Boundary continuity effects on development of membrane actions for composite beam-slab sub-assemblages at elevated temperature

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NANYANG
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**BOUNDARY CONTINUITY EFFECTS ON DEVELOPMENT OF
MEMBRANE ACTIONS FOR COMPOSITE BEAM-SLAB SUB-
ASSEMBLAGES AT ELEVATED TEMPERATURE**

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**SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING
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2016

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ASSEMBLAGES AT ELEVATED TEMPERATURE**

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Liu Junxian

Abstract

Composite beam-slab systems have been widely used in the modern building construction as they not only reduce the constructional cost, but also expedite construction schedule compared to traditional reinforced concrete slab systems. However, steel profiled decking and supporting steel beams are vulnerable under fire condition. It is necessary to provide sufficient fire protection materials to keep these components under certain temperature for a period of time. Fire protection is costly and sometimes not necessary.

Through several large-scale fire tests conducted in Europe, despite of unprotected interior supporting steel beams, it is observed that composite beam-slab systems have a superior fire performance due to mobilisation of tensile membrane action. Therefore, the British Research Establishment (BRE) published SCI-P288 and SCI-P390 design guidelines for steel framed composite slabs by taking account of tensile membrane action at fire limit state.

Since these two design guidelines were only validated through limited fire tests and focused on isolated composite slab panels, the proposed work seeks to examine these guides more deeply, especially with regard to deflection limit validation and continuity effects on the development of tensile membrane action.

Three series of nine specimens with one-quarter scale composite slabs were fabricated and tested at elevated temperature in Nanyang Technological University. The main investigated parameters included structural continuity effect and aspect ratios (1 and 1.5).

Compared with SCI-P288 and SCI-P390, the test results showed that this design method was conservative. Even at the defined deflection limits in the SCI-P288 and SCI-P390, there were still reserves for the load-bearing capacity of these specimens. To find out the development of membrane actions, numerical models were built up by ABAQUS. From the analyses, both compressive and tensile membrane forces were

developed and the equilibrium was achieved by forming a tensile membrane zone in the middle surrounded by a peripheral compressive ring.

Based on membrane forces distribution from the numerical analyses, a revised analytical model was proposed by incorporating the boundary restraint forces. Compared with the test results, the author's analytical model showed good predictions of the load-bearing capacity for composite slabs at the fire limit state by incorporating edge beams forces.

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List of symbols

a is the aspect ratio.

A_s is the area of reinforcement.

b is the parameter controls the failure mode of the laterally-unstrained slab.

B is the width of the steel section.

b_{eff} is the effective width of the slab.

C is the compressive force in the compressive ring.

D is the bending stiffness of the slab.

d is the distance from the centroid of the tension steel rebar to the extreme concrete compression fibre.

d_c and d_t are the damage factors for compressive and tensile behaviour of concrete, respectively.

e is the enhancement factor due to tensile membrane action.

E_a is the elastic modulus of the reinforcing steel.

$E_{a,\theta}$ is the elastic modulus of steel at temperature θ .

E_{cm} is the secant modulus of elasticity of concrete.

f_c is the compressive cylinder strength of concrete.

f_{cm} is the mean compressive cylinder strength of concrete.

f_{cu} is the compressive cube strength of concrete.

$f_{p,\theta}$ is the proportional limit at temperature θ .

f_t is the tensile stress of concrete.

f_y is the yield strength of the tension steel rebar.

$f_{y,\theta}$ is the effective yield strength of steel at elevated temperature.

h is the depth of the concrete slab.

H is the depth of the steel section.

I is the second moment of area.

K is the coefficient of reinforcement orthotropy in x and y directions.

k is the magnitude of the compressive forces as compared with tensile membrane forces.

KT_0 is the yield force per unit length of the reinforcement in the short span.

$k_{u,\theta}$ is the reduction factor for stud strength.

$k_{y,\theta}$ is the reduction factor for steel strength.

l and L are the short and long spans of the slab panel. (Bailey's expression)

l_x and l_y are length along x and y of a slab, respectively.

M_P is the plastic moment resistance for a unit length of yield line.

$p_{elastic}$ is the pressure limit for elastic bending.

p_{yield} is the yield line load.

q is applied load.

Q_x and Q_y are face shear forces in x and y directions, respectively.

R is the lateral restraint forces from surrounding slab panels.

S is the in-plane shear force between different elements.

T_0 is the yield force per unit length of the reinforcement in the long span.

T_1 is the tensile force in the middle crack.

T_2 is the tensile force in the diagonal crack.

t_f is the thickness of the flange.

t_w is the thickness of the slab.

W_I and W_E are the internal work dissipation and External work, respectively.

α is the coefficient of thermal expansion.

δ is the deflection.

ε_{cl} is the strain at the peak compressive stress.

$\varepsilon_{p,\theta}$ is the strain at the proportional limit at temperature θ .

$\varepsilon_{t,\theta}$ is the limiting strain for yield strength at temperature θ .

ε_{th} is the thermal strain.

$\varepsilon_{u,\theta}$ is the ultimate strain at elevated temperature θ .

$\varepsilon_{y,\theta}$ is the yield strain at temperature θ .

θ_a is the temperature of the steel cross-section.

Δ is the axial extension.

ν is the Poisson's ratio.

ξ is the distance from half of d to the compression block of concrete cross-section over d .

ξ_0 is the distance from half of d to the compression block of concrete cross-section over d when the axial force equals to zero.

τ is face shear stress.

Chapter 1 Introduction

1.1 General

Composite beam-slab systems have been widely used in various modern construction projects in Singapore, such as Marina Bay Sands Sky Park, National Library, and Fusionopolis etc. as shown in **Fig 1.1**.

(a) Marina Bay Sands Integrated Resort, Sky Park

(b) National Library

(c) Fusionopolis @ One North (Phase 1)

Fig 1.1 Typical composite beam-slab systems buildings in Singapore (Manual)

Composite beam-slab systems not only reduce the constructional cost of an overall project, but also expedite the whole construction schedule compared to traditional reinforced concrete slab system as shown in **Fig 1.2**. Six major advantages have been identified by Wright (1987):

- a) The steel deck acts as a permanent shuttering for in-situ cast concrete slabs.
- b) The steel deck provides a working platform during construction.
- c) The steel deck acts as tensile reinforcement after concrete has hardened.
- d) The steel deck geometry can reduce about 30% of concrete in terms of mass.
- e) Cellular geometry of the deck permits M&E ducting of building services.
- f) Thin-gauged steel sheeting is convenient for handling and placing due to its light weight.

(a) Cut-away view of composite slab (b) composite slab construction site

Fig 1.2 A typical composite floor cut-away view and site casting concrete onto profile deckings (Newman, Robinson et al.)

Estimations based on the composite beam-slab systems and traditional reinforced concrete buildings are shown in **Fig 1.3**. The major difference lies in the duration for constructing the steel frames and composite decks. Since for composite beam-slab construction, steel frames can be erected easily and profile decks can be used as temporary working platforms. The form of construction shortens the construction duration significantly compared with traditional RC construction.

(a) Composite slab construction schedule

(b) RC structure construction schedule

Fig 1.3 Comparison for construction durations (in weeks) for (a) composite beam-slab

system and (b) traditional concrete reinforced building construction (steel 2013)

1.2 Problems

Composite beam-slab systems with profiled steel sheeting and supporting steel beams are vulnerable under fire condition. Thus, fire safety is the foremost consideration in composite beam-slab systems design. The aim of composite beam-slab systems design is to ensure that structures do not collapse when subject to high temperature in fire.

Traditional prescriptive method based on fire resistance testing, requires steel members to be kept below a critical temperature, typically 550° C. This leads to extensive use of passive fire protection materials, such as boards, sprays and intumescent paintings, to limit the increased of temperature of structural members at considerable cost.

Current prescriptive design method of composite structures in fire, such as BS5950-8 (BSI 1990) and EN1994-1-2 (CEN 1994), relies on single-member behaviour through fire resistance tests, which may not be able to reflect the whole structural behaviour under fire conditions. Interactions among all members of a structure in the region of fire and any cooler members outside the boundary should be included in structural fire analysis to optimise structural design. Unfortunately, none of these prescriptive design codes is able to deal with that.

1.3 Fire protection materials

For the prescriptive approach, the fire resistance period is determined for buildings with different occupancies. Additionally, the selection of appropriate fire protection materials is also important.

There are many fire protection materials in the market, including:

- Intumescent paint — this is the dominant fire protection method in the UK as shown in **Fig 1.4**. A thin film coating is painted on the fire-exposed surface of steel members. After being heated up, the coating can swell at high temperature and insulate surfaces of steel members from fire. Less than 1mm thick coating can

provide one hour fire resistance and higher fire resistance can also be achieved but at a higher price.

Fig 1.4 (a) Market share of fire protection system and (b) intumescent paint insulation effect (steel 2013)

- Boards — those are often applied to exposed surfaces of columns. They provide a clean and boxed appearance, which can be decorated with suitable finishes.
- Cementitious spray — the finishes are not visually appealing. Moreover, during the spraying process, surrounding areas need to be sealed off, which increases the construction time. The dry form of cementitious spray is more advanced and preferred by applicants.

In most of composite slab designs, intumescent painting is widely used for supporting steel members, like beams and columns, to meet the requirements from aesthetic point of views.

1.4 Cost comparisons

In Singapore, the cost of construction for a warehouse in 2011 was around S\$1200 per square metre of internal areas according to an international construction survey as shown in **Table 1.1**.

Table 1.1 Singapore – international building costs per m² of internal area (Townsend 2012)

Warehouses/year	2008	2009	2010	2011
Warehouse/factory unit-basic	1400	1200	1200	1200

Large warehouse distribution centre	1600	1500	1700	1800
High-tech factory/laboratory	3000	2800	2800	3000

For a typical 27m x 27m composite beam-slab 3-storey building with 4m height for each floor as shown in **Fig 1.5**, the total construction cost is calculated based on the construction estimation:

$$\text{Total construction cost} = 27\text{m} \times 27\text{m} \times \$\$1200/\text{m}^2 \times 3 = \$\$2.6\text{million}$$

Fig 1.5 Composite slab layouts for (a) fully protected and (b) partially protected steel beams

The current market price for intumescent paint is about S\$100/m² (including steel beam treatment like sand blasting). If all beams and columns are painted with a 1mm thick intumescent coating for the warehouse, total cost for fire protection can be estimated as:

$$\text{Total fire protection cost} = 27 \times 27 \times 100 \times 3 = \$\$220\text{k}$$

Therefore, the estimated cost of fire protection for this 27m by 27m composite slab building is around 10% of total construction cost, which is relatively high compared with the total construction cost.

Interestingly, several fire tests conducted by the Building Research Establishment (BRE) on a large scale 8-storey building at Cardington in 1990s, showed that the actual structural performance of steel framed composite beam-slab systems tests was much better than that of isolated members at elevated-temperature, even when some of the

interior secondary steel beams were not fire-protected. It can also be shown that there are large reserves in load-bearing capacity of modern steel-framed buildings under fire conditions, so providing fire protection materials to all steel beam members may not be necessary.

If the protection scheme does not include interior secondary steel beams, fire protection cost will be reduced to 5% of total construction cost, which is a great saving on the total budget.

1.5 Objectives and novelties

This project is funded by Agency for Science, Technology and Research (A*Star Singapore) and in collaboration with Singapore Civil Defence Force (SCDF). From the current prescriptive Singapore fire code (2013), all steel members must be fire protected. The main purpose of this project is to investigate the behaviour of composite beam-slab systems with partial fire protection at elevated temperature. From literature reviews, although some researchers (Bailey 2001) proposed analytical models for isolated composite slabs at elevated temperature, deformations of boundary beams and boundary continuities were not considered.

Therefore, the objective of this project are to:

1. validate the defined deflection limit in SCI-P288 and SCI-P390;
2. assess the effect of boundary beam deflections at elevated temperature on the load-bearing capacities of composite beam-slab systems;
3. study the boundary continuity effects on the development of membrane actions.

This project is divided into two parts. The first part is covered by Nguyen Tuan Trung (2014), by considering deformations of boundary beams; the second part (by the author) is focused on the composite beam-slab systems with different boundary continuities and aspect ratios on the development of membrane forces at elevated temperature.

1.6 Scope of works

The scope of research works can be classified into three categories as shown below. This research is mainly focused on the experimental tests for composite beam-slab systems under fire conditions and the experimental results can also be used to validate numerical models. Apart from the major parameters (boundary continuities and aspect ratios) in the tests, some other parameters, such as sectional forces, width of compressive ring, tensile zone, can be investigated by using the FEM software, which is more economical and time-saving. Finally, an effective analytical model will be developed based on experimental results and finite element analysis for engineering purpose. The results of each of these domains can be validated by the other two, as shown in **Fig 1.6**.

Fig 1.6 Three stages approach of research works

1.7 Outline of thesis

This thesis is divided into nine chapters:

The first chapter introduces some typical structures and some advantages of composite beam-slab systems. However, several shortcomings related to fire conditions are presented for composite beam-slab systems. The research objectives have been proposed based on these inherent weaknesses of composite beam-slab systems under fire conditions.

Chapter 2 covers one of the most well-cited experimental fire test programme:

Cardington fire tests, which showed the structural performance of steel framed composite beam-slab systems was much better than that of isolated members from traditional fire resistance tests. In addition, many researchers (Park and Gamble 2000) pointed out that the enhancement of composite beam-slab systems is due to tensile membrane action. A new design approach (BRE design method) has been introduced by taking account of tensile membrane action in composite beam-slab systems at fire limit state. Some other small-scale and large-scale tests were discussed to validate this new design method.

The third chapter introduces a finite element modelling software ABAQUS, with concrete damage plasticity (CDP) model, to simulate the concrete slab behaviour at ambient and elevated temperature conditions. Additionally, a full scale FRACOF composite beam-slab fire test was used to validate the robustness of this CDP model. Chapter 4 describes the specimen design philosophy. Three series of nine specimens have been proposed to cover different boundary conditions and aspect ratios. Based on the FEM analysis, different boundary stiffness values for a prototype building were obtained and compared with those for the scale-down specimens. It was found that the boundary restraint stiffness was within the reasonable range, which was subsequently applied in numerical studies.

Chapter 5 shows the experimental setup and specimen details of isolated Series I. Temperature profiles and displacements were recorded and discussed. In addition, parametric studies were performed to investigate the development of membrane forces at elevated temperature. Furthermore, in-plane equilibrium was achieved through measured column lateral forces, supporting steel beam forces (from FEM), compressive membrane forces around the perimeter and tensile membrane forces in at centre of composite slabs from numerical analysis.

The sixth chapter studies the boundary restraint effect on Series II and III specimens. Series II had three specimens with an aspect ratio of 1 and Series III had three specimens with an aspect ratio of 1.5. Three types of boundary restraints were studied,

which represented internal, edge and corner composite slabs.

In Chapter 7, a new type of experimental instrumentation — photogrammetry is discussed and compared with direct measurements for displacements. This method was applied to capture the displacement profiles of composite slab specimens at elevated temperature.

Chapter 8 reviews several existing analytical models for reinforced concrete (RC) slabs at ambient and elevated temperature. Furthermore, some defects of these analytical models have been pointed out and a new approach has been proposed to capture the whole load-deflection relationship for RC slabs at ambient temperature. Moreover, an amplification factor for the deflections limit by considering of boundary restraints is proposed and validated through test results. A revision on Bailey's model is made based on different boundary conditions, and this revised analytical model was validated with test results and found to be conservative when used to predict the ultimate load-bearing capacity of composite beam-slab systems at elevated temperature.

The last chapter summarises findings and recommendations based on current studies and gives some suggestions for future research.

Chapter 2 Literature review

2.1 Introduction

After highlighting the deficiencies of isolated member tests and the performance enhancement of composite beam-slab systems in fire, this chapter covers the background and the evolution of the design method for composite beam-slab systems. Firstly at the elastic stage, the plate theory from Timoshenko (1959) is discussed. When a reinforced concrete (RC) slab is overloaded and the reinforcement yields, yield lines are formed which can be used to predict the ultimate load of a slab panel.

However, it has been found the load-bearing capacity of a RC slab is much higher than the predicted yield line load through slab tests. This is due to the development of tensile membrane action in the slab panel, which enhances the load-bearing capacity of RC slabs.

To investigate tensile membrane action further, several fire tests are discussed and a new design approach — the BRE-Bailey design guide is introduced. This approach takes advantage of the development of tensile membrane action for composite slabs at the fire limit state. Lastly, several large-scale fire tests are presented and compared, which show the conservatism of this design approach.

2.2 Definition for composite beam-slab systems

In EN 1994-1-1 (CEN 1994) clause 1.5.2.6, a composite slab is defined as “*a slab in which profiled steel sheets are used initially as permanent shuttering and subsequently combined structurally with the hardened concrete and act as tensile reinforcement in the finished floor*”. However, EN 1994-1-1 does not mention the “beam-slab systems” concept. Generally, in “beam-slab systems” construction, structural interactions between the concrete slab and supporting steel beams through shear connections are

considered, so that they represent the whole composite structural behaviour. With interactions between slabs and beams, membrane actions can be mobilised under certain conditions. In the next section, the different analysis methods for slabs from previous studies will be reviewed and discussed.

2.3 Conventional slab analysis theory

2.3.1. Elastic plate theory

Back to 1950s, Timoshenko (1959) neglected any strain in the middle plane during bending, and considered a small cubic element with a unit length at small deflection as shown in **Fig 2.1**.

Fig 2.1 Demonstration for a small element under bending and shear (Timoshenko and Woinowsky-Krieger)

After integration, face shear is obtained from **Eq 2.1**:

$$Q_x = \int_{-1/2}^{1/2} \tau_{xz} dz \quad Q_y = \int_{-1/2}^{1/2} \tau_{yz} dz \quad \text{Eq 2.1}$$

Projecting all the forces acting on the element onto the z axis:

$$\frac{\partial Q_x}{\partial x} dxdy + \frac{\partial Q_y}{\partial y} dydx + qdxdy = 0 \quad \text{Eq 2.2}$$

After simplifying **Eq 2.2**, **Eq 2.3** can be obtained as shown below:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \quad \text{Eq 2.3}$$

Taking moment about x axis

$$\frac{\partial M_{xy}}{\partial x} dxdy - \frac{\partial M_y}{\partial y} dydx + Q_y dxdy = 0 \quad \text{Eq 2.4}$$

$$\frac{\partial M_{xy}}{\partial x} - \frac{\partial M_y}{\partial y} + Q_y = 0 \quad \text{Eq 2.5}$$

$$\frac{\partial M_{yx}}{\partial y} - \frac{\partial M_{xy}}{\partial x} + Q_x = 0 \quad \text{Eq 2.6}$$

Eqs 2.3, 2.5 and **2.6** define the equilibrium of the element. It is possible to combine these three equations into **Eq 2.7**:

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} - \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q \quad \text{Eq 2.7}$$

It is noticed that: $M_{xy} = -M_{yx}$ and $\tau_{xy} = \tau_{yx}$

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_{yx}}{\partial x \partial y} - 2 \frac{\partial^2 M_{xy}}{\partial x \partial y} = -q \quad \text{Eq 2.8}$$

For pure bending or under lateral loading, the plate theory neglects the effect of shearing forces Q_x and Q_y and compressive stress σ_z on bending, so that:

$$M_x = -D \left(\frac{\partial^2 \delta}{\partial x^2} + \nu \frac{\partial^2 \delta}{\partial y^2} \right) \quad \text{Eq 2.9}$$

$$M_y = -D \left(\frac{\partial^2 \delta}{\partial y^2} + \nu \frac{\partial^2 \delta}{\partial x^2} \right) \quad \text{Eq 2.10}$$

$$M_{xy} = -M_{yx} = D(1 - \nu) \frac{\partial^2 \delta}{\partial x \partial y} \quad \text{Eq 2.11}$$

where D is the bending stiffness of the slab, and $D = \frac{Eh^3}{12(1 - \nu^2)}$.

Substituting **Eqs 2.9, 2.10** and **2.11** into **Eq 2.8**, **Eq 2.12** can be obtained:

$$\frac{\partial^4 \delta}{\partial x^4} + \frac{\partial^4 \delta}{\partial y^4} + 2 \frac{\partial^4 \delta}{\partial x^2 \partial y^2} = q/D \quad \text{Eq 2.12}$$

For a simply supported rectangular slab with two edges a and b , it assumes the deflection and the applied load follow the double sine approximation as shown by **Eq 2.13** and **Eq 2.14**:

$$\delta = C \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad \text{Eq 2.13}$$

$$q = q_o \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad \text{Eq 2.14}$$

$$\frac{\partial^4 \delta}{\partial x^4} = C \frac{\pi^4 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{a^4} \quad \text{Eq 2.15}$$

$$\frac{\partial^4 \delta}{\partial y^4} = C \frac{\pi^4 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{b^4} \quad \text{Eq 2.16}$$

$$\frac{\partial^4 \delta}{\partial x^2 \partial y^2} = C \frac{\pi^4 \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}}{a^2 b^2} \quad \text{Eq 2.17}$$

Substituting **Eqs 2.15, 2.16** and **2.17** into **Eq 2.12**, **Eq 2.18** can be obtained and coefficient C can be calculated. Considering uniform distributed load $q = q_o$, **Eq 2.13** can be expressed as follows.

$$\pi^4 \left(\frac{1}{a^2} + \frac{1}{b^2} \right)^2 C = \frac{q_o}{D} \quad \text{Eq 2.18}$$

$$\delta = \frac{q_o}{\pi^4 D \left(\frac{1}{a^2} + \frac{1}{b^2} \right)^2} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad \text{Eq 2.19}$$

After obtaining **Eq 2.19**, it is possible to capture the load-deflection relationship for the elastic stage.

It assumes that the applied load is a function of x and y , and uses double sine series to represent $f(x, y)$ in the form of **Eq 2.21**:

$$q = f(x, y) \quad \text{Eq 2.20}$$

$$f(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad \text{Eq 2.21}$$

It is noticed that from **Eq 2.22** and **Eq 2.23**:

$$\int_0^b \sin \frac{n\pi y}{b} \sin \frac{n'\pi y}{b} dy = 0 \quad \text{when } n \neq n' \quad \text{Eq 2.22}$$

$$\int_0^b \sin \frac{n\pi y}{b} \sin \frac{n'\pi y}{b} dy = \frac{b}{2} \quad \text{when } n = n' \quad \text{Eq 2.23}$$

Based on Fourier series, coefficients a_{mn} can be obtained from **Eq 2.24**:

$$a_{mn} = \frac{4}{ab} \int_0^a \int_0^b f(x, y) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx dy \quad \text{Eq 2.24}$$

Substituting **Eq 2.24** into **Eq 2.21** and upon simplification, one obtains:

$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad \text{Eq 2.25}$$

$$\delta = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}$$

For uniformly distributed load:

$$f(x, y) = q_o \quad \text{Eq 2.26}$$

Equation **Eq 2.25** can be simplified as **Eq 2.27**:

$$a_{mn} = \frac{4q_o}{ab} \int_0^a \int_0^b \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} dx dy = \frac{16q_o}{\pi^2 mn} \quad \text{Eq 2.27}$$

Substituting **Eq 2.27** back into **Eq 2.25**, the general load-versus-deflection relationship at the elastic stage can be expressed as:

$$\delta = \frac{16q_o}{\pi^6 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}}{mn \left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)^2} \quad \text{Eq 2.28}$$

By taking a few terms from the double sine series, **Eq 2.28** can be simplified as **Eq 2.29** for a square slab with length of a :

$$\delta = \frac{4q_o a^4}{\pi^6 D} = 0.00416 \frac{q_o a^4}{D} \quad \text{Eq 2.29}$$

Similarly, for slabs (length a) with all four built-in edges and subject to uniformly distributed load q_o , the load-deformation relationship is given by **Eq 2.30**:

$$\delta = 0.00126 \frac{q_o a^4}{D} \quad \text{Eq 2.30}$$

Comparing **Eq 2.29** with **Eq 2.30**, it can be concluded that for a certain deflection, a slab with all edges built-in has a higher load-carrying capacity than a simply supported slab provided all dimensions remain the same. Unfortunately, the Timoshenko model is only suitable for uncracked RC slabs at elastic stage. When slabs reach the flexural capacity, flexural cracks occur and stiffness of slabs will be reduced. All of these

derivations will no longer be valid unless the corresponding tangent elastic modulus can be determined with development of cracks.

2.3.2. Yield line theory

If a slab is overloaded and fails in flexure, the reinforcement will yield first in a zone of large bending moment and this portion of the slab acts like a plastic hinge. When the load is further increased, the plastic region rotates plastically and additional loads are redistributed to adjacent regions. Occurrences of these yielding bands are referred to as yield lines, which divide the slab into different elastic plates.

A yield line analysis incorporates rigid plastic theory to compute the failure loads for a given plastic moment resistance of the slab section. However, it does not give any information about deflections. In 1940s, Johansen (1962) developed a modern yield line theory based on experimental tests for slabs and gave good predictions of ultimate loads when yield line pattern was formed in the slab as shown in **Fig 2.2**.

(a) Yield lines in the test

(b) 3D view of yield lines

Fig 2.2 Yield lines in a simply supported slab test with yield line sketch in 3D
(Wight and MacGregor 2009)

For a yield line that runs at right angles to the reinforcement, the ultimate moment of resistance per unit width can be estimated based on yielding of the reinforcement by **Eq 2.31**.

$$M_P = A_s \left(d - 0.59 A_s \frac{f_y}{f_c} \right) \quad \text{Eq 2.31}$$

where A_s is the area of tension steel rebar per unit width, f_y is the yield strength of

the tension steel rebar, d is the distance from the centroid of the tension steel rebar to the extreme concrete compression fibre and f_c is the compressive cylinder strength of concrete.

For a simply supported isotropic reinforced concrete slab, the yield line pattern can be determined as shown in **Fig 2.3**.

Fig 2.3 The yield line pattern for a simply supported RC slab with isotropic reinforcement (Hsueh 1966)

Since this RC slab is simply supported around its boundary and is subjected to a uniformly distributed load q , the collapse mode must be symmetrical and the central fracture line must be parallel to the long edges since the corresponding axes of rotations never meet. The only unknown is the angle Φ .

Considering part A first, if the maximum deflection is assumed to be unity and the axes of reference x and y are parallel to the long and short edges, respectively, rotation angle (about axis y) θ_y is equal to $\frac{1}{\frac{l_x}{2} \tan \Phi}$. Similarly, for part B, $\theta_x = \frac{2}{l_x}$.

From energy conservation, internal work dissipation can be expressed as:

$$W_1 = 2 \left(M_P \cdot l_x \cdot \frac{2}{l_x \tan \Phi} + M_P \cdot l_y \cdot \frac{2}{l_x} \right) = 4M_P \left(\frac{1}{\tan \Phi} + \frac{l_y}{l_x} \right)$$

where M_P is the plastic moment resistance for a unit length of yield line.

External work can also be calculated as follows:

$$W_E = \frac{1}{3}ql_x^2 \tan\Phi + \frac{1}{2}ql_x(l_y - l_x \tan\Phi)$$

After equating internal work W_I and external work W_E , the value of M_P is calculated from:

$$M_P = \frac{ql_x^2}{8} \left(\frac{\frac{l_y}{\tan\Phi} - \frac{\tan\Phi}{3}}{\frac{1}{\tan\Phi} + \frac{l_y}{l_x}} \right) \quad \text{Eq 2.32}$$

Let $\tan\Phi = t$ and $\frac{l_y}{l_x} = \alpha$ (aspect ratio), **Eq 2.32** can be simplified follows:

$$M_P = \frac{ql_x^2}{8} \left(\frac{\alpha - \frac{t}{3}}{\frac{1}{t} + \alpha} \right) \quad \text{Eq 2.33}$$

There is one value of t for which q is the minimum, or the required plastic moment of resistance M_P is the maximum. By differentiating M_P respect to t ($t = \tan\Phi$):

$$\frac{dM_P}{dt} = 0 \text{ leads to } -\frac{1}{3} \left(\frac{1}{t} + \alpha \right) - \left(\alpha - \frac{t}{3} \right) \left(-\frac{1}{t^2} \right) = 0$$

By solving the value t , **Eq 2.34** can be obtained:

$$t = \sqrt{3 + \frac{1}{\alpha^2}} - \frac{1}{\alpha} \quad \text{Eq 2.34}$$

Substituting **Eq 2.34** into **Eq 2.32**, the final expression for the yield line moment M_P is obtained.

$$M_P = \frac{ql_x^2}{24} \left(\sqrt{3 + \left(\frac{l_x}{l_y} \right)^2} - \frac{l_x}{l_y} \right)^2 \quad \text{Eq 2.35}$$

Based on the upper bound approach, the ultimate load of a slab is estimated by postulating a collapse mechanism, which is compatible with the boundary condition. Through the principle of virtual work, the ultimate load is determined in terms of the moments at the plastic yield lines. General solutions for uniformly loaded slabs with various boundary conditions are presented in **Eq 2.36**, **Table 2.1** and **Fig 2.4**.

$$q = \frac{12}{l_y^2 [3(\frac{l_x}{l_y}) - 1]} \left(2 \left(M_{Px} + \frac{l_x}{l_y} M_{Py} \right) + M_{P1} \right)^2 \quad \text{Eq 2.36}$$

where q is the ultimate load; l_x and l_y are the respective length of the slab in x and y direction, respectively; M_{Px} , M_{Py} , M'_{Px} and M'_{Py} are the positive and negative

yield line moment capacities per unit length in x and y directions, respectively.

Table 2.1 M_{P1} values for various boundary conditions (Park and Gamble 2000)

Case	M_{P1}	Case	M_{P1}	Case	M_{P1}
1	0	4	$M'_{Px} + \frac{l_x}{l_y} M'_{Py}$	7	$2M'_{Px} + \frac{l_x}{l_y} M'_{Py}$
2	M'_{Px}	5	$2M'_{Px}$	8	$M'_{Px} + 2\frac{l_x}{l_y} M'_{Py}$
3	$\frac{l_x}{l_y} M'_{Py}$	6	$2\frac{l_x}{l_y} M'_{Py}$	9	$2(M'_{Px} + \frac{l_x}{l_y} M'_{Py})$

Fig 2.4 Various cases of uniformly loaded slabs at four edges (Park and Gamble 2000)

Theoretically, for the upper bound approach, all the possible collapse mechanisms for a slab must be examined to ensure that the load-carrying capacity of the slab is not overestimated. This is only possible for slabs with simply-supported conditions and with regular slab shapes, so designers may face the uncertainty of whether other

alternative mechanisms exist for some unusual boundary conditions or shapes. In this case, it is an unconservative approach.

2.4 Development of membrane actions in slabs

Although yield line theory gives a good indication of the ultimate load when the yield line pattern forms without the development of in-plane membrane forces, membrane forces are often presented and co-existed. Two kinds of membrane actions are discussed as follows.

For a uniformly loaded two-way reinforced concrete (RC) slab with laterally-restrained edges, a typical load-versus-central deflection curve is shown in **Fig 2.5(a)**.

Fig 2.5 (a) Load-deflection relationship for two-way reinforced concrete slabs with edge restrained against lateral movement versus laterally unrestrained (b) Compressive membrane action (or arch effect) (Park and Gamble 2000)

At the elastic stage, when the applied load increases, the slab deforms proportionally and the behaviour is well described by Timoshenko (1959). Due to lateral restraints or compressive membrane forces, this two-way RC slab can obtain its enhanced flexural strength at point B.

Compressive membrane action can be developed at small deflection stage when the slab is laterally restrained, which means the edges cannot move horizontally. The slab edge segments are restrained from moving outwards as shown in **Fig 2.5(b)**.

Compressive membrane forces occur at the bottom surface of supports and at the top

surface of the mid-span. Therefore, flexural cracks appear in the tension zones. Beyond point B with greater development of tension cracks, compressive membrane forces start reducing due to a lack of lateral restraint up to point C. Thus, from B to C the load carried by the slab decreases rapidly with increasing slab deflection.

Once point C is reached, the net axial forces in the slab at the mid-span start changing from compression to tension. The deflection at this turning point is approximately one effective depth of the slab. This will be shown in the Section 8.2. Beyond point C, it is reasonable to assume that only reinforcement is taking all the tensile forces, acting as a tensile membrane. Then the load-deflection relationship after point C can be approximate by a straight line. Finally, failure is defined as reinforcement fracture at point D. This load-deflection curve is applicable for laterally restrained RC slabs and only the single slab panel is considered for analysis.

Fig 2.6 (a) Load-deformation curves for slabs with all edges restrained (b) Load versus deflection curve (Park and Gamble 2000)

Park (2000) conducted tests of eight slabs with all edges restrained. These slabs had ultimate loads of 1.5 to 2.9 times the Johansen yield line load. Additionally, after passing through point C in **Fig 2.5** (a), a large stretch in the middle portion of slab surface had cracks penetrating through the slab thickness and load was mainly carried by the reinforcing steel. Beyond point C, the region of tensile membrane action gradually spread towards to the edge (from the centre). From C to D, this softening range indicates that tensile membrane action can prevent catastrophic failure of slabs.

2.5 Previous fire incidents and experimental tests of composite slabs at elevated temperature

The fire code (SCDF 2013) states: “the building shall be designed and constructed so that, in the event of fire, its stability will be maintained for a reasonable period.” This requirement can be interpreted in different ways. In the prescriptive approach such as EN1994-1-2, the interpretation emphasises on single member of the structure, which requires that the load-bearing elements should be capable of withstanding the fire effects for a period without losing their stability.

However, a structure usually has redundancy that allows alternative load paths to take place so that in a fire, some members may lose stability but the overall stability of the structure will not be affected. Therefore, in the next section, several famous fire incidents or fire tests were discussed in details.

2.5.1 Broadgate building fire

One of the most quoted fire incidents happened on 23rd June 1990, when a fire developed in a partially completed fourteen-storey building in Broadgate development, London. The structure of the building was a steel framed composite slab and was only partially fire protected at this stage of construction.

The fire began on the first level of the building and the estimated fire temperature was above 1000°C. The direct fire loss was in excess of £25 million but only less than £2 million was attributed to the structural frame and floor damage.

The floor was constructed using composite long lattice trusses and composite beams supporting a composite slab. The composite slab was designed to have 90 minutes of fire resistance. At the time of the fire, the building was under construction and there was only partial fire protection to the steel members. Moreover, the sprinkler system was not installed yet.

After the fire, through a metallurgical investigation, it was concluded that the

steelwork temperature did not exceed 600°C and the maximum temperature of the bolts used in the steel connections was only 540°C .

(a) A buckled unprotected steel beam (b) A buckled unprotected steel column

Fig 2.7 Structural deformations in the Broadgate fire(Steel 1999)

The maximum measured steel beam deflection was about 270mm and it showed the evidence of local buckling of the bottom flanges and webs near their supports in **Fig 2.7(a)**. Therefore, it was concluded that high boundary restraint had great effects on the behaviour of beams due to restrained thermal expansion.

The steel columns were unprotected and had shortened by approximately 100mm as shown in **Fig 2.7(b)**. This resulted in buckling of the column head which was due to restrained thermal expansion from the upper and the lower level of the building.

The floor slab consisted of a standard profile decking, concrete and reinforcing mesh. The maximum vertical displacement of the floor was 600mm. Fracture of the reinforcement was observed and in some areas steel decking was debonded from the concrete slab. This was due to steam released from the concrete at elevated temperature. Although large deformation and buckling were observed, the structure withstood the fire onslaught and there were no imminent collapse of any of the columns, beams or floors even though steel members were only partially protected from fire.

2.5.2 Cardington fire test

Prompted by the evidence from Broadgate, BRE (Building Research Establishment) conducted a series of large-scale building fire tests at Cardington. A full scale, eight-

storey steel framed building was built in 1995. The building was designed as a non-sway frame according to BS5950 (BSI 2001) including lateral bracing provided by a central core and two stair-wells as shown in **Fig 2.8**. The light weight in-situ concrete composite floor had nominal A142 anti-crack mesh in both directions.

To comply with the UK Building Regulation (Regulations 2000), this type of building would require 90 minutes of fire resistance. The required fire protection thickness was determined by Approved Document B, which was typically in the form of a board, spray or intumescent coating.

Seven large-scale fire tests were conducted at various locations within the experimental building as shown in **Fig 2.8**. The principal results of these tests are summarized in **Table 2.2** and **Table 2.3**.

Fig 2.8 Layout of the Cardington test frame showing location of the fire test (Wald, Simões da Silva et al.)

Table 2.2 Fire test on steel structure in Cardington laboratory (Bailey, Lennon et al. 1999)

No.	Test	Fire compartment		Load	
		size, m	area m	Fire	Mechanical
1	One beam heated by gas	8 x 3	24	Gas	30%
2	One frame heated by gas	21 x 2,5	53	Gas	30%
3	Corner compartment	10 x 7	70	45 kg/m ²	30%
4	Corner compartment	9x 6	54	45 kg/m ²	30%
5	Large compartment	21 x 18	342	40 kg/m ²	30%
6	Office – Demonstration	18 x 9	136	46 kg/m ²	30%

7	Structural integrity	11 x7	77	40 kg/m ²	56%
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Table 2.3 Summary of results from major fire tests in Cardington laboratory (Wald, Simões da Silva et al. 2002)

No.	Org.	Level	Time, min to max. temp.	Reached temperature °C		Measured deformations	
				gas	steel	maximal	rezidual
1	BS*	7	170	913	875	232	113
2	BS	4	125	820	800	445	265
3	BS	2	75	1020	950	325	425
4	BRE**	3	114	1000	903	269	160
5	BRE	3	70	-	691	557	481
6	BS	2	40	1150	1060	610	-
7	ČVUT***	4	55	1108	1088	> 1000	925

*BS- British Steel (now Corus); **SCI – Steel Construction Institute; ***ČVUT – collaborative research proposed by Czech Technical University

Since this research is mainly focused on composite slab behaviour, only the demonstration of the office test is discussed below in detail. The purpose of this office fire test was to simulate a realistic fire scenario of an office when subject to a building fire.

(a) View of a typical office before the fire (b) Final deformation after the fire

Fig 2.9 Internal view of the office compartment before and after the fire test (Steel 1999)

This office compartment was 18m wide and 10m deep with a floor area of 135m². The design of this compartment was to represent an open plan office and contained a number of work stations and filing cabinets as shown in **Fig 2.9(a)**.

In order to identify a wide range of scenarios with respect to fire size, ventilation of openings, ignition, fire growth and spread, the test conditions were specifically designed to create the most severe fire situation:

- Single panels of aluminum glazing windows were provided with the total area equal to 20% of total floor area (which met the minimum requirements in the UK building Regulations).
- To ensure sufficient oxygen supply during the test, some of the glazing panels were omitted. The initial opening was required to generate a hot gas layer of 500-600°C to cause flashover.
- The total fire loading was equivalent to 46 kg wood/m² of floor area. This quantity of fire-combustible material was in excess of 95% fractile for office fire loading.

Fig 2.10 (a) Maximum displacement and temperature of 9m secondary beam and (b) cracking of the composite slab around the internal column (Steel 1999)

The maximum atmosphere temperature exceeded 1213 °C and the recorded unprotected steel temperature was 1150 °C. The calculated heat release rate at the height of the fire was about 58 MW. The maximum vertical deflection of an unprotected secondary beam was 640mm after 62 minutes as shown in **Fig 2.9(b)** and **Fig 2.10**.

Even though extensive cracking occurred around the internal columns, the overall integrity of the composite beam-slab system was maintained. Investigation of this area showed that the mesh reinforcement was not lapped properly. Therefore, care should be exercised to ensure that the reinforcement mesh has adequate lapping to mobilise tensile membrane action.

It clearly showed that steel-framed composite structures had much higher reserves of fire resistance as compared to single component test when interior and edge supporting beams were not fire protected.

It was identified that the membrane characteristics of composite slabs had beneficial effects on structural performance during a fire. With increased displacement, the slab acted as a tensile membrane through the mesh reinforcement as shown in **Fig 2.11**. If there is no horizontal restraint at the supports, a compressive membrane ring will form in the peripheral to balance the central tensile membrane action.

Fig 2.11 Qualitative representation of the behaviour of the concrete floor slab (Bailey 1998)

Although the boundary beams were fire protected and assumed to be rigid during the fire condition, large deflection was observed in the supporting beams after the fire. Nevertheless, the whole integrity of the structure was maintained and no collapse occurred during and after the test.

The Cardington tests gave very comprehensive documentation on the development of tensile membrane action for laterally-restrained composite steel framed beam-slab system by adopting proper reinforcement continuity and overlapping.

2.5.3 Small scale tests

Although the full-scale Cardington tests implied that the current design method was not addressing the true structural behaviour of composite slabs during a fire, it was difficult to obtain conclusive evidence of enhancement due to complexity of this series of full-scale fire tests. In particular it was not possible to determine the formation of the compressive ring around the perimeter of the composite slab, which allowed the tensile membrane action occurred in the centre.

Fig 2.12 A summarised test procedure and details of the tested specimen (Bailey, White et al. 2000)

To investigate tensile membrane action for laterally-unrestrained slabs, an independent 9.5mx6.5m composite slab was fabricated and tested on the same frame of the Cardington test at ambient temperature.

In order to compare with previous Cardington fire tests, it was necessary to represent as close as possible the behaviour of the composite slab during a fire. Since the steel deck reached over 1100°C in the previous fire tests, the steel decking of the tested specimen was removed before load was applied onto the specimen as shown in **Fig 2.12**.

There were three stages of the test, which consisted of:

- Removal of the steel deck from the concrete slab,
- Placing the loading system on the top surface of the slab,
- Increasing the applied load until failure of the slab.

The central vertical deflection of the slab over the duration of the test is shown in **Fig 2.13**.

Fig 2.13 Maximum vertical displacement of the slab throughout the test (Bailey, White et al. 2000)

The maximum deflection of the slab during the test is around 700mm. Failure was governed by a large central crack forming through the full depth of the slab, across the short span as shown in **Fig 2.14**.

Additionally, the strain gauges attached on the reinforcement showed that the compressive ring did form near the perimeter of the slab but with a much smaller width. Failure of the slab occurred at a uniformly distributed load of 4.81 kN/m^2 , which was slightly greater than twice the load capacity of the slab based on the classic yield line analysis (2.3 kN/m^2). Therefore, it can be concluded that tensile membrane action can enhance the load-bearing capacity of the slab compared with current design method.

Fig 2.14 Diagram and photo showing the full depth crack forming in the slab

To further investigate the membrane mechanism, in 2003, Foster (2004) conducted 15 small scale unrestrained RC slab tests subject to large vertical displacements at ambient temperature.

The slabs were tested using a twelve-point loading rig and placed on a supporting frame on four edges of the slab as shown in **Fig 2.15**. The slabs had aspect ratio of 2.09 or 1.55 with either isotropic or orthotropic reinforcement.

Fig 2.15 Illustration of (a) testing rig and crack pattern at failure from (b) top surface and (c) bottom surface (Foster, Bailey et al. 2004)

All these test results showed a far greater load-carrying capacity than the design capacity based on yield line theory. For slabs with unrestrained boundary conditions, in-plane compressive membrane forces were formed around its perimeter, and these forces anchored the development of in-plane tensile membrane forces in the centre

region of the slab as shown in **Fig 2.16**. The supporting compressive ring could only occur if the vertical displacements around the perimeter beams remained small under increasing load.

Fig 2.16 Membrane action in a horizontally unrestrained concrete slab (Bailey 2001)

To investigate the failure modes further and validate with a simple design model, Toh and Bailey (2007) carried out a series of tests on unrestrained slabs at ambient and elevated temperatures with moderate load ratios and larger span/depth ratios.

A typical concrete slab specimen as shown in **Fig 2.17**, was laterally-unrestrained and simply supported all round on the C-channel steel beams at four edges. The air-bag was used to apply uniform pressure on the top surface of the RC slab to simulate uniformly distributed load for the ambient temperature test.

Fig 2.17 A typical slab specimen from Bailey's test (Bailey and Toh 2007)

The details of a total number of 14 M-series specimens are shown in **Table 2.4**. Different reinforcement ratios were designed and studied. The reinforcement ratio is

defined by **Eq 2.37**, which represents the ultimate tensile force of reinforcement relative to the compressive strength of concrete across the slab thickness.

$$\rho = \frac{1}{2} \left\{ \left(\frac{f_{u,1} A_{a,1}}{0.8 f_{cu} d_1} \right)_{\text{long span}} + \left(\frac{f_{u,2} A_{a,2}}{0.8 f_{cu} d_2} \right)_{\text{short span}} \right\} \quad \text{Eq 2.37}$$

where $A_{a,i}$ is the area of reinforcement per unit length; d_i is the effective depth of reinforcement. The subscripts $i = 1$ or 2 are for long and short span, respectively.

Table 2.4 Details of mild steel reinforced slab specimen (M Series) (Bailey and Toh 2007)

slab	Dimension $L \times B \times H$ (mm)	Long span bar			short span bar			Bar spacing (mm)	f_{cu} (Mpa)	ρ	P (kN/m ²)	P_{test} (kN/m ²)	e	Δ_{fire} (mm)
		Φ (mm)	f_y/f_u (Mpa)	Duct (%)	Φ (mm)	f_y/f_u (Mpa)	Duct (%)							
M1	1700x1100x18.2	2.42	732/756	1.49	2.42	757/777	3.26	50.8	41.3	0.198	8.52	20.74	2.44	72.5
M2	1100x1100x19.1	2.42	732/756	1.49	2.42	757/777	3.26	50.8	38	0.198	13.80	26.99	1.96	60.4
M3	1700x1100x22.0	1.53	451/487	6.39	1.49	454/495	8.98	25.4	35.3	0.079	6.35	12.28	1.93	85.4
M4	1100x1100x20.1	1.53	451/487	6.39	1.49	454/495	8.98	25.4	35.3	0.090	8.17	18.29	2.24	65.2
M5	1700x1100x18.9	1.47	406/500	9.89	1.48	435/473	9.87	12.7	37.9	0.174	8.69	17.92	2.06	68.1
M6	1100x1100x21.6	1.47	406/500	9.89	1.48	435/473	9.87	12.7	38.6	0.140	15.72	27.03	1.72	48.0
M7	1700x1100x19.0	0.84	599/653	3.30	0.85	604/679	2.91	12.7	41.6	0.061	5.11	8.65	1.69	49.7
M8	1100x1100x22.0	0.84	599/653	3.30	0.85	604/679	2.91	12.7	42.9	0.065	6.68	10.70	1.60	29.8
M9	1700x1100x19.7	0.66	450/470	1.30	0.68	402/454	3.93	6.35	37.6	0.052	5.07	7.35	1.45	22.0
M10	1100x1100x18.6	0.66	450/470	1.30	0.68	402/454	3.93	6.35	37.3	0.061	6.63	9.89	1.56	19.6
M11	1700x1100x18.6	2.42	732/756	1.49	2.42	757/777	3.26	50.8	39.8	0.313	5.11	8.89	1.74	72.4
M12	1100x1100x19.4	2.42	732/756	1.49	2.42	757/777	3.26	50.8	40.7	0.274	8.96	16.17	1.80	65.4
M13	1700x1100x36.6	2.42	732/756	1.49	2.42	757/777	3.26	50.8	42.0	0.082	21.94	29.12	1.33	54.4
M14	1100x1100x35.9	2.42	732/756	1.49	2.42	757/777	3.26	50.8	40.2	0.088	31.03	41.18	1.33	46.0

Fig 2.18 Two kinds of failure modes from the ambient test results: (a) and (b) in compressive failure at the corners; (c) and (d) tensile failure at the reinforcement (Bailey and Toh 2007)

Based on the test results, two modes of failure were identified as compressive failure of concrete at the corners and fracture of reinforcement along the short span as shown in **Fig 2.18**.

Fig 2.19 Enhancement versus reinforcement ratio for square slabs(Bailey and Toh 2007)

Fig 2.20 Details of the test set up for elevated temperature test (Bailey and Toh 2007)

Considering square slabs for example as shown in **Fig 2.19**, when the reinforcement ratio is less than 0.1, failure is governed by fracture of the reinforcement. For a higher reinforcement ratio, compressive failure occurs instead. Therefore, in the specimen design, minimum reinforcement ratio (0.4%) was adopted to achieve tensile failure.

To compare the modes of failure with similar slabs at ambient temperature, ten specimens were tested at elevated temperature and the test set up is shown in **Fig 2.20**.

The details of the MF-series specimens are shown in **Table 2.5**. The geometry of the

MF-series specimens was similar to that of the M-series specimens with supporting frame around the perimeter of slab specimens. By applying the weights on the top surface of slab specimens, a uniform distributed working load P_{test} was kept constant during the test at elevated temperature.

Table 2.5 Details of MF reinforced slab specimens at elevated temperature (Bailey and Toh 2007)

slab	Dimension $L \times l \times H$ (mm)	Long span bar			short span bar			Bar spacing (mm)	f_{cu} (Mpa)	ρ	P (kN/m ²)	P_{test} (kN/m ²)	LR= P_{test}/P	Failure temperature (°C)			P_{Tc} (kN/ m ²)	P_{test}/P_{Tc}	A_{bar} (mm)
		Φ (mm)	f_y/f_u (Mpa)	Duct (%)	Φ (mm)	f_y/f_u (Mpa)	Duct (%)							T_{Bot}	T_{Mesh}	T_{Top}			
MF1	1700x1100x19.7	2.43	695/727	2.86	2.41	722/742	3.46	50.8	43.2	0.159	9.52	5.28	0.55	840	765	672	1.16	4.55	181
MF2	1100x1100x23.1	2.41	684/736	3.19	2/43	780/795	1.07	50.8	43.4	0.129	19.00	5.52	0.29	826	747	604	2.32	2.38	140
MF3	1700x1100x19.0	1.54	336/404	18.76	1.54	349/420	12.57	25.4	39.1	0.078	4.07	3.66	0.90	801	727	-	0.49	7.47	131
MF4	1100x1100x19.8	1.54	336/404	18.76	1.54	349/420	12.57	25.4	39	0.073	6.36	5.43	0.85	787	700	575	0.79	6.87	124
MF5	1700x1100x20.1	1.51	402/463	12.84	1.52	467/498	6.53	12.7	37.1	0.170	13.47	5.28	0.39	814	722	634	1.33	3.97	137
MF6	1100x1100x19.5	1.51	402/463	12.84	1.52	467/498	6.53	12.7	38.5	0.172	14.35	7.90	0.55	840	782	654	1.72	4.59	144
MF7	1700x1100x18.8	0.85	405/444	5.40	0.83	470/486	3.65	12.7	43.8	0.045	3.36	4.46	1.33	602	557	344	1.75	2.55	94
MF8	1100x1100x20.9	0.85	405/444	5.40	0.83	470/486	3.65	12.7	43.5	0.039	5.65	4.65	0.82	749	654	667	1.43	3.25	100
MF9	1700x1100x21.6	0.71	371/382	3.40	0.68	449/455	1.44	6.35	47.1	0.042	5.23	3.70	0.71	723	652	553	1.36	2.72	152
MF10	1100x1100x21.0	0.71	371/382	3.40	0.68	449/455	1.44	6.35	40.4	0.050	7.18	5.49	0.77	751	686	560	1.17	4.69	104

The observed failure modes of MF-series specimens were governed by fracture of reinforcement with a full-depth crack along the short span as shown in **Fig 2.21**. Interestingly, none of the specimens had a compressive failure at the corner compared with the M-series specimens at ambient temperature. This might be due to lower corner temperature compared to the high reinforcement temperature at the slab centre.

(a) Failure for MF1 with an aspect ratio of 1.5 (b) Failure for MF2 with an aspect ratio of 1

Fig 2.21 Failure mode of MF-series specimens at elevated temperature (Bailey and Toh 2007)

Additionally, for MF-series specimens, failure reinforcement temperature versus load ratio is plotted in **Fig 2.22**. It can be concluded that with increasing load ratios, failure temperature of reinforcement declined correspondingly.

Fig 2.22 Relationship of failure reinforcement temperature with load ratio (Foster, Bailey et al. 2004)

Based on small-scale slab tests, Bailey developed an analytical model taking account of tensile membrane action of the slab. The enhancement factor is defined based on slab geometries and reinforcement details. This method will be discussed in the next section in details.

2.6 BRE-Bailey design method

The BRE-Bailey design method was mainly based on observation and analysis of Cardington large-scale building fire tests conducted during that period of time (1995-1996). The buildings are applicable for non-sway steel-framed buildings with composite floor construction. The floors were constructed using shallow composite

slabs with profiled steel decking attached by shear studs to downstand steel beams. As compared with normal single member design, Cardington tests showed that structural stability could be maintained even when secondary interior supporting steel beams were unprotected, as shown in **Fig 2.23**.

Compared to existing prescriptive design code, it is necessary to ensure that there should be no increased risk to life safety of occupants, fire fighters and others in the vicinity of the building. Furthermore, when the floor is exposed to fire, excessive deformation should not cause failure of compartmentation, i.e. the fire will be contained within its compartment of origin and should not spread horizontally or vertically.

This guideline does not apply to:

- Floors constructed using precast concrete slabs;
- Internal floor beams that have been designed to act non-compositely (beams at the edge of the floor slab may be non-composite);
- Beams with service openings, *i.e.* composite floor with cellular beams;
- Flat slab systems.

The BRE design method is only applicable for:

- braced, non-sway and simple connection frame;
- rectangular composite floor slabs bounded on all sides by supporting beams;
- all columns should be restrained by at least one fire-protected beam in each direction (no internal column).

Fig 2.23 Two possible fire protection schemes (a) and (b) for composite slab panels design (Bailey 2001)

Basis of design

Compared to current design practice, there should be no increased risk to life safety of occupants, fire fighters and others in the vicinity of the building. Furthermore, when the floor is exposed to fire, excessive deformation should not cause failure of compartmentation, i.e. the fire will be contained within its compartment of origin and should not spread horizontally or vertically.

Fire resistance

Both natural fires and ISO-834 standard fire tests have been conducted in Cardington. Although some of these tests did not follow the standard ISO-834 temperature-time curve that has been used to define the fire resistance duration in the building regulations, the non-standard temperature-time curves have been interpreted in terms of the standard fire resistance curve as shown in

Table 2.6. Generally, the higher the building, the long is duration of time needed for people to evacuate, which indicates that the stability, integrity and insulation should also be maintained under fire condition.

Table 2.6 Summary of fire resistance requirements from Approved Document B (Newman, Robinson et al. 2006)

	Fire resistance (mins) for height of top storey (m)				<p>Height of top storey excludes roof-top plant areas</p> <p>Height of top storey measured from upper floor surface of top floor to ground level on lower side of building</p>
	<5	≤18	≤30	>30	
Residential	30	60	90	120	
Office	30	60	90	120*	
Shops, commercial, assembly and recreation	30	60	90	120*	
Industrial and storage	60	90	120	120*	
Closed car parks	30	60	90	120*	
Open-sided car parks	15	15	15	60	
<p>Installing sprinklers will often allow fire resistance periods to be reduced from 60 to 30 minutes or from 90 to 60 minutes.</p> <p>*Sprinklers are required, but the fire resistance of the floor may be 90 minutes only.</p>					

Design principles

The structural model for BRE design method has been developed by Bailey, initially at BRE and later at the University of Manchester. His model combined the residual bending resistance of composite beams with the contribution of composite slabs, using a combined yield-line approach and tensile membrane action.

The position of reinforcing mesh may vary according to different fire resistance requirements for the structure. For 30-60 minutes fire resistance, the mesh should ideally be placed as low in the slab as practicable, whereas for 90-120 minutes fire resistance the mesh should be shifted higher to the slab soffit with an adequate top cover. To ensure membrane action in the slab, care must be taken to ensure that the reinforcing mesh is properly lapped. This is especially important in the region of unprotected beams and around columns.

The BRE design method is based on the following assumptions:

- The load supported by the flexural behaviour of composite beams within the floor design zone is added to the lower-bound mechanism for the composite slabs, and the beams are assumed to be simply supported.
- The load supported by the composite slab in bending is calculated based on the

lower-bound yield-line mechanism, assuming that the internal beams have zero bending resistance.

- An enhancement due to membrane action in the composite slab is added to obtain the total resistance of the yield-line mechanism of the slab.
- The resistances of the composite beams and slabs are added to obtain the resistance of the total complete system.

Joint

The joint models given in P288 assume that bending moments are not transferred through the joints. The joints are known as ‘simple joints’.

There are three types of beam-to-column joints that may be considered as ‘simple joint’, *viz.* flexible end plates, fin plates and web cleats as shown in **Fig 2.24**.

(a) Flexible end plates

(b) Fin plates

(c) Web cleats

Fig 2.24 Illustrations of different types of simple connections

The steel frame building tested at Cardington contained flexible end plate and fin plate connections. Partial and full failures of some of the joints were observed during the cooling phase of the Cardington fire tests. However, these failures did not result in collapse of the structure.

Fig 2.25 Boundary beams subjected to additional load (Newman, Robinson et al. 2006)

Additional load to beams at the boundaries of the floor design zone

Normal design assumes that the floor loads are supported by secondary beams which are themselves supported on primary beams. In the BRE design model at the fire limit state, the slab transfers a proportion of the load directly to surrounding beams by tensile membrane action. The load supported by the boundary beams that are parallel to the internal beams is increased as shown in **Fig 2.25**. The reason for ignoring internal unprotected steel beams is that steel strength is very vulnerable at high temperature. Once the stiffness of internal beams is reduced, the load carried by the internal beams will be transferred to the boundary beams.

Effect of additional load on boundary beams can be calculated as follows:

$$LR_{\text{eff}} = LR_0 + \frac{q_a L}{8M_c} \quad \text{Eq 2.38}$$

where: LR_{eff} is the effective load ratio of the boundary beam;

LR_0 is the load ratio based on the normal distribution of load;

q_a is the additional loading applied on the slab;

M_c is the bending resistance of the boundary beam at ambient temperature;

L is the beam span.

Design of edge beams

It is a common practice for beams at the edge of slabs to be designed as non-composite. This is because the cost of meeting the requirements for transverse shear reinforcement is greater than that for installing a slightly heavier steel beam. However, for fire design, it is important that the floor slab is adequately anchored to the edge beams.

For this purpose, if the edge beams are designed as non-composite, they must have shear connectors at not more than 300 mm centres and the reinforcing mesh must be hooked over the shear connectors.

The Cardington tests showed consistently that the edge beams above windows are not heated to the same extent as internal beams. Typically, the temperature rise of the edge beam is 25% lower than the highest internal beam temperature rise.

Load ratio and limiting temperature

As defined in BS5950-8, the load ratio is a non-dimensional measure of the load resisted by a member at the fire limit state, which is shown in **Eq 2.39**.

$$\text{Load ratio} = \frac{\text{Load or moment at the fire limit state}}{\text{Member resistance at } 20^{\circ}\text{C}} \quad \text{Eq 2.39}$$

For most practical composite beams controlled by serviceability conditions, the load ratio will be in the range 0.45 to 0.55. The load ratio determines the limiting temperature of the member. The lower the load ratio is, the greater will be the limiting temperature.

Reinforcing mesh

The design method can be used with either isotropic or orthotropic reinforcing mesh, that is, meshes with either the same or different reinforcement areas in orthogonal directions. However, in P288 only isotropic reinforcement was considered. Bailey (2003) pointed out that load-carrying capacity of the floor slab, when subjected to a fire, was increased by 23% by rearranging a given area of reinforcement, such that more reinforcing bars were placed in the long span. However, for simplicity, only isotropic design tables are given in the design guide. Additionally, the reinforcing mesh

should be sufficiently overlapped and anchored. The details of the lapping and anchorage lengths are shown in **Table 2.7**.

Table 2.7 Recommended tension laps and anchorage lengths for welded mesh(O. and B. 2014)

(LC = Light weight concrete NC = Normal weight concrete)

Reinforcement type	Wire/Bar type	Concrete class (Grade)					
		LC	NC	LC	NC	LC	NC
		25/28	25/30	28/31	28/35	32/35	32/40
Grade 500 Bar of diameter d	Ribbed	$50d$	$40d$	$70d$	$38d$	$44d$	$35d$
6 mm wires	Ribbed	300	250	300	250	275	250
7 mm wires	Ribbed	350	300	350	275	325	250
8 mm wires	Ribbed	400	325	400	325	350	300
10 mm wires	Ribbed	500	400	475	400	450	350

Calculation procedure

The main concept for the BRE-Bailey method is well described by **Eq 2.40**, where $q_{p,\theta}$ stands for the load-bearing capacity for the floor system at a specific temperature θ and the e is the enhancement factor due to membrane action in the composite slab. The details of the calculation of the enhancement factor e will be discussed in Section 8.3.

$$q_{p,\theta} = e \frac{\text{Internal work done by composite slab in bending}}{\text{External work done by the floor system per unit load}} + \frac{\text{Internal work done by the beam(s) in bending}}{\text{External work done by the floor system per unit load}}$$

Eq 2.40

Therefore, to determine the load-bearing capacity of the composite beam-slab system, the moment capacity of the unprotected beam(s), flexural capacity of the slab panel and the enhancement factor e due to membrane actions have to be calculated. A brief design procedure is presented below:

- Calculation of moment capacity of unprotected interior beam(s) in fire

Under real fire scenarios, the resistance of the unprotected interior beam(s) fails first and is converted into a load-carrying capacity $q_{beam,\theta}$, which is expressed as

resistance over the supporting area kN/m^2 . The calculation of moment capacity of the unprotected interior beam(s) is provided in EN1994-1-2.

However, it has been found that in the BRE method, the interior beam is treated as the composite beam, which means the slab effect is already considered together with the steel beam. However, when calculating the enhanced load-bearing capacity of the slab, both the bending effect and the tensile membrane effect are included in the enhancement factor e . Therefore, the total capacity of the whole system actually considers the slab capacity. Hence, in the following calculation, only the bare steel beam capacity is considered.

b). Calculation of flexural strength of composite slabs

The flexural strength of composite slabs $q_{slab,\theta}$ can be obtained by **Eq 2.41** and derived from yield line theory based on assumptions with the failure mechanism (Wood 1961), where moment capacity of the slab M_p is dependent on the thickness of the slab, reinforcement details, the temperature profile of the composite slabs and l and L are short and long spans of the slab, respectively (**Eq 2.31**).

$$q_{slab,\theta} = \frac{24M_p}{l^2} / \left(\sqrt{3 + \left(\frac{l}{L}\right)^2} - l/L \right)^2 \quad \text{Eq 2.41}$$

c). Calculation of the enhancement factor e due to the tensile membrane action

The enhancement factor e is related to the maximum deflection of the composite slab. Therefore, a deflection limit $\delta_{slab,\theta}$ in **Eq 2.42** is defined first to estimate the maximum central deflection of the composite slab for different durations under the standard fire curve.

$$\delta_{slab,\theta} = \frac{\alpha(T_2 - T_1)l^2}{19.2h} + \sqrt{\left(\frac{0.5f_y}{E}\right) \frac{3L^2}{8}} \quad \text{Eq 2.42}$$

After that the bottom surface temperature T_2 and the top surface temperature T_1 can be estimated. The first term $\frac{\alpha(T_2 - T_1)l^2}{19.2h}$ represents the deflection due to thermal bowling under free expansion, where α is the coefficient of thermal expansion (12×10^{-6} for normal weight concrete; h is the average depth of the concrete slab; l and L are the short and long spans of the slab panel; f_y and E_s are the yield strength and

elastic modulus of the reinforcing steel at ambient temperature.

The second term $\sqrt{\left(\frac{0.5f_y}{E}\right) \frac{3L^2}{8}}$ represents mechanical strains in reinforcement by assuming a parabolic deflected shape of slab. Although these two deflection limits are not based on the same boundary support condition, it is a good approximation and convenient for engineers to calculate these deflection limits based on thermal and mechanical effects.

Therefore, a general design procedure for BRE-Bailey method is shown in **Fig 2.26**. Taking the composite slab panels for an example, it is necessary to determine the fire resisting time for this particular member first according to its usage. Then the total applied load at the fire limit state (FLS) can be determined. After that, numerical scheme for the composite slab can be chosen and optimised.

Corresponding to the previous three steps, the load-bearing resistance of the slab can be calculated. If the applied load at the fire limit state is larger than its resistance, it is necessary to increase the thickness of the slab, mesh reinforcement size or beam size and re-evaluate until the slab meets all the requirements.

This design method will be validated by published fire resistance tests such as FRACOF, COSSFIRE and Mokrsko fire tests, etc. which will be discussed in the next section.

Fig 2.26 General BRE-Bailey design procedure (Bailey)

2.7 Summary of analytical models

Since there were several different approaches studying membrane effects of composite

slabs, **Table 2.8** summarises previous analytical models related to membrane actions.

Table 2.8 Summarised previous analytical models related to membrane actions

Time/Author	Topics/Findings	Reference
1960s R. Park	A theory was developed for the load carried at large deflections by tensile membrane action of uniformly loaded rectangular slabs with edges restrained against all translation.	Park, R. and W. L. Gamble (2000). Reinforced concrete slabs, John Wiley & Sons.
1970s A. Sawczuk B. Hayes	A new energy method and an equilibrium method were presented to estimate the membrane forces.	Sawczuk, A. and L. Winnicki (1965). "Plastic behavior of simply supported reinforced concrete plates at moderately large deflections." International Journal of Solids and Structures 1(1): 97-110.
1990 F. J. Vecchio K. Tang	The formation and influence of compressive membrane action in RC slabs is discussed. High axial compressive forces, which resulted in a significant increase in flexural stiffness and load capacity.	Vecchio, F. and K. Tang (1990). "Membrane action in reinforced concrete slabs." Canadian Journal of Civil Engineering 17(5): 686-697.
2003 Z.H. Huang I. W. Burgess R. J. Plank	A nonlinear layered finite element procedure for predicting the structural response of reinforced concrete slabs subjected to fire is described.	Huang, Z., I. W. Burgess and R. J. Plank (2003). "Modeling membrane action of concrete slabs in composite buildings in fire. I: Theoretical development." Journal of Structural Engineering 129(8): 1093-1102
2006 G. M. Newman J. T. Robinson C. G. Bailey	SCI P288 SCI P390-TSLAB: In these two design guides, recommendations were given for buildings up to 120 minutes standard fire resistance. Additionally, they allowed partially-unprotected interior steel beams to achieve the enhancement due to the tensile membrane action.	Newman, G. M., J. T. Robinson and C. G. Bailey "Fire safety design: A new approach to multi-storey steel framed buildings (second edition)." (The Steel Construction Institute).
2014 I. W. Burgess S.S. Huang	An analytical model was developed based on a kinematically admissible deflection from a large-deflection plastic analysis.	Burgess, I., S. Huang and S. Staikova (2014). A re-examination of the mechanics of tensile membrane action in composite floor slabs in fire. 8th International Conference on Structures in Fire.

2.8 Recent large-scale fire test

2.8.1 FRACOF test

The test specimen consisted of four secondary beams, two primary beams, four short columns and a 155mm thick composite slab. This specimen was designed to have 120 minutes of fire resistance. Only the interior two steel beams were left unprotected and the remaining beams and columns were fire protected as shown in **Fig 2.27**. Beam-column joints were connected by flexible endplate connections and beam-beam joints

were fabricated from double angle cleats. The composite slab was constructed with 0.75mm thick COFRAPLUS60 steel decking, which was widely used in French market.

Fig 2.27 The specimen test set up for FRACOF (Zanon, Somnavilla et al. 2011)

During the fire test, mechanical loading on the floor was effected by fifteen sand bags uniformly distributed as shown in **Fig 2.28**. The equivalent uniform load was 3.87 kN/m^2 which was slightly greater than a fire design load of 3.75 kN/m^2 from EN 1991-1-2 (CEN 1991).

Fig 2.28 (a) Loading pattern plan view (b) Loading of floor with sand bags (Zanon, Somnavilla et al. 2011)

The test lasted 120 minutes and the overall test continued until 900 minutes when the integrity failure of the slab was detected. Investigation showed that the final crack was caused by the failure of a welded joint between two steel reinforcing meshes as shown

in **Fig 2.29**. In this case, the reinforcing meshes were unable to reach the ultimate fracture stress at the centre of the slab. Therefore, proper tension laps should have been provided at all mesh joints.

(a) Welded reinforcement joint

(b) Crack of the reinforcement joint

Fig 2.29 Joint of reinforced steel meshes before and after test (Zanon, Somnavilla et al. 2011)

2.8.2 COSSFIRE fire test

Similar to the FRACOF fire test, a composite slab specimen was fabricated as shown in **Fig 2.30**.

Fig 2.30 Steel frame details and the overall view of the tested specimen (Zanon, Somnavilla et al. 2011)

The deflection of the specimen increased rapidly and the central deflection reached around 500 mm after 120 minutes. However, concrete crushing occurred at the mid-

span of one edge beam as shown in **Fig 2.31**, which indicated the collapse of this edge beam. Nevertheless, this failure did not lead to the collapse of the global composite beam-slab system due to load redistribution through membrane effect as shown in **Fig 2.32**.

Fig 2.31 Collapse of the edge beam after the fire test (Zanon, Somlavilla et al. 2011)

Fig 2.32 The tested specimen during and after the fire (Zanon, Somlavilla et al. 2011)

2.8.3 Mokrsko fire test

Another large-scale fire test of one floor 18m by 12m administrative building was conducted by Czech Technical University in 2008. The composite slab consisted of

castellated beams of a span 9 to 12m and the slab thickness was 60mm over the rib 120mm with Cofraplus 0.75mm profile decking. The reinforcement was $\Phi 5$ mm smooth mesh at 100mm spacing in both directions and placed 20mm below the concrete slab surface. Two 250mm thick concrete walls were constructed to provide lateral stiffness as shown in **Fig 2.33**.

(a) Plan view dimensions (b) Steel-framed construction with shear wall
Fig 2.33 The details of the fire compartment and the erection of the steel frame
 (Wald 2010)

The applied load represented the actual administrative building design load of around 4 kN/m^2 . This load was applied through sand bags placed above the composite slab. The design characteristic fire load was created by igniting 15 m^3 of wooden cribs and the estimated fire load was about 420 MJ/m^2 .

Damage of the ceiling started in the southeast corner and the slab lost its resistance at 62 minute of the experiment due to connection failure as shown in **Fig 2.34**.

Fig 2.34 The overall view of the structure after the fire test (Wald 2010)

Severe spalling was observed in a concrete wall and a column at the south east corner. Therefore, the bolted connection of the primary box girder was exposed to torsion, which led to shear failure of the bolts as shown in **Fig 2.35**. It can be concluded that the fire resistance of the connection plays a very crucial role maintaining the overall stability of structures under fire condition.

Fig 2.35 Spalling of the concrete wall and column with breaking beam-column connection (Wald 2010)

2.9 Summary

This chapter reviewed some traditional analytical models at the beginning. The Broadgate building fire incident and Cardington fire tests were introduced. Based on these fire tests, researchers started to pay attention to membrane effects in composite slabs. Thereafter, Bailey developed his method and applied it in the composites slab design. Some other large scale fire tests were included in Section 2.7 to verify Bailey's method. After reviewing the history and development of published literatures for membrane forces in composite slabs as shown in **Table 2.9**, it can be concluded that lateral restraints can enhance the load-bearing capacity of the slab by mobilising compressive membrane action at the beginning stage. At larger deformation stage, tensile membrane action together with peripheral compressive ring takes place and governs the failure mechanism. Although the tensile membrane action has been studied for isolated composite slabs at elevated temperature, very little research was conducted

on lateral restraints of composite slabs at elevated temperature. Hence, an initial numerical study was conducted in the next chapter by using ABAQUS to investigate the boundary effect on the development of membrane actions for the initial small deformation stage and subsequent large deformation stage.

Table 2.9 Summarised previous fire tests related to membrane actions

Time/Author	Events	Findings
1990s BRE Cardington fire tests	An 8-storey steel framed with composite slabs building was tested with partially protected steel beams at elevated temperature.	The tests showed that unprotected composite slabs on steel decking did not collapse after a compartment burnout. Large displacements were observed but the overall integrity was maintained.
2000 C. Bailey	Series of tests on small-scale reinforced concrete slabs at ambient and elevated temperature.	Based on Hayes's model, Bailey proposed an analytical model of slabs at large displacement to incorporate the enhancement due to tensile membrane action.
2006 G. Q. Li	Experimental tests on reinforced concrete slabs at elevated temperature.	An analytical model for calculating the enhanced ultimate load-capacity of floor slabs by considering the effect of tensile membrane action was proposed and validated with test results.
2008 FRACOF	A composite beam-slab system was tested at elevated temperature.	The test lasted for 120 minutes but large cracks occurred due to insufficient overlapping of reinforcing meshes at the mid-span of the composite slab. Proper overlapped reinforcing mesh was emphasised to mobilise tensile membrane action.
2008 COSSFIRE	A composite beam-slab specimen similar to FRACOF was tested at elevated temperature.	The edge beam buckled at high temperature but structural stability was maintained. An additional edge beam check was necessary by considering the load redistribution after interior beams failure.
2008 Mokrsko	A composite slab structure representing the administrative building of size 18m by 12m was tested at elevated temperature.	The slab lost its resistance in compression at 62 minutes of the experiment due to spalling of top concrete column, which led to the bolted connection exposed to fire. Final failure was governed by the shear failure of the connection.
2010 S. Guo C. Bailey	One-way composite slab strips were laterally restrained and tested at ambient and elevated temperature.	At ambient temperature, the restrained forces were measured and failure was governed by the strength of steel meshes. The mesh strains were also instrumented and the results showed that the reinforced meshes were in compression at the beginning stage due to thermal expansion and quickly changed to tension at large displacements.

Chapter 3 Methodology of numerical simulations

3.1 Introduction

This chapter introduces some general information about numerical simulations of RC slabs through ABAQUS. An in-built concrete damage plasticity (CDP) model will be explained with defined damage parameters. These damage parameters will be used to describe the reduced elasticity for concrete material in the softening range.

The main objectives in this chapter are: (1) to investigate the feasibility of using CDP model to simulate the behaviour of tensile membrane action in composite slabs at ambient and elevated temperature; (2) to find out the effects of boundary stiffness to the development of tensile membrane action. Additionally, mesh sensitivity studies will also be performed.

3.2 ABAQUS

As a commercial finite element modelling (FEM) software ABAQUS, it provides a good collection of multi-physics capabilities, such as fully coupled thermal-stress analysis and so on. In addition, ABAQUS can also take account of material and geometric non-linearity effects for composite slabs at ambient and elevated temperatures.

3.2.1 Solvers comparison

There are two main solvers in ABAQUS, which are ABAQUS/standard and ABAQUS/explicit as shown in **Table 3.1**. Generally, the explicit dynamic integration method is also known as the forward Euler or central difference algorithm. The

ABAQUS\explicit solver is applied in numerical analyses, since there are no force/equilibrium convergence checks in explicit analyses. Convergence criterion is not applicable for explicit analyses. In ABAQUS/Explicit analyses, only the total step time needs to be specified and time increment size is chosen automatically so that it always satisfies the stability limit.

Hence, composite slabs at elevated temperature involve large displacements as discussed from Cardington test in Section 2.5.2. Both geometry and material nonlinearities in numerical model will be encountered. ABAQUS\explicit is designed to overcome convergence problems due to degradation or cracking of concrete at elevated temperature. The ABAQUS/explicit solver will be applied in further numerical study.

Table 3.1 Comparison for Standard and Explicit solvers in ABAQUS

Solver	Advantages	Disadvantages
Standard	<ul style="list-style-type: none"> It provides a more efficient solution for very large problems 	<ul style="list-style-type: none"> It may not be possible to obtain an efficient solution with ABAQUS/Standard if there are significant discontinuities in the solution.
Explicit	<ul style="list-style-type: none"> It can deal with large displacement, nonlinear, and quasi-static problems ABAQUS/Explicit contains many modelling capabilities that do not exist in ABAQUS/Standard. 	<ul style="list-style-type: none"> It may be very time consuming when calculating highly discontinuous problems

3.2.2 Steel properties input

EN 1994-1-2 provides stress and strain relationship for steel reinforcement and structural steel at ambient and elevated temperature. The stress-strain relationship is provided as shown in **Fig 3.1** and **Table 3.2**.

Fig 3.1 Stress-strain relationship for carbon steel at elevated temperatures

Table 3.2 Stress-strain relationship for steel at ambient temperature (CEN 1994)

Strain range	Stress σ	Tangent modulus
I / elastic $\varepsilon \leq \varepsilon_{ap,\theta}$	$E_{a,\theta} \varepsilon_{a,\theta}$	$E_{a,\theta}$
II / transit elliptical $\varepsilon_{ap,\theta} < \varepsilon \leq \varepsilon_{ay,\theta}$	$(f_{ap,\theta} - c) + \frac{b}{a} \sqrt{a^2 - (\varepsilon_{ap,\theta} - \varepsilon_{a,\theta})^2}$ <p>with</p> $a^2 = (\varepsilon_{ay,\theta} - \varepsilon_{ap,\theta})(\varepsilon_{ay,\theta} - \varepsilon_{ap,\theta} + \frac{c}{E_{a,\theta}})$ $b^2 = E_{a,\theta}(\varepsilon_{ay,\theta} - \varepsilon_{ap,\theta})c + c^2$ $c = \frac{(f_{ay,\theta} - f_{ap,\theta})^2}{E_{a,\theta}(\varepsilon_{ay,\theta} - \varepsilon_{ap,\theta}) - 2(f_{ay,\theta} - f_{ap,\theta})}$	$\frac{b(\varepsilon_{ay,\theta} - \varepsilon_{a,\theta})}{a\sqrt{a^2 - (\varepsilon_{ay,\theta} - \varepsilon_{a,\theta})^2}}$
III / plastic $\varepsilon_{ay,\theta} < \varepsilon \leq \varepsilon_{au,\theta}$	$f_{ay,\theta}$	0

where $f_{y,\theta}$ is the effective yield strength of steel at elevated temperature;

$f_{p,\theta}$ is the proportional limit at temperature θ ;

$E_{a,\theta}$ is the elastic modulus of steel at temperature θ ;

$\varepsilon_{p,\theta}$ is the strain at the proportional limit at temperature θ ;

$\varepsilon_{y,\theta}$ is the yield strain at temperature θ ;

$\varepsilon_{t,\theta}$ is the limiting strain for yield strength at temperature θ ; and

$\varepsilon_{u,\theta}$ is the ultimate strain at elevated temperature θ .

However, in EN1994-1-2, the strength reduction factors at elevated temperature for structural steel and reinforcing steel are slightly different as shown in **Table 3.3** and **Table 3.4**, respectively.

Table 3.3 Reduction factors for stress-strain relationship of structural steel at elevated temperatures (CEN 1994)

Steel temperature θ_a [C°]	$k_{E,\theta} = \frac{E_{a,\theta}}{E_a}$	$k_{p,\theta} = \frac{f_{ap,\theta}}{f_{ay}}$	$k_{y,\theta} = \frac{f_{ay,\theta}}{f_{ay}}$	$k_{u,\theta} = \frac{f_{au,\theta}}{f_{ay}}$
20	1.00	1.00	1.00	1.25
100	1.00	1.00	1.00	1.25
200	0.90	0.807	1.00	1.25
300	0.80	0.613	1.00	1.25
400	0.70	0.420	1.00	
500	0.60	0.360	0.78	
600	0.31	0.18	0.47	
700	0.13	0.075	0.23	
800	0.09	0.050	0.11	
900	0.0675	0.0375	0.06	
1000	0.0450	0.0250	0.04	
1100	0.0225	0.0125	0.02	
1200	0	0	0	

Table 3.4 Reduction factors for stress-strain relationship of reinforcing steel at elevated temperatures (CEN 1994)

Steel temperature θ_a [C°]	$k_{E,\theta} = \frac{E_{a,\theta}}{E_a}$	$k_{p,\theta} = \frac{f_{ap,\theta}}{f_{ay}}$	$k_{y,\theta} = \frac{f_{ay,\theta}}{f_{ay}}$
20	1.00	1.00	1.00
100	1.00	0.96	1.00
200	0.87	0.92	1.00
300	0.72	0.81	1.00
400	0.56	0.63	0.94
500	0.40	0.44	0.67
600	0.24	0.26	0.40
700	0.08	0.08	0.12
800	0.06	0.06	0.11
900	0.05	0.05	0.08
1000	0.03	0.03	0.05
1100	0.02	0.02	0.03

1200	0	0	0
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It can be seen from **Table 3.3** and **Table 3.4** that when temperature exceeds 500°C, mechanical properties such as yield strength and elastic modulus reduce significantly for both structural steel and reinforcing steel. However, the proportional limit $f_{ap,\theta}$ for structural steel is more sensitive than that of reinforcing steel at elevated temperature. On the contrary, the elastic modulus $E_{a,\theta}$ and the yield strength $f_{y,\theta}$ for reinforcing steel reduces more rapidly than those of structural steel. A typical Grade S355 structural steel stress-strain relationship and a Grade S460 reinforcing steel stress-strain curve are plotted as shown in **Fig 3.2** and **Fig 3.3**, respectively.

Fig 3.2 Engineering stress-strain curve of Grade S355 structural steel

Fig 3.3 Engineering stress-strain curve of Grade S460 reinforcing steel

However, for steel members subject to increasing load, the cross sectional areas vary due to Poisson's ratio effect. Therefore, it is necessary to define the true stress-strain relationships in ABAQUS to simulate the realistic stress-strain behaviour of steel elements. The conversion from the engineering stress-strain curve to the true stress-strain relation is defined by **Eq 3.1** and **Eq 3.2**.

$$\sigma_{\text{True}} = \sigma_{\text{Engineering}}(1 + \varepsilon_{\text{Engineering}}) \quad \text{Eq 3.1}$$

$$\varepsilon_{\text{True}} = \text{LN}(1 + \varepsilon_{\text{Engineering}}) \quad \text{Eq 3.2}$$

Hence, the revised ABAQUS inputs for structural steel and reinforcing steel are shown in **Fig 3.4** and **Fig 3.5**, respectively. It is note-worthy that the true limiting strain for yield strength changes from 0.15 to 0.14 and the ultimate true strain changes from 0.2 to 0.182 based on **Eq 3.1** and **Eq 3.2**.

Fig 3.4 True stress-strain curve of Grade S355 structural steel

Fig 3.5 True stress-strain curve of Grade S460 reinforcing steel

From the literature review in Section 2.5.2, since buckling of steel supporting beams were observed from experimental tests, it is necessary to define thermal expansion for steel material at elevated temperature. In EN1994-1-2, thermal strain $\varepsilon_{th}(\theta)$ at different temperature θ_a is given as follows:

$$\varepsilon_{th}(\theta) = -2.416 \times 10^{-4} + 1.2 \times 10^{-5}\theta_a + 0.4 \times 10^{-8}\theta_a^2 \quad \text{for } 20^\circ\text{C} \leq \theta < 750^\circ\text{C}$$

$$\varepsilon_{th}(\theta) = 1.1 \times 10^{-2} \quad \text{for } 750^\circ\text{C} \leq \theta < 860^\circ\text{C}$$

$$\varepsilon_{th}(\theta) = -6.2 \times 10^{-3} + 2 \times 10^{-5}\theta_a \quad \text{for } 860^\circ\text{C} \leq \theta \leq 1200^\circ\text{C}$$

3.2.3 Concrete properties input

Since convergence is always a challenge when conducting non-linear analysis of concrete composite slabs, traditional techniques like Newton-Raphson approach that reduce the stress increment may not be sufficient.

In the concrete damage plasticity (CDP) model, there are two failure mechanisms defined as tensile cracking and compressive crushing of concrete material. In addition, damaged plasticity are defined to characterise the uniaxial tensile and compressive responses of concrete stress-strain relationships. According to Kmiecik et al.'s study (2011), parameters such as dilatation angle, eccentricity and ratio of strength can be defined to simulate multi-axial stress state based on uniaxial stress. Moreover, viscosity parameter μ allows plastic flow to regularise constitutive equations in the CDP model.

The damage factors have been introduced to describe the elasticity degradation when strains exceed corresponding strains at peak compressive and tensile stresses. However, there have not been many studies on the damage factors at elevated temperature. Therefore, in this section, a general approach of the damage plasticity models at both ambient and elevated temperature are described and verified with previous experimental test results.

The stress-strain relationship for normal weight concrete follows EN 1992-1-1 for ambient temperature. The uniaxial compressive stress σ_c and strain ε_c is defined as shown in **Eq 3.3**.

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta} \quad \text{Eq 3.3}$$

where $\eta = \varepsilon_c / \varepsilon_{c1}$ and ε_{c1} is the strain at the peak compressive stress; $k = 1.05E_{cm} \times |\varepsilon_{c1}| / f_{cm}$; E_{cm} and f_{cm} are the secant modulus of elasticity and the mean cylinder strength of concrete, respectively.

In **Fig 3.6(a)**, it can be seen that the elastic limit is defined as $0.4f_{cm}$ and the corresponding strain is called elastic strain $\varepsilon_{elastic}$. In ABAQUS, the inelastic strain

$\varepsilon_{inelastic}$ is defined as total strain ε_c minus elastic strain $\varepsilon_{elastic}$. It assumes that before reaching the elastic limit, there is no damage to the concrete stiffness E_{cm} .

(a)The compressive stress-strain relation for concrete at ambient temperature

(b)The tensile stress-strain relation for concrete at ambient temperature

Fig 3.6 The input uniaxial concrete properties in CDP model (Manual)

However, once the strain is beyond the elastic limit, for example at point A, when it is

unloaded, the stiffness will be reduced to $(1 - d_c)E_{cm}$ with the unloaded residue plastic strain $\varepsilon_{plastic}$. The compressive damage factor d_c describes the degradation of concrete stiffness when cyclic loading and unloading are applied. According to the work (Birtel and Mark 2006), the ratio b_c is defined as $\varepsilon_{plastic}/\varepsilon_{inelastic}$ and this value is suggested to be 0.7 based on various experimental cyclic tests (Sinha, Gerstle et al. 1964).

Unfortunately, up till now in the Eurocode, there is no stress-strain relationship of concrete in tension. Therefore, based on Guo's model (2003), the stress-strain relationship for tensile behaviour is adopted as shown in **Fig 3.6(b)**.

It can be seen that before tensile stress σ_t reaches the maximum tensile stress f_t , the stress-strain relationship is still linear for elastic modulus E_{cm} . Once f_t attains the maximum stress, tensile strain ε_t is defined as crack opening strain ε_{cro} and the degradation of tensile stress is given by the equation $y = \frac{x}{\alpha_t(x-1)^{1.7}+x}$, where $x = \varepsilon_t/\varepsilon_{cro}$, $y = \sigma_t/f_t$ and $\alpha_t = 0.312f_t^2$.

Similarly, the tensile damage factor is defined in the same way as the compressive damage factor. For example, when the specimen is unloaded, the stiffness will be reduced to $(1 - d_t)E_{cm}$ with the unloaded plastic strain $\varepsilon_{plastic}$ as shown in **Fig 3.6(b)** at Point B. Unlike compressive inelastic strain under unloading, cracking strain ε_{crack} is used instead and is equal to total tensile strain ε_t minus crack opening strain ε_{cro} . Based on experimental test results (Reinhardt and Cornelissen 1984), the ratio b_t is defined as $\varepsilon_{plastic}/\varepsilon_{cro}$ and is suggested to be 0.1.

When temperature increases, the concrete characteristic compressive strength $f_{c,\theta}$, the peak compressive strain $\varepsilon_{c1,\theta}$ and the crushing strain $\varepsilon_{cu1,\theta}$ at elevated temperature are changed based on the respective definitions in EN 1994-1-2. A summary of compressive and tensile properties of concrete is shown in **Fig 3.7**. Additionally, the compressive and tensile damage factors at elevated temperature are plotted in **Fig 3.8**.

(a) Compressive stress-strain relationship at elevated temperature

(b) Tensile stress-strain relationship at elevated temperature

Fig 3.7 Summarised compressive and tensile stress-strain relationship for concrete C25/30 at elevated temperature

(a) Compressive damage factors at elevated temperature

(b) Tensile damage factors at elevated temperature

Fig 3.8 Summarised compressive and tensile damage factors for concrete C25/30 at elevated temperature

However, none of the design codes gives information for concrete elastic modulus at elevated temperature. After reviewing several mechanical concrete models, a great scatter in the experimental results for initial elasticity of concrete was observed from

Youssef and Moftah (2007). For simplicity, in the numerical models, the author adopted the elastic modulus of concrete at elevated temperature from Li and Purkiss (2005) as shown below:

$$E_{ciT} = \frac{800-T}{740} E_{ci} \leq E_{ci}$$

where E_{ci} is the initial elastic modulus of concrete material;

E_{ciT} is the elasticity of concrete at elevated temperature T .

3.3 FE model validation for RC slabs at ambient temperature

Firstly, the Series M specimens from Section 2.5.3 were used to validate the numerical models at ambient temperature. The details of Series M specimens are shown in **Table 2.4**.

In ABAQUS/explicit, the 4-node doubly curved thin shell element SR4 with reduced integration, was used to simulate two-way spanning concrete slabs as shown in **Fig 3.9**. The slabs were simply supported around their four edges and were subjected to applied pressure on the surface.

Fig 3.9 A typical concrete slab model M1 in ABAQUS

The normalised loads versus normalised deflections for M-series are plotted in **Fig 3.10** from (a) to (f). Reinforcing mesh was modelled as a smeared layer within the concrete shell elements by defining the relevant rebar size, spacing and location. Compared with the test results, it can be seen that the CDP model is able to trace the

overall behaviour of RC slabs. Large discrepancies have been observed and these differences resulted from boundary restraints. In Bailey's small scale tests, 4 corners of the slabs were slightly clamped. However, in numerical modelling, it was difficult to quantify the definition of "slightly clamped". Additionally, fracture of reinforcement during the tests cannot be captured in numerical simulations.

Fig 3.10 Comparison between the FEM model with Series MF experimental test results at ambient temperature

Fig 3.11 Illustration for different mesh sizes

Additionally, different mesh sizes were investigated to find out the most efficient mesh size for further study. In **Fig 3.11**, three different mesh sizes for specimen M1 were constructed. Numerical simulations were compared with test results as shown in **Fig 3.12**.

Fig 3.12 Comparison between test results and different mesh sizes

It can be seen that the three numerical models are stiffer than the experimental curve at the beginning. This is because the four corners of slab M1 were fixed in the test while four pin supports were adopted in the numerical model. Additionally, it can be seen that a mesh size of 0.1m is more rigid than a mesh size of 0.05m and 0.025m.

The width of compressive membrane zones has been identified as one-eighth of the span (in Section 5.9). Since the sizes of specimens ranged from 1.1 m to 1.5 m, using 0.05 m mesh size in the models is capable of capturing compressive membrane forces around the compressive ring. Hence, this 0.05 m mesh size is sufficient for numerical

validations and specimen analyses.

3.4 FE model validation for RC slabs at elevated temperature

Series MF specimens (Section 2.5.3) were used to validate the CDP model at elevated temperature. Since this research was only focused on the mechanical behaviour of slabs at elevated temperature, thermal analysis was not conducted.

To incorporate the temperature effect into the numerical models, a temperature gradient was assigned across the thickness based on the actual measured temperature. The details of MF Series specimens are given in **Table 2.5**. Only six specimens were selected to compare with experimental results at elevated temperature as shown in **Fig 3.13**. Obviously, from the comparison study in **Fig 3.13(a)** to (f), this CDP model can be applied at elevated temperature. Therefore, this numerical model is robust and will be used in further numerical investigations.

(a) Specimen MF1

(b) Specimen MF2

(c) Specimen MF3

(d) Specimen MF4

Fig 3.13 Comparison between the FEM model with Series MF experimental test results at elevated temperature

3.5 FE model validation for beam-slab systems at elevated temperature

As discussed in Section 2.7.1, the FRACOF composite beam-slab specimen was used to validate the behaviour of composite slabs at elevated temperature. An ABAQUS model was built up as shown in **Fig 3.14**.

Fig 3.14 Numerical model for FRACOF test specimen with dimension details

The FRACOF composite slab consisted of four protected edge beams and two

unprotected interior beams. The slab was 155mm thick with 0.75mm thick COFRAPLUS60 steel decking.

The bonding between the composite slabs and beams is not modelled. Since the composite slabs in the test were designed with full shear connections with adequate number of shear studs, concrete slab elements and supporting beam elements are assumed to be fully tied together, with no relative movements between them.

Fig 3.15 The temperature profiles for composite slab with supporting beams

The temperature gradients for the composite slab and supporting steel beams were separately assigned according to the measured temperatures from the test results as shown in **Fig 3.15**. It should be noted that the temperature of unprotected beams were over 1000°C and the temperature of protected edge beams were below 300°C.

Fig 3.16 Numerical results for deflection versus experimental deformation

Numerical results were compared with the test results as shown in **Fig 3.16**. The D1 and D2 test curves give the central deflection of the composite slab and D3 is the edge secondary beam mid-span deflection. It can be seen that the ABAQUS simulations are close to the test results.

The cross-sectional forces along the short span are plotted with respect to mesh temperature as shown in **Fig 3.17**. The sectional forces at the room temperature were in compression at the beginning. When the mesh temperature increased to 107°C, tensile membrane forces occurred at the mid-span. With increased mesh temperature, the tensile membrane zone expanded outwards towards the perimeter of the composite slab. It is noteworthy that the width of the compressive ring is around one-sixth of the span in both long and short spans.

Fig 3.17 Sectional forces at mid-span along short span versus mesh temperature

The resultant in-plane forces were integrated along the mid-span and a summary is given in **Table 3.5**. With increased mesh temperature, the compressive membrane forces were increased due to thermal expansion but they decreased after the mesh temperature reached around 100°C. Meanwhile, the tensile membrane forces also increased and the tensile membrane zone expanded towards the perimeter of the slab. Overall, in-plane-equilibrium was achieved after examining the net equilibrium force. Therefore, it can be concluded that the membrane mechanism consists of a central tensile region surrounded by peripheral compressive ring bordering the four edges of the slab.

Table 3.5 In-plane forces from FEM analysis of FRACOF test

Mesh temperature (MT)	30°C	107°C	211°C	395°C
Σ tensile membrane	0.0 kN	365.9 kN	269.5 kN	215.1 kN
Σ compressive membrane	-1332.7 kN	-1844.9 kN	-907.3 kN	-655.3 kN
Supporting beam force	-206.5 kN	-1275.6 kN	-1827.2 kN	-1596.3 kN
Column force	1536.2 kN	2583.4 kN	2490.1 kN	2100.7 kN
Net equilibrium force	-3.0 kN	-171.1 kN	25.2 kN	64.3 kN

3.6 Summary

In this chapter, numerical modelling of composite slabs using ABAQUS was introduced. Both concrete and steel material input properties at ambient and elevated

temperature were presented.

Additionally, the concrete damage plasticity (CDP) model was used to simulate the overall behaviour of concrete slabs at ambient and elevated temperature. After validating the models with small scale experimental tests from Section 2.5.3, this CDP model was able to capture the behaviour of concrete slabs at ambient and elevated temperature.

In order to validate the composite beam-slab systems at fire conditions, the FRACOF test results were compared with the finite element modelling.

After assigning the temperature profiles to respective steel beams and the composite slab according to test results, the predicted slab central deflections and the edge beam deflections were quite close to the measured deflections. Therefore, it could be concluded that the deflection profiles from the FEM simulations could approximately represent the actual composite slab deformations.

Moreover, the sectional forces along the long span were analysed and plotted at different mesh temperature. The sectional forces were in compression at the beginning stage. When the mesh temperature reached around 100°C, the sectional forces in the middle of the slab changed from compression to tension. Additionally, with increased mesh temperature, both the compressive and tensile forces were reduced due to material degradation at elevated temperature. Moreover, the in-plane equilibrium was achieved by a compressive ring and a tensile zone at the centre of the slab.

In the next chapter, the specimen design principle would be discussed and parametric studies were carried out for different boundary conditions and aspect ratios.

Chapter 4 Specimens design and boundary condition studies

4.1 Introduction

The BRE-Bailey design method (P288/P390) (Newman, Robinson et al. 2000), mentioned in Section 2.6, was used to design composite beam-slab specimens by taking advantage of tensile membrane effects. Since there was hardly any research work on the boundary continuity effects on tensile membrane action, this chapter will cover the design of test specimens by incorporating boundary continuity.

Firstly, a prototype of 27 m by 27 m composite slab panel was designed based on the BRE-Bailey design method. The boundary continuity for a typical 9m by 9m composite slab panel was investigated through ABAQUS. The principle for designing small-scale specimens was discussed and the specimens' in-plane and rotational stiffness values along the boundary were calculated and compared with the prototype structure. It showed that the design stiffness values of boundary restraints were within the same order of magnitude compared to a full-scale typical composite slab. Therefore, the experimental boundary conditions for restrained specimens were effective and similar to a full-scale typical composite slab.

4.2 Prototype composite slab design

For a prototype 27m x 27m x 130mm thick composite beam-slab system, the size of supporting edge beams and protection material thickness were determined according to the BRE design method. Additionally, the load-bearing capacity for one hour fire rating was designed to satisfy the requirements in BRE design guide. A typical 4-storey steel framed building was considered as the prototype with one hour fire resistance

according to SCI P288/P390.

The composite floor slab consisted of Cofraplus 60 trapezoidal metal decking with an overall thickness of 130mm, grade C25/30 normal weight concrete and a single layer of $\Phi 10$ mm reinforcement mesh at 150mm spacing in both directions as shown in **Fig 4.1**.

Fig 4.1 A typical layout for a 9mx9m composite slab (unit in mm)

The floor loading consisted of:

- Variable action due to imposed load: 4 kN/m²
- Variable action due to light weight partitions: 1 kN/m²
- Permanent action due to ceilings and services: 0.7 kN/m²
- Self-weight of beams: 0.5 kN/m²

The design procedure was demonstrated in the following steps:

Step 1: calculation of applied load on the slab at fire limit state

The applied load on the slab in case of fire limit state with a self weight of 2.28 kN/m² for the slab could be determined by:

$$q_{fi, sd} = G + 0.5Q = (2.28 + 0.7 + 0.5) + 0.5 \times (4 + 1) = 5.98 \text{ kN/m}^2$$

Step 2: calculation of heat transfer into the composite slab Cofraplus 60

From Annex D of EN1994-1-2, the effective thickness of the slab can be expressed by:

$$h_{eff} = h_1 + 0.5h_2 \left(\frac{l_1+l_2}{l_1+l_3} \right) = 72 + 0.5 \times 58 \times \left(\frac{101+62}{101+106} \right) = 95 \text{ mm}$$

Based on one-hour standard fire, the temperatures of slab bottom surface, reinforcing mesh and slab top surface could be determined based on EN1992-1-2 (CEN 1993) as: $\theta_{bottom}=831^\circ\text{C}$; $\theta_{mesh}=288^\circ\text{C}$; $\theta_{surface}=99^\circ\text{C}$.

Therefore, there was no reduction of the mesh steel strength and the insulation criterion was satisfied since the increment of the surface temperature was less than 140K.

$$f_{sy,\theta} = 500 \text{ MPa} , \gamma_{M,fi,s} = 1.0 \text{ and } \gamma_{c,fi,c} = 1.0$$

Step 3: Calculation of the moment resistance of the slab section $M_{fi,0}$

$$(g_0)_1 \text{ or } (g_0)_2 = 1 - \frac{2A_s f_{sy,\theta} / \gamma_{M,fi,s}}{0.85 f_c / \gamma_{c,fi,c} d} = 1 - \frac{2 \times \frac{523}{1000} \times 500}{0.85 \times 25 \times 30} = 0.17$$

The positive moment resistance of the slab section was:

$$M_{fi,0} = A_s f_{sy,\theta} d \frac{3+g_0}{4} = \frac{523}{1000} \times 500 \times 30 \times \frac{3+0.17}{4} = 6217 \text{ Nmm/mm}$$

The parameters from the yield line theory were calculated from:

$$n = \frac{1}{2\alpha^2} (\sqrt{3\alpha^2 + 1} - 1) = 0.5$$

Step 4: Determination of load-bearing capacity of slab

The load-bearing capacity of slab could be determined from:

$$p_{fi} = 6 \frac{M}{n^2 \alpha^2 l^2} = 6 \times \frac{6217}{0.5^2 1^2 9000^2} = 1.84 \text{ kN/m}^2$$

Step 5: Determination of deflection limit for membrane action

$$\begin{aligned} \delta &= \min \left\{ \frac{\alpha(\theta_{bottom} - \theta_{surface}) l^2}{19.2 h_{eff}} + \min \left[\sqrt{\frac{0.5 f_{sy}}{E_a \gamma_{M,fi,s}} \frac{3L^2}{8}}; \frac{l}{30} \right]; \frac{L+l}{30} \right\} \\ &= \min \left\{ \frac{1.2 \times 10^{-5} (831 - 99) \times 9000^2}{19.2 \times 95} + \min \left[\sqrt{\frac{0.5 \times 500}{210000} \times \frac{3 \times 9000^2}{8}}; \frac{9000}{30} \right]; 600 \right\} \\ &= \min \{ 390 + \min [190; 300]; 600 \} = 580 \text{ mm} \end{aligned}$$

Step 6: Calculation of parameters to determine membrane action

$$\alpha = \frac{2g_0}{3+g_0} = 0.107 \quad \beta = \frac{1-g_0}{3+g_0} = 0.261 \quad k = \frac{4n\alpha^2(1-2n)}{4n^2\alpha^2+1} + 1 = 1$$

$$A = \frac{1}{2(1+k)} \left[\frac{l^2}{8n} - \left(\frac{1-2n}{2n} + \frac{1}{3(1+k)} \right) ((nL)^2 + (l/2)^2) \right] = 3.375$$

$$B = \frac{k^2}{2(1+k)} \left[\frac{nL^2}{2} - \frac{k}{3(1+k)} ((nL)^2 + (l/2)^2) \right] = 3.375$$

$$C = \frac{l^2}{16n} (k - 1) = 0$$

$$D = \frac{L^2}{8} (1 - 2n)^2 = 0$$

$$b = \frac{l^2}{8K(A+B+C-D)} = 1.5$$

Step 7: Calculation of enhancement factors for membrane action

$$e_{1b} = 2n \left(1 + ab \frac{k-1}{2} - \frac{\beta b^2}{3} (k^2 - k + 1) \right) + (1 - 2n)(1 - ab - \beta b^2) = 0.8$$

$$e_{1m} = \frac{4b}{3+g_0} \frac{\delta}{d} \left((1 - 2n) + n \frac{2+3k-k^3}{3(1+k)^2} \right) = 6 \quad e_1 = e_{1m} + e_{1b} = 6.8$$

$$e_{2b} = 1 + abK \frac{k-1}{2} - \frac{\beta b^2 K}{3} (k^2 - k + 1) = 0.8$$

$$e_{2m} = \frac{4bK}{3+g_0} \frac{\delta}{d} \frac{2+3k-k^3}{6(1+k)^2} = 6 \quad e_2 = e_{2m} + e_{2b} = 6.8$$

The enhancement factor e was determined by:

$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu\alpha^2} = 6.9$$

Step 8: Total bearing capacity of slab in fire condition

$$q_{fi,Rd,slab} = e \times p_{fi} = 12.7 \text{ kN/m}^2$$

Step 9: Bearing capacity of slab considering unprotected composite beams

Firstly, it was necessary to calculate the section factor of unprotected steel section UB305x102x25 kg/m. The calculated values were summarised in

Table 4.1. From EN1994-1-2, it was possible to determine the temperature of unprotected composite beams as shown in **Table 4.2.**

After that, the moment resistance of the unprotected interior beam was included in **Table 4.3.** Therefore, the bearing capacity of the slab from the interior beam(s) contribution could be obtained from:

$$q_{fi,Rd,ub} = \frac{8M_{fi,Rd}}{L_1^2} \frac{1+n_b}{L_2} = 0.42 \text{ kN/m}^2$$

Table 4.1 Section factor of unprotected composite beam

Steel section member	k_{sh} $= 0.9 \left(\frac{H + 0.5B}{H + 1.5B - t_w} \right)$	$\left(\frac{A_i}{V_i} \right) \text{ (m}^{-1}\text{)}$	$k_{sh} \left(\frac{A_i}{V_i} \right)$
Lower flange	0.709	305	216
Web		345	244
Upper flange		305	216
Notes: H is the depth of the steel section; B is the width of the steel section; t_w is the thickness of the slab; t_f is the thickness of the flange.			

Table 4.2 Temperature of an unprotected steel cross-section under ISO fire(O. and B. 2014)

Resulting section factor $k_{sh} \left(\frac{A_i}{V_i} \right)$	Temperature of the steel cross-section $\theta_a(^{\circ}\text{C})$				
	30 min	60 min	90 min	120 min	180 min
20	432	736	942	1030	1101
30	555	835	987	1039	1104
40	637	901	995	1042	1106
50	691	923	997	1043	1106
60	722	931	999	1044	1107
70	734	934	1000	1045	1107
80	742	936	1001	1046	1108
90	754	937	1001	1046	1108
100	768	938	1002	1046	1108
110	782	939	1002	1047	1108
120	793	940	1003	1047	1108
130	802	940	1003	1047	1109
140	810	940	1003	1047	1109
150	815	941	1003	1047	1109
200	829	942	1004	1048	1109
500	838	944	1005	1048	1109

Step 10: Total bearing capacity of slab in fire conditions and verification of fire resistance of slab

The total bearing capacity of the slab consisted of the composite slab and the interior

unprotected beam(s):

$$q_{fi,Rd} = q_{fi,Rd,ub} + q_{fi,Rd,slab} = 12.7 + 0.42 = 13.12 > q_{fi,sd} = 5.98 \text{ (OK)}$$

Therefore, this design satisfied the BRE design method by taking account of the tensile membrane effect for one-hour fire rating. It should be noted that the interior beam capacity was insignificant compared with the tensile membrane action from the composite slab. This is because the temperature of the unprotected interior beam was high after one-hour exposure and the strength of the steel section was reduced to 5% of its original value.

Table 4.3 Moment resistance for unprotected composite beams

Parameter	Calculated values
Effective width of the slab	$b_{eff} = \frac{9000}{4} = 2250 \text{ mm}$
Area of the steel section A_i	$A_i = 3160 \text{ mm}^2$
Reduction factor for steel strength	$k_{y,\theta} = 0.052$
Reduction factor for stud strength	$k_{u,\theta} = 0.17$
Thickness of slab in compression in fire situation	$h_u = \frac{\Sigma A_i f_y k_{y,\theta} / \gamma_{M,fi,a}}{b_{eff} f_{cd} / \gamma_{M,fi,c}} = 1 \text{ mm}$
Connection degree of the beam at 20 °C	$n_{c,20^\circ\text{C}} = 0.51$
Connection degree of the beam in fire situation	$n_{c,\theta} = \frac{n_{c,20^\circ\text{C}} k_{u,\theta} \gamma_{M,v}}{k_{y,\theta} \gamma_{M,fi,v}} = 2.09 > 1$ So full shear connection
Positive moment resistance	$M_{fi,Rd} = \frac{A_i f_y k_{y,\theta}}{\gamma_{M,fi,a}} \left(\frac{H}{2} + h_c - \frac{h_u}{2} \right) = 6.35 \text{ kNm}$

4.3 Scale-down composite slab specimen design

For experimental test, one-quarter scale was adopted due to laboratory space constraint. The scale-downed principle is to keep the same yield stress for steel reinforcement and the same slab over beam stiffness ratio. The prototype and the scale-downed test specimens are shown in **Table 4.4**.

Table 4.4 Comparison between prototype and experimental specimen

Specifications	Prototype	Experiment
Length	9 m	2.25 m
Thickness	130 mm	60 mm
Supporting Beam size	UB 305x102x25 kg/m	Joist 102x102x23 kg/m
Slab/Beam Stiffness $\frac{E_c h^3}{12(1-\nu^2)} / E_s I_{beam}$	$492E/12(1-\nu^2)$	$444E/12(1-\nu^2)$
Steel reinforcement (Same reinforcement ratio = 0.4%)	10 mm diameter mesh with 150 mm spacing in both x and y direction	6 mm diameter mesh with 120 mm spacing in both x and y direction

where E_c is the concrete elasticity, E_s is the steel elasticity, E is the relative elasticity equals to E_c/E_s , ν is the Poisson's ratio of concrete material, h is the thickness of the slab and I_{beam} is second moment area of steel beam section.

This research is focusing on the development of membrane forces for composite beam-slab systems at elevated temperature. The compressive membrane action is mainly related to the stiffness values of the slab and the supporting beams and the governing factor for tensile membrane action is yield stress of steel reinforcement. Therefore, by keeping the same slab/beam stiffness and yield stress of steel reinforcement, it is possible to capture similar compressive and tensile membrane behaviour for scale-down specimens.

Fig 4.2 Typical reinforced concrete slab cross section

Considering a typical reinforced concrete slab cross section with effective depth d in **Fig 4.2**, it assumes that tensile force F_t from reinforcement is balanced by compression $f_c d(1-\xi_0)/2$ from concrete block as shown by **Eq 4.1**. Taking moment between tension and compression, bending moment M_0 can be obtained through **Eq 4.2**. It should be

noted that bending moment is proportional to the $q_0 l^2$ term, where α is a coefficient depended on the boundary supporting conditions, q_0 is the applied pressure on the surface of RC slabs and l is the clear span in **Eq 4.3**.

$$\frac{f_c h (1 - \xi_0)}{2} = F_t \quad \text{Eq 4.1}$$

$$M_0 = F_t \left(d - \frac{1 - \xi_0}{4} d \right) = \frac{A_s f_y d}{4} (3 + \xi_0)(1 - \xi_0) \quad \text{Eq 4.2}$$

$$M_0 = \alpha q_0 l^2 = \frac{A_s f_y d}{4} (3 + \xi_0)(1 - \xi_0) \quad \text{Eq 4.3}$$

After simplifying **Eq 4.3**, the yield stress of steel reinforcement is given by **Eq 4.4**.

$$f_y = \frac{4 \alpha q_0 l^2}{A_s d (3 + \xi_0)(1 - \xi_0)} \quad \text{Eq 4.4}$$

From **Table 4.4**, the specimen has been scale-downed one-quarter for its span, one-half for its effective depth and one-half of the reinforcement area. In order to keep the same yield stress level from **Eq 4.4**, the applied uniform distributed load q_0 on the specimen should be 4 times the load applied on the prototype composite slab.

Fig 4.3 Comparison for rebar stress between the prototype and the scale-downed composite slab specimen

By keeping the same slab/beam stiffness ratio $\frac{E_{slab} I_{slab}}{E_{beam} I_{beam}}$, it is possible to simulate the

bending behaviour of full scale specimens by using the scale-down specimens. Comparing the prototype and the scale-downed composite slab specimens, the stresses of steel reinforcement are shown in **Fig 4.3**.

Since the actual connections were over designed (as shown in Appendix) and there was no failure or damages in the connections, rigid connections were defined both in experiment and numerical analyses. In the FEM analysis, the applied uniformly distributed load (UDL) on the scale-down 2250x2250x60mm thick specimen was four times greater than that for the 9000x9000x130mm thick prototype structure. However, it was noted that the prototype and scale-downed specimens showed similar reinforcement stresses with respect to the normalised deflection (deflection over effective depth). Therefore, it could be concluded that this scale-downed principle for test specimens could represent similar stress patterns for prototype structure.

4.4 Boundary stiffness study

For comparison purpose, this 27m by 27m composite slab panels discussed in Section 4.3 was used to study boundary restraint effects as shown in **Fig 4.4**. There were total nine composite slab panels with 130mm thick concrete slab with protected UB305x102x25 kg/m edge beams, unprotected UB305x102x25 kg/m interior beams and protected UC356x368x129 kg/m columns and beam-to-column connections.

Three types of boundary continuities were studied which included internal, edge and corner composite slab panels with four, three and two continuous edges to adjacent panels, respectively.

To investigate the continuity effect for different boundary conditions, a numerical model of this 27 m by 27 m composite slab was built up by ABAQUS. The dashed lines drawn in **Fig 4.4** represented the span/8 extension from a 9 mx9 m slab panel. Two kinds of boundary restraints were identified as axial restraints and rotational restraints along dashed lines. When pressure was applied on composite slab surfaces, the corresponding lateral forces over the lateral displacements and the boundary

moments over the rotation angles were used to calculate the axial restraint stiffness and the rotational restraint stiffness for different types of composite slab panels as shown in **Table 4.5**.

(a) Plan view for a 27 m x 27 m slab panels

(b) ABAQUS modelling

Fig 4.4 A typical 3x3 composite slab panels with FEM simulation

Due to restraints from surrounding panels, the internal composite slab panel (a) has the greatest axial stiffness in both the primary and secondary beams directions. However, there are two types of edge boundary conditions to be considered, *viz.* continuity on primary beams or secondary beams, respectively (see **Table 4.5**).

For edge panel (b) that is continuous in secondary beams, the axial restraint stiffness is similar to that for the internal composite slab panel (a) on the primary beams, but much smaller on the secondary beams due to one discontinuous edge. On the contrary, for the edge panel (c) continuous in primary beams, the axial stiffness is similar to that for the internal composite slab panel on the secondary beams but much smaller on the primary beams due to one discontinuous edge along the primary beams. Clearly, the axial and rotational stiffness values for the corner composite slab panel (d) are much smaller than the other types of boundary conditions due to discontinuity in both directions.

Table 4.5 Comparison of different type of composite slab panels with boundary continuities

	(a)	(b)	(c)	(d)
Types of composite slabs
Axial stiffness @ secondary beam	4580 kN/m	992 kN/m	3200 kN/m	765 kN/m
Rotational stiffness @ secondary beam	69 kNm/rad	22 kNm/rad	75 kNm/rad	22 kNm/rad
Axial stiffness @ primary beam	18882 kN/m	18063 kN/m	4187 kN/m	5616 kN/m
Rotational stiffness @ primary beam	258 kNm/rad	268 kNm/rad	264 kNm/rad	276 kNm/rad

Moreover, for this particular configuration, the axial stiffness along the dashed line of the primary beam shows much greater value compared to that of the secondary beam due to continuity in interior beams. The internal slab panel has approximately three to four times greater axial stiffness and rotational stiffness than that of edge or corner slab panel.

4.5 Details of specimens

Three series of test specimens were designed to take into account for boundary continuities and aspect ratios as shown in **Fig 4.5**. Series I focused on isolated composite beam-slab systems, and no boundary continuity was considered. For Series II & III, various boundary conditions were investigated, such as internal (four continuous edges), edge (three continuous edges) and corner (only two adjacent continuous edges) panels as indicated in **Fig 4.5**. For Series II and III, different aspect ratios (1 and 1.5) were considered. Material properties of Series I, II and III are shown

in **Table 4.6**. For series I, **ISOCS** stands for **ISOLated Composite Slab** panel and the numerical digit represents the aspect ratio of **1** or **1.5**. Hence, **ISOCS1NI** is an **ISOLated Composite Slab** panel with an aspect ratio of **1** with **No Interior beam**. For **ISOCS1**, it is **ISOLated Composite Slab** panel with an aspect ratio of **1** with unprotected interior beam while **ISOCS1.5** is **ISOLated Composite Slab** panel with an aspect ratio of **1.5** also with unprotected secondary beam (UPSB).

Fig 4.5 Details for total nine test specimens

Similarly, for Series II and III, the specimen **ICS1** and **ICS1.5** are **I**nternal **C**omposite **S**lab panels with aspect ratios of **1** and **1.5**. For **ECS1** and **ECS1.5**, they are **E**dge **C**omposite **S**lab panels with aspect ratios of **1** and **1.5** while **CCS1** and **CCS1.5** are **C**orner **C**omposite **S**lab panels with aspect ratios of **1** and **1.5**.

The edges composite slab panel was purposely chose to be continuous along the secondary beams direction since the higher axial restraint stiffness was obtained from numerical study (**Table 4.5** type (b)). Except for interior secondary beams, the rest of beams and columns were painted with intumescent coating for one-hour fire rating.

Table 4.6 Geometries and material properties details for test specimens

	<i>Geometry</i>			<i>Concrete slab</i>			<i>Reinforcement</i>			<i>Steel beam & column</i>		
	<i>h</i> mm	<i>d_x</i> mm	<i>d_y</i> mm	<i>f_{cm}</i> MPa	<i>f_{ctm}</i> MPa	<i>E_{cm}</i> GPa	<i>f_y/f_u</i> MPa	<i>Elongation</i> , %	<i>Modulus</i> <i>E_s</i> , GPa	<i>f_y/f_u</i> MPa	<i>Elongation</i> , %	<i>Modulus</i> <i>E_s</i> , GPa
<i>ISOCSINI</i>	61	34	28	33.2	2.7	30.2	573 /636	20.2	202	394 /485	23.1	211.3
<i>ISOCS1</i>	60	33	27	32.5	2.9	32.4	589 /658	22.0	210	410 /507	27.0	215.1
<i>ISOCS1.5</i>	62	35	29	34.1	2.6	31.6	559 /647	21.5	206	424 /498	22.3	208.7
<i>ICS1</i>	60	32	26	30.8	2.5	28.4	551 /617	21.5	205	368 /432	21.2	207.5
<i>ECS1</i>	61	34	28	31.5	2.6	30.9	599 /674	23.1	207	380 /468	25.1	211.1
<i>CCS1</i>	58	35	29	32.7	2.6	31.2	563 /652	22.8	201	363 /449	23.6	204.9
<i>ICS1.5</i>	59	33	27	33.6	2.9	32.8	532 /620	20.8	209	375 /444	23.1	215.4
<i>ECS1.5</i>	62	32	26	31.1	2.4	29.6	545 /655	25.0	202	382 /432	21.6	207.3
<i>CCS1.5</i>	61	35	29	32.9	2.6	30.4	539 /643	23.4	208	370 /452	22.7	201.2

Test specimens were extended out for 400 mm to achieve boundary continuities and reinforcing mesh was welded to 10 mm thick boundary steel plates around the slab. In addition, five M25 bolts were installed in each side of composite slab (with embedment length of 100 mm). The contacting area of these five M25 bolts with concrete slab was greater than the contacting area of $\Phi 6$ mesh (120mm spacing) with concrete slab. The purpose was to make sure lateral forces in the mesh was able to transfer to the bolts without any debonding between bolts and concrete slab. Hence, the overall design of total nine specimens are shown in **Fig 4.5** and details of each specimen are listed in **Table 4.6**.

slab systems can be verified.

Fig 4.7 Strain gauges attached on the CHS columns to measure the reactions

4.6.2 Thermocouples

Thermocouples were attached at various positions to monitor temperature in specimens such as reinforcing mesh and supporting steel beams as shown in **Fig 4.8**. Thermocouples were bent to certain shape in the supporting steel beams to prevent them from moving at large deflection.

Fig 4.8 Thermocouples attached on the (a) reinforcement and (b) supporting steel beams

To measure the bottom slab surface temperature, a special welder was used to fix thermocouples on the bottom surface as shown in **Fig 4.9(a)**. This welding material can work very well at high temperature to make sure thermocouples will not be detached at elevated temperature. The final position of thermocouple on the bottom

surface is shown in **Fig 4.9(b)**.

(a) Welding thermocouple on bottom of slab (b) After installation of thermocouple

Fig 4.9 Welding thermocouples on the bottom surface of composite slab

4.7 Loading system

To simulate uniformly distributed load on top surface of composite slabs, the setup took after Foster et al.'s experiment (2004) and a twelve-point loading system was designed as shown in **Fig 4.10**. Triangular plates with three point loads were very stable even at very large deformation of the slab. This was because when one of the three points deflected downwards, movements of the other two points would not affect the distribution of vertical loads if a point load was applied at the centroid of the triangular plate.

(a) Triangular plates on the surface of slab (b) Actual loading system

Fig 4.10 Twelve point loading system

In order to validate this twelve point loading system, a numerical model of

2250x2250x60mm thick specimen was built up. A uniformly distributed load (UDL) and a twelve-point concentrated load were applied on the surface of the composite slab model separately. Numerical results for these two cases were shown in **Fig 4.11**.

The UDL and the twelve-point loading systems showed a similar load/deflection trend. The major difference between point load and uniformly distributed load is the load direction. For UDL, forces are applied perpendicular to slab surfaces and more uniformly distributed within slabs compared with point load. This is why the point load deformation curve showed much larger displacement compared with that of UDL at the end of analyses. However, in actual test, only applied/yield line load around one was considered. The differences between UDL and point load are not so obvious. Hence, this twelve-point loading system can be considered to represent the UDL in the actual test.

Fig 4.11 Central deflection comparison between UDL and twelve point loading system

Additionally, three distinctive stages can be identified with increasing applied load. Initially, the relationship between applied load and deflection is linear, which represents the elastic plate behaviour of composite slab. However, once the applied load exceeded yield line limit, softening occurs due to formation of cracks. When the slab deformation reaches around one effective depth, the slope of load/deflection curve increases due to mobilisation of tensile membrane action. The details of the derivations

of this three-stage analytical model will be discussed in the Section 8.2.

4.8 Summary

In Chapter 4, the design procedure for a prototype 4-storey 27mx27m composite slab structure was explained in detail based on BRE-Bailey design guide. Due to laboratory space constraint, a scale-down principle was adopted for test specimens. Reinforcement stresses for the full-scale structure and the scale-down test specimen were compared through numerical analysis and similar stress patterns were observed for the pure flexural bending stage.

Additionally, the boundary stiffnesses including axial and rotational were also checked for the prototype composite slab structure. Three types of boundary continuities, internal, edge and corner, were investigated and these boundary continuities would be further discussed in details in chapter 6.

A twelve-point loading system was applied on the top surface of composite slabs to simulate a uniformly distributed load (UDL). Through numerical analyses, this twelve-point load system was found effective to represent applied UDL pressure.

In the next chapter, test results for Series I isolated composite slabs would be discussed. Both temperature profiles and deformation measurements were compared with numerical results. Additionally, sectional forces across mid-span were extracted from numerical models and the development of membrane forces were studied and validated with experimental measurements.

Chapter 5 Experiment and analyses for Series I of isolated composite beam-slab system

5.1 Introduction

In this chapter, Series I isolated series of three specimens are discussed. Since in the BRE-Bailey design method, composite slabs are designed based on isolated slab panels and boundary continuity is not taken into account. Additionally, the BRE-Bailey design guide assumes that in-plane equilibrium is maintained between tensile membrane zone in the central region and compressive membrane ring around the perimeter of the slab. Specimens of this series are used to validate the reliability of the BRE design method. Additionally, numerical analyses are conducted to find out the development of membrane forces at elevated temperature.

5.2 Specimens details for Series I

The Series I tests consisted of three specimens. Specimen ISOCS1NI was an ISOLated Composite Slab panel with an aspect ratio of 1 with No Interior beam. ISOCS1 was an ISOLated Composite Slab panel with an aspect ratio of 1 and with one unprotected interior beam, while ISOCS1.5 was an ISOLated Composite Slab panel with an aspect ratio of 1.5 and one unprotected secondary beam (UPSB). The main beams (MB), secondary edge beams (PSB) and columns were painted with intumescent coating for one-hour fire rating as shown in **Fig 5.1**.

For purpose of comparison, all the composite slabs were cast with concrete grade of C25/30, reinforcement grade of S460, supporting beams/columns with steel grade of S355 and profile steel decks with steel grade of S500. These three specimens were

designed with the same slab thickness h of 60mm. $\Phi 6$ mm mesh reinforcement at 120mm spacing was located at the middle layer of the concrete slab in both X-and Y-directions indicated in **Fig 5.1**. The details of geometrical and material properties for the three specimens are provided in **Table 5.1**.

Fig 5.1 Specimens for isolated composite beam-slab sub-assembly (unit in mm)

Table 5.1 Geometries and material properties for tested specimens

	Geometry			Concrete slab			Reinforcement			Steel beam & column		
	h mm	d_x m m	d_y mm	f_{cm} MPa	f_{ctm} MPa	E_{cm} GPa	f_y/f_u MPa	Elong ation, %	Modulus E_s , GPa	f_y/f_u MPa	Elong ation, %	Modulus E_s , GPa
ISOCS1NI	61	34	28	33.2	2.7	30.2	573 /636	20.2	201.4	394 /485	23.1	211.3
ISOCS1	60	33	33	32.5	2.9	32.4	589 /658	22.0	210.1	410 /507	27.0	215.1
ISOCS1.5	62	35	29	34.1	2.6	31.6	559 /647	21.5	206.3	424 /498	22.3	208.7

5.3 Test setup for Series I

A 3m wide by 3m long by 1m deep electrical heating furnace was installed and supported by two I-beams as shown in **Fig 5.2**. To simplify the calculations of bending moments in the columns, pin supports were installed and secured onto the strong floor. Since strain gauges could not function under fire conditions, circular hollow sections (CHS) were bolted onto the pin-support steel plates with strain gauges attached to the exterior of the furnace.

Fig 5.2 Test set up for Series I isolated composite beam-slab system

CHS was selected to connect the universal column sections to the pin support because of its ability to reduce stress concentration effectively (Pham and Tan 2013). Therefore, the CHS acted as an extension of the columns, giving greater accuracy in strain measurements.

Since these three specimens were smaller than the dimensions of the furnace, insulation panels were placed on top of the furnace wall to cover any gaps between the composite slab and the furnace so as to minimise heat loss during heating.

5.4 Applied loading

After the test setup was completed, a twelve-point loading system (discussed in Section 4.7) was installed on the top surface of composite slab specimens. From the literature review (Park and Gamble 2000), the tensile membrane action stage begins when the applied load exceeds the yield line capacity of the slab. Hence, the applied load from the hydraulic jack was kept at 100 kN during the test, which was equivalent to the yield line load of the composite slab specimens. The load cell was installed right below the hydraulic jack to measure the applied load.

In addition, reaction forces were monitored and calculated from strain gauges attached onto four supporting CHS columns. **Fig 5.3** showed the measured reaction forces in these four columns were in good agreement with total external applied load at the beginning stage. However, the measured reaction forces in the four columns deviated from the applied loading after 30 minutes. This was because four pin-supported columns rotated when the composite slab deformed. Therefore, to maintain equilibrium in the vertical direction, corresponding column forces increased to allow for rotating angles of the pin-supported columns.

Fig 5.3 Illustration of applied loading system with measured reaction forces for isolated series specimen

5.5 Temperature profiles

Due to constraint of power supply in the fire laboratory, the temperature of electrical heating furnace was increased linearly to 1000°C over a one-hour period and was maintained at that temperature until failure of the specimens occurred. Typical cross-sectional temperature profiles of specimens ISOCS1NI, ISOCS1 and ISOCS1.5 are shown in **Fig 5.4**, **Fig 5.5** and **Fig 5.6**, respectively.

Fig 5.4 Temperature profile and the FEM temperature input for ISOCS1NI

Fig 5.5 Temperature profile and the FEM temperature input for ISOCS1

Fig 5.6 Temperature profile and the FEM temperature input for ISOCS1.5

It is worthy to note that the mesh temperature of three specimens was kept around 100°C for a long period of time. This was because when the concrete was heated up to 100°C, steam was trapped in the concrete and vapour barrier was formed to prevent heat from transferring to the top surface of concrete slabs. After cracks had occurred at large deformation, heat could then be transferred to the top surface of the slab. This phenomenon was reflected by a sudden gradient change from the mesh temperature at around 100°C.

Fig 5.7 Temperature distribution at bottom of specimen ISOCS1NI

Fig 5.8 Temperature distribution in the mesh layer for specimen ISOCS1NI

Fig 5.9 Temperature distribution in the mesh layer for specimen ISOCS1NI

To check the uniformity of planar temperature distribution, the bottom, mesh and top surface temperatures at different locations are plotted as shown in **Fig 5.7**, **Fig 5.8** and **Fig 5.9**, respectively. The temperature from the central position is slightly higher than those in other positions, but the overall temperature distributions are quite uniform. In numerical simulations, shell elements were used to model the composite slab with a layer of steel mesh in the middle and beam elements were used to simulate protected main beams and secondary beams. Temperature gradient was linearly distributed through the shell thickness according to the measured temperature profiles during the tests. Since the main factors governing structural behaviour of composite slabs under fire condition included the mesh temperature and the slab's bottom surface temperature, these two temperatures were imported into the numerical model and the temperature difference between the top and the bottom surfaces was normalised by the slab thickness yielding the desired temperature gradient. Moreover, temperature of

supporting steel beams was assigned as a uniform values due to high thermal conductivity of steel.

5.6 Failure criteria

There are three criteria to determine failure in the fire resistance test, *viz.* load-bearing capacity, integrity and thermal insulation as mentioned in EN 1363-1 for standard fire resistance tests. The failure criteria for the three isolated composite beam-slab sub-assembly tests were governed by the load-bearing capacity as shown in **Fig 5.10**. In each test, a hydraulic jack was maintained constant at 100 kN on the slab surface and the furnace started heating up from room temperature to 1000°C under an hour and was maintained at 1000°C. However, applied load started reducing at 86 minute, 100 minute and 84 minute for ISOCS1NI, ISOCS1 and ISOCS1.5, respectively. Effort spent on pumping the jack back to 100 kN was in vain and the time corresponding to this scenario was defined as the failure time for each specimen. Severe cracks of around 10 mm width, and fracture of reinforcement through the thickness of the slab was observed in the tests. Once wide cracks occurred around the perimeter beams, the applied load on the composite slab could not be maintained.

(a) Load versus Time relationship (b) Cracks in detail after test

Fig 5.10 Governing failure criterion of load-bearing capacity

Further investigation into the formation of the crack patterns had been conducted. **Fig**

5.11(a) to (c) show that all three specimens exhibited a similar crack formation sequence at the beginning stage when the mesh temperature was around 50°C and concrete crushing occurred near the four corners of the beam-column joints, indicative of very high compression. When the mesh temperature increased to 80°C, more cracks appeared at the four corners of ISOCS1NI and they propagated inwards towards the centre of the slab. The unprotected interior steel beams of ISOCS1 and ISOCS1.5 retained their strength at 80°C, with cracks developing at both ends of the interior unprotected beams. The major difference between Figure 5.11(a) and Figure 5.11(b) is in the interior supporting beam. Some cracks developed around the corners of the interior steel beam at the initial stage for ISOCS1.

(a) Development of cracks for ISOCS1NI

(b) Development of cracks for ISOCS1

(c) Development of cracks for ISOCS1.5

- (d) Cutting bottom protecting steel sheet and observing cracks from slab bottom after test

Fig 5.11 Development of crack patterns at different elevated mesh temperature

When the mesh temperature reached 100°C , the temperature of the interior unprotected beam was around 800°C , which indicated that the strength reduction of the interior unprotected steel beam was more than 90%. Excessive deformations at the mid-span were observed and cracks along the interior beam started propagating towards the four corners. With increasing mesh temperature and deformation, more cracks were formed around the perimeter of the edge beams at the final stage. With further material degradation at a higher mesh temperature, the composite slabs could not resist peripheral hogging moment and perimeter cracks around the edge beams widened further until the mesh reinforcement fractured around the perimeter.

Interestingly, only a few cracks had appeared at the mid-span of the composite slabs as shown in **Fig 5.11(d)**. Once the bottom 1mm thick profile decking had been cut out, it was observed that numerous cracks appeared at the central slab region. These bottom cracks were a result of tensile membrane action and this was verified in numerical models.

5.7 Displacement

Traditional displacement measurements using LVDTs (linear variable displacement transducers) is a great challenge at elevated temperature. Usually, the LVDTs are placed on the top surface of slabs where the temperature is around $100\text{--}200^{\circ}\text{C}$ as shown in **Fig 5.12(a)**. However, some plastic parts of the LVDT may melt at high temperature. Thus, contact measurement was not ideal.

Therefore, a roller system was installed right above the measured point and the first roller had to be kept vertical to ensure the line transducer system measure the vertical deformation. Additionally, the roller base and the line transducer box were placed outside the furnace by connecting it through a second roller as shown in **Fig 5.12(b)**. Hence, all measuring instruments were located outside the furnace and they were

prevented from heat onslaught.

(a) LVDT on the slab surface

(b) Line transducer system

Fig 5.12 Illustrations of displacement measurement

A comparison of the slab's centre deflection for the three specimens was plotted with respect to mesh temperature as shown in **Fig 5.13**. For the slab with an aspect ratio of one, specimen ISOCS1NI (without interior secondary beam) deformed more than ISOCS1 (with internal beam) at the beginning stage. However, after 250°C, the deformation rates (deformation/temperature) of these two specimens were rather similar and this was probably due to a significant strength reduction of the interior beam from ISOCS1 as indicated in **Fig 5.11**. When the mesh temperature reached 250°C, the average temperature of the unprotected internal beam was about 800°C. According to EC4 part 1-2, bare structural steel at 800°C has a reduction factor of more than 90%. Therefore, the behaviour of ISOCS1 was similar to ISOCS1NI after the interior beam temperature had attained 800°C.

Fig 5.13 Centre deformation comparison for ISOCS1NI, ISOCS1 and ISOCS1.5

Based on the plate theory, analytical solutions for a rectangular plate elastically supported on four edge beams showed that the elastic deformation was about 10mm and the starting point of the tensile membrane was around 60mm from a previous study (Liu and Tan 2012). It can be seen that the centre deflection for ISOCS1.5 (larger aspect ratio of 1.5) was similar to ISOCS1 before 100°C, which was about one effective depth when tensile membrane action began. Subsequently, ISOCS1.5 deflected much more than ISOCS1NI and ISOCS1 for the same mesh temperature because ISOCS1.5 with an aspect ratio of 1.5 behaved more like a one-way slab, unlike ISOCS1NI and ISOCS1 with an aspect ratio of one. The failure mechanism for ISOCS1.5 changed from two-directional tensile membrane action into one-way catenary action when a crack at the centre of the slab traversed along the short span, as observed from **Fig 5.11(c)**. Therefore, the load-carrying capacity for ISOCS1.5 was much lower than ISOCS1NI and ISOCS1.

In numerical simulations, shell elements were used to model the composite slab with a reinforced steel mesh layer in the middle; beam elements were used to simulate the main and secondary beams as indicated in **Fig 5.14**. The slab elements were fully tied with the supporting beam elements assuming full composite shear connection.

(a) Separate slab and beams (b) Steel frame (c) Fully tied to each other

Fig 5.14 ABAQUS modelling of the composite beam-slab sub-assembly

From the numerical analyses of these three specimens, some discrepancies had been observed at around the mesh temperature of 100°C . This could be due to evaporation of water inside the concrete slab, but the overall trend of the numerical curves in **Fig 5.13** agrees well with the test results.

Additionally, column lateral movements were also monitored during the test. To avoid damaging the instrumentation at elevated temperature, line transducer boxes were installed outside the furnace and the measuring points were attached at the top of the columns. Moreover, the line transducers were aligned at 45° degrees with respect to the edge beams. The lateral movements of columns 3 and 4 were recorded as shown in **Fig 5.15**. Due to interference from other instruments, the column measurements were not working very well for specimen ISOCS1. Therefore, only the measurements from ISOCS1NI and ISOCS1.5 are discussed in this section.

(a) Location of column measurement (b) Column lateral movement for ISOCS1NI

(c) Column lateral movement for ISOCS1.5

Fig 5.15 The lateral movements of column 3 and 4 (negative = outward/positive = inward)

Negative sign indicates outward movement of columns at 45 degree with respect to the edge, while positive sign indicates inward movement. It can be seen that due to thermal expansion, the columns initially moved outwards at around 150-200 °C of the mesh temperature. Subsequently, due to pulling-in forces of the slab at large deflection, tensile membrane action stage began. Unlike ISOCS1NI, ISOCS1.5 showed similar column movement trend until the mesh temperature was about 100°C, before lateral movement changed from outwards into inwards. The earlier occurrence of tensile membrane stage for ISOCS1.5 could be due to greater one-way action compared to ISOCS1NI. This behaviour was entirely consistent with the FEM predictions.

5.8 Column lateral forces

To calculate the induced column lateral forces, strain gauges were attached at the CHS column positions as indicated in **Fig 5.16**. Each level had four strain gauges which were in accordance with the global X- and Y-axes. According to Wald et al's method (2009), the measured strains at each position were extrapolated up to the beam-column joint level as shown in **Fig 5.16**.

Fig 5.16 An illustration of Circular Hollow Section (CHS) columns attached with strain gauges

Since the strains of CHS were within the elastic range, the bending moment M along X- and Y-axes could be estimated through **Eq 5.1**.

$$M = \frac{E\varepsilon I}{y} \quad \text{Eq 5.1}$$

where E is elastic modulus of steel, ε is the extrapolated strain at the beam-column joint level, I is the second moment of area of the CHS and y is the distance from the neutral axis to the extreme fibre of the CHS.

Moment M could be balanced by lateral forces F multiplied by the overall column height H . Hence, the lateral forces could be estimated in both X- and Y-axes.

In **Fig 5.17**, letter “C” indicates Columns, “1” to “4” represent the different columns and “X- or Y-” indicate X- or Y-directions as shown in **Fig 5.15**. It can be seen that lateral forces started with compressive actions due to thermal expansion. However, when average mesh temperature reached around 150°C , lateral forces changed to tension forces because the edge beams started to pull the columns inwards due to large deflections of the composite slab. It is noteworthy that the turning point for lateral forces matched the measured column movements very well.

(a) Column lateral forces for ISOCS1NI (b) Column lateral forces for ISOCS1.5

Fig 5.17 Calculated lateral forces at the beam-column joints based on strain gauges measurement

A similar set of calculation was performed for specimen ISOCS1.5. Unlike ISOCS1NI, the turning point from compression to tension for ISOCS1.5 occurred earlier at around a mesh temperature of 100°C. Compared with the column movements in **Fig 5.15**, the turning point coincided with the transition for column movements (from outwards to inwards). The cross-sectional forces for ISOCS1.5 in the X-direction (long span) were dominated in tension but those in the Y-direction (short span) were in compression. Additionally, the lateral forces for columns were found out to be quite significant. Therefore, if columns are designed without any lateral restraint, careful checking on column bending capacities and connections should be incorporated by considering lateral forces due to outwards thermal expansion and inwards pulling forces.

5.9 In-plane membrane forces

Additionally, net cross-sectional forces (per unit width) at the centre, one-quarter, and the edge along the long span in the middle cross sections were plotted with respect to the mesh temperature as shown in **Fig 5.18**. The net cross-sectional forces were in compression due to applied load at the initial stage. With increasing temperature, compression forces increased due to thermal expansion. However, when the mesh temperature reached around 100-150°C, net cross-sectional forces shifted from compression to tension, indicating the start of tensile membrane action. This was the

main reason behind the abrupt change in gradient of the deflection-versus-mesh temperature around 100-150°C in **Fig 5.13**.

Fig 5.18 Section forces from numerical analysis for three different locations along the long spans

Comparing the two slabs on respective aspect ratio of 1 and 1.5, the net cross-sectional tensile forces in specimen ISOCS1.5 along the long span were much greater than that of specimens ISOCS1NI and ISOCS1 (with an aspect ratio of one). This indicated that the tensile membrane action in the long span direction was more effective than that in the short span and more tensile forces were distributed along the long span through tensile membrane action for ISOCS1.5. Moreover, the tensile membrane action for ISOCS1.5 started earlier than the specimens with an aspect ratio of 1.

It was noted that without the interior secondary beam, the magnitude of cross-sectional compressive forces for ISOCS1NI were smaller than specimen ISOCS1. This was a result of thermal expansion of the unprotected interior beam for ISOCS1.

A free body diagram of one-quarter of the specimen ISOCS1NI is shown in **Fig 5.19**. Due to symmetry, membrane forces in only one direction were considered. Since there was no interior beam in ISOCS1NI, the tensile membrane forces, compressive membrane forces, supporting beam forces and column forces were in equilibrium. In order to obtain these membrane forces, the cross-sectional forces from the FEM across the one-quarter cross section from the edge to the centre were extracted and plotted out at different mesh temperatures (MT). Therefore, the resultant membrane forces could be obtained by integrating the results from FEM along the cross section. In addition, the supporting beam forces are shown in **Fig 5.18**. Therefore, a summary of the calculated membrane forces is shown in **Table 5.2**.

(a) One-quarter free body diagram

(b) Sectional forces along A-O

Fig 5.19 In-plane membrane forces equilibrium with the section forces from FEM at elevated temperature

It can be seen that with increasing temperature, tensile membrane forces started increasing before decreasing at $MT=340^{\circ}\text{C}$ due to reduction in material strength at elevated temperature. Compressive membrane forces increased at the beginning due to thermal expansion and vanished at 340°C . Therefore, the peripheral compressive ring became smaller at elevated temperature. To balance the tensile membrane forces developed at the centre of the slab, compressive forces in the supporting beams increased accordingly. Finally, the total tensile membrane forces in the slab along the long or short span were balanced by the compressive forces in the supporting beams until tensile cracks appeared between the slab and the edge beams, leading to fracture

of reinforcement above the edge beams. In addition, the horizontal forces in columns were compared with measured lateral forces in **Fig 5.17** and good agreement was observed between them.

Table 5.2 Equilibrium column lateral forces from FEM compared with measured column lateral forces for ISOCS1NI

Mesh temperature (MT)	20°C	48°C	143°C	340°C
Σ tensile membrane	5.80 kN	72.87 kN	79.57 kN	52.43 kN
Σ compressive membrane	-1.66 kN	-62.76 kN	-23.92 kN	0.00 kN
Supporting beam force	-5.94 kN	-24.53 kN	-76.87 kN	-43.61 kN
Equilibrium column force	1.81 kN	14.42 kN	21.21 kN	-8.82 kN
Measured column force	1.36 kN	8.31 kN	7.86 kN	-7.4 kN

(a) A free body diagram for one-quarter ISOCS1.5

(b) Sectional forces along A-O

Fig 5.20 In-plane membrane forces equilibrium with section forces from FEM

Similarly, the cross-sectional forces in the one-quarter specimen ISOCS1.5 are shown

in **Fig 5.20**. Unlike ISOCS1NI, the membrane forces along the long span were in tension and small values of compressive membrane forces were observed from the FEM analysis as indicated in **Table 5.3**. Since there was only one interior secondary beam, due to symmetry, only half of the interior beam force was considered. The measured column forces were consistently in agreement with numerical predictions.

Table 5.3 Equilibrium column lateral forces from FEM compared with measured column lateral forces for ISOCS1.5 (positive in tension and negative in compression)

<i>Mesh temperature (MT)</i>	20°C	47°C	112°C	228°C
<i>Σtensile membrane</i>	17.91kN	138.90 kN	78.00 kN	51.38 kN
<i>Σcompressive membrane</i>	0.00 kN	0.00 kN	0.00 kN	0.00 kN
<i>Supporting beam force</i>	-21.14 kN	-126.98 kN	-79.74 kN	-49.41 kN
<i>Equilibrium column force</i>	3.23 kN	-11.92 kN	1.74 kN	-1.97 kN
<i>Measured column force</i>	1.26 kN	-10.94 kN	1.89 kN	-1.26 kN

5.10 Summary

Three isolated composite slabs were designed based on the BRE design guide. It was assumed that there was no boundary continuity of composite slab panels. Although large deformations occurred, the overall integrity of these three specimens was maintained. Therefore, it can be concluded that this BRE design approach is safe and conservative as it accounts for the enhancement factor from tensile membrane action. In addition, the ABAQUS model incorporating the concrete damage plasticity model at elevated temperature gave convergent predictions which agreed well with test results. It was found that tensile membrane stage began at around one effective depth of composite slab thickness. This has been verified through both the experimental and numerical results based on cross-sectional forces obtained from FEM analyses.

The lateral forces acting on the columns have been estimated through the CHS strain readings, which agreed well with the measured column movements. The column movements affect the capacity of composite beam-slab systems. When columns

moved inwards, a compressive ring formed around the peripheral of composite slabs with a tensile zone in the centre. The equilibrium mechanism was established and the capacity of composite beam-slab systems was enhanced when columns moved inwards and vice versa.

Compressive membrane forces increased at the beginning due to thermal expansion and reduced to zero at failure for ISOCS1NI. However, for ISOCS1.5, small compressive membrane forces along the long span had been observed. The tensile membrane forces for both specimens increased before decreasing due to reduction of material strength at elevated temperature.

Most importantly, the tensile membrane forces were balanced by the compressive membrane forces if the aspect ratio was one. For a larger aspect ratio (say 1.5), the tensile membrane forces could only be balanced by the compressive forces in the perimeter beams at elevated temperature. The peripheral compressive ring could not be formed for a larger aspect ratio and this indicated that the two-way tensile membrane action was not effective for an aspect ratio of 1.5.

In the next chapter, the tests on boundary restraint Series II and III specimens would be discussed and compared with Series I. The boundary restraint forces were monitored and imported into the analytical model for Series II and III specimens.

Chapter 6 Experiment test and analyses for Series II&III of boundary restraint composite beam-slab systems

6.1 Introduction

Chapter 5 on the isolated composite beam-slab sub-assembly showed the development of membrane forces at elevated temperature. By considering tensile membrane action, the BRE-Bailey design method introduces a new composite slab design approach to reduce fire protection for interior steel beams. However, this proposed model is only based on isolated slab panels which do not have boundary continuity. In actual structures, when temperature increases, thermal expansion of composite slabs inevitably takes place. High compressive membrane forces will be induced from surrounding unheated slab panels and tensile membrane action at large deformations will also be affected. In addition, for different boundary continuities of slab panels, *viz.* internal, edge and corner panels, boundary restraints have direct effects on the development of membrane actions. To date, very few researchers have studied the effect of boundary continuities on the development of membrane action for composite beam-slab system at elevated temperature.

Therefore, in this chapter, various boundary conditions will be discussed and validated through numerical simulations. Additionally, the development of membrane forces for internal, edge and corner composite slabs will be investigated at elevated temperature.

6.2 Specimen details for Series II

Series II consisted of three composite slabs with three types of boundary continuities

as shown in **Fig 6.1**. Specimen **ICS1** was an **I**nternal **C**omposite **S**lab panel with an aspect ratio of **1**. **ECS1** was an **E**dge **C**omposite **S**lab panel with an aspect ratio of **1**, while **CCS1** is a **C**orner **C**omposite **S**lab panel with an aspect ratio of **1**.

Fig 6.1 Specimens Series II for composite beam-slab sub-assemblages with different boundary conditions (unit in mm)

The edge composite slab panel was chosen to be continuous along the primary beam direction since it would have a greater axial restraint stiffness (discussed in Section 4.4). Apart from interior secondary beams, the remaining beams and columns were painted with intumescent coating for one-hour fire rating similar to isolated composite slab series.

For comparison purpose, the geometries and material properties for Series II specimens were similar to those of Series I. The details of Series II specimens are included in

Table 6.1.

The major difference between the boundary restraint series specimens was an extension of the slab edges to span/8 in both directions. For continuous slab design, it is commonly considered that the span/8 is the contraflexure point for bending moment. Therefore, if the edge of the specimen extends out for span/8, the bending moment at this contraflexure point is negligible. Besides, the embedded bolts along the four edges of the specimen were connected to surrounding A-frames to provide lateral restraints in the plane of the slab.

Table 6.1 Geometries and material properties for Series II specimens

	Geometry			Concrete slab			Reinforcement			Steel beam & column		
	h	d_x	d_y	f_{cm}	f_{ctm}	E_{cm}	f_y/f_u	Elong	Modulus	f_y/f_u	Elong	Modulus
	mm	m m	mm	MPa	MPa	GPa	MPa	ation, %	E_s , GPa	MPa	ation, %	E_s , GPa
ICSI	60	32	26	30.8	2.5	28.4	551 /617	21.5	205	368 /432	21.2	207.5
ECSI	61	34	28	31.5	2.6	30.9	599 /674	23.1	207	380 /468	25.1	211.1
CCSI	58	35	29	32.7	2.6	31.2	563 /652	22.8	201	363 /449	23.6	204.9

6.3 Specimen details for Series III

Similar to Series II tests, specimens of Series III were fabricated with an aspect ratio of 1.5. Specimens in Series III were smaller with overall dimensions of 2.25m by 1.5m from column centre to column centre. To cover the gap between the smaller specimens, the edges along the short span were extended out by 700 mm and 400 mm for the long span. The material and geometry properties for Series III specimens are shown in **Table 6.2** and **Fig 6.2**.

Table 6.2 Geometries and material properties for Series III specimens

	Geometry			Concrete slab			Reinforcement			Steel beam & column		
	h	d_x	d_y	f_{cm}	f_{ctm}	E_{cm}	f_y/f_u	Elong	Modulus	f_y/f_u	Elong	Modulus
	mm	m m	mm	MPa	MPa	GPa	MPa	ation, %	E_s , GPa	MPa	ation, %	E_s , GPa
ICSI.5	59	33	27	33.6	2.9	32.8	532 /620	20.8	209	375 /444	23.1	215.4
ECSI.5	62	32	26	31.1	2.4	29.6	545 /655	25.0	202	382 /432	21.6	207.3
CCSI.5	61	35	29	32.9	2.6	30.4	539 /643	23.4	208	370 /452	22.7	201.2

ICS1.5 (Internal Composite Slab aspect ratio =1.5) ECS1.5 (Edge Composite Slab aspect ratio =1.5) CCS1.5 (Corner Composite Slab aspect ratio =1.5)

Fig 6.2 Specimens Series III for composite beam-slab sub-assemblages with different boundary conditions (unit in mm)

6.4 Test setup for boundary Series II and III

Different from the isolated series of specimens, various boundary continuities were introduced in Series II and III. To achieve boundary continuity, the slab specimens had an overhang of 400 mm slab in both directions and wire mesh ($\Phi 6\text{mm}$ at 120mm spacing) was welded to 10 mm thick boundary steel plates placed on four edges of around the slab.

Fig 6.3 Schematic view for the continuous edge with five M25 bolts

In addition, five M25 bolts were installed on each side of composite slab with a cast-in embedment length of 100 mm as shown in **Fig 6.3**. The contact length between each M25 bolt and concrete slab was larger than that of $\Phi 6$ mesh (120mm spacing). The purpose was to ensure membrane forces developed in the mesh (at tensile membrane action stage) could be transmitted through the cast-in bolts without debonding from concrete slab.

The M25 bolts protruded through a picture frame comprising 100 x 50 x 5mm rectangular hollow sections (RHS) and were threaded to the flange of stiff perimeter H-beams (200x200x10mm thick) to provide horizontal restraints as shown in **Fig 6.4**. Finally, the perimeter restraint beams were in turn connected to the column of the A-frame, with the base fixed onto the laboratory strong floor (**Fig 6.5**).

Fig 6.4 A typical connection between the composite slab and the perimeter beams

Fig 6.5 The connection between the restrained beams and the A-frame

To affect different boundary conditions, different number of restraint edge beams is connected to the A-frames. For the internal composite slab, four continuous edges were restrained by connecting four restraint beams to A-frames (**Fig 6.6(a)**). In the same way, the edge and the corner composite slabs were connected with three and two edges to the external restraint beams as shown in **Fig 6.6(b)** and (c), respectively.

(a) Internal composite slab test setup

(b) Edge composite slab setup

(c) Corner composite slab setup

Fig 6.6 Typical setup for the different boundary conditions

6.5 Boundary restraints validation

To verify the effectiveness of these boundary conditions, a numerical model was built up as shown in **Fig 6.7**. Since there were five bolts on each edge, in the numerical analysis, five axial springs coupled with rotational springs were assigned on each edge to study both the axial and rotational stiffnesses. The braced diagonal angles for each A-frame were slightly different due to physical constraint in the laboratory. A summary of numerical predictions for each spring is shown in **Table 6.3**. Compared with the prototype 9m by 9m composite slab panels (**Table 4.5**), the boundary restraint systems showed very close stiffness values for both axial and rotational springs. These spring stiffness values would be input as realistic boundary restraints for parametric study.

Fig 6.7 Numerical studies on the restrained stiffnesses for each restrained boundary A-frame

Table 6.3 Boundary spring stiffnesses for restrained frames

<i>Stiffness</i>	<i>Main beam 1</i>	<i>Secondary beam 2</i>	<i>Main beam 2</i>	<i>Secondary beam 1</i>
Axial spring 1 (kN/m)	10330.4	6121.4	7131.0	6121.4
Axial spring 2 (kN/m)	79143.4	25577.3	49997.3	25577.3
Axial spring 3 (kN/m)	59655.2	37260.6	Pinned	37260.6
Axial spring 4 (kN/m)	43432.2	25577.3	49997.3	25577.3
Axial spring 5 (kN/m)	8440.7	6121.4	7131.0	6121.4
Rotational spring 1 (kNm/rad)	19.0	19.0	19.0	19.0
Rotational spring 2 (kNm/rad)	298.0	297.7	297.7	297.7
Rotational spring 3 (kNm/rad)	43.2	43.2	43.2	43.2
Rotational spring 4 (kNm/rad)	298.0	297.7	298.0	297.7

Rotational spring 5 (kNm/rad)	19.0	19.0	19.0	19.0
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Fig 6.8 Load equilibrium for laterally restrained composite beam-slab system

In addition, strain gauges were attached on four supporting circular hollow section (CHS) columns and A-frames to calculate total reaction forces. These measured vertical forces were compared with total applied load to ensure the loading system was in equilibrium; the sub-assembly system with rather complex boundary restraint conditions was externally statically determinate. An example of calculated total reaction forces from supporting CHS columns and A-frames is shown in **Fig 6.8**.

Total reaction forces consisted of forces from the four supporting columns within the furnace and external forces from the two laterally restrained A-frames on each side. For internal composite slab specimen ICS1, the measured total vertical reaction forces were balanced with the applied load (100 kN) as shown in **Fig 6.9**.

Fig 6.9 Measured reaction forces versus time for ICS1

When temperature increased, due to thermal expansion of the composite slab, the diagonal braces were in compression to prevent the composite slab from expanding outwards. Struts of the A-frame were in tension since they had to resist the boundary beams of the composite slab from lifting upwards. After 60 minutes, the reaction forces from the strut of A-frame decreased due to pulling-in forces from the composite slab. Since these pulling-in forces contributed to tensile membrane action of the composite slab systems, the starting point of reduction of axial forces in the strut marked the beginning of tensile membrane action in the slab.

The forces in the diagonal brace consisted of two components: vertical and horizontal brace forces. It can be seen that the vertical brace forces reduced after 60 minutes. The horizontal brace force representing the external axial restraint applied to the composite slab was around 100 kN in the X-direction and 75 kN in the Y-direction, since the axial restraint in the X- direction was stiffer than that in the Y-direction.

Similarly, measured column vertical reaction forces agreed well with the applied load at the beginning for ECS1 as shown in **Fig 6.10**. With increased temperature, the tensile forces in the struts were increased due to thermal expansion but reduced after around 60 minutes when the tensile membrane action was mobilised.

Unlike ICS1, the horizontal in-plane forces for ECS1 in the X-direction were larger than those in the Y-direction, due to one discontinuous edge for ECS1 along the Y-direction. In general, the in-plane restraint forces for specimen ECS1 were smaller than those in ICS1.

Fig 6.10 Measured reaction forces versus time compared with applied load for ECS1

For corner specimen CCS1, the whole system was in equilibrium initially since the measured reaction forces in the supporting columns agreed well with the applied load. However, much smaller tensile forces were measured in the strut compared to those in ICS1 and ECS1, since there were two adjacent discontinuous edges. Moreover, the in-plane horizontal restraint forces were also much smaller but similar in magnitude to the compressive forces in the isolated series. Therefore, the corner composite slab specimen could be considered as isolated composite slab.

Fig 6.11 Measured reaction forces versus time compared with applied load for CCS1. For Series III specimens with an aspect ratio of 1.5, similar reaction forces were observed for ICS1.5. The initial column reaction forces tallied with the applied vertical load very well but increased due to thermal expansion as shown in **Fig 6.12**. High in-plane compressive forces were measured in both X- and Y-directions, which indicated effective boundary restraints were applied to the specimen.

Fig 6.12 Measured reaction forces versus time compared with applied load for ICS1.5

Fig 6.13 Measured reaction forces versus time compared with applied load for ECS1.5

Similar to specimen ICS1.5, the applied load was resisted by four columns at the beginning stage. After temperature increased, both the strut and diagonal brace forces increased due to thermal expansion. However, high compressive forces were observed in the X-direction because of lateral restraints. There were negligible in-plane forces in the Y-direction due to one discontinuous edge as shown in **Fig 6.13**.

Fig 6.14 Measured reaction forces versus time compared with applied load for CCS1.5

Interestingly, the restraint forces for specimen CCS1.5 in **Fig 6.14** were much smaller than those in ICS1.5 and ECS1.5 since the corner composite slab CCS1.5 had two adjacent discontinuous edges. Therefore, the in-plane forces in both global X and Y directions were negligible compared with those in ICS1.5 and ECS1.5 and the boundary condition for specimen CCS1.5 could be considered as an isolated slab panel.

6.6 Temperature profiles

The temperature profiles for Series II and III specimens were plotted out as shown in **Fig 6.15** and **Fig 6.16**, respectively. The temperatures of electric furnace, interior unprotected steel beam, slab bottom surface, mesh reinforcement and slab top surface were monitored. The unprotected interior steel web and bottom flange temperatures were very close due to high thermal conductivity and the average temperature was used as inputs in the numerical analysis. Generally, the cross-section temperature profiles were very consistent for Series II and III specimens during heating.

(a) Temperature profile for ICS1

(b) Temperature profile for ECS1

(c) Temperature profile for CCS1

Fig 6.15 Temperature profiles for Series II specimens with aspect ratio of 1

(a) Temperature profile for ICS1.5

(b) Temperature profile for ECS1.5

(c) Temperature profile for CCS1.5

Fig 6.16 Temperature profiles for Series III specimens with aspect ratio of 1.5

6.7 Crack propagation and failure criteria

For boundary Series II specimens with an aspect ratio of one as shown in **Fig 6.17**, diagonal cracks started developing from the four corners of the slab at 45 degrees with respect to the edges when the mesh temperature reached 50°C. For ICS1 (**Fig 6.17(a)**), once the mesh temperature reached 100°C, more cracks were emanated from the four columns as well as the perimeter beams. When the mesh temperature achieved 260°C, cracks from the four corners propagated to the centre of the slab. Finally, the crack widths around the corners and at the mid-span steadily grew larger until the specimen could not maintain the applied load (100 kN), which violated the load-bearing capacity criterion.

Fig 6.17 The development of cracks for boundary Series II (aspect ratio =1) at elevated mesh temperature

For edge specimen ECS1 (**Fig 6.17(b)**), it had three continuous edges. Similar to ICS1, more cracks were formed at the extended edges around the panel at 100°C. However, middle cracks appeared and traversed perpendicular to the discontinuous edge above 150°C. This was because tensile membrane action was mainly developed in the direction parallel to two opposite restraint edges. On the contrary, tensile membrane action was not so effective for the discontinuous edge as observed from the crack patterns.

Specimen CCS1 in **Fig 6.17(c)** only had two adjacent continuous edges. After cracks had propagated from the four corners towards to central region at 150°C, middle cracks appeared which ran perpendicular to one of the discontinuous edges.

Although the specimens in Series II showed a similar trend of crack formation at the beginning stage, due to different boundary conditions, crack patterns showed different

failure mechanisms at the final stage. For internal composite slab ICS1, failure was governed by the two-way tensile membrane action and the cracks were well distributed around the perimeter beams and propagated towards the centre. While for edge composite slab ECS1, only one-way tensile membrane action had been mobilised and middle cracks ran perpendicular to the discontinuous edge. For corner composite slab CCS1, the crack pattern showed that folding mechanism occurred due to two adjacent discontinuous edges.

Fig 6.18 Crack development for boundary Series III (aspect ratio=1.5)

Similarly, for boundary Series III specimens with an aspect ratio of 1.5 in **Fig 6.18**, cracks occurred across the columns diagonally and cracks above the long supporting beams were also observed around the mesh temperature of 50 °C . When the temperature reached 100 °C , diagonal cracks propagated towards the centre of the composite slab and cracks also could be found above the short perimeter beams.

Different from boundary Series II specimens with an aspect ratio of one, Series III specimens with an aspect ratio of 1.5 had many cracks perpendicular to the long edges around 150°C , which showed that tensile membrane action developed along the long span more effectively. In addition, middle cracks formed towards the end of the tests, symbolising some kind of folding mechanism.

For Series II and III specimens, the failure was governed by load-bearing capacity as shown in **Fig 6.19**. Overall equilibrium was maintained through forming a compressive ring around the perimeter and a tensile zone at the centre of the slab. Once cracks had occurred around the perimeter of Series II and III specimens (at a crack width of around 10mm), applied load could not be sustained, which led to a sudden drop of the applied load.

(a) Applied load versus Time for Series II (b) Applied load versus Time for Series III

(c) Cracks around the perimeter beams at width of around 10mm

Fig 6.19 Governing failure criterion of load-bearing capacity(cracks around the perimeter)

6.8 Displacements

For Series II specimens with different boundary conditions, the central deflections are plotted versus mesh temperature as shown in **Fig 6.20**.

Tensile membrane action stage occurred at one effective depth (Liu and Tan 2012), which was around 60 mm. Clearly, specimens ICS1, ECS1 and CCS1 behaved similarly before reaching the tensile membrane action stage. However, once the central deflection exceeded one effective depth, the deflection rate for internal specimen ICS1 was much smaller than the edge and the corner specimens. In addition, corner composite slab CCS1 deflection rate was the greatest compared to ECS1 and ICS1.

Fig 6.20 Test results for centre deformation comparison for ICS1, ECS1, CCS1 and ISOCS1

The reason for the different deflection rates in the four specimens is that once the tensile membrane stage has been mobilised, internal slab ICS1 has greater capability to redistribute applied load compared to the edge (ECS1) and the corner (CCS1) composite slabs, since the former has four continuous edges, which could evenly transfer applied loads in two directions. On the other hand, for the edge composite slab ECS1 after reaching the tensile membrane action stage, loads could only be spread out more in the two opposite continuous edges. This is the reason that the middle crack pattern ran perpendicular to the discontinuous edge as shown in **Fig 6.17(b)**. For the

corner composite slab CCS1, the ability to redistribute applied load was even more limited than ECS1. Therefore, the deflection rate of CCS1 was the greatest among these three specimens.

Interestingly, the isolated composite slab panel ISOCS1 has been used to benchmark the deflection behaviour with boundary restraint Series II specimens. It can be seen that the deflection rate of ISOCS1 was much smaller than the boundary restrained specimens. The main reason is due to high compression arising from restrained thermal expansion. For boundary restraint Series II specimens with increasing temperature, high compressive membrane forces enhanced the deflections and the effect was similar to thermal bowing with restraint boundary condition.

Most importantly, once boundary restraints are considered, the tensile membrane forces was slightly reduced due to high compressive membrane forces (**Table 8.8** and **8.10**). However, the edge beam deflections were smaller than those from ISOCS1 due to continuity effect. The enhanced load-bearing capacities were calculated by tensile membrane forces multiply by the relative deflections. Therefore, ISOCS1, with higher tensile membrane forces but a smaller relative deflection, performed as well as ICS1.

A similar deflection trend was observed for Series III specimens as shown in **Fig 6.21**. At the beginning stage, three specimens, ICS1.5, ECS1.5 and CCS1.5 experienced similar deformation-temperature relationship. However, after the mesh temperature reached 350 °C , ICS1.5 (internal composite slab) had the smallest deformation compared to ECS1.5(edge composite slab) and CCS1.5(corner composite slab) at a specific mesh temperature.

Fig 6.21 Test results for centre deformation comparison for ICS1.5, ECS1.5 and CCS1.5

In addition, the deformation differences among these three specimens in Series III were not as obvious as those in Series II. For Series III specimens with a higher aspect ratio of 1.5, tensile membrane action unevenly mobilised in two directions. However, for Series II specimens with an aspect ratio of one, tensile membrane action could be developed more efficiently compared with Series III with an aspect ratio of 1.5.

In addition, numerical models were built up with defined axial and rotational spring values mentioned in **Table 6.3**. The test results of central deformations were compared with numerical analyses as shown in **Fig 6.22**. Clearly, good agreement is achieved between numerical predictions and test results.

Fig 6.22 Comparisons of centre deflections between test results and numerical analyses for Series II and III

For Series II specimens with boundary restraint, thermal expansion induced large compressive membrane forces in the composite slab, which accelerated the deflection behavior at the beginning stage. This could be clearly observed from the numerical analyses. The legend “FEM fixed” represents the numerical model with fixed boundary conditions, which gives the greatest deflection due to high compression. However, for edge and corner composite slabs, partial thermal expansion occurred due to lack of restraints for two adjacent discontinuous edges. Hence, there was negligible difference between the finite element predictions with fixed boundary conditions or with spring stiffness properties.

The enhanced deflection was detrimental to the composite slab behaviour from serviceability point of view, but larger slab deflections also implied greater mobilisation of tensile membrane action at a subsequent stage. Therefore, it was difficult to comment whether restraint boundary conditions were beneficial or detrimental to the composite slab behaviour at high temperature, as it depended on whether the slab was at the initial compressive membrane action stage, or subsequent tensile membrane action stage. Provided the slab did not fail prematurely at the four corners at the initial stage, greater central deflections would give rise to greater mobilisation of tensile membrane action in the latter stage.

6.9 Column lateral movements and forces

Column lateral movements have been monitored during the tests. Line transducers were attached at the top of the columns as shown in **Fig 6.23**. In addition, the line transducers were orientated at 45 degree with respect to the edges. Lateral movements of columns were recorded as shown in **Fig 6.24**. Negative sign indicates outward movements of columns while positive sign means inward movements of columns.

Unlike isolated composite slab series, in Series II specimens the outward thermal expansions were greatly restrained for specimens ICS1 and ECS1 by boundary continuities. But for specimen CCS1, since only two adjacent edges were restrained, there were outward movements due to thermal expansion. Columns for all specimens moved inwards at 100 °C average mesh temperature due to pulling-in forces from the slab. This marked the starting point of the tensile membrane action stage, which was consistent with the discussions on central deflections in Section 6.8.

Although the corner restrained specimen CCS1 should have behaved similar to ISOCS1, the movement of corner columns varied a lot (unfortunately, the measurement for column movements of ISOCS1 were not working well, just use ISOCS1NI to represent ISOCS1), which created instability for CCS1. Hence, the deflection rate for CCS1 is much more than ISOCS1 (**Fig 6.20**).

For Series III specimens with an aspect ratio of 1.5, inward movements of columns along the longer spans were much larger than those from Series II specimens with an aspect ratio of one. This was resulted from the folding mechanism occurred which across the long span, inducing much larger column movements.

(a) Line transducer attached on column head (b) Line transducer layout
Fig 6.23 Line transducers measuring the lateral movement of columns for Series II and III specimens

(a) Specimen ICS1

(b) Specimen ECS1

(c) Specimen CCS1

(d) Specimen ICS1.5

Fig 6.24 Lateral column movements (positive=inward) for Series II&III specimens

Similar to Section 5.8, column lateral forces were estimated based on extrapolation from strain gauges attached at the positions indicated in **Fig 6.25** on the CHS surface.

Fig 6.25 Illustration of Circular Hollow Section (CHS) columns attached with strain gauges

For Series II specimens as shown in **Fig 6.26**, it can be seen that higher compressive forces were observed due to thermal expansion and lateral restraint. However, when the average mesh temperature reached around 100°C , lateral compressive forces changed to tension because of large central deflection, which pulled the columns inwards. The transition of column lateral forces from compression to tension coincided with the measured column movements, as well as the commencement of mobilisation of tensile membrane action in the composite slab.

(a) Lateral column forces for ICS1

(b) Lateral column forces for ECS1

(c) Lateral column forces for CCS1

Fig 6.26 Calculated lateral forces at the beam-column joints for Series II specimens

(a) Lateral column forces for ICS1.5

(b) Lateral column forces for ECS1.5

(c) Lateral column forces for CCS1.5

Fig 6.27 Calculated lateral forces at the beam-column joints for Series III specimens

Similar to Series I and II, for Series III specimens with an aspect ratio of 1.5 as shown in **Fig 6.27**, the X-direction was along the long span and the Y-direction along the short span. It can be seen that lateral forces for columns along the X-direction were small at the beginning. However, when the mesh temperature increased to 100°C, lateral forces changed to tension and the tensile forces along the X-direction were larger than those in the Y-direction. It can be concluded that the tensile membrane action developed more effectively along the long span compared with the short span.

Additionally, after the mesh temperature reached 100°C, lateral forces for all columns of ICS1.5 were in tension. But for ECS1.5 and CCS1.5, only column 3 was being pulled inwards.

6.10 Sectional forces

Unlike Series I specimens, lateral restraints from the A-frames were taken into account for the boundary restraint Series II specimens. Sectional forces for ICS1 along the X- and Y-axes at the mid-span were plotted from the edge towards the centre (A to O and B to O) for different mesh temperatures (MT) in **Fig 6.28**. At the beginning stage when mesh temperature was around 100°C, much higher compressive membrane forces were observed due to restrained thermal expansion compared with those in Series I. However, with increased mesh temperature above 100°C, tensile membrane zone grew larger and expanded towards the edge beams. It was worthy to note that both the compressive and tensile membrane forces were greatly reduced due to material degradation at elevated temperature. In addition, the width compression ring was around one-sixth of the clear span, similar to that observed in Series I.

Fig 6.28 Sectional forces from numerical analysis for ICS1

Similarly, ECS1 and CCS1 were also studied and sectional forces are shown in **Fig 6.29**. Different from ICS1, ECS1 had one unrestrained edge and CCS1 had two adjacent discontinuous edges. For ECS1, since translations in the Y-direction for Segment 1 were unrestrained, the compressive membrane forces were smaller than those of ICS1 (along A-O as shown in **Fig 6.20(c)**). However, for ECS1 Segment 2, Y-direction movements were restrained and compressive membrane forces were larger than those of Segment 1 (along B-O as shown in **Fig 6.20(d)**). On the contrary, the X-direction movements for the CCS1 Segment 1 were restrained but those of Segment 2 were unrestrained. Compressive membrane forces were slightly larger in Segment 1 along A-O compared with those in Segment 2 along B-O.

(a) Locations of A, O and B points for ECS1 (b) Locations of A, O and B points for CCS1

(c) Sectional forces along A-O for ECS1 (d) Sectional forces along B-O for ECS1

(e) Sectional forces along A-O for CCS1 (f) Sectional forces along B-O for CCS1

Fig 6.29 Sectional forces from numerical analysis for ECS1 and CCS1 at elevated temperature

For Series III specimen ICS1.5 (**Fig 6.30**), the cross section was in compression in both directions at low mesh temperature due to restrained thermal expansion. Once the mesh temperature reached about 118 °C, tensile membrane action occurred in the

central part of the slab. With increased mesh temperature, the sectional forces were in tension 0.75m from the centre O towards A (**Fig 6.30(b)**), which was similar to specimen ISOCS1.5.

However, along B-O in **Fig 6.30(c)**, the sectional forces between O and A were all in tension when the mesh temperature above 212°C. This was similar to the numerical results from specimen ISOCS1.5.

Fig 6.30 Sectional forces from numerical analysis for ICS1.5

For ECS1.5 with one discontinuous edge as shown in **Fig 6.31**, at low mesh temperature, the whole cross sections were in compression. However, with increased mesh temperature (above 300°C), tensile membrane forces occurred across the whole section of the slab (along A-C in **Fig 6.31(b)**). However, the sectional forces along X-axis (From A-B in **Fig 6.31(c)**) showed a similar pattern as compared with that along

the X-axis.

For CCS1.5 across the short span as shown in **Fig 6.32**, since there were two discontinuous adjacent edges, compressive membrane forces were much smaller than ICS1.5 and ECS1.5. Additionally, high tensile membrane forces had been observed along the Y-direction (A-B in **Fig 6.32(a)**), which indicated that the majority of the tensile membrane forces were distributed along the long span.

(a) Locations of A, B and C across the Y-axis (b) Sectional forces along B-A

(c) Locations of D, E and O across the X-axis (d) Sectional forces along D-O

Fig 6.31 Sectional forces from numerical analysis for ECS1.5

(a) Location of A, C and B along the Y-axis (b) Sectional forces along A-B

Fig 6.32 Sectional forces from numerical analysis for CCS1.5

(c) Location of D, E along the X-axis (b) Sectional forces along D-E

Fig 6.35 Sectional forces from numerical analysis for CCS1.5 (continue)

For CCS1.5 cross-section along the X-axis, much lower sectional forces (along D-E in **Fig 6.32(c)**) were observed due to discontinuities, which was similar to those from isolated specimen ISOCS1.5.

After analysing all nine specimens, the section forces showed similar patterns at elevated temperature. At the beginning, with increased temperature, high compressive membrane forces were initiated in the whole cross section. After deformations increased, tensile membrane forces occurred at the central region of the slab. With a further increase in the mesh temperature, corresponding tensile and compressive membrane forces were reduced. However, tensile membrane forces and compressive membrane forces were linearly distributed along the mid-span cross sections. This assumption will be applied in the analytical model (Section 8.5) when calculating the

load-bearing capacity of the slabs at the ultimate limit state.

6.11 In-plane forces equilibrium comparison for Series I, II and III

To compare in-plane forces, the summations for compressive membrane forces and tensile membrane forces, supporting beam forces and measured lateral column forces for specimen ISOCS1NI are given in **Table 6.4**.

Table 6.4 Summarised in-plane forces from FEM for ISOCS1NI

Mesh temperature (MT)	20°C	48°C	143°C	340°C
Σ tensile membrane	5.80 kN	72.87 kN	79.57 kN	52.43 kN
Σ compressive membrane	-1.66 kN	-62.76 kN	-23.92 kN	0.00 kN
Supporting beam force	-5.94 kN	-24.53 kN	-76.87 kN	-43.61 kN
Equilibrium column force	1.81 kN	14.42 kN	21.21 kN	-8.82 kN
Measured column force	1.36 kN	8.31 kN	7.86 kN	-7.4 kN

In-plane forces for specimen ISOCS1 and ISOCS1.5 are provided in **Table 6.5** and **Table 6.6**, respectively. For specimen ISOCS1.5, when extracting the sectional forces along the long span as shown in **Fig 6.33(b)**, there were negligible compressive membrane forces. Compressive membrane forces around the four corners of the composite slab are indicated in blue in **Fig 6.33**.

Table 6.5 Summarised in-plane forces from FEM for ISOCS1

Mesh temperature (MT)	38°C	108°C	338°C	480°C
Σ tensile membrane	2.2 kN	30.1 kN	40.3 kN	20.4 kN
Σ compressive membrane	-1.7 kN	-5.7 kN	-6.4 kN	-10.2 kN
Supporting beam force	-1.1 kN	-22.3 kN	-34.2 kN	-8.1 kN
Equilibrium column force	0.6 kN	-2.1 kN	0.3 kN	-2.1 kN
Measured column force	N.A.	N.A.	N.A.	N.A.

Table 6.6 Summarised in-plane forces for ISOCS1.5 along the short span

Mesh temperature (MT)	20°C	47°C	112°C	228°C
Σ tensile membrane	17.91 kN	138.9 kN	78 kN	51.38 kN
Σ compressive membrane	0	0	0	0
Supporting beam force	-21.14 kN	-126.98 kN	-79.74 kN	-49.41 kN
Equilibrium column force	3.23 kN	-11.92 kN	1.74 kN	-1.97 kN
Measured column force	1.26 kN	-10.94 kN	1.89 kN	-1.26 kN

Fig 6.33 Illustration of in-plane section forces from FEM

Due to interference from other equipment, the column lateral movements for ISOCS1 were not recorded and thus, the lateral column forces could not be calculated. In conclusion, the overall in-plane equilibrium was achieved.

However, for boundary restrained Series II, to balance the in-plane membrane forces, lateral restrained forces were monitored through strain gauges attached on the A-frame as shown in **Fig 6.34**. A summary of in-plane forces is given in

Table 6.7.

It can be seen that high compressive membrane forces occurred at the beginning due to extended restraints from A-frames. All the forces were reduced when the mesh temperature increased. In addition, the developed tensile membrane forces were much smaller compared with those in Series I specimens. Overall, by considering the measured frame forces and column lateral forces, the in-plane forces were in equilibrium.

(c) Typical sectional force distribution for a one-quarter slab

Fig 6.34 Illustration for measurements for lateral restrained A-frame forces**Table 6.7** Equilibrium forces from FEM analysis compared with measured forces for ICS1 along both the X and Y directions for a one-quarter composite slab panel

Mesh temperature (MT)	Along A-O (kN)				Along B-O(kN)			
	100°C	207°C	385°C	526°C	100°C	207°C	385°C	526°C
Σtensile membrane	0.0	15.6	20.4	37.1	0.0	0.0	7.4	25.2
Σcompressive membrane	-384.5	-164.5	-92.3	-115.1	-400.3	-140.6	-83.1	-109.0
Supporting beam force	-245.3	-123.0	-49.7	-67.6	-298.3	-79.6	-67.8	-72.3
Measured restrained force	-128.9	-30.7	-28.1	-16.2	-89.4	-56.9	-12.8	-29.4
Measured column force	-12.7	4.6	5.3	5.6	-9.8	-0.5	2.2	2.5
Equilibrium force	2.3	0.3	0.6	0.3	-2.8	-3.5	-4.7	-9.8

Similarly, ECS1 and CCS1 were also studied and the results were included in **Table 6.8** and **Table 6.9**, respectively. Different from ICS1, ECS1 had one unrestrained edge and CCS1 had two adjacent discontinuous edges. For ECS1, since the Y-direction movements for Segment 1 were unrestrained, compressive membrane forces were smaller than those of ICS1, which were balanced by the tension in the supporting beam. However, for ECS1 Segment 2, the Y-direction movements were restrained and compressive membrane forces were larger than those of Segment 1.

Table 6.8 Equilibrium forces from FEM analysis compared with measured forces for ECS1 along Y-direction for Segments 1 and 2

Mesh temperature (MT)	Segment 1 along A-O (kN)				Segment 2 along B-O(kN)			
	45.7°C	110°C	180°C	315°C	45.7°C	110°C	180°C	315°C
Σtensile membrane	0.0	0.0	33.0	15.7	0.0	0.0	67.8	29.3
Σcompressive membrane	-223.4	-179.8	-68.4	-46.5	-460.1	-361.9	-149.5	-102.4
Supporting beam force	-205.3	-159.4	-40.8	-36.7	-342.1	-265.7	-27.2	-24.4
Measured restrained force	0.0	0.0	0.0	0.0	-112.6	-87.5	-54.3	-59.8
Measured column force	-7.5	-9.8	5.2	9.8	-7.5	-11.8	0.0	2.1
Equilibrium force	-10.7	-10.6	0.3	-4.0	2.0	3.1	-0.3	9.0

On the contrary, the X-direction movements for CCS1 Segment 1 were restrained and those for Segment 2 were unrestrained. Thus, the compressive membrane action was larger in Segment 1 along the X-direction compared with that of Segment 2. Therefore, for Segment 2 in the X-direction, since there were no restraint forces, a higher supporting beam force was expected compared to that for Segment 1 in the X-direction.

Table 6.9 Equilibrium forces from FEM analysis compared with measured forces for CCS1 along X-direction for Segments 1 and 2

Mesh temperature (MT)	Segment 1 along A-O (kN)				Segment 2 along B-O (kN)			
	46°C	110°C	180°C	254°C	46°C	110°C	180°C	254°C
Σtensile membrane	0.0	0.0	23.0	17.4	0.0	0.0	29.4	34.1
Σcompressive membrane	-306.2	-229.2	-103.1	-85.8	-216.1	-131.5	-61.9	-46.6
Supporting beam force	-190.5	-144.8	-72.3	-58.5	-213.0	-126.5	-41.1	-22.9
Measured restrained force	-113.7	-80.7	-15.2	-12.3	0.0	0.0	0.0	0.0
Measured column force	-6.4	-8.6	0.5	1.8	-6.4	1.3	3.6	4.1
Equilibrium force	-8.3	-12.3	7.8	4.2	-9.4	-3.7	12.2	14.5

For specimens with an aspect ratio of 1.5 as shown in **Table 6.10**, a similar trend was observed. Unlike Series II specimens, compressive membrane forces were larger in the short span than those in the long span. On the contrary, tensile membrane forces in the short span were smaller than those in the long span. Additionally, the larger restraint forces in the long span were equilibrated by the larger compressive membrane forces and supporting beam forces.

Since the overall behaviours of ECS1.5 and CCS1.5 were similar to ICS1.5, sectional forces were not shown in this section and the overall load-bearing capacities will be discussed in Sections 8.3 and 8.4.

Table 6.10 Equilibrium forces from FEM analysis compared with measured forces for ICS1.5 along both X and Y directions

Mesh temperature (MT)	Along A-O (kN)				Along B-O (kN)			
	38.7°C	118°C	212°C	350°C	38.7°C	118°C	212°C	350°C
ΣTensile membrane	0.0	40.5	84.5	27.8	0.0	10.6	20.8	12.4
Σcompressive membrane	-140.9	-111.4	-78.8	-31.5	-86.7	-180.3	-100.1	-54.7
Supporting beam force	-245.3	-123.0	-98.2	-58.7	-200.3	-100.4	-77.8	-61.2
Measured restrained force	-402.5	-200.5	-118.9	-80.4	-300.6	-250.5	-155.4	-124.0
Measured column force	-12.7	4.6	5.3	5.6	-9.8	-0.5	2.2	2.5
Equilibrium force	-3.7	-11.2	-31.7	-23.6	-3.8	20.1	-0.5	-23.0

6.12 Summary

In this chapter, the test set-up was discussed first. Lateral restrained A-frames were installed around the perimeter of the test specimens to simulate different restraint boundary conditions. The experimental lateral restraint stiffness values were quite close to the actual full-scale boundary stiffness values.

Temperature profiles across the thickness of composite slabs and supporting beams were measured using thermocouples. These temperature profiles were input into the numerical models. Additionally, the measured displacements were compared with numerical predictions, which showed good agreement between the experimental and numerical results. Hence, the robustness of the CDP model was verified.

Distribution of the membrane forces were plotted along the mid-span at elevated temperature. The membrane forces from the numerical analyses were discussed.

Compressive membrane forces were much smaller for Series I specimens when compared with boundary restraint Series II and III specimens due to restrained thermal expansion. In addition, the column movements for boundary series specimens were much smaller than those for Series I specimens.

For specimens with a larger aspect ratio of 1.5, tensile membrane forces along the long span were larger than those in the short span, which indicated that the distribution of compressive membrane forces were also larger in the long span in order to balance the tensile membrane forces. Additionally, the width of the compressive ring was estimated to be around one-sixth of the span and the compressive and tensile membrane forces could be approximated as linearly distributed along the mid-span in both directions.

In the next chapter, to explore the deflection profiles of composite slabs, an innovative instrumentation method, *viz.* photogrammetry, will be discussed and compared with direct measurements. The proposed deflection profile curves are estimated based on actual measured displacements.

Chapter 7 Photogrammetry

7.1 Introduction

Instrumentation at elevated temperature is always a challenge for researchers. Traditional method involving line transducers and ceramic rods connected to external LVDTs (linear variable differential transformer) have been used in the fire tests to minimise the damage of instrumentation equipment at high temperature. However, all of these methods can only obtain information at discrete points and in a fixed direction. In addition, this direction may be changed due to rotation of the measuring point. Conventional approach does not allow the measurements of displacements in three dimensions. Besides, researchers are also interested in the displacement profile along a cross section. Conventional method does not have the added flexibility for researchers to study other locations which may be more critical than the measured points. Displacement profiles are crucial in that it allows to study the collapse mechanism of the test specimen and to make comparisons with finite element modelling at the analysis stage.

Therefore, the photogrammetry is introduced to monitor displacement profiles of the composite slab at elevated temperature, which can not only capture the marked interested points, but also the overall profile of the deformation shape for the composite slab surface at elevated temperature.

7.2 Camera calibration

For this new approach, the calibration of camera is the most important step at the beginning of the test. In **Fig 7.1(a)**, a 10x10 dots pad with four controlling points was used to calibrate the camera setting configuration, such as the focal length, lens distortion and quality values as shown in **Fig 7.1(b)**. Information was processed by the

software PhotoModeler(Manual). The information of those 10x10 dots as well as camera locations was stored in the 3-D space for reference as shown in **Fig 7.1(c)**. In addition, the maximum residual values should be controlled to around unity to ensure good quality of images obtained through photogrammetry.

(a) Calibration based on 10x10 dots with 4 controlling points (b) Calibrated configuration for camera

(c) Calibrated 10x10 dots in the 3D space with camera locations

Fig 7.1 Calibration procedure to obtain camera information

7.3 Automatic photo capture system

A general procedure of applying photogrammetry for a structural test at ambient

temperature is shown in **Fig 7.2**. Since the actuator can be paused at any time at a certain displacement, photographs can be taken from different angles but not necessary at the same time. Therefore, after the information is processed by the software, the points at the final deformation could be compared with those in the original configuration. The relative displacements were drawn in yellow as indicated in **Fig 7.2**.

Fig 7.2 The measured displacements for a beam deformation test (Barazzetti and Scaioni 2011)

However, it is more difficult to apply the photogrammetry technique at elevated temperature. Generally, the applied load remains unchanged with increasing temperature. It is impossible to pause during the fire test and take photographs since the slab would continue to deform under elevated temperature. Hence, it is necessary to take photographs from different angles at the same time.

Fig 7.3 Illustration of auto-control system

To facilitate this, a special macro programme was designed to take three photographs from three different angles at the same time as shown in **Fig 7.3**. Each camera will take

a photograph at one-minute interval. In the next step, these three photographs from different angles can be used to determine the movements in a 3-D space. The principle for photogrammetry is to capture locations of interest by two other cameras mounted at similar heights but from different perspectives. The angle of view between two cameras at the interested points should be larger than 60 degrees based on the user-manual.(Manual).

7.4 Coordinates setup

Fig 7.4 shows two photographs that have been taken from two different angles at the same time. The origin was defined by three black dots on top of the furnace cover, which would remain fixed during the slab deformation. Based on this fixed origin, coordinates of all measuring points could be calculated. As temperature increased, the composite slabs deformed more. Three connected cameras were programmed to take photographs at the same time. Hence, the relative displacement could be calculated by simple subtraction from the previous coordinate information in three independent directions.

Fig 7.4 Applying photogrammetry to monitor slab surface deformation at elevated temperature

It was necessary to ensure that these painted points would not fade away with increasing temperature. Those marked points were painted with high-temperature paint up to 1300°C as shown in Fig 7.5.

Fig 7.5 High temperature paint

The interested points painted on the surface of composite slabs were automatically detected and marked by the software from three different angles as indicated in **Fig 7.6(a)**. In addition, every three photographs from different angles taken at the same time were used to build up a 3D model at a certain time. At the same time, the average mesh temperature was also recorded through thermocouples. Therefore, the deformation-versus-mesh temperature could be plotted out. In this manner, the final deformation model of the composite slab is shown in **Fig 7.6(b)**.

(a) The setup of the photogrammetry model at the beginning stage

(b) Final deformation of the photogrammetry model

Fig 7.6 Self-detecting system for photogrammetry by using the PhotoModeler

7.5 Validation with experimental results

In this section, only specimens ISOCS1 and ICS1 were discussed, which represented a typical specimen in Series I (Section 5.2) and a specimen in the boundary restraint Series II (Section 6.2), respectively. To verify the accuracy of this approach, the measured centre deformations and edge beam deformations by line transducers were compared with the photogrammetry method. **Fig 7.7** shows that there is good agreement between the direct measurements and photogrammetry at large displacement stage but the accuracy dips for small displacement stage at the beginning of the test.

Fig 7.7 Experimental displacements versus photogrammetry measurement

Generally, for small-deformation measurements, it is suggested to use a long-focus lens to obtain detailed information of the movement; for large-deformation stage, a short-focus lens with wider angle should be applied in order to obtain the overall view of the deformation. Therefore, in this research, three cameras with short-focus lens and wide angle have been installed to capture abroad-range of deformations. However, a short-focus camera cannot capture small deflections very well if the interested point is far away from the monitored points. This is the reason that at small deformation the error is up to 20% while at large deformation the discrepancy is only about 5%. Hence, this approach is robust and flexible depending on the purpose and the range of lens used.

Moreover, the restraint edge beam deflection for internal composite slab ICS1 is much

smaller than that of ISOCS1 due to boundary continuity. This will be discussed further in the next section for 3D-deformation profiles.

7.6 3D-deformation profiles

One of the most attractive features of photogrammetry is to obtain displacements of interested points in the defined X-, Y- and Z-directions. Due to obstructed views from the triangular loading plates on the top surface of composite slabs, only half of the centre line cross section and one perimeter edge beam deformation profiles can be obtained from photogrammetry for ISOCS1 as shown in **Fig 7.8**.

Fig 7.8 3-D profiles for half of the centre line and edge beam by photogrammetry for ISOCS1

Additionally, the X-, Y-, Z-coordinates of interested points on the edge beam indicated in **Fig 7.8** were extracted from photogrammetry and plotted out as shown in **Fig 7.9**. The cross marks represented the measurements from photogrammetry and the solid line was the deflection line for these cross marks applying a four order polynomial function to the cross marks.

Fig 7.9 The edge line measurement from photogrammetry compared with the simply supported analytical method for ISOCS1

The reason to focus on the edge deformation is that according to the composite beam-slab systems design guides *P288* and *P390*, it assumes there is no deformation for boundary edge beams at elevated temperature. However, from the actual large-scale fire test of an eight-storey composite steel framed building at Cardington in the 1990s, large deformations had been observed from supporting edge beams at elevated temperature. It is necessary to quantify the displacements of supporting edge beams and their effect on tensile membrane action of the composite slab at elevated temperature.

Moreover, traditional measuring technique is unable to capture horizontal movements of interested points within a plane. With photogrammetry, the points painted on the edge beam of the specimen ISOCS1 from the mid-span to the corner were monitored and compared at different mesh temperature of the slab in the global Y-axis. The global Y direction is indicated in **Fig 7.8**.

Fig 7.10 shows the lateral movements of the edge beam from the mid-span to the corner in global Y-axis of specimen ISOCS1. At the ambient temperature, the points along the edge beam remained straight at the beginning. The positive Y-direction indicates moving outwards and negative value represents moving inwards. It can be seen that at mesh temperature of 100°C, the points at the mid-span of edge beam moved

outwards and those near the corner of the specimen displaced inwards. The outward movement around the mid-span region was resulted from thermal expansion. With increased mesh temperature, the points at the corner of the specimen moved inwards due to pulling forces arising from tensile membrane action of the slab.

Fig 7.10 Lateral displacement along edge beam in global Y-axis for ISOCS1

Fig 7.11 3-D profiles for the centre line and edge beam by photogrammetry for ICS1. For specimen ICS1, the marked points and the processed images were shown in **Fig 7.11**. The points along the edge beam and half of the centre line were marked in PhotoModeler. The global coordinate system was changed for convenience of locating the black dots. The Z-axis represented vertical direction and the Y-axis the direction along the marked centre line.

Fig 7.12 The edge line measurement from photogrammetry compared with the fixed support analytical method for ICS1 at mesh temperature of 500°C

The final deformation profile of the edge beam for specimen ICS1 was plotted as shown in **Fig 7.12**. Through curve fitting represented by the solid line, the edge beam deformation for specimen ICS1 was much smaller than that of specimen ISOCS1, which was due to restraints from surrounding boundary conditions.

Based on fourth order polynomial approximation, it was found that the analytical solution for a fixed-fixed beam deflection under uniformly distributed load was close to representing the deflection shape for this boundary restrained edge beam at elevated temperature. A simplified approximation at the mid-span of the restraint edge beam deformation is shown in **Eq 7.1**.

$$W_{\text{restraint beam}} = \frac{qL^4}{384E_{\theta}I} \quad \text{Eq 7.1}$$

Moreover, the deflection profile for ICS1 was more like a double curvature shape,

which was reasonable for this fixed-fixed boundary assumption.

Fig 7.13 The edge line movements in Y-axis at different mesh temperature for specimen ICS1

A similar trend for this edge beam deformation in the global Y-axis is shown in **Fig 7.13**. The edge beam moved outwards due to thermal expansion at the beginning stage and displaced inwards due to pulling forces from the tensile membrane action when the mesh temperature exceeded 100°C.

Unlike isolated specimen ISOCS1, the lateral movement of the edge beam along the Y-axis displaced at the same time for ICS1; for ISOCS1, the points at the mid-span displaced inwards and those points at the corner moved outwards, which induced some torsion effects on the edge beam. The reason is due to the restraint effect from surrounding slab panels and restraint frames. Therefore, the lateral movements of edge beams were greatly affected by the boundary conditions.

From the direct measured column lateral movements as shown in **Fig 7.14**, line transducers were attached on top of the columns at 45 degrees with respect to the edges. It can be seen that columns C1 and C2 (indicated in **Fig 5.15**) moved outwards at the mesh temperature of 100°C and moved inwards after that. Hence, the measured lateral column movements were consistent with those points along the edge beam based on photogrammetry.

Fig 7.14 Measured column lateral movements (C1 and C2) for specimen ICS1

Since only one-half of the centre line was marked with black dots due to obstruction from loading plates, the vertical displacement profiles from the centre of the composite slab ICS1 to the edge beam were plotted at different mesh temperatures as shown in **Fig 7.15**. At the ambient temperature, it was assumed that there was no initial vertical deformation. With increasing mesh temperature, the corresponding points along the centre line were compared with the initial positions at ambient temperature. Therefore, the centre line profiles at different mesh temperatures could be obtained.

Fig 7.15 The vertical displacement of centre line at different mesh temperature for specimen ICS1

Fig 7.16 Comparison for the undeformed shape and the deformed profile for specimen ICS1

Finally, a comparison was made to capture the undeformed and deformed profiles as shown in **Fig 7.16**. The edge beam and the centre line deformation profiles were plotted with the same origin and the global axes. Hence, deformations of any point along these lines could be obtained in three dimensions.

7.7 Discussion and conclusions

The merits of using this photogrammetry method were to obtain the 3D deformation profiles of a cross section and to measure displacements in all X-, Y- and Z-direction. However, there were also limitations for applying this method if some points have been obstructed from the cameras. For example, rising smoke from heating obscured some points (in the circle of angle 1) as shown in **Fig 7.17** but these points could still be traced through camera 2 and camera 3 with lower accuracy.

With further increased deformation of a slab panel, the points on the edge beam from camera2 (in red circle) could not be captured due to obstruction by insulation material. Fortunately, these points could still be captured through camera 1 and camera 3. Hence, it is necessary install cameras from different angles and as many as possible to take account of large deformation effect.

Fig 7.17 Method of finding the obstructed points through related photos

Fig 7.18 Failure of marking points due to obstruction from smoke

If the points were only captured in camera 3 and could not be captured by the other two cameras as shown in **Fig 7.18**, then these three photographs could not build up the 3-D model for those points at that instant. Additionally, the accuracy of measurements of the marked points would be adversely affected by rising smoke. In conclusion, the photogrammetry method requires sufficient lighting condition for the three cameras to capture the marked points from three different angles at the same time. The sharper the images, the better quality the measurements would be. Through monitoring the edge beam deflection profiles, approximations for isolated and restrained edge beam

deflections at elevated temperature were proposed. Overall, the photogrammetry measurement system is very promising for fire engineering applications with high accuracy compared with direct measurement system using line transducers.

In Chapter 8, several analytical models will be discussed. Based on the results from the photogrammetry, the edge beam deflection shape can be approximated and will be adopted in the new analytical model proposed by the author. According to in-plane equilibrium, compressive ring around the perimeter and tensile zone in the central region can be simplified and the load-bearing capacity of composite slabs can be predicted at elevated temperature.

Chapter 8 Analytical models

8.1 Introduction

Chapters 5 and 6 described the experimental tests and numerical analyses for Series I, II and III composite slab specimens at elevated temperature. Photogrammetry technique was applied and compared with the direct measurement system introduced in Chapter 7, and good agreement was obtained between photogrammetry and direct measurements.

To find out the development of tensile membrane action in composite slabs, in this chapter, a three-stage approach for RC slabs under increasing applied pressure is presented in Section 8.2, which can capture the load-deformation relationships of RC slabs at elastic, transition and tensile membrane stages.

In Section 8.3, the published Bailey-BRE analytical approach for composite beam-slab systems at elevated temperature will be discussed and proposed revisions to the Bailey-BRE model to take account of deflections of perimeter beams on tensile membrane action are proposed in Section 8.4.

By taking account of in-plane membrane forces distributions from the FEM analyses in Sections 5.9 and 6.10, the author proposed a revised model incorporating boundary restraints. This model is verified with experimental test results in Section 8.5 and very good agreement with test results is obtained.

8.2 Lateral-unrestrained RC-slabs under UDL at ambient temperature

8.2.1 Assumptions

To find out the development of membrane forces, lateral unrestrained reinforced

concrete (RC) slabs under uniformly distributed load (UDL) were studied first. Three stages have been identified to describe the behaviour of RC slabs with increasing UDL as shown in **Fig 4.11**: elastic, transitional or tensile membrane stage.

The author made a few assumptions for this three-stage model:

At the elastic stage, flexural rigidity governs the initial behaviour for both laterally restrained and unrestrained slabs. Therefore, the gradients of load-versus-deflection relationships for both laterally restrained and unrestrained RC slabs are the same at the elastic stage.

After reaching the flexural capacity, the RC slabs go into transitional stage. At this stage, when the slab vertical deflections increase, the edges expand outwards. Therefore, for laterally restrained slabs, compressive membrane action (or arch effect) enhances the load-bearing capacity. For laterally-unrestrained slabs, a compressive ring is formed along the periphery of the slab surrounding the tensile membrane zone at the centre. With further increase in central deflections, tensile cracks start forming around the edges of slabs due to hogging moment. Subsequently, compressive membrane forces are reduced and load-bearing resistance begins to drop for laterally-restrained slabs. However, when the net cross sectional forces at the mid-span of the slab switch from compression to tension, tensile membrane action stage begins, which marks the end of transitional stage. Finally, at tensile membrane action stage, the applied load is assumed to be solely resisted by the steel mesh in the slabs. With the same reinforcement ratio, it is assumed that both laterally-restrained and unrestrained slabs have the same load-deflection gradients at the final stage.

8.2.2 Elastic stage

According to **Eq 8.1** derived from elastic analysis in Section 2.3.1, the load-deflection relationship of a square RC slab with a clear span l can be used for simply supported RC slabs at the elastic stage, where q is the applied pressure and D is the flexural rigidity of slabs as defined by $\frac{E_c h^3}{12(1-\nu^2)}$, where E_c is the elastic modulus of concrete,

h is the thickness of the slab and ν is the Poisson's ratio for concrete material. Based on equilibrium, it is assumed that the compressive force $f_c d(1-\xi_0)/2$ is balanced by the tensile rebar force F_t in **Fig 8.1**, where f_c is the compressive strength of concrete, d is the effective depth of the RC slab and $\xi_0 d/2$ is the distance from one-half of d to the edge of compression block in the slab as shown in **Fig. 8.1**.

$$w_{\max} = \frac{4q_0 a^4}{\pi^6 D} = 0.00416 \frac{q_0 a^4}{D} \quad \text{Eq 8.1}$$

Fig 8.1 Schematic view for RC slabs cross section in equilibrium

The flexural capacity M_0 is equal to the product of the compression force and the lever arm that starts from the centre of the compression block to the location of tensile force F_t . For a simply-supported slab, the pressure limit $p_{elastic}$ for elastic bending can be calculated from **Eq 8.2**.

$$M_0 = F_t \left(d - \frac{1-\xi_0}{4} d \right) = \frac{A_s f_y d}{4} (3 + \xi_0)(1 - \xi_0) = \frac{p_{elastic} l^2}{8} \quad \text{Eq 8.2}$$

After determining the yield line load p_{yield} by **Eq 2.31**, the normalised elastic limit is given as $p_{elastic}/p_{yield}$, which indicates the end of the elastic stage.

8.2.3 Tensile membrane action stage

At the tensile membrane action stage, it is assumed that concrete elements cannot take any tensile forces and only steel reinforcement is assumed to resist further tensile forces. An equivalent thickness of steel plate, which equals the area of reinforcement

per unit length, is used to replace the reinforcing mesh. Besides, three assumptions are made as shown in **Fig 8.2**:

- only the tensile forces are applied to the elements;
- vertical loads remain perpendicular to the deformed surface of the slab;
- there are no bending, torsion and shear forces in the slab elements at the tensile membrane action stage.

According to Park's research(2000), based on vertical equilibrium as shown by **Eq 8.3**, the vertical load is resisted by a "net-like" equivalent steel plate structure. Essentially, this method assumes membrane forces in two directions, viz. x and y directions. (lower case x - and y -directions are local axes, but upper case X - and Y -directions are global axes)

In **Fig 8.2**, T is the equivalent tensile force with respect to the yield strength of steel reinforcement; direction z is perpendicular to the x - y plane and pointing inwards into the plane of paper.

Fig 8.2 Equivalent reinforcing meshes by steel plate

$$-Tdy \frac{\partial z}{\partial x} + Tdy \frac{\partial(z + \frac{\partial z}{\partial x} dx)}{\partial x} - Tdx \frac{\partial z}{\partial y} + Tdx \frac{\partial(z + \frac{\partial z}{\partial y} dy)}{\partial y} + qdxdy = 0 \quad \text{Eq 8.3}$$

Upon simplifying **Eq 8.3**:

$$T\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) + q = 0 \quad \text{or} \quad \nabla^2 z = -\frac{q}{T} \quad \text{Eq 8.4}$$

For a simply-supported slab with boundary condition $w_s = 0$ when $x = \pm l_x$, $y = \pm l_y$, by solving **Eq 8.4**:

$$\delta = \frac{q}{2T} \left[\frac{l_y^2}{4} - y^2 - \frac{8l_y^2}{\pi^3} \sum_{m=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{m-1}{2}} \cosh \frac{m\pi x}{l_y} \cosh \frac{m\pi y}{l_y}}{m^3 \cosh \frac{m\pi l_x}{2l_y}} \right] \quad \text{Eq 8.5}$$

where q , δ and T are defined as uniformly distributed load, central deflection of slab and tension force, respectively.

The gradient of the load-deflection relationship for the tensile membrane action stage has been obtained from **Eq 8.5**. However, it is necessary to determine the starting point for this tensile membrane action stage. Therefore, the following section is going to illustrate how to use energy method (Sawczuk and Winnicki 1965) to determine the starting point of the tensile membrane action stage. The details of energy method are explained in Appendix A.

Suffice to say, when the slab deforms, it is necessary to find out the relationship between the axial extension and the bending curvature. From **Fig 8.3**, the centre line of the slab deforms from OB to OA assuming simply-supported condition.

Fig 8.3 Details of a half slab under large deformation

Let $\theta =$ rotation angle $\angle BOC = \angle CDA$;

δ = vertical deformation of the slab(BC);

c =length of the centre line ($\overline{OB}$ and $\overline{OA}$);

$h/2$ = half depth of slab;

$\xi h/2 = \overline{CD}$ represents the distance from the centre line of the slab to the compression zone;

A = axial extension ($\overline{CA}$).

$\therefore \overline{OA} = \overline{OB}$ and $\angle DBO = \angle DAO = 90^\circ$

$\therefore \overline{DB} = \overline{DA}$

For small θ , $\overline{DC} = \overline{DA} \therefore \overline{BD} = \overline{DC} = \xi h/2$

$$\therefore \theta = \delta/c \quad \xi h/2 = A/\theta = \delta/2A = \delta^2/2c \quad \text{Eq 8.6}$$

Therefore, from **Eq 8.6**, the net axial force is related to $\xi h/2$ as has been derived in Appendix A. Therefore, when ξ equals to ξ_0 , the net axial force is zero, and when ξ equals to 1, there is no compression zone in the slab and the net axial force is governed by tensile forces in reinforcement, as shown in **Eq 8.7**.

$$n = \frac{N}{N_0} = \begin{cases} 1 - \frac{1-\xi}{1-\xi_0}, & \xi_0 \leq \xi \leq 1 \\ 1 & \xi \geq 1 \end{cases} \quad \text{Eq 8.7}$$

When the net axial forces change from compression to tension, it is the starting point of the tensile membrane action stage, which indicates ξ equals to ξ_0 . At this moment, the central deflection $\delta = \xi_0 h$. Therefore, it can be concluded that the starting point of the tensile membrane action stage is when the deformation reaches one effective depth of the slab. Hence, the three-stage approach can be determined and will be validated through some published test results.

8.2.4 Validations by test results

The small scale tests (Bailey and Toh 2007) for reinforced concrete slabs at ambient temperature were used to validate the author's analytical model. These specimens were simply supported on four edges of the slabs. The details of specimen M14 are shown in **Table 8.1**. The three-stage approach will be explained step by step through specimen M14. Additionally, the starting point for each stage will be identified and plotted on the load-deflection curves. By connecting these three points together, the load-deflection curve can be generated.

Table 8.1 Specification for test specimen M14

Dimension	1100x1100x35.9 mm
Concrete strength	$f_{cu} = 40.2$ MPa
Reinforcement ratio	$\rho = \frac{1}{2} \left\{ \left(\frac{f_{u,1}A_{s,1}}{0.8f_{cu}d_1} \right)_{\text{long}} + \left(\frac{f_{u,2}A_{s,2}}{0.8f_{cu}d_2} \right)_{\text{short}} \right\} = 0.088$
Reinforcement Diameter	$\Phi = 2.42$ mm
Reinforcement strength	$f_y/f_u = 745/767$ MPa
Supporting condition	simply supported with no lateral restraint

Elastic stage

From **Eq8.1**, the load-deflection relationship at the elastic stage is a straight line and is indicated as a blue dotted line in **Fig 8.4**.

Based the slab geometry and reinforcing details from M14, the following parameters are calculated according to zero net axial force to determine the elastic limit point A:

$$\xi_0 = \frac{1-A_s f_y}{H f_c} = 1 - \frac{87.8 \times 10^{-6} \times 777 \times 10^6}{0.015 \times 38 \times 10^6} = 0.88$$

$$N_0 = A_s f_y = f_c d (1 - \xi_0) / 2 = 87.8 \times 10^{-6} \times 777 \times 10^6 = 68 \text{ kN}$$

$$M_0 = f_c d (1 - \xi_0) (d - \frac{1-\xi_0}{4} d) / 2 = 1.934 \text{ kNm/m}$$

$$p_0 = \frac{6M_0}{\alpha^2 \eta^2} = 38.4 \text{ kPa}$$

$$M_0 = \frac{f_c d (1 - \xi_0) \left(d - \frac{1 - \xi_0}{4} d \right)}{2} = \frac{p_{elastic} l^2}{8}$$

$$p_{elastic} = 12.78 \text{ kPa}$$

Fig 8.4 Normalised load-deflection relationship at elastic stage for M14

According to yield line analysis in Section 2.3.2, p_{yield} can be determined as follows:

$$A_{xx} = 87.81 \text{ mm}^2/\text{m} = 87.81 \times 10^{-6} \text{ m}^2/\text{m} \quad d_x = 25 \text{ mm} = 0.025 \text{ m}$$

$$\text{Yield line moment can be determined: } M_{Px} = A_{sx} \left(d_x - 0.59 A_{sx} \frac{f_y}{f_c} \right) = 1.56 \text{ kNm/m}$$

Then, the applied pressure to form the yield line pattern is calculated as:

$$p_{yield} = \frac{12}{l_y^2} [m_{ux} + m_{uy}] = 31 \text{ kN/m}^2$$

$$\text{Normalised load: } p_{elastic} / p_{yield} = 0.4$$

Therefore, the elastic limit is obtained and plotted as shown by point A, and it is assumed that at this stage there are no tensile cracks yet and no yield lines exist in the slab. According to the experimental results from M14, it shows that after reaching point A, softening occurs for the load-deflection curve due to the formation of cracks and yield lines after the elastic limit.

Tensile membrane stage

Substituting the slab dimensions and material properties from **Eq 8.5**, the load-deflection profile at the tensile membrane stage is plotted in **Fig 8.4**.

$$\delta = \frac{q}{2T} \left[\frac{1.1^2}{4} - \frac{8 \times 1.1^2}{\pi^3} \times 0.3978 \right]$$

Determining the starting point of tensile membrane action

Based on Sawczuk's model as included in Appendix A, the following coefficients are determined:

$$A=1$$

$$B=0$$

$$F=1.3 \times 10^{-3}$$

$$G=0.5158$$

$$\lambda=1.03$$

$$p=1.03p_0=33$$

$$\delta_0 = \xi_0 h$$

$$\delta=h= \text{slab effective depth}$$

Hence, the starting point of the tensile membrane stage is defined at point B **Fig 8.5**, which indicates that the stage starts after reaching the yield line load and the central deflection is around one effective depth.

Fig 8.5 Determining the starting point of the tensile membrane stage for M14

Finally, the full-range analytical normalised load-deflection relationship for M14 can be obtained by joining the origin, point A, point B and the line with the same gradient as shown in **Fig 8.6**. The lines joining the origin to point A, point A to B, and point B

to the end as described in **Eq 8.5** represent the elastic, transitional and tensile membrane stages, respectively. Clearly, this three-stage approach shows good agreement with the test result from M14.

Fig 8.6 Analytical normalised load-deflection relationship for M14

Fig 8.7 Analytical normalised load-deflection relationship for M3

Fig 8.8 Analytical normalised load-deflection relationship for M4

Fig 8.9 Analytical normalised load-deflection relationship for M8

In addition, **Fig 8.7**, **Fig 8.8** and **Fig 8.9** show the normalised load-deflection relationships for other specimens such as M3, M4 and M8, which have been analysed using the same procedure as M14. All the analytical models are consistent with the test results, so it can be concluded that this three-stage approach can predict the load-deformation relationships for laterally-unrestrained slabs under uniformly distributed load.

8.3 Lateral unrestrained composite slab under UDL at elevated temperature at the deflection limit (BRE-Bailey model)

8.3.1 Review on the BRE-Bailey method

Based on limitation of yield-line theory in predicting the collapse loads of reinforced concrete slabs, Hayes (1968) proposed a new analytical model incorporating membrane forces to predict the plastic behaviour of reinforced concrete slabs.

Fig 8.10 Mode of failure following membrane action in unrestrained concrete slab (Bailey 2003)

From small scale tests (Bailey and Toh 2007), a common failure mode of short span,

full-depth cracks for an unrestrained concrete slab has been observed and proposed as shown in **Fig 8.10**.

Fig 8.11 Typical failure of the reinforcement (a) compressive and tensile membrane action (b) in-plane forces distribution (c) details of element 1 (Bailey and Toh 2007)

Based on symmetry, only elements 1 and 2 are considered in **Fig 8.11**. The interface forces are described by parameters T_1 (tensile membrane forces between elements 1 and 3), T_2 (tensile membrane forces between elements 1 and 2), C (compressive

membrane forces between elements 1 and 2) and S (shear forces between elements 1 and 2), and φ is the angle between the yield line and the shorter span.

Two failure modes of laterally-unrestrained slabs had been observed from the tests, which included tensile cracks across the shorter span of a rectangular slab and crushing of concrete compressive zones at the slab corners. Based on equilibrium, it is assumed that the compressive membrane action forms along the perimeter of the slab and tensile membrane action occurs at the central region of the slab, as shown in **Fig 8.11(a)** and the parameters are defined as follows in **Fig 8.11(b)** and (c).

Considering the equilibrium about element 1 in **Fig 8.11(b)** both in vertical and horizontal directions:

$$S \sin \varphi = (C - T_2) \cos \varphi$$

$$-S \cos \varphi = (C - T_2) \sin \varphi - T_1 / 2$$

$$\text{Hence } (C - T_2) = \frac{T_1}{2} \sin \varphi \quad \text{Eq 8.6}$$

where T_0 is the yield force per unit length of the reinforcement in the long span; KT_0 is the yield force per unit length of the reinforcement in the short span; $T_1 = bKT_0(1 - 2n)L$ represents the tensile force of the reinforcement for a continuous crack in the long span; $T_2 = \frac{bKT_0}{2(1+k)} \sqrt{(nL)^2 + \frac{l^2}{4}}$ and $C = \frac{kbKT_0}{2(1+k)} \sqrt{(nL)^2 + \frac{l^2}{4}}$ are the resultant tensile membrane forces and compressive membrane forces along the yield line 1-2 in **Fig 8.11 (c)**.

From yield line analysis as shown in Section 2.3.2, parameter n that determines the yield line pattern can be calculated as:

$$n = \frac{1}{2Ka^2} (\sqrt{3Ka^2 + 1} - 1) \quad \text{Eq 8.7}$$

where a is the aspect ratio (L/l); K is the coefficient of reinforcement orthotropy in x and y directions.

Substituting T_1 , T_2 and C into **Eq 8.6**, parameter k representing the magnitude of the compressive forces as compared with tensile membrane forces can be determined shown by **Eq 8.8**:

$$\frac{kbKT_0}{2(1+k)}\sqrt{(nL)^2 + \frac{l^2}{4}} - \frac{bKT_0}{2(1+k)}\sqrt{(nL)^2 + \frac{l^2}{4}} = bKT_0(1-2n)L\frac{nL}{2\sqrt{(nL)^2 + \frac{l^2}{4}}}$$

$$\left(\frac{k-1}{1+k}\right)(4(nL)^2 + l^2) = 4(1-2n)nL^2$$

$$k = 1 + \frac{4n^2a^2(1-2n)}{1+4n^2a^2} \quad \text{Eq 8.8}$$

It is noted that parameter b from **Fig 8.12** controls the failure mode of the laterally-unrestrained slab. It can be seen that bKT_0 is the controlling factor for fracture of reinforcement, and $kbKT_0$ is the controlling factor for compressive failure at the corners of the slab. By taking moment about point E:

$$\frac{1.1KT_0l}{2} \times \frac{l}{4} + T_1 \times \frac{\left(\frac{l}{2} - nL\right)}{2} + T_2 \times \left(\sqrt{(nL)^2 + \frac{l^2}{4}} - \frac{L}{2} \frac{nL}{\sqrt{(nL)^2 + \frac{l^2}{4}}} - \frac{1}{3} \frac{\sqrt{(nL)^2 + \frac{l^2}{4}}}{(1+k)}\right) =$$

$$S \times \frac{L}{2} \frac{l/2}{\sqrt{(nL)^2 + \frac{l^2}{4}}} + C \times \left(\sqrt{(nL)^2 + \frac{l^2}{4}} - \frac{L}{2} \frac{nL}{\sqrt{(nL)^2 + \frac{l^2}{4}}} - \frac{1}{3} \frac{\sqrt{(nL)^2 + \frac{l^2}{4}}}{(1+k)}\right) \quad \text{Eq 8.9}$$

Fig 8.12 Calculation for parameter b by taking the moment about Point E

Substituting T_1 , T_2 , S and C into **Eq 8.9**, the following equation is obtained:

$$\begin{aligned} & \frac{1.1l^2}{8} + b(1-2n)L \times \frac{\left(\frac{L}{2}-nL\right)}{2} + \frac{b}{2(1+k)} \times \left(\left((nL)^2 + \frac{l^2}{4} \right) - \frac{nL^2}{2} - \frac{1}{3} \frac{\left((nL)^2 + \frac{l^2}{4} \right)}{(1+k)} \right) = \\ & \frac{lb}{4nL} (k-1) \times \frac{Ll}{4} + \frac{kb}{2(1+k)} \times \left(\left((nL)^2 + \frac{l^2}{4} \right) - \frac{nL^2}{2} - \frac{1}{3} \frac{\left((nL)^2 + \frac{l^2}{4} \right)}{(1+k)} \right) \end{aligned}$$

Therefore, with further simplification, the parameter b can be expressed by **Eq 8.10**:

$$b = \frac{1.1l^2}{8(A+B+C+D)} \quad \text{Eq 8.10}$$

where

$$A = \frac{1}{2} \frac{1}{1+k} \left[\frac{l^2}{8n} - \frac{(L/2-nL)}{nL} \left((nL)^2 + \frac{l^2}{4} \right) - \frac{1}{3} \frac{1}{1+k} \left((nL)^2 + \frac{l^2}{4} \right) \right]$$

$$B = \frac{1}{2} \frac{k^2}{1+k} \left[\frac{nL^2}{2} - \frac{1}{3} \left(\frac{k}{1+k} \right) \left((nL)^2 + \frac{l^2}{4} \right) \right]$$

$$C = \frac{1}{2} (k-1) \frac{l^2}{8n} = \frac{l^2}{16n} (k-1)$$

$$D = \left(\frac{L}{2} - nL \right) \left(\frac{L}{4} - \frac{nL}{2} \right)$$

To calculate the enhancement of load-carrying capacity due to membrane forces, it is assumed that the central deflection is δ , and taking moments for all in-plane forces about the supports as shown in **Fig 8.13**, M_{1m} can be obtained as follows:

$$M_{1m} = bKT_0L\delta \left((1-2n) + \frac{n(3k+2)}{3(1+k)^2} - \frac{nk^3}{3(1+k)^2} \right)$$

Fig 8.13 Schematic view for calculating membrane enhancement

The enhancement factor with respect to yield line load is given by:

$$e_{1m} = \frac{M_{1m}}{M_0 L} = \frac{4b}{3+g_1} \frac{\delta}{d_1} \left((1-2n) + \frac{n(3k+2)}{3(1+k)^2} - \frac{nk^3}{3(1+k)^2} \right) \quad \text{Eq 8.11}$$

Following a similar derivation, the enhancement factor for element 2 can be shown as:

$$e_{2m} = \frac{M_{2m}}{M_0 L} = \frac{4bK}{3+g_2} \frac{\delta}{d_2} \left(\frac{2+3k-k^3}{6(1+k)^2} \right) \quad \text{Eq 8.12}$$

Eqs 8.11 and **8.12** are the enhancement from tensile membrane effects. Apart from that, the enhancement may also come from bending effects due to axial forces. It was pointed out by Sawczuk (1965) that when axial forces appear in the slab, the bending capacity can be increased and calculated as follows:

$$\frac{M_i}{M_0} = 1 + \alpha \left(\frac{N}{T_0} \right) - \beta \left(\frac{N}{T_0} \right)^2$$

where $\alpha = \frac{2g_i}{3+g_i}$ and $\beta = \frac{1-g_i}{3+g_i}$ indicated in **Fig 8.14**.

Fig 8.14 Assumed stress for pure bending (Bailey 2001)

The enhancement of bending capacity due to membrane forces can be calculated as:

$$e_{1b} = \frac{M}{\mu M_0 L} = 2n \left[1 + \frac{\alpha_1 b}{2} (k-1) - \beta_1 (k^2 - k + 1) + (1-2n)(1 - \alpha_1 b - \beta_1 b^2) \right] \quad \text{Eq 8.13}$$

$$e_{2b} = \frac{M}{M_0 L} = 1 + \frac{\alpha_2 b K}{2} (k-1) - \frac{\beta_2 b^2 K}{3} (k^2 - k + 1) \quad \text{Eq 8.14}$$

Eq 8.11, **Eq 8.12**, **Eq 8.13** and **Eq 8.14** give the enhancements above the yield-line capacity of the slab due to membrane and bending effects. The net enhancement is calculated by combining these effects for elements 1 and 2, respectively:

$$e_1 = e_{1m} + e_{1b}$$

$$e_2 = e_{2m} + e_{2b}$$

Normally, e_1 and e_2 are not equal. Hayes (1968) suggests that the overall enhancement can be expressed as:

$$e = e_1 - \frac{e_1 - e_2}{1 + 2\mu a^2}$$

where a is the aspect ratio defined by $a = L/l$ and μ is the ratio of reinforcement orthotropy in x - and y - directions.

8.3.2 Deflection limit validation

To calculate the reinforced concrete slab enhancement, the deflection limit has to be defined first. There are two deflection limits mentioned in P288 and P390, which relate to thermal bowing and mechanical deflections. It should be noted that both deflection limits are calculated based on **Eq 8.15**. The distinction lies in the temperature difference term ($T_2 - T_1$). SCI P288 assumes that ($T_2 - T_1$) is equal to 770°C for fire exposure below 90 minutes and 900°C thereafter, while SCI P390 determines T_2 and T_1 from thermal analysis at the bottom and the top surfaces of the slab, respectively.

$$\delta = \frac{\alpha(T_2 - T_1)l^2}{19.2h} + \sqrt{\left(\frac{0.5f_y}{E_s}\right)_{20^\circ\text{C}} \frac{3L^2}{8}} < \delta_{\max} = \frac{\alpha(T_2 - T_1)l^2}{19.2h} + \frac{l}{30} \quad \text{Eq 8.15}$$

where α is the coefficient of thermal expansion (12×10^{-6} for normal weight concrete); h is the average depth of the concrete slab; l and L are respectively the short and long spans of the slab panel; f_y and E_s are respectively the yield strength and elastic modulus of the reinforcing steel at ambient temperature.

In addition, the first term $\frac{\alpha(T_2 - T_1)l^2}{19.2h}$ in **Eq 8.15** is based on thermal bowing effect.

From an analytical approach, when a temperature gradient appears at the bottom and the top surfaces of a slab (bottom temperature = T_2 and top surface temperature = T_1), the centre deflection due to thermal bowing effect can be calculated as follows:

$$\delta_{\text{thermal bowing}} = \frac{\alpha(T_2 - T_1)l^2}{8h} \quad \text{Eq 8.16}$$

Due to difficulty of defining thermal restraint to the heated slab, Bailey and Moore

(2000) introduced a parameter Ψ in **Eq 8.17**:

$$\delta_{tb} = \frac{\alpha(T_2 - T_1)l^2}{\Psi 8h} \quad \text{Eq 8.17}$$

This Ψ represents a partial factor of safety due to varying temperature gradients in compartment fire. They conducted a comparison study with the displacements obtained in the Cardington tests and a conservative factor of 2.4 was assumed.

The second term mechanical deflection $\sqrt{\left(\frac{0.5f_y}{E_s}\right)_{20^\circ\text{C}} \frac{3L^2}{8}}$ takes account of fracture of reinforcement at ambient temperature. Since the reinforcing mesh is located close to the top concrete slab surface, the mesh temperature is normally below 300°C , which is the starting point of strength reduction of reinforcing steel. Therefore, the fracture of reinforcement can be considered at ambient temperature only.

For example, through tests from ISOCS1 and ICS1, a comparison study is shown in **Fig 8.15**. ISOCS1 without boundary restraint can be considered as free expansion and ICS1 can be treated as a fully restrained slab. The deflection rate for ICS1 is greater than that of ISOCS1. It can be concluded that boundary restraints will increase slab deflections at elevated temperature due to high compression. This phenomenon can also be verified through a numerical analysis. In **Fig 8.15**, “ICS1 FEM” is the numerical model incorporating boundary restraint spring stiffness in **Table 4.5**, while “ICS1 FEM fixed” is the numerical model with fully rigid boundary condition. It can be seen that with fully rigid boundary condition, the deflection rate is much higher than that of ICS1 and ISOCS1.

Fig 8.15 Comparison of isolated and boundary restrained specimens

Therefore, Bailey's thermal deflection limit based on free expansion is much smaller than that of a pin-pin support (restrained) condition. The predicted load-bearing capacity of the slab is smaller which leads to a more conservative (smaller) enhancement factor for tensile membrane action.

In addition, since specimens were designed for one-hour fire rating, based on P390, the one-hour standard thermal analysis across the slab thickness was performed and the actual temperature differences were obtained and input into **Eq 8.15**. The deflection limits from P288 and P390 are summarised in **Table 8.2**. It can be seen that the deflection limits defined in SCI P390 are smaller than those in SCI P288, which indicates that the deflection limit for SCI P390 is more conservative.

Moreover, the monitored deflections in the tests are much larger compared with the calculated deflection limits (P288 & P390) as shown in **Table 8.2**, which again confirms that BRE-Bailey's deflection limit is conservative.

However, the amplified deflection limits are only applicable for restrained composite slab specimens. Even with amplified deflection limits, the composite beam-slab systems show superior fire resistant and conservatism.

Table 8.2 Comparison for deflection limits from P288 and P390

Test	Slab	Deflection	Deflection	P288	P390	Max.	P288	P390
	depth	due to	due to	deflection	deflection	deflection	deflection	deflection
		thermal	mechanical	limit	limit	in test*	limit / Test	limit / Test
	mm	curvature	strain	(Eq 5.14)			deflection	deflection
ISOCS1NI	61	33	37	70	67	178	0.393	0.376
ISOCS1	60	34	39	73	69	208	0.351	0.332
ISOCS1.5	62	16	37	53	52.7	189	0.280	0.279
ICS1	60	35	36	71	68	184	0.386	0.370
ECS1	61	35	35	70	68.3	190	0.368	0.359
CCS1	58	34	38	72	69.8	179	0.402	0.390
ICS1.5	59	17	37	54	52	153	0.353	0.340
ECS1.5	62	19	39	58	54.5	158	0.367	0.345
CCS1.5	61	18	39	57	54.1	160	0.356	0.338

8.3.3 Verification of bending resistance of unprotected interior beam

Furthermore, **Table 8.3** shows the results of unprotected interior beams (USB) in which the bending resistance of USB ($M_{Rd,\theta}$) is calculated based on steel sections and not as a composite beam. P288 and P390 calculate the load-bearing resistance for interior beam as a composite beam, which means the slab effect is considered with the steel beam as a reinforced concrete T-flange steel I-section. When calculating the enhanced load-bearing capacity of the slab, both the bending effect and the tensile membrane effect are included by the enhancement factor e . Therefore, the total capacity of the whole system counts the reinforced concrete T-flange twice. To be correct, when calculating the interior unprotected beams, they should only be treated as bare steel beams. Then the enhanced slab effect can be added to the interior beam capacity. After that, temperatures of the interior beams and the slab are taken at the time when the deflection limit (calculated by **Eq 8.15** in P390) is reached.

Table 8.3 Verification of the bending resistance of unprotected interior beam based on steel section (at the deflection limit in P390)

Series	Specimen	Time <i>min</i>	USB <i>°C</i>	Mesh <i>°C</i>	Slab top <i>°C</i>	Slab bottom <i>°C</i>	$M_{Ed, \theta}$ <i>kN/m²</i>	$M_{Rd, \theta}$ <i>kN/m²</i>	Check USB	$p_{b, \theta}$ <i>kN/m²</i>
<i>I</i> <i>Isolated</i>	ISOCS1NI	55	N.A.	330	84	672	N.A.	N.A.	N.A.	N.A.
	ISOCS1	60	807	295	97.6	680	6.25	2.96	<i>failed</i>	0
	ISOCS1.5	59	812	200	98.5	621	6.25	3.4	<i>failed</i>	0
<i>II</i> <i>Aspect ratio 1</i>	ICS1	75	802	250	98.7	654	6.25	3.0	<i>failed</i>	0
	ECS1	62	796	204	75.8	663	6.25	3.0	<i>failed</i>	0
	CCS1	57	798	191	91.2	689	6.25	3.16	<i>failed</i>	0
<i>III</i> <i>Aspect ratio 1.5</i>	ICS1.5	58	790	156	89.3	621	6.25	3.99	<i>failed</i>	0
	ECS1.5	63	800	147.6	99	710	6.25	3.6	<i>failed</i>	0
	CCS1.5	58	810	138.7	75.5	670.2	6.25	3.44	<i>failed</i>	0

Since the bending resistance $M_{Rd, \theta}$ of USB in all specimens is smaller than the design bending moment $M_{Ed, \theta}$, the unprotected interior beams have failed. Therefore, the increased slab load-bearing capacity due to flexural strength of USB, $p_{b, \theta}$, is equal to zero. In other words, the slab alone is to resist the entire applied load.

8.3.4 Verifications of load-bearing capacity of composite beam-slab systems at deflection limit

Previous calculated deflection limit in P390 (**Table 8.2**) are used to compute the enhancement factor e due to tensile membrane effect. The total capacity of the composite beam-slab system equals the yield line capacity of the slab $p_{y, \theta}$ multiplies the enhancement factor e as shown in **Table 8.4**. Compared with the applied load p_{test} , the *prediction/test load* ratios for all specimens are smaller than 1.0. Therefore, it can be concluded that the Bailey-BRE prediction is conservative if the deflection limit (**Eq 8.15**) is used.

Table 8.4 Calibration of the BRE-Bailey method for Series I, II and III at the deflection limit (P390)

Series	Specimen	p_{test} kN/m^2	$p_{y,\theta}$ kN/m^2	Slab deflection w_m mm	Enhancement factor e	Total capacity $ep_{y,\theta}$ kN/m^2	Prediction / Test
<i>I</i> <i>Isolated</i>	ISOCS1NI	20	10.68	67	1.56	16.67	0.83
	ISOCS1	20	10.69	69	1.58	16.9	0.85
	ISOCS1.5	30	19.24	52.7	1.35	26	0.87
<i>II</i> <i>Aspect</i> <i>ratio 1</i>	ICS1	20	10.64	68	1.57	16.72	0.84
	ECS1	20	10.67	68.3	1.57	16.74	0.84
	CCS1	20	10.6	69.8	1.58	16.8	0.84
<i>III</i> <i>Aspect</i> <i>ratio 1.5</i>	ICS1.5	30	19.25	52	1.34	25.9	0.86
	ECS1.5	30	19.21	54.5	1.36	26.3	0.88
	CCS1.5	30	19.25	54.1	1.36	26.4	0.88
$M =$							0.85

8.4 Laterally unrestrained composite beam-slabs under UDL at elevated temperature at the ultimate limit state (revised BRE-Bailey model)

In the Bailey-BRE method, the deflection limit is defined to calculate the enhancement factor due to tensile membrane action. Usually, this deflection limit in design (span/20) is much smaller compared with the measured deflection in experiment at the ultimate limit state (**Table 8.2**). Therefore, it is necessary to calibrate the load-bearing capacity at the ultimate limit state. Series I, II and III specimens were used and compared with the Bailey-BRE method at the failure state (by violating the load-bearing capacity criterion) as shown in **Table 8.5**.

Table 8.5 Calibration of the BRE-Bailey method for Series I, II and III at the failure point using the absolute slab central deflection

Series	Specimen	p_{test} kN/m^2	$p_{y,\theta}$ kN/m^2	Slab deflection w_m mm	Enhancement factor e	Total capacity $ep_{y,\theta}$ kN/m^2	Predicti on / Test
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<i>I</i> <i>Isolated</i>	ISOCS1NI	20	9.94	178	2.66	26.47	1.32
	ISOCS1	20	7.89	208	2.96	23.34	1.17
	ISOCS1.5	30	19.25	189	2.47	47.62	1.59
<i>II</i> <i>Aspect ratio 1</i>	ICS1	20	7.06	184	2.72	19.2	0.96
	ECS1	20	8.72	190	2.77	24.13	1.21
	CCS1	20	9.5	179	2.66	25.27	1.26
<i>III</i> <i>Aspect ratio 1.5</i>	ICS1.5	30	17.48	153	2.18	38.05	1.27
	ECS1.5	30	17.97	158	2.21	39.87	1.33
	CCS1.5	30	19.21	160	2.24	42.94	1.43

$$M = 1.28$$

It can be seen that the total predicted capacity is greater than the test load for all specimens except ICS1. This is because the Bailey-BRE method assumes that all the four edges are rigid without any deformation. However, deformations at four protected edge beams were observed during the tests. Therefore, the relative slab deflection should be used instead to calculate the enhancement factor e as discussed in Section 8.3. It can be concluded that the Bailey-BRE method may not be conservative if the load-bearing capacities are predicted based on absolute deflection measurements at the failure point.

Therefore, the author suggested using relative slab deflection w_r which is the difference between the absolute central deformation w_c and the average deformations of the four edge beams as shown in **Fig 8.16** and **Eq 8.18**.

Fig 8.16 Proposed revision to Bailey-BRE method by considering the relative deformation of boundary beams at ultimate limit state

$$\delta_r = \delta_c - \frac{1}{4}(\delta_{MB1} + \delta_{MB2} + \delta_{PSB1} + \delta_{PSB2}) \quad \text{Eq8.18}$$

Similarly, by applying this relative slab deflection δ_r , the enhancement factor and total slab capacity for all the tested specimens can be re-evaluated as shown in **Table 8.6**. Since relative central deformation is used instead of absolute deflection, the calculated enhancement factors are smaller than those in **Table 8.5**. It has been observed that the prediction/test ratios for all specimens are smaller than 1.0. Therefore, the Bailey-BRE method is conservative if relative slab central deflection is used at the ultimate limit state.

Table 8.6 Calibration of the BRE-Bailey method for Series III, IV and V at failure using the relative slab central deflection

Series	Specimen	p_{test}	p_{yield} line, θ	Slab deflection w_m	Average beam deflection	Relative slab deflection	Enhance- ment factor	Total capacity $e p_{y,\theta}$	Predictio n / Test
		kN/m^2	kN/m^2	mm	mm	mm	e	kN/m^2	
<i>I</i> <i>Isolated</i>	ISOCS1N I	20	9.94	178	83	95	1.84	18.28	0.91
	ISOCS1	20	7.89	208	80	128	2.16	17.09	0.85
	ISOCS1.5	30	19.25	189	130	59	1.4	27.03	0.90
<i>II</i> <i>Aspect ratio 1</i>	ICS1	20	7.06	184	24	140	2.29	16.12	0.81
	ECS1	20	8.72	190	58	122	2.10	18.29	0.91
	CCS1	20	9.5	179	81	98	1.86	17.68	0.88
<i>III</i> <i>Aspect ratio 1.5</i>	ICS1.5	30	17.48	153	68	85	1.61	28.03	0.93
	ECS1.5	30	17.97	158	83	75	1.54	27.6	0.92
	CCS1.5	30	19.21	160	91	69	1.47	28.2	0.94

$$M = 0.89$$

8.3.5 Comparison of enhancement factors

Additionally, when comparing the enhancement factors, specimen ICS1 has the highest enhancement factor due to the largest relative slab central deflection induced by lateral constraints.

For specimens with an aspect ratio of one, the comparison for the enhancement factor e is as follows:

$$ICS1(2.29) > ISOCS1(2.16) > ECS1(2.1) > CCS1(1.86) > ISOCS1NI(1.84)$$

It can be seen that the enhancement factor for edge composite slab (ECS1) is larger than that for corner composite slab (CCS1). However, the enhancement factor for isolated composite slab (ISOCS1) with one interior beam is greater than the edge composite slab (ECS1). This could be due to unevenly distributed membrane forces for ECS1 and CCS1 since there were discontinuous edges for these two specimens. The membrane forces for ISOCS1 were more uniform in both directions, which has greater effect on the development of membrane forces.

It can also be noticed that the enhancement factors for ISOCS1NI and CCS1 are similar. Since corner composite slab CCS1 had two discontinuous edges, its overall behaviour was similar to isolated composite slab ISOCS1NI.

For a larger aspect ratio of 1.5, a similar trend in the comparison of e can be expressed as follows:

$$ICS1.5(1.61) > ECS1.5(1.54) > CCS1.5(1.47) > ISOCS1.5(1.4)$$

In general, the enhancement factors for specimens with a larger aspect ratio were not as effective compared with aspect ratio of one due to unevenly distributed tensile membrane forces.

8.5 Laterally restrained composite beam-slabs under UDL at elevated temperature at the ultimate limit state

8.5.1 Limitations for incorporating boundary restraints in the Bailey-BRE model

Based on the BRE-bailey's model discussed in Section 8.3, a laterally restrained slab model is proposed and shown in **Fig 8.17**. The major differences for this proposed new

model include lateral restraint forces R_1 and R_2 for element 1 and element 2, respectively. These lateral restraint forces can be estimated through thermal expansion in **Eq 8.19** and **Eq 8.20**, where α_c is thermal expansion coefficient for concrete, E_{cT} is concrete elasticity modulus at elevated temperature, L and l are long and short span, respectively, t_{c1} and t_{c2} represent thicknesses of compression zone along long and short span, respectively.

Fig 8.17 A revised Bailey's model by considering the lateral restrained forces

$$R_1 = \alpha_c E_{cT} L t_{c1} \Delta T_{\text{compression zone}} \quad \text{Eq 8.19}$$

$$R_2 = \alpha_c E_{cT} l t_{c2} \Delta T_{\text{compression zone}} \quad \text{Eq 8.20}$$

The remaining parameters are kept the same as those mentioned in the Bailey's model in Section 8.3, as shown below:

$T_1 = bKT_0(1 - 2n)L$ represents the tensile force in the middle crack;

$T_2 = \frac{bKT_0}{2(1+k)} \sqrt{(nL)^2 + \frac{l^2}{4}}$ indicates the tensile force in the diagonal crack;

$C = \frac{kbKT_0}{2(1+k)} \sqrt{(nL)^2 + \frac{l^2}{4}}$ is the compressive force in the compressive ring;

$S = \frac{bKT_0 l}{4nL} (k - 1) \sqrt{(nL)^2 + \frac{l^2}{4}}$ symbolises the in-plane shear force between element 1 and element 2.

For element 1, equilibrium can be established as shown in **Eq 8.21** and the simplified equation is shown in **Eq 8.22**:

$$C - T_2 = \frac{T_1 + R_1}{2} \sin \varphi \quad \text{Eq 8.21}$$

$$\frac{kbKT_0}{2(1+k)}\sqrt{(nL)^2 + \frac{l^2}{4}} - \frac{bKT_0}{2(1+k)}\sqrt{(nL)^2 + \frac{l^2}{4}} = bKT_0(1-2n)L\frac{nL}{2\sqrt{(nL)^2 + \frac{l^2}{4}}} + \frac{R_1}{2}\frac{nL}{\sqrt{(nL)^2 + \frac{l^2}{4}}}$$

$$\frac{(k-1)bKT_0}{2(1+k)}\left((nL)^2 + \frac{l^2}{4}\right) = bKT_0(1-2n)nL^2 + R_1nL \quad \text{Eq 8.22}$$

Similarly for element 2, equilibrium is achieved (Eq 8.23) and simplified in Eq 8.24:

$$T_1\cos\varphi + S\sin\varphi = T_2\cos\varphi - R_2/2 \quad \text{Eq 8.23}$$

$$bKT_0(1-2n)L\frac{\frac{l}{2}}{\sqrt{(nL)^2 + \frac{l^2}{4}}} + \frac{bKT_0l}{4nL}(k-1)\sqrt{(nL)^2 + \frac{l^2}{4}}\frac{nL}{\sqrt{(nL)^2 + \frac{l^2}{4}}} =$$

$$\frac{bKT_0}{2(1+k)}\sqrt{(nL)^2 + \frac{l^2}{4}}\frac{\frac{l}{2}}{\sqrt{(nL)^2 + \frac{l^2}{4}}} - R_2/2$$

$$bKT_0(1-2n)L\frac{1}{\sqrt{(nL)^2 + \frac{l^2}{4}}} + \frac{bKT_0}{2nL}(k-1)nL = \frac{bKT_0}{2(1+k)} - R_2/l \quad \text{Eq 8.24}$$

After simplification, non-linear Eq 8.22 and Eq 8.24 have two unknowns k and b .

Therefore, solutions for k and b can only be obtained numerically.

In Section 8.3.2, the same specimen ICS1 is used to calculate relevant k and b numerically as shown in Table 8.7.

Table 8.7 Numerical solution for k and b for ICS1

δ_i	Mesh temperature	R_1	k	b	T_2	C
71mm	500°C	136kN	1.36	16	383.6kN	522.6kN

It can be seen that both the T_2 and C are increased significantly through the amplification factor b by 10 times compared with no boundary restrained forces ($k = 1$ and $b = 1.65$). Additionally, the calculated compression and tension forces are too large exceeding reasonable range. Hence, some other sources of forces should present to compensate restrained forces .

8.5.2 Simplified in-plane equilibrium model for isolated composite beam-slab panel

A new analytical model as shown in **Fig 8.18** is proposed based on in-plane membrane forces distributions from the FEM analyses as described in Sections 5.9 and 6.10. Different from Bailey's model, some assumptions have been made according to the FEM analyses from Sections 5.9 and 6.10:

- Tensile and compressive forces are linearly distributed along the mid-span;
- The point of the contraflexure is about span/3 from the centre;
- For continuous edges, the extension length can be estimated as span/8;
- Edge beam forces are considered in the in-plane equilibrium.

Fig 8.18 A revised in-plane equilibrium model

The restraint forces can be estimated by taking into account of thermal expansion. From a previous researcher (Nguyen 2014), the deflection of the edge beams $\delta_{\text{edge beam}}$ at elevated temperature can be calculated through **Eq 8.25**, where q is the equivalent uniform distributed load applied on the edge beam, L is the beam length, E_{θ} and I are the Young's modulus at elevated temperature and second moment area of

the steel beam, respectively. This edge beam deflection was verified by photogrammetry as discussed in Section 7.6.

$$\delta_{edge\ beam} = \frac{5qL^4}{384E_{\theta}I} \quad \text{Eq 8.25}$$

Once the deformation of the edge beam is determined, mechanical strain $\epsilon_{beam\ mechincal}$ can be calculated by assuming a parabolic deflection shape through **Eq 8.26**:

$$\epsilon_{beam\ mechincal} = \frac{8\delta_{beam}^2}{3L^2} \quad \text{Eq 8.26}$$

Another component of strain in the steel beam comes from thermal effect. The thermal strain $\epsilon_{beam\ thermal} = \alpha\Delta T$, where α is the thermal expansion factor and ΔT is the temperature increment in the steel beam. Therefore, total strain $\epsilon_{beam\ total}$ is expressed by **Eq 8.27**. The induced edge beam force EB can be computed by **Eq 8.28**.

$$\epsilon_{beam\ total} = \epsilon_{beam\ mechincal} + \epsilon_{beam\ thermal} \quad \text{Eq 8.27}$$

$$EB = \epsilon_{beam\ total}E_{\theta}A_{beam} \quad \text{Eq 8.28}$$

By equating equilibrium in both directions, the four unknowns T_1 , T_2 , C_1 and C_2 can be solved. By taking moment with respect to the corresponding edges, the total enhanced load-bearing capacity can be estimated through **Eq 8.29** and **Eq 8.30**.

$$M = T(\delta_{centre} - \delta_{edge\ beam}) \quad \text{Eq 8.29}$$

$$p_{fi} = 6 \frac{M}{n^2\alpha^2l^2} \quad \text{Eq 8.30}$$

Failure load predictions of Series I isolated composite beam-slab system specimens are summarised in **Table 8.8**. These two specimens ISOCS1NI and ISOCS1 are isolated composite slabs with aspect ratio of one. Due to the symmetry, the estimated tensile and compressive forces are identical along the long and short span. Additionally, since the interior beam is not fire protected at elevated temperature, the interior beam force is ignored due to high reduction of the steel beam strength at elevated temperature.

Table 8.8 Calculated in-plane forces for ISOCS1NI and ISOCS1

Specimen	$W_{edge\ beam}$	W_{centre}	EB_1	EB_2	C_1	T_1	C_2	T_2	p_{fi}
ISOCS1NI	49mm	178mm	-24kN	-24kN	-24kN	48kN	-24kN	48kN	16.9kN/m ²

ISOCS1	69mm	208mm	-14.8kN	-14.8kN	-14.8kN	29.5kN	-14.8kN	29.5kN	19.3kN/m ²
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However, for ISOCS1.5 with an aspect ratio of 1.5, the cross-sectional forces along the long and short spans are different and the edge beam deflections along the long and short spans vary. Hence, it is necessary to compute the load-bearing capacity in two directions separately as shown in **Table 8.9**. Besides, the average load-bearing capacity $\overline{p_{fi}}$ can be estimated through both directions p_{fi1} and p_{fi2} as $\overline{p_{fi}} = p_{fi1} - \frac{p_{fi1} - p_{fi2}}{1 + 2\alpha^2}$, which can be explained by the effects of vertical shear or in-plane shear (Hayes 1968). It can also be seen that more in-plane tensile forces are distributed in the long span compared to the short span. The membrane forces in the long span are much greater than those in the short span, which indicates that the membrane forces develops in the long span. As compared with the applied load of 30kN/m², this analytical model gives conservative predictions of the load-bearing capacity at the ultimate limit state.

Table 8.9 Calculated in-plane forces for ISOCS1.5

Specimen	Direction	Wedge beam	W _{centre}	EB	C	T	M	p_{fi}	$\overline{p_{fi}}$
ISOCS1.5	Long	120mm	189mm	-60.5kN	-60.5kN	121kN	8.45kNm	28.3kN/m ²	26.2kN/m ²
	Short	27mm	189mm	-10.6kN	-10.6kN	40kN	21.2kNm	25.7kN/m ²	

8.5.3 Simplified in-plane equilibrium model for boundary restrained composite beam-slab panel

Unlike the isolated composite slab series, for specimens with boundary restraints, it is necessary to introduce a parameter R expressed as $R = \alpha_c E_{cT} L h_c \Delta T_{\text{compression zone}}$ to represent the restraint forces at continuous boundary conditions, where α_c and E_{cT} are the expansion factor and elastic modulus for concrete at elevated temperature, L represents the edge length, h_c is the slab thickness in compression, $\Delta T_{\text{compression zone}}$ is the temperature of the compression zone which is close to the bottom surface of the slab.

Generally, the first step is to determine the temperature gradient at a specific period for a composite slab at elevated temperature. Based on different boundary conditions,

hogging moment around the slab edges can be obtained. After that, the thickness of the compression zone can be calculated from equilibrium and the boundary restraint forces can be obtained. The restraint beam deflection $\delta_{\text{restraint beam}}$ can be estimated by **Eq 8.31** from photogrammetry in Section 7.6.

$$\delta_{\text{restrained beam}} = \frac{qL^4}{384E_{\theta}I} \quad \text{Eq 8.31}$$

From **Table 8.10**, it can be seen that due to boundary restraints, the edge beam deflection is smaller than that in the isolated series. But due to thermal expansion effect, the compressive force in the edge beam is greater than that in the isolated series. In addition, the tensile membrane forces compared to the isolated series are much smaller.

Table 8.10 Calculated in-plane forces for ICS1

Specimen	$Temp_{\text{bottom}}$	$t_{\text{compression}}$	R	$w_{\text{edge beam}}$	w_{centre}	EB	C	T	p_{fi}
ICS1	750°C	2.8mm	-124kN	22mm	184mm	-112kN	-12kN	24kN	18.8kN/m ²

Moreover, for the specimen ECS1 with one discontinuous edge as shown in **Table 8.11** and **Fig 8.19**, a similar membrane force pattern is assumed. Since edge beam 1 ($EB1$) is discontinuous, the deflection for $EB1$ is larger than $EB2$. However, due to lateral restraints, the force in $EB2$ is larger than that in $EB1$. In addition, the tensile membrane force is almost identical to that in specimen ICS1.

Table 8.11 Calculated in-plane forces for ECS1

Specimen	w_{EB1}	w_{EB2}	w_{centre}	EB_1	EB_2	R	C_1	T	C_2	p_{fi}
ECS1	110mm	22mm	190mm	-80kN	-112kN	-180kN	-11.5kN	23kN	-23kN	18.4kN/m ²

Fig 8.19 The in-plane membrane forces distribution for ECS1

Since the corner composite slab panel can be treated either as an isolated or an interior panel, the calculation is similar to the previous procedure (**Table 8.8**), which will not be shown here.

For an aspect ratio other than unity, it is necessary to calculate the membrane forces in the long and short span separately. It can be seen from **Table 8.12** that the tensile membrane forces are larger in the long span than those in the short span, which indicates that majority of tensile membrane forces are distributed in the long span. Different from the isolated series, tensile membrane forces are reduced due to the presence of lateral restraint forces.

Table 8.12 Calculated in-plane forces for ICS1.5

Specimen	Direction	Wedge beam	W _{centre}	<i>EB</i>	<i>R</i>	<i>C</i>	<i>T</i>	p_{fi}	$\overline{p}_{fi}$
ICS1.5	Long	23mm	153mm	-141kN	-124kN	-29kN	58kN	25.5kN/m ²	27.2kN/m ²
	Short	5mm	153mm	-85kN	-98kN	-12kN	24kN	27.5kN/m ²	

8.6 Summary

The analytical approach for simply supported RC slabs without lateral restraints was proposed by adopting three stages, which included the elastic stage, the transitional stage and the tensile membrane stage. Additionally, the starting point of the tensile membrane action stage was around one effective depth of the RC slab. After determining the critical points for each stage, the overall load-versus-deflection curve could be obtained. Compared with published experimental test results (Bailey and Toh 2007), this three-stage approach showed good agreement with RC slabs by considering tensile membrane action.

Three series of test specimens were studied by taking into account of boundary conditions and different aspect ratios. All predicted load-bearing capacities of the nine specimens were lower than the applied loads at the deflection limit, which indicated that the Bailey-BRE method was conservative (**Table 8.2**).

Moreover, the load-bearing capacity at the ultimate limit state was also investigated.

By taking into account deflections of the edge beams, the relative central deflection was used to compute the enhancement factor and the predicted load-bearing capacities of the composite slabs were conservative when compared with applied loads (**Table 8.6**).

Additionally, the enhancement factors for different boundary conditions and different aspect ratios were compared separately. The specimens with boundary continuities had greater compressive membrane forces due to lateral restraints, which caused larger deformation at the slab centre. This resulted in higher enhancement factors for specimens with boundary restraint. For specimens with larger aspect ratios, the enhancement factors were much smaller than those with aspect ratio of unity due to unevenly distributed membrane forces.

From the numerical analyses from Sections 5.9 and 6.10, the development of membrane forces was studied and estimated. For isolated composite slab specimens, it can be seen that the compressive membrane forces around the edges together with the compressive edge beam forces were in equilibrium with the tensile membrane forces in the central region of the slab.

For isolated composite slabs, the compression in the edge beams plus the compressive membrane forces balanced the central tensile membrane forces with a smaller deflection. For boundary restrained composite slabs, high restraint forces compensated the compressive membrane effect with less tensile membrane forces but compensated by greater deflections. When calculating the overall load bearing capacity, these two systems have similar load-bearing capacities but with different in-plane forces distribution.

For specimens with different aspect ratios, the tensile membrane forces along the long span were larger than those in the short span. To balance the unequal forces, the corresponding edge beam forces and compressive membrane forces are changed to achieve in-plane equilibrium, respectively (**Table 8.12**).

Bailey's model is derived based on one piece of composite slab without considering

boundary edge beams.

As mentioned in Section 8.6, this is the beauty of the composite beam-slab systems. The induced restraint forces are balanced by compression in the edge beams as shown in **Fig 8.18**.

Since the tensile membrane forces are governed by reinforcement and the enhancement is roughly calculated by tensile membrane forces multiply by deflections. Therefore, the final tensile membrane action resistances seem to be very similar for Bailey's method and the author's approach.

Therefore, it has been proved that Bailey's model is still applicable for restraint boundary conditions, although Bailey's approach does not consider boundary restraint forces.

The author's derivation on the boundary restraint forces can be another simplified approach to calculate the membrane forces distribution within composite slabs.

Chapter 9 Conclusions and future work

9.1 Conclusions

This research is based on SCI P288/P390 “the new approach to multi-storey steel-framed building for fire safety design”. From the literature reviews on the most famous Cardington fire test, it had been found out the fire performance of the steel-framed composite slab systems were much better than those for isolated structural members. Furthermore, this enhancement for the composite beam-slab systems was identified as the development of tensile membrane action. As a result, this new design approach SCI P288/P390 was proposed and implemented in the UK as non-contradictory complementary information.

The design concept for SCI P288/P390 takes the advantage of the development of tensile membrane action by allowing some interior secondary steel supporting beams to remain unprotected without compromising the fire safety of the whole structure, which results in reduction in the fire protection cost.

Although SCI P288/P390 has been widely used in the UK, this design approach is still unfamiliar to most of engineers outside the UK by taking into account for the membrane effects. It is necessary to validate the conservativeness of this design approach.

Therefore, nine specimens were designed according to this design guideline to study the effect of different boundary conditions and different aspect ratios (1 and 1.5) on the development of membrane actions.

These nine specimens were divided into three series, which were explained in Chapter 5 as isolated composite slab Series I, boundary restraint composite slab Series II with an aspect ratio of 1 and series III with an aspect ratio of 1.5 in Chapter 6.

Compared with the SCI P288/P390, the predicted load-bearing capacities from nine specimens were smaller than the actual applied load at the defined deflection limit, which showed the conservativeness of applying this new approach to the steel-frame composite slab design.

Additionally, a new measuring technique Photogrammetry was applied to capture the deformation profile of composite slabs at elevated temperature. Compared with the direct measurement from the line transducers, the photogrammetry showed the reliability by taking photographs from different angles at the same time. Although this new non-contacting measuring technique is very promising for fire tests, blocking images from smoke or other objects during the deformation can also make the post-processing very difficult.

To investigate the development of the membrane forces at elevated temperature, the Concrete Damage Plasticity (CDP) model was introduced and implemented in the ABAQUS to take account for the damage of concrete material at elevated temperature. After validating the FEM results with previous simply supported RC slab tests, the predicted deformations at both ambient and elevated temperature were very close to the experimental test results mentioned in Chapter 3.

Therefore, by adopting this CDP model, the numerical models of nine specimens were established and verified with test results, which gave good predictions of the overall behaviour for composite beam-slab sub-assembly systems at elevated temperature. Moreover, the outputs for sectional membrane forces from the numerical analysis were investigated. It had been found out that the in-plane forces were in equilibrium through compressive membrane forces, compressive forces in the steel supporting beams, the lateral restrained forces and the tensile membrane forces.

Additionally, the compressive membrane forces and the compression in the steel beams increased at the beginning stage due to thermal expansion. After the central deformation reached about one effective depth, the tensile membrane forces occurred

in the central region of composite slabs. With further increasing temperature and deformation, the tensile membrane zone moved towards to the edge of the composite slab. The width of the compressive membrane ring was reduced to one-sixth of the span at the failure state. Compared with the isolated composite slab Series I, the compressive membrane forces and the steel beam forces in the boundary restraint Series II and III were much higher due to thermal expansion. In order to achieve the in-plane equilibrium, the lateral restraint forces increased accordingly to compensate the increments in the compressive membrane forces in composite slabs. However, both the compressive membrane forces and the tensile membrane forces were reduced due to material strength reduction at elevated temperature.

In Section 8.5.1, a revised Bailey's model is assumed by considering the boundary restrained forces (**Fig 8.18**). From solving non-linear equations numerically, larger values of k and b are obtained (**Table 8.7**), which implies that restrained boundary conditions increase central tension forces and surrounding compression forces. However, the calculated compression and tension forces are too large and exceed reasonable range. Hence, some other sources of forces (such as edge beam forces) should be present to compensate these restrained forces.

A simplified in-plane equilibrium model was developed to predict the ultimate load-bearing capacity of composite slabs at elevated temperature by incorporating edge beam forces. Compared with experimental test results, this new model showed good estimation and conservativeness by adopting linear membrane forces distribution along the mid-span of composite slabs.

Interestingly, compared the author's approach and the Bailey's method, the final membrane action resistances seem to be very similar. The main reason is induced thermal restraint forces are compensated by restraint forces of edge beams (**shown in Fig 8.18**). Although the tensile membrane forces are slightly reduced for boundary restrained cases, the overall load-bearing capacities for both approaches (author's and

Bailey's) are quite similar, since the resistances are roughly calculated by tensile membrane forces multiplying deflections.

In another way, Bailey's model can also be applied to boundary restrained composite slabs at elevated temperature. However, Bailey's model does not take boundary restraint forces into account. The author's approach explains the actual restrained behaviour of composite slabs at ultimate limit state.

9.2 Future work

Bailey's method is mainly based on results from Cardington tests. With certain in-plane forces assumptions, an equilibrium is established and the load-bearing capacity of composite slabs can be calculated. However, it is quite difficult to measure these in-plane membrane forces at elevated temperatures, which prevents other researchers to revise BRE-Bailey model. Currently, high temperature strain gages are possible solutions to measure in-plane mesh strains, but the cost of the high temperature strain gages is quite expensive and sometimes their qualities are not so reliable if debonding occurs.

Furthermore, the author would like to apply the photogrammetry technique in some other tests. Compared with traditional measuring method, the advantage of photogrammetry is that displacements in three-dimensions can be captured. Therefore, the photogrammetry can be applied for structure with three-dimensional displacements. One of the examples for three-dimensional displacement is shown in **Fig 9.1**. When the column moved down, connected beams experienced bending and torsion. It is difficult for traditional equipment to measure. However, with the help from the photogrammetry, not only the beam deformation profile, but also the joint rotation can be monitored.

(a) 3-D beam joint deformation

(b) Marked points in 3-D space

Fig 9.1 Application for photogrammetry in the 3-D beam joint test in NTU

(a) Beam-column joint cyclic test

(b) Marked points in 3-D space

Fig 9.2 Application for photogrammetry in the beam-column joint test in NTU

Furthermore, this technique was also applied to the beam-column joint under cyclic loading. The rotation angles on beams and columns were monitored and good agreement were achieved between measured values and results from photogrammetry.

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Appendix A Sawczuk's analytical model

Basic relationship

At large deflection of the slab, material and geometry non-linearity effects influence the behaviour beyond the plastic collapse load. It is assumed that the yield condition is the combination of axial forces and bending moment as described in **Eq A.1**.

$$F(M_{\alpha\beta}, N_{\alpha\beta}) = \text{constant} \quad \text{Eq A.1}$$

From conservation of energy principle, internal energy dissipation should be equal to external work done. The internal energy dissipation results from a kinematic change in the extension and curvature of the slab as expressed in **Eq A.2**. Then the overall equilibrium is established by **Eq A.3**.

$$d = M_{\alpha\beta} \dot{\kappa}_{\alpha\beta} + N_{\alpha\beta} \dot{\lambda}_{\alpha\beta} \quad \text{Eq A.2}$$

$$\text{Overall: } \int_A p \dot{W} dA \geq \int_A (M_{\alpha\beta} \dot{\kappa}_{\alpha\beta} + N_{\alpha\beta} \dot{\lambda}_{\alpha\beta}) dA \quad \text{Eq A.3}$$

where d is the internal energy dissipation; $M_{\alpha\beta}$ and $N_{\alpha\beta}$ are the bending moments and axial forces, respectively; $\dot{\kappa}_{\alpha\beta}$ and $\dot{\lambda}_{\alpha\beta}$ represent the bending curvature changing rate and axial strain rate; p is the external vertical load; $\dot{W}$ is the rate of deformation under the vertical load p .

It is assumed that there are localized n general hinges at the plastically deforming regions. Substituting all relationships into **Eq A.3**, a general load-deflection relationship that incorporates extension and curvature can be obtained as shown in **Eq A.4**, which shows that the external work done should be equal to, or larger than the internal energy dissipation. The reason is that there may be energy loss resulted from heat generation due to unquantifiable friction effect. In the next section, the M - N

interaction relationship is to be determined to obtain the load-versus-deflection relationship.

$$\int_A p \dot{W} dA \geq \sum_1^n (N \dot{\Lambda}_i + M \dot{\theta}_i) l_i \quad \text{Eq A.4}$$

where l_i is the length of the plastic hinge line.

Axial force and bending moment interaction curves

In order to obtain the load-deflection relationship as derived in **Eq A.4**, it is necessary to obtain the relationship between axial force N and bending moment M relationship. Firstly, from equilibrium condition, **Eq A.5** is used to describe the equilibrium state that the compression zone is balanced by tensile forces (reinforcement). N_0 denotes zero net axial force under force equilibrium, and ξ_0 denotes the distance from centre line to the compression zone at the zero axial force state.

$$N_0 = A_s \sigma_y = R_c H (1 - \xi_0) \quad \text{Eq A.5}$$

Fig.A.1 Slab under large deformation (a) Details of deformed slab cross section (b) Slab subjected to compression and tension resulted from large deformation (Sawczuk and Winnicki 1965)

where R_c = compressive strength of concrete

A_s = reinforcement area per unit width

σ_y = yield strength of the reinforcement

ξ_0 = distance from centre line to the compression zone at the zero net axial force

From **Fig.A.1**, the net axial force is related to ξH . So when ξ equals to ξ_0 , the net axial force is zero, and when ξ equals to 1, there is no compression zone in the slab, so the net axial force is governed by tensile forces or reinforcement, as shown in **Eq A.6**.

$$n = \frac{N}{N_0} = \begin{cases} 1 - \frac{1-\xi}{1-\xi_0}, & \xi_0 \leq \xi \leq 1 \\ 1 & \xi \geq 1 \end{cases} \quad \text{Eq A.6}$$

For the zero net axial force and the pure reinforcement governing axial force, the bending moment is calculated for these two limiting cases (**Eq A.6**), respectively, as shown in **Eq A.7**.

$$m = \frac{M}{M_0} = \begin{cases} \frac{1}{3+\xi_0} \left(2 + \frac{1-\xi^2}{1-\xi_0} \right), & \xi_0 \leq \xi \leq 1 \\ \frac{2}{3+\xi_0} & \xi \geq 1 \end{cases} \quad \text{Eq A.7}$$

where $M_0 = R_c H (1 - \xi_0) (2H - \frac{1-\xi_0}{2} H)$

$$2M_0 = R_c H^2 (1 - \xi_0) (3 + \xi_0)$$

When $N = 0$, $M = M_0$, it represents the flexural capacity. In addition, ξ_0 depends on the percentage of reinforcement and the properties of concrete materials that can be calculated by **Eq A.8**.

$$\xi_0 = 1 - A_s \sigma_y / H R_c \quad \text{Eq A.8}$$

The minimum deflection for tensile membrane action to develop is when net axial force n is greater than zero, which is $\xi > \xi_0$ indicated in **Eq A.9**:

$$W_0 = 2H\xi_0 \quad \text{Eq A.9}$$

Eliminating ξ from Equation **Eq A.8** and **Eq A.9**, the M - N interaction curve can be obtained from **Eq A.10**. Then, the internal energy dissipation can be simplified by substituting **Eq A.6** and **Eq A.7** into **Eq A.11**. Furthermore, for the two limiting cases, viz. zero net axial force and zero compression zone, **Eq A.12** can be derived according to **Eq A.6** and **Eq A.7**.

$$m + 2n \frac{\xi_0}{3 + \xi_0} + n^2 \frac{1 - \xi_0}{3 + \xi_0} - 1 = 0 \quad \text{Eq A.10}$$

$$d_i = (NW + M) \dot{\theta}_i = M_0 \left(m + \frac{2n}{3 + \xi_0} \frac{W}{H} \right) \dot{\theta}_i \quad \text{Eq A.11}$$

$$d_i = \begin{cases} \left(1 + \frac{4\delta - 1 - \xi_0}{(3 + \xi_0)(1 - \xi_0)} (\delta - \xi_0) \right) M_0 \dot{\theta}_i & \xi_0 \leq \xi \leq 1 \\ \frac{2}{3 + \xi_0} (1 + 2\delta) M_0 \dot{\theta}_i & \xi \geq 1 \end{cases} \quad \text{Eq A.12}$$

where $\delta = W/2H$ denotes a dimensionless deflection.

After obtaining the M - N relationship, the internal energy dissipation can be defined by two parameters in **Eq A.12**, which are δ and $\dot{\theta}_i$ that stand for normalise deflection and the rate of rotation of the deformed slab, respectively. Furthermore, based on these deformed configurations, the possible collapse modes are proposed, and the internal energy dissipation **Eq A.12** can be simplified further.

Collapse modes

For a given aspect ratio $\alpha = L/l$ (long span versus short span), the corresponding yield line pattern can be determined as shown in **Fig.A.2**. For a simply-support slab, the yield line pattern is determined from **Eq A.13**.

$$\eta = \frac{1}{2\alpha^2} \left[\sqrt{1 + 3\alpha^2} - 1 \right] \quad \text{Eq A.13}$$

Fig.A.2 Typical yield line pattern for RC slabs (Sawczuk and Winnicki 1965)

At the central deflection $W = 2H\xi_0$, tensile membrane forces starts to dissipate energy. Three zones have to be distinguished according to deflection W :

- a) For $0 \leq W \leq 2H\xi_0$, only bending moments do work as defined by ψ_0 and β_0 :

$$D_0 = M_0 \frac{L \dot{\theta}_{AE}}{2} + M_0 \frac{l \dot{\theta}_{AB}}{2} = M_0 \dot{W}_0 \left(\alpha + \frac{1}{2\eta\alpha} \right) \quad \text{Eq A.14}$$

- b) For $2H\xi_0 \leq W \leq 2H$, bending moments and tensile membrane forces co-exist:

$$D = M_0 (\psi_0 L \dot{\theta}_{AE} + \beta_0 l \dot{\theta}_{AB}) + M_0 \left(\dot{\theta}_{AE} + \frac{l}{2\eta L} \dot{\theta}_{AB} \right) \times \int_{\psi_0 a}^{\eta a} \left[1 + \frac{4\delta - \xi_0 - 1}{(3 + \xi_0)(1 - \xi_0)} (\delta - \xi_0) \right] dx \\ + M_0 \dot{\theta}_{AE} \left(\frac{\alpha}{2} - \eta\alpha \right) \left[1 + \frac{4\delta - \xi_0 - 1}{(3 + \xi_0)(1 - \xi_0)} (\delta - \xi_0) \right]$$

After simplification:

$$D = M_0 W_0 \left\{ \frac{1 + 4\eta^2 a^2}{2\eta\alpha} \left[\frac{(3 - \xi_0) - \frac{\xi_0^2}{\delta_0} (3 - \xi_0) \frac{1}{6} - \delta_0 (5\xi_0 + 1) \frac{1}{2} + \frac{4}{3} \delta_0^2}{(3 + \xi_0)(1 - \xi_0)} \right] + \right. \\ \left. (\alpha - 2\eta\alpha) \left[\frac{(3 - \xi_0) - \delta_0 (5\xi_0 + 1) + 4\delta_0^2}{(3 + \xi_0)(1 - \xi_0)} \right] \right\} \quad \text{Eq A.15}$$

For $W > 2H$, only tensile membrane forces do work, and this is the reason that when the central deflection exceeds one effective depth of the slab, the tensile membrane action can be fully mobilised. (H is the half effective depth according to Sawczuk's notation)

Thus, in **Eq A.14** and **Eq A.15**, ξ_0 and δ_0 are used to determine the internal energy dissipation, which are representing the distance from the centre line to the compression zone and the normalized deflection when the net axial forces are equal to zero. M_0 and W_0 can be obtained through **Eq A.7** and **Eq A.9**, respectively. Finally, the internal energy term D can be determined. Then, it is possible to equate the internal energy to the external work done.

Load-deflection relationship

Beyond the bending collapse, it is assumed that the applied load and internal energy dissipation are proportional to each other as shown by **Eq A.16**.

$$\lambda = \frac{p}{p_0} = \frac{D}{D_0} \quad \text{Eq A.16}$$

where p_0 is the bending collapse load given by **Eq A.17**, which can be derived based on **Eq A.6** when the net axial force equals to zero.

$$p_0 = \frac{6M_0}{a^2 \eta^2} \quad \text{Eq A.17}$$

Then, **Eqs A.13, 14, 15** and **17** are substituted into **Eq A.16**. For three different stages, λ can be calculated and simplified to **Eqs A.18, 19** and **20**. All the coefficients can be obtained from 错误!未找到引用源。 and 错误!未找到引用源。.

$$\lambda = 1 \quad \delta \leq \xi_0 \quad \text{Eq A.18}$$

$$\lambda = C - D(1+B)\delta_0 + E(1+2B)\delta_0^2 - CA \frac{\xi_0^2}{6\delta_0} \quad \xi_0 \leq \delta_0 \leq 1 \quad \text{Eq A.19}$$

$$\lambda = G + G(1+B)\delta_0 - AGF \frac{1}{\delta_0} \quad \delta_0 \geq 1 \quad \text{Eq A.20}$$

The intersection point before the mobilisation of pure tensile membrane action stage and the co-existence of bending moment and the tensile membrane action stage are determined from **Eq.C.15** and **Eq.C.16**. So $\delta_0 = 1$ is the starting point for the tensile membrane action when the net axial forces changes from compression to tension.

Table A.1 Coefficients of α for calculating λ (Sawczuk and Winnicki 1965)

α	1.0	1.2	1.4	1.6	1.8	2.0
$A = \frac{1+4\eta^2\alpha^2}{1+2\eta\alpha^2}$	1.000	0.947	0.897	0.843	0.794	0.750
$B = \frac{2\eta\alpha^2 - 4\eta^2\alpha^2}{1+2\eta\alpha^2}$	0.000	0.054	0.104	0.158	0.206	0.250

Table A.2 Coefficients of ξ_0 for calculating λ (Sawczuk and Winnicki 1965)

ξ_0	0.0	0.6	0.7	0.8	0.9	1.0
$C = \frac{3-\xi_0}{(3+\xi_0)(1-\xi_0)}$	1.000	1.667	2.070	2.875	5.380	∞
$D = \frac{1+5\xi_0}{2(3+\xi_0)(1-\xi_0)}$	0.167	1.398	2.030	3.290	7.050	∞
$E = \frac{4}{3(3+\xi_0)(1-\xi_0)}$	0.444	0.926	1.200	1.750	3.420	∞
$F = \frac{1-2\xi_0+\xi_0^2}{12}$	0.0833	0.0133	0.0075	0.0033	0.0008	0

$G = \frac{2}{3 + \xi_0}$	0.667	0.555	0.540	0.527	0.513	0.500
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Different values of λ at different stages can be calculated based on **Eq A.13**, **Eq A.15** and **Eq A.16**. The corresponding deflections according to applied loads can also be obtained. It is noted that λ is related to the aspect ratio α , the distance ξ_0 from the centre line of the slab to the compression zone and the normalised deflection δ_0 . So after determining the λ value, the corresponding deflection can be calculated.

Appendix B Experimental drawing in details

Technical Specification

This contract includes fabrication, erection, delivery, setting-up, dismantling and disposal of three composite slabs. Before commencing work, the Contractor needs to check carefully all the dimensions of specimens in the drawing to make sure that they are conformed to testing rig in Construction Annex Lab. of NTU. If necessary, correction of the dimensions and minor adjustment to the designed size can be considered. Nevertheless, any modification to the specimens must obtain approval from NTU beforehand.

Part 1: Work scope, Specification and Conditions

1. Work scope and drawing

1.1. The overall work includes

1.1.1. Material purchasing

1.1.2. Erecting steel beam-column frame and formwork with steel mesh (reinforcement)

1.1.3. Concrete casting

1.1.4. Curing

1.1.5. Providing reinforcing bars, steel coupons and concrete cylinders for testing, includes delivery to Construction Annex Lab. of NTU

1.1.6. Delivery of specimens to Construction Annex Lab. of NTU

1.1.7. Setting up of specimens for tests one at a time at the Construction Annex Lab. of NTU

1.1.8. Dismantling and disposal of tested specimens

1.2. The drawings provided for this part of work are listed below:

Elevation, plan view and finishing detail for the nine composite slabs (total 09 drawings)

<i>No</i>	<i>Drawing No</i>	<i>Contents</i>	<i>Notes</i>
1	D1-01	Details of composite slab specimen ISOCS1NI	For fabrication
2	D1-02	Details of composite slab specimen ISOCS1	For fabrication
3	D1-03	Details of composite slab specimen ISOCS1.5	For fabrication
4	D2-01	Details of composite slab specimen ICS1	For fabrication
5	D2-02	Details of composite slab specimen ECS1	For fabrication
6	D2-03	Details of composite slab specimen CCS1	For fabrication
7	D2-04	Details of composite slab specimen ICS1.5	For fabrication
8	D2-05	Details of composite slab specimen ECS1.5	For fabrication
9	D2-06	Details of composite slab specimen CCS1.5	For fabrication

2. Test specimens properties***2.1. Dimensions***

The 1st series of three specimens are all isolated composite slabs with aspect ratio 1 and 1.5. The spacing for column to column is about 2.25 m or 1.5 m and the column height is around 1.25 m. The thickness of the concrete slab is 60 mm thick with 1mm steel plate attached and cast together on the bottom of the concrete slab. The 2nd and 3rd series of six specimens are boundary series specimen with the aspect ratio of 1 and 1.5, respectively.

2.2. Materials

- a) Concrete: C30/37 $f_{ck} = 30$ MPa and $f_{cu} = 37$ MPa.
- b) Aggregate: Siliceous sand and gravel are used. The maximum size of aggregate is 20mm.
- c) **Structural Steel:** UC **152x152x23** for column , **Joist 102x102x23** for perimeter beams and **Joist 76x76x15** for internal beam. All steel Grade is **S355**.
- d) **Reinforcing rebar:** 6 mm diameter and 120 mm spacing at the middle layer of concrete slab with yield strength $f_y = 500$ MPa.
- e) **Shear stud:** 13 diameter x40 mm height cut from normal shear stud with grade **S275**.
- f) **Steel sheeting:** 1mm thick with yield strength $f_y = 500$ MPa.
- g) Intumescent painting is used as fire protection material on surface of columns (UC 152x152x23) and perimeter beams (Joist 102x102x23) but not on internal beams (Joist 76x76x15). The contractor shall decide the thickness of intumescent paint in order to keep protected members at fire under 550°C for 60 min.

2.3. Concrete cylinders

Concrete cylinders should be tested at 28-day strength and test day. It is important that the difference of achieve strengths does not exceed by more 10 MPa in both casts.

2.4. Welding

2.4.1. 6mm fillet weld shall be used for the welding indicated in the drawing.

2.4.2. Welding shall comply with BS 8110: Part 1 and BS 5950: Part 1.

2.5. Formwork

2.5.1. Formwork and its supports shall maintain their correct position and ensure the correct dimensions and profile of the columns as required in the drawings. Tolerance on dimensions shall not be greater than ± 1 mm. NTU reserves the right to reject any formwork that does not comply with the dimensions.

2.5.2. Formwork shall be sufficiently rigid and tight to prevent loss of grout from the fresh concrete at all stage of placing and compacting of concrete.

2.5.3. Immediately before concrete is placed, all rubbish, dirt or loose matter shall be removed from the interior of the forms. The surfaces of the forms in contact with concrete shall be cleaned and treated with form oil to facilitate easy demoulding. No reinforcement and structural steel sections shall be coated with oil and these shall not any accumulation of oil at the bottom of the formwork.

2.5.4. Formwork shall be removed from the columns until 7 days after casting.

2.5.5. Marine ply shall be 19mm thick with smooth surface.

2.5.6. Formwork shall be prepared on leveled wooden platform laid on hard ground

2.5.7. Metal rods to tighten the wooden formwork for columns shall be at a spacing of 250mm.

2.5.8. Steel moulds for standard (150mm×300mm) and small (100mm×200mm) concrete cylinder shall be well-prepared with sufficient amount. In case the moulds are provided by NTU, they must be returned in good condition.

2.6. Concrete curing

2.6.1. Exposed surfaces of concrete shall be kept moist by frequent watering and covered for seven (7) days after concreting, using a material such as polythene sheet or with an absorbing material.

2.6.2. Concrete cylinders shall be cured under the identical conditions.

2.7. Setting up of specimens

For erection of the specimens, a gantry crane in Construction Annex Lab of NTU is available for use, but other facilities and tools shall be provided by the Contractor. When setting up the specimens, there is a requirement for the contractor to send an

adequate number of his workers to NTU to erect the test rig and specimens properly.

2.8. Dismantling and disposal

The Contractor is required to dispose tested RC columns out of NTU.

3. Other conditions

3.1. The Contractor shall inform NTU of the supplier of the ready-mixed concrete and steel,. Then the contractor obtains approval from NTU before commencing work.

3.2. Before placing concrete, the steel rebar layer and steel frame must be checked by NTU.

3.3. Tolerance of dimension shall not be greater than $\pm 2\text{mm}$.

3.4. No striking and demoulding of vertical formwork up to at least 24 hours is allowed.

3.5. The Contractor has the responsibility to make sure that the composite slab will eventually be able to placed conforming to the testing frame in Construction Annex Lab of NTU.

3.6. The Contractor has the duty to ensure all specimens are well kept in good condition before erecting to test rig.

3.7. The exposed portion of reinforcement bars shall be protected from all kinds of potential corrosion action before erecting to the testing rig.

3.8. The specimens shall be fabricated, delivered and erected on time in accordance with testing schedule.

3.9. To handle the specimens, an elevated crane in Construction Annex Lab. of NTU is available and can be used with free of charge. Contractor shall provide other facilities and tools needed.

3.10. All the work must be carried out strictly according to these technical specification and drawings.

3.11. The Contractor shall be willing to help in any other reasonable tasks that are not stated here.

3.12. NTU reserves the right to eject the specimens that are not prepared in accordance with specifications and drawings. Damage to installed thermal-couples caused by the Contractor will also cause the specimens to be rejected. The Contractor has responsibility to repair or even re-fabricate the specimen, which is not up to the standard, without any extra payment.

4. Additional notes

4.1 No high tensile bar to be used as lifting hook.

4.2 Delivery of specimen to Construction Annex

Before delivery of any member/specimen to NTU, Contractor must seek approval from NTU at least 3 days in advance by submitting permit-to-work form.

4.3 Risk Assessment

4.3.1 The Awarded Vendor is required to conduct Risk Assessment for the test set-up. No installation works will be allowed until the Risk Assessment has been submitted to Construction Lab.

4.3.2 Risk Assessments should closely follow the guidelines given by MOM. For more information, kindly refer to MOM website (www.MOM.gov.sg > Workplace Safety and Health > Maintaining a Safe and Healthy Workplace > Safety and Health Management System > Risk Management).

4.3.3 Only licensed lifting supervisor to operate crane for the test set-up. However, in the laboratory, the lab staff can help to monitor the crane.

4.3.4 If the contractor like to use own lifting supervisor to operate the crane, please bring a copy of the license of the lifting supervisor certificate to the lab.

4.4 Appendix I- Pre-Qualification and Evaluation of Contractor

The contractor has to complete Appendix I and submit together with the quotation in GeBIZ.

Part 2: Expected working schedule

The exact dates are subjected to change by NTU. NTU anticipates to commencing of the fabrication in May, 2012

By week 1 and 2: Steel beam-column frame are completed and erected.

By week 3: The formwork and mesh reinforcement shall be finished and approved so that concrete casting shall be conducted.

By week 4: Installation of thermal thermocouples by NTU shall be completed.

By week 5: All work related to fabrication and delivery of specimens shall be completed. Setting up the first specimen in Construction Annex Lab.

By week 6: All the disposal work including the tested specimens and wastes shall be completed.

Delivery of composite slabs will be coordinated and informed by NTU.

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