Design, modeling, and optimization of nonlinear piezoelectric energy harvesters

Upadrashta, Deepesh

2017

Upadrashta, D. (2017). Design, modeling, and optimization of nonlinear piezoelectric energy harvesters. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/69544>

<https://doi.org/10.32657/10356/69544>

NANYANG
TECHNOLOGICAL
UNIVERSITY

DESIGN, MODELING, AND OPTIMIZATION
OF
NONLINEAR PIEZOELECTRIC
ENERGY HARVESTERS

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

2016

**DESIGN, MODELING, AND OPTIMIZATION
OF
NONLINEAR PIEZOELECTRIC ENERGY HARVESTERS**

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School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

2016

Dedicated
to
my father and grandmother

ACKNOWLEDGEMENTS

First and foremost, I would like to extend my deepest gratitude to my supervisor, Associate Professor Yang Yaowen for his valuable guidance over the last four years. He has been more than my academic advisor and was always available for helping me in resolving the technical or non-technical issues throughout my PhD study. He provides pleasant and stress-free environment for doing research and knows how to communicate with the students. He has been a great mentor and always available for discussing personal problems as well. He has given me freedom in selecting research directions and solving the problems in my own way. His valuable technical advice always inspires me to come up with fresh ideas. My last four years as a PhD student under the supervision of Prof. Yang is a memorable time and I cannot think of a better research experience.

I would also like to express my sincere gratitude to Professor Soh Chee Kiong for sharing his knowledge and vast experience. He has always been helpful and supportive of my ideas. His great sense of humor always cheered up the entire research group. I would also like to express my sincere gratitude to Dr. Tang Lihua, my senior, for his valuable help and thoughtful assistance during my initial days of PhD study and for sharing his knowledge on energy harvesting. Many thanks to Mr. Panduranga Vittal Avvari, Dr. Zhao Liya, Dr. Wu Hao, Dr. Dai Huliang and Mr. Lu Xubin for being great colleagues and friends. I have learnt many things during the discussions and while doing the experiments with them. They have been very supportive in both research and personal life and I enjoyed working with them.

Many thanks to the technicians in Protective Engineering Laboratory and Construction Technology Laboratory for their valuable assistance. Without the help of Mr. Chelladurai Subasanran, Mr. Phua Kok Soon, Mr. Tui Cheng Hoon David, Mr. Goh Choon Kiang, Mr. Chan Chiew Choon, Mr. Ho Yaow Chan, Mr. Jee Kim Tian Jeffrey and Mr. Cheng Weng Kong, I wouldn't have finished the experiments in time.

I am also very grateful to the School of Civil and Environmental Engineering, Nanyang Technological University, Singapore, for providing me the opportunity to conduct the research and I feel my honor to start my academic career in NTU.

Finally, I want to thank my mother, brothers Jagadish and Latish, and my wife Pavani. Nothing is possible without your unwavering love in each day of my life. My dreams are coming true because of your support and all my achievements are dedicated to you. I love you all forever.

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ABBREVIATIONS

DOF: Degree Of Freedom

ECM: Equivalent Circuit Modeling

FEA: Finite Element Analysis

FFT: Fast Fourier Transform

LPEH: Linear Piezoelectric Energy Harvester

MEMS: Micro Electro Mechanical Systems

MFC: Macro Fiber Composite

NPEH: Nonlinear Piezoelectric Energy Harvester

PEH: Piezoelectric Energy Harvester

PZT: Lead (Pb) – Zirconate – Titanate

RMS: Root Mean Square

SCE: Synchronous Charge Extraction

S-DOF: Single – Degree Of Freedom

SHM: Structural Health Monitoring

SSHI: Synchronized Switch Harvesting on Inductor

WSN: Wireless Sensor Node

SUMMARY

In the last decade, the emphasis on structural and environmental monitoring using wireless sensor nodes has been increased to improve the safety, serviceability, and reliability of infrastructure systems. With advancement in smart materials, low-power-consuming electronic devices and integrated circuits, energy harvesting has emerged as a potential technology for powering wireless sensor nodes. Several energy sources are available in nature for energy harvesting, such as solar, wind, thermal gradient, and vibration. The ubiquitous nature of vibration energy has encouraged researchers to use it for energy harvesting through certain transduction mechanisms. Among various transduction mechanisms, piezoelectric energy harvesting has garnered a lot of academic interest because of its higher power density and ease of application.

Conventional cantilever-type linear piezoelectric energy harvesters (PEHs) suffer from narrow operational bandwidth. Various techniques have been employed to broaden the bandwidth of linear harvester, such as oscillator arrays, multi-modal oscillators, passive or active resonance tuning, and nonlinear techniques. Magnetic interaction has been largely utilized for broadband energy harvesting and various configurations with magnetic interaction have been proposed. However, less accurate dipole-dipole approximation has been used to evaluate the magnetic force in most of the nonlinear analytical models reported so far. In this thesis, a nonlinear analytical model of the harvester with better formulation for evaluating the magnetic force is presented. While attempting to improve the performance of harvesters, the material strength has yet been considered as critical constraint in the design process. In this work, an optimal nonlinear PEH with magnetic oscillator is designed within the allowable limits of strain on macro fiber composite (MFC) which is used as an energy conversion material. Parametric study is carried out by varying the beam length and the distance between magnets to harvest more energy and to widen the operation bandwidth. Experimental results indicate that, when compared to the linear configuration, the optimal nonlinear PEH achieves higher power output and wider bandwidth with maximum strain on MFC below the allowable limit.

Analytical models of nonlinear PEHs have been proposed predominantly for simple mechanical structures like cantilever beams. It is very challenging or sometimes not possible to derive accurate analytical models for complex harvester designs and inclusion of magnetic interaction further raises the difficulty of modeling. A novel approach to model the magnetic interaction using nonlinear spring elements in ANSYS finite element software without

inclusion of the magnetic module is presented in this thesis. Firstly, a finite element model of monostable PEH with cantilever configuration is developed and the results are validated with predictions from very accurate theoretical model. Later, the proposed technique of finite element modeling is extended to a complex 2-degree of freedom nonlinear PEH for which an analytical model that can capture the exact dynamics of the system is difficult to derive. The performance of nonlinear PEH predicted using finite element simulation is compared with the experimental results. It is observed that the proposed finite element modeling technique is able to accurately analyze the dynamics of nonlinear PEHs with magnetic interaction.

The performance of PEHs operating in vibration environments depends upon several nonlinearities existing in the dynamic system. In this thesis, the effects of geometric, material, and damping nonlinearities on the dynamic response of harvester are investigated. Firstly, the finite element modeling is used for analyzing the effect of geometric nonlinearity on the response of harvester. Simulations are carried out using ANSYS for different harvester configurations to assess the influence of geometric nonlinearity on the harvester's performance. A parameter is proposed to determine the inclusion/exclusion of geometric nonlinearity for analyzing PEHs. Subsequently, a nonlinear electromechanical model considering material and damping nonlinearities is derived for a cantilever type PEH. In the earlier works, the nonlinear elastic and damping coefficients were determined by matching the analytical responses (e.g., displacement or voltage) with a set of experimental responses, which is an indirect method and might result in erroneous estimation of the unknown nonlinear coefficients as pointed out in the present work. In this thesis, the nonlinear material behavior of MFC is directly obtained from tensile testing. The experimentally obtained nonlinear stress-strain behavior of MFC is fitted to a higher-degree polynomial and is included in the nonlinear analytical model to study the dynamics of harvester. Energy harvesting experiments are conducted and the harvester's response is compared with predictions from the proposed nonlinear model. Moreover, the damping parameters are identified using the energy balance method and it is shown that nonlinear damping is essential for accurate modeling of PEHs.

An array of nonlinear piezoelectric energy harvesters (NPEHs) is proposed for scavenging electrical energy from broadband vibrations with low amplitudes. The array consists of monostable NPEHs combined to generate useful power output over wide bandwidth. The nonlinearity in each of NPEHs is induced by the magnetic interaction between an embedded magnet in the tip mass of cantilever-type harvester and a magnet clamped to the rigid platform. The dynamic responses of two NPEHs; one with attractive configuration and the other with

repulsive configuration are combined to achieve wide bandwidth. A parametric study is carried out to obtain the suitable gap distances between the magnets. Experiments are performed to validate the proposed idea and the theoretical predictions, and to demonstrate the advantage of array of NPEHs over array of linear piezoelectric energy harvesters. The experiments have clearly shown the advantage of NPEH array over its linear counterpart under both harmonic and random excitations. Furthermore, the performance of NPEH array with series and parallel connections between individual harvesters using the standard AC/DC interface circuit is also investigated and compared with its linear counterpart.

Finally, this thesis presents an experimental investigation of fatigue in MFC due to cyclic electromechanical loads. The MFC has been extensively used in actuator/sensor/harvester applications. Fatigue due to cyclic high electric fields in actuator applications has been studied extensively. However, the fatigue failure of MFC due to high stress or strains in energy harvesting applications has attracted little attention. In this study, the upper limit of dynamic strain on MFC that can be used as a failure limit in designing the harvesters is evaluated experimentally. Energy harvesting tests are conducted at various base accelerations and the voltage output and maximum strain on MFC are measured. Severe loss in the performance of the harvester is observed within million cycles of testing at high strain amplitudes. Hence several reliability tests for extended periods of time are carried out at various strain amplitudes. Degradation in the voltage output, change in the natural frequency and formation of cracks are considered as failures. The observations from this study are also applicable to MFC bending actuators undergoing cyclic strains.

CHAPTER 1 INTRODUCTION

1.1 Background of vibration energy harvesting

Structural health monitoring (SHM) has become indispensable to ensure safety and serviceability of bridges, dams, buildings, large machinery, turbines, vessels, etc. SHM involves real time data acquisition using network of sensors and data transmission to a base station. Normally the sensors are connected to a global hose wire which powers them. Over the years, wireless sensor nodes (WSNs) powered by chemical batteries have been extensively used in the SHM applications for accessing remote locations and to reduce the weight and cost of wiring. But expensive periodic maintenance, limited lifespan and disposal of chemical wastage are the severe drawbacks of using chemical batteries for powering WSNs. This scenario has prompted researchers to seek alternate energy sources to power WSNs. The reduction in power consumption of integrated circuits and the advancement in smart materials have encouraged researchers to use ambient energy sources such as solar, wind, thermal gradient, and vibration as an alternative to chemical batteries for powering WSNs or portable electronic devices.

Solar and wind energies have been widely utilized for large scale power production. Wave energy is another potential choice for large-scale power generation. However, harsh marine and offshore environments pose critical challenges for the maintenance and longevity of wave energy converters. Many researchers have successfully tapped energy from thermal gradients available in industrial plants, heat pipes, humans, etc., using thermoelectric and pyroelectric materials. However, the thermal gradients available in the common environment are not sufficient for harvesting useful power. Ambient vibrational energy is not suitable for large scale power production. However, its pervasive nature is suitable for powering small electronic devices such as WSNs used in the SHM applications. The ultimate goal of the research is to power WSNs from ambient vibrations to create intelligent, autonomous, and eco-friendly sensing and monitoring systems.

The idea of vibration-to-electricity for small scale devices is first put forth by Williams and Yates [1]. As stated by Williams and Yates, the three basic vibration-to-electric energy conversion mechanisms are the electromagnetic, electrostatic, and piezoelectric transductions. The transduction mechanisms generate electricity by using mechanical strain or relative motion

in the system. Among the three mechanisms, piezoelectric transduction received a lot of attention because of its high power density and ease of application [2]. The piezoelectric effect was first demonstrated by the Currie brothers in 1880. Piezoelectric materials exhibit coupling between mechanical and electrical domains and the coupling is usually referred as direct and converse effects. When a piezoelectric material is mechanically strained, electric charge accumulates at the electrodes located on its surface. This is called the direct piezoelectric effect. If a potential difference is applied across the electrodes, the piezoelectric material deforms mechanically. This is called the converse piezoelectric effect. The direct piezoelectric effect is the reason for power generation in the energy harvesting applications. But it is important to remember that both direct and converse piezoelectric effects always coexist in the piezoelectric material.

Energy can be harvested from vibrations either by directly bonding the piezoelectric material to the host structure or installing the harvesters as add-on systems (usually cantilever configurations with bonded piezoelectric elements) on the vibrating host structure. The cantilever-type piezoelectric energy harvesters (PEHs) have one or two piezoceramic layers covering the beam entirely or partially. Usually, a tip mass is attached to adjust the natural frequency of harvester. The PEH is mounted on the host structure and the dynamic strain induced in the piezoelectric layers due to the base vibration generates alternating voltage. In most of the theoretical and experimental studies, it is a common practice to use a resistor for representing the electric load. However, in real life applications, it is required to convert the alternating voltage output into DC voltage output using a rectifier (AC-DC converter) for charging either a small rechargeable battery or a capacitor. Lot of research has been done in designing the harvesting circuits to improve the efficiency of energy harvesting.

The ambient vibration sources are usually frequency variant or random in nature with energy distributed over a frequency range. But the conventional cantilever-type PEH gives maximum power only when excited at its resonant frequency and hence is not suitable for the practical applications. Therefore, many attempts have been made to enhance operating bandwidth of harvesters. Various techniques such as arrays of linear oscillators, multi-modal oscillators, active, and passive frequency tuning methods have been adopted for broadband energy harvesting. Furthermore, nonlinear concepts such as monostable duffing, impact, and bistable oscillator designs have been proposed to broaden the bandwidth. Although many attempts have been made to improve the performance of PEHs, the analytical models have not been fully developed to accurately study the dynamics of the harvesters. Some of the existing nonlinear

models have employed indirect methods to identify the unknown coefficients which compromise or restrict the validity of the proposed models. Furthermore, analytical solutions have been derived for simple mechanical configurations such as cantilever-type PEHs only. Capability of finite element commercial software for modeling the complex PEHs has not been tested so far. Although, volume and mass of PEH system have received lot of research interest as critical constraints on the design of PEH for practical applications, the strengths of piezoelectric and base materials are largely neglected. Furthermore, the influence of fatigue due to the cyclic electromechanical loads on the harvester's performance also requires detailed investigation.

1.2 Research objectives and scope

The ultimate goal of the research is to develop accurate modeling methods to assist in the design of nonlinear PEHs working in the anticipated vibration environments. In this thesis, efforts have been made to develop analytical and finite element models to design and optimize the nonlinear piezoelectric energy harvesters specifically for low frequency ($< 50\text{Hz}$) and low amplitude ($< 3\text{m/s}^2$) vibration sources.

- A) It is essential to design PEHs with higher power output and wider operating bandwidth to be able to power wireless sensors using the wideband ambient random vibrations. The practical constraints on the design of harvester are the system mass, volume, and strength of the materials. While attempting for enhancing the performance of PEHs, the material strength has yet been considered as a constraint in the design process although system volume and mass received a lot of research interest. This thesis focuses on the design optimization of the nonlinear PEH with magnetic oscillator within the limits of allowable strain on the piezoelectric material (macro fiber composite). Furthermore, a more accurate charge model is used for evaluating the magnetic force rather than the commonly employed dipole-dipole approximation.
- B) Various techniques have been used by researchers for broadband energy harvesting. Particularly, magnetic nonlinearity has been extensively used in several designs to improve the bandwidth of PEH. It is not always possible to develop accurate analytical models for complex PEH designs and inclusion of the magnetic interaction complicates the modeling of harvester even more. A new approach to model the magnetic interaction in ANSYS Multiphysics using the nonlinear spring element has been developed to study the behavior of complex nonlinear PEH designs.

- C) The dynamic response of PEH depends on various nonlinearities existing in the system. Although few works have been reported in the literature on the modeling of inherent nonlinearities in PEHs, most of them use the indirect methods for identifying the unknown nonlinear coefficients and thus limiting the validity of proposed models. In this thesis, the geometric, material, and damping nonlinearities in PEHs are modeled more accurately using finite element and analytical models.
- D) Despite of extensive research, still it remains a great challenge to harvest sufficient energy over wide bandwidth to power WSNs. An array of nonlinear piezoelectric energy harvesters (NPEHs) is proposed for scavenging the electrical energy from broadband vibrations with low amplitudes. The array consists of monostable harvesters and their dynamic response is combined to achieve uniform voltage output over large bandwidth. The advantage of array of NPEHs over the array of linear piezoelectric energy harvesters (LPEHs) is demonstrated experimentally under both harmonic and random excitations.
- E) Fatigue due to the cyclic electromechanical loads is a critical issue when designing the harvesters for real life applications because of the inherent brittleness of the piezoelectric materials. Most of the literature focuses on the fatigue behavior of piezoelectric materials for actuator applications where strong electric fields are used. However, the effect of high stress or strain fields which is the case in PEH applications is not fully investigated. Reliable performance over very long periods of time is a requirement for practical implementation of PEHs to replace the chemical batteries. This thesis presents the experimental study for evaluating the upper limit of strain amplitude on macro fiber composite (MFC) to assure reliable performance over extended periods of time without any failure.

1.3 Original contributions

The original contributions of this research are summarized here:

- 1) Enhancing the performance of nonlinear PEHs by optimizing the geometric parameters is not meaningful without taking strength of the materials into consideration. In this work, the performance of a PEH with magnetic oscillator is optimized while imposing constraint on the maximum strain induced in the piezoelectric material (MFC). A parametric study is carried out by varying the beam length and the distance between magnets to seek the optimal configuration of the nonlinear PEH while keeping the maximum strain on MFC below the failure limit. Furthermore, the analytical model of

nonlinear harvester is improved by using very accurate charge model for evaluating the force between the permanent magnets in place of commonly used dipole-dipole approximation. Experiments are carried out and validated the predictions from theoretical modeling.

- 2) So far in the literature of piezoelectric energy harvesting, finite element analysis (FEA) is limited for studying the linear harvesters only. FEA has yet been used for analyzing the nonlinear PEHs because of the difficulty involved in modeling the dynamic magnetic interaction and the high computational demand for solving the coupled electrical-mechanical-magnetic problem. In this thesis, the nonlinear PEHs are accurately analyzed using FEA by modeling the magnetic force with a nonlinear spring element and thus eliminating the necessity of magnetic module. A monostable nonlinear PEH is modeled by using the proposed method and the results are validated with the existing analytical model. Later, the proposed modeling technique is extended to a 2-DOF nonlinear PEH with complex geometry for which an accurate theoretical model is difficult to derive. The FEA simulation successfully captures the dynamics of harvester and the predictions from FEA are in good agreement with experiments indicating the applicability of proposed method for modeling complex harvester designs. The accuracy and the low computational effort involved in using the nonlinear spring element to model the magnetic force is an effective way for FEA modeling of the nonlinear harvesters.
- 3) The influence of geometric, material, and damping nonlinearities on the dynamic response of PEHs is investigated in this thesis. FEA approach is undertaken to understand the effect of geometric nonlinearity by analyzing different PEH configurations. Based on the observations, a parameter is proposed to assess the degree of geometric nonlinearity in PEHs. Most of the works on the nonlinear modeling of piezoelectric harvesters utilized data fitting methods to determine the unknown coefficients in the analytical model which might lead to erroneous estimation of coefficients. In this work, a nonlinear dynamic model of cantilever-type PEH is derived from the first principles accounting for material and damping nonlinearities. The nonlinear stress-strain behavior of MFC is obtained using tensile testing and incorporated in the analytical model. Moreover, the unknown damping coefficients at different base excitations are identified using the energy balance method. The comparison between responses from modeling and experiment shows very good agreement.

- 4) An array of NPEHs for broadband energy harvesting is proposed in this thesis. Its advantage over the LPEH array in terms of broadening the operation bandwidth has been proved experimentally. The theoretical study is carried out with two NPEHs and a wide operation bandwidth is achieved by proper selection of the gap distance between the magnets. For comparison purpose, the prototypes of both NPEH and LPEH arrays have been fabricated and tested under harmonic and random excitations. The rectified voltage response from harvesters is combined using series and parallel connections. Under harmonic base excitation, the NPEH array exhibits 100% improvement in the operation bandwidth over the LPEH array when operating at high energy states. Under random excitation, the NPEH array using series connection stores 80% more energy compared to its linear counterpart. The proposed idea can be extended to multiple NPEHs to cover the frequency spectrum of wideband vibration sources.
- 5) In this thesis, the reliability of MFC is investigated over extended periods of time under the cyclic loading similar to the operation conditions in the energy harvesting applications. Energy harvesting tests are carried out on PEHs comprising MFC and significant shift in the resonant frequency is observed when the base acceleration is increased. Moreover, sudden drop in the voltage output is noticed within million cycles of testing at high strains. It is found out from the tensile testing that the nonlinear stress-strain behavior of MFC below $1000\mu\epsilon$ is responsible for the observed shift in resonant frequency. The results indicate that the MFC doesn't exhibit clear linear elastic limit. Several long duration reliability tests are conducted at different strain amplitudes and frequencies to estimate the upper limit of dynamic strain on MFC below which reliable performance of harvester is guaranteed. It is observed that MFC gives high reliable performance under dynamic strain amplitude of $600\mu\epsilon$. Below $600\mu\epsilon$, harvesters didn't show any degradation in the voltage output and tip displacement over 60 million cycles of testing.

1.4 Organization of the report

The thesis is organized into eight chapters including this introductory chapter. The present chapter gives a brief overview on the area of energy harvesting with a special focus on energy harvesting from ambient vibrations using piezoelectric materials. The research objectives, scope, and original contributions of this thesis are presented in this chapter. The second chapter provides comprehensive literature survey on vibration based piezoelectric energy harvesting with emphasis on nonlinear techniques used for broadband energy harvesting. Chapter three

presents accurate theoretical modeling of nonlinear PEH with magnetic oscillator. It focuses on the optimization of harvester for achieving high power and wide bandwidth within the limits of allowable strain on MFC to ensure structural integrity. However, it is not always possible to derive accurate analytical solutions for the harvesters specially in case of complex geometries. In such cases, FEA can be used for modeling of harvesters. Chapter four presents a novel method to model the magnetic force using nonlinear spring element in ANSYS Multiphysics without inclusion of the magnetic module. In this chapter, a monostable nonlinear PEH and a complex 2-DOF nonlinear PEH are modeled using the proposed novel technique. In fifth chapter, the influence of geometric, material, and damping nonlinearities on the dynamic response of PEH is analyzed using both analytical and finite element modeling approaches. In chapter six, a nonlinear harvester array is designed where knowledge and modeling methods from previous chapters are used. The harvester is specifically designed to operate at low frequencies (14Hz-24Hz) and low amplitudes ($\leq 2\text{m/s}^2$) to avoid influence of nonlinearities and to ensure structural integrity. Chapter seven investigates the performance reliability of MFC under cyclic electromechanical loads. Reliability tests are carried out on the harvesters to examine their performance over extended periods of time which is crucial for practical implementation of harvesters. Finally, chapter eight summarizes the research presented in this thesis and provides recommendations for the future work.

CHAPTER 2 LITERATURE REVIEW

2.1 Overview on vibration-based energy harvesting

Over the past decade, the usage of WSNs has been drastically increased due to the advances in micro-electro-mechanical systems (MEMS) technology, wireless communications, and digital electronics. The low-power and low-cost WSNs have been employed in various applications ranging from monitoring of structures/environment/vehicles to healthcare. However, the battery powered WSNs in environmental and structural health monitoring applications pose certain challenges. In monitoring applications where long-term operation life is expected, replacement/recharge of the chemical batteries is difficult, or not possible in certain situations, requires high maintenance cost and involves environmental hazard also.

“Energy harvesting” is generating electrical energy from ambient energy sources surrounding the system to power small electronic devices or to store for the future use. In the last decade, enormous research interest has been focused on the area of energy harvesting to build self-powered systems for distributed or portable electronics systems, such as wireless sensing networks. There are several energy sources existing in the environment suitable for energy harvesting such as solar, wind, thermal gradient, and vibration. Various methods have been developed to harvest energy from different ambient energy sources. Energy harvesting aims at powering electronic devices such as WSNs using electrical energy harnessed from ambient sources. The harvested energy is usually in the range of milliwatts or even less. However, the phenomenal reduction in power consumption of electric circuitry due to the recent advancements enables small-scale energy harvesting as a possible alternative for replacing batteries in electronic devices.

2.1.1 Available sources for energy harvesting

Solar energy harvesting through photo-voltaic conversion mechanism provides highest power density values when compared with other ambient sources. Outdoor solar energy at noon has the potential to provide power density up to $15000\mu\text{W}/\text{cm}^2$, which makes it an ideal modality for powering an embedded system that requires several milliwatts of power [3]. However, the power density decreases by a factor of 100 on the overcast days. Nonetheless, the solar cells are appropriate solutions to the outdoor applications that operate primarily during the daytime. The solar energy is not suitable for harvesting in indoor environments as the power density values reduce to as low as $10\text{-}20\mu\text{W}/\text{cm}^2$ [4].

Thermal gradient is another source for energy harvesting, which is based on the phenomenon of flow of heat caused by the temperature difference. Thermoelectric generators are based on the principle of thermoelectric effect discovered by Thomas Johann Seebeck in 1821. The phenomenon of creating electric potential with a temperature difference and vice-versa is termed as thermoelectricity. Lawrence and Snyder [5] described a method to generate electrical energy using the temperature difference between the air and soil. Strasser et al. [6] reported a micro-scale thermoelectric generator for converting waste heat into electrical energy. A peak power of $1\mu\text{W}$ achieved at a temperature difference of 5K using a generator of 1cm^3 size. However, the level of power output from the thermoelectric generators is still considerably lower than the harvesters using other ambient sources.

Table 2-1 Ambient vibration sources (Roundy et al. [7])

Vibration source	Peak Acceleration (m/s^2)	Frequency of peak (Hz)
Base of a 5 HP 3-axis machine tool	10	70
Kitchen blender casing	6.4	120
Clothes dryer	3.5	120
Door frame just after door closes	3	125
Small microwave oven	2.25	120
HVAC vents in office building	0.2 - 1.5	60
Wooden deck with people walking	1.3	385
Bread maker	1.03	120
Windows (0.6m x 1m) next to a busy street	0.7	100
Notebook computer while CD is being read	0.6	75
Washing machine	0.5	109
Second story floor of a wood frame office building	0.2	100
Refrigerator	0.1	240

Among all the available energy sources, mechanical vibration is the most ubiquitous in our daily life and has attracted intense academic interest over the years. The ultimate goal of the vibration energy harvesting is the practical implementation of self-powered electronic devices by scavenging energy from ambient vibration sources. There are many sources for low amplitude vibrations such as large commercial buildings, automobiles, aircraft, ships, trains, industrial environments, etc. However, high amplitude vibrations usually occur in large machinery. A list of vibration sources along with peak acceleration amplitude and frequency at which that peak occurs is given in Table 2.1. It can be observed from the table that vibration frequency for most of the sources is below 400Hz and the peak amplitude is less than 3m/s^2 . It was estimated from combination of theoretical studies and experiments that approximately $300\mu\text{W/cm}^3$ (power density) could be generated from ambient vibrations [7].

Recently, many efforts have been made to harvest small-scale wind energy through the aeroelasticity phenomena. When structures are subjected to wind flows, aeroelastic instabilities such as vortex-induced vibration, galloping, flutter, turbulence-induced vibration, wake galloping, and buffeting will occur. The structural vibrations induced by aeroelastic instability can be converted to electricity by using the vibration-based piezoelectric energy harvesting approach [8]. Apart from above mentioned sources, stray magnetic fields that are generated by AC devices can be converted into electric energy. Electromagnetic energy around the current carrying wires in typical home and office areas can be tapped by placing an inductor near the vicinity of conductors. However, magnetic field strength from the power lines is strong only in their proximity which limits the placement of harvesters close to AC wires. Additionally, a high efficient power transfer circuit is required to charge a super-capacitor with minimum losses [9]. Recently, research focus is increased on the radio frequency (RF) energy harvesting where RF signals are converted to electricity. However, the preliminary studies indicate that the harvested energy is insufficient to directly power the devices but can be stored in a super-capacitor or micro-batteries [10].

2.1.2 Transduction mechanisms

Williams and Yates [1] and Roundy et al. [7] stated in their early works on vibration energy harvesting that the three basic vibration to electrical energy conversion mechanisms are the piezoelectric, electromagnetic, and electrostatic transductions. Over the last decade, many articles have been published on the use of above three mechanisms for harvesting electrical energy from ambient vibrations. However, the piezoelectric transduction received the greatest

research interest over the other two transduction mechanisms. Recently, dielectric elastomers and magnetostrictive materials also have been used for the energy harvesting.

Electrostatic Transduction

The electrostatic transduction mechanism uses a variable capacitor for energy conversion. The voltage across the variable capacitor increases when the capacitance decreases by constraining the electrical charge in the capacitor. On the contrary, the electric charge flows out of the variable capacitor as net current when the capacitance decreases if the voltage on the capacitor is fixed. A typical variable capacitor used in the energy harvesting applications usually consists of two conductors separated by a dielectric material. The relative motion between the conductors due to the vibration results in variation of the capacitance and thus the mechanical energy is converted into electrical energy. Roundy et al. [7] reported three types of electrostatic generators, differentiating them based on the direction of the relative movement between the conductors as shown in Figure 2-1. The main advantage of electrostatic transduction mechanism is its ease of scalability to smaller dimensions and therefore can be readily integrated into MEMS applications. However, this transduction mechanism requires an external power source for initiation of the harvesting process and also suffers from low energy conversion efficiency when compared to the other transduction mechanisms [11].

Figure 2-1 Three types of electrostatic energy harvesters (a) in-plane overlap converter, (b) in-plane gap closing converter, and (c) out-of-plane gap closing converter (Roundy et al. [7])

Electromagnetic Transduction

Electromagnetic transduction is based on the Faraday's law of electromagnetic induction. The relative movement between a coil and a magnet or the change in magnetic field induces current in the coil. A typical electromagnetic energy harvester consists of a coil, permanent magnet, and an oscillating element such as spring or cantilever beam. Either the coil or the magnet is attached to the oscillating element to achieve the relative motion between them. The electromagnetic energy harvester is capable of generating high-level current output and external voltage source is not required. However, this mechanism suffers from low voltage putout ($< 1\text{V}$) and thus the output is required to be transformed into usable voltage levels for practical use [2]. Originally, El-Hami et al. [12] used an electromagnetic energy harvester of volume 240mm^3 to generate 0.53mW of power at a vibration frequency of 320Hz .

Figure 2-2 Power density versus voltage output comparison of various harvesting mechanisms (Cook Chennault et al. [2])

Piezoelectric transduction

Piezoelectric materials transform strain energy into electrical energy and vice-versa. Energy harvesting using piezoelectric materials has received greatest attention in the past few years and it is evident by high number of publications appeared on this transduction mechanism when compared with others. The main advantages of the piezoelectric energy harvesting are the higher power density and ease of application. The power density versus voltage comparison for various conversion mechanisms was given by Cook Chennault et al. [2] and is shown in

Figure 2-2. It is conspicuous from the figure that piezoelectric energy harvesting covers the largest portion of the graph and the power density values are comparable to the thermoelectric generators and lithium-ion batteries. The voltage output from the piezoelectric materials is much higher than the output obtained from the electromagnetic energy harvesting and can be used directly for charging a storage component. Generally, piezoelectric energy harvesters (PEHs) are in the form of cantilever beams comprising a substrate layer and one or two piezoelectric material layers are bonded to it. The harvester beam is hosted on a vibrating structure and the dynamic strain induced in the piezoelectric material produces an alternating voltage output which is rectified and used for charging a small battery or capacitor. Over the years, various techniques have been developed to enhance the performance of PEHs, which will be discussed in the following sections.

Dielectric elastomers

Dielectric elastomer is a soft rubber like polymer material sandwiched between two conductive electrodes. The energy harvesting mechanism using dielectric elastomer is similar to that of electrostatic transduction. Dielectric material is suitable for large deformations and is capable of converting the mechanical work of the deformation into electricity [13]. Dielectric materials have been studied widely for actuator applications in robotics to design the moveable components [14]. Recently, Koh et al. [15] investigated the potential of dielectric elastomers for power generation over a wide range of strains. However, the application of dielectric materials is limited because of their material characteristics such as material rupture, loss of tension, electrical breakdown, and electromechanical instability. Moreover, the dielectric elastomers are only suitable to the large amplitude low-frequency applications like human motion or oceanic waves because of their low stiffness.

Magnetostrictive materials

Magnetostrictive materials have been used for actuator and sensor applications as they have the capability to convert magnetic energy into kinetic energy and vice-versa. The change in the susceptibility of magnetostrictive materials when subjected to vibratory force induces alternating current in the pick-up coils. The magnetostrictive materials possess high magneto-mechanical coupling coefficient, high flexibility, and are suitable for high-frequency vibrations [16]. However, it is difficult to integrate these materials in MEMS devices due to the bulky

dimensions of the pick-up coils. Moreover, the low voltage output requires voltage transformation in the post-processing circuit.

2.1.3 Fundamentals of piezoelectric energy harvesting

The term “piezoelectricity” is used to describe the coupling between mechanical and electrical domains exhibited by the piezoelectric materials. The phenomenon of piezoelectricity was first discovered by Pierre and Paul-Jacques Curie in 1880, in certain crystalline minerals such as quartz, tourmaline, and Rochelle salt. They found these crystals can develop electric charge on their surface under mechanical deformation, which is termed as *direct piezoelectric effect*. Conversely, when the piezoelectric materials are subjected to an electric field, the materials exhibited mechanical deformation in proportion to the strength of the applied electric field, which is named as *converse piezoelectric effect*. There has been an upsurge in the popularity of piezoelectricity with the discovery of piezoceramics, like Barium Titanate in 1940s and Lead Zirconate Titanate (PZT) in 1950s.

Figure 2-3 Electric dipoles (a) before (b) during, and (c) after the poling process of the piezoelectric ceramics [17]

The piezoelectric effect is caused by aligning the orientation of dipoles in the piezoceramic materials. Usually, the dipoles are randomly oriented as shown in Figure 2-3(a) and no overall macroscopic piezoelectric behavior is observable. Upon applying intense electric fields as high as 1kV/mm at a temperature slightly below the Curie point, the randomly oriented dipoles in the piezoceramics will be aligned as depicted in Figure 2-3(b). When the electric field is removed, most of the dipoles are locked and show roughly, but not complete alignment giving a poling direction to the material as illustrated in Figure 2-3(c). The material now shows strong permanent remnant polarization and has the capability of strong piezoelectric coupling. It is worth noting that the material can be de-poled by subjecting it to a very high electric field oriented in the opposite direction to the existing poling direction or by exposing to a temperature higher than the Curie temperature of the material.

2.1.3.1 Piezoelectric constitutive relations

Assuming that the mechanical perturbations are small and the piezoelectric material is under small electric field conditions, the general constitutive equations for a piezoelectric material can be written as [18]:

$$\begin{bmatrix} S \\ D \end{bmatrix} = \begin{bmatrix} s^E & d \\ d & \epsilon^T \end{bmatrix} \begin{bmatrix} T \\ E^f \end{bmatrix} \quad \mathbf{d}\text{-form} \quad (2.1)$$

$$\begin{bmatrix} T \\ D \end{bmatrix} = \begin{bmatrix} c^E & -e \\ e & \epsilon^S \end{bmatrix} \begin{bmatrix} S \\ E^f \end{bmatrix} \quad \mathbf{e}\text{-form} \quad (2.2)$$

$$\begin{bmatrix} T \\ E^f \end{bmatrix} = \begin{bmatrix} c^D & -h \\ -h & \beta^S \end{bmatrix} \begin{bmatrix} S \\ D \end{bmatrix} \quad \mathbf{h}\text{-form} \quad (2.3)$$

$$\begin{bmatrix} S \\ E^f \end{bmatrix} = \begin{bmatrix} s^D & g \\ -g & \beta^T \end{bmatrix} \begin{bmatrix} T \\ D \end{bmatrix} \quad \mathbf{g}\text{-form} \quad (2.4)$$

where $\mathbf{D}$ (C/m²) is the electric displacement tensor; $\mathbf{S}$ is the strain tensor; $\mathbf{E}^f$ (V/m) is the applied electric field tensor; $\mathbf{T}$ (N/m²) is the stress tensor; ϵ (F/m) is the dielectric constant tensor; β (m/F) is the dielectric impermeability tensor; $\mathbf{d}$ (m/V), $\mathbf{e}$ (C/m²), $\mathbf{h}$ (V/m), and $\mathbf{g}$ (m²/C) are the four forms of the piezoelectric coefficient tensor; $\mathbf{c}$ (N/m²) is the elastic constant tensor; $\mathbf{s}$ (m²/N) is the elastic compliance tensor; and the superscripts $(\ )^T$, $(\ )^S$, $(\ )^D$, and $(\ )^E$ indicate that the quantity is measured at constant stress, constant strain, constant electric displacement, and constant electric field, respectively. Among the four forms of constitutive equations, the $\mathbf{d}$ -form and $\mathbf{e}$ -form equations are widely used for the system modeling and analysis in the literature. The manufacturers of piezoelectric materials usually provide $\mathbf{d}$ -form or $\mathbf{e}$ -form piezoelectric coefficient values in the datasheets. In each form of the constitutive relations, the coupling term in the second equation indicates the electrical output resulted from the mechanical domain, corresponding to the *direct piezoelectric effect*. The coupling effect is thus termed as *forward electromechanical coupling*. The coupling term in the first equation of the constitutive relations depicts the backward effects in the mechanical domain from the electrical domain, corresponding to the *converse piezoelectric effect*. Such coupling effect is termed as *backward electromechanical coupling*.

2.1.3.2 Piezoelectric materials

The most commonly used piezoelectric material for the energy harvesting is Lead Zirconate Titanate (PZT) and it is shown in Figure 2-4(a). PZT transducers exhibit characteristics such as high elastic modulus, brittleness, and low tensile strength similar to the ceramics. The extreme brittle nature of PZT transducers makes it hard to handle and bond to host structures,

especially on curved surfaces. These disadvantages prompted researchers to develop alternate materials. One solution to overcome the brittleness of PZTs is using a composite material consisting of piezoceramic fibers embedded in a polymeric matrix phase [19]. The polymer matrix provides a protective layer around the piezoelectric material thus increases the flexibility of piezoceramic fibers and makes it conformable to the curved surfaces. Commercially available piezocomposite materials are active fiber composite (AFC) actuators developed by Bent and Hagood [20] at MIT and macro fiber composite (MFC) actuators constructed at NASA Langley Research Center [21]. The polymer layer in MFC provides excellent flexibility to withstand strain up to 4500ppm [22]. Polyvinylidene fluoride or simply PVDF is another piezoelectric material with excellent mechanical flexibility but with low piezoelectric coefficient [23]. In 1997, Park and Shrout [24] reported that PT-doped PMN, which is a relaxor-based ferroelectric material has excellent electromechanical coupling over PZT. Compared to PZT, the piezoelectric coefficient of PMN-PT materials can be increased by a factor of 3. There has been considerable interest in using non-ferroelectric materials AlN and ZnO at MEMS level or the nano scale levels of energy harvesting [25].

Figure 2-4 Conventional PZT ceramics [17], (b) Macro fiber composites [26], and (c) PVDF [27]

Piezoelectric materials are anisotropic in nature and thus their properties depend upon the orientation. Normally there are two modes of piezoelectric materials used for energy harvesting: d_{33} -mode (stack actuators) and d_{31} -mode (unimorph and bimorphs). The first subscript refers to the direction of applied or generated voltage and second subscript refers to the generated or applied stress direction. In d_{33} -mode, the direction of applied stress (force) and generated voltage is same, while in d_{31} -mode the stress is applied in axial direction but the voltage is obtained from perpendicular direction as shown in Figure 2-5.

Figure 2-5 Operating modes of piezoelectric transducer (Priya and Inman [4])

2.1.4 Piezoelectric Energy Harvesting Systems

The vibration based energy harvesting devices can be classified broadly into two groups, i.e., resonant and non-resonant (displacement dependent) energy harvesters [28]. The resonant energy harvesters are mostly employed where the input vibrations are regular, frequencies are high, and the input vibration amplitude is smaller than the device critical dimensions, like vibration from the machinery. On the other hand, the non-resonant energy harvesters are more efficient when the input vibration motion is irregular or of low frequency, but with amplitudes much larger than the device critical dimensions, such as human motions. Usually, two different installation schemes are employed for harvesting energy from vibrating structures as shown in Figure 2-6 [29]. In the first scheme, the piezoelectric element is bonded to the surface of host structure and the performance of harvester depends on the deformation of the host structure (non-resonant). It is difficult to improve the harvester's performance without modifying the host structure which is not usually preferred. The later scheme is to install an add-on system comprising a cantilever beam with bonded piezoelectric elements onto the vibrating host structure. In this scheme, the PEH's performance can be enhanced by modifying the add-on structure. The response of add-on system which is a cantilever beam configuration greatly depends on its resonant frequencies. When the frequency of vibrating structure matches with the resonances of add-on system, the cantilever beam vibrates with significant amplitude resulting in higher power generation from the piezoelectric elements. Hence, the cantilever beam configuration is adopted for the piezoelectric energy harvesting in most of the research works. A typical PEH is shown in Figure 2-6(b), which comprises of a cantilever substrate and tip mass, with piezoelectric layers bonded onto the cantilever beam at its fixed end.

Figure 2-6 Different piezoelectric energy harvesting schemes (a) surface bonded (b) add-on system (Liang and Liao [29])

A practical energy harvesting circuit for the post-processing of electrical output from PEH is shown in Figure 2-7. The dynamic strain induced in the piezoelectric elements because of base excitation gives alternating voltage output at the electrodes. It is common practice in theoretical studies and experimentations of PEHs to use a resistive load to represent the external electronic load. However, the electronic components usually require DC voltage and regulated power output. Hence, the AC voltage output needs to be converted to a stable rectified voltage which can be achieved through a combination of rectifier bridge (AC-DC converter) and a secondary stage regulator (DC-DC converter) as illustrated in Figure 2-7. Over the years, various circuit techniques have been developed by researchers which will be discussed in the later sections.

Figure 2-7 Practical energy harvesting circuit of PEH

A system level block diagram of wireless sensing system powered by an energy harvesting module is shown in Figure 2-8(a). The energy harvesting module powers bi-directional RF communication link, microcontroller, and sensors. Microcontroller normally manages power to the sensors and data acquisition elements, as well as interacts with the base station. A wireless sensing node integrated with the harvesting module developed by Arms et al. [30] is shown in Figure 2-8(b). It was demonstrated that the piezoelectric generator was capable of powering the sensor node without any kind of batteries at input acceleration of 0.1g (60Hz)

with low duty cycle wireless updates. However, the ambient vibrational energy is normally scattered over a wide frequency range. The performance of linear harvester drastically decreases away from its resonant frequency and therefore may not produce sufficient power when operating farther away from the resonant frequency.

(a)

(b)

Figure 2-8 (a) System-level block diagram for wireless sensing node with data logging and RF communications capabilities powered by energy harvesting module (b) Integrated piezoelectric vibration energy harvester and wireless sensing node (Arms et al. [30])

2.2 Modeling of vibration based piezoelectric energy harvesters

Over the years, various techniques have been adopted for the modeling of PEHs such as analytical models, equivalent circuit method, and finite element approach. Originally, Williams and Yates [1] and Roundy and Wright [31] proposed un-coupled and coupled lumped-parametric S-DOF (single degree of freedom) models, respectively, where the dynamic response of mass-spring-damper system equivalent of PEH was studied. Later, Erturk and Inman [32] derived the correction factors to compensate the inaccuracy in the existing lumped-parameter model and also developed coupled distributed-parameter models of cantilevered PEHs. Yang and Tang [33] proposed an equivalent circuit approach for the modeling of PEHs that can provide single representation to both mechanical and electrical domains and thus facilitating better understanding and visualization. Some researchers investigated the performance of harvesters using finite element approach by deriving theoretical solutions. However, finite element modeling using commercial software packages is particularly useful for modeling complex designs of PEHs for which deriving theoretical solutions is difficult or not possible. All the above modeling techniques are discussed in detail in the following sections.

2.2.1 Mathematical modeling

2.2.2.1 Simplified S-DOF model

The S-DOF mass-spring-damper model is one common approach undertaken by many researchers to obtain the response of harvester system. Williams and Yates [1] modeled a micro-generator as S-DOF system (Figure 2-9) and treated the backward coupling effect as viscous electrical damping. The governing equation of micro-generator system is given below.

$$m\ddot{w} + c\dot{w} + kw = -m\ddot{w}_b \quad (2.5)$$

where m is the seismic mass; k is the spring constant; c is the damping coefficient including structural and electric damping; w_b is the base excitation amplitude; w is the displacement of the seismic mass relative to the base excitation; and a dot above a variable represents differentiation respect to time. Total damping c includes electrical damping induced by the generator (c_e) which is treated as viscous damping. The available power (P) for energy harvesting is evaluated by the dissipated viscous electrical damping as

$$\left| \frac{P}{\ddot{w}_b^2} \right| = \left| \frac{c_e \dot{w}^2}{\ddot{w}_b^2} \right| = \frac{m}{\omega_n} \frac{\zeta_e \Omega^2}{[1 - \Omega^2]^2 + [2\zeta\Omega]^2} \quad (2.6)$$

where ζ_e is the electrically induced damping ratio; ζ is the total damping ratio; ω_n is the natural frequency; and $\Omega = \omega/\omega_n$ is the dimensionless frequency. This is a fairly accurate model for analyzing the electromagnetic harvester as the backward coupling effect of electromagnetic energy harvesting in the mechanical domain is usually proportional to the velocity [34].

Figure 2-9 Schematic of an electromagnetic generator (Williams and Yates [1])

2.2.2.2 Coupled S-DOF model

A S-DOF model of PEH working in d_{33} -mode (**Figure 2-10**) with inclusion of backward coupling effect was developed by DuToit et al. [28]. The proposed model was used for studying a MEMS piezoelectric vibration energy harvester of cantilever configuration. The governing equations of the coupled electromechanical model are given below.

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\ddot{w}_b \quad (2.7)$$

$$R_l C^s \dot{v} + v + m_{eff} R_l d_{33} \omega_n^2 \dot{w} = 0 \quad (2.8)$$

where w is the displacement of the proof mass relative to the base; w_b is the base displacement of proof mass; ζ_m is the mechanical damping ratio; ω_n is the natural frequency; v is the voltage across the load resistance R_l ; R_p is the leakage resistance; d_{33} is the piezoelectric constant; C^s is the capacitance of piezoelectric transducer; and m_{eff} is the effective mass. While equation (2.7) describes the dynamics of harvester in the mechanical domain, equation (2.8) represents the electrical domain. The terms with piezoelectric constant (d_{33}) in equations (2.7) and (2.8) establish the coupling between mechanical and electrical domains. For harmonic base excitation, power output (P) from the coupled S-DOF model is given as

$$\left| \frac{P}{\ddot{w}_b^2} \right| = \frac{m_{eff}}{\omega_n} \frac{r k_e^2 \Omega^2}{[1 - (1 + 2\zeta_m r)\Omega^2]^2 + [(1 + k_e^2)r\Omega + 2\zeta_m \Omega - r\Omega^3]^2} \quad (2.9)$$

where k_e is the alternative electromechanical coupling coefficient ($k_e^2 = d_{33}^2 c_{33}^E / \epsilon_{33}^S$); and $r = \omega_n R_l C^s$ is the dimensionless electrical load of the system. Using the coupled S-DOF model, DuToit et al. [28] observed the difference between open and short circuit frequencies, which the uncoupled model couldn't predict.

Figure 2-10 Schematic of S-DOF coupled PEH (DuToit et al. [28])

S-DOF correction factors

Erturk and Inman [32] pointed out that both the uncoupled and coupled S-DOF models (equations (2.5) to (2.9)) underestimates the performance of PEH for low proof mass (M_t) to beam mass (mL) ratios. The error in the prediction of displacement and voltage outputs is due to the inaccurate estimation of contribution of the beam mass to the excitation amplitude. Based on the single mode distributed solution, they proposed a correction factor to the excitation term in the governing equation of mechanical domain. Correction factor approaches unity for high tip to beam mass ratios indicating that tip mass inertia dominates the distributed inertia of beam. The modified mechanical equation for the coupled S-DOF model with correction factor is given as

$$\ddot{w} + 2\zeta_m \omega_n \dot{w} + \omega_n^2 w - \omega_n^2 d_{33} v = -\mu_1 \ddot{w}_b \quad (2.10)$$

where μ_1 is the correction factor for transverse vibrations and is given below

$$\mu_1 = \frac{(M_t/mL)^2 + 0.603 (M_t/mL) + 0.08955}{(M_t/mL)^2 + 0.4637 (M_t/mL) + 0.05718} \quad (2.11)$$

Then, the power output of coupled S-DOF model (2.9) is multiplied by μ_1^2 .

2.2.2.3 Coupled distributed parameter model

The S-DOF model considers only the fundamental mode of cantilever beam. If higher modes are also contributing to the system response, the distributed parameter model is required to analyze the system accurately. Erturk and Inman [34] derived the analytical solution for a cantilever-type PEH connected to a resistive load vibrating under harmonic excitation. The model is based on the Euler-Bernoulli beam theory which is a reasonable assumption as many of the harvester designs use fairly thin cantilever beams. Deformations are assumed to be small so that strains induced in both substrate and piezoelectric materials are within their respective linear elastic limits. The coupled governing equation of the mechanical domain for a cantilever-type PEH (with tip mass) under base excitation is given as

$$\begin{aligned} YI \frac{\partial^4 w_{rel}(x, t)}{\partial x^4} + c_s I \frac{\partial^5 w_{rel}(x, t)}{\partial x^4 \partial t} + c_a \frac{\partial w_{rel}(x, t)}{\partial t} + m \frac{\partial^2 w_{rel}(x, t)}{\partial t^2} \\ + \theta V(t) \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] \\ = -[m - M_t \delta(x-L)] \frac{\partial^2 w_b(x, t)}{\partial t^2} - c_a \frac{\partial w_b(x, t)}{\partial t} \end{aligned} \quad (2.12)$$

where YI is the bending stiffness; $w_b(x, t)$ and $w_{rel}(x, t)$ are the base excitation and the deflection relative to the base motion, respectively; c_s and c_a are the strain rate damping coefficient and viscous air damping coefficient, respectively; L is the length of the beam; m is the mass per unit length; M_t is the tip mass; θ is the electromechanical coupling coefficient; $V(t)$ is the voltage output of the energy harvester; $\delta(x)$ is the Dirac delta function; and x is the longitudinal coordinate. The vibratory motion of the beam can be represented by an absolutely and uniformly convergent series of Eigen functions as

$$w_{rel}(x, t) = \sum_{r=1}^{\infty} \phi_r(x) \eta_r(t) \quad (2.13)$$

Substituting equation (2.13) into (2.12) and applying the orthogonality conditions of the Eigen functions yields the electromechanically coupled ordinary differential equation for the modal response of the harvester and is given below.

$$\begin{aligned} \frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r \frac{d\eta_r(t)}{dt} + \omega_r^2\eta_r(t) + \chi_r V(t) \\ = -\frac{\partial^2 w_b(x,t)}{\partial t^2} \left(m \int_0^L \phi_r(x) dx + M_t \phi_r(L) \right) \end{aligned} \quad (2.14)$$

where ω_r and ζ_r are the natural frequency and damping ratio of the r^{th} mode, respectively; and χ_r is the modal electromechanical coupling coefficient and is defined as

$$\begin{aligned} \chi_r = \int_0^L \theta \left[\frac{d\delta(x)}{dx} - \frac{d\delta(x-L)}{dx} \right] \phi_r(x) dx = \int_0^L \theta \frac{d^2\phi_r(x)}{dx^2} dx \\ = \theta \frac{d\phi_r(x)}{dx} \Big|_{x=L} \end{aligned} \quad (2.15)$$

The governing equation representing the electrical domain is

$$\frac{V(t)}{R_l} + C^s \frac{dV(t)}{dt} - \sum_{r=1}^{\infty} \chi_r \frac{d\eta_r(t)}{dt} = 0 \quad (2.16)$$

where R_l is the load resistance and C^s is the capacitance of the piezoelectric material. Upon expressing the harmonic base excitation in the form of $u_g(t) = Ae^{j\omega t}$ (j is the unit imaginary number and ω is the excitation frequency), the steady state voltage response across the resistive load can be expressed as

$$V(t) = i(t)R_l = \frac{\sum_{r=1}^{\infty} \frac{j\omega\chi_r f_r}{\omega_r^2 - \omega^2 + j2\zeta_r\omega_r\omega}}{\sum_{r=1}^{\infty} \frac{j\omega\chi_r^2}{\omega_r^2 - \omega^2 + j2\zeta_r\omega_r\omega} + j\omega C^s + \frac{1}{R_l}} A\omega^2 e^{j\omega t} \quad (2.17)$$

$$f_r = \int_0^L m\phi_r(x) dx + M_t \phi_r(L) \quad (2.18)$$

where $i(t)$ is the current through the resistive load.

Figure 2-11 shows the comparison between the distributed model predictions and experimental measurements at a load resistance of 995k Ω . The distributed model predicts the tip velocity and voltage output of the harvester with very good accuracy. Moreover, the proposed analytical model estimates the open-circuit and closed circuit resonant frequencies quite close to the experimental values with error below 0.04%. Therefore, this model is regarded as a bench mark for the distributed parameter models developed for the cantilever-type PEHs. However, the

distributed model presented here is applicable only for rectangular beams with uniform material properties. In some research works, the cross-section of the harvester beam has been varied along the length or the piezoelectric material is irregularly distributed for improving the performance of harvester [35, 36]. In such cases, the derivation of the analytical solution is often very difficult. For such situations, Hagood et al. [37] first introduced the Rayleigh-Ritz formulation which is an approximated distributed parameter model to study the piezoelectric transducers used for vibration control. Sodano et al. [38] and Elvin and Elvin [39] also reported similar Rayleigh-Ritz solutions for PEHs. In all of the aforementioned works, a simple resistor was used for representing the external load and analytical expressions for the optimal resistance and power output were derived. Shu and Lien [40] derived solutions for the steady-state response of PEH connected to a standard rectifier AC-DC circuit. It was shown that the harvester power depends on the input vibration characteristics, the mass, the electrical load, the resonant frequency, the damping ratio, and the electromechanical coupling coefficient of the system.

Figure 2-11 Comparison between distributed model predictions and experimental measurements (a) Voltage FRF (b) Tip velocity FRF at $R_l = 995 \text{ k}\Omega$ (Erturk and Inman [41])

2.2.2.4 Modeling of nonlinearities in piezoelectric energy harvesters

The lumped-parameter and distributed parameter models discussed so far assumes linear relationships in the constitutive equations (2.1) to (2.4) under the assumptions of small deformation and weak electric fields. It is reasonable to assume weak electric fields for PEH applications as the induced electric field across the thickness of piezoceramic patches is usually well below 1kV/mm. Moreover, most of the proposed models assumed geometrically linear vibration of harvester beams. However, when the oscillator vibrates at large amplitudes with

high induced strains, the geometric, material (elastic), and piezoelectric (electroelastic) nonlinearities are manifested in the system and the linear models cannot predict the response of harvesters accurately in such scenarios. A prominent feature normally observed in the response of PEHs under the influence of intrinsic nonlinearities is the shift in the resonant frequency at high excitation amplitudes. For instance, the response of an oscillator with cubic elastic nonlinearity is shown in Figure 2-12. The reduction in resonant frequency or softening effect is evident by examining the backbone curve, which traces the peaks of frequency response curves at increasing excitation amplitudes.

Figure 2-12 Response of an oscillator with cubic elastic nonlinearity. Frequency response curves at different excitations are shown in solid blue lines. The backbone curve is shown in dotted red color (Leadenham and Erturk [42])

Maugin [43] carried out early investigations on the relevance and influence of piezoelectric nonlinearities in transducer, resonator, sensor, and signal processing applications. Aurelle et al. [44] studied the power transducers driven at high voltages using a model based on the nonlinear piezoelectric constitutive equations extended up to the second order. Wagner and Hagedorn [45] studied the dependence of resonant frequency on the excitation voltage amplitude in piezo-beam systems excited by weak electric fields. The effect of material nonlinearity on the mechanical response of piezoelectric rods was studied by Birman and Suhir [46] and it was shown that the neglecting nonlinear stress-strain behavior results in significant errors in the estimation of mechanical behavior. Stanton et al. [47] investigated the nonlinear response of bimorph PEHs comprising PZT-5A and PZT-5H laminates. In their modeling, the effects of nonlinear elastic, electroelastic and dissipation were considered and the manifestation of the effect of nonlinearities was observed at base accelerations as low as 0.5g. However, the nonlinear coefficients were obtained by fitting the experimental data to the steady-state mechanical amplitude. Besides, a single nonlinear damping coefficient was evaluated for a

wide range of accelerations (0.05g-2g) from the experiments, which is another limitation as damping varies with the excitation amplitude. Similar approach was used in the other works on the nonlinear modeling of PEHs [48-50]. A global nonlinear distributed-parametric model for a unimorph PEH accounting for geometric, inertial, piezoelectric, and fluid drag nonlinearities was developed by Abdelkefi et al. [51]. Theoretical study was performed to evaluate the effects of various nonlinearities on the harvester's response. The results indicated that the nonlinear stress-strain coefficients significantly impact the softening or hardening behavior. Recently, Leadenham and Erturk [42] proposed a unified nonlinear non-conservative model to investigate the dynamic behavior of a bimorph cantilever for excitation levels normally used in the energy harvesting, sensing, and actuation applications. In this thesis, the geometric, material, and damping nonlinearities in PEH systems are modeled and issues in the existing nonlinear models are discussed comprehensively in Chapter 5.

2.2.2 Finite element modeling

Finite element analysis (FEA) is an advanced tool for solving difficult problems across many fields of physics and also has the capability of solving the coupled-field problems. Analytical solutions are available in the literature for PEHs with simple structures for which parameters like the effective mass, effective stiffness, and electromechanical coupling coefficient can be easily derived. For complex structures, it is difficult and time consuming to derive accurate analytical models and sometimes not even possible to derive. FEA is a robust and reliable modeling method for analyzing such complex designs of energy harvesting systems. Commercial software packages like ANSYS Multiphysics, COMSOL, ABAQUS, ATILA FEA, etc., have advanced capabilities to perform complex piezoelectric analyses.

Zhu et al. [52] presented a finite element coupled piezoelectric-circuit model to analyze the power output of PEH. A numerical model of the harvester was developed using ANSYS to investigate the effect of load resistance on the performance of energy harvester. Using the developed numerical model, a design study was carried out to generate more power by varying the geometric parameters of the beam and tip mass [53]. Abdelkefi et al. [54] compared the results from analytical and finite element models of a cantilever-type PEH with the experimental measurements. The prototype of the harvester and its finite element model representation are shown in Figure 2-13(a) and (b), respectively. The open-circuit and short-circuit frequencies are predicted by FEA with an error of 0.11% and 0.035%, respectively, when compared with the experimental measurements. However, the power output measured

across the resistor is slightly overestimated by the FEA model. Arafa et al. [55] evaluated the performance of a cantilevered piezoelectric beam with the dynamic magnifier using ANSYS. The voltage output predicted by FEA was in good agreement with the experimental measurements at high load resistance but deviated considerably at low load resistance. In all of the above works, a simple resistor is used for representing the electrical load. However, it is quite hard to analyze the energy harvesting system with the practical circuit interface including the nonlinear electric components using FEA modeling. Moreover, FEA is only used for studying the linear harvesters so far. In this thesis, the nonlinear harvesters with magnetic interaction are modeled using FEA approach and the methodology is presented in Chapter 4.

Figure 2-13 (a) Prototype of cantilever-type PEH (b) Finite element representation of harvester
(Abdelkefi et al. [54])

2.2.3 Equivalent circuit modeling

It is very difficult or sometimes not possible to derive the mathematical models for complex mechanical structures. Though FEA can model the complex structures, it lacks the ability to simulate the complex circuitry. Yang and Tang [33] and Elvin and Elvin [39] employed the equivalent circuit modeling (ECM) method to overcome the challenges in studying PEHs using traditional approaches. ECM method uses the analogy between mechanical and electrical domains to analyze PEHs. The proposed model can be used for including any number of the vibration modes and can also simulate any type of circuit topology used for the energy harvesting. For a piezoelectric energy harvesting system, the parameters in the mechanical domain can be related to the electrical domain as listed in Table 2-2. Once the equivalent system parameters are determined, the performance of harvesting system can be evaluated using any commercially available circuit simulator such as SPICE. Multi-mode equivalent circuit model of PEH studied by Yang and Tang [33] is shown in Figure 2-14. Each branch in the circuit network comprises an inductor, a capacitor, a resistor, an ideal voltage source, and

an ideal transformer. The r^{th} circuit branch represents the r^{th} vibration mode of harvester. In this way, the multi-mode response of harvester can be easily evaluated. It is observed that the results from ECM simulation are consistent with the analytical predictions and experimental measurements. Hence, ECM can be used as a generic solution for analyzing the complex structures and circuit interfaces.

Table 2-2 Analogy between electrical and mechanical domains (Yang and Tang [33])

Equivalent circuit parameters at r^{th} mode	Mechanical counterparts
Charge $q_r(t)$	Modal coordinate $\eta_r(t)$
Current $i_r(t)$	Modal velocity $d\eta_r(t)/dt$
Inductance L_r	1
Resistance R_r	$2\zeta_r\omega_r$
Capacitance C_r	$1/\omega_r^2$
Voltage source $V_r(t)$	$-f_r\ddot{u}_g(t)$
Ideal transformer ratio N_r	Electromechanical coupling χ_r

Figure 2-14 Multi-mode equivalent circuit model of PEH (Yang and Tang [33])

2.3 Enhancing performance of energy harvesting systems

Despite of significant reduction in the power consumption of electronic devices, the power generated from the energy harvesting solutions is not sufficient for replacing the batteries in WSNs. Moreover, the conventional linear harvesters suffer from narrow operational bandwidth while the ambient vibrations are scattered over a frequency range. Many research attempts have been reported for performance and efficiency enhancement of harvesters, which can be categorized into four groups: (1) advanced piezoelectric and structural configurations; (2) sophisticated energy harvesting circuits; (3) frequency up-conversion in ultra-low-frequency scenarios; and (4) broadband energy harvesting.

2.3.1 Advanced piezoelectric and structural configurations

The performance of harvesters can be greatly increased by modifying the structural configuration. Anderson and Sexton [56] studied the relationship between the system performance and material/geometrical parameters of a cantilever bimorph with proof mass. They reported that among the proof mass, length, and width of bimorph, changes in the proof mass had the significant effect on the energy harvesting performance while that of the width was minimal. Inclusion of proof mass reduces the natural frequency of harvester and increases the power output. However, large proof mass results in low power density and high stress on the piezoelectric material. Kim et al. [57] investigated the effect of proof mass's rotary inertia on the performance of PEH. It has been reported that a small change in the geometry of proof mass not only affects the resonant frequency but also changes the strain distribution along the length of harvester beam. Shafer et al. [58] optimized the thickness of piezoelectric layers in bimorphs for maximizing the power output, while constraining the system mass and natural frequency of the harvester.

For a conventional cantilever-type PEH, the strain varies along the length of beam with maximum occurring at the clamped end and gradually decreasing towards the free end. Hence, the piezoelectric material in conventional PEH is not optimally utilized for energy harvesting. Some alternate configurations have been proposed for achieving uniform distribution of the strain over the piezoelectric layer to maximize the harvested power. Roundy et al. [59] proposed a trapezoidal cantilever which results in more evenly distributed strain along the beam length compared to a rectangular geometry (Figure 2-15). It was concluded that the trapezoidal cantilever could supply more than twice the energy per unit volume of PZT than the rectangular

beam. Goldschmidtboeing et al. [60] and Dietl et al. [35] have also carried out optimization studies on the beam shape for maximizing the performance of harvester.

Figure 2-15 Different geometries of cantilever beam and corresponding strain profiles

(a) Rectangular (b) Trapezoidal (c) Triangular (Roundy et al. [59])

Very recently, Ayed et al. [61] investigated the effects of the shape variations of cantilever beam on the performance of harvester. Linear and quadratic shape functions for the beam were considered for designing the PEHs operating at low frequencies and to maximize their harvested energy. Though the idea of beam geometry variation to achieve better performance seems promising, the brittle nature of piezoelectric materials poses manufacturing difficulties. Xu et al. [62] proposed a right-angle cantilever configuration in which the moment created by the motion of auxiliary beam and proof mass was responsible for inducing strain in the piezoelectric material bonded to the main beam. Experimental results have shown that power output was doubled when compared with linear PEH. Platt et al. [63] investigated the harvesting performance of PZT wafer stack. They reported that the PZT stack and the monolithic PZT of the same geometry provided equal power when the impedance matching was achieved. However, the matching load for the stack configuration was in $k\Omega$ range, while that of the monolithic PZT was in $G\Omega$ range. Therefore, the stack configuration is more attractive in terms of impedance matching.

Figure 2-16 (a) Standard energy harvesting circuit and (b) associated power form (Nachibvute et al. [64])

2.3.2 Sophisticated energy harvesting circuits

Over the years, many researchers have implemented various circuit topologies to improve the efficiency of the energy harvesting systems. Simple circuits such as pure resistive load and standard rectifier circuit (Figure 2-16) are not efficient for the practical implementation of energy harvesters. The standard energy harvesting interface circuit is passive in nature (no need of control) and therefore it is easier to implement compared to the active interfaces but lacks efficiency. From the waveform representation of power output shown in Figure 2-16(b), it is clear that most of the mechanical energy in the cycle is converted to the electrical energy and it is represented by the positive power. However, in some intervals, the power is negative indicating that the energy flows back from the electrical domain to the mechanical domain. This phenomenon severely affects the energy conversion efficiency of the standard energy harvesting circuit (Liang and Liao [65]).

Impedance adaptation interface

The principle of impedance adaptation is to tune the impedance of electrical load to get high power transfer. The matching load impedance for a simple resistance load can be shown to be equal to the internal capacitive impedance of the harvester using the uncoupled model (i.e., $R_l^{opt} = 1/\omega C^S$). Usually, in the energy harvesting process, the electrical output from the harvester is rectified and then stored in a battery (or a capacitor), rather than directly connecting to a

resistive load and thus dissipating the energy. A resistor is usually used to simulate the battery but it is a variable during battery charging process. Ottman et al. [66, 67] and Lefeuvre et al. [68] proposed to use DC-DC converters to implement the impedance adaptation interface circuits to achieve optimal power flow in battery charging process. A typical configuration of adaptive interface circuit with DC-DC converters is shown in Figure 2-17. Ottman et al. [66] proposed an adaptive circuit using a buck DC-DC converter to maximize the power flow from PEHs. By using this circuit, 400% increase in the power output has been achieved when compared with the standard circuit.

Figure 2-17 Impedance adaptation interface (Ottman et al. [66])

Synchronized Switch Harvesting Interface

Synchronized switch harvesting (SSH) techniques are the latest and most promising approaches employed in the design of nonlinear harvesting circuits to enhance the conversion efficiency of PEHs. The concept of synchronized switch harvesting interface (SSHI) is originated from synchronized switch damping (SSD) and synchronized switch damping on inductor (SSDI) topologies. SSD and SSDI concepts were previously developed to address the problem of vibration damping on the structures [69-71]. The SSH techniques basically utilize an electrical switch (typically a combination of a digital switch and an inductor) that enables nonlinear power processing. The switch is triggered on the both extremes of the displacement, and the inversion of the voltage through an oscillation process takes place in short interval of time. The SSHI technique can be implemented either in parallel or series configuration as shown in Figure 2-18. Typical waveforms for parallel and series SSHI are shown in Figure 2-19. Notice that voltage inversion is not ideal as part of the energy stored in piezoelectric capacitance is lost in switching. The nonlinear approach adopted in the synchronized switching techniques has been demonstrated to enhance the power output substantially in piezoelectric systems. In some instances, the improvement is up to ten times the power output compared to the standard rectifier interface circuit as reported by Nechibvute et al. [64].

Figure 2-18 (a) Parallel SSHI and (b) Series SSHI configurations (Liang and Liao [72])

Figure 2-19 Typical waveforms of (a) Parallel SSHI and (b) Series SSHI configurations (c) Inversion of voltage at the extreme displacement (Liang and Liao [72])

In most of the aforementioned literature on the nonlinear interface circuits, the power consumption for switching was included but the self-powered switching is not investigated. Lallart and Guyomar [73] and Liang and Liao [72] have implemented the self-powered switching interfaces. Liang and Liao [72] studied self-powered SSHI with the emphasis on three aspects: open circuit voltage, switching phase lag, and intermediate voltage among different phases. Unlike the ideal SSHI, they found that the self-powered SSHI only outperformed the standard interface when the excitation was above certain level. Despite of improvement in the efficiency due to implementation of nonlinear harvesting circuits, the overall system level efficiency still remains low because of the ineffective electromechanical energy transfer. Future research should focus on improving the conversion efficiency, power management, and harvester utilization.

2.3.3 Broadband energy harvesting

Conventional linear PEHs are well suited for narrowband excitation near their natural frequency but are less efficient when the input vibration energy is distributed over a wide spectrum. Furthermore, the vibrational energy may change over the time and is predominantly available at low frequencies. These factors motivated the researchers to explore methods for broadening the bandwidth of linear harvesters such as oscillator arrays, multi-modal oscillators, active or passive frequency-tuning methods, and nonlinear techniques.

2.3.3.1 Multimodal techniques

Traditionally, only the fundamental vibration mode of the cantilever-type PEH has been utilized for the energy harvesting. The higher modes of cantilever beam are well separated from the fundamental mode, thus not useful for broadband energy harvesting. Some researchers investigated theoretically, the possibility of using the translational and rotational modes of a rigid body (Jang et al. [74]) or the multiple translation modes of lumped mass systems (Aldraihem and Baz [75]) for energy harvesting. However, the multi-modal energy harvesters can only be implemented practically either by exploiting the multiple bending modes of a single beam or by using a cantilever array configuration.

Figure 2-20 Schematic of (a) L-shaped PEH (Erturk et al. [76]) (b) cantilever PEH with dynamic magnifier (Arafa et al. [55])

Roundy et al. [59] suggested numerous ways in their work for improving the performance of PEHs to meet the power demands of WSNs. The idea of multi-DOF systems comprising multiple beams and proof masses was introduced as one of the methods for broadening the bandwidth. Erturk et al. [76] designed a L-shaped piezoelectric structure (Figure 2-20(a)) with power output of 4.9mW/g^2 and 2.92mW/g^2 at first two resonant frequencies 16.4Hz and 49.3Hz, respectively. The intended application of this structure was in unmanned air vehicles

as a landing gear and the main focus was on powering sensors and global positioning units with harvested power. Arafa et al. [55] presented a 2-DOF cantilever-type PEH (Figure 2-20 (b)) in which a dynamic magnifier was added to increase the strain on piezoelectric elements to harvest more power. Though the power output was magnified, the working frequencies weren't close enough unless a huge magnifier was employed. Adding such huge magnifier lowers the system performance in terms of power density.

Kim et al. [77] proposed a 2-DOF piezoelectric energy harvesting device in which two piezoelectric cantilevers are connected by a rigid mass as shown in Figure 2-21(a). Translational and rotational displacements of the rigid mass were exploited and the corresponding two natural frequencies could be tuned sufficiently close to each other. The 2-DOF device exhibited two peaks around 75V/g and about 280% increase in the bandwidth at a voltage level of 55V/g when compared to a conventional S-DOF device. Wu et al. [78] proposed a compact 2-DOF energy harvester comprising one main cantilever beam and an inner secondary cantilever beam with attached proof masses as shown in Figure 2-21(b). Proof masses of both beams were adjusted to achieve wider bandwidth.

Figure 2-21 Schematics of 2-DOF energy harvester proposed by (a) Kim et al. [77] (b) Wu et al. [78]

Array of harvester beams with multiple resonant frequencies has been employed for broadband energy harvesting. With proper selection of the geometry of beams and tip masses, close resonant peaks can be achieved for broadened operation bandwidth. Sharuz [79] proposed the idea of using band-pass filter which is an ensemble of beams and masses for energy scavenging. Later, Xue et al. [80] investigated numerically the integration of multiple bimorphs with different thicknesses for broadband energy harvesting as shown in Figure 2-22(a). For the case of multiple-bimorphs in series, not only the power output increased but also the bandwidth is widened (Figure 2-22(b)). It was also reported that the bandwidth could be shifted to dominant

frequency range by changing the number of bimorphs connected in parallel. Ferrari et al. [81] used a piezoelectric multi-frequency converter array (MFCA) for powering battery less sensor module and its charging capacity was investigated at base acceleration around 1g. They reported that MFCA has shown overall improvement in the conversion effectiveness with modest increase in the system size.

Figure 2-22 (a) Illustration of energy harvesting using multiple-bimorphs (b) Power output from single bimorph and multiple-bimorph cases (Xue et al. [80])

Figure 2-23 (a) PEH connected to a standard circuit (b) Equivalent circuit model of the system (c) Equivalent impedance model of the system (d) Power output from array of three PEHs under different load resistances (Lien and Shu [82])

Lien and Shu [82] studied the electrical behavior of an array of linear PEHs using the equivalent impedance method. The schematic of PEH with a standard AC/DC interface circuit and its equivalent circuit model are shown in Figure 2-23(a) and (b), respectively. For analyzing the system, an equivalent impedance for piezoelectric capacitance and circuit elements was added as shown in Figure 2-23(c). Three PEHs with standard circuits were connected and the combined system was analyzed using equivalent impedance approach. When the deviation in the mass from harvester to harvester was large, three different resonant peaks were observed in the frequency response along with improvement in the bandwidth as illustrated in Figure 2-23(d). Similar analyses were also carried out on array of harvesters using parallel/series SSHI circuit interfaces.

Figure 2-24 (a) Schematic of branched-beam piezoelectric energy harvesting device (b) Frequency response of voltage output from harvesting device (Zhang and Hu [83])

Litak et al. [84] and Kucab et al. [85] also carried out analytical examination of energy harvesting system comprising two nonlinear magnetopiezoelectric beams for broadband energy harvesting. Though the aforementioned designs using array of harvester beams provide multiple peaks with wide operation bandwidth, their power densities are quite low because of excessive usage of the piezoelectric material. Zhang and Hu [83] proposed a harvesting device where several branches of beams with proof mass at their ends were connected to a main cantilever beam with a patch of piezoelectric material bonded to its root. The schematic of the above branched-beam energy harvesting device is shown in Figure 2-24(a). The device uses resonance of branched beams for generating multiple peaks in the frequency response as illustrated in Figure 2-24(b). It is evident from figure that the performance of branched-beam harvester is better than the conventional PEH. The power density of the device is also better

than the other designs using array of harvester beams as a single patch of piezoelectric material was used for generating multiple peaks. However, the deep valleys between the adjacent peaks in the frequency response are not suitable for efficient broadband energy harvesting.

2.3.3.2 Resonant frequency tuning methods

The linear PEH has a single peak of power at its resonant frequency and narrow half-power bandwidth. But the energy from ambient vibrations is usually distributed over a range of frequencies and thus rendering the linear harvester not suitable for practical implementation. In resonant frequency tuning methods, the resonant frequency of harvester is matched with the excitation frequency to achieve higher power output. Usually frequency tuning methods can be classified into active or passive methods based on the nature of power consumption (continuous or intermittent) for the tuning process. Tuning methods can also be categorized based on the tuning mechanisms as mechanical, magnetic, and piezoelectric methods.

Mechanical tuning methods

In mechanical tuning methods, the stiffness or the mass of the structure was modified to tune the resonant frequency. In most of the works, the stiffness of the cantilever beam of linear PEH was modified by applying axial preload [86-88]. A design of tunable harvester proposed by Eichhorn et al. [86] with the preload applied at the free end of beam is shown in Figure 2-25(a). The tuning is achieved by applying the axial load to the tip of the beam through screw and spring mechanism. By applying compressive force, 22% shift in the resonant frequency was achieved while tensile force resulted in just 4% of shift. Instead of the bending mode, Morris et al. [89] and Youngsman et al. [90] utilized the extensional mode for tuning the resonant frequency. The design proposed by Morris et al. [89] was shown in Figure 2-25(b). It comprises of a seismic mass suspended with two piezoelectric membranes and pre-tension was applied to membranes by a rigid link. The resonant frequency tuning was achieved by adjusting the link length. Shift in the resonant frequency from 80Hz to 235Hz was observed by changing the pre-tension displacement of the rigid link around 1.25mm. Adjusting the center of gravity of the tip mass is another alternate mechanical way to modify the resonance frequency of harvester. Wu et al. [91] proposed a cantilever-type energy harvester with a movable proof mass for resonance tuning. The proof mass in the above design comprises a fixed part and a movable part. The center of gravity of the whole proof mass can be adjusted by driving the movable screw. In their design, the resonant frequency was tuned between 130Hz and 180Hz by moving the gravity center of the tip mass up to 21mm. All the above discussed tuning methods are

passive in nature. However, the power requirement for the tuning process and possibility for the practical implementation in real life scenarios were not investigated.

Figure 2-25 Energy harvesters with mechanical frequency tuning mechanisms (a) Design by Eichhorn et al. [86] (b) Design by Morris et al. [89]

Magnetic Methods

Figure 2-26 (a) Schematic of resonant frequency tunable PEH using magnets (b) Experimental values of power output versus resonant frequency (Challa et al. [92])

The effective stiffness of the harvesting beam can be modified by applying magnetic force and consequently the natural frequency of the harvester. Originally, Challa et al. [92] used the magnetic force to alter the stiffness of the beam for tuning the natural frequency of the device. Figure 2-26(a) shows the schematic of energy harvester in which two magnets were attached to the tip of the cantilever beam, while two more magnets are fixed to the enclosure containing the device. By adjusting the distance between fixed and moving magnets, the resonant frequency was tuned between 22Hz and 32Hz and a power output of 240~280 μ W was achieved

in that frequency range as shown in Figure 2-26(b). Al-Ashtari et al. [93] proposed a similar design in which fixed and movable magnets were positioned end to end at the tip of the beam.

In the active resonance tuning methods presented in the literature, the mechanical strain induced by the magnetic force was used to adjust the natural frequency of the harvester to adaptively match with the excitation frequency. However, a large portion of the harvested power was utilized for the detection of frequency change and for automatic tuning [92, 94]. Recently, Aboulfotoh et al. [95] designed a self-tuning resonator which autonomously adjusts the gap between the two magnets so as to match the harvester frequency to the excitation frequency. In this study, tuning of frequency from 4.7Hz to 9Hz was achieved by varying the gap between magnets from 66 to 16.7mm. However, the beat phenomenon and reduction in the power output were observed with slight positioning error of the magnets. Nonlinearities induced by the strong magnetic interaction can also be used for broadband energy harvesting. The details of broadband energy harvesting using magnetic nonlinearity will be discussed later.

Piezoelectric Methods

Figure 2-27 Experimental setup of tunable piezoelectric energy harvesting system (Wu et al. [96])

The stiffness of piezoelectric material can be varied by applying an electric load. Furthermore, the piezoelectric actuator can be used for modifying the stiffness of the system. Both these methods were used for the frequency tuning of energy harvesters. Wu et al. [96] presented a piezoelectric bimorph generator in which the upper piezoelectric layer was used for tuning purpose by connecting to multiple capacitive loads, while the lower layer was used for charging a supercapacitor. The schematic of the experimental setup is shown in Figure 2-27. The tunable bandwidth of the generator was 3Hz from 91.5Hz to 94.5Hz, which was achieved by switching in different capacitive loads. The real-time tuning system achieved 1.53mW and 1.92mW of peak power when the device was excited under chirp and random vibrations, respectively, in

the frequency range 80Hz-115Hz. The power required for active controlling was in μW level. Peters et al. [97] reported another tunable resonator in which two piezoelectric actuators were used to create moments to stiffen the structure of energy harvester. A tuning range of $\pm 15\%$ around 78Hz was achieved by modifying the voltage applied to the actuators.

2.3.3.3 Frequency up-conversion methods

Figure 2-28 (a) Schematic of two-stage vibration harvesting design (Rastegar et al. [98]) (b) Two-stage rotary generator employing magnetic interfaces (Rastegar and Murray [99])

For some practical cases where ambient vibrations occur at very low frequencies such as those caused by the wave heaving motions or human motions, it is difficult to design energy harvester with low resonant frequency. In such scenarios, frequency of the source vibration needs to be amplified for energy harvesting. Rastegar et al. [98] proposed a concept of two-stage harvesting device as shown in Figure 2-28(a) for scavenging energy from ultra-low frequency ($< 1\text{Hz}$) vibration sources. The mechanical teeth under base vibration moves and impacts on the piezoelectric cantilever beam which vibrates freely at its natural frequency. Thus, the low-frequency base vibration from primary vibration unit (i.e., the mass) is transferred to high-frequency secondary unit (i.e., the cantilever beam). However, this design suffers from losses due to the impact and friction. The above technique was pursued by Rastegar and Murray [99] in the design of two-stage energy harvester (Figure 2-28(b)) for low-speed rotary machinery applications. The proposed design was targeted to harvest energy from windmills and turbo machinery in the tidal flows. The primary system in the design receives energy input from harvesting environment (wind, tidal flow, etc.). The low speed or frequency motion of primary system was transferred to the secondary system which comprises an array of piezoelectric

beams, thus generating energy at higher frequencies. The underlying challenge in this design was the mechanical loss from the impact when transferring motion from primary to secondary system. Wickenheiser and Garcia [100] used magnetic excitation to induce impulses on the harvester beams to avoid contact losses. Pillatsch et al. [101] studied a frequency up-converter device with rotary proof mass for harvesting energy from human body movements. The basic model of the device is shown in Figure 2-29(a). The proposed device comprises an eccentric proof mass which is free to rotate about its axis and it carries a magnet. A piezoelectric beam with a magnet at its tip is fixed to the enclosure (not shown in figure). Under the excitation from human motions, the rotary mass will move and swing its magnet past the tip of harvester beam giving it an initial deflection. Then, the harvester beam vibrates at its natural frequency and generates the voltage output as shown in Figure 2-29(b). The study reported a peak power output of $43\mu\text{W}$ when the harvester was excited around 2Hz.

Figure 2-29 (a) Piezoelectric frequency up-converting energy harvester with rotating proof mass for human body applications (b) Voltage output from piezoelectric beam when actuated at 2Hz and 0.5g acceleration, attached to a 150k Ω load (Pillatsch et al. [101])

2.3.3.4 Nonlinear configurations

It has been pointed out earlier that the nonlinear stiffness can be induced in the energy harvesting systems with strong magnetic interaction. Many researchers have incorporated magnetic interaction to widen the operational bandwidth. Some of the recent advancements in designing the broadband energy harvesters with nonlinear stiffness will be discussed in this section. Energy harvesters can be classified into mono-stable and bi-stable depending on the number of equilibrium states existed in the system after incorporating the magnetic

nonlinearity. Monostable configuration consists of a single stable equilibrium state and can be represented analytically by widely used Duffing's equation,

$$\ddot{x} + c\dot{x} - ax + bx^3 = f(t) \quad (2.19)$$

where x represents the oscillator position; c represents the viscous damping; $f(t)$ is the ambient vibration force. For $a \leq 0$, the equation represents a monostable system. $b > 0$ results in a hardening system response, while $b < 0$ gives a softening response. For $a > 0$, equation (2.19) represents a bistable nonlinear system.

Mono-stable nonlinear harvesters

Stanton et al. [102] proposed a monostable PEH, which consists of a cantilever piezoelectric beam with a magnetic end mass interacting with the field of oppositely poled stationary magnets as shown in Figure 2-30(a). By tuning the nonlinear magnetic interactions around the end mass, both hardening and softening responses were achieved, thus broadening the bandwidth bi-directionally. The frequency response of voltage output from the harvester under harmonic excitation for hardening and softening configurations is shown in Figure 2-30(b) and (c), respectively. The nonlinear behavior is apparent at higher excitation levels, while the frequency response is similar to the linear harvester at low excitation. As shown in the figure, there are two states: stable state (solid line) and unstable state (dotted line). A linearly continuous decreasing or increasing frequency sweep (refer to softening configuration or hardening configuration, respectively) can capture the high energy stable state. Unfortunately, such condition cannot always be guaranteed in practical applications.

Sebald et al. [103] carried out an experimental study on a PEH exhibiting strong nonlinearity. The examined structure is shown in Figure 2-31(a). The harvester comprises a piezoelectric cantilever beam and a magnetic end mass which interacts with two stationary magnets. The open-circuit voltage output from the harvester for frequency up and down sweeps is shown in Figure 2-31(b). It is evident from figure that the nonlinear harvester shows high operational bandwidth when the frequency was swept upwards. It was reported that the nonlinear harvester exhibited 450% increase in the bandwidth when compared with the linear system, while the maximum power output was decreased by 60%. Wu et al. [104] incorporated the magnetic interaction into a linear 2-DOF PEH to further broaden the bandwidth. The schematic of the harvester device is shown in Figure 2-32(a). The fast Fourier transform (FFT) of the voltage output obtained from the linear and nonlinear harvesters under random excitation is shown in

Figure 2-32(b) and (c), respectively. The peaks in the response of linear 2-DOF harvester are well separated. By inclusion of the magnetic nonlinearity and proper selection of proof masses attached to the inner and outer beams, the two peaks were brought close to each other and thus improving the performance of harvester. Su et al. [105] also analyzed a similar configuration of harvester comprising inner and outer cantilever beams but used three patches of the piezoelectric material. An interface circuit was designed to combine output from all patches and the combined performance showed significant improvement over the conventional PEH in terms of bandwidth.

Figure 2-30 (a) PEH with permanent magnet end mass interacting with oppositely poled stationary magnets. Response amplitudes of output voltage for (b) softening and (c) hardening configuration with different excitation levels. Solid and dotted lines represent higher and lower energy states, respectively (Stanton et al. [102])

Figure 2-31 (a) Piezoelectric duffing oscillator (b) Open-circuit voltage output from the duffing oscillator at 1 m/s^2 (Sebald et al. [103])

Figure 2-32 (a) Illustration of nonlinear 2-DOF PEH (all dimensions are in mm). FFT of voltage output from (b) 2-DOF linear harvester (c) 2-DOF nonlinear harvester (Wu et al. [104])

Tang and Yang [106] proposed a nonlinear PEH with magnetic oscillator as shown in Figure 2-33(a). The harvester comprised a piezoelectric cantilever beam with magnetic tip mass and a sub-system of spring-magnetic mass. During the vibration of cantilever beam and sub-system under base excitation, energy transfers through the magnetic interaction. Note that this configuration utilizes energy transfer through the magnetic interaction for broadening the bandwidth rather than the nonlinearity. The prototype of nonlinear harvester is shown in Figure 2-33(b). The magnetic oscillator was realized using an aluminum cantilever beam with a magnetic tip mass. The magnetic oscillator vibrating at its resonance transfers energy to the main piezoelectric beam and thus resulting in a second peak as illustrated in Figure 2-33(c). The study reported a 100% increase in the bandwidth and 41% improvement in the power output when compared with linear harvester at an excitation level of 2.8m/s^2 .

Figure 2-33 (a) Schematic of PEH with magnetic oscillator (b) Prototype of the harvester (c) Power output of harvester during upward and downward frequency sweeps at 1.4m/s^2 and 2.8m/s^2 (Tang and Yang [106])

Bi-stable nonlinear harvesters

Bi-stable oscillators have a unique double-well restoring potential as shown in Figure 2-34(a) and exhibits three distinct dynamic operating regions. A periodically excited oscillator can exhibit various dynamic responses including chaotic oscillation, large-amplitude periodic oscillation, and small-amplitude periodic oscillation depending upon the input excitation amplitude [107]. When the inertial mass oscillates around the stable-equilibrium i.e., region a,

the resulting small-amplitude intrawell displacement-time response is shown in Figure 2-34(b). The bi-stable oscillator can be excited to an extent to exhibit chaotic vibration as illustrated in Figure 2-34(c) when operating between the wells (region b). When sufficiently large excitation is applied, the oscillator operates in region c and exhibits large-amplitude interwell periodic vibrations as shown in Figure 2-34(d). The stochastic forcing can also induce transitions between the stable equilibria of bi-stable system and therefore resulting in large-amplitude oscillations.

Figure 2-34 (a) Double-well restoring potential of bi-stable oscillator (b) Intrawell oscillations (c) Chaotic vibrations (d) Interwell oscillations (Harne and Wang [107])

Erturk et al. [108] presented a nonlinear piezomagnetoelastic energy harvesting device (Figure 2-35(a)). The harvester comprised a clamped ferroelectric beam and two permanent magnets placed near the free end of the cantilever. Depending on the magnet spacing, the beam may have five (with three stable), three (with two stable), or one (stable) equilibrium positions. For the case with three equilibrium positions, the governing lumped-parameter equation of motion has the form of the Duffing equation (2.19). A large amplitude response for the harvester was obtained at an acceleration of 0.5g by giving a hand impulse. As shown in Figure 2-34(b), the bistable harvester has better voltage output over the linear harvester in the frequency range

4.5Hz-8Hz when excited at 0.5g. A similar design was proposed by Zhou et al. [109] with a magnet at the tip of harvester beam and the magnets located near to the free end of cantilever beam were rotated. By changing the orientation of magnets from 0° to 90° , large amplitudes of voltage output was achieved between 4Hz and 22Hz.

Figure 2-35 (a) Schematic of piezomagnetoelastic power generator (b) Open-circuit voltage output over a frequency range showing the broadband advantage of piezomagnetoelastic power generator (Erturk et al. [108])

A physically feasible design of bistable energy harvester termed as snap-through mechanism was proposed by Ramlan et al. [110]. The design consisted of two linear inclined springs connected to a mass and a damper, yielding a non-linear restoring force. This mechanism has the effect of steepening the displacement response of the mass as a function of time, resulting in a higher velocity for a given input excitation. Numerical results have shown that the proposed snap-through mechanism could provide much better performance than the linear mechanism when the excitation frequency was much smaller compared to the natural frequency. Sneller et al. [111] investigated the behavior of buckled piezoelectric beam with a center mass under harmonic excitation. The schematic and prototype of the harvester are shown in Figure 2-36(a) and (b), respectively. The buckled structure exhibited the snap-through behavior where it rapidly deforms from one stable configuration to other resulting in large-amplitude oscillations. As illustrated in Figure 2-36(c), addition of a small mass to the center of the beam significantly increased the bandwidth of the harvester and the power output. Zhu and Zu [111] employed magnetic force on a buckled beam to enable snap-through motions under low frequency ($<30\text{Hz}$) in the acceleration range 0.2g-0.8g and reported significant improvement in the voltage output and bandwidth. Arrieta et al. [112] exploited the nonlinear behavior of a bistable composite plate with bonded piezoelectric patches for broadband energy harvesting. Other

efforts on bi-stable nonlinear harvesters can be found in the comprehensive review articles by Tang et al. [113], Pellegrini et al. [114], and Harne and Wang [107].

Figure 2-36 (a) Schematic of buckled piezoelectric beam harvester with center mass (b) Prototype of the harvester (c) Voltage response from harvester during forward frequency sweep using different center masses at an excitation of 2.25g (Sneller et al. [111])

2.3.4 Multi-directional energy harvesting

Most of the research work in energy harvesting usually assumes input excitation in a single direction i.e., perpendicular to the cantilever beam. However, this may not be always true in practical scenarios where the ambient vibration may include components from multiple directions as concluded in a survey by Reilly et al. [115]. For instance, Statasys 3-D printer produces vibration in three perpendicular directions with three resonant peaks below 45Hz. Washing machine and HVAS roof exhibit two dimensional vibration characteristics. Hence, it is important to design harvesters which can harness power from multi-directional input excitations.

Andò et al. [116] reported a two-directional bi-stable piezoelectric harvester as shown in Figure 2-37(a). The device consists of two conventional cantilever harvesting beams in perpendicular directions coupled with the magnetic interaction. Fan et al. [117] also proposed a similar design of two-dimensional nonlinear harvester and the schematic of the design is shown in Figure 2-37(b). Both designs are capable of harvesting energy when the direction of ambient vibration

is not aligned with the beam axes. The magnetic coupling improves the efficiency of harvesting and also transmits vibrations induced in one beam to the other. However, the observed improvement in the performance due to the magnetic coupling between the orthogonal beams was not significant.

Figure 2-37 Schematics of bi-directional nonlinear PEHs (a) Design by Andò et al. [116](b) Design by Fan et al. [117]

Figure 2-38 (a) Schematic of 2-D PEH (b) Voltage output from harvester working in three different orientations at an acceleration of 2m/s^2 (Yang et al. [118])

Yang et al. [118] proposed a frame configuration as shown in Figure 2-38(a) to achieve 2-directional vibration energy harvesting. The frame configuration utilizes its vertical and horizontal vibration modes for harvesting power. A central mass was attached to the harvester and 8 patches of MFC were bonded to the frame. The structural parameters of harvester were tuned to capture the vibration energy from arbitrary directions in 2-D plane. The voltage output from each MFC when the harvester was working in 0° , 45° , and 90° orientations at an acceleration of 2m/s^2 is shown in Figure 2-38(b). The response from harvester shows a single peak when working in 0° and 90° orientations, but shows two peaks at 45° which emphasizes the obvious advantage of the proposed harvester under multi-directional vibrations.

Figure 2-39 (a) Schematic of piezoelectric cantilever-pendulum harvester (b) Prototype of harvester (c) Voltage output from harvester when excited in 3 directions at an acceleration of $0.008g$. Red asterisk: forward frequency sweep; blue dot: backward frequency sweep (Xu and Tang [119])

Xu and Tang [119] investigated a piezoelectric cantilever-pendulum design for multi-directional energy harvesting. The schematic and prototype of the harvester are shown in Figure 2-39(a) and (b), respectively. A pendulum is attached to the tip of cantilever-type PEH. The nonlinear coupling between pendulum motion in 3-D space and beam vibration at resonance was effectively utilized for energy harvesting. It was shown experimentally that the internal resonance can be induced with the proper selection of system parameters and thus enabling multi-directional energy harvesting. The voltage output from harvester when excited

in three different directions at an excitation level of 0.008g is shown in Figure 2-39(c) and the results clearly demonstrate the capability of harvester to harness energy from multi-directional excitations.

2.3.5 Hybrid energy harvesters

So far, various efforts for improving the performance of stand-alone PEHs have been discussed. However, some researchers proposed hybrid energy harvesters for harnessing more power from ambient vibrations. The voltage output from PEHs can be up to several tens of volts, but their current output is only few microamperes. On the other hand, the electromagnetic energy harvesters generate up to hundreds of microamperes, but their voltage output is too small to meet the needs of electronic devices. Therefore, the piezoelectric and electromagnetic harvesting mechanisms can be combined to make use of advantages of both mechanisms simultaneously.

Figure 2-40 (a) Schematic of coupled piezoelectric-electromagnetic harvester (b) Prototype of harvester (c) Power output comparison between hybrid harvester and stand-alone harvesters (Challa et al. [120])

All the proposed hybrid energy harvesters comprise a primary piezoelectric energy harvesting device with a permanent magnet attached to it which oscillates and interacts with a stationary coil to generate additional power through the electromagnetic transduction mechanism. Challa et al. [120] proposed a coupled piezoelectric-electromagnetic harvester and matched the total electrical damping to the mechanical damping in the system to harvest more power. The

schematic and prototype of the proposed harvester are shown in Figure 2-40(a) and (b), respectively. The power output from coupled hybrid harvester and stand-alone systems is compared in Figure 2-40(c). It is evident from the results that the coupled hybrid harvester performs better than the stand-alone harvesters. The coupled hybrid harvester generates a peak power output of $332\mu\text{W}$, compared to $257\mu\text{W}$ and $244\mu\text{W}$ obtained from the optimized, stand-alone piezoelectric and electromagnetic harvesters, respectively, showing a 30% increase in the power output.

Tadesse et al. [121] studied a multi-modal harvesting device that combines electromagnetic and piezoelectric energy harvesting mechanisms. The schematic of the hybrid harvester is shown in Figure 2-41(a). Six piezoelectric patches were bonded to a cantilever beam and the voltage output from top and bottom row of piezoelectric patches was combined using parallel connection. The electromagnetic harvester has low resonant frequency ($\sim 20\text{Hz}$) while the piezoelectric harvester shows higher resonant frequencies ($\sim 300\text{Hz}$) which allows the device to harvest power over wide range of frequencies as illustrated in Figure 2-41(b). However, the power output was reported at rather impractical acceleration magnitudes of $35g$. Additionally, the matching loads for electromagnetic and piezoelectric systems were separated by a huge margin requiring separate energy extraction interfaces.

Figure 2-41 (a) Schematic of multimodal energy harvester (b) Power output over a frequency range from piezoelectric and electromagnetic systems (Tadesse et al. [121])

Wu et al. [122] proposed a design of hybrid electromagnetic and piezoelectric MEMS energy scavenger as shown in Figure 2-42(a). The design consists of a proof mass in the center and it was attached to static frame by piezoelectric serpentine springs. Proof mass also carries permanent magnets and a static copper coil was placed in the middle of proof mass. When the

proof mass vibrates under excitation, AC voltage will be generated in both copper coil and piezoelectric springs. A power density of $1.18\text{mW}/\text{cm}^3$ was reported when the design was optimized for harvesting energy from car vibrations. Another MEMS scale hybrid energy harvester was proposed and studied by Yang et al. [123]. The schematic of the harvester is shown in Figure 2-42(b). The piezoelectric cantilever carries magnets which interact with the coils on a PCB substrate. The design of harvester was optimized for the poling direction of magnets, number of magnets, and the relative position of magnets and coils. Despite of extensive optimization, the power output and power density of the electromagnetic harvester were much less compared to piezoelectric harvester.

Figure 2-42 Hybrid electromagnetic and piezoelectric MEMS energy scavenger (a) Design by Wu et al. [122] (b) Design by Yang et al. [123]

2.4 Summary

This chapter reviews the state-of-the-art technologies in the piezoelectric energy harvesting. Firstly, the overview of piezoelectric energy harvesting is briefly given. The piezoelectric materials and their constitutive relations are presented later. The modeling methods to evaluate the performance of PEHs are reviewed, including analytical models, FEA, and ECM approaches. However, these modeling approaches have studied only linear harvesters with simple geometry. It is difficult to derive exact analytical solutions for complex electro-mechanical systems. But, FEA and ECM methods can be extended to analyze nonlinear piezoelectric harvesters. Moreover, very few works have studied the influence of material, damping, and geometric nonlinearities on the performance of harvesters and these few works used data fitting techniques to evaluate the unknown nonlinear coefficients which can be erroneous. Accurate nonlinear analytical models need to be derived using the piezoelectric constitutive relationships obtained from tensile testing.

Subsequently in this chapter, the techniques used for enhancing the performance of harvesters, including advanced piezoelectric and structural configurations, sophisticated energy harvesting circuit interfaces, frequency up-conversion techniques and broadband energy harvesting techniques are thoroughly discussed. Various designs of multi-directional and hybrid harvesters are also presented. Despite of numerous efforts, the enhancement of operational bandwidth of harvesters is not significant. Some studies have used high base accelerations and harmonic excitations to achieve wideband energy harvesting. However, such high base accelerations are unrealistic from ambient sources and the excitations are usually random in nature. More efforts need to be focused towards designing harvesters that can achieve wideband operation with low amplitude random vibrations.

Structural integrity and durability of PEHs are important criteria that have been overlooked in the past studies. The material strength of various harvester materials is not taken into consideration in designing the harvesters. Various geometric parameters of harvesters are optimized through parametric studies to achieve higher power and wider bandwidth while ignoring the strength of materials. Substrate and piezoelectric materials of harvesters undergo cyclic electromechanical loading which results in fatigue failure. Additionally, the performance of materials may deteriorate over time due to cyclic electromechanical loading. Neither past studies nor supplier data sheets provide information about the reliability of performance of harvester materials. PEHs are expected to generate power for very long periods of time and hence the above issues should be addressed.

Most of the previous works aim at improving mechanical or electrical domains of PEHs separately. Further research needs to focus on system level integration of PEH, harvesting circuit and WSN. Despite of extensive research on improving the performance of harvesters, the proposed designs, efficiency enhancement schemes and harvesting circuits still have limitations and pose challenges for the practical implementation of the technology. More efforts are required before PEHs can be widely deployed in practical applications for powering WSNs and portable electronic devices.

CHAPTER 3 MATERIAL STRENGTH CONSIDERATION IN THE DESIGN OPTIMIZATION OF NONLINEAR PIEZOELECTRIC ENERGY HARVESTER

3.1 Introduction

The practical constraints on the design of the energy harvester are the system mass, volume, and strength of the materials. Though the system mass and volume have been considered in the design optimization of harvesters, the strength of materials has not received academic focus. Structural integrity and durability over long duration are important criteria in designing the energy harvesters for real life applications. The cyclic electromechanical loading might cause abrupt failure of the harvesters because of the brittle nature of the piezoelectric materials. In all of the efforts discussed in section 2.3.3 on the design and development of nonlinear energy harvesters for broadband energy harvesting, the material strength has not been considered in the design process. Erturk et al. [108] and Zhou et al. [109] used very high magnitudes of base acceleration in their works which can cause premature failure of the harvester. On the contrary, an unchecked design may result in a scenario where the maximum stress or strain is well below the allowable limit leading to inefficient utilization of the material. Wickenheiser [124] attempted to design an energy harvester with fundamental frequency of 1.95Hz using vibrations from human walking. He arrived at a beam length of 570mm to meet the maximum strain requirements of $< 0.1\%$ and reported that designing a compact resonant harvester from the vibrations of human walking is not possible. Shafer and Garcia [125] derived an expression for the maximum tolerable input acceleration that a linear energy harvester can sustain based on the ultimate strength of piezoelectric material and used this expression to obtain the maximum harvested power corresponding to that acceleration. However, a more pragmatic approach would be optimizing the design of harvester to generate maximum power from the available input vibration until the stress or strain in the material reaches its acceptable limit.

Strength estimates of commonly used piezoelectric and substrate materials are given in Table 3-1. Fatigue strength values given in the table are the peak values of the stress allowed on the material when subjected to alternate fields or stresses. It is clearly evident from the table that the strength of piezoelectric materials rapidly decreases in case of the cyclic loading and the values are much lower compared to the strength of substrate materials such as steel, aluminum,

brass, etc. This fact accentuates the importance of the strength of piezoelectric material in designing the energy harvester. Many researchers investigated the fatigue behavior of piezoelectric materials. The failure of piezoelectric material depends on its chemical composition, degree of brittleness (hard or soft), poling direction, and also on the material used for the electrodes [126]. Fett et al. [127] investigated the behavior of soft PZT material under tensile, compression, and 4-point bending tests. Narita et al. [128] studied experimentally the influence of electric fields, poling direction, and electrode material on the cyclic fatigue behavior of PZT material. Anton et al. [129] carried out 3-point bending tests on monolithic piezoceramics (PZT-5A and PZT-5H), single crystal piezoelectric PMN-PZT, and commercially packaged QuickPack devices and reported the estimates of their strengths. The results of their study can be used as the basis for the design of PEHs.

Table 3-1 Material strength of commonly used piezoelectric and substrate materials

Material	Manufacturer	Product specification	Material strength	
MFC patch	Smart materials	M2807-P2	4500μϵ [fr]	
PZT wafer	MIDE Vulture	V22BL	800μϵ [b]	
PZT Bimorph	Morgan Advanced Materials	PZT 5A series	75.8MPa [s]	27.6MPa [f]
PZT ceramics	Piezo Systems, Inc.	PSI-5A4E	140.4MPa [b] [129]	
		PSI-5H4E	114.8MPa [b] [129]	
PZT composite	MIDE QuickPack	QP10N	206.4MPa [b] [129]	
PZT patch	Channel Industries, Inc.	PZT 5804 Navy III	82.7MPa [s]	48.3MPa [f]
PMN-PZT	Ceracomp Co., Ltd.	CPSC 160-95	44.9MPa [b] [129]	
Substrate materials				
Aluminum	--	AL 6061 T6	276MPa [y]	96.5MPa [f]
Brass	--	230 BRASS H08	435MPa [y]	
Steel	--	AISI 1050 rolled steel	415MPa [y]	

Note: [s]=static; [f]=fatigue; [b]=bending; [y]=yield; [f]=fracture

This chapter presents the optimization of nonlinear PEH with magnetic oscillator to achieve higher power and wide bandwidth while constraining the maximum strain induced in the piezoelectric material (MFC). First, an electromechanical model of the linear PEH is derived using Rayleigh-Ritz formulation. This model is extended for the nonlinear PEH with magnetic oscillator. In this nonlinear model, a better formulation for estimating the magnetic force is used in place of the commonly used dipole-dipole formulation. Furthermore, the maximum strain on the cantilever beam is derived assuming that the induced strains in the materials are within their linear elastic limits. A parametric study is conducted to obtain the optimal configuration of energy harvester by varying the beam length and the distance between magnets. Finally, experimental results are presented to validate the predictions from theoretical models.

3.2 Theoretical modeling of piezoelectric energy harvesters

3.2.1 Modeling of linear PEH

Figure 3-1 Schematic representation of linear cantilever-type PEH

A typical PEH is designed as a cantilever beam with a patch of piezoelectric material bonded at the fixed end and a proof mass is attached at the free end as shown in Figure 3-1. In this study, aluminum (A7075-T6) is used as substrate material and a patch of MFC (Smart Materials Corp., model: M2807-P2) is bonded to the beam at its root. The portion of the aluminum beam with the patch of MFC will be referred to as composite section throughout this thesis. The energy harvesting circuit comprising a simple resistive load R_L is considered in this study as the main focus is on the structural aspects of energy harvester. The governing electromechanical equations for the PEH subjected to base motion $u_0(t)$ are derived considering the single-mode approximation and using the Hamiltonian principle following the procedure described by Hagood et al. [37], Sodano et al. [38], and Elvin and Elvin [39].

$$M \ddot{u}(t) + C \dot{u}(t) + K u(t) + \Theta V(t) = F(t) \quad (3-1)$$

$$\frac{V(t)}{R_L} + C^s \dot{V}(t) - \Theta \dot{u}(t) = 0 \quad (3-2)$$

In equations (3-1) and (3-2), $V(t)$ is the voltage across the resistor R_L ; $u(t)$ is the modal displacement in the single mode approximation, where displacement $w(x,t)=u(t)\phi(x)$; M , K , C , and Θ are the modal mass, stiffness, damping, and electromechanical coupling terms, respectively; C^s is the clamped capacitance of the piezoelectric patch; and $F(t)$ is the mechanical forcing due to the base acceleration. The damping coefficient C is usually measured experimentally. The definitions of other terms are given below. Throughout this chapter subscripts s , p , and c corresponds to substrate, piezoelectric material, and composite sections, respectively.

$$M = \int_0^L m_*(x) \phi(x)^2 dx + M_t \phi(L)^2 \quad (3-3)$$

$$K = \int_0^L EI_*(x) \phi''(x)^2 dx \quad (3-4)$$

$$\Theta = - \int_{v_p} y \phi''(x) e_{31} \psi(y) dv_p \quad (3-5)$$

$$F(t) = - \left(M_t \phi(L) + \int_0^L m_*(x) \phi(x) dx \right) \ddot{u}_0(t) \quad (3-6)$$

Here, m and EI are the mass per unit length and the flexural rigidity of beam, respectively. The '*' corresponds to the specific portion of beam, either composite section or substrate. M_t is the tip mass. e_{31} is the piezoelectric stress constant. The spatial variable x is measured from the root and y is measured from the neutral axis of beam, as shown in Figure 3-1. L is the length of beam. The function $\psi(y)$ defines the electric field over the piezoelectric element which is assumed to be constant [38]. The integral in the equation (3-5) is evaluated over the volume of the piezoelectric material (v_p) in the composite section. $\phi(x)$ is the mode shape normalized to unity corresponding to the first natural frequency of beam and expressed in two parts as the cantilever beam is covered partially by the piezoelectric material.

$$\phi(x) = \begin{cases} \phi_c(x) & \text{for } 0 \leq x \leq L_p \\ \phi_s(x) & \text{for } L_p \leq x \leq L \end{cases} \quad (3-7)$$

Differential eigenvalue problem for this configuration is

$$\begin{cases} (EI)_c \phi_c^{iv} - m_c \omega^2 \phi_c = 0 \\ (EI)_s \phi_s^{iv} - m_s \omega^2 \phi_s = 0 \end{cases} \quad (3-8)$$

The solution form for the mode shape is

$$\begin{cases} \phi_c(x) = A_1 \cos(\beta_1 x) + B_1 \sin(\beta_1 x) + C_1 \cosh(\beta_1 x) + D_1 \sinh(\beta_1 x) \\ \phi_s(x) = A_2 \cos(\beta_2 x) + B_2 \sin(\beta_2 x) + C_2 \cosh(\beta_2 x) + D_2 \sinh(\beta_2 x) \end{cases} \quad (3-9)$$

with the following geometric and natural boundary conditions to determine the values of β_1 and β_2 which give non-trivial $\phi(x)$:

$$\begin{cases} \phi_c(0) = 0, \phi'_c(0) = 0, \phi_c(L_p) = \phi_s(L_p), \phi'_c(L_p) = \phi'_s(L_p), \\ (EI)_c \phi''_c(L_p) = (EI)_s \phi''_s(L_p), (EI)_c \phi'''_c(L_p) = (EI)_s \phi'''_s(L_p), \\ (EI)_s \phi''_s(L) = I_t \omega^2 \phi'_s(L), (EI)_s \phi'''_s(L) + \omega^2 M_t \phi_s(L) = 0 \end{cases} \quad (3-10)$$

where I_t is the rotational inertia of the tip mass M_t . The equations (3-9) and (3-10) are nonlinear for which the analytical solutions cannot be derived. These equations are numerically solved and plots of mode shapes for different beam configurations are added in Appendix A.1.

3.2.2 Modeling of nonlinear PEH

Tang and Yang [106] proposed a nonlinear energy harvester in which the conventional PEH was aided by a magnetic oscillator to achieve both wider bandwidth and higher power output. As shown in Figure 3-2, a subsystem of magnetic mass-spring-damper was introduced in the enclosure carrying a PEH with a magnet at its tip. The dynamics of PEH is altered by the presence of the magnetic oscillator, resulting in improvement of bandwidth and power. For theoretical study, the magnetic oscillator is assumed as an additional cantilever with a tip mass carrying magnet. The distance d between the two embedded magnets strongly influences the nonlinear characteristics of PEH. The dynamic equations of this nonlinear harvester are given below:

$$M_1 \ddot{u}_1(t) + C_1 \dot{u}_1(t) + K_1 u_1(t) + \theta V(t) - F_{mag} = F_1(t) \quad (3-11)$$

$$M_2 \ddot{u}_2(t) + C_2 \dot{u}_2(t) + K_2 u_2(t) + F_{mag} = F_2(t) \quad (3-12)$$

$$\frac{V(t)}{R_L} + C^s \dot{V}(t) - \theta \dot{u}_1(t) = 0 \quad (3-13)$$

where subscripts 1 and 2 refer to the parameters of PEH and magnetic oscillator, respectively. The effective modal terms for each beam are determined following the procedure explained in the previous section. F_{mag} is the magnetic force which varies with the relative displacement between PEH and oscillator.

Figure 3-2 Schematic representation of nonlinear PEH with magnetic oscillator

3.2.3 Magnetic force formulation

For analyzing nonlinear energy harvesters with magnetic interaction, the dipole-dipole approximation has been used extensively in the literature [92, 102, 106]. The force between the magnetic dipoles is given by:

$$F_{dipole} = \frac{3 \mu_0 m_1 m_2}{2\pi d^4} \quad (3-14)$$

where m_1 and m_2 are the moments of magnetic dipoles; μ_0 is the vacuum permeability. For two cylindrical magnets with radius R and height h , with their magnetic dipole aligned and the distance between them greater than a certain limit, the force can be approximated by [130]

$$F(d) = \frac{\pi \mu_0}{4} M^2 R^4 \left[\frac{1}{d^2} + \frac{1}{(d + 2h)^2} - \frac{2}{(d + h)^2} \right] \quad (3-15)$$

where M is the magnetization of the magnets and d is the distance between them. The dipole model is accurate only when the distance between magnets is sufficiently large. The approximate force expression given in equation (3-15) is accurate when the distance between magnets is 2-3 times more than the height of magnet. As shown in the later part of this section, equations (3-14) and (3-15) predict quite erroneous results within the distances normally used in the energy harvester designs. Some researchers have carried out experiments to establish accurate relationship between the magnetic force and the separation distance [95, 109, 131]. The experimental results were curve fitted and then used in the governing equations to study the behavior of the harvester. Though this method evaluates the magnetic interaction accurately, conducting experiments is time consuming. Experiments have to be repeated if magnets of different dimensions and strength are used. Thus, it is highly desired to have a good formulation to evaluate the magnetic force. In this section, a closed-form expression available in the literature for computing the force between cylindrical magnets is presented and compared with the extensively used dipole model. Additionally, other available solutions for evaluating

the interaction between permanent magnets are discussed in the view of growing interest in the usage of magnetic interaction for broadband energy harvesting.

There are various techniques to analyze permanent magnets such as representing magnets by equivalent magnetic charges or currents, application of the Maxwell's stress tensors, the principle of virtual work, and the use of Kelvin's formula [132]. The closed-form expression of the interaction force between two identical and axially aligned magnetic disks was developed by Furlani [133] to calculate the levitation force for the radial and axial bearing configurations. In his study, EP Furlani modeled the magnets as volumes of distributed magnetic charges and the force between the two charge distributions was computed [133]. In this model, the magnets are assumed to be of fixed and uniform polarization. Integral expressions in the model to solve for the magnetic field and force are evaluated by discretizing the magnets into mesh of elements and the contribution from each element is summed up. Due to the geometric symmetry of magnets, the equations for magnetic force and field are derived in cylindrical coordinates. The force between two identical and axially aligned magnetic disks is given as:

$$F_{mag} = \frac{\mu_0}{4\pi} J^2 \left\{ \frac{\pi(R_2^2 - R_1^2)}{N_r N_\theta} \right\}^2 N_\theta \sum_{q=0}^2 \sum_{i=1}^{N_r} \sum_{i'=1}^{N_r} \sum_{j'=1}^{N_\theta} \frac{(-1)^q \alpha_q d_q}{[r_i^2 + r_{i'}^2 - 2r_i r_{i'} \cos(\theta_{j'}) + h_q^2]^{3/2}} \quad (3-16)$$

where J is the saturation magnetization; R_1 and R_2 are the inner and outer radius of magnet, respectively; N_r and N_θ are the number of elements in the radial and circumferential directions, respectively. Other parameters are defined as follows:

$$\theta_{j'} = (j' - 1)\Delta\theta + \frac{\Delta\theta}{2} \quad (j' = 1, 2, \dots, N_\theta),$$

$$\Delta\theta = \frac{2\pi}{N_\theta},$$

$$r_i = \frac{R(i+1) + R(i)}{2} \quad (i = 1, 2, \dots, N_r),$$

$$r_{i'} = \frac{R(i'+1) + R(i')}{2} \quad (i' = 1, 2, \dots, N_r),$$

$$R(i+1) = \sqrt{R(i)^2 + \frac{R_2^2 - R_1^2}{N_r}} \quad (i = 1, 2, \dots, N_r),$$

$$R(i' + 1) = \sqrt{R(i')^2 + \frac{R_2^2 - R_1^2}{N_r}} \quad (i' = 1, 2, \dots, N_r),$$

$$d_q = d + q t_q \quad (q=0, 1, 2),$$

$$\alpha_q = \begin{cases} 1 & q = 0, \\ 2 & q = 1, \\ 1 & q = 2. \end{cases}$$

where t_q is the thickness of magnet. Since the above solution is obtained through discretization, its accuracy depends on the number of elements used. Figure 3-3 shows the convergence in the prediction of force between magnets as the number of elements used for the meshing increases. It is found out that with mesh resolution of $N_r=25$ and $N_\theta=25$, the discretization error in the evaluation of magnetic force is of the order 0.001N. Higher mesh resolution unnecessarily increases the computational time as the magnetic force is repeatedly calculated in solving the governing equations numerically.

Figure 3-3 Influence of mesh size on the prediction of magnetic force

The magnetic force calculated using various models is compared with the predictions from finite element analysis (FEA). The FEA is carried out with the magnetostatic module in COMSOL Multiphysics using 3D models of magnets. Cylindrical Neodymium (NdFeB)

magnets of radius 2mm, thickness 3mm and magnetization 828KA/m are used for the simulation. The effective magnetic moment computed by assuming magnets as dipoles is 0.0192Am^2 . Force values computed using equations (3-14) to (3-16) and FEA for a series of separation distances are compared in Figure 3-4. From Figure 3-4, it is very clear that the closed-form solution given by the charge model matches well with the FEA results whereas the dipole model predicts much higher values when the separation distance becomes smaller. The force values predicted by dipole model start deviating from the results of FEA considerably when the spacing between magnets is around 7mm. At 5mm of separation distance between magnets, the error from the dipole model is almost 200%. Similarly, Vokoun's approximation (equation (3-15)) deviates from FEA results below 6mm. Note that, the separation distance d shown in Figure 3-2 is the distance between magnets at rest. During the motion of beams under base acceleration, tip masses along with embedded magnets oscillate and interact at even closer distances. Higher base acceleration in the experiments would result in closer spacing of magnets around 5mm or less. The dipole model and Vokoun's approximate expression in such case would predict erroneous force values. Hence it is recommended that the dipole model and Vokoun approximate expression should not be used when the distance between magnets is close during the motion (less than 7mm) so as to avoid inaccurate prediction of the response of energy harvester. Moreover, the force expression (3-16) derived in terms of material properties, dimensions, and spacing of magnets is convenient to use and facilitates rapid parametric studies.

Figure 3-4 Force between cylindrical magnets versus separation distance

Cylindrical and cuboidal magnets in various configurations have been used for broadband energy harvesting. There are other analytical solutions in the literature for computing the force between permanent magnets. Agashe and Arnold [132] derived analytical expressions for the axial and lateral forces between the permanent magnets as functions of separation distance and dimensions of magnets for cylindrical and cuboidal shapes. Ravaut et al. [134] and Robertson et al. [135] presented simplified force expressions for cylindrical magnets. Akoun and Yonnet [136] derived an expression for the force exerted between two cuboidal magnets.

Table 3-2 Parameters of nonlinear PEH with magnetic oscillator

Parameter	Piezoelectric	Substrate
Material	MFC 2807-P2	Aluminum
Length (L_p, L) (mm)	28	70
Width (B_p, B_s) (mm)	7	10
Thickness (T_p, T_s) (mm)	0.2	0.6
Elastic modulus (E_p, E_s) (GPa)	30.336	68.9
Mass density (ρ_p, ρ_s) (kg/m ³)	5440	2700
Piezoelectric constant (e_{31}) (C/m ²)	-5.16	--
Capacitance (C^S) (nF)	12.4	--

3.3 Performance evaluation of linear and nonlinear harvesters

3.3.1 Power output from harvesters

The closed-form expression for the magnetic force presented in the last section is incorporated in the equations (3-11) to (3-13) to study the dynamics of nonlinear energy harvester. The governing differential equations are written in the state-space form (Appendix A.2) and solved by numerical integration in MATLAB. The geometric and material properties used for simulation are given in Table 3-2. Attractive configuration of magnets is considered for studying the behavior of the harvester. It is assumed that the rotation of tip masses is small and both the magnets are axially aligned at all times. The tip masses weighing 4.6g and 6g are used to achieve natural frequencies of 27.8Hz and 24.4Hz for the PEH and magnetic oscillator, respectively. While deriving the modal terms, the tip mass is assumed as point mass with $I_t=0$. The simulation is carried out at a base acceleration a_{rms} of 2m/s² and load resistance of 400k Ω .

The nonlinear behavior of harvester is studied when magnets are separated by 8mm and 10mm. The separation distance (8mm or 10mm) refers to the distance between magnets when the beams are stationary. Smaller distances ($< 8\text{mm}$) are not considered because of the snapping action of magnets. Usually, the magnets snap each other (sudden movement) at a certain critical distance. In this design, the attraction force between magnets is resisted by the deflection of beams ($K*\delta$). Here, K is the stiffness of beam and δ is the tip deflection. During the vibration of beams under base excitation, the tip masses with embedded magnets operate close to each other. When the force between magnets is more than the resistance from beams, the magnets snap each other. This makes the energy harvester useless and a large force is required to separate the magnets. Therefore, it is very crucial to design the harvesters such that the relative spacing between magnets during vibration is sufficiently large enough to avoid the snapping action of magnets.

Figure 3-5 Power output from the linear and the nonlinear energy harvesters at $a_{rms}=2\text{m/s}^2$ from simulation

Figure 3-5 shows the power output from the linear and nonlinear harvesters. Power delivered to the electrical load is given by V_{rms}^2/R_L , where V_{rms} is the RMS voltage output obtained from solving the governing equations (3-11) to (3-13). It is conspicuous that the PEH with magnetic oscillator provides wider bandwidth and higher output power when compared to the linear PEH. The main features of nonlinear harvester are the increased power output at the resonant frequency of PEH and the improved bandwidth because of the second power peak near the resonance frequency of magnetic oscillator. The maximum power output from the nonlinear PEH is $1579\mu\text{W}$, showing 46% increase when compared to the linear PEH ($1080\mu\text{W}$). The bandwidth corresponding to $100\mu\text{W}$ power output has also shown 30% increase from the linear

PEH (3Hz) to the nonlinear PEH (3.9Hz). The reason for enhancement in the performance is due to the dynamics of added magnetic oscillator to the system. When the frequency is swept near resonant frequencies of beams (27.8Hz and 24.4Hz), energy from one beam (oscillator) is transferred to the other (harvester) through the magnetic interaction, resulting in better performance. Subsequently, the power and bandwidth are further increased through intensification of the magnetic nonlinearity by reducing the spacing between magnets from 10mm to 8mm. For $d=8\text{mm}$ nonlinear configuration, the maximum power output of $2137\mu\text{W}$ and the bandwidth of 4.3Hz have been achieved, showing 98% and 43% increase, respectively, compared to the linear configuration. It can be observed that the resonant frequency of PEH is reduced when the magnetic oscillator is added to the system because the attractive magnetic force reduces the linear stiffness of the beam. Moreover, the shift in the resonant frequency increases with closer spacing of the magnets.

The power output using the dipole formulation is also plotted for the purpose of comparison. For the case $d=10\text{ mm}$, the maximum power output value computed using the dipole model ($1879\mu\text{W}$) is much higher compared to the value computed using the charge model ($1579\mu\text{W}$). The power output from the case $d=8\text{mm}$ using dipole model is not plotted as the model predicts higher magnetic force, resulting in higher tip deflections. Near the resonant frequency of PEH or oscillator, the initial spacing between magnets (8mm) is closed by relative displacement of magnets (u_1-u_2) and resulted in sticking of the magnets. It is because higher tip displacements are predicted by the governing equations when the dipole model was used for evaluating the magnetic force. For the same configuration, the governing equations using the charge model predict a maximum relative displacement of 3.36mm and maximum power output of $2137\mu\text{W}$. The large errors in the prediction of power output and displacement values conclude that the dipole model cannot be used when the magnets are interacting at close distances.

3.3.2 Maximum strain in harvester materials

In this section, we derive the maximum strain in piezoelectric material bonded on the cantilever beam. The prediction of power output would be meaningless if the maximum strain in the harvester materials exceeds the allowable limit. Generally, piezoceramics have lower strength than the substrate materials and are more susceptible to failure due to their brittleness. Therefore, we evaluate the maximum strain in the piezoelectric material for the material strength consideration. Assuming that the mechanical perturbations are small and the material

is under small electric field conditions, the constitutive relations for the piezoelectric material are [18]

$$\begin{bmatrix} \epsilon \\ D \end{bmatrix} = \begin{bmatrix} s^E & d^c \\ d^d & \epsilon^\sigma \end{bmatrix} \begin{bmatrix} \sigma \\ E^f \end{bmatrix} \quad (3-17)$$

In the above equation, ϵ is the strain which depends on both the stress (σ) and electric field (E^f). The strain and electric displacement (D) are related to the stress and electric field through the elastic compliance (s^E), dielectric permittivity (ϵ^σ), and piezoelectric constants (d^c , d^d). The superscripts on the elastic compliance and dielectric permittivity denote that these constant are evaluated at constant electric field and constant stress, respectively. The superscripts on the piezoelectric constants refer to direct and converse effect of piezoelectric materials but the constants are numerically the same.

In deriving the maximum strain, we assume that the aluminum substrate and MFC patch are perfectly bonded together and the effect of epoxy layer between them is neglected. The axial strain in the cantilever beam can be written from the constitutive equations (3-17) as

$$\epsilon_x = s^E \sigma_x + d_{31} E^f \quad (3-18)$$

The first term on the right-hand side in the above equation refers to the strain contribution from the deflection of beam. The compliance is equal to the inverse of elastic modulus ($s^E = 1/E_x$), and the bending stress is given by the flexure formula ($\sigma_x = -M_z y/I$). The sign convention of the bending moment (M_z) is that the positive moment produces positive curvature. The bending moment can be approximated as $K\delta(L-x)$. The tip deflection of the beam (δ) can be calculated as $u(t)\phi(L)$. Using these relationships, the first term in the equation (3-18) can be written as

$$s^E \sigma_x = \frac{-Ku(t)\phi(L)y(L-x)}{(EI)_x} \quad (3-19)$$

The second term in the equation (3-18) is the strain induced by the electric field across the piezoelectric element. d_{31} is the piezoelectric constant for 31 mode and given as $d_{31} = e_{31}/E_p$, where E_p is the elastic modulus of piezoelectric material in the axial direction. The electric field across the piezoelectric element is assumed to be uniform and given as $E^f = -V(t)/T_p$ [38]. The maximum strain occurs on the farthest layer from the neutral axis and at the root of the beam. By substituting the strain contribution from the electric field in the equation (3-18), the expression for the maximum strain is given as:

$$\epsilon_{max} = \frac{-Ku_{max}\phi(L)(h+T_p)L}{(EI)_c} - \frac{e_{31}V}{E_p T_p} \quad (3-20)$$

where

$$h = \frac{T_s}{2} \left[\frac{1 - \frac{E_p B_p}{E_s B_s} \left(\frac{T_p}{T_s} \right)^2}{1 + \frac{E_p B_p T_p}{E_s B_s T_s}} \right]$$

$$(EI)_c = \frac{1}{3} E_s B_s (T_s - h)^3 + \frac{1}{3} E_s B_s h^3 + \frac{1}{3} E_p B_p (T_p + h)^3 - \frac{1}{3} E_p B_p h^3$$

The experiments performed at NASA Langley Research Center has shown no reduction in the performance of MFC actuator up to 100 million cycles under in-plane strain amplitudes of 2000ppm [22]. Though MFC can sustain up to tensile strains of 4500ppm, the durability of material under cycling loading decreases rapidly with higher amplitudes of strain. For instance, the functional life time of MFC actuator reduces from 10^9 cycles to 10^7 cycles when the applied voltage increases from 1kV to 2kV (peak-to-peak). Furthermore, the assumptions in using the piezoelectric constitutive equations and in deriving the maximum strain on the beam require the material to be operating under the elastic limit. Considering all the above, 1000ppm (both in tension and compression) is taken as the upper limit of allowable strain in MFC. The fatigue strength of aluminum (AL7075-T6) substrate used in this study is 1400ppm which is higher than the strength of MFC. According to the Euler-Bernoulli beam theory, the strain is proportional to the distance from the neutral axis. For the beam configuration considered in this study, the aluminum layers are close to the neutral axis than the layers of MFC. Hence, aluminum undergoes less strain than the MFC and operates within its allowable limit. It is advisable to use specially treated metal sheets that have higher fatigue strength for substrates.

The maximum strain in MFC bonded on the cantilever beams of the linear and nonlinear harvesters is shown in Figure 3-6. It is clear from figure that the MFC in both linear harvester configuration and the nonlinear harvester configuration with 10mm magnetic spacing is operating within the allowable limit on strain. But, the maximum strain in MFC in case of the nonlinear harvester with 8mm magnet spacing exceeds the upper limit. Closer spacing of magnets (8mm) provides better performance in terms of power output and bandwidth (Figure 3-5), but it increases the strain in the MFC to an unacceptable value. On the contrary, the nonlinear PEH with 10mm magnet spacing is safe but inferior in the performance. Therefore, a parametric study is required to seek an optimal configuration of the nonlinear PEH that gives better performance and at the same time operates within the allowable strain limit.

Figure 3-6 Maximum strain in MFC of linear and nonlinear harvesters excited at $a_{rms}=2m/s^2$ from simulation

3.3.3 Parametric study for optimal configuration

In this parametric study, the performance characteristics of the nonlinear PEH for various beam lengths are evaluated. For each length of the beam, the tip masses are adjusted such that the natural frequencies of PEH (27.8Hz) and magnetic oscillator (24.4Hz) are kept unchanged. Again, two cases of separation distance between magnets i.e., 8mm and 10mm are considered in the simulation and the base acceleration is kept to $2m/s^2$ (RMS). The values of damping ratio, required to investigate the behavior of harvesters, are experimentally measured for different beam lengths using the logarithmic decrement method [137]. The experimentally measured damping ratios are plotted in Figure 3-7.

Figure 3-7 Experimentally measured damping ratios for different beam lengths

Table 3-3 Summary of results of parametric study

	Linear harvester			Nonlinear harvester, $d=10$ mm			Nonlinear harvester, $d=8$ mm			Tip mass (g)	
L (mm)	Max power (μ W)	Band width (Hz)	Strain (ppm)	Max power (μ W)	Band width (Hz)	Strain (ppm)	Max power (μ W)	Band width (Hz)	Strain (ppm)	Main beam	Oscillator beam
75	1153.6	2.7	748.3	1868.6	3.5	975.9	2315.7	3.6	1214.8	3.63	4.79
70	1079.9	3	737.0	1579.2	3.9	906.3	2137.5	4.3	1119.0	4.61	6.06
65	1085.3	3.5	753.8	1465.2	5.8	884.8	1926.0	7.1	1065.6	5.93	7.70
60	1076.7	3.9	768.4	1349.2	5.8	862.8	1725.9	6.7	1019.5	7.74	10.1
55	1108.4	4.7	801.5	1313.4	5.9	872.4	1600.0	6.7	997.4	10.3	13.4
50	1119.7	5.5	832.5	1261.8	6.3	880.6	1462.1	6.8	977.1	14.1	18.2

Figure 3-8 (a) Power and (b) strain curves for various configurations of nonlinear PEHs excited at $a_{rms}=2m/s^2$ and with $d=8mm$ and $10mm$

The power output and strain over the frequency range obtained from the harvesters with various beam lengths are plotted in Figure 3-8(a) and (b), respectively. For the sake of legibility, the

results of nonlinear PEH with $d=8\text{mm}$ and $d=10\text{mm}$ are only plotted. For better comparison, the results of linear PEH and two cases of nonlinear PEH with different beam lengths are also summarized in Table 3-3. An interesting finding from Figure 3-8(a) is that the peak power reduces with the shortening of beam, so is the maximum strain on beam. This behavior can be attributed to the diminishing influence of the magnetic force on the increased stiffness of shorter beams. Another noticeable feature in Figure 3-8(a) is the increase in the average power near the resonant frequency of the magnetic oscillator with the shortening of beam. In addition, longer beams have more significant shift in the natural frequencies of PEH and magnetic oscillator because of the magnetic interaction, resulting in a dip of power output in between the peaks. As the beam length reduces, the shift in resonant frequencies is not so significant. Moreover, slightly higher damping of shorter beams helped in broadening the bandwidth and thus results in achieving better power output without having a dip in between the two peaks. The bandwidth given in Table 3-3 corresponds to the frequency range with more than $100\mu\text{W}$ of power output (required by a wireless sensor) [113]. With shortening of beams, the trade-off for the reduction in maximum power output is the increase in bandwidth. The improvement in bandwidth is minimal for beam lengths less than 60mm . As shown in Figure 3-8(b), the maximum strain in the piezoelectric material increases with the beam length. For the beam length of 50mm and 55mm , the maximum strain is within the acceptable limit. Out of these two alternatives, the beam length of 55mm with 8mm spacing between magnets is taken as the optimal configuration because of its better combination of peak power and bandwidth values. Observe that the configuration $L=75\text{ mm}$, $d=10\text{ mm}$ has higher power output than the optimal configuration ($L=55\text{ mm}$, $d=8\text{ mm}$) but has much lower bandwidth.

A constraint on the system mass affects the performance of the harvester and imposes an upper limit on the harvested energy [57]. The tip masses required for various lengths of the beams to keep the natural frequencies of main beam and oscillator beam at 27.8 and 24.4 Hz , respectively, are also given in Table 3-3. It is quite evident from the table that the tip mass value increases with the shortening of beam to keep same natural frequencies for the PEH and oscillator. In this study, while choosing the optimal configuration ($L=55\text{ mm}$, $d=8\text{ mm}$), only constraint on the material strength is considered. In case, the constraint on the system mass is also imposed and if the considered geometric parameters do not meet the criteria of both system mass and material strength, a more comprehensive parametric study is required. In such scenario, various thicknesses of beams along with various lengths should also be considered in

the parametric study to choose the optimal configuration. However, in the present work, only the constraint on the material strength of piezoelectric material is taken into consideration.

3.4 Experimental validation

A nonlinear PEH is fabricated with the optimal parameters determined from the parametric study, i.e., $L=55\text{mm}$ and $d=8\text{mm}$. For comparison purpose, experiment is also carried out on $d=10\text{mm}$ and linear PEH. The overview of the experimental setup is shown in Figure 3-9(a). The base excitation to the harvester is provided by the combination of a vibration exciter (APS 113), a function generator (RIGOL DG1011), and a power amplifier (APS model 116). The prototype of nonlinear PEH with magnetic oscillator is shown in Figure 3-9(b). The magnets are embedded in the tip masses attached to the end of the beams. Figure 3-9(b) shows the strain gauge (FLA 5-23-5L) bonded on MFC for measuring the strain. The strain data is acquired using the dynamic strain meter (DRA 101C) at a resolution of 0.2ms. Experiments are conducted at a harmonic base acceleration of 2m/s^2 (RMS). The RMS voltage is measured across the resistor while sweeping the frequency and the power delivered to the resistor is calculated using the formula V_{rms}^2/R_L .

Figure 3-9 (a) Experimental setup (b) Prototype of nonlinear energy harvester with magnetic oscillator

In Figure 3-10, the power output from the linear and nonlinear harvesters is plotted. Results of the optimal configuration ($L=55\text{mm}$) are compared with the results of previous study where beam length of 70mm was used [106]. The power output from the linear configurations is

plotted in Figure 3-10(a). The optimal configuration has 21% higher power output in the linear case. The power output from the linear harvester at resonance is proportional to the mass, the electromechanical coupling coefficient, and inversely proportional to the damping [34]. It is obvious that the beam with $L=55\text{mm}$ has higher tip mass than the beam with $L=70\text{mm}$ to match the natural frequency (27.8Hz). Additionally, the coupling coefficient increases as the length of the beam decreases because of the increased slope of mode shape in the composite section. Though the optimal beam has higher damping (Figure 3-7), the combination of higher tip mass and electromechanical coupling coefficient resulted in the increased power output.

(a)

(b)

Figure 3-10 Measured power output from (a) linear and (b) nonlinear energy harvesters excited at $a_{\text{rms}}=2\text{m/s}^2$

The power output from the nonlinear configurations is plotted in Figure 3-10(b). It is obvious from Figure 3-10(b) that the optimal configuration has higher peak power and wider bandwidth compared to the harvester configuration with $L=70\text{mm}$. The optimal configuration with $L=55\text{mm}$ and $d=8\text{mm}$ produces a peak power of $1658\mu\text{W}$ with a useful bandwidth of 6Hz , and has higher power output than the configuration of $L=70\text{mm}$ and $d=10\text{mm}$ for most of the bandwidth. For $L=55\text{mm}$, the natural frequency of linear harvester is observed at 27.7Hz compared to the simulated value of 27.8Hz . The presence of magnetic nonlinearity shifted the natural frequency from 27.7Hz to 27.3Hz and to 26.3Hz for $d=10\text{mm}$ and 8mm cases, respectively. The experimentally measured strain in MFC for the linear and nonlinear harvesters is plotted in Figure 3-11. The maximum strain (1023ppm) for the optimal configuration is just above the allowable limit (1000ppm) which is acceptable.

Figure 3-11 Measured strain on MFC of linear and nonlinear energy harvesters excited at $a_{\text{rms}}=2\text{m/s}^2$

Finally, the comparison between theoretical and experimental power outputs from the optimal configuration for both linear and nonlinear harvester cases is shown in Figure 3-12. The power outputs from the theoretical modeling and experiment are in good agreement for the linear PEH with 10% error in the prediction of maximum power output. For the nonlinear PEH, the overall trend and power output is captured quite well by the theoretical modeling. However, there is an error of 0.9Hz in predicting the resonant frequency.

Figure 3-12 Experimental and theoretical power outputs of linear and nonlinear energy PEHs excited at $a_{rms}=2m/s^2$

The peak power ($1658\mu W$) and maximum strain ($1023ppm$) of optimal configuration from the experiment are slightly higher than the theoretical predictions ($1600\mu W$ and $997ppm$). This is because of slightly higher tip mass used in the experiment than in the theoretical modeling to compensate for the additional stiffness due to the epoxy layer in between the substrate and piezoelectric material. The thickness of epoxy layer increases the distance of extreme piezoelectric layer from the neutral axis, which results in excess of strain. In theoretical modeling, the damping ratio measured for the linear case has also been used for nonlinear case. That is not exactly true as the damping ratio value changes for nonlinear case because of higher amplitudes and changes in the stiffness of beam because of the magnetic interaction. Though extreme caution has been taken in conducting the experiments, a slight error in the alignment of magnets or the gap between magnets could affect the results. Moreover, the assumption of axial alignment (magnetization axis) of magnets during vibration is not so accurate because of considerable rotations of tip masses at the free end of cantilever beams. Additionally, the tip mass is assumed to be point mass without rotational inertia which is not the case in experiment. Considering these sources of error, we can say that the experimental results are in very good agreement with theoretical predictions.

Su et al. [105] designed and experimentally tested a bistable dual-cantilever PEH which consists of an inner and an outer beam with magnets attached to their tips. Under a base acceleration of $0.3g$, the inner beam has produced useful power output of $100\mu W$ between

14Hz and 16Hz, with a peak power output of approximately 0.8mW. The outer beam has useful power output between 9Hz and 11Hz and a peak power output of 1mW. Another design by Zhu and Zu [111], a buckled-beam PEH with midpoint magnetic force produced an open circuit voltage of 5V (0-peak) between 20Hz and 23Hz at a base acceleration of 0.2g. When compared with the above designs, the proposed optimal nonlinear harvester with magnetic oscillator generates useful power output between 23Hz and 29Hz with a much higher peak power of 1.66mW under a base excitation of $0.2g_{\text{rms}}$ (2.83 m/s^2).

The response of nonlinear harvester is shown only for the up-sweep in all the figures in this chapter. The down-sweep has shown slightly better bandwidth when compared with the up-sweep for the beam configurations with low stiffness. However, the high energy state vibration in the down-sweep cannot be always guaranteed practically. Therefore, the up-sweep is used for fair comparison between the nonlinear harvester and the linear harvester. For the optimal configuration, the responses in up-sweep and down-sweep are exactly similar. The stiffness of the beam is high and the nonlinearity induced by small magnets ($\varnothing 4\text{mm} \times 3\text{mm}$) is not sufficient to induce significant difference between the up-sweep and down-sweep. Under high base excitations, beams vibrate with significant amplitude and magnets will operate at closer distances resulting in high nonlinearity. In such cases, the down-sweep of harvester gives much wider bandwidth than the up-sweep. However, magnets snap each other (sticking) at a critical distance and a large force is required to separate them. Snapping of magnets will make harvester useless. Owing to the above reason, the prototype is not tested for base accelerations above 2m/s^2 (RMS) neither for distances between magnets less than 8mm.

Although material strength is considered in designing the harvester, the reliability of harvester's performance needs to be checked over long durations of time. The crack formation in piezoelectric and substrate materials due to cyclic electro-mechanical loading severely limits the integrity and durability of harvesters. The degradation in the performance of harvesters over the time should be studied for available piezoelectric materials. Chapter 7 presents experimental study on the performance reliability of piezoelectric harvesters comprising macro fiber composite. Furthermore, durability of piezoelectric energy harvesters is limited by the growing influence of ferroelastic hysteresis behavior which reduces the amplitude of harvester over the time and consequently its power output [138]. Piezoelectric materials also suffer from depolarization and debonding of PZT fibers from epoxy matrix. The charge output, strain and electroelastic coefficients of PZT films are temperature dependent and the ambient temperature variations can influence the dynamics of harvester [139]. The above mentioned issues need to

be considered in designing the piezoelectric energy harvesters for durability in practical applications.

3.5 Summary

This chapter presented the optimization of a nonlinear energy harvester with magnetic oscillator while imposing constraint on the strain induced in the piezoelectric material (MFC). Electromechanical models of both linear and nonlinear PEHs are derived using Rayleigh-Ritz formulation. The charge model is employed to calculate the force between permanent magnets and it is shown that the dipole model would give highly erroneous results when the distance between magnets is 7mm or less. The maximum strain on the cantilever beam is evaluated assuming small mechanical and electrical perturbations. A parametric study is carried out by varying the beam length and spacing between magnets of PEH and magnetic oscillator to seek the optimal configuration of nonlinear energy harvester. Experiments have shown good agreement with predictions from the theoretical modeling. At a base acceleration of 2m/s^2 (RMS), the optimal energy harvester generates a peak power of $1658\mu\text{W}$ with a useful bandwidth of 6Hz and the maximum strain in the piezoelectric material is 1023 ppm.

CHAPTER 4 FINITE ELEMENT MODELING AND ANALYSIS OF NONLINEAR PIEZOELECTRIC ENERGY HARVESTERS

4.1 Introduction

Magnetic interaction has been extensively used for broadband energy harvesting. Various designs of PEHs where the magnetic interaction is used for enhancing the performance have been presented in Section 2.3.3. In all those works, the analytical models have been derived only for simple designs in which a magnet at the free end of a cantilever beam interacts with other stationary magnets [92, 105, 106, 109]. However, it is difficult to develop the accurate analytical models for harvesters with complex structures and inclusion of the magnetic interaction further increases the difficulty of modeling. Su et al. [105], Andò et al. [116], and Wu et al. [104] proposed harvesters with complex designs and incorporated magnetic interaction for widening the bandwidth. It is difficult, or sometimes not possible to derive accurate analytical models for such complex designs of harvesters. In such cases, it is better to model and analyze the energy harvesters using commercial finite element analysis (FEA) packages like ANSYS, ABAQUS, COMSOL, ATILA FEA, etc. In Section 2.2.2, the works related to FEA modeling of PEHs have been discussed. It is evident from the discussion that the FEA modeling has been used primarily for analyzing the linear harvesters only. The main difficulties in extending the FEA modeling to analyze the nonlinear harvesters are the modeling of mesh required for the magnetic interaction in the dynamic problems and the high demand on computational resource needed for solving the coupled electrical-mechanical-magnetic problem.

In this chapter, a novel technique is proposed for modeling the nonlinear PEHs with magnetic interaction using FEA. The magnetic interaction is modeled using the nonlinear spring element available in ANSYS. Firstly, a monostable cantilever-type nonlinear harvester is modeled using the proposed technique and validated with the theoretical model. The proposed method is then extended to a rather complex 2-DOF nonlinear PEH and the predictions from FEA are compared with the experimental measurements. Finally, the applicability and limitations of the proposed technique are discussed.

4.2 Simulation of a monostable nonlinear energy harvester

4.2.1 Finite element modeling of magnetic interaction with nonlinear spring

As mentioned earlier, the nonlinearities can be introduced in the linear harvesting systems to achieve wider bandwidth and the magnetic interaction has been extensively exploited for such purpose. A cantilever beam with a magnet at its tip, interacting with a stationary magnet, is a typical configuration to achieve wider bandwidth. In such design, the magnetic force not only changes the linear stiffness, but also introduces nonlinear stiffness into the system. However, the nonlinear behavior is apparent only when the harvester vibrates with significant amplitude. Challa et al. [92] placed a magnet at the tip of the beam and a stationary magnet above it. The axial force between the magnets during the vibration of beam resulted in nonlinear stiffness and hence achieved higher bandwidth. In some other works, magnets are placed end-to-end at the tip of cantilever beam [102, 140]. In such cases, it is the lateral force between the moving magnet and stationary magnet that is responsible for widening the bandwidth. In this section, a monostable configuration with magnets placed end-to-end is modeled and analyzed using FEA. The predictions from FEA will be compared with the results from the theoretical model.

Figure 4-1 Schematic representation of monostable nonlinear energy harvester

The schematic of monostable nonlinear energy harvester is shown in Figure 4-1. The system comprises of a cantilever beam with a tip mass at its free end and a patch of piezoelectric material is bonded to its root. The substrate beam is assumed to be made of aluminum and MFC is used as the piezoelectric material. A resistor is connected across MFC to measure the electrical output. The tip mass holds a cylindrical magnet which interacts with a stationary magnet placed end-to-end. The magnetization direction is along the beam axis and both the magnets are assumed to be perfectly aligned when the beam is at rest. When the beam vibrates

under base excitation, the lateral force (F_y) affects the stiffness of beam. The mechanical, electrical, and magnetic domains of the nonlinear harvester are indicated in Figure 4-1. The previous works have used FEA for analyzing only the coupled electromechanical problems. The FEA modeling of this nonlinear harvester requires the inclusion of magnetic module along with structural and electrical modules, thus making the problem exceedingly difficult to solve using FEA.

The linear energy harvesters excited under harmonic base excitation can be easily solved in FEA packages using the harmonic analysis, which is essentially a linear analysis determining the steady-state response of a structure to a sinusoidally varying load. The harmonic analysis ignores the transient effects occurring at the beginning of vibration. But when nonlinearity such as magnetic force is introduced into the system, the problem cannot be solved using the harmonic analysis. Such type of nonlinear problems should be solved using the time-history or transient analysis. In the transient analysis, any form of external loading on the structure should be given as time-dependent loads in a series of small time steps and the response of system needs to be evaluated over a period of time. A typical coupled field problem such as piezoelectric transient analysis takes considerable amount of computational time depending upon the number of elements used in the model, the size of time step, and the total time over which the response of structure is evaluated.

When the magnetic module is added to the FEA model, the problem becomes nonlinear and computationally expensive. The complexity and high computational cost involved in modeling the magnetic domain in the dynamic analysis of energy harvesters can be understood easily through the modeling of a simple magneto-static problem. Figure 4-2 shows a magneto-static analysis of two permanent magnets which are separated axially. Figure 4-2(a) shows the meshed model of two cylindrical magnets along with the air surrounding them. The FEA modeling of permanent magnets requires meshing of the medium surrounding them which is typically air as shown in the figure. In this simulation, cylindrical magnets (NdFeB, N35 grade) of dimensions $\varnothing 10\text{mm} \times 5\text{mm}$ are modeled with a separation distance of 10mm between them. A total of 125,169 elements are used for modeling both the magnets and surrounding air. The resulting magnetic flux density contour plot is shown in Figure 4-2(b), from which the force between magnets can be evaluated. When similar kind of meshing for magnets and surrounding air is used for the dynamic analysis of nonlinear harvesters where one magnet is moving, the elements representing the surrounding air should be able to absorb the displacement of magnet

without getting extremely distorted. This is very difficult to achieve as the displacement of magnet is usually much larger than the size of elements (air). If mesh updating is used after every load step, then the computational time required to solve such a complex problem will be extremely high. Till now, no work has been reported on solving the coupled electrical-mechanical-magnetic problem using FEA in the area of piezoelectric energy harvesting. In this thesis, a novel method is proposed to model the magnetic force using a nonlinear spring element, thus eliminating the difficulty of modeling the magnetic interaction.

Figure 4-2 Magneto-static analysis of two permanent magnets separated axially (a) Meshed model (b) Contour plot of the magnetic flux density (Tesla)

COMBIN39 is a nonlinear spring element available in ANSYS. The element has the capability of simulating the nonlinear force-displacement behavior as shown in Figure 4-3. The element is defined by two nodes and a generalized force-displacement curve. This curve is defined by piece-wise linear segments using points (D1, F1), (D2, F2), etc., and it is required to pass through the origin. More information on this element is available in ANSYS documentation. The COMBIN39 element can be used to simulate the magnetic interaction in the FEA model of nonlinear harvester by approximating the nonlinear force between the magnets with piece-wise linear segments. The calculation of magnetic force and its piece-wise linear approximation is described in the following section.

Figure 4-3 Generalized nonlinear force-deflection behavior of element COMBIN39 (ANSYS® Academic Research, Release 14.0, Help System, COMBIN39 Geometry, ANSYS, Inc)

4.2.2 Evaluation of magnetic force

Various analytical solutions existing in the literature for calculating the forces between permanent magnets are discussed in Section 3.2.3. For the present study, two cylindrical permanent magnets are used in the design of nonlinear PEH. Therefore, the analytical solutions in the literature to calculate the force between cylindrical magnets are reviewed here. Furlani [133] has given the analytical expression for calculating the axial force between cylindrical or disk type permanent magnets. Vokoun et al. [130] derived an expression for the axial force between two laterally displaced cylindrical magnets assuming uniform magnetization. In another work, Vokoun et al. [141] have given the expressions for the axial and radial forces in a shell-cylinder magnet system to study the operation of a magnetic gun. Following the same approach, Avvari et al. [142] expressed the axial and lateral forces between cylindrical magnets in the integral form, which were used to study the broadband PEHs. In this study, the integral equations proposed by Vokoun et al. have been used for calculating the force between permanent cylindrical magnets. Subsequently, the variation of magnetic force is approximated with series of linear segments for giving it as input to the nonlinear spring element in ANSYS.

Figure 4-4 (a) Schematic of two cylindrical magnets with axial and lateral separation (b) Variation of magnetic force (F_y) with lateral displacement (r) for fixed $s=10$ mm

The configuration of cylindrical magnets with axial and lateral separation is shown in Figure 4-4(a). The integral equations for the axial and lateral forces between two identical cylindrical magnets derived by Vokoun et al. [141] are given below.

$$F_x = -4\pi\epsilon\mu_0 M^2 R^2 \int_0^\infty \frac{J_0\left(\frac{rq}{R}\right)}{q} J_1^2(q) \sinh^2\left(\frac{qt}{2R}\right) e^{-\left(\frac{qs}{R}\right)} dq \quad (4-1)$$

$$F_y = 4\pi\epsilon\mu_0 M^2 R^2 \int_0^\infty \frac{J_1\left(\frac{rq}{R}\right)}{q} J_1^2(q) \sinh^2\left(\frac{qt}{2R}\right) e^{-\left(\frac{qs}{R}\right)} dq \quad (4-2)$$

In the above equations, M is the magnetization; R is the radius; t is the thickness of the magnet; r is the lateral separation; s is the axial separation between the magnets; J_0 and J_1 are the Bessel functions of first kind of the order 0 and 1, respectively; μ_0 is the permeability of vacuum; ϵ is +1 for the attractive case and -1 for the repulsive case; q is the dimensionless parameter defining the shape function of magnetic particle (cylinder) and magnetic vector potential in Fourier space. The integrals in equations (4-1) and (4-2) are originally spatial integrals and they are converted into Fourier space for ease of calculation. The integrals can be evaluated using software MATLAB or MATHEMATICA. Vokoun et al. [130] compared the force between two axially aligned NdFeB magnets calculated using the equation (4-1) with the experimental measurements. The calculated force is in excellent agreement with the measurements with deviation below 5%. For illustration of the accuracy of above equations, the contact force between two magnets is compared here. From the manufacturer's data sheet, the contact force between two NdFeB cylindrical magnets of size Ø10mm x 5mm with a magnetization value of 891kA/m is 2.5kg. With $r=0$ and $s=0$, both equations (4-1) and (4-2) give a contact force value of 2.512kg (24.64N), matching closely with the measured value. Therefore, the above integral expressions are accurate and preferred over the other existing solutions to calculate the magnetic force.

The interaction between the static magnet and the moving magnet placed at the tip of beam in the monostable nonlinear harvester is similar to the configuration of magnets presented in Figure 4-4(a) if one magnet is fixed and the another one is moving. Though both F_x and F_y act on the beam, it is the lateral force F_y that changes the stiffness of beam and is responsible for the broadband energy harvesting. The variation of F_y with the lateral displacement r for a fixed axial separation distance (s) of 10mm is shown in Figure 4-4(b) for both attractive and repulsive cases between the magnets. The magnitude of force is same for both attractive and repulsive cases but opposite in direction. Additionally, the force is symmetric when the magnet is laterally moved to the either side of the fixed magnet. To represent this nonlinear force in FEA using the COMBIN39 nonlinear spring element, the force needs to be approximated with

piecewise linear segments as discussed in the previous section. The piecewise linear approximation of the magnetic force done using optimization tools in MATLAB is also shown in Figure 4-4(b). The optimization algorithm provides the displacement locations and the corresponding approximated magnetic force values. The lines joining these points ensure the minimum error throughout the interval when compared with the exact force values. The calculated force values using the equation (4-2) and the approximated values are given in Table 4-1. It is clear that the nonlinear magnetic force is approximated using linear segments with a maximum deviation of 0.002N. This set of linear approximation points are given as the force-deflection behavior of the nonlinear spring in ANSYS FEA model.

Table 4-1 Piecewise linear approximation of nonlinear magnetic force

Displacement, r (mm)	Force, F_y (N)		Displacement, r (mm)	Force, F_y (N)	
	Calculated	Approximated		Calculated	Approximated
0.0	0.000	0.000	5.751	0.468	0.470
1.594	0.178	0.180	6.855	0.484	0.486
2.751	0.292	0.294	7.855	0.478	0.479
3.751	0.372	0.374	8.855	0.457	0.458
4.751	0.431	0.433	10.00	0.422	0.422

4.2.3 Finite element model of monostable PEH

Figure 4-5 Finite element model of monostable nonlinear PEH

Table 4-2 Parameters of monostable nonlinear energy harvester

Parameter	Piezoelectric	Substrate
Material	MFC 2807-P2	Aluminum
Length (mm)	28	68
Thickness (mm)	0.3	0.64
Width (mm)	7	10
Elastic modulus (GPa)	30.336	68.9
Poisson's ratio	0.31	0.33
Mass density (kg/m ³)	5440	2700
Piezoelectric constant (C/m ²)	-5.16	--
Capacitance (nF)	12.4	--
Resistance (k Ω)	1000	--
Tip mass (g)	9	--
Damping ratio (%)	1	--
Spacing between magnets, s (mm)	10	--

The finite element modeling and analysis is performed using ANSYS® Academic Research 14.0. Figure 4-5 shows the FEA model of monostable nonlinear harvester presented in Figure 4-1. The geometric and material properties required for the FEA modeling are given in Table 4-2. Since the harvester uses only the d31 mode for energy harvesting, the material and tensorial constants related to this mode are given as input to the piezoelectric material model in ANSYS. Other non-active modes are suppressed by assigning zero value to the respective constants. The piezoelectric material MFC is bonded to the root of aluminum beam and a tip mass is attached at the free end of beam. The Aluminum beam and tip mass are modeled with 3-D 20 node hexahedral elements (SOLID186) having three translational DOFs at each node in X, Y, and Z directions. MFC is modeled with 3-D 20 node piezoelectric element (SOLID226) which has one additional DOF of voltage for each node along with three translational DOFs. A resistor element (CIRCU94) is connected across MFC to measure the electrical output. One node of the resistor element is grounded and the voltage generated from the harvester is measured at the other node. A damper element (COMBIN14) is connected to the tip mass which acts like a dashpot. It is a common practice to assume viscous damping in the piezoelectric analyses, but ANSYS has a wide range of options to incorporate the damping

in the model such as the Rayleigh damping, modal damping, material-dependent damping, etc. However, in this study, a damper element is used and the damping coefficient is given as input. The calculation of the damping coefficient will be explained in Section 4.2.4.

The nonlinear spring (COMBIN39) is used to model the magnetic force with one node of the spring connected to the tip mass and the other node fixed. The lateral force between the moving magnet (embedded in the tip mass) and the stationary magnet (fixed to the vibrating structure) is calculated and approximated with piecewise linear segments as presented in the previous section. The linearly approximated force-deflection points in Table 4-1 are given as input for the nonlinear spring element. Both the attractive and repulsive cases can be simulated with proper assignment of sign to the magnitude of force. Using this method for modeling the magnetic force, the meshing problems involved in modeling the magnets and surrounding air are eliminated.

As already pointed out, the loading needs to be applied as a function of time for solving the nonlinear problems. The nonlinear problem has to be solved for a series of time steps to obtain the system response. In this work, the harmonic base excitation is considered on the harvester. Each cycle of the base acceleration at a given exciting frequency is divided into certain number of equal time steps and the corresponding load is applied on the structure. The system is solved for several such cycles of input acceleration till the steady state response is achieved. The computational effort depends on the number of elements used, the size of time step, and the time interval over which the response of harvester is evaluated. The finite element mesh needs to be adequately fine to capture the dynamics of the system but too fine mesh will result in excessive computational effort. For instance, when meshing the beam of monostable harvester, relatively less number of elements is recommended in the direction of width when compared with the directions of length and thickness because of the absence of strain variation in the width direction. For the beam model shown in Figure 4-5, the numbers of elements in length, width, and thickness directions are determined as 25, 6, and 3, respectively, after a few trial runs. The time step should be small enough to capture the response of structure but too small time step will result in excessive computational effort. It is recommended in ANSYS documentation that 20 divisions per each cycle of loading gives very good accuracy. The time required to achieve the steady state response needs to be decided by experience and with some trial runs. Normally, for typical piezoelectric energy harvesting applications in the frequency range 10Hz-100Hz, the steady state can be achieved within 5s. Hence, the response of the nonlinear harvester is evaluated for a time interval of 5s in ANSYS.

(a)

(b)

Figure 4-6 (a) A cycle of input acceleration on FEA model and the resulting voltage output
 (b) Time-history voltage response of PEH from FEA. The harvester is excited at base acceleration 2m/s^2 (RMS) and frequency 17Hz

Figure 4-6(a) shows one cycle of the applied harmonic base acceleration on the harvester and the resulting voltage across resistor. A base acceleration of 2m/s^2 (RMS) is applied at a frequency of 17Hz. Each cycle of base acceleration is divided into 30 equal time steps for better accuracy and the resulting time step size is 0.00196s. The simulation is carried out for 5s which has a total of 2550 time steps. The time-history voltage response of the harvester is shown in

Figure 4-6(b) and it can be observed that the steady state is achieved around 2s. The total computational time for this transient analysis on a computer with 16GB RAM, 8 core i7 processor is about 1100s. Similar analyses need to be run over a frequency range to evaluate the harvester's performance. In the next section, the accuracy of the FEA model is verified by comparing with the theoretical model of monostable nonlinear harvester.

4.2.4 Theoretical modeling of monostable energy harvester

The governing electromechanical equations of monostable energy harvester are derived using the Hamiltonian principle following the procedure described by Hagood et al. [37], Sodano et al. [38], and Elvin and Elvin [39] and the equations are given below:

$$M \ddot{u}(t) + C \dot{u}(t) + K u(t) + \Theta V(t) + F_{mag} = F(t) \quad (4-3)$$

$$\frac{V(t)}{R_L} + C^s \dot{V}(t) - \Theta \dot{u}(t) = 0 \quad (4-4)$$

where $V(t)$ is the voltage across the resistor R_L ; $u(t)$ is the modal displacement in the single mode approximation with displacement $w(x,t)=u(t)\phi(x)$; M , K , C , and Θ are the modal mass, stiffness, damping, and electromechanical coupling terms, respectively, corresponding to the fundamental mode $\phi(x)$; C^s is the clamped capacitance of MFC patch; and $F(t)$ is the mechanical forcing due to the base acceleration $\ddot{u}_0(t)$. The definitions of other terms are given below.

$$M = \int_0^L m_*(x) \phi(x)^2 dx + M_t \phi(L)^2 \quad (4-5)$$

$$K = \int_0^L EI_*(x) \phi''(x)^2 dx \quad (4-6)$$

$$\Theta = - \int_{v_p} y \phi''(x) e_{31} \psi(y) dv_p \quad (4-7)$$

$$F(t) = - \left(M_t \phi(L) + \int_0^L m_*(x) \phi(x) dx \right) \ddot{u}_0(t) \quad (4-8)$$

$$C = 2\xi\omega_n M \quad (4-9)$$

Here, m and EI are the mass per unit length and flexural rigidity of beam, respectively. The subscript '*' corresponds to the specific portion of beam, either composite section or substrate. M_t is the tip mass and e_{31} is the piezoelectric stress constant. The spatial variable x is measured from the root of beam and y is measured from the neutral axis. L is the length of the beam. The function $\psi(y)$ defines the electric field over the piezoelectric element which is assumed to be constant [38]. $\phi(x)$ is the mode shape normalized to unity corresponding to the first natural

frequency (ω_n) of the harvester. ξ is the damping ratio and is usually measured from the experiments. In this theoretical model, the damping ratio is assumed to be 1%. The damping coefficient C is calculated by using the equation (4-9) and the same value is given as input to the damper element in the FEA model. For the considered harvester configuration (Figure 4-1; Table 4-2), the system parameters are calculated using the above equations and tabulated below (Table 4-3). Using these system parameters, the governing equations (4-3) and (4-4) of the monostable nonlinear harvester are solved for the tip displacement and voltage across resistor.

Table 4-3 Calculated system parameters of monostable PEH

Parameters	Values
Mass, M (kg)	9.258E-3
Stiffness, K (N/m)	227.815
Damping coefficient, C (Ns/m)	0.029
Coupling coefficient, θ (N/V)	1.833E-4
Capacitance, C^s (nF)	12.4
Resistance, R_L (ohm)	1E6

4.2.5 Validation of FEA model

The output voltage from FEA is validated with the predictions from the theoretical analysis for three different cases (Figure 4-7). A harmonic base acceleration of 2m/s^2 (RMS) is used for the analysis. The voltage output of the linear harvester (without magnets at the tip) is compared in Figure 4-7(a). Both the peak voltage and resonant frequency from the FEA match very well with the corresponding values from the theoretical analysis. The voltage output from the nonlinear harvesters is compared in Figure 4-7(b) and 7(c), when the end magnets are placed in the repulsive and attractive configurations, respectively. For the nonlinear harvester configurations, the response is evaluated when the excitation frequency is swept in both forward and backward directions. The magnetic force modifies the linear stiffness of the beam. The repulsive magnetic force softens the beam while the attractive force stiffens it. Consequently, the operating frequency range of the nonlinear harvester is shifted to the left of the resonant frequency of linear harvester for the repulsive configuration, and to the right for the attractive configuration.

(a)

(b)

(c)

Figure 4-7 Comparison between output voltages from FEA and theoretical analysis (a) Linear harvester (no magnet) (b) Nonlinear harvester with repulsive configuration (c) Nonlinear harvester with attractive configuration. Fwd=Forward; Bwd=Backward

It is evident from the Figure 4-7 that wider bandwidth is achieved using the nonlinear configurations when compared with the linear harvester. However, the nonlinear harvester achieves wider bandwidth only in one direction of the frequency sweep. The reason is the existence of two stable states for the nonlinear configuration in certain frequency range. When coexisting stable states exist, the response of nonlinear system depends on the initial conditions [102, 140]. For the nonlinear harvester with repulsive configuration (Figure 4-7(b)), the initial conditions in the forward frequency sweep captures the high energy state and the voltage output increases till 19.4Hz where it suddenly drops, resulting in the jump phenomenon. The backward sweep cannot capture the high energy state, resulting in lower bandwidth. Similar phenomena can be observed for the attractive configuration (Figure 4-7(c)). For this case, the backward frequency sweep results in slightly higher bandwidth than the forward sweep.

It is evident from the results that the FEA modeling is able to capture the dynamics of the nonlinear harvester accurately for both the forward and backward frequency sweeps. The voltage output of the harvester from the FEA agrees well with the theoretical analysis. The jump phenomenon in the forward frequency sweep is observed at 19.5Hz in the theoretical analysis for the repulsive configuration whereas FEA captures the phenomenon at 19.4Hz (Figure 4-7(b)). Similar slight difference of 0.1Hz in the frequency response is observed for the backward sweep and also in the responses of harvester with attractive configuration. The maximum discrepancy is 3% in the estimation of peak voltage from FEA and theoretical analyses. This could be due to that the presented theoretical model approximates the harvester dynamics with the fundamental mode, while FEA solves the full system (in the form of matrices) to compute the transient response. The nonlinear harvester operates in a frequency range away from the fundamental frequency of the linear harvester. Hence, the theoretical model based on the fundamental mode approximation may not capture the dynamics of harvester as accurately as FEA. Based on the results presented, it can be concluded that the proposed FEA model of nonlinear harvester where the nonlinear spring element is used for simulating the magnetic interaction is very accurate and useful for analyzing similar designs of nonlinear harvesters.

It should be noted that the same damping value is used for analyzing the three different cases presented in Figure 4-7 which may not be true practically. The damping changes with the amplitude and frequency of vibration, and the presence of magnets also affects the damping. However, assuming constant damping does not affect the validity of a comparison study like this. For the repulsive configuration (Figure 4-7(b)), the performance of harvester is evaluated

using FEA at 20 discrete frequency values from 17Hz to 21Hz with a total computational time of 7hours. For the attractive configuration, the computational time is about 10 hours for evaluating the performance at 20 discrete frequencies from 28.5Hz to 31.5Hz. In each simulation, the response of the structure is evaluated for 5s to achieve the steady state. For the higher excitation frequencies (attractive configuration), the number of cycles to be simulated in 5s time interval are higher when compared with the low excitation frequencies (repulsive configuration), resulting in higher computational time. In the subsequent section, the proposed FEA modeling technique will be extended to a nonlinear 2-DOF harvester where the advantages of using FEA modeling can be clearly distinguished.

4.3 Simulation of a broadband nonlinear 2-DOF harvester

4.3.1 FEA modeling of nonlinear 2-DOF harvester

The reason for choosing a simple cantilever-type monostable nonlinear harvester in the previous section is to illustrate the validity of the proposed FEA modeling technique by comparing it with the theoretical model which can be derived for such a simple configuration. The advantage of the proposed FEA modeling method can be realized when it is applied to a design for which deriving an accurate theoretical model is difficult or not possible. Wu et al. [78] proposed a 2-DOF PEH comprising an outer beam and an inner beam with attached tip masses. By varying the tip masses, the first two resonant frequencies were tuned close to each other to achieve wider bandwidth. In their later work, Wu et al. [104] incorporated the magnetic interaction to further broaden the bandwidth. A lumped parameter model was derived for the parametric study on resonances and bandwidth tuning. Though the lumped parameter model predicts the general trend of the harvester's performance, there is considerable discrepancy between the theoretical results and the experimental measurements. A distributed parametric model using first two modes of the harvester would give more accurate estimation of the performance of harvester, but it is very difficult to derive such model. Another source of error in their work is using the dipole-dipole approximation for evaluating the magnetic force when the magnets are placed very close to each other. In this section, the above mentioned 2-DOF nonlinear PEH will be modeled and analyzed using the proposed FEA technique and the estimated voltage from the FEA simulation will be compared with the experimental measurements.

(a)

(b)

(c)

Figure 4-8 2-DOF nonlinear energy harvester (a) Schematic representation (b) Prototype of harvester clamped on shaker (c) Developed finite element model

The schematic of the 2-DOF nonlinear harvester is shown in Figure 4-8(a) along with the geometric parameters. The outer beam is clamped at one end and the inner beam is attached to the outer beam using a bolted joint. The tip mass of inner beam holds a magnet which interacts with a stationary magnet fixed to the clamped end. Two NdFeB magnets (N35 grade) of dimensions $\varnothing 10\text{mm} \times 5\text{mm}$ were used in the experiment with a separation distance (D) of 10mm. A patch of MFC material (M2814-P2) is bonded to the inner beam near to its bolted connection to the outer beam. The developed FEA model to analyze this nonlinear harvester is shown in Figure 4-8(c). The nonlinear spring is attached to the tip mass of inner beam to simulate the magnetic force. One damper is attached to each of the tip masses. The damping coefficients for the two dampers are calculated using the equation (4-9) with a damping ratio of 0.8%. The other parameters used in the FEA modeling are given in Table 4-4.

Table 4-4 Parameters used for modeling of 2-DOF nonlinear energy harvester

Parameters	Values
MFC LxWxT (mm)	28x14x0.25
Capacitance (nF)	25.7
Tip mass of outer beam (g)	12.6
Tip mass of inner beam (g)	6.7
Magnetization (kA/m)	891
Resistance (ohm)	1E6
Damping (%)	0.8

4.3.2 Validation of FEA modeling of nonlinear 2-DOF harvester

The voltage output from FEA is compared with the experimental measurements in Figure 4-9 for the base acceleration $2\text{m/s}^2(\text{RMS})$. The responses of harvester during the forward frequency sweep are compared in the figure. The FEA is performed for three different values of the spacing between the magnets (D), which are 10, 9.5, and 9mm. It can be observed that the trend of voltage output from FEA with D=9mm closely matches with the experimental measurements which are conducted with D=10mm. One of the reasons for the discrepancy could be the experimental error in maintaining the spacing between the magnets. Moreover, the clamped end of the outer beam and the bolted joint used for fixing the inner beam to the outer beam are not completely rigid in the experimental study especially when the beams vibrate with significant amplitude. But in FEA, the boundary conditions are rigid regardless of the

amplitude. So, the fixed boundary conditions in FEA make the beams slightly stiffer than the experimental specimen, thus requiring slightly closer spacing of the magnets to have similar effect of nonlinearity as in the experiment. However, FEA is capable of modeling flexible clamping boundary conditions (not 100% rigid) too. The harvester clamped between the shaker surface and acrylic block using the threaded bolt connection is shown in Figure 4-8(b). The practical clamping boundary condition can be simulated in FEA if the torque applied on hexagonal face of nut and the friction between contact surfaces is known.

The main features in the response of nonlinear harvester are the anti-resonance around 13.5Hz, the dip in the output voltage around 17Hz because of the internal resonance, and the jump phenomenon around 20Hz [78, 104]. It is evident from the figure that all these features are successfully captured by FEA modeling with $D=9\text{mm}$. The results from FEA show slightly higher peak at 15Hz and the jump phenomenon is also observed at higher frequency (21Hz) when compared with the experiment.

Figure 4-9 Comparison of voltage output of 2-DOF nonlinear harvester from FEA with experimental measurements for base acceleration of $2\text{m/s}^2(\text{RMS})$

Figure 4-10 Comparison of experimental voltage output from 2-DOF nonlinear harvester with FEA results obtained with different damping values

Although the dynamics of harvester are captured well in the FEA modeling with $D=9\text{mm}$, the location (the frequency value) where the jump phenomenon occurs is not predicted accurately. This could be attributed to the damping ratio. In the original work of Wu et al. [104], the damping ratio 0.8% was measured at $1\text{m/s}^2(\text{RMS})$ but the voltage response was measured at $2\text{m/s}^2(\text{RMS})$. Thus, in FEA simulation, the damping ratio should be altered to have a fair comparison with the experimental results obtained at 2m/s^2 . Figure 4-10 shows the FEA results for $D=9\text{mm}$ with different damping ratios. It is shown that the simulation with damping ratio 1% matches well with the experiment. This is expected as the damping ratio at $2\text{m/s}^2(\text{RMS})$ should be slightly higher than the value 0.8% measured at $1\text{m/s}^2(\text{RMS})$. The adverse effect of the damping on the bandwidth at high energy states can also be observed from the figure. The higher damping ratio results in lower bandwidth of the nonlinear harvester. It should be noted the damping ratio referred above is of the inner beam. The damping in outer beam which has resonance at 15Hz is not given in the original work of Wu et al. [104]. So constant damping is assumed for the entire frequency range in FEA. This is also the reason for the discrepancy in the predicted output voltage at resonance of outer beam at 15Hz. The dynamics of nonlinear harvester can be predicted well with accurate measurement of damping over the frequency range. The influence of varying damping with frequency on the performance of harvester can also be modeled in FEA, provided that the data are available.

It should be noted that the actual values of material properties vary from the values mentioned in data sheets. For instance, up to 20% variation in the clamped capacitance value of MFC patches (measured using multi-meter) is observed in various experiments conducted in this research. So it is suggested to use the measured material properties for accurately modeling the harvester. However, values from data sheet are considered in this FEA modeling of 2-DOF nonlinear harvester because of lack of the experimental data which might resulted in slight discrepancy. The glue used for bonding the MFC to aluminum beam is not modeled in FEA, thus perfect bonding is assumed between the MFC and substrate. From these results, we can conclude that FEA is able to predict the performance of nonlinear harvesters having complex design with very good accuracy. The FEA simulation is carried out for this design at 40 discrete frequencies between 10Hz and 25Hz and the total computational time is about 16 hours.

In the design of energy harvesters for real life applications, structural integrity is one important concern. The higher strain (or stress) in any of the harvester materials than the allowable limit will result in premature failure. Hence, the strain in the harvester materials needs to be evaluated beforehand. The theoretical models in the literature have mainly focused on the electrical output from the harvester while ignoring the strain experienced by the harvester materials. FEA provides the advantage of monitoring the strain due to the electromechanical loading even for the complex geometries of the harvester. Figure 4-11 shows the contour plot of the strain in harvester materials from the FEA simulation performed with $D=9\text{mm}$ and $\xi=1\%$. Harvester materials undergo both tension and compression during cyclic electromechanical loading. The state of maximum strain of materials is evaluated at the frequency corresponding to the peak voltage of nonlinear harvester (Figure 4-10) and is plotted in Figure 4-11. The strain distribution in the aluminum structure along with the tip masses is shown in Figure 4-11(a). The maximum strain is 1108ppm in compression and 1106ppm in tension. These predictions are slightly higher than the experiment as the connection of inner beam to the outer beam is assumed to be rigid in FEA rather than the bolted connection in the experiment (Figure 4-8(a)). The reasons for the sharp increase in the strain near this edge can be attributed to the very small gap (2mm) between the inner and outer beams and the stress concentration due to sharp corners. The fatigue limit of the general purpose hardened aluminum alloy Al6061-T6 is 1400ppm. The maximum strain in aluminum is lower than the allowable limit, indicating a safe operation. Moreover, filleting the sharp corners would further reduce the peak strain and improves the operating life time of the structure.

(a)

(b)

Figure 4-11 Strain distribution in harvester materials for base acceleration $2\text{m/s}^2(\text{RMS})$ (a)

Aluminum structure with tip masses (b) MFC patch

As shown in Figure 4-11(b), the MFC is in tensile state and the maximum strain is 904ppm. The strain reaches its maximum at the end towards the outer tip mass and decreases along the length of MFC towards the inner tip mass. Although the MFC can withstand strains up to 4500ppm, its linear elastic limit is 1000ppm. The harvester materials undergo fatigue due to the cyclic electromechanical loading which can result in abrupt failure when the strain is well below the ultimate strength. In this case, the strain in MFC patch is below the linear elastic limit, suggesting a safe operation. In this way, FEA can be used for the failure analysis during

the conceptual design stage. The results presented in this section corroborate the accuracy and the advantage of using FEA for the performance evaluation of nonlinear energy harvesters. The same modeling technique can also be applied for FEA simulation of nonlinear PEHs employing the axial force between magnets for the broadband energy harvesting [106] against to the lateral force used in the designs considered in this study.

4.4 Applicability and limitations of FEA modeling

So far, two designs of nonlinear PEHs which use the magnetic interaction for broadband energy harvesting are modeled using FEA. The magnetic force is modeled using the nonlinear spring element available in ANSYS. However, the proposed technique for modeling the magnetic force is valid only for the configurations in which both the magnets interact in a single plane where the magnetic force is a function of single spatial variable. It can be observed that in the two designs of harvesters presented in this chapter, the magnetic force is a function of lateral distance. This gives the possibility to express the variation of nonlinear magnetic force with the deflection of nonlinear spring. If the magnetic interaction depends on more than one spatial variable, the variation of force cannot be expressed by the nonlinear spring. It should also be noted that the rotation of magnets is neglected while simulating the magnetic force with the nonlinear spring. The rotation of tip masses which hold the magnets will be typically below 10^0 for the common base excitations used in the energy harvesting applications. Ellies and Lemarquand [143] have given the formula for the axial and lateral forces between two rotated cuboidal permanent magnets. For the magnets used in this work, the percentage change in the magnetic force due to rotation is almost proportional to the rotation angle for values below 10^0 . Hence, the error in the calculation of magnetic force caused by neglecting the rotation is acceptable for the low base accelerations.

Only the linear elastic material models are allowed in ANSYS for the piezoelectric materials. Normally, the piezoceramics exhibit nonlinear material behavior at sufficiently high base accelerations [42]. However, the ambient vibrations available for common energy harvesting applications is normally found to be below 3m/s^2 (0-pk) [31, 115]. This work shows that the proposed FEA model predicts the behavior of harvester with good accuracy for the base acceleration 2.83m/s^2 , indicating that linear material model is sufficient for the considered piezoelectric material. ANSYS has a number of nonlinear material models available for modeling the substrate materials. However, the strain in the substrate material is usually below the elastic limit as previously shown in Figure 4-11(a). So the linear elastic material model is

sufficient for both the substrate and the piezoelectric materials. Hence the proposed finite element modeling technique along with the linear material model can be used to study the most of energy harvesters. In addition, ANSYS has an option for programming a user defined element with nonlinear material behavior. For high-amplitude base excitations, a user defined piezoelectric element with nonlinear material modeling capability can be developed in ANSYS.

Theoretical models have been extensively used for parametric and optimization studies for simple linear harvesters. For more practical and complex linear or nonlinear harvesters for which the theoretical models are not available, the proposed FEA method is an effective tool for their performance evaluation with moderate computational effort. This work has presented only FEA modeling of the monostable harvesters. In the future, the possibility of extending the proposed technique to bi-stable energy harvesters will be explored. The challenge lies in successfully capturing the chaotic behavior at various stages in the operation of bi-stable harvester. However, the response of bi-stable harvester needs to be evaluated for much longer time periods, thus the advantage of using FEA in such situations will be compromised.

ANSYS has various elements to model the electrical components ranging from resistors to diodes. However, only the linear elements like resistor, inductor, and capacitor are compatible with the piezoelectric analysis. Therefore, system-level FEA cannot be performed for the harvesters using practical energy harvesting circuits involving nonlinear electrical components. Nonetheless, FEA will still be useful in the design stages where the maximum power output or the harvested energy needs to be evaluated before integration with the practical harvesting circuits.

4.5 Summary

In the most of reported works on the piezoelectric energy harvesting, the use of FEA is limited for the linear harvesters only. FEA has yet been used for analyzing the nonlinear harvesters because of the difficulty involved in modeling the magnetic interaction and the heavy computational effort required for solving the coupled electrical-mechanical-magnetic problem. In this chapter, the nonlinear energy harvesters are accurately analyzed using FEA by modeling the magnetic force with a nonlinear spring element. The magnetic force is calculated over a range of separation distances either axially or laterally. The calculated magnetic force is then approximated using piecewise linear segments and given as the force-displacement behavior of the nonlinear spring element. Firstly, a monostable nonlinear energy harvester is modeled

and analyzed using the proposed technique. The voltage output from FEA over the frequency range matches well with the results from theoretical model with 3% discrepancy in the peak voltage. Later, a 2-DOF nonlinear harvester with complex geometry for which an accurate theoretical model is difficult to derive has been modeled and analyzed using FEA. The FEA simulation successfully captures the important dynamics of nonlinear harvester including the anti-resonance, internal resonance, and jump phenomenon. The results from the FEA are in good agreement with the experimental measurements indicating the applicability of proposed method for modeling complex harvester designs. Furthermore, the strain induced in the harvester materials is evaluated using FEA to verify the structural integrity of harvester. The accuracy and low computational effort suggest that the proposed technique of using nonlinear spring element to model the magnetic force is an effective way for the FEA modeling of the nonlinear PEHs.

CHAPTER 5   MODELING OF GEOMETRIC, MATERIAL, AND DAMPING NONLINEARITIES IN PIEZOELECTRIC ENERGY HARVESTERS

5.1 Introduction

The performance of harvesters operating in the anticipated vibration environments depends upon various nonlinearities existing in the electromechanical dynamic system. The linear models of PEHs have been extensively studied and experimentally validated by many researchers and are comprehensively discussed in Section 2.2. The behavior of piezoelectric materials can be modeled with linear assumptions in the constitutive relations only under low electric and strain fields. However, piezoelectric materials exhibit a wide range of nonlinear behavior when strong electric and/or large stress fields are applied. The nonlinear behavior under high electric fields is typically caused by the hysteresis between electric field and polarization. Moreover, piezoceramics exhibit nonlinear butterfly hysteresis between electric field and strain when a strong electric field is applied. These phenomena which usually occur in the actuator applications and have been extensively studied. On the other hand, piezoelectric energy harvesting applications are characterized by weak electric fields and strong stress fields. Despite of extensive research on vibration based PEHs, very few works have been reported on the nonlinear modeling of energy harvesters.

Wagner et al. [45] observed the dependence of natural frequency on the excitation voltage amplitude in the piezo-beam systems. The effect of material nonlinearity on the mechanical response of piezoelectric rods was studied by Birman and Suhir [46] and it was shown that neglecting the nonlinear stress-strain behavior results in significant discrepancy in the estimation of mechanical behavior. Stanton et al. [47] investigated the nonlinear response of bimorph PEHs comprising PZT-5A and PZT-5H laminates. In their modeling, the effects of nonlinear elastic (material), electroelastic (coupling), and dissipation (damping) were considered. In their study, the influence of nonlinearities was observed at base accelerations as low as 0.5g. However, the nonlinear coefficients were obtained by fitting the experimental data to the steady-state mechanical amplitude obtained from the theoretical model. Besides, a single nonlinear damping coefficient was evaluated for a wide range of accelerations (0.05g-2g) from the experiments, which is another limitation as damping varies with the excitation amplitude. Similar approach was undertaken in the other works on nonlinear modeling of PEHs [48-50].

A global nonlinear distributed-parametric model for a unimorph harvester accounting for geometric, inertial, piezoelectric, and fluid drag nonlinearities was developed by Abdelkefi et al. [51]. Theoretical study was carried out to evaluate the effects of various nonlinearities on the harvester's response. The results indicated that the nonlinear stress-strain coefficients significantly impact the softening or hardening behavior of the harvester. Recently, Leadenham and Erturk [42] proposed a unified nonlinear nonconservative model to investigate the dynamic behavior of a bimorph cantilever for energy harvesting, sensing, and actuation applications.

In most of the above works, high resonant frequencies were considered and the harvester beams were assumed to be geometrically linear, which is acceptable because of the low mechanical response at such high frequencies. However, for harvesters working at low frequencies where the deformations are significant, the linear geometric assumption may not be valid. In this chapter, the effect of geometric nonlinearity and the validity of linear geometric assumption will be studied using finite element modeling. Furthermore, the influence of material and damping nonlinearities on the cantilever-type PEHs will be studied using a nonlinear dynamic model. The material nonlinearity is modeled using experimentally obtained nonlinear stress-strain curve and the damping coefficients are evaluated using the energy balance method.

5.2 Finite element modeling of geometric nonlinearity in energy harvesters

In this section, a comprehensive parametric study is carried out on five different configurations of harvesters to investigate the effect of geometric nonlinearity. Moreover, a parameter is defined and a marginal value of the parameter is considered to decide on the inclusion/exclusion of geometric nonlinearity in analyzing the harvester. The schematic of a conventional cantilever-type PEH with a patch of piezoelectric material bonded to its root is shown in Figure 5-1(a). A tip mass is attached to the free end of cantilever beam to tune the natural frequency. In this study, aluminum (Al6061-T6) is used for the cantilever substrate and MFC (Smart Materials Corp., model: M2807-P2) is used as the piezoelectric material. DP-460 Scotch Weld structural epoxy adhesive is used for bonding the MFC to the substrate. A resistor is connected across MFC to measure the voltage.

Figure 5-1 (a) Schematic representation of cantilever type PEH (b) Finite element model of PEH

The finite element model of PEH is shown in Figure 5-1(b). The finite element modeling and analysis is performed using ANSYS® Academic Research 14.0. The cantilever beam, epoxy, and tip mass are modeled with 3D 20-node hexahedral elements (SOLID186) having three translational DOFs at each node. The MFC patch is modeled with 3D 20-node piezoelectric element (SOLID226) which has one additional DOF of voltage at each node along with the three translational DOFs. A resistor element (CIRCU94) is connected across the MFC with one of its nodes grounded, and the voltage is measured at the other node. ANSYS offers a wide variety of damping models such as the Rayleigh damping, material damping, modal damping, element damping, and constant damping ratio and so on. However, the constant damping ratio option is only applicable to the linear analyses. The Rayleigh proportional damping is opted in this study to induce damping in the system, which is commonly used in the nonlinear analysis. The Rayleigh damping is specified as $\alpha=0$ and $\beta=2\xi/\omega_n$, where α and β are the proportionality constants in the Rayleigh damping model, ξ is the damping ratio, and ω_n is the natural frequency

corresponding to the fundamental mode. More details on the available damping models in ANSYS can be found in documentation. The geometric parameters and material properties required for the FEA modeling are given in Table 5-1. Tensorial properties of d31 mode of the piezoelectric material are given as input and the other non-active modes are suppressed by assigning null value to the respective constants.

Table 5-1 Geometric parameters and material properties of PEH

Parameter	Piezoelectric	Substrate	Epoxy
Material	MFC 2807-P2	Aluminum	DP-460
Length (mm)	28	70	28
Thickness (mm)	0.3	0.67	0.1
Width (mm)	7	10.6	7
Elastic modulus (GPa)	30.336	68.9	2.7
Poisson's ratio	0.31	0.33	0.4
Mass density (kg/m ³)	5440	2700	1100
Piezoelectric constant (C/m ²)	-5.16	--	--
Capacitance (nF)	9.5	--	--
Damping ratio (%)	1	--	--
Tip mass (g)	7.3	--	--

Finite element analysis is performed for both geometrically linear and nonlinear cases to compare the harvester's response. The linear case is performed using the harmonic analysis option in ANSYS. For the geometrically nonlinear case, large-deflection analysis is activated using NLGEOM option. Transient analyses are carried out to investigate the behavior of harvester for the nonlinear cases. The procedure for performing transient analysis is outlined in Section 4.2.3. The voltage response from the harvester over a frequency range is compared in Figure 5-2(a) for geometrically linear and nonlinear cases at base accelerations 0.5g, 1g, and 1.5g. The resonant frequency of the linear harvester is observed at 30.2Hz. It can be clearly inferred from figure that the effect of geometric nonlinearity is almost negligible for base accelerations below 1g. But the frequency shift due to the hardening behavior caused by the geometric nonlinearity is noticeable at 1.5g. The peak of voltage response is shifted to the right by 0.1Hz with a dip of the peak value. The peak voltage is reduced because the amplitude of

vibration decreases with increasing resonant frequency for a constant base acceleration and vice-versa.

The similar trend of negligible influence of geometric nonlinearity cannot be assumed for different configurations of harvesters for low base accelerations. To further investigate the influence of geometric nonlinearity, simulations are carried out with different substrate beam lengths and thicknesses. In this way, the effect of geometric nonlinearity can be evaluated for a range of beam configurations that are generally used in the energy harvesting applications. The resonant frequencies of all configurations of PEHs are tuned to 30.2Hz by adjusting the tip mass. Simulations are carried out for different geometries at their corresponding optimal resistances where the harvester generates maximum power.

Firstly, simulations are carried out with substrate beam lengths of 50mm, 70mm, and 100mm, each with thickness of 0.67mm. The discrepancy between mechanical responses of geometrically linear and nonlinear cases at resonant frequency for different base accelerations is shown in Figure 5-2(b). The ratio of tip displacement to length of the beam (δ/l_s) is used for comparison rather than voltage as the former gives better indication of occurrence of large deflections. It is evident from the figure that geometrically linear assumption is valid for δ/l_s values below 0.1. The beams of length shorter than 50mm are not considered in this parametric study as they are characterized by higher stiffness and therefore require much higher base accelerations to induce large deflections.

Another set of simulations are carried out with different beam thicknesses of 0.3mm, 0.67mm, and 1mm, each with beam length of 70mm and the results are shown in Figure 5-2(c). Beams thicker than 1mm are not studied as heavier tip masses are required to tune the resonant frequency, which is not practical. Beams below 0.3mm thickness are not analyzed as they are characterized by less strain and consequently less power. It can be observed from Figure 5-2 (b) and (c) that none of the beams require inclusion of geometric nonlinearity at accelerations below 0.5g. The maximum discrepancy between the geometrically linear and nonlinear cases for all the considered configurations is below 2% for base accelerations below 0.5g. Moreover, the amplitude of vibration sources available for energy harvesting applications is mostly below 0.3g or 3m/s^2 [7, 115]. It is apparent from these observations that the geometrically linear assumption is valid for the harvesters working under the available ambient vibrations.

Figure 5-2 (a) Influence of geometric nonlinearity on voltage response of PEH for beam with 70mm length and 0.67mm thickness at base accelerations of 0.5g, 1g, and 1.5g (b) Comparison of mechanical responses of beams of different lengths (thickness=0.67mm) (c) Beams of different thicknesses (length=70mm) considering geometrically linear and nonlinear cases for various base accelerations. Solid lines represent geometrically linear cases and lines with circles represent geometrically nonlinear cases

From Figure 5-2(b) and (c), it can be concluded that the geometrically linear model is accurate for δ/l_s values below 0.1, beyond which the modeling of PEHs requires the inclusion of geometric nonlinear effect. The error between the linear and nonlinear cases is less than 5% for cases where δ/l_s is below 0.1. The maximum discrepancy is induced between the linear and nonlinear cases for the beam with 0.3mm thickness and 70mm length. It is obvious that the thin beam has lower stiffness resulting in large deflection when compared with other configurations. It should be noted that 1% damping ratio is used in all the simulations. Normally, damping is more than 1% at higher base accelerations, which results in lower amplitude than the predictions from these simulations. Though aluminum is used as substrate in this study, the above observations are applicable to other substrate materials such as brass and steel also as they have higher elastic modulus resulting in higher stiffness and lower mechanical response. Hence the marginal value 0.1 of δ/l_s for choosing the appropriate model is conservative and applicable to most of the commonly used geometries, substrate materials, and excitation amplitudes in PEH applications. From the above results, it is concluded that it is necessary to include geometric nonlinearity in analytical or FEA models when δ/l_s ratio exceeds 0.1. Elvin and Elvin [144] studied the large deflection effect on PVDF unimorph for static and dynamic loadings for a single geometric configuration. However, high damping ratio (4.3%) was used in the simulations and a parametric study such as in the present study was not carried out.

5.3 Modeling of material and damping nonlinearities in piezoelectric energy harvesters

In the previous section, the geometric nonlinearity is modeled in ANSYS finite element package. However, only linear material models are considered for both piezoelectric and substrate materials in obtaining the response of harvester. Though various nonlinear material models exist for commonly used substrate materials in ANSYS, the strain in substrate materials normally does not exceed their linear elastic range because of their superior strength when compared to the piezoelectric materials. Hence in PEH applications, we can assume linear elastic material behavior for the substrate but not for the piezoelectric material. However, nonlinear piezoelectric material models do not exist in any of currently available finite element software packages. Hence an analytical model is presented in this section considering higher order elastic terms in the stress-strain behavior of piezoelectric material but with the assumption of linear electromechanical coupling. Prior to that, issues in the existing nonlinear

models of PEHs are discussed. Later, the details of tensile testing of MFC are presented, followed by nonlinear modeling of PEH and damping estimation.

5.3.1 Issues in the nonlinear models of PEHs

Most of the nonlinear material models reported in the literature of piezoelectric energy harvesting take the form of higher order polynomials, and the data fitting techniques are used to determine the unknown coefficients in the polynomials. The responses from the nonlinear analytical model are fitted to the experimental data such as voltage or displacement response at certain operation conditions (excitation frequency, acceleration, and load resistance) to identify higher order elastic coefficients in the material model. Similar procedure is also applied to identify the damping coefficients. Using the identified nonlinear coefficients, the analytical nonlinear model is validated with another set of experimental data like voltage, tip displacement, velocity or current at the same or slightly different operation conditions. It is obvious that the analytical models using the fitted coefficients obtained from one set of experiments would match reasonably well with another set of experimental data. For example, a model validated by the voltage response would predict well the displacement response in the same frequency range. However, this is an indirect method of validating the proposed models and its accuracy is questionable beyond the validated frequency range or excitation level. The uniqueness or validity of the determined coefficients should be properly investigated. It will be shown shortly that the unknown nonlinear coefficients identified using data fitting methods without proper investigation might be highly erroneous.

Table 5-2 Nonlinear material models of piezoelectric materials proposed in various studies

Author	Material	Constitutive relation	Coefficients (Pa)
Stanton et al. [47]	PZT-5H	$\sigma = A_1\varepsilon + A_2\varepsilon^3 + A_3\varepsilon^5$	$A_1=60.6\text{E}9, A_2=-9.61\text{E}17, A_3=9.695\text{E}25$
	PZT-5A	$\sigma = B_1\varepsilon + B_2\varepsilon^3 + B_3\varepsilon^5$	$B_1=61\text{E}9, B_2=-9.77\text{E}17, B_3=1.47\text{E}26$
Mak et al. [48]	PZT-5A4E	$\sigma = C_1\varepsilon + C_2\varepsilon^2$	$C_1=66\text{E}9, C_2=-2.55\text{E}14$
Patel et al. [145]	PZT	$\sigma = D_1\varepsilon + D_2\varepsilon^2$	$D_1=62.1\text{E}9, D_2=-3.5\text{E}14$
Leadenham & Erturk [42]	PZT-5A	$\sigma = E_1\varepsilon + E_2\varepsilon^2$	$E_1=66\text{E}9, E_2=-60\text{E}12$

Figure 5-3 Nonlinear stress-strain behavior of piezoelectric materials estimated using curve fitting techniques in previous works

The nonlinear elastic coefficients identified using data fitting techniques in some of the previous works [42, 47, 48, 145] are given in Table 5-2 and the corresponding nonlinear stress-strain responses are plotted in Figure 5-3. Stanton et al. [47] studied the softening response in PEHs comprising PZT-5A and PZT-5H laminates. The nonlinear resonance equation was derived and used in conjunction with the experimental data to fit the unknown parameters. The exact fit between the analytical and experimental voltage and displacement responses was achieved with fitted parameters. However, the stress-strain curve in Figure 5-3 plotted using the determined coefficients shows hardening response at strains as low as $50\mu\epsilon$ and the stress values rise steeply for strains above $200\mu\epsilon$. In their work, a bimorph of dimensions $24.06\text{mm} \times 6.4\text{mm} \times 0.67\text{mm}$ using different tip masses was excited at base accelerations ranging from 0.145g to 1.7g between 200Hz and 600Hz . A simple FEA on such beam configuration with 0.787g tip mass has shown a maximum strain of $65\mu\epsilon$ on the bimorph for a base acceleration of 0.56g . It is obvious that the bimorph undergoes much higher strain when operated at base acceleration of 1.7g and also shows the nonlinear softening behavior. However, the stress-

strain behavior in Figure 5-3 shows the hardening response beyond $50\mu\epsilon$ and thus indicating error in the analysis.

Mak et al. [48] proposed a quadratic term to represent the material nonlinearity of piezoceramics. The measured tip displacement data at short circuit condition was compared with the theoretical response for identifying the nonlinear elastic coefficient. The response of nonlinear model using the identified coefficient was validated with the experimental data obtained at load resistances of $1k\Omega$, $4k\Omega$, and $60k\Omega$ which are close to the short circuit conditions but no comparison at higher load resistances was presented. The identified nonlinear stress-strain behavior gives parabolic variation of stress versus strain which is inconsistent with generally observed material behavior. As shown in Figure 5-3, the compressive stress is predicted for tensile strains beyond $200\mu\epsilon$. Similar trend can also be observed in the works of Patel et al. [145] and Leadenham and Erturk [42].

In contrast to the above works, Goldschmidtboeing et al. [138] attributed the softening response of similar bimorph as used by Stanton et al. [47] under comparable operating frequency and vibration amplitude to the ferroelastic hysteresis. The constants in the hysteresis model were determined again by matching the theoretical voltage and tip displacement responses to the experimental data. Therefore, the softening response of PZT was attributed to the material and damping nonlinearities by Stanton et al. [47] while Goldschmidtboeing et al. [138] attributed it to the ferroelastic hysteresis, resulting in confusion on which mechanism truly causes the observed softening behavior. The problem in these works lies in assuming specific sources for the nonlinearity observed in the experiments and identifying the unknown coefficients in the nonlinear model using data fitting. It can be concluded from above discussion that the nonlinear material model and its coefficients determined using data fitting techniques might be erroneous beyond the specific frequency range and excitation level and require thorough investigation. Hence in this work, we carry out the tensile tests on MFC to obtain the actual nonlinear stress-strain behavior and use this physical behavior to derive the governing electromechanical differential equations.

5.3.2 Tensile testing of macro fiber composite

Tensile tests are carried out on MFC specimens of dimensions $85\text{mm} \times 7\text{mm} \times 0.3\text{mm}$ in the axial direction. The gauge length between the grips is kept to 28mm , similar to the length of MFC used for the PEH (see Table 5-1). A high endurance strain gauge DSFLA-5-350 is attached to the middle portion of specimen and the strain values are measured using the NI-

9237 data acquisition strain measuring module. The MFC with bonded strain gauge is shown in Figure 5-4(a). The MFC specimens are tested using the MTS C-42 tensile testing machine with 250N load cell and screw grips. The load cell has very high accuracy with a tolerance of $\pm 0.5\%$ of the applied load value. The specimens are loaded with a crosshead speed of 0.1mm/min up to strain of $4500\mu\epsilon$ which is given by the supplier as the maximum limit.

(a)

(b)

Figure 5-4 (a) MFC with bonded strain gauge used for tensile testing (b) Multiple cracks across MFC specimen after tensile testing. Crack sites are highlighted with black rectangles

The stress-strain behavior obtained from five test specimens is plotted in Figure 5-5. The stress is calculated by dividing the applied load with the cross sectional area of MFC specimen (2.1mm^2). The feature that can be observed from Figure 5-5 in the context of current study is the notable nonlinear stress-strain response for lower strains (e.g. $<1000\mu\epsilon$). Earlier, Williams et al. [146] fitted the experimental data using the Ramberg-Osgood model as shown in Figure 5-5 and proposed $1000\mu\epsilon$ as the linear elastic limit for MFC by ignoring the slight nonlinearity below $1000\mu\epsilon$.

Figure 5-5 The experimental data of stress-strain behavior of MFC obtained from tensile testing

The distinct difference between the present study and earlier work is the value of ultimate stress that MFC can withstand. While tests performed by Williams et al. [146] have shown a higher ultimate stress of 43MPa, MFCs in our tests could withstand only 36.5MPa. The difference is due to the size of specimens used for the testing. Williams et al. used much larger MFCs with dimensions 88.9mm x 25.4mm x 0.302mm. The size of specimens normally would not influence the material strength. However, this is not the case for MFCs which are made of PZT fibers embedded in the epoxy matrix. MFC specimen with a wider lateral dimension has a larger number of PZT fibers, which help in better bonding between the PZT fibers and epoxy matrix. Therefore, bigger MFCs are able to withstand higher stress.

It can be clearly seen in the results of present study that slightly higher degree of nonlinearity is observed for strains below 1000 $\mu\epsilon$. Though some deviation is observed in the trend at lower strains from specimen to specimen, the overall behavior confirms the nonlinearity at lower strains. The reason could be due to the smaller size of specimens with lower strength as discussed earlier. Furthermore, 1000kN load cell was used in the earlier study where the maximum applied load on MFC is just about 0.035% (350N) of the capacity of load cell. The accuracy of load cell in reading such a small load is debatable. In our experiment, a 250N load

cell is used and the maximum applied load on the MFC is 77N. It should also be noted that in our experiments, the variation in strain gauge readings is about 0-75 $\mu\epsilon$ and the load cell value varied between 0N and 2N ($\sim$ 1MPa) in the unloaded condition. Another noticeable feature in Figure 5-5 is the dips in the stress-strain behavior above 1000 $\mu\epsilon$, which are due to the formation of cracks. Once the cracks are formed, stress is reduced and increased again gradually indicating the hardening behavior. However, two specimens failed below 1000 $\mu\epsilon$ without showing any strain hardening behavior. Multiple cracks on one of the tested specimen are shown in Figure 5-4(b), where the cracks are formed across the MFC, damaging both the PZT fibers and epoxy matrix.

$$\sigma = C_1\varepsilon + C_2\varepsilon^3 + C_3\varepsilon^5 + C_4\varepsilon^7 + C_5\varepsilon^9 + C_6\varepsilon^{11} + C_7\varepsilon^{13} \quad (5-1)$$

To model the nonlinear behavior of MFC shown in Figure 5-5, equation (5-1) is proposed, which contains seven polynomial terms to fit the experimental data up to strains as large as 4000 $\mu\epsilon$. The corresponding coefficients are given in Table 5-3. A higher number of polynomial terms are required to accurately capture the sharp variation of stress with strain around 1000 $\mu\epsilon$. However, less number of polynomial terms can be used if the expected maximum strain in energy harvesting applications is not as high as 4000 $\mu\epsilon$. The odd degree terms are only considered to replicate the symmetrical stress-strain behavior in tension and compression regions. Including even order degrees in the constitutive stress-strain equations makes the material to follow different trends for tension and compressive stresses.

Table 5-3 Polynomial coefficients for MFC stress-strain variation

Coefficient	Value (Pa)	Coefficient	Value (Pa)
C_1	3.589E10	C_5	1.926E31
C_2	-1.074E16	C_6	-7.143E35
C_3	2.242E21	C_7	1.085E40
C_4	-2.747E26		

Figure 5-6 shows the polynomial fit for stress-strain relationship and the variation of elastic modulus with strain. The elastic modulus is evaluated using the derivative of stress (polynomial) w.r.t strain. The elastic modulus decreases by approximately 65% between 0 and 1000 $\mu\epsilon$ due to the material nonlinearity. The mean values of elastic modulus calculated from experimental data points are 35.2GPa and 12.2GPa at 25 $\mu\epsilon$ and 1025 $\mu\epsilon$, respectively. The

values of elastic modulus evaluated from the polynomial fit match closely with the experimental values indicating that the polynomial fit replicates the physical phenomenon observed in tensile testing. The observed material nonlinearity influences mainly the resonant frequency and consequently the dynamics of harvester when operating at different strain amplitudes. To understand and quantify the influence of material nonlinearity on the harvester's performance, the nonlinear governing equations have to be solved as explained in the following section. Note that the wavy trend of elastic modulus beyond $1500\mu\epsilon$ is due to the inherent oscillatory nature of polynomial fit.

Figure 5-6 Variation of stress and elastic modulus with strain

5.3.3 Nonlinear modeling of PEH

The influence of material nonlinearity of MFC on the harvester's performance was never investigated so far. The nonlinear stress-strain behavior obtained from tensile testing is incorporated in the nonlinear model to analyze the dynamics of harvester which is more accurate method than using the data fitting techniques. The governing equations of the PEH system (Figure 5-1(a)) can be obtained by employing the Lagrange's equations as described by Preumont [147]

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\eta}} \right) - \frac{\partial \mathcal{L}}{\partial \eta} = -c_{f1} \dot{\eta} - c_{f2} |\dot{\eta}| \dot{\eta} \quad (5-2)$$

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\lambda}} \right) - \frac{\partial \mathcal{L}}{\partial \lambda} = -\frac{V}{R} \quad (5-3)$$

where $\mathcal{L}$ is the Lagrangian of system, given as

$$\mathcal{L} = T - U \quad (5-4)$$

where T and U are the kinetic and potential energies of piezoelectric system. V is the voltage across the resistor R . $\eta(t)$ is the modal coordinate in the single mode approximation of beam's deflection $u(x, t) = \phi(x) \eta(t)$. $\phi(x)$ is the normalized fundamental mode shape of the cantilever beam. Throughout this work, the subscripts s , p , and g refer to the substrate, piezoelectric material, and epoxy glue, respectively. c_{f1} and c_{f2} are the damping coefficients and their details will be discussed later. The kinetic energy of the system is given by the summation of individual contributions from the substrate, piezoelectric material, and tip masses.

$$T = \frac{1}{2} m_s \int_0^{l_s} [\dot{u}(x, t) + \dot{Y}(t)]^2 dx + \frac{1}{2} (m_p + m_g) \int_0^{l_p} [\dot{u}(x, t) + \dot{Y}(t)]^2 dx + \frac{1}{2} m_t [\dot{u}(l_s, t) + \dot{Y}(t)]^2 \quad (5-5)$$

where $Y(t)$ is the base motion. m is the mass per unit length corresponding to the respective sections of beam and m_t is the tip mass. l_s and l_p are the lengths of substrate and MFC, respectively. The potential energy of system is the sum of contributions from the substrate, epoxy glue, and piezoelectric material.

$$U = U_s + U_g + U_p \quad (5-6)$$

Following the conclusions from Section 5.2, the geometric nonlinearity is neglected for the base accelerations considered in this study. By assuming the substrate and epoxy as linear elastic materials and applying the Euler-Bernoulli beam theory for evaluating the strain, the sum of potential energies of the substrate and epoxy glue layers is expressed as

$$U_s + U_g = \left(\frac{1}{2} E_s \frac{b_s(y_2^3 - y_1^3)}{3} \int_0^{l_s} [\phi''(x)]^2 dx + \frac{1}{2} E_g \frac{b_g(y_3^3 - y_2^3)}{3} \int_0^{l_p} [\phi''(x)]^2 dx \right) \eta(t)^2 \quad (5-7)$$

where E_* and b_* are the elastic modulus and width of the corresponding layers, respectively. y_* is the distance of different material layers from the neutral axis, as described in Appendix A.3.

Using equation (5-1) which describes the nonlinear stress-strain relationship of MFC, the constitutive equations of MFC are postulated as

$$\sigma_1 = C_1 \varepsilon_1 + C_2 \varepsilon_1^3 + C_3 \varepsilon_1^5 + C_4 \varepsilon_1^7 + C_5 \varepsilon_1^9 + C_6 \varepsilon_1^{11} + C_7 \varepsilon_1^{13} - e_{31} E_3 \quad (5-8)$$

$$D_3 = e_{31} \varepsilon_1 + \epsilon_{33} E_3 \quad (5-9)$$

Using the above constitutive equations, the total potential energy of piezoelectric layer is given by

$$U_p = \frac{1}{2} \int \left(C_1 \varepsilon_1^2 + \frac{1}{2} C_2 \varepsilon_1^4 + \frac{1}{3} C_3 \varepsilon_1^6 + \frac{1}{4} C_4 \varepsilon_1^8 + \frac{1}{5} C_5 \varepsilon_1^{10} + \frac{1}{6} C_6 \varepsilon_1^{12} + \frac{1}{7} C_7 \varepsilon_1^{14} - 2e_{31} \varepsilon_1 E_3 - \epsilon_{33} E_3^2 \right) dV_p \quad (5-10)$$

After substituting all the energy expressions into the equation (5-4), the Lagrangian of the system is given below.

$$\begin{aligned} \mathcal{L} = & \left(\frac{M_1}{2} \dot{\eta}(t)^2 + \frac{M_2}{2} \dot{\eta}(t) \dot{V}(t) + M_3 \dot{V}(t)^2 \right) \\ & - \left(\frac{K_1}{2} \eta(t)^2 + \frac{K_2}{4} \eta(t)^4 + \frac{K_3}{6} \eta(t)^6 + \frac{K_4}{8} \eta(t)^8 \right. \\ & + \frac{K_5}{10} \eta(t)^{10} + \frac{K_6}{12} \eta(t)^{12} + \frac{K_7}{14} \eta(t)^{14} + \Theta \eta(t) V(t) \\ & \left. - \frac{C^s}{2} V(t)^2 \right) \end{aligned} \quad (5-11)$$

Upon performing differentiations in the Lagrangian equations (5-2) and (5-3) and readjusting the terms, the governing electromechanical equations of piezoelectric system are written as

$$\begin{aligned} M_1 \ddot{\eta} + c_{f1} \dot{\eta} + c_{f2} |\dot{\eta}| \dot{\eta} + K_1 \eta + K_2 \eta^3 + K_3 \eta^5 + K_4 \eta^7 + K_5 \eta^9 + K_6 \eta^{11} \\ + K_7 \eta^{13} + \Theta V = -M_2 \ddot{V} \end{aligned} \quad (5-12)$$

$$\frac{V}{R} - \Theta \dot{\eta} + C^s \dot{V} = 0 \quad (5-13)$$

The coefficients in equations (5-12) and (5-13) are

$$M_1 = m_s \int_0^{l_s} [\phi(x)]^2 dx + (m_g + m_p) \int_0^{l_p} [\phi(x)]^2 dx + m_t [\phi(l_s)]^2 \quad (5-14)$$

$$M_2 = m_s \int_0^{l_s} \phi(x) dx + (m_g + m_p) \int_0^{l_p} \phi(x) dx + m_t \phi(l_s) \quad (5-15)$$

$$M_3 = \frac{(m_g + m_p)l_p + m_s l_s + m_t}{2} \quad (5-16)$$

$$\theta = \frac{e_{31} b_p (y_4^3 - y_3^3)}{2 t_p} \int_0^{l_p} \phi''(x) dx \quad (5-17)$$

$$C^s = \frac{\mathcal{E}_{33} b_p l_p}{t_p} \quad (5-18)$$

$$K_1 = E_s \frac{b_s (y_2^3 - y_1^3)}{3} \int_0^{l_s} [\phi''(x)]^2 dx + E_g \frac{b_g (y_3^3 - y_2^3)}{3} \int_0^{l_p} [\phi''(x)]^2 dx \quad (5-19)$$

$$+ E_p \frac{b_p (y_4^3 - y_3^3)}{3} \int_0^{l_p} [\phi''(x)]^2 dx$$

$$K_i = C_i \frac{b_p (y_4^{(2i+1)} - y_3^{(2i+1)})}{(2i+1)} \int_0^{l_p} [\phi''(x)]^{2i} dx \quad \text{For } i = 2 \text{ to } 7 \quad (5-20)$$

These governing differential equations are written in the state-space form and solved in MATLAB. The response of PEH obtained from solving the above equations is compared with the experimental results to validate the proposed model. In the next section, the method for estimating the damping coefficients is presented.

5.3.4 Damping estimation by energy balance method

Accurate estimation of the damping in the vibrating structures is by no means an easy task as many mechanisms in the system contribute to the damping. It is pointed out in Section 5.3.1 that in various works, a single damping coefficient was used for a wide range of base accelerations, which is not accurate. Therefore, in this section, one of the existing damping estimation methods is presented and extended to present study. Subsequently, the damping values estimated using this method are used to validate the proposed nonlinear model. Damping coefficients are identified for each tested base acceleration.

Numerous methods are presented in the literature for identifying viscous and non-viscous damping in the vibrating structures [148]. The most commonly used method for determining the viscous damping in a S-DOF system is the logarithmic decrement method based on the transient response of the system. Alternatively, the damping can be measured from half power limit and bandwidths. It can also be calculated as the ratio of energy lost in a cycle to input

energy. Though most of the techniques are employed for S-DOF systems, they can be extended to various modes of multiple-DOF system. Identifying the non-viscous damping in the structures is very challenging and the system specific methods are best for such scenarios. Liang and Feeny [149] used the energy balance method to identify the damping in mechanical vibration systems. The method involves balancing of the input energy given to the real system to the energy dissipated in the theoretical model to identify the damping parameters. The same methodology was later extended to identify the damping parameters in multiple-DOF vibration systems [150]. The main idea of this method is briefly presented below for the mechanical vibration systems and later extended to the piezoelectric systems.

Consider a mass-spring-damper system with external forcing and its equation of motion is given as

$$m\ddot{x} + kx + F(x, \dot{x}) = a(t) \quad (5-21)$$

where x is the displacement of mass m , k is the stiffness, and $a(t)$ is the periodic excitation. $F(x, \dot{x})$ represents the damping in the system. Multiplying the equation (5-21) with velocity ($\dot{x}$) on both the sides and integrating it over one cycle of periodic motion (T_1), we have

$$\int_t^{t+T_1} (m\ddot{x} + kx)\dot{x} dt + \int_t^{t+T_1} F(x, \dot{x}) \dot{x} dt = \int_t^{t+T_1} a(t) \dot{x} dt \quad (5-22)$$

When integrated over one time period of cycle, the contribution of conservative components of the system " $m\ddot{x} + kx$ " is zero and the following energy balance can be obtained to identify the damping parameters.

$$\int_t^{t+T_1} F(x, \dot{x}) \dot{x} dt = \int_t^{t+T_1} a(t) \dot{x} dt \quad (5-23)$$

This equation suggests that the energy dissipated by damping forces is the energy supplied to the oscillator via excitation. This method requires experimental measurement of the velocity of oscillator to evaluate the damping parameters. More details about this method can be found in Liang and Feeny [149].

In the present study, the above energy balancing method is opted for identifying the linear (C_{f1}) and nonlinear (C_{f2}) damping coefficients in the equation (5-12). It is assumed that the MFC is nonlinear elastic, suggesting that there is no energy loss even when the material is operating at high cyclic strains. Following the approach described above, the energy balance in the piezoelectric energy harvesting system can be expressed as

$$\int_t^{t+T_1} (c_{f1}\dot{\eta} + c_{f2}|\dot{\eta}|\dot{\eta}) \dot{\eta} dt = \int_t^{t+T_1} (-\Theta V - M_2 \ddot{Y}) \dot{\eta} dt \quad (5-24)$$

Integration of the above equation requires experimental measurement of the time-history response of voltage across the resistor and tip velocity ($\dot{u}$). Note that $\dot{u}(l_s, t) = \phi(l_s)\dot{\eta}(t)$ and $\phi(l_s) = 1$. The algebraic equations obtained after performing the integration can be solved for evaluating the damping coefficients. In the next section, the response of harvester is studied for two cases of damping, one with linear damping ($c_{f2}=0$) and the other with nonlinear damping.

5.4 Validation of the nonlinear model with experiments

Figure 5-7 Experimental setup of piezoelectric energy harvesting tests

After confirming the negligible influence of geometric nonlinearity on the performance of harvester for low base accelerations, a dynamic nonlinear electromechanical model of PEH is derived in the previous section considering the material and damping nonlinearities. Experiments are carried out to validate the proposed nonlinear model for the geometric parameters given in Table 5-1. The test specimen is made of aluminum (AL6061-T6) with dimensions 70mm x 10.6mm x 0.67mm and a patch of MFC (28mm x 7mm x 0.3mm) is bonded to its root. A tip mass of 7.3g made of acrylic is attached at the free end of cantilever beam. A strain gauge DSFLA-5-350 of dimensions 5mm x 2mm is bonded on the top of MFC at a distance of 2mm from the clamped end. The experimental setup for the performance evaluation

of PEH is shown in Figure 5-7. The base acceleration is given to the mounted harvester through APS shaker (APS 113), waveform generator (ALP1020), and amplifier (APS 125) combination. A Polytec laser vibrometer is used for measuring the displacement and velocity of the tip mass. The base acceleration, voltage, strain, displacement, and velocity are acquired through high precision NI data acquisition modules. The experiments are conducted at identified optimal resistance of $410\text{k}\Omega$. The experiments consist of frequency sweep tests at six different accelerations ranging from 1m/s^2 to 6m/s^2 . The reason for not testing at higher base acceleration is explained later.

Figure 5-8 Comparison between experimental and predicted voltage responses of PEH for accelerations ranging from 1m/s^2 to 6m/s^2 . Solid lines represent theoretical model predictions and dotted lines with circles represent experimental data

The root mean square voltage (V_{rms}) response of the harvester from the experiments is compared with the predictions from the nonlinear model in Figure 5-8. Nonlinear damping is assumed in the theoretical model for predicting the harvester's response. The damping coefficients at different accelerations evaluated using the energy balance approach are given in Table 5-4. The resonant frequency of harvester is observed at 31.1Hz for 1m/s^2 , and it decreases gradually as the base acceleration increases. The resonant frequency is dropped to 30.4Hz for the base acceleration 6m/s^2 , resulting in 0.7Hz of shift in the resonant frequency. The nonlinear

behavior in the form of frequency shift is apparent at all the tested excitations, indicating the absence of linear elastic limit for MFC. For example, for low accelerations of 1m/s^2 and 2m/s^2 , the resonance shifts from 31.1Hz to 30.9Hz. The nonlinear stress-strain behavior of MFC (Figure 5-5) even at low strain values is the reason for the observed resonant frequency shift.

The proposed nonlinear model displays the resonant frequency at 31Hz for the base acceleration of 1m/s^2 , showing only 0.1Hz difference from the experimental value. It is highly difficult to predict the same values as the experiment because of the assumptions in the proposed nonlinear model, experimental errors, and discrepancy in the material properties. For example, though the active length of MFC patch is 28mm, the length of encompassing kapton layers extends up to 36mm which is not taken into account in the theoretical modeling. Similarly, the thickness of epoxy glue (0.1mm) used for bonding MFC to substrate is an average value of three measurements taken by micrometer. Though uniform thickness of epoxy is assumed in the modeling, it may not be true in the experiment. Furthermore, the acrylic tip mass is assumed as point mass in evaluating the mode shape of harvester beam and the rotary inertia of tip mass is neglected. Considering these sources of error, the discrepancy between the experimental and predicted resonant frequencies is admissible. The responses of harvester predicted by the nonlinear model also show drop in the resonant frequency with increase in the base acceleration. However, when the acceleration is increased from 1m/s^2 to 6m/s^2 , the model predicts only 0.5Hz of frequency shift compared to 0.7Hz observed in the experiment. The magnitude of peak voltage at different base accelerations is generally well estimated by the nonlinear model with deviation below 5%.

Table 5-4 Damping coefficients estimated using energy balance method for linear and nonlinear cases

Base acceleration (m/s^2)	Linear damping	Nonlinear damping	
	c_{f1} (Ns/m)	c_{f1} (Ns/m)	c_{f2} (Ns^2/m^2)
1	0.0315	0.0287	0.0211
2	0.0348	0.0321	0.0217
3	0.0391	0.0309	0.0228
4	0.042	0.0241	0.0383
5	0.0447	0.0237	0.0364
6	0.0471	0.0233	0.0367

Figure 5-9 Comparison between experimental and predicted strain values on MFC for accelerations ranging from 1m/s^2 to 6m/s^2 . Solid lines represent theoretical model predictions and dotted lines with circles represent experimental data

The bending strain on MFC obtained from the experiments and theoretical model is compared in Figure 5-9. The strain is evaluated using the expression $\varepsilon = -y\phi''(x)\eta(t)$ where y is the distance of the top layer of MFC from the neutral axis and x is the axial location of the center of bonded strain gauge. The strain gauge is 5mm in length and bonded at 2mm from the clamped end and hence the strain is evaluated from theoretical model at $x=4.5\text{mm}$. The general trend of strain variation over the frequency range is well captured by the nonlinear model and the peak values of strain at different excitations are close to the experimental results. PEH undergoes cyclic reversals of tension and compression during its operation and the values shown in the figure are the amplitudes of strain either tension or compression. It is already pointed out earlier that the strain readings vary between $0\text{--}75\mu\epsilon$ at zero loading and the other sources of error are discussed. From figures Figure 5-8 and Figure 5-9, it can be concluded that the predictions from the theoretical model show very good agreement with the experiments. It is concluded from the present work that MFC doesn't exhibit clear linear elastic limit and PEHs need to be carefully designed for their resonance as it depends on the operating strain amplitude. It is known that the linear PEHs suffer from narrow half power bandwidth. While

matching the harvester resonant frequency with the excitation frequency in the design process, ignoring the nonlinear stress-strain behavior of MFC at lower strain amplitudes will induce the mismatch of frequencies resulting in power loss.

Figure 5-10 Voltage response of PEH over the time when excited at base acceleration of 6m/s^2

It is worth noting that the peak strain on MFC at 6m/s^2 is $1273\mu\epsilon$ (Figure 5-9), which is close to the region where the formation of cracks is observed during tensile testing. It is observed that voltage response of harvester is not consistent when excited at the resonance frequency corresponding to 6m/s^2 and has shown continuous drop with time. Figure 5-10 shows the variation of voltage response of harvester for 20 million cycles when excited at base acceleration 6m/s^2 . The PEH was tested approximately for 8 days to record the data for 20 million cycles. The performance of harvester was deteriorated gradually over the time and the voltage output was reduced by approximately 40%. The sudden drops visible in the voltage response could be attributed to the formation of cracks in the PZT fibers due to the high strain amplitude. The harmonic nature of loading caused the cracks to undergo repeated cycles of tension and compression. Sometimes crack closure could occur mainly because of compressive stresses, resulting in the recovery of voltage as evident in multiple instances during testing. However, the degradation in the voltage output is significant even after recovery. This

significant performance reduction is not acceptable for energy harvesting applications where high operating life with reliable performance is expected. The observation emphasizes the need for detailed investigation into the reliability of the performance of PEH at high strains or excitations, especially when nonlinear and broadband techniques are frequently adopted to increase the vibration amplitude and/or bandwidth of the harvesters. Owing to the continued performance reduction, the harvester is not tested at accelerations above 6m/s^2 .

Figure 5-11 Comparison between experimental and predicted tip displacement of PEH for accelerations ranging from 1m/s^2 to 6m/s^2 . Solid lines represent theoretical predictions with nonlinear damping, dotted lines represent theoretical predictions with linear damping, and dotted lines with circles represent experimental data

The displacement of tip mass obtained from the theoretical model is compared with the experiments in Figure 5-11. The theoretical study is carried out for two cases, one with linear damping and the other with nonlinear damping. The damping coefficients for both cases are given in Table 5-4. One set of time-history data of voltage and velocity measured at the resonant frequency is sufficient for determining the value of linear damping coefficient. But when the nonlinear damping is assumed, the energy balance results in an algebraic equation

with two unknown damping coefficients (C_{f1} and C_{f2}). In such case, two sets of voltage and velocity data are required for obtaining two linear algebraic equations to solve for the unknown damping coefficients. For this case, the voltage and velocity responses are recorded at accelerations $\pm 5\%$ away from the mean base excitation level. These responses are used in identifying the damping coefficients assuming that the damping does not change in that $\pm 5\%$ range of mean base excitation. As expected, the value of linear damping increases with the base acceleration. This trend clearly points out the mistake of using single damping coefficient for wide range of base accelerations in earlier works and its possible influence on evaluating the higher order elastic coefficients inaccurately.

In Figure 5-11, the displacement predicted with the nonlinear damping is closer to the experimental values than its counterpart with the linear damping. Slightly lower values of tip displacement are predicted with the nonlinear damping, which resulted in better agreement with the experimental observations. The difference between the responses from linear and nonlinear damping is almost negligible at base acceleration 1m/s^2 . It is clear that the discrepancy between the two cases increases with the base acceleration, indicating the growing influence of the nonlinear damping. The predictions from both linear and nonlinear damping models do not show much difference when compared to the experimental results because of the limited range of base accelerations ($\leq 6\text{m/s}^2$) tested in this study. Ideally, significant discrepancy could have been obtained if the experiments were carried out at higher base accelerations. But the harvesters are not tested beyond 6m/s^2 due to the formation of cracks in MFC as pointed out earlier.

The nonlinear elastic behavior for MFC assumed in the energy balance method to determine the damping parameters may not be valid when the strain amplitude on MFC is high. Nonetheless, the error observed in the prediction of harvester's response is minimal and hence the assumption is admissible. It is worth mentioning that the exact match between any of theoretical and experimental responses (voltage, strain or tip displacement) is not achieved in the present study because the responses are not fitted over the frequency range like in the previous works. For instance, the voltage and tip velocity data at a single frequency point (31.1Hz) is used to determine the damping coefficient at 1m/s^2 rather than fitting the data over a frequency range. This approach based on the energy balance is more realistic in simulating the dynamic behavior of harvester. Furthermore, very good matching between analytical predictions and experimental results is observed throughout the tested frequency range (29Hz - 32.5Hz) indicating minimal variation of the damping within this interval. However, the

damping cannot be used for prediction of the harvester's response much far away from the frequency where it is identified because damping depends on the frequency and amplitude in general. The value of δ/l_s at 6m/s^2 is 0.06 which is much less than the proposed marginal value 0.1 indicating the absence of geometric nonlinearity. The observed discrepancy between theoretical and experimental responses could be due to the discarded mechanisms in the present study such as electroelastic nonlinearities, ferroelastic hysteresis or other sources.

The harvester beam is tested essentially around its fundamental frequency (31.1Hz) as it is the case for almost all the designs of piezoelectric harvesters. So, it is reasonable to assume single mode approximation for the nonlinear modeling of harvester. The second and third modal frequencies of the harvester are 407.6Hz and 1073.8Hz, respectively, which are very far away from the fundamental frequency. Hence, the higher modes would not have any influence on the beam response. Erturk and Inman [41] compared the responses from the single mode and multi-mode distributed models and concluded that the single mode solution using the fundamental vibrational mode is valid over a wide frequency range around the first resonant frequency. Zhao et al. [151] also carried out a similar study on the cantilever-type galloping PEH and reported negligible influence of the higher modes on the response of harvester around fundamental frequency.

Though the proposed model predicts the performance of harvester with good accuracy, the complete agreement is not displayed. Barring experimental errors, the sources for discrepancy between the predicted and experimental responses could be due to ferroelastic hysteresis or nonlinear electromechanical coupling or other sources. In energy harvesting applications where piezoelectric material undergoes millions of cycles of mechanical stresses in life time, the ferroelastic hysteresis plays significant role and it is vital to understand its influence on the performance of harvester. Though weak electric field is a good assumption for modeling PEH, the importance of stronger influence of nonlinear S_1E_3 terms in the piezoelectric constitutive equations for higher mechanical strains cannot be ignored.

5.5 Summary

In this chapter, the influence of geometric, material, and damping nonlinearities on the dynamic response of harvesters is investigated. A finite element approach is undertaken to analyze the influence of geometric nonlinearity on different configurations of harvesters. It is found that the geometric nonlinearity has negligible influence at low base accelerations. A marginal value of 0.1 is proposed for the ratio of tip deflection to beam length (δ/l_s), below which the

geometrical linearity is a valid assumption and beyond which the modeling of geometric nonlinearity is necessary. Most of the past work in energy harvesting used data fitting methods to determine the unknown coefficients in the nonlinear models. It is shown in this work that the data fitting methods without proper investigation can lead to erroneous estimates of the nonlinear coefficients. A nonlinear dynamic model of cantilever-type PEH is derived from the first principles accounting for material and damping nonlinearities. The nonlinear stress-strain behavior of MFC is obtained from the tensile testing and it is incorporated into the piezoelectric constitutive equations. Moreover, the unknown damping coefficients at different base excitations are identified through the energy balance method. The nonlinear governing differential equations are solved using the state-space form in MATLAB and the predicted responses are compared with the experimental values. The comparison demonstrates very good agreement between the responses from modeling and experiment. The material nonlinearity of MFC is responsible for the softening behavior and shift in the resonant frequency of harvester. Furthermore, continuous degradation in the performance of the harvester is observed when it is excited at higher base accelerations due to the formation of cracks and possible debonding between the PZT fiber and epoxy matrix in MFC. Future studies will focus on the nonlinearity due to ferroelastic hysteresis and electroelasticity.

CHAPTER 6   **NONLINEAR PIEZOELECTRIC HARVESTER ARRAY FOR BROADBAND ENERGY HARVESTING**

6.1 Introduction

In the last decade, the piezoelectric energy harvesting from ambient vibrations has received significant academic focus due to its potential to replace/recharge the chemical batteries in wireless sensor nodes. However, the conventional linear piezoelectric energy harvesters (LPEHs) generate useful power only in the narrow frequency bandwidth while the ambient vibrations are normally scattered over wideband. Many techniques such as multi-modal, active or passive resonant tuning, nonlinear techniques, and oscillator arrays have been proposed for widening the operation bandwidth of harvesters and are extensively discussed in Section 2.3.3. Among these solutions, the array of harvester beams provides multiple resonant peaks which facilitates broadband energy harvesting. The resonant peaks and operation bandwidth can be tuned to match the excitation source frequency range with proper selection of the geometry of beams and tip masses. Originally, Sharuz [79] proposed the idea of using band-pass filter which is an ensemble of beams and masses for energy scavenging. Later, Xue et al. [80] investigated numerically the integration of multiple bimorphs for broadband energy harvesting. They reported that the operating frequency band of a harvesting structure can be broadened by connecting multiple bimorphs with different aspect ratios in series. Ferrari et al. [81] used a piezoelectric multi-frequency converter array for powering a battery less sensor module. The converter array was used to supply power to an autonomous sensor module that intermittently reads the signal from a passive sensor and sends the data via RF transmission. Lien and Shu [82] studied the electrical behavior of an array of LPEHs working with several interfacing circuits, including the standard AC/DC circuit and parallel/series SSHI circuits.

So far in the literature, only the array of LPEHs connected to an interfacing circuit has been considered for wideband operation. In this chapter, an array of monostable nonlinear piezoelectric energy harvesters (NPEHs) is proposed for broadband energy harvesting particularly for low amplitude vibrations ($\leq 2\text{m/s}^2$). The monostable NPEHs are realized with magnetic interaction at the tip of harvester beam and their dynamic response depends on the beam parameters, distance between the interacting magnets, and their polarity. Two monostable NPEHs, one with attractive configuration and the other with repulsive configuration, are connected to a standard AC/DC interface circuit. The gap distances between

the magnets are chosen so as to achieve favorable dynamics of the harvesters for broadband energy harvesting. The superiority of NPEH array over the LPEH array is demonstrated experimentally under both harmonic and random excitations. Both series and parallel connections between the two NPEHs are investigated and compared. Though the theoretical study and experiments are carried out with only two NPEHs, the idea can be extended for multiple NPEHs and thus the proposed idea is entitled as NPEH array.

6.2 Modeling of Nonlinear Piezoelectric Energy Harvester Array

6.2.1 Conceptual design of NPEH array

The conventional LPEH generates maximum voltage output at the resonance frequency only and suffers from narrow operation bandwidth. Although the array of LPEHs provides wider operation bandwidth, the deep valleys between the successive peaks in the electrical response are not favorable for efficient energy harvesting. If the parameters of individual LPEHs in the array are designed to achieve close resonant peaks to avoid the deep valleys, the overall bandwidth reduces. NPEHs with magnetic interaction have been thoroughly studied for broadband energy harvesting and experimentally validated. This study proposes an array of NPEHs to achieve more uniform electrical response over a wide bandwidth. Firstly, a monostable NPEH will be introduced in this section followed by the description of NPEH array.

A cantilever beam with a magnet at its tip, interacting with another stationary magnet is a typical configuration of monostable NPEH (Figure 6-1(a)). The magnetization direction is along the beam axis and both magnets are assumed to be perfectly aligned when the beam is at rest. When the beam vibrates under base excitation, the lateral force between the two magnets affects the stiffness of beam and also induces nonlinearity in the system. The voltage response of such monostable NPEH is shown in Figure 6-1(b). The resonant frequency of LPEH is shifted as the linear stiffness of beam is affected because of the magnetic interaction. The attractive configuration increases the linear stiffness of beam resulting in higher resonant frequency, while the repulsive configuration reduces the stiffness of beam resulting in lower resonant frequency. It can be observed from Figure 6-1(b) that the NPEH configurations have two stable states (higher and lower) in certain frequency range. When coexisting stable states present, the response of nonlinear system depends on the initial conditions [102, 140]. In the figure, solid and dotted lines indicate higher and lower energy states, respectively. For the NPEH with repulsive configuration under harmonic base excitation, initial conditions in the

forward frequency sweep capture the higher energy state while the backward frequency sweep results in the lower energy state and the NPEH with attractive configuration performs conversely.

Figure 6-1 (a) Schematic of monostable NPEH (b) Voltage response of monostable NPEH under harmonic excitation for different magnetic interactions (c) Schematic of NPEH array (d) Voltage response of NPEH array under harmonic excitation. Solid and dotted lines indicate high and low energy states, respectively

In this study, two monostable NPEHs; one with attractive configuration and the other with repulsive configuration are combined to achieve favorable electrical response over wide bandwidth. The schematic of the proposed NPEH array is shown in Figure 6-1(c) where one NPEH is attractive magnetic configuration and the other is repulsive configuration. The rationale behind choosing the opposite magnetic configurations for two harvesters can be understood from the voltage response of individual NPEHs as shown in Figure 6-1(d). With the opposite magnetic configurations and suitable system parameters such as beam length, tip mass, and distance between the magnets, two peaks in the electrical response can be achieved over wideband. Furthermore, the electrical response from one harvester is gradually decreasing when frequency is swept towards the resonant frequency of the other harvester. This trend provides an opportunity to achieve better electrical output over wider bandwidth by combining the electrical responses from individual harvesters. As shown in figure, the voltage response from the NPEH array is more uniform without a significant valley which is advantageous in the frequency-variant vibration environment. It will be shown subsequently through the experiments that the NPEH array performs better than the LPEH array. The theoretical modelling of NPEHs and parametric study for obtaining the gap distance between magnets for achieving wide bandwidth are presented in the next section followed by experimental results.

6.2.2 Theoretical modeling of nonlinear energy harvesters

The considered NPEH in this study comprises of an aluminum beam (AL7075-T6) and a patch of MFC (M2807-P2) bonded near the clamped end of the beam. The geometric parameters of two NPEHs are defined in Figure 6-2(a) and (b) and the numerical values of the parameters used in this study and the material properties are given in Table 6-1. The governing electromechanical equations of NPEH are given below.

$$M \ddot{u}(t) + C \dot{u}(t) + K u(t) + \Theta V(t) + F_{mag} = F(t) \quad (6-1)$$

$$\frac{V(t)}{R_L} + C^s \dot{V}(t) - \Theta \dot{u}(t) = 0 \quad (6-2)$$

where $V(t)$ is the voltage across the resistor R_L ; $u(t)$ is the modal displacement in the single mode approximation with displacement $w(x,t)=u(t)\phi(x)$; M , K , C , and Θ are the modal mass, stiffness, damping, and electromechanical coupling terms, respectively, corresponding to the fundamental mode $\phi(x)$; C^s is the clamped capacitance of the piezoelectric patch; and $F(t)$ is the mechanical forcing due to the base excitation. These modal terms can be evaluated from the expressions given in the Section 4.2.4 using the material properties and geometrical

parameters given in the Table 6-1. The mode shape of beam is evaluated using ANSYS 14.0 finite element package and used in the calculation of modal terms in the governing equations. The lateral force (F_{mag}) between the cylindrical magnets is evaluated using the very accurate closed form expression given by Vokoun et al. [141]. Refer to the Section 4.2.2 for the details on the calculation of magnetic force. Cylindrical NdFeB magnets (N35H grade) of 10mm diameter and 5mm thickness are used in this study. The governing equations (6-1) and (6-2) are written in the state-space form and solved for the voltage $V(t)$ across resistor and tip displacement $u(t)$ in MATLAB for respective NPEHs.

The theoretical open-circuit voltage output from each NPEH for different gap distances (D) between the magnets under harmonic base excitation of 2m/s^2 is shown in Figure 6-3. It is worth mentioning that the theoretical study is carried out with different resistors for each harvester to find the gap distance D for the respective configurations. The voltage responses are combined using circuitry only in the experiments. The resonant frequencies of harvesters without magnets are 15.6Hz and 28.1Hz, respectively because of the different lengths of substrates (107mm and 73.5mm). All the other geometric parameters are identical for both harvesters. When the magnetic interaction is induced, the peak response of NPEH with attractive configuration moves towards higher frequencies while that of NPEH with repulsive configuration moves towards lower frequencies as shown in Figure 6-3.

Figure 6-2 (a) Parametric representation of NPEH (b) Harvester cross-section details

Table 6-1 Geometric parameters and material properties of nonlinear piezoelectric harvesters

Geometric parameters		
Parameter	NPEH-1	NPEH-2
Magnetic configuration	Attractive	Repulsive
$L_s \times B_s \times T_s$ (all in mm)	107 x 18 x 0.65	73.5 x 18 x 0.65
$L_p \times B_p \times T_p$ (all in mm)	28 x 7 x 0.3	28 x 7 x 0.3
L (mm)	10	10
Tip mass, M_t (g)	8.2	8.2
Gap distance, D (mm)	17	9.5
Material properties		
Property	Aluminum	MFC
Density (kg/m^3)	2700	5440
Elastic modulus (GPa)	69.5	30.336
Poisson's ratio	0.33	0.31
Piezoelectric constant (C/m^2)	-	-5.157
Capacitance (nF)	-	14.3

Figure 6-3 Theoretical prediction of voltage output of two NPEHs with different gap distances (D) between the magnets. Atr = Attractive; Rep = Repulsive

A parametric study is carried out by changing the gap distance D for each harvester until the voltage responses from both harvesters are sufficiently close. From the results of parametric study shown in Figure 6-3, the voltage responses from attractive and repulsive configurations are sufficiently close for $D=17\text{mm}$ and 9.5mm , respectively. At these distances, peak voltages of $35V_{\text{rms}}$ and $28.5V_{\text{rms}}$ are obtained at 17Hz and 20.2Hz , respectively. It can also be observed that the responses from both configurations intersect at 18Hz with a voltage output of 10V from

each harvester. When the responses are combined, the expected output voltage would be more than 10V between 17Hz and 20.2Hz even after accounting for the losses during the rectification process in circuitry. Experiments are carried out on the prototype of NPEH array and findings are presented in the following section.

6.3 Experimental results and discussion

An array of NPEHs consisting of two harvesters is fabricated and experimentally tested. The overall experimental setup is shown in Figure 6-4(a) and the prototype mounted on the shaker is shown in Figure 6-4(b). The platform of the prototype is firmly attached to the shaker arm (APS116) and the base excitation to the shaker is provided by a controller (VR9500) and a power amplifier (APS125). MFC is bonded to the aluminum substrate using DP-460 epoxy adhesive at 10mm away from the clamping end. Tip masses are made of acrylic material and magnets are embedded in them. Provisions are made on the platform to slide the holders containing the fixed magnets to vary the gap distance D between the fixed and vibrating magnets. The voltage response is acquired through an NI data acquisition module NI-9229.

Figure 6-4 (a) Overall experimental setup (b) Prototype of NPEH array

6.3.1 Harmonic excitation

Figure 6-5 (a) Voltage output of two NPEHs from experiments with various gap distances (D) between magnets (b) Comparison between voltage outputs from experiments and theoretical predictions. Atr = Attractive; Rep = Repulsive; Exp = Experiment; The = Theoretical

Experimentally obtained open-circuit voltage response from both NPEHs for different gap distances (D) is shown in Figure 6-5(a) and compared with the theoretical predictions in Figure 6-5(b). It is evident from figures that the experimental results are in good agreement with the theoretical predictions. The NPEH with attractive configuration and with the gap distance $D=17\text{mm}$ achieves peak voltage of $34.8V_{\text{rms}}$ at 16.8Hz experimentally compared to the theoretical prediction of $35V_{\text{rms}}$ at 17Hz . The parametric study predicts $D=9.5\text{mm}$ as the optimal gap distance between magnets for the NPEH with repulsive configuration, but the experiments show favorable voltage response at a distance of 10mm . The repulsive configuration achieves peak voltage of $27V_{\text{rms}}$ at 20.1Hz as compared to the theoretical prediction of $28.5V_{\text{rms}}$ at 20.2Hz .

The damping ratio values used in the theoretical study are measured using the amplitude decay method [137] and the values are 0.77% and 0.85% for the attractive and repulsive configurations, respectively. For the linear cases i.e., when the magnets are absent, the damping ratio values are 0.74% and 0.62% at 15.6Hz and 28Hz, respectively. Overall, the discrepancy between the theoretical and experimental voltage responses is below 6%, indicating good accuracy of the theoretical model. While the experimental errors are unavoidable, the simplified theoretical study also contributes to the discrepancy, where the single mode approximation is adopted. Since both harvesters operate away from their respective fundamental frequencies because of the magnetic interaction, some error in the theoretical predictions is admissible. Finally, slight discrepancy in the material properties and measured damping ratios also contribute to the overall error.

Figure 6-6 NPEH array (a) series connection (b) parallel connection

The voltage responses from both NPEHs are combined after rectification to avoid the charge cancellation. Both series and parallel connections (Figure 6-6) between the harvesters are investigated and compared. A filtering capacitor (C_f) is added to smooth the DC voltage output and a resistor (R) is used as terminal load. The combined voltage response from two NPEHs using both series and parallel connections is shown in Figure 6-7. Clearly, a more uniform voltage output is achieved over wideband using the NPEH array which is desirable for broadband energy harvesting. However, significant difference in the voltage response is not observed between series and parallel connections. Experiments have also been carried out on the LPEH array for comparison purpose.

Figure 6-7 Combined voltage response from two NPEHs using series and parallel connections

Two LPEHs with substrate lengths (L_s) of 99.5mm and 95mm have been used to obtain the response in the same frequency spectrum as the NPEH array. All other system parameters have been kept the same. The comparison between the open-circuit voltage responses of NPEH and LPEH arrays using both series and parallel connections at harmonic base acceleration of 1m/s^2 and 2m/s^2 is shown in Figure 6-8(a) and (b), respectively. It can be clearly seen that the NPEH array has much wider operation bandwidth than the LPEH array especially when the NPEH array operates at the high energy states. The NPEH array shows 70% and 100% improvement in the bandwidth over its linear counterpart at 1m/s^2 and 2m/s^2 , respectively. Even at the low energy states, the bandwidth of NPEH array is still larger than the LPEH array. The NPEH array achieves a minimum voltage output of $10V_{dc}$ throughout the bandwidth. It is observed that the rectification losses are between $0\text{-}2V_{dc}$ for each harvester. As already pointed out, there is minimal difference between the open-circuit voltage output from series and parallel connections. The performance of NPEH array can be further enhanced using nonlinear SSHI and SCE circuits, which is beyond the scope of this article.

Figure 6-8 Comparison between NPEH and LPEH arrays. Open-circuit voltage at (a) $1m/s^2$ (b) $2m/s^2$. Power output at (c) $1m/s^2$ (d) $2m/s^2$. Solid and dotted lines indicate high and low energy states, respectively

The power outputs from NPEH and LPEH arrays are compared in Figure 6-8(c) and (d) for base excitation of $1m/s^2$ and $2m/s^2$, respectively. The power output is measured at the optimal

resistance values obtained for the respective configurations. The optimal load resistances for series and parallel connections of the NPEH array are $750\text{k}\Omega$ and $600\text{k}\Omega$, respectively and those of LPEH array are $950\text{k}\Omega$ and $700\text{k}\Omega$, respectively. The NPEH array achieves a minimum of $100\mu\text{W}$ and $200\mu\text{W}$ over wider bandwidth at 1m/s^2 and 2m/s^2 , respectively. Clearly, the NPEH array achieves useful power output over wider bandwidth when compared with the LPEH array. It is worth mentioning that depending upon the minimum power required for a specific application, the distance between the magnets in NPEH array can be adjusted accordingly. Minimum power output from NPEH array can be increased by adjusting the gap distances between the magnets but at the loss of bandwidth. As shown in Figure 6-8(c) and (d), the LPEH array achieves higher peak power outputs but the bandwidth is less and it also suffers from deep valleys in between the peaks. NPEH array generates a peak power output of $1146\mu\text{W}$ and more than $200\mu\text{W}$ of power output is harvested over a bandwidth of 2.8Hz under harmonic excitation of 2m/s^2 . On the other hand, LPEH array generates a peak power output of $1328\mu\text{W}$ at 2m/s^2 which is more than the NPEH array but the bandwidth corresponding to power output of $200\mu\text{W}$ is only 1.3Hz . Above results clearly indicate the obvious advantage of the NPEH array over the LPEH array. The results of experiments on prototype tested with random excitation are presented in the next section.

6.3.2 Random excitation

Majority of the ambient vibration sources are random in nature with their energy usually distributed over a wide frequency range. Hence, the proposed NPEH array is also tested for random excitations within the frequency range 14Hz - 24Hz and at amplitudes of 1m/s^2 and 2m/s^2 . Again, the LPEH array is tested for comparison purpose. The input signal to both NPEH and LPEH arrays for random excitation level 2m/s^2 in the frequency range 14Hz - 24Hz is shown in Figure 6-9(a). The voltage response from each harvester is rectified and then combined using parallel connection between the harvesters. The time-history of voltage responses from NPEH and LPEH arrays for 30s is plotted in Figure 6-9(b) and their fast Fourier transforms (FFTs) are plotted in Figure 6-9(c). It is evident from Figure 6-9(c) that the NPEH array has better operation bandwidth over the LPEH array. It is worth noting that the plotted FFT has peaks in the frequency range 32Hz - 42Hz . It is because FFT of a rectified signal gives amplitudes at even multiples of the frequency components in the original signal along with a DC offset value. Since the excitation frequency is in the range 14Hz - 24Hz , the FFT comprises amplitudes at 28Hz to 48Hz and its higher even multiples. The DC offset voltage values are 9.7V and 8.4V for the NPEH and LPEH arrays, respectively.

More experiments are carried out to evaluate the energy storing capacity of various configurations. The voltage output from each harvester is rectified and then combined using series or parallel connection to charge a capacitor. The accumulated energy on a 510 μ F capacitor under random excitation in the frequency range 14Hz-24Hz using different configurations is compared in Figure 6-9(d) and (e) at excitation levels of 1m/s² and 2m/s², respectively. The capacitor is charged for sufficiently long time (200s) and the voltage across the capacitor is monitored continuously. Each configuration is tested for 3 times and their average is used to evaluate the accumulated energy. It is evident from figures that the performance of NPEH array is superior to the LPEH array in terms of accumulated energy because of its wider operation bandwidth. At 1m/s², the NPEH array with series and parallel connections stores 80% and 30% more energy than its linear counterpart, respectively. At higher random excitation level of 2m/s², the NPEH array stores more energy than the LPEH array and charges the capacitor much faster. For instance, at 2m/s² excitation level, the LPEH array takes approximately 100s and 140s to store 3mJ of energy using parallel and series connections, respectively. However, the NPEH array takes only 80s and 110s to achieve the same, respectively, showing the obvious advantage of NPEH array.

From Figure 6-9(d) and (e), it is evident that the parallel connection stores higher energy than the series connection during the initial period of charging but the series connection wins back as the charging continues. Note that the capacitor is not charged at its optimal impedance throughout the charging period. The series connection has higher optimal resistance values than the parallel connection as notified in the previous section. When the capacitor is charged, the initial stage is close to the short-circuit condition (low impedance) and the fully charged stage resembles the open-circuit condition (high impedance). Hence, the impedance matching is favorable to the parallel connection during the initial period of charging, thus it stores higher amount of energy. As the charging continues, the impedance values gradually become more favorable to the series connection, therefore the series connection charges the capacitor more efficiently at the later stage. However, the accumulated energy from both connections gradually decreases after sometime as the harvesters are not able to harvest sufficient energy to charge the capacitor.

Figure 6-9 (a) Input signal to shaker-controller for random excitation level of $2m/s^2$ (b) Time-history voltage responses and (c) Fast Fourier transform of time-history voltage responses from NPEH and LPEH arrays with parallel connection under random excitation in the frequency range 14Hz-24Hz at $2m/s^2$. Accumulated energy on 510 μF capacitor under random excitation levels of (d) $1m/s^2$ and (e)

$2m/s^2$

From the above experimental results, it is quite evident that the NPEH array has better performance over the LPEH array under both harmonic and random excitations. For implementation of this idea in real life applications, the geometric parameters of harvesters, the dimensions of magnets, and the gap distance between the magnets need to be carefully selected to match the bandwidth of harvester with the frequency spectrum of vibration source. Although this study is limited to two NPEHs, the idea can be extended to multiple NPEHs to cover much wider frequency spectrum of the vibration source. It is also worth mentioning that only low amplitude base excitations (1m/s^2 and 2m/s^2) are considered in this study contrary to high amplitudes in many studies as high amplitude ambient vibration sources are rare. Since the proposed NPEH array outperforms the LPEH array at low amplitude vibrations, a much better performance can be expected at high amplitude vibrations. Similarly, the proposed idea is also applicable to vibration sources with higher frequencies. Contrary to the active resonance tuning techniques where tuning requires a lot of energy, the proposed NPEH array is passive in nature and has good potential for practical implementation.

6.4 Summary

This chapter proposes an array of NPEHs for broadband energy harvesting. Its advantage over the LPEH array in terms of broadening the operation bandwidth has been proved experimentally. The theoretical study is carried out with two NPEHs and a wide operation bandwidth is achieved by proper selection of the gap distance between the magnets. For comparison purpose, the prototypes of both NPEH and LPEH arrays have been fabricated and tested under harmonic and random excitations. The rectified voltage responses from harvesters are combined using series and parallel connections. Under harmonic base excitation, the NPEH array exhibits 100% improvement in the operation bandwidth over the LPEH array when operating at high energy states. At optimal load resistance, the NPEH array generates a peak power output of $1145\mu\text{W}$ and a bandwidth of 2.8Hz corresponding to power output of $200\mu\text{W}$ is achieved under harmonic excitation of 2m/s^2 . Under random excitation level of 1m/s^2 , the NPEH array using series connection stores 80% more energy compared to its linear counterpart. The proposed idea can be extended to multiple NPEHs to cover the frequency spectrum of wideband vibration sources. The passive nature of NPEH array in achieving the wider bandwidth is advantageous for practical implementation.

CHAPTER 7   EXPERIMENTAL INVESTIGATION OF PERFORMANCE RELIABILITY OF MACRO FIBER COMPOSITE FOR PIEZOELECTRIC ENERGY HARVESTING APPLICATIONS

7.1 Introduction

Structural integrity and reliable performance over extended periods of time are important considerations in the design and practical implementation of harvesters. The piezoelectric materials are brittle in nature and very susceptible to fatigue failure due to the cyclic electromechanical loading on the harvester. Normally, a PEH is comprised of a piezoelectric material bonded to a substrate such as aluminum, brass, steel, etc. Strength of some of the commercially available piezoelectric materials and commonly used substrate materials is given in Table 3-1. The values represent the strength of materials under various testing conditions such as quasi-static (tensile), fatigue (dynamic), bending, yielding, and fracture as notified in the supplier's data sheet. The material strength is specified either in $\mu\epsilon$ (micro strain) or MPa (stress). The fatigue strength of the piezoelectric materials which is crucial information required for designing highly durable harvesters is not provided in most of the instances. From the tabulated data, it is conspicuous that the dynamic strength of piezoelectric materials is much less when compared to the static and bending strength. Though macro fiber composite (MFC) has very high fracture strength, normally the material fails at much lower strain values under cyclic loading. Moreover, the information on the reliability of performance of materials such as the number of cycles without any degradation is not available.

It is also evident from the table that the strength of the commonly used substrate materials is much superior to the piezoelectric materials, indicating the possibility of early fatigue failure in the piezoelectric materials. However, very few works have taken the material strength into consideration in the design of PEHs. Anton et al. [129] carried out 3-point bending tests on monolithic piezoceramics (PZT-5A and PZT-5H), single crystal piezoelectric PMN-PZT, and commercially packaged QuickPack devices and reported their strengths to be used as the basis for the design of PEHs. These results are also included in the Table 3.1 and it can be observed that they are among the highest as they are obtained from the quasi-static bending tests. However, fatigue strengths are required for the design of harvesters rather than static strengths.

Shafer and Garcia [125] derived an expression for the maximum tolerable input acceleration that a linear energy harvester can sustain based on the ultimate strength of the piezoelectric material and used this expression to determine the maximum harvested power corresponding to that acceleration.

The fatigue behavior of piezoelectric materials has been extensively studied in the literature for actuator applications. Lupascu and Rödel [152] studied the microscopic mechanisms causing the fatigue phenomenon in the bulk PZT actuator materials under cyclic loading. The fatigue behavior of an adaptive structure comprising graphite/epoxy laminate with embedded PZT actuator was investigated by Mall and Hsu [153] for the mechanical and electromechanical cyclic loading conditions. However, only tensile stresses were applied in both cyclic loading conditions. Significant drop in the actuator's voltage was observed within one million cycles and higher voltage drop was noticed as the mechanical stress was increased. A similar study was carried out by Yocum et al. [154] on a composite laminate with embedded piezoelectric actuator under fully reversed electromechanical cyclic loading. Chaplya et al. [155] studied the influence of electro-thermo-mechanical loading conditions on the actuation capabilities of five commercial piezoelectric stack actuators with emphasis on the durability performance. Experimental results indicated the strong dependence of the piezoelectric properties and power requirements on both mechanical loading and temperature. Wang et al. [156] evaluated the reliability of piezoelectric actuators under high cyclic electric fields with different magnitudes of mechanical preload. The results demonstrated a monotonic decrease in the charge density and mechanical strain as loading cycles increased and the degradation was dependent on the preload stress. Wang et al. [157] also studied the piezoelectric and dielectric performances of a poled PZT subjected to the electric cyclic loading conditions similar to the high-field actuator applications. Bhattacharyya and Arockiarajan [158] investigated the influence of electrical fatigue on PZT for different loading frequencies and observed appreciable reduction in the piezoelectric and dielectric coefficients for high number of cycles.

In 2002, NASA Langley Research Center developed the MFC which is a low-cost piezo composite actuator with high flexibility [22]. The MFC consists of a single layer of rectangular PZT fibers embedded in the epoxy matrix and kapton shell. MFC has high strain energy density, directional actuation, conformability, and durability. It was reported that the MFC is capable of enduring large strains about $2000\mu\epsilon$ (peak to peak) up to 100 million cycles without any degradation in the free-strain performance. However, approximately 5% degradation in the performance was observed when the MFC was operated under cyclic loading of 1500V (peak

to peak), +300V bias at 500 Hz. MFC has also been used in the special applications apart from typical actuators/sensors/harvester devices. Tarazaga et al. [159] successfully tested the integration, packaging, deployment, thermal rigidization in vacuum, and operation of MFC in rigidizable inflatable boom application. The MFC was also used in the health monitoring of the wing skin-to-spar joints of unmanned aerial vehicles [160].

As pointed out earlier, the previous studies have evaluated the fatigue behavior of piezoelectric materials mainly for the actuator applications. While strong electric fields are common in the actuator applications, the piezoelectric materials in the harvester applications experience high stress or strain fields. Furthermore, they undergo completely reversed cyclic loading, meaning equal amplitudes of tensile and compressive strains with zero mean. Previous studies on the actuators used either tensile strains or biased voltages which resulted in lower compressive strains. Though bulk piezoceramics exhibit good compressive strength [161], thin or film like patches used in the actuators/sensors/harvesters do not have the capability to withstand high compressive loading. Moreover, high durability and structural integrity are required for the piezoceramics for practical applications of PEHs. However, neither previous works nor supplier's data sheets provide the estimates of material strength below which the piezoelectric materials perform with high reliability over extended periods of time.

In this chapter, the MFC is tested for reliability of its performance at different strain amplitudes with varied loading frequencies. Two types of experiments are performed for evaluating the performance of MFC. Firstly, the energy harvesting tests are carried out with cantilever-type PEH to estimate the voltage output and strain on MFC. Subsequently, several long duration tests are conducted with increasing strain amplitudes at different frequencies to evaluate the reliability of harvester's performance. Based on the results, a safe dynamic strain limit on the MFC is proposed for reliable operation.

7.2 Energy harvesting tests

The schematic of PEH used for the energy harvesting tests is shown in Figure 7-1(a). The PEH is comprised of a cantilever beam made of aluminum (AL7075) with a patch of MFC (M2807-P2) bonded to its root using DP-460 epoxy glue. A tip mass made of acrylic was attached to the free end of the cantilever beam to tune its resonant frequency. The tip mass has dimensions 25mmx15mmx10mm and a weight of 7.3g. A strain gauge DSFLA-5-350 of dimensions 5mm x 2mm was bonded on the top of MFC at distance of 2mm from the clamped end. A resistor was connected across MFC to measure the voltage output. The experimental setup of energy

harvesting tests is shown in Figure 7-1(b). The geometric parameters and material properties of the harvester used in the experiments are given in Table 7-1. The PEH was clamped to the shaker arm and the base acceleration was given through the combination of APS shaker (APS 113), waveform generator (ALP1020), and amplifier (APS 125). The Polytec laser vibrometer was used for measuring the displacement of tip mass. Data of base acceleration, voltage, strain, and displacement were acquired through high precision NI data acquisition modules. The experiments were conducted at identified optimal resistance of 410k Ω . The frequency sweep tests were conducted at six different base accelerations ranging from 1m/s² to 6m/s².

Figure 7-1 (a) Schematic representation of cantilever-type PEH (b) Experimental set up of energy harvesting tests

Table 7-1 Geometric parameters and material properties of PEH

Parameter	Piezoelectric	Substrate	Epoxy
Material	MFC 2807-P2	Aluminum	DP-460
Length (mm)	28	70	28
Thickness (mm)	0.3	0.67	0.1
Width (mm)	7	10.6	7
Elastic modulus (GPa)	30.336	68.9	2.7
Poisson's ratio	0.31	0.33	0.4
Mass density (kg/m ³)	5440	2700	1100
Piezoelectric constant (C/m ²)	-5.16	--	--
Capacitance (nF)	9.5	--	--
Optimal Resistance (k Ω)	410	--	--
Tip mass (g)	7.3	--	--

The root mean square voltage (V_{rms}) response of the harvester during the frequency sweep for different accelerations is plotted in Figure 7-2(a). The resonant frequency of harvester is observed at 31.1Hz for the base acceleration $1m/s^2$ and decreases gradually as the base acceleration increases. The resonant frequency is reduced to 30.4Hz for the base acceleration $6m/s^2$ and resulting in 0.7Hz of frequency shift. The maximum bending strain on the MFC is shown in Figure 7-2(b). The values indicate the amplitude of strain (either tension or compression) at the root of top layer of MFC. As already pointed out that the strain gauge is 5mm in length and bonded at 2mm from the clamped end, hence the strain is measured at a distance of 4.5mm from the clamped end where the maximum strain is induced. According to the Euler-Bernoulli beam theory, the variation of bending strain along the length of the beam is proportional to the curvature of beam. Using the above principle and by evaluating the mode shape of beam, it is determined that the measured strain is approximately 0.93 times of the maximum strain at the root for the above beam configuration. Hence, the maximum strain plotted in Figure 7-2(b) is calculated by dividing the measured strain by 0.93.

(a)

(b)

Figure 7-2 Experimental responses of PEH for base accelerations ranging from 1m/s^2 to 6m/s^2 (a) RMS voltage output across resistor (b) Maximum strain measured on MFC

The supplier's data sheet indicates $1000\mu\epsilon$ as the linear elastic limit of MFC. However, it is clear from Figure 7-2(a) and (b) that shifts in the resonant frequency is observed even from low

strain amplitudes. For example, at low accelerations 1m/s^2 and 2m/s^2 , with corresponding maximum strains of $340\mu\epsilon$ and $535\mu\epsilon$ in the MFC, the resonance shifts from 31.1Hz to 30.9Hz . Similar nonlinear behavior in the form of resonant frequency shift is apparent at all the tested excitations, indicating the absence of linear elastic limit for MFC contrast to the supplier's data. The reason for the shift in the resonant frequency with increase in the base acceleration is the nonlinear stress-strain behavior of MFC. The results of tensile tests carried out on MFC to understand the nonlinear stress-strain behavior are presented here. For complete details on the tensile testing of MFC, refer to Section 5.3.2.

(a)

(b)

Figure 7-3 (a) Nonlinear stress-strain behavior of MFC obtained from tensile testing (b) Variation of elastic modulus and stress with strain

The stress-strain behavior obtained from the tensile tests on five MFC specimens is plotted in Figure 7-3(a). The stress is calculated by dividing the applied force with the cross sectional

area of MFC specimen (2.1mm^2). A polynomial fit is constructed using the MATLAB curve fitting tool box from the experimental data. The slope of the polynomial fit which represents the elastic modulus is plotted in Figure 7-3(b). The important feature that can be observed from Figure 7-3(a) and (b) in the context of present study is the nonlinearity in the stress-strain behavior of MFC for lower strains (e.g. $<1000\mu\epsilon$). The elastic modulus decreases by approximately 55% between 0 to $1000\mu\epsilon$ due to the material nonlinearity and this explains the resonant frequency shift observed in the energy harvesting tests. Note that the wavy trend of elastic modulus beyond $1500\mu\epsilon$ is due to the inherent oscillatory nature of the polynomial fit.

Figure 7-4 (a) Finite element model of PEH (b) Strain variation on the harvester materials due to the weight of tip mass

The harvester materials experience some mean strain due to the weight of the tip mass. However, the magnitude of induced mean strain is very low and within the range of measuring accuracy of the strain gauge ($0\text{--}75\mu\epsilon$). Therefore, the mean strain on the PEH due to the weight of tip mass cannot be measured by the strain gauge and thus evaluated using simple finite element static analysis. Finite element model of tested PEH is shown in Figure 7-4(a) and the evaluated strain on the harvester materials is shown in Figure 7-4(b). MFC undergoes a maximum strain of $90\mu\epsilon$ at the clamped end. Note that, the resistor is not included in the finite element analysis as it is a static analysis. The strain gauge reading is normally calibrated to zero before commencing the dynamic tests on the harvester and hence the results in Figure 7-2(b) indicate only the strain amplitudes. Throughout this chapter, the strain values are rounded to nearest multiples of $5\mu\epsilon$. The true state of strain on MFC can be calculated by adding the mean strain value to the amplitude. The measured strain amplitude at the resonance for the base acceleration 6m/s^2 is $1370\mu\epsilon$. Hence, the state of strain on MFC varies between $-1280\mu\epsilon$ and $1460\mu\epsilon$ at resonance of 6m/s^2 .

The energy harvesting tests were not conducted at higher base accelerations as the voltage response at resonance corresponding to 6m/s^2 shows continuous degradation. The response of the harvester was recorded for half million cycles to estimate the performance degradation. In Figure 7-5(a), the variation of voltage output with the number of cycles is plotted for the base acceleration 6m/s^2 . The resonant frequency was checked approximately after every 25k cycles and the excitation frequency was matched with the new resonant frequency. Essentially, the harvester was tuned to vibrate at its resonant frequency throughout the tested period of 0.5 million cycles. The strain on MFC and resonant frequency are plotted in Figure 7-5(b). It can be observed that the voltage output is reduced to 50% within 0.25 million cycles of operation which is about 2.5 hours. The resonant frequency is also reduced, indicating the decrease in the beam stiffness possibly due to the excessive deformation or cracking in MFC or debonding between PZT fibers and epoxy matrix. The sudden rise in strain on the MFC between 0.2 and 0.35 million cycles indicates possible failure of the specimen. The sharp reduction in the voltage output can also be observed during that interval in Figure 7-5(a).

It is worth mentioning that the loss in the performance of harvester is not due to the fatigue under electrical loading but solely because of the fatigue due to alternating bending strain. Assuming a uniform electric potential across the PZT layer (0.18mm thickness) in MFC, the maximum value of induced electric field in the MFC is 0.29kV/mm only. This induced electric field in PEH is much lower as compared to the actuator applications. Hence, the performance

degradation in MFC due to cyclic electrical loading can be neglected. These results show that the MFC cannot withstand high strains for a long period of time.

(a)

(b)

Figure 7-5 Variation of PEH response over time at base acceleration of 6m/s^2 (a) RMS voltage (b) Strain and resonant frequency

The stress-strain behavior in Figure 7-3(a) also shows noticeable dips above $1000\mu\epsilon$ which are due to the formation of cracks. Once the cracks are formed, stress is reduced and increased again gradually indicating the hardening behavior. Several such trends can be easily observed in Figure 7-3(a), indicating the formation of multiple cracks. During the energy harvesting

tests, the voltage output of PEH is reduced sharply at 6m/s^2 because of formation of cracks as the strain amplitude is much higher than $1000\mu\epsilon$ (Figure 7-5(b)). Though the MFCs withstood strains up to $4500\mu\epsilon$ without fracture, it is obvious that their performance degrades even with the formation of a single crack. From Figure 7-3(a), $1000\mu\epsilon$ can be considered as the upper limit of MFC for quasi-static loading conditions. It will be shown in the subsequent section through the reliability tests that MFC shows appreciable degradation even at strains below $1000\mu\epsilon$ under dynamic loading.

7.3 Reliability tests of macro fiber composite

The piezoelectric materials in actuator/sensor/harvester applications undergo dynamic loading. Fatigue failure due to the cyclic stresses or strains on the materials is the cause for damage in more than 90% of mechanical structures [162]. In the context of present study, not only the permanent damage or fracture in the piezoelectric material, but considerable degradation in either mechanical or electrical response of the piezoelectric material can be considered as failure. For example, 30% reduction in the voltage output across the resistor load of harvester results in 51% drop of the power output which might render the harvester useless. Therefore, it is very important to understand the influence of dynamic loading on the performance of piezoelectric materials. To the best of authors' knowledge, neither previous works nor manufacturer's data sheets provide specific upper limit on the dynamic stress or strain below which a high reliable performance of MFC is expected.

In this section, the results of reliability tests on PEHs comprising MFC are presented. It is very difficult to conduct the conventional fatigue tests on the thin piezoceramic patches where repeated cycles of axial tension-compression are applied. It is pointed out earlier that the piezoceramics normally exhibit higher compressive strength than the tensile strength. However, as the size of specimen decreases, compressive load would lead to significant lateral movement of the specimen resulting in premature failure. It is worth mentioning that because of the susceptibility of thin PZT patches for failure in compression, most of the reliability tests conducted in the previous studies used biased voltages which induced lower compressive stress when compared to the tensile stress. For instance, MFC which consists of $180\mu\text{m}$ thin PZT fibers embedded in the epoxy matrix was reported to endure $2000\mu\epsilon$ (peak to peak) [22]. However, the applied voltage on the MFC actuator varied from -500V to 1500V which resulted in a state of strain approximately between $-500\mu\epsilon$ to $1500\mu\epsilon$.

In the present study, PEHs are opted for investigating the reliability of MFC under complete reversed bending cycles. The PEHs were tested for long duration of time at different base accelerations corresponding to certain strain amplitudes and the responses were recorded continuously. All the tests were conducted at the initial resonant frequency of harvester corresponding to the base acceleration, i.e., the base excitation frequency did not change while the resonance of harvester slightly shifted in the course of testing due to the material degradation. The typical operation frequency of piezoelectric harvesting applications ranges from 5Hz to 200Hz. Hence two sets of tests are conducted; one set with low resonant frequency around 30Hz and the other with high resonant frequency around 135Hz. The reliability tests conducted with low resonant frequency are carried out for 20 million cycles and those with high resonant frequency are carried out for 60 million cycles. The configuration of tested PEHs at low frequency for the performance evaluation of MFC is the same as the specimen used for the energy harvesting tests (see Section 7.2). However, the tip mass was removed in the performance tests conducted at high frequency (~135Hz). All the tests were conducted at experimentally determined optimal load resistances as shown in Figure 7-6. The optimal resistances values were 410k Ω and 82k Ω at the low and high frequencies, respectively. Although the aluminum beam and MFC dimensions were same for the both cases, the presence/absence of tip mass influenced the optimal resistance value as shown in the figure.

Figure 7-6 Variation of power output with load resistance for performance evaluation tests carried out at low and high frequencies

The results of first set of experiments carried out at low resonant frequencies ($\sim 30\text{Hz}$) are plotted in Figure 7-7. The voltage responses of ten PEHs tested over 20 million cycles at various base accelerations are plotted in the figure. Two specimens were tested at each base acceleration for better understanding of the material behavior and to reduce the margin of error in the findings. The resonant frequencies of tested harvesters varied between 29Hz and 31.1Hz depending on the base acceleration and also because of slight variation in the specimen preparation. From Figure 7-7(a), it is clear that the harvesters did not show any signs of degradation in the voltage output at 1m/s^2 and 2m/s^2 where strain amplitudes are below $600\mu\epsilon$. However, the voltage output from two harvesters tested at base acceleration of 3m/s^2 with strain amplitude of $765\mu\epsilon$ shows significant dips, indicating possible formation of cracks. The voltage output is recovered in both cases possibly due to the closure of cracks but has shown continuous degradation over the time in case of PEH-6.

The harvesters' performance at 4m/s^2 and 6m/s^2 is plotted in Figure 7-7(b). The results show multiple dips and severe degradation as the harvesters were tested at high strain amplitudes more than $1000\mu\epsilon$. A significant decrease of 55% in the voltage output of PEH-10 was observed after 20 million cycles. At high strain amplitude, dips in the voltage output were observed much earlier (less than 2 million cycles), indicating the early formation of cracks. The performance degradation of harvesters tested at high strain amplitude also indicates change in the resonant frequency as observed in Figure 7-5(b). The observed drift in resonant frequency during the operation of harvester is not acceptable as it may result in significant power loss. Though few active tuning techniques have been proposed in the literature, none of them are feasible as they dissipate most or all of the harvested power. Hence the degradation in the voltage output and change in the resonant frequency are considered as potential failures. From the results presented in Figure 7-7(a) and (b), it can be concluded that the performance of MFC is affected considerably beyond $750\mu\epsilon$ of strain amplitude.

The reliability of MFC performance was further investigated at high frequency to understand the influence of high strain rate. The same harvester configuration as in the previous tests is used without the tip mass, which possesses a high resonant frequency of about 135Hz . Degradation was observed around strain amplitude of $750\mu\epsilon$ in the previous tests conducted at low resonant frequencies. So, two tests were conducted at high frequency; one with $605\mu\epsilon$ and the other with $825\mu\epsilon$ of strain amplitude on the MFC.

(a)

(b)

Figure 7-7 Reliability tests for performance evaluation of harvesters at low resonant frequency ($\sim 30\text{Hz}$) and at different base accelerations (a) 1m/s^2 , 2m/s^2 , and 3m/s^2 (b) 4m/s^2 and 6m/s^2 . Measured maximum strain on MFC for each specimen is mentioned in micro strain ($\mu\epsilon$)

The strain gauge data from two tests is plotted in Figure 7-8. Strain amplitude of $560\mu\epsilon$ was obtained on the MFC of PEH-11 at resonant frequency of 136.9Hz under base acceleration of 2.69g and the corresponding maximum strain on MFC is $605\mu\epsilon$. High strain amplitude of $765\mu\epsilon$ on PEH-12 was attained at 132.6Hz under base acceleration of 4.31g and the corresponding maximum strain on MFC is $825\mu\epsilon$. The mean strain should be zero as these two harvesters do not have any tip mass. But small mean strain can be observed in Figure 7-8 due to the experimental error which is unavoidable. Note that the resonant frequencies for the above two cases are slightly apart just because of variation in the specimen preparation. Higher base accelerations were used in these tests conducted at high frequencies when compared to tests conducted at low frequencies. It is because the amplitude of vibration decreases with increasing resonant frequency for a constant base acceleration and vice-versa. Hence, high base accelerations are required to achieve sufficient amplitude to induce high strains on the MFC. It is worth mentioning that, such high base accelerations are not practically available; they were used in the present study to understand the influence of high strain rate.

Figure 7-8 Measured strain on MFCs of PEH-11 and PEH-12 during reliability tests at high resonant frequencies

Figure 7-9 Normalized voltage and displacement responses of PEH-11 and PEH-12 harvesters tested at high resonant frequency. Measured maximum strain on MFC for each specimen is mentioned in micro strain ($\mu\epsilon$)

The voltage and tip displacement were recorded for 60 million cycles which is approximately six days of testing. The voltage and tip displacement are normalized with their initial values and plotted in Figure 7-9. The results of PEH-11 with maximum strain of $605\mu\epsilon$ show reliable performance over the duration of testing with $\pm 3\%$ variation from the mean values. No significant degradation in the performance is observed during the entire testing period. However, the performance of PEH-12 under higher strain amplitude ($825\mu\epsilon$) shows steady decline over the time because of the material degradation. When the PEH is operated at high strain amplitudes, the stiffness of MFC decreases over the time because of the formation of cracks and consequently the resonant frequency of harvester reduces. When the resonant frequency reduces with time, the mismatch between the resonant frequency and excitation frequency increases as the excitation frequency is kept constant during the test. As the harvester is operated gradually farther away from its resonant frequency, its voltage output and tip displacement steadily decrease. Approximately 15% reduction in the voltage output is observed after 55 million cycles which results in 28% drop in the harvested power. These results are analogous to the trends observed in the previous tests conducted at low frequency, indicating that the frequency does not have significant effect on the fatigue behavior of MFC at least

within the range considered in this work. The strain data is not recorded as the durability of strain gauges cannot be as high as 60 million cycles.

Figure 7-10 Microscopic images of cracks induced in MFC specimens during reliability tests. Figures (a), (b), and (c) are captured at 100x, 500x, and 5000x magnification, respectively

The images of cracks induced in the MFC specimens during the reliability tests are captured using a scanning electron microscope and are shown in Figure 7-10. The protective polyimide film on the specimen is removed and a thin layer of platinum (in order of nm) is coated. The cracks shown in Figure 7-10(a), (b), and (c) are captured at 100x, 500x, and 5000x magnification, respectively. The cracks are formed perpendicular to the axial direction because of high strains induced in the MFC and the cyclic nature of strain due to the dynamic motion of harvester further assists in the crack propagation. The parallel slots visible in Figure 7-10(a) are the imprints of interdigitated electrodes sandwiched between PZT fibers and polyimide film. One or more cracks are observed when the MFC was tested above strain amplitude of

700 $\mu\epsilon$. From these results, it can be concluded that the MFC shows high reliable performance under completely reversed strain amplitudes below 600 $\mu\epsilon$ only.

It is observed that the performance of harvesters is not degraded for accelerations up to 2m/s². These results imply the safety and durability of prototype designed in Chapter 6. The nonlinear harvester array is designed to harvest energy from wideband spectrum at low amplitude vibrations in the range available from ambience ($\leq 2\text{m/s}^2$). Therefore, high reliable performance from nonlinear array can be expected. The MFC supplier's data sheet specifies 4500 $\mu\epsilon$ as the maximum operational tensile strain. However, it is found in the present study that formation of cracks in MFC is initiated as early as 1000 $\mu\epsilon$ during the tensile tests. Moreover, it is observed that high reliable performance of MFC can only be achieved under completely reversed strain amplitude of 600 $\mu\epsilon$ for energy harvesting applications. Thus, a dynamic strain limit of 600 $\mu\epsilon$ is proposed here for MFC, which can be used in the design of MFC based PEHs for practical applications. MFC has exhibited nonlinear stress-strain behavior below 1000 $\mu\epsilon$ during tensile testing which resulted in resonant frequency shift when base acceleration increases (Chapter 5). So, it is essential to consider material nonlinearity in design of harvesters if the base excitations vary a lot. In this study, approximately 100 days were spent to carry out tests on MFC for evaluating its reliability at various strain amplitudes and two different frequencies.

It is a common practice to use a factor of safety in engineering designs, which is a ratio of material failure limit to the actual stress or strain in the material. The use of factor of safety in designing PEHs will provide sufficient margin between the operating strain on the harvester materials and the dynamic strain limit, thus ensuring very reliable operation. The proposed dynamic strain limit is also applicable to MFC bending actuators undergoing completely reversed strains. It should be ensured that the applied electric fields to the actuators do not induce strain in excess of 600 $\mu\epsilon$. Similar studies on the other piezoelectric materials like PMN-PT, PVDF, and BaTiO₃ shall be performed to ascertain their dynamic strain limits for designing reliable PEHs in the practical applications.

7.4 Summary

The reliability of MFC's performance at various strain amplitudes under reversed cyclic loading as found in the energy harvesting applications is investigated in this chapter. Firstly, the energy harvesting tests are carried out on a PEH comprising MFC, and significant shift in the resonant frequency of PEH is observed for strains below 1000 $\mu\epsilon$. Moreover, more than 50%

degradation in the voltage output within half million cycles is noticed when the harvester is excited at strain amplitude around $1370\mu\epsilon$. The nonlinear stress-strain behavior of MFC below $1000\mu\epsilon$ is responsible for the observed drop in the resonant frequency with increasing strain amplitude in the energy harvesting tests. These results indicate that the MFC does not have a clear linear elastic limit. In addition, it is noticed from the tensile tests that the crack formation is initiated in the MFC at strains as low as $1000\mu\epsilon$. Finally, several long duration reliability tests are conducted at both low and high resonant frequencies to estimate the upper limit of dynamic strain in the MFC, below which the material gives reliable performance. It is concluded that the MFC gives high reliable performance under dynamic strain of $600\mu\epsilon$ and does not show any degradation in the voltage output and tip displacement over 60 million cycles of testing. The results of reliability tests also indicate that the frequency considered in the typical energy harvesting applications does not have significant effect on the fatigue behavior of MFC.

CHAPTER 8 CONCLUSIONS AND FUTURE WORK

The ultimate goal of the research is the practical implementation of vibration-based piezoelectric energy harvesters (PEHs) for powering wireless sensors or portable electronic devices. Efforts are made in this thesis to design, model, and optimize the harvesters aiming to practically implement this technology. This thesis presented an accurate analytical model of nonlinear PEHs for analyzing the performance of harvesters. The magnetic interaction in nonlinear harvesters is precisely modeled using accurate analytical expressions of the force between permanent magnets. More importantly, material strength is considered in the design optimization of harvester to emphasize the importance of structural integrity. Subsequently, a novel approach is proposed to model complex designs of nonlinear harvesters using finite element modeling approach. This methodology facilitates analysis of complex piezoelectric systems with low computational effort. Later, the influence of geometric, material, and damping nonlinearities on the dynamics of PEH is studied. It is understood from this study that the nonlinear modeling is essential to capture the resonance shift and to estimate the voltage/displacement amplitude accurately under high base excitations. A nonlinear PEH array is proposed for broadband energy harvesting from low-frequency and low-amplitude vibrations. The performance of harvester array is evaluated using the modeling techniques developed in the previous chapters. It is shown that the harvester array generates sufficient amount of energy to power a WSN from ambient low-amplitude random vibrations. The reliability of harvester's performance is crucial for practical implementation of PEHs in real life applications where harvesters are intended to operate for long periods of time. Therefore, experiments are carried out to study the harvester's performance over extended periods of time. From the experimental results, an upper limit to the dynamic strain induced in MFC is given to achieve high reliable performance. The proposed limit on dynamic strain can be used as a design guideline in future studies. The conclusions from the thesis work are presented below in detail.

8.1 Conclusions

- (1) The material strength of piezoelectric material (MFC) is considered in the design optimization of the nonlinear piezoelectric energy harvester (PEH) with a magnetic oscillator. The performance of harvester is optimized while imposing the constraint on the maximum strain induced in the MFC. Analytical models are derived for the linear and nonlinear harvesters using Rayleigh-Ritz formulation. The charge model is employed to

calculate the force between the permanent magnets in place of the commonly used dipole-dipole approximation. It is shown that the dipole approximation gives erroneous estimation of the magnetic force when the magnets are close and consequently results in imprecise estimation of the response of harvester. Furthermore, a parametric study is carried out by varying the beam length and the spacing between the magnets of harvester and magnetic oscillator. From the results of parametric study, optimal parameters for the design of nonlinear harvester are chosen for achieving both wider bandwidth and higher power output with the maximum strain on the MFC below the allowable limit. Experiments are carried out with the optimal configuration and good agreement with the theoretical predictions is observed. At a base acceleration of 2m/s^2 (RMS), the optimal nonlinear PEH generates a peak power of $1658\mu\text{W}$ with a useful bandwidth of 6Hz and the maximum strain in MFC is 1023 ppm.

- (2) So far in the literature, the finite element models are developed only for the linear piezoelectric energy harvesters. Finite element analysis (FEA) has not been used for analyzing the nonlinear harvesters because of the difficulty involved in the modeling of magnetic interaction and the heavy computational effort required for solving the coupled electrical-mechanical-magnetic problem. In this thesis, the magnetic interaction in the nonlinear harvesters is modeled with the nonlinear spring element available in ANSYS FEA package. The magnetic force is calculated over a range of separation distances either axially or laterally. The calculated magnetic force is then approximated using the piecewise linear segments and given as the force-displacement behavior of the nonlinear spring element. The proposed modeling technique is validated by comparing the response of monostable nonlinear harvester obtained from FEA with the theoretical predictions. Later, the advantages of the proposed technique are demonstrated by modeling a complex 2-DOF nonlinear harvester for which deriving an accurate analytical model is difficult. It is shown that the results from FEA modeling are in good agreement with the experiments.
- (3) The influence of geometric, material, and damping nonlinearities on the dynamic response of harvesters is investigated. Firstly, the FEA modeling is used for analyzing the effect of geometric nonlinearity on the response of harvesters. The cantilever-type PEH configurations with various beam lengths and thicknesses are studied for understanding the degree of geometric nonlinearity at different base accelerations. A marginal value of 0.1 is proposed for the ratio of tip deflection to beam length (δ/l_s), below which the geometrical linearity is a valid assumption and beyond which the modeling of geometric nonlinearity is necessary. Subsequently, it is demonstrated that the data fitting techniques

used in many of the past research works for determining the unknown nonlinear coefficients can lead to erroneous estimates unless thoroughly investigated. Unlike the other works, the nonlinear stress-strain behavior of the piezoelectric material (MFC) and the corresponding nonlinear coefficients are obtained from the tensile testing. The nonlinear dynamic model of the cantilever-type PEH is derived from the first principles accounting for the material and damping nonlinearities. Experiments are conducted at different base excitations to validate the derived nonlinear model. The unknown damping coefficients at different base excitations are identified through the energy balance method. It is concluded that the material nonlinearity of MFC is responsible for the softening behavior and shift in the resonant frequency of harvester. Furthermore, continuous degradation in the performance of harvester is observed when excited at higher base accelerations due to the formation of cracks.

- (4) An array of nonlinear piezoelectric energy harvesters (NPEHs) is proposed for broadband energy harvesting. The theoretical study is carried out with two monostable NPEHs and their rectified voltage output is combined in the experiments using series or parallel connection between the harvesters. The geometry of the beams, spacing between the magnets and their polarity is chosen so as to achieve favorable response from both harvesters. A uniform voltage output over a wideband is achieved when the response from both harvesters is combined. Experiments are conducted on the prototype of NPEH array under harmonic and random excitations. An array of linear piezoelectric energy harvesters (LPEHs) is also tested for the purpose of comparison. Under the harmonic base excitation of 2m/s^2 , the NPEH array exhibits 100% improvement in the operation bandwidth over the LPEH array. At optimal load resistance, the NPEH array generates a peak power output of $1145\mu\text{W}$ and a bandwidth of 2.8Hz corresponding to power output of $200\mu\text{W}$ is achieved. The NPEH array shows superior performance over the LPEH array under the random excitation also. At 1m/s^2 , the NPEH array using the series connection stores 80% more energy compared to its linear counterpart. Moreover, the NPEH array charges the capacitor in the harvesting circuit much faster compared to the LPEH array. The passive nature of the NPEH array in achieving the wider bandwidth renders itself as a possible alternative for the chemical batteries in the wireless sensor node applications.
- (5) The reliability of the performance of macro fiber composite (MFC) at various strain amplitudes under the reversed cyclic loading is investigated. It is observed from the earlier study in the thesis that the degradation in the voltage output from MFC is severe at high strain amplitudes. Energy harvesting tests are conducted with increasing base acceleration

from 1m/s^2 to 6m/s^2 and a gradual decrease in the resonant frequency of the harvester is observed. It is found that the nonlinear stress-strain behavior below $1000\mu\epsilon$ is responsible for the reduction in the resonant frequency. Moreover, 50% degradation in the voltage output within half million cycles is noticed when the harvester is excited at the resonance corresponding to 6m/s^2 . Therefore, several long duration tests are conducted at different strain amplitudes and at different loading frequencies to investigate the reliability of the performance of MFC. From the results, it is concluded that the MFC gives high reliable performance below $600\mu\epsilon$ of strain amplitude. When the harvester is tested at $605\mu\epsilon$ with high resonant frequency, no sign of degradation in the voltage output and tip displacement is observed after 60 million cycles of testing. The results of reliability tests also indicate that the frequency considered in the typical energy harvesting applications does not have significant effect on the fatigue behavior of MFC.

8.2 Future work

Piezoelectric energy harvesting from ambient vibrations is suitable for powering small-scale, portable electronic devices and wireless sensor nodes. With the continuous reduction in the power consumption of electronic circuitry, piezoelectric energy harvesting is a possible alternative for replacing the chemical batteries across many applications in near future. Although considerable research has been done in this area over the years, including the work presented in this thesis, still there are many challenges for the practical implementation of this technology. Especially, more efforts are needed in improving the power output from harvesters to match the power consumption of the electronic devices. The operational bandwidth of harvesters needs to be further improved to harvest more energy from wideband ambient vibrations. The work presented in this thesis can be further extended as follows:

- (1) In this thesis, an array of nonlinear piezoelectric energy harvesters is presented and experimentally demonstrated its superiority over the array of linear harvesters. However, only two nonlinear harvesters are considered in the theoretical and experimental studies and the prototype is tested in the frequency range of 14Hz-24Hz only. However, the idea can be extended to multiple harvesters to cover the wide range of ambient vibrations ($> 20\text{Hz}$). Moreover, significant loss in the voltage output is observed when using the standard AC-DC harvesting circuit. Therefore, the performance of nonlinear harvester array needs to be improved by using SSHI or SCH circuits. A system level study needs to be carried out by integrating the harvester and its circuit with the wireless sensor node.

- (2) In this work, the prime focus is on the design and modeling of the monostable nonlinear harvesters only. Over the last few years, the bi-stable configurations have shown potential to greatly improve the bandwidth as well as the efficiency, provided the external excitation is large enough to create high-amplitude oscillations. Although many designs of the bi-stable harvesters are proposed, the power output is less when the harvesters cannot snap-through the barrier between the potential wells specially at low excitations. The author is investigating the potential active or passive mechanisms for achieving the snap-through motion at relatively low base excitations.
- (3) Very few works have used multiple modes of the system for energy harvesting. Although, some works employed the multi-modal techniques for broadband energy harvesting, either the modes are widely separated in the frequency response or the power output from the higher modes is much less when compared to the fundamental mode. The author is investigating a multi-modal harvester design where the spring-mass systems are attached as add-on systems to the conventional cantilever-type harvester. The schematic of the proposed multi-modal harvester design is shown in Figure 8-1. Closer peaks in the frequency response of the harvester can be achieved by modifying the geometric and structural parameters of the beam and spring-mass systems.

Figure 8-1 Schematic of the multi-modal harvester design

APPENDIX

A.1 Mode shapes of beam configurations

Figure A.1 Mode shapes of beam configurations considered in parametric study. Length of beam varies from 50mm to 75mm

A.2 State-space form of the governing equations of nonlinear piezoelectric harvester with magnetic oscillator

Assuming variable substitution as below:

$$q_1(t) = u_1(t), \quad q_2(t) = \dot{u}_1(t),$$

$$q_3(t) = u_2(t), \quad q_4(t) = \dot{u}_2(t),$$

$$q_5(t) = V(t)$$

Using above variable substitution, the governing equations (3.11) to (3.13) can be written in state space form as

$$\dot{q}_1(t) = q_2(t),$$

$$\dot{q}_2(t) = -\frac{C_1}{M_1} q_2(t) - \frac{K_1}{M_1} q_1(t) - \frac{\theta}{M_1} q_5(t) + \frac{F_{mag}}{M_1} + \frac{F_1(t)}{M_1},$$

$$\dot{q}_3(t) = q_4$$

$$\dot{q}_4(t) = -\frac{C_2}{M_2} q_4(t) - \frac{K_2}{M_2} q_3(t) - \frac{F_{mag}}{M_2} + \frac{F_2(t)}{M_2},$$

$$\dot{q}_5(t) = \frac{\theta}{C_s} q_2(t) - \frac{1}{RC_s} q_5(t)$$

A.3 Distances of various material layers from neutral axis

Figure A.3 Representation of transverse distances of various material layers from neutral axis

Y_1 and Y_2 are the distances of top and bottom layers of aluminum from the neutral axis, respectively.

Y_3 is the distance of top layer of epoxy glue from the neutral axis.

Y_4 is the distance of top layer of MFC from the neutral axis.

AUTHOR'S PUBLICATIONS

Journal papers

- (1) **Upadrashta, Deepesh**, Yaowen Yang, and Lihua Tang. "Material strength consideration in the design optimization of nonlinear energy harvester." *Journal of Intelligent Material Systems and Structures* 26.15 (2015): 1980-1994.
- (2) **Upadrashta, Deepesh**, and Yaowen Yang. "Finite element modeling of nonlinear piezoelectric energy harvesters with magnetic interaction." *Smart Materials and Structures* 24.4 (2015): 045042.
- (3) Yang, Yaowen, and **Deepesh Upadrashta**. "Modeling of geometric, material and damping nonlinearities in piezoelectric energy harvesters." *Nonlinear Dynamics* 84.4 (2016): 2487-2504.
- (4) **Upadrashta, Deepesh**, and Yaowen Yang. "Experimental investigation of performance reliability of macro fiber composite for piezoelectric energy harvesting applications." *Sensors and Actuators A: Physical* 244 (2016): 223-232.
- (5) **Upadrashta, Deepesh**, and Yaowen Yang. "Nonlinear piezomagnetoelastic harvester array for broadband energy harvesting." *Journal of Applied Physics* 120.5 (2016): 054504.

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