Advanced seismic signal processing techniques for detection and bearing estimation of footsteps and vehicular signals

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Advanced Seismic Signal Processing Techniques for Detection and Bearing Estimation of Footsteps and Vehicular Signals

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To *Amma* and *Daddy*.

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Abstract

This thesis is aimed towards the development of algorithms for improved detection and bearing estimation of human activities in urban surveillance systems. Human footsteps and vehicles are used as sources and seismic signals, generated from these sources, are recorded using a (buried) tri-axial geophone. The performance of existing energy-based metrics deteriorate significantly with increasing source-sensor distance due to poor signal-to-noise ratio. The three-dimensional (3D) trajectory traced by the tri-axial signal, acquired from near-field seismic sources, is elliptically polarized. This thesis takes as its point of departure the direction attribute of the elliptically polarized 3D trajectory to detect and estimate the direction of arrival of surface seismic sources.

The smoothness of the 3D trajectory is used as a feature and instantaneous metrics using consecutive snapshots of the normalized trajectory are defined. Furthermore, the 3D trajectory is analyzed using two different representations of the hypercomplex quaternions. Using the vector-quaternion representation a quaternion generalized Gaussian distribution (QGGD) is derived. The QGGD is parameterized by a shape parameter and a quaternion augmented covariance matrix to capture non-Gaussianity and inter-channel correlation, respectively, of the tri-axial signals. The orientation-quaternion representation is used to parameterize the instantaneous orbital phase of the trajectory. The concentration and direction of the orbital phase are used to detect the presence of elliptical polarization and to estimate the bearing of seismic surface sources, respectively.

The proposed trajectory-based metrics were found to improve the detection range of human footstep signals by 5 m to 7 m and vehicular signals by 25 m to 100 m, compared with existing energy-based detection metrics. The orientation-quaternion representation of the 3D trajectory enhanced bearing estimation accuracy, in addition to addressing the $\pm 90^\circ$ ambiguity that occurred in existing methods.

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List of Acronyms

3D	Three-dimensional
CGGD	Complex Generalized Gaussian Distribution
CinH	Concentration in Hypersphere
CMA	Covariance Matrix Analysis
CPA	Closest Point of Approach
CRLB	Cramer-Rao Lower Bound
DOA	Direction-of-Arrival
DOI	Degree-of-Q-Improperness
DOP	Degree Of Polarization
DRV	Directional Random Vectors
EEC	Eigenangle-Eigenaxis Concentration
GA	Great Arc
GGaD	Generalized GAMMA Distribution
GS	Geometric Sequence
HTA	Hilbert Transform Analysis
knn	k Nearest Neighbour
NED	Neighbourhood Euclidean Distance
NR	Neighbourhood Ratio
OPA	Orbital Phase Analysis
pdf	probability density function
PE	Polarization-cum-Energy
QGGD	Quaternion Generalized Gaussian Distribution
QRM	Quaternion Rotational Model
SLERP	Spherical Linear interPolation
SNR	Signal to Noise Ratio
TDOA	Time Difference Of Arrival
TGR	Tilted Generalized Rayleigh
vMF	von Mises-Fisher (Distribution)
VR	Velocity-to-Rotation (Transform)

List of Notations

Chapters 2 & 3

$\mathbf{x}(n)$	Tri-axial signal (vector)
$x_1(n)$	x -axis signal
$x_2(n)$	y -axis signal
$x_3(n)$	z -axis signal
Θ	Azimuth angle
Φ	Elevation angle
Ψ	Orientation angle
Ξ	Ellipticity
$\chi(\Theta)$	Horizontal radial power
$\hat{\mathbf{C}}$	3×3 Covariance matrix
$\hat{\mathbf{U}}$	3×3 Eigenvector matrix
$\hat{\mathbf{V}}$	3×3 (diagonal) matrix with eigenvalues
$\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3$	Eigenvalues
$\mathcal{H}(\cdot)$	Hilbert transform
$\hat{E}_c(l)$	Cumulated energy metric
$\hat{K}_c$	Cumulative Kurtosis metric
$\hat{\Theta}_{\text{CMA}}$	Bearing angle estimated using covariance matrix analysis
$\hat{\Theta}_{\text{VS-MUSIC}}$	Bearing angle estimated using vector-sensor MUSIC
$\hat{\Theta}_{\text{RA}}$	Bearing angle estimated using rotational analysis
$\hat{\Theta}_{\text{HTA}}$	Bearing angle estimated using Hilbert transform analysis
$\hat{\Theta}_{\text{SWA}}$	Bearing angle estimated using surface wave analysis
$\bar{\mathbf{x}}(n)$	Normalized tri-axial signal (vector)
$\text{NED}_{\bar{\mathbf{x}}}(n)$	Neighbourhood Euclidean Distance metric
$\text{NR}_i(n)$	Neighbourhood Ratio metric using i th channel signal
$e_{\text{gs}}(n)$	Geometric Sequence (GS) model-fit error
$e_{\text{ga}}(n)$	Great Arc (GA) interpolation error
$e_{\text{PE}}(l)$	Polarization-cum-Energy metric
$\tau_{\text{gs}}(b), \tau_{\text{ga}}(b), \tau_{\text{PE}}(b)$	Threshold for GS-based, GA-based and PE metric
$\gamma_{\text{gs}}, \gamma_{\text{ga}}, \gamma_{\text{PE}}$	Threshold scaling parameter for GS-based, GA-based and PE metric
σ_n^2	Noise variance
$\rho_{\bar{\mathbf{x}}}(l)$	Temporal correlation of the normalized signal with lag l
W_{CorrDiff}	Windowed normalized correlation difference metric

Chapter 4

$\mathbf{x}(n)$	Tri-axial signal (vector)
$\mathbb{R}(\cdot)$	Real component
$\mathbb{I}(\cdot)$	Imaginary component
$\mathbf{x}(n)$	Vector-quaternion representation of $\mathbf{x}(n)$
$\mathbf{x}^{\imath}(n)$	Involution of $\mathbf{x}$ with respect to $\imath$
Θ	Azimuth angle
ϕ	Elevation angle
$\mathbf{q}$	Orientation quaternion
ρ	Real magnitude
$\mathbf{\hat{z}}$	Eigenaxis (vector-quaternion)
θ	Eigenangle
$\mathcal{U}$	Uniform distribution
$\mathbb{H}$	Hypercomplex space
$\Gamma(\cdot)$	Gamma function
c	Shape parameter
$\hat{c}$	Shape parameter (estimated)
$\Phi_{\mathbb{Q}}$	$\mathbb{Q}$ -coherence matrix
$\mathbf{q}_{\mathbf{r}}^{\mathbf{a}}(n)$	Quaternion augmented vector
$\mathbb{C}_{\mathbf{q}_{\mathbf{r}}}^{\mathbf{a}}$	Quaternion augmented covariance matrix
$\hat{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}}$	Quaternion augmented covariance matrix (estimated)
$\tilde{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}}$	Quaternion augmented covariance matrix with $\mathbb{Q}$ -proper structure
$\gamma(l)$	Threshold

Chapter 5

$\mathbb{R}^p$	p -dimensional real space
$\mathbf{R}(n)$	Instantaneous 3×3 rotation matrix
ϕ_x, ϕ_y, ϕ_z	Eüler angles
$\bar{\mathbf{x}}(n)$	Vector-quaternion representation of $\mathbf{x}(n)$
$F_p^v(\cdot)$	von Mises-Fisher pdf
η	Concentration (variance) of vMF pdf
$\check{\boldsymbol{\mu}}$	Mean of vMF pdf
$C_p(\eta)$	Normalizing constant of vMF pdf
$\mathbb{S}^{p-1}$	p -1-dimensional hypersphere space
$\mathbf{d}^{(p)}$	p -dimensional DRV
$\hat{\mathbf{q}}(n)$	Instantaneous orientation quaternion (estimated)
$\hat{\mathbf{z}}(n)$	Instantaneous eigenaxis (estimated)
$\hat{\theta}(n)$	Instantaneous eigenangle (estimated)
$\hat{\eta}$	Concentration parameter (estimated)
$\mathcal{M}_{\text{EEC}}$	Eigenangle-Eigenaxis Concentration metric
$\mathcal{M}_{\text{CinH}}$	Concentration in Hypersphere metric
$\hat{F}^k$	knn density
$\mathcal{H}(\cdot)$	Entropy
$\psi(k)$	Digamma function
$\Psi(\phi_r)$	Surface area of the hyperspherical cap
Φ_p	Total surface area of the hypersphere $\mathbb{S}^{p-1}$
$B_{\cos^2 \phi_r}$	Regularized incomplete Beta function
$\hat{\Omega}(l)$	Degree-of-directivity metric
$\hat{\omega}(l)$	Angular momentum metric
$\mathbf{q}_{\text{sim}}(n)$	Simulated DRV

Chapter 6

$\hat{\mathbf{q}}(n)$	Instantaneous orientation quaternion (estimated)
$\hat{\mathbf{z}}(n)$	Instantaneous eigenaxis as vector-quaternion (estimated)
$\hat{\theta}(n)$	Instantaneous eigenangle (estimated)
$\hat{\tilde{\mathbf{z}}}(n)$	Instantaneous eigenaxis as a vector (estimated)
Θ	Ground truth bearing angle
$\hat{\Theta}_{\text{CMA}}$	Bearing angle estimated using covariance matrix analysis
$\hat{\Theta}_{\text{HTA}}$	Bearing angle estimated using Hilbert transform analysis
$\hat{\Theta}_{\text{OPA}}$	Bearing angle estimated using orbital phase analysis

Essentially, all models are wrong, but some are useful.

-George E.P. Box.

Chapter 1

Introduction

1.1 Background

Geophones are vibration sensors capable of detecting vibrations generated from seismic sources of varying degree of magnitude - ranging from earthquakes and explosions to human footsteps. These geophones are transducers that convert the Earth's particle displacement into electrical signals and the voltage induced is proportional to the Earth's (soil) particle velocity, based on the principle of electromagnetic induction [1]. Seismic signals are recorded and processed in a variety of applications, which include:

- **Seismological Study** - The propagation of elastic waves through the Earth's medium caused by tremors or fracture of geological faults are analyzed to understand the effects of natural seismic sources that such as shifting of tectonic plates, volcanoes, oceanic and atmospheric activities, landslides and cracking of glaciers [2–6].
- **Exploration Geophysics** - Reflection seismology is used to estimate the impulse response of Earth's sub-surface from reflected seismic waves [7–10]. The seismic waves are generated by controlled seismic sources such as dynamite, airgun or

seismic vibrator on the Earth's surface. The seismic waves reflected by the different layers of the Earth's sub-surface are captured either on the surface or at different depths from the surface.

- **Unattended Ground Sensor** - They are used in surveillance systems to monitor military activity such as firing of artillery shells in war zones or to detect human activity in large outdoor fields such as airports or cross-country borders [11–14].

This thesis is aimed towards building a surveillance system to monitor human activities in large outdoor enclosures. The system records seismic signals that are generated from human activities such as human footstep and vehicular movement.

1.2 Motivation and objectives

Surveillance is defined as the continuous or periodic monitoring of person, vehicles, places or objects to obtain information concerning the activities and/or identities of the individual for the purpose of managing or protecting them. Urban surveillance systems were initially established as vision-based intelligent transport systems [15–17] or intelligent indoor/outdoor surveillance system [18–20] using network of cameras. Subsequently, distributed sensor based surveillance systems were built for variety of applications including landslide detection [21, 22], avalanche detection [23], forest intrusion detection [24], perimeter security [25] etc. Most of these sensor systems used seismic sensors because of their ability to detect ground movements caused by both natural sources such as landslides, earthquakes and man-made sources such as human footsteps, vehicles etc. Furthermore, seismic signals are less susceptible to change in lighting conditions or acoustic interference that may affect video cameras and microphones, respectively [26]. Seismic sensors are combined with one or more sensing technologies such as microphones [27], radar [28], video cameras [29], passive infrared, infrasound [30], ultrasonic, magnetome-

ters [24] depending on the surveillance application. Hence, there is a necessity for improved algorithms that are aimed towards processing seismic signals for surveillance applications.

This thesis is focussed towards the development of algorithms for seismic signals aimed at improved detection and direction-of-arrival estimation of surface sources. Conventional single-axis geophones are capable of capturing vibration only along a single direction and are always planted vertically on the ground. However in this work, the seismic signals are acquired using a tri-axial geophone. A tri-axial geophone consists of three geophones co-located along the three orthogonal directions - x , y and z axes of the Cartesian coordinate system. The tri-axial geophone is also referred to as vector-sensor as it capture a vector (3×1 vector in this case) at each time instant. As opposed to single-axis geophone, a vector-sensor has the ability to acquire the polarization of the seismic signal impinging the sensor.

Seismic waves and wavefields have been well researched for several decades by seismologists. Consequently, majority of the effort has been towards modeling far-field seismic signals. The focus of this thesis is, however, towards a better comprehension of the polarization of the tri-axial seismic signals generated by near-field surface sources. The vector-valued signals acquired by a tri-axial geophone comprises of two instantaneous properties - energy and direction. The energy quantifies the magnitude of the signal in a particular direction. Conventional detection methods exploit the energy, i.e, magnitude of the signal. In this work, the direction of the vector-valued tri-axial signal is exploited by analyzing the 3D trajectory. In line with this objective, the acquired tri-axial signal is normalized at each time instant by its magnitude to form the normalized trajectory. Instead of extending single-axis metrics to tri-axial data, trajectory-based metrics using the three-channel signal are proposed to quantify the instantaneous movement of the tri-axial signal. At the fundamental level, the thesis is focussed towards answering the research questions - *How can we define metrics that exploit the direction attribute of*

tri-axial signals to detect elliptically polarized footstep/vehicular signals? Do these directional metrics improve the detection range and bearing estimation accuracy for seismic surface sources compared to conventional energy-based metrics?

1.3 Contributions of this thesis

This section contains a brief overview of the proposed approaches and contributions made in this thesis. Research in this thesis includes two main fields of study. First, research is aimed towards *detection* of surface seismic sources and later, focus is shifted to *bearing estimation* of the detected sources.

Detection of seismic sources is achieved by identifying elliptical polarization in the acquired tri-axial signal. Elliptical polarization is quantified using three different features of the tri-axial signal. In the first approach, the smoothness of the tri-axial trajectory is used as a feature to identify elliptically polarized signals. Towards this purpose, the first contribution is the definition of two instantaneous metrics - neighbourhood Euclidean distance (NED) and neighbourhood ratio (NR). These metrics quantify variations of the smoothness of the normalized tri-axial trajectory which, in turn, is exploited to detect the presence of elliptically polarized signals generated by human footsteps. Although neighbourhood metrics detect elliptically polarized signals, they do not exploit the normalized trajectory lying on the surface of a unit sphere. Hence in the second contribution, the great-arc model is employed to fit the instantaneous variations of the normalized trajectory and the deviation error is used to detect polarized footstep signals. The great-arc error metric is then combined with the energy of the tri-axial signal to define a polarization-cum-energy metric that describes both elliptical polarization and transient characteristics of human footsteps.

In the second approach, the inter-channel correlation of the tri-axial signal is used to detect the presence of elliptically polarized signals. Instead of denoting the instant-

neous snapshots of the tri-axial signal as a 3×1 vectors, it is expressed as hypercomplex vector-quaternions. The vector-quaternion representation naturally accounts for the intrinsic relationship between the orthogonal signal components. As the third contribution, a quaternion generalized Gaussian distribution (QGGD), parameterized by a shape parameter and a quaternion augmented covariance matrix, is derived. The derived density is then used to quantify the dissimilarity between elliptically polarized footstep and random noise signals through the inter-channel correlation (via the degree-of-improperness of the quaternion augmented covariance matrix) and non-Gaussianity (via the shape parameter of the QGGD).

In the third approach, the rotational characteristic of the traced orbit is used as a feature to identify elliptical polarization. Instead of vector-quaternions, orientation-quaternion representation is used to model the instantaneous rotation of the 3D trajectory. In the fourth contribution, a velocity-to-rotation (VR) transform is proposed to estimate the instantaneous rotational parameters of eigenangle, eigenaxis and orientation quaternions. The estimated rotational parameters are then fitted to directional distributions. The presence of polarized footstep signals is characterized by higher concentration of the directional random vectors in their respective hyperspheres. The concentration parameter is estimated using both parametric and non-parametric techniques.

After the discussion on detection of seismic surface sources, the second half of the thesis is dedicated to estimating the direction-of-arrival of the detected source. Towards this purpose, the performance of the covariance matrix analysis (CMA) for estimating the bearing of near-field surface sources is studied in detail. The CMA is applied to footstep signals collected from different walking patterns and the existence of two ambiguities - an energy (180°) ambiguity and wave polarization (90°) ambiguity, are reported.

Extending on the rotational characteristic of the tri-axial trajectory, a trajectory-based approach is proposed, as the fifth contribution, to estimate the bearing angle of surface seismic sources. This approach is motivated by the fact that in near-field, the

wavefront impinging the sensor is transversely elliptical due to the simultaneous arrival of Love and Rayleigh waves. The bearing angle is estimated using the instantaneous eigenaxis of the tri-axial trajectory.

The organization of this thesis is as follows: Chapter 2 introduces the fundamentals of seismic signal processing. It also reviews existing detection and bearing estimation algorithms. Chapters 3, 4 and 5 address the problem of detection of human footstep and vehicular signals and Chapter 6 address the problem of bearing estimation of the detected footstep and vehicular signals. In Chapter 3, the smoothness of the normalized tri-axial trajectory is used as a feature to identify elliptically polarized signals. This feature is used for detection of seismic sources and two different approaches are proposed to quantify the smoothness of the 3D trajectory generated by human footstep and vehicular motion. In Chapter 4, the tri-axial signal is represented using hypercomplex vector-quaternions. A quaternion distribution is derived to model vector-quaternion signals and the dissimilarity in the statistics between footstep and noise vector-quaternion signal is used to detect the presence of footsteps. In Chapter 5, a quaternion rotational model is proposed to model the orbit traced by the tri-axial signal. The orientation quaternions are used to parameterize the instantaneous 3D rotation of the acquired trajectory. A velocity-to-rotation transform is proposed to estimate the instantaneous 3D rotations using the instantaneous eigenangle and eigenaxis. The estimated rotational parameters are found to be p -dimensional directional random vectors in $\mathbb{S}^{p-1}$ hyperspace. The difference in the concentration of the directional random vectors (DRV) between the presence and absence of a footstep is quantified using parametric and non-parametric approaches. After defining trajectory-based detection metrics, Chapter 6 starts with an analysis on the short-comings of existing bearing estimation algorithms that employ inter-channel correlated energy. A trajectory-based approach is proposed to estimate the bearing angle of seismic sources using the instantaneous eigenaxis evaluated from the velocity-to-rotation transform. Chapter 7 concludes the thesis with possible future directions.

1.4 Statement of Originality and Related Publications

To the best of the candidates knowledge, the following contributions of this thesis are original:

- (i) The definition of neighbourhood metrics, viz., neighbourhood Euclidean distance (NED) and neighbourhood ratio (NR) using normalized tri-axial trajectory and its application for the detection and denoising of human footstep signals. The publication related to this work is [C1].
- (ii) The definition of great-arc interpolation error using the spherical linear interpolation (SLERP) technique and polarization-cum-energy metric for detection of human footstep signals. This contribution has resulted in publication [C2].
- (iii) The derivation of quaternion generalized Gaussian distribution (QGGD) parameterized by a shape parameter and quaternion augmented covariance matrix to fit vector-quaternion signals. The shape parameter estimate and degree-of-Q-improperness of the quaternion augmented covariance matrix were employed for detecting human footstep signals. The publication related to this work is [C4].
- (iv) The definition of velocity-to-rotation transform based on a novel quaternion rotational model that parameterized the instantaneous movement of the tri-axial signal as instantaneous rotations in to three-dimensions. The derivation of both a parametric and a non-parametric approach to quantify the difference in concentration of the rotational parameters using directional statistics. The parametric and non-parametric algorithms were used to detect human footsteps in the recorded seismic signal. This work has been submitted as [J1].
- (v) The detailed study of the ambiguities that exist in the covariance matrix anal-

ysis (CMA) estimation of the bearing angle for near-field footstep signals. The publication related to this work is [C3].

- (vi) The use of the instantaneous eigenaxis to estimate the bearing angle of near-field surface sources that generate coupled polarization. This work is to be submitted as [J2].

Chapter 2

Fundamentals of Seismic Signal Processing

The foundation necessary for processing of seismic signals is laid in this chapter. The different types of seismic wave polarization is introduced in Section 2.1. In Section 2.2, the sub-components of the seismic signal acquisition system, used to record the seismic signals, are described. In this work, human footsteps and commercial vehicles are used as sources to generate seismic signals. A literature review of existing approaches and metrics used to detect impulsive human footstep signals and vehicular signals is presented in Section 2.5. The polarization model used to represent far-field tri-axial measurements is analyzed in Section 2.6. Using the polarization model, existing bearing estimation algorithms, using a single tri-axial geophone, are reviewed in Section 2.7.

2.1 Seismic Wave Propagation

Seismic waves are elastic waves that propagate due to elasticity of the Earth medium. Any pressure impact which may be contributed by major earthquakes, underground explosions, human footsteps or vehicular movement, causes the Earth medium to compress

Figure 2.1: Summary of seismic waves (after [31]).

and/or shear. This results in volume or shape distortion of the medium. Since the Earth (soil) is elastic, the medium attempts to return to its original volume/shape once the pressure load ceases to act. For transient seismic sources, the medium exhibits high degree of elasticity resulting in the propagation of seismic waves.

The seismic energy propagates through the heterogeneous Earth via multiple polarization modes [32]. Each mode has a specific orientation of particle oscillation with respect to the direction of propagation of the wavefront. Seismic waves are broadly classified into body and surface waves. Body waves propagate across the Earth's crust and consist of longitudinal P-waves and transverse S-waves. On the other hand, surface waves propagate on the Earth's surface and consist of elliptically polarized Rayleigh waves and transverse 'snake-like' Love waves. Among the different modes, the compressional P-waves have the highest propagation speed in the range of 1 to 14 km/s and the S-waves have a relatively lower velocity in the range of 1 to 8 km/s. The amplitude or energy

Figure 2.2: Pictorial representation of the propagating pattern of - (a) direct P-waves, (b) Love waves, and (c) Rayleigh waves, generated by a surface source and travelling towards the sensor.

of the P- and S-waves varies inversely with the distance of propagation. Love waves are transverse surface waves with particles oscillating perpendicular to the direction of propagation in the horizontal plane. These waves have a velocity range of 2 to 6 km/s. Rayleigh waves are generated by the interaction of P- and S-waves on the surface of the earth and their velocities range from 1 to 5 km/s. Rayleigh waves cause the soil particles to move in small elliptical orbits in a plane normal to the ground and parallel to the propagating wavefront [31]. These elliptical orbits can be traced in two directions - if the direction of the elliptical trace is in the direction of propagation, they are known as 'prograde', otherwise if the elliptical trace is in the direction opposite to the direction of propagation, they are called 'retrograde'. The different modes of polarization of seismic waves are summarized in Fig. 2.1.

Any seismic event generates all polarization modes but in varying proportions. Hence, the propagation of seismic vibrations through the heterogeneous Earth medium is com-

plex. It is known that a normal load on the Earth's surface is estimated to generate 67% Rayleigh waves, 26% shear (Love) or transverse waves and 7% compressional waves [33]. On the other hand, a tangential load generates 18% Rayleigh waves, 76% shear waves and 6% P waves [34]. Since an exerted force on the Earth's surface due to human activities can be considered as a combination of normal and tangential load due to the nature of impact, the signal acquired by a seismic sensor (geophone) is dominated by Love and Rayleigh waves. The propagating patterns of the different polarization modes for a surface impact are illustrated in Fig. 2.2.

2.2 Seismic Signal Acquisition System

A data acquisition system was built to record seismic signals in outdoor environments. The three main factors considered while designing the hardware were:

- (i) Simultaneous sampling of the three channels of a tri-axial geophone;
- (ii) No phase distortions from the pre-amplifying circuits;
- (iii) Operating the geophone under critically damped conditions.

The hardware system used for seismic signal acquisition is shown in Fig. 2.3. Features of the individual components are elaborated in detail below.

Tri-axial geophone GS32CT

Geophones are passive velocity sensors that convert displacement into electrical voltage. A geophone works on the principle of electromagnetic induction. It consists of a coil wound on a highly permeable magnetic core suspended by a leaf spring into the field of a permanent magnet. The coupling of a geophone, buried in soil, results in an induced voltage when impinged by a seismic wave. The coil inside the geophone oscillates with approximately the same velocity as that of the Earth's particles and induces an electrical signal proportional to the particle velocity.

Figure 2.3: Block diagram of the seismic signal acquisition system.

Figure 2.4: Internal configuration of tri-axial geophone GS32CT.

The tri-axial geophone model GS32CT shown as Component (1) in Fig. 2.3, was supplied by GeoSpace Technologies Inc. The three orthogonal placed geophones were packaged inside a Nail 3C marsh enclosure [35]. Figure 2.4 shows the internal placement of the three orthogonally co-located geophones. When buried, the first two geophones (on the left of Fig. 2.4) record vibrations in the horizontal x and y direction and the third geophone captures vibrations in the vertical z direction. A tri-axial geophone is therefore capable of recording the three-dimensional particle velocity of the wavefront impinging the sensor. A tap test was conducted by gently thumping the two sides of the Nail 3C casing to identify the two horizontal axes directions and these directions were labeled ‘H1’ and ‘H2’.

Two important specifications of this geophone are its sensitivity of 0.275 V/cm/sec and its natural frequency of 11 Hz (as shown in the frequency response plot in Fig. 2.5). It can be observed that the geophone response attenuates the signals below the natural frequency and is constant for all the signal frequencies above 11 Hz. Although GS32CT is a commercial grade tri-axial geophone, it is sold independently and without any accompanying software since it can be used in a variety of applications including seismological studies and exploration geophysics.

Analog-to-digital convertor

The Presonus Firestudio-2626 [36], a multi-channel analog-to-digital (ADC) convertor shown as the third component in Fig. 2.3, was used to record the seismic signals because it had the following characteristics:

- (i) Flat frequency response between 5 Hz - 20 kHz;
- (ii) Simultaneous sampling across eight channels;
- (iii) Bit resolution of 24 bit with maximum sampling frequency of 96 kHz per channel;
- (iv) Easy scaling of the number of channels up to 24, using Presonus Digimax [37];

Figure 2.5: Frequency response of tri-axial geophone GS32CT (after [35]).

- (v) Preamplifier phantom power for each channel;
- (vi) Easy interface to a laptop via the firewire interface.

The digital signals recorded by the ADC were saved as ‘.wav’ files for further processing.

Power management circuitry

A vital component in a portable outdoor data acquisition system is a power source to operate the analog-to-digital convertor (ADC), laptop computer and preamplifier circuits. For obtaining 220 V, 50 Hz AC supply, two car batteries were connected in parallel and supplied as input to an inverter as shown in Components 4 and 5 in Fig. 2.3. The inverter output was used to power the ADC and the computer. The 12 V DC from the car battery was used to power the preamplifier circuits.

Grounding and preamplifier circuits

During the initial experiments, a strong interference of 50 Hz and its harmonics were encountered in the recorded data. Further analysis attributed the problem to ground

Figure 2.6: Internals of the (a) preamplifier and (b) data injection circuits.

loops that arose due to differences in the ground-state of the powered components of the data acquisition system. The problem was solved by connecting all components to a common ground.

In order to enhance the detection range, the signal strength had to be amplified before it was fed to the ADC. The amplification increased the noise floor of the acquired seismic signal to be greater than the interference noise added by the ADC. A pre-amplifier circuit, with a flat frequency response from 0-20 kHz, was designed using the instrumentation amplifier IC INA126. Figure 2.6(a) shows the preamplifier circuit along with the voltage regulator-converter ICL7660 which provided the 12 V power supply. The gain G of the instrumentation amplifier INA126 [38] was controlled by a single resistor, R_G and evaluated as

$$G = 5 + \frac{80 \text{ k}\Omega}{R_G}. \quad (2.1)$$

The preamplifier gain G was set to 25 for all the three channels. As shown in Fig. 2.6(a), Pin 3 of IC INA126 is connected to the ground via a 47 k Ω resistor in order to provide a path for the input bias current whereas 48 k Ω is used on Pin 3 to provide shunt

Figure 2.7: Footstep signal acquired by the z -axis of the geophone under (a) overdamped and (b) critically damped, conditions.

resistance for optimal operation of the geophone with a damping coefficient of 0.7. Figures 2.7(a) and (b) show the difference in the signal acquired from an overdamped and a critically damped geophone, respectively. It can be observed that when the geophone was overdamped, the acquired signal has a tendency to over-swing once triggered and takes a longer time to return to its equilibrium state. Figure 2.6(a) shows the complete preamplifier circuitry. The amplified seismic signal is balanced using a data injection (DI) circuit before digital conversion.

Data injection (DI) circuit

In order to improve the robustness of the signal against interference for long-distance transmission, a data injection (DI) circuit is built as shown in Fig. 2.6(b). The DI circuit converts the unbalanced signal into a balanced signal. The data injection process is performed after signal amplification and before the signal is transmitted to the recording station. Three pre-amplifier and DI circuits, for the three channels of a tri-axial geophone,

Figure 2.8: Internals of the (a) DI box (housing the preamplifier and DI circuits) and the (b) power distribution box.

were mounted compactly within an aluminium chassis, as shown in Fig. 2.8(a). The chassis allows for easy handling during outdoor experiments. Finally, a power distribution system, as shown in Fig. 2.8(b), was built to scale the system for experiments that employ multiple tri-axial geophones.

2.3 Experiment Setup for Seismic Signal Acquisition

The seismic signal acquisition system was used to record seismic signals generated by both human footsteps and vehicular movement. Most of the experiments were conducted within the NTU campus on a grassy field of size $75 \text{ m} \times 75 \text{ m}$. Experiments were also conducted outside NTU at two locations - (i) an urban training facility where vehicles operated on a mud road with the geophone buried on the side of the road and (ii) a remote bus terminus where vehicles operated on an asphalt road with the geophone buried on the grassy side walk. The geophone was buried with good coupling at a pre-determined location $(0,0)$ designated as the origin, with its orthogonal sensors aligned with the x ,

y and z axes of the Cartesian coordinate system. During the field experiments, footstep signals were recorded using eight different human subjects walking normally with soft-sole shoes and vehicular signals were recorded using three different vehicles viz., a bus, a medium-sized commercial lorry and a small-sized goods carrier. Although different sources have been used to record seismic signals during the several field experiments, each seismic recording is conducted with only one active source, either a single human walking or a single vehicle in motion.

Although existing footstep [13, 39] and vehicular [40] detection algorithms, discussed later in this chapter, define a detection range for the baseline metrics, it is highly susceptible to the diversity of the medium, the coupling of geophone to the ground, the intensity of footsteps and factors such as vehicle size, weight, wheel, speed, road composition and other environmental factors. Hence in this thesis, the detection range of the baseline metrics will be determined using the acquired dataset. The performance of both baseline and proposed metrics, for detection and bearing estimation, will be evaluated based on its averaged performance over at least thirty different experiment trails.

2.4 Review of Urban Surveillance Systems

Urban surveillance systems initially started as intelligent transport systems [15–17] and intelligent indoor/outdoor surveillance system [18–20] using only vision-based sensors, i.e., cameras. The aim of vision-based surveillance systems, used in advanced traffic management systems, is to extract descriptions of several moving vehicles from video data. In [15] motion and contour estimation of the vehicle objects are performed by linear Kalman filters based on an affine motion model and occlusion detection is achieved by intersecting the depth ordered regions associated to the objects. The symbolic reasoning of the traffic scene based on the extracted tracks is performed using a belief network because of its inherent ability to model uncertainties. Alternatively, a vision-

based surveillance system is also used in monitoring the status of senior people towards the development of a fall detection emergency alarm system [20].

Subsequently, distributed sensor based surveillance systems were built for different types of surveillance applications including landslide detection [21, 22], avalanche detection [23], forest intrusion detection [24], perimeter security [25] etc. The forest intrusion detection system in [24] uses geophones, accelerometers and magnetometers to detect vehicles entering into or moving around in a protected area. The geophone and accelerometers detect seismic vibrations caused by vehicles from a distance of 2-10 m. In order to increase reliability, the system contains large number of sensors connected over a network to detect vehicles. The system is focused on fusion algorithms that reduce false alarm and estimate the location, speed and direction of the intruder vehicle. The border security system in [25] describes a feature-based classification for unattended ground sensors with each node containing both a tri-axial accelerometer and a tri-axial geophone. The system uses a tree-based classifier for robust discrimination of humans and animals in the field using wavelet statistics, spectral statistics, power spectral density, cadence and kurtosis as features. The acousto-seismic air detection (ASAD) system in [27] consists of several nodes with each node having five microphones and three geophones. The system is used to detect, track and classify low flying aircrafts that are used to smuggle illicit drugs and illegal immigrants across the border. The ground target classification system in [41] uses vertical geophones to classify tracked and wheeled vehicles via a robust feature extraction method that employs mel-frequency cepstral coefficients.

Seismic sensors are gaining popularity in surveillance systems because of their ability to detect ground movements caused by both natural sources such as landslides, earthquakes and man-made sources such as human, vehicles etc. There is a necessity for improved algorithms that are aimed towards processing seismic signals for surveillance applications. This thesis is focussed towards the development of algorithms using seismic signals for better detection and bearing estimation of human surveillance in outdoor envi-

ronments. In the following sections, existing detection and direction-of-arrival estimation algorithms will be reviewed.

2.5 Review of Detection Algorithms

Using the seismic acquisition system described in Section 2.2, seismic signals were recorded over several outdoor field experiments. As an illustrative example, Fig. 2.9 shows a train of footsteps recorded in the three channels of the tri-axial geophone when a human subject walks radially away from the geophone. The three-dimensional (3D) vector representation of the recorded signal at a particular time instant n is expressed as

$$\mathbf{x}(n) = [x_1(n) \ x_2(n) \ x_3(n)]^T, \quad (2.2)$$

where the first two elements correspond to signals received at the two horizontal channels while the third element corresponds to the signal recorded in the vertical channel.

This section reviews metrics that can be used to detect the presence of seismic sources, namely, human footsteps or vehicular movement. It is important to note that although detection algorithms have been developed for both single-channel [13, 42] and tri-axial signals [43], there are relatively less publications that describe these algorithms in detail. This can be attributed to the fact that such algorithms were developed by the military and hence remain proprietary.

2.5.1 Detection of Footstep Signals

Detection of human footsteps is challenging due to the dynamic nature of a footstep and the medium through which the signal propagates [14]. Footstep signature may vary based on factors such as the nature of walk (normal, stealthy) and the propagating medium (grassy, clay soil, pebbled ground). The detection of footsteps can be formulated using

Figure 2.9: Plot of footstep signal acquired in the (a) x –, (b) y – and (c) z – axis of the tri-axial geophone.

four different features such as:

- (i) Transient-based detection;
- (ii) Energy-based detection;
- (iii) Spectrum-based detection;
- (iv) Trajectory-based detection.

Existing detection approaches are reviewed along with their merits and demerits.

2.5.1.1 Transient-based detection

Detection of transient signals, a short-lived burst of energy caused by a sudden change of state of a system, has been achieved in a number of applications (eg., SOund Navigation And Ranging (SONAR), RAdio Detection And Ranging (RADAR) and machine fault detection). Statistical detection of a signal embedded in noise has been well researched

where noise is assumed to be random and Gaussian distributed. In the presence of a transient, the statistics are expected to deviate from Gaussian characteristics. Hence, evaluating non-Gaussianity provides a natural way for transient detection. Footstep signals can also be considered as transient signals since they are short-lived and transient detection metrics can be used to detect human footsteps. Non-Gaussianity metrics used in literature for transient detection are kurtosis, Kolmogorov-Smirnov test and bispectrum analysis [44].

Kurtosis

The kurtosis K , is defined as the ratio of the fourth-order moment to the second-order moment of the data. The mathematical equation used to estimate kurtosis is given by [13]

$$\hat{K} = \frac{\frac{\sum_{n=0}^{N-1} (x(n) - \mu)^4}{N-1}}{\left(\frac{\sum_{n=0}^{N-1} (x(n) - \mu)^2}{N-1} \right)^2}, \quad (2.3)$$

where μ is the mean of the received signal $\mathbf{x}(n)$ of frame length N which extends over the signal transient. Kurtosis quantifies the peakyness of the signal. The presence of an impulsive signal within a given time window results in a heavy-tailed distribution with $\hat{K} > 3$. On the contrary, Gaussian distributed background noise yields a kurtosis value of $\hat{K} = 3$. In [13], kurtosis is computed on seismic signals recorded using a single-channel geophone and footsteps are detected when $\hat{K} > 6$.

As opposed to the single-channel metric used in [13], kurtosis for multi-channel data can be defined by summing the kurtosis over the three channels to obtain the cumulative kurtosis metric

$$\hat{K}_c = \hat{K}_1 + \hat{K}_2 + \hat{K}_3, \quad (2.4)$$

where each $\hat{K}_i$ is computed from acquired i th channel signal $\mathbf{x}_i(n)$, using (2.3). Since the kurtosis values are added, the detection threshold is increased to $\hat{K}_c > 10$ for a positive

footstep detection.

Kolmogorov-Smirnov test

The Kolmogorov-Smirnov (K-S) test is a nonparametric goodness-of-fit test, which compares the probability distribution of the data with a reference distribution, and Gaussian distribution is often used as the reference. If $X_1, X_2, \dots, X_N$ are independent Gaussian random variables, then

$$Y = \sum_{i=1}^N X_i^2 \quad (2.5)$$

is chi-square distributed with N degrees of freedom. The test for non-Gaussianity determines the number of degrees of freedom available in the recorded signal. In [45], acquired time-domain signal is transformed into the wavelet domain using a continuous wavelet transform. In the wavelet domain, transient signals exhibit a higher non-Gaussianity measure compared to noise which exhibits a uniform distribution. The K-S test is performed on the wavelet coefficients to detect transients.

Bispectrum analysis

Bispectrum is the Fourier transform of bi-covariance or the third-order cumulant. The third-order cumulant is defined as

$$C_{3X}(k, l) = E\{x(n)x(n+k)x(n+l)\}, \quad (2.6)$$

where the subscript denotes the third-order cumulant of a signal $x(n)$ and k, l are the lag parameters. Defining $j = \sqrt{-1}$, the corresponding bispectrum is given by

$$S_{3X}(f_1, f_2) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} C_{3X}(k, l) e^{-j2\pi f_1 k} e^{-j2\pi f_2 l}. \quad (2.7)$$

The bispectrum is a function of two frequencies f_1 and f_2 . If a transient signal is added to

stationary noise, the observed signal becomes nonstationary in the active window. This results in a shift of mean and skewness of the signal distribution. The detection technique proposed in [46] uses bispectrum to distinguish noise segments from those containing the transients.

All the metrics discussed above detect transients by exploiting their leptokurtic nature and hence are effective only when the source is close to the sensor. Their detection performance may deteriorate when the spread of the transients increases with increasing source-sensor distances. This spreading of time-domain waveform can be attributed to the dispersion and multi-path effects of the propagation medium. Among the non-Gaussianity measures, kurtosis remains the most popular technique for footstep detection.

2.5.1.2 Energy-based detection

Human footstep signals generated by a subject walking at constant pace can be characterized as a periodic train of impulsive transient signals. A threshold applied on an energy metric computed over sliding windows is a classical method to detect them.

Cumulated energy

Multi-channel data has been applied for detection of ground vibrations caused by rock motions using cumulated energy metric [3]. The cumulated energy metric over a frame of length N is estimated as

$$\hat{E}_c = \sum_{n=0}^{N-1} [x_1^2(n) + x_2^2(n) + x_3^2(n)], \quad (2.8)$$

where $x_1(n)$, $x_2(n)$ and $x_3(n)$ are the signals corresponding to the x , y and z axes of the tri-axial geophone. By applying a suitable threshold on the cumulated energy metric, detection is achieved. Although the advantage of the cumulated energy metric is its simplicity, the accuracy of the metric deteriorates at low signal-to-noise ratio (SNR)

conditions.

Wavelet energy

In the wavelet domain, the energy of an impulsive function is concentrated in a few wavelet coefficients while the background noise energy is spread across all the coefficients. A nonlinear threshold applied on the wavelet coefficients suppresses the lower energy coefficients. This process results in a denoised signal after the application of an inverse wavelet transform [47].

In [42], footstep signals acquired by a single-channel geophone is transformed to the wavelet domain. For each scale, the eigenvalues of the corresponding autocorrelation matrix are evaluated. An adaptive threshold is then computed for eigenvalues in each scale and detection is performed. Furthermore, denoising is performed by discarding those smaller eigenvalues corresponding to noise and the signal is reconstructed in the time domain by applying the inverse wavelet transform. Experiments in [42] highlight that the denoising algorithm improved the SNR of the footstep signal recorded at farther source-sensor distances. In order to detect footsteps under lower SNR conditions, a wavelet energy feature using the db4 wavelet is estimated as

$$\hat{E}_{w_{l,n}} = \sum_k (w_{l,n,k}^2), \quad (2.9)$$

where w is the wavelet coefficient, l is the wavelet scale, n defines the wavelet shift and k denotes the index of the sub-bands. Using the energy of the three predominant energy subbands, the wavelet energy feature is demonstrated to be effective for footstep detection under low SNR conditions. Alternatively, the Haar wavelet has been used in [48] to extract features for footstep detection. Higher-order statistics are combined with wavelet packet transform in [49] to detect transient signals for industrial applications. It is useful to note that the wavelet-based metrics are susceptible to the choice of the wavelet and

its performance deteriorates when the wavelet fails to approximate the signature of the transient signal.

Eigenvalues of covariance matrix

Detection of a hammer strike on the ground [50] is achieved by evaluating the largest eigenvalue of the covariance matrix formed from overlapping windows of the tri-axial data. A frame of signal received from a tri-axial geophone can be represented as

$$\mathbf{x}_i(n : n + N - 1) = [x_i(n) \ x_i(n + 1) \ \dots \ x_i(n + N - 1)]^T, \quad (2.10)$$

where “ $n : n + N - 1$ ” denotes the time index from n to $n + N - 1$ and the channel indices $i = 1, 2$ correspond to the horizontal x, y components while $i = 3$ corresponds to the vertical z component. A $3 \times N$ data matrix is constructed as

$$\mathbb{X}(n : n + N - 1) = \begin{bmatrix} \mathbf{x}_1^T(n : n + N - 1) \\ \mathbf{x}_2^T(n : n + N - 1) \\ \mathbf{x}_3^T(n : n + N - 1) \end{bmatrix}. \quad (2.11)$$

The corresponding covariance matrix is estimated as

$$\hat{\mathbf{C}} = \frac{1}{N} \mathbb{X}(n : n + N - 1) \mathbb{X}^T(n : n + N - 1). \quad (2.12)$$

The eigenvalue decomposition of this symmetric covariance matrix yields

$$\hat{\mathbf{C}} = \hat{\mathbf{U}} \hat{\mathbf{V}} \hat{\mathbf{U}}^T, \quad (2.13)$$

where $\hat{\mathbf{V}} = \text{diag}\{\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3\}$ is a matrix containing the eigenvalues of $\hat{\mathbf{C}}$ with $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \hat{\lambda}_3$ and $\hat{\mathbf{U}} \in \mathbb{R}^{3 \times 3}$ is the unitary matrix consisting of the eigenvectors of $\hat{\mathbf{C}}$. A plot of the largest eigenvalue $\hat{\lambda}_1$ against overlapping windows reflects the energy of the principal

Figure 2.10: Spectrum-based footstep detection algorithm (after [39]).

component as a function of time and is used to detect the presence of a seismic source.

2.5.1.3 Spectrum-based detection

Human footsteps corresponding to a subject walking/running at a constant pace are periodic and hence spectrum-based techniques can be used for detection. Cadence, defined as the time interval between two events in a signal, is a good indicator for footstep detection from recorded footstep trains [13]. Spectrum-based detection algorithms exploit the fact that the interval between footsteps is approximately 0.5 s (2 Hz). Cadence is usually computed over a sliding time window of approximately 4 s for automatic detection of human footsteps. An extension of the cadence technique is the spectrum-based technique reported in [39]. Since the information of footsteps primarily resides in the regular pattern of impulse arrival times and not in the impulse themselves, spectrum analysis technique is intuitively less error prone than other transient-based footstep detection techniques. Figure 2.10 shows the frequency based footstep detection process in [39].

The acquired seismic signals are subsequently bandpass-filtered and an envelope detector is then used to process the signals before downsampling to 40 Hz. Spectrum analysis is performed on the downsampled signal using a Hanning window with 0.95 overlapping factor. This spectrum is further passed through a split-window two-pass normalizer [51] on a line-by-line basis to improve the SNR of the received signal. The spectrogram of the processed signals corresponding to the improved SNR values strongly depicts the footstep harmonics for an SNR greater than 15 dB. The criteria to determine

the existence of footsteps are [39]

- (i) A primary frequency occurrence in the 0.5 to 3 Hz range;
- (ii) The primary frequency must have a second or third harmonic present;
- (iii) The primary frequency must be greater than 11 dB SNR and the harmonic greater than 7 dB or vice versa.

It is important to note that the latency in detection is apparent in the frequency-based detection metrics since measuring the periodicity requires a train of footsteps. Moreover, determining the exact location of each footstep in the recorded signal is challenging for these metrics.

2.5.1.4 Trajectory-based detection

In order to understand the azimuthal response of the tri-axial geophone GS32CT using seismic signals generated by human footsteps, similar to [52], an outdoor experiment was conducted (on a grassy field) using two geophones G1 and G2 buried as shown in Fig. 2.11(a). Human footsteps were created at (0,0). Each footstep signal exhibited approximately two complete cycles in each the three channel of the tri-axial geophone. The cycles correspond to the heel striking against the ground followed by the forefoot hitting the ground [53].

Since the geophones G1 and G2 are placed diametrically opposite to the location of impact, the seismic signals arriving at them must be 180° phase shifted with respect to each other. In order to observe this phase shift, a single cycle of z -axis signal arriving at G1 and G2 are plotted against y -axis signal in Figs 2.11(b) and 2.11(c), respectively. We observe from the indication of the arrow that the particle motion are in opposite direction, i.e., a 180° phase shift exists between the wavefront arriving at G1 and G2. Unlike [52], where the usage of sledge hammer generated linearly polarized body waves, this experiment generated elliptically polarized waves as shown in Fig. 2.11.

Figure 2.11: Plot of (a) experiment setup and z -axis vs y -axis signal received at (b) G1 and (c) G2.

Since seismic signals generated by a human footstep and acquired by a tri-axial geophone are elliptically polarized, the evolution of the 3D velocity trajectory can be used as a characteristic for detection. The tri-axial signal trajectory within a small time segment can be formulated as a group of independent particles in motion. In the context of our application, the direction of evolution of $\mathbf{x}(n)$ is estimated as the direction vector [54]

$$\hat{\mathbf{v}}(n) = \frac{\mathbf{x}(n+1) - \mathbf{x}(n)}{\|\mathbf{x}(n+1) - \mathbf{x}(n)\|}, \quad (2.14)$$

where $\mathbf{x}(n)$ is defined in (2.2). An elliptical path traced by a polarized signal can be detected by estimating the angular momentum

$$\hat{\omega}(l) = \frac{1}{N} \left\| \sum_{n=1}^N \left[\frac{\mathbf{x}(lN - l_o + n) - \boldsymbol{\mu}(l)}{\|\mathbf{x}(lN - l_o + n) - \boldsymbol{\mu}(l)\|} \times \hat{\mathbf{v}}(lN - l_o + n) \right] \right\|, \quad (2.15)$$

where $\boldsymbol{\mu}(l)$ is the 3×1 vector defining the temporal mean of $\mathbf{x}(n)$ in the frame and $\times$ denotes the vector cross product. The estimated $\hat{\omega}(l)$ quantifies the amount of rotation about the frame's mean direction [54]. While tracing an elliptical orbit, $\hat{\mathbf{v}}(n)$ aligns in a particular direction for short piecewise time segments within a footstep signal. This

Figure 2.12: Plot of vehicular signal acquired in the (a) x –, (b) y – and (c) z – axis of the tri-axial geophone.

continuous alignment of $\hat{\mathbf{v}}(n)$ is estimated as the degree-of-directivity [54]

$$\hat{\Omega}(l) = \frac{1}{N} \left\| \sum_{n=1}^N \hat{\mathbf{v}}(lN - l_o + n) \right\|. \quad (2.16)$$

Other than the energy-based metrics $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$ defined after (6.2), trajectory-based metrics $\hat{\omega}(l)$, $\hat{\Omega}(l)$ will be used as baselines for comparing the detection performance of newly developed metrics in the subsequent chapters.

2.5.2 Detection of Vehicular Signals

All vehicles including commercial vehicles such as car, jeep, sport utility vehicle (SUV) and military vehicles such as self-propelled howitzer, armoured vehicle, tracked vehicles, exert a force on the Earth's surface when in motion. This force causes the Earth's particles to vibrate and a geophone buried in the vicinity of vehicular traffic is capable of converting

these vibrations into electrical signals. Seismic sensors have the advantage of sensing signals beyond line-of-sight. Besides vehicles moving on the ground, targets operating in other mediums like air (helicopters and aircrafts) and water (ships and underwater vehicles) also induce vibrations on the Earth. Aircrafts, for example, mostly generate acoustic waves in air, but they also cause coupled seismic and hydro-acoustic waves in the surrounding ground and water medium, respectively. As an example, Fig. 2.12 illustrates the seismic signals generated by a commercial mid-sized vehicle on the three channels of the tri-axial geophone.

Vehicular detection systems that employ geophones often focus on vehicular classification using feature vectors. Since vehicular signals are non-stationary and wide-band, it is challenging to detect peaks in the frequency spectrum. Hence, time-frequency based feature vectors using short time fourier transform (STFT) coefficients [55], or wavelet coefficients [56] were developed. A variety of feature vectors such as spectral statistics and wavelet coefficients characterization (SSWCC) [56], which capture the amplitude statistics, shape statistics, wavelet coefficients and peak locations in the spectrum have been proposed to classify vehicular data. Once the feature vectors are identified, different types of neural networks are used in their classification [57,58]. It is useful to note that majority of work focus on the detection of military vehicles [59–61]. Military vehicles have distinct seismic signature due to tracked wheels. The seismic signature of a variety of sources have been studied in [62] and it has been shown that every target has a specific seismic signature which can be used to uniquely characterize them.

The seismic signature of a moving vehicle depends strongly on factors such as type of road pavement, vehicle speed and characteristics of vehicular movement such as acceleration, constant speed movement, braking etc. These factors are often reflected in the spectrum of the seismic signal and its variations with time. Frequency-domain algorithms generally operate on frequency range 5-250 Hz because the signature of a vehicle is generated from two main sources - engine and propulsion gear [63]. Figure 2.13(a)

Figure 2.13: Plot of (a) signal spectrum and (b) features used in vehicular signal detection and classification (after [63]).

shows a typical spectrum of vehicular signal with the corresponding contributing factors for features in different frequency ranges identified. The basic hierarchy of features for vehicular detection and classification is illustrated in Fig. 2.13(b). In a spectral-based vehicular detection algorithm, the broadband energy in the dominant frequency range of 5 to 250 Hz is continuously monitored through band-averaged energy levels in bandwidths of 20 Hz [63]. Vehicles are detected if there is a change in the energy of any of the frequency bands with respect to the background noise level. This spectral-based algorithm will be used as the baseline to compare the performance of the newly proposed trajectory-based metrics.

2.6 Review of A Far-field Polarized Signal Model

A polarized model is defined for seismic waves in far-field, i.e., waves generated from point-source and acquired by point-like sensor, with constant polarization in the fre-

quency range of the sensor. The unit vector pointing towards the source direction is given by [64]

$$\mathbf{u}(\Theta, \Phi) = \begin{bmatrix} \cos \Theta \cos \Phi \\ \sin \Theta \cos \Phi \\ \sin \Phi \end{bmatrix}, \quad (2.17)$$

where $\Theta \in (-\pi, \pi]$ and $\Phi \in [-\pi/2, \pi/2]$ are the azimuth and elevation angles, respectively. The polarization parameters of orientation angle Ψ and ellipticity Ξ are introduced to model a tilted generalized Rayleigh (TGR) wave. The measurement model for a tri-axial signal is given by

$$\dot{\mathbf{x}}(t) = \dot{\mathbf{p}}(t) + \dot{\mathbf{e}}(t) = \dot{A}(\theta)\dot{\mathbf{s}}(t) + \dot{\mathbf{e}}(t), \quad (2.18)$$

where

- $\dot{\mathbf{x}}(t) \in \mathbb{C}^{3 \times 1}$ denotes the complex envelope of acquired vector;
- $\dot{\mathbf{p}}(t) \in \mathbb{C}^{3 \times 1}$ denotes the complex envelope corresponding to the noise-free particle displacement vector;
- $\dot{\mathbf{s}}(t)$ is the complex envelope of the source signal;
- $\dot{A}(\theta)$ is the response vector of the sensor;
- $\dot{\mathbf{e}}(t)$ is the measurement noise.

The response of the sensor $\dot{A}(\theta)$ is expressed in terms of azimuth, elevation angles of the source and orientation angle and ellipticity of the polarization ellipse.

Using the measurement model, a TGR wave is modeled as

$$\dot{\mathbf{x}}(t) = \mathbf{V}_{\text{TGR}} \mathbf{Q} \dot{\mathbf{w}} \dot{\mathbf{s}}(t) + \dot{\mathbf{e}}(t), \quad (2.19)$$

where

$$\mathbf{V}_{\text{TGR}} = \begin{bmatrix} \cos \Theta \cos \Phi & -\cos \Theta \sin \Phi \\ \sin \Theta \cos \Phi & -\sin \Theta \sin \Phi \\ \sin \Phi & \cos \Phi \end{bmatrix} = [\mathbf{u} \ \mathbf{v}_2] \quad (2.20)$$

is the polarization ellipse that is confined to the plane containing the source direction $\mathbf{u}$ and an orthogonal direction $\mathbf{v}_2$. For TGR wave, the direction of propagation is located in the plane spanned by the ellipse major and minor axes as shown in Fig. 2.14. The sign of Ξ is used to denote the spin direction (prograde or retrograde), whereas the magnitude denotes the eccentricity of the ellipse. The orientation angle Ψ defines the tilt of the ellipse with respect to the unit vector $\mathbf{s}(\Theta, \Phi)$. Hence, $\mathbf{Q}$ and $\dot{\mathbf{w}}$ are given by

$$\mathbf{Q} = \begin{bmatrix} \cos \Psi & \sin \Psi \\ -\sin \Psi & \cos \Psi \end{bmatrix}, \quad (2.21)$$

$$\dot{\mathbf{w}} = \begin{bmatrix} \cos \Xi \\ i \sin \Xi \end{bmatrix}, \quad (2.22)$$

where the matrix $\mathbf{Q}$ performs a rotation of angle Ψ of the ellipse that is traced by $\dot{\mathbf{w}}$. The linearly polarized P-wave and elliptically polarized generalized Rayleigh wave are modeled as a special form of TGR with $\Psi = 0$, $\Xi = 0$ and $\Phi = 0$, $\Psi = 0$, respectively. For plane waves, a single three-component sensor is capable of identifying a single generalized Rayleigh wave where $\Psi = 0$.

On the other hand, transverse waves are modeled as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \dot{\mathbf{A}}_{\text{T}}(\Theta, \Phi, \Psi, \Xi) \dot{\mathbf{s}}(t) + \dot{\mathbf{e}}(t) \\ &= \mathbf{V}_{\text{T}} \mathbf{Q} \dot{\mathbf{w}} \dot{\mathbf{s}}(t) + \dot{\mathbf{e}}(t), \end{aligned} \quad (2.23)$$

Figure 2.14: Polarization ellipse parameters for tilted generalized Rayleigh wave.

where the subscript ($\cdot_T$) denotes transverse waves and

$$\mathbf{V}_T = \begin{bmatrix} -\sin \Theta & -\cos \Theta \sin \Phi \\ \cos \Theta & -\sin \Theta \sin \Phi \\ 0 & \cos \Phi \end{bmatrix} = [\mathbf{v}_1 \ \mathbf{v}_2] \quad (2.24)$$

spans the orthogonal space of the vector $\mathbf{u}(\Theta, \Phi)$ in the direction of the source. This is because the particle displacement for a transverse wave lies in a plane perpendicular to the direction of propagation.

The above time-invariant polarized seismic wave model can be used to approximate the tri-axial signals acquired from footstep or vehicular movement using the TGR wave model. However, the polarization properties cannot be assumed to be constant and time-invariant. Unlike far-field conditions, polarization modes tend to arrive at the sensor simultaneously in the near-field because of the short propagation distance of the wavefront that is insufficient for the wavefront to decouple into the different modes. Furthermore, error in the exact perpendicular placement of the vertical channel of the geophone to the Earth's surface and combined influence of all the different polarization modes makes it

a challenge to apply the above measurement model for tri-axial signal generated from near-field surface sources.

2.7 Review of Bearing Estimation Algorithms

Bearing estimation refers to the estimation of the azimuth angle of a source with respect to the sensor. Conventionally, bearing estimation is performed using time-difference-of-arrival (TDOA) [11] between the multiple sensors deployed as an linear array. However in this section, bearing estimation algorithms using a single tri-axial geophone, a vector-sensor, are reviewed.

Covariance Matrix Analysis

Covariance matrix analysis (CMA) is a classical algorithm used for estimating the bearing of earthquake signals using a single tri-axial geophone [2, 65]. The covariance matrix evaluates the second-order characteristics of a signal and the construction of the covariance matrix varies from application to application. For spectral estimation, temporal snapshots are rearranged to obtain the covariance matrix. In array processing, on the other hand, the covariance matrix is formed from the data acquired from the array elements to capture the spatial correlation. In a linear array, the azimuth angle is estimated using the phase difference of the signals arriving at the sensors, i.e., sensors are spatially separated. For a vector-sensor, the azimuth angle is estimated using the phase difference of the signals arriving at the orthogonally oriented and co-located sensors, i.e., sensors are polarization diverse.

From the $3 \times N$ data matrix constructed in (2.11), the covariance matrix is estimated as

$$\begin{aligned}\hat{\mathbf{C}} &= \mathbf{E}\{\mathbf{x}(n)\mathbf{x}^T(n)\} \\ &\approx \frac{1}{N}\mathbb{X}(n : n + N - 1)\mathbb{X}^T(n : n + N - 1).\end{aligned}\tag{2.25}$$

The matrix $\hat{\mathbf{C}}$ can also be expressed as

$$\hat{\mathbf{C}} = \begin{bmatrix} \sigma_{\mathbf{x}_1}^2 & \sigma_{\mathbf{x}_1\mathbf{x}_2} & \sigma_{\mathbf{x}_1\mathbf{x}_3} \\ \sigma_{\mathbf{x}_2\mathbf{x}_1} & \sigma_{\mathbf{x}_2}^2 & \sigma_{\mathbf{x}_2\mathbf{x}_3} \\ \sigma_{\mathbf{x}_3\mathbf{x}_1} & \sigma_{\mathbf{x}_3\mathbf{x}_2} & \sigma_{\mathbf{x}_3}^2 \end{bmatrix},$$

where

$$\sigma_{\mathbf{x}_i}^2 = \text{E}[\mathbf{x}_i^2(n : n + N - 1)]$$

and

$$\sigma_{\mathbf{x}_i\mathbf{x}_j} = \text{E}[\mathbf{x}_i\mathbf{x}_j],$$

where $i, j = 1, 2, 3$ and $i \neq j$. The covariance matrix obtained from spatially separated sensors preserve the path delay across sensors which is dependent on the source direction [11]. Likewise, $\hat{\mathbf{C}}$ obtained from orthogonally co-located sensors preserves the energy of the polarized signal which, with the aid of the polarization model, is used to estimate the direction of maximum signal energy arriving at the sensor.

Principal component analysis (PCA) is a widely used technique which employs orthogonal transformation to obtain uncorrelated principal components. Eigenvalue decomposition of $\hat{\mathbf{C}}$ yields

$$\hat{\mathbf{C}} = \hat{\mathbf{U}}\hat{\mathbf{V}}\hat{\mathbf{U}}^T, \quad (2.26)$$

where $\hat{\mathbf{V}} = \text{diag}\{\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3\}$ is the diagonal matrix containing the eigenvalues and

$$\hat{\mathbf{U}} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = [\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3] \quad (2.27)$$

is the unitary matrix consisting of the eigenvectors of $\hat{\mathbf{C}}$. The orthogonal eigenvectors

represent the principal directions of the signal $\mathbf{x}(n)$, while the eigenvalues represents the magnitude of the corresponding eigenvector. From eigen-decomposition, the direction of maximum energy or equivalently the bearing angle can be estimated as the azimuth angle

$$\hat{\Theta}_{\text{CMA}} = \tan^{-1} \left(\frac{e_{21}}{e_{11}} \right). \quad (2.28)$$

The corresponding elevation angle is estimated as

$$\hat{\Phi}_{\text{CMA}} = \tan^{-1} \left(\frac{e_{31}}{\sqrt{e_{11}^2 + e_{21}^2}} \right) \quad (2.29)$$

where e_{11} , e_{21} and e_{31} are the components of the principal eigenvector $\mathbf{e}_1$ corresponding to the largest eigenvalue $\hat{\lambda}_1$.

In applications such as oil exploration or the study of earthquake activities, the seismic waves arrive from different layers of the Earth's crust. Localization of such events include both the azimuth angle as well as the elevation angle from the surface. However, for surface seismic sources such as human footsteps and vehicles a 2D covariance matrix estimated using data of only the two horizontal axes $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are sufficient to estimate the bearing angle using (2.28).

Vector-Sensor MUSIC

Multiple Signal Classification (MUSIC) is an algorithm for estimating the direction-of-arrival of electromagnetic sources using an uniform linear array. Unlike the frequency dependent steering vector of a linear array, for a vector-sensor array, the unit vector pointing towards the source is approximated as [64]

$$\mathbf{u}(\Theta, \Phi) = \begin{bmatrix} \cos \Theta \cos \Phi \\ \sin \Theta \cos \Phi \\ \sin \Theta \end{bmatrix}, \quad (2.30)$$

where $\Theta \in (-\pi, \pi]$ is the azimuth angle and $\Phi \in [-\pi/2, \pi/2]$ is the elevation angle. For a seismic source triggered on the surface of the ground, the elevation angle $\Phi = 0$. Therefore the array manifold reduces to

$$\mathbf{u}(\Theta) = \begin{bmatrix} \cos \Theta \\ \sin \Theta \end{bmatrix}.$$

This can be used in vector-sensor MUSIC [66] to compute the azimuth angle of the source. The estimator of azimuth angle is obtained as the inverse of the projection to the noise subspace given by

$$\hat{\Theta}_{\text{VS-MUSIC}} = \frac{1}{\| \Pi_{\text{Noise}} \mathbf{u}(\Theta, \Phi) \|}, \quad (2.31)$$

where Π_{Noise} is the noise subspace. The noise subspace is estimated as the second and third column of the eigenvector matrix $\hat{\mathbf{U}}$ corresponding to noise eigenvalues $\hat{\lambda}_2$ and $\hat{\lambda}_3$ as

$$\Pi_{\text{Noise}} = \begin{bmatrix} e_{12} & e_{13} \\ e_{22} & e_{23} \\ e_{32} & e_{33} \end{bmatrix} = [\mathbf{e}_2 \ \mathbf{e}_3]. \quad (2.32)$$

Rotational Analysis

A bearing estimation algorithm using rotational analysis (RA) has been proposed in [67] for finding the direction of a glacier crack in ice-mounted regions using a tri-axial geophone. For a seismo-acoustic wave incident at an azimuth angle Θ and elevation angle Φ , the horizontal radial motion at any instant of time is defined as

$$\mathbf{r}(n) = \mathbf{x}_1(n) \cos(\Theta) + \mathbf{x}_2(n) \sin(\Theta), \quad (2.33)$$

where $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are the particle velocity recorded by the x and y axes of the tri-axial geophone. For waves with particle motion confined in the radial-vertical direction,

the azimuth angle Θ can be estimated from the horizontal components by maximizing the rotational power estimated as

$$\begin{aligned}\chi(\Theta) &= \sum_{n=1}^N \mathbf{r}^2(n) \\ &= \cos^2 \Theta \sum_{n=1}^N \mathbf{x}_1^2(n) + \sin^2 \Theta \sum_{n=1}^N \mathbf{x}_2^2(n) + 2 \cos \Theta \sin \Theta \sum_{n=1}^N \mathbf{x}_1(n) \mathbf{x}_2(n).\end{aligned}\tag{2.34}$$

The azimuth angle is estimated as

$$\hat{\Theta}_{\text{RA}} = \frac{1}{2} \tan^{-1} \left[\frac{2 \sum_{n=1}^N \mathbf{x}_1(n) \mathbf{x}_2(n)}{\sum_{n=1}^N \mathbf{x}_1^2(n) + \sum_{n=1}^N \mathbf{x}_2^2(n)} \right].\tag{2.35}$$

However, when significant non-radial components exists in the propagating waves, polarization filters are defined using the vertical channel signal $\mathbf{x}_3(n)$ to achieve better bearing estimate. Seismic source bearings are estimated using rotational analysis on two sets of filtered signals. The rotational functions for the two filtered signals can be computed as follows:

$$\chi_{x_3}(\Theta) = \sum_{n=1}^N \mathbf{r}_z^2(n),\tag{2.36}$$

$$\chi_{x'_3}(\Theta) = \sum_{n=1}^N \mathbf{r}_{z'}^2(n),\tag{2.37}$$

$$\begin{aligned}\chi_z(\Theta) &= \cos^2 \Theta \sum_{n=1}^N (\mathbf{x}_1(n) \mathbf{x}_3(n))^2 + \sin^2 \Theta \sum_{n=1}^N (\mathbf{x}_2(n) \mathbf{x}_3(n))^2 \\ &\quad + 2 \cos \Theta \sin \Theta \sum_{n=1}^N (\mathbf{x}_1(n) \mathbf{x}_3(n)) (\mathbf{x}_2(n) \mathbf{x}_3(n)),\end{aligned}\tag{2.38}$$

$$\begin{aligned} \chi_{z'}(\Theta) = & \cos^2 \Theta \sum_{n=1}^N (\mathbf{x}_1(n) \mathbf{x}_3'(n))^2 + \sin^2 \Theta \sum_{n=1}^N (\mathbf{x}_2(n) \mathbf{x}_3'(n))^2 \\ & + 2 \cos \Theta \sin \Theta \sum_{n=1}^N (\mathbf{x}_1(n) \mathbf{x}_3'(n)) (\mathbf{x}_2(n) \mathbf{x}_3'(n)), \end{aligned} \quad (2.39)$$

where $\mathbf{x}_3'(n) = \frac{d}{dn}(\mathbf{x}_3(n))$. The two rotational functions can be combined as

$$\chi_{zz'}(\Theta) = \frac{\chi_z(\Theta)}{\min(\chi_z(\Theta))} + \frac{\chi_{z'}(\Theta)}{\min(\chi_{z'}(\Theta))}. \quad (2.40)$$

The rotational functions exhibits a 180° ambiguity that is inherent in the definition of $\chi(\Theta)$ defined in terms of the horizontal particle motion. This ambiguity is resolved by considering propagation in the radial-vertical plane, i.e., by determining if the data is dominated by prograde or retrograde particle motion. This is achieved by evaluating

$$\varsigma = \sum_{n=2}^{n=N-1} \mathbf{x}_3(n) (\mathbf{r}(n+1) - \mathbf{r}(n-1)). \quad (2.41)$$

If $\varsigma > 0$, the impinging wave is considered to have dominant prograde motion, otherwise if $\varsigma < 0$, the wavefront is considered retrograde. The angle corresponding to prograde motion is chosen as the azimuth angle $\hat{\Theta}_{\text{RA}}$.

Hilbert Transform Analysis

In [14], the particle velocity components in the x , y and z directions for a Rayleigh wave is expressed as

$$\begin{aligned} \tilde{\mathbf{x}}_1(t) &= a \sin \Theta e^{ik_x x + ik_y y - i\omega t}, \\ \tilde{\mathbf{x}}_2(t) &= a \cos \Theta e^{ik_x x + ik_y y - i\omega t}, \\ \tilde{\mathbf{x}}_3(t) &= A e^{ik_x x + ik_y y - i\omega t + i\pi/2}, \end{aligned} \quad (2.42)$$

where a , A are the amplitude of the radial in-phase horizontal and vertical component, respectively. The variables k_x , k_y are the wavenumber in the x and y direction, respectively and ω is the angular frequency. The phase of the vertical component is 90° ahead of the horizontal components. Hence, Hilbert transform $\mathcal{H}(\cdot)$ is applied to the acquired vertical component $\mathbf{x}_3(n)$ as follows

$$\mathbf{x}_{3h}(n) = \mathcal{H}(\mathbf{x}_3(n)), \quad (2.43)$$

to ensure the vertical component is in-phase with the horizontal components. The phase-corrected $\mathbf{x}_{3h}(n)$ is cross-correlated with the in-phase horizontal components. The bearing angle is then estimated as

$$\hat{\Theta}_{\text{HTA}} = \tan^{-1} \left(\frac{\text{E}\{\mathbf{x}_{3h}(n)\mathbf{x}_1^T(n)\}}{\text{E}\{\mathbf{x}_{3h}(n)\mathbf{x}_2^T(n)\}} \right). \quad (2.44)$$

Surface-Wave Analysis

Similar to Hilbert transform based bearing estimation, Stafsudd et al., proposed a bearing estimation algorithm based on the analysis of surface wave propagation [50]. As shown in Fig. 2.2(c), Rayleigh waves exhibit an elliptical motion in the vertical-radial plane while Love waves exhibit a rectilinear motion in a direction perpendicular to the direction of propagation. The surface wave analysis algorithm computes two correlation coefficients

$$\begin{aligned} \sigma_{zx} &= \mathbf{x}_3(n) \frac{d}{dn} (\mathbf{x}_1(n))^T, \\ \sigma_{zy} &= \mathbf{x}_3(n) \frac{d}{dn} (\mathbf{x}_2(n))^T, \end{aligned} \quad (2.45)$$

where $\frac{d}{dn}(\mathbf{x}_1(n))$, $\frac{d}{dn}(\mathbf{x}_2(n))$ refer to the acceleration of $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ in the x and y

direction, respectively. The azimuth is then estimated as

$$\hat{\Theta}_{\text{SWA}} = \tan^{-1} \frac{\sigma_{zx}}{\sigma_{zy}}. \quad (2.46)$$

The underlying assumption is that the tri-axial geophone is planted exactly perpendicular to the ground ($z=0$) such that there exists no leakage between the vertical channel and the ground plane when the Rayleigh wave is dominant.

2.8 Signal Model Used in This Thesis

An impact of a surface source, such as human footstep or vehicular movement, produces seismic energy that propagates via multiple polarization modes as explained in Section 2.1. The tri-axial geophone (GS32CT) captures the Earth particle velocity along the x , y and z directions. The analog particle velocity, captured by the geophone, is sampled using the Presonus Firestudio ADC, described in Section 2.2, at 44.1kHz. The velocity of surface waves is estimated using seismic signals, generated by hammer strikes, and acquired by two geophones spatially separated by 10 m. Using time-difference-of-arrival algorithm [11], the velocity of surface wave is approximated to be 180-250 m/s.

As shown in Section 2.5.1.4, surface sources generate elliptically polarized waves that cause the acquired tri-axial signal to trace an elliptical path in a 3D space. The thesis aims to detect either human footsteps or vehicular movement by identifying the time segments that contain elliptical polarization. This is achieved by developing models that parameterize the evolution of the 3D trajectory. Algorithms developed in this thesis, therefore, exploit the evolution of the 3D trajectory to identify elliptically polarized signals. Since the algorithms are trajectory-based, the recorded signal is resampled at a relatively high sampling rate of 8 kHz in order to examine the instantaneous movement of the trajectory. The resampled signal, described as $\mathbf{x}(n)$, is elliptically polarized in

the time segments containing a seismic event, due to the presence of Rayleigh and Love waves, and unpolarized in the absence of a seismic event, due to the presence of high frequency components in the band 0 – 4 kHz.

Using the parameterizing models, dissimilarity in the trajectory of the acquired signal between the presence and absence of a seismic source is analyzed. As opposed to the conventional additive noise model [64], the acquired signal $\mathbf{x}(n)$, is modeled independently in the presence and absence of a seismic event as follows:

$$\mathbf{x}(n) \sim \begin{cases} \mathbf{x}_{\text{seismic-event}}(n) & - \text{Elliptically polarized signal,} \\ \mathbf{x}_{\text{noise}}(n) & - \text{Randomly polarized noise.} \end{cases} \quad (2.47)$$

Using this signal model, the following specific terminologies will be used throughout this thesis. The term ‘signal’ will refer to $\mathbf{x}(n)$, i.e, the real-valued signal recorded by the tri-axial geophone. The ‘signal’ may or may not contain a seismic event, which denotes either a human footstep or movement of a vehicle. The term ‘footstep/vehicular signal’ refers to the segment of the tri-axial signal containing the corresponding seismic event, viz., footstep or vehicle. On the other hand, the term ‘noise’ refers to background noise-only segment of the acquired signal, i.e., absence of a seismic event.

Since this thesis is aimed towards increasing the detection range for surface seismic sources, we next analyze the rate at which the signal-to-noise ratio (SNR) varies with increasing source-sensor distances. Radial walking footsteps, recorded when a person moves away from the sensor from a distance of 3 m to 30 m, is used. The SNR is estimated as

$$\begin{aligned} \text{SNR} &= 10 \log_{10} \frac{\text{Signal Power}}{\text{Noise Power}} \\ &= 10 \log_{10} \frac{\widehat{E}_c(\mathbf{x}_{\text{footstep}}(n))}{\widehat{E}_c(\mathbf{x}_{\text{noise}}(n))} \end{aligned} \quad (2.48)$$

Figure 2.15: Plot of SNR vs source-sensor distance.

where the signal and noise power is computed using the cumulated energy metric defined in (2.8) within a time window containing a footstep signal, denoted as $\mathbf{x}_{\text{footstep}}(n)$, and background noise, denoted as $\mathbf{x}_{\text{noise}}(n)$, respectively. The noise power is estimated using a window of length equal to the footstep window but approximately 0.2 s after the footstep signal. Figure 2.15 plots the SNR evaluated using (2.48) against source-sensor distance, averaged over ten walking experiments. It can be observed that as a human walks away from the sensor, the SNR of the footstep signal decreases because the power (amplitude) of the footstep signal impinging the sensor reduces with increased propagation distance through the Earth medium. Other seismic sources in the environment such as the thumping of the bulldozer at a far-field construction site or heavy trucks passing on the freeway near-by affect the background noise power. It can be observed from the plot that beyond 23 m, the SNR does not decrease significantly. This is due to the fact that the signal power is comparable to the background noise power and causes the energy-based detection metrics, discussed in Section 2.5.1, to fail at farther source-sensor

distances.

2.9 Conclusion

The review of detection metrics showed that majority of the existing techniques are defined for single-channel geophone. Even the multi-channel metrics quantify the magnitude of the vector-valued signal acquired by the tri-axial geophone. Hence, there is a need to explore the three-dimensional signal trajectory and define metrics that better characterize elliptical polarization. Although trajectory-based metrics have not been fully exploited for seismic signals, they were investigated in Section 2.5.1.4. The chapter concluded with a signal model, where the presence of a seismic source is defined with respect to polarization of the acquired tri-axial signal. In the subsequent chapters, 3D trajectories in the presence and absence of human activity will be analyzed to develop parameterization models that enable the detection of elliptically polarized signals.

Chapter 3

Exploiting Smoothness of 3D Trajectory For Seismic Source Detection

In this chapter, the direction attributes of the tri-axial signal, in the presence and absence of a seismic surface source, are examined in detail. Towards this purpose, the acquired signal is normalized to obtain the normalized trajectory. It will be demonstrated that in the presence of a seismic source, the normalized trajectory traces a smooth elliptical path compared to a signal segment containing only background noise. Based on this key observation, smoothness of the trajectory on the unit sphere is quantified using the uniformity of particle motion in order to identify the presence of elliptical polarization.

This chapter is divided into two segments. In the first segment, trajectories of footstep signals, characterized by transient behaviour, are examined. Neighbourhood and model-based metrics are proposed to quantify smoothness of the 3D trajectory. Extending on the trajectory-based footstep detection metrics, in the second segment, 3D trajectory of vehicular signals are analyzed and detection metrics quantifying long-term smoothness are proposed.

Figure 3.1: 3D plots of (a) $\mathbf{x}(n)$ and $\bar{\mathbf{x}}(n)$ for (b) a footstep, (c) absence of a footstep.

3.1 Detection of Polarized Footstep Signals

The voltage recorded in the three orthogonal channels of the tri-axial geophone is proportional to the particle velocity that is generated by the impinging wavefront in the x , y and z directions. When no seismic events occur, the induced voltage in the three channels of the geophone are uncorrelated. The polarization characteristics of seismic waves generated by near-field human activities is complex and time varying. The augmented effect of multiple polarization modes results in the acquired tri-axial signal tracing a wobbly elliptical orbit in three-dimension [68].

As an illustrative example, the time evolution of the signal $\mathbf{x}(n)$ recorded by a tri-axial geophone in the presence of a footstep is plotted in Fig. 3.1(a). The data acquired by the tri-axial geophone is resampled at a relatively high sampling rate of 8 kHz in order to better examine its trajectory. It is observed that $\mathbf{x}(n)$ exhibits a constantly varying elliptical trajectory. In order to distinguish the difference in the velocity trajectory in the presence of a footstep, $\mathbf{x}(n)$ is normalized at each time instant giving a unit-norm

Figure 3.2: 3D plot of simulated (normalized) (a) elliptically polarized signal (b) unpolarized Gaussian noise (after [70]).

vector

$$\begin{aligned}\bar{\mathbf{x}}(n) &= \frac{\mathbf{x}(n)}{\|\mathbf{x}(n)\|} \\ &= [\bar{x}_1(n) \quad \bar{x}_2(n) \quad \bar{x}_3(n)]^T,\end{aligned}\tag{3.1}$$

where $\|\cdot\|$ is the L_2 -norm. The normalization ensures the velocity vector is independent of power and enables better signal conditioning and visualization, since the trajectory now lies on the surface of a unit sphere. Figures 3.1(b) and (c) illustrate the normalized trajectory $\bar{\mathbf{x}}(n)$ in the presence and absence (noise-only) of a footstep signal, respectively [69]. It can be seen that $\bar{\mathbf{x}}(n)$ traces a smooth elliptical trajectory in the presence of a footstep and is randomly distributed in its absence.

A similar phenomenon has been shown on simulated data in [70]. A linearly polarized signal produces two distinct points on a unit sphere. On the other hand, an elliptical orbit is inscribed on the circumference of the sphere for elliptically polarized signal as shown in Fig. 3.2(a) and the data is randomly distributed on the unit sphere for unpolarized random noise as shown in Fig. 3.2(b). It is therefore straightforward to infer that footstep signals generate elliptically polarized seismic waves.

3.1.1 Proposed Neighbourhood Metrics

Any variations in the smoothness of the normalized trajectory between noise and footstep signal segment can be quantified by determining the instantaneous movement of the 3D trajectory. In this regard, two instantaneous metrics are proposed. The neighbourhood Euclidean distance (NED) defined as [71]

$$\text{NED}_{\bar{\mathbf{x}}}(n) = \|\bar{\mathbf{x}}(n+1) - \bar{\mathbf{x}}(n)\|_2 \quad (3.2)$$

quantifies the instantaneous Euclidean distance between two consecutive snapshots of the normalized trajectory $\bar{\mathbf{x}}(n)$. In the presence of a polarized footstep signal, $\text{NED}_{\bar{\mathbf{x}}}(n)$ is a low value due to the smooth trajectory traced. On the other hand, the absence of a footstep results in a random trajectory yielding a higher value of $\text{NED}_{\bar{\mathbf{x}}}(n)$. Therefore, the variance of $\text{NED}_{\bar{\mathbf{x}}}(n)$ estimated across overlapping time windows can be used to detect the presence of a footstep.

Figure 3.3(b) shows how $\text{NED}_{\bar{\mathbf{x}}}(n)$ varies with time for an impulsive signal corresponding to a hammer strike plotted in Figure 3.3(a). The hammer is used to simulate an impact similar to a footstep. As expected, $\text{NED}_{\bar{\mathbf{x}}}(n)$ is high in the noise-only time segments, whereas for time segments where the source is active, $\text{NED}_{\bar{\mathbf{x}}}(n)$ varies less significantly. The variance σ_{NED}^2 , evaluated using overlapped frames of $\text{NED}_{\bar{\mathbf{x}}}(n)$ is plotted in Fig. 3.3(c). This plot shows that the variance does not show significant distinction between the presence of a footstep signal and absence. Since the maximum Euclidean distance between any two points on the unit sphere is two, the NED metric is constrained within the range $[0, 2]$. This limits the capability of $\text{NED}_{\bar{\mathbf{x}}}(n)$ to detect the presence of a footstep.

In order to address the above limitation, a neighbourhood ratio (NR) metric [71]

Figure 3.3: Plot of (a) $\frac{x_3(n)}{\max[x_3(n)]}$, (b) $\text{NED}_{\bar{x}}(n)$ and (c) σ_{NED}^2 evaluated with $N = 50$ ms and $l_0 = 42.5$ ms.

defined as

$$\text{NR}_i(n) = \begin{cases} \bar{x}_i(n+1)/\bar{x}_i(n) & \bar{x}_i(n) > \delta \\ 1 & \text{otherwise,} \end{cases} \quad (3.3)$$

has been proposed. The variable $\bar{x}_i(n)$ is the normalized data from i th axis and δ is a small value to avoid dividing by a small number. It is important to note that three different neighbourhood ratios corresponding to the three channels are obtained. When a polarized footstep signal is present, $\text{NR}_i(n)$ will be close to unity while for noise-only segments this ratio varies randomly. The variance of the NR metric for each of the three channel are shown in Fig. 3.4(a-c) along with the normalized acquired signal. From Fig. 3.4, it is observed that $\sigma_{\text{NR}_i}^2(l)$ reduces close to zero when the footstep signal is present and increases to a high value in the noise-only segments. This affirms that the NR metric provides a better detection of the footstep signal than the NED metric. The NR metric is also used to denoise the signal by truncating the acquired signal $\mathbf{x}(n)$ using

Figure 3.4: Plot of (a) $\frac{x_1(n)}{\max[x_1(n)]}$ and σ_{NR1}^2 , (b) $\frac{x_2(n)}{\max[x_2(n)]}$ and σ_{NR2}^2 , (c) $\frac{x_3(n)}{\max[x_3(n)]}$ and σ_{NR3}^2 .

segments of low variance of the NR metric [71].

3.2 Proposed Model-based Footstep Detection

The technique of nonlinear regression [72] is often employed to model a smooth curve. In order to regress the 3D normalized data as a function of time, a polynomial of certain degree is required for each of the three channels. Such a regression is based on the mean-squared error between the polynomial and normalized trajectory. However, the normalization process leads to an arbitrary scaling of the signal at each time instant, thereby making the polynomial-fit error non-stationary. This violates the Gauss-Markov conditions [72], and therefore a detection criteria based on statistical detection theory cannot be applied directly. Moreover the performance of nonlinear regression is susceptible to the order of the polynomial chosen.

Figure 3.5: The proposed model-based detection framework using 3D trajectory.

Due to limitations of non-linear regression, consecutive snapshots of the normalized signal $\bar{\mathbf{x}}(n)$ are fitted to geometric sequence and great arc models. The model-fit error is then used to quantify the presence of elliptical polarization. In the presence of a footstep, the model-fit error is low since the model is able to characterize the continuum pattern formed by the particle motion in a three-dimensional space. In the absence of footsteps, however, the model-fit error is expected to be high due to the randomness of the particle motion caused by background noise. A threshold is then applied on either the model-fit error or framewise variance of the model-fit error to detect human footsteps. Figure 3.5 summarizes the proposed detection framework.

3.2.1 Geometric Sequence Model and Instantaneous Euclidean Error

The geometric sequence (GS) is proposed as a model to fit the tri-axial trajectory. Consider a frame of three consecutive snapshots

$$\mathbf{X} = [\mathbf{x}(n) \ \mathbf{x}(n+1) \ \mathbf{x}(n+2)], \quad (3.4)$$

where $\mathbf{x}(n)$ is the tri-axial signal at time instant n . The corresponding normalized data frame is given by a 3×3 matrix

$$\begin{aligned}\bar{\mathbf{X}} &= [\bar{\mathbf{x}}(n) \ \bar{\mathbf{x}}(n+1) \ \bar{\mathbf{x}}(n+2)] \\ &= \begin{bmatrix} \bar{\chi}_1^T(n:n+2) \\ \bar{\chi}_2^T(n:n+2) \\ \bar{\chi}_3^T(n:n+2), \end{bmatrix}\end{aligned}\quad (3.5)$$

where

$$\begin{aligned}\bar{\chi}_i(n:n+2) &= \left[\frac{x_i(n)}{\|\mathbf{x}(n)\|_2} \ \frac{x_i(n+1)}{\|\mathbf{x}(n+1)\|_2} \ \frac{x_i(n+2)}{\|\mathbf{x}(n+2)\|_2} \right]^T \\ &= [\bar{x}_i(n) \ \bar{x}_i(n+1) \ \bar{x}_i(n+2)]^T.\end{aligned}\quad (3.6)$$

Since the normalized signal traces a smooth curve in the presence of a polarized signal, the inter-element ratio across consecutive columns of $\bar{\mathbf{X}}$ is expected to exhibit a small variation. When consecutive samples follow a geometric sequence,

$$\mathbf{u}_i(n:n+1) = \begin{bmatrix} \frac{\bar{x}_i(n+1)}{\bar{x}_i(n)} \ \frac{\bar{x}_i(n+2)}{\bar{x}_i(n+1)} \end{bmatrix} \quad (3.7)$$

$$\begin{aligned}&= [u_i(n) \ u_i(n+1)]^T \\ &= r \mathbf{1}_{2 \times 1},\end{aligned}\quad (3.8)$$

where r is the geometric ratio while $\mathbf{1}_{2 \times 1} = [1 \ 1]^T$. This gives rise to

$$\begin{aligned}u_i(n) &= u_i(n+1) = r \\ \Rightarrow \frac{\bar{x}_i(n+1)}{\bar{x}_i(n)} &= \frac{\bar{x}_i(n+2)}{\bar{x}_i(n+1)} = r,\end{aligned}\quad (3.9)$$

which implies that the trajectory represented by $\bar{\mathbf{X}}$ is smooth and indicates the presence of a footstep. However, in the absence of a polarized wave, it is expected that the

Figure 3.6: Plot of (i) z -axis signal along with (a) GS model - (ii) error $e_{\text{gs}}(n)$ and (iii) variance $\sigma_{\text{gs}}^2(l)$ and (b) GA model - (ii) error $e_{\text{ga}}(n)$ and (iii) variance $\sigma_{\text{ga}}^2(l)$.

normalized signal will deviate from the smooth trajectory and hence the large deviation can be quantified by the difference between the two elements of $\mathbf{u}_i(n : n + 1)$, i.e.,

$$\begin{aligned} \epsilon_i(n) &= u_i(n) - u_i(n + 1) \\ &= \frac{\bar{x}_i(n + 1)}{\bar{x}_i(n)} - \frac{\bar{x}_i(n + 2)}{\bar{x}_i(n + 1)}. \end{aligned} \quad (3.10)$$

The deviation $\epsilon_i(n)$ from the three channels are squared and accumulated to obtain the unified error signal

$$e_{\text{gs}}(n) = \sum_{i=1}^3 \epsilon_i^2(n), \quad (3.11)$$

where the subscript ‘gs’ denotes for the geometric sequence.

It is useful to establish the link between the proposed instantaneous geometric sequence with that of NR and NED metrics defined in Section 3.1.1. This is achieved by

defining a concatenated ratio vector across the three channels as

$$\underline{\mathbf{u}}(n) = [u_1(n) \ u_2(n) \ u_3(n)]^T. \quad (3.12)$$

The cumulative error across the three channels can be re-written as

$$e_{\text{gs}}(n) = \|\underline{\mathbf{u}}(n) - \underline{\mathbf{u}}(n+1)\|_2. \quad (3.13)$$

This cumulative error is equivalent to evaluating NR followed by the NED metrics. When the normalized signal $\bar{\mathbf{x}}(n)$ is subjected to NR processing, $\underline{\mathbf{u}}(n)$ is obtained, from which the geometric sequence error $e_{\text{gs}}(n)$ is computed by estimating NED on $\underline{\mathbf{u}}(n)$.

The performance of the geometric sequence based model is illustrated by plotting $e_{\text{gs}}(n)$ in Fig. 3.6(a)(ii) for a train of footsteps, recorded as shown in Fig. 3.6(i). It is observed that $e_{\text{gs}}(n)$ reduces in the presence of a footstep while its value remains consistently high in the noise-only segments, thus quantifying the deviation of $\bar{\mathbf{x}}(n)$ from a smooth trajectory. It can also be observed that fluctuations in $e_{\text{gs}}(n)$ also decreases significantly during periods in which a polarized footstep signal exists. Such fluctuations can be quantified using the variance computed as

$$\sigma_{\text{gs}}^2(l) = \frac{1}{N} \mathbf{e}_{\text{gs}}^T(l) \mathbf{e}_{\text{gs}}(l), \quad (3.14)$$

where $\mathbf{e}_{\text{gs}}(l) = [e_{\text{gs}}((l-1)l_0+1), \dots, e_{\text{gs}}((l-1)l_0+N)]^T$, while l , N and l_0 are the frame index, analysis window size and window shift length, respectively. Figure 3.6(a)(iii) shows the variation of $\sigma_{\text{gs}}^2(l)$ with time for the footstep signal in Fig. 3.6(a)(i). We note that $\sigma_{\text{gs}}^2(l)$ reduces in the presence of a footstep signal while its value is significantly high in the noise-only segments.

It is also important to note that the statistical characteristics of $\sigma_{\text{gs}}^2(l)$ are distorted by the normalization process. Therefore, identifying a detection threshold is not trivial. A

sub-optimal approach is to estimate the long-term average of $\sigma_{\text{gs}}^2(l)$ as the noise variance $\sigma_{\text{n}}^2(b)$ and the proposed detection threshold is defined as

$$\tau_{\text{gs}}(b) = \gamma_{\text{gs}} \sigma_{\text{n}}^2(b), \quad (3.15)$$

where $0 < \gamma_{\text{gs}} < 1$ is the scaling factor that regulates the trade-off between the probability of detection and false alarm while $\sigma_{\text{n}}^2(b)$ tracks changes in the background noise thereby making the threshold dependent of the noise statistics.

3.2.2 Great Arc Model and Instantaneous Angular Error

Although the proposed GS-based error metric detects polarized footsteps, it does not capitalize on the fact that the normalized trajectory lies on the unit sphere. Therefore, the great arc (GA) on the surface of a unit sphere is proposed as a model to quantify the deviation error for detection of polarized footstep signals.

A great circle is defined as a circle that circumscribes a sphere such that its center coincides with the center of the sphere and its radius is equivalent to that of the sphere [73]. An arc on a great circle is known as the great arc. For a segment within a footstep signal, the normalized signal is assumed to trace a great arc while for a signal segment in the absence of a footstep, the normalized data of background noise is randomly polarized and does not follow a well-defined pattern. Hence, the great arc is proposed to fit $\bar{\mathbf{x}}(n)$, so that deviation of the trajectory from the great arc, measured on the surface of a sphere, is expected to be low during the presence of a footstep segment compared to its absence.

For any two arbitrarily defined points $\mathbf{p}_0$ and $\mathbf{p}_2$ lying on the surface of a unit sphere, a point $\hat{\mathbf{p}}_1$ can be interpolated on the surface of the sphere using Spherical Linear intERpolation (SLERP) [74] defined by

$$\hat{\mathbf{p}}_1 = \frac{\sin(1 - \alpha)\Omega}{\sin \Omega} \mathbf{p}_0 + \frac{\sin \alpha\Omega}{\sin \Omega} \mathbf{p}_2, \quad (3.16)$$

where $\Omega = \cos^{-1}(\mathbf{p}_0^T \mathbf{p}_2)$ is the angle between $\mathbf{p}_0$ and $\mathbf{p}_2$ inscribed at the center of the sphere and $\alpha \in [0, 1]$ is the interpolation parameter. The points $\mathbf{p}_0, \hat{\mathbf{p}}_1, \mathbf{p}_2$ form a great arc on the unit sphere. In the context of footstep detection, for the normalized data frame $\bar{\mathbf{X}}$ defined in (3.5), a point

$$\hat{\bar{\mathbf{x}}}(n+1) = \frac{\sin 0.5\Omega}{\sin \Omega} \bar{\mathbf{x}}(n) + \frac{\sin 0.5\Omega}{\sin \Omega} \bar{\mathbf{x}}(n+2) \quad (3.17)$$

is interpolated between $\bar{\mathbf{x}}(n)$ and $\bar{\mathbf{x}}(n+2)$ on the arc of the great circle using SLERP with $\alpha = 0.5$ and $\Omega = \cos^{-1}(\bar{\mathbf{x}}^T(n) \bar{\mathbf{x}}(n+2))$. If the acquired signal follows a smooth trajectory due to the presence of a footstep, the interpolated signal $\hat{\bar{\mathbf{x}}}(n+1)$ is expected to approximate the normalized signal snapshot $\bar{\mathbf{x}}(n+1)$. On the other hand, in the absence of a footstep, the separation distance between the points $\hat{\bar{\mathbf{x}}}(n+1)$ and $\bar{\mathbf{x}}(n+1)$ will be significant. The proximity between these two points on a sphere can be quantified by the length of the great arc between the two points. The arc length on a sphere is given by $s = r\theta$, where r is the radius while θ is the angle (in radians) inscribed at the center of the sphere. With $r = 1$ in our application, the arc length is therefore equivalent to the angle between $\hat{\bar{\mathbf{x}}}(n+1)$ and $\bar{\mathbf{x}}(n+1)$ inscribed at the center. In contrast to the Euclidean error for GS based model, the proposed GA interpolation error is computed as

$$e_{\text{ga}}(n) = \cos^{-1}(\hat{\bar{\mathbf{x}}}^T(n+1) \bar{\mathbf{x}}(n+1)). \quad (3.18)$$

The variation of $e_{\text{ga}}(n)$ corresponding to a footstep signal, characterized by elliptical polarization is expected to be low while for randomly polarized noise segments it is expected to be significantly high.

For illustration, the same footstep train shown in Fig. 3.6(i) is subjected to SLERP interpolation described in (3.17) followed by the evaluation of GA interpolation error using (3.18). Figure 3.6(b)(ii) shows the variation of $e_{\text{ga}}(n)$ with time, evaluated for every

Figure 3.8: Plot of (a) z -axis signal, (b) $e_{ga}(n)$, (c) $\hat{E}_c(n)$ and (d) $e_{PE}(n)$ for stationary footsteps at a distance of 5 m from the sensor.

defined as

$$\tau_{ga}(b) = \gamma_{ga} \sigma_n^2(b), \quad (3.20)$$

where $0 < \gamma_{ga} < 1$ is the scaling factor, chosen to regulate between the probability of detection and false alarm.

3.2.3 Proposed Polarization-cum-Energy Metric

A polarization-cum-energy (PE) metric is proposed by combining the polarization and transient nature of human footsteps. The motivation for employing the proposed polarization-cum-energy metric is derived from the presence of correlated noise which may arise from

footsteps generated by stationary marching of a human subject relatively close to the geophone. Such footsteps result in standing waves which induce signals that are polarized. As a consequence, $e_{\text{ga}}(l)$ is reduced even in noise-only segments between two actual footsteps, giving false detection. Although the magnitude of a standing wave is insignificant (due to their polarized nature), this results in false alarms in the GA interpolation error metric. In order to address this problem, the cumulated energy estimated as

$$\widehat{E}_c = \sum_{n=0}^{N-1} [x_1^2(n) + x_2^2(n) + x_3^2(n)] \quad (3.21)$$

is combined with the GA polarization metric to define the polarization-cum-energy (PE) metric as

$$e_{\text{PE}}(l) = \frac{e_{\text{ga}}(l)}{\widehat{E}_c(l)}, \quad (3.22)$$

where $\widehat{E}_c(l)$ is computed across overlapping frames of length 0.2 s and $e_{\text{ga}}(l)$ is the GA interpolation error defined in (3.18). Using the proposed PE metric defined in (3.22), a polarized signal with high energy will result in a low value of $e_{\text{PE}}(l)$. This is due to the high value of $\widehat{E}_c(l)$ in the denominator of (3.22) which, in turn, results in the detection of a footstep. Any correlated polarized interference signal with a low value of $\widehat{E}_c(l)$ will induce a high value of $e_{\text{PE}}(l)$ resulting in reduced false alarms.

The detection performance of the PE metric for stationary footstep train at a distance of 5 m from the sensor is shown in Fig. 3.8. Subplot (a) shows the signal acquired by the z -axis of the geophone while subplots (b), (c) and (d) illustrate the variation of $e_{\text{ga}}(l)$, $\widehat{E}_c(l)$ and $e_{\text{PE}}(l)$, respectively, with time. It can be seen that although $E_c(l)$ provides a good distinction between the presence and absence of footsteps, $\widehat{E}_c(l)$ is not expected to achieve similar performance compared to $e_{\text{ga}}(l)$, which utilizes the polarization of signal, for larger source-sensor distances. More importantly, comparing Figs. 3.8(b) and

3.6(b)(ii), we note that for source-sensor distance of 5 m shown in Fig. 3.8(b), the presence of correlated noise due to standing waves has resulted in less significant reduction of $e_{\text{ga}}(l)$ corresponding to segment containing the footstep signal. This effect can be seen in Fig. 3.8(b) at approximately 3.6 s where $e_{\text{ga}}(l)$ exhibits a low value despite the absence of a footstep signal. This, in turn, leads to an increase in false alarm rate for the GA-based error metric when stationary footsteps are generated. However the proposed PE metric $e_{\text{PE}}(l)$ shows a clear distinction between the footstep and noise-only segments due to the added weight of $\widehat{E}_c(l)$. Therefore the PE metric is more robust against to false alarms when the source is at a distance close to the geophone. In addition, the PE metric has shown improved detection performance at farther source-sensor distances.

Similar to the proposed GS and GA based metrics, the detection threshold based on the noise statistics can be estimated from the variance of the PE metric across a long time window. The detection threshold is chosen as

$$\tau_{\text{PE}}(b) = \gamma_{\text{PE}} \frac{\sigma_n^2(b)}{\widehat{E}_c(l)}, \quad (3.23)$$

where $\sigma_n^2(b)$ is the long-time average of the variance $\sigma_{\text{ga}}^2(l)$ computed as described in Section 3.2.2 and $\widehat{E}_c(l)$ is computed over a frame size of 0.2 s in order to encapsulate entire footstep. As before, the threshold scaling parameter $0 < \gamma_{\text{PE}} < 1$, regulates the trade-off between the probability of detection and false alarm.

3.2.4 Experiment Results and Discussions for Footstep Signal Detection

The detection performance of the proposed model-based instantaneous detection metrics are analyzed using footstep signals collected from outdoor experiments. The window length and time offset for evaluating the model-based error metrics are chosen to be 0.08 s and 0.032 s, respectively, in order to achieve multiple analysis windows within

single footstep impulse. The noise variance $\sigma_n^2(b)$ is computed using a longer window of 4 s in order to adapt the detection thresholds to the changing noise conditions. The proposed model-based detection metrics are compared with the cumulative kurtosis metric $\hat{K}_c$, and the largest eigenvalue metric $\hat{\lambda}_1$, discussed in Chapter 2. The window length and time-frame offset for evaluating the $\hat{\lambda}_1$ metric is 0.08 s and 0.032 s, respectively. On the other hand, the $\hat{K}_c$ metric is evaluated using a window length of 0.2 s since it is a statistical measure and requires the window to cover the entire footstep duration. The presence of footstep signals are detected when $\hat{K}_c > 10$. The detection threshold for the $\hat{\lambda}_1$ metric is computed using the entire footstep train.

We now evaluate the selection of detection threshold for the proposed metrics. Detection thresholds for the model-based instantaneous metrics are dependent on scaling factors γ_{gs} , γ_{ga} , and γ_{PE} as defined in (3.15), (3.20), (3.23), respectively. In order to gain insights into the effect of scaling factors on the performance of the metrics, the detection rate R_d and false alarm rate R_{fa} defined as

$$R_d = \frac{\text{no. of footsteps detected}}{\text{total no. of actual footsteps}}, \quad (3.24)$$

$$R_{fa} = \frac{\text{no. of false detections}}{\text{total no. of actual footsteps}} \quad (3.25)$$

are employed. The detection and false alarm rates of the newly proposed metrics were evaluated for scaling factors $\gamma_{gs} = \gamma_{ga} = \gamma_{PE} = 0.25$ and $\gamma_{gs} = \gamma_{ga} = 0.4$, $\gamma_{PE} = 0.5$. It was found that, for the scaling factor 0.25, the R_d of the proposed GS-based error metric decreases at farther source-sensor distances compared to that of the PE metric. On the other hand, the R_d of GS-based error metric decreases when the range is approximately 25 m but increases beyond that distance. The results also showed that the false alarm rate decreases with reducing scaling parameters and, as expected, a reduction in R_{fa} occurred at the cost of a modest reduction in the detection rate. Therefore, as with any detection

Figure 3.9: Plot of (i) z -axis signal, footstep detection decision using (ii) cumulative kurtosis $\hat{K}_c$, (iii) largest eigenvalue $\hat{\lambda}_1$, (iv) $e_{gs}(n)$, (v) $e_{gs}(n)$, and (vi) $e_{PE}(n)$ metrics, for source-sensor range (a) 3 - 10 m, (b) 20 - 30 m.

algorithm, the threshold offers a tradeoff between R_d and R_{fa} . More importantly, we note that both R_d and R_{fa} are modestly sensitive to the variation of the scaling factors for the proposed GA and PE algorithms. In view of this, $\gamma_{gs} = \gamma_{ga} = 0.4$ and $\gamma_{PE} = 0.5$ are chosen for the evaluation of the corresponding metrics.

We next evaluate the detection range of the proposed model-based smoothness metrics. In these experiments, the subject walks radially from a distance of 3 to 30 m away. For clarity of presentation, Figs. 3.9(a)(i) and (b)(i) illustrate the z -axis signal for two source-sensor distance range of 3 to 10 m and 20 to 30 m, respectively. The detection decision of the baseline metrics $\hat{K}_c$, $\hat{\lambda}_1$, and the newly proposed trajectory-based metrics using $e_{gs}(n)$, $e_{ga}(n)$ and $e_{PE}(n)$ and are shown in subplots (ii)-(vi), respectively. From these results, we observe that all the metrics are capable of detecting footsteps when the subject is within 10 m from the tri-axial geophone. However, as observed from Fig. 3.9(b) for distance greater than 18 m, the trajectory-based metric successfully detected most footsteps while the performance energy-based metrics $\hat{K}_c$, and $\hat{\lambda}_1$, reduce significantly. This phenomenon is observed because the signal energy as well as its peakiness reduce with distance, whereas the signal polarization is preserved by the propagating waves even at farther distances. Therefore, exploiting the polarization of the footstep signal results in an increased detection range compared to the energy-based metrics. The proposed PE metric is observed to detect all the footsteps compared to GS and GA-based detection since PE exploits both polarization and energy for detection. These results indicate that the smoothness of the 3D trajectory deteriorates at a rate slower than the peakiness (function of $\hat{K}_c$) and SNR (function of $\hat{\lambda}_1$) of the incoming signal for increasing source-sensor distance.

It is important to study the performance of trajectory-based metrics on footstep signals originating at different azimuths since the trajectory traced by the impinging wave on the vector-sensor varies for different source azimuths. In this experiment, the subject was

 Figure 3.10: Comparison of R_d vs source azimuth angle.

made to walk radially away from the sensor at various azimuths measured with respect to the positive x -axis. The detection rate R_d is evaluated for a maximum radial distance of 23 m in each direction and averaged over three independent trials. Figure 3.10 shows the variation of R_d with the azimuth angle for all the multi-component footstep detection metrics. It is observed that the proposed trajectory-based metrics achieved consistently higher average detection rate of 97% across different azimuth angles compared to 93% and 87% for $\hat{K}_c$ and $\hat{\lambda}_1$ respectively. This can be attributed to the fact that the polarization generated by a footstep signal is composite, therefore resulting in smooth normalized trajectory irrespective of the source azimuth.

In the next experiment, the subject was made to march at a fixed location of 11 m along the x -axis, i.e., parallel to the ‘H1’ axis of the geophone. We now evaluate the performance of the algorithm on marching footsteps. The results of various detection metrics for the signal acquired is shown in Fig. 3.11. It is observed that the proposed polarization-cum-energy metric is capable of detecting all the footsteps whereas the cumulated energy and cumulative kurtosis metrics can detect only a few of them. This is attributed to the fact that standing waves resulting from stationary footsteps leads to the spread of the footstep transients. This, in turn, reduces the peakyness of the

Figure 3.11: Plot of (a) z -axis signal, (b) cumulated energy $\hat{E}_c$ and footstep detection decision for (c) cumulative kurtosis $\hat{K}_c$ and (d) proposed PE metric, for stationary footsteps at 11 m from the sensor.

signal resulting in a reduction in $\hat{K}_c$. Since the proposed polarization-cum-energy metric exploits the benefits of both the cumulated energy and polarization, it can detect the arrival of each of the footstep signals at the sensor.

The performance of the proposed detection metrics are now evaluated as a function of source-sensor distance. It is important to note that the detection range is highly susceptible to the diversity of the medium, coupling of geophone to the ground as well as the intensity of footsteps. Hence, a comparative study of the detection and false alarm rate as a function of source-sensor distance is performed for all detection metrics.

Figure 3.12(a) shows the plot of R_d with source-sensor distance for all detection metrics. It is important to note that the newly proposed trajectory-based metrics have extended the detection capabilities of energy-based metrics. The detection rate of the largest eigenvalue metric progressively decreases beyond 14 m, whereas the baseline cumulative kurtosis metric ceases to detect footsteps beyond 23 m. In comparison, the polarization-based metrics can detect footsteps of up to 30 m with acceptable R_d . The

 Figure 3.12: Plot of (a) R_d (b) R_{fa} , vs source-sensor distance.

evaluation of false alarm rate R_{fa} along with R_d provides a comprehensive evaluation of the detection performance. Figure 3.12(b) shows that the R_{fa} for the baseline cumulative kurtosis metric increases from 20 to 25 m. The increase in false alarm beyond 20 m is due to the spread of footstep signals and they occur in segments immediately following a footstep signal. The false alarm rate beyond 25 m is not shown due to the limited detection range of the $\hat{K}_c$ metric. With limited detection range, the largest eigenvalue method also shows a very low false alarm rate. However, R_{fa} for the proposed GS-based error metric increases considerably with distance. This undesired false detection arise due to the non-stationarity of the metric. The GA-based error metric demonstrates its advantage with a reduced R_{fa} of 0.2 compared to 0.7 for GS when the source-sensor distance is greater than 20 m. This improvement in performance is due to the exploitation of the intrinsic trajectory of the normalized signal on the unit sphere in order for GA to distinguish between the footstep and background noise. For source-sensor distances greater than 20 m, the false alarm rates of the GS, GA -based and PE metrics are 33%, 15%, 9%, respectively. This clearly illustrates that PE metric has the least false alarm

Figure 3.13: 3D plot of $\bar{\mathbf{x}}(n)$ for (a) noise-only segment, (b) vehicle at a distance of 150 m and (c) vehicle at a distance of 50 m.

rate, and hence outperforms both GA-based and GS-based metrics. Comparing results in Fig. 3.12, the proposed trajectory-based footstep detection metrics achieved a 5 to 8 m improvement in the detection range compared to existing baseline metrics. The integration of cumulated energy with trajectory-based GA interpolation error, employed in PE, is observed to possess an advantage in terms of improved detection range and reduced false alarm rate.

3.3 Detection of Polarized Vehicular Signals

Similar to footstep signals, polarized seismic waves are also generated on Earth's surface due to the presence of vehicles. The plot of $\bar{\mathbf{x}}(n)$ corresponding to vehicular signal, using data sampled at 8 kHz, is shown in Fig. 3.13(b) and (c) for vehicular activity at approximately 150 m and 50 m from the sensor, along with that of background noise (without any interference) in Fig. 3.13(a). We note that the normalized trajectory corresponding to background noise is random and does not have a specific pattern. On the other hand, the normalized trajectory corresponding to the vehicular signal is elliptical in nature. The closer the vehicle is to the sensor, the smoother is the normalized trajectory. Unlike footstep signals discussed in Section 3.1, the elliptical trajectory generated by vehicular

Figure 3.14: Plot of (i) z -axis vehicular signal and (ii) $\rho_{\bar{\mathbf{x}}}(l)$ for vehicle at (a) 150 m, (b) 70 m and (c) 5 m (CPA), from the sensor.

activity is consistent for a longer time duration due to the continuous movement of the vehicle on the Earth's surface.

Since vehicular signals generate a consistently smooth normalized trajectory over a long duration, the temporal correlation of the normalized signal is evaluated as

$$\begin{aligned} \rho_{\bar{\mathbf{x}}}(l) &= E\{\bar{\mathbf{x}}^T(n)\bar{\mathbf{x}}(n+l)\} \\ &\approx \frac{1}{N} \sum_{n=1}^N \bar{\mathbf{x}}^T(n)\bar{\mathbf{x}}(n+l), \end{aligned} \quad (3.26)$$

where $\bar{\mathbf{x}}(n)$ is the normalized vector defined in (3.1), l and N refer to the lag parameter and window length, respectively, over which the correlation is computed. Figure 3.14(ii) shows the plot of $\rho_{\bar{\mathbf{x}}}(l)$ with respect to the lag parameter l , evaluated over a frame length of 1 s, for different segments of vehicular signals as indicated in Fig. 3.14(i). The data is acquired by a tri-axial geophone buried close to a mud road, inside an urban training facility, on which the vehicles were operated. The initial 10 s of the recorded signal does not have any vehicular activity and is used as the background noise profile. The

correlation of the corresponding normalized signal segment shown in Fig. 3.14(ii)(a) and is observed to be random. As any correlation function, the maximum correlation is at zero lag $l = 0$ and correlation reduces for higher lags. When the vehicle moves closer to the sensor, the correlation of the normalized trajectory as seen in Fig. 3.14(ii)(c) corresponding to vehicle at 70 m, is smoother and increases to a high value (note the y -axis) compared to the noise correlation in Fig. 3.14(a) when the vehicle is approximately at a distance of 150 m from the sensor. Figure. 3.14(ii)(c) shows the correlation near the closest point of approach (CPA) of the vehicle to the sensor and is observed to be very high. From the above discussion, it is evident that a metric using the proposed correlation of the normalized signal $\rho_{\mathbf{x}}(l)$ for non-zero lags can be used to detect the presence of vehicles in the vicinity of the sensor.

3.3.1 Proposed Windowed Normalized Correlation Difference

In order to quantify the extent of smoothness in the acquired 3D signal, a sum of correlation differences over a specific range of lag values is used. The windowed normalized correlation difference (WNCD) metric defined as

$$W_{\text{CorrDiff}} = \sum_{k=-K}^K \left| \rho_{\mathbf{x}}(0) - \rho_{\mathbf{x}}(k) \right| \quad (3.27)$$

is proposed, where $K > 1$ is the number of lags considered for the metric. The term within the summation corresponds to the difference between the correlation of the 0th and k th lag. A lower value indicates higher correlation, which occurs due to polarized waves impinging the sensor. A high value reflects randomness in the data snapshots, which is attributed to noise or the absence of a seismic event.

3.3.2 Experiment Results and Discussions for Vehicular Signal Detection

Vehicular detection capability of the proposed WNCD metric is demonstrated on data collected from recordings in two diverse environmental conditions. In this experiment, seismic signals were recorded while a medium commercial lorry travelled at different velocities on an isolated mud road. The tri-axial geophone was buried next to the road with closest point of approach (CPA) of 5 m. In the second setup, seismic recordings were generated via the movement of a small-sized goods carrier vehicle on an asphalt road and the geophone was buried with a CPA of 2 m. In both cases, the acquired signal is sampled at 8 kHz. The correlation window was chosen to be $N = 2$ s with 30% overlap. The detection threshold was evaluated by averaging the W_{CorrDiff} metric across 30 s background noise profile. The performance of the detection metrics are illustrated by plotting the signal against source-sensor distance. The vehicle is assumed to approach and reced the sensor along the positive and negative x -axis, respectively.

Figure 3.15(a) shows data acquired from the z -axis of the sensor in the first experiment setup. The vehicle approaches the sensor at a constant speed of 30 km/h. The corresponding W_{CorrDiff} metric evaluated for the data is shown in Fig. 3.15(b). We note that as the vehicle approaches the sensor, correlation of the normalized trajectory increases and hence results in a decrease of the W_{CorrDiff} metric. Figure 3.16(c) illustrates the detection performance of the proposed W_{CorrDiff} metric in comparison with the baseline spectral-based detection method (where energy in the frequency range of 5 to 250 Hz is used to detect the presence of a vehicle [63]). From the results, we note that the WNCD metric achieved an increased detection range up to 200 m, compared to 165 m detection range for the spectral-based algorithm.

In the following experiment, signal is recorded when the vehicle is at 100 m from the CPA. Figure 3.16 shows the performance of the WNCD metric in comparison to

Figure 3.15: Plot of (a) z -axis signal, (b) W_{CorrDiff} , along with (c) vehicular detection decision for spectral-based and WNCD metrics, vs source-sensor distance.

the spectral-based algorithm for the vehicle travelling at 40 km/h. It is observed that a change in the medium to asphalt (from mud in the previous experiment) results in a reduction in the detection range. However, the WNCD metric based on the correlation of the normalized trajectory achieved a detection range up to 62 m, which is nearly twice compared to the 33 m detected by spectral-based algorithm.

It was observed from the different experiments that the detection range depended on the velocity at which the vehicle travelled. We now evaluate the detection performance of the polarization-based metric as a function of the vehicular velocity. Figure 3.17 shows the detection range as a function of the vehicle velocity for vehicular signals recorded using the second experimental setup. We note that the detection range increases with increasing vehicular velocity. This could be attributed to the nature of seismic polarization modes generated by the impact of the vehicle tyre (travelling at different velocity) against the Earth's surface. From the results, we note that at higher velocity these waves

Figure 3.16: Plot of (a) z -axis signal, (b) W_{CorrDiff} , along with (c) vehicular detection decision for spectral-based and WNCd metrics, vs source-sensor distance.

are capable of travelling farther distances on the Earth's surface. However, increased vehicular velocity does not show significant energy difference in the frequency band of operation, resulting in poor performance of the spectral-based method. This could be due to signal-attenuation on the asphalt medium compared to the mud road.

We now evaluate the performance of the proposed algorithm to detect both footstep and vehicular activities. The two variables in the evaluation of the proposed WNCd metric are the window length N and the number of lags K over which the difference in correlation is summed. Variation in the values of K and N regulates the detection performance and reveal the possibility of detecting both footstep signals and vehicular signals and footstep signals in the background of vehicular activity. The estimation of WNCd using a small value of N results in the computation of temporal correlation over smaller time windows which is suitable for the identification of impulsive seismic event such as footsteps, hammer strikes, etc. Although the use of smaller time windows does

Figure 3.17: Plot of detection range (in meters) vs vehicle velocity (in km/h.)

not affect the detection performance for vehicular signals, it results in high computational load. The selected value of K is dependent on the degree of smoothness of the trajectory of the acquired tri-axial signal. At the CPA, since the seismic source is close to the sensor, the value of K can be large as the temporal correlation is consistent over a larger lag of the signal. However as the source moves away from the sensor, the value of K is decreased in order to capture the polarization of the signal. Therefore, both N and K need to be regulated to detect both impulsive (caused by footstep) and non-impulsive (caused by vehicle) seismic events.

In order to illustrate the capability of the WNCD metric for detection of both footstep and vehicles, the metric is evaluated on vehicular and footstep signals with varying window length of $N = 1$ s and $N = 200$ ms. The results are shown in Fig. 3.18(a) and (b) respectively for $K = 6$. It is observed that the correlation difference metric starts to decrease as the vehicle approaches the CPA. Hence, we infer that the correlation of the signal trajectory increases with decreasing sensor-vehicle distance. In the case of footstep signals, it can be seen that correlation difference metric decreases in the presence of a footstep compared to its absence. The detection of both footstep and vehicles can there-

Figure 3.18: Plot of (a) vehicular and (b) footstep signals - (i) z -axis signal and (ii) W_{CorrDiff} metric.

fore be accomplished using the W_{CorrDiff} metric. However, there is a need for a threshold to be applied on W_{CorrDiff} to make the detection decision. This threshold can be obtained by evaluating the metric for background noise profile as in (3.20).

Based on the results obtained in Fig. 3.18, an effective method to detect the presence of footsteps in vehicular activities, would be to first evaluate WNCD metric over larger time windows of $N = 1$ s. This detects a longer duration of correlation in the normalized trajectory. Subsequently, WNCD metric is evaluated over smaller windows to detect the presence of footsteps. However, in the absence of vehicular signals, the acquired data need to be scanned for footstep signals using smaller value of N .

Figures 3.19(a) and (b) show two sets of footstep train recorded with and without vehicular signal in the background, respectively. The WNCD metric evaluated on the signal with $N = 1$ s shows a dip corresponding to vehicular signal, i.e., the normalized trajectory resulting in longer correlation, in Fig. 3.19(b). After presence of a vehicle is detected, the signal between 6-12 s is used to evaluate the WNCD metric using a smaller

Figure 3.19: Plot of (i) z -axis signal and (ii) W_{CorrDiff} metric for (a) no vehicle in the background and W_{CorrDiff} evaluated using $N=1$ s, (b) vehicle in the background and W_{CorrDiff} evaluated using $N=1$ s and (c) vehicle in the background and W_{CorrDiff} evaluated using $N=150$ ms.

time frame of $N = 150$ ms. A threshold value evaluated using the vehicular signal as the background noise is used to detect the footsteps embedded as shown in Fig. 3.19(c).

3.4 Conclusion

In this chapter, smoothness-based metrics evaluated using the normalized tri-axial trajectory were employed to quantify the dissimilarity in the 3D trajectory traced between the presence and absence of a seismic source such as human footsteps or vehicular motion. Two different approaches were proposed to detect smoothness of the normalized trajectory. For footstep signals, the instantaneous deviation of the acquired signal to geometric sequence and great arc based signal were evaluated and the framewise variance of the model-fit error was used as a detection metric. Instead of evaluating the instantaneous deviation, the correlation of the normalized trajectory over a larger time window was

used to detect the presence of vehicular signals.

Experiment results on recorded footstep signals and vehicular signals showed that the trajectory-based metrics exploited the uniform continuity of the normalized trajectory and achieved better performance than the energy-based metrics, especially at farther source-sensor distances when the signal-to-noise ratio (SNR) of the signal deteriorates. The polarization-cum-energy metric was observed to achieve a good trade-off between detection rate and false alarm rates for different footstep walking patterns. The WNCD metric was used to detect both vehicular and footstep signals by regulating the values of N and K to capture either the long-term or the short-term correlation, respectively.

Chapter 4

Exploiting Vector-Quaternion Statistics For Footstep Detection

Quaternions are hypercomplex numbers which are often used to present vector-sensor data. In this chapter, a vector-quaternion representation of tri-axial signals is introduced. Real-world seismic signals cannot be completely described using existing Gaussian models [75] and hence a quaternion generalized Gaussian distribution (QGGD) is derived as an extension of complex generalized Gaussian distribution (CGGD) [76,77] to model near-field tri-axial signals. The novelty of this approach lies in the use of quaternion-based statistics to quantify the dissimilarity between footstep signal and noise. The QGGD parameterizes non-Gaussianity of the vector-sensor signal using a shape parameter and inter-channel correlation of the tri-axial data using a quaternion augmented covariance matrix.

4.1 Vector-Quaternion For Tri-axial Data

In this section, tri-axial signals will be represented as vector-quaternions. Quaternions are 4D hypercomplex numbers given by

$$\mathbf{q} = \underbrace{q_0}_{\mathbb{R}(\mathbf{q})} + \underbrace{q_1\iota + q_2j + q_3\kappa}_{\mathbb{I}(\mathbf{q})}, \quad (4.1)$$

where $\mathbb{R}(\cdot)$ and $\mathbb{I}(\cdot)$ denote, respectively, the scalar real part and the vector imaginary part consisting of three imaginary components. The variables ι , j and κ are imaginary numbers such that $\iota j \kappa = \iota^2 = j^2 = \kappa^2 = -1$, $\iota j = \kappa$, $j \kappa = \iota$, $\kappa \iota = j$ and $q_0, q_1, q_2, q_3 \in \mathbb{R}$. When $q_0 = 0$, $\mathbf{q}$ is known as *pure* or vector-quaternion and is often used to represent a 3×1 vector [70].

The signal acquired by a tri-axial geophone at time instant n is given by

$$\mathbf{x}(n) = [x_1(n) \ x_2(n) \ x_3(n)]^T. \quad (4.2)$$

This 3×1 vector containing data from the three orthogonal channels can be represented as a (pure) vector-quaternion

$$\mathbf{r}(n) = x_1(n)\iota + x_2(n)j + x_3(n)\kappa. \quad (4.3)$$

The corresponding polar representation of $\mathbf{r}(n)$ is given by

$$\mathbf{r}(n) = \rho(n)e^{\mathbf{j}(n)\theta(n)}, \quad (4.4)$$

Figure 4.1: Plot of (i) acquired signal and (ii) distribution, for (a) noise only (absence of footstep) and (b) footstep signal, for each channel of the tri-axial geophone.

where

$$\begin{aligned}\rho(n) &= \|\mathbf{x}(n)\|_2 \\ &= \sqrt{x_1^2(n) + x_2^2(n) + x_3^2(n)}\end{aligned}\quad (4.5)$$

defines the magnitude, $\mathbf{z}(n) = \mathbf{x}(n)/\|\mathbf{x}(n)\|_2$ is the *eigenaxis* and $\theta(n) = \pi/2$ is the *eigenangle* [70] of the vector-quaternion. It is useful to note that the normalized vector $\bar{\mathbf{x}}(n)$, defined in (3.1), contains the coefficients of the eigenaxis

$$\mathbf{z}(n) = \bar{x}_1(n)\imath + \bar{x}_2(n)\jmath + \bar{x}_3(n)\kappa. \quad (4.6)$$

A vector-quaternion representation of tri-axial data retains the non-linear relation between the orthogonal components, compared to long-vector [64] or component-wise

Figure 4.2: 3D plot of (a) noise, (b) footstep signal, on Poincare sphere.

processing. The orthogonal components of the vector-quaternion signal corresponding to noise only segment (i.e., absence of footstep signal) and to the presence of a footstep signal, along with their distributions are plotted in Fig. 4.1(a) and (b), respectively. In the absence of a footstep, the distribution tends towards Gaussian, while in its presence, the distribution approaches super-Gaussianity. In addition, it has been described in [33] that footsteps generate multiple modes of polarization that are dominated by elliptically polarized Rayleigh waves. The time evolution of the noise and footstep signals, shown in Fig. 4.1, is plotted in 3D in Fig. 4.2 (a) and (b), respectively. As opposed to the absence of a footstep shown in Fig. 4.2(a), it is noted from Fig. 4.2(b) that the tri-axial signal in the presence of a footstep traces a wobbly elliptical orbit [68].

There exist primary differences between the Poincare sphere representation [78], shown in Fig. 4.2, and the normalized trajectory representation, illustrated in Fig. 3.1 of Chapter 3.1. A Poincare sphere is a graphical tool used to describe polarized signals in a real 3D space where the polarization state is represented by a point, given by the three Stokes parameter, on or within a unit sphere centered on the Cartesian coordinate system [78]. The degree-of-polarization (DOP) is defined as the distance of the point from the origin of the coordinate system and the nature of polarization is represented by the position of the point on the surface of the sphere. Linearly polarized signals

are represented by points on the equator of the sphere while circularly polarized waves are represented by points at the poles of the sphere. Elliptical polarization is therefore identified by points distributed between the equator and the poles. A value of zero for DOP corresponds to a point at the origin denoting unpolarized signal, whereas points on the surface of the sphere exhibit unity DOP corresponding to complete polarization [79]. When polarization is expressed using the normalized trajectory, unpolarized signal is characterized by random points on the surface of the sphere. However, linearly polarized signal maps onto two points on the surface of the sphere while elliptically polarized signal forms a smooth elliptical trajectory on the surface the sphere. In the normalized trajectory representation, the energy of the signal is ignored because of the projection onto the surface of the sphere.

In Fig. 4.2, the subplots (a) and (b) show the continuous evolution of polarization state on a Poincare sphere for a time segment when footstep is absent (noise-only) and present, respectively. The sphere is scaled to 0.1 in order to better visualize the difference. It can be observed that a random noise segment is characterized by points close to the origin and hence are considered unpolarized signals or partially polarized with insignificant degree-of-polarization. However, a footstep segment is characterized by partial elliptical polarization with relatively higher degree-of-polarization. This is consistent with the well-known fact that footstep signals are dominated by elliptically polarized Rayleigh waves [14, 80].

Elliptical polarization results in correlation between the signals in the three channels when a footstep is present. Along with this correlation, the difference in the signal distributions across orthogonal channels of the geophone, as observed in Fig. 4.1, gives rise to impropriety in the vector-quaternion signal. This is preserved in the quaternion augmented statistics. Therefore, presence of a footstep is reflected both as super-Gaussianity in the individual axes as illustrated in Fig. 4.1, and as correlation between the channels of the vector-sensor signal as illustrated in Fig. 4.2.

4.2 Proposed Quaternion Generalized Gaussian Distribution

In this section, a quaternion generalized Gaussian distribution (QGGD) is derived using quaternion augmented statistics in order to detect elliptically polarized footstep signals. Quaternions can be considered as an extension of complex numbers with three imaginary components, ι , j and κ . For the quaternion signal $\mathbf{x}(n)$ defined in (4.3), the 4×1 quaternion augmented vector [81] is defined as

$$\mathbf{q}_{\mathbf{x}}^{\mathbf{a}}(n) = [\mathbf{x}(n) \quad \mathbf{x}^{\iota}(n) \quad \mathbf{x}^j(n) \quad \mathbf{x}^{\kappa}(n)]^T, \quad (4.7)$$

where the superscript ‘a’ denotes for the augmented vector and $\mathbf{x}^{\iota}(n) = -\iota \star \mathbf{x}(n) \star \iota$ denotes the quaternion involution with respect to ι . Quaternion involutions defined over the imaginary axis $\eta \in \iota, j, \kappa$ represent a rotation by an angle π in the imaginary plane orthogonal to $\{1, \eta\}$. The corresponding quaternion augmented covariance matrix is evaluated as [81]

$$\mathbb{C}_{\mathbf{q}_{\mathbf{x}}}^{\mathbf{a}} = \mathbb{E} \left\{ \mathbf{q}_{\mathbf{x}}^{\mathbf{a}}(n) (\mathbf{q}_{\mathbf{x}}^{\mathbf{a}})^H(n) \right\}, \quad (4.8)$$

where $\mathbb{E}\{\cdot\}$ and the superscript $(\cdot)^H$ are the expectation and Hermitian operators, respectively.

The quaternion augmented covariance matrix $\mathbb{C}_{\mathbf{q}_{\mathbf{x}}}^{\mathbf{a}}$ and the tri-variate covariance matrix evaluated as

$$\mathbb{C} = \mathbb{E} \left\{ \mathbf{x}(n) \mathbf{x}^T(n) \right\}, \quad (4.9)$$

are related by the identity [81]

$$\mathbf{x}^T(n) \mathbb{C}^{-1} \mathbf{x}(n) = (\mathbf{q}_{\mathbf{x}}^{\mathbf{a}})^H(n) (\mathbb{C}_{\mathbf{q}_{\mathbf{x}}}^{\mathbf{a}})^{-1} \mathbf{q}_{\mathbf{x}}^{\mathbf{a}}(n). \quad (4.10)$$

The augmented covariance matrix $\mathbb{C}_{\mathbf{q}_r}^a$ completely describes the second-order statistics of the vector-quaternion signal. With the above definitions, the probability density function (pdf) for a QGGD is derived similar to its complex counterpart CGGD [76] by representing the vector-quaternion in its polar form using (4.4) with $\theta(n) = \pi/2$.

The shape of the vector-quaternion distribution can be described by its magnitude $\rho(n)$ defined in (4.5). In [76], the magnitude of the CGGD is described by a modified Gamma variate and hence, in a similar manner, $\rho(n)$ is assumed to be a generalized Gamma distribution (GGaD) with shape parameters $(3/k, k)$ and unit scale, i.e., $\rho(n) \sim \text{GGaD}(3/k, k, 1)$. The pdf of $\rho(n)$ is then given by [82]

$$p(\rho) = k\rho^2(n)e^{-\rho^k(n)} / \Gamma(3/k), \quad (4.11)$$

where $\Gamma(3/k) = \int_0^\infty e^{-t} t^{3/k-1} dt$ is the Gamma function.

Similarly extending on [76], the eigenaxis $\mathbf{z}(n)$ is assumed to be uniformly distributed on the surface of a sphere and described in terms of an azimuth angle $\Theta(n) \sim \mathcal{U}(0, 2\pi)$ and elevation angle $\phi(n) \sim \mathcal{U}(0, \pi)$. Assuming $\rho(n)$, $\Theta(n)$ and $\phi(n)$ are independent, the joint pdf (in polar form) is given by

$$p(\rho, \Theta, \phi) = p(\rho)(1/2\pi)(1/\pi). \quad (4.12)$$

The vector-quaternion random variables can alternatively be expressed in terms of the Cartesian tri-variate random variables using $x_1(n) = \rho(n) \sin \phi(n) \cos \Theta(n)$, $x_2(n) = \rho(n) \sin \phi(n) \sin \Theta(n)$ and $x_3(n) = \rho(n) \cos \phi(n)$ where $x_i(n)$, $i = 1, 2, 3$ are the received signals defined in (4.2). The joint pdf of the Cartesian tri-variate data in terms of its polar representation is then given by

$$p(x_1, x_2, x_3) = \frac{1}{|\mathbf{J}|} p(\rho, \Theta, \phi), \quad (4.13)$$

such that $\rho = \sqrt{x_1^2 + x_2^2 + x_3^2}$, $\Theta = \tan^{-1}(x_2/x_1)$ and $\phi = \tan^{-1}(x_3/\sqrt{x_1^2 + x_2^2})$, where the time dependency ‘(n)’ is temporarily omitted for compactness in notation and the determinant of the Jacobian matrix $\mathbf{J}$ is given by

$$\begin{aligned} |\mathbf{J}| &= \begin{vmatrix} \sin \phi \cos \Theta & -\rho \sin \phi \sin \Theta & \rho \cos \phi \cos \Theta \\ \sin \phi \sin \Theta & \rho \sin \phi \cos \Theta & \rho \cos \phi \sin \Theta \\ \cos \phi & 0 & -\rho \sin \phi \end{vmatrix} \\ &= \rho^2 \sin \phi. \end{aligned} \quad (4.14)$$

Substituting (4.11), (4.12) and (4.14) into (4.13), the pdf of the generalized Gaussian distribution is obtained as

$$\begin{aligned} p(x_1, x_2, x_3) &= \frac{1}{2\pi} \frac{1}{\pi} \frac{1}{\rho^2 \sin \phi} \frac{k \rho^2}{\Gamma(3/k)} e^{-(x_1^2 + x_2^2 + x_3^2)^{k/2}} \\ &= \frac{c}{\pi^2 \Gamma(3/2c) \sin \phi} e^{-(x_1^2 + x_2^2 + x_3^2)^c}, \end{aligned} \quad (4.15)$$

where $c = k/2$. Figure 4.3(a) shows the distribution of $\sin \phi$ corresponding to a uniform distribution of elevation angle ϕ illustrated in Fig. 4.3(b). It is observed that $\sin \phi = 1$ occurs with the highest probability. With $\sin \phi \approx 1$ the generalized Gaussian pdf is reduced to

$$p(x_1, x_2, x_3) \approx c e^{-(x_1^2 + x_2^2 + x_3^2)^c} / (\pi^2 \Gamma(3/2c)). \quad (4.16)$$

For $c = 1$, the pdf $p(x_1, x_2, x_3)$ degenerates to a Gaussian distribution which is analogous to the CGGD for complex signals [76].

Since the three orthogonal sensors of the geophone are co-located, it can be assumed

 Figure 4.3: Plot of histogram of (a) $\sin \phi$ and (b) $\phi \sim \mathcal{U}(0, \pi)$.

that $E\{x_1^2\} = E\{x_2^2\} = E\{x_3^2\}$ and the second-order moment is given by

$$\begin{aligned}
 \xi(c) &\triangleq E\{x_1^2\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_1^2 p(x_1, x_2, x_3) dx_1 dx_2 dx_3 \\
 &= \int_0^{\pi} \int_0^{2\pi} \int_0^{\infty} \rho^2 \sin^2 \phi \cos^2 \Theta \frac{c e^{(\rho^2)^c} (\rho^2 \sin \phi d\rho d\Theta d\phi)}{\pi^2 \Gamma(\frac{3}{2c}) \sin \phi} \\
 &= \frac{\Gamma(5/c)}{2\Gamma(3/2c)}. \tag{4.17}
 \end{aligned}$$

Similar to the approach in [76], the random variable $x_i(n)$ is normalized with $\xi(c)$ to have unit variance using the linear transform $\mathbf{v}(n) = \mathbf{D}\mathbf{x}(n)$, where $\mathbf{D} = \sqrt{1/\xi(c)}\mathbf{I}_3$ and $\mathbf{I}_3$ is a 3×3 identity matrix. This transform is applied to (4.16) to obtain

$$\begin{aligned}
 p(\mathbf{v}) &= \frac{1}{|\mathbf{D}|} p(\mathbf{D}^{-1}\mathbf{v}(n)) \\
 &= \beta(c) e^{-\xi(c)[\mathbf{v}^T(n)\mathbf{v}(n)]^c}, \tag{4.18}
 \end{aligned}$$

where $p(\mathbf{D}^{-1}\mathbf{v}(n)) = p(x_1, x_2, x_3)$, $\beta(c) = c \frac{\Gamma(5/c)}{(2\pi^2 \Gamma(3/2c))^2}$ and $E\{\mathbf{v}(n)\mathbf{v}^T(n)\} = \mathbf{I}_3$ for $c > 0$.

Next, $\mathbf{v}(n)$ is expressed as an augmented quaternion vector $\mathbf{q}_v^a(n)$ using (4.7) and the corresponding quaternion pdf is derived using (4.10) as

$$p(\mathbf{q}_v^a) = \beta(c) e^{-\xi(c)[(\mathbf{q}_v^a)^H \mathbf{q}_v^a]^c}. \tag{4.19}$$

In order to extend this pdf for any arbitrary quaternion augmented vector $\mathbf{q}^a$ with quaternion augmented covariance matrix $\mathbb{C}_q^a$, the transformation $\mathbf{q}^a = \mathbb{T}_q^a \mathbf{q}_v^a$, where $\mathbb{T}_q^a = \sqrt{\mathbb{C}_q^a}$ is applied. Since the diagonal elements of $\mathbb{C}_q^a$ are real, the pdf of the QGGD is given by

$$p(\mathbf{q}^a) = \left(\sqrt{|\mathbb{C}_q^a|} \right)^{-1} \beta(c) e^{-[\xi(c)((\mathbf{q}^a)^H (\mathbb{C}_q^a)^{-1} \mathbf{q}^a)]^c}, \quad (4.20)$$

where $\mathbf{q}^a$ and $\mathbb{C}_q^a$ are defined similar to (4.7) and (4.8) respectively. The parameter c defines the shape of the vector-quaternion distribution while $\mathbb{C}_q^a$ is the covariance matrix which preserves the inter-channel correlation between the orthogonal components of the vector-quaternion.

The log-likelihood of the QGGD using N independent and identically distributed samples can be expressed in terms of the tri-variate signal $\mathbf{x}(n)$ and covariance matrix $\mathbb{C}$ as

$$\mathcal{L}(\mathbf{x}(n)) = N \ln(\beta(c)) - \frac{N}{2} \ln(|\mathbb{C}|) - \xi^c(c) \sum_{n=1}^N \left((\mathbf{x}(n)^T (\mathbb{C})^{-1} \mathbf{x}(n)) \right)^c \quad (4.21)$$

Differentiating $\mathcal{L}(\mathbf{x}(n))$ with respect to c , we get

$$\frac{\partial \mathcal{L}(\mathbf{x}(n))}{\partial c} = \frac{N \beta'(c)}{\beta(c)} - \xi^c(c) \ln(\xi(c)) \xi'(c) \sum_{n=1}^N m^c(n) - \xi^c(c) \sum_{n=1}^N m^c(n) \ln(m(n)), \quad (4.22)$$

where $m(n) = \mathbf{x}(n)^T (\mathbb{C}^{-1}) \mathbf{x}(n)$. The Cramer-Rao lower bound (CRLB) for the shape estimator, i.e., lower bound on the variance of the estimator, can be obtained by differentiating (4.22) again with respect to c as

$$\begin{aligned} \frac{\partial^2 \mathcal{L}(\mathbf{x}(n))}{\partial^2 c} &= N \left[\frac{-(\beta'(c))^2}{(\beta(c))^2} + \frac{\beta''(c)}{\beta(c)} \right] - [\xi^c(c) \ln^2(\xi(c)) - \xi'(c)(\xi(c))^{c-1} - \xi''(c) \xi^c(c) \ln(\xi)] \times \\ &\quad \sum_{n=1}^N m^c(n) + \xi^c(c) \ln(\xi) \xi'(c) \sum_{n=1}^N m^c(n) \ln(m(n)) - \\ &\quad \xi^c(c) \left[\ln(\xi(c)) \sum_{n=1}^N m^c(n) \ln(m(n)) + \sum_{n=1}^N m^c(n) \ln^2(m(n)) \right], \end{aligned} \quad (4.23)$$

where $\beta'(c) = \frac{\partial\beta(c)}{\partial c}$, $\xi'(c) = \frac{\partial\xi(c)}{\partial c}$, $\beta''(c) = \frac{\partial\beta'(c)}{\partial c}$ and $\xi''(c) = \frac{\partial\xi'(c)}{\partial c}$. From (4.23), we note that the maximum likelihood estimate of the parameters of QGGD do not result in a closed form expression. Hence, numerical methods have to be used for estimating these parameters. In the following sections, frames of the vector-quaternion tri-axial signal will be fitted to the derived QGGD pdf and the presence of footsteps is identified using two proposed techniques: (a) estimating the shape parameter $\hat{c}$ and (b) evaluating the degree-of-Q-improperness of the estimated augmented quaternion covariance matrix $\hat{\mathbb{C}}_q^a$.

4.2.1 Shape Parameter $\hat{c}$: Non-Gaussianity Measure

The shape of the vector-quaternion distribution, as illustrated in Fig. 4.1, is reflected in the c parameter of the QGGD in (4.20). This shape parameter is estimated from the magnitude $\rho(n)$, which follows a GGaD pdf given in (4.11). A heuristic method [83] that maps the GGaD pdf to a Gamma distribution using the power transformation identity, i.e., when $\rho(n) \sim \text{GGaD}(3/2c, 2c, 1)$ then $\rho^{2c}(n) \sim \text{Gamma}(3/2c, 1^{2c})$, is used to estimate the shape parameter c . This routine iteratively loops through values $[0.1, 2]$ to estimate $\hat{c}$ as the best chi-square fit between $\rho^c(n)$ and an empirical Gamma distribution with the same parameters [83]. Therefore, $\hat{c}$ is expected to be approximately equal to unity for Gaussian distributed noise and less than unity for a super-Gaussian signal. Detection of footstep signals is therefore expressed as a binary hypothesis test using $\hat{c}$ as

$$H_0 : \hat{c} \approx 1 \text{ for Gaussian noise;}$$

$$H_1 : \hat{c} < 1 \text{ for Super - Gaussian footstep signal.}$$

4.2.2 Degree-of- $\mathbb{Q}$ -improperness of $\hat{\mathbb{C}}_q^a$: Inter-channel Correlation Measure

The augmented covariance matrix of the QGGD is estimated using a frame of N consecutive augmented quaternion vectors $\mathbf{q}_r^a(n)$, evaluated using (4.7), as

$$\hat{\mathbb{C}}_q^a = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{q}_r^a(n) \mathbf{q}_r^{aH}(n). \quad (4.24)$$

The estimated $\hat{\mathbb{C}}_q^a$ has a structure

$$\hat{\mathbb{C}}_q^a = \begin{bmatrix} \hat{\mathbb{C}}_{qq} & \hat{\mathbb{C}}_{qq^\iota} & \hat{\mathbb{C}}_{qq^j} & \hat{\mathbb{C}}_{qq^\kappa} \\ \hat{\mathbb{C}}_{q^\iota q} & \hat{\mathbb{C}}_{q^\iota q^\iota} & \hat{\mathbb{C}}_{q^\iota q^j} & \hat{\mathbb{C}}_{q^\iota q^\kappa} \\ \hat{\mathbb{C}}_{q^j q} & \hat{\mathbb{C}}_{q^j q^\iota} & \hat{\mathbb{C}}_{q^j q^j} & \hat{\mathbb{C}}_{q^j q^\kappa} \\ \hat{\mathbb{C}}_{q^\kappa q} & \hat{\mathbb{C}}_{q^\kappa q^\iota} & \hat{\mathbb{C}}_{q^\kappa q^j} & \hat{\mathbb{C}}_{q^\kappa q^\kappa} \end{bmatrix}, \quad (4.25)$$

where the off-diagonal elements are the pseudo-covariances

$$\hat{\mathbb{C}}_{qq^\iota} = \frac{1}{N} \sum_{n=0}^{N-1} q(n)(q^\iota)^H(n). \quad (4.26)$$

The augmented covariance matrix quantifies the correlation between the orthogonal axes of the geophone which is dependent on the signal power. The matrix $\hat{\mathbb{C}}_q^a$ is [84, 85]

- (i) $\mathbb{Q}$ -proper if and only if the pseudo-covariances with respect to ι , j and κ vanish;
- (ii) $\mathbb{Q}$ -improper if the pseudo-covariances do not vanish.

Random Gaussian noise with comparable power in all the three channels of the geophone results in an unpolarized $\mathbb{Q}$ -proper diagonal structure since there exists no correlation between the orthogonal axes. However, as seen in Fig. 4.2(b), footstep signals are expected to be $\mathbb{Q}$ -improper due to the presence of elliptically polarized signals and the amount of correlation is described using the degree-of- $\mathbb{Q}$ -improperness (DOI). The locally most powerful invariant test [86] is used to quantify the DOI and is defined as the Frobenius

norm $||\Phi_{\mathbb{Q}}||$ of the $\mathbb{Q}$ -coherence matrix

$$\Phi_{\mathbb{Q}} = (\tilde{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}})^{-1/2} \hat{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}} (\tilde{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}})^{-1/2}, \quad (4.27)$$

where $\hat{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}}$ is the estimated augmented matrix and $\tilde{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}}$ is the augmented matrix with a $\mathbb{Q}$ -proper diagonal structure.

In practical scenarios, perfectly unpolarized $\mathbb{Q}$ -proper signals are challenging to obtain due to the presence of standing waves, Earth vibrations in the distant environment and differences in the channel gain [69]. In this work, $\tilde{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}} = \mathbf{I}_4$ is used to evaluate $||\Phi_{\mathbb{Q}}||$ by preserving the energy of the vector-quaternion signal. The evaluated $||\Phi_{\mathbb{Q}}||$ is expected to be high in the presence of a polarized footstep and low in the absence of it. Footstep detection can therefore be defined as a binary hypothesis test using the estimated DOI as

$$\begin{aligned} H_0 & : ||\Phi_{\mathbb{Q}}|| < \gamma \text{ for } \mathbb{Q}\text{-Proper noise;} \\ H_1 & : ||\Phi_{\mathbb{Q}}|| > \gamma \text{ for } \mathbb{Q}\text{-Improper footstep signal,} \end{aligned} \quad (4.28)$$

where γ is a threshold. Since footsteps are intermittent and impulsive, the noise statistics is estimated by evaluating the DOI over long-time window, denoted as $\text{DOI}_{\mathbf{n}}$. An adaptive detection threshold is evaluated as

$$\gamma(l) = \eta(\text{DOI}_{\mathbf{n}}), \quad (4.29)$$

where the scaling parameter $\eta > 0$ regulates the trade-off between the probability of detection and false alarm.

4.3 Experiment Results and Discussions

In this section, we first define a procedure to simulate QGGD samples for a give shape parameter and augmented covariance matrix. The simulated QGGD samples are used to analyze the performance of the heuristic shape estimator defined in Section 4.2.1. Using (4.24) and (4.28), the acquired tri-axial signal, in the absence and presence of a footstep signal, is modeled as

$$\mathbf{x}(n) \sim \begin{cases} \mathbf{QGGD}(\hat{c} = 1, \hat{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}} \text{ with proper structure}) & - \text{absence of footstep;} \\ \mathbf{QGGD}(\hat{c} < 1, \hat{\mathbb{C}}_{\mathbf{q}}^{\mathbf{a}} \text{ with improper structure}) & - \text{presence of footstep.} \end{cases}$$

Using this signal model, the ability of the proposed shape parameter estimate $\hat{c}$ and degree-of-Q-improperness metric $\|\Phi_{\mathbb{Q}}\|$, to detect human footsteps is analyzed by fitting real tri-axial signal to the QGGD pdf.

4.3.1 Performance of the Shape Estimator

The procedure to generate QGGD samples is defined using the derivation of the QGGD pdf in Section 4.20. Random QGGD samples, for a given shape parameter c and quaternion augmented covariance matrix $\mathbb{C}_{\mathbf{q}}^{\mathbf{a}}$, are simulated using the following steps:

Procedure 1

(i) Generate N random eigenaxis, $\mathbf{z}_{\text{rand}}(n)$, on the surface of the sphere.

(ii) Using $\mathbf{z}_{\text{rand}}(n)$, vector-quaternions are generated as

$$\mathbf{v}(n) = (\text{gamrnd}(3/(2c), 1, 1, N))^{1/(2c)} e^{\mathbf{z}_{\text{rand}}(n) * \pi/2},$$

where `gamrnd` is a MATLAB function to generate Gamma random numbers.

(iii) The vector-quaternions are normalized as $\mathbf{v}(n) = \frac{\mathbf{v}(n)}{\sqrt{\xi(c)}}$, where $\xi(c) = \frac{\Gamma(5/c)}{2\Gamma(3/2c)}$.

Figure 4.4: Plot of mean $\hat{c}$ vs c for (a) proper $\mathbb{C}_q^a$ and (b) improper $\mathbb{C}_q^a$, evaluated for different sample size N .

- (iv) A quaternion augmented vector is formed using the normalized vector-quaternions and their involutions as $\mathbf{v}^a = [\mathbf{v}(n) \quad \mathbf{v}^i(n) \quad \mathbf{v}^j(n) \quad \mathbf{v}^k(n)]$.
- (v) A transform matrix is calculated from the given quaternion augmented covariance matrix $\mathbb{C}_q^a$ using $\mathbb{T}^a = \text{sqrtm}(\mathbb{C}_q^a)$, where sqrtm is a MATLAB function that computes matrix square root. The transform matrix is then applied to the quaternion augmented vector as $\mathbf{q}^a = \mathbb{T}^a \mathbf{v}^a$.

The performance of parameter estimators corresponding to the pdf of a distribution is theoretically studied by deriving its CRLB [87]. However, from (4.22) and (4.23) we note that the derivatives of the log-likelihood of the QGGD pdf did not result in a closed form expression. Since the acquired tri-axial signal is modeled as (4.30), we use an approach similar to [76] to analyze the performance of the heuristic shape estimator by simulating random QGGD samples using the steps described in procedure (1). First, QGGD samples are generated for different shape values $c \in [0.2, 1.8]$ and both proper and improper structure of the augmented covariance matrix. For proper $\mathbb{C}_q^a$,

Figure 4.5: Plot of standard deviation of $\hat{c}$ vs c for sample size $N = 500$, over 500 runs.

$$\mathbb{C}_q^a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ is used and for improper } \mathbb{C}_q^a$$

$$\mathbb{C}_q^a = \begin{bmatrix} 1 & 0.9 & 0.9 & 0.9 \\ 0.9 & 1 & 0.9 & 0.9 \\ 0.9 & 0.9 & 1 & 0.9 \\ 0.9 & 0.9 & 0.9 & 1 \end{bmatrix} \text{ is used.}$$

The magnitude of the vector-quaternions generated in step (iii) of procedure (1) is subjected to the heuristic routine described in Section 4.2.1 to estimate the shape. The mean of the shape estimate $\hat{c}$, averaged across 500 runs, is compared to the true shape c for different sample size N . Figures 4.4 (a) and (b), plot the estimated shape $\hat{c}$ against the true shape c for proper and improper structure of quaternion augmented covariance matrix, respectively. From the results, we observe that the shape estimator flattens into a non-linear function for $c > 1.2$, irrespective of the structure of $\mathbb{C}_q^a$.

The standard deviation of the shape estimate for $N = 500$ is plotted in Fig. 4.5(a) and (b) for proper and improper quaternion augmented covariance matrix, respectively.

Figure 4.6: Plot of (a) z -axis signal, (b) cumulative kurtosis $\hat{K}_c$, (c) shape parameter $\hat{c}$ and (d) $\|\Phi_Q\|$, for radial walk from 9 m to 30 m.

We note that the standard deviation of the shape estimate corresponding to a proper augmented covariance matrix is lower than that corresponding to an improper $\mathbb{C}_q^a$. The graphs also show that the spread of shape estimate $\hat{c}$ reduces for increasing sample size N . The heuristic shape estimator based on a power transformation rule, is not consistent as it does not converge to the true shape for increasing number of N . However, in the context of footstep detection, the shape estimator is to only distinguish between the absence of a footstep, corresponding to $c \approx 1$, and presence of a footstep, corresponding to $c < 1$, from the vector-quaternion signal. We note that the proposed heuristic shape estimator is capable of distinguishing difference in shape of the distribution for $c < 1.2$.

4.3.2 Performance of the Proposed Metrics

We evaluate this performance for a human subject walking radially away from the sensor from 2 m to 30 m and marching at fixed distance of 11 m from the sensor. These ex-

periments evaluate the robustness of the metrics under practical conditions of increasing source-sensor distances and in the presence of standing waves. Experiments were conducted on a grassy field with a single person walking with hard sole shoes. The estimated metrics $\hat{c}$ and $||\Phi_{\mathbb{Q}}||$ are compared with cumulative kurtosis $\hat{K}_c$, which is evaluated as the sum of kurtosis of the three-channel signal defined in Chapter 2 [69]. The metrics are evaluated using a sliding window of length 200 ms with 50% overlap on data sampled at 8 kHz.

Figure 4.6(a) shows the signal acquired by the z -axis of the geophone for the radial-walk experiment. As the source moves away from the sensor the leptokurtic nature of the footstep impulse reduces owing to a longer propagation in the medium. This effect is directly reflected in the deterioration of the detection performance of $\hat{K}_c$ illustrated in Fig. 4.6(b), especially for source-sensor distances greater than 25 m. Although the estimated $\hat{c}$, plotted in Fig. 4.6(c) quantifies the super-Gaussianity of the signal, it shows higher superiority in terms of the ability to detect footstep segments than $\hat{K}_c$ at farther source-sensor distances. The DOI metric $||\Phi_{\mathbb{Q}}||$ plotted in Fig. 4.6(d) shows consistent detection of footsteps at farther distances and does not show any spurious peaks as the value of the metric corresponding to a footstep segment is distinctly higher than that of noise.

Figure 4.7 shows the detection metrics for stationary marching experiment in the presence of a moving bus in the background. The existence of standing waves combined with increased noise level results in the spread of the source impulse. This causes inconsistency in both $\hat{K}_c$ and $\hat{c}$ whereas the corresponding DOI metric is consistent even in the presence of noise.

The detection performance averaged over thirty experiments involving random, radial and zig-zag walk with $\eta = 0.98$ for $||\Phi_{\mathbb{Q}}||$ is compared in Table 4.1 for detection rate R_d

Figure 4.7: Plot of (a) z -axis signal, (b) $\hat{K}_c$, (c) shape parameter $\hat{c}$ and (d) $\|\Phi_Q\|$, for stationary footsteps at 11 m from sensor.

and false alarm rate R_{fa} , defined as

$$\begin{aligned}
 R_d &= \frac{\text{no. of footsteps detected}}{\text{total no. of actual footsteps}}, \\
 R_{fa} &= \frac{\text{no. of false detections}}{\text{total no. of actual footsteps}}.
 \end{aligned} \tag{4.30}$$

From the table, it is observed that for stationary marching at source-sensor distance less than 10 m, the performance of the cumulative kurtosis is comparable to that of DOI metric. However, the shape parameter is found to exhibit high false alarm rate. This could be attributed to the presence of standing waves, which made the threshold estimation on $\hat{c}$ a challenge. As the source-sensor distance increased, the proposed DOI metric outperformed baseline cumulative kurtosis metric. On the other hand, for radial walk, there are nearly no false alarms for footsteps at a distance less than 10 m. With increasing source-sensor distance, the DOI metric showed a better detection rate than $\hat{K}_c$ with reduced false alarm rate.

Table 4.1: Comparison of detection performance

Walking Style	Distance	$\widehat{K}_c$		$ \Phi_{\mathbb{Q}} $		$\widehat{c}$	
		R_d	R_{fa}	R_d	R_{fa}	R_d	R_{fa}
Stationary Marching	< 10 m	0.99	0.005	0.98	0.005	0.93	0.2
	10-20 m	0.7	0.06	0.95	0.08	0.8	0.17
Radial Walking	< 10 m	1	0	1	0	1	0
	10-30 m	0.9	0.1	0.94	0.02	0.8	0.15

Although both $\widehat{K}_c$ and $\widehat{c}$ measure non-Gaussianity, $\widehat{K}_c$ is estimated using the fourth order statistics whereas $\widehat{c}$ is estimated by fitting a generalized Gamma distribution to the power of the signal. Due to the limited range of $\widehat{c}$, it is observed that $\widehat{K}_c$ performs better than $\widehat{c}$. However it is clear from these results that the proposed inter-channel correlation is a better metric to quantify elliptical polarization in the acquired signal.

4.4 Conclusion

In contrast to parameterizing elliptical polarization as the smoothness of the trajectory, elliptical polarization is parameterized as non-Gaussianity and inter-correlation of the tri-axial signal. The inter-channel correlation is further quantified using quaternion augmented statistics. It is found experimentally that the proposed degree-of- $\mathbb{Q}$ -improperness metric showed a good trade-off between detection rate (94% for radial walk up to 30 m and 95% for stationary marching up to 20 m) and false alarm rate (2% for radial walk up to 30 m and 8% for stationary marching up to 20 m.) This implies that the elliptical polarization of the tri-axial data serves as a better metric than that based on the shape of the signal distribution.

Chapter 5

Exploiting Orientation-Quaternion Statistics For Seismic Source Detection

In the previous chapter, elliptical polarization of a seismic wave was analyzed by representing the instantaneous snapshot of the tri-axial signal as a vector-quaternion [70], which naturally accounted for the correlation between the orthogonal components. Polarization was analyzed as the correlation between the tri-axial signals using quaternion generalized Gaussian distribution subsequently quantified using the degree-of-Q-improperness of the augmented quaternion covariance matrix. However, the vector-quaternion representation does not capture the instantaneous orbital phase of the 3D trajectory. In this chapter, the hypercomplex orientation-quaternion representation is used to parameterize the instantaneous directional attribute of the tri-axial signals.

A quaternion based rotational model is firstly developed as a framework to analyze the tri-axial trajectory as instantaneous 3D rotations. Each 3D rotation is defined by an eigenangle and an eigenaxis. Using the rotational model, a velocity-to-rotation transform is used to estimate the instantaneous rotational parameters from the acquired signal.

Figure 5.1: Block diagram illustrating the effect of noise in $\mathbb{R}^3$ modeled as fluctuations in the hypersphere $\mathbb{S}^{p-1}$ for $p = 2, 3, 4$.

It will be shown that the estimated rotational parameters correspond to p -dimensional directional random vectors (DRVs). The aim of this work is towards modeling the *effect* of noise in the acquired tri-axial signal in terms of fluctuations in the estimated rotational parameters as shown in Fig. 5.1. It is useful to note, at this juncture, that noise in the acquired tri-axial signal will affect and cause random fluctuations in the estimated rotational parameters.

Elliptical polarization is defined using the instantaneous rotational parameters that parameterize the orbital phase of the real-valued tri-axial trajectory. The difference in the distribution of the DRV between the presence and absence of a seismic source is determined using both parametric and non-parametric approaches. These directional statistics based detection metrics detect the presence of seismic source signals via phase-synchronized segments of the tri-axial signal [88, 89]. The framework and the detection metrics are derived using human footsteps as the seismic source. Nonetheless, the algorithm can be applied to vehicular signals, as will be illustrated in the experiment section.

5.1 Orientation-Quaternion for Tri-axial Signals

The propagation medium, i.e., the Earth, induces attenuation and phase shift in the propagating wavefront. The magnitude of a velocity-vector determines the amount of attenuation, whereas evolution of the trajectory determines the phase of the wavefront. In Chapter 3.2, elliptical polarization was characterized using the smoothness of the trajectory and quantified in terms of the proximity between consecutive snapshots of 3D velocity vector. Although detection metrics proposed in that chapter quantified the smoothness of the trajectory, they did not exploit the instantaneous rotational characteristics of the trajectory. In this chapter, the smoothness of the elliptically polarized trajectory is quantified using its instantaneous rotational characteristics.

A quaternion rotational model is proposed to quantify the instantaneous orbital phase of the real-valued tri-axial trajectory using orientation-quaternions. It is important to note that this model does not exploit the ellipticity of the trajectory and is mainly aimed towards parameterizing the rotational evolution of the particle velocity rather than to exactly synthesize a unique seismic signature. The instantaneous movement of the tri-axial trajectory can be expressed as a instantaneous 3D rotation as

$$\mathbf{x}(n) = \mathbf{R}(n)\mathbf{x}(n-1), \quad (5.1)$$

where the 3×1 vector at time instant n is expressed as the rotation of $\mathbf{x}(n-1)$, at the previous snapshot. Since the focus of this work is on the directional attribute of the vector-valued signal, (5.1) can be rewritten in two steps

$$\begin{aligned} \bar{\mathbf{x}}(n) &= \mathbf{R}(n)\bar{\mathbf{x}}(n-1), \\ \mathbf{x}(n) &= \rho(n)\bar{\mathbf{x}}(n), \end{aligned} \quad (5.2)$$

where the first equation represents the instantaneous rotation of the normalized signal

$\bar{\mathbf{x}}(n)$ defined in (3.1) and the second equation represents the scaling of the normalized signal using a scalar $\rho(n)$, representing the instantaneous magnitude. The 3×3 rotation matrix is defined as

$$\mathbf{R}(n) = \begin{bmatrix} \mathcal{C}_y \mathcal{C}_z - \mathcal{S}_x \mathcal{S}_z + \mathcal{S}_x \mathcal{S}_y \mathcal{C}_z & \mathcal{S}_x \mathcal{S}_z + \mathcal{C}_x \mathcal{S}_y \mathcal{C}_z & \\ \mathcal{C}_y \mathcal{S}_z & \mathcal{C}_x \mathcal{S}_z + \mathcal{S}_x \mathcal{S}_y \mathcal{C}_z & -\mathcal{S}_x \mathcal{S}_z + \mathcal{C}_x \mathcal{S}_y \mathcal{S}_z \\ -\mathcal{S}_y & \mathcal{S}_x \mathcal{C}_y & \mathcal{C}_x \mathcal{C}_y \end{bmatrix}, \quad (5.3)$$

where $\mathcal{C}_x = \cos \phi_x$, $\mathcal{S}_x = \sin \phi_x$ while ϕ_x , ϕ_y , ϕ_z are the Euler angles about the x , y and z -axis. It is useful to know that Euler angles are limited by the gimbal lock problem [90]. This problem describes the occasional loss of one degree of freedom thereby locking the system in a degenerated two-dimensional (2D) space. Quaternion representation, on the other hand, addresses this problem by representing rotation in three-dimensional space as a single rotation about an eigenaxis instead of three Euler angle rotations about each of the coordinate axes [90]. Their compact representation and ease of renormalizing lead to faster and accurate computations involving rotations [91].

As defined in Chapter 4.1, quaternions are hypercomplex numbers written as

$$\mathbf{q} = \underbrace{q_0}_{\mathbb{R}(\mathbf{q})} + \underbrace{q_1\iota + q_2j + q_3\kappa}_{\mathbb{I}(\mathbf{q})}, \quad (5.4)$$

where $\mathbb{R}(\cdot)$ is the real scalar part and $\mathbb{I}(\cdot)$ is the vector imaginary part consisting of three orthogonal imaginary components ι , j and κ , such that $\iota j \kappa = \iota^2 = j^2 = \kappa^2 = -1$, $\iota j = \kappa$, $j \kappa = \iota$, $\kappa \iota = j$. When $q_0 = 0$, $\mathbf{q}$ is referred as *pure* or vector-quaternion and is used to represent a 3×1 vector as shown in Chapter 4.1. However, when $q_0 \neq 0$ it is referred as a full quaternion. Additionally when $\|\mathbf{q}\|=1$, the full quaternion is used to represent rotation in three-dimension. The full quaternions will be referred as *orientation-quaternion* as opposed to vector-quaternion to distinguish between the two representations. Quaternions

Figure 5.2: Quaternion rotational model for polarized 3D velocity trajectory.

are also used in a number of applications including polarimetric analysis of synthetic aperture radar (SAR) images [92, 93], inverse SAR [94], 3D beam theory [95], multicomponent velocity analysis [96] and in orientation estimation [97].

The polar representation describe how an orientation-quaternion can be used to parameterize a rotation in 3D. The orientation-quaternion defined in (5.4) is alternatively expressed as [70]

$$\mathbf{q} = e^{\mathbf{z}\theta/2} = \cos \theta/2 + \mathbf{z} \sin \theta/2, \quad (5.5)$$

where

$$\mathbf{z} = \frac{q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}}{\sqrt{q_1^2 + q_2^2 + q_3^2}} \quad (5.6)$$

is defined as the *eigenaxis* (axis of rotation) and

$$\theta = 2 \tan^{-1} \left(\frac{\sqrt{q_1^2 + q_2^2 + q_3^2}}{q_0} \right), \quad (5.7)$$

defines the *eigenangle* (angle of rotation). Orientation-quaternion with $\|\mathbf{q}\| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$ is used to rotate a 3D vector about an eigenaxis $\mathbf{z}$ through an eigenangle θ .

Instead of the vector multiplication in (5.1) using $\mathbf{R}(n)$, rotation of a 3D vector using an orientation-quaternion involves sandwiching the vector quaternion (formed from the 3D vector) between the orientation-quaternion and its conjugate as

$$\bar{\mathbf{x}}(n) = \mathbf{q}(n) \star \bar{\mathbf{x}}(n-1) \star \mathbf{q}^{-1}(n), \quad (5.8)$$

where $\bar{\mathbf{x}}(n) = \bar{x}_1(n)\iota + \bar{x}_2(n)j + \bar{x}_3(n)\kappa$, $\mathbf{q}^{-1} = \mathbf{q}^*/\|\mathbf{q}\| = q_0 - q_1\iota - q_2j - q_3\kappa$ and $\star$ denotes the quaternion product. The sandwiching operation using a unit quaternion and its conjugate combines two half-angle rotations to rotate the vector $\bar{\mathbf{x}}(n-1)$ through an angle $\theta(n)$ [91]. The magnitude of the tri-axial velocity trajectory is modeled using a real excitation signal $\rho(n)$, i.e., $\mathbb{I}(\rho(n)) = 0$, such that

$$\mathbf{x}(n) = \rho(n) \star \bar{\mathbf{x}}(n). \quad (5.9)$$

Using (5.5), (5.8) and (5.9), a piecewise segment of $\mathbf{x}(n)$ of length N is understood as a sequence of successively scaled 3D rotations

$$\bar{\mathbf{x}}(n) = e^{\mathfrak{z}(n)\theta(n)/2} \star \bar{\mathbf{x}}(n-1) \star \left(e^{\mathfrak{z}(n)\theta(n)/2}\right)^{-1}, \quad (5.10)$$

$$\mathbf{x}(n) = \rho(n) \star \bar{\mathbf{x}}(n), \quad (5.11)$$

for $n = 1, 2, \dots, N$, such that $\bar{\mathbf{x}}(0)$ will be the initial point on the trajectory and that $\theta(n)$ and $\mathfrak{z}(n)$ are the instantaneous angle and axis of rotation, respectively. The eigenangle $\theta(n)$ and eigenaxis $\mathfrak{z}(n)$ therefore model the magnitude and direction of the instantaneous orbital phase, while $\rho(n)$ models the energy of the trajectory. Figure 5.2 is a pictorial representation of the quaternion rotational model.

Different trajectories can be simulated by varying $\theta(n)$, $\mathfrak{z}(n)$ and $\rho(n)$. A linearly polarized velocity trajectory can be traced using $\theta(n) \approx \pi$ with $\mathfrak{z}(n)$ pointing in any direction perpendicular to $\bar{\mathbf{x}}(0)$. An elliptically polarized trajectory, as in our application,

Figure 5.3: Row 1: Plot of (a) excitation signal $\rho(n)$, (b) $\bar{\mathbf{x}}(n)$ and (c) $\mathbf{x}(n)$, simulated using $\theta(n) = \pi/10$, $\mathbf{z}(n) = 1\mathbf{i} + 0\mathbf{j} + 1\mathbf{k}$, $\bar{\mathbf{x}}(0) = 0\mathbf{i} + 1\mathbf{j} + 0\mathbf{k}$ for $N = 100$. Row 2: 3D plot of $\bar{\mathbf{x}}(n)$ with fluctuations in (d) $\theta(n)$, (e) $\mathbf{z}(n)$ and (f) $\mathbf{q}(n)$ generated as a DRV using vMF with finite concentration $\eta < \infty$ for $p = 2, 3, 4$, respectively.

can be described using $\theta(n) = \beta\pi, 0 < \beta \leq 1$. As an illustrative example, an elliptically polarized velocity trajectory is generated using an excitation signal as shown in Fig. 5.3(a) with $\theta(n) = \pi/10$, $\mathbf{z}(n) = 1\mathbf{i} + 0\mathbf{j} + 1\mathbf{k}$ and $\bar{\mathbf{x}}(0) = 0\mathbf{i} + 1\mathbf{j} + 0\mathbf{k}$ for $N = 100$. The initial point $\bar{\mathbf{x}}(0)$ together with $\mathbf{z}(n)$ determine the plane in which the trajectory is traced, i.e., the plane of polarization (POP.) The 3D plot of the normalized trajectory $\bar{\mathbf{x}}(n)$ and simulated signal $\mathbf{x}(n)$ are plotted in Figs. 5.3(b) and (c), respectively. From Fig. 5.3(b), it is observed that the smooth evolution that is consistent with that of a recorded footstep signal is exhibited. However, the elliptical path traced by a real footstep signal is wobbly due to the accumulation of composite polarization modes. Noise in the acquired signal cause randomness in the smooth trajectory traced on the surface of the sphere. This randomness can be modeled as fluctuations in the rotational parameters $\theta(n)$, $\mathbf{z}(n)$ and $\mathbf{q}(n)$. Directional distributions can be used to introduce fluctuations in the rotational parameters to model the effect of noise in the acquired signal.

Figure 5.4: Plot of DRV for $p = 3$ generated using vMF distribution for $\check{\boldsymbol{\mu}} = [1 \ 0 \ 0]^T$ and (a) $\eta = 0$, (b) $\eta = 10$ and (c) $\eta = 100$.

5.1.1 Directional Distribution

Fluctuations in the rotational parameters can be modeled with the help of directional distributions. The probability density function (pdf) of the von Mises-Fisher (vMF) distribution (a directional distribution [98] for an arbitrary p -dimensional directional random vector $\mathbf{d}^{(p)}$ on the hypersphere $\mathbb{S}^{p-1} = \{\mathbf{d}^{(p)} \in \mathbb{R}^p \mid \|\mathbf{d}^{(p)}\| = 1\}$) is defined as

$$F_p^v(\mathbf{d}^{(p)}; \check{\boldsymbol{\mu}}, \eta) = C_p(\eta) e^{\eta \check{\boldsymbol{\mu}}^T \mathbf{d}^{(p)}}, \quad \mathbf{d}^{(p)} \in \mathbb{S}^{p-1}. \quad (5.12)$$

The variable $\check{\boldsymbol{\mu}} \in \mathbb{S}^{p-1}$ is the mean direction, $\eta \geq 0$ is known as the concentration parameter and the superscript ‘v’ stands for vMF distribution. For $\eta = 0$ the DRV is uniformly distributed on the surface of $\mathbb{S}^{p-1}$ whereas for $\eta \rightarrow \infty$ the DRV is concentrated around $\check{\boldsymbol{\mu}}$. The normalizing constant $C_p(\eta)$ is defined as

$$C_p(\eta) = \frac{\eta^{p/2-1}}{(2\pi)^{p/2} I_{p/2-1}(\eta)}, \quad (5.13)$$

where $I_{p/2-1}(\eta)$ is the modified Bessel function of the first kind of order $p/2 - 1$ [99].

Although there exists other circular models such as the projected normal distribution and the cardioid distribution [98], the vMF is a versatile distribution that can be used to generate DRVs lying on the hypersphere $\mathbb{S}^{p-1}$ for dimensions $p = 2, 3, 4$ [98]. Fluctuations in $\theta(n)$, $\mathfrak{z}(n)$ and $\mathfrak{q}(n)$ can be introduced by generating them as DRVs using vMF with finite concentration $0 < \eta < \infty$, as opposed to having constant values ($\eta \rightarrow \infty$) as shown in Fig. 5.3(b). As an example to illustrate the role of concentration metric η in the pdf, directional random vectors are generated using vMF for $p = 3$ and plotted in Fig. 5.4(a), (b) and (c) for increasing value of η . It can be observed that as η increase, the concentration of the DRV increases.

Using the quaternion rotational model, the effect of fluctuations on the normalized 3D trajectory is next analyzed. Figures 5.3 (d), (e) and (f) illustrate variations in the trajectory $\bar{\mathbf{x}}(n)$ simulated when $\theta(n)$, $\mathfrak{z}(n)$ and $\mathfrak{q}(n)$ are generated using vMF pdf with $\eta < \infty$, respectively. The fluctuations in $\theta(n)$ alter the amount of rotation between each time snapshot while fluctuations in $\mathfrak{z}(n)$ cause a change in the POP of the trajectory. However, fluctuations in $\mathfrak{q}(n)$ change both instantaneous rotation angle and rotation axis. Comparing Fig. 5.3(d), (e) and (f), it is noted that the fluctuations in $\theta(n)$ do not change the POP, whereas fluctuations in $\mathfrak{z}(n)$ and $\mathfrak{q}(n)$ do. Using the quaternion rotational model, a velocity-to-rotation transform is proposed in the following section to estimate the rotational parameters from the acquired tri-axial signal. These estimated rotational parameters can be fitted to directional distributions to examine the amount of fluctuations in them.

5.1.2 Proposed Velocity-to-Rotation Transform

A transform is proposed to estimate these rotational parameters from the acquired signal $\mathbf{x}(n)$. The velocity-to-rotation (VR) transform converts the evolution of the tri-axial data into a sequence 3D rotations, representing the instantaneous orbital phase in two

steps. First, $\widehat{\theta}(n)$ and $\widehat{\mathbf{z}}(n)$ are estimated from the consecutive snapshots of $\mathbf{x}(n)$. The instantaneous 3D rotation is then estimated as $\widehat{\mathbf{q}}(n)$ using $\widehat{\theta}(n)$ and $\widehat{\mathbf{z}}(n)$. Since the aim of the transform is to quantify the instantaneous rotation of the velocity trajectory, tri-axial data is sampled at a high sampling rate of 8 kHz before evaluating the VR transform.

The instantaneous rotation angle is the angular distance between consecutive snapshots of $\mathbf{x}(n)$ and is evaluated using the dot product

$$\widehat{\theta}(n) = \cos^{-1} \left(\frac{\mathbf{x}(n-1) \cdot \mathbf{x}(n)}{\|\mathbf{x}(n-1)\| \|\mathbf{x}(n)\|} \right). \quad (5.14)$$

The instantaneous rotation axis, on the other hand, is normal to the plane containing the path traced and hence evaluated as a vector cross product

$$\begin{aligned} \widehat{\mathbf{z}}(n) &= \mathbf{x}(n-1) \times \mathbf{x}(n) \\ &= [\widehat{z}_1(n) \ \widehat{z}_2(n) \ \widehat{z}_3(n)]^T, \end{aligned} \quad (5.15)$$

which can further be expressed as a vector quaternion

$$\widehat{\mathbf{z}}(n) = \frac{\widehat{z}_1(n)}{\|\widehat{\mathbf{z}}\|}i + \frac{\widehat{z}_2(n)}{\|\widehat{\mathbf{z}}\|}j + \frac{\widehat{z}_3(n)}{\|\widehat{\mathbf{z}}\|}k. \quad (5.16)$$

The variables $\widehat{\theta}(n)$ and $\widehat{\mathbf{z}}(n)$ are then used to construct the instantaneous 3D rotation

$$\widehat{\mathbf{q}}(n) = \cos \frac{\widehat{\theta}(n)}{2} + \widehat{\mathbf{z}}(n) \star \sin \frac{\widehat{\theta}(n)}{2}. \quad (5.17)$$

The relationship between $\bar{\mathbf{x}}(n-1)$ and $\bar{\mathbf{x}}(n)$ is then given by

$$\bar{\mathbf{x}}(n) = \widehat{\mathbf{q}}(n) \star \bar{\mathbf{x}}(n-1) \star \widehat{\mathbf{q}}^{-1}(n), \quad (5.18)$$

for $n = 1, 2 \dots N$.

Figure 5.5: Plot of distributions of eigenangle $\hat{\theta}(n)$ in the (a) presence, (d) absence of a footstep; eigenaxis $\hat{\mathbf{z}}(n)$ in the (b) presence and (e) absence of a footstep; $\mathbb{I}(\hat{\mathbf{q}}(n))$ in the (c) presence and (f) absence of a footstep.

The distribution of estimated rotational parameters in the presence and absence of a footstep, recorded at a distance of 3 m from the tri-axial geophone, is illustrated in Fig. 5.5. Due to the high sampling rate of 8 kHz, $\hat{\theta}(n)$ is distributed on the first quadrant of $\mathbb{S}^1$, i.e., $0 < \hat{\theta}(n) < \pi/2$. More importantly, as seen from Fig. 5.5(a) and (d), $\hat{\theta}(n)$ corresponding to the presence of a footstep signal is incrementally small compared to the absence of a footstep. This can be attributed to the nature of elliptically polarized velocity-vector of a footstep which traces a smooth path with small rotation angles. The eigenaxis $\hat{\mathbf{z}}(n)$ is distributed on the surface of a unit sphere $\mathbb{S}^2$ and Figs. 5.5(b) and (e), respectively, illustrate its distribution corresponding to the presence and absence of a footstep. As can be seen, the elliptical orbit traced by $\bar{\mathbf{x}}(n)$ in the presence of a footstep results in a high concentration of $\hat{\mathbf{z}}(n)$ when compared to random background noise. The orientation-quaternion $\hat{\mathbf{q}}(n)$ on $\mathbb{S}^3$ is visualized using two solid and one empty sphere shells [90]. Since $0 \leq \hat{\theta}(n) \leq \pi/2$, $\mathbb{R}(\hat{\mathbf{q}}(n)) > 0$, only the imaginary component $\mathbb{I}(\hat{\mathbf{q}}(n))$ is plotted in the presence and absence of a footstep in Figs. 5.5(c) and (f), respectively. It is observed that the distribution of $\hat{\mathbf{q}}(n)$ is concentrated in the presence of a footstep signal

compared to its distribution in its absence. Hence, footstep signals can be characterized by an increased concentration of the rotational parameters in their respective hypersphere due to the presence of elliptical polarization.

The estimated eigenaxis $\hat{\mathbf{z}}(n)$ is orthogonal to the trajectory traced, which results in $\hat{\boldsymbol{\theta}}(n)$, $\hat{\mathbf{z}}(n)$ and the instantaneous rotation $\hat{\mathbf{q}}(n)$ being independently generated. In addition, as observed from Fig. 5.5, the rotational parameters estimated from a piecewise segment of $\mathbf{x}(n)$ can be fitted to the vMF pdf defined in (5.12) for dimensions $p = 2, 3, 4$. Hence, $\hat{\boldsymbol{\theta}}(n)$, $\hat{\mathbf{z}}(n)$ and $\hat{\mathbf{q}}(n)$ can be assumed to be independent and identically distributed (i.i.d) DRVs lying on hypersphere $\mathbb{S}^{p-1}$ for $p = 2, 3, 4$, respectively. In the context of our application, the DRV $\mathbf{d}^{(p)}$ can take any of the following forms such as

$$\begin{aligned}\mathbf{d}^{(2)} &= [\cos \hat{\theta} \ \sin \hat{\theta}]^T \in \mathbb{S}^1 \text{ for } p = 2, \\ \mathbf{d}^{(3)} &= \hat{\mathbf{z}}(n) \in \mathbb{S}^2 \text{ for } p = 3, \\ \mathbf{d}^{(4)} &= \hat{\mathbf{q}}(n) \in \mathbb{S}^3 \text{ for } p = 4,\end{aligned}\tag{5.19}$$

where $\hat{\mathbf{z}}(n)$ is the vector representation of $\hat{\mathbf{z}}(n)$ defined in (5.15) and

$$\hat{\mathbf{q}}(n) = [\hat{q}_0(n) \ \hat{q}_1(n) \ \hat{q}_2(n) \ \hat{q}_3(n)]^T\tag{5.20}$$

is the vector representation of $\hat{\mathbf{q}}(n)$.

5.2 Proposed Detection Metrics for Elliptically Polarized Trajectory

The differences in the distribution of the DRV between the presence and absence of footsteps is quantified using two approaches. The first proposed approach is a parametric approach where the difference is quantified by estimating the concentration parameter

Figure 5.6: Proposed detection framework for elliptically polarized footstep signals.

η using overlapping frames of the estimated rotational parameters. The second proposed approach is a non-parametric approach where Kullback-Liebler divergence (KLD) is computed between the directional densities evaluated using the estimated rotational parameters and a reference distribution. In both approaches, the estimated rotational parameters $\hat{\theta}(n)$, $\hat{\mathbf{z}}(n)$ and $\hat{\mathbf{q}}(n)$ are statistically analyzed in their respective hypersphere $\mathbb{S}^{p-1}$ for $p = 2, 3, 4$, respectively. The proposed framework for detection of human footsteps exploiting elliptical polarization is shown in Fig. 5.6.

5.2.1 Parametric Approach: Estimation of η

As observed from Fig. 5.5, the detection of an elliptically polarized trajectory of a footstep signal can be defined as a hypothesis test

$$\begin{aligned}
 H_0: \mathbf{d}_{\text{fluctuation}}^{(p)} &\sim F_p^v(\check{\boldsymbol{\mu}}, \eta \rightarrow 0) - \text{absence of footstep} \\
 H_1: \mathbf{d}_{\text{signal}}^{(p)} &\sim F_p^v(\check{\boldsymbol{\mu}}, 0 < \eta < \infty) - \text{presence of footstep}.
 \end{aligned} \tag{5.21}$$

Hence, in the parametric approach, overlapping frames of the rotational parameters $\hat{\theta}(n)$, $\hat{\mathbf{z}}(n)$ and $\hat{\mathbf{q}}(n)$ are fitted to the vMF density defined in (5.12) and the concentration parameter η is estimated. The estimated $\hat{\eta}$ is expected to be high in the presence of elliptically polarized footsteps signal and remain low for random background noise.

In order to estimate η , let $\mathbf{d}_1^{(p)}, \mathbf{d}_2^{(p)} \dots \mathbf{d}_N^{(p)} \in \mathbb{S}^{p-1}$ denote a set of N i.i.d DRVs following a vMF density $F_p^v(\mathbf{d}^{(p)}; \check{\boldsymbol{\mu}}, \eta)$ defined in (5.12). To estimate η , logarithm is

applied to their combined likelihood giving [99]

$$\mathcal{L}(\mathbf{d}^{(p)}; \check{\boldsymbol{\mu}}, \eta) = N \ln(C_p(\eta)) + \eta \check{\boldsymbol{\mu}}^T \tilde{\mathbf{d}}^{(p)}, \quad (5.22)$$

where $\tilde{\mathbf{d}}^{(p)} = \sum_{n=1}^N \mathbf{d}_n^{(p)}$ is the resultant vector. Differentiating (5.22) w.r.t η results in

$$\frac{NC'_p(\eta)}{C_p(\eta)} + NL(\mathbf{d}^{(p)}) = 0, \quad (5.23)$$

where

$$L(\mathbf{d}^{(p)}) = \frac{1}{N} \left\| \sum_{n=1}^N \mathbf{d}_n^{(p)} \right\| \quad (5.24)$$

is the mean resultant length and

$$C'_p(\eta) = \frac{(p/2 - 1)\eta^{p/2-2}}{(2\pi)^{p/2} I_{p/2-1}(\eta)} - \frac{\eta^{p/2-1} I'_{p/2-1}(\eta)}{(2\pi)^{p/2} I_{p/2-1}^2(\eta)} \quad (5.25)$$

is the partial derivative of $C_p(\eta)$ defined in (5.13) w.r.t η . Dividing (5.25) with (5.13) and using the property

$$\eta I_{p/2}(\eta) = \eta I'_{p/2-1}(\eta) - (p/2 - 1) I_{p/2-1}(\eta), \quad (5.26)$$

to substitute for the partial derivative $I'_{p/2-1}(\eta)$, results in

$$-\frac{C'_p(\eta)}{C_p(\eta)} = \frac{I_{p/2}(\eta)}{I_{p/2-1}(\eta)} \triangleq A_p(\eta) \quad (5.27)$$

where $A_p(\eta)$ is defined as the ratio of modified Bessel functions. Substituting (5.27) in (5.23) the maximum-likelihood estimate (MLE) of η for an arbitrary DRV $\mathbf{d}^{(p)}$ is given by

$$\hat{\eta}(\mathbf{d}^{(p)}) = A_p^{-1}(L(\mathbf{d}^{(p)})), \quad (5.28)$$

where $A_p^{-1}(L(\mathbf{d}^{(p)}))$ is the inverse of the ratio of modified Bessel functions. Since there exists no closed-form solution for evaluating $A_p^{-1}(L(\mathbf{d}^{(p)}))$, numerical methods are employed to approximate a solution. For $p = 2, 3$ the MLE is evaluated as [98]

$$\hat{\eta}(\mathbf{d}^{(p)}) = pL(\mathbf{d}^{(p)}). \quad (5.29)$$

For $p > 3$, the MLE is evaluated as [99]

$$\hat{\eta}(\mathbf{d}^{(p)}) = \frac{L(\mathbf{d}^{(p)})p - L^3(\mathbf{d}^{(p)})}{1 - L^2(\mathbf{d}^{(p)})}. \quad (5.30)$$

In the context of footstep detection, the concentration is estimated by computing the mean resultant lengths of $\hat{\theta}(n)$, $\hat{\mathbf{z}}(n)$ and $\hat{\mathbf{q}}(n)$ across overlapping frames using (5.24). The mean resultant length of $\hat{\theta}(n) \in \mathbb{S}^1$ is evaluated, across N samples, using the directional cosines as [98]

$$L(\hat{\theta}) = \sqrt{\left[\sum_{n=1}^N \cos \hat{\theta}(lN - l_o + n) \right]^2 + \left[\sum_{n=1}^N \sin \hat{\theta}(lN - l_o + n) \right]^2}. \quad (5.31)$$

The mean resultant length of $\hat{\mathbf{z}}(n) \in \mathbb{S}^2$ is evaluated as

$$L(\hat{\mathbf{z}}) = \frac{1}{N} \left\| \sum_{n=1}^N \hat{\mathbf{z}}(lN - l_o + n) \right\|, \quad (5.32)$$

where $\hat{\mathbf{z}}(n)$ is defined in (5.15). Similar to $\hat{\mathbf{z}}(n)$, the mean resultant length of $\hat{\mathbf{q}}(n) \in \mathbb{S}^3$ is given by

$$L(\hat{\mathbf{q}}) = \frac{1}{N} \left\| \sum_{n=1}^N \hat{\mathbf{q}}(lN - l_o + n) \right\| \quad (5.33)$$

where $\hat{\mathbf{q}}(n)$ has been defined in (5.20).

After computing the mean resultant lengths, the concentrations $\hat{\eta}(\hat{\theta})$ and $\hat{\eta}(\hat{\mathbf{z}})$ are estimated using (5.29) and $\hat{\eta}(\hat{\mathbf{q}})$ is estimated using (5.30). Using (5.21), footstep detection

is then defined as a hypothesis test using $\hat{\eta}(\mathbf{d}^{(p)})$ given by

$$\begin{aligned} H_0 &: \hat{\eta}(\mathbf{d}^{(p)}) < \gamma - \text{absence of footstep;} \\ H_1 &: \hat{\eta}(\mathbf{d}^{(p)}) > \gamma - \text{presence of footstep,} \end{aligned} \quad (5.34)$$

where γ is a suitable threshold. For real applications, the null hypothesis H_0 is rejected for large values of $\hat{\eta}$ since noise is not completely unpolarized and random.

The Cramer-Rao lower bound (CRLB) for the concentration estimator $\hat{\eta}(\mathbf{d}^{(p)})$ is derived to understand the relationship between the variance of the estimator and true concentration η of the sample DRVs. The CRLB is derived as

$$\text{var}(\hat{\eta}(\mathbf{d}^{(p)})) \geq \frac{1}{-\mathbb{E}\left\{\frac{\partial^2 \mathcal{L}(\mathbf{d}^{(p)})}{\partial \eta^2}\right\}}, \quad (5.35)$$

where $\text{var}(\cdot)$ is the variance, $\mathbb{E}(\cdot)$ is the expectation operator and $\mathcal{L}(\mathbf{d}^{(p)})$ is the log-likelihood of the vMF pdf defined in (5.22). Differentiation (5.22) with respect to η , we get

$$\begin{aligned} \frac{\partial \mathcal{L}(\mathbf{d}^{(p)})}{\partial \eta} &= -N A_p(\eta) + N L(\mathbf{d}^{(p)}), \\ \frac{\partial^2 \mathcal{L}(\mathbf{d}^{(p)})}{\partial \eta^2} &= A'_p(\eta) = \left(\frac{I_{p/2}(\eta)}{I_{p/2-1}(\eta)} \right)'. \end{aligned} \quad (5.36)$$

Using the property (5.26) and

$$\eta I'_{p/2}(\eta) = \eta I_{p/2-1}(\eta) - p/2 I_{p/2}(\eta), \quad (5.37)$$

the second derivative in (5.36) can be simplified into

$$\frac{\partial^2 \mathcal{L}(\mathbf{d}^{(p)})}{\partial \eta^2} = A'_p(\eta) = 1 - (A_p(\eta))^2 - \frac{p-1}{\eta} A_p(\eta). \quad (5.38)$$

Thus the CRLB for the concentration estimator is given by

$$\text{var}(\hat{\eta}(\mathbf{d}^{(p)})) \geq \frac{1}{1 - (A_p(\eta))^2 - \frac{p-1}{\eta} A_p(\eta)}. \quad (5.39)$$

The variable $A_p(\eta)$ is a non-linear function and increases from 0 to 1 as η increases from 0 to ∞ . This implies that as the value of true concentration η increases, the estimated concentration $\hat{\eta}$ has a higher variance or lower precision. In the context of our application, $\hat{\eta}(\mathbf{d}^{(p)})$ is expected to have high variance when estimated from signal segments containing elliptically polarized signal than when estimated from random background noise.

To illustrate the feasibility of the above proposed approach, the concentration metrics are evaluated for a footstep signal recorded when a human subject walks from 2.5 to 6 m away from the tri-axial geophone. Figure 5.7(a) shows the signal from the z -axis of the geophone while Figs. 5.7(b) and (c) illustrate $\hat{\theta}(n)$ and its corresponding $\hat{\eta}(\hat{\theta})$, for overlapping frames of length $N = 200$ ms (with 50% overlap), respectively. It is observed that $\hat{\theta}(n)$ decreases in the presence of a footstep due to the smooth trajectory traced by elliptically polarized signals which, in turn, results in the increase of $\hat{\eta}(\hat{\theta})$. Figure 5.7(d) illustrates the concentration of eigenaxis $\hat{\eta}(\hat{\mathbf{z}})$ evaluated using a frame length $N = 100$ ms with a 50% overlap. The concentration parameter $\hat{\eta}(\hat{\mathbf{z}})$ is estimated over a shorter frame length than that used for $\hat{\eta}(\hat{\theta})$. This is because the instantaneous eigenangle $\hat{\theta}(n)$ (that parameterizes the instantaneous magnitude of the orbit), is small over the entire duration of the footstep signature whereas the corresponding instantaneous eigenaxis $\hat{\mathbf{z}}(n)$ (that parameterizes the instantaneous direction of the orbital phase) is concentrated over short segments within a single footstep signature. Comparing Figs. 5.7(c) and (d), it is observed that $\hat{\eta}(\hat{\mathbf{z}})$ exhibits a better distinction between footstep and noise compared to $\hat{\eta}(\hat{\theta})$. This is due to the significant difference in the distribution of $\hat{\mathbf{z}}(n)$ on $\mathbb{S}^2$ (as shown in Figs. 5.5(b) and (e)) compared to the distribution of $\hat{\theta}(n)$, which is limited to only the first quadrant of $\mathbb{S}^1$ (as observed in Figs. 5.5(a) and (d)).

In order to estimate the overall concentration of the orbital phase, both $\hat{\eta}(\hat{\theta})$ and $\hat{\eta}(\hat{\mathbf{z}})$ is jointly realized resulting in the eigenangle-eigenaxis concentration (EEC) metric proposed as

$$\mathcal{M}_{\text{EEC}} = \hat{\eta}(\hat{\theta}) \cdot \hat{\eta}(\hat{\mathbf{z}}), \quad (5.40)$$

where $\hat{\eta}(\hat{\theta})$ and $\hat{\eta}(\hat{\mathbf{z}})$ are estimated using (5.29). The concentration of $\hat{\mathbf{q}}(n)$ can also be directly estimated in $\mathbb{S}^3$ using (5.30) and (5.33) and is proposed as

$$\mathcal{M}_{\text{CinH}} = \hat{\eta}(\hat{\mathbf{q}}), \quad (5.41)$$

where the subscript ‘CinH’ denotes for the proposed concentration in hypersphere metric. Figures 5.7(e) and (f) plot the $\mathcal{M}_{\text{EEC}}$ and $\mathcal{M}_{\text{CinH}}$ metrics, respectively. Comparing these results, it is observed that as the dimension p of the rotational parameter increases, the ability of the corresponding concentration to distinguish the presence of elliptical polarization improves. These results are consistent with observations derived from Fig. 5.5, where footsteps exhibited higher concentration (compared to noise) when the dimension of the hypersphere p is increased. In addition, the time segment over which $\hat{\eta}$ increases becomes shorter with increasing dimension. This can be attributed to the multiple modes of polarization impinging the tri-axial geophone resulting in a consistent elliptical trajectory over a section of the footstep signature. Furthermore in line with the CRLB derived in (5.39), the variance of the estimated concentration $\hat{\eta}$ is higher in the presence of elliptically polarized signal segments than in the presence of random noise. The $\mathcal{M}_{\text{CinH}}$ metric shows the best distinction between the presence and absence of a footstep as it intrinsically combines the statistical variations of both $\hat{\theta}(n)$ and $\hat{\mathbf{z}}(n)$.

Figure 5.7: Plot of (a) z -axis signal and the directional statistics based metrics, (b) $\hat{\theta}(n)$, (c) $\hat{\eta}(\hat{\theta})$, (d) $\hat{\eta}(\hat{\mathbf{z}})$, (e) $\mathcal{M}_{\text{EEC}}$, (f) $\mathcal{M}_{\text{CinH}}$.

5.2.2 Non-parametric Approach: Hyperspherical KLD

In the second proposed approach, a non-parametric k-nearest neighbour density (knn) is exploited to model the distribution of estimated rotational parameters. The Kullback-Liebler divergence (KLD) is then evaluated between the knn density estimated using the DRVs derived from the VR transform and the knn density estimated using a reference set of DRVs corresponding to unpolarized random noise. In the presence of elliptically polarized footstep signals, the KLD is expected to be high due to the high concentration of the DRVs resulting in a significant deviation in the distribution of the DRVs with respect to the reference set. On the other hand, in the absence of a footstep, the KLD remains low since the distribution of the DRVs is close to the reference distribution.

Let $\mathbf{d}_1^{(p)}, \mathbf{d}_2^{(p)} \dots \mathbf{d}_N^{(p)} \in \mathbb{S}^{p-1}$ denote a set of N i.i.d DRVs in $\mathbb{S}^{p-1}$. The knn density

centered at a particular DRV $\mathbf{d}_r^{(p)}$ is estimated as [100]

$$\widehat{F}_p^k(\mathbf{d}_r^{(p)}) = \frac{k/N}{\Psi(\phi_r)}, \quad (5.42)$$

where the superscript ‘k’ denotes for the knn pdf. The denominator

$$\Psi(\phi_r) = \frac{1}{2}\Phi_p \left[1 - \text{sgn}(\cos \phi_r) B_{\cos^2 \phi_r} \left(\frac{1}{2}, \frac{p-1}{2} \right) \right] \quad (5.43)$$

defines the surface area of the hyperspherical cap. Defining Γ as the Gamma function, the surface area of this hyperspherical cap is computed in terms of the total surface area of the hypersphere $\mathbb{S}^{p-1}$ given by $\Phi_p = 2\pi^{p/2}/\Gamma(\frac{p}{2})$. The term $B_{\cos^2 \phi_r}(\frac{1}{2}, \frac{p-1}{2})$ denotes the regularized incomplete Beta function where $\phi_r = \cos^{-1}(\mathbf{d}_r^{(p)T} \mathbf{d}_k^{(p)})$ is the angular distance between $\mathbf{d}_r^{(p)}$ and its k -nearest neighbours in the set of size N . The knn pdf is constructed as the average hyperspherical area enclosed by the k closest DRVs. Therefore, when the DRVs are concentrated on $\mathbb{S}^{p-1}$, the surface area of the hyperspherical cap containing the selected k nearest neighbour DRVs is small whereas when the DRVs are uniformly distributed, the corresponding area is large. For this knn non-parametric directional density $\widehat{F}_p^k(\mathbf{d}_r^{(p)})$, the entropy is estimated as an average across all the N DRVs in the set using [101]

$$\begin{aligned} \mathcal{H}(\widehat{F}_p^k) &= \mathbb{E}\{-\ln \widehat{F}_p^k(\mathbf{d}_r^{(p)})\} \\ &\approx \frac{1}{N} \sum_{r=1}^N \ln[N\Psi(\phi_r)] - \psi(k), \end{aligned} \quad (5.44)$$

where $\psi(k) = \Gamma'(k)/\Gamma(k)$ is a digamma function.

Given another set of M reference DRVs $\mathbf{d}_1^{(p)\text{ref}}, \mathbf{d}_2^{(p)\text{ref}}, \dots, \mathbf{d}_M^{(p)\text{ref}}$ i.i.d in $\mathbb{S}^{p-1}$, the knn

Figure 5.8: Plot of (a) z -axis signal and the directional statistics based metrics (b) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ (c) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ (d) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$.

density $F_p^k(\mathbf{d}_r^{(p)\text{ref}})$ is estimated similar to (5.42) as

$$\hat{F}_p^k(\mathbf{d}_r^{(p)\text{ref}}) = \frac{k/M}{\Psi(\phi_r^{\text{ref}})}, \quad (5.45)$$

where ϕ_r^{ref} is the angular distance between $\mathbf{d}_r^{(p)\text{ref}}$ and its k -nearest neighbours in the set.

The cross entropy between $\hat{F}_p^k(\mathbf{d}_r^{(p)})$ and $\hat{F}_p^k(\mathbf{d}_r^{(p)\text{ref}})$ is evaluated as [100]

$$\mathcal{H}(\hat{F}_p^k(\mathbf{d}_r^{(p)}), \hat{F}_p^k(\mathbf{d}_r^{(p)\text{ref}})) = \frac{1}{N} \sum_{r=1}^N \ln(N\Psi(\phi_r^{\text{ref}})) + \ln M - \psi(k). \quad (5.46)$$

The KLD metric that quantifies the deviation between $\hat{F}_p^k(\mathbf{d}_r^{(p)})$ and $\hat{F}_p^k(\mathbf{d}_r^{(p)\text{ref}})$, estimated

from DRVs of size N and M , respectively, is then computed using

$$\begin{aligned}\mathcal{M}_{\text{KLD}}(\mathbf{d}^{(p)}) &= \mathcal{H}(\widehat{F}_p^{\text{k}}(\mathbf{d}_r^{(p)}), \widehat{F}_p^{\text{k}}(\mathbf{d}_r^{(p)\text{ref}})) - \mathcal{H}(\widehat{F}_p^{\text{k}}(\mathbf{d}_r^{(p)})) \\ &= \frac{1}{N} \sum_{r=1}^N \ln \frac{\Psi(\phi_r^{\text{ref}})}{\Psi(\phi_r)} + \ln \frac{M}{N}.\end{aligned}\tag{5.47}$$

In the context of footstep detection, $\widehat{F}_p^{\text{k}}(\mathbf{d}_r^{(p)})$ is evaluated using overlapping frames of the DRVs (estimated from the VR transform) and $\widehat{F}_p^{\text{k}}(\mathbf{d}_r^{(p)\text{ref}})$ is evaluated using the DRVs generated from $F_p^{\text{v}}(\mathbf{d}^{(p)}; \boldsymbol{\mu}, 0)$, which corresponds to unpolarized random noise since $\eta = 0$. The proposed KLD metric $\mathcal{M}_{\text{KLD}}(\mathbf{d}^{(p)})$ is then evaluated using (5.47) for each DRV in their respective hyperspherical space. Footstep detection is then defined as a hypothesis test given by

$$\begin{aligned}H_0 &: \mathcal{M}_{\text{KLD}}(\mathbf{d}^{(p)}) < \gamma - \text{absence of footstep}; \\ H_1 &: \mathcal{M}_{\text{KLD}}(\mathbf{d}^{(p)}) > \gamma - \text{presence of footstep},\end{aligned}\tag{5.48}$$

where γ is a suitable threshold.

Figure 5.8 shows the comparison of the proposed KLD based metric for DRVs $\widehat{\theta}(n)$, $\widehat{\mathbf{z}}(n)$ and $\widehat{\mathbf{q}}(n)$ corresponding to a footstep train shown in Fig. 5.8(a). The knn density is estimated with $k = 400$ from the DRVs using frames of length 100 ms with 50% overlap. As expected, the KLD based metrics $\mathcal{M}_{\text{KLD}}(\widehat{\theta})$, $\mathcal{M}_{\text{KLD}}(\widehat{\mathbf{z}})$ and $\mathcal{M}_{\text{KLD}}(\widehat{\mathbf{q}})$ increase in the presence of a footstep. This is due to the difference in distribution of the DRVs in the presence of elliptically polarized signals compared to background noise as illustrated in Fig. 5.5. We note that, in the absence of a footstep $\mathcal{M}_{\text{KLD}}(\widehat{\mathbf{z}})$ is closer to zero than for $\mathcal{M}_{\text{KLD}}(\widehat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\widehat{\mathbf{q}})$. This implies that the distribution of $\widehat{\mathbf{z}}(n)$ in the absence of a footstep is closer to the distribution reference DRVs in $\mathbb{S}^2$. On the other hand, $\widehat{\theta}(n)$ is restricted to the first quadrant of $\mathbb{S}^1$ which deviates from the reference DRVs, distributed uniformly in $\mathbb{S}^1$. In addition, as described by (6.15), the distribution of $\widehat{\mathbf{q}}(n)$

is dependent on $\hat{\theta}(n)$. Hence, non-zero values of $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ are exhibited in the absence of a footstep. Further experiments also revealed that $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ did not exhibit any distinction between a footstep signal and noise for $k < 400$. This is due to the modest variation in the distribution of $\hat{\mathbf{z}}(n)$ for the presence and absence of a footstep signal as shown in Fig. 5.5(b) and (e), respectively, when the number of nearest neighbours selected is small.

Comparing the two proposed approaches, in the parametric approach, $\hat{\eta}(\hat{\theta})$ shows the least distinction in the presence of elliptical polarization, while in the non-parametric approach $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ shows the least distinction. However in both approaches, the metrics evaluated using $\hat{\mathbf{q}}(n)$ exhibits the best distinction between the presence and absence of elliptically polarized footstep signal, since it depends on the distributions of both $\hat{\theta}(n)$ and $\hat{\mathbf{z}}(n)$. It is important to note that the directional statistics based metrics rely on the rotational characteristics of the 3D trajectory. In the event that one of the channel malfunctions, the directional statistics based metrics will not be able to quantify elliptical polarization since the 3D trajectory will degenerate into a 2D trajectory.

5.3 Experiment Results and Discussions

The performance of the proposed directional statistics based detection metrics is illustrated first on simulated footstep-like DRVs followed by actual footstep signals acquired using a tri-axial geophone.

5.3.1 Detection on Simulated DRVs

Since the focus of the thesis is on modeling the effect of noise in tri-axial signal (in $\mathbb{R}^3$) in terms of random fluctuations in estimated rotational parameters (in $\mathbb{S}^3$), in this section, footstep-like DRVs are simulated in $\mathbb{S}^3$ to evaluate the proposed detection metrics. Using the von Mises-Fisher pdf defined in (5.12), the difference in the distribution of the DRV

Figure 5.9: Plot of (a) η and proposed parametric algorithms (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\xi})$, (d) $\hat{\eta}(\mathbf{q}_{\text{sim}})$ for $\mathbf{q}_{\text{sim}}(n)$ generated with random mean $\check{\boldsymbol{\mu}}(\mathbf{q})$. Footstep-like DRVs are simulated with (i) $\eta = [1000, 50]$ and (ii) $\eta = [40, 1]$ and noise DRVs with $\eta = 0$.

between the presence of a footstep signal and its absence is expressed as

$$\begin{aligned} \mathbf{d}_{\text{signal}}^{(p)} &: F_p^v(\check{\boldsymbol{\mu}}, 0 < \eta < \infty) - \text{presence of footstep} \\ \mathbf{d}_{\text{fluctuation}}^{(p)} &: F_p^v(\check{\boldsymbol{\mu}}, \eta = 0) - \text{absence of footstep.} \end{aligned} \quad (5.49)$$

In order to generate a train of footsteps, noise DRVs (900 samples in length) and footstep-like DRVs (100 samples in length) are generated using the vMF pdf with $\eta = 0$ and $0 < \eta < 1000$, respectively. The mean orientation $\check{\boldsymbol{\mu}}(\mathbf{q})$ is randomly generated in $\mathbb{S}^3$. The concentration η used to simulate the footstep-like DRVs is chosen in the range $[1000, 1]$, where consecutive footstep impulse in the train is generated with a decreasing value of η . The noise and footstep-like DRVs are appended alternatively to simulate a footstep train $\mathbf{q}_{\text{sim}}(n)$, where the subscript “sim” denotes simulated DRV.

The eigenaxis, $\hat{\xi}(n)$, and eigenangle, $\hat{\theta}(n)$, are evaluated from $\mathbf{q}_{\text{sim}}$ using (5.6) and (5.7), respectively. The proposed parametric and non-parametric algorithms are applied

Figure 5.10: Plot of (a) η and proposed non-parametric algorithms (b) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ (c) $\mathcal{M}_{\text{KLD}}(\hat{\xi})$ (d) $\mathcal{M}_{\text{KLD}}(\mathbf{q}_{\text{sim}})$ for $\mathbf{q}_{\text{sim}}(n)$ generated with random mean $\check{\boldsymbol{\mu}}(\mathbf{q})$. Footstep-like segments are simulated with (i) $\eta = [1000, 50]$ and (ii) $\eta = [40, 1]$ and noise segments with $\eta = 0$.

on $\mathbf{q}_{\text{sim}}(n)$ and the results are plotted in Figs. 5.9 and 5.10, respectively. The left and right column displays the result from footstep-like DRVs for $\eta = [1000 \ 900 \ 800 \ 700 \ 600 \ 500 \ 400 \ 300 \ 200 \ 100 \ 90 \ 80 \ 70 \ 60 \ 50]$ and $\eta = [40 \ 30 \ 20 \ 10 \ 9 \ 8 \ 7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1]$, respectively. The η used to simulate each impulse is plotted in Figs. 5.9(a) and 5.10(a). From these figures, we observe that both the parametric and the non-parametric approach are capable of detecting the presence of footstep-like signals, where $\eta > 0$. Comparing these plots, we observe that the variance of $\hat{\eta}(\hat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, using the eigenangle $\hat{\theta}(n)$, is high in the noise segments where $\eta = 0$. The detection performance of the two metrics deteriorate for impulses with $\eta < 10$. The descending value of η (used in the simulation) is best captured in $\hat{\eta}(\mathbf{q}_{\text{sim}})$ or $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\mathbf{q}_{\text{sim}})$. This can be attributed to the process by which $\mathbf{q}_{\text{sim}}(n)$ is simulated where variations in concentration of the footstep-like DRVs are introduced in the hyperspace $\mathbb{S}^3$.

In the next simulation, with results depicted in Figs. 5.11 and 5.12, instead of gener-

Figure 5.11: Plot of (a) η and proposed parametric algorithms (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathbf{z}})$, (d) $\hat{\eta}(\mathbf{q}_{\text{sim}})$ for $\mathbf{q}_{\text{sim}}(n)$ generated with $\check{\boldsymbol{\mu}}(\mathbf{q}) = \cos \theta/2 + \mathbf{z} \star \sin \theta/2$ for $\theta = \pi/20$ and randomly generated $\mathbf{z}$. Footstep-like segments are simulated with (i) $\eta = [1000, 50]$ and (ii) $\eta = [40, 1]$ and noise segments with $\eta = 0$.

ating the mean orientation randomly for each run, we used $\check{\boldsymbol{\mu}}(\mathbf{q}) = \cos(\theta/2) + \mathbf{z} \star \sin(\theta/2)$, where $\theta = \pi/20$ and $\mathbf{z}$ was randomly generated using vMF distribution for $p = 3$. Footsteps-like DRVs are then generated using vMF pdf for $p = 4$ and $0 < \eta < 1000$ similar to the first simulation. A footstep train is generated in the hypersphere $\mathbb{S}^3$ by appending footstep and noise segments successively. Figures 5.11 and 5.12 illustrate the performance of the parametric and non-parametric algorithms, respectively, on the DRVs generated using the above approach.

As opposed to the first simulation, we observe that the metrics using the eigenaxis, $\hat{\eta}(\hat{\mathbf{z}})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$, are noisy and their performance deteriorates for $\eta < 30$. This can be attributed to the random distribution of the eigenaxis while generating $\mathbf{q}_{\text{sim}}(n)$. Since concentration variation is introduced in $\mathbb{S}^3$, we observe that $\hat{\eta}(\mathbf{q}_{\text{sim}})$ captures the variation in the concentration in $\mathbf{q}_{\text{sim}}(n)$. From the simulation experiments, we observe that the proposed metrics using the orientation quaternion $\mathbf{q}_{\text{sim}}(n)$ are capable of distinguishing

Figure 5.12: Plot of (a) η , (b) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ (c) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ (d) $\mathcal{M}_{\text{KLD}}(\mathbf{q}_{\text{sim}})$ for $\mathbf{q}_{\text{sim}}(n)$ generated with $\check{\boldsymbol{\mu}}(\mathbf{q}) = \cos \theta/2 + \mathbf{z} \star \sin \theta/2$ for $\theta = \pi/20$ and randomly generated $\mathbf{z}$. Footstep-like segments are simulated with (i) $\eta = [1000, 50]$ and (ii) $\eta = [40, 1]$ and noise segments with $\eta = 0$.

small variations in concentration and $\hat{\eta}(\mathbf{q}_{\text{sim}})$ and $\mathcal{M}_{\text{KLD}}(\mathbf{q}_{\text{sim}})$ achieve the best detection performance.

Next, the performance of the proposed algorithms is evaluated on actual footstep signals recorded by a tri-axial geophone buried in a grassy field with good coupling. These footstep signals are recorded when a single human subject walks normally with soft-sole shoes. Seismic signals for different walking patterns including radial walking, circular walking, diagonal walking and stationary marching are used to evaluate the performance of the proposed metrics. The proposed metrics are evaluated on data sampled at 8 kHz.

5.3.2 Detection Range

The detection performance of the directional statistics based metrics is compared with conventional energy-based metrics $\hat{K}_c(l)$, $\hat{\lambda}_1(l)$ and trajectory-based metrics $\hat{\omega}(l)$, $\hat{\Omega}(l)$, described in Chapter 2.5.1, for increasing source-sensor distances. For clarity of presen-

Figure 5.13: Plot of (a) z -axis signal, (b) cumulative kurtosis $\hat{K}_c$ [69], (c) largest eigenvalue $\hat{\lambda}_1$ [50], (d) angular momentum $\hat{\omega}$ [54], (e) degree-of-directivity $\hat{\Omega}$ [54], for source-sensor distance 2 to 28 m.

tation, the baseline algorithms are plotted in Fig. 5.13 while the proposed algorithms are plotted in Fig. 5.14. For both figures, subplot (a) illustrate the z -axis data of the tri-axial geophone for a subject walking radially from 2 to 28 m, away from the sensor. The SNR corresponding to footstep signal shown in Fig. 5.13 was estimated to be 16, 10.5 and 6.5 dB at source-sensor distance of 10, 15 and 20 m, respectively. This SNR was estimated by evaluating the signal power as the cumulated tri-axial energy [3] over a frame of length 180 ms around a footstep signal. The noise power is estimated similarly using a signal frame approximately 0.3 s after the footstep window, i.e., between two footsteps. The baseline metrics $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$ are evaluated on signal bandpass filtered between 20 to 200 Hz, in order to reduce high frequency noise [13], using frames of length 250 ms with 50% overlap. These metrics are plotted in Figs. 5.13(b) and (c), respectively. It is observed that, as the source moves away from the sensor the footstep signature spreads, resulting in the deterioration of $\hat{K}_c(l)$. With increasing source-sensor distance the signal

energy also reduces, resulting in a significant reduction of $\hat{\lambda}_1(l)$. Additional tests showed that it may be possible to apply the 3D analytic signal proposed in [102] to tri-axial data and a footstep detection metric can be defined using the root mean-square amplitude of the model. However, the root mean-square amplitude is also an energy-based metric and was found to exhibit similar detection performance as $\hat{\lambda}_1(l)$.

Figures 5.13(d) and (e) illustrate the performance of the baseline metrics $\hat{\omega}(l)$ and $\hat{\Omega}(l)$, respectively. The metric $\hat{\omega}(l)$ is evaluated using the same frame length as $\hat{K}_c(l)$, whereas $\hat{\Omega}(l)$ is evaluated using shorter frames of length 100 ms with 50% overlap. Since the path of $\mathbf{x}(n)$ varies rapidly even within a footstep segment, $\hat{\Omega}(l)$ is computed over smaller time frames compared to $\hat{\omega}(l)$. From these results, we observed that $\hat{\omega}(l)$, which defines the degree of rotation of the tri-axial signal, fails to distinguish the presence of footsteps. Since this metric is a function of both $\mathbf{x}(n)$ and $\hat{\mathbf{v}}(n)$, its poor performance can be attributed to the random curvature of $\hat{\mathbf{v}}(n)$ (caused by composite modes of seismic polarization) and the decrease in signal energy with increasing source-sensor distance. On the other hand, $\hat{\Omega}(l)$ is noisy and the distinction of footsteps from the background noise segments is not significant. An increase in the value of $\hat{\Omega}(l)$ corresponding to the presence of footstep is evident only between 11-18 m. It is interesting to note that the degree-of-directivity $\hat{\Omega}(l)$ is the mean resultant length of $\hat{\mathbf{v}}(n)$ and that both $\hat{\mathbf{v}}(n)$ and $\hat{\mathbf{z}}(n)$ are on $\mathbb{S}^2$. For an elliptical trajectory that is traced, the eigenaxis points in the direction normal to the plane of the trajectory resulting in high value for $L(\hat{\mathbf{z}})$ due to the concentration of $\hat{\mathbf{z}}(n)$ as shown in Fig. 5.5(b). On the other hand, $\hat{\mathbf{v}}(n)$ points in the direction tangential to the trajectory traced by $\mathbf{x}(n)$, which constantly varies due to the changing curvature, resulting in a low concentration of $\hat{\mathbf{v}}(n)$. However, even for noisy tri-axial data, $\hat{\mathbf{z}}(n)$ exhibits higher concentration than $\hat{\mathbf{v}}(n)$. This implies that $\hat{\mathbf{z}}(n)$ is more clustered than $\hat{\mathbf{v}}(n)$ in $\mathbb{S}^2$ and hence it is expected that $\hat{\eta}(\hat{\mathbf{z}})$ will outperform $\hat{\Omega}(l)$. Due to the inconsistent performance of $\hat{\omega}(l)$ and $\hat{\Omega}(l)$, only $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$ will be considered in our subsequent analysis.

Figure 5.14: Plot of (a) z -axis signal, and the directional statistics based metrics (b) $\mathcal{M}_{\text{EEC}}$, (c) $\mathcal{M}_{\text{CinH}}$, (d) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, (e) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$, (f) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$, for source-sensor distance 2 to 28 m.

The proposed directional statistics based metrics are evaluated using sliding windows of length 100 ms with 50% overlap in order to extract rotational characteristics of the rapidly varying 3D velocity trajectory. Figures 5.14(b) and (c) illustrate the detection results for the proposed $\mathcal{M}_{\text{EEC}}$ and $\mathcal{M}_{\text{CinH}}$, respectively. It is observed that these metrics increase consistently in value in the presence of a footstep. This high performance is due to the fact that the rotational parameters, estimated using the VR transform, effectively parameterize the trajectory traced by the 3D velocity vector. While $\mathcal{M}_{\text{EEC}}$ exhibits noisy fluctuations beyond 23 m, $\mathcal{M}_{\text{CinH}}$ continues to peak in the presence of footsteps up to 28 m since it takes into account concentration of $\hat{\mathbf{q}}(n)$ in $\mathbb{S}^3$.

Figures 5.14(d), (e) and (f) illustrate the performance of the proposed KLD-based metrics $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$, respectively. Due to the short frame length used, the rotational characteristic of the trajectory in the presence of a footstep, all the directional statistics based metrics exhibit narrow and sharp peaks with each peak

being approximately in the middle of the footstep signature. Comparing within the proposed metrics, it is observed that $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ exhibit higher ability to detect footsteps, for all source-sensor distances, compared to $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$. This is due to the fact that with increasing source-sensor distance, $\hat{\mathbf{z}}(n)$ is more prone to background noise which causes an increased spreading of $\hat{\mathbf{z}}(n)$ in $\mathbb{S}^2$. Due to the relatively poor performance of $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$ compared to the other directional metrics, it will not be used in further comparisons. From these results, it is noted that the quaternion rotational model effectively parameterizes the 3D trajectory thus enabling the concentration of the 3D rotations to outperform conventional trajectory-based metrics such as $\hat{\omega}(l)$ and $\hat{\Omega}(l)$. Furthermore, concentration in $\mathbb{S}^3$ distinguishes elliptically polarized footstep signal from unpolarized background noise better than variance of the tri-axial signal in $\mathbb{R}^3$.

5.3.3 Rate of Decline with Source-Sensor Distance

The rate at which the directional statistics based metrics vary with source-sensor distance is next investigated. In order to have a common scale for comparison, each detection metric, evaluated over an entire footstep train acquired for a subject walking from 2 to 30 m away from the sensor, is normalized between $[0, 1]$. The distinguishing capability of a detection metric $\mathcal{V}(l)$ at each footstep is quantified using

$$\delta(m) = \bar{\mathcal{V}}_m(l) - \frac{1}{F} \sum_{l=1}^F \bar{\mathcal{V}}(l), \quad m = 1, \dots, M, \quad (5.50)$$

where m denotes the footstep index (out of M total number of footsteps in the received signal), $\bar{\mathcal{V}}(l)$ is the normalized metric evaluated over F frames across the entire received signal and $\bar{\mathcal{V}}_m(l)$ is the value of the metric corresponding to the m th footstep. The variable $\delta(m)$ therefore computes the difference between the value of a metric corresponding to each of the M footsteps and the mean value across the whole footstep train including background noise.

Figure 5.15: Plot of δ vs source-sensor distance for (a) parametric $\mathcal{M}_{\text{EEC}}$ and $\mathcal{M}_{\text{CinH}}$ metrics with $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$; (b) KLD-based $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, $\mathcal{M}_{\text{KLD}}(\hat{q})$ metrics with $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$.

Figures 5.15(a) and (b) illustrate how $\delta(m)$ varies with source-sensor distance (averaged over five walking experiments with different bearing angles), where the subject walks from 2 to 30 m away from the geophone with $M = 42$ footsteps. For clarity of presentation, $\delta(m)$ is plotted for alternate footsteps. From Fig. 5.15(a), it is observed that $\hat{\lambda}_1(l)$ reduces most significantly with increasing source-sensor distance followed by $\hat{K}_c(l)$. While $\hat{\lambda}_1(l)$ starts reducing right after the first footstep, $\hat{K}_c(l)$ fluctuates until approximately 12 m, after which it reduces at a lower rate due to the spread of the foot-step signature beyond some distance. It is also observed that while the proposed metrics $\mathcal{M}_{\text{EEC}}$ and $\mathcal{M}_{\text{CinH}}$ reduce with increasing source-sensor distance, they are consistently higher than $\hat{\lambda}_1(l)$ and $\hat{K}_c(l)$. From Fig. 5.15(b), it is observed that the $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\hat{q})$ metrics exhibit the slowest rate of decline with distance. This implies that the proposed directional statistics based metrics are less sensitive to source-sensor distance compared to conventional energy-based metrics.

5.3.4 Detection Threshold

The threshold γ applied for footstep detection decision using the directional statistics based metrics is dependent of the corresponding noise statistics. The background noise (i.e., absence of a footstep) is not completely random in real data collected from the field. Seismic sources such as the thumping of a bulldozer at a far-field construction site, heavy trucks passing by on the freeway near-by causes the background data to be mildly polarized. Since footsteps signals are intermittent and impulsive, a sub-optimal approach to estimate the noise statistics will be to compute the mean of the evaluated metric over long time windows. The proposed detection threshold is hence defined as

$$\gamma = \alpha \left(\frac{1}{N} \sum_{l=1}^N \mathcal{V}(l) \right), \quad (5.51)$$

where $\mathcal{V}(l)$ denotes the detection metric evaluated over N frames across long time periods, i.e., 3-4 s of the acquired signal. It was determined from various experiments that a value of $\alpha = 1.2$ exhibits robust detection. As an illustrative example, Fig. 5.16 illustrates the binary footstep detection decision plot corresponding to the $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ metrics, where the estimated instantaneous rotation $\hat{\mathbf{q}}(n)$ is used. The adaptive threshold is evaluated using (5.51) for footsteps recorded from 16 to 28 m from the sensor. Of the 18 footsteps, it is observed that the $\mathcal{M}_{\text{CinH}}$ metric detects 17 footsteps with 2 false alarms, whereas the $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ detects 16 footsteps with no false alarm.

Figure 5.16: Plot of (a) z -axis signal, and footstep detection decision for (b) $\mathcal{M}_{\text{CinH}}$ and (c) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ metrics, for source-sensor distance from 16 to 28 m.

5.3.5 Detection Rate vs False Alarm Rate

To evaluate the performance of the footstep detection the detection rate R_d and false alarm rate R_{fa} for footstep detection as

$$\begin{aligned} R_d &= \frac{\text{no. of footsteps detected}}{\text{total no. of actual footsteps}}, \\ R_{fa} &= \frac{\text{no. of false detections}}{\text{total no. of actual footsteps}}. \end{aligned} \quad (5.52)$$

For decreasing values of γ , R_d and R_{fa} are evaluated and the receiver operating characteristic curves (ROCs) are plotted in Fig. 5.17. These results show that $\mathcal{M}_{\text{CinH}}$ achieves the best operating performance followed by $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$. The energy-based metrics achieve poor performance compared to the directional statistics based metrics. The degree of accuracy of the detection metrics defined as [103]

$$\text{Accuracy} = \frac{\text{no. of footsteps detected} - \text{no. of false detection}}{\text{total no. of footsteps}} \quad (5.53)$$

Figure 5.17: Plot of ROC curves for the directional statistics based metrics $\mathcal{M}_{\text{CinH}}$, $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$, $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, $\mathcal{M}_{\text{EEC}}$ with energy-based $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$.

is computed next. It was found that $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ achieve the highest accuracy of 97% while $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ and $\mathcal{M}_{\text{EEC}}$ achieve 95% and 92%, respectively. On the other hand, the baseline metrics $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$ achieve only a modest (average) accuracy of 82%. These results show that footstep detection at increasing source-sensor distance can be achieved by quantifying the elliptical polarization of the tri-axial trajectory using the quaternion rotational model.

5.3.6 Detection of Footstep Signals for Different Walk Patterns

We further evaluate the performance of the proposed detection metrics on seismic signals from additional walking experiments such as circular walking, diagonal walking and stationary marching. For circular walking, a subject is made to walk around the geophone, buried at (0,0), along the circumference of a circle of radius of 6 m. Figure 5.18(a) shows the z -axis signal received by the sensor and we observe that the received signal is noisy. A total of 56 footsteps are recorded in this experiment. From these plots, we observe that the $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, plotted in Fig. 5.18(f), can better distinguish between the presence

Figure 5.18: Plot of (a) z -axis signal, (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathfrak{z}})$, (d) $\mathcal{M}_{EEC}$, (e) $\mathcal{M}_{CinH}$, (f) $\mathcal{M}_{KLD}(\hat{\theta})$, (g) $\mathcal{M}_{KLD}(\hat{\mathfrak{z}})$, (h) $\mathcal{M}_{KLD}(\hat{\mathfrak{q}})$ for signals received at (0,0) corresponding to circular walking at a radius of 6 m.

and absence of footsteps compared to $\hat{\eta}(\hat{\theta})$, plotted in Fig. 5.18(b). Compared to radial walking footsteps, we observe that the concentration of the eigenaxis measured parametrically as $\hat{\eta}(\hat{\mathfrak{z}})$ and non-parametrically as $\mathcal{M}_{KLD}(\hat{\mathfrak{z}})$ peaks in the presence of a footstep. Due to the noise in the acquired signal, the peaks in the proposed metrics, corresponding to a footstep signal, are not evident between 25-29 s.

Figures 5.19(d) and (e) illustrate the binary footstep detection decision corresponding to the proposed $\mathcal{M}_{CinH}$ and $\mathcal{M}_{KLD}(\hat{\mathfrak{q}})$ using an adaptive threshold evaluated using (5.51). The detection decision for the baseline energy-based metrics $\hat{K}_c$ and $\hat{\lambda}_1$ are plotted in Figs. 5.19(b) and (c), respectively. From the detection plots, we observe that the cumulative kurtosis $\hat{K}_c$, exhibit false alarms due to the presence of noise, which affects the peakyness of the acquired signal. On the other hand, the $\hat{\lambda}_1$ detects the footsteps accurately, except between 22-30 s, where few footsteps are not detected. This could be

Figure 5.19: Plot of (a) z -axis signal and footstep detection using (b) $\hat{K}_c$, (c) $\hat{\lambda}_1$, (d) $\mathcal{M}_{\text{CinH}}$, (e) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ for signals received at (0,0) corresponding to circular walking at a radius of 6 m.

attributed to the low energy of the corresponding footsteps. Comparing the proposed metrics, we observe that both $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ detect circular footsteps consistently compared to the baseline metrics. However, $\mathcal{M}_{\text{CinH}}$ tends to introduce false alarms in the detection compared to $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$.

For diagonal-walk footsteps, four different footstep trains are recorded when a person walks diagonally from (5,-5) to (-5,5), from (-5,5) to (5,-5), from (-5,-5) to (5,5) and from (5,5) to (-5,-5). For all these experiments, the seismic signals are acquired by a tri-axial geophone buried at (0,0). Figures 5.20 and 5.21 illustrate the performance of the proposed parametric and non-parametric detection metrics, respectively. We observe that the proposed metrics using the estimated orientation quaternion, $\hat{\mathbf{q}}(n)$, combine the two footsteps near the sensor, i.e., when the signal energy is maximum. This is because, when footsteps pass the geophone sensor very closely, polarized vibrations exist between the footsteps. These vibrations affect the polarization of the signal impinging the sensor,

Figure 5.20: Plot of (a) z -axis signal and proposed parametric metrics (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathbf{z}})$, (d) $\mathcal{M}_{EEC}$, (e) $\mathcal{M}_{CinH}$ for seismic signals received at (0,0) corresponding to diagonal walk from (i) (5,-5) to (-5,-5), (ii) (-5,5) to (5,-5), (iii) (-5,-5) to (5,5), (iv) (5,5) to (-5,-5).

which in turn affects the proposed trajectory-based metrics. However, the concentration of the eigenaxis estimated as $\hat{\eta}(\hat{\mathbf{z}})$ and $\mathcal{M}_{KLD}(\hat{\mathbf{z}})$ detect the two footsteps near the origin separately. Similar to circular walking, the non-parametric $\mathcal{M}_{KLD}(\hat{\theta})$ shows a better distinction between the presence and absence of a footstep for the estimated eigenangle $\hat{\theta}(n)$ than $\hat{\eta}(\hat{\theta})$.

We next evaluate the performance of the proposed algorithms for stationary sources at multiple sensor locations. In the first experiment, a hammer is used as the seismic source and impulsive signals are generated by striking the hammer against the ground at (5,-10) with geophones that are buried at five different Cartesian coordinates, as shown in Fig. 5.22. The average SNR corresponding a hammer strike signal, acquired by geophones G0, G1, G2, G3, G4, is estimated to be 35 dB, 22.7 dB, 22.8 dB, 23.82 dB, 65.32 dB, respectively. This SNR is estimated by evaluating the signal power as the tri-axial cumulated energy using (2.8) over a frame of length 180 ms around the hammer strike signal and the noise power as the cumulated energy over a frame of the same length

Figure 5.21: Plot of (a) z -axis signal and proposed non-parametric metrics (b) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, (c) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$, (d) $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ for seismic signals received at (0,0) corresponding to diagonal walk between (i) (5,-5) to (-5,5), (ii) (-5,5) to (5,-5), (iii) (-5,-5) to (5,5), (iv) (5,5) to (-5,-5).

in between two hammer strikes. The SNR of the signal at G4 is the maximum due to the close source-sensor proximity.

Figures 5.23 and 5.24 illustrate the performance of the proposed parametric and non-parametric detection metrics, respectively, corresponding to seismic signals from hammer strikes. We observe that the concentration of $\hat{\theta}(n)$ between the presence and absence of a hammer strike is evident in the non-parametric algorithms, similar to previous experiments. Since the range of the estimated eigenangle $\hat{\theta}(n)$, is limited to a single quadrant of the circle, distinction in concentration between the presence and absence of a seismic source fails to influence the parametric algorithms. On the other hand, the non-parametric approach captures this distinction due to its comparison against a reference uniform distribution. From these results, we observe that concentration of the estimated eigenaxis of the tri-axial trajectory distinguishes the presence and absence of the seismic source both in the parametric $\hat{\eta}(\hat{\mathbf{z}})$ and non-parametric $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$. The estimated eige-

Figure 5.22: Setup for the stationary hammer strike experiment.

naxis $\hat{\mathbf{z}}(n)$, is dependent on the nature of seismic polarization generated by the hammer strike impinging the sensor. Due to the sharp impact of the source, the trajectory traced by the acquired signal could be completely elliptical, causing the concentration of the eigenaxis to peak in the presence of the source. Comparing the concentration of the estimated orientation quaternion $\hat{\mathbf{q}}(n)$, we observe that $\mathcal{M}_{\text{CinH}}$ shows the best distinction between the presence and absence of the source across the SNR of the signal received.

In the last experiment, a subject marches at (16,0), i.e., walk at the same spot, and seismic signals are acquired by two geophones - G1 at (3,0) and G2 at (-3,0). Figures 5.25 and 5.27 illustrate the performance of the proposed parametric and non-parametric detection metrics using signals acquired at G1 and G2, respectively. We observe from these results, that the frequency of marching footsteps are slightly higher than the frequency of footsteps generated while walking. Unlike the hammer strike experiment, the concentration of the estimated eigenaxis $\hat{\mathbf{z}}(n)$, measured as $\hat{\eta}(\hat{\mathbf{z}})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{z}})$, shown in Figs. 5.25(c) and 5.25(g), does not increase significantly in the presence of the footstep signal. This is attributed to the fact that human footstep may not be as impactful on the ground as a hammer strike. As shown previously, a footstep signal traces an unsteady and wobbly elliptical trajectory in 3D resulting in the estimated eigenaxis to be scattered. In addition, stationary footsteps generate standing waves that result in correlated

Figure 5.23: Plot of (a) z -axis signal and proposed parametric metrics (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathbf{z}})$, (d) $\mathcal{M}_{EEC}$, (e) $\mathcal{M}_{CinH}$ corresponding to signals received at (i) G0, (ii) G1, (iii) G2, (iv) G3, (v) G4 for stationary hammer strikes at (5,-10)

Figure 5.24: Plot of (a) z -axis signal and proposed non-parametric metrics (b) $\mathcal{M}_{KLD}(\hat{\theta})$, (c) $\mathcal{M}_{KLD}(\hat{\mathbf{z}})$, (d) $\mathcal{M}_{KLD}(\hat{\mathbf{q}})$ for signals received at (i) G0, (ii) G1, (iii) G2, (iv) G3, (v) G4, corresponding to stationary hammer strikes at (5,-10).

noise that is polarized. This explains the variance of the proposed algorithms in the noise segments.

The signals received at G2, plotted in Fig. 5.27, have a lower SNR than the signals received at G1 due to the increased source-sensor distance. Figures 5.26 and 5.28 illustrate the footstep detection decision for the proposed $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ along with the energy-based metrics of $\hat{K}_c$ and $\hat{\lambda}_1$ corresponding to seismic signals received at G1 and G2, respectively. Within the proposed algorithms, it is observed that $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ achieves an improved detection rate compared to $\mathcal{M}_{\text{CinH}}$. Additionally, we note that $\mathcal{M}_{\text{CinH}}$ has an increased false alarm rate which is due to the high variance of the proposed metric in the noise segments. Comparing against the energy-based metric, we observe that the largest eigenvalue metric $\hat{\lambda}_1$ has the best detection performance for signals received at both G1 and G2. As the SNR deteriorates, the detection rate of $\hat{K}_c$ reduces. From the result we note that for stationary footsteps, the energy-based metrics perform better than the proposed algorithms. Due to the correlated noise caused by the standing waves, the false alarm rate of the proposed algorithms, especially $\mathcal{M}_{\text{CinH}}$, increases rapidly.

5.3.7 Detection of Vehicular Signals

Lastly, the proposed $\mathcal{M}_{\text{CinH}}$ metric is used to identify synchronized orbital phase from a tri-axial signal recording of vehicular motion. Figure 5.29 shows the experiment setup for a lorry travelling from a distance of 250 m at 30 km/h parallel to the x -axis of the sensor. From the figure, we note that the vehicle moves along the third and fourth quadrant with respect to the orientation of the sensor. The z -axis signal is shown in Fig. 5.30(a). It is known that vehicular signals exhibit long and consistent elliptical trajectory due to the nature of the source. The plot of $\mathcal{M}_{\text{CinH}}$, evaluated on frames of length 100 ms with 50% overlap, is shown in Fig. 5.30(b) and the detected segment of elliptically polarized trajectory, identified by applying a threshold of 0.75 of the mean

Figure 5.25: Plot of (a) z -axis signal, (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathfrak{z}})$, (d) $\mathcal{M}_{\text{EEC}}$, (e) $\mathcal{M}_{\text{CinH}}$, (f) $\mathcal{M}_{\text{KLD}}(\hat{\theta})$, (g) $\mathcal{M}_{\text{KLD}}(\hat{\mathfrak{z}})$, (h) $\mathcal{M}_{\text{KLD}}(\hat{\mathfrak{q}})$ for signals received at G1 at (3,0), corresponding to stationary footsteps at (16,0).

Figure 5.26: Plot of (a) z -axis signal and footstep detection using (b) $\hat{K}_c$, (c) $\hat{\lambda}_1$, (d) $\mathcal{M}_{\text{CinH}}$, (e) $\mathcal{M}_{\text{KLD}}(\hat{\mathfrak{q}})$ for signals received at G1 at (3,0), corresponding to stationary footsteps at (16,0).

Figure 5.27: Plot of (a) z -axis signal, (b) $\hat{\eta}(\hat{\theta})$, (c) $\hat{\eta}(\hat{\mathbf{z}})$, (d) $\mathcal{M}_{EEC}$, (e) $\mathcal{M}_{CinH}$, (f) $\mathcal{M}_{KLD}(\hat{\theta})$, (g) $\mathcal{M}_{KLD}(\hat{\mathbf{z}})$, (h) $\mathcal{M}_{KLD}(\hat{\mathbf{q}})$ for signals received at G2 at $(-3,0)$, corresponding to stationary footsteps at $(16,0)$.

Figure 5.28: Plot of (a) z -axis signal and footstep detection using (b) $\hat{K}_c$, (c) $\hat{\lambda}_1$, (d) $\mathcal{M}_{CinH}$, (e) $\mathcal{M}_{KLD}(\hat{\mathbf{q}})$ for signals received at G2 at $(-3,0)$, corresponding to stationary footsteps at $(16,0)$.

Figure 5.29: Outdoor setup for recording vehicular signals.

value of $\mathcal{M}_{\text{CinH}}$ estimated across the entire signal, is shown Fig. 5.30(c). It is observed that the elliptical polarization is dominant in the time segment when the vehicle is near the closest point of approach (CPA) of the sensor, which is approximately 5 m from the sensor. From the results, we note that the detection range of the proposed $\mathcal{M}_{\text{CinH}}$ metric is approximately 95 m from the sensor. However, it is observed that the detection range is slightly higher when the vehicle is moving away from the sensor (second half of the signal) compared to when the vehicle is moving towards the sensor.

5.4 Conclusion

In conclusion, a new VR transform was proposed to convert the time evolution of the 3D velocity trajectory into instantaneous rotational parameters using the quaternion rotational model. The estimated rotational parameters showed increased concentration in their respective hypersphere space, in the presence of elliptically polarized signals. Foot-step detection was then formulated using parametric and non-parametric approaches employing directional statistics. Of the proposed directional statistics based metrics, $\mathcal{M}_{\text{EEC}}$, $\mathcal{M}_{\text{CinH}}$, $\mathcal{M}_{\text{KLD}}(\hat{\theta})$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ metrics exhibit lower rate of decline with increasing source-sensor distances compared to the energy-based metrics $\hat{K}_c(l)$ and $\hat{\lambda}_1(l)$. The proposed $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ metrics achieved the best operating characteristics

Figure 5.30: Plot of (a) z axis signal, (b) $\mathcal{M}_{\text{CinH}}$ and (c) detected elliptically polarized segment for a vehicular signal;

with a detection accuracy of 97%. The proposed trajectory-based metrics also achieve increased detection rate for circular and diagonal footsteps compared to the energy-based metrics. Due to the presence of standing waves, they suffer from increased false alarm rate for stationary footsteps. Quantifying the concentration in $\mathbb{S}^{p-1}$ therefore resulted in robust detection of elliptically polarized footstep signals that outperformed existing energy-based and trajectory-based metrics. The concentration of orientation-quaternions, $\mathcal{M}_{\text{CinH}}$ was also shown to detect vehicular signals.

Chapter 6

Exploiting Orientation-Quaternion Statistics For Bearing Estimation

The research focus is now shifted from detection of surface sources to estimating the bearing angle of the detected sources. Bearing estimation is an important attribute of outdoor surveillance systems since it provides source localization and tracking capabilities [11, 104, 105]. Most of these security systems utilize multiple geophones and achieve localization either by evaluating the signal energy to determine the vicinity of the source to a particular sensor node [106] or by evaluating the time-difference-of-arrival between the sensors [11].

Prior research works model seismic waves from earthquakes, oil exploration surveys and other geophysical activities [102, 107, 108]. Under these far-field conditions, different polarization modes of the propagating wavefront de-couple (due to variations in their propagation velocities) and arrive at the sensor at different time instances. However under near-field conditions, waves generated from surface sources arrive at the sensor simultaneously resulting in composite polarization. Since Love and Rayleigh waves generate orthogonal oscillations on the horizontal plane, their superposition results in a transversely elliptical wavefront. Existing bearing estimation algorithms using a single

Figure 6.1: Tri-axial signal as 3D trajectory.

tri-axial geophone assume either point source/sensor in far-field [65, 66] or dominance of Rayleigh waves [14]. These algorithms exploit the inter-channel correlation energy to estimate the direction-of-arrival (DOA) of the source. However, when applied to near-field sources, the estimated bearing suffers a $\pm 90^\circ$ phase shift relative to the true source bearing. In Section 6.1, we will use the existing covariance matrix analysis [65], an energy-based algorithm, to show the $\pm 90^\circ$ offset that occurs in the estimated bearing angle.

The left panel of Fig. 6.1 shows signals received by a tri-axial geophone, where $x_1(n)$ and $x_2(n)$ correspond to signals in the horizontal x and y -axis, while $x_3(n)$ denotes the vertical axis signal. The 3D trajectory traced by the acquired tri-axial signal, as shown in the right panel, depends on factors such as the direction of the source relative to the sensor and the relative proportion of different polarization modes in the impinging wavefront. In near-field, multiple polarization modes arrive at the sensor simultaneously, resulting in a wobbly elliptical trajectory [68, 69]. We propose an orbital phase analysis (OPA) algorithm that utilizes the instantaneous movement of the 3D trajectory to estimate the bearing of surface sources. Exploiting the presence of coupled transversely elliptical wavefront in near-field, the OPA algorithm estimates the source bearing to be

in the direction perpendicular to the plane containing the tri-axial trajectory. This addresses the $\pm 90^\circ$ phase shift that occur in existing algorithms that employ inter-channel correlation energy. Using the quaternion rotational model proposed in Chapter 5, the instantaneous orbital phase of the 3D trajectory is compactly parameterized via a rotation angle (*eigenangle*) and a rotation axis (*eigenaxis*) through the use of hypercomplex quaternions. In this chapter, we show that for an elliptically polarized trajectory the estimated instantaneous eigenaxis is concentrated in the direction normal to the plane containing the trajectory. The spherical mean of the instantaneous eigenaxis is then used to estimate the source bearing.

6.1 Insight into Bearing Angle Ambiguities

Covariance matrix analysis (CMA), which has been proposed to estimate the DOA of seismic surface sources, is used as the baseline algorithm to evaluate the bearing of footstep signals [2, 50, 65]. With reference to Chapter 2.7, the CMA algorithm is reproduced here for ease and clarity of presentation. The CMA bearing angle is estimated as the azimuth corresponding to the direction of maximum energy. This is achieved by first evaluating the 3×3 covariance matrix

$$\begin{aligned}\hat{\mathbf{C}}(l) &= \mathbb{E}\{\mathbf{x}(n)\mathbf{x}^T(n)\} \\ &\approx \frac{1}{N}\mathbf{X}(l)\mathbf{X}^T(l),\end{aligned}\tag{6.1}$$

where $\mathbb{E}(\cdot)$ is the expectation operator and $\mathbf{X}(l)$ is the $3 \times N$ data matrix corresponding to the l th frame of the tri-axial signal $\mathbf{x}(n)$ of length N defined in (2.11) of Chapter 2.7. Eigen-analysis is performed such that $\hat{\mathbf{C}}(l)$ is decomposed as

$$\hat{\mathbf{C}}(l) = \hat{\mathbf{U}}(l)\hat{\mathbf{\Lambda}}(l)\hat{\mathbf{U}}^T(l),\tag{6.2}$$

Figure 6.2: Pictorial representation of (a) circular, (b) diagonal and (c) ‘V’-shaped walking path.

where $\hat{\mathbf{\Lambda}}(l) = \text{diag}\{\lambda_1(l), \lambda_2(l), \lambda_3(l)\}$ is the 3×3 matrix containing the eigenvalues $\lambda_1(l) \geq \lambda_2(l) \geq \lambda_3(l)$ and $\hat{\mathbf{U}}(l)$ is the unitary matrix containing the eigenvectors. The orthogonal eigenvectors correspond to the principal directions of the tri-axial signal $\mathbf{x}(n)$, while the eigenvalues correspond to the magnitude. Therefore, the eigenvector corresponding to the largest eigenvalue $\lambda_1(l)$, i.e., the first column of $\hat{\mathbf{U}}(l) = [e_{11} \ e_{21} \ e_{31}]^T$ points in the direction of maximum energy. The source bearing is then evaluated as the azimuth angle from the Cartesian vector using

$$\hat{\Theta}_{\text{CMA}} = \tan^{-1} \left(\frac{e_{21}}{e_{11}} \right). \quad (6.3)$$

A 2×2 covariance matrix formed using the horizontal channel data $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ can be used to estimate the bearing similar to (6.3) [2]. We now use footstep signals recorded from three different walking experiments to illustrate ambiguities that exists in the bearing angle estimated using CMA.

6.1.1 Energy (180°) Ambiguity

In the first experiment, a human subject was made to walk on the circumference of a circle of radius 2 m inscribed about the origin starting from $(-2, 0)$ m in a clockwise direction as illustrated in Fig. 6.2(a). Twenty-nine footsteps were recorded and the bearing angle estimated using CMA is plotted along with the corresponding ground truth angle Θ in Fig. 6.3. We observe that the estimated bearing angle approximately follows the true source bearing in quadrants Q2 and Q3. However when the source is in Q1, we note that the estimated bearing is in Q3, although the slope of the estimated bearing across the footsteps in Q1 is parallel to the ground truth angle. A similar phenomenon is observed when the source is in Q4 and $\hat{\Theta}_{\text{CMA}}$ is in Q2. When footsteps are in Q3, CMA once again estimates the source in the correct quadrant. The jump in the last footstep is attributed to the wrapping of the estimated angle. From Fig. 6.3, we observe that footsteps that are 180° apart (i.e, in Q2 and Q4) result in the same estimated bearing. This 180° ambiguity is referred to as the *energy ambiguity*.

In order to understand the reason behind the existence of the 180° energy ambiguity, consider a polarized signal generated by a source at position (c, d) in quadrant Q1, impinging on the sensor buried at the origin. Without any loss of generality, Rayleigh wave propagation is assumed, although the discussion holds true for other types of waves. The vector formed using only the horizontal channels of the acquired data is expressed as

$$\mathbf{x}_{\text{Q1}}(n) = [\mathbf{x}_1(n) \ \mathbf{x}_2(n)]^T, \quad (6.4)$$

where the subscript ‘Q1’ denotes for the quadrant in which the source is located. However, when the source with the same polarization parameters is shifted to coordinates $(-c, -d)$ in Q3, the prograde Rayleigh polarization induces a signal with opposite phase in the

Figure 6.3: Plot of $\hat{\Theta}_{\text{CMA}}$ and Θ vs footstep index for a human subject walking along the circumference of a circle shown in Fig. 6.2(b).

two horizontal channels giving raise to

$$\mathbf{x}_{Q3}(n) = [-\mathbf{x}_1(n) \quad -\mathbf{x}_2(n)]^T. \quad (6.5)$$

We therefore note that the 2×2 covariance matrices evaluated from $\mathbf{x}_{Q1}(n)$ and $\mathbf{x}_{Q3}(n)$ are identical and is given by

$$\hat{\mathbf{C}}_{Q1} = \hat{\mathbf{C}}_{Q3} = \begin{bmatrix} \sigma_{\mathbf{x}_1}^2 & \sigma_{\mathbf{x}_1\mathbf{x}_2} \\ \sigma_{\mathbf{x}_1\mathbf{x}_2} & \sigma_{\mathbf{x}_2}^2 \end{bmatrix}, \quad (6.6)$$

where

$$\sigma_{\mathbf{x}_i}^2 = E[\mathbf{x}_i^2], \quad (6.7)$$

$$\sigma_{\mathbf{x}_i\mathbf{x}_j} = E[\mathbf{x}_i\mathbf{x}_j], \quad (6.8)$$

Figure 6.4: Plot of $\hat{\Theta}_{\text{CMA}}$, Θ vs footstep index for a human subject walking along a (a) diagonal path and (b) ‘V’-shaped path, as shown in Fig. 6.2(b) and (c), respectively.

with $i = 1, 2$ denoting the x and y channel data of the tri-axial geophone. This implies that $\hat{\mathbf{C}}(l)$ in 6.1 is insensitive to the polarized seismic signals from two sources that are 180° apart. Since the covariance matrix is a measure of the signal power, it is insensitive to the phase of the wave impinging the sensor from diametrically opposite quadrants resulting in a 180° energy ambiguity.

6.1.2 Wave-polarization (90°) Ambiguity

In the next experiment, a human subject was made to walk along a diagonal path starting from the third quadrant to the first quadrant as shown in Fig. 6.2(b). The $\hat{\Theta}_{\text{CMA}}$ is plotted in Fig. 6.4(a) along with the true bearing Θ . From the plot, we observe that during the first segment of the diagonal walk, CMA estimates the source in Q2. This is a $+90^\circ$ phase shift relative to the true source bearing. During the second segment of the diagonal walk a -90° phase shift is introduced in $\hat{\Theta}_{\text{CMA}}$. Furthermore, the change in quadrant when

Figure 6.5: Polarization ellipse parameters for tilted generalized Rayleigh wave.

the source passes through the origin is not reflected in $\hat{\Theta}_{\text{CMA}}$. As opposed to the previous experiment, there exists a $\pm 90^\circ$ ambiguity.

Similar behaviour is observed when CMA is used to estimate the bearing corresponding to a human walking along a ‘V’-shaped path starting from the second quadrant towards the origin and diagonally into Q1 as shown in Fig. 6.2(c). The bearing angle estimated by CMA is plotted in Fig. 6.4(b) along with the true bearing angle. From Fig. 6.4(b), except for a few estimation errors, it is observed that CMA estimates the bearing to be $+90^\circ$ phase-shifted with respect to the true bearing. During the first segment of the ‘V’ path walk, CMA estimates the source in Q3 whereas during the second segment CMA estimates the source in Q2. This $\pm 90^\circ$ phase shift in the estimated bearing angle relative to the true source bearing is referred to as the *wave-polarization ambiguity*.

We next analyze the reason for the existence of these ambiguities in the bearing angle estimated using CMA. It is known that seismic waves propagate through several modes of polarization such as longitudinal P-waves, transverse S-waves, Rayleigh and Love waves [31]. Due to near-field conditions, it is foreseeable that the signals received by the orthogonal sensors are coupled with multiple modes which result in ambiguities

in the estimated bearing angles. In the circular walking experiment shown in Fig. 6.2(a) and Fig. 6.3, the subject walks in a tangential direction with respect to the geophone whereas in subsequent experiments, the subject walks radially towards and away from the geophone. This difference in the walk pattern significantly varies the proportion of the different polarization modes in the seismic waves impinging on the sensor. In order to gain insights into the $\pm 90^\circ$ ambiguity, we refer to the polarized seismic model in Fig. 6.5, that is reproduced from Chapter 2.6. Polarized seismic signals are parameterized using the azimuth angle Θ , elevation angle Φ , orientation angle Ψ and ellipticity Ξ . From the above experiment results, we infer that the angle estimated by CMA is *not* an estimate of the source azimuth but an estimate of the angle of particle vibration, i.e., the orientation angle Ψ . Unless direct P-waves impinge the sensor, the estimated $\hat{\Theta}_{\text{CMA}}$ does not correspond to the source azimuth angle. Since the proportion of direct P-waves generated from a surface source is insignificant [33,34], the estimated $\hat{\Theta}_{\text{CMA}}$ corresponds to the orientation angle of the ellipse in the horizontal plane, which is dependent on the proportion of Rayleigh and Love modes in the propagating wavefront [64]. Furthermore, in the presence of coupled polarization $\hat{\mathbf{C}}(l)$ is influenced, to a large extent, by Love waves due to its presence in the horizontal plane. Hence, in the presence of horizontally transverse waves, the bearing estimated using CMA has a $\pm 90^\circ$ phase shift with respect to the true source bearing.

The Hilbert transform analysis (HTA) reviewed in Chapter 2.7, on the other hand, alters the phase of the z -axis signal in order for $\hat{\Theta}_{\text{HTA}}$ to point in the radial direction. However, in the presence of significant transverse oscillations, the energy of the signal acquired in the horizontal axes is no longer limited to the radial direction. Under these conditions, the bearing angle estimated using HTA also suffers from a $\pm 90^\circ$ phase shift with respect to the true source bearing. Other techniques such as rotational analysis [67] and surface wave analysis [50] reviewed in Chapter 2.7, also estimate the bearing using the correlation energy between the radial (using the horizontal channel data $\mathbf{x}_1(n)$ and $\mathbf{x}_1(n)$)

Figure 6.6: Pictorial representation of the superposition of Love and Rayleigh modes forming transverse elliptically polarized wavefront.

and vertical channel data $\mathbf{x}_3(n)$. These algorithms exploit the energy of the vector-valued signal to estimate the bearing angle on the plane of polarization of the tri-axial trajectory. In the presence of coupled polarization, the energy of the impinging wavefront is no longer oriented in the source-sensor radial direction. As a result, the estimated bearing suffers from $\pm 90^\circ$ offset with respect to the true source direction. In this chapter, a trajectory-based approach is proposed to address this $\pm 90^\circ$ wave-polarization ambiguity.

6.2 Proposed Orbital Phase Analysis Algorithm

The proposed algorithm exploits coupled polarization that arises due to the simultaneous presence of both Rayleigh and Love waves in near-field. A perpendicular load on the Earth's surface generates a greater proportion of Rayleigh (67%) compared to Love (26%) waves, whereas a tangential load generates a greater proportion of shear (Love) waves (76%) than Rayleigh (18%) waves [33, 34]. Since seismic surface sources such as

vehicles and human footsteps exert a combination of normal and tangential forces on the ground [53], they generate both Rayleigh and Love waves. This is illustrated in Fig. 6.6, where the source generates both elliptically polarized Rayleigh waves (denoted by the black lines) and horizontally transverse Love waves (denoted by the grey lines.) In near-field, both polarization modes impinge on the sensor (black dot at the origin) simultaneously resulting in composite transversely elliptical waves. The presence of such transverse elliptically polarized waves causes the 3D trajectory of the acquired tri-axial signal $\mathbf{x}(n)$ (denoted by dotted line), to be traced in a plane perpendicular to the radial source-sensor direction. The proposed algorithm exploits coupled polarization of Rayleigh and Love waves, in near-field, to estimate the source bearing as the direction perpendicular to the plane of polarization of the 3D trajectory. This normal direction is estimated using the quaternion rotational model, proposed in Chapter 5, which modelled the instantaneous orbital phase of the tri-axial trajectory.

6.2.1 Bearing Angle Estimation

Instead of using the individual channels of the tri-axial signal to evaluate the orbital phase of the 3D trajectory, the time evolution of the acquired tri-axial signal $\mathbf{x}(n)$, as described in Chapter 5.1, is expressed in two steps as

$$\bar{\mathbf{x}}(n) = \mathbf{q}(n) \star \bar{\mathbf{x}}(n-1) \star \mathbf{q}^{-1}(n), \quad (6.9)$$

$$\mathbf{x}(n) = \rho(n) \star \bar{\mathbf{x}}(n), \quad (6.10)$$

where the first step defines the *instantaneous rotation* of the normalized 3D trajectory from $\bar{\mathbf{x}}(n-1)$ and the second step defines the *instantaneous scaling* of the normalized trajectory with an excitation signal $\rho(n)$ such that $\mathbb{I}(\rho(n)) = 0$. The instantaneous 3D rotation using orientation-quaternion $\mathbf{q}(n)$ involves sandwiching the vector-quaternion between $\mathbf{q}(n)$ and its conjugate $\mathbf{q}^{-1}(n) = \mathbf{q}^*(n)/\|\mathbf{q}(n)\| = q_0(n) - q_1(n)\imath - q_2(n)\jmath - q_3(n)\kappa$,

Figure 6.7: Orientation-quaternion parameterization of instantaneous orbital phase of the 3D trajectory.

where $\star$ denotes the quaternion product [91]. This operation combines two half-angle rotations in order to rotate the 3D vector through an angle $\theta(n)$. The variables $\bar{\mathbf{x}}(n)$ and $\bar{\mathbf{x}}(n-1)$ are vector-quaternions constructed using the acquired tri-axial signal $\mathbf{x}(n)$ such that

$$\bar{\mathbf{x}}(n) = \frac{x_1(n)}{\|\mathbf{x}(n)\|}i + \frac{x_2(n)}{\|\mathbf{x}(n)\|}j + \frac{x_3(n)}{\|\mathbf{x}(n)\|}k, \quad (6.11)$$

where $\|\mathbf{x}(n)\| = \sqrt{x_1^2(n) + x_2^2(n) + x_3^2(n)}$ and $\bar{\mathbf{x}}(n)$ denotes, respectively, the instantaneous magnitude and instantaneous direction of the tri-axial signal. From (6.9), we note that the instantaneous orbital phase of the tri-axial trajectory is quantified by the 3D rotation $\mathbf{q}(n)$, which is in turn parameterized by the eigenangle $\theta(n)$ and eigenaxis $\mathbf{z}(n)$ as illustrated in Fig. 6.7.

Given $\mathbf{x}(n)$, the instantaneous orbital phase of the 3D trajectory $\hat{\mathbf{q}}(n)$ can be estimated using the velocity-to-rotation transform. As described in Chapter 5.1.2, the instantaneous 3D rotation consists of two components - the *magnitude*, specified by the eigenangle $\theta(n)$ and the *direction*, specified by the eigenaxis $\mathbf{z}(n)$. The instantaneous eigenangle is

estimated as the angle between consecutive samples using the dot product

$$\hat{\theta}(n) = \cos^{-1} \left(\frac{\mathbf{x}(n-1) \cdot \mathbf{x}(n)}{\|\mathbf{x}(n-1)\| \|\mathbf{x}(n)\|} \right). \quad (6.12)$$

Since the rotation axis is in the direction normal to the 3D trajectory, the instantaneous eigenaxis is evaluated as a unit-norm vector using the cross product

$$\begin{aligned} \hat{\mathbf{z}}(n) &= \frac{\mathbf{x}(n-1) \times \mathbf{x}(n)}{\|\mathbf{x}(n-1) \times \mathbf{x}(n)\|} \\ &= [\hat{z}_1(n) \ \hat{z}_2(n) \ \hat{z}_3(n)]^T. \end{aligned} \quad (6.13)$$

which can further be expressed in vector quaternion form as

$$\hat{\mathbf{z}}(n) = \hat{z}_1(n)\imath + \hat{z}_2(n)\jmath + \hat{z}_3(n)\kappa. \quad (6.14)$$

Using (6.12) and (6.14), the instantaneous orbital phase is evaluated as

$$\hat{\mathbf{q}}(n) = \cos \frac{\hat{\theta}(n)}{2} + \hat{\mathbf{z}}(n) \star \sin \frac{\hat{\theta}(n)}{2}. \quad (6.15)$$

The instantaneous eigenaxis of $\hat{\mathbf{q}}(n)$, evaluated as the cross product between consecutive snapshots of the signal trajectory, points in the direction perpendicular to the instantaneous movement of the tri-axial trajectory. Furthermore, for elliptically polarized waves generated by surface sources, $\hat{\mathbf{z}}(n)$ is concentrated. In this work, we use the estimated eigenaxis to determine the direction of the source.

As an illustrative example, we use seismic signals acquired by a tri-axial geophone when a lorry travels at 30 km/h parallel to the x -axis of the sensor. The lorry approaches the sensor from the left and crosses over to the right before moving away. Figure 6.8(a) shows the z -axis signal when the vehicle is in transit between -40 m and 60 m relative to the sensor. The 3D plot of the normalized trajectory $\bar{\mathbf{x}}(n)$ along with the eigenaxis vector

Figure 6.8: (a) Plot of z -axis signal vs distance of source relative to the sensor and 3D plot of (b) $\bar{x}(n)$ and $\hat{z}(n)$ over a small 50 ms signal window, (c) $\mu(\hat{z})$ evaluated across overlapping frames, for vehicle moving parallel to the x -axis.

$\hat{z}(n)$, over a small frame of 50 ms is shown in Fig. 6.8(b). From Fig. 6.8(b), we observe that for an elliptical trajectory, $\hat{z}(n)$ (shown with circular markers) is concentrated in the direction perpendicular to the plane containing the signal trajectory.

The direction of the source is estimated by evaluating the mean direction of the concentrated eigenaxis. The spherical mean of $\hat{z}(n)$ over a segment of length N is evaluated

as [98]

$$\begin{aligned}\boldsymbol{\mu}(\hat{\mathbf{z}}(n)) &= \frac{1}{N} \left(\frac{\sum_{n=1}^N \hat{\mathbf{z}}(n)}{\left\| \sum_{n=1}^N \hat{\mathbf{z}}(n) \right\|} \right) \\ &= [\hat{\mathbf{z}}_{\mu 1} \quad \hat{\mathbf{z}}_{\mu 2} \quad \hat{\mathbf{z}}_{\mu 3}].\end{aligned}\tag{6.16}$$

The spherical mean of the concentrated eigenaxis in Fig. 6.8(b) is denoted by a black arrow which points in the direction of the source. The bearing angle is then estimated as the azimuth corresponding to the spherical mean of the eigenaxis evaluated in (6.16) using

$$\hat{\Theta}_{\text{OPA}} = \tan^{-1} \left(\frac{\hat{\mathbf{z}}_{\mu 2}}{\hat{\mathbf{z}}_{\mu 1}} \right).\tag{6.17}$$

The subscript ‘OPA’ denotes for orbital phase analysis. It is important to note that unlike the baseline algorithms where the estimated bearing is dependent of the inter-channel correlation energy of the impinging signal, the proposed $\hat{\Theta}_{\text{OPA}}$ is dependent on the orbital phase of the 3D trajectory. Since the proposed algorithm exploits the instantaneous evolution of the tri-axial trajectory, the acquired signal is sampled at a high sampling rate of 8 kHz.

Unlike $\hat{\Theta}_{\text{CMA}}$ and $\hat{\Theta}_{\text{HTA}}$, the proposed $\hat{\Theta}_{\text{OPA}}$ is estimated using the direction normal to the plane of the tri-axial trajectory. Thus, the proposed algorithm addresses $\pm 90^\circ$ offset (with respect to the true source bearing) that occur in the existing algorithms due to the presence of coupled polarization. Although the instantaneous eigenaxis is normal to the tri-axial trajectory, it is insensitive to the clockwise or counter-clockwise direction of the trace. However, this 180° offset with respect to the true source bearing in the proposed $\hat{\Theta}_{\text{OPA}}$ can be corrected by evaluating the propagation direction in the radial-vertical plane using (2.41) in Chapter 2.7.

Under the assumption that the estimated eigenaxis follows von Mises-Fisher dis-

tribution, the CRLB for the mean estimator $\hat{\boldsymbol{\mu}}$ is derived to understand the effect of concentration η on the variance of the estimator. The CRLB is evaluated as

$$\text{var}(\hat{\boldsymbol{\mu}}) \geq \frac{1}{-\text{E}\left\{\frac{\partial^2 \mathcal{L}(\mathbf{d}^{(p)})}{\partial \boldsymbol{\mu}^2}\right\}}, \quad (6.18)$$

where $\text{var}(\cdot)$ is the variance, $\text{E}(\cdot)$ is the expectation operator and $\mathcal{L}(\mathbf{d}^{(p)})$ is the log-likelihood of the vMF distribution. As shown in Chapter 5, the log-likelihood of the vMF distribution is given by

$$\mathcal{L}(\mathbf{d}^{(p)}; \boldsymbol{\mu}, \eta) = N \ln(C_p(\eta)) + \eta \boldsymbol{\mu}^T \tilde{\mathbf{d}}^{(p)}. \quad (6.19)$$

Differentiating (6.19) with respect to $\boldsymbol{\mu}$, we get

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \boldsymbol{\mu}} &= \eta \boldsymbol{\mu} N L(\mathbf{d}^{(p)}) \\ \frac{\partial^2 \mathcal{L}}{\partial \boldsymbol{\mu}^2} &= \eta A_p(\eta). \end{aligned} \quad (6.20)$$

The CRLB for the mean parameter estimator is given by

$$\text{var}(\hat{\boldsymbol{\mu}}) \geq \frac{1}{\eta A_p(\eta)}. \quad (6.21)$$

From (6.21), we note that the estimated mean has a higher precision (or lower variance) when estimated from DRVs corresponding to high concentration, $\eta \rightarrow \infty$ than when estimated from DRVs tending towards uniform distribution i.e., $\eta \rightarrow 0$. This is intuitively in line with the algorithm, as only elliptically polarized signal segments where concentration of instantaneous eigenaxis is high is used to estimate the source bearing, as shown in Fig. 6.8(b).

6.2.2 Insights into the Proposed Algorithm

In order to understand why $\hat{\mathbf{z}}(n)$ can be used to address the $\pm 90^\circ$ phase shift that occurs in existing algorithms, we derive the mathematical expression of the instantaneous eigenaxis. While tri-axial signals acquired from real-world experiments consist of multiple frequencies, for clarity of presentation, we express a real-valued tri-axial signal using a single frequency for each channel given by

$$\mathbf{x}(n) = \begin{bmatrix} \cos(\phi_x n + \phi_1) \\ \cos(\phi_y n + \phi_2) \\ \cos(\phi_z n + \phi_3) \end{bmatrix}. \quad (6.22)$$

Here ϕ_x, ϕ_y, ϕ_z denote the instantaneous phase corresponding to the dominant frequency in the three orthogonal channels and ϕ_1, ϕ_2, ϕ_3 denote the corresponding initial phase. The elliptical trajectory traced by the tri-axial signal is a function of these phase values. Since the focus is on the direction of the eigenaxis, we ignore the scaling in (6.13) to

derive the expression as

$$\widehat{\mathbf{z}}'(n) = \mathbf{x}(n-1) \times \mathbf{x}(n) \quad (6.23a)$$

$$= \begin{bmatrix} \cos(\phi_x n - \phi_x + \phi_1) \\ \cos(\phi_y n - \phi_y + \phi_2) \\ \cos(\phi_z n - \phi_z + \phi_3) \end{bmatrix} \times \begin{bmatrix} \cos(\phi_x n + \phi_1) \\ \cos(\phi_y n + \phi_2) \\ \cos(\phi_z n + \phi_3) \end{bmatrix} \quad (6.23b)$$

$$= \begin{bmatrix} \cos(\phi_z n + \phi_3) \cos(\phi_y n + \phi_2) (\cos \phi_y - \cos \phi_z) - \\ \cos(\phi_y n + \phi_2) \sin(\phi_z n + \phi_3) \sin \phi_z + \\ \cos(\phi_z n + \phi_3) \sin(\phi_y n + \phi_2) \sin \phi_y \\ \cos(\phi_z n + \phi_3) \cos(\phi_x n + \phi_3) (\cos \phi_x - \cos \phi_z) - \\ \cos(\phi_x n + \phi_1) \sin(\phi_z n + \phi_3) \sin \phi_z + \\ \cos(\phi_z n + \phi_3) \sin(\phi_x n + \phi_1) \sin \phi_x \\ \cos(\phi_x n + \phi_1) \cos(\phi_y n + \phi_2) (\cos \phi_x - \cos \phi_y) - \\ \cos(\phi_x n + \phi_1) \sin(\phi_y n + \phi_2) \sin \phi_y + \\ \cos(\phi_y n + \phi_2) \sin(\phi_x n + \phi_1) \sin \phi_x \end{bmatrix}. \quad (6.23c)$$

For a small piecewise 3D elliptical trajectory, the instantaneous phase shift across the three channels of the tri-axial signal are synchronized. Since our focus is only on the phase of the tri-axial signal and not its amplitude, such an elliptical trajectory can be assumed to be traced when $\phi_x \approx \phi_y \approx \phi_z$. Furthermore, when $\mathbf{x}(n)$ is sampled at a high rate, the instantaneous phase shifts ϕ_x , ϕ_y and ϕ_z can be assumed small. Applying small angle approximations, $\sin \phi_x = \sin \phi_y = \sin \phi_z \approx \phi$ and $\cos \phi_x = \cos \phi_y = \cos \phi_z \approx 1 - \frac{\phi^2}{2}$

to the eigenaxis expression in (6.23c), we get

$$\hat{\mathbf{z}}(n) \approx \begin{bmatrix} \sin(\phi_2 - \phi_3) \\ \sin(\phi_1 - \phi_3) \\ \sin(\phi_1 - \phi_2) \end{bmatrix}. \quad (6.24)$$

From the above equation, we note that for elliptically polarized signals the instantaneous eigenaxis have a compact representation independent of time and points in the direction perpendicular to the plane of polarization of the tri-axial signal. As described in the beginning of Section 6.2, seismic surface sources generate both Rayleigh and Love waves resulting in a transversely elliptical seismic wavefront. We know from Section 6.1.2 that in the presence of significant transverse oscillations existing CMA and HTA algorithms suffer a $\pm 90^\circ$ offset with respect to the true bearing. The proposed $\hat{\Theta}_{\text{OPA}}$, estimated from the eigenaxis which is perpendicular to the plane the 3D trajectory, corrects for the 90° offset in the presence of transverse elliptically polarized waves.

It is useful to note that the normal vector of a tri-axial signal can alternatively be defined as the cross product between the signal and its Hilbert transformed signal (applied across each individual channel) [102]. However, sufficient duration of the tri-axial signal is required to estimate a reliable Hilbert transform. Furthermore, the Hilbert transform introduces spurious oscillations at both ends of the transformed signal due to leakage of the discrete Fourier transform (DFT) [109, 110] and this will result in an erroneous estimate of the normal vector especially when estimated over small time frames.

6.2.3 Signal Window Selection for Human Footstep DOA Estimation

As shown in Fig. 6.8(a), seismic signals generated by vehicular movement are active for a prolonged time period due to continuous contact of the tires with the ground. Hence, the

Figure 6.9: Plot of (i) TFA of (a) $\mathbf{x}_1(n)$, (b) $\mathbf{x}_2(n)$ and (c) $\mathbf{x}_3(n)$ along with (i) selected signal window using (a) $\mathbf{x}^f(n)$ with vertical energy and (b) $\mathbf{x}(n)$ with $\hat{E}_c(l)$.

proposed bearing estimation metric is estimated across overlapping frames of the detected signal. On the other hand, a typical human footstep signal is transient in nature since the source is active only for limited duration at each footstep impact. The segment over which the bearing angle is estimated needs to be prescriptively chosen.

The proposed algorithm exploits the presence of coupled transversely elliptical trajectory as shown in Fig. 6.6. Hence, the algorithm requires a segment of the impinging wavefront that traces a consistent elliptical orbit with dominant energy vertical energy. In order to find the segment with dominant vertical energy, the cumulated energy metric defined as [3]

$$\hat{E}_c(l) = \sum_{i=1}^N x_1^2(i) + x_2^2(i) + x_3^2(i), \quad (6.25)$$

was used. The $\hat{E}_c(l)$ was evaluated over small overlapping frames within a footstep signal to select a window over which the signal energy was high. However, horizontal energy

from $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ dominates the value of $\hat{E}_c(l)$ compared to the vertical energy from $\mathbf{x}_3(n)$. This resulted in the selected window tracing a trajectory in the horizontal plane. For such a trajectory, $\hat{\Theta}_{\text{OPA}}$ approximated to $\pm 90^\circ$, i.e, the eigenaxis lying on the z -axis. Thus cumulated energy was found to be unsuitable for selecting the window.

Since footstep signals are impulsive, the time-frequency analysis (TFA) of the tri-axial signal provided a better energy map of the signal compared to $\hat{E}_c(l)$. In this work, TFA is performed by applying short-time Fourier transform (STFT) to signals received by the individual channels of the tri-axial geophone to obtain $\underline{\mathbf{X}}_i(n, f)$, where $i = 1, 2, 3$ corresponds to the x , y and z axis, respectively. The absolute value of the STFT coefficients $|\underline{\mathbf{X}}_i(n, f)|$ for $i = 1, 2, 3$, corresponding to a footstep signal, is shown in Fig. 6.9(i) (a), (b) and (c), respectively. In order to obtain a tri-axial signal containing the dominant frequency in each channel, the signal in the i th channel is band-pass filtered with a frequency band containing 70% of the maximum absolute value of the STFT coefficients $|\underline{\mathbf{X}}_i(n, f)|$, for $i = 1, 2, 3$. The band-pass filtered tri-axial signal $\mathbf{x}^f(n)$ is plotted in Fig. 6.9(ii)(a). Using $\mathbf{x}^f(n)$, the time duration over which the z -axis signal is significant is chosen for estimating $\hat{\Theta}_{\text{OPA}}$ as indicated by the black window in Fig. 6.9(ii)(a). For comparison, Fig. 6.9(ii)(b) illustrates the selected frame using $\hat{E}_c(l)$ on the acquired signal when the cumulated energy decays to 50% of the maximum value within the footstep signal. It can be observed that using $\hat{E}_c(l)$ fails to select the window where the vertical channel energy is high.

6.3 Experiment Results and Discussions

The proposed trajectory-based bearing estimation algorithm is evaluated on seismic signals acquired by a GS32-CT tri-axial geophone. The geophone is buried with good coupling at a pre-determined location $(0, 0)$ designated as the origin, with its orthogonal sensors aligned with the x, y and z axes of the Cartesian coordinate system. The per-

formance of the algorithm is evaluated on seismic signals generated from two different surface sources - vehicular motion and human footsteps. Vehicular signals are generated by a medium-sized lorry whereas footstep signals are generated by a human subject wearing soft-sole shoes and walking at a normal pace. Since the proposed bearing estimation algorithm is trajectory-based, the acquired signal is sampled at a relatively high sampling rate of 8 kHz, to capture the instantaneous changes in the orbital phase of impinging wavefront.

The performance of the bearing estimation algorithms is validated using the mean angular error (MAE) and circular variance metrics. The MAE is estimated as

$$\mu(\Delta\hat{\Theta}) = \angle \left\{ \frac{1}{N} \sum_{i=1}^N (\cos \Delta\hat{\Theta}_i + \imath \sin \Delta\hat{\Theta}_i) \right\}, \quad (6.26)$$

where $\Delta\hat{\Theta}$ is the angular error between the estimated bearing $\hat{\Theta}$ and true bearing Θ . The value of N in (6.26) denotes the number of overlapping frames for vehicular signal and the number of footsteps for recorded footstep train. The spread of the angular error is quantified using the circular variance estimated by

$$\sigma(\Delta\hat{\Theta}) = 1 - \sqrt{\frac{1}{N} \left[\left(\sum_{i=1}^N \cos \Delta\hat{\Theta}_i \right)^2 + \left(\sum_{i=1}^N \sin \Delta\hat{\Theta}_i \right)^2 \right]}. \quad (6.27)$$

We first estimate the bearing of seismic signals generated by a lorry travelling at a velocity of 30 km/h parallel to the x -axis of the sensor as shown in Fig. 6.8(a). The bearing angle is estimated using overlapping frames of length 100 ms with a 50% overlap. The lorry approaches the sensor from the third quadrant and crosses over to the fourth quadrant before moving away. The true source bearing Θ , evaluated using the closest point of approach (CPA) distance of 5 m and the speed of the vehicle, increases from -180° to 0° . The proposed OPA algorithm estimated an increasing bearing angle trace from 0° to 180° . The 180° phase shift with respect to the true bearing was corrected

Figure 6.10: Plot of $\hat{\Theta}_{OPA}$, $\hat{\Theta}_{CMA}$, $\hat{\Theta}_{HTA}$ along with ground truth bearing Θ vs distance from the sensor.

after evaluating the direction of propagation in the radial-vertical plane. The $\hat{\Theta}_{OPA}$ is plotted in Fig. 6.10 along with Θ . From this figure, we observe that $\hat{\Theta}_{OPA}$ follows the ground truth, especially when the vehicle crosses the sensor. As the vehicle moves away to a distance of 20 m from the sensor, fluctuations are observed in $\hat{\Theta}_{OPA}$. This can be attributed to the tangential impact of the vehicle resulting in significant energy on the horizontal channels [34]. This results in $\hat{\Theta}_{OPA}$ being estimated to be approximately -90° in signal frames when the vertical energy is not significant.

The bearing estimated using the baseline CMA and HTA algorithms are also plotted in Fig. 6.10 for comparison. It can be seen that the CMA algorithm estimates the bearing to approximately 90° and it does not show any variation that is in line with the actual movement of the vehicle. On the other hand, $\hat{\Theta}_{HTA}$ traces the source to be travelling a path approximately from 180° to 0° , i.e., from the second quadrant to the first quadrant. As opposed to the theoretical case where HTA exhibits a consistent 180°

Figure 6.11: Plot of $\hat{\Theta}_{OPA}$, $\hat{\Theta}_{CMA}$, $\hat{\Theta}_{HTA}$ along with ground truth Θ vs footstep index, for circular walking.

phase shift with respect to the source, the trace of $\hat{\Theta}_{HTA}$ corresponds to the true bearing flipped about the x -axis. The MAE and circular variance for $\hat{\Theta}_{OPA}$, $\hat{\Theta}_{CMA}$ and $\hat{\Theta}_{HTA}$, estimated across overlapping frames of the vehicular signal, are $(30.2^\circ, 6^\circ)$, $(99.2^\circ, 14.7^\circ)$ and $(51.3^\circ, 13^\circ)$, respectively. From the error statistics, we observed that exploitation of rotational characteristics of the 3D trajectory better estimates the bearing of vehicular source.

Bearing estimation algorithms are next evaluated on tri-axial signals generated by human footsteps. In the first experiment, a human subject walks along the circumference of a circle of radius 2 m inscribed about the origin in counter-clockwise direction starting from $(2, 0)$. A total of twenty-three footsteps are recorded. Figure 6.11 illustrates the estimated bearing for each footstep using the proposed algorithm along with the baseline

algorithms. Compared to CMA and HTA, we note that the proposed trajectory-based algorithm follows the true bearing consistently, thus achieving higher bearing accuracy. The $\hat{\Theta}_{\text{OPA}}$ corresponding to the 5th and 11th footstep index are approximately 90° . This is attributed to the low energy in the horizontal channels of the tri-axial signal. Although both Love and Rayleigh waves are generated, the trajectory traced by the tri-axial signal also dependent on the Earth (physical) channel through which the signal propagates.

In comparison, the CMA estimates the bearing of majority of the footsteps to be approximately 180° , irrespective of the actual source direction. For better visualization, the estimated $\hat{\Theta}_{\text{CMA}}$ is phase shifted by $\pm 360^\circ$ to align with the sign of the true source bearing. The only distinct variation in CMA is between the 13th and 14th footstep when $\hat{\Theta}_{\text{CMA}}$ jumps from approximately -120° to $+120^\circ$. This variation corresponds to the point when the footstep transits from the third to the fourth quadrant. The bearing estimated using HTA corresponds to a circle in the clockwise direction starting from $(-2,0)$ and this corresponds to true bearing flipped about the y -axis. The bearing estimated using HTA corresponds to the true source bearing only when the source is at $\pm 90^\circ$. In terms of angular error statistics, CMA and HTA have an MAE of 92.9° and 78.0° , respectively while the proposed OPA algorithm achieves the lowest MAE of 18.6° . The circular variance of the angular error for the proposed algorithm is 1.8° compared to circular variance of 15.6° and 18.6° corresponding to $\hat{\Theta}_{\text{CMA}}$ and $\hat{\Theta}_{\text{HTA}}$, respectively. From the MAE of CMA and HTA, we see that both algorithms on an average estimate a bearing that is $\pm 90^\circ$ with respect to the true bearing.

We next evaluate the performance of the algorithms using seismic signal generated when a human subject walks along a diagonal path starting from $(-5,5)$ in the third quadrant, passed through origin $(0,0)$ and walks toward $(5,-5)$ in the fourth quadrant. Figure 6.12 plots the estimated $\hat{\Theta}_{\text{OPA}}$ along with bearing estimated by CMA and HTA against footstep indices. Again for better visualization, both $\hat{\Theta}_{\text{CMA}}$ and $\hat{\Theta}_{\text{HTA}}$ are phase shifted by $\pm 360^\circ$ have the same sign as the true source bearing. From the graph, we note

Figure 6.12: Plot of $\hat{\Theta}_{\text{OPA}}$, $\hat{\Theta}_{\text{CMA}}$, $\hat{\Theta}_{\text{HTA}}$ along with ground truth Θ vs footstep index, for diagonal walking.

that the bearing estimated using the proposed OPA algorithm follows the true source bearing Θ . The CMA estimates majority of the footsteps to be approximately -135° . This shows that CMA suffers from $\pm 90^\circ$ ambiguity and is not capable of distinguishing the quadrant change when the human subject crosses the origin. On the other hand, HTA exhibits a consistent -90° phase shift with respect to the true bearing. The proposed OPA algorithm is able to determine exactly when the source crosses the origin, whereas the HTA algorithm determines the crossover one footstep later. The MAE of $\hat{\Theta}_{\text{OPA}}$, $\hat{\Theta}_{\text{CMA}}$ and $\hat{\Theta}_{\text{HTA}}$ is 17.5° , 75.6° and 76.7° , respectively, for this experiment. It is important to note that the proposed OPA algorithm takes into account the transverse oscillations of the seismic wave present in the impinging wavefront, thus addressing the $\pm 90^\circ$ offset that existed in the energy-based baseline algorithms.

In the final experiment, we investigate the performance of the bearing estimation algorithms for radial footsteps. Seismic signals are recorded while a human subject

Figure 6.13: Comparison of (a) $\hat{\Theta}_{OPA}$, (b) $\hat{\Theta}_{CMA}$, (c) $\hat{\Theta}_{HTA}$, for radial footsteps at (I) 30° (away from the sensor) (II) 30° (towards the sensor) (III) 60° (away from the sensor) (IV) 60° (towards the sensor). In every plot, the black line corresponds to the ground truth source bearing.

walks radially at azimuth angles 30° and 60° , both towards and away from the sensor. Approximately twenty-two footsteps are recorded during each of the walking experiment. Results of bearing estimation using the proposed $\hat{\Theta}_{\text{OPA}}$ are plotted as a circular histogram for better visualization in Fig. 6.13. In each plot, the bold line indicates true source bearing Θ . From the graphs, we observe that the proposed OPA algorithm estimates the bearing angle in the same quadrant as that of the source. Furthermore, it is observed that CMA exhibits a consistent 90° offset for majority of the footsteps. On the other hand, the HTA algorithm displays a consistent offset of 60° relative to the true bearing, for radial walk at $\Theta = 30^\circ$. For radial walk towards the sensor at $\Theta = 60^\circ$, $\hat{\Theta}_{\text{HTA}}$ exhibits high variance. With regard to the angular error statistics, OPA achieved the least MAE of 22.14° , 14.3° for radial footsteps moving away from the sensor at 30° and 60° , respectively. Furthermore, the MAE corresponding to radial walking towards the sensor is approximately 30° for both azimuth angles. The poor performance of the baseline algorithms can be attributed to the fact that these algorithms estimate the bearing using inter-channel correlation energy and hence are affected by the presence of the horizontally transverse waves. In the presence of coupled polarization, exploiting the rotational characteristics of the 3D trajectory results in a better estimate of the source direction for near-field vehicular and footstep sources.

6.4 Conclusion

Existing inter-channel correlation energy based DOA algorithm suffer from energy and wave-polarization ambiguity. In order to address the $\pm 90^\circ$ wave-polarization ambiguity, a trajectory-based orbital phase analysis algorithm was proposed. The OPA algorithm exploited the simultaneous presence of both radial and transverse particle oscillations in the horizontal plane, containing the seismic source and the tri-axial geophone sensor, resulting in a coupled transversely elliptical wavefront. The OPA algorithm shows higher

performance compared to CMA and HTA in terms of bearing accuracy for both vehicular and human footstep signals for different walking patterns considered in this chapter. For vehicular signals, the mean angular error for OPA is approximately 30° and 70° less than that of HTA and CMA, respectively. On the other hand, for footstep signals the MAE for OPA is approximately 19° (18.6° , 17.5° and 22.1° for circular, diagonal and radial walk, respectively.) This MAE is lower compared to the MAE of approximately 80° for HTA and CMA. The results presented showed that the proposed OPA algorithm addresses the $\pm 90^\circ$ offset relative to the true source bearing that is caused by the presence of the horizontally transverse waves in near-field.

Chapter 7

Conclusions and Future Research Directions

In this thesis, a trajectory-based approach was proposed to exploit the direction attribute of the tri-axial signals acquired by a vector-sensor geophone. Since seismic activity is caused by near-field surface sources such as human footsteps or vehicular motion, the acquired signals are elliptically polarized. Instead of employing energy-based algorithms that measured the *magnitude* of the acquired signal, this work focussed on modeling elliptical polarization using the *direction* attribute embedded in the evolution of the 3D trajectory.

7.1 Conclusions

Chapter 2 of this thesis discussed the fundamentals necessary for the understanding of near-field seismic wave polarization. Firstly, specifications of the different components of the seismic signal acquisition system employed for collecting seismic data, were described. Furthermore, the characteristics of the seismic signals due to the presence of footsteps and vehicles were analyzed. Existing algorithms for detection of human footstep and

vehicular movement were reviewed independently. The chapter ended with a review of bearing estimation algorithms using data acquired from a single tri-axial geophone.

In Chapter 3, the distinction in 3D normalized trajectory traced between elliptically polarized signal (in the presence of footsteps/vehicles) and random noise (in the absence of a source) was observed. Elliptical polarization was modeled as the smoothness of the 3D normalized trajectory. To quantify the smoothness, instantaneous metrics such as neighbourhood Euclidean distance (NED) and neighbourhood ratio (NR) were defined. Furthermore, geometric sequence (GS) and great-arc (GA) models were used to fit the normalized trajectory of footstep signals. The model-fit error was used to quantify smoothness and detect human footstep signals. Experiment results showed that the proposed trajectory-based smoothness metrics deteriorated at a slower rate compared to energy-based metrics for source-sensor distance greater than 20 m. In order to extend the trajectory-based metrics for stationary marching footsteps, the GA interpolation error was combined with tri-axial signal energy to propose a polarization-cum-energy (PE) metric. It was observed that the PE metric effectively combined the trajectory-based smoothness (that improved detection range for radial walking at farther source-sensor distances) and the tri-axial energy (that improved detection robustness for stationary marching). Extending the smoothness measure to vehicular signal trajectory, the windowed normalized correlation difference (WNCD) metric was proposed. It was shown that the WNCD metric improved the vehicular detection by approximately 25-100 m at different vehicular velocities. The window length N and the number of lags K of the WNCD metric were varied to detect both vehicular and footstep signals.

In Chapter 4, tri-axial signals were represented using hypercomplex vector-quaternions. Using this representation, elliptical polarization was modeled as non-Gaussianity and inter-channel correlation of the vector-quaternion signal. A quaternion generalized Gaussian distribution (QGGD) was derived using vector-quaternion based augmented statistics. The shape parameter of the QGGD determined non-Gaussianity whereas the degree-

of-improperness (DOI) of the quaternion augmented covariance matrix determined inter-channel correlation. By fitting the vector-quaternion signal to QGGD, a shape metric and a DOI metric were estimated. These metrics were subsequently proposed for human footstep detection. It was shown that the DOI metric improved the detection range and robustness for both radial and stationary marching footsteps compared to the shape metric.

In Chapter 5, the instantaneous orbital phase of a tri-axial signal was quantified using hypercomplex orientation-quaternion. Elliptical polarization was modeled as the instantaneous rotational characteristic of the 3D trajectory. Instead of exploiting the variance of the tri-axial signal, the concentration of rotational parameters in their respective hypersphere was used to quantify the rotational characteristics of the elliptically polarized trajectory. A velocity-to-rotation transform was proposed to estimate instantaneous rotational parameters of the 3D trajectory from the acquired tri-axial signal. Furthermore, the concentration of estimated rotational parameters was estimated using both parametric and non-parametric approaches. Experiment results showed that distinction between the presence of elliptically polarized signal and random noise was higher when concentration was estimated using rotational parameter at higher hypersphere dimension. The estimated concentration using orientation quaternion (parameterizing the instantaneous orbital phase of the trajectory) $\mathcal{M}_{\text{CinH}}$ and $\mathcal{M}_{\text{KLD}}(\hat{\mathbf{q}})$ in $\mathbb{S}^3$ achieved the best performance compared to the estimated concentration using the eigenaxis and eigenangle in $\mathbb{S}^2$ and $\mathbb{S}^1$, respectively.

In Chapter 6, focus was shifted to the estimation of direction-of-arrival of near-field surface sources. A trajectory-based approach was proposed and this approach exploited the simultaneous presence of both radial and transverse particle oscillations in the horizontal plane to estimate the source bearing. Instead of using *concentration* of the orientation-quaternion which parameterized the orbital phase of the 3D trajectory, the proposed orbital phase analysis (OPA) utilized the *direction* of the orientation-quaternion,

i.e., eigenaxis, to estimate the source bearing. The performance of the proposed OPA algorithm was evaluated on both vehicular and human footstep signals and was observed to improve bearing accuracy. In addition, the proposed algorithm addresses the $\pm 90^\circ$ phase shift relative to the true source bearing.

This thesis modeled elliptical polarization of the acquired tri-axial signal using multiple trajectory-based models. It was shown that the trajectory-based metrics which exploited the direction or orientation of the tri-axial signal not only detected the presence of near-field surface source but also increased the detection range for radial footsteps by 5-7 m and vehicular signals by 25-100 m, compared to energy-based metrics.

7.2 Future Work

There are two directions in which the work presented in this thesis can be extended. This thesis demonstrated the detection and bearing estimation of near-field surface sources using a *single* tri-axial geophone. The orbital phase analysis algorithm can be extended to estimate the bearing of surface sources using a vector-sensor array. Different configurations such as linear and circular array can be used to better understand the spatial diversity of seismic wave polarization in near-field. The orbital phase analysis of the individual elements of the vector-sensor array could be aggregated to resolve the differences between prograde/retrograde propagation of Rayleigh waves which, in turn, could address the 180° ambiguity.

On the other hand, statistics of the hypercomplex quaternions can further be exploited to model near-field polarization. Quaternions can be used to represent the inter-footstep polarization change at different source-sensor distances for a radial walk along a particular azimuth direction using perspective projection from the sensor [91]. This representation could enable better understanding of the channel effect on the polarization of the impinging seismic wave.

Furthermore, existing algorithms do not take into account the characteristics of the propagating medium. The acquired tri-axial signal can be interpreted as the response of a wave travelling through a vector-channel, which can be modeled as a quaternion. This representation will enable characterization of the propagation medium which, in turn, could lead to better detection and DOA estimation of surface sources in near-field.

Author's Publications

Conference papers

[C1] Vinod Reddy, Divya Venkatraman, Andy W. H. Khong and B. P. Ng, "Footstep detection and denoising using a single triaxial geophone," *IEEE Asia Pacific Conf. Circuits and Systems*, pp.1171-1174, KualaLumpur, Malaysia, Dec. 2010.

[C2] Divya Venkatraman, Vinod Reddy, Andy W. H. Khong and B. P. Ng, "Polarization-cum-energy metric for footstep detection using vector-sensor," *IEEE Intl. Conf. Technologies for Homeland Security*, pp.196-201, Greater Boston, Nov. 2011.

[C3] Divya Venkatraman, Vinod Reddy and Andy W. H. Khong, "A study of the ambiguity problem in footstep bearing estimation using tri-axial geophone," *8th Intl. Conf. Information, Comm. and Signal Process.*, pp.1-5, Singapore, Dec. 2011.

[C4] Divya Venkatraman, Vinod Reddy and Andy W. H. Khong, "On the use of quaternion generalized Gaussian distribution for footstep detection," *38th IEEE Intl. Conf. Acoustics, Speech, and Signal Process.*, pp.6521-6525, Vancouver, May. 2013.

Provisional Patent

A.W.H Khong, D. Venkatraman and V. Reddy, "Instantaneous orbital phase using orientation quaternions for footstep detection", US 61/729,431. 2012.

Journal paper under review

[J1] Divya Venkatraman and Andy W. H. Khong, "Directional Statistics Based Approach On Instantaneous Rotational Parameters Of Tri-axial Trajectories for Footstep Detection", under review at *Circuits, Systems, and Signal Processing*.

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