Attitude determination system of nano-satellite

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ATTITUDE DETERMINATION SYSTEM OF NANO-SATELLITE

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ENGINEERING**

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ATTITUDE DETERMINATION SYSTEM OF NANO-SATELLITE

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“To my brother, I feel your blessing from above”

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ABSTRACT

In recent years, the industry of miniature satellite based on cubesat standard has been growing rapidly. This growing industry has changed the direction of miniature satellite from educational and research purposes to application focused. As most of the implemented applications require target pointing, precise attitude determination is highly crucial to the mission success.

There are several established attitude determination methods namely the TRIAD algorithm, QUEST algorithm and Kalman filter. They share a common shortcoming of requiring at least two observer-reference pairs as the input to the algorithm. This has caused an issue for nano-satellite's attitude determination system as the magnetic field generated by the magnetic torquers corrupt the magnetometer reading while performing momentum dumping. To overcome this problem, a modified extended Kalman filter algorithm has been developed in this thesis such that it can work even with the presence of only a single observer-reference pair by trading off attitude knowledge accuracy. From the study, it was noticed that the accuracy of the attitude knowledge drops drastically with time when utilizing only a single observer-reference pair.

To further improve the performance, an alternative approach has been developed such that the attitude knowledge accuracy can be retained even when magnetic torquers are switched on. The alternative approach used a hybrid optimization method based on particle swarm optimization and iterated extended Kalman filter to calibrate the magnetometer in-orbit. Beside calibrates for the coupling effects of magnetic torquers, the proposed magnetometer calibration method also calibrates for the in-orbit variation that affects the measurement accuracy of the magnetometer. Both simulation and experimental results show that the measurement accuracy of magnetometer is enhanced

and the problem of magnetometer reading corruption while performing momentum dumping is resolved.

MEMS based gyroscope is typically used in a nano-satellite. Its drawback is the low accuracy. While there are studies to improve the performance of MEMS based gyroscope, they either require an array of MEMS gyroscopes or a high complexity algorithm and thus not suitable for nano-satellite applications. To address this issue, a low complexity Kalman filter for MEMS based gyroscope has been introduced in this thesis. Besides having a lower computational load, it also reduces the MEMS based gyroscope noise in all three-axes without the implementation of extra gyroscopes.

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CHAPTER 1

INTRODUCTION

1.1 Background

The industry of miniature satellite based on cubesat standard (i.e. satellites that weigh less than 5 kg) has been growing in recent years due to its low cost, short developmental time and can be launched in multiples using excess capacity of launch vehicle. SpaceWorks' nano/microsatellite market assessment as shown in Fig. 1.1 indicates that there are around 180 nano/microsatellites requiring a launch in the year 2020 [1].

Fig. 1.1 SpaceWorks nano-satellite market assessment

The growing industry of miniature satellite has also changed the direction of users from educational and research purposes to application focused [2]. Among the applications implemented on miniature satellite are earth imaging, weather forecasting and inter-satellite communication. The Satellite Research Centre (SaRC) of Nanyang

Technological University (NTU) has the vision to be a world class centre for advanced research and training in innovative space technologies for small satellite system. Presently, SaRC has 6 satellites orbiting in space which include two of the VELOX series nano-satellites named VELOX-I and VELOX-II.

VELOX-I [3] is the third satellite missions of SaRC. It is a 3U nano-satellite equipped with a fault-tolerant optical payload whereas VELOX-II [4] is a 6U nano-satellite which demonstrates the technology of Inter-Satellite Data Relay System (IDRS) via a communication payload. As most of the implemented applications require target pointing, precise attitude determination is highly crucial to the mission success.

1.2 Motivations

Attitude determination for conventional satellites can be performed at high accuracy as they are equipped with high performance attitude sensors such as the star tracker, fluxgate magnetometer, digital sun sensor, and high precision gyroscope. Due to the space, mass, budget, and power constraint of nano-satellite, precise attitude determination for nano-satellite has faced many challenges. Typically, microelectromechanical systems (MEMS) based attitude determination system (ADS) which consists of three-axis magnetometer (TAM) and gyroscope together with analog sun sensor [5] are used in nano-satellite with the trade-off of lower accuracy.

It is well known that the TAM measurements are often affected by the thermal variation, mechanical mis-alignment, and on-board electronics time-varying bias which lower the attitude knowledge accuracy. In addition, analysis on the whole orbit data (WOD) of VELOX-II shows that the attitude knowledge accuracy falls significantly when the magnetic torquers are switched on as it disrupts the TAM reading.

In view of these problems, the objective of this thesis involved studying the characteristics of attitude sensors and system modelling of the attitude sensors. By having a precise model, filtering algorithm for noise removal, sensor fusion algorithm for precise attitude estimation and optimization algorithm for sensors calibration can be developed. This will greatly enhance the attitude knowledge accuracy of nano-satellites such that precise target tracking can be executed.

1.3 Objectives

The objectives of this thesis are as follows:

- To investigate and model attitude sensors used in nano-satellite.
- To develop a sensor fusion algorithm that accurately estimates the orientation of a nano-satellite.
- To develop an in-orbit TAM calibration method to eliminate the coupling effect of magnetic torquer.
- To develop an attitude sensor's noise removal algorithm for a better performance.
- Setup an ADS experimental test bench to test, calibrate and verify the performance of attitude sensors.

1.4 Main Contribution

The main contributions of the thesis are as follows:

- An attitude determination algorithm based on extended Kalman filter (EKF) has been developed. The algorithm uses the data from the Sun model, International Geomagnetic Reference Field (IGRF) model, sun sensor reading, TAM reading and gyroscope reading. The attitude estimation

performance has been compared with QUEST algorithm and shows a better performance with a mean attitude knowledge error of 0.39° .

- An attitude-independent in-orbit TAM calibration method has been developed by using a hybrid optimization based on particle swarm optimization (PSO) and iterated extended Kalman filter (IEKF). The proposed calibration method considers the effect of hard iron bias, mechanical mis-alignment, on-board electronics time-varying bias, and the coupling effect of magnetic torquers. As such, accurate attitude knowledge can be retained even when the magnetic torquers are switched on. Experimental and simulation results show that the proposed method converged faster when compared with the PSO and was able to solve for large initialization error.
- A low complexity Kalman filter (LCKF) for improving microelectromechanical systems (MEMS) based attitude determination system has been developed. The algorithm focused on reducing the MEMS gyroscope noise which is crucial to the satellite's sun tracking performance. This approach utilizes both the angular rate and angular acceleration for gyroscope drift reduction. Low complexity was achieved by expressing the state transition matrix and the observation matrix into sparse matrices with non-zero diagonal elements. Experimental results show that the LCKF is capable of reducing the gyroscope noise in all axes and reduced the computational time by 40.81%.

1.5 Outline of Thesis

The thesis is organized as follows:

Chapter 2 presents a literature review of nano/pico-satellite's attitude determination method, attitude-independent in-orbit TAM calibration method and MEMS gyroscope noise removal algorithm.

Chapter 3 presents a modified EKF algorithm for attitude determination such that it can work even with the presence of only a single observer-reference pair. The performance of the modified EKF algorithm when utilizing two observer-reference pairs and a single observer-reference pair has been investigated. In addition, the shortcomings of the modified EKF algorithm when utilizing only a single observer-reference pair has been discussed.

Chapter 4 proposes the attitude-independent in-orbit TAM calibration approach by using a hybrid optimization method based on PSO and IEKF. Experimental and simulation results show that the proposed method has a fast convergent rate despite large initialization error.

Chapter 5 presents the proposed LCKF for improving MEMS based gyroscope performance. Experimental results show that the LCKF is capable of reducing the gyroscope noise in all axes and reduced the computational time by 40.81%.

Chapter 6 concludes the research findings and discusses the future works that can be conducted to improve the attitude determination performance.

CHAPTER 2

LITERATURE REVIEW

2.1 Attitude Determination Methods for Nano/pico-satellites

Attitude determination method for satellite has been researched for a long history. It typically uses a combination of sensors and mathematical models to collect vector components in the satellite body frame and inertial reference frame [6]. The vector components are then used by various algorithms to determine the attitude typically in the form of quaternion [7]. Attitude sensors that are commercially available include the star sensor, sun sensor, earth sensor, magnetometer and gyroscope. For the attitude control, actuators such as magnetic torquer, reaction wheel and thruster are typically used [8]. For attitude determination and control application of nano/pico-satellite, the sensors and actuators that are commonly used are the sun sensor, magnetometer, gyroscope, reaction wheel and magnetic torquer [5].

There are two general classes of satellite attitude determination methods namely the point-by-point attitude determination whereby the sensors provide vectors measurement in a single point of time and a recursive estimation which uses information from successive time steps [9]. The most common point-to-point attitude determination methods are the TRIAD algorithm and the Quaternion estimator (QUEST) whereas the most common recursive attitude determination method is the EKF. These methods have been implemented in various nano/pico-satellites. For example, the TRIAD algorithm was implemented in PolySat CP3 built by California Polytechnic State University [10]. QUEST was modified and used by the Norwegian University of Science and Technology (NTNU) Cubesat [11]. Numerous Cubesats have implemented the EKF

such as VELOX-I [12] and VELOX-II [13] built by Nanyang Technological University and Illinisat built by the University of Illinois [14]. Table 2.1 provides a summary of attitude determination methods and the nano/pico-satellites that had implemented them. Detailed discussion on the attitude determination method will be covered in the following section.

Table 2.1 Summary of cubesat attitude determination methods

<i>Attitude determination method</i>	<i>Nano/pico-satellite</i>
<i>TRIAD algorithm</i>	<ul style="list-style-type: none"> i. <i>PolySat CP3 [10]</i> ii. <i>Small Astronomy Satellite</i> iii. <i>Atmospheric Explorer Mission</i> iv. <i>CubeSat TestBed 1 [15]</i> v. <i>UWE-3 [16]</i>
<i>QUEST</i>	<ul style="list-style-type: none"> i. <i>NTNU Cubesat [11]</i> ii. <i>Cute-1.7 + APD II [17]</i>
<i>EKF</i>	<ul style="list-style-type: none"> i. <i>VELOX-I [12]</i> ii. <i>VELOX-II [13]</i> iii. <i>Illinisat [14]</i> iv. <i>HokieSat [18]</i> v. <i>SERPENS [19]</i> vi. <i>CANX [20]</i> vii. <i>CONASAT [21]</i>

2.1.1 TRIAD algorithm

The TRIAD algorithm [22] requires the knowledge of two vectors in the observer frame and reference frame to estimate the attitude of satellite. These observer-reference vector pairs depend on the attitude determination sensor equipped in the satellite which can be sun vector pairs, magnetic field vector pairs, star vector pairs etc. The attitude of the satellite is then determined by transforming the observer-reference vector pairs into the reference and observed triad of three orthogonal unit vectors such that an orthogonal matrix that relate them can be found by solving the Wahba's problem.

Fig. 2.1 Observer-reference pairs

Let the two observer-reference pairs denoted by $\vec{V}_{1obs}$, $\vec{V}_{1ref}$ and $\vec{V}_{2obs}$, $\vec{V}_{2ref}$ as illustrated in Fig. 2.1. The TRIAD algorithm starts with the formulation of the reference triad orthogonal unit vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ by utilizing $\vec{V}_{1ref}$ and $\vec{V}_{2ref}$ as shown

$$\vec{r}_1 = \vec{V}_{1ref} \quad (2.1)$$

$$\vec{r}_2 = \frac{\vec{V}_{1ref} \times \vec{V}_{2ref}}{|\vec{V}_{1ref} \times \vec{V}_{2ref}|} \quad (2.2)$$

$$\vec{r}_3 = \vec{r}_1 \times \vec{r}_2 \quad (2.3)$$

Next, the observer triad orthogonal unit vectors $\vec{s}_1$, $\vec{s}_2$ and $\vec{s}_3$ are formulated as

$$\vec{s}_1 = \vec{V}_{1obs} \quad (2.4)$$

$$\vec{s}_2 = \frac{\vec{v}_{1obs} \times \vec{v}_{2obs}}{|\vec{v}_{1obs} \times \vec{v}_{2obs}|} \quad (2.5)$$

$$\vec{s}_3 = \vec{s}_1 \times \vec{s}_2 \quad (2.6)$$

Let the reference triad orthogonal matrix $\mathbf{M}_{ref}$ be

$$\mathbf{M}_{ref} = [\vec{r}_1 : \vec{r}_2 : \vec{r}_3] \quad (2.7)$$

and the observer triad orthogonal matrix $\mathbf{M}_{obs}$ be

$$\mathbf{M}_{obs} = [\vec{s}_1 : \vec{s}_2 : \vec{s}_3] \quad (2.8)$$

then using (2.7) and (2.8), the attitude matrix is calculated by

$$\mathbf{A}_{ref}^{obs} = \mathbf{M}_{obs} \mathbf{M}_{ref}^T \quad (2.9)$$

where $\mathbf{A}_{ref}^{obs}$ is a 3×3 attitude matrix that rotate from reference frame to observer frame.

The TRIAD algorithm is easy to implement. However, it is sensitive to the order at which the algorithm receives the two vector pairs [23]. This is because the TRIAD solution is not symmetric and part of the information contained in the second vector pairs will be discarded. Therefore, the TRIAD solution will be more accurate when the observer-reference pair which has the greater accuracy is processed first [22]. In addition, the TRIAD algorithm can only solve for two observer-reference pairs. This will cause a great loss in accuracy if more measurements are available.

2.1.2 QUEST

QUEST algorithm was developed for MAGSAT in 1979 [24] which carried the mission to map the earth magnetic field with an accuracy of 6 nT. In order to carry out the mission, the earth magnetic field measurement has to be spaced very closely thus it was necessary to measure the magnetic field frequently. The magnetic field

measurement would only be meaningful if the attitude is updated every time a measurement is taken. However, the computer was not fast enough to process the data using the q-Method at that time. Thus the QUEST algorithm was developed [24]. QUEST algorithm is a modification of the q-Method algorithm by utilizing an iterative method to approximate the maximum eigenvalue [25]. This is because q-method is less computationally efficient as it needs to calculate the exact eigenvalue.

Let $\bar{\mathbf{v}}_{k,\text{obs}}$ and $\bar{\mathbf{v}}_{k,\text{ref}}$ be the observer-reference vector pairs where $k=1,2,3\dots$ depending on the number of vector pairs available. The optimal eigenvalue is approximated by

$$\lambda = \sum_{k=1}^N w_k \quad (2.10)$$

where w_k is define to be the weightage of each of the observer-reference pair. Once the eigenvalue is estimated, the corresponding eigenvector which is the quaternion that gives the optimal attitude estimate is calculated as

$$\mathbf{p} = [(\lambda + \sigma)\mathbf{I}_{3 \times 3} - \mathbf{S}]^{-1} \mathbf{Z} \quad (2.11)$$

where $\mathbf{Z}$ is a 3x1 vector calculated as

$$\mathbf{Z} = \begin{bmatrix} \mathbf{B}_{23} - \mathbf{B}_{32} \\ \mathbf{B}_{31} - \mathbf{B}_{13} \\ \mathbf{B}_{12} - \mathbf{B}_{21} \end{bmatrix} \quad (2.12)$$

$\mathbf{S}$ is define to be the sum of $\mathbf{B}$ and $\mathbf{B}^T$:

$$\mathbf{S} = \mathbf{B} + \mathbf{B}^T \quad (2.13)$$

and σ is define to be the sum of diagonal element of $\mathbf{B}$:

$$\sigma = \mathbf{B}_{11} + \mathbf{B}_{22} + \mathbf{B}_{33} \quad (2.14)$$

where $\mathbf{B}$ is a 3×3 matrix which sums up the multiplication of w_k with its respective observer-reference pair:

$$\mathbf{B} = \sum_{k=1}^N w_k \left(\hat{\mathbf{v}}_{k,\text{obs}} \hat{\mathbf{v}}_{k,\text{ref}}^T \right) \quad (2.15)$$

The output quaternion is calculated using (2.11) as

$$\mathbf{q}_{\text{ref} \rightarrow \text{obs}} = \frac{1}{\sqrt{1 + \mathbf{p}^T \mathbf{p}}} \begin{bmatrix} 1 \\ \mathbf{p} \end{bmatrix} \quad (2.16)$$

where $\mathbf{q}_{\text{ref} \rightarrow \text{obs}}$ is the quaternion from the reference to the observer frame in the form of

$$\begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_v \end{bmatrix}. \quad q_s \text{ is the scalar part of the quaternion and } \mathbf{q}_v \text{ is the vector of the quaternion.}$$

The advantages of QUEST algorithm is that it can solve two or more observer-reference pairs which can improve the attitude estimation accuracy. Unlike the TRIAD algorithm, the QUEST algorithm is not sensitive to the order at which the algorithm receives the two vector pairs. Instead the reliability of each observer-reference pair can be predefined via w_k . Another significant advantage of QUEST algorithm is that it preserves the attitude quaternion's normalization. However, the QUEST algorithm can only deal with vector type measurements and simple dynamic model. In other word, it is a point-to-point attitude determination method [26]. The QUEST algorithm however requires at least two observer-reference pairs to perform attitude determination.

2.1.3 Extended Kalman filter

Kalman filter is a recursive method for linear state estimation. However, Kalman filter only applies for linear dynamics system which is not the case for attitude determination. As a result, the EKF is formulated for attitude determination by using a

discrete model consisting of a nonlinear state-space model which includes process noise and measurement noise [27].

As the EKF algorithm can be altered by changing the state vector, many have since modified the EKF algorithm for attitude determination based on the satellite application. For instance, the unscented Kalman filter (UKF) [28] method which approximates a Gaussian probability function by characterizing a set of sigma points instead of linearizing a nonlinear model is shown to have a higher accuracy but higher computational load [29] as compared to EKF. In addition, the robust Kalman filtering formulated by modify the existing EKF and UKF has shown to be robust against measurement malfunctions [30].

EKF is highly robust and accurate [9]. Besides, the EKF algorithm can be altered based on satellite requirement and characteristics. However, EKF is computational inefficient as compared to the QUEST and TRIAD algorithm and it is difficult to model as it is subjected to the constraint $\mathbf{q}\mathbf{q}^T = 1$ in order to maintain orthogonality of the quaternion, where $\mathbf{q}$ is the quaternion from the reference to the observer frame in the form of $\begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_v \end{bmatrix}$. $\mathbf{q}_s$ is the scalar part of the quaternion and $\mathbf{q}_v$ is the vector of the quaternion. Besides, gyroscope is required in order to implement the EKF on small satellites. In addition, it requires at least two observer-reference pairs for the algorithm to work.

2.1.4 Summary on existing attitude determination methods

Table 2.2 provides a summary of the advantages and disadvantages of the attitude determination method as discussed in the previous sections. From Table 2.2, it can be observed that all the three methods possessed the same disadvantage, i.e. require at least

two observer-reference pairs to perform the algorithm. In Chapter 3, a modified EKF algorithm will be discussed such that it can work even with the presence of only one observer-reference pair.

Table 2.2 Advantages and disadvantages of attitude determination methods

<i>Attitude determination method</i>	<i>Advantages</i>	<i>Disadvantages</i>
<i>TRIAD algorithm</i>	i. Easy to implement	i. Sensitive to the order at which the algorithm receive the two vector pairs ii. Can only solve for two observer-reference pair
<i>QUEST</i>	i. Can solve two or more observer-reference pair ii. Not sensitive to the order at which the algorithm receive the two vector pairs iii. Preserved the attitude quaternion's normalization	i. Can only deal with simple dynamics models ii. Requires at least two observer-reference pair
<i>EKF</i>	i. Highly robust and accurate ii. Can be altered based on satellite characteristics	i. Computational inefficient and difficult to model ii. Requires gyroscope iii. Requires at least two observer-reference pair

2.2 Nano-satellite Magnetometer Calibration Methods

As mentioned in Section 2.1, one of the common attitude sensor for satellite is TAM. However, TAM measurement is often affected by thermal variation, mechanical mis-alignment, and on-board electronics time-varying bias which lower the attitude knowledge accuracy. In addition, our analysis on the whole orbit data (WOD) of VELOX-II shows that the attitude knowledge accuracy falls significantly when the magnetic torquers are switched on as it disrupts the TAM reading.

Several studies [31-37] have been conducted to improve the measurement accuracy of TAM in-orbit. In [31], a survey has been conducted to study a number of existing attitude-independent in-orbit magnetometer bias determination methods. It was reported

that the most robust and accurate of all the algorithm is the TWOSTEP algorithm. In [32], the TWOSTEP algorithm is further extended to estimate the magnetometer soft iron scale factors and non-orthogonality factors. In [33, 34], a different approach of attitude-independent in-orbit TAM calibration method has been reported such that real time implementation can be realised. In [33], TAM calibration using EKF and UKF has been introduced. It was reported that UKF is more robust to large initial condition error with an expense of higher computational load. In [34], a TAM calibration method for spinning projectile by using sliding window approach has been presented. In [36, 37], a reconfigurable UKF which consists of two stages has been introduced whereas in [35] an iterative calibration technique based on multi-parameter algorithm is presented.

Nonetheless, these studies do not consider the coupling effect of magnetic torquers as the satellites might not equipped with magnetic torquers. For a conventional satellite, this problem could be solved by replacing the magnetic torquers with thrusters, utilizing a deployable magnetometer boom or incorporating other attitude sensors that are not affected by magnetic fields such as the star tracker. The space, mass, budget, and power constraint of a nano-satellite however inhibits such implementation.

A common solution to prevent the coupling effect of magnetic torquers is to turn off the torquers when measuring the magnetic field. However, the magnetic torquers might contain magnetic remanence. In [38, 39], TAM calibration which takes the coupling effect of magnetic torquers into consideration have been reported. In [38], TAM calibration is performed by using a least square method. However, calibration was only performed in the laboratory by using the Helmholtz cage. On the other hand, [39] calibrated the TAM in-orbit using PSO algorithm but it does not consider the time required to estimate the calibration constants. In this thesis, a robust and fast converging

attitude-independent in-orbit TAM calibration with the presence of magnetic torquer will be proposed in Chapter 4.

2.3 MEMS Gyroscope Noise Removal Methods

Due to the size, computational power and weight constraint of nano-satellite, a MEMS based attitude determination system such as the MEMS gyroscope is typically used at the expense of accuracy [40]. In general, MEMS gyroscope is noisy which affects the accuracy of attitude estimation. To reduce the effect of gyroscope noise, one of the widely implemented method is the virtual gyroscope technology [41]. The idea is to combine multiple identical low precision gyroscopes to form an array of measurements. Subsequently, the optimal gyroscope measurement is estimated by using an optimal filter. Ref. [42] fuses two gyroscope measurements with a Kalman filter (KF) which is able to achieve a significant improvement under static condition. In [43], multiple MEMS gyroscopes are used and the rate signal is modelled by using the first-order Markov process. The optimal rate is then obtained by KF. The experimental results indicated a reduction factor of about 3.4 in the gyroscope angular random walk. Although these methods could reduce the gyroscope noise, it is at the expense of cost, mass and power consumption.

Ref. [44] estimated the noise variance of multiple MEMS gyroscopes individually by Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to improve the gyroscope model. Although the GARCH model is able to reduce the noise level by 56%, the better gyroscope performance is compromised with a higher computational cost, which is impractical for nano-satellite application. By applying the Square Root Unscented Kalman filter (SRUKF) to the gyroscope measurement [45], the gyroscope model accuracy can be improved as compared to KF. However, this is at the

expense of a higher computational memory. Different from the previous reported approaches, a low complexity Kalman filter (LCKF) based approach to lower the MEMS gyroscope noise will be presented in Chapter 5.

2.4 Summary

In this chapter, a literature review of nano-satellites attitude determination methods, TAM calibration methods and MEMS gyroscope noise removal methods have been discussed. Nano-satellites attitude determination methods are first presented to provide an overall understanding of nano-satellite ADS. The advantages and disadvantages of all the existing attitude determination methods have been discussed. In Chapter 3, a modified EKF algorithm is presented such that it overcomes the disadvantages of existing attitude determination methods.

The measurement accuracy of TAM is affected by in-orbit variation as well as coupling effect of magnetic torquer. Among all the methods presented in literature review, TAM calibration which includes the torquer effect by using PSO has been reported to improve the TAM measurement accuracy even when magnetic torquer is turned on. However, the time requires to estimate the TAM calibration constants are not considered. In Chapter 4, a fast hybrid optimization method for TAM calibration which includes the coupling effect of magnetic torquer will be proposed.

MEMS gyroscope has a high output noise which affect the control and estimation performance of nano-satellite in-orbit. Numerous methods presented has been reported to reduce the MEMS gyroscope noise by a large margin. However, the discussed methods either requires the implementation of complex algorithm or multiple gyroscopes. In Chapter 5, a LCKF which lower the MEMS gyroscope noise will be presented.

CHAPTER 3

MODIFIED EKF FOR ATTITUDE DETERMINATION

3.1 Overview

As mentioned in Chapter 1, one of the main challenge in developing a precise ADS of a nano-satellite is the usage of MEMS based attitude sensor which has a lower accuracy. In addition, Chapter 2 explained that the existing EKF approach requires at least two observer-reference pairs to perform attitude determination. Typically, the two observer-reference pairs of a nano-satellite are the sun vector and the Earth magnetic field vector in the inertial frame and body frame respectively. This has caused a significant drop in attitude knowledge accuracy when the nano-satellite is performing momentum dumping as the magnetic torquers corrupt the magnetometer measurement. In this chapter, an EKF for attitude determination based on attitude sensors (sun sensor, magnetometer and gyroscope) equipped in a nano-satellite is developed. The EKF algorithm is modified such that it is able to work even with the presence of only one observer-reference pair. This is accomplished by propagating the satellite's attitude from the previous time step using gyroscope measurement and update the propagated attitude using the sole available observer-reference pair. This allows the satellite to perform the desired mission during eclipse whereby the sun vector is absence or when the satellite is undergoing momentum dumping by using magnetic torquers. Simulation study has been conducted to verify the performance of the proposed modified EKF. The simulation results show that the mean attitude knowledge error in the worst case is 0.62° and 0.13° in the best case with the presence of 2 observer-reference pairs. In addition,

the simulation results show that the proposed modified EKF can even work when utilizing only one observer-reference pair.

3.2 EKF Algorithm

In Section 2.1.3, it was mentioned that the spacecraft attitude determination is a non-linear dynamics system. The EKF algorithm for attitude determination is formulated by altering the standard Kalman filter state propagation equation

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A}\hat{\mathbf{x}}_k + \mathbf{w}_k \quad (3.1)$$

and the covariance propagation equation

$$\mathbf{P}_{k+1} = \mathbf{A}\mathbf{P}_k\mathbf{A}^T + \mathbf{Q} \quad (3.2)$$

where $\hat{\mathbf{x}}_k$ is the state of the system, $\mathbf{A}$ is the state transition matrix, $\mathbf{w}_k$ is the process noise, $\mathbf{P}_k$ is the state error covariance and $\mathbf{Q}$ is the process noise covariance.

The EKF for attitude determination [46] begins with the propagation of the initial attitude information by using the inertial measurement unit (IMU) gyroscope measurement via the quaternion kinematics model

$$\mathbf{q}_{k+1} = e^{0.5\Omega\Delta t}\mathbf{q}_k \quad (3.3)$$

where

$$e^{0.5\Omega\Delta t} = \begin{bmatrix} \cos(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_x}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_y}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_z}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) \\ \frac{\omega_x}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & \cos(0.5|\boldsymbol{\omega}|\Delta t) & \frac{\omega_z}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_y}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) \\ \frac{\omega_y}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_z}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & \cos(0.5|\boldsymbol{\omega}|\Delta t) & \frac{\omega_x}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) \\ \frac{\omega_z}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & \frac{\omega_y}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & -\frac{\omega_x}{|\boldsymbol{\omega}|}\sin(0.5|\boldsymbol{\omega}|\Delta t) & \cos(0.5|\boldsymbol{\omega}|\Delta t) \end{bmatrix}$$

and Δt is the time step, q_{k+1}^- is the propagated quaternion, $\boldsymbol{\omega} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$ is the gyroscope

reading in rad/s. Quaternion from the reference to the observer frame with the notation

$\mathbf{q} = \begin{bmatrix} \mathbf{q}_s \\ \mathbf{q}_v \end{bmatrix}$ is being used as the attitude representation in the EKF as it is computational

efficient and able to satisfy the orthogonality constraint of the quaternion.

Similarly, the error state covariance matrix, $\mathbf{P}_k$ is also propagated by using the IMU gyroscope measurement as

$$\dot{\mathbf{P}}_{k+1} = [-\boldsymbol{\omega} \times] \mathbf{P}_k + \mathbf{P}_k [-\boldsymbol{\omega} \times]^T + \sigma_v^2 \mathbf{I}_{3 \times 3} \quad (3.4)$$

where

$$[\boldsymbol{\omega} \times] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

$$\mathbf{I}_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\dot{\mathbf{P}}_{k+1}^-$ is the first derivative of the propagated error state covariance matrix, σ_v is the gyroscope angular random walk in $rad/\sqrt{s}$.

With the assumption that Δt is small enough such that the gyroscope reading remain unchanged for that specific period of time, the first derivative of the propagated error state covariance matrix $\dot{\mathbf{P}}_{k+1}^-$ is then solved using a 4th order Runge-Kutta [47] to yield

$$\mathbf{P}_{k+1} = \mathbf{P}_k + \Delta t \left(\frac{1}{6} \mathbf{k}_1 + \frac{1}{3} \mathbf{k}_2 + \frac{1}{3} \mathbf{k}_3 + \frac{1}{6} \mathbf{k}_4 \right) \quad (3.5)$$

where

$$\begin{aligned} \mathbf{k}_1 &= [-\boldsymbol{\omega} \times] \mathbf{P}_k + \mathbf{P}_k [-\boldsymbol{\omega} \times]^T + \sigma_v^2 \mathbf{I}_{3 \times 3} \\ \mathbf{k}_n &= [-\boldsymbol{\omega} \times] \left[\mathbf{P}_k + \frac{1}{2} (\Delta t) \mathbf{k}_{n-1} \right] + \left[\mathbf{P}_k + \frac{1}{2} (\Delta t) \mathbf{k}_{n-1} \right] [-\boldsymbol{\omega} \times]^T + \sigma_v^2 \mathbf{I}_{3 \times 3}, \text{ for } n=2,3,4 \end{aligned}$$

Next, the Kalman gain $\mathbf{K}_{k+1}$ is updated using the propagated error state covariance matrix $\mathbf{P}_{k+1}^-$ calculated from (3.5) as

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^- \mathbf{H}_{k+1}^T (\mathbf{H}_{k+1} \mathbf{P}_{k+1}^- \mathbf{H}_{k+1}^T + \mathbf{R})^{-1} \quad (3.6)$$

where $\mathbf{H}_{k+1}$ is formulated by

$$\mathbf{H}_{k+1} = \begin{bmatrix} (\mathbf{A} [\mathbf{q}_{k+1}^-] \mathbf{v}_{1\text{ref}}) \times \\ (\mathbf{A} [\mathbf{q}_{k+1}^-] \mathbf{v}_{2\text{ref}}) \times \end{bmatrix} \quad (3.7)$$

given that $\mathbf{A} [\mathbf{q}_{k+1}^-]$ is a 3x3 rotation matrix converted from the propagated quaternion

$\mathbf{q}_{k+1}^-$ in (3.3) as

$$\mathbf{A} [\mathbf{q}_{k+1}^-] = \begin{bmatrix} q_s \mathbf{I}_{3 \times 3} + [\mathbf{q}_v \times] \\ -\mathbf{q}_v^T \end{bmatrix}^T \begin{bmatrix} q_s \mathbf{I}_{3 \times 3} - [\mathbf{q}_v \times] \\ -\mathbf{q}_v^T \end{bmatrix} \quad (3.8)$$

where $\mathbf{v}_{1\text{ref}}$ and $\mathbf{v}_{2\text{ref}}$ are the sun vector and Earth magnetic field vector in the inertial frame obtained from the Sun Model and the International Geomagnetic Reference Field (IGRF) model [48] respectively and $\mathbf{R}$ is formulated by

$$\mathbf{R} = \begin{bmatrix} \sigma_1^2 \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \sigma_2^2 \mathbf{I}_{3 \times 3} \end{bmatrix} \quad (3.9)$$

where σ_1 and σ_2 are the standard deviations of sun sensor measurement error and the magnetometer measurement error respectively. Noise characteristic of each sensor has to be evaluated such that optimal attitude estimation can be achieved by utilizing the EKF algorithm.

The error quaternion vector, $\Delta \mathbf{x}_{k+1}^+$ is then corrected using $\mathbf{K}_{k+1}$ by

$$\Delta \mathbf{x}_{k+1}^+ = \mathbf{K}_{k+1} \left[\begin{bmatrix} \mathbf{v}_{1\text{obs}} \\ \mathbf{v}_{2\text{obs}} \end{bmatrix} - \begin{bmatrix} \mathbf{A}[\mathbf{q}_{k+1}^-] \mathbf{v}_{1\text{ref}} \\ \mathbf{A}[\mathbf{q}_{k+1}^-] \mathbf{v}_{2\text{ref}} \end{bmatrix} \right] \quad (3.10)$$

where $\mathbf{v}_{1\text{obs}}$ and $\mathbf{v}_{2\text{obs}}$ are the sun vector and Earth magnetic field vector in the body frame obtained from sun sensor and magnetometer. $\Delta \mathbf{x}_{k+1}^+$ is a 3×1 matrix which represent the vector part, $\mathbf{q}_v$ of the error quaternion. The error quaternion, $\mathbf{q}_{\text{error}}$ is therefore

$$\mathbf{q}_{\text{error}} = \begin{bmatrix} \mathbf{1} \\ \frac{\Delta \mathbf{x}_{k+1}^+}{2} \end{bmatrix} \quad (3.11)$$

The term error quaternion is being used here because $\mathbf{q}_{\text{error}}$ is the quaternion error between the propagated quaternion $\mathbf{q}_{k+1}^-$ and the actual quaternion output, $\mathbf{q}_{k+1}$ of the EKF algorithm.

The propagated quaternion $\mathbf{q}_{k+1}^-$ is then multiply with error quaternion vector $\mathbf{q}_{\text{error}}$ through quaternion multiplication $\otimes$ [49].

$$\mathbf{q}_{k+1} = \mathbf{q}_{k+1}^- \otimes \mathbf{q}_{\text{error}} \quad (3.12)$$

where $\otimes$ is denoted by

$$\mathbf{q}_1 \otimes \mathbf{q}_2 = \begin{bmatrix} q_{1s}q_{2s} - \mathbf{q}_{1v} \cdot \mathbf{q}_{2v} \\ q_{1s}\mathbf{q}_{2v} + q_{2s}\mathbf{q}_{1v} + \mathbf{q}_{1v} \times \mathbf{q}_{2v} \end{bmatrix}$$

Finally, the propagated error state covariance matrix $\mathbf{P}_{k+1}^-$ is updated using the following equation

$$\mathbf{P}_{k+1} = (\mathbf{I}_{3 \times 3} - \mathbf{K}_{k+1}\mathbf{H}_{k+1})\mathbf{P}_{k+1}^- \quad (3.13)$$

Equation (3.13) sums up the EKF algorithm for attitude estimation. Fig. 3.1 shows the flow chart of the EKF algorithm from (3.3) to (3.13).

Fig. 3.1 EKF flow chart

3.3 EKF Simulation Results

Simulation study has been conducted using MATLAB to verify the performance of the formulated EKF algorithm. For the simulation, the Two Line Element (TLE) of VELOX-II satellite which is orbiting in a near equatorial orbit is used as an illustrative example. The simulation runs for a whole orbit of 90 minutes. For the simulation, it is assumed that the satellite is operating in the normal mode with the satellite body rate lower than $\pm 5^\circ/\text{s}$ (i.e. the satellite has de-tumbled). In addition, performance noise, σ_v and measurement noise σ_1 and σ_2 from (3.4) and (3.9) is simulated based on the performance of the sun sensor and IMU (magnetometer and gyroscope) used in

VELOX-II attitude determination and control system (ADCS) flight hardware. Table 3.1 summarizes the values used in the simulation.

Table 3.1 IMU and sun sensor technical specifications

<i>Attributes</i>	<i>Value</i>	<i>Remarks</i>
σ_v	0.9 °/s	IMU gyroscope output noise
σ_1	0.8°	Sun sensor angular error
σ_2	18	IMU magnetometer signal to noise ratio

The performance of the EKF algorithm is first compared with QUEST algorithm with two different sampling times of 0.1s and 1s. The results are shown in Figs. 3.2 - 3.5.

Fig. 3.2 EKF quaternion error in x direction with 1s sampling time

Fig. 3.3 QUEST quaternion error in x direction with 1s sampling time

Fig. 3.4 EKF quaternion error in x direction with 0.1s sampling time

Fig. 3.5 QUEST quaternion error in x direction with 0.1s sampling time

The boundary curves in Fig. 3.2 and Fig. 3.4 represent the error bound of the EKF which is the 3σ error bound calculated from the diagonal element of the error state covariance matrix $\mathbf{P}_{k+1}$. From Fig. 3.2 and Fig. 3.3, it is observed that the quaternion error for both EKF algorithm and QUEST algorithm are similar with a sampling time of 1s. However, Fig. 3.4 shows that there is a significant reduction in the quaternion error for EKF with a 0.1s sampling time whereas the results in Fig. 3.5 show that the QUEST algorithm does not improve with a lower sampling time. This result is expected because the QUEST algorithm is a point-by-point attitude determination method which is time independent.

To further verify the effect of sampling time on EKF algorithm and QUEST algorithm, the simulation study is conducted with the sampling time varied from 0.1s to 2s with an incremental time step of 0.1s. The results are shown in Fig. 3.6.

Fig. 3.6 RMS error of EKF and QUEST at different sampling time

From Fig. 3.6, it is observed that EKF could lower the root mean square (RMS) error.

To study the effect of the attitude sensors noise characteristics on the attitude estimation performance of the EKF, multiple scenarios has been run with various factors that affect the attitude knowledge accuracy. The factors are sun sensor angular error σ_{sun} , magnetometer signal to noise ratio σ_{mag} and gyroscope noise σ_{gyro} . The factors are listed in Table 3.2 as shown

Table 3.2 Factors that affect the attitude knowledge accuracy

σ_{sun}	0.4°, 0.8°, and 1.2°
σ_{mag}	10, 12, 14, 16, 18 and 20
σ_{gyro}	0.1°/s, 0.27°/s, 0.5°/s, 0.7°/s, 0.9°/s and 1.1°/s

The simulations were conducted with a sampling time of 0.2s. Maximum attitude knowledge error occurred when σ_{sun} is 1.2°, σ_{mag} is 10 and σ_{gyro} is 1.1°/s which gives

a mean attitude error of 0.62° . The minimum attitude knowledge error occurred when σ_{sun} is 0.4° , σ_{mag} is 20 and σ_{gyro} is $0.1^\circ/\text{s}$ which gives a mean attitude error of 0.13° . The expected attitude error occurred when σ_{sun} is 0.8° , σ_{mag} is 18 and σ_{gyro} is $0.9^\circ/\text{s}$ which resemble the sun sensor and IMU equipped in VELOX-II and it gives a mean attitude error of 0.39° . The simulation results for worst and best case scenario are presented in Table 3.3.

Table 3.3 Extended simulation results of attitude knowledge

<i>Attributes</i>	<i>Worst Case</i>	<i>Best Case</i>	<i>Expected Case</i>
Maximum attitude knowledge error	1.49°	0.37°	0.82°
Mean attitude knowledge error	0.62°	0.13°	0.39°
σ_{sun}	1.2°	0.4°	0.8°
σ_{mag}	10	20	18
σ_{gyro}	$1.1^\circ/\text{s}$	$0.1^\circ/\text{s}$	$0.9^\circ/\text{s}$

Figs. 3.7 – 3.9 show the results for all the scenarios. From Figs. 3.7 - 3.9, it can be observed that the quaternion error increases when either the gyroscope noise level or the sun sensor angular error increases. In addition, the quaternion error also increases under the effect of a lower magnetometer signal to noise ratio. However, it can be observed that the gyroscope noise has the largest impact on the attitude knowledge accuracy.

Fig. 3.7 0.4 degree sun sensor error with 0.2s sampling time

Fig. 3.8 0.8 degree sun sensor error with 0.2s sampling time

Fig. 3.9 1.2 degree sun sensor error with 0.2s sampling time

3.4 Modified EKF Equation Derivation

As mentioned in Section 1.2, the main concern for the attitude determination of nano/pico-satellite is the loss of attitude knowledge during momentum dumping as the magnetic field generated by the magnetic torquer will disrupt the magnetometer reading. As such, the EKF presented in Section 3.2 is modified such that it works even with the presence of only one observer-reference pair with a lower attitude knowledge accuracy.

The modified EKF follows the same order from (3.3) to (3.13) in Section 3.2 with the modifications on (3.7), (3.9) and (3.10). Under the circumstances whereby the magnetic torquer is turned on, the magnetometer reading will be corrupted. Thus, there will be only 1 observer-reference vector pairs available which is the sun sensor measurement. As a result, the observation matrix $\mathbf{H}_{k+1}$ in (3.7) can only utilize the non-corrupted reference vector as follows

$$\mathbf{H}_{k+1} = \left[(\mathbf{A} [\mathbf{q}_{k+1}^-] \mathbf{v}_{1\text{ref}}) \times \right] \quad (3.14)$$

From (3.14), it can be observed that the order of the observation matrix $\mathbf{H}_{k+1}$ is reduced from 6x3 matrix to 3x3 matrix. Because only one observer-reference vector pair will be utilized, the measurement noise matrix $\mathbf{R}$ in (3.9) only accounts for the standard deviation of the observer-reference vector pairs that will be utilized. In this scenario, σ_1 is the standard deviation of sun sensor measurement error.

$$\mathbf{R} = \left[\sigma_1^2 \mathbf{I}_{3 \times 3} \right] \quad (3.15)$$

Similarly, the order of the measurement noise matrix $\mathbf{R}$ is also reduced from 6x6 matrix to 3x3 matrix. As both the matrix $\mathbf{H}_{k+1}$ and $\mathbf{R}$ has been reduced to 3x3 matrix, the calculated Kalman gain matrix $\mathbf{K}_{k+1}$ from (3.6) will be a 3x3 matrix.

$\Delta \mathbf{x}_{k+1}^+$ from (3.10) is then being corrected by only 1 pair of observer-reference vector pairs as follows

$$\Delta \mathbf{x}_{k+1}^+ = \mathbf{K}_{k+1} \left[\mathbf{v}_{1\text{obs}} - \mathbf{A} [\mathbf{q}_{k+1}^-] \mathbf{v}_{1\text{ref}} \right] \quad (3.16)$$

3.5 Modified EKF Simulation Results

MATLAB simulation study has been conducted for the modified EKF with the assumption that the magnetic torquer was turned on for 15 minutes under the sampling time of 0.1s, 0.2s and 0.5s. The results are shown in Figs. 3.10 – 3.12.

Fig. 3.10 Quaternion error in x direction with 0.5s sampling time and torquer on

Fig. 3.11 Quaternion error in x direction with 0.2s sampling time and torquer on

Fig. 3.12 Quaternion error in x direction with 0.1s sampling time and torquer on

The results from Figs. 3.10 – 3.12 suggest that the duration whereby the magnetic torquer is turned on plays a major role in the attitude knowledge error. This can be observed from the figures whereby the quaternion error increases significantly with time when only one pair of observer-reference vector is being utilized. Besides, the attitude knowledge error also depends on the sampling time with 0.1s having the lowest maximum quaternion error. In order to further verify the relationship between torquer turned on duration, sampling time and maximum attitude knowledge error, the simulation study is conducted at various time steps and turn on duration of torquer. The results are shown in Fig. 3.13.

Fig. 3.13 Maximum quaternion error at different sampling time and torquer turned on duration

Fig. 3.13 shows that the maximum quaternion error increases when either the sampling time or the torquer on duration increases. Moreover, it was observed in Fig. 3.13 that the torquer on duration has a greater impact on the maximum quaternion error. The results also show that even at a sampling time of 0.1s, the attitude knowledge error can go up to 15° which is not a satisfactory accuracy for satellite mission. Due to the constraint of the electronics runtime and the fact that the attitude knowledge error increases gradually with torquer on duration as shown in Fig. 3.13, the modified EKF is not the most ideal solution for the loss of attitude knowledge when the satellite is performing momentum dumping using the magnetic torquer. Therefore, an attitude-independent in-orbit TAM calibration approach which includes the coupling effect of magnetic torquer will be investigated. It is desire that accurate attitude knowledge can be retained even when the magnetic torquers are turned on. Detailed of the proposed approach will be discussed in Chapter 4.

3.6 Summary

In this chapter, a modified EKF algorithm for attitude determination has been presented. The proposed modified EKF algorithm can work even with the presence of only one observer-reference pair. Simulation results have shown that the expected mean attitude knowledge error with the presence of two observer-reference pair is 0.39° . From the study, it is noted that the gyroscope noise has the most impact on the accuracy of attitude knowledge error among the three attitude sensors.

Simulation by utilizing only a single observer-reference pair has also been conducted. Results have suggested that the system sampling time and the duration of utilizing a single observer-reference pair for attitude estimation affect the attitude knowledge accuracy. To overcome these limitations, a new solution to retain accurate attitude knowledge even when momentum dumping is being performed will be presented in Chapter 4.

CHAPTER 4

ATTITUDE-INDEPENDENT IN-ORBIT TAM CALIBRATION

4.1 Overview

As mentioned in Chapter 3, the two observer-reference pairs of a nano-satellite are the sun vector and the Earth magnetic field vector. However, analysis on the whole orbit data (WOD) of VELOX-II shows that the accuracy of magnetometer measurement falls significantly when momentum dumping is performed as shown in Fig. 4.1. From Fig. 4.1, a large discrepancy between the VELOX-II normalized magnetometer measurement and the normalized IGRF magnetic field can be observed. This is due to the disruption of the TAM reading by the external magnetic field generated by the magnetic torquers when it is switched on for momentum dumping purposes.

Fig. 4.1 Comparison of VELOX-II normalized magnetometer reading with respect to normalized

IGRF magnetic field

Simulation results in Chapter 3 suggested that the performance of modified EKF deteriorates with time when utilizing only a single observer-reference pair. This has caused an issue for near equatorial orbit satellite because it is necessary for momentum dumping to be performed throughout the non-eclipse period to prevent saturation of the satellite's reaction wheels. In this chapter, an alternative approach has been investigated such that the accuracy of attitude knowledge can be retained even when the magnetic torquers are turned on.

Besides corruption of TAM measurement from the magnetic torquers, TAM measurement is often affected by the thermal variation, mechanical mis-alignment, and on-board electronics time-varying bias which lower the attitude knowledge accuracy. In view of these problems, an attitude-independent in-orbit TAM calibration with the presence of magnetic torquer is proposed in this chapter. The proposed calibration method includes the effect of hard iron bias, mechanical mis-alignment, on-board electronics time-varying bias, and the coupling effect of magnetic torquers.

The proposed approach uses a hybrid optimization method comprises of particle swarm optimization (PSO) and iterated extended Kalman filter (IEKF). Experimental and simulation results show that accurate attitude knowledge can be the retained even when the magnetic torquers are switched on and it will be presented in Fig. 4.16. Moreover, the results also show that the TAM is accurately calibrated with a fast convergent rate despite large initialization error. With the proposed technique, the TAM calibration can be carried up in real time while the satellite is in orbit.

4.2 System Modelling of TAM

The TAM has to be precisely modelled such that all the factors that affect the TAM measurement in space is taken into account. In [50], a TAM model was presented which

includes the effect of hard iron bias, mechanical mis-alignment, and on-board electronics time-varying bias. The model can be written as follows:

$$\bar{B}_x = a_1 B_x + x_0 + v_x \quad (4.1)$$

$$\bar{B}_y = a_2 [B_y \cos(\rho) + B_x \sin(\rho)] + y_0 + v_y \quad (4.2)$$

$$\bar{B}_z = a_3 [B_x \sin(\lambda) + B_y \sin(\phi) \cos(\lambda) + B_z \cos(\phi) \cos(\lambda)] + z_0 + v_z \quad (4.3)$$

where B_x , B_y , and B_z are the true magnetic field of each axis, $\bar{B}_x$, $\bar{B}_y$, and $\bar{B}_z$ are the onboard measured magnetic field of each axis, v_x , v_y , and v_z are the magnetometer measurement output noise of each axis, a_1 , a_2 , and a_3 are the scaling factors of each axis, x_0 , y_0 , and z_0 are the three axes offset biases. The mechanical mis-alignment is expressed relative to the actual sensor axes with the sensor x-axis being the reference axis. Thus, ρ is the angle from actual y-axis in the x-y plane, ϕ is the angle from actual z-axis in the y-z plane, and λ is the angle from actual z-axis in the x-z plane. The mis-alignment angle ρ , ϕ and λ are shown in Fig. 4.2.

Fig. 4.2 Mechanical mis-alignment expressed relative to the actual magnetometer axes

In order to account for the presence of magnetic torquers, the coupling effect of magnetic torquers is added into the model in this study. The coupling effect is dependent on the distance between TAM and magnetic torquers as well as the magnetic field strength generated by the torquers. As the magnetic torquers are current carrying loop, the generated magnetic field strength B can be calculated by using the Biot-Savart law [51] as

$$B = \frac{\mu_0 I}{2\pi R} \quad (4.4)$$

where μ_0 is the permeability of free space, I is the current flows across the magnetic torquer, and R is the radius of the magnetic torquer. It is noticed that the magnetic field strength generated by the magnetic torquer decreases with distance. Given that the distance between TAM and magnetic torquer is fixed, the magnetic field B_{tqr} generated by the magnetic torquer at the location of TAM can be modified from (4.4) as

$$B_{tqr} = dI \quad (4.5)$$

where d is the torquer calibration constant taking into account of the TAM-magnetic torquer distance and $\frac{\mu_0}{2\pi R}$ in (4.4).

The TAM model which takes the coupling effect of the magnetic torquers is then modified from (4.1), (4.2) and (4.3) as

$$\bar{B}_x = a_1 B_x + x_0 + s_1 I_x + s_2 I_y + s_3 I_z + v_x \quad (4.6)$$

$$\bar{B}_y = a_2 [B_y \cos(\rho) + B_x \sin(\rho)] + y_0 + t_1 I_x + t_2 I_y + t_3 I_z + v_y \quad (4.7)$$

$$\begin{aligned} \bar{B}_z = a_3 [& B_x \sin(\lambda) + B_y \sin(\phi) \cos(\lambda) \\ & + B_z \cos(\phi) \cos(\lambda)] + z_0 + u_1 I_x + u_2 I_y + u_3 I_z + v_z \end{aligned} \quad (4.8)$$

where s_n , t_n , and u_n for $n=1,2,3$ are the torquer calibration constants of the magnetic field generated by the magnetic torquers in each axis as shown in (4.5) whereas I_x , I_y , and I_z are the currents flow across the magnetic torquers in each axis measured onboard. When the torquers are turned off, I_x , I_y , and I_z are equal to zero. Rewriting (4.6), (4.7), and (4.8) in terms of B_x , B_y , and B_z give

$$\begin{aligned} B_x &= \frac{1}{a_1} (\bar{B}_x - x_0 - s_1 I_x - s_2 I_y - s_3 I_z - v_x) \\ &= A_1 (\bar{B}_x - v_x) - X_0 - S_1 I_x - S_2 I_y - S_3 I_z \end{aligned} \quad (4.9)$$

$$\begin{aligned} B_y &= \frac{1}{a_2 \cos(\rho)} (\bar{B}_y - y_0 - t_1 I_x - t_2 I_y - t_3 I_z - v_y) - B_x \tan(\rho) \\ &= \frac{1}{\cos(\rho)} [A_2 (\bar{B}_y - v_y) - Y_0 - T_1 I_x - T_2 I_y - T_3 I_z] - B_x \tan(\rho) \end{aligned} \quad (4.10)$$

$$\begin{aligned} B_z &= \frac{\sec(\phi) \sec(\lambda)}{a_3} (\bar{B}_z - z_0 - u_1 I_x - u_2 I_y - u_3 I_z - v_z) - B_x \sec(\phi) \tan(\lambda) - B_y \tan(\phi) \\ &= \sec(\phi) \sec(\lambda) [A_3 (\bar{B}_z - v_z) - Z_0 - U_1 I_x - U_2 I_y - U_3 I_z] - B_x \sec(\phi) \tan(\lambda) - B_y \tan(\phi) \end{aligned} \quad (4.11)$$

where X_0 , Y_0 , Z_0 , ρ , ϕ , λ , A_n , S_n , T_n , U_n , for $n=1,2,3$ are the 18 TAM calibration constants that need to be estimated by minimizing the following cost function

$$J = \sum_{j=1}^m \{ [\|B_{IGRF,j}\|^2 - f(x) + \varepsilon]^2 \} \quad (4.12)$$

$$f(x) = B_{x,j}^2 + B_{y,j}^2 + B_{z,j}^2 \quad (4.13)$$

where $j=1,2,3,...,m$ given that m is the total number of onboard measurements,

$\|B_{IGRF,j}\|$ is the normalized Earth magnetic field in the Earth-centered, Earth-fixed

(ECEF) frame obtained from the 12th-order International Geomagnetic Reference Field (IGRF) model [48], $B_{x,j}$, $B_{y,j}$, and $B_{z,j}$ are the magnetic field components of each axis calculated from (4.9), (4.10), and (4.11). Assuming that ν_x , ν_y , and ν_z are statistically similar, they can be represent by ν . The effective measurement noise ε is then formulated as

$$\begin{aligned} \varepsilon = & -2\nu[A_1 B_{x,j} + \frac{A_2}{\cos(\rho)} B_{y,j} + A_3 \sec(\phi) \sec(\lambda) B_{z,j}] + \\ & \nu^2[A_1^2 + (\frac{A_2}{\cos(\rho)})^2 + (A_3 \sec(\phi) \sec(\lambda))^2] \end{aligned} \quad (4.14)$$

The effective measurement noise ε is assumed approximately Gaussian with a mean μ and a variance σ^2 given by

$$\mu = E\{\varepsilon\} \quad (4.15)$$

$$\sigma^2 = E\{\varepsilon^2\} - \mu^2 \quad (4.16)$$

4.3 Hybrid Optimization Algorithm

In Section 2.2, a literature review on attitude-independent in-orbit TAM calibration has been presented. Among them, only [38] and [39] have taken the coupling effect of magnetic torquers into consideration. In [38], the TAM calibration experiment was conducted on-ground by utilizing a Helmholtz cage and the effect of orbit variation on the TAM measurement accuracy was not considered. In [39], it utilized a PSO algorithm to perform in-orbit TAM calibration without considering the time required to estimate the calibration constants. In view of these shortcomings, a hybrid optimization method to calibrate the TAM in-orbit is proposed in this research to improve the accuracy and convergent speed.

The proposed hybrid optimization method is a fusion algorithm of the PSO algorithm and the IEKF. PSO is a stochastic global optimization method based on swarm intelligence [52]. Although PSO has proven to be effective in various applications including satellites [53-59], lacking the mathematical proof on the convergence and the speed of the convergence show that its theoretical foundation is weak [60]. Kalman filter and its variant have also been used in various satellite applications [12, 61-65]. IEKF linearizes the measurement function around the updated state estimate and performs well to process non-linear measurement [66]. However, both EKF and IEKF perform the first order approximation on the measurement function and therefore they are inadequate to capture the large initialization error [33]. By combining both PSO and IEKF, the proposed hybrid method takes advantages of both PSO and IEKF, allowing it to greatly improve the convergence speed and ability to solve for large initialization error.

The formulation starts with the coarse estimation of the TAM calibration constants using PSO. PSO first randomly assigned the position and velocity of all particles within a predefined range. In this study, the particles' position is defined to have 18 members, which are the TAM calibration constants derived from the model (4.9) - (4.11):

$$\mathbf{x}_i^k = [A_n \quad X_0 \quad Y_0 \quad Z_0 \quad S_n \quad T_n \quad U_n \quad \rho \quad \phi \quad \lambda]^T \quad (4.17)$$

where $n = 1, 2, 3$. At every iteration, each particle keeps track on the best solution it has achieved so far, known as *pbest* based on the evaluation from the cost function, (4.12). Besides, the particle with the optimal fitness, known as *gbest* obtained up to the current iteration is also being stored. Then, the position and velocity of each particle are updated by

$$\mathbf{v}_i^k = w\mathbf{v}_i^{k-1} + c_1 r_{i1}^{k-1} (\mathbf{p}_i^{k-1} - \mathbf{x}_i^{k-1}) + c_2 r_{i2}^{k-1} (\mathbf{p}_g^{k-1} - \mathbf{x}_i^{k-1}) \quad (4.18)$$

$$\mathbf{x}_i^k = \mathbf{x}_i^{k-1} + \mathbf{v}_i^k \quad (4.19)$$

where k is the index of iteration, N is the total number of particles $i = 1, 2, 3, \dots, N$, c_1 and c_2 are acceleration coefficients with $c_1 = c_2 = 2$, r_{i1}^{k-1} and r_{i2}^{k-1} are random numbers uniformly distributed between 0 and 1, $\mathbf{p}_i^{k-1}$ is the local best (*pbest*), $\mathbf{p}_g^{k-1}$ is the global best (*gbest*), $\mathbf{v}_i^k$ is the particles' velocity, $\mathbf{x}_i^k$ is the particles' position shown in (4.17) and w is the inertial weight factor, calculated by

$$w = w_{max} - (w_{max} - w_{min}) \frac{k}{k_{max}} \quad (4.20)$$

with $w_{max} = 0.9$, $w_{min} = 0.4$ and k_{max} being the maximum number of iteration [67]. Coarse estimation stops when the evaluation on the *gbest* value met the coarse estimation stopping criterion.

The *gbest* information obtained from the coarse estimation is further optimized by fine estimation using IEKF. IEKF first initializes the initial error state covariance matrix, $\mathbf{P}_j^1$. Since there is no process noise appears in the TAM model as shown in (4.6), (4.7), and (4.8), the propagation steps for the state vector and error state covariance matrix can be neglected [33]. The first iteration of the IEKF is then formulated as

$$\mathbf{H}_j^1 = \left. \frac{\delta f(x)}{\delta x} \right|_{\mathbf{x}=\mathbf{x}_j^1} \quad (4.21)$$

$$\mathbf{K}_j^1 = \mathbf{P}_j^1 \mathbf{H}_j^{1T} (\mathbf{H}_j^1 \mathbf{P}_j^1 \mathbf{H}_j^{1T} + \mathbf{R}^1)^{-1} \quad (4.22)$$

$$\mathbf{x}_{j+1}^1 = \mathbf{x}_j^1 + \mathbf{K}_j^1 (\|B_{IGRF,j}\|^2 - f(\mathbf{x}_j^1)) \quad (4.23)$$

$$\mathbf{P}_{j+1}^l = (\mathbf{I} - \mathbf{K}_j^l \mathbf{H}_j^l) \mathbf{P}_j^l \quad (4.24)$$

where $j = 1, 2, 3, \dots, m$ and m is the total number of onboard measurements, $f(x)$ is given in (4.13), $\mathbf{x}_j^l$ is the *gbest* obtained from the coarse estimation of PSO, $\|B_{IGRF,j}\|$ is given in (4.12), $\mathbf{I}$ is the identity matrix, and $\mathbf{R}$ is the measurement noise covariance matrix given by

$$\mathbf{R}^k = \sigma^2(\mathbf{x}_j^k) \quad (4.25)$$

which is calculated by using (4.14), (4.15), and (4.16). The first iteration ends when all the m sets of onboard measurements have been utilized to update the state. Further iteration is performed to improve the state's accuracy until a fine estimation stopping criterion is met based on the evaluation from the cost function, (4.12). The iteration is as follows:

$$\mathbf{H}_j^k = \left. \frac{\delta f(x)}{\delta x} \right|_{\mathbf{x}=\mathbf{x}_j^k} \quad (4.26)$$

$$\mathbf{K}_j^k = \mathbf{P}_j^l \mathbf{H}_j^{kT} (\mathbf{H}_j^k \mathbf{P}_j^l \mathbf{H}_j^{kT} + \mathbf{R}^k)^{-1} \quad (4.27)$$

$$\mathbf{x}_{j+1}^k = \mathbf{x}_j^k + \mathbf{K}_j^k (\|B_{IGRF,j}\|^2 - f(\mathbf{x}_j^k)) \quad (4.28)$$

where k is the index of iteration. Once the fine estimation stopping criterion is met, the iteration stops and the error state covariance matrix is updated as

$$\mathbf{P} = (\mathbf{I} - \mathbf{K}_j^k \mathbf{H}_j^k) \mathbf{P}_j^l \quad (4.29)$$

The proposed hybrid optimization algorithm is presented in the form of pseudocode as shown in Table 4.1. The algorithm begins with the initialization of IEKF and PSO parameters. Subsequently, coarse estimation of the TAM calibration constants is

performed by PSO. Once the TAM calibration constants meet the coarse estimation stopping criterion, the accuracy of the TAM calibration constants is further improved by fine estimation using IEKF until a fine estimation stopping criterion has been met.

Table 4.1 Pseudocode of the proposed method

<p>Step 1: Initialization.</p> <p>Load the onboard measured whole orbit data: $\bar{B}_{x,j}$, $\bar{B}_{y,j}$, $\bar{B}_{z,j}$, $I_{x,j}$, $I_{y,j}$, $I_{z,j}$, and $\ B_{IGRF,j}\$. ($j = 1, 2, 3, \dots, m$)</p> <p>Let $k = 1$.</p> <p>PSO initialization: For a population of N particles, randomly generate the particle's position, $\mathbf{X}_i^k$ as shown in (4.17) and velocities, $\mathbf{V}_i^k$. ($i = 1, 2, 3, \dots, N$)</p> <p>Let pbest equal to gbest, $\mathbf{p}_i^k = \mathbf{p}_g^k$. ($i = 1, 2, 3, \dots, N$)</p> <p>IEKF initialization: Initialize $\mathbf{P}_j^k$.</p> <p>Step 2: Coarse estimation using PSO.</p> <p>Update $pbest$ and $gbest$.</p> <p>For $i=0$; $i \leq N$; $i++$</p> <p> for $j=0$; $j \leq m$; $j++$</p> <p> Evaluate cost function, J_i^k using (4.9), (4.10), (4.11), (4.12), and (4.13).</p> <p> end</p> <p> if ($J_i^k < J(\mathbf{p}_i^k)$) $\mathbf{p}_i^k = \mathbf{x}_i^k$</p> <p> end</p> <p> $\mathbf{p}_g^k = \min(\mathbf{p}_i^k)$</p> <p>Step 3: Position and velocity update.</p> <p>Update the particles' position and velocities using (4.18), (4.19), and (4.20).</p>
--

Step 4: Check for coarse stopping criterion.

if ($J(\mathbf{p}_g^k) < \text{coarse stopping criterion}$)

$\mathbf{x}_j^k = \mathbf{p}_g^k$ and proceed to Step 5.

else

Update index of iteration, k and repeat Step 2 to Step 4.

Step 5: Fine estimation using IEKF (first iteration).

Let $k = 1$.

for $j=0; j \leq m; j++$

Update $\mathbf{R}^k$, $\mathbf{x}_j^k$, and $\mathbf{P}_j^k$ using (4.13), (4.21), (4.22), (4.23), (4.24), and (4.25).

end

for $j=0; j \leq m; j++$

Evaluate cost function, J^k using (4.9), (4.10), (4.11), (4.12), and (4.13).

end

$k++$

Step 6: Fine estimation using IEKF (second iteration and beyond).

while ($J^{k-1} > \text{fine stopping criterion}$)

for $j=0; j \leq m; j++$

Update $\mathbf{R}^k$ and $\mathbf{x}_j^k$ using (4.13), (4.25), (4.26), (4.27), and (4.28).

end

for $j=0; j \leq m; j++$

Evaluate cost function, J^k using (4.9), (4.10), (4.11), (4.12), and (4.13).

end

$k++$

end

Update $\mathbf{P}$ using (4.29).

4.4 Simulation Results

In this section, simulation results of the proposed method are presented. The simulation study has been conducted using MATLAB with a personal computer operating at 3.4 GHz. The spacecraft used in this simulation is based on VELOX-II.

VELOX-II is a near-equatorial orbit nano-satellite whereby the Earth magnetic field vector has a low variation. Unlike the polar orbit nano-satellite such as VELOX-I which has a high variation in the Earth magnetic field that allows the magnetic torquer to maintain a high momentum dumping efficiency [68], VELOX-II implemented the strategy of constant momentum dumping by turning on the magnetic torquer throughout the non-eclipse period to prevent the saturation of reaction wheels unless the pointing mission is performed. The orbital elements of VELOX-II used in the simulation are presented in Table 4.2.

Table 4.2 Orbital elements of VELOX-II

Orbital elements	Value
Inclination	14.9883°
Right ascension of the ascending node	15.9307°
Eccentricity	0.000874
Argument of perigee	345.3365°
Mean anomaly	14.6713°
Mean motion	15.07784195 rev/day
Epoch	16012.22580441

The simulation is executed for one complete orbit and the on-board measurement is sampled at 60 seconds based on the on-board data handling (OBDH) design of VELOX-II. The TAM measurement noise is assumed to be white and Gaussian with an output noise of 0.125 μ T based on the datasheet of ADIS16405, which is the IMU equipped in VELOX-II. In addition, VELOX-II is assumed to be de-tumbled with an angular rate less than 5 °/s.

The performance of the proposed method is benchmark with the standard PSO algorithm and IEKF algorithm in terms of the speed of convergence and the ability to

solve for large initialization error. As the convergence speed of PSO is dependent on the population size, simulation of the standard PSO algorithm with the population size varied from 50 particles to 800 particles with an incremental population size of 50 is first conducted to select the optimal population size, N for both the proposed method and the standard PSO algorithm. The results are shown in Fig. 4.3. The initial TAM calibration constants in (4.17) are randomly initialize within a predefined range as shown in Table 4.3. For each of the population size, 50 simulations have been repeated with k_{max} equals to 10000.

Table 4.3 Initialization range for TAM calibration constants

Calibration constants	Range
A_n ($n = 1, 2, 3$)	0.85 ~ 1.15
X_0, Y_0, Z_0	-60 ~ 60
S_n, T_n, U_n ($n = 1, 2, 3$)	-1 ~ 1
ρ, ϕ, λ	-1 ~ 1

Fig. 4.3 Best fitness versus iteration for $N=50$ to 800

From Fig. 4.3, it is observed that the PSO estimation is unable to converge for N of 50 and 100. For N equal or greater than 150, the PSO estimation converged to 0.125 μT . In the sequel, $N=300$ is selected for both the proposed and the standard PSO algorithm. Moreover, it is noticed that the PSO estimation converged in less than 4000 iterations. Thus k_{max} is selected to be 4000.

With N and k_{max} determined, simulation for both the proposed and the standard PSO algorithm was executed for 50 times. For a fair comparison, both methods used the same initialize constants for each run. Simultaneously, simulation for the IEKF algorithm was also conducted to demonstrate its inability to solve for large initialization error. The initial error state covariance matrix, $\mathbf{P}_j^1$ for both the proposed method and IEKF is obtained by calculating the variance of all the random initialized calibration constants for every run with respect to the truth value. The 3σ bound for A_n is 0.25, 100 for X_0 , Y_0 , and Z_0 , and 1.7 for ρ , ϕ , λ , S_n , T_n , U_n . The initial error state covariance matrix, $\mathbf{P}_j^1$ for both the proposed method and IEKF is thus estimated as follows

$$\mathbf{P}_j^1 = \begin{bmatrix} 0.008\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 12} \\ \mathbf{0}_{3 \times 3} & 1200\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 12} \\ \mathbf{0}_{12 \times 3} & \mathbf{0}_{12 \times 3} & 0.34\mathbf{I}_{12 \times 12} \end{bmatrix} \quad (4.30)$$

The simulation results of the IEKF algorithm are shown in Fig. 4.4.

Fig. 4.4 Best fitness versus time elapsed for 50 run using IEKF

From Fig. 4.4, it was observed that IEKF has a fast average convergence speed of 106 seconds but is not consistent in terms of its estimation performance due to large initialization error. For the proposed hybrid optimization method, the coarse stopping criterion was chosen such that fine estimation using IEKF is not affected by the large initialization error. Besides, the coarse stopping criterion has to be as large as possible to reduce the lengthy coarse estimation time by PSO. To select the most optimal coarse stopping criterion for the proposed method, a study has been conducted to analyze the impact of initialization error on final convergence value by using IEKF. Fig. 4.5 shows the results of the study.

Fig. 4.5 Final convergence value versus initialization error using IEKF

In Fig. 4.5, the dotted lines refer to the 4 μT initialization error and 0.125 μT convergence value. From Fig. 4.5, it is observed that for initialization error larger than 5 μT , the performance of IEKF is not consistent as only some scenarios are able to converge. For initialization error that is less than 4.5 μT , all the scenarios managed to converge within the simulated magnetometer output noise of 0.125 μT . Therefore, the coarse stopping criterion for the proposed method is selected to be 4 μT .

The results for the convergent trend for standard PSO algorithm and the proposed method are shown in Fig. 4.6 and Fig. 4.7 respectively. Figs. 4.8 – 4.11 show the comparison of convergence speed for the calibration constants from one of the random selected simulation run. The mean estimated calibration constants with 3σ bound for 50 run using the proposed method and the standard PSO algorithm are summarized in Table 4.4.

Fig. 4.6 Best fitness versus time elapsed for 50 run using PSO

Fig. 4.7 Best fitness versus time elapsed for 50 run using hybrid method

Fig. 4.8 Comparison of A_1 estimates versus time elapsed

Fig. 4.9 Comparison of X_0 estimates versus time elapsed

Fig. 4.10 Comparison of S_1 estimates versus time elapsed

Fig. 4.11 Comparison of ρ estimates versus time elapsed

Table 4.4 Calibration constants from simulation

Calibration constants	Truth	Estimated with 3σ	
		Hybrid	PSO
A_1	0.976	0.9733 ± 0.0004	0.9754 ± 0.0011
A_2	0.932	0.9326 ± 0.0004	0.9307 ± 0.0016
A_3	0.897	0.8954 ± 0.0005	0.8972 ± 0.0008
X_0 (μT)	18.47	18.3922 ± 0.0199	18.4577 ± 0.0454
Y_0 (μT)	-4.73	-4.7514 ± 0.0050	-4.7388 ± 0.0178
Z_0 (μT)	-22.16	-22.1199 ± 0.0153	-22.1913 ± 0.0380
S_1	0.36	0.3597 ± 0.0002	0.3599 ± 0.0007
S_2	0.0026	0.0031 ± 0.0003	0.0023 ± 0.0006
S_3	0.0014	0.0001 ± 0.0003	0.0009 ± 0.0006
T_1	0.0029	0.0017 ± 0.0003	0.0018 ± 0.0005
T_2	0.45	0.4496 ± 0.0005	0.4483 ± 0.0011
T_3	0.0007	-0.0007 ± 0.0002	-0.00001 ± 0.0007
U_1	0.00017	0.0002 ± 0.0001	0.0002 ± 0.0007
U_2	0.00024	0.0008 ± 0.0002	0.0005 ± 0.0006
U_3	0.19	0.1893 ± 0.0002	0.1897 ± 0.0007
ρ ($^\circ$)	0.132	0.1161 ± 0.0232	0.2123 ± 0.3921
ϕ ($^\circ$)	0.036	0.0773 ± 0.0194	0.0759 ± 0.4095
λ ($^\circ$)	0.089	0.1261 ± 0.0292	0.1308 ± 0.4478

From Fig. 4.6, Fig. 4.7, and Table 4.4, it can be observed that both the proposed method and the standard PSO algorithm are able to solve for large initialization error and estimate all the 18 calibration constants accurately. However, the proposed method is much faster with an average convergence speed of 102 seconds whereas the standard PSO algorithm has an average convergence speed of 2808 seconds. In addition, the proposed method also has a lower 3σ bound. Overall, the proposed method improves

the convergence speed by over 90% as compared to the standard PSO algorithm. This is because the estimation performance of PSO is much slower with each iteration requires 1.03 seconds for $N=300$ and one orbit data. On the other hand, each iteration for IEKF only requires 0.03 seconds. Besides, the optimization efficiency of PSO falls as the best fit value is approaching the final stopping criterion. In contrast, the proposed method switches to fine estimation using IEKF when the best fit value reaches 4 μT . This avoids the slow estimation performance of PSO.

In summary, IEKF requires the lowest computational time for each iteration but it is unable to solve for large initialization error whereas PSO requires the most computational time for each iteration but is robust in terms of solving large initialization error. The proposed method takes the advantages of both PSO and IEKF, allowing it to solve for large initialization error at a fast convergence speed.

4.5 Experimental Results

The proposed hybrid optimization method has been evaluated using the flight data of VELOX-II. The flight data was obtained at a sampling rate of 60 seconds throughout 5 orbits. During the non-eclipse period, VELOX-II performed in sun-tracking mode by using the reaction wheels and simultaneously momentum dumping is conducted by turning on the magnetic torquer. During the eclipse period, VELOX-II is in the momentum hold mode whereby the reaction wheels maintain its speed. Similarly, the experiment is conducted using MATLAB with a personal computer operating at 3.4 GHz. The value of N , k_{max} , coarse stopping criterion and initial $\mathbf{P}_j^l$ used in the experiment are the same as in Section 4.4. The experimental results are shown in Figs. 4.12 – 4.17. Figs. 4.12 – 4.15 show the comparison of the convergence speed between PSO and the proposed hybrid optimization method. The proposed method has a

convergence speed of 267 seconds whereas the PSO has a convergence speed of 8331 seconds.

Fig. 4.12 Comparison of VELOX-II A_1 estimates versus time elapsed

Fig. 4.13 Comparison of VELOX-II X_0 estimates versus time elapsed

Fig. 4.14 Comparison of VELOX-II S_I estimates versus time elapsed

Fig. 4.15 Comparison of VELOX-II ρ estimates versus time elapsed

Fig. 4.16 Comparison of VELOX-II in-orbit calibrated (hybrid method) and pre-launch calibrated magnetometer reading with respect to normalized IGRF magnetic field

Fig. 4.17 Comparison of VELOX-II in-orbit calibrated (hybrid method) and pre-launch calibrated normalized residual

Experimental results show that the TAM of VELOX-II is calibrated with a root-mean-square (RMS) error of $0.5678 \mu\text{T}$ by using the proposed method and a RMS error of $0.5548 \mu\text{T}$ by using PSO. This has validated the simulation results in Section 4.4 which suggested that both the proposed hybrid optimization method and PSO are able to solve for large initialization error with the proposed method having a faster convergence speed. The convergence speed of the hybrid method in the experiment is however slower than the average convergence speed in the simulation because the experiment has much more data to be processed. For $N=300$ and 5 orbit data, PSO requires 5.27 seconds for each iteration whereas fine estimation using IEKF only requires 0.13 seconds. Fig. 4.16 shows the comparison of VELOX-II in-orbit (hybrid method) and pre-launch calibrated normalized TAM measurements with respect to the normalized IGRF magnetic field. The pre-launch TAM calibration of VELOX-II was performed by using a 30 arc-second accuracy turn table and a Helmholtz cage. However, the VELOX-II pre-launch calibration is only conducted for the effect of hard iron bias. Fig. 4.17 shows the comparison of VELOX-II in-orbit (hybrid method) and pre-launch calibrated normalized residual. From Fig. 4.17, it can be observed that the normalized residual for the in-orbit calibrated TAM are bounded between $\pm 2 \mu\text{T}$. The pre-launch calibrated and in-orbit calibrated TAM calibration constants of VELOX-II by using the proposed hybrid method and PSO are shown in Table 4.5.

Table 4.5 VELOX-II TAM calibration constants

Calibration constants	Value		
	Hybrid	PSO	Pre-launch calibrated
A_1	0.9984	1.0042	-
A_2	1.0346	1.0256	-
A_3	1.0255	1.0239	-
X_0 (μ T)	-6.2909	-6.3621	-5.9
Y_0 (μ T)	26.6289	26.4074	30.8
Z_0 (μ T)	-18.8309	-18.9293	-19.95
S_1	0.0024	0.0021	-
S_2	-0.0054	-0.0053	-
S_3	0.0406	0.0400	-
T_1	0.0003	-0.0003	-
T_2	-0.0406	-0.0403	-
T_3	-0.0118	-0.0122	-
U_1	0.0590	0.0586	-
U_2	-0.0089	-0.0089	-
U_3	0.0162	0.0169	-
ρ ($^\circ$)	-0.7383	-0.7404	-
ϕ ($^\circ$)	-0.6042	-0.5922	-
λ ($^\circ$)	1.4010	1.3712	-

From Table 4.5, both the in-orbit and pre-launch calibrated value of X_0 , Y_0 , and Z_0 are similar, which further verified the calibration performance of TAM using the proposed method. The improvement in convergence speed is however at the expense of an extra 6584 byte of code size when implemented using a C8051 microcontroller that was used in the ADCS of VELOX-II.

4.6 Summary

In this chapter, a fast converging attitude-independent in-orbit TAM calibration method has been presented. The proposed calibration method uses a hybrid optimization of PSO and IEKF. It includes the effect of hard iron bias, mechanical mis-alignment, on-board electronics time-varying bias, and the coupling effect of magnetic torquers. Simulation results have shown that both the proposed method and the standard PSO algorithm are able to solve for large initialization error and accurately estimate the TAM calibration constants. However, the proposed method converged faster than the standard PSO algorithm by 2706 seconds and has a lower 3σ bound. TAM calibration using the proposed method was also carried out using the VELOX-II flight data. Results show that the calibration is accurately conducted with a RMS error of $0.5678 \mu\text{T}$ and a convergence speed of 267 seconds. The improvement is achieved at the expense of larger code size and a more complicated algorithm. With the proposed attitude-independent in-orbit TAM calibration method, the problem of TAM reading disruption when performing momentum dumping using magnetic torquers can be resolved and accurate attitude knowledge can be retained.

CHAPTER 5

A LOW COMPLEXITY KALMAN FILTER FOR IMPROVING MEMS BASED GYROSCOPE PERFORMANCE

5.1 Overview

As mentioned in Section 2.3, the size and weight constraint of nano-satellite has resulted in the use of microelectromechanical systems (MEMS) based attitude determination system at the expense of accuracy. In addition, the capabilities of nano-satellite are also limited by lower computational power as compared to the traditional satellites. For VELOX series nano-satellite built in NTU, the sun tracking algorithm is achieved based on an observer free quaternion error correction method. This method does not require satellite attitude and position information. However, its performance is highly susceptible to gyroscope's noise.

To overcome this limitation, a low complexity Kalman filter (LCKF) based approach will be presented in this chapter. This approach utilizes both the angular rate and angular acceleration for gyroscope drift reduction. Low complexity has been achieved by expressing the state transition matrix and the observation matrix into sparse matrices with non-zero diagonal elements. By taking advantages of the sparse matrices, matrix decomposition can be used to reduce the number of arithmetic operations and iterations. Furthermore, the sparse matrices formulation leads to inverse of diagonal matrix which further simplifies the matrix calculations. Its performance has been evaluated experimentally using the VELOX's ADCS flight hardware. The result shows a 40.81% reduction in computational time.

5.2 Standard Kalman Filter Formulation

In order to use the Kalman filter (KF) for improving the gyroscope readings, the formulation [69] requires the knowledge of the MEMS based gyroscope system. MEMS gyroscopes use the Coriolis acceleration effect on a vibrating mass to detect the angular rotation. The gyroscope measures the angular rate which is linear to the rate of rotation. It responds quickly and accurately and the rotation can be computed by time-integrating the gyroscope output [70]. In general, the MEMS gyroscope can be modelled as

$$\boldsymbol{\omega}_{k+1} = \boldsymbol{\omega}_k + \dot{\boldsymbol{\omega}}_k \Delta t + \boldsymbol{\sigma}_{\text{arw}} \quad (5.1)$$

$$\dot{\boldsymbol{\omega}}_{k+1} = \dot{\boldsymbol{\omega}}_k + \dot{\boldsymbol{\sigma}}_{\text{arw}} \quad (5.2)$$

where $\boldsymbol{\omega}_k$ is the angular rate, $\dot{\boldsymbol{\omega}}_k$ is the angular acceleration, $\boldsymbol{\sigma}_{\text{arw}}$ is the gyroscope angular random walk, $\dot{\boldsymbol{\sigma}}_{\text{arw}}$ is the first order of gyroscope angular random walk and Δt is the sampling time. Let $\hat{\mathbf{x}}_k = \begin{bmatrix} \boldsymbol{\omega}_k^T & \dot{\boldsymbol{\omega}}_k^T \end{bmatrix}^T$ be the state of the system, then (5.1) and (5.2) can be expressed in a state space model as

$$\hat{\mathbf{x}}_{k+1} = \mathbf{A} \hat{\mathbf{x}}_k + \mathbf{w}_k \quad (5.3)$$

where the state transition matrix $\mathbf{A}$ is

$$\mathbf{A} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} \Delta t \\ \mathbf{0}_{3 \times 3} & \mathbf{I}_{3 \times 3} \end{bmatrix} \quad (5.4)$$

and the process noise $\mathbf{w}_k$ is

$$\mathbf{w}_k = \begin{bmatrix} \boldsymbol{\sigma}_{\text{arw}} \\ \dot{\boldsymbol{\sigma}}_{\text{arw}} \end{bmatrix} \quad (5.5)$$

In (5.4), $\mathbf{I}_{3 \times 3}$ is the 3x3 identity matrix whereas $\mathbf{0}_{3 \times 3}$ is the 3x3 zero matrix.

The covariance propagation for the linear system in (5.3) is

$$\mathbf{P}_{k+1}^- = \mathbf{A}\mathbf{P}_k\mathbf{A}^T + \mathbf{Q} \quad (5.6)$$

where $\mathbf{P}_k$ is the state error covariance and $\mathbf{Q}$ is the process noise covariance

$$\mathbf{Q} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{q}_2 \end{bmatrix} \quad (5.7)$$

where $\mathbf{q}_1 = \mathbf{I}_{3 \times 3} \boldsymbol{\sigma}_{\text{arw}} \boldsymbol{\sigma}_{\text{arw}}^T$ and $\mathbf{q}_2 = \mathbf{I}_{3 \times 3} \dot{\boldsymbol{\sigma}}_{\text{arw}} \dot{\boldsymbol{\sigma}}_{\text{arw}}^T$.

The Kalman gain $\mathbf{K}$ is then calculated by

$$\mathbf{K} = \mathbf{P}_{k+1}^- \mathbf{H}^T (\mathbf{H} \mathbf{P}_{k+1}^- \mathbf{H}^T + \mathbf{R})^{-1} \quad (5.8)$$

where $\mathbf{H} = [\mathbf{I}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}]$ is the observation matrix, $\mathbf{R}$ is the measurement noise covariance

with $\mathbf{R} = E\{\mathbf{v}_k \mathbf{v}_k^T\}$ and $\mathbf{v}_k$ is the MEMS gyroscope output noise.

Let $\mathbf{z}_{k+1} = \boldsymbol{\omega}_k + \mathbf{v}_k$ be defined as the gyroscope measurement. Then the predicted

state $\hat{\mathbf{x}}_{k+1}^-$ is corrected by the gyroscope measurement and Kalman gain $\mathbf{K}$ by

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1}^- + \mathbf{K}(\mathbf{z}_{k+1} - \mathbf{H}\hat{\mathbf{x}}_{k+1}^-) \quad (5.9)$$

The propagated state error covariance, $\mathbf{P}_{k+1}^-$ is updated as follows

$$\mathbf{P}_{k+1} = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}_{k+1}^- \quad (5.10)$$

Equation (5.3) to (5.10) form the standard KF formulation for the noise removal of MEMS based gyroscope.

5.3 Low Complexity Kalman Filter

Matrix calculations of the standard KF formulation mentioned in Section 5.2 require high amount of computational load as the state $\hat{\mathbf{x}}_k$ is a 6×1 vector. Equation (5.3) required a multiplication of a 6×6 matrix $\mathbf{A}$ and a 6×1 vector $\hat{\mathbf{x}}_k$. However, $\mathbf{A}$ is a sparse matrix with the value 1 in all of its diagonal elements. Thus, expanding (5.3) yields

$$\hat{\mathbf{x}}_{k+1}^- = \begin{bmatrix} \mathbf{w}_k + \dot{\mathbf{w}}_k \Delta t \\ \dot{\mathbf{w}}_k \end{bmatrix} \quad (5.11)$$

The matrix multiplication $\mathbf{A}\mathbf{P}_k\mathbf{A}^T$ in (5.6) can be simplified with matrix decomposition due to the sparse matrix property of $\mathbf{A}$. After simplification, (5.6) is obtained as

$$\mathbf{P}_{k+1}^- = \begin{bmatrix} D(\mathbf{P}_{k,1}) + D(\mathbf{P}_{k,4})\Delta t^2 + \mathbf{q}_1 & D(\mathbf{P}_{k,4})\Delta t \\ D(\mathbf{P}_{k,4})\Delta t & D(\mathbf{P}_{k,4}) + \mathbf{q}_2 \end{bmatrix} \quad (5.12)$$

where $\mathbf{P}_k = \begin{bmatrix} \mathbf{P}_{k,1} & \mathbf{P}_{k,2} \\ \mathbf{P}_{k,3} & \mathbf{P}_{k,4} \end{bmatrix}$ and $D(\mathbf{X}) = \text{diag}(\mathbf{X})$ which returns a diagonal matrix with diagonal elements of the matrix $\mathbf{X}$.

It can be observed that the outcome $\mathbf{P}_{k+1}^-$ of (5.12) is always a sparse matrix. Therefore, the complexity of (5.8) can be greatly reduced. As both the matrix $\mathbf{P}_{k+1}^-$ and $\mathbf{H}$ are sparse matrices with non-zero diagonal elements, the matrix multiplication $\mathbf{H}\mathbf{P}_{k+1}^-\mathbf{H}^T$ in (5.8) produces a diagonal matrix. The addition of the resultant diagonal matrix with diagonal matrix $\mathbf{R}$ leads to the inverse of a 3×3 diagonal matrix. The inverse of a 3×3 diagonal matrix is much computationally efficient than the inverse of a 3×3 matrix as the former is just the reciprocal of each of the diagonal elements of the

diagonal matrix. The multiplication of the matrix $\mathbf{H}^T$ with the inverse of the diagonal matrix can be further simplified and (5.8) can be reduced to

$$\mathbf{K} = \mathbf{P}_{k+1}^* \begin{bmatrix} \frac{1}{D(\mathbf{P}_{k+1,1}^*) + \mathbf{R}} \\ \mathbf{0}_{3 \times 3} \end{bmatrix} \quad (5.13)$$

The multiplication of $\mathbf{H}\hat{\mathbf{x}}_{k+1}^*$ in (5.9) and multiplication of $\mathbf{KH}$ in (5.10) can be expanded to further reduce the computational load. Thus, (5.9) and (5.10) can be rewritten as:

$$\hat{\mathbf{x}}_{k+1} = \hat{\mathbf{x}}_{k+1}^* + \mathbf{K}(\mathbf{z}_{k+1} - \mathbf{w}_{k+1}^*) \quad (5.14)$$

$$\mathbf{P}_{k+1} = (\mathbf{I} - [\mathbf{K} \quad \mathbf{0}_{6 \times 3}])\mathbf{P}_{k+1}^* \quad (5.15)$$

Equation (5.15) completes the LCKF formulation. By comparing the equations of the standard KF and the LCKF, it clearly shows that the LCKF has significantly lower number of multiplication and division operations. The performance comparisons of the LCKF and the standard KF are further verified in the next section.

5.4 Computational Load Comparisons

The matrix computation complexity method in [71] was used to compare the computational load between the standard KF and the LCKF. Let the variable n denotes the total number of estimated states and m denotes the total number of measurements. In this study, $n = 6$ and $m = 3$. The multiplications required by each equation in the standard KF and LCKF are compared in Table 5.1.

Table 5.1 Computational load of KF and LCKF

Equations		Number of multiplications required	
Standard KF	LCKF	Standard KF	LCKF
(5.3)	(5.11)	n^2	$n/2$
(5.6)	(5.12)	$2n^3$	$2n$
(5.8)	(5.13)	$2n^2m+2nm^2+m^3+m^2$	n^2m+m
(5.9)	(5.14)	$2nm$	nm
(5.10)	(5.15)	n^2m+n^3	n^3

Table 5.1 shows a significant reduction in the multiplications required in all stages of the Kalman process loop using the proposed approach. The total number of multiplications required for standard KF is 1188 whereas for LCKF is 360. Equations (5.12) and (5.13) of LCKF have the most decrement in the multiplication load. Matrix decomposition of the spare matrices in (5.12) reduces the number of multiplications from 432 multiplications to 12 multiplications. In (5.13), the matrix decomposition along with the inverse of the diagonal matrix reduces the number of multiplications from 360 multiplications to 111 multiplications.

By using MATLAB running on a 3.4 GHz personal computer, the average time of each Kalman process loop from (5.3) to (5.10) and from (5.11) to (5.15) required by both the standard KF and the LCKF when operating at a sampling time of 0.2s was recorded. The standard KF requires 0.049034ms on the average while the LCKF only requires 0.029023ms on the average. The result indicates a 40.81% reduction in computational time required by the LCKF.

5.5 Experimental Setup

To test the noise removal performance of the LCKF algorithm on the MEMS gyroscope as well as the observer free sun tracking performance with the

implementation of LCKF, an experimental setup which consists of a two-axis turn table and a sun simulator is used. The experimental setup is shown in Fig. 5.1.

Fig. 5.1 ADS test bench

5.5.1 Two-axis turn table

The two-axis turn table is a commercial product from Ideal Aerosmith. It allows the rotation in both inner axis and outer axis with an accuracy of 30 arc-second. Besides, the two-axis turn table includes additional features such as configurable rate of rotation, allowing it to be used as a rate table system. It can handle payloads up to 18.1 kg. The two-axis turn table is used to calibrate the MEMS gyroscope offset bias, the in-house built sun sensor as well as the TAM.

To prevent the inducing of offset bias in the MEMS gyroscope reading when conducting the LCKF noise removal experiment, the MEMS gyroscope equipped in the spacecraft simulator which will be presented in Section 5.6.2 has to be calibrated and verified by using the two-axis turn table. The MEMS gyroscope is initially kept

stationary and the three-axis gyroscope output reading is being logged. Fig. 5.2 shows the non-calibrated MEMS gyroscope experimental reading.

Fig. 5.2 Non-calibrated MEMS gyroscope reading

The ideal gyroscope output reading when it is being kept stationary should be 0 °/s. However, Fig. 5.2 shows that the three-axis gyroscope output is fluctuating with an offset bias. By taking the average of the gyroscope raw reading for the entire duration, the constant offset bias can be obtained and is shown in Table 5.2.

Table 5.2 MEMS gyroscope offset bias

Constant offset bias	Value (°/s)
x	-0.00299
y	0.003635
z	-0.02078

To verify if the MEMS gyroscope was calibrated properly, the MEMS gyroscope verification had been conducted in all three axis. The two-axis turn table was set to rotate

along its inner axis at either 5 °/s or -5 °/s, depending on the mounting configuration of the spacecraft simulator. The MEMS gyroscope verification results are shown in Figs. 5.3 – 5.5.

Fig. 5.3 Calibrated MEMS gyroscope reading with x-axis rotates at -5°/s

Fig. 5.4 Calibrated MEMS gyroscope reading with y-axis rotates at 5°/s

Fig. 5.5 Calibrated MEMS gyroscope reading with z-axis rotates at $-5^{\circ}/s$

From Fig. 5.3, it can be observed that when the x-axis of the MEMS gyroscope is rotated along the turn table at $-5^{\circ}/s$, the x-axis MEMS gyroscope reading fluctuates about $-5^{\circ}/s$ whereas y-axis and z-axis gyroscope reading fluctuates about $0^{\circ}/s$. The results are similar for y-axis rotation and z-axis rotation shown in Fig. 5.4 and Fig. 5.5 respectively. Thus it can be concluded that the gyroscope constant offset bias has been calibrated properly.

5.5.2 Sun simulator

The Sun simulator model S13-575WC from Optical Energy Technologies Inc is used in the experimental setup. It has a 575W metal halide arc lamp which produces a light source equal to one solar constant. The lamp is placed at the focus of paraboloid mirror which allows it to produce a collimated light beam. The sun simulator is being used to calibrate and verified our in-house built sun sensors as well as to verify the

VELOX series nano-satellite observer free sun tracking algorithm which will be presented in Section 5.6.2.

To conduct the observer free sun tracking experiment by using the spacecraft simulator, the 60° and 120° FOV sun sensors equipped in the spacecraft simulator has to be calibrated. First of all, the two-axis turn-table is instructed to rotate in both inner axis and outer axis from -30° to 30° with an incremental step size of 5° for the 60° FOV sun sensors. For the 120° FOV sun sensors, the inner axis is instructed to rotate from -60° to 60° whereas the outer axis is instructed to rotate from -45° to 45° both with an incremental step size of 5°. Simultaneously, the photodiode reading is being logged at each permutation of inner axis and outer axis angle. Once all the reading has been logged, the calibration constants is calculated based on the optimization of the least square algorithm.

With the calculated calibration constants, the verification of the sun sensors performance is conducted. Similarly, the two-axis turn-table is instructed to rotate in both inner axis and outer axis as describe in the calibration process. At the same time, the sun vector reading is being logged and compared with the true sun vector at each permutation of inner axis and outer axis angle. The verification results is shown in Table 5.3, Fig. 5.6 and Fig. 5.7.

Table 5.3 Sun sensors verification results

	60FOV	120FOV
RMS (°)	0.3635	0.8188
Mean (°)	0.2860	0.6867
Std (°)	0.2245	0.4477

Fig. 5.6 60° FOV sun sensor verification

Fig. 5.7 120° FOV sun sensor verification

From the sun sensor verification results, it can be observed that the 60° FOV sun sensor has a RMS error of about 0.4° whereas the 120° FOV sun sensor has a RMS error about 0.8°. Besides, it can also be observed that high error occurred at the edge which is not within the sun sensor FOV. With all the attitude sensors calibrated and verified, the noise removal performance of the LCKF algorithm on the MEMS gyroscope can be evaluated.

5.6 Experiment Results

5.6.1 Kalman filter experiment result

The experiment was conducted to compare the MEMS gyroscope performance with and without the implementation of the proposed LCKF by using the MEMS gyroscope equipped in the spacecraft simulator. The tuning of the KF process noise and measurement noise were based on the technical datasheet of the ADIS16405 MEMS gyroscope from Analog Devices. Table 5.4 presents the sampling time, tuning parameters and initialization parameters for the system.

Table 5.4 Kalman filter parameters

Parameters	Value
Δt	5 Hz
θ_0	0 rad/s
$\dot{\theta}_0$	0 rad/s ²
q_1	0.00026 rad/s ²
q_2	0.0065 rad/s ⁴
$P_{0,1}$	0.0003 rad/s ²
$P_{0,2}$	0 rad/s ³
$P_{0,3}$	0 rad/s ³
$P_{0,4}$	0.0003 rad/s ⁴
R	0.00314 rad/s ²

The experiment was first conducted in static condition. The filtered and non-filtered gyroscope angular rate of all 3-axis were logged and compared. The results are shown in Figs. 5.8 – 5.10.

Fig. 5.8 Comparison of x-axis filtered and non-filtered angular rate at static condition

Fig. 5.9 Comparison of y-axis filtered and non-filtered angular rate at static condition

Fig. 5.10 Comparison of z-axis filtered and non-filtered angular rate at static condition

From the experimental results, it is observed that the filtered angular rate has a lower output noise in all axes with the non-filtered angular rate and the filtered angular

rate having a Root Mean Square (RMS) error of 0.32 °/s and 0.23 °/s respectively. To further verify the performance of LCKF, the experiment is repeated in dynamic condition by rotating the z-axis of the MEMS gyroscope from 1 °/s to 10 °/s. The RMS error of filtered and non-filtered gyroscope angular rate was calculated and the results are shown in Figs. 5.11 – 5.13.

Fig. 5.11 Comparison of x-axis filtered and non-filtered angular rate at dynamic condition

Fig. 5.12 Comparison of y-axis filtered and non-filtered angular rate at dynamic condition

Fig. 5.13 Comparison of z-axis filtered and non-filtered angular rate at dynamic condition

Figs. 5.11 – 5.13 show that the performance between the LCKF and the standard KF is similar. Moreover, the filtered gyroscope angular rate in all axes has lower RMS

error at any given rotation rate. Thus, it can be concluded that the LCKF is able to reduce the gyroscope noise in both static and dynamic conditions.

5.6.2 Observer free sun tracking simulation and experiment

VELOX series nano-satellite uses an observer free sun tracking algorithm based on error quaternion. This allows it to perform sun tracking in the absence of its attitude knowledge. As the gyroscope reading is one of the states in the attitude control algorithm, any additional noise in the state would degrade the overall pointing performance. For the attitude control system, the Model Predictive Control (MPC) approach is used. MPC is a model based optimization approach that has been used in various applications such as power source [72], precision stage [73] and satellite [74]. The general linearized control law for the observer free sun tracking algorithm is given as [74],

$$\mathbf{T}_{RW} = \mathbf{G} \begin{bmatrix} \boldsymbol{\varepsilon}_e^T & \boldsymbol{\omega}^T \end{bmatrix}^T \quad (5.16)$$

where $\mathbf{T}_{RW}$ is the torque applied by the reaction wheels, $\boldsymbol{\varepsilon}_e$ is the vector component of error quaternion and $\boldsymbol{\omega}$ is the body rate of the satellite. $\mathbf{G}$ is the control gain matrix that is obtained using Model Predictive Control (MPC) strategy by tuning its prediction horizon.

To study the effect of gyroscope noise on the performance of the observer free sun tracking algorithm, MATLAB simulation has been conducted by varying the gyroscope output noise from 0.1 °/s to 1 °/s. The RMS pointing error is calculated and the results are shown in Fig. 5.14.

Fig. 5.14 Sun tracking simulation at different gyroscope noise

The simulation results show that the pointing performance is best at low gyroscope noise and degrades as the gyroscope noise increases. From the comparison of filtered and non-filtered angular rate at dynamic conditions as shown in Figs. 5.11 – 5.13, the results show that the LCKF is able to lower the overall gyroscope noise. Thus it is expected that a lower RMS pointing error could be obtained thru the integration of LCKF into the observer free sun tracking algorithm.

Subsequently, a sun tracking experiment has been carried out using the in-house build spacecraft simulator [13] to validate the simulation results. The spacecraft simulator utilized the VELOX's ADCS flight hardware. The Attitude Control System (ACS) incorporates three reaction wheels and three magnetic torquers to provide 3-axis stabilization and pointing for the satellite. Fig. 5.15 shows the in-house build spacecraft simulator and Fig. 5.16 shows the experimental setup.

Fig. 5.15 Spacecraft simulator

Fig. 5.16 Sun tracking experiment setup

The experiment was conducted by allowing the spacecraft simulator to perform sun tracking. The sun tracking error is logged for both filtered angular rate using LCKF and non-filtered scenarios when the spacecraft simulator reached its sun tracking steady state. The sun tracking error is computed based on the cosine angle (or dot product) between the expected sun sensor reading and the measured sun sensor reading. The

expected sun sensor reading is dependent on the satellite operation. For example, if the +x face of the satellite is commanded to face the sun, the expected sun sensor reading will be $[1; 0; 0]$. The results are shown in Fig. 5.17.

Fig. 5.17 Sun tracking error at steady state

The experimental results show that the steady state sun tracking error significantly reduces with the implementation of the LCKF. The RMS steady state tracking error without LCKF is 1.6717° and reduces to 0.6217° with the implementation of LCKF

5.7 Summary

In this chapter, a LCKF for MEMS gyroscope has been presented. Its computational load as compared with the standard KF shows a 40.81% reduction in computational time. Experiment using VELOX's ADCS flight hardware has also been conducted under both static and dynamic conditions. The results indicate that the LCKF is able to reduce the gyroscope noise under both static and dynamic condition in all 3-axis. Simulation and experimental study has been conducted to determine the effect of the gyroscope

noise on the performance of the observer free sun tracking algorithm. With LCKF, the steady state sun tracking error has been reduced from 1.6717° to 0.6217° .

CHAPTER 6

CONCLUSION AND FUTURE WORKS

6.1 Conclusion

In this thesis, a literature review has been conducted on the existing attitude determination methods. It was found that all the existing attitude determination algorithms require at least two observer-reference pairs to estimate the attitude of the satellite. This leads to poor attitude determination of a nano-satellite when it is performing momentum dumping using the magnetic torquers as the magnetometer measurement is corrupted by the torquers.

To overcome this problem, a modified EKF algorithm has been derived in order for it to work with either one or two observer-reference pairs. Simulation results show that the mean attitude knowledge error is 0.62° and 0.13° in the worst and best case respectively with the presence of two observer-reference pairs. However, the attitude knowledge accuracy drops with respect to the time lapse when utilizing only one observer-reference pair. The error can reach as high as 15° . The necessity of the near equatorial orbit satellite to perform momentum dumping constantly during non-eclipse period such that the saturation of the reaction wheels can be prevented has lead the author to develop a hybrid optimization method for TAM calibration with the presence of magnetic torquers.

The proposed TAM calibration method used a fusion algorithm comprising of PSO and IEKF. Simulation results show that it converged faster by over 90% when compared with PSO and is able to solve for large initialization error. Besides, experimental results using the flight data of VELOX-II show that the proposed TAM calibration managed to

calibrate the TAM with an error of $0.5678 \mu\text{T}$ and a convergence speed of 243 seconds. In addition, the proposed TAM calibration method takes the coupling effect of the magnetic torquers into consideration, allowing accurate attitude knowledge to be retained even when magnetic torquers are turned on. Thus, the problem of TAM reading corruption when performing momentum dumping is resolved.

A LCKF has also been presented to lower the noise of MEMS based gyroscope. Simulation studies show that the LCKF is 40.81% more computational efficient as compared to the standard KF algorithm. Moreover, experimental results show that the LCKF managed to lower the gyroscope noise in all three-axes under both static and dynamic conditions. The performance of the LCKF is further verified by conducting observer free sun tracking experiment. Experimental results show that the steady state sun tracking error reduces significantly with the implementation of LCKF on the MEMS gyroscope. Hence, the pointing and estimation performance can be improved with a less noisy gyroscope reading.

6.2 Future Works

In this thesis, the in-orbit variation such as thermal variation, mechanical misalignment, and on-board electronics time-varying bias of TAM has been considered to improve the TAM measurements as well as to reduce the coupling effect of magnetic torquers. However, the in-orbit variation of MEMS gyroscope has not been considered. Study has suggested that the MEMS gyroscope measurements are prone to thermal variation as well as on-board electronics time-varying bias. In addition, MEMS gyroscope measurement are also subjected to gyroscope bias drift with respect to time. As Chapter 3 suggested that the gyroscope measurement has the highest impact on the

attitude knowledge accuracy, an in-orbit gyroscope calibration will be highly beneficial for improving the attitude determination performance.

LIST OF PUBLICATIONS

- **J. W. Chia**, M. S. C. Tissera, K. S. Low, S. T. Goh, and Y. T. Xing, "A low complexity Kalman filter for improving MEMS based gyroscope performance," presented at the IEEE Aerosp. Conf., Big Sky, MT, Mar. 2016.
- **J. W. Chia** and K. S. Low, "A hybrid optimization approach for satellite's in-orbit magnetometer calibration," *IEEE Trans. Aerosp. Electron. Syst.* (Submitted on 2nd June 2016)
- M. S. C. Tissera, **J. W. Chia**, K. S. Low, and Y. T. Xing, "A novel simulator for measuring the performance of nanosatellite's attitude control system," presented at the IEEE Aerosp. Conf., Big Sky, MT, Mar. 2016.
- S. T. Goh, **J. W. Chia**, S. T. Chin, K. S. Low, and L. S. Lim, "Performance Analysis on NORAD TLE Pre-Process Method for Low Cost ADCS MCU," presented at the 34th International Electrical Propulsion Conference & 6th Nano-Satellite Symposium, Kobe, Japan, July. 2015.
- S. T. Goh, **J. W. Chia**, S. T. Chin, K. S. Low, and L. S. Lim, "A pre-processed orbital parameters approach for improving cubesat orbit propagator and attitude determination," Transactions of JSASS. (**Accepted**)
- S. T. Goh, **J. W. Chia**, K. S. Low, and L. S. Lim, "In-orbit VELOX-II orbit propagator performance study using pre-processed two-line element method," AIAA Space and Astronautics Forum and Exposition. (**Accepted**)

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