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Electromagnetic Wave Excitation and Propagation in Novel Photonic Materials

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Lists of Publications

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1. **X. Shi**, X. Lin, F. Gao, H. Xu, Z. Yang, and B. Zhang*, Caustic graphene plasmons with Kelvin angle, **Phys. Rev. B**, vol. 92, 081404(Rapid Communication), Aug. 2015.
2. **X. Shi**, F. Gao, X. Lin, and B. Zhang*, Electromagnetic Detection of a Perfect Carpet Cloak, **Sci. Rep.**, vol. 5, 10401, May 2015.

Others

3. F. Gao, Z. Gao, **X. Shi**, Z. Yang, X. Lin, H. Xu, J. D. Joannopoulos, M. Soljacic, H. Chen*, L. Lu, Y. Chong*, and B. Zhang*, Probing topological protection using a designer surface plasmon structure, **Nature Comm.**, vol. 7, 11619, May 2016.
4. Z. Gao, F. Gao, Y. Zhang, **X. Shi**, Z. Yang, and B. Zhang*, Experimental demonstration of high-order magnetic localized spoof surface plasmons, **Appl. Phys. Lett.**, vol. 107, 041118, Jul. 2015.
5. Z. Yang, F. Gao, **X. Shi**, X. Lin, Z. Gao, Y. Chong, and B. Zhang*, Topological Acoustics, **Phys. Rev. Lett.**, vol. 114, 114301, Mar. 2015.
6. H. Xu, **X. Shi**, F. Gao, H. Sun* and B. Zhang*, Ultra-thin three-dimensional thermal cloak, **Phys. Rev. Lett.**, vol. 112, 054301, Feb. 2014.

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Chapter 1

Introduction

We report in this thesis the new wave phenomena from the interaction of swift sources (electrons and a rotating dipole), which have relativistic speeds, with graphene, metals and a carpet cloak from the point of classical electrodynamics. We carry out these studies not only because of scientific curiosity, but also with the motivation to provide theories for some experiments carried out before and find some new physical phenomena for the future experiments of plasmonics.

Graphene plasmons (GPs) have attracted a lot of scientific interests in recent years [1–4]. Because of the large momentum mismatch between GPs and incident electromagnetic waves, the optical excitation of GPs has a low efficiency [3, 5]. The swift electrons provide an alternative source for GPs, as the electron field can be regarded as a supercontinuum source of evanescent waves [6–8]. We study in Chapter 2 and 3 the excitation of GPs with swift electrons.

The studies of GPs have made great improvements from basic wave properties, like static images detection [9, 10], propagation length [2, 4], to more complex wave phenomena, like refraction [11] and plasmonic Cherenkov radiation (CR) [12].

In Chapter 2, we show that the complex wave phenomena in hydrodynamics and optics can find counterparts in GPs, and the well developed techniques in hydrodynamics and optics can also be applied to study GPs.

Our theoretical studies can also give explanations and guides for the experiments conducted before and in the future. In Chapter 3, we study a swift electron passing through graphene. This setup has been used in studying GPs with the electron energy loss spectroscopy (EELS) [13], but there was no thoroughly studies of the physical process in theory, especially the energy coupled to GPs. In the previous studies, the electron energy loss spectra was equivalent to the energy spectra of GPs excited by the swift electrons. But our study shows that it is valid only under the lossless GPs condition. In Chapter 4, we use a half-space (half air and half metal) model to clarify the controversy in the past about the origin of the emission when electrons pass through metal plates – whether it came from transition radiation (TR) or radiative bulk plasmons [14–16]. We show in Chapter 4 that both TR and radiative bulk plasmons contribute to the emission. We find that the experiments of light interaction with nano-structures conducted should be modelled by moving dipoles [17, 18] while only fixed in position dipole models have been studied in theory. We present, in Chapter 5, the Doppler effect in the GPs excitation by a swift dipole.

The interaction of swift electrons with materials has been widely used to study the properties of materials since the first electron microscope invented in 1931 [19]. Before the development of electron microscopes, optical microscopes contributed to this end. However, the resolution of optical microscopes is limited by the Rayleigh criterion, which is about half of the light wavelength. Since the de Broglie wavelength for electrons is much smaller than that of light, which is

below $0.1 \text{ \AA}$ for swift electrons at energy of 20keV, the electron microscopes can greatly improve the spacial resolution to less than $1 \text{ \AA}$ [20].

More than acquiring static images of higher resolution by electron microscopes, the interaction of swift electrons with materials can be used to study the intrinsic properties of materials [6], such as finding the collective modes sustained by the materials [21, 22], and the corresponding electromagnetic dispersion relationship [22, 23]. The electron field can be regarded as a supercontinuum source of evanescent waves. One advantage of evanescent waves is that they lie outside the light cone which is formed by only propagating waves [6, 24]. Therefore, the evanescent waves can provide larger wave-vector than that of propagating waves of the same frequency, and excite the collective modes which are unachievable with lasers in materials.

From about 100 years ago to the present, the interactions of swift electrons with metals have contributed a lot in discovering and characterizing electron gas plasmons [25–30], including bulk plasmons [21], and surface plasmon polaritons (SPPs) [31, 32]. Plasmons are quasi-particles representing the collective oscillations of electron gas plasmas, which is the classical model of metals and graphene. Compared to bulk plasmons, SPPs are confined and propagating at the surface of metals or semiconductors.

Several kinds of signal can be detected to probe the interaction of swift electrons with materials. One kind is the near-field signal which consists evanescent waves and decays fast in the direction normal to the surface. The near-field signal can be collected and measured by the sub-wavelength nano-probe of a near-field scanning optical microscopy (NSOM) [33, 34], or in another name scattering-type scanning near-field optical microscope (SNOM) [35]. The near-field nano-imaging

develops fast in recent years and is especially useful in detecting SPPs and GPs which are evanescent in the direction normal to the surface. This technique has been used to provide real-space images of GPs field distribution [9, 10].

Another widely used signal is the electron energy loss spectra. Since the first systematic measurement of characteristic electron energy loss of swift electrons scattered by metals conducted by Rudberg in 1936 [36], electron energy loss spectra have become a powerful tool in studying the collective modes sustained by materials [6, 29, 37, 38]. The discoveries, both in experiments and theories, of bulk plasmons and surface plasmons supported by metals were attributed to electron energy loss spectra of swift electrons scattered by metal foils [25–28, 36]. The electron energy loss spectra can be used to retrieve the dispersion relationship of plasmons and verify the constitutive parameters of materials [22, 23].

The third kind of signal is the far-field radiation, or cathodoluminescence (CL). The CL includes coherent emission and incoherent emission, which correspond to the linear and nonlinear signals, respectively [6]. However, the incoherent emission is very weak in metals. The coherent emission results from multiple physical processes, like CR, transition radiation (TR), or the leaky waves from both bulk and surface plasmons. SPPs can also be decoupled into emission when encountering gratings [39] or nano-particles like quantum dots [40].

CR will be excited when a swift electron travels faster than the phase velocity of light in that medium [41]. It was discovered almost simultaneously with the discovery of the plasmon mode in metals excited by swift electrons [36, 42]. CR was explained later by Frank in classical electromagnetic wave theory [41]. CR is widely used to study the velocity of swift electrons in particle physics. Additionally, CR can also be used to detect the properties of materials [43, 44], and can work

as a light source [7, 24]. TR was discovered later, and unlike CR which requires superluminal electrons, TR is produced when a swift electron passes through an inhomogeneous medium [45], of which the simplest model is the half-space model comprising two different homogeneous materials [46].

The interaction of swift electrons with materials is classified as direct contact and indirect contact. The direct contact is that swift electrons impinge on materials, like passing through or being scattered back from a thin plate [25, 27, 36, 47]. The indirect contact is that swift electrons move close to, but not pass through or be scattered from, materials. For example, swift electrons moves parallel to the surface of materials [29, 48–53] or along a parabolic trajectory [54] above the surface. In the indirect contact cases, the electrons interact with the material via evanescent fields. Two instructive situations to study the interaction of swift electrons with materials are shown in Fig. 1.1. In one case, a swift electron moves parallel to the planar interface separating the two media, with the impact parameter b relative to the interface. In another case, a swift electron passes through the interface along a normal trajectory.

Except for studying the properties of materials, the interaction of swift electrons with materials provides a good source for light and SPPs [7, 24, 55, 56]. Because of the large momentum mismatch between SPPs and light, lasers cannot excite SPPs directly. The momentum ratio of SPPs in metal and light in vacuum is slightly above one. The momentum of light can be increased with the help of prisms and gratings [57], and thereby being coupled to that of surface plasmons in metals. But the low efficiency strongly limit the development of plasmonics with metals. The momentum mismatch between GPs and light in vacuum is large with the ratio being dozens [2, 3, 58], making the traditional methods like

Figure 1.1: There are two homogeneous materials with relative permittivities of ϵ_1 and ϵ_2 . In the left part, a swift electron moves parallel to the interface with the impact parameter b . In the right part, a swift electron passes through the interface along a perpendicular trajectory. In both cases, the swift electrons move with a constant velocity v . Adopted from [6].

using prisms and gratings to increase the momentum almost impossible. Swift electrons, on the other hand, provide evanescent waves within a wide range of frequencies. In the recent study of GPs excited by swift electrons, the coupling efficiency is relatively high because of the large quality factor, great confinement, thus facilitating strong light-matter interaction in GPs [58,59]. Furthermore, the size of optic devices are limited by the diffraction limit of light. However, electronic devices can be designed with several nanometers, as the wavelengths of de Broglie waves are much smaller than that of light.

Graphene physics has become a vibrant field of investigation since the discovery of graphene [60,61]. People found later that it greatly overlaps with plasmonics [1,3,62]. Graphene possess intrinsic plasmons working at terahertz (THz) [59,63]. GPs are the coupling between electrons' motion and electromagnetic fields, therefore the tunable and adjustable electrical properties of graphene [64] would contribute to the manipulation of GPs [65]. Graphene also has greater

carrier concentrations when grating or doped [3, 66]. Graphene plasmonics are attractive alternatives to traditional metal plasmonics, because of the relatively long lifetimes, strong confinement, and high localization to volumes (of the order of 10^6 times smaller than the diffraction limit of light) of GPs [67], thus facilitating strong light-matter interactions and great efficiency [58]. The combination of graphene with conventional plasmonic nanostructures and metamaterials can further increase the versatility of GPs [4, 65, 67, 68], like providing sensitive sensors [69], and increasing the working frequency from THz to visible frequency [70].

The dynamical physical processes of the interaction between swift electrons and monolayer graphene are worth investigating for the future applications of graphene plasmonics [63]. We use the classical electrodynamics to investigate the interaction processes. Classical theory is enough to describe the physical phenomena with dimensions above 10nm [6]. Furthermore, classical electrodynamics can give us the dynamical evolution, which is necessary for exploring a myriad of applications of GPs in plasmonics [6]. We consider two instructive models to study the interaction of swift electrons with monolayer graphene. One is that a swift electron moves close and parallel to a monolayer graphene, the other is that a swift electron penetrates a monolayer graphene with the normal trajectory. In the first case, Cherenkov-type [17, 71] GPs are excited. The dynamical processes are explained by the Kelvin model [72], which was developed by Lord Kelvin in 1887 for ship wakes. The second is the extension of TR theory [45] to graphene. In the study of TR, a swift electron penetrates a free standing monolayer graphene in air.

We also use classical electrodynamics to study the case of a swift electron passing from one half space occupied with air to another half space occupied

with metal. The metal is assumed to be the homogeneous electrical gas described by Drude model [73, 74]. In the previous studies of the radiation excited by a swift electron passing through a thin metal plate, there was controversy about the origin of the radiation [16, 31, 75]. Some believed that the radiation came from the radiative bulk plasmons [76–78], and others supported that it came from TR [79]. There was also a paper claiming the equivalent of the two kinds of explanation [15]. In our work, we use a half-space structure to clarify the controversy. We use classical electrodynamics to show the dynamical physical processes when a swift electron passes from air into metal. The dynamical processes clearly show that the radiation is contributed by both radiative plasmons and TR. The radiations from the two channels are quite different—the radiation from TR dominates in intensity, but that from radiative plasmons lasts much longer, especially in low damping electronic gases.

Swift electrons carry only linear momentum, but rotating electrical dipoles can provide angular momentum [18]. The rotating dipole model can be used to describe some light-metamaterial interactions, like the cases where circular polarized lights are incident onto some nano-structures [17, 18, 80, 81]. In the previous theoretical studies of the excitation of guide waves from a rotating dipole, the dipole remains in the same position [18, 82]. However, in the experiments carried out recently [17, 18], the sources are better to be modelled by the swift moving dipoles.

When a rotating dipole is fixed in position, the guide waves are strengthened on one side, depending on the helicity of the rotating dipole. In our work, we study the case of a swift rotating dipole moving above and parallel to a free-standing monolayer graphene. We choose graphene as the substrate for the guided waves

because of the much low phase velocity of GPs, which is of the magnitude of $1/100c$, with c the phase velocity of light in vacuum [3]. Therefore, the Cherenkov-type guided waves are relatively easy to be realized in graphene. Based on the asymmetrical wave pattern of the guided waves excited by a rotating dipole, it is expected that the Cherenkov-type GPs excited by a swift rotating dipole would also be strengthened on one side. However, our calculation shows that the GPs pattern's intensity distribution is determined by the combination of the helicity and the speed of the swift dipole. At relatively low speed, the pattern's intensity distribution is mainly affected by the helicity. But as the speed increases, the Doppler effect would contribute to the asymmetry of the GPs in a way that offsets the contribution from the helicity of the rotating dipole.

We also calculate the radiation from a swift electron passing through a carpet cloak [83–85]. Cloaks have attracted many scientific interests in recent years because of their ability to guide light rays around the hidden objects [86, 87]. Cloaks are not observable to eyes or other detectors that receive electromagnetic signals. Cloaks are designed based on the principles of guiding optical rays. When a photon is incident on the cloak, the photon will not detect the existence of the cloak, therefore, no reflection or scattering would happen. However, when a swift electron is incident on the cloak, it will interact with the cloak and give out radiation [88], making the cloak visible. This is the only method developed so far to make a perfect cloak observable to eyes.

The thesis is organized as follows. Chapter 2 and 3 are about the interaction of a swift electron with a free-standing monolayer graphene, with the trajectory parallel and perpendicular to the graphene sheet. Chapter 4 discusses the case where a swift electron is incident on a bulk metal modelled by free electron gas.

In this chapter, the contributions to the radiation and surface plasmons come from both radiative plasmons and TR. In Chapter 5, the directional excitation of plasmonic CR is excited by a swift and rotating dipole is presented. Both the helicity from the rotation and the Doppler effect of the dipole near-field affect the asymmetrical GPs pattern. In Chapter 6, the radiation excited by a swift electron passing through a carpet cloak is discussed.

Caustic graphene plasmons with Kelvin Angle

Graphene, which is strictly a two-dimensional (2D) crystalline material [60, 89] showing remarkable electrical and optical properties [1, 61, 90], promises a variety of applications [4, 65, 68, 70]. One branch is graphene plasmons (GPs) [9, 10, 65, 91–94], the typical 2D plasmons confined on a 2D surface [95]. As the first real 2D crystalline material, graphene supports GPs with many useful advantages, such as high quantum efficiency, small compact sizes, being tunable and adjustable, high confinement, and broad spectral bandwidth [3, 11, 58, 96], etc.

One of the techniques used to study GPs is electron energy-loss spectroscopy (EELS) [6, 92, 97–99]. Two particular situations in studying planar surface with EELS are that a swift electron is moving parallel and perpendicular to the surface [6]. These two configurations have been extensively studied both experimentally and theoretically for graphene [8, 13, 92, 100] recently, and other materials before [49, 101–106].

It is well known that 2D plasmons in the long wavelength limit have the same dispersion as that of deep-water waves [3, 6, 94, 107]. Therefore, some of the deep-water wave phenomena can find counterparts in GPs. The well developed mathematical models and techniques in hydrodynamic can also be applied to GPs. Recently, one famous hydrodynamic phenomenon, the Kelvin angle of ship wakes [108, 109] was re-investigated [110, 111]. Lord Kelvin made the prediction that the semi-angle of ship wakes on deep-water surface is fixed at 19.5° [72], independent of the ship's velocities. But it was recently challenged that the semi-angle of ship wakes would gradually transit from the Kelvin angle to the Mach angle as the ship's velocity increases [110, 111]. Kelvin made his prediction based on the assumption that the water waves excited by a ship would propagate to all directions. But it is found recently that the water waves' propagating directions are affected by the Frouder number, the ratio of the ship's dimension to its velocity, and therefore the water waves have preferred propagating directions.

Caustic wave theory is adopted to analyze the GPs excited by a swift electron moving above a free standing graphene sheet. Caustics are the places where the rays accumulate, and thereby the field intensities swell [112]. The recently generated arbitrary bending surface plasmon beams on metal surfaces [113, 114] are the results of waves' caustic effects. Yet caustics were treated predominantly on the level of geometrical optics in the previous studies, and have never been discussed in GPs. In our model, the GPs excited by a swift electron moving parallel to a graphene sheet also have the caustics. We find that the GPs accumulate along the V-shape boundaries of the GPs pattern when the electron's velocity is relatively slow ($\sim 0.1c$ or smaller, where c is the phase velocity of light in vacuum). The semi-angle of the V-shape is also 19.5° . The boundaries are the loci of fold

caustics with cusps wave fronts [112]. Each point inside the boundary is covered twice by rays, whereas there are no rays outside. Detailed calculation provides a complete field distribution in the neighborhood of caustics. The field does not stop abruptly in the boundary, though from the geometrical optics point, the rays make a sharp stop there. It was claimed that there are requirements for the caustics to be deemed real, i.e., observed or detected. One is that the amplitude on the caustic is at least a few times the field value elsewhere. We find that as the velocity of the electron increases, the GPs arriving at the caustics become weak. Therefore, the caustics get blur and vanish eventually as the electron's velocity increases. When the velocity increases, the effective semi-angle of the pattern approaches the Mach angle, similar to the recent studies of ship wakes in fluid mechanics [110, 111].

2.1 The model setup

The theoretical model is illustrated in Fig. 2.1. An electron with charge q is moving along the z direction with a uniform speed v , and it is moving parallel to a monolayer graphene sheet with the horizontal distance of d . The current density of the swift electron is,

$$\mathbf{J}(\mathbf{r}, t) = \hat{z} q v \delta(x) \delta(y) \delta(z - vt). \quad (2.1)$$

Figure 2.1: The schematic of the calculation model. The GPs excited by a electron moving along z direction with a constant velocity v . The vertical distance between the graphene sheet and the swift particle is d .

In the calculation, we expand all the quantities $\mathbf{J}, \mathbf{E}$ and $\mathbf{H}$ by the Fourier transform in time,

$$\begin{aligned}\mathbf{J}(\mathbf{r}, t) &= \int \mathbf{J}(\mathbf{r}, \omega) \exp(i\omega t) dt \\ \mathbf{E}(\mathbf{r}, t) &= \int \mathbf{E}(\mathbf{r}, \omega) \exp(i\omega t) dt \\ \mathbf{H}(\mathbf{r}, t) &= \int \mathbf{H}(\mathbf{r}, \omega) \exp(i\omega t) dt\end{aligned}\tag{2.2}$$

The Fourier transform of Eq. (2.1) is,

$$\mathbf{J}(\mathbf{r}, \omega) = \hat{z} \frac{q}{4\pi^2 \rho} e^{i\omega z/v} \delta(\rho),\tag{2.3}$$

where $\rho = \sqrt{(x^2 + y^2)}$.

The separation distance is set to be $d = 1\mu\text{m}$. The free standing graphene sheet is assumed to have a chemical potential of $\mu_c = 0.15\text{eV}$, a scattering rate

of $\Gamma = 0.11\text{meV}$ at room temperature $T = 300\text{K}$ [66]. The graphene is modelled by an infinity thin sheet with frequency dependent conductivity $\sigma(\omega)$ from Kubo formula [66, 115], $\sigma(\omega, \mu_c, \Gamma, T) = \sigma_{\text{intra}} + \sigma_{\text{inter}}$, with

$$\sigma_{\text{intra}} = i \frac{e^2 k_B T}{\pi \hbar^2 (\omega + i2\Gamma)} \left(\frac{\mu_c}{k_B T} + 2 \ln(e^{-\mu_c/k_B T} + 1) \right), \quad (2.4)$$

and

$$\sigma_{\text{inter}} = \frac{-ie^2 (\omega + i2\Gamma)}{\pi \hbar^2} \int_0^\infty \frac{f_d(-\varepsilon) - f_d(\varepsilon)}{(\omega + i2\Gamma)^2 - 4(\varepsilon/\hbar)^2} d\varepsilon. \quad (2.5)$$

where $f_d(\varepsilon) = (e^{(\varepsilon - \mu_c)/k_B T} + 1)^{-1}$. Since the separation is $d = 1\mu\text{m}$, the calculation shows the frequencies of the evanescent waves from the swift electron are mostly below 50THz , and the surface conductivity σ is dominated by Eq. (2.4).

2.2 Derive the graphene plasmons distribution

When the electron moves with the constant velocity v in air, no CR will be excited since the swift electron cannot be superluminal. But it excites evanescent waves in the near region, which can be coupled to GPs. Since only transverse magnetic (TM) components of the evanescent waves can be effectively coupled to GPs, we use E_y to represent the electron field and the GPs. The total field in region 1 ($y < d$) is

$$E_y^1(r, \omega) = \frac{q}{8\pi^2 v \varepsilon_0} e^{i\omega z/v} \int dk_x (e^{ik_y y} + R e^{2ik_y d} e^{-ik_y y}) e^{ik_x x} \quad (y < d), \quad (2.6)$$

and in region 2 ($y > d$) is

$$E_y^2(r, \omega) = \frac{q}{8\pi^2 v \varepsilon_0} e^{i\omega z/v} \int dk_x T e^{ik_y y} e^{ik_x x} \quad (y > d), \quad (2.7)$$

ε_0 is the permittivity of air, which is assumed to equal to vacuum permittivity. R and T are the reflection and transmission coefficients of the E_y component from

the graphene sheet. They can be derived from the boundary conditions,

$$\begin{cases} H_z^2(y=d) - H_z^1(y=d) = \sigma(\omega)E_x^1(y=d) \\ E_x^2(y=d) = E_x^1(y=d) \end{cases}. \quad (2.8)$$

The result is

$$R = \frac{\sigma k_y}{2\omega\varepsilon + \sigma k_y}, \quad T = \frac{2\omega\varepsilon}{2\omega\varepsilon + \sigma k_y}. \quad (2.9)$$

The GPs distribution can be obtained from the Sommerfeld pole contribution in Eq. (2.6) or (2.7). In Eq. (2.6), it can be shown that the pole comes from the singularity of the reflection coefficient (Eq. 2.9), i.e., $2\omega\varepsilon + \sigma k_y = 0$. This is also the dispersion relationship of GPs. After taking the residue of the pole from Eq. (2.6), we get the E_y component of the GPs distribution,

$$E_y(\omega) = \frac{-iq}{4\pi v\varepsilon_0} \frac{k_y^2}{k_x} e^{ik_y d + ik_x x + ik_z z}, \quad (2.10)$$

where $k_y = -2\omega\varepsilon_0/\sigma(\omega)$ represents the confinement of GPs, $k_z = \omega/v$ and $k_x = \sqrt{\omega^2\varepsilon_0\mu_0 - k_y^2 - k_z^2}$ are the wave vectors of GPs. The GPs distribution in time domain is the Fourier transform of Eq. (2.10),

$$E_y(y=d, t) = \int \frac{-iq}{4\pi v\varepsilon_0} \frac{k_y^2}{k_x} e^{ik_y d} e^{ik_x x + ik_z z - i\omega t} d\omega \quad (2.11)$$

At first, we let the swift electron's velocity to be $v = 0.1c$, and we plot the GPs distribution with the numerical quadrature of Eq. (2.11), as shown in Fig. 2.2. The electron is in the position pointed by the white arrow. The left part of the figure is the absolute value of E_y , and the right is the absolute of the total electrical field E_{tot} , which is almost equal to the root of the energy stored in GPs. The GPs form a V-shape pattern, similar to the ship wakes on deep-water surface. From the figure, we can see that the semi-angle is around 19.5° . Later, we will use caustic theory to find the boundary of the V-shape, and thereby calculating

the semi-angle. Inside the V-shape boundary, there are plane-like waves moving forward following the swift electron.

Figure 2.2: The GPs distribution excited by a swift electron with the velocity $v = 0.1c$, where c is the phase velocity of light in vacuum. The left part is represented by the absolute of the E_y field, and the right is the absolute of the total electrical field, which represents the energy distribution of the GPs. The electron moves to the position pointed by the tip of the white arrow. The semi-angle of the pattern is 19.56° .

As mentioned before, the surface conductivity σ is dominated by the intraband contribution, i.e., $\sigma \cong \sigma_{intra}$ (Eq. 2.4). We neglect the scattering rate in Eq. (2.4) to facilitate the analytic derivation. The σ is rewritten as,

$$\sigma(\omega) = i \frac{G(\mu_c, T)}{\omega}, \quad (2.12)$$

where $G(\mu_c, T)$ is a real function of μ_c and T . With this simplification, we can

rewrite the GPs vectors in Eq. (2.11) as,

$$k_y = \frac{2\omega\varepsilon_0}{\sigma} = \frac{2i\omega^2\varepsilon_0}{G(\mu_c, T)} \quad (2.13)$$

representing the confinement of GPs, and

$$\begin{cases} k_z &= \frac{\omega}{v} \\ k_x &= \sqrt{\frac{\omega^2}{c^2} - \frac{\omega^2}{v^2} + \frac{4\omega^4\varepsilon_0^2}{G^2(\mu_c, T)}} \end{cases}, \quad (2.14)$$

which are the two components of the wavevectors of GPs,

$$k_{gp} = \sqrt{k_x^2 + k_z^2} = \omega \sqrt{\frac{1}{c^2} + \frac{4\omega^2\varepsilon^2}{G^2}}. \quad (2.15)$$

We follow the approaches developed by Lord Kelvin for ship wakes to study the GPs pattern. As shown in Fig. 2.3, we assume the electron moves from the point A to the point B with a constant velocity. It excites waves propagating to all the directions with different phase velocities. $v_p = \frac{\omega}{k_{gp}} = \frac{\omega/v}{k_{gp}/v} = \frac{k_z}{k_{gp}}v = v\cos\theta$, where $\cos\theta = k_z/k_{gp}$ representing the GPs' propagating direction. Since $v_p = v\cos\theta$, the wave pattern is stationary in the reference of the moving electron [108, 109]. The phases of the GPs excited at point A in Fig. 2.3 would reach the dashed line circle when the electron arrives at point B. But the GPs move not with phase velocity, but instead with group velocity. The following calculation shows that the group velocity is only one-half of the phase velocity,

$$\begin{aligned} v_g &= \frac{\partial\omega}{\partial k_{gp}} = \frac{k_{gp}}{\omega} \frac{1}{\left(\frac{1}{c^2} + \frac{8\omega^2\varepsilon^2}{G^2}\right)} = \frac{k_{gp}}{\omega} \frac{1}{\left(2\left(\frac{k_{gp}}{\omega}\right)^2 - \frac{1}{c^2}\right)} \\ &= \frac{v_p k_{gp}^2}{2k_{gp}^2 - k_0^2} \cong \frac{v_p}{2}, \end{aligned} \quad (2.16)$$

for the free space wavevector k_0 is much smaller than that of GPs k_{gp} . The GPs excited would only form a small solid line circle AEDE' in Fig. 2.3 with the radius

Figure 2.3: Analysis of the GPs pattern with Kelvin model. The swift electron moves from the point A to B with a constant velocity v . The swift electron excites GPs at each point it passes by. When it arrives at the point B, the GPs excited at A would form a circle. The phases of the GPs excited at A form the dashed line blue circle, but the GPs reach the smaller solid blue circle AEE'. The interference of the waves excited at each point along line AB would form the GPs pattern in Fig. 2.2

only one half the dashed line circle. The GPs pattern is the interference of the waves excited by the electron at all previous instants. In Fig. 2.3, the boundary is the tangent line from point B to the solid line circles. The semi-angle of the GPs is determined by the geometrical relationship, and is $\theta = \arcsin(1/3) = 19.5^\circ$

2.3 Ray tracing theory for graphene plasmons

Equation (2.11) can be rewritten as

$$E_y(y = d, t) = \int e_y(\omega) e^{ik_y d} e^{i\psi(\omega)t} d\omega, \quad (2.17)$$

where $e_y(\omega) e^{ik_y d} = \frac{-iq}{4\pi v \epsilon_0} \frac{k_y^2}{k_x} e^{ik_y d}$ and $\psi(\omega) = k_x \frac{x}{t} + k_z \frac{z}{t} - \omega$ are the amplitude and phase of each wave component. In ray optics theory, each frequency component represents a ray. The amplitude and the phase determine the intensity and the velocity of the rays respectively.

The stationary point of each frequency component is defined by

$$\frac{d\psi(\omega)}{d\omega} = \frac{dk_x}{d\omega} \frac{x}{t} + \frac{dk_z}{d\omega} \frac{z}{t} - 1 = 0. \quad (2.18)$$

To have a deep understanding of Eq. (2.18), we add one parameter Ω , of which the physical meaning will be explained in the following, on both sides of the equation,

$$\frac{dk_x}{d\omega} x + \frac{dk_z}{d\omega} z - \frac{\Omega}{v} = t - \frac{\Omega}{v}, \quad (2.19)$$

and

$$\frac{dk_x}{d\omega} \frac{x}{t - \frac{\Omega}{v}} + \frac{dk_z}{d\omega} \frac{z - \frac{\Omega}{v}}{t - \frac{\Omega}{v}} = 1. \quad (2.20)$$

The total differential of k_x and k_z are expressed as,

$$\begin{cases} \frac{dk_x}{d\omega} = \frac{\partial k_x}{\partial \omega} + \frac{\partial k_x}{\partial k_z} \frac{dk_z}{d\omega} = \frac{\partial k_x}{\partial \omega} - \frac{k_z}{k_x} \frac{dk_z}{d\omega} \\ \frac{dk_z}{d\omega} = \frac{\partial k_z}{\partial \omega} + \frac{\partial k_z}{\partial k_x} \frac{dk_x}{d\omega} = \frac{\partial k_z}{\partial \omega} - \frac{k_x}{k_z} \frac{dk_x}{d\omega}. \end{cases} \quad (2.21)$$

Combining Eq. (2.21) and Eq. (2.20), we have

$$\begin{aligned} & \left(\frac{\partial k_x}{\partial \omega} - \frac{k_z}{k_x} \frac{dk_z}{d\omega} \right) \frac{x}{t - \Omega/v} + \frac{dk_z}{d\omega} \frac{z - \Omega/v}{t - \Omega/v} = 1 \\ \Rightarrow & \frac{\partial k_x}{\partial \omega} \frac{x}{t - \Omega/v} + \frac{dk_z}{d\omega} \left[\frac{z - \Omega/v}{t - \Omega/v} - \frac{k_z}{k_x} \frac{x}{t - \Omega/v} \right] = 1 \end{aligned} \quad (2.22)$$

and

$$\frac{\partial k_z}{\partial \omega} \frac{z - \Omega}{t - \Omega/v} + \frac{dk_x}{d\omega} \left[\frac{z - \Omega}{t - \Omega/v} - \frac{k_z}{k_x} \frac{x}{t - \Omega/v} \right] = 1. \quad (2.23)$$

The combination of Eq. (2.22) and Eq. (2.23) arrives at

$$\frac{\partial k_x}{\partial \omega} \frac{x}{t - \Omega/v} = 1 \quad (2.24)$$

$$\frac{\partial k_z}{\partial \omega} \frac{z - \Omega}{t - \Omega/v} = 1 \quad (2.25)$$

$$\frac{z - \Omega}{t - \Omega/v} - \frac{k_z}{k_x} \frac{x}{t - \Omega/v} = 0. \quad (2.26)$$

Since $\partial k_x / \partial \omega = 1/v_{gx}$ and $\partial k_z / \partial \omega = 1/v_{gz}$, the physical meanings of Eqs. (2.24)-(2.26) are apparent. As shown in Fig. 2.4, when the electron arrives at position $z = \Omega$, it excites GPs that are moving with group velocity v_{gx} and v_{gz} . The GPs will arrive at point (x, z) at time t . $t - \Omega/v$ is the time taken for the GPs to travel. Equation (2.26) represents the GPs' propagating direction.

It has been shown that $v_g \cong v_p/2$, therefore Eqs. (2.24) and (2.25) can be rewritten as

$$\begin{cases} \frac{x}{t - \Omega/v} = \frac{1}{2}v \cos \theta \sin \theta \\ \frac{z - \Omega}{t - \Omega/v} = \frac{1}{2}v \cos^2 \theta. \end{cases} \quad (2.27)$$

The wave front at each point is perpendicular to the ray's propagating direction, as shown by the tangent solid line in Fig. 2.4, making an angle $\pi - \theta$ with the x -axis. Therefore, the wave front satisfies the equation $dx/dx = -\tan \theta$. Associating this with Eq. (2.27), we can derive the wave front pattern equations,

$$\begin{cases} x = \frac{1}{2}\Xi \cos^2 \theta \sin \theta \\ z = vt - \Xi \cos \theta + \frac{1}{2}\Xi \cos^3 \theta \end{cases}, \quad (2.28)$$

where $\Omega = vt - \Xi \cos \theta$ and Ξ ranges from 0 to vt . The wave front pattern is contributed by rays excited by the electron at all previous instants. The wave

Figure 2.4: The schematic of the derivation of the wave front pattern of GPs with ray tracing theory. A swift electron excites a ray at point $z = \Omega$ propagating with an angle θ to the z axis. At time t ($t = 0$ when the electron moves to the origin), the ray moves to the point (x, z) . The wave front pattern is perpendicular to the rays.

front pattern in Fig. 2.5 (the blue solid lines) is plot with Eq. (2.28) by varying Ξ from 0 to vt , and θ from $-\pi/2$ to $\pi/2$. All the wave fronts have cusps at the caustic boundary. This is one peculiarity of the fold caustics [112].

2.4 Caustic graphene plasmons

Numerical quadrature of Eq. (2.11) can be used to plot the GPs pattern in Fig. 2.2. The integral of Eq. (2.11) can be evaluated asymptotically with the stationary phase method [109] to further analyze the field distribution. The stationary phase frequency $\omega = \omega_s$ is defined by Eq. (2.18). We expand the

Figure 2.5: The wave front pattern excited by a swift electron with the velocity $v = 0.1c$. The blue curves represent the wave front. The black solid lines represent the caustic boundary. The solid and dashed red lines represent the maximum and minimum field distribution in the neighbourhood of caustics. The field has distribution beyond the caustic boundary. We use the dashed black line to represent the positions where the field becomes $1/e$ of its maximum value in the caustic zone. We call it the field boundary of the GPs pattern.

phase ψ in Eq. (2.11) around ω_s ,

$$\psi(\omega) = \psi(\omega_s) + \frac{1}{2}\psi''(\omega_s)(\omega - \omega_s)^2 + O(|\omega - \omega_s|^3). \quad (2.29)$$

By choosing the steepest descent path [116] for the integral of Eq. (2.11), we approximate the integral of Eq. (2.11) as,

$$E_y(y = d, t) = 2\text{Re} \left[e_y(\omega_s) \frac{\sqrt{2\pi}}{\sqrt{\psi''(\omega_s)} t} e^{ik_y(\omega_s)d + i\psi(\omega_s)t + i\pi/4} \right]. \quad (2.30)$$

It can be shown from Eq. (2.30) that there is divergence when $\psi''(\omega) = 0$. It does not mean the field diverges there, but results from the wrong assumption in Eq. (2.29) that the second derivation of ψ at ω_s is not zero. It also means that the steepest descent path for the integral of ω in Eq. (2.11) chosen is not the “steepest descent”. Another steepest descent path has to be chosen to deal with the divergence. We use ω_c to represent the result of

$$\frac{d^2\psi(\omega)}{d\omega^2} = \frac{d^2k_x}{d\omega^2} = 0. \quad (2.31)$$

Following the derivation from Eqs. (2.18) to (2.26), it is easy to prove that $\psi''(\omega_c) = 0$ is equivalent to $v'_{gx}(\omega_c) = 0$. The rays of frequency ω_c have the same group velocity along the x direction, and would pile up in the points which are called caustics. The loci of the caustics form the caustic boundary.

The positions of the caustic boundary are determined by combining Eq. (2.18) and (2.31),

$$2\sqrt{2}\sqrt{1 - \left(\frac{v}{c}\right)^2} \frac{x}{vt} + \frac{z}{vt} = 1. \quad (2.32)$$

The results are shown by the black lines in Fig. 2.5. When $v = 0.1c$, the exact angle of the caustic boundary is $\alpha = 19.56^\circ$.

The field distribution in the neighborhood of caustics cannot be approximated with Eq. (2.30), which results from the Gaussian integral. In the neighborhood of caustics, we bring in the Airy integral, which plays the same role as the Gaussian integral in the derivations from Eqs. (2.17) to (2.30). We first expand the phase ψ around ω_c to the third order, instead of the second order in Eq. (2.29),

$$\psi(\omega) = \psi(\omega_c) + \psi'(\omega_c)(\omega - \omega_c) + \frac{1}{6}\psi'''(\omega_c)(\omega - \omega_c)^3 + O((\omega - \omega_c)^4). \quad (2.33)$$

The first derivative of phase ψ is almost, but not exactly, equal to zero in the

neighborhood of caustics. Substituting Eq. (2.33) into Eq. (2.17), we have

$$E_y(y = d, t) = 2\text{Re} \left[2\pi e_y(\omega_c) e^{ik_y(\omega_c)d + i\psi(\omega_c)t} \int e^{i[\psi'(\omega_c)(\omega - \omega_c) + \frac{1}{6}\psi'''(\omega_c)(\omega - \omega_c)^3]} t d\omega \right]. \quad (2.34)$$

Let

$$\omega - \omega_c = \left[\frac{1}{2} t \psi'''(\omega_c) \right]^{-1/3} s \quad (2.35)$$

$$X = \left[\frac{1}{2} t \psi'''(\omega_c) \right]^{-1/3} t \psi'(\omega_c), \quad (2.36)$$

then Eq. (2.34) becomes

$$\begin{aligned} E_y(y = d, t) &= 2\text{Re} \left[e_y(\omega_c) e^{ik_y(\omega_c)d + i\psi(\omega_c)t} \left[\frac{1}{2} t \psi'''(\omega_c) \right]^{-1/3} \int e^{iXs + \frac{i}{3}s^3} ds \right] \\ &\simeq 2\text{Re} \left[e_y(\omega_c) e^{ik_y(\omega_c)d + i\psi(\omega_c)t} \left[\frac{1}{2} t \psi'''(\omega_c) \right]^{-1/3} \text{Ai}(X) \right], \end{aligned} \quad (2.37)$$

where $\text{Ai}(X)$ is the Airy integral function of X . The distribution of $\text{Ai}(X)$ around $X = 0$ is plotted in Fig. 2.6.

Figure 2.6: Airy function $\text{Ai}(X)$ profile in the neighborhood of $X = 0$.

When $X = 0$, Eq. (2.36) is equivalent to the caustic boundary equation Eq. (2.32). When $X > 0$, Eq. (2.36) represents the caustic shadow region.

Because the fields decay exponentially as a result of the exponential decay of the Airy function shown in Fig. 2.6. The Airy function reaches $1/e$ of its maximum around $X = 0$. We use the line $X = 0.66$ to represent the boundary of the caustic shadow, as shown by the dashed black line in Fig. 2.5. The Airy function reaches its local maximum (around $X = 0$) at $X = -1.02$, representing by the red solid line in Fig. 2.5. The region between $X = 0$ and $X = -1.02$ is called caustic zone. The fields focus in the region, and this is the peculiarity of caustics [112]. We can observe the field enhancement in the caustic zone from Fig. 2.2. The Airy function vanishes at $X = -2.34$, representing by the dashed red line in Fig. 2.5. It means the fields have a minimum value there, which can also be observed in Fig. 2.2. The dark region in Fig. 2.2 separates the neighborhood of caustics from the regions where geometrical optic theory applies. The field in the neighborhood of caustics decays more slowly, with $t^{-1/3}$ as shown in Eq. (2.37), than the field in the region where geometrical optics theory applies, with $t^{-1/2}$ as shown by Eq. (2.30).

2.5 Change from the Kelvin angle to the Mach angle

We then increase the swift electron's velocity to $0.3c$, and plot the GPs pattern as shown in Fig. 2.7. The solid and white lines represent the caustic boundary defined by Eq. (2.32) and the caustic shadow defined by Eq. (2.36) with $X = 0.66$. There are many differences between Fig. 2.7 and Fig. 2.2. First, there are no plane-like waves moving forward following the swift electron. Second, the caustic shadow region, where is the region beyond the caustics, expands in area. Third,

the semi-angle of the pattern increases slightly.

Figure 2.7: The GPs distribution excited by a swift electron with the velocity $v = 0.3c$, where c is the phase velocity of light in vacuum. The left part is represented by the absolute of the E_y field, while the right is the absolute of the total electrical field, which represents the energy distribution of the GPs. The electron moves to the position pointed by the tip of the white arrow. The white solid line is the caustic boundary and the white dashed line is the line of the caustic boundary

To understand the differences when we increase the velocity from $0.1c$ to $0.3c$, we derive the angular spectrum representation of E_y [116]. We rewrite the integral

of Eq. (2.17) over the propagation angle θ , and arrive at

$$\begin{aligned} E_y(t) &= 2\text{Re} \left[\int e_y e^{ik_y d} e^{i\psi(\omega)t} \frac{d\omega}{d\theta} d\theta \right] \\ &= \text{Re} \left[\int E_y(\theta) e^{i\psi(\omega)t} d\theta \right], \end{aligned} \quad (2.38)$$

where

$$E_y(\theta) = 2e_y e^{ik_y d} \frac{d\omega}{d\theta}. \quad (2.39)$$

The angular spectrum representations of $|E_y(\theta)|$ with different electron's velocities are plotted in Fig. 2.8. It can be seen that when the velocity is $0.1c$,

Figure 2.8: The angular spectrum representation of GPs under different electron's velocities. The horizontal axis is the ray's angle with respect to the electrons's trajectory. The arrows represent the angle of the rays that would contribute to the caustics

$|E_y(\theta)|$ is in the range 0° to 60° . But as the velocity increases to $0.3c$, $0.5c$ and $0.7c$, $|E_y(\theta)|$ moves to the high angle direction. We also calculate the propagating angles of the GPs that contribute to the caustics. They are $\theta = 35.1^\circ$,

$\theta = 34.0^\circ, \theta = 31.5^\circ$ and $\theta = 26.8^\circ$, corresponding to velocities of $0.1c$, $0.3c$, $0.5c$ and $0.7c$, respectively. The angles are marked by the arrows in Fig. 2.8 at the horizontal axis, with the same color as that of the curves. It shows that when the velocity is $0.3c$, the GPs propagating to the caustics are very weak. As pointed in the beginning of this chapter, one of the requirements for the caustics to be recorded is that the field amplitudes at the caustics cannot be very small. But from Fig. 2.8, when we increase the velocity above $0.3c$, the wave components propagating to the caustics are very weak, therefore the caustics get blur or even disappear.

We plot the GPs pattern in Fig. 2.9. Comparing with Figs. 2.2, 2.7 and 2.9, we find that the GPs pattern shrinks as the electron's velocity increases. That is because, from the angular spectrum representation of GPs in Fig. 2.8, the GPs propagate with large propagation angle well above the corresponding caustic angle. From the Kelvin model we used to analyse the GPs shown in Fig. 2.3, if the GPs' propagating angles are above the caustic angles as shown in Fig. 2.8, the effective boundary is determined by the smallest propagating angle, like θ shown in Fig. 2.3. Not like Fig. 2.2, the semi-angle cannot be determined by the caustic boundary. We use numerical methods to determine the boundary of the GPs at large electron's swift velocity, and thereby determining the semi-angle.

In Fig. 2.2, we calculate the $|E_{tot}(r, t)|$ field distribution along the line $z = 0$, and find that the total electrical field at the caustic point is 0.61 of the maximum along the line $z = 0$. We use this as a criterion to numerically determine the semi-angle of the GPs pattern with different electron's velocities. The results are shown in Fig. 2.10. The horizontal line $\alpha = 19.47$ comes from the semi-angle of the caustics determined by Eq. (2.32) when the velocity v tenders to zero. The

Figure 2.9: The GPs distribution excited by a swift electron with the velocity $v = 0.5c$ (left) and $v = 0.7c$ (right), where c is the phase velocity of light in vacuum. The left part is represented by the absolute of the E_y field, while the right is the absolute of the total electrical field, which represents the energy distribution of the GPs. The electron moves to the position pointed by the tip of the white arrow.

oblique line $\alpha = \frac{4.52}{v}$ comes from the fitting of the equation $\alpha \propto \frac{1}{v}$ with the wake angle when the velocity of the electron is $v = 0.9c$. It shows that the semi-angle gets closer to the line as the electron's swift velocity increases. From Fig. 2.10, we can see the semi-angle of GPs pattern changes from the Kelvin angle to the Mach angle as the velocity increases.

Figure 2.10: The variation of the semi-angle from the Kelvin angle to the Mach angle as the swift electron's velocity increases. At relatively small velocity, the angle is around the Kelvin angle $\alpha = 19.5^\circ$. But as the particle's velocity increases, the angle decrease behaving as the Mach angle.

2.6 Energy loss of the swift electron via coupling to the graphene plasmons

We estimate the energy coupled to GPs by numerically calculating the energy flux from the grey plane shown in Fig. 2.11. The plane is very close to the electron with a horizontal distance of $1\mu\text{m}$. One more plane is needed on the other side. Then we calculate the energy loss when the electron travels for $100\mu\text{m}$ above the graphene plane. Figure 2.12 shows the ratio of energy loss per $100\mu\text{m}$ travel distance to the kinetic energy of the electron. It can be shown the energy loss is negligible compared to the kinetic energy of the electron, therefore it is reasonable to assume the electron's velocity remains constant in our calculation.

Figure 2.11: Schematic of the calculation model of the energy loss from a swift electron to GPs.

2.7 Conclusion

In this chapter, we apply the Kelvin mathematical model developed for ship wakes to study the GPs excited by a swift electron. It has been reported before that GPs have the similar dispersion as that of deep-water waves, but for the first time, we apply the well developed techniques for deep-water waves to study GPs. We find that the semi-angle of the GPs pattern is the Kelvin angle at low swift velocity ($\sim 0.1c$), but changes to the Mach angle as the velocity increases. We also for the first time apply the caustic wave theory to study GPs. Caustics are well studied in fluid mechanics, but only the geometrical properties of caustics in plasmons were studied before. We apply the caustic wave theory to calculate the field distribution in the neighbourhood of caustics from the wave point. The work has been published in **Phys. Rev. B**, vol. 92, 081404(Rapid Communication).

Figure 2.12: Ratio of the energy loss of the electron per $100\mu\text{m}$ travel distance to the kinetic energy of the electron. The horizontal axis is the velocity of the swift electron.

Dynamic Mechanism of Two-Dimensional Plasmons Launching by Swift Electrons

We show in this chapter the dynamics of two-dimensional (2D) plasmons launching by a swift electron passing through graphene. Not only graphene plasmons (GPs), but also free propagating waves (the so called "transition radiation (TR)" [45, 46]) are excited. Generally, in calculating electron-energy-loss spectra for a very thin metal foil or other 2D electron system, energy loss from TR can be omitted because of its negligible amount compared to that from plasmons [6, 13]. Historically, the concepts of formation zone and formation time have caused profound discussions in classical electromagnetics, quantum mechanics, and high-energy physics [45, 46, 117–120]. Yet even in the well-established frame of classical electromagnetic, many aspects of TR still require further studies [46]. It is thus reasonable that, in the context of 2D plasmons launched by swift electrons, despite its long research history, the dynamical spatio-temporal processes

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need to be further investigated. The launching of 2D GPs is an analogue of deep-water hydrodynamic splashing. Particularly, a picosecond process of a jet-like rise of excessive charge concentration, which is analogous to the "Worthington jet" or "Rayleigh jet" in hydrodynamic splashing [121, 122], lies between the impact of the electron and the launching of 2D plasmons, whose duration is consistent with the formation time of TR. Although we adopt graphene as the platform, our analysis can apply to other 2D electron systems. Note that while the formal similarity of the dispersion between hydrodynamic water waves [109] and 2D electron systems [123–127, 127] (including GPs) [3, 93, 128] has been investigated with some hydrodynamic-wave-like phenomena predicted [129–132, 132], the analogue to the splashing phenomenon has never been discussed. We demonstrate within the framework of classical electrodynamics that the launching of 2D plasmons by an electron's impact is delayed after a hydrodynamic splashing-like process, which occurs during the plasmonic "formation time" when the electron traverses the "formation zone". The only way to tell what happens in space and time during the formation time of TR is to solve Maxwell's equations strictly. Soon after Frank and Tamm successfully explained Cherenkov radiation in the framework of classical electrodynamics [133], it is Fermi who moved one step further to describe electron-matter interaction using classical electrodynamics [134]. Shortly, Ginzburg and Frank extended related theories to account for TR when swift electrons pass an interface between different media [45, 46]. Following this tradition, our rigorous analytical calculations are based on an extension of Ginzburg and Frank's theory on TR within the framework of macroscopic electrodynamics. Note that in addition to the TR, the passage of a swift electron through graphene will possibly excite electron-hole pairs, excitons, secondary electrons or

Auger electrons. The accompanied decay processes can give rise to the incoherent cathodoluminescence emission or secondary electron emission [6], increasing the complexity of this scenario. There is also a trivial probability for the swift electron to have heavy encounters with graphene atoms, which will generate the so-called bremsstrahlung radiation [6]. However, since the coherent cathodoluminescence emission (TR in our case) among electron-induced radiation is dominant in metals, and doped graphene layer effectively behaves as an ultrathin metallic layer, we treat the TR as the dominant radiation process here.

3.1 The model setup

The calculation model is schematically shown in Fig. 3.1. We consider a swift electron with charge q , moving with a velocity $\bar{v} = \hat{z}v = \hat{z}\beta c$, where c is the light speed in vacuum. Since the energy loss of the electron coupled to GPs and TR is much lower than the electron's kinetic energy, the electron's velocity is treated to be constant. The space-time dependence of the electron's motion is classically described as $\delta(\bar{r} - \bar{v}t)$. A free-standing monolayer graphene layer locates at the interface between medium 1 ($z \leq 0$) with relatively permittivity ε_1 and medium 2 ($z \geq 0$) with permittivity ε_2 . Graphene is treated as one infinitely thin layer with surface conductivity modelled by Kubo formula [66, 115], which agrees well in the low-energy frequency band with that calculated by the nonlocal model based on random-phase approximation and number conserving approximation [2, 128]. The Kubo formula has been described by Eqs. (2.4)-(2.5) in Chapter 2. In our work, we set the temperature $T = 300\text{K}$, the relaxation time $\tau = 0.5\text{ps}$, and the chemical potential in graphene $\mu_c = 0.4\text{eV}$ (close to the experimentally achievable

Figure 3.1: Schematic of 2D plasmons launching with a swift electron penetrating through a free-standing graphene monolayer with a constant velocity. Forward, backward radiation and GPs are excited. L_{f1} and L_{f2} are the length of the formation zone in the region above and below the graphene layer, respectively.

value [135]).

The moving electron is modelled with current density as in Chapter 2

$$\mathbf{J}(\mathbf{r}, t) = \hat{z} q v \delta(x) \delta(y) \delta(z - vt). \quad (3.1)$$

3.2 Derivation of the transition radiation

In Chapter 2, the case is homogeneous in the z direction, therefore, $k_z = \omega/v$ remains invariant for both the incident and reflect (including the surface waves coupled to GPs) waves from the graphene. But in this chapter, the medium is not homogeneous in the z direction. Therefore, the reflected and transmitted waves (TR) from the graphene layer must have different k_z compared with that from the swift electron's self-generated evanescent fields, based on the boundary match condition.

We expand all quantities $\mathbf{J}, \mathbf{E}$ and $\mathbf{H}$ with Fourier transform in time and in the transverse component of $\mathbf{r}$, since the problem is homogeneous both in time and in the transverse direction.

$$\begin{aligned}\mathbf{J}(\mathbf{r}, t) &= \int \mathbf{J}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_{\perp} - i\omega t) d\omega d\boldsymbol{\kappa} \\ \mathbf{E}(\mathbf{r}, t) &= \int \mathbf{E}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_{\perp} - i\omega t) d\omega d\boldsymbol{\kappa} \\ \mathbf{H}(\mathbf{r}, t) &= \int \mathbf{H}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_{\perp} - i\omega t) d\omega d\boldsymbol{\kappa}\end{aligned}\quad (3.2)$$

where $\boldsymbol{\kappa} = \hat{x}k_x + \hat{y}k_y$, $\mathbf{r}_{\perp} = \mathbf{x} + \mathbf{y}$. The Fourier transform of the current density Eq. (3.1) is

$$\mathbf{J}_{\kappa, \omega}(z) = \frac{q}{(2\pi)^3} \exp\left(i\frac{\omega}{v}z\right). \quad (3.3)$$

We make use of dyadic Green function [136] to calculate the electron self-generated fields in homogeneous medium, i.e., air in our model,

$$\mathbf{E}_{\omega, \kappa}^q = -\frac{iq}{(2\pi)^3 \omega \varepsilon} \frac{\exp\left(i\frac{\omega}{v}z\right)}{k^2 - \kappa^2 - \left(\frac{\omega}{v}\right)^2} \left(-k_x \frac{\omega}{v} \hat{x} - k_y \frac{\omega}{v} \hat{y} + \left(k^2 - \frac{\omega^2}{v^2}\right) \hat{z} \right), \quad (3.4)$$

$$H_{\omega, \kappa, x}^q = -\frac{iq}{(2\pi)^3} \frac{k_y \exp\left(i\frac{\omega}{v}z\right)}{k^2 - \kappa^2 - \left(\frac{\omega}{v}\right)^2}, \quad (3.5)$$

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and

$$H_{\omega,\kappa,y}^q = \frac{iq}{(2\pi)^3} \frac{k_x \exp\left(i\frac{\omega}{v}z\right)}{k^2 - \kappa^2 - \left(\frac{\omega}{v}\right)^2}, \quad (3.6)$$

where ε is the permittivity of air and $k^2 = \omega^2 \varepsilon \mu$. We use q in the superscript to represent the electron-self generated fields.

We assume that the scattering fields from the graphene satisfy the equation,

$$E_{\omega,k,z}^s = \frac{iq}{(2\pi)^3 \omega \varepsilon} \begin{cases} a_1 e^{-ik_z z} & z > 0 \\ a_2 e^{ik_z z} & z < 0 \end{cases}, \quad (3.7)$$

where $k_z = \sqrt{k^2 - \kappa^2}$ and $a_{1,2}$ are dimensionless coefficients to be determined from the boundary condition. The s in the superscript represents the scattering field. Since the electron is moving along the z direction and the model is rotationally symmetrical with respect to the $z = 0$ line, there is no z component of the scattering magnetic field, i.e., $H_{\omega,k,z}^s = 0$. The other components of the scattering field can be derived from Eq. (3.7) and $H_{\omega,k,z}^s = 0$,

$$E_{\omega,\kappa,x}^s = \frac{ik_x}{\kappa^2} \frac{\partial}{\partial z} E_{\omega,\kappa,z}^s = \frac{iq}{(2\pi)^3 \omega \varepsilon} \frac{k_x k_z}{\kappa^2} \begin{cases} a_1 e^{-ik_z z} & z > 0 \\ -a_2 e^{ik_z z} & z < 0, \end{cases} \quad (3.8)$$

$$H_{\omega,\kappa,x}^s = \frac{iq}{(2\pi)^3} \frac{k_y}{\kappa^2} \begin{cases} a_1 e^{-ik_z z} & z > 0 \\ a_2 e^{ik_z z} & z < 0 \end{cases}, \quad (3.9)$$

and

$$H_{\omega,\kappa,y}^s = -\frac{iq}{(2\pi)^3} \frac{k_x}{\kappa^2} \begin{cases} a_1 e^{-ik_z z} & z > 0 \\ a_2 e^{ik_z z} & z < 0 \end{cases}. \quad (3.10)$$

The dimensionless coefficients a_1 and a_2 can be determined from the boundary

conditions

$$\begin{aligned}
E_{\omega,\kappa,y}^q(z=0^+) + E_{\omega,\kappa,y}^s(z=0^+) &= E_{\omega,\kappa,y}^q(z=0^-) + E_{\omega,\kappa,y}^s(z=0^-) \\
[H_{\omega,\kappa,y}^q(z=0^+) + H_{\omega,\kappa,y}^s(z=0^+)] - [H_{\omega,\kappa,y}^q(z=0^-) + H_{\omega,\kappa,y}^s(z=0^-)] \\
&= \sigma(\omega) [E_{\omega,\kappa,y}^q(z=0^+) + E_{\omega,\kappa,y}^s(z=0^+)]
\end{aligned} \tag{3.11}$$

The results are

$$\begin{cases} a_2 &= -a_1 \\ a_1 &= \frac{-\frac{\sigma}{v\epsilon}\kappa^2}{\left(k^2 - \kappa^2 - \frac{\omega^2}{v^2}\right)\left(2 + \frac{\sigma}{\omega\epsilon}k_z\right)}. \end{cases} \tag{3.12}$$

Since the electron moves along the z direction and the case is rotationally symmetrical to the line $z = 0$, the magnetic field should follow the azimuthal direction. $H_\varphi = -H_x \sin \varphi + H_y \cos \varphi$, where φ is the azimuth angle with $\varphi = 0$ along the x axis. Therefore, the H_φ component for both the electron self-generated field and the scattering field can be represented with the combination of Eqs. (3.5),(3.6),(3.9) and (3.10),

$$\begin{aligned}
H_{\omega,\kappa,\varphi}^q &= -H_{\omega,\kappa,x}^q \sin \varphi + H_{\omega,\kappa,y}^q \cos \varphi \\
&= \frac{iq}{(2\pi)^3} \frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} (\sin \varphi k_y + \cos \varphi k_x) \\
&= \frac{iq}{(2\pi)^3} \frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} \kappa (\sin \varphi \sin \alpha + \cos \varphi \cos \alpha) \\
&= \frac{iq}{(2\pi)^3} \frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} \kappa \cos(\alpha - \varphi),
\end{aligned} \tag{3.13}$$

and

$$\begin{aligned}
 H_{\omega,\kappa,\varphi}^s &= -H_{\omega,\kappa,x}^s \sin \varphi + H_{\omega,\kappa,y}^s \cos \varphi \\
 &= \frac{iq}{(2\pi)^3} \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} (-\sin \varphi k_y - \cos \varphi k_x) \\
 &= -\frac{iq}{(2\pi)^3} \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \kappa (\sin \varphi \sin \alpha + \cos \varphi \cos \alpha) \\
 &= -\frac{iq}{(2\pi)^3} \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \kappa \cos (\alpha - \varphi),
 \end{aligned} \tag{3.14}$$

where $k_y = k \sin \alpha$, $k_x = k \cos \alpha$. The total H field in the frequency domain is the inverse Fourier expansion of the combination of Eqs. (3.13) and (3.14),

$$\begin{aligned}
 H_\varphi(\omega) &= -\frac{iq}{(2\pi)^3} \int dk_x dk_y \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) \kappa \cos (\alpha - \varphi) e^{i k_x x + i k_y y} \\
 &= -\frac{iq}{(2\pi)^3} \int \kappa d\kappa d\alpha \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) \kappa \cos (\alpha - \varphi) e^{i \kappa \rho \cos(\alpha - \varphi)} \\
 &= -\frac{iq}{(2\pi)^3} \int d\kappa \kappa^2 \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) \int d\alpha \cos (\alpha - \varphi) e^{i \kappa \rho \cos(\alpha - \varphi)} \\
 &= -\frac{iq}{(2\pi)^3} \int d\kappa \kappa^2 \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) \int d\alpha \frac{d e^{i \kappa \rho \cos(\alpha - \varphi)}}{d(i \kappa \rho)} \\
 &= -\frac{q}{(2\pi)^3} \int d\kappa \kappa^2 \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) \frac{d}{d(\kappa \rho)} \int d\alpha e^{i \kappa \rho \cos(\alpha - \varphi)} \\
 &= -\frac{q}{(2\pi)^3} \int d\kappa \kappa^2 \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right) (-2\pi J_1(\kappa \rho)) \\
 &= \frac{q}{(2\pi)^2} \int d\kappa \kappa^2 J_1(\kappa \rho) \left(-\frac{e^{-i\omega z/v}}{k^2 - \kappa^2 - \omega^2/v^2} + \frac{a_{1,2} e^{\mp i k_z z}}{\kappa^2} \right).
 \end{aligned} \tag{3.15}$$

Similar results can be got for E_z and E_ρ .

3.3 Dynamics of graphene plasmons launching

Now we discuss the dynamics of GPs launching and radiation emission during the electron's penetration through the monolayer graphene, from the perspective

Figure 3.2: Temporal evolution of the magnetic field $H_\varphi(r, t)$ by a swift electron perpendicularly penetrating through a graphene monolayer. The green dashed line represents the monolayer graphene. GPs and freely propagating waves are both excited when the electron moves across the graphene plane and are clearly separated as the electron moves away from the graphene layer. The electron has a velocity of $v = 0.8c$, and the monolayer graphene has a chemical potential of $\mu_c = 0.4\text{eV}$.

of classical electrodynamics. We consider an electron with $v = 0.8c$ (or kinematic energy 340 keV) which can be realized by modern electron microscope systems [6, 137]. (This relatively large velocity further guarantees that the time for the electron to cross over the graphene layer is negligible compared to the formation time.) As shown in Fig. 3.2, we can divide the radiation process into two stages, one for time $t < 0$ and the other for $t > 0$ ($t = 0$ when the electron is located at the graphene plane). When the incident electron is far above the graphene layer as shown in Fig. 3.2a, the radiation field is almost zero. As the electron moves closer to the interface (Fig. 3.2b), the evanescent electron fields interact

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with the graphene layer, expelling the conduction electrons in graphene from the vicinity of the incident electron's trajectory. Due to electromagnetic induction, the induced surface current density in turn obstructs the penetration of a large proportion of the evanescent electron field. As a consequence, at the moment just before the incident electron crosses over graphene (Fig. 3.2c), a large portion of magnetic fields has accumulated on the upper side of graphene, while only a moderate portion has leaked into the lower side of graphene. Immediately after the emergence of the electron from the lower side of the graphene layer (Fig. 3.2d), the magnetic fields previously accumulated on the upper side of the graphene layer have lost direct interaction with the electron, and the insufficient magnetic fields on the lower side of graphene need to recover their strength in presence of the electron. Under these effects, a part of magnetic fields will be shaken off into the ambient space and thus form TR. At the same time, the perturbed conduction electrons in graphene will tend to move back to their equilibrium places and start a subsequent subtle process (analogous to the hydrodynamic splashing, as will be discussed later) after which their electromagnetic energy will be carried away by GPs (Fig. 3.2e-f). The emitted radiation freely propagates in the surrounding space with a sphere-like shape and azimuthal angle-dependent energy distribution. When the electron moves further away, the originally mixed evanescent electron field, freely propagating radiation, and GPs will gradually separate from each other (Fig. 3.2h).

Now we discuss the same process from the perspective of electron motion on graphene. We plot in Fig. 3.3 the dynamics of the deviation of electron density from its average value $\delta n(r, t)$ (will be elaborated later) on graphene during the penetration of the swift electron. The electron density variation on the graphene

Figure 3.3: Temporal evolution of the deviation of the electron density from its average value on graphene plane $\delta n(r, t)$ when a swift electron penetrates through a graphene monolayer. In (a-c), when the incident electron approaches the graphene plane, a crater for the density of conduction electron in graphene is formed. In (d-e), when the incident electron leaves graphene, the conduction electrons will rush back to fill the crater, and form a jet-like rise for the density of conduction electron in the center of the crater. In (f-i), when the central jet-like rise of electron density falls down, GPs disperse like hydrodynamic ripples.

monolayer can be got by matching the boundary condition at the layer $z = 0$,

$$\rho_s(r, t) = \varepsilon_0 (E_z(t, z = 0^+) - E_z(t, z = 0^-)). \quad (3.16)$$

ρ_s is the net surface charge density. The deviation of the electron density is $\delta n(r, t) = -\rho_s(r, t)/e$, as shown in Fig. 3.3.

Figure 3.3 is similar to the hydrodynamic splashing scenario caused by a little

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droplet falling on a liquid surface, as will be elaborated in the following. Firstly, in hydrodynamics [121, 122], when a small droplet impacts a calm deep-water surface, a crater is formed first and water splashes to the side. As the water rushes back to fill the crater, a central jet column, called "Rayleigh jet" due to its instability [121] or "Worthington jet" because A. M. Worthington performed extensive observations back to 1908 [121, 122], will rise above the initial water surface along the collapse axis. When it comes to charge motion on graphene, a "crater" of the density of conduction electron in graphene is first formed when the incident electron approaches the graphene layer (Fig. 3.3a-c). Immediately following the penetration of the electron through the graphene layer, a rebound of the density of conduction electrons is formed with a sharp central jet-like rise (Fig. 3.3d-e), being analogous to the hallmark of "Rayleigh jet" or "Worthington jet" in hydrodynamics.

Secondly, after the central jet-like rise falls down, ripples of GPs propagate outward as concentric circles, as shown in Fig. 3.3f-i. The excited GPs, which cover a broad spectrum of frequencies, gradually spread into a sequence of plasmonic ripples where longer-wavelength plasmons stay at the periphery and shorter-wavelength plasmons stay close to the inner boundary (Fig. 3.3i). This is because the dispersion of GPs, i.e., $\omega \propto \sqrt{\kappa}$ (where κ is the in-plane wavevector of GPs), which is formally analogous to the dispersion of deep-water waves [109, 128, 138], determines that longer waves travel faster. Note that these plasmonic ripples correspond to what were observed as GPs in previous studies.

3.4 Analysis of the spatio-temporal process

Figure 3.2 shows the spatio-temporal process of the GPs launching and TR excitation processes. It shows that the radiation is not generated immediately when the electron hits the graphene layer. Historically, the concepts of "formation time" and "formation zone" (i.e., "it takes a relatively long time and therefore a long distance for an energetic electron to create a photon," as retailed in [139]) was firstly presented by Ter-Mikaelian in Landau's seminar in 1952 [140]. Landau at first strongly opposed these concepts [140], but soon realized their correctness and significance, and further developed them by laying his name in the Landau-Pomeranchuk-Migdal effect, which was experimentally confirmed forty years later. Indeed, the existence of "formation time" and "formation zone" describes our ignorance on the exact moment and location at which a photon is generated. In fact, in the early days of quantum uncertainty principle, Bohr already commented on the impossibility of describing electron-photon interaction "without considering a finite space-time region" [141]. Later, it is Ginzburg and colleagues who included the "formation time" and "formation zone" with mathematical form into the classical framework of TR (established by Ginzburg and Frank in 1945), while still admitting that "comparatively little is known" [45, 46].

Ginzburg estimated the length of formation zone (denoted as "formation length" L_f) for photon emission in TR based on the following considerations [46]. Inside the formation zone, the total energy of the fields $\mathbf{E}_{\kappa, \omega}^q + \mathbf{E}_{\kappa, \omega}^s$ is proportional to $(\mathbf{E}_{\kappa, \omega}^q + \mathbf{E}_{\kappa, \omega}^s)^2$. The length L_f describes a boundary at which the total energy becomes practically equal to the sum of the energy of the electron field, i.e., $(\mathbf{E}_{\kappa, \omega}^q)^2$, and the energy of the radiation field, i.e., $(\mathbf{E}_{\kappa, \omega}^s)^2$, meaning that the electron field and the radiation field are separated from each other. In other

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words, the interference term $(\mathbf{E}_{\kappa,\omega}^q \cdot \mathbf{E}_{\kappa,\omega}^s)^2$ must play a trivial role. Ginzburg thus set the phase difference of 2π between $\mathbf{E}_{\kappa,\omega}^q$ and $\mathbf{E}_{\kappa,\omega}^s$ to determine the length L_f . In other words, $\frac{\omega L_f}{v} \pm \frac{\omega L_f}{c\sqrt{1-\frac{\kappa_\perp^2 c^2}{\omega^2}}} = 2\pi$, which gives

$$L_{f1} = \frac{2\pi}{\left| \frac{\omega}{v} + \omega/c\sqrt{1-\frac{\kappa_\perp^2 c^2}{\omega^2}} \right|}, L_{f2} = \frac{2\pi}{\left| \frac{\omega}{v} - \omega/c\sqrt{1-\frac{\kappa_\perp^2 c^2}{\omega^2}} \right|} \quad (3.17)$$

where L_{f1} and L_{f2} correspond to the length of the formation zones in regions of $z < 0$ and $z > 0$, respectively (see Fig. 3.1). Apparently, the formation length depends on not only the frequency of emitted photons and the swift electron's velocity, but also the emission angle of photons. The bandwidth of calculation is from 0 to 20THz. At the central frequency 10THz, the formation time is 0.06ps for the backward radiation in the normal direction($\kappa_\perp = 0$), consistent with Fig. 3.2b-c. The formation time for the forward radiation is $\frac{L_{f2}}{v} = 0.5\text{ps}$ in the normal direction($\kappa_\perp = 0$), as can be seen from Fig. 3.2c-f. The electron's velocity is assumed to be $v = 0.8c$.

Ginzburg's estimation only applies to photon emission with the condition $\kappa_\perp^2 < \omega^2/c^2$ in order to validate the square root in Eq. (3.17). Yet it is well-known that surface plasmons have $\kappa_\perp^2 > \omega^2/c^2$. Compared to the electron-photon interaction as analyzed by Ginzburg, in the interaction between the electron and surface plasmons, only the electron field has phase variation along the z axis, while the surface plasmons have zero phase variation. Therefore, along the line of Ginzburg's thought, the phase difference of 2π between $\mathbf{E}_{\kappa,\omega}^q$ and $\mathbf{E}_{\kappa,\omega}^s$ should only come from $\mathbf{E}_{\kappa,\omega}^q$, i.e., $\frac{\omega L_f}{v} = 2\pi$. This gives

$$L_{f1} = L_{f2} = \frac{2\pi v}{\omega}. \quad (3.18)$$

Numerically, it means that it will take the electron about $(L_{f1} + L_{f2})/v = 0.2\text{ps}$

to launch a 2D plasmon at 10 THz. This 0.2ps is considered as the "formation time" for the 2D plasmon at 10THz.

We plot in Fig. 3.4a the time evolution of total radiated photon energy by numerically monitoring the Poynting power going through an imaginary sphere with a large radius R centered at the origin and then shifting the time axis backwards by R/c . The total radiated photon energy eventually saturates and approaches a value of $0.171 \times 10^{-4}\text{eV}$ that is analytically obtained by letting $t \rightarrow \infty$. It can be seen that most photon energy has been radiated out before $t = 0.15\text{ps}$, which is the moment when the jet-like rise of charge reaches its maximum (Fig. 3.3e), as most photon energy is "shaken off" by the charge jet. We also plot the time evolution of "energy" of induced fields without considering the interference fields of the swift electron. Note that 2D plasmons have not been generated before their formation times. So this "energy" should be treated as a parameter related to the field strength, $(E_{\kappa,\omega}^{1,2})^2$, of induced charges, but not as the real energy. It can be seen that even before most photon energy is radiated out, the field strength of $(E_{\kappa,\omega}^{1,2})^2$ has already started to drop.

Now we calculate the real energy of generated GPs by checking the electromagnetic energy for each frequency component immediately after the time $t = L_{f2}/v$. For the calculation spectrum of 2D plasmons from 0 to 20 THz, the corresponding formation time $(L_{f1} + L_{f2})/v$ varies from ∞ to 0.1ps, whose duration is comparable with the propagation time ($\sim 0.5\text{ps}$) for GPs. The energy loss during the formation time cannot be taken into account by simply resorting to the propagation time or propagation length of 2D plasmons that are widely used to characterize the propagation of GPs, because this energy loss happens before the generation of 2D plasmons themselves.

Only in the ideal lossless situation can one equal the electron energy loss spectroscopy (EELS) spectrum to the energy spectrum of generated 2D plasmons (the energy of emitted photons generally occupies only $< 5\%$ of the energy loss of the swift electron and thus is negligible). We plot in Fig. 3.4b the spectrum of 2D plasmons in the ideal lossless situation by taking $t \rightarrow \infty$, which can be considered equivalent to the EELS calculation, and that for the realistic lossy graphene after formation time is taken into account by taking $t = L_{f2}/v$. The comparison between the lossless and lossy cases shows that a significant portion of energy (over 25% total energy from 0 to 20 THz) has already been dissipated before 2D plasmons are generated.

Figure 3.4: Energy dissipation during plasmonic formation time. (a) Time evolution of emitted photon energy and the induced field strength $(E_{\kappa,\omega}^{1,2})^2$. (b) Energy spectra of GPs by taking $t \rightarrow \infty$ in the lossless case and by taking $t = L_{f2}/v$ in the lossy case.

3.5 Conclusion

In this chapter, we apply the TR theory to study the GPs launching by a swift electron passing through a free-standing monolayer graphene perpendicularly. With the classical electrodynamic approaches, we successfully show the dynamic processes of the GPs launching. We find that the rebound of the density of conduction electrons is formed with a sharp centra jet-like rise, being analogous to the "Rayleigh jet" or "Worthington jet" in hydrodynamics. We predict the existences of such jet in 2D plasmons for the first time. We also calculate the formation time for GPs, and find that not all the energy coupled to the graphene sheet is converted to that of GPs, as part of the energy is dissipate before the formation of GPs.

The Combination of Transition Radiation and Radiative Plasmons

The bombarding of a thin metal plate plays a vital role in the study of bulk plasmons and surface plasmons polaritons (SPPs) [6, 21–23, 25–32, 36]. A lot of studies come from the analysis of the electron energy loss spectra [6, 29, 37, 38]. However, the electron energy loss spectra can only give us static images of the energy transfer from swift electrons to the collective modes supported by the materials. This is only the first step of the whole physical process, and electron energy loss spectra cannot tell the dynamic evolution. A part of the energy from swift electrons is immediately converted to that of transition radiation (TR), bulk plasmons and SPPs when it penetrates through a metal plate. But after the swift electrons leave the metal plate, there are still energy exchange between these modes, even in the ideal lossless metal cases.

In history, there was controversy about the origin of the emission excited by swift electrons penetrating through a metal plate. One group believed that it

came from TR [79] which was developed by Ginzburg [45]. Another group insisted that it resulted from the radiative decay of the bulk plasmons [76–78]. There were also people claiming that the two kinds of theories were equivalent [15]. According to our literature review, the controversy has not been settled yet. The judgement is easy to be settled if we show the whole dynamic physical processes.

In our calculation, we use a half space model (half air and half metal) instead of a metal plate model. In the half space model, we can focus on the energy conversion from the bulk plasmons to the SPPs and radiation, without considering the coupling of the SPPs in thin metal plate which further complicates the whole physical processes. In our calculation, we assume the velocity of a swift electron is constant, which is often used to study the interaction between swift electrons with materials [41, 45]. Our calculation shows that the bombarding of the metal surface with a swift electron would excite SPPs and radiation, as predicted by the TR theory. Bulk plasmons inside the bulk metal are also excited, and bulk plasmons work as a source for both radiation and SPPs when the swift electron moves away from the interface. In a low loss metal model, the bulk plasmons could last over a long period of time.

Furthermore, the energy loss of swift electrons comes from the difference of two large quantities, each of which exceeds the quantity of interest by a factor of the order of above 10^3 [78]. With optical methods that study the emission excited by a swift electron penetrating the metal plate, we can measure the photons energy distribution directly. Our calculation shows that, both TR and radiative plasmons contribute to the radiation, but not within the same time period. The emission from TR is excited almost immediately (after the formation-time), while the emission from radiative bulk plasmons is excited later, but can last over a

long period of time.

4.1 The model setup

Figure 4.1 depicts the half space model for the calculation. A swift electron with charge q penetrates from medium 1 (air) into medium 2 (metal) with a uniform velocity v following a straight trajectory along $-z$ direction. The current density of the swift electron is

$$\mathbf{J}(\mathbf{r}, t) = -\hat{z}qv\delta(x)\delta(y)\delta(z + vt). \quad (4.1)$$

The air locates in the up-half space ($z \geq 0$), while the metal is in the low-half space ($z \leq 0$). They all have no magnetism, and therefore the relative permeabilities for the up and bottom space are $\mu_{1,2} = 1$. The air is assumed to have the same permittivity with vacuum (the relative permittivity $\varepsilon_1 = 1$). The relative permittivity of metal is described by the Drude model Eq. (4.2)),

$$\varepsilon_2(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\omega\gamma}. \quad (4.2)$$

4.2 Derivation of the field distribution

We first expand all quantities $\mathbf{J}$, $\mathbf{E}$ and $\mathbf{H}$ by the Fourier transform in time and the transverse component of $\mathbf{r}_\perp$, as in section 3.2 of Chapter 3,

$$\begin{aligned} \mathbf{J}(\mathbf{r}, t) &= \int \mathbf{J}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_\perp - i\omega t) d\omega d\boldsymbol{\kappa} \\ \mathbf{E}(\mathbf{r}, t) &= \int \mathbf{E}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_\perp - i\omega t) d\omega d\boldsymbol{\kappa} \\ \mathbf{H}(\mathbf{r}, t) &= \int \mathbf{H}_{\omega, \kappa}(z) \exp(i\boldsymbol{\kappa} \cdot \mathbf{r}_\perp - i\omega t) d\omega d\boldsymbol{\kappa}, \end{aligned} \quad (4.3)$$

Figure 4.1: Schematic depicting a swift electron penetrating from air into metal with a constant velocity v . The swift electron would excite radiation, SPPs and bulk plasmons

where $\boldsymbol{\kappa} = \hat{x}k_x + \hat{y}k_y$, $\mathbf{r}_\perp = \mathbf{x} + \mathbf{y}$. Then we follow the same steps from Eq. (3.3) to (3.11) in Chapter 3, but with two changes. One is that in Chapter 3, medium 1 and 2 have the same relative permittivities, while in this chapter, they are different. The other difference is the boundary condition. In Chapter 3, there is a sharp variation of the horizontal component of magnetic field because of the surface conductivity. But in this chapter, the horizontal component of magnetic

field is continuous across the plane. The boundary condition becomes,

$$\begin{aligned} E_{\omega,\kappa,y}^q(z=0^+) + E_{\omega,\kappa,y}^s(z=0^+) &= E_{\omega,\kappa,y}^q(z=0^-) + E_{\omega,\kappa,y}^s(z=0^-) \\ H_{\omega,\kappa,y}^q(z=0^+) + H_{\omega,\kappa,y}^s(z=0^+) &= H_{\omega,\kappa,y}^q(z=0^-) + H_{\omega,\kappa,y}^s(z=0^+) \end{aligned} \quad (4.4)$$

After tedious calculation, we obtain the field distribution in the following.

The electron self-excited fields are,

$$\mathbf{E}_{\omega,\kappa}^q = \frac{iq}{(2\pi)^3 \omega \varepsilon_0 \varepsilon_{1,2}} \frac{e^{-i\frac{\omega}{v}z}}{(k^2 - \kappa_x^2 - \kappa_y^2 - \frac{\omega^2}{v^2})} \left(\kappa_x \frac{\omega}{v} \hat{x} + \kappa_y \frac{\omega}{v} \hat{y} + \left(k^2 - \frac{\omega^2}{v^2} \right) \hat{z} \right), \quad (4.5)$$

and

$$\mathbf{H}_{\omega,\kappa}^q = \frac{iq}{(2\pi)^3} \frac{e^{-i\frac{\omega}{v}z}}{k^2 - \kappa_x^2 - \kappa_y^2 - \frac{\omega^2}{v^2}} (\kappa_y \hat{x} - \kappa_x \hat{y}). \quad (4.6)$$

The scattering fields are

$$\mathbf{E}_{\omega,\kappa}^s(z) = \frac{iq\hat{z}}{(2\pi)^3 \omega \varepsilon_0} \begin{cases} \frac{a_1}{\varepsilon_1} e^{ik_{1z}z} & z > 0 \\ \frac{a_2}{\varepsilon_2} e^{-ik_{2z}z} & z < 0, \end{cases} \quad (4.7)$$

$$\mathbf{E}_{\omega,\kappa}^s(z) = \frac{iq}{(2\pi)^3 \omega} \frac{\kappa_x \hat{x} + \kappa_y \hat{y}}{\kappa^2 \varepsilon_0} \begin{cases} -\frac{a_1}{\varepsilon_1} k_{1z} e^{ik_{1z}z} & z > 0 \\ \frac{a_2}{\varepsilon_2} k_{2z} e^{-ik_{2z}z} & z < 0, \end{cases} \quad (4.8)$$

and

$$\mathbf{H}_{\omega,\kappa}^s(\omega) = \frac{iq}{(2\pi)^3} \frac{\kappa_y \hat{x} - \kappa_x \hat{y}}{\kappa^2} \begin{cases} a_1 e^{ik_{1z}z} & z > 0 \\ a_2 e^{-ik_{2z}z} & z < 0. \end{cases} \quad (4.9)$$

The coefficients are

$$\begin{cases} a_1 = \frac{\kappa^2 \varepsilon_1}{k_{1z} \varepsilon_2 + k_{2z} \varepsilon_1} \left(\frac{\frac{\omega}{v} \varepsilon_2 - k_{2z}}{k_1^2 - \kappa^2 - \frac{\omega^2}{v^2}} - \frac{\frac{\omega}{v} - k_{2z}}{k_2^2 - \kappa^2 - \frac{\omega^2}{v^2}} \right) \\ a_2 = \frac{\kappa^2 \varepsilon_2}{k_{1z} \varepsilon_2 + k_{2z} \varepsilon_1} \left(\frac{\frac{\omega}{v} + k_{1z}}{k_1^2 - \kappa^2 - \frac{\omega^2}{v^2}} - \frac{\frac{\omega}{v} \varepsilon_1 + k_{2z}}{k_2^2 - \kappa^2 - \frac{\omega^2}{v^2}} \right) \end{cases} \quad (4.10)$$

Next, we change the $\mathbf{E}$ and $\mathbf{H}$ field from the cartesian coordinates to the spherical coordinates. The azimuthal angle ϕ and α are in the position space and

Figure 4.2: The coordinate transform between cartesian and cylindrical coordinate in the a) position space and b) k space.

k space as defined in Fig. 4.2. With this definition, we can change the horizontal components of $\mathbf{E}$ field (Eq. (4.8)), and $\mathbf{H}$ field (Eq. (4.9)) to their cylindrical coordinate representations,

$$E_{\omega, \kappa, \rho}^s(z) = \frac{iq}{(2\pi)^3 \omega \varepsilon_0} \frac{\cos(\alpha - \phi)}{\kappa} \begin{cases} -\frac{a_1}{\varepsilon_1} k_{1z} e^{ik_{1z}z} & z > 0 \\ \frac{a_2}{\varepsilon_2} k_{2z} e^{-ik_{2z}z} & z < 0 \end{cases}, \quad (4.11)$$

and

$$H_{\omega, \kappa, \rho}^s(\omega) = -\frac{iq}{(2\pi)^3} \frac{\cos(\alpha - \phi)}{\kappa} \begin{cases} a_1 e^{ik_{1z}z} & z > 0 \\ a_2 e^{-ik_{2z}z} & z < 0 \end{cases}. \quad (4.12)$$

Submit Eq. (4.7), (4.11) and (4.12) into Eqs. (4.3), we have

$$E_z^s(r, t) = \frac{iq}{(2\pi)^2 \varepsilon_0} \int_{-\infty}^{\infty} d\omega \frac{\exp(-i\omega t)}{\omega} \int_0^{\infty} d\kappa \kappa J_0(\kappa \rho) \begin{cases} \frac{a_1}{\varepsilon_1} e^{ik_{1z}z} & z > 0 \\ \frac{a_2}{\varepsilon_2} e^{-ik_{2z}z} & z < 0 \end{cases}, \quad (4.13)$$

$$E_\rho^s(r, t) = \frac{q}{(2\pi)^2 \varepsilon_0} \int_{-\infty}^{\infty} d\omega \frac{\exp(-i\omega t)}{\omega} \int_0^{\infty} d\kappa \kappa J_1(\kappa \rho) \begin{cases} \frac{a_1}{\varepsilon_1} k_{1z} e^{ik_{1z}z} & z > 0 \\ \frac{a_2}{\varepsilon_2} k_{2z} e^{-ik_{2z}z} & z < 0 \end{cases}, \quad (4.14)$$

and

$$H_\varphi^s(r, t) = \frac{q}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \exp(-i\omega t) \int_0^{\infty} d\kappa J_1(\kappa \rho) \begin{cases} a_1 e^{ik_{1z}z} & z > 0 \\ a_2 e^{-ik_{2z}z} & z < 0 \end{cases}. \quad (4.15)$$

The other components are zero. The integral identity for the Bessel function

$$J_n(\xi) = \frac{1}{2\pi} \int_0^{2\pi} d\alpha e^{i\xi \cos \alpha + in\alpha - in\frac{\pi}{2}} \quad (4.16)$$

is used during the derivation.

The swift electron's self-excited fields are

$$E_z^q(r, t) = \int_{-\infty}^{\infty} \frac{q\omega}{8\pi\epsilon_0} \left(\frac{1}{c^2} - \frac{1}{v^2\epsilon^{1,2}(\omega)} \right) e^{-i\frac{\omega}{v}z} H_0^{(1)}(\kappa\rho) d\omega, \quad (4.17)$$

$$E_\rho^q(r, t) = \int_{-\infty}^{\infty} \frac{iq}{8\pi v\epsilon_0} \frac{\kappa}{\epsilon^{1,2}} H_1^{(1)}(\kappa\rho) e^{-i\frac{\omega}{v}z} d\omega, \quad (4.18)$$

and

$$H_\varphi^q(r, t) = -\frac{iq}{8\pi} \int_{-\infty}^{\infty} \kappa H_1^{(1)}(\kappa\rho) e^{-i\frac{\omega}{v}z} d\omega. \quad (4.19)$$

The total field should be the sum of the scattering fields and particle self excited fields.

4.3 Dynamics of plasmons launching and radiation excitation

The dynamics of the physical processes of a swift electron penetrating from air into a metal described by Drude model (Eq. (4.2)) can be shown by the numerical quadrature of Eq. (3.2).

At first, we fit the Drude model Eq. (4.2) to the optical constants of silver [142], and get the result $\epsilon_\infty = 3.7$, $\omega_p = 1.39 \times 10^{16}$ Hz, and $\gamma = 3.1 \times 10^{-14}$ sec. The variation of H_ϕ , E_z and E_ρ are shown in Figs. 4.3, 4.4 and 4.5, respectively.

As shown in Fig. 4.3, when a swift electron penetrates from air to metal, it would excite emission and SPPs almost immediately. When the electron moves

Figure 4.3: Temporal evolution of the magnetic field H_ϕ excited by a swift electron penetrating from air into low-loss metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

inside the metal, it would excite bulk plasmons, as shown in Figs. 4.4 and 4.5. The bulk plasmons excited are local instead of propagating waves, since the bulk plasmons have only the electrical field (Figs. 4.4, 4.5).

The bulk plasmons frequency ω_{bp} is determined by $\varepsilon_2(\omega) = 0$ [46]. The loss $1/\gamma$

Figure 4.4: Temporal evolution of the electrical field E_z excited by a swift electron penetrating from air into low-loss metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

is two orders of magnitude smaller than the bulk plasmon frequency. Therefore, the bulk plasmons can last over a long period of time, as can be seen from Fig. 4.4 f. The bulk plasmons provide a source for both the radiation and SPPs.

To further demonstrate the contribution of the bulk plasmons, we increase

Figure 4.5: Temporal evolution of the magnetic field E_ρ excited by a swift electron penetrating from air into low-loss metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

the loss by decreasing the mean free travel time γ in Eq. (4.2) by two order of magnitude, arriving at $\gamma = 3.1 \times 10^{-16}$ sec. Then the variations of H_ϕ, E_z and E_ρ over time are shown in Figs. 4.6, 4.7 and 4.8. From Figs. 4.8e f and 4.7e f, we can see that the bulk plasmons decay much faster than those in Figs. 4.5e f and

Figure 4.6: Temporal evolution of the magnetic field H_ϕ excited by a swift electron penetrating from air into metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

4.4e f, and there is no long lasting emission and SPPs.

We calculate the radiation at point $(y = z = 1\mu\text{m})$ in both cases, and compare them in Fig. 4.9. We use H_ϕ component to represent the far field radiation. $t = 0$ is the time the swift electron reaches the interface separating the air and

Figure 4.7: Temporal evolution of the electrical field E_z excited by a swift electron penetrating from air into high-loss metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

the metal. It can be shown that at about $t = 5\text{fs}$, there are sharp peaks in both lines. They represent the TR excited when the swift electron passing through the interface. Afterwards, in the low loss case, where $\gamma = 10^{-14}\text{sec}$, there is a continues but slowly decaying radiation. However, in the high loss case, where

Figure 4.8: Temporal evolution of the electrical field E_ρ excited by a swift electron penetrating from air into high-loss metal. The dashed white line indicates the interface between air and metal. In each figure, the swift electron moves to the position pointed by the tip of the white arrow.

$\gamma = 10^{-16}$ sec, the radiation only last a short time period (4 – 7fs). The smooth red line after $t = 7$ fs comes from the static magnetic field decay resulting from the swift electron.

Figure 4.9: Temporal evolution of the far-field($y = z = 1\mu m$) magnetic component H_ϕ excited by a swift electron penetrating from air into metal. The blue and red line indicate the cases where the metal has low loss ($\gamma = 3.1 \times 10^{-14}\text{sec}$) and high loss ($\gamma = 3.1 \times 10^{-16}\text{sec}$), respectively.

4.4 Conclusion

It was believed that, when electrons pass from one medium to another, the emission mainly comes from TR and Cherenkov radiation (if the electron's speed exceeds the light speed in the medium). We show in this chapter that there is one more channel, the decay of the radiative bulk plasmons, where the emission comes from. We show the dynamic processes of the coupling from the radiative bulk plasmons to the emission with half-space model. The results show that the radiative bulk plasmons will be converted to emission in a quite long period of time after the electron leaves the boundary that separates the two half spaces.

Directional Coupling of Plasmonic Cherenkov radiation: NOT Always One-Sided

Near field interference is essential for controlling the propagation direction of surface waves [17, 80, 81, 143–145]. The previous studies of the excitation of asymmetrical surface waves use a rotating but fixed in position dipole to model the emitter which directional excites surface waves [18, 82]. We demonstrate that when the rotating dipole is in motion, a new kind of interference manner would arise as a result of the relativistic Doppler effect. If without this effect, the two-dimensional (2D) plasmonic Cherenkov radiation (CR) from the swift rotating dipole would be strengthened only at one side of its branch (Fig. 5.1b), following the same interference principle as that when the dipole is fixed in position (Fig. 5.1a). However, we show in this chapter that the plasmonic CR excited by a

swift rotating dipole is not always strengthened at one side (Fig. 5.1c). Furthermore, several experiments conducted before are better to be modelled by a swift, instead of the adopted fixed in position, dipole model [17, 18, 82].

It has been shown recently that a circularly polarized dipole's near-field is directionally coupled to the surface waveguide modes on a plane with the direction determined by the helicity of the rotating dipole [18, 82]. As a result, the surface waves on the plane are non-symmetric with the intensity on one side greater than that on the other side. The non-symmetry comes from the interference of the two orthogonal oriented dipole components of the rotating dipole. After taking a Fourier transform of the dipole's near-field, we can see that the superposition of the near-field from the two components is constructively on one side and destructively on the other side [18, 82].

We let the rotating dipole moves along its rotating axis. When the dipole's velocity is greater than the phase velocity of the surface wave, a plasmonic CR will be excited [17, 71]. If the swift rotating dipole follows the same interference manner as that when the dipole is fixed in position, which side of the plasmonic CR has stronger intensity distribution is determined by the helicity of the rotating dipole. In this chapter, we find that at relatively slow swift velocity, like $0.1c$ where c is the phase velocity of light in vacuum, the plasmonic CR has only one bright branch, as shown in Fig. 5.1b. But as the dipole's swift velocity increases to a relatively large value, like $0.3c$, the pattern has two bright branches on both sides, as shown in Fig. 5.1c. Therefore, the interference is not totally determined by the helicity of the rotating dipole [18, 82]. When the dipole is in motion, the relativistic Doppler effect reverses the horizontal phase velocity of the dipole's evanescent waves, which affect the interference from the two dipole components.

Figure 5.1: The GPs excited by rotating dipoles. a) The rotating dipole remains in the same position above the graphene with distance d . b) The rotating dipole moves along z direction with a constant velocity $v = 0.1c$. c) The rotating dipole moves along z direction with a constant velocity $v = 0.3c$.

5.1 The derivation of the plasmonic Cherenkov radiation

We choose free-standing graphene as the platform to study the plasmonic CR because of the relatively slow phase velocity of GPs [3] (about $1/100c$, where c is the phase velocity of light in vacuum). The free-standing graphene locates in the $y = d = 1\mu\text{m}$ plane. We assume it has a chemical potential of $\mu_c = 0.15\text{eV}$, and a scattering rate of $\Gamma = 0.11\text{meV}$ at room temperature $T = 300\text{K}$. The frequency dependent complex conductivity $\sigma(\omega)$ of the isolated graphene is modelled with Kubo formula [66,115], as shown in Eqs. (2.4),(2.5).

As shown in Fig. 5.1, the circularly polarized dipole moves along z direction with a constant velocity v parallel to an free standing monolayer graphene in

the lab frame S . In the dipole frame S' , the circularly polarized dipole can be decomposed into two orthogonal dipole components in the $x' - y'$ direction with a phase difference of $\pi/2$. The dipole momentum in the frame S' can be represented as,

$$\mathbf{P}'(\mathbf{r}', t') = \text{Re} \left\{ (p_x \hat{x} + ip_y \hat{y}) e^{-i\omega_0 t'} \right\} \delta(\mathbf{r}'), \quad (5.1)$$

where p_x and p_y are the two linearly polarized dipole momentums, ω_0 is the self-vibration frequency of the dipole in the frame of S' . The current density generated by the dipole in the frame S' is

$$\begin{aligned} \mathbf{J}'(\mathbf{r}', t') &= \frac{\partial \mathbf{P}'(\mathbf{r}', t')}{\partial t'} \\ &= \text{Re} \left\{ -i\omega_0 (p_x \hat{x} + ip_y \hat{y}) e^{-i\omega_0 t'} \right\} \delta(\mathbf{r}') \quad . \\ &= (-\omega_0 p_x \sin \omega_0 t' \hat{x} + \omega p_y \cos \omega_0 t' \hat{y}) \delta(\mathbf{r}') \end{aligned} \quad (5.2)$$

We then apply the Lorentz transform to the current and find that, as the relative movement between the two frames is along the z direction, the current in the frame S remains the same form as that in the frame S' , i.e., $\mathbf{J}(\mathbf{r}, t) = \mathbf{J}'(\mathbf{r}'(\mathbf{r}), t'(t))$. The space-time transform between the two frames is $t' = \gamma t - \gamma v z / c^2$ and $z' = -\gamma v t + \gamma z$, with the other coordinates remain unchanged, where v is the velocity of the dipole and $\gamma = 1/\sqrt{1 - (v/c)^2}$. By substituting these space-time transformations back to Eq. (5.2), we get the current density in the frame S ,

$$\begin{aligned} \mathbf{J}(\mathbf{r}, t) &= \omega_0 \left\{ -p_x \sin \left[\omega_0 \left(\gamma t - \gamma \frac{vz}{c^2} \right) \right] \hat{x} + p_y \cos \left[\omega_0 \left(\gamma t - \gamma \frac{vz}{c^2} \right) \right] \hat{y} \right\} \times \\ &\quad \delta(x) \delta(y) \delta(\gamma z - \gamma v t) \\ &= \frac{\omega_0}{\gamma} \left\{ -p_x \sin \left[\omega_0 \left(\gamma t - \gamma \frac{vz}{c^2} \right) \right] \hat{x} + p_y \cos \left[\omega_0 \left(\gamma t - \gamma \frac{vz}{c^2} \right) \right] \hat{y} \right\} \times \\ &\quad \delta(x) \delta(y) \delta(z - vt) \end{aligned} \quad (5.3)$$

Its Fourier transform in the frequency domain is

$$\begin{aligned}\mathbf{J}(\mathbf{r}, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \mathbf{J}(\mathbf{r}, t) e^{i\omega t} \\ &= \frac{\omega_0 \delta(\rho)}{4\pi^2 \rho \gamma v} \left(-p_x \sin\left(\frac{\omega_0 z}{\gamma v}\right) \hat{x} + p_y \cos\left(\frac{\omega_0 z}{\gamma v}\right) \hat{y} \right) e^{i\omega z/v},\end{aligned}\quad (5.4)$$

where $\rho = \sqrt{x^2 + y^2}$. In the frame S , the dipole's self-vibration frequency becomes ω_0/γ , because of the relativistic time dilation effect. The evanescent waves from the swift circularly polarized dipole can excite GPs.

Transverse-magnetic (TM) GPs dominate under the prescribed parameters of graphene. Each frequency component of the GPs can be exactly derived by taking the residue of the Sommerfeld pole [116, 146]. Here we use the vertical component of the electric field E_y to represent the GPs,

$$E_y(\omega) = \pm \frac{\omega \omega_0 \varepsilon}{2\pi \gamma v \sigma^2} e^{ik_y d} \left\{ \left[\frac{k^2 - k_y^2}{k_{x1} k_y} p_y - i p_x \right] e^{ik_{x1} x + i(\omega + \omega_0/\gamma)z/v} + \left[\frac{k^2 - k_y^2}{k_{x2} k_y} p_y + i p_x \right] e^{ik_{x2} x + i(\omega - \omega_0/\gamma)z/v} \right\}, \quad (5.5)$$

where ε is the vacuum permittivity, $k = \omega/c$, $k_y = -2\omega\varepsilon/\sigma$ representing the confinement of GPs, $k_{x1} = \pm \sqrt{k^2 - k_y^2 - (\omega + \omega_0/\gamma)^2/v^2}$ and $k_{x2} = \pm \sqrt{k^2 - k_y^2 - (\omega - \omega_0/\gamma)^2/v^2}$. The positive sign applies to the $x > 0$ region and negative applies to the $x < 0$ region. Referring to Eq. (5.5), the GPs are decomposed into two waves at each frequency component.

5.2 Plasmonic Cherenkov radiation distribution under different dipole's swift velocities

When the dipole is fixed in position in the lab frame S , as shown in Fig. 5.1a, the GPs excited is strengthened at the left-hand side as we choose a clockwise rotating

dipole. It is similar to the one-sided dielectric waveguide mode and the surface plasmon polaritons studied before [18,82]. Next we let the rotating dipole moves forward along z direction with a constant velocity v . Because of the Doppler effect, the GPs propagating to different direction have different frequencies. When the swift velocity of the dipole is much faster than the phase velocity of GPs [17,71], the V-shape plasmonic CR would be excited. Because of the interference between the two dipole components, the field distribution is not be symmetrical to the source's moving direction as studied before. We choose two velocities, $v = 0.1c$ and $0.3c$, to study the excited plasmonic CR. The results are shown in Figs. 5.2a and b, respectively. When the velocity is $0.1c$, the plasmonic CR is strengthened on the left-side with a bright branch at the boundary, similar to the case when the dipole is fixed in position. If we change the dipole's rotating direction from clockwise to anticlockwise, the bright branch in Fig. 5.2a will occur on the right side. But as the velocity increases to $0.3c$, the fields are strengthened at both sides, generating two different bright branches. Therefore, when the dipole is in motion, the interference from the two components of the rotating dipole is not completely determined by the helicity of the rotating dipole.

5.3 Analysis of the asymmetrical plasmonic Cherenkov radiation pattern

When the dipole remains fixed in position, the GPs excited by the rotating dipole have stronger intensity distribution at the left-hand side (Fig. 5.1a). The calculation shows that the GPs excited by the horizontal oriented and vertical oriented dipole components interference constructively on the $-k_x$ propagating direction

Figure 5.2: The asymmetrical plasmonic CR pattern excited by a circular polarized dipole moving above graphene with velocity $v = 0.1c$ and $0.3c$. We use $|E_y(r, t)|$ to represent the field distribution when the dipole moves to the point $z = 40\mu\text{m}$ pointed by the tip of the white arrow. The bright branches are the caustics.

and destructively on the $+k_x$ propagating direction. As the dipole moves forward along z direction with the velocity v , we derive the GPs distribution in the frequency domain in the following,

$$E_y^1(\omega) = \pm \frac{ik_y}{8\pi v \varepsilon} e^{ik_y d} \left(\frac{\frac{\omega'}{\gamma} k^2 - k_y^2}{k_{x1}} p_y + ik_y p_x \right) e^{ik_{x1} x + i(\omega - \omega'/\gamma)z/v} \quad (5.6)$$

$$E_y^2(\omega) = \pm \frac{ik_y}{8\pi v \varepsilon} e^{ik_y d} \left(\frac{\frac{\omega'}{\gamma} k^2 + k_y^2}{k_{x2}} p_y + ik_y p_x \right) e^{ik_{x2} x + i(\omega + \omega'/\gamma)z/v}, \quad (5.7)$$

where ε is the vacuum permittivity, $k = \omega/c$, $k_y = (-2\omega\varepsilon)/\sigma$ represents the confinement of GPs, $k_{x1} = \pm \sqrt{k^2 - k_y^2 - (\omega - \omega_0/\gamma)^2/v^2}$ and $k_{x2} = \pm \sqrt{k^2 - k_y^2 - (\omega + \omega_0/\gamma)^2/v^2}$. The positive sign applies to the $x > 0$ region, and the negative sign applies to the $x < 0$ region. The plasmonic CR patterns shown in Fig. 5.2 are plotted with the numerical Fourier integral of the sum of Eqs. (5.6)-(5.7), i.e., $E_y(t) = \int d\omega (E_y^1(\omega) + E_y^2(\omega)) e^{-i\omega t}$.

As the dipole moves forward, the frequency of the near-field emitted by the rotating dipole varies with the propagating direction. The frequency and wave vectors in the two frames S and S' are related by

$$\omega = \gamma\omega' + \gamma v k'_z \quad (5.8)$$

$$k_z = \frac{\omega - \omega'/\gamma}{v}, \quad (5.9)$$

where the parameters with prime and without prime are in the frames S' and S , respectively. The other wave vectors, k_x and k_y , remain unchanged under Lorentz transformation. For propagating waves, k'_z cannot be smaller than $-\omega'/c$, therefore the frequencies ω in the lab frame S are always positive. However, in the near-field, there is no limitation of k'_z . Following Eq. (5.8), the frequency of the near-field with k'_z being smaller than $-\omega'/v$ is shifted to a negative value in the lab frame. Since the horizontal wave vectors remain unchanged under

Lorentz transform, when the frequency go to a negative value, the evanescent waves emitted from the dipole with negative k_x have a positive horizontal phase velocity, and they will be coupled to the GPs propagating to $+x$ direction.

When the dipole is fixed in position as shown in Fig. 5.2a, the excited GPs with negative k_x have greater intensity, and the GPs with negative k_x are propagating to the $-x$ direction. Therefore, the fields are strengthened at the x negative part. But as the dipole moves forward, some of the excited GPs with negative k_x have positive phase velocities, and are propagating to the x positive direction. The Lorentz Doppler effect manifest itself only when the dipole's velocity is fast enough. When the dipole's velocity is $0.1c$, the GPs in Fig. 5.1a are still strengthened at the left-hand side, for only a small part of the waves with negative k_x value reverse the propagating direction from $-x$ to $+x$ direction. But as the velocity increases to $0.3c$, the strong Lorentz effect reverses a large part of the waves' propagating direction and causes the GPs be strengthened at both sides. In the derivation of Eqs. (5.6)-(5.7), we assume the frequency ω being positive. It can be proved that the second part of Eqs. (5.6)-(5.7) are equivalent to the negative frequency spectrum caused by the Lorentz Doppler effect.

We further analyse the field intensity distribution with the propagating angle by converting Eqs. (5.6) and (5.7) to its angular spectrum representation, i.e., $E_y^{1,2}(\theta) = E_y^{1,2}(\omega) \frac{d\omega}{d\theta}$, with θ determined by $\frac{\omega \pm \omega_0/\gamma}{v} = k_{gp} \cos\theta$, where k_{gp} is the wavenumber of GPs. $\theta = 0$ when GPs propagate along the z direction, and is positive for $\text{Re}(k_x) > 0$, negative for $\text{Re}(k_x) < 0$ (since ω in Eqs.(5.6) and (5.7) are positive). We plot the distribution of the absolute value of $E_y^{1,2}(\theta)$ in Fig. 5.3.

Equation (5.6) has greater intensity distribution of the waves propagating to

Figure 5.3: The angle spectrum distribution of the asymmetrical plasmonic CR.

a) The angle spectrum distribution of the plasmonic CR when the velocity is $0.1c$.

The black and red line corresponds to the two different spectrum equations. b)

The angle spectrum distribution of the plasmonic CR when the velocity is $0.3c$.

the left. That part is determined by the helicity of the rotating dipole, for the Lorentz Doppler effect does not reverse the propagating direction of the waves with negative k_x . Equation (5.7) shows the fields are strengthened at the x positive direction. As discussed above, Eq. (5.7) represents the waves that have opposite horizontal propagating direction in the dipole frame S' and the lab frame S . Though in the dipole frame S' , the field has greater value with negative k_x , it

is coupled to the GPs in the lab frame with positive propagating direction.

The angular spectrum distribution in Fig. 5.3 shows that when the dipole's swift velocity increases from $0.1c$ to $0.3c$, the field intensity of the waves propagating to the x positive direction gets stronger than the waves propagating to the x negative direction. But it cannot explain the appearance of the bright branches in Fig. 5.3. The bright branches appear in the neighbourhood of caustics, where the rays pile up if we treat each frequency component as a ray. The positions of the rays at any time t and where they pile up depend all on the phase of Eqs. (5.6) and (5.7), which can be written as $\psi = k_x x/t + k_z x/t - \omega$. The vanish of first derivative of ψ shows the position of each ray, and the vanish of the second derivative of ψ shows the frequencies of the rays that will pile up along the caustics. We can further calculate the propagating angles of the waves that contribute to the caustics. When the swift velocity of the dipole is $0.1c$, they are $\theta_1 = \pm 3\pi/5$ for E_y^1 and $\theta_2 = \pm \pi/4$ for E_y^2 , respectively. Referring to Fig. 5.3a, only waves that propagate at the angle $\theta_1 = -3\pi/5$ have large intensity to make the caustics at x negative part detectable. When the swift velocity of the dipole increases to $0.3c$, the waves that propagate at angles $\theta_1 = \pm 0.64\pi$ and $\theta_2 = \pm 0.27\pi$ would pile up along the caustics. Referring to Fig. 5.3b, the waves propagating at angles $\theta_1 = -0.64\pi$ and $\theta_2 = 0.27\pi$ have great intensity to let the caustics detectable, which are the two bright branches in Fig. 5.2b. We plot the caustic line as shown by the black solid lines in Fig. 5.4.

Figure 5.4: The field distribution around caustics. The dipole's swift velocity is a) $0.1c$ and b) $0.3c$. The black solid line represents the caustic boundary, where the rays emitted from the dipole pile up. Inside the caustics, the fields have a maximum and a minimum value. Beyond the caustics the field decays exponentially. At the dashed black line, the field decay to $1/e$ of its maximum value.

5.4 The field distribution in the neighbourhood of caustics

In the ray optics theory, the waves cannot cross the caustic line. But from the wave theory, the field cannot have a sharp boundary. We calculate the field

distribution in the neighbourhood of caustics. As the rays get close to the caustics, the strong interference between the rays make the fields have a minimum and maximum value inside the caustics. The region between the caustics and the place of maximum intensity is called the caustic zone, the region with the bright branch in Fig. 5.2. The region beyond the caustics is called the caustic shadow. Though the rays cannot reach the caustic shadow region from ray optics theory, the field decays exponentially in the caustic shadow. From the field distribution equations in the neighbourhood of caustics, we can find the positions of the minimum and maximum intensity value, and the boundary of caustic shadow where the field decays to $1/e$ of the maximum value. These places are plotted in Fig. 5.4, the region between the black solid line and the red solid line is the caustic zone, where the field have great intensity. The derivation is as follows.

The time dependent phase of Eqs. (5.6) and (5.7) is $\psi = k_x x/t + k_z x/t - \omega$. The combination of the equation $d\psi/d\omega = 0$, which determines the position of each ray at time t , and $d^2\psi/d\omega^2 = 0$, which determines the caustic frequency ω_c of the rays that contribute to the caustics, will determine the caustic boundary. The field distribution in the neighbourhood of caustics can be calculated with the modified stationary phase method. The field distribution at any time can be calculated by $E_y(r, t) = \int d\omega e_y(\omega) e^{i\psi(\omega)t}$. To approximate the field distribution in the neighbourhood of caustics, we need to carry out the integral analytically. First, we need to expand the phase $\psi(\omega)$ around the caustic frequency ω_c to the third order, i.e., $\psi(\omega) = \psi(\omega_c) + \psi'(\omega_c)(\omega - \omega_c) + \frac{1}{6}\psi'''(\omega_c)(\omega - \omega_c)^3 + O((\omega - \omega_c)^4)$. Second, we substitute the phase back to the integral, and get,

$$\begin{aligned} E_y(r, t) &\simeq e_y(\omega_c) \int d\omega e^{i[\psi(\omega_c) + \psi'(\omega_c)(\omega - \omega_c) + \frac{1}{6}\psi'''(\omega_c)(\omega - \omega_c)^3]t} \\ &\simeq e_y(\omega_c) e^{i\psi(\omega_c)t} \left[\frac{1}{2}t\psi'''(\omega_c) \right]^{-1/3} \text{Ai}(X) \end{aligned} \quad , \quad (5.10)$$

where $\text{Ai}(X)$ is the Airy function, and $X = [\frac{1}{2}t\psi'''(\omega_c)]^{-1/3}t\psi'(\omega_c)$. $X = 0$ is equivalent to $\psi'(\omega_c) = 0$, which represents the caustic boundary. Around $X = 0$, the Airy function has a maximum and a minimum at $X = -1.02$ and $X = -2.34$, respectively. $X > 0$ represents the region where the rays cannot reach from the ray optics point, but the calculation of the field distribution shows that the waves decay exponentially away from the caustics. The region beyond the caustics is termed as caustic shadow. When $X = 0.66$, the Airy function reach e^{-1} of its maximum value at $X = -1.02$, we use the line $X = 0.66$ to represent the boundary of the caustic shadow. The four lines, which are caustic shadow, caustics, the maximum and the minimum from outside to inside, are shown in Fig. 5.4.

5.5 Conclusion

We study in this chapter the plasmonic CR excited by a swift rotating dipole. It has been shown before that, when the rotating dipole is fixed in position, the 2D plasmons excited by the vertical and parallel components of the dipole interfere differently on the left-hand side and right-hand side of the dipole, resulting in the 2D plasmons being stronger on one side and weaker on the other side. However, when the rotating dipole moves relative to the substrate, the Doppler effect would change the 2D plasmons' interference manner. Therefore, when the swift velocity of the dipole is relatively high ($\sim 0.3c$), the 2D plasmonic CR has two bright branches on both sides.

Electromagnetic Radiation from Uniformly Moving Sources through a Carpet Cloak

We show in this chapter the radiation from an electron uniformly passing through a carpet cloak, which can work in broadband frequency. Transformation Optics (TO) theory is used by deriving the electromagnetic field distribution in the virtual space first, then we change the field back to the physical space. This method offers clearly the physical insight of the wave generation and propagation inside TO materials. The result shows that transition radiation (TR) is generated when the electron passes through the boundary, and Cherenkov radiation (CR) is also excited inside the cloak when the electron's velocity is above the threshold.

The interaction of swift electrons with materials will generate different kinds of radiation under different conditions. CR is generated when an electron moves inside a transparent material with the velocity greater than the phase velocity of

light in that material, while TR is emitted when an electron passes through the boundary of two media with no requirement for its speed [41]. The importance of these radiations lie in their widely applications, from the Cherenkov counters and TR counters used in the study of nuclear and electron reactions [147], detection of cosmic gamma-ray [148], to the new generation of laser [149]. Therefore, the radiations by uniformly swift electrons have been extensively studied in novel materials [44, 150–154], bearing the hope that some new phenomena will be found to benefit applications. TO materials, in which light rays can be manipulated arbitrarily to some extent [86, 87], have attracted a lot of research interests in recent years, and are forecasted with promising prospects [155]. In this chapter, we consider a carpet cloak, instead of a complete one [88] which can only work in narrow frequency band. The carpet cloak makes the broadband cloaking one step further since it does not need resonant elements [83, 156–160]. It has also been realised with natural materials at optical frequencies [84, 85].

In this chapter, we consider the radiation from a uniformly swift electron passing through a two-dimensional (2D) carpet cloak. First, TO theory is used by deriving the current from the swift electron in the virtual space (the definition of virtual and physical space can be found in papers about TO theory, such as [87, 161]). Then we investigate the radiation from the current in the virtual space. In the end, the radiation in the physical space is obtained by applying coordinate transformation according to TO theory. Though similar work has been done before [88], the work reported in this chapter starts with evaluating the radiation in the virtual space first, giving us intuitional insights of the wave generation and propagation in TO material. Furthermore, the velocity threshold for CR can be defined with the help of virtual space. Most importantly, the

spherical cloak discussed in [88] can only work at certain frequency because of the singularity of the permittivity and permeability [87]. However, the waves emitted from a swift electron contains a quiet board range of spectrum. The carpet cloak, on the contrary, can work at a rather board frequency band, which makes the work discussed about here verifiable in experiments.

6.1 Transformation optics theory of the carpet cloak

Figure 6.1 represents the carpet cloak (left) and its virtual space (right). The cloak is made by squeezing the virtual flat vacuum space proportional along the y axis following the coordinate transformation [158, 160],

$$\begin{cases} x' = x \\ y' = \kappa y + \tau(a - |x|) \\ z' = z \end{cases} \quad , \quad (6.1)$$

where (x', y', z') and (x, y, z) are the coordinates in the physical and virtual space, respectively, $\kappa = [\tan(\alpha + \beta) - \tan(\beta)] / \tan(\alpha + \beta)$ and $\tau = \tan \beta$. α, β are shown in Fig. 6.1. Photons incident on the cloak will bypass the cloak region, as shown in Fig. 6.1a, but restore to their undisturbed trajectories after passing through the cloak. Their corresponding trajectories in the virtual space (Fig. 6.1b) are straight lines.

The electrical density, current density, electromagnetic fields, constitutive parameters in the physical space (shown in Fig. 6.1a and represented with $'$) and

Figure 6.1: Schematic picture depicting the swift electron, represented by the red dots following the dashed black line, and light ray represented by red solid line, passing through the cloak region in the physical space (left) and the virtual space (right). The gridding represents the electromagnetic space which is curved in the physical space and flat in the virtual space. The grey region in the physical space is invisibility for electromagnetic wave and the yellow region is the cloak. The dashed black line represents the trajectory of the swift electron.

virtual space (Fig. 6.1b) are related by the TO theory equations,

$$\begin{aligned}
 \rho'(r') &= |\det \Lambda|^{-1} \rho(r) \\
 J^{i'}(r') &= |\det \Lambda|^{-1} \Lambda_j^{i'} j^j(r) \\
 E_{i'}(r') &= (\Lambda^{-1})_{i'}^j E_j(r) \\
 H_{i'}(r') &= (\Lambda^{-1})_{i'}^j H_j(r) \\
 \epsilon^{i'j'}(r') &= |\det \Lambda|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \delta^{ij} \epsilon_0 \\
 \mu^{i'j'}(r') &= |\det \Lambda|^{-1} \Lambda_i^{i'} \Lambda_j^{j'} \delta^{ij} \mu_0
 \end{aligned} \tag{6.2}$$

Thereafter, we use the up case $\mathbf{J}$ and low case $\mathbf{j}$ to represent the current densities in the physical space and virtual space, respectively. Λ is the Jacobian

transformation matrix,

$$[\Lambda_j^{i'}] = \left(\frac{\partial x^{i'}}{\partial x^j} \right) = \begin{pmatrix} 1 & 0 & 0 \\ \mp \tau & \kappa & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (6.3)$$

where the minus sign applies to the $x > 0$ part, while positive to the $x < 0$ part.

With Eqs.(6.2) - (6.3), the relative permittivity and permeability for the cloak are

$$\varepsilon_r = \mu_r = \begin{pmatrix} 1/\kappa & \mp \tau/\kappa & 0 \\ \mp \tau/\kappa & \kappa + \tau^2/\kappa & 0 \\ 0 & 0 & 1/\kappa \end{pmatrix}, \quad (6.4)$$

the minus sign applies to the $x > 0$ part, while positive to the $x < 0$ part. The background here is choose to be $\varepsilon_b = 2.1\varepsilon_0$ and $\mu_b = \mu_0$, where ε_0 and μ_0 are the permittivity and permeability of vacuum. The angle $\alpha = 57.5^\circ$ and $\beta = 20.7^\circ$, similar to the geometry in [84, 85]. The length of the bottom of the cloak is $2a = 16\mu\text{m}$. The carpet cloak is scaleable.

6.2 Electron radiation through a perfect carpet cloak

For simplicity we consider the 2D case, where all quantities are invariant along the third dimension. We let a 2D line electron passes through the cloak with a constant velocity v along the straight trajectory $y' = y_0$. As shown by the blue dots in Fig. 6.1, the electron impinges on the cloak at point A' , and leaves at point C' . According to Eq. (6.4), in the virtual space Fig. 6.1b, the electron should follow the trajectory $y = y_0/\kappa - \tau a/\kappa + \tau|x|/\kappa$. As the time intervals

of passing through the physical and virtual space are the same, the velocity of the electron should be increased by a factor of $\sqrt{1 + (\tau/\kappa)^2}$ in the virtual space compared to that in the physical space. If its speed in the virtual space is greater than the phase velocity of light in that space, the electron will excite CR. Other than CR, the electron would also excite radiation when it changes velocity. The radiation excited in the virtual space would be transformed to the radiation in the physical space, which is excited when the electron passes through the cloak. With the help of the homogeneous virtual space, we are relieved from calculating the tedious Green function in the anisotropic cloak.

The uniformly swift electron with the charge q and the velocity v passing through the cloak is modelled by the current density,

$$\mathbf{J}(\mathbf{r}', t) = \hat{x}qv\delta(y' - y_0)\delta(x' - vt), \quad (6.5)$$

$t = 0$ when the electron is located at $x' = 0$, the point separating the left the right part of the cloak. After Fourier transform, the current density in the frequency domain is

$$\begin{aligned} \mathbf{J}(\mathbf{r}', \omega) &= \frac{1}{2\pi} \int \mathbf{J}(\mathbf{r}', t) e^{i\omega t} dt \\ &= \hat{x} \frac{q}{2\pi} \delta(y' - y_0) e^{i\omega x'/v}. \end{aligned} \quad (6.6)$$

The line electron would excite only transverse magnetic (TM) waves, therefore, we choose $\mathbf{H}$ to represent the radiation hereafter. With the help of dyadic Green function, we can calculate the magnetic field excited by the source Eq. (6.6) in a homogeneous medium with permittivity ε and permeability μ , the result is

$$\mathbf{H}_1(\mathbf{r}') = \pm \frac{q}{4\pi} e^{i\omega x'/v \pm ik_y(y' - y_0)} \hat{z}, \quad (6.7)$$

where the positive sign applies to $y > y_0$ region, and negative sign to the $y < y_0$ region.

The current densities of the segments AB and BC in the frequency domain in the virtual space can be derived according to transformation rules,

$$\begin{aligned} \mathbf{j}(\mathbf{r}, \omega) &= \det(\Lambda) \Lambda^{-1} \cdot \mathbf{J}(\mathbf{r}', \omega) \\ &= \frac{q}{2\pi} \begin{pmatrix} 1 \\ \pm\tau/\kappa \\ 0 \end{pmatrix} \delta\left(y - \frac{y_0 - \tau a}{\kappa} - \frac{\tau}{\kappa}|x|\right) e^{i\omega x/v}, \end{aligned} \quad (6.8)$$

where the positive sign applies to the segments AB and negative sign to the segment BC.

It can be seen from Fig. 6.1 that the current in the virtual space is separated into several pieces, $-\infty \rightarrow A$, AB , BC , $C \rightarrow \infty$. It is not easy to apply the Green function to the finite currents, therefore, we make use of the technique used in [153, 154] to analyze the radiation by the current – the finite current segment is modelled with dipole arrays. Equation (6.7) represents the fields by the current along a infinite long straight line. To get the field from the current line along the bend trajectory in Fig. 6.1b, we need to subtract from Eq. (6.7) the $\mathbf{H}$ generated by the current segment AC, and then add that from the current segments AB and BC. The current densities from the combination of the dipole arrays in segments AC, AB, and BC are,

$$\mathbf{j}_{AC}(\mathbf{r}, \omega) = \hat{x} \sum_n^{AC} \frac{ql}{2\pi} \delta(x - x_n) \delta(y - y_0) e^{i\omega x_n/v}, \quad (6.9)$$

and

$$\mathbf{j}_{ABC}(\mathbf{r}, \omega) = \sum_n^{ABC} \frac{ql}{2\pi} \begin{pmatrix} 1 \\ \pm\tau/\kappa \\ 0 \end{pmatrix} \delta(x - x_n) \delta(y - y_n) e^{i\omega x_n/v}, \quad (6.10)$$

where $(x_n, y_0), (x_n, y_n)$ are the coordinates of the dipoles along the segments

AC, AB and BC, $y_n = (y_0 - \tau a) / \kappa + \tau |x_n| / \kappa$. The positive and negative signs applies to AB and BC, respectively. With dyadic Green function [136], the magnetic fields from the dipole arrays along the segments AC and ABC are,

$$\mathbf{H}_{AC}(\mathbf{r}) = \hat{z} \sum_n^{AC} \frac{ikql(y - y_0)}{8\pi r_n} H_1^{(1)}(kr_n) e^{i\omega x_n/v}, \quad (6.11)$$

where $r_n = \sqrt{(x - x_n)^2 + (y - y_0)^2}$ and $H_1^{(1)}(kr_n)$ is the first order Hankel function of the first kind, and

$$\mathbf{H}_{ABC}(\mathbf{r}) = -\hat{z} \sum_n^{ABC} \frac{ikql}{8\pi r_n} H_1^{(1)}(kr_n) \left[(x - x_n) \frac{\tau}{\kappa} - (y - y_n) \right] e^{i\omega x_n/v}, \quad (6.12)$$

where $r_n = \sqrt{(x - x_n)^2 + (y - y_n)^2}$. The field by the current density in the virtual space should be

$$\mathbf{H}_{total}(\mathbf{r}) = \mathbf{H}(\mathbf{r}) - \mathbf{H}_{AC}(\mathbf{r}) + \mathbf{H}_{ABC}(\mathbf{r}). \quad (6.13)$$

The bottom of the cloak is covered with perfect electric conductor (PEC) [84,85], therefore, we need to take into account of the reflection from the ground plane in the virtual space. We use the equivalent image sources to calculate the reflection. The image 2D line currents of Eqs. (6.6),(6.9) and (6.10) are,

$$\begin{aligned} \mathbf{J}^{image}(\mathbf{r}, \omega) &= -\hat{x} \frac{q}{2\pi} \delta(y + y_0) e^{i\omega x/v} \\ \mathbf{j}_{AC}^{image}(\mathbf{r}, \omega) &= -\hat{x} \sum_n^{AC} \frac{ql}{2\pi} \delta(x - x_n) \delta(y + y_0) e^{i\omega x_n/v} \\ \mathbf{j}_{ABC}^{image}(\mathbf{r}, \omega) &= \sum_n^{ABC} \frac{ql}{2\pi} \begin{pmatrix} -1 \\ \pm\tau/\kappa \\ 0 \end{pmatrix} \delta(x - x_n) \delta(y + y_n) e^{i\omega x_n/v} \end{aligned} \quad (6.14)$$

The magnetic field from these current densities are recorded as $\mathbf{H}^{image}(\mathbf{r})$, $\mathbf{H}_{AC}^{image}(\mathbf{r})$ and $\mathbf{H}_{ABC}^{image}(\mathbf{r})$, respectively. Add the reflected magnetic fields to Eq. (6.13), we

have

$$\begin{aligned} \mathbf{H}_{total}(\mathbf{r}) = & \mathbf{H}(\mathbf{r}) + \mathbf{H}^{image}(\mathbf{r}) - \mathbf{H}_{AC}(\mathbf{r}) \\ & - \mathbf{H}_{AC}^{image}(\mathbf{r}) + \mathbf{H}_{ABC}(\mathbf{r}) + \mathbf{H}_{ABC}^{image}(\mathbf{r}). \end{aligned} \quad (6.15)$$

In the end, we apply the TO Eqs. (6.4) to the total magnetic field Eq. (6.15), can get the total radiation in the physical space.

To plot the radiation pattern, we need to numerical carry out the Fourier integral of Eq. (6.15). During the numerical calculation, we use a Gaussian pulse $\frac{1}{\sigma\sqrt{2\pi}}e^{-(x'-vt)^2/2\sigma^2}$ to replace the point source $\sigma(x' - vt)$ in Eq. (6.5). When δ goes to zero, the Gaussian pulse returns to the point source. We use the Gaussian pulse because the high frequency components in suppressed. Therefore, the Fourier integral of Eq. (6.15) becomes a finite integral. In our calculation, σ is chosen to be $0.1\mu\text{m}$, corresponding to a upper integral limit of 1000THz .

6.3 Analysis of the radiation

We have shown that the speed of the electron in the virtual space increases by a factor of $\sqrt{1 + (\tau/\kappa)^2}$ compared to that in the physical space. In the virtual space, when the speed is greater than c_b , where c_b is phase velocity of light in the virtual space, CR would be excited. The corresponding speed limit of CR in the physical space is $c_b/\sqrt{1 + (\tau/\kappa)^2} = 0.90c_b$. We choose two different velocities, $0.81c_b$ and $0.98c_b$, in the physical space, to study the radiation excited by the electron passing through the cloak. The results are shown in Figs. 6.2 and 6.3. The electron moves along the line $y = 4\mu\text{m}$ in both cases. We separate the whole process into four small processes – the electron moves close to the cloak, the electron impinges on the cloak and travels in the left part of the cloak, the

electron passes through the interface of the cloak in the end leaves that separates the left and right part of the cloak, and in the end leaves the cloak. In both Figs. 6.2 and 6.3, we can see there is no self-excited radiation outside the cloak, as the velocities of the electron ($0.81c_b$ and $0.98c_b$) are below c_b . When the electron enters the cloak from the left boundary, TR is excited in both figures. We mark them as $T1$. From the figure, we can see the radiation wave front pattern forms a loop, and propagates to all directions. The TR in the physical space corresponds to the abrupt velocity change at point A in the virtual space (Fig. 6.2a). In Fig. 6.3b, when the electron with velocity $0.98c_b$ moves inside the cloak, two bright branches move forward following the electron, this is CR, marked as $C1$. The radiation pattern in Fig. 6.3 is the combination of TR and CR. As the electron continues to move forward, and crosses the interface line $x = 0$, another TR is excited for the left and right part of the cloak have different permittivities. In the virtual space, it corresponds to the sudden velocity change at point B in Fig. 6.1b. We mark the radiation as T2 in Figs. 6.2 and 6.3. At this time, the radiation moving downwards excited before reaches the bottom and is totally reflected. Though the left and right part of the cloak have different permittivities, they are impedance matched. Therefore, radiation will go from the left part to the right part without being affected by the material interface. The interface only affects the swift electron, and causes the TR. In Fig. 6.3c, two more branches of CR are excited, marked as $C2$. When the electron moves out of the cloak from the right boundary, TR is also excited which is marked as $T3$. This radiation comes from the sudden velocity change at point C in the virtual space in Fig. 6.1b.

Figure 6.2: The magnetic field of radiation emitted by an electron passing through the carpet cloak with the velocity $0.81c_b$, where c_b is the phase velocity of light in the background. The electron moves to different positions in a-d. T1, T2 and T3 are the emitted TR.

6.4 Experimental verification of radiation excited by the swift electron

We would like to verify the results analyzed above with a full-pfrequency component parameter implementable carpet cloak in reality. Though it is difficult to observe the radiation excited by a swift electron, we can demonstrate the results by a microwave experiment by monitoring only one frequency component. Eq. (6.4) shows the constitutive parameters of the cloak. Because of the 2D nature

Figure 6.3: The magnetic field of radiation emitted by an electron passing through the carpet cloak with the velocity $0.98c_b$, where c_b is the phase velocity of light in the background. The electron moves to different positions in a-d. T1, T2 and T3 are the emitted TR. C1 and C2 are the CR emitted by the electron inside the carpet cloak.

of the case discussed above, we need to construct μ_z, ε_x and ε_y only. We use a layered structure from homogeneous and isotropic materials to build the cloak, as shown in Fig. 6.4. We build the carpet cloak with air layer and Zn-Ni-Fe composite with $\varepsilon_2 = 12 + 0.24i$ and $\mu_2 = 2.4 + 0.09i$ [162]. The background is Teflon with $\varepsilon_1 = 2.1$ and $\mu_1 = 1$. The thickness for the air layer and the Zn-Ni-Fe composite layer are $d_1 = 1\text{cm}$ and $d_2 = 0.29\text{cm}$, respectively. With the thickness ratio of the air and the Zn-Ni-Fe composite being $1/0.29$, it is easy to get the

effective parameters for the layered structure in the $u-v-z$ orthogonal coordinates, $\varepsilon_u = 1.26 + 0.0006i$, $\varepsilon_v = 3.49 + 0.05i$ and $\mu_z = 1.32 + 0.02i$. The angle between the horizontal plane and the normal line to the layer is 55.2° . The left part and the right part are symmetric to each other. We verify the cloak effect by simulating a TM plane wave with 1GHz passing through the cloak. The results are shown in Fig. 6.4. Because of the loss from the Zn-Ni-Fe composite, the field of the plane wave on the right gets a little weak.

Figure 6.4: The designed carpet cloak with a layered structure.

We verify the excited radiation by letting a single frequency current Eq. (6.6) with frequency $f = 1\text{GHz}$ pass through the cloak. The single frequency current is realizable in the experiment by antenna array, of which the phase difference of adjacent dipoles represents the phase velocity of the single frequency current Eq. (6.6) [153, 154]. Figure 6.5a,b show the radiation angular distribution in the far field. The phase velocities in Fig. 6.5a and b are $0.81c_b$ and $0.98c_b$, respectively. The proposed experiment setup is shown in the inset of Fig. 6.5, the cloak is

Figure 6.5: The angular distribution of the scattering field along a semicircle far from the cloak excited by the current with frequency 1GHz, and the phase velocity a) $0.81c_b$ and b) $0.98c_b$.

scaled for demonstrate at 1GHz. The current passes through the cloak along the red dotted lines at $y = 0.4m$. The detector is placed at a far region. The red line comes from the radiation of the current passing through a perfect cloak with

the constitutive parameters defined in Eq. (6.4). The blue lines represent the radiation of the current passing through the layered structure cloak as described before. The profiles show several peculiarities of the radiation angular spectrum. One is that the radiation has greater intensity in the forward direction, which is also consistent with the TR theory developed before [119]. The second is that the total radiation intensity increases a lot as the phase velocity of the current changes from $0.81c_b$ to $0.98c_b$. That is mainly because in the second case, CR is excited. The third is that there are two peaks in Fig. 6.5b at about $\pi/8$ and $\pi/3$. Referring to Fig. 6.3, we find that the radiation at $\pi/8$ comes from the overlapped part of the upper branch of C1 and the lower branch of C2. The radiation peak at $\pi/3$ is mainly the combination of the lower branch of C1 and the upper branch of C2.

6.5 Conclusion

We study in this chapter the cloak detection by letting a swift electron passing through a carpet cloak. The cloak can guide the light rays around the hidden objects, but cannot guide the electrons. Therefore, when a swift electron passes through the carpet cloak, it will generate both Cherenkov radiation and TR, making the cloak visible. Full parameters carpet cloak has been realised in the experiment, therefore the theory studied in this chapter can be verified in the experiment. We propose the experiment setup in the end to verify our theory with one frequency current. The work has been published in **Sci. Rep.**, vol. 5, 10401, May 2015.

Summary

Swift electrons are excellent sources for the excitation of graphene plasmons (GPs), for their sizes are much smaller than the diffraction limit of light. The understanding of the dynamical interaction between swift electrons with graphene is vital for the application of GPs in the integrated plasmonic devices. We have shown the dynamical processes of the GPs excitation and propagation when swift electrons move parallel and perpendicular to a free-standing monolayer graphene. When a swift electron moves parallel to graphene, the GPs pattern with the Kelvin angle are excited. The GPs pattern is strongly related to the intrinsic electronic properties of graphene. We use the Kelvin model which was developed for ship-wakes to explain the mechanism of the pattern generation. At the boundary of the GPs pattern, waves accumulate generating the caustic phenomenon. For the first time, we discuss the caustic phenomenon of GPs with wave theory.

When a swift electron penetrates a free-standing monolayer graphene, radiation and GPs are excited because of transition radiation (TR). We show the dynamical process and calculate the time for the GPs launching. Because of the

large carrier concentration of graphene, a electron jet is excited after the swift electron passing through the graphene sheet, and the electron jet contributes to the GPs launching.

We also calculate the interaction of swift electrons with metal. When a swift electron passes from air to metal, radiation, surface plasmon polaritons (SPPs) and bulk plasmons are excited. There were controversy about the origin of the radiation and the SPPs – whether they came from the radiative bulk plasmons or TR. We use the dynamical process to show that both the radiative plasmons and TR contribute to the radiation and SPPs. The contribution from TR happens at a relatively short time (formation-time), while the contribution from radiative plasmons can last over a relative long period of time in low-loss metals.

Asymmetrical plasmonic Cherenkov radiation (CR) is excited by a swift and rotating dipole above graphene. The contribution from the helicity of the rotating dipole is well studied. When the dipole is fixed in position, the GPs excited is intensified at only one side, because of the interference from the two orthogonal dipole components. But when the dipole moves relative to the graphene, the Doppler effect would change the interference manner from the two dipole components. In the result, when the dipole has a relative large swift velocity, the plasmonic CR pattern generated will be strengthened on both sides.

In the last chapter, we study the radiation by a swift electron passing through a carpet cloak. The radiation is the combination of CR and TR. The radiation from the electron passing through cloak not only can work as a tool to detect the cloak, but also can reveal the properties of the cloak.

Future Works

1, Experimental verification of the theoretical studies in the thesis. We can verify the theories in the microwave frequency with the dipole arrays and metasurface simulating the swift sources and the free-standing monolayer graphene, respectively. Though the swift electrons and rotating dipoles excite continuous electromagnetic waves, we can use dipole arrays to model their single frequency components [17, 163]. In Chapter 2 and 3, the phenomenon is related to the long-wavelength ($q \ll k_F$, where q and k_F are the wavenumbers of GPs and the Fermi wavenumber, respectively) graphene plasmons dispersion law, $\omega_{gp} \propto \sqrt{q}$. The difficulty lies in finding a metasurface with the similar dispersion law. In Chapter 5, the swift dipole can be modelled by a circularly polarized laser incident at the nearly grazing angle onto a slit of a metasurface, which simulates the free-standing monolayer graphene.

2, The Smith-Purcell radiation excited by a swift electron traveling above the graphene-based or metal-based periodic structures. In our previous studies of the interaction between swift electrons with free-standing monolayer graphene or

metal, the two dimensional surface is homogeneous. In some real graphene-based or metal-based plasmonics, there are periodic structures. When a swift electron moves above the periodic structures, it will excite the Smith-Purcell radiation. The Smith-Purcell radiation can work as a light source, by coupling the swift electron's near-field to propagating waves. By analytical calculation, we can have a deeper understanding of how the radiations are excited, and thereby maximizing the radiation intensity.

3, The field enhancement excited by a swift electron passing by touching metal columns or metal spheres. It has been proved that the touching metal column or metal sphere can have light enhancement effect by incident electromagnetic waves. We want to analyze the field enhancement effect on the touching metal columns or metal spheres by swift electrons. The analytical derivation of the field excitation and propagation can let us have a deep and thorough physical insight of the field enhancement in the touching point. This model setup is quite challenging for the analytical derivation. At first, we need to derive the field coupling from a swift electron to a single metal column or metal sphere. Next, we let two metal columns or metal spheres move close to each other, can then calculate the coupling effect between the two columns or spheres.

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