microns, the interest of spintronics scientific community switched to a search of suitable material among a group of III-V semiconductors [69]. In a short time, a subsequent study has been stopped on GaAs, since it has relatively long coherence length as most of III-V semiconductor materials and its growth technology is also well-studied.

Speaking about current accomplishments in spintronics, the most known example of a spin-based semiconductor device is Datta-Das transistor [70]. The principal complexity of other spin-based devices comes from various fundamental and commercial problems arising, for instance, from the influences of material impurities and defects or impacts of external fields on spin transport, which have to be resolved before semiconductor spintronics will take the central place in the modern industry. For that reason, fundamental mechanisms governing spin dynamics and methods of spin manipulation in semiconductor structures are thoroughly studied nowadays. Therefore, in the following section, we will highlight the key effects acting on the carrier spin alignment in semiconductors leaving aside the impacts of external fields on the system properties.

2.2.2 Spin-orbit coupling in two-dimensional semiconductor materials

Following the intention at the end of the previous section, it seems reasonable to start the study of internal spin effects from the overview of the effect of spin-orbit interaction (SOI). As the name implies, it comes from the coupling between the spin of the particle and its orbital motion in matter. Generally speaking, the emergence of SOI in solids is attributed to the group of purely relativistic quantum-mechanical effects, which results in an additional relativistic correction to the energy spectrum of the system and manifests in the lift of energy band degeneracy. Note that the value of this correction can be accurately found from the first principles using co-variant Dirac equation for the spin particle, however, the general picture of spin-orbit interaction can be intuitively understood in terms of semi-classical physics. For qualitative description of this effect, start from the fact that in solids during the propagation through the crystal lattice electrons experience the influence of an electric field induced by unscreened atomic nucleus of the crystal core. From the electron frame of reference, the crystal lattice creates moving electric field and, as a consequence, the internal magnetic field B_{eff} . Following foregoing idea, it is easy to understand that direction and magnitude of induced field entirely depend on characteristics of moving particle, such as traveling direction and velocity, in the chosen material. In return, recall the quantum nature of the carrier. For that reason induced magnetic field interacts with propagating electron by acting on its intrinsic magnetic moment or spin. Basically, this interaction manifests in the precession of the carrier spin about the direction of the magnetic field, B_{eff} , with precession vector $\Omega = g\mu_b B_{\text{eff}}/\hbar$, where g is the particle g-factor and μ_b is the magnetic constant, which subsequently results in the splitting of the energy bands. It should be stressed one more time that presented model is not accurate and was only used to familiarize the reader with the process of SOI. Therefore despite the fact that we have considered the effect of spin-orbit interaction

as an effective momentum-dependent magnetic field, it acts only on the spin property of the particle, meaning that charged carrier does not experience the influence of corresponding Lorentz force.

Mathematically the effect of spin precession in the effective magnetic field, B_{eff} , is introduced in the Hamiltonian of the system, $\hat{H}$, by an additional term: $\hat{H}_{SO} = \hbar\Omega \cdot \sigma/2$, where σ is the set of Pauli matrices, which in general case changes the eigenvalues of the system. For that reason, performing the simple calculation of the eigenvalue problem, one can obtain that the effect of SOI modifies the energy dispersion of the electron and particularly results in the splitting of conduction band into two branches shifted along the momentum axis with the energy difference $E_{SO} = 2\hbar|\Omega|$ (see Fig. 2.5).

Figure 2.5: Sketch of the effect of spin-orbit interaction on conduction band, where it results in the splitting in two bands by the value $2k_{SO}$ and the energy E_{SO} .

Quantitatively the effect of SOI might vary significantly from one semiconductor to another. For example, because of the symmetry of a crystal lattice in bulk Si semiconductor, the interaction between the spin and the orbital moment is insignificant and the energy of spin-orbit splitting reaches only 0.044 eV [71, 72]. This minor difference between the energy states in conduction band gives rise to relatively long period of time while spin remains unaffected, which is called spin relaxation time of the conduction electron. For example, it reaches 60 μs at low temperature [73] and decreases to 20 ns at room temperature [74], respectively. Therefore, at the first blush, the use of Si structures for spintronics purposes has strong applied potential. However, in the next place, one can notice that it meets a series of difficulties mainly arising from the problem of manipulation of selected

spin direction by external fields. Consider GaAs semiconductor, where the spin-orbit splitting is stronger in contrast to silicon, which mainly originates from the asymmetry of GaAs crystal lattice [75]. In analogy to Si material, spin-orbit interaction also exerts the influence on the spin relaxation times in GaAs but in comparison to the materials with symmetric lattices it makes the spin relaxation time shorter meaning that it only reaches 100 ps at room temperature [76]. Recalling the disadvantage of the relatively small energy value of spin-orbit interaction in Si, one can also expect that presence of strong SOI in GaAs material opens the way for relatively simple spin control by external fields, which makes GaAs more favorable structure for various spin-based purposes [77].

In the previous paragraph, we connected the effect of spin-orbit interaction with the appearance of the intrinsic effective momentum-dependent magnetic field coming from the lattice asymmetry of a bulk material. However, it should be noted that there are other structural effects, related to creation of structural asymmetry in semiconductor systems like quantum confinement in low-dimensional systems, which lead to formation of additional effective fields apart bulk contribution. Then, generally speaking, total internal effective magnetic field, which affects spin dynamics in the material, should be treated as a combination of several impacts dominating one over other and resulting in various exciting spin dynamics effects [78, 79].

Nowadays depending on the composition of pure semiconductor heterostructure three principal sources of system asymmetry have been highlighted:

1. Bulk inversion asymmetry (BIA), arising from the intrinsic property of the bulk crystal.
2. Structure inversion asymmetry (SIA), coming from the lack of symmetry of the confined system.
3. Neutral inversion asymmetry (NIA), induced by the configuration of heterostructure boundaries.

Graphically these basic types of inversion asymmetry are plotted in Fig. 2.6. Since each of them has a different nature, in the following sections we will discuss their influences on the carrier behavior separately and in more details.

Figure 2.6: The origins of inversion asymmetry in pure semiconductor systems:

(a) bulk inversion asymmetry associated with the crystal structure of the bulk material, (b) structural inversion asymmetry coming from the effective fields created on the edge of semiconductor heterostructure, (c) native interface asymmetry, which occurs because of the presence of non-common atom on the edge between two contacting semiconductors. In panel (c), heterostructure of InAs/GaAs is shown as an example of NIA, since the chemical bonds at the left interface of heterostructure are rotated 90° from the chemical bonds at the right interface.

2.2.2.1 Dresselhaus spin-orbit coupling

As previously mentioned, bulk inversion asymmetry occurs due to the lattice structure of selected crystal. Then following this definition, one can see that BIA is negligibly small in centrosymmetric crystals, such as Si or Ge, but performs in semiconductors with the lack of central lattice symmetry, like crystals with zinc-blended lattice, which is typical for GaAs and other III-V semiconductors. The initial idea of the effect of spin-orbit interaction caused by an asymmetric crystal potential belongs to G. Dresselhaus, and since then, the SOI induced by a bulk inversion asymmetry is named in his honor [80]. Mathematically term, describing the spin-orbit interaction in bulk materials caused by the Dresselhaus magnetic field, is given by:

$$\hat{H}_{SO}^{BIA(3D)} = \beta [k_x(k_y^2 - k_z^2)\sigma_x + k_y(k_z^2 - k_x^2)\sigma_y + k_z(k_x^2 - k_y^2)\sigma_z], \quad (2.7)$$

where β is the Dresselhaus spin-splitting coefficient, which contains the information about the strength of the Dresselhaus spin-orbit interaction and depends on material composition. From Eq. (2.7) one can notice the condition when this effect vanishes. Working in the Cartesian coordinates it happens during the propagation of the particle strictly along one of the principal axes (x, y, z) in the bulk crystal.

Switching to the case of a low-dimensional system, consider the quantum well with $[001]$ growth axis. Then from the general sense, one can rightly expect that confinement of the system will lead to the change of the spin-orbit interaction term (2.7). In the first approximation let us replace k_z component in Eq. (2.7) by its expected value, which is equal to $\langle k_z \rangle = 0$ and $\langle k_z^2 \rangle = (\pi/d)^2$, where d is the thickness of a QW. Accordingly, the term of spin-orbit interaction in a 2D system turns to:

$$H_{SOI}^{BIA(2D)} = \beta[k_x(k_y^2 - \langle k_z^2 \rangle)\sigma_x + k_y(\langle k_z^2 \rangle - k_x^2)\sigma_y], \text{ for } [001] \text{ growth axis} \quad (2.8)$$

Recall that typically k_z component in 2D systems is greater than both in-plane wave vector components, k_x and k_y , of the particle. Then following this approach several the terms in Eq. (2.8), which are cubical in in-plane components of k -vector, might be neglected. As a result in a two-dimensional system term corresponding to the Dresselhaus spin-orbit interaction can be approximated by the expression linear in $\mathbf{k}$ -vector:

$$\hat{H}_{SOI}^{BIA(2D)} = \beta \langle k_z^2 \rangle (-k_x \sigma_x + k_y \sigma_y), \text{ for } [001] \text{ growth axis.} \quad (2.9)$$

According to Eq. (2.9) the Dresselhaus field is oriented in the plane of a QW. Speaking about the effect of this induced field on the spin dynamics, one can remember that it might cause the change in the initial direction of the spin vector if it is not aligned along the field. For example, in the systems with $[001]$ growth axis spin direction loss appears when spin of the particle is initially oriented perpendicularly to a QW plane or lays in the plane of

a QW, but inclined to the direction of the field. For other configurations of the system, for example, the 2D structures with $[110]$ or $[111]$ growth axes, the directions of induced Dresselhaus field modify, see Fig. 2.7 and mathematically given by the following expressions:

$$\begin{aligned}\hat{H}_{SOI}^{BIA(2D)} &= \frac{2\beta}{\sqrt{3}} (k_y\sigma_x - k_x\sigma_y), \text{ for } [111] \text{ growth axis,} \\ \hat{H}_{SOI}^{BIA(2D)} &= 2\beta k_x\sigma_z, \text{ for } [110] \text{ growth axis.}\end{aligned}\tag{2.10}$$

Figure 2.7: Symmetry of the effective magnetic field induced by the bulk inversion asymmetry in a 2D quantum well on the Fermi surface, shown by a circle. The black arrows are introduced to illustrate the direction and magnitude of the field depending on the electron wave vector. The cases for $[001]$, $[111]$ and $[110]$ growth direction are shown in panels (a), (b) and (c), respectively. Note, on panels (a) and (b) effective magnetic field lie in the plane of the structure, while the field for $[110]$ case is oriented perpendicularly to the plane and its magnitude changes along the Fermi surface.

2.2.2.2 Rashba spin-orbit coupling

In addition to the impact of the bulk inversion asymmetry on spin dynamics, the emergence of spin-orbit interaction and subsequent tuning of the spin direction of the particle might also be connected with the lack of structural symmetry, which, for example, can be achieved by applied external electric field. Analogously to an external electric field, the crystal field in two-dimensional system, confining electrons in a QW plane, can be treated as a quasi-electric field leading to the creation of structural-inversion asym-

metry in the heterostructure as well. Following conventional terminology, both external and internal quasi-electric fields causing structural-inversion asymmetry in the system, refer to the process called the Rashba spin-orbit interaction and result in the energy band splitting. Leaving aside the influence of external electric fields, let us focus on the effect of internal effective magnetic field caused by SIA in a 2D system. Note that since quantum confinement is a purely internal effect, the magnitude of this crystal field depends only on the properties of the structure. Then mathematical model of a 2DEG with the Rashba coupling reads:

$$\hat{H} = \frac{\hbar^2 k^2}{2m^*} + \alpha(\mathbf{k} \times \boldsymbol{\sigma}) \cdot \mathbf{z} = \frac{\hbar^2 k_x^2 + \hbar^2 k_y^2}{2m^*} + \alpha(\sigma_x k_y - \sigma_y k_x), \quad (2.11)$$

where α is the parameter of the system, which depends on the properties of a QW.

From the Hamiltonian (2.11) the energy dispersion of conduction electrons with the Rashba coupling can be easily calculated:

$$E(\mathbf{k}) = \frac{\hbar^2 \mathbf{k}^2}{2m^*} \pm \alpha|\mathbf{k}|, \quad (2.12)$$

where $\pm$ sign corresponds to two spatially separated energy branches with the magnitude of the splitting proportional to the absolute value of $\mathbf{k}$ -vector of the particle. Schematically effect of Rashba SOI is shown in Fig. 2.8.

In agreement with those mentioned above, conceptual difference between internal effective magnetic fields induced by the bulk and the structural inversion asymmetries lies in the direction of induced magnetic field with respect to the crystal lattice structure. Compare the Dresselhaus magnetic field, which changes with the configuration of the crystal lattice, and the Rashba field, which does not depend on the lattice structure of the bulk material and relies only on the direction of confining potential, meaning that it is always oriented in-plane of a quantum well. Generally speaking, effects of both the Rashba and the Dresselhaus spin-orbit interactions might

Figure 2.8: Symmetry of the effective magnetic field induced by structural inversion asymmetry in a 2D quantum well on the Fermi surface, shown by a circle. Here we recall that effective magnetic field acts on the electron spin and results in the splitting of spin states. Then since states with opposite spin directions have different energies, Fermi circle is a set of two concentric circles with opposite direction of the effective field. The black arrows are introduced to illustrate the direction and the magnitude of the effective field depending on the electron wave vector.

be potentially present in 2D systems and affect the spin properties of the particle. For that reason in past years attention of scientific community was also focused on the study of interplay between these induced effective magnetic fields. It was found that in most studied multi-layered structures with a quantum well composed by a narrow-gap III-V material, the effect of SIA usually dominates over the effects of impurities or BIA, which for the case of a high-quality two-dimensional system allows to approximate the impact of spin-orbit effects by leaving only SIA contribution (or the Rashba coupling term) in the Hamiltonian without accuracy loss [81, 82, 83].

2.2.2.3 Spin-orbit coupling induced by native inversion asymmetry

As the last effect of the internal influence of the heterostructure on the spin dynamics of the propagating particle, which we would like to mention here, let us consider the impact of the edge of the heterostructure. In principle, an additional effective magnetic field might occur if the atomic layer on the boundary between two different materials consists of uncompensated number of either anions or cations without common atoms on the interface. The governing asymmetry was called native interface asymmetry (NIA). Since it comes from chemical bonds of the multi-layer structure, it is clear that NIA

strongly depends on the direction of heterostructure growth. Besides that talking about limiting cases, recall that it is a surface effect. Then it is easy to understand that the influence of NIA on spin properties of the particle increases with decreasing of thicknesses of composing layers and, moreover, reduces with the degradation of the interface, because of the component mixture on the boundary, coming from the imperfections of the interface.

Regarding the quantitative analysis of the effect caused by NIA, it should be mentioned that it is connected to the fundamental problem of isolation of NIA effect in the system. Therefore, microscopic parameters of the vector field, created by NIA in the heterostructures, are much less known, in comparison to BIA and SIA, and the importance of this effect is uncertain at present. Nevertheless, speaking about a good example of the system with energy splitting dominated by NIA, one can address to the heterostructure, composed of InAs/GaSb layers with perfect interface [84]. However, note that in the widely-implemented GaAs/AlGaAs heterostructures the effect of state splitting caused by NIA is absent, because of the presence of common atom at the edge between a quantum well and barrier layers.

2.2.3 Spin relaxation mechanisms in semiconductors

Another fundamental difficulty of spin-based device realization comes from the fact that specially prepared carrier spin alignment does not match an equilibrium state of the system and, therefore, remains in the system only for a finite amount of time. For the quantitative description of the spin dynamics inside of a real material the physical quantity, called spin relaxation time, was introduced, which following conventional definition displays the time required to reduce the number of aligned spin states by a factor of $1/e$. In the matter of the value of spin relaxation time in real structures, note that it might vary from several picoseconds to nanoseconds, which is mainly governed by the properties of a semiconductor material and operating temperature.

Therefore, it seems to be appropriate to begin the study of the spin dynamics in low-dimensional systems from the short overview of the main mechanisms of spin relaxation interplaying inside of semiconductor structures. From the general sense, it is clear that presence of any magnetic field in the system might potentially control the behavior of the carrier spin and, consequently, cause the loss of its initial alignment. Bring to mind the discussion from the previous sections and the central idea, that along with external real magnetic fields, built-in effective magnetic fields originated from the spin-orbit interaction also have an authority over spin evolution and, therefore, might lead to the spin information loss as well. Up to now apart the influence of an external magnetic field scientific community distinguishes four relevant mechanisms governing the evolution of specially prepared spin states in semiconductor, called the Elliott-Yafet, the D'yakonov-Perel', the Bir-Aronov-Pikus and the hyperfine interaction, which are mainly named in honor of the pioneer scientists. For clarity in the following sections, we will highlight the physical origin of each of these mechanisms and the limits of their applicability.

2.2.3.1 Elliott-Yafet spin relaxation

Starting from the Elliott-Yafet (EY) mechanism of spin relaxation, it should be initially mentioned, that fundamentally it is connected to the effects of spin-orbit interaction and reflects the case, when the loss of initial spin orientation is driven by the processes of particle scatterings on phonons or impurities and accompanied by the change in the particle spin alignment. A schematic picture of the process of spin relaxation, guided by the Elliott-Yafet mechanism is shown in Fig. 2.9.

From historical retrospect, the initial idea of this process belongs to R. J. Elliott, who in 1954 showed that since the eigenvectors of the particle in the crystal lattice consist of indecomposable combinations of spin and orbital components, carrier spin states should be treated as mixed states, due to

Figure 2.9: Picture of the Elliott-Yafet mechanism of spin relaxation, where the driving effect of initial spin alignment loss comes from inelastic scattering collisions.

the presence of spin-orbit interaction effects [85]. Then following this basic idea, he stated that spin-independent scattering events, such as interactions with phonons or impurities, accompanied by the random change in the direction of carrier momentum, simultaneously result in the switch of the carrier spin direction. In a short time later Elliott [85] and Yafet [86] determined this effect as one of the governing processes of spin dynamics, which depends on the strength of spin-orbit coupling and the number of impurities in the structure. Particularly, the dependence on the number of particle conclusions means that this mechanism of spin relaxation becomes dominant in systems with large amount of impurities or high value of spin-orbit strength. Mathematically this dependence has been written proportionally to the carrier momentum scattering time as:

$$\frac{1}{\tau_s^{EY}} = A \left(\frac{E_k}{E_g} \right)^2 \left(\frac{\Delta_{SO}}{E_g + \Delta_{SO}} \right)^2 \frac{1}{\tau_p}, \quad (2.13)$$

where A is a dimensionless constant, Δ_{SO} is the value of spin-orbit splitting in the selected material, E_k is the electron energy, E_g is the value of band gap, and τ_p is the carrier momentum scattering time.

As one can notice from the expression (2.13), the Elliott-Yafet mechanism of spin relaxation is present in most common semiconductors and has a relatively high impact on the spin dynamics of the particle in narrow band gap semiconductors with a large value of spin-orbit splitting, Δ_{SO} , while the electron momentum scattering time, τ_p , is relatively short.

2.2.3.2 D'yakonov-Perel' spin relaxation

The second effect, governing the process of spin relaxation in crystals, was called the D'yakonov-Perel' (DP) mechanism. Following the definition, the loss of initial spin orientation in this regime appears due to the spin vector precession around intrinsic effective magnetic field arising from an asymmetric lattice arrangement [87]. Qualitatively the process of the D'yakonov-Perel' spin relaxation is shown in Fig. 2.10.

Figure 2.10: Picture of the D'yakonov-Perel' mechanism of spin relaxation, which comes from the influence of an effective magnetic field in the system. Generally speaking, the loss of initial spin direction can be explained by the spin precession along the direction of momentum-dependent field.

The origin of spin direction randomization in the DP regime can be intuitively understood as follows. Consider the electron with some initial wave vector, $\mathbf{k}$, and spin vector, $\mathbf{s}$, where the last precesses along the direction of an internal magnetic field $B(\mathbf{k})$. Next, imagine that after a single scattering event, electron wave vector was changed to $\mathbf{k}'$ and because of the connection between $\mathbf{k}$ and $\mathbf{s}$ vectors, electron spin $\mathbf{s}$ immediately starts to precess around a new direction, corresponding to the direction of a magnetic field $B'(\mathbf{k}')$. Repeating this procedure, multiple scattering events lead to the loss of initial spin direction of the selected particle and, more generally speaking, result in a decrease in the total number of carriers with initial spin alignment.

As one can notice a key ingredient of the D'yakonov-Perel' spin relaxation mechanism lies in the presence of the effective magnetic field in the structure. Recall the basic internal sources leading to the creation of built-in effective magnetic field. They are mainly related to the presence of any asymmetry in the system, like BIA, SIA and NIA. This conclusion brings us to the fact

that the D'yakonov-Perel' regime of spin relaxation is not applicable to the bulk symmetric semiconductors, such as Si or Ge, but it might potentially exist in the systems with the lack of inversion symmetry, for example, in GaAs heterostructures, where the D'yakonov-Perel' and the Elliott-Yafet mechanisms of spin relaxation coexist together, but with different strengths.

Mathematically the D'yakonov-Perel' relaxation time is given by:

$$\frac{1}{\tau_s^{DP}} = Q\alpha^2 \left(\frac{E_k^2}{\hbar^2 E_g} \right) \tau_p, \quad (2.14)$$

where Q and α are constants, related to the spin-orbit interaction strength.

According to Eq. (2.14), one can see that the D'yakonov-Perel' regime of spin relaxation is prevalent over the Elliott-Yafet mechanism in wide band gap semiconductors with weak momentum scatterings. Comparing these two regimes, it is important to stress that the main difference between the Elliott-Yafet and the D'yakonov-Perel' mechanisms lies in their dependence on the momentum scattering time. In particular, the Elliott-Yafet relaxation mechanism becomes more effective under the increase in the number of scattering events, while the effectiveness of the D'yakonov-Perel' regime of spin relaxation decreases.

2.2.3.3 Bir-Aronov-Pikus spin relaxation

Another aforecited regime of spin information loss, known as the Bir-Aronov-Pikus (BAP) mechanism, is based on the effect of electron-hole exchange interaction, which gives rise to an additional fluctuating effective magnetic field in crystals [88]. Generally speaking, the effect of spin relaxation in the Bir-Aronov-Pikus regime is pretty similar to the D'yakonov-Perel' regime with the only difference that unlike in the last, the effective magnetic field in the Bir-Aronov-Pikus regime is produced by holes. The change of spin direction or spin flip in the BAP regime can be understood as the pronounced overlap between the electron and the hole wave functions, which is presented schematically in Fig. 2.11.

Figure 2.11: Illustration of the Bir-Aronov-Pikus mechanism of spin relaxation, where the probability of spin flip occurs, due to pronounced overlap between the wave functions of the electron (solid line) and the hole (dotted line).

According to the definition, the process of spin relaxation induced by exchange interaction with holes mainly depends on to the concentration of unpolarised holes in the system, which makes this mechanism most likely dominating at low temperatures in highly doped semiconductors with relatively large amount of unpolarized holes.

2.2.3.4 Hyperfine interaction spin relaxation

The last process affecting the spin dynamics of the particle, which will be mentioned here, is called the hyperfine interaction (HFI). Speaking about the origin of this effect, it is interesting to note that similarly to the Bir-Aronov-Pikus mechanism of spin relaxation it comes from the overlap between the wave functions. However in the hyperfine interaction regime, it occurs from the overlap between the electron and the nuclei wave functions [89]. Leaving aside the accurate derivation of the spin relaxation time, governed by the hyperfine interaction mechanism, from the definition one can find that its effect on spin dynamics is proportional to the value of the overlapping between two wave functions [90]. From the quantitative analysis it was obtained that this mechanism is rather weak to become dominating in real bulk semiconductors while it becomes more relevant in the confined systems, but still concedes to the previous mechanisms of spin relaxation.

2.3 Strong coupling regime in two-dimensional systems

Recalling the motive of this thesis to use alternative spin particles for spintronics purposes, let us return to the effect of electron excitation under the illumination of a semiconductor material.

As it was mentioned previously, one of the interesting fundamental results in solid state physics was connected to the observation of a distinctive electron-hole coupled state under the optical excitation in semiconductor system, called an exciton. It should be noted, that despite the scientific prove of this effect, that time several subsequent optical experiments showed unexpected result at odds with developed exciton theory again. For the theoretical explanation of a new unknown phenomenon S. I. Pekar in 1957 (see, Ref. [91]) was compelled to introduce additional eigenvectors of the system, which gave a start to further theoretical studies. In the same year, two independent scientists V. M. Agranovich [92] and J. J. Hopfield [93] explained this nameless behavior of the system by an idea, that in the presence of external light system can potentially transfer to a new coupled state, originated from the coupling between the photon and the exciton. This novel quasi-particle with extraordinary properties coming from its combined light-matter nature was called an exciton-polariton. For simplicity of the following discussion, let us name this light-matter coupled state a polariton, keeping in mind about the existence of other types of polartions, which will not be covered in this thesis.

Speaking about the setup for experimental study of polariton states, it was intuitively clear that observation of distinct polariton states required the improvement of the coupling between the exciton and the photon, which can be achieved by the increase of the photon lifetime inside of the structure. Without going deep into details, let us note here that relatively long lifetime of the photon in the system can be achieved via the chain processes of

photon absorption and photon emission by selected semiconductor medium. Therefore, following this intention the choice of the optical devices, called resonators, among other various types of the systems, seemed fairly reasonable. Note that potential ability of photon reabsorption in resonators originates from their basic configuration, which leads to the confinement of photons inside of the structure. Generally speaking, the resonator is a system, formed by two mirrors placed at the distance equal to multiple integers of the photon half-wavelength. It should be stressed that regarding the use of the resonators for experiments involving creation of exciton-polaritons, the medium of the resonator should be made of a semiconductor material, where the exciton states might be created under the effect of photon absorption. The last point brings us to the main technical difficulty of the polariton system realization originated from the properties of resonator mirrors. It was found that the usage of metallic mirrors faces several problems, such as the fundamental problem of relatively low reflectivity of metals in UV range and underlying technological problem, arising from the difficulty of the growth of semiconductor material on the surface of metallic mirror. For that reason an alternative model of the resonator with reflecting mirrors formed by multilayer semiconductor structures, or distributed Bragg reflectors (DBRs), was proposed.

Single DBR is a periodic stack of semiconductor bilayers with different values of refractive index, where the physical origin of high reflection comes from the optical effect of the interference inside of the multilayer system. Recalling the optical laws, one can infer that high DBR reflection can be achieved when the optical thicknesses of alternate semiconductor layers match to the quarter of photon wavelength. From this conclusion, the operating limit and the suitability of DBRs for resonator purposes can be estimated. In particular, this restriction shows that, firstly, the lifetime of the photon inside of resonator strongly depends on the number of layers in DBR and, secondary, only photons with selected values of wavelength are able to remain inside

of the system. Remembering the effect of particle-wave duality, the last outcome can be also interpreted as a process of electromagnetic field quantization along the direction of structure growth, which allows to treat the resonator formed by DBRs, as photonic analog of a quantum well, where the effect of quantum confinement comes from the total internal reflection at the boundary. Returning to DBR properties, let us introduce a valuable parameter defining its operating range. Imagine the case, when the wavelength of the photon is slightly tuned from the optimal value of DBR. Then according to the previous discussion, it is clear that reflection of DBR should decrease with the wavelength detuning but remain relatively high. The range of the photon wavelengths, which allows obtaining relatively high values for resonator reflectivity, is called a DBR stopband, which has a great technological importance for mirror classification.

Since we briefly discuss the properties of principal elements of the resonator, it is also necessary to consider the influence of an embedded semiconductor material on the resonator reflection spectrum. Along with the study of this class of systems, it was shown that resonator medium has a potential ability to modify the reflectance of the system and create a narrow window in the DBR stopband range. Following the optical rules, the behavior of the reflection spectrum of the system at different values of the photon wavelength can be easily simulated. For example, the reflectance of AlGaAs/GaAs system is shown in Fig. 2.12.

In the conclusion, let us introduce a particular type of the resonators, so-called planar microcavity, where the energy of the confined photon is comparable to the exciton energy of the resonator medium. Generally, a microcavity is a system with a quantum well, placed inside of the resonator and formed by semiconductor material with bandgap close to the energy of confined photon. Principal scheme of this setup is plotted in Fig. 2.13.

Figure 2.12: Reflectivity profile of a DBR microcavity, formed by a top and bottom mirror with 15 and 21 repeats of GaAs/AlAs with 240 nm-thick GaAs QW sandwiched between two DBRs. The appearance of the narrow pick in reflectance spectrum of DBR comes from the presence of GaAs quantum well. Note that without it, there will be a reputable picture of the stopband of DBR [94].

Figure 2.13: Geometry of a microcavity formed by Bragg mirrors with a quantum well inside, where the excitonic states can be created by illumination.

2.3.1 Exciton-polaritons in semiconductor microcavities

Consider the behavior of the photon, which is confined inside of the microcavity, formed by two DBR mirrors placed at the distance L_{cav} from each other, in more details. Recalling the particle-wave duality, let us discuss the effect of the photon confinement in terms of the wave vector of the propagating electromagnetic wave, $\mathbf{k} = k_{\parallel} + k_z$, where $k_{\parallel}$ denotes the component of the wave vector oriented parallel to the mirror plane and k_z denotes the projection of the wave vector oriented perpendicularly to DBR mirrors. Note that in the microcavity propagation of the photon and corresponding electromagnetic wave is not confined in the direction parallel to DBR mirrors and, therefore, its behavior can be still treated as a free propagation of electromagnetic wave in the medium without any restriction on the component $k_{\parallel}$. Consider the behavior of the k_z photon component, oriented perpendicularly to the mirrors. Recalling the effect of photon confinement, which imposes a restriction on the allowed wavelengths inside of the system, and the relation between the wavelength and the wave vector of the particle, possible values of the k_z component in the microcavity turn to a discrete set, which following the requirement of the interference reads: $k_z = n\pi/L_{cav}$, where n is the number of a photonic mode in the system. Limiting our consideration by the photonic mode with the lowest energy, factor n turns to 1 and, subsequently, photon component k_z becomes: $k_z = \pi/L_{cav}$.

According to above discussion, it is clear that the photon confinement also affects the energy dispersion of the photon in the microcavity. New energy spectrum of the photon can be found from the linear dispersion of a free photon by substituting the allowed value of the photon wave vector, k_z . Performing basic mathematics, the energy spectrum of a confined photon

reads:

$$E_C(k_{\parallel}) = \frac{\hbar c}{n_{cav}} \sqrt{k_z^2 + k_{\parallel}^2} \approx \frac{\hbar c k_z}{n_{cav}} \left(1 + \frac{k_{\parallel}^2}{2k_z^2} \right) = E_C(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_{cav}}, \quad (2.15)$$

where n_{cav} is the refractive index of the cavity medium, $E_C(0) = \hbar c \pi / n_{cav} L_{cav}$, and $m_{cav} = \hbar n_{cav} / (c L_{cav})$ is the effective photon mass.

From Eq. (2.15), one can extract two limiting cases of the cavity photon dispersion with respect to the value of the wave vector $k_{\parallel}$, which can be introduced as follows. At large values of the k_z component the energy spectrum of the cavity photon still obeys the linear dependence on its wave vector and, therefore, can be approximated by the expression for $E_C(0)$ [95, 96]. Otherwise, at small values of the k_z component the energy of the cavity photon degenerates into a nearly parabolic function $E_C(k_{\parallel}) \propto k_{\parallel}^2$, which in analogy to the energy dispersion of a free electron can be rewritten using the factor, called the effective mass of the photon in the microcavity, m_{cav} , remembering the massless nature of the photon.

Speaking about the creation of a novel quasi-particle in the microcavity, we should also note that generally microcavity can operate in two regimes, depending on the strength of the interaction between the cavity photon and the exciton in the microcavity medium. One of them is called a weak coupling regime, where photons and excitons are treated independently and do not interact with each other, and in return, another regime is known as a strong coupling regime, where photons and excitons start to interact, which results in the formation of exciton-polaritons. The qualitative difference between the prevalent operating regime is based on the amount of time corresponding to the lifetime of the photon inside the system. Regarding the last parameter, the definition of the operating regime in the microcavity can be represented as following. The weak coupling regime occurs in the system, where the photon escapes from the microcavity faster than it might be re-absorbed by the medium, while the strong coupling regime takes place in the system, where the photons formed by the exciton recombination in

the medium remain in the microcavities for a time sufficient for the multiple re-absorption by matter. These operating regimes have been proved by the optical experiments of the light reflection and transmission, where the weak coupling regime performed by a single peak in the reflection spectrum of the system, corresponding to the value of the exciton energy in the medium and the strong coupling regime appeared as an unusual signal given by two optical resonances separated by the value, called the vacuum-field Rabi splitting.

The formation of an exciton-polariton state in the microcavity has been described within a relatively simple theoretical model of the two coupled oscillator Hamiltonians, which is applicable when the composite nature of the excitons is neglected. Physically this assumption can be interpreted via the exciton concentration in the system, meaning that this model is only valid, when the concentration of the excitons is relatively low, which along with remaining strong exciton binding energy leads to a decrease of the effect of Coulomb interaction between electrons and holes composing different excitons. Otherwise, in the case when this requirement is not fulfilled, and the concentration of electrons and holes is relatively high, the Coulomb interaction between the particles starts to play the key role and results in the vanishing of the exciton states and subsequent exciton-photon coupling.

Working in the limit of a low particle concentration, the Hamiltonian of the system can be written using the secondary quantization creation-annihilation operators as:

$$\hat{H} = \sum_{k_{\parallel}} E_C(k_{\parallel}) a_{k_{\parallel}}^{\dagger} a_{k_{\parallel}} + \sum_{k_{\parallel}} E_X(k_{\parallel}) b_{k_{\parallel}}^{\dagger} b_{k_{\parallel}} + \sum_{k_{\parallel}} \hbar\Omega (a_{k_{\parallel}}^{\dagger} b_{k_{\parallel}} + b_{k_{\parallel}}^{\dagger} a_{k_{\parallel}}) \quad (2.16)$$

where operators $a^{\dagger}, a$ and $b^{\dagger}, b$ are the creation and the annihilation operators for the photons and the excitons with in-plane wave vector $k_{\parallel}$, respectively, and Ω_R is the constant of the exciton-photon coupling.

In Eq. (2.16) the first term corresponds to the energy of the cavity photons, which have the parabolic dispersion (2.15) according to the discussion

in the previous section, the second term is the superposition of allowed exciton energy states in the microcavity, which are characterized by the quadratic dependence on the wave vector $k_{\parallel}$ as well, and the last term corresponds to the effect of exciton-photon coupling.

Recall that in real systems the exciton and the photon states have finite lifetimes inside the microcavity. Then for reliability of the model of two coupled oscillators, the process of the particle decay might be taken into account using an additional term in the dispersion relations for the photons and the excitons. The energy spectrum of the photon with finite lifetime (2.15) should be written as:

$$E_C(k_{\parallel}) = E_C(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} - i \frac{\hbar \Gamma_C(k_{\parallel})}{2}, \quad (2.17)$$

where $\Gamma_C(k_{\parallel})$ is the photon decay rate, which originates from the fact that photons might leave the microcavity through the DBR mirrors.

In analogy, the energy dispersion for the exciton should be rewritten as:

$$E_X(k_{\parallel}) = E_X(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_X} - i \frac{\hbar \Gamma_X(k_{\parallel})}{2}, \quad (2.18)$$

where m_X is the exciton mass, and $\Gamma_X(k_{\parallel})$ is the exciton non-radiative decay rate, which shows the finite width of the exciton resonance.

Coming back to the effect of the exciton-photon coupling in the Hamiltonian (2.16) and recalling the approach of two coupled oscillators, the interaction constant, Ω_R , can be found using the relation:

$$\Omega_R = \frac{d}{2\pi a_B^{2D}} \sqrt{\frac{E_X}{\hbar^2 n_{cav} \lambda_0}}, \quad (2.19)$$

where a_B^{2D} is the 2D exciton Bohr radius, d is the dipole matrix element of the radiative exciton transition in the microcavity, and λ_0 is the resonant wavelength of the cavity.

Since we have introduced the major effects, which occur in the microcavity, now we become able to calculate the eigenstates of this system by

solving the eigenvalue problem with the Hamiltonian (2.16) represented in the matrix form by the expression:

$$\begin{vmatrix} E_X(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_X} - i\frac{\hbar\Gamma_X}{2} - E(k_{\parallel}) & \frac{\hbar\Omega_R}{2} \\ \frac{\hbar\Omega_R}{2} & E_C(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} - i\frac{\hbar\Gamma_C}{2} - E(k_{\parallel}) \end{vmatrix} = 0. \quad (2.20)$$

Using the well-known procedure and introducing the parameter δ , which reflects the detuning between the cavity and the exciton modes $\delta = E_C(0) - E_X(0)$, the energy spectrum performs:

$$E^{UP(LP)}(k_{\parallel}) = \frac{1}{2} \left(\frac{\hbar^2 k_{\parallel}^2}{2m_X} + \frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} + \delta - i\hbar \frac{\Gamma_X + \Gamma_C}{2} \right) \quad (2.21)$$

$$\pm \frac{1}{2} \sqrt{\left[\frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} - \frac{\hbar^2 k_{\parallel}^2}{2m_X} + \delta - i\hbar \left(\frac{\Gamma_C + \Gamma_X}{2} \right) \right]^2 + (\hbar\Omega_R)^2}, \quad (2.22)$$

where “ $\pm$ ”- sign denotes to two eigenvalues of the system. Note that following the conventional terminology, the energy $E^{UP}(k_{\parallel})$ with “+”- sign is called the upper polariton branch and the energy $E^{LP}(k_{\parallel})$ with “-”- sign is called the lower polariton branch, respectively.

In order to analyze the obtained dispersion relation (2.22), for simplicity lets limit our consideration by the case when both parameters of the decay, Γ_C and Γ_X , are equal to zero, meaning that there are no relaxation processes in the system. Therefore, Eq. (2.22) reads:

$$E^{UP(LP)}(k_{\parallel}) = \frac{1}{2} \left(\frac{\hbar^2 k_{\parallel}^2}{2m_X} + \frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} + \delta \right) \pm \frac{1}{2} \sqrt{\left[\frac{\hbar^2 k_{\parallel}^2}{2m_{cav}} - \frac{\hbar^2 k_{\parallel}^2}{2m_X} + \delta \right]^2 + (\hbar\Omega_R)^2}. \quad (2.23)$$

From Eq. (2.23), an ability of the system to perform in two different regimes depending on the value of exciton-photon coupling, Ω_R , becomes clear. Assume that the parameter Ω_R is relatively small in comparison to the energy of a bare exciton and a photon or even is equal to zero. Then the

dispersion relation $E^{UP(LP)}(k_{\parallel})$ degenerates into two independent solutions corresponding to the energy spectrum of a bare exciton and a photon, which according to above definition allows us to refer this condition to the weak coupling regime. Alternatively, when the impact of the parameter Ω_R is significant in comparison to the energy of an exciton and a photon, the dispersions of a bare cavity photon and a exciton rearrange into the new states, corresponding to the creation of a polariton with the energy spectrum defined by the upper and the lower polariton branches. For that reason, the last condition can rightfully be referred to the strong coupling regime in the microcavity. This dependence of the dispersion relation on the strength of the exciton-photon coupling defines the transfer between the weak and strong coupling regimes, called the cross-over, which is schematically depicted in Fig. 2.14(a). From the first sign, one can say that this illustration contradicts the before mentioned dispersion relation of an exciton, which should have a parabolic dependence on $k_{\parallel}$, but has a flat shape in Fig. 2.14(a). The qualitative explanation of this difference lies in dissimilarity of an exciton and a photon effective masses. Indeed, estimating the values of the exciton and the photon masses in the realistic semiconductor, one can find that the exciton mass, $m_X = m_e^* + m_h^*$, is of the order of a free electron mass, which is five orders of magnitude greater than the cavity photon mass. Therefore, the exciton dispersion is broader than the photon dispersion and looks flat on the typical scales of the polariton system.

Including the effect of detuning between the modes, δ , one can notice that along with the influence of coupling term, Ω_R , on the system dispersion, the effect of detuning between the exciton and photon modes has a crucial impact on the non-parabolic shape of polariton dispersion, as it is shown in Fig. 2.14(b, c).

Speaking about the distance between the upper and the lower polariton branches, it should be mentioned that when the energy of the exciton mode is equal to the energy of the cavity photon (or $\delta=0$), the splitting between

Figure 2.14: Polatiron dispersion in the microcavity. The bare exciton and photon dispersions are shown by red lines and the polariton dispersion is plotted by a blue line. Panel (a) corresponds to the case when the energy of the exciton transition in the structure is equal to the photon energy. Panels (b) and (c) shows the polariton dispersion when energy values of the exciton and the photon differ by parameter $\delta = -10$ meV and $\delta = 10$ meV, respectively. Here UP is the upper polariton branch, and LP is the lower polariton branch.

polariton modes, coming from the interaction between quantum confined excitons and the cavity mode, is minimal and equals to $\hbar\Omega_R$. In analogy to energy splitting at atomic resonance in atomic physics, this splitting was called vacuum-field Rabi splitting [97, 98], which varies from the few meV in GaAs semiconductor microcavities to hundred of meV in organic microcavities.

Remember that for simplicity of the discussion in Eq. (2.22) we roughly assumed that the cavity photons and the excitons remain in the system for an infinite time, which is incorrect in the real systems. Therefore, taking into account the effects of the exciton recombination and the finite photon lifetime caused by the mirror quality in the microcavity, polariton eigenstates become complex-valued functions with additional terms corresponding to the dissipation effects. Then as it is clearly seen from Eq. (2.22) polariton dispersion starts to depend on the relation between the strength of the coupling, Ω_R , and dissipation effects. Analyzing the result, note that if the interaction between modes is smaller than the correlation between mode losses ($\hbar\Omega_R < \Gamma_X + \Gamma_C$), the dispersion of the system remains unaffected and performs as a system of weakly interacting independent photon and exciton modes. Alternatively, if the expression under the square root is positive

($\hbar\Omega_R > \Gamma_X + \Gamma_C$), the exciton-photon coupling is prevalent, which leads to the strong coupling between these modes and creation of a polariton state.

Note the significant properties of the polaritons coming from their dual light-matter nature. Indeed, from the photonic part polaritons have small effective mass and long coherence length, which allows them to propagate rapidly in the structure and to be measured optically, and from the excitonic nature polaritons can interact with each other. Speaking about the polariton lifetime, it should be mentioned that in modern microcavities photons can stay in the system for a few picoseconds while the exciton state lives longer and can reach a nanosecond range, which quantitatively reflects that the polariton lifetime is mainly governed by the photonic component and lies in the picoseconds range.

Summarizing the above, it sounds intriguing to study unique features of polaritons for the further scientific applications in more details. Therefore, in the following sections, one can find a brief overview of the selected topics in polaritonics, which will be necessary for better understanding of the subsequent chapters.

2.3.2 Spin property of exciton-polaritons

Note that along with finite lifetimes optically created polaritons from the photonic components constituting these quasi-particles also adopt their spin directions. According to the conservation laws, it is easy to understand that exciton-polaritons can be only created under the coupling between the heavy-hole excitons with the total angular momentum $J = \pm 1$, which following conventional terminology of quantum optics is referred as exciton spin, and photons with the same spin value, which reflects the polarization of the pumping light. Indeed, recall that generally the total exciton angular momentum has projections ± 1 and ± 2 on the structure axis. Keeping in mind that the photons with spin $+1$ and -1 are referred to the process of system illumination by a right, σ^+ , circular polarized light and a left, σ^- ,

circular polarized light, respectively, while a linearly, π , polarized light creates both photon states, and the fact that spin should be conserved during the process of photon absorption, one can see that the dark exciton states with $J = \pm 2$ can not be optically created and only the bright exciton states with $J = \pm 1$ have an ability to couple with light. Therefore, polaritons can be represented as a two-level system with the spin directions, corresponding to the states with the total angular momentum $J = -1$ and $J = +1$.

In order to describe the polariton spin state, it seems reasonable to introduce the polariton spin alignment similarly to the electron spin by 2×2 density matrix $\mathcal{S}_{\mathbf{k}}$ as:

$$\mathcal{S}_{\mathbf{k}} = \frac{N_{\mathbf{k}}}{2} \hat{I} + S_{\mathbf{k}} \cdot \sigma_{\mathbf{k}}, \quad (2.24)$$

where $\hat{I}$ is the identity matrix operator, $N_{\mathbf{k}}$ is the number of polaritons, $S_{\mathbf{k}}$ is the mean polariton spin vector, characterized by the wave vector $\mathbf{k}$, and σ is the vector of Pauli matrices.

Note that further we will discuss the spin states in terms of circularly polarized states, S_z , and their linear combinations defined by S_x and S_y spin components, yield to a linearly polarized light emission process. In particular, we will associate S_x polariton spin state oriented parallel to x -axis to x -linearly polarized light and oriented anti-parallel to x -axis to y -linearly polarized light and S_y polariton spin state oriented along the y -axis to diagonal linear polarizations. Thence the polariton spin states can be introduced in the quantum analogy of the Stokes vector of the light, where the mean value of the spin operator matches the Stokes vector of the partially polarized light and can be represented by the point on the surface of the sphere, which is shown in Fig. 2.15. This sphere is also called Poincaré sphere [96] and its radius is equal to the total intensity of the polarized light. According to this representation the polariton spin vector of the coherent light is a vector, which begins in the central point of the sphere and ends on its surface, while the direction of this vector defines the polarization of

emitted light. From the Poincare sphere (see, Fig. 2.15) it becomes clear that for sufficient polarization picture we only need to include four quantities, such as the total intensity, the intensity of two linear polarizations and the intensity of a circularly polarized light component.

Figure 2.15: Graphic representation of the polariton spin on the Poincaré sphere, in analogy to the light polarization representation.

For mathematical description of the polariton state with the wave vector $\mathbf{k}$, remember that if polariton concentration is low, we can treat polaritons as a bosons with corresponding bosonic annihilation operators $a_{\mathbf{k}\uparrow}$ and $a_{\mathbf{k}\downarrow}$. Consequently, the components of the polariton spin vector, $\mathcal{S}_{\mathbf{k}}$, can be written using the well-known expressions, where these physically observable quantities are given by products of creation and annihilation operators with opposite spin projections [99, 100, 101]:

$$N_{\mathbf{k}} = N_{\mathbf{k}\uparrow} + N_{\mathbf{k}\downarrow} = \langle a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\uparrow} \rangle + \langle a_{\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\downarrow} \rangle, \quad (2.25)$$

$$S_{z,\mathbf{k}} = s_{z,\mathbf{k}} N_{\mathbf{k}} = \frac{1}{2} \left(\langle a_{\mathbf{k}\uparrow}^\dagger a_{\mathbf{k}\uparrow} \rangle - \langle a_{\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\downarrow} \rangle \right), \quad (2.26)$$

$$S_{x,\mathbf{k}} = s_{x,\mathbf{k}} N_{\mathbf{k}} = \text{Re} \langle a_{\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\uparrow} \rangle, \quad (2.27)$$

$$S_{y,\mathbf{k}} = s_{y,\mathbf{k}} N_{\mathbf{k}} = \text{Im} \langle a_{\mathbf{k}\downarrow}^\dagger a_{\mathbf{k}\uparrow} \rangle, \quad (2.28)$$

where $a_{\mathbf{k}\uparrow}$ and $a_{\mathbf{k}\downarrow}$ are annihilation operators of a right and a left circularly polarized light and brackets $\langle \rangle$ correspond to the mean values.

Speaking about polariton spin dynamics, we need to stress that in real systems polariton spin does not retain its initial direction for infinitely long time. Sooner or later it will be changed by various spin relaxation effects, like

the interaction with an effective magnetic field or scatterings. Remember that for non-interacting polariton systems, temporal evolution of the spin density matrix, $\mathcal{S}_{\mathbf{k}}$, is given by Liouville-von Neumann equation $i\hbar\partial_t\mathcal{S}_{\mathbf{k}} = [H, \mathcal{S}_{\mathbf{k}}]$, where H is the Hamiltonian, which includes the effects acting on polariton spin.

Since polaritons are the composite particles, it is clear that they are exposed to the mechanisms of spin relaxation applicable to the electrons and the holes in semiconductor systems, such as the Elliott-Yafet mechanism, the D'yakonov-Perel' mechanism, the Bir-Aronov-Pikus mechanism and other. Following profound theoretical studies, at the end of last century, it was found that the Bir-Aronov-Pikus mechanism of the spin relaxation caused by the long-range electron-hole interaction is the dominant mechanism of the polariton spin relaxation among other spin relaxation processes in low-dimensional systems [102]. Generally speaking, it manifests in the energy splitting between polariton states with non-zero momentum and a dipole moments oriented perpendicular (TE) and parallel (TM) to the wave vector. This splitting is also called the longitudinal-transverse (LT) splitting of the exciton-polaritons and can be introduced in the Hamiltonian by the term:

$$H_{LT} = A \begin{pmatrix} 0 & (k_x - ik_y)^2 \\ (k_x + ik_y)^2 & 0 \end{pmatrix}, \quad (2.29)$$

where the parameter A denotes the properties of the selected system, such as the value of LT splitting in the material.

Mathematically presence of non-zero off-diagonal terms in Hamiltonian H_{LT} (2.29) leads to the polariton spin flips, created by an effective in-plane magnetic field in the system, also known as Maialle field (shown, in Fig. 2.16). From the first sign, one can say that polariton spin precession about the Maialle field is similar to the electron spin precession about the Rashba effective magnetic field, introduced in the previous sections. However, the

physical origin of these fields is different. Remember that the Rashba field is created by the spin-orbit interaction in asymmetric quantum wells and the Maialle field comes from the long-range electron-hole interaction. Besides that note that in contrast to the Rashba effective field, which is oriented perpendicularly to the electron wave vector, this effective magnetic field is an anisotropic momentum-dependent field, which makes a double turn with respect to the propagation direction of the particle, defined by its wave vector $\mathbf{k}$.

Figure 2.16: Illustration of the TE-TM mechanism of polariton spin relaxation, which results in the creation of four-lobed spin pattern in $\mathbf{k}$ -space. Here the direction of polariton spin is introduced by blue arrows, the direction of the induced effective field is shown by red arrows, and the effect of pseudospin precession around this field is marked by yellow circles.

Speaking about polariton spin transport, it should be mentioned that polaritons can propagate ballistically for over a few picoseconds in current high-quality microcavities, which gives an opportunity to observe an effect of polariton spin precession under the Maialle field experimentally. In particular, it was found that this polariton spin precession results in so-called optical spin Hall effect [103, 104, 105, 106], which as the name implies, performs in the spatial separation of the polaritons with opposite spin directions, and governed by the Rayleigh scattering of light in the microcavity. Consider the process of polariton spin evolution under the Maialle field in more details. For definiteness, let us select the system of coordinates, where x and y axes lie in the cavity plane and discuss the case when the microcavity is irradiated by x -linearly polarized light. Then this linearly polarized light may excite the polaritons with the wave vector components (k_x, k_y) and S_x

spin direction, which is parallel to x -axis and coincides with an effective field direction. For that reason at the beginning polariton spin does not experience any precession. Consider now a scattering effect, which brings the polariton to the new state with the wave vector $\mathbf{k}' = (k'_x, k'_y)$. Recalling the classical theory of Rayleigh scattering, assume that the polariton spin does not change under the scattering, in what follows, that after the scattering event the polariton spin is still oriented in the x -direction. However, generally speaking, in the point of the momentum space with coordinates (k'_x, k'_y) an effective field is no longer parallel to the polariton spin, which leads to the spin precession along a new field direction, which is shown in Fig. 2.16. Here we come to the key part of optical spin Hall effect, which states that under precession the polariton spin acquires the z -component resulting in the formation of the circular polarized signal in the system. Eventually according to effective field alignment, these spin precession areas start to form a quadrupole pattern of separated right and left circular polarized domains, called the Langbein cross (see, Fig. 2.17) [107].

Figure 2.17: Spatial imaging of polariton propagation for various times after the excitation. Left picture corresponds to the signal detected collinearly polarized to the excitation direction, and right picture corresponds to the signal cross-linearly polarized to the excitation direction [107].

In Fig. 2.17 we show the experimental result, which confirmed the formation of the Langbein cross in the system plane, where bright areas correspond to the right and the left circular polarized light signals. Note that in this

experiment, the effective field in the $x, -x, y, -y$ points is parallel to the polariton spin direction, but in the quarters, the effective field is inclined to the polariton spin, which results in the spin precession and appearance of circularly polarized signal.

3. Spin and transport properties of two-dimensional electrons dressed by laser field

Motivated by the study of spin properties of alternative particles, in the following chapter, we will discuss the behavior of a two-dimensional gas in the presence of strong external electromagnetic field. Mainly we will show theoretically that spin and transport properties of a 2DEG drastically change under the influence of a high-frequency electromagnetic field. For that reason, this chapter is organized as follows.

In the beginning, we will introduce the basic concept of the system, where a 2DEG is subjected to a high-frequency electromagnetic field and highlight the main factors affecting the carrier spin and transport properties in a low-dimensional system. Following the discussion, we will demonstrate theoretically that a strong external electromagnetic field allows to control transport properties of a 2DEG and, in particular, results in a giant increase in the system conductivity and a suppression of the effect of weak localization in the absence of an external magnetic field.

Subsequently, we will sophisticate the problem of a 2DEG modulated by an external electromagnetic field by a discussion of the influence of the Rashba spin-orbit coupling on its spin properties. Solving the conventional equations of spin dynamics in diffusive regime, we will consider the effect of an external non-absorbable electromagnetic field on spin dynamics of a 2DEG and show that a strong electromagnetic field provides an opportunity to tune the prevalent spin relaxation mechanism among so-called the Elliott-Yafet and the D'yakonov-Perel' regimes of the spin relaxation and to

control the spin dynamics of a 2DEG, which from the manufacturing point of view can be found useful in spintronics for the creation of novel light-tuned devices.

3.1 Electrons strongly coupled to a high-frequency electromagnetic field

3.1.1 Introduction

Along with the development of light-emitter technology, an external electromagnetic field became a standard instrument used to control the transport properties of carriers, which attracted a lot of attention in past decades. However most of the interest was centered on the regime of weak light-matter interaction, corresponding to the condition when the electron energy spectrum remains unperturbed under irradiation, and the interaction between electrons and an external light manifested only in the electron transfer between the energy states, accompanied by absorption or emission of the photons [36, 108, 109]. Speaking about the weak coupling regime followed by the processes of photon absorption by the conduction electrons in the semiconductor, it should be mentioned that it performs in various experimentally proved electronic transport phenomena, such as effect of high-frequency conductivity, photoelectric and photovoltaic effects and others.

However, in a little while, it was found that the influence of a high-frequency electromagnetic field on the semiconductor has a distinct impact on the system behavior. Therefore, the effects connected with irradiation of semiconductor by a strong high-frequency electromagnetic field were attributed to another regime of light-matter interaction, where unlike the weak coupling regime the interaction between the field and the electron cannot be treated as perturbation and should be introduced as a principal operand describing the bound between the electron and the electromagnetic field, called “electron dressed by light” or just, “dressed elec-

trons". In last years effects of the strong coupling in solids started to attract more attention of scientific community [110, 111], which resulted in a series of experimentally observed phenomena, such as the dynamics Stark effect [112, 113, 114, 115, 116] and the effects of modification in electron scattering processes [117, 118, 119, 120, 121]. Consequently it established a new field of physics, called quantum optics, which studies the behavior of a novel states in solids, like the dressed electrons in quantum wires [122] and graphene [123, 124, 125] and the exciton-polaritons in microcavities [126, 127, 128, 129, 130, 131, 132, 133, 134, 135], which were found perspective for applied technological purposes. However, the behavior of a 2DEG modulated by an electromagnetic field was not intensively studied till now and, therefore, motivated by the previous promising studies in quantum optics in the following chapter transport and spin properties of a 2DEG irradiated by a strong electromagnetic field will be theoretically investigated.

3.1.2 Model of a dressed electron

Primarily, it should be noted that subsequent discussion might be generalized to different system configurations, but for definiteness, we will limit our consideration to two cases of a 2DEG lying in the (x, y) coordinate plane at $z = 0$. In the first instance we will examine a 2DEG system subjected to a homogeneous linearly polarized electromagnetic field $\mathbf{E}(t) = \underline{\mathbf{e}}E_0 \sin \omega t$, oriented perpendicularly to the quantum well plane, where E_0 and ω are the amplitude and the frequency of a linearly polarized electromagnetic field, respectively, and $\underline{\mathbf{e}}$ is the unit vector, which denotes to the field polarization direction. In the second instance we will consider system of a 2DEG irradiated by a circularly polarized light propagating perpendicularly to the 2DEG plane along the z axis with field dependence $\mathbf{E}(t) = \mathbf{e}_x E_{0\omega} \sin \omega t - \mathbf{e}_y E_{0\omega} \cos \omega t$, where $E_{0\omega}$ is the amplitude of a circularly polarized light and $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors. Schematic pictures of these systems are shown in Figs. 3.1(a) and 3.1(b), respectively.

Figure 3.1: Sketch of the system of a 2DEG lying in the (x, y) plane and irradiated by (a) a linearly polarized and (b) a circularly polarized high-frequency field $\mathbf{E}(t)$, propagating along the z direction.

To study the behavior of these systems, let us describe them using the fundamental principals of quantum mechanics. Start from well-known statement that neglecting the electron interactions and the scattering events, the behavior of a 2DEG in the presence of a electromagnetic field should satisfy the Schrödinger equation with Hamiltonian $\hat{H}_0$:

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}_0 \psi, \quad (3.1)$$

$$\text{where } \hat{H}_0 = \frac{1}{2m^*} (\hat{\mathbf{p}} - e\mathbf{A}(t))^2,$$

m^* is the effective mass of the electron, $\hat{\mathbf{p}} = (\hat{p}_x, \hat{p}_y)$ is the electron momentum operator, e is the value of the electron charge, and $\mathbf{A}(t)$ is the general expression of a vector potential of the electromagnetic field, which for selected cases reads:

$$\mathbf{A}(t)^{lin} = \underline{\mathbf{e}} \frac{E_0}{\omega} \cos \omega t, \quad (3.2)$$

$$\mathbf{A}(t)^{circ} = \mathbf{e}_x \frac{E_{0\omega}}{\omega} \cos \omega t + \mathbf{e}_y \frac{E_{0\omega}}{\omega} \sin \omega t. \quad (3.3)$$

It should be noted that expression (3.1) describes the system of a 2DEG under the influence of an external electromagnetic field without any restriction on the system operating regime. Therefore, since we are interested only in a dressed electron states, recall that formation of a dressed state occurs when the frequency of the field is out of the resonant electron frequencies corresponding to the possible electron transitions between the energy states. This idea brings us to the conclusion that we should set a limit on allowed

frequencies of dressing field, ω , to achieve the strong coupling regime in the system. This restriction can be formulated using well-known expression for the process of collisional field absorption within the classical Drude theory [136, 137]:

$$\mathcal{E} = \frac{1}{T} \int_0^T \mathbf{j}(t) \mathbf{E}(t) dt = \frac{E_0^2}{2} \frac{\sigma_0}{1 + (\omega\tau)^2}, \quad (3.4)$$

where $\mathcal{E}$ is the period-averaged energy of the field absorbed by the electrons per unit time and volume, $\mathbf{j}(t)$ is the current density induced by the influence of the strong electromagnetic field $\mathbf{E}(t)$, σ_0 is the static Drude conductivity with $\sigma_0 = e^2 n \tau / m^*$, where n is the electron density of the selected material and τ is the electron relaxation time. From (3.4) it is easy to see that the process of photon absorption is negligibly small at condition $\omega\tau \gg 1$. Then since we are interested in the case of a 2DEG irradiated by non-absorbable electromagnetic field, we can impose the same requirement ($\omega\tau \gg 1$) to the frequency of a dressing field.

In general finding a solution of the system with the Hamiltonian (3.1) is a tricky task. However, following the problem conditions posed in Eqs. (3.2) and (3.3), one can notice that the vector potential $\mathbf{A}(t)$ does not have a coordinate dependence, which makes the problem easier to solve. Namely, the transport properties of the dressed electrons can be accurately found from the time-dependent Schrödinger equation (3.1) using conventional quantum mechanical approach. To find the wave function of the dressed electron, assume that it can be represented in the following form:

$$\psi_{\mathbf{k}}(\mathbf{r}, t) = e^{-iF(t)} \varphi_{\mathbf{k}}(\mathbf{r}), \quad (3.5)$$

with the function $\varphi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$, which denotes to the wave function of the plane wave of the electron, and where V is the normalization volume, $\mathbf{k} = (k_x, k_y)$ is the electron wavevector, $\mathbf{r} = (x, y)$ is the electron radius-vector, and $F(t)$ is the unknown time-dependent function, which refers to

the presence of a strong electromagnetic field.

Whereupon using the presumptive wave function (3.5) and substituting it to the Schrödinger equation (3.1), expression for unknown function $F(t)$ reads:

$$\hbar \frac{dF(t)}{dt} = \frac{\hbar^2 \mathbf{k}^2}{2m^*} - \frac{e\hbar}{m^*} \mathbf{k} \mathbf{A}(t) + \frac{e^2}{2m^*} \mathbf{A}^2(t). \quad (3.6)$$

Performing the integration over the time in Eq. (3.6), one can easily find the expression for $F(t)$, which in turn allows writing an explicit expression for the wave function of the dressed electron:

$$\psi_{\mathbf{k}}(\mathbf{r}, t)^{lin} = \exp \left[-i \left(\frac{\epsilon_{\mathbf{k}} t}{\hbar} + \frac{e^2 E_0^2 t}{4m^* \omega^2 \hbar} + \frac{e^2 E_0^2}{8m^* \omega^3 \hbar} \sin 2\omega t - \frac{e E_0 \mathbf{e} \mathbf{k}}{m^* \omega^2} \sin \omega t \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}) \quad (3.7)$$

$$\psi_{\mathbf{k}}(\mathbf{r}, t)^{circ} = \exp \left[-i \left(\frac{\epsilon_{\mathbf{k}} t}{\hbar} + \frac{e^2 E_{0\omega}^2 t}{2m^* \omega^2 \hbar} - \frac{e E_{0\omega} k_y}{m^* \omega^2} \sin \omega t + \frac{e E_{0\omega} k_x}{m^* \omega^2} \cos \omega t \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}) \quad (3.8)$$

where $\epsilon_{\mathbf{k}} = \frac{\hbar^2 \mathbf{k}^2}{2m^*}$ is the energy spectrum of the electron without field. In order to simplify expression (3.8), let us also introduce the polar coordinate system $(|\mathbf{k}|, \phi)$, where projections of the electron wave vector reads: $k_x = |\mathbf{k}| \cos \phi$ and $k_y = |\mathbf{k}| \sin \phi$. Then the wave function (3.8) is given by:

$$\psi_{\mathbf{k}}(\mathbf{r}, t)^{circ} = \exp \left[-i \left(\frac{\epsilon_{\mathbf{k}} t}{\hbar} + \frac{e^2 E_{0\omega}^2 t}{2m^* \omega^2 \hbar} + \frac{e E_{0\omega} k_x}{m^* \omega^2} \cos(\omega t + \phi) \right) \right] \varphi_{\mathbf{k}}(\mathbf{r}). \quad (3.9)$$

To verify the results obtained for the dressed electrons and summarized in Eqs. (3.7) and (3.9), it seems reasonable to compare the wave functions of the dressed electron with the wave function of a bare electron in the crystal. Thence we set the electromagnetic field amplitude to zero ($E_0 = 0$) in Eqs. (3.7) and (3.9). As one can get both wave functions of the dressed electrons take the form:

$$\psi_{\mathbf{k}}^{(0)}(\mathbf{r}, t) = \exp \left[-i \frac{\epsilon_{\mathbf{k}} t}{\hbar} \right] \varphi_{\mathbf{k}}(\mathbf{r}), \quad (3.10)$$

which coincides with the solution for a free electron in solids.

Speaking about the influence of a high-frequency electromagnetic field on the transport properties of the dressed electron, it is interesting to note that an external field does not directly affect the electron dynamics, meaning that electron velocity does not change in the presence of a strong electromagnetic field. Indeed, the averaged velocity of a bare electron reads:

$$\mathbf{v}^{(0)}(\mathbf{k}) = \left\langle \psi_{\mathbf{k}}^{(0)}(\mathbf{r}, t) \left| \frac{\hat{\mathbf{p}}}{m^*} \right| \psi_{\mathbf{k}}^{(0)}(\mathbf{r}, t) \right\rangle = \frac{\hbar \mathbf{k}}{m^*}, \quad (3.11)$$

while the velocity of the dressed electron averaged over the period of a electromagnetic field $T = 2\pi/\omega$ is:

$$\mathbf{v}(\mathbf{k}) = \frac{1}{T} \int_0^T \left\langle \psi_{\mathbf{k}}(\mathbf{r}, t) \left| \frac{\hat{\mathbf{p}} - e\mathbf{A}(t)}{m^*} \right| \psi_{\mathbf{k}}(\mathbf{r}, t) \right\rangle dt = \frac{\hbar \mathbf{k}}{m^*}, \quad (3.12)$$

which equals to the velocity of a bare electron (3.11).

Although we did not find the straightforward transformation in the transport properties of a 2DEG under the influence of a high-frequency field, the difference between the wave functions of the bare and the dressed electrons leads to the conclusion that a strong electromagnetic field has an ability to change transport properties of the conduction electrons via modulation of their scattering processes in the system. Therefore, it seems interesting to analyze the effects of multiple scatterings of the dressed electrons in a 2D system, which we will do in the next section.

3.2 Transport properties of a dressed 2DEG

3.2.1 Scattering probability of a dressed 2DEG

Extending the study of the transport properties of the dressed electrons, consider a 2D disordered system, where electrons propagate in the presence of a strong electromagnetic field with the vector potential (3.2) or (3.3). Accounting multiple scattering events we need to modify the Schrödinger

equation (3.1) by an additional term $U(\mathbf{r})$:

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = [\hat{H}_0 + U(\mathbf{r})]\Psi(\mathbf{r}, t), \quad (3.13)$$

where $U(\mathbf{r})$ is the total scattering potential coming from the presence of a disorder in the system.

Following conventional methods, let us treat the term $U(\mathbf{r})$ as a small perturbation in comparison to the Hamiltonian $\hat{H}_0$ [138]. Thus according to the time-dependent perturbation theory the wave function of the conduction electrons interacting with the electromagnetic field in the presence of scattering events can be represented as a set of basis wave functions of unperturbed Hamiltonian (3.7) or (3.9):

$$\Psi(\mathbf{r}, t) = \sum_{\mathbf{k}'} a_{\mathbf{k}'}(t) \psi_{\mathbf{k}'}(\mathbf{r}, t), \quad (3.14)$$

with the coefficients $a_{\mathbf{k}'}(t)$, which can be found from the initial conditions. For accuracy, let us assume that initially an electron is in the state with the wave vector $\mathbf{k}$ and defined by the wave functions (3.7) or (3.9). This condition simultaneously claims that at $t = 0$ the rest coefficients $a_{\mathbf{k}'}(0)$ with the wave vectors $\mathbf{k}'$ different from $\mathbf{k}$, are equal to zero, which mathematically can be formulated by the expression $a_{\mathbf{k}'}(0) = \delta_{\mathbf{k}'\mathbf{k}}$ using the Kronecker symbol ($\delta_{\mathbf{k}'\mathbf{k}}$).

In order to find the coefficients $a_{\mathbf{k}'}(t)$ at some period of time t , let us substitute the wave function of the dressed electrons (3.14) with the wave vector $\mathbf{k}'$ into modified Schroedinger equation (3.13). Then, if the wave vector $\mathbf{k}'$ differs from the wave vector $\mathbf{k}$, the coefficient of the wave function of the electron modulated by linear or circular electromagnetic field $a_{\mathbf{k}'}(t)$

reads:

$$a_{\mathbf{k}'}(t)^{lin} = -i \frac{U_{\mathbf{k}'\mathbf{k}}}{\hbar} \int_0^t \exp \left[i \left(\frac{(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}})t'}{\hbar} + f_{\mathbf{k}'\mathbf{k}}^{lin} \sin \omega t' \right) \right] dt', \quad (3.15)$$

$$a_{\mathbf{k}'}(t)^{circ} = -i \frac{U_{\mathbf{k}'\mathbf{k}}}{\hbar} \int_0^t \exp \left[i \left(\frac{(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}})t'}{\hbar} + f_{\mathbf{k}'\mathbf{k}}^{circ} \sin(\omega t' + \frac{\phi + \phi'}{2}) \right) \right] dt', \quad (3.16)$$

$$\text{with } f_{\mathbf{k}'\mathbf{k}}^{lin} = \frac{e\mathbf{e}E_0(\mathbf{k} - \mathbf{k}')}{m^*\omega^2} \text{ and } f_{\mathbf{k}'\mathbf{k}}^{circ} = \frac{2eE_0k}{m\omega^2} \sin\left(\frac{\theta}{2}\right),$$

where $U_{\mathbf{k}'\mathbf{k}} = \langle \phi_{\mathbf{k}'}(\mathbf{r}) | U(\mathbf{r}) | \phi_{\mathbf{k}}(\mathbf{r}) \rangle$ is the matrix element of the scattering potential and $\theta = \phi - \phi' = \widehat{(\mathbf{k}, \mathbf{k}')}$ is the scattering angle between the electron wave vectors $\mathbf{k}$ and $\mathbf{k}'$.

As one can notice the expressions (3.15) and (3.16) have a complex dependence on the frequency of the electromagnetic field ω , which makes the analysis of the eigenfunctions of the dressed electrons a complicated task. For simplicity of the study let us apply the Jacobi-Anger expansion, which in general form is given by [139]:

$$e^{iz \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(z) e^{in\theta}, \quad (3.17)$$

where $J_n(z)$ is the n -th order Bessel function of the first kind.

Accordingly we become able to rewrite Eqs. (3.15) and (3.16) in the form:

$$|a_{\mathbf{k}'}(t)^{lin}|^2 = \frac{|U_{\mathbf{k}'\mathbf{k}}|^2}{\hbar^2} \left| \sum_{n=-\infty}^{\infty} J_n(f_{\mathbf{k}'\mathbf{k}}) \exp \left[\frac{i\mathcal{K}t}{2\hbar} \right] \int_{-t/2}^{t/2} \exp \left[\frac{i\mathcal{K}t'}{\hbar} \right] dt' \right|^2, \quad (3.18)$$

$$|a_{\mathbf{k}'}(t)^{circ}|^2 = \frac{|U_{\mathbf{k}'\mathbf{k}}|^2}{\hbar^2} \left| \sum_{n=-\infty}^{\infty} J_n(f_{\mathbf{k}'\mathbf{k}}) \exp \left[i \left(\frac{\mathcal{K}t}{2\hbar} + \frac{n(\phi + \phi')}{2} \right) \right] \int_{-t/2}^{t/2} \exp \left[\frac{i\mathcal{K}t'}{\hbar} \right] dt' \right|^2, \quad (3.19)$$

where $\mathcal{K} = (\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} + n\hbar\omega)$.

Speaking about the time dependence in equations (3.18) and (3.19) it is also interesting to note that under transition to long times integrals trans-

form into the delta-functions, which by the definition read:

$$\delta(\epsilon) = \frac{1}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} e^{i\epsilon t'/\hbar} dt' \quad (3.20)$$

and, therefore, the coefficients $a_{\mathbf{k}'}(t)^{lin}$ and $a_{\mathbf{k}'}(t)^{circ}$ can be represented by:

$$|a_{\mathbf{k}'}(t)^{lin/circ}|^2 = 4\pi^2 |U_{\mathbf{k}'\mathbf{k}}|^2 \sum_{n=-\infty}^{\infty} J_n^2(f_{\mathbf{k}'\mathbf{k}}^{lin/circ}) \delta^2(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} + n\hbar\omega). \quad (3.21)$$

It should be stressed that after the transformation we got a square of the delta-function $\delta^2(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} + n\hbar\omega)$, which using the conventional technique (see, e.g. Ref. [140]) can be replaced by the single delta-function using relation:

$$\delta^2(\epsilon) = \delta(\epsilon)\delta(0) = \frac{\delta(\epsilon)}{2\pi\hbar} \lim_{t \rightarrow \infty} \int_{-t/2}^{t/2} e^{i0t'/\hbar} dt' = \frac{\delta(\epsilon)t}{2\pi\hbar}. \quad (3.22)$$

Summarizing our previously stated assumptions and recalling that probability of the electron scattering between the electron states with the wave vectors $\mathbf{k}$ and $\mathbf{k}'$ per unit time is defined by the coefficient $a_{\mathbf{k}'}$, the scattering probability of the dressed electron is given by:

$$W_{\mathbf{k}'\mathbf{k}}^{lin/circ} = \frac{d|a_{\mathbf{k}'}|^2}{dt} = \frac{2\pi}{\hbar} |U_{\mathbf{k}'\mathbf{k}}|^2 \sum_{n=-\infty}^{\infty} J_n^2(f_{\mathbf{k}'\mathbf{k}}^{lin/circ}) \delta(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}} + n\hbar\omega). \quad (3.23)$$

In order to analyze the scattering effects in a 2DEG irradiated by a strong electromagnetic field, let us look closely at expression for the scattering probability (3.23). One can notice that since the terms with $n \neq 0$ are related to events, accompanied by the processes of collisional photon absorption or emission, which are forbidden by the definition of a dressing field, without accuracy loss we can leave only one term in $W_{\mathbf{k}'\mathbf{k}}^{lin/circ}$ corresponding to the case $n = 0$ in Eq. (3.23). Then the final expression for the scattering probability of the dressed electron reads:

$$W_{\mathbf{k}'\mathbf{k}}^{lin/circ} = \frac{2\pi}{\hbar} |U_{\mathbf{k}'\mathbf{k}}|^2 J_0^2(f_{\mathbf{k}'\mathbf{k}}^{lin/circ}) \delta(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}}). \quad (3.24)$$

Compare it to well-known result for the scattering probability of the bare electron:

$$W_{\mathbf{k}'\mathbf{k}}^{(0)} = \frac{2\pi}{\hbar} |U_{\mathbf{k}'\mathbf{k}}|^2 \delta(\epsilon_{\mathbf{k}'} - \epsilon_{\mathbf{k}}). \quad (3.25)$$

Mathematically the difference between the scattering probabilities of the bare and the dressed electrons lies in the presence of the Bessel function factor $J_0^2(f_{\mathbf{k}'\mathbf{k}})$, which originates from the strong coupling between the electron and the electromagnetic field. To verify the obtained result (3.24), consider the case when there is no dressing field in the system. Then the field amplitude is equal to zero, $E_0 = 0$, and the Bessel function is unity, $J_0(0) = 1$, which gives the matching of the results for the dressed electron (3.24) and the bare electron (3.25).

According to the recent theoretical studies presence of an oscillating factor in the scattering probability of the dressed electrons, in particular, results in the vanishing of the backscattering effects for the values of the field amplitudes, which satisfy the equation $J_0(f_{\mathbf{k}'\mathbf{k}}^{lin/circ}) = 0$ [141]. This non-trivial dependence of the scattering probability on the value of the dressing field leads us to the conclusion that irradiation of a 2DEG by a strong electromagnetic field might affect the experimentally observable quantity, called conductivity of the system. Therefore, it seems interesting to study the dependence of the system conductivity on the parameters of a dressing field.

3.2.2 Conductivity of a dressed 2DEG

Consider the system of a 2DEG irradiated by a high-frequency electromagnetic field with a stationary electric field $\mathbf{E}$ applied along the 2DEG plane (see Fig. 3.2).

Limiting our consideration by the condition of a zero temperature, electric current density, $\mathbf{J}$, can be found from the conventional Boltzmann equation

Figure 3.2: Schematic picture of a 2DEG dressed by (a) a linearly polarized and (b) a circularly polarized strong electromagnetic field $\mathbf{E}(t)$ in the presence of a stationary in-plane electric field $\mathbf{E} = (E_\parallel, E_\perp)$.

for conduction electrons [142]

$$\mathbf{J} = \frac{e^2}{2\pi^2} \int_{\mathbf{k}} [\mathbf{E} \cdot \mathbf{v}(\mathbf{k})] \tau(\mathbf{k}) \mathbf{v}(\mathbf{k}) \delta(\epsilon_{\mathbf{k}} - \epsilon_F) d^2\mathbf{k}, \quad (3.26)$$

where $\mathbf{v}(\mathbf{k}) = \nabla_{\mathbf{k}} \epsilon_{\mathbf{k}} / \hbar$ is the electron velocity and $\tau(\mathbf{k})$ is the relaxation time, which depends on the scattering probability of the electron $W_{\mathbf{k}'\mathbf{k}}$ as [143]

$$\frac{1}{\tau(\mathbf{k})} = \sum_{\mathbf{k}'} \left[1 - \frac{\tau(\mathbf{k}') \mathbf{E} \cdot \mathbf{v}(\mathbf{k}')}{\tau(\mathbf{k}) \mathbf{E} \cdot \mathbf{v}(\mathbf{k})} \right] W_{\mathbf{k}'\mathbf{k}}. \quad (3.27)$$

Remembering the requirement to a dressing field, which states that the frequency of an electromagnetic field should be far from the electron resonance frequencies, one can come to the conclusion that a dressing field does not induce the change in the electron equilibrium distribution function. In what follows that the process of sample illumination might only affect the electron stationary transport via renormalization of the dressed electron scattering probability (3.24). Otherwise, if above requirement is not fulfilled, the photon absorption will result in the heating of the system and its transition to the non-equilibrium state. Then electron transport should be treated using an alternative non-equilibrium approaches, which will not be covered in this thesis.

Substituting the expression of the scattering probability of the dressed electron (3.24) to the anisotropic relaxation time (3.27) and, subsequently, to equation for the electric current density (3.26), we become able to find

the conductivity of the dressed 2DEG using the relation $\sigma_{ij} = J_i/E_j$, where $i, j = (x, y, z)$.

In order to simplify the following calculation, let us consider the process of the electron scattering within s-wave approximation, meaning that we will neglect the angular dependence $\theta_{\mathbf{k}'\mathbf{k}}$ in the matrix element $U_{\mathbf{k}'\mathbf{k}}$ [138]. Firstly, examine the case of a 2DEG irradiated by a linearly polarized light. Then we should distinguish two cases, when the vector of applied electric field $\mathbf{E}$ is oriented parallel ($\parallel$) and when it is oriented perpendicularly ($\perp$) to the vector of the strong electromagnetic field $\mathbf{E}(t)$. Thus write the conductivity of the system as a tensor $\sigma_{ij} = (\sigma_{\parallel}, \sigma_{\perp})$ with the components

$$\frac{\sigma_{\parallel}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\parallel}(\theta)}{\tau_0} \cos^2 \theta d\theta \quad (3.28)$$

and

$$\frac{\sigma_{\perp}}{\sigma_0} = \frac{1}{\pi} \int_0^{2\pi} \frac{\tau_{\perp}(\theta)}{\tau_0} \sin^2 \theta d\theta, \quad (3.29)$$

where σ_0 is the conductivity of an unirradiated 2DEG, $\tau_{\parallel}(\theta)$ and $\tau_{\perp}(\theta)$ are the relaxation times of the dressed electron at the Fermi energy for selected value of a stationary electric field $\mathbf{E} = (E_{\parallel}, 0)$ and $\mathbf{E} = (0, E_{\perp})$, respectively, and θ is the angle between the electron wave vector and the direction of an applied field $\theta = \widehat{(\mathbf{k}, \mathbf{E})}$.

Secondary, study the conductivity of a 2DEG irradiated by circularly polarized light, σ_c , which is isotropic and, performing the previously discussed approach, given by:

$$\frac{\sigma_c}{\sigma_0} = 2\pi \left[\int_0^{2\pi} [1 - \cos \theta] J_0^2 \left(\frac{2eE_0\omega k_F}{m\omega^2} \sin \frac{\theta}{2} \right) d\theta \right]^{-1}. \quad (3.30)$$

In order to find the value of the conductivity of an irradiated 2DEG at the selected parameters of the electromagnetic field one only needs to perform a simple numerical calculation of Eqs. (3.28), (3.29) or (3.30). For instance, let us consider a GaAs quantum well irradiated by a field with the frequency $\omega = 10^{11}$ rad/s. Performing the calculation of the conductivity, we found

the dependence of the conductivity on the irradiation intensity, which is graphically shown in Fig. 3.3.

Figure 3.3: The conductivity of a 2DEG in a GaAs quantum well irradiated by a dressing electromagnetic field with the frequency $\omega = 10^{11}$ rad/s at zero temperature and the Fermi energy $\epsilon_F = 10$ meV.

From Figure 3.3 it is easy to see that presence of the dressing field leads to a giant change in the system conductivity, which in comparison to the conductivity of a 2DEG without irradiation can be increased by 10 times depending on the field intensity. Speaking about the physical explanation of this phenomenon we need to stress that it only lies in the strong dependence of the scattering probability on the field intensity, which was scrupulously studied in the previous section. From the mathematical point of view, the difference between the unirradiated and the dressed states comes from the additional factor $J_0(f_{\mathbf{k}'\mathbf{k}})$ in the scattering probability (3.24), which results in the oscillating behavior of the system conductivity. Regarding the difference between the parallel, $\sigma_{\parallel}$, and the perpendicular, $\sigma_{\perp}$, components of the conductivity tensor, remember the anisotropic nature of a 2DEG system under the irradiation by a linearly polarized light, which was highlighted in Eq. (3.24). One can also notice a large quantitative contrast between the cases of system irradiation by a linearly and a circularly polarized light. Analyzing the result it is easy to understand that this difference is related to the argument of Bessel function, $J_0(f_{\mathbf{k}'\mathbf{k}})$, in the scattering probability,

$W_{\mathbf{k}'\mathbf{k}}$. Focusing on the dependence of a 2DEG conductivity on the intensity of the dressing field with linear polarization, we found that it also has an oscillating nature, but the amplitude of these oscillations is relatively small in comparison to the case of a 2DEG dressed by circularly polarized light and, therefore, cannot be clearly seen in the simulation. For this reason, talking about the experimental observation of this nontrivial oscillating behavior of the system conductivity, it seems more appropriate to choose the circularly polarized light as a source of an external high-frequency electromagnetic field.

3.2.3 Weak localization of a dressed 2DEG

In the previous section, we have calculated the conductivity of a 2DEG system using the conventional Boltzmann transport theory, where propagation of the electron between the scattering events was treated as a purely classical motion. In order to highlight the limit of this approach, we need to say that its applicability is restricted by the condition that the electron mean free path, l , should be much larger than the electron wavelength, λ , ($l \gg \lambda$). Otherwise, when these quantities are about the same order, $\lambda \sim l$, the electron transport becomes non-classical, due to the process of electron localization [144], which on the purpose of the theoretical study of the electron transport requires taking into account the quantum nature of the electron. This quantum correction was explained by the developed theory of weak localization (WL), which introduces the process of self-interference of the electron waves propagating along closed trajectories in mutually opposite directions (clockwise and counterclockwise) in the presence of disorder.

Historically the effect of the weak localization was firstly discussed by Abrahams in his paper on the scaling theory of a 2DEG in 1979 [145], and in a short period incipient interest in the field of quantum transport started to grow rapidly and developed into a detailed study of the quantum correction to the Boltzmann conductivity in various systems [146, 147, 148, 149].

In particular, it was found that the most powerful method for experimental observation of the weak localization effect comes from the study of the system response under the influence of an external magnetic field. Following profound study, it was also concluded that although the process of weak localization exists in the systems with different dimensionality, the most experimentally clear image of weak localization effect can be achieved in a two-dimensional systems, where, for example, the value of correction to the conductivity can reach a few percents [150, 151].

In order to continue theoretical study of the transport properties of the dressed electrons in 2D semiconductor systems, let us extend developed theory and focus on the correction to the conductivity originated from the effect of carrier quantum self-interference within the intuitively clear quasi-classical approximation, which will be introduced in the following paragraph [151, 152, 153].

Recall that in addition to the classical representation motion of the conduction electron can be simultaneously described as the propagation of the wave packet with scales of order $v\tau_\varphi$ and $\lambda = h/vm^*$ in longitudinal and transverse directions, respectively, where τ_φ and v are the effective phase breaking time and the velocity of the electron, respectively, and h is the Planck constant. Therefore, multiple scattering events, which give rise to randomization of the propagating electron path, can potentially induce the effects of interference between the wave packets of the electrons moving along different random paths. Following the well-known fact that generally two electrons, which propagate along uncorrelated paths, have different phases, one can conclude that this quantum effect nulls due to the distractive interference between two randomly selected wave packets and the only non-zero quantum impact can be achieved from the self-interfering paths. Consider the wave packet moving along the closed loop in two opposite directions, which is graphically shown in Fig. 3.4).

Following this intent, one can notice the relation between the quantum

Figure 3.4: Diffusion paths of the conduction electron in the presence of disorder, resulting in the appearance of the self-interference effect. Here the electron propagation is marked by both dashed and solid lines, which differ only by the direction of electron propagation - clockwise and counterclockwise.

correction to the conductivity induced by scatterings and the probability to find the electron in the initial point after the multiple scattering events. Accurately, the weak localization correction was introduced by the expression [153]:

$$\frac{\Delta\sigma}{\sigma} = - \int_{\tau_F}^{\tau_\varphi} \tilde{W}(t) \lambda v dt = - \int_{\tau_F}^{\tau_\varphi} \tilde{W}(t) \frac{h}{vm^*} v dt = - \frac{h}{m^*} \int_{\tau_F}^{\tau_\varphi} \tilde{W}(t) dt, \quad (3.31)$$

where $\tilde{W}(t)$ is the disorder-averaged probability of finding an electron at the initial point of its propagation path at the time t , τ_F is the mean free time of the conduction electron at the Fermi energy, and τ_φ is the effective phase breaking time of the electron.

Studying equation (3.31), note that as the lower limit of the integral we use the mean free time of the propagating electron, τ_F , since it defines the shortest path among other possible electron paths and corresponds to the propagation of the electron without any scattering event. By definition, the value of the mean free time entirely depends on the system properties and can be found as

$$\frac{1}{\tau_F} = \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'}, \quad (3.32)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are the initial and the final wave vectors of the electron,

respectively, and $W_{\mathbf{k}\mathbf{k}'}$ is the carrier scattering probability.

Recalling the motive to find the impact of WL correction to the conductivity of the dressed electrons, note that the mean free time of the dressed electron, τ_F , is given by the scattering probability of the dressed electron, which was calculated in the previous section (see, Eq. (3.24)). Since the scattering probability of a 2DEG irradiated by a linearly polarized dressing field is an anisotropic function, the mean free time (3.32) also depends on the direction of the wave vector of the dressed electron $\mathbf{k}$. Therefore we should introduce two mean free times, $\tau_{F\parallel}$ and $\tau_{F\perp}$, corresponding to the propagation of the dressed electron with the wave vector oriented parallel to the electromagnetic field vector, $\mathbf{k} \parallel \mathbf{E}(t)$, and perpendicular to the electromagnetic field vector, $\mathbf{k} \perp \mathbf{E}(t)$, respectively. Consequently, these times give two values of WL correction, $\Delta\sigma_{\parallel}$ and $\Delta\sigma_{\perp}$, which describe the behavior of the electron in the 2D system plotted in Fig. 3.1. Speaking about the case of a 2DEG irradiated by a circularly polarized light, note that since the electron transport is isotropic in this system, WL correction, $\Delta\sigma_c$, does not dependent on the direction of the wave vector of a 2DEG as well.

Regarding the parameter τ_{φ} used as the upper limit of the integral in (3.31), the origin of this time scale can be understood as follows. Recall that in disordered systems various scattering events also tend to destroy the phase coherence between the time-reversed paths of the propagating conduction electron, which results in the destructive interference of self-intersecting electron paths [154, 155, 156, 157, 158, 159]. Thence we introduce the process of phase information loss by the temperature-dependent parameter τ_{φ} . Taking into account the presence of an external magnetic field in the system, the phase breaking time of the dressed electron without spin-orbit interactions can be written as a combination of two main processes, such as the effect of inelastic scatterings and interaction with an external magnetic field, $1/\tau_{\varphi} = 1/\tau_{\varphi\epsilon} + 1/\tau_{\varphi B}$, where $\tau_{\varphi\epsilon}$ is the characteristic phase breaking time, coming from inelastic processes and $\tau_{\varphi B} = \hbar/(4eDB)$ is the phase break-

ing time, caused by an external magnetic field, B , where D is the diffusion constant [158].

The only function that escaped our attention so far is $\tilde{W}(t)$, corresponding to the probability to find an electron in the initial point at time t . Following conventional theory of weak localization, it is proportional to modulus squared of the electron propagation amplitude $K(t)$. To write this function, firstly consider the more general case of a bare electron propagation in disordered system, when the conduction electron initially stays at the point described by vector $\mathbf{r}_i$ at time t_i and finishes at the point with the vector $\mathbf{r}_f$ at time t_f . Then using Feynman representation, particle propagation amplitude can be written as a path integral [160]:

$$K(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i) = \int \mathcal{D}\mathbf{r} \exp\left(\frac{i}{\hbar} S[\mathbf{r}]\right), \quad (3.33)$$

where the functional integral includes all possible trajectories $\mathbf{r}$, which start at point $\mathbf{r}_i$ and time t_i and finish at point $\mathbf{r}_f$ and time t_f , and $S[\mathbf{r}]$ is the action given by the integral

$$S[\mathbf{r}] = \int_{t_i}^{t_f} L(\dot{\mathbf{r}}, \mathbf{r}, t) dt, \quad (3.34)$$

with total Lagrangian of the system $L(\dot{\mathbf{r}}, \mathbf{r}, t)$.

For simplicity, let us limit our consideration by quasi-classical approach and rewrite Lagrangian in the form: $L(\dot{\mathbf{r}}, \mathbf{r}, t) = L_0(\dot{\mathbf{r}}, \mathbf{r}) + L_1(\dot{\mathbf{r}}, \mathbf{r}, t)$, where the time-independent term $L_0(\dot{\mathbf{r}}, \mathbf{r}) = \mathbf{p}^2/(2m^*) - V_R(\mathbf{r})$ is a combination of a kinetic energy of a bare electron and a random potential $V_R(\mathbf{r})$ induced by system disorder and term $L_1(\dot{\mathbf{r}}, \mathbf{r}, t)$ includes the rest effects in the system, such as interaction with external fields. Within the boundaries of these assumptions we are allowed to replace the path integral (3.33) by a sum over all possible classical trajectories $\mathbf{r}^{cl}$, which fulfill the requirement that the action, $S[\mathbf{r}]$, remains stationary. Therefore the propagation amplitude

$K(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$ reads:

$$K(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i) = \sum_{\mathbf{r}^{cl}} A[\mathbf{r}^{cl}] \exp \left(\frac{i}{\hbar} S[\mathbf{r}^{cl}] \right), \quad (3.35)$$

where $A[\mathbf{r}^{cl}]$ is the factor accounting the Gaussian fluctuations along the classical electron trajectory.

Recalling that probability $\mathcal{W}(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$ is proportional to modulus squared of propagation amplitude $K(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$, one can write it as:

$$\begin{aligned} \mathcal{W}(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i) &= \frac{1}{2N(0)} \times \\ &\times \sum_{\mathbf{r}^{cl}} \sum_{\mathbf{s}^{cl}} A[\mathbf{r}^{cl}] A^*[\mathbf{s}^{cl}] \exp \left[\frac{i}{\hbar} (S[\mathbf{r}^{cl}] - S[\mathbf{s}^{cl}]) \right] [\delta(\epsilon[\mathbf{r}^{cl}] - \epsilon_F) + \delta(\epsilon[\mathbf{s}^{cl}] - \epsilon_F)], \end{aligned} \quad (3.36)$$

where $N(0)$ is the density of states per spin direction at the Fermi surface, $\mathbf{r}^{cl}$ and $\mathbf{s}^{cl}$ are two different classical paths with energies $\epsilon[\mathbf{r}^{cl}]$ and $\epsilon[\mathbf{s}^{cl}]$, respectively, and ϵ_F is the electron Fermi energy.

From Eq. (3.36) it is clear that major amount of terms vanish in the presence of disorder, because of the effect of destructive interference between the propagating wave packages. As a consequence we can leave only self-interference terms in the expression for $\mathcal{W}(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$:

$$\mathcal{W}(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i) = \frac{1}{2N(0)} \sum_{\mathbf{r}^{cl}} |A[\mathbf{r}^{cl}]|^2 \delta(\epsilon[\mathbf{r}^{cl}] - \epsilon_F). \quad (3.37)$$

Speaking about the reverse electron propagation path, meaning that the electron initially stays at the point $\mathbf{r}_f$ at time t_i and finishes at the point $\mathbf{r}_i$ at time t_f , note that the probability amplitude $K(\mathbf{r}_i, \mathbf{r}_f; t_f, t_i)$ is represented by the same expression as $K(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$ in Eq. (3.37), with the only difference that the classical path $\mathbf{r}^{cl}$ should be replaced by the time-reversed path $\mathbf{r}_R^{cl}$. For that reason the behavior of two self-interference paths $\mathcal{W}'(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i)$ is

given by the expression for the probability

$$\mathcal{W}'(\mathbf{r}_f, \mathbf{r}_i; t_f, t_i) = \frac{1}{N(0)} \sum |A[\mathbf{r}^{cl}]|^2 \delta(\epsilon[\mathbf{r}^{cl}] - \epsilon_F) \exp(i\phi[\mathbf{r}^{cl}]), \quad (3.38)$$

where we neglected the difference between the factors $A[\mathbf{r}^{cl}]$ and $A[\mathbf{r}_R^{cl}]$ and the argument $\phi[\mathbf{r}^{cl}]$ stands for the phase difference between the pair of time-reversed paths.

In order to find the difference between the electron phases, note that by definition time-independent term $L_0(\dot{\mathbf{r}}, \mathbf{r})$ does not depend on the direction of the electron propagation. Hence, the only term, which might affect the electron coherence is $L_1(\dot{\mathbf{r}}, \mathbf{r}, t)$. Therefore $\phi[\mathbf{r}^{cl}]$ can be expressed by:

$$\begin{aligned} \hbar\phi[\mathbf{r}^{cl}] &= S[\mathbf{r}^{cl}] - S[\mathbf{r}_R^{cl}] = \int_{-t_0/2}^{t_0/2} dt (L(\dot{\mathbf{r}}^{cl}, \mathbf{r}^{cl}, t) - L(-\dot{\mathbf{r}}_R^{cl}, \mathbf{r}_R^{cl}, t)) = \\ &= \int_{-t_0/2}^{t_0/2} dt (L_1(\dot{\mathbf{r}}^{cl}, \mathbf{r}^{cl}, t) - L_1(-\dot{\mathbf{r}}_R^{cl}, \mathbf{r}_R^{cl}, t)). \end{aligned} \quad (3.39)$$

Coming back to the effect of weak localization, the electron probability to return to the initial point can be easily found from Eq. (3.37), where the final point $\mathbf{r}_f$ is replaced by the initial point $\mathbf{r}_i$, and reads $\mathcal{W}'(\mathbf{r}_i, \mathbf{r}_i; t_f, t_i)$. One can notice that there is no explicit dependence on random potential of the system, V_R , in equations for the probability $\mathcal{W}'(\mathbf{r}_i, \mathbf{r}_i; t_f, t_i)$. In order not to mislead the reader, we should stress that presence of disorder will be taken into account by the averaging of all possible trajectories in the expression for the probability $\mathcal{W}'(\mathbf{r}_i, \mathbf{r}_i; t_f, t_i)$.

The functional of the probability (3.38), which includes all classical paths at selected initial point and specified time with generalized disorder potential, is given by:

$$\mathcal{P}[\mathbf{R}] = \frac{1}{N(0)} \left\langle \sum |A[\mathbf{r}^{cl}]|^2 \delta(\epsilon[\mathbf{r}^{cl}] - \epsilon_F) \right\rangle_R. \quad (3.40)$$

This expression brings us to the final result, which defines the probability

of the particle to return to the vicinity near the initial point $\tilde{\mathcal{W}}(t_f, t_i) = \int d[\mathbf{R}] \mathcal{P}[\mathbf{R}]$. Note that within quasi-classical approach classical electron paths in the presence of disorder are similar to the Brownian paths with the probability given by Wiener measure [161]. Therefore the impurity averaged probability, $\tilde{\mathcal{W}}(t_f, t_i)$, can be rewritten as a path integral:

$$\tilde{\mathcal{W}}(t_f, t_i) = \int \mathcal{D}\mathbf{r} \exp \left(- \int_{t_i}^{t_f} dt F(\dot{\mathbf{r}}, \mathbf{r}, t) \right), \quad (3.41)$$

where $F(\dot{\mathbf{r}}, \mathbf{r}, t) = \frac{1}{4D} \dot{\mathbf{r}}^2 - \frac{i}{\hbar} L_1(\dot{\mathbf{r}}, \mathbf{r}, t) + \frac{i}{\hbar} L_1(-\dot{\mathbf{r}}, \mathbf{r}, t)$.

Using the previously discussed approach, we become able to study the effect of weak localization in the system of a 2DEG modulated by a strong electromagnetic field by the change of Hamiltonian of a bare electron to Hamiltonian of a dressed electron (3.1). Recalling the procedure, firstly one should represent Hamiltonian of the dressed electron (3.1) as a sum of the isotropic term, representing the kinetic energy of a bare electron and the term, corresponding to the electron interaction with an external dressing electromagnetic field. For that reason we apply the unitary transformation $\hat{U} = \exp \left(\frac{i}{\hbar} e \mathbf{r} \mathbf{E}(t) \omega \right)$ to the Hamiltonian $\hat{H}$ and performing a well-known formalism, where $\hat{H}' = U^\dagger \hat{H} U - i \hbar U^\dagger \frac{\partial}{\partial t} U$, arrive to the new unitary transformed Hamiltonian of the dressed electron, $\hat{H}'$, which is physically equal to the initial Hamiltonian, $\hat{H}$:

$$\hat{H}' = \frac{\hat{\mathbf{p}}^2}{2m^*} - e \mathbf{r} \mathbf{E}(t). \quad (3.42)$$

From Eq. (3.41) the probability of the dressed electron to return to the vicinity near the initial point $\tilde{\mathcal{W}}(t_f, t_i)$ is given by expression:

$$\tilde{\mathcal{W}}(t_f, t_i) = \int \mathcal{D}\mathbf{r} \exp \left[- \left(\int_{t_i}^{t_f} \frac{\dot{\mathbf{r}}^2}{4D} - \frac{i}{\hbar} e \mathbf{r} [\mathbf{E}(t') - \mathbf{E}(-t')] dt' \right) \right] \quad (3.43)$$

where the integration is performed over all closed electron trajectories in a 2D plane.

Consider the first term in the exponent of the integrand in Eq. (3.43). According to our previous discussion, it describes the kinetic energy of the electron with diffusion constant D , which in terms of the electron transport in a 2D system modulated by the dressing field unfolds into two diffusion constants corresponding to the propagation of the electron in the direction parallel, $D_{\parallel}$, and perpendicular, $D_{\perp}$, to the direction of the dressing field polarization. Since by definition diffusion constant depends on the value of the electron Fermi velocity, v_F , and the scattering time, $\tau_{F\parallel}$ or $\tau_{F\perp}$, in the selected direction (see, e.g., Ref. [158]), we introduce two diffusion constants: $D_{\parallel} = v_F^2 \tau_{F\parallel} / 2$ and $D_{\perp} = v_F^2 \tau_{F\perp} / 2$, respectively. Note that since in the system of a 2DEG irradiated by a circularly polarized electromagnetic field the scattering times $\tau_{F\parallel}$ and $\tau_{F\perp}$ and, subsequently, the values of the diffusion constants $D_{\parallel}$ and $D_{\perp}$ are equal, we will use the value of the parameter $D_{\parallel}$, in order not to confuse the reader by a new variable.

Recalling that the translational invariance in time is broken in a selected systems, it seems useful to represent the temporal behavior of the dressed electron by the sum and the difference coordinates $T = (t_i + t_f)/2$ and $t_0 = t_f - t_i$. Performing the standard procedure of the path integration in Eq. (3.43), which can be found for example in Ref. [160], the probabilities $\tilde{W}^{lin/circ}(t_0, T)$ for the cases of irradiation by a linearly and a circularly polarized dressing fields are given by:

$$\tilde{W}^{lin}(t_0, T) = \frac{1}{4\sqrt{D_{\parallel}D_{\perp}}\pi t_0} \exp \left[-\frac{2(eE_{0\omega})^2 D_{\parallel} \mathcal{K}^{lin}}{\hbar^2 \omega^4 t_0} \right], \quad (3.44)$$

$$\tilde{W}^{circ}(t_0, T) = \frac{1}{4D_{\parallel}\pi t_0} \exp \left[-\frac{2(eE_0)^2 D_{\parallel} \mathcal{K}^{circ}}{\hbar^2 \omega^4 t_0} \right], \text{ where} \quad (3.45)$$

$$\mathcal{K}^{lin} = \sin^2 \omega T [\omega t_0 (\omega t_0 + \sin \omega t_0) - 4(1 - \cos \omega t_0)],$$

$$\mathcal{K}^{circ} = \omega t_0 (\omega t_0 + \sin \omega t_0) - 4(1 - \cos \omega t_0).$$

An arbitrary shift in time for a linearly polarized case, shown in Eq. (3.44), can be avoided by integration over the period of time T . Performing

the integration, the probability $\tilde{W}^{lin}$ reads:

$$\tilde{W}^{lin}(t_0) = \frac{1}{4\sqrt{D_{\parallel}D_{\perp}}\pi t_0} \exp[-\mathcal{K}'] I_0(\mathcal{K}'), \quad (3.46)$$

where $\mathcal{K}' = \frac{(eE_{0\omega})^2}{\hbar^2\omega^4 t_0} D_{\parallel} [\omega t_0 (\omega t_0 + \sin \omega t_0) - 4(1 - \cos \omega t_0)]$

and I_0 is the modified Bessel function.

For quantitative analysis of the effect of illumination on the transport properties of a 2DEG introduced by Eqs.(3.45) and (3.46), let us continue the study of a 2DEG in GaAs quantum well at zero temperature with the Fermi energy $\epsilon_F=10$ meV and characteristic phase breaking time $\tau_{\varphi\epsilon}=80$ ps. Performing simple numerical calculation with given parameters, one can find the dependence of weak localization correction to the system conductivity on the intensity of the dressing field, which is shown in Fig. 3.5.

From the obtained numerical results, one can see that irradiation of a 2DEG by a high-frequency electromagnetic field reduces the effect of electron weak localization, which is explained by the suppression of scattering events in a dressed 2D systems. As to the small oscillatory behavior in the value of weak localization correction on the system conductivity, $\Delta\sigma_c/\sigma$, under the influence of the dressing field, it has the same origin as the oscillations in the dressed electron conductivity (Fig. 3.3) and comes from the modification of the scattering probability of the dressed electron. It should be noted that for a 2D system dressed by linearly polarized electromagnetic field, we only introduce the dependence of WL correction component $\Delta\sigma_{\parallel}/\sigma$ on irradiation intensity, which generally should differ from the value of $\Delta\sigma_{\perp}/\sigma$ component. However, using the numerical calculation, one can see that the value of weak localization correction in the direction parallel and perpendicular to the direction of the dressing electromagnetic field polarization is varied within one percent and cannot be easily observed experimentally. For this reason, we can roughly say that in a 2D GaAs systems they are equal $\Delta\sigma_{\parallel} \approx \Delta\sigma_{\perp}$.

It should be stressed that the nature of WL suppression in the dressed 2D

Figure 3.5: The dependence of weak localization correction to the conductivity of a 2D dressed electron gas on the intensity of the dressing electromagnetic field. Here we perform the calculation for a 2DEG realized in a 2D GaAs system with the parameters $\epsilon_F = 10$ meV, $\tau_{\varphi\epsilon} = 80$ ps at $T = 0$. In the insert we show the dependence of WL correction on the external magnetic field applied perpendicularly to the 2D plane in the presence of the dressing field with intensity $I = 3$ mW/cm².

system differs from the well-known effect of WL suppression under the intraband photon absorption by conduction electrons, where the last is caused by the break in the electron phase during inelastic electron-photon interactions [162, 163, 164].

Including the effect of an external magnetic field oriented perpendicularly to the system plane, recall that the influence of the magnetic field of the transport properties of the dressed electrons can be taken into account by an additional term $\tau_{\varphi B}$ in the total phase breaking time τ_{φ} . In order to find the value of WL correction in the presence of a magnetic field, we repeated the calculation of WL correction using above approach and found that the impact of WL correction to the conductivity for the dressed 2DEG is smaller than one for the bare 2DEG, which can be clearly seen in the insert Fig. 3.5. It is interesting to note that as in the case of the bare electrons, WL correction of the dressed electrons also obeys the logarithmic law on the magnetic field with the only difference in the value of this correction, which reduces in the presence of irradiation.

Summarizing this section, we have considered the effect of the strong

coupling between a 2DEG and a high-frequency electromagnetic field, which leads to a drastic change in the transport properties of the dressed 2DEG. In particular, we showed that it performs in a giant increase of the system conductivity, the appearance of the oscillating behavior of the conductivity on the intensity of a dressing field and the suppression of weak localization effect. From the applied point of view, we should also state that obtained results open an alternative way to control transport properties of a 2DEG by the external electromagnetic field, which can give a start for a novel optoelectronic device realization.

3.3 Spin properties of a dressed 2DEG

Recall that spintronics is promising research field, which proposes to use the spin property of the carrier as an alternative basic element in various high-speed devices. Therefore, from the fundamental point of view, one can conclude that the spin relaxation time is the one of the most important characteristics of spin-based systems, which shows the evolution of the spin in the selected system and subsequently reflects the efficiency of the proposed spin system.

In order to continue the study of a potentially interesting system of a 2DEG modulated by light, remember that the high-frequency dressing electromagnetic field has an ability to suppress the scattering of the dressed electrons. For that reason, one can expect that the spin relaxation time of a 2DEG dressed by a high-frequency electromagnetic field might be strongly affected by this field and perform differently in comparison to the well-known result of a bare electron in a two-dimensional system. Additionally, we need to mention that the process of spin relaxation depends on the influence of scattering events and the effect of spin-orbit interaction, which we should also include in the developed model for the physically accurate study of the spin dynamics in a dressed 2D system.

3.3.1 Spin Hamiltonian of a dressed 2DEG

Let us limit the study of the spin dynamics of the dressed electrons by a discussion of the system of a 2DEG with the Rashba spin-orbit interaction, which is irradiated by a high-frequency linearly polarized electromagnetic field propagating perpendicularly to the 2DEG plane (see, Fig. 3.1(a)). Note that in order to stay in the limit of dressing effect, parameters of the electromagnetic field should satisfy the requirements to the value of the wave frequency, ω , introduced in the previous section.

In the beginning, consider the case when there are no scattering effects in the system. Therefore, the wave function of the dressed electron with the Rashba spin-orbit interaction can be found from the non-stationary Schrödinger equation with Hamiltonian $\hat{\mathcal{H}}$:

$$\hat{\mathcal{H}} = \frac{1}{2m^*} (\hbar \mathbf{k} - e \mathbf{A})^2 + \alpha [\boldsymbol{\sigma} \times (\hbar \mathbf{k} - e \mathbf{A})]_z, \quad (3.47)$$

where $\mathbf{A} = (\mathbf{E}/\omega) \cos \omega t$ is the vector potential of a linearly polarized along y -axis electromagnetic field with the amplitude E_0 , $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is the Pauli matrix vector, and α is the Rashba spin-orbit coupling constant.

For simplification of subsequent calculation, we subject the Hamiltonian (3.47) to the unitary transformation:

$$\hat{U} = \frac{1}{\sqrt{2}} e^{i \left(\frac{k_y e E_0}{m^* \omega^2} \sin \omega t - \frac{e^2 E_0^2 t}{4m^* \omega^2 \hbar} - \frac{e^2 E_0^2}{8m^* \omega^3 \hbar} \sin 2\omega t \right)} \begin{pmatrix} e^{i \frac{\alpha e E_0}{\hbar \omega^2} \sin \omega t} & e^{-i \frac{\alpha e E_0}{\hbar \omega^2} \sin \omega t} \\ e^{i \frac{\alpha e E_0}{\hbar \omega^2} \sin \omega t} & -e^{-i \frac{\alpha e E_0}{\hbar \omega^2} \sin \omega t} \end{pmatrix} \quad (3.48)$$

Following the conventional method, transformed Hamiltonian, $\hat{\mathcal{H}}'$, reads:

$$\hat{\mathcal{H}}' = \begin{pmatrix} \frac{\hbar^2 k^2}{2m^*} + \alpha \hbar k_y & -i \alpha \hbar k_x e^{-i \frac{2\alpha e E_0}{\hbar \omega^2} \sin \omega t} \\ i \alpha \hbar k_x e^{i \frac{2\alpha e E_0}{\hbar \omega^2} \sin \omega t} & \frac{\hbar^2 k^2}{2m^*} - \alpha \hbar k_y \end{pmatrix}. \quad (3.49)$$

Regarding the wave function of the dressed electron, solutions of non-stationary Schrödinger equation: $i\hbar \frac{\partial \psi}{\partial t} = \hat{\mathcal{H}}' \psi$ can be represented as a spinor, $\psi =$

$(a_+, a_-)^T$. Substituting it to the Hamiltonian (3.49), one arrives to the system of differential equations

$$i\dot{a}_\pm = \left(\frac{\hbar k^2}{2m^*} \pm \alpha k_y \right) a_\pm \mp i\alpha k_x a_\mp e^{\mp i \frac{2\alpha e E_0}{\hbar \omega^2} \sin \omega t}. \quad (3.50)$$

From a mathematical point of view, the system of dynamical equations (3.50) describes a two-level quantum system with periodical external pumping, which is discussed in details in various fundamental books on quantum mechanics (see, for e.g. Ref. [138]). For simplicity of the subsequent analytical calculation, let us apply the Jacobi-Anger expansion to the exponents in Eqs. (3.50), as we did in the previous section for the calculation of the eigenfunctions of the dressed electrons without spin-orbit interaction.

It should be noted that if the energy of the photons, $\hbar\omega$, induced by a dressing field with the frequency ω is much larger than both the energy of a bare electron in the system and the energy of the spin-orbit interaction, $\alpha\hbar k$, the high-frequency harmonics $e^{in\omega t}$ with non-zero parameter n , so-called non-resonant terms, have negligibly small impact on the final solutions of the system of dynamical equations (3.50) and therefore, can be omitted without a qualitative change in the wave function of the dressed electron. Thence staying in the limit of a high-frequency dressing field, Eqs. (3.50) can be rewritten as

$$i\dot{a}_\pm = \left(\frac{\hbar k^2}{2m^*} \pm \alpha k_y \right) a_\pm \mp i\alpha k_x a_\mp J_0 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right), \quad (3.51)$$

where $J_0(x)$ is the zeros order Bessel function of the first kind.

Using the conventional Floquet theory and seeking the solutions of the system of Eqs. (3.51) in the form: $a_\pm(t) = C_\pm \exp(-i\varepsilon_\pm(\mathbf{k})t/\hbar)$, where $C_\pm$ are the constants, one arrives to two spin-splitted branches of the quasi-energy spectrum of the dressed 2DEG:

$$\varepsilon_\pm(\mathbf{k}) = \frac{\hbar^2 k^2}{2m^*} \pm \alpha \hbar \sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right) k_x^2}. \quad (3.52)$$

Accordingly the system of equations (3.51) can be solved trivially and eigenfunctions of the dressed electron read:

$$\psi_{\pm}(\mathbf{k}) = \begin{pmatrix} \left[\frac{\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right) k_x^2 \pm k_y}}{2\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right) k_x^2}} \right]^{\frac{1}{2}} \\ \pm i \left[\frac{\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right) k_x^2 \mp k_y}}{2\sqrt{k_y^2 + J_0^2 \left(\frac{2\alpha e E_0}{\hbar \omega^2} \right) k_x^2}} \right]^{\frac{1}{2}} \end{pmatrix} e^{-\frac{i\varepsilon_{\pm}(\mathbf{k})t}{\hbar}}. \quad (3.53)$$

3.3.2 Spin dynamics of a dressed 2DEG

3.3.3 Analytical approach of diffusive transport

Previously we have considered the properties of the dressed electron with the Rashba coupling in the absence of disorder. It seems the time has come to introduce the effect of disorder and discuss the diffusive transport of the dressed electrons in 2D systems. Speaking about the fundamental difference between ballistic transport and diffusive transport, it should be noted that disorder drastically affects the electron dynamics by multiple random scattering events, in what follows that diffusive transport cannot be longer described by the single-particle wave function.

From the general sense, it is reasonable to start calculation of the spin dynamics of the dressed electrons in diffusive regime from the averaging of particle properties over disorder. However, recalling that under multiple scattering event electron phase becomes randomized, simple disorder-averaging of the electron wave function will give a zero. This conclusion implies that unlike in ballistic regime, the wave function of the electron does not solely define the particle evolution in diffusive regime and thence we need to implement an alternative approach for studying the spin dynamics of the dressed electrons in the disordered system.

Accordingly apply a widely adopted formalism for the description of charge and spin evolution in a disordered system based on diagrammatic technique, where the scattering events are introduced by the perturbative ladder Feynman diagrams [165, 166, 167, 168, 169, 170, 171, 172, 173]. Start

the study of the spin dynamics of the dressed electrons by the overview of this approach. Let us begin with the definition of the single-particle density matrix, $\rho(\mathbf{r}, t)$, which contains the information about single particle evolution. Following standard notation the probability density to find a particle at selected place and certain time, $I(\mathbf{r}, t)$, is proportional to the particle density, $\rho(\mathbf{r}, t)$, and reads: $I(\mathbf{r}, t) = \text{Tr}[\rho(\mathbf{r}, t)]$, where $\text{Tr}[\rho(\mathbf{r}, t)]$ denotes the trace of the matrix $\rho(\mathbf{r}, t)$ with respect to the spin indexes. Similarly, the spin densities are given by:

$$\begin{aligned} S_x(\mathbf{r}, t) &= \text{Tr}[\rho(\mathbf{r}, t)\sigma_x], \\ S_y(\mathbf{r}, t) &= \text{Tr}[\rho(\mathbf{r}, t)\sigma_y], \\ S_z(\mathbf{r}, t) &= \text{Tr}[\rho(\mathbf{r}, t)\sigma_z]. \end{aligned} \tag{3.54}$$

Remembering the definition of the function $\rho(\mathbf{r}, t)$, it is easy to see that quantities I, S_x, S_y and S_z do not null after the averaging over disorder and hence can be applied to the study of diffusive transport. To make further discussion more compact, let us combine these quantities in a vector $\mathcal{S}(\mathbf{r}, t)$ in the coordinate space and a vector $\mathcal{S}(\mathbf{q}, t)$ in the momentum space, respectively:

$$\mathcal{S}(\mathbf{r}, t) = [I(\mathbf{r}, t), S_x(\mathbf{r}, t), S_y(\mathbf{r}, t), S_z(\mathbf{r}, t)]^T, \tag{3.55}$$

$$\mathcal{S}(\mathbf{q}, t) = [I(\mathbf{q}, t), S_x(\mathbf{q}, t), S_y(\mathbf{q}, t), S_z(\mathbf{q}, t)]^T, \tag{3.56}$$

where symbol T denotes the transpose of the matrix.

Looking ahead, note that frequency representation of the components of vector $\mathcal{S}(\mathbf{r}, t)$ can be found using temporal Fourier transformation:

$$\mathcal{S}^T(\mathbf{r}, \omega) = \int d\omega \exp(i\omega t) \mathcal{S}^T(\mathbf{r}, t). \tag{3.57}$$

Subsequently introduce the function, called the response function in the coordinate space, $\Phi(\mathbf{r}, t)$, or alternatively $\Phi(\mathbf{q}, t)$ in the momentum space, which is a 4×4 matrix, containing whole information about the particle and

the spin densities and their correlation at selected time t . Then since the disordered system is translational invariant after the averaging over disorder, the vector $\mathcal{S}(\mathbf{q}, t)$ in momentum space can be found from the response function $\Phi(\mathbf{q}, t)$ at given initial conditions $\mathcal{S}(\mathbf{q}, t = 0)$ as:

$$\mathcal{S}(\mathbf{r}, t) = \int \frac{d\mathbf{q}}{(2\pi)^2} \exp(i\mathbf{q}\mathbf{r}) \Phi(\mathbf{q}, t) \mathcal{S}(\mathbf{q}, t = 0). \quad (3.58)$$

Speaking about the effects in the system of a 2DEG dressed by light, which act the particle dynamics and perform in the response function, $\Phi(\mathbf{q}, t)$, let us limit our study of the particle propagation through the system with impurities by the case, where the processes of quantum interference are neglected, and system disorder is caused by so-called “white noise” potential, which does not explicitly affect the spin behavior. Note that “white noise” disorder only performs in the loss of initial direction of the carrier momentum, which leads to even distribution of the particles along the elastic scattering circle. Following these assumptions, the time response function, $\Phi(\mathbf{q}, t)$ and the frequency response function, $\Phi(\mathbf{q}, \omega)$, in momentum space are defined by:

$$\Phi(\mathbf{q}, t) = \exp\left(\frac{t}{\tau} D^{-1}(\mathbf{q})\right), \quad (3.59)$$

$$\Phi(\mathbf{q}, \omega) = \frac{1}{i\omega + \tau^{-1} D^{-1}(\mathbf{q})}, \quad (3.60)$$

where $D^{-1}(\mathbf{q}) = 1 - I_{ij}$ is the inverse of the operator, called diffuson, which contains the information about multiple scattering events as a sum of the single scattering events I_{ij} , schematically shown in Fig. 3.6.

Following above definitions, evolution of the density matrix $\mathcal{S}(\mathbf{q}, t)$ within the diffusion equation is given by expression:

$$\frac{\partial \mathcal{S}(\mathbf{q}, t)}{\partial t} = \Phi(\mathbf{q}, t) \mathcal{S}(\mathbf{q}, t). \quad (3.61)$$

To find the laws of system evolution, we should find the exact form of the

diffusion operator $D^{-1}(\mathbf{q})$.

Figure 3.6: Diagram illustration of the process of diffusion caused by the scattering events, marked by dashed lines. Here functions G^A and G^R define the time evolution of the carriers, which start to correlate at the moment of scattering.

Speaking about mathematical representation of the diffuson, recall the well-known expression for the scattering operator, I_{ij} , introduced by the disorder-averaged single-particle advanced and retarded Green's functions G^A and G^R , respectively:

$$I_{ij} = \frac{\hbar}{4\pi\nu\tau} \int d\mathbf{k} \text{Tr}[G^A(\mathbf{k} - \mathbf{q}/2, E)\sigma_i G^R(\mathbf{k} + \mathbf{q}/2, E)\sigma_j], \quad (3.62)$$

where the trace of the matrix product is taken over the spin indexes in the Green's functions, σ_i and σ_j are the Pauli matrices, $\mathbf{q}$ is the diffuson wave vector, and ν is the density of states, and the advanced and retarded Green's functions G^A and G^R of the particle with the wave function ψ in the system with the Hamiltonian $\hat{\mathcal{H}}$:

$$G^{A,R} = \frac{|\psi\rangle\langle\psi|}{E - \hat{\mathcal{H}} \mp \frac{i\hbar}{2\tau}}. \quad (3.63)$$

Summarizing the above mentioned, this formalism allows not only to calculate the relaxation time of each spin component of the propagating particle in the system, introduced by diagonal terms in the response function $\Phi(\mathbf{q}, t)$ at $i = j$, but also to determine the correlation between them, shown by off-diagonal matrix elements of response function $\Phi(\mathbf{q}, t)$ at $i \neq j$. Recalling that we are interested in the dependence of the spin relaxation time of a 2DEG on intensity of the dressing field and we should perform the calculation of three scattering integrals I_{ij} , which define to the dynamics of x, y and z spin components at $(i, j) = (1, 1), (2, 2), (3, 3)$, respectively. Following the

definition of the scattering time within this approach, we are able to rewrite expression (3.61) as

$$\frac{\partial S_i}{\partial t} = -\frac{\tau}{\tau_i} S_i, \quad (3.64)$$

where τ is the scattering time of the particle and τ_i is the spin relaxation time of i -component, which is given by the response function. Therefore in order to find the spin relaxation time of the dressed electron, we should find the components of the scattering operator, I_{ii} , for the selected system of the dressed electron with the Rashba interaction, which consequently will drive us to required response function, Φ , and spin relaxation time, τ_i .

3.3.4 Results and discussion

Speaking about the analytical calculation of the spin relaxation time, note that since the eigenvalues of a dressed electron with the Rashba spin-orbit coupling are sophisticated functions (3.53), the straightforward calculation of the scattering integrals I_{ii} is a difficult task. However according to the problem assumptions, it becomes clear that the argument of the Bessel function is much less than one, and therefore the wave function (3.53) can be simplified by the Taylor expansion. In addition, let us introduce two more restrictions to the system of the dressed electrons. Firstly, assume that the scattering disorder is relatively weak and satisfies the non-equality $\hbar/\tau\varepsilon_F \ll 1$, and secondary set the value of the spin-orbit interaction small in comparison to the energy value of the dressed electron, $\alpha\hbar k_F/\varepsilon_F \ll 1$, where k_F and ε_F denote the Fermi wave vector and the energy of the dressed electron, respectively.

Performing the integration of the product of the Green's functions in the scattering operator (3.62), we get to the point of the analytical calculation of the matrix elements of the diffuson within the second-order approximation in the spin-orbit coupling. For spatially uniform initial electron distribution the spin relaxation rate corresponding to the selected spin component S_i

with index $i = (x, y, z)$ reads:

$$\tau/\tau_x = \frac{2\zeta^2}{1+4\zeta^2} \left(1 + \frac{\eta}{4}\right), \quad (3.65)$$

$$\tau/\tau_y = \frac{2\zeta^2}{1+4\zeta^2} \left(1 - \frac{\eta}{4}\right), \quad (3.66)$$

$$\tau/\tau_z = \frac{\zeta^2}{1+4\zeta^2} \left(4 + \frac{3\eta}{2}\right), \quad (3.67)$$

where the parameters $\eta = (2\alpha e E_0/\omega^2)^2$ and $\zeta = \tau/\tau_{SO}$, τ is the scattering time of a dressed electron given by the expression (3.27), and $\tau_{SO} = 1/(\alpha k_F)$ is the time of the spin precession at the Fermi level induced by the spin-orbit interaction.

Note that correctness of the calculation can be verified by the limiting case of a 2DEG without external irradiation, where field parameters are set to zero, $E_0 = 0$ and $\eta = 0$. In this instance our result simulating the process of the spin relaxation of the dressed electron reduces to a well-known problem of a 2DEG with the Rashba spin-orbit coupling, discussed in details in various articles (see, for e.g. Refs. [174, 175]).

Without generality loss, subsequently, consider the dynamics of S_x spin component with the spin relaxation time τ_x , since the dynamics of S_y and S_z spin projections is similar to S_x spin projection. Analyzing the result (3.65) one can see that the spin relaxation time, τ_x , strongly depends on the parameter ζ , which describes the ratio between the time of a single scattering event, τ , and the spin precession time, τ_{SO} . When the time of a single scattering event is less than the spin precession time, $\zeta \ll 1$, the spin relaxation time is directly proportional to the time of spin precession, $\tau_x \sim \tau_{SO}$, and, otherwise, when the time of a single scattering event is greater than the spin precession time, $\zeta \gg 1$, time of spin relaxation is governed by the scattering time in the form $\tau_x \sim \tau$. Graphically this dependence of the spin relaxation time τ_x on the parameter ζ plotted in Fig. 3.7.

From Eq. (3.65) we arrive at a decision that this extraordinary result is induced by the presence of different mechanisms of spin relaxation in the

Figure 3.7: Dependence of the spin relaxation time on the value of spin precession time in the system of the dressed electrons with the Rashba coupling, performing in two regimes: so-called the Elliott-Yafet and the D'yakonov-Perel' regimes of spin relaxation.

system of a 2DEG dressed by light, which in general terms were introduced in the first chapter of this thesis. Indeed, recall that the case of $\zeta \gg 1$ defines the mechanism of the Elliott-Yafet spin relaxation, which alone results in $\tau_x \sim \tau$, while the case of $\zeta \ll 1$ refers to the D'yakonov-Perel spin relaxation mechanism, leading to the non-monotonic dependence of the spin relaxation time τ_x on the ratio τ/τ_{SO} . For that reason, one can come to the idea that the value of the spin relaxation time of the dressed electron can be tuned by the change in the ratio between the scattering and the spin precession times, introduced by the parameter ζ .

In the system of a 2DEG without dressing, values of time scales τ_{SO} and τ depend only on the material properties and cannot be easily altered [176]. Shifting to the system of a 2DEG dressed by light, the scattering time τ depends on the amplitude E_0 and frequency ω of dressing field (see, Eq. (3.27)), in what follows that it can be varied by an external field and at realistic parameters of the system can be strongly different from the scattering time of unirradiated 2DEG, τ_0 . Discussed dependence allows us to conclude that prevalent regime of the spin relaxation in a 2DEG can be tuned by a high-frequency electromagnetic field, which from applied point of view might be used as a promising method for spin relaxation time control.

For quantitative analysis, consider the numerical simulation of this effect

and mainly dependence of the scattering time τ_x on the intensity of a linearly polarized dressing field given by well-known expression: $I = \epsilon_0 E_0^2 c / 2$. It should be stressed that mathematically this effect is explained by the presence of the Bessel function in the scattering probability of the dressed electron (3.41) and from the physical point of view it comes from the fact that the value of the scattering time of the dressed electron increases with increasing of an external electromagnetic field intensity. To calculate the impact of the dressing field on spin relaxation time, continue the study of a 2D GaAs system with the parameter of the spin-orbit coupling $\alpha = 3.3 \times 10^3$ m/s [177]. Since the scattering time of unirradiated electron is the governing parameter responsible for the preferential relaxation regime, choose several reasonable values of the electron scattering time: $\tau_0 = 0.2, 0.4$ and 1.0 ps. Note that if the initial scattering time of the electron in a 2D system without irradiation, τ_0 , is large enough (for e.g., $\tau_0 = 1.0$ ps), in the absence of external fields the process of spin relaxation governs by the Elliott-Yafet spin relaxation mechanism.

Since the presence of the dressing field leads to increase in the value of electron scattering time, from Eq. (3.65) we are able to state that an external high-frequency electromagnetic field results in the conservation of the Elliott-Yafet mechanism as a dominant regime of the dressed electron spin relaxation, which qualitatively performs in the monotonic growth of the spin relaxation time, τ_x , under the increase of the dressing field intensity shown by dashed line in Fig. 3.8. Otherwise, if the scattering time in unirradiated system, τ_0 , is small enough, the D'yakonov-Perel' mechanism is the prevalent regime of spin relaxation in the absence of dressing field. Applying the dressing field, which leads to the increase of the electron scattering time we predict the transition from the D'yakonov-Perel' spin relaxation mechanism to the Elliott-Yafet spin relaxation mechanism, governed by the intensity of the dressing field (solid and dot-dashed lines in Fig. 3.8).

Generalizing the idea of switching between the D'yakonov-Perel' and the

Figure 3.8: Dependence of the spin relaxation time in a 2D dressed system on the intensity of a dressing electromagnetic field with the frequency $\omega = 100$ GHz. A 2DEG is realized in a GaAs quantum well with the Fermi energy $\epsilon_F = 10$ meV, the spin-orbit coupling constant $\alpha = 3.3 \times 10^3$ m/s and at different values of the scattering time τ_0 .

Elliott-Yafet mechanisms, perform the dependence of the spin relaxation time in a dressed 2DEG on the intensity of this dressing field by a 3D plot with respect to the initial scattering time τ_0 (see, Fig. 3.9). Thence it is clear that the process of the spin relaxation, τ_x , in the 2D systems in the presence of a high-frequency electromagnetic field depends on the quality of the system introduced by the scattering time τ_0 , and at specific values of τ_0 one can achieve the transfer between the D'yakonov-Perel' and the Elliott-Yafet regimes of the spin relaxation. Additionally, one can notice that both regimes in Fig. 3.9 are modulated by oscillations, which according to the theory of the dressed electrons originate from the influence of an electromagnetic field on the scattering probability, $w_{\mathbf{k}'\mathbf{k}}$, introduced by the Bessel function J_0 .

Concluding those described above, in this section we have demonstrated theoretically that the spin properties of a 2DEG can be controlled by an external high-frequency electromagnetic field. In particular, we have found that irradiation of the system by a strong electromagnetic field can lead to the switch between preferential spin relaxation mechanism in a 2DEG and at specific parameters of the system can result in the increase of the spin relax-

Figure 3.9: Generalized dependence of spin relaxation time of a 2DEG in the GaAs system on a dressing field intensity.

ation time in a 2DEG dressed by light. Until now the spin control was only implemented by low-frequency or stationary magnetic and electric fields, in what follows that presented effect can be used in the applied problem of light-tuned spintronics device as an alternative source of spin control.

4. Spin transport of polaritons in diffusive regime

Current chapter denotes the study of the spin dynamics of an alternative quasi-particle, called a polariton, in semiconductor microcavities. Recalling the polariton features mentioned in Introduction, we will discuss the spin transport properties of the polaritons in diffusive regime and compare calculations with well-known results for the spin evolution of the carrier in the electron and the hole-doped 2D systems in the presence of multiple scattering events.

In order to make the study self-sufficient and clear for the reader, we will start this chapter with the definition of the electron, the polariton and the hole-doped systems using conventional model Hamiltonians and introduce the parameters, which might be used for comparison of the diffusive spin transport in these systems. We will focus on the discussion of possible effects governing the spin dynamics in 2D disordered systems, and in particular, as potential sources acting on the spin evolution of the polaritons we will introduce the effect of TE-TM splitting and the Zeeman splitting, induced by applied external magnetic field.

Thereafter we will show that if initially these particles are spatially uniform distributed in the diffusive system, after the dimensional rescaling the dynamics of polariton spins is similar to the electron and the hole spins with the distinctive spin pattern organization in a 2D plane of the system. For that reason we will examine diffusion equations, which provide the information about the dominant interaction effects in these systems, and show that in the presence of external perpendicularly oriented magnetic field polariton spin patterns evolve and tend to match with well-known results for the polariton spin orientation in the ballistic regime. Moreover we will see that in-plane and out-of-plane spin signals of the polaritons can be achieved from

unpolarized initial pump and specially prepared spin signals can transfer to other spin signals under the diffusion processes.

Speaking about experimental implementation note that along with an opportunity to identify the dominant mechanism of spin-orbit interaction this study might be found useful for the prediction of scattering times and the magnitude of spin-orbit coupling in various systems.

4.1 Introduction to the problem

At the beginning of the discussion of polariton dynamics, recall that polaritons can be experimentally observed in a quantum well, which along with two Bragg mirrors form a system called microcavity. For clarity, set a system of coordinates as follows. Assume that the x - and y - axes lie in the plane of a quantum well, and the z -axis is oriented perpendicularly to it. Remember that in analogy to the electron states, polariton states are also defined by the quantum number, called spin, which originates from the polarization direction of its photonic component. Since in the previous chapter we have determined the electron state by the density matrix $\mathcal{S} = [N, S_x, S_y, S_z]^T$ with the matrix elements, corresponding to the charge density, N , and the densities of spin components, S_x, S_y, S_z , introduce the polariton state by the vector similar to the electron vector $\mathcal{S}$, composed by the polariton density N and the polariton spin densities S_x, S_y and S_z , where S_x and S_y denote the linear polarizations of the photonic component of the polariton, oriented parallel to x axis and diagonal to (xy) plane, respectively, and S_z denotes the circular polarization of the polariton photonic component.

Note that like in the electron systems, the presence of spin-orbit coupling in the clean polariton systems results in the precession of polariton spin vector and, consequently, in the transfer from one spin state to another. However, we should remind the reader one of the polariton features, which claims that even though the axis of polariton precession rotates over 360 degrees, the change in the particle momentum occurs under the rotation at

180 degrees. This behavior is unique and does not match with the behavior of the electron systems, where the electron precession axis is linked to the momentum vector. For that reason one can expect that initial S_x or S_y polariton spin state will not evolve as the electron spin into a two-lobed spatial pattern of spin states, but will perform in a quadrupole pattern of S_z states, where in terms of polariton polarization half of lobes correspond to a σ^+ and other half correspond to a σ^- circular polarizations. Recently this effect of polariton spin evolution, called Langbein cross, was deeply studied both theoretically and experimentally in the clean microcavities [103, 178, 104, 107, 179, 180]. In particular, experimental results showed that under the influence of external perpendicularly oriented magnetic field this spatial pattern starts to spiral and turns into a set of concentric circles [181, 182].

Nowadays polariton spin transport was mainly studied in the clean GaAs microcavities, where polaritons were able to propagate without a single scattering on the scale of millimeters [95, 183, 184, 185]. According to the previous results, it was also found that in this, so-called ballistic, regime, polaritons propagate radially from the polariton pump and evolve into a quadrupole pattern formed by opposite S_z spin projections, which change selected spin direction twice for each precession length, l_{prcsn} . Graphically it can be understood as concentric circles with different spin signs superimposed on a quadrupole pattern, which remains until polariton decay [180].

Therefore, it seems interesting to consider a different scenario of system evolution, where polariton motion is affected by diffusion. To achieve this diffusive regime, we should set the length of the system and time of our interest to be long enough in comparison to parameters of a single scattering event. Speaking about the previous studies, it should be mentioned that calculation of polariton dynamics in the equilibrium of a steady-state spatially uniform system in diffusive regime has been done previously using a classical kinetic equation [186, 187]. Motivated by these results, to extend the study of spin transport in polariton systems in the current chapter we are going to

consider polariton dynamics in non-uniform systems within standard perturbation technique used in the diffusive electron systems in the previous chapter.

Before we move to the study of the diffusive spin transport of the polaritons, it seems reasonable to pick up several well-known systems, which can be used for comparison of the spin transport processes in the 2D systems. For that reason, let us choose the electron and the hole-doped systems, created in a two-dimensional GaAs semiconductor, where the Fermi level is doped into the valence band with $J = 3/2$ rather than the conduction band. Remember that the electrons and the holes are defined by real spin number and dynamics of their spin vectors coincide with the precession vector, which results in the switch of spin sign under rotation of momentum to 180 degrees. Speaking about the hole-doped system, it should be also noted that as a consequence of valence band degeneracy in a 2D structure, the spin direction of the hole can be reversed by rotation of its momentum to 60 degrees as well, which can result in the transformation of the initial spin state into a sextuple pattern in a 2D plane with time [188]. Note that the diffusive regime is typical for 2D electron and hole systems, but in the polariton systems, it can be created by the fabrication of nanopillars, embedded in the polariton sample in analogy to optical systems, where the disorder is induced by growing nanocolumns [189, 190].

4.2 Quantum mechanical description of the systems with electrons, holes, and polaritons

4.2.1 The microscopic Hamiltonian

For clarity, let us introduce generalized Hamiltonian suitable for the electron, the polariton, the hole two-dimensional systems, which takes into account

effects of spin-orbit interaction and an external magnetic field. Recall that fundamental difference among these systems lies in the form of the spin-orbit coupling term, which is linear for electrons, quadratic for polaritons and cubic for holes. For that reason, unifying the notation of the Hamiltonian $\hat{H}$, we introduce parameter N , where $N = 1$ for electrons, $N = 2$ for polaritons and $N = 3$ for holes. Then generalized Hamiltonian, $\hat{H}$, is given by:

$$\hat{H}^N = \frac{\hbar^2 k^2}{2m} + b_z \sigma_z + \alpha \begin{bmatrix} 0 & (k_x - ik_y)^N \\ (k_x + ik_y)^N & 0 \end{bmatrix}, \quad (4.1)$$

where the first term is the kinetic energy of selected particle, the second term, called the Zeeman splitting, denotes the system evolution induced by a magnetic field oriented along the z -axis, and the last term refers to the effect of momentum-dependent spin-orbit coupling. Note, that here m is the effective mass of the particle, $\vec{\sigma} = [\sigma_x, \sigma_y, \sigma_z]$ is the vector of Pauli matrices, $\mathbf{k}$ is the wave vector of the particle with components k_x and k_y lying in a 2D plane, b_z is the amplitude of a magnetic field, and α is the parameter of the spin-orbit coupling in the system.

For simplicity, let us rewrite the Hamiltonian (4.1) using the relation between strengths of the Zeeman and the spin-orbit coupling terms, introduced by the angle $\theta_B = \arctan(\alpha k_F^N / b_z)$ and define parameters $\mathcal{S}_B = \sin \theta_B$, and $\mathcal{C}_B = \cos \theta_B$. Then the Hamiltonian (4.1) reads:

$$H^N = \frac{\hbar^2 k^2}{2m} + \mathcal{C}_B E_{prcsn} \sigma_z + \mathcal{S}_B E_{prcsn} k_F^{-N} \begin{bmatrix} 0 & (k_x - ik_y)^N \\ (k_x + ik_y)^N & 0 \end{bmatrix} \quad (4.2)$$

It might be also found useful to express the parameters $\mathcal{C}_B$ and $\mathcal{S}_B$ by the parameter of the energy of splitting between the energy levels caused by the interactions $E_{prcsn} = \sqrt{(\alpha k_F^N)^2 + b_z^2}$ and set:

$$\mathcal{C}_B = \cos \theta_B = \frac{b_z}{E_{prcsn}}, \quad \mathcal{S}_B = \sin \theta_B = \frac{\alpha k_F^N}{E_{prcsn}}, \quad (4.3)$$

where k_F is the magnitude of wave vector $\mathbf{k}$ the particle with energy E . Note that in the electron and the hole systems energy E is the Fermi energy of the structure, controlled by system properties.

Speaking about polariton systems with $N = 2$, we should stress that the Hamiltonian (4.1) is only applicable to the systems where polariton density is low, which allows to neglect the effects of polariton mutual interactions. Otherwise, the Hamiltonian (4.1) should be supplemented by the term responsible for polariton-polariton interaction, which will drastically affect polarization pattern creation and lead to the vanishing of particular spin state formation in system plane [191, 192]. For that reason, it seems reasonable to limit our consideration to the system with low polariton concentration. Then polariton energy E is equal to the energy of the polariton at the lower polariton branch, E_p , with the constant value along an elastic scattering circle in momentum space at fixed magnitude of wave vector $\mathbf{k}$. Setting the zero energy at the bottom of the lower polariton branch, polariton energy value can be esteemed by a fraction of Rabi splitting, $\hbar\Omega_R$. Since in GaAs systems the value of Rabi splitting reaches 6 meV [193, 194, 195], it seems reasonable to choose the value of E_p in the range $E_p=[0, 2]$ meV. Note that variation of this parameter only affects the values of polariton scattering length and time, therefore, we fix our consideration by the value $E_p = 2.0$ meV. Speaking about the influence of interaction on the polariton state, it should be mentioned that experiments showed that values of both terms in Eq. (4.1) can reach 0.2 meV in GaAs microcavities, which is comparable to the value of total polariton energy E_p and therefore potentially might affect the polariton dynamics [193, 196, 197, 198, 199, 200].

4.2.2 Comparative scales

In order to compare the behavior of charged particles - electrons and holes and uncharged polaritons, let us ignore the electrical and magnetic potentials affecting charged particles. Note that this approximation holds for

the systems where cyclotron radius of the particle is greater than both the particle scattering and dephasing lengths, which experimentally feasible in the samples with impurities and at high temperatures in order to break the self-interference effects.

Since transport properties of the particle, such as diffusion length and scattering times are mainly governed by the particle nature along with system properties, it seems reasonable to introduce the dimensionless ratios formed by combination of these parameters. Therefore let us summarize the physically relevant scales, which will be necessary for the following discussion.

- Particle wavelength, l_p , and velocity, v_p :

Particle wavelength is one of the fundamental parameters, which defines dynamics of the particle with mass m and energy E , which is equal to the pumping energy for polaritons and the Fermi energy for electrons and holes and reads:

$$l_p = \frac{h}{\sqrt{2mE}}. \quad (4.4)$$

Accordingly following well-known expressions particle velocity in ballistic regime is given by:

$$v_p = \sqrt{\frac{2E}{m}}, \quad (4.5)$$

which in terms of particle characterization allows us to shift from energy to length scales. For example, in polariton system with energy $E = 2.0$ meV, the wavelength and velocity are $l_p = 4$ μm and $v_p = 4$ $\mu\text{m}/\text{ps}$, respectively.

- The precession length, l_{prcsn} :

Remembering that both Zeeman and spin-orbit interactions lead to spin precession in the system, we can introduce the precession length

as:

$$l_{prcsn} = \frac{\hbar v_p}{E_{prcsn}}, \text{ where } E_{prcsn} = \sqrt{(\alpha k_F^N)^2 + b_z^2}. \quad (4.6)$$

Recall that spin-orbit term is relatively small in comparison to kinetic term in the Hamiltonian (4.1). In what follows that in terms of length scales, this relation fulfills when precession length, l_{prcsn} , is larger than particle wavelength, l_p .

- The scattering length, l :

Following the definition of diffusive regime, it is clear that scattering length, l , should be greater than the particle wavelength to assure an existence of multiple scattering events and, simultaneously, prevent the creation of Anderson-localized states. Note that according to the Ioffe-Regel criterion this requirement mathematically reads: $l_p/l\pi \ll 1$. For example, considering polariton system with $l_p = 4 \mu\text{m}$ let us choose $l = 16 \mu\text{m}$, which gives the polariton scattering time $\tau = l/v_p = 4 \text{ ps}$.

- The decay length, l_{decay} :

Since number of polaritons in the system tends to decrease with time, due to the finite polariton lifetime, τ_{decay} , it seems reasonable to introduce the length scale called the decay length, l_{decay} . Note that it is only applicable for polariton systems because in electron and hole systems decay time is infinite due to charge conservation. Let us interpret the polariton lifetime as the maximum propagation distance of the polariton, l_{decay} , which in diffusive regime reads:

$$l_{decay} = \sqrt{D\tau_{decay}}, \quad (4.7)$$

where D is diffusion constant, which in two-dimensional systems is given by expression $D = lv_p/2$. Thence it is easy to see that to achieve diffusive regime in polariton system, the decay length, l_{decay} , should

exceed the value of scattering length, l . Speaking about realistic parameters for a GaAs system it seems reasonable to choose the micro-cavity where $l_{decay} = 80 \text{ } \mu\text{m}$ or in time scales $\tau_{decay} = 200 \text{ ps} = 50\tau$, which is accessible nowadays in high-quality systems [183, 184]. As it will be shown further, spin transport effects mainly occur at times of order 10τ , in what follows that our discussion will not depend on the polariton decay effects.

Following above discussion, one can notice that above scales can be expressed in terms of scattering length, l , and time, τ . Therefore, in order to generalize our discussion of the spin dynamics in electron, polariton and hole-doped systems, let us introduce four basic dimensionless quantities:

1. The disorder strength $\xi = l_p/l\pi = \hbar/E\tau \ll 1$;
2. The angle $\theta_B = \arctan(\alpha k_F^N/b_z)$;
3. The dimensionless diffusion length $r_0/l = \sqrt{t/2\tau}$, controlling the diffusive propagation of the particles, where r_0 is averaged displacement of the particle, which according to the diffusive law reads $r_0 = \sqrt{Dt}$;
4. The dimensionless energy splitting between states with opposite spin alignment $\zeta = l/l_{prcsn} = E_{prcsn}\tau/\hbar$, which determines the prevalent mechanism of spin relaxation. Following discussion in Introduction, one can notice that this quantity defines the governing mechanism of spin evolution among Elliott-Yafet regime with $\zeta \gg 1/2$ and D'yakonov-Perel regime, where $\zeta \ll 1/2$.

4.3 Analytical calculation of carrier transport in diffusive regime

In order to find spin dynamics in introduced 2D systems in diffusive regime, let us apply the standard method based on diagrammatic technique for dis-

ordered systems, which was used in the previous chapter for the problem of dressed electrons.

Remember that in Eq. (3.60) we used function $\Phi(\mathbf{q}, t)$, called the response function, which contains the information about the magnitude of system response in time and can be found using the diffusion operator $D^{-1}(\mathbf{q}) = 1 - I_{ij}$, related to the scattering operator I_{ij} in Eq. (3.62). Therefore let us start our study of polariton diffusion from the calculation of the scattering operator I_{ij} , which is formed by products of retarded G^R and advanced G^A Green's functions of selected system (3.63). Using the Hamiltonian (4.1), one can easily obtain the expressions for advanced and retarded Green's functions and, for simplification, let us resolve them into separate contributions depending on the spin state $s = \pm 1$ with energy $E(\mathbf{k}, s)$:

$$G^{R,A}(\mathbf{k}, E) = \frac{1}{2} \sum_s \frac{1 + s\mathbf{X}(\mathbf{k}) \cdot \boldsymbol{\sigma}}{E - E(\mathbf{k}, s) \pm i\hbar/2\tau}, \text{ where } \mathbf{X} = [X_1, X_2, X_3] \quad (4.8)$$

$$X_1 = \frac{\alpha \text{Re}[(k_x + ik_y)^N]}{\sqrt{(\alpha|\mathbf{k}|^N)^2 + b_z^2}}, X_2 = \frac{\alpha \text{Im}[(k_x + ik_y)^N]}{\sqrt{(\alpha|\mathbf{k}|^N)^2 + b_z^2}}, X_3 = \frac{b_z}{\sqrt{(\alpha|\mathbf{k}|^N)^2 + b_z^2}}.$$

Note that using expression (4.8) highly accurate derivation of the scattering operator I_{ij} in Eq. (3.62) can be performed numerically in different modern software. However, in the following paragraph, we will not discuss its program algorithm, instead, we will point basic steps of this approach and show how expressions for diffuson $D^{-1}(\mathbf{q})$ can be developed analytically.

Recall that I_{ij} is a 4×4 matrix. Then without generality loss, let us limit our through consideration by matrix elements I_{0i} of the scattering operator matrix I_{ij} . Substituting the Green's functions (4.8) to expression (3.62), one can straightforwardly perform a product of the matrices and find the trace of resulting 2×2 matrix. After that to determine the matrix element I_{0i} , we should sum obtained terms over spin indexes s and s' , which from the first sign seems to be a difficult task. To overcome this difficulty, let us apply the

identity $\frac{1}{ab} = \left(\frac{1}{a} - \frac{1}{b}\right) \left(\frac{1}{b-a}\right)$. Then matrix element I_{0i} reads:

$$\begin{aligned}
 I_{0i} &= \frac{\hbar}{8\pi\nu\tau} \int d\mathbf{k} \sum_{s,s'} (sX_i(\mathbf{k} - \mathbf{q}/2) + s'X_i(\mathbf{k} + \mathbf{q}/2))Y, \text{ where} \\
 Y &= \frac{1}{E_p - E(\mathbf{k} - \mathbf{q}/2, s) - i\hbar/2\tau} \frac{1}{E_p - E(\mathbf{k} + \mathbf{q}/2, s') + i\hbar/2\tau} \\
 &\approx \frac{\pi\tau}{\hbar} \left[\frac{\delta(E_p - E(\mathbf{k} - \mathbf{q}/2, s))}{1 + i\tau(E(\mathbf{k} + \mathbf{q}/2, s') - E_p)/\hbar} + \frac{\delta(E_p - E(\mathbf{k} + \mathbf{q}/2, s'))}{1 + i\tau(E_p - E(\mathbf{k} - \mathbf{q}/2, s))/\hbar} \right].
 \end{aligned} \tag{4.9}$$

Here the last line was obtained by replacement of fraction $(E_p - E(\mathbf{k} - \mathbf{q}/2, s) - i\hbar/2\tau)^{-1}$ by its imaginary part, which is taken to be the Dirac delta function $i\pi\delta(E_p - E(\mathbf{k} - \mathbf{q}/2, s))$. Note that this standard approximation is fulfilled, when the scattering time is long enough comparing to the kinetic time scale, i.e. $\hbar/(E_p\tau) \ll 1$. Physically it means that under this transition we limit our consideration by processes, which occur only on the Fermi surface of electron or hole systems and on the elastic scattering circle of polariton system. From Eq. (4.9) it is interesting to note that dependence of scattering operator, I_{0i} , on scattering time, τ , only performs in the denominator of function Y , which in addition states that since dimensionless parameter of energy splitting $\zeta = E_{prcsn}\tau/\hbar$ depends on time τ it may come to the spin diffusion equation only from the denominator of function Y .

Returning to calculation of I_{0i} , further we need to multiply the integrand $(sX_i(\mathbf{k} - \mathbf{q}/2) + s'X_i(\mathbf{k} + \mathbf{q}/2))$ by function Y . Shifting the wave vector $\mathbf{k} \rightarrow \mathbf{k} \pm \mathbf{q}/2$ the scattering operator I_{0i} reads:

$$\begin{aligned}
 I_{0i} &= \\
 &\frac{1}{8\nu} \int d\mathbf{k} \sum_{s,s'} s \left[\frac{\hbar X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{\hbar + i\tau(E(\mathbf{k} + \mathbf{q}, s') - E(\mathbf{k}, s))} + \frac{\hbar X_i(\mathbf{k} - \mathbf{q})\delta(E_p - E(\mathbf{k}, s'))}{\hbar + i\tau(E(\mathbf{k}, s') - E(\mathbf{k} - \mathbf{q}, s))} \right] + \\
 &+ \frac{1}{8\nu} \int d\mathbf{k} \sum_{s,s'} s' \left[\frac{\hbar X_i(\mathbf{k} + \mathbf{q})\delta(E_p - E(\mathbf{k}, s))}{\hbar + i\tau(E(\mathbf{k} + \mathbf{q}, s') - E(\mathbf{k}, s))} + \frac{\hbar X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s'))}{\hbar + i\tau(E(\mathbf{k}, s') - E(\mathbf{k} - \mathbf{q}, s))} \right].
 \end{aligned} \tag{4.10}$$

After that expand the function X_i in powers of diffuson wave vector $\mathbf{q}$.

As one can see each power of $\mathbf{q}$ is accompanied by a power of factor $1/k_F$. Following the definition of dimensionless parameters introduced previously, it is easy to get that product of these factors can be rewritten as a small parameter ξql . Note that since ξ is a small parameter and there are no compensating τ factors in X_i , at first approximation we are allowed to expand X_i only to the leading order in ξ . Then let us decompose function X_i as $X_i(\mathbf{k} + \mathbf{q}) = X_i(\mathbf{k}) + \mathbf{q} \cdot \nabla X_i(\mathbf{k})$, which will give us two separated diffusion matrix elements $I_{0i} = I_{0i}^{(0)} + I_{0i}^\xi$, where

$$I_{0i}^{(0)} = \frac{1}{8\nu} \int d\mathbf{k} \sum_{s,s'} (s + s') \left[\frac{X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{1 + i\tau(E(\mathbf{k} + \mathbf{q}, s') - E(\mathbf{k}, s))/\hbar} + \frac{X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s'))}{1 + i\tau(E(\mathbf{k}, s') - E(\mathbf{k} - \mathbf{q}, s))/\hbar} \right], \quad (4.11)$$

$$I_{0i}^\xi = \frac{1}{8\nu} \int d\mathbf{k} \sum_{s,s'} \left[s \frac{-\mathbf{q} \cdot \nabla X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s'))}{1 + i\tau(E(\mathbf{k}, s') - E(\mathbf{k} - \mathbf{q}, s))/\hbar} + s' \frac{\mathbf{q} \cdot \nabla X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{1 + i\tau(E(\mathbf{k} + \mathbf{q}, s') - E(\mathbf{k}, s))/\hbar} \right]. \quad (4.12)$$

Note that this rearrangement gives us a significant result. In particular, it became obvious that terms with $s \neq s'$ give null contribution to $I_{0i}^{(0)}$, which allows us to perform additional simplification

$$I_{0i}^{(0)} = \frac{1}{4\nu} \int d\mathbf{k} \sum_s s \left[\frac{X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{1 + i\tau(E(\mathbf{k} + \mathbf{q}, s) - E(\mathbf{k}, s))/\hbar} + \frac{X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{1 + i\tau(E(\mathbf{k}, s) - E(\mathbf{k} - \mathbf{q}, s))/\hbar} \right] = \frac{1}{2\nu} \int d\mathbf{k} \sum_s s \frac{X_i(\mathbf{k})\delta(E_p - E(\mathbf{k}, s))}{1 + i\tau\mathbf{q} \cdot \nabla E(\mathbf{k}, s)/\hbar}, \quad (4.13)$$

where we have expanded the denominator in powers of $\mathbf{q}$.

It should be stressed that presence of variations in the Dirac delta-function with spin index s reflects the difference between the density of states corresponding to the Fermi surface with the spin state $s = +1$ and the density of states on the Fermi surface with the spin $s = -1$. In analogy, the energy gradient $\nabla E(\mathbf{k}, s)$ shows the difference between the Fermi velocity of the particles on the Fermi surfaces with opposite spin alignments and variation

of X_i reflects the difference in the angular orientation on these surfaces originated from the spin-orbit interaction. Therefore to analyze their impacts on the particle dynamics, we suggest to calculate the spin splitting of these surfaces as a function of the angular variable ϕ_k . Limiting calculation of the scattering operator by the first order in the ratio between the energy of spin splitting and the Fermi energy $E_{prcsn}/E_p = \xi\zeta$, contribution of scattering operators I_{0i} and I_{0i}^ζ to the system dynamics reads:

$$I_{0i}^{(0)} = \int \frac{d\phi_k}{4\nu} \sum_s \rho(\phi_k, s) \frac{X_i(\phi_k, s)}{1 + i\tau \mathbf{q} \cdot \nabla E(\mathbf{k}, s)/\hbar}. \quad (4.14)$$

$$I_{0i}^\zeta = \frac{1}{8\nu} \int \frac{d\phi_k}{16\pi} \mathbf{q} \cdot \nabla X_i(\mathbf{k}) \sum_N (-i\tau v_F(\mathbf{k} \cdot \mathbf{q}))^{N-1} \sum_{\pm} \sum_s \frac{\mp s}{(1 \mp 2is\zeta)^N}, \quad (4.15)$$

Summarizing above, we found that to obtain matrix elements of the scattering operator I_{ij} one only needs to perform the calculation of several angular integrals, similar to expressions (4.14) and (4.15). However, it should be mentioned that straightforward analytical calculation of these integrals is still a difficult task. For that reason recall a direct relation between the particle position in momentum and coordinate spaces, introduced by the Fourier transformation in Eq. (3.58). Thence one can understand that factor iq_x in momentum space corresponds to the spatial derivative ∂/∂_x in coordinate space. Thus, since we are interested in the effects, which occur at length scales longer than the scattering length, l , we can apply a Taylor series expansion of the scattering operator I_{ij} with respect to ql and limit our consideration by the second order terms, which from mathematical point of view gives us two spatial derivatives in diffusion matrix in coordinate space. Note that this limit agrees with a conventional representation of diffusion in the presence of random scattering events, shown in various fundamental works on diffusive transport [167, 174].

Eventually remembering initial conditions of our problem and performing Taylor expansion in the integrand of scattering operator I_{ij} , we become able

4. Transport of polaritons in diffusive regime

to make an integration over the Fermi surface of electron or hole systems and elastic scattering circle of polariton system and find the diffusion matrix, which reads:

$$\begin{aligned}
 \tau D^{-1}(q) = & \tau \partial_t + \begin{bmatrix} \frac{(ql)^2}{2} & 0 & 0 & D_{0z,N} \frac{(ql)^2}{2} \\ 0 & \frac{\tau}{\tau_{xx}} + D_{xx,N} \frac{(ql)^2}{2} & \frac{\tau}{\tau_{xy}} + D_{xy,N} \frac{(ql)^2}{2} & 0 \\ 0 & -\frac{\tau}{\tau_{xy}} - D_{yx,N} \frac{(ql)^2}{2} & \frac{\tau}{\tau_{xx}} + D_{yy,N} \frac{(ql)^2}{2} & 0 \\ D_{0z,N} \frac{(ql)^2}{2} & 0 & 0 & \frac{\tau}{\tau_{zz}} + D_{zz,N} \frac{(ql)^2}{2} \end{bmatrix} \\
 & - (\imath ql)^N \begin{bmatrix} 0 & \gamma_N \cos N\theta_q & \gamma_N \sin N\theta_q & 0 \\ \gamma_N \cos N\theta_q & 0 & 0 & f_N \cos N\theta_q \\ \gamma_N \sin N\theta_q & 0 & 0 & f_N \sin N\theta_q \\ 0 & f_N \cos N\theta_q & f_N \sin N\theta_q & 0 \end{bmatrix} - \\
 & - (\imath ql)^N \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_N \sin N\theta_q \\ 0 & 0 & 0 & g_N \cos N\theta_q \\ 0 & g_N \sin N\theta_q & -g_N \cos N\theta_q & 0 \end{bmatrix}, \tag{4.16}
 \end{aligned}$$

where for simplicity we have introduced parameters:

– related to spin evolution of a spatially uniform particle distribution:

$$\frac{\tau}{\tau_{xy}} = 2\mathcal{C}_B \frac{\zeta}{1+4\zeta^2}, \quad \frac{\tau}{\tau_{xx}} = 2(1+\mathcal{C}_B^2) \frac{\zeta^2}{1+4\zeta^2}, \quad \frac{\tau}{\tau_{zz}} = 4\mathcal{S}_B^2 \frac{\zeta^2}{1+4\zeta^2};$$

- related to diffusive transport of selected spin component

$$\begin{aligned}
D_{zz} &= \left(\mathcal{C}_B^2 + \mathcal{S}_B^2 \frac{1 - 12\zeta^2}{(1 + 4\zeta^2)^3} \right), \\
D_{xy,2} &= D_{xy,3} = D_{yx,2} = D_{yx,3} = \frac{2\mathcal{C}_B\zeta(-3 + 4\zeta^2)}{(1 + 4\zeta^2)^3}, \\
D_{xx,2} &= D_{yy,2} = D_{xx,3} = D_{yy,3} = \frac{1}{2} \left(\mathcal{S}_B^2 + (2 - \mathcal{S}_B^2) \frac{1 - 12\zeta^2}{(1 + 4\zeta^2)^3} \right), \\
D_{xx,1} &= D_{xx,2} + e \cos(2\theta_q), \quad D_{yy,1} = D_{yy,2} - e \cos(2\theta_q), \\
D_{xy,1} &= D_{xy,2} + e \sin(2\theta_q), \quad D_{yx,1} = D_{yx,2} - e \sin(2\theta_q), \\
D_{0z,N} &= -\chi\zeta\mathcal{C}_B^3 + (N - 1)\chi\zeta\mathcal{C}_B\mathcal{S}_B^2 + \chi\zeta\mathcal{C}_B\mathcal{S}_B^2 N/(1 + 4\zeta^2)^2;
\end{aligned}$$

- related to the interaction between the components of $\mathcal{S}$ particle vector

$$\begin{aligned}
e &= \frac{2\zeta^2\mathcal{S}_B^2(3 + 6\zeta^2 + 8\zeta^4)}{(1 + 4\zeta^2)^3}, \\
\gamma_N &= \chi\zeta\mathcal{S}_B \times [(1 + \mathcal{C}_B^2)/4, \mathcal{C}_B^2/2, (3/16)(2\mathcal{C}_B^2 - \mathcal{S}_B^2)] + \\
&\quad + \chi\zeta\mathcal{S}_B(1 + \mathcal{C}_B^2)/(1 + 4\zeta^2)^N \times [-1/4, -1/2, -(9/16)(1 - 4\zeta^2/3)], \\
f_N &= \frac{\mathcal{C}_B\mathcal{S}_B\zeta^2}{(1 + 4\zeta^2)^{N+1}} \times [-(6 + 8\zeta^2), 6 + 12\zeta^2 + 16\zeta^4, -(5 + 10\zeta^2 + 32\zeta^4 + 32\zeta^6)], \\
g_N &= \frac{\zeta\mathcal{S}_B}{(1 + 4\zeta^2)^{N+1}} \times [2, (-3 + 4\zeta^2)/2, 1 - 4\zeta^2].
\end{aligned}$$

In order to make our analysis more clear to the reader, let us represent matrix (4.16) as a sum of following terms: $D^{-1} = -(\lambda - \Delta + \kappa_{0l} + \kappa_{lc} + \tau/\tau_{decay})$, where:

- matrix λ gives the spin relaxation rates;
- matrix Δ shows the coupling between the number and S_z spin density component;
- matrix κ_{0l} demonstrates the coupling between the number and S_x or S_y spin density component;
- matrix κ_{lc} corresponds to the coupling between spin components;

- matrix τ/τ_{decay} is a diagonal matrix, introduced phenomenologically in order to take into account effect of polariton decay in the system.

Note that expression for diffusion matrix (4.16) is an interposition of two limiting cases of spin relaxation. Indeed, let us represent above matrices in the Elliott-Yafet regime, when $\zeta \gg 1$, and in the D'yakonov-Perel' regime, when $\zeta \ll 1$, and analyze their features, defining evolution processes in 2D systems.

Start our study from the matrix λ . Using the limits introduced above, matrix λ^{DP} in the D'yakonov-Perel' regime and matrix λ^{EY} in the Elliott-Yafet regime read:

$$\lambda^{DP} = 2\zeta \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \zeta(1 + \mathcal{C}_B^2) & \mathcal{C}_B & 0 \\ 0 & -\mathcal{C}_B & \zeta(1 + \mathcal{C}_B^2) & 0 \\ 0 & 0 & 0 & 2\zeta\mathcal{S}_B^2 \end{bmatrix}, \lambda^{EY} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1+\mathcal{C}_B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{1+\mathcal{C}_B^2}{2} & 0 \\ 0 & 0 & 0 & \mathcal{S}_B^2 \end{bmatrix} \quad (4.17)$$

Recall that diagonal terms of matrix λ show the relaxation rates for particle number and each spin component of the particle in selected system. Thence, both matrix elements $\lambda^{DP}(1,1)$ and $\lambda^{EY}(1,1)$ in these regimes of spin relaxation equal zero, which means that a total number of particles in the system is conserved during the process of elastic scattering. To check the correctness of our result, let us study the dependence of spin relaxation time on the scattering time τ in both the DP and the EY regimes. After simple manipulation, one can see that in the D'yakonov-Perel' regime relaxation rate of selected spin component is proportional to square of ζ , in what follows that spin relaxation time $\tau_{x,y,z} \sim \tau/\zeta^2 \sim 1/\tau$, while in the Elliott-Yafet regime spin relaxation times are linearly proportional to the scattering time τ , which agrees with their fundamental principals mentioned in Introduction.

Speaking about the effect of off-diagonal terms in matrix λ , we need to mention that they show the precession of in-plane spin components around the magnetic field oriented perpendicularly to 2D system plane. As one can

see from the values of off-diagonal terms in the DP and the EY regimes, our result coincides with conventional theory of spin evolution, which claims that in the DP regime spin precesses many times $O(\zeta^{-1})$ before the decay, while in the EY regime precession is not visible at diffusive scales, because each scattering event randomizes the spin orientation, which performs in nulling of off-diagonal terms in λ^{EY} .

In addition note that λ^{DP} and λ^{EY} does not have a dependence on parameter N , responsible for the particle properties, which means that spin relaxation processes in electron, polariton and hole systems follow similar laws.

Secondary, consider matrix Δ , which reflects the diffusion processes in the system:

$$\Delta^{DP} = -\frac{(ql)^2}{2} \begin{bmatrix} 1 & 0 & 0 & D_{0z} \\ 0 & 1 & -6\zeta\mathcal{C}_B & 0 \\ 0 & 6\zeta\mathcal{C}_B & 1 & 0 \\ D_{0z} & 0 & 0 & 1 \end{bmatrix}, \Delta^{EY} = -\frac{(ql)^2}{2} \begin{bmatrix} 1 & 0 & 0 & D_{0z} \\ 0 & \frac{\mathcal{S}_B^2}{2} & 0 & 0 \\ 0 & 0 & \frac{\mathcal{S}_B^2}{2} & 0 \\ D_{0z} & 0 & 0 & \mathcal{C}_B^2 \end{bmatrix}$$

$$\text{where } D_{0z,N} = -\xi\zeta\mathcal{C}_B + \xi\zeta\mathcal{C}_B\mathcal{S}_B^2N - \xi\zeta/(1 + 4\zeta^2)^2\mathcal{C}_B\mathcal{S}_B^2N. \quad (4.18)$$

It is easy to see that matrix Δ nulls at zero wave vector $q = 0$, which means that it does not affect spatially uniform particle distribution. Therefore focusing on the case of non-uniform particle distribution, we find that matrix elements $\Delta(1,4)$ and $\Delta(4,1)$ are equal to D_{0z} , which show the presence of coupling between the S_z spin component and the particle density. Note that in terms of polariton systems, non-zero values of $\Delta(1,4)$ and $\Delta(4,1)$ matrix elements reflect the process of S_z signal generation from initially unpolarized polariton state, which is entirely controlled by diffusion with factor $\xi\zeta = l_p/\pi l_{prcsn}$. Analyzing the dependence of this term on the value of magnetic field, one can notice that term D_{0z} equals zero in the absence of external magnetic field $b_z = 0$ in the system. It explains the reason why this coupling term has not been obtained in the previous fundamental works

on diffusion with a Rashba spin-orbit interaction term [167, 174]. Also, note that resonance behavior of matrix element D_{0z} differs from one system to another. In particular, D_{0z} term in electron systems with linear spin-orbit interaction does not depend on the strength of spin-orbit interaction and only vanishes at $b_z = 0$. However, in polariton and hole-doped systems, this coupling term also vanishes, when the magnitude of external magnetic field $|b_z|$ is tuned to the value of spin-orbit interaction term $|\alpha k_F^2|$ and $\sqrt{2}|\alpha k_F^3|$, respectively. Therefore speaking about a practical interest of this result we suggest to use this effect for the measurement of the strength of spin-orbit coupling in polariton and hole-doped systems.

It should also be mentioned that during the calculation of the scattering operator I_{ij} , we have neglected the terms, which were proportional to $(ql)^{2N}$, since their contributions in polariton and hole systems are smaller than $(ql)^2$. However, speaking about electron systems with Rashba interaction it is clear that impact of these additional terms is comparable with the effect of matrix Δ . Therefore, for accuracy let us take them into account and represent by matrix Δ^1 :

$$\Delta^1 = -e' \frac{(ql)^2}{2} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \cos(2\theta_q) & \sin(2\theta_q) & 0 \\ 0 & \sin(2\theta_q) & -\cos(2\theta_q) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.19)$$

where parameter $e' = (2\zeta^2 \mathcal{S}_B^2 (3 + 6\zeta^2 + 8\zeta^4)) / (1 + 4\zeta^2)^3$.

As one can see, in the limit of the D'yakonov-Perel' regime, $\zeta \ll 1/2$, matrix elements of matrix Δ^1 are proportional to ζ^2 , which means that their effects on electron spin components are small and can be neglected. However, in the limit of the Elliott-Yafet regime, $\zeta \gg 1/2$, Δ^1 tends to $\mathcal{S}_B^2/4$ and results in the formation of a quadrupole spin pattern with angular dependence $2\theta_q$ in the system plane. Note that recently presence of the terms with similar angular dependence in diffusion matrix has been found in the

problem of spin conduction on the surface of a 3D topological insulators [166, 201].

Summarizing the discussion of matrices Δ and Δ^1 , let us compare our result with Ref. [174], where the value of spin splitting was very small, and therefore diffusion terms were approximated by the constant values at $\zeta = 0$. Note that in the current problem we kept terms with all orders of ζ , which allows us to consider the spin diffusion with the rates different from the evolution of charge or number densities proposed in several previous theoretical works [202, 203, 204]. Analyzing the impact of our complexity, it is interesting to note that matrix element $\Delta(1,1)$, which controls the evolution of total number/charge density of the particles, is equal to $(ql)^2/2$ and independent on ζ value, while the rest terms are ζ -dependent. For instance, expanding $\Delta(4,4)$ matrix element to the second order in ζ , we get that it is equal to $(ql)^2(1 - 24\zeta^2)/2$, which means that spatial inhomogeneity in number or charge particle density reduces faster than spatial inhomogeneities of spin components in selected system.

Speaking about third matrix κ_{0l} , recall that physically it is related to the effect of the S_x or the S_y spin component generation from the charge/number density:

$$\kappa_{0l} = -(ql)^N \gamma_N \begin{bmatrix} 0 & \cos N\theta_q & \sin N\theta_q & 0 \\ \cos N\theta_q & 0 & 0 & 0 \\ \sin N\theta_q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (4.20)$$

$$\begin{aligned} \gamma_N = & \chi\zeta\mathcal{S}_B \times \{(1 + \mathcal{C}_B^2)/4, \mathcal{C}_B^2/2, (3/16)(2\mathcal{C}_B^2 - \mathcal{S}_B^2)\} + \\ & + \chi\zeta/(1 + 4\zeta^2)^N \mathcal{S}_B(1 + \mathcal{C}_B^2) \times [-1/4, -1/2, -(9/16)(1 - 4\zeta^2/3)]. \end{aligned}$$

Following expression (4.20) κ_{0l} implies that in the presence of spin-orbit coupling, for example, in polariton system, initially unpolarized particle state can evolve into S_x and S_y polariton spin states under the diffusion process with the strength, governed by factor $\xi\zeta = l_p/\pi l_{prcsn}$. According

to the angular dependence $N\theta_q$ in the matrix elements of κ_{0l} , note that this effect is very sensitive to the nature of the particle, which performs in angular-dependent spin pattern formation in the system plane. For example, in electron systems with $N = 1$ it results in a dipole pattern, in polariton systems with $N = 2$ in a quadrupole pattern and in hole-doped systems with $N = 3$ in a sextupole pattern configurations.

In addition, we found that the magnitude of resulting signal has a strong dependence on the dimensionless factor ql at N -th power. Therefore, since we are looking for the signal response at length scales much larger than the scattering length l , the probability to observe a diffusive spin pattern experimentally is higher in electron systems and reduces under transition to hole-doped systems.

Speaking about matrix κ_{0l} , let us show the relation between matrix elements and the Zeeman term. Start with the fact that it is different in electron, polariton and hole systems and more visible in the Elliott-Yafet regime of spin relaxation. For that reason considering the limit of the EY regime, we obtain that presence of the Zeeman term changes the value of a spin signal at most in two times. Regarding the polariton systems, note that absence of Zeeman splitting, $\mathcal{C}_B = 0$, vanishes this effect, in what follows that presence of external magnetic field is necessary requirement for the creation of selected spin polarization from an unpolarized pump in polariton systems. Besides that, note S_x and S_y states can be set to zero by the magnetic field with the value satisfying the relation $|b_z| = |\alpha|/\sqrt{2}$, which can also be used for the measurement of the spin-orbit strength in these systems.

Finally, turn to matrix elements of the scattering operator I_{ij} related to the coupling between S_x or S_y and S_z spin component, which are written in

matrix κ_{lc} :

$$\kappa_{lc} = -(\imath ql)^N \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & f_N \cos N\theta_q \\ 0 & 0 & 0 & f_N \sin N\theta_q \\ 0 & f_N \cos N\theta_q & f_N \sin N\theta_q & 0 \end{bmatrix} - (\imath ql)^N \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -g_N \sin N\theta_q \\ 0 & 0 & 0 & g_N \cos N\theta_q \\ 0 & g_N \sin N\theta_q & -g_N \cos N\theta_q & 0 \end{bmatrix} \quad (4.21)$$

$$DP : f_N = 0, \quad g_N = \zeta \mathcal{S}_B \times [2, -3, 1],$$

$$EY : f_N = -\mathcal{C}_B \mathcal{S}_B (-2)^{-N}, \quad g_N = 0.$$

Physically these coupling terms state that presence of spin-orbit interaction in the system causes the creation of the out-of-plane spin component from the initial S_x or S_y signals, and vice versa. Following the definition of parameters f_N and g_N , it is easy to see that in the D'yakonov-Perel' regime transition between in-plane and out-of-plane spin components only requires the presence of spin-orbit coupling in the system, while in the Elliott-Yafet regime this transition is only possible, when both effects of spin-orbit interaction and external magnetic field are present in the system. Note that in the DP regime this transition is called bona fide spin precession, because of the contribution of factor g_N , which makes matrix κ_{lc} anti-Hermitian and results in the possibility of oscillations between S_x or S_y and S_z spin components, while in the EY regime these oscillations are not possible, because f_N term is dominant and makes matrix κ_{lc} Hermitian.

Comparing the magnitude of this effect in the system, we should mention that, in analogy to the dependence of matrix κ_{0l} on particle nature, precession between spin states also depends on parameters of the system and quantitatively stronger in electron system and weaker in hole-doped systems.

Finally, speaking about the coordinate representation, like κ_{0l} , matrix κ_{lc} performs in the angular dependence of spin state distribution introduced by a dipole, quadrupole and sextupole patterns in the system planes for the electron, polariton and hole systems, respectively.

4.3.1 Summary of analytical results

Summarizing the above, we need to stress that analytical results for diffusion matrix have been calculated in momentum space, which using the Fourier transformation can be found in the coordinate space representation. According to our result, let us make several conclusions on the particle distribution in the system.

Firstly, we have shown that for spatially homogeneous particle distribution, systems of electrons, polaritons and holes have similar behavior with identical spin relaxation and precession rate dependencies. Additionally, we have found that these systems do not have coupling between neither spin components, nor number/charge density and spin projections, which coincides with fundamental results on diffusive spin transport in electron systems.

Secondary, shifting to the case of spatially non-uniform charge and spin density distributions we have seen more enthralling and promising results. Speaking about the S_z spin component, we observed that dynamics of its magnitude is similar to electron, polariton and hole systems. However, we have noticed that for polariton and hole systems in the Elliott-Yafet regime S_z spin component can be vanished by external magnetic field, which satisfies the resonance requirement on the spin-orbit coupling strength. Note that in analogy to the dependence of S_z spin component on the magnetic field, in the Elliott-Yafet regime other spin projections might be tuned by applied magnetic field, however, it occurs only in hole-doped systems.

Finally, we have found that starting from specially prepared spin state or even unpolarized state, spin system is able to evolve and under the dif-

fusion process can transfer to another spin state. It should be mentioned, that effect of transfer between spin projections depends on particle nature. In particular, it is stronger in electron systems and relatively weaker in hole-doped systems and speaking about the coordinate representation, this transition can be seen as a formation of a distinctive dipole, quadrupole and sextupole patterns in 2D system plane for electrons, polaritons, and holes, respectively.

In order to show the quantitative impact of these effects on spin dynamics, in the next section, we will show numerical results of the response function Φ in real space and spin effects produced by the spatially localized pumps.

4.4 Numerical calculation of carrier transport in diffusive regime

In the previous section we have calculated analytically matrix elements of diffusion matrix $D^{-1}(\mathbf{q})$ in momentum space. As it was discussed in Chapter 2, in order to simulate evolution of the spin components in disordered system, one needs to find either the frequency response function, $\Phi(\mathbf{q}, \omega)$, in momentum space, which is given by the expression $i\omega + \tau^{-1}D^{-1}(\mathbf{q})$, or the time-dependent response function, $\Phi(\mathbf{q}, t)$, in momentum space from relation $\exp[tD^{-1}(\mathbf{q})/\tau]$. Note that for the study of system dynamics in the coordinate space, these response functions can be represented as a real-space functions $\Phi(\mathbf{r}, \omega)$ or $\Phi(\mathbf{r}, t)$ using conventional Fourier transformation.

Since the expressions for the matrix elements of diffuson are known, before numerical calculation we should introduce an additional macroscopic scale, which defines the system sizes. Let us assume that length of the sample is much larger than the scattering length of the particle l , which will allow us to neglect the effects of the boundary on diffusion process.

4.4.1 Effects of spin evolution: precession and decay

In order to simulate the evolution of polariton spin components in diffusive systems, start from the momentum representation of the diffusion equation and examine the processes of spin decay in the systems with spin-orbit interaction and external magnetic field at fixed value of the wave vector.

Assume that initially polaritons are spatially uniform distributed in the system, $\mathbf{q} = 0$, and characterized by the spin S_x . Thence, performing the calculation of the response function, one can see that under diffusion initial S_x spin state starts to evolve to S_y spin component and oscillates between these in-plane components with time, as it is shown in Fig. 4.1(a). Generally speaking, the physical origin of these oscillations lies in the effect of spin precession around the Zeeman field, oriented perpendicularly to the system plane. This conclusion can be easily verified by neglecting the spin-orbit coupling term in the diffusion matrix. Indeed, at $\alpha = 0$ we found that S_x and S_y spin components, shown by the orange and the red dashed lines in Fig. 4.1(a) respectively, still perform the oscillatory behavior with decay coming from the finite polariton lifetime. Note that during the study of spin transport in electron and hole-doped systems, we have obtained identical spin relaxation dynamics with the only difference that the total particle density, I , is conserved with time, because of the infinite lifetimes of the electrons and the holes. Next, consider the process of S_z spin signal generation from the initial in-plane S_x spin state. Recalling that it cannot be achieved from spatially uniform polariton distribution, let us study the case when polaritons are non-uniformly distributed with nonzero wave vector $\mathbf{q} = \mathbf{e}_x/l\sqrt{10}$. To make our analysis clear, one can find useful to introduce alternative functions s_x, s_y and s_z , called the polariton spin degrees, proportional to the S_x, S_y and S_z spin densities, respectively, but divided by the total polariton density I . Then performing the similar procedure, we found the result shown in Fig. 4.1(b). As one can notice the largest value of spin degree s_z can be achieved at large time scales ($t \gg \tau$). For example, at time $t = 50\tau$

Figure 4.1: Dependence of polariton spin precession on time in diffusive system, where the black lines show the polariton density; the orange, the red and the blue lines correspond to S_x , S_y and S_z polariton spin components, respectively; solid lines show behavior of the system with both spin-orbit and Zeeman terms ($\theta_B = 5\pi/8$) and dashed lines show spin evolution in the absence of spin-orbit coupling ($\theta_B = 0$). Panel (a) denotes the evolution of polariton spin density at spatially uniform particle distribution with wave vector $q = 0$ and panel (b) denotes the evolution of polariton spin degree at particle distribution with the wave vector $ql = 1/\sqrt{10}$. Note that panel (a) reflects the effect of in-plane spin precession between S_x and S_y spin components while panel (b) shows creation of out-of-plane S_z spin component under diffusion process. In order to obtain this result, we used following parameters of the system: $\zeta = 0.1$, $\xi = \hbar/E_p\tau = 1/4\pi$, $\tau_{decay} = 50\tau$, and $\tau = 4$ ps.

the total polariton density reduces in about 3% from the initial value while initial s_x spin degree reverses sign and reduces in 2 times and the value of generated s_z spin degree equals 13% from the initial spin state.

According to our results for spin relaxation in polariton systems with the disorder, it is easy to conclude that initial particle distribution has a significant influence on polariton spin dynamics. Thence it seems interesting to check the dependencies of spin decay times, τ_p , on the dimensionless parameter ζ , which are shown in Fig. 4.2 at four values of the dimensionless parameter θ_B . Analyzing the result, we found that in the absence of spin-orbit coupling in the system $\theta_B = 0$ decay times of S_x and S_z spin components τ_p are equal and do not change with parameter ζ . Basically, this relation means that at $\theta_B = 0$ time of spin decay coincides with the time of polariton decay, τ_{decay} , in the system, in what follows, that we can expect to have a significant accumulation of S_z spin component when the value of spin-orbit coupling is small.

Figure 4.2: Dependence of spin decay time, τ_p , on the value of a dimensionless parameter of spin splitting ζ , where solid lines correspond to S_z spin projection and dashed lines correspond to S_x spin projection. We show the dependence of spin decay time on ζ for four values of the parameter $\theta_B = 0, \pi/6, \pi/3, \pi/2$ in the system. Note that at the EY regime, when $\zeta \gg 1/2$, spin decay time is constant and at the DP regime, when $\zeta \ll 1/2$, it scales with ζ^{-2} .

4.4.2 Spatial spin profile

To find the results, which can be confirmed experimentally, let us focus on the dynamics of spin components in the coordinate space. Performing the calculation of the system response function in the coordinate space, $\Phi(\mathbf{r}, t)$, one can see that dynamics of the electron, polariton, and hole systems are qualitatively identical. Therefore, in the following section, we will only discuss the radial dependence of polaritons, created by a spatially localized pump keeping in mind that similar results can be obtained in the electron and the hole-doped systems.

Let us consider the response of polariton system under both transient and oscillating initial pumps and study the temporal system response after the pulse at times $t = 5\tau, 20\tau, 80\tau$, shown in the left panels of Fig. 4.3 and the frequency response to an oscillating signal with periods $T = 5\tau, 20\tau, 80\tau$, shown in the right panels of Fig. 4.3, where we plot the averaged polarization degrees $\langle s_x, s_y, s_z \rangle_\phi$, which are calculated by the rms-average of selected polarization S_x, S_y, S_z by the rms-average of I . Note that for simplicity of graphical representation, we also use the coordinate axis rescaling in the each panel of Fig. 4.3.

Speaking about dynamics of polariton density, it is interesting to note that according to the panels 4.3(a) and 4.3(b) the polatiron density, I , decays quadratically with respect to the distance, r , which is related to the Gaussian distribution, and corresponding frequency response curves decay exponentially and coincide with each other after the axis rescaling $r_0 = \sqrt{DT}$. It should be mentioned that the shape of these response functions is similar to the results for response functions of the particle without spin-orbit interaction or spin precession in diffusive regime. Indeed, following conventional studies the last case is represented by the diffusion matrix element $D\tau q^2$ in the momentum space. Then performing the Fourier transformation, the time response function in the coordinate space, $\Phi(\mathbf{r}, t)$, is an expanding Gaussian function: $(2\pi Dt)^{-1} \exp(-r^2/2Dt)$ and the frequency response function

Figure 4.3: Radial distribution of polariton density and spin degree in the system with disorder, where the left panels show the time response $\mathcal{S}(\mathbf{r}, t)$ at time t for a transient pulse and the right panels show the frequency response $\mathcal{S}(\mathbf{q}, t)$ to an oscillating signal with period T using the rescaled by diffusion length radial coordinate $r_0 = \sqrt{Dt}$. Note that all panels show the signal response after the rms average over the polar angle ϕ . In particular, the upper panels show the dynamics of polariton density $\langle I \rangle_\phi$ produced by an unpolarized pump, the middle panels show the creation of $\langle s_z \rangle_\phi$ spin signal from linearly polarized pump, corresponding to s_x polariton spin degree, and the lower panels show the dynamics of $\langle s_x \rangle_\phi$ and $\langle s_y \rangle_\phi$ polariton spin degrees produced by unpolarized pump. Here parameters of polariton system are: $\zeta = 0.1$, $\xi = \hbar/E_p\tau = 1/4\pi$, $\theta_B = 5\pi/8$, $\tau_{decay} = 50\tau$, and $\tau = 4$ ps.

in the coordinate space, $\Phi(\mathbf{r}, \omega)$, is given by expression: $2\pi|K_0(r\sqrt{i\omega/D})|$, where K_0 is a modified Bessel function, which decays exponentially with r . Therefore comparing these analytical results to Fig. 4.3(a)-(b), it is clear that the dynamics of the particle density without spin-orbit interaction is very similar to our calculation.

In the middle panels 4.3(c) and 4.3(d) we demonstrate the precession between initially linear x -polarized pump and the circularly polarized s_z polariton state. From the general sense, it is easy to understand that this rate decreases with time and can be slightly tuned by the parameters θ_B and ζ . Quantitatively from the time representation picture, we see that the conversion rate between them can reach the 10^{-1} range at times $t = 5\tau$ and 20τ , which decays at longer times. It should also be mentioned that the value of s_z polariton spin degree increases with radius, but at the cost of reducing the total polariton density.

Talking about the lower panels 4.3(e) and 4.3(f), they show the effect of linearly polarized s_x polariton response under the irradiation by an unpolarized light. Note that the shape of this response is very similar to the results in (c) and (d) panels, with a quantitative difference in order of signal magnitude. Mathematically the difference between the middle and the lower panels is scaled by a factor of ξ , which comes from the presence of factor ξ in the coupling matrix κ_{0l} and shows the relation between the energy splitting and the pumping energy E_p . Since we set the parameter $\xi = 1/4\pi \approx 0.08$, one can check that the rate of these oscillations is lower than response signal shown in the middle panels, by the value ξ .

Analyzing the results, it is interesting to note that time and frequency response functions show that the polariton spin degree $\langle s \rangle_\phi$, which by definition is a ratio of the polariton spin signal density and the total polariton density, increases with distance r . From the first sight, it seems to be a surprising result, but its explanation is pretty simple. Recall that independently both polariton spin signal and total density decrease with distance

r , however, the rate of selected spin signal decay is smaller than the rate of number/charge density decay, which results in the increase of spin degree with distance r . At relatively large r , it even can exceed 1, which is shown in panel 4.3(d). We should stress that this unphysical effect is a pure mathematical artifact of our calculation method, which can be neglected since it occurs when the total polariton density is reduced by a factor of 10^{-4} .

4.4.3 Angular distribution in real space

Speaking about angular spin distribution in the coordinate space, it is interesting to compare our numerical results with the theory of multiple pattern formation in electron, polariton, and hole-doped systems and study their behavior on the parameter ζ .

Firstly, consider the polariton system in the diffusive regime, which is irradiated by a linearly polarized light, and study the evolution of circular polarization degree s_z in coordinate space. Following our analytical results, recall that in the EY regime conversion between linear S_x and circular S_z polariton densities is only allowed in the systems with spin-orbit coupling in the presence of an external magnetic field. Therefore, let us choose dimensionless parameter $\theta_B = 5\pi/8$, to make the impacts of these both effects strong.

Remembering the effect of quadrupole S_z pattern formation in the systems without disorder from the linearly polarized pump, which under magnetic field transforms into a spiral with polarization governed by the coordinate vector r , let us study the s_z polarization dynamics in the diffusive regime. Performing the simulation, we also found that initial s_x polariton state evolves into a quadrupole pattern with s_z polarizations in the diffusive system and the form of this pattern strongly depends on the value of parameter ζ (see, Fig. 4.4).

According to our analytical results for the diffusion matrix κ_{lc} , recall that the parameter ζ defines the preferential regime of spin evolution, called the

Figure 4.4: Angular distribution of circular s_z polariton polarization degree in the plane of the system with disorder at two values of parameter ζ and two values of time t . The left panels show the spin distribution at $t = 5\tau$ and the right panels show the spin distribution at $t = 20\tau$. At the upper panels parameter $\zeta = 0.25$ and at the lower panels parameter $\zeta = 0.5$. Note that initially system was pumped by light with linear x -polarization. It is clear that with time quadrupole patterns start to rotate and deform into a spiral, and the increase of parameter ζ leads to the rotation of quadrupole distribution. Here parameters of polariton system are: $\xi = \hbar/E_p\tau = 1/4\pi$, $\theta_B = 5\pi/8$, $\tau_{decay} = 50\tau$, and $\tau = 4$ ps.

Elliott-Yafet and the D'yakonov-Perel' regimes of spin relaxation and has an ability to rotate the quadrupole pattern by angle $\pi/4$. Therefore let us choose two values of dimensionless parameter ζ from the range of these limiting cases, defined by $\zeta \ll 1/2$ and $\zeta \gg 1/2$, and set the parameter $\zeta = 0.25$ (panel 4.4(a)) and 0.5 (panel 4.4(c)).

As a consequence, performing the simulation of dynamics of circular s_z polariton polarization degree induced by linear x -polarized pump, we found clear quadrupole S_z pattern formation at time $t = 5\tau$, shown in Fig. 4.4(a,c) and confirmed that polarization lobes shifts under the variation of the parameter ζ .

It should be noted that in contrast to the dynamics of the system in ballistic regime, where S_z spin polarization starts to spiral into a concentric circles in the presence of magnetic field, we got that in disordered systems s_z polarization component does not twist at $t = 5\tau$ and the angular dependence is still present in the system plane. For that reason, we decided that the time

t was not sufficient to observe this spiral effect and chose longer time scale (see, Fig. 4.4(b,d)). At $t = 20\tau$ we found that both polarization patterns start to twist, in analogy to ballistic regime.

Summarizing the above, we need to stress that circular S_z polarization pattern in both ballistic and diffusive regimes has a quadrupole shape in the plane of the system, which starts to twist in the presence of magnetic field oriented perpendicularly to it. However unlike in ballistic regime, the orientation of this pattern in disordered systems has a strong dependence on the parameters ζ and t , which particularly performs in the shift of quadrupole poles and the absence of twisting at time $t = 5\tau$. Therefore, speaking about the application of this surprising effect of spiral pattern evolution, we suggest that it can be used, for example, in disordered polariton systems for the measurement of the value of the scattering time τ .

Using the same procedure, let us study the angular distribution of s_z spin component in the electron and the hole systems. As a result, we found the angular distribution of spin s_z component, which is shown in Fig. 4.5.

Figure 4.5: Angular distribution of s_z spin degree in the plane of the electron (the upper panels) and the hole (the lower panels) systems with the disorder. The left and the right panels show the spin degree distribution at $t = 5\tau$ and $t = 20\tau$, respectively. We see that the electrons show a dipole pattern and holes show a sextupole pattern formation. It is clear that with time these patterns start to rotate and deform into a spiral in response to the influence of external magnetic field. Here parameters of polariton system are: $\zeta = 0.25$, $\xi = \hbar/E_p\tau = 1/4\pi$, $\theta_B = 5\pi/8$, $\tau_{decay} = \infty$, and $\tau = 4$ ps.

Recalling our discussion at the beginning of this chapter, it is clear that

in 2D plane electrons will form a dipole pattern (upper panels Fig.4.5) and holes will display a sextupole pattern formation (lower panels Fig. 4.5). In analogy to the polariton systems, we can compare time evolution of these patterns at times $t = 5\tau$ and $t = 20\tau$. We found that as time increases these spin patterns start to spiral in the presence of magnetic field as well. Following their analytical dependence on ζ represented by matrix κ_{lc} , it should be noted that, like in polariton systems, the angular position of these patterns also depends on the value of parameter ζ . Then speaking about experimental implementation, we propose to use these signature patterns as a basic element for determining a dominating type of spin-orbit coupling in selected systems.

4.4.4 Temporal dependence of system dynamics

In the previous section, we studied the spatial spin distributions in the presence of magnetic field. Now let us consider the evolution of the spin s_x and s_z degrees in both the D'yakonov-Perel' and the Elliott-Yafet regimes in more details.

Results of our calculation are shown in Fig. 4.6, where the left panels correspond to the case of the D'yakonov-Perel' regime and the right panels refer to the Elliott-Yafet regime. To compare the spin dynamics in different systems, it seems interesting to repeat the simulation of spin dynamics in electron, polariton and hole systems, represented by the rows in Fig. 4.6 respectively, where in an effort to make our discussion compact, linear s_x polarization is marked by dashed line, circular s_z component is marked by solid line, the blue color of the line shows the process of spin s_x and s_z degrees creation from unpolarized pump and the orange color of the line shows the process of specific polarization degree creation under polarized system irradiation. Note that here we have introduced the impact of both the Zeeman and the spin-orbit interaction terms on the spin dynamics in the system, where the relation between them can be expressed by the dimen-

sionless parameter $\theta_B = 5\pi/8$. It should be mentioned that at this value of θ_B they both have significant, but not precisely equal effect on spin dynamics. According to the previous discussion, one can see that this value of θ_B leads to the precession of both in-plane and out-of-plane s_x and s_z spin components, which results in larger signal strengths in comparison to the effects at limiting cases, such as $\theta_B = 0$ or $\theta_B = \pi/2$.

It is intuitively clear that time-dependence of the spin evolution should be determined by the scattering time τ . Analyzing the signal strength, one can see that in the DP regime maximum of the system response occurs at time $t = 4\tau$ and in the EY regime at time $t = 6\tau$. Speaking about the spin processes in the DP regime, it is clear that process of s_z component generation from an initially polarized light is the dominant (see, the orange lines in Fig. 4.6) and exceeds 61% in the electron, 43% in the polariton, and 23% in the hole systems. However, speaking about the spin processes in the EY regime, one can notice that two processes of spin signal generation, coming from an unpolarized state and a polarized state have similar magnitudes and peak of response signal reaches 32% in the electron systems, 8% in the polariton systems, and 11% in the hole-doped systems.

Concluding the numerical results of this section, we should stress that the strength of final signals is different in the DP and the EY regimes, and the value of the signal in the DP regime is much larger than in the EY regime. In addition, one can notice the uniform decrease of the spin degree in these regimes, performed by a merging of solid and dashed lines in Fig. 4.6. Thence we come to the interesting conclusion, which states that at the moment of time, when the system loses its initial polarization direction, the only effect of original polarization intensity remains in the system and affects the polarization dynamics. This idea of memory loss can also be spread on the prevalent mechanism of spin generation. Indeed, it can be proved by the left panels in Fig. 4.6, where one can notice that in the beginning process of out-of-plane spin component generation is dominating mechanism in the

Figure 4.6: Temporal dependence of spin components in the electron systems (the upper panels), the polariton systems (the middle panels) and the hole systems (the lower panels). Left panels reflect the spin dynamics at $\zeta = 0.1$ and right panels show the spin dynamics at $\zeta = 4$. The blue lines denote the polarization degree produced by an unpolarized pump, and the orange lines show the polarization degree caused by out-of-plane precession from a polarized pump. Dashed lines show the linear s_x polarization degree and solid lines show circular s_z polarization degree. For polariton system we set $\tau_{decay} = 50\tau$ and for electrons and holes $\tau_{decay} = \infty$. Here we keep ratio $r/\sqrt{Dt}$ fixed at 3, $\theta_B = 5\pi/8$, $\xi = \hbar/E_p\tau = 1/4\pi$, and $\tau = 4$ ps.

system, however, after a while it switches to the process of polarization generation from an unpolarized pump.

Speaking about characteristic timescales of memory loss, note that in the EY regime it is equal to 10τ , because as we discussed at the beginning this mechanism defines the tight coupling between the spin and the particle momentum, which randomizes at every scattering event and leads to rapid spin information loss. Recalling from Introduction, that in the DP regime spin loss is governed by the spin precession around the effective magnetic field, it is reasonable to expect longer times of memory loss, which agrees with our numerical simulation and reaches 100τ scale for selected parameters of the systems.

It should also be mentioned that after complete initial alignment loss all spin signals start to decay as a power of the inverse time (straight lines on log-log plots), which allows the selected spin degree to remain in the system for a quite long time. For example, in the DP regime at $t = 100\tau$ s_z spin degree reaches 7% and 0.5% from initial value for the electrons and the polaritons, respectively, and in the EY regime at $t = 100\tau$ system response is smaller, but still reaches 2% for the electron system.

4.4.5 Influence of interaction effects on spin properties

Now let us examine the dependence of the spin degree on the value of energy splitting, which can be introduced by the dimensionless parameter ζ describing the relation between the scattering length, l , and the precession length, l_{prcsn} . According to our analytical results, one can see that the value of spin response depends linearly on the parameter ζ for both the DP and the EY regimes. Since numerical simulation is a powerful approach for quantitative analysis of the response magnitudes, it seems interesting to consider the case when the system is in the intermediate state between the EY and the DP regimes.

Let us set the parameter $\theta_B = 5\pi/8$, to include the processes of both in-

plane and out-of-plane polarization degrees generation with a large strength of the signal and fix the time $t = 20\tau$, which will keep our calculation in the safe range of diffusion approach. Then performing the numerical simulation, one can find the signal dependence on parameter ζ , which is graphically shown in Fig. 4.7.

Following numerical results, one can see that transition between the DP and the EY regimes of spin relaxation starts from $\zeta = 0.1$ and finishes at value $\zeta = 0.5$. According to the definition of the parameter ζ , it is easy to understand that at small parameters, $\zeta < 0.1$, evolution of the system is purely governed by the DP regime and at large parameters $\zeta > 0.5$ evolution of the system is defined by the EY regime. From Fig. 4.7 note that in the DP limit s_z spin signals, shown by orange lines, are very similar to each other and grow linearly with the parameter ζ . Comparing this result with analytical terms of diffusion matrix, one can notice that this increase agrees with our analytical terms of system response, represented by linear-circular coupling matrix κ_{lc} . Note that processes of selected components s_x and s_z creation from an unpolarized pump (blue lines) are smaller, but linear on ζ , which corresponds with matrix terms in κ_{0l} and d_{0z} matrices in the DP regime. After the transfer to the EY regime with $\zeta > 0.5$, one can see that response signals start to grow with ζ again, which agrees with our analytical result on the coupling matrices κ_{0l} , κ_{lc} and d_{0z} in the EY regime as well.

4.4.6 Dependence of signal on the balance between SOI and Zeeman terms

Now let us study the dependence of spin degree on the parameter θ_B showing the relation between the spin-orbit interaction and the Zeeman splitting terms. Recalling the definition, it is easy to see that the parameter $\theta_B = 0$ in a 2D system without spin-orbit interaction, exposed to external magnetic field and $\theta_B = \pi/2$ in the system with spin-orbit interaction in the absence of the external magnetic field. To highlight our analytical results, we set

Figure 4.7: Dependence of spin components on the dimensionless parameter of energy splitting $\zeta = l_p/l_{prcsn}$ in the electron systems (the upper panel), the polariton systems (the middle panel) and the hole systems (the lower panel). The blue lines denote the polarization degree produced by an unpolarized pump, and the orange lines show the polarization degree caused by out-of-plane precession from a polarized pump. Dashed lines show the linear s_x polarization degree and solid lines show circular s_z polarization degree. This result is a rms-average of the polar angle ϕ . Here we keep ratio $r/\sqrt{Dt}$ fixed at 3, $\theta_B = 5\pi/8$, $\xi = \hbar/E_p\tau = 1/4\pi$, and $\tau = 4$ ps.

the time $t = 20\tau$ and found the following dependence of the signal on the parameter θ_B (see, Fig. 4.8).

Figure 4.8: Dependence of spin degree on the balance between the spin-orbit interaction and the Zeeman splitting terms, introduced by the parameter θ_B in the electron systems (the upper panels), the polariton systems (the middle panels) and the hole systems (the lower panels). The left panels correspond to the case when $\zeta = 0.1$ (the DP regime) and the right panels show the case when $\zeta = 4$ (the EY regime). The blue lines denote the polarization degree produced by an unpolarized pump, and the orange lines show the polarization degree caused by out-of-plane precession from a polarized pump. The dashed lines show the linear s_x polarization degree and the solid lines show circular s_z polarization degree. This result is a rms-average of the polar angle ϕ . Here we keep ratio $r/\sqrt{Dt}$ fixed at 3, $t = 20\tau = 80$ ps, $\xi = \hbar/E_p\tau = 1/4\pi$, and $\tau = 4$ ps.

At the first sign, one can notice the strong sensitivity of the polarization

signal on the parameter θ_B . Remember that during the above discussion we set $\theta_B = 5\pi/8$, which according to these relations allowed us to avoid the low values in spin degree components. Additionally, from the right panels in Fig. 4.8, which describe the system evolution in the EY regime, one can see that both the solid and the dashed lines of different colors coincide at most values of the parameter θ_B , which again shows us that after long time ($t = 20\tau$) systems completely forget their initial states and evolve similarly. Therefore, it can be found interesting to highlight the basic features of system signal response on the parameter θ_B , which were introduced in the analytical section during the discussion of the coupling matrices κ_{0l} , κ_{lc} and d_{0z} .

Firstly note that linearly polarized signal can not be achieved without the spin-orbit coupling interaction (see, the dashed lines at $\theta_B = 0$), which is a common feature for electron, polariton, and hole systems. It should be stressed that presence of only Zeeman splitting is not enough for the creation of the signal because it does not break the rotation symmetry of the particle in the system plane.

Secondary, one can notice that in the DP regime (see, left panels in Fig. 4.8), initial s_x signal leads to the generation of s_z spin degree regardless the value of the parameter θ_B and it can also be achieved by I signal at the parameter $\theta_B = 0$, defining the system without of spin-orbit coupling.

It seems interesting to mention that in the polariton systems at the EY regime (right middle panel in Fig. 4.8) polarization signals vanish in the absence of magnetic field $\theta_B = \pi/2$, which contradicts the results in the electron and the hole systems, where presence of magnetic field is not necessary requirement for creation of s_x signal (dashed lines), while it is still necessary for s_z component generation. Simultaneously, following the graphical representation of s_z signal response generated by I signal (solid blue lines), note that it nulls in both the EY and the DP regimes at $\theta_B = \pi/2$, in what follows, that the magnetic field is essential for this process of signal transfer.

Discussing the sharp dips of the circular s_z polarization component (the solid lines) in the polariton and the hole systems at the EY regime (right panels), we should mention that they reflect the resonant conditions discussed in the analytical section, corresponding to the cases when the magnitudes of the spin-orbit interaction and the Zeeman terms are equal ($|b_z| = |\alpha k_F^2|$ for the polaritons, $|b_z| = \sqrt{2}|\alpha k_F^3|$ for the holes). Note that in agreement with our analytical result, there is one more resonant condition for the hole-doped systems, which nulls the generation of s_x component from unpolarized I state (dashed lines). For that reason speaking about experimental implementation, we suggest that this feature can be found useful for the measurement of the value of spin-orbit interaction by an external magnetic field in the polariton and hole systems in the EY regime.

4.4.7 Summary of numerical analysis

In this section, we have studied the spin dynamics in electron, polariton and hole-doped systems with disorder using numerical methods. From the plot on the spatial dependence of the number/charge particle density, we saw that the system response always decays with time and follows exponential laws in the real space, while the spin component densities perform a more unusual behavior. In particular, we found that in diffusive regime electrons, polaritons, and holes have an ability to form a dipole, quadrupole and sextupole patterns in the system plane, respectively, which have a strong correlation with the value of spin-orbit interaction in the selected systems. In addition, we showed that under external magnetic field these patterns start to spiral depending on the value of scattering time, τ , in the selected system. In agreement with the obtained evolution plots of signal response functions, we also concluded that spin degree components can perform in two phases, which refer to the cases before and after the time when the information about initial state will be lost. Following our result, we saw that the first phase is known by the domination of the transition process between

the s_x or s_y spin degree components and the s_z spin degree and contrariwise and the second phase determines the decrease in spin density, where the dominant mechanism governing the spin alignment is the process of direct conversion from the number/charge density I .

4.5 Conclusions

Summarizing our discussion, in this chapter we discussed the spin transport properties of various particles in diffusive regime. In particular, we showed that spatially homogeneous distributions of electrons, polaritons, and holes act identically in the diffusive regime. They all have the similar expressions for the spin relaxation times and the spin precession rates. From the first sign, this identical behavior can be found disappointing, since one might expect the difference between the fermionic electrons and holes and the bosonic polaritons. To explain this similarity, let us stress that we have considered our systems in the limit of low particle density and neglected mutual interactions between the particles.

However, studying the processes of spin response generation from the spatially localized pump we found the large difference between these three particles, where the most obvious point lies in the spatial pattern formation, which has a two-lobed shape for electrons, four-lobed shape for polaritons and six-lobed shape for holes. Following this result, we concluded that this spin distribution provides an opportunity to determine the dominating term of spin-orbit interaction in the system, which might be found useful in applied problems.

We also showed that external magnetic field acts differently on the system response signal in diffusive regime. We saw that polaritons and holes in the EY regime of spin relaxation have a resonance peak of the response signal at specific values of magnetic field strength, which performs by the vanishing of s_z degree response from the unpolarized state. For that reason, we suggested to use this feature to measure the strength of spin-orbit coupling and to check

the sort of the prevalent particles in selected two-dimensional systems.

5. Polaritons spin Rabi oscillations

Following an intense study of polariton spin dynamics in the previous chapter, now let us extend it by a discussion of an alternative method of spin control in the polariton systems, which from applied point of view provides an opportunity to generate a response signal with tunable polarization in the polariton microcavities. From the very beginning note that we will limit our consideration to typical clean systems, where the polariton evolution is not affected by disorder and, therefore, the standard quantum mechanical approaches will be valid in this type of semiconductor systems.

To familiarize the reader with the procedure, we will start this chapter with an overview of a basic idea of polarization dynamics in the polariton microcavities under irradiation by two counter-polarized pumps, which is governed by the evolution of polariton spin vector in the microcavity and particularly relies on the effect of interference between specially prepared polariton states, also known as the light-matter Rabi oscillations. Thus, we will show that the time scale of the final emitted pulse along with the speed of signal polarization precession can be easily tuned by variation of the time delay between external pulses. Speaking about the feasibility of proposed approach, we will continue the study of polarization control in the polariton systems by a discussion of experimental results, which show the perfect agreement with theoretical predictions. In the conclusion of this chapter, we will summarize the benefits of this system and show the possibilities of experimental implementation of this model.

5.1 Polarization control of emitted light

Recently an aspiration to control and to manipulate the light properties have been developed into a novel topic of optics, called “polarization shaping”, where light is defined not only by the parameters of its amplitude and phase but also by the state of polarization [205, 206]. Recalling that the interaction between the light and the matter is sensitive to polarization degree of light, it was concluded that the light polarization has a significant effect on the material properties and can be found valuable for various industrial needs. Speaking about a direct application, an ability to control the light polarization can be potentially utilized in laser micro-processing, for example, to improve the efficiency and the quality of silicon wafer processing [207], drilling holes for fuel-injection nuzzles [208] and manufacturing of medical stent devices [209]. Thus, it seems beneficial to find a way to generate all polarization states in one selected system.

Lately, several independent ideas of light emitting systems with controlled polarization have been developed. However, currently existing models come with respective difficulties and limitations. For instance, since conventional solid-state lasers typically emit the light with a fixed linear polarization [210] and optical nano-antennas radiate the light with fixed polarization degree, determined by their geometrical structures [211], preparation of the signal with desired light polarization in time [205] or space [212] is a complicated and enthralling task, which requires the use of complex setups consisting of liquid crystals, spatial light modulators, interferometers and computer resources with construction algorithms imposing the restrictions of system operating regime [213].

For that reason, let us propose an alternative method of light polarization control based on the fundamental phenomenon of the Rabi oscillations in polariton microcavities [214]. Note that comparing to the current complex devices, the suggested setup will not require any external signal processing

and, as in many pulse shaping counterparts, we expect that dynamics of polarization control should occur in the microcavity in a femtosecond range.

5.2 The formalism of the Rabi oscillations in polariton microcavities

Firstly let us discuss the proposed model theoretically. According to the system conditions, polarization dynamics can be introduced by a coherent superposition of the photon fields with opposite polarizations, where each of them is in the Rabi oscillating regime. Generally speaking, there are four fields in the polariton system: two of them correspond to the light field with selected polarization and each of them oscillates with the exciton field of corresponding polarization while fields with opposite polarization do not couple with each other. For that reason, in the linear regime, we can define the polariton system using four parameters, which are complex amplitudes of the selected field with a specific polarization. For clarity, let us introduce the amplitude of the photon field by the parameter α_p and the amplitude of the exciton field by the parameter β_p , where index $p = \odot, \oslash$ denotes to the left ($\odot$) and the right ($\oslash$) circular polarizations of light.

For the description of polariton system focus on the quantum formalism instead of the classical approach. The reasons for this choice can be understood as follows. Firstly, remember that quantum approach allows treating the polariton system in a more general way by unifying both regimes with the only difference in the definition of the functions α_p and β_p . Secondary, recall that processes of incoherent pumping and dephasing can be more conveniently implemented within the quantum formalism rather than the statistical theory of classical formalism.

Following quantum mechanical representation, the amplitudes of fields in the microcavity define the probability of the system to be in the coherent

state with the wave function

$$|\psi(t)\rangle = |\alpha_p(t), \beta_p(t)\rangle \otimes |\alpha_q(t), \beta_q(t)\rangle, \quad (5.1)$$

where $|\alpha_p\rangle = \exp\left(-\frac{|\alpha_p|^2}{2}\right) \sum_{k=0}^{\infty} \frac{\alpha_p^k}{\sqrt{k!}} |k\rangle$,

and p and q are the orthogonal polarization indexes.

From Eq. (5.1), one can see that this wave function does not perform any quantum effect and superposition features. For that reason, we can represent polarization states using the conventional Stokes vectors in matrix form by:

$$\begin{pmatrix} \alpha_{\leftrightarrow}(t) \\ \alpha_{\uparrow\downarrow}(t) \end{pmatrix} = \frac{\alpha_{\odot}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} + \frac{\alpha_{\ominus}}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}. \quad (5.2)$$

In rotating frame approximation dynamics of two uncoupled in polarization fields is introduced in the Hamiltonian by the term $\sum_p g(a_p^\dagger b_p + a_p b_p^\dagger)$ for the photon a_p and the exciton b_p operators and the coupling strength g , which may lead to formation of two new modes in the system, called the upper polaritons, $u_p = (a_p + b_p)/\sqrt{2}$, and the lower polaritons, $l_p = (a_p - b_p)/\sqrt{2}$.

Therefore, the relationships for the photon operators read:

$$\begin{aligned} a_{\leftrightarrow} &= \frac{a_{\odot} + a_{\ominus}}{\sqrt{2}}, \\ a_{\uparrow\downarrow} &= i \frac{a_{\odot} - a_{\ominus}}{\sqrt{2}}, \\ a_{\nearrow} &= \frac{(1-i)a_{\odot} + (1+i)a_{\ominus}}{2}, \\ a_{\searrow} &= \frac{(1+i)a_{\odot} + (1-i)a_{\ominus}}{2}. \end{aligned} \quad (5.3)$$

To study the polarization of emitted light, it is useful to recall the expression for the intensity of emitted photons $\langle a_p^\dagger a_p \rangle$ and its relation to the light polarization degree, which by the definition reads:

$$S_p = \frac{\langle a_p^\dagger a_p \rangle - \langle a_q^\dagger a_q \rangle}{\langle a_p^\dagger a_p \rangle + \langle a_q^\dagger a_q \rangle}. \quad (5.4)$$

Additionally limit the study by leaving aside the fact that at the moment of time $t = 0$ selected state is created by a pulsed excitation and assume that initially, the system is defined by the field amplitudes $\alpha_{p,q}(0)$ and $\beta_{p,q}(0)$, which within the conventional formalism evolves as [215]:

$$\alpha_p(t) = \alpha_p(0) \cos(gt) - i\beta_p(0) \sin(gt). \quad (5.5)$$

Therefore, one can consider the process of system evolution under the two counter-polarized external pulses using the conventional equations of motion. Qualitatively calculated result of the Rabi oscillations between two polarization signals can be understood as follows (see, Fig. 5.1). The evolution of the system is initiated by the pulse, which excites both polarizations and thus triggers the Rabi oscillations with the same circular polarization. Recalling that the excitation transfers from the photon to the exciton states and back with time, it becomes clear that this intermittent transfer between the photon and the exciton field results in a temporary switch-off of the cavity light emission. Thus, the considered system emits light with constant polarization and oscillating intensity, represented in Fig. 5.1 till $t = 3$ ps. Subsequently, under the irradiation by the second pulse with opposite polarization at selected time $t = 3$ ps, which corresponds to the out-of-phase Rabi oscillations triggered by the first pulse, one can obtain the reverse situation of the emitting light with a constant intensity and oscillating polarization.

Speaking about the experimental realization of these oscillations, it was decided to use essentially spatially homogeneous profiles in order to focus on the time dynamics of the system. Performing the experiment, which will be introduced in the following sections, we also found the presence of the Rabi oscillations in the polarization signal, shown in Figs. 5.2 and 5.3.

It should be noted that behavior of the exciton field is similar to the photon field, however, the explicit expressions for the exciton field will not be introduced in this chapter since we are only interested in the time dynamics of the photon field governing the system response.

Figure 5.1: Theoretical prediction of the Rabi oscillations in the polariton system, resulting in the beats between two polarization signals. In the beginning, one can see only Rabi oscillations in one polarization with oscillating intensity $|\alpha_{\odot}|^2 + |\alpha_{\odot}|^2$, marked by a purple line, and constant polarization (a yellow line). At $t = 3$ ps the system transforms and the Rabi oscillations in two polarizations with reverse oscillation period occur.

Figure 5.2: Experimental observation of the effect of the Rabi oscillations of polarized beams through the cavity field with a left (red) and a right (blue) circular polarizations as a false color plot, where the decay of the signal comes from the leakage of the photons in the microcavity.

Figure 5.3: Spatial distribution of the density and the polarization at 200 fs time intervals, during one of the initial cycles after the second pulse interval. It is easy to see that while the intensity of emitted light is approximately constant (shown by the height of the peak signal), the resulting polarization is strongly affected by the Rabi oscillations.

5.3 Dynamics of the polarization induced by the Rabi oscillations

5.3.1 Theoretical description of polarization spanning

In order to study more complicated cases of polarization dynamics in the polariton microcavity, continue working in the circular polarization basis, which can be achieved under the tracing over the matter: $|\psi_\alpha(t)\rangle = |\alpha_\circ(t)\rangle |\alpha_\circ(t)\rangle$. In the following we are able to introduce the parameter, called the relative optical phase ϕ_α between two photon fields at $t = 0$ and the relative optical phase ϕ_β between two exciton fields at $t = 0$, which will have a great importance in subsequent discussion:

$$\phi_\alpha = \frac{\alpha_\circ(0)|\alpha_\circ(0)|}{\alpha_\circ(0)|\alpha_\circ(0)|}, \quad \phi_\beta = \frac{\beta_\circ(0)|\beta_\circ(0)|}{\beta_\circ(0)|\beta_\circ(0)|}. \quad (5.6)$$

As we have mentioned earlier, dissipations are the crucial factors affecting

the polariton dynamics, and in particular, the Rabi oscillations in the polariton systems. To include their impacts, let us modify the master equation and express the effect of dissipation in so-called Lindblad form:

$$\frac{\partial \rho}{\partial t} = i[\rho, H] + \mathcal{L}\rho, \quad (5.7)$$

where ρ is the density matrix, which includes the effects of four fields in the system, and $\mathcal{L}\rho$ is a sum of super-operators $\mathcal{L}_c\rho = 2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c$ for generic operator c .

Apart the radiative decay, introduced by the decay rate $\gamma_{a,b}$ of each field, and the implementation of the incoherent pumping P_b from the exciton reservoir, one should also include less common terms, which will directly describe the polariton states, $\mathcal{L}_{u_p}$ and $\mathcal{L}_{u_p^\dagger u_p}$. Note that they correspond to the effect of dephasing of the upper polaritons represented by radiative decay γ_U^R or pure dephasing γ_U^ϕ of the polaritons. Then Liouvillian $\mathcal{L}$ reads [216]:

$$\mathcal{L}\rho = \sum_{p=\odot, \ominus} \left[\frac{\gamma_a}{2} \mathcal{L}_{a_p} + \frac{\gamma_b}{2} \mathcal{L}_{a_b} + \frac{P_a}{2} \mathcal{L}_{b_p^\dagger} + \frac{\gamma_U^R}{2} \mathcal{L}_{U_p} + \frac{\gamma_U^\phi}{2} \mathcal{L}_{u_p^\dagger u_p} \right] \rho, \quad (5.8)$$

where the values of the relaxation parameters can be found experimentally.

It should be stressed that for any similar dynamics, the field amplitudes can be calculated in the closed form. Particularly, the solution for the amplitude of the photon field is entirely defined by the complex amplitudes:

$$\alpha_p(t) = \left[\alpha_p(0) \cosh\left(\frac{Rt}{4}\right) - \left(\frac{\beta_p(0)G + \alpha_p(0)\Gamma}{R}\right) \sinh\left(\frac{Rt}{4}\right) \right] \exp\left(-\frac{\gamma t}{4}\right) \quad (5.9)$$

where for simplicity we introduced the following parameters: $\gamma = \gamma_a + \gamma_b + \gamma_U - P_b$, $\Gamma = P_b - \gamma_b + \gamma_a$, $G = i4g + \gamma_U$, $R = \sqrt{G^2 + \Gamma^2}$, and $\gamma_U = \gamma_U^R + \gamma_U^\phi$.

These simple expressions give rise to the interesting dynamics of the system, which will be shown in the following sections.

5.3.2 Experimental setup for implementation of the effect of the Rabi oscillations in the microcavities

Start the study of polarization dynamics from the discussion of the experimental setup. To achieve the effect of the Rabi oscillations in the polariton system, we set it as follows. We use a GaAs polariton microcavity in the regime of the Rabi oscillations, with the lower and the upper polariton branches at 836.2 nm and 833.2 nm, respectively, at $T = 13$ K.

The sketch of the setup is shown in Fig. 5.4. Generally speaking, it is based on the principles of the time-resolved digital holography [216], while an additional information can be found in Refs. [216, 217]. Note that because of the setup configuration the delay between two femtosecond pulses with selected polarization exciting the system is adjustable. Additionally, we set the energy spread of the pulses to overlap with both polariton branches with a radiative weight that can be tuned to the pulse mean energy, which triggers the Rabi oscillations between the excitons and the photons.

5.3.3 Dynamics of the polarization vector in the polariton microcavity from two external pulses

Recall that in linear regime dynamics of the Rabi oscillations for each polarization component in isolation can be found from the expressions (5.5) and (5.9) for both conservative and dissipative regimes. As one remembers, we set that polarizations of the fields are uncoupled, however, as we will show further, a judicious combination of the fields with different polarizations can result in nontrivial results for the combined light signal.

Working in the circular polarization basis, polarization vector of the system can be defined by two angles (θ_ξ, Φ_ξ) on the Poincaré sphere, where $\xi = \alpha, \beta$ and R_α and R_β are the ratios of polarization $R_\alpha = \alpha_\circ/\alpha_\circ$ and $R_\beta = \beta_\circ/\beta_\circ$ for the photon α and the exciton β fields at $t = 0$. Therefore,

Figure 5.4: Setup for experimental observation of the Rabi oscillations in the polariton system. This effect is achieved by a time-resolved digital holography setup under counter-polarized double pulse experiments. Firstly the femtosecond pulse is split in a reference beam ($\uparrow$) and an excitation beam ($\leftrightarrow$). The latter beam is further divided into twin pulses of equal $\leftrightarrow$ linear polarizations, where only one, upon double passage onto a $\lambda/4$ plate, becomes $\uparrow$ linear. After rejoining their paths, the twins become counter-circular polarized using a second $\lambda/4$ plate. Their mutual delay (Delay 2) can be tuned on the timescale of the Rabi period with Φ or the optical period with ϕ_a . Subsequently the emission from the microcavity sample (MC) is filtered in polarization and interfere with the reference signal on the detecting CCD camera, before digital elaboration. The evolution of the emission can be tracked by tuning the time of interference through the delay stage (Delay 1).

using Eq. (5.5) parameters (θ_ξ, Φ_ξ) can be straightforwardly expressed by a geometric construction:

$$\theta_\xi = 2 \arccos \left(\frac{1}{\sqrt{1 + |R_\xi|^2}} \right), \quad (5.10)$$

$$\Phi_\xi = \phi_\xi + \arg R_\xi.$$

Additionally, introduce a polariton feature, which is usually neglected in polariton systems, but in our case, will transform into a mechanism, which governs a new type of emitted signal. For that reason, recall the experimental fact, that there is a significant lifetime imbalance between two polariton states in the microcavities, meaning that the upper polaritons, $|U_p\rangle = |\alpha_p, \alpha_p\rangle$, have much shorter lifetime in comparison to the lower polaritons with $|L_p\rangle = |\alpha_p, -\alpha_p\rangle$ regardless the polarization p [216]. Quantitatively speaking, in the real systems, the value of the lifetime of the upper polariton state is usually of the order of 2 ps while the lifetime of the lower

polariton state is of the order of 10 ps. Note that commonly this effect is attributed to the vicinity of the interband absorption edge and the phonon-assisted relaxation of polaritons from the bottom of the upper branch to high- k states of the lower branch [218, 219]. This imbalance can be phenomenologically introduced by an additional term in the Lindblad equation (5.7), expressed by polariton operators.

Summarizing above idea, one become able to parametrize the final state of the polarization in analogy to the initial state representation by a set of angles $(\theta'_\alpha, \Phi'_\alpha)$ describing the rotation of the state in the polariton basis. Recall the expressions for the description of the final states $\alpha'_p = (\alpha_p - \beta_p)/\sqrt{2}$ and $R'_\alpha = \alpha'_\circ/\alpha'_\circ$. Then the angles read:

$$\begin{aligned}\theta'_\alpha &= 2 \arccos \left(\frac{1}{\sqrt{1 + |R'_\alpha|^2}} \right), \\ \Phi'_\alpha &= \phi_\alpha + \arg R'_\alpha.\end{aligned}\tag{5.11}$$

According to these expressions, it is easy to see that any polarization trajectory can be described by three parameters: two at the initial moment (R_α and R_β) and one at the final point (R'_α).

Return to the problem of the pulsed excitation of the system into the state $|\alpha_p, \beta_p\rangle$, which gives rise to the spanning of the light vector on the polarization sphere. Following the definition of the ratios of polarization, one can easily get the expressions describing the photon states themselves for any nonzero $\alpha_\circ$:

$$\begin{aligned}\beta_\circ &= \frac{R_\alpha - R'_\alpha}{R_\beta - R'_\beta} \alpha_\circ, \\ \beta_\circ &= R_\beta \frac{R_\alpha - R'_\alpha}{R_\beta - R'_\beta} \alpha_\circ, \\ \alpha_\circ &= R_\alpha \alpha_\circ.\end{aligned}\tag{5.12}$$

Generally speaking, any state in a two-level system can be created experimentally by the control of the amplitude of the exciting pulse [220]. To simplify the model, assume that the light pulse prepares coherent states of

two coupled harmonic oscillators, which at resonance can be introduced in the Hamiltonian by the term describing a simple Gaussian pulse: $H_L = P_\alpha \exp((t - t_0)^2 \sigma^2) \exp(i\omega_L^p t) a_p + \text{h.c.}$, where σ is the width of the pulse in frequency representation pinned at time t_0 with the energy ω_L^p , which creates the state $|\alpha_p, \beta_p\rangle$:

$$\omega_L^p = \frac{\sigma^2}{2g} \ln \frac{|\alpha_p + \beta_p|}{|\alpha_p - \beta_p|}. \quad (5.13)$$

Figure 5.5: Theoretical calculation of polarization dynamics of the emitted light, shown as a vector on the Poincaré sphere. The green arrows, defined by the angles θ_α and θ_β , fix the polarization circle in the absence of system decay ($\gamma = 0$) by intersecting the meridian of azimuthal angle ϕ_α . The red arrows, defined by the angle θ'_α , fix the point of long time polarization. In the presence of decay ($\gamma \neq 0$), the trajectory of the polarization, marked by blue lines, drifts from the green circle to the red final point. In panel (a), we show the span of the northern hemisphere of the Poincaré sphere in circular polarization, by setting $R_\alpha = 1 - \epsilon$, $R_\beta = -1$, and $R'_\alpha \rightarrow \infty$ with $\epsilon \rightarrow 0$. In panel (b) we show the span of the full Poincaré sphere from the anti-diagonal to the diagonal pole of the sphere, by setting $R_\alpha = 1 - \epsilon$, $R_\beta = 1 + \epsilon$, and $R'_\alpha = -1$ for $\epsilon \rightarrow 0$. In panel (c) we demonstrate the span of the Poincaré sphere excluding a spherical cap of anti-diagonal polarization, by setting $R_\alpha = 0.41$, $R_\beta = 2.41$, and $R'_\alpha = -1$. In panel (d), we show distorted spanning of the sphere by choosing close initial and final points, by setting $R_\alpha = 0$, $R_\beta = -2/3$, and $R'_\alpha = 1$ and $\Phi_\alpha = \pi/2$. Here we set parameters: for the blue trajectory $\gamma = 1$ and for the purple trajectory $\gamma = 3$.

It should be stressed that the second pulse must be sent at the specific frequency (5.13) in order to complete the state preparation in other polarization and only the relative phase difference between the two polarized states

affects system dynamics, which is defined by a parameter of a time delay between the pumps, $\Delta\tau = R - \alpha/(2g|R_\alpha|)$. Speaking about quantitative characteristics, note that the time delay should be small enough so that the first polarization does not shift notably from the circle induced by the first pulse, which is always possible whenever the optical frequency is much higher than the Rabi frequency.

5.3.4 Experimental observation of the effect of polarization shaping by polariton oscillations

Following the above procedure, it seems interesting to realize experimentally this polarization spanning of the signal, predicted by theory (5.5). In particular, we decided to perform experimentally two limiting cases. In Figs. 5.6 and 5.7 we present the detected polarization in two cases, when two pulses start with the same phase and the opposite phase, respectively. In Figures, we introduce the experimental data of the signal dynamics under the picosecond polarized pulses by points and theoretical calculation data for the intensity of light emitted by each component $\langle a_p^\dagger a_p \rangle$ and the corresponding degree of polarization S_p by solid lines. Remember that the initial conditions were specially selected, in order to achieve two special effects, namely, the Rabi oscillations in both polarizations either in-phase or anti-phase. Generally speaking, it can be realized by an equal overlap of the polariton branches with both pulses and setting the delay between them to obtain peak-to-peak correspondence, shown in Fig. 5.6(c) with delay $\Delta\tau \approx 2\pi/g$ between the circular polarized pulses, and peak-to-node correspondence, shown in Fig. 5.7(c) with delay $\Delta\tau \approx 3\pi/(2g)$ between the circular polarized pulses. According to our results, in the first instance we observe an oscillating intensity with an essentially fixed polarization and in the second instance, we observe oscillating polarization with almost constant intensity (except the decay in the system).

The consistency of proposed model can be checked by the fitting polar-

Figure 5.6: Experimental observation of the effect of the polarization spanning in polariton systems from two external pulses starting with the same phase. Here we show results of the Rabi in-phase experiment, where in panels (a-c) black dots denote to the experimental data and solid lines denote to the theoretical predictions, for field amplitudes $I = |\alpha_{p,q}|^2$ and degrees of polarization S_p . In panel (d) we show the dynamics of polarization using the Poincaré sphere, where one can see the rapid transition to the linear polarized state. In panel (e) we demonstrate the evolution of the polarization flow over a 10 ps time span, where the surface represents the envelope of the electric field and the color is scaled on the instantaneous field amplitude. In order to achieve the agreement between the experimental and the theoretical results, we choose the following fitting parameter values: $g = 4.02 \text{ ps}^{-1}$, $\gamma_a = 0.2 \text{ ps}^{-1}$, $\gamma_U = 0.43 \text{ ps}^{-1}$, $\omega_L^p = 829 \text{ nm}$, $\sigma = 1/0.15 \text{ ps}^{-1}$, and $\Delta\tau = 1.55 \text{ ps}$.

Figure 5.7: Experimental observation of the effect of the polarization spanning in polariton systems from two external pulses starting with the opposite phase. Here we show results of the Rabi anti-phase experiment, where in panels (a-c) black dots denote to the experimental data and solid lines denote to the theoretical predictions, for field amplitudes $I = |\alpha_{p,q}|^2$ and degrees of polarization S_p . In panel (d) we show the dynamics of polarization using the Poincaré sphere, where one can see the spanning of the full hemisphere polarization. In panel (e) we demonstrate the evolution of the polarization flow over a 10 ps time span, where the surface represents the envelope of the electric field and the color is scaled on the instantaneous field amplitude. In order to achieve the agreement between the experimental and the theoretical results, we choose the following fitting parameter values: $g = 4.02 \text{ ps}^{-1}$, $\gamma_a = 0.2 \text{ ps}^{-1}$, $\gamma_U = 0.43 \text{ ps}^{-1}$, $\omega_L^p = 829 \text{ nm}$, $\sigma = 1/0.15 \text{ ps}^{-1}$, and $\Delta\tau = 1.13 \text{ ps}$.

ization of the experimental results in all bases and reconstruction from one basis only. Recalling that experimentally we can only obtain the results for the photonic field, we should stress that developed theory allows reaching the dynamics of the exciton field as well. For that reason, we are able to reconstruct the polarization dynamics as shown in Figs. 5.6-5.7(d-e) with a 3D representation of the polarization evolution.

Speaking about the application, consider the case when a molecule in the path of the beam scatters light for a given type of its polarization. Then the use of full Poincaré beam would ultimately provide it, thereby offers an optimized exciting light, even for a gas of randomly oriented emitters. This effect could also be used for temporal tagging, in a similar way that full Poincaré beams have been used for sub-wavelengths localization. According to Figs. 5.6 and 5.7 one can notice that the microcavities provide an opportunity of ultra-fast light sweeping of a hemisphere of the Poincaré sphere with a fast transition toward the final point 5.6 and sweeping of a hemisphere of the Poincare sphere with a slow whirling round, spanning half the sphere in agreement with theoretical results (see, Fig. 5.5(d)). As one can see, all considered cases show the excellent agreement between theory and experiment.

5.4 Conclusions

We have proposed and implemented a novel system, which can generate a light pulse of any polarization in the time duration of each pulse. Qualitatively, this effect relies on the dynamics of the Rabi oscillations in the polariton systems in the linear regime. We have proved this model experimentally and have concluded that this system may allow to disseminate the use of time-dependent polarization beams and additionally may allow extending this model to the quantum regime by powering the same mechanism with quantum Rabi oscillations.

In order to extend our idea, note that a spatially dependent polarization

signal may be combined with the temporal dynamics. Space patterning can be achieved under the sending the second pulse with a slight angle of incidence (inducing the Δk_α) with respect to the first pulse, which is beyond the scope of this work, but gives a hint how to reach the patterns with selected polarization texture and introduce the dynamics of polariton fluids [221].

6. Summary

In conclusion, in the current thesis, the various aspects of the spin dynamics in two-dimensional systems in the strong-coupling regime have been considered. Particularly we were focused on the study of the spin evolution of the dressed electrons and the polaritons in ballistic and diffusive regimes.

Starting from the concept of the dressed electron states, we extended it by the description of the dressed electron transport properties in the presence of disorder. Subsequently, within the conventional formalism of the density response function, we discussed the spin dynamics of the dressed electrons in the presence of the Rashba spin-orbit interaction. We found that it could be drastically changed by a high-frequency electromagnetic field, where this dependence is enclosed in the transfer between the governing mechanism of spin relaxation, known as the Elliott-Yafet and the D'yakonov-Perel' regimes. Regarding experimental implementation, we concluded that this phenomenon could be used in various modern problems, in order to control the spin properties of a 2DEG by a high-frequency electromagnetic field.

Further, we applied a similar approach to the problem of the spin transport in polariton systems in diffusive regime, where for accuracy, we included the effect of spin-orbit interaction, called TE-TM splitting, and the influence of external magnetic field. We found that alike in ballistic regime, polariton spins in disordered systems are also able to form a distinct multi-pole spin patterns in the plane of the system, which can be found useful for identification of the dominating impact governing spin evolution in the different systems and for the estimation of the spin relaxation times.

Finally, we introduced a novel method for generation of the light with controlled polarization, based on the effect of the Rabi oscillations in the polariton systems. We showed theoretically that under the excitation by two counter-polarized and delayed in time pulsed system can reduce to different polariton spin state guiding the polarization of emitted light. We also

proved the feasibility of this idea experimentally and concluded that it might be found attractive to an industrial realization of the multitasking devices, where polarization spanning is required.

List of publications

1. A. A. Pervishko, T. C. H. Liew, V. M. Kovalev, I. G. Savenko, and I. A. Shelykh,
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Opt. Express, 21 (15183), 2013.
2. A. A. Pervishko, T. C. H. Liew, A. V. Kavokin, and I. A. Shelykh,
“Bistability in bosonic terahertz lasers”,
J. Phys. Cond. Matt., 26 (085303), 2014.
3. S. Morina, O. V. Kibis, A. A. Pervishko, and I. A. Shelykh,
“Transport properties of a two-dimensional electron gas dressed by light”, *Phys. Rev. B*, 91 (155312), 2015.
4. D. Colas, L. Dominici, S. Donati, A. A. Pervishko, T. C. H. Liew, I. A. Shelykh, D. Ballarini, M. de Giorgi, A. Bramati, G. Gigli, E. del Valle, F. P. Laussy, A. V. Kavokin and D. Sanvitto,
“Polarization shaping of Poincar beams by polariton oscillations”,
Light: Science & Applications, 4 (e350), 2015.
5. A. A. Pervishko, O. V. Kibis, S. Morina, and I. A. Shelykh,
“Control of spin dynamics in a two-dimensional electron gas by electromagnetic dressing”, *Phys. Rev. B*, 92 (20), 2015.
6. V. Sacksteder IV, A. A. Pervishko, and I. A. Shelykh,
“Spin response to localized pumps: Exciton polaritons versus electrons and holes”, *Phys. Rev. B*, 93 (8), 2016.

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