Optimal structural design for offshore wind energy system

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OPTIMAL STRUCTURAL DESIGN FOR OFFSHORE
WIND ENERGY SYSTEM

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Abstract

Offshore wind industry has been gaining much momentum over the last decade due to increasing energy demand and global appeal to create a balanced energy mix. Current wind energy research and development has been focusing on driving down the cost of energy. Support structures of offshore wind turbines are site-specifically designed and can be optimized for cost reduction. However, the task is computationally expensive due to the highly-constrained, non-convex and non-linear nature of the design problem. A good depth of detail in the problem formulation can give useful insights in the practical design process, but may also compromise the efficiency. Therefore, the main objective of this thesis is *to investigate an efficient, effective and practical automated structural optimization framework for designing the support structures of offshore wind turbines*, where efficiency relates to the convergence speed to attain the optimal design; effectiveness measures the degree of success to converge to the correct optimal solution reliably and practicality represents the quality of a design problem being mathematically formulated to fit to the industrial requirements.

The thesis begins with the development of an in-house *modeling and structural analysis module* for offshore wind turbine system, which is based on a decoupled aero-servo loading acting onto a linear hydro-elastic model. Code comparison and verification were carried out against the fully-coupled non-linear simulations in commercial software FEDEM Windpower. Under the normal operating wind-wave conditions, the partially decoupled approach was able to give accurate response analysis and fatigue assessment, as the bottom-fixed offshore wind structures behave linearly. However, when subjected to severe wind and

wave loads, non-linearity within aerodynamic and hydrodynamic drag forces become prominent, and the partially decoupled approach tended to generate conservative results in the extreme load analysis.

Secondly, the thesis studied *analytical analysis in the dynamic sensitivity computation* to eliminate the inaccuracies related to finite difference approximations. Analytical formulae were developed to calculate gradients for various design assessment criteria prescribed by the industry, including ultimate limit state (ULS) checks and fatigue limit state (FLS) checks in both time- and frequency-domains. Comparison studies against finite difference schemes for simulated stress data show that the analytical gradients could avoid numerical artifacts that typically occurred in the analysis of extreme load constraints for beams that experienced alternating tension and compression modes. Moreover, the fatigue damage gradients were very sensitive to response sensitivities, where factors like step sizes, time steps and locations of assessment could influence the accuracies of finite difference approximations greatly.

Thirdly, an *integrated analysis and optimization framework* was developed for the design optimization of offshore wind turbine support structures. Case studies were performed on the UpWind monopile, OC4 jacket and UpWind jacket. Results show that the framework was reliable and consistent in delivering superior efficiency and accuracy in the optimization study, as compared with the conventional optimization method that is based on finite difference gradients. The global optimum could be achieved in the design optimization process, where the large number of design constraints implemented can possibly lead the optimizer to find the global optimum. Among the list of design constraints implemented, the buckling and fatigue load constraints had significant influence over the design, where each component is oriented to maximize the utilization against the prescribed limit state functions. The framework has enabled the design process of the entire wind turbine support structure system, including the

pile penetration depth to be carried out simultaneously in an integrated manner, hence could possibly lead to a more optimized design solution.

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Nomenclature

List of Abbreviations

ASD	Allowable stress design
BEM	Blade element - momentum
CFD	Computational fluid dynamics
CPC	Collective pitch controllers
CTSE	Complex Taylor series expansion
DDM	Direct differentiation method
DEL	Damage equivalent loads
DLC	Design load case
DOF	Degrees of freedom
EB	Euler-Bernoulli
EOG	Extreme operating gust
FEM	Finite element method
FID	Final investment decision
FK	Froude-Krylov
FLS	Fatigue limit state
GPU	Graphics processing unit
HAWT	Horizontal axis wind turbines
HRPC	High-rise-pile-caps
HSS	Hot spot stresses
IEC	International Electrotechnical Commission
IPC	Individual pitch controllers
JST	Jacket sizing tool
LCOE	Levelized cost of energy
LRFD	Load and resistance factor design
LSD	Limit state design
MCS	Monte Carlo simulations
MDO	Multidisciplinary design optimization
MDOF	Multi-degree-of-freedom
MSL	Mean sea level
NLP	Non-linear programming
NREL	National Renewable Energy Laboratory
NSS	Normal stochastic sea state
NTM	Normal atmospheric turbulence
NTNU	Norwegian University of Science and Technology
NTU	Nanyang Technological University
O&M	Operation and maintenance
OC3	Offshore Code Comparison Collaboration
OC4	Offshore Code Comparison Collaboration Continuation
OWT	Offshore wind turbine
PBWE	Performance-based wind engineering

PDF	Probability density function
PSD	Power spectral density
RBDO	Reliability-based design optimization
RBO	Robust design optimization
RNA	Rotor-nacelle assembly
SCF	Stress concentration factor
SPSA	Simultaneous perturbation stochastic approximation
SQP	Sequential quadratic programming
SSS	Severe sea state
SWH	Severe deterministic design wave
SWL	Still water level
SWPP	Saturation wind power potential
ULS	Ultimate limit state
WESE	Wind Energy Systems Engineering

List of Greek Symbols

α	Wind gradient exponent [-]
$\delta(x - x_i)$	Heaviside function
δt	Time step [s]
ϵ_n	Wave random phase of n -th component [-]
η	Wave surface elevation [m]
γ	Peak shape parameter [-]
γ'	Effective soil unit weight [N/m ³]
γ_m	Partial safety factor for material [-]
λ	Wave length [m]
λ_s	Reduced shell slenderness [-]
∇g_0	Gradient of objective function
∇g_i	Gradient of constraint function i
ν_f	Fatigue usage factor [-]
ω	Wave angular frequency [rad/s]
ω_i	i -th structural eigenfrequency [rad/s]
ω_p	wave peak frequency [rad/s]
Φ	Angle of internal friction [°]
ρ_b	Material density of beam [kg/m ³]
ρ_w	Seawater density [kg/m ³]
ρ_{mg}	Density of marine growth [kg/m ³]
σ	Spectral width parameter [-]
$\sigma_{ac,Sd}$	Design axial compressive stress [Pa]
$\sigma_{at,Sd}$	Design axial tensile stress [Pa]
$\sigma_{c,Sd}$	Design maximum combined compressive stress [Pa]
σ_{HSS}	Hot spot stress [Pa]
$\sigma_{j,Sd}$	Design combined stress [Pa]
$\sigma_{my,Sd}$	Design bending stress about member y-axis [Pa]
σ_{my}	Nominal stress due to bending in-plane [Pa]
$\sigma_{mz,Sd}$	Design bending stress about member z-axis [Pa]
σ_{mz}	Nominal stress due to out-of-plane [Pa]
$\sigma_{p,Sd}$	Design hoop stress [Pa]

$\sigma_{q,Sd}$	Design capped-end compressive stress [Pa]
σ_{U_y}	Lateral wind turbulence [m/s]
σ_{U_z}	Vertical wind turbulence [m/s]
σ_U	Longitudinal wind turbulence [m/s]
σ_x	Nominal stress due to axial load [Pa]
τ_{Sd}	Design membrane shear stress due to torsional moment and shear forces [Pa]
$\Delta\sigma_{u,HSS}$	u -th HSS stress range [MPa]
ζ^A	Aerodynamic damping ratio [%]
ζ^S	Structural damping ratio [%]
ζ^{sys}	System damping ratio [%]
ζ_i	Target damping ratio for i -th eigenfrequency [%]

List of Latin Symbols

$[B]$	Strain-displacement matrix
$[C^A]$	Aerodynamic damping matrix
$[C^H]$	Hydrodynamic damping matrix
$[C^{sys}]$	System damping matrix
$[C^S]$	Structural damping matrix
$[C_{el}]$	Element damping matrix
$[D]$	Constitutive matrix
$[K^P]$	Foundation secant stiffness matrix
$[K^{sys}]$	System stiffness matrix
$[K^S]$	Structural stiffness matrix
$[K_{EB}]$	Stiffness matrix of EB beam element
$[K_{eg}]$	Element stiffness matrix in global coordinates
$[K_{el}]$	Element stiffness matrix in local coordinates
$[M^H]$	Hydrodynamic mass matrix
$[M^{sys}]$	System mass matrix
$[M^S]$	Structural mass matrix
$[M_{EB}]$	Mass matrix of EB beam element
$[M_{el}]$	Element mass matrix in local coordinates
$[N]$	Shape function matrix
$[T_g]$	Transformation matrix
$\bar{a}$	Intercept of the design S-N curve with log N axis [-]
$\ddot{z}$	Structural acceleration vector
$\dot{v}_w$	Water particle acceleration vector
$\dot{z}$	Structural velocity vector
$\hat{a}_{i=x,y,z}$	Unit vector in local beam x-, y- and z- directions
$\hat{g}$	Unit gravitational vector
b^{max}	Upper bound of design variable vector
b^{min}	Lower bound of design variable vector
b	Design variable vector
$F(\omega)$	Force vector in the frequency-domain
f^A	Aerodynamic force vector
f^B	Buoyant force vector
f^G	Gravitational force vector

$\mathbf{f}^H$	Hydrodynamic force vector on fixed structure
$\mathbf{f}_{el}^{int}$	Element internal force vector
$\mathbf{f}_{dyn}^{sys}$	System dynamic force vector
$\mathbf{f}_{sta}^{sys}$	System static force vector
$\mathbf{f}^{sys}$	System external force vector
$\mathbf{f}_D$	Hydrodynamic damping force vector
$\mathbf{f}_{EB}$	Equivalent nodal force vector of EB beam element
$\mathbf{f}_{el}$	Element force vector in local coordinates
$H(\omega)$	Transfer function
$H(\mathbf{b}^k)$	Second-order derivative of Langragian function $L(\mathbf{b}, \lambda)$
$\mathbf{q}^H$	Unit length hydrodynamic force vector
$\mathbf{R}^D$	Damping force vector
$\mathbf{R}^E$	External force vector
$\mathbf{R}^I$	Inertia force vector
$\mathbf{R}^S$	Structural reaction force vector
$\mathbf{S}_F(\omega)$	Spectral matrix of force vector
$\mathbf{S}_X(\omega)$	Spectral matrix of response vector
$\mathbf{v}_w$	Water particle velocity vector
$\mathbf{Z}(\omega)$	Response vector in the frequency-domain
$\mathbf{z}_{dyn}$	Structural dynamic displacement vector
$\mathbf{z}_{eg}$	Element displacement vector in global coordinates
$\mathbf{z}_{el}$	Element displacement vector in local coordinates
$\mathbf{z}_{sta}$	Structural static displacement vector
$\mathbf{z}_{sta}$	Structural total displacement vector
$\mathbf{z}$	Structural displacement vector
$\dot{u}$	Wave horizontal acceleration [m/s ²]
$\dot{w}$	Wave vertical acceleration [m/s ²]
$\Im z$	Imaginary part of complex variable z
$\Re z$	Real part of complex variable z
A	Cross-sectional area [m ²]
a_0	Constant of proportionality to mass matrix [-]
a_1	Constant of proportionality to stiffness matrix [-]
A_l	Loading factor [-]
A_n	Wave amplitude of n -th component [m]
c^A	Aerodynamic damping coefficient [Ns/m]
C_A	Added mass coefficient [-]
C_D	Drag coefficient [-]
C_M	Inertial coefficient [-]
C_{my}	Reduction factor in member y-axis [-]
C_{mz}	Reduction factor in member z-axis [-]
D	Outer diameter [m]
d	Water depth [m]
D_f	Total fatigue damage [-]
d_s	Soil depth [m]
E	Young's modulus [Pa]
$E[P]$	Expected number of peaks per unit time [s ⁻¹]

f	Frequency of occurrence for each event [-]
F_s	Sampling frequency [Hz]
f_U	Wind speed frequency [Hz]
f_{ae}	Elastic buckling strength for axial force [Pa]
f_{cle}	Characteristic elastic hoop buckling strength [Pa]
$f_{h,Rd}$	Design hoop buckling strength [Pa]
f_{he}	Elastic buckling strength for hoop stress [Pa]
f_{ksd}	Design shell buckling strength [Pa]
f_{me}	Elastic buckling strength for bending moment [Pa]
$f_{\tau e}$	Elastic buckling strength for shear force [Pa]
G	Shear modulus [Pa]
g	Gravitational acceleration [m/s ²]
g_0	Objective function
g_i	Constraint function
g_{deform}	Deformation constraints
g_{eigen}	Eigenfrequency constraint
g_{FLS}	Fatigue load constraints
g_{size}	Sizing constraint
g_{ULS}	Extreme load constraints
H	Wave height [m]
H_s	Significant wave height
I_y	Second moment of area in local y-direction [m ⁴]
I_z	Second moment of area in local z-direction [m ⁴]
J	Number of samples within the windows [-]
J_x	Second moment of area in local x-direction [m ⁴]
k	Wave number [m ⁻¹]
$K_{x_i=1,2}^{p-y}$	Secant stiffness for lateral bearing force at node i [N/m ²]
$K_{x_i=1,2}^{Q-z}$	Secant stiffness for tip bearing force [N/m]
$K_{x_i=1,2}^{t-z}$	Secant stiffness for axial shaft friction at node i [N/m ³]
k_0	Initial modulus of subgrade reaction [N/m ³]
k_T	Thickness exponent [-]
K_{modal}	Modal spring constant [N/m]
L	Length [m]
L_n	Length of n -th member [m]
L_U	Integral length scale of wind speed [m]
L_{if}	Length portion of beam flooded with seawater [m]
L_{mg}	Length portion of beam covered with marine growth [m]
m	Negative inverse slope of the S-N curve [-]
m_n	n -th order spectral moment
M_x	Nodal torsional moment in local yz-direction [Nm]
m_x	Distributed twisting load around local x-direction [Nm]
M_y	Nodal bending moment in local xz-direction [Nm]
M_z	Nodal bending moment in local xy-direction [Nm]
M_{Rd}	Design resistance for bending moment [Nm]
$M_{y,Rdj}$	Design resistance of tubular joints for bending moment about member y-axis [Nm]

$M_{y,Sd}$	Design bending moment about y-axis [Nm]
$M_{z,Rdj}$	Design resistance of tubular joints for bending moment about member z-axis [Nm]
$M_{z,Sd}$	Design bending moment about z-axis [Nm]
nP	Blade passing frequency [Hz]
n_u	Number of stress cycles in $\Delta\sigma_{u,HSS}$ [-]
N_x	Nodal axial force in local x-direction [N]
$N_{c,Rd}$	Design resistance for axial compression [N]
$N_{c,Sd}$	Design axial compressive force [N]
$N_{cl,Rd}$	Design resistance for local buckling [N]
N_E	Design resistance for Euler buckling [N]
$N_{t,Rd}$	Design resistance for axial tension [N]
$N_{t,Sd}$	Design axial tensile force [N]
$p(\Delta\sigma_{HSS})$	Probability density function of HSS range
p_u	Ultimate lateral bearing capacity [N/m]
P_v	Probability of v -th event [-]
$P_{\sigma_{HSS}}(f)$	Spectral density of HSS [(MPa) ² /Hz]
Q	Number of window segments [-]
Q_p	Total end bearing capacity [N]
q_x	Distributed axial load in local x-direction [N]
q_y	Distributed normal load in local y-direction [N]
q_z	Distributed normal load in local z-direction [N]
Q_f	Chord action factor [-]
Q_u	Chord strength factor [-]
$S(\omega)$	Power spectral density [m ² s]
SCF_{AC}	Stress concentration factor for axial load at the crown [-]
SCF_{AS}	Stress concentration factor for axial load at the saddle [-]
SCF_{MIP}	Stress concentration factor for in-plane moment [-]
SCF_{MOP}	Stress concentration factor for out-of-plane moment [-]
T	Thickness of tubular beam [m]
t	Time [s]
T_p	Wave peak period [s]
t_{max}	Maximum unit skin friction capacity [Pa]
T_{mg}	Thickness of marine growth [m]
T_{ref}	Reference thickness [-]
t_{tot}	Total time [s]
TI	Turbulence intensity [-]
u	Wave horizontal velocity [m/s]
U_R	Ursell number [-]
U_{10}	10-minute averaged wind speed [m/s]
U_{hub}	Mean wind speed at hub height [m/s]
V_y	Nodal shear force in local y-direction [N]
V_z	Nodal shear force in local z-direction [N]
V_{Rd}	Design resistance for shear force [N]
V_{Sd}	Design shear force [N]
W	Window function

w	Wave vertical velocity [m/s]
1P	Rotor rotational frequency [Hz]

Chapter 1

Introduction

1.1 Background

The world has been facing waves of energy crises since the 1970s, when the global economy was heavily hit by fuel shortage, real and perceived, over-consumption of energy, market manipulation as well as elevated oil prices, etc. Severe energy demand growth is inevitable in the coming years due to rapid industrial development and population increase particularly in the developing nations. At the same time, greenhouse gas emission from burning of fossil fuels have been identified as the primary cause of global warming and climate change ([Edenhofer et al., 2014](#)). The current situation creates the “energy trilemma” that entails aspects of energy security, energy equity and environmental sustainability whereby countries are required to diversify their energy resources and to manage the demand of energy through reliable, affordable and sustainable energy-efficiency programmes ([WEC, 2013b](#)). Wind has been one of the most promising sustainable energy sources. It has an estimated global saturation wind power potential (SWPP)¹ of about 253 TW over the entire global surface which is sufficient to power 21 times the total world energy consumption in 2012 ([Jacobson and Archer, 2012](#); [IEA, 2014b](#)). About 72 percent of the SWPP is distributed in the offshore region. People are looking at tapping into this richer wind resource that

¹SWPP is defined as the maximum theoretical wind power potential that can be extracted upon increasing the number of wind turbines over a large geographical region, with the consideration of “atmospheric efficiency” ([Jacobson and Archer, 2012](#)).

is capable of providing a good energy supply to the populated cities mainly at the coastal region without affecting the human habitat onshore.

Offshore wind power has set foot in the energy industry as early as 1991 when the 4.95-MW Vindeby wind farm began operating in Danish waters. However, it was only over the last decade that the offshore wind industry started to boom (mainly in Europe), when more focus is given to promote a clean and diverse energy mix, in view of the environmental impacts caused by fossil fuels (Esteban et al., 2011). The global cumulative offshore wind capacity has reached 8.771 GW at the end of 2014, making up 2.37 percent of the total global wind capacity (GWEC, 2015). In 2015, the industry is going to set a record for installing the most number of offshore wind turbines (OWTs) (Smith et al., 2015). It is estimated that the total cumulative capacity should reach 28 GW globally by 2018, mainly contributed by the United Kingdom, Germany, Denmark and Belgium in Europe, China, Japan, Korea and Taiwan in Asia and the United States (IEA, 2013). Nevertheless, strong support from different stakeholders i.e., governments, turbine manufacturers, suppliers and contractors, etc. on tackling challenges faced by the industry in technology deployment, market readiness and general acceptance is required in order to realize this production target (Musial and Ram, 2010).

Among all, the levelized cost of energy (LCOE)² for offshore wind power has to be reduced further in order for the industry to stay sustainable and competitive against other sources of energy (WEC, 2013a). On the one hand, the investment costs for offshore wind projects can be two to three times higher than onshore projects, mainly due to the additional expenses incurred for sup-

²The LCOE is defined as the present value (price of energy) for which the revenue generated can break-even the total costs incurred over the lifetime of the wind farm projects, that includes the capital expenditure (C_i), operational expenditure (O_i) and decommissioning expenditure (D_i), as given by
$$\text{LCOE} = \frac{\sum_{i=1}^n ((C_i + O_i + D_i) / (1 + W_i)^i)}{\sum_{i=1}^n (E_i / (1 + W_i)^i)}$$
 where E_i is the energy produced in year i [MWh], W_i is the weighted average cost of capital [%] and n is the operating lifetime (Willows and Valpy, 2015).

port structures, offshore electrical infrastructures, offshore installation and commissioning, etc. (IEA, 2013). On the other hand, the costs for operation and maintenance (O&M) and project financing are also higher in offshore projects, with the former estimated to double or triple the onshore projects, whereas the latter depends on risk perception and project uncertainties (Musial and Ram, 2010). Even though greater energy is produced in the offshore environment due to higher and stable wind speed, the current LCOE for offshore wind farms constructed between 2010 and 2012 are in the range of USD 190/MWh to USD 260/MWh and are still much more expensive than the onshore projects which can be as low as USD 60/MWh (IEA, 2014a). Hence, many leading countries in the industry have set goals to reduce the present LCOE to GBP 100/MWh for projects (in 2012 terms) with final investment decision (FID) in 2020, but preferably more (The Carbon Trust, 2008; The Crown Estate, 2012; BTM Consult ApS, 2013).

For a 500-MW offshore wind farm to be built in the United States, recent studies have estimated that the capital costs are in the order of USD 5,000/kW to USD 6,000/kW, where the support structures can contribute up to 22 percent of the total capital costs and 36 percent of the total installation and commissioning costs (Navigant Consulting, 2013; WEU, 2015b). They have been identified as one of the key areas for cost reduction (The Crown Estate, 2012). As offshore wind farms venture to deeper waters along with larger and heavier turbines installed for greater energy extraction, upscaling of current support structure designs to accommodate the increasing wind and wave dynamic loads can be economically and technologically infeasible. Development of novel concepts and evolution of existing designs are necessary to increase the structural strength-to-mass ratios for such applications, as well as to streamline the manufacturing and installation processes (IEA, 2013; Willows and Valpy, 2015). Complex support structure designs such as space frame structures, suction bucket foundations and

floating structures have been introduced as alternatives to monopiles. Through demonstration in pilot projects, some of these technologies have been proven feasible and further commercialized for large-scale deployments, such as the jackets. Moreover, industries also advised against standardization on the design of primary support structures as it is more cost-effective to have them purpose-built to fit to the specific projects (WEU, 2015a). As a result, structural optimization is often carried out to fulfill this customization requirement.

The study of structural optimization for OWT support structures is so far very limited. This may be partly due to the fact that traditional onshore wind turbine support structures mainly consist of the tubular monopiles and are relatively simple in design (Muskulus and Schafhirt, 2014). Besides, dynamic analysis of wind turbine systems is usually carried out using specialized software that supports integrated structural modeling and simulation with consideration of wind and wave loading effects. Since hundreds to thousands of design load cases are assessed in the design phase and they have to be repeated for each iteration in the optimization study, the overall process can be computationally very expensive. In the industry, manual optimization techniques are usually practiced since they only take a few number of iterations. The methods depend more on engineers' experience when improving the structural design. As such, the "true optimal solutions" are generally difficult to achieve and the design space may not be fully explored since the solutions can be sometimes non-obvious to the human designers. Furthermore, turbines (as well as towers) and foundations are typically designed and supplied by different manufacturers in an offshore wind project. Foundation contractors tend to perform optimization studies in the early design phase by applying the damage equivalent loads (DEL) obtained from the turbine manufacturers on the tower base along with a constant aerodynamic damping ratio of 4 percent based on the crude engineering estimation (Weijtjens et al., 2014). This decoupled approach, although simple to imple-

ment, may not give very accurate representations of the structural dynamics due to the simplified modeling that typically neglects the mass distribution of the rotor blades as well as the conservative usage of aerodynamic damping and DELs. In addition, the exclusion of towers in the support structure design can also undermine the full potential of cost reduction through structural optimization. Through development of methodology that enables efficient, reliable and accurate design optimization of OWT support structures at the early design phase of the projects, cost reduction in both initial capital costs and long term operational costs of the plants can be attained.

1.2 Research objectives

Based on the previous discussion, this Doctoral Thesis has attempted to address the existing research gap on:

How to develop an efficient, effective and practical automated structural optimization framework for designing the support structures of offshore wind turbines?

The research emphasizes on three important characteristics that ought to be considered for the proposed optimization framework, namely efficiency: the convergence speed to attain the optimal design; effectiveness: the degree of success to converge to the correct optimal solution reliably; and practicality: the quality of a design problem being mathematically formulated to fit to the industrial requirements. The work conducted in this thesis consists of the following:

1. Studies of methodologies to perform multi-disciplinary dynamic analysis and design assessment of bottom-fixed OWT support structures.
2. Investigation of methodologies to calculate sensitivity derivatives analytically for various design assessments performed in the offshore wind struc-

tural analysis.

3. Development and demonstration of an analytical gradient-based optimization algorithm for the design of OWT support structures.

1.3 Original contributions of present study

In the course of this PhD work, the thesis has made contributions in the following areas.

1. The thesis has developed a *partially decoupled multi-disciplinary modeling and analysis tool* for the OWT system. The method gives good accuracy in comparison with the fully-coupled analysis and can be integrated effectively and efficiently with the optimization framework.
2. The thesis has developed *exact analytical formulae* to calculate the *dynamic sensitivity derivatives* for fatigue damages, DELs in the time- and frequency-domains and extreme load utilization. The method avoids numerical errors that typically occur when using the finite difference approach and eliminates the dependency on step sizes, time-steps, orders of finite difference and locations of assessments.
3. The thesis has formulated a *novel optimization framework* to solve the optimal design problem of an *integrated support structure system for OWTs* that includes towers, substructures and foundations. A comprehensive list of design assessment criteria prescribed by the industry, such as eigen-frequency analysis, extreme load analysis and fatigue load analysis are included. The methodology has demonstrated fast and accurate convergence of solutions, thanks to the analytical gradients.

1.4 Organization of the thesis

The overall structure of the thesis is as follows:

Chapter 1 Introduction to *research motivations* and *gaps*, and highlights on *objectives* and *contributions* of the thesis.

Chapter 2 Literature review on the *OWT support structure concepts* and the *structural optimization methodologies* applied to the design of OWT support structures.

Chapter 3 Description of theories used in the *modeling and simulation* of OWT systems and the *industrial code checks* performed when designing the support structures.

Chapter 4 Detailed explanation of the proposed *integrated analysis and optimization framework*.

Chapter 5 Case studies for *verification* and *capability demonstration* of the proposed optimization framework.

Chapter 6 *Conclusions* and *future work*.

1.5 Publications as a result of the current work

Research outcomes of the PhD have been published and presented in various journals and conferences, as listed below:

Journal publications

1. Chew, K.-H., Tai, K., Ng, E.Y.K., Muskulus, M., 2016. Analytical gradient-based optimization of offshore wind turbine substructures under fatigue and extreme loads. *Marine Structures*. *Marine Structures* 47, pp. 23-41.

2. Chew, K.-H., Tai, K., Ng, E.Y.K., Muskulus, M., 2015. Optimization of offshore wind turbine support structures using an analytical gradient-based method. *Energy Procedia* 80, pp. 100-7.
3. Chew, K.H., Ng, E.Y.K., Tai, K., Muskulus, M., Zwick, D., 2014. Offshore wind turbine jacket substructure: A comparison study between four-legged and three-legged designs. *J. Ocean Wind Energy* 1 (2), pp. 74-81.

Conference publications

1. Chew, K.-H., Muskulus, M., Srikanth, N., Tai, K., Ng, E.Y.K., 2015. Fatigue sensitivity analysis of offshore wind turbine structures. In: *Proc WCSMO-11*. Sydney, Australia, pp. 1339:1-6.
2. Chew, K.H., Muskulus, M., Zwick, D., Ng, E.Y.K., Tai, K., 2013. Structural Optimization and Parametric Study of Offshore Wind Turbine Jacket Substructure. In: *Proc. 23rd Int. Offshore Polar Eng. Conf. Vol. 1*. ISOPE, Anchorage, AK, USA, pp. 203-10.

Chapter 2

Design optimization of offshore wind turbine support structures

2.1 Support structure concepts for offshore wind turbines

The development of offshore wind energy research has promised to meet the demand for larger wind turbine systems with higher wind capacities. Despite the introduction of new wind extraction device concepts, three-bladed horizontal axis wind turbines (HAWT) remain still as the main choice in offshore wind implementation presently, due to the technology maturity that it gains from onshore application. While upscaling of turbine design is feasible ([Ashuri and Zaaier, 2010](#)), the design of support structures is more challenging and hardly a one-size-fits-all concept exists for any large scale offshore wind farm project due to the high dependency on site conditions (i.e., metocean and soil conditions).

“A support structure is the structure that supports the turbine and holds it in place and transfers the loads from the turbine to the ground” ([de Vries, 2011](#), pp 15). According to [DNV \(2011b\)](#), a support structure typically comprises the following parts ([Fig. 2.1](#)):

1. A tower

The component onto which the wind turbine is installed. The conventional design follows a tubular shape and is usually supplied by turbine manufacturers.

Figure 2.1: Components of support structures for offshore wind turbines

2. A substructure or floater

- a) Substructure: The component of a bottom-fixed wind turbine that extends from the bottom of tower to the seabed and is mainly submerged below the mean sea level (MSL). Some concepts require an additional transition piece to connect the tower to the substructure.
- b) Floater: The component of a floating wind turbine which the tower is sitting onto. It provides buoyancy to the structure.

3. A foundation or mooring system

- a) Foundation: The bottom-most component of a bottom-fixed wind turbine which penetrates partially or wholly into the soil to transfer loads from the upper part to the soil.
- b) Mooring system: The component that connects the floater to the seabed, for seakeeping and/ or stability purposes.

There is a variety of support structure concepts being developed, tested and installed in the offshore wind industry, as shown in Fig. 2.2. They can

be classified based on different criteria: the deployment water depth, the types of foundation or the operating natural frequencies. The current state of OWT substructure designs are generally divided into shallow, transitional and deep water technologies, where the water depth bands are approximated in the ranges of 0-30 m, 30-60 m and beyond 60 m, respectively (Musial and Ram, 2010). These terminologies are specific to OWTs that can be distinct from oil and gas technologies. On the other hand, the foundation designs for bottom-fixed OWTs mainly consist of driven piles, gravity-bases and suction buckets. The monopiles, gravity foundations and high-rise-pile-caps (HRPC) are the most widely used concepts to date, installed in nearshore or calm waters with shallow water depths (WEU, 2015b). Besides, tripods and tripiles have been deployed in a few wind farms that are furthest from shore with an average distance of 94.9 km out to sea (Fraunhofer IWES, 2014). In the transitional waters, jackets are widely perceived as the best candidate for OWT substructures while facing fierce competition posed by the extra-large (XL) monopiles presently. Depending on the soil condition, suction buckets can possibly be used as alternatives to the piled structures, as the former can eliminate driving noise and simplify the installation process (Valpy and English, 2014).

With installation sites migrating towards deeper water depths, heavier and bigger structures are required to maintain structural rigidity against dynamic loads (Musial and Ram, 2010). At some depths where rigid structures are no longer economical, compliant structures are more appealing for they are designed to react to environmental loads through structural flexibility instead of structural resistance (de Vries, 2009). The fundamental natural frequencies of rigid bottom-fixed structures are intended to lie in the soft-stiff or stiff-stiff regions whereas compliant structures are designed to operate in the soft-soft frequency range (see Fig 2.3). In fact, the compliant structures can be further subdivided into bottom-fixed and floating designs. They are derived from the

Figure 2.2: Various support structure concepts for offshore wind turbines, namely (a) monopiles, (b) tripods, (c) tripiles, (d) four-legged jackets, (e) three-legged jackets, (f) full lattice towers, (g) spars, (h) semi-submersibles, (i) barges and (j) tension-leg platforms.

oil and gas technologies, where the floating OWT concepts have been piloted in several projects, such as the Hywind in Norway, the WindFloat in Portugal and the Fukushima Shimpuu in Japan.

The UpWind Work Package 4 Task 2 has investigated the feasibility of different support structure designs, which include both existing and new concepts for large OWTs that are in the capacity range of 5 MW to 20 MW and to be deployed in water depths between 25 m and 70 m (de Vries, 2011). In general, the preliminary study found that monopiles are most effective for application in waters of less than 25 m deep, whereas the monopile-truss hybrids are suitable for the water depth between 25 m and 40 m and the jackets are for the water depth around 50 m. Furthermore, the INNWIND.EU project has been initiated between 2012 and 2017 as the successor of the UpWind project and is currently tasked to study various innovative concepts for beyond-state-of-the-art 10-20 MW offshore wind turbines. These include truss towers, smart jackets integrated with tuned mass dampers, jackets with hybrid members and jacket-bucket-concepts (Kuhnle et al., 2014).

In this thesis, focus is given to the bottom-fixed OWT support struc-

Figure 2.3: Soft to stiff frequency intervals for variable speed three-bladed wind turbines

tures. Among different concepts highlighted, the monopiles and jackets are selected for the case studies in this thesis.

Monopiles. Monopile towers have been widely deployed at wind farms in shallow waters. The deep penetrated foundations transfer lateral loads due to bending and shear of the tower to the soil. They require relatively large outer diameters to provide sufficient structural stiffness. Several projects under construction use monopiles of more than 6.5 m and weigh more than 800 ton in water depths up to 30 m. In return, the structures also experience higher hydrodynamic loads. In general, the design may not be economically attractive for deep water applications due to the fabrication and pile driving limitations (de Vries, 2011). However, recent upgrades of fabrication facilities and installation vessel capacities can probably extend the technology further to deployment at 30-35-m water depths by using the XL monopiles with outer diameters of up to 8-11 m (Seidel, 2014). So far the monopile technology has taken up 77 percent of the operating turbine foundation market share and 65.2 percent of those under construction (WEU, 2015b), including the world largest London Array windfarm in the United Kingdom with a nameplate capacity of 630 MW, Denmark’s Horns

Rev 1 and 2 windfarms, and China's Donghai Bridge windfarm, etc.

Jackets and full lattice towers. Space frame structures, i.e., jackets and lattice towers consist of three or more legs that are connected by braces in sections. The legs are typically tapered outwards to form a larger base and the foundation is typically pile-driven, either using pre-piling or post-piling methods. The advantages of space frame structures include the high transparency to wave loads, the resistance to global vibration, soil uncertainty and scouring issues, and being less susceptible to wind-wave misalignment as compared with monopiles (Cordle, 2010; Muskulus, 2012; Chew et al., 2013). Furthermore, the designs are also less likely to be constrained by fabrication and pile driving limitations while the manufacturing expertise is widely available from the oil and gas sector which is not yet in support for the serial production. As for the disadvantages, they are more complex and expensive to fabricate since more welded joints are involved (Muskulus, 2012). They are also easily affected by load directionalities due to the axisymmetry of the design, thus generating higher fatigue loads at the affected legs (Chew et al., 2013). When subjected to high frequency sources, local vibrations of braces can be excited and may have high impact on the fatigue life (Seidel and Ostermann, 2009). Nevertheless, the space frames are more appealing for intermediate water depth applications as the required structural mass generally increases linearly with respect to the water depth instead of quadratically as for the monopiles (de Vries, 2011). To date, only the jackets are deployed in the offshore wind farms, but not the full lattice structures. They comprise 5.3 percent of the market share and 11.7 percent of those under construction presently. Some of the examples include the OWEC Quattropods in Ormonde (30 units), Thornton Bank II (48 units), Alpha Ventus (6 units), Beatrice wind farms (2 units), etc.

For the next decade, it is widely perceived that the offshore wind industry is likely to continue adopting monopiles and jackets, which is somewhat aligned with the study conducted in the UpWind WP 4.2 (Seidel, 2014; WEU, 2015a). The monopiles are proven technology that can get financing easily and the breakthrough in manufacturing and installation capabilities pushes the sizes and water depth limitations, whereas the jackets are perceived as the best alternatives to monopiles when the monopiles are no longer feasible or too expensive to be deployed.

2.2 Design optimization challenges

The field of structural optimization, albeit having been extensively developed and studied in the subjects of aerospace, automotive, civil, mechanical engineering, etc., is still undergoing rapid changes in methodology development and applications. The classification of different optimization techniques can be generally perceived from the types of optimization problems (e.g., convex or non-convex optimization, deterministic or probabilistic optimization), the types of design variables (e.g., continuous or discrete optimization, sizing or topology optimization), the types of solution convergence (e.g., local or global optimization), etc. (Boyd and Vandenberghe, 2004; Ohsaki, 2010; Karadeniz, 2013). When implementing these techniques for the design of OWT support structures, the following challenges have to be addressed, one way or another.

Firstly, OWTs are complex and unique dynamical systems. They experience irregular and highly fluctuating environmental loads not only from the wind, but also from the waves and currents. Unlike the civil engineering systems in offshore oil and gas industry, OWTs comprise both mechanical and structural elements. In particular, the operating frequencies of the rotating wind turbines have to be avoided as they can potentially cause dynamic amplification to the system responses and degrade the structural integrity. Hence, the optimal de-

sign problem for OWT support structures has to be formulated to accommodate the necessary multi-domain knowledge and multiple-physics analyses involved for inter-structural interaction with the offshore environment. Besides, the dynamics of OWT systems contains various aspects of non-linearity, for instance the aerodynamic loads due to the unsteady flow, the non-linear wave loads (e.g., slamming force of breaking waves), the soil-structure interaction (e.g., p-y curves for lateral force-displacement relationships), etc. The individual subsystems also exhibit strong couplings and interactions in the dynamics despite that different isolated forces are experienced. For instance, the dampening effects arising from the rotating turbines and the viscous drag forces of waves or currents flowing through slender structures can counteract the motions due to waves and wind, respectively. As a result, it is difficult to apply the efficient separate and modular design approach to each subsystem in the optimization process directly, if high accuracy is sought ([Muskulus and Schafhirt, 2014](#)).

Secondly, multiple design checks have to be performed for structural evaluations, hence they need to be included as part of the design constraints in the optimization study. The structures have to comply with codes and standards imposed by the offshore and wind industries to ensure safe and reliable operations throughout the design lifetime ([IEC, 2009](#); [DNV, 2011b](#)). Primarily they cover the ultimate limit state (ULS) assessments which perform yield and buckling checks under extreme loads and the fatigue limit state (FLS) assessments which study failure due to quasi-periodic motions caused by wind and wave loads. The fatigue failure analysis is crucial for bottom-fixed OWT structures and the damage is cumulative over the response histories. Hence, a large number of design load cases (DLCs) subject to different combinations of wind-wave conditions have to be simulated to account for the long-term variability of the environmental loads as well as the different turbine operating conditions. By including these assessments in the problem formulation, practical insights

can be given on how the optimal structure performs against the relevant design considerations, but in return demands higher computational effort for design evaluations.

Thirdly, the analysis of OWTs is commonly carried out using specialized software that is equipped with capabilities to perform aero-elastic and hydro-elastic simulations in an integrated manner. The downside of this demand is that often the software is neither customizable nor extendable due to the lack of access to the source program. Therefore, engineers are hard pressed to use the software as black boxes and integrate with the external optimization solvers. This may result in difficulties or limit ways to construct efficient and reliable optimization methodologies for designing the OWT support structures. Furthermore, the structural analysis of OWT systems is generally performed using the finite element method (FEM) or the multibody-FEM method in the time-domain to account for system non-linearity. They are rather different from the traditional structural analysis that is carried out in the frequency-domain for offshore structures as well as the modal analysis for onshore wind turbines. Both FEM and multibody-FEM methods can provide higher simulation fidelity but are also more computationally expensive.

Different optimization frameworks have tried to address the above-mentioned challenges by adopting different combinations of methodologies for modeling, structural analysis, failure assessment and optimization, which generally decide the overall efficiency, effectiveness and practicality of the optimization framework. Their implementation in the optimization studies is discussed next.

2.3 Current state-of-the-art optimization frameworks for wind turbine support structures

2.3.1 Implementation of modeling and structural analysis methods

Various modeling and structural analysis methods have been employed to solve the dynamic responses for OWT support structures in the optimization studies. They can be classified based on:

Couplings of aero-elastic and hydro-elastic modules. Strong couplings exist between the two modules. Various coupling methodologies are described in Section 3.3.2. Long and Moe (2012) applied the *superposition method* when calculating the resultant response spectrum, by adding the response spectra excited by wind and waves. The aerodynamic damping was included as an additional structural damping in the analysis, while the hydrodynamic damping was not accounted for. The *superposition method* is very efficient as it can be implemented in the *frequency-domain* but high accuracy has to be forgone due to missing couplings and missing non-linearity effects (refer to Section 3.3.1 for explanation on the time- and frequency-domain analysis). King et al. (2013) applied different coupling methods in different phases of the optimization study: a simplified *superposition method* that excluded both aerodynamic and hydrodynamic damping in the first-pass jacket design phase, followed by a *fully integrated analysis* in the detailed design phases. Moreover, the way that aerodynamic forces are extracted from the aero-elastic tool for the analysis of support structures was shown to have great influence on the accuracy of the decoupled analysis (Ong et al., 2014). Direct application of reaction forces from coupled simulations would cause excitation when applied to the hydro-elastic analysis directly (Muskulus, 2015; Passon and Branner, 2013). Chew et al. (2015b) has performed the optimization study using the *partial integrated approach*

in the *time-domain analysis*, where the aerodynamic forces were obtained from a fixed turbine and applied onto the hub node together with a linear damper that accounts for the aerodynamic damping, and the entire rotor-nacelle assembly (RNA) was modeled to simulate the accurate dynamic properties.

Static and dynamic loading conditions. Despite that the OWT is a highly dynamic system, *static load analysis* is still commonly employed for design optimization process within the industry. [Uys et al. \(2007\)](#) applied this approach when investigating the minimal material and manufacturing costs for an onshore monopile tower. The operational aerodynamic load on the rotor was represented as simplified damage equivalent forces and moments applied at the tower bottom, whereas the distributed wind load on the tower could be calculated analytically using the empirical formula prescribed in the Eurocode Part 1-4 ([European Standard, 2005a](#)). Even though treated as constant in this study, DELs are supposed to be updated during the optimization iterations, as they vary with respect to the fundamental eigenfrequencies of the structures along with the number of stress cycles and locations where they are made reference to. Stresses are computed from the internal loads to determine the fatigue damage for offshore structure design, as opposed to the methodology proposed for the turbine blade testing by [Freebury and Musial \(2000\)](#). On the other hand, when subject to *dynamic loading* conditions, the optimal design problem will be solved using the dynamic response optimization approach. Transient loads introduce time-dependent constraints, which can be treated in a discretized time-domain by various methods as reviewed in the literature ([Kang et al., 2006](#)) (see Section 3.2 for the transient loads experienced by OWTs).

Recent innovative developments. Yang et al. (2015) has implemented the Kriging model to solve the structural responses when studying the reliability-based optimization of OWT support structures. Kriging is one of the *metamodeling* techniques that can provide approximations to the output parameters by using regression analysis of the sampled points selected within the design space during the computer experiments (Wang and Shan, 2007). Even though it is computationally expensive to construct the meta-models (e.g., 100 design points are used in this study), the overall efficiency can be attained when a great number of simulation tasks (e.g., finite element structural analysis) have to be repeated during the optimization process, such as the Monte Carlo simulations (MCS) carried out in the simulation-based optimization. In addition, to the best of the author's knowledge, there are also various advancements in methodologies that were developed for the OWT structural analysis but have not been applied to the optimization studies. For instance, Voormeeren et al. (2014) has further improved the work by Hald and Høgedal (2005) and developed the methodology which combined both *superelement modeling* and *modal truncation augmentation* to enable efficient and accurate modeling of reduced OWT substructures which can account for the distributed hydrodynamic loads experienced by the entire submerged superelement. Furthermore, Schafhirt et al. (2015) has combined the *substructuring technique* that was based on the principle of superposition together with the graphics processing unit (GPU) parallel computing, in solving the dynamic responses of OWT systems and has significantly reduced the computational time taken.

2.3.2 Implementation of failure assessment methods

Optimization studies in the literature might have implemented different failure assessment criteria for the evaluation of OWT support structures. On the one

hand, variations can occur due to differences in *formulations applied by codes and standards* or due to *simplifications* adopted to suit the modeling and structural analysis. For instance, buckling checks of offshore structures could be carried out in compliance with recommended practices or standards published by DNV GL (DNV, 2010a), Standards Norway (Standards Norway, 2013) or European Standard (European Standard, 2007), as found in (Thiry et al., 2011), (Chew et al., 2015b) and (Haghi et al., 2014), respectively. As for the fatigue failure assessment, most of the literature referred to DNV-RP-C203 (DNV, 2011c). The variations of formulas used in code checks as well as the different load and resistance factors prescribed gave different failure assessment results, where some may deem more conservative than others, as investigated in Tuen (2012). Furthermore, Long and Moe (2012) has simplified the Hot Spot Stresses (HSS) calculation in the fatigue failure analysis for tubular joints by just taking the absolute values of the HSS and hence reducing the eight HSS around the weld to three. Structural optimization is commonly used for the purposes of preliminary design, therefore such simplifications can enhance the efficiency, even though they are not as accurate as the complete code checks.

On the other hand, the variations of failure assessment criteria are typically dependent on the *types of failure mode* covered in the structural evaluation process. AlHamaydeh and Hussain (2011) studied the design of onshore wind turbine foundations considering the *eigenfrequency constraints*. By ensuring a sufficient separation between the natural and excitation frequencies, the optimization study eliminated the need for reinforced concrete encasement in various foundation designs with varying radius of pile group, grillage beams, pile sizes and stiffening methods. Moreover, Karadeniz et al. (2009) applied the eigenfrequency constraint to minimize the dynamic responses of offshore tripods and consequently the risk of fatigue failure. Even though the method did not strictly guarantee a safe design against fatigue failure, it could avoid the fatigue evalu-

ation procedure which requires full dynamic response analysis and can be very expensive. As for the *extreme load constraints* (see Section 3.4.1), the failure criteria may also vary with respect to the types of structures. For instance, Long et al. (2012) examined the structural strength against yielding for tubular members and joints as well as the column buckling of members when optimizing the full lattice towers, whereas Uys et al. (2007) evaluated the shell buckling for unstiffened monopile towers, the ring buckling for stiffened towers and the local buckling for flat ring-stiffeners. In additions, the code checking process should also include the fatigue failure assessment when designing the offshore structures. *Fatigue load constraints* could be implemented in the frequency-domain by using the Dirlik's method or in the time-domain using the Rainflow counting method (see Section 3.4.2 for details). Since the FLS assessment activities are computationally very expensive, the selection between these two methods is often decided in conjunction with the modeling and analysis approaches. For instance, the former was adopted by Long and Moe (2012) and the latter was used by Zwick et al. (2012).

2.3.3 Implementation of optimization strategies

When preparing the optimization strategy for a design problem, the mathematical problem formulation and the optimization algorithm used for solving the optimum have to be determined.

Problem formulation. In wind energy research, the common goal tends to focus on the reduction of LCOE. To date, majority of the research carried out aimed at minimizing the mass or volume of the support structures when designing the OWT support structures (Fylling and Berthelsen, 2011; Zwick et al., 2012; Yang and Zhu, 2015; Yang et al., 2015). This is a simplified objective or cost function that represents the main part of capital costs and is easy to implement. The School for Structural Optimization in University

of Miskolc has extensively studied the fabrication cost of steel structures for the assembly, rolling, cutting, grinding, welding and painting processes (Farkas and Jármai, 2013). These mathematical cost models were used in optimizing the fixed offshore towers (Jarmai et al., 2006) as well as the on-shore wind turbine towers (Uys et al., 2007). Apart from minimizing the structural weight, different problem formulations could also be adopted for similar purposes. Negm and Maalawi (2000) have investigated another four optimization strategies with different objective functions when designing the wind turbine towers, i.e., maximizing the stiffness, maximizing the stiffness-to-mass ratio, minimizing the vibration using the frequency-placement criterion and minimizing the vibration using the maximum frequency criterion. The study considered only the simplified yield and buckling checks using the static analysis and has shown that the last strategy gave the best results for both mass and stiffness improvement while attaining the global optimal of design solutions. Additionally, by integrating the control strategies into the optimization framework, the utilization of OWT support structures could be enhanced together with the reduction of load effects exerted on the structures. This was performed by Fischer et al. (2012) who considered different operational and dynamic control strategies and by King et al. (2013) who studied the comparative effects of collective pitch controllers (CPC) and individual pitch controllers (IPC). Furthermore, the optimal design problem could also be formulated at the system level. National Renewable Energy Laboratory (NREL) has initiated the Wind Energy Systems Engineering (WESE) framework to optimize the overall wind farm system that includes wind turbines and components, wind plants, turbine-to-turbine interaction, grid integration, etc. towards a configuration that attains the minimum LCOE (Dykes et al., 2011). Within the WESE, a jacket sizing tool (JST) was developed to

perform preliminary design optimization on the space-frame substructures and piles based on the eigenfrequency and static analysis (Damiani and Song, 2013; Song et al., 2013). Furthermore, system level optimization of OWTs could also be carried out using the integrated multidisciplinary design optimization (MDO) approach which combined the design of wind turbine rotors and monopile towers concurrently, as investigated by Ashuri et al. (2014, 2016).

All the literature discussed so far was based on the deterministic method and did not consider uncertainties. In the OWT system, uncertainties exist in the forms of wind, wave and soil characterization, aerodynamic and hydrodynamic load calculation, material properties, finite element modeling, manufacturing errors, simulation errors, etc. (Veldkamp, 2008; Dong et al., 2012; Negro et al., 2014; Zwick and Muskulus, 2015). In order to account for these uncertainties, the optimal design problem of OWT support structures has been formulated using the probabilistic methods, such as the extended performance-based wind engineering (PBWE) (Ciampoli and Petrini, 2010), the robust design optimization (RBO) (Yang and Zhu, 2015) and the reliability-based design optimization (RBDO) (Yang et al., 2015).

Optimization algorithms. The selection of optimization algorithms depends very much on the problem formulations. Generally, the design of OWT support structures is a non-linear programming (NLP) problem, i.e., the objective or constraint functions are not linear, but not known to be convex. Solving the NLP problems can be extremely challenging as the problem can possibly become intractable when a large number of design variables are involved (Boyd and Vandenberghe, 2004). Nevertheless, it is relatively straightforward to apply the local optimization methods to solve the NLP since only the gradients of the objective and constraint functions

are required. Some of the commonly used local optimization algorithms for the OWT application include the sequential quadratic programming (SQP) (Fylling and Berthelsen, 2011) and the interior point method (Haghi et al., 2014), etc. Zwick et al. (2012) have performed an iterative search for optimal full lattice tower design by assuming that weak couplings exist between the individual members and locality effects exist in the member loads, hence the method is very efficient but may compromise on the accuracy.

Most of the work performed so far has utilized the finite difference method in estimating the sensitivity derivatives for the gradient-based optimization procedure. Alternative complex variables sensitivity methods such as the complex Taylor series expansion (CTSE) (Squire and Trapp, 1998) and the Fourier differentiation (Henrici, 1979) can also be used to obtain the derivatives at different orders accurately since the imaginary step sizes selected can be made arbitrarily small without the concern of increasing round-off errors. Besides, the direct differentiation method and the adjoint method can be employed to calculate the analytical gradients. The computational speeds of both methods are often limited by the number of design variables and the number of constraints, respectively. Jørgensen and Nissen (2013); Oest et al. (2015, 2016) attempted to calculate the gradients for fatigue damages using the aggregated adjoint method which can reduce the large number of design constraints to one by aggregation functions. So far these methods have been applied to the quasi-static sensitivity analysis where the differential changes in the size of each stress cycle is evaluated based on changes in the geometry and quasi-static loading only.

On the other hand, global optimization algorithms can be applied to search for the global optimum of the design problem. Due to the intensive computational requirements, they are generally applicable to problems with

a small number of design variables, where computing time is less critical and the value of finding the true global solution is very high (Boyd and Vandenberghe, 2004). Since the convexity of the problem formulation is not guaranteed for the design of OWT support structures, single-solution metaheuristic optimization methods such as the adaptive simulated annealing and population-based metaheuristic methods such as the genetic algorithm and particle swarm optimization have been implemented in studies conducted by Yang and Zhu (2015), Yang et al. (2015) and Karpát (2013), respectively. The metaheuristic methods could also be used for the optimization studies involving combined discrete and continuous variables, as performed by Yoshida (2006) in optimizing the monopile tower dimensions as well as the locations of the flanges and access doors to navigation lights. In addition, Molde et al. (2014) performed global optimization on the full lattice OWT structures by adopting the simulation-based optimization approach that was based on the simultaneous perturbation stochastic approximation (SPSA). The method could save time in assessing the pseudo-gradients, but conversely required a large number of iterations to converge to optimal solutions.

Chapter 3

Offshore wind turbine simulation and analysis

3.1 Offshore environmental conditions

3.1.1 Wind conditions

Wind, from which the turbine harvests its energy, varies in speed and direction with time and space. When defining the wind speed statistics, the averaging time and reference height have to be specified. For instance, a 10-min averaged wind speed U_{10} and a standard deviation σ_U at the reference height of 10 m are typically used to measure wind intensity and variability about the mean, respectively. Typically, both U_{10} and σ_U refer to the longitudinal direction recorded by wind vanes in a 10-min average. The overall power spectrum of wind speeds collated for a site generally looks like Fig. 3.1. Empirical wind data to be used as design basis must cover a sufficiently long period of time, preferably a 10-year period or more (DNV, 2010c). The range of interest for load simulations falls between several days and a few seconds (Vorpahl et al., 2013). In the mesometeorological range with low-frequency oscillations, the synoptic peak results from changes due to crossing weather phenomena while the diurnal peak represents day and night changes; whereas in the micrometeorological range, the turbulent peak is due to high-frequency fluctuation of wind speeds. Lying between the two ranges is the spectral gap which has a very low energy content. As a result, time series with a given average wind speed over a period of 10 min to 1 hour are usu-

Figure 3.1: Wind speed fluctuation spectrum from days to seconds (Vorpahl et al., 2013)

ally used to describe the random nature of the wind in the simulations, where the wind can be assumed as a stationary stochastic process in this frequency range.

The actual wind speed distribution is shown in Fig. 3.2. It consists of two components: the mean wind speed and the turbulence. The mean wind speed varies across the height above the ground or still water level (SWL). When the terrain condition and the atmospheric condition are not complex, the mean wind speed can be fitted to simple models such as the logarithmic profile, the power law, the Frøya profile, etc. As for the wind speed turbulence in longitudinal direction σ_U , it follows a lognormal distribution when conditioned on U_{10} (Det Norske Veritas and RISO National Laboratory, 2002). The turbulence intensity $TI = \sigma_U / U_{10}$ is also commonly used as a measurement of wind speed variability. On the other hand, the lateral wind turbulence σ_{U_y} can be taken as a value between $0.75 \sigma_U$ and $0.80 \sigma_U$, whereas the vertical wind turbulence σ_{U_z} can be taken as $0.50 \sigma_U$ (DNV, 2010c). Since the turbulence is highly dependent on the local roughness condition, extrapolation of wind data from adjacent location to estimate the local turbulence is encumbered with ambiguity. Therefore, conservatively high values for σ_U are commonly used for design purposes.

Figure 3.2: Actual wind speed distribution in global X-direction (van Der Tempel, 2006)

The short-term stationary wind condition may be described by a wind spectrum, i.e., the power spectral density (PSD) of wind speeds, using either the site-specific spectrum based on measured data or the model spectrum that has been extensively calibrated to wind data obtained over land or over water. Some of the commonly used model spectra in the industry include the Mann model, the Kaimal spectrum, the empirical Simiu and Leigh spectrum, etc. (DNV, 2010c). The IEC Kaimal spectrum used in this thesis is based on the UpWind design basis and is given by

$$S_{Kaimal}(f_U) = \sigma_U^2 \frac{6.868 \frac{L_U}{U_{10}}}{\left(1 + 10.32 \frac{f_U L_U}{U_{10}}\right)^{5/3}} \quad (3.1)$$

where f_U is the wind frequency. The integral length scale L_U is specified by IEC 61400-1 to be independent to the terrain roughness (IEC, 2010),

$$L_u = \begin{cases} 3.33z & \text{for } z < 60 \text{ m} \\ 200 \text{ m} & \text{for } z \geq 60 \text{ m} \end{cases}$$

where z denotes the height above the ground or sea level. In wind turbine load simulations, stochastic three-dimensional wind fields can be realized in the time-domain from the wind spectra and the coherence functions using the methods as described in [Veers \(1988\)](#) and [Kelley \(1993\)](#). The coherence function is a frequency-dependent measure of the spatial connectivity of wind speeds that decays with respect to the increasing separation, and is represented by model spectra such as the Davenport coherence model, the IEC coherence model and the von Karman coherence model. Each spectrum makes use of different flow turbulence assumption and may offer varying level of accuracies when predicting the turbulence coherence for longitudinal, lateral and vertical components under the longitudinal and lateral separations ([DNV, 2010c](#)).

While describing the long-term wind climate, the probability density distributions (PDFs) for the U_{10} fit the Weibull function relatively well at most sites. It can also be represented using a scatter diagram that provides the frequency of occurrence P_v for discretized pairs of (U_{10}, σ_U) . The U_{10} and σ_U are usually binned in a gap of 2 ms^{-1} and the corresponding wind directional distributions for each bin can be visualized using the wind roses. As for the extreme wind conditions and gusts, since measured data over the necessarily long time period is rarely available, an extreme value distribution (e.g., power-law or Gumbell function) is usually employed to approximate the distribution for maximum mean wind speeds in a given returning period (1, 50 or 100 years).

3.1.2 Wave conditions

Ocean waves are random and irregular in shape and velocity of propagation. There are two main types of waves, i.e., wind-generated waves and swell, where the former are generated by local wind while the latter is formed by the distant weather system. Stochastic modeling of sea surfaces and wave kinematics is required to represent a sea state. In application, the sea state is usually assumed

to be a stationary random process. The standard period for wave measurement is normally 3 hours per registration, but the period of stationarity can range from 30 minutes to 10 hours (DNV, 2010c).

When modeling regular waves, the three main wave parameters required for wave characterization are wave height, wave period and water depth. Several regular wave theories have been developed, with each having a different range of validity governed by any two of these non-dimensional parameters: wave steepness parameter, shallow water parameter and Ursell number U_R (American Petroleum Institute, 2007). Linear wave theory or Airy theory is the simplest wave theory that models the wave surface elevation to be sinusoidal, i.e., the wave crest height is equal to the wave trough height, under the assumption that the wave height is much smaller than both the wave length and the water depth (Chakrabarti, 2005). Therefore, the surface elevation η ; the horizontal and vertical velocities u and w ; and the horizontal and vertical accelerations $\dot{u}$ and $\dot{w}$ measured along the horizontal x-axis and the vertical z-axis at t seconds are given by

$$\eta(x, t) = \frac{H}{2} \cos(kx - \omega t), \quad (3.2)$$

$$u(x, z, t) = \frac{gkH}{2\omega} \frac{\cosh[k(z + d)]}{\cosh kd} \cos(kx - \omega t), \quad (3.3)$$

$$w(x, z, t) = \frac{gkH}{2\omega} \frac{\sinh[k(z + d)]}{\cosh kd} \sin(kx - \omega t), \quad (3.4)$$

$$\dot{u}(x, z, t) = \frac{gkH}{2} \frac{\cosh[k(z + d)]}{\cosh kd} \sin(kx - \omega t), \text{ and} \quad (3.5)$$

$$\dot{w}(x, z, t) = -\frac{gkH}{2} \frac{\sinh[k(z + d)]}{\cosh kd} \cos(kx - \omega t), \text{ respectively,} \quad (3.6)$$

where H is the wave height; k is the wave number; ω is the wave angular frequency; d is the water depth and g is the gravitational acceleration (see Fig. 3.3 for the description of the Airy wave properties). The wave parameters also follow the dispersion relationship, i.e.,

$$\omega^2 = gk \tanh(kd). \quad (3.7)$$

Figure 3.3: Regular Airy wave properties (DNV, 2010c)

Meanwhile, Stokes wave theory expands the surface elevation in the powers of the linear wave height to simulate the asymmetric surface elevation profile which has steeper crests and wider troughs. Stokes 5th order wave theory is applicable to regular steep waves of up to $U_R = 30$, while the maximum crest-to-wave-height ratio for a Stokes wave is 0.635 (DNV, 2010c). When modeling a much steeper wave with higher U_R , Cnoidal wave theory (Wiegel, 1959) or a more generic stream function wave theory (Dean, 1965) should be used instead.

Nevertheless, the actual sea state is formed by irregular random waves. The simplest way to model irregular waves is summing up the sinusoidal wave components (Airy waves) separated by random phases ϵ_n that are mutually independent and uniformly distributed between 0 and 2π , i.e.,

$$\eta(x, t) = \sum_{n=1}^N A_n \cos(k_n x - \omega_n t + \epsilon_n). \quad (3.8)$$

The random wave amplitudes A_n follow the Rayleigh distribution with the mean square value given by

$$A_n^2 \approx E[A_n^2] = 2S(\omega_n) \Delta\omega_n \quad (3.9)$$

where $\Delta\omega_n = \omega_n - \omega_{n-1}$ is the difference between the successive frequencies and $S(\omega_n)$ is the mean amplitude of the wave spectral density within $\Delta\omega_n$. In this case, the linear superposition also applies to the wave kinematics and wave dynamics.

The short term sea surface can be described using a wave spectrum, given either in a tabulated form for the measured spectrum or in a parameterized analytic formula. Some of the commonly used spectrum model include the Pierson-Moskowitz (PM) spectrum $S_{PM}(\omega)$ and the JONSWAP spectrum $S_J(\omega)$:

$$S_{PM}(\omega) = \frac{5}{16} H_s^2 \omega_p^4 \omega^{-5} \exp \left[-\frac{5}{4} \left(\frac{\omega}{\omega_p} \right)^{-4} \right] \quad \text{and} \quad (3.10)$$

$$S_J(\omega) = [1 - 0.287 \ln(\gamma)] S_{PM}(\omega) \gamma^{\exp \left[-\frac{1}{2} \left(\frac{\omega - \omega_p}{\sigma \omega_p} \right)^2 \right]} \quad (3.11)$$

where ω_p is the peak frequency (inverse of peak period T_p); H_s is the significant wave height; γ is the non-dimensional peak shape parameter and σ is the spectral width parameter. When $\gamma = 1$, the PM spectrum proposed for fully-developed seas is reduced from the JONSWAP spectrum that is developed for fetch limited seas. Typically, the number of wave components N to be used for modeling the irregular waves should be at least 200, preferably, 1000. The cut-off angular frequencies ω_1 and ω_N ought to cover a sufficiently wide range of wave frequencies.

The wave kinematics for linear wave theory can only be applied up to the SWL. Hence, modification is required to extend the wave kinematics up to the free surface and Wheeler's stretching method is widely used for that purposes (Wheeler, 1970). The instantaneous vertical coordinate z in Eqs. 3.3-3.6 is

3.2 Aero-hydro-servo-elastic simulations

The general dynamic equilibrium behavior of an OWT model can be written as

$$\mathbf{R}^{\mathbf{I}}(\mathbf{z}, \ddot{\mathbf{z}}, t) + \mathbf{R}^{\mathbf{D}}(\mathbf{z}, \dot{\mathbf{z}}, t) + \mathbf{R}^{\mathbf{S}}(\mathbf{z}, t) = \mathbf{R}^{\mathbf{E}}(\mathbf{z}, \dot{\mathbf{z}}, t) \quad (3.13)$$

where $\mathbf{R}^{\mathbf{I}}$ refers to the inertia force vector contributed by the structural mass and the hydrodynamic added mass; $\mathbf{R}^{\mathbf{D}}$ is the damping force vector that depends on the internal structural damping and the hydrodynamic radiation of the floating or submerged body; $\mathbf{R}^{\mathbf{S}}$ is the internal structural reaction force vector that correlates with the stiffness matrix of the system; and $\mathbf{R}^{\mathbf{E}}$ is the external force vector that accounts for gravitational loads, aerodynamic loads, hydrostatic loads, hydrodynamic loads, and soil reaction on loaded piles, while $\mathbf{z}$, $\dot{\mathbf{z}}$ and $\ddot{\mathbf{z}}$ refer to the structural displacement, velocity, and acceleration, respectively (Karimirad, 2012). This general form of equations of motion allows the inclusion of non-linear dynamics within the OWT system.

When the dynamics of OWTs is reduced to a linear system, Eq. 3.13 can be simplified to

$$[\mathbf{M}^{\text{sys}}] \ddot{\mathbf{z}}(t) + [\mathbf{C}^{\text{sys}}] \dot{\mathbf{z}}(t) + [\mathbf{K}^{\text{sys}}] \mathbf{z}(t) - \mathbf{f}^{\text{sys}}(t) = \mathbf{0} \quad (3.14)$$

where $[\mathbf{M}^{\text{sys}}]$, $[\mathbf{C}^{\text{sys}}]$ and $[\mathbf{K}^{\text{sys}}]$ refer to the system matrices for mass, damping and stiffness, respectively, and $\mathbf{f}^{\text{sys}}$ is the external force vector. In this case, $\mathbf{f}^{\text{sys}}$ is different from $\mathbf{R}^{\mathbf{E}}$, because the former is not state dependent, similarly for $[\mathbf{M}^{\text{sys}}]$, $[\mathbf{C}^{\text{sys}}]$ and $[\mathbf{K}^{\text{sys}}]$. See Section 4.1 for detailed descriptions of various terms in Eq. 3.14. In the subsequent sections, the fundamental dynamics of the OWT system is discussed.

3.2.1 Aero-servo dynamics

The aerodynamics of a wind turbine concerns primarily the interaction between wind forces and rotor blades. The aerodynamic loads comprise mainly the lift, skin friction and pressure viscous drag. They can be categorized into different types of loads, depending on loading conditions and turbine operation. However, it is worth to note that the loading types may overlap one another in some cases ([Manwell et al., 2009](#)):

1. Static load, that is generated by steady wind flowing through a parked or idling wind turbine.
2. Steady load, that is generated by steady wind flowing through a rotating wind turbine.
3. Cyclic load, that can be induced by a rotating wind turbine passing through wind shear or by yaw motion, yaw error, gravitational load on the blades, etc.
4. Transient load, the temporal unsteady load that can be resulted from the controller dynamic effects, such as brake application, release of the drive train during turbine shut down or start up procedures, feathering of blades, teetering of a two-bladed wind turbine and dynamic stall that results in or delays the stall; or from the gust passing through a wind turbine.
5. Impulsive load, that exerts a significant peak magnitude load within a short duration. It can be generated when a downwind turbine blade passing a wake of tower.
6. Stochastic load, due to the turbulent wind force acting on the blades.
7. Resonance induced load, that arises when the blade natural frequencies are close to the excitation force.

8. Inertial load, that includes the centrifugal load of rotating blades, the gyroscopic load when a turbine yaws and the breaking load when a turbine accelerates or decelerates.

The aero-servo dynamic model has to be sufficiently robust in modeling the deterministic and stochastic aerodynamic loads while subject to the induced wake generated by rotating turbines, the dynamic interaction of rotor blades with the unsteady air flow as well as the influence of control system on rotor operation, etc.

One of the most widely used aerodynamic models in the wind industry is the blade element - momentum (BEM) theory developed originally by [Glauert \(1935\)](#). The theory assumes that there is no radial interaction of air flows between the adjacent annuli, and therefore the forces of a blade element are responsible solely for the change of momentum of the air passing through the swept annulus. The rotor torque and thrust are calculated from the flow induction factors, which are then solved in an iterative process based on the local two-dimensional airfoil characteristic that are non-linear functions of the angle of attack ([Burton et al., 2011](#)). The set of airfoil data needs to be further corrected considering the three-dimensional and rotational effects, before it can be used for the BEM computation for the entire blade. In addition, the classical BEM theory is further improved to include corrections for tip and hub losses, yaw or tilt of the rotor as well as heavily loaded rotor ([Sørensen, 2012](#)). The current BEM method has been extended to the unsteady BEM model to account for the unsteadiness of the wind seen by the rotor, as well as the dynamic wake and the dynamic stall effects which are particularly important for transient load analysis ([Hansen, 2008](#)). The BEM method can generally provide good aerodynamic load modeling for HAWT, besides requiring less computational effort since the air flow and wake are not numerically solved in the process. Moreover, various sophisticated numerical-based aerodynamic models have been developed in the

recent years, which enable the study of the underlying rotor-wake interaction physics. Some of the prominent models are the vortex model, the numerical actuator line and disk models, the full Navier-Stokes computational fluid dynamics (CFD) model (Sørensen, 2012; Hansen and Aagaard Madsen, 2011).

Control and supervisory systems are employed by wind turbines to maximize the power production while reducing the structural loads on turbines. Passive power control can be achieved by exploiting the blade stall effects to diminish the lifting force when the wind speed becomes too high and hence does not require any moving parts for the rotor. On the other hand, active power control requires external power for actuation purposes (usually run by hydraulic actuators) when regulating the rotor rotational speed. For variable-speed wind turbines commonly used nowadays, the active control strategy varies with respect to the operating wind regime. In each operating region, different controllers can be activated to meet the wind turbine performance objectives and to ensure smooth transition between the regions (Pouliezos, 2012). Some of the common active control system include the pitch controller that can operate either collectively or individually, generator control that regulate the torque on the rotor shaft and yaw control. Operating wind turbines with active control systems experience aerodynamic damping. Aerodynamic damping occurs as the rotor blades encounter an increase or decrease of wind speeds due to forward or backward motion of the tower top, hence responding aerodynamically (such as pitching the blades) to counteract this motion (Salzmann and Van Der Tempel, 2005). This damping effect plays a significant role in reducing fatigue damage on the support structures of OWTs.

3.2.2 Hydrodynamics

Hydrodynamic forces are experienced at the submerged parts of an OWT. They can be induced by wave diffraction and wave radiation based on linear poten-

tial flow theory, as well as hydrostatic forces, inertial loads, non-linear viscous drag and current loads. The following classification of hydrodynamic loads may overlap one another in some cases.

1. Wave diffraction, occurs when the undisturbed incident wave potential (Froude-Krylov) is superimposed with the scattering potential that is created when the incident wave is disturbed by the fixed body (Newman, 1977). The wave diffraction component gives the wave excitation force and moment that are proportional to the incident-wave amplitude.
2. Wave radiation, occurs when the fluid motion is generated by rigid body motion of the structural body, which is forced to oscillate at the degrees of freedom (DOF) excited by the external mechanism in the absence of incident waves (Newman, 1977). The wave radiation component contributes to the added mass and damping forces.
3. Hydrostatic forces includes the buoyancy force from Archimedes principle and the linear restoring force due to the effect of water-plane and center of buoyancy.
4. Inertial load, arises due to the pressure gradient associated with the relative accelerations of the structural components and the surrounding fluids (Wilson, 2003).
5. Viscous drag load, the non-linear force terms that arise as a result of flow separation induced by the relative velocities between fluid particles and structures (Wilson, 2003).
6. Current load, can be modeled in Morison's formula by combining the kinematics of both currents and waves into the viscous drag and inertial terms. Various current profiles are available and they may depend on tidal, local

wind, Stokes drift, ocean circulation, storm surge, density-driving effects, etc. (Faltinsen, 1990).

Hydrodynamic forces exerted on offshore structures can be calculated by various methods, such as the panel method, the pressure integration method, Morison's formula, or any combination of these methods. (Faltinsen, 1990). For a slender cylindrical structural member that has an outer diameter D of less than one-fifth of the wave length λ , the changes of fluid particle kinematics when flowing through the member can be assumed negligible and Morison's load formula is applicable. The relative dominance between drag and inertial terms depends on the flow regimes (Sumer and Fredsøe, 2006). For fixed structures, the flow becomes inertial-dominant when $0.5 < D/H < 1.0$ where H is the wave height, and the inertial load comprises components of hydrodynamic added mass forces and FK forces (Sumer and Fredsøe, 2006). On the other hand, viscous drag is not included in the linear hydrodynamic model yet it dominates the damping effect when the characteristic diameter and motion of the structure are very small, i.e., $D/H < 0.1$. The values of drag coefficients C_D and inertial coefficients C_M were determined in the past based on experimental data, and they vary with respect to the Reynolds number, the Keulegan-Carpenter number and the surface roughness (DNV, 2010c). Furthermore, wave diffraction effect becomes important for large volume structures, when the diameter of the structure is in the same order as the wavelength of waves, i.e., $D/\lambda > 0.2$. The diffraction forces can be calculated from multiplying FK forces by the experimental modified flow coefficients, on the basis of diffracted wave potential developed by MacCamy and Fuchs (1954).

Therefore, it is imperative to consider both aspects of wave kinematics and wave hydrodynamics when performing load analysis on offshore structures (Faltinsen, 1990; Newman, 1977). In general, linear wave theory is most applicable to deep water applications, but it may not be sufficiently accurate for

shallow water waves. The second-order non-linear irregular waves were shown to describe better the shallow water waves while applied in the reliability study of offshore wind turbines ([Agarwal and Manuel, 2009](#)). Besides, higher order waves also introduce additional effects such as mean drift, slow varying frequency and sum frequency forces, drift-added masses, and damping [Karimirad \(2012\)](#). These non-linearities are mainly active in resonant responses at low natural frequencies, which may be prominent in compliant and floating wind turbine systems.

3.2.3 Pile-soil dynamics

To date, monopiles, jackets and gravity-based foundations are the three main foundation concepts commonly employed by bottom-fixed offshore wind turbines. Pile driven foundations, such as monopiles and jackets are designed to have the external loads applied absorbed by the soil through lateral and axial resistance, whereas for gravity-based foundations, the combined loads are balanced off by bearing forces and sliding resistance at the foundation-soil interface. In this section, the dynamics of the piled-foundation is discussed.

Pile-soil dynamics depends much on the soil properties, the geometry of the pile and the embedment into the soil. Various factors such as the soil types, the soil permeability, the relative density, the soil compressibility and the past loading history can have great influence on the soil properties ([Bartrop and Adams, 2013](#)). Soils are generally very non-linear in their behavior, as it is caused by the soil particles moving relative to one another when the load is applied. As a result, geotechnical investigation of the site and laboratory tests for soil strength are usually carried out to determine the soil characteristics for the detailed design of piles.

Piles experience axial resistance from the soil, due to the accumulated skin friction and the tip end bearing. The axial soil reaction changes with

the pile deformation and it can be modeled as a series of springs with varying stiffness attached on the pile. The axial resistance-displacement relationships are generally derived from the skin friction-displacement ($t - z$) curves and the tip load-displacement ($Q - z$) curves. In addition, when piles are loaded laterally, the soil resists the deformation, of which the lateral soil reaction to the piles can be represented as a set of non-linear springs ([International Standard, 2007](#)). The lateral load-displacement ($p - y$) curves associate the soil resistance with the pile deflection under the lateral loading, which account for the plastic behavior of soils, such as clay or sand ([Det Norske Veritas and RISO National Laboratory, 2002](#)). The curves are estimated empirically by fitting non-linear curves to t_{max} , Q_p and p_u , which denote the maximum unit skin friction capacity, the maximum end bearing capacity and the ultimate lateral bearing capacity, respectively. However, the empirical formulae derived for clay and sand may lack of agreement among the literature due to the scarcity of high quality experimental data. As for pile groups, the pile-soil-pile interaction exists but can be neglected if they are spaced far enough to one another (at least four- to five-pile-diameter spacing) ([Reese, 2003](#)).

Apart from the conventional method mentioned above, [Zaaijer \(2006\)](#) studied different simplified foundation modeling approaches for modeling the OWT foundations, which include the effective fixity depth model, the stiffness matrix method and the Winkler FEM model. [Van Buren and Muskulus \(2012\)](#) improved the traditional method by implementing the dynamic $p - y$, $T - z$ and $Q - z$ models proposed by [Naggar and Novak \(1994\)](#) and [Naggar and Bentley \(2000\)](#) to have the inner-field spring-damper-elements to model the non-linear stiffness and hysteretic damping of the soil and the far-field elements to model the wave propagation away from the pile.

Furthermore, scour of seabed surfaces may occur around the offshore piles when the flow velocity induced by the presence of the piles and turbulence

is sufficiently high to lift and carry the seabed sediments. Scour can reduce the axial capacity of the soil as the buried length of the pile is reduced, hence the piled-foundations may require scour protection to avoid destabilization (Boon et al., 2004).

3.2.4 Structural dynamics

Structural dynamics of OWTs studies the behavior of OWT structures when subjected to static and dynamic loads. It can be represented using the following models (Cordle, 2010):

Modal representation This method is widely used in the onshore wind turbine simulation codes. The fundamental mode shapes and frequencies of the structure are first computed, usually using a finite element pre-processor, followed by a linear superposition of the eigenmodes to calculate the over-all dynamic responses of the structure. This method is computationally efficient, but at the same time poses limitations such as low fidelity analysis due to low number of DOF, unsupported for complex wind turbine configuration, as well as only account for linear responses. Therefore, this method may not be able to accurately model the dynamic responses for compliant or floating wind turbines that expect significant translational and rotational motions.

Multibody system The multibody system method generally treats the over-all system as a number of bodies that are interconnected by joints, and obtains the dynamic responses by solving the coupled equations of motion. The individual bodies can be modeled as rigid or flexible and each joint has to be defined by boundary constraints. Therefore, this method supports non-linear response analysis by allowing large translational and rotational displacements at the bodies and joints. Besides, higher fidelity simulations can be accounted for by introducing more DOF compared to

modal representation, but still relatively less than a full finite element modeling in general. The equations of motion are usually derived from the Newton-Euler equation or the Lagrange's equation.

Finite element model The finite element analysis discretizes the wind turbine structure into a mesh of finite elements that are interconnected at nodes, whereby the ordinary and partial differential equations of motion for each element are then numerically solved. The elements can be modeled as linear, or even higher order one-dimensional lines, two-dimensional surfaces, or three-dimensional solids. Besides, it also supports material non-linearity, which is important in soil-interaction or mooring line modeling. Nevertheless, this method requires very high computational effort and may not be as efficient as multibody approach in modeling the wind turbine's large displacement motions. Therefore, some of the wind turbine modeling codes combine both multibody method and FEM.

Specifically, the gravitational force plays an important design consideration in the turbine blade fatigue checks, particularly for the large wind turbines nowadays. It introduces cyclic shear forces and bending moments on the turbine blades during operation. Besides, it also creates bending moments in both edge-wise and flapwise directions when the turbine blades pitch. Furthermore, the nacelle and rotor weights generally decide the static loading condition for the support structure design and the pile penetration depth. They also determine the stability criteria for floating wind turbines. Finally, the weights of individual wind turbine parts ought to be assessed for safe operation during the transportation, installation and assembly processes.

3.3 Dynamic load analysis

3.3.1 Time- and frequency-domain solvers

In preceding sections, stochastic loading processes due to winds and waves were mentioned. There are two main approaches to solve the dynamic responses of offshore structures. In the frequency-domain, spectral dynamic analysis using the concepts of PDFs and PSDs can be employed. This approach is commonly used in the oil and gas industry and has been extended to the analysis of offshore wind turbines as well. In general, the system dynamics is assumed to be linear while the general equations of motion Eq. 3.13 can be simplified to Eq. 3.14. For a multi-degree-of-freedom (MDOF) system, assuming that $\mathbf{f}(t)$ can be represented as $\mathbf{F}(\omega) \exp(i\omega t)$, the response vector in the frequency-domain $\mathbf{Z}(\omega)$ is solved using

$$\mathbf{Z}(\omega) = \mathbf{H}(\omega) \mathbf{F}(\omega) \quad (3.15)$$

where $\mathbf{F}(\omega)$ denotes the force vector in the frequency-domain and the transfer function matrix $\mathbf{H}(\omega)$ is given as

$$\mathbf{H}(\omega) = \left[-\omega^2 [\mathbf{M}^{\text{sys}}] + i\omega [\mathbf{C}^{\text{sys}}] + [\mathbf{K}^{\text{sys}}] \right]^{-1}. \quad (3.16)$$

The spectral matrix of vector force processes $\mathbf{S}_{\mathbf{F}}(\omega)$ is often used to describe the stochastic loading processes and contains information about the variance and cross spectra of all component processes for the forces. The spectral matrix for the vector response processes $\mathbf{S}_{\mathbf{X}}(\omega)$ can then be calculated by

$$\mathbf{S}_{\mathbf{X}}(\omega) = \mathbf{H}(\omega) \mathbf{S}_{\mathbf{F}}(\omega) \mathbf{H}(\omega)^T. \quad (3.17)$$

For details about the spectral dynamic analysis and the methodologies to estimate $\mathbf{S}_{\mathbf{F}}(\omega)$ and $\mathbf{H}(\omega)$ for wind and wave action, see Naess and Moan (2013) and Barltrop and Adams (2013). The response spectrum $\mathbf{S}_{\mathbf{X}}(\omega)$ in Eq.

3.17 is commonly referred to as the response spectrum of internal forces or internal stresses in the members, which are then used in the code checks.

The linear spectral analysis method can be very efficient as compared with the time-domain approach in solving the dynamic problems. However, the drawback is that the method is unable to incorporate transient events or non-linear dynamics in the analysis directly (Cordle, 2010). On the load dynamics, the load spectra can be tricky and time-consuming to estimate, since strictly speaking the rotor loads are cyclo-stationary and depend on the rotor azimuth while the wave loads such as Morison's forces are stationary but non-Gaussian. Linearization of force terms (e.g., viscous drag) for example is commonly adopted to simplify the load dynamics. On the structural dynamics, large structural deformations and geometrical updating of the model are also difficult to implement (Karimirad, 2012). The couplings between support structures and rotors are missed out and the analysis is carried out separately subjected to wind and wave loads (Matha, 2009). Furthermore, the bandwidth of aerodynamic responses is very wide and the spectral methods used for fatigue analysis tend to be very conservative and overestimate the fatigue damages.

Another approach to solve the dynamic problem is by utilizing the time-domain method. The method is useful for general response analysis as it is capable of modeling non-linearity effects in the formulation, such as the material non-linearity, the geometry non-linearity and the contact non-linearity. The force vector $\mathbf{f}^{\text{sys}}(t)$ must be expressed in a deterministic form and is usually realized into time series from the load spectrum.

In structural dynamics, there are several techniques available for the time-domain analysis (see Fig. 3.5) (Barltrop and Adams, 2013). The convolution integral method considers the response at any time to be the summation of responses resulted from a series of impulses representing the preceding force time history. The time stepping method analyzes responses at t_{j+1} considering

Figure 3.5: Different time-domain methods for solving structural dynamics problems

equilibrium either at t_j (explicit method) or t_{j+1} (implicit method), where the equations of motion are solved iteratively for all elements or bodies in order to obtain convergent results in the element kinematics and dynamics. Both methods are generally suitable for analyzing any kind of loading conditions, but the structure has to behave linearly for convolution integral approach because it is derived from the principle of linear superposition.

The Newmark-beta approach is widely used and the finite difference approximation is given as

$$\mathbf{z}(t_{j+1}) \approx \mathbf{z}(t_j) + \delta t (\dot{\mathbf{z}}(t_j)) + \delta t^2 \left[\left(\frac{1}{2} - \beta \right) \ddot{\mathbf{z}}(t_j) + \beta \ddot{\mathbf{z}}(t_{j+1}) \right] \text{ and} \quad (3.18)$$

$$\dot{\mathbf{z}}(t_{j+1}) \approx \dot{\mathbf{z}}(t_j) + \delta t [(1 - \gamma) \ddot{\mathbf{z}}(t_j) + \gamma \ddot{\mathbf{z}}(t_{j+1})], \quad (3.19)$$

where δt is the time step, β and γ are the parameters that measure how much of the acceleration at the end of the time interval $\ddot{\mathbf{z}}(t_{j+1})$ enters into the relations

for displacement $\mathbf{z}(t_{j+1})$ and velocity $\dot{\mathbf{z}}(t_{j+1})$, respectively (Newmark, 1959). The approach becomes implicit and unconditionally stable when $\beta = 1/4$ and $\gamma = 1/2$, where the acceleration within the time interval $[t_j, t_{j+1})$ is presumed to be constant. However, this does not guarantee the accuracy of solutions. When $\beta = 1/6$ and $\gamma = 1/2$, the Newmark-beta method is identical to the linear acceleration method. The method can also be combined with the Newton-Raphson iterative algorithm to include non-linear restoring forces, etc.

The second-order ordinary differential equations of motion Eq. 3.14 can be rewritten in the state space representation that is a first-order ordinary differential equation as follows

$$\frac{d}{dt} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -[\mathbf{M}^{\text{sys}}]^{-1} [\mathbf{K}^{\text{sys}}] & -[\mathbf{M}^{\text{sys}}]^{-1} [\mathbf{C}^{\text{sys}}] \end{bmatrix} \begin{bmatrix} \mathbf{z} \\ \dot{\mathbf{z}} \end{bmatrix} + \begin{bmatrix} 0 \\ [\mathbf{M}^{\text{sys}}]^{-1} \end{bmatrix} \mathbf{f}^{\text{sys}}(t) \quad (3.20)$$

The Runge-Kutta methods are a family of solvers that can solve a general system of non-homogeneous first-order ordinary differential equations. The famous ones include the explicit fixed-step fourth-order Runge-Kutta solver and the explicit variable-step fourth-fifth-order Runge-Kutta solver.

The time-domain method is capable of simulating accurate representation of complex OWT dynamics which the frequency domain method fails to do so. However, it demands much higher computational effort due to the numerical integration process as well as the long simulation runs required for realizing the stochastic characteristics embedded within the environmental loads, not to mention the large set of DLCs that has to be carried out. The duration of simulations for each DLC should be at least 1 hour according to IEC standard or at least 3 hours for wave loads according to the recommended practice in offshore engineering. Shorter simulation runs will introduce larger uncertainties to the fatigue lifetime estimate and structural capacity utilization (Zwick and Muskulus, 2015). With the great advancement in computing power nowadays,

the current commercial and research developed wind turbine design tools are based on, if not gearing towards the fully coupled time-domain analysis ([Cordle, 2010](#)).

3.3.2 Decoupled and coupled design approaches

The design practice of OWT support structures is often determined by the commercial and technical constraints of a project, as the RNA-tower and substructure-foundation are typically supplied by the turbine manufacturers and the offshore engineering companies, respectively. The RNA offers a very few number of project-specific adaptations, mainly on the supervisory control and data acquisition or dynamic control devices, which are designed and checked at the beginning of the design process based on preliminary data and later at the end based on project design data ([Fischer et al., 2012](#)). Therefore, the main design scopes focus on the support structures, which have to be designed specifically for the given site. There are a variety of design approaches adopted in the industry and they can be classified as follows:

Superposition approach. Couplings between aerodynamic forces and hydrodynamic forces are neglected. This approach is normally implemented in the frequency-domain, where the final responses, i.e., displacements or internal loads, are summed up linearly from dynamic analysis carried out subjected to wind and wave loads separately. Another variation was mentioned in [King et al. \(2013\)](#), where an aero-elastic simulation is performed in the time-domain to calculate the aerodynamic load at the interface point, such as the tower base, while the substructure and foundation are modeled using equivalent stiffness, mass and damping matrices. The reaction force at the interface is commonly represented as the DEL (see [Fig. 3.6](#)). Subsequently, it is fed into a hydro-elastic tool to determine the internal loads for code checks, where in this phase the support structure

Figure 3.6: Superposition approach, adapted from King et al. (2013)

is modeled in details. However, dynamic amplification of responses can occur in this approach because the reaction force applied contains force components that excite at the natural frequencies of the structures, as demonstrated in Ong et al. (2014). In general, the superposition approach generates conservative results since the dynamic coupling that can dampen the responses is absent.

Partial integrated approach. This approach is widely practised in the industry, which considers the dynamic couplings partially or in a simplified manner. One of which is the sequential approach (see Fig. 3.7). The foundation designer provides the turbine supplier with a superelement model that contains equivalent matrices of the foundation and an equivalent wave load time series at the interface. The wave load is studied on a fixed substructure, as the motion of the structure is unknown. The wave load and

superelement are then imported into the aero-elastic tool and a wind-wave dynamic analysis is run to generate the corresponding aerodynamic load at the interface. In this case, the extracted reaction load will contain the aerodynamic damping force. The aerodynamic load is then fed back to the hydro-elastic tool and a coupled analysis is rerun on the flexible substructure, combining with the hydrodynamic load (Seidel et al., 2005). The accuracy of the results depends on how well the superelement can accurately represent the dynamic model used by the foundation designer. This can vary with respect to the substructuring approaches being used, for instance Guyan reduction, Craig-Bampton method, etc., as well as the degree of flexibility being accounted for, such as the number of internal modes (Klerk et al., 2008; Voormeeren et al., 2011). On the other hand, the partial integrated approach can also be carried out in an opposite manner, where the aerodynamic load is first generated on an isolated turbine in fixed position using the aero-elastic tool. The load is then transferred to a hydro-elastic tool and a combined wind-wave dynamic simulation is carried out (Ong et al., 2014). As such, the hydrodynamic damping force is included. Besides, the aerodynamic damping can be modeled as a separate damper acting on the hub or tower top. The aerodynamic damping can be estimated using the method in Salzmann and Van Der Tempel (2005). Kaufer et al. (2009) further extended the sequential method to be repeated at each time step, by using the dynamic link library to transfer kinematic and force information between the aero-elastic and hydro-elastic modules. This method is almost similar to the fully integrated approach.

Fully integrated approach This approach is able to capture dynamic couplings between the aerodynamic and hydrodynamic loads by simulating the entire OWT model using an integrated aero-hydro-servo-elastic tool.

Figure 3.7: Sequential partial integrated approach

The wind and wave loads are applied to the finite element model simultaneously when solving the equations of motion in the time-domain (see Fig. 3.8). As a result, this approach gives the most accurate representation of the OWT dynamics. Some of the current state-of-the-art tools that provide such simulation capabilities include Bladed by DNV GL, FEDEM Windpower by FEDEM AS, FAST by NREL, etc. These tools were validated and compared against each other for applications to different kinds of support structure concepts in the Offshore Code Comparison Collaboration (OC3) and Offshore Code Comparison Collaboration Continuation (OC4) exercises (Passon et al., 2007; Jonkman and Musial, 2010; Jonkman et al., 2012)

Figure 3.8: Fully integrated approach

3.4 Limit state design of offshore wind turbine structures

There are different design philosophies and design methods that can be applied for structural design, including limit state design (LSD), allowable stress design (ASD), design assisted by testing and probability-based design. The LSD methodology was adopted in this thesis in accordance with the offshore standard DNV-OS-J101 (DNV, 2011b). The standard applies load and resistance factors to determine the design load effect and the design resistance for each limit state, while qualifying for the design to the normal safety class with a nominal annual probability of failure of 10^{-4} . Therefore, LSD is also known as load and resistance factor design (LRFD).

3.4.1 Ultimate limit states

Structural design in the ULS evaluates the ultimate strength capacity (yield and buckling) of all structural components against excessive yielding and buckling.

The check is important to ensure that the structure can operate safely under the extreme environmental loading conditions, where the local peak stress can cause profound geometrical changes and structural failure. It is fundamentally assumed that the failure modes are sufficiently ductile such that the structures will behave according to the anticipated model used for the response analysis. Besides, in the case of linear elastic structural analysis, any cyclic yielding deformation is assumed shaken down without further checks when combined with the resistance formulations set down in this standard. The checks are also performed based on the net undeformed sectional properties. In general, the ULS assessment criteria vary with respect to the loading conditions and the types of structure. Under the complicated environmental conditions, offshore structures are potentially exposed to any combination of axial tension, axial compression, bending, shear and hydrostatic loads. The following lists some of the design codes and recommended practices that are generally used within the offshore wind industry when performing the ULS assessments.

NORSOK N-004. This standard covers detailed ULS assessment criteria for tubular members, tubular joints and conical transitions ([Standards Norway, 2013](#)).

DNV-RP-C202. This recommended practice provides detailed guidelines for studying buckling stability of shell structures, whereby the assessment of interaction between shell buckling and column buckling under the effect of external pressure is applicable to tubular structures ([DNV, 2010a](#)).

API RP 2A-LRFD or ISO 19902. Both codes can provide recommended design criteria for fixed offshore structures, but are often used for the specific provisions on the design of steel cylindrical (pipe) pile foundations. Note: [American Petroleum Institute \(1993\)](#) has been withdrawn and any amendment on the LRFD formulations can be referred to [International Standard](#)

(2007).

EN 1993-1-1 and EN 1993-1-6. The former provides design guidelines on general steel structures, while the latter is devoted to detailed assessment of shell structures (European Standard, 2005b, 2007).

See Section 4.3.2 for structural analysis procedure in ULS adopted in this thesis.

3.4.2 Fatigue limit states

Offshore wind turbine structures are subjected to periodic motion when excited by environmental loads. Hence, fatigue strength analysis has to be carried out to ensure that each structural component possesses adequate resistance against fatigue failure and survives over the intended service life, which is usually 20 years if not specified. The recommended practice DNV-RP-C203 has been widely adopted by the industry to assess fatigue failure of offshore steel structures in the high cycle region, using either S-N data or fracture mechanics (DNV, 2011c). The high cycle fatigue loading is normally understood to exceed 10^4 cycles during the design life, whereas stress responses from wind and wave actions are typically in the order of 10^8 cycles over 20- to 30-year lifetime. The recommended practice prescribes empirical formulae to determine stress concentration factors (SCFs) for different types of joint connections, e.g., tubular joints, non-tubular joints and plated structures, which will be used for calculating the HSS. The corresponding Wöhler curves or S-N curves are developed based upon the mean-minus-two-standard-deviation experimental results obtained from fatigue tests, hence the curves are associated with 97.7 percent probability of survival. All significant stress ranges contributed by the HSS should be considered in the fatigue analysis, whereas the mean stress influences are neglected due to the residual stresses existing in welded joints.

Stress range histograms or stress range PDFs can be estimated using a cycle counting method in the time-domain (e.g., Rainflow counting procedure)

or a spectral method in the frequency-domain (e.g., Dirlik's method), respectively ([Ragan and Manuel, 2007](#)). Albeit being more accurate, a large number of simulations may be required for the time-domain approach to realize the rarely occurred stress cycles and to obtain an adequate resolution for the stress range histograms. On the other hand, the frequency-domain method tends to overestimate the fatigue damage prediction while the stochastic analysis of OWTs can be challenging due to the non-linear rotor dynamics. Both approaches can perform fatigue analysis based on linear accumulation methods such as the Palmgren-Miner's rule to calculate the total accumulated fatigue damage. In addition, fracture mechanics can be used to supplement fatigue analysis carried out by S-N data, particularly in assessing acceptance criteria for defects and fabrication, and planning for in-service inspection. Generally, fatigue life computed using fracture mechanics is shorter than by using S-N data, as the crack initiation phase is not accounted for in the former. The crack propagation and fatigue life can be computed using the Paris Law (see [Anderson \(1995\)](#) for more information).

Besides, fatigue damage can be induced on the piles during the installation process. This ought to be considered in the pile design in addition to the operational fatigue damage explained above. Generally, it is challenging to evaluate pile driving fatigue and their implications in the butt welds, since the effective damage is depending on the residual stresses retained in the weld toes and weld roots ([Chung et al., 2013](#)).

See Section [4.3.2](#) for structural analysis procedure in FLS adopted in this thesis.

3.4.3 Design load cases

The IEC has published the IEC 61400-3 standard providing guidelines to define all aspect of load cases that a wind turbine may experience in its life cycle

and recommends thirty five types of DLCs to be used for wind turbine design analysis. These DLCs comprise different combinations of winds, waves, wind-wave directionality, sea currents, water levels, etc., for simulating various design situations to test the wind turbine structural integrity in ULS or FLS ([IEC, 2009](#)).

1. Power production (DLC 1.1 to 1.6)

This design situation represents the general normal operational case, where the OWT is running with an active control while connected to an electrical load. DLC 1.1 and 1.2 simulate loads resulting from a combined normal atmospheric turbulence (NTM) and normal stochastic sea state (NSS), for fatigue and ultimate limit state analyses, respectively. DLC 1.3 assumes an extreme turbulence condition in ultimate loading analysis. DLC 1.4 and 1.5 embody the transient requirements for ultimate loads when subjected to critical events, i.e., an extreme coherent gust with directional change and an extreme wind shear, respectively. Lastly, DLC 1.6 models ultimate loading analysis caused by a NTM wind and a severe sea state (SSS) or by a severe deterministic design wave (SWH).

2. Power production plus fault condition (DLC 2.1 to 2.4)

This design situation simulates a transient event triggered by a fault or a loss of electrical network connection, while the wind turbine is in normal operation. These faults include the control function or the loss of electrical network connection in DLC 2.1, rare events related to the protection function or internal electrical system failure in DLC 2.2, and the combined significant extreme operating gust (EOG) with internal or external electrical system faults in DLC 2.3. DLC 2.1-2.3 simulate the ultimate loading condition, whereas DLC 2.4 analyzes the fatigue loads on wind turbines during a fault.

3. Start-up (DLC 3.1 to 3.3)

This design situation involves all transient events when the turbine changes from an idling or standstill state to power production. The number of occurrences shall be obtained based on the control system behavior. DLC 3.3. includes the transient change in wind direction during the start-up.

4. Normal shutdown (DLC 4.1 to 4.2)

Contrary to the start-up design situation, this involves all transient events when the turbine transits from a power production state to an idling or standstill condition.

5. Emergency shutdown (DLC 5.1)

This design situation embodies ultimate loading in emergency shutdown during the normal turbulent wind and normal sea state phase.

6. Parked (DLC 6.1 to 6.4)

This design situation covers the important aspects of dynamic loading analysis when the turbine is idling or in standstill. In DLC 6.1-6.3, ultimate loads analysis is carried out subjected to extreme wind and wave conditions of 50 year recurrence period for DLC 6.1 and 6.2 whereas 1 year recurrence for DLC 6.3. DLC 6.4 studies the fatigue loading condition. In addition, the wind-wave misalignment and the turbine yaw misalignment ought to be considered during the load analysis.

7. Parked plus fault condition (DLC 7.1 to 7.2)

This design situation includes the deviated behavior of a parked wind turbine, due to faults on the electrical network or the wind turbine itself.

8. Transport, assembly, maintenance, and repair (DLC 8.1 to 8.3)

This design situation states the loading analysis requirements during the processes of transport, assembly on site, maintenance and repair of OWTs.

The maximum wind and wave conditions that can occur in these processes should be applied with sufficient safety margins in DLC 8.1 analysis. Besides, the impact studies on support structures by service vessels as well as loads on wind turbines during helicopter operation are also carried out in DLC 8.1. DLC 8.2 simulates these operations under the extreme sea states, whereby they can last for longer than a week while the turbines are partially or fully deployed at this stage without electrical network connection. DLC 8.3 complement the earlier analysis by carrying out fatigue analysis, considering the expected number of hours for non-power production during the construction prior to the connection to the electrical grid for each wind speed and sea state.

Chapter 4

An integrated analysis and optimization framework

In this chapter, the integrated analysis and optimization tool developed for design of OWT support structures is explained. The general framework is shown in Fig. 4.1.

The in-house tool was developed in Matlab (Ver. 2014b, The Mathworks, Inc.)¹ and consists of three main modules. In the FEM module, the entire OWT system and its inter-structural interaction with the offshore environment are modeled using the FEM method. The aerodynamic force vector $\mathbf{f}^A$ and aerodynamic damping ratio ζ^A are calculated using external aero-servo-elastic software, whereas the modeling of structural dynamics, hydrostatics, hydrodynamics and soil-pile interaction are performed within the in-house code (see Section 4.1).

In the linear dynamics and failure analysis module, structural responses and internal forces are computed for the OWT system by utilizing the in-house static and dynamic solvers, in order to evaluate the structural performance according to the limit state functions prescribed by design standards and recommended practices (see Section 4.2). Both modules form a standalone modeling and analysis tool for OWTs, which can also be integrated into the sensitivity analysis and optimization module when performing optimization of OWT support structures.

¹www.mathworks.com

Gradient vectors of the objective function ∇f and constraint functions ∇g_i are calculated in the design sensitivity analysis from the system matrices, force vectors and system responses derived in the FEM and liner dynamics modules, while the optimizer searching for the best direction for improvement in the successive iterations until the final design solution for $\mathbf{b}_{\text{final}}$ converged (see Section 4.3).

4.1 Finite element modeling module

The in-house FEM module simulates the OWT system as a linear-hydro-elastic finite element (FE) model acted with a decoupled aero-servo loading. The OWT system comprises five structural parts, namely RNAs, towers, transition pieces, substructures and piles. The FE models for individual parts can be described using matrices and vectors generated according to the relevant inherent dynamics, as shown in Fig. 4.2. The details of the FE approach and implementation with all element matrices and vectors are explained in the next sections. All matrices and vectors are eventually assembled to form the global system matrices and vectors before proceeding to the [Linear dynamics and failure analysis module](#). See [Cook \(1995\)](#) for detailed explanation on the FEM.

Figure 4.2: Workflow of the finite element modeling module. $[M^{sys}]$: system mass matrix; $[K^{sys}]$: system stiffness matrix; $[C^{sys}]$: system damping matrix; f_{sta}^{sys} : system static force vector; and f_{dyn}^{sys} : system dynamic force vector.

4.1.1 FEM of structural parts

All parts within the OWT system are basically formed by three-dimensional uniform Euler-Bernoulli (EB) beams that support axial (1 translational DOF), bending (2 translational and 2 rotational DOF) and St. Venant torsional deformations (1 rotational DOF). The element stiffness matrix is formulated for each deformation in the strong sense, followed by conversion to the weak form and approximation of solution through finite element discretization. The element stiffness matrix $[\mathbf{K}_{\text{EB}}]$ of an EB beam is given by [Przemieniecki \(1985\)](#) as a function of the cross-sectional area A , second moment of area in local x-, y- and z-directions, J_x , I_y and I_z , length of the beam L , shear modulus G and Young's modulus E , i.e.,

$$[\mathbf{K}_{\text{EB}}] = \begin{bmatrix} \frac{EA}{L} & & & & & & & & & & & \\ 0 & \frac{12EI_z}{L^3} & & & & & & & & & & \\ 0 & 0 & \frac{12EI_y}{L^3} & & & & & & & & & \\ 0 & 0 & 0 & \frac{GJ_x}{L} & & & & & & & & \\ 0 & 0 & -\frac{6EI_y}{L^3} & 0 & \frac{4EI_y}{L} & & & & & & & \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & & & & & & \\ -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & & & & & \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & & & & \\ 0 & 0 & -\frac{12EI_y}{L^3} & 0 & \frac{6EI_y}{L^2} & 0 & 0 & 0 & \frac{12EI_y}{L^3} & & & \\ 0 & 0 & 0 & -\frac{GJ_x}{L} & 0 & 0 & 0 & 0 & 0 & \frac{GJ_x}{L} & & \\ 0 & 0 & -\frac{6EI_y}{L^2} & 0 & \frac{2EI_y}{L} & 0 & 0 & 0 & \frac{6EI_y}{L^2} & 0 & \frac{4EI_y}{L} & \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix} \quad \text{Sym.} \quad (4.1)$$

Meanwhile, the element mass matrix $[\mathbf{M}_{\text{EB}}]$ of an EB beam is given by

[illegible]

where ρ_b is the material density of the beam. Equations 4.1 and 4.2 assume that the ratios of radii of gyration to element lengths are small in comparison with unity, and are applicable to slender beams that neglect the effect of shear deformations. In addition, marine growth surrounding the tubular beams as well as internal fluid for flooded beams can be incorporated into $[\mathbf{M}_{\mathbf{EB}}]$ by modifying the density ρ_b to $\rho_{b,mg}$ and $\rho_{b,if}$, respectively, which are given by

$$\rho_{b,mg} = \rho_b + \rho_{mg} \frac{\pi T_{mg} (D + T_{mg})}{A} \frac{L_{mg}}{L} \quad (4.3)$$

and

$$\rho_{b,if} = \rho_b + \rho_w \frac{\pi (D - 2T)^2}{4A} \frac{L_{if}}{L} \quad (4.4)$$

where D and T are the outer diameter and thickness of the beam; T_{mg} and ρ_{mg} are the thickness and density of the marine growth; ρ_w is the seawater density; and L_{mg} and L_{if} are the length portion of the beam that is covered with marine growth and flooded with the seawater, respectively. Besides, internal fluid also introduces an additional static gravity load that cancels out the hydrostatic force acting on the submerged beam (see Section 4.1.2), whereas the marine growth influences not only the gravitational and buoyant loads but also the hydrodynamic loads (see Section 4.1.4). Nevertheless, the stiffness of the beam element that associates with the restoring strength is not affected by internal fluid or marine growth (DNV, 2010b).

Equations 4.1 and 4.2 are generalized for uniform beams with any cross-sectional design. They can be applied readily in modeling of support structures. However, as for wind turbine blades, since the mechanical properties such as stiffness that correspond to the axial, edgewise bending, flapwise bending and torsional modes, and mass density vary along the blade with respect to the aerofoil design, the EA , EI_y , EI_z , GJ_x in Eq. 4.1 and the unit length mass m/L have to be specified for each section of blades.

Moreover, Rayleigh damping is included for all structural parts by superimposing the mass- and stiffness-proportional damping. The structural damping matrix $[\mathbf{C}^S]$ is given by

$$[\mathbf{C}^S] = a_0 [\mathbf{M}^S] + a_1 [\mathbf{K}^S] \quad (4.5)$$

where a_0 and a_1 are the constants of proportionality to the structural mass matrix $[\mathbf{M}^S]$ and the structural stiffness matrix $[\mathbf{K}^S]$, respectively. Typically, the structures are designed to attain specific target damping ratios ζ_i for the i -th structural eigenfrequencies ω_i . Hence, a_0 and a_1 can be solved using

$$\frac{1}{2} \begin{bmatrix} \frac{1}{\omega_i} & \omega_i \\ \frac{1}{\omega_j} & \omega_j \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \end{Bmatrix} = \begin{Bmatrix} \zeta_i \\ \zeta_j \end{Bmatrix}. \quad (4.6)$$

4.1.2 FEM of gravitational and hydrostatic forces

Loads are applied to the beam element, either as nodal forces (e.g., point loads), body forces (e.g., weight or buoyancy), or distributed external loads (e.g., wave forces). In the FE model, these forces are represented as equivalent nodal forces $\mathbf{f}_{EB}$, by using

$$\begin{aligned} \mathbf{f}_{EB} &= \left\{ F_1 \ F_2 \ F_3 \ M_4 \ M_5 \ M_6 \ F_7 \ F_8 \ F_9 \ M_{10} \ M_{11} \ M_{12} \right\}^T \\ &= \int_0^L [\mathbf{N}]^T \begin{Bmatrix} q_x \\ q_y \\ q_z \\ m_x \end{Bmatrix} dx + \left[[\mathbf{N}]^T \begin{Bmatrix} -N_x \\ V_z \\ V_y \\ -M_x \end{Bmatrix} + [\mathbf{B}]^T \begin{Bmatrix} 0 \\ -M_y \\ -M_z \\ 0 \end{Bmatrix} \right]_0^L \end{aligned} \quad (4.7)$$

where $[\mathbf{N}]$ and $[\mathbf{B}]$ are the shape function matrix and its derivative against x , i.e., the longitudinal length along the beam axis (see Appendix A.1); N_x , V_y ,

V_z , M_x , M_y and M_z denote the nodal axial force applied in the x-direction, the nodal shear forces applied in the local y- and z-directions, and the nodal torsion and bending moments applied in the yz-, xz- and xy-planes, respectively; and q_x , q_y , q_z and m_x are the distributed axial load exerted in the local x-direction, the distributed normal loads applied in the y- and z-directions, and the distributed twisting load, respectively.

For an OWT system, gravitational forces are experienced by all structural members while hydrostatic forces can be computed as the weight of displaced liquid for structural parts that are submerged in water. The unit length weight $q_{i=x,y,z}^g$ and the unit length buoyancy $q_{i=x,y,z}^b$ in the local x-, y- and z-directions are

$$q_{i=x,y,z}^g = \rho_b Ag |\hat{\mathbf{g}} \cdot \hat{\mathbf{a}}_{i=x,y,z}| \quad (4.8)$$

and

$$q_{i=x,y,z}^b = \rho_w Ag |-\hat{\mathbf{g}} \cdot \hat{\mathbf{a}}_{i=x,y,z}| \quad (4.9)$$

where $\hat{\mathbf{g}}$ is the unit gravitational vector applied in the global Z-direction and $\hat{\mathbf{a}}_{i=x,y,z}$ is the unit vector applied in the local beam directions: x- refers to the longitudinal direction, y- and z- refer to the transverse directions. The effective beam mass density ρ_b can be corrected to account for internal fluid and marine growth (see Section 4.1.1). By substituting Eqs. 4.8 and 4.9 into Eq. 4.7, the gravitational force vector $\mathbf{f}^G$ and the buoyancy vector $\mathbf{f}^B$ for individual EB beam can be determined.

4.1.3 FEM of aerodynamic forces

The aero-servo dynamics is implemented as a decoupled loading acting onto the linear hydro-elastic OWT model. It can be separated into aero-servo loads and aerodynamic damping (see Fig. 4.3).

The decoupled aero-servo force can be obtained from any aero-servo-

elastic code. In this thesis, it was calculated externally using FEDEM Windpower Version R7.1.1² developed by FEDEM Technology AS, through a co-simulation with AeroDyn and TurbSim by National Renewable Energy Laboratory (NREL). FEDEM Windpower is an integrated multibody-FEM solver that supports control modeling (Fedem Technology AS, 2014b). Aerodyn calculates the aerodynamic forces using either the full Blade Element Momentum (BEM) or the Generalized Dynamic Wake (GDW) methods that account for tip or hub loss corrections (Laino and Hansen, 2002; Moriarty and Hansen, 2005). TurbSim is the pre-processing tool used to generate the turbulent wind input files (Jonkman, 2009). Simulations under various wind states as specified in the DLCs were performed on a numerical wind turbine model which is clamped at the yaw node, in order to exclude the flexibility of the support structure from the simulations. After subtracting the baseline results modeled in a still wind condition, forces and moments in six DOF experienced at the hub node were extracted and imported into the in-house FEM module as the aerodynamic force vector $\mathbf{f}^A$ (see Fig. 4.1).

The aerodynamic damping is included as a linear damper applied onto the hub node or the tower top node. Non-linear time-domain simulations are carried out for each wind speed within the DLCs, where the full OWT system is subjected to a pulse load while simulated under the turbulent wind fields. Then, the free decay vibration of the hub motions is measured and the system damping ratio ζ^{sys} is assessed using

$$\Lambda = \frac{1}{n} \ln \left(\frac{u_i}{u_{i+n}} \right) \quad (4.10)$$

and

$$\zeta^{sys} = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}} \quad (4.11)$$

²<http://www.fedem.com/>

Figure 4.3: (a) Decoupled aero-servo loads and (b) linear aerodynamic damping model applied onto the hub node

where Λ is the logarithmic decrement and u_i is the i -th peak response of the tower top motion. The linear aerodynamic damping ratio ζ^A can be estimated by subtracting the overall structural damping ratio ζ^S from ζ^{sys} , i.e.,

$$\zeta^A = \zeta^{sys} - \zeta^S, \quad (4.12)$$

as described in [Salzmann and Van Der Tempel \(2005\)](#). The aerodynamic damping matrix $[\mathbf{C}^A]$ is formed by calculating the aerodynamic damping coefficient c^A for the fore-aft DOF of the hub node as

$$c^A = \frac{2\zeta^A K_{modal}}{\omega_1} \quad (4.13)$$

where K_{modal} is the modal spring constant and ω_1 is the first mode fore-aft eigenfrequency. The K_{modal} can be determined from $\frac{F}{u(\mathbf{b})}$, when applying an arbitrary horizontal force within the elastic limit F onto the hub node and obtaining the corresponding horizontal static deflection $u(\mathbf{b})$ using the FEM code.

4.1.4 FEM of hydrodynamic forces

For offshore structural members that are slender and satisfy $D < \frac{\lambda}{5}$, Morison's load formula is applicable (see Section 3.2.2). Morison's load equation is given by

$$d\mathbf{f}_{\text{Morison}} = \left[\frac{\pi\rho_w D^2}{4} (C_M \dot{\mathbf{v}} - C_A \ddot{\mathbf{z}}) + \frac{\rho_w D}{2} C_D (\mathbf{v} - \dot{\mathbf{z}}) |\mathbf{v} - \dot{\mathbf{z}}| \right] dL \quad (4.14)$$

where C_M is the inertial coefficient ($C_M = 1 + C_A$); C_A is the added mass coefficient; C_D is the drag coefficient; ρ_w is water density; and $\mathbf{v}_w$ and $\dot{\mathbf{v}}_w$ are the water particle velocity and acceleration, respectively (Morison et al., 1950). The term $-\frac{\pi\rho_w D^2 dL}{4} C_A \ddot{\mathbf{z}}$ contributes to the hydrodynamic added mass. For a three-dimensional EB beam, the hydrodynamic added mass matrix $[\mathbf{M}^H]$ is given by

$$[\mathbf{M}^H] = \frac{\pi\rho_w D^2 C_A}{4} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13L}{32} & 0 & 0 & \frac{3L}{32} & 0 \\ 0 & 0 & \frac{13L}{32} & 0 & 0 & \frac{3L}{32} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{11L^2}{192} & 12 & 0 & 0 & -\frac{5L^2}{192} & 12 \\ 0 & \frac{11L^2}{192} & 0 & \times & 0 & \frac{5L^2}{192} & 0 & \times \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 \\ 0 & \frac{3L}{32} & 0 & \text{null} & 0 & \frac{13L}{32} & 0 & \text{null} \\ 0 & 0 & \frac{3L}{32} & 0 & 0 & 0 & \frac{13L}{32} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{5L^2}{192} & 0 & 0 & 0 & \frac{11L^2}{192} & 0 \\ 0 & -\frac{5L^2}{192} & 0 & 0 & -\frac{11L^2}{192} & 0 & 0 & 0 \end{bmatrix} \quad (4.15)$$

where D and L are the effective diameter and length of the member. Secondly, the quadratic drag force $\frac{\rho_w D dL}{2} C_D (\mathbf{v} - \dot{\mathbf{z}}) |\mathbf{v} - \dot{\mathbf{z}}|$ can be expanded into $-\left[\mathbf{C}^H\right] \dot{\mathbf{z}} + \mathbf{f}_D$ by assuming that $\dot{\mathbf{z}} \ll \mathbf{v}_w$, where $[\mathbf{C}^H]$ is the linearized hydro-

dynamic damping matrix and $\mathbf{f}_D$ is the non-linear hydrodynamic drag force for fixed structures. $[\mathbf{C}^H]$ is given by

$$[\mathbf{C}^H] = \frac{\rho_w D C_D}{2} \cdot \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{13L}{32} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{13L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{3L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} \\ 0 & \frac{13L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{13L}{32} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{3L}{32} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{11L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & -\frac{11L^2}{192} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & 12 & -\frac{5L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & -\frac{5L^2}{192} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & 12 \\ 0 & \frac{11L^2}{192} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{11L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \times & \frac{5L^2}{192} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{5L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \times \\ 0 & 0 & 0 & 4 & 0 & 0 & 3 \\ 0 & \frac{3L}{32} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \text{null} & \frac{13L}{32} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{13L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \text{null} \\ 0 & \frac{3L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & & \frac{13L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{13L}{32} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & \\ 0 & 0 & 0 & & 0 & 0 & \\ 0 & \frac{5L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{5L^2}{192} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & & \frac{11L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{11L^2}{192} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & \\ 0 & -\frac{5L^2}{192} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & -\frac{5L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & & -\frac{11L^2}{192} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & -\frac{11L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \end{bmatrix} \quad (4.16)$$

where v_2 and v_3 denote the water particle velocities in the local y- and z-coordinates for end node 1 of the beam while v_8 and v_9 are for end node 2.

Thirdly, the remaining FK force $\frac{\pi \rho_w D^2 dL}{4} C_M \dot{\mathbf{v}}$ and $\mathbf{f}_D$ form the external hydrodynamic force applied onto fixed structures $\mathbf{f}^H$. Since wave varies randomly across the beam element, the distribution of wave kinematics can be better represented when more data points are used for the integration of beam equivalent nodal forces. In this thesis, the procedures below are followed for the calculation of the force vector $\mathbf{f}^H$:

1. The wave kinematics u , w , $\dot{u}$ and $\dot{w}$ are calculated in the global coordinates

for each integration point that are distributed equally across the beam element based on the number of pseudo-section N defined, using Eqs. 3.3-3.6.

2. The global wave kinematics are then decomposed to the local y-and z-axes that are perpendicular to the beam axis, for each integration point.
3. The unit length hydrodynamic force $\mathbf{q}^{\mathbf{H}}$ are calculated using

$$\mathbf{q}^{\mathbf{H}} = \frac{\pi \rho_w D^2 C_M}{2} \begin{Bmatrix} 0 \\ \dot{v}_2 \\ \dot{v}_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \frac{\rho_w D C_D}{2} (v_2^2 + v_3^2) \begin{Bmatrix} 0 \\ \frac{v_2}{\sqrt{v_2^2 + v_3^2}} \\ \frac{v_3}{\sqrt{v_2^2 + v_3^2}} \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.17)$$

where $\dot{v}_2$ and $\dot{v}_3$ are the water particle accelerations in the local y-and z-coordinates for end node 1.

4. Subsequently, the nodal equivalent hydrodynamic force vector $\mathbf{f}^{\mathbf{H}}$ is obtained through integrating $\mathbf{q}^{\mathbf{H}}$ along the beam element, i.e.,

$$\begin{aligned} \mathbf{f}^{\mathbf{H}} = & \int_0^L [\mathbf{N}_{2-3}]^T \begin{bmatrix} \delta(x) & \cdots & \delta\left(x - \frac{(2n-1)L}{2N}\right) & \cdots & \delta\left(x - \frac{(2N-1)L}{2N}\right) \\ 0 & \cdots & 0 & \cdots & 0 \end{bmatrix} dx \cdot \\ & \left\{ q_{y,0}^H \cdots q_{y,n}^H - q_{y,n-1}^H \cdots q_{y,N}^H - q_{y,N-1}^H \right\}^T + \\ & \int_0^L [\mathbf{N}_{2-3}]^T \begin{bmatrix} 0 & \cdots & 0 & \cdots & 0 \\ \delta(x) & \cdots & \delta\left(x - \frac{(2n-1)L}{2N}\right) & \cdots & \delta\left(x - \frac{(2N-1)L}{2N}\right) \end{bmatrix} dx \cdot \\ & \left\{ q_{z,0}^H \cdots q_{z,n}^H - q_{z,n-1}^H \cdots q_{z,N}^H - q_{z,N-1}^H \right\}^T \end{aligned} \quad (4.18)$$

where $\delta(x - x_i)$ is the heaviside function while q_y^H and q_z^H are the unit length hydrodynamic force applied in the local y- and z-directions that

correspond to the second and third row vectors in Eq. 4.17, respectively.

Refer to Appendix A.2 for the detailed derivation of the Eqs. 4.15-4.18.

4.1.5 FEM of pile-soil stiffness

When modeling the pile-soil interaction, the following assumptions are made:

1. The soil damping is not included in the modeling.
2. The soil inertia is negligible in the modeling.

According to the conventional design practice adopted by the industry, global dynamic analysis of OWT structures with consideration of the pile-soil interaction can be performed by modeling the foundation as a series of non-linear springs applied onto the pile nodes. Generally, the soil resistance-pile displacement relationships at any level on a pile are independent from the deflections at any other levels, i.e., no interaction between the strata of the soil. The foundation follows the $p - y$ and $t - z$ curves for lateral bearing and axial shaft friction, respectively, that vary along the pile and the $Q - z$ curve for axial end bearing at pile tips. The $p - y$ curve mainly determines the dynamic characteristics of the OWTs due to lateral environmental loading while the $t - z$ and $Q - z$ curves provide resistance against dead loads.

In this thesis, the resistance-displacement relations were linearized from the non-linear curves prescribed in API RP 2A ([American Petroleum Institute, 1993](#)). The foundation secant stiffness matrix $[\mathbf{K}^P]$ was derived using the nodal equivalent force vector on beams, treating the soil resistance as distributed loads applied on the pile instead of point loads on the nodes (see Appendix A.3 for detailed derivation). This method was first studied by [Griffiths \(1989\)](#) where the “consistent soil stiffness” was assumed to be distributed uniformly or linearly-varying along the pile depth, in comparison with the “lumped soil stiffness” such as the Winkler model. The formulation is further extended here to combine

with the linearized API RP 2A resistance-displacement curves. The $[\mathbf{K}^P]$ for an unplugged vertical cylindrical pile element is therefore given by

$$[\mathbf{K}^P] = \begin{bmatrix} \frac{3L}{4} K_{x_1}^{t-z} \pi (D - T) & 0 & 0 & \frac{L}{4} K_{x_2}^{t-z} \pi (D - T) & 0 & 0 \\ 0 & \frac{13L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{3L}{32} K_{x_2}^{p-y} & 0 \\ 0 & 0 & \frac{13L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{3L}{32} K_{x_2}^{p-y} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{11L^2}{192} K_{x_1}^{p-y} & 0 & 0 & -\frac{5L^2}{192} K_{x_2}^{p-y} \\ 0 & \frac{11L^2}{192} K_{x_1}^{p-y} & 0 & 0 & \frac{5L^2}{192} K_{x_2}^{p-y} & 0 \\ \frac{L}{4} K_{x_1}^{t-z} \pi (D - T) & 0 & 0 & \frac{3L}{4} K_{x_2}^{t-z} \pi (D - T) & 0 & 0 \\ 0 & \frac{3L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{13L}{32} K_{x_2}^{p-y} & 0 \\ 0 & 0 & \frac{3L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{13L}{32} K_{x_2}^{p-y} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{5L^2}{192} K_{x_1}^{p-y} & \frac{5L^2}{192} K_{x_1}^{p-y} & 0 & -\frac{11L^2}{192} K_{x_2}^{p-y} & 0 \end{bmatrix} \quad (4.19)$$

where $K_{x_1}^{t-z}$ and $K_{x_2}^{t-z}$ are the secant stiffness for axial shaft friction calculated at node 1 and node 2 of the beam; $K_{x_1}^{p-y}$ and $K_{x_2}^{p-y}$ are the secant stiffness for lateral bearing force calculated at node 1 and node 2; D , T and L are the outer diameter, thickness and length of the pile. For pile tips, the secant stiffness for tip bearing force K^{Q-z} has to be added into $[\mathbf{K}^P]$ corresponding to the node number. Unlike the Winkler soil model, Eq. 4.19 contains soil stiffness terms that link one pile to another and also captures the couplings between lateral forces and bending moments due to pile deflection. The plastic deformation of soil is not mobilized in the secant stiffness method. As a result, the pile deflections should be kept within the reasonable limits by imposing additional design constraints (see Section 4.3.2) so as to reduce errors due to the linearization of foundation stiffness.

In the complete dynamic analysis, a single linearization is normally used for all wind-wave load cases by employing the most sensible case that causes the most fatigue damage. The general procedure adopted in this thesis for the tabulation of $[\mathbf{K}^P]$ matrix is as follows:

1. Depending on the depths of the pile nodes, the stratum at which the pile element lies is identified and the soil properties are determined.
2. The K^{t-z} and K^{Q-z} are determined from the equations below based on the soil type,

$$K^{t-z} = \begin{cases} \frac{100t_{max}}{D} & \text{for clay} \\ 393.7t_{max} & \text{for sand} \end{cases} \quad (4.20)$$

and

$$K^{Q-z} = \frac{10Q_p}{D} \quad (4.21)$$

where t_{max} is the maximum unit skin friction capacity and Q_p is the total end bearing capacity. Both can be determined from [American Petroleum Institute \(1993\)](#).

3. The K^{p-y} is determined from the equations below based on the soil type,

$$K^{p-y} = \begin{cases} \frac{0.24p_u}{y_c} & \text{for clay} \\ \frac{k_0 d_s}{\cosh^2\left(\frac{k_0 d_s y_0}{A_l p_u}\right)} & \text{for sand} \end{cases} \quad (4.22)$$

where p_u is the ultimate lateral bearing capacity at soil depth d_s ; y_c is the displacement corresponding to the strain that occurs at one-half the maximum stress on laboratory unconsolidated undrained compression tests of the soil; and k_0 is the initial modulus of subgrade reaction; and A_l is the loading factor. For sand, when $y_0 = 0$, the linearized $p - y$ curve generated is basically the tangent stiffness taken at the neutral position, i.e., $K^{p-y} = k_0 d_s$ that theoretically gives an over-stiff foundation property. The approximation of K^{p-y} can be improved by subsequently utilizing $y_0 = \frac{y_{max}}{2}$, after obtaining the maximum lateral displacements y_{max} from an initial simulation that applies $K^{p-y} = k_0 d_s$.

4. Substitute K^{t-z} , K^{Q-z} and K^{p-y} into Eq. 4.19.

4.1.6 Global transformation of FE matrices and vectors

In the previous sections, matrices and vectors are generated in the local coordinate systems for individual beams. They need to be transformed to the global reference frame before the assembly of FE matrices can be performed. A transformation matrix $[\mathbf{T}_g]$ allows a vector to be transformed from one coordinate system to another. Given a beam element in the local xyz-frame, its element displacement vector $\mathbf{z}_{el}$ and element stiffness matrix $[\mathbf{K}_{el}]$ are related to the corresponding $\mathbf{z}_{eg}$ and $[\mathbf{K}_{eg}]$ in the global XYZ-frame as

$$\mathbf{z}_{el} = [\mathbf{T}_g]^T \mathbf{z}_{eg} \quad (4.23)$$

and

$$[\mathbf{K}_{eg}] = [\mathbf{T}_g] [\mathbf{K}_{el}] [\mathbf{T}_g]^T. \quad (4.24)$$

Similarly, Eq. 4.23 is applicable to the element force vector $\mathbf{f}_{el}$ while Eq. 4.24 can be used for the element mass matrix $[\mathbf{M}_{el}]$ or the element damping matrix $[\mathbf{C}_{el}]$. The full transformation matrix $[\mathbf{T}_g]$ for a beam element is given by

$$[\mathbf{T}_g] = \begin{bmatrix} [\mathbf{T}] & 0 & 0 & 0 \\ 0 & [\mathbf{T}] & 0 & 0 \\ 0 & 0 & [\mathbf{T}] & 0 \\ 0 & 0 & 0 & [\mathbf{T}] \end{bmatrix} \quad (4.25)$$

where $[\mathbf{T}] = \begin{bmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \end{bmatrix}$; and $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$ and $\hat{\mathbf{a}}_z$ are the unit vectors describing the local xyz-coordinate system for the beam element.

Then, the matrices and vectors for individual beams in the global coordinates are assembled to form the system matrices and vectors, following the process as depicted in Fig. 4.2.

4.2 Linear dynamics and failure analysis module

In this module, the system matrices and vectors generated in FEM module are processed to obtain the response outputs as well as to check for design performance, as illustrated in Fig. 4.4. Failure analysis of individual structural components is carried out based on code checking procedures of ULS (tubular or shell structures) and FLS assessment. Together with the response outputs, the limit state functions form the design constraints of the optimization problem, which are then inputted to [Sensitivity analysis and optimization module](#).

When solving the response dynamics, Eq. 3.14 is broken down into the static and dynamic problems, i.e.,

Figure 4.4: Workflow of the linear dynamics and failure analysis module

$$[\mathbf{K}^{sys}] \mathbf{z}_{sta} - \mathbf{f}_{sta}^{sys} = \mathbf{0} \quad (4.26)$$

and

$$\begin{aligned} [\mathbf{M}^{sys}] \ddot{\mathbf{z}}_{dyn}(t_j) + [\mathbf{C}^{sys}] \dot{\mathbf{z}}_{dyn}(t_j) + \\ [\mathbf{K}^{sys}] \mathbf{z}_{dyn}(t_j) - \mathbf{f}_{dyn}^{sys}(t_j) = \mathbf{0} \end{aligned} \quad (4.27)$$

where $[\mathbf{M}^{sys}]$ is the system mass matrix given by $[\mathbf{M}^{sys}] = [\mathbf{M}^S] + [\mathbf{M}^H]$; $[\mathbf{C}^{sys}]$ is the system damping matrix given by $[\mathbf{C}^{sys}] = [\mathbf{C}^S] + [\mathbf{C}^A] + [\mathbf{C}^H]$; $[\mathbf{K}^{sys}]$ is the system stiffness matrix given by $[\mathbf{K}^{sys}] = [\mathbf{K}^S] + [\mathbf{K}^P]$; $\mathbf{f}_{sta}^{sys}$ is the system static force vector given by $\mathbf{f}_{sta}^{sys} = \mathbf{f}^G + \mathbf{f}^B$ and $\mathbf{f}_{dyn}^{sys}$ is the system dynamic force vector given by $\mathbf{f}_{dyn}^{sys}(t_j) = \mathbf{f}^A(t_j) + \mathbf{f}^H(t_j)$.

Newmark-beta integration method was used to solve Eq. 4.27 to obtain the dynamic response $\mathbf{z}_{dyn}(t_j)$. The total response of the OWT $\mathbf{z}_{tot}(t_j)$ is

calculated as

$$\mathbf{z}_{\text{tot}}(t_j) = \mathbf{z}_{\text{sta}} + \mathbf{z}_{\text{dyn}}(t_j)$$

where $\mathbf{z}_{\text{sta}}$ is the static response solved in Eq. 4.26. See [Chopra \(1995\)](#) for the linear Newmark-beta integration procedure. Next, the total responses are transformed back to the local coordinates and the element internal force vector $\mathbf{f}_{\text{el}}^{\text{int}}$ is recovered using

$$\begin{aligned} \mathbf{f}_{\text{el}}^{\text{int}} &= \left\{ N_x^{(1)} \ V_y^{(1)} \ V_z^{(1)} \ M_x^{(1)} \ M_y^{(1)} \ M_z^{(1)} \ N_x^{(2)} \ V_y^{(2)} \ V_z^{(2)} \ M_x^{(2)} \ M_y^{(2)} \ M_z^{(2)} \right\}^T \\ &= [\mathbf{D}] [\mathbf{B}] \mathbf{z}_{\text{el}} \end{aligned} \quad (4.28)$$

where $N_x^{(1)}$, $V_y^{(1)}$, $V_z^{(1)}$, $M_x^{(1)}$, $M_y^{(1)}$ and $M_z^{(1)}$ denote the internal axial force applied in the x-direction, the internal shear forces applied in the local y- and z-directions, and the internal torsion and bending moments applied in the yz-, xz- and xy-planes, respectively and the superscripts refer to the node number (e.g., (1) refers to node 1); $[\mathbf{D}]$ is the constitutive matrix, formed by EA , EI_y , EI_z and GJ_x ; $[\mathbf{B}]$ is the strain-displacement matrix; and $\mathbf{z}_{\text{el}}$ is the total nodal displacement vector in the local coordinate system. Next, the beams are evaluated in the limit state analyses based on the types of structures.

4.2.1 ULS analysis of tubular structures

The ultimate limit state (ULS) analysis of tubular support structures is performed to check if structural strength and stability requirements are fulfilled in the design process when subjected to extreme loading conditions, which includes yield checks on beams and joints as well as the buckling assessments. In this study, the extreme load constraints are formed in accordance with the limit state functions specified in the NORSOK N-004 ([Standards Norway, 2013](#)). This standard is applicable to tubular designs with diameter-to-thickness ratios

of below 120 and thicknesses of greater than 6 mm (see Table 6.8). Under the complicated environmental conditions, the offshore tubular members are potentially exposed to any combination of axial tension, axial compression, bending, shear and hydrostatic loads. The hydrostatic pressure also generates an additional axial compressive force if the ends are capped. The limit state functions for tubular members under combined loads are given by

1. Combined axial tension and bending:

$$g_{ULS1} = \left(\frac{N_{t,Sd}}{N_{t,Rd}} \right)^{1.75} + \frac{\sqrt{M_{y,Sd}^2 + M_{z,Sd}^2}}{M_{Rd}} - 1.0 \leq 0 \quad (4.29a)$$

$$g_{ULS1} = \frac{\sigma_{at,Sd}}{f_{th,Rd}} + \frac{\sqrt{\sigma_{my,Sd}^2 + \sigma_{mz,Sd}^2}}{f_{mh,Rd}} - 1.0 \leq 0 \quad (4.29b)$$

2. Combined axial compression and bending, where column buckling is studied in Eq. (4.30a) and (4.30b):

$$g_{ULS2} = \frac{N_{c,Sd}}{N_{c,Rd}} + \frac{1}{M_{Rd}} \left[\left(\frac{C_{my}M_{y,Sd}}{1 - \frac{N_{c,Sd}}{N_{Ey}}} \right)^2 + \left(\frac{C_{mz}M_{z,Sd}}{1 - \frac{N_{c,Sd}}{N_{Ez}}} \right)^2 \right]^{0.5} - 1.0 \leq 0 \quad (4.30a)$$

$$g_{ULS2} = \frac{\sigma_{ac,Sd} - \sigma_{q,Sd}}{f_{ch,Rd}} + \frac{1}{f_{mh,Rd}} \left[\left(\frac{C_{my}\sigma_{my,Sd}}{1 - \frac{\sigma_{ac,Sd} - \sigma_{q,Sd}}{f_{Ey}}} \right)^2 + \left(\frac{C_{mz}\sigma_{mz,Sd}}{1 - \frac{\sigma_{ac,Sd} - \sigma_{q,Sd}}{f_{Ez}}} \right)^2 \right]^{0.5} - 1.0 \leq 0 \quad (4.30b)$$

$$g_{ULS3} = \frac{N_{c,Sd}}{N_{cl,Rd}} + \frac{\sqrt{M_{y,Sd}^2 + M_{z,Sd}^2}}{M_{Rd}} - 1.0 \leq 0 \quad (4.31a)$$

$$g_{ULS3} = \frac{\sigma_{ac,Sd}}{f_{cl,Rd}} + \frac{\sqrt{\sigma_{my,Sd}^2 + \sigma_{mz,Sd}^2}}{f_{mh,Rd}} - 1.0 \leq 0 \quad (4.31b)$$

$$g_{ULS4} = \frac{\sigma_{c,Sd} - 0.5 \frac{f_{he}}{\gamma_m}}{\frac{f_{cle}}{\gamma_m} - 0.5 \frac{f_{he}}{\gamma_m}} + \left(\frac{\sigma_{p,Sd}}{\frac{f_{he}}{\gamma_m}} \right)^2 - 1.0 \leq 0 \quad (4.32)$$

3. Interaction shear, bending moment and torsional moment:

$$g_{ULS5} = \begin{cases} \frac{M_{Sd}}{M_{Red,Rd}} - \sqrt{1.4 - \frac{V_{Sd}}{V_{Rd}}} \leq 0 & \text{for } \frac{V_{Sd}}{V_{Rd}} \geq 0.4 \\ \frac{M_{Sd}}{M_{Red,Rd}} - 1.0 \leq 0 & \text{for } \frac{V_{Sd}}{V_{Rd}} < 0.4 \end{cases} \quad (4.33)$$

4. Hoop buckling:

$$g_{ULS6} = \frac{\sigma_{p,Sd}}{f_{h,Rd}} - 1.0 \leq 0 \quad (4.34)$$

where $N_{t,Sd}(\sigma_{at,Sd})$, $N_{c,Sd}(\sigma_{ac,Sd})$, $M_{y,Sd}(\sigma_{my,Sd})$, $M_{z,Sd}(\sigma_{mz,Sd})$, V_{Sd} are the design axial tensile force, compressive force, bending moments about member y-axis (in-plane) and z-axis (out-of-plane) and shear force, respectively; $N_{t,Rd}(f_{th,Rd})$, $N_{c,Rd}(f_{ch,Rd})$, $N_{cl,Rd}(f_{cl,Rd})$, $N_{Ey}(f_{Ey})$, $N_{Ez}(f_{Ez})$, $M_{Rd}(f_{mh,Rd})$ and V_{Rd} are the design resistance for axial tension, axial compression, local buckling, Euler buckling (member y- and z- axes), bending and shear, respectively; $\sigma_{p,Sd}$, $\sigma_{q,Sd}$ and $\sigma_{c,Sd}$ are the design hoop stress, capped-end compressive stress and maximum combined compressive stress, respectively; and $f_{h,Rd}$, f_{he} , f_{cle} , C_{my} , C_{mz} and γ_m are the design hoop buckling strength, elastic hoop buckling strength, characteristic elastic local buckling strength, reduction factors (member y- and z-axes) and partial safety factor for material, respectively. The symbols in parentheses refer to the corresponding stress terms that account for the external hydrostatic pressure effects. Eqs. (4.29a), (4.30a), (4.31a) and (4.33) are applicable to the members which are free of hydrostatic pressure, such as, beams above the mean sea level (MSL) or flooded internally, whereas Eqs. (4.29b), (4.30b), (4.31b), (4.32) and (4.34) are pertinent to the submerged members which experience hydrostatic pressure externally.

Similarly, the resistance of tubular joints N_{Rdj} , $M_{y,Rdj}$ and $M_{z,Rdj}$ is checked to conform to the interaction equation for combined axial force and bending moments in the braces:

$$g_{ULS7} = \frac{N_{Sd}}{N_{Rdj}} + \left(\frac{M_{y,Sd}}{M_{y,Rdj}} \right)^2 + \frac{M_{z,Sd}}{M_{z,Rdj}} - 1.0 \leq 0 . \quad (4.35)$$

The characteristic resistance depends on the strength factor Q_u and the chord action factor Q_f , which vary with respect to the joint configuration (i.e., joint type and joint dimension), material strength, as well as the presence of factored actions within the chords.

4.2.2 ULS analysis of shell structures

Offshore wind turbine shell structures such as towers and monopiles are assessed for their structural strength and stability requirements. Equations (4.29a-4.34) are applied to evaluate the yield strength and column buckling strength of shell structures above the seabed. Besides, the shell buckling strength f_{ksd} needs to be checked against the design combined stress $\sigma_{j,Sd}$ according to DNV-RP-C202 (DNV, 2010a), i.e.,

$$g_{ULS8} = \frac{\sigma_{j,Sd}}{f_{ksd}} - 1.0 \leq 0. \quad (4.36)$$

The parameters f_{ksd} and $\sigma_{j,Sd}$ can be calculated as

$$f_{ksd} = \frac{f_y}{\gamma_m \sqrt{1 + \bar{\lambda}_s^4}} \quad (4.37)$$

and

$$\sigma_{j,Sd} = \sqrt{(\sigma_{ac,Sd} + \sigma_{m,Sd})^2 - (\sigma_{ac,Sd} + \sigma_{m,Sd}) \sigma_{p,Sd} + \sigma_{p,Sd}^2 + 3\tau_{Sd}^2} \quad (4.38)$$

where γ_m is the partial material safety factor, f_y is the material yield strength; $\bar{\lambda}_s$ is the reduced shell slenderness; $\sigma_{ac,Sd}$, $\sigma_{m,Sd}$, $\sigma_{p,Sd}$ and τ_{Sd} are the design axial compressive stress, design bending stress, design hoop stress and design

shear stress, respectively. The reduced shell slenderness is given as a function of external stresses and elastic buckling strengths attributed to axial forces (f_{ae}), bending moments (f_{me}), hoop stresses (f_{he}) and shear forces ($f_{\tau e}$), i.e.,

$$\bar{\lambda}_s^2 = \frac{f_y}{\sigma_{j,Sd}} \left[\frac{\sigma_{ac,Sd}}{f_{ae}} + \frac{\sigma_{m,Sd}}{f_{me}} + \frac{\sigma_{p,Sd}}{f_{he}} + \frac{\tau_{Sd}}{f_{\tau e}} \right]. \quad (4.39)$$

As for the portion of monopiles below the mudline, general column buckling needs not be considered as the pile is laterally supported by the surrounding soil. Apart from providing sufficient stability to the overall wind turbine system, piles should be designed to withstand the hammer weight and driving force during the installation. In general, D/T ratios of the entire pile should be small to avoid local buckling at stresses up to the yield strength of the pile material. Assuming that the installation process involves hard driving, [American Petroleum Institute \(2007\)](#) recommends that a minimum piling wall thickness T (m) with respect to the pile outer diameter D (m) as follows,

$$D - 100T + 0.635 \leq 0. \quad (4.40)$$

4.2.3 FLS analysis of steel structures

Fatigue failure often occurs around the weld toes at member connections due to high stress concentration in long term cyclic loads. It is another important aspect to be analyzed when designing offshore structures. This failure mode is cumulative along the response histories. Often the fatigue damage is calculated on the response data available for a certain simulation period, for instance 1 hour simulation time for wind turbine structures, followed by a projection to the entire design lifespan (typically 20 years or more). Figure 4.5 depicts the procedure to perform fatigue damage assessments in the time-domain or frequency-domain ([Ragan and Manuel, 2007](#)).

Internal nominal stresses, in the form of time series which are recovered

from system responses, are pre-multiplied with Stress Concentration Factors (SCF) and superimposed to obtain the Hot Spot Stresses (HSS). The HSS are evaluated at eight different spots around the circumference of joint intersections as shown in Fig. 4.6 for both brace and leg sides. The HSS σ_{HSS} at individual locations are given by

$$\begin{aligned}
 \sigma_{HSS1} &= SCF_{AC}\sigma_x + SCF_{MIP}\sigma_{my} \\
 \sigma_{HSS2} &= \frac{1}{2}(SCF_{AC} + SCF_{AS})\sigma_x + \frac{1}{2}\sqrt{2}SCF_{MIP}\sigma_{my} - \frac{1}{2}\sqrt{2}SCF_{MOP}\sigma_{mz} \\
 \sigma_{HSS3} &= SCF_{AC}\sigma_x - SCF_{MOP}\sigma_{mz} \\
 \sigma_{HSS4} &= \frac{1}{2}(SCF_{AC} + SCF_{AS})\sigma_x - \frac{1}{2}\sqrt{2}SCF_{MIP}\sigma_{my} - \frac{1}{2}\sqrt{2}SCF_{MOP}\sigma_{mz} \\
 \sigma_{HSS5} &= SCF_{AC}\sigma_x - SCF_{MIP}\sigma_{my} \\
 \sigma_{HSS6} &= \frac{1}{2}(SCF_{AC} + SCF_{AS})\sigma_x - \frac{1}{2}\sqrt{2}SCF_{MIP}\sigma_{my} + \frac{1}{2}\sqrt{2}SCF_{MOP}\sigma_{mz} \\
 \sigma_{HSS7} &= SCF_{AC}\sigma_x + SCF_{MOP}\sigma_{mz} \\
 \sigma_{HSS8} &= \frac{1}{2}(SCF_{AC} + SCF_{AS})\sigma_x + \frac{1}{2}\sqrt{2}SCF_{MIP}\sigma_{my} + \frac{1}{2}\sqrt{2}SCF_{MOP}\sigma_{mz}
 \end{aligned} \tag{4.41}$$

where σ_x , σ_{my} and σ_{mz} are the nominal stresses due to axial load, bending in-plane and out-of-plane, respectively; SCF_{AS} and SCF_{AC} are the SCF for axial load at the saddle and crown, respectively ; SCF_{MIP} is the SCF for in-plane moment and SCF_{MOP} is the SCF for out-of-plane moment. The SCF can be computed using empirical formulae (e.g., Eftymiou's formulae) prescribed by Det Norske Veritas, given the joint class, geometry and dimensions (DNV, 2012). It was assumed here that each joint is given a specific joint class depending on the joint design, instead of being determined by the actual axial forces distributed among the braces.

The S-N curves are based on fatigue test data collected from experiments performed at constant amplitude cyclic loads. As the HSS output is variable in amplitudes, post-processing is required to estimate the stress ranges before one can proceed to calculate the fatigue damage. This can be achieved either by

Figure 4.5: Procedure to calculate fatigue damage either using time-domain or spectral method

Figure 4.6: Superposition of HSS (DNV, 2011c)

using cycle counting or spectral techniques. ASTM's rainflow counting algorithm is a commonly used cycle counting method which identifies the stress ranges and associated number of cycles (half or full cycles) for the HSS time series by pairing the peaks and valleys in analogy with rain flowing down a pagoda roof (ASTM, 2011). Alternatively, Dirlik's method obtains the stress range PDF from a spectrum. It assumes that the stress range PDF is a weighted combination of an exponential and two Rayleigh distributions; and is intended for both wide- and narrow-band processes (Dirlik, 1985). Dirlik's method is frequently used in conjunction with dynamic analysis performed in the frequency domain, where the PSD of HSS is calculated using transfer functions (see Section 3.3.1). The individual stress ranges of the histogram or PDF are then compared against the S-N curves to determine the fatigue damage. By applying the Palmgren- Miner's rule, the damage for each stress range is summed up linearly, which gives the total accumulated damage.

For a fatigue damage assessment in the time-domain that involves several FLS load cases, the total fatigue damage D_f can be computed as

$$g_{FLS} = D_f = \sum_{v=1}^V \left[P_v \sum_{u=1}^U \frac{1}{\bar{a}\eta_f} \left(\frac{T}{T_{ref}} \right)^{mk_T} n_u (\Delta\sigma_{u,HSS})^m \right] \leq 1.0 \quad (4.42)$$

where $\bar{a}$ is the intercept of the design S-N curve with log N axis; m is the negative inverse slope of the S-N curve; $\Delta\sigma_{u,HSS}$ is the u -th HSS stress range; n_u is the number of stress cycles in $\Delta\sigma_{u,HSS}$; U is the total number of stress ranges; η_f is the fatigue usage factor; T , T_{ref} and k are the member thickness, reference thickness and thickness exponent, respectively; P_v is the probability of v -th event; and V is total number of FLS events.

Directly analogous to Eq 4.42, the expected fatigue damage in the frequency-domain (without consideration of the thickness effect) can be esti-

mated using

$$g_{FLS} = E[D_f] = \sum_{v=1}^V \left[P_v \cdot \frac{t_{tot}}{a} E[P] \cdot \int_0^\infty \Delta\sigma_{HSS}^m \cdot p(\Delta\sigma_{HSS}) d\Delta\sigma_{HSS} \right] \leq 1.0 \quad (4.43)$$

where $p(\Delta\sigma_{HSS})$ is the HSS range PDF; t_{tot} is the total time; and $E[P]$ is the expected number of peaks per unit time. If numerical integration is carried out, Eq. 4.43 can be re-written as

$$E[D_f] = \sum_{v=1}^V \left[P_v \cdot \frac{t_{tot}}{a} E[P] \cdot \sum_{k=1}^K (S_{HSS,k}^m \cdot p(S_{HSS,k}) \Delta S_{HSS,k}) \right] \leq 1.0 \quad (4.44)$$

where S_{HSS} is used to replace $\Delta\sigma_{HSS}$ to avoid confusion when expressing the discretized HSS stress range ΔS_{HSS} .

The empirical distribution weight factors of $p(\Delta\sigma_{HSS})$ and $E[P]$ are governed by the 0th, 1st, 2nd and 4th spectral moments, as the n -th order spectral moment m_n is given by

$$m_n = \sum_{l=1}^L [f_l^n \cdot P_{\sigma_{HSS}}(f_l) df_l] \quad (4.45)$$

where the spectral density $P_{\sigma_{HSS}}(f)$ is obtained by Fourier transform of HSS $\sigma_{HSS}(t)$.

4.3 Sensitivity analysis and optimization module

This module provides the platform that integrates both FEM module and linear dynamics and failure analysis module together in solving the optimization problem, as illustrated Fig 4.1. Besides, it also serves the main functions to perform dynamic sensitivity analysis (see Sections 4.3.3-4.3.6) and to carry out gradient-based optimization procedure (see Section 4.3.7).

The dynamic response optimization of OWT support structures is a constrained non-linear programming (NLP) problem, which can be illustrated

using the following expression:

$$\text{Find } \mathbf{b} \tag{4.46}$$

$$\text{to minimize } g_0(\mathbf{b}) \tag{4.47}$$

$$\text{subject to } g_i(\mathbf{b}, \mathbf{z}(t_j), t_j) \leq 0; \quad i = 1, \dots, p, \quad j = 1, \dots, q \tag{4.48}$$

$$\text{and } [\mathbf{M}^{\text{sys}}(\mathbf{b})] \ddot{\mathbf{z}}(t_j) + [\mathbf{C}^{\text{sys}}(\mathbf{b})] \dot{\mathbf{z}}(t_j) + [\mathbf{K}^{\text{sys}}(\mathbf{b})] \mathbf{z}(t_j) - \mathbf{f}^{\text{sys}}(\mathbf{b}, t_j) = \mathbf{0} \tag{4.49}$$

where g_0 is the objective function; g_i is the i -th constraint function; $\mathbf{b}$ is the design variable vector; $\mathbf{z}(t_j)$, $\dot{\mathbf{z}}(t_j)$ and $\ddot{\mathbf{z}}(t_j)$ are the displacement, velocity and acceleration vectors; t_j is the j -th time step; $[\mathbf{M}^{\text{sys}}(\mathbf{b})]$, $[\mathbf{C}^{\text{sys}}(\mathbf{b})]$ and $[\mathbf{K}^{\text{sys}}(\mathbf{b})]$ are the system mass, damping and stiffness matrices, respectively; and $\mathbf{f}^{\text{sys}}(\mathbf{b}, t_j)$ is the force vector. Note: Equation 4.49 refers to the general equation of motion, which consists of a static subproblem and a dynamic subproblem (see 4.2).

4.3.1 Objective function

In general, the support structure can be modeled as an assemblage of beam elements, connected through nodes in resemblance to the actual structures, e.g., monopile, spaceframe and tripod structures. In this thesis, the objective function to be minimized is the structural mass and is given by

$$g_0 = \sum_{n=1}^N \rho_b A_n(\mathbf{b}) L_n \tag{4.50}$$

where ρ_b is the material density; and $A_n(\mathbf{b})$ and L_n are the cross-sectional area and length the n -th member. This is a simplified representation of the cost function since other cost components that are incurred in the design life cycle of OWT support structures, such as manufacturing, installation and maintenance costs are excluded. For sizing optimization of tubular and shell structures, $A_n(\mathbf{b}) = \pi (D_n T_n - T_n^2)$ and $\mathbf{b} = [D_1, T_1, D_2, T_2, \dots, D_{M/2}, T_{M/2}]$, where D_n and T_n are the diameter and thickness of n -th member; and M is the total number of design variables.

4.3.2 Design constraint functions

The design constraints implemented in the study include various limit state functions as prescribed by the design standards and recommended practices used within the offshore and wind industries (IEC, 2009; DNV, 2011a; Standards Norway, 2013; American Petroleum Institute, 1993). They are based upon the load and resistance factor design (LRFD) method, and can be generally classified into sizing, eigenfrequency, deformation, extreme load and fatigue load constraints.

1. Sizing constraints g_{size} . Sizing constraints define the lower and upper bounds of design variables $\mathbf{b}$ as well as the geometrical relationships among the variables, e.g., D_n/T_n , etc. They can be expressed as

$$g_{size1} = \mathbf{b}^{\min} \leq \mathbf{b} \leq \mathbf{b}^{\max} \quad (4.51)$$

$$g_{size2} = [\mathbf{A}_{ineq}] \mathbf{b} - \mathbf{c}_{ineq} \leq \mathbf{0} \quad (4.52)$$

where $\mathbf{b}^{\min}$ is the lower bound and $\mathbf{b}^{\max}$ is the upper bound of $\mathbf{b}$; $[\mathbf{A}_{ineq}]$ and $\mathbf{c}_{ineq}$ are the linear inequality matrix and vector. The matrix $[\mathbf{A}_{ineq}]$ and vector $\mathbf{c}_{ineq}$ can be established from the validity range of $\mathbf{b}$ provided for the prescribed limit state functions.

2. Eigenfrequency constraints g_{eigen} . The natural frequencies of the overall

OWT system have to be assessed against the excitation frequency zones caused by environmental loads (e.g., winds and waves) and rotor rotations (see Fig. 4.7), so as to avoid (or minimize) the risk of resonance. In modern variable speed turbines, the main rotor excitation frequencies exist as 1P and nP frequency bands, where 1P is associated with the rotor rotational revolutions per minute (RPM) while nP is due to the blade passing frequency, i.e., n -th multiple of the rotor rotational RPM where n is the number of blades. Hence for a three-bladed rotor, the first mode eigenfrequencies f_1 are usually designed to lie in the soft-stiff region as bounded between $1P_U$ and $3P_L$, i.e.,

$$g_{eigen1} = 1P_U \leq f_1 \leq 3P_L \quad (4.53)$$

where $1P_U$ is the upper bound of 1P range and $3P_L$ is the lower bound of 3P range. This is a more economical approach to design a bottom-fixed support structure that withstands the excitation through structural resistance (van Der Tempel, 2006).

Figure 4.7: Environmental and rotor excitation frequencies, adapted from van der Tempel (2002)

3. Deformation constraint g_{deform} . A pile and its interaction with the sur-

rounding soil should possess sufficient stiffness to limit the deformations under the influences of combined loadings. Typically, the deformation constraints are imposed on the pile head at the mudline level and the pile tip. In this thesis, maximum deflections of 100 mm and 20 mm were adopted for the pile head and pile tip, respectively (Krolis et al., 2010; de Vries, 2011). Besides, the axial displacements of the pile nodes should all be maintained below $0.01D$ for sands to ensure that the plastic deformation of sands is not mobilized. This helps to reduce the error caused by the linearized foundation stiffness model.

4. Extreme load constraints g_{ULS} . Extreme load constraints are built on the ULS checks, which vary with respect to the types of support structures or the loading types (see section 4.2). Since the extreme load constraints are time-dependent, they are supposed to be satisfied at all time steps. Several methods are available to handle this type of constraint as reviewed by Kang et al. (2006), and the worst case approach was implemented in this study. The method identifies the maximum violated limit state values in time as design constraints; while gradients are calculated for the constraints active at those time points.
5. Fatigue load constraints g_{FLS} . Fatigue load constraints are built on the FLS checks, which are basically cumulative over the entire simulation period (see section 4.2.3).

4.3.3 Dynamic response sensitivity analysis

Design sensitivity analysis is a process that calculates the rate of change of a performance measure with respect to the design variables. The performance measures in this context refer to the objective and constraint functions. The rate of change or simply gradient is required by the optimizer to decide the best

direction for improvement during the optimization process. In this study, the gradients were calculated using the direct differentiation method (DDM) as

$$\nabla g_0 = \frac{\partial g_0}{\partial \mathbf{b}} \quad (4.54)$$

and

$$\nabla g_i = \frac{\partial g_i}{\partial \mathbf{b}} + \frac{\partial g_i}{\partial \mathbf{z}} \frac{d\mathbf{z}}{d\mathbf{b}} \quad (4.55)$$

where ∇g_0 is the gradient of objective function and ∇g_i is the gradient of i -th constraint function (Park, 2007). The calculations of ∇g_0 and ∇g_{size2} were straightforward.

However, since the extreme load and fatigue load constraints are time-dependent and are determined from post-processing the dynamic responses, their sensitivities are dependent on the response sensitivities. The dynamic response sensitivities $\frac{d\mathbf{z}}{d\mathbf{b}}$ is the sum of derivatives for static and dynamic displacements, i.e.,

$$\frac{d\mathbf{z}}{d\mathbf{b}} = \frac{d\mathbf{z}_{sta}}{d\mathbf{b}} + \frac{d\mathbf{z}_{dyn}}{d\mathbf{b}} . \quad (4.56)$$

Both derivatives are solved using

$$\frac{d\mathbf{z}_{sta}}{d\mathbf{b}} = [\mathbf{K}^{sys}]^{-1} \left(\frac{d\mathbf{f}_{sta}^{sys}}{d\mathbf{b}} - \frac{d[\mathbf{K}^{sys}]}{d\mathbf{b}} \mathbf{z}_{sta} \right) \quad (4.57)$$

and

$$\begin{aligned} & [\mathbf{M}^{sys}] \frac{d^2}{dt^2} \left(\frac{d\mathbf{z}_{dyn}}{d\mathbf{b}} \right) + [\mathbf{C}^{sys}] \frac{d}{dt} \left(\frac{d\mathbf{z}_{dyn}}{d\mathbf{b}} \right) + [\mathbf{K}^{sys}] \left(\frac{d\mathbf{z}_{dyn}}{d\mathbf{b}} \right) = \\ & \frac{d\mathbf{f}_{dyn}^{sys}}{d\mathbf{b}} - \frac{d[\mathbf{M}^{sys}]}{d\mathbf{b}} \ddot{\mathbf{z}}_{dyn} - \frac{d[\mathbf{C}^{sys}]}{d\mathbf{b}} \dot{\mathbf{z}}_{dyn} - \frac{d[\mathbf{K}^{sys}]}{d\mathbf{b}} \mathbf{z}_{dyn} , \end{aligned} \quad (4.58)$$

respectively. Eq. (4.58) is a second-order differential equation obtained from differentiating Eq. (4.27) with respect to $\mathbf{b}$.

The matrices $\frac{d[\mathbf{M}^{\text{sys}}]}{d\mathbf{b}}$, $\frac{d[\mathbf{C}^{\text{sys}}]}{d\mathbf{b}}$ and $\frac{d[\mathbf{K}^{\text{sys}}]}{d\mathbf{b}}$ are the derivatives of system matrices $[\mathbf{M}^{\text{sys}}]$, $[\mathbf{C}^{\text{sys}}]$ and $[\mathbf{K}^{\text{sys}}]$ that include the aero-hydro-elastic contributions, as mentioned earlier. Meanwhile, $\frac{d\mathbf{f}_{\text{sta}}^{\text{sys}}}{d\mathbf{b}}$ denotes the derivative of static forces, i.e., $\mathbf{f}^{\text{B}}$ and $\mathbf{f}^{\text{G}}$, while $\frac{d\mathbf{f}_{\text{dyn}}^{\text{sys}}}{d\mathbf{b}}$ refers to the derivative of dynamic forces, i.e., $\mathbf{f}^{\text{H}}$. Note that the $\mathbf{f}^{\text{A}}$ does not vary with $\mathbf{b}$ in the decoupled method, as $\mathbf{f}^{\text{A}}$ is simulated from the aerodynamic loads exerted onto the wind turbine fixed rigidly at the yaw.

4.3.4 Extreme load sensitivity analysis

The computation of extreme load constraints ∇g_{ULS} is split in two parts. First is the $\frac{d\mathbf{z}}{d\mathbf{b}}$ in Eq. (4.55), which are solved using the methods as presented in section 4.3.3. Secondly, the partial derivatives $\frac{\partial g_i}{\partial \mathbf{b}}$ and $\frac{\partial g_i}{\partial \mathbf{z}}$ in Eq. (4.55) can be evaluated readily using the analytical DDM.

This is because the design loads and design resistance in the constraint functions g_{ULS1} to g_{ULS7} , i.e., the numerator and denominator terms, can be expressed as functions of internal forces and geometric variables. The internal forces as given by Eq. 4.28 are generally explicit functions of $\mathbf{b}$ and $\mathbf{z}$.

4.3.5 Fatigue load sensitivity analysis

With regards to the fatigue load constraint, methods are developed to calculate gradients for fatigue damage D_f in the time-domain and the frequency-domain.

For the time-domain method, differentiation of Eq. (4.42) against $\mathbf{b}$ gives the sensitivity of fatigue damage as

$$\begin{aligned}
\nabla g_{FLS} &= \nabla D_f \\
&= \sum_{v=1}^V \left[P_v \sum_{u=1}^U \frac{m}{\bar{a}\eta_f} \left(\frac{T}{T_{ref}} \right)^{mk_T} n_u (\Delta\sigma_{u,HSS})^{m-1} \frac{d\Delta\sigma_{u,HSS}}{d\mathbf{b}} \right] + \\
&\quad \sum_{v=1}^V \left[P_v \sum_{u=1}^U \frac{mk_T}{\bar{a}\eta_f} \left(\frac{1}{T_{ref}} \right)^{mk_T} T^{mk_T-1} \frac{dT}{d\mathbf{b}} n_u (\Delta\sigma_{u,HSS})^m \right] \quad (4.59)
\end{aligned}$$

where $\Delta\sigma_{u,HSS}$ here refers to the individual stress range without binning into blocks, while the stress cycle n_u corresponds to either a half or full cycle for the $\Delta\sigma_{u,HSS}$. By doing so, the $\frac{dn_u}{d\mathbf{b}}$ term could be neglected in the formulation. Among the terms, the derivative of stress range $\frac{d\Delta\sigma_{u,HSS}}{d\mathbf{b}}$ was calculated by taking the difference of HSS sensitivities $\frac{d\sigma_{u,HSS}}{d\mathbf{b}}$ at $t_j = t_{u,1}$ and $t_j = t_{u,2}$:

$$\frac{d\Delta\sigma_{u,HSS}}{d\mathbf{b}} = \begin{cases} \frac{d\sigma_{u,HSS}(t_j)}{d\mathbf{b}} \Big|_{t_j=t_{u,1}} - \frac{d\sigma_{u,HSS}(t_j)}{d\mathbf{b}} \Big|_{t_j=t_{u,2}} & \text{for } \sigma_{u,HSS}(t_{u,1}) > \sigma_{u,HSS}(t_{u,2}) \\ \frac{d\sigma_{u,HSS}(t_j)}{d\mathbf{b}} \Big|_{t_j=t_{u,2}} - \frac{d\sigma_{u,HSS}(t_j)}{d\mathbf{b}} \Big|_{t_j=t_{u,1}} & \text{for } \sigma_{u,HSS}(t_{u,2}) > \sigma_{u,HSS}(t_{u,1}) \end{cases} \quad (4.60)$$

where $t_{u,1}$ and $t_{u,2}$ are the times of initial and reversal points for $\Delta\sigma_{u,HSS}$, respectively; which could be identified during the rainflow counting process (see Fig. 4.8).

Meanwhile, the derivative of HSS $\frac{d\sigma_{u,HSS}}{d\mathbf{b}}$ can be determined from

$$\begin{aligned}
\frac{d\sigma_{u,HSS}}{d\mathbf{b}} &= c_x \left(\frac{dSCF_A}{d\mathbf{b}} \sigma_{u,x} + SCF_A \frac{d\sigma_{u,x}}{d\mathbf{b}} \right) + \\
&\quad c_{my} \left(\frac{dSCF_{MIP}}{d\mathbf{b}} \sigma_{u,my} + SCF_{MIP} \frac{d\sigma_{u,my}}{d\mathbf{b}} \right) + \\
&\quad c_{mz} \left(\frac{dSCF_{MOP}}{d\mathbf{b}} \sigma_{u,mz} + SCF_{MOP} \frac{d\sigma_{u,mz}}{d\mathbf{b}} \right). \quad (4.61)
\end{aligned}$$

The constants c_x , c_{my} and c_{mz} are determined based on the HSS locations (See Eq. 4.41). Since the SCFs are generally governed by the joint

Figure 4.8: Rainflow counting process. In this example, E-F-E' is counted as one full stress cycle, where $\Delta\sigma_{u,HSS}$ is the u th HSS range, $t_{u,1}$ and $t_{u,2}$ are the corresponding time steps at E (initial point) and F (reversal point), respectively.

dimensions, the derivatives $\frac{dSCF}{db}$ can be calculated using the DDM as well. As for the nominal stresses $\sigma_{u,x}$, $\sigma_{u,my}$ and $\sigma_{u,mz}$, the derivatives could be evaluated using the same method as explained in section 4.3.4.

As for the spectral method, the gradient of expected fatigue damage $E[D_f]$, by using DDM on Eq. 4.44 gives

$$\begin{aligned} \nabla g_{FLS} &= \nabla E[D_f] \\ &= \sum_{v=1}^V \left[P_v \cdot \frac{t_{tot}}{a} \frac{dE[P]}{db} \cdot \sum_{k=1}^K \left(S_{HSS,k}^m \cdot p(S_{HSS,k}) \Delta S_{HSS,k} \right) \right] + \\ &\quad \sum_{v=1}^V \left[P_v \cdot \frac{t_{tot}}{a} E[P] \cdot \sum_{k=1}^K \frac{d(S_{HSS,k}^m \cdot p(S_{HSS,k}) \Delta S_{HSS,k})}{db} \right] \end{aligned} \quad (4.62)$$

where the empirical distribution weight factors of $p(S_{HSS})$ and $E[P]$ are dependent on the spectral moments m_n , which in turn requires Fourier transform to be performed on $\sigma_{HSS}(t)$ (see Eq. 4.45). The Welch method (modified periodogram) with Hamming windows and overlaps is typically used for this purpose as the taper reduces leakage from the spectral density near the large peaks in

the spectrum (Ragan and Manuel, 2007).

The sensitivity of Welch's method with respect to $\mathbf{b}$ is derived as

$$\frac{d}{d\mathbf{b}} P_{\sigma_{HSS}}(f) = \frac{4}{QJUF_s} \sum_{q=1}^Q \left[\Re(A_q(f)) \cdot \Re(A'_q(f)) + \Im(A_q(f)) \cdot \Im(A'_q(f)) \right] \quad (4.63)$$

where

$$A_q(f) = \sum_{j=0}^{J-1} \left[\sigma_{HSS,q}(j) \cdot W(j) \cdot \exp\left(\frac{-2\pi i j f}{J}\right) \right],$$

$$A'_q(f) = \sum_{j=0}^{J-1} \left[\frac{d\sigma_{HSS,q}(j)}{d\mathbf{b}} \cdot W(j) \cdot \exp\left(\frac{-2\pi i j f}{J}\right) \right] \text{ and}$$

$$U = \frac{1}{J} \sum_{j=0}^{J-1} W^2(j).$$

$\Re(z)$ and $\Im(z)$ refer to the real and imaginary parts of a complex variable z ; Q is the number of window segments; J is the number of samples within the window; W is the windowing function (e.g., Hamming), F_s is the sampling frequency; and $i = \sqrt{-1}$. Note: A scaling factor of $\frac{1}{J}$ to be used for samples at zero frequency and Nyquist frequency. See Welch (1967) for more information about the Welch method.

4.3.6 Eigenfrequency sensitivity analysis

Lastly, the gradient of eigenfrequencies ∇g_{eigen} is computed using

$$\nabla g_{eigen} = \frac{1}{4\pi} \lambda_l^{-\frac{1}{2}} \frac{\partial \lambda_l}{\partial \mathbf{b}} \quad (4.64)$$

$$\text{and } \frac{\partial \lambda_l}{\partial \mathbf{b}} = \phi_l^T \left(\frac{\partial [\mathbf{K}^{\text{sys}}]}{\partial \mathbf{b}} - \lambda_l \frac{\partial [\mathbf{M}^{\text{sys}}]}{\partial \mathbf{b}} \right) \phi_l \quad (4.65)$$

where λ_l is the eigenvalue of l -th mode; and ϕ_l is the eigenvector of l -th mode, as derived in (Fox and Kapoor, 1968).

4.3.7 Optimization algorithm

The constrained NLP problem is solved using the Sequential Quadratic Programming (SQP) algorithm, whereby the general problem is reformulated as QP subproblems at each iteration as given by

$$(\text{QP}) = \begin{cases} \min & g_0(\mathbf{b}^{(k)}) + \nabla g_0(\mathbf{b}^{(k)})^T (\mathbf{b} - \mathbf{b}^{(k)}) + \frac{1}{2} (\mathbf{b} - \mathbf{b}^{(k)})^T \mathbf{H}(\mathbf{b}^{(k)}) (\mathbf{b} - \mathbf{b}^{(k)}) \\ \text{s.t.} & g_i(\mathbf{b}^{(k)}) + \nabla g_i(\mathbf{b}^{(k)})^T (\mathbf{b} - \mathbf{b}^{(k)}) \leq 0, \quad i = 1, \dots, l \\ & \mathbf{b} \in \{\mathbf{b} \in \mathbb{R}^n \mid b_j^{\min} \leq b_j \leq b_j^{\max}, \quad j = 1, \dots, n\} \end{cases} \quad (4.66)$$

where $\mathbf{H}(\mathbf{b}^k)$ denotes a second-order derivative of the Lagrangian function $L(\mathbf{b}, \lambda) = g_0(\mathbf{b}) + \sum_{i=1}^m \lambda_i g_i(\mathbf{b})$ (Christensen and Klarbring, 2009).

The integrated optimization code implements the SQP subroutine available from the Matlab optimization toolbox (MathWorks Inc, 2014). Although $\mathbf{H}$ may be computed as the actual Hessian, approximation is usually used. For the Matlab SQP subroutine, $\mathbf{H}$ is calculated using the modified Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula, as given by

$$\mathbf{H}^{(k+1)} = \mathbf{H}^{(k)} + \frac{\mathbf{q}^{(k)} (\mathbf{q}^{(k)})^T}{(\mathbf{q}^{(k)})^T \mathbf{s}^{(k)}} - \frac{\mathbf{H}^{(k)} \mathbf{s}^{(k)} (\mathbf{H}^{(k)})^T}{(\mathbf{s}^{(k)})^T \mathbf{H}^{(k)} \mathbf{s}^{(k)}} \quad (4.67)$$

where

$$\mathbf{s}^{(k)} = \mathbf{b}^{(k+1)} - \mathbf{b}^{(k)} \quad (4.68)$$

and

$$\mathbf{q}^{(k)} = \nabla L(\mathbf{b}^{(k+1)}, \lambda) - \nabla L(\mathbf{b}^{(k)}, \lambda). \quad (4.69)$$

This approximation guarantees positive definite Hessian matrices and ensures that the subproblems are strictly convex. The solution to the QP subproblem produces a vector $\mathbf{d}^{(k)}$, which is then used to form a new iterate $\mathbf{b}^{(k+1)}$ through

line search, i.e.,

$$\mathbf{b}^{(k+1)} = \mathbf{b}^{(k)} + \varepsilon \mathbf{d}^{(k)} \quad (4.70)$$

where a step length parameter ε is inserted to produce a sufficient decrease in the merit function ([MathWorks Inc, 2014](#)). In SQP, the bounds are strictly respected in all iterations. This is beneficial to problems where the non-linear constraint functions might be undefined outside the bounds, which is relevant to our case.

More details about the SQP method can be found in [Nocedal and Wright \(2006\)](#); [Arora \(2011\)](#).

Chapter 5

Modeling and simulation verification

5.1 Description of numerical offshore wind turbine models

Two types of support structures were investigated in this thesis, namely the monopile and jacket structures. The former is based on the reference monopile design developed in the UpWind Project under Work Package 4, whereas the latter is referred to the reference jacket design used within the IEA Task 30 OC4 Phase I Project. The NREL 5-MW baseline wind turbine was adopted as the wind turbine model for both systems (see Section 5.1.3). The general dimensions about support structure systems are summarized in Table 5.1 .

5.1.1 UpWind monopile reference design

The UpWind monopile reference support structure consists of a tower, a transition piece and a monopile. The schematic drawing of the overall model is shown in Fig. 5.1. The tower top where the yaw bearing is placed is situated 82.76 m

Table 5.1: General dimensions of the reference monopile and jacket support structures

Dimension	Monopile support structure	Jacket support structure
Reference design	UpWind monopile model	OC4 jacket model
Hub height [m]	85.16	90.55
Tower top [m]	82.76	88.15
Tower base [m]	14.76	20.15
Water depth [m]	25.0	50.0
Pile depth [m]	24.0	No pile model

above the MSL. The nodal coordinates of the support structure can be referred

Figure 5.1: A finite element model of OC3 OWT with monopile structures

to [de Vries \(2011\)](#). In actual design, the tower base is bolted onto the transition piece while the transition piece is grouted to the protruding pile at the elevation between -3.4 m and 5 m. The grout section has been simplified in the FEM model where a tubular beam that accounts for thicknesses of the pile, transition piece and grout was used. The overall support structure was modeled using flexible EB elements. The monopile is penetrated 24 m below the mudline. The foundation stiffness was represented using the $p - y$ curves for lateral pile-soil interaction and the $t - z$ and $Q - z$ curves for shaft friction and tip end bearing. The torsional degree of freedom of the pile is constrained for all the nodes along

the pile. The seabed was assumed to be sands with a soil profile as defined in Table 5.2. Rayleigh damping was used to model structural damping for the support structure. A critical damping ratio of 1 percent was assumed for the first and second eigenmodes of the overall support structures.

Table 5.2: Soil conditions for soft soil profile (Fischer et al., 2010). γ' : effective soil unit weight and Φ : angle of internal friction.

Depths [m]	γ' [N/m ³]	Φ [°]
0-3	10000	36
3-5	10000	33
5-7	10000	26
7-10	10000	37
10-15	10000	35
15-50	10000	37.5

5.1.2 OC4 jacket reference design

The OC4 support structure is formed by a monopile tower, a transition piece and a jacket substructure. The overall finite element model is shown in Fig. 5.2. The overall elevation on which the RNA is installed was modified from the original design according to (Vorpahl et al., 2011), where the hub height was shifted to 88.15 m. As for the monopile tower, it was represented using 18 segments of Euler-Bernoulli beam elements with different cross-sectional areas. The transition piece that is supposed to be a rigid concrete block, was modeled using a lumped mass system which gives the same mass and inertia distributions in X-, Y- and Z-directions as the actual model. The Young's modulus of beam elements within the lumped mass model was adjusted to 10^4 -fold the actual value to generate high rigidity. Pile members below the seabed were not included in the study and the turbine was assumed to be fixed at the ground, in order to be comparable with the published results of the OC4 project. Similarly, Rayleigh damping was used to model structural damping for the support structure. A critical damping ratio of 1 percent was assumed for the first and second

eigenmodes of the support structures, which includes the tower, transition piece and jacket substructure.

Figure 5.2: A finite element model of the OC4 OWT with jacket substructures

5.1.3 NREL 5-MW baseline turbine

The reference NREL 5-MW wind turbine model is a horizontal axis three-bladed turbine, of which the properties are summarized in Table 5.3. The model was built according to Jonkman et al. (2009), where the structural parts were discretized into customized beam elements following the distributed blade structural properties. Moreover, the hub heights of the turbine models were adapted

Table 5.3: Properties of NREL 5-MW baseline turbine ([Jonkman et al., 2009](#))

Parameters	Values
Rotor Orientation, Configuration	Upwind, 3 blades
Controller	Variable speed, collective pitch
Drivetrain	High speed, multiple-stage gearbox
Rotor, Hub Diameter [m]	126, 3
Cut-In, Rated, Cut-Out Wind Speed [m/s]	3, 11.4, 25
Cut-In, Rated Rotor Speed [rpm]	6.9, 12.1
Rated Tip Speed [m/s]	80

to the UpWind and OC4 reference designs, respectively. A generator-torque controller and a rotor-collective blade-pitch controller were implemented to regulate the rotor rotational speed ([Bossanyi and Witcher, 2009](#)).

The decoupled aero-servo rotor loads were generated by running simulations on the finite element RNA in FEDEM Windpower, while being fixed at the yaw node. The aerodynamic modeling utilized the generalized dynamic wake model without considering the tower influence. The rotor excitation frequency varies with respect to the rotor rotational speed, as given in Fig. 5.3. The operating rotor speed of NREL baseline turbine falls between 6.9 rpm and 12.1 rpm, where the intersections with the 1P, 3P, 6P curves correspond to frequency ranges that should be avoided when designing the support structure. Considering a safety margin of 10 percent on the maximum and minimum rotor speed, the allowable frequency range between $1P_U$ and $3P_L$ is between 0.222 Hz and 0.31 Hz, as shown in Fig. 5.4

5.2 Verification of in-house modeling and analysis tool

The following sections discussed the verification studies carried out on the in-house modeling and analysis tool against FEDEM Windpower software. Multiple sets of simulations were performed to verify different aspects of the in-house tool so as to ensure that it can provide accurate modeling and analysis of the OWT dynamics.

Figure 5.3: Campbell diagram of NREL 5-MW baseline turbine (Jonkman, 2013)

Figure 5.4: Allowable natural frequency range for the support structure (Fischer et al., 2010)

Figure 5.5 depicts the nodal and beam locations at the OC4 jacket substructure of which outputs were extracted for code verification. The load cases used in the case studies for code verification are summarized in Table 5.4.

The comparison of results obtained using the in-house code y_{Matlab} and FEDEM Windpower y_{FEDEM} was measured in the percentage difference, as given by

$$\text{Percentage difference} = \frac{y_{Matlab} - y_{FEDEM}}{y_{FEDEM}} \times 100 \%. \quad (5.1)$$

5.2.1 Verification of mass

Table 5.5 summarizes the comparison study of mass calculated in the in-house tool and FEDEM Windpower. The results matched well with those published for the OC4 project in Jonkman et al. (2012). This is essential to ensure that the structural mass that is used as the main objective function in the optimization study is correctly evaluated.

5.2.2 Verification of foundation stiffness

The foundation stiffness $[\mathbf{K}^P]$ was derived using the consistent secant stiffness formulation described in Section 4.1.5 and compared with the lumped secant stiffness formulation (linear Winkler model). The UpWind monopile reference wind turbine was modeled with the pile section below mudline, i.e., between -25 m and -49 m elevation, was discretized into different number of equal length beam elements. A static force $F_X = 790 \text{ kN}$ and moment $M_Y = 3300 \text{ kNm}$ were applied at the wind turbine hub node and static analysis was performed to obtain the structural deformation.

When comparing the pile deflection curves in Fig. 5.6, results show that both foundation stiffness models basically give very similar stiffness strength for the element size of 0.5 m. This is expected since an accurate representation

Figure 5.5: Output locations at node 34 (K-joint), node 50 (X-joint) and beam 21

Table 5.4: Description of load cases used for code verification. U_{hub} : mean wind speed at hub height; TI : turbulence intensity (ratio of standard deviation to mean wind speed) ; α : wind gradient exponent; H_s : significant wave height; T_p : peak spectral period and γ : peak shape parameter.

Load case	Enabled DOF	Wind conditions	Wave conditions
LC 1.0a	All	No wind	Still water
LC 1.0b	None, rigid model	No wind	NSS (PM) $H_s = 1.1$ m, $T_p = 2.88$ s, $\gamma = 1.0$
LC1.0c	All	No wind	ESS (JONSWAP) $H_s =$ 9.4 m, $T_p = 13.70$ s, $\gamma = 3.3$
LC 1.0d	All	NTM (Kaimal) $U_{hub} = 12.00$ m/s, $TI = 14.6$ %, $\alpha = 0.14$	Still water
LC 2.0a	All	NTM (Kaimal) $U_{hub} = 12.00$ m/s, $TI = 14.6$ %, $\alpha = 0.14$	NSS (PM) $H_s = 1.7$ m, $T_p = 5.88$ s, $\gamma = 1.0$
LC 2.0b	All	ETM (Kaimal) $U_{hub} =$ 42.73 m/s, $TI = 11.0$ %, $\alpha = 0.11$	ESS (JONSWAP) $H_s =$ 9.4 m, $T_p = 13.70$ s, $\gamma = 3.3$

Table 5.5: Comparison of mass calculated using the in-house code and FEDEM Windpower

Part	Matlab [kg]	FEDEM Windpower [kg]	Percentage Difference [%]
Rotor	3.50E+05	3.50E+05	0.00
Tower	2.16E+05	2.16E+05	0.00
Transition piece	6.66E+05	6.66E+05	0.00
Jacket	6.74E+05	6.74E+05	0.00
Internal fluid	2.05E+05	2.05E+05	0.03
Marine growth	1.81E+05	1.81E+05	0.00

of FE modeling can be attained with the smaller mesh sizes. However, as the element size increases, the differences of pile deflection at both ends become noticeable between the two formulation. The errors are more significant in the lumped foundation stiffness, where the percentage errors of the deflections at the pile head and pile tip gave 49 percent and 55 percent, in comparison with the consistent foundation stiffness that yielded percentage errors of 13 percent and 7 percent, respectively.

Figure 5.6: Comparison of pile deflections in global X-direction using different FEM methods for soil stiffness matrices. Consistent SS: consistent soil stiffness; Lumped SS: lumped soil stiffness. The dimensions (0.5 m, 1 m, 2 m, 4 m, 6 m) refer the element sizes and the corresponding number of elements are 48 elements, 24 elements, 12 elements, 6 elements and 4 elements, respectively.

Therefore, the consistent foundation stiffness approach is shown to be more robust and less sensitive to the element size variations, similar to the results presented in Griffiths (1989). This is mainly because $[\mathbf{K}^P]$ derived from the

nodal equivalent force formula captures the coupling between the nodal bending moment and translational displacement. Besides, under the dynamic loading, it is also recommended that the element sizes selected for pile modeling has to be carefully controlled. Large elements tend to induce a filtering effect on stress waves in the higher frequency, which can be important when high frequency process such as earthquake is involved (Barltrop and Adams, 2013).

5.2.3 Verification of eigenanalysis

This section verified the accuracies of the eigenfrequencies solved using the in-house code. The study not only helped to check the capabilities of the code to produce the mass and stiffness matrices accurately, but also ensured that the dynamic properties of the OWT system are correctly modeled. Four important aspects were investigated. Firstly, the in-house code had to capture the additional hydrodynamic added mass experienced by the submerged parts. Secondly, the hollow beams that were designed to flood with seawater, for instance the legs or braces of the jackets, had to account for the additional internal fluid mass. Thirdly, marine growth surrounding the structural components were typically necessary for consideration of offshore structural design, which also have influence on the hydrodynamic added mass. Lastly, although not explicitly available for input in the code, corrosion effect could be considered by reducing the member thickness based on the Offshore Standards. Table 5.6 summarize the first 20 eigenfrequencies for the overall OWT structural system calculated using the in-house code and FEDEM Windpower, considering ‘with’ and ‘without’ the marine growth.

The eigenfrequencies generated by both solvers basically agreed very well, as the maximum percentage difference for the first 20 modes was less than 4 percent. When considering marine growth, the eigenfrequencies dropped as additional mass was created. In the FE formulation, the in-house code applied

Table 5.6: Comparison of eigenfrequencies solved using the in-house code and FEDEM Windpower

Mode	Without marine growth			With marine growth		
	In-house code [Hz]	FEDEM Wind-power [Hz]	Percentage difference [%]	In-house code [Hz]	FEDEM Wind-power [Hz]	Percentage difference [%]
1	0.312	0.307	1.49	0.312	0.307	1.49
2	0.313	0.308	1.47	0.312	0.308	1.46
3	0.619	0.623	-0.76	0.619	0.623	-0.76
4	0.656	0.654	0.20	0.655	0.654	0.20
5	0.682	0.680	0.36	0.682	0.680	0.35
6	0.939	0.933	0.67	0.935	0.929	0.67
7	1.067	1.065	0.20	1.066	1.064	0.19
8	1.083	1.071	1.14	1.083	1.070	1.22
9	1.164	1.143	1.82	1.144	1.125	1.69
10	1.259	1.236	1.88	1.240	1.219	1.77
11	1.602	1.613	-0.64	1.599	1.609	-0.63
12	1.876	1.874	0.13	1.875	1.872	0.14
13	1.940	1.943	-0.16	1.940	1.943	-0.17
14	2.651	2.601	1.93	2.574	2.529	1.77
15	3.374	3.293	2.46	3.203	3.109	3.05
16	3.547	3.414	3.91	3.259	3.144	3.68
17	3.654	3.555	2.79	3.476	3.411	1.90
18	3.985	3.873	2.88	3.739	3.639	2.77
19	4.030	3.928	2.58	3.917	3.863	1.39
20	4.084	4.026	1.45	4.038	3.945	2.35

the classical Euler-Bernoulli beam theory that uses the 1st order shape function for axial and torsional modes and the 3rd order shape functions for the bending modes. On the other hand, FEDEM Windpower was based on the Euler-Bernoulli's beam theory with quadratic shape functions and continuous first derivatives (Fedem Technology AS, 2014a), hence causing slight differences in the FE modeling and the eigenfrequencies. On top of that, the in-house code modeled the transition piece as assemblage of rigid beam elements whereas FEDEM modeled it as cubic structure formed by FE tetrahedrons. This might also have effects on the mass and stiffness distribution within the OWT system. However, the effect should be very minimal as the transition piece was modeled as rigid bodies in both cases.

5.2.4 Verification of static analysis

In the static analysis, the OWT system was studied for the responses when subjected to LC 1.0a (see Table 5.4), i.e., the gravitational and buoyancy forces were active in the simulation. Tables 5.7 and 5.8 present static responses obtained at the tower top and node 50, respectively. The responses z_X , z_Z and θ_Y mainly experienced the most deformations as compared to the others because the location of the RNA's central of gravity is not situated at the yaw node, but rather slightly negative in the X-axis. The results for both solvers matched well at these key DOF, giving percentage difference of less than 10 percent. The differences were due to the usage of different FE formulation for beam elements. As for the z_Y , θ_X and θ_Z , the large percentage difference were negligible since the response measurements for these DOF should be very minimal (at a few orders of magnitude smaller than the key DOF) while the OWT model is symmetrical along XZ- plane.

Table 5.7: Comparison of static responses in global coordinates at tower top solved using the in-house code and FEDEM Windpower

Response	In-house code	FEDEM Windpower	Percentage difference [%]
z_X	-1.50E-02 m	-1.40E-02 m	7.21
z_Y	-2.23E-06 m	7.16E-08m	-3215.50
z_Z	-1.08E-02 m	-1.10E-02 m	-1.95
θ_X	7.03E-08 rad	-1.99E-09 rad	-3629.13
θ_Y	-4.74E-04 rad	-4.38E-04 rad	8.24
θ_Z	4.31E-06 rad	-4.56E-09 rad	-94603.56

Table 5.8: Comparison of static responses in global coordinates at node 50 solved using the in-house code and FEDEM Windpower

Response	In-house code	FEDEM Windpower	Percentage difference [%]
z_X	-1.56E-03 m	-1.52E-03 m	2.55
z_Y	-9.10E-06 m	-4.14E-09 m	219829.53
z_Z	-5.12E-03 m	-5.17E-03 m	-0.93
θ_X	6.55E-07 rad	3.98E-07 rad	64.81
θ_Y	-3.51E-05 rad	-3.70E-05 rad	-5.01
θ_Z	2.02E-06 rad	-6.04E-04 rad	-3439.73

5.2.5 Verification of hydrodynamic forces

The verification of Morison's loads comprised two parts. Firstly, the hydrodynamic force experienced by fixed structures, i.e., $\mathbf{f}^H$ in Eq. 4.18 was tested. The corresponding Morison's load in FEDEM Windpower was simulated through modeling the FE model as rigid parts. While subjected to LC 1.0b which consisted of small waves, both in-house code and FEDEM Windpower were still able to show good agreements, despite that the wave seed used for wave modeling in FEDEM Windpower was unable to be reproduced in the in-house code, as shown in Fig. 5.7.

The second verification was conducted on Morison's force applied on moving structures, which is basically the complete Eq. 4.17. For the in-house code, upon solving the equation of motion, structural responses were substituted back into the linearized Eq. 4.17 for Morison's force calculation. Figure 5.8 compares the drag and inertial forces computed using the in-house code and FEDEM Windpower while subjected to the LC 1.0c. The inertial force was

Figure 5.7: PDF of Morison's force on fixed structures (global X-direction) experienced at beam 21 under LC 1.0b

Figure 5.8: PDFs of (a) drag force and (b) inertia force for moving structures (global X- direction) experienced at beam 21 under DLC 1.0c

dominant over the drag force in this flow region. However, as the in-house code used only the linearized hydrodynamic damping, larger structural response was generated due to less dampening effect. Therefore, the drag and inertial forces calculated using the in-house code gave larger spread.

5.2.6 Verification of dynamics analysis

Figure 5.9 depicts an exemplary result of the response PSD obtained at node 50 (see Fig. 6.6(c)) by using different dynamic solvers under LC 2.0a. Results show that the in-house dynamic solver exhibited good agreements with FEDEM Windpower, not only in identifying the critical excitation and global eigenfrequencies, but also in matching the spectral power contents. The fully coupled aero-hydro-servo-elastic simulations carried out in FEDEM Windpower give the most accurate response analysis, since the coupling effects and non-linearity are accounted for. At low frequencies between 0.1-1.0 Hz, the inconspicuous differences between the codes could be due to different finite element models adopted instead of the coupling and aerodynamic non-linearity, as suggested by the excellent agreement between the coupled and partially decoupled simulations in FEDEM Windpower. FEDEM Windpower implements the Euler-Bernoulli beam theory with quadratic shape functions, whereas classical 3rd order shape functions are used for the in-house code. Besides, fatigue damage estimations correlate strongly with the dynamic response PSD as it is used in the spectral FLS approach. Therefore, the result gives good indications that the fatigue analysis can be carried out reliably using the linear decoupled method. The method works well for bottom fixed substructures that are relatively rigid, as they exhibit little non-linear characteristics. However, care must be taken when applying to the method to softer support structures, such as compliant towers.

Fig. 5.10 depicts the PDFs of structural responses obtained at node 34 (see Fig. 6.6(c)) under LC 2.0b. In general, slight deviations were observed

Figure 5.9: Dynamic response PSD of node 50 (X-joint) in global X-direction under LC 2.0a

Figure 5.10: Dynamic response PDF of node 34 (K-joint) in global X-direction under LC 2.0b

among the PDF, with the in-house solver yielding the widest distributions in most DOF, followed by the FEDEM decoupled model. These differences can be explained in part by the fact that the in-house solver considers both aerodynamic damping and hydrodynamic damping to be linear, while the FEDEM decoupled model uses only the former. When subjected to severe wind and wave loads, the non-linearity within aerodynamic and hydrodynamic drag forces becomes prominent, thus generating larger dampening effects. As for the aerodynamic damping, it is usually relatively small for the extreme load case, since the turbine is idling. Nevertheless, the deviations are deemed acceptable for the in-house code, as a larger spread of response tends to generate more conservative results in the extreme load analysis.

In addition, the load simulations of OWT jacket substructures can be sensitive to the joint modeling techniques. In this study, LJF was omitted for simplification purposes. More realistic representation of tubular joints using the advanced superelement method for instance can attain a higher level of accuracy when studying the global and local dynamics of the structures, and therefore can have impacts on the load distribution path and the fatigue analysis of the tubular joints (Tu and Vorpahl, 2014).

5.2.7 Verification of dynamic analysis with consideration of pile-soil dynamics

Figure 5.11 depicts the dynamic response obtained from the tower top node when the UpWind monopile reference turbine was subjected to LC 1.0d. Pile-soil interaction was modeled in the in-house solver using the consistent secant stiffness approach; whereas in FEDEM Windpower, it was modeled using the lumped non-linear spring relations applied at the pile nodes. The length of each pile element was set as 1 m in the analysis.

Generally, the in-house solver managed to simulate the overall dynamic

Figure 5.11: Response time series of tower top node for Upwind monopile OWT under LC 1.0d.

response of the support structure accurately when comparing with the fully coupled non-linear analysis performed in FEDEM Windpower. The primary deformation in global X-direction basically followed a very similar pattern in both time-series, where the response PDF and PSD also matched well like in Figs. 5.9 and 5.10 (not shown here). The differences in the first 50 seconds of the time series are due to the different initial conditions applied in both tools, where FEDEM allows equilibrium iteration to be solved at the initial position before the dynamic analysis commences. Nonetheless, the simulation for the first 60-100 seconds is usually removed from the analysis to allow this transient effect to decay.

Chapter 6

Case studies in sensitivity analysis and structural design optimization

6.1 Assessment of analytical and finite difference sensitivity analysis

In this section, computation of sensitivity derivatives were carried out using the exact analytical analysis and the conventional finite difference method, on simulated responses and failure assessment results obtained for the OWT system. The derivatives were compared between two approaches and evaluated against different factors for their influence on the accuracy of finite difference sensitivities.

6.1.1 Description of case study

The numerical simulations were performed on the OC4 OWT jacket substructure. See section 5.1 for the model description. In section 6.1.3, the HSSs were evaluated at six distinct locations. They were the middle K-joints facing upwind (K2U) and downwind (K2D), the bottom K-joints facing upwind (K3U) and downwind (K3D), the X-joints at 2nd bay (X2) and 4th bay (X4) from top, as shown in Fig. 6.1.

Design variables. The design variables selected for this study were the diameters and thicknesses of the jacket members, which were distinct for various

Figure 6.1: OC4 jacket substructure model and locations identified for simulated HSS data output

bays and member types, either legs or braces. The design variable vector $\mathbf{b}$ consisted of 22 design variables in total, i.e., where the odd and even numbered design variables referred to the member diameters and thicknesses, respectively. The sensitivities were evaluated at the initial jacket dimensions.

Design load cases. The OWT model was subject to combined wind and wave loads in the simulations. The wind and wave conditions were modeled as three-dimensional turbulent wind fields according to the Kaimal spectral model, and as Wheeler stretched irregular waves according to the JON-SWAP wave spectrum, respectively. The wind and wave parameters are summarized in Table 6.1.

Table 6.1: Description of simplified load cases used for case study in sensitivity analysis. NTM: normal turbulent wind; NSS: normal sea state; EWM: extreme wind model; ESS: extreme sea state; U_{hub} : mean wind speed at hub height; TI : turbulence intensity (ratio of standard deviation to mean wind speed); α : wind gradient exponent; H_s : significant wave height; T_p : peak spectral period and γ : peak shape parameter.

Load case	Wind conditions	Wave conditions
FLS	NTM (Kaimal spectrum)	NSS (JONSWAP spectrum)
	$U_{hub} = 8.00 \text{ ms}^{-1}$	$H_s = 1.31 \text{ m}$
	$TI = 10.00 \%$	$T_p = 5.67 \text{ s}$
	$\alpha = 0.14$	$\gamma = 1.0$
ULS	EWM (Kaimal spectrum)	ESS (JONSWAP spectrum)
	$U_{hub} = 42.73 \text{ ms}^{-1}$	$H_s = 9.40 \text{ m}$
	$TI = 10.00 \%$	$T_p = 13.70 \text{ s}$
	$\alpha = 0.11$	$\gamma = 3.3$

6.1.2 Evaluation of accuracy of sensitivity analysis

The following compares selected design sensitivities calculated using the DDM and the finite difference method. The results were measured in the normalized root-mean-square deviations (NRMSD) as

$$NRMSD = \frac{\sqrt{\frac{\sum_{i=1}^n (y_{finite,i} - y_{analyt,i})^2}{n}}}{y_{analyt,max} - y_{analyt,min}} \times 100 \% \quad (6.1)$$

where y_{finite} is the finite difference sensitivities; y_{analyt} is the analytical sensitivities; and n is the total number of sensitivity data. These sensitivities were evaluated at each node of the OWT model for both methods, while numerical integration was carried out at time steps of 0.010 s and 0.025 s. As for the finite difference method, both forward and central difference schemes were investigated. They can be computed as

$$\left(\frac{dg_i}{db} \right)_{forward} = \frac{g_i^+ - g_i}{b_n^+ - b_n} \quad (6.2)$$

and

$$\left(\frac{dg_i}{db}\right)_{central} = \frac{g_i^+ - g_i^-}{b_n^+ - b_n^-} \quad (6.3)$$

where g_i^+ , g_i and g_i^- are the design constraints evaluated at b_n^+ , b_n and b_n^- , respectively while b_n^+ and b_n^- are the positively and the negatively perturbed values of b_n (n -th design variable) that can be calculated using $b_n^+ = b_n + \Delta b_n$ and $b_n^- = b_n - \Delta b_n$, respectively. The perturbation step size Δb_n applied in the finite difference methods is 10^{-3} .

Table 6.2 shows that the central difference method attained good agreements with the DDM in estimating the displacement derivatives in six DOF with reference to the global coordinate system for both ULS and FLS load cases, which were generally less than 2.5 percent NRMSD; whereas the forward difference scheme yielded higher discrepancies of up to 6 percent NRMSD. The forward difference method generally experiences larger truncation errors in comparison with the central difference scheme, while using the same step size in the sensitivity analysis. The deviations have become more prominent when considering the nodal displacements for entire OWT system, reaching NRMSD of 12 percent. These large deviations occurred at the vertical displacement fields (global Z-axis) of the structure when varying the jacket leg dimensions. The truncation errors cascade upwards from the jacket to the other structural parts, e.g., transition piece, tower and nacelle, thereby causing large discrepancies. Similar trends could be observed in the eigenfrequency constraint derivatives when comparing between the finite difference methods, as shown in Table 6.3. The central difference approximation generally performed better than the forward difference scheme, with much smaller NRMSD, except for $\frac{dg_3}{db_4}$.

As for the sensitivities of extreme load constraints in tubular beams, it can be seen from Table 6.4 that the NRMSD were generally larger than the corresponding ULS displacement derivatives. The NRMSD could reach as high as 22.4 percent, when analyzing against the forward difference approximation

Table 6.2: Summary of NRMSD of the displacement sensitivities computed using the DDM and finite difference methods at the jacket substructure nodes. Values in parentheses refer to the NRMSD of sensitivities calculated for the overall OWT system.

Derivatives	Time steps (method)	$\frac{dg_i}{db_3}$ [%]	$\frac{dg_i}{db_4}$ [%]	$\frac{dg_i}{db_{15}}$ [%]	$\frac{dg_i}{db_{16}}$ [%]
Displacement under ULS load case	$\Delta t = 0.025$ s	2.64	2.84	5.90	5.72
	(Forward)	(2.46)	(2.39)	(7.41)	(6.79)
	$\Delta t = 0.025$ s	2.17	2.41	1.04	0.91
	(Central)	(1.82)	(2.00)	(1.16)	(0.97)
	$\Delta t = 0.010$ s	1.49	1.51	5.37	5.24
	(Forward)	(1.47)	(1.31)	(6.83)	(6.30)
Displacement under FLS load case	$\Delta t = 0.010$ s	0.88	0.97	0.49	0.40
	(Central)	(0.73)	(0.81)	(0.54)	(0.43)
	$\Delta t = 0.025$ s	3.63	2.95	4.36	4.29
	(Forward)	(4.39)	(2.71)	(11.9)	(8.92)
	$\Delta t = 0.025$ s	2.44	2.47	1.49	1.22
	(Central)	(2.10)	(2.17)	(1.48)	(1.24)
	$\Delta t = 0.010$ s	2.46	1.59	3.89	3.90
	(Forward)	(3.27)	(1.54)	(11.25)	(8.45)
	$\Delta t = 0.010$ s	0.96	0.97	0.611	0.50
	(Central)	(0.84)	(0.86)	(0.67)	(0.54)

Table 6.3: Summary of NRMSD of the eigenfrequency sensitivities computed using the DDM and finite difference methods at jacket substructure nodes

Derivatives	Method	$\frac{dg_i}{db_3}$ [%]	$\frac{dg_i}{db_4}$ [%]	$\frac{dg_i}{db_{15}}$ [%]	$\frac{dg_i}{db_{16}}$ [%]
Eigenfrequency	Forward difference	0.94	1.02	2.11	2.08
	Central difference	0.15	2.41	0.03	0.04

Table 6.4: Summary of NRMSD of the extreme load constraint sensitivities for beams computed using the DDM and finite difference methods at jacket substructure nodes.

Derivatives	Time steps (method)	$\frac{dg_i}{db_3}$ [%]	$\frac{dg_i}{db_4}$ [%]	$\frac{dg_i}{db_{15}}$ [%]	$\frac{dg_i}{db_{16}}$ [%]
Extreme load constraint (beam)	$\Delta t = 0.025$ s (Forward)	18.49	22.39	7.54	8.46
	$\Delta t = 0.025$ s (Central)	17.93	1.58	7.52	8.44
	$\Delta t = 0.010$ s (Forward)	18.26	1.79	7.46	8.38
	$\Delta t = 0.010$ s (Central)	0.78	1.01	7.33	8.24

with a 0.025 s time step. Upon investigation, it was found that these huge deviations took place at the beams which experienced alternating tension and compression modes in the sectional normal force or stress. The usage of different formulae between tension (see Eqs. 4.29a-4.29b) and compression modes (see Eqs. 4.30a-4.32) has caused the ULS state functions to be discontinuous at these time steps, as shown in Fig. 6.2. As a result, the discontinuities result in numerical errors when performing the finite difference calculations, due to the mismatches of ULS values at these affected time steps. In some cases, the discrepancies could be eliminated either when a smaller time step was utilized, or when the central difference method, with higher order of accuracy, was used, or both, since these methods might have removed the mismatches.

Regarding the extreme load constraints in tubular joints, the derivatives tallied well between the DDM and central difference method, giving NRMSD of below 2.5 percent, as shown in Table 6.5. When employing the forward difference method in estimating the sensitivities against leg dimensions, the NRMSD were found to be relatively large and the results seemed to correlate with the displacement sensitivities under the ULS load case. Since the interaction equation applies the same formula for both axial tension and compression modes, it avoids the discontinuity problem.

Figure 6.2: Comparison of ULS derivatives (beam) against b_{15} using the central difference and analytical methods. b_{15}^+ and b_{15}^- : positively and negatively perturbed b_{15} when calculating the central difference ULS gradient.

Table 6.5: Summary of NRMSD of the extreme load constraint sensitivities for joints computed using the DDM and finite difference methods at jacket substructure nodes

Derivatives	Time steps (method)	$\frac{dg_i}{db_3}$ [%]	$\frac{dg_i}{db_4}$ [%]	$\frac{dg_i}{db_{15}}$ [%]	$\frac{dg_i}{db_{16}}$ [%]
Extreme load constraint (joint)	$\Delta t = 0.025$ s (Forward)	3.91	2.37	6.16	7.35
	$\Delta t = 0.025$ s (Central)	0.86	0.92	2.06	2.11
	$\Delta t = 0.010$ s (Forward)	3.57	2.30	4.77	6.22
	$\Delta t = 0.010$ s (Central)	0.44	0.33	2.24	2.38

Finally, Table 6.6 compares between the analytical derivatives and the central difference approximations for fatigue load constraints yielded satisfying results that the NRMSD were well below 5 percent. However, no significant correlation was found between the results and the FLS displacement sensitivities, as the fatigue sensitivity analysis involves complicated calculation procedures and it probably gets significant contributions from other derivatives, such as SCF derivatives. In general, smaller time steps used in the numerical integration are found to diminish the numerical errors in the sensitivity estimations for both finite difference schemes. Additionally, the accuracy of the finite difference method also depends on the step size used in the sensitivity analysis. The two main sources of errors, i.e. truncation errors and condition errors, are positively and inversely proportional to the step size, respectively. The optimal step size can be determined from further analysis to minimize the total numerical error, yet this will introduce additional computational burdens (Iott et al., 1985).

Table 6.6: Summary of NRMSD of the fatigue load constraint sensitivities computed using the DDM and finite difference methods at jacket sub-structure nodes

Derivatives	Time steps (method)	$\frac{dg_i}{db_3}$ [%]	$\frac{dg_i}{db_4}$ [%]	$\frac{dg_i}{db_{15}}$ [%]	$\frac{dg_i}{db_{16}}$ [%]
Fatigue load constraint	$\Delta t = 0.025$ s (Forward)	6.96	1.68	2.70	1.97
	$\Delta t = 0.025$ s (Central)	4.55	1.96	1.88	0.88
	$\Delta t = 0.010$ s (Forward)	4.84	2.03	2.40	1.81
	$\Delta t = 0.010$ s (Central)	3.81	1.44	1.70	0.56

In this section, the DDM and central difference method are shown to give comparable results in calculating the derivatives. The forward difference method, on the contrary is not recommended due to the large discrepancies in estimating the sensitivities. Furthermore, the DDM method is twice as efficient in the calculation process, since only $M + 1$ number of system analyses are

required instead of $2M + 1$ for the central difference method, for every function and gradient evaluation, where M is the number of design variables.

6.1.3 Fatigue sensitivity analysis of offshore wind turbine structures

Design optimization of offshore wind turbine structures is generally fatigue driven, yet the fatigue sensitivities with respect to design variables are commonly approximated using finite differences, leading to inefficiency and unreliable information, as shown in previous section. Next, further investigation was carried out to study the influence of various factors on the accuracy of fatigue sensitivities and to highlight the possible measures to reduce these discrepancies.

Step sizes and time steps. Figure 6.3 depicts the variation of fatigue damage sensitivities with respect to step sizes and time steps used in the sensitivity analysis. The design variables were perturbed in steps of tenfold increments from 10^{-7} to 10^{-1} , while two different time steps, 0.025 s and 0.010 s, were used. Both central difference (CD) and forward difference (FD) schemes followed different patterns, with each attaining minimum numerical errors at different step sizes. The ‘optimal’ step sizes also varied for different joint and hot spot locations (not shown here), resulting in difficulties to determine a ‘good’ step size to be used in the overall design sensitivity analysis. The numerical errors consist of truncation and condition errors, which are positively and inversely proportional to the step sizes, respectively. The CD generally yielded smaller numerical errors as compared with the FD due to a higher order of approximation. However, in some cases, the CD could be more erroneous, as indicated in Fig. 6.4. Similarly, the finite difference approximations improved when smaller time steps were used. The analytical solutions for fatigue damage sensitivities between the time domain and spectral methods came closer when a smaller time step was implemented. The spectral method was more sensitive to

the time step change, since the method is based on the Fourier transform which is more susceptible to the quality of input signals. Besides, the semi-finite difference methods could help to enhance the numerical sensitivity analysis. In this method, the HSS sensitivities were estimated numerically using finite difference schemes while the final fatigue sensitivity analysis was performed using analytical formulations. The results of this approach matched well with the finite difference methods. However, in some cases, they could avoid numerical artifacts, as shown for the FD in Fig. 6.3. The above mentioned findings have demonstrated the subtle characteristics of fatigue sensitivity analysis with regard to the response sensitivities. Therefore, it is imperative to make sure that the quality of response sensitivities is sufficient when calculating the numerical gradients for fatigue damage. Often the analytical solutions are not available for the response sensitivities due to software constraints. In such case, the semi-finite difference approach can be suggested.

Joint locations and hot spot locations. Figure 6.4 summarizes the fatigue damage sensitivities calculated using the analytical, finite difference and semi-finite difference methods at various joint and hot spot locations. The last two were taken at step sizes which resulted in the smallest numerical errors. The percentage errors of numerical sensitivities y_{finite} against the analytical solutions y_{analyt} (1st bars) were listed above and below the respective bars, for the time-domain and spectral methods, which can be computed as

$$\text{percentage error} = \left| \frac{y_{finite} - y_{analyt}}{y_{analyt}} \right| \times 100 \%. \quad (6.4)$$

In Fig. 6.4(a), the fatigue damage sensitivities at specific joint locations are shown to be localized, i.e., to be influenced most by the design variables

Figure 6.3: Comparison of fatigue damage sensitivities varying against design variable step sizes and time steps in the numerical sensitivity analysis, for (a) spectral and (b) time domain methods. Sensitivities were calculated against b_{13} at hot spot location 1 of K3U (leg side) joint.

Figure 6.4: Comparison of fatigue damage sensitivities at various (a) joint locations and (b) hot spot locations. (a) Sensitivities were calculated against b_3 at hot spot location 1 for various joints and (b) Sensitivities were calculated against b_8 at K2U (leg side) for 8 hot spot locations using time domain ('time') and spectral ('freq') methods.

which the joint was directly connected to. The finite difference schemes also gave smaller errors when evaluating fatigue sensitivities at locations where the design variable was directly connected to. On the contrary, the associated errors at joint locations where the design variables were not in connection with could reach values as high as 1700 percent or could be wrong in sign. Whereas Fig. 6.4(b) indicates that the fatigue damage sensitivities were distinct at various hot spot locations. Although the time-domain and spectral methods differ in estimating the damage sensitivities (of which the accuracy depends on the quality of response sensitivities), the percentage errors of their finite difference counterparts were generally similar in scale.

Stress concentration effects. So far all the fatigue damage sensitivities discussed earlier have considered the SCF variations against the design variables. However, since the empirical formulae to estimate the SCF are laborious to use, it raised the interest to investigate the contributions of SCF derivatives in the overall fatigue sensitivity analysis. HSS are geometric stresses that account for stress concentration effects occurring at the joint regions. The sensitivities depend on both the sensitivities of nominal stress as well as the sensitivities of SCF with respect to design variables. Fig. 6.5 shows the analytical fatigue damage sensitivities, performed with and without consideration of the SCF derivatives. The SCF were treated as constants in the latter case. Results indicate that the SCF derivatives exerted significant influences in the calculation, for all joint types. In some cases, the sensitivities could be inverted in sign. Therefore, it is important to include the SCF variations with respect to the design variables in the gradient assessment of fatigue damage, when HSS are used.

Figure 6.5: Comparison of analytical fatigue damage sensitivities with and without consideration of SCF derivative. Sensitivities were calculated against b_i at hot spot location 1 for various joints using time domain ('time') and spectral ('freq') methods.

6.2 Performance evaluation of design optimization framework

In this section, the integrated analysis and optimization framework was evaluated for its efficiency, effectiveness and practicality when used in solving the design problem of OWT support structures.

6.2.1 Description of case studies

In the following case studies, two different types of OWT systems were investigated, where the detailed models are described in Section 5.1.

6.2.1.1 Case study 1: Jacket substructure design

The interest of this case study is to optimize the dimensions of the jacket substructure, where the results are discussed in Sections 6.2.2, 6.2.3 and 6.2.4. While the RNA, tower and transition piece were maintained the same, two different jacket models were tested: the OC4 and the UpWind jackets (Vorpahl et al.,

2011; Vemula, 2010). The former is a simplified model adapted from the latter that has been studied in previous sections, whereas the latter comprises joint cans for reinforced structural performance around the tubular joints. The joint cans were modeled explicitly for the UpWind model by introducing more design variables, which can be modified independently from the tubular members.

The tubular members were represented using Euler-Bernoulli beam elements whereas the tubular joints were modeled as members connected rigidly at the intersection points of the members' centerlines, i.e., the overlap in members and the local joint flexibility (LJF) at the tubular joints were not accounted for in this study. The legs of the jacket were assumed to be free flooded by seawater while the braces were not. The substructure between water depths of -2 m and -40 m was covered with a layer of marine growth that has a thickness of 0.1 m and a density of 1100 kg/m^3 , following the (Vorpahl et al., 2011).

The material properties of the steel jacket are summarized in Table 6.7. Bi-linear S-N curves were used in the fatigue analysis. All components above the MSL referred to the S-N curves in air whereas the submerged parts used the S-N curves in seawater with cathodic protection (DNV, 2012).

Table 6.7: Material properties of steel used for the jacket substructure and monopile support structure

Parameter	Notation [units]	Numerical values
Density	ρ_s [kg/m^3]	7.8E+3
Young's modulus	E_s [Pa]	2.1E+11
Yield strength	f_{ys} [Pa]	2.6E+8
Poisson's ratio	ν	0.3
S-N curve 1 in air ($N \leq 10^7$ cycles)	m_1	3.0
	$\log \bar{a}_1$	12.164
S-N curve 2 in air ($N > 10^7$ cycles)	m_2	5.0
	$\log \bar{a}_2$	15.606
S-N curve 1 in seawater ($N \leq 10^6$ cycles)	m_1	3.0
	$\log \bar{a}_1$	11.764
S-N curve 2 in seawater ($N > 10^6$ cycles)	m_2	5.0
	$\log \bar{a}_2$	15.606

Design variables. A total number of 22 and 38 design variables were defined for

each model as indicated in Figs. 6.6(a) and (b), respectively. The odd and even indexed variables denote the member outer diameters and thicknesses, respectively, i.e., $[b_1, b_2, b_3, b_4, \dots, b_{M-1}, b_M] = [D_1, T_1, D_2, T_2, \dots, D_{M/2}, T_{M/2}]$. Five sets of initial dimensions $\mathbf{b}^{(0)}$ were investigated on the OC4 jacket (ini-

Figure 6.6: Design variables defined for (a) the OC4 jacket and (b) the UpWind jacket (proportion of the structure not to scale)

tial designs 1-5) and one set for the UpWind jacket (initial design 1). The initial design 1 refers to the original OC4 jacket dimensions, while others are randomly generated. The lower and upper bounds $\mathbf{b}_{min}$ and $\mathbf{b}_{max}$ were set to 30 percent and 300 percent of the OC4 jacket dimensions, respectively. Fig. 6.7 explains the geometrical terms for tubular joints as defined in this study, while Table 6.8 summarizes the validity range of various siz-

ing dimensions with reference to the limit state functions implemented. For D_{chord}/T_{chord} and D_{can}/T_{can} , a smaller bound between 20 and 64 was used instead, upon combining the recommendations from both standards.

Figure 6.7: Geometrical definitions of tubular joints

Table 6.8: Validity range of various sizing dimensions for a tubular joint

Variables	Lower bound	Upper bound	Reference
D_{stub}/D_{can}	0.2	1.0	(Standards Norway, 2013; DNV, 2012)
T_{stub}/T_{can}	0.2	1.0	(DNV, 2012)
T_{can}/T_{chord} and T_{stub}/T_{brace}	-	2.0	(DNV, 2012)
D_{chord}/T_{chord}	16	64	(DNV, 2012)
and D_{can}/T_{can}	20	100	(Standards Norway, 2013)
D_n/T_n	-	120	(Standards Norway, 2013)

Design load cases. The wind and wave data was gathered from a 3-hour average for a period of 22 years at the K13 platform in the Dutch North Sea (Fischer et al., 2010). Tables 6.9 and 6.10 summarize the extreme and fatigue DLCs used in the optimization simulations. They correspond to the partial load cases carried out for the final design phase in the UpWind

project (Vemula, 2010). The wind condition was modeled as a three-dimensional turbulent wind field according to the Kaimal spectrum, while the wave condition was modeled as a Wheeler stretched irregular wave following the JONSWAP spectrum based on the linear wave theory.

Table 6.9: Description of extreme design load cases used for case study in design optimization. U_{hub} : mean wind speed at hub height; TI : turbulence intensity (ratio of standard deviation to mean wind speed); α : wind gradient exponent; H_s : significant wave height; T_p : peak spectral period and γ : peak shape parameter.

DLC	U_{hub} [m/s]	TI [%]	α [—]	H_s [m]	γ [—]	T_p [s]
6.1a	42.73	11.0	0.11	9.40	3.3	13.70

Table 6.10: Description of fatigue design load cases used for case study in design optimization. U_{hub} : mean wind speed at hub height; TI : turbulence intensity (ratio of standard deviation to mean wind speed); α : wind gradient exponent; H_s : significant wave height; T_p : peak spectral period and γ : peak shape parameter and P_v : event probabilities.

DLC	U_{hub} [m/s]	TI [%]	α [—]	H_s [m]	T_p [s]	γ [—]	P_v [%]
6.4a	2.0	29.2	0.14	1.07	6.03	1.0	4.958
1.2a	4.0	20.4	0.14	1.10	5.88	1.0	9.985
1.2b	6.0	17.5	0.14	1.18	5.76	1.0	11.333
1.2c	8.0	16.0	0.14	1.31	5.67	1.0	13.491
1.2d	10.0	15.2	0.14	1.48	5.74	1.0	12.287
1.2e	12.0	14.6	0.14	1.70	5.88	1.0	12.982
1.2f	14.0	14.2	0.14	1.91	6.07	1.0	9.995
1.2g	16.0	13.9	0.14	2.19	6.37	1.0	8.729
1.2h	18.0	13.6	0.14	2.47	6.71	1.0	5.723
1.2i	20.0	13.4	0.14	2.76	6.99	1.0	3.836
1.2j	22.0	13.3	0.14	3.09	7.40	1.0	3.304
1.2k	24.0	13.1	0.14	3.42	7.80	1.0	1.489
6.4b	30.0	11.8	0.14	4.46	8.86	1.0	1.701

The DLC 6.1a simulated extreme winds and waves with a 50-year returning period where the OWT was modeled in idling mode. Regarding the FLS DLCs, both DLCs 1.2 and 6.4 were combined into a reduced (lumped) scatter diagram that segregates the events based on different U_{hub} bins and P_v . The DLC 1.2 modeled the turbine in power production mode that

operates in the normal turbulent wind and wave conditions; whereas the DLC 6.4 modeled the turbine to be idling under the wind conditions below the cut-in and above the cut-out wind speeds. All turbine blades at idling were feathered to the pitch angle of 90° to neutralize the torque on the rotor shaft, without applying a brake. Both winds and waves were assumed to be co-directional and applied in the global X-direction towards the side of the jacket where the turbine was facing to without any yaw error. Besides, no current model was included in the simulations. For each DLC, a dynamic analysis with a simulation period of 660 seconds was carried out. The first 60 seconds of the simulation results were discarded to allow initial transients to decay. This has been abbreviated from the simulation time recommended by the IEC standard or the offshore recommended practice, in order to shorten the computational time required for the case study. Furthermore, a time step of 0.025 seconds was used to generate the rotor load time series in FEDEM Windpower and also for the numerical integration in the Matlab dynamic solver. During the optimization process, the same sets of turbulent wind seeds and irregular wave seeds were retained for all iterations.

6.2.1.2 Case study 2: Monopile support structure design

The overall monopile support structure including the tower, transition piece and monopile was optimized in this case study. The monopile support structure was modeled using EB beam elements with constant cross-sectional areas connected concentrically from the pile tip to the tower top. The structural part below the MSL was assumed to be non-flooded and marine growth was no included in the analysis. The support structure was made of steel with material property summarized in Table 6.7. Besides, the monopile was embedded in sand soils with soil properties simplified from Table 5.2, where $\gamma' = 10000 \text{ N/m}^3$ and $\Phi = 36^\circ$

for all depths.

Design variables. A total number of 27 design variables were defined for the monopile support structure as illustrated in Fig. 6.8. They consisted of 26 sizing design variables $[b_1, b_2, b_3, b_4, \dots, b_{25}, b_{26}]$ that correspond to the diameters and thicknesses of each section, i.e., $[D_1, T_1, D_2, T_2, \dots, D_{13}, T_{13}]$ and a shape design variable b_{27} , which refers to the embedded pile depth, i.e., the pile tip Z-coordinate. The pile section below the mudline was discretized into 24 equal length elements, where the length and nodal coordinates of each element are varying with respect to b_{27} during the course of iteration. The initial dimensions $\mathbf{b}^{(0)}$ implemented in the optimization analysis were adapted from the UpWind reference monopile design, whereas the lower and upper bounds $\mathbf{b}_{\min}$ and $\mathbf{b}_{\max}$ were set to values described in Table 6.11.

Table 6.11: Lower and upper bounds of design variables $\mathbf{b}$

$\mathbf{b}$	$\mathbf{b}_{\min}$	$\mathbf{b}_{\max}$
b_1, b_3, b_5, b_7	5.5	7.5
b_2, b_4, b_6, b_8	0.02	0.10
b_9, b_{11}, b_{13}	4.5	7.0
b_{10}, b_{12}, b_{14}	0.01	0.10
b_{15}, b_{17}, b_{19}	4.0	7.0
b_{16}, b_{18}, b_{20}	0.01	0.10
b_{21}, b_{23}, b_{25}	3.0	6.0
b_{22}, b_{24}, b_{26}	0.01	0.10
b_{27}	-53.0	-45.0

Design load cases. The design load case implemented in this case study is the DLC 6.1a as detailed in Table 6.9. Ultimate limit state checks on shell structures comprising $g_{ULS1} - g_{ULS6}$ and g_{ULS8} were examined in the optimization process, along with the eigenfrequency constraint g_{eigen} and deformation constraint g_{deform} . It should be noted that the DLC 6.1a based on Fischer et al. (2010) was originally adapted for deep water study

Figure 6.8: Design variables defined for monopile support structure (proportion of the structure not to scale)

(50 m). It was applied in this case study to investigate the optimization framework in solving the shape optimization problem, as explained in Section 6.2.5.

6.2.2 Performance of integrated optimization framework

The optimization of the OC4 jacket model was carried out by applying the proposed analytical method and the conventional finite difference approach for the design sensitivity analysis. Fig. 6.9 displays the variations of objective functions and maximum constraint violations during the optimization process. It is apparent from this figure that the analytical gradient-based method was more efficient than the finite difference approach in searching for the optimal design, as fewer

iterations were required to attain the convergence of solutions. The findings are consistent with previous research, which found that the finite difference sensitivity information could be unreliable and inaccurate when applied in the dynamic problems for offshore wind turbine system, and therefore resulting in slow convergence. Moreover, the improved efficiency also extends to sensitivity analysis, as summarized in Table 6.12. The run time required for each computation phase varies, but phases 1 and especially phase 3 take most of the computation time in general. On one hand, the analytical gradient-based method can recycle the $[\mathbf{M}]$, $[\mathbf{C}]$ and $[\mathbf{K}]$ matrices as well as the eigenvectors (eigenvalues) solved in Eq. (4.27) for the gradient evaluations in Eq. (4.58). This saves the computation time required for assembling the finite element models and solving the eigenvalue problems. On the other hand, the computation at each iteration only requires $M + 1$ number of system analyses, as compared with $2M + 1$ for the central difference (CD) method, for every function and gradient evaluation, where M is the number of design variables. The number of iterations taken for the analytical gradient-based method to converge varied moderately among the 5 initial designs tested, ranging between 22 and 31 iterations.

Table 6.12: A comparison of computation time required for analytical and central difference sensitivity analysis for every functional and gradient evaluation, where M is the number of design variables

Computation phase	Analytical method	Central difference method
Determine $[\mathbf{M}]$, $[\mathbf{C}]$, $[\mathbf{K}]$ matrices, $\mathbf{f}$ vector and eigenvectors (eigenvalues)	1	$2M + 1$
Determine $\frac{d[\mathbf{M}]}{db}$, $\frac{d[\mathbf{C}]}{db}$, $\frac{d[\mathbf{K}]}{db}$ and $\frac{d\mathbf{f}}{db}$	M	Not required
Solving equations of motion	$M + 1$	$2M + 1$

Another important observation made is that the accuracy of the optimization procedure was affected, since the finite difference gradient-based method converged to a different solution which was heavier and infeasible. Figs. 6.10 and 6.11 show the dimensions and performance of the initial and optimized jacket de-

Figure 6.9: Variation of design objective functions and maximum constraint violations during the optimization of the OC4 jacket substructure (initial design 1). The numbers in the legend refer to the corresponding values at the final iteration. Analy: analytical method. CD: central difference method.

signs presented in the study. In general, the diameter-to-thickness ratios of legs and braces were smaller for the optimal design given by the analytical gradient-based approach. Both optimal designs have shown improvements in diminishing the constraint violations occurring at the X-braces. However, being misled by the incorrect approximation of constraint gradients, the thicknesses of leg members were reduced too much to maintain the required buckling strengths at the central bays. As the OWT system is exposed to high cycle fatigue loads, the accuracy of finite difference approximations can be susceptible to the step sizes used in perturbing the design variables, the time steps applied in the numerical integration, the joint or HSS locations where the gradients are evaluated, etc. (Chew et al., 2015a). Besides, for tubular beams that experience alternating tension-compression modes, numerical artifacts can occur when calculating the extreme load constraint sensitivities, due to the possible mismatches of ULS values at the affected time steps (Chew et al., 2015c). As some of these factors

can be interdependent, it would be very computationally taxing to determine the “optimal” values to use, especially when a large number of design load constraints are implemented.

It is also interesting to note that the mathematical formulae for the sensitivity analysis, i.e., Eqs. (4.54)-(4.65) can be generalized to include variations with respect to member lengths, brace angles and leg distances. This allows for studying more general changes in the configuration of OWT jacket substructures during optimization. However, the needed sensitivity derivatives will be more complicated to calculate, and it is presently unclear if the gradient-based optimization algorithm will be effective in solving such more general configuration optimal design problems.

6.2.3 Local and global optimal solutions

In Figs. 6.10 and 6.11, all the initial designs 1-5 converged to the same optimal OC4 jacket solution using the analytical gradient-based method. The first-order optimality, i.e., a necessary but not sufficient measurement of the solution being at minimum, fell below the bound of 10^{-6} ; while the maximum constraint violation was less than the constraint tolerance of 10^{-4} , for all cases. For this design problem, the local uniqueness of solution is guaranteed since the external loads, initial conditions and boundary conditions applied do not vary independently with respect to the design variables during the iterations. A check on the nearby points of the solution or a multi-start analysis as performed did not find any better solution, thus strongly indicating that an optimum is found. However, as the overall problem formulation is non-convex, the proposed gradient-based algorithm may only locate a local optimum instead of the global optimum during the optimization process. Multiple repetitions of the same local optimization procedures were carried out from different starting points, in an attempt to search in more than one possible basin of attraction. Interestingly, it seems

Figure 6.10: Dimensions of initial and optimized designs for the jacket substructures. Analy: analytical method. CD: central difference method.

Figure 6.11: Structural performance of initial and optimized designs for the jacket substructures. The coordinates (x-utilization factor, y-height) of several points that are outside the plot limits are labeled in (a) and (c). Analy: analytical method. CD: central difference method.

possible that the large number of design constraints formulated in the problem has constrained the feasible regions significantly enough for the global optima to be attained in this sizing optimization problem. Although it is not the main interest of this study to prove the convexity of the design optimization problem, it can be observed that the mass reduction objective function which is a hyperbolic paraboloid (non-convex), for instance, has become convex within the sizing constraints implemented. Nevertheless, it is still very challenging to determine the situation for the dynamic load constraints. All the starting points used in the study were sampled from the design space that fulfills the sizing constraints (but not the load constraints); while the SQP algorithm applied strictly obeys the bounds at every iteration.

6.2.4 Influence of design constraints on structural design and optimization process

The influence of design constraints on the structural design and behavior is quite revealing in several ways. Comparing Figs. 6.10 and 6.11, it can be seen that the mass reduction attained in the optimized design depends strongly on how well each structural component is utilized against the prescribed limit state functions. The trade-offs among the leg and brace dimensions to maximize the utilization are non-trivial and complex. Competition exists among the design constraints to be the critical design factor, while the criticality can change during the course of optimization. For instance, in the UpWind jacket configuration, the buckling load constraints on the leg members overtook the fatigue load constraints at the TYK-joints (leg side) to be the critical constraints for the topmost leg section. As for the braces, the X-joints are normally more critical than the TYK-joints, since higher SCFs are produced around the X-joints. Furthermore, being higher in fidelity, the UpWind jacket allowed better utilization of structural strengths for the members and joints. As a result, it yielded an optimal design that was

22 percent lighter in comparison with the OC4 counterpart. The inclusion of joint cans in the design modeling spread the geometry of legs and braces more evenly, where the outer diameters of legs were decreasing gradually from bottom to top, while the thicknesses of braces were following the opposite.

Besides, a more realistic optimal design would require more load cases and longer simulation times when formulating the design constraints. In fact, hundreds and thousands of load simulations considering different DLCs and loading directions ought to be carried out, while each simulation should run at least 1 hour according to IEC standard or at least 3 hours for wave loads according to the recommended practice in offshore engineering. Nevertheless, by incorporating the critical DLCs in the optimization process as attempted here, the integrated methodology can be used to generate preliminary designs that are lighter and better utilized. The use of shorter simulations is generally sufficient for optimization purposes, since it is more important to obtain accurate estimates of the changes in the design constraint than of the actual design constraint itself. For jackets substructures, it was also found that wind loads contribute most to the uncertainty in the fatigue lifetime estimate compared to wave loads, and therefore it seems not necessary to use full 3 hours simulations for the dynamic analysis ([Zwick and Muskulus, 2015](#)). Of course the final, converged design needs to be verified and fine-tuned with further analysis and longer simulation runs in the detailed design stage.

On the other hand, Figs. [6.12](#) and [6.13](#) display the variations of eigenfrequency, extreme load and fatigue load constraints during the optimization process. The first mode side-to-side and fore-aft eigenfrequencies varied in a similar pattern as the structural mass, implying that a lighter support structure correlates to “softer” design. The eigenfrequencies fluctuated within the allowable limits and finally converged to approximately 0.28 Hz. This finding is somehow different from the case study of a monopile tower which was investi-

gated in (Haghi et al., 2014), whereby the first eigenfrequency converged to the $1P_U$. The jacket substructures are generally stiffer than the monopile towers, therefore they do not allow significant dips in the eigenfrequencies, particularly when only sizing optimization is considered. In addition, multiple eigenfrequency checks can be included in the optimization design studies to avoid local vibrations at the overhanging X-braces, for instance, which can occur at high excitation frequencies, i.e., multiples of blade passing frequencies. Among all the design load constraints, the buckling and compressive load constraint g_{ULS2} and g_{ULS3} , as well as the fatigue load constraint g_{FLS} were found active during the optimization process (see Fig. 6.13). The tensile state function g_{ULS1} and extreme load constraint for tubular joints g_{ULS7} were relatively more active in fluctuating within the allowable limit. In contrast, the variation of g_{ULS5} and g_{ULS6} were very small at all iterations. Long tubular members generally experience minimal shear force and torsional moment in frames whereby the diameters of members are also relatively too small for hoop buckling to occur. Moreover, g_{ULS4} is a conditional design constraint, and it was not activate in most of the iterations. As highlighted previously, g_{ULS2} and g_{FLS} remained as the active constraints until the final iterations, where g_{ULS2} tended to gain influence when the design was gradually tuned to be less fatigue prone towards the end. Nevertheless, g_{FLS} is generally the final design constraint to converge during the optimization of the OC4 jacket substructure.

Importantly, the results signify that both design constraints should be included concurrently in the optimization procedure, instead of handling them sequentially. For instance, although infeasible, the “optimized” OC4 jacket solution produced by the CD gradient-based method in Fig. 6.11 shows that the buckling failure mode could be possibly more prominent than the fatigue failure mode for offshore wind turbine frame structures, which goes against the general belief within the industry that the structures should be fatigue driven.

Figure 6.12: Variation of eigenfrequency constraint functions during the optimization of the OC4 jacket substructure (initial design 1)

Figure 6.13: Variation of extreme and fatigue load constraint functions during the optimization of the OC4 jacket substructure (initial design 1)

6.2.5 Shape optimization of wind turbine support structures

Figure 6.14 shows the variation of the objective function and constraint violation when optimizing the monopile support structure in Case Study 2. Results show that the optimization procedure took more iterations in comparison with Case Study 1 to converge to an optimal design. The support structure has basically attained a solution that is close to optima in the 30-th iteration, where the design variables were subsequently adjusted gradually to fulfill the design constraints.

Figure 6.15 depicts the dimensions of the initial and optimized monopile support structures. Lighter support structure solution was yielded with the reduction of outer diameters and thicknesses. The pile section below the mudline converged to the minimum bound defined while the substructure section increased in outer diameters and thicknesses with the water depth. Maximum outer diameter was attained at the transition piece which was then tapered to the tower top. The distribution of mass and structural stiffness was optimized to keep the eigenfrequencies outside of the rotor and blade passing frequency zones. Besides, as the load being transferred from the turbine to the soil higher shear and moment will be experienced at the bottom parts of the support structure. Nevertheless, the transition of optimal dimensions between the substructure and transition piece was affected by the hydrodynamic added mass and buoyancy introduced to the system dynamics, therefore giving the sudden increase in the outer diameter above the MSL.

The embedded pile length eventually converged to -46.07 m, where the maximum pile head deflection was 83 mm. This result matched the findings in Krolis et al. (2010), where the deformation constraint can be fulfilled with embedded pile between $5.0D$ and $3.3D$ for 5.0 MW turbines. In this study, the check on soil strength due to pile deformation was not included as a design constraint, while the pile deformation was set to be constrained within the soil elastic zone. Having said, further possible reduction of structural mass can be

Figure 6.14: Variation of design objective function and maximum constraint violations during the optimization of the monopile support structure. The numbers in the legend refer to the corresponding values at the final iteration.

attained if the utilization of soil strength is further extended to the plastic zone. Conventionally, the pile dimensions are designed and optimized in a static analysis before the design of wind turbine structures. As a result, the design might not be optimal under the entire support structure standpoint. The result presented has demonstrated the capability of the proposed optimization framework in solving this shape optimization problem.

Figure 6.15: Dimensions of initial and optimized designs for the monopile support structure, represented as variation of outer diameters of monopile sections (X-axis) with respect to the tower height (Y-axis)

Chapter 7

Conclusion and recommendations

7.1 Conclusion

This thesis has investigated the structural optimization problem related to off-shore wind energy system. An integrated analysis and optimization framework was developed, utilizing the analytical gradient-based optimization approach while implementing the dynamical system as a decoupled aero-servo loading acting on a linear hydro-elastic OWT. Case studies were carried out to verify the performance of the framework, whereby findings and implications are summarized in three aspects as follows:

Modeling and simulation. The partially decoupled method has shown to demonstrate reliable and conservative response analyses which is comparable to the fully-coupled non-linear simulation performed in FEDEM Windpower. However, it should be noted that this method might only work well for bottom fixed substructures, as they exhibit little non-linear characteristics. Besides, the method could be well suitable for the initial design phase of support structures in actual industrial applications where typically the foundation designers do not have access to the information required for the fully-coupled analysis to be performed. In this case, the turbine manufacturers just need to provide the foundation designers a set of hub height load time series generated for the relevant design load cases. While inte-

grated into the proposed design framework, the optimization process can be automated since the aero-servo load time series is reused for each iteration, therefore eliminating the need for the turbine manufacturer to rerun the load analysis when the foundation design is updated.

Sensitivity analysis. Exact solutions to calculate the dynamic sensitivity derivatives for various design checks that are commonly evaluated in offshore wind industry were derived. The analytical sensitivity analysis could eliminate numerical artifacts related to finite difference approximations that could result in significant impacts on the gradients of ULS and FLS constraints. When using the finite difference method in assessing the extreme load constraint gradients, numerical errors could occur due to the discontinuities of ULS utilization factors when the beams are experiencing internal forces or stresses switching between tension and compression modes. Besides, fatigue sensitivity analysis was very susceptible to the quality of response sensitivities. Several recommendations which could help improve the overall quality of numerical sensitivities include using sufficiently small step sizes when perturbing the design variables; implementing more robust time domain methods when evaluating the fatigue damage; or adopting the semi-finite difference methods. When the hot spot stress methodology is used for the fatigue assessment, the derivatives of SCF exerted significant influences in determining the fatigue damage sensitivities, therefore should always be considered.

Overall optimization framework. The overall optimization framework could converge to optimum more efficiently and effectively in comparison with the conventional finite difference gradient-based method. A considerable amount of computation time can be saved in the optimization iterations, the sensitivity analysis, the matrix assembly and solving of the eigenvalue

problems. The optimization framework was also shown to be reliable for the convergence to true optimum and robust against the change of starting points. Besides, in cases where the design problem of the wind turbine structures was heavily constrained, global optimum could probably be found by the proposed local optimizer, as inferred by the exact convergence of all design solutions in the multi-start analysis in the case study. As the design checks were implemented concurrently, the framework could easily study trade-offs among all structural components to maximize the utilization and the competition between the design constraints to be the critical design driver. Additionally, further cost savings could be attained through designing the offshore wind turbine support structure as a whole instead of separating the design of piles and substructures, as demonstrated in the shape optimization of monopiles. This could be well applied to the industry since offshore wind turbine support structures are typically designed specifically for a wind farm site.

7.2 Recommendations

In addition, many issues related to the design optimization of wind turbine system remain open, which ought to be investigated further. We list below some plausible future directions in this area of research.

Life-cycle design optimization. Design optimization of offshore wind turbine support structures should be further extended to consider the overall life-cycle costs. These include expenses that are incurred throughout the wind turbine service life, spanning from production, commissioning, operation and maintenance, to decommissioning of wind turbines. The assessment of life-cycle costs becomes important as the key design driver for each phase

could be different and therefore lead to a “biased” optimal design solution instead. For instance, designing a support structure that minimizes the structural mass (capital cost) might in return cause more maintenance activities to be planned in the later operational stage. Therefore, with proper consideration of life-cycle cost analysis, the design process will produce a more robust design solution for the offshore wind turbine structures.

Design optimization under uncertainties. Wind turbine dynamics is very much affected by the variability and uncertainty of wind, wave and soil characterization, aerodynamic and hydrodynamic load calculation, and material properties, etc. As a result, design optimization of offshore wind turbine structures should be performed considering these uncertainties, where the probabilistic design optimization techniques ought to be adopted. Currently, these techniques are often limited by the large computational requirement in assessing the structural reliability. With better understanding gained on the impacts of the uncertainties to the structural reliability, we might be able to simplify the failure probability model and solve the optimization problem in a more efficient way.

Non-linear dynamic sensitivity analysis. System non-linearities become more prominent with the increasing wind turbine sizes and more flexible wind turbine support structures being introduced to the wind industry. In cases where gradient-based optimization is preferred for designing the structures (mainly due to the speed of convergence), accurate and efficient methods to perform non-linear dynamic sensitivity analysis have to be developed. Given that the sensitivities of offshore wind turbine dynamics is highly susceptible to numerical errors due to the perturbation step sizes of structural dimensions used in the finite difference method for instance, a good balance between accuracy and efficiency is required when adapting the

non-linear sensitivity analysis techniques to this application. As an alternative, investigation of using some advanced finite difference techniques such as the complex Taylor series expansion for approximation purposes or solving the analytical design sensitivities from non-linear dynamics can be carried out.

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Appendix A

Formulation of the finite element method

A.1 Basic FE matrices for EB beams

This section summarizes the basic FE matrices adopted in the in-house code.

1. The shape function matrix $[\mathbf{N}]$ for the EB beam element is given by

$$[\mathbf{N}] = \begin{bmatrix} 1 - \xi & 0 & 0 & 0 \\ 0 & 1 - 3\xi^2 + 2\xi^3 & 0 & 0 \\ 0 & 0 & 1 - 3\xi^2 + 2\xi^3 & 0 \\ 0 & 0 & 0 & 1 - \xi \\ 0 & 0 & -(\xi - 2\xi^2 + \xi^3)L & 0 \\ 0 & (\xi - 2\xi^2 + \xi^3)L & 0 & 0 \\ \xi & 0 & 0 & 0 \\ 0 & 3\xi^2 - 2\xi^3 & 0 & 0 \\ 0 & 0 & 3\xi^2 - 2\xi^3 & 0 \\ 0 & 0 & 0 & \xi \\ 0 & 0 & (\xi^2 - \xi^3)L & 0 \\ 0 & -(\xi^2 - \xi^3)L & 0 & 0 \end{bmatrix}^T \quad (\text{A.1})$$

where ξ is x/L ; x is the distance from the end node 1 along the beam axis, $x \in [0, L]$ and L is the beam length. The 1st, 2nd, 3rd and 4th rows of $[\mathbf{N}]$ correspond to the axial, bending and torsional DOF.

2. The strain-displacement matrix $[\mathbf{B}]$ for the EB beam element is obtained by differentiating $[\mathbf{N}]$ with respect to x , once for the 1st and 4th rows and twice for the 2nd and 3rd rows, i.e.,

$$[\mathbf{B}] = \begin{bmatrix} -\frac{1}{L} & 0 & 0 & 0 \\ 0 & -\frac{6}{L^2} + \frac{12x}{L^3} & 0 & 0 \\ 0 & 0 & -\frac{6}{L^2} + \frac{12x}{L^3} & 0 \\ 0 & 0 & 0 & -\frac{1}{L} \\ 0 & 0 & \frac{4}{L} - \frac{6x}{L^2} & 0 \\ 0 & -\frac{4}{L} + \frac{6x}{L^2} & 0 & 0 \\ \frac{1}{L} & 0 & 0 & 0 \\ 0 & \frac{6}{L^2} - \frac{12x}{L^3} & 0 & 0 \\ 0 & 0 & \frac{6}{L^2} - \frac{12x}{L^3} & 0 \\ 0 & 0 & 0 & \frac{1}{L} \\ 0 & 0 & -\frac{6x}{L^2} + \frac{2}{L} & 0 \\ 0 & \frac{6x}{L^2} - \frac{2}{L} & 0 & 0 \end{bmatrix}^T. \quad (\text{A.2})$$

A.2 FE formulation of Morison's force

For a slender structural beam of length dL in a general fluid flow, if the beam possesses cross-sectional dimension that is sufficiently small, i.e., wave length of the flow $> 5 \times$ effective diameter of the beam D , the gradients of the flow velocity and acceleration can be assumed not influenced by the submerged beam. Therefore, Morison's formula is applicable and the hydrodynamic force can be calculated in the relative velocity form as

$$d\mathbf{f}_{\text{Morison}} = \left[\frac{\pi \rho_w D^2}{4} (C_M \dot{\mathbf{v}} - C_A \ddot{\mathbf{z}}) + \frac{\rho_w D}{2} C_D (\mathbf{v} - \dot{\mathbf{z}}) |\mathbf{v} - \dot{\mathbf{z}}| \right] dL \quad (\text{A.3})$$

Figure A.1: A three-dimensional beam section and the associated dynamic responses and wave kinematics in local coordinates

where ρ_w is the water density; $\mathbf{v}$ and $\dot{\mathbf{v}}$ are the velocity and acceleration of the water particle; $\dot{\mathbf{z}}$ and $\ddot{\mathbf{z}}$ are the velocity and acceleration of the structural member; and C_M , C_A and C_D are the hydrodynamic coefficients for inertia, added mass and drag, respectively. In general, the hydrodynamic force acting on a submerged beam can be decomposed to a normal force, a tangential force and a lift force. The normal hydrodynamic force is most significant among the three and will be considered in this subsequent formulation. Hence, $\mathbf{v}$, $\dot{\mathbf{v}}$, $\dot{\mathbf{z}}$ and $\ddot{\mathbf{z}}$ are corresponding to the velocity and acceleration components normal to the beam axis, as shown in Fig. A.1.

Given a three-dimensional beam section of length dL subjected to Morison's load, Eq. A.3 can be split into three parts:

1. $\frac{\pi\rho_w D^2}{4}C_M\dot{\mathbf{v}}dL$ denotes the FK force applied on stationary objects, which can be represented in exact vector form as

$$d\mathbf{f}_{\mathbf{FK}} = \frac{\pi\rho_w D^2 C_M dL}{2} \begin{Bmatrix} 0 & \dot{v}_2 & \dot{v}_3 & 0 & 0 & 0 \end{Bmatrix}^T; \quad (\text{A.4})$$

2. $-\frac{\pi\rho_w D^2}{4}C_A\ddot{\mathbf{z}}dL$ denotes the hydrodynamic added mass as given by

$$d\mathbf{f}_{\mathbf{AM}} = -\frac{\pi\rho_w D^2 C_A dL}{4} \begin{Bmatrix} 0 & \ddot{z}_2 & \ddot{z}_3 & 0 & 0 & 0 \end{Bmatrix}^T; \text{ and} \quad (\text{A.5})$$

3. $\frac{\rho_w D}{2}C_D(\mathbf{v} - \dot{\mathbf{z}})|\mathbf{v} - \dot{\mathbf{z}}|dL$ denotes the hydrodynamic drag that can be derived in the vector form (magnitude multiply by unit vector) as

$$\begin{aligned} d\mathbf{f}_{\mathbf{D}} &= \frac{\rho_w D}{2}C_D(\mathbf{v} - \dot{\mathbf{z}})|\mathbf{v} - \dot{\mathbf{z}}|dL \\ &= \frac{\rho_w D C_D dL}{2} \cdot A_1 \cdot \begin{Bmatrix} 0 & \frac{v_2 - \dot{z}_2}{\sqrt{A_1}} & \frac{v_3 - \dot{z}_3}{\sqrt{A_1}} & 0 & 0 & 0 \end{Bmatrix}^T \end{aligned} \quad (\text{A.6})$$

where $A_1 = (v_2 - \dot{z}_2)^2 + (v_3 - \dot{z}_3)^2$. The denominator of the unit vector $\frac{1}{\sqrt{A_1}}$ can be rewritten as

$$\frac{1}{\sqrt{v_2^2 + v_3^2} \sqrt{1 + \left(-\frac{2v_2\dot{z}_2 + 2v_3\dot{z}_3}{v_2^2 + v_3^2}\right)}}. \quad (\text{A.7})$$

For bottom-fixed offshore structures, the structural movements are relatively insignificant as compared with the wave kinematics, i.e., v_2, v_3 are $\sim V$ and $\dot{z}_2, \dot{z}_3$ are $\sim \epsilon V$, therefore $\left|\frac{2v_2\dot{z}_2 + 2v_3\dot{z}_3}{v_2^2 + v_3^2}\right| \ll 1$. By applying Taylor series expansion on Eq A.7, it gives

$$\frac{1}{\sqrt{v_2^2 + v_3^2}} \left[1 + \frac{v_2\dot{z}_2 + v_3\dot{z}_3}{v_2^2 + v_3^2} + \frac{3}{2} \left(\frac{v_2\dot{z}_2 + v_3\dot{z}_3}{v_2^2 + v_3^2} \right)^2 + O(\epsilon^3) \right]. \quad (\text{A.8})$$

Furthermore, the magnitude of drag force in Eq. A.6, while combining with the numerator of the unit vector, can be expanded to

$$A_1(v_2 - \dot{z}_2) = v_2^3 + v_2v_3^2 - (3v_2^2 + v_3^3)\dot{z}_2 - 2v_2v_3\dot{z}_3 + O(\epsilon^2) \quad (\text{A.9})$$

and

$$A_1(v_3 - \dot{z}_3) = v_3^3 + v_2^2 v_3 - 2v_2 v_3 \dot{z}_2 - (v_2^3 + 3v_3^2) \dot{z}_3 + O(\epsilon^2), \quad (\text{A.10})$$

respectively. Assuming that truncation errors of $O(\epsilon^2)$ and above are ignored in Eqs. A.8-A.10, Eq. A.6 can then be simplified to

$$\text{d}\mathbf{f}_D = \frac{\rho_w D C_D dL}{2} (v_2^2 + v_3^2) \begin{Bmatrix} 0 \\ \frac{v_2}{\sqrt{v_2^2 + v_3^2}} \\ \frac{v_3}{\sqrt{v_2^2 + v_3^2}} \\ 0 \\ 0 \\ 0 \end{Bmatrix} - \frac{\rho_w D C_D dL}{2} \begin{Bmatrix} 0 \\ \frac{2v_2^2 + v_3^2}{\sqrt{v_2^2 + v_3^2}} \dot{z}_2 + \frac{v_2 v_3}{\sqrt{v_2^2 + v_3^2}} \dot{z}_3 \\ \frac{v_2 v_3}{\sqrt{v_2^2 + v_3^2}} \dot{z}_2 + \frac{v_2^2 + 2v_3^2}{\sqrt{v_2^2 + v_3^2}} \dot{z}_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (\text{A.11})$$

where the first term refers to the drag force applied on stationary objects in the exact form while the second term is the hydrodynamic damping force experienced by the submerged beam.

Subsequently, in order to formulate Morison's force for a finite element beam, we assume that the beam element is applied with the normal hydrodynamic distributed loads q_y and q_z , and the nodal equivalent force vector is given by

$$\begin{aligned} \mathbf{f}^{\text{Eq}} &= \left\{ F_1 \ F_2 \ F_3 \ M_4 \ M_5 \ M_6 \ F_7 \ F_8 \ F_9 \ M_{10} \ M_{11} \ M_{12} \right\}^T \\ &= \int_0^L [\mathbf{N}^*]^T \begin{Bmatrix} q_y \\ q_z \end{Bmatrix} dx \end{aligned} \quad (\text{A.12})$$

where $[\mathbf{N}_{2-3}]$ here comprises only the shape functions that associate with the bending modes, i.e., 2^{nd} and 3^{rd} rows of $[\mathbf{N}]$. Assuming that the finite element beam is further discretized into N pseudo-sections as shown in Fig. A.2, then there will be $N + 1$ integration points created for each element (including the

end nodes).

Figure A.2: Illustration of pseudo-section and integration point concept adopted for generating the Morison's load vector for a finite element beam. In this diagram, $N = 3$ and only the plane x-y is shown for simplification purposes.

As the wave kinematics vary randomly across the beam, using more integration points will give better accuracy in the calculation of the hydrodynamic force. This feature is especially important for the computation of the FK force and the drag force. The unit length loads $q_{x_1}, q_{x_2}, \dots, q_{x_{N+1}}$ evaluated using the wave kinematics obtained at the $1^{st}, 2^{nd}, \dots, (N+1)^{th}$ integration points that are distributed equally, respectively. The unit length loads are treated uniform within the pseudo-sections $x_{1-2}, x_{2-3}, \dots, x_{N+1-N+2}$, hence can be expressed as

$$q = \left\{ \delta(x - x_1) \quad \delta(x - x_2) \quad \cdots \quad \delta(x - x_N) \quad \delta(x - x_{N+1}) \right\} \begin{pmatrix} q_{x_1} \\ q_{x_2} - q_{x_1} \\ \vdots \\ q_{x_N} - q_{x_{N-1}} \\ q_{x_{N+1}} - q_{x_N} \end{pmatrix} \quad (\text{A.13})$$

$$\text{where the heaviside piecewise function } \delta(x - x_i) = \begin{cases} 0 & x < x_i \\ 1/2 & x = x_i \\ 1 & x > x_i \end{cases} .$$

Therefore, substituting Eq. A.13 into Eq. A.12 gives

$$\begin{aligned} \mathbf{f}^{\text{Eq}} = & \int_{x_1}^{x_{N+2}} [\mathbf{N}_{2-3}]^T \begin{bmatrix} \delta(x - x_1) & \delta(x - x_2) & \cdots & \delta(x - x_N) & \delta(x - x_{N+1}) \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} dx \cdot \\ & \left\{ q_{y,x_1} \quad q_{y,x_2} - q_{y,x_1} \quad \cdots \quad q_{y,x_N} - q_{y,x_{N-1}} \quad q_{y,x_{N+1}} - q_{y,x_N} \right\}^T + \\ & \int_{x_1}^{x_{N+2}} [\mathbf{N}_{2-3}]^T \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \delta(x - x_1) & \delta(x - x_2) & \cdots & \delta(x - x_N) & \delta(x - x_{N+1}) \end{bmatrix} dx \cdot \\ & \left\{ q_{z,x_1} \quad q_{z,x_2} - q_{z,x_1} \quad \cdots \quad q_{z,x_N} - q_{z,x_{N-1}} \quad q_{z,x_{N+1}} - q_{z,x_N} \right\} . \end{aligned} \quad (\text{A.14})$$

For an EB beam that consists of only one pseudo-section ($N = 1$), the integration points are established at the end nodes while the heaviside functions are formed at $x_1 = 0$ and $x_2 = L/2$, where L is the full length of the EB beam. Performing the integration on Eq. A.14 gives a general formula for the nodal

equivalent force vector:

$$\mathbf{f}^{\text{Eq}} = \begin{bmatrix} 0 & 0 \\ \frac{L}{2} & \frac{3L}{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L^2}{12} & \frac{5L^2}{192} \\ 0 & 0 \\ \frac{L}{2} & \frac{13L}{32} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{L^2}{12} & -\frac{11L^2}{192} \end{bmatrix} \begin{Bmatrix} q_{y,x_1} \\ q_{y,x_2} - q_{y,x_1} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{L}{2} & \frac{3L}{32} \\ 0 & 0 \\ -\frac{L^2}{12} & -\frac{5L^2}{192} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L}{2} & \frac{13L}{32} \\ 0 & 0 \\ \frac{L^2}{12} & \frac{11L^2}{192} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} q_{z,x_1} \\ q_{z,x_2} - q_{z,x_1} \end{Bmatrix} \quad (\text{A.15})$$

The non-zero vector terms in Eqs. A.4, A.5 and A.11, when divided by dL , are the unit length forces applied at the local y - and z -directions, i.e., q_y and q_z , respectively. Firstly, we apply Eq. A.5 to A.15 and obtain

$$\begin{aligned}
 \mathbf{f}_{\text{AM}}^{\text{Eq}} &= -\frac{\pi\rho_w D^2 C_A}{4} \left(\begin{bmatrix} 0 \\ \frac{13L}{32}\ddot{z}_2 + \frac{3L}{32}\ddot{z}_8 \\ 0 \\ 0 \\ 0 \\ \frac{11L^2}{192}\ddot{z}_2 + \frac{5L^2}{192}\ddot{z}_8 \\ 0 \\ \frac{3L}{32}\ddot{z}_2 + \frac{13L}{32}\ddot{z}_8 \\ 0 \\ 0 \\ 0 \\ -\frac{5L^2}{192}\ddot{z}_2 - \frac{11L^2}{192}\ddot{z}_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{13L}{32}\ddot{z}_3 + \frac{3L}{32}\ddot{z}_9 \\ 0 \\ -\frac{11L^2}{192}\ddot{z}_3 - \frac{5L^2}{192}\ddot{z}_9 \\ 0 \\ 0 \\ \frac{3L}{32}\ddot{z}_3 + \frac{13L}{32}\ddot{z}_9 \\ 0 \\ 0 \\ \frac{5L^2}{192}\ddot{z}_3 + \frac{11L^2}{192}\ddot{z}_9 \\ 0 \end{bmatrix} \right) \\
 &= -[\mathbf{M}^{\text{H}}] \ddot{\mathbf{z}}
 \end{aligned}$$

where $\ddot{\mathbf{z}}$ is the acceleration vector of the EB beam and the hydrodynamic added mass matrix $[\mathbf{M}^{\text{H}}]$ is given by

$$[\mathbf{M}^{\mathbf{H}}] = \frac{\pi \rho_w D^2 C_A}{4} \begin{bmatrix} 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & \frac{13L}{32} & 0 & & 0 & \frac{3L}{32} & 0 \\ 0 & 0 & \frac{13L}{32} & & 0 & 0 & \frac{3L}{32} \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & -\frac{11L^2}{192} & 12 & 0 & 0 & -\frac{5L^2}{192} & 12 \\ 0 & \frac{11L^2}{192} & 0 & \times & 0 & \frac{5L^2}{192} & 0 & \times \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 \\ 0 & \frac{3L}{32} & 0 & \text{null} & 0 & \frac{13L}{32} & 0 & \text{null} \\ 0 & 0 & \frac{3L}{32} & & 0 & 0 & \frac{13L}{32} \\ 0 & 0 & 0 & & 0 & 0 & 0 \\ 0 & 0 & \frac{5L^2}{192} & & 0 & 0 & \frac{11L^2}{192} \\ 0 & -\frac{5L^2}{192} & 0 & & 0 & -\frac{11L^2}{192} & 0 \end{bmatrix}. \quad (\text{A.16})$$

Similarly, when applying the second term in Eq. A.11 to Eq. A.15, the hydrodynamic damping force vector can be derived as $\mathbf{f}_D^{\mathbf{Eq}} = -[\mathbf{C}^{\mathbf{H}}] \dot{\mathbf{z}}$, where $\dot{\mathbf{z}}$ is the velocity vector of the beam element and the hydrodynamic damping

matrix $[\mathbf{C}^{\mathbf{H}}]$ is

$$\begin{aligned}
 & [\mathbf{C}^{\mathbf{H}}] = \\
 & \frac{\rho_w D C_D}{2} \cdot \left[\begin{array}{ccccc}
 0 & 0 & 0 & 0 & 0 \\
 0 & \frac{13L}{32} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{13L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{3L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} \\
 0 & \frac{13L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{13L}{32} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{3L}{32} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} \\
 0 & 0 & 0 & 0 & 0 \\
 0 & -\frac{11L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & -\frac{11L^2}{192} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & 12 & -\frac{5L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & -\frac{5L^2}{192} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & 12 \\
 0 & \frac{11L^2}{192} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{11L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \times & \frac{5L^2}{192} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{5L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \times \\
 0 & 0 & 0 & 4 & 0 & 0 & 3 \\
 0 & \frac{3L}{32} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \text{null} & \frac{13L}{32} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & \frac{13L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \text{null} \\
 0 & \frac{3L}{32} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{3L}{32} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & & \frac{13L}{32} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{13L}{32} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & \\
 0 & 0 & 0 & & 0 & 0 & \\
 0 & \frac{5L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & \frac{5L^2}{192} \frac{v_2^2+2v_3^2}{\sqrt{v_2^2+v_3^2}} & & \frac{11L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} & \frac{11L^2}{192} \frac{v_8^2+2v_9^2}{\sqrt{v_8^2+v_9^2}} & \\
 0 & -\frac{5L^2}{192} \frac{2v_2^2+v_3^2}{\sqrt{v_2^2+v_3^2}} & -\frac{5L^2}{192} \frac{v_2 v_3}{\sqrt{v_2^2+v_3^2}} & & -\frac{11L^2}{192} \frac{2v_8^2+v_9^2}{\sqrt{v_8^2+v_9^2}} & -\frac{11L^2}{192} \frac{v_8 v_9}{\sqrt{v_8^2+v_9^2}} &
 \end{array} \right].
 \end{aligned}
 \tag{A.17}$$

Both $[\mathbf{M}^{\mathbf{H}}]$ and $[\mathbf{C}^{\mathbf{H}}]$ are combined into the system mass and damping matrices, respectively. Finally, the hydrodynamic force vector $\mathbf{f}^{\mathbf{H}}$ sums up the wave loads exerted onto a stationary object, which includes the FK force (Eq. A.4) and the drag force component (first term of Eq. A.11). The unit length

hydrodynamic force vector is given by

$$\mathbf{q}^{\mathbf{H}} = \mathbf{q}_{\mathbf{FK}} + \mathbf{q}_{\mathbf{D}} = \frac{\pi \rho_w D^2 C_M}{2} \begin{Bmatrix} 0 \\ \dot{v}_2 \\ \dot{v}_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \frac{\rho_w D C_D}{2} (v_2^2 + v_3^2) \begin{Bmatrix} 0 \\ \frac{v_2}{\sqrt{v_2^2 + v_3^2}} \\ \frac{v_3}{\sqrt{v_2^2 + v_3^2}} \\ 0 \\ 0 \\ 0 \end{Bmatrix}, \quad (\text{A.18})$$

and is applied to Eq. A.14 to determine the $\mathbf{f}^{\mathbf{H}}$. In this case, the number of pseudo-section N should be adjusted to obtain an accurate $\mathbf{f}^{\mathbf{H}}$.

A.3 FE formulation of foundation stiffness

When piles are displaced during the static and dynamic loading processes, the soil deforms and exerts reaction forces onto the piles through skin friction, end bearing pressure and lateral bearing pressure. The resistance-displacement relationships for these forces are normally described using non-linear $t-z$, $Q-z$ and $p-y$ curves, respectively, which vary with respect to the soil type and properties, the pile dimension and the depth of assessment (American Petroleum Institute, 1993). Barltrop and Adams (2013) has discussed several methods to determine the piled foundation stiffness for global structural analysis. When subjected to dynamic loading, the linearized foundation secant stiffness is recommended. The following explains the formulation of the FE stiffness matrix for piled foundations.

1. Linearization of axial shaft resistance-displacement ($t-z$) curves. The API RP 2A prescribes the non-linear empirical $t-z$ curves for clay and sand to be based on $\left(\frac{z}{D}, \frac{t}{t_{max}}\right)$ pairs and $\left(z, \frac{t}{t_{max}}\right)$ pairs, respectively, where z is the local axial pile deflection [m]; D is the pile diameter [m], t is the mobilized

soil pile adhesion [Pa] and t_{max} is the maximum unit skin friction capacity [Pa]. The secants of the $t-z$ curves can be approximated by connecting the origins to the points where the curves first reach t_{max} , i.e., when $\frac{z}{D} = 0.01$ for clay and $z = 2.54$ mm for sand. The linearized $t-z$ curve is then represented as

$$t = K^{t-z} z \quad (\text{A.19})$$

where K^{t-z} is the secant stiffness for axial shaft friction [Pa/m]. Besides, the maximum axial displacements of the pile nodes should be maintained $\leq 0.01D$ for clay and ≤ 2.54 mm for sand, in order to reduce the errors of the linearized approximation.

2. Linearization of axial tip bearing-displacement ($Q-z$) curves. The API RP 2A prescribes the non-linear empirical $Q-z$ curves for sand and clay to be based on $\left(\frac{z}{D}, \frac{Q}{Q_p}\right)$ pairs, where Q is the mobilized end bearing capacity [N] and Q_p is the total end bearing capacity[N]. Generally, relatively large pile tip movements are required to mobilize the full end bearing resistance, which is about 10 percent of the pile diameter. Similarly, the secants of the $Q-z$ curves are formed by connecting the origin to the point where $z = 0.1D$, therefore giving more flexible foundation characteristics than the original non-linear curves. The linearized $Q-z$ curve is expressed as

$$Q = K^{Q-z} z \quad (\text{A.20})$$

where K^{Q-z} is the secant stiffness for axial tip bearing force[N/m].

3. Linearization of lateral bearing-displacement ($p-y$) curves. The API RP 2A prescribes the non-linear empirical $p-y$ curves for clay to be based on

Figure A.3: Axial pile resistance-displacement ($t-z$) curves (American Petroleum Institute, 1993)

$\left(\frac{y}{y_c}, \frac{p}{p_u}\right)$ pairs, whereas for sand the following equation is used:

$$p = A_l p_u \tanh\left(\frac{k_0 H}{A_l p_u} y\right) \quad (\text{A.21})$$

where p is the mobilized lateral bearing capacity [N/m], p_u is the ultimate bearing capacity [N/m] at depth H [m], y is the local lateral displacement [m]; y_c is the displacement corresponding to the strain that occurs at one-half the maximum stress on laboratory unconsolidated undrained compression tests of the soil [m]; k_0 is the initial modulus of subgrade reaction [N/m³]; and A_l is the loading factor [–]. The secants of the $p - y$ curves can be approximated for clay according to the linearization approach described earlier, and gives

$$p = K^{p-y} y \quad (\text{A.22})$$

where K^{p-y} is the secant stiffness for lateral bearing force [N/m²]. As for sand, an initial p_0 is computed from Eq. A.21 by substituting $y = y_0$, thereafter the secant stiffness is determined from $K^{p-y} = \frac{p_0}{y_0}$. However, the selection of an appropriate y_0 can be challenging to give an accurate approximation of K^{p-y} . The other method proposed is to make use of the tangent formula of Eq. A.21, i.e.,

$$K^{p-y} \approx \left. \frac{dp}{dy} \right|_{y=y_0} = \frac{k_0 H}{\cosh^2\left(\frac{k_0 H y_0}{A_l p_u}\right)}. \quad (\text{A.23})$$

When $y_0 = 0$, the linearized $p - y$ curve generated is basically the tangent stiffness taken at the neutral position, i.e., $K^{p-y} = k_0 H$ which theoretically gives an over-stiff foundation property. The secant stiffness for lateral bearing can be improved by utilizing $y_0 = \frac{y_{max}}{2}$ in Eq. A.23, after obtaining the maximum lateral displacements y_{max} from an initial simulation that

applies $K^{p-y} = k_0 H$.

Next, the piled foundation stiffness matrix is derived from the nodal equivalent force vector (Eq. A.12) following the procedure described in Appendix A.2. Given a small section of vertical unplugged cylindrical pile that is represented as a EB beam element, the unit length axial friction resistance is computed as $q_x^P = -2\pi (D - T) K^{t-z} z$ since friction is exerted on both the outer and internal cylindrical surfaces of the pile. On the other hand, the unit length lateral bearing resistance is basically $q_{y,z}^P = -K^{p-y} y$. The “negative signs” in $q_{x,y,z}^P$ are due to that the soil reaction force is always applied in the opposite direction to the pile displacement. However, if non-linear $p - y$ curves are to be adopted, the resultant lateral resistance cannot be resolved to q_y^P and q_z^P directly as shown here. Assuming that $N = 1$ is used and the coupling between axial and lateral deformations is ignored for the soil, the equivalent soil load applied onto the pile is given by

$$\begin{aligned}
 \mathbf{f}^P = & - \int_0^L [\mathbf{N}_{1-3}]^T \begin{bmatrix} \delta(x) & \delta(x - \frac{L}{2}) \\ 0 & 0 \\ 0 & 0 \end{bmatrix} dx \cdot \\
 & \left\{ \begin{array}{c} 2\pi K_{x_1}^{t-z} (D - T) z_1 \\ 2\pi K_{x_2}^{t-z} (D - T) z_2 - 2\pi K_{x_1}^{t-z} (D - T) z_1 \end{array} \right\} - \\
 & \int_0^L [\mathbf{N}_{1-3}]^T \begin{bmatrix} 0 & 0 \\ \delta(x) & \delta(x - \frac{L}{2}) \\ 0 & 0 \end{bmatrix} dx \cdot \left\{ \begin{array}{c} K_{x_1}^{p-y} z_2 \\ K_{x_2}^{p-y} z_2 - K_{x_1}^{p-y} z_2 \end{array} \right\} - \\
 & \int_0^L [\mathbf{N}_{1-3}]^T \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \delta(x) & \delta(x - \frac{L}{2}) \end{bmatrix} dx \cdot \left\{ \begin{array}{c} K_{x_1}^{p-y} z_3 \\ K_{x_2}^{p-y} z_3 - K_{x_1}^{p-y} z_3 \end{array} \right\} \quad (\text{A.24})
 \end{aligned}$$

where $[\mathbf{N}_{1-3}]$ is the shape function includes the axial mode (1^{st} row) and bending modes (2^{nd} and 3^{rd} rows) of $[\mathbf{N}]$; and $\mathbf{z} = \{z_1, \dots, z_{12}\}^T$ denotes the beam displacement vector in the local beam coordinate. Do not confuse with “ z ” in Eqs. A.19 and A.20. Upon integration, $\mathbf{f}^P$ can be represented in the form of $-\left[\mathbf{K}^P\right] \mathbf{z}$ where the foundation secant stiffness matrix $\left[\mathbf{K}^P\right]$ is given by

$$\begin{aligned}
 [\mathbf{K}^P] = & \begin{bmatrix}
 \frac{3L}{4} K_{x_1}^{t-z} \pi (D-T) & 0 & 0 & \frac{L}{4} K_{x_2}^{t-z} \pi (D-T) & 0 & 0 \\
 0 & \frac{13L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{3L}{32} K_{x_2}^{p-y} & 0 \\
 0 & 0 & \frac{13L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{3L}{32} K_{x_2}^{p-y} \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & -\frac{11L^2}{192} K_{x_1}^{p-y} & -\frac{11L^2}{192} K_{x_1}^{p-y} & 12 & 0 & -\frac{5L^2}{192} K_{x_2}^{p-y} 0 \\
 0 & \frac{11L^2}{192} K_{x_1}^{p-y} & 0 & \times & \frac{5L^2}{192} K_{x_2}^{p-y} & 0 \\
 \frac{L}{4} K_{x_1}^{t-z} \pi (D-T) & 0 & 0 & 3 \frac{3L}{4} K_{x_2}^{t-z} \pi (D-T) & 0 & 0 \\
 0 & \frac{3L}{32} K_{x_1}^{p-y} & 0 & \text{null} & \frac{13L}{32} K_{x_2}^{p-y} & 0 \\
 0 & 0 & \frac{3L}{32} K_{x_1}^{p-y} & 0 & 0 & \frac{13L}{32} K_{x_2}^{p-y} \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{5L^2}{192} K_{x_1}^{p-y} & 0 & 0 & \frac{11L^2}{192} K_{x_2}^{p-y} \\
 0 & -\frac{5L^2}{192} K_{x_1}^{p-y} & 0 & 0 & -\frac{11L^2}{192} K_{x_2}^{p-y} & 0
 \end{bmatrix}.
 \end{aligned}
 \tag{A.25}$$

Note: For the pile tip nodes, the axial tip bearing stiffness K^{Q-z} can be added directly to the $(1, 1)$ or $(7, 7)$ term in $[\mathbf{K}^P]$, according to the node number.