Towards a sustainable and resilient fleet management

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2017

Tong, Y. (2017). Towards a sustainable and resilient fleet management. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/69830>

<https://doi.org/10.32657/10356/69830>

NANYANG TECHNOLOGICAL UNIVERSITY

**Towards a Sustainable
and Resilient Fleet Management**

Ph.D. Dissertation

by

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Acknowledgments

Firstly, I would like to express my sincere gratitude to my supervisor Prof. Mao Jianfeng for the continuous support of my Ph.D study and related research, for his patience, motivation, and immense knowledge. His guidance provided insights and expertise that helped me in all the time of research and writing of this thesis. I could not have imagined having a better advisor and mentor for my Ph.D study. I am also extremely grateful for scholarship and education provided by Nanyang Technological University. Last but not the least, I would like to thank my parents for supporting me spiritually throughout this thesis and my my life in general.

Contents

Acknowledgments	2
List of Figures	4
List of Tables	6
Abstract	8
1 Introduction	9
1.1 Background	9
1.2 Emerging Factors Affecting International Shipping	11
1.2.1 Oil prices and transportation costs	11
1.2.2 Energy issues and environmental sustainability	13
1.2.3 Resilience	13
1.2.4 Summary	14
1.3 Research Issues and Scope	14
1.4 Aims and Contributions	16
1.5 Organization of this Work	18
2 Literature Review	20
2.1 Literatures Related to Ship Operations	20
2.1.1 Liner shipping	21
2.1.2 Industrial shipping	22
2.2 Routing and Scheduling Models	23
2.2.1 Routing	24
2.2.2 Scheduling	26
2.2.3 Comparison between the two approaches	29
2.3 Energy Efficiency in Ship Operations	30
2.3.1 The effect of oil price on ship speed	30
2.3.2 Ship speed as a variable in fleet management models	32
2.4 Summary	32

3	Basic Model Development and Algorithms	34
3.1	Capacitated Vehicle Routing Problem with Time Windows	34
3.1.1	Problem description	34
3.1.2	Notations	35
3.1.3	Constraints	37
3.1.4	Formulation	40
3.2	Enhanced Model with Fuel Management	41
3.2.1	Problem description	41
3.2.2	Notations	42
3.2.3	Three-stage formulation	43
3.2.4	Equivalent aggregated model with linear constraints	47
3.3	A Heuristic	51
3.3.1	Problem structure	51
3.3.2	Heuristic algorithm	52
3.4	Numerical Results	55
3.4.1	Sensitivity analysis	55
3.4.2	Computational time	63
3.5	Summary	67
4	Fleet Management with Simultaneous Pickup and Delivery	68
4.1	Problem Description	68
4.2	Problem Formulation	70
4.2.1	Notations	70
4.2.2	Formulation	73
4.3	Heuristics	75
4.3.1	Single ship problems	76
4.3.2	Neighborhood Search	78
4.3.3	Heuristic based on Most Promising Area	79
4.3.4	Improvements	82
4.4	Numerical Results	84
4.4.1	Effects of weight and bunkering	84
4.4.2	Step-by-step procedure for MPA	86
4.4.3	Computational time and Optimality gap	88
4.5	Case study	95
4.6	Summary	99

5	Fleet Management with general pickup and delivery	100
5.1	Problem Description	101
5.2	Problem Formulation 1	101
5.3	Problem Formulation 2	104
5.4	Case Study	107
5.5	Summary	111
6	Fleet Management with disruptions	112
6.1	Problem Description	113
6.2	Problem Formulation	113
6.3	Simulation Results	116
6.4	Summary	121
7	Conclusions and Future Work	122
7.1	Conclusions	122
7.2	Future Work	124
7.2.1	Fuel consumption models	124
7.2.2	General pickup and delivery problems	124
7.2.3	Cargo type and positioning	125
	References	126

List of Figures

1.1	International seaborne trade (millions of tons loaded)[1]	10
1.2	Installed base of fleet management units(EU27 +2 2009-2015)[2] . .	10
1.3	Bunkerworld index from 2008-12-01 to 2012-04-12[3]	12
1.4	Average crude oil price in Brent may 1987 to Jan 2012[4]	12
1.5	Indices for global container, tanker, and major dry bulk volumes, 2007-2011[5]	15
1.6	Share of new ships with ballast water treatment systems[6]	15
1.7	Development of models	19
3.1	An example of sub-tour	38
3.2	Problem Structure	51
3.3	Optimal routing	57
3.4	Optimal routing when $d_2 = 2500$	59
3.5	Optimal profit with respect to change in d_2	60
3.6	Optimal routing when $d_4 \leq 1300$	61
3.7	Optimal routing when $r_5 \geq 209$	61
3.8	Optimal routing when $p_5 \leq 610$	62
3.9	Optimal routing when $l_2 = 35(d_2 = 2500)$	63
3.10	Optimal routing when $l_2 = 35(d_2 = 2000)$	63
3.11	Computational times by CPLEX	65
3.12	Computational times by heuristic	65
3.13	Optimality Gaps	66
4.1	True optimal routing	85
4.2	Optimal routing under variate conditions	86
4.3	CPLEX Computational times for small cases	89
4.4	MPA Computational times for small cases	90
4.5	NS Computational times for small cases	90

4.6	Performance comparison between MPA and NS, as percentage of MPA solutions	91
4.7	Computational times for two heuristics	92
4.8	Computational times when $k=2$	92
4.9	Computational times when $k=3$	93
4.10	Computational times when $k=4$	93
4.11	MPA Computational times	94
4.12	NS Computational times	94
5.1	Optimal routing	108
5.2	Optimal routing with loose time windows and less demands	109
5.3	Optimal routing with 4 orders	110
6.1	Routing with no penalty cost for lateness	116
6.2	Routing with time windows	117
6.3	disruption on (1,5)	118
6.4	disruption on (3,6)	119
6.5	disruption on (4,7)	119
6.6	disruption on (2,7) and (6,3)	121
7.1	Fuel consumption rate in bow waves[7]	124

List of Tables

1.1	Models in this work	19
3.1	Procedures for heuristic: stage 1	54
3.2	Procedures for heuristic: stage 2	56
3.3	Ports information	57
3.4	Optimal result table	57
3.5	Number of variables	64
3.6	Optimality gaps	66
4.1	Calculating optimal profit for a given assignment Y	78
4.2	Heuristic based on neighborhood search	79
4.3	Heuristic based on Most Promising Area	80
4.4	Ports information	85
4.5	Computational times (in sec)	89
4.6	Optimality gap	91
4.7	Parameters for case study	95
4.8	Information of ports	96
4.9	Optimal routing and fuel strategy: Ship 1	97
4.10	Optimal routing and fuel strategy: Ship 2	97
4.11	Optimal routing and fuel strategy: Ship 3	98
4.12	Optimal routing and fuel strategy: Ship 4	98
4.13	Optimal routing and fuel strategy: Ship 5	98
5.1	Orders: case 1	107
5.2	Optimal routing and fuel strategy: Ship 1	108
5.3	Optimal routing and fuel strategy: Ship 2	108
5.4	Orders: case 2	109
5.5	Orders: case 3	110

6.1	Optimal routing with disruption on (1,5)	117
6.2	Optimal routing with disruption on (3,6)	118
6.3	Optimal routing with disruption on (4,7)	118
6.4	Optimal routing with disruption on (2,7) and (6,3)	120

Abstract

While petroleum is still a major energy source, the vast majority of world trade is carried by the sea shipping industry. To reduce greenhouse gas emissions, it is urgent for both industry and government to promote green shipping. Considering the surges in fuel prices and concerns on environmental issues in recent years, researches should be done to move shipping towards a more environmentally and financially sustainable future. Unlike most of previous work where fleet deployment and bunkering management are considered separately, we develop an integrated fleet deployment and bunker management system in this work. The fleet deployment refers to the decisions on which ships to operate on which route, at what speed and in what cargos to be transported. The bunker management refers to the decision on which ships to bunker on which port and at what volume. Numerical results show that they should not be decomposed as current practices. A more realistic fuel consumption function that considers not only cruising speed but also freight tonnage onboard is adopted in the proposed models, which can further improve its effectiveness in practice. In this thesis, we explore on four such integrated models to cope with different business modes covering feeder service, industry shipping and tramp services. Mixed Integer Programming models with nonlinear constraints are formulated. We also extended the model from deterministic problem to stochastic version, where uncertain disruptions are taken into consideration during fleet planning. This work can be extended in the future to study the disruption management in tramp services, and to deal the positioning and loading sequence of cargos.

Chapter 1

Introduction

1.1 Background

The rapid growth of world population and people's living standards require international exchanges of resources, which increases the dependence of world economy on international trading. The majority of international trading is achieved by sea shipping, which is the main provider of large volume transportation over long distances, as well as low cost mass transportation over short distances. The world seaborne trade in 2013 amounted to 9841 million tons, a 195.5% increase over 3330 million tons in 1985 (see Fig. 1.1) [1].

Fleet management plays a significant role in the planning of shipping activities, allowing companies who rely on transportation to minimize the risks associated with vehicle investment, improving efficiency, productivity and reducing overall transportation costs [8]. These functions can be dealt with by either an personal management department or an out-sourced fleet management provider. According to market research from the independent analysis firm Berg Insight, the number of fleet management units deployed in commercial fleets in Europe has increased in past years, and is expected to keep rising for the next few years, as presented in Fig. 1.2. It is predicted that the number will grow at a compound annual growth rate of 15.1% from 4.40 million units at the end of 2014 to 8.90 million by 2019. [2].

Figure 1.1: International seaborne trade (millions of tons loaded)[1]

Figure 1.2: Installed base of fleet management units(EU27 +2 2009-2015)[2]

During the one year time of my work, two companies have approach us for fleet management consulting. One operates a global network of multi-purpose liner services, and the other is a leading global company in the offshore industry which provides integrated offshore construction and support services for shallow water oil and gas field development. Leading companies like these realize the importance of smart fleet management and the profit it may bring.

1.2 Emerging Factors Affecting International Shipping

1.2.1 Oil prices and transportation costs

Shipping heavily relies on oil, and is not yet in a position to adopt alternative energy resources. The rapid growth in global trade over the past few decades was powered by easily available and affordable oil. The market price of Fuel Oil 380 in Singapore significantly increased from below \$300 to above \$700 per ton in three years from 2009 to 2012, and has been above \$600 since then.

Fig. 1.3 shows the fuel price index from 01 December 2008 to 12 April 2012 [3]. The index is used in the Bunkerworld website to illustrate the trends of fuel prices. It is a weighted daily index made up of 20 key bunkering ports, and thus represents the fundamentals for the fuel market price changes. As another example, Fig. 1.4 shows the average of crude oil prices in Brent from May 1987 to Jan 2012. Not many of us were paying much attention to oil prices until 2004, since the average trading price is between \$22 to \$26. In 2004, the surge in 2004 reached \$50 average in 2005, \$71 average in 2010 and above \$90 average in 2011. In shipping industries, the bunker fuel is now a considerable expense.

The cost of operating vessel fleet is always a main concern in the planning of shipping activities for shipping companies. As evidenced by surges in oil prices [9], one main challenge faced by shipping companies is how to plan for fuel savings in order to cut on transportation costs.

Figure 1.3: Bunkerworld index from 2008-12-01 to 2012-04-12[3]

Figure 1.4: Average crude oil price in Brent may 1987 to Jan 2012[4]

1.2.2 Energy issues and environmental sustainability

Since environmental concerns have gained much attentions recently, reducing oil consumption and greenhouse gas emissions has been a key issue in all industries. The importance of energy efficiency monitoring has become more and more significant to countries and organizations as a result of higher oil prices and global warming. In the European Union, for example, member states are expected to achieve an overall national indicative energy savings target of equal to or higher than 9% for the nine year period of application of Directive 2006/32/EC [10], referred to as the Energy Services Directive (ESD). In most countries, transportation and services contribute to a large portion of national energy using [11]. At the price of 135 USD per ton, bunker fuel costs constituted about half the operating cost of a large containership company [12]. And when fuel price reaches 500 USD per ton, bunker fuel costs will be as much as about three quarters of the operating cost of a large containership company [13]. International Maritime Organization (IMO) predicted that international shipping will be generating at least 3% of global carbon emissions by 2050 [5].

1.2.3 Resilience

While cutting transportation costs and environmental issues are crucial to fleet management, building the resilience of the transport system to cope with unexpected situations is equally important. Redundant resources are necessary for this purpose. Yet, resources planned for adaptation to uncertainties and emergencies remain inadequate [14].

When consulting with a global shipping company, they pointed out that for the last few years, oil consumption and cost has been quite a main issue in the planning of service lines. Due to the economic crisis these two years (from mid 2015 to 2016), a slump has shown in the shipping demands, and their focus is now fulfilling more demands, where resilience has been a more important concern in the

picture, since building up resilience helps in dealing with demands with uncertain time and amount.

1.2.4 Summary

New regulations require shipping industries to invest in environmentally-friendly technologies, covering issues such as emissions, waste and ballast water treatment. The investments are not only beneficial for the environment, but also may result in long term cost savings [6]. Appropriate marine fuel management strategy will not only save the environment by less greenhouse gas emission, but also benefit shipping companies by less operational cost. Hence, the first two concerns are gradually taken into consideration as main concerns for ship operators, and they keep adapting their strategies in terms of these factors. A perfect example can be provided by Maersk Line of the triple E-class 18000 TEUs ships, representing economy of scale, energy efficiency and environmental improvement [15]. However, there is still lack of concern on resilience management.

1.3 Research Issues and Scope

Fleet management by global shipping companies typically include a range of functions, such as vehicle financing, vehicle maintenance, vehicle tracking and diagnostics, vehicle routing and scheduling, driver management, speed management, fuel management and health and safety management. The work in this report will be focused on vehicle routing and scheduling, speed management and fuel management. Building resilience to fleet will also be mentioned.

Nowadays, container trade volumes have experienced an unexpected rise. Containerized cargos made up 56 percent of all dry cargos, and they expanded at an average rate of 8.2 percent between 1990 and 2011, as shown in Fig. 1.5. On the other hand, containerships gain more attention in the current shipping companies in aspect of investment on environmental technologies. Fig. 1.6 illustrates the

Figure 1.5: Indices for global container, tanker, and major dry bulk volumes, 2007-2011[5]

increasing introduction of ballast water treatment system, making use of new technologies. Due to the market share and concerns of container trade, the problems studies in this report will be restricted to container shipping. This assumption will bring us the advantage of uniform units and allowing cargo splitting.

Sea shipping operators have adopted certain ways to implement fleet deployment to determine appropriate route and speed for each ship to reduce fuel con-

Source: UNCTAD secretariat, based on data supplied by Clarksons Research.
Note: Propelled seagoing merchant vessels of 1,000 GT and above.

Figure 1.6: Share of new ships with ballast water treatment systems[6]

sumption and associated financial hemorrhage [16] (laying up or chartering out ships are also viable options when facing insufficient demands). Generally, slow steaming saves fuel consumption and releases fewer greenhouse gas. However, lower speed may lengthen the traveling time at the risk of violating service schedule. Hence, it is necessary to carefully tune cruising speed for each ship on each voyage to trade off operational cost and service quality. On the other hand, fuel prices vary a lot at different ports, as shown in Fig. 1.4, oil prices can be very fluctuating in a certain port. Sea shipping operators can take this advantage to further enhance the profitability through bunker management, i.e., where and how much to bunker, which is also critical to reduce the total operational cost.

In general, the operational costs are mainly influenced by both fleet deployment and bunker management. The fleet deployment refers to the decisions on which ships to operate on which route, at what speed and what cargos to be transported. The bunker management refers to the decision on which ships to bunker on which port and at what volume. These two types of decisions are separately considered in practice and most of literatures, which is quite helpful in reducing the complexity in decision making. However, they are tightly coupled by planned routes. Separately considering them may not achieve efficient solutions. To further enhance green shipping, we will develop an integrated fleet deployment and bunker management model to derive better solutions.

The fuel consumption estimation itself is another crucial issue in this problem, which is mainly related to the voyage distance and the resistance a ship overcomes in water. The approximation on fuel consumption rate in most literatures may result in an inferior solution and degrade the effectiveness in practice. A more realistic fuel consumption function will be adopted in this work.

1.4 Aims and Contributions

As mentioned previously, the main concerns in current fleet management is on economic profits as well as environmental sustainability. In adhering to these con-

cerns, the planning in shipping activities is expected to achieve efficiency, quality of service, which at the same time taking into account the cost generated. This work is aimed at developing models that can solve shipping planning problems with the above objectives under different business modes.

Energy Efficiency

We will adopt energy efficiency in classical vehicle routing problems. Unlike most of previous work where fleet deployment and bunkering management are considered separately, our work focus on the integration of these two decisions. Numerical results can show that they should not be decomposed as current practices(as illustrated in Chapter 4).

Additionally, most fuel consumption rate is currently considered as proportional to the cube of cruising speed. Refined model use two coefficients to model the rate to more precisely formulate the fuel consumption. However, the weight ships carry also affect the fuel consumption rate by affecting the displacement. The factor of displacement is considered in Chapter 4 and afterwards.

Heuristics

Vehicle Routing Problem itself is proved to be NP-hard. Considering fleet deployment together with bunkering management leads to a rapid increase in the complexity of the problem. True optimal solutions can only be obtained for small size problems in our numerical experiments. Two heuristic algorithms are developed so that solutions can be derived within reasonable time with acceptable optimality gaps. Moreover, one of the algorithms guarantees global optimal solution with probability 1.

Stochastic factors and general pickup and delivery problems

We extend our work to plan for fleet management where disruptions are possible on some voyages, while disruptions refer to delays due to weather, port operation,

etc. A scenario based formulation is developed.

An exact formulation is proposed for the fleet management with general pickup and delivery, where cargo routing should be considered. We alternatively convert the problem to a similar problem as in Chapter 4 with little procedures, and reformulated the problem into a similar model.

1.5 Organization of this Work

This work contains seven chapters. Chapter 2 to 7 are as following:

- (i) Chapter 2 introduces and summarizes some related work in shipping management.
- (ii) In Chapter 3, we start by a simple fleet deployment model, which can be seen as a improvement model on classical problems.
- (iii) In Chapter 4, bunkering decisions are considered together with fleet deployment, with pickup and delivery demands simultaneous existing on each customer ports. Two heuristics are proposed, and one of them is proved to be able to converge to global optimal solution with probability 1.
- (iv) In Chapter 5, general pickup and delivery demands are considered. 1-M-1 model refers to a problem where ships set out from a depot, visiting customer ports, and return to depot. If there is not such depot, the operation can be described as a M-M model.
- (v) Stochastic factors are introduced in Chapter 6, considering fleet management under possible disruptions.
- (vi) In Chapter 7, the conclusions and future work are discussed.

In summary, four models are formulated in this work from chapters 3 to 6. Chapter 3 is a base model, on which the other three model are built. The development can be illustrated by Fig. 1.7. The three models are able to solve planning

Figure 1.7: Development of models

Table 1.1: Models in this work

	1-M-1	M-M
deterministic	Chapter 4	Chapter 5
stochastic	Chapter 6	

problems in different business modes, and the concerns are shown in Table 1.1. The basic model is Chapter 3 can be adopted to plan for feeder services, the model in chapters 4 and 6 is perfect for industrial shipping, and the one with general pickup and delivery as in Chapter 5 is suitable for tramp service planning.

Chapter 2

Literature Review

2.1 Literatures Related to Ship Operations

Although transportation problems have been widely discussed in previous works, researches on quantitative analysis of shipping systems are relatively few before 1980s [17]. Some of the reasons are as follows.

- After thousands of years of ocean shipping, the earliest way of international trade, there has been a branch of traditions in this industry, and it had been operated mainly by experience. Such an industry is conservative and hard to be open to new ideas.
- When international trading was not as convenient as today, most domestic trading took place by truck or rail in early years, especially in big countries. At that time, ships are a minor transportation method. This is also the reason why most researches on shipping originated in Europe, where shipping is more necessary and common.
- Uncertainties such as weather conditions, mechanical problems always cause more severe consequences on shipping operations. This makes it difficult to precise and quantitative analysis.

Despite these reasons, there are still some works on ship routing and scheduling. The research on shipping industry, as early as the system developed by Olson, Sorenson and Sullivan [18], and the analysis done by Naslund [19], first began with practical cases, which were usually arisen by shipping companies or organizations. The studies were mainly based on some real situation and the model was only suitable for the specific environment that the study was carried on. A survey paper is first provided by Ronen [17] in ship routing and scheduling problems and related models. Ronen [20] and Christiansen et al. [21] continued the review on the same topic covering the literatures published during 1982-1992 and 1993-2003 respectively.

There are mainly three categories of shipping operations: liner, tramp and industrial [22]. A liner operation is like a bus, which operates vessel between fixed ports on a strict timetable. A tramp operation resembles a taxi. The objective of these two modes of shipping operations is usually to maximize the profit rather than minimizing the operating cost from a business perspective [17]. On the other hand, because the fleet owner owns the cargo at the same time, industrial shipping operations aims to achieve an efficient transportation with lowest cost.

Due to the less concern from large companies, there have been relatively few researches on tramp operations. Liner and industrial operations, on the other hand, have attracted more attention on research. While the liner operators focus on the revenue which highly depend on the quality of services, the primary purpose of industrial operation is usually to ensure transportation continuity of cargos and reduce cost. We will discuss the literatures for liner and industrial transportation system respectively.

2.1.1 Liner shipping

The volume of containers carried by the top 20 liner shipping companies increased 12.4% in 2010 [5]. Despite the importance of liner shipping, there are scarce

researches on routing and scheduling in liner operations [21]. And most early studies on liner shipping were based on modeling with simulation and heuristic decision rules, due to the uncertainties on cargo availability.

Boffey et al. [23] provided a description of the realistic ship scheduling problem, and built an interactive computer program which used heuristics for scheduling. A fine formulation was provided by Jaramillo and Perakis [24], which not only discussed the mathematical modeling of the scheduling problem, but also provided a way to estimate all the necessary parameters from available data. The problem was formulated using linear programming, and some heuristic analysis is included in the solution approach. Although as mentioned above, the objective is usually to maximize the profit in liner operations, this article provides a good example of using minimizing cost as the objective. Fagerholt [25] considered a multi-ship pickup and delivery problem with soft time windows, where time is controlled by inconvenience cost, and provided a more realistic model. Meng and Wang [26] proposed a chance constrained programming model to deal with a short term liner ship fleet planning problem with cargo shipment demand uncertainty for a container shipping company.

There are some other researches which used other mathematical techniques, on various shipping problems. Rana and Veckson [27], as well as Fagerholt [28], used a model with integer programming to make decisions on fleet size and vessel routing. Rana and Vickson [29] developed another model using non-linear programming in order to decide the deployment. Hersh and Ladany [30], as well as Ting and Tzeng [31], on the other hand, used dynamic programming for the scheduling for cruise passengers.

2.1.2 Industrial shipping

As early as 1954, Dantzig and Fulkerson [32] discussed a tanker scheduling problem where dates of loading and discharging are specified, and the objective is to minimize the total traveling distance, with the assumption of a single loading

and a single discharging port per voyage. The problem is solved considering another objective of minimizing the total distance with empty tanker by Flood [33]. Briskin [34] extended the problem by allowing multiple discharging ports. These are the starting of researches on industrial and tramp shipping.

Some computer-aided systems were then developed to help making decisions. Miller [35] developed a system with mixed integer programming model and a network flow model in order to generate schedules. The model not only considered the fleet routing and scheduling problem, but the inventory resupply problem as well.

In terms of mathematical techniques, Brown et al. [36] used integer programming techniques for solving the optimal scheduling and cruising speed with the objective of minimizing the total operating cost. Boykin and Levary, on the other hand, analyzed the scheduling problem based on simulation [37]. Miller [35] and Ronen [38] combined integer programming techniques with heuristic analysis to get the optimal solutions.

While planning for liner shipping are more on strategy level, researches on industrial shipping focus more on operation level. The improvement in the research for industrial operations is mainly focused on the modeling part and considering more issues. For example, empty container repositioning is an issue getting some concern in some literatures. Shintani et al. designed for both ship routing and empty container repositioning using a two-stage formulation [39]. Imai et al. compared services in two kinds of port networks, taking empty container management into consideration [40].

2.2 Routing and Scheduling Models

Ronen specified the definition of routing and scheduling as following. “Routing” means specifying sequences of ports of call to ships, while “scheduling” is routing with times (or time windows) attached to the calls from the ports [17].

2.2.1 Routing

The routing problem aims to determine the optimal visiting sequence of ports. A well-defined class of problem, vehicle routing problem (VRP), defined as the problem of optimally designing routes for a set of vehicles, describes such problems. It is a generalization of the Traveling Salesman Problem (TSP), and hence is NP-hard. Much progress has been made on this area since Dantzig and Ramser published the first article on truck dispatching [41]. A survey can be found in the book by Toth and Vigo [42].

There are generally two kinds of approaches in the literatures.

Approach 1

The first approach is to formulate the problem directly as a mathematical model, usually as a mixed integer programming problem. The solution shows the selection of a set of edges that traverse required customers. The first formulation was first proposed by Laporte and Nobert [43], which sets the integer variables as the number of times that each edge is traveled, and the problem is solved by a branch and bound algorithm. Another paper by Laporte [44] extended and improved the work by involving capacity and distance constraints simultaneously while taking advantage of the structural similarities.

Rana and Vickson's work proposed a nonlinear model where partial demand fulfillment is considered [29]. Another model which is similar to knapsack problem is adopted by Shintani et al [39]. Alvarez presented a comprehensive model with transshipment and partial demand fulfillment taken into consideration [45].

The work presented in [44] is known as the classical VRP. A fixed fleet of m identical vehicles with limited capacity at a depot are to be assigned to customer ports so that

- (i) Each city, except the depot(node 1), must be visited exactly once and by a single vehicle

- (ii) Each vehicle starts and ends its journey at the depot.

A solution of the VRP can be interpreted as m cycles sharing the same vertex at the depot. The formulation is written as:

$$\min \sum_{i,j \in V} c_{ij} x_{ij} \quad (2.1)$$

$$\text{s.t.} \quad \sum_{j \in V^-} x_{1j} = 2m \quad (2.2)$$

$$\sum_{i < k} x_{ik} + \sum_{j > k} x_{kj} = 2, \quad k \in V^- \quad (2.3)$$

$$\sum_{i,j \in S} x_{ij} \leq |S| - r(S), \quad S \subseteq V^-, |S| \geq 3 \quad (2.4)$$

$$x_{ij} = 0, 1, 2, \quad i = 1, j \in V^- \quad (2.5)$$

$$x_{ij} = 0, 1, \quad \text{otherwise} \quad (2.6)$$

In this formulation, x_{ij} is a binary variable indicating whether there is a voyage between port i and j . Constraints in 2.4 ensure that there is no occurrence of illegal sub-tours in the solution, i.e. there are no ports disconnected from the depot. Alternative sub-tour elimination constraints have been studied [46, 47]. And it is proved that the constraint can also be expressed as $\sum x_{ij} \geq 2r(S)$, $\forall i \in S, j \notin S$, (or) $i \notin S, j \in S$.

In classical VRP, the vehicles are assumed to travel with constant speed, so that traveling distances and traveling cost are considered as synonymous [48].

Approach 2

In this approach, two phases are usually involved. Feasible routes are first generated, and the routes are given as input to an integer programming problem in the second phase to be selected, and assigned to vehicles. The contributions of researches focus on new and efficient route generation algorithms for this approach.

An early method for route generation proposed by Clarke and Wright is based on the concept of saving [49], where an estimate of the cost reduction obtained by

serving two customers in the same route, rather than in two separate ones. Laporte and Semet showed that the algorithm was fast, but might have a poor performance [50]. Modifications were made to the algorithm to improve its computing efficiency by Paessens [51]. Another development based on the savings approach is done by Wark and Holt [52], who implemented route merges by a matching algorithm. It provides a generally better result, while requires much more computational times.

Another widely used route construction method is sequential insertion algorithm [53]. The extra distance resulted from insertion of an un-routed customer is evaluated as the criteria for selection and insertion. A 2-opt procedure is carried out to further improve the current route. This method is adopted in real problem in Ref. [28].

Foster and Ryan [54] and another paper by Ryan [53] developed heuristic rules for clustering customers based on some properties of optimal routes. It results in the family of petal algorithms. Renaud extended the algorithm as a 2-petal heuristic by embedding routes and intersecting routes in Ref. [55].

The second phase of this approach is then carried out to select among the generated routes. A simple example was presented in an early work by Balinski and Quandt [56], where integer programming is used to minimize the total cost, with binary variables indicating whether a route is selected. Extensions can be found in Ref. [57, 58, 59]. Powell and Perakis presented in Ref. [60] a model to determine the assignment of available routes to ships. Fagerholt in Ref. [28] allowed multi-trip for one vehicle as long as the total operating time does not exceed the limit.

2.2.2 Scheduling

In the scheduling problem, the time to visit each customer is determined under time window and precedence constraints. Since the process is highly related to routing information, algorithms can be also generally categorized into two families respectively.

Approach 1

Corresponding to the first approach in routing problem, visiting time is determined by including time window constraints in mathematical model formulation. The vehicle routing problem with the time windows [61], as a generalization of the classical VRP, aims at determining routing and scheduling in which service at each customer i must start within a given time window $[a_i, b_i]$. A widely used formulation is presented in Ref. [62] in which the model assigns routes to vehicles, and time window constraints are considered for each vehicle and each edge. An improvement on the strengthening of the time window constraints is proposed by Desrochers and Laporte later [63]. Similar formulations can be found in Ref. [64], where a capacitated vehicle routing problem with pickups, deliveries and time deadlines is formulated.

A new model is presented by Fagerholt et al. in Ref. [65] to determine an assignment of vessels to predefined voyages, as well as the sequence of voyages to be traveled. The binary decision variables show continuity between each two voyages.

Agarwal and Ergun solved ship scheduling and cargo routing problem simultaneously under a weekly service frequency constraint [66], which can apply to liner shipping industry. The mixed integer programming model is formulated based on a space-time network. As each node in the network represent the port and the time of the week, arcs between nodes show routing and scheduling at the same time. The same technique is adopted in [67]. Multiple layers of space-time networks are constructed to represent the routing for each ship and the flow of containers.

Approach 2

Time constraints can be considered as a criterion for feasibility in the process of route generation when plan using this approach.

Boffey et al proposed a heuristic optimizing model for scheduling and compared the results with an interactive computer program [23]. A hill climbing heuristic

method is adopted in the generation of routes, and ports are inserted according to determined criteria. Route practicability is considered with regard to leading to no delays.

Fagerholt and Lindstad considered a real problem in Nowegian sea [68]. Due to the small problem size, all feasible schedules are enumerated, with time and capacity constraints considered. Fagerholt and Christiansen proposed an algorithm to generate feasible schedules by solving a shortest path problem, with time and allocation information included in the nodes [69]. An extension is then given in Ref. [25] by introducing soft time windows, when inconvenience costs are imposed.

Hennig et al. introduced a path flow model where paths are ship routes, in order to determine ship routing, arrival and departure times for each port [70]. Time window constraints are the main concern in route generation, early arrival of ships is also limited due to the allowed waiting time.

Minimum operation time or operation cost is usually considered for optimality. Mourao et al. presented an integer programming model where inventory cost occurred by transshipment at hub port, together with ship operation cost, is considered in the objective function [71].

Other scheduling problems and approaches

Ting and Tzeng proposed a dynamic programming model for ship scheduling to minimize the total expected variations in time between available berth time windows and estimated berth time [31]. Instead of an objective function, the operational cost is analyzed only to help planners make better decision. Similar objective can be found in Ref. [72], where the objective is to minimize the total inconvenience in the schedule, defined as a weighted sum of excess ride time and delivery deviations.

Lee et al. presented a resource allocation scheduling model to schedule loads with delivery times [73]. Minimizing the total number of vehicle used, as well as the associated cost with tardiness and earliness, are the two objectives considered in the problem simultaneously.

2.2.3 Comparison between the two approaches

Both approaches discussed previously have been widely used researches. Some of the advantages and disadvantages can be summarized as follows:

- The first approach defines binary variables based on edges, and thus the number of binary variables is almost proportional to the number of edges. However, the number of constraints can be exponentially increasing, and hence lead to high complexity with larger problem sizes. For example, in classical Vehicle Routing Problem, a constraint is constructed for every subset of the nodes, and thus the number of constraints for sub-tour elimination is exponential with respect to the number of ports considered. On the other hand, the second approach has the advantage of less constraint, whose number is usually proportional to the number of ports or ships, while the number of binary variables, which represent the number of potential feasible routing and scheduling, can be disastrous in some cases.
- Generating of possible routes is usually done using heuristics and the processes are mostly rule-based, it is relatively easy to take additional constraints into consideration. Since route feasibility is considered in the route generation phase, approach 2 is more adaptable to various cases and concerns. On the other hand, adding some concerns in the problem sometimes requires significant changes in the formulation for the first approach.
- The improvement in solution and computational times for a certain problem is mostly on using optimization techniques to solve of the mixed integer programming in approach 1, while the improvement for approach 2 usually focuses on development of more efficient heuristic algorithms for route generations. There have been various families of heuristics for route generation and route improvement, including saving-based algorithms [49, 52], sweep algorithm [74], petal algorithm [55], and some well development algorithms such as annealing [75] and genetic search [76, 77, 78].

2.3 Energy Efficiency in Ship Operations

As mentioned in last chapter, bunker fuel cost constitutes a large portion of total operational cost of shipping companies. A recent study by UNCTAD shows that a 10% increase in oil price would lead to 1.9% to 3.6% of increase in cost of shipping a container [9]. Hence, the effect of fuel consumption on operational cost should be examined.

2.3.1 The effect of oil price on ship speed

Ronen [79] has developed a mathematical model to study the optimal ship speed considering the trade-off between savings and voyage extension. In his paper, he discussed ship speeding problem according to three kinds of trips, and in each scenario, he gave a precise mathematical presentation of the objective function.

(i) Income generating leg

$$\max\{Z = \frac{R - D_p C - \frac{LC}{24V} - (\frac{V}{V_0})^3 F_0 F_c \frac{L}{24V}}{D_p + \frac{L}{24V}}\} \quad (2.7)$$

(ii) Positioning leg

$$\min\{Z = \frac{L}{24V} C_a + (\frac{V}{V_0})^3 F_0 F_c \frac{L}{24V}\} \quad (2.8)$$

(iii) Speed related income

$$\max\{Z = \frac{R + \frac{KL}{24}(\frac{1}{V_R} - \frac{1}{V}) - D_p C - \frac{LC}{24V} - (\frac{V}{V_0})^3 F_0 F_c \frac{L}{24V}}{D_p + \frac{L}{24V}}\} \quad (2.9)$$

with the following notations:

R	Income from the leg
D_p	Port days in the leg
L	Leg distance in nautical miles
V	Actual cruising speed
V_0	Nominal (maximal) cruising speed
C	Daily cost of vessel
F	Actual daily fuel consumption
F_0	Daily Fuel consumption at nominal speed
F_c	Cost of bunker fuel(\$/ton)

Note that in the second model, where only the cost is considered, the optimal cruising speed is not a function of the length of the trip L . It is suggested that the same result can be applicable to round trip calculations. For the second mode, setting $dZ/dV = 0$ gives $V^* = V_0(\frac{C_a}{2F_0F_c})^{1/3}$. Hence, the fuel consumption of a vessel is almost proportional to the third power of the speed.

However, the fuel consumption rate is also related to the resistance a ship overcomes in water. Theoretically, not only cruising speed but also displacement contributes to the resistance, since the resistance a ship overcomes is proportional to the wetted surface and the square of cruising speed [80]. The wetted surface is directly related to the displacement, and hence the total weight of the ship. The fuel consumption rate is proportional to the power of ship, which can be expressed as Fv , and hence is proportional to the displacement and v^3 . As the deadweight of a large cargo ship can be several times the lightweight, the effect of load on fuel consumption rate should not be ignored. However, in most literatures, the factor of displacement is ignored or assumed to be constant throughout voyages in most of literatures. The approximation may result in an inferior solution and degrade the effectiveness in practice. Since the pickup and delivery service order may result in quite fluctuating load on board, it is important and beneficial to consider this effect on fuel consumption from rate from the changing of weights on board when deciding shipping service orders.

2.3.2 Ship speed as a variable in fleet management models

The property that fuel consumption rate is proportional to cube of velocity, leads to some insights for reducing fuel consumption by slow steaming strategy. For example, cruising speed is included as a decision variable in ship scheduling problem by Brown et al. [36], and Schrady and Wadsworth developed a computerized system to estimate fuel consumption of naval combat vessels [81]. However, a review chapter by Christiansen et al. reveals that relatively little research has been done regarding optimizing operation cost by adjusting the speed of vessels [82].

Alvares presented a model to address the routing problem and deployment problem jointly [45]. Ship operation speed selection is considered when generating parameters for the fuel cost in objective function. The third power relationship is assumed in the model. The speed cannot be optimized since it is considered as a parameter rather than a decision variable. Fagerholt et al. proposed a method to determine the cruising speed for a given fixed route consisting of a sequence of ports with time windows [83]. Arrival times are discretized and the problem is solved as a shortest path problem on a directed acyclic graph. Yao et al. considered a bunker fuel management strategy study for a single ship on a liner service with fixed routing, where they highlighted the importance of joint optimization of bunkering ports selection, bunkering amount determination, and ship speed adjustment [84]. However, the decision are limited to the bunkering and speed, with given routing information.

2.4 Summary

The models in this work are based on Approach 1, as the formulation is straightforward in terms of showing routing and scheduling. Approach 2 has certainly shed some lights on the development of heuristics. The heuristic algorithms discussed later in this thesis are a combination of these two approaches.

We first list some critical references and issues on which our model is based.

- (i) The classical VRP [44]
- (ii) Scheduling problem for hub-and-spoke model(single depot) [85]
- (iii) Pure scheduling problem [31]
- (iv) Vehicle routing problem with time window [86]
- (v) Bunkering strategy [84]

The models we developed in this thesis are based on the vehicle routing problem with time windows, while considering more real life factors including demand split and partial demand fulfillment. Such problems have been partially studied in paper, such as in Ref. [44, 85, 86]. The optimal control of cruising speed and refueling policy is relatively new issue in papers, which is discussed in Ref. [84], will also be studied in our models.

Chapter 3

Basic Model Development and Algorithms

3.1 Capacitated Vehicle Routing Problem with Time Windows

3.1.1 Problem description

In this section, a simple problem is presented for a fixed fleet of K identical ships $V = 1, 2, \dots, K$ with limited capacities to plan for shipping services on a set of ports $1, 2, \dots, N$, among which port 1 is the depot, and the other ports are customer ports. The shipping network can be modeled as a graph $G = (P, E)$, where $P = 1, 2, \dots, N$ is the set of ports and $E = (i, j) : i, j \in P$ is the set of feasible directed voyages between ports.

Each customer port has a deterministic demand that the fleet needs to deliver from the depot to the customer. Splitting demands such that the delivery of certain loads is completed in multiple trips rather than one trip results in opportunities for a reduction in cost and the number of vehicle used, by the efficient use of excess capacities [87], this model allows that the demands for a certain customer port can be fulfilled by more than one ship. It is assumed that the total capacity of the fleet is no less than the total demands from all customer ports.

Each customer port has a pre-defined time window in which ships can serve. Ships must arrive at the port within the time window. For simplicity, the waiting at the depot or anywhere outside customer ports is assumed to incur no cost for all ships. The port processing time at each port, including entry time, unloading time, idle time and exiting time, is assumed to be known or can be estimated with reasonable accuracy, and hence is assumed to be fixed in the model.

The planning must consider the following requests, while fulfilling all demands from customers with the objective as minimizing total operating cost:

- (i) Each customer port is to be visited at least once, and its demand must be fulfilled. It is allowed that a port is visited by several ships, which means demand can be split.
- (ii) Each ship starts and ends its journey at the depot.
- (iii) The total amount of cargos that a certain ship carries cannot exceed its capacity.
- (iv) Ships must arrive at customer ports within the scheduled time windows. And all ships should return to the depot before a requested operating cycle time.
- (v) It is assumed in this model that the ships are always traveling at a fixed speed, so that the traveling cost on each voyage can be represented by the traveling distances.

3.1.2 Notations

We define notations in the model as follows,

Sets

P Set of all ports. Ports are indexed by i, j

P^- Set of all customer ports, $P^- = P - \{1\}$

S Subset of P^-

V Set of all ships. k indicates a certain ship

Parameters

N	The number of ports. For ease notation, Port $N + 1$ is defined as the index of a virtual port with the same location as depot
d_{ij}	The traveling cost (distance) between port i and j
D_i	The demand from port i , where $i \in P^-$
Q^k	Capacity of ships
T^k	Operating cycle time for ship k
v	Traveling speed
s_i^k	Port processing time for ship k at port i
e_i, l_i	Time window at port i

Decision variables

x_{ij}^k	Binary variable indicating whether directed arc $\{i, j\}$ is traveled by ship k . $x_{ij}^k = 1$ if ship k travels from port i to j , and 0 otherwise. Note that voyages from depot to customer ports are defined as $x_{1j}^k, j \in P^-$, and voyages from customer ports to depot are defined as $x_{i,N+1}^k, i \in P^-$
A_i^k	Arrival time of ship k at port i
q_i^k	The amount of cargos ship k delivers to port i

3.1.3 Constraints

Constraint set 1: Routing

The first set of constraints, based on the classical VRP developed by Laporte et al. [44], describes the flow of all ships as follows.

$$\sum_{j \in P^-} x_{1j}^k = 1, \forall k \in V; \quad (3.1)$$

$$\sum_{i \in P^-} x_{i,N+1}^k = 1, \forall k \in V; \quad (3.2)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \forall i \in P^-, k \in V; \quad (3.3)$$

$$\sum_k \sum_{j \in P} x_{ji}^k \geq 1, \forall i \in P^-, k \in V; \quad (3.4)$$

$$\sum_{i,j \in S} x_{ij}^k \leq |S| - 1, S \subseteq P^-, \forall k \in V \quad (3.5)$$

Constraints in 3.1 ensure that each ship sets out from the depot and equations in 3.2 force ship to return to depot after services. Equation 3.3 is a set of flow conservation constraints, which states that for each customer port, the number of voyages into the port is the same as the number of voyages out from the port. Note that the flow conservation also holds for the depot, which can be derived from the first two constraints. The fourth constraints in 3.4 state that each customer port is visited at least once.

Sub-tour refers to the case that there exists a sub-graph G' of G , such that G' is not connected to $G - G'$. Figure 3.1 is an simple example of sub-tour occurrence. The arrows in black and red represent the voyages for two ships respectively. It can be easily shown that the first four constraints hold for the red voyages, while there is no way for the ship to serve ports 6, 7 and 8. However, for any subset S of customer ports P^- , no loops should be contained, as every loop in the feasible solution should contain the depot, which is not in S . Hence, the maximum number of voyage legs with both starting and ending ports within S is $|S| - 1$, which is

Figure 3.1: An example of sub-tour

stated by constraints in 3.5. If there is a sub-tour in the solution, take Figure 3.1 for example, we can always find a subset S which contains all the ports in the sub-tour (port 6, 7 and 8 in this case). The number of voyages within S is $|S|$, and thus the constraints in 3.5 will be violated.

As a matter of fact, constraints in 3.4 and 3.5 are not necessary in the problem with time windows, as they can be implied by other constraints.

Constraint set 2: Assignment

This set of constraints describes the assignment of ships to customer ports so that all demand can be met without violating the capacity constraints.

$$\sum_{k \in V} q_i^k = D_i, \forall i \in P^-; \quad (3.6)$$

$$\sum_{i \in P^-} q_i^k \leq Q^k, \forall k \in V; \quad (3.7)$$

$$q_i^k \leq M \sum_{j \neq i} x_{ij}^k, \forall i \in P^-, k \in V \quad (3.8)$$

Constraints in 3.6 indicate that the amount of cargos delivered to a port by all ships fulfills its demand, and thus the demand of every port is met. Constraints in 3.7 make sure that the total amount of cargos that a certain ship carries to all ports does not exceed its capacity.

Constraints in 3.8 are the indicator constraints, so that a ship delivers cargo to a port only if the port is on its planned journey. If a ship k goes through port i , there must be a voyage leg starting from port i , and hence the right hand side is equal to M , which is a large enough number. And hence, q_i^k can be positive. On the other hand, if ship k does not go through the port, the right hand side is 0, and q_i^k is forced to be 0.

As mentioned in the Constraint set 1, constraints in 3.4 are not necessary because of the existence of 3.6 and 3.8. Since the demands of all ports are met, and demand can only be met when at least one ship goes to the depot, constraint in 3.4 can be implied.

Constraint set 3: Scheduling

The last set of constraints defines the scheduling of all ships, so that the arrival times of all ships to all ports are within the pre-defined time windows. These constraints can also be found in the model of Vehicle Routing Problem with Time Windows (VRPTW) [86].

$$A_j^k \geq A_i^k + s_i^k + \frac{d_{ij}}{v} - M(1 - x_{ij}^k), \forall i, j, k; \quad (3.9)$$

$$e_i \leq A_i^k \leq l_i, \forall i \in V^-, k \in V; \quad (3.10)$$

$$A_1^k = 0, A_{N+1}^k \leq T^k, \forall k \in V; \quad (3.11)$$

Constraints in 3.9 describe that if a ship travels from port i to port j , the time constraint should be met. As mentioned previously, the sub-tour elimination constraints are not necessary because of this set of constraints. As all $d_{ij} > 0$, $A_j^k > A_i^k$ for all voyages legs such that $x_{ij}^k = 1$. If a loop within the customer ports exists, $i_1, i_2, \dots, i_t, i_1$ for example, where i_s is the s th visiting port, it can be derived that $i_1 > i_t > \dots > i_2 > i_1$, which is a contradiction. As the time window constraints imply the sequence of visiting ports, the occurrence of sub-tours can be prevented.

Constraints in 3.10 ensure that the arrival times of ships at customer ports are within the time windows. The set out time from the depot is assumed to be 0, and

the return time for each ship must be before its cycle time T^k , which is indicated by constraints in 3.11.

3.1.4 Formulation

The objective function is to minimize the total traveling distances of all ships. Using the defined binary variables x_{ij}^k , and the parameter d_{ij} , the objective function can be simply expressed as follows,

$$\min \sum_{k \in V} \sum_{i,j} d_{ij} x_{ij}^k \quad (3.12)$$

Combining with all the constraints discussed above, the planning model for a fleet from depot to multiple customers with time windows is formulated as follows, where the binary decision variables are defined in constraints in 3.23.

$$\min \sum_{k \in V} \sum_{i,j} d_{ij} x_{ij}^k \quad (3.13)$$

$$\sum_{j \in P^-} x_{1j}^k = 1, \forall k \in V; \quad (3.14)$$

$$\sum_{i \in P^-} x_{i,N+1}^k = 1, \forall k \in V; \quad (3.15)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \forall i \in P^-, k \in V; \quad (3.16)$$

$$\sum_{k \in V} q_i^k = D_i, \forall i \in P^-; \quad (3.17)$$

$$\sum_{i \in P^-} q_i^k \leq Q^k, \forall k \in V; \quad (3.18)$$

$$q_i^k \leq M \sum_{j \neq i} x_{ij}^k, \forall i \in P^-, k \in V; \quad (3.19)$$

$$A_j^k \geq A_i^k + s_i^k + \frac{d_{ij}}{v} - M(1 - x_{ij}^k), \forall i, j, k; \quad (3.20)$$

$$e_i \leq A_i^k \leq l_i, \forall i \in P^-, k \in V; \quad (3.21)$$

$$A_1^k = 0, A_{N+1}^k \leq T^k, \forall k \in V; \quad (3.22)$$

$$x_{ij}^k = 0 \text{ or } 1, \forall i, j, k \quad (3.23)$$

3.2 Enhanced Model with Fuel Management

3.2.1 Problem description

In this problem, each customer port brings different revenues to the fleet for every unit of demand fulfilled. It is possible that the total demands from all customer ports are larger than the total capacity of the fleet. The fuel prices are different at each port. As incremental quantity price discounts are commonly applied [88], and bunkering ports usually provide price discounts under contractual agreement [84], it is assumed in this model that ports provide price discounts on incremental quantity of fuel bunkered by a ship. The planning must consider the following requests, in addition to the first basic model:

- (i) Each ship can adjust its cruising speed on every voyage within a requested velocity range. The speed should make sure that the ship can arrive at service ports within the pre-determined time windows.
- (ii) For each ship, it can choose to refuel at any port including the depot. Note that fuel consumption rate is related to the ship speed, the refueling amount should be made sure to be enough for traveling using the planned speed to the next refueling port.
- (iii) The fuel amount a ship carries at any time cannot exceed its fuel capacity, and cannot be lower than a pre-determined amount (safety level).

We will develop a planning model to determine routing and scheduling, as well as the optimal fuel management strategy, with the objective of maximizing the total profit. The revenues come from customer ports with demand fulfilled, and the traveling cost is considered as fuel cost. As choosing demand to serve is a concern in this problem, maximizing the profit is considered as a more proper objective than minimizing the total cost, as in the first model.

3.2.2 Notations

In addition to the notations in last model, we need to define some other notations as follows.

Parameters

I_m	Fuel capacity of ship k
r_i	Revenue for each unit port i 's demand fulfilled
k_1, k_2	Fuel consumption coefficients
v_{min}, v_{max}	Minimum and maximum ship cruising speed
p_i^1	Unit fuel price at port i
p_i^2, p_i^3	Discounted unit fuel prices at port i , $p_i^3 < p_i^2 < p_i^1$

Decision variables

v_{ij}^k	Cruising speed of ship k from port i to j
I_i^k	Residual fuel in ship k when it arrives at port i
B_i^k	Bunkering volume of ship k at port i
y_i^k	Binary variable to indicate whether to refuel ship k at port i
$\alpha_{i1}^k, \alpha_{i2}^k, \alpha_{i3}^k, \alpha_{i4}^k$	Variables used for piecewise fuel cost for ship k at port i
$\beta_{i1}^k, \beta_{i2}^k, \beta_{i3}^k$	Binary variables used for piecewise fuel cost for ship k at port i

It is assumed that the fuel price discount is set such that the unit fuel price at port i is p_i^1 for the first refueling amount $1/3 I_m$, p_i^2 for the second $1/3 I_m$, and p_i^3 for the amount more than $2/3 I_m$. The bunkering cost at port i can then be expressed by a piecewise linear function as follows.

$$\left\{ \begin{array}{ll} B_i^k p_i^1 & B_i^k \in [0, {}^1/{}_3 I_m] \\ {}^1/{}_3 I_m p_i^1 + (B_i^k - {}^1/{}_3 I_m) p_i^2 & B_i^k \in ({}^1/{}_3 I_m, {}^2/{}_3 I_m] \\ {}^1/{}_3 I_m p_i^1 + {}^1/{}_3 I_m p_i^2 + (B_i^k - {}^2/{}_3 I_m) p_i^3 & B_i^k \in ({}^2/{}_3 I_m, I_m] \end{array} \right. \quad (3.24)$$

An empirical model to describe the relationship between fuel consumption rate and ship speed provided in Ref. [84] is adopted in this model, which stated that $F = k_1 V^3 + k_2$ is a better model to express the relationship than Ronen's model.

3.2.3 Three-stage formulation

We first propose a three-stage model to determine the following issues in each stage respectively.

- (i) Ports to visit and quantities to deliver for each ship;
- (ii) The routing and scheduling for each ship;
- (iii) Fuel management strategy, including refueling port selection, bunkering amount determination and ship speed adjustment.

Stage 1: Quantities

In the first stage, an assignment problem is solved to maximize the total profit, where decision is made on the amount of cargos delivering to each customer port by each ship.

$$\begin{aligned} \max_q \quad & \sum_{k \in V} \sum_i r_i q_i^k - \sum_{k \in V} G_k(q_i^k) \\ \text{s.t.} \quad & \sum_{k \in V} q_i^k \leq D_i, \quad \forall i \in P^- \end{aligned} \quad (3.25)$$

$$\sum_{i \in P^-} q_i^k \leq Q^k, \quad \forall k \in V \quad (3.26)$$

The objective function of this stage is to maximize the total profit by delivering the cargos. The first term is the total revenue gained from all delivered cargos. The second term is the optimal total traveling cost for all ships to fulfill those demands, expressed as the sum of cost from each ship, which is the objective function of the second stage.

Constraints in 3.25 describe amount of cargos delivered to each customer port. Note that when the fleet capacity is not enough to fulfill all demands, these inequalities instead of equalities 3.6, together with the objective of maximizing the total profit, are able to handle such situations and choose the most profitable ports to visit. Constraints 3.26 are capacity constraints for each ship.

Stage 2: Routing

The second stage solves a Traveling Salesman Problem (TSP) for a single ship. In the first stage, we have decided the ports the ship needs to visit. The sequence of visiting those ports is to be determined in this stage with the objective to minimize the traveling cost, expressed as the sum of fuel cost at each port the ship passes.

$$G_k(q) = \min_x \sum_i H_{ik}(x, q) \quad (3.27)$$

$$\text{s.t. } q_i^k \leq M \sum_{j \neq i} x_{ij}^k \quad (3.28)$$

$$\sum_{j \in P^-} x_{1j}^k = 1 \quad (3.29)$$

$$\sum_{i \in P^-} x_{i,N+1}^k = 1 \quad (3.30)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k \quad (3.31)$$

$$\sum_{i,j \in S} x_{ij}^k \leq |S| - 1, \quad S \subseteq P^- \quad (3.32)$$

$$x_{ij}^k = 0 \text{ or } 1, \quad \forall i, j \quad (3.33)$$

Constraints in 3.28 are the indicator constraints to make sure that the ship goes to those ports as determined in the first stage. Equations 3.29 and 3.30

indicate that the ship starts and ends at the depot. Equation 3.31 is the flow conservation constraints for each customer port, and constraint in 3.32 is the sub-tour elimination constraints. And constraints in 3.33 define the binary property of the variables.

Stage 3: Bunker strategy

In the third stage, decisions are made for each ship at each port on whether the ship should refuel at the port, and how much to refuel, with the objective to minimize the total bunkering cost.

$$H_{ik}(x, q) = \min \alpha_{i2}^k \frac{I_m}{3} p_i^1 + \alpha_{i3}^k \left(\frac{I_m}{3} p_i^1 + \frac{I_m}{3} p_i^2 \right) + \alpha_{i4}^k \left(\frac{I_m}{3} p_i^1 + \frac{I_m}{3} p_i^2 + \frac{I_m}{3} p_i^3 \right) \quad (3.34)$$

$$\text{s.t. } e_i \leq A_i^k \leq l_i \quad (3.35)$$

$$A_1^k = 0, a_{N+1}^k \leq T^k \quad (3.36)$$

$$A_j^k \geq A_i^k + d_{ij}/v_{ij}^k + s_i - M(1 - x_{ij}^k), \forall j \quad (3.37)$$

$$A_j^k \leq A_i^k + d_{ij}/v_{ij}^k + s_i + M(1 - x_{ij}^k), \forall j \quad (3.38)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij}/v_{ij}^k [k_1 (v_{ij}^k)^3 + k_2] - M(1 - x_{ij}^k), \forall j \quad (3.39)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij}/v_{ij}^k [k_1 (v_{ij}^k)^3 + k_2] + M(1 - x_{ij}^k), \forall j \quad (3.40)$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i, k \quad (3.41)$$

$$B_i^k = \alpha_{i1}^k 0 + \alpha_{i2}^k 1/3 I_m + \alpha_{i3}^k 2/3 I_m + \alpha_{i4}^k I_m \quad (3.42)$$

$$B_i^k \geq y_i^k 20\% I_m \quad (3.43)$$

$$B_i^k \leq y_i^k I_m \quad (3.44)$$

$$\alpha_{i1}^k \leq \beta_{i1}^k \quad (3.45)$$

$$\alpha_{ij}^k \leq \beta_{i(j-1)}^k + \beta_{ij}^k, j = 2, 3 \quad (3.46)$$

$$\alpha_{i4}^k \leq \beta_{i3}^k \quad (3.47)$$

$$\alpha_{i1}^k + \alpha_{i2}^k + \alpha_{i3}^k + \alpha_{i4}^k = 1 \quad (3.48)$$

$$\beta_{i1}^k + \beta_{i2}^k + \beta_{i3}^k = 1 \quad (3.49)$$

$$\alpha_{ij}^k \geq 0, \quad j = 1, 2, 3, 4 \quad (3.50)$$

$$\beta_{ij}^k = 0 \text{ or } 1, \quad j = 1, 2, 3 \quad (3.51)$$

$$I_i^k \geq 5\%I_m \quad (3.52)$$

$$I_i^k + B_i^k \leq I_m \quad (3.53)$$

$$y_i^k = 0 \text{ or } 1 \quad (3.54)$$

$$v_{min} \leq v_{ij}^k \leq v_{max}, \quad \forall j. \quad (3.55)$$

Constraints in 3.35 to 3.38 are the time window constraints. Inequality 3.35 is the pre-defined time window, and 3.36 define the set out and return times at the depot. If ship k travels from port i to j , i.e. $x_{ij}^k = 1$, the equation $A_j^k = A_i^k + d_{ij}/v_{ij}^k + s_i$ should hold, which is expressed by inequalities 3.37 and 3.38 together. As discussed in the first model, the sub-tour elimination constraints in 3.32 are not necessary.

Similarly, constraints in 3.39 and 3.40 describe the fuel consumption on each voyage. If ship k travels from port i to j with cruising speed at v_{ij}^k , the fuel consumption, described as $I_i^k + B_i^k - I_j^k$, should be equal to the fuel consumption on the (i, j) voyage, expressed as the product of the fuel consumption rate $k_1(v_{ij}^k)^3 + k_2$ and the traveling time d_{ij}/v_{ij}^k .

Constraints in 3.41 to 3.54 describe the bunkering activity at each port. Constraint in 3.41 is the indicator constraint to ensure that the ship can only refuel at ports that it passes. The bunkering amount, which is expressed using constraint in 3.42, must be within a pre-defined range, with the lower bound expressed by 3.43, and upper bound as 3.44. Note that constraint in 3.44 also serves as an indicator constraint, so that if $y_i^k = 0$, ship k should not bunker at port i , and thus $B_i^k = 0$. Constraints in 3.45 to 3.51 are the piecewise linearization constraints. Equation 3.49 with binary variables ensures that the bunkering amount falls in

one of the three intervals. Constraints in 3.45 to 3.48 make sure that only the fraction variables α s at the two ends of chosen interval are active, and they add up to 1. Hence, the fuel cost, as expressed in the objective function, can be viewed as a linear combination of refuel cost at the two breakpoints which define the interval. And the bunkering amount is a linear combination of the two breakpoints with the same fraction. Constraint in 3.52 ensures the minimum amount of fuel the ship carries at any time no less than a certain percentage of the fuel capacity of the ship, which is set at 5% arbitrarily in this model. Constraint in 3.53 defines the upper bound of the fuel amount to be I_m , so that the fuel capacity of the ship should not be violated. At last, constraint in 3.54 defines the binary variables y_i^k .

Constraints in 3.55 define the bounds for the velocity.

3.2.4 Equivalent aggregated model with linear constraints

Although the three-stage model discussed above gives a clear map of the problem, it is hard to use existing software to solve such an optimization problem for two reasons:

- (i) There are some non-linear terms in the formulation due to the fact that the fuel consumption rate is proportional to the cubic of the cruising speed.
- (ii) There is no solver that can solve a multiple-stage optimization problem.

In order to provide a solvable formulation, we will first make some changes to the constraints with non-linear terms, i.e. 3.37 to 3.40. We define $t_{ij}^k = 1/v_{ij}^k$, and replace v_{ij}^k with $1/t_{ij}^k$ in the constraints. Then 3.37 to 3.40 become

$$A_j^k \geq A_i^k + d_{ij}t_{ij}^k + s_i - M(1 - x_{ij}^k), \forall j \quad (3.56)$$

$$A_j^k \leq A_i^k + d_{ij}t_{ij}^k + s_i + M(1 - x_{ij}^k), \forall j \quad (3.57)$$

$$I_j^k \geq I_i^k + B_i^k - k_1 d_{ij}(t_{ij}^k)^{-2} - k_2 d_{ij}t_{ij}^k - M(1 - x_{ij}^k), \forall j \quad (3.58)$$

$$I_j^k \leq I_i^k + B_i^k - k_1 d_{ij}(t_{ij}^k)^{-2} - k_2 d_{ij}t_{ij}^k + M(1 - x_{ij}^k), \forall j \quad (3.59)$$

There is only one non-linear term in the last two new constraints. We can use a piecewise linear function to approximate the term $(t_{ij}^k)^{-2}$. For simplicity, we divide $[1/v_{max}, 1/v_{min}]$ into n intervals, with break points at $t_2, t_3, \dots, t_n$, $1/v_{max} = t_1 < t_2 < t_3 < \dots < t_n < t_{n+1} = 1/v_{min}$. Using similar techniques introduced in stage 3, we define new binary variables $b_{ij1}^k, b_{ij2}^k, \dots, b_{ijn}^k$ for each interval, such that

$$b_{ijp}^k = 1 \text{ only if } t_p \leq t_{ij}^k < t_{p+1}, \quad p = 1, 2, \dots, n-1 \quad (3.60)$$

$$b_{ijn}^k = 1 \text{ only if } t_n \leq t_{ij}^k \leq t_{n+1} \quad (3.61)$$

$$b_{ij1}^k + b_{ij2}^k + \dots + b_{ijn}^k = 1, \quad \forall i, j, k \quad (3.62)$$

And define $[0, 1]$ continuous variables $a_{ij1}^k, a_{ij2}^k, \dots, a_{ij(n+1)}^k$ at each breakpoint, such that

$$a_{ij1}^k + a_{ij2}^k + \dots + a_{ij(n+1)}^k = 1, \quad \forall i, j, k \quad (3.63)$$

$$a_{ij1}^k \leq b_{ij1}^k, \quad \forall i, j, k \quad (3.64)$$

$$a_{ijp}^k \leq b_{ij(p-1)}^k + b_{ijp}^k, \quad p = 2, 3, \dots, n, \forall i, j, k \quad (3.65)$$

$$a_{ij(n+1)}^k \leq b_{ijn}^k, \quad \forall i, j, k \quad (3.66)$$

Using these variables, t_{ij}^k , and the non-linear term, $(t_{ij}^k)^{-2}$, can be expressed as

$$t_{ij}^k = \sum_{p=1}^{n+1} a_{ijp}^k t_p, \quad \forall i, j, k \quad (3.67)$$

$$(t_{ij}^k)^{-2} = \sum_{p=1}^{n+1} a_{ijp}^k t_p^{-2} = \sum_{p=1}^{n+1} a_{ijp}^k v_p^2, \quad \forall i, j, k \quad (3.68)$$

where $v_p = 1/t_p$. Note that using this approximation, the boundary constraints in 3.55 are not necessary anymore. Using this expression, 3.37 to 3.40 can be

approximated as

$$A_j^k \geq A_i^k + d_{ij} \sum_{p=1}^{n+1} a_{ijp}^k t_p^k + s_i - M(1 - x_{ij}^k), \forall j \quad (3.69)$$

$$A_j^k \leq A_i^k + d_{ij} \sum_{p=1}^{n+1} a_{ijp}^k t_p^k + s_i + M(1 - x_{ij}^k), \forall j \quad (3.70)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij} \sum_{p=1}^{n+1} (k_1 v_p^2 + k_2 t_p) a_{ijp}^k - M(1 - x_{ij}^k), \forall j \quad (3.71)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij} \sum_{p=1}^{n+1} (k_1 v_p^2 + k_2 t_p) a_{ijp}^k + M(1 - x_{ij}^k), \forall j \quad (3.72)$$

Then we can aggregate all the three stages and the linear approximation as follows.

$$\begin{aligned} \max \quad & \sum_k \sum_i r_i q_i^k \\ & - \sum_k \sum_i \alpha_{i2}^k \frac{I_m}{3} p_i^1 + \alpha_{i3}^k \left(\frac{I_m}{3} p_i^1 + \frac{I_m}{3} p_i^2 \right) + \alpha_{i4}^k \left(\frac{I_m}{3} p_i^1 + \frac{I_m}{3} p_i^2 + \frac{I_m}{3} p_i^3 \right) \end{aligned} \quad (3.73)$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{k \in V} q_i^k \leq D_i, \forall i \in P^- \\ & \sum_{i \in P^-} q_i^k \leq Q^k, \forall k \in V \end{aligned} \quad (3.74)$$

$$q_i^k \leq M \sum_{j \neq i} x_{ij}^k, \forall i \in P^-, k \in V \quad (3.75)$$

$$\sum_{j \in P^-} x_{1j}^k = 1, \forall k \in V \quad (3.76)$$

$$\sum_{i \in P^-} x_{i,N+1}^k = 1, \forall k \in V \quad (3.77)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \forall i \in P^-, k \in V \quad (3.78)$$

$$x_{ij}^k = 0 \text{ or } 1, \forall i, j, k \quad (3.79)$$

$$e_i \leq A_i^k \leq l_i, \forall i \in P^-, k \in V \quad (3.80)$$

$$A_1^k = 0, a_{N+1}^k \leq T^k, \forall k \in V \quad (3.81)$$

$$A_j^k \geq A_i^k + d_{ij} \sum_{p=1}^{n+1} a_{ijp}^k t_p^k + s_i - M(1 - x_{ij}^k), \forall j \quad (3.82)$$

$$A_j^k \leq A_i^k + d_{ij} \sum_{p=1}^{n+1} a_{ijp}^k t_p^k + s_i + M(1 - x_{ij}^k), \forall j \quad (3.83)$$

$$b_{ij1}^k + b_{ij2}^k + \dots + b_{ijn}^k = 1, \forall i, j, k \quad (3.84)$$

$$a_{ij1}^k + a_{ij2}^k + \dots + a_{ij(n+1)}^k = 1, \forall i, j, k \quad (3.85)$$

$$a_{ij1}^k \leq b_{ij1}^k, \forall i, j, k \quad (3.86)$$

$$a_{ijp}^k \leq b_{ij(p-1)}^k + b_{ijp}^k, p = 2, 3, \dots, n, \forall i, j, k \quad (3.87)$$

$$a_{ij(n+1)}^k \leq b_{ijn}^k, \forall i, j, k \quad (3.88)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij} \sum_{p=1}^{n+1} (k_1 v_p^2 + k_2 t_p) a_{ijp}^k - M(1 - x_{ij}^k), \forall j \quad (3.89)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij} \sum_{p=1}^{n+1} (k_1 v_p^2 + k_2 t_p) a_{ijp}^k + M(1 - x_{ij}^k), \forall j \quad (3.90)$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i, k \quad (3.91)$$

$$B_i^k = \alpha_{i1}^k 0 + \alpha_{i2}^k 1/3 I_m + \alpha_{i3}^k 2/3 I_m + \alpha_{i4}^k I_m \quad (3.92)$$

$$B_i^k \geq y_i^k 2\% I_m \quad (3.93)$$

$$B_i^k \leq y_i^k I_m \quad (3.94)$$

$$\alpha_{i1}^k \leq \beta_{i1}^k \quad (3.95)$$

$$\alpha_{ij}^k \leq \beta_{i(j-1)}^k + \beta_{ij}^k, j = 2, 3 \quad (3.96)$$

$$\alpha_{i4}^k \leq \beta_{i3}^k \quad (3.97)$$

$$\alpha_{i1}^k + \alpha_{i2}^k + \alpha_{i3}^k + \alpha_{i4}^k = 1 \quad (3.98)$$

$$\beta_{i1}^k + \beta_{i2}^k + \beta_{i3}^k = 1 \quad (3.99)$$

$$\alpha_{ij}^k \geq 0, j = 1, 2, 3, 4 \quad (3.100)$$

$$\beta_{ij}^k = 0 \text{ or } 1, j = 1, 2, 3 \quad (3.101)$$

$$I_i^k \geq 5\% I_m \quad (3.102)$$

$$I_i^k + B_i^k \leq I_m \quad (3.103)$$

$$y_i^k = 0 \text{ or } 1 \quad (3.104)$$

Figure 3.2: Problem Structure

This mixed-integer formulation can be then solved using CPLEX.

3.3 A Heuristic

It has been well proved that TSP is NP-complete even if its instances are restricted to be realizable by sets of points on the Euclidean plane [89]. Finding an exact solution for TSP using dynamic programming requires a computational complexity of $O(c^n)$. It is obvious that a TSP is contained in the problem we are trying to solve, as in stage 2. With the formulation is of extreme large size, that it is very difficult and time consuming to effectively solve suing an exact algorithm. A simple heuristic is then proposed here to solve the problem.

3.3.1 Problem structure

We will first dig into the problem to find out some insights which can help simplify the problem. The structure we are going to discuss are actually conclusions of the sensitivity analysis from a small problem, which supported by the discussions on numerical results in the next section.

The decisions to be made using the formulation we developed in section 3.2 can be summarized as follows,

- (i) Set of ports that each ship is going to visit
- (ii) Quantities of cargos that the ship is going to deliver to the ports
- (iii) Routing and scheduling, including the sequence of visiting ports and arrival times at each port
- (iv) The ports where a ship bunkers
- (v) The bunkering amount

As a matter of fact, if one of the first three decisions is made, we can easily derive the other decisions. For example, for each ship, given a set of ports to visit (decision(i)), it will choose the most profitable cargos to deliver as long as its capacity allows. Decision(ii), the shipping quantities can be determined by unit revenues. On the other hand, as the ports that a ship visits are fixed, for each single ship, it becomes a problem that is similar to traveling salesman problem with time window, while the objective is to minimize the total fuel consumption instead of traveling distances. Moreover, the routing and bunkering policy for each ship is independent of the others. The routing and scheduling, as well as the bunkering decisions, which solely depend on routing and scheduling, can be determined as long as the time window are known. The refueling activities will be done at those from visiting ports with lowest fuel prices. This structure can be shown in Fig. 3.2.

3.3.2 Heuristic algorithm

According to the problem structure, the decisions on shipping quantities, routing and bunkering policy can be made based on the decision on visiting ports. A heuristic is proposed here based on this idea. Some rules will be defined to prioritize all the potential customer ports, and some assignments of ports to ships will

be obtained. For each assignment, the decisions on shipping quantities, routing and bunkering policy are made, and the optimal profit under the assignment is computed. The heuristic will be divided into 2 stages:

- (i) Prioritize customer ports, and choose the ports to visit under capacity constraint
- (ii) Divide chosen ports into groups, and assign to ships

Stage 1: Choose ports to visit for the fleet

This stage aims at finding roughly the most profitable ports by computing an index for each port. The index has a brief reflection on the unit profit the fleet can get by delivering the demand directly to the port. It is defined as

$$index = \text{unit revenue} - \frac{\text{fuel price} * \text{fuel consumption of a roundtrip}}{\text{demand}}$$

where the fuel consumption refers to the minimum amount of fuel the ship uses to travel from depot to the port, arrive at the port within time window, and return to depot. If the ship cannot arrive at the port within time window, the port is considered as an infeasible customer port. The unit fuel price is taken as the lowest price among ports nearby. To find nearby ports, we will first sort all customer ports in clockwise order (or anti-clockwise order) with depot as the center. If c is the average number of ports that a ship can visit, the nearby ports of port k are defined as set of ports from $k - (c - 1)$ to $k + (c - 1)$.

All feasible customer ports are then sorted according to the respective indices, and the ports are then sorted as $p_1, p_2, p_3, \dots, p_{N-1}$. The higher the index is, the higher the priority the port has. Practically, a port is profitable and worth serving if it provides high unit revenue, large demand, relatively near to the depot, and has a neighbor port with low fuel price. Ports with these characteristics are more likely to obtain higher indices to be chosen. Ports are then chosen according to

Table 3.1: Procedures for heuristic: stage 1

Step 1	Arrange all ports in angular order
Step 2	Compute index for every port
Step 3	Prioritize ports according to descending order of indices, $p_1, p_2, p_3, \dots, p_{N-1}$
Step 4	Determine k such that $\sum_{i=1}^{k-1} D_{p_i} < KQ$, and $\sum_{i=1}^k D_{p_i} \geq KQ$
Step 5	Take sets of chosen ports as $\{p_1, p_2, p_3, \dots, p_{k-1}\}$, $\{p_1, p_2, p_3, \dots, p_k\}$, and $\{p_1, p_2, p_3, \dots, p_{k+1}\}$, into stage 2, select the best set of serving ports.

the priority until the total demand is larger than the total capacity of the fleet.

The number of ports chosen is k such that

$$\sum_{i=1}^{k-1} D_{p_i} < KQ, \sum_{i=1}^k D_{p_i} \geq KQ \quad (3.105)$$

where Q is the capacity of ships. Three cases are then explored, with the set of visiting ports as $\{p_1, p_2, p_3, \dots, p_{k-1}\}$, $\{p_1, p_2, p_3, \dots, p_k\}$, and $\{p_1, p_2, p_3, \dots, p_{k+1}\}$ respectively. Stage 1 will give three sets of ports, with each set as an input, to stage 2, to examine the optimal profit of the given set of chosen ports. The ports have been sorted into angular order, and the chosen ports are picked out, remaining in angular order.

Table 3.1 shows stage 1 procedures in detail.

Stage 2: Divide set of visiting ports and assign demands to ships

With the assumption that a ship will serve a set of ports that are angularly adjacent to each other, we can divide the whole space into K fan-shaped sectors, and assign the K ships to each sector. The routing and scheduling for each single ship can be solved by mixed integer programming using CPLEX, as the problem size for each ship is relatively small.

The question is then to decide the cutting of the space to obtain the highest profit. The chosen ports take turns to be selected as the first port after the starting

cut. Possible optimal cuttings for each kind of starting cutting are enumerated. Ports are sequentially assigned to ship in angular order until the total demands exceed the ship capacity. It is then to determine the demands that the ship will fulfill. In the sense that a ship will choose to deliver cargos with higher revenue to gain higher profit under the condition of fixed routing and scheduling, we assume in this heuristic that the demands from the port with lowest unit revenue will be sacrificed. Optimal fuel cost for the current ship is determined by solving a mixed integer programming problem similar to TSPTW using CPLEX. Hence, the profit obtained by the current ship can be easily got.

It is assumed that only the demand from the port at the cutting edge can be shared by two ships. Hence, when the port partially fulfilled by one ship is at the end of its cut, two situations are considered by the next ship, with the port at the end of first cut included or not.

Stage 2 can be carried out using the steps in Table 3.2.

3.4 Numerical Results

As the second model we built can be thought as a comprehensive version of the first one, considering more factors, we will do the analysis based on the second model.

3.4.1 Sensitivity analysis

First we will show an example solved by the model, with parameters on ports in Table 3.3. Two ships with capacity of 6000TEU are to be assigned to serve these ports. Results are shown in the table below. And routing is visually shown in Fig. 3.3.

Since the total demands from all ports are larger than the total capacity of the fleet in this case, it is predictable that there will be some lost demand. Demands on port 4 and 5 are not fully met in the optimal solution. 1500 TEUs and 1000

Table 3.2: Procedures for heuristic: stage 2

Step 1	<p>For $i = 1$ to N', where N' is the number of chosen ports.</p> <p>Starting port = p_i, cut = p_i. Go to Step 2.</p> <p>Select the best cutting with the maximum total profit.</p>
Step 2	<p>From cut = p_t, choose ports sequentially in angular order for a ship until p_s,</p> <p>such that $\sum_{j=t}^{s-1} d_{p_j} < Q$, and $\sum_{j=t}^s d_{p_j} \geq Q$, or p_{s+1} is the starting port</p> <p>The set of visiting ports for the ship is $VP = \{p_t, p_{t+1}, \dots, p_s\}$</p>
Step 3	<p>Locate p_u among VP with the minimum unit revenue r_{p_u}.</p> <p>If $D_{p_u} \leq \text{total demand} - \text{capacity}$, delete p_u from VP. Repeat Step 3.</p> <p>Else, $\text{residual} = \text{total demand} - \text{capacity}$. $\text{Partial} = p_u$.</p> <p>Compute the total revenue from all demand fulfilled by the ship.</p>
Step 4	<p>Compute the minimum fuel consumption FC to travel all ports in VP by</p> <p>CPLEX. Choose ports in VP with lowest fuel price to bunker. Compute the</p> <p>fuel cost and profit for the ship.</p>
Step 5	<p>If $p_{s+1} = \text{starting port}$, return current profit.</p> <p>Else, cut = p_{s+1}. Go to Step 2, get Pf, profit for remaining ports.</p> <p>If $p_u \neq p_s$, return profit = current profit + Pf</p> <p>Else, cut = p_s, adjust D_{p_s} to residual. Go to Step 2, get Pf'.</p> <p>Return profit = current profit + $\max\{Pf, Pf'\}$.</p>

Table 3.3: Ports information

Port	Location	Demand	Unit revenue	Deadline	Fuel price
1	(0,0)	N.A.	N.A.	N.A.	677.5
2	(-800,200)	2000	200	120	679.5
3	(400,1000)	2500	215	90	673.5
4	(-400,-600)	2000	170	80	629
5	(500,-300)	2000	195	40	650
6	(-200,900)	2500	205	120	635
7	(-400,200)	2500	190	120	655.5

Table 3.4: Optimal result table

Ship	Port	A	Q	B	v	F
1	1	0				
	7	23.2	500		24	120.2
	6	59.6	3000	764.6	20	196.8
	3	90	2500		20	164.5
	1	153.4			17	283.1
2	1	0				
	4	42.4	2500	647.4	24	189.6
	2	95	2000		17	235.1
	7	118.6	1500		17	105.1
	1	144.9			17	117.6

Figure 3.3: Optimal routing

TEUs are delivered to them respectively while they both requested for a batch of 2000 TEUs.

We will show the effect of change in demand, revenue, fuel price and time window on the result.

3.4.1.1 Change in demand

The change in demand will mainly have an effect in the first stage of the three-stage formulation, in terms of constraint in 3.25. The following two types of changes will have a potential effect on the optimal solution.

- Change of d_i in the active constraints. Practically, for those ports whose demand is fully fulfilled, changing the demand may cause the ship to carry a different amount of cargos to the port, and consequently change the cargos delivering to other ports.
- Changing, specifically decreasing of d_i in the inactive constraints. Practically, when the demand from a partially fulfilled port drops to lower than the amount the ship is carrying, the decision may be affected.

In this case, the active constraints correspond to ports 2, 3, 6, 7, and ports 4 and 5 are partially fulfilled. Demands on ports 2 and 4 will be adjusted for illustration.

Change in demand in active constraint

The change in demand from port 2 will have an effect on the optimal routing. When d_2 increases, the ship will carry more cargos to port 2, and demands from some other ports will be lost. When it decreases, the space capacity may be utilized by other partially fulfilled ports. For example, when d_2 increases to 2500, demands from port 5 will be lost, and the optimal routing will be shown as in Figure 3.4.

The program is repeatedly solved for different values of d_2 from 1100 to 4100. The optimal delivery amounts to customer ports and corresponding optimal objective values changes according to its demand as shown in Fig. 3.5. Port 2 has

 Figure 3.4: Optimal routing when $d_2 = 2500$

a relative higher unit profit compared with all customer ports. Hence, if d_2 is no less than 1204, the demand from port 2 will be always fulfilled by the visiting ship. While $d_2 \leq 1203$, despite the high unit profit, the quantity is too small that the total profit brought by the port is not competitive. Hence it is not profitable to visit port 2, and the change in d_2 would no longer have effect on the optimal solution. The first breakpoint at 1204 corresponds to the change in optimal solution in terms of visiting port 2 or not. And the routing will be changed.

The next change in optimal routing will occur at the critical point of 2017. When d_2 is no more than 2016, one ship visits ports 2, 4 and 7. The ship capacity is not utilized when $1204 \leq d_2 < 1500$. When d_2 increases to more than 1500, due to the capacity constraint, the demand from port 4 will be sacrificed as it provides a lower profit. Breakpoint at 2017 corresponds to the change in terms of visiting port 5 or not, and when d_2 increases above 2017, abandoning port 5 will lead to less traveling distance, and hence less operating costs. The ship is not able to fulfill all demands from ports 2, 4 and 7 due to capacity. As port 7 is the nearest port to the routing of the other ship, the demand from port 7 will be shared by the two ships.

Figure 3.5: Optimal profit with respect to change in d_2

The demands from all customer ports except port 5 can be fulfilled by the ship capacities when d_2 is no more than 2500. Part of port 4 demand will be sacrificed with the increasing afterwards until it is not profitable to go to port 4 anymore, which occurs after d_2 increases to 3283. A ship will then visit only two ports, 2 and 7, while the other ship visits 3, 5 and 6. The increasing in the cargo amount delivered to port 2 is achieved with the decreasing for port 7 when the total demands from these two ports exceed the ship capacity after d_2 increases above 3500. When the sacrifice of port 7 demand is too much after d_2 is above 3963, the other ship will abandon port 5 to compensate for the lost demand from port 7.

Change in demand in inactive constraint

Due to inactiveness, the change in d_4 will not have any effect on the optimal on the optimal solution as long as d_4 is above 1500, which is the amount delivered to port 4 in the optimal solution. However, when d_4 drops, the ship carries less cargo, causing the change in the total profit. When it drops below 1321, it is not profitable to go to port 4 anymore, and the routing will be as Fig. 3.6.

 Figure 3.6: Optimal routing when $d_4 \leq 1300$

 Figure 3.7: Optimal routing when $r_5 \geq 209$

3.4.1.2 Change in unit revenue

Suppose that d_2 is 2500TEUs, with the optimal routing as Fig. 3.4, port 5 is not visited due to its low revenue. When r_5 is increased such that it is more profitable than other ports, it will be taken into consideration. Hence, increasing the unit revenue of unvisited port might affect the final decision on routing.

When the unit price increases up to \$209, port 5 becomes more profitable than port 4. Hence port 4 will be abandoned in the planning, as shown in Fig. 3.7.

Figure 3.8: Optimal routing when $p_5 \leq 610$

3.4.1.3 Change in fuel price

If a port has lower fuel price compared with other ports, it is high possibility that ships will go through the port to refuel. For example, when the unit fuel price at port 5 is $p_5 = 610$, the optimal routing is changed to Fig. 3.8. In fact, before p_5 decreases to \$613 per ton, the optimal routing remains the same as Fig. 3.4.

3.4.1.4 Change in time windows

The change in time windows will mainly have effect on the sequence of visiting ports and speed control. For example, if the time window for port 2 becomes tighter, the routing will remain the same with the voyage speed adjusted accordingly, until the deadline l_2 decreases to 36 hours. The sequence of visiting ports will change when $l_2 = 35$, as shown in Fig. 3.9. Port 2 has to be the first port to visit after setting out from depot, due to the tight deadline.

The change in demand together with the change in time window may cause significant changes in routing solution. If d_2 is 2000, tightening the deadline to be 35 will give another result with the routing in Fig. 3.10. Instead of changing the sequence in the previous case, the port is abandoned. The decreased demand leads to decreased profit, and it no longer justify for the increasing in traveling

Figure 3.9: Optimal routing when $l_2 = 35(d_2 = 2500)$

distances.

3.4.2 Computational time

The computational time using CPLEX for true optimal solutions increases significantly with the increment on the problem size, represented by the number of ports in consideration. When there are N ports to be served by K ships, the

Figure 3.10: Optimal routing when $l_2 = 35$ ($d_2 = 2000$)

Table 3.5: Number of variables

Variables	Number
x_{ij}^k	$KN(N - 1)$
A_i^k	$K(N + 1)$
q_i^k	$K(N - 1)$
B_i^k	KN
y_i^k	KN
I_i^k	KN
$\alpha_{ip}^k, \beta_{ip}^k$	$7KN$
a_{ijp}^k, b_{ijp}^k	$7KN(N - 1)$

number of variables can be computed as Table 3.5. The total number of variables is $8KN^2 + 4KN$, with $4KN^2$ being binary, which means the increasing is quadratic with respect to the problem size. The computation complexity determined the rapid growth of the computational time.

For number of ports from 4 to 10, 100 samples are randomly generated with random demands and time windows. Two ships are assigned to them to get the maximum profit. The average solution time using CPLEX are plotted as in Figure 3.11.

The heuristic proposed, on the other hand, performs much better in terms of computation time. Same problems are tested on the heuristic, and the average times used are shown in Figure 3.12. The problem with 10 ports can be solved within 2.8 seconds on average, which is a big improvement compared with 2416s using CPLEX. The curve of computation time with respect to the number of ports is still in a convex shape, which indicates that the speed of growth in computation time is increasing, as a problem similar to TSPTW is solved using CPLEX. Furthermore, the speed of growth in time is much smaller than that using CPLEX. The computation time for larger scale problem would be acceptable. As a matter of fact, the average computation time for the problems with 20 ports is 18.1453 seconds.

Figure 3.11: Computational times by CPLEX

Figure 3.12: Computational times by heuristic

Table 3.6: Optimality gaps

Number of ports	min	Average	max
5	0	7.487%	37.301%
6	0	10.568%	37.735%
7	0	6.169%	31.588%
8	0	7.784%	26.739%
9	0	9.770%	27.024%

Figure 3.13: Optimality Gaps

On the other hand, the significant improvement in computation time does not sacrifice the optimality of solution. Although the real optimal solution may not be obtained using the heuristic, the solution is considerably acceptable. Table 3.6 shows the minimum, average and maximum optimality gap in the experiment for each number of ports considered. For all the problems tested in the experiment, the results obtained using the heuristic is 8.35% less than the real optimal value on average. The optimality gaps are shown in Figure 3.13, where the points on the middle line represent the average difference, and the points on the other lines are the maximum and minimum differences respectively.

3.5 Summary

In this chapter, we first built a mixed integer problem for simple capacitated vehicle routing problem with time windows. The model is extended so that cruising speed and refuel strategy can be included in the decisions. Simulations are carried out using the second formulation, and sensitivity analysis is done in order to get an insight on the problem structure. A heuristic algorithm is also proposed based on the problem structure we have observed. The heuristic prioritize customer ports using an easy-to-compute index, and group them according to geometric locations. The heuristic showed great advantage in terms of computational time, without sacrificing much optimality.

Chapter 4

Fleet Management with Simultaneous Pickup and Delivery

We talked with someone in a company in the offshore industry which provides integrated offshore construction and shallow water oil and gas field development. The company has bases in two cities in west India along Arabian Sea, and it needs periodically to send people, materials and devices from the two bases to some near sites and four working sites on Indian Ocean. The transportation of workers and product from sites back to base cities are also necessary.

Although the model we develop in this chapter cannot solve exactly the problem for this company, we can make very little adjustment so that it can be adopted.

The problem we consider in this chapter can be considered as a more generalized and realistic version of the model 2 of Chapter 3. We will start the research by exact formulation, followed by two heuristics.

4.1 Problem Description

In this model, demands for pickup and delivery may simultaneously exist for each customer port. Each unit of fulfilled demand would bring some revenue to the

company. Ships start their journey from the depot, visit a sequence of customer ports, and return to the depot eventually. A ship can also be chartered out (or just laid up) if the potential profit is less than some threshold due to insufficient demands or high operational cost. (The threshold can be estimated as the profit by chartering out the vessel or some operating overhead.)

Each customer port may have some visiting time window in which ships can serve. The port processing times at each port, including entry time, unloading and loading time, idle time and exiting time, is assumed to be known or can be estimated with reasonable accuracy, and hence is assumed to be fixed in this model. The sailing time between any two ports is determined by the cruising speed on that voyage. Although the speed is practically variable along time, the average cruising speed can be utilized to determine the schedule and fuel consumption in the tactical level. So the speed is assumed to be constant within each voyage.

The company receives higher revenue by fulfilling more demands, which in turn raises fuel consumption and increases greenhouse gas emissions due to larger amount of freight tonnage on board or longer traveling distance. The fleet deployment and bunker management are applied to improve profitability and reduce operational cost through two ways respectively: less fuel consumption and lower bunkering cost. In the fleet deployment, the fuel consumption rate is proportional to the total weight and the third power of speed with the assumption that the displacement is approximately proportional to the total weight of a ship. In the bunker management, the fuel prices are different at each port. As incremental quantity price discounts are commonly applied [90], and bunkering ports usually provide price discounts under contractual agreement [84], it is assumed in this problem that ports provide price discounts on incremental quantity of fuel bunkered by a ship.

The shipping company tries to maximize the total profit earned by the fleet, where the operational cost is considered as the total bunker fuel costs. Although shipping service incurs some other costs, such as the administrative costs, capital

costs, container costs, etc., these costs can be considered approximately constants with the number of ships and total cycle time fixed.

The characteristics of the problem can be summarized as follows:

- (i) Pickup and delivery demands may simultaneously exist at every customer port.
- (ii) The company is allowed to choose ports to serve according to demands received.
- (iii) Partial fulfillment is allowed for each serving port.
- (iv) Demands at each port can be split and served by multiple ships.
- (v) It is a joint optimization of routing, scheduling, speed, pickup and delivery quantities, bunkering ports and volume for each ship (laying up or chartering out ships are viable options).
- (vi) The freight tonnage on board is considered as an important factor in estimating fuel consumption rate, instead of cruising speed as the only factor. This assumption leads to more precise estimation while produces nonlinear constraints.
- (vii) When determining bunkering policies, different fuel prices at each port is taken into consideration. The incremental quantity price discounts are also considered.

4.2 Problem Formulation

4.2.1 Notations

First we will define the variables and parameters.

Decision variables

Q_{i1}^k	Delivery quantities from depot to port i by ship k
Q_{i2}^k	Pickup quantities from port i to depot by ship k
t_{ij}^k	Traveling time of ship k per unit distance from port i to j
x_{ij}^k	Whether ship k travels from port i to j .
	Note that $x_{1,N+1}^k = 1$ indicates that ships k stays at depot.
B_i^k	Bunkering volume of ship k at port i
y_i^k	Whether to bunker ship k at port i

State variables

I_i^k	Residual fuel volume of ship k when entering port i
W_i^k	Displacement of ship k when leaving port i
A_i^k	Arrival time of ship k at port i .
	The starting time A_i^k can be specified by user.

Parameters

N	Number of ports including the depot.
	For ease of notation, $N + 1$ is defined as the index for a virtual port with the same location as the depot.
K	Number of ships
R^k	Threshold revenue for ship k
r_{i1}	Unit delivery revenue at port i
r_{i2}	Unit pickup revenue at port i
D_{i1}	Total delivery demands at port i
D_{i2}	Total pickup demands at port i
I_m^k	Maximum fuel volume of ship k
e^k	Minimum bunkering level of ship k
c^k	Safety fuel level of ship k

W_0^k	Lightweight of ship k , i.e. the weight of empty ship
$W_{\max}^k$	Deadweight of ship k , i.e. the weight when fully loaded
w	weight per unit cargo
d_{ij}	Distance between port i and j
s_i^k	Port processing time of ship k at port i
M	A large enough number

Sets

P	Set of all port. $P = \{1, 2, \dots, N\}$
P^-	Set of all customer port. $P^- = \{2, \dots, N\}$
P^+	For ease of notation, $P^+ = \{2, 3, \dots, N + 1\}$
V	Set of all ships, indexed by k .

The following functions are nonlinear, which can be approximated by piece-wise linear function later:

- $f_i(B_i^k)$: Fuel cost function considering increment quantity price discounts at port i and is defined as

$$f_i(B_i^k) = \begin{cases} p_i^1 B_i^k & B_i^k \in [0, v_1] \\ p_i^1 v_1 + p_i^2 (B_i^k - v_1) & B_i^k \in [v_1, v_2] \\ p_i^1 v_1 + p_i^2 (v_2 - v_1) + p_i^3 (B_i^k - v_2) & B_i^k \in [v_2, v_3] \end{cases} \quad (4.1)$$

- $g_k(W_i^k, t_{ij}^k)$: Fuel consumption of ship k per unit distance and defined as

$$g_k(W_i^k, t_{ij}^k) = C^k W_i^k / (t_{ij}^k)^2 \quad (4.2)$$

where C^k is a constant for a specific ship k .

4.2.2 Formulation

With these notations, the problem can be formulated as the following Mixed Integer Programming (MIP) Problem.

$$\max \quad \sum_{k \in V} \sum_{i \in P^-} (r_{i1} Q_{i1}^k + r_{i2} Q_{i2}^k) + \sum_{k \in V} R^k x_{1,N+1}^k - \sum_{k \in V} \sum_{i \in P} f_i(B_i^k) \quad (4.3)$$

$$\text{s.t.} \quad \sum_{k \in V} Q_{i1}^k \leq D_{i1}, \sum_{k \in V} Q_{i2}^k \leq D_{i2}, \forall i \in P^-; \quad (4.4)$$

$$Q_{i1}^k \leq M \sum_{j \neq i} x_{ij}^k, Q_{i2}^k \leq M \sum_{j \neq i} x_{ij}^k, \forall i \in P^-, k \in V; \quad (4.5)$$

$$\sum_{j \in P^-} x_{1j}^k = 1 - x_{1,N+1}^k, \forall k \in V; \quad (4.6)$$

$$\sum_{i \in P^-} x_{i,N+1}^k = 1 - x_{1,N+1}^k, \forall k \in V; \quad (4.7)$$

$$\sum_{j \neq i, j \in P^-} x_{ij}^k \leq 1 - x_{1,N+1}^k, \forall i \in P^-, k \in V; \quad (4.8)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \forall i \in P^-, k \in V; \quad (4.9)$$

$$W_1^k = w \sum_{i \in P^-} Q_{i1}^k + W_0^k, \forall k \in V; \quad (4.10)$$

$$W_j^k \leq W_i^k + w(Q_{j2}^k - Q_{j1}^k) + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (4.11)$$

$$W_j^k \geq W_i^k + w(Q_{j2}^k - Q_{j1}^k) - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (4.12)$$

$$W_i^k \leq W_{\max}^k, \forall i \in P, k \in V; \quad (4.13)$$

$$I_1^k = I_{N+1}^k, \forall k \in V; \quad (4.14)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij} g_k(W_i^k, t_{ij}^k) + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, \quad (4.15)$$

$$i \neq j, k \in V;$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij} g_k(W_i^k, t_{ij}^k) - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, \quad (4.16)$$

$$i \neq j, k \in V;$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i \in P, k \in V; \quad (4.17)$$

$$e^k I_m^k y_i^k \leq B_i^k \leq I_m^k y_i^k, \forall i \in P, k \in V; \quad (4.18)$$

$$I_i^k \geq c^k I_m^k, \forall i \in P \cup \{N+1\}, k \in V; \quad (4.19)$$

$$I_i^k + B_i^k \leq I_m^k, \forall i \in P, k \in V; \quad (4.20)$$

$$A_j^k \geq A_i^k + d_{ij} t_{ij}^k + s_i^k - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (4.21)$$

$$A_j^k \leq A_i^k + d_{ij} t_{ij}^k + s_i^k + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (4.22)$$

$$e_i \leq A_i^k \leq l_i, \forall i \in P^-, k \in V; \quad (4.23)$$

$$A_{N+1}^k \leq T^k, \forall k \in V; \quad (4.24)$$

$$Q_{i1}^k \geq 0, Q_{i2}^k \geq 0, \forall i \in P^-, k \in V; \quad (4.25)$$

$$x_{ij}^k = 0 \text{ or } 1, \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (4.26)$$

$$y_i^k = 0 \text{ or } 1, \forall i \in P, k \in V. \quad (4.27)$$

The objective function 4.3 is to maximize the total profit, where first term is the total revenue from fulfilling demands, second term is the profit if a ship is chartered out, and third term is the total refueling cost.

Constraints in 4.4 simply means that the pickup or delivery quantity should not exceed the demands for each customer port. Constraints in 4.5 are indicative constraints to make sure that a ship can serve a port only if the port is visited by the ship.

Constraints in 4.6 to 4.9 defines the flows of ships. If a ship is not laid up ($x_{i,N+1} \neq 1$), it will set out from the depot (specified by 4.6), go through some customer ports (specified by 4.9), and return to the depot (specified by 4.7). Inequalities 4.8 suggest that if a ship is laid up, no customer port can be visited by it.

Constraints in 4.10 to 4.13 monitor the changes in the tonnage on board. A ship will set out with all the cargos that it need to deliver from the depot (as in 4.10). At each customer port it passes, it will unload the deliver demand, and load pickup demand. Inequalities 4.11 and 4.12 together serve as a conditional equation.

Constraints in 4.14 to 4.20 describe the fuel consumptions and bunkering activities. Equation 4.14 can be considered as an assumption that all the fuel consumption will be refueled along its trip, so that the fuel consumption can be measured using fuel cost. Under this assumption, the fuel a ship carries when leaving the depot, will be the same as that when it returns, which is 4.14. Inequalities 4.15 and 4.16 are conditional equation for the fuel consumption on voyage from port i to j if the voyage is traveled.

Constraints in 4.21 to 4.24 measure the arrival times. Inequalities 4.21 and 4.22 are conditional equation for the traveling time along voyage (i, j) if the voyage is traveled.

4.3 Heuristics

The problem is NP-hard since it is a generalization of the classical VRP. When there are N ports (including depot) and K ships in the problem, there are $N(N-1)$ voyages to be considered. The number of variables is $15KN^2 + 5KN$, in which $9KN^2 + K$ of them are binary variables. The complexity leads to the rapid growth of computational time. Only small problems can be solved by exact algorithms using the formulated MIP. Hence, heuristic algorithms are necessary.

We tried to follow the idea of the heuristic in last chapter, but the prioritizing process is more complicated as we consider more factors in this problem. The decision variables are tightly coupled. The routing decision determines the set of visiting ports and fuel consumption for each ship, and hence the bunkering ports and bunkering amounts depend on routing and scheduling. On the other hand, the routing has to be adjusted in order to reduce total fuel consumption and bunkering costs. It is desirable for a heuristic to keep the advantage of the joint optimization of these decisions.

Note that if the sets of visiting ports for each ship are known, the decisions can be made independently for each single ship. The original problem can then

be partially decomposed into K single-vehicle problems. We can still use the formulation to solve each of them, since it contains only one ship and potentially smaller number of ports. Even for large-scale problems, the single-vehicle problems can be still efficiently solved using optimization tools. On the other hand, the coupling of decision variables within each ship is kept. Hence, we propose a two-stage heuristic. In the first stage, the assignment of ports to ships is generated, and in the second stage decisions are made using optimization tools for each single ship based on the assignment. The two stages are iteratively carried out to find the best assignment. The main challenge is then to find an efficient way of searching a promising assignment.

The heuristic is also reasonable from a managerial perspective. Determining the assignment can be seen as a tactic level decision, while routing, scheduling and profit management can be treated as operational level decisions.

We first define a set of binary vectors $y_k = [y_k(1), y_k(2), \dots, y_k(N-1)]$, $k = 1, 2, \dots, K$, in which $y_k(i) = 1$ indicates that customer port i is served by ship k , and 0 otherwise. $Y \in \Theta$ is then defined based on y_k : $Y = [y_1, y_2, \dots, y_K]$. Y can be thought as a variable which assigns customer ports to ships. Given a specific assignment, a simple optimization problem can be solved for each single ship. Let $h(Y)$ be the total optimized profit when customer ports are assigned to ships as Y .

4.3.1 Single ship problems

Given an assignment Y , the optimal decisions can be made for each single ship independently. The following formulation can efficiently solve each single ship

problem with the number of visiting ports as n .

$$\max \quad \sum_{i=2}^n (r_{i1}Q_{i1} + r_{i2}Q_{i2}) - \sum_{i=1}^n f_i(B_i) \quad (4.28)$$

$$\text{s.t.} \quad Q_{i1} \leq D_{i1}, Q_{i2} \leq D_{i2}, \forall i; \quad (4.29)$$

$$\sum_{j \neq i} x_{ij} = 1, \forall i; \quad (4.30)$$

$$\sum_{j \neq i} x_{ji} = 1, \forall i; \quad (4.31)$$

$$W_1 = w \sum_{i=2}^n Q_{i1} + W_0; \quad (4.32)$$

$$W_j \leq W_i + w(Q_{j2} - Q_{j1}) + M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.33)$$

$$W_j \geq W_i + w(Q_{j2} - Q_{j1}) - M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.34)$$

$$W_i \leq W_{\max}, \forall i; \quad (4.35)$$

$$I_1 = I_{n+1}; \quad (4.36)$$

$$I_j \leq I_i + B_i - d_{ij}g(W_i, t_{ij}) + M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.37)$$

$$I_j \geq I_i + B_i - d_{ij}g(W_i, t_{ij}) - M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.38)$$

$$eI_m y_i \leq B_i \leq I_m y_i, \forall i; \quad (4.39)$$

$$I_i \geq cI_m, \forall i; \quad (4.40)$$

$$I_i + B_i \leq I_{\max}, \forall i; \quad (4.41)$$

$$A_j \geq A_i + d_{ij}t_{ij} + s_i - M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.42)$$

$$A_j \leq A_i + d_{ij}t_{ij} + s_i + M(1 - x_{ij}), \forall i, j, i \neq j; \quad (4.43)$$

$$e_i \leq A_i \leq l_i, \forall i; \quad (4.44)$$

$$A_{n+1} \leq T; \quad (4.45)$$

$$Q_{i1} \geq 0, Q_{i2} \geq 0, \forall i; \quad (4.46)$$

$$x_{ij} = 0 \text{ or } 1, \forall i, j, i \neq j; \quad (4.47)$$

$$y_i = 0 \text{ or } 1, \forall i. \quad (4.48)$$

Note that constraints in 4.30 and 4.31 ensures that the ship visits every customer ports that are assigned to it.

Using this formulation, we can calculate the optimal profit of any given assignment Y as in Table 4.1. Note that in Step 4, we will examine whether the profit by serving customer ports is larger than the profit of chartering out the ship. If not, the ship will not set out.

Table 4.1: Calculating optimal profit for a given assignment Y

Step 1:	$k = 1$, set the demands at each customer ports as the real demands;
Step 2:	For ship k , check the $(k - 1) * (N - 1) + 1$ to $k * (N - 1)$ digits of Y , and determine the visiting ports;
Step 3:	Solve a single ship problem for the visiting ports, with optimal profit p_k^* and pickup and delivery amount Q_{i1}^*, Q_{i2}^* ;
Step 4:	If the single ship profit exceeds the chartering profit for the ship, delete the fulfilled amount from the ports' demands. Otherwise, set this ship as chartered out, and $p_k^* = R_k$. If $k < K$, set $k = k + 1$, and go to step 2;
Step 5:	Calculate the optimal profit $h(Y) = \sum_{k=1}^K p_k^*$.

The key problem is then to find an optimized assignment Y for ports. We will propose the application of two methods in this work.

4.3.2 Neighborhood Search

There are some existing heuristics to search for a local optimum within discrete variables. The most common one is neighborhood search. For chosen assignment, its neighborhood is examined. If a better assignment is found, that assignment is chosen and the procedure is repeated. But first of all, we need to define neighborhood.

The distance between two assignment variable Y_1 and Y_2 , $dist(Y_1, Y_2)$ is defined as the number of different digits, i.e. the number of i 's such that $Y_1(i) \neq Y_2(i)$. Let $N(Y) = \{Z : Z \in \Theta \text{ and } dist(Y, Z) = 1\}$ be the local neighborhood of $Y \in \Theta$.

Y is a local optimum if $h(Y) \geq h(Z)$ for all $Z \in N(Y)$. Let Φ denote the set of local optima in Θ .

Based on this definition, the neighborhood search algorithm is shown in Table 4.2.

Table 4.2: Heuristic based on neighborhood search

Step 1:	Generate an initial solution $Y_0 \in \Theta$, calculate $h(Y_0)$. Set $Y^* = \{Y_0\}$ and $h^* = h(Y_0)$;
Step 2:	Examine neighborhood of Y^* by changing its digits once at a time, if there is $Y_1 \in N(Y^*)$ such that $h(Y_1) > h(Y^*)$, set $Y^* = \{Y_1\}$ and $h^* = f(Y_1)$, go to Step 2;
Step 3:	Y^* is a local optimal solution.

However, the size of neighborhood increases with the increment of number of ports and ships. A more efficient way of converging is still needed for large problems.

4.3.3 Heuristic based on Most Promising Area

Hong and Nelson [91] proposed an algorithm termed convergent optimization via most-promising-area stochastic search (COMPASS), based on a unique neighborhood structure. The neighborhood is defined in each iteration, and is fully adaptive. The idea that the most promising area is defined as the set of feasible solutions that are at least as close to the current best solution as they are to other visited ones, can be utilized to solve the fleet problem in this paper. Our algorithm is however not the same as COMPASS. We will not use Euclidean distance as the definition of distances between points, which is used in COMPASS. Since we are only dealing with binary variables, we will define distance using our own definition. Although COMPASS is proposed for stochastic problems, but the ways of searching neighborhood is also suitable for the deterministic problem in this paper.

The definition of local neighborhood is the same as that in neighborhood search algorithm.

In this algorithm, we use R_r to denote the set of all solutions visited through iteration r , and use Y_r^* to denote the solution with the largest total profit among all $Y \in R_r$. At the end of iteration r , we construct the most promising area $\zeta_r = \{Y : Y \in \Theta \text{ and } \text{dist}(Y, Y_r^*) \leq \text{dist}(Y, Z), \forall Z \in R_r \text{ and } Z \neq Y_r^*\}$. The set ζ_r includes all feasible solutions that are at least as close to Y_r^* as to other solutions in R_r . At iteration $r + 1$, m solutions will be randomly generated from ζ_r uniformly and independently. The algorithm based on Most Promising Area is shown in Table 4.3.

Table 4.3: Heuristic based on Most Promising Area

Step 1:	Set iteration count $r = 0$. Generate an initial solution $Y_0 \in \Theta$, set $R_0 = \{Y_0\}$ and $Y_0^* = Y_0$. Calculate $h(Y_0)$. $\zeta_0 = \Theta$;
Step 2:	Let $r = r + 1$. Generate $Y_{r1}, Y_{r2}, \dots, Y_{rm}$ from ζ_{r-1} uniformly and independently, and calculate $h(Y_{ri})$ for every newly generated assignment;
Step 3:	Let $R_r = R_{r-1} \cup \{Y_{r1}, Y_{r2}, \dots, Y_{rm}\}$, and $Y_r^* = \arg \max_{Y \in R_r} h(Y)$. Construct ζ_r . If Y_r^* has not changed for certain iterations, stop. Else, and go to step 2.

Other terminal conditions can be adopted. More iterations can be examined for precision. Or the algorithm can be stopped whenever all of the computational budget is consumed.

It is worthwhile to note that this algorithm is efficient even for large scale problems. The number of points generated in each iteration is fixed, and will not be affected by the size of neighborhood. The computational time is hence proportional to the number of iterations, number of points generated in each iteration, the number of ships, and the time used for each single ship problem. The computational complexity is then mainly dependent on that of single ship problems. Due to time and capacity constraints, the number of ports assigned to a ship is limited, so each single ship problem can be solved efficiently.

Of course we have to make sure that the algorithm converges.

Theorem 4.1 *The infinite sequence $\{Y_0^*, Y_1^*, \dots\}$ generated by the algorithm converges with probability 1 to the set Φ , i.e. $P\{Y_r^* \in \Phi \text{ i.o.}\} = 1$.*

Proof: For any infinite sequence $\{Y_0^*, Y_1^*, \dots\}$ generated by the algorithm, $A = R_\infty$ exists since $R \in \Theta$ and Θ is a finite set. Since $|R_\infty| < \infty$, there exists a $\kappa > 0$ such that $R_r = R_\infty = A$ for all $r \geq \kappa$. Then for $r \geq \kappa$,

$$h(Y_r^*) = \max_{Y \in R_r} h(Y) = \max_{Y \in R_\infty} h(Y) \quad (4.49)$$

Hence

$$P\{\lim_{r \rightarrow \infty} h(Y_r^*) = \max_{Y \in R_\infty} h(Y) | R_\infty = A\} = 1 \quad (4.50)$$

Note that

$$P\{Y_r^* \in \Phi \text{ i.o.}\} = \sum_A P\{Y_r^* \in \Phi \text{ i.o.} | R_\infty = A\} P\{R_\infty = A\} \quad (4.51)$$

Then proving the theorem is equivalent to proving $P\{Y_r^* \in \Phi \text{ i.o.} | R_\infty = A\} = 1$ for any nonempty A such that $P\{R_\infty = A\} > 0$. Since $Y_r^* = \arg \max_{Y \in R_r} h(Y)$, there exists a Y such that $Y \in A, Y \notin \Phi$ and $Y_r^* = Y$. If $Y_r^* \notin \Phi \text{ i.o.}$, it is not a local optimum, and there exists a $Z \in N(Y)$ and $h(Z) > h(Y)$. For any other solution $Y' \in R_r$, $\text{dist}(Z, Y) = 1 \leq \text{dist}(Z, Y')$. According to the definition of most promising area, $Z \in \zeta_r$. As solutions are generated uniformly in ζ_r , we have

$$P\{Z \in R_{r+1} | Y_r^* = Y \text{ and } Z \notin R_r\} = \frac{1}{|\zeta_r|} \geq \frac{1}{|\Theta|} > 0 \quad (4.52)$$

Hence, $P\{Z \in A | R_\infty = A, Y_r^* = Y\} = 1$. Note that if $Z \in A$, then Y is not a local optimum since $h(Z) > h(Y)$. By equation 4.50 we know that with probability 1, Y_r^* can only equal to Y finitely many times, which contradicts with "infinitely often". Therefore, $Y_r^* \in \Phi \text{ i.o.}$ with probability 1, which proves the theorem.

4.3.4 Improvements

As the generation of assignments is actually a random generation of a vector of binary variables, it is highly possible that there are few or even no remaining demands for a port after visited by other ships. Although there are enough demands, the ship may not do any services for the port due to capacity constraints. In these two situations, the ship will still visit the ports although there is no service activity there. We will show that it is not desirable for a ship to visit a port if no service and no bunkering happens there, under the assumption that the travelling distance between any two ports i, k is no larger than that the distance with another port j visited between them, i.e. $d_{ik} \leq d_{ij} + d_{jk}$.

We will first proof a lemma that same velocity should be used on the voyages to and from the no-service port.

Lemma 4.1 *If a ship visits port i, j, k in sequence, while there is no pickup, delivery and bunkering activities at port j , the cruising speed is equal on two voyages, i.e. $v_{ij} = v_{jk}$.*

Proof: If $v_{ij} \neq v_{jk}$, WLOG, assume $v_{ij} < v_{jk}$. Since the function $f(v) = (d_{ij} + d_{jk})/v$ is continuous and monotonically decreasing on its domain, and

$$\frac{d_{ij} + d_{jk}}{v_{jk}} < \frac{d_{ij}}{v_{ij}} + \frac{d_{jk}}{v_{jk}} < \frac{d_{ij} + d_{jk}}{v_{ij}} \quad (4.53)$$

Hence, there exists a $v^* \in [v_{ij}, v_{jk}]$ such that

$$\frac{d_{ij} + d_{jk}}{v^*} = \frac{d_{ij}}{v_{ij}} + \frac{d_{jk}}{v_{jk}} \quad (4.54)$$

The fuel consumption on the two voyages is $CW_{ij}d_{ij}v_{ij}^2 + CW_{jk}d_{jk}v_{jk}^2$, with C as a constant, and W_{ij} as the weight of ship on voyage ij . Since there is no service and no bunkering happening at port j , the weight of ship will not change, $W_{ij} = W_{jk} = W$. Hence the fuel consumption is $CW(d_{ij}v_{ij}^2 + d_{jk}v_{jk}^2)$. On the other

hand, if the ship travel through the two voyage with speed v^* , the fuel consumption is $CW(d_{ij} + d_{jk})v^{*2}$. We want to show that the amount of fuel consumption is less with speed v^* , which is equal to show that

$$d_{ij}v_{ij}^2 + d_{jk}v_{jk}^2 > (d_{ij} + d_{jk})v^{*2} \quad (4.55)$$

with

$$v^* = \frac{(d_{ij} + d_{jk})v_{ij}v_{jk}}{d_{ij}v_{jk} + d_{jk}v_{ij}} \quad (4.56)$$

The inequality shown in 4.55 can be derived as

$$d_{ij}(2v_{ij}^3v_{jk} + v_{ij}^4) + d_{jk}(v_{ij}^4 + 2v_{ij}v_{jk}^3) > 3d_{ij}v_{ij}^2v_{jk}^2 + 3d_{jk}v_{ij}^2v_{jk}^2 \quad (4.57)$$

Since $2v_{ij}^3v_{jk} + v_{ij}^4 - 3v_{ij}^2v_{jk}^2 = v_{jk}(v_{ij} - v_{jk})^2(2v_{ij} + v_{jk}) > 0$, $d_{ij}(2v_{ij}^3v_{jk} + v_{ij}^4) > 3d_{ij}v_{ij}^2v_{jk}^2$. Similarly, $d_{jk}(v_{ij}^4 + 2v_{ij}v_{jk}^3) > 3d_{jk}v_{ij}^2v_{jk}^2$. Hence, the inequality 4.57 holds, and 4.55 also holds, which indicates that fuel consumption is less using one speed. Since the total travel time on the two voyages is the same according to the definition of v^* , v^* is a feasible and better solution.

Using this fact, we will show that any port without service and bunkering should not be included in voyages for maximum profit.

Proposition 1 *Suppose $d_{ij} \leq d_{ik} + d_{kj}$ holds for any set of ports. A ship visits port j , while there is no pickup, delivery and bunkering activities, then port j should not be assigned to the ship for maximum profit.*

Proof: If port j is visited, the voyages from i to j , and from j to k is contained as a part of the optimal routing. According to Lemma 4.1, the cruising speed on the two voyages are the same, i.e. $v_{ij} = v_{jk} = v$. We can always construct a new routing for the ship with port i to port k directly using speed v , and the other part of the cycle remains the same. With the same speed and less traveling distance, traveling time is reduced, and the deadline will not be violated, which indicates that the new routing is a feasible one.

Since no service is done, the weight of ship is not changed. With the same weight and velocity, the amount of fuel consumption is proportional to the traveling distance. Since $d_{ik} \leq d_{ij} + d_{jk}$, the fuel consumption is less in the new routing. Since visiting port j does not produce revenues, the new routing will result in more profit.

When we get a solution of a single ship problem for a given assignment, we can always examine whether there are ports with no service and bunkering. If such ports exist, we should delete these ports from the assignment, and re-optimize the routing for that ship. The two-stage heuristic algorithm can be improved to a three-stage algorithm, with the third step of refining solutions. This step gives an opportunity to find a better solution for assignment in each iteration, and help faster convergence to local optimum.

4.4 Numerical Results

4.4.1 Effects of weight and bunkering

In this section, a simple example will be given, in which 7 ports are to be considered. The proposed model is a very complex formulation since several factors are comprehensively considered in the optimization. The example will be used to illustrate the necessity of considering these factors by examining the effect on the optimal routing if a factor is not included.

The experiment is carried out based on ports information illustrated in Table 4.4. The exact optimal profit obtained by the model is 1.6356×10^6 with routing shown in Fig. 4.1.

In the first variation case, the weight of ships are fixed throughout their journey. Hence, the fuel consumption rate is only related to cruising speed and the effect of weight is not considered, the optimal routing is then as shown in Fig. 4.2(a). Port 2 is serviced instead of port 5 due to high demand and relatively high revenue of port 2. However, the demand from port 2 is all pickup cargos, the higher demand

Table 4.4: Ports information

Port	Location	Delivery demand	Unit delivery revenue	Pickup demand	Unit pickup revenue	Dead-line	Fuel price
1	(0,0)	N.A.	N.A.	N.A.	N.A.	168	677.5
2	(-800,200)	0	130	3600	127	120	629
3	(400,1000)	3000	158	2700	133	90	673.5
4	(-400,-600)	2500	105	2500	160	80	679.5
5	(470,-300)	2700	120	0	95	40	630
6	(-200,900)	3200	132	0	120	120	635
7	(-400,200)	0	90	3500	133	120	655.5

Figure 4.1: True optimal routing

indicates a voyage with large carrying weight. Furthermore, the distance between port 2 and the depot is relatively far, hence, in Fig. 4.1, where weight is considered, the increase in revenue is compensated by the increase in fuel consumption if weight contributes to fuel consumption rate. In this case, if we use the original ports information, the obtained decision variables will results in a profit of 1.486×10^6 , with a 9.15% decrease. As a matter of fact, the experiment shows an average difference of 8.71% based on the sample of 100 random cases.

On the other hand, if information of fuel prices at different ports are not utilized, the model might also give different solutions. In this example, if we have no information of fuel prices, and assume that the fuel prices at all ports are set at

Figure 4.2: Optimal routing under variate conditions

the average level, the optimal routing will be as shown in Fig. 4.2(b). Port 5 has a competitive fuel price but a lower revenue compared with nearby ports. Hence, visiting port 5 provides a benefit in the fuel cost. While all ports share the same fuel price, the port is not as profitable as other ports. If we have full information of the prices, the obtained decision variables will give a profit of 1.600×10^6 , with a decrease of 2.17%. An average difference of 5.74% is obtained in the experiment of 100 random cases.

4.4.2 Step-by-step procedure for MPA

To better understand the procedures of the heuristic based on MPA, we will take the example problem in section 4.4.1, and show the steps of two iterations as an example.

First, we will initialize by setting iteration count $r = 0$, and randomly generate an initial solution Y_0 . The initial solution may determine the number of iterations before a optimum can be found. For an illustration, we will just assign port 2,3,4 to ship 1, and port 5,6,7 to ship 2, which means $Y_0 = \{1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1\}$, $R_0 = \{Y_0\}$ and ζ_0 is the set of all feasible solutions.

Since this is just an simple demonstration, we will generate only 2 solutions in every iteration. In iteration 1, we will generate Y_{11}, Y_{12} randomly from ζ_0 , and calculate $h(Y_{11}), h(Y_{12})$.

$$Y_{11} = \{1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 1, 1\}$$

$$Y_{12} = \{1, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 0, 1\}$$

After running through single ship problems, we may find that Y_{11} has the minimum cost among these three solutions. Thus, $R_1 = \{Y_0, Y_{11}, Y_{12}\}$, $Y_1^* = Y_{11}$. The most promising area in this iteration will become the solutions which are closer to Y_{11} than to Y_0 and Y_{12} , which means $\zeta_1 = \{Y : Y \in \theta, \text{dist}(Y, Y_{11}) \leq \text{dist}(Y, Z), \forall Z \in R_1 \setminus Y_{11}\}$.

Similarly, in iteration 2, we will generate 2 solutions randomly from ζ_1 .

$$Y_{21} = \{1, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1\}$$

$$Y_{21} = \{1, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1\}$$

After running of single ship problems, we can know that Y_{12} is the solution with the minimum cost. Hence, $Y_2^* = Y_{12}$, and $R_2 = \{Y_0, Y_{11}, Y_{12}, Y_{21}, Y_{22}\}$. The most promising area will become the solutions which are closer to Y_{12} , which can be expressed as $\zeta_2 = \{Y : Y \in \theta, \text{dist}(Y, Y_{21}) \leq \text{dist}(Y, Z), \forall Z \in R_1 \setminus Y_{21}\}$.

Similarly, we can continue to do several iterations before we can get the optimal result. In each iteration, we will do the following steps:

- (i) Generate solutions from most promising area
- (ii) Update record set R_r
- (iii) Run single ship problems, update Y^* and most promising area

In my program, after another 17 iterations, the optimal solution is given. As this is a very small case, I set the termination to be 10 iterations without optimal solution changed, which means that the program hit the optimal solution at 9th iteration.

4.4.3 Computational time and Optimality gap

The formulation is able to help fleet managers make decision in aspects including choosing serving ports, routing and scheduling, as well as refueling policy. Such a comprehensive formulation leads to rapidly increasing number of variables and constraints.

The proposed two-stage heuristic, on the other hand, performs better in terms of computational time, especially when N is large. The advantage of heuristic in terms of computational time can be significantly seen when $N \geq 8$. Although the subproblems are also solved using CPLEX in the heuristic, a single ship problem is solved each time, which reduces the complexity to a large extent. As a matter of fact, using CPLEX may consume up the memory of computer when N is large, while the heuristic algorithm can still provide answers when it occurs.

The experiment is carried out such that ports information is generated randomly, and three ways are used to solve the problem. The experiment is all run using Matlab2010b in 64-bit Windows 7, on PC with Intel(R) Core(TM) i7-4770 CPU @ 3.40GHz, and RAM of 8GB. Since CPLEX in Matlab can only guarantee to successfully solve the problems for $N < 10$, samples are generated for $N = 7, 8, 9$, and $K = 2, 3$. Table 4.5 shows the average computational times as well as the longest time it takes to solve a problem of that size. Comparison is made between the time used by (1) CPLEX (exact optimal solution), (2) MPA and (3) heuristic based on neighborhood search (later referred to as NS) respectively. They can also be shown by box plots as in Fig. 4.3 to 4.5.

The computational time for CPLEX increased dramatically. For these size of problems, which is relatively small, MPA works better than NS, which can clearly shown comparing their average times. For all three methods, there are occasionally outlier cases which it takes very long time to solve. Solving for the true optimal solution is still workable averagely though, the performance in terms of computational time is unstable. For the size of $N = 9, K = 3$, the worst case used over 20000 seconds. The average time should not be the only criteria when

Table 4.5: Computational times (in sec)

N	K	CPLEX	worst	MPA	worst	NS	worst
7	2	11.69	58.15	5.99	15.29	13.89	34.30
7	3	656.53	10883	9.01	21.04	21.53	49.09
8	2	26.13	90.51	11.65	21.49	18.48	37.35
8	3	882.47	6333.9	20.18	40.71	41.03	84.84
9	2	89.08	449.99	21.38	45.95	23.95	56.59
9	3	3716.7	20052	32.09	64.58	42.02	105.08

Figure 4.3: CPLEX Computational times for small cases

evaluating algorithms, which is why box plot can be helpful. MPA works better than NS also in terms of the outlier cases.

Table 4.6 shows the performances of the two heuristics by the optimality gaps. Results on worst cases are also listed. MPA shows an obvious advantage over NS in terms of optimality gap, and even in worst cases, MPA can still generate a reasonable good solution.

With the increasing of the number of ports, compared with the rapid increment in computational time to get exact optimal solutions, the growth of time for both heuristics is not too steep, with reasonably close optimal solutions. We will then

Figure 4.4: MPA Computational times for small cases

Figure 4.5: NS Computational times for small cases

Table 4.6: Optimality gap

N	K	MPA(average)	MPA(max)	NS(average)	NS(max)
7	2	0.34%	3.42%	4.78%	40.62%
7	3	0.98%	8.30%	6.58%	35.96%
8	2	0.81%	9.25%	10.14%	44.83%
8	3	1.68%	12.24%	7.59%	24.63%
9	2	1.34%	12.69%	8.85%	57.22%
9	3	1.19%	8.39%	6.31%	25.28%

Figure 4.6: Performance comparison between MPA and NS, as percentage of MPA solutions

test on the two heuristics alone for larger problem sizes, where $N = [7, 15]$ and $K = 2, 3, 4$.

Fig. 4.6 is a box plot on difference between the result from MPA and NS, as a percentage of MPA solution. It can be clearly shown that in most cases, MPA is able to get a more optimal solution. Moreover, neighborhood search may sometimes stuck into local optimum, and give very poor result.

Fig. 4.7 shows the average computational times used by the two heuristics with respect to number of ports. The comparison between them is shown in Fig. 4.8

(a) Computational times for MPA

(b) Computational times for NS

Figure 4.7: Computational times for two heuristics

 Figure 4.8: Computational times when $k=2$

to 4.10 for different number of ships. Although MPA was faster for small size of problems, it obviously increases faster than NS with the increasing of number of ports. At the level of $N = 15$, the average computational time for MPA is almost twice as the time for NS.

However, comparing the worst cases, MPA seems to be more stable. For example, as shown in Fig. 4.12, the computational time almost reached 3000s in one case when $N = 15, K = 4$, while the average is only less than 300s. Considering that MPA can always give a more optimal solution, we will use MPA in the case

Figure 4.9: Computational times when $k=3$

Figure 4.10: Computational times when $k=4$

Figure 4.11: MPA Computational times

study.

Figure 4.12: NS Computational times

Table 4.7: Parameters for case study

Parameter	Value
Number of ports of calls	18
Number of ships	5
Ship size	6000 TEU
Ship bunker fuel capacity	3000 ton
Ship speed interval	(14,24) knots
Bunker fuel price discount	10%,20%
Bunker fuel consumption parameter	$C_k = 7.55 \times 10^{-7}$

4.5 Case study

In this case study, we will apply our model and solution approach to a specific company with an depot settled in Busan, who has major clients on Asia and America. The company provides a set of shipping services to four major ports in North America (such as Vancouver, Seattle, Oakland and San Pedro) and many ports in Asia (such as Singapore, Hong Kong, Ningbo, Qingdao, Shanghai, Xingang, Yantian, Xiamen, Kaosiung, Chiwan and Yokohama). During the visits to some of these ports, ships will collect some raw materials and returned products. In order to decide the routing and bunkering strategies, we collected the relevant parameters including distances between ports, demand, revenues, and fuel prices. The heuristic algorithm is then carried out to solve the problem. 19 ports and 5 ships are included in the problem, and some parameters are shown in Table 4.7. And information of ports are in Table 4.8. Since MPA algorithm is able to get a more optimal solution compared with NA algorithm, as discussed in section 4.4.2, we will use MPA algorithm to solve the problem in this case study.

Note that in reality, it is usually not desirable to assign many ports to one ship, due to the constraints of capacities and cycle times. Hence, when solving these real problems with our heuristic algorithms, only small sized single-ship problems are to be solved in each iteration. With 7 or 8 customer ports, a single-ship problem

Table 4.8: Information of ports

Port	delivery demand	delivery revenue	pickup demand	pickup revenue	fuel price
Busan					645
Singapore	4210	1036	3054	1107	632
Seattle	3585	1416	2900	1396	NA
Vancouver	2320	1502	0	1538	633
Washington	1410	1671	850	1501	619
San Pedro	4692	1806	2930	1177	674
Oakland	3497	1646	1994	1558	695
Yokohama	3398	1362	6147	1294	634
Kaosiung	4288	1117	6445	1228	609
Chiwan	2506	840	3548	1460	662
Shanghai	2078	914	5483	1583	690
Yantian	644	928	2245	1540	689
HongKong	2654	992	1469	1593	641
Xiamen	81	1661	2092	1563	636
Dalian	412	1054	1037	1503	657
Qingdao	537	1241	1241	1473	674
KWY	108	1007	0	1046	695
Xingang	40	1195	437	1497	681
Ningbo	0	898	359	1828	624

Table 4.9: Optimal routing and fuel strategy: Ship 1

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Busan				0	
Yokohama	3398		14.0	46.55	
Vancouver	2320		15.8	318.26	
Seattle	282		17.0	339.55	
Hongkong		158	19.8	628.95	
Ningbo		359	20.0	665.3	2467.9
Shanghai		5483	17.0	672.74	
Busan			14.0	705.6	

Table 4.10: Optimal routing and fuel strategy: Ship 2

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Busan				0	
Oakland	1308		16.9	292.28	1545.6
San Pedro	4692		18.5	314.24	
Xingang		352	23.2	570.41	
Hongkong		1311	21.0	637.62	
Yantian		2245	24.0	639.32	
Xiamen		2092	19.6	653.36	2850
Busan			17.0	705.6	

can be solved within seconds. Hence, our heuristic algorithm shows great efficiency when solving real problems.

The optimal solution for each ship is shown in Table 4.9 - 4.13 respectively. From the results of the study, we can draw some insights. Ships tend to do delivering services first followed by pickup activities, even when sometimes it causes geographically more traveling distances. Apart from the capacity concern, the savings on fuel consumption due to less weight of ships may outperform the cost of a longer voyage.

Although 17 ports of calls are considered in the problem, only 15 ports are actually visited if the proposed solution is implemented. Large demands exist in Washington, but the prolonged traveling distances causes outstanding costs. A small port near Pusan is also excluded from the solution due to small demands.

Table 4.11: Optimal routing and fuel strategy: Ship 3

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Busan				0	
Kaosiung	3811	1076	17.0	54.68	
Oakland	2189	1994	18.2	374.47	1797.6
San Pedro		2930	20.0	394.84	
Busan			17.0	705.6	

Table 4.12: Optimal routing and fuel strategy: Ship 4

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Busan				0	
Qingdao	537	691	17.1	30.58	
Shanghai	2078		17.0	69.33	
Xiamen	81		17.0	101.52	1467.6
Yantian	644		18.2	116.08	
Chiwan	2506	3548	17.0	120.00	
Yokohama		1761	17.0	216.28	
Busan			17.0	254.62	

Table 4.13: Optimal routing and fuel strategy: Ship 5

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Busan				0	
Hongkong	1313		14.0	81.71	
Singapore	4210		14.0	185.81	1224.2
Kaosiung	477	1064	14.0	305.2	
Yokohama		4936	14.0	402.54	
Busan			14.0	449.09	

4.6 Summary

In this chapter, we extended and generalized the model from last chapter. Pickup and delivery may now exist simultaneously on each customer port. Tonnage onboard is considered in fuel consumption rate to get a more realistic model. Simulation is carried out to show the necessity to integrate tonnage and bunker decisions into the model. As such a complex formulation requires a long time to solve, a two-stage heuristic is proposed with two kinds of generating schemes, both showed advantage in computational time compared with solving with CPLEX directly. The one based on Most Promising Area is proved to converge to a local optimum with probability 1. It showed much better performance in terms of optimality gaps, but requires longer time than neighborhood search. At last, a case study is done to test its performance in real cases.

Chapter 5

Fleet Management with general pickup and delivery

Until now, we have explored on problems such that ships bring cargo to from depot to customer ports, and collected from customers back to depot. We call this kind of model 1-M-1 modes, which is mostly seen in industrial shipping or offshore constructing companies. In the formulation, we categorized cargos into two categories: delivery demands and pickup demands. All cargos in a category are treated as identical, due to the fact that all the cargos to be delivered are from the depot, and all cargos picked up are sent back to depot. As long as the ship set out from depot and return to depot, the order is automatically completed.

In tramp services, on the other hand, demands are from a customer port to another. We can describe this kind of service as M-M modes. Our model in Chapter 4 is not able to solve these problems since the cargo routing need to taken care of. The 1-M-1 model can be considered as a simplified model of M-M, with the origin or destination of orders as the depot.

In this chapter, a model to solve general pickup and delivery problem will be developed.

5.1 Problem Description

A set of m shipping orders is coming to a shipping company with the origin o_i , destination d_i , quantity to be shipped D_i , and unit revenue for fulfilling each unit r . A fleet of K ships will set out from a depot, fulfill some of the demands, and come back to depot before a fixed cycle time.

5.2 Problem Formulation 1

In the previous models, we could categorize the demands from customer ports into two kinds. And the cargo routing can be captured by the two variables Q_{i1}^k and Q_{i2}^k , which represents the delivery and pickup ones respectively, as when origin or destination is defined, the other one is set as depot. However, in the general pickup and delivery problem, we need to add a new set of variables to describe the cargo routing. The following decision variables are able to describe the quantity of cargos on each voyage carried by each ship. Monitoring of this variable will give us a picture of flow of cargos.

λ_{ijd}^k The quantity of cargo with d as destination that ship k carries from port i to port j

Q_{ij}^k The demand with origin as port i and destination as j fulfilled by ship k

And the parameter is added to show the quantity of demands between each pair of ports:

D_{ij} The total demand with origin as port i and destination as j

The key idea of this formulation is that once a cargo is on board, the only thing we care is what the destination is. This is also reflected in the new decision

variable λ_{ijd}^k . Then we can formulate the problem as follows:

$$\max \quad \sum_{k \in V} \sum_{j \in P^-} \sum_{i \neq j} r Q_{ij}^k + \sum_{k \in V} R_k x_{1,N+1}^k - \sum_{k \in V} \sum_{i=1}^N f_i(B_i^k) \quad (5.1)$$

$$\text{s.t.} \quad \sum_{j \in P^-} x_{1j}^k \leq 1, \quad \forall k \in V; \quad (5.2)$$

$$\sum_{i \in P} x_{i,N+1}^k = 1, \quad \forall k \in V; \quad (5.3)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \quad \forall i \in P^-, k \in V; \quad (5.4)$$

$$\sum_j \sum_d \lambda_{jid}^k + \sum_d Q_{id}^k - \sum_o Q_{oi}^k = \sum_j \sum_d \lambda_{ijd}^k, \quad \forall i \in P^-, k \in V; \quad (5.5)$$

$$\lambda_{ijd}^k \leq M x_{ij}^k, \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.6)$$

$$\sum_{i \neq j} \lambda_{ijj}^k = \sum_i Q_{ij}^k, \quad \forall j \in P^+, k \in V; \quad (5.7)$$

$$\sum_k Q_{ij}^k \leq D_{ij}^k, \quad \forall i \in P, j \in P^+, i \neq j; \quad (5.8)$$

$$A_j^k \geq A_i^k + d_{ij} t_{ij}^k + s_i^k - M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.9)$$

$$A_j^k \leq A_i^k + d_{ij} t_{ij}^k + s_i^k + M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.10)$$

$$e_i \leq A_i^k \leq l_i, \quad \forall i \in P^-, k \in V; \quad (5.11)$$

$$A_{N+1}^k \leq T^k, \quad \forall k \in V; \quad (5.12)$$

$$W_1^k = W_0^k, \quad \forall k \in V; \quad (5.13)$$

$$W_j^k \leq W_i^k + w(Q_{j2}^k - Q_{j1}^k) + M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.14)$$

$$W_j^k \geq W_i^k + w(Q_{j2}^k - Q_{j1}^k) - M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.15)$$

$$W_i^k \leq W_{\max}^k, \quad \forall i \in P, k \in V; \quad (5.16)$$

$$I_1^k = I_{N+1}^k, \quad \forall k \in V; \quad (5.17)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij} g_k(W_i^k, t_{ij}^k) + M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.18)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij} g_k(W_i^k, t_{ij}^k) - M(1 - x_{ij}^k), \quad \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (5.19)$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i \in P, k \in V; \quad (5.20)$$

$$e^k I_{max}^k y_i^k \leq B_i^k \leq I_m^k y_i^k, \forall i \in P, k \in V; \quad (5.21)$$

$$I_i^k \geq c^k I_{max}^k, \forall i \in P \cup \{N+1\}, k \in V; \quad (5.22)$$

$$I_i^k + B_i^k \leq I_{max}^k, \forall i \in P, k \in V. \quad (5.23)$$

The constraints on routing, time, displacements and bunkering are similar to the previous model. Hence, we will only explain the constraints on cargo routing in this section.

Equation 5.5 describes the cargo flow within one customer port i . The inflow is represented by $\sum_j \sum_d \lambda_{jid}^k$, the cargo ship k carries when entering port i , and $\sum_d Q_{id}^k$, the cargo ship k loads from port i . The outflow can similarly be represented by $\sum_j \sum_d \lambda_{ijd}^k$, the quantity it takes when departing from port i , and $\sum_o Q_{oi}^k$, the ship drops at port i . The cargo flow is balanced using equation 5.5. Inequality 5.6 indicates that there will be no cargo flow on voyage (i, j) if the ship does not travel on the voyage. Equation 5.7 is saying that the cargo with port j as destination that ship k carries will be all unloaded at port j . Constraint in 5.8 gives an upper bound on the quantity of demands that the fleet can fulfill.

Although the number of visits to a customer port i by a ship k , represented by $\sum_{j \neq i} x_{ij}^k$ is not limited, the actual number of visits is constrained to 1 due to the existence of the variable Q_{ij}^k and the constraint in 5.7 in this model. In the case of 1-M-1, there is no point to visit a customer port more than once, since all deliveries are from depot, and all pickups are to depot. Take delivery orders for example, if there are more than one order to be fulfilled on a certain customer port by one ship, it is more profitable to visit once, since the ship need to take all the delivery cargo when setting out. However, in M-M operations, a customer port may serve as both a origin and a destination. Due to capacity, it may be necessary to visit a port more than once to complete orders. For example, there is an order from port i to j , and another order from j to i . A ship can travel to i to pickup the first order, complete it at j , pickup the second order, and go back to i to unload.

In this simple example, port i need to be visited twice, while this formulation will not provide a reasonable solution to achieve this goal.

This formulation is enlightened by the formulation in Ref. [45], in which a liner service was discussed. Since each port is visited once in one cycle under that case, it was a suitable model for that story. Nevertheless, this model we just developed is still a workable formulation if the business do not require multiple visits in one cycle.

5.3 Problem Formulation 2

Due to the limitations of the last formulation, we will develop another model, which allows multiple visits to customer port.

First, we will build a map with $2m + 1$ virtual nodes. In other words, node 1 is the depot, nodes $O = \{2, 4, \dots, 2m\}$ correspond to the origins $o_1, o_2, \dots, o_m$, and nodes $D = \{3, 5, \dots, 2m + 1\}$ correspond to the destinations $d_1, d_2, \dots, d_m$ respectively. For simple indexing, we will define $N = 2m + 1$ to be these nodes, among which nodes with even numbers are origins. Hence, nodes with even number will only have one pickup order, and nodes with odd number except 1 will only have one delivery order. Note that some nodes may coincide in position if a port is the origin or destination of multiple orders. We will define the set $N = O \cup D \cup \{1\}$ to be the set of all nodes, $N^- = O \cup D$ to be the nodes with orders, and $N^+ = O \cup D \cup \{N + 1\}$ for convenience index.

We need to define some parameters based on this virtual map.

r_i Unit revenue for fulfilling demand at node i

D_i The quantity of demand at node i .

Note that there is only pickup demands at nodes with even number,
and only delivery demands at nodes with odd number.

We also need define a new variable,

Q_i^k Quantity of demand fulfilled at port i by ship k .

Note that if $i \in O$, a pickup is implied. Similarly, if $i \in D$, Q_i^k is a delivery.

Using these definitions, we can formulate the general pickup and delivery problem as following:

$$\max \quad \sum_{k \in V} \sum_{i \in D} r_i Q_i^k + \sum_{k \in V} R_k x_{1,N+1}^k - \sum_{k \in V} \sum_{i \in N} f_i(B_i^k) \quad (5.24)$$

$$\text{s.t.} \quad \sum_{j \in N^-} x_{1j}^k \leq 1, \quad \forall k \in V; \quad (5.25)$$

$$\sum_{i \in N} x_{i,N+1}^k = 1, \quad \forall k \in V; \quad (5.26)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \quad \forall i \in N^-, k \in V; \quad (5.27)$$

$$\sum_k Q_i^k \leq D_i, \quad \forall i \in N^-; \quad (5.28)$$

$$Q_i^k = Q_{i+1}^k, \quad \forall k \in V, i \in O; \quad (5.29)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{i+1,j}^k, \quad \forall k \in V, i \in O; \quad (5.30)$$

$$A_j^k \geq A_i^k + d_{ij} t_{ij}^k + s_i^k - M(1 - x_{ij}^k), \quad \forall i \in N, j \in N^+, i \neq j, k \in V; \quad (5.31)$$

$$A_j^k \leq A_i^k + d_{ij} t_{ij}^k + s_i^k + M(1 - x_{ij}^k), \quad \forall i \in N, j \in N^+, i \neq j, k \in V; \quad (5.32)$$

$$e_i \leq A_i^k \leq l_i, \quad \forall i \in N^-, k \in V; \quad (5.33)$$

$$A_{N+1}^k \leq T^k, \quad \forall k \in V; \quad (5.34)$$

$$A_{i+1}^k \geq A_i^k, \quad \forall i \in O, k \in V; \quad (5.35)$$

$$W_1^k = W_0^k, \quad \forall k \in V; \quad (5.36)$$

$$W_j^k \leq W_i^k + w Q_j^k + M(1 - x_{ij}^k), \quad \forall i \in N, j \in O, i \neq j, k \in V; \quad (5.37)$$

$$W_j^k \geq W_i^k + w Q_j^k - M(1 - x_{ij}^k), \quad \forall i \in N, j \in O, i \neq j, k \in V; \quad (5.38)$$

$$W_j^k \leq W_i^k - wQ_j^k + M(1 - x_{ij}^k), \forall i \in N, j \in D, i \neq j, k \in V; \quad (5.39)$$

$$W_j^k \geq W_i^k - wQ_j^k - M(1 - x_{ij}^k), \forall i \in N, j \in D, i \neq j, k \in V; \quad (5.40)$$

$$W_i^k \leq W_{\max}^k, \forall i \in N, k \in V; \quad (5.41)$$

$$I_1^k = I_{N+1}^k, \forall k \in V; \quad (5.42)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij}g_k(W_i^k, t_{ij}^k) + M(1 - x_{ij}^k), \forall i \in N, j \in N^+, i \neq j, k \in V; \quad (5.43)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij}g_k(W_i^k, t_{ij}^k) - M(1 - x_{ij}^k), \forall i \in N, j \in N^+, i \neq j, k \in V; \quad (5.44)$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i \in N, k \in V; \quad (5.45)$$

$$e^k I_{\max}^k y_i^k \leq B_i^k \leq I_m^k y_i^k, \forall i \in N, k \in V; \quad (5.46)$$

$$I_i^k \geq c^k I_{\max}^k, \forall i = 1, 2, \dots, N+1, k \in V; \quad (5.47)$$

$$I_i^k + B_i^k \leq I_{\max}^k, \forall i \in N, k \in V. \quad (5.48)$$

We will explain the constraints on pickup and delivery activities and displacements, as they are the key constraints of this formulation.

Constraints in 5.28 ensure that the fulfilled quantity cannot exceed the demand at each node. The set of constraints in 5.29 make sure that the quantity picked up at origin is the same as the quantity delivered to the destination. The origin and destination of an order will be both visited or both unvisited as indicated by equation 5.30. It also force a ship to visit the origin of an order before the destination, using the time window constraint as 5.35.

Constraints in 5.37 and 5.38 together model the change of carrying weight at origins where ships pickup cargos. Similarly, inequations 5.39 and 5.40 describe the change of displacement at delivery nodes.

Using this formulation, the model allows multiple visit to a certain port if it is involved in several orders, since it is treated as several virtual port in this model. Hence, the solution will decide the order it is fulfilling at each visit, and bunker at which time it visits if it is a refueling port.

Table 5.1: Orders: case 1

order	Origin	Destination	Quantity	Deadline	Revenue
1	Marsden Point	Napier	3500	180	680
2	Brisbane	Darwin	3800	180	760
3	Lyttelton	Nelson	4700	110	720

However, as the number of virtual ports is related to the number of orders, instead of actual number of ports, the complexity of this formulation is based on the number of orders. N can be very large if the number of orders is large when planning for a long cycle time, which may lead to a very long computational time.

5.4 Case Study

In this section, we will apply our second formulation on a small case for a company who serve ports in New Zealand and along the east coast of Australia. The company has a main depot at Geelong. We used the coordinates of the actual ports, and convert them into coordinates in nautical miles, and the $(0, 0)$ point is set as the depot Geelong.

We firstly plan for three orders using two ships with capacity 600TEU each. The detail of the orders is presented in Table 5.1. The result provide a solution with the detailed operations and bunkering instructed as in Table 5.2 and 5.3. Fig. 5.1 is an abstract illustration of the routing. It is not surprise that a ship is serving second order alone, since these two are in Australia. The routing of Ship 2 requires a longer traveling distances compared with circle-like routing due to two reasons: (1)The deadline for order 3 is tight such that it has to be completed before visiting Marsden Point and Napier; and (2)The capacity of the ship is not enough to pickup both orders before sending them to their destinations.

If we change order 3 so that the orders will be as shown in Table 5.4. The optimized solution will provide a routing as in Fig. 5.2 instead. The total demand

Table 5.2: Optimal routing and fuel strategy: Ship 1

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Geelong				0	
Brisbane		3800	14.0	63.07	2580
Darwin	3800		14.0	176.72	
Geelong			14.0	313.31	

Table 5.3: Optimal routing and fuel strategy: Ship 2

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
Geelong				0	
Lyttelton		4700	15.92	99.82	2453.92
Nelson	4700		15.92	110	
Marsden Point		3500	14.0	137.24	
Napier	3500		14.0	157.17	
Geelong			14.0	283.33

Figure 5.1: Optimal routing

Table 5.4: Orders: case 2

order	Origin	Destination	Quantity	Deadline	Revenue
1	Marsden Point	Napier	3500	180	680
2	Brisbane	Darwin	3800	180	760
3	Lyttelton	Nelson	2700	180	720

Figure 5.2: Optimal routing with loose time windows and less demands

of order 1 and 3 is still a bit larger than the capacity of the ship, but the loss in the revenue can be compensated by the shorter traveling distance.

We then test the model on the third case where four orders are to be fulfilled as in Table 5.5. The routing will then shown as in Fig. 5.3. Note that Nelson is visited twice in this case to fulfill order 3 and 4 respectively.

This data from this case study is provided by a company who would like a free demonstration of this program. Although the offshore construction case is not of a big size, the result of this program can provide every details of operational decisions, including cruising speed, bunkering amount, materials to be carried.

Figure 5.3: Optimal routing with 4 orders

Table 5.5: Orders: case 3

order	Origin	Destination	Quantity	Deadline	Revenue
1	Marsden Point	Napier	3500	180	680
2	Brisbane	Darwin	3800	180	760
3	Lyttelton	Nelson	4700	110	720
4	Napier	Nelson	2000	200	680

5.5 Summary

The problem with general pickup and delivery is studied in this chapter. A formulation is first proposed, while it has a limitation on the number of visits at each customer port. Another formulation is given based on a virtual map so that the key ideas in the model in Chapter 4 can still be utilized. Small case simulations are then done to show the result by using the second model.

Chapter 6

Fleet Management with disruptions

The problems we have considered till now always assume deterministic information on ports and cargos, as well as the traveling time between ports. However, a lot of components are random in real planning. There have been some researches on stochastic vehicle routing problems, and the existing literatures mostly focus on three cases in which the randomness are presence of customer, demand and the time (service time or traveling time) respectively. In this section, we will mainly focus on the random factor on the time including the following scenarios:

- (i) Random available time of cargos and demands. Agrali et al.[92] studied the dynamics of a logistics center where shipment requests arrive randomly with various destinations. As the arrival time can be uncertain, we will model the delay of demands.
- (ii) Stochastic edges. Due to various uncontrollable conditions such as weather, some edges may be temporarily unavailable for voyages.
- (iii) Stochastic processing time on ports.

6.1 Problem Description

In this section, we will consider a stochastic version of the problem in Chapter 4. A potential disruption is described by the edge on which it occurs, the delay it results in, and the probability of its occurrence. Situations below can be modelled:

- (i) Delay in the traveling time on a certain edge (i, j) . The direction from i to j , as well as j to i are both affected.
- (ii) Delay in the arrival of demand on port i . All edges with i as origin are affected.
- (iii) Longer processing time on ports i . All Edges with i as origin are affected.
- (iv) Full occupation of all berths on port i . All edges with i as destination are affected.

6.2 Problem Formulation

We will first define an event as a disruption on one edge with a certain probability, described by four parameters (i, j, t, P) , indicating a delay of t hours on the voyage from port i to j with a probability of P . Note that if a disruption on an edge affect both directions, two events are added to the model. If a delay may occur on a certain port, $N - 1$ events are added to the model. With the definition of scenarios, we can solve the stochastic problem using a scenario-based formulation.

Hard time constraints are not suitable anymore. Soft time constraints will be proposed in this model so that lateness will be taken into consideration.

First we need to introduce some more parameters:

P_ω Probability that scenario ω occurs

C_d Cost for each hour of lateness at each port, which is assumed to be identical for all ports and cargos in this model

$\tau_{ij\omega}$ The delay on voyage (i, j) under scenario ω

Some state variable will be added:

$A_{i,\omega}^k$ Arrival time at port i of ship k under scenario ω .

$\delta_{i,\omega}^k$ Lateness by ship k at port i under scenario ω .

If $A_{i,\omega}^k \leq l_i$, then $\delta_{i,\omega}^k = 0$, otherwise $\delta_{i,\omega}^k = A_{i,\omega}^k - l_i$

With these notations, along with those defined in Chapter 4, we can formulate the problem with disruptions as following.

$$\begin{aligned} \max \quad & \sum_{k \in V} \sum_{i \in P^-} (r_{i1} Q_{i1}^k + r_{i2} Q_{i2}^k) + \sum_{k \in V} R_k x_{1,N+1}^k \\ & - \sum_{k \in V} \sum_{i \in P} f_i(B_i^k) - C_d \sum_{\omega \in \Omega} \sum_{i \in V^+} P_\omega \delta_{i,\omega}^k \end{aligned} \quad (6.1)$$

$$\text{s.t.} \quad \sum_{k \in V} Q_{i1}^k \leq D_{i1}, \sum_{k \in V} Q_{i2}^k \leq D_{i2}, \forall i \in P^-; \quad (6.2)$$

$$Q_{i1}^k \leq M \sum_{j \neq i} x_{ij}^k, Q_{i2}^k \leq M \sum_{j \neq i} x_{ij}^k, \forall i \in P^-, k \in V; \quad (6.3)$$

$$\sum_{j \in V^-} x_{1j}^k = 1 - x_{1,N+1}^k, \forall k \in V; \quad (6.4)$$

$$\sum_{i \in V^-} x_{i,N+1} = 1 - x_{1,N+1}^k, \forall k \in V; \quad (6.5)$$

$$\sum_{j \neq i, j \in V^-} x_{ij}^k \leq 1 - x_{1,N+1}^k, \forall i \in P^-, k \in V; \quad (6.6)$$

$$\sum_{j \neq i} x_{ij}^k = \sum_{j \neq i} x_{ji}^k, \forall i \in P^-, k \in V; \quad (6.7)$$

$$W_1^k = w \sum_{i=2}^N Q_{i1}^k + W_0^k, \forall k \in V; \quad (6.8)$$

$$W_j^k \leq W_i^k + w(Q_{j2}^k - Q_{j1}^k) + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (6.9)$$

$$W_j^k \geq W_i^k + w(Q_{j2}^k - Q_{j1}^k) - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (6.10)$$

$$W_i^k \leq W_{\max}^k, \forall i \in P, k \in V; \quad (6.11)$$

$$I_1^k = I_{N+1}^k, \forall k \in V; \quad (6.12)$$

$$I_j^k \leq I_i^k + B_i^k - d_{ij}g_k(W_i^k, t_{ij}^k) + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (6.13)$$

$$I_j^k \geq I_i^k + B_i^k - d_{ij}g_k(W_i^k, t_{ij}^k) - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, i \neq j, k \in V; \quad (6.14)$$

$$y_i^k \leq \sum_{j \neq i} x_{ij}^k, \forall i \in P, k \in V; \quad (6.15)$$

$$e^k I_{\max}^k y_i^k \leq B_i^k \leq I_{\max}^k y_i^k, \forall i \in P, k \in V; \quad (6.16)$$

$$I_i^k \geq c^k I_{\max}^k, \forall i \in P \cup \{N+1\}, k \in V; \quad (6.17)$$

$$I_i^k + B_i^k \leq I_{\max}^k, \forall i \in P, k \in V; \quad (6.18)$$

$$A_{j,\omega}^k \geq A_{i,\omega}^k + d_{ij}t_{ij}^k + s_i^k + \tau_{ij\omega} - M(1 - x_{ij}^k), \forall i \in P, j \in P^+, \\ i \neq j, k \in V, \omega \in \Omega; \quad (6.19)$$

$$A_{j,\omega}^k \leq A_{i,\omega}^k + d_{ij}t_{ij}^k + s_i^k + \tau_{ij\omega} + M(1 - x_{ij}^k), \forall i \in P, j \in P^+, \\ i \neq j, k \in V, \omega \in \Omega; \quad (6.20)$$

$$\delta_{1,\omega}^k = \max \{0, A_{1,\omega}^k - l_i\}, \forall i \in P^+, k \in V, \omega \in \Omega; \quad (6.21)$$

$$Q_{i1}^k \geq 0, Q_{i2}^k \geq 0, \forall i \in P^-, k \in V; \quad (6.22)$$

$$x_{ij}^k = 0 \text{ or } 1, \forall i \in V, j \in P^+, i \neq j, k \in V; \quad (6.23)$$

$$y_i^k = 0 \text{ or } 1, \forall i \in P, k \in V. \quad (6.24)$$

The last term of the objective function is the expected cost due to lateness under all scenarios. Constraints in 6.19 and 6.20, as soft time window constraints, describe the effect of disruptions on arrival times at each port under scenarios. Equation 6.21 give the definition of lateness on each port $\delta_{i,\omega}^k$. This set of constraints can be equivalently replaced by the following two sets of inequations:

$$\delta_{i,\omega}^k \geq 0, \forall i \in P^+, k \in V, \omega \in \Omega; \quad (6.25)$$

$$\delta_{i,\omega}^k \geq A_{i,\omega}^k - l_i, \forall i \in P^+, k \in V, \omega \in \Omega \quad (6.26)$$

Figure 6.1: Routing with no penalty cost for lateness

Note that this replacement is valid since the lateness is to be minimized in the objective function.

Note that the purpose of this formulation is a pro-active planning of the fleet deployment, considering all possible scenarios. When information on the disruptions become known, deterministic model as in Chapter 4, with constraints included to reflect the disruptions, should be applied to solve for the updated deployment and bunkering strategies.

6.3 Simulation Results

In this section, we will use the same toy example as in Chapter 4 to illustrate the effect of disruptions and penalty cost on the planning of the fleet deployment.

To show the effect of time windows and disruptions, we will first work out a solution under the situation where no penalty or very low penalty for lateness. The routing is shown in Fig. 6.1. Since lateness is not a concern, every port is visited to maximize the fulfilling demands, so that revenue is maximized.

When penalty cost is increased, we can obtain a solution similar to the optimal

Figure 6.2: Routing with time windows

Table 6.1: Optimal routing with disruption on (1,5)

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
depot		4000		0	
4	2500	2500	18.785	38.39	786.47
2	1000	1000	17.0	91	
7	500	2500	14.0	119.57	
depot	6000		14.0	151.52	
depot		6000		0	
6	3200	2800	17.05	54.24	877.72
3	2800	2000	17.0	90.03	
depot	4800		14.0	166.96	

solution under hard time window constraints as we had in Chapter 4 (Fig. 6.2). We note from several times of simulation that when penalty is not very high, the optimal result will always provide routing solution the same as the one with hard time constraints, but the cruising speed is usually higher. So we will fix the unit penalty cost at a relatively high level, to show the change of routing due to disruptions.

We now consider one disruption at a time. Suppose there is a possible disruption on voyage 1, 5. To test, the probability of the event is adjusted from 0.1 to

Figure 6.3: disruption on (1,5)

Table 6.2: Optimal routing with disruption on (3,6)

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
depot		6000		0	
5	2700	500	17.0	34.31	
4	2500	2500	19.78	82.26	1274.32
2	800	3000	17.81	132.48	
depot	6000		17.0	181	
depot		3700		0	
7	500	2500	14.0	31.94	
6	3200	2800	14.0	83.94	539.12
depot	5300		14.0	149.80	

Table 6.3: Optimal routing with disruption on (4,7)

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
depot		6000		0	
5	2700	500	17.0	34.31	
4	2500	2500	19.78	82.26	1274.32
2	800	3000	17.81	132.48	
depot	6000		17.0	181	
depot		6000		0	
6	3200	2800	17.0	54.24	877.72
3	2800	2000	17.0	90.03	
depot	4800		14.0	166.96

Figure 6.4: disruption on (3,6)

Figure 6.5: disruption on (4,7)

Table 6.4: Optimal routing with disruption on (2,7) and (6,3)

Port	Delivery (TEU)	Pickup (TEU)	Velocity (knots)	Arrival time	Bunkering amount
depot		5200		0	
5	2700	1000	17.0	34.3	
4	2500	2500	16.73	91	645.53
depot	3500		14.0	142.51	
depot		3700		0	
7	500	2500	14.0	31.94	539.12
6	3200	2800	14.0	83.94	
depot	5300		14.0	149.80	

0.9 with 0.1 step size, and the routing changes not until the probability reaches 0.7. Even at the probability level of 0.7, the routing is still affected by the hours of delay. The routing is changed when the delay is not less than 26 hours, and speed adjustment is the way to achieve low penalty cost when delay is not high enough. Fig. 6.3 and Table 6.1 shows the optimal routing when there is a long disruption with high possibility on the voyage (1,5). Note that the arrival time for port 2 and port 3 are slightly later than the time window, since a soft time window is adopted in this model, and the penalty of lateness can be covered by the fuel consumption savings.

Although we have tested quite a few scenarios with ranges of parameters, we will only show those with high enough probability and long enough delay, to illustrate the change in the routing.

Similarly, Fig. 6.4 and Table 6.2 is an illustration of the routing solution when there is a disruption on the voyage (3,6). Port 3 is not visited by ship 1 in this case, and port 7 is included in its journey instead. Fig. 6.5 and Table 6.3 shows the routing when disruption may occur on voyage (4,7). The ship will serve port 2 instead of 7, since it need to pass port 4 just before the visit to 7 in the no disruption case.

We will then consider the case where two disruptions may occur at the same time. Fig. 6.6 and Table 6.4 is the routing when there is a disruption on both voyages (2,7) and (3,6). Demands on two ports are not fulfilled in this case. This

Figure 6.6: disruption on (2,7) and (6,3)

illustrates the situation where in short distance shipping, if the weather causes bad sea state in a large range, the business may shut down for some period.

From the simulations, we can see the effect of disruptions on the optimal routing. Of course, the result is affected by the lateness, penalty cost and the probability of the event. The trade off between the expected lateness penalties and revenues is achieved using this formulation.

6.4 Summary

In this chapter, we explored on the disruption management in fleet management. A model is built to handle the situations that disruptions may occur in the story in Chapter 4. We managed to convert the stochastic problem into a scenario-based formulation, so that we can still solve the problem in a mixed integer programming. The model is then verified by some simulations.

Chapter 7

Conclusions and Future Work

7.1 Conclusions

In this thesis, we developed four integrated fleet management models to support fleet deployment and bunker management, with the aim of a more environmentally and financially sustainable future of shipping industry. The case studies are also carried out using the models for verification purposes. The models are able to reduce fuel cost with the consideration of high fuel prices nowadays. The reduction in fuel consumption is also vital in saving of non-regenerated energy resources.

The four models are summarized as follows,

- A capacitated vehicle routing problem with time windows is formulated. As most of the existing works, cruising speed is taken as the only factor influencing fuel consumption rate. A simple heuristic based on priority and geographical locations is proposed, with assigning each port a comprehensive index to indicate the overall profitability.
- The effect of tonnage onboard on fuel consumption rate is included in the second model. Pickup and delivery demands may exist simultaneously on customer ports. The corresponding NP-hard optimization problem is formulated as a Mixed Integer Programming problem with nonlinear terms. Although the problem can be linearly approximated and solved using CPLEX,

the complexity still leads to a rapid increase in computational time, and very high computational time is needed for large scale problems. A two-stage method with two possible generating algorithms is proposed, among which a time-efficient heuristic based on Most Promising Area is recommended. The MPA Algorithm is proven to converge to local optimum with probability 1, while the computation time decreased on a large extent compared with exact optimization. An improvement step is also proposed so that each solution in the two-stage algorithm can be examined and further optimized.

- General pickup and delivery problems are considered in the third model, so that the origins and destinations of orders are all customer ports. The formulation is built based on a virtual map, and cargo routing can be modeled.
- The last model includes stochastic factor so that the planning can consider the possible disruptions in the planning cycle. The problem is solved using a scenario based formulation, which turns the stochastic problem into a deterministic one.

A more realistic fuel consumption estimation function is proposed and adopted to enhance the effectiveness of our models in practice, in which not only cruising speed but freight tonnage onboard as well are taken into account when estimating ship fuel consumption rate. The effect of displacement on fuel consumption rate, and hence the routing decisions has been proved by simple numerical results.

Of course, there are some limitations in each model. Although it can be solved efficiently using proposed heuristic, the second model can only be applied to 1-M-1 business models, such as feeder services and industrial shipping. This is also the limitation for the stochastic model in Chapter 6. On the other hand, the general pickup and delivery problem discussed in Chapter 5 are more widely applicable to most shipping operations. But as the formulation is built based on a virtual map, the number of variables and constraints are related to number of tasks, which can be very large, and cannot be solved efficiently.

Figure 7.1: Fuel consumption rate in bow waves[7]

7.2 Future Work

7.2.1 Fuel consumption models

Fuel consumption rate can be very challenging to precisely estimate, as it is affected by various factors, among which the cruising speed is a main determinant. The introduction of displacement into the model of fuel consumption rate has been proved to affect the consumption, and hence lead to different routing solutions. As a matter of fact, the fuel consumption rate can be significantly affected by some other factors including weather and sea conditions which may influence the consumption rate. Figure 7.1 shows the fuel consumption rate of a 13000TEU container ship in different waves[93]. The empirical model for fuel consumption rates can be further refined, and examine the influence on optimal solutions.

7.2.2 General pickup and delivery problems

Fleet management with general pickup and delivery, as discussed in Chapter 5, is a main concern in tramp businesses. The need to formulate cargo routing

has resulted in an increase in the complexity of the problem. Although we have developed a workable formulation, it is not efficient enough to solve for large scale problems. More work can be done to develop a suitable heuristic algorithm based on the formulation, or other ways of formulation can be explored. When the problem can be solved efficiently, case studies can be carried out to further utilize the model.

On the other hand, stochastic factors can be taken into consideration to bring out a new model for disruption management in tramp service planning.

7.2.3 Cargo type and positioning

Our models in this thesis are all based on the assumption that we are dealing with containerships, which give us the advantage of uniform cargo unit. Bulk as another main cargo type in industrial and tramp shipping, requires a different model to plan for.

During the discussion with shipping companies, they point out three dimensions that they give much concern: (1) routing and scheduling, (2) fuel management, and (3) sequence and repositioning of cargos. They have ships that bulk cargos are stored inside the ship under the deck, and with containers above the deck. In this case, the sequence of pickup and delivery become a main issue in the planning. As repositioning of cargos may lengthen the operating time on port to a large extent, shipping companies wish to plan for services such that little repositioning need to be done to finish their jobs. Although it does not have to be strictly First In Last Out, the overall sequence should be to reduce cargo repositioning. The consideration for cargo sequence can be a challenging work, as the cargos are deemed as identical in our models currently. Efforts can be made on such formulations with cargo repositioning as a constraint.

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