Interference management in wireless networks with limited channel information at the transmitters

Anand, Kushal

2017

Anand, K. (2017). Interference management in wireless networks with limited channel information at the transmitters. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/69486>

<https://doi.org/10.32657/10356/69486>

Interference Management in Wireless Networks with Limited Channel Information at the Transmitters

KUSHAL ANAND

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2017

To My Parents

Acknowledgments

First of all, I would like to express my deep gratitude to my advisor and Guru, Professor Erry Gunawan for his valuable guidance and encouragement throughout the course of my PhD research. As my mentor, he was always there to provide his insightful inputs to my research problems. Thanks a lot for the constant encouragement, kindness and the many useful discussions. I am also very fortunate to have Professor Yong Liang Guan as my co-advisor. I am greatly indebted to him for giving me several interesting ideas in our various meetings and his constant encouragement to think something bigger.

Another person I would like to thank is Dr. Chui Chee Cheon from the DSO National Laboratories, Singapore for the various useful discussions and the critical comments which really helped me a lot. I am also grateful to the FSTD-DSO-grant (grant reference DSOCL14095) and the ONR-G grant (grant reference N62909-12-1-7015) for their support of my PhD study at NTU. Special thanks to Prof. Guan and Dr. Chui for constantly renewing my scholarship. Also, I am highly thankful to Prof Erry for supporting me financially in the last few months of my PhD study through his grant from the Ministry of Education.

Next, I would like to thank Professor Yuen Chau from the Singapore University of Technology and Design (SUTD) for being on my thesis advisory committee. Despite being busy and in a different university, he agreed to advise me during my research. I thank him for his useful comments during the various meetings at

Acknowledgments

NTU and at SUTD. Special thanks to Dr. Francois Chin from A*STAR, Singapore for giving me the opportunity to work on some really interesting topics in wireless communications apart from my main thesis research.

Thanks to all my wonderful friends and colleagues, especially, Professor Cenk M. Yetis, Dr. Liu Zilong, Dr. Yong Zeng, Dr. Xiaoli, Praveen, Dr. Nguyen Thanh Hieu and Dr. Kun Zhong for the many useful suggestions and brainstorming sessions. I would also like to sincerely thank my friends Dr. Rajendra, Ankit, Dr. Akshay, Dr. Arun, Ravi Kishore, Ian, Dr. Sibtain, Dr. Kaushik and Munir who always helped and supported me during the duration of my PhD. Thanks to the administrative and technical staff of my Lab, Ng Teng Kwee and Thida Than for their immense help and support in administrative matters.

Finally, I would like to mention special gratitude to my parents Sri Balanand and Smt. Kalpana, my wife Sonal, my brother Kunal and my entire family for their love, patience and encouragement through all these years. This thesis is dedicated to my parents. Nothing could have been possible without their unconditional love and support.

Kushal Anand

August 2016, Singapore

Contents

Acknowledgments	i
Contents	iii
Abstract	viii
List of figures	xi
List of tables	xv
List of abbreviations	xvi
List of symbols	xviii
1 Introduction	1
1.1 Motivation	1
1.2 Objectives	4
1.3 Major Contributions of the work	6
1.4 Organization of the work	9
2 Literature Survey	11

Contents

2.1	Interference and X channels	12
2.1.1	Interference channels	12
2.1.2	X channels	13
2.2	Interference Management in Wireless Networks	14
2.2.1	Interference Alignment with Perfect CSIT	15
2.2.2	Interference Alignment with Imperfect CSIT	17
2.2.2.1	IA with channel feedback errors	18
2.2.2.2	IA with delayed CSIT	21
2.2.2.3	Blind Interference Alignment	22
2.2.3	Relay-aided interference management with no CSIT	24
3	On Degrees of Freedom of wireless networks with hybrid CSI at the transmitters	28
3.1	DoF of SISO-X channel with hybrid CSIT	30
3.1.1	System Model	30
3.1.2	Main results: Achievable DoFs	32
3.1.3	Proofs of the main results	32
3.1.3.1	Proof of Theorem 3.1	32
3.1.3.2	Proof of Theorem 3.2	37
3.1.4	Discussion and Comments	39
3.1.4.1	Scalability	39
3.1.4.2	Channel feedback overhead and Alternating CSIT framework	40
3.1.4.3	Computational Complexity	43

3.1.4.4	Number of Channel uses	44
3.1.5	Summary of Section 3.1	44
3.2	DoF of 4-user SISO-IC with hybrid CSIT	45
3.2.1	System Model	46
3.2.2	Main Result	47
3.2.3	Discussion and Comments	55
3.2.3.1	Channel feedback overhead	55
3.2.3.2	Computational complexity	56
3.2.4	Summary of Section 3.2	56
3.3	A semi-blind interference alignment scheme	57
3.3.1	System model	58
3.3.2	Main results	59
3.3.3	Proofs of the main results	60
3.3.3.1	Two-user 2×1 MISO-BC: Proof of Corollary 3.1	60
3.3.3.2	Proof of Theorem 3.4	63
3.3.3.3	Proof of Corollary 3.2	66
3.3.3.4	Proof of Corollary 3.3	67
3.3.4	Summary of Section 3.3	68
4	Finite-SNR Precoder Design for the Interference Relay Channels	
	without CSI at the source Transmitters	69
4.1	System Model	71
4.2	Precoder designs at the relay	73
4.2.1	The MSMSE precoder and decoder designs	73

Contents

4.2.2	Computational complexity	78
4.2.3	Convergence of the MSMSE algorithm	79
4.2.4	Relay power control and Scaled IA initialization	80
4.2.4.1	Power control for IA precoder design	81
4.2.4.2	Scaled-IA precoder initialization	82
4.2.5	Weighted sum rate maximizing precoder design	84
4.3	Numerical results	86
4.4	Summary	97
5	Precoder Designs for the relay-aided X Channel without source CSI	99
5.1	System model	101
5.2	IA feasibility conditions for Relay-aided MIMO-X channels	103
5.3	Relay-aided SISO-X channel (SISO-XRC)	107
5.3.1	System model for SISO-XRC	107
5.3.2	Power constraint at the relay	109
5.4	IA under limited power at the relay	110
5.5	MMSE Precoder design	114
5.5.1	Precoder design	114
5.5.2	Precoder initialization and power constraint issues	121
5.5.3	Computational complexity	123
5.6	Sum-rate and BER simulation Results	124
5.7	Summary	132

6	Dual-hop Interference Channels with Limited Channel Information	135
6.1	System description	137
6.1.1	System model	137
6.1.2	Channel feedback model	139
6.2	Feasibility conditions and Interference neutralization (with perfect CSI)	141
6.3	Power scaling law	144
6.4	Rate loss analysis	146
6.4.1	Rate loss upper bound	146
6.4.2	Bit scaling law	149
6.4.3	Adaptive bit allocation	151
6.5	Robust MMSE precoder design with imperfect CSI	152
6.5.1	Computational complexity of the precoder design	157
6.6	Numerical results and discussion	157
6.7	Summary	164
7	Conclusions and Future Work	170
7.1	Conclusion	170
7.2	Future Work	174
	List of publications	178
	Bibliography	179

Abstract

In this thesis, we investigate various methods of interference mitigation techniques in wireless networks under limited or imperfect channel state information (CSI) at the transmitters (CSIT). We consider various useful networks models, especially the interference channels (ICs), the X channels (XCs) and a special case of broadcast channel. Since relays can help in greatly reducing the channel feedback overhead to the source transmitters, we also consider relay-aided interference and X channels (without any CSI feedback to the source transmitters) where the relay aids in interference alignment (IA) or interference neutralization (IN) to maximize the network degrees of freedom (DoF).

First of all, we consider the single-input-single-output (SISO) ICs and XCs (without relays) under hybrid CSIT framework where each transmitter has access to delayed global and limited amount of local instantaneous CSI. Construction schemes are proposed for these networks which achieve higher DoF than the completely delayed-CSIT based schemes using much smaller coding lengths. For the three and four-user SISO-IC, we design systematic space-time coding which can achieve both secure as well as higher DoF as compared to the existing schemes.

Next, we propose a semi-blind interference alignment scheme for the broadcast network where one receiver has a reconfigurable antenna capable of creating staggered channel patterns while all other receivers are normal single antenna nodes. We propose a coding scheme for this model that attains a network DoF (optimal

DoF in certain cases) which is higher than that achieved by TDMA, and requires reasonable coding lengths which is suitable for time-varying channels.

In our third and fourth works, we design digital precoding algorithms for the relay-aided IC and XC respectively where the source transmitters are completely blind, and the precoding schemes are designed based on the CSI which is available only at the relay. For the relay-aided IC, we design the precoder at the relay and the receive filters at the receivers using the minimum-sum mean square error (MMSE) and the sum-rate maximization criteria. Suitable initialization method is proposed for the iterative precoder designs which not only achieves better BER performance than the IA algorithm but also attains the maximum network DoF while consuming much lesser relay power.

Next, for the relay-assisted multi-input-multi-output (MIMO) X channel with no or limited CSI at the source transmitters, we derive the number of antennas required at the relay to achieve the maximum network DoF using interference alignment. Our proposed DoF achievability scheme attains the maximum network DoF for both time-varying and time-invariant channels and under arbitrarily low power at the relay node. Finally, the MMSE precoders are derived for the case of relay-aided K -user SISO-X channel which show much better BER performance than the conventional IA precoder even under limited relay power.

In our final work, we consider a dual-hop SISO interference channel aided by multiple relays where direct links do not exist between the transmitters and the receivers. This type of network is applicable to the situations where the relays aid in range and coverage extension. Here, we consider no CSI feedback to the source transmitters and imperfect (quantized) CSI feedback to the relays based on which the relays perform the precoding design. We derive the interference neutralization feasibility conditions and show that in order to attain the maximum network DoF, the overall relay power must scale at a certain minimum rate with respect to the source transmitters' powers.

Abstract

We derive the rate-loss upper bound for this type of network due to the quantized channel information at the relays. Based on our analysis, we propose bit scaling law and an adaptive bit allocation strategy for channel feedback by the relays and the receivers so that the overall rate loss is minimized. Finally, we propose a stochastically robust MMSE precoder which exploits the adaptive bit allocation and the corresponding channel quantization error statistics to significantly improve the network sum-rate compared to all other algorithms under limited channel resolution.

List of Figures

2.1	K -user Interference channel	12
2.2	$A \times B$ X channel	12
2.3	Interference alignment for a 3-user interference channel	16
2.4	Limited CSI feedback model in which the k^{th} RX quantizes its local links, i.e., $Q(h_{kj}), j \in \{1, 2\}$ and feeds back to its TX. The TXs share the channels using a backhaul link. The solid yellow and the dashed red lines denote the desired and the interfering links respectively. . .	19
2.5	Relay-aided interference channel with direct links between the TXs and the RXs	26
2.6	Dual-hop Relay-aided interference channel (no direct links between the TXs and the RXs)	26
3.1	A three-user SISO-X network with hybrid (delayed and local instantaneous) CSIT. Feedback is shown for 1 st user only for clarity. . . .	31
3.2	DoF scaling with the number of users for various schemes	40
3.3	CSI feedback in the 3-user SISO-X channel for <i>Scheme-2</i> . N, D and P beside the RXs denote no feedback, delayed feedback (i.e., the CSI fed back in the n^{th} time-slot will be used for precoding after “n”), and instantaneous feedback respectively.	42
3.4	Fraction of time that a receiver assumes a particular channel state (N, D or P) for a k -user ($k \in \mathcal{K}$) SISO-X channel for <i>Scheme-2</i> . . .	43
3.5	A 2-user MISO-BC with one supernode and one naïve receiver. . . .	57

List of Figures

3.6	Channel staggering pattern due to antenna switching at the supernode in Theorem 3.4.	64
4.1	A relay-aided SISO interference channel with direct link between the transmitters and receivers	72
4.2	Sum-rate comparison of various algorithms for a three-user relay-aided SISO-IC.	88
4.3	Uncoded BER performances of the algorithms for various constellation sizes.	90
4.4	Data rate vs. uncoded BER performances of the algorithms.	92
4.5	Sum-rate performances of algorithms vs. number of users at various SNR values.	93
4.6	Sum rates of algorithms in asymmetric network when the relay is close to the receivers.	94
4.7	CDFs of the power consumed at the relay for a symmetric network. Total available power at the relay, P_t is unity or 0 dB for MSMSE algorithm.	95
4.8	BER performances of MSMSE and IA algorithms for different values of scaled-IA initializations. 16-QAM modulation is used.	96
4.9	Sum rate convergence behaviour (averaged over 100 channel realizations) for MSMSE(blue dashed) and WSR(red solid) algorithms, for various SNR values and for IA and random precoder initializations. The IA and random initializers are marked in the figure for 30 dB and 40 dB, they are not shown for 10 and 20 dB as their sum rate values are very close.	97
5.1	A K -user X channel aided by a MIMO relay	101
5.2	SMSE convergence behaviour for the MMSE algorithm at 21.76 dB total SNR	121

5.3	Sum rate comparison showing the effectiveness of IA-LFB algorithm for a three-user “symmetric” SISO-XRC. Here, “symmetric” implies equal average link strengths of the desired and interfering links at a user.	126
5.4	Sum rate comparison for a three-user SISO-XRC.	128
5.5	Power consumed at the relay due to various algorithms	130
5.6	BER comparison for a three-user SISO-XRC. QPSK modulation is used.	131
5.7	BER performance of the algorithms (QPSK modulation is used), and power savings at the TXs in the 4 th and 5 th time slots (blue squares with values on the right y-axis), as a function of the total relay power. Maximum allowable total source transmit power is 31.76 dB.	132
6.1	A dual-hop SISO interference channel aided by multiple MIMO relays. Prior to data transmission, the relay and the receivers estimate their local channels perfectly, quantize the channel directions, and feedback the quantized vector channel direction and scalar channel gains to the central station (CS). The CS computes the relay precoders based on quantized channels, and broadcasts the precoders to all the relays.	137
6.2	Sum rate for a $3 \times 2 \times 3$ dual-hop SISO-IC. Each MIMO relay has 2 antennas. P is the source TX’s power and P_t is the maximum power available at the relay.	146
6.3	Rate-loss upper bound vs. Actual rate loss due to channel quantization	150
6.4	A $3 \times 2 \times 3$ dual-hop SISO-IC. The path loss values are given as: $\alpha_{11}^{(h)}(T_1 X_1) = 1$, $\alpha_{12}^{(h)}(T_2 X_1) = 0.001$, $\alpha_{13}^{(h)}(T_3 X_1) = 0.001$, $\alpha_{21}^{(h)}(T_1 X_2) = 0.001$, $\alpha_{22}^{(h)}(T_2 X_2) = 1$, $\alpha_{23}^{(h)}(T_3 X_2) = 1$, $\alpha_{11}^{(g)}(X_1 R_1) = 1$, $\alpha_{21}^{(g)}(X_1 R_2) = 1$, $\alpha_{31}^{(g)}(X_1 R_3) = 0.01$, $\alpha_{12}^{(g)}(X_2 R_1) = 0.01$, $\alpha_{22}^{(g)}(X_2 R_2) = 1$, $\alpha_{32}^{(g)}(X_2 R_3) = 1$	158
6.5	Sum rate for a $3 \times 2 \times 3$ dual-hop SISO-IC with quantized CSI based precoding	162

List of Figures

6.6	A $4 \times 2 \times 4$ dual-hop SISO-IC with each relay having three antennas.	162
6.7	Sum-rate performances for a $4 \times 2 \times 4$ dual-hop SISO-IC for various precoding schemes	163

List of Tables

3.1	Comparison of DoF and channel uses of various schemes	44
3.2	SBIA coding for two-user $K \times 1$ MISO-BC	64

List of Abbreviations

BCD	Block coordinate descent
BIA	Blind interference alignment
CDI	Channel direction information
CS	Central Station
CSI	Channel state information
CSIT	Channel state information at the transmitters
DoF	Degree(s) of freedom
FDMA	Frequency division multiple access
i.i.d.	independent and identically distributed
IA	Interference alignment
IA-LFB	Interference alignment with limited feedback
IA-PC	Interference alignment with power control
IC	Interference channels
IN	Interference neutralization
IRC	Interference relay channels
LI-CSIT	Local instantaneous CSIT
MIMO	Multi-input-multi-output
MIMO-IC	Multi-input-multi-output interference channels
MIMO-XC	Multi-input-multi-output X channels
MISO	Multi-input-single-output
MISO-BC	Multi-input-single-output broadcast channels

List of Abbreviations

MISO-IC	Multi-input-single-output interference channels
MSE	Mean-square-error
MSMSE	Minimum-sum-mean-square-error
PIN	Partial interference nulling
RX	Receiver
SBIA	Semi-blind interference alignment
sDoF	Secure degrees of freedom
SINR	Signal-to-interference and noise ratio
SISO	Single-input-single-output
SISO-IC	Single-input-single-output interference channels
SISO-IRC	Single-input-single-output interference relay channels
SISO-XC	Single-input-single-output X channels
SISO-XRC	Single-input-single-output X relay channels
SNR	Signal-to-noise-ratio
TDMA	Time division multiple access
TX	Transmitter
WSR	Weighted sum rate
WSMSE	Weighted sum mean square error
XC	X channels
XRC	X relay channels

List of Symbols

$\mathcal{C}_{\Sigma}(SNR)$	Total network capacity as a function of SNR.
$\mathcal{O}(1)$	Big-O Landau notation
$o(g(x))$	Little-o Landau notation
$\mathbf{X}$	matrix.
$\mathbf{x}$	vector.
x	scalar.
$\mathbb{C}$	set of complex numbers.
$\mathbb{R}$	set of real numbers.
$\mathbb{N}$	set of natural numbers.
$\min.$	minimum operator.
$\max. \text{ or } \text{maximize}$	maximum operator.
$\text{span}(\mathbf{X})$	subspace spanned by the columns of the matrix $\mathbf{X}$.
$\text{null}(\mathbf{X})$	null space of the matrix $\mathbf{X}$.
$\mathbf{X}^{-1}$	inverse of the square matrix $\mathbf{X}$.
$(\cdot)^H$	complex conjugate of a matrix or vector.
$(\cdot)^T$	transpose of a matrix or vector.
$\sim$	distributed as.
$\mathcal{CN}(a, b)$	complex Gaussian distribution with mean a and variance b.
$\ \mathbf{x}\ \text{ or } \ \mathbf{x}\ _2$	two-norm of the vector $\mathbf{x}$.
$\mathbb{E} \{ \cdot \}$	expectation operator.

List of Symbols

$rank(\mathbf{X})$	rank of the matrix $\mathbf{X}$.
$Tr(\mathbf{X})$	trace of the matrix $\mathbf{X}$.
$\mathbf{I}_N$	identity matrix of dimension N .
$V^{min}(\mathbf{X})$	eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{X}$.
$vec(\mathbf{X})$	operator which stacks the columns of $\mathbf{X}$ into a long column vector.
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product between the matrices $\mathbf{A}$ and $\mathbf{B}$.
$\mathcal{R}\{x\}$	real part of the variable x .
$u^{[k \mathcal{S};j]}$	In the context of XC, denotes a symbol transmitted by TX- k , available at RX- j , and intended for RXs in the set $\mathcal{S}$.
$u^{[k \mathcal{S}]}$	In the context of XC, denotes a symbol transmitted by TX- k , and intended for RXs in the set $\mathcal{S}$.
$u_n^{[k j]}$	In the context of XC, denotes the n^{th} symbol transmitted by TX- k and intended for RX- j .
$u^{[k k,j]}$	In the context of IC, denotes a symbol transmitted by TX- k , intended for RX- k but decodable at both RXs k (desired RX) and j (undesired RX).
$u_j^{(k)}$	In the context of IC, denotes the j^{th} private symbol of the k^{th} user.
$\mathcal{B}(x, y)$	Beta function defined as $\int_0^1 t^{x-1} (1-t)^{y-1} dt$
$\Gamma(t)$	Gamma function defined as $\int_0^\infty x^{t-1} \exp^{-x} dx$

Note - The symbols $u^{[k|\mathcal{S};j]}$, $u^{[k|\mathcal{S}]}$, $u_n^{[k|j]}$, $u^{[k|k,j]}$ and $u_j^{(k)}$ are found in Chapter 3 only whereas the symbols $\mathcal{B}(x, y)$ and $\Gamma(t)$ are found in Chapter 6 only. Rest of the above mentioned symbols are used throughout the thesis.

Chapter 1

Introduction

1.1 Motivation

Interference is a major bottleneck in the wireless networks which can severely limit the multiplexing gain or degrees of freedom (also called DoF or equivalently, the number of interference-free data streams) of the network. With the proliferation of wireless devices and network densification in the recent and coming years, interference management in the multi-user wireless networks becomes a very important and challenging problem. To this end, interference alignment (IA) based coordinated beamforming was proposed for the two-user multi-input-multi-output X (MIMO-X) channels [1] where every transmitter has to send a message to every receiver in the network. A major breakthrough was reported in [2] where IA was proposed as a systematic beamforming scheme which can attain the DoF outer bound [3] of $\frac{K}{2}$ for the K -user interference channel (IC).

However, like other beamforming methods, IA requires the complete channel state information (CSI) of the network at every transmitter. While the lack of channel state information at the transmitter (CSIT) does not incur any loss of DoF in single-user systems, this loss is quite pronounced in the multi-user systems [4]. Thus, in a multi-user system such as an IC, the DoF gains of IA cannot

be realized unless *global* CSI is available at all the transmitters with a *very high* accuracy [4–6]. In practice, the channel feedback link from the receivers to the transmitters is bandwidth-limited and therefore, accurate feedback of the CSI is not possible [7], leading to DoF or sum-rate degradation.

To this end, several algorithms have been proposed [6, 8–17] to increase the system throughput under the effects of limited channel information at the transmitters. However, unless the CSI resolution is sufficiently high, there is still significant DoF loss. In order to reduce the CSI feedback to the transmitters, blind interference alignment (BIA) was proposed in [18–21] which obviates the need of short-term CSI feedback to the transmitters. Although a promising approach, BIA requires the use of extra hardware in the form of reconfigurable antennas at *every* receiver which is practically difficult.

Another remarkable result towards interference alignment and cancelation under imperfect CSIT was reported in [22] for the memoryless multi-input-single-output broadcast channel (MISO-BC) where it was shown that even using completely outdated channel information, significant DoF gains can be obtained as compared to the conventional orthogonal transmission schemes such as time-division-multiple-access (TDMA) and frequency-division-multiple-access (FDMA). Unfortunately, when the delayed CSI based precoding scheme is applied to the networks with distributed transmitters such as ICs and X channels (XCs) [23], it takes very long coding lengths to get significant DoF gains even for small and moderate size networks which make the coding approach impractical.

Keeping the above points in mind, we are motivated to find practical transmission schemes for the wireless networks which can increase the network throughput even with imperfect CSI. Imperfect CSI can broadly encompass several channel imperfections such as channel errors due to estimation, quantization, noisy feedback and delayed CSI availability. In this thesis, we are going to address some of the important CSI imperfections in the context of various wireless networks.

One important question that we ask is: Can we get significant DoF gains in the ICs and XCs with somewhat less restricted yet practical CSI requirements, for example, when the transmitter does not only have completely outdated CSI but also some amount of instantaneous and perfect CSI available to it *locally* for a fraction of the channel coherence time? Note that some amount of instantaneous and perfect CSI can be easily available to the transmitters through their local links. This type of practical CSI feedback scenario is referred to as the hybrid CSI feedback. While some works have been reported recently for the DoF of broadcast channels under hybrid CSI [24–27], very limited works exist on hybrid CSI-based DoF for the distributed networks such as ICs and XCs [28]. Moreover, the methods proposed in these works require long coding lengths and do not provide secure DoF. Thus, obtaining secure DoFs using shorter coding lengths under hybrid CSIT for the XCs and ICs is the motivation for the first chapter of the thesis.

Next, we propose a new hybrid CSI framework for the MISO-BC in which one of the receivers is special because it chooses its channel values from a finite set and feeds back its CSI to the transmitter in a delayed (or instantaneous) manner. All other receivers are normal single-antenna receivers with *no* feedback to the transmitter. Thus, in this case, the CSI heterogeneity is across the users and not across time. The main motivation for the above model and scheme comes from the blind IA (BIA) model in [19] which required every receiver to possess reconfigurable antennas which is difficult in practice. This led us to the question whether DoF gains can still be obtained if only one (or few) receiver(s) possesses reconfigurable antennas with the added feature such as feedback to the transmitter from this (these) receiver(s). We show that our proposed semi-blind IA (SBIA) scheme indeed provides DoF gains under the proposed hybrid CSIT framework.

Yet another approach to improve the network DoF for any generic SISO-IC is possible by the use of relays as first proposed in [29]. Note that in the relay-aided IC, high quality CSI can be acquired at a central relay whereas no CSI feedback is

needed at the source transmitters. However, the scheme in [29] is DoF-optimal at high SNR only and not in the finite SNR regions. In this context, we are motivated to design finite-SNR precoders for the relay-aided IC with blind transmitters so that the system throughput is increased in the more practical low-mid SNR regime.

A closely related network model to the relay-aided ICs is the relay-aided X channels. However, unlike the ICs, maximum DoF gains using IA was possible only in *time-varying* SISO-X channels, and it was not known whether we can apply IA precoding to the *time-invariant* case also. In our work, we address this issue by designing a systematic DoF achievability scheme for the symmetric MIMO-XC which, unlike earlier works, can attain the maximum network DoF in both time-varying and time-invariant channels and under arbitrarily low power at the relay. Further, on similar lines as the relay-aided SISO-IC, finite-SNR precoder designs are proposed for the generic K -user SISO-XCs.

Finally, we consider digital precoding for the multiple relay-aided dual-hop interference channels where the source transmitters are blind but imperfect CSI (due to channel quantization) based precoding is done at the relays. While several transmission schemes (such as [30] and the references therein) have been proposed for the relay-aided IC, they mostly assume perfect CSI at the transmitters. However, in practice, the CSI at the relay can be imperfect due to channel estimation and quantization errors. This motivates us to address the general relay-aided IC with quantized CSI feedback to the relays.

1.2 Objectives

The objectives of this work are summarized below.

1. Degrees of freedom (DoF) of wireless networks with hybrid CSIT: In this work, our first objective is to analyse the DoF of the single-input-single-output IC (SISO-IC) and single-input-single-output XC (SISO-XC) with hy-

brid CSIT. While hybrid CSIT models have only recently gained attention for the broadcast channels, their analysis for the ICs and XCs has been largely unexplored. Our work provides some important theoretical construction schemes in this direction.

Our second objective is to investigate hybrid CSIT for the MISO-BC with CSI heterogeneity across users (instead of time). This hybrid CSIT model seems similar to that in [25] with one significant difference that one of the receivers possesses reconfigurable antennas that restricts it to choose its channel values from a finite set only instead of continuous channel values in [25]. Channel fluctuations due to reconfigurable antennas was first proposed for the blind IA scheme in [19] but it required reconfigurable antennas at every receiver which is practically difficult. Our hybrid CSIT is practically more viable where we attempt to answer the question whether DoF gains can be obtained with reconfigurable antennas only at one special receiver (with a feedback link to the transmitter) while all other receivers are naïve single-antenna nodes with *no* feedback to the transmitter.

2. Finite-SNR precoder design for relay-aided IC: In this work, our objective is to design precoder for the ICs with completely blind transmitters, and aided by a multi-antenna relay. While IA precoder has already been proposed for such networks in [31], design of precoders in the practical SNR regions is more imperative. The main objective is to obtain higher throughput performance and better bit error rate (BER) performance under limited relay power and in the low-mid SNR operating regions of the network.
3. Feasibility conditions and precoder designs for the relay-aided X channels: In this work, our objectives are two-fold. First, to investigate whether relay-aided X channels can achieve the maximum network DoF when the channel is time-invariant. Note that while the maximum network DoF can be attained

for the time-varying X channels (both with and without relays), the time-invariant case was still an open issue which we address in this thesis. Our second objective is to design the finite-SNR precoders for the relay-aided XCs which can improve the network sum-rate in the low-mid SNR regions under arbitrarily low power at the relay.

4. Relay-aided dual-hop interference channels with imperfect CSI: This work is a natural extension of our previous works where our objective is to investigate beamforming schemes for the dual-hop interference channels assisted by multiple half-duplex MIMO relays. We wish to design precoders at the relay with imperfect (specifically, quantized) CSI at the relays and no CSI at the source transmitters. Feasibility conditions, relay power scaling laws and feedback bit scaling laws are investigated which, to the best of our knowledge, have not been considered for this type of channel before.

1.3 Major Contributions of the work

1. We investigate the DoF of SISO XCs and SISO ICs under hybrid CSI at the transmitters. We show that with the help of some local instantaneous CSI, in addition to delayed CSIT, we can obtain a DoF greater than that achievable using existing results based on completely delayed CSIT. Moreover, our proposed construction schemes require much smaller coding lengths as compared to the completely delayed CSIT scenario and is more suitable for fast time-varying channels.

We propose a new hybrid CSI framework for MISO-BC where only one of the receivers possesses reconfigurable antennas and has a feedback mechanism to the transmitter. A semi-blind interference alignment (SBIA) scheme is proposed for this model which achieves a DoF of $\frac{4}{3}$ and $\frac{3}{2}$ for the two-user and three-user MISO-BC respectively. This scheme is also readily applicable

to the $K \times 2$ X channel where the optimal DoF of $\frac{2K}{K+1}$ is attained.

2. We consider a SISO-IC aided by a MIMO relay which obviates the need of CSI feedback to the source transmitters, thereby reducing the channel feedback overhead. For this network, we propose a finite SNR based relay precoder design that seeks to minimize the sum mean square error of the received symbols of all the users in the network. The proposed precoder achieves higher throughput in the low-mid SNR regions, and also attains the maximum DoF in the high SNR region. We also propose a scaled-IA precoder initialization for our iterative precoder design which significantly saves power at the relays and yet attains better BER than the conventional IA precoder which operates without any power control at the relays.

Finally, we propose a weighted sum-rate maximizing precoder that seeks to maximize the system throughput when different users are given different priorities. By setting equal priorities for all the users, a sum-rate maximizing relay transmit filter is obtained which shows higher throughput compared to the minimum-sum mean square error (MMSE) filter and the conventional IA filter in all the SNR regions.

3. We consider a relay-aided X channel which is a more general network than the IC where every transmitter has a message for every receiver. Here, first of all, we investigate the IA feasibility conditions and show that for a K -user MIMO-XC with M antennas at every transmitter and every receiver, $(K-1)M$ antennas at the relay are sufficient to attain the maximum network DoF for both time-invariant and time-varying channels.

Unlike the relay-aided SISO-IC as mentioned above, under limited power constraint at the relay, alignment of interference is not possible in the relay-aided XC. To solve this problem, we propose a limited-feedback IA algorithm (called the “IA-LFB” algorithm) which can attain IA at the cost of minimal

feedback overhead to the source transmitters. Thus, the maximum network DoF can be attained even under arbitrarily low relay power.

We propose MMSE precoder and decoder designs for a generic K -user SISO-XC aided by a MIMO relay with $(K-1)$ antennas which significantly improves the sum-rate and the BER over a wide SNR range as compared to conventional IA precoding. Moreover, we propose to use the IA-LFB as the initializer for the iterative MMSE algorithm which not only improves the sum-rate and BER of the network but also retains the network DoF.

4. We investigate K -user dual-hop interference networks with multiple relays, no CSI at the source transmitters, and imperfect CSI based precoding at the relays. Firstly, we provide the sufficient conditions for interference neutralization (IN) for our network model and we show that unlike broadcast and interference channels, channel gains (and not only channel directions) affect the network DoF significantly.

Next, we propose the so-called *power scaling law* according to which, in addition to IN, the overall sum power of all the relays must scale with the source transmitters' powers at a certain rate so that the maximum network DoF is attained.

We derive the rate-loss upper bound for the aforementioned dual-hop network due to imperfect CSI feedback and provide the *bit scaling law* which tells how the feedback bits at the relays and the receivers should scale with the source transmit SNR so that the maximum network DoF is attained. Based on the rate-loss analysis, we propose a network-adaptive feedback bit allocation method. Precoders based on this proposed bit allocation show higher sum-rate (especially in the high SNR region) than the ones based on the naïve approach in which every link is allocated equal number of feedback bits.

Finally, we design *stochastically robust* MMSE precoder based on quantized CSI which improves the network sum-rate significantly as compared to the non-robust precoders.

1.4 Organization of the work

The organization of the remainder of the thesis is as follows. Since interference alignment (IA) underlies many of the proposed transmission schemes in this thesis, Chapter 2 provides a literature survey of IA schemes for various wireless networks. We begin our discussion with IA under perfect and global CSI availability at the transmitters. Next, we discuss various kinds of CSI imperfections and the various methods in literature to deal with them such as reduced CSI and delayed CSI based precoding, blind IA and relay-aided transmission techniques.

In Chapter 3, first, we propose the DoF achievability schemes for the SISO-XC and SISO-IC when hybrid CSI, i.e., a mix of delayed and local instantaneous CSI is available to the transmitters. Two kinds of local feedback are considered for the 3-user SISO-XC and one of them is found to be scalable to the K -user case also. For the SISO-IC, we propose systematic coding schemes for the 4-user case with hybrid CSI feedback. In fact, our coding scheme can be applied to both 4 and 3-user SISO-IC, and can achieve *secure* as well as higher DoF as compared to the delayed-CSIT based schemes.

Next, we consider a hybrid CSI framework for the MISO-BC with CSI heterogeneity across users. We propose semi-blind IA-based space-time coding for our proposed framework where only one of the receivers possesses reconfigurable antennas and can feedback information to the transmitter. In other words, the transmitter receives CSI feedback from only one receiver while it is blind to the other receivers. The proposed method achieves higher DoF than the orthogonal transmission schemes using smaller coding lengths as compared to the conventional BIA.

In the next three chapters, we investigate the use of relaying to reduce the channel overhead in interference and X networks. In Chapter 4, we consider the SISO-IC aided by a MIMO relay and also with direct links between the transmitters and the receivers. Finite-SNR precoders are designed which achieve higher throughput and better bit error rates than the conventional IA precoders. We show that with appropriate precoder initialization for the proposed iterative precoder designs, significant relay power can be saved while still obtaining better BER performances than the IA precoders.

Chapter 5 discusses a more general X network aided by a MIMO relay. We discuss the feasibility conditions for IA on the MIMO-XC for both time-varying and time-invariant channels. We propose a limited feedback based precoder design which obtains the network DoF under any relay power constraint. Finally, MMSE precoders are designed with appropriate initialization techniques so that both the sum rate and the BER are improved in the low-mid SNR regions.

In Chapter 6, we further extend the network model in previous chapters to address the dual-hop interference channels assisted by multiple MIMO relays and no direct links between the transmitters and the receivers. First, we investigate the interference neutralization feasibility conditions, followed by power scaling law assuming perfect CSI at the relays. Next, we derive the rate-loss upper bound due to the limited resolution channel feedback. We propose an adaptive bit allocation method for channel quantization at the relays and the receivers so that the overall rate-loss is minimized. A stochastically robust MMSE precoder is designed which takes into account the channel quantization error statistics to improve the sum-rate of the network.

Finally, Chapter 7 discusses the concluding remarks and future research directions.

Chapter 2

Literature Survey

In this chapter, we describe briefly the state-of-the-art techniques on interference management in multi-user wireless networks where multiple transmissions occur over a common transmission medium. Multi-user wireless networks encompass various network types, notable among them being the broadcast channels (BCs), the interference channels (ICs) and the X channels (XCs). In this chapter, first, we review two very important wireless networks, namely, the ICs and the XCs which form the essential building blocks of various communication networks¹.

Next, we provide a comprehensive review of the interference management strategies in the wireless networks, in particular, interference alignment (IA) which, of late, has come up as a promising transmission technique that can achieve the optimal degrees of freedom (DoF or equivalently, multiplexing gain) of various wireless networks. Specifically, we focus on the application of IA to ICs with perfect CSI at the transmitters (CSIT).

However, in order to design IA beamforming, heavy channel overhead together with very high channel accuracy is desired, thereby rendering the application of IA difficult in practice [7]. We discuss the various research works in this direction

¹Note that most of the content in this thesis deals with beamforming strategies in IC and XC with and without relay(s).

which address IA under partial, imperfect or no CSIT. Finally, we review the relay-aided techniques that greatly help in reducing the CSI feedback overhead and help achieve the network DoF in ICs or XCs in finite number of channel uses without long symbol extensions.

2.1 Interference and X channels

2.1.1 Interference channels

Figure 2.1 shows a K -user single-input-single-output IC (SISO-IC) where each single-antenna transmitter transmits to its intended single-antenna receiver at the same time and at the same frequency band as the other users, and hence, creates interference to other users in the network. In short, IC is a network where several input terminals communicate with their respective output terminals on a common channel.

Figure 2.1: K -user Interference channel

Figure 2.2: $A \times B$ X channel

Capacity characterization of the ICs is a long standing problem that has been of interest to the researchers for more than three decades [32–39]. In fact, the exact capacity regions of even very simple SISO-IC models such as the two-user Gaussian SISO-IC is unresolved till now [38]. Among other works, capacity region to within a constant have been reported for the very special case of many-to-one and one-to-many Gaussian ICs [40]. Capacity characterization of multiple-input-multiple-output IC (MIMO-IC) even to within some constant number of bits is

unknown.

In recent years, much of the research focus has shifted towards one particular (yet important) scenario, i.e., the high-SNR case which ignores the effects of noise. Although high-SNR analysis does not yield the exact capacity results, it gives the first-order capacity characterization or degrees of freedom (DoF) of the network, and thus, provides very useful insights into the fundamental limits of the network.

For the K -user IC, the network capacity is defined as²

$$C_{\Sigma}(SNR) = d(\log(SNR)) + \mathcal{O}(\log(SNR)) \quad (2.1)$$

where $SNR = \frac{\text{Total transmit power of the network}}{\text{Noise variance at the receiver}}$. The pre-log factor in the capacity expression, d is the DoF of the network. The DoF is defined as the rate at which the network sum rate scales with the total SNR of the network at high SNR and is given as $DoF, d = \lim_{SNR \rightarrow \infty} \frac{C_{\Sigma}(SNR)}{\log(SNR)}$. Essentially, the DoF represents the number of interference-free data streams that can be transmitted over the network at high SNR. The DoF outer bound for the K -user SISO-IC was first mentioned in [3] but its achievability was reported in [2] using the novel IA beamforming scheme as reviewed briefly in Section 2.2.

2.1.2 X channels

Figure 2.2 shows an $A \times B$ SISO-X channel in which each of the A single-antenna transmitters sends a distinct message to each of the B single-antenna receivers. It is worth noting that the X channel is a more general form of wireless network that encompasses all communication scenarios in a single-hop distributed wireless network, i.e., BCs, multiple-access channels (MAC) and even ICs. Similar to the IC, capacity of the XC is also largely unexplored except in some special cases [41] or within a constant gap for a simple 2×2 XC [28].

²In fact, this definition applies to any wireless network.

Like the case of IC, DoF is a comparatively easier metric that characterizes the high-SNR performance of the XC. DoF of the SISO-XC was first reported in [1] for the two-user case where the idea of IA was first introduced. DoF region of the two-user MIMO-X channel was established in [42]. However, the DoF outer bound and its achievability for the general $A \times B$ SISO-XC was reported in [43] using IA as the underlying principle. Note that for small and moderate sized networks, the DoF of the symmetric XC (i.e., $A = B = K$), $\frac{K^2}{2K-1}$ is significantly higher than its K -user IC counterpart $\frac{K}{2}$ but as the network size grows (i.e., large K), this DoF gain disappears, i.e., $\lim_{K \rightarrow \infty} \frac{K^2}{(2K-1)} \rightarrow \frac{K}{2}$.

DoF of the general multi-input-multi-output XC (MIMO-XC) with each transmitter and receiver having different number of antennas is still an unsolved problem. For the MIMO-XC case, the best known DoF result so far has been reported for the $A \times B$ XC in which every node has M antennas where the DoF simply scales with M , i.e., a DoF of $\frac{MAB}{A+B-1}$ is achieved.

2.2 Interference Management in Wireless Networks

As mentioned in the previous section, DoF approximates the network capacity of interference or X networks at high SNR. However, at high SNR, the major challenge in characterizing the DoF lies in the effective interference management. Interference is a major bottleneck in the multi-user communication systems that arises naturally due to the broadcast and superposition nature of the wireless signals. Traditionally, interference can be dealt with using *interference avoidance*, i.e., orthogonalization of time/frequency resources (clearly, leading to DoF loss), or treating interference as noise (applicable in the *weak interference regime*). Another approach to handle interference is using *interference decoding* in which the interference is decoded along with the desired signals and is generally applicable

in the *strong or very strong* interference regime.

The main challenge in handling interference lies in the *moderate interference region* when the power of the interfering signals is roughly equal to the desired signals' power. The last scenario is becoming more common nowadays due to the heavy densification of networks. In this case, coordinated transmission among the users is necessary in order to mitigate the detrimental effects of interference. To handle the moderate interference scenario, a coordinated beamforming technique called interference alignment (IA) was proposed in [1, 2] which can attain the network DoF of interference and X networks.

2.2.1 Interference Alignment with Perfect CSIT

IA is a form of coordinated beamforming technique [1, 2] to manage interference at the users whereby all the interfering signals at the receiver are made to align in a minimum common subspace as seen by the receiver, so that maximum subspace is available at the receivers for the desired signals. The interfering signals can then be cancelled using an orthogonal receive filter. For example, consider the 1st receiver of the 3-user IC in Figure 2.3. Here, the unintended signals from the TXs 2 and 3, after passing through the channels $\mathbf{H}_{12}$ and $\mathbf{H}_{13}$ respectively, fall in the same subspace whereas the desired signal from the 1st TX falls in a different subspace. Since RX-1 has 2 subspaces (due to 2 spatial dimensions provided by the two antennas), it can easily apply *zero-forcing* receive filter to cancel the aligned interfering signals and extract the desired signal.

It is to be noted that IA was initially proposed for the time-varying or frequency-selective SISO-IC using asymptotic signal expansion or symbol extension using the so-called “signal-space IA” as discussed above. The concept of signal-space IA was extended to MIMO-IC together with *channel extension in time* which attains the optimal DoF for a large class of networks [44]. However, for the time-invariant IC, signal-space IA is successful to a certain extent only, particularly, when asymmetric

Figure 2.3: Interference alignment for a 3-user interference channel

complex signaling (ACS) [45, 46], together with symbol extension, is used which leads to a DoF of 1.2 for almost any K -user Gaussian SISO-IC. The ACS also leads to the optimal DoF of $\frac{4}{3}$ for the specific case of the two-user SISO-X channel. Unfortunately, ACS, in general, does not cause DoF scaling with the number of users in IC or XC as was possible using IA in the channels with time/frequency variations.

A new signaling scheme called the “signal-scale IA” (which makes use of lattice codes) was introduced in [39] to attain IA in the time-invariant channels. However, [39] and other later works [47–49] that used signal-scale IA, were restricted to specific cases only. Finally, in [50, 51], a signal-scale IA scheme based on number-theoretic properties of rational and irrational numbers was introduced which can attain the maximum network DoF for almost all cases of time-invariant ICs.

Note that signal-space IA exploits channel variations for alignment of interference, and therefore, requires coding over long channel extensions. On the other hand, although signal-scale IA can attain the DoF without channel extensions, it requires very precise CSI which is practically infeasible. So, a natural question is

whether IA is beneficial for the time-invariant (or slowly time-varying) channels without channel extensions if finite spatial (instead of time) dimensions (specifically, multiple antennas) are considered at the transmitters and the receivers. For this case, it was shown in [52, 53] that IA can indeed provide DoF benefits over traditional TDMA scheme but its DoF is bounded (except in some specific cases [54]) depending on the number of antennas. Further, it was reported in [52, 53, 55–58] that for finite number of antennas at the nodes, there is a constraint on the maximum number of users that can be accommodated in the system.

Practical IA and IA-inspired precoder and combiner designs for the time-invariant MIMO-IC which satisfy the feasibility constraints [52, 53, 55–58] were proposed in several works [54, 59–70] which targeted different objectives such as overall interference minimization and network sum-rate maximization. IA and IA-inspired algorithms with perfect CSIT were also designed for other network types such as the interfering broadcast channels [71, 72], the interfering multiple-access channels [73] and the general cellular networks [74, 75]. Broadly speaking, these works deal with transceiver design in distributed networks with IA as the underlying principle. Except for some special network configurations, most of these works propose iterative transceiver designs (where several iterations between transmit and receive filter updates are performed) to obtain locally optimal transmit and receive filters.

2.2.2 Interference Alignment with Imperfect CSIT

As discussed in the previous section, IA forms the core component of interference management in the multi-user wireless networks such as ICs and XCs. However, the main practical hindrance in achieving the DoF gain promised by IA is the lack of precise CSI availability at the transmitters. The imperfect CSIT may be due to the channel estimation errors at the receivers, channel quantization errors, delay in feedback from the receivers to the transmitters (which affects the

sum rate significantly for fast time-varying channels), and the limited bandwidth of the channel feedback link.

2.2.2.1 IA with channel feedback errors

While IA achieves the maximum network DoF, it does so at the cost of large channel feedback overhead to each of the distributed transmitters (both in IC or X channel) [7]. The feedback size grows dramatically with the increase in the network size and the number of users. Moreover, IA is highly sensitive to the CSIT resolution [4, 5]. In practice, due to limited CSIT, the designed beamformers are not perfect and there is significant interference leakage at the users. The leakage power increases as the transmitted power of the users increases, i.e., at high SNR. As a result, the sum rate of the network saturates to a constant value in the high SNR region, thereby making the DoF of the system to zero.

To this end, various transmission strategies have been designed [6–10, 13–15, 76–79] in order to increase the network throughput under limited channel resolution. Most of the works consider a finite rate feedback link from the receivers to the transmitters for feeding back the channel information as shown in Figure 2.4. Finite rate feedback schemes have been dealt with for the multi-user downlink networks in [16, 80–82], for the MISO-IC case in [11, 13, 76, 77] and for the MIMO-IC in [8, 9, 12]. Further, under a certain method of channel feedback scheme, it has been derived in [5] and [4] as to how much feedback bits should be used to quantize the CSI so that the network DoF is the same as the case with perfect CSIT. For example, in [4, 83], it has been shown that $(\min N, M)^2 K(RL - 1) \log(P)$ minimum number of bits must be required in order to quantize the CSI so that IA is obtained perfectly. Here, N and M are the number of antennas at the receivers and the transmitters respectively, P is the total transmit power of the network, K is the number of users in the network, L is the number of taps for the frequency-selective channel and R is defined as $R = \left\lfloor \frac{\max(N, M)}{\min(N, M)} \right\rfloor$ where $\lfloor \cdot \rfloor$ is the floor function.

Figure 2.4: Limited CSI feedback model in which the k^{th} RX quantizes its local links, i.e., $Q(h_{kj}), j \in \{1, 2\}$ and feeds back to its TX. The TXs share the channels using a backhaul link. The solid yellow and the dashed red lines denote the desired and the interfering links respectively.

The so-called bit scaling law which says how much feedback bits are required in order to maintain the network DoF, has also been derived for other topologies of interference networks with quantized CSIT [79]. It is apparent from the bit scaling law in [4] that the amount of feedback necessary is very large even for the small and moderate sized networks. It is worth noting that there are other directions of works [6, 15, 77] that intend to increase the system sum-rate in the low-mid SNR regions and under limited channel resolution (i.e., no feedback bit scaling applies). However, in these works, DoF of the network still remains close to zero (i.e., the sum-rate saturates at high SNR) as the feedback budget does not scale with the SNR.

In [79], several feedback topologies have been suggested, and the topology which reduces the feedback by the largest amount is suggested as the proposed method of feedback. For example, an efficient method is to consider a central unit which collects the CSI from all the receivers, computes the beamformers, quantizes them based on its limited feedback budget and then sends them to the respective

transmitters. In this approach, each transmitter need not know the CSI of all the links, and thus the feedback overhead at the transmitters is significantly reduced at the expense of a central CSI collecting node. In [9], sub-optimal algorithms for partitioning of users into groups has been suggested for large networks so as to reduce the overhead costs. In [6], a two-way approach involving both dynamic bit allocation to the channel links at the receivers, as well as transmit beamformer computation at the transmitters based on channel quantization error statistics, has been suggested in order to improve the system sum rate significantly. Performance analysis of IA with imperfect CSIT under different scenarios have been addressed in [84–87]. Other works such as [15] suggest “effective channel” creation at the receiver end using a suitable design of the receive filters. This effective channel, when quantized using the limited number of feedback bits at the receiver and fed back to the transmitters, gives higher throughput compared to the actual channel feedback schemes. However, a drawback of the scheme in [15] is its high computational complexity. Other approaches to reduce the channel feedback overhead using modified channel feedback have been mentioned in [88, 89].

Another line of work that deals with making IA practically feasible is the Opportunistic IA (OIA) [90–97]. OIA is particularly applicable in a cellular system where it exploits the inherent user diversity to select only those users to form an IC whose channel conditions satisfy the IA criteria. Note that OIA does not require complete CSI feedback from the users but a much lesser amount of feedback such as the user index or other smaller overhead. To this end, several OIA schemes have been suggested in [98, 99] which significantly reduce the feedback overhead compared to standard OIA schemes. Since they involve lesser feedback and are applicable to cellular networks where user diversity can be exploited, the OIA schemes are particularly promising. However, OIA requires multiple users in order to select the best users from the network.

Another challenge in IA implementation in networks is the case when the users

are mobile, i.e., the time-varying channels scenario. In this case, if the channels are varying fast, the channel becomes outdated by the time it is obtained by the transmitters. However, if the channel variations are slow enough, beamformer design can be done based on predictive algorithms for the slowly varying time-correlated channels. To this end, various predictive IA beamforming schemes have been suggested in literature such as the adaptive beam tracking for IA [100–102], and the Grassmannian differential limited feedback schemes for IA [12].

2.2.2.2 IA with delayed CSIT

If the channels are varying fast, CSI may become completely outdated soon. For this scenario, a remarkable scheme was proposed by Motahari et al in [22, 103] for the *memoryless* broadcast networks where it was surprisingly shown that DoF gains greater than unity can be obtained based on completely delayed CSIT. Specifically, for a fully time-uncorrelated K -user multiple-input-single-output broadcast channels (MISO-BC), a DoF of $\frac{K}{1+\frac{1}{2}+\frac{1}{3}+\dots+\frac{1}{K}}$ can be obtained. Inspired by [22], delayed CSIT was exploited in [104] to obtain a DoF of $\frac{9}{8}$ and $\frac{8}{7}$ for the three-user SISO-IC and two-user SISO-XC respectively. Further improvements were reported in [105] and [23] for the SISO-XC and SISO-IC. However, it is conjectured in [23] that the DoFs of the SISO-IC and XC with delayed CSIT are bounded due to the distributed nature of the transmitters. Moreover, the best known delayed-CSIT based schemes in [23] require extremely long coding lengths to attain slight DoF improvements over TDMA scheme. Other delayed-CSIT based results inspired from the scheme in [22] include [106, 107] and the references therein.

While completely delayed CSIT (for the case of temporally uncorrelated channels) and instantaneous CSIT are two different extremes, for the realistic case of temporally correlated channels, the DoF region can be further enlarged considerably as compared to [22] using a hybrid scheme that provides a trade-off between the scheme in [22] for the uncorrelated channels and the zero-forcing design based

on the channel correlation feedback such as in [108, 109]. Another type of hybrid CSIT configuration was considered in [26, 28, 110] where a combination of delayed CSIT with some local instantaneous CSIT (also referred to as moderately delayed CSIT if coding is done across several channel coherence blocks) can provide significant DoF benefits for some special cases of broadcast, interference and X networks.

While the above hybrid CSIT configurations considered hybrid CSI feedback over time (i.e., delayed CSI for a fraction of the channel coherence duration and instantaneous CSI for the remaining duration), [25] considered hybrid CSI across users, i.e., a receiver can feedback either delayed or instantaneous or no CSI at all. In [25], complete *linear* DoF characterization has been done for the three-user case with different CSI feedback possibilities by the different receivers. An interesting result was reported in [24] for the two-user MISO-BC where it was shown that when the CSI states at the receivers alternate (with a certain probability) between different states, i.e., delayed CSI, instantaneous CSI or no CSI, space-time coding can be done across these *alternating* states to obtain significant DoF gains. These DoF gains are strictly higher as compared to the case when the CSI feedback from the receivers assume static channel states. Similar results were reported for the two-user and K -user SISO-XC in [111] and [110] respectively showing the *synergistic* benefits of alternating CSIT.

2.2.2.3 Blind Interference Alignment

A promising space-time IA approach was proposed in [18] for the MISO-BC where the transmitter does not require any short-term channel information, yet significant DoF gains can be obtained. This space-time coding scheme is called the Blind IA (BIA) because the transmitters are blind to the CSI (except the channel auto-correlation function knowledge). However, for the BIA beamforming scheme, the channels at the users need to follow a certain coherence structure which is

almost impossible in practice. To address this issue, it was shown in [19] that the required channel structure can be created artificially using extra hardware at each receiver so that the receiver can assume different channel modes by switching its antennas. Specifically, each receiver is equipped with reconfigurable antennas to fluctuate its channels in a certain pattern (thereby, creating a staggered channel pattern). Next, space-time transmit beamforming is performed (using only identity and zero matrices and no channel information) over this artificially generated channel so that the interfering signals intended for the other users overlap in a minimum possible subspace at the desired receiver.

Note that with this approach, $\frac{KM}{K+M-1}$ DoF can be obtained for a $M \times K$ MISO-BC, where K is the number of users in the network, and M is the number of antennas at the transmitter. Also, the obtained DoF was proved to be optimal for the proposed BIA in [19]. Later, BIA using reconfigurable antennas was proposed in [112] for the three-user Gaussian IC with complete sum-DoF characterization. A BIA scheme for the K -user IC was proposed in [113] where it was shown that a DoF of $\frac{2K}{K+2}$ is achievable. A better BIA code was reported very recently in [114] which shows that the network DoF of the IC grows sub-linearly with the users instead of being bounded by 2 as in [113]. The concept of BIA was further applied to cellular networks in [115] and [116], leading to significant DoF gains over TDMA.

While BIA provides DoF gains for various network types, one major drawback of BIA is the requirement of reconfigurable antennas at every receiver which is practically difficult. To this end, a semi-blind IA was proposed in [117] which requires only one of the receivers to have reconfigurable antennas while all other receivers are normal single-antenna receivers, and yet, significant DoF gains are obtained. Similar to IA, BIA does not show good sum-rate/BER performance in the low-mid SNR region. The sum-rate/BER performance of BIA in the low-mid SNR region is further worsened because of the zero-forcing receive filter due to which the noise power gets enhanced. The increment in the noise power is more

severe when the number of users is large. Another major concern with BIA scheme in [19] is that the space-time code length or the supersymbol length increases exponentially with the increase in the number of users in the network. For example, for a three-user MISO-BC with three antennas at the transmitter, the supersymbol length of each user extends over 20 time slots. This incurs huge delay in the decoding of the symbols at the receiver end. This problem was solved to some extent in [118] using a so-called hierarchical BIA which reduces the BIA code length at the cost of DoF gains.

Although BIA was originally proposed for the heterogenous block fading networks (due to different staggered channel patterns at different users), BIA codes have been designed for homogenous networks as well to give certain DoF gains [119]. BIA codes for a class of line-of-sight ICs have been mentioned in [120]. Further, the low-mid SNR problem of BIA codes has been addressed to some extent in [121, 122]. Since BIA requires special receivers, another important line of work that aims to eliminate CSI feedback to the transmitters has come up recently which exploits the network topology (and not the exact channel information) to obtain DoF gains. The first detailed topology-based interference management techniques with no CSIT (except the topology knowledge only at the transmitters) were proposed in [123]. This was followed by several other works [124–128] which exploit the network topology and require neither CSIT nor reconfigurable antennas at the receivers.

2.2.3 Relay-aided interference management with no CSIT

As discussed above, CSI overhead at the transmitters is a major bottleneck towards the attainment of IA for the multi-user wireless networks. BIA, topological interference management and delayed-CSIT based IA are some important techniques that can achieve DoF gains for the ICs and XCs. However, these methods suffer from drawbacks such as requirement of extra hardware at every receiver,

specific network connectivity and long coding lengths. In this section, we discuss another practical approach to eliminate interference in the wireless networks using relays. Relays are already included in the future generation wireless architectures [129, 130] as potential techniques to increase the network data rate and for coverage extension. An important use of relays is to reduce the CSI overhead in the IC and XC, especially when the CSI needs to be fed back to the relays only (and not to the transmitters which are essentially blind) and yet, the maximum network DoF can be attained [29, 31]. Since the relays are located in between the transmitters and the receivers, the CSI acquisition at the relays is more accurate as compared to the case when the CSI is fed back from the receivers to the transmitters which are farther away. Furthermore, relay-aided transmission requires very few channel uses to attain the network DoF as compared to long coding lengths required without using relays.

Relay-aided transmission includes *two scenarios*, first, when apart from the indirect links via relays, direct links between the transmitters and the receivers also exist as shown in Figure 2.5. In this case, relays do not increase the network DoF as compared to the case without relays [131] but provide other advantages such as CSI overhead reduction, reduced channel uses and more accurate CSI for precoding. Second scenario is the case when no direct links exist between the transmitters and the receivers as shown in Figure 2.6. These networks are also called dual-hop layered networks [132] in which the transmitters transmit their messages to the relay(s) in the first channel use (or first few channel uses), and the relays either decode (if they have enough antennas), amplify or precode the received signals, and forward to the receivers in the next channel use (or next few channel uses). In this thesis, we consider both these scenarios using half-duplex relaying³.

Note that in some cases of dual-hop relaying (i.e., no direct links) such as a

³Full-duplex relaying is not considered in this thesis and so, not discussed here.

Figure 2.5: Relay-aided interference channel with direct links between the TXs and the RXs

Figure 2.6: Dual-hop Relay-aided interference channel (no direct links between the TXs and the RXs)

$2 \times 2 \times 2$ or $K \times K \times K$ network with each node having single antennas, it was recently shown that a network DoF of 2 or K (i.e., the network cut-set bound) can be achieved using the so-called *Aligned interference neutralization* [133] or *Aligned network diagonalization* [134] respectively. While these and other related methods [135, 136] attain the maximum possible network DoF (or the network cut-set bounds), they require long coding lengths and CSI at all the nodes including the source transmitters, making the proposed schemes useful mostly from an information-theoretic perspective only.

Another line of works [30, 132, 137–143] focuses on practical scenarios in which finite number of time slots are required for data transmission. While some of these works consider CSI feedback to the source transmitters, others consider CSI feedback only to the relays and the source transmitters are blind. Works such as [140, 144–148] address the signal processing challenges for improving the network throughput or attaining a certain quality-of-service (QoS) for the end users under source and/or relay power constraints. Note that the network throughput or QoS problems are difficult to solve under different network constructs, and optimization techniques ranging from second-order cone programming (SOCP) [140] and semi-definite programming (SDP) [139, 145, 149] to difference of convex (DC) programs [148] have been applied extensively in the recent years. In [30], various relay-

aided cooperative beamforming strategies have been proposed for the dual-hop interference channels with multiple antennas at every node.

However, in all the above works, perfect CSI is required either at the source transmitters or the relays or both. This incurs heavy channel overhead even for the case of two-time-slot based transmission schemes, especially for the time-varying channels. We note that very few works exist on the relay-aided multi-user networks under imperfect CSI and have started receiving attention only recently. [150, 151] and [146] are very recent works that address the relay-aided transmission for the broadcast and the ICs respectively under imperfect channel information.

In this thesis, we consider half-duplex relay-aided transmission for the interference and X networks (refer to chapters 4-5) which attains the network DoF using IA precoding at the relay and without CSI requirement at the source transmitters. In Chapter 6, we consider dual-hop SISO-IC without CSI at the source transmitters and assisted by multiple MIMO relays which design the precoders based on quantized channel information.

Chapter 3

On Degrees of Freedom of wireless networks with hybrid CSI at the transmitters

In this chapter, we consider the degrees of freedom (DoF) of single-input-single-output (SISO) X channels, interference channels and a special case of broadcast channel under hybrid CSI at the transmitters (CSIT). Hybrid CSI implies that some part of the CSI at the transmitters may be delayed while some part may be instantaneous. This is a reasonable practical assumption because although CSI feedback to the transmitters can be delayed yet some of the CSI can be instantaneously accessible, especially if the CSI is fed back to the transmitters through its *local* links only. Furthermore, in another realistic scenario, the channel may not be varying too fast, and it remains (almost) constant during its coherence interval. For such channels, the CSI fed back to the transmitters may be delayed for a certain portion of the coherence time but once it is received at the transmitters, it can act as the instantaneous CSI for the rest of the coherence time (since the channel is still unchanged) [26, 28]. Thus, a mix of delayed and instantaneous CSI (hybrid CSI) based space-time coding across several coherence blocks can be used

to improve the network DoF.

The hybrid CSIT model has gained considerable attention recently, especially for the broadcast channels [24–26], and some special cases of interference and X channels [28, 111, 152] and is an important topic of information-theoretic research. In this work, first, we consider the three-user SISO-X channels under two different types of hybrid CSIT frameworks. We propose coding schemes for these two feedback types that obtain higher DoF than all the previously reported cases. Further, the second scheme is also scalable with the number of users.

Second, we address the special case of four-user interference channel under hybrid CSIT and propose coding schemes that achieve higher DoF than the completely delayed CSIT based scenario [23, 105] and in much reduced channel uses. Moreover, the obtained DoF using our proposed method is *secure* unlike the scheme in [28]. Our proposed scheme can also be applied to the three-user case where it is more systematic and obtains secure DoF (unlike the three-user case in [28]).

While our proposed schemes address the DoF of only three and four user networks, they provide useful construction methods that achieve significant DoF gains over orthogonal channel access schemes, e.g., TDMA, using delayed and local instantaneous CSIT only. Note that the systematic space-time IA code designs proposed here may be extended to the general K -user network. However, in a K -user network, the interfering signals scale with the number of users, and thus the DoF achievability schemes may require higher order symbol generation and long coding lengths. The extension to the general case may be considered in the future works.

Finally, we consider a broadcast channel where one receiver can assume finite channel states with feedback to the transmitters (this receiver is referred to as “supernode”) while the other receivers have no feedback to the transmitters. Note that this type of feedback model also falls under the class of hybrid CSIT [24, 26]

as mentioned above. Essentially, we use reconfigurable antennas (similar to [19]) to generate the finite channel states of the supernode. We propose a semi-blind interference alignment (SBIA) coding for the above proposed model which achieves higher DoF than traditional orthogonal channel access schemes.

Note that all the three network models (i.e., the interference channel, the X channel, and the broadcast channel) considered in this chapter fall under the broad class of interference networks where eliminating the multi-user interference is the main challenge. While our proposed DoF achievability schemes for these networks are different, the underlying principle to handle the interference is based on the common idea of “interference creation” in one phase and “interference resurrection and cancellation” in another phase, as described in detail later.

3.1 DoF of SISO-X channel with hybrid CSIT

3.1.1 System Model

A three-user SISO-X network consists of 3 transmitters and 3 receivers as shown in Figure 3.1 where every transmitter and every receiver has a single antenna, and every transmitter sends a message for every receiver in the network. Thus, the j^{th} transmitter, $j \in \mathcal{K} = \{1, 2, 3\}$ transmits a message W_{kj} which is intended for the k^{th} ($k \in \mathcal{K}$) receiver.

A complex baseband channel is considered and the input-output model is described as

$$y_k[n] = \sum_{i=1}^3 h_{ki}[n]x^{(i)}[n] + z^{(k)}[n], \quad k \in \mathcal{K} = \{1, 2, 3\}, \quad (3.1)$$

where $h_{ki}[n] \in \mathbb{C}$ denotes the channel coefficient from the i^{th} transmitter to the k^{th} receiver in the n^{th} time-slot and is distributed as $h_{ki}[n] \sim \mathcal{CN}(0, 1)$. $x^{(i)}[n]$ is the symbol transmitted by the i^{th} transmitter in the n^{th} time-slot, and $z^{(k)}[n]$ is the circularly symmetric additive white Gaussian noise (AWGN) at the k^{th} receiver

Figure 3.1: A three-user SISO-X network with hybrid (delayed and local instantaneous) CSIT. Feedback is shown for 1st user only for clarity.

and is distributed as $z^{(k)}[n] \sim \mathcal{CN}(0, \sigma^2)$. We assume that the channel coefficients are i.i.d. across space and time, and the transmitted symbols are assumed to have a power constraint as $\mathbb{E} \left\{ |x^{(i)}[n]|^2 \right\} \leq P$. The DoF of the network is defined as the pre-log factor of the network sum-rate (see the standard definition in [43]).

Let $R_{kj}(P)$ denote the rate of the message W_{kj} at a given power constraint. Then the rate tuple $\{R_{kj}(P), k, j \in \mathcal{K}\}$ is achievable if a $(2^{nR_{kj}(P)}, n)$ code of block length n exists for every (j, k) transmitter-receiver pair such that the average probability of error can be made arbitrarily small for sufficiently large n . The capacity region of the X network is defined as the set of all achievable rate tuples $\{R_{kj}(P)\}, k, j \in \mathcal{K}$ and the corresponding DoF region $\mathcal{D}$ is the set of all tuples $\{d_{kj}\}, k, j \in \mathcal{K}$ where $d_{kj} \in \mathbb{R}_+$ is defined as $d_{kj} = \lim_{P \rightarrow \infty} \frac{R_{kj}(P)}{\log P}$. The DoF of the network, which also represents the number of interference-free data streams that can be transmitted reliably over the network, is defined as $\mathcal{D} = \max_{\mathcal{D}} \sum_{k,j \in \mathcal{K}} d_{kj}$.

As shown in the figure, delayed CSI of the network is available to all the transmitters. Specifically, at time instant n , every transmitter has access to $\{h_{kj}(n')\}_{n'=1}^{n-1}$, $k, j \in \mathcal{K}$. Further, for our first result *which is more restrictive*, we assume that each TX has a perfect CSI feedback link from its *own* RX only, and for our second result, we assume that each TX has access to perfect CSI from all the receivers

connected to it *locally*, as shown in Figure 3.1. Finally, each receiver is assumed to have access to perfect global¹ CSI similar to [24, 111].

3.1.2 Main results: Achievable DoFs

Theorem 3.1. *For the three-user symmetric² SISO-X network under hybrid CSIT model, with each transmitter having access to delayed global CSI and perfect instantaneous CSI from its own receiver only, a DoF of $\frac{4}{3}$ is achievable almost surely in 9 channel uses.*

Theorem 3.2. *For the three-user symmetric SISO-X network, with each transmitter having access to delayed global CSI and perfect instantaneous CSI from all the receivers connected to it locally, a DoF of $\frac{3}{2}$ is achievable almost surely only in 6 channel uses.*

3.1.3 Proofs of the main results

Notations: Similar to the notation used in [23], the transmitted symbol $u^{[k|\mathcal{S};j]}$ denotes a symbol which is available at TX_k and RX_j and is intended to be decoded at every receiver in the set $\mathcal{S}$. $u^{[k|\mathcal{S}]}$ simply indicates that the symbol is available at TX_k only and is intended for the receivers in the set $\mathcal{S}$. An order-1 symbol transmission implies that the symbol is intended only for a single receiver, i.e., cardinality of $\mathcal{S}$, $|\mathcal{S}| = 1$ whereas an order-2 symbol transmission implies that the symbol is intended for 2 receivers, i.e., $|\mathcal{S}| = 2$.

3.1.3.1 Proof of Theorem 3.1

Our transmission consists of 4 main phases. The 1st 3 phases are dedicated to order-1 symbol transmission. Each of these 3 phases consists of 2 channel uses (or time slots) of *redundancy transmission* (RT), followed by *partial interference*

¹Please note that while the global CSI assumption is much more flexible, on careful examination, we find that in our schemes, only RXs 2 and 3 need perfect CSI (that too *partially*) only of each other's local links while RX_1 requires only delayed network CSI.

²“symmetric” implies that equal number of data streams are transmitted to every user.

nulling (PIN) (the terminologies RT and PIN are similar to [23]). These 3 phases are essentially the “interference creation” phases in which each RX receives uncoordinated interference in addition to the desired signals. However, the 4th phase is the “interference resurrection” phase in which the interfering signals received earlier are recreated using coordinated precoding, and then cancelled from the previously received signals to obtain *only* the desired signals. The 4th phase comprises of the order-2 symbol transmission and it consists of 3 time slots.

Phase-1: In this phase, every transmitter transmits fresh information symbols intended for RX_1 .

Redundancy Transmission (RT): In the 1st time-slot of the 1st phase, transmitters 1, 2 and 3 transmit the symbols $u_1^{[1|1]}$, $u_1^{[2|1]}$ and $u_1^{[3|1]}$ respectively intended for RX_1 . In the 2nd time-slot, TX_1 transmits fresh symbol again, i.e., $u_2^{[1|1]}$ while the transmitters 2 and 3 simply repeat the symbols $u_1^{[2|1]}$ and $u_1^{[3|1]}$ respectively. Thus, the symbols received at RX_k during the 2 time slots are given as follows (Note that we neglect the noise terms since we consider only the DoF analysis).

$$y_k[1] = h_{k1}[1]u_1^{[1|1]} + h_{k2}[1]u_1^{[2|1]} + h_{k3}[1]u_1^{[3|1]}. \quad (3.2)$$

$$y_k[2] = h_{k1}[2]u_2^{[1|1]} + h_{k2}[2]u_1^{[2|1]} + h_{k3}[2]u_1^{[3|1]}. \quad (3.3)$$

At this point, using $k=1$ in (3.2) and (3.3), note that RX_1 has 2 equations already, and requires 2 more linear combinations of any of its desired symbols, i.e., $u_1^{[1|1]}$, $u_2^{[1|1]}$, $u_1^{[2|1]}$ or $u_1^{[3|1]}$ so that 4 linearly independent equations are obtained for 4 variables. Thus, RX_1 will be able to decode its desired variables. As shown next, linear combinations of the desired symbols for RX_1 will be generated at RXs 2 and 3 using PIN and then supplied to RX_1 in phase-4.

Partial interference nulling (PIN): Using $k=2$ in (3.2) and (3.3), we can see that RX_2 eliminates $u_1^{[3|1]}$ to obtain a linear combination of $u_1^{[1|1]}$, $u_2^{[1|1]}$ and $u_1^{[2|1]}$ as

shown below.

$$\begin{aligned} \frac{h_{23}[2]}{h_{23}[1]}y_2[1] - y_2[2] &= \underbrace{\left(\frac{h_{23}[2]}{h_{23}[1]}h_{21}[1]\right)}_{\beta_{21}}u_1^{[1|1]} + \underbrace{(-h_{21}[2])}_{\gamma_{21}}u_2^{[1|1]} \\ \underbrace{\left(\frac{h_{23}[2]}{h_{23}[1]}h_{22}[1] - h_{22}[2]\right)}_{\alpha_{21}}u_1^{[2|1]} &=: u^{[1|1,2;2]} + u^{[2|1,2;2]}, \end{aligned} \quad (3.4)$$

where $u^{[1|1,2;2]} = \beta_{21}u_1^{[1|1]} + \gamma_{21}u_2^{[1|1]}$, and $u^{[2|1,2;2]} = \alpha_{21}u_1^{[2|1]}$. Note that a linear combination of $u^{[1|1,2;2]}$ and $u^{[2|1,2;2]}$ is desired at RX_1 and will be provided to RX_1 in phase-4. Further, $u^{[1|1,2;2]}$ and $u^{[2|1,2;2]}$ are also useful at RX_2 in order to align and cancel the interference, as shown in phase-4. Here, $\{\beta_{21}, \gamma_{21}\}$ and $\{\alpha_{21}\}$ are the delayed channel information that are fed back to TXs 1 and 2 respectively.

Similar to above, substituting $k=3$ in (3.2) and (3.3) and using PIN, RX_3 eliminates $u_1^{[2|1]}$ to obtain $(u_1^{[1|1,3;3]} + u_1^{[3|1,3;3]})$, or equivalently, $(\beta_{31}u_1^{[1|1]} + \gamma_{31}u_2^{[1|1]} + \alpha_{31}u_1^{[3|1]})$, where $u_1^{[1|1,3;3]} = \beta_{31}u_1^{[1|1]} + \gamma_{31}u_2^{[1|1]}$ and $u_1^{[3|1,3;3]} = \alpha_{31}u_1^{[3|1]}$, and $\{\beta_{31}, \gamma_{31}\}$ and $\{\alpha_{31}\}$ are the delayed channel information that need to be fed back to the TXs 1 and 3 respectively. A linear combination of $u_1^{[1|1,3;3]}$ and $u_1^{[3|1,3;3]}$ will be supplied to RX_1 in phase-4.

Phase-2: In this phase, every TX transmits fresh information symbols intended for RX_2 . Specifically, TXs 1, 2 and 3 transmit $u_1^{[1|2]}$, $u_1^{[2|2]}$ and $u_1^{[3|2]}$ respectively in the 3rd time-slot, and $u_1^{[1|2]}$, $u_2^{[2|2]}$ and $u_1^{[3|2]}$ respectively in the 4th time-slot. Thus, RX_2 has obtained 2 equations in terms of its 4 desired symbols. Now, using PIN (similar to phase-1), the *unintended* receivers, i.e., RX_1 and RX_3 construct linear combinations of the symbols desired by RX_2 denoted as $(u^{[1|1,2;1]} + u^{[2|1,2;1]})$ and $(u^{[2|2,3;3]} + u^{[3|2,3;3]})$ respectively at the end of 4th time-slot (i.e., the end of phase-2). Finally, a linear combination of $u^{[1|1,2;1]}$ and $u^{[2|1,2;1]}$, and another linear combination of $u^{[2|2,3;3]}$ and $u^{[3|2,3;3]}$ will be supplied to RX_2 in phase-4.

Phase-3: In this phase, every TX transmits fresh information symbols in-

tended for RX_3 . Specifically, TXs 1, 2 and 3 transmit $u_1^{[1|3]}$, $u_1^{[2|3]}$ and $u_1^{[3|3]}$ respectively in the 5^{th} time-slot, and $u_1^{[1|3]}$, $u_1^{[2|3]}$ and $u_2^{[3|3]}$ respectively in the 6^{th} time-slot. Thus, RX_3 has obtained 2 equations in terms of its 4 desired symbols. Now, using PIN (similar to phase-1), the *unintended* receivers, i.e., receivers RX_1 and RX_2 construct linear combinations of the symbols desired by RX_3 denoted as $(u^{[1|1,3;1]} + u^{[3|1,3;1]})$ and $(u^{[2|2,3;2]} + u^{[3|2,3;2]})$ respectively at the end of 6^{th} time-slot (i.e., the end of phase-3). Finally, a linear combination of $u^{[1|1,3;1]}$ and $u^{[3|1,3;1]}$, and another linear combination of $u^{[2|2,3;2]}$ and $u^{[3|2,3;2]}$ will be supplied to RX_3 in phase-4.

Phase-4: In this phase, 3 order-2 symbols are transmitted during 3 time slots, i.e., $n = 7, 8, 9$. During the 7^{th} time-slot, we wish that RXs 1 and 2 obtain one linear combination each of their desired symbols. Thus, TXs 1 and 2, based on their delayed channel information and local instantaneous CSIT (LI-CSIT), construct and transmit the following order-2 symbols.

$$TX_1 : x_1[7] = u^{[1|1,2;2]} + \delta_{21}u^{[1|1,2;1]}, \quad (3.5)$$

$$TX_2 : x_2[7] = u^{[2|1,2;1]} + \delta_{12}u^{[2|1,2;2]}, \quad (3.6)$$

where $u^{[1|1,2;2]}$ and $u^{[2|1,2;2]}$ can be constructed by TX_1 and TX_2 respectively, based on the delayed CSI information $\{\beta_{21}, \gamma_{21}, \alpha_{21}\}$ (see phase-1). Similarly, $u^{[1|1,2;1]}$ and $u^{[2|1,2;1]}$ are constructed by TX_1 and TX_2 respectively based on delayed CSI generated during phase-2. δ_{21} and δ_{12} are the complex scalar precoding coefficients based on the LI-CSIT value and will be chosen as shown next.

The received signals at RX_1 and RX_2 in the 7^{th} time-slot are given as

$$y_1[7] = h_{11}[7]x_1[7] + h_{12}[7]x_2[7], \quad (3.7)$$

$$y_2[7] = h_{21}[7]x_1[7] + h_{22}[7]x_2[7]. \quad (3.8)$$

Since RX_1 already has the interference terms $(u^{[1|1,2;1]} + u^{[2|1,2;1]})$ from phase-2, it

can subtract $h_{12}[7] (u^{[1|1,2;1]} + u^{[2|1,2;1]})$ from $y_1[7]$ to obtain

$$h_{11}[7]u^{[1|1,2;2]} + h_{12}[7]\delta_{12}u^{[2|1,2;2]} + \underbrace{\delta_{21}h_{11}[7]u^{[1|1,2;1]} - h_{12}[7]u^{[1|1,2;1]}}_{\mathcal{I}_1}. \quad (3.9)$$

At this point, note that if we choose $\delta_{21} = \frac{h_{12}[7]}{h_{11}[7]}$, the interference term in (3.9), i.e., $\mathcal{I}_1 = 0$ (equivalently, interference alignment and cancelation) and RX_1 obtains a linear combination of its desired symbols as:

$$\tilde{y}_1[7] = h_{11}[7]\beta_{21}u_1^{[1|1]} + h_{11}[7]\gamma_{21}u_2^{[1|1]} + h_{12}[7]\delta_{12}\alpha_{21}u_1^{[2|1]}. \quad (3.10)$$

Thus, TX_1 must set δ_{21} as $\frac{h_{12}[7]}{h_{11}[7]}$ in (3.5) based on the instantaneous channel information $\frac{h_{12}[7]}{h_{11}[7]}$. Note that $\frac{h_{12}[7]}{h_{11}[7]}$ can be computed at RX_1 based on its local CSI and then fed back to its local transmitter only, i.e., TX_1 .

Similarly, using $\delta_{12} = \frac{h_{21}[7]}{h_{22}[7]}$ in (3.6) for precoding by TX_2 , and based on its interference term constructed in the past, RX_2 can cancel its *already known* interference, i.e., $(u^{[1|1,2;2]} + u^{[2|1,2;2]})$ (see phase-1) from $y_2[7]$ in (3.8), and obtains a linear combination of its desired symbols, i.e., $u_1^{[2|2]}$, $u_2^{[2|2]}$ and $u_1^{[1|2]}$.

Next, during the 8th time-slot, TXs 1 and 3 transmit the following symbols based on delayed and LI-CSIT.

$$TX_1 : x_1[8] = u^{[1|1,3;3]} + \delta_{31}u^{[1|1,3;1]}, \quad (3.11)$$

$$TX_3 : x_3[8] = u^{[3|1,3;1]} + \delta_{13}u^{[3|1,3;3]}, \quad (3.12)$$

where $\delta_{31} = \frac{h_{13}[8]}{h_{11}[8]}$ and $\delta_{13} = \frac{h_{31}[8]}{h_{33}[8]}$. During the 9th time-slot, TXs 2 and 3 transmit the following symbols.

$$TX_2 : x_2[9] = u^{[2|2,3;3]} + \delta_{32}u^{[2|2,3;2]}, \quad (3.13)$$

$$TX_3 : x_3[9] = u^{[3|2,3;2]} + \delta_{23}u^{[3|2,3;3]}, \quad (3.14)$$

where $\delta_{32} = \frac{h_{23}[9]}{h_{22}[9]}$ and $\delta_{23} = \frac{h_{32}[9]}{h_{33}[9]}$. Similar to the above analysis, at the end of 8^{th} time-slot, it can be easily checked that both RXs 1 and 3 have obtained one linear combination each of their desired symbols whereas at the end of 9^{th} time-slot, both RXs 2 and 3 have obtained one linear combination of their desired symbols.

Finally, i.e., after 9 channel uses, it can be verified (details omitted) that RX_k ($k \in \mathcal{K}$) has obtained 4 linearly independent equations (2 each from phase- k and phase-4) in terms of its 4 unknown desired symbols. *Thus, 12 symbols (4 symbols per user) are transmitted reliably over 9 time slots, achieving a DoF of $\frac{4}{3}$.* ■

3.1.3.2 Proof of Theorem 3.2

Here, our transmission scheme consists of 2 phases, with 3 time slots per phase. As discussed earlier, Phase 1 is the “interference creation” phase whereas Phase 2 is the “interference resurrection” phase.

Phase-1: It consists of 3 time slots. In the i^{th} ($i \in \mathcal{K}$) time-slot, every TX transmits a message intended for the i^{th} RX, i.e., TX_l ($l \in \mathcal{K}$) transmits $u^{[l|i]}$. As a result, the received signal at RX_k ($k \in \mathcal{K}$) in the i^{th} ($i \in \mathcal{K}$) time-slot (ignoring the noise) is given as

$$y_k[i] = h_{k1}[i]u^{[1|i]} + h_{k2}[i]u^{[2|i]} + h_{k3}[i]u^{[3|i]}. \quad (3.15)$$

At this point, we define the linear combination of the symbols *intended for* the m^{th} user *but received at* RX_i as $\mathcal{L}^{[i|m]}(u^{[1|m]}, u^{[2|m]}, u^{[3|m]})$, or simply $\mathcal{L}^{[i|m]}$ for brevity. Using the above notation, at the end of i^{th} time-slot, RX_i obtains a useful linear combination $\mathcal{L}^{[i|i]}$ of its desired symbols, i.e., $u^{[l|i]}$ ($l \in \mathcal{K}$) while all other receivers RX_j ($j \neq i$) receive the interference $\mathcal{L}^{[j|i]}$. Note that no precoding is required at any TX upto the 3^{rd} time-slot.

It can be shown using $k = 1$ and $i = 1, 2, 3$ in (3.15) that RX_1 obtains a

linear combination, say $\mathcal{L}^{[1|1]}$, of its desired symbols (i.e., $u^{[1|1]}$, $u^{[2|1]}$ and $u^{[3|1]}$) in the 1st time-slot, as well as the interfering terms, $\mathcal{L}^{[1|2]}$ and $\mathcal{L}^{[1|3]}$ in the 2nd and 3rd time slots respectively. Similarly, RX_2 obtains a desired linear combination $\mathcal{L}^{[2|2]}$ in the 2nd time-slot and the interfering terms $\mathcal{L}^{[2|1]}$ and $\mathcal{L}^{[2|3]}$ in the 1st and 3rd time slots respectively, and RX_3 obtains a desired linear combination $\mathcal{L}^{[3|3]}$ in the 3rd time-slot and the interfering terms $\mathcal{L}^{[3|1]}$ and $\mathcal{L}^{[3|2]}$ in the 1st and 2nd time slots respectively. *Now, each RX needs 2 more linear combinations of its desired symbols which is accomplished in phase-2 as explained next.*

Phase-2: This phase also consists of 3 time slots, i.e., $t \in \mathcal{T} = \{4, 5, 6\}$. Here, the TXs use the perfect and instantaneous CSI (acquired through their local links) to precode the symbols such that in each time-slot, every RX in a chosen set of two RXs receives a linear combination of its desired symbols as well as the interference term that it already received in phase-1. This allows each RX in the chosen set to cancel the *known* interference from its newly received signal and retrieve a useful combination of its desired symbols.

Specifically, let the pair of RXs targeted during the n^{th} ($n \in \mathcal{T}$) time-slot be denoted by the set $\mathcal{S}_n$. Then, we choose $\mathcal{S}_4 = \{1, 2\}$, $\mathcal{S}_5 = \{1, 3\}$ and $\mathcal{S}_6 = \{2, 3\}$. Based on the delayed channel information upto the 3rd time-slot and the perfect CSI in the n^{th} time-slot, the signals transmitted by TX_i ($i \in \mathcal{K}$) in the n^{th} ($n \in \mathcal{T}$) time-slot are designed as:

$$x_i[n] = \left(\frac{h_{li}[m]}{h_{li}[n]} \right) u^{[i|m]} + \left(\frac{h_{mi}[l]}{h_{mi}[n]} \right) u^{[i|l]}, \quad \{m, l\} = \mathcal{S}_n. \quad (3.16)$$

Now, in the 4th time-slot, i.e., $n = 4$, RX_1 obtains $y_1[4] = \sum_{i=1}^3 h_{1i}[4]x_i[4]$, which can be shown using (3.16), to be a summation of the interfering term $\mathcal{L}^{[1|2]}$ and another useful linear combination of its desired symbols, say, $\mathcal{G}^{[1|1]}$. Since RX_1 already has $\mathcal{L}^{[1|2]}$ from phase-1, RX_1 can subtract it from $y_1[4]$ (essentially, interference alignment and cancelation occurs) to obtain the desired term $\mathcal{G}^{[1|1]}$.

Similarly, RX_2 also receives one more linear combination of its desired symbols denoted as $\mathcal{G}^{[2|2]}$. Following similar analysis, it can be shown that RXs 1 and 3 receive one desired linear combination each in the 5^{th} time-slot, and RXs 2 and 3 receive one desired linear combination each in the 6^{th} time-slot.

Finally, at the end of the 6^{th} time-slot, each RX has 3 linearly independent combinations of its 3 desired symbols, and therefore, each RX can decode its desired symbols. Thus, overall, 9 distinct symbols are transmitted over the network in 6 time slots, providing a DoF of $\frac{3}{2}$. ■

3.1.4 Discussion and Comments

3.1.4.1 Scalability

It can be verified that the scheme-1 (Theorem 3.1), if trivially extended to the K -user case, yields a constant DoF of $\frac{4}{3}$, i.e., the DoF does not scale with the number of users. Note that it still requires much lesser channel uses and feedback volume as compared to the completely-delayed CSIT schemes [23] (please refer to Section 3.1.4.4 and Table 3.1).

On the other hand, it can be shown that for the scheme-2, the DoF scales with the number of users. Specifically, for the K -user case, (3.15) can be slightly modified as $y_k[i] = \sum_{l=1}^K h_{kl}[i]u^{[l|i]}$, $i, k \in \mathcal{K} = \{1, 2, \dots, K\}$. Following similar approach as in the proof of Theorem 3.2, K time slots are consumed in phase-1, and each RX obtains one linear combination of its K desired symbols. In phase-2, we can modify (3.16) accordingly to design the transmitted signals. Thus, 2 out of K RXs are selected in every time-slot starting from $n = (K+1)$ in phase-2. It can be verified that in every time-slot, each RX in the selected set obtains one useful linear combination. In the above manner, ${}^K C_2$ receiver sets are considered in ${}^K C_2$ time slots in phase-2. Finally, each RX obtains K desired symbols in $K + {}^K C_2$ time slots, giving a total DoF of $\frac{K^2}{K + {}^K C_2}$ or $\frac{2K}{K+1}$. It is worth noting that the DoF

is bounded by 2 as $K \rightarrow \infty$. Figure 3.2 shows the DoF scaling of the proposed scheme-2 with the number of users as compared to the existing schemes based on completely-delayed CSIT.

Figure 3.2: DoF scaling with the number of users for various schemes

3.1.4.2 Channel feedback overhead and Alternating CSIT framework

In this section, we discuss the channel feedback overhead for schemes 1 and 2. It is worth noting that in both the schemes, the instantaneous or perfect channel feedback is *local* to the transmitters. Moreover, the receivers feed back the CSI only for a fraction of the total feedback duration. During each feedback time-slot, every receiver has 3 feedback states, namely, (a) it does not feedback (state **N**) (b) it feeds back CSI after a delay (state **D**) and (c) it feeds back CSI instantaneously or perfectly (state **P**).

Scheme-1: In scheme-1, the TXs do not do any precoding in the 1st 3 phases, i.e., in the 1st 6 time slots (refer to the proof of Theorem 3.1 in Section 3.1.3.1). In fact, in the odd-numbered time-slots, TXs do not get any feedback from the RXs. It is only in the even time-slot that the interfered RXs can construct the useful linear combination which is intended for the desired receiver. For example, RXs 2

and 3 compute the desired combinations for RX_1 , i.e., $u^{[1|1,2;2]} = \beta_{21}u_1^{[1|1]} + \gamma_{21}u_2^{[1|1]}$ (please see (3.4)) and $u^{[1|1,3;3]}$ respectively in the 2^{nd} time-slot and feed back the required coefficients (e.g., β_{21} , γ_{21} and α_{21} by RX_2) to the TXs in the 2^{nd} time-slot. These coefficients are used by the TXs to construct the order-2 symbols after the 6^{th} time-slot, i.e., after some delay.

Thus, if we denote the no CSIT, delayed CSIT and perfect (i.e., instantaneous, in this case) CSIT states by N, D and P respectively, then for the scheme-1, we can write N, N, N or (NNN) corresponding to *no* feedback by RXs 1, 2 and 3 respectively (similar to [24, 25]) in the 1^{st} time-slot. In the 2^{nd} time-slot, only RXs 2 and 3 construct the linear combination of symbols for the 1^{st} user, and the 1^{st} RX does not need to compute or feed back anything at this stage. Thus, RX_1 's state is N while the states of other receivers (which need to feed back CSI in a delayed manner) is denoted by D. Thus, for the 2^{nd} time-slot, the channel state is (NDD). Similarly, the CSI states from the 1^{st} to the 9^{th} time slots are given as (NNN, NDD, NNN, DND, NNN, DDN, PPN, PNP, NPP). The bold letter corresponds to the CSI state for RX_1 and it can be found that RX_1 assumes “N” state for 5 out of 9, “P” and “D” states each for 2 out of 9 time slots. This also holds for other RXs due to symmetry. Note that the proposed scheme-1 shows an example of DoF gain due to *synergistic benefits of alternating CSIT* (refer to [24] for details) where every RX is alternating between 3 feedback states.

Denoting by $(\lambda_N, \lambda_D, \lambda_P)$ respectively as the fraction of time that the particular CSI states (N,D,P) exist for a given receiver, it can be verified that $(\lambda_N, \lambda_D, \lambda_P) = (\frac{5}{9}, \frac{2}{9}, \frac{2}{9})$. Also, note that overall, 6 complex coefficients are fed back locally and *instantaneously* for scheme-1.

Scheme-2: It can be verified from the proof of Theorem 3.2 in Section 3.1.3.2 that 18 complex coefficients need to be fed back locally and instantaneously to the TXs in scheme-2. According to the coding scheme (3.16) in the proof of Theorem 3.2, Figure 3.3 shows the various feedback states by the various receivers

in all the 6 time slots. In this case, similar to the analysis for scheme-1, we get $(\lambda_N, \lambda_D, \lambda_P) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$. In particular, the CSIT states for the 6 time slots are given as (NDD, DND, DDN, PPN, PNP, NPP). *It is easy to see that channel states are alternating across the time slots, and thus, we get synergistic benefits of alternating CSIT [24, 111] for a 3-user SISO-X channel.*

Figure 3.3: CSI feedback in the 3-user SISO-X channel for *Scheme-2*. N, D and P beside the RXs denote no feedback, delayed feedback (i.e., the CSI fed back in the n^{th} time-slot will be used for precoding after “n”), and instantaneous feedback respectively.

As mentioned above, the proposed scheme is easily scalable to the K -user case. For the K -user case, we get a DoF of $\frac{2K}{K+1}$ and it can be shown that $(\lambda_N, \lambda_D, \lambda_P) = \left(\frac{K^2-3K+4}{K(K+1)}, \frac{2(K-1)}{K(K+1)}, \frac{2(K-1)}{K(K+1)} \right)$. For example, the number of time slots in which a RX feeds back delayed CSI (i.e., D) can be shown to be $(K-1)$ out of total $(K + {}^K C_2)$ time slots which means $\lambda_D = \frac{2(K-1)}{K(K+1)}$. An interesting point to note is that for the K -user case in *scheme-2*, $\lim_{K \rightarrow \infty} \lambda_N = 1$ and $\lim_{K \rightarrow \infty} \lambda_P = \lambda_D = 0$. This means that as $K \rightarrow \infty$, negligibly small (yet finite) amount of delayed or perfect CSI needs to be fed back to attain a gain of 2 $\left(\because \lim_{K \rightarrow \infty} \frac{2K}{K+1} = 2 \right)$. Although interesting, this result is not surprising because it is only the fraction of P or D that is decreasing with K , and a finite amount of P or D feedback is always required from every RX. Figure 3.4 shows $(\lambda_N, \lambda_D, \lambda_P)$ as a function of K .

Figure 3.4: Fraction of time that a receiver assumes a particular channel state (N, D or P) for a k -user ($k \in \mathcal{K}$) SISO-X channel for *Scheme-2*.

3.1.4.3 Computational Complexity

It is to be noted that both schemes 1 and 2 proposed above involve only linear precoding techniques. Since Scheme 1 involves only three users and finite coding length, only finite number of simple arithmetic operations (addition, subtractions and multiplications) are involved (refer to (3.4)-(3.14)).

On the other hand, since Scheme 2 is scalable with the number of users, K , its computational complexity grows linearly with K as shown next. In the Scheme 2, no precoding is involved in the first K channel uses (refer to Section 3.1.4.1). However, in the next $^K C_2$ channel uses, one scalar addition and two multiplication operations are involved during precoding at each of the K transmitters (refer to (3.16)). According to the proposed scheme, the above mentioned three arithmetic operations are required at each transmitter K times because each transmitter is selected K times out of $^K C_2$ combinations. Thus, exactly $3K$ arithmetic operations are required at each transmitter, i.e., the computational complexity at each transmitter scales linearly with the number of users.

At the receiver end, note that at the end of the transmission in Scheme 2, each receiver has K linear equations in K unknowns (symbols to be decoded). Thus, the computational complexity at each receiver is of the order $c \times \mathcal{O}(K^3)$ (where c is some positive constant) because a $K \times K$ matrix inversion is involved while solving the K linear equations.

3.1.4.4 Number of Channel uses

The proposed schemes in this chapter require much lesser channel uses as compared to the completely delayed CSIT case in [23]. Note that while the DoF of scheme-1 does not scale with the number of users, it still achieves higher DoF than the completely outdated CSIT-based DoF in [105] for all K , and better DoF than that in [23] for $K = 3, 4$. Moreover, note that the channel uses consumed by scheme-1 is much lesser as compared to [23, 105]. Table 3.1 shows the comparison of time slots consumed by each scheme.

Table 3.1: Comparison of DoF and channel uses of various schemes

Schemes ↓	DoF			No. of channel uses		
	3 users	4 users	5 users	3 users	4 users	5 users
Delayed CSI [23]	1.2857	1.329	1.354	70	632	2326
Delayed CSI [105]	1.25	1.2727	1.2857	12	22	35
Scheme-1 (proposed)	1.33	1.33	1.33	9	18	30
Scheme-2 (proposed)	1.5	1.6	1.67	6	10	15

3.1.5 Summary of Section 3.1

In Section 3.1, two new achievability results are obtained for the DoF of the three-user SISO-X network with hybrid CSIT, i.e., a mix of delayed network CSIT and limited amount of local instantaneous CSIT (LI-CSIT). It is shown that in addition to delayed CSIT, limited LI-CSIT, which can be available in practice, provides a strict DoF gain over all existing schemes which make use of delayed CSIT only. Furthermore, the proposed schemes use much fewer channel uses to attain a higher DoF than the existing delayed-CSIT-based schemes.

3.2 DoF of 4-user SISO-IC with hybrid CSIT

In this section, DoF of the SISO-IC with hybrid CSIT is investigated. Note that the SISO-IC is a special case of the SISO-XC [43] but a trivial application of the coding scheme for the SISO-XC to the SISO-IC results in DoF loss, and hence suitable code designs for the SISO-IC are required. Note that in Section 3.1, we proposed space-time coding schemes based on “interference creation” in one phase and “interference resurrection” in the next phase for the SISO-XC. Since the SISO-IC is a special case of the SISO-XC, we utilize similar ideas to construct space-time coding scheme for the SISO-IC with hybrid CSIT.

In particular, we consider the 4-user SISO-IC with hybrid CSIT. Note that the 3-user SISO-IC with hybrid CSIT has been addressed in [28] but its extension to 4-user case is not trivial. Also, the scheme in [28] does not achieve secure DoF (i.e., the user’s messages are not completely confidential) for the 3-user case. In this work, we provide construction scheme which achieves a DoF of 1.2 in only half the number of channel uses as compared to a trivial extension of the scheme proposed for the 3-user case in [28]. Moreover, our scheme is more systematic and more easily amenable for extension to the general K -user case, and provides *secure* DoF (sDoF) unlike [28] where some messages are *common* to all the users. Also, it is worth noting that our scheme can be applied to the 3-user SISO-IC in a systematic manner and can achieve an sDoF of 1.2.

Note that our scheme, if applied to the 3-user case, achieves the sDoF upper bound [153] using only local instantaneous and delayed CSIT and without any *real* interference alignment (IA) scheme [51] based on global CSIT as in [153]. Thus, our scheme is DoF-optimal for the 3-user case. We do not discuss the 3-user case explicitly because the underlying principle for the 3-user case is same as that proposed for the 4-user case in this work. For the 4-user case as described in this chapter, our scheme is still short of the sDoF upper bound $\frac{12}{7}$ [153]. However, it

must be noted that we consider only local CSIT and *signal-space based* IA which are much more practically useful as compared to the real-IA (or signal-level based IA) based on global CSIT and long channel extensions. Furthermore, a tight sDoF outer bound for the 4-user SISO-IC (or for K -user SISO-IC for that reason) with only local CSIT is still an open problem.

3.2.1 System Model

Consider a 4-user SISO-IC (see Figure 2.1) where every TX intends to send a message to its RX while creating interference to other users operating on the same channel use. The baseband input-output model for the k^{th} RX is given as:

$$y_k[n] = \sum_{i \in \Omega} h_{ki}[n]x_i[n] + z_k[n], \quad k \in \mathcal{K} = \{1, 2, \dots, K\}, \quad \Omega \subseteq \mathcal{K}, \quad (3.17)$$

where $h_{ki}[n]$ denotes the channel coefficient from the i^{th} TX to the k^{th} RX in the n^{th} channel use and is distributed as $\mathcal{CN}(0, 1)$, $x_i[n]$ denotes the symbol transmitted by the i^{th} TX and has a power constraint of P , i.e., $\mathbb{E}\{|x_i[n]|^2\} \leq P$, and $z_k[n] \sim \mathcal{CN}(0, \sigma^2)$ is the circularly symmetric AWGN at the k^{th} RX. Note that only a subset $i \in \Omega$ of all the users are actively transmitting while $i' \in \mathcal{K} \setminus \Omega$ are silent.

The DoF of the network is the pre-log factor of the network sum-rate as defined in [2]. Note that delayed CSI of the whole network is available to all the TXs whereas global CSI is available at the RXs ³ as assumed in [24, 103]. Specifically, at time instant n , every TX has access to $\{h_{kj}(n')\}_{n'=1}^{n-1}$, $k, j \in \mathcal{K}$. Further, each TX has access to perfect or instantaneous CSI available to it through its local links, i.e., the k^{th} TX has access to $\{h_{jk}[n]\}$, $j \in \mathcal{K}$ at the n^{th} time instant.

³In fact, on careful analysis, it can be found that the delayed CSIT of the entire network and the global CSI availability at all the RXs are indeed not required but these details are avoided in this work.

3.2.2 Main Result

Theorem 3.3. *The 4-user SISO-IC, with each TX having access to global delayed CSIT and perfect instantaneous CSI from all the RXs connected to it locally, can achieve a secure DoF of 1.2 in 10 channel uses only.*

Proof. Notations: $u^{[k|k,j]}$ denotes a symbol which is transmitted by TX- k (see k on the left side of “|” in $u^{[k|k,j]}$), *intended* for RX- k , but has to be decoded by both the RXs k and j . Note that this is like an order-2 “common” symbol [23] which needs to be decoded by two receivers. However, in an interference channel (IC), it is *useful* only to RX- k although RX- j can also decode it. On the other hand, $u_j^{(k)}$ denotes the j^{th} “private” (also called order-1 symbol because it is intended only for one receiver) symbol of the k^{th} user.

In this section, we prove that 12 desired symbols can be transmitted over the 4-user SISO-IC in 10 channel uses only, with no user being able to decode other user’s symbols, thereby achieving a secure DoF of 1.2. Our proposed scheme makes use of two principles. Firstly, a linear combination of the user’s private (order-1) symbols is done at the TX to create an order-2 symbol. This order-2 symbol is a *common* symbol which is decodable by a chosen pair of users, one intended and the other unintended RX. However, the unintended RX can decode only the common symbol but it cannot decode the individual order-1 symbols which constitute it, thus, ensuring confidentiality of the *intended* user’s messages. Secondly, we apply interference alignment based on delayed and local instantaneous CSI to cancel the interference at the users.

Generation of order-2 symbols by encoding the private messages:

The k^{th} ($k \in \mathcal{K} = \{1, 2, 3, 4\}$) TX intends to transmit 3 private symbols, i.e., $\{u_1^{(k)}, u_2^{(k)}, u_3^{(k)}\}$ (total 12 private symbols) which must be decodable only by the intended, i.e., k^{th} RX. However, before the transmission, the k^{th} TX performs a linear encoding (a linear combination) of its private symbols to generate an order-2

symbol. It is this order-2 symbol that is actually transmitted. Unlike the order-1 private symbols, this order-2 symbol is also decodable at one more user- j , $j \neq k$, and is denoted as $u^{[k|k,j]}$. $u^{[k|k,j]}$ is written as

$$u^{[k|k,j]} = \sum_{l=1}^3 \alpha_l^{[k|k,j]} u_l^{(k)}, \quad j \neq k, \quad j, k \in \mathcal{K}, \quad (3.18)$$

where $\alpha_l^{[k|k,j]}$ are the random coefficients for the user pair (k, j) , $j \neq k$ which are known beforehand to the TXs and the RXs. From (3.18), note that overall 12 order-2 symbols (3 per user) are generated. Now, we describe the transmission of these order-2 symbols.

Order-2 symbols transmission: Our transmission consists of 2 phases. Phase-1 consists of 6 channel uses from $n = 1$ to $n = 6$ and Phase-2 consists of 4 channel uses from $n = 7$ to $n = 10$. Note that in each channel use, the TXs transmit order-2 symbols of the form $u^{[k|k,j]}$ which are constructed from order-1 or private symbols as shown in (3.18), *but* our ultimate aim is to decode the private symbols for each user and at the same time, ensure that it cannot be decoded by an *unintended* user. The decoding of order-1 or private symbols is done after Phase-2 as explained later.

Phase-1: Consider a pair of users in the 4-user network. Note that 4C_2 or 6 such combinations can be chosen.

1) In the 1st channel use, i.e., $n = 1$, user pair $(1, 2)$ is selected. Here, the users 1 and 2 are the “common” users and other users such as 3 and 4 are the “interfered” users. TX-1 transmits $u^{[1|1,2]}$ intended for RX-1, and TX-2 transmits $u^{[2|1,2]}$ intended for RX-2. However, both these symbols are “common” symbols which can be decoded by both RXs 1 and 2. The interfered RXs 3 and 4 obtain (or overhear) a linear combination of the transmitted symbols. This linear combination will be utilized later on for interference alignment in Phase-2. Using $\Omega = \{1, 2\}$ in (3.17), the signals received at the RXs (neglecting the noise terms because only

DoF analysis is done) are given as

$$\begin{aligned}
 RX-1 : y_1[1] &= h_{11}[1]u^{[1|1,2]} + h_{12}[1]u^{[2|1,2]}, \\
 RX-2 : y_2[1] &= h_{21}[1]u^{[1|1,2]} + h_{22}[1]u^{[2|1,2]}, \\
 RX-3 : y_3[1] &= h_{31}[1]u^{[1|1,2]} + h_{32}[1]u^{[2|1,2]}, \\
 RX-4 : y_4[1] &= h_{41}[1]u^{[1|1,2]} + h_{42}[1]u^{[2|1,2]}.
 \end{aligned} \tag{3.19}$$

2) At $n = 2$, user pair $(1, 3)$ is selected. TX-1 transmits $u^{[1|1,3]}$ and TX-3 transmits $u^{[3|1,3]}$. The signals received are given as

$$\begin{aligned}
 RX-1 : y_1[2] &= h_{11}[2]u^{[1|1,3]} + h_{13}[2]u^{[3|1,3]}, \\
 RX-2 : y_2[2] &= h_{21}[2]u^{[1|1,3]} + h_{23}[2]u^{[3|1,3]}, \\
 RX-3 : y_3[2] &= h_{31}[2]u^{[1|1,3]} + h_{33}[2]u^{[3|1,3]}, \\
 RX-4 : y_4[2] &= h_{41}[2]u^{[1|1,3]} + h_{43}[2]u^{[3|1,3]}.
 \end{aligned} \tag{3.20}$$

3) At $n = 3$, user pair $(1, 4)$ is selected. TX-1 and TX-4 transmit $u^{[1|1,4]}$ and $u^{[4|1,4]}$ respectively. The signals received are given as

$$\begin{aligned}
 RX-1 : y_1[3] &= h_{11}[3]u^{[1|1,4]} + h_{14}[3]u^{[4|1,4]}, \\
 RX-2 : y_2[3] &= h_{21}[3]u^{[1|1,4]} + h_{24}[3]u^{[4|1,4]}, \\
 RX-3 : y_3[3] &= h_{31}[3]u^{[1|1,4]} + h_{34}[3]u^{[4|1,4]}, \\
 RX-4 : y_4[3] &= h_{41}[3]u^{[1|1,4]} + h_{44}[3]u^{[4|1,4]}.
 \end{aligned} \tag{3.21}$$

4) At $n = 4$, user pair $(2, 3)$ is selected. TX-2 and TX-3 transmit $u^{[2|2,3]}$ and $u^{[3|2,3]}$ respectively. The signals received are given as

$$\begin{aligned}
 RX-1 : y_1[4] &= h_{12}[4]u^{[2|2,3]} + h_{13}[4]u^{[3|2,3]}, \\
 RX-2 : y_2[4] &= h_{22}[4]u^{[2|2,3]} + h_{23}[4]u^{[3|2,3]}, \\
 RX-3 : y_3[4] &= h_{32}[4]u^{[2|2,3]} + h_{33}[4]u^{[3|2,3]}, \\
 RX-4 : y_4[4] &= h_{42}[4]u^{[2|2,3]} + h_{43}[4]u^{[3|2,3]}.
 \end{aligned} \tag{3.22}$$

5) At $n = 5$, user pair $(2, 4)$ is selected. TX-2 and TX-4 transmit $u^{[2|2,4]}$ and $u^{[4|2,4]}$ respectively. The signals received are given as

$$\begin{aligned} RX-1 : y_1[5] &= h_{12}[5]u^{[2|2,4]} + h_{14}[5]u^{[4|2,4]}, \\ RX-2 : y_2[5] &= h_{22}[5]u^{[2|2,4]} + h_{24}[5]u^{[4|2,4]}, \\ RX-3 : y_3[5] &= h_{32}[5]u^{[2|2,4]} + h_{34}[5]u^{[4|2,4]}, \\ RX-4 : y_4[5] &= h_{42}[5]u^{[2|2,4]} + h_{44}[5]u^{[4|2,4]}. \end{aligned} \quad (3.23)$$

6) At $n = 6$, user pair $(3, 4)$ is selected. TX-3 and TX-4 transmit $u^{[3|3,4]}$ and $u^{[4|3,4]}$ respectively. The signals received are given as

$$\begin{aligned} RX-1 : y_1[6] &= h_{13}[6]u^{[3|3,4]} + h_{14}[6]u^{[4|3,4]}, \\ RX-2 : y_2[6] &= h_{23}[6]u^{[3|3,4]} + h_{24}[6]u^{[4|3,4]}, \\ RX-3 : y_3[6] &= h_{33}[6]u^{[3|3,4]} + h_{34}[6]u^{[4|3,4]}, \\ RX-4 : y_4[6] &= h_{43}[6]u^{[3|3,4]} + h_{44}[6]u^{[4|3,4]}. \end{aligned} \quad (3.24)$$

Note that at the end of Phase-1, every user has obtained 3 linearly independent equations in terms of its desired order-2 symbols. However, each user needs to decode all of its 6 order-2 symbols (i.e., 3 *desired* order-2 symbols and 3 *undesired* order-2 symbols) if it wants to decode its desired symbols. Thus, it requires 3 more linearly independent equations in terms of its desired symbols. For example, RX-1 has obtained $y_1[1]$ which is a linear combination of $u^{[1|1,2]}$ and $u^{[2|1,2]}$, $y_1[2]$ which is a linear combination of $u^{[1|1,3]}$ and $u^{[3|1,3]}$, and $y_1[3]$ which is a linear combination of $u^{[1|1,4]}$ and $u^{[4|1,4]}$. However, in order to decode its desired symbols $u^{[1|1,2]}$, $u^{[1|1,3]}$ and $u^{[1|1,4]}$, it needs to eliminate the *undesired* symbols $u^{[2|1,2]}$, $u^{[3|1,3]}$ and $u^{[4|1,4]}$ for which it requires 3 more linearly independent equations.

Phase-2: This phase consists of 4 channel uses. In each channel use, a set of 3 users is selected out of 4 users. Thus, 4 sets, viz., $(1,2,3)$, $(1,2,4)$, $(1,3,4)$ and $(2,3,4)$ are possible. In each channel use of Phase-2, a given set of users is

selected, and the idea is to design the precoding at every TX (in the selected set) such that an already known interference term (which is obtained from Phase-1) for every RX in the set is re-constructed at that RX. By doing so, every RX in the set can subtract the interference term from Phase-1 to obtain a linear combination in terms of its desired symbols. Since every user appears thrice when all the 4 sets are consumed, 3 linear combinations are generated for every user in the set by the end of Phase-2. Detailed description is given below.

7) At $n = 7$, user triplet (1,2,3) is selected. TXs 1, 2 and 3 transmit $x_1[7]$, $x_2[7]$ and $x_3[7]$ respectively by precoding their symbols based on the delayed and locally available instantaneous CSIT.

$$\begin{aligned} x_1[7] &= \frac{h_{31}[1]}{h_{31}[7]} u^{[1|1,2]} + \frac{h_{21}[2]}{h_{21}[7]} u^{[1|1,3]}, \\ x_2[7] &= \frac{h_{32}[1]}{h_{32}[7]} u^{[2|1,2]} + \frac{h_{12}[4]}{h_{12}[7]} u^{[2|2,3]}, \\ x_3[7] &= \frac{h_{23}[2]}{h_{23}[7]} u^{[3|1,3]} + \frac{h_{13}[4]}{h_{13}[7]} u^{[3|2,3]}. \end{aligned} \quad (3.25)$$

Using the above precoding, the signal received at RX-1 at $n = 7$ is given as

$$\begin{aligned} y_1[7] &= h_{11}[7]x_1[7] + h_{12}[7]x_2[7] + h_{13}[7]x_3[7] \\ &= \frac{h_{11}[7]h_{31}[1]}{h_{31}[7]} u^{[1|1,2]} + \frac{h_{11}[7]h_{21}[2]}{h_{21}[7]} u^{[1|1,3]} \\ &\quad + \frac{h_{12}[7]h_{32}[1]}{h_{32}[7]} u^{[2|1,2]} + \frac{h_{13}[7]h_{23}[2]}{h_{23}[7]} u^{[3|1,3]} \\ &\quad + \underbrace{h_{12}[4]u^{[2|2,3]} + h_{13}[4]u^{[3|2,3]}}_{=y_1[4]} \end{aligned} \quad (3.26)$$

Thus, using suitable precoding, interference term $y_1[4]$ obtained at RX-1 in Phase-1 is re-constructed at $n = 7$, or in essence, *interference alignment* is achieved. RX-1 can now subtract $y_1[4]$ from $y_1[7]$ to obtain a linear combination of its desired

symbols, i.e.,

$$\begin{aligned}
 y_1[7] - y_1[4] &= \frac{h_{11}[7]h_{31}[1]}{h_{31}[7]}u^{[1|1,2]} + \frac{h_{11}[7]h_{21}(2)}{h_{21}[7]}u^{[1|1,3]} \\
 &+ \frac{h_{12}[7]h_{32}[1]}{h_{32}[7]}u^{[2|1,2]} + \frac{h_{13}[7]h_{23}[2]}{h_{23}[7]}u^{[3|1,3]}
 \end{aligned} \tag{3.27}$$

Similarly, it can be checked that other RXs in the selected set, i.e., RXs 2 and 3 also obtain one linear combination each of their desired symbols. For brevity, only the case of user-1 is discussed. Analysis of other users is very similar and therefore, not shown.

8) At $n = 8$, user triplet (1,2,4) is selected. TXs 1, 2 and 4 transmit $x_1[8]$, $x_2[8]$ and $x_4[8]$ respectively by precoding their symbols based on the locally available CSIT.

$$\begin{aligned}
 x_1[8] &= \frac{h_{41}[1]}{h_{41}[8]}u^{[1|1,2]} + \frac{h_{21}[3]}{h_{21}[8]}u^{[1|1,4]}, \\
 x_2[8] &= \frac{h_{42}[1]}{h_{42}[8]}u^{[2|1,2]} + \frac{h_{12}[5]}{h_{12}[8]}u^{[2|2,4]}, \\
 x_4[8] &= \frac{h_{24}[3]}{h_{24}[8]}u^{[4|1,4]} + \frac{h_{14}[5]}{h_{14}[8]}u^{[4|2,4]}.
 \end{aligned} \tag{3.28}$$

Using the above precoding, and after subtracting the aligned interference term from the received signal (as done earlier), RX-1 obtains a linear combination of its desired symbols as

$$\begin{aligned}
 y_1[8] - y_1[5] &= h_{11}[8]x_1[8] + h_{12}[8]x_2[8] + h_{14}[8]x_4[8] - y_1[5] \\
 &= \frac{h_{11}[8]h_{41}[1]}{h_{41}[8]}u^{[1|1,2]} + \frac{h_{11}[8]h_{21}(3)}{h_{21}[8]}u^{[1|1,4]} \\
 &+ \frac{h_{12}[8]h_{42}[1]}{h_{42}[8]}u^{[2|1,2]} + \frac{h_{14}[8]h_{24}[3]}{h_{24}[8]}u^{[4|1,4]}.
 \end{aligned} \tag{3.29}$$

Similarly, it can be checked that other RXs in the set (1,2,4), i.e., RXs 2 and 4 also obtain one linear combination each of their desired symbols.

9) At $n = 9$, user triplet (1,3,4) is selected. TXs 1, 3 and 4 transmit $x_1[9]$, $x_3[9]$ and $x_4[9]$ respectively by precoding their symbols based on the locally available

CSIT.

$$\begin{aligned}
 x_1[9] &= \frac{h_{41}[2]}{h_{41}[9]} u^{[1|1,3]} + \frac{h_{31}[3]}{h_{31}[9]} u^{[1|1,4]}, \\
 x_3[9] &= \frac{h_{43}[2]}{h_{43}[9]} u^{[3|1,3]} + \frac{h_{13}[6]}{h_{13}[9]} u^{[3|3,4]}, \\
 x_4[9] &= \frac{h_{34}[3]}{h_{34}[9]} u^{[4|1,4]} + \frac{h_{14}[6]}{h_{14}[9]} u^{[4|3,4]}.
 \end{aligned} \tag{3.30}$$

Using the above precoding, and after subtracting the aligned interference term from the received signal (as done earlier), RX-1 obtains a linear combination of its desired symbols as

$$\begin{aligned}
 y_1[9] - y_1[6] &= h_{11}[9]x_1[9] + h_{13}[9]x_3[9] + h_{14}[9]x_4[9] - y_1[6] \\
 &= \frac{h_{11}[9]h_{41}[2]}{h_{41}[9]} u^{[1|1,3]} + \frac{h_{11}[9]h_{31}[3]}{h_{31}[9]} u^{[1|1,4]} \\
 &\quad + \frac{h_{13}[9]h_{43}[2]}{h_{43}[9]} u^{[3|1,3]} + \frac{h_{14}[9]h_{34}[3]}{h_{34}[9]} u^{[4|1,4]}.
 \end{aligned} \tag{3.31}$$

Similarly, it can be checked that other RXs in the set (1,3,4), i.e., RXs 3 and 4 also obtain one linear combination each of their desired symbols. Note that by this time, RX-1 has already obtained 6 linearly independent equations (see (3.19)-(3.21), (3.27), (3.29), (3.31)) in terms of its 6 unknown order-2 symbols (3 desired symbols, i.e., $u^{[1|1,2]}$, $u^{[1|1,3]}$ and $u^{[1|1,4]}$, and 3 undesired symbols, i.e., $u^{[2|1,2]}$, $u^{[3|1,3]}$ and $u^{[4|1,4]}$), and therefore, can decode all of its desired order-2 symbols. On the other hand, RXs 2, 3 and 4 need one more linear combination each of their desired symbols which is accomplished in the channel use $n = 10$.

10) At $n = 10$, user triplet (2,3,4) is selected. TXs 2, 3 and 4 transmit $x_2[10]$, $x_3[10]$ and $x_4[10]$ respectively by precoding their symbols based on the locally

available CSIT.

$$\begin{aligned}
 x_2[10] &= \frac{h_{42}[4]}{h_{42}[10]} u^{[2|2,3]} + \frac{h_{32}[5]}{h_{32}[10]} u^{[2|2,4]}, \\
 x_3[10] &= \frac{h_{43}[4]}{h_{43}[10]} u^{[3|2,3]} + \frac{h_{23}[6]}{h_{23}[10]} u^{[3|3,4]}, \\
 x_4[10] &= \frac{h_{34}[5]}{h_{34}[10]} u^{[4|2,4]} + \frac{h_{24}[6]}{h_{24}[10]} u^{[4|3,4]}.
 \end{aligned} \tag{3.32}$$

Using the above, it can be shown that RXs 2, 3 and 4 receive one linear combination each of their desired symbols (not shown here). Finally, at the end of Phase-2, every RX has obtained its required linear combinations in order to decode its symbols. Thus, overall, 12 order-2 symbols (3 desired symbols per user) are transmitted over the network in 10 channel uses.

Decoding of the private messages: Once the order-2 symbols are obtained as described above, each RX needs to decode its private symbols from these order-2 symbols. Note that RX-1 has obtained its desired order-2 symbols, i.e., $u^{[1|1,2]}$, $u^{[1|1,3]}$ and $u^{[1|1,4]}$. From (3.18), it can be seen that each of $u^{[1|1,2]}$, $u^{[1|1,3]}$ and $u^{[1|1,4]}$ is an independent linear combination of the 3 order-1 private symbols, i.e., $u_1^{(1)}$, $u_2^{(1)}$ and $u_3^{(1)}$. Thus, RX-1 can easily decode its 3 private symbols.

On the other hand, note that $u^{[1|1,2]}$, $u^{[1|1,3]}$ and $u^{[1|1,4]}$ have also been decoded by the undesired RXs, i.e., RXs 2, 3 and 4 respectively. However, none of these RXs can decode RX-1's private symbols because each RX is able to obtain only one linear combination of RX-1's 3 private symbols. Essentially, RX-1's private symbols are completely confidential. The same explanation applies to decoding of private symbols of other users also. Thus, 12 private (order-1) symbols are transmitted over the network securely in 10 channel uses, achieving a secure DoF of 1.2. ■

Remark 3.1. Note that the 3-user SISO-IC in [28] can also attain a DoF of 1.2 in 5 channel uses. However, a trivial extension of the scheme in [28] to the 4-user case requires user-scheduling in which 3 out of 4 users will be selected as a group

on which the scheme in [28] can be applied. Since 4 such groups can be selected, 20 channel uses are required in the trivial extension case.

Remark 3.2. Note that the scheme in [28] requires any pair of users to share some “common” messages, i.e., the obtained DoF is not secure. In fact, every user decodes 2 additional symbols in addition to decoding its own 2 symbols. However, in our proposed method in Theorem 3.3, any user’s message is *completely confidential*, and no user can decode any symbol of any other user.

Remark 3.3. In our proposed scheme, note that the local instantaneous CSIT is required only in the last 4 out of 10 channel uses, i.e., only 40% of the transmission duration. Moreover, it achieves a secure DoF of 1.2 which is higher than the DoF of 1.1842 using completely delayed CSIT in [23]. Also, our scheme achieves its DoF in 10 channel uses only as against extremely large coding lengths in [23]. Thus, our scheme is more suited to fast time-varying channels.

3.2.3 Discussion and Comments

3.2.3.1 Channel feedback overhead

As described above, our proposed transmission scheme for the four-user SISO-IC network model in Section 3.2.2 requires delayed global and only *locally* available instantaneous CSIT to achieve the DoF gains. Furthermore, the local instantaneous CSIT is required only in the last four channel uses whereas delayed or no CSI are fed back in the first six channel uses. As discussed in Section 3.1.4.2 for the X channels, each RX has three feedback states, i.e., no CSI feedback (**N**), delayed CSI feedback (**D**) or instantaneous or perfect CSI feedback (**P**). On careful analysis of the equations (3.25), (3.28), (3.30) and (3.32), it can be shown that the fraction of time that *each* of the four RXs feeds back no, delayed or instantaneous CSI to the TXs, i.e., $\{\lambda_N, \lambda_D, \lambda_P\}$ is given by $\{\frac{4}{10}, \frac{3}{10}, \frac{3}{10}\}$ respectively. Note that while local instantaneous CSI is fed back for the last four channel uses, each RX feeds back the instantaneous CSI for only $\frac{3}{10}$ of the total transmission time.

3.2.3.2 Computational complexity

Since our proposed scheme uses only linear precoding and finite number of channel uses, finite number of arithmetic operations are involved in the precoder and decoder designs. Specifically, during the precoder design at each TX, first, the order-2 symbols are generated in (3.18) which requires two addition and three multiplication operations. Next, we note that no precoding is done in Phase-1 whereas in Phase-2, a careful analysis of the equations (3.25), (3.28), (3.30) and (3.32) shows that one addition and two multiplication/division operations are required for precoding by each TX during three channel uses (see for example, $n = 7$, $n = 8$ and $n = 9$ in (3.25), (3.28), and (3.30) respectively for TX-1).

On the other hand, complexity analysis at the RXs involves two steps. First, as described in our proposed method, each RX has to decode six order-2 unknown symbols from six linearly independent equations. Once the order-2 symbols are decoded at the RX, each RX still needs to decode the three desired private symbols from the three equations (refer to (3.18)) in the second step. Thus, approximately $c_1 \times 6^3$ and $c_2 \times 3^3$ operations (where c_1 and c_2 are some positive constants) are required to solve the linear equations in the steps one and two respectively.

3.2.4 Summary of Section 3.2

In Section 3.2, we presented a space-time coding scheme for the 4-user SISO-IC with hybrid CSIT, i.e., delayed and locally instantaneous CSIT. We showed that we can achieve a secure DoF of 1.2 which is higher than the existing result using completely-delayed CSIT scenario [23]. Further, due to limited amount of local instantaneous CSIT, our proposed method requires much smaller number of channel uses as compared to [23]. Our scheme can also be systematically applied to the 3-user SISO-IC with hybrid CSIT for which it is secure-DoF-optimal.

3.3 A semi-blind interference alignment scheme

In this section, we show that the idea of “interference creation” in one phase and “interference resurrection” in another phase (considered in the previous sections) can also be applied to another important class of network, namely, the multi-input-single-output broadcast channels (MISO-BC) with hybrid CSIT configuration. For the MISO-BC model considered here, we assume that the CSI is heterogeneous across the users. In particular, we consider a special case of MISO-BC with hybrid CSI configuration [24, 154, 155] where the transmitter receives channel feedback from only one receiver (referred to as *supernode*) whereas the other receivers do *not* feed back any CSI to the transmitter. The supernode is called so because it has reconfigurable antennas (capable of switching modes or taking a *finite set of channel values*) as well as a feedback mechanism to the transmitters (Please refer to Figure 3.5 for a 2-user MISO-BC). All other receivers neither have reconfigurable antennas nor provide any kind of feedback to the transmitter.

Figure 3.5: A 2-user MISO-BC with one supernode and one naïve receiver.

A significant difference of our proposed framework from other hybrid CSIT models is that the supernode’s channel assumes its values from a *finite* set of channel modes (instead of continuous values) due to the presence of reconfigurable antennas. We provide a linear precoding scheme using semi-blind interference alignment (SBIA) for this new framework, and show that the DoF gain is higher

than the traditional orthogonal transmission schemes such as TDMA.

Note that completely blind IA (BIA) was proposed in [19] where none of the receivers need to feedback the short-term channel coefficients to the transmitter. However, a major drawback of this scheme is that every receiver needs to have reconfigurable antennas which is difficult in practice. The proposed model in this work is more suitable for practical scenarios where it is not possible for every receiver to have reconfigurable antennas, yet the whole network can gain (in terms of DoF) from the extra hardware at just one receiver. A suitable application scenario is the case when the supernode can be a user in a quasi-static indoor environment or a slowly fading environment where the reconfigurable antennas can fluctuate the channels as desired and can also feedback the channel information quickly and accurately to the transmitters. On the other hand, other receivers can be chosen to be any outdoor user with fast varying channels and no feedback. While there will be a DoF trade-off as compared to the BIA in [19], the DoF achieved using the proposed SBIA coding schemes is higher than the TDMA scheme for all the considered cases in this work. Another advantage of the proposed scheme is the much reduced supersymbol length which implies that our scheme is applicable to much faster time-varying channels as compared to the BIA in [19].

3.3.1 System model

Consider a K -user $M \times 1$ MISO-BC in which a transmitter or base station (BS) with M antennas transmits distinct data to K receivers (users), in which the 1st receiver is a supernode which has a reconfigurable antenna capable of switching between fixed number of finite channel states, and also has a feedback link to the transmitter. All other receivers are simple single-antenna receivers also called naïve receivers which neither possess reconfigurable antennas nor have any feedback mechanism to the transmitter. A two-user 2×1 example is shown in Figure 3.5.

Let $\mathcal{K}$ denote the set of users in the network and $x^{[j]}[n]$ denote the symbol

transmitted from the j^{th} transmit antenna in the n^{th} time-slot which is subject to an average power constraint of P . Then, the signal received at the 1^{st} and the k^{th} , $k \in \mathcal{K} \setminus \{1\}$ receivers in the n^{th} time-slot are given in (3.33) and (3.34) respectively.

$$y^{[1]}[n] = \sum_{j=1}^M h_{1j}(m[n])x^{[j]}[n] + z^{[1]}[n], \quad (3.33)$$

$$y^{[k]}[n] = \sum_{j=1}^M h_{kj}[n]x^{[j]}[n] + z^{[k]}[n], \quad k \in \mathcal{K} \setminus \{1\}, \quad (3.34)$$

where $h_{1j}(m[n])$ and $h_{kj}[n]$ ($k \in \mathcal{K} \setminus 1$) denote the channel from the j^{th} transmit antenna to the 1^{st} and the k^{th} ($k \in \mathcal{K} \setminus 1$) receivers respectively in the n^{th} time-slot, and $z^{[k]}[n] \sim \mathcal{CN}(0, 1)$ denotes the AWGN at the k^{th} receiver in the n^{th} time-slot. Note that the channel to the 1^{st} receiver can assume only finite states (or *modes*) depending on the switching of the reconfigurable antennas, and its mode at time-slot n is given by $m[n]$. Further, the transmission duration for one SBIA code consists of a few time slots (or channel uses) which is called *supersymbol* similar to [19]. For example, in Figure 3.5, the supersymbol length is 3 time slots. The values of $h_{kj}[n]$ are i.i.d. across space and supersymbol blocks, and drawn from a continuous Rayleigh distribution but are assumed to remain constant in the supersymbol during the transmission duration. Similarly, the 1^{st} receiver's channel states (corresponding to its various modes) also remain constant during the supersymbol duration but it can change its modes according to a pre-defined switching pattern, thereby changing its effective channel.

3.3.2 Main results

We provide some definitions followed by the main results.

Definition 3.1. A *supernode* is defined as a receiver which has both reconfigurable antennas as well as a feedback link to the transmitter for feeding back its channel

information. A *naïve* receiver is defined as one which has neither reconfigurable antennas nor any feedback link to the transmitter. A *mode* is defined as a channel state at any receiver with finite channel states [19]. For example, a receiver with a reconfigurable antenna which can switch between 3 channel states has 3 modes.

Theorem 3.4. *In a two-user $K \times 1$ MISO-BC, with one receiver being a supernode having K modes and another receiver being naïve, a DoF of $\frac{2K}{K+1}$ is achievable using linear IA beamforming.*

Corollary 3.1. *For the two-user 2×1 MISO-BC, a DoF of $\frac{4}{3}$ is achievable which is also the DoF attained for the two-user 2×1 MISO-BC in [19] with reconfigurable antennas at both the receivers.*

Corollary 3.2. *The above proposed SBIA is also applicable to the $K \times 2$ X channel⁴ with one supernode receiver and one naïve receiver. Further, the achieved DoF is optimal in this case.*

Corollary 3.3. *In an N -user $K \times 1$ MISO-BC with $3 \leq N \leq (K+1)$, one receiver being a supernode having K modes and other receivers being naïve, a DoF of $\frac{2K}{K+1}$ is achievable in $K+1$ channel uses, with each naïve RX receiving at least one data stream which is distinct from the other users.*

3.3.3 Proofs of the main results

We begin with the *special case* of two-user 2×1 MISO-BC which is also the proof of Corollary 3.1. Then, we provide a proof of Theorem 3.4 followed by the proof of Corollary 3.2 which is closely related to Theorem 3.4. Finally, a short proof of Corollary 3.3 is provided.

3.3.3.1 Two-user 2×1 MISO-BC: Proof of Corollary 3.1

Proof. Consider a two-user MISO-BC as shown in Figure 3.5 where one receiver is a supernode while the other is a naïve receiver. The supernode has a reconfigurable

⁴In a $K \times 2$ X channel, each of the K TXs transmits distinct data to each of the 2 RXs.

antenna which can switch between 2 modes. On the other hand, the naïve receiver can assume any channel value drawn randomly from a continuous distribution. An assumption about the naïve receiver in the presented scheme is that its channel coherence time constitutes 3 time slots which is not quite demanding even for a reasonably fast time-varying channel.

In the proposed coding scheme, the supersymbol duration for the SBIA code is 3 time slots (please see Figure 3.5). Note that the 1st user's channel changes from mode-1 to mode-2 in the 2nd time-slot due to antenna switching, and then remains in the 2nd mode even during the 3rd time-slot. On the other hand, the 2nd or the naïve user's channel remains constant during all the three time slots because its channel coherence time is assumed to be 3 time slots.

Let $u_i^{[z]}$ denote the l^{th} symbol intended for the i^{th} user. The transmission is done in 3 time slots. In the 1st time-slot, 1st TX antenna transmits $x^{[1]}[1] = (u_1^{[1]} + u_1^{[2]})$ and the 2nd antenna transmits $x^{[2]}[1] = (u_2^{[1]} + u_2^{[2]})$. In the 2nd time-slot, 1st and 2nd TX antennas transmit $x^{[1]}[2] = u_1^{[1]}$ and $x^{[2]}[2] = u_2^{[1]}$ respectively. Note that till the 2nd time-slot, the TX does not use any form of channel information for precoding. In the 3rd time-slot, 1st and 2nd TX antennas transmit $x^{[1]}[3] = \alpha[3]u_1^{[2]}$ and $x^{[2]}[3] = \beta[3]u_2^{[2]}$ respectively, where $\alpha[3]$ and $\beta[3]$ are given as

$$\alpha[3] = \frac{h_{11}[1]}{h_{11}[2]}, \quad \beta[3] = \frac{h_{12}[1]}{h_{12}[2]}. \quad (3.35)$$

Note that the precoding in the 3rd time-slot is done based on the fed back channel information of the 1st user only, and the TX is still blind to the 2nd user. Using the above SBIA coding scheme, the signal received at the 1st RX (the noise terms

are neglected as they do not affect the DoF) is given as

$$\mathbf{y}_1 = \underbrace{\begin{bmatrix} h_{11}[1] & h_{12}[1] \\ h_{11}[2] & h_{12}[2] \\ 0 & 0 \end{bmatrix}}_{\text{desired signal}} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} + \underbrace{\begin{bmatrix} h_{11}[1] & h_{12}[1] \\ 0 & 0 \\ \alpha[3]h_{11}[2] & \beta[3]h_{12}[2] \end{bmatrix}}_{\text{interference}} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} \quad (3.36)$$

Using (3.35) in (3.36), it is easy to see that the interfering signals $u_1^{[2]}$ and $u_2^{[2]}$ fall in a common subspace. It can also be checked that the rank of the desired signals subspace is 2, and the desired and the interfering signals' subspaces do not overlap. Thus, the desired symbols for the 1st user can be reliably decoded by *zero-forcing* the 1-D interference term.

Similarly, using the above SBIA scheme, the signal received at the 2nd RX is given as

$$\mathbf{y}_2 = \underbrace{\begin{bmatrix} h_{21}[1] & h_{22}[1] \\ 0 & 0 \\ \alpha[3]h_{21}[1] & \beta[3]h_{22}[1] \end{bmatrix}}_{\text{desired signal}} \begin{bmatrix} u_1^{[2]} \\ u_2^{[2]} \end{bmatrix} + \underbrace{\begin{bmatrix} h_{21}[1] & h_{22}[1] \\ h_{21}[1] & h_{22}[1] \\ 0 & 0 \end{bmatrix}}_{\text{interference}} \begin{bmatrix} u_1^{[1]} \\ u_2^{[1]} \end{bmatrix} \quad (3.37)$$

Similar to the explanation for the 1st user above, 2nd user's symbols can also be decoded reliably. Thus, 4 symbols are transmitted reliably over the network in 3 time slots, thereby achieving a DoF of $\frac{4}{3}$. ■

Remark 3.4. An alternating CSI framework has been proposed for the 2-user MISO-BC in [24] in which various combinations of hybrid CSI considerations are discussed with the channel states of the users alternating over certain fraction of channel uses, achieving significant DoF gains. Unfortunately, this interesting alternating CSIT approach cannot be applied to the cases where one receiver does not have any feedback mechanism to the transmitter.

Remark 3.5. An interesting point to note from (3.35) is that $\alpha[3]$ and $\beta[3]$ values

to be used in precoding in the 3^{rd} time-slot depend on the channel states only up to the 2^{nd} time-slot. Thus, we conclude that with *delayed* CSI feedback from a supernode and no feedback from a naïve RX, we can obtain $\frac{4}{3}$ DoF for our network. This is similar to the (D,N) channel state discussed in Section 3.1.4.2 where “D” and “N” denote the *delayed* and *no* channel state respectively of the RXs 1 and 2.

Remark 3.6. Note that during decoding of the desired symbols at the 2^{nd} RX, the precoding values $\alpha[3]$ and $\beta[3]$ should be known to the 2^{nd} RX (please see (3.37)). However, as discussed above, these values can be provided to the 2^{nd} RX in a *delayed* manner similar to the case in [103]. We want to emphasize that even though these precoding coefficients are computed based on the current (and not necessarily delayed) channel values, these may still not be a problem if proper user-scheduling is done. For example, the supernode can always be chosen to be a slowly varying channel, e.g., an indoor RX, whose channel remains constant for a much longer time as compared to the other naïve users. On the other hand, the channel of the naïve users need to remain constant only for *few* time slots (only 3 time slots in the two-user case) well-suited to a reasonably fast time-varying channel, and they can vary afterwards. In the above scenario, note that the $\alpha[3]$ and $\beta[3]$ precoding values will not change for a long time because the 1^{st} user’s channel remains constant for a long time but these same precoding values can be used to decode their symbols at the naïve users even though their channel changes. The precoding values can be periodically updated based on the channel coherence time of the 1^{st} user, and these updated values (which are only few scalars) can be made available to the naïve receivers.

3.3.3.2 Proof of Theorem 3.4

Proof. Consider a two-user $K \times 1$ MISO-BC where one receiver is a supernode while the other receiver is naïve. The supernode has K modes and has a feedback link to the transmitter while the naïve receiver has neither reconfigurable antennas

nor any feedback link to the transmitter. This model is similar to Figure 3.5 except that the TX has K antennas and the supersymbol duration is $(K+1)$ time slots.

We assume that the channels of the naïve RX remain constant for $K+1$ time slots. On the other hand, the 1^{st} RX changes its mode from 1^{st} time-slot to 2^{nd} time-slot, remains in the same mode in the 2^{nd} and 3^{rd} time slots, and then changes its modes every time-slot beginning from 4^{th} to $(K+1)^{th}$ time-slot as shown in Figure 3.6. Note that this channel staggering pattern may not be unique.

Figure 3.6: Channel staggering pattern due to antenna switching at the supernode in Theorem 3.4.

Using similar notations as in the proof of Corollary 3.1, we provide the SBIA coding scheme for the two-user $K \times 1$ case in Table 3.2, where the precoding values based on the channel information fed back from the 1^{st} receiver only are given as

$$\alpha_n^{[l]} = \frac{h_{1l}[1]}{h_{1l}[n-1]}, \quad l \in \{1, 2, \dots, K\}, n \in \{3, \dots, K+1\}, \quad (3.38)$$

where $\alpha_n^{[l]}$ denotes the precoding coefficient of the l^{th} antenna at time-slot n .

Table 3.2: SBIA coding for two-user $K \times 1$ MISO-BC

TX antenna (l) $\downarrow$	n=1	n=2	n=3	n=4	...	n=k	...	n=K+1
1	$u_1^{[1]} + u_1^{[2]}$	$u_1^{[1]}$	$\alpha_3^{[1]} u_1^{[2]}$	$u_1^{[1]} + \alpha_4^{[1]} u_1^{[2]}$	...	$u_1^{[1]} + \alpha_k^{[1]} u_1^{[2]}$	...	$u_1^{[1]} + \alpha_{K+1}^{[1]} u_1^{[2]}$
2	$u_2^{[1]} + u_2^{[2]}$	$u_2^{[1]}$	$\alpha_3^{[2]} u_2^{[2]}$	$u_2^{[1]} + \alpha_4^{[2]} u_2^{[2]}$	...	$u_1^{[1]} + \alpha_k^{[2]} u_1^{[2]}$	...	$u_1^{[1]} + \alpha_{K+1}^{[2]} u_1^{[2]}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
k	$u_k^{[1]} + u_k^{[2]}$	$u_k^{[1]}$	$\alpha_3^{[k]} u_k^{[2]}$	$u_k^{[1]} + \alpha_4^{[k]} u_k^{[2]}$	...	$u_1^{[1]} + \alpha_k^{[k]} u_1^{[2]}$	...	$u_1^{[1]} + \alpha_{K+1}^{[k]} u_1^{[2]}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	...	$\vdots$	...	$\vdots$
K	$u_K^{[1]} + u_K^{[2]}$	$u_K^{[1]}$	$\alpha_3^{[K]} u_K^{[2]}$	$u_K^{[1]} + \alpha_4^{[K]} u_K^{[2]}$	...	$u_1^{[1]} + \alpha_k^{[K]} u_1^{[2]}$	...	$u_1^{[1]} + \alpha_{K+1}^{[K]} u_1^{[2]}$

For the above scenario, we intend to transmit K data streams, $u_i^{[1]} (i \in \mathcal{K})$, $\mathcal{K} = \{1, 2, \dots, K\}$, to the 1^{st} user and K data streams, $u_i^{[2]} (i \in \mathcal{K})$ to the 2^{nd} user, thereby transmitting $2K$ data streams over the network in $K+1$ time slots. Using

the SBIA coding scheme in Table 3.2, the signal received at the k^{th} ($k \in \mathcal{K}$) RX at high SNR (neglecting the noise terms since they do not affect the DoF) is given as

$$\mathbf{y}_k = \mathbf{H}_1^{[k]} \mathbf{u}^{[1]} + \mathbf{H}_2^{[k]} \mathbf{u}^{[2]}, \quad (3.39)$$

where $\mathbf{u}^{[k]} = [u_1^{[k]}, u_2^{[k]}, \dots, u_K^{[k]}]^T$, $k \in \{1, 2\}$, and the channel matrices $\mathbf{H}_l^{[k]}$, $k, l \in \{1, 2\}$ are given in (3.40)-(3.41).

Decoding at the receivers: Now, consider the 1^{st} user, i.e., $k = 1$ in (3.39). Using SBIA in Table 3.2 and (3.40), it can be checked that all the interfering signals intended for the 2^{nd} user are aligned into one-dimensional subspace at the RX-1, i.e., $\text{rank}(\mathbf{H}_2^{[1]}) = 1$. Thus, RX-1 is left with K subspaces to hold its desired signals, i.e., $\mathbf{u}_i^{[1]} (i \in \mathcal{K})$.

$$\mathbf{H}_1^{[1]} = \begin{bmatrix} h_{11}[1] & h_{12}[1] & \dots & h_{1k}[1] & \dots & h_{1K}[1] \\ h_{11}[2] & h_{12}[2] & \dots & h_{1k}[2] & \dots & h_{1K}[2] \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{11}(K) & h_{12}(K) & \dots & h_{1k}(K) & \dots & h_{1K}(K) \end{bmatrix},$$

$$\mathbf{H}_2^{[1]} = \begin{bmatrix} h_{11}[1] & h_{12}[1] & \dots & h_{1k}[1] & \dots & h_{1K}[1] \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \alpha_3^{[1]} h_{11}[2] & \alpha_3^{[2]} h_{12}[2] & \dots & \alpha_3^{[k]} h_{1k}[2] & \dots & \alpha_3^{[K]} h_{1K}[2] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{K+1}^{[1]} h_{11}(K) & \alpha_{K+1}^{[2]} h_{12}(K) & \dots & \alpha_{K+1}^{[k]} h_{1k}(K) & \dots & \alpha_{K+1}^{[K]} h_{1K}(K) \end{bmatrix}. \quad (3.40)$$

$$\mathbf{H}_2^{[2]} = \begin{bmatrix} h_{21}[1] & h_{22}[1] & \dots & h_{2k}[1] & \dots & h_{2K}[1] \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \alpha_3^{[1]} h_{21}[1] & \alpha_3^{[2]} h_{22}[1] & \dots & \alpha_3^{[k]} h_{2k}[1] & \dots & \alpha_3^{[K]} h_{2K}[1] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha_{K+1}^{[1]} h_{21}[1] & \alpha_{K+1}^{[2]} h_{22}[1] & \dots & \alpha_{K+1}^{[k]} h_{2k}[1] & \dots & \alpha_{K+1}^{[K]} h_{2K}[1] \end{bmatrix},$$

$$\mathbf{H}_1^{[2]} = \begin{bmatrix} h_{21}[1] & h_{22}[1] & \dots & h_{2k}[1] & \dots & h_{2K}[1] \\ h_{21}[1] & h_{22}[1] & \dots & h_{2k}[1] & \dots & h_{2K}[1] \\ 0 & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ h_{21}[1] & h_{22}[1] & \dots & h_{2k}[1] & \dots & h_{2K}[1] \end{bmatrix}. \quad (3.41)$$

It can be further checked that $\text{rank}(\mathbf{H}_1^{[1]}) = K$, and the subspaces occupied by the desired and the interfering signals do not overlap, i.e., $\text{rank}(\mathbf{H}_1^{[1]}, \mathbf{H}_2^{[1]}) = K + 1$. Thus, the desired signals can be reliably decoded at RX-1 by *zero-forcing* the 1-D interference term.

Similarly, it is easy to check that at the RX-2, the interfering signals from the 1st user are aligned into a 1-D subspace, i.e., $\text{rank}(\mathbf{H}_1^{[2]}) = 1$. Also, $\text{rank}(\mathbf{H}_2^{[2]}) = K$ and $\text{rank}(\mathbf{H}_1^{[2]}, \mathbf{H}_2^{[2]}) = K + 1$. Thus, the desired signals can be reliably decoded at RX-2. Thus, overall, $2K$ data streams are transmitted over the network in $K+1$ time slots, thereby achieving a DoF of $\frac{2K}{K+1}$. ■

3.3.3.3 Proof of Corollary 3.2

Proof. Note that in the $K \times 2$ X channel, there is no cooperation among the TX antennas. Thus, there are K distributed single-antenna transmitters, each of which intends to send a message to each of the two single-antenna receivers. Let each RX receive its k^{th} ($k \in \mathcal{K} = \{1, 2, \dots, K\}$) message from the k^{th} ($k \in \mathcal{K}$) TX. Then, following the same convention as in the proof of Theorem 3.4, symbols intended for the RXs 1 and 2, i.e., $u_k^{[1]}$ and $u_k^{[2]}$ respectively, are available at TX- k ($k \in \mathcal{K}$). Using a similar transmission scheme as for the $K \times 2$ MISO-BC in the proof of Theorem 3.4, it can be shown that the j^{th} ($j \in \{1, 2\}$) RX is able to decode its K

desired symbols $u_k^{[j]}$ ($k \in \mathcal{K}$) in $(K+1)$ time slots. Since we know that the DoF of the $K \times 2$ X channels is $\frac{2K}{K+1}$ even with perfect CSIT, the proposed SBIA scheme is already DoF-optimal. ■

3.3.3.4 Proof of Corollary 3.3

Proof. Clearly, the DoF of the N -user $K \times 1$ ($3 \leq N \leq (K+1)$) MISO-BC is lower bounded by the DoF of the two-user $K \times 1$ MISO-BC in Theorem 3.2. For the N -user $K \times 1$ ($3 \leq N \leq (K+1)$) network, the K data streams for the naïve RX in Theorem 3.4 can be equivalently replaced by N naïve users, with K data streams distributed among them such that each naïve RX receives at least one distinct data stream. This follows from the statistical equivalence of the naïve RXs. Note that for $N > (K+1)$, the data streams at any two RXs cannot be distinct anymore. ■

Remark 3.7. From Corollary 3.3, note that for the 3-user symmetric MISO-BC, we obtain a DoF of $\frac{3}{2}$ (which is lesser than $\frac{9}{5}$ in [19] but still 50% higher than the TDMA scheme) only in 4 channel uses as compared to 20 channel uses in [19] with reconfigurable antennas at every RX. Thus, our scheme is more suitable for faster time-varying channels.

Remark 3.8. For the two-user $K \times 1$ MISO-BC, note that the optimal DoF of $\frac{2K}{K+1}$ is obtained with reconfigurable antennas at both the RXs in (K^2-1) channel uses [19]. However, in our proposed SBIA, the same DoF can be obtained in $(K+1)$ channel uses only.

Remark 3.9. We compare our scheme for the 3-user MISO-BC to the one in [25] in which one user feeds back perfect CSI (which can assume any value drawn from a continuous distribution) while other two feed back no CSI to the transmitter. Note that the maximum DoF using linear schemes for the abovementioned model in [25] is unity. However, in our case, using similar structure but finite channel states (using reconfigurable antennas) at RX-1, we get a DoF of at least $\frac{3}{2}$.

3.3.4 Summary of Section 3.3

In Section 3.3, we presented a new framework for semi-blind IA (SBIA) for MISO-BC where only one receiver possesses reconfigurable antennas and has a feedback link to the transmitter (this receiver is called the supernode) while the other receiver(s) do not have any reconfigurable antennas or feedback link. We obtained a DoF of $\frac{2K}{K+1}$ for the two-user $K \times 1$ MISO-BC. When applied to the $K \times 2$ X channel with one supernode and one naïve receiver, our scheme is DoF-optimal. Note that the above proposed model is a special case of hybrid CSIT feedback model where one of the RXs feeds back perfect or delayed CSIT while the other RXs do not feed back any CSI to the TX. As a possible future work, our results can be extended to a general K -user case with more than one supernode.

Chapter 4

Finite-SNR Precoder Design for the Interference Relay Channels without CSI at the source Transmitters

In this chapter, we design the precoders for a relay-aided single-input-single-output interference channel (SISO-IC), also referred to as the SISO interference relay channel (SISO-IRC). A K -user SISO-IRC is a network in which K single-antenna transmitters transmit data to their intended single-antenna receivers simultaneously and at the same frequency, but their transmission is aided by a relay. The motivation for using relay-aided IC is that for the conventional K -user SISO-IC mentioned in [2], we require long symbol extensions (or long coding lengths) together with global CSI feedback over time-varying or frequency-selective channels in order to obtain the maximum DoF of $K/2$. Thus, heavy CSI feedback overhead is required from the receivers to every transmitter which is practically impossible. On the other hand, relay-aided transmission over the IC can greatly reduce the CSI overhead and still achieve the maximum DoF, as mentioned next.

To attain the maximum DoF of $K/2$ for the SISO-ICs, relay-aided IA was first proposed for the interference networks in [54] which required only two channel uses. However, in [54], the channel still needs to be time-varying/frequency-selective and furthermore, the global CSI needs to be available at all the nodes in the network. Note that the DoF of the SISO-IC cannot be increased beyond the $K/2$ outerbound even with the aid of the relay [131], and hence, in the best case, this outer bound should be targeted. Further progress in this direction was made in the work [29, 31] in which it was shown that $K/2$ DoF can be obtained in a K -user SISO-IC in two time slots even without any CSIT. This means that there need not be any channel feedback to the source transmitters from the receivers. However, in this case, the relay must have sufficient number of antennas and needs to know the global CSI of the network.

As mentioned in Chapter 2, the CSI quality is very important for IA precoding and even slight CSI imperfections can result in severe DoF degradation. A major advantage of the scheme proposed in [29] is that since the relay is located closer to the receivers than the transmitters, therefore the CSI quality acquired at the relays is much more accurate than if the channel were fed back to the transmitters. Furthermore, the relays needed for aiding IA for the interference networks are available in practice to coordinate the data transmission, for example, Type-II relays [129] have been proposed for the LTE-Advanced whose purpose is to aid the main base stations to increase the data rate and/or diversity.

Due to the aforementioned reasons, relay-aided transmission is promising for practical implementation of IA. In [29, 31], DoF of the network is the performance metric. As we know that DoF is the system performance measure only in the high SNR region, and in this case, noise enhancement at the relays and practical low-mid SNR region performance are not taken into account. To this end, we propose two finite SNR precoder designs for the relay aided IA, one of which is based on minimizing the sum mean square error (SMSE) of all the received symbols in the

network whereas the other aims to maximize the sum-rate of the network. The system model and the proposed algorithms are mentioned below.

4.1 System Model

A K -user SISO-IC is considered, aided by a MIMO half-duplex relay with $L = (K-1)$ antennas in between the transmitters and the receivers, as shown in Figure 4.1. Note that since the relay has only $(K-1)$ antennas, decode-and-forward (DF) relaying cannot be performed in this case. However, it has been shown in [29] that amplify-and-forward (AF) relaying can be applied easily using linear beamforming techniques such as IA so as to reap the maximum DoF of the network. Each transmitter directly communicates to its corresponding receiver, and at the same time creates interference to other users in the network. Further, since we consider a two-hop communication, all the transmitters send their messages to a common relay in the first time slot. The relay then applies precoding to the received signals and transmits them to all the receivers in the second time slot. Thus, both the direct and the indirect links (via a relay) exist between the transmitters and the receivers and hence, a two-dimensional subspace is observed by the receivers. In the first time slot, the data symbol $d_k \sim \mathcal{CN}(0, 1)$ is transmitted by the k^{th} transmitter, thus the received signal at the k^{th} receiver in the first time slot is given as

$$y_k(1) = \sum_{i=1}^K h_{ki}(1)d_i + n_k(1), \quad (4.1)$$

where, $h_{ki}(1)$ denotes the channel from the i^{th} transmitter to the k^{th} receiver in the first time-slot, and $n_k(1)$ denotes the complex circularly symmetric additive white Gaussian noise (AWGN) distributed as $n_k(1) \sim \mathcal{CN}(0, \sigma_{k1}^2)$. The received signal

Figure 4.1: A relay-aided SISO interference channel with direct link between the transmitters and receivers

at the relay $\mathbf{y}_R(1) \in \mathbb{C}^{L \times 1}$ in the first time slot is given as

$$\mathbf{y}_R(1) = \sum_{i=1}^K \mathbf{h}_{Ri}(1)d_i + \mathbf{n}_R(1), \quad (4.2)$$

where $\mathbf{h}_{Ri}(1) \in \mathbb{C}^{L \times 1}$ denotes the channel from the i^{th} transmitter to the relay in the first time slot, and $\mathbf{n}_R(1) \in \mathbb{C}^{L \times 1}$ denotes the AWGN at the relay, and is distributed as $\mathbf{n}_R(1) \sim \mathcal{CN}(0, \sigma_R^2 \mathbf{I})$. Finally, the relay precodes its received signal $\mathbf{y}_R$ as $\mathbf{x}_R = \mathbf{U}\mathbf{y}_R$ where $\mathbf{U} \in \mathbb{C}^{L \times L}$ is the IA precoder. The relay broadcasts $\mathbf{x}_R$ in the second time-slot to all the receivers with a power constraint on the signal given as $\mathbb{E}\{\|\mathbf{x}_R\|^2\} \leq P_t$. Thus, the signal received at the k^{th} receiver after combining the signals from the direct path in the first time-slot and the indirect path (via the relay) in the second time-slot, is given as

$$\mathbf{y}_k = \begin{bmatrix} y_k(1) \\ y_k(2) \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^K h_{ki}(1)d_i + n_k(1) \\ \mathbf{h}_{kR}^T(2)\mathbf{U} \sum_{i=1}^K \mathbf{h}_{Ri}(1)d_i + \mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{n}_R(1) + n_k(2) \end{bmatrix}, \quad (4.3)$$

where $\mathbf{h}_{kR}(2) \in \mathbb{C}^{L \times 1}$ denotes the channel from the relay to the k^{th} receiver in the second time-slot, and $n_k(2)$ is the AWGN at the k^{th} receiver in the second

time-slot, and is distributed as $n_k(2) \sim \mathcal{CN}(0, \sigma_{k2}^2)$.

4.2 Precoder designs at the relay

In this section, we design the minimum-sum mean square error (MSMSE) and weighted sum-rate (WSR) maximizing precoders for the relay-aided SISO-IC model. While the WSR precoder directly aims to maximize the network sum-rate, it does not yield good BER performance because its design is based on the Gaussian inputs [156] instead of the actual finite constellation inputs. On the other hand, although the MSMSE precoder design is not exactly related to the sum-rate, it considers the noise powers and the desired signal links in its design, and improves the sum-rate significantly in the low-mid SNR regions. Also, its sum-rate performance can be further improved over wide SNR regions using suitable initializations (as shown later). In addition, the MSMSE precoder intends to reduce the errors in the received symbols, and thus definitely reduces the BER in all the SNR regions (Note that the MSMSE objective is closely related to the BER of the network [157])¹.

4.2.1 The MSMSE precoder and decoder designs

In this section, we describe the MSMSE precoder and decoder designs which aim to minimize the SMSE of all the received symbols in the network. Following the system model in Section 4.1, the received signal at the k^{th} receiver in (4.3) can

¹Note that the precoder designs which directly aim to minimize the system BER for finite constellation inputs can be quite difficult to design primarily due to the intractability of the analytical BER expression for multi-user networks. On the other hand, MSMSE precoder impacts the BER directly and its related minimum sum MSE objective minimization admits highly tractable solutions based on solving convex optimization problem using block coordinate descent methods.

be re-written as

$$\mathbf{y}_k = \sum_{i=1}^K \underbrace{\begin{bmatrix} a_{ki} & \mathbf{0}_{1 \times R^2} \\ 0 & \mathbf{b}_{ki}^T \end{bmatrix}}_{\mathbf{A}_{ki}} \underbrace{\begin{bmatrix} 1 \\ \mathbf{u} \end{bmatrix}}_{\bar{\mathbf{u}}} d_i + \underbrace{\begin{bmatrix} c_k & \mathbf{0}_{1 \times R^2} \\ 0 & \mathbf{d}_k^T \end{bmatrix}}_{\mathbf{C}^{(k)}} \underbrace{\begin{bmatrix} 1 \\ \mathbf{u} \end{bmatrix}}_{\bar{\mathbf{u}}} + \underbrace{\begin{bmatrix} 0 \\ e_k \end{bmatrix}}_{\mathbf{e}}, \quad (4.4)$$

where, $a_{ki} = h_{ki}(1)$, $\mathbf{b}_{ki} = \mathbf{h}_{Ri}(1) \otimes \mathbf{h}_{kR}(2)$, $c_k = n_k(1)$, $\mathbf{d}_k = \mathbf{n}_R(1) \otimes \mathbf{h}_{kR}(2)$, $e_k = n_k(2)$.

$$\therefore \mathbf{y}_k = \sum_i \mathbf{A}_{ki} \bar{\mathbf{u}} d_i + \mathbf{C}^{(k)} \bar{\mathbf{u}} + \mathbf{e} = \mathbf{A}_k \bar{\mathbf{u}} + \mathbf{C}^{(k)} \bar{\mathbf{u}} + \mathbf{e}, \quad (4.5)$$

where $\mathbf{A}_k = \sum_i \mathbf{A}_{ki} d_i$, $\bar{\mathbf{u}} = [1, \text{vec}(\mathbf{U})]^T$. After being processed by the receive filter $\mathbf{w}_k \in \mathbb{C}^{2 \times 1}$ at the k^{th} receiver, the received symbol is given as

$$\hat{d}_k = \mathbf{w}_k^H \mathbf{y}_k = \mathbf{w}_k^H (\mathbf{A}_k \bar{\mathbf{u}} + \mathbf{C}^{(k)} \bar{\mathbf{u}} + \mathbf{e}). \quad (4.6)$$

The sum mean square error (SMSE) of the received symbols of all users is given as

$$SMSE = \sum_{k=1}^K \mathbb{E} \left\{ \left| d_k - \hat{d}_k \right|^2 \right\} \quad (4.7)$$

Substituting $\hat{d}_k$ in (4.6) into (4.7), and using the equality $\mathbb{E}\{d_i^* d_j\} = 0$ for $i \neq j$, SMSE is obtained as

$$\mathcal{S} = \underbrace{\sum_k 1 - 2\mathcal{R} \{ \bar{\mathbf{u}}^H \mathbf{A}_{kk}^H \mathbf{w}_k \} + \mathbf{w}_k^H \left(\sum_j \mathbf{A}_{kj} \bar{\mathbf{u}} \bar{\mathbf{u}}^H \mathbf{A}_{kj}^H + \mathbf{C}_u^{(k)} + \mathbf{Q}_e \right) \mathbf{w}_k}_{\varepsilon_k} \quad (4.8)$$

where,

$$\begin{aligned} \mathbf{C}_u^{(k)} &= \mathbb{E}\{\mathbf{C}^{(k)} \bar{\mathbf{u}} \bar{\mathbf{u}}^H \mathbf{C}^{(k)H}\} = \text{diag} \left(\left[\sigma_{k1}^2, \sum_{i=2}^{L^2+1} |\bar{\mathbf{u}}(i)|^2 \sigma_R^2 \left| \tilde{\mathbf{h}}_{kR}^{(i-1)}(2) \right|^2 \right] \right), \\ \tilde{\mathbf{h}}_{kR}^{(i)}(2) &= \mathbf{h}_{kR}^{[(i-1) \bmod L] + 1}(2), \quad \mathbf{Q}_e = \text{diag}([0, \sigma_{k2}^2]), \end{aligned} \quad (4.9)$$

and $\tilde{\mathbf{h}}_{kR}^{(l)}(n)$ denotes the $(\text{mod}(l-1, L) + 1)^{th}$ element of the channel vector $\mathbf{h}_{kR}(n)$.

After simplifications, SMSE (4.8) can also be written as

$$\begin{aligned} \mathcal{S} = & \sum_{k=1}^K \left[1 + |\mathbf{w}_k(1)|^2 \sum_{i=1}^K |a_{ki}|^2 + |\mathbf{w}_k(2)|^2 \sum_{i=1}^K |\mathbf{b}_{ki}^T \mathbf{u}|^2 + 2\Re \left\{ \mathbf{w}_k(1) \mathbf{w}_k^*(2) \sum_{i=1}^K a_{ki}^* \mathbf{b}_{ki}^T \mathbf{u} \right\} \right. \\ & \left. + \left(|\mathbf{w}_k(1)|^2 + |\mathbf{w}_k(2)|^2 \right) \sigma_k^2 + \mathbb{E} \left\{ |\mathbf{w}_k^*(2) \mathbf{g}_k^T \mathbf{u}|^2 \right\} - 2\Re \left\{ \mathbf{w}_k^*(1) a_{kk} + \mathbf{w}_k^*(2) \mathbf{b}_{kk}^T \mathbf{u} \right\} \right], \end{aligned} \quad (4.10)$$

where $\mathbf{u} = \text{vec}(\mathbf{U}) \in \mathbb{C}^{L^2 \times 1}$, and the noise variances at the k^{th} receiver are assumed to be same over two time slots, $\sigma_{k1}^2 = \sigma_{k2}^2 = \sigma_k^2$.

The power transmitted by the relay can be obtained as

$$\mathbf{u}^H \underbrace{\left[\left(\sum_{i=1}^K \mathbf{h}_{Ri}(1) \mathbf{h}_{Ri}^H(1) + \sigma_R^2 \mathbf{I}_R \right) \otimes \mathbf{I}_R \right]}_{\mathbf{D}} \mathbf{u}. \quad (4.11)$$

Note that $\mathbf{D}$ is a positive-definite matrix (please refer to Appendix at the end of the chapter). Thus, the power in (4.11) is a convex quadratic function of $\mathbf{u}$. Using (4.11) and the power constraint P_t at the relay, the MSMSE optimization problem can be written as

$$\begin{aligned} & \underset{\mathbf{u}, \mathbf{w}_k \forall k \in \{1, 2, \dots, K\}}{\text{minimize}} \quad \mathcal{S} \\ & \text{subject to} \quad \mathbf{u}^H \mathbf{D} \mathbf{u} \leq P_t. \end{aligned} \quad (4.12)$$

Note that the SMSE objective in (4.8) is not jointly convex with respect to (w.r.t.) the transmit and receive filters $\mathbf{u}$ and $\mathbf{w}_k$. Therefore, global optimal solution of the above problem is difficult to obtain. However, if either $\mathbf{u}$ or $\mathbf{w}_k \forall k$ is kept fixed, the objective SMSE is partially convex w.r.t. $\mathbf{w}_k \forall k$ or $\mathbf{u}$ respectively. In this case, locally optimal solutions are possible by alternating minimization of the

SMSE objective over the relay transmit filter $\mathbf{u}$ (keeping all the receive filters $\mathbf{w}_k$ fixed) and then the receive filters $\mathbf{w}_k \forall k$ (keeping the relay transmit filter $\mathbf{u}$ fixed) and so on in an iterative manner, as discussed later. The alternating optimization can be proved to be convergent as discussed in Section 4.2.3. The algorithm can be stopped when fixed number of iterations is reached or when the decrement in SMSE is not significant.

The above sum mean square error (SMSE) minimization problem can be solved by forming the Lagrange function, incorporating the power constraint at the relay, and is given as

$$\mathcal{L} = \mathcal{S} + \lambda (\mathbf{u}^H \mathbf{D} \mathbf{u} - P_t), \quad (4.13)$$

The Karush-Kuhn-Tucker (KKT) conditions to be satisfied at the optimal points are given as

$$\begin{aligned} \frac{\partial L}{\partial \mathbf{u}^H} &= 0, \quad \frac{\partial L}{\partial \mathbf{w}_k^H} = 0, \\ \lambda &\geq 0, \quad \mathbf{u}^H \mathbf{D} \mathbf{u} \leq P_t, \quad \lambda \left\{ \mathbf{u}^H \mathbf{D} \mathbf{u} - P_t \right\} = 0, \end{aligned} \quad (4.14)$$

Using the above KKT condition $\frac{\partial L}{\partial \mathbf{u}^H} = 0$, we get the following expression for the relay precoder.

$$\mathbf{u} = (\mathbf{T} + \lambda \mathbf{D})^{-1} \mathbf{t}, \quad (4.15)$$

where,

$$\mathbf{T} = \sum_{k=1}^K \sum_{i=1}^K |\mathbf{w}_k(2)|^2 \mathbf{b}_{ki}^* \mathbf{b}_{ki}^T + \sum_{k=1}^K |\mathbf{w}_k(2)|^2 \tilde{\mathbf{J}}_k, \quad (4.16)$$

$$\tilde{\mathbf{J}} = \text{blkdiag}(\mathbf{J}_k, \mathbf{J}_k), \mathbf{J}_k^T = \mathbf{h}_{kR} \mathbf{h}_{kR}^H \sigma_{nR}^2, \quad (4.17)$$

$$\mathbf{t} = - \sum_{k=1}^K \sum_{i=1}^K \mathbf{w}_k^*(1) \mathbf{w}_k(2) a_{ki} \mathbf{b}_{ki}^* + \sum_{k=1}^K \mathbf{w}_k(2) \mathbf{b}_{kk}^* \quad (4.18)$$

Note that in (4.16), $\mathbf{T}$ is an invertible matrix. This can be easily seen because the first term in $\mathbf{T}$ consists of summation of K^2 different rank-1 matrices (since $\mathbf{b}_{ki}$ is a vector and each of its elements are chosen from a continuous distribution, and further each of the $\mathbf{b}_{ki}$'s are independent of each other). So, even if λ is 0, the matrix $\mathbf{T} + \lambda \mathbf{D}$ is full-rank and hence invertible.

Proposition 4.1. *The power transmitted by the relay f is a decreasing function of λ and is given as*

$$f = \sum_{i=1}^K \frac{[\mathbf{X}]_{ii}}{([\boldsymbol{\Sigma}]_{ii} + \lambda)^2}, \quad (4.19)$$

where $\boldsymbol{\Sigma}$ is the singular value matrix formed by the SVD decomposition of $\mathbf{D}^{-1} \mathbf{T}$, and $\mathbf{X}$ is

$$\mathbf{X} = \mathbf{V}_L^{-1} \mathbf{D}^{-1} \mathbf{t} \mathbf{t}^H \mathbf{V}_R \quad (4.20)$$

with the matrices $\mathbf{V}_L$ and $\mathbf{V}_R$ formed by the left and right singular vectors of $\mathbf{D}^{-1} \mathbf{T}$, respectively.

Proof: Please refer to Appendix at the end of the chapter.

Thus the lagrange multiplier λ in (4.15) can be computed easily by solving the equation $f = P_t$ using bisection search method [158]. If λ is negative, it is set to zero so as to satisfy the non-negativity of λ in the KKT condition (4.14).

In this case, the transmit power of the relay is lesser than the maximum power limit P_t . Similarly, from the KKT conditions in (4.14) and (4.8), the receive filter $\mathbf{w}_k \forall k \in \mathcal{K} = \{1, 2, \dots, K\}$ can be obtained as

$$\mathbf{w}_k = (\mathbf{A}_d + \mathbf{C}_u^{(k)} + \mathbf{Q}_e)^{-1} \mathbf{A}_{kj} \bar{\mathbf{u}}, \quad (4.21)$$

where $\bar{\mathbf{u}} = [1 \ \mathbf{u}]^T$, $\mathbf{A}_d = \sum_{j=1}^K \mathbf{A}_{kj} \bar{\mathbf{u}} \bar{\mathbf{u}}^H \mathbf{A}_{kj}^H$, and the remaining variables are defined in (4.4) and (4.9). The SMSE minimization problem to find the locally optimal precoder and decoders are mentioned in Algorithm 1. Here, $\mathbf{u}^{(l)}$ and $\mathbf{w}_k^{(l)}$ denote the precoder and the decoder in the l^{th} iteration.

Algorithm 1 MSMSE Precoder design algorithm

- 1: Initialize the relay precoder $\mathbf{u}^{(0)}$ to a suitable vector of dimension $L^2 \times 1$.
 - 2: Set a pre-defined counter limit, lim . Set threshold $\tau = 10^{-6}$.
 - 3: $i \leftarrow 1$
 - 4: **while** $i \leq lim$ or $\Delta SMSE > \tau$ **do**
 - 5: Compute the receive filters $\mathbf{w}_k^{(i)} \forall k \in \mathcal{K}$ using (4.21).
 - 6: Compute the Lagrange multiplier λ using $f = P_t$ from (4.19).
 - 7: Update the relay precoder $\mathbf{u}^{(i)}$ (while the receive filters $\mathbf{w}_k$ are fixed) using (4.15).
 - 8: $\Delta SMSE = SMSE^{(i-1)} - SMSE^{(i)}$
 - 9: $i \leftarrow i + 1$
 - 10: **end while**
 - 11: Compute the final receive filters $\mathbf{w}_k^{(final)} \forall k \in \mathcal{K}$ using (4.21).
-

4.2.2 Computational complexity

The main computational complexity of the MSMSE precoder design proposed above comes from the matrix inverse operation in (4.15). In (4.15), it can be noted that the matrix $\mathbf{T} + \lambda \mathbf{D}$ is of size $L^2 \times L^2$, where L is the number of relay antennas. Thus, the matrix inverse operation in (4.15) requires $\mathcal{O}((L^2)^3) \approx \mathcal{O}(L^6)$ arithmetic floating point operations [159]. Next, it can be shown that the number of operations involved in computing $\mathbf{T}$ and $\mathbf{t}$ in (4.16) and (4.18) respectively are of the order $K^2 \mathcal{O}(L^4)$ and $K^2 \mathcal{O}(L^2)$ respectively. Since $L = (K - 1)$, the complexity in computing $(\mathbf{T} + \lambda \mathbf{D})^{-1}$, $\mathbf{T}$ and $\mathbf{t}$ become of the order $\mathcal{O}(K^6)$, $\mathcal{O}(K^6)$ and $\mathcal{O}(K^4)$

respectively. Moreover, the multiplication of the inverted matrix $(\mathbf{T} + \lambda \mathbf{D})$ with $\mathbf{t}$ in (4.15) involves complexity of the order $\mathcal{O}(K^4)$. Combining all the above major operations in computing $\mathbf{u}$ in (4.15), the overall complexity is dominated by $\mathcal{O}(K^6)$ operations.

Furthermore, as shown in Algorithm 1, the precoder computation also requires the computation of λ (refer to (4.19)) which, in turn, requires the computation of $\mathbf{X}$ in (4.20). Note that λ in (4.19) is solved by simple bisection search, and the major complexity comes from the SVD operation of $\mathbf{D}^{-1}\mathbf{T}$ (refer to Proposition 4.1) which requires $\mathcal{O}(K^6)$ operations. Finally, note that the above computations are performed for a single iteration of the MSMSE algorithm. If n iterations are involved in Algorithm 1, the overall complexity in the precoder design is given by $n \times \mathcal{O}(K^6)$.

Next, we consider the complexity of the k^{th} user's receive filter design (refer to (4.21)). Note that in this case, the complexity is not dominated by the matrix inversion term in (4.21) because $(\mathbf{A}_d + \mathbf{C}_u^{(k)} + \mathbf{Q}_e)$ in (4.21) is a 2×2 matrix, and is independent of the number of users K . Here, the main computational complexity comes from the computation of the inner term $\mathbf{A}_d$ which requires $\mathcal{O}(K^3)$ operations (counting the matrix multiplications and additions). Finally, the overall complexity of the receive filter design at each user can be shown to be $n \times \mathcal{O}(K^3)$, where n is the number of iterations involved in the MSMSE algorithm.

4.2.3 Convergence of the MSMSE algorithm

It is easy to see that the SMSE objective in (4.10) decreases with every iteration and is thus, convergent since it is also bounded below by zero. However, the iterative MSMSE algorithm, which is a block coordinate descent (BCD) method [158, 160], gives a local optimal solution if a useful stationary point is obtained at every iteration [30, 161]. Note that in (4.12), the power constraint involves only one relay precoder variable and there are no other coupling constraints [161, 162].

Thus, the SMSE objective in (4.10) is differentiable with separable constraint set in $\mathbf{u}$ and $\mathbf{w}_k$. Furthermore, the convergence to a stationary point also requires that the minimizer at every BCD step is uniquely attained [158] which follows from the positive definite (PD) property of the matrices $\mathbf{T}$ and $\mathbf{D}$ (as discussed earlier). Similarly, the decoder expression in (4.21) gives a unique solution since it can be easily shown that $(\mathbf{A}_d + \Upsilon_k + \mathbf{Q}_e)$ is PD and invertible. From the above conditions, the iterative MSMSE algorithm gives us a useful stationary solution at every iteration.

As mentioned in Algorithm 1, there are two ways to terminate the above iterative algorithm, i.e., either fix the number of iterations to a certain number, or stop the algorithm after the decrement in the SMSE objective over two consecutive steps becomes significantly small. It should be noted that a large number of iterations leads to high computational complexity. However, we show next that with suitable initialization, we can get good solutions with very few iterations, that give higher throughput and better BER than the IA precoder design.

4.2.4 Relay power control and Scaled IA initialization

For the iterative precoder design proposed in the previous section, we need to initialize the precoder at the beginning of the algorithm. Since the proposed MSMSE precoder is only locally optimal in the sense of system throughput, the precoder initialization affects its sum-rate and BER performances significantly. For example, when the precoder is initialized randomly, it is observed that the MSMSE algorithm requires a lot of iterations to attain the maximum DoF of $K/2$. In [30, 160], it has been suggested that multiple initializations be performed in the context of interference and interfering broadcast channels and finally, the precoder initialization that gives the best sum-rate solution is selected. However, this approach is computationally very expensive and does not guarantee the maximum network DoF or the maximum sum-rate at high SNR. Next, we present a modified

IA algorithm under power constraint at the relay. Based on the power-constrained IA, we propose to use scaled-IA initialization for the MSMSE algorithm which attains better BER performance than the conventional IA precoding while consuming much lesser relay power.

4.2.4.1 Power control for IA precoder design

From (4.3), the IA conditions [31] for the k^{th} receiver (after neglecting the noise terms) can be written as

$$\frac{\mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Ri}(1)}{h_{ki}(1)} = \frac{\mathbf{h}_{kR}^T(2)\mathbf{U}\mathbf{h}_{Rj}(1)}{h_{kj}(1)}, \text{ for } k \neq i, j; i \neq j; i, j, k \in \mathcal{K}. \quad (4.22)$$

Accumulating the IA equations for all the users, and incorporating the relay power control, the optimization problem can be written as

$$\begin{aligned} & \underset{\mathbf{u}_{IA}}{\text{minimize}} \quad \|\mathbf{H}\mathbf{u}_{IA}\| \\ & \text{subject to} \quad \mathbf{u}_{IA}^H \mathbf{D} \mathbf{u}_{IA} = P_t, \end{aligned} \quad (4.23)$$

where $\mathbf{u}_{IA} = \text{vec}(\mathbf{U})$, $\mathbf{H} \in \mathbb{C}^{K(K-2) \times L^2}$, and is defined as

$$\mathbf{H} = \begin{bmatrix} \mathbf{i}_{13}; \mathbf{i}_{14}; \dots; \mathbf{i}_{1K}; \mathbf{i}_{23}; \mathbf{i}_{24}; \dots; \mathbf{i}_{2K}; \dots; \mathbf{i}_{K2}; \mathbf{i}_{K3}; \dots; \mathbf{i}_{K,(K-1)} \end{bmatrix}, \quad (4.24)$$

and $\mathbf{i}_{kj} \in \mathbb{C}^{1 \times L^2}$ are defined as

$$\mathbf{i}_{kj} = \frac{\mathbf{h}_{Rm}(1)^T \otimes \mathbf{h}_{iR}^T(2)}{h_{km}(1)} - \frac{\mathbf{h}_{Rj}(1)^T \otimes \mathbf{h}_{kR}^T(2)}{h_{kj}(1)}, \quad k \in \mathcal{K}; \quad m=2 \text{ if } k=1, m=1 \text{ otherwise}; \quad j \neq m, k. \quad (4.25)$$

For details, readers are referred to [31]. Note that in (4.23), we seek to minimize the so-called interference misalignment term $\|\mathbf{H}\mathbf{u}_{IA}\|$ under the relay power constraint, P_t . The inequality constraint cannot be applied here since it would

result in a trivial solution of all zeros vector for the relay precoder, and so the constraint involves power equality constraint. Note that for any arbitrary number of antennas at the relay, (4.23) is a generalized eigenvalue problem whose solution is given as

$$\mathbf{u}_{IA}^{(opt)} = \sqrt{P_t} \mathbf{D}^{-\frac{1}{2}} \mathbf{V}^{\min}(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}}) \quad (4.26)$$

where $\mathbf{V}^{\min}(\mathbf{X})$ denotes the eigenvector corresponding to the minimum eigenvalue of the matrix $\mathbf{X}$, and $\tilde{\mathbf{H}} = \sqrt{P_t} \mathbf{H} \mathbf{D}^{-\frac{1}{2}}$. However, for the IA-feasible network [31, 52], i.e., $L = (K-1)$ as in our case, it is easy to see from (4.23) that $\mathbf{u}_{IA}$ lies in the null-space of $\mathbf{H}$, and the objective and the constraints become separable. *In this case, it can be seen from (4.22) that simply scaling the relay precoder $\mathbf{U}$ to $\alpha \mathbf{U}$ for some $\alpha > 0$ so as to satisfy a power constraint at the relay, still satisfies the IA conditions.*

4.2.4.2 Scaled-IA precoder initialization

As mentioned earlier, MSMSE algorithm can attain only locally optimal solution which does not guarantee the maximum network DoF or even better BER performance as compared to IA precoding. However, in an IA-feasible network [31, 52], an alternative option is to initialize the precoder as the IA precoder. In this case, as the MSMSE algorithm updates its solution locally in the vicinity of the IA precoder, it almost retains the network DoF, and at the same time, ensures better BER performance as compared to the IA algorithm. However, it is observed through our numerical results that although the MSMSE algorithm imposes a power constraint on the relay, it always operates close to the maximum available relay power with both random and IA initializations. To solve this problem, we propose a *scaled*-IA initialization which saves the relay power significantly for the MSMSE algorithm, and yet performs better than the conventional IA in terms of the average BER of the network and preserves the network DoF.

Essentially, a scaled-IA does not affect the alignment of interference but only scales the power of the precoder. The scaled-IA precoder with power constraint P_t at the relay can be obtained as

$$\mathbf{u}_{PC} = \frac{\sqrt{P_t} \mathbf{u}_{IA}}{\sqrt{\mathbf{u}_{IA}^H \mathbf{D} \mathbf{u}_{IA}}}. \quad (4.27)$$

The idea is to initialize the precoder in Algorithm 1 to a scaled-IA precoder where the power scaling depends on the amount of power we want to save at the relay. This is explained as follows.

First, define $\mathcal{U}$ as the set of all IA precoders with power constraints ranging from 0 to ∞ as

$$\mathcal{U} = \{\mathbf{u}_{IA} | \mathbf{u}_{IA}^H \mathbf{D} \mathbf{u}_{IA} = P, P \in (0, \infty)\}. \quad (4.28)$$

Any precoder in $\mathcal{U}$ (including the case when $P \rightarrow 0$) is DoF-optimal, and hence, in order to save the relay power, we just need to choose a precoder $\mathbf{u}_{IA, P_c} \in \mathcal{U}$ satisfying a suitable power constraint P_c , i.e., $\mathbf{u}_{IA, P_c}^H \mathbf{D} \mathbf{u}_{IA, P_c} = P_c$. While choosing a very low power P_c still maintains the DoF, it will result in significant sum-rate loss. Thus, a desirable value of P_c should be chosen so that DoF is maintained without much loss in the sum-rate.

For example, let the maximum power limit at the relay be P_t . We choose an IA precoder $\mathbf{u}_{IA, \frac{P_t}{2}}$ from the set $\mathcal{U}$ which consumes half the maximum available power, i.e., $\mathbf{u}_{IA, \frac{P_t}{2}}^H \mathbf{D} \mathbf{u}_{IA, \frac{P_t}{2}} = \frac{P_t}{2}$. The MSMSE algorithm in Algorithm 1 is then initialized with this scaled-IA precoder $\mathbf{u}_{IA, \frac{P_t}{2}}$ and starts searching for the locally optimal solution in the vicinity of $\mathbf{u}_{IA, \frac{P_t}{2}}$. The algorithm, because of its limitation to get trapped into the local solution after few iterations, converges towards a solution which expends only slightly more power than $\frac{P_t}{2}$ (but still much lesser power than the maximum allowable power P_t) while showing very little DoF loss.

It is important to note that the maximum allowable relay power is still kept at P_t so that the algorithm has the flexibility to utilize the full transmission power when necessary such as in the noise-limited regions where increasing the power also increases the system sum-rate. The scaled-IA initialization method's reaping significant power savings are illustrated via numerical results in Section 4.3. Also, in Section 4.3, we discuss how to choose an appropriate value of the initial power, P_c .

4.2.5 Weighted sum rate maximizing precoder design

In this section, the weighted sum rate (WSR) maximizing precoder is designed for the relay-aided SISO-IC which aims to maximize the network sum-rate when the users are given different priorities (or weights). The sum-rate expression for the relay-aided SISO-IC is given as

$$S = \frac{1}{2} \sum_{k=1}^K \underbrace{\log_2(1 + \rho_k)}_{S_k}, \quad (4.29)$$

where ρ_k denotes the signal-to-interference plus noise ratio (SINR) of the k^{th} user and can be obtained from (4.5)-(4.6) as

$$\rho_k = \frac{|\mathbf{w}_k^H \mathbf{A}_{kk} \bar{\mathbf{u}}|^2}{\mathbf{w}_k^H \left(\sum_{i \neq k} \mathbf{A}_{ki} \bar{\mathbf{u}} \bar{\mathbf{u}}^H \mathbf{A}_{ki}^H + \Upsilon_k + \mathbf{Q}_e \right) \mathbf{w}_k}. \quad (4.30)$$

WSR is simply defined by modifying (4.29) as $\mathcal{R} = \frac{1}{2} \sum_{k=1}^K \alpha_k S_k$ where $\alpha_k > 0$ denotes the weight associated with the k^{th} user. Design of weighted sum rate maximizing precoder is a celebrated problem in literature. Various techniques have been proposed for WSR maximization in different networks [160, 163–167] but it is an NP-hard problem in general [168]. Recently, by establishing connection between the weighted sum mean square error (WSMSE) minimization and WSR

maximization problems, locally optimal solutions have been derived for the broadcast, interference and interfering broadcast networks in [163, 164], [165–167], and [160] respectively. We utilize this property to solve the WSR maximization in the context of our network model. Similar to the WSR expression, WSMSE can be written as $\mathcal{E} = \sum_k \beta_k \varepsilon_k$, where $\beta_k > 0$ denotes the weight associated with the MSE of the k^{th} user. Next, we show that a similar relationship between WSR and WSMSE exists for the IRC also, and the so-designed locally optimal WSR precoder achieves higher sum-rate than the IA and the MSMSE precoders as shown in Section 4.3.

By using (4.8) and (4.30), it can be shown that the SINR of the k^{th} user and its MSE ε_k are related by $\frac{1}{\varepsilon_k} = 1 + \rho_k$. Considering the power constraint at the relay, the Lagrange function for the WSR and the WSMSE are given in (4.31) and (4.32), respectively.

$$\mathcal{L}_{WSR} = \mathcal{R} + \lambda(\tilde{\mathbf{u}}^H \tilde{\mathbf{D}} \tilde{\mathbf{u}} - P_t) + \mu(\tilde{\mathbf{1}}^T \tilde{\mathbf{u}} - 1), \quad (4.31)$$

$$\mathcal{L}_{WSMSE} = \mathcal{E} + \lambda(\tilde{\mathbf{u}}^H \tilde{\mathbf{D}} \tilde{\mathbf{u}} - P_t) + \mu(\tilde{\mathbf{1}}^T \tilde{\mathbf{u}} - 1), \quad (4.32)$$

where λ and μ are the Lagrange multipliers, $\tilde{\mathbf{1}} = [1 \ \mathbf{0}_{1 \times L^2}]^T$, $\tilde{\mathbf{1}}^T \tilde{\mathbf{u}} = 1$ indicates that the first element of $\tilde{\mathbf{u}}$ is 1 (refer to (4.5)), and

$$\tilde{\mathbf{D}} = \begin{bmatrix} \mathbf{0}_{1 \times (R^2+1)} \\ \mathbf{0}_{R^2 \times 1}, \quad \mathbf{D} \end{bmatrix}.$$

By setting the gradients of (4.31) and (4.32) w.r.t. $\tilde{\mathbf{u}}^H$ to zero [169], and using the equality $\frac{1}{\varepsilon_k} = 1 + \rho_k$, we get

$$\underbrace{\frac{\log_2 e}{2}}_{\delta} \sum_k \frac{\alpha_k}{\varepsilon_k} \frac{\partial \varepsilon_k}{\partial \tilde{\mathbf{u}}^*} + \lambda \mathbf{D} \tilde{\mathbf{u}} = 0, \quad (4.33)$$

$$\sum_k \beta_k \frac{\partial \varepsilon_k}{\partial \tilde{\mathbf{u}}^*} + \lambda \mathbf{D} \tilde{\mathbf{u}} = 0, \quad (4.34)$$

respectively. Using (4.33) and (4.34), the stationary points of the WSMSE and WSR optimization problems are the same if the weights of the MSE, β_k are set as

$$\beta_k = \delta \alpha_k \varepsilon_k^{-1}, \quad (4.35)$$

where δ is shown in (4.33). By setting the weights appropriately as obtained in (4.35), we can maximize the WSR by solving the WSMSE minimization problem equivalently [165]. The WSMSE problem can be solved iteratively in a similar manner as the MSMSE algorithm with alternate optimization of the relay precoder, the decoders and the weights, β_k (4.35) as described in Algorithm 2. Note that the relay precoder $\mathbf{u}$ for the WSMSE can be obtained using (4.15) with β_k weighting each of the summands in $\mathbf{T}$ and $\mathbf{t}$ in (4.16) and (4.18), respectively. The receive filters $\mathbf{w}_k$ can again be obtained from (4.21) without any change. Algorithm 1 can be modified for the WSR precoder as follows. The steps 5 and 6 in Algorithm 1 are replaced with steps 5' and 6' in Algorithm 2 while all the other steps remain the same. Convergence of the WSR algorithm can be proved for the current relay-

Algorithm 2 WSR Precoder design algorithm

- 5': Compute the receive filters $\mathbf{w}_k^{(i)} \forall k \in \mathcal{K}$ using (4.21), and also, compute the weights $\beta_k^{(i)}$ (at the i^{th} iteration) using (4.35).
 - 6': Compute the modified $\mathbf{T}$ and $\mathbf{t}$ from (4.16) and (4.18) respectively by multiplying each summand in (4.16) and (4.18) by $\beta_k^{(i)}$, and then compute the Lagrange multiplier λ using $f = P_t$ from (4.19).
-

assisted SISO-IC on similar lines as in [163].

4.3 Numerical results

In this section, the simulation results under various scenarios for the IA, MSMSE and the WSR precoder design algorithms are presented. Each user has a single antenna and a MIMO relay is placed between the transmitters and the receivers. Since each transmitter has only one antenna, it transmits only one da-

ta stream per channel use. Further, all transmitters transmit with equal powers and no precoding is involved at the transmitters as they do not have any channel state information (blind transmitters). Perfect CSI at the relay and the receivers is assumed.

We assume a time-invariant channel in which all channel coefficients are i.i.d. and drawn from a complex normal distribution with mean zero and unit variance. We consider two types of network in our simulations, symmetric and asymmetric networks. In symmetric networks, the average desired and interfering signal strengths at each receiver are assumed to be the same while in asymmetric networks, the average desired and interfering signals are assumed to be of unequal strengths as in many practical scenarios which may be due to different path losses and shadowing effects of various links. The sum-rate and BER are computed by averaging over 1000 channel realizations. For the alternating precoder and decoder updates, 50 iterations are allowed unless stated otherwise. Throughout the results, SNR denotes the transmit SNR of the user given as $\text{SNR} = \frac{\text{transmit power}}{\text{noise power at the receiver}}$. In all simulations, the weight of each user in the WSR algorithm, α_k is set to unity, i.e., all the users are given equal priorities. The transmit power at each transmitter is assumed to be unity. Also, for the IA-PC, MSMSE and WSR algorithms, the transmit power at the relay is constrained by unity. With IA initializations for the iterative algorithms, the initial power P_c (refer to Section 4.2.4.2) is set to unity in all our simulations except in Figure 4.8 where simulation results are shown for different values of P_c . The noise powers at the receivers and the relay are assumed to be the same.

The sum rates of the IA, MSMSE and the WSR algorithms are presented in Figure 4.2 for a symmetric network. Note that the “IA-noPC” plot is the sum-rate curve for the conventional IA algorithm [31] without power constraint at the relay while “IA-PC” plot is the sum-rate curve for IA with power control as introduced in Section 4.2.4.1. Two types of initializations are considered for the WSR and the

MSMSE precoders, IA and random precoder initializations. The WSR algorithm with IA initialization gives the best sum-rate in the finite SNR range, followed by the MSMSE and the IA algorithms. In the high SNR range, its sum-rate converges to the IA-PC sum-rate curve as seen in Figure 4.2. On the other hand, with random precoder initialization, both the MSMSE and WSR algorithms give better sum rates than the IA algorithm in the low-mid SNR range but their performances degrade in the high SNR range.

Figure 4.2: Sum-rate comparison of various algorithms for a three-user relay-aided SISO-IC.

It is worth noting that the rate of convergence of the WSR algorithm with random initializations becomes slow after few initial iterations and therefore, 1000 iterations are allowed for WSR with random initializations so that significant sum-rate improvement can be obtained (although at the cost of much higher computational complexity). However, we note that it still achieves lower sum-rate than the WSR with IA initialization. This is because with random initialization, the WSR algorithm starts optimizing its objective in an iterative manner, and eventually gets trapped in a local solution in the neighbourhood of the random point. Since the random initialization is not DoF-optimal, it is highly likely that the final local solution does not align the interference and the DoF degrades severely due

to the large interference leakage. However, with IA or scaled-IA initialization, the algorithm finally converges to a local point in the vicinity of the IA solution, which retains the DoF-optimality. Interestingly, WSR with IA initialization converges to near-optimal local solution only in few iterations (e.g., 10 iterations suffice in general), and shows little improvement after 50 iterations.

Additionally, we show the sum-rate plots for scenarios when the relay has three antennas. In these cases, the relay can adopt the decode-and-forward (DF) approach to decode the symbols received from all the users and then broadcast them to their intended receivers, effectively resulting in a MISO-BC if the symbols are decoded perfectly. Figure 4.2 shows that using the minimum-sum-mean-squared error minimization of the received symbols for the MISO-BC, one additional antenna can give around 2 bits per sec per Hertz sum-rate gain compared to the two-antenna MSMSE scheme in the high SNR region. However, note that the DoF of the network remains the same.

There are few points worth mentioning here. First, using only two antennas (as in our network model) clearly saves one antenna at the relay, providing the same DoF and comparable sum-rate in the finite SNR regions. This extra antenna could be used for transmitting a data stream to another user or for providing diversity. Another advantage of our model is that AF relaying avoids the complexity (due to decoding) of the DF relaying. However, the downside of our model is that the two-antenna case requires the availability of the CSI of all the links at the relay. Second, our simulation of the three-antenna case is overly optimistic in the sense that perfect decoding of all the symbols is assumed at the relay. For example, in the case of asymmetric network in which the relay is far from the transmitters, the links from the transmitters to the relay are weak resulting in large decoding errors at the relay. Thus, apart from saving an antenna resource, the usefulness of the MSMSE schemes with two antennas at the relay becomes evident in the asymmetric networks. In Figure 4.2, we also show the sum-rate performance for

the three-antenna MISO-BC using zero-forcing (ZF) precoding at the relay.

Figure 4.3 shows the BER performance of the algorithms for various constellation sizes. It is observed that for the proposed model with the relay having only two antennas, and for low to mid constellation sizes, the MSMSE shows better BER performance in the entire SNR range. The better BER performance of the MSMSE can be explained as it takes the desired channel also into account for the precoder design and additionally, it accounts for the noise at the receivers and the noise added at the relay. However, it is observed that as the data rate is increased, the BER performance of IA becomes closer to or better than the MSMSE algorithm, especially at high SNR. This may be because the MSMSE algorithm is unable to decode the desired symbol from the high data rate interfering signals.

Figure 4.3: Uncoded BER performances of the algorithms for various constellation sizes.

Interestingly, although the WSR algorithm is better than others in terms of sum-rate, it has poorer BER performance ($\sim 2 - 3$ dB SNR loss at a BER of 10^{-3} or lower for QPSK modulation) than the MSMSE precoder, especially for the lower-order constellations. The mismatch in the trends of sum-rate and BER for WSR is due to the fact that the WSR and all other precoders are designed based on Gaussian input signaling [170]. However, the BER simulations are obtained using

finite alphabet signaling. As shown in [170] for the IC, if all the precoders were derived based on finite alphabet signaling, WSR should give the best performance both in terms of sum-rate as well as BER. In fact, even IA beamforming (which is DoF-optimal for Gaussian inputs) is found to be sub-optimal in terms of sum-rate for finite alphabet signaling as compared to the optimal sum-rate maximizing precoder design specific to the signal constellation. This is because for finite alphabets, the sum-rate is bounded. Thus, providing extra power to the saturated signal [170] and reserving half of the signal subspace explicitly for interference signals are not the optimal transmission strategies. Based on the above arguments, while design of the finite alphabet based WSR precoder could indeed give the best sum-rate as well as the best BER results, such a design may be much more involved for the relay interference channels and may be considered as an interesting future work.²

Similar to Figure 4.2, we compare the algorithms in our system model with a MISO-BC with the relay having three antennas viz. “BC-ZF” i.e. MISO-BC with ZF precoding and “BC-MSE” i.e. MISO-BC with minimum mean-square-error precoding. Clearly, adding one additional antenna shows significant BER improvement compared to our best MSMSE scheme (with two antenna case), especially above 20 dB and higher. Also, the performance gap increases as the constellation size increases. Thus, there is a trade-off between the number of antennas and the BER performance in IRC.

Figure 4.4 shows a comparison of the sum-rate versus the uncoded average BER of the users based on Figures 4.2 and 4.3. It can be observed that at lower data rates, MSMSE outperforms IA significantly. However, when the data rate increases, BER performance of the IA approaches close to that of the MSMSE

²Another interesting point to note from Figures 4.2 and 4.3 is that as the constellation size grows (for example, 32-PSK), the mismatch between the sum-rate and BER trends of the WSR precoder reduces, as compared to smaller constellations’ scenario. One possible explanation is that the large constellations are closer to Gaussian signaling [171], and hence, Gaussian-input based precoders approach closer to the optimal precoder performance. However, in this work, we obtain only locally optimal precoders and therefore, the exact reasons for the abovementioned higher constellation behavior needs deeper investigations in the future.

algorithm and at very high data rates, the uncoded BER of the IA may even become better than the MSMSE algorithm. This is because when the data rate increases, interference reduction becomes a more prominent factor and since IA precoder is specifically entailed to reduce the interference leakage, its performance becomes close to the MSMSE algorithm and even outperforms at higher data rates and higher SNR. Further, note that the WSR algorithm which aims to maximize the sum-rate, shows slightly better BER performance than the others at high data rates and high SNR. Thus, in applications requiring higher data rates, IA algorithm is more suitable as it shows similar BER performance with much lower computational complexity (because no iterations are involved in IA precoder design).

Figure 4.4: Data rate vs. uncoded BER performances of the algorithms.

Figure 4.5 shows the sum-rate comparison of the algorithms at different SNRs with varying number of users in the network. As the number of users K in the network increases, we use $(K-1)$ antennas at the relay correspondingly. A similar performance trend as in Figure 4.2 is verified through our simulations where the WSR algorithm shows the best sum-rate performance among all the discussed algorithms, followed by the MSMSE and the IA algorithms in the low-mid SNR regions. Also note that in the mid-SNR regime, the sum-rate gain by the proposed

algorithms significantly outperforms IA as the number of users increases.

Figure 4.5: Sum-rate performances of algorithms vs. number of users at various SNR values.

Figure 4.6 shows the sum rates of an asymmetric network where the relay is located closer to the receivers than the transmitters. Thus, the links from the relay to the receivers are strong while the links from the transmitter to the relay are weak. We assume a scenario where the transmitters and the receivers are located far away from each other, and thus the link strengths from the transmitters to the receivers are weak due to the large path losses. In this case, the IA-PC algorithm performs better than the conventional IA-noPC algorithm. This is because the links from the transmitters to the relay are weak and therefore, the power of the symbol received by the relay in the 1st time slot is low. Furthermore, unlike the IA-PC algorithm which uses the full power transmission, the IA-noPC algorithm does not have the flexibility to scale the power during the broadcast phase in the 2nd time slot. Note that MSMSE and WSR algorithms have the flexibility to utilize the powers at the relay upto the maximum allowable power and hence, achieve higher sum-rate compared to other methods.

In Figure 4.7, we compare the power consumption at the relay for the various schemes in symmetric scenario, with random and IA precoder initializations. The

Figure 4.6: Sum rates of algorithms in asymmetric network when the relay is close to the receivers.

empirical cumulative distribution function (CDF) of power consumed at the relay is plotted for each channel realization at 40 dB SNR. The random initializer is taken to be any vector $\mathbf{u} \in \mathbb{C}^{L^2 \times 1}$ with each element chosen randomly from a normal distribution with zero mean and unit variance. Note that the power consumption of the IA-noPC algorithm shows large fluctuations. With random initialization, the power consumption of the MSMSE algorithm assumes values lower than or equal to unity. However, with IA initialization, the power consumption is almost always close to unity for the MSMSE algorithm. Note that the power is always fixed at unity for the IA-PC algorithm.

Figure 4.8 shows the BER comparison of the benchmark IA and the best algorithm, i.e., MSMSE algorithm for different power values of scaled-IA initializations using 16-QAM modulation. Note that the BER performances of the scaled-IA based MSMSE algorithm are superior to those of the IA algorithm at all SNR values while consuming much lesser power. But it is not immediately clear what should be the appropriate power initialization, i.e., P_c (refer to Section 4.2.4.2) for the MSMSE algorithm so that a specified value of BER is attained at a given

Figure 4.7: CDFs of the power consumed at the relay for a symmetric network. Total available power at the relay, P_t is unity or 0 dB for MSMSE algorithm.

SNR. This is because it is difficult to characterize the BER expression due to the iterative nature of the algorithm. However, here we adopt a practical approach to decide an approximate value of P_c . Note from Figure 4.8 that if the MSMSE algorithm is initialized with $P_c = -6$ dB, at 50 dB transmit SNR, it almost overlaps the BER of the IA algorithm with $\mathbf{u}^H \mathbf{D} \mathbf{u} = 0$ dB and the final power consumed by the MSMSE algorithm (at the end of the iteration) is found to be -5.88 dB. Thus, for the same BER performance at 50 dB, 5.88 dB power savings is attained with the MSMSE precoder.

To decide a good value of P_c , the BER curve of the IA algorithm is taken as the benchmark. We consider the IA BER at high SNR, say 50 dB and note its value. Now, for the MSMSE algorithm, we start with $P_c = 0$ dB, -2 dB, -4 dB, -6 dB etc. and for each P_c , we compute the MSMSE precoder and compute the BER at 50 dB SNR. From the above P_c values, we choose the “best” value, i.e., which gives closest to the IA BER value at 50 dB while expending the least power, $P_c^{(min)}$ (i.e. minimum P_c value). Note that for all the SNR values below 50 dB, BER of MSMSE is definitely better than IA BER because MSMSE is inherently superior in the noisy region. Thus, better BER is guaranteed for this value of P_c

Figure 4.8: BER performances of MSMSE and IA algorithms for different values of scaled-IA initializations. 16-QAM modulation is used.

for all SNR values upto 50 dB. Hence $P_c^{(min)}$ is an approximate good value for power initialization for the scaled-IA based MSMSE precoder. Note that in the abovementioned method, we need to choose the “best” P_c from a number of values as mentioned above and MSMSE precoder and BER needs to be computed for each of these values. However, this needs to be done only at a particular SNR value, e.g., 50 dB in the above example, and better than IA BER performance is guaranteed at all of the lower SNR values with considerable power savings, especially in the mid-to-high SNR regions. In the low SNR region, MSMSE consumes close to 0 dB (i.e., equal to the power used by the IA algorithm) in order to overcome the detrimental effects of noise.

Figure 4.9 shows the convergence of the algorithms at various SNR values. As discussed in Section 4.2.4, the convergence of the algorithms depends heavily on the precoder initialization. While it takes lot of iterations to converge to a stable local solution with random initializations, it takes very few, i.e., only around 2-10 iterations to reach the locally optimal point with IA initialization. Also, note that with random initialization, the WSR shows the fastest convergence rate as observed from the figure. However, with IA initialization, the convergence rate of

Figure 4.9: Sum rate convergence behaviour (averaged over 100 channel realizations) for MSMSE (blue dashed) and WSR (red solid) algorithms, for various SNR values and for IA and random precoder initializations. The IA and random initializers are marked in the figure for 30 dB and 40 dB, they are not shown for 10 and 20 dB as their sum rate values are very close.

the MSMSE and the WSR algorithms are almost the same. Clearly, IA or scaled-IA initialization save much of the computational complexity due to the lesser number of iterations.

4.4 Summary

In this chapter, we designed minimum-sum-mean-square-error (MSMSE) and weighted sum rate (WSR) maximizing precoders for a relay-aided SISO interference channel with blind transmitters and global CSI at the relay. These precoders outperform the IA precoders in terms of sum-rate in the low and mid SNR regions. Also, their BER performances are greatly improved for the fixed data rates, especially for the low to mid constellation sizes. Further, we proposed a scaled-IA precoder initialization for our iterative algorithms which not only shows better BER performance than IA precoder but also shows faster convergence compared to random initializations and saves the relay power significantly.

Appendix

Positive-definiteness of $\mathbf{D}$ in (4.11)

Note that $\mathbf{D}$ in (4.11) can be written as

$$\mathbf{D} = \sum_{i=1}^K \mathbf{B}_i^H \mathbf{B}_i + \sigma^2 \mathbf{I}_{K^2}, \quad (4.36)$$

where $\mathbf{B}_i = \mathbf{h}_{Ri}^T \otimes \mathbf{I}_K$, and $\mathbf{B}_i^H \mathbf{B}_i \succeq 0$ due to its structure [172]. Thus, $\mathbf{D}$ has a summation of positive semi-definite (PSD) matrices. Note that for PSDs chosen randomly (as in our case), the probability $Pr\{\exists z \mid z^* \mathbf{B}_i^H \mathbf{B}_i z = 0 \wedge z^* \mathbf{B}_j^H \mathbf{B}_j z = 0, i \neq j\} = 0$. Hence, in our case, summation of PSDs is almost always a positive definite matrix. The noise matrix summand in (4.36) only strengthens the positive definiteness property. ■

Proof of Proposition 4.1

The power transmitted by the relay can be written as

$$\begin{aligned} \mathbf{u}^H \mathbf{D} \mathbf{u} &= \mathbf{t}^H (\mathbf{T} + \lambda \mathbf{D})^{-1H} \mathbf{D} (\mathbf{T} + \lambda \mathbf{D})^{-1} \mathbf{t} \\ &\stackrel{(a)}{=} \text{tr}[\mathbf{t}^H (\mathbf{D}^{-1} \mathbf{T} + \lambda \mathbf{I})^{-2} \mathbf{D}^{-1} \mathbf{t}] \end{aligned} \quad (4.37)$$

$$\stackrel{(b)}{=} \text{tr}[\mathbf{t}^H (\mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^H + \lambda \mathbf{V}_L \mathbf{V}_L^H)^{-2} \mathbf{D}^{-1} \mathbf{t}] \quad (4.38)$$

$$\begin{aligned} &= \text{tr}[\mathbf{V}_L^{-1} \mathbf{D}^{-1} \mathbf{t} \mathbf{t}^H \mathbf{V}_R (\mathbf{\Sigma} + \lambda \mathbf{I})^{-1}] \\ &\stackrel{(c)}{=} \sum_i \frac{[\mathbf{V}_L^{-1} \mathbf{D}^{-1} \mathbf{t} \mathbf{t}^H \mathbf{V}_R]_{ii}}{[\mathbf{\Sigma}]_{ii} + \lambda} \end{aligned} \quad (4.39)$$

where (a) is obtained using the fact that $\mathbf{D} = \mathbf{D}^H$ and $\mathbf{T} = \mathbf{T}^H$ and some manipulations. (b) is obtained using SVD decomposition $\mathbf{D}^{-1} \mathbf{T} = \mathbf{V}_L \mathbf{\Sigma} \mathbf{V}_R^H$ where $\mathbf{\Sigma}$ is a diagonal matrix consisting of the singular values, and $\mathbf{V}_L$ and $\mathbf{V}_R$ are the matrices consisting of the left and the right eigenvectors respectively, corresponding to the singular values. (c) is obtained after some manipulations to (4.38). Here $[\mathbf{X}]_{ii}$ denotes the element in the i^{th} row and i^{th} column of the matrix $\mathbf{X}$. ■

Chapter 5

Precoder Designs for the relay-aided X Channel without source CSI

In this chapter, we consider the relay-aided wireless X channels with limited or no CSI at the source transmitters. The X channel (XC) is more generalized form of wireless network (as compared to the interference channels (IC) in Chapter 4) where every transmitter sends a message to every receiver in the network. A typical X network scenario is the multi-cell coordinated transmission where every base station has a distinct message for its own receiver as well as receivers in other cells also. It has been shown in [42, 43] that the DoF of the XC is higher than the IC having the same network topology, and the DoF gap is more pronounced for small and moderately-sized network topologies. Moreover, similar to the DoF achievability scheme in IC, interference alignment (IA) beamforming [1, 43] can achieve the maximum network DoF of certain classes of X networks.

However, in all the above cases, perfect CSI is assumed at all the distributed transmitters which incurs heavy channel overhead. Furthermore, IA precoding is

possible only with infinitely long channel extensions over time-varying or frequency-selective channels only. Note that these problems of channel overhead, requirement of channel variations and long coding lengths are similar to the SISO-IC case, and therefore, relay-aided transmission (as in IC) seems a possible solution to tackle the above challenges. Recently, relay-aided transmission using IA beamforming was proposed for the time-varying X channels [31, 173] where the maximum network DoF can be attained only in finite number of time slots (or channel uses), thereby, greatly reducing the CSI feedback overhead to the transmitters. However, precoder designs in the noise-limited relay-aided X networks, irrespective of channel variations (i.e., both time-varying and time-invariant channels), and under relay power constraints, are practical problems of interest that have not been addressed before.

We address the above issues in this work where we derive the IA feasibility conditions (specifically, the *sufficient* conditions on network dimensions needed for IA) and DoF achievability scheme for a relay-aided symmetric MIMO-XC (also referred to as MIMO-X relay channels or “MIMO-XRC”) without any CSI at the source transmitters and without any necessity of channel variations. Next, we propose an IA-based transmitter-relay *joint* beamforming approach with very limited feedback to the transmitters (referred to as “IA-LFB” in this paper) which can attain the maximum network DoF under arbitrarily low relay power. Finally, we propose an iterative (but having closed-form designs at every iteration) minimum-sum mean-square-error (MMSE) precoder design for the general K -user relay-aided SISO-XC (also referred to as SISO-X relay channels or “SISO-XRC”) for throughput and BER improvement in the finite signal-to-noise ratio (SNR) region. Since the MMSE precoder can achieve only locally optimal solutions, we propose to initialize the iterative MMSE algorithm with the IA-LFB precoder which, unlike random initializations, not only improves the throughput in the finite SNR regions but also preserves the network DoF even under limited relay power.

5.1 System model

In this section, we describe the system model for the symmetric¹ MIMO-XRC. While this system model will be used in proving our first main result for a more general MIMO-XRC in Section 5.2, we will focus on the special case of SISO-XRC for our proposed precoder² designs in Sections 5.4 and 5.5. Please note that the results for the SISO-XRC in the subsequent sections can also be extended to the symmetric MIMO-XRC model but this extension is not considered in our current work. The system model for SISO-XRC with necessary modifications and other details, is provided in Section 5.3.

We consider a relay-aided K -user MIMO-X channel with no CSIT as shown in Figure 5.1 where each transmitter and each receiver has M antennas and the relay has L antennas. Since the transmitters are blind, the CSI is made available only to the relay. A $(2K-1)$ -phase transmission scheme similar to [174] is adopted but with variations in the last $(K-1)$ phases (or time slots) due to which time variations of the channel is not a necessity, and IA can be achieved under extremely limited power at the relay.

Figure 5.1: A K -user X channel aided by a MIMO relay

In the 1^{st} K time-slots, every transmitter broadcasts its data while the receivers

¹Symmetric implies that every transmitter and every receiver has same number of antennas.

²precoder is also referred to as transmit filter in this work.

and the relay listen. In the i^{th} time-slot, $i \in \mathcal{I} = \{1, 2, \dots, K\}$, the j^{th} transmitter, $j \in \mathcal{K} = \{1, 2, \dots, K\}$ transmits its data streams for the i^{th} receiver, i.e., $\mathbf{x}_{ij} \in \mathbb{C}^{M \times 1}$, $\mathbf{x}_{ij} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. The symbols received by the k^{th} receiver, $k \in \mathcal{K}$, and the relay in the i^{th} time-slot are given in (5.1) and (5.2) respectively.

$$\mathbf{y}_k(i) = \sum_{j=1}^K \mathbf{H}_{kj}(i) \mathbf{x}_{ij} + \mathbf{n}_k(i), \quad i \in \mathcal{I} = \{1, 2, \dots, K\}, \quad (5.1)$$

$$\mathbf{y}_R(i) = \sum_{j=1}^K \mathbf{H}_{Rj}(i) \mathbf{x}_{ij} + \mathbf{n}_R(i), \quad i \in \mathcal{I} = \{1, 2, \dots, K\}, \quad (5.2)$$

where $\mathbf{H}_{kj} \in \mathbb{C}^{M \times M}$ and $\mathbf{H}_{Rj} \in \mathbb{C}^{L \times M}$ denote the channel matrices from the j^{th} transmitter to the k^{th} receiver and the relay respectively, and $\mathbf{n}_k(i) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$ and $\mathbf{n}_R(i) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_L)$ denote the AWGN vectors. In the next $(K-1)$ time-slots, for each time-slot $i \in \mathcal{I}' = \{K+1, K+2, \dots, 2K-1\}$, the relay precodes the signals received over the previous K time-slots as

$$\mathbf{x}_R(i) = \sum_{n=1}^K \mathbf{U}_{in} \mathbf{y}_R(n), \quad i \in \mathcal{I}' = \{K+1, 2, \dots, 2K-1\}, \quad (5.3)$$

where $\mathbf{U}_{in} \in \mathbb{C}^{L \times L}$ denotes the precoding matrix in the i^{th} time-slot corresponding to the data in the n^{th} time-slot, and broadcasts $\mathbf{x}_R(i)$ to the receivers. In the same i^{th} ($i \in \mathcal{I}'$) time-slot, the $\tilde{k}^{th}$ transmitter, where $\tilde{k} = (i-K)$, transmits the following symbol,

$$\mathbf{x}_{\tilde{k}}(i) = \epsilon \sum_{j=1}^K \mathbf{x}_{j\tilde{k}}, \quad (5.4)$$

where the real scalar value $\epsilon > 0$ will be used for IA under power constraint at the relay, and its use will become evident in Section 5.4.

The signal received by the k^{th} receiver in the i^{th} ($i \in \mathcal{I}'$) time-slot is given as

$$\mathbf{y}_k(i) = \mathbf{H}_{k\tilde{k}} \mathbf{x}_{\tilde{k}}(i) + \mathbf{H}_{kR}(i) \mathbf{x}_R(i) + \mathbf{n}_k(i), \quad i \in \mathcal{I}', \quad (5.5)$$

where $\mathbf{H}_{k\tilde{k}} \in \mathbb{C}^{M \times M}$ and $\mathbf{H}_{kR} \in \mathbb{C}^{M \times L}$ denote the channel matrices to the k^{th} receiver from the $\tilde{k}^{th}$ transmitter and the relay respectively, and $\mathbf{n}_k(i) \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_M)$. Finally, the signals received at the k^{th} receiver over the $(2K-1)$ time-slots can be written as $\mathbf{y}_k = [\mathbf{y}(1), \mathbf{y}(2), \dots, \mathbf{y}(2K-1)]^T$

$$= \underbrace{\sum_{i=1}^K \mathbf{F}_{ki}^{(k)} \mathbf{x}_{ki}}_{\text{Desired signals}} + \underbrace{\sum_{\substack{j=1 \\ j \neq k}}^K \sum_{i=1}^K \mathbf{F}_{ji}^{(k)} \mathbf{x}_{ji}}_{\text{Interfering signals}} + \tilde{\mathbf{n}}_k, \quad (5.6)$$

where $\tilde{\mathbf{n}}_k \in \mathbb{C}^{(2K-1)M \times 1}$ is the noise vector which includes the noise amplified at the relay and the noise at the k^{th} receiver (note that we do not elaborate on the noise here because it is not required in the DoF analysis), and the effective channel matrix $\mathbf{F}_{ji}^{(k)} \in \mathbb{C}^{(2K-1)M \times M}$, $j \in \mathcal{K}$, $i \in \mathcal{I}$, is given as

$$\begin{bmatrix} \mathbf{0}_{(j-1)M \times M} \\ \mathbf{H}_{ki}(j) \\ \mathbf{0}_{(K-j)M \times M} \\ \epsilon \delta_{t_{K+1}, i} \mathbf{H}_{kt_{K+1}}(K+1) + \mathbf{H}_{kR}(K+1) \mathbf{U}_{(K+1)j} \mathbf{H}_{Ri}(j) \\ \vdots \\ \epsilon \delta_{t_{2K-1}, i} \mathbf{H}_{kt_{2K-1}}(2K-1) + \mathbf{H}_{kR}(2K-1) \mathbf{U}_{(2K-1)j} \mathbf{H}_{Ri}(j) \end{bmatrix}, \quad (5.7)$$

where $t_j = j-K$, and $\delta_{j,i} = 1$ if $j = i$, $\delta_{j,i} = 0$ otherwise.

5.2 IA feasibility conditions for Relay-aided MIMO-X channels

Before moving on to our joint beamforming design under limited power at the relay, and the finite-SNR precoder designs for the XRC, we extend the scheme for the SISO-XRC in [31] to the symmetric MIMO-XRC case and derive a sufficiency condition on the number of antennas required at the relay for aligning the interference in MIMO-XRC. As mentioned in the system model in Section 5.1, in addition

to the IA feasibility analysis, we modify the original scheme in [31] in two more important ways. *Firstly*, in the last $(K-1)$ time slots, the transmitters take turns in transmitting the messages (see (5.4)) which enable us to apply our scheme to both time-varying and time-invariant XRC (unlike [31] where time variation of the channel is a necessity). *Secondly*, the transmitters transmit suitably scaled messages (with the help of a single scalar feedback from the relay to the transmitters, as shown in Section 5.4) so that maximum DoF using IA is attained even when the relay has very low power budget. We present the main result in Theorem 1 as follows.

Theorem 5.1. *For a K -user MIMO-X channel with M antennas at every transmitter and every receiver, and aided by a MIMO relay and no CSI at the source transmitters, $(K-1)M$ antennas at the relay are sufficient to attain the maximum system DoF $\frac{K^2M}{2K-1}$ in either time-varying or time-invariant channels.*

Proof. Consider the system model defined in Section 5.1 (see (5.1)-(5.7)). As seen from (5.6) and (5.7), for alignment of interference at the k^{th} receiver, the interfering signals intended for the other $(K-1)$ receivers, i.e., $\mathbf{x}_{ji}, j \neq k, i \in \mathcal{I}$ should be confined in a $(K-1)M$ dimensional subspace as observed by the k^{th} receiver so that KM dimensions are left for the k^{th} user's desired signals, i.e., $\mathbf{x}_{ki}, i \in \mathcal{I}$. One possible way for IA is that all the K interfering signals intended for the receiver- $j, j \neq k$, should span at most M dimensions at the k^{th} receiver. For this case, a sufficient condition for IA is given as (see (5.6))

$$\text{span} \left(\mathbf{F}_{ji}^{(k)} \right) = \text{span} \left(\mathbf{F}_{jl}^{(k)} \right), k \neq j; l \neq i. \quad (5.8)$$

Note that from the expression of $\mathbf{F}_{ji}^{(k)}, \mathbf{H}_{ki}(j)$ in (5.7) is almost always a full-rank matrix for both time-varying and time-invariant channels, and hence, invertible. Therefore, post-multiplication of $\mathbf{F}_{ji}^{(k)}$ by $\mathbf{H}_{ki}(j)$ does not change its signal subspace

and re-writing (5.8), we get the following IA conditions.

$$\text{span}\left(\mathbf{F}_{ji}^{(k)} \mathbf{H}_{ki}^{-1}(j)\right) = \text{span}\left(\mathbf{F}_{jl}^{(k)} \mathbf{H}_{kl}^{-1}(j)\right), k \neq j; l \neq i. \quad (5.9)$$

As can be seen from (5.7) and (5.9), one sufficient condition for IA is that starting from the $(M(K + m' - 1) + 1)^{th}$ term, $m' = 1, 2, \dots, (K - 1)$, each sub-matrix (of dimension $M \times M$) of the matrix $\mathbf{F}_{ji}^{(k)} \mathbf{H}_{ki}^{-1}(j)$ must be equal to the corresponding sub-matrix term of the matrix $\mathbf{F}_{jl}^{(k)} \mathbf{H}_{kl}^{-1}(j)$, i.e., for the k^{th} receiver, $k \in \mathcal{K}, j \neq k, i \neq l$, and $m \in \mathcal{I}'$,

$$\begin{aligned} & \epsilon \delta_{t_m, i} \mathbf{H}_{kt_m}(m) \mathbf{H}_{ki}^{-1}(j) + \mathbf{H}_{kR}(m) \mathbf{U}_{mj} \mathbf{H}_{Ri}(j) \mathbf{H}_{ki}^{-1}(j) \\ &= \epsilon \delta_{t_m, l} \mathbf{H}_{kt_m}(m) \mathbf{H}_{kl}^{-1}(j) + \mathbf{H}_{kR}(m) \mathbf{U}_{mj} \mathbf{H}_{Rl}(j) \mathbf{H}_{kl}^{-1}(j). \end{aligned} \quad (5.10)$$

Applying the $\text{vec}()$ operator on both sides of (5.10) and re-arranging the terms, we get the following for $k \neq j, i \neq l$.

$$\left[\tilde{\mathbf{H}}_{ji}^{(k)}(m) - \tilde{\mathbf{H}}_{jl}^{(k)}(m) \right] \mathbf{u}_{mj} = \mathbf{c}_{jil}^{(k)}(m), \quad (5.11)$$

where $\tilde{\mathbf{H}}_{ji}^{(k)}(m) = [\mathbf{H}_{Ri}(j) \mathbf{H}_{ki}^{-1}(j)]^T \otimes \mathbf{H}_{kR}(m)$, $\mathbf{c}_{jil}^{(k)}(m) = \epsilon \delta_{t_m, l} \text{vec}(\mathbf{H}_{kt_m}(m) \mathbf{H}_{kl}^{-1}(j)) - \epsilon \delta_{t_m, i} \text{vec}(\mathbf{H}_{kt_m}(m) \mathbf{H}_{ki}^{-1}(j))$, $\mathbf{u}_{mj} = \text{vec}(\mathbf{U}_{mj})$. Note that for every $\mathbf{u}_{mj}$, there are $(K - 1)^2$ matrix equations as in (5.11). Now, using $i = K$ in (5.11), define the following matrices.

$$\mathbf{M}_j^{(k)}(m) = \begin{bmatrix} \tilde{\mathbf{H}}_{jK}^{(k)}(m) - \tilde{\mathbf{H}}_{j1}^{(k)}(m) \\ \tilde{\mathbf{H}}_{jK}^{(k)}(m) - \tilde{\mathbf{H}}_{j2}^{(k)}(m) \\ \vdots \\ \tilde{\mathbf{H}}_{jK}^{(k)}(m) - \tilde{\mathbf{H}}_{j(K-1)}^{(k)}(m) \end{bmatrix}, \quad \mathbf{c}_j^{(k)}(m) = \begin{bmatrix} \mathbf{c}_{jK1}^{(k)}(m) \\ \mathbf{c}_{jK2}^{(k)}(m) \\ \vdots \\ \mathbf{c}_{jK(K-1)}^{(k)}(m) \end{bmatrix}. \quad (5.12)$$

Collecting all the equations in (5.11) which are associated with $\mathbf{u}_{mj}$ for a given

$j \in \mathcal{I}$ and $m \in \mathcal{I}'$, ($k \neq j, l \neq i$), and using (5.12), we get the following.

$$\underbrace{\begin{bmatrix} \mathbf{M}_j^{(1)}(m) \\ \vdots \\ \mathbf{M}_j^{(j-1)}(m) \\ \mathbf{M}_j^{(j+1)}(m) \\ \vdots \\ \mathbf{M}_j^{(K)}(m) \end{bmatrix}}_{\mathbf{M}_j(m)} \mathbf{u}_{mj} = \underbrace{\begin{bmatrix} \mathbf{c}_j^{(1)}(m) \\ \vdots \\ \mathbf{c}_j^{(j-1)}(m) \\ \mathbf{c}_j^{(j+1)}(m) \\ \vdots \\ \mathbf{c}_j^{(K)}(m) \end{bmatrix}}_{\mathbf{c}_j(m)}. \quad (5.13)$$

Note that $\mathbf{M}_j(m) \in \mathbb{C}^{(K-1)^2 M^2 \times L^2}$, $\mathbf{u}_{mj} \in \mathbb{C}^{L^2 \times 1}$, and $\mathbf{c}_j(m) \in \mathbb{C}^{(K-1)^2 M^2 \times 1}$. Thus, a solution to (5.13) exists if $L^2 \geq (K-1)^2 M^2$ so that the number of variables is greater than or equal to the number of equations. Also, since the channel matrices are drawn randomly from a continuous distribution, it can be shown that the matrix $\mathbf{M}_j(m)$ is full-rank almost surely even when L is exactly equal to $(K-1)M$. $\mathbf{c}_j(m)$ is a non-zero vector, and hence, the precoder solution $\mathbf{u}_{mj}$ is a non-zero vector. Further, it can be shown that the subspaces occupied by the desired signals at the k^{th} receiver, i.e., $\mathbf{F}_{ki}^{(k)} \forall i \in \mathcal{I}$ do not overlap among one another and with the interfering signal subspaces, i.e., $\mathbf{F}_{ji}^{(k)} \forall j \neq k, i \in \mathcal{I}$. Thus, the desired signals can be decoded at every receiver by simply zero-forcing the interference signals.

Thus, $L = (K-1)M$ antennas at the relay are sufficient to attain the maximum DoF of the network. ■

Since $j \in \mathcal{I}, m \in \mathcal{I}'$, and $|\mathcal{I}| = K, |\mathcal{I}'| = K-1$, there are $K(K-1)$ precoders in total and each precoder can be derived using its corresponding equation in (5.13).

Remark 5.1. The maximum DoF of a symmetric K -user X network was established recently in [175] which showed the so-called “spatial-scale invariance” property of a symmetric MIMO-X network, according to which, providing M antennas to every node in an X network scales the DoF of the SISO-X network by a factor of M . While adding relay(s) cannot increase the DoF of an X network beyond its maximum DoF without the use of relays [131], the proposed scheme achieves the

maximum network DoF $\frac{K^2 M}{2K-1}$ [175] only in $(2K-1)$ time slots and is, therefore, DoF-optimal for the symmetric MIMO-XRC.

Remark 5.2. Note that while the $(K-1)M$ antennas at the relay with blind transmitters is a sufficient condition for achieving the maximum DoF of the network, the necessary condition on the minimum number of antennas at the relay under finite time-slot transmission, and the corresponding transmission scheme, are still unsolved problems.

5.3 Relay-aided SISO-X channel (SISO-XRC)

5.3.1 System model for SISO-XRC

In this section, we consider the K -user SISO-X network aided by a MIMO relay with $(K-1)$ antennas and blind transmitters. We refer to Figure 5.1 again where each transmitter and receiver has only one antenna. The same $(2K-1)$ -slot transmission scheme as mentioned in Section 5.2 holds except that the channel matrices, $\mathbf{H}_{Rj}$, $\mathbf{H}_{jR}$ and $\mathbf{H}_{ij}$ are now replaced by the channel vectors $\mathbf{h}_{Rj}$, $\mathbf{h}_{jR}$ and the scalar channel h_{ij} respectively because only single antenna is considered at the transmitters and the receivers. Also, the vector data symbols $\mathbf{x}_{ij}$ in the previous section are now replaced by the scalar data symbols x_{ij} . Similarly, the equations (5.1)-(5.7) hold with the abovementioned changes in the channel matrices and the data symbols.

Based on (5.6), the $(2K-1)$ -dimensional signal received at the k^{th} receiver for the SISO-XRC model is given as

$$\mathbf{y}_k = \underbrace{\sum_{i=1}^K \tilde{\mathbf{F}}_{ki}^{(k)} x_{ki}}_{\text{Desired signals}} + \underbrace{\sum_{\substack{j=1 \\ j \neq k}}^K \sum_{i=1}^K \tilde{\mathbf{F}}_{ji}^{(k)} x_{ji}}_{\text{Interfering signals}} + \underbrace{\tilde{\mathbf{n}}_k}_{\text{total noise}}, \quad (5.14)$$

where $\tilde{\mathbf{F}}_{ji}^{(k)}$ is similar to $\mathbf{F}_{ji}^{(k)}$ in (5.7) with changes for the SISO-XRC model as

mentioned above, and is re-written as

$$\tilde{\mathbf{F}}_{ji}^{(k)} = \begin{bmatrix} \mathbf{0}_{(j-1)1 \times 1} \\ h_{ki}(j) \\ \mathbf{0}_{(K-j)1 \times 1} \\ \epsilon \delta_{t_{K+1},i} h_{kt_{K+1}}(K+1) + \mathbf{h}_{kR}(K+1) \mathbf{U}_{(K+1)j} \mathbf{h}_{Ri}(j) \\ \vdots \\ \epsilon \delta_{t_{2K-1},i} h_{kt_{2K-1}}(2K-1) + \mathbf{h}_{kR}(2K-1) \mathbf{U}_{(2K-1)j} \mathbf{h}_{Ri}(j) \end{bmatrix}.$$

At this point, define $\mathbf{G}_{ji}^{(k)}$ and $\mathbf{u}_i$ as

$$\mathbf{G}_{ji}^{(k)} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times (K-1)(K^2-2K+2)} \\ h_{ki}(j) & \mathbf{0}_{1 \times (K-1)(K^2-2K+2)} \\ 0 & \mathbf{0}_{1 \times (K-1)(K^2-2K+2)} \\ \epsilon \delta_{t_{K+1},i} h_{kt_{K+1}}(K+1) & \mathbf{0}_{1 \times (K-1)(K-2)^2} & \mathbf{h}_{Ri}(j)^T \otimes \mathbf{h}_{kR}(K+1) & \mathbf{0}_{1 \times (K-1)^2} \\ \vdots & & & \\ \epsilon \delta_{t_{2K-1},i} h_{kt_{2K-1}}(2K-1) & \mathbf{0}_{1 \times (K-1)(K^2-3K+3)} & \mathbf{h}_{Ri}(j)^T \otimes \mathbf{h}_{kR}(2K-1) \end{bmatrix},$$

$$\mathbf{u}_i = [\mathbf{1}_{K \times 1}, \mathbf{u}_{(K+1)i}, \dots, \mathbf{u}_{(2K-1)i}]^T \quad (5.15)$$

Now, writing $\tilde{\mathbf{F}}_{ji}^{(k)} = \mathbf{G}_{ji}^{(k)} \mathbf{u}_i$, and *explicitly expanding the noise term* in (5.14) (noise term can be derived from (5.1)-(5.6) by setting $M = 1$, and its derivation is omitted here), (5.14) can be re-written as

$$\mathbf{y}_k = \sum_{j=1}^K \sum_{i=1}^K \mathbf{G}_{ji}^{(k)} \mathbf{u}_i x_{ji} + \mathbf{\Delta}_j^{(k)} \mathbf{u}_j + \mathbf{n}^{(k)}, \quad (5.16)$$

where $\mathbf{\Delta}_j^{(k)}$ and $\mathbf{n}^{(k)}$ are the noise matrices due to noise amplified at the relay, and

noise vectors due to the noise at the receivers respectively, defined as

$$\Delta_j^{(k)} = \begin{bmatrix} \mathbf{0}_{K \times (K+(K-1)^3)} \\ \mathbf{0}_{1 \times K} & \mathbf{n}_R(j)^T \otimes \mathbf{h}_{kR(K+1)} & \mathbf{0}_{1 \times (K-1)^2} \\ \vdots & \vdots & \\ \mathbf{0}_{1 \times K+(K-1)^2} & \mathbf{n}_R(j)^T \otimes \mathbf{h}_{kR(2K-1)} & \end{bmatrix},$$

$$\mathbf{n}^{(k)} = [n_k(1) \ n_k(2) \ \dots \ n_k(2K-1)]^T, \quad (5.17)$$

where $\mathbf{n}_R(j) \in \mathbb{C}^{(K-1) \times 1}$ is the noise vector at the relay in the j^{th} time-slot and is distributed as $\mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{(K-1)})$, and $n_k(j) \sim \mathcal{CN}(0, \sigma^2)$ is the circularly symmetric additive white Gaussian noise (AWGN) at the k^{th} receiver in the j^{th} time-slot.

5.3.2 Power constraint at the relay

Note that there is a power constraint P_t at the relay that must be taken care of while designing the precoders in sections 5.4 and 5.5. The power consumed by the relay (see (5.3)) is given as

$$P_R = \sum_{i=1}^{K-1} \mathbb{E} \{ \|\mathbf{x}_R(i+K)\|^2 \} \quad (5.18)$$

Using (5.2) and (5.3), P_R can be derived as

$$\sum_{m=1}^{K-1} \sum_{t=1}^K \sum_{i=1}^K \mathbf{h}_{Ri}^H(t) \mathbf{U}_{\tilde{m}t}^H \mathbf{U}_{\tilde{m}t} \mathbf{h}_{Ri}(t) + \sigma^2 \sum_{m=1}^{K-1} \sum_{i=1}^K \text{tr} \{ \mathbf{U}_{\tilde{m}i}^H \mathbf{U}_{\tilde{m}i} \}, \quad (5.19)$$

where $\tilde{m} = (m+K)$. Note that in (5.19), $\mathbf{h}_{Ri}^H(t) \mathbf{U}_{\tilde{m}t}^H \mathbf{U}_{\tilde{m}t} \mathbf{h}_{Ri}(t)$ is a scalar and can be written as $\text{tr} \{ \mathbf{h}_{Ri}^H(t) \mathbf{U}_{\tilde{m}t}^H \mathbf{U}_{\tilde{m}t} \mathbf{h}_{Ri}(t) \} = \text{tr} \{ \mathbf{U}_{\tilde{m}t}^H \mathbf{U}_{\tilde{m}t} \mathbf{h}_{Ri}(t) \mathbf{h}_{Ri}^H(t) \}$. Now, using $\text{tr} \{ \mathbf{A}^H \mathbf{B} \} = [\text{vec}(\mathbf{A})]^H \text{vec}(\mathbf{B})$ [176], and taking $\mathbf{A}$ as $\mathbf{U}_{\tilde{m}t}$ and $\mathbf{B}$ as

$\mathbf{U}_{\tilde{m}t} \mathbf{h}_{Ri}(t) \mathbf{h}_{Ri}^H(t)$, (5.19) can be further simplified as

$$\sum_{m=1}^{K-1} \sum_{t=1}^K \mathbf{u}_{\tilde{m}t}^H \underbrace{\left[\sum_{i=1}^K (\mathbf{h}_{Ri}(t) \mathbf{h}_{Ri}^H(t))^T \otimes \mathbf{I}_L + \sigma^2 \mathbf{I}_{L^2} \right]}_{\mathbf{D}} \mathbf{u}_{\tilde{m}t}, \quad (5.20)$$

where $L = (K-1)$. Please note that $\mathbf{D}$ is a positive-definite matrix because it can be expressed as a summation of $\mathbf{B}_i^H \mathbf{B}_i \succeq 0$ where $\mathbf{B}_i = \mathbf{h}_{Ri}^T(t) \otimes \mathbf{I}_L$, and the noise matrix.

5.4 IA under limited power at the relay

In this section, we propose IA with limited feedback (also called the “IA-LFB”) to the transmitters, for the case of time-invariant SISO-XRC. From (5.13), the basic IA equation for the SISO-XRC model, corresponding to the precoder $\mathbf{u}_{mj}$, is re-written as

$$\mathbf{M}_j(m) \mathbf{u}_{mj} = \mathbf{c}_j(m).$$

For this type of precoder, it can be observed that in order to attain interference alignment, a minimum amount of power $P_{IA} = \sum_{m \in \mathcal{I}'} \sum_{t \in \mathcal{I}} P_{IA,mt}$ must be available at the relay, where $P_{IA,mt} = \mathbf{u}_{mt}^H \mathbf{D} \mathbf{u}_{mt}$ is the power expended due to the IA precoder $\mathbf{u}_{mt}$ (see (5.20)). If power at the relay is less than this limit, alignment of interference is not possible and there will be some leakage interference after receiver processing, resulting in DoF loss. This is a limitation of the scheme in [31, 174] which is removed using our proposed scheme, especially for the case of time-invariant channels.

Considering *time-invariant* channels and the scaling of the symbol by a real scalar ϵ in (5.4), it can be shown that $\mathbf{c}_j(m)$ and $\mathbf{M}_j(m)$ in (5.13) depend only on m and j respectively, where $m \in \mathcal{I}'$ and $j \in \mathcal{I}$. Thus, $\mathbf{c}_j(m)$ and $\mathbf{M}_j(m)$ can be

denoted as $\mathbf{c}(m)$ and $\mathbf{M}_j$ respectively. $\mathbf{c}_j(m)$ can then be written as

$$\mathbf{c}_j(m) \equiv \mathbf{c}(m) = [\mathbf{1}_{K-1}^T \otimes \tilde{\epsilon}_m^T]^T, \quad (5.21)$$

where $\mathbf{1}_{K-1}$ denotes a $(K-1) \times 1$ vector with every element as 1, and $\tilde{\epsilon}_m$ denotes a $(K-1) \times 1$ vector containing ϵ at the $(m-K)^{th}$ position and 0's everywhere else. Next, we propose a scheme which guarantees IA at any arbitrary relay power.

Proposition 5.1. *Let P_t be the maximum power available at the relay. Then the value of ϵ in (5.4) that guarantees IA, is given as*

$$\epsilon = \sqrt{\frac{P_t}{\sum_{j \in \mathcal{I}} \sum_{\substack{m', n' \in \mathcal{S}_m \\ \forall m \in \mathcal{I}'}} [\mathbf{\Gamma}_j]_{m', n'}}} \quad (5.22)$$

where the set

$$\begin{aligned} \mathcal{S}_m = \{ & (m-K), (m-K) + (K-1), (m-K) + 2(K-1), \dots \\ & , (m-K) + (K-2)(K-1) \}, \end{aligned} \quad (5.23)$$

$$\mathbf{\Gamma}_j = \mathbf{M}_j^H \mathbf{D} \mathbf{M}_j. \quad (5.24)$$

Proof. The power expended at the relay (see (5.20)) can be re-written as

$$P_R = \sum_{m \in \mathcal{I}'} \sum_{j \in \mathcal{I}} \mathbf{c}_j^H(m) \mathbf{\Gamma}_j(m) \mathbf{c}_j(m), \quad (5.25)$$

where $\mathbf{c}_j(m)$ or $\mathbf{c}(m)$ and $\mathbf{\Gamma}_j(m)$ or $\mathbf{\Gamma}_j$ are defined in (5.21) and (5.24) respectively. Equating P_R to P_t and substituting the value of $\mathbf{c}(m)$ from (5.21), we get the “IA-attaining” value of ϵ in (5.22). ■

Accordingly, if the power available at the relay is not sufficient for achieving interference alignment, the source transmitters must send an ϵ -scaled version of the symbols as shown in (5.4) in the last $(K-1)$ time slots. Note that the relay

knows the global CSI of the network. Hence, the relay can compute the ‘ ϵ ’ value and feedback to the transmitters so that they will transmit the correctly scaled symbols in the last $(K-1)$ time slots. Thus, with a very limited feedback in the form of a real scalar value from the relay to the transmitters, IA is restored and the maximum network DoF is attained.

Remark 5.3. It is worth noting that while an ϵ -scaled transmission restores IA even under limited relay power, it is possible that for some channel realizations, the transmit power of the source transmitters is increased due to the scaling of its symbols, thereby violating the transmitter’s power constraint.

In order to take care of the power constraints at the source transmitters also, the following proposition gives a modified IA-LFB precoder design.

Proposition 5.2. *Under power constraint at both the relay and the transmitters, the optimal IA-restoring strategy is to use conventional IA without any feedback from the relay to the transmitters when ϵ in (5.22) exceeds unity. In other words, set $\epsilon = 1$ when $\epsilon > 1$. On the other hand, if ϵ in (5.22) < 1 , use IA-LFB, i.e., the transmitter performs ϵ -scaled transmission if $\epsilon < 1$.*

Proof. Please refer to Appendix at the end of the chapter. ■

At this point, we introduce the following definition.

Definition 5.1. *Interference misalignment (IM) is defined as the interference leakage when the relay precoder power is not sufficient to align the interference. Mathematically, IM due to the precoder $\mathbf{u}_{mj}$, $IM = \|\mathbf{M}_j(m)\mathbf{u}_{mj} - \mathbf{c}_j(m)\|_2^2$.*

Now, we discuss the case when there is abundant power available at the relay, i.e., more than that required for achieving IA. In this case, no scaling of symbols is required at the source transmitters in the last $(K-1)$ time slots, i.e., $\epsilon = 1$. Here, we note that naïve scaling up of the relay power destroys the alignment

of interference, and thus, is harmful from a DoF perspective. The misalignment leakage power, due to naïve power scaling, is quantified below.

Proposition 5.3. *Let $P_{IA,mt}$ be the power of the relay precoder $\mathbf{u}_{mt}$ which is just sufficient to achieve IA. Let $P_u > P_{IA,mt}$ be the naive power scaling applied to the relay precoder $\mathbf{u}_{mt}$ ($m \in \mathcal{I}', t \in \mathcal{I}$). Then, the misalignment leakage power due to that precoder, ζ_{mt} is given as*

$$\zeta_{mt} = 2 \left(\sqrt{\frac{P_u}{\mathbf{u}_{mt}^H \mathbf{D} \mathbf{u}_{mt}}} - 1 \right)^2. \quad (5.26)$$

Proof. Naïve scaling of the power of the relay precoder $\mathbf{u}_{mt}$ gives the new precoder $\tilde{\mathbf{u}}_{mt} = \mathbf{u}_{mt} \frac{P_u}{\sqrt{\mathbf{u}_{mt}^H \mathbf{D} \mathbf{u}_{mt}}}$. Substituting this and the value of $\mathbf{c}_j(m)$ in the IM term, we get the result in (5.26). ■

Next, we give the following important remarks.

Remark 5.4. The value of ϵ in Proposition 5.1 can be utilized only in time-invariant channels. This is because ϵ is calculated based on the channel conditions from the transmitters to the relay from the $(K+1)^{th}$ to the $(2K-1)^{th}$ time slot. But the transmitters need to send the scaled symbols in the $(K+1)^{th}$ time-slot as well which requires non-causal information at the transmitters. Hence, the IA-LFB scheme is applicable to the time-invariant or slowly varying channels only.

Remark 5.5. For the time-varying channels, a modified transmission strategy under limited power at the relay is that the relay first computes the total power expended due to the IA precoders from the $(K+1)^{th}$ time-slot to the $(2K-2)^{th}$ time-slot, i.e., $\tilde{P}$. If this power is less than the maximum power P_t available at the relay, then the relay can use a similar strategy proposed in Proposition 5.1 to design the precoder $\mathbf{u}_{(2K-1)t}$ in the $(2K-1)^{th}$ time-slot where the remaining power, i.e., $(P_t - \tilde{P})$ will be used instead of P_t in (5.22), and the denominator in (5.22) will be substituted by $\sum_{j \in \mathcal{I}} \sum_{\substack{m', n' \\ \in \mathcal{S}_{2K-1}}} [\mathbf{\Gamma}_j]_{m', n'}$. However, if $\tilde{P} \geq P_t$, then IA cannot be

achieved for the time-varying channels.

5.5 MMSE Precoder design

5.5.1 Precoder design

In this section, we propose closed-form minimum-sum mean square error (MMSE) precoder and receive filter designs for the K -user relay-aided SISO-XC model described in Section 5.3.

Note that for a K -user XRC, the k^{th} receiver has to decode K desired symbols, i.e., $\mathbf{x}_{kj}$, $j \in \mathcal{K}$. Thus, the k^{th} receiver applies a receive filter $\mathbf{w}_{kj} \in \mathbb{C}^{(2K-1) \times 1}$ to extract the k^{th} data symbol as $\hat{x}_{kj} = \mathbf{w}_{kj}^H \mathbf{y}_k$, where $\mathbf{y}_k$ is defined in (5.16). To design the MMSE precoders, the sum-mean square error (SMSE) of all the received symbols in the network must be minimized. The SMSE of the network is derived as

$$\mathcal{S} = \sum_{k=1}^K \sum_{j=1}^K \mathbb{E}\{|x_{kj} - \hat{x}_{kj}|^2\} = \sum_{k=1}^K \varepsilon_k, \quad (5.27)$$

where

$$\begin{aligned} \varepsilon_k = & \sum_j \sum_i \mathbf{w}_{kj}^H \left\{ \sum_l \mathbf{G}_{il}^{(k)} \mathbf{u}_i \mathbf{u}_i^H \mathbf{G}_{il}^{(k)H} + \mathbf{\Upsilon}_i^{(k)} \right\} \mathbf{w}_{kj} \\ & + \sigma^2 \sum_j \|\mathbf{w}_{kj}\|^2 - 2 \sum_j \Re \left\{ \mathbf{w}_{kj}^H \mathbf{G}_{kj}^{(k)} \mathbf{u}_k \right\} + K. \end{aligned} \quad (5.28)$$

$\mathbf{\Upsilon}_i^{(k)}$ may be further written as

$$\begin{bmatrix} \mathbf{0}_{K \times K} & \mathbf{0}_{K \times (K-1)} \\ \mathbf{0}_{(K-1) \times K} & \mathbf{R}_{ki} \end{bmatrix}, \quad (5.29)$$

where $\mathbf{R}_{ki} \in \mathbb{C}^{(K-1) \times (K-1)}$, and its $(v, \tau)^{th}$ element is defined as

$$r_{v\tau} = \sigma^2 \mathbf{u}_{(\tau+K)i}^H \left[\mathbf{I}_{(K-1)} \otimes \mathbf{h}_{kR}^H(\tau + K) \mathbf{h}_{kR}(v + K) \right] \mathbf{u}_{(v+K)i}. \quad (5.30)$$

Now, the MMSE optimization problem, with power constraint at the relay, can be written as

$$\begin{aligned} \text{(P1)} \quad & \underset{\substack{\mathbf{u}_{\tilde{m}t}, \mathbf{w}_{kj} \\ \forall k, j, t \in \mathcal{K}, \tilde{m} \in \mathcal{I}'}}{\text{minimize}} \quad \mathcal{S} \\ & \text{subject to} \quad \sum_{m=1}^{K-1} \sum_{t=1}^K \mathbf{u}_{\tilde{m}t}^H \mathbf{D} \mathbf{u}_{\tilde{m}t} \leq P_t. \end{aligned}$$

We note that while the SMSE expression in (5.27) will be useful in deriving the receive filters as shown later, it is not readily amenable to precoder optimization. This is because in (5.27), the precoders $\mathbf{u}_{mi}$ ($m \in \mathcal{I}', i \in \mathcal{I}$) are stacked together into a single vector $\mathbf{u}_i$ which involves concatenation with extra 1's (see (5.15)). In order to remove the extra 1's, we need to suitably re-write the SMSE expression. As shown next, we re-write the SMSE expression (see (5.32)) which enables us to optimize each precoder $\mathbf{u}_{\tilde{m}i}$ separately using a low-complexity alternative minimization approach. Based on the SMSE expression in (5.32), we obtain the closed-form solutions to the precoder designs for the general K -user SISO-XRC model.

First, let us define the following terms.

$$\begin{aligned} p_{mi}^{kj} &= [\mathbf{w}_{kj}]_m^* h_{ki}(m) + \epsilon(1 - \delta_{iK}) [\mathbf{w}_{kj}]_{(i+K)}^* h_{ki}(i + K), \\ \mathbf{q}_{mi,l}^{kj} &= [\mathbf{w}_{kj}]_l^* (\mathbf{h}_{Ri}^T(m) \otimes \mathbf{h}_{kR}(l)), \\ \kappa_{mn}^{kj} &= \sigma^2 [\mathbf{w}_{kj}]_m [\mathbf{w}_{kj}]_n^*, \\ \mathbf{C}_{mn}^{(k)} &= \mathbf{I}_{(K-1)} \otimes \mathbf{h}_{kR}^H(m) \mathbf{h}_{kR}(n), \\ \phi_{mi}^{kj} &= p_{mi}^{kj} + \sum_{l \in \mathcal{I}'} \mathbf{q}_{mi,l}^{kj} \mathbf{u}_{lm}. \end{aligned} \quad (5.31)$$

After some manipulations, the SMSE in (5.27) for the K -user SISO-XRC can be expressed explicitly in terms of the precoders $\mathbf{u}_{mi}$, ($m \in \mathcal{I}'$, $i \in \mathcal{I}$) as³

$$\begin{aligned} \mathcal{S} = & \sum_{k,j,m,i} \phi_{mi}^{kj} \phi_{mi}^{kj*} + \sum_{k,j} \sum_{\substack{m,n \\ \in \mathcal{I}'}} \sum_i \kappa_{mn}^{kj} \mathbf{u}_{mi}^H \mathbf{C}_{mn}^{(k)} \mathbf{u}_{ni} \\ & - 2 \sum_{k,j} \Re \left\{ \phi_{kj}^{kj} \right\} + \sigma^2 \sum_{k,j} \|\mathbf{w}_{kj}\|^2 + K^2. \end{aligned} \quad (5.32)$$

Note that for the K -user SISO-XRC, we have $K(K-1)$ precoders (or transmit filters) $\mathbf{u}_{mj}$ and K^2 receive filters $\mathbf{w}_{kj}$, and thus, the SMSE needs to be optimized over $K(2K-1)$ variables. Also, note that the SMSE objective in (5.32) is not jointly convex w.r.t. the transmit and the receive filters. Therefore, globally optimal solution of the SMSE minimization problem is very difficult to obtain. However, from (P1) and (5.32), it is easy to see that the problem (P1) is a quadratically constrained quadratic problem with respect to (w.r.t.) one variable if all others are fixed. Therefore, locally optimal solution of the above problem can be targeted by optimizing the objective over one variable while keeping all others fixed, i.e., we adopt the block coordinate descent (BCD) method [158], as discussed in Algorithm 3.

In the following, we provide closed-form expressions for the precoders and receive filters at every iteration. Since the variables are alternately optimized, the problem can be simplified by allocating certain power constraint to every precoder. One possible way is to allocate equal power to every precoder, i.e., $P_t/(K(K-1))$. Note that this gives us a sub-optimal solution to (P1) where the overall power constraint is still satisfied but the solution involves much lesser computational complexity. It can be observed from our numerical results that even this sub-optimal solution provides much better BER results than the conventional IA precoder upto

³The summation limits such as k, j, m, i in the above expression and other expressions here onwards (their ranges are not explicitly mentioned to make the equations compact), assume the values as $k, j, m, i \in \{1, 2, \dots, K\}$ unless explicitly mentioned in the expression.

a wide SNR range. The modified sub-optimal problem is defined as

$$\begin{aligned}
 \text{(P2)} \quad & \underset{\substack{\mathbf{u}_{\tilde{m}t}, \mathbf{w}_{kj} \\ \forall k, j, t \in \mathcal{K}, \tilde{m} \in \mathcal{I}'}}{\text{minimize}} \quad \mathcal{S} \\
 & \text{subject to } \mathbf{u}_{\tilde{m}t}^H \mathbf{D} \mathbf{u}_{\tilde{m}t} \leq P_{\tilde{m}t} \quad \forall \tilde{m} \in \mathcal{I}', t \in \mathcal{K},
 \end{aligned}$$

where $P_{\tilde{m}t}$ is the power constraint on the precoder $\mathbf{u}_{\tilde{m}t}$ and will be discussed in a later section. The SMSE minimization problem in (P2) is solved using the Lagrange multiplier method where the Lagrange function is defined as

$$\mathcal{L} = \mathcal{S} + \sum_{\substack{\tilde{m} \in \mathcal{I}' \\ t \in \mathcal{K}}} \lambda_{\tilde{m}t} (\mathbf{u}_{\tilde{m}t}^H \mathbf{D} \mathbf{u}_{\tilde{m}t} - P_{\tilde{m}t}) \quad (5.33)$$

As mentioned above, (P2) is partially convex w.r.t. one variable (transmit or receive filter) while all others are fixed. Moreover, during each BCD step, we impose individual power inequality constraint on the transmit filter and no constraint on the receive filters. Then, it can be verified that the feasible set of solutions (in every step) is closed and satisfies Slater's condition [158]. Thus, we can use the Karush-Kuhn-Tucker (KKT) conditions to get the stationary solutions. The KKT conditions for the problem (5.33) are given as

$$\begin{aligned}
 & \frac{\partial \mathcal{L}}{\partial \mathbf{u}_{\tilde{m}t}^H} = 0, \quad \frac{\partial \mathcal{L}}{\partial \mathbf{w}_{kj}^H} = 0, \\
 & \lambda_{\tilde{m}t} \geq 0, \quad \mathbf{u}_{\tilde{m}t}^H \mathbf{D} \mathbf{u}_{\tilde{m}t} \leq P_{\tilde{m}t}, \quad \lambda_{\tilde{m}t} \left\{ \mathbf{u}_{\tilde{m}t}^H \mathbf{D} \mathbf{u}_{\tilde{m}t} - P_{\tilde{m}t} \right\} = 0.
 \end{aligned} \quad (5.34)$$

Using $\frac{\partial \mathcal{L}}{\partial \mathbf{u}_{\tilde{m}t}^H} = 0$ in (5.34), and the results in [169], we get the closed-form expression for the precoders as

$$\mathbf{u}_{\tilde{m}t} = (\mathbf{A}_{\tilde{m}t} + \lambda_{\tilde{m}t} \mathbf{D})^{-1} \tilde{\mathbf{p}}_{\tilde{m}t}, \quad (5.35)$$

where

$$\mathbf{A}_{\tilde{m}t} = \sum_{k,j,i} \left(\mathbf{q}_{ti,\tilde{m}}^{kj} \right)^H \mathbf{q}_{ti,\tilde{m}}^{kj} + \sum_{k,j} \kappa_{\tilde{m}\hat{m}}^{kj} \mathbf{C}_{\tilde{m}\hat{m}}^{(k)}, \quad (5.36)$$

$$\begin{aligned} \tilde{\mathbf{p}}_{\tilde{m}t} = & - \sum_{k,j,i} p_{ti}^{kj} \left(\mathbf{q}_{ti,\tilde{m}}^{kj} \right)^H - \sum_{k,j,i} \sum_{\substack{\hat{m} \in \mathcal{I}' \\ \hat{m} \neq \tilde{m}}} \left(\mathbf{q}_{ti,\tilde{m}}^{kj} \right)^H \mathbf{q}_{ti,\hat{m}}^{kj} \mathbf{u}_{\hat{m}t} \\ & - \sum_{k,j} \sum_{\substack{\hat{m} \in \mathcal{I}' \\ \hat{m} \neq \tilde{m}}} \kappa_{\tilde{m}\hat{m}}^{kj} \mathbf{C}_{\tilde{m}\hat{m}}^{(k)} \mathbf{u}_{\hat{m}t} + \sum_j \left(\mathbf{q}_{tj,\tilde{m}}^{tj} \right)^H, \end{aligned} \quad (5.37)$$

and $\tilde{m}, \hat{m} \in \mathcal{I}'$, $t \in \mathcal{I}$.

Note that $\mathbf{A}_{\tilde{m}t}$ is an invertible matrix. This is because the first term in $\mathbf{A}_{\tilde{m}t}$ consists of the summation of K^3 different rank-1 matrices (It is easy to see this because $\mathbf{q}_{ti,l}^{kj}$ is a vector and all its elements are drawn independently from a continuous distribution). The second term in $\mathbf{A}_{\tilde{m}t}$ further strengthens the full-rank property of the matrix. Thus, even if $\lambda_{\tilde{m}t} = 0$, $(\mathbf{A}_{\tilde{m}t} + \lambda_{\tilde{m}t} \mathbf{D})$ is full-rank and invertible.

Proposition 5.4. *The power transmitted by the relay is a monotonically decreasing function of $\lambda_{\tilde{m}t}$, and is given as*

$$f = \sum_{i=1}^{L^2} \frac{[\mathbf{X}]_{ii}}{([\boldsymbol{\Sigma}]_{ii} + \lambda_{\tilde{m}t})^2}, \quad (5.38)$$

where $\boldsymbol{\Sigma}$ is the singular value matrix formed by the SVD decomposition of $\mathbf{D}^{-1} \mathbf{A}_{\tilde{m}t}$, and $\mathbf{X} = \mathbf{V}_L^{-1} \mathbf{D}^{-1} \tilde{\mathbf{p}}_{\tilde{m}t} \tilde{\mathbf{p}}_{\tilde{m}t}^H \mathbf{V}_R$ with the matrices $\mathbf{V}_L$ and $\mathbf{V}_R$ formed by the left and right singular vectors of $\mathbf{D}^{-1} \mathbf{A}_{\tilde{m}t}$, respectively.

The proof of above proposition is similar to that of Proposition 4.1 (refer to Chapter 4). The Lagrange multiplier $\lambda_{\tilde{m}t}$ in (5.35) can be computed by solving the equation $f = P_{\tilde{m}t}$ via bisection search method. Using the SMSE expression in

(5.27) and the KKT condition $\frac{\partial \mathcal{L}}{\partial \mathbf{w}_{kj}^H} = 0$ in (5.34), we get the decoder expression as

$$\mathbf{w}_{kj} = \left(\sum_i \sum_j \mathbf{G}_{ij}^{(k)} \mathbf{u}_i \mathbf{u}_i^H \mathbf{G}_{ij}^{(k)H} + \mathbf{\Upsilon}_i^{(k)} + \sigma^2 \mathbf{I}_5 \right)^{-1} \mathbf{G}_{kj}^{(k)} \mathbf{u}_k, \quad (5.39)$$

where $\mathbf{\Upsilon}_i^{(k)}$ is defined in (5.29) and $\mathbf{u}_i$ is defined in (5.15). Note that the above expression gives the MMSE decoder (or receive filter) which is the optimal information-lossless receiver under the assumption of Gaussian signaling and Gaussian interference [166]. The alternating optimization algorithm for the precoder and decoder design is given in Algorithm 3. $\mathbf{u}_{mt}^{(i)}$, $\mathbf{w}_{kj}^{(i)}$ and $\mathcal{S}^{(i)}$ denote the precoder, the decoder and the SMSE respectively, in the i^{th} iteration.

Algorithm 3 Alternating optimization algorithm for the precoder and decoder designs

- 1: Initialize the relay precoders $\mathbf{u}_{\tilde{m}t}^{(0)}$ $\tilde{m} \in \mathcal{I}' = \{(K+1), \dots, (2K-1)\}$, $t \in \mathcal{I} = \{1, \dots, K\}$ to suitable vector of dimension $(K-1)^2 \times 1$. The initialization can be random or IA initialization using (5.13).
 - 2: Set a pre-defined counter limit, *limit*. Set threshold $\tau = 10^{-6}$. Set the initial SMSE, $\mathcal{S}^{(0)}$ as a large positive value, and $\Delta \mathcal{S}$ as any value $> \tau$.
 - 3: $i \leftarrow 1$
 - 4: **while** $i \leq \text{limit}$ or $\Delta \mathcal{S} > \tau$ **do**
 - 5: Compute the receive filters $\mathbf{w}_{kj}^{(i)} \forall k, j \in \mathcal{K}$ using (5.39) while keeping the precoders fixed.
 - 6: **for** $t = 1$ to K **do**
 - 7: **for** $\tilde{m} = (K+1)$ to $(2K-1)$ **do**
 - 8: Compute the Lagrange multiplier $\lambda_{\tilde{m}t}$ using $f = P_{\tilde{m}t}$ from (5.38).
 - 9: Update the relay precoder $\mathbf{u}_{\tilde{m}t}^{(i)}$ (while the other precoders $\mathbf{u}_{mj}$, $m \neq \tilde{m}$ and the decoders $\mathbf{w}_{kj}$ are fixed) using (5.35).
 - 10: **end for**
 - 11: **end for**
 - 12: Compute SMSE, $\mathcal{S}^{(i)}$ using (5.27).
 - 13: $\Delta \mathcal{S} = \mathcal{S}^{(i-1)} - \mathcal{S}^{(i)}$
 - 14: $i \leftarrow i + 1$
 - 15: **end while**
 - 16: Compute the final receive filters $\mathbf{w}_{kj}^{(i)} \forall k, j \in \mathcal{K}$ using (5.39).
-

Note that $K(K-1)$ precoders need to be optimized in the above algorithm. Since the precoders are optimized one-by-one, there can be various orders in which the optimization of these variables can be performed. The above algorithm shows

one such order in which the precoders are optimized using a cyclic update rule [158], e.g., the order of optimization is $\mathbf{u}_{41}, \mathbf{u}_{51}, \mathbf{u}_{42}, \mathbf{u}_{52}, \mathbf{u}_{43}, \mathbf{u}_{53}$ for a 3-user SISO-XRC. Since all of these orders can result in different locally sub-optimal solutions, it is difficult to predict the best order for the optimization. However, through extensive simulation results on various orders of optimization, it is found that the throughput difference due to these orders are only slightly different from one another, and furthermore, this behaviour may change depending on the channel realizations. Instead, we found that the *precoder initialization* and *power constraint allocation* are the important factors that essentially enhance the system performance, as discussed in the next sub-section. Also, from our numerical results, we observe that the chosen order indeed gives better throughput and BER performance as compared to the conventional IA algorithm.

Similar to the MMSE precoder design in Chapter 4, it can be shown that the proposed sub-optimal SMSE minimization problem (P2) is convergent, and its locally optimal solutions are guaranteed. Figure 5.2 shows the convergent behaviour of the MMSE algorithm for the 3-user SISO-XRC when the SNR per symbol is 10 dB. Since 15 symbols (1 symbol per user per time-slot for the first 3 time slots, and 3 symbols by users 1 and 2 each in the last 2 time slots) are transmitted overall, the maximum transmit SNR is 21.76 dB.

Remark 5.6. It is to be noted that the MMSE algorithm utilizes the channel information in all the time slots to design the precoders and the decoders. This implies non-causal channel information availability at the relay, e.g., for the 3-user case, CSI of the 5^{th} time-slot is required for the precoder design in the 4^{th} time-slot. Hence, optimization of all the precoders is not possible for the time-varying channels. However, if IA algorithm is used for designing the precoders up to the $(2K-2)^{th}$ time-slot, MMSE algorithm can still be applied for the precoder design in the $(2K-1)^{th}$ time-slot, which still reduces the SMSE of the network and shows slightly better performance than conventional IA as also verified through

our simulation results. On the contrary, the MMSE algorithm can be applied to the design of all the precoders for the time-invariant and also, for the frequency-selective time-variant channels if precoding is done across frequency.

Figure 5.2: SMSE convergence behaviour for the MMSE algorithm at 21.76 dB total SNR

5.5.2 Precoder initialization and power constraint issues

Note that the MMSE algorithm can provide multiple locally optimal solutions depending on the precoder initialization (see Algorithm 3). However, with unsuitable initializations, the MMSE algorithm often results in solutions which are not DoF-optimal. Further, since the precoders are optimized alternately, each precoder needs to have its own power constraint (see problem (P2)), i.e., $P_{\tilde{m}t}$ and depending on the $P_{\tilde{m}t}$ value, the BER performance of the algorithm varies. Thus, deciding suitable initial precoders and their respective power constraints are important issues in the MMSE precoder design so that DoF optimality is preserved together with good BER performance.

Precoder initialization

Due to multiple local solutions possible, suitable precoder initialization for the iterative MMSE algorithm is desirable. If IA initialization is used, the MMSE algorithm searches for the local solutions in the vicinity of the IA precoders, thereby preserving the DoF of the network. Any random initialization results in severe DoF loss as also verified through our numerical results. However, when sufficient power is not available at the relay to align the interference (see section 5.4), we propose to use IA-LFB precoders as the initializers for the MMSE algorithm which preserves the network DoF (due to interference alignment restoration) and also shows better BER (due to MSE minimization) than the conventional IA algorithm.

Precoder Power constraints

In problem (P2), the power constraint to every precoder, $P_{\tilde{m}t}$, can be fixed to any suitable value. Note that joint power allocation to the precoders for throughput maximization or MSE minimization is difficult and computationally intensive. In fact, joint power allocation may be done by combining all the precoder vectors in a single large vector. However, we note that the MMSE precoder design requires matrix inversion (see (5.35)). For the case of 3-user SISO-XRC, 6 precoders can be combined into a long vector. This requires the inversion of a 24×24 matrix at each iteration of the MMSE algorithm which results in a complexity of $\mathcal{O}(24^{2.3728})$ even using the best known optimized CW-like algorithm [177]. In contrast, individual power constraint on each precoder requires a far lesser total complexity of $6 \times \mathcal{O}(4^{2.3728})$ only.

Therefore, we put individual power constraint on every precoder. However, finding suitable power constraints for every precoder is also challenging. In fact, different power constraints $P_{\tilde{m}t}$ allow different search space for the MMSE optimization problem and give different BER performances. A suitable option is to

allocate the same power as that expended by the IA precoders $\mathbf{u}_{\tilde{m}t,IA}$. This means that we note the power expended due to each relay precoder when conventional IA algorithm is used for transmission, and assign this as the power constraint to the respective precoders for the MMSE design, i.e., $P_{\tilde{m}t} = \mathbf{u}_{\tilde{m}t,IA}^H \mathbf{D} \mathbf{u}_{\tilde{m}t,IA}$. This enables us to compare the MMSE precoder against the benchmark IA precoder performance under similar power constraints. However, we note that the powers expended due to the IA precoders are very high (especially in the low-mid SNR regions). Another option is to allocate equal power constraint to each precoder, i.e., $P_{\tilde{m}t} = \frac{P_t}{K(K-1)}$. Using this approach, together with IA initialization, it is observed that the MMSE algorithm can save a lot of power and yet show better BER performance than the conventional IA precoder upto a wide SNR regime.

Again, for the case when sufficient power is not available at the relay to align the interference, IA can be restored using IA-LFB and we can assign equal power constraint to each precoder, i.e., $\frac{P_t}{K(K-1)}$, or allocate the same power as that expended by the IA-LFB precoders. Note that for the iterative MMSE algorithm, while both the precoder initialization and the power constraint allocation are important, the former is a more crucial design issue. This is because *without* IA or IA-LFB initialization, both the DoF and the BER performances are degraded due to interference leakage, and choosing suitable power constraint is not of much help in this case. The throughput and BER performances for the various precoder initializations and power constraints are shown in the numerical results.

5.5.3 Computational complexity

The computational complexity of the MMSE precoder design in (5.35) is mainly due to the computation of $\mathbf{A}_{\tilde{m}t}$, $\tilde{\mathbf{p}}_{\tilde{m}t}$, inversion of the matrix $(\mathbf{A}_{\tilde{m}t} + \lambda_{\tilde{m}t} \mathbf{D})$ and multiplication of $(\mathbf{A}_{\tilde{m}t} + \lambda_{\tilde{m}t} \mathbf{D})^{-1}$ with $\tilde{\mathbf{p}}_{\tilde{m}t}$. On analysing the components of $\mathbf{A}_{\tilde{m}t}$ in (5.36) and $\tilde{\mathbf{p}}_{\tilde{m}t}$ in (5.37), it is found that their computational complexities are of the order $\mathcal{O}(L^6)$ and $K^3(K-2)\mathcal{O}(L^4)$ respectively. On the other hand, matrix

inversion and multiplication of $(\mathbf{A}_{\tilde{m}t} + \lambda_{\tilde{m}t}\mathbf{D})^{-1}$ with $\tilde{\mathbf{p}}_{\tilde{m}t}$ in (5.35) require $\mathcal{O}(L^6)$ and $\mathcal{O}(L^4)$ operations respectively. Using $L = (K - 1)$ and the fact that the computational complexity of $\tilde{\mathbf{p}}_{\tilde{m}t}$ is greater than all other computations involved, the overall complexity of the MMSE precoder design is of the order $\mathcal{O}(K^8)$ asymptotically as K becomes large. Furthermore, since $K(K - 1)$ precoders are designed at the relay, the total complexity of the precoders' design is of the order $\mathcal{O}(K^{10})$. When n iterations are used, the overall complexity is $n \times \mathcal{O}(K^{10})$. Note that the computation of λ and SVD operation involved in Proposition 5.4 are of lower complexity than the computation of $\tilde{\mathbf{p}}_{\tilde{m}t}$ and can be neglected in comparison as K becomes large.

Similarly, the complexity of the receive filter (refer to (5.39)) at each receiver is found to be $\mathcal{O}(K^6)$. Note that one of the reasons for the higher complexity of the receive filter design for the relay-aided XC as compared to the relay-aided IC in Chapter 4 is that the receive filter for the relay-aided XC is a function of K because it is designed over $K(K - 1)$ channel uses.

5.6 Sum-rate and BER simulation Results

In this section, numerical results are shown for the algorithms proposed in this chapter. For presenting our main idea, we consider a three-user SISO-X network aided by a MIMO relay having two antennas for our simulations. Based on the achievability scheme in Theorem 5.1, we can attain the maximum DoF of 1.8 in 5 time slots. All the transmitters transmit with equal power and no precoding is involved at the transmitters because they have no CSI (except the IA-LFB case where only one scalar feedback is needed). Perfect CSI is assumed at the relay and the receivers. Also, the relay is a central station that knows the global CSI of the network and designs all the precoders.

While our schemes can be applied to the time-varying channels with some

modifications as pointed out in Remarks 5.5 and 5.6, we present our results for the time-invariant channels (note that the achievability scheme presented in Theorem 5.1 applies to time-invariant channels also). All the channel coefficients are i.i.d. and drawn from a complex normal distribution with zero mean and unit variance. The sum rate and BER results are plotted by averaging over 1000 channel realizations. For the alternating precoder and decoder updates, 10 iterations are allowed. Throughout the paper, SNR denotes the maximum total transmit SNR of all the transmitted symbols in five time slots, and is given as $SNR = \frac{\text{maximum total power of all the transmitted symbols}}{\text{noise power at the receiver}}$. We assume that each symbol is transmitted with a maximum power P without loss of generality. In this five-slot transmission scheme, each transmitter transmits its symbol for the 1st three time slots, thereby consuming a total power of $9P$. In the 4th and 5th time-slot each, a sum of three symbols is transmitted, each symbol with a maximum power of P . Thus, a total maximum power of $15P$ is consumed in the entire transmission process⁴.

The sum-rate expression is given as

$$\mathcal{S} = \frac{1}{2K-1} \sum_{k=1}^K \sum_{j=1}^K \log_2(1 + \gamma_{kj}), \quad (5.40)$$

where γ_{kj} denotes the SINR of the data stream desired from the j^{th} transmitter by the k^{th} receiver, and is given as

$$\frac{\left| \mathbf{w}_{kj}^H \tilde{\mathbf{F}}_{kj}^{(k)} \right|^2}{\underbrace{\sum_{\substack{l=1 \\ l \neq j}}^K \left| \mathbf{w}_{kj}^H \tilde{\mathbf{F}}_{kl}^{(k)} \right|^2 + \sum_{\substack{m=1 \\ m \neq k}}^K \sum_{l=1}^K \left| \mathbf{w}_{kj}^H \tilde{\mathbf{F}}_{ml}^{(k)} \right|^2}_{\text{interference}} + \underbrace{\sum_{j=1}^K \left| \mathbf{w}_{kj}^H \Delta_j^{(k)} \mathbf{u}_j \right|^2 + \left| \mathbf{w}_{kj}^H \mathbf{n}^{(k)} \right|^2}_{\text{noise}}}, \quad (5.41)$$

⁴Alternately, a transmit power constraint P on every transmit antenna could be assumed in which case the symbol powers in the last two time slots would be scaled accordingly to satisfy the transmit power constraint. Please note that the alternate method does not change either the sum-rate or the BER trend of the curves.

$\tilde{\mathbf{F}}_{kl}^{(k)}$ is defined below (5.14), $\Delta_j^{(k)}$ and $\mathbf{n}^{(k)}$ are defined in (5.17).

In Figure 5.3, we show that IA can be achieved even with limited power at the relay using the “IA-LFB” method proposed in Section 5.4. Here, “IA (unconstrained)” denotes the conventional IA algorithm with no power constraint at the relay. When the power at the relay is limited, we employ a naive power scaling on top of the conventional IA precoder (see Section 5.4) to meet the power constraint. However, it is observed that with this method, the DoF falls down to zero as shown in the “IA-naive-scaled” plots. Also, note that if there is abundant power at the relay, providing extra power to the precoders using naive power scaling distorts the alignment of interference and the DoF degrades sharply. It can be observed from (5.26) that by providing more power to the precoders, there is more misalignment of interference and the performance is further degraded. This is clear from the plot where the “IA-naive-scaling” at 30 dB shows the worst sum rate performance. *In contrast*, the proposed “IA-LFB” algorithm attains the maximum DoF even when arbitrarily low power is available at the relay, e.g. at 10 dB.

Figure 5.3: Sum rate comparison showing the effectiveness of IA-LFB algorithm for a three-user “symmetric” SISO-XRC. Here, “symmetric” implies equal average link strengths of the desired and interfering links at a user.

Figure 5.4 shows the sum-rate plot for a general asymmetric network. Here, “asymmetric” implies that the desired and the interfering links at a receiver are assumed to have different channel link strengths which is possible due to large scale fading effects. *In general*, it is observed that with suitable initializations (as discussed next), the proposed MMSE precoder shows better sum rate than the IA or IA-LFB algorithms up to around 40 dB. Note that depending on the precoder initialization, multiple local optima points are possible for the MMSE algorithm. When randomly initialized, the MMSE algorithm may get stuck in any of these local points and this may lead to a deterioration of the DoF as shown in the figure. Therefore, the MMSE algorithm should be initialized with the DoF-optimal, i.e., IA or the IA-LFB precoder. With DoF-optimal initialization, the MMSE algorithm iteratively converges towards the local optimal point in the vicinity of the IA or IA-LFB precoder. Thus, it is observed that while the MMSE algorithm improves the sum rate in the finite SNR region, it also maintains the DoF of the network. In the figure, “IAinit” or “IA-LFBinit” indicate that the MMSE algorithm has been initialized with conventional (unconstrained) IA precoder or IA-LFB precoders, respectively. Note that the “IA-LFBinit” shows almost the same sum-rate performance as the “IAinit” but with much lesser relay power consumption (please see Figure 5.5).

As discussed in Section 5.5, power constraint allocation to each precoder is also an important design issue. In the figure, “(equal power)” indicates that the power constraint on each relay precoder is set to be equal, i.e., $\frac{P_t}{6}$, where P_t is the total relay power constraint. Similarly, “(IA-based-power)” and “(IA-LFB-based-power)” denote that the relay power constraint for each precoder is set to be the same as that of the IA or IA-LFB precoder’s power. For the “MSE-IAinit (equal power)”, the maximum relay power constraint is fixed at 30 dB. Note that with both the IA and the IA-LFB-based precoder initializations, “(equal power)” constraint shows slightly higher sum-rate in the low SNR regions than the “IA (or

IA-LFB)-based-power” initialization. This is because IA (or IA-LFB)-based power initialization very much restricts the search space for each precoder in the MSE iterative algorithm around the IA (or IA-LFB) precoder. However, while equal power allocation may not perfectly align the interference for some precoders, it increases the search space in terms of power allocation in general. Since appropriate power allocation and not IA is more important in the noise-limited regions, “(equal power)” allocation has better performance in the finite SNR region. Note that in this process, the “(equal power)” precoders also consume more power to improve the sum-rate in the noise-limited regions (please see Figure 5.5).

Figure 5.4: Sum rate comparison for a three-user SISO-XRC.

Figure 5.5 shows the power consumed by the relay (refer to (5.20)) with different precoder initializations and power constraint allocations for the MMSE algorithm. The power consumption in this plot correspond to the same channel realizations and the same set of precoders as in Figure 5.4. Note that the “MSE-IAinit (IA-based-power)” scheme consumes only slightly lower relay power as compared to the conventional (unconstrained) IA because here the precoder initialization is done with the IA precoder and also, power constraint allocation to each precoder is the same as that expended by the corresponding IA precoders.

Since the MMSE algorithm converges in few iterations around the IA precoder, the DoF (as seen in Figure 5.4) and the power expended by the relay are not affected much (although the sum rate is improved in the finite SNR range). With IA initialization and *equal power constraint allocation* to each precoder (under a total power constraint of 30 dB at the relay), the MMSE algorithm gets a different search space for power allocation which saves $\sim 1.4 - 1.5$ dB relay power as compared to the conventional IA (especially at high total transmit SNR) while also preserving the network DoF.

Further, with the proposed ϵ -scaled transmission scheme (i.e., IA-LFB) to restore IA for the case of low relay power, we can attain IA under any power constraint at the relay. This enables us to save significant relay power while also preserving the DoF, as seen in the figure. For example, the “MSE-IA-LFBinit (IA-LFB-based-power)” algorithm saves about 4.18 dB at 21.76 dB total transmit SNR and at the same time, shows about 0.56 bits/sec/Hz (see Figure 5.4) sum-rate gain as compared to the conventional IA algorithm. Further, note from Figure 5.5 that the “(equal power)” allocation, together with IA-LFB initialization, consumes more relay power as compared to the “(IA-LFB-based power)” allocation in the low SNR range as discussed previously.

Figure 5.6 shows the BER comparison of the various algorithms for an asymmetric network corresponding to the same set of channel realizations and precoders as in Figure 5.4. Clearly, all variations of the MMSE algorithm outperform the IA and IA-LFB schemes for a wide SNR range, giving about 3 dB SNR gain. The maximum available relay power in “MSE-IAinit (equal power)” is taken to be 30 dB which is divided equally into 6 equal parts to be allotted as the power constraint on each precoder. For clarity, the BER curves for the MMSE algorithm corresponding to “(IA/IA-LFB-based power)” allocation are not shown as they almost overlap with the “(equal power)” allocation curves.

In addition, we show the BER performances of the proposed algorithms in a

Figure 5.5: Power consumed at the relay due to various algorithms

fully-connected *symmetric*⁵ system where the desired and interfering signal strengths at a receiver are assumed to be the same. In the symmetric system, “IA-LFB, $P_t=24.77$ dB” shows the proposed IA algorithm with limited feedback (LFB) when the maximum power available at the relay is 24.77 dB. The maximum available relay power in “MSE-IAinit (IA-based-power)” is taken to be 30 dB. It is observed that with the MMSE algorithm coupled with the IA-LFB algorithm, and “IA-LFB-based-power” constraint on the precoders, we obtain almost the same BER performance as the “MSE-IAinit (IA-based-power)” algorithm and better BER performance than all other algorithms with reduced relay power consumption.

Figure 5.7 shows the BER performance of the IA and the MMSE algorithms in a symmetric network as a function of the relay power at a maximum total source transmit SNR of 31.76 dB. Note that the BERs of the IA-LFB and the MMSE algorithms based on IA-LFB initialization improve as the relay power increases. As expected, BER of the IA-LFB coincides with the conventional IA (or IA-unconstrained where $\epsilon = 1$) at the total relay power of 27.71 dB. Note that the IA-LFB not only restores interference alignment for any relay power (unlike

⁵*symmetric* here should not be confused with the *symmetric* on the footnote of page 101.

Figure 5.6: BER comparison for a three-user SISO-XRC. QPSK modulation is used.

conventional IA precoder) but also provides the *added* benefit of reduced power consumption at the *source* TXs also. This is because when the relay power is not sufficient, the source TXs scale their symbols in the last two time slots using a scaling factor $\epsilon < 1$. The power savings⁶ at the source TXs is computed as $\mathbb{E} \{-10\log_{10}\epsilon^2\}$.

Another important point to observe from this plot is that the “MSE-IA-LFB (IA-LFB-based-power)” shows almost the same BER of 1.16×10^{-2} as the conventional IA algorithm but consumes about 16 dB lesser relay power (see the dashed black horizontal line). At the same time, since the MMSE is initialized with IA-LFB precoder, it saves more than 5 dB power at the source transmitters also (see the dashed black vertical line at 10.87 dB). The conventional IA precoder can operate only above a threshold relay power as discussed earlier, and therefore, its plot is shown only at the threshold relay power of 27.71 dB. Note that if we apply IA-LFB when the relay power is greater than 27.71 dB and remove the power constraint at the source TXs (i.e., ϵ can assume any value in (5.42)), we can further

⁶The power savings and BER are shown on the same plot to immediately visualize the advantage of the proposed algorithms.

improve the BER as seen from Figure 5.7 (please note the BER of IA-LFB at a relay power of 30 dB). However, in this case, the source TXs spend ~ 12 dB more power (equivalently, the maximum allowable total source transmit power of 31.76 dB is exceeded) at the relay power of 30 dB (see the blue square in Figure 5.7), as compared to the conventional IA (when $\epsilon = 1$).

Figure 5.7: BER performance of the algorithms (QPSK modulation is used), and power savings at the TXs in the 4th and 5th time slots (blue squares with values on the right y-axis), as a function of the total relay power. Maximum allowable total source transmit power is 31.76 dB.

5.7 Summary

In this chapter, IA feasibility is investigated for a MIMO-X channel aided by a MIMO relay. A sufficient condition on the number of antennas needed at the relay is derived which can attain the maximum DoF of the network even in time-invariant channels. While power constraint at the relay may restrict alignment of interference, we propose a novel IA-LFB scheme which makes use of a simple scalar feedback to the source transmitters to attain the maximum system DoF even under extremely limited power at the relay. Next, a finite-SNR closed-form MMSE precoder design is proposed. It minimizes the sum-mean-square-error of

the network, attains better sum rate in the finite SNR range, and also better BER performance than the conventional IA algorithm. We note that the iterative MMSE algorithm intends to attain only locally-optimal solution and its performance can severely degrade if it is not suitably initialized. To this end, we further propose to use IA-LFB solution as the initialization seed for the MMSE algorithm which maintains the maximum DoF even under limited relay power.

Appendix

Proof of Proposition 5.2

P_R in (5.25) can be re-written as $\epsilon^2 \tilde{P}$, where $\tilde{P} = \sum_{j \in \mathcal{I}} \sum_{\substack{m', n' \in S_m \\ \forall m \in \mathcal{I}'}} [\Gamma_j]_{m', n'}$. Now, in order to minimize the power at the relay, and at the same time, ensuring that the power expended by the source transmitters from the $(K+1)^{th}$ to $(2K-1)^{th}$ time-slot does not exceed its maximum allowable limit, we have the optimization problem:

$$\epsilon^* = \arg \min_{\epsilon} \left| \epsilon^2 \tilde{P} - P_t \right| \text{ s.t. : } 0 < \epsilon \leq 1. \quad (5.42)$$

Note that the the transmitters scale their symbols in the last $(K-1)$ time slots by ϵ so that the power transmitted by a transmitter in each of the last $(K-1)$ time slots becomes $\epsilon^2 KP$, where P is the maximum power allowed per symbol. The constraint $\epsilon \leq 1$ comes from the fact that the transmitted power in each of the last $(K-1)$ time slots should be less than or equal to KP . The constraint $\epsilon > 0$ ensures joint beamforming of the transmitter together with the relay *otherwise IA is not feasible*.

In the above problem, clearly, the *unconstrained* minimum value is 0 which is attained at $\epsilon^* = \sqrt{\frac{P_t}{\tilde{P}}}$. However, if $\epsilon^* > 1$, this means that the source transmitter power is exceeded in each of the last $(K-1)$ time slots (since the transmitters scale their symbol by $\epsilon > 1$). Thus, when $\epsilon > 1$ for the unconstrained problem,

we enforce the source transmitter power constraint $\epsilon \leq 1$ in (5.42), and it can be shown that the minimum objective value is attained at $\epsilon^* = 1$ (under the feasible constraint set $\epsilon \leq 1$), i.e., *we should use conventional IA which does not require any feedback to the source transmitters*. Also, in this case, it can be verified that the power consumed at the relay, $\tilde{P} < P_t$, i.e., conventional IA consumes less than the maximum available relay power. However, if $\epsilon^* < 1$ for the *unconstrained* problem (i.e., the constraint $\epsilon < 1$ is already satisfied), relay consumes the maximum available power to restore IA. *Thus, we should simply use IA-LFB in this case*. A side advantage of $\epsilon^* < 1$ is that the symbols transmitted by the source transmitters utilize less than their maximum available powers in the last $(K-1)$ time slots (please see Figure 5.7 for the 3-user case).

■

Chapter 6

Dual-hop Interference Channels with Limited Channel Information

The relay-aided networks have received considerable attention in the recent years [129, 130, 137–140, 144, 178–183] where layered (no direct connection between the transmitters and the receivers) as well as non-layered topologies (direct connections exist between the transmitters and the receivers) have been considered. Cooperation using relaying in the non-layered topology (also called “fully-connected” network) was discussed in chapters 4 and 5, and is useful in attaining the network DoF and reducing the number of transmission channel uses.

However, in many scenarios, there are no direct connections between the transmitters and the receivers due to large path-losses or shadowing effects, and the main purpose of the relays is to extend the range of the signals transmitted by the source transmitters to their respective receivers while handling the inter-user interference at the same time (especially, in multi-user dual-hop networks). In this chapter, we investigate such layered relay-aided SISO interference channels (also referred to as

“dual-hop SISO-IC”). Note that this chapter essentially considers the relay-aided interference network in Chapter 4 (where every transmitter intends to transmit a distinct message to its own receiver only) *but* with the additional conditions of multiple relays and no direct links between the transmitters and the receivers. In addition, we considered perfect CSI based precoding in our previous relay-aided network models in chapters 4 and 5 but in this chapter, we consider the practical constraint of imperfect CSI based precoding at the relays.

While there has been significant research recently for beamforming in the relay-aided multi-user dual-hop networks [140–144, 164], these works consider perfect CSI availability at the transmitters or the relays or both. On the contrary, in this work, we consider the dual-hop SISO-IC with no CSI at the transmitters and quantized CSI at the relays which, to the best of our knowledge, has not been addressed before.

In this chapter, first, we study the feasibility conditions for interference neutralization (IN) for a dual-hop IC and show that the unlike the single-hop multi-user networks such as broadcast or interference networks, channel gains, and not only channel directions, affect the network DoF significantly. Further, we show that the overall power of the relays must scale at a certain rate with respect to the source transmitters’ power in order to attain the network DoF. Next, for the abovementioned IN-feasible dual-hop SISO-IC with sufficient relay power, we perform rate-loss analysis of the network under the effects of quantized CSI based beamforming. Based on the minimization of the derived rate-loss upper bound, we propose an adaptive bit allocation strategy to improve the network sum-rate. Finally, we design a stochastically robust minimum-sum mean square error (MMSE) precoder which exploits the CSI quantization error statistics to improve the network sum-rate significantly as compared to the non-robust precoders.

6.1 System description

6.1.1 System model

We consider a K -user dual-hop single-input-single-output IC (SISO-IC) assisted by M multi-input-multi-output (MIMO) relays (as shown in Figure 6.1) which perform the amplify-and-forward (AF) relaying. The m^{th} ($m \in \mathcal{M} = \{1, 2, \dots, M\}$) relay has L_m antennas and each relay operates in the half-duplex mode. The relays and the receivers are connected to a central station (CS) through limited-capacity wireless feedback links. We assume that each relay and each receiver estimates its local links perfectly. Also, each relay and each receiver has a limited feedback bits budget which it uses to quantize its local link's channel direction information, i.e., CDI, and feed back the quantized CDI to the CS via the limited capacity feedback link. On the other hand, the scalar channel gains are assumed to be fed back to the CS perfectly¹.

Figure 6.1: A dual-hop SISO interference channel aided by multiple MIMO relays. Prior to data transmission, the relay and the receivers estimate their local channels perfectly, quantize the channel directions, and feedback the quantized vector channel direction and scalar channel gains to the central station (CS). The CS computes the relay precoders based on quantized channels, and broadcasts the precoders to all the relays.

Note that in the dual-hop IC, each transmitter transmits a message for its own receiver only. The transmission takes place in two time slots. In the 1^{st} time-slot,

¹In this work, we assume that the real gain values can be fed back with sufficient accuracy using much fewer number of feedback bits as compared to the CDI vector quantization, and they are assumed to be perfect. However, note that for our dual-hop IC, channel gains are also important, and their quantization and feedback can be important topics of future research.

the k^{th} ($k \in \mathcal{K} = \{1, 2, \dots, K\}$) transmitter transmits the symbol x_k intended for the k^{th} receiver, where $\mathbb{E}\{x_k^H x_k\} = 1$, and the power of the k^{th} transmitter is constrained by P_k . Thus, the signal received by the m^{th} relay in the 1^{st} time-slot is given as²

$$\mathbf{y}_m^{(R)} = \sum_{i=1}^K \sqrt{\alpha_{mi}^{(h)}} \mathbf{h}_{mi} \sqrt{P_i} x_i + \mathbf{n}_m^{(R)}, \quad (6.1)$$

where $\alpha_{mi}^{(h)}$ and $\mathbf{h}_{mi} \in \mathbb{C}^{L_m \times 1}$ denote the average path-loss and the Rayleigh flat-fading short-term channel coefficient vector respectively from the i^{th} TX to the m^{th} relay, and $\mathbf{n}_m^{(R)} \in \mathbb{C}^{L_m \times 1}$ is the additive white Gaussian noise (AWGN) vector at the relay such that $\mathbf{n}_m^{(R)} \sim \mathcal{CN}(\mathbf{0}, \sigma_{R,m}^2 \mathbf{I})$. The m^{th} relay precodes the received signal as $\mathbf{x}_m = \mathbf{U}_m \mathbf{y}_m^{(R)}$ using the precoder $\mathbf{U}_m \in \mathbb{C}^{L_m \times L_m}$, and broadcasts $\mathbf{x}_m$ to the receivers in the 2^{nd} time-slot. The signal received by the k^{th} receiver in the 2^{nd} time-slot is given as

$$y_k = \sum_{m=1}^M \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} \mathbf{x}_m + n_k, \quad (6.2)$$

where $\alpha_{km}^{(g)}$ and $\mathbf{g}_{km} \in \mathbb{C}^{1 \times L_m}$ denote the average path-loss and the Rayleigh short-term channel vector respectively from the m^{th} relay to the k^{th} receiver, and $n_k \in \mathbb{C}$ is the AWGN at the k^{th} receiver, distributed as $\mathcal{CN}(0, \sigma_k^2)$.

Using $\mathbf{x}_m = \mathbf{U}_m \mathbf{y}_m^{(R)}$ in (6.2), the received signal at the k^{th} receiver can be

²The system model is applicable to both time-varying and time-invariant channels and the channel use index is omitted for brevity.

written as

$$\begin{aligned}
 y_k = & \underbrace{\sum_{m=1}^K \sqrt{\alpha_{mk}^{(h)}} \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} \mathbf{U}_m \mathbf{h}_{mk} \sqrt{P_k} x_k}_{\text{desired signal}} + \underbrace{\sum_{\substack{i=1 \\ i \neq k}}^K \sum_{m=1}^M \sqrt{\alpha_{mi}^{(h)}} \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} \mathbf{U}_m \mathbf{h}_{mi} \sqrt{P_i} x_i}_{\text{interference}} \\
 & + \underbrace{\sum_{m=1}^M \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} \mathbf{U}_m \mathbf{n}_m^{(R)}}_{\text{overall noise}} + n_k. \tag{6.3}
 \end{aligned}$$

The signal-to-interference and noise ratio (SINR) of the k^{th} user is given as:

$$\rho^{(k)} = \frac{P_k \left| \sum_{m=1}^M \bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk} \right|^2}{\sum_{l \neq k} P_l \left| \sum_{m=1}^M \bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{ml} \right|^2 + \sigma_{R,m}^2 \sum_{m=1}^M \|\bar{\mathbf{g}}_{km} \mathbf{U}_m\|^2 + \sigma_k^2}, \tag{6.4}$$

where $\bar{\mathbf{g}}_{km} = \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km}$ and $\bar{\mathbf{h}}_{mk} = \sqrt{\alpha_{mk}^{(h)}} \mathbf{h}_{mk}$, and the sum-rate of the network is given as $\frac{1}{2} \sum_{k=1}^K \log_2(1 + \rho^{(k)})$.

6.1.2 Channel feedback model

As mentioned above, a CS collects the quantized CSI from all the relays and the receivers and then designs the relay precoders. In practice, since the feedback link from each node (i.e., relay or receiver) to the CS is bandwidth-constrained, each node can quantize its links only using a fixed amount of feedback bits [6]. We write $\bar{\mathbf{h}}_{mk}$ and $\bar{\mathbf{g}}_{km}$ as

$$\bar{\mathbf{h}}_{mk} = \sqrt{\alpha_{mk}^{(h)}} \mathbf{h}_{mk} = \sqrt{\alpha_{mk}^{(h)}} \sqrt{\delta_{mk}^{(h)}} \tilde{\mathbf{h}}_{mk}, \tag{6.5}$$

$$\bar{\mathbf{g}}_{km} = \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} = \sqrt{\alpha_{km}^{(g)}} \sqrt{\delta_{km}^{(g)}} \tilde{\mathbf{g}}_{km}, \tag{6.6}$$

where $\delta_{mk}^{(h)} = \|\mathbf{h}_{mk}\|^2$ and $\delta_{km}^{(g)} = \|\mathbf{g}_{km}\|^2$. We assume that the long-term channel information, i.e., $\alpha_{km}^{(g)}$ and $\alpha_{mk}^{(h)}$, and the channel gains $\delta_{mk}^{(h)}$ and $\delta_{km}^{(g)}$ are known perfectly to the CS. However, the CDI, i.e., $\tilde{\mathbf{h}}_{mk}$ and $\tilde{\mathbf{g}}_{km}$ are quantized by the m^{th}

relay and the k^{th} receiver using B_{mk} and $\tilde{B}_{km}$ bits respectively, and then fed back to the CS over the limited capacity feedback links (Please refer to Figure 6.1).

Specifically, the m^{th} relay has a codebook, viz., $\mathbf{W}_m^{(R)}$ which consists of codewords $\mathbf{w}_i \in \mathbb{C}^{L_m \times 1}$ such that $\|\mathbf{w}_i\| = 1$, $i = \{1, 2, \dots, 2^{B_{mj}}\}$, where $2^{B_{mj}}$ is the size of the codebook used for quantizing the channel direction from the j^{th} transmitter to the m^{th} relay using B_{mj} bits. The relay chooses that codeword from its codebook which is nearest to the CDI, i.e., $\tilde{\mathbf{h}}_{mk}$, and feeds back its index to the CS. This is the well-known chordal-distance (CD) minimization based quantization [184–186], and the quantized channel direction for the CDI, $\hat{\mathbf{h}}_{mj}$ is given as

$$\hat{\mathbf{h}}_{mj} = \arg \min_{\mathbf{w}_i \in \mathbf{W}_m^{(R)}} \underbrace{\left(1 - \left|\tilde{\mathbf{h}}_{mj}^H \mathbf{w}_i\right|^2\right)}_{z_{mj}}. \quad (6.7)$$

Further, in the CD based model, the CDI vector can be written as [184],

$$\tilde{\mathbf{h}}_{mj} = \sqrt{1 - z_{mj}} \hat{\mathbf{h}}_{mj} + \sqrt{z_{mj}} \overline{\Delta \mathbf{h}}_{mj}, \quad (6.8)$$

where z_{mj} is the CDI quantization error as given in (6.7), and $\overline{\Delta \mathbf{h}}_{mj}$ is the unit-norm error vector isotropically distributed in the $(L_m - 1)$ -dimensional nullspace of the vector $\hat{\mathbf{h}}_{mj}$.

We assume that CS also knows the relays' codebooks and thus, it can easily decode the codeword sent by the relay. Similarly, the k^{th} receiver also quantizes its CDI, i.e., $\tilde{\mathbf{g}}_{km}$ to the unit-norm vector $\hat{\mathbf{g}}_{km}$ using its codebook $\mathbf{W}_k$, and feeds back the codeword index to the CS.

Before proceeding to the rate-loss analysis, in the next two sections, we provide the feasibility conditions and power scaling law for interference neutralization in the dual-hop SISO-IC.

6.2 Feasibility conditions and Interference neutralization (with perfect CSI)

Theorem 6.1. *For the dual-hop K -user SISO-IC with no CSI at the source transmitters, assisted by M half-duplex MIMO relays with abundant relay power³, the m^{th} relay having L_m antennas, and all channel coefficients being independent and drawn from a continuous distribution, a sufficient condition to achieve the network DoF of $\frac{K}{2}$ using linear IN beamforming without symbol extensions (i.e., in two channel uses), is given as*

$$\sum_{m=1}^M L_m \geq K, \quad (6.9a)$$

$$\sum_{m=1}^M L_m^2 - 1 \geq K(K - 1), \quad (6.9b)$$

$$|\bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk}| = \kappa, \quad (6.10)$$

where $\mathbf{U}_m \in \mathbb{C}^{L_m \times L_m}$ denotes the precoder matrix at the m^{th} relay, and κ is some positive constant.

Proof. The condition (6.9a) comes from the cut-set bound. (6.9b) is obtained by counting the number of equations and valid variables and establishing the *properness* criterion similar to [52, 132] by stating that the number of variables involved in the resultant polynomials should be greater than or equal to the number of constraints or equations involved.

It is easy to verify that for the single antenna transmitters and receivers as in the above model, if (6.9b) is satisfied, the properness condition for every subset $K' \subseteq \{1, 2, \dots, K\}$ is also automatically satisfied. An IN-based achievability scheme

³Note that the relay power must scale sufficiently with the source transmit power as shown in Section 6.3.

that establishes the condition in (6.9b) is given next.

Finally, (6.10) guarantees that the effective desired link of every user, i.e., $\bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk}$ is not zero.

IN Achievability scheme

From the interference term in (6.3), for IN, the following condition needs to be satisfied.

$$\sum_{m=1}^M \sqrt{\alpha_{mi}^{(h)}} \sqrt{\alpha_{km}^{(g)}} \mathbf{g}_{km} \mathbf{U}_m \mathbf{h}_{mi} = 0, \quad \forall k, i \in \mathcal{K}, i \neq k. \quad (6.11)$$

Defining $\psi_m^{(ki)} \triangleq \sqrt{\alpha_{mi}^{(h)}} \sqrt{\alpha_{km}^{(g)}} \sqrt{\delta_{mi}^{(h)}} \sqrt{\delta_{km}^{(g)}}$, and $L \triangleq \sum_{m=1}^M L_m^2$, (6.11) can be written in a compact form as

$$\mathbf{H} \mathbf{u} = \mathbf{0}_{K(K-1) \times 1}, \quad (6.12)$$

where $\mathbf{H} \in \mathbb{C}^{K(K-1) \times L}$ is defined as

$$\left[(\psi^{(12)} \mathbf{H}_{(12)})^T, \dots, (\psi^{(1K)} \mathbf{H}_{(1K)})^T, \dots, (\psi^{(K1)} \mathbf{H}_{(K1)})^T, \dots, (\psi^{(K(K-1))} \mathbf{H}_{(K(K-1))})^T \right]^T, \quad (6.13)$$

and $\psi^{(ki)} \in \mathbb{C}^{1 \times M}$, $\mathbf{H}_{(ki)} \in \mathbb{C}^{M \times L}$, and $\mathbf{u} \in \mathbb{C}^{L \times 1}$ are defined as

$$\begin{aligned} \psi^{(ki)} &= [\psi_1^{(ki)}, \psi_2^{(ki)}, \dots, \psi_M^{(ki)}], \\ \mathbf{H}_{(ki)} &= \text{blkdiag} \left[(\tilde{\mathbf{h}}_{1i}^T \otimes \tilde{\mathbf{g}}_{k1}), (\tilde{\mathbf{h}}_{2i}^T \otimes \tilde{\mathbf{g}}_{k2}), \dots, (\tilde{\mathbf{h}}_{Mi}^T \otimes \tilde{\mathbf{g}}_{kM}) \right], \\ \mathbf{u} &= [\text{vec}^T(\mathbf{U}_1), \text{vec}^T(\mathbf{U}_2), \dots, \text{vec}^T(\mathbf{U}_M)]^T. \end{aligned} \quad (6.14)$$

Clearly, from (6.12), $\mathbf{u}$ definitely has a non-trivial solution given as $\mathbf{u} \in \mathcal{N}(\mathbf{H})$ (where $\mathcal{N}()$ denotes the null space) if $L > K(K-1)$. Further, if $\mathbf{u}$ is a solution, $\frac{1}{\mathbf{u}(1)} \mathbf{u}$ (where $\mathbf{u}(1)$ denotes the 1st element of $\mathbf{u}$) is also a solution (by eliminating

the superfluous variable [52]). Thus, there are $K(K-1)$ equations and $(L-1)$ valid variables, and a solution exists for (6.12) if the condition in (6.9b) is satisfied. ■

Remark 6.1. Note that [132] mentions the properness criteria for the case of CSI at every node and establishes the necessity of the properness criteria by proving that improperness implies unsolvability of the resultant polynomial equations. However, for the MIMO relay case (as in our model), the necessity of the properness criteria for IN is still an open issue. In Theorem 6.1, although the condition (6.9b) is obtained from the properness criteria, we do not claim it to be a necessary condition for IN⁴.

Proposition 6.1. *For the dual-hop SISO-IC, accurate channel gains also, and not only channel directions, are necessary to attain the DoF of $\frac{K}{2}$ using IN.*

The proof immediately follows from (6.13) where channel gains $\psi^{(ki)}$ are required. Intuitively, the net interfering signal at any receiver is the weighted (weighted by the channel gains) combination of the interfering signals (from the non-intended transmitters) via all the relays. If the weights are not accurate (e.g., for the case of imperfect channel gains feedback), the net interference at the receiver is non-zero and DoF degrades. This is in contrast to the single-hop broadcast or interference networks where channel directions are sufficient for beamforming (such as zero-forcing or interference alignment).

Remark 6.2. If IN is to be achieved using CDI only, the condition for IN is given as $\tilde{\mathbf{g}}_{km} \mathbf{U}_m \tilde{\mathbf{h}}_{mi} = 0 \ \forall k, m, i, \ i \neq k$ (see (6.11)). Thus, the number of equations is $K(K-1)M$ and the number of valid variables is still $(L-1)$. The sufficiency condition for IN is given as $(L-1) \geq K(K-1)M$. In this case, it is interesting to note that no matter how many relays are used, and every relay has up to $(K-1)$ antennas, i.e., $L_m = (K-1) \ \forall m$, the sufficiency condition is *not* satisfied. On

⁴Note that in general, properness does not imply sufficiency also. For example, for the $2 \times 2 \times 2$ dual-hop network with 3 antennas each at the 1st transmitter, 1st relay and the 1st receiver, and 2 antennas at all other nodes, if the 1st and the 2nd transmitters send 3 and 2 data streams each, the system is proper [132] yet IN-infeasible due to *non-redundant* nature of the network [135].

the other hand, if the relays have K antennas, the sufficiency condition is *always* satisfied irrespective of the number of relays, i.e., even a single K -antenna relay is sufficient to achieve the network DoF using only CDI ⁵.

6.3 Power scaling law

Theorem 6.2. *For the dual-hop SISO-IC aided by multiple MIMO relays, and given the IN sufficiency conditions are satisfied, the network DoF of $\frac{\beta K}{2}$ is attained if the overall relay power scales as $\frac{P^\beta}{f(P)}$ such that $\log(f(P)) = o(\log(P))$, where P is the power of each source transmitter.*

Proof. Assuming every transmitter transmits with the same power, i.e., $P_i = P$ and $\sigma_{R,m}^2 = \sigma_R^2 \forall m$, the overall power spent by the m^{th} relay can be shown to be

$$P \mathbf{u}_m^H \left[\underbrace{\sum_{i=1}^K (\mathbf{h}_{mi}^* \mathbf{h}_{mi}^T \otimes \mathbf{I}_{L_m})}_{\mathbf{D}_m} \right] \mathbf{u}_m + \sigma_{R,m}^2 \mathbf{u}_m^H \mathbf{u}_m, \quad (6.15)$$

where $\mathbf{u}_m = \text{vec}(\mathbf{U}_m)$, and the overall power expended by all the relays can be written as

$$P \mathbf{u}^H \mathbf{D} \mathbf{u} + \sigma_R^2 \mathbf{u}^H \mathbf{u}, \quad (6.16)$$

where $\mathbf{D} = \text{blkdiag}[\mathbf{D}_1, \dots, \mathbf{D}_M]$ and $\mathbf{u}$ is defined in (6.14). Since DoF is defined at high SNR, the 2nd term in (6.16) can be neglected and the overall relay power $\sim P \mathbf{u}^H \mathbf{D} \mathbf{u}$. Note that $\mathbf{u}$ can be scaled as $\tilde{\mathbf{u}} = \eta \mathbf{u}$ without affecting the IN condition (see (6.12)). Also, assuming that the channel values do not take up infinite values, it can be seen that $\mathbf{u}^H \mathbf{D} \mathbf{u}$ is a finite value, say, ϕ . Then, after scaling the precoder so that the maximum relay power P_t is consumed, the power consumed by the scaled relay precoder $\tilde{\mathbf{u}}$ at high SNR can be written as $P_t \triangleq \eta^2 \phi P$.

⁵Note that the second-hop becomes the broadcast network when a single MIMO relay is used.

Now, consider the SINR of the k^{th} user, i.e., $\rho^{(k)}$ (see (6.4)). Since the precoder $\mathbf{u}$ is scaled to $\tilde{\mathbf{u}}$ in order to satisfy the maximum relay power constraint P_t , it can be noted that each of the individual relay precoder, i.e., $\mathbf{U}_m$ is also scaled as $\eta \mathbf{U}_m$. Then, $\rho^{(k)}$ at high SNR can be written as $\eta^2 P \kappa$ (Note that the interference term in (6.4) becomes zero due to IN and the noise terms are neglected at high SNR), where $\kappa \triangleq \left| \sum_{m=1}^K \bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk} \right|^2$ is a constant because $\|\mathbf{U}_m\| \leq 1$.

Thus, the DoF attained by the k^{th} user is given as

$$DoF_k = \lim_{P \rightarrow \infty} \frac{\log_2(\eta^2 P \kappa)}{\log_2(P)} \quad (6.17)$$

Using $\eta^2 P = \frac{P_t}{\phi}$ as mentioned above, (6.17) can be written as $DoF_k = \lim_{P \rightarrow \infty} \frac{\log_2(\frac{\kappa P_t}{\phi})}{\log_2(P)}$.

Note that κ and ϕ are finite values, and therefore, in order to attain a DoF_k of β , $\beta > 0$, P_t must scale as $\frac{P^\beta}{f(P)}$ such that $\lim_{P \rightarrow \infty} \frac{\log(f(P))}{\log(P)} \rightarrow 0$. $f(P) \triangleq (\log(P))^\epsilon$, $\epsilon \geq 0$ is one such function, and thus, if the maximum relay power remains equal to or above $\frac{P^\beta}{(\log(P))^\epsilon}$, the k^{th} user attains a DoF of β . Similar analysis applies to other users also, and the proposed power scaling attains the network DoF of $\frac{\beta K}{2}$. ■

Figure 6.2 shows the sum-rate plots for a 3-user dual-hop SISO-IC using different powers available at the relays. It can be verified that the conventional IN achieves a DoF of $\frac{3}{2}$ whereas if the relay power P_t scales as $P^{0.8}$ or $\frac{P^{0.8}}{(\log P)^{0.5}}$, the network DoF is $\frac{3 \times 0.8}{2} = 1.2$, as stated in Theorem 6.2 above. Also, with fixed relay power of 30 dB, the DoF drops to 0.

Remark 6.3. Note that the above result is unlike the non-layered SISO-IC [31, 143] where the maximum network DoF is attained at any relay power due to the direct connectivity between the transmitters and the receivers. However, for the layered dual-hop as shown above, the relay power must scale sufficiently with the power of the source transmitters in order to attain the network DoF. If the relay

Figure 6.2: Sum rate for a $3 \times 2 \times 3$ dual-hop SISO-IC. Each MIMO relay has 2 antennas. P is the source TX's power and P_t is the maximum power available at the relay.

power is fixed, it can be verified that the DoF of every user drops to zero.

6.4 Rate loss analysis

6.4.1 Rate loss upper bound

In this section, we derive the upper bound for the rate loss of the network when imperfect CSI is available at the relays. As mentioned in the system model, each relay or receiver quantizes its CDI and feeds back to the CS which then computes the precoders for the relays denoted as $\hat{\mathbf{U}}_m \forall m$. Since these precoders are based on quantized CSI, they incur significant sum-rate loss as characterized next.

The expected network rate-loss is given as

$$\Delta R = \sum_{k=1}^K \mathbb{E} \left\{ \Delta R^{(k)} \right\} = \sum_{k=1}^K \mathbb{E} \left\{ R^{(k)} - \hat{R}^{(k)} \right\}, \quad (6.18)$$

where $\Delta R^{(k)}$ denotes the rate-loss for the k^{th} user, $R^{(k)}$ and $\hat{R}^{(k)}$ denote the sum-rate of the k^{th} user for the perfect and quantized CSI respectively, and at high SNR (neglecting the noise terms), using the approximation $\log(1+x) \approx \log x$, and

assuming that the noise powers are unity, they are given as

$$R_k \approx \frac{1}{2} \log_2 \left(P_k \left| \sum_{m=1}^K \bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk} \right|^2 \right), \quad (6.19)$$

$$\hat{R}_k \approx \frac{1}{2} \log_2 \left(\frac{P_k \left| \sum_{m=1}^K \bar{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{mk} \right|^2}{\sum_{l \neq k} P_l \left| \sum_{m=1}^K \bar{\mathbf{g}}_{lm} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml} \right|^2 + \sum_{m=1}^M \|\bar{\mathbf{g}}_{km} \mathbf{U}_m\|^2 + 1} \right). \quad (6.20)$$

Using (6.19) and (6.20), $\mathbb{E} \{ \Delta R^{(k)} \}$ can be written as

$$\frac{1}{2} (\mathcal{I}_{des}^{(k)} + \mathcal{I}_{int}^{(k)}),$$

where $\mathcal{I}_{des}^{(k)}$ and $\mathcal{I}_{int}^{(k)}$ are the rate losses for the k^{th} user due to the channel quantization on the *effective* desired and the *effective* interfering signals respectively, and are given as

$$\mathcal{I}_{des}^{(k)} = \mathbb{E} \left\{ \log_2 \left(P_k \left| \sum_{m=1}^K \bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk} \right|^2 \right) \right\} - \mathbb{E} \left\{ \log_2 \left(P_k \left| \sum_{m=1}^K \bar{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{mk} \right|^2 \right) \right\}, \quad (6.21)$$

$$\mathcal{I}_{int}^{(k)} = \mathbb{E} \left\{ \log_2 \left(\sum_{l \neq k} P_l \left| \sum_{m=1}^K \bar{\mathbf{g}}_{lm} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml} \right|^2 + \sum_{m=1}^M \|\bar{\mathbf{g}}_{km} \mathbf{U}_m\|^2 + 1 \right) \right\}. \quad (6.22)$$

We provide the upper bounds for $\mathcal{I}_{int}^{(k)}$ and $\mathcal{I}_{des}^{(k)}$ in Lemma 6.1 and Lemma 6.2 respectively.

Lemma 6.1.

$$\begin{aligned} \mathcal{I}_{int}^{(k)} \leq \mathcal{I}_{int}^{(k),UB} &= \log_2 \left(1 + \eta_k + \sum_{l \neq k} \sum_m P_l \alpha_{km}^{(g)} \alpha_{ml}^{(h)} L_m^2 \Gamma \left(\frac{L_m}{L_m - 1} \right) 2^{-\frac{B_{ml}}{L_m - 1}} \right. \\ &\quad \left. + \sum_{l \neq k} \sum_m P_l \alpha_{km}^{(g)} \alpha_{ml}^{(h)} L_m^2 \Gamma \left(\frac{L_m}{L_m - 1} \right) 2^{-\frac{\tilde{B}_{km}}{L_m - 1}} \right), \end{aligned} \quad (6.23)$$

where $\alpha_{km}^{(g)}$ and $\alpha_{ml}^{(h)}$ are the path-loss factors defined in Section 6.1.1, and $\eta_k \triangleq \sum_{m=1}^M L_m \alpha_{km}^{(g)}$. B_{ml} and $\tilde{B}_{km}$ are the number of bits used to quantize the CDI of the TX-relay and relay-RX links, and are defined in Section 6.1.2.

Proof. Please refer to Appendix at the end of the chapter. ■

Lemma 6.2.

$$\mathcal{I}_{des}^{(k)} \leq \mathcal{I}_{des}^{(k),UB} = \sum_m \epsilon_{km} \Gamma \left(\frac{L_m}{L_m - 1} \right) 2^{-\frac{\tilde{B}_{km}}{L_m - 1}} + \sum_m \epsilon_{km} \Gamma \left(\frac{L_m}{L_m - 1} \right) 2^{-\frac{B_{mk}}{L_m - 1}}, \quad (6.24)$$

where ϵ_{km} is the weighting coefficient of the desired signal links, and is defined as

$$\epsilon_{km} = \mathbb{E} \left\{ \frac{|\bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk}|}{\sum_m |\bar{\mathbf{g}}_{km} \mathbf{U}_m \bar{\mathbf{h}}_{mk}|} \right\}, \quad (6.25)$$

and $\mathbf{U}_m$ is the precoder obtained using a given algorithm (e.g., IN precoding) considering perfect CSI availability at the relay.

Proof. Please refer to Appendix at the end of the chapter. ■

Remark 6.4. Note that ϵ_{km} in (6.25) is a function of perfect CDI values. This can be easily obtained using training at the CS by finding the expectation over multiple randomly generated channel values and need to be computed only once. Also, $\epsilon_{km} < 1$ and gives us tighter upper bounds.

Based on Lemma 6.2 and Lemma 6.1, we state the following theorem.

Theorem 6.3. *The expected rate-loss of the overall network is bounded as*

$$\Delta R \leq \Delta R^{UB} = \frac{1}{2} \sum_{k=1}^K \left(\mathcal{I}_{des}^{(k),UB} + \mathcal{I}_{int}^{(k),UB} \right), \quad (6.26)$$

where $\mathcal{I}_{des}^{(k),UB}$ and $\mathcal{I}_{int}^{(k),UB}$ are defined in Lemma 6.2 and Lemma 6.1 respectively.

We consider an illustrative example for a $3 \times 2 \times 3$ dual-hop SISO-IC network (please refer to Figure 6.4 in Section 6.6) where three single antenna TXs transmit data to their respective single antenna RXs via two MIMO relays having two antennas each. 10 bits are used for quantizing the channel direction information (CDI) of every channel link at each relay (or RX), and IN precoding scheme (discussed in Section 6.2) is applied. The rate loss of the network is computed due to IN precoding based on quantized CDI and the corresponding rate-loss upper bound in Theorem 6.3 is plotted in Figure 6.3. Figure 6.3 shows that the actual rate-loss is upper bounded by the analytical expression.

Note that for fixed feedback bits quantization (10 bits per link in this case), the rate loss scales as $\log(P)$ at high SNR (this is evident from (6.23)). Since our rate loss analysis has been done for high SNR and high channel resolution, it becomes more accurate in the high SNR regions and when the channel quantization bits increase.

6.4.2 Bit scaling law

Given sufficient power at the relays (see Theorem 6.2), the channel resolution must also be sufficient enough in order to achieve the network DoF of $\frac{K}{2}$. Note that for the case of dual-hop SISO-IC, channel values of two hops are involved, viz., from transmitters to the relays and from relays to the receivers. Thus, in order to attain a DoF of $\frac{1}{2}$ at every receiver, the bit allocation (equivalently, the channel resolution) to every channel direction at each of the relays and the receivers must be sufficiently high so that the interference at every receiver is bounded.

Figure 6.3: Rate-loss upper bound vs. Actual rate loss due to channel quantization

The following theorem provides a sufficient condition on the number of feedback bits needed for channel quantization so that the network DoF is attained.

Theorem 6.4. *In order to preserve the network DoF of $\frac{K}{2}$ for the dual-hop SISO-IC, a sufficient condition for feedback bits allocation to the channel directions at the relays and the receivers is given as*

$$B_{ml} \geq (L_m - 1) \log(P) + c'_1, \quad (6.27)$$

$$\tilde{B}_{km} \geq (L_m - 1) \log(P) + c'_2, \quad (6.28)$$

where

$$c'_1 = (L_m - 1) \log \left(\frac{\alpha_{ml}^{(h)} \left(\sum_{k \neq l} \alpha_{km}^{(g)} \right) L_m^2 \Gamma\left(\frac{L_m}{L_m - 1}\right)}{c_1} \right), \quad (6.29)$$

$$c'_2 = (L_m - 1) \log \left(\frac{\alpha_{km}^{(g)} \left(\sum_{l \neq k} \alpha_{ml}^{(h)} \right) L_m^2 \Gamma\left(\frac{L_m}{L_m - 1}\right)}{c_2} \right), \quad (6.30)$$

and c_1 and c_2 are some finite positive values.

Proof. From Lemma 6.1, the overall interference upper bound is given as $\sum_{k=1}^K \mathcal{I}_{int}^{(k)}$.

Note that the interference upper bound is a function of both B_{ml} and $\tilde{B}_{km}$ which are coupled together through the log function. Since we intend to investigate the bit scaling at the individual node (relay or receiver), we derive the bit scaling law using a loose upper bound. Using $\log(1+x) < x$, we obtain the overall interference upper bound as

$$\sum_{k=1}^K \mathcal{I}_{int}^{(k)} \leq \sum_{k=1}^K \sum_{\substack{l=1 \\ l \neq k}}^K \sum_{m=1}^M P_l \alpha_{km}^{(g)} \alpha_{ml}^{(h)} L_m^2 \Gamma \left(\frac{L_m}{L_m - 1} \right) \times \left(2^{-\frac{B_{ml}}{L_m - 1}} + 2^{-\frac{\tilde{B}_{km}}{L_m - 1}} \right) + \sum_{k=1}^K \eta_k. \quad (6.31)$$

Choosing a particular CDI from the l^{th} transmitter to the m^{th} relay, i.e., $\tilde{\mathbf{h}}_{ml}$, we get the interference bound due to limited feedback bit allocation (i.e., B_{ml}) to this link as

$$P_l \left(\sum_{k \neq l} \alpha_{km}^{(g)} \right) \alpha_{ml}^{(h)} L_m^2 \Gamma \left(\frac{L_m}{L_m - 1} \right) 2^{-\frac{B_{ml}}{L_m - 1}} \quad (6.32)$$

Using $P_l = P$, we can get a sufficient condition to guarantee no DoF loss by bounding the interference upper bound in (6.32) by a finite positive value, c_1 . Thus, we get the bit scaling law in (6.27). Similarly, we can obtain the bit scaling law in (6.28) (details are omitted for brevity). ■

Remark 6.5. From (6.27) and (6.28), note that if the feedback bits for every link at every node scale as the logarithmic function of the source transmit power, there is no DoF loss. However, if bit allocation to *any* of the links in the network does not scale as $\log(P)$, it incurs DoF loss.

6.4.3 Adaptive bit allocation

Based on the rate-loss analysis, we propose an adaptive bit allocation for the dual-hop SISO-IC similar to the idea used in [6]. However, in our current model, the bit allocation has to be done to the channel links both at the relay as well as at

the receivers. *It is to be noted that unlike [6], the bit allocation at a node depends not only on its local link but also on other links in the network as discussed later in the numerical results in Section 6.6.*

Essentially, we solve the optimization problem to minimize the rate-loss as given below.

$$\begin{aligned}
 & \min_{\substack{B_{mk}, \tilde{B}_{km}, \\ \forall m \in \mathcal{M}\{1,2,\dots,M\}, k \in \mathcal{K}\{1,2,\dots,K\}}} \Delta R^{UB}, \\
 & s.t. \quad \sum_{k=1}^K B_{mk} = B_m^{tot}, \quad B_{mk} \geq 0 \quad \forall m \in \mathcal{M}, \\
 & \quad \sum_{m=1}^M B_{km} = \tilde{B}_k^{tot}, \quad \tilde{B}_{km} \geq 0 \quad \forall k \in \mathcal{K},
 \end{aligned} \tag{6.33}$$

where ΔR^{UB} is defined in (6.26), B_m^{tot} and $\tilde{B}_k^{tot}$ denote the total number of feedback bits available at the m^{th} relay and the k^{th} receiver.

Note that the above problem is an integer program which is NP-hard [186]. While closed-form approximate solutions can be obtained for some special cases similar to [77], they may be considered as future research directions. In this work, exhaustive search is used to find out the optimal bit allocation at each node (relay or receiver). As expected, on solving this problem, more feedback bits are allotted to the stronger links. The numerical results in Section 6.6 show the effectiveness of the above adaptive bit allocation optimization problem.

6.5 Robust MMSE precoder design with imperfect CSI

In this section, we design stochastically robust minimum-sum mean square error (MMSE) relay precoder designs for the dual-hop SISO-IC. Apart from their inherent ability to improve the network sum-rate in the noise-limited regions, the proposed MMSE precoders exploit the channel quantization error statistics to fur-

ther improve the sum-rate as compared to the IN precoders. When combined with the adaptive bit allocation policy, the robust precoders show even more sum-rate improvement in the high SNR regions.

The MMSE precoders are designed at the CS based on the quantized CDI received from the relays and the receivers. After the precoder designs, they are broadcasted to the relays via error-free feed-forward control channels from the CS to the relays. The signal received at the k^{th} receiver under imperfect channel information is similar to (6.3) except that $\mathbf{U}_m$ is replaced by the precoder $\hat{\mathbf{U}}_m$ which is based on quantized CDI as discussed next. (6.3) can be re-written as

$$\hat{y}_k = \mathbf{e}^T \mathbf{A}_k \hat{\mathbf{u}} + \hat{\mathbf{b}}^T \hat{\mathbf{u}} + n_k, \quad (6.34)$$

where $\hat{\mathbf{u}} \in \mathbb{C}^{L \times 1} = [\text{vec}^T(\hat{\mathbf{U}}_1), \text{vec}^T(\hat{\mathbf{U}}_2), \dots, \text{vec}^T(\hat{\mathbf{U}}_M)]$, $\mathbf{e} = \mathbf{1}_{K \times 1}$, and $\hat{\mathbf{b}}$ and i^{th} row of $\mathbf{A}_k \in \mathbb{C}^{K \times L}$, i.e., $[\mathbf{A}_k]_i$ are defined as

$$\begin{aligned} \hat{\mathbf{b}} &= [(\mathbf{n}_{R,1}^T \otimes \mathbf{g}_{k1}), \dots, (\mathbf{n}_{R,M}^T \otimes \mathbf{g}_{kM})] \\ [\mathbf{A}_k]_i &= [(\mathbf{h}_{1i}^T \otimes \mathbf{g}_{k1}), (\mathbf{h}_{2i}^T \otimes \mathbf{g}_{k2}), \dots, (\mathbf{h}_{Mi}^T \otimes \mathbf{g}_{kM})] \sqrt{P_i} x_i. \end{aligned} \quad (6.35)$$

After being filtered by the receive filter w_k at the k^{th} receiver, the received data is given as

$$\hat{x}_k = w_k^H \hat{y}_k, \quad (6.36)$$

and the sum mean square error (SMSE) of the network, expected over the codebooks at all the relays and the receivers, can be written as

$$\mathcal{S} = \sum_{k=1}^K \mathbb{E}_{\mathbf{w}_k, \mathbf{w}_m^{(R)}} \{|\hat{x}_k - x_k|^2\} \quad (6.37)$$

Assuming high channel resolution for ease of tractability (i.e., sufficiently large

number of feedback bits are used to quantize the CDI at the relay and the receiver nodes), (6.37) can be further written as⁶

$$\mathcal{S} = K + \sum_{k=1}^K |w_k|^2 (\hat{\mathbf{u}}^H \mathbf{D}_{k1} \hat{\mathbf{u}} + \hat{\mathbf{u}}^H \mathbf{D}_{k2} \hat{\mathbf{u}} + \sigma_k^2) - 2\mathcal{R} \left\{ \sum_{k=1}^K \sqrt{P_k} w_k^H \mathbf{d}_k \hat{\mathbf{u}} \right\}, \quad (6.38)$$

where $\mathbf{D}_{k1}$ is given as

$$\mathbf{D}_{k1} = \sum_{i=1}^K P_i \mathbf{Q}^{(ki)}, \quad (6.39)$$

and $\mathbf{Q}^{(ki)} \in \mathbb{C}^{L \times L}$ is the covariance matrix of the effective link to the k^{th} receiver from the i^{th} transmitter via M different paths through the M relays as defined next. Denote $\tilde{\Upsilon}_{kl} \triangleq \Gamma(\frac{L_l}{L_l-1}) 2^{-\frac{\tilde{B}_{kl}}{L_l-1}}$ and $\Upsilon_{li} \triangleq \Gamma(\frac{L_l}{L_l-1}) 2^{-\frac{B_{li}}{L_l-1}}$, and define a matrix $\tilde{\mathbf{Q}}^{(ki)} \in \mathbb{C}^{L \times L}$, whose $(m, n)^{th}$ sub-matrix element (at high feedback bits channel resolution) is given as

$$[\tilde{\mathbf{Q}}^{(ki)}]_{mn} \approx \begin{cases} (1 - \Upsilon_{mi})(1 - \tilde{\Upsilon}_{km})(\hat{\mathbf{h}}_{mi}^T \otimes \hat{\mathbf{g}}_{km})^H (\hat{\mathbf{h}}_{mi}^T \otimes \hat{\mathbf{g}}_{km}) \\ + (\frac{1}{L_m}) \tilde{\Upsilon}_{km} (1 - \Upsilon_{mi})(\hat{\mathbf{h}}_{mi}^c \hat{\mathbf{h}}_{mi}^T \otimes \mathbf{I}_{L_m}) \\ + (\frac{1}{L_m}) \Upsilon_{mi} (1 - \tilde{\Upsilon}_{km})(\mathbf{I}_{L_m} \otimes \hat{\mathbf{g}}_{km}^H \hat{\mathbf{g}}_{km}) + (\frac{1}{L_m^2}) \tilde{\Upsilon}_{km} \Upsilon_{mi} \mathbf{I}_{L_m^2}, & \text{if } m = n, \\ \left(1 - \frac{\Upsilon_{mi}}{2} - \frac{\tilde{\Upsilon}_{km}}{2} - \frac{\Upsilon_{ni}}{2} - \frac{\tilde{\Upsilon}_{kn}}{2}\right) \times (\hat{\mathbf{h}}_{mi}^T \otimes \hat{\mathbf{g}}_{km})^H (\hat{\mathbf{h}}_{ni}^T \otimes \hat{\mathbf{g}}_{kn}), & \text{if } m \neq n. \end{cases}$$

Then, the $(m, n)^{th}$ element of the matrix $\mathbf{Q}^{(ki)}$ is given as

$$[\mathbf{Q}^{(ki)}]_{mn} = (\alpha_{mi}^{(h)} \alpha_{km}^{(g)} \alpha_{ni}^{(h)} \alpha_{kn}^{(g)})^{\frac{1}{2}} (\delta_{mi}^{(h)} \delta_{km}^{(g)} \delta_{ni}^{(h)} \delta_{kn}^{(g)})^{\frac{1}{2}} [\tilde{\mathbf{Q}}^{(ki)}]_{mn}. \quad (6.40)$$

$\mathbf{D}_{k2} \in \mathbb{C}^{L \times L}$ in (6.38) is a block-diagonal matrix with the non-diagonal matrix

⁶The derivation details are omitted for brevity.

elements as zeros, and the m^{th} diagonal block, i.e., $[\mathbf{D}_{k2}]_{mm}$ given as

$$\sigma_{R,m}^2 \alpha_{km}^{(g)} \delta_{km}^{(g)} \left[\mathbf{I}_{L_m} \otimes \left((1 - \tilde{\Upsilon}_{km}) \hat{\mathbf{g}}_{km}^H \hat{\mathbf{g}}_{km} + \frac{\tilde{\Upsilon}_{km}}{L_m} \mathbf{I}_{L_m} \right) \right], \quad (6.41)$$

and $\mathbf{d}_k \in \mathbb{C}^{1 \times L}$ in (6.38) is given as $\mathbf{d}_k = [\mathbf{d}_{k,1}, \dots, \mathbf{d}_{k,M}]$, where $\mathbf{d}_{k,m} \in \mathbb{C}^{1 \times L_m^2}$ is derived as

$$(\alpha_{mk}^{(h)} \alpha_{km}^{(g)})^{\frac{1}{2}} (\delta_{mk}^{(h)} \delta_{km}^{(g)})^{\frac{1}{2}} \left(1 - \frac{1}{2} \Upsilon_{mk} - \frac{1}{2} \tilde{\Upsilon}_{km} \right) (\hat{\mathbf{h}}_{mk}^T \otimes \hat{\mathbf{g}}_{km}) \quad (6.42)$$

Note that at high CDI resolution, $\Upsilon_{li} \rightarrow 0$ and $\tilde{\Upsilon}_{kl} \rightarrow 0$, $\hat{\mathbf{h}}_{mk} \rightarrow \tilde{\mathbf{h}}_{mk}$ and $\hat{\mathbf{g}}_{km} \rightarrow \tilde{\mathbf{g}}_{km}$, and the SMSE in (6.38) becomes the SMSE for the perfect CSI case.

The overall power expended by the relays expected over the random codebooks can be derived as

$$P_R = \hat{\mathbf{u}}^H \hat{\mathbf{D}} \hat{\mathbf{u}}, \quad (6.43)$$

where $\hat{\mathbf{u}}$ is defined in (6.34), $\hat{\mathbf{D}} = \text{blkdiag}[\hat{\mathbf{D}}_1, \dots, \hat{\mathbf{D}}_M]$, and $\hat{\mathbf{D}}_m$ ($m \in \mathcal{M} = \{1, 2, \dots, M\}$) is defined as

$$\sum_{i=1}^K P_i \left[\alpha_{mi}^{(h)} \delta_{mi}^{(h)} \left((1 - \Upsilon_{mi}) \hat{\mathbf{h}}_{mi}^c \hat{\mathbf{h}}_{mi}^T + \frac{\Upsilon_{mi}}{L_m} \mathbf{I}_{L_m} \right) \right] \otimes \mathbf{I}_{L_m} + \sigma_{R,m}^2 \mathbf{I}_{L_m^2} \quad (6.44)$$

Note that while designing the precoder, the CS cannot compute the exact power expended by the relays due to lack of perfect CSI, and can compute only the expected relay power based on the quantized CDI and the CDI quantization error statistics as given in (6.43). We form the Lagrangian for the SMSE $\mathcal{S}$ under the expected overall relay power constraint as

$$\mathcal{L} = \mathcal{S} + \lambda(P_R - P_t). \quad (6.45)$$

Note that the SMSE expression in (6.38) is a convex function of the variable $\hat{\mathbf{u}}$, and thus, on similar lines as in [66], in order to minimize the SMSE under the expected relay power constraint, the relay precoder $\hat{\mathbf{u}}$ is derived by finding the critical point as $\frac{\partial \mathcal{L}}{\partial \hat{\mathbf{u}}^H} = 0$, and can be obtained as

$$\hat{\mathbf{u}} = \left(\sum_{k=1}^K |w_k|^2 (\mathbf{D}_{k1} + \mathbf{D}_{k2}) + \lambda \hat{\mathbf{D}} \right)^{-1} \left(\sum_{k=1}^K \mathbf{d}_k^H w_k \right). \quad (6.46)$$

λ can be obtained by satisfying the power constraint $P_R \leq P_t$ (see (6.43)). Moreover, by scaling the precoder $\hat{\mathbf{u}}$ as $\frac{\hat{\mathbf{u}}\sqrt{P_t}}{\sqrt{\hat{\mathbf{u}}^H \hat{\mathbf{D}} \hat{\mathbf{u}}}}$ to full available relay power (similar to the Max-SINR algorithm [54, 84, 187, 188]), the sum-rate of the network can be further improved over a wide SNR range. Similar to above, the MMSE decoder at the k^{th} receiver can be derived using $\frac{\partial \mathcal{L}}{\partial w_k^H} = 0$, and is given as

$$w_k = \frac{\sqrt{P_k} \mathbf{d}_k^H \hat{\mathbf{u}}}{\hat{\mathbf{u}}^H (\mathbf{D}_{k1} + \mathbf{D}_{k2}) \hat{\mathbf{u}} + \sigma_k^2} \quad (6.47)$$

An alternate minimization procedure [60, 66] is adopted to minimize the SMSE by alternately updating the precoder and the decoder expressions using the above-mentioned expressions.

It is to be noted from (6.46) (and the related equations (6.39)-(6.42)) that the proposed precoder takes into account the CDI quantization error statistics to improve the network sum-rate. The CDI quantization can be done at the relays and the receivers in two ways, firstly, using the naïve approach, by allocating equal number of bits (called the “equal bit allocation (EQB)”) to each of its links, and secondly, using the adaptive bit allocation (called the “ABA”) proposed in the Subsection 6.4.3.

Given fixed feedback budget at a node, the ABA approach at the node allocates more bits to the stronger links while the weaker links get lesser number of bits. Moreover, it can be observed from (6.46) (and the related equations (6.39)-(6.42))

that during the precoder design, the CS gives more importance to the stronger (i.e., channel link with the higher resolution) links due to the correction factors, e.g., see $(1 - \Upsilon_{mi})(1 - \hat{\Upsilon}_{km})$ above $[\tilde{\mathbf{Q}}^{(ki)}]_{mn}$ in (6.40). Note that the above two ingredients of our proposed method (i.e., ABA and the quantization error-aware robust precoder design at the CS) jointly improve the network sum-rate significantly.

6.5.1 Computational complexity of the precoder design

The computational complexity of the robust MMSE precoder design in (6.46) comes from three main operations, namely, the complexity in computing the matrix $\bar{\mathbf{D}} \triangleq \left(\sum_{k=1}^K |w_k|^2 (\mathbf{D}_{k1} + \mathbf{D}_{k2}) + \lambda \hat{\mathbf{D}} \right)$ and its inverse, the matrix $\left(\sum_{k=1}^K \mathbf{d}_k^H w_k \right)$, and the product of the matrices $\bar{\mathbf{D}}^{-1}$ and $\left(\sum_{k=1}^K \mathbf{d}_k^H w_k \right)$. Again, the computational complexity of the term $\bar{\mathbf{D}}$ is mainly due to the complexity of the term $\mathbf{D}_{k1}$ in (6.39) which is found to be $\mathcal{O}(\sum_{m=1}^M L_m^2) + \mathcal{O}(\sum_{m=1}^M \sum_{n \neq m} L_m L_n) + KL^2$. Since $\bar{\mathbf{D}}$ is of size $L \times L$, its inverse requires complexity of the order $\mathcal{O}(L^3)$. In a similar way, $\left(\sum_{k=1}^K \mathbf{d}_k^H w_k \right)$ requires $\mathcal{O}(KL)$ arithmetic operations, and product of the matrices $\bar{\mathbf{D}}$ and $\left(\sum_{k=1}^K \mathbf{d}_k^H w_k \right)$ involves $\mathcal{O}(L^2)$ operations. Using the IN condition (6.9b), $L \approx \mathcal{O}(K^2)$, and the fact that the inverse matrix operation dominates all other operations, the precoder design complexity requires $\mathcal{O}(K^6)$ operations. Finally, if n iterations are involved in our alternating algorithm, the overall computational complexity is approximately given as $n \times \mathcal{O}(K^6)$.

6.6 Numerical results and discussion

We consider a $3 \times 2 \times 3$ dual-hop SISO-IC where each TX and RX has a single antenna while each of the two relays has 2 antennas. The network connectivity is shown in Figure 6.4 where the solid lines show the strong links while the dashed links show the weak links. Without loss of generality, path loss of each strong link is assumed to be unity and the path losses of the weaker links are assumed to be

much less than unity (the exact values are shown in the figure). Each source TX transmits with equal power P .

Figure 6.4: A 3x2x3 dual-hop SISO-IC. The path loss values are given as: $\alpha_{11}^{(h)}(T_1X_1)=1$, $\alpha_{12}^{(h)}(T_2X_1)=0.001$, $\alpha_{13}^{(h)}(T_3X_1)=0.001$, $\alpha_{21}^{(h)}(T_1X_2)=0.001$, $\alpha_{22}^{(h)}(T_2X_2)=1$, $\alpha_{23}^{(h)}(T_3X_2)=1$, $\alpha_{11}^{(g)}(X_1R_1)=1$, $\alpha_{21}^{(g)}(X_1R_2)=1$, $\alpha_{31}^{(g)}(X_1R_3)=0.01$, $\alpha_{12}^{(g)}(X_2R_1)=0.01$, $\alpha_{22}^{(g)}(X_2R_2)=1$, $\alpha_{32}^{(g)}(X_2R_3)=1$

The short-term channel fading values are drawn from a Rayleigh distribution and the simulation is carried out over 100 channel realizations and 100 random codebooks. The overall relay power constraint for the MMSE precoder at any given SNR is taken to be the same as the average power consumed by the IN precoder at that SNR. This ensures fairness between the MMSE and the IN precoding schemes in terms of relay power consumption so that the MMSE precoder cannot consume more power than the IN precoder.

Each relay has 9 feedback bits to distribute among its three local links (see Figure 6.4 for “local” links), one from each of the TXs, while each receiver has 8 feedback bits to allocate to its two local links from each of the relays to the receiver. In the “EQB” approach, each relay allocates 3 bits to each of its local links while each receiver allocates 4 bits to each of its local links without considering the long-term strengths of the links. On the other hand, using the “ABA” approach, the relay as well as the receiver allocate the bits to their respective local links based on the long-term channel strengths (or path losses). Essentially, the long-term path loss factors do not vary over a long time and are assumed to be known to the central station (CS). Based on this information, the CS solves the optimization problem

mentioned in (6.33) and obtains the optimal number of feedback bits for each relay and receiver. The optimal bits information is then broadcast to the relays and the receivers prior to the data transmission. Now, each relay and receiver quantizes its CDI adaptively by allocating the optimal number of bits to its links.

It is worth noting that the rate-loss upper bound is derived for high channel resolution, and becomes more accurate when large number of feedback bits are used at the relay and the receivers for CDI quantization. However, as the number of feedback bits increases, the codebook size at the relay or the receiver also increases and the search complexity to find the codeword closest to the CDI increases significantly. For example, if B_{ml} bits are used for quantizing the CDI $\hat{\mathbf{g}}_{ml}$ at the m^{th} relay, the relay has to search for the closest codeword from a codebook of size $2^{B_{ml}}$. *In the $3 \times 2 \times 3$ case in Figure 6.4, we use only 8 and 9 bits as the feedback budget at the RX and the relay respectively so that the codebook search complexity is not very high.*

Discussion

In Figure 6.4, the optimal bit allocation values (based on the rate-loss upper bound minimization) are shown alongside the links. The solid blue line between two nodes indicates a strong link whereas a dotted line indicates a weak link. All the strong links are assumed to be equally strong with a path loss factor of unity without any loss of generality. The average link strengths (path losses) of the weaker links relative to the strong link are given in the caption of the plot.

Note that the bit allocation in the dual-hop network is quite different from that in the single-hop IC [6, 77]. The bit allocation depends not only on its local link but also on other links in the network. For example, consider the bit allocation at RX_1 where the upper link, i.e., the link from relay-1 is stronger (solid blue line) and the lower link, i.e., the link from relay-2 is weaker (dotted black line). Still, 5 bits are allocated to the lower link X_2R_1 and 3 bits to the upper link X_1R_1 as

explained next.

Note that the bit allocation is done at the RX based on two criteria, viz., the interference to the RX is reduced, and the desired signal strength is maximized. First, consider the interference through the link X_1R_1 which is coming from TXs 2 and 3 via relay X_1 . Thus, $T_2X_1R_1$ and $T_3X_1R_1$ are the two interfering links to RX_1 . However, the links T_2X_1 and T_3X_1 are very weak, effectively causing very little interference to RX_1 even though the final *local* link X_1R_1 is strong. On the other hand, the interference to RX_1 via the paths $T_2X_2R_1$ and $T_3X_2R_1$ are *effectively* stronger because the links T_2X_2 and T_3X_2 are very strong while the link X_2R_1 is not so weak (see the path loss values in Figure 6.4). Essentially, we need to give more weight to that link through which the *effective* interference is stronger. Since the link X_2R_1 receives stronger interference compared to X_1R_1 , it gets more bits even though it seems weak to RX_1 locally.

This is also apparent from the interference bound in Lemma 6.1 where the coefficients for the link from the m^{th} relay to the k^{th} RX contains the factor $\left(\alpha_{km}^{(g)} \sum_{l \neq k} P_l \alpha_{ml}^{(h)} \right)$ which considers the *effective* interference through the link X_mR_k .

Apart from the interference minimization part as discussed above, the overall rate-loss is also based on rate-loss due to quantization of the links responsible for the desired signal strength maximization, i.e., $\mathcal{I}_{des}^{(k),UB}$ (see Lemma 6.2 and Theorem 6.3). Here, the bit allocation for the local link from the m^{th} relay to the k^{th} RX, i.e., $\tilde{B}_{km}$, depends on the weighting coefficient ϵ_{km} . Note that there are two paths of the desired links also through the two relays, e.g., $T_1X_1R_1$ and $T_1X_2R_1$ for RX_1 . Intuitively, based on the *effective* link that carries the desired signals, the bit allocation should depend on the strengths of the links $T_1X_1R_1$ and $T_1X_2R_1$. (6.25) confirms this by providing a more accurate expression where the rate-loss and the bit allocation for a particular desired link depends on the ratio of the absolute value of the strength of that particular link to the summation of the absolute values of all the desired links. For example, the bit allocation for the

link X_1R_1 involves both the paths $T_1X_1R_1$ and $T_1X_2R_1$, and depends on the ratio as defined by (6.25).

As per the explanation for the desired signal maximization, it can be verified that the link X_1R_1 should be given more feedback bits because the link $T_1X_1R_1$ is effectively stronger than the other desired link path, i.e., $T_1X_2R_1$. However, the net rate-loss and the bit allocation depends both on the desired signal strength maximization and the interference minimization. In fact, in our bit allocation using (6.33), if we consider only $\mathcal{I}_{int}^{(k),UB}$, we get the bits (2, 6) for the links (X_1R_1, X_2R_1) but since the desired signal strength maximization advocates more bits to the link X_1R_1 , the overall effect is that one more bit is allocated to X_1R_1 and finally, we get (3, 5) as shown in Figure 6.4.

Finally, Figure 6.5 shows the sum-rate of the network using various algorithms. Clearly, the IN precoding using network adaptive bit allocation shows better sum-rate than the “EQB” approach in the high SNR region. The proposed stochastically robust MMSE precoders show better sum rates than the IN algorithms over a wide SNR range. Moreover, MMSE using “ABA” shows better sum-rate than MMSE using “EQB” in the high SNR region. This is because the rate-loss upper bound and the adaptive bit allocation are derived for the high SNR region. While our high channel resolution based analysis works significantly well even for the small and moderate feedback bit budget (as shown through simulations), we believe that more accurate results can be obtained with higher number of feedback bits budget at the relays and the receivers.

Next, we consider a $4 \times 2 \times 4$ dual-hop SISO-IC as shown in Figure 6.6 where each of the two relays has three antennas. We assume that the channel links from the TXs to the relays are estimated and fed back perfectly to the central station (CS) for precoder computation. Similarly, RX-1 also feeds back its local links perfectly to the CS. However, the RXs 2, 3 and 4 have limited feedback budget, and therefore, each of these RXs feeds back quantized CDI to the CS for precoder

Figure 6.5: Sum rate for a $3 \times 2 \times 3$ dual-hop SISO-IC with quantized CSI based precoding

computation. Specifically, each of RXs 2, 3 and 4 has 12 feedback bits to quantize its local links using “EQB” or the proposed “ABA” bit-partitioning strategy.

Furthermore, as shown in Figure 6.6, RXs 1 and 2 are closer to relay-1, and thus the links from relay-1 to RXs 1 and 2, i.e., X_1R_1 and X_1R_2 are stronger (20 dB stronger in our example) as compared to the links from the relay-2 to RXs 1 and 2, i.e., X_2R_1 and X_2R_2 respectively. Similarly, the strong and weak links of other RXs are shown using solid and dashed lines respectively in Figure 6.6.

Figure 6.6: A $4 \times 2 \times 4$ dual-hop SISO-IC with each relay having three antennas.

Figure 6.7 shows the sum-rate of the above network for various precoding schemes. Similar to the previous example, for both the MMSE and the IN precoders, adaptive bit allocation (“ABA”) shows better sum rates as compared to the equal bit allocation (“EQB”) strategy. Also, the robust MMSE precoders signifi-

cantly improve the network sum-rate as compared to the IN precoding techniques. It is worth noting that in this example, the sum-rate scales with $\log(P)$ (unlike Figure 6.5 where the sum-rate tends to saturate at high SNR). Specifically, the sum-rate slope at high SNR (or the DoF) is approximately 0.5 for the IN precoding. This DoF gain is due to the DoF of RX-1 which feeds back perfect CSI as mentioned earlier.

This example also gives us a useful insight that if the links from the TXs to the relays are fed back perfectly (i.e., perfect CSI feedback for the first hop), DoF for a specific RX depends on its local links only. Thus, RX-1 attains its DoF of 0.5 (one data stream transmitted from its corresponding TX, i.e., TX-1) because it feeds back its local links perfectly. Note that the increasing sum-rate of the overall network with SNR is largely due to the increasing sum-rate of RX-1. Due to imperfect CSI feedback by other RXs, their sum rates tend to saturate at high SNR.

Figure 6.7: Sum-rate performances for a $4 \times 2 \times 4$ dual-hop SISO-IC for various precoding schemes

6.7 Summary

In this chapter, we considered the dual-hop multiple relay-assisted SISO-IC under imperfect (quantized) CSI at the relays and no CSI at the source transmitters. Given abundant power at the relays, we provided the sufficiency conditions on the number of relays and relay antennas for complete interference neutralization (IN). We then showed that the overall relay power should scale with the source transmitter power in order to attain the maximum network DoF. However, the relay power can scale slowly as compared to the source transmitters' powers. We derived the rate-loss upper bound for the network with quantized channel information based precoding at the relays. Based on the rate-loss analysis, bit scaling law was derived, and a network adaptive bit allocation strategy was proposed. Our analysis showed that the bit allocation strategy for the dual-hop network is quite different from the single-hop networks because in our case, the bit allocation depends not only on the local links of the node but also on other links of the network.

Appendix

Proof of Lemma 6.1

Using (6.5) and (6.6), (6.22) can be written as⁷

$$\mathcal{I}_{int}^{(k)} = \mathbb{E} \left\{ \log \left[1 + \sum_{m=1}^M \|\bar{\mathbf{g}}_{km} \mathbf{U}_m\|^2 + \sum_{l \neq k} P_l \left| \sum_{m=1}^M \psi_m^{(kl)} \tilde{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \tilde{\mathbf{h}}_{ml} \right|^2 \right] \right\}. \quad (6.48)$$

Using Jensen's inequality, (6.48) can be upper bounded as

$$\mathcal{I}_{int}^{(k)} \leq \log \left[1 + \eta_k + \mathbb{E} \left\{ \sum_{l \neq k} P_l \left| \sum_{m=1}^M \psi_m^{(kl)} \tilde{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \tilde{\mathbf{h}}_{ml} \right|^2 \right\} \right] \quad (6.49)$$

$$\text{where } \eta_k = \mathbb{E} \left\{ \sum_{m=1}^M \|\bar{\mathbf{g}}_{km} \mathbf{U}_m\|^2 \right\} = \sum_{k=1}^K L_m \alpha_{km}^{(g)}.$$

Denoting $\tilde{\mathcal{I}}_{int}^{(k)} \triangleq \mathbb{E} \left\{ \sum_{l \neq k} P_l \left| \sum_{m=1}^M \psi_m^{(kl)} \tilde{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \tilde{\mathbf{h}}_{ml} \right|^2 \right\}$, $\mathcal{I}_{int}^{(k)}$ in (6.49) can be further bounded by substituting the upper bound of $\tilde{\mathcal{I}}_{int}^{(k)}$ in (6.49). The upper bound of $\tilde{\mathcal{I}}_{int}^{(k)}$ is derived as shown next. Using the CDI quantization in (6.8), i.e., $\tilde{\mathbf{h}}_{ml} = \sqrt{1 - z_{ml}^h} \hat{\mathbf{h}}_{ml} + \sqrt{z_{ml}^h} \Delta \bar{\mathbf{h}}_{ml}$, and $\tilde{\mathbf{g}}_{km} = \sqrt{1 - z_{km}^g} \hat{\mathbf{g}}_{km} + \sqrt{z_{km}^g} \Delta \bar{\mathbf{g}}_{km}$ in the expression of $\tilde{\mathcal{I}}_{int}^{(k)}$, expanding it, and neglecting the terms which involve $\sqrt{z_{km}^g} \sqrt{z_{ml}^h}$ due to high channel resolution assumption, we obtain

$$\tilde{\mathcal{I}}_{int}^{(k)} = \mathbb{E} \left\{ \log \left[\sum_{l \neq k} P_l \left| \sum_m a_m^{(kl)} + \sum_m b_m^{(kl)} + \sum_m c_m^{(kl)} \right|^2 \right] \right\}, \quad (6.50)$$

where

$$\begin{aligned} a_m^{(kl)} &= \psi_m^{(kl)} \sqrt{1 - z_{km}^g} \sqrt{1 - z_{ml}^h} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{ml}, \\ b_m^{(kl)} &= \psi_m^{(kl)} \sqrt{1 - z_{km}^g} \sqrt{z_{ml}^h} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \Delta \bar{\mathbf{h}}_{ml}, \\ c_m^{(kl)} &= \psi_m^{(kl)} \sqrt{z_{km}^g} \sqrt{1 - z_{ml}^h} \Delta \bar{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml}. \end{aligned} \quad (6.51)$$

⁷Unless otherwise stated, the expectations are taken over the multiple random codebooks, and log denotes base-2 logarithm.

$\tilde{\mathcal{I}}_{int}^{(k)}$ in (6.50) can be bounded as

$$\leq \mathbb{E} \left\{ \log \left[\sum_{l \neq k} P_l \left(\left| \sum_m a_m^{(kl)} \right| + \left| \sum_m b_m^{(kl)} \right| + \left| \sum_m c_m^{(kl)} \right| \right)^2 \right] \right\} \quad (6.52)$$

$$\begin{aligned} &\leq \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \left| \sum_m a_m^{(kl)} \right|^2 + \left| \sum_m b_m^{(kl)} \right|^2 + \left| \sum_m c_m^{(kl)} \right|^2 \right. \right. \\ &\quad \left. \left. + 2 \left| \sum_m a_m^{(kl)} \right| \cdot \left| \sum_m b_m^{(kl)} \right| + 2 \left| \sum_m a_m^{(kl)} \right| \cdot \left| \sum_m c_m^{(kl)} \right| + 2 \left| \sum_m b_m^{(kl)} \right| \cdot \left| \sum_m c_m^{(kl)} \right| \right\} \right] \end{aligned} \quad (6.53)$$

$$= \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \sum_m \left| a_m^{(kl)} \right|^2 + \sum_m \left| b_m^{(kl)} \right|^2 + \sum_m \left| c_m^{(kl)} \right|^2 \right\} \right] \quad (6.54)$$

$$\approx \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \sum_m \left| b_m^{(kl)} \right|^2 + \sum_m \left| c_m^{(kl)} \right|^2 \right\} \right] \quad (6.55)$$

$$\begin{aligned} &\approx \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \left| \sum_m \psi_m^{(kl)} \sqrt{z_{ml}^h} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}} \overline{\Delta \mathbf{h}_{ml}} \right|^2 \right\} \right. \\ &\quad \left. + \sum_{l \neq k} P_l \mathbb{E} \left\{ \left| \psi_m^{(kl)} \sqrt{z_{km}^g} \overline{\Delta \mathbf{g}_{km}} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{ml} \right|^2 \right\} \right] \end{aligned} \quad (6.56)$$

$$\begin{aligned} &\leq \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \left(\sum_m \left| \psi_m^{(kl)} \sqrt{z_{ml}^h} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \overline{\Delta \mathbf{h}_{ml}} \right| \right)^2 \right\} \right. \\ &\quad \left. + \sum_{l \neq k} P_l \mathbb{E} \left\{ \left(\sum_m \left| \psi_m^{(kl)} \sqrt{z_{km}^g} \overline{\Delta \mathbf{g}_{km}} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml} \right| \right)^2 \right\} \right] \end{aligned} \quad (6.57)$$

$$\begin{aligned} &= \log \left[\sum_{l \neq k} P_l \mathbb{E} \left\{ \sum_m \left| \psi_m^{(kl)} \sqrt{z_{ml}^h} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \overline{\Delta \mathbf{h}_{ml}} \right|^2 \right\} \right. \\ &\quad \left. + \sum_{l \neq k} P_l \mathbb{E} \left\{ \sum_m \left| \psi_m^{(kl)} \sqrt{z_{km}^g} \overline{\Delta \mathbf{g}_{km}} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml} \right|^2 \right\} \right] \end{aligned} \quad (6.58)$$

$$\begin{aligned} &= \log \left[\sum_{l \neq k} P_l \sum_m \mathbb{E} \left\{ (\psi_m^{(kl)})^2 \right\} \mathbb{E} \left\{ z_{ml}^h \right\} \mathbb{E} \left\{ \left| \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \overline{\Delta \mathbf{h}_{ml}} \right|^2 \right\} \right. \\ &\quad \left. + \sum_{l \neq k} P_l \sum_m \mathbb{E} \left\{ (\psi_m^{(kl)})^2 \right\} \mathbb{E} \left\{ z_{km}^g \right\} \mathbb{E} \left\{ \left| \overline{\Delta \mathbf{g}_{km}} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml} \right|^2 \right\} \right], \end{aligned} \quad (6.59)$$

where (6.52) is obtained using triangle inequality, (6.53) is obtained by expanding (6.52) and using Jensen's inequality, (6.54) is obtained using the independence of the terms $\left| \sum_m a_m^{(kl)} \right|$ and $\left| \sum_m b_m^{(kl)} \right|$ and so on, (6.55) is obtained as at high channel

resolution, $\left| \sum_m a_m^{(kl)} \right|^2 \rightarrow \left| \sum_m \psi_m^{(kl)} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{ml} \right|^2 = 0$ due to interference neutralization. Note that even for the MMSE precoder with initialization using IN precoder, the local solution remains in vicinity of IN solution and the above equality approximately holds. (6.56) is obtained from (6.55) by substituting the values from (6.51) and using $\sqrt{1 - z_{km}^g}, \sqrt{1 - z_{ml}^h} \rightarrow 1$ at high channel resolution. (6.57) is obtained using triangle inequality, (6.58) is obtained using independence of the summands, and (6.59) is also obtained due to independence of the terms. Finally, substituting $\mathbb{E}\left\{(\psi_m^{(kl)})^2\right\} = L_m^2$, $\mathbb{E}\left\{z_{ml}^h\right\} \leq \Gamma\left(\frac{L_m}{L_m-1}\right)2^{-\frac{B_{ml}}{L_m-1}}$, $\mathbb{E}\left\{z_{km}^g\right\} \leq \Gamma\left(\frac{L_m}{L_m-1}\right)2^{-\frac{\bar{B}_{km}}{L_m-1}}$ [184], $\mathbb{E}\left\{\left|\hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \bar{\Delta} \bar{\mathbf{h}}_{ml}\right|^2\right\} \leq \|\mathbf{U}_m\|^2 \leq 1$, and $\mathbb{E}\left\{\left|\bar{\Delta} \bar{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \bar{\mathbf{h}}_{ml}\right|^2\right\} \leq \|\hat{\mathbf{U}}_m\|^2 \leq 1$ (using Cauchy-Schwartz inequality), and using (6.49), we get the interference upper bound in (6.23). $\blacksquare$

Proof of Lemma 6.2

Before proceeding to the proof of Lemma 6.2, we need the following proposition.

Proposition 6.2.

$$\mathbb{E}\left\{\log\left[P_k \left|\sum_m \psi_m^{(kk)} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{mk}\right|^2\right]\right\} = \mathbb{E}\left\{\log\left[P_k \left|\sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \mathbf{U}_m \tilde{\mathbf{h}}_{mk}\right|^2\right]\right\}, \quad (6.60)$$

where P_k is the power of the k^{th} source transmitter, $\psi_m^{(kk)} = \sqrt{\alpha_{mk}^{(h)}} \sqrt{\alpha_{km}^{(g)}} \sqrt{\delta_{mk}^{(h)}} \sqrt{\delta_{km}^{(g)}}$ (see below (6.11)), $\hat{\mathbf{g}}_{km}, \hat{\mathbf{h}}_{mk}$ are the quantized CDI, $\hat{\mathbf{U}}_m$ is the m^{th} relay's precoder based on quantized CDI. Similarly, $\tilde{\mathbf{g}}_{km}, \tilde{\mathbf{h}}_{mk}$ are the actual CDI, $\mathbf{U}_m$ is the m^{th} relay's precoder based on actual CDI. The expectation on the left-hand side is taken over the random codebooks used for CDI quantization of all the links in the network, and the expectation on the right-hand side is taken over all the actual channel links in the network.

Proof. Note that on the left hand side, the precoder $\hat{\mathbf{U}}_m$ is based on the quantized channel directions which are codewords drawn from a random distribution on the

unit sphere. This is similar to the actual channel directions which are unit-norm vectors drawn from a random distribution based on which the precoder $\mathbf{U}_m$ on the right-hand side is designed. Thus, $\left| \sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \mathbf{U}_m \tilde{\mathbf{h}}_{mk} \right|^2$ and $\left| \sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{mk} \right|^2$ should follow a similar distribution and (6.60) follows. ■

Now, we proceed to the proof of Lemma 6.2. $\mathcal{I}_{des}^{(k)}$ in (6.21) can be written as

$$\mathcal{I}_{des}^{(k)} = \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \mathbf{U}_m \tilde{\mathbf{h}}_{mk} \right|^2 \right] \right\} - \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \tilde{\mathbf{h}}_{mk} \right|^2 \right] \right\} \quad (6.61)$$

Denoting $\mathcal{J} \triangleq \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \psi_m^{(kk)} \tilde{\mathbf{g}}_{km} \mathbf{U}_m \tilde{\mathbf{h}}_{mk} \right|^2 \right] \right\}$, and using (6.8), (6.61) can be written as

$$\begin{aligned} \mathcal{I}_{des}^{(k)} &= \mathcal{J} - \mathbb{E} \left\{ \log \left[P_k \left| \psi_m^{(kk)} \left(\sqrt{1 - z_{km}^g} \hat{\mathbf{g}}_{km} + \sqrt{z_{km}^g} \overline{\Delta \mathbf{g}}_{km} \right) \right. \right. \right. \\ &\quad \left. \left. \times \hat{\mathbf{U}}_m \left(\sqrt{1 - z_{mk}^h} \hat{\mathbf{h}}_{mk} + \sqrt{z_{mk}^h} \overline{\Delta \mathbf{h}}_{mk} \right) \right|^2 \right] \right\} \end{aligned} \quad (6.62)$$

$$\approx \mathcal{J} - \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \psi_m^{(kk)} \left(1 - \frac{z_{km}^g}{2} - \frac{z_{mk}^h}{2} \right) \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{mk} \right|^2 \right] \right\}, \quad (6.63)$$

where (6.63) is obtained by expanding (6.62) and assuming high channel resolution so that the second order terms of the quantization errors can be neglected and $\left(\sqrt{1 - z_{km}^g} \sqrt{1 - z_{mk}^h} \right) \approx \left(1 - \frac{z_{km}^g}{2} - \frac{z_{mk}^h}{2} \right)$. Denoting $\phi_{km} \triangleq \psi_m^{(kk)} \hat{\mathbf{g}}_{km} \hat{\mathbf{U}}_m \hat{\mathbf{h}}_{mk}$, (6.63) can be further written as:

$$\mathcal{I}_{des}^{(k)} = \mathcal{J} - \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \phi_{km} - \frac{1}{2} \sum_m z_{km}^g \phi_{km} - \frac{1}{2} \sum_m z_{mk}^h \phi_{km} \right|^2 \right] \right\} \quad (6.64)$$

$$= \mathcal{J} - \mathbb{E} \left\{ \log \left[P_k \left| \sum_m \phi_{km} \right|^2 \cdot \left| 1 - \frac{1}{2} \sum_m \frac{z_{km}^g \phi_{km}}{\sum_m \phi_{km}} - \frac{1}{2} \sum_m \frac{z_{mk}^h \phi_{km}}{\sum_m \phi_{km}} \right|^2 \right] \right\} \quad (6.65)$$

At this point, we refer to Proposition 6.2. Using (6.60), (6.65) can be written as

$$\mathcal{I}_{des}^{(k)} = -\mathbb{E} \left\{ \log \left[\left| 1 - \underbrace{\frac{1}{2} \sum_m \frac{z_{km}^g \phi_{km}}{\sum_m \phi_{km}}}_{e_1} - \underbrace{\frac{1}{2} \sum_m \frac{z_{mk}^h \phi_{km}}{\sum_m \phi_{km}}}_{e_2} \right|^2 \right] \right\} \quad (6.66)$$

In (6.66), $|e_1|$ and $|e_2|$ become very small at high channel resolution. For this case, using the relation $\left|1 - e_1 - e_2\right|^2 \geq 1 - 2\mathcal{R}\{e_1\} - 2\mathcal{R}\{e_2\} \geq 1 - 2|e_1| - 2|e_2|$, we obtain

$$\mathcal{I}_{des}^{(k)} \leq -\mathbb{E}\left\{\log\left[1 - \frac{\left|\sum_m z_{km}^g \phi_{km}\right|}{\left|\sum_m \phi_{km}\right|} - \frac{\left|\sum_m z_{mk}^h \phi_{km}\right|}{\left|\sum_m \phi_{km}\right|}\right]\right\} \quad (6.67)$$

Further, using $-\log(1 - x) \approx x$ for small x , and $\left|\sum_m z_{km}^g \phi_{km}\right| \leq \sum_m z_{km}^g |\phi_{km}|$, and $\left|\sum_m z_{mk}^h \phi_{km}\right| \leq \sum_m z_{mk}^h |\phi_{km}|$ using triangle inequality, (6.67) can be approximated at high channel resolution (i.e., small quantization errors) as

$$\mathcal{I}_{des}^{(k)} \leq \sum_m \mathbb{E}\left\{\frac{|\phi_{km}|}{\left|\sum_m \phi_{km}\right|}\right\} \left(\mathbb{E}\{z_{km}^g\} + \mathbb{E}\{z_{mk}^h\}\right). \quad (6.68)$$

Again, similar to the argument in Proposition 6.2, $\mathbb{E}\left\{\frac{|\phi_{km}|}{\left|\sum_m \phi_{km}\right|}\right\} = \epsilon_{km}$ in (6.25), and therefore, given the path-loss factors at the CS, the CS can obtain the coefficient ϵ_{km} by averaging over multiple randomly generated channel realizations. Further, $\mathbb{E}\{z_{km}^g\} = \mathcal{B}\left(2^{\tilde{B}_{km}}, \frac{L_m}{L_m-1}\right)$ and $\mathbb{E}\{z_{mk}^h\} = \mathcal{B}\left(2^{B_{mk}}, \frac{L_m}{L_m-1}\right)$ [184] (Here $\mathcal{B}(x, y)$ denotes the beta function ⁸[189]) which are tightly bounded by $\Gamma\left(\frac{L_m}{L_m-1}\right)2^{-\frac{\tilde{B}_{km}}{L_m-1}}$ and $\Gamma\left(\frac{L_m}{L_m-1}\right)2^{-\frac{B_{mk}}{L_m-1}}$ respectively. Substituting these values into (6.68), we get the upper bound in (6.24). ■

⁸ $\mathcal{B}(x, y) = \int_0^1 t^{x-1} (1-t)^{y-1} dt.$

Chapter 7

Conclusions and Future Work

7.1 Conclusion

In this thesis, we propose various interference management techniques for the multi-user wireless networks, especially, the interference channels (ICs) and the X channels (XCs) under imperfect or limited CSI at the source transmitters. Recently, interference alignment (IA) was proposed as a coordinated beamforming technique that can attain or improve the degrees of freedom (DoF) of ICs, XCs and various other multi-user networks. Unfortunately, highly accurate and large volume of channel information need to be fed back to the transmitters in order to achieve the DoF gains promised by IA.

We consider two main strategies for reducing the channel overhead and yet achieve DoF gains for the ICs and XCs. First, we consider a hybrid CSI model, i.e., a mix of delayed CSI as well as locally available instantaneous CSI is available at the transmitters. This scenario is particularly useful because in practical situations, some portion of CSIT may indeed be outdated by the time they are acquired by the transmitters, but at the same time, the CSI which is local to the transmitters may be available instantaneously. Second, we consider the use of relays to design the IA and IA-inspired precoders so that the network sum-rate is improved over a wide SNR range. Note that the relays (with sufficient antennas) not only help in DoF-optimal precoding and forwarding of information but also obviate the CSI

feedback to the source transmitters.

In Chapter 3, we propose space-time coding schemes for DoF improvement for the 3-user SISO-X channel with delayed and local instantaneous CSIT. We consider two types of hybrid CSI feedback scenarios. In the first case, the CSI to each transmitter is fed back from its *own* receiver only whereas in the second case, the CSI to each transmitter is fed back through the *local* links from all the receivers connected to it. The proposed schemes achieve DoF gains in both the cases as compared to the completely delayed CSI cases reported earlier. Furthermore, our second scheme is also scalable with the users, and in the presence of large number of users, its DoF gain approaches to double that of the traditional TDMA scheme. An interesting aspect of both the schemes is that they require much fewer channel uses as compared to the completely delayed CSI case in [23] and thus, significantly reduce channel overhead and long decoding time at the receivers. Also, note that our proposed methods show the synergistic benefits of an alternating CSIT framework first proposed for the two-user broadcast channel in [24].

We also address the specific case of 4-user SISO-IC with hybrid CSIT. In this case, our proposed space-time IA coding scheme achieves a DoF of 1.2 (i.e., 20% DoF gain over the TDMA scheme) in only 10 channel uses. Furthermore, unlike the coding scheme in [28], our proposed scheme achieves *secure* DoF for both the 3 and 4 user cases, i.e, the user's messages are completely confidential, thanks to the *second-order* symbol generation combined with the hybrid CSIT based IA coding approach. Note that our proposed method provides a systematic coding approach and is *secure*-DoF-optimal for the 3-user SISO-IC with local CSIT.

Next, we consider a special practical case of hybrid CSI configuration for the MISO-BC. We propose a new framework for the MISO-BC where one of the receivers is a special receiver (also called the supernode) because it has a reconfigurable antenna capable of switching between various fixed channel modes and has a feedback link to the transmitter. On the other hand, other receivers are normal re-

ceivers without any extra hardware or even feedback mechanism to the transmitter. We propose a semi-blind IA (SBIA) coding scheme for the $K \times 2$ MISO-BC with one supernode which provides significant DoF benefits over the TDMA scheme. Note that the proposed SBIA scheme does not require reconfigurable antennas at all the receivers as in [19] and achieves the DoF gain in much fewer channel uses comparatively (although with some DoF loss). The SBIA can also be applied to the $K \times 2$ SISO-X channel for which it is DoF-optimal.

In Chapter 4, we consider a K -user SISO-IC assisted by a MIMO relay and without CSI at the source transmitters. We propose minimum-sum mean square error (MMSE) and sum-rate maximizing transmit and receive filter designs for this model which significantly improve the sum-rate of the network in the low-mid SNR region as compared to the conventional IA precoding. The numerical results show that although the sum-rate maximizing precoder shows the highest sum-rate, the best BER performance is shown by the MMSE algorithm. This is because the sum-rate maximization in our work is based on Gaussian input signaling whereas for practical simulations, finite input constellations are used which are far from Gaussian.

Note that the proposed algorithms involve iterative transmit and receive filter updates, and can yield locally optimal solutions only. Since multiple local solutions are possible, suitable initializations are required otherwise the proposed algorithms show significant DoF degradation (i.e., lower sum-rate at high SNR) and poor BER performances at high SNR. To address this issue, we propose a scaled-IA based precoder initialization for our iterative algorithms which not only improves the sum-rate in the finite-SNR regions but also preserves the network DoF (unlike random initializations) and saves the relay power significantly. Further, the proposed scaled-IA initialization requires very few iterations to converge to the local optimal solution, and thus, reduces the computational complexity significantly as compared to other works where the best precoder initialization is chosen out of

several random initializations.

In Chapter 5, we consider the relay-aided X network without CSI requirement at the source transmitters. First, we propose IA feasibility conditions for a MIMO-relay-aided symmetric MIMO-X channel where sufficient number of relay antennas are derived so that IA is achieved in both time-varying and time-invariant channels. We also propose a limited feedback (only a single complex coefficient is fed back to the source transmitters) based IA precoder (called the “IA-LFB” precoder) design at the relay so that alignment of interference is achieved at every user under *arbitrarily low relay power*. Finally, we propose iterative MMSE precoder designs (with closed-form precoder and combiner designs at each iteration) which improve the network sum-rate substantially over a wide SNR range as compared to IA precoders. We propose to use IA-LFB precoder as the initializer for the iterative algorithm so that in addition to sum-rate improvement in the finite-SNR region, the network DoF is also preserved and good BER performance is obtained even at high SNR.

In Chapter 6, we consider a more generic relay-aided SISO-IC without CSI at the source transmitters, and multiple multi-antenna relays. A significant difference from the earlier chapters is that there are no direct links between the transmitters and the receivers, i.e., a dual-hop network is considered. Furthermore, precoding at the relays is based on quantized channel information (unlike perfect CSI at the relays as in earlier works). First, we derive the interference neutralization (IN) feasibility condition for this network which gives the number of relays and relay antennas required to eliminate the interference at every user. We further show that in addition to the sufficient spatial dimensions provided by the relays, the overall power of all the relays must scale at a minimum rate with respect to the source transmitter’s power in order to guarantee the maximum network DoF.

Next, we perform the rate-loss analysis for this model and derive the rate-loss upper bound due to channel quantization at the relays and the receivers. Based on

the rate-loss bound minimization, we propose an adaptive bit allocation algorithm using which each relay or receiver, which has a limited feedback budget, allocates the feedback bits to its channel links adaptively. Essentially, the *effective* stronger links are allocated more quantization bits. The IN precoder based on the adaptive bit allocation shows higher sum-rate than the precoder based on the naïve equal bit allocation. Interestingly, the bit allocation and the rate-loss bound in the dual-hop model depend not only on the local links of the node (as in single-hop interference networks) but also on other links in the network. Finally, based on the proposed adaptive bit allocation and the resultant channel quantization error statistics, we propose a stochastically robust MMSE precoder design. The proposed precoder shows the best throughput as compared to other precoding strategies under limited channel resolution.

7.2 Future Work

While IA has been firmly established as an important building block for interference management in multi-user wireless networks, its practical implementation still poses challenges mainly in the form of channel feedback overhead. While we addressed some of these issues in this thesis, specifically, through the use of hybrid CSI and relay-aided transmission, many future research directions are possible.

1. In this thesis, we considered hybrid CSI models for the distributed networks such as ICs and XCs. However, these works are limited to some specific cases such as only the 3 or 4-user SISO-IC. While the hybrid CSI has been addressed in much detail recently for the 3-user broadcast networks, its application in the context of ICs and XCs is mostly unexplored. In fact, DoF of ICs or XCs with local and instantaneous CSIT is also unresolved till date. If this DoF is found out, it can serve as the upper bound on the DoF of these networks with delayed or hybrid CSIT. Alternately, space-time coding schemes can be devised for the ICs or XCs for the general K -user case which

can attain higher secure DoF as compared to the completely delayed CSIT scenario.

The proposed SBIA scheme in this thesis is DoF-optimal for the $K \times 2$ SISO-XC but it is not optimal for the MISO-BC. Thus, DoF-optimal SBIA coding scheme for the MISO-BC under the proposed framework can be an important research direction. Moreover, the SBIA scheme proposed in Chapter 3 can be extended to the general $M \times K$ MISO-BC with more than one supernodes.

2. In this thesis, we considered relay precoding for the relay-aided SISO-IC and SISO-XC with direct links between the transmitters and the receivers but with blind source transmitters and perfect global CSI at the relay. However, the CSI feedback to the relay will not be perfect or instantaneous in practical scenarios. Two possible research directions are possible here. First, from an information-theoretic perspective, we can investigate DoF of the above networks with any fixed number of relay antennas and delayed CSI feedback to the relays. Second, from a signal processing perspective, beamformer designs for the above networks under quantized channel information at the relays can be other useful research directions. Multiple relay-aided transmission for the above networks can also be investigated with delayed and/or quantized CSI feedback to the relays.
3. Dual-hop interference networks with quantized CSI was investigated in Chapter 6. However, imperfect CSI for the relay-aided dual-hop interference networks has not been addressed in detail till now and several research directions are possible emanating from our work. For example, we considered single-antenna transmitter and receiver nodes. Finding the feasibility conditions for extension to the generic case of M and N antennas at each transmitter and receiver respectively is challenging, and obtaining the necessary and sufficient conditions for this generic case can be an important future work.

In our work, power control based on limited CSI feedback was not considered. Thus, joint power control together with adaptive feedback bit allocation for the dual-hop network can be another interesting and practically useful problem. Since channel gains are important for interference neutralization (as discussed in Chapter 6), it will be useful to investigate the channel gain quantization together with the channel direction quantization.

In our model, we assumed perfect channel estimation and feedback error in the form of quantized CSI direction was considered. While this provided useful insights into the network performance and enabled us to design robust precoders, we should also consider channel estimation errors for more realistic situations. Further, similar to our quantized CSI based analysis, rate-loss analysis for analog CSI feedback (refer to [8]) together with pilot power and training length optimization under training resource constraints can be other useful research directions. Using more advanced optimization algorithms and precoder designs for quality-of-service issues under quantized and delayed CSI considerations are other possible extensions.

List of Publications

Journals

- [1] K. Anand, E. Gunawan, C. M. Yetis, and Y. L. Guan, “Finite-SNR Precoder Designs for the Interference Relay Channel Without CSI at the transmitters,” *IEEE Trans. Commun.*, vol. 63, no. 12, pp. 4754-4767, Dec. 2015.
- [2] K. Anand, E. Gunawan, and Y. L. Guan, “Degrees of Freedom of Three-User SISO-X Networks With Hybrid CSI at the Transmitters”, *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 1002-1005, May 2016.
- [3] K. Anand, E. Gunawan, and Y. L. Guan, “Semiblind Interference Alignment: A New Framework”, *IEEE Signal Process. Lett.*, vol. 23, no. 5, pp. 580-584, May 2016.
- [4] K. Anand, E. Gunawan, and Y. L. Guan, “Precoder Designs for the Relay-Aided X Channel without source CSI”, *IEEE Trans. Signal Process*, vol. 65, no. 1, pp. 41-55, Jan 2017.
- [5] K. Anand, E. Gunawan, and Y. L. Guan, “Relay-Aided Interference Channels with Limited Channel Information”, submitted to *IEEE Trans. Signal Process*.
- [6] K. Anand, E. Gunawan, and Y. L. Guan, “On Degrees of Freedom of 4-user SISO Interference Channel with Hybrid CSIT and receiver cooperation”, to be submitted.

Conferences

- [1] K. Anand, C. M. Yetis, E. Gunawan, and Y. L. Guan, "Precoder design for the interference relay channels with blind transmitters," *IEEE Wireless Networking and Communications Conference (WCNC'2014)*, pp. 810-814, April, 2014.
- [2] K. Anand, E. Gunawan, and Y. L. Guan, "Degrees of freedom of three-user SISO interference channels under hybrid CSI at the transmitters," *International Conference on Information, Communications and Signal Processing (ICICS'2015)*, pp. 1-4, Dec. 2015.
- [3] K. Zhong, K. Anand, N. T. Hieu, E. Gunawan, and Y. L. Guan, "A blind and partial interference alignment scheme based on pulse shaping for 3-user SISO interference channel," *International Conference on Information, Communications and Signal Processing (ICICS'2015)*, Dec. 2015.
- [4] C. M. Yetis, K. Anand, A. H. Kayran, Y. L. Guan, and E. Gunawan, "Implementation aspects for interference alignment," *IEEE International Conference on Digital Signal Processing (DSP'2015)*, pp. 580-584, July 2015.

Bibliography

- [1] M. Maddah-Ali, A. Motahari, and A. Khandani, “Communication over MI-MO X channels: Interference alignment, decomposition, and performance analysis,” *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3457–3470, Aug. 2008.
- [2] V. R. Cadambe and S. A. Jafar, “Interference alignment and degrees of freedom of the K -user interference channel,” *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3425–3441, Aug. 2008.
- [3] A. Host-Madsen and A. Nosratinia, “The multiplexing gain of wireless networks,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 2065–2069, Sep. 2005.
- [4] R. T. Krishnamachari and M. K. Varanasi, “Interference alignment under limited feedback for mimo interference channels,” *IEEE Trans. Signal Process.*, vol. 61, no. 15, pp. 3908 – 3917, Aug. 2013.
- [5] H. Bolcskei and I. J. Thukral, “Interference alignment with limited feedback,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 1759–1763, Jun. 2009.
- [6] K. Anand, E. Gunawan, and Y. Guan, “Beamformer design for the mimo interference channels under limited channel feedback,” *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3246–3258, Aug. 2013.
- [7] O. El Ayach, S. Peters, and J. Heath, R.W., “The practical challenges of interference alignment,” *IEEE Wireless Commun. Mag.*, vol. 20, no. 1, pp. 35–42, Feb. 2013.

- [8] O. Ayach and R. Heath, "Interference alignment with analog channel state feedback," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 626–636, Feb. 2012.
- [9] S. Peters and R. Heath, "User partitioning for less overhead in mimo interference channels," *IEEE Trans. Wireless Commun.*, vol. 11, no. 2, pp. 592–603, Feb. 2012.
- [10] B. Nosrat-Makouei, J. Andrews, and R. Heath, "Mimo interference alignment over correlated channels with imperfect csi," *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2783–2794, Jun. 2011.
- [11] R. Bhagavatula and R. Heath, "Predictive limited feedback for cooperative transmission," *Signals, Systems and Computers (ASILOMAR), 2010 Conference Record of the Forty Fourth Asilomar Conference on*, pp. 1237–1241, Nov. 2010.
- [12] O. El Ayach and R. Heath, "Grassmannian differential limited feedback for interference alignment," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6481–6494, Dec. 2012.
- [13] R. Bhagavatula and R. Heath, "Adaptive limited feedback for sum-rate maximizing beamforming in cooperative multicell systems," *IEEE Trans. Signal Process.*, vol. 59, no. 2, pp. 800–811, Feb. 2011.
- [14] K. Huang and R. Zhang, "Cooperative precoding with limited feedback for mimo interference channels," *IEEE Trans. Wireless Commun.*, vol. 11, no. 3, pp. 1012–1021, Mar. 2012.
- [15] J. Kim, S. Moon, S. Lee, and I. Lee, "A new channel quantization strategy for mimo interference alignment with limited feedback," *IEEE Trans. Wireless Commun.*, vol. 11, no. 1, pp. 358–366, Jan. 2012.

- [16] T. Yoo, N. Jindal, and A. Goldsmith, “Multi-antenna downlink channels with limited feedback and user selection,” *IEEE J. Sel. Areas Commun.*, vol. 25, no. 7, pp. 1478–1491, Sep. 2007.
- [17] X. Rao, L. Ruan, and V. Lau, “Limited feedback design for interference alignment on mimo interference networks with heterogeneous path loss and spatial correlations,” *IEEE Trans. Signal Process.*, vol. 61, no. 10, pp. 2598–2607, May 2013.
- [18] S. Jafar, “Blind interference alignment,” *IEEE J. Sel. Signal Process.*, vol. 6, no. 3, pp. 216–227, Jun. 2012.
- [19] T. Gou, C. Wang, and S. Jafar, “Aiming perfectly in the dark-blind interference alignment through staggered antenna switching,” *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2734–2744, Jun. 2011.
- [20] C. Wang, “Degrees of freedom characterization: The 3-user siso interference channel with blind interference alignment,” *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 757–760, May 2014.
- [21] A. Alaa and M. Ismail, “Achievable degrees-of-freedom of the k-user siso interference channel with blind interference alignment using staggered antenna switching,” *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1–1, May 2016.
- [22] M. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4418–4431, Jul. 2012.
- [23] M. Abdoli, A. Ghasemi, and A. Khandani, “On the degrees of freedom of k-user siso interference and x channels with delayed csit,” *IEEE Trans. Inf. Theory*, vol. 59, no. 10, pp. 6542–6561, Oct. 2013.

- [24] R. Tandon, S. A. Jafar, S. Shamai (Shitz), and H. V. Poor, “On the synergistic benefits of alternating csit for the miso broadcast channel,” *IEEE Trans. Inf. Theory*, vol. 59, no. 7, pp. 4106–4128, Jul. 2013.
- [25] S. Lashgari, R. Tandon, and S. Avestimehr, “MISO broadcast channel with hybrid CSIT: beyond two users,” *CoRR*, vol. abs/1504.04615, 2015.
[Online]. Available: <http://arxiv.org/abs/1504.04615>
- [26] L. Lee and R. W. Heath, “Space-time interference alignment and degree-of-freedom regions for the miso broadcast channel with periodic csi feedback,” *IEEE Trans. Inf. Theory*, vol. 60, no. 1, pp. 515–528, Jan. 2014.
- [27] B. Rassouli, C. Hao, and B. Clerckx, “Dof analysis of the mimo broadcast channel with alternating/hybrid csit,” *IEEE Trans. Inf. Theory*, vol. 62, no. 3, pp. 1312–1325, Mar. 2016.
- [28] N. Lee, R. Tandon, and R. W. Heath, “Distributed space-time interference alignment with moderately delayed csit,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 2, pp. 1048–1059, Feb. 2015.
- [29] Y. Tian and A. Yener, “Relays can provide alignment for the k-user interference channel without channel state information at the transmitters,” *Proc. IEEE Signal Processing Advances in Wireless Communications (SPAWC)*, pp. 254–258, Jun. 2012.
- [30] K. Truong, P. Sartori, and R. Heath, “Cooperative algorithms for MIMO amplify-and-forward relay networks,” *IEEE Trans. Signal Process.*, vol. 61, no. 5, pp. 1272 – 1287, Mar. 2013.
- [31] Y. Tian and A. Yener, “Guiding blind transmitters: Degrees of freedom optimal interference alignment using relays,” *IEEE Trans. Inf. Theory*, vol. 59, no. 8, pp. 4819–4832, Aug. 2013.

- [32] A. Carleial, “Interference channels,” *IEEE Trans. Inf. Theory*, vol. 24, no. 1, pp. 60–70, Jan. 1978.
- [33] T. Han and K. Kobayashi, “A new achievable rate region for the interference channel,” *IEEE Trans. Inf. Theory*, vol. 27, no. 1, pp. 49–60, Jan. 1981.
- [34] I. Sason, “On achievable rate regions for the gaussian interference channel,” *IEEE Trans. Inf. Theory*, vol. 50, no. 6, pp. 1345–1356, Jun. 2004.
- [35] X. Shang, G. Kramer, and B. Chen, “A new outer bound and the noisy-interference sum-rate capacity for gaussian interference channels,” *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 689–699, Feb. 2009.
- [36] A. S. Motahari and A. K. Khandani, “Capacity bounds for the gaussian interference channel,” *IEEE Trans. Inf. Theory*, vol. 55, no. 2, pp. 620–643, Feb. 2009.
- [37] V. S. Annapureddy and V. V. Veeravalli, “Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region,” *IEEE Trans. Inf. Theory*, vol. 55, no. 7, pp. 3032–3050, Jul. 2009.
- [38] R. H. Etkin, D. N. C. Tse, and H. Wang, “Gaussian interference channel capacity to within one bit,” *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5534–5562, Dec. 2008.
- [39] G. Bresler, A. Parekh, and D. N. C. Tse, “The approximate capacity of the many-to-one and one-to-many gaussian interference channels,” *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4566–4592, Sep. 2010.
- [40] G. Bresler, P. A., and D. Tse, “The approximate capacity of the many-to-one and one-to-many Gaussian interference channels,” ArXiv pre-print cs.IT/0809.3554. <http://arxiv.org/abs/0809.3554>.

- [41] K. V. P. and S. Bhashyam, “On the sum capacity of the gaussian X channel in the mixed interference regime,” *CoRR*, vol. abs/1505.06317, 2015. [Online]. Available: <http://arxiv.org/abs/1505.06317>
- [42] S. A. Jafar and S. Shamai (Shitz), “Degrees of freedom region of the MIMO X channel,” *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 151–170, Jan. 2008.
- [43] V. R. Cadambe and S. A. Jafar, “Interference alignment and the degrees of freedom of wireless X networks,” *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3893–3908, Sep. 2009.
- [44] T. Gou and S. A. Jafar, “Degrees of freedom of the k user $m \times n$ mimo interference channel,” *IEEE Trans. Inf. Theory*, vol. 56, no. 12, pp. 6040–6057, Dec. 2010.
- [45] V. R. Cadambe, S. A. Jafar, and C. Wang, “Interference alignment with asymmetric complex signaling - settling the Host-Madsen-Nosratinia conjecture,” *IEEE Trans. Inf. Theory*, vol. 56, no. 9, pp. 4552 – 4565, Sep. 2010.
- [46] Z. K. M. Ho and E. Jorswieck, “Improper gaussian signaling on the two-user iso interference channel,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 9, pp. 3194–3203, Sep. 2012.
- [47] V. R. Cadambe, S. A. Jafar, and S. Shamai, “Interference alignment on the deterministic channel and application to fully connected gaussian interference networks,” *IEEE Trans. Inf. Theory*, vol. 55, no. 1, pp. 269–274, Jan. 2009.
- [48] R. H. Etkin and E. Ordentlich, “The degrees-of-freedom of the k-user gaussian interference channel is discontinuous at rational channel coefficients,” *IEEE Trans. Inf. Theory*, vol. 55, no. 11, pp. 4932–4946, Nov. 2009.
- [49] A. S. Motahari, S. O. Gharan, and A. K. Khandani, “On the degrees of freedom of the 3-user gaussian interference channel: The symmetric case,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 1914–1918, Jun. 2009.

- [50] —, “Real interference alignment with real numbers,” ArXiv pre-print at <http://arxiv.org/abs/0908.1208>, Aug. 2009.
- [51] A. S. Motahari, S. Oveis-Gharan, M. A. Maddah-Ali, and A. K. Khandani, “Real interference alignment: Exploiting the potential of single antenna systems,” *IEEE Trans. Inf. Theory*, vol. 60, no. 8, pp. 4799–4810, Aug. 2014.
- [52] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, “On feasibility of interference alignment in MIMO interference networks,” *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4771 – 4782, Sep. 2010.
- [53] L. R., V. K. N. Lau, and M. Z. W., “The feasibility conditions for interference alignment in mimo networks,” *IEEE Trans. Signal Process.*, vol. 61, no. 8, pp. 2066–2077, Sep. 2013.
- [54] K. Gomadam, V. R. Cadambe, and S. A. Jafar, “A distributed numerical approach to interference alignment and applications to wireless interference networks,” *IEEE Trans. Inf. Theory*, vol. 57, no. 6, pp. 3309–3322, May 2011.
- [55] H. Ning, C. Ling, and K. Leung, “Feasibility condition for interference alignment with diversity,” *IEEE Trans. Inf. Theory*, vol. 57, no. 5, pp. 2902 – 2912, May 2011.
- [56] G. Bresler, D. Cartwright, and D. Tse, “Settling the feasibility of interference alignment for the MIMO interference channel: the symmetric square case,” 2011, ArXiv pre-print cs.IT/1104.0888v1. <http://arxiv.org/abs/1104.0888>.
- [57] C. M. Yetis, T. Gou, S. A. Jafar, and A. H. Kayran, “Feasibility conditions for interference alignment,” *Proc. IEEE Global Telecom. Conf. (GLOBECOM)*, pp. 1–6, 2009.
- [58] O. Gonzalez, C. Beltran, and I. Santamaria, “A feasibility test for linear interference alignment in mimo channels with constant coefficients,” *IEEE Trans. Inf. Theory*, vol. 60, no. 3, pp. 1840–1856, Mar. 2014.

- [59] S. W. Peters and R. W. Heath Jr., “Interference alignment via alternating minimization,” *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Process.*, pp. 2445–2448, Apr. 2009.
- [60] —, “Cooperative algorithms for MIMO interference channels,” *IEEE Trans. Veh. Technol.*, vol. 60, no. 1, pp. 206 – 218, Jan. 2011.
- [61] H. Sung, K. J. Lee, S. H. Park, and I. Lee, “Linear precoder designs for K -user interference channels,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 1, pp. 291–301, Jan. 2010.
- [62] Z. K. M. Ho and D. Gesbert, “Balancing egoism and altruism on the interference channel: The MIMO case,” May 2010.
- [63] Z. K. Ming Ho and D. Gesbert, “Balancing egoism and altruism on MIMO interference channel,” ArXiv pre-print cs.IT/0910.1688. <http://arxiv.org/abs/0910.1688>.
- [64] C. Shi, D. A. Schmidt, R. A. Berry, M. L. Honig, and W. Utschick, “Distributed interference pricing for the MIMO interference channel,” *Proc. IEEE Int. Conf. on Commun.*, pp. 1–5, Jun. 2009.
- [65] S. Ye and R. S. Blum, “Optimized signaling for MIMO interference systems with feedback,” *IEEE Trans. Signal Process.*, vol. 51, no. 11, pp. 2839–2848, Nov. 2003.
- [66] H. Shen, B. Li, M. X. Tao, and X. D. Wang, “MSE-based transceiver designs for the MIMO interference channel,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3480–3489, Nov. 2010.
- [67] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, “Minimum mean squared error interference alignment,” *Proc. IEEE Asilomar Conference on Signals, Systems and Computers (ACSSC)*, pp. 1106–1110, Nov. 2009.

- [68] I. Santamaria, O. Gonzalez, S. W. Peters, and R. W. Heath, “Maximum sum-rate interference alignment algorithms for MIMO channels,” *Proc. IEEE Global Telecom. Conf. (GLOBECOM)*, pp. 1–6, Dec. 2010.
- [69] Y. Heejung and S. Youngchul, “Least squares approach to joint beam design for interference alignment in multi-input multi-output interference channels,” *IEEE Trans. Signal Process.*, vol. 58, no. 9, pp. 4960 – 4966, Sep. 2010.
- [70] J. Park, Y. Sung, and H. V. Poor, “On beamformer design for multiuser MIMO interference channels,” 2010, [Online]. Available: <http://arxiv.org/abs/1011.6121>.
- [71] J. Tang and S. Lambotharan, “Interference alignment techniques for mimo multi-cell interfering broadcast channels,” *IEEE Trans. Commun.*, vol. 61, no. 1, pp. 164–175, Jan. 2013.
- [72] H. H. Lee, M. J. Kim, and Y. C. Ko, “Transceiver design based on interference alignment in mimo interfering broadcast channels,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6474–6483, Nov. 2014.
- [73] H. J. Yang, W. Y. Shin, B. C. Jung, and A. Paulraj, “Opportunistic interference alignment for mimo interfering multiple-access channels,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2180–2192, May 2013.
- [74] C. Suh, M. Ho, and D. N. C. Tse, “Downlink interference alignment,” *IEEE Trans. Commun.*, vol. 59, no. 9, pp. 2616–2626, Sep. 2011.
- [75] V. Ntranos, M. A. Maddah-Ali, and G. Caire, “Cellular interference alignment: Omni-directional antennas and asymmetric configurations,” *IEEE Trans. Inf. Theory*, vol. 61, no. 12, pp. 6663–6679, Dec. 2015.
- [76] K. Huang, V. Lau, and D. Kim, “Stochastic control of event-driven feedback in multiantenna interference channels,” *IEEE Trans. Signal Process.*, vol. 59, no. 12, pp. 6112–6126, Dec. 2011.

- [77] R. Bhagavatula and R. Heath, “Adaptive bit partitioning for multicell inter-cell interference nulling with delayed limited feedback,” *IEEE Trans. Signal Process.*, vol. 59, no. 8, pp. 3824–3836, Aug. 2011.
- [78] R. Tresch and M. Guillaud, “Cellular interference alignment with imperfect channel knowledge,” *IEEE Int. Conf. on Commun. Workshops*, pp. 1–5, Jun. 2009.
- [79] S. Cho, K. Huang, D. K. Kim, V. Lau, H. Chae, H. Seo, and B. Kim, “Feedback-topology designs for interference alignment in mimo interference channels,” *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6561–6575, Dec. 2012.
- [80] B. Khoshnevis and W. Yu, “Bit allocation laws for multiantenna channel feedback quantization: Multiuser case,” *IEEE Trans. Signal Process.*, vol. 60, no. 1, pp. 367–382, Jan. 2012.
- [81] S. Jing, D. Tse, J. Soriaga, J. Hou, J. Smee, and R. Padovani, “Multicell downlink capacity with coordinated processing,” *EURASIP J. Wireless Commun. Netw.*, vol. 2008, no. 1, 2008.
- [82] P. Ding, D. Love, and M. Zoltowski, “Multiple antenna broadcast channels with shape feedback and limited feedback,” *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3417–3428, Jul. 2007.
- [83] R. T. Krishnamachari and M. K. Varanasi, “Interference alignment under limited feedback for mimo interference channels,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 619–623, Jun. 2010.
- [84] S. Razavi and T. Ratnarajah, “Performance analysis of interference alignment under CSI mismatch,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 9, pp. 4740–4748, Nov. 2014.

- [85] X. Chen and C. Yuen, “Performance analysis and optimization for interference alignment over mimo interference channels with limited feedback,” *IEEE Trans. Signal Process.*, vol. 62, no. 7, pp. 1785–1795, Apr. 2014.
- [86] —, “On interference alignment with imperfect csi: Characterizations of outage probability, ergodic rate and ser,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 1, pp. 47–58, Jan. 2016.
- [87] B. Xie, Y. Li, H. Minn, and A. Nosratinia, “Adaptive interference alignment with csi uncertainty,” *IEEE Trans. Commun.*, vol. 61, no. 2, pp. 792–801, Feb. 2013.
- [88] X. Rao, L. Ruan, and V. Lau, “Csi feedback reduction for mimo interference alignment,” *IEEE Trans. Signal Process.*, vol. 61, no. 18, pp. 4428–4437, Sep. 2013.
- [89] M. Rezaee and M. Guillaud, “Interference alignment with quantized grassmannian feedback in the k-user mimo interference channel,” *CoRR*, vol. abs/1207.6902, 2012.
- [90] H. J. Yang, W.-Y. Shin, B. C. Jung, and A. Paulraj, “Opportunistic interference alignment for mimo interfering multiple-access channels,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 2180–2192, May 2013.
- [91] B. C. Jung and W.-Y. Shin, “Opportunistic interference alignment for interference-limited cellular tdd uplink,” *IEEE Commun. Lett.*, vol. 15, no. 2, pp. 148–150, Feb. 2011.
- [92] J. H. Lee and W. Choi, “Opportunistic interference aligned user selection in multiuser mimo interference channels,” *Proc. IEEE Global Telecom. Conf. (GLOBECOM)*, pp. 1–5, Dec. 2010.

- [93] S. M. Perlaza, M. Debbah, S. Lasaulce, and J.-M. Chaufray, “Opportunistic interference alignment in mimo interference channels,” *CoRR*, vol. abs/0806.3653, 2008.
- [94] J. H. Lee and W. Choi, “Opportunistic interference alignment by receiver selection in a k -user 1×3 simo interference channel,” *Proc. IEEE Global Telecom. Conf. (GLOBECOM)*, pp. 1–5, Dec. 2011.
- [95] J. H. Lee, W. Choi, and D. J. Love, “Achievable dof and its user scaling law for opportunistic user selection in a k -transmitter simo interference channel,” *CoRR*, vol. abs/1111.1051, 2011.
- [96] J. Jose, S. Subramanian, X. Wu, and J. Li, “Opportunistic interference alignment in cellular downlink,” *Proc. Allerton Conf. on Commun., Control and Computing*, pp. 1529–1534, Oct. 2012.
- [97] C. ki Cho, J. W. Kang, and S.-H. Kim, “Opportunistic maximum rate user selection with low complexity in mimo interference channel,” pp. 732–637, Sep. 2012.
- [98] H. Gao, J. Leithon, C. Yuen, and H. A. Suraweera, “New uplink opportunistic interference alignment: An active alignment approach,” *Wireless Communications and Networking Conference (WCNC), 2013 IEEE*, pp. 3099–3104, Apr. 2013.
- [99] J. Leithon, C. Yuen, H. Suraweera, and H. Gao, “A new opportunistic interference alignment scheme and performance comparison of mimo interference alignment with limited feedback,” *Globecom Workshops (GC Wkshps), 2012 IEEE*, pp. 1123–1127, Dec. 2012.
- [100] H. Yu, Y. Sung, H. Kim, and Y. H. Lee, “Adaptive beam tracking for interference alignment for multiuser time-varying mimo interference channels,” *Proc.*

- IEEE Int. Conf. on Acoustics, Speech, and Signal Process.*, pp. 3086–3089, Mar. 2010.
- [101] ———, “Beam tracking for interference alignment in slowly fading mimo interference channels: A perturbations approach under a linear framework,” *IEEE Trans. Signal Process.*, vol. 60, no. 4, pp. 1910–1926, Apr. 2012.
- [102] J. Lee, H. Yu, Y. Sung, and Y. H. Lee, “Adaptive beam tracking for interference alignment in time-varying mimo interference channels: Conjugate gradient approach,” *Acoustics, Speech and Signal Processing (ICASSP), 2011 IEEE International Conference on*, pp. 3364–3367, May 2011.
- [103] M. Maddah-Ali and D. Tse, “Completely stale transmitter channel state information is still very useful,” *Proc. Allerton Conf. on Commun., Control and Computing*, pp. 1188–1195, Sep. 2010.
- [104] H. Maleki, S. Jafar, and S. Shamai, “Retrospective interference alignment over interference networks,” *IEEE J. Sel. Signal Process.*, vol. 6, no. 3, pp. 228–240, Jun. 2012.
- [105] A. Ghasemi, A. S. Motahari, and A. K. Khandani, “On the degrees of freedom of x channel with delayed csit,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 767–770, Jul. 2011.
- [106] C. Hao and B. Clerckx, “Achievable sum dof of the k-user MIMO interference channel with delayed CSIT,” *CoRR*, vol. abs/1605.01641, 2016.
[Online]. Available: <http://arxiv.org/abs/1605.01641>
- [107] S. Lashgari, A. S. Avestimehr, and C. Suh, “Linear degrees of freedom of the x -channel with delayed csit,” *IEEE Trans. Inf. Theory*, vol. 60, no. 4, pp. 2180–2189, Apr. 2014.
- [108] X. Yi, D. Gesbert, S. Yang, and M. Kobayashi, “The dof region of the

- multiple-antenna time correlated interference channel with delayed csit,” *CoRR*, vol. abs/1204.3046, 2012.
- [109] P. de Kerret, X. Yi, and D. Gesbert, “On the degrees of freedom of the k-user time correlated broadcast channel with delayed csit,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 624–628, Jul. 2013.
- [110] K. Anand, E. Gunawan, and Y. L. Guan, “Degrees of freedom of three-user siso-x networks with hybrid csi at the transmitters,” *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 1002–1005, May 2016.
- [111] A. Wagdy, A. El-Keyi, T. Khattab, and M. Nafie, “A degrees of freedom-optimal scheme for siso x channel with synergistic alternating csit,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 376–380, Jun. 2014.
- [112] C. Wang, “Degrees of freedom characterization: The 3-user siso interference channel with blind interference alignment,” *IEEE Commun. Lett.*, vol. 18, no. 5, pp. 757–760, May 2014.
- [113] A. Alaa and M. Ismail, “Achievable degrees-of-freedom of the k-user siso interference channel with blind interference alignment using staggered antenna switching,” *IEEE Trans. Veh. Technol.*, vol. PP, no. 99, pp. 1–1, May 2016.
- [114] M. Johnny and M. R. Aref, “Sum degrees of freedom of the \$k\$-user interference channel with blind CSI,” *CoRR*, vol. abs/1605.04042, 2016. [Online]. Available: <http://arxiv.org/abs/1605.04042>
- [115] M. Morales-Cspedes, J. Plata-Chaves, D. Toumpakaris, S. A. Jafar, and A. G. Armada, “Blind interference alignment for cellular networks,” *IEEE Trans. Signal Process.*, vol. 63, no. 1, pp. 41–56, Jan. 2015.
- [116] F. C. Kavasoglu, Y. Huang, and B. D. Rao, “Semi-blind interference alignment techniques for small cell networks,” *IEEE Trans. Signal Process.*, vol. 62, no. 23, pp. 6335–6348, Dec. 2014.

- [117] K. Anand, E. Gunawan, and Y. L. Guan, “Semiblind interference alignment: A new framework,” *IEEE Signal Process. Lett.*, vol. 23, no. 5, pp. 580–584, May 2016.
- [118] H. Yang, W. Shin, and J. Lee, “Hierarchical blind interference alignment over interference networks with finite coherence time,” *IEEE Trans. Signal Process.*, vol. 64, no. 5, pp. 1289–1304, Mar. 2016.
- [119] Q. Zhou, Q. Zhang, and F. Lau, “Diophantine approach to blind interference alignment of homogeneous k-user 2x1 miso broadcast channels,” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 10, pp. 2141–2153, Oct. 2013.
- [120] S. H. Chae and S.-Y. Chung, “Blind interference alignment for a class of k-user line-of-sight interference channels,” *IEEE Trans. Commun.*, vol. 60, no. 5, pp. 1177–1181, May 2012.
- [121] C. Wang, H. Papadopoulos, S. Ramprasad, and G. Caire, “Improved blind interference alignment in a cellular environment using power allocation and cell-based clusters,” *Proc. IEEE Int. Conf. on Commun.*, pp. 1–6, Jun. 2011.
- [122] —, “Design and operation of blind interference alignment in cellular and cluster-based systems,” *Proc. IEEE Information Theory and Application Workshop*, pp. 1–10, Feb. 2011.
- [123] S. A. Jafar, “Topological interference management through index coding,” *IEEE Trans. Inf. Theory*, vol. 60, no. 1, pp. 529–568, Jan. 2014.
- [124] N. Naderializadeh and A. S. Avestimehr, “Interference networks with no csit: Impact of topology,” *IEEE Trans. Inf. Theory*, vol. 61, no. 2, pp. 917–938, Feb. 2015.
- [125] X. Yi and D. Gesbert, “Topological interference management with transmitter cooperation,” *IEEE Trans. Inf. Theory*, vol. 61, no. 11, pp. 6107–6130, Nov. 2015.

- [126] H. Sun, C. Geng, and S. A. Jafar, “Topological interference management with alternating connectivity,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 399–403, Jul. 2013.
- [127] C. Geng, H. Sun, and S. A. Jafar, “Multilevel topological interference management,” *Proc. IEEE Information Theory Workshop*, pp. 1–5, Sep. 2013.
- [128] P. Aquilina and T. Ratnarajah, “On the degrees of freedom of interference broadcast channels with topological interference management,” *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1477–1489, Apr. 2016.
- [129] Y. Yang, H. Hu, J. Xu, and G. Mao, “Relay technologies for wimax and lte-advanced mobile systems,” *IEEE Commun. Mag.*, vol. 47, no. 10, pp. 100–105, Oct. 2009.
- [130] S. W. Peters and R. W. Heath, “The future of WiMAX: Multihop relaying with IEEE 802.16j,” *IEEE Commun. Mag.*, vol. 47, no. 1, pp. 104–111, Jan. 2009.
- [131] V. Cadambe and S. Jafar, “Degrees of freedom of wireless networks with relays, feedback, cooperation, and full duplex operation,” *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2334–2344, May 2009.
- [132] X. Chen, S. H. Song, and K. B. Letaief, “Interference alignment in dual-hop mimo interference channel,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1274–1283, Mar. 2014.
- [133] T. Gou, S. A. Jafar, C. Wang, S. W. Jeon, and S. Y. Chung, “Aligned interference neutralization and the degrees of freedom of the $2 \times 2 \times 2$ interference channel,” *IEEE Trans. Inf. Theory*, vol. 58, no. 7, pp. 4381–4395, Jul. 2012.
- [134] I. Shomorony and A. S. Avestimehr, “Degrees of freedom of two-hop wireless networks: Everyone gets the entire cake,” *IEEE Trans. Inf. Theory*, vol. 60, no. 5, pp. 2417–2431, May 2014.

- [135] D. Wu, C. Yang, and Z. Xiong, "On maximum linearly achievable degrees of freedom region of $2 \times 2 \times 2$ interference network with arbitrary antenna configurations," *IEEE Trans. Signal Process.*, vol. 63, no. 14, pp. 3770–3782, Jul. 2015.
- [136] C. S. Vaze and M. K. Varanasi, "Beamforming and aligned interference neutralization achieve the degrees of freedom region of the $2 \times 2 \times 2$ mimo interference network," *Proc. IEEE Information Theory and Application Workshop*, pp. 199–203, Feb. 2012.
- [137] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 2, pp. 379–389, Feb. 2007.
- [138] M. R. A. Khandaker and Y. Rong, "Interference MIMO relay channel: Joint power control and transceiver-relay beamforming," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6509–6518, Dec. 2012.
- [139] Y. Cheng and M. Pesavento, "Joint optimization of source power allocation and distributed relay beamforming in multiuser peer-to-peer relay networks," *IEEE Trans. Signal Process.*, vol. 60, no. 6, pp. 2962–2973, Jun. 2012.
- [140] M. Fadel, A. El-Keyi, and A. Sultan, "Qos-constrained multiuser peer-to-peer amplify-and-forward relay beamforming," *IEEE Trans. Signal Process.*, vol. 60, no. 3, pp. 1397–1408, Mar. 2012.
- [141] Z. Zhou and B. Vucetic, "A cooperative beamforming scheme in mimo relay broadcast channels," *IEEE Trans. Wireless Commun.*, vol. 10, no. 3, pp. 940–947, Mar. 2011.
- [142] K. Truong and R. Heath, "Joint transmit precoding for the relay interference broadcast channel," *IEEE Trans. Veh. Commun.*, vol. 62, no. 3, pp. 1201–1215, Mar. 2013.

- [143] K. Anand, E. Gunawan, C. M. Yetis, and Y. L. Guan, “Finite-snr precoder designs for the interference relay channel without csi at the transmitters,” *IEEE Trans. Commun.*, vol. 63, no. 12, pp. 4754–4767, Dec. 2015.
- [144] S. Fazeli-Dehkordy, S. ShahbazPanahi, and S. Gazor, “Multiple peer-to-peer communications using a network of relays,” *IEEE Trans. Signal Process.*, vol. 57, no. 8, pp. 3053–3062, Aug. 2009.
- [145] A. Phan, H. Tuan, H. Kha, and H. Nguyen, “Beamforming optimization in multi-user amplify-and-forward wireless relay networks,” *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1510–1520, Apr. 2012.
- [146] J. Yang, B. Champagne, Y. Zou, and L. Hanzo, “Joint optimization of transceiver matrices for mimo-aided multiuser af relay networks: Improving the qos in the presence of csi errors,” *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1434–1451, Mar. 2016.
- [147] B. Chalise and L. Vandendorpe, “Optimization of MIMO relays for multipoint-to-multipoint communications: Nonrobust and robust designs,” *IEEE Trans. Signal Process.*, vol. 58, no. 12, pp. 6355–6368, Dec. 2012.
- [148] U. Rashid, H. D. Tuan, and H. H. Nguyen, “Relay beamforming designs in multi-user wireless relay networks based on throughput maximin optimization,” *IEEE Trans. Commun.*, vol. 61, no. 5, pp. 1739–1749, May 2013.
- [149] A. H. Phan, H. D. Tuan, H. H. Kha, and H. H. Nguyen, “Iterative d.c. optimization of precoding in wireless mimo relaying,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 4, pp. 1617–1627, Apr. 2013.
- [150] J. Liu, F. Gao, and Z. Qiu, “Robust transceiver design for downlink multiuser mimo af relay systems,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2218–2231, Apr. 2015.

- [151] Z. Peng, W. Xu, L. C. Wang, and C. Zhao, “Achievable rate analysis and feedback design for multiuser mimo relay with imperfect csi,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 2, pp. 780–793, Feb. 2014.
- [152] K. Mohanty, C. S. Vaze, and M. K. Varanasi, “The degrees of freedom region of the mimo interference channel with hybrid csit,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 1837–1848, Apr. 2015.
- [153] J. Xie and S. Ulukus, “Secure degrees of freedom of k -user gaussian interference channels: A unified view,” *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2647–2661, May 2015.
- [154] S. Amuru, R. Tandon, and S. Shamai, “On the degrees-of-freedom of the 3-user miso broadcast channel with hybrid csit,” *2014 IEEE International Symposium on Information Theory*, pp. 2137–2141, Jun. 2014.
- [155] B. Rassouli, C. Hao, and B. Clerckx, “Dof analysis of the mimo broadcast channel with alternating/hybrid csit,” *IEEE Trans. Inf. Theory*, vol. 62, no. 3, pp. 1312–1325, March 2016.
- [156] C. Xiao, Y. R. Zheng, and Z. Ding, “Globally optimal linear precoders for finite alphabet signals over complex vector gaussian channels,” *IEEE Trans. Signal Process.*, vol. 59, no. 7, pp. 3301–3314, Jul. 2011.
- [157] A. J. Tenenbaum, “Mse-based linear transceiver designs for multiuser mimo wireless communications,” *Doctoral Thesis*, 2011.
- [158] D. Bertsekas, *Nonlinear programming*. Athena Scientific, 1999.
- [159] R. Hunger, “Floating point operations in matrix-vector calculus,” *Technical Report: Technische Universitat Munchen Associate Institute for Signal Processing*, 2007.

- [160] Q. Shi, M. Razaviyayn, Z.-Q. Luo, and C. He, “An iteratively weighted MMSE approach to distributed sum-utility maximization for a MIMO interfering broadcast channel,” *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4331–4340, Sep. 2011.
- [161] M. V. Solodov, “On the convergence of constrained parallel variable distribution algorithms,” *SIAM Journal on Optimization*, vol. 8, no. 1, pp. 187–196, 1998.
- [162] M. Hong, T.-H. Chang, X. Wang, M. Razaviyayn, M. Shiqian, and Z.-Q. Luo, “A block coordinate descent method of multipliers: Convergence analysis and applications,” *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Process.*, pp. 7689–7693, May 2014.
- [163] S. Christensen, R. Agarwal, E. Carvalho, and J. Cioffi, “Weighted sum-rate maximization using weighted mmse for mimo-bc beamforming design,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 4792–4799, Dec. 2008.
- [164] C. Wei Tan, M. Chiang, and R. Srikant, “Maximizing sum rate and minimizing MSE on multiuser downlink: Optimality, fast algorithms and equivalence via max-min SIR,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 2669–2673, Jun. 2009.
- [165] J. Shin and J. Moon, “Weighted-sum-rate-maximizing linear transceiver filters for the K-user MIMO interference channel,” *IEEE Trans. Commun.*, vol. 60, no. 10, pp. 2776–2783, Oct. 2012.
- [166] F. Negro, S. Shenoy, I. Ghauri, and D. T. M. Slock, “On the mimo interference channel,” *Proc. IEEE Information Theory and Application Workshop*, pp. 1–9, Jan. 2010.
- [167] D. Schmidt, C. Shi, R. Berry, M. Honig, and W. Utschick, “Comparison of

- distributed beamforming algorithms for MIMO interference networks,” *IEEE Trans. Signal Process.*, vol. 61, no. 13, pp. 3476–3489, Jul. 2013.
- [168] Z.-Q. Luo and S. Zhang, “Dynamic spectrum management: Complexity and duality,” *IEEE J. Sel. Signal Process.*, vol. 2, no. 1, pp. 57–73, Feb. 2008.
- [169] A. Hjørungnes and D. Gesbert, “Complex-valued matrix differentiation: Techniques and key results,” *IEEE Trans. Signal Process.*, vol. 55, no. 6, pp. 2740–2746, Jun. 2007.
- [170] Y. Wu, C. Xiao, X. Gao, J. Matyjask, and Z. Ding, “Linear precoder design for mimo interference channels with finite-alphabet signaling,” *IEEE Trans. Commun.*, vol. 61, no. 9, pp. 3766–3780, Sep. 2013.
- [171] Y. Wu and S. Verdu, “The impact of constellation cardinality on gaussian channel capacity,” *Proc. Allerton Conf. on Commun., Control and Computing*, pp. 620–628, Sep. 2010.
- [172] H. Lütkepohl, *Handbook of Matrices*. Wiley, 1997.
- [173] Y. Tian and A. Yener, “Relay-aided interference alignment for the x channel with limited csi,” *Proc. IEEE Wireless Commun. Netw. Conf.*, pp. 465–469, Apr. 2012.
- [174] —, “Guiding blind transmitters: Relay-aided interference alignment for the x channel,” *Proc. IEEE Int. Symp. on Inf. Theory*, pp. 1513–1517, Jul. 2012.
- [175] H. Sun, C. Geng, T. Gou, and S. A. Jafar, “Degrees of freedom of MIMO X networks: Spatial scale invariance and one-sided decomposability,” *IEEE Trans. Inf. Theory*, vol. 59, no. 12, pp. 8377–8385, Dec. 2013.
- [176] R. Horn and C. Johnson, *Topics in Matrix Analysis*. Cambridge University Press, 1994.

- [177] F. Le Gall, “Powers of tensors and fast matrix multiplication,” *Proceedings of the 39th International Symposium on Symbolic and Algebraic Computation*, pp. 296–303, 2014.
- [178] H. Bolcskei, R. Nabar, O. Oyman, and A. J. Paulraj, “Capacity scaling laws in MIMO relay networks,” *IEEE Trans. Wireless Commun.*, vol. 5, no. 6, pp. 1433–1444, Jun. 2006.
- [179] M. Gastpar and M. Vetterli, “On the capacity of large Gaussian relay networks,” *IEEE Trans. Inf. Theory*, vol. 51, no. 3, pp. 765–779, Mar. 2005.
- [180] V. Genc, S. Murphy, Y. Yu, and J. Murphy, “IEEE 802.16J relay-based wireless access networks: An overview,” *IEEE Wireless Commun. Mag.*, vol. 15, no. 5, pp. 56–63, Oct. 2008.
- [181] G. Kramer, M. Gastpar, and P. Gupta, “Cooperative strategies and capacity theorems for relay networks,” *IEEE Trans. Wireless Commun.*, vol. 51, no. 9, pp. 3037–3063, Sep. 2005.
- [182] A. Host-Madsen and J. Zhang, “Capacity bounds and power allocation for wireless relay channels,” *IEEE Trans. Inf. Theory*, vol. 51, no. 6, pp. 2020–2040, Jun. 2005.
- [183] I. Maric and R. D. Yates, “Bandwidth and power allocation for cooperative strategies in Gaussian relay networks,” *IEEE Trans. Inf. Theory*, vol. 56, no. 4, pp. 1880–1889, Apr. 2010.
- [184] N. Jindal, “Mimo broadcast channels with finite-rate feedback,” *IEEE Trans. Inf. Theory*, vol. 52, no. 11, pp. 5045–5060, Nov. 2006.
- [185] C. K. Au-yeung and D. J. Love, “On the performance of random vector quantization limited feedback beamforming in a miso system,” *IEEE Trans. Wireless Commun.*, vol. 6, no. 2, pp. 458–462, Feb. 2007.

- [186] S. Cho, K. Huang, D. Kim, V. K. N. Lau, H. Chae, and H. Seo, “Feedback-Topology designs for interference alignment in MIMO interference channels,” *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6561–6575, Dec. 2012.
- [187] C. Wilson and V. V. Veeravalli, “A convergent version of the max SINR algorithm for the MIMO interference channel,” *IEEE Trans. Wireless Commun.*, vol. 12, no. 6, pp. 2952–2961, Jun. 2013.
- [188] D. A. Schmidt, C. Shi, R. A. Berry, M. L. Honig, and W. Utschick, “Minimum mean squared error interference alignment,” *Proc. IEEE Asilomar Conference on Signals, Systems and Computers (ACSSC)*, pp. 1106–1110, Nov. 2009.
- [189] I. S. Gradshteyn and I. M. Ryzhik, *Table of integrals, series, and products*. Elsevier/Academic Press, Amsterdam, 2007.