Adaptive nonlinear modeling procedure for progressive collapse analysis of reinforced concrete frame structures

Weng, Jian

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**ADAPTIVE NONLINEAR MODELING
PROCEDURE FOR PROGRESSIVE COLLAPSE
ANALYSIS OF REINFORCED CONCRETE
FRAME STRUCTURES**

WENG JIAN

SCHOOL OF CIVIL & ENVIRONMENTAL ENGINEERING

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**Adaptive Nonlinear Modeling Procedure
for Progressive Collapse Analysis of
Reinforced Concrete Frame Structures**

Weng Jian

School of Civil & Environmental Engineering

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ABSTRACT

Due to increasing threats from terrorism in the past decades, progressive collapse modeling of buildings is gaining popularity with objective of simulating collapse process of the whole or partial structural system, in order to give useful insight to improve existing design for the structure against progressive collapse. In this thesis, a novel modeling framework for progressive collapse of reinforced concrete (RC) frame structures is proposed.

A set of damage assessment criteria to identify and quantify flexural, shear and axial damages and failures of RC members is suggested. This set of damage criteria incorporates axial-shear-flexural interactions of the structure during the analysis and it is capable of tracing cracking and crushing of concrete, yielding and fracture of reinforcement as well as final failures of cross-sections within RC members.

Direct member removal algorithm is developed to simulate the process of collapse at member level, based on combined flexural/shear/axial failures of RC members. With developing a specially designed searching scheme, a new algorithm to monitor substructure collapse is also proposed. Locations and magnitudes of impact loads of the collapsed partial structure are also identified and calculated, according to rigid-body kinematics and energy principle. In addition, inelastic and oblique impact effects are properly considered.

To efficiently simulate progressive collapse of large-scale buildings, superelement technique which separates the structure into linear and nonlinear zones is employed to reduce the computational time. As buildings in progressive collapse may undergo large rigid-body rotations of the structure, a rigid-body rotation

correction to improve conventional superelement formulation is proposed for a more accurate large deformation analysis using superelement.

Further, in order to consider nonlinearity spread during progressive collapse analysis, an adaptive superelement algorithm to automatically identify the propagation of nonlinear zone and then update the definition of superelement is proposed. By incorporating direct member removal algorithms as well as model regeneration procedure, an efficient adaptive superelement modeling procedure is developed for progressive collapse of RC frame structures.

Numerical examples are given to show the advantages and effectivity of the suggested damage criteria and modeling procedures, through comparison with the results obtained from physical tests or standard nonlinear finite element analysis.

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LIST OF SYMBOLS

$\{ \}$	Vector
$[\]$	Matrix
T	Transpose of vector or matrix
$^{-1}$	Inverse of matrix
Σ	Summation
$\int$	Integration
$\ \ \ $	Norm
K	Stiffness matrix
u	Displacement vector
P	Load vector
E	Young's modulus
G	Shear modulus
Π	Strain energy norm
D_f	Flexural damage index
D_s	Shear damage index
D_a	Axial damage index
J	Joint
E	Edge
E_{cont}	Continuous edge
E_{fr}	Fractured edge
J_{st}	Starting joint
Set_{st}	Starting joint set
J_{in}	Internal joint

Set_{in}	Internal joint set
E_{in}	Internal edge
C_{str}	Constraint
D_{gr}	Degree
J_{in}^{crnt}	Current internal joint
J_{adj}	Adjacent joint
Set_{adj}	Adjacent joint set
IAF	Impact amplification factor
λ	Load factor
$\oplus$	Combination of load vectors
$\mathbf{K}^*$	Condensed stiffness matrix
$\mathbf{P}^*$	Condensed load vector
$\mathbf{k}_e$	Original element stiffness matrix
$\mathbf{k}'_e$	Rotation-corrected element stiffness matrix
$\mathbf{R}_0$	Rigid-body rotation matrix
$\delta\mathbf{\Theta}$	Rigid-body rotation variation matrix
W	Strain energy density
$\mathbf{e}$	Strain energy error of cross-section
η	Error tolerance of adaptive analysis

CHAPTER 1 INTRODUCTION

1.1 Research background

Progressive collapse is an extreme failure phenomenon of building structures and it can be characterized as ‘the spread of local damage, from an initiating event, from element to element resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it’ (ASCE 7-05). In the past decades, progressive collapse occurred in various buildings and caused huge losses of life and property world-wide (Georgakopoulos 2005). The most famous progressive collapse event is the collapse of World Trade Center towers (Figure 1.1) on 11 September, 2001. In this event, more than 2752 people died and the total economic losses exceeded \$3 trillion (Shan 2015). Since then, interest in resisting progressive collapse for building structures has been intensified among structural engineering communities in many countries. Most recently, the collapse of Rana Plaza commercial building (Figure 1.2) which caused the death of 1129 people and more than 2515 people injured (Alam and Hossain 2013) indicates further necessity and significance on investigating progressive collapse resistance of building structures.

Figure 1.1 Progressive collapse of WTC towers (McAllister *et al.* 2002)

Figure 1.2 Progressive collapse of Rana Plaza building (Mollah *et al.* 2013)

To improve structural behavior against progressive collapse, design considerations have been incorporated into building codes and design guidelines (ACI318-05 2005; GSA 2003; DoD 2010) to enhance the integrity, robustness and resilience of structures. Specific design methodologies, classified into indirect and direct methods, are presented in these codes and guidelines to provide a higher

level of protection for buildings resisting progressive collapse. The indirect method specifies a minimum level of connectivity among various structural components to ensure ductility of the structure. It aims to enable the development of tensile membrane action and catenary action, without further structural analysis needed. This method has been widely used in practice to reduce the likelihood of progressive collapse (NIST 2007). However, its practical effect is rarely verified by realistic disasters or test results.

The direct design methodology consists of Specific Local Resistance (SLR) method and Alternate Load Path (ALP) method (Ellingwood and Leyendecker 1978). The former requires a specific normative load (or threat) and seeks for sufficient strength to prevent a specific level of damage from this threat. However, there is no assurance that the building can perform adequately for the threats, other than the one specifically considered (Ellingwood *et al.* 2009). The ALP method provides alternate load paths to dissipate the localized damage and to avert the initial collapse. This method enables a formal check of structural resistance against the removal of key structural components, such as columns or load-carrying walls. For RC frame structures, a good means to assess progressive collapse resistance is by investigating structural behaviors under a column removal scenario, so as to attain an improvement of existing design against progressive collapse.

Both of the SLR and ALP design methods need verification by test results or numerical simulation. A few experimental studies on this verification were reported in recent literature (Sasani *et al.* 2007; Yi *et al.* 2008; Yu and Tan 2013). Unfortunately, the actual behavior of complete structures in progressive collapse was not studied by full-scale tests, due to physical constraints of scale, equipment and expenses. Therefore, numerical modeling, which is able to provide an integral assessment of structural behavior, can be an effective way to investigate

progressive collapse resistance of building structures.

So far, four typical analysis procedures (DoD 2010) have been proposed for progressive collapse assessment, i.e., linear-elastic static, nonlinear static, linear-elastic dynamic and nonlinear dynamic procedures. Linear analysis requires the material response to remain in linear-elastic range and second-order effects as well as instabilities of the structure are ignored in the analysis. This limits use of linear analysis procedure to small displacements and often results in conservative design in order to prevent invalidating assumptions (Byfield *et al.* 2013). Nonlinear dynamic procedure is potentially able to provide more acceptable prediction results, but in practice it is prone to inappropriate assumptions and modeling errors due to its characteristic of high complexity (Kwasniewski 2010). In addition, significant computational demand limits the application of dynamic analysis, especially for modeling large-scale buildings. Nowadays, nonlinear static analysis approach is still a major modeling technique in engineering application due to its efficiency and ease of use.

While several numerical studies have been conducted for progressive collapse analysis (Fu 2010; Valipour and Foster 2010; Long 2013; Tsai and Huang 2013), there is very limited research at a systematic framework in modeling progressive collapse of buildings, especially of RC frame structures. This framework requires damage assessment criteria to identify failures and collapse modes, removal algorithms to delete the collapsed members/substructures when collapse occurs, and model regeneration procedures to update loads and model information for subsequent analysis. Damage assessment criteria for progressive collapse analysis of RC frames were proposed by Kaewkulchai (2003). However, catenary action in RC beams under large deformations was not considered in his work. A direct element removal algorithm was developed in the literature (Talaat and Mosalam 2009), while the proposed element removal criteria based on flexural failure of

RC beams could be too conservative. In addition, member removal due to partial substructure collapse in the analysis was not considered.

Since progressive collapse occurs in multi-story buildings (as Figure 1.2) or even in tall buildings (as Figure 1.1), computational efficiency will be a significant concern for progressive collapse modeling of large analytical models. To model progressive collapse of the World Trade Centre towers, more than two million refined elements were employed with taking huge computational effort for the analysis (Hoffmann 2005). This is definitely not suitable to use general computational programs to run a full-process analysis of collapse of large-scale buildings. Towards this end, an efficient modeling approach is essential for nonlinear progressive collapse of buildings, especially for large-scale building structures.

1.2 Objective and scope of this research

The objective of this research is to develop a precise and efficient nonlinear modeling procedure for progressive collapse analysis of RC frame structures. To achieve this objective, three aspects of research work are conducted as follows:

- (i) **Damage assessment criteria:** To develop a set of effective damage assessment criteria to monitor stiffness and strength deterioration of RC frames so that damages, failures and collapse modes of the structure can be traced and identified during the analysis. To account for complicate response of the structure in progressive collapse, damage effects of flexural-shear-axial interactions in the structure are also considered in the proposed damage criteria.
- (ii) **Progressive collapse modeling procedure:** To develop direct removal algorithms for member/substructure collapse as well as automatic model

regeneration procedure to update computational model for subsequent analysis after collapse. A removal algorithm at member level is performed based on combined axial/shear/flexural failures of elements in a member. In addition, a removal algorithm at substructure level is implemented based on a specially designed searching scheme for collapsed substructures. After removal of collapsed part (members or substructures), model regeneration procedure to generate a new model is carried out, where model information and new loads after collapse are updated.

- (iii) Adaptive superelement modeling procedure: To develop a new superelement formulation with rigid-body rotation correction for efficient progressive collapse analysis of RC frame structures. In order to reduce the computational cost of the modeling procedure while maintaining the modeling accuracy, an effective adaptive superelement scheme is proposed to trace nonlinearity spread in the structure and update superelement for new linear zone during collapse analysis.

As the first trial to apply adaptive superelement technique in simulation of progression of collapse of RC frames, the proposed modeling procedure in this thesis has restrictions as follows:

- (i) Only progressive collapse of two dimensional (2D) RC beam-column frames is studied. This implies that three dimensional (3D) space as well as the integrated slab effects of the structure are not considered. However, the proposed adaptive modeling procedure has indicated a viable and valuable extension of progressive collapse simulations for 3D RC beam-slab-column frame systems.
- (ii) A quasi-static progressive collapse analysis is carried out. Needless to say, the dynamic analysis will be much closer to the reality as it can take velocity

(damping force) and acceleration (inertia force) effects into account and the dynamic behavior of materials will also be considered. Normally, actions in dynamic analysis will be increased compared to static analysis, but the resistance of dynamic analysis can also be enhanced due to strain rate effect of materials. However, the advantage of quasi-static analysis is that it requires much lower computational resources and the modeling accuracy of nonlinear analysis can still remain reasonable and acceptable when both geometric and material nonlinearities are included (Menchel *et al.* 2009; Alashker *et al.* 2011). In addition, the main dynamic effect in progressive collapse – ‘impact’ has been considered in the present work based on an appropriate equivalent static load approach (AASHTO 2009). Hence, the proposed approach is rational with acceptable accuracy.

1.3 Novelty of this research

The novelty of this research includes the following:

- (i) A set of simple yet effective damage assessment criteria is proposed to monitor stiffness and strength deterioration of RC frames in progressive collapse analysis. Damage effect of flexural-shear-axial interactions of the structure is considered in order to accommodate actual force conditions within progressive collapse.
- (ii) A direct removal algorithm is developed to identify and remove failed members during the progressive collapse analysis based on a combined flexural/shear/axial failure criteria. An effective removal algorithm at substructure level is proposed with a specially designed searching scheme to identify and remove failed elements in partial collapsed substructures.
- (iii) An updated scheme for collapse loads is proposed. Locations of collision

points and magnitudes of impact loads are identified and calculated based on rigid-body kinematics and energy principle with both inelastic and oblique impact effects considered.

- (iv) A rigid-body rotation correction formulation of superelement is developed to enhance modeling precision of large deformation analysis with superelement. Numerical examples demonstrate that new superelement formulation can be more accurate than conventional superelement approach, without a significant loss of computational efficiency.
- (v) An adaptive superelement algorithm is developed to automatically update new superelement when nonlinearity spreads to the linear zone during the analysis. It is found that by adopting the proposed adaptive superelement strategy, the computational effort in modeling the whole process of progressive collapse can be reduced obviously, without a significant loss of modeling accuracy.

1.4 Layout of this thesis

The thesis is organized into eight chapters. Following this introduction, Chapter 2 presents a literature review on progressive collapse modeling of building structures. In Chapter 3, the Total Lagrangian formulation of nonlinear beam elements, on which the present modeling procedure is based, is introduced.

In order to provide an appropriate scheme to identify failures and collapse of RC frame, a set of effective damage assessment criteria is established in Chapter 4. Chapter 5 presents a nonlinear modeling procedure for simulation progression of collapse of 2D RC frames. Based on damage assessment criteria presented in Chapter 4, direct member/substructure removal algorithms, load update scheme as well as model regeneration procedure, are developed for modeling the progression of collapse of RC frames.

In Chapter 6, a superelement formulation considering rigid-body rotation correction is proposed. The developed rotation correction algorithm for linear superelement enables more accurate large deformation analysis using superelement, without significant increase of computational effort. However, it is found that the spread of nonlinearity in progressive collapse analysis, which starts from a small localized zone and expands to a larger portion, will make conventional superelement technique inapplicable. Therefore, an adaptive superelement modeling procedure, which traces the shrinkage of linear zone and redefines new superelement during collapse analysis, is proposed in Chapter 7. Conclusions of this thesis are made in Chapter 8, where future works are also suggested.

CHAPTER 2 LITERATURE REVIEW

2.1 Numerical studies for progressive collapse of building structures

Earlier numerical research on progressive collapse may date back to 1983, when steel frames subject to a column loss scenario were first simulated (Gross and McGuire 1983). A stress resultant plasticity method with concentrated plasticity at member ends was used. Catenary action was mobilized to carry the loads after formation of plastic hinges at the mid-span and ends of the beam. This is an advanced approach in modeling progressive collapse, although it only employed the lumped plasticity model and the progress of collapse was not considered.

Pretlove (1986) studied the dynamic effects that occur in the progressive failure of a simple uniaxial tension structure. It was found that a structure that is safe under static load redistribution may actually be unsafe if a dynamic analysis is carried out. This study implies the significance in consideration of dynamic effects during progressive collapse analysis.

Hakuno and Meguro (1993) proposed an extended distinct element method for fracture analysis of concrete structures. In order to model a concrete member, a large number of rigid elements and spring elements were employed. With this method, the authors simulated collapse of a concrete frame under seismic loading. However, modeling accuracy was sacrificed in order to save computational cost using only a few elements for cross-sections of concrete members.

Abedi and Parke (1996) examined the behavior of single layer braced domes which are prone to progressive collapse due to localized damage propagation initiated by member or joint instability. Elastic perfectly plastic Timoshenko beam elements were used to model tube-type members and Euler-Bernoulli beam elements were employed for members with solid cross-section. Static analysis followed by dynamic analysis was conducted for a dome with a span of 1m and a rise of 40 mm. It was shown that dynamic snap-through can result in progressive collapse.

Luccioni *et al.* (2004) studied the progressive collapse of an RC building under blast loading with a refined finite element model. The spread of shock wave, formation of debris as well as contact problems were considered. However, computational cost for the entire progressive collapse analysis was expensive due to a large number of refined elements used.

Vlassis *et al.* (2008) proposed a design-oriented methodology for progressive collapse assessment of floor systems within multi-storey buildings subject to impact from an above failed floor. The conceptual basis of the proposed framework is that the structural resistance of the lower floor during an impact event. Three principal independent stages are employed, including: (1) determination of the nonlinear static response of the impacted floor system, (2) dynamic assessment using a simplified energy balance approach, and (3) ductility assessment at the maximum level of dynamic deformation attained upon impact. The application study illustrates the extremely onerous conditions imposed on the impacted floor resulting in an increased vulnerability to progressive collapse for structures of this type. In addition, the likelihood of shear failure modes beyond inadequate ductility supply under combined bending/axial actions is also identified.

Izzuddin *et al.* (2008) proposed a simplified framework for progressive collapse assessment of multi-storey buildings, considering sudden column loss as a design scenario. The proposed framework offers a practical means for assessing structural robustness at various levels of structural idealization. A major feature of this approach is its ability to accommodate simplified as well as detailed models of the nonlinear structural response, with the additional benefit of allowing incremental assessment over successive levels of structural idealization. The conceptual clarity of the proposed framework sheds considerable light on the adequacy of commonly advocated measures and indicators of structural robustness, culminating in the proposal of a single rational measure of robustness that is applicable to building structures subject to sudden column loss.

Kim *et al.* (2009) developed a simplified finite element model for progressive collapse analysis. The post-failure behavior of elements was described by reduction of material parameters and the degrees of freedom of failed elements were released based on the proposed condensation approach. This model may be suitable to deal with the failure of single members in the structure but is unable to identify and handle collapse of partial substructure during the analysis.

Formisano *et al.* (2015) presented two assessment approaches to evaluate the robustness of steel frame structures under catastrophic events. The robustness index of the structure was calculated by considering the influence of both catenary action in the beams and the behavior of beam-column connections. However, since the robustness index is mainly dependent on the number of plastic hinges rather than the ultimate state of collapse which includes fracture as well as buckling of critical cross-sections, the proposed approaches could not be able to evaluate actual collapse modes during progressive collapse analysis.

2.2 Damage assessment for reinforced concrete structures

In progressive collapse towards failures of one or more elements, stiffness and strength deteriorate with damage accumulation in the system. One needs to develop damage assessment criteria to trace degradation mechanisms during deformations of the structure.

Mazars and Pijaudiercabot (1989) proposed a concrete damage model based on continuum damage mechanics. The material was assumed isotropic. A scalar damage index was introduced to assess the behavior of concrete damage. Based on this model, Sanches and Venturini (2007) developed a modified damage model for reinforced concrete beams, where the reduction of stiffness and ultimate load-carrying capacity was evaluated by considering the damage effect of shear components.

The damage model proposed by Park and Ang (1985) has been adopted frequently for seismic damage analysis. This model evaluated structural damages with a linear combination of the maximum deformation and the cumulative strain energy of the structural system. Although this model is a good to express damage by the combined effect of deformation and energy dissipation, there are uncertainties about damage parameters and different applications may generate different results on damage assessment.

A simplified damage assessment method was suggested by Cipollina *et al.* (1995) to incorporate stiffness and strength deterioration of member hinges in nonlinear analysis. The tangent stiffness matrix $\mathbf{K}_t$ of a member was obtained by inverting its flexibility matrix $\mathbf{F}_t$ using a damage flexibility matrix $\mathbf{f}_d$, which is given by

$$\mathbf{f}_d = \begin{bmatrix} \frac{D_a \cdot F_{e11}}{(1-D_a)} & 0 & 0 \\ 0 & \frac{D_i \cdot F_{e22}}{(1-D_i)} & 0 \\ 0 & 0 & \frac{D_j \cdot F_{e33}}{(1-D_j)} \end{bmatrix} \quad (2.1)$$

where D_a is axial damage index, D_i and D_j are flexural damage indices at hinge i and j , F_{ekk} are diagonal terms of the elastic flexibility matrix of member. The effect of strength degradation of a member is captured by the modified yield function as follows:

$$f = \left| \frac{M}{1-D} \right| \cdot M_y \quad (2.2)$$

This approach accounts for damages from both deformation and energy and the degradation of stiffness and strength can be quantified with damage indices. However, it was developed based on lumped plasticity model and hence plastic damage propagation besides specific plastic hinges at member ends cannot be identified and considered.

Based on a two-parameter damage model, Saetta *et al.* (1999) defined an expression of local damage indices for concrete and steel materials. The damage variables in this model were derived from the principle of equivalent strain energy, but they were highly dependent of constitutive law of the material, especially of definition of the ultimate damage state. In addition, numerical integrations in calculating local damage index require extensive computational effort, which is not suitable for large-scale structural analysis.

Wahab *et al.* (1999) reported a three-parameter damage model for reinforced concrete beams by introducing a reduction function of flexural rigidity of cross-section. With this model, element damage index was derived from average flexural rigidity (Zhu *et al.* 2009). The expression of damage index for element i can be written as

$$\alpha_i = 1 - \frac{K^i}{K_0^i} \quad (2.3)$$

where K^i and K_0^i are the residual and original flexural rigidities of the element, respectively.

Williamson and Hjelmstad (2001) developed a damage formulation that linearly depends on the maximum deformation and accumulated plastic energy. The damage index used is given as

$$D = \alpha U(\delta) + \beta W(\delta) \quad (2.4)$$

where α and β are constant parameters, $U(\delta)$ is a function associated with the maximum deformation, and $W(\delta)$ is a function of the accumulated plastic energy. Through adjusting the value of α and β , one can represent different rates of damage accumulation. To account for both axial and rotational components, this model is extended by a modification of damage index for plastic hinges, defined as (Kaewkulchai 2003)

$$D_i = \alpha_i \left(\frac{\theta_{mi}}{\theta_{yi}} + \frac{\delta_{ma}}{\delta_{ya}} + \frac{\theta_{mi}}{\theta_{yi}} \frac{\delta_{ma}}{\delta_{ya}} \right) + \beta_i \left(\frac{\sum E_{pi}}{E_{oi}} + \frac{\sum E_{pa}}{E_{oa}} + \frac{\sum E_{pi}}{E_{oi}} \frac{\sum E_{pa}}{E_{oa}} \right) \quad (2.5)$$

where θ_{mi} and θ_{yi} are the maximum and yield rotations for plastic hinge i , δ_{ma} and δ_{ya} are the maximum and yield axial displacements, respectively. E_p and E_o are the plastic energy dissipation in one cycle and the initial elastic energy prior to yielding, respectively.

In Equation 2.5, the first two terms within each set of the parentheses represent a basic extension of the Park and Ang model (Park and Ang 1985) in which damage is assumed to vary linearly as a function of the maximum deformation accumulated and the hysteretic energy dissipated. The last term within each set of parentheses accounts for the coupling between axial and flexural behavior, which is consistent with the constitutive model used to describe the behavior of plastic hinges.

Although axial-flexural coupling was explicitly expressed in this work, axial-shear and flexural-shear interactions of the structure were not considered. However, it is essential to consider these interactions during structural analysis under complex force conditions, such as progressive collapse analysis of building structures.

Talaat (2007) defined axial and flexural damage indices of cross-section with a fiber model for progressive collapse analysis of RC frames. In addition, zero-length elements with a shear model were introduced to assess the extent of shear damage. In this work, flexural-axial and shear-axial interactions were considered separately while flexural-shear interaction was ignored. Further, the proposed approach did not consider catenary action in RC beams as it was assumed that the beam will be removed when a threshold value of flexural damage is attained. This approximation may attain conservative assessment of collapse ultimate state of the structure, and thus the predicted collapse mode may largely differ from the actual collapse situation.

Bui *et al.* (2014) proposed a dynamic measurement approach to assess the localized scale damage of RC frames. This approach is useful in structural monitoring of existing buildings by identifying the stiffness of plastic hinges. It was shown that a structural element will lose its bearing capacity when 30%-60% of stiffness is lost and a structure will be fully damaged when it loses around 20% of the overall stiffness. However, this conclusion may need further validation by more experimental studies (Bui *et al.* 2014). In addition, a more powerful optimization algorithm will be required if the approach is applied to a whole structure with the number of model parameters to be identified is rapidly increased.

2.3 Progressive collapse modeling of reinforced concrete frames

There were limited publications (Kaewkulchai 2003, Talaat 2007) on comprehensive and systematic modeling progressive collapse of RC frame structures in the past decades, due to the complex material properties of reinforced concrete and the complicated damages and failures of the structure. It is noteworthy that non-homogeneous behavior of reinforced concrete materials may cause convergence difficulties in large deformation analysis, especially for ultimate deformation analysis of progressive collapse.

Casciati and Faravelli (1984) discussed progressive failure for seismic reliability analysis of 2D moment-resisting RC frames. The element formulation for beams and columns was based on lumped plasticity hinge model of member ends. Two damage parameters were introduced to model damages under monotonic and cyclic effects. A Geometrical stiffness matrix was used to model $P-\Delta$ effects and failures of RC members were considered through removing their contribution from global stiffness matrix when the damage parameter attained a threshold value. It was concluded that a column failure initiates the global failure and progressive collapse coincides with the failure of the first column. However, the use of lumped plasticity model in this work ignored the distributed plasticity in the structure and may affect prediction accuracy of damage propagation in modeling. In addition, the forming of a plastic hinge mechanism has been considered as occurrence of ultimate failure in this approach. Therefore, catenary action in RC beams under large deformations was ignored. This would, in turn, affect the effective identification of collapse modes.

Kaewkulchai and Williamson (2004) presented an effective beam element formulation and solution procedure for dynamic progressive collapse analysis of 2D frame structures. A macro-level damage index was defined to predict the

collapse of yield beam-column elements, based on the maximum deformation and accumulated plastic energy. Upon a threshold value of damage index, failed elements were removed from the structural system. External nodal forces were applied at the end-nodes using a step function to represent the effect of the redistributed internal forces. The assumption in this work that plastic hinge failure at member ends would cause separations from adjacent members will lead to conservative assessment of progressive collapse resistance, without consideration of catenary action in RC beams. Accordingly, the prediction for the next phase of collapse may differ from the actual collapse situation. Collision and impact in collapse were investigated in this study, while few details were illustrated for the progress of collapse of frame structures.

Talaat (2007) developed an analytical approach with finite element modeling for time-dependent analysis of RC structures under sudden removal of structural members, in terms of dynamic force equilibrium. Flexural and axial damage indices were defined for damage assessment and their threshold values were used to identify the occurrence of element failures. Direct element removal algorithms were developed to remove the failed elements in the model for subsequent analysis. This element removal algorithm was tested numerically with a benchmark structural system of trussed cantilever beams for progressive collapse analysis, but was not validated by typical RC frame structures. In addition, combined flexural, shear and axial failures of the structure that could result in collapse at both member and substructure levels were also not considered.

Iribarren *et al.* (2011) employed layered beam elements to study structural dynamic response of a multi-story planar frame subject to a sudden column removal. Systematic parametric studies were carried out to investigate the influence of ultimate strain of concrete and steel, reinforcement ratio, column removal time and locations as well as load combinations on structural behavior. It

was assumed that when the ultimate strain of steel is attained, such as rebar fracture, the frame is regarded as collapse. From this work, it was found that the higher ultimate strain of steel and reinforcement ratio can attain the greater progressive collapse resistance. It was also found that the scenario of corner column removal is more collapse-prone than interior column removal, since the latter can make use of catenary action in beams against progressive collapse. However, this modeling did not identify the collapsed part and carry out member or substructure removal for subsequent analysis after collapse. Consequently, progression of collapse cannot be traced as damages, failures and collapse modes in progressive collapse were not identified.

Long (2013) developed a component-based 2D beam-column joint model for progressive collapse analysis of RC frames subject to a column removal scenario. Analytical models were proposed respectively to calibrate the shear-panel and bond-slip components. An empirical model was summarized for the interfacial shear component based on extensive experimental results and design regulations. It can be concluded that the predicted structural response of RC frames under column removal scenario would be consistent between the modeling with and without joint elements, when no obvious damages were found in beam-column joints.

2.4 Superelement technique

The superelement technique is used widely to improve computational efficiency for large-scale structural analysis with local nonlinearity. Progressive collapse of buildings, which initiates a small localized damage (as Figure 2.1), is a typical problem that can be simulated with superelement. As one of the most common ways, static condensation is broadly adopted to establish superelement formulation before starting the nonlinear analysis.

Figure 2.1 Initial localized damage for progressive collapse (Griffiths *et al.* 1968)

2.4.1 Static condensation

The condensation technique to eliminate unwanted degrees of freedom (DOFs) was first proposed by Guyan (1965). As dynamic effect is ignored in the condensation, this method is usually referred to as static condensation. Since the late 1960s, this technique has been used extensively in many static and dynamic problems, including component mode synthesis, eigenvalue problems and mechanical vibrations etc. (Weaver and Johnston 1987; Bathe 1996; Ju and Choo 2005).

Static condensation is always conducted for a linear zone, as shown in Figure 2.2, where the effect of all internal nodes (yellow nodes) in the linear zone is condensed to the boundary nodes (red nodes) so that the total number of DOFs in the model is reduced significantly and hence computational effort is saved. The matrix equilibrium equations for the linear zone can be given as

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_i \\ \mathbf{P}_b \end{Bmatrix} \quad (2.6)$$

where $\mathbf{K}_{xx}$ is the stiffness submatrix of substructures corresponding to internal DOFs and boundary DOFs of the linear zone; $\mathbf{u}_i$, $\mathbf{P}_i$, $\mathbf{u}_b$ and $\mathbf{P}_b$ are displacements and external forces at the internal and boundary nodes, respectively.

Figure 2.2 Boundary and internal nodes of a linear zone (Felippa *et al.* 2005)

The static condensation method releases the internal DOFs from Equation 2.6 to obtain the equilibrium equation in terms of boundary DOFs as follows:

$$\mathbf{K}^* \mathbf{u}_b = \mathbf{P}^* \quad (2.7)$$

where the condensed stiffness matrix is given by

$$\mathbf{K}^* = \mathbf{K}_{bb} - \mathbf{K}_{bi} \mathbf{K}_{ii}^{-1} \mathbf{K}_{ib} \quad (2.8)$$

and the condensed load vector is

$$\mathbf{P}^* = \mathbf{P}_b - \mathbf{K}_{bi} \mathbf{K}_{ii}^{-1} \mathbf{P}_i \quad (2.9)$$

The internal displacements are obtained from

$$\mathbf{u}_i = \mathbf{K}_{ii}^{-1} \mathbf{P}_i - \mathbf{K}_{ib} \mathbf{u}_b \quad (2.10)$$

The matrix inversion in Equations 2.8 to 2.10 is not done explicitly. An efficient matrix decomposition technique (Han and Abel 1984) can be employed to achieve the process of static condensation.

Equation 2.8 implies a symmetric condensed stiffness matrix which ensures the global stiffness matrix in the nonlinear zone is symmetric. Since only matrix transformation is carried out in the condensation process, no errors will be generated if the elements in the linear zone remain linear-elastic without rigid-body rotations. In other words, static condensation is exact for linear-elastic analysis when rigid-body rotation effect is ignored in the analysis.

2.4.2 Rigid-body rotation effect of superelement

Although progressive collapse of building structures usually initiates a small portion of localized damage and the major portion of the structure remains in linear-elastic state, it may undergo large rotations during the collapse, as shown in Figure 2.3. In this case, the condensed stiffness matrix $\mathbf{K}^*$ and load vector $\mathbf{P}^*$ in Equations 2.8 and 2.9 are required to be modified by considering large rigid-body rotation correction. An application of superelement for progressive collapse analysis is carried out by Li and Hao (2013), but it did not consider the rigid-body rotation effect in superelement formulation. In fact, so far, only limited publications have been reported on considering large rigid-body rotations in superelement analysis.

Figure 2.3 Collapse with large rigid-body rotations (RMS 2000)

Cardona and Grardin (1992) proposed a superelement formulation capable of handling large finite rotations during the analysis. The standard component mode approach with a lumped mass inertia approximation was adopted for linear vibration analysis of mechanism members. Limitations of this study may include the limited application of this program for special element types, and the computation of inertia forces only relied on the information contained in the reduced mass matrix over the component mode basis.

The commercial program ABAQUS/Standard (ABAQUS 2011) evaluates large rigid-body rotation effect of superelement by picking the reference nodes in the linear substructure, in order to determine the overall rigid-body rotations of superelement during the analysis. In this method, it is assumed that the substructure exhibits only small strains. Hence, the original and current positions of two (or three) reference nodes can be used to determine the rotation matrix of the substructure for 2D (or 3D) problems. However, this method is not ideal and has the disadvantages that (1) the ambiguity in selection of reference nodes, and (2) the relatively large errors when using global rigid-body rotations to replace

respective rigid-body rotations of each individual element in the linear substructure.

Long *et al.* (2013) proposed an integrated superelement concept for an efficient structural analysis. The superelement is formulated by applying a unit-load to each DOF of one selected boundary node and one selected DOF of an additional node in the linear zone. The rigid-body rotation effect of superelement was considered by the resultant rotation matrix of all boundary nodes, based on the compound rotation formulation proposed by Crisfield (1990). While this approach could be considered to be a more appropriate way to carry out a general superelement analysis, some points may still need to be further discussed: (1) Due to a non-standard treatment of condensation, the superelement stiffness matrix obtained is non-symmetric. Therefore, more memories are required to store the non-zero stiffness terms during the process of condensation and recovery. In addition, non-conservative character of moments about fixed axes may lead to a series of difficulties culminating in non-symmetric stiffness matrices of elements (Argyris 1982); (2) it may be unreliable that using the compound rotation effect of boundary nodes evaluates the global rigid-body rotations of the superelement. This is because the rigid-body rotations of individual nodes in an element may not be consistent with the rigid-body rotations of this element. Likewise, it may not be accurate to evaluate the rigid-body rotation effect of the superelement with the compound rotations of its boundary nodes; and (3) the compound rotation formulation (Crisfield 1990) adopted is non-commutative. Hence, its application to predict the compound rotation effect of boundary nodes may not be suitable for 3D problems.

2.4.3 Adaptive superelement technique

Conventional superelement analysis requires the analyst to define the linear and

nonlinear zones in advance before the analysis. However, in some problems, the analyst may not know the linear zone *a-priori*, and thereby cannot define the superelement appropriately. In addition, nonlinearity may initiate in a relatively small portion of the structure but spread gradually and finally expand to different locations that cannot be predefined. Progressive collapse is such a typical problem that difficult for the analyst to predefine the eventual range of the nonlinear zone for superelement analysis. Consequently, conventional superelement approaches, which need to know the linear zone in advance, are not applicable directly for progressive collapse analysis. Fortunately, adaptive superelement technique can give an effective way to handle this problem. When nonlinearity spreads to the linear zone, the analysis program identifies new linear and nonlinear zones and reformulates new superelement for subsequent analysis. However, only few publications (Han and Abel 1985; Radermacher and Reese 2014) on adaptive superelement technique have been presented, and even no report of this technique applied to progressive collapse analysis of RC structures has been found so far.

Han and Abel (1985) proposed an adaptive substructure algorithm for finite element elastoplastic analysis. In this work, the adaptation scheme was established based on the effective stress level at the integration points of each element. Empirical formulae were provided to determine three types of effective stress level in the vicinity of yield stress so that the nonlinear zone is sufficiently large and remains valid for several load steps. From this study, it was concluded that the results obtained from the adaptive substructure analysis were nearly identical to that from fully nonlinear analysis, and when nonlinearity initiated in a small subgroup of elements and expanded gradually during the loading process, the adaptive substructure analysis was generally more efficient than the fully nonlinear analysis. However, this approach may not be available for progressive collapse analysis of RC structures because (1) its adaptation scheme is not

suitable for reinforced concrete material which is non-homogeneous and has no specific yield stress level; (2) it did not consider rigid-body rotation effect of superelement during static condensation; and (3) it considered the material nonlinearity but did not consider Geometrical nonlinearity in the analysis.

Radermacher and Reese (2014) proposed an adaptive Proper Orthogonal Decomposition (POD) method for metal forming analysis. The adaptation scheme was activated as the accumulated effective plastic strain of the element attains a prescribed threshold value during the analysis. The threshold values of plastic strain at the level of 10^{-2} , 10^{-3} and 10^{-4} were selected for error comparison analysis among the adaptive, conventional and selective POD methods. The results obtained showed that the adaptive POD method was more accurate than the conventional POD method and was more efficient than the selective POD method. However, this approach may not be suitable for progressive collapse analysis as it cannot consider the rigid-body rotation effect of the linear substructure, which is critical to ensure computational accuracy in large deformation analysis. In addition, the adaptation scheme proposed, based on effective plastic strain, is not applicable for reinforced concrete materials, due to the brittleness of concrete which may crack or crush suddenly even when steel rebar is still linear-elastic.

Without using superelement formulation, an efficient adaptive approach was proposed for nonlinear analysis of RC framed structures (Karayannis *et al.* 1994; Izzuddin and Smith 2000). Details of the automatic refinement of elastic quartic elements into elasto-plastic cubic elements were illustrated, including: (1) the construction of an effective interaction surface between the axial force and biaxial moments which defines the elastic limit, (2) the determination of the generalized cross-section stress state along an elastic element, (3) the checking of a cross-section stress state for exceeding the elastic limit, and (4) the automatic subdivision of an elastic element into an appropriate number of elastic and

elasto-plastic elements. The proposed nonlinear adaptive analysis method is able to enhance computational efficiency without significant loss of modeling accuracy.

2.5 Conclusions

In this chapter, the current status of research on four aspects, namely, numerical studies of progressive collapse, damage assessment of RC structures, progressive collapse modeling of RC frames and superelement technique is reviewed. Three research gaps in progressive collapse modeling, i.e., *comprehensive* damage and failure criteria to identify actual collapse mechanism, simulation with collapse sequence considered and *efficient* analysis of large scale model for progressive collapse modeling of RC frame structures are found from the existing research work, which give the impetus to carry out the work in this thesis.

CHAPTER 3 FORMULATION OF NONLINEAR BEAM ELEMENTS FOR LARGE DEFORMATION ANALYSIS

3.1 Introduction

A 3-node Total Lagrangian (TL) Timoshenko beam element formulation (Dvorkin *et al.* 1988) is presented. A fiber-discretized section model is incorporated into the TL beam element to characterize the cross-section properties of RC beams/columns during the analysis. The nonlinear constitutive laws of concrete and steel used in the fiber model are introduced. Some assumptions are stated as follows for large deformation analysis:

- The beam element is initially straight.
- The cross-section of a beam element is initially a plane and it remains plane after deformation (plane-section assumption).
- Transverse shear stress and strain are assumed uniformly distributed across the cross-section of the element.

To capture the nonlinear response of the structure, the generalized displacement control method (Yang and Shieh 1990) is employed. In addition, energy convergence criterion is adopted as $\Delta \mathbf{u}^T \cdot \Delta \mathbf{Q} \leq tol \cdot \mathbf{u}^T \cdot \lambda \mathbf{Q}$, where the tolerance can be automatically adjusted within 10^{-4} - 10^{-6} according to the number of the current iteration step. Two typical numerical examples are given to demonstrate the validity of the beam element formulation for large deformation analysis.

3.2 Total Lagrangian formulation of nonlinear beam elements

3.2.1 Kinematics of beam elements

In order to describe the position vector of any point inside the beam element, a natural coordinate system (ξ, η, ζ) is given as shown in Figure 3.1. Three orthogonal vectors $({}^0\mathbf{V}_\xi^k, {}^0\mathbf{V}_\eta^k, {}^0\mathbf{V}_\zeta^k)$ are introduced at node k in the initial configuration, where the vector ${}^0\mathbf{V}_\xi^k$ is defined by the tangent of the centroidal line of the element. ${}^0\mathbf{V}_\eta^k$ and ${}^0\mathbf{V}_\zeta^k$ are determined by two auxiliary points located at the direction of the major inertia axis of cross-section. Therefore, the initial position can be expressed as

$${}^0\mathbf{x}(\xi, \eta, \zeta) = \sum_{k=1}^3 \left[h_k(\xi) {}^0\mathbf{x}_k + \frac{\eta}{2} \cdot h_k(\xi) a_k {}^0\mathbf{V}_\eta^k + \frac{\zeta}{2} \cdot h_k(\xi) b_k {}^0\mathbf{V}_\zeta^k \right] \quad (3.1)$$

Figure 3.1 Beam element configuration

where a_k and b_k are the size of the cross-section in the direction of η and ζ , respectively. The shape functions $h_k(\xi)$ is represented by

$$h_1(\xi) = \frac{1}{2}(1 - \xi) - \frac{1}{2}(1 - \xi^2), h_2(\xi) = 1 - \xi^2, h_3(\xi) = \frac{1}{2}(1 + \xi) - \frac{1}{2}(1 - \xi^2) \quad (3.2)$$

Similarly, the position vector of any point inside the beam element at time t is

$${}^t\mathbf{x}(\xi, \eta, \zeta) = \sum_{k=1}^3 \left[h_k(\xi) {}^t\mathbf{x}_k + \frac{\eta}{2} \cdot h_k(\xi) a_k {}^t\mathbf{V}_\eta^k + \frac{\zeta}{2} \cdot h_k(\xi) b_k {}^t\mathbf{V}_\zeta^k \right] \quad (3.3)$$

where the orthogonal vectors ${}^t\mathbf{V}_\eta^k, {}^t\mathbf{V}_\zeta^k$ are calculated by

$${}^t\mathbf{V}_i^k = {}^t\mathbf{R}^k {}^0\mathbf{V}_i^k, \quad i = \eta, \zeta \quad (3.4)$$

${}^t\mathbf{R}^k$ is the rotation matrix of node k at time t corresponding to the initial configuration. The expression is given by

$${}^t\mathbf{R}^k = \mathbf{I}_3 + \frac{\sin {}^t\theta^k}{{}^t\theta^k} {}^t\boldsymbol{\Theta}^k + \frac{1}{2} \left[\frac{\sin({}^t\theta^k/2)}{{}^t\theta^k/2} \right]^2 ({}^t\boldsymbol{\Theta}^k)^2 \quad (3.5)$$

where $\mathbf{I}_3$ is a 3×3 identity matrix, and

$${}^t\boldsymbol{\Theta}^k = \begin{bmatrix} 0 & -{}^t\theta_3^k & {}^t\theta_2^k \\ {}^t\theta_3^k & 0 & -{}^t\theta_1^k \\ -{}^t\theta_2^k & {}^t\theta_1^k & 0 \end{bmatrix} \quad (3.6)$$

${}^t\boldsymbol{\Theta}^k$ refers to the spin matrix corresponding to the rotational axial vector ${}^t\boldsymbol{\theta}^k$ at node k , given by

$${}^t\boldsymbol{\theta}^k = {}^t\theta^k \mathbf{e}^k, \quad {}^t\theta^k = \sqrt{({}^t\theta_1^k)^2 + ({}^t\theta_2^k)^2 + ({}^t\theta_3^k)^2} \quad (3.7)$$

3.2.2 Constitutive description

The covariant basis vector $\mathbf{g}_i$ can be defined as

$$\mathbf{g}_i = \frac{\partial \mathbf{x}}{\partial i}, \quad i = \xi, \eta, \zeta \quad (3.8)$$

Hence, the components of Green-Lagrangian strain tensor at time t is

$$\begin{cases} {}^t\tilde{\varepsilon}_{\xi\xi} = \frac{1}{2} ({}^t\mathbf{g}_\xi \cdot {}^t\mathbf{g}_\xi - {}^0\mathbf{g}_\xi \cdot {}^0\mathbf{g}_\xi) \\ {}^t\tilde{\varepsilon}_{\xi\eta} = \frac{1}{2} ({}^t\mathbf{g}_\xi \cdot {}^t\mathbf{g}_\eta - {}^0\mathbf{g}_\xi \cdot {}^0\mathbf{g}_\eta) \\ {}^t\tilde{\varepsilon}_{\xi\zeta} = \frac{1}{2} ({}^t\mathbf{g}_\xi \cdot {}^t\mathbf{g}_\zeta - {}^0\mathbf{g}_\xi \cdot {}^0\mathbf{g}_\zeta) \end{cases} \quad {}^t\mathbf{g}_i = \frac{\partial {}^t\mathbf{x}}{\partial i}, \quad {}^0\mathbf{g}_i = \frac{\partial {}^0\mathbf{x}}{\partial i} \quad (3.9)$$

The other three components are zero on account of the deformation hypotheses. Considering the equilibrium configuration of the system at time $t + \Delta t$, the principle of virtual work gives

$$\int_{_0V} {}^{t+\Delta t} \tilde{S}^{ij} \delta {}^{t+\Delta t} \tilde{\varepsilon}_{ij} d^0V = {}^{t+\Delta t} \mathfrak{R} \quad (3.10)$$

where ${}_0V$ is the volume measured in the initial configuration, ${}^{t+\Delta t} \tilde{S}^{ij}$ are the contravariant components of 2nd Piola-Kirchhoff stress tensor in the convected coordinate system at time $t + \Delta t$, and ${}^{t+\Delta t} \mathfrak{R}$ is the virtual work expression of total external loads acting on the body at time $t + \Delta t$.

The virtual work equation 3.10 can be rewritten in an incremental matrix form:

$$\int_{_0V} \delta {}_0 \tilde{\mathbf{e}}^T {}_0 \tilde{\mathbf{C}} {}_0 \tilde{\mathbf{e}} d^0V + \int_{_0V} \delta {}_0 \tilde{\boldsymbol{\eta}}^T {}_0 \tilde{\mathbf{S}} d^0V = {}^{t+\Delta t} \mathfrak{R} - \int_{_0V} \delta {}_0 \tilde{\mathbf{e}}^T {}_0 \tilde{\mathbf{S}} d^0V \quad (3.11)$$

where

$${}_0 \tilde{\mathbf{e}} = \begin{bmatrix} {}_0 \tilde{\varepsilon}_{\xi\xi} & 2 {}_0 \tilde{\varepsilon}_{\xi\eta} & 2 {}_0 \tilde{\varepsilon}_{\xi\zeta} \end{bmatrix}^T = {}_0 \tilde{\mathbf{B}}_L \mathbf{U} \quad (3.12)$$

${}_0 \tilde{\boldsymbol{\eta}}$ is the vector which includes all the quadratic terms of strain. ${}_0 \tilde{\mathbf{C}}$ is the constitutive matrix in the orthogonal curvilinear coordinate frame for the current iteration. Its components ${}_0 \tilde{C}^{ijkl}$ are transformed from the components in the Cartesian coordinate frame by

$${}_0 \tilde{C}^{ijkl} = \frac{1}{\sqrt{g_{ii}g_{jj}g_{kk}g_{ll}}} C_{ijkl} \quad (3.13)$$

where g_{ij} is the covariant component of metric tensor. For orthogonal curvilinear coordinate frame, one obtains

$$g_{ij} = \mathbf{g}_i \cdot \mathbf{g}_j = \begin{cases} 0 & (i \neq j) \\ \|\mathbf{g}_i\|^2 & (i = j) \end{cases} \quad (3.14)$$

The constitutive matrix in the Cartesian coordinate frame is given as

$${}_0\mathbf{C} = \begin{bmatrix} E & 0 & 0 \\ 0 & \kappa G & 0 \\ 0 & 0 & \kappa G \end{bmatrix} \quad (3.15)$$

where E is Young's modulus; G is shear modulus; κ is shape coefficient. Hence

$${}_0\tilde{\mathbf{C}} = \begin{bmatrix} \frac{E}{\| {}^0\mathbf{g}_\xi \|^4} & 0 & 0 \\ 0 & \frac{\kappa G}{\| {}^0\mathbf{g}_\xi \|^2 \| {}^0\mathbf{g}_\eta \|^2} & 0 \\ 0 & 0 & \frac{\kappa G}{\| {}^0\mathbf{g}_\xi \|^2 \| {}^0\mathbf{g}_\zeta \|^2} \end{bmatrix} \quad (3.16)$$

The components of Green-Lagrangian strain tensor transformed from the orthogonal curvilinear frame into the Cartesian coordinate frame are given as

$${}^t\tilde{\varepsilon}_{kl} = \| {}^0\mathbf{g}_k \| \cdot \| {}^0\mathbf{g}_l \| {}^t\varepsilon_{kl} \quad (3.17)$$

The component transformation of 2nd Piola-Kirchhoff stress tensor is

$${}_0^t\tilde{S}^{kl} = \frac{1}{\| {}^0\mathbf{g}_k \| \cdot \| {}^0\mathbf{g}_l \|} {}^t\tau^{kl} \quad (3.18)$$

where ${}^t\tau^{kl}$ is the Cartesian components of stress tensor.

For the present beam element formulation

$${}_0^t\tilde{\mathbf{S}} = [{}_0^t\tilde{S}^{\xi\xi} \quad {}_0^t\tilde{S}^{\xi\eta} \quad {}_0^t\tilde{S}^{\xi\zeta}]^T \quad (3.19)$$

3.2.3 Stiffness matrix and internal force vector

Substituting Equation 3.12 into the first integral of Equation 3.11, one obtains

$$\int_{\circ_V} \delta {}_0\tilde{\mathbf{e}}^T {}_0\tilde{\mathbf{C}} {}_0\tilde{\mathbf{e}} d^0V = \delta \mathbf{U}^T \left(\int_{\circ_V} {}_0^t\tilde{\mathbf{B}}_L^T {}_0\tilde{\mathbf{C}} {}_0^t\tilde{\mathbf{B}}_L d^0V \right) \mathbf{U} = \delta \mathbf{U}^T {}_0^t\mathbf{K}_L \mathbf{U} \quad (3.20)$$

where

$${}_0^t\mathbf{K}_L = \int_{\circ_V} {}_0^t\tilde{\mathbf{B}}_L^T {}_0\tilde{\mathbf{C}} {}_0^t\tilde{\mathbf{B}}_L d^0V \quad (3.21)$$

The second integral of Equation 3.11 can be written in the following form:

$$\int_{\text{o}_V} \delta \text{o}_0 \tilde{\boldsymbol{\eta}}^T \text{o}_0^t \tilde{\mathbf{S}} d^0V = \delta \mathbf{U}^T \text{o}_0^t \mathbf{K}_{NL} \mathbf{U} \quad (3.22)$$

where $\text{o}_0^t \mathbf{K}_{NL}$ is derived from Equation 3.23 where the expressions of matrix $\mathbf{B}$ is

$$\begin{bmatrix} \tilde{\mathbf{B}}_\xi^T \tilde{\mathbf{B}}_\xi + 2 \text{o}_0^t \tilde{\mathbf{B}}_{\xi\xi}^2 & \tilde{\mathbf{B}}_\xi^T \tilde{\mathbf{B}}_\eta + \tilde{\mathbf{B}}_\eta^T \tilde{\mathbf{B}}_\xi + 2(\text{o}_0^t \tilde{\mathbf{B}}_{\xi\eta}^2 + \text{o}_0^t \tilde{\mathbf{B}}_{\eta\xi}^2) & \tilde{\mathbf{B}}_\xi^T \tilde{\mathbf{B}}_\zeta + \tilde{\mathbf{B}}_\zeta^T \tilde{\mathbf{B}}_\xi + 2(\text{o}_0^t \tilde{\mathbf{B}}_{\xi\zeta}^2 + \text{o}_0^t \tilde{\mathbf{B}}_{\zeta\xi}^2) \end{bmatrix}$$

which can be found in the literature (Dvorkin *et al.* 1988).

$$\text{o}_0^t \mathbf{K}_{NL} = \int_{\text{o}_V} \mathbf{B}_0^t \tilde{\mathbf{S}} d^0V \quad (3.23)$$

Substituting Equation 3.12 into the last integral of Equation 3.11, we have

$$\int_{\text{o}_V} \delta \text{o}_0 \tilde{\mathbf{e}}^T \text{o}_0^t \tilde{\mathbf{S}} d^0V = \delta \mathbf{U}^T \left(\int_{\text{o}_V} \text{o}_0^t \tilde{\mathbf{B}}_L^T \text{o}_0^t \tilde{\mathbf{S}} d^0V \right) = \delta \mathbf{U}^T \text{o}_0^t \mathbf{F} \quad (3.24)$$

where $\text{o}_0^t \mathbf{F}$ is the internal force vector of the element, which can be expressed as

$$\text{o}_0^t \mathbf{F} = \int_{\text{o}_V} \text{o}_0^t \tilde{\mathbf{B}}_L^T \text{o}_0^t \tilde{\mathbf{S}} d^0V \quad (3.25)$$

Note that the virtual work due to external loads can be expressed by

$$\text{o}_0^{t+\Delta t} \mathfrak{R} = \delta \mathbf{U}^T \text{o}_0^{t+\Delta t} \mathbf{P} \quad (3.26)$$

Using Equations 3.20, 3.22, 3.24 and 3.26 for the virtual work equation 3.11, we have

$$\delta \mathbf{U}^T (\text{o}_0^t \mathbf{K}_L + \text{o}_0^t \mathbf{K}_{NL}) \mathbf{U} = \delta \mathbf{U}^T (\text{o}_0^{t+\Delta t} \mathbf{P} - \text{o}_0^t \mathbf{F}) \quad (3.27)$$

Hence

$$(\text{o}_0^t \mathbf{K}_L + \text{o}_0^t \mathbf{K}_{NL}) \mathbf{U} = \text{o}_0^{t+\Delta t} \mathbf{P} - \text{o}_0^t \mathbf{F} \quad (3.28)$$

where $\text{o}_0^t \mathbf{K} = \text{o}_0^t \mathbf{K}_L + \text{o}_0^t \mathbf{K}_{NL}$ is the stiffness matrix of the element.

3.3 Fiber model

As shown in Equations 3.21, 3.23 and 3.25, a volume integral is needed for calculation of both stiffness matrix and internal force vector for each element. This volume integral can be separated effectively into a surface integral over the cross-section and a linear integral along the element length. The linear integral is carried out by an exact Gaussian integration, while the surface integral is performed with the fiber model. Figure 3.2 shows a typical fiber cross-section in a beam element. The cross-section at each Gaussian integration point of the element is discretized by several fibers to capture nonlinear behavior at the material level.

Figure 3.2 Fiber beam element

Based on the plane-section assumption, a complete trace of nonlinear behavior for each fiber is performed. This ensures an accurate prediction of the stiffness matrix and internal force vector of the cross-section during deformations. The advantages of using fiber model include: (1) it is able to be implemented in terms of the 1D material constitutive law of each fiber, which can be simply obtained from physical tests; and (2) The fiber discretization enables an effective combination of multiple materials in one cross-section and this is especially suitable for RC structural analysis. Empirically, for a regular RC cross-section, 40 fiber segments are capable of attaining a reasonably accurate prediction from the analysis.

Assumptions made in the fiber model are stated as follows:

- (i) Uniaxial material stress-strain relations are adopted for each fiber. Accordingly, the multi-axial nonlinear behavior of cross-section will be described by the superposition of uniaxial nonlinear behavior.
- (ii) The strain distribution in each fiber remains uniform during deformations, that is, the strain at the Geometrical center of a fiber is employed as the average strain of that fiber for numerical integrations over the cross-section.

3.4 Material nonlinearity

3.4.1 Concrete

The modified Kent-Park model is adopted to provide tensile and compressive constitutive laws for concrete fibers. It was originally established by Kent and Park (1971) and then extended by Scott *et al.* (1982). The monotonic envelope curve of concrete in compression is described as

$$\sigma_c = \begin{cases} K f'_c \left[2 \left(\frac{\varepsilon_c}{\varepsilon_0} \right) - \left(\frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] & \varepsilon_c \leq \varepsilon_0 \\ K f'_c [1 - Z \cdot (\varepsilon_c - \varepsilon_0)] \geq 0.2 K f'_c & \varepsilon_c > \varepsilon_0 \end{cases} \quad (3.29)$$

$$\left. \begin{aligned} K &= 1 + \frac{\rho_s f_{yh}}{f'_c} \\ \varepsilon_0 &= 0.002 K \\ Z &= 0.5 / \left(\frac{3 + 0.29 f'_c}{145 f'_c - 1000} + 0.75 \rho_s \sqrt{h'/s_h} - \varepsilon_0 \right) \end{aligned} \right\} \quad (3.30)$$

where, ε_0 is concrete strain at the maximum stress. K is a factor accounting for the strength increase due to stirrup confinement. Z is the strain softening slope. f'_c is the concrete compressive cylinder strength. f_{yh} is the yield strength of stirrup. ρ_s is the ratio of the stirrup volume. h' is the width of the concrete core measured to the outside of stirrups. s_h is the central to central spacing of stirrups. For the confined concrete model, the ultimate compressive strain ε_u is determined by

$$\varepsilon_u = 0.004 + 0.9\rho_s(f_{yh}/300) \quad (3.31)$$

The envelop curve of concrete in tension is described as follows:

$$\sigma_c = \begin{cases} E_t \varepsilon_c & \varepsilon_c \leq \varepsilon_{0t} \\ f_t \left[1 - \mu \left(\frac{\varepsilon_c}{\varepsilon_{0t}} - 1 \right) \right] \geq 0 & \varepsilon_{0t} < \varepsilon_c \leq \varepsilon_{cr} \\ 0 & \varepsilon_c > \varepsilon_{cr} \end{cases} \quad (3.32)$$

where $\varepsilon_{0t} = f_t/E_t$ with $E_t = 2Kf_c/\varepsilon_0 = 1000f_c$. μ is a factor that accounts for the effect of tensile hardening, and the cracking strain is determined by

$$\varepsilon_{cr} = (1 + 1/\mu)\varepsilon_{0t} \quad (3.33)$$

For the hysteretic behavior of concrete material, the following rules proposed by Spacone *et al.* (1996) are employed.

1. Unloading from a point on the envelop curve takes place along a straight line connecting the point ε_r at which unloading starts to a point ε_p on the strain axis given by the equations

$$\frac{\varepsilon_p}{\varepsilon_0} = 0.145 \cdot \left(\frac{\varepsilon_r}{\varepsilon_0} \right)^2 + 0.13 \cdot \frac{\varepsilon_r}{\varepsilon_0} \quad \text{for } \frac{\varepsilon_r}{\varepsilon_0} < 2 \quad (3.34)$$

$$\frac{\varepsilon_p}{\varepsilon_0} = 0.707 \cdot \left(\frac{\varepsilon_r}{\varepsilon_0} - 2 \right) + 0.834 \quad \text{for } \frac{\varepsilon_r}{\varepsilon_0} \geq 2 \quad (3.35)$$

where ε_0 is the strain level corresponding to the maximum stress in compression.

2. The concrete stress is equal to zero for strains smaller than the strain at complete unloading (open crack) since tensile resistance is neglected.
3. On reloading in compression the stress is zero as long as the strain is smaller than the strain at complete unloading. Once the concrete strain becomes larger than that value, reloading continues along the previous unloading path. In reality, unloading and reloading follow nonlinear paths which together form a hysteresis loop.

The complete concrete stress-strain curves for both confined and unconfined

concrete are shown in Figure 3.3.

Figure 3.3 Stress-strain curves for confined and unconfined concrete

3.4.2 Steel

A bilinear strain-hardening model is employed for steel fibers, as shown in Figure 3.4. This model is used for steel in both tension and compression. The expression is given by

$$\begin{cases} \varepsilon = \varepsilon_e + \varepsilon_p \\ \sigma_n = \sigma_{n-1} + E_s d\varepsilon_e \end{cases} \quad (3.36)$$

where ε is the total normal strain of a steel fiber. ε_e is the elastic strain and ε_p is the plastic strain. σ_n is the desired normal stress. σ_{n-1} is the normal stress at the latest equilibrium state. E_s is the Young's modulus and $d\varepsilon_e$ is the elastic incremental strain. In this thesis, the value of the hardening modulus E'_s (in Figure 3.4) is determined based on the rebar tensile test results for those numerical examples with the test validation. And for the other numerical examples, as the real value is unknown, a lower 5% characteristic value has been assumed for this elastic-plastic steel model.

Figure 3.4 Stress-strain relationship for steel

3.5 Validation

3.5.1 Example 3.1: bending of cantilever beam

A straight cantilever beam is subjected to a concentrated end-moment M_0 . The cantilever is uniformly discretized into five 3-node TL beam elements, and its details are given in Figure 3.5. The load factor and displacements at the free end of the beam is listed in Table 3.1, while the deformation shapes of this beam are depicted in Figure 3.6. As this example is a simple cantilever beam subjected to pure bending at the free end, the accurate rotation expression of end-bending moment M can be given as $\theta = ML/EI = \lambda M_0 L/EI$. Substituting the given value of $M_0 = 2\pi EI/L$, the theoretical solution of rotation $\theta = \lambda \cdot 2\pi$. Hence, the simulation results shown in Figure 3.6 and Table 3.1 justify that the analysis can capture the theoretical solution very well and indicate that there is a very good accuracy of this TL beam element formulation for large rotation analysis.

Figure 3.5 Cantilever beam and its finite element model

Table 3.1 Load and displacement characteristics of the cantilever beam

Load factor λ	u_x	u_y	θ
0.1	-6.5323	30.5772	0.6323
0.2	-24.5206	55.1676	1.2623
0.3	-49.9159	69.5606	1.8935
0.4	-76.9703	71.9314	2.5219
0.5	-100.4556	63.3680	3.1560
0.6	-115.9030	47.4685	3.7881
0.7	-121.6544	29.3151	4.4140
0.8	-118.7397	13.3579	5.0447
0.9	-109.9743	3.0985	5.6809
1.0	-99.4345	0.0106	6.3189

Figure 3.6 Deformed shapes of the cantilever beam

3.5.2 Example 3.2: cantilever 45° bend subject to fixed-end load

A cantilever 45° bend is subjected to a fixed-end load as shown in Figure 3.7. The bend has a radius of 100 units and a unit square cross-section. Young's modulus $E = 10^7$ and shear modulus $G = 5 \times 10^6$. The same problem has been studied by Bathe and Bolourchi (1979). In their analysis, eight 2-node beam elements were employed. In this analysis, six 3-node TL beam elements were used and the tip displacements under different load levels were predicted, as listed in Table 3.2, from which it can be found that the results predicted by the present TL beam elements agree well with that from Bathe's analysis.

Figure 3.7 Cantilever 45° bend subjected to an end load

Table 3.2 Comparison of tip displacements with Bathe's analysis

Load level	Present (Dvorkin <i>et al.</i> 1988)			Bathe and Bolourchi (1979)		
	u	v	w	u	v	w
300	22.9	59.7	39.8	22.5	59.2	39.5
450	18.7	52.8	48.1	-	-	-
600	16.1	47.5	53.5	15.9	47.2	53.4

3.6 Conclusions

A 3-node TL Timoshenko beam element formulation is presented. This beam element formulation will be employed to model RC beam-column frame structures. Two classical examples have demonstrated the effectiveness of the beam element formulation for large deformation analysis, providing the basis of

this study for progressive collapse modeling of RC frames. Fiber-discretized section model is introduced. This model will be incorporated into TL beam element formulation to capture material nonlinear behavior of RC cross-sections during the analysis. Also, this model will lay the foundation to achieve a set of effective damage assessment criteria for progressive collapse analysis of RC frames, by considering axial-shear-flexural interactions during structural deformations.

CHAPTER 4 DAMAGE ASSESSMENT FOR REINFORCED CONCRETE FRAMES SUBJECT TO PROGRESSIVE COLLAPSE

4.1 Introduction

A set of damage assessment criteria for progressive collapse analysis of RC frame structures is proposed in this chapter. Based on fiber-discretized finite beam element formulation (Dvorkin *et al.* 1988), flexural, shear and axial damage indices are respectively proposed with consideration of flexural, shear and axial coupling in the analysis. Flexural failure of RC cross-sections is indicated by complete loss of flexural rigidity, and shear/axial failures are identified as all material fibers across cross-section attain their ultimate tensile/compressive strains.

Three RC moment-resisting frames were tested to investigate structural behavior of RC frames under progressive collapse. They were used to verify the proposed flexural/axial damage criteria. The results obtained show an accurate capturing of critical damage characteristics (cracking/crushing of concrete, fracture/buckling of rebar and final failure/collapse) of RC frames during deformation, even at the late stage of catenary action. In addition, three shear-dominant damaged tests were studied to demonstrate the validity of the proposed shear damage criteria.

4.2 Flexural, shear and axial damage criteria for RC members

4.2.1 Overview of proposed damage criteria

For simulating the behavior of RC beam/column members, a fiber beam element

formulation is often preferred to obtain reasonable accuracy with acceptable computational cost (Felice 2009; Miha *et al.* 2014) compared to 3D solid formulation. With such formulation, a reasonably detailed strain or stress profile of cross-section can be described by fiber beam elements using a fiber model at the section level. Nonlinear behavior at the material level is characterized through a fiber discretization of the cross-section at integration points of each element. Figure 3.2 (in Section 3.3) shows a typical fiber beam element in an RC member. In this study, based on this fiber beam element model, flexural, shear and axial damages of an RC member are quantified by introducing appropriate damage indices in terms of deterioration of flexural rigidity, shear and axial strain energy of cross-sections. For different types of damages, the effect from flexural/shear/axial interaction is also considered. Figure 4.1 shows an overall framework for the proposed damage assessment scheme which provides the basis for identification of element failure and removal due to collapse. As shown in Figure 4.1, the proposed flexural damage index will take the damage effects from shear and axial forces into account. Also, the shear and axial damage indices will consider the flexural/axial and flexural/shear damage effects during the analysis, respectively.

E_i , ε_i and A_i are the elastic modulus, normal strain and area of fiber i ; ε_0 is the normal strain at the centroid of the cross-section; y_i is the distance from the center of fiber i to the neutral plane of the cross-section. Furthermore, the flexural damage index D_i describes the degradation of tangent modulus of fiber i . For concrete fibers in compression, D_i is given by $D_i = (E_i - E_{ri})/E_i$ where E_{ri} represents the residual tangent modulus. The tangent modulus is negative on the descending portion of the stress-strain relationship of concrete, which means $D_i = 1$ for the descending portion. For concrete fibers in tension, E_{ri} is set as the initial elastic modulus before cracking and zero after cracking. For steel fibers, the damage index is defined as $D_i = 0$ for damage state before yielding and $D_i = 1$ after yielding.

4.2.2.2 Shear effect on flexural damage

In the uniaxial constitutive laws adopted in the present formulation, shear is independent of bending. But from the truss models (Park and Paulay 1975; Collins and Mitchell 1991; Collins 1996; Kim and Mander 1999), shear force will increase tensile strain of longitudinal reinforcement when shear cracks occur. As shown in Figure 4.2, additional tensile strain ε_{SV} of longitudinal steel bars due to shear can be expressed as (Collins 1996)

$$\varepsilon_{SV} = \frac{0.5V \cot \theta}{E_s A_s} \quad (4.4)$$

where, V is the shear force of cross-section; E_s and A_s are the elastic modulus and area of steel; θ is the inclination angle of shear crack, which are aligned with the direction of principle stresses. In this study, θ is set to be 45° for simplicity so that no iteration to find θ is needed. It should be mentioned that a constant inclination angle of shear crack is an approximation since the real inclination angle may be different. It depends on several physical factors including aggregate size,

stirrup spacing, structural type and level of deformation etc. Considering all these factors is definitely a complicated work and not practical in actual application so we assume a constant inclination angle of shear crack for simplicity. Indeed, many studies and design codes (Park and Ang 1985; ACI318-05; Bazant 1997; McKenzie 2013) have adopted such an assumption for shear analysis and this assumption could be considered to be conservative (Bentz *et al.* 2006) in prediction of shear resistance. It is also worthy to emphasize that in Equation (4.4) the modulus E_s is initially elastic as the longitudinal steel bar is assumed to remain in the linear elastic range during the analysis.

Figure 4.2 Shear effect on flexural damage

For confined concrete, the variation of normal strain ε_{cV} is determined by force equilibrium as

$$\varepsilon_{cV} = \frac{E_s \varepsilon_{sV} A_s}{E_c A_{c0}} \quad (4.5)$$

where E_c and A_{c0} are the initial elastic modulus and area of confined concrete.

The flexural damage of fiber i considering shear effect is obtained by superimposing ε_{sVi} and ε_{cVi} onto the normal strain ε_i in Equations 4.2 and 4.3. Flexural failure of a cross-section occurs when its flexural damage index D_f attains unity. Since flexural failure of RC structures depends mainly on tensile yielding of reinforcement and concrete crushing, it is assumed that flexural failure of a cross-section will take place once all the steel bars in tension yield and all the extreme concrete fibers in compression crush in that cross-section.

4.2.3 Shear damage assessment

4.2.3.1 Bilinear shear model with flexural and axial effects

A bilinear shear model is developed to describe the shear behavior of RC structures, taking into account of flexural and axial effects. As shown in Figure 4.3, the key parameters V_y , K , K_h , γ_y and γ_u are needed to be defined. V_y is the vertical shear strength of cross-section; K and K_h are the shear rigidity of cross-section before and after shear yielding; γ_y and γ_u are the yield and ultimate shear strain of cross-section, respectively.

Figure 4.3 A bilinear shear model

By using a variable angle truss model (Kim and Mander 1999) for RC structures under shear and considering that shear distortion distributes uniformly along a shear-dominant damaged element, the combined shear-flexural mechanism in that element can be expressed by the relationship between shear force V and shear strain γ as follows

$$\gamma = \frac{V}{K} = \frac{K_{\gamma s} + K_{\gamma f}}{K_{\gamma s} \cdot K_{\gamma f}} \cdot V \quad (4.6)$$

where $K_{\gamma s}$ and $K_{\gamma f}$ are shear rigidity contributions from shear and flexural actions, deriving from virtual work principle in accordance with the variable angle truss model. The expressions (Kim and Mander 1999) are

$$K_{\gamma s} = \int_0^1 \frac{E_c A_g \cot^2 \alpha}{\frac{1}{\rho_{sw} n} + 2(1+x^2 \cdot \cot^2 \alpha)^2 + 2[1+(1-x)^2 \cdot \cot^2 \alpha]^2} dx \quad (4.7)$$

$$K_{\gamma f} = \frac{E_s A_{sl} \tan^2 \alpha}{\zeta} \quad (4.8)$$

where $\cot \alpha = a/h$ which is the shear span to the height of cross-section; A_g is the gross area of cross-section; A_{sl} is the area of longitudinal steel bar; ρ_{sw} refers to transverse reinforcement ratio; $n = E_s/E_c$ is the modulus ratio of steel to concrete; $\zeta = 0.5704$ is a constant coefficient. Two-point Gaussian integration is employed to solve Equation 4.7. For a shear-critical damaged element, yielding of transverse reinforcement should occur after peak shear. Hence, the post-yield shear rigidity can be predicted by the yielding of transverse reinforcement. Substituting the post-yield (hardening) modulus E_h of transverse reinforcement into Equation 4.7 will lead to the shear rigidity $K_{\gamma s}$. As E_h is usually small compared to the pre-yield modulus, the value of $K_{\gamma s}$ is small. Thus, from Equation 4.6 the post-yield shear rigidity is expressed as $K_h = \frac{K_{\gamma s} \cdot K_{\gamma f}}{K_{\gamma s} + K_{\gamma f}}$ which is also a small value, as shown in Figure 4.3.

The shear strength V_y suggested by Sezen *et al.* (2008) is adopted as

$$V_y = k \frac{f_{yw} A_{sw} d}{s} \sqrt{f'_c} A_g + k \left(\frac{0.5 \sqrt{f'_c} d}{a} \sqrt{1 + \frac{P}{0.5 \sqrt{f'_c} A_g}} \right) \cdot 0.8 A_g \quad (4.9)$$

where k is defined by displacement ductility; f'_c is the compressive strength of concrete; d is the depth of cross-section; a is the shear span; P is the axial load (positive for compression and negative for tension); s is the transverse reinforcement spacing; f_{yw} and A_{sw} are the yield strength and area of transverse reinforcement.

The yield shear strain can then be calculated from

$$\gamma_y = \frac{V_y}{K} \quad (4.10)$$

and the ultimate shear strain γ_u is obtained by (Gerin and Adebar 2004)

$$\gamma_u = \left(4 - 12 \frac{v_y}{f'_c}\right) \cdot \gamma_y \quad (4.11)$$

where coefficient v_y is given as (ACI318-05 2005)

$$v_y = 0.25\sqrt{f'_c} + \rho_{sw} \cdot f_{yw} \quad (4.12)$$

From this model, it can be found that the flexural effect on shear behavior of the element is incorporated into the shear rigidity K with K_{yf} , while the yield shear strain γ_y varies with shear strength V_y which is affected by axial load P .

4.2.3.2 Shear damage index

Based on the above shear model, shear damage index D_s is introduced to assess the damage growth of cross-section in terms of shear strain energy dissipation as

$$D_s = \frac{\Pi_s}{\Pi_{smax}} \quad (4.13)$$

where $\Pi_s = \left(\int_0^{\bar{\gamma}} \tau(\gamma) d\gamma\right)^{1/2}$ is the current shear strain energy norm and $\Pi_{smax} = \left(\int_0^{\gamma_u} \tau(\gamma) d\gamma\right)^{1/2}$ is the maximum shear strain energy norm. The current shear strain $\bar{\gamma}$ is determined by calculating weighted average for the fiber shear strain using the following equation.

$$\bar{\gamma} = \frac{\sum \gamma_i A_i}{\sum A_i} \quad (4.14)$$

where, γ_i and A_i are respectively the shear strain and area of fiber i . The term D_s is set as unity once $\bar{\gamma}$ is greater than γ_u when shear failure occurs. Considering the irreversibility of shear damage, it is assumed that unloading of shear forces during load redistribution process will not reduce shear damage.

4.2.4 Axial damage assessment

Under large deformation of RC beams in progressive collapse, catenary action may be mobilized when sufficient restraints are maintained at both beam-ends. In this case, axial damage of cross-sections dominates the eventual failure of the beam. For RC columns, axial failure can be caused by either the fracture of tensile reinforcement or the buckling of compressive reinforcement. These two scenarios for RC beams or columns should be considered in axial damage assessment.

4.2.4.1 Axial damage index

With fiber-discretized beam element formulation, the axial damage index of a cross-section is defined by summing up the axial damage of each fiber in the cross-section as

$$D_a = \frac{\sum D_{ai} A_i}{\sum A_i} \quad (4.15)$$

$$D_{ai} = \frac{\Pi_{ai}}{\Pi_{amax}} \quad (4.16)$$

where D_{ai} is the axial damage index of fiber i which is determined from the axial strain energy dissipation. The term $\Pi_{ai} = \left(\int_0^{\varepsilon_i} \sigma(\varepsilon) d\varepsilon \right)^{1/2}$ represents the current axial strain energy norm of fiber i ; ε_i is the normal strain of fiber i ; $\Pi_{amax} = \left(\int_0^{\varepsilon_u} \sigma(\varepsilon) d\varepsilon \right)^{1/2}$ is the maximum axial strain energy norm of the fiber. For concrete, ε_u refers to the cracking strain and ultimate compressive strain when it is subjected to tension and compression, respectively; for steel bars, ε_u is the fracture strain in tension, or the buckling strain $\varepsilon_{buckling}$ in compression.

Excessive compression in an RC member may cause buckling of reinforcement determined by spacing between stirrups and it in turn marks the deformation limit of the member. The buckling strain of reinforcement can be determined based on

the equilibrium of bending moment of reinforcement cross-section as (Pantazopoulou 1998)

$$\varepsilon_{buckling} = \frac{4\pi^2 I}{L^2 A} + \frac{\beta k L^2}{EA} \leq \varepsilon_y \quad (4.17)$$

where ε_y is the yield strain of longitudinal reinforcement; E , A and I are the elastic modulus, cross-sectional area and the second moment of longitudinal reinforcement, respectively; β is a constant and is equal to 0.0875; the terms k and L are given by $k = K/s$ and $L = (4\pi^2 E_h I / \beta k)^{1/4}$ where E_h is the hardening modulus of reinforcement, K and s are the effective axial stiffness and spacing of transverse reinforcement.

For RC structures, due to confinement of surrounding concrete, it is assumed that the reinforcement will not buckle before concrete crushing. Hence, the buckling strain $\varepsilon_{buckling}$ will be given by $\max\{\varepsilon_{cu}, \varepsilon_{buckling}\}$, where ε_{cu} is the ultimate compressive strain of concrete around the reinforcement.

4.2.4.2 Axial failure due to rebar fracture

It should be mentioned that based on the axial damage criteria given in Section 4.2.4.1, axial failure of a RC member should only occur when the axial damage index attaining unity (i.e. $D_a=1$ which indicates that all the concrete and the steel fibers are failed at that cross-section) at one cross-section of the member. However, it has been observed in many practical cases that axial failure, or almost complete loss of stiffness, of an RC member could occur when the axial damage indices at two or more different cross-sections in the member are close to but still lower than 1 (i.e. $D_a < 1$). An example of such case is shown in Figure 4.4.

Figure 4.4 Axial failure due to rebar fracture

In Figure 4.4, the RC beam has practically failed due to the complete fracture of tensile reinforcing bars in the member at 3 different cross-sections, but the axial damage indices at these locations have not reached (but close to) unity as the fracture of reinforcement occurs at *different* cross-sections. Such phenomenon can often be observed at the last stage of catenary action of RC beams under a missing column scenario, when all the tensile reinforcing bars at both the mid-span (bottom reinforcing bars) and the beam-end (top reinforcing bars) have fractured by axial tension. In this case, the stiffness of the member will only be a very small fraction of the stiffness of the undamaged member and the member can be considered to be failed completely in practice. In addition, such conditions may also cause numerical instability or convergent problems during progressive collapse analysis. Hence, during the progress collapse analysis, to ensure the stability of the numerical model and to avoid non-conservative prediction (very weak members hanging with very little stiffness), it is necessary to identify such axial failure case in an RC member and removed it from the model.

It is assumed that axial failure have occurred in a member when *all* its top and bottom rows of rebars have fractured, regardless whether the fracture is at the same cross-section of the member or not. After such an RC member is identified, the cross-section with the maximum axial damage index is considered to have failed

axially. The key steps to identify such axial failure mode are listed in Box 4.1.

- (i) At each equilibrium step, inspect axial damage indices of every member. For each member, check and identify any cross-section that all the top or bottom bars are fractured.
- (ii) If it is found for a given member that both the top and bottom bars have fractured (the fractured cross-sections can be different for the top and bottom bars), the member is considered to fail axially. The failed cross-section of the member is at where the axial damage index is the maximum.

Box 4.1 Identification of axial failure due to rebar fracture

4.3 Validation of proposed damage criteria

In order to validate the proposed flexural and axial damage assessment criteria, an experimental study with three scaled RC frames was conducted under a middle column removal scenario (Lim *et al.* 2015) and compared with numerical models. Since the specimens are specially designed as moment-resisting frames, shear damage is not evidently observed during the tests. Therefore, three shear-dominant damaged tests from the literature (Martinelli 2008) are modeled to verify the proposed shear damage criteria.

4.3.1 RC moment-resisting frames under column removal scenario

4.3.1.1 Experimental program

The RC moment-resisting frame is designed based on EC2 (2004), with dead loads and imposed loads equal to 5.0kN/m^2 and 7.1kN/m^2 , respectively. To investigate the structural mechanism of the frame under progressive collapse, beam-column sub-assemblages for the middle-bay and the end-bay were extracted at the locations of the contra-flexural points of bending moments as shown in Figure 4.5. The test program studied the influence of three types of boundary effect on RC

sub-assemblages with middle column removal scenario.

Figure 4.5 Locations of test specimens in the prototype structure

Accordingly, three one-third scaled specimens were designed and tested under a middle column removal scenario. Full Restraint (FR) and Full Restraint-Seismic (FR-S) specimens from the middle-bay frame were designed with normal and seismic detailing to investigate the effect of closer stirrup spacing on progressive collapse resistance. Partial Restraint (PR) specimen is extracted from an end-bay frame and designed with normal detailing. Reinforcement details of the three specimens are shown in Figure 4.6 and their geometrical and material properties are listed in Table 4.1.

columns. To investigate the behavior of the frame during progressive collapse when the middle column has been removed, an actuator was applied at the top of the middle joint until the specimen completely failed (Lim *et al.* 2015). Horizontal steel rods at both sides of the frame provided axial restraints for the FR and FR-S specimens (Figure 4.7a) so that the full restraint conditions of an internal frame (Figure 4.5a) are reproduced. Tension-compression load cells were installed to monitor horizontal reactions during the test. In addition, load pins were installed at the base of both columns to measure support reactions. Note that for the PR specimen, one horizontal support is removed (Figure 4.7b) so that the boundary condition effects of the end-bay frame (Figure 4.5b) is reproduced accordingly.

Figure 4.7 Support and boundary conditions of test setup

The instrumentation scheme shown in Figure 4.8 was used to measure internal forces of cross-section in the specimens. Six linear variable differential transformers (LV_1 to LV_6) were arranged to measure vertical displacements along the beam, upon which, rotational angles of the beam could be determined by linear interpolation (Lim *et al.* 2015). Thus, equilibrium relationships of the frame as a free body (as seen in Figure 4.8) could be obtained as follows.

$$M_i = H_3 \cdot (L_t + \Delta_i) + H_2 \cdot \Delta_i + H_1 \cdot (\Delta_i - L_b) - V_1 \cdot L_i$$

$$V_i = V_1 \cdot \cos \theta_i - (H_1 + H_2 + H_3) \cdot \sin \theta_i$$

$$N_i = V_1 \cdot \sin \theta_i + (H_1 + H_2 + H_3) \cdot \cos \theta_i$$

Figure 4.8 Layout of instrumentation

4.3.1.2 Numerical modeling of test specimens

Fiber-based finite element models adopted in the proposed damage assessment criteria are employed to simulate the failure of all three specimens. Figure 4.9a shows the numerical model for Specimens FR and FR-S and in Figure 4.9b Spring 4 is removed for the Specimen PR in order to simulate the effect of an end bay. 3-node TL Timoshenko beam element formulation (Dvorkin *et al.* 1988) is employed to capture the nonlinear response of RC frames under large deformations. Beam-column elements of different lengths are employed in different parts of the structure in accordance with longitudinal reinforcement ratio and stirrup spacing. Horizontal restraints from four tension-compression load cells are modeled by four bilinear springs. The properties of each spring in Table 4.2 are determined from the relationship of reaction forces and displacements measured by each load cell. To model the steel plate connections between the top of the column and the horizontal load cell, two pure steel elements are inserted as Elements 23 and 24 (Elements 22 and 23 for Specimen PR), as shown in Figure 4.9. A concentrated load is applied at

the mid-span between elements 15 and 16 until the frame has failed completely. Prior to this load, two point loads of $0.3f'_c A_g$ (i.e. the axial compression ratio is assumed to be 0.3) are applied at the top of both columns to simulate gravity loads from above stories and their values are kept constant during the whole test (Lim *et al.* 2015). Under extreme loading, generalized displacement control method (Yang and Shieh 1990) with superior convergence performance through limit points and snap-back points is applied to ensure numerical stability in the large deformation analysis.

As severe damages are expected to occur at both the mid-span and the two beam-ends, four Gauss-point cross-sections are selected to trace the crucial damage propagation as shown in Figure 4.9, in which, ‘L1’ and ‘L2’ represent the cross-sections near the beam-end and the mid-span of the left-side beam; ‘R1’ and ‘R2’ are the cross-sections near the beam-end and the mid-span of the right-side beam.

Table 4.2 Bilinear spring properties

Specimen	Spring	Property			Legend
		K_1 (N/mm)	gap (mm)	K_2 (N/mm)	
FR	1	636	2.4	18355
	2	244	3.1	27677	
	3	337	3.7	10566	
	4	455	2.6	85010	
FR-S	1	2715	2.2	20641	
	2	716	4.5	23667	
	3	160	7.8	101894	
	4	2073	3.9	4080	
PR	1	1359	2.9	27860	
	2	396	1.9	5917	
	3	2695	1.7	27606

Figure 4.9 Finite element models for Specimen (a) FR/FR-S; (b) PR

4.3.1.3 Validation of flexural and axial damage criteria

Before verifying the proposed damage assessment criteria, general results from numerical analysis are compared with those from experimental study to show the accuracy and validity of the present modeling. Note that reasonably accurate but not perfect predictions of the test results are expected. As in any numerical model, it is impossible to consider all factors that may affect the modeling accuracy. For example, in the present numerical model, factors such as splicing of reinforcement and discontinuity of cracking are not considered. However, it is expected that if a reasonably accurate numerical model is available, a reliable and effective damage assessment scheme should be able to give useful indication to the damage status of structural members of the RC frames.

(a) Comparison between numerical analysis and experimental results

Figure 4.10 shows the predicted and measured applied forces and displacements relationships at the mid-span for Specimens FR, FR-S and PR. In general, good agreement can be found between the two sets of results in terms of peak resistance of test frames at both the flexural and catenary action phases. It demonstrates the validity of the proposed damage assessment criteria.

Figure 4.10 Relationships of the applied force and displacement for Specimen (a) FR; (b) FR-S; (c) PR

As shown in Figures 4.10a and 4.10b, the first sudden drop of the applied force for FR and FR-S is caused by the fracture of bottom steel bars at the mid-span. Such occurrences only result in minor deviation between the numerical models and

actual experiments. For the second drop, it is found that consistent fracture of the top bars for Specimen FR occurs at the mid-span, but there are differences in the fracture locations of the top bars of Specimen FR-S for the experiment and the model. The actual fracture occurs near Section R1 while the model predicts its occurrence at Element 14. This difference may result from the finite element assumption of continuous displacement field which could lead to deviations in predictions of localized fracture of reinforcement where large cracking or spalling of concrete take place, as seen in the actual tests. Nevertheless, the top rebar fracture of both cases implies the eventual failure of the structure due to axial tension at catenary action stage. As the difference only surfaces at the last stage, it will not affect the accuracy of predictions. The behavior of Specimen PR with one less horizontal restraint is different from FR and FR-S. Catenary action could not further develop after the fracture of bottom bars at the mid-span. Numerical model agrees well with the test results for Specimen PR, as shown in Figure 4.10c.

In summary, although there are some differences in the prediction of the location of rebar fracture forwards the end stage, load-displacement curves for both flexural and catenary action stages of each specimen are in good agreement with test results. Hence, it can be concluded that the numerical model can provide good inputs for the validation of the proposed damage assessment criteria.

(b) Flexural damage identification

The beam-end and mid-span cross-sections of the more severely damaged beam for each specimen are chosen to demonstrate the proposed damage assessment criteria. Sections 'L1' and 'L2' are selected for Specimen FR-S and Sections 'R1' and 'R2' are for Specimens FR and PR. The relationships between the bending moment/flexural damage index and the mid-span displacement for the test frames are shown in Figure 4.11. From the moment-displacement relationships of FR and

FR-S, it can be seen that consistently good results are predicted by the two numerical models, showing effective and reliable predictions for flexural damage, especially for the yield displacement and the peak flexural resistance. A slightly larger error of predicted peak bending moment is found in Specimen PR. This is probably due to non-symmetric boundary conditions.

The flexural damage index D_f is used to identify the flexural damage state of beam sections, as labeled in Figure 4.11. The first sudden increase of D_f reflects the cracking of extreme concrete fibers in tension. The tensile stiffness of concrete fibers reaches zero when cracks occur, leading to a sharp reduction of cross-sectional flexural stiffness. The second sudden increase of D_f is caused by rebar yielding, which fits well with the yield point obtained from test results. Once the rebar has yielded, the value of D_f shoots up from about 0.4 to 0.8 or even beyond. This implies a close correlation between rebar yielding and flexural damage mechanism. After that, D_f attains unity and this corresponds to eventual crushing of extreme concrete fibers which marks flexural failure of cross-section. Then a plastic hinge forms at the corresponding cross-section reflected by the constant value of $D_f = 1$. Therefore, the proposed flexural damage criterion is able to predict well the yielding of reinforcing bars and the cracking and crushing of concrete, providing an effective trace of flexural damage propagation. In addition, flexural failure after rebar yielding will be more reasonable than the conventional prediction for plastic hinges using rebar yielding, which mostly occurs before attaining the peak flexural resistance, as shown in Figure 4.11.

Figure 4.11 Bending moment/flexural damage index D_f versus displacement at critical sections

(c) Axial damage identification

Figure 4.12 shows the relationships of the axial force/axial damage index to the mid-span displacement for the tested frames. For Specimen FR and FR-S, consistent trends of axial force resulting from tests and numerical models can be seen, while relatively larger difference for PR is observed in Figures 4.12e and 4.12f. The reason is that the column joint near Section R1 is severely damaged during the test due to the non-symmetric boundary conditions, but in the numerical model, no special joint elements are employed to capture joint damage effects. Figure 4.13 shows the eventual damage configurations of the joint near Section R1 of the three tests conducted. It can be seen that more severe damages occur in Specimen PR than in FR and FR-S due to non-symmetric test setup of PR. In essence, when damages propagate in a joint, their effects on lap length between reinforcing bars and concrete as well as dowel action inside the joint may lead to relatively inaccurate prediction of axial forces near the joint. Hence, developing effective joint elements to properly predict the behavior of RC joints during progressive collapse is an important research topic, but it is outside the scope of the present study. Suffice to say, the proposed axial failure criteria indicated by fiber elements attaining the ultimate strain of reinforcing bars provide reliable predictions for the ultimate state of collapse.

Figure 4.12 Axial force/axial damage index D_a versus displacement at critical sections

Figure 4.13 Damage patterns of the joint near Section R1 for Specimen (a) FR; (b) FR-S; (c) PR

Similar to flexural damage curve, two sharp growths of axial damage index D_a can be found in each axial damage curve (Figure 4.12). The first sudden increase corresponds to the cracking of concrete in tension which leads to axial tensile failure of concrete fibers. The second jump is due to gradual yielding of steel bars, which results in a rapid growth of fiber strain. For each plot in Figure 4.12, it is found that the value of axial damage index at rebar yielding (around 0.4) is consistently smaller than the flexural damage index at yielding (around 0.8), which implies that the ultimate capacity of a cross-section is eventually dominated by axial damage.

As shown in Figure 4.12b, the predicted axial failure occurs at Section R2 for Specimen FR when all reinforcing bars have fractured at this section. Axial failure of Specimen FR is marked by the entire failure of the mid-span section, as shown in Figure 4.14a. This demonstrates a good prediction of axial failure for Specimen FR with based on present axial failure criteria. Figure 4.12d shows that the axial failure of Specimen FR-S occurred at Section L2 with a sudden jump of D_a to unity. This is because the axial failure took place when all the steel bars fractured in the left beam while the fractured sections (Section in Element 14 and Section L2) are different for the top and bottom bars. According to the proposed axial failure criteria (Section 4.2.4.2), Section L2 is predicted to attain axial failure with a greater axial damage index. Compared with the test results, the ultimate failure of

Specimen FR-S occurs at mid-span with almost complete fracture of reinforcement over the cross-section, although the section has not been severed completely as shown in Figure 4.14b. It indicates good agreement of axial failure between the experiment and the simulation for Specimen FR-S. Generally, it has been observed from Figure 4.14 that good predictions on the ultimate state of collapse are achieved for both specimens. For Specimen PR, the maximum axial damage index attained is less than 1.0 since the test was terminated at a nearly buckling state of the right column due to safety concern and displacement constraint of the test rig. In addition, it should also be noted that similar trends for the variation of axial damage index can be found when compared with Specimen FR and FR-S.

In short, the proposed axial damage criteria are able to trace axial damage propagation of RC members beyond the stage of catenary action until the eventual axial failure of the structure. The yielding and complete fracture of reinforcing bars, as well as the final collapse status can be identified based on the present axial assessment procedure. This gives useful indication to the damage state of RC members after flexural action and provides reasonable evidence to identify collapse mechanisms under very large deformations.

Figure 4.14 The ultimate state of collapse for Specimens

4.3.2 Validation of shear damage criteria

Since the three specimens in Section 4.3.1 are designed as moment-resisting frames, shear damage is not critical and shear failure is avoided. Therefore, in this section, three shear tests SBV1, SBV2 and SBV3 from the literature (Martinelli 2008) are modeled to demonstrate the validity of the proposed shear damage criteria. All the specimens are designed as cantilever short column structures for investigation of shear-dominant damage during the tests. Geometrical and material properties for each specimen are given in Table 4.3. Three equal-length finite beam elements, which were corresponding to the minimum numbers of element needed to model the test results with reasonable accuracy, are employed along each column. A horizontal load is applied at the top of each specimen until final failure of the structure.

Table 4.3 Geometrical and material properties for shear tests

Term	Legend	Section (mm ²)	L (mm)	Strength (MPa)			Reinforcement	
				Concrete	Longitudinal bar	Transverse bar	Longitudinal	Transverse
SBV1		300×300	570	39.4	566.0	514.0	12Ø12	2Ø6@80
SBV2			660					
SBV3			750					

Figures 4.15a-4.15c show the shear force-drift ratio curves for the tested structures from numerical analysis and test results. Consistent peak shear force for each specimen can be found between experiment and simulation, showing good predictions for shear resistance of the specimens. The predicted shear yielding occurs in a strongly nonlinear shear state before attaining peak shear. This implies an acceptable degree of accuracy of the present modeling. As seen in Figure 4.15, the proposed bilinear shear model has provided reasonable predictions of shear behavior for each specimen, which gives good inputs for shear damage assessment.

The relationships between the shear damage index D_s and the drift ratio for the structures are also shown in Figure 4.15. The value of D_s hovers around 0.7 for each test when shear yielding occurs and it continues to unity upon reaching the ultimate shear strain. Adopting a bilinear shear model, shear strain before yielding is linearly proportional to shear force. Hence, shear damage index defined by the concept of shear strain energy norm is increased in proportion. Shear failure from modeling has been underestimated for all three specimens in terms of shear ductility calculated from Equation 4.11, indicating conservative predictions of shear failure for these tests based on present shear damage criteria. However, considering brittle property of shear, shear failure would usually occur soon after shear yielding. Therefore, the proposed damage criteria give reasonable predictions

of shear failure of these tested specimens.

Figure 4.15 Shear damage assessment for Specimen (a) SBV1; (b) SBV2; (c) SBV3

4.4 Conclusions

A set of damage assessment criteria to separately monitor flexural, shear and axial damage propagations is proposed for progressive collapse modeling of RC frames. The proposed damage criteria consider the damage effects from axial-shear-flexural coupling in progressive collapse. In addition, a numerical study is conducted and compared with the test results obtained on three scaled RC frames. Comparisons of test results with models show reasonable agreement. Clearly, provided that reasonable modeling results are available, the proposed damage criteria are able to capture damage characteristics of the tested moment-resisting frames and specimens with dominant shear behavior. The proposed flexural damage criteria predict well the critical damage characteristics (cracking and crushing of concrete, yielding of rebar etc.) of RC frames at the stage of flexural action. Moreover, flexural failure is identified after rebar yielding. This provides a better means to evaluate the initiation of plastic hinges. The axial damage criteria

are capable of assessing the ultimate capacity of RC members after attaining flexural failure while the sub-assemblages underwent catenary action and rebar fracture. Furthermore, a failure criterion to identify axial failure of RC members due to rebar fracture is proposed and it is capable of predicting actual collapse modes. For shear damage analysis, three shear-dominant damage tests were simulated. The results obtained show that the proposed shear damage criteria are able to simulate shear damage and predict shear failure of RC structures with acceptable accuracy.

As the proposed damage assessment scheme can effectively identify the flexural, shear and axial failures of RC structures and could be easily implemented with a cost-effective fiber beam-column finite element formulation, it can provide a good basis to identify collapse mechanisms in progress collapse modeling. Towards this end, one possible immediate extension of the presented work is to combine proposed damage assessment scheme with an effective algorithm to trace the progression of the collapse for RC frames so that an automatic and practical progressive collapse prediction tool could be developed.

CHAPTER 5 NONLINEAR MODELING PROCEDURE FOR PROGRESSIVE COLLAPSE OF 2D REINFORCED CONCRETE FRAMES

5.1 Introduction

Based on the damage and failure criteria for RC members proposed in Chapter 4, in this chapter, a new modeling procedure for progressive collapse analysis of 2D RC frames subject to single or multiple column removal scenarios is developed. Different from conventional pushdown analysis of progressive collapse, the proposed modeling procedure incorporates the effects of service loads *before* column removal into the analysis. In order to trace the collapse sequence of the structure, a removal algorithm based on combined actions of flexural/shear/axial failures is employed for single member removal. For detecting substructure collapse mechanism, a specially designed searching algorithm is developed. Furthermore, the locations and magnitudes of collapse impacts are respectively determined by rigid-body kinematics and energy principle with both inelastic and oblique impact effects considered. Numerical examples corresponding to different loading and column removal scenarios are given to validate the suggested damage assessment procedure, failure identification, collapse searching algorithms and to demonstrate the effectiveness of the proposed modeling approach.

5.2 Basic terminologies and modeling framework

5.2.1 Basic terminologies

In order to give a clear description of the proposed modeling procedure, a list of self-explanatory terminologies used in this study are summarized in Table 5.1.

Table 5.1 Terminologies and notations used to describe the structural system

Term	Used in description of the model and the whole algorithm
Element	A finite element of the model.
Member or Edge	A beam or column of the frame. A member (e.g. beam or column) can be discretized into several elements.
Joint	A junction that connects two members (e.g. beam or column) or a member at a support of the structure.
Substructure	Part of the whole structure that consists of more than one member.
Structure	The entire system which includes all the members and joints (and hence all the elements in the model).
Failure	At section/element level, it refers to flexural/shear/axial failures; at member/substructure level, it refers to the ultimate collapse state.
Collapse	The ultimate state of a member or a substructure, at which the collapsed part would fall down and collide to the lower floor.

5.2.2 Modeling framework

The modeling framework proposed in this study is based on the standard nonlinear quasi-static analysis procedure in which the total loads in the structure are proportionally applied at each incremental step to the structure during analysis. A load factor ($0 \leq \lambda \leq 1$) is employed to control the loading level during the incremental analysis such that $\lambda=1.0$ indicates that the full load is applied to the structure. In order to give an overall view of this modeling framework, a flowchart of the analysis is provided in Figure 5.1.

Figure 5.1 An overall summary of the present modeling procedure

As shown in Figure 5.1, the whole framework can be divided into two main parts

corresponding to (1) service load analysis before column removal and (2) progressive collapse analysis after column removal. First, as shown in Figure 5.2a, service loads are applied to the intact structure, where the load factor λ_1 will be increased gradually from 0 to 1.0 and the damage of the structure under the action of this service load will be determined. The member stress level is usually not at all critical if the structure has been properly designed to satisfy code provisions. . After the full service loading is applied, progressive collapse analysis will commence by removing the single or multiple columns predefined by the user (Figure 5.2b). In Figure 5.2b, the solid-line loads denote the equilibrium loads applied at the moment of column removal, while the dash-line loads are reactions to the internal forces at the ends of the removed column. With the equilibrium loads in place, the dash-line loads will then be proportionally applied to the new model (with the column removed) using a new load factor λ_2 to simulate column removal. During this process, damage assessment will be performed at each load increment. Member removal and model regeneration will be conducted when at least one member has failed in the model. Figure 5.2c shows a partial collapse of a substructure in which the corresponding members are identified and removed. Additional loads created after the partial collapse including released forces of removed members and impact loads from collision of members will be determined during model regeneration. In order to continue the analysis, the newly created loads (indicated as dash-line loads in Figure 5.2d) will also be proportionally applied to the new model by using another new load factor λ_3 (from $\lambda_3=0$) while the equilibrium loads from Figure 5.2b remain unchanged. The above process will continue until either (1) all the loads in the new model (λ_3 attaining unity) are fully applied, or (2) the whole structure has collapsed.

Figure 5.2 Modeling RC frame under column removal scenario

In this study, Total Lagrangian beam element formulation (Dvorkin *et al.* 1988; Weng *et al.* 2015) is employed to model RC members for large deformation analysis. Fiber discretization for cross-section at each integration point of elements is carried out to characterize the nonlinear behaviors of concrete and steel during the analysis. To predict the incremental step to reflect stiffening or softening of the structure, generalized displacement control method (Yang and Shieh 1990) is adopted.

5.3 Failure criteria and collapse modes

The set of damage assessment criteria of RC frames stated in Chapter 4 is used to quantify flexural, shear and axial damages and failures, considering flexural-shear-axial interactions during deformation. Critical damages at RC cross-sections, such as concrete cracking or crushing, rebar fracture in tension or buckling in compression between stirrup locations as well as final failure, can be identified. In the following, the flexural/shear/axial failure criteria for RC frames are briefly reviewed. Then the collapse modes of the structure, which are determined based on a combination of flexural/shear/axial failures of members, are presented. The failure criteria and collapse modes of the structure provide the firm foundation for developing removal algorithms and tracing collision and impact after each partial collapse. New features associated with the developed searching and removal algorithms for the collapsed members or substructure as well as collapse-and-impact procedure will be highlighted in Sections 5.4 and 5.5.

5.3.1 Collapse modes

5.3.1.1 Combined action of flexural/shear/axial failures

Member *failure* in RC structures can be broadly classified into three categories: flexural failure, shear failure and axial failure. Flexural failure leads to the loss of flexural rigidity at a cross-section where a plastic hinge will be formed. Shear and axial failures may lead to separation of cross-sections from adjacent structures due to severe diagonal shear cracks or rebar fracture. Figure 5.3 shows the respective failure modes of flexure, axial and shear in RC structures. Failure or collapse of a member can be caused by combined flexural/shear/axial failures of the members at a single or multiple cross sections. As shown in Figure 5.4a, flexural failure at the fixed end may cause collapse in a cantilever member. Axial failures at two sections

may lead to localized collapse of a beam segment with the two end segments remaining intact (Figure 5.4b). A combination of axial and flexural failures may result in collapse of a member (Figure 5.4c). More collapse scenarios can be observed in Figure 5.4d in which combined axial/shear or axial/flexural failures cause multiple failures of the members.

Figure 5.3 Typical failure modes in RC structures: (a) Flexural failure (Abbasi and Hogg 2006); (b) Axial failure (Stover and Coffman 1993); (c) Shear failure (Yarandi *et al.* 2004).

Figure 5.4 Different collapse modes due to combined flexural/shear/axial failures

Figure 5.5 Direct and rotational collapse modes (Department of LOS (US) 2013)

5.3.1.2 Rotational collapse and direct collapse

In general, when a member fails, two different collapse modes can be identified. *Rotational collapse* involves plastic hinge rotation due to flexural failure (e.g. in Figures 5.4a, 5.4c and 5.4d) while *Direct collapse* is caused by the loss of boundary restraints at two ends due to either axial or shear failure (e.g. Figures 5.4b and 5.4d). In practice, collapse may occur in a *member* or in a *substructure* under specific combined failure modes. The failed members or substructures will have different *subsequent motion trajectories* which can be employed to determine their impact points and loads on the surviving structure. Figure 5.5 shows examples of rotational and direct collapses for RC beams in a structure that has undergone major collapse. Figure 5.5 clearly shows that different collapse modes will generate different effects to the surviving structure. Hence, when tracing the collapse sequence, it is important to identify the collapsed members and to consider their impact effects on the remaining structure. Toward this end, an effective algorithm to identify

rotational and direct collapses for both members and substructures are developed in this study.

5.4 Member and substructure collapse searching algorithms

5.4.1 Single member collapse

5.4.1.1 Element removal in single beam

The simplest collapse is the failure of a single beam. In general, more than one element may be involved in the case of rotational collapse or direct collapse of a beam. As shown in Figure 5.6, elements 5 and 6 between the two axial/shear failed elements 4 and 7 will directly fall onto the beam below; elements 5 and 6 will in turn be removed after the failure and removal of elements 4 and 7. With removal of elements 4 to 7, elements 1 to 3 become a cantilever beam and rotational collapse will occur if flexural failure occurs in element 1. Thus, element 8 will be the only surviving element of the beam member. Detection and identification of a single member collapse can be easily achieved by a searching algorithm which is summarized in Box 5.1. It should be noted that under large deformation, no collapse will occur for the beam shown in Figure 5.7; although the beam is now a Mechanism, continuity is still maintained at both ends, where catenary action (Yu and Tan 2013) can be mobilized to provide additional tensile resistance so long as axial/shear failure does not occur at the beam ends.

Figure 5.6 Collapse scenarios in a member

- (i) At each step, scan all existing elements in the model one by one to check for the occurrence of flexural/shear/axial failures.
- (ii) If axial/shear failure takes place, identify the failed elements and remove them first, and record the surviving beam with removed element(s) as cantilever.
- (iii) For each cantilever beam, check whether there is more than one element that fails in axial/shear. If yes, remove the elements in between two axial/shear failed elements.
- (iv) For each cantilever beam, search elements from two beam ends and check whether there is flexural failure in these elements. If yes, remove the elements in between the first element that fails in flexure and the element that fails in axial/shear from beam ends.

Box 5.1 Beam elements removal

5.4.1.2 Element removal in columns

For columns, axial/shear failure at *any* cross-section will result in separation of the members, (e.g. Figures 5.3b and 5.3c). In this case, the column is assumed to have failed as it can no longer be able to resist any vertical loading applied to it. Monitoring of column failure can be easily implemented by checking all existing columns one-by-one in each load increment. Once an axial/shear failed element is detected, the column and all its elements will be removed. It is worthy to point out that after the occurrence of shear failure, there is a possibility that the columns will continue carrying axial loads through the shear failure plane, which could lead to a

conservative evaluation if a column is removed upon the detection of shear failure only.

Figure 5.7 Three hinges in a beam under large deformation

5.4.2 Substructure collapse

During tracing of progressive collapse of RC frames, besides the collapse of single members, local substructure may also collapse due to either loss of boundary restraints or mechanism formation arising from rotational instability. Similar to single member collapse, substructure collapse can also be classified as either direct or rotational collapse. A substructure which loses all its boundary restraints or connections will undergo direct collapse (Figure 5.8a) while rotational collapse occurs when a substructure is only connected to the remaining part of the frame via a single plastic hinge (Figure 5.8b). In general, detecting substructure collapse and performing the corresponding member removal requires the development of a more sophisticated searching algorithm. The basic approach is to first map the 2D frame structure into a simple graph (Wilson 1996) by treating all beam-column connections as ‘joints’ and beam/column members as ‘edges’. This simple graph which defines the topological connections between joints and edges can then be employed to identify collapsed substructures. Obviously, if none of the members (column or beam) of the RC frame has failed or removed, then the entire frame will be represented as a *single connected* graph (Wilson 1996). In the searching algorithm, if a member has failed and subsequently removed, the corresponding

edge will be deleted and removed from the original graph. Based on such a concept, detecting direct substructure collapse will be equivalent to identification of a *disconnected subgraph* or *disconnected component* (as shown in Figure 5.8a) formed due to edge deletions (or member removals). For the detection of rotational collapse of a substructure (Figure 5.8b), it is equivalent to the formation of a subgraph with a single edge (or member) that is *only* connected to the main graph via a plastic hinge. The problem of searching for a disconnected subgraph or component within a simple graph is a well-studied topic in graph theory (Wilson 1996) and many efficient algorithms have been developed (Sedgewick and Wayne 2011). In the current study, searching of direct substructure collapse will always commence from the joint that is originally connected to the last failed column. Similarly, for searching rotational substructure collapse, another algorithm is developed. Figure 5.9 shows two simple examples in which direct and rotational substructure collapses are identified. In Figure 5.9, direct substructure collapse 1 is identified by starting with joint 12 and searching is stopped when disconnected edges E42 and E43 are found. For rotational collapse 2, searching begins from joint 16 until 7 more joints (17, 18, 22, 23, 24, 29 and 30), 3 disconnected edges (E37, E39 and E43) and edge E35 with only one plastic hinge are found. Detailed illustrations of this process are shown in Appendix A.

Figure 5.8 Direct and rotational collapses of substructures

Figure 5.9 Examples for identification of collapsed substructures

5.5 Loads generated from member and substructure collapse

Whenever a member or a substructure collapse occurs, additional loads will be generated and they will be transferred to the new numerical model after the corresponding collapsed members and substructures are removed. These additional loads include (1) released loads that act at the common boundary between the removed members/substructures and the surviving structure, (2) impact loads generated from the crashing of the members onto the remaining structures and (3) remaining loads from the old model for surviving elements.

5.5.1 Internal forces released by member or substructure removal

For the internal loads released due to member or substructure removal, the released loads acting at the common boundary between the removed member/substructure can be easily extracted from the old model and transferred to the new model. An example is given in Figure 5.10 in which three member/substructure collapses are shown in Figure 5.10a. The regenerated model of the surviving frame after removal of collapsed elements/members/substructures together with the released loads applied at the boundary joints is shown in Figure 5.10b.

Figure 5.10 Internal forces released at common joints after collapse

5.5.2 Impact loads generated by failed members or substructures

For impact loads generated from the crashing of the fallen members onto the remaining structure, it is required to identify both the impact points and the magnitudes of impact forces. In this study, the impact point will be identified by treating the collapsed member/substructure as a rigid-body while the magnitude of impact force will be obtained by using a simple energy conservation principle.

Figure 5.11 Impact of collapsed members in frame structures

5.5.2.1 Determination of impact points

In Figure 5.11, a few examples are given to indicate how impact points can be identified for different collapse scenarios. For direct collapse of a beam (direct collapse 1) or a substructure (direct collapse 2), the direct collapse length L_d is simply equal to the length of the elements removed when the beam or the substructure directly falls onto the floor below. For rotational collapse of a single beam (rotational collapse 1) or a substructure (rotational collapse 2), the rotational collapsed length L_r is equal to the length of elements removed in the failed member or substructure. As the story height is known, the location of the impact point can be easily calculated by Pythagoras Theorem. Note that if L_r is less than the story's height (e.g. rotational collapse 3 in Figure 5.11), no impact point will be found. In this case, it is assumed that all existing loads from the collapsed members/substructures will be uniformly added onto the member right below.

For the more complicated scenario of rotational collapse of substructures with

arbitrary shape, it is assumed that any outstanding elements from the failed edge (Figure 5.12) are deemed to be too weak to resist subsequent impact force and will be ignored when determining the impact point. In order to identify the exact impact location, all boundary joints of the collapsed substructure and the surviving structure should be checked for potential impact. However, in most cases, the number of potential impact joints can be reduced by considering a few convex joints of the collapsed substructure and tracing their trajectories during rotational collapse. The frame shown in Figure 5.13a is employed to demonstrate how in practice the impact point is identified. In this frame, some members have been removed and the substructure enclosed in dash line will undergo a rotational collapse. The substructure will collide with the lower story by undergoing a rigid-body rotation about the plastic hinge which connects it with the surviving structure. In this case, five convex joints 1 to 5 can be easily identified (the cubic dots in Fig 5.13a). The actual impact point can be located by calculating and comparing the rotation angles associated with all convex joints when the impact occurs. In this example, convex joint 3 is the actual impact joint as the substructure will undergo the minimum rotation angle. It is worthy to mention that in some cases it is required to check and compare *all* boundary joints in order to identify the correct impact point. An example is shown in Figure 5.13b.

Figure 5.12 Joints for collision identification

The spring mass system shown in Figure 5.14 is employed to estimate the value of IAF . The mass corresponding to the total loads from collapse is denoted as m_1 , while v_1 is the impact velocity. The terms m_2 and v_2 refer to the mass and velocity of the hit member at the moment of impact, respectively. After impact, the initial velocity of the system is evaluated based on conservation of momentum (Kaewkulchai and Williamson 2006) such that

$$v = (m_1 v_1 + m_2 v_2) / m \quad (5.1)$$

where $m = m_1 + m_2$ is the total mass of the system. Compared with m_1 , the value of m_2 is usually small and can be ignored as it only refers to the self-weight of the target member (e.g. the beam), while m_1 represents the whole loads from the above floors (including both gravity and imposed loads) when collapse occurs. In addition, if v_2 is close to zero (i.e. the member below is stationary before the impact), one obtains $m = m_1$ and $v = v_1$. By applying the energy conservation principle, one has arrives at

$$\frac{1}{2} m v^2 = \frac{1}{2} k \Delta^2 = \frac{1}{2} \frac{P^2}{k} \quad (5.2)$$

In Equation 5.2, k is the stiffness at the location of impact. The terms Δ and P are the equivalent elastic deformation and impact load of the member when the final combined velocity is reduced to zero, respectively. From Equation 5.2, the equivalent impact load P is given by

$$P = v \sqrt{m k} = v_1 \sqrt{m_1 k} \quad (5.3)$$

Hence, IAF is obtained as

$$IAF = P / m_1 g = \frac{v_1}{g} \sqrt{\frac{k}{m_1}} \quad (5.4)$$

where $g = 9.8 \text{ m/s}^2$ is the gravity acceleration. In practice, the hit member is usually assumed to be supported by idealized fixed-fixed restraints. Hence, the

system stiffness k can be taken as the elastic flexural stiffness of the member at the impact point. The velocity v_1 for direct collapse can be calculated by using free fall assumption as

$$v_1 = \sqrt{2gh} \quad (5.5)$$

where h is the falling distance of the collapsed member. As an example, considering a fixed-fixed RC beam with span $L = 6.0\text{m}$, cross-section width=200mm and depth=300mm, and modulus of elasticity $E = 3.25 \times 10^4 \text{MPa}$. It is impacted at its mid-span by a point mass with $m_1 = 50 \times 10^3 \text{kg}$ which is dropped from a height $h = 4.0\text{m}$. In this case, $v_1 = \sqrt{2gh} = 8.85 \text{m/s}$, $k = 192 \cdot EI/L^3 = 1.3 \times 10^7 \text{N/m}$ so that $IAF = v_1/g \cdot \sqrt{k/m_1} = 14.5$. However, the actually impact load of this example is overestimated by completely ignoring inelastic energy dissipation. Towards this end, an idealized elastic plastic model is introduced to consider inelastic impact effect.

In elastic impacts, no energy is dissipated in the form of yielding or frictional resistance. In practice, plastic deformation is often incorporated in structural design as “cushion” against impact. Idealized elastoplastic model (AASHTO 2009) is a simple but effective means for predicting inelastic impact effect. As shown in Figure 5.15, the actual impact response curve (the green curve) is idealized as a bilinear model. The ultimate displacement Δ_u can be expressed in terms of the yield displacement Δ_y as $\Delta_u = \mu \Delta_y$, where μ is the *ductility coefficient* of system.

Based on the principle of conservation of energy, the energy dissipation based on elastic assumption (area under the line $OP_e = \frac{1}{2} \frac{P_e^2}{k}$ in Figure 5.15), and from idealized elastic plastic assumption (area under the line OP_rG in Figure 5.15) should both be equal to the energy dissipation of the real inelastic response (area under the green curve). Thus,

$$\frac{1}{2} \frac{P_e^2}{k} = \frac{1}{2} \frac{P_I^2}{k} + P_I (\Delta_u - \Delta_y) = \frac{1}{2} \frac{P_I^2}{k} + (\mu - 1) \cdot \frac{P_I^2}{k} \quad (5.6)$$

Therefore, the predicted impact loads P_I can be expressed as

$$P_I = P_e / \sqrt{2\mu - 1} \quad (5.7)$$

By using P_I instead of P in Equation 5.3, the IAF for the inelastic impact loads P_I can be expressed as

$$IAF = \frac{v_1}{g\sqrt{2\mu-1}} \sqrt{\frac{k}{m_1}} \quad (5.8)$$

Value of the ductility coefficient μ can be obtained from the literature (Lee and Pan 2003). For this example, assuming a value of $\mu = 4.0$ for typical RC members, the inelastic IAF is calculated as $IAF = 14.5/\sqrt{7} = 5.48$. It should be noteworthy to point out that in the present study the energy dissipation of damping as well as heat consumption etc. during the impact process has been ignored.

Figure 5.14 Spring mass system under elastic collision

Figure 5.15 Idealized elastoplastic model for inelastic impact

5.5.2.3 Oblique impact

As shown in Figure 5.16, rotational collapse will lead to an oblique impact with an impact velocity $v_1 = \omega l$ where ω and l are the angular velocity and radius of rotation of the rotational collapse, respectively. Assuming that the collapse loads are distributed uniformly over the length l , the principle of conservation of angular momentum is employed.

$$M = \int_0^l \bar{l} \cos \bar{\theta} g \lambda d\bar{l} = J \cdot \alpha = \left(\frac{1}{3} m l^2\right) \cdot \left(\frac{\bar{\omega}}{d\bar{\theta}} d\bar{\omega}\right) \quad (5.9)$$

where M is the angular momentum of the system and $\lambda = m/l$ is the linear density of mass along the member. $\bar{\theta}$ and $\bar{\omega}$ are dummy variables of rotational angle and angular velocity, respectively. By rewriting Equation 5.9, one obtains

$$\int_0^{\theta} \frac{3g}{2l} \cos \bar{\theta} d\bar{\theta} = \int_0^{\omega} \bar{\omega} d\bar{\omega} \quad (5.10)$$

Integrating both side of Equation 5.10 gives the velocity v_1 so that

$$v_1 = \omega l = \sqrt{3gl \sin \theta} \quad (5.11)$$

Hence, the vertical impact velocity v_{1y} is calculated as

$$v_{1y} = v_1 \cos \theta \quad (5.12)$$

where the impact angle θ can be easily determined from the Geometrical configuration. Assuming the *IAF* at vertical direction can be represented by the corresponding velocity component, the vertical *IAF* can then be obtained by substituting v_1 by v_{1y} in Equation 5.8.

For the tangential *IAF*, since it is difficult to identify the impact mass in tangential direction, the *tangential* impact loads are estimated by an experimental study based on a friction model (Lewis and Rogers 1988). It was found that the impact ratio ζ (tangential load to vertical load ratio) can be expressed as

$$\zeta = \begin{cases} 0.00448\theta & 0 < \theta < 40^\circ \\ 0.179 & \theta \geq 40^\circ \end{cases} \quad (5.13)$$

The tangential impact load P_t is then calculated as $P_t = \zeta \cdot P_v$ where $P_v = v_1 \cos \theta \sqrt{m_1 k / (2\mu - 1)}$ is the corresponding vertical impact load.

Figure 5.16 Oblique impact due to rotational collapse

5.5.3 Remaining loads from the previous model

In the present modeling procedure, total loads applied to the model are known

a-priori and are applied proportionally using an incremental approach. This implies that in the progressive collapse analysis, if the structure can sustain the applied loads for a load factor $\lambda = 1.0$ without any partial (member or substructure) collapse, the structure is robust enough and has adequate resistance against progressive collapse after the predefined column is removed. However, if a partial collapse occurs when $\lambda < 1.0$, a new load factor λ_2 will be employed in subsequent analysis and the surviving structure will be required to sustain the remaining loads too. Assuming that a partial collapse occurs at a specific load factor where $\lambda_0 < 1.0$, the remaining loads are represented as $(1 - \lambda_0)P'$ where P' is the load applied to the surviving elements from the previous model. To illustrate this loading procedure clearly, an example is given in Figure 5.17. Figure 5.17a shows the initial model of the structure. Two service loads are applied on the structure, viz. one acting on the top cantilever floor and another two stories below the top floor. For ease of illustration, it is assumed that two partial collapses eventually occur in this model. Figure 5.17b shows the new model after the first partial collapse. In Figure 5.17b, the loads P_1 and P_2 respectively represent the impact load and the released load due to the first collapse of the cantilever floor on the floor beneath it. In Figure 5.17b, P_3 denotes the load applied on the lower floor. Therefore, the total loads applied to the model after the first collapse can be represented as $P_1 \oplus P_2 \oplus P_3$, where the symbol $\oplus$ is the load combination operator. The combined load will be proportionally applied to the frame by using a load factor λ starting with $\lambda = 0$ in subsequent analysis. It is assumed that when $\lambda = \lambda_0$, the proportional load $\lambda_0(P_1 \oplus P_2 \oplus P_3)$ causes the second direct collapse on second floor where $\lambda_0 P_3$ is applied (Figure 5.17c). At this moment, the new impact and released loads P_4 and P_5 due to the second direct collapse are generated and the remaining (or uncompleted) loads from the previous model is equal to $(1 - \lambda_0)P_1 \oplus (1 - \lambda_0)P_2$. In this case, as shown in Figure 5.17d, the new

total loads for the model after the second collapse can be represented as $P_4 \oplus P_5 \oplus (1 - \lambda_0)P_1 \oplus (1 - \lambda_0)P_2$. During subsequent analysis, this new loads will be applied proportionally as $\lambda' (P_4 \oplus P_5 \oplus (1 - \lambda_0)P_1 \oplus (1 - \lambda_0)P_2)$, starting from $\lambda' = 0$ where λ' is a new load factor of the new model.

Figure 5.17 Remaining loads during collapse analysis

Figure 5.18 Ultimate failure scenarios of RC member sections

5.6 Numerical examples

5.6.1 Adjustment of axial damage index

The axial damage indices given in Section 4.2.4.1 are designed to detect a *complete* loss of axial resistance of the cross-section. That is, an axial damage index of 1.0 will only be attained when the RC member experiences tension, tension-compression or compression failure (Figure 5.18). . However, strict compliance to this definition causes numerical issues during the progressive collapse analysis when a very weak or ‘nearly-failed’ cross-section is formed due to severe degradation of reinforced concrete materials. If *all* the fibers at a cross-section reach the ultimate state (dash line of the constitutive curve in Figure 5.18), numerical singularity will occur. In practice, such exact numerical singularity seldom occurs due to round off errors and discretization of the model. However, when almost of all the fibers reach their ultimate state, the stiffness of the corresponding member at a cross section will deteriorate to a very small fraction (e.g. <1%) of its original value. This may cause numerical instability and greatly slow down the convergence process so that more iterations are needed for searching the equilibrium configuration. Furthermore, in practice, when a member loses 90% to 95% of its original axial stiffness, local failure is usually considered to happen in the form of very large deformation. Hence, in the current modeling procedure, a relaxation of ultimate failure criterion is suggested. A maximum allowable axial damage index $D_{amax} < 1$ is adopted to improve the numerical stability of the progressive collapse analysis. In the current study, a section will be considered to have failed axially whenever the axial damage index at any section is $D_a \geq D_{amax}$. Besides improving the stability of the modeling procedure and increasing the convergence speed of the analysis, the use of D_{amax} allows the user to obtain more conservative modeling results. Obviously, the lower the value of D_{amax} , the earlier

an axial damaged member will be removed and the modeling results will be more conservative. Numerical studies show that when using a low value of D_{amax} , say $D_{amax}=0.85$, it will result in overly conservative prediction with many members removed prematurely. The optimal value of D_{amax} is within the range of [0.90, 0.95]. In all the numerical examples given in the following sections a value of $D_{amax}=0.95$ is adopted.

5.6.2 Example 5.1: an L-shape RC frame subject to UDL

This example is employed to demonstrate the differences between the proposed progressive collapse analysis procedure and the more commonly used pushdown analysis approach (Khandelwak and Tawil 2008). It is a simple L-shape frame shown in Figure 5.19a. The geometrical properties, reinforcement details together with material strengths used are listed in Table 5.2. A UDL is acting on the 6m horizontal beam and collapse is caused by removing the 4m high column. In the more commonly used pushdown analysis, analysis will be conducted on an undamaged model *without* the removed column (Figure 5.19c) while the UDL is incrementally applied till either the frame collapses or the full loading has been applied. Obviously, this modeling approach neglects the deformation and damage effects of the initial loading *before* the column removal (Figure 5.19b). In the proposed modeling approach, the service loads will be first applied to the intact frame by using the first load factor λ_1 and the damage of the members (in terms of damage indices) generated will be recorded. After this, the prescribed “missing column” is removed from the partially damaged structure (Figure 5.19d) and the analysis will restart with a new load factor λ_2 . In this example, for both analysis approaches, six equal-length nonlinear fiber beam elements (Weng *et al.* 2015; Dvorkin *et al.* 1988) are employed for the beam and four elements are used for the column. To study the effect of the initial service load, two different UDLs of

27.5kN/m and 11.0kN/m are applied.

Figure 5.19 Example 5.1: Two modeling approaches for a frame under column removal

Table 5.2 Geometrical and material properties of RC frames in numerical examples

Example	Width×Depth (mm)		Longitudinal reinforcement			Material strength (MPa)	
	Beam	Column	Beam top	Beam bottom	Column	Concrete	Reinforcement
1	250×450	450×450	3T25	3T20	8T20	30.0	550.0
2	250×450	450×450	3T20	3T20	8T20	30.0	550.0
3	250×450	450×450	3T25	3T20	8T20	30.0	550.0
4, Normal design	250×450	450×450	3T25	3T20	8T20	30.0	550.0
4, Improved design	250×450	450×450	3T28	3T25	8T20	30.0	550.0
5	250×450	450×450	3T25	3T20	8T20	30.0	550.0

Figure 5.20 shows the load-displacement paths at the beam-column connection (Point A in Figure 5.19a) for both approaches. From Figure 5.20a, it is found that for small UDL, both approaches result in very similar predictions and a load factor >1 can be sustained indicating that the frame will not collapse after the column removal. However, Figure 5.20b shows that when the larger service load is

applied (27.5kN/m), while both approaches predict that collapse will occur, there is a deviation between the results from the pushdown analysis and the present approach. In particular, the pushdown analysis predicts a higher resistance compared to the present approach. This deviation between the two analyses is mainly caused by the initial service load effects before column removal, which inflict some damage in the beam before the column is removed. In fact, for $UDL=27.5\text{kN/m}$ when full service load is applied, a flexural damage index of 0.13 is attained at the mid-span of the beam and it will affect resistance of the frame after column removal. It should be noted that for the lower UDL case, the flexural damage index at the mid-span is only equal to 0.009 and this explains why the difference between the two approaches is small. In practice, for RC frames, the full service loads (dead and imposed loads) are not always negligible and in some cases may range between 30%-40% of the ultimate design strength of the frame. Therefore, it is generally not conservative to ignore the initial damage generated by the service loads and the present approach will be more accurate and reliable than the conventional pushdown analysis.

(a) under $UDL=11.0\text{kN/m}$

(b) under UDL=27.5kN/m

Figure 5.20 Load-displacement paths at Point A

5.6.3 Example 5.2: a 1-bay-by-2-story RC Frame subject to direct collapse

To illustrate a direct collapse simulation using the proposed modeling procedure, a two-story frame subject to a large UDL of $q = 249.0\text{kN/m}$ on its second floor is studied (Figure 5.21). The column height is 4.0m and the beam length is 6.0m and its Geometrical and material properties are shown in Table 5.2. It is noted that 1 m long elements are used for the whole frame in the analysis. Figure 5.22 shows the load-displacement paths at Point B (mid-span of the second floor) and Point C (mid-span of the first floor). Due to the large UDL applied, axial failure first occurs at two ends of the second floor beam and the first direct collapse occurs (Figure 5.21b) when the load factor is equal to 0.31 (load displacement curve for Point B in Figure 5.22). The collapse impact loads generated are then transferred into the first floor with an *IAF* of 2.52. Then the analysis continues with the new load factor corresponding to the collapse loads and the second direct collapse occurs when the new load factor attains a value equal to 0.11. All the loads are then transferred to the

ground (Figure 5.21c). Finally, only two cantilever columns survive as no column failure is detected.

Figure 5.21 Example 5.2: Direct collapse of a 1-bay-by-2-story RC frame

Figure 5.22 Load-displacement paths for the collapsed beams

5.6.4 Example 5.3: a 2-bay-by-2-story RC Frame under side-column removal

In this example, a perimeter RC frame (Figure 5.23) with a floor height of 4m is studied. An imposed load of 15.0kN/m and a dead load including self-weight of 21.3kN/m are applied. Geometrical properties and reinforcement details of the beam and column are listed in Table 5.2 while the whole frame is discretized by

using 1m long elements. Figure 5.24 shows released load-displacement curve at the top of the removed column (Point D in Figure 5.23). Detailed failures and progression of collapse mechanism of this frame are shown in Figure 5.25. The first peak in Figure 5.24 corresponds to the application of total service loads on the intact structure. The column is then removed and loadings are re-applied using a new load factor. After this, the first flexural failure is detected at 132.5mm displacement which corresponds to the flexural failure 1 in Figures 5.24 and 5.25a. It can be seen that the structure has not reached its peak resistance then. As the load factor increases, flexural failures gradually develop within the substructure above the removed column (Figure 5.25a). The first sudden drop of the load-displacement curve occurs at around 400mm which is due to deterioration of stiffness after flexural failure 5 appears. However, the analysis continues because no rebar fracture has occurred yet. Further flexural failures (flexural failures 6-8 in Figures 5.24 and 5.25a) occur and a drop in load resistance at a displacement of 600mm is due to rebar yielding as well as concrete crushing. The final drop arises from rebar fracture at sections f1 and f2 (Figure 5.25b). At the same time, axial failure is detected at section f1. As a result, a member and a substructure collapse are identified. In this case, all the impact loads are transferred to the ground and only the released forces are applied at the beam ends (Figure 5.25c). Since the released loads (dash line loads in Figure 5.25c) are in equilibrium with internal forces, the load factor reaches unity and the analysis is completed. Finally, as shown in Figure 5.25d, half of the frame remains. From Figure 5.24, it can be seen that the maximum value of load factor reached is about 0.4. This implies that this frame has inadequate structural resistance against progressive collapse. Although no failure of column is detected during the analysis, the one above the missing column is removed due to loss of boundary restraints.

Figure 5.23 Example 5.3: a 2-bay-by-2-story frame under side-column removal

Figure 5.24 Load-displacement paths at Point D

Figure 5.25 Damage propagation and collapse modes of the frame

5.6.5 Example 5.4: 4-bay-by-5-story RC frames under mid-column removal

In this example, a 4-bay by 5-story RC frame subject to middle-column removal scenario is investigated. The configuration of this frame is shown in Figure 5.26. The same dimensions, loadings, material properties and element arrangement from Example 5.3 are applied. In this example, two design scenarios, namely, normal and improved designs are studied (Table 5.2). The main difference between the normal design and the improved design is an increase in all beam reinforcement.

Figure 5.26 Example 5.4: 4-bay by 5-story RC frame under mid-column removal

Figure 5.27 Load-displacement paths at Point E

Figure 5.27 shows the relationship of the released vertical force versus vertical displacement at the top of the missing column (Point E in Figure 5.26) for both designs. Due to an increase of reinforcement ratio, the first flexural failure and plastic hinge mechanism in the middle two-bay beams has been delayed in the improved design. After the formation of plastic hinges in the middle two-bay frame, catenary action appears in both frames. Progressive collapse occurs in the normal design while no collapse is found in the improved design. Different from the normal design, flexural failures are observed in the columns supports of the improved design. This is because the beams with an increase in steel content have enhanced flexural resistance and can sustain higher bending moments and axial forces at the beam-ends, which in turn increase bending moments, shear and axial forces at the corresponding column-ends. Figure 5.27 shows the collapse process (from c1 to c8) is short, especially, for c4 to c8 collapses, they occur at almost the same time (only one incremental step interval) and with very small load vector (close to zero). This example also shows the progression of collapse is usually formed in an unstable structure in a very short time. However, the collapse sequence predicted for the first floor beam is later. This is probably because the current failure criteria to predict collapse mode are sensitive for members with similar damages (corresponding to c5-c8 collapses).

Figure 5.28 Damage propagation and collapse modes of the frame

Figure 5.28 shows the failure sequence of the frame for both normal and improved designs. First, flexural failures occur at two far-ends of mid-bay beams (flexural failure 1 in Figure 5.28). After all the flexural failures near the middle columns take place (flexural failure 2 in Figure 5.28), plastic hinge mechanisms are formed in the middle two-bay frame. For normal design, the first collapse occurs in the third span beams of the first and second stories (axial failure c1 in Figures 5.27 and 5.28a). However, it is noteworthy that the collapse sequence is not exactly symmetric and this is due to round-off and numerical errors during analysis. After collapse occurs at section c1, further collapses occur in those beams right above the c1 beam (axial failures c2, c3 and c4 in Figure 5.28a). This is because after the first collapse at c1, the third span of this frame becomes much weaker than the second span. Then collapses at sections c5 to c8 occur. It should also be noted that axial failure c8 corresponds to a substructure collapse. The collapse sequence of the normal design is shown in Figure 5.29. It is found that the middle two-bay sub-frame has collapsed completely at a small displacement increment after the first collapse at c1, as shown in Figure 5.27.

Figure 5.29 Sequence of collapse of the frame

5.6.6. Example 5.5: a 4-bay-by-5-story RC frame under two-column removal

In this example, as shown in Figure 5.30, the normal design frame used in Example 5.4 is investigated under a two-column removal scenario. Figure 5.31 shows the variation of released vertical force/load factor with vertical displacement at the top of a removed side column (Point F in Figure 5.30). Figure 5.32 shows the complete collapse sequence of this frame during the analysis. In Figure 5.31, the first sudden drop in resistance occurs when flexural failure appears at all end-elements of the cantilever substructure (Figure 5.32a). The second drop of resistance is due to rebar fracture at the end-element of the top beam at section f1 (Figure 5.32b). Thereafter, within a small amount of deflection, rebar fracture occurs again at another beam-end element (Section f2 in Figure 5.32b). Meanwhile, axial failure at section f2 takes place and this leads to the first collapse under combined flexural-axial failures at c1 (Figure 5.32c). The model is then regenerated by removing the failed elements. Released forces and impact loads are calculated and a new load factor is

employed. As the analysis continues, the second collapse occurs when rebar fracture appears at c2 and axial failure is identified. As a result, the collapsed beam (Figure 5.32d) is removed. At the same time, the rotational substructure collapse marked by a dash box in Figure 5.32d is identified. Therefore, the released force-displacement curve shown in Figure 5.31 ends with collapse of the measured point. For the surviving structure after this collapse, impact points are identified and impact loads magnitudes are calculated. Another new model (Figure 5.32e) for the remaining structure is generated for subsequent analysis. Eventually, three more member collapses are identified (Figure 5.32f). Eventually, the analysis comes to an end with the final surviving structure remains stable as shown in Figure 5.32g.

Figure 5.30 Example 5.5: 4-bay by 5-story RC frame subject to two-column removal

Figure 5.31 Load-displacement paths at Point F

Figure 5.32 Sequence of collapse of the frame for Example 5.5

5.7 Conclusions

In this chapter, a new modeling framework for progressive collapse analysis of 2D RC frames subject to column removal scenarios is proposed. Distinctly different

from the more conventional pushdown analysis approach, the proposed framework incorporates the effects and damages caused by the initial service loads on the structure before the prescribed column removal is carried out. In order to trace the collapse sequence of the progressive collapse process, a member removal algorithm is developed based on combined flexural/shear/axial failure criteria. In addition, an effective searching algorithm to identify substructure collapse is proposed. These algorithms can accurately identify both direct and rotational collapses which are commonly observed in many progressive collapse events. Furthermore, the locations of impact points and the magnitude of impact loads due to collapse are respectively calculated based on rigid-body kinematics and energy conversation principle with inelastic and oblique impact effects considered. Numerical examples presented demonstrate the proposed modeling approach can trace the collapse sequence and identify failures and collapses of 2D RC frames under various column removal scenarios. Single member and complex substructure collapse as well as impacts after collapse are detected and correctly predicted by the proposed modeling procedure. The modeling results can therefore provide new insight to assess damages, failures and collapses during progressive collapse analysis of 2D RC frame structures. In addition, the modeling results can provide engineers much useful information for improving structural design against progressive collapse. Moreover, many of the modeling approaches and algorithms presented in this study can be extended for 3D modeling of RC frames.

CHAPTER 6 SUPERELEMENT FORMULATION WITH RIGID-BODY ROTATION CORRECTION FOR LARGE DEFORMATION ANALYSIS

6.1 Introduction

It is recognized that existing progressive collapse events often occurred in large scale buildings, such as three famous progressive collapse buildings: Ronan Point building, Murrah Federal building and World Trade Center towers (Georgakopoulos 2005). In modeling progressive collapse of these large scale buildings, computational efficiency would be critical for structural analysts. Superelement technique is a good way to handle large scale modeling to save analysis time. Conventional superelement approach is used in small-strain and small-rotation configurations. However, progressive collapse of buildings may undergo large rigid-body rotations during deformation, as seen in Figure 2.3. Hence, to enhance analysis efficiency of progressive collapse modeling, a rotation correction based superelement formulation will be required.

In this chapter, an effective superelement formulation with rigid-body rotation correction is proposed. Rigid-body rotation matrix is derived based on minimum energy functional (Felippa *et al.* 2005). Matrix decomposition technique proposed by Han and Abel (1984) is employed to carry out static condensation to form superelement formulation. Three numerical examples are given to demonstrate the effectiveness of the proposed superelement approach. It is found that compared to conventional superelement analysis, the present superelement approach can

provide more accurate modeling results while maintaining almost the same computational effort.

6.2 Conventional superelement formulation

6.2.1 Static condensation

The principle of static condensation has been reviewed in Section 2.4.1. Key points of static condensation include: (1) solving condensed stiffness matrix (Equation 2.8) and load vector (Equation 2.9), and (2) recovering internal displacements of the linear zone (Equation 2.10) after the analysis of nonlinear zone. For ease of reading, Equations 2.8 to 2.10 are rewritten as follows:

$$\mathbf{K}^* = \mathbf{K}_{bb} - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \quad (6.1)$$

$$\mathbf{P}^* = \mathbf{P}_b - \mathbf{K}_{bi}\mathbf{K}_{ii}^{-1}\mathbf{P}_i \quad (6.2)$$

$$\mathbf{u}_i = \mathbf{K}_{ii}^{-1}\mathbf{P}_i - \mathbf{K}_{ib}\mathbf{u}_b \quad (6.3)$$

6.2.2 Matrix decomposition for condensation process

The force equilibrium equations for a linear substructure is given as

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i \\ \mathbf{u}_b \end{Bmatrix} = \begin{Bmatrix} \mathbf{P}_i \\ \mathbf{P}_b \end{Bmatrix} \quad (6.4)$$

The modified decomposition method (Han and Abel 1984) breaks down the stiffness matrix in Equation 6.4 into the following matrix product:

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} = \begin{bmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{R} & \mathbf{K}^* \end{bmatrix} \begin{bmatrix} \mathbf{L}^T & \mathbf{R}^T \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \quad (6.5)$$

where $\mathbf{L}$ is a lower triangular matrix, $\mathbf{R}$ is a rectangular matrix, $\mathbf{0}$ is a null matrix, $\mathbf{I}$ is an identity matrix and $\mathbf{K}^*$ is a symmetric matrix. From Equations 6.1-6.3 and 6.5, the following relations can be obtained by matrix multiplication:

$$\mathbf{K}_{ii} = \mathbf{L}\mathbf{L}^T \quad (6.6)$$

$$\mathbf{K}_{bi} = \mathbf{R}\mathbf{L}^T \quad (6.7)$$

$$\mathbf{K}^* = \mathbf{K}_{bb} - \mathbf{R}\mathbf{R}^T \quad (6.8)$$

$$\mathbf{P}^* = \mathbf{P}_b - \mathbf{R}\mathbf{L}^{-1}\mathbf{P}_i \quad (6.9)$$

$$\mathbf{u}_i = \mathbf{L}^{-T}\mathbf{L}^{-1}\mathbf{P}_i - \mathbf{R}^T\mathbf{u}_b \quad (6.10)$$

To determine the matrices of $\mathbf{L}$ and $\mathbf{R}$, Choleski decomposition is adopted as follows:

$$\left. \begin{aligned} L_{ij} &= 0 & i < j \\ L_{11} &= (K_{11})^{1/2}, L_{ij} = K_{ij}/L_{11} & j = 1, i > 1 \\ L_{ii} &= \left(K_{ii} - \sum_{k=1}^{i-1} L_{ik}^2 \right)^{1/2} & 1 < i \leq m \\ L_{ij} &= \left(K_{ij} - \sum_{k=1}^{j-1} L_{ik}L_{jk} \right)/L_{jj} & i \leq m, j < i \\ R_{ij} &= \left(K_{ij} - \sum_{k=1}^{j-1} R_{ik}L_{jk} \right)/L_{jj} & i > m, j \leq m \end{aligned} \right\} \quad (6.11)$$

where m is the number of internal DOFs.

A forward reduction is performed on $\mathbf{P}_i$ to obtain $\mathbf{P}'_i = \mathbf{L}^{-1}\mathbf{P}_i$ as

$$\left. \begin{aligned} P'_i &= P_i/L_{11} & i = 1 \\ P'_i &= \left(P_i - \sum_{k=1}^{i-1} L_{ik}P'_k \right)/L_{ii} & 1 < i \leq m \end{aligned} \right\} \quad (6.12)$$

and the following formulas are used to obtain $\mathbf{K}^*$:

$$\left. \begin{aligned} K_{ij}^* &= K_{ij} - \sum_{k=1}^m R_{ik}R_{jk} & i > m, j > m, j < i \\ K_{ii}^* &= K_{ii} - \sum_{k=1}^m R_{ik}^2 & i > m \end{aligned} \right\} \quad (6.13)$$

According to Equation 6.9, the condensed load vector $\mathbf{P}^*$ can be computed from

$$P_i^* = (P_i - \sum_{k=1}^m R_{ik}P'_k)/L_{ii} \quad i > m \quad (6.14)$$

Once the boundary displacements $\mathbf{u}_b$ are solved, the internal displacements $\mathbf{u}_i$

can be determined from Equation 6.10, by matrix multiplication, matrix subtraction, followed by backward substitution as below:

$$\left. \begin{aligned} u'_{bi} &= \sum_{k=1}^n R_{(m+k)i} u_{bk} & i = 1, m \\ u''_{bi} &= P'_i - u'_{bi} & i = 1, m \\ u_{im} &= u''_{bm} / L_{mm} & i = m \\ u_{ii} &= \left(u''_{bi} - \sum_{k=i+1}^m u_{ik} L_{ki} \right) / L_{ii} & i < m \end{aligned} \right\} \quad (6.15)$$

To improve computational efficiency, the stiffness matrix is stored in a skyline form (Lo 1992) such that all summations in Equations 6.11 to 6.15 are not computed from the first term. Instead, the computation starts from the first non-zero product.

6.3 Proposed rigid-body rotation correction

According to Equations 6.1 and 6.2, it can be seen that $\mathbf{K}^*$ and $\mathbf{P}^*$ are directly obtained by the stiffness submatrices $\mathbf{K}_{ii}$, $\mathbf{K}_{bb}$, $\mathbf{K}_{ib}$ and $\mathbf{K}_{bi}$ of the linear substructure from the assemblage of stiffness matrix $\mathbf{k}_e$ of each individual element. Under the situation of large rotations, the linear-elastic element stiffness matrix $\mathbf{k}_e$ in the substructure will be inaccurate and unreliable, if the effect of rigid-body rotations is neglected during the analysis. Towards this end, in this section, a rigid-body rotation correction in superelement formulation is proposed.

6.3.1 Rigid-body rotation matrix

The classical co-rotational configuration for large displacement analysis is achieved through separating the total motion into the rigid-body motion and the deformation motion, as shown in Figure 6.1. The deformed displacement $\mathbf{u}_d$ of any point in an element can be extracted from the total displacement $\mathbf{u}$ as

$$\mathbf{u}_d = \mathbf{u} - \mathbf{c} + (\mathbf{I} - \mathbf{R}_0)\mathbf{X} \quad (6.16)$$

where $\mathbf{X}$ is the coordinates of this point in the element; vector $\mathbf{c}$ is the element centroid translations; $\mathbf{I}$ is an identity matrix; $\mathbf{R}_0$ is the rigid-body rotation matrix of the element, expressed as follows (Argyris 1982):

$$\mathbf{R}_0 = \frac{1}{1 + \omega_x^2 + \omega_y^2 + \omega_z^2} \begin{bmatrix} 1 + \omega_x^2 - \omega_y^2 - \omega_z^2 & 2(\omega_x\omega_y - \omega_z) & 2(\omega_x\omega_z + \omega_y) \\ 2(\omega_x\omega_y + \omega_z) & 1 - \omega_x^2 + \omega_y^2 - \omega_z^2 & 2(\omega_y\omega_z - \omega_x) \\ 2(\omega_x\omega_z - \omega_y) & 2(\omega_y\omega_z + \omega_x) & 1 - \omega_x^2 - \omega_y^2 + \omega_z^2 \end{bmatrix} \quad (6.17)$$

where ω_x , ω_y and ω_z are three rotation parameters to be determined.

Figure 6.1 Corotational configuration for rigid-body motion (Felippa *et al.* 2005)

When the rigid-body rotations of the element are infinitely small, the rotation matrix $\mathbf{R}_0$ tends to the identity matrix $\mathbf{I}$. Therefore, the initial element stiffness matrix $\mathbf{k}_e$ is sufficiently accurate to formulate the superelement for structural analysis. However, if the magnitudes of the rotations are large, rigid-body rotation matrix $\mathbf{R}_0$ will diverge from $\mathbf{I}$. In this case, it is required to correct element stiffness matrix $\mathbf{k}_e$ using rigid-body rotation matrix $\mathbf{R}_0$ as follows:

$$\mathbf{k}'_e = \mathbf{R}_0^T \mathbf{k}_e \mathbf{R}_0 \quad (6.18)$$

6.3.2 Rigid-body rotation correction

The minimum energy functional proposed by Felippa and Haugen (2005) is used to determine the rigid-body rotation matrix $\mathbf{R}_0$, which is defined as

$$J[\mathbf{c}, \mathbf{R}_0] = \frac{1}{2} \int \mathbf{u}_d^T \mathbf{u}_d dV = \frac{1}{2} \int [\mathbf{u} - \mathbf{c} + (\mathbf{I} - \mathbf{R}_0)\mathbf{X}]^T [\mathbf{u} - \mathbf{c} + (\mathbf{I} - \mathbf{R}_0)\mathbf{X}] dV \quad (6.19)$$

The variation with respect to $\mathbf{R}_0$ gives the following condition

$$\frac{\partial J[\mathbf{c}, \mathbf{R}_0]}{\partial \mathbf{R}_0} \delta \mathbf{R}_0 = \int \mathbf{X}^T \delta \mathbf{R}_0 [\mathbf{u} - \mathbf{c} + (\mathbf{I} - \mathbf{R}_0)\mathbf{X}] dV = 0 \quad (6.20)$$

Replacing $\delta \mathbf{R}_0 = \delta \boldsymbol{\Theta} \mathbf{R}_0$ and noting that $\mathbf{X}^T \delta \boldsymbol{\Theta} \mathbf{X} = 0$ and $\int \mathbf{X}^T \delta \mathbf{R}_0 \mathbf{c} dV = 0$ yields

$$\int \mathbf{X}^T \delta \boldsymbol{\Theta} \mathbf{R}_0 (\mathbf{X} + \mathbf{u}) dV = \int \mathbf{X}^T [\delta \boldsymbol{\Theta} \times \mathbf{R}_0 (\mathbf{X} + \mathbf{u})] dV = 0 \quad (6.21)$$

where the auxiliary variation matrix $\delta \boldsymbol{\Theta}$ is given by $\delta \boldsymbol{\Theta} = \begin{bmatrix} 0 & -\delta \theta_z & -\delta \theta_y \\ \delta \theta_z & 0 & -\delta \theta_x \\ \delta \theta_x & \delta \theta_y & 0 \end{bmatrix}$

with the angular rotation variation $\delta \boldsymbol{\Theta} = (\delta \theta_x \quad \delta \theta_y \quad \delta \theta_z)^T$, and θ_x , θ_y and θ_z are the rigid-body rotation angles for x, y and z directions, respectively. The terms θ_x , θ_y and θ_z are related to the rotational parameters ω_x , ω_y and ω_z by the following equation:

$$(\omega_x \quad \omega_y \quad \omega_z) = \frac{\tan(\theta/2)}{\theta} (\theta_x \quad \theta_y \quad \theta_z), \quad \theta = \sqrt{\theta_x^2 + \theta_y^2 + \theta_z^2} \quad (6.22)$$

Applying Equation 6.21 into each angular variation $\delta \theta_x$, $\delta \theta_y$ and $\delta \theta_z$ at x, y and z directions, in turn, provides three nonlinear scalar equations (Appendix B) to determine the three parameters that define $\mathbf{R}_0$. To solve these three nonlinear equations, gradient descent method (Avriel 2003) is employed. After $\mathbf{R}_0$ is computed, the stiffness matrix $\mathbf{k}_e$ of each element in the linear substructure can

be modified by Equation 6.18.

6.3.3 Implementation of the correction algorithm

The proposed rigid-body rotation correction algorithm is incorporated into static condensation process to modify superelement formulation for nonlinear analysis.

A summary to implement this superelement procedure is provided as shown in

Box 6.1.

- (i) For all elements in the substructure, differentiate the internal and boundary nodes based on element connectivity in the model.
- (ii) Reorder the numbering of internal nodes. Renumber the boundary nodes continuously after the internal nodes.
- (iii) Determine the load vector for new node numbering of the substructure based on node mapping relationship.
- (iv) Read the linear stiffness terms for each element in the substructure from the direct-access file, and store them in terms of new node numbering.
- (v) Modify stiffness terms for each element in the substructure with rigid-body rotation correction algorithm stated in Section 6.3.2.
- (vi) Carry out static condensation (for both the stiffness and load terms) to obtain the modified superelement formulation according to the matrix decomposition algorithm given in Section 6.2.2.

Box 6.1 Summary for rigid-body rotation correction of superelement

6.4 Numerical examples

Three numerical examples are provided in this section. Nonlinear analyses of each example are conducted three times with (1) fully nonlinear analysis without superelement (FullNL analysis), (2) conventional superelement analysis without considering rigid-body rotation effect (ConvSup analysis), and (3) the present superelement analysis with rigid-body rotation correction (CorrSup analysis). The results from fully nonlinear (FullNL) analysis are taken as the benchmark for

comparison of accuracy and efficiency.

6.4.1 Example 6.1: a cantilever beam subject to end-bending

As shown in Figure 6.2, the cantilever beam structure is subject to a bending moment M at the free end. This is a typical large rotational problem. The beam section is W8×40 and the Young's modulus E is 207GPa. In the simulation, the beam is uniformly discretized into six 3-node TL beam elements, where elements 5 and 6 are defined as the linear zone of the superelement.

Figure 6.2 A cantilever steel beam subject to a free-end moment

The beam deflected profiles are showed in Figure 6.3. The conventional superelement used for the analysis (ConvSup analysis) has introduced incremental errors with increasing rotations of the beam. When rigid-body rotation effect of superelement is considered in ConvSup analysis, the accuracy in modeling is improved by showing a closer deflected profile to the benchmark. Meanwhile, the CPU time consuming for CorrSup analysis is comparative with ConvSup analysis, as shown in Table 6.1. This implies that the proposed rigid-body rotation correction can improve the accuracy without a significant loss of computational efficiency.

Figure 6.3 Deformation profiles of the beam at $M = 1390$ kNm

Table 6.1 CPU time at different load steps for Example 6.1

Load steps	CPU time (s)		
	FullNL analysis	ConvSup analysis	CorrSup analysis
150	5.4	3.8	3.9
300	10.0	7.0	7.3

6.4.2 Example 6.2: a canopy grillage with UDL at its end

In this example, a 3-by-4 grid grillage that is subject to UDL at its free-end is studied, as shown in Figure 6.4. The length of each grid is 3.6m in both directions. The cross-section and material properties used are the same as Example 6.1. Five identical elements are arranged for each beam that made up the frame. Part of the grillage that is enclosed by the dashed lines in Figure 6.4 is defined as the linear zone of the superelement.

Figure 6.4 A canopy grillage subject to UDL at the free-end

Figure 6.5 shows the load factor versus vertical displacement at mid-point of the free-end of the grillage for different analyses. For this more complex large displacement problem, it is found that CorrSup analysis is more accurate than ConvSup. From CPU time in Table 6.2, it is found that CorrSup analysis can significantly improve computational efficiency compared to FullNL analysis, and it is as good as ConvSup in terms of analysis time.

Figure 6.5 Load-displacement paths at mid-point of the free-end

Table 6.2 CPU time at different load steps for Example 6.2

Load steps	CPU time (s)		
	FullNL analysis	ConvSup analysis	CorrSup analysis
250	327.5	30.8	31.4
500	655.0	61.1	62.6

6.4.3 Example 6.3: an RC frame under a mid-column removal

The planar frame test conducted by Yi *et al.* (2008) has been widely used for validation of progressive collapse analysis of RC frames. In this example, the proposed superelement formulation with rigid-body rotation correction is employed to model this test frame.

Figure 6.6 The test frame under a middle column removal

As shown in Figure 6.6, all beams have the same length of 2.667m while the column height for the first story is 1.567m and 1.1m for the other stories. The cross-section size and reinforcement configuration of beams and columns are also described in Figure 6.6. The material properties are listed in Table 6.3. Four identical beam elements are used for each beam in all stories as well as each column in the first story, and three for the columns in other stories. To simulate the removal of the middle column at the first story, a vertical point load is applied at the top of the middle column above the third story. The dash portion in Figure 6.6 is

defined as the linear zone for the superelement.

Table 6.3 Material properties of the test frame

Concrete Strength f_c (MPa)	Concrete Modulus E_c (GPa)	Rebar Strength f_y (MPa)		Rebar Modulus E_s (GPa)
		Longitudinal	Transverse	
25	30	416	370	200

Figure 6.7 shows the load-displacement paths at the loading point from the physical test, FullNL analysis, ConvSup analysis and CorrSup analysis. Compared with experimental results, all three analyses with or without superelement are effective to attain good predictions. However, ConvSup analysis loses certain degree of accuracy in pursuit of shorter computational cost, as shown in Table 6.4. For CorrSup analysis, it is found that better agreement between the numerical simulation and experimental results is achieved. In addition, the CPU time of CorrSup analysis is comparable with ConvSup analysis but much less than FullNL analysis, as seen in Table 6.4. This good comparison with FullNL and test results has shown the feasibility and reliability of the proposed superelement approach (CorrSup) for progressive collapse analysis of RC frames.

Figure 6.7 Load-displacement paths at the loading point

Table 6.4 CPU time at different load steps for Example 6.3

Load steps	CPU time (s)		
	FullNL analysis	ConvSup analysis	CorrSup analysis
1000	1723.4	480.1	502.4
2000	3003.8	851.3	889.9

6.5 Conclusions

In modeling progressive collapse problems with superelement, large rigid-body rotations of the structure can lead to significant computational errors when conventional superelement technique is used. In this chapter, a new superelement approach with rigid-body rotation correction is proposed. Numerical examples of a cantilever beam and a long-cantilever grillage demonstrate that the present superelement formulation has better accuracy while maintaining comparable computational efficiency compared to conventional superelement formulation. In addition, a typical RC frame under middle column removal scenario is modeled by the proposed superelement approach and the results show the applicability of CorrSup analysis in modeling progressive collapse of RC frames.

CHAPTER 7 ADAPTIVE SUPERELEMENT MODELING PROCEDURE FOR PROGRESSIVE COLLAPSE OF REINFORCED CONCRETE FRAMES

7.1 Introduction

In Chapter 5, a nonlinear modeling procedure for progressive collapse analysis of 2D RC frames is illustrated. For this modeling procedure, the superelement formulation with rigid-body rotation correction proposed in Chapter 6 can be incorporated to reduce computational time and cost of progressive collapse analysis. However, conventional superelement approach (Section 6.2) requires the analyst to define the linear and nonlinear zones before conducting the nonlinear analysis, which is often unrealistic for progressive collapse modeling since the location of the linear zone is unknown in advance and the final damage mechanism or collapse mode of the structure cannot be identified *a-priori*. In practice, to keep the predefined superelement is unchanged or ‘stationary’ throughout the entire progressive collapse analysis, accuracy of superelement formulation is not examined as nonlinearity often initiates from the designated nonlinear zone and propagate into the linear zone. Hence, conventional superelement approach would lead to great errors in modeling progressive collapse due to the linear-elastic assumption of material property assumed in the initial stationary superelement. These give an impetus to figure out an approach to simulate the shrinkage of the linear zone as well as refinement of superelement when nonlinearity spreads during progressive collapse analysis.

An effective way to handle this is to develop an *adaptive superelement technique*

in which the size of the linear zone will be updated as the analysis progress by monitoring the magnitudes of errors of the elements in the linear zone which may undergo nonlinear response. These errors refer to deviations of stiffness or strain energy of the assumed linear elements from the actual nonlinear behavior during the spread of nonlinearity. This technique is aiming to strike a balance between accuracy and computational efficiency for nonlinear modeling of large scale buildings. To develop this technique, an error estimator is required to estimate the above-mentioned errors and compare them to the error tolerance that the user predefined. The adaptive algorithm will then identify which elements in the linear zone are affected by nonlinearity spread and need to conduct superelement refinement for subsequent analysis. With incorporating the developed adaptive superelement approach into the modeling procedure (Chapter 5) for progressive collapse analysis of RC frames, the automatic adaptive superelement modeling procedure is implemented and its effectiveness will be demonstrated by several numerical examples.

7.2 Proposed adaptive superelement approach

7.2.1 Adaptive superelement refinement scheme

To illustrate the adaptive superelement refinement scheme for progressive collapse analysis of RC frames, progressive collapse process of a planar frame subject to column removal is given in Figure 7.1. This process is divided into three stages for clarity of illustration as follows.

Figure 7.1 Nonlinearity spread of a planar frame under column-removal

Stage I: Before the conduct of analysis, a nonlinear zone with the initial nonlinear elements is defined in terms of the removed column, and the remaining portion of the structure is the linear zone including the initial linear elements. Since the initial damages always commence near the removed column, the adjacent members above the removed column are defined as the initial nonlinear zone, as shown in Figure 7.1a. The superelement is formed by static condensation of the linear zone. The nodes of superelement consist of the boundary nodes between the linear and nonlinear zones, as ‘circle dots’ shown in Figure 7.1a.

Stage II: With load applied, nonlinearity propagates from the initial nonlinear zone due to increase of structural deformation. At this time, using the predefined

linear zone (in Figure 7.1a) would lead to error of modeling since some elements in the linear zone which are assumed linear-elastic may actually have undergone nonlinear response due to nonlinearity spread. The magnitude of this error (or the degree of modeling inaccuracy) can be specified by introducing an error estimator to identify the unacceptable accuracy and thereby to carry out the superelement refinement for subsequent analysis. The error estimator used for RC frames in this study is proposed in Section 7.2.2. Once the error estimator is employed, the spread of nonlinearity will be monitored and hence updating of the linear zone for new superelement will be performed. In the deformation state of Figure 7.1b, the members in the shadow part are placed to the nonlinear zone from the initial linear zone due to nonlinearity spread. Meanwhile, a shrinking linear zone is updated to form new superelement for a more accurate subsequent analysis.

Stage III: With further load increment, spread of nonlinearity to the linear zone continues to be identified based on the error estimator (Section 7.2.2), as shown in Figure 7.1c. If partial collapse of the structure occurs (as the pink box in Figure 7.1c), the model will be regenerated in subsequent analysis by removing the collapsed part, as shown in Figure 7.1d. Due to the effect of collapse collision, nonlinearity will rapidly expand to the members carrying the collapse impact loads (as ‘arrow’ in Figure 7.1d) and further refinement of superelement will be carried out.

7.2.2 Error estimator for superelement refinement

Using superelement technique in the modeling, errors can be introduced due to the linear-elastic assumption of the elements in the linear zone which may have undergone nonlinear response. The magnitudes of the errors are determined by the degrees of nonlinearity of the assumed linear elements during nonlinearity spread. The nonlinearity degree of an element may be evaluated by stress, strain, stiffness

or strain energy, etc. Effective stress (Han and Abel 1985) or effective plastic strain (Radermacher and Reese 2014) of an element is usually used as an error indicator in finite element analysis. These two indicators are suitable for refined FE analysis based on solid elements which only consist of one homogeneous material with easily specified plastic stress or strain state.

However, for RC members, it is difficult to define the yield state or the extent of plasticity during the analysis as general macro beam/column elements with *composite materials* of steel and concrete are employed. Given an initial cracking of concrete as a flag of approaching nonlinearity for an element, the stress state of steel will be far from yielding and thus update of the linear zone will be carried out frequently. As a result, computational efficiency can be affected significantly due to frequent refinement of superelement as well as a large portion of nonlinearity spread. On the other hand, if steel yielding is used as the initiation of plasticity of an element, concrete damages would be severe and thus lead to large errors of superelement formulation. Therefore, an error estimator considering the *combined nonlinearity effects* of steel and concrete would be more appropriate for adaptive superelement analysis of RC structures.

An example of a cantilever beam shown in Figure 7.2 is used to illustrate the nature of errors in superelement analysis. A point load is applied at the end of the cantilever, and two elements in the linear zone are defined as a superelement. Figure 7.3 shows different responses of the material (concrete or steel) according to linear and nonlinear constitutive laws at different strain states (ϵ_0 and ϵ_1).

Figure 7.2 Comparison of uperelement and nonlinear analyses by a cantilever

Figure 7.3 Material responses based on linear and nonlinear constitutive laws

In the process of the analysis with a stationary superelement, if the maximum strain of the elements in the linear zone is small (as attaining a linear-elastic strain ϵ_0 in Figure 7.3), no difference will be found between superelement analysis and fully nonlinear analysis, as the deformation profiles at the strain of ϵ_0 in Figure 7.2. This is because no errors have been introduced as the actual responses of all the elements in the linear zone remain linear-elastic during fully nonlinear analysis.

However, once some elements in the linear zone are affected by nonlinearity spread such that the strains of material fibers in these elements are large (as attaining a nonlinear strain ϵ_1 in Figure 7.3), inaccuracy of superelement analysis will be

found, as the deformation profiles at the strain of ϵ_1 in Figure 7.2. This is resulted from large errors of stiffness (E_s vs E_{NL} in Figure 7.3) or strain energy (ΔW in Figure 7.3) in these elements, which are introduced by linear-elastic assumption. Therefore, stiffness or energy error of element can be used as indicator for superelement refinement when nonlinearity spreads during the analysis. Since energy-based error estimator which was employed in many adaptive mesh refinement analyses has shown to be stable for various adaptive problems (Zienkiewicz and Zhu 1987), an estimator in terms of strain energy error of element would be more appropriate for adaptive superelement refinement of RC frame structures.

To consider the combined nonlinearity effects of steel and concrete in RC members, a fiber model (as Figure 3.2 of Section 3.3) describing the nonlinear behavior of RC cross-sections is employed. In the present study, 36 fibers for concrete are used by dividing the whole cross-section into 6×6 mesh as small occupation of rebar on concrete is negligible. For steel, each reinforcing bar is considered as one fiber to simulate the behavior of steel over the cross-section. A representative description of fiber discretization for one cross-section with four reinforcing bars is shown in Figure 7.4.

Figure 7.4 Fiber discretization of a cross-section with four bars

Based on the fiber model, an energy-based error estimator at *element* level for adaptive superelement refinement of RC frames is proposed. Definition of this error estimator is based on the following steps:

1) To compute strain for material fibers at specific cross-sections

At each load step, nodal displacements of each element in the linear zone are extracted from displacement recovery of superelement by Equation 6.3. Displacements of the boundary nodes are obtained directly from the analysis results of the nonlinear zone. Based on fiber model, the strain levels ($\boldsymbol{\varepsilon}$) of material fibers *over the cross-section at each integration point of each element in the linear zone* are obtained according to Green-Lagrange strain description. It should be pointed out that in this study, two-point Gauss integration is employed for each beam element.

2) To compute strain energy density for each material fiber i

For each material fiber i stated in Step 1, strain energy density at the current load step is calculated respectively based on the initial *elasticity* modulus (corresponding to the linear-elastic assumption of superelement formulation) and the *nonlinear* constitutive law (corresponding to the actual nonlinear response) of the material. The strain energy density with respect to the initial elasticity modulus is given by

$$W_{si} = \frac{1}{2}(\boldsymbol{\sigma}^T \mathbf{D}^{-1} \boldsymbol{\sigma})_i \quad (7.1)$$

while with respect to the nonlinear constitutive law it is given by

$$W_{NLi} = (\int \boldsymbol{\sigma}^T d\boldsymbol{\varepsilon})_i \quad (7.2)$$

Where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are stress and strain vectors; $\mathbf{D}$ is constitutive matrix of material.

3) To compute error of strain energy for the cross-section

For the cross-section at each integration point of each element in the linear zone, the total error of strain energy density is obtained by summing up the contribution of all fibers in this cross-section:

$$\|\mathbf{e}\| = [\sum_i (W_{si} - W_{NLI})]^{1/2} \quad (7.3)$$

The corresponding relative error at this integration point thus can be calculated by

$$\frac{\|\mathbf{e}\|}{\|\mathbf{u}\|} = \left[\sum_i \frac{W_{si} - W_{NLI}}{W_{si}} \right]^{1/2} \quad (7.4)$$

4) *To define the error estimator of the element*

The relative error of an element is defined by the maximum relative error among all the integration point cross-sections over this element. Therefore, the *error estimator of the element* is given as

$$\max \left\{ \left(\frac{\|\mathbf{e}\|}{\|\mathbf{u}\|} \right)_j \right\} \leq \eta \quad (7.5)$$

where $\left(\frac{\|\mathbf{e}\|}{\|\mathbf{u}\|} \right)_j$ denotes the relative error of integration point cross-section j of this element. η is a user-defined element error tolerance for adaptive superelement analysis of RC structures. If Equation 7.5 is violated, the corresponding element will be removed from the linear zone and added to the nonlinear zone in order to ensure modeling accuracy for subsequent analysis. The value of η that is prescribed by the user in advance gives the accuracy demand of the analysis.

7.3 Adaptive superelement modeling of RC frames subject to progressive collapse

To simulate the process of progressive collapse of RC frames, partial collapses of members or substructures of the structure should be identified and traced in the process. Towards this end, an earlier work on progressive collapse modeling (in Chapter 5) is employed, where key procedures for damage assessment, collapse identification and removal, and model regeneration after collapse are involved. These procedures are combined with the proposed adaptive superelement algorithm to achieve a complete adaptive superelement modeling procedure for

progressive collapse analysis of RC frames. For the sake of clarity, an overall flowchart to implement this modeling procedure is summarized in Figure 7.5.

Figure 7.5 Flowchart for adaptive superelement modeling procedure

As shown in Figure 7.5, after the basic data input, the procedure starts from an initial definition of the linear and nonlinear zones. The linear zone is condensed to superelement formulation using static condensation (Section 6.2.1) and the superelement will be assembled into the nonlinear zone after the model for the nonlinear zone is regenerated. FE analysis will then be carried out for the nonlinear

zone, and the nodal displacements in the linear zone will be obtained by displacement recovery using Equation 6.3 (in Section 6.2.1). At each equilibrium step, all the elements in the linear zone will be checked by error estimator (Section 7.2.2) to see if the linear zone need shrink or not. On the other hand, damage assessment of elements in the model will be carried out to find out the (flexural/shear/axial) failures of elements. After that, member/substructure collapse searching algorithms (Section 5.4) will be performed to check if collapse has occurred. It is worthy to emphasize that collapse identification of a member or a substructure will be carried out in the *whole* structure including the linear zone, according to member connectivity of the whole structure and the combined (flexural/shear/axial) failure modes of elements in the nonlinear zone. If collapse occurs *or* the linear zone shrinks, the linear and nonlinear zones will be updated and the model will be regenerated for subsequent analysis. If both member/substructure collapse and shrinkage of linear zone are not found, program termination will be checked by the value of load factor λ . If $\lambda \geq 1$, then the loads in the model have been fully applied and thus stop the analysis. Otherwise, the next incremental analysis will commence without regeneration of the model.

In this process of adaptive superelement modeling, it should be noted that both collapse and the shrinkage of linear zone will lead to model regeneration. If only collapse occurs, the collapsed part will be removed from the model and thus the nonlinear zone is reduced for a new model, although the superelement formulation may not be modified. However, if rigid-body rotation correction (Section 6.3.2) is carried out in the linear zone, the superelement formulation need be rebuilt even no shrinkage of the linear zone occurs. If only linear zone shrinks, it is necessary to redefine the superelement for new linear zone. At the meantime, the nonlinear zone has spread for a new model for subsequent FE analysis.

For damage assessment of the model, it may not be necessary to conduct damage assessment for the elements in the linear zone, although collapse may also occur in this zone. This is because based on the error estimator all the elements in the linear zone will neither undergo serious material damages nor failures. As a result, collapse of the linear zone is always caused by loss of boundary restraints when some elements adjacent to the linear zone but in the nonlinear zone have failed and resulted in partial collapse of a member or a substructure of the structure. Example 7.5 in Section 7.4.5 will show a partial collapse in the linear zone (as members 3, 4, 5 and 33, 34, 35 in Figure 7.20a), which is identified by substructure collapse searching algorithm (Section 5.4.2), although no failures of members in the linear zone are found.

7.4 Numerical examples

Five numerical examples are provided in this section. Nonlinear analyses of all five examples are conducted with (1) fully nonlinear analysis without superelement (FullNL analysis), (2) conventional superelement analysis with fixed (or stationary) linear and nonlinear zones (ConvSup analysis), (3) adaptive superelement analysis *without* rigid-body rotation correction (AdapSup analysis), and (4) adaptive superelement analysis with the proposed rigid-body rotation correction (AdaptSupCorr analysis). The results of fully nonlinear (FullNL) analysis are taken as the *benchmark* for comparison of accuracy and efficiency. For adaptive superelement analyses, error tolerance η of 5% and 10% are adopted for the RC beam in Example 7.1. It is found that the analysis within a tolerance of 5% takes significant computational effort. Therefore, an error tolerance of 10% is selected for comparison of the adaptive analyses in Examples 2 to 5. It should be mentioned that the configuration of the computer used for all the examples is ‘Intel Core i7-4770 CPU @ 3.4GHz processor with 16GB on board memory’. The software

environment for the program development is ‘Intel Visual Fortran 11.0’.

7.4.1 Example 7.1: a fixed-fixed RC beam subject to mid-span point loading

A fixed-fixed RC beam subject to a concentrated load in the mid-span is given as the first example. The beam length is 7.2m. On account of symmetry, one half-structure of the beam is modeled and is uniformly discretized into six three-node TL beam elements (Dvorkin *et al.* 1988), as shown in Figure 7.6. Geometrical and material properties are listed in Table 7.1. For superelement analyses, Elements 2 to 5 are defined as the initial linear zone as shown in Figure 7.6. The objective of this example is to study the tolerance effect (5% vs 10%) and correction for rigid-body rotation on prediction accuracy based on the fully nonlinear analysis.

Figure 7.6 Half-structure model of an RC fixed-fixed beam

Table 7.1 Geometrical and material properties of Example 7.1

Longitudinal reinforcement ratio			Concrete	Reinforcement	
Beam top	Beam bottom	Column	f_c (MPa)	f_{sy} (MPa)	f_{su} (MPa)
1.55%	1.01%(2T10)	1.53%	30.0	505.0	605.0
(3T10)		(4T10[corner]+4T6)			

Figure 7.7 shows the relationship of load factor versus mid-span displacement for different analyses. It is found from Figure 7.7 that ConvSup analysis has the largest error among all the superelement analyses and it cannot capture catenary action in the beam. Besides, ConvSup analysis has over predicted the peak capacity in flexural action stage. Compared with ConvSup, AdapSup(10%) analysis has

achieved higher accuracy at the stage before yielding, as well as at the first peak resistance in flexural action. This shows the significance of adaptive superelement which can consider nonlinearity spread to the linear zone during the analysis. As seen in Table 7.3, number of nonlinear elements gradually increases at different stages of the adaptive analyses to enhance the accuracy of modeling. However, AdapSup(10%) analysis still has relatively large deviation from the benchmark under large deformations (after a displacement of 250mm) and catenary action is only slightly mobilized, as shown in Figure 7.7. This is because the neglect of rigid-body rotation correction in AdapSup(10%) has resulted in large modeling errors especially at large deformation state. Incorporating rigid-body rotation effect of superelement, AdapSupCorr(10%) analysis performs better than AdapSup(10%) analysis. The former simulation nearly coincides with the benchmark (FullNL) except for small differences at the occurrence of rebar fracture as listed in Table 7.2, where the mid-span bottom rebar fracture has been delayed in AdapSupCorr(10%) analysis compared to the benchmark. This is because a large error tolerance of 10% has delayed the increase of rebar strains of elements in the linear zone during the adaptive process. AdapSup(5%) analysis has the highest accuracy among all the superelement analyses, due to an early occurrence of superelement refinement under a much smaller error tolerance (5%).

Figure 7.7 Load-displacement paths at mid-span loading point

Table 7.2 Rebar fracture sequence in Example 7.1

Analysis scenario	Rebar fracture		
	f1	f2	f3
FullINL analysis	end-top + mid-bot	end-bot + mid-top	-
ConvSup analysis	mid-bot	end-top	-
AdapSup(5%) analysis	end-top + mid-bot	end-bot + mid-top	-
AdapSup(10%) analysis	end-top	element5-bot	-
AdapSupCorr(10%) analysis	end-top	mid-bot	end-bot

End: beam-end; mid: mid-span; top: top rebar fracture; bot: bottom rebar fracture; e.g., end-top: top rebar fracture at the beam-end.

Table 7.3 Adaptive update of superelement for Examples 7.1-7.4

Ex.	Analysis scenario	Term	Adaptive update						
			Initial	1 st	2 nd	3 rd	4 th	5 th	6 th
1	AdapSup (5%)	NL element	1,6	2,5	3,4	-	-	-	-
		d (mm)	0	1.64	4.68	-	-	-	-
	AdapSup (10%)	NL element	1,6	2,5	-	-	-	-	-
		d (mm)	0	11.7	-	-	-	-	-
	AdapSupCorr (10%)	NL element	1,6	2,5	3,4	-	-	-	-
		d (mm)	0	11.4	22.0	-	-	-	-
2	AdapSup (10%)	NL element	1,21,41,61	2,22,42,62	3,23,43,63	-	-	-	-
		d (mm)	0	84.07	103.15	-	-	-	-
	AdapSupCorr (10%)	NL element	1,21,41,61	2,22,42,62	3,23,43,63	4,24,44,64	5-14,25-34,45-54,65-74	-	-
		d (mm)	0	77.68	95.26	109.5	169.44	-	-
	AdapSup (10%)	NL element	21-24,31-36	30,43	42,44,48	47	45,46	-	-
		d (mm)	0	7.24	10.12	15.64	18.10	-	-
3	AdapSup (10%)	NL element	21-24,31-36	30,43	42,44,48	47	45,46	-	-
		d (mm)	0	6.92	9.73	15.57	17.63	-	-
4	AdapSup (10%)	NL member	19,20,24,25,36,37,40,41,44,45	35,39,43	34,38,42	32,33	31	29	28
		d (mm)	0	6.09	13.04	872.4	-	-	-
	AdapSupCorr (10%)	NL member	19,20,24,25,36,37,40,41,44,45	35,39,43	34,38,42	32,33	31	29	28
		d (mm)	0	6.10	13.08	870.3	-	-	-

‘NL’ refers to Nonlinear; ‘d’ refers to ‘Displacement’ which is the specific displacement from the corresponding load-displacement path when adaptive update occurs. ‘1st-6th’ denotes the sequence of adaptive update with *new* nonlinear elements/members identified; ‘-’ denotes no further update.

Figure 7.8 shows a comparison of CPU time for different analyses. For this problem, AdapSup(5%) analysis, rather than FullNL analysis, has taken the most time. This is because AdapSup(5%) analysis has switched to fully nonlinear analysis occurs very early in the analysis (at a displacement of 4.68mm, as shown in Table 7.3), and the adaptive process has led to a slight difficulty in convergence during incremental iterations. ConvSup analysis has the minimum computational

time among all the analyses, but its accuracy is unacceptable as can be seen in Figure 7.7. At a load step of 250, there is comparative computational cost between AdapSup(10%) and AdapSupCorr(10%) analyses. However, with an increase of load steps even AdapSupCorr(10%) analysis takes less time. A possible reason is that superelement formulation without rigid-body rotation correction may cause deteriorating convergence performance for this example, which requires much more steps for equilibrium at some load steps. Compared with the benchmark, AdapSupCorr(10%) analysis has reduced computational cost but the reduction is not so obvious in such a small-scale problem.

Figure 7.8 CPU time of the analyses in Example 7.1

7.4.2 Example 7.2: a canopy RC grillage subject to UDL at free end

In this example, a 3-by-4 bay long-cantilever canopy RC grillage subject to uniformly distributed loads (UDL) at its free-end is studied, as shown in Figure 7.9. The length of each bay is 3.6m in both directions. The sectional and material properties used are the same as in Example 7.1. Five elements were employed for

each beam that made up the RC grillage. Member numbering is shown in Figure 7.9 and thus the numbering of the i^{th} element of member j can be denoted by $(j - 1) \times 5 + i$. For superelement analyses, the initial nonlinear zone is defined by four fixed-end elements, i.e., elements 1, 21, 41 and 61, as shown in Figure 7.9.

Figure 7.9 A canopy RC grillage subject to UDL at the free end

Figure 7.10 shows the load-displacement paths at mid-point of free-end of the frame (Point A in Figure 7.9) for these analyses. ConvSup analysis incurs the largest error among all the superelement analyses. Even at the stage before yielding, its deviation from the benchmark is large due to the neglect of rigid-body rotations and nonlinearity spread of the linear zone. The sudden drop of load is a result of rebar fracture at the fixed-ends, indicating the collapse of the grillage. As seen in Figure 7.10, the collapse predicted by ConvSup analysis occurs (at a displacement of 4000mm) much earlier compared to the benchmark (at around 7000mm), which shows an unreasonable assessment of load-carrying capacity and structural ductility using conventional superelement. Compared with ConvSup, AdapSup(10%) analysis has higher accuracy. The pre-yield stiffness and ultimate deformation of the grillage are better predicted by AdapSup(10%). As shown in Table 7.3, eight elements (2, 22, 42, 62 and 3, 23, 43, 63) in the initial linear zone have been moved to the nonlinear zone through two adaptive updates so as to

improve the prediction accuracy. However, compared to the benchmark, there are still large errors in structural stiffness and peak resistance in AdapSup(10%) analysis, as seen in Figure 7.10. This is because the neglect of rigid-body rotations in the linear zone has led to a large error of superelement formulation for this nonlinear analysis. From Figure 7.10, AdapSupCorr(10%) analysis gives the most reasonable prediction among all the superelement analyses. Before a displacement of 3000mm, it has nearly the same load-displacement path as the benchmark. This implies the significance of rigid-body rotation correction in superelement formulation, especially for the analysis of long-cantilever grillage, where large rigid-body rotation is critical. Although all the three superelement analyses have attained consistent collapse mode (a complete fracture at the four supporting ends) of this RC grillage with FullNL analysis, the prediction from AdapSupCorr(10%) analysis is most acceptable.

Figure 7.10 Load-displacement paths at Point 'A'

Figure 7.11 shows a comparison of CPU time for the four analyses. Both ConvSup and AdapSup(10%) analyses take much less time, but significant and very large errors in stiffness, resistance and ductility can be found even at the beginning of the analysis (Figure 7.10). Although AdapSupCorr(10%) analysis takes more time, its

accuracy is reasonable as shown in Figure 7.10. From Figure 7.11, at a load step of 250, the computational cost of AdapSupCorr(10%) analysis is only one quarter of the benchmark, while at a step of 500 and 1000 there is a gradual decrease of computational efficiency, due to shrinkage of the linear zone with spread of nonlinearity from the fixed-ends to the free-ends of the grillage. It can be seen that at a step of 1000, there is a 50% reduction of computational cost using AdapSupCorr(10%) analysis, without a significant loss of accuracy.

Figure 7.11 CPU time of the analyses in Example 7.2

7.4.3 Example 7.3: a 2-bay-by-2-story RC frame under one-column removal

In this example, a 2-bay-by-2-story RC frame subject to a side-column removal scenario is studied. This is a perimeter frame of a commercial building, of which the stories are equally 4.0m high and the spans at two orthogonal directions are 6.0m each. This building is designed based on EC2 (2004). The imposed load is 15.0kN/m and the dead load including self-weight is 21.3kN/m. Geometrical properties and reinforcement details are listed in Table 7.4. In this model, four

equal-length beam elements are employed for each column and six for each beam. Numbering of members in this frame is shown in Figure 7.12. Therefore, the i^{th} element number of member j can be denoted by $(j - 1) \times 4 + i$ for $j \leq 6$ (column) and by $24 + (j - 6) \times 6 + i$ for $j > 6$ (beam). After applying the service loads to the beam for the intact structure, member 5 is removed as the initial trigger for progressive collapse analysis. For the superelement analyses, members 6 and 8 are predefined to be the initial nonlinear zone as shown in Figure 7.12.

Figure 7.12 2-bay-by-2-story RC frame subject to side-column removal

Table 7.4 Geometrical and material properties of the frame in Example 7.3

Width×Depth (mm)		Longitudinal reinforcement			Material strength (MPa)	
Beam	Column	Beam top	Beam bottom	Column	Concrete	Reinforcement
250×4	450×450	3T25	3T20	8T20	30.0	550.0

Figure 7.13 shows load-displacement paths at the top of the removed column (Point B in Figure 7.12) for each analysis. Among all the analyses, ConvSup analysis has the worst accurate result compared to the benchmark. After column removal, the load factor attains unity at a small displacement of 100mm as seen in Figure 7.13. This is because under the missing column scenario, member 10 has undergone severe downward deformations similar to member 8, but the former is always included in the linear zone during ConvSup analysis. Such unreasonable linear-elastic approximation for member 10 results in large modeling errors.

Compared with ConvSup, AdapSup(10%) and AdapSupCorr(10%) analyses have achieved much higher accuracy. Yielding and final collapse of the structure have been identified, as shown in Figure 7.13.

Figure 7.13 Load-displacement paths at Point 'B'

Figure 7.14 shows the collapse mode of this frame with FullNL analysis where rotational collapses of member 10 and a substructure consisting of members 6 and 8 occur at the same load step. The same collapse mode is also found in AdapSup(10%) and AdapSupCorr(10%) analyses. Unlike Example 7.2, AdapSup(10%) and AdapSupCorr(10%) give almost the same load-displacement path for this problem. This is because after adaptive updates of the two analyses in Table 7.3, the superelement which belongs to a partial-frame consisting of members 1-4, 7 and 9 has not undergone large rotations during the analysis. Therefore, rigid-body rotation effect of the linear zone is not critical in this example.

Figure 7.14 Collapse mode of the frame in Example 7.3

Figure 7.15 shows a comparison of CPU time for all four analyses. ConvSup analysis takes the least time because the linear zone of this model is the smallest (only members 6 and 8 are included). The analysis terminates at a step of 153 such that the CPU time in Figure 7.15 after step 153 is not shown. Compared with AdapSup(10%), AdapSupCorr(10%) analysis requires even less time. A possible reason is that rotation correction for the elements in the linear zone has improved the accuracy of superelement formulation, which in turn enhances the rate of convergence during load increment. At a step of 600, AdapSupCorr(10%) analysis takes 70% of computational time of FullNL analysis.

Figure 7.15 CPU time of the analyses in Example 7.3

7.4.4 Example 7.4: a 4-bay-by-5-story RC frame under two-column removal

In this example, a 4-bay-by-5-story RC frame subject to a two-column removal scenario is studied, as shown in Figure 7.16. For simplicity, structural dimensions, material properties and load scenarios are adopted from Example 7.3. Four elements are used for columns and six for beams. Numbering of members is given in Figure 7.16. After applying the service loads for the intact structure, members 18 and 23 are removed as the initial trigger for progressive collapse analysis of this structure. Different from the last examples, this example will indicate collapse progression after the initial collapse of partial structure. To avoid too frequent updates of superelement, in this example, the spread of nonlinearity will be identified at member level, i.e., a member in the linear zone will be enlisted in the nonlinear zone if there is at least one element in this member violating the error estimator given in Equation 7.5. In this example, the members above the removed two columns are defined as the initial nonlinear zone for superelement analyses, as the dash box in Figure 7.16.

Figure 7.16 A 4-bay-by-5-story RC frame under two-column removal

Figure 7.17 shows the load-displacement paths at the top of member 23 (Point C in Figure 7.16). From Figure 7.17, ConvSup analysis gives worst accurate prediction, even at an early stage before the yielding of members. This implies original linear-elastic stiffness matrix of superelement cannot ensure adequate accuracy for nonlinear analysis and therefore adaptive superelement technique for a more accurate analysis is needed. Compared to ConvSup analysis, the adaptive superelement analyses of AdapSup(10%) and AdapSupCorr(10%) yield more accurate predictions of stiffness and ductility of the structure as shown in Figure 7.17. Similar to Example 7.3, they have almost the same load-displacement path due to negligible rigid-body rotations in the linear zone and their adaptive processes of superelement are similar as seen in Table 7.3.

Figure 7.17 Load-displacement paths at Point 'C'

Figure 7.18 shows the progression of collapse mechanisms of this RC frame. Note that only three analyses, i.e., FullNL, AdapSup(10%) and AdapSupCorr(10%) analyses, are able to achieve the whole process of collapse and they give consistent collapse mode due to the same identification of failures of elements. It is not possible to continue ConvSup analysis after the collapse of c2 (Figure 7.18c) due to

vanishing of the nonlinear zone. In addition, it is not reasonable that members 32 and 33 are always in the linear zone even when they are subject to the collapse impact from c2. However, as seen in Figure 7.18, the correct tracing and monitoring of collapse modes by both AdapSup and AdapSupCorr analyses demonstrate a distinct advantage of prediction accuracy using the proposed adaptive superelement approach.

Figure 7.18 Collapse modes for FullNL, AdapSup and AdapSupCorr analyses

Figure 7.19 shows a comparison of computational time for the four analyses. ConvSup analysis takes the least computational time since the linear zone is stationary throughout the analysis. However, it terminates at a step of 236 due to unreasonable linear and nonlinear zones after the collapse of c2. Compared with AdapSup(10%), AdapSupCorr(10%) analysis requires slightly more time due to

rotation correction procedure. At a step of 1000, AdapSupCorr(10%) takes around 60% of time of the benchmark case in consequence of an extensive spread of nonlinearity.

Figure 7.19 CPU time of the analyses in Example 7.4

7.4.5 Example 7.5: a 7-bay-by-5-story RC frame subject to different column removal scenarios

In this example, a 7-bay-by-5-story RC frame subject to two-column (members 1 and 31), side-column (member 1) and mid-column (member 31) removal scenarios is studied, as shown in Figure 7.20. For simplicity, member dimensions, material properties and load scenarios as well as element arrangement for this example are adopted the same from Example 7.4. For the superelement analyses, the members adjacent to the removed member are defined as the initial nonlinear zone, as the two dash boxes shown in Figure 7.20.

(a) Two-column (members 1 and 31) removal scenario

(b) Side-column (member 1) removal scenario

(c) Mid-column (member 31) removal scenario

Figure 7.20 A 7-bay-by-5-story RC frame under column removal scenarios

Figure 7.21 shows the load-displacement paths at the top of member 1 (Point D in Figure 7.20). It is found that for two-column (Figure 7.21a) and side-column (Figure 7.21b) removal scenarios, ConvSup analysis is the least accurate. Even at a very small displacement ($<20\text{mm}$) the load factor has attained one and the analysis terminates. This shows that a stationary superelement for the initial linear zone in ConvSup fails to evaluate accurate deflections of the members above a removed column, and hence the prediction results from ConvSup analysis is unacceptable. However, both AdapSup(10%) and AdapSupCorr(10%) analyses yield good results as the benchmark, as shown in Figure 7.21. In addition, these two analyses are capable of tracing the whole process of collapse after the trigger of column removal, as shown in Figure 7.23. Similar to Example 7.4, consistent prediction results between AdapSup(10%) and AdapSupCorr(10%) analyses can be found from Figure 7.21 due to negligible rigid-body rotations of the elements in the linear zone. From Figures 7.21a and 7.21b, it is found that there are relatively larger errors of adaptive analysis for the scenario of two-column removal compared to the side-column removal scenario, indicating a decrease of accuracy of superelement

modeling due to a large spread of nonlinearity. For column removal (member 31) scenario, similar characteristics of load-displacement relationship (Point E) are found in Figure 7.22.

(a) Two-column (members 1 and 31) removal scenario

(b) Side-column (member 1) removal scenario

Figure 7.21 Load-displacement paths at Point 'D'

(a) Two-column (members 1 and 31) removal scenario

(b) Mid-column (member 31) removal scenario

Figure 7.22 Load-displacement paths at Point 'E'

Figure 7.23 shows the progressive collapse modes of this frame subject to two-column (members 1 and 31) removal scenario. In Figure 7.23, a number in a parenthesis refers to a bay numbering otherwise it denotes a member number. For FullNL analysis (Figure 7.23a), the first collapse c1 occurs at member 41 of Bay 1, followed by the collapse of members 48, 67 and 74 (collapse c2), where members

67 and 74 belong to Bay 6. This shows a close occurrence of collapses at Bay 1 and Bay 6 during the analysis. This arises from the uniform consistent resistance of substructures above members 1 and 31 to their corresponding load increment. After that, collapse c3 occurs when members 55, 62 and 69 fail in Bay 1, as seen in subplot a4 of Figure 7.23a. Due to a loss of boundary restraint, the columns in Bay 1 collapse without any material failure. Subsequent collapse continues in Bays 6 and 7 until the two bays completely fall down and only Bays 2-5 survive finally, as shown in subplot a6 of Figure 7.23a.

For adaptive superelement analysis (Figure 7.23b), the first collapse c1 refers to an overall collapse of Bay 1 due to axial failures at members 48, 55, 62 and 69. Member 41 and the columns in Bay 1 collapse at the same increment due to a rotational collapse, as shown in subplot b2 of Figure 7.23b. Subsequent collapses occur in Bays 6 and 7 with a similar collapse mode in FullNL analysis. From Figure 7.23, it is found that there are some differences of collapse sequence among FullNL, AdapSup(10%) and AdapSupCorr(10%) analyses. This is mainly because the approximation of modeling using superelement formulation which may introduce some deviation of stress/strain distributions over the linear and nonlinear zones, such that the failure sequence of nonlinear elements can be affected during the analysis. However, this deviation should be acceptable as the final collapse modes of the structure are consistent among these three analyses, as seen in subplots a6 and b6 of Figure 7.23. In fact, from Figure 7.22, it can be found that the collapses c2 and c3 in Bay 1 have occurred very fast after collapse c1 due to large deformations of members above the removed column.

(a) FullNL analysis

(b) AdapSup(10%) and AdapSupCorr(10%) analyses

Figure 7.23 Collapse modes for two-column (members 1 and 31) removal scenario

Figure 7.24 shows the collapse modes of the frame under a side-column (member 1) removal scenario. Similar to two-column removal scenario, there are slight differences in collapse sequence during the analysis but they give the same final collapse mode in all three analyses. Bays 2-7 remain stable after the complete collapse of Bay1, as seen in subplot a4 and b4 of Figure 7.24. For the mid-column (member 31) removal scenario, consistent collapse modes for Bays 6 and 7 have been attained and only Bays 1-5 remain stable after collapse, as shown in Figure 7.25.

(a) FullINL analysis

(b) AdapSup(10%) and AdapSupCorr(10%) analyses

Figure 7.24 Collapse modes for side-column (member 1) removal scenario

(a) FullNL analysis

(b) AdapSup(10%) and AdapSupCorr(10%) analyses

Figure 7.25 Collapse modes for mid-column (member 31) removal scenario

Figure 7.26 shows the CPU time of the analyses for this frame. For the three removal scenarios, ConvSup analysis takes the least computational time but terminates at an early stage as the load factor has attained unity and the prediction of ConvSup analysis is not applicable, as seen in Figures 7.21 and 7.22. As shown in Figure 7.26, AdapSupCorr(10%) analysis requires more time than AdapSup(10%) analysis for all removal scenarios due to rotation correction procedure. However, the increase of cost is not significant. It is also found that for both AdapSup(10%) and AdapSupCorr(10%) analyses, the computational time of side-column removal scenario is much shorter than the two-column and the mid-column removal scenarios. This is due to a much larger spread of nonlinearity occurred in both two-column and mid-column removal scenarios (as dash-circled

members in Figures 7.20a and 7.20c), whereby the advantage of reducing of DOFs using superelement is gradually lost. But for the side-column removal scenario, at a step of 400, AdapSupCorr(10%) only takes 18.1% of time of the benchmark as shown in Figure 7.26b. This shows a very significant saving of computational cost for this larger-scale frame analysis, without substantial loss of modeling accuracy.

(a) Two-column (members 1 and 31) removal scenario

(b) Side-column (member 1) removal scenario

(c) Mid-column (member 31) removal scenario

Figure 7.26 CPU time of the analyses in Example 7.5

7.5 Conclusions

This chapter presents an automatic adaptive superelement modeling procedure for progressive collapse analysis of RC frame structures. Two main works have been conducted in this chapter, which include: (1) an adaptive superelement algorithm developed to update the linear zone of the RC frame for new superelement during nonlinear analysis, and (2) an adaptive superelement modeling procedure for simulating progression of collapse of RC frame structures. In addition, rigid-body rotation correction procedure proposed in Section 6.3.2 has also been incorporated into the present modeling procedure. Numerical examples are given to demonstrate the proposed modeling approach and procedure. It is shown that: (1) the suggested rigid-body rotation correction can improve the accuracy of conventional superelement formulation without a significant increase of computational effort, (2) the proposed automatic adaptive superelement analysis approach can automatically identify nonlinearity spread of RC frames during the analysis and thus the

modeling accuracy can be improved significantly, (3) the developed adaptive superelement modeling procedure is able to trace and simulate the whole process of progressive collapse of RC frames subject to single or multiple column removal scenarios, and (4) the present adaptive superelement analysis can improve computational efficiency without a significant loss of accuracy, especially for large-scale progressive collapse problems.

CHAPTER 8 CONCLUSIONS AND FUTURE WORK

8.1 Conclusions

In this thesis, a new adaptive nonlinear modeling procedure has been developed for progressive collapse analysis of RC frame structures. Focus of the current work is devoted to three essential ingredients for the whole modeling procedure for RC frames subject to progressive collapse: damage assessment criteria, nonlinear modeling for progression of collapse, and adaptive superelement modeling procedure.

8.1.1 Damage assessment criteria

The originality of this part is the development of a set of damage criteria for RC frames subject to progressive collapse. Flexural, shear and axial damages and failures are separately quantified and identified, with consideration of axial-shear-flexural interactions during the analysis. The characteristics of this set of damage criteria are listed as follows:

1) Monitoring of critical damage features of RC frames

The suggested flexural damage criterion is capable of capturing cracking and crushing of concrete, as well as yielding and fracture of reinforcement during the analysis. The proposed axial and shear damage criteria can indicate ultimate state of collapse through the axial or shear damage index attaining unity. Collapse does not occur when only plastic hinge mechanism in RC beams is formed. Thereafter, further structural resistance by catenary action of RC beams is involved until the

final collapse of the structure.

2) Identification of collapse from fracture of reinforcement

For structural collapse at member level, an additional axial failure criterion is suggested in terms of fracture of reinforcement in the member. When fracture of all reinforcement occurs in the member, regardless whether the fracture occurs at the same cross-section or not, full loss of axial capacity of a member has been attained. An effective algorithm is developed to identify this type of axial failure that can often be found at the stage of catenary action in RC beams.

3) Identification of collapse modes

Based on the proposed damage criteria, flexural, shear and axial failures can be determined for each element in the RC frame. Therefore, two types of collapse modes (*rotational* and *direct* collapse modes) at both member and substructure levels can be identified, according to a combination of flexural, shear and axial failures of elements. This provides the basis for development of member/substructure removal algorithms in progressive collapse modeling.

8.1.2 Nonlinear modeling for progression of collapse

The contribution of this part is the development of nonlinear modeling procedure for collapse progression of RC frames. Distinctly different from conventional structural analysis, the present program models the whole process of progressive collapse. New model for the surviving residual structure is regenerated after each partial collapse has taken place and the analysis continues on. Key characteristics of this modeling procedure are as follows:

1) Member/substructure removal after collapse

In this study, the failed elements are directly deleted from the model, which

eliminates continued accumulation of very small stiffness terms in the global stiffness matrix of the model and provides a more robust updated model for subsequent analysis after each partial collapse. New removal algorithms are developed at both member and substructure levels so as to identify authentic collapse modes and trace collision and impact after collapse properly.

2) Collision and Impact after collapse

Actual structural behavior during collapse progression or under collapse impact will provide useful insight to improve existing design against progressive collapse. In this thesis, the locations and magnitudes of collapse impact are identified and evaluated based on rigid-body kinematics and energy principle with inelastic and oblique impact effects considered for a more realistic prediction of collapse impact.

8.1.3 Adaptive superelement modeling procedure

The originality of this work is the development of a new adaptive superelement modeling procedure for efficient progressive collapse analysis of RC frames. The proposed adaptive superelement approach can consider rigid-body rotation effect in superelement and nonlinearity spread of nonlinearity in RC frames during progressive collapse analysis. The characteristics of this new superelement technique are summarized as follows:

1) Superelement formulation with rigid-body rotation correction

Based on minimum energy functional, rigid-body rotation correction is carried out for all elements in the linear zone to improve superelement formulation during subsequent collapse analysis. It is found that the proposed superelement formulation with rigid-body rotation correction is reasonable and effective, without significant increase of computational cost.

2) Adaptive superelement modeling of progressive collapse

The proposed adaptation scheme based on error estimate of strain energy can characterize combined nonlinearity of concrete and reinforcement at cross-section level for RC members. Incorporating with damage assessment, identification and removal of collapsed members/substructures and model regeneration modules, an adaptive superelement modeling procedure is established, which can give accurate and yet more efficient numerical predictions for progressive collapse of RC frames.

8.2 Future work

With regard to further work of this study, several issues remain to be investigated and extended in the future:

1) Extension to 3D models for RC frame structures

The progressive collapse modeling procedure developed in this study is only concentrated on simulation of 2D RC beam-column frames. Resistance of RC slabs, walls and 3D space effects are not considered in the present model. Therefore, in the future work, development of nonlinear modeling procedure for progressive collapse analysis of 3D RC beam-slab-column structures is necessary. Towards this end, the current algorithms for collapse and impact predictions based on 2D problems will need to consider kinematics in 3D space.

2) Extension to dynamic analysis model

In this study, the modeling procedure is developed only for nonlinear static analysis. Although obvious advantages are involved in static analysis such as cost saving and convenience for structural applications, dynamic analysis can provide closer-to-actual predictions for progressive collapse analysis as time and dynamic effects are directly considered in the model. Therefore, more complex dynamic

modeling procedure will be worthy and necessary. In this regard, algorithms for identifying and tracking dynamic collapse process need to be developed. In addition, dynamic condensation technique should be applied to superelement formulation to reduce computational cost for progressive collapse analysis of large-scale structures.

3) Use of advanced RC joint model

In the current study, only second order TL beam elements are used to model RC beam-column frame structures. Hence, damages and failures of RC beam-column connections are not investigated. As severe damage or failure of RC connections may give rise to different structural nonlinear behavior and collapse modes, it is worthy to develop and incorporate advanced RC joint models for more reasonable and reliable progressive collapse analysis of RC frame structures.

4) Development of global buckling assessment procedure for progressive collapse

In this thesis, only localized buckling of reinforcement at cross-section level is investigated, while global buckling of a member or partial substructure is not considered, which may usually cause structural collapse as a result of global instability. Therefore, in the future work, development of global buckling assessment procedure at both member and substructure levels for further identification of collapse mechanisms due to global buckling is important for progressive collapse analysis.

5) Development of complete damage assessment criteria under axial, shear, flexural and *torsional* interactions

The damage assessment criteria developed in this study has not considered torsional damage and failure of RC frames subject to progressive collapse. In view of complex force conditions of progressive collapse, it is worthy to develop complete damage assessment criteria which can quantify and identify combined

damages and failures under axial, shear, flexural and torsional interactions.

6) Use of advanced XFEM technique

The present beam element formulation is based on continuous displacement field assumption within and between the elements, while actual structural damages may cause discontinuous displacements when plastic hinge, cracking or disconnection of cross-sections occur during large deformation. Such errors that result from standard finite element formulation are not treated in the current computational program. Therefore, in the future work, advanced extended finite element methods (XFEM) can be applied to progressive collapse analysis of RC frames, which offer particular advantages to model non-smooth deformations in progressive collapse.

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APPENDIX A: EXAMPLES FOR IDENTIFICATION OF COLLAPSED SUBSTRUCTURES

To demonstrate the specially designed algorithm for identification of collapsed substructures, simple examples are used and illustrated as follows. To give a clear description of this identification algorithm, a number of terminologies and notations are used and summarized in Table A.1.

One direct collapse (*collapse 1*) and one rotational collapse (*collapse 2*) of substructures are included in the dash line boxes shown in Figure 5.9. The proposed searching algorithm (in Box A.1) is used to identify these two collapse scenarios. First of all, Step (i) of Box A.1 is carried out to obtain the starting joint set $Set_{st} = (12, 16, 22, 29)$ corresponding to the top joint numbers of removed columns in the system. Thus, the search starts from the first starting joint $J_{st} = 12$ (Step (ii)). Then only one internal joint 12 and its corresponding removed edges E42 and E43 are identified for *collapse 1* by Steps (iii), (iv) and (vi), due to the complete loss of all boundary restraints of the joint 12 ($C_{str} = 0$). Now the search for the collapsed substructure with starting joint 12 is finished. Then a new search kicked in as the following for the next starting joint $J_{st} = 16$ by Step (ix).

From Step (iii), the internal joint set $Set_{in} = (16)$ and the current internal joint $J_{in}^{crnt} = 16$. Carrying out Steps (iv) and (v), the constraint C_{str} (the number of boundary restraints of the substructure corresponding to Set_{in}) for the substructure involving joint 16 is 3 which is greater than 1. Thus, the adjacent joint set Set_{adj} for $J_{in}^{crnt} = 16$ is determined by Step (x): $Set_{adj} = (10, 17, 22)$. Then the degree D_{gr} , which is defined to identify new internal joint from adjacent joint set, is calculated as: $D_{gr}(10) = 1, D_{gr}(17) = 2$ and $D_{gr}(22) = 3$. The adjacent joint

with the highest degree has the priority to be the next internal joint based on its loss of restraints (the number of fractured edges) and its compactness to Set_{in} (the number of continuous edges joining to the internal joints). Hence, we obtain $J_{in}^{crnt} = 22$ and $Set_{in} = (16,22)$ by Step (xii). Again, recalculate C_{str} for the substructure corresponding to the current Set_{in} , the value is 3 which is still greater than 1 (Step (v)). Thus, search for the next internal joint by Steps (x), (xi) and (xii) and continues subsequent rounds of search by Steps (iv)→(v)→(x)→(xi)→(xii)→(iv). Finally, as Joint 29 is identified and assigned into Set_{in} , the value of C_{str} for the substructure corresponding to $Set_{in} = (16,22,23,17,18,24,30,29)$ has attained 1 (Steps (iv), (vii)). Meanwhile, flexural failure has been found in the surviving edge E35 (Step (viii)). Therefore, rotational collapse of this substructure (*collapse 2*) is identified, involving the internal joints 16, 22, 23, 17, 18, 24, 30 and 29. This one-by-one search process can be shown as the dash arrows in Figure 5.9. As the remaining starting joints (Joints 22 and 29) have been removed in collapse 2, there is no new starting joint in the system. Therefore, the searching procedure ends by Step (ix).

Table A.1 Terminologies and notations used

No	Term	Used in search of partial collapsed substructures	Notation
1	A continuous edge	An edge is continuous if no element is removed in that edge.	E_{cont}
2	Adjacent	Two joints are adjacent if there is a continuous edge (E_{cont}) joining them. An edge is adjacent to a joint if it connects to this joint and is continuous.	adj
3	A fractured edge	An edge is fractured if there is at least an element removed in the edge.	E_{fr}
4	A starting joint	A joint below which the column has been removed (A starting joint will be selected for search of a potentially collapsed substructure).	J_{st}
5	Starting joint set	The set of joint that includes all the starting joints in model system.	Set_{st}
6	An internal joint	A joint that has been included in a potentially collapsed substructure.	J_{in}
7	Internal joint set	The set of joint that includes all the internal joints.	Set_{in}
8	An internal edge	An edge that connects two internal joints.	E_{in}
9	Constraint	The total number of adjacent edges of all the internal joints; these edges do not belong to internal edges.(constraint is denoted as an integer)	C_{str}
10	Degree of a joint	Summation of the number of fractured edges connecting to this joint plus the number of the continuous edges joining this joint and an internal joint (J_{in}).(degree is denoted as an integer)	D_{gr}
11	Current internal joint	The first current internal joint is the starting joint, otherwise it is the joint with the highest degree among all the joints in adjacent joint set (Set_{adj}).	$J_{\text{in}}^{\text{crnt}}$
12	An adjacent joint	A joint that is adjacent to the current internal joint but not belongs to internal joint set (Set_{in}).	J_{adj}
13	Adjacent joint set	The set of joint that includes all the adjacent joints.	Set_{adj}

- (i) Scan all existing joints, assign the joints with loss of the lower column into *starting joint set* (Set_{st}).
- (ii) Start with the first *starting joint* (J_{st}) based on ascending order of joint number.
- (iii) Assign J_{st} into *internal joint set* (Set_{in}) and set it as the first *current internal joint* (J_{in}^{crnt}).
- (iv) Calculate *constraint* (C_{str}) for checking the substructure that involves all the joints in Set_{in} .
- (v) If $C_{str} > 1$, go to step (x).
- (vi) If $C_{str} = 0 \rightarrow$ direct collapse occurs: identify and remove corresponding elements; go to step (ix).
- (vii) If $C_{str} = 1$, identify the unique *continuous edge* E_{cont} and check whether flexural failure occurs in this edge or not.
- (viii) If flexural failure occurs $\rightarrow$ rotating collapse appears: identify and remove corresponding elements; go to step (ix). Else go to step (x).
- (ix) If flexural failure occurs $\rightarrow$ rotating collapse appears: identify and remove corresponding elements; go to step (ix). Else go to step (x).
- (x) Scan for the next J_{st} from Set_{st} : For a new J_{st} go to step (iii), otherwise quit this procedure.
- (xi) Identify each *adjacent joint* (J_{adj}) of J_{in}^{crnt} and assign them into *adjacent joint set* (Set_{adj}).
- (xii) Calculate *degree* (D_{gr}) for each J_{adj} in Set_{adj} .
- (xiii) Identify the joint with the highest D_{gr} in Set_{adj} , assign it into Set_{in} , and reset J_{in}^{crnt} with it; go to (iv).

Box A.1 Identification of collapsed substructures

APPENDIX B: NONLINEAR SCALAR EQUATIONS FOR DETERMINATION OF $\mathbf{R}_0$

Let

$$\mathbf{X} = \{X_x \quad X_y \quad X_z\}^T \quad (\text{B.1})$$

$$\delta\boldsymbol{\theta} = \{\delta\theta_x \quad \delta\theta_y \quad \delta\theta_z\}^T \quad (\text{B.2})$$

$$\mathbf{X} + \mathbf{u} = \mathbf{X}^R = \{X_x^R \quad X_y^R \quad X_z^R\}^T \quad (\text{B.3})$$

From Equation 6.17 and Equation B.3, one obtains

$$\begin{aligned} & \mathbf{R}_0(\mathbf{X} + \mathbf{u}) \\ &= \begin{pmatrix} \frac{(1 + \omega_x^2 - \omega_y^2 - \omega_z^2)X_x^R + 2(\omega_x\omega_y - \omega_z)X_y^R + 2(\omega_x\omega_z + \omega_y)X_z^R}{1 + \omega_x^2 + \omega_y^2 + \omega_z^2} \\ \frac{2(\omega_x\omega_y + \omega_z)X_x^R + (1 - \omega_x^2 + \omega_y^2 - \omega_z^2)X_y^R + 2(\omega_y\omega_z - \omega_x)X_z^R}{1 + \omega_x^2 + \omega_y^2 + \omega_z^2} \\ \frac{2(\omega_x\omega_z - \omega_y)X_x^R + 2(\omega_y\omega_z + \omega_x)X_y^R + (1 - \omega_x^2 - \omega_y^2 + \omega_z^2)X_z^R}{1 + \omega_x^2 + \omega_y^2 + \omega_z^2} \end{pmatrix} \\ &= \{a_x \quad a_y \quad a_z\}^T \end{aligned} \quad (\text{B.4})$$

Thus we have

$$\begin{aligned} & \delta\boldsymbol{\theta} \times \mathbf{R}_0(\mathbf{X} + \mathbf{u}) \\ &= \{\delta\theta_y a_z - \delta\theta_z a_y \quad \delta\theta_z a_x - \delta\theta_x a_z \quad \delta\theta_x a_y - \delta\theta_y a_x\}^T \end{aligned} \quad (\text{B.5})$$

and

$$\begin{aligned}
& \int \mathbf{X}^T [\delta \boldsymbol{\theta} \times \mathbf{R}_0 (\mathbf{X} + \mathbf{u})] dV \\
&= \int [X_x (\delta \theta_y a_z - \delta \theta_z a_y) + X_y (\delta \theta_z a_x - \delta \theta_x a_z) + X_z (\delta \theta_x a_y - \delta \theta_y a_x)] dV \\
&= \int (X_z a_y - X_y a_z) dV \delta \theta_x + \int (X_x a_z - X_z a_x) dV \delta \theta_y + \int (X_y a_x - X_x a_y) dV \delta \theta_z
\end{aligned} \tag{B.6}$$

From Equation 6.21, $\int \mathbf{X}^T [\delta \boldsymbol{\theta} \times \mathbf{R}_0 (\mathbf{X} + \mathbf{u})] dV = 0$. Hence

$$\left. \begin{aligned}
& \int (X_z a_y - X_y a_z) dV = 0 \\
& \int (X_x a_z - X_z a_x) dV = 0 \\
& \int (X_y a_x - X_x a_y) dV = 0
\end{aligned} \right\} \tag{B.7}$$

Equation B.7 provides three nonlinear scalar equations which include three unknown parameters ω_x , ω_y and ω_z to determine $\mathbf{R}_0$.

APPENDIX C: PUBLICATIONS BASED ON THIS THESIS TO BE PUBLISHED

J.Weng, C. K. Lee and K. H. Tan (2015). Damage Assessment for Reinforced Concrete Frames Subject to Progressive Collapse, *the 5th International Conference on Design and Analysis of Protective Structures, Singapore*. (Runner-up of the best paper award)

J.Weng, K. H. Tan and C. K. Lee. (2015). On Nonlinear Progressive Collapse Analysis Using Superelement with Large Rotation Correction, *the 2nd Australasian Conference on Computational Mechanics, Brisbane*.

J.Weng, C. K. Lee and K. H. Tan (2016). Damage Assessment for Reinforced Concrete Frames Subject to Progressive Collapse, *Engineering Structures*, In press.

J.Weng, K. H. Tan and C. K. Lee (2016). Modeling Progressive Collapse of 2D Reinforced Concrete Frames Subject to Column Removal Scenario, Submitted.

J.Weng, K. H. Tan and C. K. Lee (2016). Adaptive Superelement Modeling for Progressive Collapse Analysis of Reinforced Concrete Frames, Submitted.