Signal processing for cooperative cognitive radio networks

Rabiee, Ramtin

2017

Rabiee, R. (2017). Signal processing for cooperative cognitive radio networks. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/69614>

<https://doi.org/10.32657/10356/69614>

SIGNAL PROCESSING FOR COOPERATIVE COGNITIVE RADIO NETWORKS

RAMTIN RABIEE

SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

**A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy**

2017

Dedicated to
my parents
and my beloved wife
Marjan

In memory of
my father-in-law and my brothers

Acknowledgments

I would like to express my sincere gratitude to my supervisor, Prof. Li Kwok Hung, whose indispensable advice has guided me to achieve this important point. This work would not have been possible without his kind guidance and continuous support. His sophisticated knowledge, professional attitude and kind personality have always inspired me to follow his manner in my future professional pursuits.

My special thanks to my nice friends Dr. Amir Shahroudy, Dr. Mehrnaz Afshang, Dr. Zahra Madadi Ardekani, Seyed Ziaeddin Alborzi and our supportive lab technician Mr. Ng Teng Kwee for giving me their kind help and technical support for running the simulations.

I would like to convey my heartfelt thanks to my family in Iran, to my parents who have always supported and encouraged me to be a successful man. Definitely, I would not have gained any of the achievements in my life without all that they have done for me. Special thanks to my lovely sister, Rabe'e, my sister-in-law, Bahar, and my brother-in-law's family, Mehrdad and Gita, for their love, and moral support.

Last but not the least, I would like to express my deepest thanks to my beloved wife, Marjan, who stood beside me all along the way. I remember the day an Italian guy told me that student life is something between a poor and a normal life. He was absolutely right. Student life is quiet challenging, especially for married people in a foreign country, but she has kept trying to smooth it for us. Definitely, it would have been difficult to go through the way without her love, patience, support, and encouragement.

At the end, I would like to dedicate this thesis to the memory of my father-in-law, Dr. Ali Hajiaghamohammadi, who always encouraged me to pursue my PhD studies, and sadly, we lost him during our stay in Singapore, and to the memory of my brothers, Ramin and Rozbeh.

Contents

Acknowledgments	i
Table of Contents	iii
List of Figures	vii
List of Tables	xi
List of Abbreviations	xiii
List of Symbols	xv
Abstract	xvii
1 Introduction	1
1.1 Cognitive radio	1
1.2 Spectrum sensing in the CRN	2
1.3 Motivation	4
1.4 Main contributions	5
1.5 Organization of the thesis	6
2 Related works and existing methods	7
2.1 Literature survey	7
2.2 Sensing improvement methods	13
2.2.1 Improved energy detector	14
2.2.2 Comparison schemes	15
2.2.2.1 Single-threshold	15
2.2.2.2 Double-threshold	16
2.2.3 Cooperative spectrum sensing	17
2.2.3.1 Single-threshold based CSS	18
2.2.3.2 Double-threshold based CSS	18

3	Deterministic-modeled secondary users in a low-traffic network	21
3.1	Detection performance by minimum sensing time	21
3.1.1	Single-threshold based analysis	22
3.1.2	Double-threshold based analysis	28
3.2	Transmission throughput	36
3.2.1	Real AWGN sensing channels	39
3.2.2	Rayleigh-faded sensing channels	44
3.3	Conclusion	50
4	Deterministic-modeled secondary users in a medium-traffic network	51
4.1	Distribution of observed energy level	52
4.1.1	CED over AWGN sensing channels	52
4.1.2	IED over Rayleigh-faded sensing channels	58
4.2	Detection performance	66
4.2.1	Probabilities formulation	66
4.2.2	Numerical results by CED over AWGN sensing channels	68
4.2.3	Numerical results by IED over Rayleigh-faded sensing channels	72
4.3	Transmission throughput	76
4.3.1	Probabilities formulation	78
4.3.2	Throughput formulation	79
4.3.3	Numerical results	81
4.4	Conclusion	85
5	Stochastic-modeled secondary users in a medium-traffic network	87
5.1	CSS with Extra sensing	90
5.1.1	Network characteristics	91
5.1.1.1	Medium-traffic primary network	91
5.1.1.2	Secondary network with random users	93
5.1.2	Detection formulation	94
5.1.2.1	Interference	96
5.1.2.2	Detection probabilities of the main sensing part	97
5.1.2.3	Detection probabilities of the extra sensing part	98
5.1.3	System performance	101
5.1.4	Numerical results	103
5.2	1-Persistent based CSS	108
5.2.1	Network characteristics	110
5.2.1.1	Medium-traffic primary network	110
5.2.1.2	Secondary network with random users	111
5.2.2	Detection formulation	112

5.2.2.1	Local detection probabilities of the main sensing part . . .	113
5.2.2.2	Local detection probabilities of the Cx sensing parts . . .	113
5.2.2.3	Local detection probabilities of the Ex sensing part	114
5.2.2.4	Cooperative spectrum sensing	115
5.2.3	System performance	115
5.2.3.1	Total throughput	115
5.2.3.2	Increased factor of sensing attempts	117
5.2.4	Numerical results	119
5.3	Conclusion	125
6	Conclusion and future work	127
6.1	Conclusion	127
6.2	Future work	129
	Appendices	131
	Appendix A Mean and variance of the IED's output over Rayleigh fading	132
	Appendix B Deterministic shape parameters when $\gamma_I \leq -5$ dB	134
	Appendix C Conditional local probability with random scale parameter	136
	Author's Publications	139
	References	141

List of Figures

2.1	The structure of secondary network's operations during PU's idle time . . .	15
2.2	Different comparison criteria of observed energy in the SU	16
2.3	Q_d vs. Q_f with $\bar{\gamma} = 5$ dB and $P_e = 1\%$ for the OR, AND and majority fusion rules in different numbers of the cooperative users	19
3.1	The total error rate vs. threshold with $p = 2$, $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for different fusion rules	26
3.2	The total error rate vs. threshold in the different p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the ISED	26
3.3	The minimum total error rate vs. p in the optimum threshold with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the ISED	27
3.4	The minimum total error rate vs. threshold in the optimum p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the ISED	27
3.5	Second order derivative of P_{err} w.r.t p vs IDED parameters with $\bar{\gamma} = 5$ dB	31
3.6	The minimum total error rate vs. p in the optimum $\Delta\lambda$ with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the proposed IDED	34
3.7	The minimum total error rate vs. $\Delta\lambda$ in the optimum p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the proposed IDED	34
3.8	The total error rate vs. $\Delta\lambda$ in different p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the proposed IDED	35
3.9	Total probabilities (Q_m , Q_f) vs. $\Delta\lambda$ with $p = 1.3$, $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the proposed IDED	35
3.10	Maximum achievable throughput vs. number of samples under perfect reporting channels ($P_e = 0$) with $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule	40
3.11	Maximum achievable throughput vs. number of samples over imperfect reporting channels as compared with the perfect one with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule	41

3.12	Optimum probabilities of different EDs vs. N over imperfect reporting channels with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule . . .	42
3.13	Normalized maximum throughput vs. average SNR over imperfect reporting channels with $P_e = 1\%$ and $K = 10$ for the OR fusion rule	43
3.14	CDF of N -sample based IED output	45
3.15	Mean square error of approximate distribution of N -sample based IED vs. p over different SNRs	46
3.16	Maximum achievable throughput vs. number of samples under perfect reporting channels ($P_e = 0$) with $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule	48
3.17	Maximum achievable throughput vs. number of samples over imperfect reporting channels as compared with the perfect one with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule	48
3.18	Optimum probabilities of different EDs vs. N over imperfect reporting channels with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule . . .	49
3.19	Normalized maximum throughput vs. average SNR over imperfect reporting channels with $P_e = 1\%$ and $K = 10$ for the OR fusion rule	50
4.1	CDF and PDF of CED outputs with $\bar{\gamma} = 0$ dB and $N = 100$ over AWGN sensing channel with medium-traffic primary network	55
4.2	CDF and PDF of CED outputs with $\bar{\gamma} = 10$ dB and $N = 100$ over AWGN sensing channel with medium-traffic primary network	55
4.3	MSE of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using CED over AWGN sensing channel with medium-traffic primary network	56
4.4	MSE of approximate distributions vs. average SNR with $N = 100$, using CED over AWGN sensing channel with medium-traffic primary network . .	56
4.5	KLD of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using CED over AWGN sensing channel with medium-traffic primary network	57
4.6	KLD of approximate distributions vs. average SNR with $N = 100$, using CED over AWGN sensing channel with medium-traffic primary network . .	57
4.7	CDF and PDF of IED outputs with $p = 2$, $\bar{\gamma} = 0$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network . . .	60
4.8	CDF and PDF of IED outputs with $p = 3$, $\bar{\gamma} = 0$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network . . .	60
4.9	CDF and PDF of IED outputs with $p = 2$, $\bar{\gamma} = 10$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network . . .	61

4.10	CDF and PDF of IED outputs with $p = 3$, $\bar{\gamma} = 10$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network . . .	61
4.11	MSE of approximate distributions vs. p with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network . . .	62
4.12	MSE of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network	62
4.13	MSE of approximate distributions vs. average SNR with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network	63
4.14	KLD of approximate distributions vs. p with $N = 100$ and $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network	63
4.15	KLD of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network	64
4.16	KLD of approximate distributions vs. average SNR with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network	65
4.17	Minimum total error rate vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ (i.e., $\mu_i = \mu_b$) by the OR fusion rule	70
4.18	Minimum total error rate vs. duty cycle with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_e = 1\%$ by the OR fusion rule	70
4.19	Minimum total error rate vs. P_e with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_b = 0.5$ by the OR fusion rule	71
4.20	Minimum total error rate vs. average SNR with $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ by the OR fusion rule	71
4.21	Minimum total error rate vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ by the OR fusion rule	74
4.22	Minimum total error rate vs. duty cycle with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $\mu_T = 0.1$ s by the OR fusion rule	74
4.23	Minimum total error rate vs. P_e with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $\mu_i = 50$ ms and $P_b = 0.5$ by the OR fusion rule	75
4.24	Minimum total error rate vs. average SNR with $N = 100$, $K = 10$, $P_e = 1\%$, $\mu_i = 50$ ms and $P_b = 0.5$ by the OR fusion rule	75
4.25	Minimum total error rate over different P_e vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_b = 0.5$ (i.e., $\mu_i = \mu_b$) for different fusion rules	77

4.26	Maximum normalized throughput vs. mean idle time with $\bar{\gamma} = 0$ dB, $K = 10$ and $P_e = 10\%$ for different duty cycles by the majority fusion rule . . .	83
4.27	Maximum normalized throughput vs. duty cycle with $\bar{\gamma} = 0$ dB, $K = 10$ and $P_e = 10\%$ for different mean idle times by the majority fusion rule . .	83
4.28	Maximum normalized throughput vs. P_e with $\bar{\gamma} = 0$ dB and $K = 10$ for different duty cycles and mean idle times by the majority fusion rule	84
4.29	Maximum normalized throughput vs. average SNR with $K = 10$, $P_b = 0.5$ and $P_e = 10\%$ for different mean idle times by the majority fusion rule . .	84
5.1	A CRN with the stochastic-modeled secondary network	88
5.2	Positioning in the stochastic-modeled secondary network	89
5.3	Frame structure of the SU's tasks in addition to the possible status changing of the PU (solid lines for $u = 0$ and dashed lines for $u = 1$) when the extra sensing part is utilized	91
5.4	Average power of aggregate interference caused by SUs to PU vs. total intensity of the SUs with $R_s = 10$ m and $P_{SU} = 15$ dBm	103
5.5	Maximum normalized throughput vs. total intensity of SUs	104
5.6	Throughput changes vs. total intensity of SUs	104
5.7	Throughput changes vs. β with $\lambda_{SU} = 0.05$	105
5.8	Throughput changes vs. number of sensing samples with $\lambda_{SU} = 0.05$ and $\beta = 0.9$	107
5.9	The algorithm used by SUs for sensing and transmission tasks, based on the 1-persistent spectrum sensing method	109
5.10	Maximum normalized throughput vs. total intensity of SUs	119
5.11	Throughput changes vs. total intensity of SUs	120
5.12	Increased factor of sensing attempts vs. total intensity of SUs	120
5.13	Maximum normalized throughput vs. β with $\lambda_{SU} = 5\%$	121
5.14	Throughput changes vs. β with $\lambda_{SU} = 5\%$	121
5.15	Increased factor of sensing attempts vs. β with $\lambda_{SU} = 5\%$	122
5.16	Maximum normalized throughput vs. β with $\lambda_{SU} = 25\%$	123
5.17	Throughput changes vs. β with $\lambda_{SU} = 25\%$	123
5.18	Increased factor of sensing attempts vs. β with $\lambda_{SU} = 25\%$	124

List of Tables

4.1	Means and variances of different cases with medium-traffic primary network over AWGN sensing channel, using CED	54
4.2	Means and variances of different cases with medium-traffic primary network over Rayleigh-faded sensing channel, using IED	59
5.1	Status of the primary user during the frame time	92
5.2	Status of the primary user during the extra sensing part	92
5.3	Means and variances of different cases for both main and extra sensing parts	95
5.4	Transmission probabilities of the PU over different situations	102
5.5	Means and variances of different cases for the ‘ $\mathfrak{S}$ ’ sensing part	112

List of Abbreviations

AWGN	Additive White Gaussian Noise
BS	Base Station
BSC	Binary Symmetric Channel
CDED	Conventional Double-threshold Energy Detector
CDF	Cumulative Distribution Function
CED	Conventional Energy Detector
CFD	Cyclostationary Feature Detector
CLT	Central Limit Theorem
CR	Cognitive Radio
CRN	Cognitive Radio Network
CSA	Concurrent Spectrum Access
CSED	Conventional Single-threshold Energy Detector
CSS	Cooperative Spectrum Sensing
CUSUM	Cumulative Sum
D2D	Device-to-Device
ED	Energy Detector
EGK	Extended Generalized- K
FC	Fusion Center
FCC	Federal Communications Commission
IDED	Improved Double-threshold Energy Detector
IED	Improved Energy Detector
ISED	Improved Single-threshold Energy Detector
KLD	Kullback-Leibler Divergence
LCFS	Least Cost First Serve
LHS	Left-Hand Side
LTE-A	Long Term Evolution Advanced
MAC	Medium Access Control
MAP	Maximum A Posteriori
ML	Maximum Likelihood
MSE	Mean Square Error

NP	Neyman-Pearson
NPRM	Notice of Proposed Rule Making
OSA	Opportunistic Spectrum Access
PBS	Primary Base Station
PCAST	President's Council of Advisors on Science and Technology
PDF	Probability Density Function
PLE	Path-Loss Exponent
PLG	Path-Loss Gain
PMF	Probability Mass Function
PP	Point Process
PPP	Poisson Point Process
ProSe	Proximity Services
PU	Primary User
ROC	Receiver Operating Characteristic
SBS	Secondary Base Station
SDR	Software-Defined Radio
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SU	Secondary User
TDMA	Time Division Multiple Access
WRAN	Wireless Regional Area Network

List of Symbols

K	Number of cooperative users
K_s	Number of sensing SUs during the extra sensing part
K_t	Number of transmitting SUs during the extra sensing part
N	Number of sensing samples
L	Number of frame samples
P_e	Error probability of BSC
P_{err}	Total error rate
P_f	Local probability of false alarm
P_{d_0}	Local probability of detecting H_0
P_d	Local probability of detection
P_m	Local probability of missed detection
Δ_0	local probability of no decision under H_0
Δ_1	local probability of no decision under H_1
Q_f	Total probability of false alarm
Q_{d_0}	Total probability of detecting H_0
Q_d	Total probability of detection
Q_m	Total probability of missed detection
σ_s^2	Average received power of the primary signal at the receiver
σ_n^2	Noise power
σ_h^2	Variance of faded sensing channel
$\sigma_{h_s}^2$	Variance of faded interference channel
$\bar{\gamma}$	Average signal-to-noise ration
p	IED exponent
T	Frame time
T_s	Sampling time
λ'	Detection threshold of the extra part
λ	Detection threshold in a single-threshold method
λ_1	Lower detection threshold in a double-threshold method
λ_2	Upper detection threshold in a double-threshold method
$\Delta\lambda$	Threshold distance in a double-threshold method

λ_{SU}	Total intensity of SUs
λ_s	Intensity of sensing SUs
λ_t	Intensity of transmitting SUs
λ_i	Rate parameter for the exponential distributed idle PU
λ_b	Rate parameter for the exponential distributed active PU
μ_i	Mean idle time of the PU
μ_b	Mean active (busy) time of the PU
μ_T	Total mean time of the PU
P_i	Probability of the idle PU in the medium-traffic primary network
P_b	Probability of the active PU in the medium-traffic primary network
β	The percentage of the total SUs who do sensing during the extra part
R_s	Radius of the secondary network
P_{PU}	Transmitted power of the primary signal
P_{SU}	Transmitted power of the secondary signal
I_k	Power of interference from k -th transmitting SU
I	Total power of interference from transmitting SUs
r_s	Distance between SBS and SU
r_p	Distance between SBS and PU
d_s	Distance between SU and another SU
d_p	Distance between SU and PU
η	PLE
$\delta_{i,j}$	Kronecker delta function
$\mathcal{L}_x(\cdot)$	Laplace transform of distribution of x
$\gamma(\cdot, \cdot)$	Lower incomplete gamma function
$J_\nu(\cdot)$	ν -th order Bessel function of the first kind

Abstract

In recent decades, cognitive radio has been introduced as a practical solution to solve the frequency scarcity problem by setting up a secondary network besides the primary one through a cognitive radio network (CRN) to increase the utilization rate of licensed frequency bands. There exist many parameters to achieve a reliable and practical CRN and it is necessary to conduct extensive signal processing studies to optimize them. It is worth mentioning that the protection of primary signals, belonging to licensed users, is a crucial requirement in establishing of a CRN. On the other hand, conducting a strict sensing method may severely reduce the chance of a secondary network to utilize a free spectrum channel, where there is no point to using a CRN anymore. Therefore, modeling a spectrum sensing mechanism complying both networks' requirements is a substantial part of the CRN design.

Since spectrum sensing is a sensitive and important part of the CRN design from the viewpoint of signal processing, it has become the matter of interest of advanced research activities. There are many approaches to optimize the detection performance, and it is worthwhile to note that some proposed methods add new parameters to the system and we need to take them into account. For example, cooperative spectrum sensing (CSS), which is suggested to solve the hidden terminal problem, entails two important parameters as the quality of reporting channels (links between secondary users and fusion center) and the number of cooperative users. Moreover, presuming the possible usage of the CRN concepts in new wireless technologies such as device-to-device (D2D) communication over Long Term Evolution Advanced (LTE-A) standard, it motivates us to think beyond the traditional applications of the CRN by considering some other parameters such as randomness in the number of available secondary users and higher traffic rate of the primary users as compared with the wireless regional area network (WRAN - IEEE 802.22).

In this dissertation, we focus on some possible complexities which may be faced when the CSS has been utilized. By considering them step-by-step, we try to reach a more realistic system model which makes our analyzed CRN more feasible. In each step, some performance parameters of the system (such as detection reliability and transmission throughput) would be studied based on several proposed methods and new ideas will be introduced to improve sensing reliability and transmission throughput of the system.

“Sensing channels”, “reporting channels”, “number of cooperative users” and “traffic rate of the primary network” are four important parameters to be focused on in this research. Note that many studies have been done under simple assumptions such as AWGN sensing channel, perfect reporting channels, deterministic number of available sensing users and a low-traffic rate of the primary network. Although they could represent a real-life case, focusing on the desired performance criterion optimization over Rayleigh-faded sensing and erroneous reporting channels is still the primary interest of this research work.

For the first step, we suppose that the primary network has a low traffic rate and the number of available cooperative users is deterministic. We aim to analyze some pre-existing methods such as improved energy detector (IED) or double-threshold comparison scheme in terms of detection performance and transmission throughput when a CSS-based detection scheme has been implemented among the secondary users. We seek to study their efficiency, especially in the case of imperfect reporting channels. Furthermore, to meet both reliability (protection of the primary signal) and spectrum utilization requirements for throughput analysis, the throughput of the secondary network is maximized over some required constraints.

We then extend our analyses to the case of medium-traffic primary networks. Although the analyzed period might be very short, it is most likely that the status of the primary user alters during the duration. This situation affects performance of the system due to higher uncertainty on the activity of the primary user.

In the final stage, by using stochastic geometry concepts, we would like to take the effect of randomness in distribution of cooperative users into account. Since the number of available users may not be fixed in a practical wireless network, a stochastic model is considered for distribution of the secondary users besides prior assumptions. This is more critical when a cooperative sensing scheme has been utilized among the secondary users. We finally propose our method to enhance the total throughput of the system when it suffers from an unstable primary network, due to a medium-traffic rate, and a random density of the secondary users, due to a stochastic-modeled secondary network, at the same time. By defining the expression for possible interference and required formulation, we demonstrate the advantages of our proposed method in terms of maximum achievable total throughput of the system in the introduced CRN model. It is worth mentioning that both reliability and spectrum utilization are implicitly considered by assuming the total throughput of the system as the desired performance parameter.

Chapter 1

Introduction

1.1 Cognitive radio

In the last decade, the spectrum scarcity problem has been considered as an important limiting factor that blocks the growth of wireless communications. This issue became more interesting when scientists found that there are many unused licensed frequency bands in many cases, resulting in the low utilization rate of licensed spectrum. Based on the Federal Communications Commission (FCC) report, 70% of allocated spectrum in the US is not utilized [1]. Another report of Shared Spectrum Company illustrated that the average occupancy over six locations of the US is just 5.2% for the frequency band of 30 MHz to 3 GHz, with the maximum of 13.1% at Republican National Convention of the New York city and the minimum of 1% at the National Radio Astronomy Observatory site in Green Bank [2]. These results show that spectrum reuse can be an applicable scheme to relieve the frequency scarcity problem. How can we do it satisfying some required parameters such as throughput performance, security and interference?

Cognitive radio (CR) was introduced as a practical approach by Mitola in 1999 [3]. The CR can be defined as a software-defined radio (SDR) based smart wireless communication system which reuses the spectrum via required awareness of its operational environment [4–7]. Subsequently, many research activities have been done on the CR concept till the FCC issued a Notice of Proposed Rule Making (NPRM) in 2003, suggesting the CR as an efficient technology which can be suitable for both negotiated and opportunistic spectrum sharing [8]. At first, spectrum sharing was considered as a potential option based on the FCC’s report. Subsequently, it has become a key strategy when the President’s Council of Advisors on Science and Technology (PCAST) issued their spectrum sharing proposal in the middle of 2012 [9]. Wireless regional area network (WRAN)

is a practical example of using CR which utilizes the white spaces in television spectrum bands without interfering with licensed users based on the IEEE 802.22 standard.

In a CR network (CRN), we set up a secondary network besides the primary one using the same frequency spectrum within a region to increase the spectrum utilization rate. Therefore, we have two types of users in a CRN: primary user (PU) and secondary user (SU). PUs have been licensed for a specific frequency band. In other words, PUs have higher priority to use the desired spectrum range, whereas SUs try to reuse licensed frequency bands either opportunistically or simultaneously without interfering with the PU. Consequently, there are two spectrum access methods to implement a CRN. In the first one, SUs occupy the unused frequency bands opportunistically in the case of idle PUs which is called the *opportunistic spectrum access* (OSA) method. On the other hand, the *concurrent spectrum access* (CSA) is another method in which SUs reuse the occupied frequency bands coexisting with active PUs by imposing some constraints on signal power and distance to avoid any excess interference to primary signals [10, 11].

Nowadays, the concept of CRN goes beyond the WRAN and one can see the signs of the CR technology in device-to-device (D2D) communication among cellular users and proximity services (ProSe) in Long Term Evolution Advanced (LTE-A) standard (3GPP Release 12 onwards) [12–16]. For instance, the spectrum reuse in inband underlay (and somehow overlay) D2D communications can be interpreted as a sort of OSA/CSA based CRNs.

1.2 Spectrum sensing in the CRN

A CRN design includes different components such as physical layer signal processing, medium access control (MAC) layer spectrum management and network layer routing control [10]. The key part of physical layer signal processing is spectrum sensing which enables SUs to identify PUs' status. The effectiveness of the CRN (in terms of both primary and secondary networks' throughputs) is directly related to the quality of spectrum sensing. Many works have been published in the field of spectrum sensing under different conditions of the CRN in terms of primary and secondary networks' properties and the quality of sensing channel (the link between SU and PU). In a wireless environment the sensing channel can be treated as either additive white Gaussian noise (AWGN) [17, 18] or faded (such as Rayleigh and Nakagami fading) [19–25].

There exist some challenging issues of achieving reliable spectrum sensing with realistic assumptions as much as possible. *Hidden terminal problem* [26] is one of them in which

an SU is shadowed or affected by severe multipath fading. Since there are more than one SU in a CRN in most cases, *cooperative spectrum sensing* (CSS) has been considered as a practical solution to decrease the effects of this problem [27]. SUs can help each other to have a more reliable decision in a cooperative sensing manner. The CSS is mainly referred to as checking the availability of a desired channel by cooperative users at the same time. However, it might be utilized for detection over different spectrum channels simultaneously to find a free channel in a shorter time and achieve better sensing efficiency [28]. Usually a fusion center (FC) is responsible to collect SUs' detection information to make the final decision based on a predefined fusion rule. The next problem appears when the SUs send their observations to the FC through noisy reporting channels (the link between SU and FC). Although having a perfect (noise-free) reporting channel might be possible [29–43], it is just an ideal assumption for analyzing the efficiency of the CSS. One may say that the perfect reporting channel is not realistic, but if we suppose a single secondary node with multiple antennas, then channels between the center and antennas can be modeled as perfect reporting channels because of low error probability. Nevertheless, the quality of reporting channels for a cooperative detection among more than one SU is not guaranteed to be perfect in a realistic wireless network. An imperfect reporting channel can be modeled as either the binary symmetric channel (BSC) [44] or faded [45–48]. Furthermore, in the CSS, it is very important to choose a suitable detection method involving a sufficient number of cooperative users (K) and an optimum fusion rule to minimize any decision error. It is more critical when the number of available SUs in the network is not deterministic.

Among different signal processing techniques [49], three detection methods have been intensively proposed for spectrum sensing: matched-filter detection, energy detection and cyclostationary feature detection [26, 50]. The inherent periodicity of the received signal with specific modulation type is exploited for detection in a cyclostationary feature detector (CFD). Despite the CFD does not need the transmitter information, its processing time and computational complexity are high. On the other hand, for matched-filter detectors, the exact information of the received signal is required with a high implementation complexity and power consumption while its processing time is low. Moreover, the matched filter is not the optimum spectrum detector in a fast-fading Rayleigh channel where the PU uses pilot signals which are known to the SU and the statistical properties of the fading channel either are known or can be estimated sufficiently accurate at the SU [51]. Although matched-filter is generally capable to maximize received signal-to-noise ratio (SNR) through coherent detection, energy detector (ED) [52–56] is studied widely

in the case of CR systems because of its independency to any prior knowledge of primary signal and sensing channel, and it is an efficient method to detect unknown signals, especially in the presence of random Gaussian noise. Furthermore, its implementation is simple as compared with other methods.

Regardless of the above-mentioned advantages, application of the ED is quite challenging due to its susceptibility to any changes in the system parameters. Albeit many methods have been proposed to dominate this shortage, it is worthwhile to note that some proposed methods add new parameters to the system and we need to take them into account. For instance, the CSS, which is discussed earlier, entails two important parameters as the quality of reporting channels and the number of cooperative users. In many works, the CSS has been applied over error-free reporting channels and/or the number of users are supposed fixed, whereas they are not guaranteed in a realistic case. Note that the amount of information sent by SUs to the FC is another challenging issue to design an ED-based CSS. Although sending total value of the measured energy by the SU (soft information) to the FC increases reliability of the final decision, it imposes more energy and bandwidth overheads to the network. Hence, the soft-decision method may not be desirable in many cases and hard-decision based CSS is considered more efficiently, when each SU makes its local decision and sends a one-bit binary data to the FC.

1.3 Motivation

As it is mentioned earlier, establishing a cooperation among SUs for their spectrum sensing tasks has been proposed to enhance the detection reliability and throughput of the CRN. Nevertheless, any scheme may add some new parameters to the system which should be considered properly. Hence, it motivates us to focus on some important aspects of the ED-based CSS from the perspective of the signal processing such as the quality of reporting channels, the fusion rule, number of cooperative users and their possible interference in addition to the existing system parameters such as the quality of sensing channel and stability of the PU's activity in the region. The possible usage of the CRN concepts in new wireless technologies, such as D2D communication in the cellular network, is another motivation to consider some other parameters such as randomness in the number of available cooperative users and higher traffic rate of the primary network. On the other hand, it is quite important to be conscious about the performance parameters to be analyzed to define the feasibility of the proposed methods. For instance, considering the

detection performance without taking the effects of the desired methods on the throughput of the system is pointless.

In this work, we mainly focus on the faded sensing channels (using Rayleigh fading) and imperfect reporting channels, which may be considered to be a realistic wireless network with distributed cooperative users in a region. Moreover, by considering the contingency of PU's status changing and randomness in number of available cooperative users, we try to model a more practical situation for a CSS-based CRN to do our performance analysis with higher competency. These conditions together degrade the performance of the CRN significantly due to higher uncertainty on the available SUs for doing CSS and status of the PU's activity in the region. Therefore, finding a method to mitigate this situation and enhance the system performance is another matter of interest in this research work.

1.4 Main contributions

In this thesis, the objective is to model and analyze the possible issues of the modern wireless networks with utilizing the CSS among SUs from the viewpoint of signal processing. Subsequently, we aim to propose some methods to abate their negative effects on throughput of the system. Therefore, the main contributions of the current research work can be summarized as follows:

1. By elucidating the efficiency of some existing methods for the CSS enhancement, improved double-threshold energy detector (IDED) is introduced as a promising method to improve the performance of the CRN in terms of both detection reliability and transmission throughput of the secondary network when reporting channels are erroneous over a CSS-based sensing scheme.
2. Considering the possible status changing of the PU during both sensing and transmission phases of the SUs is another contribution of this work when SUs are collaborating in a CSS manner over imperfect reporting channels. This part includes finding a proper approximate distribution for observed energy levels of the received signal at sensing SUs as well as analyzing the effect of the medium-traffic rate primary network on both detection performance and transmission throughput of the secondary network. We show that the gamma approximation works better, especially for throughput analysis in which it is required to consider all possible values of number of sensing samples (as low as only one sample). To evaluate the accuracy of the approximate distributions, both mean square error (MSE) and Kullback-Leibler divergence (KLD) of approximate probability functions are analyzed.

3. Using Poisson point process (PPP), a stochastic model is assumed for the distribution of SUs within a secondary network over the medium-traffic primary network. We aim to introduce a CRN model which is quite possible in new wireless networks. To improve the proficiency of the system, two spectrum sensing methods are proposed as ‘CSS with Extra sensing’ and ‘1-Persistent based CSS’. The required interference expression, probabilities and throughput formulations are derived for evaluating the ability of the proposed methods to enhance the performance of the CRN in this particular system model. We show how these methods can improve the total throughput of the system in a CRN with a high level of uncertainty in the conditions of the users at both networks. We also evaluate the possible cost which might be paid to achieve the throughput enhancement. For example, the amount of energy consumption overhead is to be analyzed in the case of 1-Persistent bases CSS. Therefore, one may consider the functionality of our proposed method based on the application.

1.5 Organization of the thesis

For the first step, we would talk about related works and explain some existing methods to improve the proficiency of the ED in Chapter 2. By considering a system model with deterministic number of available SUs over a low-traffic rate primary network, as many current works, we introduce IDED method in Chapter 3 and analyze its proficiency to enhance the system performance, subsequently. We then extend our work to a system model with medium-traffic rate primary network in Chapter 4. As the last stage, using the concept of stochastic geometry, we define a CRN with stochastic cooperative users distributed in the region of a medium-traffic primary network in Chapter 5. By proposing our methods, we will show that how effective they can improve the total throughput of the system in this particular system model. Finally, we conclude our recent research activities and present some open areas along this way as future work in Chapter 6.

Chapter 2

Related works and existing methods

2.1 Literature survey

As discussed in Chapter 1, ED-based detectors have been studied widely in the current works in which many schemes have been suggested (such as the CSS) to improve the quality of the spectrum sensing and system performance.

In [57], the author introduced another enhancement to the ED called *improved energy detector* (IED) by changing the power operation of signal amplitudes from two in the *conventional energy detector* (CED) to an arbitrary value such that the optimum exponent in different channel conditions is not necessarily “two” in terms of the detection performance. In [18–20], the authors analyzed the IED based sensing in a CSS scheme. The important difference of their utilized IED with [57] is that they used just one sample for each sensing attempt. They first considered a simple AWGN sensing channel with perfect reporting channels in [18] and then extended it to a Rayleigh-faded sensing channel in the case of imperfect reporting channels [19]. Finally, they equipped SUs with multiple antennas to achieve more reliable local decisions [20]. The authors in [58] added a censoring scheme to the multiple-antenna method of [20] to enhance the reliability of received local decisions in the FC and analyzed it under the majority fusion rule.

Another improvement in the ED with the double-threshold method was introduced in [59], in which SUs compare the estimated energy level of the received signal with two thresholds and avoid sending any decision when it lies between them due to the uncertainty in the presence or absence of the PU. This method improves the efficiency by reducing the average number of bits sent by SUs to the FC. The authors in [34] introduced a fusion rule, called ‘ n ratio’ logic, and showed that this logic outperforms the OR fusion rule in the double-threshold based ED in terms of detection reliability while the network

overhead is remained low. They represented that there is an optimum n under different channel conditions to achieve a minimum detection error rate. References [29–31] used the double-threshold comparison method with the difference that SUs send the statistical information of estimated energy when it lies between two thresholds in order to solve the *sensing failure problem* (the situation in which all SUs do not send any decision to the FC) at the cost of increased overhead. Furthermore, a new softened hard combination of observed energy was proposed in [30] to reduce the additional overhead. The main idea to use the double-threshold method in [60] is to identify the presence of PUs in two different operating areas (regions). Thus, SUs can choose either a full or partial access mode based on PU's operating region. By using the double-threshold scheme besides the IED method, a two-stage detection scheme was considered in [61] to improve the detection reliability of the system as compared with the conventional double-threshold energy detector (CDED). They analyzed the sensing performance of an IDDED-based system over the AWGN sensing channels, while the approximate probability density function (PDF) of the N -sample based IED was employed as derived in [57]. However, the effect of the number of sensing samples (N) is not analyzed. Since the sensing duration is related to N , the throughput should also be considered and evaluated. In [36], the authors proposed another two-stage detector to increase the chance of obtaining a reliable final decision. At the first stage, a single-threshold detector evaluates the presence of the PU. If the PU is found idle, the system uses a double-threshold detector in the second stage to ensure that the decision of the first stage is correct. Nonetheless, the sensing duration of the proposed method might be higher than that of a single-stage based double-threshold scheme. The authors in [38–40] applied the multi-threshold scheme in the CSS with perfect reporting channels. They achieved better detection performance by using more thresholds. Note that a higher number of thresholds imposes extra overhead to the system in terms of both bandwidth and energy consumption. There are also some works in which CFD is used besides ED as a two-stage detector, at the expense of system complexity with higher power and processing time consumption of the CFD [62–64]. In [62], it is shown that the ED has a SNR wall (minimum acceptable SNR) to achieve the desired detection accuracy when there is an uncertainty in estimation of the noise power and they have tried to use CFD just in the case of low SNR to reduce source consumption of the whole two-stage detector. The authors in [62, 63] considered a non-cooperative scheme while [64] proposed a CSS-based two-stage method. However, the reporting channels are assumed perfect in [64].

In all of above-mentioned works, the authors supposed a low-traffic primary network. That is, the status of the PU has been assumed to be fixed during the sensing time, while it

is an oversimplification in many cases. We should mention that it is acceptable whenever the mean holding (either idle or active) time of the PU is high enough as compared with the test duration. Nevertheless, the reliability of the ED would be degraded if the PU changes its status during the sensing duration. The authors in [65–71] considered a higher traffic rate for the primary network where PUs either arrive at or depart from the network during the sensing time. It is called the medium-traffic rate where the status changing happens only once (either one arrival or one departure) within the watching time (either sensing period or whole frame time). An improvement is introduced in [66] by using the maximum likelihood (ML) method in the hypothesis testing step, and by considering the Neyman-Pearson (NP) criterion they showed that it outperforms the CED in both gamma and Gaussian distributed ED's outputs. In [67], the authors analyzed the sensing accuracy when there are multiple PUs in different arrival and departure rates and different numbers of PUs. They showed that the sensing performance is degraded when the arrival and departure rates increase (idle and active durations decrease) and this degradation is reduced when the number of PUs increases. Finally, the authors in [70] proposed a detection method in which a side detector helps the main one by producing a rough estimation of PU's presence and its appearance time to discard the received noise power in the absence of PU, resulting in a more reliable decision.

There are different performance criteria which are considered as optimization problems in the literature. In some of them the authors have tried to decrease total error rate (P_{err}) which is the summation of the weighted total probability of missed detection (Q_m) and the weighted total probability of false alarm (Q_f). Some studies maximized the total probability of detection (Q_d) in a fixed Q_f based on the NP criterion. In [42–44], the authors analyzed the detection diversity of the CSS where it is improved by using a transmit and relay based CSS in [44]. One of the important performance criteria in a CRN is transmission throughput of SUs, which is affected by sensing time. A comprehensive study of sensing-throughput trade-off has been done in [72]. Firstly, the authors analyzed it for the single SU case, and continued by considering a multi-slot sensing manner in which the sensing slot in each frame is split into multiple discontinuous mini-slots. Finally, it was extended to the multiple SUs through the CSS scheme with the perfect reporting channels. In [73], a time division multiple access (TDMA) based CSS has been assumed. By considering reporting duration in which all SUs send their local decisions to the FC by TDMA, the number of cooperative SUs was optimized to achieve the maximum CRN's throughput subject to constraints on energy consumption of the each SU and the total probability of detection. The authors in [64] analyzed the throughput of the

CSS over Rayleigh-faded sensing channels. They considered a multiple-channel primary network with identical statistics in which there always exist at least one available channel for SUs. It means that SUs switch from one channel to another, once the PU becomes active in current channel. The perfect reporting channels are supposed to be embedded to the same transmission channel. Thus, they showed that there is a trade-off between the throughput and the overhead caused due to the number of cooperative users, and subsequently optimized the number of SUs as well as sensing time. As mentioned earlier, they extended it to a cooperative two-stage sensing mechanism that combines energy detection with feature detection. If all SUs determine the idleness of the PU on a specific channel, they will access the channel. Otherwise, the second-stage fine sensing will be performed by the base station (BS) or a designated SU. In [74], the throughput of the secondary network has been analyzed over imperfect reporting channels. They also showed that there is an optimum K due to the Q_f lower bound in the case of OR fusion rule through erroneous reporting channels. Furthermore, the throughput of the double-threshold based ED scheme was investigated in [32] in the case of perfect reporting channels. By considering a medium-traffic primary network, the authors in [65] showed that the achievable throughput of the secondary network is reduced when the random arrival and departure of the PU are taken into account, and the optimal sensing time may also be changed. Based on their non-CSS analysis, the performance degradation is related to the traffic intensity of the PU as well as the quality of the sensing channel.

In the above-mentioned throughput evaluation works, the sensing time is optimized over a fixed frame time. There are also some works in which the transmission time (and subsequently the frame time) is optimized over a fixed sensing time [75–77], or both sensing and transmission times are optimized [71, 78, 79].

In [79], the authors used maximum *a posteriori* (MAP) energy detector to consider *a posteriori* probabilities of the PU becomes active or idle in the detection probabilities over the low-traffic primary network. They conducted a multiple-band spectrum sensing through the CSS with perfect reporting channels. The transmission and sensing times are optimized besides the number of cooperative users subject to a constraint on maximum tolerable interference by the PU. They introduced a framework in which BSs first determine the optimum parameters for all channels. SUs choose the best channel for sensing and configure own themselves based on the optimum parameters. If anything is changed, BSs optimize parameters again and inform SUs for update. In order to have multi-channel sensing, they used sequential sensing where each sensing transceiver monitors only a single spectrum band at a time. They next introduced the least cost first

serve (LCFS) algorithm for sensing scheduling of the spectrum bands. Each SU selects a channel for sensing based on the opportunity cost of each spectrum band. They showed that the optimum parameters and sensing performance are more sensitive to the activity rate of the PU than its idleness rate.

The non-CSS throughput optimization is considered in [71,75–78]. The authors in [71] supposed a medium-traffic primary network over the frame time and analyzed it under two constraints, the interference to the primary signal and the lost opportunity of the SU. The optimization problem is defined as the ratio of the transmission duration over the entire frame time. The PU status changing during the sensing period exhibits a random duty cycle. Therefore, for the ED, they defined a test statistic conditioned on the duty cycle of the PU during sensing time and averaged it over the duty cycle distribution numerically. In [75], the frame time is optimized over a constraint on the collision probability when the detection is assumed to be perfect (no false alarm and no missed detection). Another assumption is that the PU becomes active at most once during the transmission time, and if it happens the PU remains active till the end of the frame. The energy detection of [75] is assumed to be an imperfect block sensing method in [76]. Again, the PU remains unchanged during the sensing time and they derive the average throughput for both quickest (sequential) [80–82] and block sensing. The quickest sensing method is the Page cumulative sum (CUSUM) algorithm [82,83] in which the SU detects the activity of the PU correctly with probability one. In the quickest method, the probability of missed detection is negligible and the sensing time is a random variable while the missed detection is not negligible and sensing time is not random in the block sensing method. They showed that the quickest scheme has higher average throughput compared to the block sensing. In [77], there is a single multiple-antenna SU besides multiple PUs which listen to all channels simultaneously. Each PU is licensed to an individual spectrum band (one-to-one) and they are assumed to be unchanged during sensing period. Although multiple-antenna SU is a kind of CSS with perfect reporting channels, the authors did not use the number of antennas as a parameter in their performance analysis. Thus, it should be considered as a non-CSS analysis. They defined the transmission throughput of the SU over both perfect (accurate sensing) and imperfect (with missed detection and false alarm) detections when the sensing channels are faded, and optimized transmission time for a fixed sensing time. It is shown that the impact of imperfect sensing on throughput is not only related to the number of coexisting PUs but also much involved with the traffic loads of a realistic CRN. Moreover, the probability of collision (to use in the throughput expression) for the cases of once and twice status changing of the PU during the transmission time is found in [78].

They optimized both sensing and frame durations so that the maximized throughput is subjected to a constraint on the probability of collision (implying interference tolerance), satisfying another assumption on the unchanged PU during the sensing time.

Total throughput (summation of both primary and secondary networks' throughputs) maximization is another matter of interest in some works with the fact that there is no point to improve the secondary throughput without any consideration about its effect on the primary one. The authors in [84] formulated two different approaches which are functions of both detection duration and threshold over a non-CSS method. One approach minimizes the mean sensing time over some constraints on the quality of detection, and the other one maximizes the aggregate throughput of both networks. In [85, 86], a multiple-user CRN (both PUs and SUs) was considered in a multiple-channel scenario. By considering a perfect detection, they maximized the total throughput of the network with respect to the number of SUs. Subsequently, for both cases of single and multi-channel sensing schemes, the maximization of the total throughput of the system was analyzed in [41] under required detection constraints of the cooperative sensing. However, an ideal CSS was utilized with error-free reporting channels.

Note that the number of users is always assumed to be deterministic in the above-mentioned works. Using stochastic geometry concepts, users can be spatially distributed randomly [87, 88]. The authors in [89] analyzed both primary and secondary throughputs while the spatial distributions of the primary and secondary transmitters followed the PPP and modified Matérn point process (PP), respectively. All possible kinds of interference (received by primary and secondary receivers) were studied through Rayleigh-faded channels to define the operating region of SUs (in terms of intensity and minimum distance) as well as detection threshold satisfying the required interference constraint which protects the PU. In [90], primary and secondary transmitters are assumed to follow two independent homogeneous PPPs. They proposed two access schemes, called primary receiver/transmitter assisted protocols, in which the distribution of active secondary transmitters does not follow the homogeneous PPP any more. Moreover, the aggregate interference of SUs in a given primary receiver has been analyzed in [91, 92] when SUs are spatially distributed by a homogeneous PPP beyond a protected region around the PU. By assuming a primary receiver located at the origin, the SUs in [91] utilized a beacon-based detector without any cooperative sensing when there is no interference consideration. In [92], it was assumed that a primary transmitter is located at the origin and SUs' distribution area (around the primary transmitter) is finite. Then, the authors studied the outage probability over Rayleigh-faded channels and showed that

this area can be considered infinite (like many other works) just in the case of high values of the path-loss gain (PLG).

A competitive (no cooperation) spectrum access method is analyzed in [93] when the distribution of the SUs has been modeled by the PPP. They considered a secondary network with multiple SUs which are working independently, i.e., each SU does its own sensing and occupies a channel when it is found free. Hence, an SU may generate interference into the received signals by other SUs which are working in their sensing part, resulting in detection performance degradation. The interference is estimated over an AWGN channel. In [94], the SUs have been distributed in a plane according to a homogeneous PPP around the FC at the origin. They analyzed the detection performance and power consumption of a soft-decision based CSS with censoring while there is no interference consideration. In their proposed method, the SU sends its test statistic to the FC if the received energy level is greater than a predetermined threshold and the required power to send the test statistic is less than a power limit. It is worth mentioning that the status of the PU has been assumed to be fixed in the explained works with random SUs.

2.2 Sensing improvement methods

In this section, we have a short review on some suggested methods to improve the quality of spectrum sensing in an ED-based system, which are considered in our works.

Suppose that a secondary network with K SUs and one FC has been established as a part of the CRN. They sense the PU's situation cooperatively in a CSS-based manner. The m -th received signal sample of the i -th SU, $r_i(m)$, under hypotheses H_0 (idle PU) and H_1 (active PU) are, respectively, $r_i^0(m)$ and $r_i^1(m)$ as follows:

$$\begin{aligned}
 H_0: \quad r_i^0(m) &= u_i(m) \sim \mathcal{CN}(0, \sigma_n^2) \\
 H_1: \quad r_i^1(m) &= \begin{cases} s(m) + u_i(m) \sim \mathcal{CN}(0, \sigma_s^2 + \sigma_n^2) & \text{AWGN Sensing Ch.} \\
 h_i(m)s(m) + u_i(m) \sim \mathcal{CN}(0, \sigma_h^2\sigma_s^2 + \sigma_n^2) & \text{Rayleigh-faded Sensing Ch.} \end{cases}
 \end{aligned} \tag{2.1}$$

$$i = 1, 2, \dots, K$$

with the average SNR of

$$\bar{\gamma} = \begin{cases} \frac{\sigma_s^2}{\sigma_n^2} & \text{AWGN Sensing Ch.} \\
 \frac{\sigma_h^2\sigma_s^2}{\sigma_n^2} & \text{Rayleigh-faded Sensing Ch.} \end{cases} \tag{2.2}$$

where $s(m)$ is the zero-mean primary signal with average power σ_s^2 , and is independent of the zero-mean $u_i(m)$. Note that $u_i(m)$ in AWGN and Rayleigh-faded channels is, respectively, a real and complex Gaussian noise with variance σ_n^2 . Moreover, Rayleigh fading is characterized by the channel coefficient, $h_i(m)$, as independent of $u_i(m)$ when $|h_i(m)|$ is Rayleigh distributed with parameter $\sqrt{\sigma_h^2/2}$.

2.2.1 Improved energy detector

The IED in [57] uses N normalized received signal samples (with respect to the noise standard deviation), $x_i(m)$, to estimate the signal energy as follows:

$$Y_i = \frac{1}{N} \sum_{m=1}^N |x_i(m)|^p, \quad p > 0 \quad (2.3)$$

in which

$$x_i(m) = \frac{r_i(m)}{\sigma_n} \quad (2.4)$$

and p is one of the parameters that can optimize the required performance criteria by finding its optimum value given other parameters. It is clear that $p = 2$ implies CED. By considering T_s as sampling time, the relation between the number of samples and sensing time, τ , can be defined as

$$N = \frac{\tau}{T_s} \quad (2.5)$$

If we assume a frame time, T , as the average idle time of PU in a low-traffic case, the transmission time of the secondary network will be $(T - \tau)$ as shown in Figure 2.1. One can see that the observed energy level can be more accurate by using more samples, and it helps SUs to have more reliable local decisions. It is obvious that the sensing time increases by using more samples (larger N) for detection and it degrades the transmission throughput of the secondary network due to the lower remaining time for transmission. Thus, N is another parameter which should be optimized in the case of N -sample based IED. Subsequently, the authors in [18–20] used a one-sample based IED as

$$Y_i(m) = |x_i(m)|^p, \quad p > 0 \quad (2.6)$$

Hence, the sensing time equals sampling time from (2.5). This method is useful whenever the minimum sensing time is required and the detection reliability is at the lower priority. Therefore, the optimization problem in this case is to minimize detection errors.

Figure 2.1: The structure of secondary network's operations during PU's idle time

2.2.2 Comparison schemes

To complete hypothesis testing execution and making decision in an ED-based detector, SUs need at least one threshold as their reference for comparison and decide the status of the PU. There are many comparison methods in current works. Despite the proficiency of some suggested methods to enhance the detection quality with higher number of thresholds, two schemes with the minimum bandwidth overhead (one bit per decision) are commonly used in the case of CR.

2.2.2.1 Single-threshold

Each SU uses a single threshold of λ to make its own local decision as shown in Figure 2.2.a and defined as

$$D_i = \begin{cases} H_0 & \lambda \geq Y_i \geq 0 \\ H_1 & Y_i > \lambda \end{cases}, \quad \lambda > 0 \quad (2.7)$$

Hence, for the *local probability of false alarm* (P_f), the *local probability of detection* (P_d), the *local probability of detecting H_0* (P_{d0}) and the *local probability of missed detection* (P_m) in the case of single-threshold comparison based ED, we have

$$P_f = \Pr \{Y_i > \lambda | H_0\} \quad (2.8)$$

$$P_d = \Pr \{Y_i > \lambda | H_1\} \quad (2.9)$$

$$P_{d0} = \Pr \{Y_i \leq \lambda | H_0\} = 1 - P_f \quad (2.10)$$

and

$$P_m = \Pr \{Y_i \leq \lambda | H_1\} = 1 - P_d \quad (2.11)$$

respectively.

Figure 2.2: Different comparison criteria of observed energy in the SU

2.2.2.2 Double-threshold

Figure 2.2.b shows the double-threshold based comparison, in which each SU performs a comparison between the observed energy level of received signal and two thresholds, and makes its own decision as [59]

$$D_i = \begin{cases} H_0 & \lambda_1 > Y_i \geq 0 \\ \text{No Decision} & \lambda_2 \geq Y_i \geq \lambda_1, \quad \lambda_2 > \lambda_1 > 0 \\ H_1 & Y_i > \lambda_2 \end{cases} \quad (2.12)$$

Note that

$$\Delta\lambda = (\lambda_2 - \lambda_1) > 0 \quad (2.13)$$

In the case of idle PU, the *local probability of false alarm* (P_f), the *local probability of detecting* H_0 (P_{d0}) and the *local probability of no decision under* H_0 (Δ_0) are given by

$$P_f = \Pr \{Y_i > \lambda_2 | H_0\} \quad (2.14)$$

$$P_{d0} = \Pr \{Y_i < \lambda_1 | H_0\} \quad (2.15)$$

and

$$\Delta_0 = \Pr \{\lambda_2 \geq Y_i \geq \lambda_1 | H_0\} = 1 - P_f - P_{d0} \quad (2.16)$$

respectively, whereas the *local probability of detection* (P_d), the *local probability of missed detection* (P_m) and the *local probability of no decision under* H_1 (Δ_1) in the situation of active PU are, respectively,

$$P_d = \Pr \{Y_i > \lambda_2 | H_1\} \quad (2.17)$$

$$P_m = \Pr \{Y_i < \lambda_1 | H_1\} \quad (2.18)$$

and

$$\Delta_1 = \Pr \{\lambda_2 \geq Y_i \geq \lambda_1 | H_1\} = 1 - P_d - P_m \quad (2.19)$$

2.2.3 Cooperative spectrum sensing

After estimating the received signal energy level by the ED method and applying hypothesis testing, each SU sends its own decision to the FC to make the global decision through a suitable fusion rule. There are different suggested fusion rules, but the OR, AND, Majority and M -of- K rules are the most popular rules among them:

- (i) **OR fusion rule:** the global decision indicates the presence of PU (H_1) when at least one of the local decisions is H_1 . This rule is very strict on the interference protection. We can use this rule when the quality of primary network's communications is more important than spectrum utilization. It means that, this rule tries to decrease the chance of missed detection (the situation in which the PU is active but SUs find it idle) more than the false alarm (the situation in which the PU is idle but SUs find it active) rate.
- (ii) **AND fusion rule:** the global decision indicates the presence of PU (H_1) when all local decisions are H_1 . In other words, the FC decides on the idle PU if it receives at least one local decision on the idleness of PU. We can use it when the utilization rate of the desired frequency band is more important than the protection of the primary signal. Thus, this rule decreases the false alarm rate as the top priority.
- (iii) **Majority fusion rule:** the majority of received local decisions specifies the global decision. This rule is a moderate one in terms of both spectrum utilization and primary signal protection as compared with OR and AND.
- (iv) **M -of- K fusion rule:** the global decision declares the presence of PU (H_1) when at least M local decision(s) from total K are H_1 . The OR, AND and Majority rules are the special cases of this rule. We can call the OR rule as the 1-of- K rule or the AND rule as the K -of- K rule. The majority fusion rule can also be expressed as the $K/2$ -of- K rule.

2.2.3.1 Single-threshold based CSS

Since all of above-mentioned fusion rules are the special cases of the M -of- K fusion rule, we define the *total probability of false alarm* (Q_f), the *total probability of detection* (Q_d), the *total probability of detecting H_0* (Q_{d_0}) and the *total probability of missed detection* (Q_m) over the M -of- K fusion rule through the imperfect reporting channels as

$$Q_f = \sum_{l=M}^K \binom{K}{l} [P_f(1 - P_e) + P_{d_0}P_e]^l [P_{d_0}(1 - P_e) + P_fP_e]^{K-l} \quad (2.20)$$

$$Q_d = \sum_{l=M}^K \binom{K}{l} [P_d(1 - P_e) + P_mP_e]^l [P_m(1 - P_e) + P_dP_e]^{K-l} \quad (2.21)$$

$$Q_{d_0} = 1 - Q_f \quad (2.22)$$

and

$$Q_m = 1 - Q_d \quad (2.23)$$

respectively, where reporting channels are considered as BSC with error probability P_e . Furthermore, using the single-threshold method, it can be proved that there is a lower bound for Q_f in the case of the OR rule over imperfect reporting channels as follows:

$$\underline{Q_f} = \lim_{P_f \rightarrow 0} Q_f \rightarrow 1 - [1 - P_e]^K \approx KP_e \quad (2.24)$$

The receiver operating characteristic (ROC) curves of single-threshold based CSS for different numbers of K over the OR, AND and majority fusion rules are represented in Figure 2.3, when the one-sample based IED is used to estimate the energy level of received signal through a Rayleigh-faded sensing channel with $\bar{\gamma} = 5$ dB and $P_e = 1\%$. The lower bound of Q_f in the case of the OR rule is obvious in this figure.

2.2.3.2 Double-threshold based CSS

Suppose that K_d of K SUs can determine a reliable decision (H_0 or H_1) based on the double-threshold scheme as described in Section 2.2.2.2 and send their local decisions to the FC. In other words, $(K - K_d)$ SUs do not send any decision. Hence, the M -of- K fusion rule for this case will be expressed as the M -of- K_d rule. For its total probabilities

Figure 2.3: Q_d vs. Q_f with $\bar{\gamma} = 5$ dB and $P_e = 1\%$ for the OR, AND and majority fusion rules in different numbers of the cooperative users

over imperfect reporting channels, we have

$$Q_f = (1 - \Delta_0^K) \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} \Delta_0^{K-K_d} [P_f(1 - P_e) + P_{d0}P_e]^l [P_{d0}(1 - P_e) + P_fP_e]^{K_d-l} \quad (2.25)$$

$$Q_d = (1 - \Delta_1^K) \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} \Delta_1^{K-K_d} [P_d(1 - P_e) + P_mP_e]^l [P_m(1 - P_e) + P_dP_e]^{K_d-l} \quad (2.26)$$

$$Q_{d0} = 1 - Q_f \quad (2.27)$$

and

$$Q_m = 1 - Q_d \quad (2.28)$$

where Δ_0^K and Δ_1^K are the *probability of sensing failure* under hypotheses H_0 and H_1 , respectively.

Chapter 3

Deterministic-modeled secondary users in a low-traffic network

As the starting stage of our study on the CSS-based schemes, we assume a simple model of the CRN which has been considered in the most of the current works. In this model, the number of available SUs (K) is deterministic and they cooperatively do the spectrum sensing over the primary network with a low traffic rate. It means that the status of the PU (idle/active) is supposed unchanged during the studied period.

3.1 Detection performance by minimum sensing time

In the first step, the one-sample based IED in (2.6), which has the minimum sensing time, has been considered as the detection scheme over a Rayleigh-faded sensing channel to keep the available transmission time for the secondary network as much as possible. Note that the PU's status is definitely unchanged (implying the low traffic rate) from the perspective of a single sample. Since we worry about the detection accuracy due to a low sensing time, we analyze the sensing reliability of SUs under different ED methods. The objective is to optimize the total error rate which is

$$P_{err} = Q_m + Q_f = 1 - Q_d + Q_f \quad (3.1)$$

We will show that the single-threshold based IED, called improved single-threshold energy detector (ISED), would not decrease P_{err} as compared with the conventional single-threshold energy detector (CSED), and there is no optimization profit for the ISED in terms of the total error rate. Then, by introducing the IDDED which is an improved version

of the CDED, we reveal that there is jointly an optimum point for the pair of $(p, \Delta\lambda)$ to minimize P_{err} .

The cumulative distribution function (CDF) of the one-sample based IED is

$$F_{Y_i}(y) = \Pr \{Y_i \leq y\} = \Pr \{|x_i|^p \leq y\} = \Pr \{|x_i|^2 \leq y^{2/p}\} = F_{|x_i|^2}(y^{2/p}) \quad (3.2)$$

and by differentiation of (3.2), the PDF is

$$f_{Y_i}(y) = \frac{2}{p} y^{(2-p)/p} f_{|x_i|^2}(y^{2/p}) \quad (3.3)$$

which can be written under two hypotheses as

$$f_{Y_i|H_s}(y) = \frac{2}{p} y^{(2-p)/p} f_{|x_i|^2|H_s}(y^{2/p}), \quad H_s \in \{H_0, H_1\} \quad (3.4)$$

From (2.1), for Rayleigh-faded sensing channels, $f_{|x_i|^2|H_0}(\cdot)$ and $f_{|x_i|^2|H_1}(\cdot)$ (PDFs of the one-sample based CED) are exponentially distributed as [19], [95]

$$\begin{cases} f_{|x_i|^2|H_0}(y) = \exp(-y) \\ f_{|x_i|^2|H_1}(y) = \frac{1}{1+\bar{\gamma}} \exp\left(-\frac{y}{1+\bar{\gamma}}\right) \end{cases}, \quad y \geq 0 \quad (3.5)$$

Finally, by substituting (3.5) into (3.4), we obtain the PDFs of the one-sample based IED outputs under both hypotheses as follows:

$$\begin{cases} f_{Y_i|H_0}(y) = \frac{2y^{(2-p)/p}}{p} \exp(-y^{2/p}) \\ f_{Y_i|H_1}(y) = \frac{2y^{(2-p)/p}}{p(1+\bar{\gamma})} \exp\left(-\frac{y^{2/p}}{1+\bar{\gamma}}\right) \end{cases}, \quad y \geq 0 \quad (3.6)$$

Now we continue the analysis over two different comparison schemes separately.

3.1.1 Single-threshold based analysis

From (2.8)-(2.11) and (3.6), the local probabilities over single-threshold based comparison method are

$$P_f = \int_{\lambda}^{\infty} f_{Y_i|H_0}(y) dy = \exp(-\lambda^{2/p}) \quad (3.7)$$

$$P_d = \int_{\lambda}^{\infty} f_{Y_i|H_1}(y) dy = \exp\left(-\frac{\lambda^{2/p}}{1+\bar{\gamma}}\right) \quad (3.8)$$

$$P_{d_0} = 1 - P_f = 1 - \exp(-\lambda^{2/p}) \quad (3.9)$$

and

$$P_m = 1 - P_d = 1 - \exp\left(-\frac{\lambda^{2/p}}{1 + \bar{\gamma}}\right) \quad (3.10)$$

Note that the total probabilities are obtained from (2.20)-(2.23).

Since our focus is to find the optimum points for the ISED parameters (p and λ) in terms of the minimum total error rate, we first differentiate P_{err} with respect to p and obtain

$$\frac{\partial P_{err}}{\partial p} = 0 \quad \Rightarrow \quad \frac{\partial Q_f}{\partial p} - \frac{\partial Q_d}{\partial p} = 0 \quad (3.11)$$

where from (2.20) and (2.21),

$$\begin{aligned} \frac{\partial Q_f}{\partial p} = \frac{\partial P_f}{\partial p} \sum_{l=M}^K \binom{K}{l} (P_f(1 - P_e) + P_{d_0}P_e)^{l-1} (P_{d_0}(1 - P_e) + P_fP_e)^{K-l} \\ \times \left[l(1 - 2P_e) + \frac{(P_f(1 - P_e) + P_{d_0}P_e)(K - l)(2P_e - 1)}{P_{d_0}(1 - P_e) + P_fP_e} \right] \end{aligned} \quad (3.12)$$

$$\begin{aligned} \frac{\partial Q_d}{\partial p} = \frac{\partial P_d}{\partial p} \sum_{l=M}^K \binom{K}{l} (P_d(1 - P_e) + P_mP_e)^{l-1} (P_m(1 - P_e) + P_dP_e)^{K-l} \\ \times \left[l(1 - 2P_e) + \frac{(P_d(1 - P_e) + P_mP_e)(K - l)(2P_e - 1)}{P_m(1 - P_e) + P_dP_e} \right] \end{aligned} \quad (3.13)$$

and from (3.7)-(3.8),

$$\frac{\partial P_f}{\partial p} = \frac{\xi \ln \xi}{p} \exp(-\xi) \quad (3.14)$$

$$\frac{\partial P_d}{\partial p} = \frac{\xi \ln \xi}{p(1 + \bar{\gamma})} \exp\left(-\frac{\xi}{1 + \bar{\gamma}}\right) \quad (3.15)$$

in which

$$\xi = \lambda^{2/p} \quad (3.16)$$

Substituting (3.12)-(3.15) into (3.11), we have

$$\sum_{l=M}^K \binom{K}{l} (1 - \exp(-\xi))^{K-l} \left[l - \frac{\exp(-\xi)(K-l)}{(1 - \exp(-\xi))} \right] - \frac{\left(\exp\left(\xi \frac{\bar{\gamma}}{1+\bar{\gamma}}\right) \right)^l}{(1+\bar{\gamma})} \left(1 - \exp\left(-\frac{\xi}{1+\bar{\gamma}}\right) \right)^{K-l} \left[l - \frac{\exp\left(-\frac{\xi}{1+\bar{\gamma}}\right)(K-l)}{\left(1 - \exp\left(-\frac{\xi}{1+\bar{\gamma}}\right)\right)} \right] = 0 \quad (3.17)$$

where we have assumed $P_e \approx 0$ to simplify the derivation. Equation (3.17) shows that the minimum P_{err} depends on the number of cooperative users (K), the minimum required number of the same decisions (M) (which is related to the fusion rule), average SNR of the sensing channel ($\bar{\gamma}$) and the parameter ξ .

Since the measured energy of the received signal is a positive value, ξ must be in the range of $[0, +\infty)$. As ξ is a small value, the left-hand side (LHS) of (3.17) is positive. On the other hand, if ξ goes to $+\infty$, the LHS of (3.17) becomes $-\infty$. Hence, there is at least one root for ξ . In other words, it is possible to find an optimum ξ given other parameters to minimize P_{err} .

From (3.16), there exists a direct relationship between p and λ for a particular ξ . Thus, there are many different pairs of (p, λ) for an optimum ξ to satisfy this equation, in which one of them is $p = 2$ (which implies CSSED). Hence, the ISED has no optimization gain as compared with CSSED in this type of the sensing channel.

Subsequently, we need to minimize P_{err} in terms of λ , as another ISED parameter, to have an adequate analysis as follows:

$$\frac{\partial P_{err}}{\partial \lambda} = 0 \quad \Rightarrow \quad \frac{\partial Q_f}{\partial \lambda} - \frac{\partial Q_d}{\partial \lambda} = 0 \quad (3.18)$$

where from (2.20) and (2.21),

$$\begin{aligned} \frac{\partial Q_f}{\partial \lambda} = \frac{\partial P_f}{\partial \lambda} \sum_{l=M}^K \binom{K}{l} & (P_f(1 - P_e) + P_{d_0}P_e)^{l-1} (P_{d_0}(1 - P_e) + P_fP_e)^{K-l} \\ & \times \left[l(1 - 2P_e) + \frac{(P_f(1 - P_e) + P_{d_0}P_e)(K-l)(2P_e - 1)}{P_{d_0}(1 - P_e) + P_fP_e} \right] \end{aligned} \quad (3.19)$$

$$\begin{aligned} \frac{\partial Q_d}{\partial \lambda} = \frac{\partial P_d}{\partial \lambda} \sum_{l=M}^K \binom{K}{l} & (P_d(1 - P_e) + P_mP_e)^{l-1} (P_m(1 - P_e) + P_dP_e)^{K-l} \\ & \times \left[l(1 - 2P_e) + \frac{(P_d(1 - P_e) + P_mP_e)(K-l)(2P_e - 1)}{P_m(1 - P_e) + P_dP_e} \right] \end{aligned} \quad (3.20)$$

and from (3.7)-(3.8),

$$\frac{\partial P_f}{\partial \lambda} = -\frac{2\xi}{p\lambda} \exp(-\xi) \quad (3.21)$$

$$\frac{\partial P_d}{\partial \lambda} = -\frac{2\xi}{p\lambda(1+\bar{\gamma})} \exp\left(-\frac{\xi}{1+\bar{\gamma}}\right) \quad (3.22)$$

By substituting (3.19)-(3.22) into (3.18), we obtain (3.17) again. It is proved that we can just find an optimum ξ given other parameters with respect to the minimum P_{err} and, from (3.16), there are many different pairs of (p, λ) for an optimum ξ . Note that $\lambda = 1$ is a special case in which ξ equals 1 for all values of p as well as very large values of p ($p \rightarrow \infty$) in which $\xi \approx 1$ for any λ . We will show that this special point does not affect our concluded result negatively due to the high value of P_{err} at this point.

As a summary, our optimization problem in the ISED which is

$$\underset{p, \lambda}{\text{minimize}} : P_{err}(p, \lambda) \quad (3.23a)$$

$$\text{subject to} : p, \lambda > 0 \quad (3.23b)$$

can be simplified to

$$\underset{\xi}{\text{minimize}} : P_{err}(\xi) \quad (3.24a)$$

$$\text{subject to} : \xi > 0 \quad (3.24b)$$

P_{err} versus the threshold (λ) is shown in Figure 3.1 for $p = 2$. It is revealed that the majority fusion rule has the minimum P_{err} in this type of sensing channel. The same conclusion is valid for other values of p . Figure 3.2 shows that the minimum P_{err} is fixed for different values of p in their optimum λ through the ISED as it was proved. There is an optimum value of λ to achieve the minimum P_{err} for each p .

Furthermore, the minimum achievable P_{err} versus p using its optimum threshold (λ) is plotted for different numbers of cooperative users (K) in Figure 3.3. We also represent the minimum achievable P_{err} versus λ using its optimum exponent (p) in Figure 3.4. They show that there is no optimization gain in using the ISED as compared with CSED in our channel model. Note that in Figure 3.4 $\lambda = 1$ is a special case in which the parameter ξ equals 1 for all values of p . In this case, we cannot find the optimized value of P_{err} . From (3.16), if we choose a very large value of p ($p \rightarrow \infty$), then we will face the same situation as $\lambda = 1$ in which $\xi \approx 1$, where Figure 3.4 shows that there is no performance improvement in this situation.

Figure 3.1: The total error rate vs. threshold with $p = 2$, $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for different fusion rules

Figure 3.2: The total error rate vs. threshold in the different p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the ISFD

Figure 3.3: The minimum total error rate vs. p in the optimum threshold with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the ISED

Figure 3.4: The minimum total error rate vs. threshold in the optimum p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the ISED

3.1.2 Double-threshold based analysis

In the previous section, it is revealed that the ISED cannot improve the detection performance more than CSED in terms of minimum P_{err} over Rayleigh-faded sensing channels. Here we use the double-threshold method besides IED, called IDED. By considering (3.6), for (2.14)-(2.19), the local probabilities over one-sample based IDED are given by

$$P_f = \int_{\lambda_2}^{\infty} f_{Y_i|H_0}(y) dy = \exp\left(-\lambda_2^{2/p}\right) \quad (3.25)$$

$$P_{d0} = \int_0^{\lambda_1} f_{Y_i|H_0}(y) dy = 1 - \exp\left(-\lambda_1^{2/p}\right) \quad (3.26)$$

$$\Delta_0 = 1 - P_f - P_{d0} = \exp\left(-\lambda_1^{2/p}\right) - \exp\left(-\lambda_2^{2/p}\right) \quad (3.27)$$

$$P_d = \int_{\lambda_2}^{\infty} f_{Y_i|H_1}(y) dy = \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \quad (3.28)$$

$$P_m = \int_0^{\lambda_1} f_{Y_i|H_1}(y) dy = 1 - \exp\left(-\frac{\lambda_1^{2/p}}{1 + \bar{\gamma}}\right) \quad (3.29)$$

$$\Delta_1 = 1 - P_d - P_m = \exp\left(-\frac{\lambda_1^{2/p}}{1 + \bar{\gamma}}\right) - \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \quad (3.30)$$

Similar to Section 3.1.1, we first analyze the effect of the IED exponent (p) in the proposed IDED method in terms of P_{err} optimization. Again, from (3.11), we differentiate P_{err} with respect to p . From (2.25) and (2.26), we have

$$\begin{aligned} \frac{\partial Q_f}{\partial p} = \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} & \left[\frac{\partial P_f}{\partial p} l P_f^{l-1} P_{d0}^{K_d-l} \Delta_0^{K-K_d} (1 - \Delta_0^K) \right. \\ & + \frac{\partial P_{d0}}{\partial p} P_f^l (K_d - l) P_{d0}^{K_d-l-1} \Delta_0^{K-K_d} (1 - \Delta_0^K) \\ & \left. + \frac{\partial \Delta_0}{\partial p} P_f^l P_{d0}^{K_d-l} \left[(K - K_d) \Delta_0^{K-K_d-1} + (K_d - 2K) \Delta_0^{2K-K_d-1} \right] \right] \end{aligned} \quad (3.31)$$

$$\begin{aligned} \frac{\partial Q_d}{\partial p} = \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} & \left[\frac{\partial P_d}{\partial p} l P_d^{l-1} P_m^{K_d-l} \Delta_1^{K-K_d} (1 - \Delta_1^K) \right. \\ & + \frac{\partial P_m}{\partial p} P_d^l (K_d - l) P_m^{K_d-l-1} \Delta_1^{K-K_d} (1 - \Delta_1^K) \\ & \left. + \frac{\partial \Delta_1}{\partial p} P_d^l P_m^{K_d-l} \left[(K - K_d) \Delta_1^{K-K_d-1} + (K_d - 2K) \Delta_1^{2K-K_d-1} \right] \right] \end{aligned} \quad (3.32)$$

in which we have assumed $P_e \approx 0$ to simplify the derivation. From (3.25)-(3.30),

$$\frac{\partial P_f}{\partial p} = \frac{\xi_2 \ln \xi_2}{p} \exp(-\xi_2) \quad (3.33)$$

$$\frac{\partial P_{d0}}{\partial p} = -\frac{\xi_1 \ln \xi_1}{p} \exp(-\xi_1) \quad (3.34)$$

$$\frac{\partial \Delta_0}{\partial p} = -\left(\frac{\partial P_f}{\partial p} + \frac{\partial P_{d0}}{\partial p}\right) = \frac{\xi_1 \ln \xi_1}{p} \exp(-\xi_1) - \frac{\xi_2 \ln \xi_2}{p} \exp(-\xi_2) \quad (3.35)$$

$$\frac{\partial P_d}{\partial p} = \frac{\xi_2 \ln \xi_2}{p(1+\bar{\gamma})} \exp\left(-\frac{\xi_2}{1+\bar{\gamma}}\right) \quad (3.36)$$

$$\frac{\partial P_m}{\partial p} = -\frac{\xi_1 \ln \xi_1}{p(1+\bar{\gamma})} \exp\left(-\frac{\xi_1}{1+\bar{\gamma}}\right) \quad (3.37)$$

$$\frac{\partial \Delta_1}{\partial p} = -\left(\frac{\partial P_d}{\partial p} + \frac{\partial P_m}{\partial p}\right) = \frac{\xi_1 \ln \xi_1}{p(1+\bar{\gamma})} \exp\left(-\frac{\xi_1}{1+\bar{\gamma}}\right) - \frac{\xi_2 \ln \xi_2}{p(1+\bar{\gamma})} \exp\left(-\frac{\xi_2}{1+\bar{\gamma}}\right) \quad (3.38)$$

where

$$\xi_j = \lambda_j^{2/p}, \quad j = 1, 2 \quad (3.39)$$

By substituting (3.31) and (3.32) into (3.11), it is found that the P_{err} optimization is related to the number of cooperative users (K), the number of cooperative users who send decision (K_d) and the minimum required number of the same decisions (M). Since our focus is to analyze the effects of local detector's parameters, we consider $K = 1$ without loss of generality and it can be written as

$$\begin{aligned} \frac{\partial P_{err}}{\partial p} &= \frac{\partial}{\partial p} (Q_m + Q_f) = \frac{\partial}{\partial p} (1 - Q_d + Q_f) = \frac{\partial}{\partial p} [1 - (1 - \Delta_1)P_d + (1 - \Delta_0)P_f] \\ &= -\frac{\partial P_d}{\partial p}(1 - \Delta_1) + \frac{\partial \Delta_1}{\partial p}P_d + \frac{\partial P_f}{\partial p}(1 - \Delta_0) - \frac{\partial \Delta_0}{\partial p}P_f \end{aligned} \quad (3.40)$$

and from the relevant expressions, the first order differential equation can be given by

$$\begin{aligned} \frac{\partial P_{err}}{\partial p} &= -\frac{2\lambda_2^{2/p} \ln(\lambda_2)}{p^2(1+\bar{\gamma})} \left[\exp\left(-\frac{\lambda_2^{2/p}}{1+\bar{\gamma}}\right) + 2 \exp\left(-\frac{2\lambda_2^{2/p}}{1+\bar{\gamma}}\right) \right] \\ &\quad + \left(\frac{2\lambda_2^{2/p} \ln(\lambda_2)}{p^2(1+\bar{\gamma})} + \frac{2\lambda_1^{2/p} \ln(\lambda_1)}{p^2(1+\bar{\gamma})} \right) \exp\left(\frac{-\lambda_1^{2/p} - \lambda_2^{2/p}}{1+\bar{\gamma}}\right) \\ &\quad + \frac{2\lambda_2^{2/p} \ln(\lambda_2)}{p^2} \left[\exp(-\lambda_2^{2/p}) + 2 \exp(-2\lambda_2^{2/p}) \right] \\ &\quad - \left(\frac{2\lambda_2^{2/p} \ln(\lambda_2)}{p^2} + \frac{2\lambda_1^{2/p} \ln(\lambda_1)}{p^2} \right) \exp(-\lambda_1^{2/p} - \lambda_2^{2/p}) \end{aligned} \quad (3.41)$$

By putting (3.41) equal to zero, the simplified equation will be

$$\begin{aligned}
 & 2 \exp(-2\lambda_2^{2/p}) + \exp(-\lambda_2^{2/p}) - \left(1 + \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{2}{p}} \frac{\ln(\lambda_1)}{\ln(\lambda_2)}\right) \exp(-\lambda_1^{2/p} - \lambda_2^{2/p}) - \frac{1}{(1 + \bar{\gamma})} \\
 & \times \left[2 \exp\left(-\frac{2\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) + \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) - \left(1 + \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{2}{p}} \frac{\ln(\lambda_1)}{\ln(\lambda_2)}\right) \exp\left(-\frac{\lambda_1^{2/p} - \lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \right] = 0
 \end{aligned} \tag{3.42}$$

Equation (3.42) shows that it depends on the average SNR of sensing channels ($\bar{\gamma}$), the IDED parameters (p, λ_1, λ_2) in addition to K , K_d and M . Furthermore, we take the second order derivative of P_{err} with respect to p to check the convexity of the function. Thus, for the second order differential equation we have

$$\begin{aligned}
 \frac{\partial^2 P_{err}}{\partial p^2} = & \left(1 + 2 \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right)\right) \frac{4\lambda_2^{2/p} \ln(\lambda_2)}{p^3(1 + \bar{\gamma})} \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \\
 & - \left(1 + 2 \exp\left(-\lambda_2^{2/p}\right)\right) \frac{4\lambda_2^{2/p} \ln(\lambda_2)}{p^3} \exp\left(-\lambda_2^{2/p}\right) \\
 & + \left(1 - \frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \frac{4\lambda_2^{2/p} \ln^2(\lambda_2)}{p^4(1 + \bar{\gamma})} \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) - \left(1 - \lambda_2^{2/p}\right) \frac{4\lambda_2^{2/p} \ln^2(\lambda_2)}{p^4} \exp\left(-\lambda_2^{2/p}\right) \\
 & + \left(1 - \frac{2\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \frac{8\lambda_2^{2/p} \ln^2(\lambda_2)}{p^4(1 + \bar{\gamma})} \exp\left(-\frac{2\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) - \left(1 - 2\lambda_2^{2/p}\right) \frac{8\lambda_2^{2/p} \ln^2(\lambda_2)}{p^4} \exp\left(-2\lambda_2^{2/p}\right) \\
 & + \left(\frac{2\lambda_2^{2/p} \ln(\lambda_2) + 2\lambda_1^{2/p} \ln(\lambda_1)}{p^2(1 + \bar{\gamma})}\right)^2 \exp\left(-\frac{\lambda_1^{2/p} - \lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \\
 & - \left[\frac{4\lambda_2^{2/p} \ln(\lambda_2) + 4\lambda_1^{2/p} \ln(\lambda_1)}{p^3(1 + \bar{\gamma})} + \frac{4\lambda_2^{2/p} \ln^2(\lambda_2) + 4\lambda_1^{2/p} \ln^2(\lambda_1)}{p^4(1 + \bar{\gamma})}\right] \exp\left(-\frac{\lambda_1^{2/p} - \lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \\
 & - \left(\frac{2\lambda_2^{2/p} \ln(\lambda_2) + 2\lambda_1^{2/p} \ln(\lambda_1)}{p^2}\right)^2 \exp\left(-\lambda_1^{2/p} - \lambda_2^{2/p}\right) \\
 & + \left[\frac{4\lambda_2^{2/p} \ln(\lambda_2) + 4\lambda_1^{2/p} \ln(\lambda_1)}{p^3} + \frac{4\lambda_2^{2/p} \ln^2(\lambda_2) + 4\lambda_1^{2/p} \ln^2(\lambda_1)}{p^4}\right] \exp\left(-\lambda_1^{2/p} - \lambda_2^{2/p}\right)
 \end{aligned} \tag{3.43}$$

Figure 3.5 shows that (3.43) is positive over different values of $(p, \Delta\lambda)$ of our working region with $K = 1$, which implies the convexity of the function for the case of single SU. Hence, there is an optimum p to minimize P_{err} given other parameters in this case. However, as a general case, we apply an exhaustive search to find the optimum p over different values of K .

Figure 3.5: Second order derivative of P_{err} w.r.t p vs IDED parameters with $\bar{\gamma} = 5$ dB

Now, we analyze P_{err} with respect to the difference between two thresholds ($\Delta\lambda$) as

$$\frac{\partial P_{err}}{\partial \Delta\lambda} = 0 \quad \Rightarrow \quad \frac{\partial Q_f}{\partial \Delta\lambda} - \frac{\partial Q_d}{\partial \Delta\lambda} = 0 \quad (3.44)$$

where from (2.25) and (2.26),

$$\begin{aligned} \frac{\partial Q_f}{\partial \Delta\lambda} = & \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} \left[\frac{\partial P_f}{\partial \Delta\lambda} l P_f^{l-1} P_{d0}^{K_d-l} \Delta_0^{K-K_d} (1 - \Delta_0^K) \right. \\ & + \frac{\partial P_{d0}}{\partial \Delta\lambda} P_f^l (K_d - l) P_{d0}^{K_d-l-1} \Delta_0^{K-K_d} (1 - \Delta_0^K) \\ & \left. + \frac{\partial \Delta_0}{\partial \Delta\lambda} P_f^l P_{d0}^{K_d-l} \left[(K - K_d) \Delta_0^{K-K_d-1} + (K_d - 2K) \Delta_0^{2K-K_d-1} \right] \right] \end{aligned} \quad (3.45)$$

$$\begin{aligned}
 \frac{\partial Q_d}{\partial \Delta \lambda} = & \sum_{K_d=1}^K \sum_{l=M}^{K_d} \binom{K}{K_d} \binom{K_d}{l} \left[\frac{\partial P_d}{\partial \Delta \lambda} l P_d^{l-1} P_m^{K_d-l} \Delta_1^{K-K_d} (1 - \Delta_1^K) \right. \\
 & + \frac{\partial P_m}{\partial \Delta \lambda} P_d^l (K_d - l) P_m^{K_d-l-1} \Delta_1^{K-K_d} (1 - \Delta_1^K) \\
 & \left. + \frac{\partial \Delta_1}{\partial \Delta \lambda} P_d^l P_m^{K_d-l} \left[(K - K_d) \Delta_1^{K-K_d-1} + (K_d - 2K) \Delta_1^{2K-K_d-1} \right] \right]
 \end{aligned} \tag{3.46}$$

in which $P_e \approx 0$ is assumed. From (3.25)-(3.30), we have

$$\frac{\partial \Delta_0}{\partial \Delta \lambda} = - \left(\frac{\partial P_f}{\partial \Delta \lambda} + \frac{\partial P_{d0}}{\partial \Delta \lambda} \right) \tag{3.47}$$

$$\frac{\partial \Delta_1}{\partial \Delta \lambda} = - \left(\frac{\partial P_d}{\partial \Delta \lambda} + \frac{\partial P_m}{\partial \Delta \lambda} \right) \tag{3.48}$$

and

$$\frac{\partial P_f}{\partial \Delta \lambda} = - \frac{\xi_2}{p \lambda_2} \exp(-\xi_2) \tag{3.49}$$

$$\frac{\partial P_{d0}}{\partial \Delta \lambda} = - \frac{\xi_1}{p \lambda_1} \exp(-\xi_1) \tag{3.50}$$

$$\frac{\partial P_d}{\partial \Delta \lambda} = - \frac{\xi_2}{p \lambda_2 (1 + \bar{\gamma})} \exp\left(-\frac{\xi_2}{1 + \bar{\gamma}}\right) \tag{3.51}$$

$$\frac{\partial P_m}{\partial \Delta \lambda} = - \frac{\xi_1}{p \lambda_1 (1 + \bar{\gamma})} \exp\left(-\frac{\xi_1}{1 + \bar{\gamma}}\right) \tag{3.52}$$

Again, by considering $K = 1$, (3.44) is simplified to

$$\begin{aligned}
 & 2 \exp(-2\lambda_2^{2/p}) + \exp(-\lambda_2^{2/p}) + \left(\left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{2}{p}-1} - 1 \right) \exp(-\lambda_1^{2/p} - \lambda_2^{2/p}) \\
 & - \frac{1}{(1 + \bar{\gamma})} \left[2 \exp\left(-\frac{2\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) + \exp\left(-\frac{\lambda_2^{2/p}}{1 + \bar{\gamma}}\right) + \left(\left(\frac{\lambda_1}{\lambda_2} \right)^{\frac{2}{p}-1} - 1 \right) \exp\left(-\frac{\lambda_1^{2/p} - \lambda_2^{2/p}}{1 + \bar{\gamma}}\right) \right] = 0
 \end{aligned} \tag{3.53}$$

Similar to (3.42), (3.53) shows that it is related to the average SNR of the sensing channels ($\bar{\gamma}$) and IDED parameters (p, λ_1, λ_2) in addition to K, K_d and M . From (2.13), λ_1 and λ_2 are related to $\Delta \lambda$. Since we want to disclose the profit of double-threshold method as compared with the single-threshold one, the optimum (λ_1, λ_2) are found with respect to the optimum λ . It means that $\lambda_{2,opt} = \lambda_{opt} + \Delta \lambda_{opt}/2$ and $\lambda_{1,opt} = \lambda_{opt} - \Delta \lambda_{opt}/2$. Hence, there is an optimum $\Delta \lambda$ to minimize P_{err} given other parameters.

As a summary, our performance criterion over the IDED scheme remains as

$$\underset{p, \Delta\lambda}{\text{minimize}} : P_{err}(p, \Delta\lambda) \quad (3.54a)$$

$$\text{subject to} : p, \Delta\lambda > 0 \quad (3.54b)$$

To justify the mathematical analysis of the proposed IDED performance, the minimum achievable P_{err} versus p (in their relevant optimum $\Delta\lambda$) and $\Delta\lambda$ (in their relevant optimum p) through the IDED method are, respectively, represented in Figures 3.6 and 3.7 for the majority fusion rule with different K . From these figures, it is clear that there exist optimum p and $\Delta\lambda$ given other parameters to achieve the minimum P_{err} in the case of the IDED. They also illustrate that the optimization power of the IDED will be improved as K increases. In addition, we can understand from Figure 3.6 that P_{err} can be decreased more by choosing a suitable p which is not necessarily two (which implies the CDED). Therefore, the IDED has an optimization gain as compared with the CED.

From Figure 3.6, the minimum P_{err} for $K = 10$ is achieved by $p = 1.3$ which is further explained in Figures 3.8 and 3.9. Figure 3.8 shows P_{err} versus $\Delta\lambda$ with $K = 10$ for different p . It is revealed that the P_{err} decreases from 10.5 % in the case of ISED ($\Delta\lambda = 0$) to 4.6 % in the case of IDED (in optimum $\Delta\lambda$) using the optimum exponent ($p = 1.3$), which is more than 50 % improvement. In Figure 3.9, we separately show Q_f and Q_m versus $\Delta\lambda$ with $p = 1.3$ for $K = 10$. It can be observed that the IDED improves both Q_f and Q_m to minimize P_{err} using the optimum values of p and $\Delta\lambda$, whereas it reduces Q_f more than Q_m . It means that this method increases spectrum utilization (which is related to Q_f) more than reliability (which is related to Q_m).

Finally, we have disclosed that the IED method can improve the minimum achievable P_{err} over Rayleigh-faded sensing channels (as compared with the CED) when it is used with the double-threshold comparison method. Thus, the proposed IDED method has an performance gain to minimize P_{err} (in terms of either p or $\Delta\lambda$) as compared with other schemes (CSED, CDED and ISED) when the sensing channel is Rayleigh-faded.

Figure 3.6: The minimum total error rate vs. p in the optimum $\Delta\lambda$ with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the proposed IDED

Figure 3.7: The minimum total error rate vs. $\Delta\lambda$ in the optimum p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ for different K of the majority fusion rule through the proposed IDED

Figure 3.8: The total error rate vs. $\Delta\lambda$ in different p with $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the proposed IDED

Figure 3.9: Total probabilities (Q_m , Q_f) vs. $\Delta\lambda$ with $p = 1.3$, $\bar{\gamma} = 5$ dB, $P_e = 1\%$ and $K = 10$ for the majority fusion rule through the proposed IDED

3.2 Transmission throughput

In this step, the performance analysis of the CRN in the case of N -sample based IED, as in (2.3), has been considered. Since the sensing period includes N samples of the primary signal, it is likely that the PU changes its activity status along detection part. Here, we first study the low-traffic primary network case in which the PU status is remained unchanged during the sensing time of the secondary network. The objective is to analyze the effects of different ED methods on the transmission throughput of the secondary network over both perfect and imperfect reporting channels. We first use the real AWGN sensing channel model in [57]. By deriving IED expression over Rayleigh-faded sensing channel, we extend our work to the Rayleigh fading case. It will be shown that the minimum sensing time (results in higher maximum achievable transmission throughput) over imperfect reporting channels is achieved by using the proposed IDED.

Suppose that the PU's probability of idleness is P_{H_0} (while its probability of activity is $P_{H_1} = 1 - P_{H_0}$) with the average idle time of T as described in Section 2.2.1. Hence, the total achievable transmission throughput of the secondary network is [72]

$$R(\tau, \Lambda) = R_0(\tau, \Lambda) + R_1(\tau, \Lambda) \quad (3.55)$$

where

$$R_0(\tau, \Lambda) = (1 - Q_f(\tau, \Lambda)) \left(1 - \frac{\tau}{T}\right) P_{H_0} C_0 \quad (3.56)$$

and

$$R_1(\tau, \Lambda) = (1 - Q_d(\tau, \Lambda)) \left(1 - \frac{\tau}{T}\right) P_{H_1} C_1 \quad (3.57)$$

are the average throughputs of SUs with channel capacities C_0 and C_1 under hypotheses H_0 and H_1 , respectively, in which

$$\Lambda = \begin{cases} \lambda & \text{conventional single-threshold energy detector (CSED)} \\ (\lambda_1, \lambda_2) & \text{conventional double-threshold energy detector (CDED)} \\ (\lambda, p) & \text{improved single-threshold energy detector (ISED)} \\ (\lambda_1, \lambda_2, p) & \text{improved double-threshold energy detector (IDED)} \end{cases} \quad (3.58)$$

Using more samples in (2.3) empowers SUs to make their local decisions with higher certainty (higher reliability of detection), while larger N results in a longer sensing time from (2.5) and degrades the average throughput of the secondary network, as shown in (3.55)-(3.57). Therefore, there is a trade-off between transmission throughput and sensing

reliability with respect to N . To optimize the transmission throughput over suitable detection reliability, we define some constraints:

- (i) $R_{\min}$: Minimum acceptable throughput
- (ii) $\overline{Q_f}$: Maximum tolerable level of Q_f (false alarm's acceptable range is $[0, \overline{Q_f}]$)
- (iii) $\underline{Q_d}$: Minimum tolerable level of Q_d (detection's acceptable range is $[\underline{Q_d}, 1]$)

Thus, the optimization problem is expressed as

$$\underset{\tau, \Lambda}{\text{maximize :}} \quad R(\tau, \Lambda) \quad (3.59a)$$

$$\text{subject to :} \quad R(\tau, \Lambda) > R_{\min} \quad (3.59b)$$

$$Q_f(\tau, \Lambda) < \overline{Q_f} \quad (3.59c)$$

$$Q_d(\tau, \Lambda) > \underline{Q_d} \quad (3.59d)$$

$$\Lambda > 0 \quad (3.59e)$$

From (3.55)-(3.57), the maximum acceptable sensing time under required constraints is given by

$$\tau_{\max} = T \left(1 - \frac{R_{\min}}{(1 - \overline{Q_f})P_{H_0}C_0 + (1 - \underline{Q_d})P_{H_1}C_1} \right) \quad (3.60)$$

and, using (2.5), the maximum acceptable number of samples can be obtained as

$$N_{\max} = \frac{T}{T_s} \left(1 - \frac{R_{\min}}{(1 - \overline{Q_f})P_{H_0}C_0 + (1 - \underline{Q_d})P_{H_1}C_1} \right) \quad (3.61)$$

Later, by introducing PDFs over AWGN and Rayleigh-faded sensing channels, we show that the total probabilities are functions of N , and there is a minimum required number of samples ($N_{\min}$) to jointly satisfy both second and third constraints. Hence, the optimization problem can be simplified as

$$\underset{N, \Lambda}{\text{maximize :}} \quad R(N, \Lambda) \quad (3.62a)$$

$$\text{subject to :} \quad N_{\min} \leq N \leq N_{\max} \quad (3.62b)$$

$$\Lambda > 0 \quad (3.62c)$$

when there is an optimum number of samples (N_{opt}) satisfying both throughput (desired by secondary network) and detection reliability (desired by primary network) based on the predetermined requirements.

Since our focus is to optimize throughput over a suitable detection performance, we use the OR fusion rule to make the global decision by FC, which is more strict about the

interference protection of the primary signal as described in Section 2.2.3. Thus, (2.20) and (2.21) of single-threshold based CSS are simplified to the OR fusion rule as

$$Q_f = 1 - [P_{d0}(1 - P_e) + P_f P_e]^K \quad (3.63)$$

$$Q_d = 1 - [P_m(1 - P_e) + P_d P_e]^K \quad (3.64)$$

and for (2.25) and (2.26) of double-threshold based CSS, we have

$$Q_f = (1 - \Delta_0^K) \left[1 - \sum_{K_d=1}^K \binom{K}{K_d} [P_{d0}(1 - P_e) + P_f P_e]^{K_d} \Delta_0^{K-K_d} \right] \quad (3.65)$$

$$Q_d = (1 - \Delta_1^K) \left[1 - \sum_{K_d=1}^K \binom{K}{K_d} [P_m(1 - P_e) + P_d P_e]^{K_d} \Delta_1^{K-K_d} \right] \quad (3.66)$$

Now, we need the PDF of IED's outputs to obtain the local probabilities of different comparison methods as described in Section 2.2.2. Hence, we continue the analysis over different models of sensing channels separately.

3.2.1 Real AWGN sensing channels

Firstly, we use the channel model as in [57]. The author derived the IED expression under real AWGN sensing channels. It means that the imaginary part of received signal in (2.1) is assumed to be zero. Thus, without loss of generality we consider $u_i(n) \sim \mathcal{N}(0, \sigma_n^2)$ in (2.1)-(2.3). Therefore, Y_i in (2.3) is Chi-squared distributed in the case of CED ($p = 2$) and it can be shown as gamma distribution as

$$f_{Y_i|H_s}(y) = \frac{1}{(\theta_{H_s})^{k_{H_s}} \Gamma(k_{H_s})} y^{k_{H_s}-1} \exp\left(-\frac{y}{\theta_{H_s}}\right), \quad y \geq 0 \quad (3.67)$$

when $H_s \in \{H_0, H_1\}$ with the shape parameters of $k_{H_0} = k_{H_1} = N/2$ and the scale parameters of $\theta_{H_0} = 2/N$ and $\theta_{H_1} = 2(1+\bar{\gamma})/N$ under hypotheses H_0 and H_1 , respectively. From (2.2), the average SNR of AWGN sensing channels is $\bar{\gamma} = \sigma_s^2/\sigma_n^2$. Note that the Chi-squared distribution is a special case of the gamma distribution. The author in [57] showed that the general PDF of the IED output cannot be expressed as Chi-squared, but it can be well approximated as gamma distribution as in (3.67) with parameters

$$k_{H_s} = \frac{\mathbb{E}^2 \{Y_i|H_s\}}{\mathbb{V}\text{ar} \{Y_i|H_s\}} \quad (3.68)$$

and

$$\theta_{H_s} = \frac{\mathbb{V}\text{ar} \{Y_i|H_s\}}{\mathbb{E} \{Y_i|H_s\}} \quad (3.69)$$

in which the means and variances under both hypotheses are, respectively,

$$\begin{aligned} \mathbb{E} \{Y_i|H_0\} &= \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \\ \mathbb{V}\text{ar} \{Y_i|H_0\} &= \frac{2^p}{N\sqrt{\pi}} \Gamma\left(\frac{2p+1}{2}\right) - \frac{2^p}{N\pi} \Gamma^2\left(\frac{p+1}{2}\right) \end{aligned} \quad (3.70)$$

and

$$\begin{aligned} \mathbb{E} \{Y_i|H_1\} &= (1 + \bar{\gamma})^{p/2} \mathbb{E} \{Y_i|H_0\} \\ \mathbb{V}\text{ar} \{Y_i|H_1\} &= (1 + \bar{\gamma})^p \mathbb{V}\text{ar} \{Y_i|H_0\} \end{aligned} \quad (3.71)$$

For our numerical analysis, we define the required constraints as $\underline{Q}_f = 10\%$, $\overline{Q}_d = 99\%$ and $R_{\min} = 1$ bps with the channel capacity of $C_0 = 2$ bps when the PU is idle with probability $P_{H_0} = 60\%$. The CRN includes 10 SUs ($K = 10$), in which they send their local decisions through BSC-modeled reporting channels with error probability of $P_e = 1\%$ in the case of erroneous. The available frame duration of the secondary network

Figure 3.10: Maximum achievable throughput vs. number of samples under perfect reporting channels ($P_e = 0$) with $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule

is $T = 20$ ms and the received signal is sampled by a period of $T_s = 1$ μ s. Firstly, we assume that the average SNR of the sensing channels is $\bar{\gamma} = 0$ dB and try to find the N_{opt} over the range of $[N_{min}, N_{max}]$. Then, we present the effect of SNR on the maximum throughput. Figure 3.10 presents the maximum achievable throughput of different ED methods versus the number of samples in the case of perfect reporting channels. It can be found that there is no optimization gain to use either IED or double-threshold methods as compared with the CED when reporting channels are error-free ($P_e = 0$).

The maximum achievable throughput of different ED methods against the number of samples in the case of imperfect reporting channels are shown in Figure 3.11. For a better comparison, the maximum throughput curve through perfect reporting channels is also included as reference. It shows that the maximum throughput is very sensitive to the condition of the reporting channels. In the case of imperfect reporting channels, the ISSED cannot improve transmission throughput any more, but double-threshold methods (CEDED and IDDED) can increase it a lot, whereas the IDDED has better performance. We observe a lower value of optimum N , achieved by the IDDED, resulting in higher throughput.

To go deeply, the maximum achievable Q_d and the minimum achievable Q_f (in their optimum thresholds) of different ED schemes are analyzed over the range of N through imperfect reporting channels in Figures 3.12. They show that the Q_f has more influence on throughput improvement as compared with the Q_d . Actually, double-threshold schemes

Figure 3.11: Maximum achievable throughput vs. number of samples over imperfect reporting channels as compared with the perfect one with $P_e = 1\%$, $\bar{\gamma} = 0\text{dB}$ and $K = 10$ for the OR fusion rule

can retrieve the lost throughput by affecting on the Q_f lower bound of single-threshold schemes in (2.24), which is apparent in Figure 3.12.c as $\underline{Q}_f \approx KP_e = 0.1$.

Furthermore, Figure 3.13 represents the normalized maximum achievable throughput of different ED methods versus average sensing channels' SNR over imperfect reporting channels. In this figure, the maximum transmission throughput over imperfect reporting channels is normalized by that of perfect channels for different SNRs. It is revealed that the normalized throughput is not changed in the CSED. It means that the SNR affects the maximum achievable transmission throughput of a CRN through the single-threshold based ED over an imperfect reporting channel as well as the perfect one. It is also clear that the IDED has a gain in SNR as compared with the CDED. Based on our analyzed results, the N_{opt} is different for each point of Figure 3.13, and it decreases from 720 in the case of $\bar{\gamma} = 0\text{dB}$ to 2 for the case of $\bar{\gamma} = 15\text{dB}$ under the IDED. It is reasonable due to the better channel condition.

 3.12.a: Maximum Q_d over single-threshold based comparison schemes

 3.12.b: Maximum Q_d over double-threshold based comparison schemes

 3.12.c: Minimum Q_f over single-threshold based comparison schemes

 3.12.d: Minimum Q_f over double-threshold based comparison schemes

 Figure 3.12: Optimum probabilities of different EDs vs. N over imperfect reporting channels with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule

Figure 3.13: Normalized maximum throughput vs. average SNR over imperfect reporting channels with $P_e = 1\%$ and $K = 10$ for the OR fusion rule

3.2.2 Rayleigh-faded sensing channels

To extend our work in the previous section to the case of Rayleigh fading, we adopt the channel model which has been used in [35, 42, 43, 96]. Since the received signal in (2.1) through a Rayleigh-faded channel is a complex Gaussian distributed random variable under both hypotheses [42, 96, 97], the PDF of Y_i in the case of CED is still Chi-squared distributed, and can be expressed as gamma distribution as in (3.67) with the shape parameters of $k_{H_0} = k_{H_1} = N$ and the scale parameters of $\theta_{H_0} = 1/N$ and $\theta_{H_1} = (1 + \bar{\gamma})/N$ under hypotheses H_0 and H_1 , respectively. Note that the average SNR of Rayleigh-faded sensing channels in (2.2) is $\bar{\gamma} = (\sigma_h^2 \sigma_s^2) / \sigma_n^2$.

Similar to Section 3.2.1, the IED's output (when $p \neq 2$) is not Chi-squared distributed. We show that it can be still approximated as gamma distribution as in (3.67), especially for large N , with the parameters as defined in (3.68) and (3.69). The difference is that the means and variances are matched to the case of Rayleigh fading under both hypotheses as

$$\begin{aligned}\mathbb{E}\{Y_i|H_0\} &= \Gamma\left(\frac{p+2}{2}\right) \\ \mathbb{V}\text{ar}\{Y_i|H_0\} &= \frac{1}{N} \left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right) \right]\end{aligned}\tag{3.72}$$

and

$$\begin{aligned}\mathbb{E}\{Y_i|H_1\} &= (1 + \bar{\gamma})^{p/2} \mathbb{E}\{Y_i|H_0\} \\ \mathbb{V}\text{ar}\{Y_i|H_1\} &= (1 + \bar{\gamma})^p \mathbb{V}\text{ar}\{Y_i|H_0\}\end{aligned}\tag{3.73}$$

They have been proven in Appendix A.

To justify the approximate density function of the N -sample based IED under Rayleigh-faded sensing channels, we have simulated it with 10^7 signal samples which seems enough to have a reliable comparison. Figure 3.14.a shows the approximate and simulated CDFs for an N -sample based IED with different p in the case of idle PU (H_0). Similarly, it is justified for the presence of PU (H_1) in Figures 3.14.b and 3.14.c with different SNRs of $\bar{\gamma} = 0$ dB and $\bar{\gamma} = -10$ dB.

For a small number of received samples ($N = 10$), it is revealed that the approximation can be more accurate under a SNR as low as $\bar{\gamma} = -10$ dB when p is equal or higher than its conventional value ($p \geq 2$). Furthermore, Figures 3.14.d-3.14.f show that the simulated CDF is closer to the approximate one with a higher number of samples which is good enough for our later throughput analysis.

For a detailed statistical analysis, the MSE of approximate distributions over H_1 is also evaluated versus different values of p for different SNRs in Figure 3.15. The MSE of approximate Gaussian distribution, using the central limit theorem (CLT), is also

 Figure 3.14: CDF of N -sample based IED output

presented as reference. Based on our analysis, the optimum p will not be greater than 4 in this particular system model. From Figure 3.15.a, for a high value of N , the MSE of the gamma approximation remains below 10^{-4} within $1 < p < 4$ while it is at least ten times greater for Gaussian approximation. Furthermore, the MSE of the gamma approximation

 3.15.a: Active PU (H_1) and $N = 100$

 3.15.b: Active PU (H_1) and $N = 10$

 Figure 3.15: Mean square error of approximate distribution of N -sample based IED vs. p over different SNRs

is not greater than 10^{-3} in the case of $N = 10$ in Figure 3.15.b. From these figures, it is observed that the approximate gamma distribution of the N -sample based IED is good enough for our further analysis while the Gaussian approximation is not.

Note that the CLT is valid just for high values of N and it is not useful to analyze those performance parameters (such as the transmission throughput) in which we need also to evaluate them for low values of N , where the gamma approximation is still acceptable. Moreover, one can see that the simulated points are exactly matched to those of the approximation in the case of CED ($p = 2$) with very low MSE.

By using justified results, we are now able to extend the transmission throughput analysis of Section 3.2.1 to the case of Rayleigh-faded sensing channels. In this part, we again consider the defined constraints, parameters and conditions in the previous section.

Similar to Section 3.2.1, Figures 3.16 and 3.17 show the maximum achievable transmission throughput of different ED methods over perfect and imperfect reporting channels, respectively. Again, it is clear that there is no difference on using suggested EDs whenever reporting channels are perfect. Figure 3.17 includes the curve of the maximum throughput over perfect reporting channels for reference. They show that the maximum throughput is still sensitive to the condition of reporting channels in the case of Rayleigh-faded as well as AWGN sensing channels, and double-threshold comparison based schemes are able to improve the degraded throughput in the case of erroneous reporting channels whereas single-threshold schemes cannot do it. Moreover, it is observed that the proposed IDED method has an optimization profit as compared with the CDED. To analyze it further, the maximum achievable Q_d and the minimum achievable Q_f of different ED methods over imperfect reporting channels versus N are shown in Figure 3.18. They reveal that the double-threshold approach affects the transmission throughput by improving Q_f more than Q_d (the same result as that of the AWGN case) and again double-threshold based methods improve maximum throughput by combating on the Q_f lower bound in the case of the OR fusion rule over single-threshold based methods. One can see that the IDED minimizes the achieved Q_f in a lower N as compared with the CDED which results in a higher throughput.

Finally, the normalized maximum achievable throughputs (with respect to the maximum throughput over perfect reporting channels) of different ED methods versus the average SNR of the sensing channels are shown in Figure 3.19. It is observed that the normalized throughput is almost fixed for the case of CSED as a single-threshold scheme. Again, this figure represents the SNR gain of the IDED as compared with the CDED. The N_{opt} changes from one point to another point in Figure 3.19, and for the IDED, it decreases from 328 to 4 when the SNR increases from $\bar{\gamma} = 0$ dB to $\bar{\gamma} = 10$ dB.

Based on the achieved results in both types of sensing channels, one can observe the IDED ability among suggested ED methods to improve the maximum transmission throughput of the secondary network over erroneous reporting channels under either AWGN or Rayleigh-faded sensing channels. It is also revealed that the IDED behaves better in the case of Rayleigh-faded sensing channels as compared with the AWGN one in terms of maximum achievable throughput by using less N_{opt} and supporting a lower range of the average SNR.

Figure 3.16: Maximum achievable throughput vs. number of samples under perfect reporting channels ($P_e = 0$) with $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule

Figure 3.17: Maximum achievable throughput vs. number of samples over imperfect reporting channels as compared with the perfect one with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule

 3.18.a: Maximum Q_d over single-threshold based comparison schemes

 3.18.b: Maximum Q_d over double-threshold based comparison schemes

 3.18.c: Minimum Q_f over single-threshold based comparison schemes

 3.18.d: Minimum Q_f over double-threshold based comparison schemes

 Figure 3.18: Optimum probabilities of different EDs vs. N over imperfect reporting channels with $P_e = 1\%$, $\bar{\gamma} = 0$ dB and $K = 10$ for the OR fusion rule

Figure 3.19: Normalized maximum throughput vs. average SNR over imperfect reporting channels with $P_e = 1\%$ and $K = 10$ for the OR fusion rule

3.3 Conclusion

In this chapter, we focused on the CSS over imperfect reporting channels when the number of available cooperative SUs is deterministic and the primary network has a low traffic rate. Considering the IED and both single- and double-threshold methods, we first analyzed detection performance of different ED-based schemes using one-sample based IED (the minimum sensing time). We observed that the IED can be able to improve the detection performance of the system just by utilizing the double-threshold method, called IDED. Finally, we considered the N -sample based IED and used the OR fusion rule (to be more strict about interference). By defining some constraints as our minimum required throughput and detection reliability, we aimed to find the optimum ED method over both perfect and imperfect reporting channels with respect to the SU's throughput. It has been shown that the maximum achievable throughput over perfect reporting channels is almost the same for all analyzed ED-based schemes while they affect differently over imperfect reporting channels. It is observed that the proposed IDED has an optimization gain as compared with the others when reporting channels are erroneous. Furthermore, the SNR gain of the IDED has been revealed in terms of maximum normalized throughput.

As a summary, we showed that the IDED (either one- or N -sample) outperforms others in terms of both detection reliability and throughput optimization in this particular CRN.

Chapter 4

Deterministic-modeled secondary users in a medium-traffic network

In the previous chapter, the status of the PU has been considered as invariant during the sensing time (either active or idle). This assumption is practical but we should mention that it is true whenever the traffic of the primary network is low and the mean idle/active time of the PU is high enough as compared with the test period.

Here, we consider a medium-traffic primary network to study a more complicated case. The PU may either arrive at or depart from the network at most once among the frame duration¹. This situation degrades the detection accuracy as well as transmission throughput of SUs due to more uncertainty on the presence of the PU.

We first need to find an appropriate distribution for the observed energy levels when the PU may switch to another state during the sensing time. We then consider the achieved results to analyze the performance of the different ED methods in terms of detection reliability and transmission throughput of the secondary network when it is suffered by a medium-traffic primary network.

The status changing process of the PU is considered as a two-state process in which the holding times are independent exponential random variables with parameters λ_i (related to the PU's idle period) and λ_b (related to the PU's busy period). Thus for their transition probabilities we have [95, p. 782]

$$P(t) = \begin{pmatrix} P_{00}(t) & P_{01}(t) \\ P_{10}(t) & P_{11}(t) \end{pmatrix} = \frac{1}{\lambda_i + \lambda_b} \begin{pmatrix} \lambda_b + \lambda_i e^{-(\lambda_i + \lambda_b)t} & \lambda_i - \lambda_i e^{-(\lambda_i + \lambda_b)t} \\ \lambda_b - \lambda_b e^{-(\lambda_i + \lambda_b)t} & \lambda_i + \lambda_b e^{-(\lambda_i + \lambda_b)t} \end{pmatrix} \quad (4.1)$$

¹Since our main concern is the availability of a desired frequency band from the perspective of the SUs, there is no difference to define the traffic rate of the primary network based on the status changing process of either single PU or multiple PUs in that frequency band. Hence, the traffic rate of the primary network is only defined based on the mean holding times of the PU(s) in the desired channel and there is no need to define any specific parameter for the number of PUs in the network.

Considering the same frame structure as in Figure 2.1, there are different situations, called H_{uv} , for status changing of the PU within the frame duration where

$$\begin{aligned} u \in \begin{cases} 0 & \text{PU is idle at the end of the sensing part} \\ 1 & \text{PU is active at the end of the sensing part} \end{cases} \\ v \in \begin{cases} 1 & \text{no status changing} \\ 2 & \text{status changing during the sensing part} \\ 3 & \text{status changing after the sensing part} \end{cases} \end{aligned} \quad (4.2)$$

We also define J_0 and J_1 as the numbers of samples in which the PU's status is constant, and thereafter it goes to the idle state (from the active state) and vice versa, respectively.

4.1 Distribution of observed energy level

In this section, we first conduct our analysis using CED over AWGN sensing channels. By employing the IED, we then extend it to the Rayleigh-faded case.

4.1.1 CED over AWGN sensing channels

For a non-average CED, the observed energy level of N normalized received samples at the i -th SU is obtained by

$$Y_i = \begin{cases} \sum_{m=1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & \tilde{H}_{u2} \\ \sum_{m=1}^{J_u} \left(\frac{|r_i^{\tilde{u}}(m)|}{\sigma_n} \right)^2 + \sum_{m=J_u+1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & H_{u2} \end{cases} \quad (4.3)$$

in which $u, \tilde{u} \in \{0, 1\}$ are the one's complement of each other, $\tilde{H}_{u2}$ means that the changing process does not happen during the sensing part, and $r_i^u(m)$ is considered as (2.1) for the case of the AWGN sensing channel. Note that both idle and busy times are exponentially distributed with means $\mu_i = 1/\lambda_i$ and $\mu_b = 1/\lambda_b$ [69, 98]. Therefore, the probabilities of being idle and active (busy) for the PU at each time instant are P_i and P_b (known as the duty cycle), respectively, which can be defined as follows:

$$\begin{aligned} P_i &= \frac{\mu_i}{(\mu_i + \mu_b)} \\ P_b &= \frac{\mu_b}{(\mu_i + \mu_b)} \end{aligned} \quad (4.4)$$

The non-average CED is considered only for understanding the distribution derivation easily. For the average energy level, we just need to divide (4.3) by N .

The observed energy level, Y_i , is gamma distributed where the state of the PU is fixed within the sensing period ($\tilde{H}_{02}$ and $\tilde{H}_{12}$). Since there is no closed-form distribution for other two cases (H_{02} and H_{12}), we need to approximate them. From (4.3), one can see that there is a summation of two gamma distributed parts whose parameters have different values in both H_{02} and H_{12} cases. Thus, one approach is to approximate them as gamma distribution as (3.67) with the shape and scale parameters provided in (3.68) and (3.69), respectively. Note that H_s is representative of the different possible situations and the means and variances of different cases are given in Table 4.1.

Applying the Gaussian approximation is another approach when the PDF of observed energy in (4.3) is [95, p. 162]

$$f_{Y_i|H_s}(y) = \frac{1}{\sqrt{2\pi\text{Var}\{Y_i|H_s\}}} \exp\left(-\frac{(y - \mathbb{E}\{Y_i|H_s\})^2}{2\text{Var}\{Y_i|H_s\}}\right), \quad y \geq 0 \quad (4.5)$$

with the same means and variances as given in Table 4.1. It is worth mentioning that the approximation, based on the CLT, is valid whenever the number of samples, N , is large enough. Hence, we have a constraint when using the CLT that limits the scope of our parameter analysis. For example, it is not appropriate to use the Gaussian approximation to analyze the throughput of the network related to the sensing and transmission durations, whereas N can be as small as one.

The CDF and PDF of the estimated energy of the received signal have been shown in Figures 4.1 and 4.2, where the PU changes its status once during the sensing time (H_{02} and H_{12}). They represent both approaches of approximation in comparison with the simulation results over different SNRs. One can see that the gamma approximation is more accurate. Moreover, the MSE of the approximate methods with respect to the simulation results is shown in Figures 4.3 and 4.4 versus different numbers of sensing samples and different SNRs, respectively. It is found that the MSE in the gamma approximation is generally all under 10^{-4} over the range of N as well as different SNRs, which is good enough. On the other hand, the MSE is higher for the Gaussian approximation. Especially in low values of N , it lies above 10^{-3} . The same results are valid for other hypotheses.

As a consequence, the gamma approximation works better than the Gaussian approximation, and we will adopt it for our further performance analysis. More specifically, its usefulness would be prominent whenever we want to analyze a performance criterion such as transmission throughput over a wide range of N , starting from one.

Note that it is also valid for the average CED when means and variances of Table 4.1 are divided by N and N^2 , respectively.

Table 4.1: Means and variances of different cases with medium-traffic primary network over AWGN sensing channel, using CED

H_s	$\mathbb{E}$	$\mathbb{V}\text{ar}$
$\tilde{H}_{02}$	N	$2N$
$\tilde{H}_{12}$	$N(1 + \bar{\gamma})$	$2N(1 + \bar{\gamma})^2$
H_{02}	$J_0(1 + \bar{\gamma}) + (N - J_0)$	$2J_0(1 + \bar{\gamma})^2 + 2(N - J_0)$
H_{12}	$J_1 + (N - J_1)(1 + \bar{\gamma})$	$2J_1 + 2(N - J_1)(1 + \bar{\gamma})^2$

Since our aim is to check the analogy between probability distributions, we need to analyze the KLD as the distance between approximate PDFs and accurate distribution. Note that the KLD for two probability distributions p and q is known as their relative entropy and can be obtained by $D(p||q) = \sum_x p(x) \log(p(x)/q(x))$ [99, p. 19], where $p(x)$ and $q(x)$ typically represent exact and approximate distributions of x , respectively. Hence, in Figures 4.5 and 4.6, the KLD of both Gaussian and gamma distributions (with respect to the PDF of the simulated received energy) are illustrated for different parameters when the entropy of the measured energy level is about 16.6 bits. They imply that the KLD of the gamma approximation is always less than that of the Gaussian one. Moreover, Figure 4.6 shows that the Gaussian approximation is sensitive to the SNR while the distance for the gamma approximation is almost fixed over different qualities of the sensing channel (in terms of the SNR). Consequently, from the viewpoint of the error level, it is found that the MSE and KLD of the gamma approximation are generally less than their Gaussian counterpart, and the gamma approach can be considered as good enough.

Figure 4.1: CDF and PDF of CED outputs with $\bar{\gamma} = 0$ dB and $N = 100$ over AWGN sensing channel with medium-traffic primary network

Figure 4.2: CDF and PDF of CED outputs with $\bar{\gamma} = 10$ dB and $N = 100$ over AWGN sensing channel with medium-traffic primary network

Figure 4.3: MSE of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using CED over AWGN sensing channel with medium-traffic primary network

Figure 4.4: MSE of approximate distributions vs. average SNR with $N = 100$, using CED over AWGN sensing channel with medium-traffic primary network

Figure 4.5: KLD of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using CED over AWGN sensing channel with medium-traffic primary network

Figure 4.6: KLD of approximate distributions vs. average SNR with $N = 100$, using CED over AWGN sensing channel with medium-traffic primary network

4.1.2 IED over Rayleigh-faded sensing channels

In the next step, we try to analyze the accuracy of the approximate gamma distribution as compared with the Gaussian distribution, when the SUs suffer a Rayleigh-faded sensing channel. Moreover, by considering IED, we will evaluate a general case for our analysis. Hence, for a non-average IED over a Rayleigh-faded sensing channel, (4.3) will be

$$Y_i = \begin{cases} \sum_{m=1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^p, & \tilde{H}_{u2} \\ \sum_{m=1}^{J_u} \left(\frac{|r_i^{\tilde{u}}(m)|}{\sigma_n} \right)^p + \sum_{m=J_u+1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^p, & H_{u2} \end{cases} \quad (4.6)$$

where $r_i^u(m)$ is considered as (2.1) for the case of the Rayleigh-faded sensing channel. Similar to the previous section, the means and variances of different cases are obtained for the case of IED over Rayleigh fading, which are given in Table 4.2.

Approximate CDF and PDF of the IED's output have been shown in Figures 4.7-4.10 as compared with the simulated results for different power exponents and SNRs, where the PU changes its status once during the sensing time (H_{02} and H_{12}). The gamma approximation is still better than the Gaussian one even if $N = 100$, in which the CLT is expected to achieve an appropriate approximation.

Figures 4.11-4.13 show the MSE of the approximate methods with respect to the simulated results versus IED exponent parameter (p), number of sensing samples (N) and average SNR of sensing channels ($\bar{\gamma}$), respectively. Note that the provided MSEs are averaged over all possible PU's status changing cases (with different J_u 's). In Fig. 4.11, it is observed that the quality of the gamma approximation is higher within a reasonable range of p and it is more significant for $p = 2$ (implies the CED) which is expected.

Figure 4.12 depicts that the MSE of the gamma approach is always less than 10^{-3} for different values of N , and it goes below 10^{-5} for the CED which is quite acceptable for throughput analysis. Furthermore, the MSE of the gamma approximation is one to two orders of magnitude lower than that of the Gaussian approximation. In Figure 4.13, the use of the gamma approximation becomes significant when the quality of sensing channel is worse (low SNRs).

Moreover, the KLD of both Gaussian and gamma distributions (with respect to the PDF of the simulated received energy) are illustrated for different parameters in Figures 4.14-4.16, when the entropy of the measured energy level is about 16.7 bits. They imply that the KLD of the gamma approximation is always less than that of the Gaussian one. Moreover, Figure 4.14 shows that the Gaussian approximation is very sensitive to the exponent value while the distance for the gamma approximation is almost fixed. The same result can be obtained from Figure 4.16, when the variation of the Gaussian

Table 4.2: Means and variances of different cases with medium-traffic primary network over Rayleigh-faded sensing channel, using IED

H_s	$\mathbb{E}$	$\mathbb{V}\text{ar}$
$\tilde{H}_{02}$	$N\Gamma\left(\frac{p+2}{2}\right)$	$N\left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right)\right]$
$\tilde{H}_{12}$	$N(1+\bar{\gamma})^{\frac{p}{2}}\Gamma\left(\frac{p+2}{2}\right)$	$N(1+\bar{\gamma})^p\left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right)\right]$
H_{02}	$\left[J_0(1+\bar{\gamma})^{\frac{p}{2}} + (N - J_0)\right]\Gamma\left(\frac{p+2}{2}\right)$	$\left[J_0(1+\bar{\gamma})^p - (N - J_0)\right]\left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right)\right]$
H_{12}	$\left[J_1 + (N - J_1)(1+\bar{\gamma})^{\frac{p}{2}}\right]\Gamma\left(\frac{p+2}{2}\right)$	$\left[J_1 + (N - J_1)(1+\bar{\gamma})^p\right]\left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right)\right]$

distribution over different SNRs is more significant in Figure 4.16.b (as an example of the IED with $p > 2$) as compared with the CED ($p = 2$) in Figure 4.16.a. One can see that the KLD is still stable for the gamma approximation over different qualities of the sensing channel (in terms of the SNR) in both figures.

As a summary, from the viewpoint of the error level, it is found that the MSE and KLD of the gamma approximation are generally less than their Gaussian counterparts, and the gamma approach can be considered as good enough, especially in the case of the CED ($p = 2$). Furthermore, from the viewpoint of the stability over different exponents of the IED, the gamma approximation is again more reliable.

Figure 4.7: CDF and PDF of IED outputs with $p = 2$, $\bar{\gamma} = 0$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.8: CDF and PDF of IED outputs with $p = 3$, $\bar{\gamma} = 0$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.9: CDF and PDF of IED outputs with $p = 2$, $\bar{\gamma} = 10$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.10: CDF and PDF of IED outputs with $p = 3$, $\bar{\gamma} = 10$ dB and $N = 100$ over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.11: MSE of approximate distributions vs. p with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.12: MSE of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.13: MSE of approximate distributions vs. average SNR with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

Figure 4.14: KLD of approximate distributions vs. p with $N = 100$ and $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

 4.15.a: $p = 2$ (CED)

 4.15.b: $p = 3$

Figure 4.15: KLD of approximate distributions vs. number of sensing samples with $\bar{\gamma} = 0$ dB, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

 4.16.a: $p = 2$ (CED)

 4.16.b: $p = 3$

 Figure 4.16: KLD of approximate distributions vs. average SNR with $N = 100$, using IED over Rayleigh-faded sensing channel with medium-traffic primary network

4.2 Detection performance

Similar to Section 3.1, for the detection performance analysis of both single- and double-threshold methods, we consider the total error rate as defined in (3.1) and try to minimize it by finding the optimum threshold(s) in each possible method. Each SU obtains the energy level of the received signal through either CED as defined in (4.3) or the IED method as stated in (4.6). We will conduct the performance analysis of both comparison methods and justify them by simulation, numerically.

4.2.1 Probabilities formulation

Considering the detection performance degradation caused by increased uncertainty due to the higher traffic rate of the primary network, we utilize the CSS through the OR fusion rules as given in (3.63)-(3.66) which is more strict about primary signal protection from interference. Now we need to find the required local probabilities.

Note that, for detection analysis, we just need to focus on the sensing phase of the frame structure in Figure 2.1. Hence, the possible status changing of the PU during the sensing period is the matter of interest in this part. It means that $\tilde{H}_{02} = H_{01}$ and $\tilde{H}_{02} = H_{01}$ in (4.3) and (4.6), based on the definition provided in (4.2).

Since the PU changes its status at most once along the sensing period, the joint probabilities that the PU goes to another situation (either idle or active) after J_0 or J_1 samples of the total N considered samples are defined as

$$\begin{aligned} P_{J_0}^N &= \Pr \{H_{02}, J_0\} = P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(N-J_0-1)} \\ P_{J_1}^N &= \Pr \{H_{12}, J_1\} = P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(N-J_1-1)} \end{aligned} \quad (4.7)$$

Subsequently, the marginal probabilities of different hypotheses are

$$\begin{aligned} P_{H_{01}} &= P_i \cdot (P_{00}(T_s))^N \\ P_{H_{11}} &= P_b \cdot (P_{11}(T_s))^N \\ P_{H_{02}} &= \sum_{J_0=1}^{N-1} P_{J_0}^N = \sum_{J_0=1}^{N-1} P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(N-J_0-1)} \\ P_{H_{12}} &= \sum_{J_1=1}^{N-1} P_{J_1}^N = \sum_{J_1=1}^{N-1} P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(N-J_1-1)} \end{aligned} \quad (4.8)$$

Using the single-threshold scheme, the conditional probabilities of false alarm and

detection under different hypotheses can be obtained by

$$\begin{aligned}
 P_{f,1} &= \Pr \left\{ H_1 | \tilde{H}_{02} \right\} = \Pr \{ Y_i > \lambda | \text{PU remains idle at interval of sensing time } (0, \tau) \} \\
 P_{f,2} &= \Pr \{ H_1 | H_{02}, J_0 \} = \Pr \{ Y_i > \lambda | \text{PU goes to the idle state after } J_0 \text{ of } N \text{ samples} \} \\
 P_{d,1} &= \Pr \left\{ H_1 | \tilde{H}_{12} \right\} = \Pr \{ Y_i > \lambda | \text{PU remains active at interval of sensing time } (0, \tau) \} \\
 P_{d,2} &= \Pr \{ H_1 | H_{12}, J_1 \} = \Pr \{ Y_i > \lambda | \text{PU goes to the active state after } J_1 \text{ of } N \text{ samples} \}
 \end{aligned} \tag{4.9}$$

Subsequently, by considering $\tilde{H}_{02} = H_{01}$ and $\tilde{H}_{02} = H_{01}$ for analysis of the detection performance, the local probabilities are

$$P_f = \frac{P_{H_{01}} P_{f,1} + \sum_{J_0=1}^{N-1} P_{J_0}^N P_{f,2}}{P_{H_{01}} + P_{H_{02}}} \tag{4.10}$$

and

$$P_d = \frac{P_{H_{11}} P_{d,1} + \sum_{J_1=1}^{N-1} P_{J_1}^N P_{d,2}}{P_{H_{11}} + P_{H_{12}}} \tag{4.11}$$

Note that the local probabilities of detecting H_0 (P_{d0}) and missed detection (P_m) are obtained from (2.10) and (2.11), respectively.

Recall from Section 2.2.2.2, for the double-threshold scheme, each SU uses two thresholds as stated in (2.12) and sends no information to the FC whenever its estimated energy level falls between two thresholds. Similar to the case of the single-threshold scheme, the conditional probabilities of the double-threshold method will be defined as

$$\begin{aligned}
 P_{f,1} &= \Pr \left\{ H_1 | \tilde{H}_{02} \right\} = \Pr \{ Y_i > \lambda_2 | \text{PU remains idle at interval of sensing time } (0, \tau) \} \\
 P_{f,2} &= \Pr \{ H_1 | H_{02}, J_0 \} = \Pr \{ Y_i > \lambda_2 | \text{PU goes to the idle state after } J_0 \text{ of } N \text{ samples} \} \\
 P_{d,1} &= \Pr \left\{ H_1 | \tilde{H}_{12} \right\} = \Pr \{ Y_i > \lambda_2 | \text{PU remains active at interval of sensing time } (0, \tau) \} \\
 P_{d,2} &= \Pr \{ H_1 | H_{12}, J_1 \} = \Pr \{ Y_i > \lambda_2 | \text{PU goes to the active state after } J_1 \text{ of } N \text{ samples} \} \\
 P_{d0,1} &= \Pr \left\{ H_0 | \tilde{H}_{02} \right\} = \Pr \{ Y_i < \lambda_1 | \text{PU remains idle at interval of sensing time } (0, \tau) \} \\
 P_{d0,2} &= \Pr \{ H_0 | H_{02}, J_0 \} = \Pr \{ Y_i < \lambda_1 | \text{PU goes to the idle state after } J_0 \text{ of } N \text{ samples} \} \\
 P_{m,1} &= \Pr \left\{ H_0 | \tilde{H}_{12} \right\} = \Pr \{ Y_i < \lambda_1 | \text{PU remains active at interval of sensing time } (0, \tau) \} \\
 P_{m,2} &= \Pr \{ H_0 | H_{12}, J_1 \} = \Pr \{ Y_i < \lambda_1 | \text{PU goes to the active state after } J_1 \text{ of } N \text{ samples} \}
 \end{aligned} \tag{4.12}$$

Eventually, local probabilities of false alarm and detection can be obtained from (4.10)

and (4.11) and for the rest of local probabilities, we have

$$P_{d_0} = \frac{P_{H_{01}}P_{d_{0,1}} + \sum_{J_0=1}^{N-1} P_{J_0}^N P_{d_{0,2}}}{P_{H_{01}} + P_{H_{02}}} \quad (4.13)$$

and

$$P_m = \frac{P_{H_{11}}P_{m,1} + \sum_{J_1=1}^{N-1} P_{J_1}^N P_{m,2}}{P_{H_{11}} + P_{H_{12}}} \quad (4.14)$$

Note that the local probabilities of no decision (Δ_0 and Δ_1) are obtained from (2.16) and (2.19).

4.2.2 Numerical results by CED over AWGN sensing channels

In this section, the detection performance of the two comparison methods will be shown for the case of medium traffic rate of the primary network when the CED is utilized over the AWGN sensing channels. We use the gamma approximation of the energy distribution observed by SUs, as described in Section 4.1.1, to analyze the total error rate related to different parameters.

Suppose that the system is working with $N = 100$ and $T_s = 100 \mu\text{s}$, which implies a sensing time of $\tau = NT_s = 10 \text{ ms}$. The CSS is applied with 10 SUs ($K = 10$) when the reporting channels are imperfect. We first conduct our analysis for $P_e = 1\%$ and $\bar{\gamma} = 0 \text{ dB}$. Then, the effect of both reporting and sensing channels' quality (P_e and $\bar{\gamma}$) on the detection performance will be evaluated. We further justify theoretical results by simulation.

Figure 4.17 presents the minimum achievable total error rate versus mean holding times when they are assumed to be equal (i.e., $\mu_i = \mu_b$). Since the status of the PU will be more stable for higher mean holding times, SUs can detect it with lower uncertainty. Thus the detection performance improves by the increase in mean holding times. It is clear that the double-threshold method has lower detection error as compared with the single one, and their difference becomes greater for higher values of mean holding times. The minimum total error rate versus the duty cycle is shown in Figure 4.18 for different values of the mean idle time. For a fixed mean idle time of the PU, a lower value of P_b means that the mean active time of the PU is smaller as compared with the whole sensing time ($\mu_b \ll \tau$) and $P_{H_{11}}$ is subsequently low (while $P_{H_{12}}$ is high), which leads to a worse detection due to the instability of the PU. On the other hand, when P_b is greater, the probability of having a stable active PU during the sensing time ($P_{H_{11}}$) is higher. Therefore, the detection performance improves. Again, the performance of the double-threshold method is better than that of the conventional single-threshold scheme.

To analyze the effect of the CSS, we evaluate the minimum achievable total error rate with respect to P_e in Figure 4.19. One can see that detection error rates are the same when $P_e = 0$. Subsequently, it is revealed that the performance gain of the double-threshold scheme dominates the single-threshold one whenever the reporting channel becomes erroneous. It is also obtained that the single-threshold method is very sensitive to the quality of the reporting channels while the variation of the double-threshold scheme is very slight and the minimum P_{err} is almost the same for different values of P_e . In other words, the detection performance of the single-threshold method degrades when reducing the quality of the reporting channels. The double-threshold scheme can relieve this degradation, which implies the performance gain of the double-threshold method in the case of the CSS with imperfect reporting channels. As a consequence, when the reporting channels are erroneous, the CSS can be considered as a useful method to improve the detection performance of the secondary network when utilized besides the double-threshold method for this particular primary network model.

Furthermore, the effect of the SNR on the minimum achievable total error rate is presented in Figure 4.20. It is reasonable to have better detection performance in a higher SNR. However, our purpose is to show the SNR gain of the double-threshold method. Its detection performance is still better than the single-threshold method, and it becomes more obvious by increasing the SNR.

Consequently, the results showed that the double-threshold method outperforms the single one by decreasing the effect of additional uncertainty in different mean holding times, duty cycles and SNRs when the reporting channels are imperfect. It was also revealed that the single-threshold method is very sensitive to the quality of reporting channels while the double-threshold method is not. One reason can be considered from the effect of the double-threshold method on the lower bound of the probability of false alarm (Q_f) in the case of the single-threshold based OR fusion rule (i.e. (2.24)) which was explained in Section 3.2.1. Hence, over imperfect reporting channels, the double-threshold based CSS is more useful in improving the detection performance for this particular cognitive radio network model. Finally, a good agreement was obtained between theoretical and simulated results. Note that the slight difference between simulation and theoretical results is anticipated due to the approximated PDF, based on our analysis. Nevertheless, it does not mutate our primary conclusion in terms of the performance of the double-threshold method over different parameters.

Figure 4.17: Minimum total error rate vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ (i.e., $\mu_i = \mu_b$) by the OR fusion rule

Figure 4.18: Minimum total error rate vs. duty cycle with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_e = 1\%$ by the OR fusion rule

Figure 4.19: Minimum total error rate vs. P_e with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_b = 0.5$ by the OR fusion rule

Figure 4.20: Minimum total error rate vs. average SNR with $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ by the OR fusion rule

4.2.3 Numerical results by IED over Rayleigh-faded sensing channels

In the next step, we consider the situation in which SUs use IED to obtain the energy level of received signals as given in (4.6). We analyze the detection performance of proposed methods (CSED, ISSED, CDED and IDDED) in terms of total error rate over Rayleigh-faded sensing channels. Considering the possible ED methods, the optimization problem can be defined as

$$\underset{\Lambda}{\text{minimize : }} \quad P_{err}(\Lambda) = Q_m(\Lambda) + Q_f(\Lambda) \quad (4.15a)$$

$$\text{subject to : } \quad \Lambda > 0 \quad (4.15b)$$

in which Λ has been interpreted in (3.58).

By using the gamma approximation for the distribution of the observed energy, as described in Section 4.1.2, and explained formulation in Section 4.2.1, we analyze the minimum total error rate with respect to different parameters.

We use the same system parameters as stated in the previous section, i.e., $K = 10$, $N = 100$ and $T_s = 100 \mu\text{s}$. Again, the CSS is applied over imperfect reporting channels (with $P_e = 1\%$) when the average SNR of the sensing channels is assumed to be $\bar{\gamma} = 0 \text{ dB}$. We then present the effect of the error probability and average SNR on the detection performance. The theoretical results are also justified by simulations.

Figure 4.21 depicts the minimum total error rate versus mean holding times when they are assumed to be equal (i.e., $\mu_i = \mu_b$). It is clear that the stability of the PU increases as its holding time increases. Thus SUs can do their sensing tasks with lower uncertainty which leads to have better detection performance. Since the status of the PU will be more stable for higher mean holding times, SUs can detect it with lower uncertainty. This figure shows that the detection error is less for double-threshold based schemes (CDED and IDDED) while the IDDED is the best one. The effect of the duty cycle on the minimum total error rate is analyzed in Figure 4.22. In a fixed total mean time (i.e., $\mu_T = \mu_i + \mu_b$), as P_b becomes low μ_b decreases while μ_i increases, which results in lower probabilities of H_{11} and H_{02} (higher probabilities of H_{01} and H_{12}). It is clear that the stability of the PU reduces as its holding time decreases. On the other hand, when P_b is very high, the probabilities of H_{01} and H_{12} decrease (while probabilities of H_{11} and H_{02} increase) due to lower μ_i and higher μ_b . In both cases, the detection performance becomes worse because changing status of the PU is more likely which leads SUs to detect the presence of the PU with higher uncertainty. This figure shows that H_{02} (effective on Q_f) affects detection error more than H_{12} (effective on Q_m) with a minimum point around $P_b = 0.2$. Again, the IDDED outperforms others.

As discussed before, we believe that there is no guarantee to have a perfect medium between SUs and FC in a realistic case. Hence, it is important to evaluate the performance of proposed methods for erroneous reporting channels. Thus we analyze the effect of the CSS in Figure 4.23, when the minimum total error rate is evaluated with respect to P_e . Similar to the case of AWGN, it shows that detection error rates of different methods are the same when $P_e = 0$. By increasing P_e , it is revealed that the single-threshold based methods are very sensitive to the quality of the reporting channel while the variation is very small for double-threshold schemes. It is again due to the false alarm wall which has been explained in the previous section. Interestingly, by considering the results in Section 3.1, the same conclusion is achieved for both low- and medium-traffic primary networks in this particular system model. That is, for Rayleigh-faded sensing channel over erroneous reporting channels, the CSS can enhance the detection reliability if it uses the double-threshold scheme for decision making.

Finally, Figure 4.24 represents the effect of SNR on the minimum total error rate. It makes sense to have better performance in a higher SNR. However, our purpose is to disclose the SNR gain of the IDED. Its detection performance is still higher than others and becomes better by increasing the SNR.

Figure 4.21: Minimum total error rate vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $P_b = 0.5$ by the OR fusion rule

Figure 4.22: Minimum total error rate vs. duty cycle with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $P_e = 1\%$ and $\mu_T = 0.1$ s by the OR fusion rule

Figure 4.23: Minimum total error rate vs. P_e with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$, $\mu_i = 50$ ms and $P_b = 0.5$ by the OR fusion rule

Figure 4.24: Minimum total error rate vs. average SNR with $N = 100$, $K = 10$, $P_e = 1\%$, $\mu_i = 50$ ms and $P_b = 0.5$ by the OR fusion rule

4.3 Transmission throughput

In this section, for analysis of transmission throughput of the secondary network, we consider the medium-traffic primary network in such a way that the PU either arrives at or departs from the network at most once during the frame duration of T . It means that the status changing of the PU during the transmission phase of the SUs must be considered as well as that of the sensing phase. Thus, by considering (4.2) for H_{uv} , there are six situations as follows:

- (i) H_{01} : PU remains idle all over the frame time,
- (ii) H_{02} : Active PU goes to the idle state during the sensing period and awaits idle up to the end of the frame,
- (iii) H_{03} : PU is idle all over the sensing period and then goes to the active state (after the sensing duration) by the end of the frame time,
- (iv) H_{11} : PU remains active all over the frame time,
- (v) H_{12} : Idle PU goes to the active state during the sensing period and awaits active up to the end of the frame,
- (vi) H_{13} : PU is active all over the sensing period and then goes to the idle state (after the sensing duration) by the end of the frame time.

From the viewpoint of the sensing part, H_{01} and H_{03} are the same (the PU is idle during the sensing time) as well as H_{11} and H_{13} (the PU is active during the sensing time). Therefore, in (4.3) and Table 4.1, we have $\tilde{H}_{02} = \{H_{01}, H_{03}\}$ and $\tilde{H}_{12} = \{H_{11}, H_{13}\}$. Again, the observed energy level of Y_i can be defined as gamma distributed accurately where the PU's status is fixed within the sensing time ($\tilde{H}_{02}$ and $\tilde{H}_{12}$), and it can be well approximated for the cases of H_{02} and H_{12} as it is evaluated in Section 4.1.1.

From Figure 4.25, one can see that the effect of double-threshold method on detection improvement is more in the case of the OR fusion rule than that of the majority one. On the other hand, the range of total error rate is less when the majority rule is utilized. Since we want to analyze the transmission throughput of the secondary network over some required detection constraints (which are defined to guarantee the protection of the primary signal), we would be able to apply the CSS through the majority fusion rule to keep detection errors as low as possible (less than 10%). Thus, we obtain the total probabilities of false alarm and detection from equations (2.20), (2.21), (2.25) and (2.26), when $M = K/2$.

 4.25.a: $P_e = 1\%$

 4.25.b: $P_e = 10\%$

Figure 4.25: Minimum total error rate over different P_e vs. mean holding times with $\bar{\gamma} = 0$ dB, $N = 100$, $K = 10$ and $P_b = 0.5$ (i.e., $\mu_i = \mu_b$) for different fusion rules

4.3.1 Probabilities formulation

With a sampling period of T_s , the total frame duration consists of $L = T/T_s$ samples. SUs need to spend a sensing time of τ included $N = \tau/T_s$ samples (from (2.5)) on defining the status of the PU. If the global decision reveals that the PU is idle, SUs begin their transmission tasks within the remained time $(T - \tau)$.

Since the PU is considered to change its status at most once during the whole frame time, the joint probabilities that the PU goes to another situation (either idle or active) after J_u ($u \in \{0, 1\}$) samples within a frame with L samples are defined as

$$\begin{aligned} P_{J_0}^L &= P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(L-J_0-1)} \\ P_{J_1}^L &= P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(L-J_1-1)} \end{aligned} \quad (4.16)$$

where J_u is within $(1, N - 1)$ and $(N, L - 1)$ in the cases of H_{u2} and H_{u3} , respectively. Subsequently, the marginal probabilities of different hypotheses are

$$\begin{aligned} P_{H_{01}} &= P_i \cdot (P_{00}(T_s))^L \\ P_{H_{11}} &= P_b \cdot (P_{11}(T_s))^L \\ P_{H_{02}} &= \sum_{J_0=1}^{N-1} P_{J_0}^L = \sum_{J_0=1}^{N-1} P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(L-J_0-1)} \\ P_{H_{12}} &= \sum_{J_1=1}^{N-1} P_{J_1}^L = \sum_{J_1=1}^{N-1} P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(L-J_1-1)} \\ P_{H_{03}} &= \sum_{J_1=N}^{L-1} P_{J_1}^L = \sum_{J_1=N}^{L-1} P_i \cdot (P_{00}(T_s))^{J_1} \cdot P_{01}(T_s) \cdot (P_{11}(T_s))^{(L-J_1-1)} \\ P_{H_{13}} &= \sum_{J_0=N}^{L-1} P_{J_0}^L = \sum_{J_0=N}^{L-1} P_b \cdot (P_{11}(T_s))^{J_0} \cdot P_{10}(T_s) \cdot (P_{00}(T_s))^{(L-J_0-1)} \end{aligned} \quad (4.17)$$

For the case of single-threshold scheme, and by considering $\tilde{H}_{02} \in \{H_{01}, H_{03}\}$ and $\tilde{H}_{12} \in \{H_{11}, H_{13}\}$ for throughput analysis, the conditional local probabilities can be defined as shown in (4.9). Therefore, the local probabilities of (4.10) and (4.11) will be, respectively, updated as

$$P_f = \frac{[P_{H_{01}} + P_{H_{03}}] P_{f,1} + \sum_{J_0=1}^{N-1} P_{J_0}^L P_{f,2}}{P_{H_{01}} + P_{H_{02}} + P_{H_{03}}} \quad (4.18)$$

and

$$P_d = \frac{[P_{H_{11}} + P_{H_{13}}] P_{d,1} + \sum_{J_1=1}^{N-1} P_{J_1}^L P_{d,2}}{P_{H_{11}} + P_{H_{12}} + P_{H_{13}}} \quad (4.19)$$

Note that P_{d_0} and P_m are given in (2.10) and (2.11), respectively.

Similarly, for the case of double-threshold method, the conditional local probabilities of the double-threshold scheme, which are given in (4.12), would be considered when $\tilde{H}_{02} \in \{H_{01}, H_{03}\}$ and $\tilde{H}_{12} \in \{H_{11}, H_{13}\}$. Subsequently, local probabilities of false alarm and detection can be obtained from (4.18) and (4.19) and the local probabilities of the detecting H_0 and missed detection in (4.13) and (4.14) will be updated as

$$P_{d_0} = \frac{[P_{H_{01}} + P_{H_{03}}] P_{d0,1} + \sum_{J_0=1}^{N-1} P_{J_0}^L P_{d0,2}}{P_{H_{01}} + P_{H_{02}} + P_{H_{03}}} \quad (4.20)$$

and

$$P_m = \frac{[P_{H_{11}} + P_{H_{13}}] P_{m,1} + \sum_{J_1=1}^{N-1} P_{J_1}^L P_{m,2}}{P_{H_{11}} + P_{H_{12}} + P_{H_{13}}} \quad (4.21)$$

Note that the local probabilities of no decision (Δ_0 and Δ_1) can be obtained as shown in (2.16) and (2.19).

4.3.2 Throughput formulation

SUs start transmitting their buffered data at the end of the sensing time only if the global decision implies that the PU is idle. The collided packets (due to the active PU) are considered as lost. The total normalized transmission throughput (with respect to the maximum capacity of the channel) of the secondary network is given by

$$R_n = \frac{R}{C_{\max}} = R_{01} + R_{02} + R_{03} + R_{11} + R_{12} + R_{13} \quad (4.22)$$

which is the summation of the average normalized throughput over different hypotheses. Note that $C_{\max}$ is the maximum capacity of a free channel when the PU is idle. Thus, we continue over six possible situations:

H_{01} : since the PU is idle all over the frame time as well as its transmission part, there is no collision and SUs have a successful transmission. Accordingly,

$$R_{01} = \left(\frac{T - \tau}{T} \right) (1 - Q_f) P_{H_{01}} \quad (4.23)$$

H_{02} : the PU has changed its status once during the sensing time. Hence, it is remained idle during the transmission time and again there is no collision. The throughput is

$$R_{02} = \left(\frac{T - \tau}{T} \right) (1 - Q_f) P_{H_{02}} \quad (4.24)$$

H_{03} : it is very probable that the PU becomes active during the transmission time. Therefore, the mean collision time is [77]

$$\bar{T}_C = T - \tau - \mu_i \left(1 - \exp \left(-\frac{T - \tau}{\mu_i} \right) \right) \quad (4.25)$$

and the probability of occurring collision during the transmission time is

$$P_{coll,H03} = \frac{\bar{T}_C}{T - \tau} = 1 - \frac{\mu_i}{T - \tau} \left(1 - \exp \left(-\frac{T - \tau}{\mu_i} \right) \right) \quad (4.26)$$

Thus, for normalized throughput of this hypothesis, we have

$$R_{03} = \left(\frac{T - \tau}{T} \right) (1 - Q_f)(1 - P_{coll,H03})P_{H_{03}} \quad (4.27)$$

H_{11} : since the PU is active within the whole frame time, all of the transmitted packets by the SUs (due to the missed detection) will be collided and the throughput is zero ($R_{11} = 0$).

H_{12} : the PU has changed its status once during the sensing time. Hence, it is remained active during the transmission time. Similar to H_{11} , the throughput is zero ($R_{12} = 0$).

H_{13} : although SUs start to transmit due to the missed detection, it is very probable that the PU becomes idle during the transmission time. Therefore, there is a mean time in which SUs transmit without collision as follows [77]:

$$\bar{T}_{NC} = T - \tau - \mu_b \left(1 - \exp \left(-\frac{T - \tau}{\mu_b} \right) \right) \quad (4.28)$$

and the probability of occurring collision during the transmission time is

$$P_{coll,H13} = \frac{T - \tau - \bar{T}_{NC}}{T - \tau} = \frac{\mu_b}{T - \tau} \left(1 - \exp \left(-\frac{T - \tau}{\mu_b} \right) \right) \quad (4.29)$$

Thus, the normalized throughput of this hypothesis is

$$R_{13} = \left(\frac{T - \tau}{T} \right) (1 - Q_d)(1 - P_{coll,H13})P_{H_{13}} \quad (4.30)$$

Obviously, the detection performance is better with a bigger sensing time (resulting in a higher N). On the other hand, the bigger sensing time decreases the transmission time, and throughput will be reduced consequently. Hence, there is a trade-off between sensing reliability and transmission throughput, and we need to find an optimum sensing time. We define some detection constraints to maximize normalized throughput satisfying

reliability as well as utilization rate. Therefore, the optimization problem is given by

$$\underset{\tau, \Lambda}{\text{maximize :}} \quad R_n(\tau, \Lambda) \quad (4.31a)$$

$$\text{subject to :} \quad Q_f(\tau, \Lambda) \leq \overline{Q_f} \quad (4.31b)$$

$$Q_d(\tau, \Lambda) \geq \underline{Q_d} \quad (4.31c)$$

where

$$\Lambda = \begin{cases} \lambda & \text{Single-threshold based spectrum sensing scheme} \\ (\lambda_1, \lambda_2) & \text{Double-threshold based spectrum sensing scheme} \end{cases} \quad (4.32)$$

$\overline{Q_f}$ and $\underline{Q_d}$ are, respectively, the maximum acceptable total probability of false alarm and the minimum tolerable level of the total probability of detection as defined in Section 3.2. We then provide the numerical optimization of the maximum transmission throughput of the secondary network, $R_{\max}$, in the next section.

4.3.3 Numerical results

The system is supposed to take samples of the received signal with sampling period of $T_s = 100 \mu\text{s}$. It is further assumed that the frame size is $L = 500$, which results in the frame duration of $T = LT_s = 50 \text{ ms}$. In order to apply the CSS, there are 10 SUs ($K = 10$) and the FC uses the majority rule (as a moderate rule to meet the primary signal's protection as well as the spectrum utilization rate) to fuse the received local decisions thorough erroneous reporting channels. We first conduct our analysis over a predetermined quality level of the sensing and reporting channels with $\bar{\gamma} = 0 \text{ dB}$ and $P_e = 10 \%$, and then present the effect of the sensing channels' SNR and the quality of reporting channels on the maximum normalized throughput ($R_{\max}$). We analyze the effect of different parameters on $R_{\max}$ over both single- and double-threshold based spectrum sensing schemes with the required detection constraints of $\overline{Q_f} = 10 \%$ and $\underline{Q_d} = 99 \%$.

The maximum normalized throughput versus mean idle time is depicted in Figure 4.26. For a fixed duty cycle, as μ_i becomes higher, the maximum throughput increases. The reason is that the mean busy time is also increasing as the mean idle time increases. Since both mean holding times become higher, the status of the PU remains more stable. Therefore, SUs can achieve suitable detection performance (satisfying sensing requirements) in a shorter sensing time. Subsequently, transmission time and throughput of the secondary network improve. By comparing two curves, it is revealed that in low value of the duty cycle ($P_b = 0.1$), throughput continuously improves as μ_i increases, while in higher value of the duty cycle ($P_b = 0.5$), the improvement speed reduces and $R_{\max}$ is almost the same

after $\mu_i = 0.35$ s. For $P_b < 0.5$ (i.e., $\mu_b < \mu_i$), although μ_b increases as μ_i becomes higher, μ_i is increasing faster than μ_b which implies that the idleness portion of the time becomes bigger. Therefore, an additional reason (higher probability of idleness) would help the first reason (status stability of the PU) to have better throughput. On the contrary, when P_b is high, the increase in μ_b is more significant than μ_i and the portion of the busy time becomes bigger which results in higher probability of collision, and the chance of secondary network for transmission reduces. Thus, throughput remains on a specific value after some improvement by the increase of mean holding times. To understand it more, the maximum normalized throughput versus duty cycle is presented in Figure 4.27. In a fixed mean idle time, we can see that the maximum throughput decreases as P_b increases. In this figure, both curves converge to a value at $P_b = 0.9$. Regardless of μ_i is whether low or high, μ_b is very high as compared to the frame time ($\mu_b \gg T$). Hence, the PU is active for the majority of the time and the probability of the channel becomes idle is very low. Even if the global decision is idle at the end of the sensing time, the probability of the collision is still high which leads to have a low throughput. On the other hand, when P_b decreases (P_i increases), the portion of the idle time becomes higher ($\mu_i > \mu_b$) and the chance of the secondary network to transmit data without collision improves. The improvement becomes more significant when μ_i is higher ($\mu_i = 0.5$ s in this figure), in which bigger mean holding times allow SUs to decide in the shorter sensing time (due to higher stability of the PU). Similar to Figure 4.26, there are two reasons (status stability of the PU and higher probability of idleness) to have better throughput.

Figure 4.28 represents the maximum achievable normalized throughput versus error probability of reporting channels for different mean idle times and different duty cycles. It shows that the single-threshold method is more sensitive to the condition of the reporting channel as compared with the double-threshold method. This sensitivity becomes more significant when the duty cycle increases ($P_b = 0.5$) in Figure 4.28.b.

In Figure 4.29, the maximum normalized throughput is analyzed versus the SNR of sensing channels. The transmission throughput improves as SNR increases. It is related to detection performance of SUs. It is obvious to have better spectrum sensing in higher SNR, when SUs spend lower sensing time to meet detection requirements. Then, there is longer transmission time available to the secondary network and throughput will be improved. In all figures, the double-threshold method has better performance than that of single-threshold as the double-threshold scheme reduces the uncertainty in the decision made by each SU. It means that they achieve the same detection performance in lower sensing time as compared with the single-threshold scheme, resulting in higher throughput.

Figure 4.26: Maximum normalized throughput vs. mean idle time with $\bar{\gamma} = 0$ dB, $K = 10$ and $P_e = 10\%$ for different duty cycles by the majority fusion rule

Figure 4.27: Maximum normalized throughput vs. duty cycle with $\bar{\gamma} = 0$ dB, $K = 10$ and $P_e = 10\%$ for different mean idle times by the majority fusion rule

Figure 4.28: Maximum normalized throughput vs. P_e with $\bar{\gamma} = 0$ dB and $K = 10$ for different duty cycles and mean idle times by the majority fusion rule

Figure 4.29: Maximum normalized throughput vs. average SNR with $K = 10$, $P_b = 0.5$ and $P_e = 10\%$ for different mean idle times by the majority fusion rule

4.4 Conclusion

In this chapter, we supposed that the SUs encountered a medium-traffic rate of the primary network, when the status of the PU may alter at most once during the studied period. We first accomplished an investigation into distribution accuracy of the observed energy levels of received primary signals by SUs and showed that the gamma approximation can be considered as good enough in terms of both MSE and KLD for our further performance analyses over both AWGN and Rayleigh-faded sensing channels.

We then conducted the performance analyses of different ED-based methods (as explained in previous chapters) in terms of detection reliability and transmission throughput. The results showed that the double-threshold based schemes (i.e., CDED and IDDED) outperform the single-threshold based cases (i.e., CSED and ISED) by decreasing the effect of additional uncertainty (due to the medium traffic rate of the primary network) in a range of different system parameters (e.g., mean holding times, duty cycles and SNRs) when the reporting channels are imperfect. It was also revealed that the single-threshold based methods are very sensitive to the quality of reporting channels while the double-threshold based methods are not. Moreover, the proposed IDDED outperforms others.

Chapter 5

Stochastic-modeled secondary users in a medium-traffic network

The traffic rate of the primary network and distribution intensity of the secondary network are two important concerns of designing a CSS-based CRN as practical as possible. In previous steps we just considered a deterministic model for the distribution of SUs when the primary network has either a low or medium traffic rate. On the other hand, we need to be cautious about the performance parameters to be optimized. In many studies, as we did in previous parts, the detection quality and/or the throughput of the secondary network are the main objectives to enhance the performance of the CRN. It is obvious that the basic reason to introduce the CR is to acquit the market demand for spectrum. The CRN employment is feasible only if the minimum throughput desired by operators has been met. There exists an active trade-off between the transmission throughput of the PU and that of SUs. It would be critical that the traffic rate of the primary network is higher. The detection improvement should be in the way that propels both sides to an acceptable compromise which reinforces the total transmission throughput of the system (summation of both primary and secondary networks' throughputs). Therefore, it motivates us to propose a method to improve the overall throughput of a system in the presence of mentioned uncertainties.

In this chapter, we consider a system model in which the effects of traffic rate of the primary network and density randomness of the secondary network are taken into account simultaneously. These conditions together degrade the performance of an ED-based CSS strongly due to higher uncertainty about both PU's activity status and available cooperative SUs.

A CRN has been assumed as shown in Figure 5.1, where a stochastic-modeled secondary network has been set up in the region of a primary network. SUs are spatially distributed within an area around secondary base station (SBS) by the homogeneous PPP with a total intensity of λ_{SU} . It is further assumed that there is always one SU in the

Figure 5.1: A CRN with the stochastic-modeled secondary network

network and secondary network zone is small enough to consider the average SNR of the sensing channels as identical for all SUs. The SBS is also acting as the FC of the CSS scheme and communicates with all K cooperative SUs. Meanwhile, the primary network has a medium traffic rate with mean idleness and activity (busy) times of μ_i and μ_b , respectively, as defined in Chapter 4. Since the quality of the PU's transmitted signal is expected to be better than that of primary base station (PBS) within the secondary network region, the SUs do detection through uplink channel and sense the presence of the PU once the communication channel is assigned to the PU by the PBS. Note that all sensing and transmission channels are Rayleigh-faded and the reporting channels (SU-FC links) are modeled as the BSC with error probability P_e .

From Figure 5.1, r_s and r_p are, respectively, the distances between SBS-SU and SBS-PU links. Since the locations of users with respect to the SBS are independent uniform random variables, their distribution is given by

$$f_{r_s}(r) = f_{r_p}(r) = \begin{cases} \frac{2\pi r}{\pi R_s^2} = \frac{2r}{R_s^2} & 0 \leq r \leq R_s \\ 0 & \text{otherwise} \end{cases} \quad (5.1)$$

where R_s is the radius of the secondary network's operating region around the SBS. Subsequently, the link distances between a desired SU to the others (SU-SU and SU-PU links), d_s and d_p , have a PDF conditioned on the location of that SU with respect to the SBS as

$$f_{d_s|r_s}(d|r) = f_{d_p|r_s}(d|r) = \frac{2d}{\pi R_s^2} \begin{cases} \pi & 0 \leq d \leq R_s - r \\ \varphi & R_s - r \leq d \leq R_s + r \\ 0 & \text{otherwise} \end{cases} \quad (5.2)$$

Figure 5.2: Positioning in the stochastic-modeled secondary network

Note that from Figure 5.2,

$$\varphi = \cos^{-1} \left(\frac{d^2 + r^2 - R_s^2}{2dr} \right) \quad (5.3)$$

5.1 CSS with Extra sensing

In this section, we propose an extra sensing part in which only a fraction of the active SUs, called sensing SUs, are supposed to interrupt their transmission tasks for another sensing attempt during the transmission phase. Since the other SUs, called transmitting SUs, remain active, they may interfere with sensing SUs during the extra sensing part, affecting the detection reliability of the sensing SUs negatively. We also compare results with the situation that all SUs participate in the extra sensing routine (that is no interference), which means that the desired frame time is just reduced. It will be shown that the smaller frame time does not help to improve the total throughput of the CRN, while our proposed method can increase the overall throughput in this particular network model.

Figure 5.3 shows a frame structure that the cooperative SUs complete their working routine within a duration of T in a described manner as follows:

1. All SUs are supposed to be engaged in a cooperative-based detection of the main sensing part to define the presence of the PU by spending the first τ second(s) of the total frame time.
2. If the global decision of the main sensing part indicates that the PU is active, SUs remain idle till the extra part to avoid causing any possible interference to the primary signal within the T_1 second(s) after the main part. Then all K SUs engage in another sensing attempt with a duration of τ' as the extra sensing part. If the global decision of the extra sensing part is still positive to the active PU, SUs remain idle up to the end of the frame time (T_2 second(s) after extra part). Otherwise (if the PU is found idle), SUs use the remained T_2 second(s) for their transmission.
3. If the global decision of the main sensing part indicates that the PU is idle, SUs occupy the channel for their transmission tasks. A subset of SUs (K_s) interrupt their transmission to do another sensing during the extra sensing part while the rest of them (K_t) continue their transmission. If the global decision of the extra sensing part is still positive to the idleness of the PU, all SUs continue their transmission. Otherwise (if the PU is found active), those K_t active SUs stop their transmission as well to prevent causing any interference to the primary signal during the remained T_2 second(s) of the frame time. Then all SUs await in the inactive state up to the next frame.

Suppose that the total sample size of the frame is L . The main sensing part uses N samples to make its own decision on the presence of the PU while the sample size of the extra sensing part is N' . By assuming L' as the number of samples come before the extra sensing part and T_s as the sampling period, the durations in Figure 5.3 are defined as $T = LT_s$, $\tau = NT_s$, $\tau' = N'T_s$, $T_1 = (L' - N)T_s$ and $T_2 = (L - L' - N')T_s$.

Figure 5.3: Frame structure of the SU's tasks in addition to the possible status changing of the PU (solid lines for $u = 0$ and dashed lines for $u = 1$) when the extra sensing part is utilized

5.1.1 Network characteristics

5.1.1.1 Medium-traffic primary network

The primary network is supposed to have a medium traffic rate, as described in Chapter 4, over the observing frame in Figure 5.3. Therefore, there are ten situations, called H_{uv} , as explained in Table 5.1, where $u \in \{0, 1\}$ denotes the status of the PU at the end of the main sensing part and $v \in \{1, 2, 3, 4, 5\}$ represents different cases. For the extra sensing part, we also define H'_{wx} as explained in Table 5.2, in which $w \in \{0, 1\}$ denotes the status of the PU at the end of the extra sensing part and $x \in \{1, 2\}$ represents different cases. Thus,

$$\begin{cases} H'_{01} = \{H_{01}, H_{02}, H_{05}, H_{13}\} \\ H'_{02} = H_{14} \\ H'_{11} = \{H_{11}, H_{12}, H_{15}, H_{03}\} \\ H'_{12} = H_{04} \end{cases} \quad (5.4)$$

Hence, we can define total hypotheses as the status of the PU at the end of the sensing time for the main and extra sensing parts as, respectively,

$$\begin{aligned} H_0 &= \{H_{01}, H_{02}, H_{03}, H_{04}, H_{05}\} \\ H_1 &= \{H_{11}, H_{12}, H_{13}, H_{14}, H_{15}\} \end{aligned} \quad (5.5)$$

and

$$\begin{aligned} H'_0 &= \{H'_{01}, H'_{02}\} = \{H_{01}, H_{02}, H_{05}, H_{13}, H_{14}\} \\ H'_1 &= \{H'_{11}, H'_{12}\} = \{H_{11}, H_{12}, H_{15}, H_{03}, H_{04}\} \end{aligned} \quad (5.6)$$

As defined earlier, J_0 and J_1 are the numbers of samples from the beginning of the frame after which the PU switches from active to idle states, and vice versa. Similarly, J'_0

Table 5.1: Status of the primary user during the frame time

H_{uv}	Description
H_{01}	PU remains idle within the whole frame interval
H_{02}	Active PU switches its status to the idle state during the main sensing part
H_{03}	Idle PU switches its status to the active state during the 1 st transmission part
H_{04}	Idle PU switches its status to the active state during the extra sensing part
H_{05}	Idle PU switches its status to the active state during the 2 nd transmission part
H_{11}	PU remains active within the whole frame interval
H_{12}	Idle PU switches its status to the active state during the main sensing part
H_{13}	Active PU switches its status to the idle state during the 1 st transmission part
H_{14}	Active PU switches its status to the idle state during the extra sensing part
H_{15}	Active PU switches its status to the idle state during the 2 nd transmission part

Table 5.2: Status of the primary user during the extra sensing part

H'_{wx}	Description
H'_{01}	PU remains idle within interval of the extra sensing part
H'_{02}	Active PU switches its status to the idle state during the extra sensing part
H'_{11}	PU remains active within interval of the extra sensing part
H'_{12}	Idle PU switches its status to the active state during the extra sensing part

and J'_1 are known as the corresponding numbers of samples from the beginning of the extra sensing part after which the PU's status changes to idle and active states, respectively. Therefore,

$$J'_u = \begin{cases} J_u - L' & J_u \geq L' \\ 0 & \text{otherwise} \end{cases}, \quad u \in \{0, 1\} \quad (5.7)$$

Considering the transition probabilities as defined in (4.1) and the PU's probabilities of idleness/activity, P_i and P_b , as given in (4.4), the marginal probabilities of different hypotheses are

$$\begin{aligned} P_{H_{01}} &= P_i \cdot (P_{00}(T_s))^L, & P_{H_{11}} &= P_b \cdot (P_{11}(T_s))^L \\ P_{H_{u2}} &= \sum_{J_u=1}^{N-1} P_{J_u}^L, & P_{H_{u3}} &= \sum_{J_{\tilde{u}}=N}^{L'-1} P_{J_{\tilde{u}}}^L \\ P_{H_{u4}} &= \sum_{J_{\tilde{u}}=L'}^{L'+N'-1} P_{J_{\tilde{u}}}^L, & P_{H_{u5}} &= \sum_{J_{\tilde{u}}=L'+N'}^{L-1} P_{J_{\tilde{u}}}^L \end{aligned} \quad (5.8)$$

in which $u, \tilde{u} \in \{0, 1\}$ and $P_{J_u}^L$ is expressed in (4.16).

5.1.1.2 Secondary network with random users

Since SUs are spatially Poisson distributed with intensity λ_{SU} , those portions of them who do either sensing or transmitting during the extra part are still Poisson distributed with intensities λ_s (sensing SUs) and λ_t (transmitting SUs). Let us define β as the parameter which gives the percentage of the total SUs who do sensing during the sensing part. Consequently, intensities λ_s and λ_t are related to λ_{SU} as follows:

$$\begin{aligned}\lambda_s &= [\alpha(\beta - 1) + 1] \lambda_{SU} \\ \lambda_t &= [\alpha(1 - \beta)] \lambda_{SU}\end{aligned}\tag{5.9}$$

in which

$$\alpha = \begin{cases} 0, & \text{Decision of the main part declares that the PU is active } (H_1) \\ 1, & \text{Decision of the main part declares that the PU is idle } (H_0) \end{cases}\tag{5.10}$$

In other words, α determines the possibility of having interference (caused by transmitting SUs) during the extra sensing part, in regard to the decision of the main sensing part. $\alpha = 0$ means no interference while $\alpha = 1$ declares that the causing interference is possible. Note that β is another parameter which must be considered to determine the presence of interference as well as its power level. $\beta = 1$ is the only case with no interference when all SUs are acting as sensing users (no secondary transmitter) during the extra sensing part.

Therefore, for a secondary network's region around the SBS with radius R_s , the number of users inside the area follows a Poisson probability mass function (PMF) as

$$f_K(k) = \frac{\Lambda^k}{k!} \exp(-\Lambda)\tag{5.11}$$

with means

$$\Lambda = \begin{cases} \lambda_{SU}\pi R_s^2 & \text{Total SUs} \\ \lambda_s\pi R_s^2 & \text{Sensing SUs (for the extra sensing part)} \\ \lambda_t\pi R_s^2 & \text{Transmitting SUs (for the extra sensing part)} \end{cases}\tag{5.12}$$

Accordingly, the average numbers for different kinds of the SUs are

$$\begin{cases} \bar{K} = \lceil \lambda_{SU}\pi R_s^2 \rceil = \bar{K}_s + \bar{K}_t & \text{Average number of total SUs} \\ \bar{K}_s = \lceil \lambda_s\pi R_s^2 \rceil & \text{Average number of sensing SUs} \\ \bar{K}_t = \lceil \lambda_t\pi R_s^2 \rceil & \text{Average number of transmitting SUs} \end{cases}\tag{5.13}$$

5.1.2 Detection formulation

Recalling equation (4.3), the i -th SU measures the energy level of the received normalized samples during each sensing part as follows:

$$Y_i = \begin{cases} \sum_{m=1}^{\dot{N}} \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & \tilde{H}_{u2}/H'_{u1} \\ \sum_{m=1}^{\dot{J}_u} \left(\frac{|r_i^{\tilde{u}}(m)|}{\sigma_n} \right)^2 + \sum_{m=\dot{J}_u+1}^{\dot{N}} \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & H_{u2}/H'_{u2} \end{cases} \quad (5.14)$$

$u, \tilde{u} \in \{0, 1\}$

in which $\tilde{H}_{u2} = \{H_{u1}, H_{u3}, H_{u4}, H_{u5}\}$ and $r_i^u(m)$ is the received sample at the m -th time instant. Note that $(\dot{J}_u, \dot{N})$ is representative of (J_u, N) and (J'_u, N') , respectively, for the cases of the main and extra sensing parts. For the main sensing part, $r_i^u(m)$ is the same as given in (2.1) for the case of the Rayleigh-faded sensing channel. Actually, the average power of the received primary signal is $\sigma_s^2 = \bar{g}_{ps} \cdot P_{PU}$ in which P_{PU} is the transmitted power of the PU's signal and $\bar{g}_{ps}$ is the average PLG of the SU-PU link.

Since sensing SUs may suffer interference from transmitting SUs during the extra sensing part, the received samples, $r_i^u(m)$, are different from that of the main sensing part as follows:

$$\begin{aligned} r_i^0(m) &= u_i(m) + \alpha \nu_i(m) \\ r_i^1(m) &= h_i(m)s(m) + u_i(m) + \alpha \nu_i(m) \end{aligned} \quad (5.15)$$

in which $\alpha \in \{0, 1\}$ and $\nu_i(m)$ is the aggregate interference from the transmitting SUs (interferers) during the extra sensing part with the total power I as

$$\nu_i(m) = \sum_{k=1}^{K_t} \nu_{i,k}(m) \Leftrightarrow \begin{cases} \nu_i(m) \sim \mathcal{CN}(0, I) \\ \nu_{i,k}(m) \sim \mathcal{CN}(0, I_k) \end{cases}, \quad I = \sum_{k=1}^{K_t} I_k \quad (5.16)$$

where the power of interference from the k -th transmitting SU can be obtained as

$$I_k = \underbrace{|h_s|^2}_{\sigma_{h_s}^2} \cdot g_{ss}(d_s) \cdot P_{SU} \quad (5.17)$$

Note that P_{SU} and $g_{ss}(d_s)$ are the SU's transmitted power and the PLG of the interference link (SU-SU), respectively. Since SUs and PU are located in the same area, the Rayleigh fading coefficient of the interference channel, $h_s \sim \mathcal{CN}(0, \sigma_{h_s}^2)$, is supposed to be equal as that of the sensing channels. That is, $\sigma_{h_s}^2 = \sigma_h^2$.

Table 5.3: Means and variances of different cases for both main and extra sensing parts

ED	H_s	$\mathbb{E}$	$\mathbb{V}\text{ar}$
Main	$\tilde{H}_{u2}$	$N\Xi_u$	$N\Xi_u^2$
	H_{u2}	$J_u\Xi_{\tilde{u}} + (N - J_u)\Xi_u$	$J_u\Xi_{\tilde{u}}^2 + (N - J_u)\Xi_u^2$
Extra	H'_{u1}	$N'\Xi_u$	$N'\Xi_u^2$
	H'_{u2}	$J'_u\Xi_{\tilde{u}} + (N' - J'_u)\Xi_u$	$J'_u\Xi_{\tilde{u}}^2 + (N' - J'_u)\Xi_u^2$

For both sensing parts, the distribution of Y_i can be approximated as the gamma distribution given in (3.67) when the shape and scale parameters of the different cases can be obtained by the related means and variances provided in Table 5.3. Note that parameter Ξ_u can be defined as

$$\Xi_0 = \begin{cases} 1, & \text{for either the main part or the extra part with } \alpha = 0 \\ \frac{\bar{\gamma}}{\gamma_I}, & \text{for the extra part with } \alpha = 1 \end{cases} \quad (5.18)$$

$$\Xi_1 = \Xi_0 + \bar{\gamma}$$

where $\bar{\gamma} = \sigma_h^2 \sigma_s^2 / \sigma_n^2$ and $\gamma_I = \sigma_h^2 \sigma_s^2 / (\sigma_n^2 + I)$ are the average SNR and signal-to-interference-plus-noise ratio (SINR) of the sensing channels, respectively.

5.1.2.1 Interference

As it is discussed earlier, sensing SUs may suffer interference from transmitting SUs during the extra sensing part when the main sensing part has declared that the PU is idle. Hence, we need to take into account the effect of interference in the mentioned situation. From (5.17), the Laplace transform of the distribution of the k -th transmitting SU's interference can be defined as

$$\begin{aligned}
 \mathcal{L}_{I_k}(s) &= \mathbb{E}_{I_k} \{ \exp(-s I_k) \} = \mathbb{E}_{d_s} \left\{ \mathbb{E}_{|h_s|^2} \left\{ \mathbb{E}_{I_k} \{ \exp(-s I_k) \mid |h_s|^2, d_s \} \right\} \right\} \\
 &= \mathbb{E}_{d_s} \left\{ \mathbb{E}_{|h_s|^2} \left\{ \exp(-s P_{SU} g_{ss}(d_s) |h_s|^2) \right\} \right\} = \mathbb{E}_{d_s} \left\{ \mathcal{L}_{|h_s|^2}(s P_{SU} g_{ss}(d_s)) \right\} \\
 &\stackrel{(i)}{=} \mathbb{E}_{d_s} \left\{ \frac{\frac{1}{\sigma_{h_s}^2}}{s P_{SU} g_{ss}(d_s) + \frac{1}{\sigma_{h_s}^2}} \right\} = \mathbb{E}_{d_s} \left\{ \frac{1}{(s P_{SU} \sigma_{h_s}^2) g_{ss}(d_s) + 1} \right\} \\
 \Rightarrow \mathcal{L}_{I_k}(s) &= \mathbb{E}_{d_s} \left\{ \frac{1}{1 + a(s) g_{ss}(d_s)} \right\}, \quad a(s) = \underbrace{(P_{SU} \sigma_{h_s}^2)}_{\text{Received faded power}} s
 \end{aligned} \tag{5.19}$$

Through the fact that $|h_s|^2 \sim \text{Exp}(1/\sigma_{h_s}^2)$, (i) can be easily obtained as the Laplace transform of the exponential distribution as follows:

$$\begin{aligned}
 \mathcal{L}_{|h_s|^2}(s P_{SU} g_{ss}(d_s)) &= \int_0^\infty \exp(-s P_{SU} g_{ss}(d_s) |h_s|^2) * \frac{1}{\sigma_{h_s}^2} \exp\left(-\frac{1}{\sigma_{h_s}^2} |h_s|^2\right) d|h_s|^2 \\
 &= \frac{1}{\sigma_{h_s}^2} \int_0^\infty \exp\left[-|h_s|^2 \left(s P_{SU} g_{ss}(d_s) + \frac{1}{\sigma_{h_s}^2}\right)\right] d|h_s|^2 \\
 &= \frac{\exp\left(-\frac{1}{\sigma_{h_s}^2}\right)}{s P_{SU} g_{ss}(d_s) + \frac{1}{\sigma_{h_s}^2}} * \exp\left[-|h_s|^2 \left(s P_{SU} g_{ss}(d_s) + \frac{1}{\sigma_{h_s}^2}\right)\right] \Big|_0^\infty \\
 &= \frac{\frac{1}{\sigma_{h_s}^2}}{s P_{SU} g_{ss}(d_s) + \frac{1}{\sigma_{h_s}^2}}
 \end{aligned} \tag{5.20}$$

Considering the PLG of the interfering channel as $g_{ss}(d_s) = (1 + d_s^\eta)^{-1}$ with η as path-loss exponent (PLE), unconditional $\mathcal{L}_{I_k}(s)$ can be calculated by using (5.1) and (5.2) (there is no closed-form). Since the secondary network area is limited, we can consider I_k as independent and identically distributed (i.i.d.) for all $(k = 1, \dots, K_t)$ interferers. Consequently, for the total received interference, we have

$$\mathcal{L}_{I|K_t}(s) = \mathbb{E}_{I_k} \left\{ \exp\left(-s \sum_{k=1}^{K_t} I_k\right) \right\} = \prod_{k=1}^{K_t} \mathbb{E}_{I_k} \{ \exp(-s I_k) \} \stackrel{i.i.d.}{=} \mathcal{L}_{I_k}^{K_t}(s) \tag{5.21}$$

On the other hand, PU may suffer interference from transmitting SUs when missed detection is happened. In the worst case, when all K SUs are transmitting, one may know that the average power of aggregate interference from SUs to the primary signal is

$$\bar{I}_{SU \rightarrow PU} = \bar{K} \sigma_h^2 \bar{g}_{ps} P_{SU} \quad (5.22)$$

Hence, it is worthwhile to mention that the amount of interference can be reduced during the extra sensing part (when $\beta \neq 0$). Thus, finding the optimum β helps us have better protection of the primary signal besides reliable detection.

5.1.2.2 Detection probabilities of the main sensing part

From the viewpoint of the main sensing part, as defined in $\tilde{H}_{u2} = \{H_{u1}, H_{u3}, H_{u4}, H_{u5}\}$ for (5.14), $\{H_{03}, H_{04}, H_{05}\}$ are the same as H_{01} due to the fixed status of the idle PU during the sensing duration, and $\{H_{13}, H_{14}, H_{15}\}$ are the same as H_{11} when the PU is active throughout the mentioned period. Hence, for local probabilities of false alarm (P_f), detection (P_d), detecting H_0 (P_{d_0}) and missed detection (P_m) at each SU, we have

$$\begin{aligned} P_f &= \frac{(P_{H_{01}} + P_{H_{03}} + P_{H_{04}} + P_{H_{05}})P_{f,1} + \sum_{J_0=1}^{N-1} P_{J_0}^L P_{f,2}}{P_{H_{01}} + P_{H_{02}} + P_{H_{03}} + P_{H_{04}} + P_{H_{05}}} \\ P_d &= \frac{(P_{H_{11}} + P_{H_{13}} + P_{H_{14}} + P_{H_{15}})P_{d,1} + \sum_{J_1=1}^{N-1} P_{J_1}^L P_{d,2}}{P_{H_{11}} + P_{H_{12}} + P_{H_{13}} + P_{H_{14}} + P_{H_{15}}} \\ P_{d_0} &= 1 - P_f \\ P_m &= 1 - P_d \end{aligned} \quad (5.23)$$

in which

$$\begin{aligned} P_{f,1} &= \Pr \{Y_i > \lambda \mid \tilde{H}_{02}\} = 1 - F_{Y_i}(\lambda; k_{\tilde{H}_{02}}, \theta_{\tilde{H}_{02}}) \\ P_{f,2} &= \Pr \{Y_i > \lambda \mid H_{02}, J_0\} = 1 - F_{Y_i}(\lambda; k_{H_{02}}, \theta_{H_{02}}) \\ P_{d,1} &= \Pr \{Y_i > \lambda \mid \tilde{H}_{12}\} = 1 - F_{Y_i}(\lambda; k_{\tilde{H}_{12}}, \theta_{\tilde{H}_{12}}) \\ P_{d,2} &= \Pr \{Y_i > \lambda \mid H_{12}, J_1\} = 1 - F_{Y_i}(\lambda; k_{H_{12}}, \theta_{H_{12}}) \end{aligned} \quad (5.24)$$

Considering the gamma CDF, $F_{Y_i}(\cdot)$, it is $1 - F_{Y_i}(\lambda; k_{H_s}, \theta_{H_s}) = \Gamma\left(k_{H_s}, \frac{\lambda}{\theta_{H_s}}\right) / \Gamma(k_{H_s})$, where λ is the detection threshold of the main sensing part.

Therefore, using the OR fusion rule as the scheme to protect the PU from interference strictly, the total probabilities of false alarm and detection are given by

$$\begin{aligned} Q_f &= \mathbb{E}_K \{Q_{f|K}\} = \mathbb{E}_K \left\{1 - [P_{d_0}(1 - P_e) + P_f P_e]^K\right\} \\ Q_d &= \mathbb{E}_K \{Q_{d|K}\} = \mathbb{E}_K \left\{1 - [P_m(1 - P_e) + P_d P_e]^K\right\} \end{aligned} \quad (5.25)$$

5.1.2.3 Detection probabilities of the extra sensing part

During the extra sensing part, the PU remains idle and active in the cases of $H'_{01} = \{H_{01}, H_{02}, H_{05}, H_{13}\}$ and $H'_{11} = \{H_{11}, H_{12}, H_{15}, H_{03}\}$, respectively. In addition, detection performance of the extra sensing part is directly related to the decision of the main sensing part, when it determines the number of sensing users which attend in the extra sensing part as well as the number of interferers (if there is). If the global decision of the main sensing part specifies that the PU is active, there is no interference ($\alpha = 0$) and all K SUs cooperate for another sensing during the extra part. On the other hand, if the PU is denoted as idle by the main sensing part, there is interference ($\alpha = 1$) and only K_s SUs do sensing. Thus, when $\alpha = 1$, the local probabilities of each sensing SU are conditioned on the number of sensing users in the main part (K) and the number of interferers (K_t) as follows:

$$\begin{aligned}
 P_{f|\alpha=0,K} &= \frac{[Q_{f|K}(P_{H_{01}} + P_{H_{02}} + P_{H_{05}}) + Q_{d|K}P_{H_{13}}]P_{f,1|\alpha=0} + \sum_{J_0=L'}^{L'+N'-1} [Q_{d|K}P_{J_0}^L]P_{f,2|\alpha=0}}{Q_{f|K}(P_{H_{01}} + P_{H_{02}} + P_{H_{05}}) + Q_{d|K}(P_{H_{13}} + P_{H_{14}})} \\
 P_{d|\alpha=0,K} &= \frac{[Q_{d|K}(P_{H_{11}} + P_{H_{12}} + P_{H_{15}}) + Q_{f|K}P_{H_{03}}]P_{d,1|\alpha=0} + \sum_{J_1=L'}^{L'+N'-1} [Q_{f|K}P_{J_1}^L]P_{d,2|\alpha=0}}{Q_{d|K}(P_{H_{11}} + P_{H_{12}} + P_{H_{15}}) + Q_{f|K}(P_{H_{03}} + P_{H_{04}})} \\
 P_{f|\alpha=1,K,K_t} &= \frac{[Q_{d_0|K}(P_{H_{01}} + P_{H_{02}} + P_{H_{05}}) + Q_{m|K}P_{H_{13}}]P_{f,1|\alpha=1,K_t} + \sum_{J_0=L'}^{L'+N'-1} [Q_{m|K}P_{J_0}^L]P_{f,2|\alpha=1,K_t}}{Q_{d_0|K}(P_{H_{01}} + P_{H_{02}} + P_{H_{05}}) + Q_{m|K}(P_{H_{13}} + P_{H_{14}})} \\
 P_{d|\alpha=1,K,K_t} &= \frac{[Q_{m|K}(P_{H_{11}} + P_{H_{12}} + P_{H_{15}}) + Q_{d_0|K}P_{H_{03}}]P_{d,1|\alpha=1,K_t} + \sum_{J_1=L'}^{L'+N'-1} [Q_{d_0|K}P_{J_1}^L]P_{d,2|\alpha=1,K_t}}{Q_{m|K}(P_{H_{11}} + P_{H_{12}} + P_{H_{15}}) + Q_{d_0|K}(P_{H_{03}} + P_{H_{04}})}
 \end{aligned} \tag{5.26}$$

For the case of $\alpha = 0$, the conditional probabilities can be found in the same way as the main sensing part, when

$$\begin{aligned}
 P_{f,1|\alpha=0} &= \Pr\{Y_i > \lambda' \mid H'_{01}, \alpha = 0\} = 1 - F_{Y_i}(\lambda'; k_{H'_{01}}, \theta_{H'_{01}}) \\
 P_{f,2|\alpha=0} &= \Pr\{Y_i > \lambda' \mid H'_{02}, J'_0, \alpha = 0\} = 1 - F_{Y_i}(\lambda'; k_{H'_{02}}, \theta_{H'_{02}}) \\
 P_{d,1|\alpha=0} &= \Pr\{Y_i > \lambda' \mid H'_{11}, \alpha = 0\} = 1 - F_{Y_i}(\lambda'; k_{H'_{11}}, \theta_{H'_{11}}) \\
 P_{d,2|\alpha=0} &= \Pr\{Y_i > \lambda' \mid H'_{12}, J'_1, \alpha = 0\} = 1 - F_{Y_i}(\lambda'; k_{H'_{12}}, \theta_{H'_{12}})
 \end{aligned} \tag{5.27}$$

in which λ' is the detection threshold of the extra part. For the case of $\alpha = 1$, the conditional probabilities are also conditioned on the total power of the received interference and they need to be averaged. From (3.68), (3.69) and Table 5.3, it is revealed that the scale parameters for all cases ($\theta_{H'_{01}}, \theta_{H'_{11}}, \theta_{H'_{02}}, \theta_{H'_{12}}$) and the shape parameters for the cases with PU's status changing ($k_{H'_{02}}, k_{H'_{12}}$) are functions of I while the shape parameters of the other cases (with fixed-status PU), $k_{H'_{01}} = k_{H'_{11}} = N'$, are found deterministic. Moreover, it can be shown that $k_{H'_{02}} \approx k_{H'_{12}} \approx N'/C$ is an acceptable approximation with correction factor C when $\gamma_I \leq -5$ dB. It has been proven in Appendix B.

For relaxing the computational complexity, we do our analysis over $\gamma_I \leq -5$ dB. Since this assumption can be seen as the worst case in the quality of the sensing channel, it does not affect negatively on our later results and conclusion. On the other hand, the calculation of mentioned probabilities are still complicated when the shape and/or scale parameters are random variables (related to interference), due to no closed-form of the unconditional PDF. If the scale parameter is just a random variable (shape parameter is deterministic), we can find the required probabilities using the Laplace transform where

$$\begin{aligned}
 \Pr \{Y_i > \lambda' | H_s\} &= \mathbb{E}_{\theta_{H_s}} \{ \Pr \{Y_i > \lambda' | H_s, \theta_{H_s}\} \} \\
 &\stackrel{(i)}{=} \mathbb{E}_{\theta_{H_s}} \left\{ \left(1 - \frac{1}{\Gamma(k_{H_s})} \int_0^\infty J_{k_{H_s}}(2\sqrt{Z}) Z^{\frac{k_{H_s}}{2}-1} e^{-\frac{\theta_{H_s} Z}{\lambda'}} dZ \right) \right\} \\
 &= 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^\infty J_{k_{H_s}}(2\sqrt{Z}) Z^{\frac{k_{H_s}}{2}-1} \mathbb{E}_{\theta_{H_s}} \left\{ e^{-\frac{\theta_{H_s} Z}{\lambda'}} \right\} dZ \\
 &= 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^\infty J_{k_{H_s}}(2\sqrt{Z}) Z^{\frac{k_{H_s}}{2}-1} \mathcal{L}_{\theta_{H_s}}(Z/\lambda') dZ
 \end{aligned} \tag{5.28}$$

in which $J_\nu(\cdot)$ is the ν -th order Bessel function of the first kind and (i) has been proven in Appendix C. Therefore, the required conditional probabilities in (5.26) can be obtained by

$$\begin{aligned}
 P_{f,1|\alpha=1,K_t} &= \mathbb{E}_{\theta_{H'_{01}}} \{ \Pr \{Y_i > \lambda' | H'_{01}, \theta_{H'_{01}}\} | K_t \} \\
 &= 1 - \frac{1}{\Gamma(k_{H'_{01}})} \int_0^\infty J_{k_{H'_{01}}}(2\sqrt{Z}) Z^{\frac{k_{H'_{01}}}{2}-1} \mathcal{L}_{\theta_{H'_{01}}|K_t}(Z/\lambda') dZ \\
 P_{f,2|\alpha=1,K_t} &= \mathbb{E}_{\theta_{H'_{02}}} \{ \Pr \{Y_i > \lambda' | H'_{02}, \theta_{H'_{02}}\} | K_t \} \\
 &= 1 - \frac{1}{\Gamma(k_{H'_{02}})} \int_0^\infty J_{k_{H'_{02}}}(2\sqrt{Z}) Z^{\frac{k_{H'_{02}}}{2}-1} \mathcal{L}_{\theta_{H'_{02}}|K_t}(Z/\lambda') dZ \\
 P_{d,1|\alpha=1,K_t} &= \mathbb{E}_{\theta_{H'_{11}}} \{ \Pr \{Y_i > \lambda' | H'_{11}, \theta_{H'_{11}}\} | K_t \} \\
 &= 1 - \frac{1}{\Gamma(k_{H'_{11}})} \int_0^\infty J_{k_{H'_{11}}}(2\sqrt{Z}) Z^{\frac{k_{H'_{11}}}{2}-1} \mathcal{L}_{\theta_{H'_{11}}|K_t}(Z/\lambda') dZ \\
 P_{d,2|\alpha=1,K_t} &= \mathbb{E}_{\theta_{H'_{12}}} \{ \Pr \{Y_i > \lambda' | H'_{12}, \theta_{H'_{12}}\} | K_t \} \\
 &= 1 - \frac{1}{\Gamma(k_{H'_{12}})} \int_0^\infty J_{k_{H'_{12}}}(2\sqrt{Z}) Z^{\frac{k_{H'_{12}}}{2}-1} \mathcal{L}_{\theta_{H'_{12}}|K_t}(Z/\lambda') dZ
 \end{aligned} \tag{5.29}$$

when

$$\begin{aligned}
 \mathcal{L}_{\theta_{H'_{01}}|K_t}(Z/\lambda') &= e^{-\frac{Z}{\lambda'}} \mathcal{L}_{I|K_t}(Z/(\lambda'\sigma_n^2)) \\
 \mathcal{L}_{\theta_{H'_{11}}|K_t}(Z/\lambda') &= e^{-\frac{(1+\bar{\gamma})Z}{\lambda'}} \mathcal{L}_{I|K_t}(Z/(\lambda'\sigma_n^2)) \\
 \mathcal{L}_{\theta_{H'_{02}}|K_t}(Z/\lambda') &= e^{-\frac{(1+\frac{J'_0}{N'})CZ}{\lambda'}} \mathcal{L}_{I|K_t}(CZ/(\lambda'\sigma_n^2)) \\
 \mathcal{L}_{\theta_{H'_{12}}|K_t}(Z/\lambda') &= e^{-\frac{(1+\frac{(N'-J'_1)}{N'})\bar{\gamma}CZ}{\lambda'}} \mathcal{L}_{I|K_t}(CZ/(\lambda'\sigma_n^2))
 \end{aligned} \tag{5.30}$$

and $\mathcal{L}_{I|K_t}(s)$ has been given by (5.19) and (5.21).

Finally, since K_s might be zero, for total probabilities of the extra sensing part, we have

$$\begin{aligned}
 Q'_f &= \mathbb{E}_K \{ Q'_{f|K} \} = \mathbb{E}_K \left\{ (1 - \delta_{K_s,0}) \left[Q'_{f|\alpha=0,K} P_{\alpha_0=0|K} + Q'_{f|\alpha=1,K} P_{\alpha_0=1|K} \right] + \delta_{K_s,0} Q_{f|K} \right\} \\
 Q'_d &= \mathbb{E}_K \{ Q'_{d|K} \} = \mathbb{E}_K \left\{ (1 - \delta_{K_s,0}) \left[Q'_{d|\alpha=0,K} P_{\alpha_1=0|K} + Q'_{d|\alpha=1,K} P_{\alpha_1=1|K} \right] + \delta_{K_s,0} Q_{d|K} \right\}
 \end{aligned} \tag{5.31}$$

in which

$$\begin{aligned}
 P_{\alpha_0=0|K} &= \Pr \{ \alpha = 0 | H'_0, K \} = \frac{Q_{f|K} (P_{H_{01}} + P_{H_{02}} + P_{H_{05}}) + Q_{d|K} (P_{H_{13}} + P_{H_{14}})}{P_{H_{01}} + P_{H_{02}} + P_{H_{05}} + P_{H_{13}} + P_{H_{14}}} \\
 P_{\alpha_1=0|K} &= \Pr \{ \alpha = 0 | H'_1, K \} = \frac{Q_{f|K} (P_{H_{03}} + P_{H_{04}}) + Q_{d|K} (P_{H_{11}} + P_{H_{12}} + P_{H_{15}})}{P_{H_{03}} + P_{H_{04}} + P_{H_{11}} + P_{H_{12}} + P_{H_{15}}} \\
 P_{\alpha_0=1|K} &= \Pr \{ \alpha = 1 | H'_0, K \} = 1 - P_{\alpha_0=0|K} \\
 P_{\alpha_1=1|K} &= \Pr \{ \alpha = 1 | H'_1, K \} = 1 - P_{\alpha_1=0|K}
 \end{aligned} \tag{5.32}$$

Considering the fact that K_s and K_t are functions of K , the conditional total probabilities in (5.31) can be obtained by

$$\begin{aligned}
 Q'_{f|\alpha=0,K} &= 1 - [(1 - P_e)P_{d_0|\alpha=0,K} + P_e P_{f|\alpha=0,K}]^{K_s} \\
 Q'_{d|\alpha=0,K} &= 1 - [(1 - P_e)P_{m|\alpha=0,K} + P_e P_{d|\alpha=0,K}]^{K_s} \\
 Q'_{f|\alpha=1,K} &= 1 - [(1 - P_e)P_{d_0|\alpha=1,K,K_t} + P_e P_{f|\alpha=1,K,K_t}]^{K_s} \\
 Q'_{d|\alpha=1,K} &= 1 - [(1 - P_e)P_{m|\alpha=1,K,K_t} + P_e P_{d|\alpha=1,K,K_t}]^{K_s}
 \end{aligned} \tag{5.33}$$

Note that $\delta_{i,j}$ is the Kronecker delta function and it can be interpreted as

$$\delta_{i,j} = \begin{cases} 1, & i = j \\ 0, & \text{otherwise} \end{cases} \tag{5.34}$$

5.1.3 System performance

Since transmission throughputs of both primary and secondary networks are important in designing a practical CRN, we try to maximize total throughput of a CRN which is summation of transmission throughputs of both secondary and primary networks. To meet the feasibility in designing of an adequate CRN, the maximum achievable throughput of the secondary network has been considered as the main approach in the most of current works. Hence, we aim to find the efficiency of the using extra sensing part on improvement of the total throughput when the maximum total throughput is subjected to the maximum achievable transmission throughput of the secondary network. We suppose that the collided packets are lost and normalized throughputs of both networks (with respect to the maximum available channel capacity) are considered. Therefore, the total normalized throughput is obtained by

$$R_T = R_{SU} + R_{PU} \quad (5.35)$$

Suppose that P_{S_1} , P_{S_2} and P_{S_3} are, respectively, the probabilities that SUs have chance to occupy T_1 , τ' and T_2 intervals for their transmission without any collision with primary samples. Hence, we can define R_{SU} as follows:

$$R_{SU} = \left(\frac{T_1}{T} \cdot P_{S_1} \right) + \left(\frac{\tau'}{T} \cdot P_{S_2} \right) + \left(\frac{T_2}{T} \cdot P_{S_3} \right),$$

$$\begin{cases} P_{S_1} = Q_{d_0} [P_{H_{01}} + P_{H_{02}} + (1 - \Upsilon_{Tr, H_{03}}) P_{H_{03}} + P_{H_{04}} + P_{H_{05}}] + Q_m (1 - \Upsilon_{Tr, H_{13}}) P_{H_{13}} \\ P_{S_2} = \dot{Q}_{d_0} [P_{H_{01}} + P_{H_{02}} + (1 - \Upsilon_{Tr, H_{04}}) P_{H_{04}} + P_{H_{05}}] + \dot{Q}_m [P_{H_{13}} + (1 - \Upsilon_{Tr, H_{14}}) P_{H_{14}}] \\ P_{S_3} = Q'_{d_0} [P_{H_{01}} + P_{H_{02}} + (1 - \Upsilon_{Tr, H_{05}}) P_{H_{05}} + P_{H_{13}} + P_{H_{14}}] + Q'_m (1 - \Upsilon_{Tr, H_{15}}) P_{H_{15}} \end{cases} \quad (5.36)$$

where $Q_{d_0} = 1 - Q_f$, $Q_m = 1 - Q_d$, $Q'_{d_0} = 1 - Q'_f$, $Q'_m = 1 - Q'_d$, $\dot{Q}_{d_0} = \bar{\beta}_T - \dot{Q}_f$ and $\dot{Q}_m = \bar{\beta}_T - \dot{Q}_d$. Since K_t is an integer value, we define $\beta_T = K_t/K$ (which might be different from $(1 - \beta)$) and $\bar{\beta}_T = \mathbb{E}_K \{\beta_T\}$. Therefore, we have

$$\begin{aligned} \dot{Q}_f &= \mathbb{E}_K \{\beta_T Q_{f|K}\} \\ \dot{Q}_d &= \mathbb{E}_K \{\beta_T Q_{d|K}\} \end{aligned} \quad (5.37)$$

Table 5.4: Transmission probabilities of the PU over different situations

H_{uv}	$\Upsilon_{Tr,H_{uv}}$
H_{02}	$\left[\mu_b \left(1 - \exp \left(-\frac{\tau}{\mu_b} \right) \right) \right] / \tau$
H_{03}	$\left[T_1 - \mu_i \left(1 - \exp \left(-\frac{T_1}{\mu_i} \right) \right) \right] / T_1$
H_{04}	$\left[\tau' - \mu_i \left(1 - \exp \left(-\frac{\tau'}{\mu_i} \right) \right) \right] / \tau'$
H_{05}	$\left[T_2 - \mu_i \left(1 - \exp \left(-\frac{T_2}{\mu_i} \right) \right) \right] / T_2$
H_{12}	$\left[\tau - \mu_i \left(1 - \exp \left(-\frac{\tau}{\mu_i} \right) \right) \right] / \tau$
H_{13}	$\left[\mu_b \left(1 - \exp \left(-\frac{T_1}{\mu_b} \right) \right) \right] / T_1$
H_{14}	$\left[\mu_b \left(1 - \exp \left(-\frac{\tau'}{\mu_b} \right) \right) \right] / \tau'$
H_{15}	$\left[\mu_b \left(1 - \exp \left(-\frac{T_2}{\mu_b} \right) \right) \right] / T_2$

Similarly, P_{P_1} , P_{P_2} , P_{P_3} and P_{P_4} are defined as probabilities that PU may utilize τ , T_1 , τ' and T_2 intervals without collision. Thus, R_{PU} is

$$R_{PU} = \left(\frac{\tau}{T} \cdot P_{P_1} \right) + \left(\frac{T_1}{T} \cdot P_{P_2} \right) + \left(\frac{\tau'}{T} \cdot P_{P_3} \right) + \left(\frac{T_2}{T} \cdot P_{P_4} \right),$$

$$\begin{cases} P_{P_1} = P_{H_{11}} + (\Upsilon_{Tr,H_{12}} P_{H_{12}}) + P_{H_{13}} + P_{H_{14}} + P_{H_{15}} + (\Upsilon_{Tr,H_{02}} P_{H_{02}}) \\ P_{P_2} = Q_d [P_{H_{11}} + P_{H_{12}} + (\Upsilon_{Tr,H_{13}} P_{H_{13}}) + P_{H_{14}} + P_{H_{15}}] + Q_f (\Upsilon_{Tr,H_{03}} P_{H_{03}}) \\ P_{P_3} = \ddot{Q}_d [P_{H_{11}} + P_{H_{12}} + (\Upsilon_{Tr,H_{14}} P_{H_{14}}) + P_{H_{15}}] + \ddot{Q}_f [P_{H_{03}} + (\Upsilon_{Tr,H_{04}} P_{H_{04}})] \\ P_{P_4} = Q'_d [P_{H_{03}} + P_{H_{04}} + P_{H_{11}} + P_{H_{12}} + (\Upsilon_{Tr,H_{15}} P_{H_{15}})] + Q'_f (\Upsilon_{Tr,H_{05}} P_{H_{05}}) \end{cases} \quad (5.38)$$

Note that K_t might be zero. Thus, $\ddot{Q}_f$ and $\ddot{Q}_d$ are

$$\begin{aligned} \ddot{Q}_f &= \mathbb{E}_K \{ \delta_{K_t,0} + (1 - \delta_{K_t,0}) Q_{f|K} \} \\ \ddot{Q}_d &= \mathbb{E}_K \{ \delta_{K_t,0} + (1 - \delta_{K_t,0}) Q_{d|K} \} \end{aligned} \quad (5.39)$$

Moreover, $\Upsilon_{Tr,H_{uv}}$ is the transmission probability of the PU over H_{uv} , and it is equal to the collision probability (in the presence of transmitting SU) as defined in Table 5.4 [77].

Figure 5.4: Average power of aggregate interference caused by SUs to PU vs. total intensity of the SUs with $R_s = 10$ m and $P_{SU} = 15$ dBm

5.1.4 Numerical results

In this section we present the numerical analysis of our proposed method and justify our results via Monte Carlo simulation method over at least 2×10^5 random numbers. Suppose that the secondary network is set up in an area with radius $R_s = 10$ m around the FC or SBS. The SUs are working based on a frame with $L = 100$ and sampling period of $T_s = 10$ ms, implying $T = 1$ s. Moreover, the sensing parts are assumed to have the same duration $N = N'$ and the extra sensing part starts at the middle of the frame time. That is $L' = L/2 = 50$. The transmitted power levels of all users (either the PU or the SU) are $P_{PU} = P_{SU} = 15$ dBm with an indoor PLE $\eta = 2$. To apply CSS, the OR fusion rule is chosen as a strict rule to protect the primary signal from interference when the error probability of reporting channels is $P_e = 1\%$. To meet the assumption of having at least one SU in the network, the total intensity is chosen as $\lambda_{SU} \geq 2.5\%$ to have $\Pr\{K = 0\} < 0.05\%$ over an area with $R_s = 10$ m. On the other hand, the PU has a duty cycle of 50% ($P_b = 0.5$) with holding times of $\mu_b = \mu_i = 5$ s, which is sufficient to satisfy the assumption of medium-traffic rate over a frame duration of $T = 1$ s.

Figure 5.4 represents the average power of aggregate interference caused by SUs to PU, $\bar{I}_{SU \rightarrow PU}$, versus the total intensity of the SUs. It is analyzed over different cases of Rayleigh fading and PLE. From this figure, it is reasonable to have more interference when SU's intensity is higher. Moreover, lower η and/or higher σ_h^2 result in less attenuation of the interfering signal received at the PU.

Figure 5.5: Maximum normalized throughput vs. total intensity of SUs

Figure 5.6: Throughput changes vs. total intensity of SUs

For both cases of the with and without extra sensing part, the maximum achievable levels of all explained throughputs (R_{PU} , R_{SU} and R_T) are depicted in Figure 5.5 versus the total intensity of the SUs. One can see that R_{PU} improves and R_{SU} reduces as λ_{SU} increases. It is obvious that higher λ_{SU} means more SUs in the area. Moreover, by using the OR fusion rule, it is more likely to find the PU in the active status when the number

 Figure 5.7: Throughput changes vs. β with $\lambda_{SU} = 0.05$

of cooperative SUs increases. Hence, higher λ_{SU} results in a lower missed detection and a higher false alarm. R_{PU} can be improved by decreasing missed detection while R_{SU} degrades by increasing false alarm. Furthermore, the effect of λ_{SU} on the throughput of SUs becomes higher when the extra sensing part is utilized. Because the number of SUs which interrupt their transmission during the extra sensing part would be more in higher λ_s . This effect is tangible in Figure 5.6. To find the efficiency of the extra sensing part, changes in percentages of the throughputs, due to the utilization of the extra sensing part, are presented in Figure 5.6 versus λ_{SU} . Although the variation is low over λ_{SU} , this figure shows that the degradation of R_{SU} becomes more by increasing λ_{SU} as described before. On the other hand, for R_{PU} , there are two phases in this figure. In the first phase, the improvement has a positive slope when $\lambda_{SU} < 0.05$, due to sensing improvement by a higher number of cooperative SUs in both main and extra sensing parts. In the second phase, we have a negative slope in the improvement of R_{PU} when $\lambda_{SU} > 0.05$. This is because the amount of improvement in the sensing quality becomes less as the number of users increases after a certain density of the secondary network. For a fixed β , although the number of sensing SUs increases, the number of transmitting SUs increases as well which leads to a higher probability of collision. Briefly, albeit Figure 5.5 shows that higher number of SUs improves R_{PU} , Figure 5.6 reveals that the amount of improvement can be dropped after an optimum intensity. Consequently, they declare a 10 % gain for R_T .

Figure 5.7 depicts the changes of the throughputs (by using the extra sensing part) versus the percentage of SUs participating in the extra sensing (β) in a fixed intensity

($\lambda_{SU} = 0.05$). For $0 < \beta < 0.1$, we have an increase in improvement of R_{PU} as well as degradation of R_{SU} as β increases. It is due to the effect of the number of sensing users on the quality of the sensing (which is explained before) in addition to this fact that the number of transmitting SUs during the extra sensing part reduces as β increases. Furthermore, for higher β , the amount of interference received by sensing users in the extra part reduces which results in better quality of detection. For $0.1 < \beta < 0.5$, the maximum improvement on R_{PU} is almost achieved and the amount of improvement by a higher number of sensing SUs is not significant while the degradation on R_{SU} is still significant due to the less number of transmitting SUs during the extra part. For $0.5 < \beta < 0.9$, since the number of sensing users is high enough, the maximum R_{SU} is achievable in a less number of sensing samples (for both sensing parts). Hence, the total required sensing duration reduces which leads to have less degradation in R_{SU} , due to more available transmission time, and less improvement in R_{PU} , due to higher probability of collision (considering the fact that the PU has a medium traffic rate). Moreover, this effect is more significant on R_{SU} than that of R_{PU} . Note that the optimum numbers of samples for $\beta = 0.1$, $\beta = 0.5$, $\beta = 0.9$ and $\beta = 1$ are 11, 9, 7 and 1, respectively, which means that the optimum number of sensing samples decreases as the number of sensing users increases, based on our analysis. It shows that the maximum achievable quality of sensing is accessible with a lower number of samples when we have enough detectors. This figure implies that the effect of sensing time is not much enough to combat the degradation of R_{SU} in $\beta < 0.5$ while it is considerable for $\beta > 0.5$. For $0.9 < \beta < 1$, as it is mentioned earlier, the optimum number of sensing samples decreases to its minimum value ($N = N' = 1$) which affects R_{PU} significantly as the chance of having collision goes higher. Note that $\beta = 1$ is the same as splitting the total frame time in Figure 5.3 to two frames with equal durations. As a consequence, the number of detectors and sensing duration are two parameters which affect both throughputs directly. In Figure 5.7, the maximum achievable total throughput with respect to these two parameters reveals that reducing the total frame time is not a good idea to improve the total throughput and we can choose an optimum number of SUs to interrupt their transmission and do another sensing (during the transmission time) when the others are still involving in their transmission tasks. Furthermore, the optimum point (for β) is not necessarily 50 % as one can see that it might be 90 % or 10 % in this particular system model.

Finally, Figure 5.8 represents the effect of the number of sensing samples on the relative changes in percentages of the throughputs for $\lambda_{SU} = 0.05$ and $\beta = 0.9$. As discussed earlier, it is obvious to have more improvement in R_{PU} and higher degradation in R_{SU} as the total sensing time (related to N and N') increases. This figure shows that after the optimum point ($N = N' = 7$) improvement of R_{PU} is not significant while R_{SU} is still facing degradation.

Figure 5.8: Throughput changes vs. number of sensing samples with $\lambda_{SU} = 0.05$ and $\beta = 0.9$

As a summary, it is shown that we can achieve higher total throughput (up to 10% improvement) by utilizing extra sensing among transmission phase when the primary signal has a medium traffic rate and the density of the secondary network is randomly distributed. That is, R_{PU} could be improved significantly (around 20%) at the expense of smaller degradation (around 10%) in R_{SU} . We also clarified that although we may have interference during the extra sensing part, the optimum number of detectors for the extra sensing part is not necessarily all of them (which means reducing frame time) or half of them. In this work, we supposed that the extra part conducted at the middle of the frame with the same duration of the main sensing part. Therefore, to achieve better results, one may consider the effect of proposed extra part when its optimum duration might be different from the main part and its optimum starting point is not necessarily at the middle of the frame.

5.2 1-Persistent based CSS

In this part, we propose a 1-persistent based spectrum sensing among the stochastic cooperative users. Note that the explained extra sensing method in Section 5.1 is also considered as part of the proposed method. We show that this method can surprisingly improve the throughput of the primary network without negative effect on the secondary throughput, results in a great enhancement in total throughput of the system. Moreover, we show that the extra sensing part can be utilized to do adjustment on desired levels of the both throughputs.

The working routine of the stochastic-modeled SUs follows a frame-based scheme and the status of the PU may alter at most once within the frame duration, T , based on the medium rate assumption. Using a 1-persistent sensing method, as shown in Figure 5.9, the sensing-transmission algorithm of the SUs is as follows:

1. At the beginning of the frame, all SUs are supposed to be engaged in a cooperative-based detection during the main sensing part.
2. If the overall decision of the main sensing part, D_m , shows that the channel is occupied (active PU), H_1 , SUs attempt for another sensing continuously till they either find that the channel is free or reach the end of the frame. From the sensing decision of the n -th continuous (Cx) sensing part, D_n , SUs occupy the channel once it is found free to use the remained time of the frame duration for their transmission tasks.
3. Contrarily, if the D_m indicates that the channel is free (idle PU), H_0 , SUs occupy it immediately for their transmission. To make sure that the channel is still free, a subset of SUs (K_s) interrupt their transmission to do another sensing during an extra (Ex) sensing part while the rest of them (K_t) continue their transmission. If the overall decision of the Ex part, D_e , is still positive to the unoccupied channel, all SUs continue their transmission. Otherwise (if the channel has been occupied by the PU), those K_t SUs stop their transmission as well to prevent further interference to the primary signal. Then, all SUs await idle up to the next frame.

The medium traffic rate of the primary network is the main reason of using the Ex part. Since we doubt that the PU may go to another state during the transmission of SUs, it is preferred to do another sensing to decrease the chance of facing interference by primary signal. On the other hand, to keep the throughput of SUs in an acceptable range, only a set of SUs are supposed to leave their transmission for sensing, as explained in Section 5.1.

By assuming T_s as sample period, frame duration consists of $L = T/T_s$ samples. All sensing parts collect N samples for their ED-based spectrum sensing whose duration would be $\tau = NT_s$. Furthermore, the Ex part is supposed to be utilized at the mid of the frame.

5.9.a: Working routine of a SU from the perspective of a frame

5.9.b: Flowchart of SU's working routine

Figure 5.9: The algorithm used by SUs for sensing and transmission tasks, based on the 1-persistent spectrum sensing method

Hence, the transmission durations before and after Ex part are $T_{tr}^m = T_{tr}^e = T/2 - \tau$. Note that a minimum required transmission time, $T_{tr,min} = N_{tr,min}T_s$, is considered for the case of Cx sensing. It means that the next Cx sensing part will start only if the remained time to the end of the frame is at least $\tau + T_{tr,min}$, as shown in Figure 5.9.b. Thus, the maximum number of Cx sensing parts would be $N_{cs} = \lfloor (L - N_{tr,min})/N \rfloor - 1$. Moreover, the remained transmission time after n -th Cx part is $T_{tr}^n = T - (n + 1)\tau$.

5.2.1 Network characteristics

5.2.1.1 Medium-traffic primary network

Considering the mentioned sensing-transmission algorithm of the SUs over a medium traffic rate of the primary network as described in Chapter 4, there are different situations, called $H_{uv}^{\mathfrak{S}}$, for the status changing of the PU within the frame duration where

$$\begin{aligned} \mathfrak{S} &\in \begin{cases} \mathfrak{m} & \text{main sensing part} \\ \mathfrak{e} & \text{Ex sensing part} \\ n & n\text{-th Cx sensing part } (n \in \{1, \dots, N_{cs}\}) \end{cases} \\ u &\in \begin{cases} 0 & \text{PU is idle at the end of '}\mathfrak{S}\text{' sensing part} \\ 1 & \text{PU is active at the end of '}\mathfrak{S}\text{' sensing part} \end{cases} \\ v &\in \begin{cases} 1 & \text{no status changing} \\ 2 & \text{status changing during the '}\mathfrak{S}\text{' sensing part} \\ 3 & \text{status changing after the '}\mathfrak{S}\text{' sensing part} \end{cases} \end{aligned} \quad (5.40)$$

For example $H_{12}^{\mathfrak{e}}$ means that the PU alters its status from idle to active during the Ex part, $H_{03}^{\mathfrak{m}}$ means that the PU has been idle within the interval of the main sensing part and goes to the active state during the transmission phase after the main part (within $T_{tr}^{\mathfrak{m}}$), and H_{02}^5 means that the PU switches from active to idle during the 5th Cx part. Note that $H_{01}^{\mathfrak{m}} = H_{01}^{\mathfrak{e}} = H_{01}^1 = \dots = H_{01}^{N_{cs}}$ and $H_{11}^{\mathfrak{m}} = H_{11}^{\mathfrak{e}} = H_{11}^1 = \dots = H_{11}^{N_{cs}}$ denote the PU is, respectively, idle and active for whole frame time. Hence, in the mentioned cases, we omit the upper index as H_{01} and H_{11} for simplicity. Note that $\tilde{u} \in \{0, 1\}$ is still the one's complement of ' u '.

By considering J_0 and J_1 as explained before, for the cases of status changing within the sensing parts (i.e., $H_{u2}^{\mathfrak{S}}$), we define $J_u^{\mathfrak{S}}$ as the number of samples from the beginning of ' $\mathfrak{S}$ ' sensing part with an unchanged PU's status where the PU switches from state ' $\tilde{u}$ ' to state ' u ' afterwards. Therefore,

$$\begin{aligned} J_u^{\mathfrak{m}} &= J_u \\ J_u^{\mathfrak{e}} &= \begin{cases} J_u - \frac{L}{2} & J_u \geq \frac{L}{2} \\ 0 & \text{otherwise} \end{cases} \\ J_u^n &= \begin{cases} J_u - nN & J_u \geq nN \\ 0 & \text{otherwise} \end{cases}, \quad n \in \{1, \dots, N_{cs}\} \end{aligned} \quad (5.41)$$

Again, by considering the transition and idleness/activity probabilities as defined in (4.1)

and (4.4), the marginal probabilities of different hypotheses can be obtained by

$$\begin{aligned}
 P_{H_{01}} &= P_i \cdot (P_{00}(T_s))^L, & P_{H_{11}} &= P_b \cdot (P_{11}(T_s))^L \\
 P_{H_{u2}^m} &= \sum_{J_u=1}^{N-1} P_{J_u}^L, & P_{H_{u3}^m} &= \sum_{J_{\bar{u}}=N}^{(L/2)-1} P_{J_{\bar{u}}}^L, \\
 P_{H_{u2}^e} &= \sum_{J_u=(L/2)}^{(L/2)+N-1} P_{J_u}^L, & P_{H_{u3}^e} &= \sum_{J_{\bar{u}}=L/2+N}^{L-1} P_{J_{\bar{u}}}^L, \\
 P_{H_{u2}^n} &= \sum_{J_u=nN}^{(n+1)N-1} P_{J_u}^L, & P_{H_{u3}^n} &= \sum_{J_{\bar{u}}=(n+1)N}^{L-1} P_{J_{\bar{u}}}^L,
 \end{aligned} \tag{5.42}$$

where $P_{J_u}^L$ has been defined in (4.16).

5.2.1.2 Secondary network with random users

The characteristics for the secondary network are the same as that of explained in Section 5.1.1.2. Note that, in the current method, the Ex part is only happened when the main sensing part indicates the idleness of the PU. Therefore, based on the definition of Ex sensing part in Section 5.1, having interference in Ex part of the current method is irresistible ($\alpha = 1$) unless all SUs attend in the Ex sensing part as $\beta = 1$ ($K_s = K$). Thus, the intensities of transmitting and sensing SUs during the Ex part, which are given in (5.9), would be simplified to

$$\begin{aligned}
 \lambda_s &= \beta \lambda_{SU} \\
 \lambda_t &= (1 - \beta) \lambda_{SU}
 \end{aligned} \tag{5.43}$$

Table 5.5: Means and variances of different cases for the ‘ $\mathfrak{S}$ ’ sensing part

H_s	$\mathbb{E}$	$\mathbb{V}\text{ar}$
$\tilde{H}_{02}^{\mathfrak{S}}$	$N\Xi_0$	$N\Xi_0^2$
$\tilde{H}_{12}^{\mathfrak{S}}$	$N\Xi_1$	$N\Xi_1^2$
$H_{02}^{\mathfrak{S}}$	$J_0^{\mathfrak{S}}\Xi_1 + (N - J_0^{\mathfrak{S}})\Xi_0$	$J_0^{\mathfrak{S}}\Xi_1^2 + (N - J_0^{\mathfrak{S}})\Xi_0^2$
$H_{12}^{\mathfrak{S}}$	$J_1^{\mathfrak{S}}\Xi_0 + (N - J_1^{\mathfrak{S}})\Xi_1$	$J_1^{\mathfrak{S}}\Xi_0^2 + (N - J_1^{\mathfrak{S}})\Xi_1^2$

5.2.2 Detection formulation

The energy level of the N received normalized samples in (5.14) can be generalized for the 1-persistent based CSS method as

$$Y_i^{\mathfrak{S}} = \begin{cases} \sum_{m=1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & \tilde{H}_{u2}^{\mathfrak{S}} \\ \sum_{m=1}^{J_u^{\mathfrak{S}}} \left(\frac{|r_i^{\tilde{u}}(m)|}{\sigma_n} \right)^2 + \sum_{m=J_u^{\mathfrak{S}}+1}^N \left(\frac{|r_i^u(m)|}{\sigma_n} \right)^2, & H_{u2}^{\mathfrak{S}} \end{cases} \quad (5.44)$$

$u, \tilde{u} \in \{0, 1\}$

in which $\tilde{H}_{u2}^{\mathfrak{S}}$ means that the changing process is not happened during the ‘ $\mathfrak{S}$ ’ sensing part, and

$$\begin{aligned} r_i^0(m) &= u_i(m) + \nu_i(m) \\ r_i^1(m) &= h_i(m)s(m) + u_i(m) + \nu_i(m) \end{aligned} \quad (5.45)$$

Again, for all sensing parts, we can approximate $Y_i^{\mathfrak{S}}$ as the gamma distribution as (3.67) when the required means and variances of different cases for the shape and scale parameters of (3.68) and (3.69) are represented in Table 5.5 with

$$\Xi_0 = \begin{cases} 1, & \text{main and Cx sensing parts} \\ \frac{\bar{\gamma}}{\gamma_I}, & \text{Ex sensing part} \end{cases} \quad (5.46)$$

$$\Xi_1 = \Xi_0 + \bar{\gamma}$$

5.2.2.1 Local detection probabilities of the main sensing part

In the main part, all SUs are supposed to attend sensing. Thus, there is no transmitting SU as interferer and $\nu_i(m)$ can be omitted from (5.45), leading to have $I = 0$ and $\gamma_I = \bar{\gamma}$.

During the main sensing part, the status of the PU would be considered as unchanged if the changing process happens either out of the frame duration, H_{u1} , or within the frame duration but after the main part. For the latter situation, and by considering the possible decisions of the main part in the algorithm of Figure 5.9, it can be seen from two ways. Hence, for $\tilde{H}_{u2}^m$ in (5.44), we have $\tilde{H}_{u2}^m \in \{H_{u1}, H_{u3}^m, H_{u2}^c, H_{u3}^c\}$ (if the main part indicates that the PU is idle) and $\tilde{H}_{u2}^m \in \{H_{u1}, H_{u2}^1, \dots, H_{u2}^{N_{cs}}, H_{u3}^{N_{cs}}\}$ (if the main part indicates that the PU is active). What we need, for the required detection probabilities, is the total probability of $\tilde{H}_{u2}^m$, $P_{\tilde{H}_{u2}^m}$, which is the summation of probabilities for all possible cases in each set. Since the summation of their probabilities is the same for both sets, we can consider either of them. Eventually, for the main part, the local probabilities of false alarm (P_f^m) and detection (P_d^m) at each SU, are

$$\begin{aligned} P_f^m &= \frac{P_{\tilde{H}_{02}^m} \Pr\{Y_i^m > \lambda \mid \tilde{H}_{02}^m\} + \sum_{J_0=1}^{N-1} P_{J_0}^L \Pr\{Y_i^m > \lambda \mid H_{02}^m\}}{P_{\tilde{H}_{02}^m} + P_{H_{02}^m}} \\ P_d^m &= \frac{P_{\tilde{H}_{12}^m} \Pr\{Y_i^m > \lambda \mid \tilde{H}_{12}^m\} + \sum_{J_1=1}^{N-1} P_{J_1}^L \Pr\{Y_i^m > \lambda \mid H_{12}^m\}}{P_{\tilde{H}_{12}^m} + P_{H_{12}^m}} \end{aligned} \quad (5.47)$$

where $\Pr\{Y_i^m > \lambda \mid H_s\} = \Gamma\left(k_{H_s}, \frac{\lambda}{\theta_{H_s}}\right) / \Gamma(k_{H_s})$ and λ is the detection threshold of the main part.

5.2.2.2 Local detection probabilities of the Cx sensing parts

In each Cx sensing part, since the channel has been found occupied in the previous part(s), there is no transmitting SU. Therefore, interference is still zero and we can drop $\nu_i(m)$. In n -th Cx part, the status of the PU can be assumed unchanged if the changing process happens either out of the frame duration or within the frame period but before/after n -th Cx sensing part. Hence, $\tilde{H}_{u2}^n \in \{H_{u1}, H_{u2}^m, H_{u2}^{n'}, H_{u3}^n\}$, $\forall n' < n$ for each $n \in \{1, \dots, N_{cs}\}$,

and we can obtain their local probabilities as follows:

$$\begin{aligned}
 P_f^n &= \frac{P_{\tilde{H}_{02}^n} \Pr\{Y_i^n > \lambda \mid \tilde{H}_{02}^n\} + \sum_{J_0=nN}^{(n+1)N-1} P_{J_0}^L \Pr\{Y_i^n > \lambda \mid H_{02}^n\}}{P_{\tilde{H}_{02}^n} + P_{H_{02}^n}} \\
 P_d^n &= \frac{P_{\tilde{H}_{12}^n} \Pr\{Y_i^n > \lambda \mid \tilde{H}_{12}^n\} + \sum_{J_1=nN}^{(n+1)N-1} P_{J_1}^L \Pr\{Y_i^n > \lambda \mid H_{12}^n\}}{P_{\tilde{H}_{12}^n} + P_{H_{12}^n}}
 \end{aligned} \tag{5.48}$$

where it is supposed to use the same threshold as the main part in $\Pr\{Y_i^n > \lambda \mid H_s\}$.

5.2.2.3 Local detection probabilities of the Ex sensing part

In the Ex part, as discussed earlier, it is assumed that some SUs (K_s), called sensing SUs, interrupt their transmission to do another sensing while other SUs (K_t), called transmitting SUs, are still active. We define $\tilde{H}_{u2}^\epsilon \in \{H_{u1}, H_{u2}^m, H_{u3}^m, H_{u3}^\epsilon\}$ by using the same logic, and local probabilities can be obtained as

$$\begin{aligned}
 P_f^\epsilon &= \frac{P_{\tilde{H}_{02}^\epsilon} \Pr\{Y_i^\epsilon > \lambda' \mid \tilde{H}_{02}^\epsilon, K_t\} + \sum_{J_0=\frac{L}{2}}^{\frac{L}{2}+N-1} P_{J_0}^L \Pr\{Y_i^\epsilon > \lambda' \mid H_{02}^\epsilon, K_t\}}{P_{\tilde{H}_{02}^\epsilon} + P_{H_{02}^\epsilon}} \\
 P_d^\epsilon &= \frac{P_{\tilde{H}_{12}^\epsilon} \Pr\{Y_i^\epsilon > \lambda' \mid \tilde{H}_{12}^\epsilon, K_t\} + \sum_{J_1=\frac{L}{2}}^{\frac{L}{2}+N-1} P_{J_1}^L \Pr\{Y_i^\epsilon > \lambda' \mid H_{12}^\epsilon, K_t\}}{P_{\tilde{H}_{12}^\epsilon} + P_{H_{12}^\epsilon}}
 \end{aligned} \tag{5.49}$$

Note that $\Pr\{Y_i^\epsilon > \lambda' \mid H_s, K_t\}$ is conditioned on K_t and the threshold (λ') is supposed different with that of the main and Cx parts due to the possible interference.

Using the same expressions as derived in Section 5.1.2, we have

$$\Pr\{Y_i^\epsilon > \lambda' \mid H_s, K_t\} = 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^\infty J_{k_{H_s}}(2\sqrt{Z}) Z^{\frac{k_{H_s}}{2}-1} \mathcal{L}_{\theta_{H_s}|K_t}(Z/\lambda') dZ \tag{5.50}$$

where

$$\begin{aligned}
 \mathcal{L}_{\theta_{\tilde{H}_{02}^\epsilon}|K_t}(Z') &= e^{-Z'} \mathcal{L}_{I|K_t}(Z'/\sigma_n^2) \\
 \mathcal{L}_{\theta_{\tilde{H}_{12}^\epsilon}|K_t}(Z') &= e^{-(1+\bar{\gamma})Z'} \mathcal{L}_{I|K_t}(Z'/\sigma_n^2) \\
 \mathcal{L}_{\theta_{H_{02}^\epsilon}|K_t}(Z') &= e^{-(1+\frac{J_0^\epsilon}{N}\bar{\gamma})CZ'} \mathcal{L}_{I|K_t}(CZ'/\sigma_n^2) \\
 \mathcal{L}_{\theta_{H_{12}^\epsilon}|K_t}(Z') &= e^{-(1+\frac{(N-J_1^\epsilon)}{N}\bar{\gamma})CZ'} \mathcal{L}_{I|K_t}(CZ'/\sigma_n^2)
 \end{aligned} \tag{5.51}$$

with $Z' = Z/\lambda'$ and $\mathcal{L}_{I|K_t}(s)$ as obtained in (5.19) and (5.21).

5.2.2.4 Cooperative spectrum sensing

Similar to Section 5.1, the number of available SUs may not be necessarily enough to protect the primary signal from the interference through a satisfying CSS. Hence, we use the OR fusion rule to make sure that the system is strict to protect the primary signal which has the first priority. Consequently, for the conditional total probabilities of false alarm ($Q_{f|K}^{\mathfrak{S}}$) and detection ($Q_{d|K}^{\mathfrak{S}}$) of the ‘ $\mathfrak{S}$ ’ sensing part, we have

$$\begin{aligned} Q_{f|K}^{\mathfrak{S}} &= 1 - [P_{d_0}^{\mathfrak{S}}(1 - P_e) + P_f^{\mathfrak{S}}P_e]^{\mathcal{K}} \\ Q_{d|K}^{\mathfrak{S}} &= 1 - [P_m^{\mathfrak{S}}(1 - P_e) + P_d^{\mathfrak{S}}P_e]^{\mathcal{K}}, \quad \mathfrak{S} \in \{\mathbf{m}, \mathbf{e}, 1, \dots, N_{cs}\} \end{aligned} \quad (5.52)$$

where $P_{d_0}^{\mathfrak{S}} = 1 - P_f^{\mathfrak{S}}$ and $P_m^{\mathfrak{S}} = 1 - P_d^{\mathfrak{S}}$ are local probabilities of detecting H_0 and missed detection, respectively, and

$$\mathcal{K} \in \begin{cases} K & \text{main and Cx sensing parts} \\ K_s = \lceil \beta K \rceil & \text{Ex sensing part} \end{cases} \quad (5.53)$$

5.2.3 System performance

5.2.3.1 Total throughput

As discussed earlier, the maximization of the secondary throughput without taking the level of primary throughput into account does not make sense and we aim to enhance the total transmission throughput of the system to meet the feasibility of an adequate CRN. Therefore, similar to Section 5.1.3, we analyze the performance of the proposed method on improvement of the total throughput when the maximum total throughput is subjected to the maximum achievable transmission throughput of the secondary network. Since the throughputs of the networks are conditioned to the number of cooperative users (due to the production of conditional total probabilities), the total normalized throughput of the system in (5.35) would be obtained by expectation over K as follows:

$$R_T = R_{SU} + R_{PU} = \mathbb{E}_K \{R_{SU|K} + R_{PU|K}\} \quad (5.54)$$

Both secondary and primary throughputs comprise that of their successful transmission when the main part indicates the idle PU ($R_{SU|K}^0, R_{PU|K}^0$), which would be affected by the Ex part, and that of their successful transmission when the main part reveals the activity of the PU ($R_{SU|K}^1, R_{PU|K}^1$), which is related to the decision(s) of Cx parts. Since the PU has the chance of successful transmission during the main part, $R_{PU|K}$ consists of

one more part as $R_{PU|K}^m$. Accordingly, we have

$$\begin{aligned}
 R_{SU|K} &= R_{SU|K}^0 + R_{SU|K}^1 \\
 R_{SU|K}^0 &= \left(\frac{T_{tr}^m}{T} P_{SU,1|K}^0 \right) + \left(\beta_T \frac{\tau}{T} P_{SU,2|K}^0 \right) + \left(\frac{T_{tr}^e}{T} P_{SU,3|K}^0 \right) \\
 R_{SU|K}^1 &= \sum_{n=1}^{N_{cs}} \frac{T - (n+1)\tau}{T} P_{SU,n|K}^1
 \end{aligned} \tag{5.55}$$

with the related utilization probabilities as

$$\begin{aligned}
 P_{SU,1|K}^0 &= Q_{d_0|K}^m [P_{H_{01}} + P_{H_{02}^m} + (1 - \Upsilon_{03}^m) P_{H_{03}^m} + P_{H_{12}^e} + P_{H_{03}^e}] + Q_{m|K}^m (1 - \Upsilon_{13}^m) P_{H_{13}^m} \\
 P_{SU,2|K}^0 &= Q_{d_0|K}^m [P_{H_{01}} + P_{H_{02}^m} + (1 - \Upsilon_{12}^e) P_{H_{12}^e} + P_{H_{03}^e}] + Q_{m|K}^m [P_{H_{13}^m} + (1 - \Upsilon_{02}^e) P_{H_{02}^e}] \\
 P_{SU,3|K}^0 &= Q_{d_0|K}^m [\delta_{K_s,0} + (1 - \delta_{K_s,0}) Q_{d_0|K}^e] [P_{H_{01}} + P_{H_{02}^m} + (1 - \Upsilon_{03}^e) P_{H_{03}^e}] \\
 &\quad + Q_{m|K}^m [\delta_{K_s,0} + (1 - \delta_{K_s,0}) Q_{d_0|K}^e] [P_{H_{13}^m} + P_{H_{02}^e}] \\
 &\quad + Q_{m|K}^m [\delta_{K_s,0} + (1 - \delta_{K_s,0}) Q_{m|K}^e] (1 - \Upsilon_{13}^e) P_{H_{13}^e} \\
 P_{SU,n|K}^1 &= Q_{f|K}^m Q_{d_0|K}^n \prod_{i=1}^{n-1} Q_{f|K}^i [P_{H_{01}} + P_{H_{02}^m} + (1 - \Upsilon_{03}^n) P_{H_{03}^n}] \\
 &\quad + Q_{d|K}^m Q_{m|K}^n \prod_{i=1}^{n-1} Q_{d|K}^i (1 - \Upsilon_{13}^n) P_{H_{13}^n} + Q_{d|K}^m Q_{d_0|K}^n \sum_{i=1}^n \prod_{j=1}^{i-1} Q_{d|K}^j \prod_{j'=i}^{n-1} Q_{f|K}^{j'} P_{H_{02}^i}
 \end{aligned} \tag{5.56}$$

and

$$\begin{aligned}
 R_{PU|K} &= R_{PU|K}^m + R_{PU|K}^0 + R_{PU|K}^1 \\
 R_{PU|K}^m &= \frac{\tau}{T} P_{PU|K}^m \\
 R_{PU|K}^0 &= \delta_{K_t,0} \left(\frac{\tau}{T} P_{PU,1|K}^0 \right) + (1 - \delta_{K_s,0}) \left(\frac{T_{tr}^e}{T} P_{PU,2|K}^0 \right) \\
 R_{PU|K}^1 &= \frac{\tau}{T} \sum_{n=1}^{N_{cs}} P_{PU,n|K}^1 + \frac{T_{tr}^{N_{cs}}}{T} P_{PU,wait|K}^1
 \end{aligned} \tag{5.57}$$

when

$$\begin{aligned}
 P_{PU|K}^m &= P_{H_{11}} + \Upsilon_{02}^m P_{H_{02}^m} + \Upsilon_{12}^m P_{H_{12}^m} + P_{H_{13}^m} + P_{H_{02}^\epsilon} + P_{H_{13}^\epsilon} \\
 P_{PU,1|K}^0 &= Q_{m|K}^m [P_{H_{11}} + P_{H_{12}^m} + \Upsilon_{02}^\epsilon P_{H_{02}^\epsilon} + P_{H_{13}^\epsilon}] + Q_{d_0|K}^m [P_{H_{03}^m} + \Upsilon_{12}^\epsilon P_{H_{12}^\epsilon}] \\
 P_{PU,2|K}^0 &= Q_{m|K}^m Q_{d|K}^\epsilon [P_{H_{11}} P_{H_{12}^m} + \Upsilon_{13}^\epsilon P_{H_{13}^\epsilon}] + Q_{d_0|K}^m Q_{d|K}^\epsilon [P_{H_{03}^m} + P_{H_{12}^\epsilon}] \\
 &\quad + Q_{d_0|K}^m Q_{f|K}^\epsilon \Upsilon_{03}^\epsilon P_{H_{03}^\epsilon} \\
 P_{PU,n|K}^1 &= Q_{d|K}^m \prod_{i=1}^{n-1} Q_{d|K}^i [P_{H_{11}} + P_{H_{12}^m} + P_{H_{13}^n} + \Upsilon_{02}^n P_{H_{02}^n}] \\
 &\quad + Q_{f|K}^m \left(\prod_{i=1}^{n-1} Q_{f|K}^i \Upsilon_{12}^n P_{H_{12}^n} + \sum_{i=1}^{n-1} \prod_{j=1}^{i-1} Q_{f|K}^j \prod_{j'=i}^{n-1} Q_{d|K}^{j'} P_{H_{12}^{i'}} \right) \\
 P_{PU,wait|K}^1 &= Q_{d|K}^m \prod_{i=1}^{N_{cs}} Q_{d|K}^i [P_{H_{11}} + P_{H_{12}^m} + \Upsilon_{13}^{N_{cs}} P_{H_{13}^{N_{cs}}}] \\
 &\quad + Q_{f|K}^m \left(\prod_{i=1}^{N_{cs}} Q_{f|K}^i \Upsilon_{03}^{N_{cs}} P_{H_{03}^{N_{cs}}} + \sum_{i=1}^{N_{cs}} \prod_{j=1}^{i-1} Q_{f|K}^j \prod_{j'=i}^{N_{cs}} Q_{d|K}^{j'} P_{H_{12}^{i'}} \right)
 \end{aligned} \tag{5.58}$$

Note that, $Q_{d_0|K}^\epsilon = 1 - Q_{f|K}^\epsilon$ and $Q_{m|K}^\epsilon = 1 - Q_{d|K}^\epsilon$ are conditional total probabilities of detecting H_0 and missed detection of the ‘ $\mathfrak{S}$ ’ sensing part, respectively. Moreover, $\delta_{K_t,0}$ and $\delta_{K_s,0}$ are the Kronecker delta functions as defined in (5.34) and $\beta_T = K_t/K$. In the case of no sensing user for Ex part ($K_s = 0$), it is worthwhile to notice that the SUs will continue their transmission tasks based on the decision of the main part (which is $D_m = 0$). Therefore, for the OR fusion rule with $K_s = 0$, we have $Q_{f|K}^\epsilon = Q_{d|K}^\epsilon = 0$ from (5.52), resulting in $P_{PU,2|K}^0 = 0$ and $Q_{d_0|K}^\epsilon = Q_{m|K}^\epsilon = 1$ in $P_{SU,3|K}^0$. It shows that we can omit $\delta_{K_s,0}$ in (5.56) and (5.57) in the case of the OR fusion rule. Moreover, $P_{PU,wait|K}^1$ is the probability that SUs wait in the idle mode during the remained time of the frame due to the decision of the last Cx part as active PU ($D_{N_{cs}} = 1$).

5.2.3.2 Increased factor of sensing attempts

Applying a 1-persistent based sensing method increases the average number of sensing attempts by SUs. Since the energy consumption is very important in some applications, we further analyze the effect of the proposed method on this parameter. Hence, one may consider the level of throughput enhancement at the expense of consumed energy where it is not crucial. Suppose that the energy consumption for each sensing attempt at each SU is known. Then, we define a factor, Ω , to determine the amount of change in the average number of required sensing attempts (which might be done by the cooperative sensing SUs during a frame time) when the proposed method is applied as compared with that of the conventional method. This factor can be seen as the increased factor of the energy

consumption and can be obtained as follows:

$$\Omega = \mathbb{E}_K \left\{ \frac{\overbrace{K(1 + \bar{N}_{Cx})}^{\text{Main} + \text{Cx}} + \overbrace{K_s(P_0^m Q_{d_0|K}^m + P_1^m Q_{m|K}^m)}^{\text{Ex}}}{\underbrace{K}_{\text{Conventional}}} \right\} \quad (5.59)$$

where

$$\begin{aligned} P_0^m &= P_{H_{01}} + P_{H_{02}^m} + P_{H_{03}^m} + P_{H_{12}^\epsilon} + P_{H_{03}^\epsilon} \\ P_1^m &= P_{H_{11}} + P_{H_{12}^m} + P_{H_{13}^m} + P_{H_{02}^\epsilon} + P_{H_{13}^\epsilon} \end{aligned} \quad (5.60)$$

and $\bar{N}_{Cx}$ is the average number of Cx sensing parts and it is given by

$$\bar{N}_{Cx} = \sum_{n=1}^{N_{cs}} n \Pr\{N_{Cx} = n\} \quad (5.61)$$

with

$$\begin{aligned} \Pr\{N_{Cx} = n\} &= Q_{d|K}^m Q_{m|K}^n \prod_{i=1}^{n-1} Q_{d|K}^i [P_{H_{11}} + P_{H_{12}^m} + P_{H_{13}^n}] \\ &\quad + Q_{f|K}^m Q_{d_0|K}^n \prod_{i=1}^{n-1} Q_{f|K}^i [P_{H_{01}} + P_{H_{02}^m} + P_{H_{03}^n}] \\ &\quad + \sum_{i=1}^n Q_{f|K}^m Q_{m|K}^n \prod_{j=1}^{i-1} Q_{f|K}^j \prod_{j'=i}^{n-1} Q_{d|K}^{j'} P_{H_{12}^i} \\ &\quad + Q_{d|K}^m Q_{d_0|K}^n \prod_{j=1}^{i-1} Q_{d|K}^j \prod_{j'=i}^{n-1} Q_{f|K}^{j'} P_{H_{02}^i} \end{aligned} \quad (5.62)$$

Figure 5.10: Maximum normalized throughput vs. total intensity of SUs

5.2.4 Numerical results

In this section we present the theoretical analysis of our proposed method numerically and justify them with simulation. It is supposed to have the same system parameters as considered in Section 5.1.4. It is also assumed that $T_{tr,min} = T_s = 10$ ms.

Figure 5.10 represents the maximum achievable throughputs versus the user intensity of the secondary network. It shows that the proposed method can improve both primary and secondary throughputs (and then the overall one) for a range of the SU's intensity. The throughput enhancement of the primary network is more significant while it is slight for the secondary network. Moreover, one can see that R_{SU} may face a small degradation when the user intensity is not enough ($\lambda_{SU} < 4\%$). Since the number of available SUs is not enough in the case of very low intensity, the optimum number of sensing samples (related to the sensing time) increases in order to keep the detection reliability in an acceptable level. Hence, SUs spend more time for sensing instead of transmission which leads to the throughput degradation. Based on our analysis, the optimum N for $\lambda_{SU} = 2.5\%$, $\lambda_{SU} = 5\%$, $\lambda_{SU} = 7.5\%$, $\lambda_{SU} = 10\%$ and $\lambda_{SU} \geq 15\%$ is $N = 7$, $N = 4$, $N = 3$, $N = 2$ and $N = 1$, respectively. Furthermore, Figure 5.10 reveals that the achieved throughputs are almost steady in the proposed method while the throughputs of the conventional method are sensitive to the intensity of the SUs. Note that the OR fusion rule is very strict to prevent the missed detection (at the expense of false alarm). Therefore, by using the OR rule in the conventional method, it is more likely to find the status of the PU in the active state when the number of cooperative users is higher.

In Figure 5.11, relative changes in percentages of the throughputs have been displayed over different values of λ_{SU} . It shows that, by utilizing the proposed method, the total

Figure 5.11: Throughput changes vs. total intensity of SUs

Figure 5.12: Increased factor of sensing attempts vs. total intensity of SUs

throughput gains around 36 % - 43 %, which increases as the SU's intensity goes higher. The slopes in the changes of R_{SU} and R_{PU} are due to the above-mentioned intensity sensitivity of the conventional method. It means that the proposed method could solve the sensitivity of the conventional method on the SU's intensity and keep all throughputs in a steady level in addition to the throughput enhancement. The increased factor of the average number of sensing attempts versus λ_{SU} is shown in Figure 5.12. It is reasonable to have higher number of sensing attempts when the number of available users is higher.

To analyze the efficacy of the Ex part deeply, Figures 5.13-5.18 represent the effect of β on the system performance over both low ($\lambda_{SU} = 5\%$) and high ($\lambda_{SU} = 25\%$)

 Figure 5.13: Maximum normalized throughput vs. β with $\lambda_{SU} = 5\%$

 Figure 5.14: Throughput changes vs. β with $\lambda_{SU} = 5\%$

values of the user intensity in the secondary network. Since the reliability of the system would be improved by doing another sensing during the transmission (especially in the case of medium-traffic rate of the PU), the optimum number of sensing samples reduces accordingly. Small N means shorter sensing time which helps SUs to save the time for their transmission tasks. Note that less N results in higher possible number of Cx parts, subsequently. However, by applying Ex part, some SUs must interrupt their transmission which degrades the secondary throughput. Therefore, in Figures 5.13 and 5.16, once the Ex part is applied ($\beta = 0.1$), we have degradation in secondary throughput, due to the effect of interruption in SU's transmission, and improvement in primary throughput, due

Figure 5.15: Increased factor of sensing attempts vs. β with $\lambda_{SU} = 5\%$

to better quality of detection. For $\beta > 0.1$, the effect of β on R_{SU} and R_{PU} has a small difference for the cases with low and high intensities. In $\lambda_{SU} = 5\%$, as shown in Figure 5.13, since optimum N is reducing, we see that R_{SU} increases and R_{PU} decreases as β increases. In $\lambda_{SU} = 25\%$, as shown in Figure 5.16, since the amount of cooperative users is high enough, the minimum value of the $N = 1$ is considered for the range of $0.1 \leq \beta \leq 1$ and R_{PU} is remained unchanged as β increases. For R_{SU} , it is valid when $0.1 \leq \beta \leq 0.9$ while an improvement can be seen in $\beta > 0.9$ due to the insignificant interference which results in the false alarm reduction. In general, Figures 5.13, 5.14, 5.16 and 5.17 reveal that the effect of Ex part on throughputs (especially total throughput) is not significant and Cx parts have the main contribution to improve the system throughput. However, we can utilize the Ex part besides Cx parts to adjust the levels of R_{SU} and R_{PU} around the desired values. In Figure 5.15, one may see an unexpected behavior when Ω reduces for $\beta > 0.75$. In this range, since the number of sensing users is high enough and the interference is negligible (implies a reliable Ex part), the optimum threshold of the main sensing part (and optimum threshold of Cx parts, subsequently) goes higher to diminish false alarm at the expense of missed detection. It is because the system has trusted on the Ex part to amend any missed detection of the main part afterwards. Higher detection threshold means that the main part may find the channel free with higher probability. It means that the probability of using Cx parts reduces. In addition, if the system goes to the Cx parts (after main part), it may find channel free in less number of Cx sensing attempts. Therefore, the average number of required sensing attempts drops significantly. It can explain the reason of having higher slope of throughput variation for $\beta > 0.75$ in Figures 5.13 and 5.14 as well.

 Figure 5.16: Maximum normalized throughput vs. β with $\lambda_{SU} = 25\%$

 Figure 5.17: Throughput changes vs. β with $\lambda_{SU} = 25\%$

The same scenario can be seen in the case of $\lambda_{SU} = 25\%$ for $\beta > 0.9$, as shown in Figures 5.16-5.18. Since the total number of SUs is higher (as compared with $\lambda_{SU} = 5\%$), the possibility of facing interference from transmitting SUs is higher and the system needs a bigger portion of SUs as sensing SUs to achieve the desired level of the confidence on the Ex part to increase the threshold. Moreover, they disclose that it is better to involve all SUs in the Ex sensing part ($\beta = 1$), in which we achieve the highest total throughput with smallest increase factor of energy consumption (in the case of using Ex part), unless we aim to adjust R_{SU} and R_{PU} around the desired levels.

In general, the results show that the 1-persistent sensing method can enhance the

Figure 5.18: Increased factor of sensing attempts vs. β with $\lambda_{SU} = 25\%$

total throughput of the system significantly by improving both primary and secondary throughputs. For example, for $\lambda_{SU} = 5\%$, R_{SU} , R_{PU} and R_T could gain, respectively, 4.5%, 34.5% and 39% just by utilizing the Cx parts (without using Ex part) at the expense of $\Omega \approx 7$ ($\beta = 0$ in Figure 5.15). Note that the energy consumption is not a big issue in many applications where the capability of the proposed method emerges more. Furthermore, Figures 5.14 and 5.17 show that the efficiency of the proposed method would be more significant as the intensity of SUs goes higher, when R_T could gain up to 39% and 42%, respectively, for the cases of $\lambda_{SU} = 5\%$ and $\lambda_{SU} = 25\%$ just by utilizing the Cx parts (i.e., $\beta = 0$). It is further obvious in Figure 5.11, in which R_T improvement increases from 36% for the case of $\lambda_{SU} = 2.5\%$ to 42.6% for the case of $\lambda_{SU} = 25\%$ by using both Cx and Ex sensing parts.

Moreover, it is shown that we can access to a fine adjustment of required throughputs by using an extra sensing part during the transmission phase.

However, we have assumed that all sensing durations are equal (N), the Ex part located at the middle of the frame and the optimum threshold is the same for the main and Cx parts. Hence, one may consider them differently to achieve better results by a joint optimization.

5.3 Conclusion

In this chapter, a CRN with medium-traffic primary network has been considered when the SUs are stochastically distributed over an area around the SBS. Since the uncertainties about the activity of the PU and the number of available cooperative SUs are high, these conditions together degrade the performance of an ED-based CSS significantly. To mitigate this situation, we first proposed an extra sensing part within the transmission phase in which a portion of SUs interrupt their transmission tasks for another sensing while others continue their transmission. Although the sensing SUs may suffer interference from the transmitting SUs during the extra sensing part, we showed that the total throughput of the system can be improved by optimizing the percentage of SUs which are attending in the extra part, β , and the optimum value is not necessarily $\beta = 1$ (i.e., all SUs).

We further proposed a 1-persistent based CSS in which the extra sensing part has been included. The results disclosed that the total transmission throughput of the system can be enhanced significantly over a range of SU's intensity when both primary and secondary throughputs are gained. Moreover, we analyzed the amount of energy consumption which may burden the system by the proposed method. Therefore, one may consider this method based on the application.

Chapter 6

Conclusion and future work

6.1 Conclusion

Since the spectrum sensing of secondary users (SUs) is an important part of a cognitive radio network (CRN) design from the viewpoint of signal processing, we focused on optimization of system performance (in terms of detection quality and throughput) in the case of cooperative spectrum sensing (CSS). ‘Number of cooperative users’, ‘quality of reporting channels’ and ‘the fusion rule’ are some parameters which would be added to the current system parameters such as ‘quality of sensing channel’ and ‘stability in the activity of the primary user (PU)’ when the CSS has been utilized. Therefore, we aimed to achieve a practical system model in which different aspects of a feasible CSS based CRN have been considered. To do so, we first focused on faded sensing and imperfect reporting channels that are quite likely in a wireless network. Note that it is firstly supposed to have a predetermined number of cooperative SUs when the PU has a fixed status during the studied period. By introducing current works in this area, we analyzed the performance of some suggested methods (such as the improved energy detector and double-threshold comparison scheme) in the improvement of the energy detector based CSS. We then introduced a new idea of the improved double-threshold energy detector (IDED) method, and showed that it can outperform others over defined channels and required constraints. The results disclosed that the improved energy detector would be able to enhance the performance of the system, in terms of both detection reliability and transmission throughput, when it is utilized besides the double-threshold method.

Nonetheless, it is quite possible that the PU changes its activity status during the considered period. Hence, we extended our analysis to a case with medium-traffic primary network in the next step, when the number of SUs is still deterministic. It is assumed that the PU may change its status at most once during the studied interval. The possible changes in status of the PU degrade detection performance of the SUs significantly and we need to take it into account. We showed that how double-threshold based methods,

particularly IDED, can improve the performance parameters of the system.

Moreover, as another important parameter of the CSS scheme, there is no guarantee to have a fixed number of cooperative users in the network to detect the activity of the either fixed-state or state-variable PU. This randomness in the number of serving users is quite possible in new technologies such as device-2-device communication among cellular users, where we can see the signs of cognitive radio concepts.

Therefore, we subsequently turned our system model to a more realistic situation where cooperative SUs are stochastically modeled over a CRN with medium-traffic primary network. It is worth mentioning that these two circumstances together can degrade the performance of a CRN extremely because the uncertainty is quite high in the activity of the PU as well as available cooperative users. By proposing extra sensing part and 1-persistent spectrum sensing scheme, we evaluated the proficiency of the suggested methods to enhance the total throughput of the system in this particular CRN.

6.2 Future work

Although we have tried to define a CRN as practical as possible for conducting our analyses with higher competency, there still exist some realistic situations to extend our current work and the proposed methods to achieve a more comprehensive study on a feasible CRN. Hence, we define some future works as follows:

1. **Faded sensing channel:** Rayleigh fading is just one type of faded sensing channel. In [22, 40], the CSS was analyzed over Nakagami- m faded channels. Moreover, a great work has been done in [25], in which the faded sensing channel is modeled by the extended generalized- K (EGK), as a general distribution which can be simplified to the most well-known fading distributions (such as Nakagami- m , Rayleigh and Weibull). Hence, one can extend our analysis over other fading types.
2. **Faded reporting channels:** in paper [46], Rayleigh and Nakagami faded versions of reporting channels were considered besides BSC channels. Working on other types of reporting channels is also an open area to extend our research activities.
3. **Traffic rate of the primary network:** Since the spectrum sensing and utilization of the spectrum by SUs should be done in a very short time interval, medium-traffic rate of the PU seems to be quite enough for consideration in analysis of spectrum sensing by SUs. Nevertheless, it is possible to have more than one status changing by PU during the working (either sensing, transmitting or both) time of the SUs based on traffic of the primary network and desired CRN characteristic, as the work in [67] has been analyzed for multiple PUs with different arrival and departure rates. Hence, it can be reasonable to extend our work to a higher traffic rate of the primary network.
4. **Stochastic model of the secondary network:** In this work the homogeneous PPP has been considered to model the distribution of SUs within the network. Definitely, other types of the PP can be considered to model a particular CRN as a real case, based on the network properties. Therefore, this work can be extended to other PPs such as Matérn hard-core PP and Binomial PP [89, 100].

Appendices

Appendix A

Mean and variance of the IED's output over Rayleigh fading

We recall the m -th received samples over the Rayleigh-faded sensing channels as described in (2.1) and its normalized version in (2.4). Since all samples are supposed i.i.d., we omit the index i and redefine them as $r_m \sim \mathcal{CN}(0, \sigma_r^2)$ and x_m without loss of generality, where

$$\sigma_r^2 = \begin{cases} \sigma_n^2 & \text{PU is idle } (H_0) \\ \sigma_h^2 \sigma_s^2 + \sigma_n^2 & \text{PU is active } (H_1) \end{cases} \quad (\text{A.1})$$

Using the same concepts as in (3.2) and (3.3), we have

$$F_{|r_m|^p}(y) = F_{|r_m|^2}\left(y^{\frac{2}{p}}\right) \iff f_{|r_m|^p}(y) = \frac{2}{p} y^{\frac{2}{p}-1} f_{|r_m|^2}\left(y^{\frac{2}{p}}\right) \quad (\text{A.2})$$

Since $|r_m|^2$ is exponentially distributed with parameter $1/\sigma_r^2$, the PDF of $|r_m|^p$ can be given by

$$f_{|r_m|^p}(y) = \frac{2}{p} y^{\frac{2}{p}-1} \times \frac{1}{\sigma_r^2} \exp\left(-\frac{y^{\frac{2}{p}}}{\sigma_r^2}\right) = \frac{2y^{(\frac{2}{p}-1)}}{p\sigma_r^2} \exp\left(-\frac{y^{\frac{2}{p}}}{\sigma_r^2}\right), \quad y \geq 0 \quad (\text{A.3})$$

Suppose that w is the observed energy level of the x_m through the IED as given in (2.3). Therefore,

$$\begin{aligned} \mathbb{E}\{w\} &= \frac{1}{N} \sum_{m=1}^N \mathbb{E}\{|x_m|^p\} = \frac{N}{N} \times \mathbb{E}\{|x_m|^p\} = \mathbb{E}\{|x_m|^p\} \\ &= \mathbb{E}\left\{\left(\frac{|r_m|}{\sigma_n}\right)^p\right\} = \frac{\mathbb{E}\{|r_m|^p\}}{\sigma_n^p} \end{aligned} \quad (\text{A.4})$$

with

$$\begin{aligned}
 \mathbb{E} \{|r_m|^p\} &= \int_{-\infty}^{+\infty} y f_{|r_m|^p}(y) dy = \int_0^{+\infty} y \frac{2y^{\left(\frac{2}{p}-1\right)}}{p\sigma_r^2} \exp\left(-\frac{y^{\frac{2}{p}}}{\sigma_r^2}\right) dy \\
 &\stackrel{(i)}{=} \frac{2}{\sigma_r^2} \int_0^{+\infty} x^{p+1} \exp\left(-\frac{x^2}{\sigma_r^2}\right) dx \stackrel{(ii)}{=} \frac{2}{\sigma_r^2} \times \frac{1}{2} \left(\frac{1}{\sigma_r^2}\right)^{-\left(\frac{p+2}{2}\right)} \Gamma\left(\frac{p+2}{2}\right) \quad (\text{A.5}) \\
 &= \sigma_r^p \Gamma\left(\frac{p+2}{2}\right)
 \end{aligned}$$

where we considered $y^{1/p} = x$ and used [101, eq. 3.462.9] (when $\mu = 2$) in (i) and (ii), respectively. Finally, by substituting (A.5) in (A.4), we have

$$\mathbb{E}\{w\} = \left(\frac{\sigma_r^2}{\sigma_n^2}\right)^{\frac{p}{2}} \Gamma\left(\frac{p+2}{2}\right) \quad (\text{A.6})$$

On the other hand, $\text{Var}\{w\}$ can be obtained as

$$\begin{aligned}
 \text{Var}\{w\} &= \mathbb{E}\{(w - \mathbb{E}\{w\})^2\} = \mathbb{E}\left\{\left(\frac{1}{N} \sum_{m=1}^N |x_m|^p - \mathbb{E}\left\{\frac{1}{N} \sum_{m=1}^N |x_m|^p\right\}\right)^2\right\} \\
 &= \mathbb{E}\left\{\frac{1}{N^2} \left(\sum_{m=1}^N (|x_m|^p - \mathbb{E}\{|x_m|^p\})\right)^2\right\} = \frac{1}{N^2} \sum_{m=1}^N \mathbb{E}\left\{(|x_m|^p - \mathbb{E}\{|x_m|^p\})^2\right\} \\
 &= \frac{N}{N^2} \times \left(\mathbb{E}\{|x_m|^{2p}\} - \underbrace{\mathbb{E}^2\{|x_m|^p\}}_{\mathbb{E}^2\{w\}}\right) = \frac{1}{N} \times (\mathbb{E}\{|x_m|^{2p}\} - \mathbb{E}^2\{w\}) \quad (\text{A.7})
 \end{aligned}$$

We can similarly find

$$\mathbb{E}\{|x_m|^{2p}\} = \mathbb{E}\left\{\left(\frac{|r_m|}{\sigma_n}\right)^{2p}\right\} = \frac{\mathbb{E}\{|r_m|^{2p}\}}{\sigma_n^{2p}} \stackrel{(i)}{=} \frac{\sigma_r^{2p} \Gamma(p+1)}{\sigma_n^{2p}} = \left(\frac{\sigma_r^2}{\sigma_n^2}\right)^p \Gamma(p+1) \quad (\text{A.8})$$

where (i) has been derived as explained in (A.5). By substituting (A.6) and (A.8) in (A.7), we have

$$\text{Var}\{w\} = \frac{1}{N} \left(\frac{\sigma_r^2}{\sigma_n^2}\right)^p \left[\Gamma(p+1) - \Gamma^2\left(\frac{p+2}{2}\right)\right] \quad (\text{A.9})$$

By using (A.6) and (A.9), we can find the means and variances of observed energy levels over a Rayleigh-faded channel for different cases. Note that the average IED is considered as expressed in (2.3). For the case of the non-average IED, as given in (4.6), the obtained mean and variance in (A.6) and (A.9) must be multiplied by N and N^2 , respectively.

Appendix B

Deterministic shape parameters when $\gamma_I \leq -5$ dB

In the case of PU's status changing during a sensing part with N number of sensing samples (H_{02} and H_{12}), J_0 and J_1 are defined as the numbers of samples after which the PU switches from active to idle and vice versa, respectively. In the presence of interference, from (3.68), (5.18) and Table 5.3, the shape parameters for H_{02} and H_{12} are functions of total power of received interference, I , as follows:

$$\begin{aligned} k_{H_{02}}(I) &= N \left[1 + \underbrace{J_0 (N - J_0) / \left(J_0 + \frac{N}{\gamma_I} \right)^2}_{\spadesuit} \right]^{-1} \\ k_{H_{12}}(I) &= N \left[1 + \underbrace{J_1 (N - J_1) / \left[(N - J_1) + \frac{N}{\gamma_I} \right]^2}_{\clubsuit} \right]^{-1} \end{aligned} \quad (\text{B.1})$$

in which γ_I is the SINR.

We would like to approximate $k_{H_{02}}$ and $k_{H_{01}}$ as deterministic, like $k_{H_{01}}$ and $k_{H_{11}}$. To do so, we differentiate $\spadesuit$ and $\clubsuit$ with respect to J_0 and J_1 to find their maximum values over different $J_{0/1}$'s. $\spadesuit$ has its maximum value in $J_0 = N / (2 + \gamma_I)$ as

$$\spadesuit_{\max} = \text{Max}_{J_0} \left\{ \frac{J_0 (N - J_0)}{\left(J_0 + \frac{N}{\gamma_I} \right)^2} \right\} = \frac{\frac{N}{2 + \gamma_I} \left(N - \frac{N}{2 + \gamma_I} \right)}{\left(\frac{N}{2 + \gamma_I} + \frac{N}{\gamma_I} \right)^2} = \frac{\gamma_I^2}{4(1 + \gamma_I)} \quad (\text{B.2})$$

and maximum $\clubsuit$ is given in $J'_1 = N'(1 + \gamma_I) / (2 + \gamma_I)$ as

$$\clubsuit_{\max} = \text{Max}_{J_1} \left\{ \frac{J_1 (N - J_1)}{\left[(N - J_1) + \frac{N}{\gamma_I} \right]^2} \right\} = \frac{N \left(\frac{1 + \gamma_I}{2 + \gamma_I} \right) \left(N - N \left(\frac{1 + \gamma_I}{2 + \gamma_I} \right) \right)}{\left[\left(N - N \left(\frac{1 + \gamma_I}{2 + \gamma_I} \right) \right) + \frac{N}{\gamma_I} \right]^2} = \frac{\gamma_I^2}{4(1 + \gamma_I)} \quad (\text{B.3})$$

From (B.2) and (B.3), one can see that $\spadesuit_{\max} = \clubsuit_{\max} < 0.02$ for $\gamma_I \leq -5$ dB. Hence, from (B.1), $k_{H_{02}} = k_{H_{12}} = N/C$ are deterministic with an error less than 1 % (in the worst case) by choosing $C = 1.01$.

Appendix C

Conditional local probability with random scale parameter

For a gamma distributed random variable, Y_i , with shape and scale parameters of k_{H_s} and θ_{H_s} over situation H_s , the probability of having Y_i bigger than a certain threshold, λ , conditioned on a random scale parameter can be defined as

$$\begin{aligned}
\Pr \{Y_i > \lambda | H_s, \theta_{H_s}\} &= 1 - F_{Y_i}(\lambda; k_{H_s}, \theta_{H_s}) \\
&= 1 - \frac{\gamma\left(k_{H_s}, \frac{\lambda}{\theta_{H_s}}\right)}{\Gamma(k_{H_s})} \\
&\stackrel{(i)}{=} 1 - \frac{1}{\Gamma(k_{H_s})} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\lambda}{\theta_{H_s}}\right)^{k_{H_s}+n}}{n! (k_{H_s} + n)} \\
&= 1 - \frac{1}{\Gamma(k_{H_s})} \sum_{n=0}^{\infty} \frac{(-1)^n \lambda^{k_{H_s}+n}}{n! (k_{H_s} + n)} \theta_{H_s}^{-(k_{H_s}+n)} \\
&\stackrel{(ii)}{=} 1 - \frac{1}{\Gamma(k_{H_s})} \sum_{n=0}^{\infty} \frac{(-1)^n \lambda^{k_{H_s}+n}}{n! \underbrace{(k_{H_s} + n) \Gamma(k_{H_s} + n)}_{\Gamma(k_{H_s} + n + 1)}} \int_0^{\infty} t^{k_{H_s}+n-1} e^{-\theta_{H_s} t} dt \\
&= 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^n (\lambda t)^{k_{H_s}+n}}{n! \Gamma(n + k_{H_s} + 1)} t^{-1} e^{-\theta_{H_s} t} dt \\
&\stackrel{(iii)}{=} 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^{\infty} \underbrace{\sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{2\sqrt{\lambda t}}{2}\right)^{2n+k_{H_s}}}{n! \Gamma(n + k_{H_s} + 1)}}_{J_{k_{H_s}}(2\sqrt{\lambda t})} (\lambda t)^{\frac{k_{H_s}}{2}} t^{-1} e^{-\theta_{H_s} t} dt \\
&\stackrel{(iv)}{=} 1 - \frac{1}{\Gamma(k_{H_s})} \int_0^{\infty} J_{k_{H_s}}(2\sqrt{Z}) Z^{\frac{k_{H_s}}{2}-1} e^{-\frac{\theta_{H_s} Z}{\lambda}} dZ
\end{aligned} \tag{C.1}$$

where $\gamma(.,.)$ is the lower incomplete gamma function, $J_\nu(.)$ is the ν -th order Bessel function of the first kind and [102, eq. 8.354.1, 8.312.2, 8.331.1, 8.440] are utilized to obtain (i), (ii), (iii) and (iv) [103].

Author's Publications

Journal papers

1. Rabiee, Ramtin; Li, Kwok Hung, "Application of extra sensing by stochastic-modeled secondary users over a medium-traffic network," *IEEE Transactions on Wireless Communications*, vol. 15, no. 8, pp. 5708-5718, Aug. 2016.
2. Rabiee, Ramtin; Li, Kwok Hung, "Performance Evaluation of Improved Double-Threshold Energy Detector over Rayleigh-faded Sensing and Imperfect Reporting Channels," *Physical Communication (Elsevier) Journal*, vol. 17, pp. 58-71, Dec. 2015.
3. Rabiee, Ramtin; Li, Kwok Hung, "A 1-persistent based spectrum sensing among the stochastic cooperative users in the presence of the state-variable primary user," *submitted to IEEE Transactions on Wireless Communications*.

Conference papers

1. Rabiee, Ramtin; Li, Kwok Hung, "Distribution of Observed Energy Level over Medium-traffic Primary Network using Improved Energy Detector," *10th International Conference on Information, Communications and Signal Processing (ICICS)*, 2-4 Dec. 2015.
2. Rabiee, Ramtin; Li, Kwok Hung, "Cooperative IDed in a Medium-traffic Primary Network over Rayleigh-faded Sensing Channels," *IEEE 20th International workshop on Computer Aided Modeling and Design of Communication Links and Networks (IEEE CAMAD)*, pp. 308-312, 7-9 Sep. 2015.
3. Rabiee, Ramtin; Li, Kwok Hung, "Cooperative Spectrum Sensing in a Medium-traffic Primary Network using Double-threshold Scheme over Imperfect Reporting Channels," *IEEE 80th Vehicular Technology Conference (VTC Fall)*, pp.15, 14-17 Sep. 2014.
4. Rabiee, Ramtin; Li, Kwok Hung, "Throughput analysis of secondary network over a mediumtraffic primary network using cooperative spectrum sensing," *11th International Symposium on Wireless Communications Systems (ISWCS)*, pp.64-69, 26-29 Aug. 2014.

5. Rabiee, Ramtin; Li, Kwok Hung, “Throughput optimization of double-threshold based improved energy detection in cooperative sensing over imperfect reporting channels,” *9th International Conference on Information, Communications and Signal Processing (ICICS)*, pp.1-5, 10-13 Dec. 2013.

References

- [1] “Spectrum Policy Task Force,” Federal Communications Commission, Tech. Rep. ET Docket No. 02-135, Nov. 2002.
- [2] M. A. McHenry, “NSF spectrum occupancy measurements project summary,” Shared Spectrum Company Report, Tech. Rep., Aug. 2005. [Online]. Available: <http://www.sharedspectrum.com>
- [3] I. Mitola, J. and J. Maguire, G.Q., “Cognitive radio: making software radios more personal,” *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.
- [4] B. A. Fette, *Cognitive Radio Technology*, 2nd ed. Academic Press/Elsevier, Mar. 2009.
- [5] S. Haykin, “Cognitive radio: brain-empowered wireless communications,” *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 2, pp. 201–220, Feb. 2005.
- [6] H. Arslan and A. Gorcin, “Cognitive radio and software defined radio: signal processing perspectives,” in *IEEE 16th Signal Processing, Communication and Applications Conference*, Apr. 2008, pp. 1–5.
- [7] J. Perez-Romero, L. Doyle, and M. C. Vuran, “Applications of cognitive radio networks [from the guest editors],” *IEEE Vehicular Technology Magazine*, vol. 7, no. 2, pp. 23–24, June 2012.
- [8] “Facilitating Opportunities for Flexible, Efficient, and Reliable Spectrum Use Employing Cognitive Radio Technologies,” Federal Communications Commission, Tech. Rep. NPRM 03-322, Dec. 2003.
- [9] “Realizing The Full Potential Of Government-Held Spectrum To Spur Economic Growth,” President’s Council of Advisors on Science and Technology, Tech. Rep., July 2012.

- [10] Y.-C. Liang, K.-C. Chen, G. Li, and P. Mahonen, "Cognitive radio networking and communications: An overview," *IEEE Transactions on Vehicular Technology*, vol. 60, no. 7, pp. 3386–3407, Sep. 2011.
- [11] J. Li, W. Zhang, and J. Yuan, "Opportunistic spectrum sharing in cognitive radio networks based on primary limited feedback," *IEEE Transactions on Communications*, vol. 59, no. 12, pp. 3272–3277, Dec. 2011.
- [12] A. Roessler, J. Schlien, S. Merkel, and M. Kottkamp, "LTE-Advanced (3GPP Rel.12) Technology Introduction." Rohde & Schwarz, June 2014, White Paper.
- [13] X. Lin, J. G. Andrews, A. Ghosh, and R. Ratasuk, "An overview of 3GPP device-to-device proximity services," *IEEE Communications Magazine*, vol. 52, no. 4, pp. 40–48, Apr. 2014.
- [14] A. Asadi, Q. Wang, and V. Mancuso, "A survey on device-to-device communication in cellular networks," *IEEE Communications Surveys Tutorials*, vol. 16, no. 4, pp. 1801–1819, Fourthquarter 2014.
- [15] D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, S. Li, and G. Feng, "Device-to-device communications in cellular networks," *IEEE Communications Magazine*, vol. 52, no. 4, pp. 49–55, Apr. 2014.
- [16] K. Doppler, M. Rinne, C. Wijting, C. B. Ribeiro, and K. Hugl, "Device-to-device communication as an underlay to LTE-advanced networks," *IEEE Communications Magazine*, vol. 47, no. 12, pp. 42–49, Dec. 2009.
- [17] J. Xie and J. Chen, "An adaptive double-threshold spectrum sensing algorithm under noise uncertainty," in *IEEE CIT 2012, 12th International Conference on Computer and Information Technology*, 2012, pp. 824–827.
- [18] A. Singh, M. Bhatnagar, and R. Mallik, "Cooperative spectrum sensing with an improved energy detector in cognitive radio network," in *National Conference on Communications (NCC) 2011*, Jan. 2011, pp. 1–5.
- [19] —, "Optimization of cooperative spectrum sensing with an improved energy detector over imperfect reporting channels," in *IEEE Vehicular Technology Conference (VTC Fall) 2011*, Sep. 2011, pp. 1–5.
- [20] —, "Cooperative spectrum sensing in multiple antenna based cognitive radio network using an improved energy detector," *IEEE Communications Letters*, vol. 16, no. 1, pp. 64–67, Jan. 2012.

- [21] C. Altay, H. Yilmaz, and T. Tugcu, "Cooperative sensing analysis under imperfect reporting channel," in *IEEE Symposium on Computers and Communications (ISCC)*, 2012, pp. 000 770–000 775.
- [22] D. Ren, J. Ge, J. Li, and Y. Ji, "Cooperative spectrum sensing using opportunistic amplify-and-forward relaying over nakagami-m fading channels," *IET Communications*, vol. 7, no. 13, pp. 1377–1382, Sep. 2013.
- [23] Jun-pyo Hong, B. Hong, T.-W. Ban, and W. Choi, "On the cooperative diversity gain in underlay cognitive radio systems," *IEEE Transactions on Communications*, vol. 60, no. 1, pp. 209–219, Jan. 2012.
- [24] S. Nallagonda, S. Roy, and S. Kundu, "Performance evaluation of cooperative spectrum sensing scheme with censoring of cognitive radios in rayleigh fading channel," *Wireless Personal Communications*, vol. 70, no. 4, pp. 1409–1424, 2013.
- [25] I. E. Atawi, O. S. Badarneh, M. S. Aloqlah, and R. Mesleh, "Spectrum-sensing in cognitive radio networks over composite multipath/shadowed fading channels," *Computers & Electrical Engineering*, vol. 52, pp. 337–348, May 2016. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0045790615004449>
- [26] D. Cabric, S. Mishra, and R. Brodersen, "Implementation issues in spectrum sensing for cognitive radios," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers 2004*, vol. 1, Nov. 2004, pp. 772–776.
- [27] I. F. Akyildiz, B. F. Lo, and R. Balakrishnan, "Cooperative spectrum sensing in cognitive radio networks: A survey," *Physical Communication*, vol. 4, no. 1, pp. 40–62, Mar. 2011.
- [28] S. Xie, Y. Liu, Y. Zhang, and R. Yu, "A parallel cooperative spectrum sensing in cognitive radio networks," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 8, pp. 4079–4092, Oct. 2010.
- [29] J. Zhu, Z. Xu, F. Wang, B. Huang, and B. Zhang, "Double threshold energy detection of cooperative spectrum sensing in cognitive radio," in *CrownCom 2008, 3rd International Conference on Cognitive Radio Oriented Wireless Networks and Communications 2008*, May 2008, pp. 1–5.
- [30] S.-Q. Liu, B.-J. Hu, and X.-Y. Wang, "Hierarchical cooperative spectrum sensing based on double thresholds energy detection," *IEEE Communications Letters*, vol. 16, no. 7, pp. 1096 –1099, July 2012.

- [31] J. Wu, T. Luo, J. Li, and G. Yue, "A cooperative double-threshold energy detection algorithm in cognitive radio systems," in *5th International Conference on Wireless Communications, Networking and Mobile Computing (WiCom '09)*, Sep. 2009, pp. 1–4.
- [32] J. Jafarian and K. Hamdi, "Throughput optimization in a cooperative double-threshold sensing scheme," in *IEEE Wireless Communications and Networking Conference (WCNC)*, 2012, pp. 1034–1038.
- [33] A. Rao and M.-S. Alouini, "Cooperative spectrum sensing over non-identical nakagami fading channels," in *IEEE VTC Spring 2011, 73rd Vehicular Technology Conference*, 2011, pp. 1–4.
- [34] S. Srivastava and A. Banerjee, "'n-ratio' logic based cooperative spectrum sensing using double threshold energy detection," in *CROWNCOM '09, 4th International Conference on Cognitive Radio Oriented Wireless Networks and Communications 2009*, June 2009, pp. 1–6.
- [35] D. Duan and L. Yang, "Cooperative Spectrum Sensing with Ternary Local Decisions," *Communications Letters, IEEE*, vol. 16, no. 9, pp. 1512–1515, 2012.
- [36] A. Bagwari and G. S. Tomar, "Two-stage detectors with multiple energy detectors and adaptive double threshold in cognitive radio networks," *International Journal of Distributed Sensor Networks*, vol. 2013, p. 8, 2013.
- [37] W. Zhang, R. Mallik, and K. Letaief, "Optimization of cooperative spectrum sensing with energy detection in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 12, pp. 5761–5766, Dec. 2009.
- [38] B. Bastami and E. Saberinia, "A low complexity multi-threshold centralized detection strategy for cooperative spectrum sensing," in *IEEE 22nd International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2011, pp. 349–353.
- [39] Y. Li, X. Zhang, and Y. Gu, "Cooperative spectrum detection based on multi-threshold in cognitive radio system," *Journal of Information and Computational Science*, vol. 8, no. 16, pp. 3827–3835, 2011.
- [40] O. E.-B. Hamed and M. Abdel-Hafez, "Multi-level collaborative spectrum sensing in nakagami fading channels," in *The Eighth International Conference on Wireless and Mobile Communications (ICWMC)*, June 2012, pp. 284–289.

- [41] J. Shen, T. Jiang, S. Liu, and Z. Zhang, "Maximum channel throughput via cooperative spectrum sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 10, pp. 5166–5175, Oct. 2009.
- [42] D. Duan, L. Yang, and J. Principe, "Cooperative diversity of spectrum sensing for cognitive radio systems," *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 3218–3227, 2010.
- [43] —, "Cooperative diversity of spectrum sensing in cognitive radio networks," in *IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2009, pp. 1–6.
- [44] W. Zhang and K. Letaief, "Cooperative spectrum sensing with transmit and relay diversity in cognitive radio networks - [transaction letters]," *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 4761–4766, 2008.
- [45] J. Lu and P. Wei, "Performance analysis for spectrum sensing over fading reporting channels," in *CECNet 2012, 2nd International Conference on Consumer Electronics, Communications and Networks*, 2012, pp. 433–436.
- [46] S. Nallagonda, S. Roy, and S. Kundu, "Performance of cooperative spectrum sensing using an improved energy detector in fading channels," in *CODEC 2012, 5th International Conference on Computers and Devices for Communication*, 2012, pp. 1–4.
- [47] G. Xiong, S. Kishore, and A. Yener, "Spectrum sensing in cognitive radio networks: Performance evaluation and optimization," *Physical Communication*, vol. 9, pp. 171–183, Dec. 2013.
- [48] D. Hamza, S. Aissa, and G. Aniba, "Equal gain combining for cooperative spectrum sensing in cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 8, pp. 4334–4345, Aug. 2014.
- [49] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Communications Surveys Tutorials*, vol. 11, no. 1, pp. 116–130, First 2009.
- [50] K. Seshukumar, R. Saravanan, and M. Suraj, "Spectrum sensing review in cognitive radio," in *International Conference on Emerging Trends in VLSI, Embedded System, Nano Electronics and Telecommunication System (ICEVENT)*, Jan. 2013, pp. 1–4.
- [51] M. Mohammadkarimi, B. Mahboobi, and M. Ardebilipour, "Optimal spectrum sensing in fast fading rayleigh channel for cognitive radio," *IEEE Communications Letters*, vol. 15, no. 10, pp. 1032–1034, Oct. 2011.

- [52] H. Urkowitz, “Energy detection of unknown deterministic signals,” *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523–531, Apr. 1967.
- [53] V. Kostylev, “Energy detection of a signal with random amplitude,” in *ICC 2002, IEEE International Conference on Communications 2002*, vol. 3, 2002, pp. 1606–1610.
- [54] F. Digham, M.-S. Alouini, and M. K. Simon, “On the energy detection of unknown signals over fading channels,” *IEEE Transactions on Communications*, vol. 55, no. 1, pp. 21–24, 2007.
- [55] —, “On the energy detection of unknown signals over fading channels,” in *IEEE International Conference on Communications (ICC)*, vol. 5, 2003, pp. 3575–3579.
- [56] H. Semlali, N. Boumaaz, A. Soulmani, A. Ghammaz, and J.-F. Diouris, “Energy detection approach for spectrum sensing in cognitive radio systems with the use of random sampling,” *Wireless Personal Communications*, vol. 79, no. 2, pp. 1053–1061, 2014.
- [57] Y. Chen, “Improved energy detector for random signals in Gaussian noise,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 2, pp. 558–563, Feb. 2010.
- [58] S. Nallagonda, S. Sudheer, S. Roy, and S. Kundu, “Cooperative spectrum sensing with censoring of cognitive radios and improved energy detector under majority logic fusion,” in *IEEE International Conference on Signal Processing, Communication and Computing (ICSPCC) 2012*, Aug. 2012, pp. 262–267.
- [59] C. Sun, W. Zhang, and K. Letaief, “Cooperative spectrum sensing for cognitive radios under bandwidth constraints,” in *WCNC 2007, IEEE Wireless Communications and Networking Conference 2007*, Mar. 2007, pp. 1–5.
- [60] X. Jiang, K.-K. Wong, Y. Zhang, and D. Edwards, “On hybrid overlay-underlay dynamic spectrum access: Double-threshold energy detection and markov model,” *IEEE Transactions on Vehicular Technology*, 2013.
- [61] S. S. Kalamkar and A. Banerjee, “Improved double threshold energy detection for cooperative spectrum sensing in cognitive radio,” *Defence Science Journal (DESI-DOC)*, vol. 63, no. 1, pp. 34–40, Jan. 2013.
- [62] P. Nair, A. Vinod, K. Smitha, and A. Krishna, “Fast two-stage spectrum detector for cognitive radios in uncertain noise channels,” *IET Communications*, vol. 6, no. 11, pp. 1341–1348, July 2012.

- [63] S. Maleki, A. Pandharipande, and G. Leus, "Two-stage spectrum sensing for cognitive radios," in *IEEE International Conference on Acoustics Speech and Signal Processing (ICASSP)*, March 2010, pp. 2946–2949.
- [64] Y.-J. Choi, W. Pak, Y. Xin, and S. Rangarajan, "Throughput analysis of cooperative spectrum sensing in rayleigh-faded cognitive radio systems," *IET Communications*, vol. 6, no. 9, pp. 1104–1110, June 2012.
- [65] L. Tang, Y. Chen, E. Hines, and M.-S. Alouini, "Effect of primary user traffic on sensing-throughput tradeoff for cognitive radios," *IEEE Transactions on Wireless Communications*, vol. 10, no. 4, pp. 1063–1068, Apr. 2011.
- [66] N. Beaulieu and Y. Chen, "Improved energy detectors for cognitive radios with randomly arriving or departing primary users," *IEEE Signal Processing Letters*, vol. 17, no. 10, pp. 867–870, 2010.
- [67] B. Zhao, Y. Chen, C. He, and L. Jiang, "Performance analysis of spectrum sensing with multiple primary users," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 2, pp. 914–918, 2012.
- [68] L. Tang, Y. Chen, E. L. Hines, and M. S. Alouini, "Performance analysis of spectrum sensing with multiple status changes in primary user traffic," *IEEE Communications Letters*, vol. 16, no. 6, pp. 874–877, June 2012.
- [69] T. Wang, Y. Chen, E. Hines, and B. Zhao, "Analysis of effect of primary user traffic on spectrum sensing performance," in *ChinaCOM 2009, Fourth International Conference on Communications and Networking in China*, 2009, pp. 1–5.
- [70] T. Shehata and M. El-Tanany, "A novel adaptive structure of the energy detector applied to cognitive radio networks," in *CWIT 2009, 11th Canadian Workshop on Information Theory*, 2009, pp. 95–98.
- [71] K. Chang and B. Senadji, "Spectrum sensing optimisation for dynamic primary user signal," *IEEE Transactions on Communications*, vol. 60, no. 12, pp. 3632–3640, December 2012.
- [72] Y.-C. Liang, Y. Zeng, E. Peh, and A. T. Hoang, "Sensing-throughput tradeoff for cognitive radio networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 4, pp. 1326–1337, 2008.
- [73] S. Maleki, S. P. Chepuri, and G. Leus, "Optimization of hard fusion based spectrum sensing for energy-constrained cognitive radio networks," *Physical Communication*, vol. 9, pp. 193–198, Dec. 2013.

- [74] W. Cui, H. Zhao, and X. Mu, "Throughput analysis in cooperative sensing networks over imperfect reporting channel," in *CMC 2011, Third International Conference on Communications and Mobile Computing*, 2011, pp. 290–293.
- [75] Y. Pei, A. T. Hoang, and Y.-C. Liang, "Sensing-throughput tradeoff in cognitive radio networks: How frequently should spectrum sensing be carried out?" in *IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Sept 2007, pp. 1–5.
- [76] S. Zarrin and T. J. Lim, "Throughput-sensing tradeoff of cognitive radio networks based on quickest sensing," in *IEEE International Conference on Communications (ICC)*, June 2011, pp. 1–5.
- [77] W. Tang, M. Shakir, M. Imran, R. Tafazolli, and M.-S. Alouini, "Throughput analysis for cognitive radio networks with multiple primary users and imperfect spectrum sensing," *IET Communications*, vol. 6, no. 17, pp. 2787–2795, Nov. 2012.
- [78] M. Guerrini, L. Rugini, and P. Banelli, "Sensing-throughput tradeoff for cognitive radios," in *IEEE 14th Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, June 2013, pp. 115–119.
- [79] W.-Y. Lee and I. Akyildiz, "Optimal spectrum sensing framework for cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 10, pp. 3845–3857, October 2008.
- [80] L. Lai, Y. Fan, and H. Poor, "Quickest detection in cognitive radio: A sequential change detection framework," in *IEEE Global Telecommunications Conference (IEEE GLOBECOM)*, Nov 2008, pp. 1–5.
- [81] H. Li, C. Li, and H. Dai, "Quickest spectrum sensing in cognitive radio," in *42nd Annual Conference on Information Sciences and Systems (CISS 2008)*, March 2008, pp. 203–208.
- [82] D. Abraham and P. Willett, "Active sonar detection in shallow water using the page test," *IEEE Journal of Oceanic Engineering*, vol. 27, no. 1, pp. 35–46, Jan 2002.
- [83] G. Lorden, "Procedures for reacting to a change in distribution," *The Annals of Mathematical Statistics*, vol. 42, no. 6, pp. 1897–1908, 12 1971. [Online]. Available: <http://dx.doi.org/10.1214/aoms/1177693055>
- [84] L. Luo and S. Roy, "Efficient spectrum sensing for cognitive radio networks via joint optimization of sensing threshold and duration," *IEEE Transactions on Communications*, vol. 60, no. 10, pp. 2851–2860, Oct. 2012.

- [85] M. Wadhwa, C. Xin, M. Song, and E. Park, "Throughput analysis for a contention-based dynamic spectrum sharing model," *IEEE Transactions on Wireless Communications*, vol. 9, no. 4, pp. 1426–1433, Apr. 2010.
- [86] A. K. Subhasree Bhattacharjee and S. Bhattacharjee, "Throughput maximization problem in a cognitive radio network," *International Journal of Machine Learning and Computing*, vol. 1, no. 4, Oct. 2011.
- [87] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 996–1019, Third 2013.
- [88] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.
- [89] A. Busson, B. Jabbari, A. Babaei, and V. Veque, "Interference and throughput in spectrum sensing cognitive radio networks using point processes," *Journal of Communications and Networks*, vol. 16, no. 1, pp. 67–80, Feb. 2014.
- [90] X. Song, C. Yin, D. Liu, and R. Zhang, "Spatial throughput characterization in cognitive radio networks with threshold-based opportunistic spectrum access," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 11, pp. 2190–2204, Nov. 2014.
- [91] A. Ghasemi, "Statistical characterization of interference in cognitive radio networks," in *IEEE 19th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Sep. 2008, pp. 1–6.
- [92] L. Vijayandran, P. Dharmawansa, T. Ekman, and C. Tellambura, "Analysis of aggregate interference and primary system performance in finite area cognitive radio networks," *IEEE Transactions on Communications*, vol. 60, no. 7, pp. 1811–1822, July 2012.
- [93] A. Makarfi and K. Hamdi, "Efficiency of energy detection for spectrum sensing in a poisson field of interferers," in *IEEE Wireless Communications and Networking Conference (WCNC)*, Apr. 2012, pp. 1023–1028.
- [94] Y. Sharkasi, M. Ghogho, D. McLernon, and S. Zaidi, "Performance analysis of co-operative spectrum sensing for cognitive radio using stochastic geometry," in *Proceedings of the 21st European Signal Processing Conference (EUSIPCO)*, Sep. 2013, pp. 1–5.

- [95] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, 2002.
- [96] S.-C. Hong, M. Vu, and V. Tarokh, “Cognitive sensing based on side information,” in *IEEE Sarnoff Symposium*, 2008, pp. 1–6.
- [97] N. O’Donoughue and J. Moura, “On the product of independent complex gaussians,” *IEEE Transactions on Signal Processing*, vol. 60, no. 3, pp. 1050–1063, 2012.
- [98] A. Ghasemi and E. Sousa, “Optimization of spectrum sensing for opportunistic spectrum access in cognitive radio networks,” in *4th IEEE Consumer Communications and Networking Conference (CCNC 2007)*, Jan. 2007, pp. 1022–1026.
- [99] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Wiley-Interscience, 2006.
- [100] S. N. Chiu, D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and Its Applications*. Wiley, 2013.
- [101] I. S. Gradshteyn, I. M. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of Integrals, Series, and Products*, 6th ed. Academic Press, 2000, previous ed.: 1994.
- [102] I.S. Gradshteyn and I.M. Ryzhik, *Table of Integrals, Series, and Products*, 7th ed. Academic Press, Mar. 2007.
- [103] A. Makarfi and K. Hamdi, “Interference analysis of energy detection for spectrum sensing,” *IEEE Transactions on Vehicular Technology*, vol. 62, no. 6, pp. 2570–2578, July 2013.