Modeling and balancing of HVAC air duct systems

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2016

Chen, H. (2016). Modeling and balancing of HVAC air duct systems. Doctoral thesis,
Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/69404>

<https://doi.org/10.32657/10356/69404>

Modeling and Balancing of HVAC Air Duct Systems

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2016

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A thesis submitted to Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

2016

Acknowledgements

First and foremost, I would like to express my sincere gratitude to my supervisor, Prof. Cai Wenjian for his patient supervision, tremendous support and invaluable guidance throughout the course of my research work. I will dedicate myself into research and creation to maximize benefit of mankind as a whole in return.

Secondly, I would like to thank all my friends in the EEE-ERI@N Joint Lab, in School of Electrical & Electronic Engineering, in Nanyang Technological University, in Singapore, in China and in the world. I will be honest and frank to them and prepare for any help I can.

Lastly, I would like to devote my deepest appreciation and love to my parents and families, for their constant understanding, company and encouragement. I will put in my best effort to protect them, take care of them and share happiness with them.

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Summary

This thesis presents the development of new tools for duct system simulation and advanced techniques for air balancing that methodically proportion the air flows through the duct system mains, branches and terminal devices, based on computational model of duct system. The new simulation tools based on differential algebraic equation solver is efficient and convenient for duct system modelling, analysis, identification and optimization. The new air balancing methods are non-iterative, efficient, easy to use and accurate, which overcome the disadvantages of traditional methods that are based on rules of thumb, inaccurate, time-consuming and costly. The contributions of this thesis include:

1. Develop the duct system model using circuit network analogy based on the pressure drop relationships of the conduits and fittings. Implement a powerful and easy-to-use block library in MATLAB/Simulink environment using Simscape physical modeling language simulation. Define a class for the duct model simulation for professional high-performance applications with flexible application programming interface. The duct model could be used to perform duct system simulation accurately with high flexibility and scalability.
2. Propose a non-iterative air balancing method, which is accurate, efficient and simple to operate. The method is implemented by three steps: 1) adjust dampers and collect measurements of pressures and flow rates; 2) identify the parameters of the duct model using the obtained data; 3) calculate the optimal damper positions corresponding to the balanced flow distribution while minimizing the fan energy consumption. A sequential damper adjustment guide using flow measurements as the indicator is proposed in case of inaccurate damper position indicator. This method requires only common measuring devices, so it is suitable for unprofessional users in balancing small to intermediate HVAC systems.
3. Establish a fan-independent air balancing method which modifies model formulation, data acquisition, parameter identification, and adjusting operations for improved accuracy and robustness. In this method, the fan outlet pressure is monitored by a pressure sensor. The duct model and the parameter identification

algorithm is modified to estimate both duct parameters and pressures simultaneously. After obtaining the balanced damper positions, a novel indicator considering the variations of fan pressures is introduced for sequential adjustment of dampers. This method is more accurate and robust against modeling error and disturbances of fans. The extra cost is acceptable given the significant performance improvement. This method is suitable for high performance applications where more accurate airflow distribution is required.

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Nomenclature

a_1, a_2	parameters of duct cross section, m
b	number of branches in duct system
f	simulation function
m	number of measurements per terminal
n	number of nodes in duct system
n_T	number of terminals in duct system
n_p	number of processes in experiments
q	volume flow rate, m^3/s
q_{max}	fan maximum flow rate, m^3/s
q_{peak}	fan flow rate at peak pressure, m^3/s
r	accumulation variable in Nesterov momentum algorithm
A	cross section area, m^2
C_f	friction coefficient, dimensionless
C	local coefficient, dimensionless
C^*	modified local pressure drop coefficient, dimensionless
D	diameter, m
D_h	hydraulic diameter, m
D_e	equivalent diameter, m
DF	damper correction factor in modified target method, dimensionless
F	objective function for optimization
FF	fan correction factor in modified target method, dimensionless
H	load, unit depends
L	length of duct, m
L_0	fully extended length of flexible duct, m
M	total mass, kg
P	perimeter of duct cross section, m
P_v	dynamic pressure, Pa
ΔP	pressure drop, Pa
P_{max}	fan maximum pressure, Pa

PF	pressure drop correction factor for flexible duct, dimensionless
Re	Reynolds number, dimensionless
V	average velocity, m/s
$\%Q$	flow ratio, dimensionless
$\mathbf{A}$	association matrix of duct system, $n \times b$ matrix, dimensionless
$\mathbf{C}$	sensor position matrix, $n_t m \times (n + b)$ matrix, dimensionless
$\mathbf{g}$	gradient of objective function
$\mathbf{H}$	Hessian matrix of objective function
$\mathbf{K}$	damper parameters, $n_t \times 1$ vector, unit depends
$\mathbf{P}$	pressures in duct system, $n \times 1$ vector, Pa
$\mathbf{q}$	airflow rates in duct system, $b \times 1$ vector, m^3/s
$\bar{\mathbf{q}}$	normalized flow rate, dimensionless
$\tilde{\mathbf{q}}$	target flow rate during balancing, m^3/s
$\mathbf{s}$	internal state of duct system, $n_t \times 1$ vector, m^3/s
$\mathbf{U}$	orthonormal basis for the null space of $\mathbf{A}$, $b \times n_t$ matrix, dimensionless
$\mathbf{v}$	velocity vector in RMSProp algorithm
$\mathbf{X}$	observable variables of duct system, $(n + b) \times 1$ vector, unit depends
$\mathbf{Z}$	measurements in experiments, variable size vector, unit depends

Subscripts

b	values in branch duct
c	values in main duct
D	values on design conditions
fan	properties related to fan
i, j	counting index depends on context
in	values at inlets
I	properties related with internal branches except terminals
min	minimum value
max	maximum value
O	values on all damper fully open conditions
out	values at outlets
P	properties related with nodal pressures

q	properties related with branch flows
SA	supply air
T	properties related with terminals

Greek symbols

ϕ	zone condition, unit depends
ρ	air density, kg/ m ³
ϵ	the roughness of duct wall, mm
ϵ	equation residuals, variable size vector, unit depends
β	duct system parameters, variable size vector, unit depends
λ	scalar factor for residuals of equation, unit depends
α	updating rate for gradient descent, dimensionless
μ	expectation of the measurements, variable size vector, unit depends
Σ	covariance matrix of the sensors, variable size matrix, unit depends
σ	sensor uncertainty, unit depends
κ	set of damper parameters, unit depends
$\hat{\beta}$	estimated model parameters, variable size vector, unit depends
η	momentum coefficient in Nesterov momentum algorithm
γ	decay rate in RMSProp algorithm

Abbreviations

ACB	active chilled beam
ACMV	air conditioning and mechanical ventilation
AHU	air handling unit
ASHRAE	American Society of Heating, Refrigeration and Air-Conditioning Engineer
BIM	building information modeling
CAD	computer aided design
CAV	constant air volume
CFD	computational fluid dynamics
CIBSE	Chartered Institution of Building Services Engineers
DAE	differential and algebraic equations
DCV	demand controlled ventilation

DOAS	dedicated outdoor air system
DOF	degree of freedom
FPB	fan pressure-based method
HVAC	heating, ventilation and air conditioning
IAQ	indoor air quality
LDDS	liquid desiccant dehumidification system
MAPE	maximum absolute percentage error
NEBB	National Environment Balancing Bureau
PFM	progressive flow method
TAB	testing, adjusting and balancing
VAV	variable air volume
VSD	variable speed drive

Chapter 1. Introduction

1.1 Background

Heating, ventilation and air-conditioning (HVAC) is the technology to provide thermal comfort and acceptable indoor air quality (IAQ) for occupants in buildings and vehicles. HVAC systems have become an indispensable part of daily life and big influential factors to public health and work efficiency, especially for urban citizens who spend over 80% of time indoors[1]. Studies have shown that children's schoolwork performance in classroom[2] and human productivity[3] in office is positively correlated with thermal comfort and indoor environment including air quality. Currently, HVAC systems are so heavily relied on that the majority of buildings are equipped with them and a significant portion of energy is consumed by them. In U.S., the space heating, cooling and ventilation together are responsible for 41.4% (16.71 Quad Btu) of energy in building sector in the 2010[4]. In Singapore, which is a tropical country with annual average temperature of 27°C and annual average relative humidity of 84%[5], the portion (cooling and ventilation only) can be as high as 70%[3].

Under the situation of increasing requirement of indoor environment quality and increasing pressure of applying green concept in buildings, the future of HVAC systems develops towards three aspects: improved IAQ, reduced energy consumption and personalized services. To improve IAQ with reasonable cost, dedicated outdoor air systems (DOAS) supply the proper ventilation air quantities into every space and prevent airborne contaminants from propagation between rooms. For better energy efficiency, demand-controlled ventilation (DCV) has been developed to supply the only necessary amount of ventilation airflow based on occupancy, especially for buildings with variable occupancy[6]. Besides, Water-based systems including chilled beams and radiant heating/cooling systems significantly reduce the volume of air transport by handling sensible load via water which only need less than 5% of the energy that is necessary for transporting same quantity via air[7]. Singapore government[8] accentuates the decoupling of ventilation and cooling by combining DOAS with water-based system. To provide personalized services, the localized ventilation system which introduces fresh air directly in the respiration area is developed to promote air quality

and personal thermal comfort using customizable air supply. HVAC systems will gradually become high performance and high efficiency in heat transfer, moisture control, air delivery and contaminant removal to fulfill dynamic and heterogeneous demands of people.

Developing next generation HVAC systems should aim at the two core functionalities of the system: air treatment and air delivery, since the ultimate medium contacting people is air. The air treatment provides conditioned air as needed for the conditioned space. The air delivery ensures proper amount of air reaches the conditioned space as desired. Natural ventilation and mechanical ventilation are the two major approaches to achieving air delivery. Natural ventilation passively uses outside air movements and pressure differences around the building, while mechanical ventilation uses fans to force air intake from outdoors and circulate in the conditioned space. Modern building with large area, complex indoor layout and high energy efficiency requirement relies majorly on mechanical ventilation. The scope of this thesis focuses on the issue of air delivery in air conditioning mechanical ventilation (ACMV) system.

1.2 Overview of ACMV system

It has been recognized that the first mechanical ventilation system was made by the engineer David Boswell Reid at St George's Hall in Liverpool in 1851, which received the first Blue Plaque award by the Heritage Group of the CIBSE in 2005[9]. At that time, four large steam-driven fans forced air passing through the plenum between brick walls and the diffuser formed by thousands of holes near the floor and entering into the conditioned space[10]. A copy of diagrams for the system is shown in Figure 1.1.

Figure 1.1: Diagrams of St George's Hall. [11]

It has been over 150 years of development that modern mechanical ventilation systems upgrade dramatically in materials, equipment and functionalities, but the principle remains unchanged, as illustrated in Figure 1.2[12]. The air is constantly inhaled by fan from outdoor or return duct, processed in air handling unit (AHU), and pressurized into the supply duct. The following pathway is all enclosed by duct, during which the airflow splits at junctions, regulated by dampers, and finally discharged into the conditioned space through diffusers, grills and other terminal units.

Figure 1.2: Modern mechanical ventilation system and the working principle

An air duct system consists of many components, including ducts, fans, elbows, junctions, dampers and terminal units. Ducts have smooth interior surfaces and well-sealed exterior, mostly constructed of sheet metals like galvanized steel, stainless steel and aluminum. Common duct conduits in ACMV systems includes round, rectangular, flat oval, and flexible ducts, demonstrated in Figure 1.3[13-16]. The round duct saves the cost of material, insulation, support and labor because of less perimeter for same cross-sectional area. It has lower frictional loss at the same air speed, stiffer, easier to insulate and good attenuation of low-frequency sound. On the other hand, the rectangle duct can be adapted to any space height constraints. They provide flat surfaces for convenient branch tap-ins as well as fittings. And shipping rectangular ducts are also easier if broken down and nested. Besides, the flat oval duct in spiral lockseam can be regarded as an intermediate alternative which shares both advantages and disadvantages with the round and rectangle ducts. The flexible duct is made of durable textiles over a metal wire coil to shape a tube and is often used to connect between terminal units and the end of rigid ductwork. The pressure loss through flexible duct is significant, especially at turns.

Figure 1.3: Duct conduits
(a) Rectangular; (b) Round; (c) Flat oval; (d) Flexible

Fans in duct system are usually centrifugal or axial, shown in Figure 1.4[17]. The centrifugal fan intakes air from the center and accelerate air radially, changing the direction of the airflow (typically by 90 degree). The kinetic energy of the impellers or the rotating blades is converted to the pressure of air, which then overcomes the resistance of duct system. It is by far the most prevalent type of fan used in ACMV system because of lower cost and simpler construction. The axial fan, on the other hand, rotates the blades and forces air to move parallel to the rotating shaft. Axial fans in ACMV system usually have lower static pressure but larger flow rate.

Figure 1.4: Fans in the duct system
(a) Centrifugal; (b) Axial

Many different fittings are commonly used to connect between ducts, as shown in Figure 1.5[18]. Elbows turn the direction of air flow (usually by 45 degree or 90 degree) without too much pressure loss. The geometric shape and interior smoothness of the elbow are the two major influential factors to the pressure loss under different airflow rates. Besides, turning vanes can be installed at the elbow in order to further reduce pressure loss as well as turbulence. Junctions connect different branches together. Common junctions can be Y-shape, T-shape and cross. Governed by complex fluid

mechanics, the quantity of airflow separation into different branches depends on the area ratio, flow condition, interior geometry and downstream resistance.

Figure 1.5: Elbows and junctions of different types in ducts

Dampers are the devices regulating and balancing the airflow rate through ducts. By partially closing the damper, the resistance of airflow through this damper increases, and the airflow rate is consequently reduced. Common dampers in ACMV systems include butterfly, opposite, parallel, vanes and iris damper, as shown in Figure 1.6[19-22]. The dampers can be manually adjusted or motorized by damper actuators.

Figure 1.6: Dampers in ducts

(a) Butterfly; (b) parallel; (c) opposite; (d) iris damper

Terminal units discharge air into the indoors. Typical terminals include grill, diffuser, swirl diffuser and chilled beam units, as shown in Figure 1.7[23-25].

Figure 1.7: Terminals in ducts
(a) grill; (b) diffuser; (c) swirl; (d) chilled beam

The duct systems constructed by the above duct components can be formed in different types of structures. The commonly used systems are radial, extended plenum, reducing trunk and perimeter loop systems[26], which are shown in Figure 1.8[27, 28]. Different types of duct systems have different strengths in versatility, performance, economy, space requirement, and appearance. Engineers should properly design the air path and duct size, within prescribed limits of velocity, noise intensity, space availability, to efficiently transmit the required flow rate of air to each space while maintaining a proper balance between investment and operating cost.

Figure 1.8: Structures of duct systems

(a) Radial; (b) Extended plenum; (c) Reducing trunk; (d) Perimeter loop

A large duct system can serve multiple spaces whose loads vary with location and time. In ACMV terminology, the *condition* is certain property of air that is concerned in ACMV system, which can be one of the temperature, humidity, CO₂, and etcetera. The *zone* is an area in which conditions are sensed and maintained near desired set-points by a single control device like thermostat. The *load* quantifies the deviation of a condition from desired set-point per unit time. Temperature could be affected by heating and sensible cooling load, and latent load corresponds to humidity. Ventilation load is currently represented by CO₂ majorly because of low sensor cost.

The change of indoor condition ϕ for a single zone satisfies $Md\phi/dt = \rho q(\phi - \phi_{SA}) + H(t)$, where M is the total mass of indoor, ρ is the air density, q is the volume rate of supply air, ϕ_{SA} is the condition of supply air, and H is the load. To satisfy the varying load $H(t)$, the air system can vary either ϕ_{SA} or q . This leads to different approaches to conditioning indoor space. A constant air volume (CAV) system adjusts ϕ_{SA} , while a variable air volume (VAV) system adjusts q . In the CAV, the condition of supply air can be changed by either reheating or using dual duct in which one is hot and the other is cold. Neither methods are energy efficient because of counteractive effects of mixing. The VAV system changes the airflow in the duct with time by means of on-off schedule or continuous control. The VAV system can also be

reheated or dual duct system, but using constant condition of supply air is the most efficient. As the development of control techniques and the increasing concern of energy saving, most buildings are now equipped with VAV systems.

For multi-zone system, one AHU must serve multiple terminals and fit with multiple varying loads simultaneously, which commonly occurs in large commercial buildings. The proportioning of airflow to different zones becomes a difficult problem. On one hand, the duct system must be properly designed to satisfy the design air flow rates to all zones. But duct design alone can not guarantee the proper amount of air to each branch accurately. Due to the space limit, noise level and fire protection considerations, some duct designs do not purpose on a balanced system, while those that do try, can only roughly approach air balance due to modeling errors and construction variations. On the other hand, the dampers in the duct perform the critical role to regulate the airflow and achieve design specifications for proper functionality of the entire mechanical ventilation system.

1.3 Motivations and objectives

The methodical proportioning of airflow through the duct system's mains, branches, and terminal devices, is known as *air balancing*. Air balancing is the last and most important means of guaranteeing the airflow distribution in the duct as design. It could fix the problem caused by unavoidable estimation error during the duct design and the construction modifications. Air balancing is commonly implemented by partially closing the dampers to regulate the excess proportions and varying fan speeds to achieve overall air flow rate as designed within tolerance. According to ASHRAE standards, the tolerance is $\pm 10\%$ for terminal air flow and $\pm 5\%$ for main duct[29].

Currently, all air balancing tasks are accomplished by standard air balancing methods, which are performed by qualified engineers during the commissioning of ACMV system. Standard air balancing methods are iterative approaches based on rules of thumb. In each iteration, dampers will be adjusted according to the current measurements of terminal airflow rates. However, adjusting a damper can affect flow rates of neighboring branches. Due to this coupling effect, the actual airflow rates for some terminals are still unsatisfied. Hence the proportioning adjustments must be repeated iteratively until all branches satisfy the design air flow rates within tolerance.

The number of iterations is difficult to be estimated as well. Standard air balancing methods are time-consuming and costly, and they greatly depend on the engineer's experience to achieve accurate balancing results. It is estimated that air balancing need 1.5 man-hour per terminal[30]. Although duct systems change with time and use, building owners are unwilling to pay high costs for regular professional testing adjusting and balancing (TAB) services and interrupt the normal service of the building. The standard air balancing methods gradually become unsuitable for advanced ACMV system as the system becomes more complicated and the requirement of accuracy increases. In fact, Okochi et al. [31] suggested that balancing and distribution of airflow in VAV system can be considered as one of the main challenge areas of research concerning VAV system control. Under this background, it is an urgent need of advanced balancing method with the following features:

- The method should be non-iterative to improve the efficiency of balancing process. The time cost is also estimable.
- The method should be more accurate in achieving desired airflow rate under current available equipment and affordable cost.
- The method should be scalable for larger and much more complex duct system within reasonable cost and time consumption
- The method should be low cost and easy to implement.

To overcome the coupling effects between branches and terminals, it is necessary to fully analyze the duct system and deeply understand the air balancing process. Considering the strong demand for air balancing, the objective of this thesis is to study duct system and develop accurate, efficient air balancing methods. More specifically, there is a one to one correspondence between topics conducted in the thesis to resolve aforementioned difficulties in the development of air balancing:

- Develop the mathematical model for a duct system and simulation tool for analysis of system behaviors as well as performance testbed of proposed air balancing algorithms. The mathematical model for the duct system helps understand the behavior of duct system as dampers and fan changes, and quantify coupling effect. Developing a simulation environment for duct system is helpful to evaluate the performance of air balancing algorithms, and has

potential for applications including optimization of working condition, design of control strategy and fault detection and diagnosis.

- Develop non-iterative air balancing method based on model that is efficient and convenient for applications where users can easily implement using common devices. This method is suitable for unprofessional users who need simple and low cost air balancing method for home or small to medium size office.
- Develop fan-independent air balancing method to reduce modelling errors and disturbances that is more accurate and robust for applications where accuracy is primary considerations. The fan-independent method is designed for high performance ACMV system which deliver airflow to each space precisely. The potential customers are willing to pay for more complicated balancing procedures in order to obtain that accuracy.

1.4 Major Contribution

The major contribution of the thesis is accordingly summarized:

- The mathematical model for a duct system is developed based on pressure drop model of conduits and fittings and circuit network analogy. The characteristics of the duct model and the degree of freedom are investigated. Numerical solving algorithm for the duct model is developed. A powerful and convenient block library in MATLAB/Simulink environment using Simscape physical modeling language is developed for simulating duct systems. A class for the duct model simulation is defined which has higher performance, more flexibility and richer application programming interface.
- An efficient air balancing method is proposed, which consists of three steps: 1) measure pressures and flow rates in duct system in specific procedure; 2) identify the parameters of the duct model using obtained data to predict system behavior; 3) calculate the optimal damper positions corresponding to the desired balanced status of the system. The optimal damper positions not only achieve desired airflow rates in all terminals, but also minimize the energy consumption of fan. In the case that users may be difficult to adjust dampers to the optimal positions accurately, a sequential damper adjustment based on airflow measurements is also proposed. Since this method is performed in a simple and clear operating

procedure and requires only pressure probes and flow measurement device, it is expected to be used by unprofessional users for their small to intermediate ACMV system in a basic air balancing applications.

- A fan-independent air balancing method based on the duct model is proposed, which has made many modifications in model formulation, measurement acquisition, parameter identification, and adjusting implementation. The fan is remodeled as a variable pressure source instead of parametric model with constant (unknown) parameters. Fan outlet pressure is measured in addition to the measurements in each terminal by a pressure sensor installed near the outlet of fan. Model identification algorithm is modified to estimate both duct parameters and fan pressures in all steps. Optimal damper positions are then calculated at specific fan pressures as optimal working condition for fan. A novel indicator for damper sequential adjustment is introduced in order to consider variations of fan pressures. The performance of this method is more accurate and robust against modeling error and disturbances of fans with an acceptable extra cost and time. This method is suitable for high performance applications like hospital and laboratory where more accurate airflow distribution is required.

1.5 Organization

The rest of the thesis is organized in 7 chapters:

Chapter 2 presents a comprehensive review of state of art in the research areas related to the means of obtaining desired air flow in the duct system, including the duct model and simulation, duct sizing, standard air balancing methods, and advanced air balancing methods. This chapter can be used as a context for understanding air balancing as well as the studies described in the following chapters.

Chapter 3 develops the mathematical model of the duct system and the simulation algorithm. Based on the model, the duct simulation tools are developed for the convenience of establishing duct model and performing simulations under various of conditions. This model is the foundation of analyzing the characteristics of duct system, developing the air balancing methods and validating the results in simulation.

Chapter 4 Establishes the procedure of model identification which consists of obtaining necessary measurements from experiments and performing optimization

algorithm on the objective function which is developed for estimating the posteriori of model parameter given the measured data. Method of evaluating model accuracy is proposed in order to verify the identified model. The model identification is one of the key steps for air balancing.

Chapter 5 proposes the model-based air balancing method to adjust dampers in an efficient, non-iterative way to achieve balance. The key algorithm is calculating the optimal damper positions and generating the sequential damper adjustment procedure to implement them. With this method, one can perform air balancing in a simple and efficient way without any professional training. The accuracy of balancing is greatly improved while the time consumption is reduced.

To further simplify the model, reduce the dependency on the type of fan, and improve the performance, **Chapter 6** proposes a fan-independent method for air balancing. Based on the assumption of replacing fan model with a variable pressure source, a series of modifications in model development, data acquisition, identification and balancing are made. Comparison studies with previous methods in varies of conditions are conducted and analyzed. The results have shown satisfactory improvements in efficiency, accuracy, stability and robustness. The fan-independent method offers the user an alternative method with higher performance in wider applications.

A conclusion of this thesis is given in **Chapter 7**, and the foreseeable research topics based on the current results are presented as well.

Chapter 2. A review of research into air balancing

2.1 Introduction

To achieve the objectives of this thesis, a comprehensive review of available information, especially the literatures, is necessary. It provides the background knowledge, avoid repeated failure and success, and inspire ideas. The development of air balancing methods should be reviewed so that the state of art air balancing methods is covered. Not only the advantages and disadvantages of different methods should be investigated, their inherent relationships are also worth to be studied. Besides, the target of air balancing is to achieve desired airflow distribution in the duct network, which is shared by duct sizing. Therefore, the duct sizing methods should be scrutinized to give some expectations about the property of the problem and the inspiration of potential solutions in similar way. Note that the duct sizing not only aim at desired airflow distribution, but also noise level control, space limitation, fire protection and cost balance. Moreover, solving the air balancing problem requires deep insight into the duct system behavior. A model for the duct network as well as convenient simulation tools for studying will be beneficial to success. Formulating the duct model and preforming simulation under various conditions could help to understand the duct system and verify any ideals about air balancing. Finally, studying the current situations of air balancing can be helpful to judge the contribution of the work and foresee the possible direction of research.

In this chapter, related areas are surveyed in literatures and available sources. For the convenience of understanding, the contents are reorganized as follow. Current duct models are investigated in Section 2.2. Simulation tools and software for ACMV systems and ducts are examined in Section 2.3. The typical duct sizing methods are reviewed in Section 2.4. The standard air balancing methods adopted by the industries are explained in Section 2.5. And the advanced air balancing methods in the literature are studied in Section 2.6. Finally, a brief summary is drawn in Section 2.7.

2.2 Modeling pressure loss in ducts

The pressure drop of duct system is contributed by the friction and dynamic losses. The frictional loss quantifies the pressure drop due to the relative motion of the duct

wall and the airflow. The dynamic loss results from flow disturbances caused by duct-mounted equipment and fittings that change the flow direction (elbows), area (transitions), and distribution (converging/diverging junctions). The overall pressure loss in any duct flow is the sum of all the frictional loss and dynamic loss together. Due to the theoretical complexity, the studies in this area are mostly empirical and experimental.

The widely used data for estimating pressure loss across ducts and fittings are those provided in ASHRAE handbook[32], the CIBSE guide[33], and the handbook by Idelchik[34]. These data are obtained by summarizing many experimental works based on ASHRAE Standard 120P[35]. However, the accuracy of these experimental data available in the handbooks has been questioned many times[36-41]. One major questionable aspect is that measurements were conducted on single, isolated duct fittings without consideration of the influence of the interaction of other fittings. Unfortunately, it is very common to have multiple fittings that are relatively close to each other. Experimental study[42] on the effect of interactions between bends have revealed coupling effects in pressure loss, which implies that summing pressure loss across each fitting together could be inaccurate.

Computational fluid dynamics (CFD) analysis on how separation and orientation of two closely connected bends affect the pressure loss has been studied[38], which points out that the system will suffer a varied pressure drop across the two bends depending on their relative orientation of the fittings. Moreover, the CFD method has been used to predict the change of flow regime and estimate the pressure loss coefficient across the duct fittings, transitions, dampers as well as orifices[40, 41, 43, 44], and junction[45, 46]. The aforementioned studies are based on the steady standard k-epsilon turbulence model[47-51]. An alternative CFD model using the large-eddy simulation has also been proposed to predict pressure losses across multiple fittings and shows consistence with the measured data[52]. However, models developed by CFD are unable to compute fast for the application of air balancing due to the limit of computational power.

2.3 Simulation of duct systems

Modeling and simulation for HVAC system have long been applied for research and engineering, and many successful software applications have been developed. Many

comprehensive review papers and reports on the building and HVAC simulation tools can be found in literature[53-56]. In these applications, the range of modelling and the simulation approaches for HVAC and other environmental control systems is greatly different. When allowing very coarse distinctions, these applications can be classified into two categories: process-based approach and equation-based approach[57].

Process-based approach establishes a duct model with explicit solving procedures to obtain the simulation results. Software applications like Energy Plus[58-62], Trane TRACE 700[63], HVACSIM+[64-66], TRANSYS[67-70] and International Building Physics Toolbox[71] are all process-based approach. The MATLAB/Simulink environment is also applied to building and HVAC simulation[72], which has grown to almost a de facto standard in non-CFD scientific computation[73]. It has integrated many computational functions, rich toolboxes in various domains and wide connections with other simulation applications, program developing environments, embedded real-time systems, and data acquisition instruments, which makes it powerful and widely accepted. However, as pointed out by de Canete et al.[74], the process-based approach of the block diagram representation is inconsistent with the nature representation of physical model.

Equation-based approach has clear separation of system definition as a set of differential and algebraic equations (DAE) and the solver. DAE approach is identified to be the modern concepts from computer science and software engineering that could be used to develop new building performance simulation software[75]. SPARK[76-79], Energy Kernel System[80, 81], Neutral Model Format[82-85], ESP-r[86-89], EQUA IDA Indoor Climate Energy[90-92], and Modelica[93-97] are the simulation software based on the DAE. The equation-based approach offers many advantages:

- Implicit coding avoids unnatural assigning of inputs and outputs;
- Object-oriented programming support modularity that makes connected components analogous to assembling objects in physical world;
- Hierarchical structure allows development of large scale system in bottom-up style with high efficiency and reliability[98];

Despite great advantages of DAE approach, it has not been widely applied yet. Several factors are contributing to this apparent lack of progress[53]: 1) Some

exploratory projects fail to deliver as expected; 2) Leading research groups have reverted back to existing solutions. 3) Multi-domain simulation is attempted by coupling of existing domain specific simulators. 4) Attentions have shifted from developing new tools to integrating existing tools. Moreover, further scrutinize the current available software applications reveals that current applications are standalone that neither provide sufficient powerful code packages for deep mathematical analysis or complex algorithm implementation nor provide rich interfaces for data exchange with developed and widely used applications.

Efforts are made [95, 99] to integrate DAE simulator into S-function block in the Simulink. The software package Building Controls Virtual Test Bed can be used as a bridge to exchange data between Modelica Buildings Library and MATLAB/Simulink environment[100-102]. However, only limited data exchange and simulation process manipulations are supported, which prevent comprehensive and in-depth co-simulation. Instead of integrating DAE software packages into MATLAB/Simulink environment, the native DAE-based toolbox in MATLAB/Simulink environment, the Simscape, has been developed for modeling and simulating multi-domain physical system. According to investigations by Schijndel[103], Modelica and Simscape are quite similar in many aspects except syntax that mutual conversion is possible. Unfortunately, no evidence yet have been reported using Simscape in HVAC system[53].

2.4 Duct sizing methods

Duct sizing determines the geometry dimension of ducts, which relates closely with selection of fan, air flow distribution, construction cost and operating cost. Duct design should use the lowest cost materials to obtain approximate pressure balancing for design airflow and minimum operating pressure for minimum operating cost. Meanwhile, duct design subjects to many constraints: 1) space, 2) noise, 3) air leakage, 4) heat transfer, 5) fire and smoke control, and 6) cost. It can be expected that designing duct for ACMV systems in large buildings can be quite complicated. In recent years, with the assists from computer aided design (CAD) and building information modeling (BIM) software, the duct sizing can be more adaptable and specific, but the basic duct design methods are still useful as a good starting point, including equal-friction, velocity reduction, static regain, and T-method.

- Velocity reduction[104]. The velocity reduction method is the simplest method. In this method, duct dimension is selected such that its cross-sectional area A_i and the design airflow rate through the duct q_i satisfy $A_i = q_i / V_i$ where V_i is the expected velocity of the duct. Starting from fan, V_i is empirically reduced over subsequent ducts, and the reductions are uniformly taken place for different branches. The major consideration in velocity reduction is the noise level control. A refinement of velocity reduction is to assign different V_i for different branch such that the available pressure at the junction is equally dissipated at all branches. The process can be repeated for subsequent junctions. However, the empirically selected velocities may be unsuitable for certain situations. Too low V_i is restricted by available installation space, and high V_i must consider the large pressure drop and the operating cost of fan. Besides, velocity reduction method does not completely balance the system.
- Equal friction[105]. The equal friction method tries to maintain constant pressure drop per unit length along the duct. Figure 2.1[106] shows the duct friction chart used for calculation which can easily obtain the corresponding duct diameter D_i given the design friction loss $\Delta P / L$ and the flow rate q_i . When different branches have widely varying pressure loss, the design pressure gradients $\Delta P / L$ are modified to be inversely proportional to the duct length such that the total pressure drops are equal, and the system is balanced. Equal friction is also simple to use. The velocity naturally decreases as the flow rate reduces near the terminal of duct system, and usually yields a better design than velocity reduction. However, equal friction leads to considerable high duct resistance and thus high operating cost. Besides, VAV system designed by equal friction method requires pressure independent control to prevent large flow rate at high pressure.

Figure 2.1: Duct pressure loss diagram

- Static regain[107]: The static regain method is commonly used for high velocity, large system. This method increases or regains the static pressure by reducing air velocity to compensate pressure loss. Considering sizing a segment of duct between two branch take-offs, the cross-section right before the first (upstream) junction is defined as section A and the cross-section right before the second (downstream) junction is defined as section B. The dynamic pressure of the two section $P_{V,A}$ and $P_{V,B}$ are calculated by $P_{V,A} = \rho V_A^2 / 2$ and $P_{V,B} = \rho V_B^2 / 2$ respectively. By selecting proper duct size, the pressure loss ΔP_{AB} between sections A and B should satisfy: $\Delta P_{AB} + P_{V,B} = P_{V,A}$. Since both $P_{V,B}$ and ΔP_{AB} is dependent on duct size, solving the solution for duct size is likely to be more time-consuming. This method is unable to deliver a duct system design that ensures total pressure balancing at the specific air flow rates[108]. An adaptation for this method is the total pressure design method which is advantageous because the intermediate system pressures and control of duct sizes and velocities are known. Because the static pressure determines the flow discharge rate through outlets, maintaining same static pressure would lead to a balanced air distribution through the main duct without dampers. The static regain method uses less fan power for air delivery and has less noise issue in take-offs. A

disadvantage of this method is that duct getting larger at it runs. The duct sizes are usually large, which may conflict with space constraints and increase initial cost.

- T-method[109-113]: The T-method optimizes the duct system to achieve lowest life cycle cost. The T-method consists of three steps: system condensation, fan selection and system expansion. By simulating the duct model in design conditions, an equivalent duct with the same pressure-flow characteristics and economic characteristics is obtained first. Fan operating condition is then optimized. Finally, the duct model is expanded to simulate the pressure distribution in the entire system to optimize duct size. T-method is the best economical duct sizing method. The advantage is the robustness and speed. However, the T-method does not treat the constraints of standard duct sizes very well. It may be unstable and produce grossly oversized ducts when calculating dynamic pressure loss at junctions and crosses which is dependent on adjacent duct sections[114]. Besides, T-method is unable to optimize a system with time variable duct flow rates or utility rates. Lee et al. [115] suggests extended T-method for loop duct system. Modification for multi-fan duct system has also been proposed for exhaust duct ventilation[116]. It is also used to analyze the pressure differences between adjacent confined spaces in a nuclear facility when a design basis accident occurs[117].

2.5 Standard air balancing methods

According to ASHRAE standard[118], the objectives of air balancing are to satisfy:

- All airflow quantities should be controlled within $\pm 10\%$ of the design airflow quantities. If the total available air cannot meet the design requirements, the air distribution system shall be proportionally balanced to within $\pm 10\%$ of the available total.
- To minimize the energy consumption, at least one terminal damper should be wide-open. In addition, if a system also consists of branch dampers, at least one branch damper should be fully open.

The proportional method or ratio method is the most widely adopted method for balancing by ASHRAE[118], CIBSE[119], NEBB[120]. It can be used on constant or

variable air volume system; low, medium or high pressure system; or single zone, multi-zone or dual duct system. The procedure of proportional method is as follows[121]:

1. Measure the total system airflow at the fan outlet and calculate the ratio of actual airflow to design airflow ($\%Q_{fan} = q_{fan} / q_{D,fan}$).
2. Adjust the fan speed such that total system airflow close to about 110% of the design airflow if needed.
3. Measure the airflows at all terminals and calculate flow ratio $\%Q_i = q_i / q_{D,i}$.
4. Arrange the ratios $\%Q_i$ in ascending order and keep the damper with lowest ratio ($\%Q_{min}$) unchanged.
5. Starting from the second lowest ratio, adjust all the other dampers to $\%Q_{min}$ sequentially.
6. Repeat procedure (a) to (e) until all terminals have been balanced within $\pm 10\%$ of the design airflow.
7. Adjust total system airflow to the design airflow.
8. Re-measure the air flow of all the terminals and make final adjustments.

The stepwise method is also commonly used, which is described by NEBB[122].

The detail procedure is presented below:

9. Measure the total system airflow at the fan outlet and calculate the ratio of actual airflow to design airflow ($\%Q_{fan} = q_{fan} / q_{D,fan}$).
10. Adjust the fan speed such that total system airflow close to about 110% of the design airflow if needed
11. Measure the airflows at all terminals and calculate flow ratio $\%Q_i = q_i / q_{D,i}$.
12. Starting from the highest $\%Q_i$, adjust the damper so that the airflow will be 10% below the designed.
13. Adjust the rest dampers until all terminals are with $\pm 10\%$ of the design airflow with at least one fully open.
14. Adjust total system airflow to the design airflow
15. Re-measure the air flow of all the terminals and make final adjustments.

The standard air balancing methods are inefficient and unsatisfactory. Guffey et al.[123] introduces a novel target computing algorithm for the proportional method to improve the convergence and reduce the times of iterations needed for balancing. Based

on the observations that the airflow rates of previously adjusted dampers and fans is changed as the latter adjustments, the new target tries to take the damper adjusting sequence and fan correction factor into account by empirical rules. Extensive trial and error simulations using ventilation design software give the sequence correction factor:

$$DF = \left(\frac{i}{b} \right)^{0.0445} \times 100 \quad (2.1)$$

where

DF = damper correction factor

i = sequence number for adjustment of the damper

b = total number of branch ducts.

The fan correction factor is given by:

$$FF = \frac{1}{2} \frac{q_{O, fan}}{q_{D, fan}} + \frac{1}{2} \min_i \frac{q_i}{q_{D, i}} \quad (2.2)$$

where

FF = fan correction factor

$q_{O, fan}$ = the fan airflow rate when all dampers fully open

$q_{D, fan}$ = the design fan airflow rate

q_i = the actual airflow rate at terminal i

$q_{D, i}$ = the design airflow rate at terminal i

During the practice of TAB, rich experience about the changes of flow rates when adjusting dampers. The primary effect of adjusting damper is the reduction of the airflow through this branch and the subsequent branches. Within a small range close to fully open, the reduction of airflow is proportional to the damper position. When the damper position increases, the airflow nonlinearly decreases. In this case, the secondary effect becomes significant that the airflow rates in other branches increase. Due to the secondary effect, during the balancing, the branch that has been adjusted properly gradually deviates from the target airflow rate. Consequently, multiple rounds of adjustments are often necessary to correct the deviated flow rates in some branches. Adjusting the airflow to the value near the lower boundary of design target partially alleviates the problem by increasing the tolerance for the secondary effect. Nevertheless,

without secondary effect elimination, the standard methods could only approximate the balanced status iteratively, and the duration is priori unpredictable.

2.6 Advanced air balancing methods

The concept of non-iterative balancing which has deterministic procedure becomes the main trend. Pedranzini et al.[124] proposed a non-iterative balancing procedure, named progressive flow method (PFM). This method is based on the theoretical analysis of the lumped parameters representations of air distribution system. The procedure of PFM considers one branch at a time and follows the backward sequence from the furthest terminal from fan to the closest terminal. At any split, the upstream part can be treated as a fan and the downstream part can be reduced to a single equivalent branch characterized by the resistance. By the progress of balancing, it only requires to adjust the i damper to obtain the design airflow rate $q_{D,i}$, because the downstream portion of the network has been already balanced. To eliminate the variation of fan airflow rate by the change of system resistance during damper adjustments, it requires an inverter in order to regulate fan speed continuously. The inverter is negative feedback controlled by airflow measuring instruments like vane, hot-wire anemometer or pitot tube installed in the terminal at the end of duct system. This method could be inconvenient when the terminal airflow rate remains lower than the design value even though the damper has been fully open. In this case, a damper or a variable geometry device shall be installed immediately downstream to the split junction to increase the resistance of downstream ducts. The PFM can significantly improve the efficiency of air balancing with high accuracy. For a system with n_t terminals and b branches, the execution of $2n_t + b - 1$ measurements can be avoided. The range of efficiency improvement varies from 57% to 67% as n approaches infinity.

As the development of variable speed drive (VSD), accurate estimation of mechanical power and rotational speed of fan without additional instrumentation becomes possible[125]. The VSD has been applied in developing fan airflow station[125, 126]. The capabilities of VSD operating condition estimation enables a new non-iterative TAB, known as the fan pressure-based method(FPB), which is developed by Tamminen et al.[125]. With help of a fan pressure sensor, this method could adjust terminals step by step by estimating fan working condition to achieve design airflow

rate. From the initial all-damper-closed status, the dampers are sequentially adjusted so that the increment of fan flow rate is equal to the design airflow rate of each damper. Comparing with PFM, the procedure of FPB is the same efficient as PFM, but the effort of balancing could be further reduced because no installations of airflow measuring instruments at the terminals are needed. However, the accuracy of this method will be affected by the common duct segments shared by multiple terminals, large duct leakage and insufficient duct pressure. Depending on the sensitivity of the fan working condition estimator, the FPB method could be applied in small to medium scale air distribution system.

To develop a non-iterative balancing method, Small[30] proposed a model-based approach that first develops the mathematical model of the system based on flow and pressure measurements, and then adjusts dampers in one time to achieve balance. The model is a hybrid model that has the same network topology as in real duct system, but the parameters in the branches that govern the characteristics of the model are all identified from experimental data. It is figured out that at least two tests from different system configurations are needed. The first test is chosen to be conducted under condition of all dampers wide-open, with the assumption of zero pressure loss for each wide-open damper. The second test is conducted by completely closing a certain damper that the airflow rate through this damper is assumed to be zero. In this way, the system parameter can be obtained. The flow and pressure measurements from these two tests can provide sufficient number of equations for solving all undetermined parameters. With the obtained parameters, the model can simulate the airflow rates and pressures in the entire system in any conditions of damper positioning coefficients. By substituting the design airflow rate for each branch, the damper positioning coefficients that correspond to the design airflow can be solved given the static pressure of the fan. Finally, the dampers are only adjusted once to the calculated value to achieve balance and no iteration is required.

All non-iterative methods eliminate the secondary effect of damper either implicitly or explicitly. Implicit secondary effect elimination methods like Pedranzini et al.[124] and Tamminen et al.[125] limit the change of flow by specific sequence of damper adjustments and precise control of fan speed. This requires variable speed drive for fan

to control the flow rate. The damper positions are initially set to fully closed, and opened successively to reach balance. This method requires additional devices to control the speed of fan which may limit the applications. The other approach is to explicitly estimate and eliminate the secondary effect using duct model. This requires a complete model for a duct system and deep understanding of the change of flow rates by the dampers. With duct model, one can predict the airflow at any given damper positions, and a mapping from the n_T dimensional phase space of damper positions to the n_T dimensional phase space for airflow rates vector can be established. This could be helpful to find the optimal damper positions corresponding to the design flow rates so that only single adjustment of damper for each terminal is needed. Unlike implicit approach, explicit secondary effect elimination methods do not rely on any additional devices other than basic measuring devices, which is convenient for practical applications.

2.7 Summary

In ducts, friction appears as pressure drop. Accurate model is necessary to estimate pressure drop and flow distribution. Due to complex interaction within the fluid, most models are empirical. However, the accuracy of these empirical models is often questioned, especially when two fittings are close to each other. CFD analysis on this coupling effect has provided another approach to estimating pressure drop in complex situations.

For applications in ACMV systems, many simulation software applications have been developed. Procedure-based approach has been developed and applied greatly in the past. Equation-based approach offers advantages in natural way of connecting components, modular design and hierarchical structure. The Simscape for modeling and simulating multi-domain physics in MATLAB/Simulink environment is one of the equation-based tools that has the potential but not yet be applied in ACMV system.

The air distribution in a duct network is govern by the duct resistance, and thus proper duct sizing is important to achieve design specifications. Duct sizing methods includes equal friction, velocity reduction, static regain and T-method. Currently, the T-method is the mostly widely used method.

Changing the airflow distribution in duct systems could be accomplished by adjusting dampers, which is known as air balancing. Standard air balancing methods do not eliminate the secondary effect of damper, which can only approximate to the balanced status iteratively. Their trail-and-error natural makes them time consuming and costly. Advanced air balancing methods eliminating the secondary effect of damper in either implicit or explicit way. Implicit methods limit the secondary effect by a specific damper adjusting order and proper control of fan speed, which requires additional devices for balancing. Explicit methods estimate and compensate the secondary effect based on model of ducts, which need no more than the basic measuring devices.

Chapter 3. Duct model development and simulation

3.1 Introduction

Friction in fluid has long been studied. In the situation where fluid is enclosed by duct wall and driven by pressure, friction appears as pressure loss along the path, which is mostly studied in hydraulics. Estimating the required driving pressure for certain amount of flow rate is one of the typical problems in pipeline transportation. In a network of pipes/ducts, estimating the flow distribution becomes the most fundamental problem. Estimating pressure drop and flow distribution has wide applications in the petroleum industry for calculations of oil and gas pipelines, in civil engineering for calculation of water supply and district heating systems, in chemical engineering, and in all fields of engineering where fluid flow can be occurred. Because of the important applications in industries, an accurate duct model is essential. The duct model establishes the pressure loss function in terms of flow rates through ducts and estimates flow distribution in a network given the boundary pressures.

In ACMV system, there is a great importance and difficulty to estimate flow distribution for both air loop and water loop. This problem is closely related with many engineering problems. Duct layout design and sizing must be based on the calculation of airflow. The control of infiltration and contaminant propagation requires accurate estimation of indoor pressures. Partial load working condition that a branch of system is completely closed could lead to inefficient operating, and optimizations relies on the estimation of flow distribution in various situations. When retrofitting the system by adding branches or upgrading ducts, the air distribution could be changed considerably without careful estimation. A powerful and convenient tool for simulation could be significantly beneficial to studying the behavior of duct system and validate the air balancing methods. The Simscape library in MATLAB/Simulink environment is a DAE-based physical modeling tool. It obtains the convenient and hierarchical structure as well as the accessibility to powerful functions for scientific computing. Unfortunately, Simscape has not yet been applied to ACMV system.

This chapter is purposed on establish the mathematical model for a duct system and develop the tools for duct simulation. First, the estimation of friction loss and dynamic

loss in the ducts are introduced in Section 3.2 and Section 3.3 respectively. Second, the mathematical model for the entire duct network is developed in Section 3.4. Then, the simulation environment based on Simscape library is introduced in Section 3.5. And the benchmark validation is demonstrated in Section 3.6. Finally, Section 3.7 gives a short summary for this chapter.

3.2 Frictional loss estimation

The Darcy-Weisbach equation[127] in fluid dynamics describes the frictional loss in any pipe flow, including both air duct, water pipe and open channel flow.

$$\Delta P = C_f \frac{L}{D_h} \frac{1}{2} \rho V^2 \quad (3.1)$$

where

C_f = the friction coefficient

D_h = the hydraulic diameter

L = duct length

ρ = flow density

V = the average flow velocity

Equation (3.1) applies based on the assumption of 1) steady state, 2) fully developed flow, 3) incompressible and subsonic flow, 4) Newtonian fluid, and 5) macroscopic scale. The friction coefficient can be variable in different situations like laminar or turbulent, whose estimation is described in the next section in detail.

The friction coefficient is the most important factor for estimating the pressure loss in laminar and turbulent region. For laminar flow, generally identified by Reynolds number $Re < 10^5$, the C_f is estimated by $C_f = 64 / Re$. However, due to complex turbulent phenomena, C_f in turbulent region depends on not only Reynolds number but also the roughness of pipe wall. In 1939, Colebrook published an estimation equation for C_f in turbulent region[128], which is based on Prandtl's work and Nikuradse's investigations[129]. The final modified version suggested by American Gas Association in the case of natural gas pipelines[130, 131] produces maximal deviation up to 3.2%, which is mostly adopted by researchers.

$$\frac{1}{\sqrt{C_f}} = -2 \log_{10} \left(\frac{\epsilon}{3.7 D_h} + \frac{2.51}{\text{Re} \sqrt{C_f}} \right) \quad (3.2)$$

where ϵ is the roughness of duct wall with unit mm.

Despite good accuracy, the implicit form of the Colebrook's equation makes it difficult for applications in engineering. It must be solved iteratively to obtain the friction coefficient. Since a whole system may contain tens of thousands of pipes, the computational cost is significant even in the era of advanced computer technology. Therefore, it becomes attractive for researchers to develop the explicit approximations to Colebrook's equation to obtain friction coefficient by the Reynolds number and roughness in one step. The widely used Moody diagram[132], shown in Figure 3.1[133], is a plot of solutions for Colebrook's equation. It can be used conveniently to solve the friction coefficient by hand. Since Moody diagram is a graph representation, explicit analytic formulae are also needed for numerical simulation. A comprehensive review of the explicit approximations can be found in [134].

Figure 3.1: Moody diagram

For non-circular duct, a hydraulic diameter is introduced to substitute the diameter in the Darcy-Weisbach equation and Colebrook's equation. The hydraulic diameter is defined as:

$$D_h = \frac{4A}{P} \quad (3.3)$$

where

D_h = hydraulic diameter, m

A = duct area, m^2

P = perimeter of cross section, m

Experimental validations showed that calculating friction of non-circular duct using the hydraulic diameter has good agreement with measured data. Huebscher[135] found that for the same hydraulic diameter and same mean velocities, experiments using the round, square and rectangular ducts had the same flow resistance for most purposes. Melling and Whitelaw [136] also tested rectangular duct data for airflow over the range typical of ACMV systems and suggested that using hydraulic diameter is satisfactory.

Huebscher[135] suggested an equivalent diameter for rectangular duct based on equal flow, resistance and length comparing to circular duct:

$$D_e = 1.30 \frac{(a_1 a_2)^{0.625}}{(a_1 + a_2)^{0.250}} \quad (3.4)$$

where

D_e = equivalent diameter, m

a_1, a_2 = lengths of the two sides of rectangular duct, m

Heyt and Diaz[137] suggested an equivalent diameter for flat oval duct based on equal flow, resistance and length comparing to circular duct:

$$D_e = 1.55 \frac{(\pi a_2^2 / 4 + a_2 (a_1 - a_2))^{0.625}}{(\pi a_2 + 2(a_1 - a_2))^{0.250}} \quad (3.5)$$

where

D_e = equivalent diameter, m

a_1 = the minor axis of flat oval duct, m

a_2 = the major axis of flat oval duct, m

For commercial systems, flexible ducts are usually used to connect duct branches with terminal units. Because of high resistance, the fully stretched length of flexible duct is usually limited at 1.5m. A 70% extended flexible duct can be three to nine times higher resistance than a fully extended one. Besides, flexible ducts exhibit considerable variation in pressure drop, which can be in $\pm 15\%$ to $\pm 25\%$ range, depending on

differences in manufacturing, materials, inner linear uniformities, installation and draw-through or blow-through applications. To obtain the pressure drop of flexible duct, Abushakra et al.[138] and Culp et al.[139] provides the pressure loss correction factor for compressed flexible ducts of medium rough category ($\epsilon = 0.9\text{mm}$). The equation for pressure loss correction factor is:

$$\text{PF} = 1 + 58 \left(1 - \frac{L}{L_0} \right) e^{-4.96D} \quad (3.6)$$

where

PF = pressure drop correction factor, dimensionless

D = the flexible duct diameter, m

L = the installed duct length, m

L_0 = the fully extended duct length, m

It should be noted that the friction loss coefficient for bends in flexible ductwork vary widely from condition to condition without uniform or consistent trends[36].

3.3 Dynamic loss estimation

The estimation of dynamic losses is discussed in detail by Idelchik et al.[34]. Dynamic losses in fittings are caused by redistributing flow pattern during the changing of airflow direction or flow path area. Theoretically, dynamic losses occur along a duct length and cannot be separated from friction loss. For convenience of calculation, dynamic losses are assumed to be concentrated at a section and to exclude friction. Therefore, dynamic loss is also named as concentrated loss in some literatures. Because of complex fluid interactions, dynamic losses are usually estimated by empirical laws. The commonly accepted empirical law is that dynamic loss is proportional to the averaged dynamic pressure of the flow[140]:

$$\Delta P = C P_v = C \frac{1}{2} \rho V^2 \quad (3.7)$$

where

C = the local loss coefficient, dimensionless

ΔP = the dynamic loss, Pa

P_v = the dynamic pressure, Pa

ρ = the air density, kg/ m³

V = the averaged velocity, m/ s

The equation (3.7) is valid to estimate pressure drop subject to the standard test conditions described by ASHRAE standard 120-2008[141]. Equation (3.7) is a generalized form of equation (3.1) where frictional coefficient is explicitly proportional to duct length. Coefficients in equation (3.7) subjects to the complicated geometry of fittings, while coefficients in equation (3.1) is determined by Reynolds number and the roughness of inner surface. Dynamic loss is based on the actual velocity in the duct, not the velocity in the hydraulic diameter. For transitions, the inlet and outlet areas are different which causes different velocity. Consequently, the local loss coefficients are also different. The equation $C_{in} V_{in}^2 = C_{out} V_{out}^2$ relates the inlet and outlet local loss coefficients.

For converging and diverging flow junctions, total pressure losses for the main and branch section can be calculated separately by equation ((3.7) with different coefficient tables. However, either main or branch local loss coefficient depends on airflow rate of each other. Moreover, there exists the entrainment effect that local loss coefficient can even be negative for converging junctions in certain conditions. An example of local loss coefficients for a converging T-junction(ED5-3) in various conditions is presented in Figure 3.2[142] and Appendix A.

Figure 3.2: Schematic diagram of T-junction ED5-3

For fittings closely coupled (less than six hydraulic diameters apart), the flow pattern entering subsequent fittings differs from the flow pattern used to determine loss

coefficients. Therefore, adding the friction coefficients of the two fittings together simply gives incorrect estimation of fitting loss. However, experimental data for these situations are often unavailable.

The ASHRAE duct fitting database includes the loss coefficients for all 235 duct fittings. By the cross-sectional area, the fittings can be divided into round, rectangular and flat oval fittings. By the functionality, the fittings can be divided into supply, exhaust and common duct. Recently, this database is available online as a software for desktop computer and iOS app for mobile devices. Design engineers can deal with various duct fittings by pictorial outlines and estimate the pressure drop by entering flow rate and fitting information. Not only the coefficient table, the available software offers full calculation of pressure loss given the fitting's size and flow condition. Details during calculation is viewable. The mobile app is more convenient when quick pressure loss computation is needed in the field. The inputs can be adjusted by touch, and installation is automatic. Figure 3.3 shows the graphic user interface of the software.

Figure 3.3: ASHRAE duct fitting database software

In general, the pressure loss coefficients are nonlinearly dependent on given parameters. Due to the limitation of experimental data, storage, and computation, discrete values are available in the form of table. For the values between adjacent values available in the table, interpolations must be adopted. The ASHRAE duct fitting database applies linear interpolation for 1D and 2D tables as shown in Figure 3.4.

However, direct linear interpolation on coefficients could lead to a serious problem when applied on T-junctions and crosses. Taking SR5-21 as an example, the flow-pressure relationship is shown in Figure 3.5. The x-axis is the flow rate ratio, and the y-axis is the pressure drop and coefficients respectively. The stars are the points explicitly recorded on the ASHRAE's database while the blue lines are the interpolated values by linear interpolation method. An unreasonable wave-like behavior can be seen when estimating the pressure drop in the junction, especially at low flow ratio. It is clearly shown that because of fast change of coefficients in low flow rate region, the pressure loss is unreasonably high. Note that the y axis is logged, and thus the linearly interpolated coefficients seems curved in graph.

Figure 3.5: Flow-pressure curve using linear interpolated coefficients

This could result in multiple match problem. For example, there are eleven possible solutions for the SR5-21 junction, shown in Figure 3.6. To visualize the results, a 3D space is created where x axis is the flow ratio of branch 1 with the main branch, y axis is the flow ratio of branch 2 with the main branch, and z axis is the pressure at common duct end. The three surfaces drawn in this 3D space as series of parallel curves colored in red, green and blue represents the flow-pressure relationships of B1, B2 and S branch correspondingly, and their common intersections are the solution. The black solid points are the solutions where all three surfaces meet. Multi-solution problem is caused by the unnatural wave-like flow-pressure curves that are not monotonically increasing or decreasing as the flow ratio changes.

Figure 3.6: Solutions for junction using linear interpolated coefficients

In order to obtain reasonable results, we propose to perform linear interpolation on pressure loss instead of friction coefficients. Introducing the modified coefficient:

$$C_b^* = C_b \left(\frac{q_b}{q_c} \right)^2 \left(\frac{A_c}{A_b} \right)^2 \quad (3.8)$$

where

C_b^* is the modified pressure loss coefficient, dimensionless

C_b is the original pressure loss coefficient, dimensionless

q_b is the branch flow rate, m^3/s

q_c is the total flow rate, m^3/s

A_b is the branch outlet area, m^2

A_c is the main inlet area, m^2

By using the modified coefficient C_b^* , the branch pressure loss is obtained by:

$$\Delta P_b = C_b^* \frac{1}{2} \rho V_c^2 \quad (3.9)$$

where

ΔP_b is the pressure drop for branch, Pa

V_c is the velocity at main inlet, m/s

Equation (3.9) subjects to the standard testing conditions described in ASHRAE's standard 120-2008[141]. The proposed interpolation method can obtain smooth flow-pressure relationship, as shown in Figure 3.7. Comparing to original interpolation

method, the revised interpolation method has more smooth pressure loss curve in small flow rate region.

Figure 3.7: Flow-pressure curve using modified coefficients

By applying the revised interpolation method, the SR5-21 test converges to one solution, as shown in Figure 3.8. In this way, the intersection converges to single point.

Figure 3.8: Solutions for junction using modified coefficients

In summary, procedures of obtaining the pressure loss is listed below:

16. For each element in the given coefficient table, calculate the revised coefficient table by $C_b^* = C_b (q_b / q_c)^2 (A_c / A_b)^2$ using corresponding q_b / q_c and A_c / A_b at the position.
17. Use linear interpolation method to find out the friction coefficient C_b^* under the given condition
18. Calculate pressure loss by $\Delta P_b = C_b^* \rho V_c^2 / 2$

3.4 Mathematical model for duct network

An exemplary four-terminal duct network is shown in Figure 3.9. This duct model assumes incompressible fluid and steady state. For analyzing transient behavior of fluid in pipe network, one may refer to [143]. Hence there exists a good analogy between duct network and circuit network that the pressure is equivalent to the voltage and the airflow rate is equivalent to the current. Similar terminology is adopted to analyze the duct network. In the duct model, each component, like fan, duct and damper, is defined as a branch with a branch ID and a positive flow direction. The total number of branches is defined as b . Any point that joints two or more branches together is defined as a node, with a node ID. The total number of nodes is defined as n . To describe the topology of the network, an association matrix $\mathbf{A}$ of dimension $n \times b$ is defined as:

$$A_{ij} = \begin{cases} -1 & , \text{direction of branch } j \text{ enters node } i \\ 0 & , \text{branch } j \text{ is disconnected to node } i \\ 1 & , \text{direction of branch } j \text{ leaves node } i \end{cases} \quad (3.10)$$

Figure 3.9: Model of four-terminal duct network

The duct network shown in Figure 3.9 has 11 branches and 7 nodes, where all branches are indicated by boxed number and nodes are shown in underlined number. The association matrix $\mathbf{A}$ is given by:

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

By this definition, the mass conservation gives the constraints for airflow rates in the duct network:

$$\mathbf{A}\mathbf{q} = \mathbf{0} \quad (3.11)$$

where $\mathbf{q}$ is the airflow rate vector for all b branches. Equation (3.11) consists of n equations.

Furthermore, the pressure difference across branches is given by

$$\Delta\mathbf{P} = \mathbf{A}^T\mathbf{P} \quad (3.12)$$

where $\mathbf{P}$ is the nodal pressure vector with n dimensions, and $\Delta\mathbf{P}$ is the vector of pressure changes across every branch whose elements are determined by equations (3.1)-(3.7). Equation ((3.12) consists of b equations.

To show that these $n+b$ equations are sufficient to solve system state variables which includes b branch flow rates and n nodal pressures, property of the equations (3.11) and (3.12) must be investigated. The degree of freedom (DOF) of the flow rates is the dimension of the null space in $\mathbf{A}$. Mathematically, the basis of null space can be obtained by using singular value decomposition. Suppose $\mathbf{U}$ is the orthonormal basis of the null space of $\mathbf{A}$ such that:

$$\mathbf{A}\mathbf{U} = \mathbf{0} \quad (3.13)$$

$$\mathbf{U}^T\mathbf{U} = \mathbf{I} \quad (3.14)$$

Then the number of columns (basis) in $\mathbf{U}$ is the dimension of null space, namely the DOF of the system state. In principle, this DOF is essentially equal to the number of terminals n_T in the duct system, because once the airflow rates of all terminals are given,

the airflow rates of the internal branches can be obtained by summing its downstream terminal airflow rates. Therefore, the dimension of $\mathbf{U}$ is $b \times n_T$. If given a n_T dimensional vector $\mathbf{s}$ representing the internal state of the system, it is always possible to recovery the flow rates and pressures by:

$$\mathbf{q} = \mathbf{U}\mathbf{s} \quad (3.15)$$

$$\mathbf{P} = (\mathbf{A}\mathbf{A}^T)^{-1} \mathbf{A}\Delta\mathbf{P}(\mathbf{q}, \boldsymbol{\beta}, \mathbf{K}) \quad (3.16)$$

where

$\Delta\mathbf{P}$ = pressure drop function

$\boldsymbol{\beta}$ = collection of duct parameters

$\mathbf{K}$ = collection of damper parameters, $n_T \times 1$ vector

Note that by left multiplying $\mathbf{U}^T$ on equation (3.12), the pressure drop equation gives n_T inherent constraints:

$$\mathbf{U}^T \Delta\mathbf{P} = \mathbf{0} \quad (3.17)$$

which is exactly the independent equations to completely constrain the DOF of the system state variables and give a unique solution. Therefore, the listed n_T equations are sufficient to solve n_T system state variables and to determine the flows and pressures.

The solving procedure starts from an initial guess of state $\mathbf{X}(0) = [\mathbf{q}^T(0) \mathbf{P}^T(0)]^T$. Since the initial guess generally does not satisfy the system equations, the values must be updated iteratively until converge to the satisfactory small residuals. Without loss of generality, for any step k , the residuals of the equations are computed by:

$$\begin{bmatrix} \mathbf{q}(0) \\ \mathbf{P}(0) \end{bmatrix} = \mathbf{X}(0) \quad (3.18)$$

$$\boldsymbol{\varepsilon}_q = \lambda_q \mathbf{A}\mathbf{q}(k) \quad (3.19)$$

$$\boldsymbol{\varepsilon}_P = \lambda_P (\Delta\mathbf{P}(\mathbf{q}(k), \mathbf{K}, \boldsymbol{\beta}) - \mathbf{A}^T \mathbf{P}(k)) \quad (3.20)$$

$$\boldsymbol{\varepsilon}_X = \begin{bmatrix} \boldsymbol{\varepsilon}_q \\ \boldsymbol{\varepsilon}_P \end{bmatrix} \quad (3.21)$$

where

$\Delta\mathbf{P}$ = pressure drop functions

$\boldsymbol{\epsilon}_q$ = residuals on continuity equations.

$\boldsymbol{\epsilon}_p$ = residuals of pressure drop equations.

λ_q = scalar factor for residuals of continuity equation

λ_p = scalar factor for residuals of pressure drop equations.

In order to minimize the residuals, the system state should be modified by gradient descent:

$$\mathbf{X}(k+1) = \mathbf{X}(k) - \alpha \left(\frac{\partial \boldsymbol{\epsilon}_\mathbf{X}}{\partial \mathbf{X}} \right)^T \boldsymbol{\epsilon}_\mathbf{X} \quad (3.22)$$

where

α = step rate of updating system state

$\partial \boldsymbol{\epsilon}_\mathbf{X} / \partial \mathbf{X}$ = Jacobian matrix of residuals with respect to the state.

The converging criteria is defined by:

$$\|\boldsymbol{\epsilon}_\mathbf{X}\| < \epsilon_R \quad (3.23)$$

where

ϵ_R = convergence radius.

For convenience, the procedure to solve equations by ((3.18))-((3.23)) can be represented as a function:

$$\mathbf{X} = f(\mathbf{K}, \boldsymbol{\beta}) \quad (3.24)$$

In order to use this model for air balancing, the model parameter $\boldsymbol{\beta}$ must be identified from measured data. The choice of $\boldsymbol{\beta}$ is based on the dimensions of branch flow-pressure characteristics. For the ducts, only the duct length is selected as the dominant parameter, while both maximum flow rate and maximum pressure of the fan must be selected because they change fan curve in two different ways. Simulation function f predicts the flow rates and pressures in the duct system given the damper parameters and the model parameters.

Based on the duct network model, a class can be defined to implement the duct model in code. The class of duct network model consists of some properties and functions for defining and computing duct network. Properties include:

- `N_node`: the number of node
- `N_branch`: the number of branch

- `A`: association matrix
- `Branch_Pdrop`: function handle array to compute pressure drop given flow rate and parameters.
- `Branch_Param_idx`: index array for the parameters used in each branch
- `N_Param`: the number of parameter
- `Param`: the vector of all parameters
- `N_Damper`: number of damper
- `Damper_Branch_idx`: index array for the branch ID of each damper
- `Damper_Param`: the vector of all damper parameters
- `Flow_Residual_Scale`: the scalar factor for residuals of continuity equation
- `Pressure_Residual_Scale`: the scalar factor for residuals of pressure drop equation

The methods include:

- Construct and deconstruct function of the class
- `AddBranch()`: add a new branch into the model by specifying the node ID for the begin and end of this branch as well as the pressure drop function in this branch.
- `EquationResidual()`: compute the residual of equation of systems and its Jacobian given the state variables of the system. The residual consists of two parts. The first part is the residual of the continuity equation, and the second part is the residual of pressure drop equations.
- `Sim()`: calculate the system state variables by minimizing the residuals using gradient descent.

3.5 Simscape library for simulation

Simscape provides an environment for modeling and simulating physical system of multiple domains in Simulink. Many fundamental building blocks have been provided in the library, and the user only need to drag and connect the blocks to establish the model. Building a model using Simscape blocks is simple and straightforward. The only step is to drag components into Simulink workspace and connect them according to physical relationships. The Simscape also support customized domain and blocks. The

MathWorks Inc. provides Simscape Language and a full set of functions to develop new blocks by either using a set of DAEs or composing existing blocks together.

In Simscape, all the variables and parameters shared among components have to be defined in the domain. Simscape provides several build-in domains like Mechanical, Electrical, Pneumatic, Thermal and etc. Generally, a domain defines through variables, across variables and some properties. Through variable follows conservation laws that sums to be zero among interconnected ports, like force and current. Across variable will be identical in these interconnected ports, like velocity and voltage. The properties will be shared by all components in the network. Gravity, gas constants and other constants can be defined as property values. For our applications on ACMV duct system, we create an HVAC air domains based on the incompressible air assumption. According to the assumptions, the volumetric flow rate is defined as through variable, and the pressure is the across variable. Note that each variable must be specified with a tuple of initial value containing both the absolute value and the unit. Simscape converts the unit from one form to another during computation if necessary. We also define the density, kinematic viscosity and specific heat capacity as constant parameters in this domain. Code for defining HVAC_air domains is shown below:

```
domain HVAC_air
    variables
        p = {10325, 'Pa'}; % pressure
    end
    variables(Balancing = true)
        q = {1e-3, 'm^3/s'}; % flow rate
    end
    parameters
        density = { 1.204 , 'kg/m^3' }; % Fluid density
        viscosity_kin = { 15.11e-6 , 'm^2/s' }; % Kinematic
    viscosity
        c_p = { 1005 , 'J/kg/K' }; % Specific heat
    at constant pressure
        range_error = { 2 , '1' }; % Pressure below
    absolute zero
    end
end
```

With this customized HVAC_air domain, we can create customized components for modeling and simulations, which will be shown as a block in the Simulink environment. Any block components must contain several parts: nodes, variables, branches, parameters, setup function, and equations. Nodes section specifies the number of ports for these components, their names and their domains. In Simscape, instead of

using unidirectional inputs and outputs, the blocks are connected by bidirectional ports of certain domain. Variables section defines the internal variables shown in the equations. Branch section specifies the linkage between any two port, the direction and the representative internal variable. Parameters section defines some configurable variables which can be changed by user for each instance. Setup function is called whenever the parameters of the block is modified by user. It is majorly used to check the legality of the parameter values. The equations section defines the behavior of this component. After compiling, the codes will be converted into blocks that can be dragged and connected in MATLAB/Simulink environment. An example of connected blocks that represents a duct system is shown in Figure 3.10.

Figure 3.10: Simscape block diagram in MATLAB/Simulink environment

The simulation environment for Simscape blocks is integrated in Simulink. The control signals and sensor signals can easily be imported and exported via physical signal convertors. Therefore, it is simple and natural to monitor the simulation results as well as apply controller to the physical system. Besides, all the variables and parameters are accessible in Simulink and MATLAB. It is easy to optimize the performance of the system by changing parameters and re-starting simulation in scripts. In summary,

Simscape has large potential of integrating DAE based modeling method with traditional algorithms. To start simulation, a solver configuration block which defines the tolerance and algorithm must be connected to the Simscape block network. Besides, the Simulink solver must be selected as ode15s or ode23t algorithm for stiff problems.

3.6 Benchmark validation

The ASHRAE[144] has created an artificial duct network (Example 8) to illustrate the duct calculation procedures. The schematic of Example 8 can be found in Appendix B. Based on the schematics, the Simscape duct network model is built. In the example, the air flow for each branch is assumed, and the pressure loss is the target of calculation. Such assumption is not valid in real duct calculation because the flows are unknown and have to be determined during the calculation. Consequently, the result of duct calculation in Example 8 showed inconsistency of pressure drop. Namely, the pressure drops from atmosphere to a common point in the duct along two independent paths are different. Therefore, the true flow pattern will be different from the assumption. Example 8 stopped before calculating the true flow rates. In this thesis, the calculation result in Example 8 is reproduced and the true flow rates in the ducts is calculated.

The original calculation results are obtained by adding some constant flow rate sources on corresponding branches to maintain desired flow rate in the branch while generating sufficient pressure drop/boost to fill in the gap in the original calculation. The actual flow rate patterns can be obtained simply by linking the components together in their natural way and output the simulation results. Lately, note that there are many dampers in the duct system for balancing purposes but none of them are assigned meaningful values. In this time, the original computation, the actual flow rates and the balanced results are calculated. The flow rate and pressure drop of each branch in all three cases are summarized in Table 3.1.

Table 3.1: Example 8 Simulation Results

ID	Original Case		Actual Case		Balanced Case		Damper
	q (L/s)	ΔP (Pa)	q (L/s)	ΔP (Pa)	q (L/s)	ΔP (Pa)	
1	700	35.0129	690.134	35.1671	702.730	51.7309	5.7139°
2	250	57.0221	212.026	35.1671	245.797	51.7309	0.3983°
3	950	111.127 ^a	902.160	101.994	948.527	110.7122	0°
4	950	29.5259	997.840	29.9911	951.473	30.6775	3.5831°
5	950	92.5145 ^b	997.840	107.170	951.473	131.765	14.0113°
6	1900	102.457	1900.00	102.457	1900.00	102.457	N.A.
7	275	32.2125	335.697	34.8530	275.496	40.8245	15.700°
8	275	40.7355	218.180	34.8530	274.628	40.8245	0.1520°
9	550	24.5789	553.876	24.9200	550.124	24.5898	N.A.
10	550	64.5241 [*]	553.876	61.4334	550.124	64.5230	0.0444°
11	475	66.3677 [*]	472.511	65.6802	474.506	68.3618	2.4564°
12	475	67.9720 [*]	466.674	65.6802	474.489	68.3618	0.6127°
13	950	55.5232 [*]	939.185	55.5261	948.995	61.5754	3.4866°
14	1500	17.4490	1493.10	17.2650	1499.10	17.4417	N.A.
15	200	55.5232 [*]	216.567	63.0497	200.418	67.9792	0°
16	200	66.9949	190.372	63.0497	200.463	67.9792	12.528°
17	400	58.0070 ^c	406.939	75.4217	400.881	79.3997	1.0215°
18	1900	176.213 [*]	1900.00	176.213	1900.00	176.213	N.A.
19	1900	95.7082 [*]	1900.00	95.7082	1900.00	95.7082	10.671°

^aDue to different value of loss coefficient, value in Example 8 is 103.

^bDue to different value of loss coefficient, value in Example 8 is 103

^cDue to the change of loss coefficient as the database updates, value in Example 8 is 79

^{*}Due to rounding error, the values are different from Example 8

The pressure changes through the duct network can be illustrated in Figure 3.11 where the blue lines, red lines and green lines represent the original, actual and balanced cases correspondingly.

Figure 3.11: Pressure drop in Example 8

The flow rates for each branch is shown by bar chart in Figure 3.12. It can be seen clearly that the calculated damper position successfully balanced the duct system (yellow bars) to the desired flowrate (blue bars). Therefore, conclusion can be drawn that the proposed methods can calculate the suitable damper position for ACMV duct balancing.

Figure 3.12: Flow distribution in Example 8

3.7 Summary

Accurate estimation of the pressure loss through ducts and the airflow distribution is the theoretical foundation for air balancing. The pressure loss contains friction loss and dynamic loss. Friction loss can be estimated by Darcy-Weisbach equation in which a key factor, the friction coefficient, is obtained by solving implicit Colebrook's equation. Explicit approximations to the Colebrook's equation are often used in engineering applications. Non-circular ducts and flexible ducts can be estimated by introducing some modifications. The dynamic loss is estimated by empirical formula that is proportional to the dynamic pressure of airflow. The proportional factor, the local loss coefficient, is obtained by experiments. ASHRAE publishes the standards for conducting the experiments and the database of local loss coefficients for over 200 common duct fittings. Although the accuracy of the estimation of dynamic loss is often questioned, especially when fittings are relatively close, these data are widely used in engineering applications for duct design. For values that are located between two available data in the given table, linear interpolation on local loss coefficient is applied. However, this thesis demonstrates that such linear interpolation could cause undesired over-estimated pressure loss and wave-like behavior, which could lead to multiple

solutions in a duct model containing junctions. Modifications on linear interpolation is given to solve such problem. The flow rates in the duct system must be solved in the duct network which connects models of all components together. The concept of node, branch and association matrix is introduced in order to define the topology of the duct network. Equations for continuity and pressure loss is established to solve the flow rates in all branches and pressures in all nodes simultaneously. Computational method for solving this system of equations is provided. Based on the duct model, a class for the duct model is implemented, and a Simscape block library is created. The Simscape block library is the native DAE solver in MATLAB/Simulink environment that is deeply integrated with MATLAB and highly customizable. The HVAC physical domain is defined and blocks for various components in the duct system is implemented. Lastly, the presented mathematical models and simulation tool are tested by the exemplary duct system provided by ASHRAE's handbook. The results of original calculation, actual flow rates and the balanced configuration demonstrates the correctness, flexibility and potential of the model and simulation tools.

Chapter 4. Duct model identification

4.1 Introduction

The duct model that has been developed in Chapter 3 can predict pressures of the nodes and flow rates of the branches given the model parameters. The values of parameters therefore have significant effect on the accuracy of the model in real applications. The duct model developed by using duct fitting database may lead to significant error in practice. One possible situation is that duct fitting database may not cover the data for the exact fittings in the system, especially for two closely located fittings. Another possibility is that original duct layout is lost or the current duct size has changed when retrofitting existing ACMV system. Developing an algorithm that can identify duct model is useful and necessary. In this case, hybrid modeling method is suitable since the duct model structure is known and only the friction coefficients are undetermined. The hybrid modeling method obtains advantages of both analytical method and data-driven method that has high accuracy in predicting system behavior while explaining the undergo process in the system with good generality.

In order to completely capture the system characteristics and properly identify parameters, the measurements taken from the system must be sufficient, robust and efficient. First, the information in the measurements should be sufficient to identify the undetermined coefficients. Second, the coefficients determined by measurements should not be sensitive to the errors or noises of the measurements. Third, the measurements should be obtained conveniently such that parameter identification is efficient, and the result should be verifiable.

With the obtained data, the model parameters should be identified such that the model can best fit with the measured data. In this sense, the curve fitting methods such as least squares are often applied. One issue should be taken carefully is the over-fitting that the model using identified parameters has high accuracy only on the experimental data but fail to generalize the predictability. The Bayesian method is a statistical approach that regards the system parameters as random variables and models all possible outcomes in all cases of model parameters. The measurements are treated as a given condition that constrain the possibilities of the parameters. Usually the set of parameters

with highest possibility will be selected. In Bayesian method, over-fitting could be alleviated by introducing prior probability on the system parameters to constrain values in reasonable range.

In this chapter, methods for acquiring data and identifying model is developed and tested. The procedure to acquire measurements is described in Section 4.2. The parameter identification algorithm is introduced in Section 4.3. The experimental duct system is introduced in Section 4.4. The experimental validation results are shown in Section 4.5. And Section 4.6 gives a short summary for this chapter.

4.2 Method of measurements acquisition

For the duct networks to be measured, it is reasonable to make the following assumptions:

- The duct system is a connected tree. There exists and only exists one path between any two nodes in the network.
- The source of duct system is unique. Multiple fans in parallel or series can be replaced by an equivalent fan.
- The undetermined parameters are observable. There is no such combination of changes in parameter that can maintain all measurements unchanged.

Suppose the duct network has n_T terminals, it has been shown[30] that the maximum number of undetermined parameters for passive branch is $2n_T - 2$, and there are two undetermined parameters for fan. In total, the maximum number of parameters to be identified is $2n_T$.

As there are n_T terminals, measurements taken from only one configuration is unable to obtain enough data for identifying model parameter. Considering the sensitivity of the change of configuration and the balance, it is best to monitoring the terminal closest to the damper that is under adjustment and alter the adjusted dampers for different measurements. Therefore, each terminal should take multiple measurements in different damper positions. In order to track the damper positions easily, the dampers that are not under adjusted should keep fully open. The number of measurements taken at different damper positions per terminal, denoted by m , must satisfy $m \geq 2$ in order to obtain at least $2n_T$ measurements to solve for all variables. For this purposes, the first two measurements should choose fully open and fully closed

damper positions to cover the maximum and minimum flow rate conditions. Furthermore, increasing m will be beneficial to obtain better result since the uncertainties caused by measurements can be reduced. Implementing the pressure sensor at the upstream side of damper and flow meter at the measured terminal, while keeping all the other dampers fully open, the measurement procedures are shown in Figure 4.1 and summarized below:

1. Set all dampers fully open, and fan operating at normal speed
2. Record the maximum flow rate and corresponding pressure in fully open case
3. Adjust the damper to $m-1$ different positions, including fully closed, and record the corresponding flow rates, pressures and damper positions.
4. Set the measured damper back to fully open position.
5. Move to the next damper and repeat procedure 2 to 4 until all dampers have been tested.

Figure 4.1: Measurement procedure for model identification

In the example shown in Figure 3.9, the duct system with four terminals can be measured in four steps. Before measuring, fully open all the dampers in the duct system and start the fan. In the first step, setup the flow sensor at branch 8 and the pressure sensor at node 4. Gradually close damper T1 and take measurements on m different damper positions from fully open to fully closed. After finished measurements in the first terminal, reset the damper T1 fully open and setup the flow sensor at branch 9 and pressure sensor at node 5. Then measuring process repeats for the second terminal that

gradually close damper T2 and take measurements at m different damper positions. This process continues for the third terminal (branch 10 and node 6) and the fourth terminal (branch 11 and node 7) to complete the measure procedure.

4.3 Parameter identification algorithm

In Bayes approach, the observations in experiment is a random vector $\mathbf{Z}$, and the system parameter β is also random, with a prior probability distribution $P(\beta)$. The probability distribution of observations $P(\mathbf{Z})$ is conditionally dependent on the system parameter β , and therefore, leads to a joint probability distribution $P(\mathbf{Z}, \beta)$, such that:

$$P(\mathbf{Z}, \beta) = P(\mathbf{Z} | \beta) P(\beta) \quad (4.1)$$

Assume the experiments consist of n_p steps, each of which is independent to the others. Moreover, each step of experiment corresponds to a unique setting, which is the damper positions, $\mathbf{K}_i$. So, the conditional probability could be further expanded as a product of conditional probability of each experiment:

$$P(\mathbf{Z} | \beta) = \prod_{i=1}^{n_p} P(\mathbf{Z}_i | \beta, \mathbf{K}_i) \quad (4.2)$$

In any step i , the system is set to certain damper positions $\mathbf{K}_i$ to reach certain pressures $\mathbf{P}_i$ and air flow rate $\mathbf{q}_i$ of the entire system, which completely determines all measurable results $\mathbf{Z}_i$. Hence, measurements $\mathbf{Z}_i$ is conditionally independent to system parameter β and damper positions $\mathbf{K}_i$ given the system variables $\mathbf{P}_i$ and $\mathbf{q}_i$.

$$P(\mathbf{Z}_i | \beta, \mathbf{K}_i) = P(\mathbf{Z}_i | \mathbf{P}_i, \mathbf{q}_i) P(\mathbf{P}_i, \mathbf{q}_i | \beta, \mathbf{K}_i) \quad (4.3)$$

The probability $P(\mathbf{Z}_i | \mathbf{P}_i, \mathbf{q}_i)$ is closely related with the sensor position and the measuring process. Each sensor measures certain pressure or flow rates in the system, with some uncertainties. The measuring process can be modeled by normal distribution:

$$P(\mathbf{Z}_i | \mathbf{P}_i, \mathbf{q}_i) \propto \exp \left[-(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} (\mathbf{Z}_i - \boldsymbol{\mu}_i) \right] \quad (4.4)$$

and

$$\boldsymbol{\mu}_i = \mathbf{C}_{P,i} \mathbf{P}_i + \mathbf{C}_{q,i} \mathbf{q}_i \quad (4.5)$$

where

$\boldsymbol{\mu}_i$ is the expectation of the measurements

Σ_i is the covariance of the sensors which describes the characteristics of measurements and the sensor accuracy.

$C_{p,i}$ is the pressure sensor position matrix of size $n_T m \times n$ that indicates the location of pressure sensor

$C_{q,i}$ is the flow sensor position matrix of size $n_T m \times b$ that indicates the location of flow sensor

For example, in the case of four terminal duct system, the number of step, or the unique configurations of experiments, is $n_p = 1 + n_T(m-1)$. In the first step, all dampers are fully open. The pressures at nodes 4, 5, 6 and 7 are measured by pressure sensors, and the flows through branches 8, 9, 10 and 11 are measured by flow sensors. Hence the observation matrix $C_{p,1}$ is an 8×7 sparse matrix that:

$$\begin{aligned} C_{p,1}(1,4) &= 1 \\ C_{p,1}(2,5) &= 1 \\ C_{p,1}(3,6) &= 1 \\ C_{p,1}(4,7) &= 1 \end{aligned}$$

The flow observation matrix $C_{q,1}$ is an 8×11 sparse matrix:

$$\begin{aligned} C_{q,1}(5,8) &= 1 \\ C_{q,1}(6,9) &= 1 \\ C_{q,1}(7,10) &= 1 \\ C_{q,1}(8,11) &= 1 \end{aligned}$$

The covariance matrix is a diagonal matrix:

$$\Sigma_1 = \text{diag} \left\{ \left[\sigma_p^2 \quad \sigma_p^2 \quad \sigma_p^2 \quad \sigma_p^2 \quad \sigma_q^2 \quad \sigma_q^2 \quad \sigma_q^2 \quad \sigma_q^2 \right] \right\}$$

where

σ_p is the sensor uncertainty for pressure sensor

σ_q is the sensor uncertainty for flow sensor

The probability $P(\mathbf{P}_i, \mathbf{q}_i | \boldsymbol{\beta}, \mathbf{K}_i)$ is characterized by the system equation that $\mathbf{A}\mathbf{q}_i = \mathbf{0}$ and $\Delta\mathbf{P}(\mathbf{q}_i, \mathbf{K}_i, \boldsymbol{\beta}) - \mathbf{A}^T \mathbf{P}_i = \mathbf{0}$. The probabilities of the system variables are proportional to the exponential of negative quadratic of the residual:

$$P(\mathbf{P}_i, \mathbf{q}_i | \boldsymbol{\beta}, \mathbf{K}_i) \propto \exp \left[-\boldsymbol{\varepsilon}_{q,i}^T \boldsymbol{\varepsilon}_{q,i} - \boldsymbol{\varepsilon}_{p,i}^T \boldsymbol{\varepsilon}_{p,i} \right] \quad (4.6)$$

where

$\varepsilon_{q,i}$ is the residuals of continuity equations in measurement i

$\varepsilon_{p,i}$ is the residuals of pressure drop equations in measurement i

In each step of the experiment, $\mathbf{K}_i$ represents the system configuration in this experiment, which is basically the damper positions. According to the measurement acquisition procedure, the number of all possible combinations is $1+4(m-1)$. For each damper j , the damper parameter $K_{i,j}$ can take m different values from the set $\kappa^{(j)}$ corresponding to m different positions.

$$\kappa^{(j)} = \{\kappa_0^{(j)}, \kappa_1^{(j)}, \dots, \kappa_{m-1}^{(j)}\} \quad (4.7)$$

where

$\kappa_0^{(j)}$ = damper parameter at fully open status

$\kappa_{m-1}^{(j)}$ = damper parameter at fully closed status

For example, in the four-terminal duct system, the first step of experiment corresponds to all damper fully open. The damper parameter $\mathbf{K}_1 = [\kappa_0^{(1)} \quad \kappa_0^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T$. For the following $m-1$ steps, the first damper is adjusted but the other are remained fully open. Therefore:

$$\begin{aligned} \mathbf{K}_2 &= [\kappa_1^{(1)} \quad \kappa_0^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T \\ \mathbf{K}_3 &= [\kappa_2^{(1)} \quad \kappa_0^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T \\ &\vdots \\ \mathbf{K}_m &= [\kappa_{m-1}^{(1)} \quad \kappa_0^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T \end{aligned}$$

From the 5 steps, the first damper is fully open, but the next damper starts to change. Therefore,

$$\begin{aligned} \mathbf{K}_{m+1} &= [\kappa_0^{(1)} \quad \kappa_1^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T \\ &\vdots \\ \mathbf{K}_{2m-1} &= [\kappa_0^{(1)} \quad \kappa_{m-1}^{(2)} \quad \kappa_0^{(3)} \quad \kappa_0^{(4)}]^T \\ \mathbf{K}_{2m} &= [\kappa_0^{(1)} \quad \kappa_0^{(2)} \quad \kappa_1^{(3)} \quad \kappa_0^{(4)}]^T \\ &\vdots \\ \mathbf{K}_{3m-2} &= [\kappa_0^{(1)} \quad \kappa_0^{(2)} \quad \kappa_{m-1}^{(3)} \quad \kappa_0^{(4)}]^T \end{aligned}$$

$$\begin{aligned}\mathbf{K}_{3m-1} &= \begin{bmatrix} \kappa_0^{(1)} & \kappa_0^{(2)} & \kappa_0^{(3)} & \kappa_1^{(4)} \end{bmatrix}^T \\ &\vdots \\ \mathbf{K}_{4m-3} &= \begin{bmatrix} \kappa_0^{(1)} & \kappa_0^{(2)} & \kappa_0^{(3)} & \kappa_{m-1}^{(4)} \end{bmatrix}^T\end{aligned}$$

Based on the above formation procedure, although there are in total $1+4(m-1)$ sets of vector $\mathbf{K}_i$, the actual number of independent variables is only $4m$ in total. By substituting equation ((4.2), ((4.3), ((4.4), and ((4.5) into ((4.1), the conditioned probability is obtained:

$$P(\boldsymbol{\beta} | \mathbf{Z}) \propto \prod_{i=1}^{N_p} \exp \left[-(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} (\mathbf{Z}_i - \boldsymbol{\mu}_i) - \boldsymbol{\varepsilon}_{\mathbf{q},i}^T \boldsymbol{\varepsilon}_{\mathbf{q},i} - \boldsymbol{\varepsilon}_{\mathbf{P},i}^T \boldsymbol{\varepsilon}_{\mathbf{P},i} \right] P(\boldsymbol{\beta}) \quad (4.8)$$

To find the maximum probability, equation ((4.8) is equivalent to minimizing the negative logarithm of the probability, hence we obtain:

$$\hat{\boldsymbol{\beta}} = \min_{\boldsymbol{\beta}, \kappa^{(j)}, \mathbf{P}_i, \mathbf{q}_i} \sum_{i=1}^{N_p} \left[(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} (\mathbf{Z}_i - \boldsymbol{\mu}_i) + \boldsymbol{\varepsilon}_{\mathbf{q},i}^T \boldsymbol{\varepsilon}_{\mathbf{q},i} + \boldsymbol{\varepsilon}_{\mathbf{P},i}^T \boldsymbol{\varepsilon}_{\mathbf{P},i} \right] - \ln P(\boldsymbol{\beta}) \quad (4.9)$$

Hence define the objective function for optimization:

$$F(\boldsymbol{\beta}, \kappa^{(j)}, \mathbf{P}_i, \mathbf{q}_i) = \sum_{i=1}^{N_p} \left[(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} (\mathbf{Z}_i - \boldsymbol{\mu}_i) + \boldsymbol{\varepsilon}_{\mathbf{q},i}^T \boldsymbol{\varepsilon}_{\mathbf{q},i} + \boldsymbol{\varepsilon}_{\mathbf{P},i}^T \boldsymbol{\varepsilon}_{\mathbf{P},i} \right] - \ln P(\boldsymbol{\beta}) \quad (4.10)$$

Optimization is a classical numerical problem which is in general extremely difficult. Extensive researches has been made on this topic and a lot of algorithms are available. Mathematically, the approaches to the optimization problem can be classified by the degree of derivatives of the objective function that has been used in the computation.

Zeros-order methods, or searching method, use only the original objective function without any derivatives. These methods basically compare the objective values at different candidate positions to determine the better one, and iteratively propose new candidates based on the historical results until converge. Typical searching methods include binary search, climbing hill method, genetic algorithm and particle swarm optimization.

First-order methods, or gradient-based method, use the information of first-order derivative of the objective function with respect to the optimized variables to help propose new candidates during numerical optimization process. The first-order derivative of objective function is the gradient vector, which is defined by:

$$\mathbf{g} = \nabla F(x) = \frac{\partial F}{\partial x} \quad (4.11)$$

The gradient vector in this parameter optimization problem is then:

$$\mathbf{g} = \left[\frac{\partial F}{\partial \boldsymbol{\beta}} \quad \frac{\partial F}{\partial \kappa^{(j)}} \quad \frac{\partial F}{\partial \mathbf{P}_i} \quad \frac{\partial F}{\partial \mathbf{q}_i} \right]^T \quad (4.12)$$

where

$$\begin{aligned} \frac{\partial F}{\partial \boldsymbol{\beta}} &= \sum_{i=1}^{N_p} \left[\boldsymbol{\varepsilon}_{\mathbf{P},i}^T \frac{\partial \boldsymbol{\varepsilon}_{\mathbf{P},i}}{\partial \boldsymbol{\beta}} \right] - \frac{1}{P(\boldsymbol{\beta})} \frac{\partial P(\boldsymbol{\beta})}{\partial \boldsymbol{\beta}} \\ \frac{\partial F}{\partial \kappa^{(j)}} &= \sum_{i=1}^{N_p} \left[\boldsymbol{\varepsilon}_{\mathbf{P},i}^T \frac{\partial \boldsymbol{\varepsilon}_{\mathbf{P},i}}{\partial \kappa^{(j)}} \right] \\ \frac{\partial F}{\partial \mathbf{P}_i} &= \sum_{i=1}^{N_p} \left[-(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} \mathbf{C}_{P,i} - \boldsymbol{\varepsilon}_{\mathbf{P},i}^T \boldsymbol{\lambda}_{\mathbf{P}} \mathbf{A}^T \right] \\ \frac{\partial F}{\partial \mathbf{q}_i} &= \sum_{i=1}^{N_p} \left[-(\mathbf{Z}_i - \boldsymbol{\mu}_i)^T \boldsymbol{\Sigma}_i^{-1} \mathbf{C}_{q,i} + \boldsymbol{\varepsilon}_{\mathbf{q},i}^T \boldsymbol{\lambda}_{\mathbf{q}} \mathbf{A} \right] \end{aligned}$$

In gradient-based method, the searching direction is specified by the gradient vector, while the searching step size can be possibly proportional to the norm of gradient vector or based on other specifications. Gradient descent method is the most typical gradient based method whose update change of positions is proportional to the gradient directly. Many variants of gradient descent exist to improve the performance. Directional search method is another gradient based method which chooses a suitable step size along the gradient direction to update the positions.

Second-order methods uses the information of second-order derivatives of the objective function, the Hessian matrix, to help determine the update of positions. Hessian matrix characterize the local curvature of the objective function at a given point, which is defined by:

$$\mathbf{H} = \frac{\partial^2 F}{\partial x_i \partial x_j} \quad (4.13)$$

The second-order methods further exploit the local information on a position to determine the searching direction and the step size of the updates in one iteration.

Newton's method and Levenberg-Marquardt method are typical second-order methods who have wide applications in engineering.

Any optimization problem whose eigenvalues of the Hessian matrix are all positive is a convex problem. The convex optimization problem has unique local minima which is the global minima, hence an unconstrained convex problem is considered simple among all optimization tasks. Unfortunately, when identifying system parameters, we must confront the general non-convex case. In this case, several prominent challenges can be possibly involved. Some challenges arise even when optimizing convex functions, and the most general one in numerical optimization is ill-conditioning of the Hessian matrix $\mathbf{H}$. Ill-conditioning can manifest by causing the gradient descent method to get stuck in the sense that even very small steps increase the objective function. Newton's method is an excellent tool for minimizing convex objective function with poorly conditioned Hessian matrix, but it requires a significant modification before it can be applied to parameter identification. At local minima, the gradient of function vanishes, the step size could reduce to zero. Local minima are good for convex problem because any local minima can be the global minima in this case. However, for non-convex problem like parameter identification, stuck in any local minima that is not globally optimal could be a serious problem. For high-dimensional non-convex problem, local minima are in fact rare compared to another type of point with zero gradient: a saddle point. Unlike local minima, the eigenvalues of Hessian matrix near saddle point contain both positive and negative values. It is clear that saddle points constitute a problem for Newton's method. There may also be wide flat regions of constant value where all gradients and Hessians vanish. Such degenerate locations pose major problems for all optimization methods. The opposite of the plateaus is the extremely steep regions resembling cliffs. On the face of a steep cliff structure, the gradient update step can move the parameters extremely far that jumps off the cliff structure altogether. This can be avoided using gradient clipping to limit the step size within a reasonable small range.

This chapter adopts the RMSProp combined with Nesterov momentum which has an adaptive descent rate. RMSProp shrinks the descent rate using an exponentially decaying average to discard history from the extreme past so that it can converge rapidly

after finding a convex bowl. Empirically, RMSProp has shown to be an effective and practical optimization algorithm. The algorithm is shown as below:

```

Given the hyper parameter: global descent rate  $\alpha$ , decay
rate  $\gamma$ , momentum coefficient  $\eta$ 

Initialize the parameter  $\beta$  by random, velocity  $\mathbf{v}$  by
random, accumulation variable  $r \leftarrow 0$ 

While stopping criterion not meet do
    Compute interim update  $\tilde{\beta} \leftarrow \beta + \eta \mathbf{v}$ 
    Compute gradient  $\mathbf{g} \leftarrow \partial F / \partial \beta|_{\tilde{\beta}}$ 
    Accumulate gradient  $r \leftarrow \gamma r + (1 - \gamma) \mathbf{g} \cdot \mathbf{g}$ 
    Compute velocity update  $\mathbf{v} \leftarrow \eta \mathbf{v} - \alpha \mathbf{g} / \sqrt{r}$ 
    Apply update  $\mathbf{x} \leftarrow \mathbf{x} + \mathbf{v}$ 

End while

```

4.4 Experimental duct system

The experimental ACMV system is located at EEE-ERI@N Joint Lab of the Nanyang Technological University in Singapore. The conditioned space is about 400 m². The system consists of one AHU, one liquid desiccant dehumidification system (LDDS) and four active chilled beam (ACB) terminals. Figure 4.2 shows the photo of AHU.

Figure 4.2: AHU in the small ACMV system

Figure 4.3 shows the LDDS in the system.

Figure 4.3: LDDS in the small ACMV system

Figure 4.4 shows the photo of ACB terminals installed on the ceiling. Figure 4.5 demonstrates the details of ACB terminal.

Figure 4.4: Ceiling installation of ACB terminal

Figure 4.5 ACB terminal in the small ACMV system

The ducts in the system is shown in Figure 4.6. The duct is made of galvanized steel. The interior roughness is 0.09mm. The exterior of duct is covered by fiberglass duct wrap insulation. The cross-section shape of the duct is rectangular. The size of ducts ranges from 200mm to 450mm. The end of duct is connected to terminals by flexible duct. The flexible duct is D160mm round duct with fiberglass insulations. The elbows in the main duct is 90 degree turn with no duct vanes. T-junction is variable size.

Figure 4.6: Ducts in the small ACMV system

Figure 4.7 shows the schematic drawings of the system.

Figure 4.7: Duct design layout of the small ACMV system

The damper is D160mm butterfly damper, shown in Figure 4.8. The damper is controlled by actuator shown in Figure 4.9.

Figure 4.8: Dampers in the small ACMV system

Figure 4.9: Damper controller in the small ACMV system

The hotwire anemometer is shown in Figure 4.10.

Figure 4.10: Hotwire anemometer in experiments

The capture hood is shown in Figure 4.11.

Figure 4.11: Capture hood in experiments

The photo for pressure sensor is shown in Figure 4.12.

Figure 4.12: Pressure sensor in experiments

The detail specifications of the sensors are listed in Table 4.1.

Table 4.1: Sensor and Data Acquisition System Specifications

Instrument	Item	Value
Dwyer MS-311 Differential Pressure Sensor	Range	0-250 Pa
	Accuracy	1%
	Output	0-10V
TSI Air Capture Hood 8380	Range	42-4250 m ³ /hr
	Accuracy	12 m ³ /hr
	Resolution	1 m ³ /hr

The photo of data acquisition and control system is shown in Figure 4.13.

Figure 4.13: Data acquisition system in experiments

4.5 Experimental validation

Based on this experimental duct system, it is necessary to verify the duct model and parameter identification algorithm. If works properly, one should be able to predict the terminal pressure and flow rates in any damper position. To validate the duct model and the parameter identification algorithm, a duct model for the experimental duct system is established, shown in Figure 4.14. This model consists of 7 nodes and 11 branches. Branch 1 is fan, which has two undetermined parameters. Branches 2, 3, 4, 6, 8, and 10 contain 1 undetermined parameter each. Branches 5, 7, 9 and 11 are Dampers 1, 3, 2 and 4 respectively. For any damper position, there is a corresponding damper parameter, but the exact mapping function is unclear. In total, the number of undetermined parameters is 8.

Figure 4.14: Duct model of the small ACMV system

This model structure provides a suitable parameter space of 8 dimensions where a best-fit model could exist to capture the pressure and flow characteristics of the real system. By applying the parameter identification algorithm, the most suitable set of parameters corresponding to the best-fit model should be obtained. This best fit model should have sufficient accuracy to predict the behavior of the duct system in any possible damper positions. Considering the accuracy of measurements in this system, a maximum of 6% tolerance is adopted. If the maximum of relative error on flow rate or pressure prediction exceed 6%, the above hypothesis should be rejected.

The test experiment is conducted in two stages both of which consists of multiple times of measurements on all terminals. In the first stage, the sampling stage, five

different configurations of damper positions are tested, denoted by A0 to A4. In A0 configuration, all dampers are fully open. From A1 to A4, each damper is closed in turns while others are remained fully open. By the end of each step, the dampers will recover to the fully open status. In the second stage, the testing stage, 20 times of continuous measurements on all terminals will be conducted, denoted by B1 to B20. Starting from all dampers fully open, one damper will be chosen randomly and adjusted to a random damper position in each step. Unlike the first stage, the damper positions will not be recovered to fully open status after measuring. Therefore, after several times of tests, the duct system will enter a configuration that all damper positions are non-zero, which will definitely not be covered in the sampling stage. The prediction accuracy by the model in this step could represents the generality of the model. The detail experimental procedure is conducted as follows:

1. Start the fan inverter and set the frequency to be 40Hz
2. Set the damper position to fully open for all dampers
3. Calibrate the capture hood and differential pressure sensor used in the system.
4. Starting from the first terminal, install the capture hood to cover the terminal completely.
5. Record the flow rate measurements after the reading is stable.
6. Insert the high-pressure end of the pressure probe into the duct's test hole and block the remaining gaps.
7. Record the pressure measurements after the reading is stable.
8. Uninstall the measurement devices and move to the next terminal until all four terminals are tested.
9. Set the damper position of the first terminal to 90 (fully closed).
10. Repeat steps 3 to 8 to get measurements for all terminals.
11. Recovery the damper position of the first terminal back to zero (fully open).
12. Repeat steps 9 to 11 for the second, third and fourth terminal to obtain data in these configurations.
13. Reset all dampers fully open.
14. Randomly generated an integer from 1 to 4 to choose the adjusted damper
15. Randomly generated a real number from 0 to 90 for the damper position.

16. Adjust the chosen damper to the appointed position.
17. Repeat steps 3 to 8 to get measurements for all terminals.
18. Record the index of the adjusted damper and the corresponding positions.
19. Repeat steps 14 to 18 for 20 times

All data obtained from experiments should be either used for identifying model or testing model accuracy but not both. The entire data obtained in A0 is used in identification. In A1, A2, A3, and A4, the data of the first, second, third and fourth terminal is used respectively. The remaining data in A1, A2, A3 and A4 are all used in testing. Based on the identification data, the model parameters are calculated. After model identification, the prediction of pressures and flow rates at the same damper positions as A0, A1, A2, A3, and A4 are calculated. Then, starting from B1, the damper parameters in the model are tuned sequentially such that the predicted air flow rates corresponding to the adjusted dampers match with the experiment records. Because these experimental data are involved in calculating predicting data, they should be marked as assistive data and should not be used for comparing predicting accuracy. The rest experimental data is compared with the predictions to estimate model accuracy.

Table 4.2: Flow and pressure data in Section A of experimental validation

	Flow (CMH)	Terminal 1	Terminal 2	Terminal 3	Terminal 4
	Pressure (Pa)				
A0		62.9	130.4	128.5	106.0
		94.1	91.9	84.4	93.2
A1		0.9	138.1	144.0	108.0
		104.2	104.5	94.4	103.6
A2		71.4	0.4	153.5	115.3
		118.8	121.1	107.2	117.6
A3		71.1	147.2	2.4	115.9
		117.6	118.0	119.5	118.8
A4		69.3	143.4	150.5	1.1
		111.8	112.3	103.0	114.0

The measurements taken in the experiments at the Section A are listed in Table 4.2. The data on grey background are used for parameter identification, the data on white background are used for model accuracy validation. The measurements of Section B are shown in Table 4.3 and Table 4.4. For convenience, the flow rates and pressures in all terminals are visualized in Figure 4.15 and Figure 4.16 correspondingly. The four lines

in different color represent the values taken from four terminals. The change of damper position for the four damper is shown in Figure 4.17.

Figure 4.15: Flow change in Section B for each terminal

Figure 4.16: Pressure change in Section B for each terminal

Figure 4.17: Damper positions in Section B for each terminal

Table 4.3: Flow data in Section B of experimental validation

Flow (CMH)	Terminal 1	Terminal 2	Terminal 3	Terminal 4	Adjusted Damper	Damper Position
B1	65.4	131.6	120.6	102.3	3	35.8
B2	2.7	145.7	134.9	115.8	1	86.4
B3	2.3	139.7	129.7	113.9	2	26.7
B4	2.0	140.2	101.7	116.8	3	52.4
B5	4.2	52.2	107.2	120.4	2	65.3
B6	2.3	52.6	109.3	106.8	4	50.7
B7	62.8	54.2	105.2	97.7	1	58.0
B8	59.4	49.4	104.3	114.9	4	33.9
B9	71.4	49.4	103.0	110.0	1	35.0
B10	72.8	54.3	104.0	91.6	4	52.5
B11	68.6	50.4	105.1	89.7	1	39.2
B12	75.1	49.8	111.4	16.7	4	75.9
B13	73.8	49.7	149.8	20.0	3	27.5
B14	72.6	49.8	155.9	15.5	1	49.2
B15	71.8	48.7	148.1	17.3	3	35.0
B16	71.8	9.9	147.2	15.9	2	79.7
B17	72.4	8.9	158.6	18.0	3	3.8
B18	63.7	142.2	144.1	14.3	2	19.2
B19	58.5	134.8	132.4	90.8	4	48.6
B20	63.2	133.8	126.1	92.1	3	34.3

Table 4.4: Pressure data in Section B of experimental validation

Pressure (Pa)	Terminal 1	Terminal 2	Terminal 3	Terminal 4	Adjusted Damper	Damper Position
B1	93.0	95.0	88.1	91.9	3	35.8
B2	108.9	103.0	97.0	105.6	1	86.4
B3	108.8	106.0	101.3	104.2	2	26.7
B4	113.0	109.4	110.4	112.4	3	52.4
B5	132.9	134.5	124.9	131.5	2	65.3
B6	137.0	136.6	130.5	135.5	4	50.7
B7	126.0	124.0	119.3	124.9	1	58.0
B8	123.4	123.1	113.7	121.7	4	33.9
B9	119.6	121.4	115.0	118.9	1	35.0
B10	123.9	124.2	121.4	125.4	4	52.5
B11	123.8	123.5	118.5	125.7	1	39.2
B12	139.3	137.1	133.4	137.9	4	75.9
B13	130.0	132.3	121.0	131.4	3	27.5
B14	132.9	131.9	119.1	131.2	1	49.2
B15	133.0	132.5	123.1	132.3	3	35.0
B16	137.2	138.5	125.8	138.6	2	79.7
B17	138.0	135.6	124.0	135.7	3	3.8
B18	111.0	109.9	102.3	110.9	2	19.2
B19	96.2	94.8	88.2	97.7	4	48.6
B20	98.5	98.0	90.0	100.5	3	34.3

Based on these data, the parameters are identified. Using the identified parameter, the pressures and flow rates can be predicted by the model. For predicting A1 to A5, the damper positions are set as either zero (fully open) or infinity (fully closed) as the configurations in experiments. For the following B1 to B20, the corresponding dampers are adjusted such that the flow rates through that terminal is equal to the record in experiment. The pressures and flow rates predictions on other three terminals will be compared with the records.

Table 4.5: Flow predictions vs. measured data in Section A

Flow (CMH)	Terminal 1	Terminal 2	Terminal 3	Terminal 4	
All fully open	62.9	130.4	128.5	106.0	Measured
	62.8	130.5	128.5	106.0	Predicted
	0.1	-0.1	0.0	0.0	Error
ACB 1 closed	0.9	138.1	144.0	108.0	
	1.0	138.9	136.7	112.7	
	-0.1	-0.8	7.3	-4.7	
ACB 2 closed	71.4	0.4	153.5	115.3	
	71.3	0.7	145.7	120.2	
	0.1	-0.3	7.8	-4.9	
ACB 3 closed	71.1	147.2	2.4	115.9	
	71.0	147.5	2.4	120.1	
	0.1	-0.3	0.0	-4.2	
ACB 4 closed	69.3	143.4	150.5	1.1	
	69.6	144.6	142.7	1.2	
	-0.3	-1.2	7.8	-0.1	
MAPE					5.17%

Table 4.6: Pressure predictions vs. measured data in Section A

Pressure (Pa)	Terminal 1	Terminal 2	Terminal 3	Terminal 4	
All fully open	94.1	91.9	84.4	93.2	Measured
	93.3	91.7	84.3	93.0	Predicted
	0.8	0.2	0.1	0.2	Error
ACB 1 closed	104.2	104.5	94.4	103.6	
	105.7	103.9	95.4	105.2	
	-1.5	0.6	-1.0	-1.6	
ACB 2 closed	118.8	121.1	107.2	117.6	
	120.3	120.3	108.5	119.6	
	-1.5	0.8	-1.3	-2.0	
ACB 3 closed	117.6	118.0	119.5	118.8	
	119.2	117.2	119.4	119.4	
	-1.6	0.8	0.1	-0.6	
ACB 4 closed	111.8	112.3	103.0	114.0	
	114.6	112.6	104.0	114.7	
	-2.8	-0.3	-1.0	-0.7	
MAPE					2.48%

Table 4.5 and Table 4.6 present the comparison between the prediction and the measured values in flow rates and pressures for Section A, respectively. Note that the

cell with grey background is the data used for identifying parameters, and thus not involved in calculating the overall accuracy.

For Section B, the comparison of flow rates and pressures for each terminal is shown in Figure 4.18 and Figure 4.19 separately. The points with a cross above it is not involved in estimating accuracy.

Figure 4.18: Flow predictions vs. measured data in Section B

Figure 4.19: Pressure predictions vs. measured data in Section B

Based on these data, the model accuracy can be computed. For Section A, the model accuracy is estimated to be 5.17%. The model accuracy according to Section B is 5.81%(MAPE), which agrees with what in Section A. Since this value is less than the tolerance, the statement that the duct model with the identification algorithm can predict behavior of the duct system is accepted.

4.6 Summary

Hybrid modeling method gains both advantages from analytical modeling and data-driven methods that obtains both accuracy and generality. In order to develop hybrid model, some measurements must be taken from experiments to determine the model parameters. The measurements are selected carefully to satisfy the criterion of sufficiency, robustness and efficiency. Based on the obtained data, the model parameter should be selected such that the discrepancy to the measurements is minimized. However, over-fitting could occur during this process that the fitted parameters loss the generality to the unseen situations. In this chapter, a Bayes approach to optimize the posteriori probability over the entire parametric space is introduced. The measurements taken from experiments are treated as certain constraints to the probability distribution of the parameters. By optimizing the probability, the system parameters are identified such that over-fitting problem could be alleviated by the prior knowledge about parameters.

The methods of acquiring data from experiments are conducted in n_T steps. Measurements are taken for all n_T terminals where each step could take m times of measurements from the fully open to fully closed, which could gain more accuracy of parameter identification.

The parameter identification algorithm is derived by Bayes law. The posterior probability of system parameter conditioned on all measurements from experiments could be separated into a product of conditioned probabilities for each step of the experiments which is independent to other steps. The system variable can be served as a bridge to connect the system parameter with the obtained measurements. By building up the measuring process and the system simulation process separately, the entire formula of the posterior probability of the system parameter can be estimated. Finally, the parameter identification can be implemented by optimizing the probability function.

The model parameter identification algorithm is validated in an experimental ACMV system in the lab. The system consists of four terminals with proper instruments to measure airflow rates and pressures and adjust dampers. The model for the system is built up and experimental data are collected. Based on the data, the system parameters are identified to predict the pressures and flows in various conditions. The testing results

have shown that this method could be able to predict system behavior with high agreements with experimental results for unseen testing conditions. In summary, the model and the model identification algorithm are suitable for the duct system.

Chapter 5. Model-based method for air balancing

5.1 Introduction

Balancing a fluid distribution system is both a physical problem and engineering problem. Flow balancing can be defined as the process to achieve desired flux within tolerance on the surface of a given control volume by adjusting parameters of the enclosed system. In particular, air balancing studies the control of airflow in ACMV duct system. The air balancing problem is a special case of fluid balancing where both space and time dimension are constrained. Air balancing is considered as one-dimensional problem because the velocity distribution on any cross section of the duct is replaced by the averaged velocity. The remaining dimension is the length along the duct, since the air movement is constrained by the boundary of ducts. This simplification is reasonable because only the gross volume rate through duct is concerned for engineering applications. On the other hand, only the steady state of the airflow distribution in the ducts is modeled, because the transient process to establish air balance to satisfy the conditions of flow-pressure relationship is significantly faster than most ACMV control process when adjusting the airflow resistance of ducts.

Altering the flow distribution in any given duct network can be accomplished by adjusting dampers. Traditional methods are iterative approach based on the rule of thumb, and have been adopted as industrial standards. Since the iterative process is time-consuming and costly because of trial-and-error natural, the idea of non-iterative air balancing becomes popular in the research of duct systems. Comparing with iterative method, the non-iterative methods eliminate the secondary effect of dampers implicitly or explicitly. Model based method explicitly estimates and compensates the change of airflow due to secondary by proper damper adjustments. This method needs a computational model of duct system, but requires no additional control devices like implicit non-iterative methods.

In this chapter, the model-based air balancing method is proposed and evaluated. The methods to obtain damper position for balancing is developed in Section 5.2. The approach to implement damper adjustment is established in Section 5.3. Simulations to investigate the effectiveness of the method are discussed in Section 5.4, and the

experiments are conducted to validate the method in Section 5.5. Finally, the summary for this method is drawn in Section 5.6.

5.2 Calculation of damper position for balancing

For any given combination of damper positions in each damper, denoted by $\mathbf{K}$, there exist a vector of flow rates at each terminal $\mathbf{q}_T$. The mapping from $\mathbf{K}$ to $\mathbf{q}_T$ can be mathematically expressed given the duct model by:

$$\mathbf{q}_T = \mathbf{B}_T f(\mathbf{K}, \boldsymbol{\beta}) \quad (5.1)$$

where function f is defined by equation (3.24) and the matrix $\mathbf{B}_T$ is a $n_T \times (n+b)$ sparse matrix where $B_{T,ij} = 1$ if the flow rate of terminal i is represented by j th variable.

The matrix $\mathbf{B}_T$ denotes the mapping from the duct system variables to the terminal airflow rates. In the example of the duct model shown in Figure 3.9, the matrix $\mathbf{B}_T$ is given by:

$$\mathbf{B}_T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

In fact, if given the terminal flow rate $\mathbf{q}_T$, any internal branch can be computed by summing the flow rates of its downstream terminal. This can be shown by the conservation equation $\mathbf{A}\mathbf{q} = \mathbf{0}$. Note that we can always rearrange the association matrix $\mathbf{A}$ to separate the terminal branch from the internal branches, and separate the terminal upstream node from internal nodes such that the last n_T columns of matrix $\mathbf{A}$ are the terminal branches and the last n_T rows are the terminal upstream nodes. Therefore:

$$\begin{bmatrix} \mathbf{A}_{II} & \mathbf{A}_{IT} \\ \mathbf{A}_{TI} & \mathbf{A}_{TT} \end{bmatrix} \begin{bmatrix} \mathbf{q}_I \\ \mathbf{q}_T \end{bmatrix} = \mathbf{0} \quad (5.2)$$

where the $\mathbf{A}_{II}$ is $(n-n_T) \times (b-n_T)$ matrix, $\mathbf{A}_{IT}$ is $(n-n_T) \times n_T$ matrix, $\mathbf{A}_{TI}$ is $n_T \times (b-n_T)$ matrix and $\mathbf{A}_{TT}$ is $n_T \times n_T$ matrix. It is easy to show that $\mathbf{A}_{TT}$ is an identity matrix and $\mathbf{A}_{IT} = \mathbf{0}$ because the terminal branches are only connected with the terminal

upstream nodes with outward direction. By solving conservation equation, internal flow rates $\mathbf{q}_I$ is given by:

$$\mathbf{q}_I = -\left(\mathbf{A}_{II}^T \mathbf{A}_{II} + \mathbf{A}_{TI}^T \mathbf{A}_{TI}\right)^{-1} \mathbf{A}_{TI}^T \mathbf{A}_{TI} \mathbf{q}_T \quad (5.3)$$

Given a small perturbation $\delta \mathbf{q}_T$ on $\mathbf{q}_T$, there is a small change of $\mathbf{q}_I$:

$$\delta \mathbf{q}_I = -\left(\mathbf{A}_{II}^T \mathbf{A}_{II} + \mathbf{A}_{TI}^T \mathbf{A}_{TI}\right)^{-1} \mathbf{A}_{TI}^T \mathbf{A}_{TI} \delta \mathbf{q}_T \quad (5.4)$$

Similarly, the pressure drop equation can be written as:

$$\begin{bmatrix} \Delta \mathbf{P}_I \\ \Delta \mathbf{P}_T \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{II}^T & \mathbf{A}_{TI}^T \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{P}_I \\ \mathbf{P}_T \end{bmatrix} \quad (5.5)$$

Given small change in $\mathbf{K}$, the flow rates and pressures in the system change according to:

$$\frac{\partial \Delta \mathbf{P}_I}{\partial \mathbf{q}_I} \delta \mathbf{q}_I = \mathbf{A}_{II}^T \delta \mathbf{P}_I + \mathbf{A}_{TI}^T \delta \mathbf{P}_T \quad (5.6)$$

$$\frac{\partial \Delta \mathbf{P}_T}{\partial \mathbf{q}_T} \delta \mathbf{q}_T + \frac{\partial \Delta \mathbf{P}_T}{\partial \mathbf{K}} \delta \mathbf{K} = \delta \mathbf{P}_T \quad (5.7)$$

Note that $\partial \Delta \mathbf{P}_I / \partial \mathbf{q}_I$ and $\partial \Delta \mathbf{P}_T / \partial \mathbf{q}_T$ are diagonal matrixes because the pressure drop of one branch could only depends on the flow through this branch and the damper parameter at a branch. By substituting the equations together, a set of equations for the $b - n_T$ internal branches are established:

$$\mathbf{M} \delta \mathbf{q}_T = \mathbf{A}_{II}^T \delta \mathbf{P}_I + \mathbf{A}_{TI}^T \frac{\partial \Delta \mathbf{P}_T}{\partial \mathbf{K}} \delta \mathbf{K} \quad (5.8)$$

where

$$\mathbf{M} = -\frac{\partial \Delta \mathbf{P}_I}{\partial \mathbf{q}_I} \left(\mathbf{A}_{II}^T \mathbf{A}_{II} + \mathbf{A}_{TI}^T \mathbf{A}_{TI}\right)^{-1} \left(\mathbf{A}_{II}^T \mathbf{A}_{II}\right) - \mathbf{A}_{TI}^T \frac{\partial \Delta \mathbf{P}_T}{\partial \mathbf{q}_T}$$

If there is no change in air flow, $\delta \mathbf{q}_T = \mathbf{0}$, it must be satisfied that:

$$\delta \mathbf{K} = -\left(\frac{\partial \Delta \mathbf{P}_T}{\partial \mathbf{K}}\right)^{-1} \left(\mathbf{A}_{TI}^T \mathbf{A}_{TI}\right)^{-1} \mathbf{A}_{TI}^T \mathbf{A}_{II} \mathbf{A}_{II}^T \delta \mathbf{P}_I \quad (5.9)$$

Therefore, if the internal pressure increases as the fan speed increases, it is always possible to adjust damper position $\mathbf{K}$ in an appropriate way such that the terminal flow rate remains same. This agrees with the engineering experience that total airflow is

controlled by the fan and the proportioning of airflow in terminal is controlled by the damper position. This means that one degree of freedom of the airflow rates should be reserved for the total airflow and the rest $n_T - 1$ DOF is used in air balancing. To eliminate the degree of freedom for total airflow, define the normalized flow:

$$\bar{\mathbf{q}}_T = \frac{\mathbf{q}_T}{\|\mathbf{q}_T\|_1} \quad (5.10)$$

Then the target of air balancing becomes the normalized design airflow rate $\bar{\mathbf{q}}_D = \mathbf{q}_D / \|\mathbf{q}_D\|_1$. Similarly, the degree of freedom of the damper should be reduced by one. According to the ASHRAE standards, at least one damper should keep fully open in order to minimize the energy consumption of the fan.

To locate the damper that should be kept fully open, the terminal airflow when all dampers are fully open can be used, which has been measured previously. Noted that each adjustment of one damper will cause flow increase in all other dampers. The last damper that should be adjusted is the one which has largest gap to the desired airflow. Therefore, the critical damper that should be kept fully open is determined by:

$$k_O = \max_i \bar{q}_{D,i} - \bar{q}_{O,i} \quad (5.11)$$

where $\bar{q}_{D,i}$ is the i element of normalized design flow rate $\bar{\mathbf{q}}_D$ and $\bar{q}_{O,i}$ is the i element of normalized full open flow rates $\bar{\mathbf{q}}_O$. Specifically, $\mathbf{q}_O = \mathbf{B}_T f(\mathbf{K}_O, \boldsymbol{\beta})$ where $\mathbf{K}_O$ is the damper parameter for fully open status.

After locate the critical damper, the optimal damper positions for the rest of dampers can be calculated to meet the target airflow rates $\mathbf{q}_D$. An optimization problem can be established to obtain the optimal damper positions as follows:

$$\mathbf{K}_D = \min_{\mathbf{K}} \frac{1}{2} f(\mathbf{K}, \hat{\boldsymbol{\beta}})^T \mathbf{B}_T^T \mathbf{B}_T f(\mathbf{K}, \hat{\boldsymbol{\beta}}) - \mathbf{q}_D^T \mathbf{B}_T f(\mathbf{K}, \hat{\boldsymbol{\beta}}) \quad (5.12)$$

subject to

$$K_{k_0} = 0$$

For this optimization problem, the trust-region method can be applied to find $\mathbf{K}_D$.

5.3 Implement damper adjustments

In practice, adjusting dampers is not always possible given only the optimal damper parameters $\mathbf{K}_D$ because the dampers are not necessary to have accurate indicators for this value. Inspired by traditional air balancing method, it is desirable if the proposed method adopts similar way to adjust dampers according to the flow rates measured at the terminals. Unlike traditional methods, the secondary effect of dampers can be estimated and compensated, if the target airflow rates during damper adjusting are properly designed. This is achieved by simulating the damper adjusting process in the duct model where dampers are sequentially tuned to the optimal damper positions. The resulting change of flow rates at the terminals will be the target for actual damper adjustments.

Initially, set all dampers fully open and let $\tilde{\mathbf{K}}(0) = \mathbf{0}$. Sorting the dampers according to their optimal damper parameters $K_{D,i}$ in descending order, and adjust the damper starting from the first one $K_{D,max}$. Substituting $K_{D,max}$ to the corresponding position in $\tilde{\mathbf{K}}(1)$ and let the other elements inherit $\tilde{\mathbf{K}}(0)$, the situation for the first step of damper adjustments can be simulated. The terminal airflow rates at this step can be obtained by:

$$\tilde{\mathbf{q}}_T(1) = \mathbf{B}_T f(\tilde{\mathbf{K}}(1), \hat{\boldsymbol{\beta}})$$

Then, the corresponding estimated flow rate $\tilde{\mathbf{q}}_T(1)$ at the first damper can be used as the indicator for adjusting this damper. Considering that secondary effect of all other dampers always result in the increase of air flow rate at this damper, the target flow rate $\tilde{q}_T$ should be smaller than the final design value q_D . Once the flow rate in actual duct system at this terminal reaches the value, this damper is considered as balanced and no more adjustments are needed.

In the following, the balancing process is an iteration from $i=1$ to n_T . In any iteration i , the status of balancing is: dampers 1, 2, ..., $i-1$ have been properly adjusted, but dampers $i, i+1, \dots, n_T$ are still fully open. Note that $i=1$ corresponds to the initial status that all dampers are fully open. Substituting the optimal damper position $K_{D,i}$ to damper i and compute the adjusting target $\tilde{\mathbf{q}}_T(i)$ by:

$$\tilde{\mathbf{q}}_T(i) = \mathbf{B}_T f(\tilde{\mathbf{K}}(i), \hat{\boldsymbol{\beta}}) \quad (5.13)$$

where

$$\tilde{K}_j(i) = \begin{cases} K_{D,j} & , j = i \\ \tilde{K}_j(i-1) & , j \neq i \end{cases}$$

With the $\tilde{\mathbf{q}}_T(i)$ as the target, damper i can be adjusted by measuring the actual terminal flow rate as an indicator and approach to the indicator. When the indicator reaches the target, the damper is considered as properly adjusted. Note that only $n_T - 1$ steps of damper adjusting are needed because there is always one fully open damper that will be skipped during this process.

5.4 Simulation studies

To verify this air balancing method, we build up a simulation environment for a duct system, which is shown in Figure 5.1. Here:

- Fan is represented by a triangle within a circle
- Damper is represented by an inclined plate rotated around a small circle
- Fan inlet is represented by a grounded sign because it is open to atmosphere
- Terminal outlet is represented by a white arrow which is also open to atmosphere
- Each node is represented by a circled number
- Each branch is represented by a link between nodes with a number arrow

Figure 5.1: Model of the duct system for simulations and validation

The models and design parameters for each branch is listed in Table 5.1. In this eight-terminal duct system, the eight solid dampers that directly connected to the eight terminals are necessary to achieve any flow distribution. In comparison to the method given in [124], this method dropped the intermediate dampers as indicated in dashed lines.

Table 5.1: Model specifications of the duct system in simulations

Branch	Model	Parameters
1	$\Delta P = P_{\max} \left(\frac{q^2}{Q_{\max}^2} - 1 \right)$	$P_{\max} = 160\text{Pa}$, $Q_{\max} = 400 \text{ l/s}$
2		$L = 2.5\text{m}$, $D = 150\text{mm}$
3	$\Delta P = C_f \left(\frac{L}{D} \right) \left(\frac{\rho}{2} V^2 \right)$	$L = 0.5\text{m}$, $D = 80\text{mm}$
5		$L = 1.5\text{m}$, $D = 80\text{mm}$
7	$q = \pi (D/2)^2 V$	$L = 2.5\text{m}$, $D = 120\text{mm}$
8		$L = 0.5\text{m}$, $D = 80\text{mm}$
10	$C_f = 8 \left[\left(\frac{8}{\text{Re}} \right)^{12} + \frac{1}{(A+B)^{1.5}} \right]^{1/12}$	$L = 1.5\text{m}$, $D = 80\text{mm}$
12		$L = 2.5\text{m}$, $D = 150\text{mm}$
13		$L = 1.5\text{m}$, $D = 80\text{mm}$

15	$A = \left[-2.475 \ln \left(\left(\frac{7}{\text{Re}} \right)^{0.9} + \frac{0.27\varepsilon}{D} \right) \right]^{16}$	$L = 2.5\text{m} , D = 120\text{mm}$
16		$L = 1.5\text{m} , D = 80\text{mm}$
18		$L = 2.5\text{m} , D = 100\text{mm}$
19		$L = 1.5\text{m} , D = 80\text{mm}$
21	$B = (37530 / \text{Re})^{16}$	$L = 3\text{m} , D = 80\text{mm}$
		$\rho = 1.204 \text{ kg/m}^3$
		$\varepsilon = 0.015 \text{ mm}$
		$\nu = 15.11 \times 10^{-6} \text{ m}^2/\text{s}$

Since the association matrix **A** is large and sparse, **Error! Not a valid bookmark self-reference.** only demonstrates the non-zero elements in the association matrix **A**.

Table 5.2: The association matrix of the duct system in simulations

Node	Association matrix A	Node	Association matrix A
1	$A_{1,1} = -1, A_{1,2} = 1, A_{1,12} = 1$	8	$A_{8,12} = -1, A_{8,13} = 1, A_{8,15} = 1$
2	$A_{2,2} = -1, A_{2,3} = 1, A_{2,5} = 1, A_{2,7} = 1$	9	$A_{9,13} = -1, A_{9,14} = 1$
3	$A_{3,3} = -1, A_{3,4} = 1$	10	$A_{10,15} = -1, A_{10,16} = 1, A_{10,18} = 1$
4	$A_{4,5} = -1, A_{4,6} = 1$	11	$A_{11,16} = -1, A_{11,17} = 1$
5	$A_{5,7} = -1, A_{5,8} = 1, A_{5,10} = 1$	12	$A_{12,18} = -1, A_{12,19} = 1, A_{12,21} = 1$
6	$A_{6,8} = -1, A_{6,9} = 1$	13	$A_{13,19} = -1, A_{13,20} = 1$
7	$A_{7,10} = -1, A_{7,11} = 1$	14	$A_{14,21} = -1, A_{14,22} = 1$

To measure data for identification, nodal pressure at nodes 3, 4, 6, 7, 9, 11, 13 and 14 superposed with a Gaussian white noise of magnitude $\sigma_p = 2\text{Pa}$ are used for each damper, and similarly, the flow rates through branches 4, 6, 9, 11, 14, 17, 20 and 22 superposed with a Gaussian white noise of magnitude $\sigma_q = 10^{-3} \text{ m}^3/\text{s}$ are used. The number of measurements for each damper is chosen as $m = 3$, which corresponds to the three different damper positions: fully open, half open and fully closed. A total of 24 sets of pressure and flow measurements are obtained, shown in Figure 5.2.

Figure 5.2: Measurements obtained in simulation
Number of measurements per terminal is $m=3$. The true pressure-flowrate curve for each damper is shown in blue, error bars for sensors are shown black and the final measured values are shown in red.

Based on the measurements, we can obtain vector $\mathbf{Z}$ by grouping these 24 flow rates and 24 pressures together. The covariance matrix $\mathbf{\Sigma}$ is constructed by using flowmeter's uncertainty $\sigma_q = 10^{-3} \text{ m}^3/\text{s}$ and pressure sensor's uncertainty $\sigma_p = 2 \text{ Pa}$ respectively. Then, by applying the trust region algorithm for optimizing the objective function of the identification problem, the estimated duct parameter is obtained and shown in Table 5.3. After identification, by comparing the fully open flow rates for each damper with the desired flow rate, damper 8 is identified as the fully open damper. Then by using equation ((5.12), the optimal damper positions are calculated, and the tuning sequence with corresponding flow rates for the sequential tuning is also listed, shown in Table 5.4.

Table 5.3: Model parameter estimated by identification

Parameters	True β	β_0	d_i	$\hat{\beta}$	Error
$P_{\max}$ (Pa)	160	150	0.3	155.04	-3.1%
$Q_{\max}$ (l/s)	400	500	0.3	403.55	0.9%
L_2 (m)	2.5	3.0	0.5	2.52	1.0%
L_3 (m)	0.5	1.0	0.5	0.485	-3.1%
L_5 (m)	1.5	2.0	0.5	1.45	-2.9%
L_7 (m)	2.5	3.0	0.5	2.37	-5.1%
L_8 (m)	0.5	1.0	0.5	0.452	-9.6%

L_{10} (m)	1.5	2.0	0.5	1.54	2.8%
L_{12} (m)	2.5	3.0	0.5	2.45	-1.9%
L_{13} (m)	1.5	2.0	0.5	1.59	6.2%
L_{15} (m)	2.5	3.0	0.5	2.41	-3.6%
L_{16} (m)	1.5	2.0	0.5	1.64	9.5%
L_{18} (m)	2.5	3.0	0.5	3.04	21.5%
L_{19} (m)	1.5	2.0	0.5	1.55	3.5%
L_{21} (m)	3	3.0	0.5	2.70	-10.1%

Table 5.4: Damper adjustment sequence in simulation

Step	Action	Target damper position	Transient target flow rate (l/s)
1	Adjust Damper 1	70.28	25.65
2	Adjust Damper 5	69.21	27.55
3	Adjust Damper 2	67.99	28.66
4	Adjust Damper 3	65.99	31.15
5	Adjust Damper 4	69.50	30.22
6	Adjust Damper 6	65.17	31.14
7	Adjust Damper 7	60.48	31.86
	Leave Damper 8 open	0	N.A.

According to the procedures in Table 5.4, the dampers are adjusted and the balancing results are shown in Table 5.5. To quantify the effectiveness of balancing, we use the definition that $MAPE = \max_i |(q_i - q_{i,d}) / q_{i,d}|$, where q_i is the normalized flow rate of the i th terminal and $q_{i,d}$ is the corresponding normalized desired flow rate. The MAPE value reveals the worst imbalance in the duct network. The proposed TAB method successfully reduces the MAPE to 3.35%, which satisfies the ASHRAE's standard (10%). Therefore, the proposed TAB method is valid for balancing duct network system.

Table 5.5: Balancing results in simulation

Terminal	θ_i	q_i (L/s)	$\bar{q}_i$	$\bar{q}_{i,t}$	Error
1	70.28	30.82	12.08%	12.5%	-3.55%
2	69.21	31.53	12.36%	12.5%	-1.13%
3	67.99	32.34	12.68%	12.5%	1.4%
4	65.99	32.90	12.90%	12.5%	3.17%
5	69.50	31.26	12.26%	12.5%	-1.96%
6	65.17	31.67	12.41%	12.5%	-0.70%
7	60.48	31.86	12.49%	12.5%	-0.11%
8	0	32.74	12.83%	12.5%	2.67%
MAPE					3.35%

It is noted that the number of measurements m at each terminal is a trade-off between accuracy and cost. To understand how balancing accuracy is affected, we perform the

previous procedures in a series of tests. The results are shown in Table 5.6. The accuracy improves as more measurements are taken. The ultimate accuracy is limited by the sensor. As the number of measurements increases, the major cost increment is the additional man-hour and effort for taking measurements. Each addition measurement costs about 30s (adjusting damper position and taking measurements). Comparing to labor cost, the computational cost is insignificant. However, due to the complexity of the system, theoretical trade-off curve for optimizing number of measurements are still under analysis. The author suggests that for general purposes ventilation system, three to five measurements are enough, and for special ventilation system in hospital, laboratory, clean room or process industry, taking seven or more measurements are possible to guarantee higher balancing accuracy.

Table 5.6: Balancing accuracy under different number of measurements

Terminal	$\bar{q}_{i,t}$	$m=2$	$m=3$	$m=5$	$m=7$	$m=10$
1	12.5%	12.10%	12.08%	12.63%	12.57%	12.58%
2	12.5%	12.36%	12.36%	12.56%	12.53%	12.53%
3	12.5%	12.71%	12.68%	12.51%	12.51%	12.54%
4	12.5%	12.90%	12.90%	12.50%	12.43%	12.42%
5	12.5%	12.00%	12.26%	12.60%	12.49%	12.51%
6	12.5%	12.26%	12.41%	12.46%	12.44%	12.42%
7	12.5%	12.27%	12.49%	12.47%	12.42%	12.38%
8	12.5%	13.40%	12.83%	12.28%	12.60%	12.61%
Overall MAPE		7.23%	3.35%	1.77%	0.78%	0.96%

To evaluate the performance and efficiency of the proposed method, comparison tests with progressive flow method [124] and conventional ratio method [122, 145] have been conducted. The progressive flow method requires a fan speed control loop and some sub-branch dampers in addition to the given test environment. It should be mentioned that tuning control loop parameter is not trivial: the time consumption for this step is not negligible, and the requirement for experienced engineers will also increase the cost. Once the control loop has been set up, the damper adjustment is very efficient because each damper is adjusted only once. But, because of the same reason, the sensor error is proportionally reflected in the final balancing result. In this test, the final balancing MAPE is 4.13%. For ratio method, it is almost unavoidable to over-dampen some terminals after first iteration of adjustments which makes the fully-open terminal changing from the smallest airflow ratio to the largest airflow ratio. If keeping

wide open the terminal to avoid waste fan power, the ratio method becomes difficult to converge. For this eight-terminal duct system, this method takes four iterations to reduce MAPE down to 9.89%. The comparison result is shown in Table 5.7. The proposed method obtains competitive balancing results in terms of low MAPE. For the operational complexity, the proposed method is also significantly simpler. Although this method measures three times for each terminal, the marginal time consumption for additional measurements is relative small because the major man-hour cost is moving from one terminal to another and setting up the measuring unit. For the damper adjustment procedure, this method is almost as simple as the progressive flow method. Overall, the proposed method is an accurate and efficient TAB method.

Table 5.7: Accuracy and efficiency comparison among different methods

Method	MAPE	Complexity
Model-based method	3.35%	8×3 initial measurements 8 damper adjustments
Progressive flow method	4.13%	Install and test fan speed control loop 8 initial measurements 8+1 damper adjustments
Ratio method	9.89%	8 initial measurements 4 iterations × 8 damper adjustments

5.5 Experimental validation

The measurements taken in the experiments are listed in Table 5.8. Based on these data, the parameters are identified and predictions of the obtained model are also listed in this table.

Table 5.8: Measured data and model predictions in the experiment

Terminal	Damper Open		Damper Closed		
	Pa	m ³ /hr	Pa	m ³ /hr	
ACB 1	88.3	62	77.2	11	Measured
	86.9	61	76.7	3	Predicted
ACB 2	68.9	131	112.6	3	
	67.6	130	112.9	3	
ACB 3	69.8	132	104.8	1	
	69.5	132	107.0	3	
ACB 4	82.5	104	86.1	6	
	82.4	104	85.7	3	

Based on the obtained model, the optimal damper positions are calculated and the instructions for air balancing are generated as below:

1. Set all dampers fully open;

2. Adjust damper 2 such that the flow rate of ACB 2 is 64.8 m³/hr;
3. Adjust damper 3 such that the pressure of ACB 3 is 70.0 m³/hr;
4. Adjust damper 4 such that the pressure of ACB 4 is 72.6 m³/hr;

Finally, to verify the balancing results, the flow rates of all ACB terminals are measured, and the results are shown in Table 5.9. Note that the maximum absolute percentage error is 4.7%, which satisfies ASHRAE's standard.

Table 5.9: Balancing results in the experiment

Terminal	Target	Final Flow Rate (m ³ /hr)	Relative Error
ACB 1	25%	78 (26.17%)	4.7%
ACB 2	25%	75 (25.17%)	0.67%
ACB 3	25%	71 (23.83%)	-4.7%
ACB 4	25%	74 (24.83%)	-0.67%
MAPE			4.7%

5.6 Summary

Air balancing is an important means to ensure proper air delivery, good indoor air quality, good indoor air quality, and high energy efficiency. Since ACMV duct system changes with time and use, air balancing must be performed regularly. The related standards have been published by organization like ASHRAE, NEBB etc. The traditional air balancing approach is largely based on rule of thumb and requires iteratively trial-and-error to approximate the design airflow rate on each terminal. The primary reason is the complex interactions between different terminals and difficulties to predict airflows in duct system. Consequently, the traditional methods become inefficient, time-consuming and costly. The ideal of non-iterative balancing arises research interests recently. Two types of approaches to non-iterative balancing are available. The first approach tries to limit or eliminate the coupling effect. To reduce the change of airflow rates in terminals that has been adjusted properly, the automatic control of variable speed drive for fan and a specific tuning sequence must be applied. The procedure of damper adjustments starts from all damper fully closed, and internal branch damper may be required if the airflow in one terminal is less than desired airflow rate even fully opened. The other approach is to estimate the coupling effect by means of modeling duct system such that the adjustment of damper is the optimal to the balancing airflow at the end of balancing. This approach requires a proper model for a duct system, an efficient measuring procedure to identify model parameter and a suitable method to calculate the optimal damper position to guide adjustments. This approach does not require special equipment for fan speed control, and has much wider applications. This chapter describes a method for air balancing based on the second approach that compute optimal damper positions.

The development of the duct model and model identification has been discussed in previous chapter. In this chapter, the terminal airflow rates can be obtained from the simulation results through the matrix B . It has been shown that by given the terminal flow rates, the system variable can be uniquely determined. The change of the state variables with respect to the change of $\mathbf{K}$ is given. This demonstrates that for any internal pressure increase by the change of fan speed, it is always possible to adjust dampers to compensate the change of airflow for each terminal. Therefore, the degree of freedom in this system break into two parts: one DOF to control the total airflow by the fan and the $n_T - 1$ DOF to adjust dampers to proportioning the airflow between terminals. To reduce the extra DOF, normalized airflow rate is defined and the damper that should be kept fully open in order to reduce fan energy consumption is located by maximizing the gap between the normalized desired airflow rates and the actual normalized airflow rates at each terminal when all dampers are fully open, which is easy to get since the data has been measured previously.

After determine the critical damper, the damper position for the rest of dampers are obtained by solving an optimization problem which has an objective function based on squared difference between the predicted terminal airflow rates and the desired airflow rates under constraints. After calculating the optimal damper position, the air balancing process can be implemented in a non-iterative approach. In order to reduce the error of adjustments and overcomes the difficulties of lack of proper indicators in practice, a sequential tuning procedure is proposed which uses airflow meter as the indicator for accurate damper adjustments. The indicator values in the sequential tuning procedure is obtained by the model using the optimal damper position. By applying the method, the adjusted terminals will gradually change the airflow values such that the desired airflow rate can be reached at the end of the sequential tuning procedure, when all terminals reach balance together.

A simulation on an eight-terminal duct system validates this method. Measurements taken from the system is superposed with noise in order to simulate the sensor error. Identification for the duct model parameters is performed with high accuracy. After identification, the critical damper is located and the optimal damper position for balancing is calculated. The sequential damper adjusting procedure is generated by the

model using the optimal damper position. The error of the airflow after balancing is 3.35% of MAPE which satisfies the ASHRAE's standard of 10%. The number of measurements that influences the accuracy and costs is investigated. It is shown clearly that the accuracy improves as more measurements are taken. The ultimate accuracy approaches to a limit which is due to sensor error. When the number of measurements increases, the marginal cost is insignificant comparing to the initial of setup the measuring devices. Hence it is suggested that for general purposes ventilation system, three to five measurements are suitable. Comparing this results with progressive flow methods and conventional ratio method, the final balancing accuracy of the proposed method is the best. The efficiency is also better than ratio method, and close to the progressive flow methods. Hence this method is expected to be attractive to TAB firms. The experimental results conducted in experimental ACMV system also shown good results. The maximum absolute percentage error is 4.7% which is acceptable by ASHRAE's standards.

Chapter 6. Fan-Independent method for balancing duct system

6.1 Introduction

In duct system, the performance curves of fan, which depend on type of fan blades, driving motors and housing design, are generally difficult to be modeled. Fans using different motors (for example Permanent Split Capacitor and Brushless Permanent Magnet) and different curved blades (forward, backward or radial) have different response in terms of changed air flow rate with changing system air flow resistance. Although fan law express the homology between two performance curves of geometrically similar series of fans[146], the physical model of fan performance curve models like quadratic model are unable to describe the performance curve in the whole range. In addition, when retrofitting existing duct system, the original performance curve of fan may be inaccessible, and thus it is difficult to obtain the model of fan. Furthermore, disturbances from upstream flow of the duct system under balancing could also affect the pressures and flow rates. Therefore, it is of great significant to develop suitable methods to balance duct system without known the pressure-flow rate relationship.

For this reason, a fan-independent method for TAB based on grey model of duct network is proposed. The advantages of the proposed method are three folds: 1) improving modeling accuracy, 2) improving robustness against disturbances from fans, and 3) improving efficiency of balancing. The fan-independent method consists of model formulation, measurement acquisition, parameter identification, balancing calculation and adjusting implementation. The conventional parametric fan model is replaced by a variable pressure source to adapt to arbitrary fan characteristics. For each damper, at least two measurements in different damper positions are taken. Each set of data is a triple of measurements taken at the terminal outlet, upstream side of the damper and the fan outlet. Model identification algorithm estimates both the duct model parameters and the fan pressures simultaneously. To handle variations of fan pressure, a new indicator for the damper adjustments is introduced. By using this method, the balancing error can be further reduced as well as the labor and time cost for air balancing.

In this chapter, the fan-independent method is proposed. Firstly, the model of fan is investigated in Section 6.2. Second, the modifications on measuring and model identification method are discussed in Section 6.3. Third, the balancing and damper adjusting procedure are developed in Section 6.4. Then the simulations and experiments are conducted for this method in Section 6.5. And a comparison study with previous model-based method is performed in Section 6.6. Finally, a summary is drawn in Section 6.7.

6.2 Fan characteristics and model

Backward and forward curved blades of the centrifugal fan as well as axial fan have different pressure-flow characteristics due to the different ways of composing velocity vector of the air leaving the blades. Figure 6.1[147] shows the velocity diagram of centrifugal fan where V_b is the velocity of blade tip, V_r is the radial velocity of air leaving the black and the V is the composed absolute velocity of air. Figure 6.2[148] shows the velocity diagram of axial fan where V_f is the blade tip speed, V_r is the relative velocity of air near blade surface, and v is the absolute velocity of air.

Figure 6.1: Velocity diagram for centrifugal fans

Figure 6.2: Velocity diagram for axial fan

Because of the different speed, attack angle and curvature, the pressure lift by the fan and the efficiency are different for different fan and different flow rates. Consequently, these fans have different suitable range of airflow and working pressure for different applications. Figure 6.3[149, 150] demonstrates the fan curve and their working range for different types of fan. The fan curves could various greatly and are difficult to be formulated by simple analytical equations.

Figure 6.3: Operating conditions for different fans

Traditional parametric model approximates fan curve by quadratic formula with constant parameters, which is not very appropriate for most types of fan. Therefore, instead of using parametric model, the fan is modeled by a variable pressure source:

$$\Delta P = -P_{fan} \quad (6.1)$$

The negative sign represents a pressure lift instead of a pressure drop. This variable depends on fan speed and the configuration of duct system. According to the previous DOF analysis, the fan pressure occupies the DOF to control total airflow rates. In this method, fan pressure is monitored and recorded separately during both measuring and adjusting process. For the convenience of expressing formula, the system flows and pressures is computed by the function:

$$\begin{bmatrix} \mathbf{q} \\ \mathbf{P} \end{bmatrix} = f(\mathbf{K}, P_{fan}, \boldsymbol{\beta}) \quad (6.2)$$

6.3 Identifying fan-independent model

The number of measurements at each terminal must be larger than or equal to two. More measurements per terminal are acceptable to reduce the uncertainty of the identification. In this method, not only the terminal flow rates and pressures are recorded, the change of P_{fan} is also measured by a pressure sensor installed near the fan outlet. Therefore, each set of measurement is a triple $(\mathbf{q}_i, \mathbf{P}_i, P_{fan,i})$. The cost of this additional pressure sensor is not significant, since only one pressure sensor is needed, and only one installation of pressure sensor is needed throughout the air balancing process.

The estimation of pressures and flow rates by the model for identifying parameter beta is given by the following equations:

$$\hat{\mathbf{q}}_i = \mathbf{C}_{q,i} f(\mathbf{K}_i, \hat{P}_{fan,i}, \boldsymbol{\beta}) \quad (6.3)$$

$$\hat{\mathbf{P}}_i = \mathbf{C}_{P,i} f(\mathbf{K}_i, \hat{P}_{fan,i}, \boldsymbol{\beta}) \quad (6.4)$$

Therefore, for each set of data in experiments at a given configuration of damper positions, the posterior probability is given by:

$$P(\mathbf{K}_i, \hat{P}_{fan,i}, \boldsymbol{\beta} | \mathbf{P}_i, \mathbf{q}_i, P_{fan,i}) \propto \exp \left[\begin{array}{c} -(\mathbf{q}_i - \hat{\mathbf{q}}_i)^T \sigma_q^{-2} (\mathbf{q}_i - \hat{\mathbf{q}}_i) \\ -(\mathbf{P}_i - \hat{\mathbf{P}}_i)^T \sigma_P^{-2} (\mathbf{P}_i - \hat{\mathbf{P}}_i) \\ -(\mathbf{P}_{fan,i} - \hat{\mathbf{P}}_{fan,i})^T \sigma_P^{-2} (\mathbf{P}_{fan,i} - \hat{\mathbf{P}}_{fan,i}) \end{array} \right] P(\boldsymbol{\beta}) \quad (6.5)$$

where equation ((6.3) and ((6.4) applies for $\hat{\mathbf{q}}_i$ and $\hat{\mathbf{P}}_i$ respectively.

The equations apply to all set of data in experiments at a given configuration of damper positions, and the objective function is obtained:

$$F(\boldsymbol{\beta}) = \sum_{i=1}^{N_p} \left[\begin{array}{c} (\mathbf{q}_i - \hat{\mathbf{q}}_i)^T \sigma_q^{-2} (\mathbf{q}_i - \hat{\mathbf{q}}_i) \\ (\mathbf{P}_i - \hat{\mathbf{P}}_i)^T \sigma_P^{-2} (\mathbf{P}_i - \hat{\mathbf{P}}_i) \\ (\mathbf{P}_{fan,i} - \hat{\mathbf{P}}_{fan,i})^T \sigma_P^{-2} (\mathbf{P}_{fan,i} - \hat{\mathbf{P}}_{fan,i}) \end{array} \right] - \ln P(\boldsymbol{\beta}) \quad (6.6)$$

The model identification is conducted by optimizing the objective function. Variables under optimizations are model parameters beta, damper parameter $\mathbf{K}_i$ and fan pressure at each case $P_{fan,i}$. The increase of DOF by introducing the variable $P_{fan,i}$ is compensated by the measured fan outlet pressures for each set of data.

6.4 Balancing for fan-independent method

Once the duct model is established, the pressures and airflow rates in the duct system under different damper positions can be estimated. Based on this model, the optimal damper positions corresponding to the desired airflow distribution can be calculated and used for balancing the duct system. The critical damper that should be fully opened is located by using the fully open status. In this process, the variation of $P_{fan,ref}$ could only affects the total terminal flow rates, which is independent to the proportioning of flow rates of terminals. To reduce the additional variations, the normalized terminal flow rates should be used, which is calculated by:

$$\mathbf{q}_T = \mathbf{B}_T f(\mathbf{K}, P_{fan,ref}, \hat{\boldsymbol{\beta}}) \quad (6.7)$$

$$\bar{\mathbf{q}}_T = \frac{\mathbf{q}_T}{\|\mathbf{q}_T\|_1} \quad (6.8)$$

One may obtain a suitable value of $P_{fan,ref}$ by averaging the previous fan pressure measurements. Once the critical damper is located, the rest of dampers positions can be obtained by solving the optimization problem similar to previous method. The objective

function is the sum squared error of the normalized terminal flow rates comparing to the normalized desired terminal flow rates. The method of solving the optimization algorithm is trust-region algorithm.

Based on the calculated optimal damper position $\mathbf{K}_D$, the duct system can be balanced by adjusting dampers. However, as mentioned previously, the actual damper position theta is incalculable. To adjust dampers to achieve $\mathbf{K}_D$, flow measurements together with fan pressure are used as a new indicator, and a sequential tuning method is applied. This sequential tuning method generates the expected values of indicator of the target terminal during the damper adjusting procedure. The procedure need n_T steps where i increases from 1 to n_T . For any step i , the status of the duct system is that the dampers 1, 2, ..., $i-1$ have been in proper positions while remaining dampers $i, i+1, \dots, n_T$ are fully open, the expected value of indicator at this step is calculated by:

$$\hat{r}(i) = \frac{\mathbf{B}_T f(\tilde{\mathbf{K}}(i), P_{fan}, \hat{\boldsymbol{\beta}})}{\sqrt{P_{fan}}} \quad (6.9)$$

where

$$\tilde{K}_j(i) = \begin{cases} K_{D,j} & j \leq i \\ 0 & j > i \end{cases}$$

Based on this indicator, the algorithm for sequential tuning is given as follows:

1. Start with all dampers open and let $i = 1$, $\tilde{\mathbf{K}}(0) = \mathbf{0}$.
2. For step i , let $\tilde{\mathbf{K}}(i) = \tilde{\mathbf{K}}(i-1)$, then let $\tilde{K}_i(i) = K_{D,i}$ as the i th element of $\mathbf{K}_D$. Calculate the expected flow rate $\hat{\mathbf{q}}_T = \mathbf{B}_T f(\tilde{\mathbf{K}}(i), \hat{P}_{fan}, \hat{\boldsymbol{\beta}})$ under given $\hat{P}_{fan}$ to obtain the indicator $r(i) = \hat{\mathbf{q}}_T / \sqrt{\hat{P}_{fan}}$.
3. Adjust damper such that the measured flow rate q_i and fan pressure $P_{fan,i}$ satisfies the indicator $q_{T,i} / \sqrt{P_{fan,i}} = r(i)$
4. Move to the next damper and repeat the steps 2 to 4 until all dampers have been adjusted properly.

6.5 Simulation and experimental validation

We use the duct system shown in Figure 5.1 to verify this method. This duct system has eight terminal outlets to distribute the processed air from central AHU to eight different rooms. The arrangement of these terminals can be categorized into two groups.

The bottom four terminals T1, T2, T3 and T4 are arranged in tree structure, which has multiple branches for each node. The top four terminals T5, T6, T7 and T8 are arranged in linear structure, which splits one terminal outlet at each junction as the main duct going through. Both arrangements are commonly used in practice. The validation experiments will evaluate the performance of the air balancing method for both arrangements. The duct segments are modeled by a function of pressure drop in terms of airflow rate through it. The detail formula and the parameters used in simulation are summarized in Table 6.1.

Table 6.1: Model parameters used in simulation

	Branch	Model	Parameters
Fan	1	$\Delta P = P_{\max} (q^2 / q_{\max}^2 - 1)$	$P_{\max} = 160 \text{ Pa}$, $Q_{\max} = 400 \text{ l/s}$
	2		$D = 150 \text{ mm}$, $L = 2.5 \text{ m}$
	3		$D = 80 \text{ mm}$, $L = 0.5 \text{ m}$
	5	$\Delta P = C_f \left(\frac{L}{D} \right) \left(\frac{\rho}{2} V^2 \right)$	$D = 80 \text{ mm}$, $L = 1.5 \text{ m}$
	7		$D = 120 \text{ mm}$, $L = 2.5 \text{ m}$
	8	$q = \pi (D/2)^2 V$	$D = 80 \text{ mm}$, $L = 0.5 \text{ m}$
	10		$D = 80 \text{ mm}$, $L = 1.5 \text{ m}$
	12	$C_f = 8 \left[\left(\frac{8}{\text{Re}} \right)^{12} + \frac{1}{(A+B)^{1.5}} \right]^{1/12}$	$D = 150 \text{ mm}$, $L = 2.5 \text{ m}$
Duct	13		$D = 80 \text{ mm}$, $L = 1.5 \text{ m}$
	15	$A = \left[-2.475 \ln \left(\left(\frac{7}{\text{Re}} \right)^{0.9} + \frac{0.27\varepsilon}{D} \right) \right]^{16}$	$D = 120 \text{ mm}$, $L = 2.5 \text{ m}$
	16		$D = 80 \text{ mm}$, $L = 1.5 \text{ m}$
	18	$B = (37530 / \text{Re})^{16}$	$D = 100 \text{ mm}$, $L = 2.5 \text{ m}$
	19		$D = 80 \text{ mm}$, $L = 1.5 \text{ m}$
	21		$D = 80 \text{ mm}$, $L = 3 \text{ m}$
	4		$D = 80 \text{ mm}$
	6	$\Delta P = \frac{1}{2} C_f \rho V^2$	$D = 80 \text{ mm}$
Damper	9		$D = 80 \text{ mm}$
	11	$q = \pi (D/2)^2 V$	$D = 80 \text{ mm}$
	14		$D = 80 \text{ mm}$
	17	C_f refers to ASHRAE duct	$D = 80 \text{ mm}$
	20	fittings database 5.0 CD 9-1	$D = 80 \text{ mm}$
	22		$D = 80 \text{ mm}$
Shared parameters			$\rho = 1.204 \text{ kg/m}^3$
			$\varepsilon = 0.015 \text{ mm}$
			$\nu = 15.11 \times 10^{-6} \text{ m}^2/\text{s}$

To simulate the measuring process, each measurement of nodal pressures or airflow rates are superposed with certain random noise. In particular, terminal airflow rates through branch 4, 6, 9, 11, 14, 17, 20 and 22 are measured, each of which is superposed with a Gaussian white noise with $\sigma_q = 10^{-3} \text{ m}^3/\text{s}$. The damper upstream pressures at

node 3, 4, 6, 7, 9, 11, 13 and 14 are monitored, superposed with a Gaussian white noise with $\sigma_p = 2 \text{ Pa}$. The fan outlet pressure at node 1 is also superposed with a σ_p . Gaussian white noise. In this experiment, each damper is measured in five different damper positions, i.e. $m=5$. In total of 40 sets of data are obtained. The result is shown in Figure 6.4.

Figure 6.4: Measurement data obtained from simulation

The red crosses indicate the measured terminal airflow rate and damper upstream pressure, with the 95% error bar in black. The vertical columns indicate the fan pressure with error bars correspondingly. The blue curves are the actual pressure-airflow response curve of the duct system.

The model parameters are obtained by parameter identification algorithm, as shown in Table 6.2. To verify the accuracy of the identified model, a random walk sampling algorithm is applied to evaluate accuracy of the predictions, shown in Figure 6.5. The random walk algorithm draws samples from $\mathbb{R}^{n_r}$ phase space of the damper position by evenly selecting a random damper and changing its damper position to a uniformly random value in each step, starting from the initial damper positions $\mathbf{K} = \mathbf{0}$. Comparing the predicted terminal airflow rates $\hat{\mathbf{q}}$ with the actual flow rate $\mathbf{q}$, the modelling error can be estimated unbiasedly by:

$$\text{RelErr}(i) = \sqrt{\frac{n_T}{n_T - 1}} \frac{\|\hat{\mathbf{q}} - \mathbf{q}\|_2}{\|\mathbf{q}\|_2} \quad (6.10)$$

Figure 6.5: Model predicting accuracy

160 steps of random walk have been tested. The variations of damper positions for all eight dampers are shown in the middle. Flow rates through each terminal are shown in the bottom. The dashed line represents the 95% boundary of relative errors, which are within 2.0%.

Table 6.2: Model parameter estimated by identification in simulation

Branch	b	Branch	b
2	606.57	13	8398.2
3	3346.5	15	1782.8
5	10408	16	10612
7	2368.2	18	5585.2
8	1964.0	19	9078.3
10	7492.7	21	2251.7
12	779.36		

Based on the identified model, the critical damper k_0 , the optimal damper parameters $\mathbf{K}^*$, and the sequential procedures of damper adjustments $\tilde{\mathbf{K}}(i)$ are computed consecutively. Table 6.3 shows the results of damper optimization.

Table 6.3: Damper adjustment sequence in simulation

Step	Action	$\mathbf{K}^*$	q L/s [†]
1	Adjust Damper 1	61888	27.7257
2	Adjust Damper 5	54159	28.8341
3	Adjust Damper 2	52055	28.3525
4	Adjust Damper 3	44122	27.8422
5	Adjust Damper 4	52466	29.2574
6	Adjust Damper 6	33777	28.9705
7	Adjust Damper 7	11083	29.2500
	Leave Damper 8 idle	0	N.A.

[†] Under fan pressure $P_{fan} = 80$ Pa

The results of air balancing following the instructions are shown in Figure 6.6 and Table 6.4 where the maximum absolute percentage error (MAPE) is used to quantify the degree of imbalance in the duct network. The MAPE has been reduced from 30% to 2.02% by the proposed TAB method, which satisfies the ASHRAE's standard (10%).

Table 6.4: Balancing results in simulation

Terminal	Actual θ	Actual q (L/s)	$\bar{q}$	$\bar{q}_d$	Relative Error
1	70.04	31.92	12.48%	12.5%	-0.02%
2	68.67	31.97	12.50%	12.5%	0.00%
3	68.52	31.79	12.43%	12.5%	-0.55%
4	66.94	31.86	12.46%	12.5%	-0.34%
5	68.74	31.88	12.47%	12.5%	-0.26%
6	64.90	31.89	12.47%	12.5%	-0.24%
7	60.44	31.80	12.44%	12.5%	-0.49%
8	0	32.61	12.75%	12.5%	2.02%
MAPE					2.02%

Figure 6.6: TAB procedures and results

The x-axis represents the steps of damper adjustments. Vertical bars represent the airflow rates of different terminals. The black curve with red circle represents the variation of balancing MAPE as dampers being adjusted. After 7 steps, the MAPE is reduced to 2.02%.

The fan-independent method has been applied in the experimental ACMV duct system. Two tests of air balancing have been conducted. In the first test, the target air flow proportioning is 1:1:1:1. The balancing procedure is shown in Figure 6.7. The final balancing accuracy is 2.6% MAPE. The second test requires a different air balancing proportioning 1:3:2:2. The balancing procedure is shown in Figure 6.8. The final balancing accuracy is 3.1% MAPE.

Figure 6.7: Balancing procedures for desired flow 1:1:1:1 in the experiment

Figure 6.8: Balancing procedures for desired flow 1:3:2:2 in the experiment

6.6 Comparison studies and discussions

To further evaluate the proposed method, the effects of fan-independent model are compared with previous parametric fan model method which assumes the following fan model:

$$\Delta P = P_{\max} \left(1 - \frac{q^2}{q_{\max}^2} \right) \quad (6.11)$$

Except fan, other parts of the model remain unchanged. Since the fan is modeled by parametric formula, sensors for fan outlet pressure is no longer used. The measuring and balancing method is different from fan-independent method. During damper adjusting, the indicator is chosen as the terminal flow rate $q_i(i)$. The measuring data used for identification in both methods are the same, shown in Figure 6.9. Using random walk algorithm, the modelling accuracy for both methods are estimated and compared in Figure 6.10. The final balancing results for both methods are shown in Figure 6.11. It can be seen clearly that using fan-independent model obtains better modeling and balancing accuracy.

Figure 6.9: Model identification data for comparison test

(a) fan-independent method, 95% Error are below 3.8% (b) parametric fan model method, 95% Error are below 7.9%.

Figure 6.11: Comparison on balancing results
(a) fan-independent method, MAPE=1.78%; (b) parametric fan model method, MAPE=4.47%.

The robustness against different types of fans and disturbances are tested in two sets of experiments for both methods. The first test uses shifted peak quadratic model for fan which satisfies:

$$\Delta P = P_{\max} \left(1 - \frac{(q - q_{\text{peak}})^2}{(q_{\max} - q_{\text{peak}})^2} \right) \quad (6.12)$$

where q_{peak} is the peak shifting parameter.

The second test adds a leaking branch $q_{\text{leak}} = \sqrt{P_{\text{fan}} / L}$ in parallel with fan where L is the leaking parameter. The diagram of duct system with leaking branch is shown in Figure 6.12. The resulting fan curves for both systems are shown in Figure 6.13.

Figure 6.12: Duct system with leakage

Figure 6.13: Equivalent fan curves in comparison experiments
 (a) the original fan $Q_{\text{peak}}=0$, $L=\text{inf}$; (b) shifted model $Q_{\text{peak}}=40$; (c) shifted model $Q_{\text{peak}}=80$; (d) leaking model $L=10^7$; (e) leaking model $L=10^6$; (f) leaking model $L=10^5$.

In each set of test, both fan-independent method and parametric fan model method are conducted. The data used for model identification are identical for both methods. The 95% boundary of modeling predicting error and the final balancing result in terms of MAPE are shown in Table 6.5. The proposed fan-independent method shows strong robustness against modeling error of fan performance curve and disturbances from leakage.

Table 6.5: Modeling and balancing accuracy under different disturbances

Case	Fan-Independent Model		Parametric Fan Model	
	Modeling Accuracy	Balancing Accuracy	Modeling Accuracy	Balancing Accuracy
Original	1.96%	1.51%	4.17%	3.70%
$Q_{peak} = 40$	1.96%	1.77%	6.74%	7.41%
$Q_{peak} = 80$	2.02%	2.00%	7.95%	12.41%
$L = 10^7$	1.92%	1.54%	5.78%	5.24%
$L = 10^6$	2.00%	1.57%	6.56%	7.17%
$L = 10^5$	2.09%	1.73%	7.74%	8.60%

To investigate how the number of measurements per terminal m affects the balancing accuracy, a series of tests on different m ranging from 3 to 15 are conducted for both fan-independent method and parametric fan model method. The result is shown in Table 6.6. It is interesting that fan-independent method is significantly better than the parametric fan model in low- m cases while parametric fan model method is more accurate in high- m cases. The fan-independent method probably exists a limit for balancing accuracy which is higher than parametric fan model given sufficient large m . Based on this result, it is reasonable to choose $m = 2$ or $m = 3$ for saving the operational costs while obtaining satisfactory accuracy.

Table 6.6: Comparison of accuracy under different number of measurements

m	Fan-Independent Model		Parametric Fan Model	
	Modeling Accuracy	Balancing Accuracy	Modeling Accuracy	Balancing Accuracy
2	1.95%	1.55%	7.56%	7.97%
3	1.96%	1.51%	4.17%	3.70%
5	1.99%	1.50%	2.86%	1.75%
7	1.97%	1.47%	2.09%	1.68%
9	2.00%	1.46%	2.06%	1.09%
15	1.85%	1.43%	1.84%	1.08%

Finally, the cost of operating time is estimated. In the initial measuring process, the engineers must approach to each terminal and take several measurements. During this process, the time used for moving instruments from one terminal to another is estimated to be five minutes, while the time of actual measuring is only thirty seconds per measurement. Traditional TAB methods have low efficiency during the measuring process that only one measurement is taken per access. However, model-based methods are more efficient in taking measurements. Fan independent method may take two or three measurements per access, and parametric fan model method may take more

measurements to obtain comparable accuracy. Comparing with parametric fan model method, the fan-independent method need to setup the pressure sensor near the fan outlet, which requires five minutes. During adjusting of dampers, proportional method needs several times of adjustments, while neither fan-independent model method nor parametric fan model method need more than once. As the duct system becomes more complex, the required times of adjustments for proportional method increases. In this thesis, it is assumed that three rounds of adjustments are taken for the proportional method in a duct system with eight terminals. After adjustments, a checking procedure is required for all methods. Table 6.7 summarizes the time consumption of these methods. Fan-independent model method shows over 50% of improvement in efficiency than proportional method. The additional time cost due to the installation of additional pressure sensor is not significant compared with the additional measurements needed for the parametric fan model method.

Table 6.7: Comparison of efficiency for different balancing methods

Step	Fan-Independent Model	Parametric Fan Model	Proportional Method
Initial	$5 + (5 + 2 \times 0.5) \times 8$	$(5 + 0.5) \times 8$	$(5 + 0.5) \times 8$
Adjusting	$(5 + 0.5) \times 8$	$(5 + 0.5) \times 8$	$(5 + 0.5) \times 8 \times 3$
Check	$(5 + 0.5) \times 8$	$(5 + 0.5) \times 8$	$(5 + 0.5) \times 8$
Sum	141 min	148 min	220 min
Efficiency	156%	148%	100%

In summary, comparing with the proportional method, the efficiency of the proposed fan-independent model method can be improved by more than 50%. Although an additional pressure sensor must be used, the cost can still be reduced by improving the operating efficiency. The fan-independent model method has potential to be applied to balance duct system in a distributed manner by regarding each branch as an independent duct system and regarding the duct pressure at the entrance as the airflow source.

6.7 Summary

Fan performance curve are generally difficult to model accurately, and often difficult to obtain when retrofitting existing duct system. Hence developing an air balancing method that requires no fan model becomes practical and useful. This chapter propose a fan-independent method for air balancing which improves accuracy, robustness and efficiency. In this method, fan model is replaced by a variable pressure source to adapt

to arbitrary fan characteristics. Hence the different pressure-flow characteristics of backward, forward curved centrifugal fan or axial fan no longer have influence on the fan model, as well as the motors. Since the fan pressure changes with flow and fan speed. A pressure sensor near fan outlet should be added to each set of measurements. Hence each set of data is a triple of measurements taken at the terminal outlet, upstream side of the damper and the fan outlet. The cost of this additional pressure sensor is insignificant because only one installation is needed through the process. The entire measurements taken from duct system must be at least two set of measurements per terminal. More measurements per terminal are acceptable to reduce uncertainty and improve the accuracy. With the measured data, one can optimize the model parameter such that the predicted flows and pressures by the model is close to the measured data. Comparing with previous model identification method, this method also needs to estimate the fan outlet pressures for each set of experimental data. The optimization algorithm applies to previous model identification methods is also applicable in this method. Once the model parameters are obtained, the optimal damper position corresponding to the desired air flow distribution can be calculated. It has been shown that the fan pressure affects only the total airflow rates of the terminals but not the proportioning of flows. By using normalized flow rates for all terminals, the critical damper that should be kept fully open can be determined by comparing the normalized flow rate at fully open status with the normalized desired flow rate. The fan pressure is set to be the average of the measured fan pressures in this process. Then the optimal damper position for the rest dampers can be calculated by minimizing the objective function using algorithms like trust-region algorithm. Based on the calculated optimal damper position $\mathbf{K}_D$, a sequential tuning procedure are generated in order to implement damper adjustment more accurately. This sequential tuning process introduces a new indicator since the fan outlet pressure can be variable. The new indicator taken both the intermediate flow rate of the terminal and the fluctuation of fan pressure into account. By adjusting the damper so that the indicator approaches to the expected values, the expected damper positions are achieved. At the end of process, all dampers are set to the expected damper position and the air balancing is completed.

Simulations are conducted to validate this method. Model structure and the parameters used in the simulation are given. The measured data, which contains terminal airflow rates, the damper upstream pressures and fan outlet pressures are collected. Measuring error is analyzed and shown as well. After identification, the model parameters are obtained. The accuracy of the model using the obtained model parameter is evaluated by the random walk algorithm which randomly chose a damper and adjusts to a specific angle. The model need to predict the terminal flow rates and compare with the actual values. Errors of the obtained model are within 2.0% for 95% of predictions. Based on the obtained model, the critical damper and the optimal damper positions are calculated by the optimization algorithm. And consequently, the damper adjusting procedure and the expected values for the indicators is generated. During the sequential tuning process, the airflow rates are monitored for all terminals, and the maximum absolute percentage error is computed. It is shown that MAPE reduces from 30% to 2.02% after complete the air balancing procedure, which satisfies the ASHRAE standards.

Comparison study to investigate the performances of fan-independent method under large fan modeling error and fan leakage is performed. The benchmark method is the parametric model-based air balancing method which uses constant parameter for fan model. The measured data for both method is the same. The model prediction accuracy for both models are evaluated by random walk algorithm, and the result has shown that fan-independent method has 3.8% relative error for 95% of data, which out-performs the parametric model-based method that has 7.9% relative error for 95% of data. The balancing result of the fan-independent model is also better than parametric model-based method. MAPE for fan-independent method is 1.78% while parametric model-based method is 4.47%. The robustness against different types of fan and the disturbances are tested for both methods. Detail testing conditions are introduced, and the measurements obtained in each test are applied equally for both methods. After model identification, the model prediction accuracy for both methods are evaluated, and the results have shown that fan-independent model is significantly better than parametric model-based method in all tests. The air balancing accuracy after adjusting dampers also shows the consistent trend that fan-independent method is better than parametric model-

based method. Another comparison test on the modeling accuracy and balancing accuracy under different number of measurements for both model is conducted. This experiment has shown that parametric model-based method could significantly reduce its modeling accuracy and balancing accuracy if given enough number of measurements per terminal. The accuracy of both methods approaches to the same value when m is larger than 7. However, the fan-independent method could achieve satisfactory accuracy at lower m than model-based method. Finally, the cost of operating time is estimated for both methods. Comparing with parametric model-based method, fan-independent method need to setup the pressure sensor near the fan outlet, but the fewer number of measurements per terminal compensates the additional time for installing sensors. However, the traditional methods need iterative adjustments of dampers and repeatedly measuring duct terminals in turns, which takes significantly amount of time. In summary, fan-independent method has close efficiency performance to the parametric model-based methods, and it is significantly better than traditional methods. The fan-independent method has potential to be applied to balance duct system in a distributed manner by regarding each branch as an independent duct system and regarding the duct pressure at the entrance as the airflow source.

Chapter 7. Conclusions and future work

7.1 Conclusions

If properly balanced, the ACMV system will have significant improvements on thermal comfort, indoor air quality, healthy of occupants, energy efficiency, and operating cost reduction. All types of CAV/VAV systems, supply/return/exhaust systems will be benefitted greatly from an accurate, efficient and convenient air balancing method. It is therefore highly desirable to study the duct model and develop a non-iterative air balancing based on deep understanding of the duct system. The present thesis has addressed the need with following contributions:

The duct model based on Darcy-Weisbach equation and circuit network analogy is developed. It was found that traditional estimation of pressure drop by the linear interpolation methods for pressure loss coefficient could cause multi-root problem for T and cross junctions when solving the flow distribution. A revised interpolation formula is proposed to estimate pressure drop for each branch of the junction such that the flows converges to a reasonable unique solution. The mass conservation law in duct system is formulated by defining the association matrix which describes the topology of the duct network. It has been shown that the degree of freedom in the duct network is equal to the number of terminals in duct system, which could be completely fixed given the pressure drop formula for all branches. Then the numerical approach to find solution of a duct system efficiently is given, together with the programming code implementation. To obtain the advanced DAE modeling approach for duct simulation, while utilizing powerful scientific computing packages provided by MATLAB/Simulink environment and convenient connectivity to various of software, a Simscape library for duct system has been developed. Validations by ASHRAE's example have demonstrates the correctness and convenience of the simulation tools.

In many cases, the exact values of model parameters are often difficult to estimate directly. The model identification method for this model has been developed and evaluated on experimental platform. Considering the cost and accuracy of taking flow and pressures measurements from duct system, a method to acquiring measurements have been developed. The proposed measuring method has some advantages: covering

the complete range from fully open to fully closed for all dampers; minimizing the number of setup of flow measurements to save time; and requiring minimum labor work and equipment. Meanwhile, the objective function has been derived for parameter identification problem based on MAP estimation by Bayes' law. Substituting the acquired measuring data and optimizing the established objective function will lead to the most likely set of parameters. Numerical approach to the optimization problem is proposed based on RMSProp algorithm with Nesterov momentum. Experiments on small-scale ACMV system have validated the measurements acquisition procedure and the model identification algorithm. A random walk algorithm is proposed to test the model accuracy in real duct system which explores the entire possible state space and compares the deviation of randomly sampled measurements with the corresponding predictions by model. It was found that the model accuracy after identification are below MAPE 6%.

The non-iterative air balancing method is proposed based on estimation of the coupling effect of terminals. This method is model-based method to estimate flow rates by the duct model which can be identified from measured data. Firstly, the critical damper which should be kept fully open is identified. Then the optimal damper positions for the other dampers are calculated by solving an optimization problem. The objective function based on normalized flow rates of each terminal have been derived. In order to reduce the balancing error caused by the inaccurate position indicator on the damper, an algorithm to generate the sequential tuning procedure which utilizes flow measurements as the indicator for damper adjustments is proposed. This sequential tuning procedure describes clearly which damper should be adjusted and what air flow readings at the terminal outlet is expected. The coupling effect on the change of airflow by adjusting neighboring damper is considered such that the desired air flow rates can be reached simultaneously at the end of sequential tuning procedure. Simulations and experimental results have shown that the proposed non-iterative air balancing method is effective. In the simulation, the final balancing accuracy is 3.35% MAPE which is better than the progressive flow method (MAPE 4.13%) and the ratio method (MAPE 9.89%). It was found that the accuracy can be improved by increasing the number of measurements up to 7 times per terminal, where the accuracy is within 1% MAPE. As the number of

measurements increases, the marginal cost is not significant. Hence it is suggested that three to five measurements are suitable for general purposes ventilation duct system. Experimental tests on small-scale ACMV system has shown that 4.7% MAPE of balancing accuracy using the model-based non-iterative method can be achieved.

In order to overcome the difficulty of modeling different types of fan, especially when retrofitting existing duct system, it is desirable to eliminate the dependences on fan model. A fan-independent method has been proposed which improves accuracy, robustness and efficiency. Fan model has been replaced by a variable pressure source to adapt to arbitrary fan characteristics. A new method is proposed to obtain sufficient data and identify the model with variable pressure source. A pressure sensor is installed to monitor the fan outlet pressure which takes only limited extra time and cost. The identification algorithm is modified to estimate fan pressures in addition to model parameters. It was found that fan pressure affects only the total airflow rate but not the proportions. Normalized flow rate for each terminal is introduced and utilized to calculate the critical damper and the optimal damper parameters for the rest dampers. A new indicator which taken the variation of fan pressure into account is introduced to implement damper adjustments in the sequential tuning procedure. Simulations and experiments has been conducted to validate this method. After model identification, the accuracy of the model is evaluated by random walk algorithm, and the errors are within 2.0% for 95% of predictions. During sequential tuning procedures, the airflow rates at all terminals are monitored, which has shown that the MAPE reduces from 30% to 2.02% after completing damper adjustments. The fan-independent method has shown good accuracy and robustness in the comparison study under fan modeling error and leaking conditions. Fan-independent method has 3.8% error for 95% of predictions while this value is 7.9% for parametric model-based method. Balancing accuracy of fan-independent method is 1.78%, which out-performed the parametric model-based method (4.47%). The robustness of fan-independent method is better than parametric model-based method in all tests. It has been investigated that fan-independent model achieves better modeling and balancing accuracy at less measurements per terminal.

7.2 Future work

Despite all the achievements in this thesis, it is still insufficient to complete the development of an accurate, efficient, simple, low cost and universally applicable air balancing methods. Further efforts on this area could be highly valuable and urgently demanded. A list of foreseeable works can be done in the near future:

- According to the previous research, fittings that are closely located in the duct could be coupled and the pressure drop is different from estimations by two distinct components. As a common situation in practice, this could affect the accuracy of the duct model. Considering the reason of the coupling effect that airflow has not yet fully developed, the approach of parameterizing the velocity distribution on the cross-sectional plane of the duct and estimating pressure drop based on development of velocity distribution passing through ducts and fittings could probably improve the model accuracy. This could be beneficial to both air balancing and duct sizing.
- The development of the duct model and simulation environment is a good foundation for accurate estimation of operating cost and fan selection. An improved T-method based on more powerful duct simulation tools could be used to optimize the duct sizing in higher accuracy and greater details.
- The current model developing method requires manually input of the duct model structure, including the types of ducts and fittings and the network topology. This may be tedious for the engineers for large and complex ACMV system and it could possibly be inconvenient if no duct design drawings are available when retrofitting buildings. Besides, it is not user friendly for customers with no engineering experience at all. In the future, it is highly desirable to develop a companion algorithm to extract model structure information from the building information model (BIM). Besides, the model identification algorithm could be modified to simultaneously identify model structure and model parameter, given sufficient data. It could greatly reduce the difficulty of using this method.
- Current air balancing methods are all based on the assumption of equal indoor pressure. For applications of nuclear industry, hospitals and laboratory where airflow leakage between zones must be properly controlled by pressurizing the

room, the air balancing must be specially designed. It is therefore necessary to take the indoor pressure and inter-zone airflow into accounts and develop a better model of the complete air loop so that air balancing methods could be more effective and accurate in these applications.

- Air balancing and VAV control is closely coupled and interacted. They share the same target of achieving desired air delivery except that air balancing deals with steady airflow while VAV changes airflow with time. In current design of ACMV system, the control of VAV box is isolated from air balancing. Although the duct has properly balanced when VAV box is not functioning, the status of balance will be broken once the VAV boxes control air flow. The control loop will also experience disturbances when neighboring VAV boxes are regulating the airflow rates. It is therefore necessary to integrate the air balancing with VAV control together to achieve variable air volume balancing in real time.
- In principle, air balancing is no different from water balancing except the density of working fluid. The methods, models and algorithms for air system can be transfer to water system with minor modifications. Considering the applications of water-based ACMV systems and district heating/cooling system, the demand of water balancing is as large as air balancing. Therefore, it is natural and necessary to extend the current air balancing methods to water system to obtain desired flow distribution in pipe network.

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Appendix A Tables of frictional coefficient for ED 5-3

ED5-3 Tee, $D_c \leq 250$ mm, Converging

		C_b Values									
		Q_b/Q_c									
A_s/A_c	A_b/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0.2	0.2	-24.56	-3.63	-0.36	0.59	0.93	1.08	1.14	1.19	1.27	
	0.3	-56.72	-9.54	-2.15	-0.01	0.78	1.10	1.23	1.30	1.39	
	0.4	-101.83	-17.86	-4.68	-0.87	0.52	1.09	1.32	1.41	1.48	
	0.5	-159.91	-28.59	-7.98	-2.02	0.17	1.05	1.40	1.51	1.51	
	0.6	-230.83	-41.68	-11.98	-3.39	-0.24	1.03	1.53	1.66	1.61	
	0.7	-314.56	-57.10	-16.68	-4.98	-0.69	1.04	1.71	1.90	1.82	
	0.8	-411.18	-74.90	-22.10	-6.82	-1.21	1.04	1.92	2.16	2.05	
	0.9	-520.69	-95.08	-28.25	-8.90	-1.81	1.04	2.15	2.45	2.31	
	1.0	-643.09	-117.63	-35.12	-11.24	-2.47	1.04	2.41	2.78	2.58	
0.3	0.2	-14.05	-1.55	0.36	0.89	1.08	1.16	1.19	1.23	1.34	
	0.3	-33.18	-4.91	-0.58	0.64	1.07	1.23	1.30	1.34	1.44	
	0.4	-60.09	-9.68	-1.94	0.24	1.00	1.29	1.39	1.42	1.47	
	0.5	-94.80	-15.89	-3.74	-0.33	0.87	1.32	1.46	1.46	1.38	
	0.6	-136.97	-23.33	-5.84	-0.92	0.81	1.45	1.65	1.66	1.53	
	0.7	-186.81	-32.14	-8.32	-1.62	0.74	1.61	1.88	1.88	1.70	
	0.8	-244.33	-42.30	-11.19	-2.43	0.65	1.78	2.14	2.13	1.88	
	0.9	-309.54	-53.82	-14.44	-3.35	0.54	1.98	2.42	2.41	2.08	
	1.0	-382.43	-66.70	-18.08	-4.39	0.42	2.19	2.74	2.72	2.29	
0.4	0.2	-8.95	-0.54	0.71	1.04	1.15	1.20	1.22	1.26	1.40	
	0.3	-21.82	-2.70	0.16	0.94	1.19	1.29	1.32	1.35	1.47	
	0.4	-39.99	-5.81	-0.67	0.73	1.19	1.35	1.39	1.39	1.41	
	0.5	-63.37	-9.82	-1.75	0.45	1.18	1.42	1.47	1.43	1.32	
	0.6	-91.72	-14.59	-2.97	0.20	1.26	1.60	1.67	1.60	1.43	
	0.7	-125.23	-20.24	-4.41	-0.10	1.34	1.81	1.90	1.81	1.56	
	0.8	-163.91	-26.77	-6.09	-0.45	1.43	2.04	2.16	2.03	1.69	
	0.9	-207.76	-34.17	-7.99	-0.85	1.53	2.30	2.44	2.28	1.82	
	1.0	-256.79	-42.45	-10.12	-1.30	1.63	2.58	2.75	2.54	1.95	
0.5	0.2	-6.03	0.04	0.91	1.13	1.20	1.22	1.24	1.29	1.44	
	0.3	-15.35	-1.46	0.56	1.09	1.25	1.30	1.32	1.35	1.46	
	0.4	-28.59	-3.67	-0.01	0.96	1.26	1.34	1.35	1.32	1.29	
	0.5	-45.45	-6.42	-0.66	0.85	1.33	1.45	1.45	1.38	1.24	
	0.6	-65.92	-9.70	-1.41	0.78	1.46	1.64	1.63	1.53	1.32	
	0.7	-90.12	-13.58	-2.29	0.69	1.61	1.86	1.85	1.70	1.39	
	0.8	-118.07	-18.07	-3.32	0.57	1.78	2.11	2.09	1.89	1.46	
	0.9	-149.75	-23.18	-4.49	0.43	1.96	2.38	2.35	2.09	1.53	
	1.0	-185.19	-28.89	-5.81	0.27	2.16	2.67	2.63	2.31	1.57	
0.6	0.2	-4.20	0.39	1.03	1.18	1.22	1.24	1.26	1.30	1.47	
	0.3	-11.33	-0.72	0.79	1.16	1.27	1.30	1.31	1.33	1.43	
	0.4	-21.57	-2.42	0.35	1.05	1.25	1.29	1.27	1.22	1.12	
	0.5	-34.29	-4.35	-0.03	1.07	1.38	1.44	1.41	1.32	1.16	
	0.6	-49.85	-6.73	-0.50	1.08	1.54	1.63	1.57	1.45	1.19	
	0.7	-68.26	-9.55	-1.06	1.09	1.71	1.83	1.76	1.58	1.21	
	0.8	-89.52	-12.81	-1.72	1.10	1.91	2.07	1.97	1.73	1.22	
	0.9	-113.64	-16.52	-2.47	1.10	2.12	2.32	2.20	1.88	1.21	
	1.0	-140.62	-20.68	-3.33	1.09	2.35	2.60	2.44	2.03	1.16	

ED5-3 Tee, $D_c \leq 250$ mm, Converging (Continued)

		C_b Values (Concluded)								
		Q_b/Q_c								
A_s/A_c	A_b/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.7	0.2	-3.00	0.62	1.10	1.21	1.23	1.24	1.26	1.31	1.49
	0.3	-8.74	-0.27	0.91	1.19	1.26	1.27	1.27	1.28	1.36
	0.4	-16.90	-1.59	0.58	1.11	1.25	1.27	1.24	1.18	1.06
	0.5	-26.99	-3.06	0.33	1.17	1.38	1.41	1.36	1.26	1.06
	0.6	-39.35	-4.86	0.02	1.22	1.54	1.57	1.50	1.35	1.05
	0.7	-53.97	-7.01	-0.35	1.29	1.72	1.76	1.65	1.45	1.02
	0.8	-70.87	-9.50	-0.79	1.35	1.91	1.97	1.82	1.54	0.96
	0.9	-90.04	-12.34	-1.31	1.41	2.12	2.19	2.00	1.64	0.86
	1.0	-111.50	-15.53	-1.89	1.46	2.34	2.43	2.19	1.73	0.72
0.8	0.2	-2.20	0.76	1.14	1.22	1.24	1.24	1.26	1.31	1.49
	0.3	-7.04	-0.01	0.95	1.18	1.23	1.23	1.23	1.22	1.27
	0.4	-13.77	-1.06	0.71	1.13	1.24	1.24	1.20	1.13	1.00
	0.5	-22.11	-2.24	0.54	1.20	1.36	1.36	1.30	1.19	0.97
	0.6	-32.33	-3.69	0.31	1.27	1.50	1.50	1.41	1.25	0.90
	0.7	-44.42	-5.41	0.04	1.34	1.66	1.65	1.53	1.30	0.81
	0.8	-58.40	-7.42	-0.29	1.42	1.83	1.83	1.65	1.35	0.67
	0.9	-74.28	-9.72	-0.67	1.49	2.01	2.01	1.78	1.38	0.49
	1.0	-92.06	-12.30	-1.12	1.56	2.21	2.20	1.92	1.40	0.24
0.9	0.2	-1.67	0.85	1.16	1.22	1.23	1.24	1.25	1.30	1.48
	0.3	-5.95	0.12	0.95	1.14	1.18	1.18	1.16	1.15	1.14
	0.4	-11.68	-0.74	0.77	1.12	1.20	1.20	1.16	1.08	0.93
	0.5	-18.85	-1.74	0.63	1.18	1.31	1.30	1.23	1.11	0.86
	0.6	-27.63	-2.98	0.44	1.24	1.42	1.41	1.31	1.13	0.75
	0.7	-38.04	-4.45	0.21	1.30	1.55	1.53	1.39	1.14	0.58
	0.8	-50.07	-6.17	-0.07	1.36	1.69	1.66	1.47	1.13	0.37
	0.9	-63.75	-8.14	-0.40	1.42	1.83	1.79	1.54	1.11	0.09
	1.0	-79.08	-10.36	-0.79	1.46	1.98	1.92	1.61	1.06	-0.26
1.0	0.2	-1.33	0.89	1.16	1.21	1.22	1.22	1.24	1.29	1.46
	0.3	-5.30	0.15	0.90	1.08	1.11	1.11	1.09	1.06	0.99
	0.4	-10.31	-0.57	0.78	1.09	1.16	1.15	1.11	1.03	0.86
	0.5	-16.71	-1.47	0.64	1.13	1.24	1.22	1.15	1.03	0.74
	0.6	-24.56	-2.59	0.46	1.17	1.32	1.30	1.20	1.01	0.57
	0.7	-33.87	-3.93	0.23	1.20	1.41	1.38	1.24	0.97	0.34
	0.8	-44.64	-5.49	-0.05	1.22	1.51	1.46	1.27	0.91	0.05
	0.9	-56.89	-7.29	-0.38	1.24	1.59	1.54	1.28	0.82	-0.33
	1.0	-70.62	-9.32	-0.77	1.24	1.68	1.61	1.28	0.69	-0.80

ED5-3 Tee, $D_c \leq 250$ mm, Converging

		C_s Values								
		Q_s/Q_c								
A_s/A_c	A_b/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.2	0.2	18.11	3.42	1.62	1.11	0.90	0.80	0.74	0.70	0.68
	0.3	12.67	2.79	1.45	1.04	0.87	0.78	0.73	0.70	0.68
	0.4	9.98	2.47	1.36	1.01	0.85	0.77	0.72	0.69	0.67
	0.5	8.39	2.27	1.30	0.98	0.84	0.76	0.72	0.69	0.67
	0.6	7.34	2.13	1.26	0.96	0.83	0.76	0.72	0.69	0.67
	0.7	6.61	2.02	1.22	0.95	0.82	0.75	0.71	0.69	0.67
	0.8	6.08	1.94	1.19	0.93	0.81	0.75	0.71	0.68	0.67
	0.9	5.68	1.87	1.17	0.92	0.80	0.74	0.70	0.68	0.66
	1.0	4.55	1.61	1.05	0.86	0.76	0.71	0.68	0.66	0.65
0.3	0.2	44.33	7.19	2.80	1.57	1.08	0.84	0.71	0.63	0.57
	0.3	29.24	5.46	2.33	1.40	1.00	0.80	0.69	0.62	0.57
	0.4	21.88	4.59	2.09	1.30	0.96	0.78	0.67	0.61	0.56
	0.5	17.62	4.06	1.93	1.24	0.92	0.76	0.66	0.60	0.56
	0.6	14.90	3.71	1.82	1.19	0.90	0.74	0.65	0.59	0.55
	0.7	13.06	3.45	1.74	1.15	0.88	0.73	0.64	0.59	0.55
	0.8	11.78	3.26	1.67	1.12	0.86	0.72	0.63	0.58	0.54
	0.9	9.02	2.64	1.41	0.97	0.77	0.66	0.59	0.54	0.51
	1.0	8.36	2.52	1.36	0.95	0.75	0.65	0.58	0.54	0.51

FD5-3 Tee, $D_c \leq 250$ mm, Converging (Continued)

		C_s Values (Concluded)								
A_f/A_c	A_b/A_c	Q_s/Q_c								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.4	0.2	78.99	12.25	4.42	2.26	1.39	0.97	0.74	0.60	0.50
	0.3	50.14	8.96	3.54	1.92	1.24	0.90	0.70	0.57	0.49
	0.4	36.26	7.32	3.08	1.74	1.16	0.85	0.67	0.56	0.48
	0.5	28.38	6.35	2.80	1.63	1.10	0.82	0.65	0.54	0.47
	0.6	23.50	5.72	2.61	1.54	1.05	0.79	0.63	0.53	0.46
	0.7	20.32	5.27	2.46	1.47	1.02	0.77	0.62	0.52	0.45
	0.8	14.94	4.13	1.98	1.21	0.85	0.65	0.53	0.46	0.40
	0.9	13.55	3.88	1.89	1.16	0.82	0.63	0.52	0.45	0.39
	1.0	12.66	3.69	1.80	1.12	0.79	0.62	0.51	0.44	0.39
0.5	0.2	114.73	17.76	6.27	3.07	1.79	1.16	0.81	0.60	0.46
	0.3	70.56	12.71	4.92	2.56	1.56	1.05	0.75	0.56	0.44
	0.4	49.68	10.24	4.23	2.29	1.43	0.98	0.71	0.54	0.42
	0.5	38.12	8.81	3.81	2.11	1.34	0.93	0.68	0.52	0.41
	0.6	31.23	7.90	3.53	1.99	1.27	0.88	0.65	0.50	0.39
	0.7	21.87	6.00	2.75	1.57	1.01	0.71	0.52	0.40	0.32
	0.8	19.30	5.57	2.59	1.49	0.96	0.67	0.50	0.38	0.30
	0.9	17.84	5.27	2.46	1.42	0.92	0.65	0.48	0.37	0.29
	1.0	17.16	5.05	2.36	1.36	0.88	0.62	0.46	0.35	0.28
0.6	0.2	142.32	22.64	8.06	3.91	2.23	1.39	0.92	0.63	0.44
	0.3	84.89	16.05	6.28	3.24	1.92	1.23	0.83	0.58	0.41
	0.4	58.43	12.90	5.39	2.88	1.75	1.14	0.78	0.55	0.39
	0.5	44.34	11.13	4.86	2.66	1.63	1.07	0.74	0.52	0.37
	0.6	29.06	8.20	3.69	2.04	1.25	0.81	0.55	0.38	0.26
	0.7	24.71	7.51	3.44	1.91	1.18	0.77	0.52	0.35	0.24
	0.8	22.56	7.06	3.26	1.81	1.11	0.72	0.48	0.33	0.22
	0.9	21.89	6.78	3.12	1.73	1.06	0.68	0.45	0.30	0.20
	1.0	22.24	6.61	3.00	1.65	1.00	0.65	0.43	0.28	0.18
0.7	0.2	152.32	25.82	9.48	4.66	2.65	1.63	1.04	0.68	0.44
	0.3	87.85	18.38	7.46	3.88	2.29	1.44	0.94	0.62	0.40
	0.4	59.34	14.92	6.47	3.48	2.09	1.33	0.87	0.58	0.37
	0.5	35.18	10.56	4.78	2.60	1.55	0.97	0.62	0.38	0.22
	0.6	28.26	9.51	4.41	2.42	1.45	0.90	0.57	0.35	0.19
	0.7	25.45	8.91	4.16	2.28	1.36	0.85	0.53	0.32	0.17
	0.8	25.21	8.60	3.99	2.18	1.29	0.79	0.49	0.28	0.14
	0.9	26.68	8.48	3.86	2.08	1.22	0.74	0.45	0.25	0.12
	1.0	29.34	8.49	3.77	2.01	1.16	0.70	0.41	0.22	0.10
0.8	0.2	136.74	26.38	10.30	5.22	3.01	1.85	1.17	0.74	0.45
	0.3	75.52	19.20	8.32	4.45	2.64	1.66	1.06	0.67	0.41
	0.4	37.55	12.79	5.92	3.23	1.91	1.17	0.72	0.42	0.21
	0.5	27.25	11.28	5.41	2.98	1.77	1.08	0.66	0.37	0.18
	0.6	24.23	10.57	5.10	2.81	1.66	1.01	0.60	0.33	0.14
	0.7	25.36	10.32	4.91	2.69	1.57	0.94	0.55	0.29	0.11
	0.8	29.09	10.37	4.80	2.59	1.50	0.88	0.50	0.25	0.08
	0.9	34.55	10.60	4.74	2.50	1.42	0.82	0.46	0.21	0.05
	1.0	41.23	10.98	4.71	2.43	1.36	0.77	0.41	0.18	0.01
0.9	0.2	90.70	23.73	10.34	5.54	3.28	2.05	1.30	0.81	0.47
	0.3	29.93	14.20	6.95	3.86	2.30	1.41	0.85	0.48	0.22
	0.4	16.27	12.21	6.28	3.55	2.12	1.29	0.77	0.42	0.18
	0.5	14.80	11.58	5.96	3.35	1.99	1.20	0.70	0.37	0.14
	0.6	19.43	11.62	5.81	3.23	1.89	1.13	0.64	0.32	0.10
	0.7	27.55	12.06	5.77	3.14	1.81	1.06	0.59	0.27	0.06
	0.8	37.84	12.73	5.79	3.07	1.74	0.99	0.53	0.23	0.02
	0.9	49.59	13.57	5.85	3.01	1.67	0.93	0.48	0.18	-0.02
	1.0	62.35	14.52	5.94	2.97	1.61	0.87	0.42	0.14	-0.06
1.0	0.2	-6.40	12.70	7.32	4.31	2.64	1.64	1.00	0.56	0.25
	0.3	-17.35	10.90	6.66	3.97	2.44	1.51	0.90	0.49	0.20
	0.4	-11.05	11.02	6.50	3.82	2.32	1.41	0.83	0.43	0.15
	0.5	2.15	11.91	6.54	3.74	2.23	1.33	0.76	0.38	0.10
	0.6	18.80	13.18	6.67	3.70	2.16	1.26	0.70	0.32	0.06
	0.7	37.42	14.67	6.86	3.68	2.09	1.19	0.63	0.26	0.01
	0.8	57.27	16.30	7.09	3.67	2.03	1.12	0.57	0.21	-0.04
	0.9	77.95	18.02	7.35	3.66	1.97	1.06	0.51	0.15	-0.09
	1.0	99.20	19.80	7.61	3.67	1.92	1.00	0.45	0.10	-0.14

ED5-3 Tee, $D_c > 250$ mm, Converging (Continued)

A_t/A_c	A_b/A_c	C_b Values									
		Q_b/Q_c									
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	
0.2	0.2	-26.08	-4.19	-0.70	0.33	0.71	0.87	0.93	0.95	0.93	
	0.3	-59.71	-10.53	-2.72	-0.43	0.43	0.78	0.91	0.95	0.91	
	0.4	-106.78	-19.39	-5.53	-1.46	0.05	0.67	0.91	0.97	0.91	
	0.5	-167.36	-30.77	-9.12	-2.78	-0.42	0.55	0.93	1.02	0.93	
	0.6	-241.50	-44.68	-13.50	-4.37	-0.97	0.42	0.96	1.10	0.98	
	0.7	-329.25	-61.15	-18.68	-6.25	-1.62	0.27	1.02	1.21	1.06	
	0.8	-430.67	-80.18	-24.67	-8.42	-2.37	0.10	1.09	1.35	1.17	
	0.9	-545.81	-101.78	-31.47	-10.89	-3.22	-0.08	1.17	1.52	1.31	
	1.0	-674.72	-125.98	-39.08	-13.64	-4.17	-0.28	1.28	1.72	1.48	
0.3	0.2	-15.50	-2.16	-0.04	0.58	0.81	0.90	0.93	0.94	0.91	
	0.3	-35.76	-5.90	-1.20	0.16	0.66	0.85	0.92	0.92	0.88	
	0.4	-64.09	-11.09	-2.78	-0.38	0.48	0.82	0.93	0.94	0.86	
	0.5	-100.54	-17.73	-4.78	-1.06	0.29	0.80	0.97	0.98	0.87	
	0.6	-145.16	-25.85	-7.21	-1.86	0.06	0.80	1.05	1.05	0.90	
	0.7	-198.01	-35.46	-10.08	-2.81	-0.19	0.82	1.15	1.16	0.96	
	0.8	-259.13	-46.56	-13.39	-3.89	-0.47	0.85	1.28	1.30	1.05	
	0.9	-328.59	-59.18	-17.15	-5.11	-0.78	0.89	1.44	1.47	1.17	
	1.0	-406.44	-73.33	-21.37	-6.48	-1.12	0.94	1.63	1.68	1.32	
0.4	0.2	-10.31	-1.18	0.26	0.69	0.84	0.91	0.93	0.93	0.90	
	0.3	-23.96	-3.65	-0.48	0.43	0.75	0.88	0.91	0.91	0.86	
	0.4	-42.98	-7.03	-1.46	0.11	0.67	0.87	0.93	0.91	0.84	
	0.5	-67.44	-11.35	-2.69	-0.26	0.59	0.90	0.97	0.94	0.84	
	0.6	-97.39	-16.60	-4.17	-0.69	0.52	0.95	1.06	1.01	0.87	
	0.7	-132.88	-22.81	-5.91	-1.17	0.46	1.03	1.17	1.11	0.92	
	0.8	-173.96	-29.99	-7.90	-1.73	0.40	1.15	1.33	1.24	1.00	
	0.9	-220.69	-38.15	-10.16	-2.35	0.35	1.29	1.51	1.40	1.11	
	1.0	-273.12	-47.31	-12.70	-3.04	0.29	1.45	1.74	1.61	1.26	
0.5	0.2	-7.26	-0.62	0.43	0.75	0.86	0.91	0.93	0.93	0.90	
	0.3	-16.99	-2.35	-0.07	0.57	0.80	0.89	0.91	0.90	0.87	
	0.4	-30.49	-4.67	-0.72	0.38	0.76	0.89	0.92	0.90	0.85	
	0.5	-47.82	-7.61	-1.50	0.19	0.75	0.93	0.97	0.93	0.85	
	0.6	-69.03	-11.17	-2.42	-0.03	0.76	1.01	1.05	0.98	0.88	
	0.7	-94.17	-15.37	-3.49	-0.26	0.80	1.13	1.17	1.07	0.93	
	0.8	-123.30	-20.22	-4.71	-0.50	0.87	1.29	1.33	1.20	1.02	
	0.9	-156.48	-25.73	-6.09	-0.77	0.96	1.48	1.53	1.36	1.13	
	1.0	-193.74	-31.92	-7.63	-1.07	1.06	1.71	1.77	1.56	1.28	
0.6	0.2	-5.28	-0.27	0.54	0.78	0.88	0.91	0.93	0.93	0.91	
	0.3	-12.43	-1.51	0.18	0.66	0.83	0.89	0.91	0.91	0.89	
	0.4	-22.29	-3.15	-0.25	0.55	0.82	0.90	0.92	0.91	0.88	
	0.5	-34.92	-5.19	-0.74	0.46	0.84	0.95	0.96	0.93	0.89	
	0.6	-50.35	-7.64	-1.30	0.38	0.91	1.05	1.04	0.98	0.93	
	0.7	-68.66	-10.52	-1.94	0.32	1.01	1.18	1.16	1.07	0.99	
	0.8	-89.89	-13.83	-2.65	0.26	1.15	1.36	1.33	1.19	1.08	
	0.9	-114.09	-17.61	-3.46	0.22	1.32	1.59	1.53	1.35	1.21	
	1.0	-141.33	-21.84	-4.35	0.18	1.54	1.85	1.77	1.54	1.37	
0.7	0.2	-3.90	-0.03	0.61	0.81	0.89	0.92	0.94	0.94	0.93	
	0.3	-9.25	-0.94	0.35	0.72	0.85	0.90	0.92	0.92	0.92	
	0.4	-16.54	-2.10	0.07	0.66	0.85	0.91	0.93	0.92	0.92	
	0.5	-25.85	-3.51	-0.22	0.64	0.90	0.97	0.97	0.94	0.95	
	0.6	-37.21	-5.18	-0.54	0.65	1.00	1.07	1.05	1.00	1.00	
	0.7	-50.68	-7.13	-0.87	0.70	1.14	1.22	1.17	1.08	1.08	
	0.8	-66.31	-9.37	-1.24	0.78	1.33	1.41	1.33	1.21	1.20	
	0.9	-84.17	-11.92	-1.64	0.89	1.56	1.65	1.53	1.36	1.34	
	1.0	-104.29	-14.78	-2.09	1.03	1.84	1.94	1.78	1.56	1.52	
0.8	0.2	-2.90	0.15	0.67	0.83	0.90	0.93	0.94	0.95	0.96	
	0.3	-6.91	-0.53	0.47	0.76	0.87	0.91	0.93	0.94	0.96	
	0.4	-12.31	-1.34	0.30	0.74	0.88	0.93	0.94	0.95	0.98	
	0.5	-19.16	-2.29	0.15	0.77	0.94	0.99	0.98	0.98	1.03	
	0.6	-27.50	-3.39	0.01	0.84	1.06	1.09	1.06	1.03	1.11	
	0.7	-37.38	-4.66	-0.11	0.97	1.23	1.24	1.18	1.12	1.21	
	0.8	-48.87	-6.11	-0.22	1.15	1.46	1.45	1.35	1.25	1.35	
	0.9	-62.01	-7.75	-0.33	1.37	1.73	1.70	1.55	1.41	1.52	
	1.0	-76.85	-9.59	-0.44	1.63	2.06	2.00	1.80	1.61	1.73	

ED5-3 Tee, $D_c > 250$ mm, Converging (Continued)

		C_b Values (Concluded)								
		Q_b/Q_c								
A_s/A_c	A_b/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.9	0.2	-2.14	0.28	0.71	0.85	0.91	0.94	0.96	0.97	0.99
	0.3	-5.14	-0.21	0.57	0.80	0.88	0.92	0.95	0.97	1.02
	0.4	-9.09	-0.76	0.47	0.80	0.91	0.94	0.96	0.98	1.06
	0.5	-14.06	-1.36	0.42	0.86	0.98	1.01	1.01	1.02	1.14
	0.6	-20.08	-2.04	0.42	0.99	1.11	1.12	1.09	1.09	1.24
	0.7	-27.21	-2.79	0.47	1.17	1.30	1.27	1.21	1.19	1.38
	0.8	-35.50	-3.63	0.55	1.42	1.55	1.49	1.38	1.32	1.55
	0.9	-45.01	-4.57	0.66	1.72	1.86	1.75	1.59	1.49	1.75
	1.0	-55.79	-5.64	0.80	2.08	2.22	2.06	1.84	1.69	1.99
1.0	0.2	-1.54	0.39	0.74	0.87	0.92	0.95	0.97	0.99	1.03
	0.3	-3.75	0.03	0.64	0.83	0.90	0.94	0.97	1.00	1.08
	0.4	-6.57	-0.32	0.61	0.85	0.93	0.97	0.99	1.03	1.16
	0.5	-10.05	-0.65	0.64	0.94	1.02	1.03	1.04	1.08	1.26
	0.6	-14.24	-0.98	0.74	1.10	1.16	1.15	1.13	1.16	1.40
	0.7	-19.20	-1.32	0.91	1.33	1.37	1.31	1.26	1.27	1.57
	0.8	-24.98	-1.69	1.14	1.63	1.63	1.53	1.43	1.41	1.78
	0.9	-31.62	-2.10	1.42	2.00	1.96	1.80	1.64	1.59	2.02
	1.0	-39.19	-2.55	1.76	2.43	2.35	2.12	1.90	1.81	2.30

ED5-3 Tee, $D_c > 250$ mm, Converging

		C_s Values								
		Q_s/Q_c								
A_s/A_c	A_b/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.2	0.2	20.43	3.28	1.45	0.98	0.81	0.73	0.69	0.66	0.64
	0.3	12.53	2.40	1.22	0.90	0.77	0.71	0.68	0.66	0.64
	0.4	8.78	1.98	1.12	0.86	0.76	0.70	0.67	0.66	0.64
	0.5	6.69	1.75	1.06	0.84	0.75	0.70	0.67	0.65	0.64
	0.6	5.43	1.61	1.02	0.83	0.74	0.70	0.67	0.65	0.64
	0.7	4.64	1.52	1.00	0.82	0.74	0.70	0.67	0.65	0.64
	0.8	4.15	1.47	0.98	0.81	0.74	0.69	0.67	0.65	0.64
	0.9	3.86	1.43	0.97	0.81	0.74	0.69	0.67	0.65	0.64
	1.0	3.71	1.42	0.97	0.81	0.73	0.69	0.67	0.65	0.64
0.3	0.2	51.24	7.11	2.49	1.33	0.90	0.70	0.60	0.54	0.50
	0.3	29.57	4.70	1.87	1.10	0.80	0.66	0.58	0.53	0.50
	0.4	19.40	3.57	1.58	1.00	0.76	0.64	0.57	0.52	0.50
	0.5	13.84	2.96	1.42	0.94	0.73	0.62	0.56	0.52	0.50
	0.6	10.58	2.59	1.32	0.90	0.72	0.62	0.56	0.52	0.49
	0.7	8.64	2.38	1.27	0.88	0.71	0.61	0.56	0.52	0.49
	0.8	7.52	2.25	1.23	0.87	0.70	0.61	0.56	0.52	0.49
	0.9	6.95	2.19	1.22	0.87	0.70	0.61	0.56	0.52	0.49
	1.0	6.76	2.17	1.21	0.86	0.70	0.61	0.55	0.52	0.49
0.4	0.2	90.30	12.10	3.91	1.85	1.08	0.74	0.55	0.45	0.38
	0.3	49.68	7.59	2.74	1.42	0.90	0.65	0.51	0.43	0.37
	0.4	30.96	5.51	2.21	1.23	0.82	0.61	0.49	0.42	0.37
	0.5	21.00	4.40	1.92	1.13	0.78	0.59	0.48	0.42	0.37
	0.6	15.43	3.78	1.76	1.07	0.75	0.58	0.48	0.41	0.37
	0.7	12.36	3.44	1.67	1.04	0.74	0.57	0.48	0.41	0.37
	0.8	10.86	3.27	1.63	1.02	0.73	0.57	0.47	0.41	0.37
	0.9	10.40	3.22	1.61	1.01	0.73	0.57	0.47	0.41	0.37
	1.0	10.67	3.25	1.62	1.02	0.73	0.57	0.47	0.41	0.37
0.5	0.2	126.36	16.99	5.39	2.42	1.32	0.81	0.54	0.38	0.28
	0.3	65.94	10.28	3.65	1.79	1.05	0.68	0.48	0.35	0.27
	0.4	38.84	7.27	2.87	1.51	0.93	0.63	0.45	0.34	0.27
	0.5	25.07	5.74	2.47	1.37	0.87	0.60	0.44	0.33	0.26
	0.6	17.98	4.95	2.27	1.29	0.84	0.58	0.43	0.33	0.26
	0.7	14.69	4.58	2.17	1.26	0.82	0.58	0.43	0.33	0.26
	0.8	13.78	4.48	2.15	1.25	0.82	0.57	0.43	0.33	0.26
	0.9	14.45	4.56	2.17	1.26	0.82	0.58	0.43	0.33	0.26
	1.0	16.24	4.76	2.22	1.28	0.83	0.58	0.43	0.33	0.26

ED5-3 Tee, $D_c > 250$ mm, Converging (Continued)

		C_s Values (Concluded)								
		Q_s/Q_c								
A_s/A_c	A_p/A_c	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.6	0.2	146.22	20.32	6.54	2.92	1.54	0.89	0.54	0.33	0.20
	0.3	70.93	11.95	4.37	2.13	1.20	0.73	0.46	0.30	0.18
	0.4	38.66	8.37	3.44	1.80	1.06	0.67	0.43	0.28	0.18
	0.5	23.61	6.70	3.00	1.64	0.99	0.64	0.42	0.28	0.18
	0.6	17.17	5.98	2.82	1.57	0.97	0.62	0.41	0.27	0.18
	0.7	15.64	5.81	2.77	1.56	0.96	0.62	0.41	0.27	0.18
	0.8	17.19	5.98	2.82	1.57	0.97	0.62	0.41	0.27	0.18
	0.9	20.79	6.38	2.92	1.61	0.98	0.63	0.42	0.27	0.18
	1.0	25.82	6.94	3.07	1.66	1.00	0.64	0.42	0.28	0.18
0.7	0.2	137.78	20.74	7.01	3.21	1.70	0.96	0.54	0.29	0.13
	0.3	58.74	11.96	4.73	2.39	1.34	0.79	0.47	0.26	0.12
	0.4	27.78	8.52	3.84	2.06	1.21	0.73	0.44	0.24	0.11
	0.5	16.04	7.21	3.50	1.94	1.15	0.71	0.43	0.24	0.11
	0.6	13.91	6.97	3.44	1.92	1.14	0.70	0.42	0.24	0.11
	0.7	17.28	7.35	3.54	1.95	1.16	0.71	0.43	0.24	0.11
	0.8	24.08	8.10	3.73	2.02	1.19	0.72	0.43	0.24	0.11
	0.9	33.17	9.11	3.99	2.12	1.23	0.74	0.44	0.25	0.11
	1.0	43.86	10.30	4.30	2.23	1.28	0.76	0.45	0.25	0.11
0.8	0.2	92.97	17.35	6.57	3.21	1.75	0.99	0.55	0.27	0.08
	0.3	26.98	10.02	4.67	2.52	1.46	0.86	0.48	0.24	0.07
	0.4	6.75	7.77	4.09	2.31	1.37	0.81	0.46	0.23	0.06
	0.5	4.83	7.56	4.03	2.29	1.36	0.81	0.46	0.23	0.06
	0.6	12.05	8.36	4.24	2.37	1.39	0.83	0.47	0.23	0.07
	0.7	24.51	9.75	4.60	2.49	1.45	0.85	0.48	0.24	0.07
	0.8	40.23	11.49	5.05	2.66	1.52	0.88	0.50	0.24	0.07
	0.9	58.13	13.48	5.57	2.85	1.60	0.92	0.51	0.25	0.07
	1.0	77.56	15.64	6.13	3.05	1.68	0.96	0.53	0.26	0.08
0.9	0.2	10.77	10.05	5.20	2.91	1.70	0.99	0.55	0.25	0.04
	0.3	-21.27	6.49	4.28	2.57	1.56	0.93	0.52	0.24	0.04
	0.4	-19.11	6.73	4.34	2.60	1.57	0.93	0.52	0.24	0.04
	0.5	-3.28	8.49	4.80	2.76	1.64	0.97	0.54	0.24	0.04
	0.6	19.39	11.01	5.45	3.00	1.74	1.01	0.56	0.25	0.04
	0.7	45.97	13.96	6.21	3.27	1.86	1.07	0.58	0.27	0.05
	0.8	74.99	17.18	7.05	3.58	1.98	1.13	0.61	0.28	0.05
	0.9	105.64	20.59	7.93	3.89	2.12	1.19	0.64	0.29	0.06
	1.0	137.43	24.12	8.85	4.23	2.26	1.26	0.67	0.31	0.06
1.0	0.2	-99.78	-0.17	3.15	2.40	1.58	0.98	0.56	0.25	0.02
	0.3	-75.42	2.54	3.85	2.65	1.69	1.03	0.58	0.26	0.03
	0.4	-38.31	6.66	4.92	3.04	1.86	1.11	0.62	0.28	0.03
	0.5	3.90	11.35	6.14	3.48	2.04	1.20	0.66	0.29	0.04
	0.6	48.66	16.32	7.43	3.94	2.24	1.29	0.70	0.31	0.04
	0.7	94.88	21.46	8.76	4.43	2.45	1.38	0.75	0.33	0.05
	0.8	142.01	26.70	10.12	4.92	2.66	1.48	0.79	0.35	0.06
	0.9	189.74	32.00	11.49	5.41	2.87	1.58	0.84	0.37	0.07
	1.0	237.90	37.35	12.88	5.92	3.08	1.68	0.88	0.39	0.07

Appendix B Schematics of ASHRAE's example