An efficient momentum dumping method for near equatorial orbit satellites

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**AN EFFICIENT MOMENTUM DUMPING METHOD FOR
NEAR EQUATORIAL ORBIT SATELLITES**

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**SCHOOL OF ELECTRICAL & ELECTRONIC
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AN EFFICIENT MOMENTUM DUMPING METHOD FOR NEAR EQUATORIAL ORBIT SATELLITES

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TABLE OF CONTENTS

ACKNOWLEDGEMENT	I
TABLE OF CONTENTS	III
ABSTRACT	VII
LIST OF FIGURES	IX
LIST OF TABLES	XII
ABBREVIATIONS	XIII
1 INTRODUCTION.....	1
1.1 Background.....	1
1.2 Objectives	3
1.3 Contribution.....	4
1.4 Outline of the thesis	5
2 LITERATURE REVIEW	6
2.1 Overview.....	6
2.2 Satellite attitude control actuators	6
2.2.1 Reaction wheels for attitude control.....	7
2.2.2 Magnetic torquer for attitude control	8
2.3 ACS performance in LEO using reaction wheels and magnetic torquers.	9
2.4 Near equatorial sun pointing satellites.....	10

2.5	Summary.....	12
3	SATELLITE ATTITUDE CONTROL USING MODEL PREDICTIVE CONTROL STRATEGY	13
3.1	Overview	13
3.2	VELOX-II attitude control system	13
3.2.1	ACS requirement for VELOX-II nanosatellite.....	15
3.3	Model Predictive Control strategy.....	15
3.3.1	Kinematic model of the system	17
3.3.2	Dynamic model of the system	18
3.3.3	State space formulation	20
3.3.4	Formulation of unconstrained MPC	21
3.3.5	Formulation of constrained MPC	24
3.3.6	Sun-pointing algorithm of VELOX-II.....	29
3.4	Simulation results and analysis of MPC control law.....	30
3.4.1	Simulation model.....	30
3.4.2	Performance criteria: mean error at steady state	31
3.4.3	Performance criteria: settling time	33
3.4.4	Performance criteria: overshoot	36
3.4.5	MPC subjected to gyroscope bounded errors	37
3.5	Selection of MPC controller parameters.	40
3.5.1	Selection based on ACS requirements.	40

3.5.2	Selection based on ACS requirements and $\mathbf{h}_{rw}$ usage.	42
3.6	Controllability and close-loop stability analysis.....	45
3.6.1	Controllability.....	46
3.6.2	Close-loop stability.....	47
3.7	In-orbit results.....	48
3.8	Summary.....	49
4	AN EFFICIENT MOMENTUM DUMPING METHOD FOR NEAR EQUATORIAL ORBIT SATELLITES	51
4.1	Overview.....	51
4.2	Momentum dumping controller.....	52
4.3	Proposed efficient momentum dumping controller	53
4.4	Simulation results	57
4.4.1	Simulation model.....	57
4.4.2	Selection of tuning parameters	58
4.4.3	Momentum dumping efficiency	63
4.4.4	Momentum dumping efficiency of fully saturated reaction wheel..	68
4.4.5	Sun-pointing performance	69
4.5	Summary.....	71
5	EXPERIMENTAL RESULTS	72
5.1	Overview.....	72
5.2	Introduction to an air bearing simulator	72

5.3	Hardware-in-the-loop spacecraft simulator.	76
5.4	Experimental results of MPC controller	77
5.5	Experimental results of MPC controller subjected to gyroscope bounded error.	81
5.6	Experimental results of momentum dumping controller	82
5.7	Experimental results of new Sun-pointing strategy	83
5.8	Summary	85
6	CONCLUSION.....	86
6.1	Summary of the research	86
6.2	Future works	88
	LIST OF PUBLICATIONS.....	89
	BIBLIOGRAPHY	91

ABSTRACT

In this research, the improvement of the reaction wheel angular momentum dumping efficiency for a near equatorial orbit satellite through a new Sun-pointing strategy will be presented.

Reaction wheels suffer from angular momentum saturation (i.e. when the speed of the reaction wheel reaches its maximum value). However, an alternative set of actuators such as magnetic torquers can be used to prevent such a saturation. It was identified that the torque produced by the magnetic torquers is limited to a plane that is only perpendicular to the geomagnetic field across the satellite body frame. Nevertheless, if the orbit inclination angle with respect to the geomagnetic equator is large, the magnetic field vector periodically changes its direction with respect to the Earth's inertial reference frame. As a result, on average over the orbital period, the magnetic torques can be applied in all directions within a desired time period. On the other hand, the Earth magnetic field vector for satellites in the near equatorial orbit (NEO) is at a much lower variation. This results in the satellite to lose its three-axis stabilization capability.

Consider VELOX-II which is a near equatorial orbit satellite, there was a need to increase the momentum dumping efficiency to improve the reliability and performance of its attitude control system. The proposed new Sun-pointing strategy aligns both the solar panel of the satellite towards the Sun as well as a secondary body frame vector with the Earth's geomagnetic field vector.

To describe the kinematics needed for the proposed method, a cost function was derived using QUEST (QUaternion ESTimation) method to obtain the error quaternion

for re-orientation of the satellite. To produce an optimal control torque for re-orientation, a model predictive control (MPC) algorithm has been formulated. The performance of the controller was analysed for varying MPC control parameters (i.e. prediction horizon and control horizon). The produced torque re-orientates the satellite while overcoming the disturbance torque produced by the magnetic torquer for momentum dumping of the reaction wheels.

To analyse the momentum dumping performance of the proposed method, the simulation results will be compared with the typical Sun-pointing strategy commonly employed by near equatorial orbit satellites. Results have demonstrated that the proposed method leads to a higher momentum dumping efficiency. Moreover, results have also shown that the proposed method outperforms the conventional method across all seasons and orbit inclination angles. Using an in-house designed hardware-in-the-loop spacecraft attitude control simulator, the new control algorithm has been validated experimentally.

LIST OF FIGURES

Fig 1-1. VELOX-II nanosatellite. Left: launch configuration. Right: in-orbit configuration.	1
Fig 2-1. Angular difference between geomagnetic plane and equatorial plane [36]. ...	10
Fig. 2-2 Typical Sun-pointing orientation of a near equatorial orbit satellite.	11
Fig. 3-1. VELOX-II attitude control system sensors and actuators: 1-60° field of view (FOV) fine Sun sensor, 2-120° FOV fine Sun sensor, 3-reaction wheels, 4-magnetic torquers, 5-inetial measurement unit (i.e. gyroscope and magnetometer)	14
Fig. 3-2. MPC performance measurement using mean error at steady state as an index	31
Fig. 3-3. Closer look at Fig. 3-2 (i.e. $N_c > 1$) for gyroscope noise of 0.9 deg/s.	33
Fig. 3-4. MPC performance measurement using settling time as an index for gyroscope noise of 0.27 deg/s	33
Fig. 3-5. MPC performance measurement using settling time as an index for gyroscope noise of 0.9 deg/s	34
Fig. 3-6. MPC performance measurement using max overshoot as an index for gyroscope noise of 0.27 deg/s	36
Fig. 3-7. MPC performance measurement using max overshoot as an index for gyroscope noise of 0.9 deg/s	36
Fig. 3-8. MPC performance measurement using mean error at steady state as an index for gyroscope noise of 0.9 deg/s subjected to different gyroscope bounded errors.	37
Fig. 3-9. Closer view of Fig. 3-8 (i.e. $N_c > 1$)	38
Fig. 3-10. Step responses of unconstrained and constrained MPC for selected tuning parameter N.	41

Fig. 3-11. Steady state normalised angular momentum for $N_c = 1$ and changing N_p .	42
Fig. 3-12. Step responses of unconstrained MPC for $N_c = 1$ and $N_p=12$.	44
Fig. 3-13. Plot of eigenvalues of closed-loop state matrix for $N_p=4$ and $N_p=12$.	47
Fig. 3-14. . Sun-pointing performance recorded from 16 th Dec. 2015 to 1 st Feb. 2016.	48
Fig. 4-1 Proposed efficient momentum dumping strategy	53
Fig. 4-2 Controller block diagram for the proposed EMD method.	57
Fig. 4-3. Sun-pointing performance for different w and N_p for the EMD method.	59
Fig. 4-4 Steady state average normalized reaction wheel angular momentum recorded for changing N_p .	59
Fig. 4-5. Plot of eigenvalues of closed-loop state matrix for $N_p=15$.	61
Fig. 4-6. Sun angle observed on the equatorial plane.	63
Fig. 4-7. Momentum dumping performance between the TSP and EMD methods for an equatorial orbit during the equinox season.	64
Fig. 4-8. Variation in $\mathbf{h}_{rw}$ for an equatorial orbit during the equinox season: (a) TSP and (b) EMD	65
Fig. 4-9. Variation $\mathbf{B}_b$ for an equatorial orbit during the equinox season: (a) TSP and (b) EMD	65
Fig. 4-10. Comparison of reaction wheel momentum dumping performance between TSP and EMD methods.	67
Fig. 4-11. Reaction wheels momentum dumping performance of EMD method.	67
Fig. 4-12. Momentum dumping performance with different saturation levels (i.e. 25%, 50%, 75% and 100%)	68
Fig. 4-13. Sun-pointing performance comparison between the TSP and the EMD methods.	70

Fig 5-1. Spherical air bearing for spacecraft simulators	73
Fig.5-2 CAD of VELOX II air bearing spacecraft simulator.	74
Fig.5-3 Integrated coarse and fine counterweight tuning mechanism. (a) Coarse tuning, (b) fine tuning, (c).Modular design to accommodate counterweights of different masses	75
Fig 5-4. Hardware-in-the-loop spacecraft simulator: 1- Helmholtz cage; 2- air bearing spacecraft simulator; 3-Two axis rate table; 4- sun simulator; 5-PC.....	76
Fig. 5-5. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=2.78$	
Fig. 5-6. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=4.79$	
Fig. 5-7. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=6.79$	
Fig. 5-8 Experimental steady state response vs simulated (with 1 mN.m torque limit) steady state response during Sun-pointing by air bearing spacecraft simulator for $N_p=2$	80
Fig. 5-9 Sun pointing performance. Section (a), using uncalibrated gyroscope. Section (b), using calibrated gyroscope.	81
Fig. 5-10 Experimental results of the momentum dumping controller.....	83
Fig. 5-11 Experimental results of the momentum dumping controller.....	83
Fig. 5-12 Experimental demonstration of the new Sun-pointing strategy,	84

LIST OF TABLES

Table 3-1. ACS actuators/sensors specifications.	15
Table 3-2. VELOX-II ACS performance requirements.....	15
Table 3-3. Formulation of actuator constraints.....	26
Table 3-4. Orbital elements of VELOX-II.....	31
Table 3-5. Summary: final selection of N for both constrained and unconstrained MPC	40
Table 3-6. Performance results of unconstrained and constrained MPC for selected tuning parameter N	41
Table 3-7. Performance results of unconstrained MPC for N_p of 12 and N_c of 1.	43
Table 3-8. Eigenvalues of closed-loop state matrix for $N_p=4$ and $N_p=12$	47
Table 3-9. In-orbit vs simulated performance of unconstrained MPC for N_p of 12 and N_c of 1.....	49
Table 4-1. Eigenvalues of closed-loop state matrix for $N_p=15$	60
Table 4-2: Pseudocode of Sun-pointing and momentum dumping controller.....	62
Table 4-3: Pseudocode of supervisor controller	63
Table 4-4. Sun-pointing performance analysis between the TSP and the EMD methods.....	71
Table 5-1. Gyroscope calibration of air bearing spacecraft simulator.....	81

ABBREVIATIONS

ACS	: Attitude control system
ADCS	: Attitude determination and control system
CAD	: Computer aided design
CG	: Centre of gravity
CoM	: Centre of mass
CoR	: Centre of rotation
COTS	: Commercial off-the-shelf
EMD	: Efficient momentum dumping
GPS	: Global positioning system
IGRF	: International geomagnetic reference field
LEO	: Low earth orbit
MPC	: Model predictive control
NEO	: Near equatorial orbit
NTU	: Nanyang Technological University
QP	: Quadratic programming
SaRC	: Satellite Research Centre
TCP/IP	: Transmission control protocol/internet protocol
TSP	: Typical Sun-pointing

CHAPTER 1

INTRODUCTION

1.1 Background

Small low-cost satellites have become important and popular with the increase of piggyback launch opportunities, e.g., the Ariane Structure for Auxiliary Pay loads (ASAP), and smaller commercial launchers like Pegasus [1]. Today, miniaturized satellites such as CubeSats and nanosatellites are becoming important tools to carry out space science and technology research for educational institutions [2-6]. They are attractive for several reasons, one of which is the much reduced cost as compared to the conventional satellites.

Fig 1-1. VELOX-II nanosatellite. Left: launch configuration. Right: in-orbit configuration.

Fig 1-1 shows the VELOX-II satellite, which is a 6U CubeSat [7] and is Nanyang Technological University's (NTU) second nanosatellite built in Satellite Research Centre (SaRC). VELOX-II was launched from the India's Satish Dhawan Space Centre on the Indian Space Research Organisation's Polar Satellite Launch Vehicle (PSLV-C29) rocket, together with VELOX-CI, a 123kg climate monitoring and

navigation satellite developed by NTU and four other Singapore satellites on 16 December 2015.

VELOX-II carries an experimental “communication-on-demand” Inter-Satellite Data Relay System (IDRS) as the primary payload [8]. It allows VELOX-II to be in contact with NTU ground station at anytime and anywhere in space. The secondary payloads include a new radiation-resistant hardware and a fast global positioning system (GPS) tracking algorithm that determines the VELOX-II position accurately within a minute.

Besides the nanosatellite VELOX-I [9, 10], NTU has also had two pico-satellites named VELOX-PII and VELOX-PIII in orbit as well as a 105 kg microsatellite named X-SAT which has been operating in space since 20 April 2011.

Among the many satellite subsystems, attitude control system is one of the key systems for a three axis stabilized nanosatellite. With advanced attitude control system (ACS) and double deployable solar panels, the nanosatellites will be able to harvest sufficient solar energy to carry up advanced payloads with high power requirements. Therefore, its ACS must be robust and reliable at all times in orbit. A failure in the ACS may result in losing its three-axis stabilization capability and eventually run out of power. Such a recent failure in the ACS was experienced by X-ray Astronomy Satellite (also known as Hitomi) launched on February 17, 2016 JST [11, 12].

The hardware and software of VELOX-II Attitude Determination and Control System (ADCS) is inherited from VELOX-I nanosatellite that was launched in to a sun-synchronous polar orbit. The research presented in this thesis identifies a challenge faced by VELOX-II when using the same hardware and software, but in a near equatorial orbit (NEO).

1.2 Objectives

There are three objectives that have been identified to improve the reliability and performance of ACS of VELOX-II:

- The Earth magnetic field vector for satellite such as VELOX-II in the near equatorial orbit (NEO) is at a much lower variation. This reduces the momentum dumping efficiency of reaction wheels by magnetic torquers and causes the satellite to lose its three-axis stabilization capability. Therefore, the main objective of this thesis will be to improve the momentum dumping capability of the reaction wheels for a satellite in the NEO. The improvement will be achieved using a new Sun-pointing strategy.
- The efficiency of the new Sun-pointing strategy strongly depends on the performance of the attitude acquisition controller. Therefore, the second objective will be to design a suitable digital controller to produce an optimal control torque for re-orientation of the satellite.
- The performance of both the digital attitude acquisition controller and the new Sun-pointing strategy will be analysed using results obtained from computer based simulations. Therefore, the final objective of this report will be to validate the computer based models and results experimentally using a suitable laboratory platform.

1.3 Contribution

The main contributions of the thesis are as follows:

- Two types of model predictive control (MPC) algorithms (i.e. unconstrained and constrained MPC) had been explored to be used as attitude controller. The analysis were subjected to different levels of gyroscope noise and bounded errors. The performance of both the controllers were evaluated under different performance criteria. A suitable MPC controller was then selected to satisfy VELOX-II ACS requirements. The simulated results of the controller were then compared with in-orbit performance of VELOX-II ACS.
- A new Sun-pointing strategy was developed to improve the reaction wheel angular momentum dumping efficiency for a near equatorial orbit satellite. The proposed method aligns both the solar panel of the satellite towards the Sun as well as a secondary body frame vector with the Earth's geomagnetic field vector. To describe the kinematics needed for the proposed method, a cost function was derived using QUEST (QUaternion ESTimation) method to obtain the error quaternion for re-orientation of the satellite. To produce an optimal control torque, the calculated error quaternion will then be used by the developed MPC controller.
- A novel air bearing spacecraft simulator was integrated with a two axes high precision rate table, a Helmholtz cage, and a sun simulator to form a hardware-in-the-loop (HIL) spacecraft simulator. The HIL was used to validate the developed MPC controller and the proposed new Sun-pointing strategy experimentally.

1.4 Outline of the thesis

The thesis is organized as follows:

- Chapter 2 gives a literature review of the spacecraft attitude control actuators highlighting the strengths and weaknesses of magnetic torquer and reaction wheel for attitude control. This will be followed by a comparison of NEO against polar orbit with regard to reaction wheel momentum dumping efficiency by the magnetic torquer.
- Chapter 3 presents a digital controller using MPC for attitude control of a satellite. It also shows how different types of MPC algorithms (i.e. unconstrained MPC and constrained MPC) perform when subjected to unfavourable conditions such as gyroscope noise and bounded errors. It gives an insight to how a suitable MPC algorithm was chosen for VELOX-II and how its tuning parameters were adjusted to meet the mission requirements.
- Chapter 4 presents the proposed new Sun-pointing strategy to improve the reaction wheel angular momentum dumping capability. The proposed strategy employs the attitude acquisition controller developed in chapter 3. The chapter also analyses the momentum dumping performance of the proposed method against the typical Sun-pointing strategy commonly employed by near equatorial orbit satellites.
- Chapter 5 presents the experimental results to validate both the developed attitude acquisition controller and the proposed new Sun-pointing strategy experimentally.
- Chapter 6 provides a summary of the research followed by future works.

CHAPTER 2

LITERATURE REVIEW

2.1 Overview

This chapter provides a review of actuators used for satellite attitude control. It includes a study of magnetic torquers and reaction wheels for attitude control highlighting their strengths and weaknesses. The performance of classical cross product control law for momentum dumping by magnetic torquers is then reviewed both for a satellite in near equatorial and a polar orbit.

2.2 Satellite attitude control actuators

Reaction wheels, control moment gyroscopes, propulsion system and magnetic torquers are the actuators available for attitude control of a satellite.

The selection of a suitable actuator depends on:

- Power available from the power subsystem of the satellite.
- Mass budget of the satellite for launch.
- Space constraints.
- Maximum torque required for attitude control.

For a three axes stabilized nanosatellite, the most commonly used attitude control actuators for low earth orbit (LEO) satellites are magnetic torquers and reaction wheels [13-15]. For VELOX-II, which is used in this study, the available power, size and mass budget limit the implementation of propulsion system and control moment

gyroscopes. Therefore, the attitude control actuators selected for VELOX-II will be reaction wheels and magnetic torquers. However both the actuators have their own strengths and weaknesses.

2.2.1 Reaction wheels for attitude control

Reaction wheels work through momentum transfer between the spacecraft body and one or more spinning wheels. A cluster of three or more are used for arbitrary three-axis spacecraft pointing. In a reaction wheel, motor torque drive the momentum wheel and this motor torque, in turn, acts equally and opposite on the spacecraft frame. Therefore, with careful control of wheel acceleration, a desired torque for general attitude corrections can be created [16, 17]

$$\mathbf{T}_{rw} = -\dot{\mathbf{h}}_{rw} \quad (2.1)$$

where $\mathbf{T}_{rw} \in \mathbb{R}^3$ is tri-axial torque produced by the reaction wheels and $\mathbf{h}_{rw} \in \mathbb{R}^3$ is the angular momentum of the reaction wheels.

Typically, reaction wheel speeds are modified in such a way as to effectively absorb the spacecraft momentum of a tumbling spacecraft [18] or to absorb all external disturbance forces experienced by the spacecraft [19]. This causes a gradual increase of their spinning wheels and may lead to momentum saturation (i.e. when the stored angular momentum of the reaction wheel reaches its maximum value). Such saturation in momentum results in the satellite losing its three-axis stabilization capability.

To reduce the stored momentum in each wheel, either propulsion systems or magnetic torquers are used to perform momentum dumping. The propulsion system that uses either electrical or liquid propulsions provides the most efficient method in momentum dumping. But the implementation of a propulsion system for such a

purpose in a nanosatellite or a microsatellite is often limited by the satellite size and mass constraints. An alternative small propulsion system, such as the micro thruster [20-22] can be considered for a nanosatellite system. However, both the power and mass requirement of VELOX-II limits the implementation of a micro-thruster. Therefore, for VELOX-II, magnetic torquers will be used to perform momentum dumping and will be discussed next.

2.2.2 Magnetic torquer for attitude control

Since the successful operation of the magnetic controllers in the early space programs [23], magnetic attitude control strategy has been used by small satellites with not-too-demanding attitude knowledge requirements and pointing performances [24-26]. This strategy saves overall cost, weight, power and reduced risk of failures as compared to other attitude control strategies [14].

In addition, The Earth's magnetic field magnitude decreases as the inverse cube of the distance from the centre of the Earth. Therefore, the magnetic torque will be several orders of magnitude smaller at high earth orbits than at low earth orbits (LEOs) [27]. This makes magnetic torquers very attractive to a small LEO satellite such as VELOX-II.

Magnetic control can be either active or passive. In the passive magnetic attitude control, hysteresis rod or a permanent magnet that does not require any power supply is used [26, 28]. In active magnetic attitude control, magnetic torquers are used as actuators. The dipole moment of the magnetic torquers can be regulated by varying their input currents, which allows the implementation of closed-loop feedback laws, based on the sensors' measurements [29].

The torque generated by a magnetic torquer , $\mathbf{T}_{mt}$ at any instant is given by the classical cross product control law [30]:

$$\mathbf{T}_{mt} = \mathbf{M} \times \mathbf{B}_b \quad (2.2)$$

where “ $\times$ ” denotes the vector cross product operation, $\mathbf{M} \in \mathbb{R}^3$ is tri-axial magnetic dipole moment of the magnetic torquer and $\mathbf{B}_b \in \mathbb{R}^3$ is the magnetic field across the satellite body frame.

A challenge with using magnetic torque rods lies in their inability to produce an arbitrary three-dimensional torque. In fact, equation (2.2) shows that a torque can only be produced perpendicular to the Earth’s magnetic field, providing only two degrees of freedom available for dumping the wheel momentum [1, 31, 32].

Nevertheless, if the satellite orbit inclination angle with respect to the geomagnetic equator is large, the magnetic field vector periodically changes its direction with respect to the Earth’s inertial reference frame. On average over the orbital period, the magnetic torques can be applied in all directions within a desired time period [33-35].

2.3 ACS performance in LEO using reaction wheels and magnetic torquers.

As explained in section 2.2, the most common attitude control actuators used for small satellite are either magnetic torquers alone or a hybrid of reaction wheels and magnetic torquers for attitude control and momentum dumping respectively. Therefore the performance of magnetic torquers reflects the overall ACS actuator performance.

Torque produced by the magnetic torquers are restricted to lie on a two-dimensional plane orthogonal to the magnetic field. Therefore only two out of three axes can be controlled at a given time instant. However, full three-axis control is available provided that the spacecraft's orbital plane does not coincide with the geomagnetic plane as the direction of the magnetic field will then change along the orbit [27, 33].

Fig 2-1. Angular difference between geomagnetic plane and equatorial plane [36].

As the geomagnetic plane is only 11° from the Earth's equator (see Fig 2-1 and [27]), this implies that the polar orbits have a higher control authority over the torque produced by magnetic torquers as compared to the NEOs. Section 2.4 will relate how the sun pointing strategy (i.e. housekeeping mode) of a NEO satellite can result in the loss of 3-axis attitude stabilization due to poor performances of the magnetic torquers.

2.4 Near equatorial sun pointing satellites

Fig. 2-2 shows the typical Sun-pointing strategy for a NEO satellite [37]. The direction of solar panels is $-\mathbf{Z}_b$. For a typical Sun-pointing orientation, $-\mathbf{Z}_b$ is aligned with respect to the Sun direction, $\mathbf{S}_b \in \mathbb{R}^3$ and the satellite's $\mathbf{X}_b$ direction is aligned

with a vector that is orthogonal to both $\mathbf{Z}_b$ and orbit normal vector, $\mathbf{h}_{\text{orbit}} \in \mathbb{R}^3$. Due to the precession movement of Earth, the direction of Sun from Earth's centre varies in different time of the year (see Fig. 2-2). The highest and lowest points of the Sun direction that is approximately 23.5° away from the equatorial plane are known as the solstices. When the Sun crosses the Earth equator plane, it is known as the equinox.

Fig. 2-2 Typical Sun-pointing orientation of a near equatorial orbit satellite.

From (2.2), it is observed that the $\mathbf{T}_{\text{mt}}$ is always on the plane perpendicular to $\mathbf{B}_b$. Thus, there will be instances where no magnetic torque is produced along one of the three-axis of the satellite which is in parallel with the earth magnetic field. From Fig. 2-2, it is noticed that variation of the Earth geomagnetic field is minimal throughout the near equatorial orbit, and is generally parallel to satellite's $\mathbf{Y}_b$ direction. Therefore,

the magnetic torquers are unable to unsaturate the $\mathbf{Y}_b$ reaction wheel. This would lead to failure of the ACS to perform three-axis stabilization.

2.5 Summary

This chapter presented a review of commonly used attitude control actuators in LEOs with regard to reaction wheel momentum dumping.

It was identified that the torque produced by magnetic torquers is limited to a plane that is only perpendicular to the geomagnetic field across the satellite body frame. Nevertheless, if the satellite orbit inclination angle with respect to the geomagnetic equator is large, the magnetic torques can be applied in all directions within a desired time period. However, momentum dumping efficiency for near equatorial orbit satellites will be reduced as a result of very low orbit inclination angles.

As VELOX-II is a NEO satellite, there is a need to increase the momentum dumping efficiency to improve the reliability and performance of its ACS. In order to overcome this challenge, attitude control using Model Predictive Control strategy will be presented in Chapter 3 followed by a new Sun-pointing strategy in Chapter 4 to improve the momentum dumping efficiency of the reaction wheels.

CHAPTER 3

SATELLITE ATTITUDE CONTROL USING MODEL PREDICTIVE CONTROL STRATEGY

3.1 Overview

The efficiency of the harvested energy in Sun-pointing mode for a satellite depends on the performance of the attitude acquisition controller. In this chapter, a suitable digital controller to produce an optimal control torque for re-orientation of the satellite will be presented.

This chapter starts with an introduction to VELOX-II attitude control system. Unconstrained and constrained MPC algorithms will then be formulated. Simulations will then be conducted to analyse the controller under different performance indexes, subjected to different levels of gyroscope noise and bounded errors. Subsequently, a suitable controller will be selected to meet the ACS requirements of VELOX-II. The controllability and closed-loop stability of the selected controller will then be evaluated. Finally, the simulated performance of the selected controller will be compared with the in-orbit performance of VELOX-II.

3.2 VELOX-II attitude control system

VELOX-II uses reaction wheels to achieve three-axis stabilization for Sun-pointing, target tracking and nadir pointing. Commercial off-the-shelf (COTS) magnetic torquers are used in VELOX-II to perform momentum dumping of the reaction wheels to prevent them from saturating due to external disturbance forces.

The standard orthogonal 3-wheel configuration was chosen due to space constraint. The reaction wheels were made in-house to provide a minimum slew rate of $3^\circ/\text{s}$ for each axis of the spacecraft. The specifications of the on-board sensors and actuators are summarized in Table 3-1. They will be used in all computer simulations presented in this thesis. VELOX-II ACS actuators and sensors are shown in Fig. 3-1.

Sun-pointing will be the default (or house-keeping) pointing mode of VELOX-II and will be presented in this chapter. Under this mode, the ACS re-orientates the satellite such that $-\mathbf{Z}_b$ (i.e. the solar panel normal vector) is constantly aligned with the sun vector measured by the sun sensors through-out the non-eclipse period with constant momentum dumping by the magnetic torquers. More details will be presented in section 3.3.6. The efficient momentum dumping pointing method (presented in Chapter 4) will be considered as the safe-hold pointing mode while target tracking and nadir pointing modes will be the mission pointing modes of the satellite.

Fig. 3-1. VELOX-II attitude control system sensors and actuators: 1-60° field of view (FOV) fine Sun sensor, 2-120° FOV fine Sun sensor, 3-reaction wheels, 4-magnetic torquers, 5-inertial measurement unit (i.e. gyroscope and magnetometer)

Table 3-1. ACS actuators/sensors specifications.

Sensor/Actuator		Specifications at 25°C
Sun Sensor	60° FOV	$\pm 0.3^0$
	120° FOV	$\pm 0.8^0$
Inertial Measurement Unit-A (experimental); P/N: ADIS16405	Gyroscope	$\pm 0.9^0/\text{sec}$
	Magnetometer	$\pm 1.25 \text{ m.Gauss}$
Inertial Measurement Unit-B (default) P/N: ADIS16448	Gyroscope	$\pm 0.27^0/\text{sec}$
	Magnetometer	$\pm 2.4 \text{ m.Gauss}$
Max. reaction wheel Angular Momentum ($h_{\text{rw-max}}$)		$\pm 5.16 \text{ m.Nms}$
Max. Dipole Moment of magnetic torquer (M_{max})		$\pm 0.2 \text{ Am}^2$
Max. allowable torque by reaction wheels ($T_{\text{rw-max}}$)		$\pm 1 \text{ m.Nm}$

3.2.1 ACS requirement for VELOX-II nanosatellite.

Table 3-2 presents the VELOX-II ACS performance requirements needed for constant Sun-pointing during housekeeping, target tracking and nadir pointing to conduct missions related to the primary payload and GPS payload respectively.

Table 3-2. VELOX-II ACS performance requirements

Sensor/Actuator	Requirement
Mean error at steady state	$< 3^0$
Settling time	$< 1 \text{ minute}$
Maximum overshoot	Zero (desirable)
Insensitive to gyroscope bounded errors	Yes

3.3 Model Predictive Control strategy

Model Predictive Control (MPC) strategy is a digital control strategy first developed for the process control industry such as arc welding and sheet/film processes. Subsequently other applications such as precision control of linear motor drive [38] and inverter [39] have been reported. It is a model based control strategy where it predicts the output of the system over a time horizon. The appropriate control

sequence is calculated to reduce the tracking error by minimizing a cost function. MPC gives good rejection against modelling errors and disturbances [38] along with many other practical advantages [40].

MPC has the following practical advantages [40, 41]:

- Straightforward formulation, based on well understood concepts.
- Explicitly handles constraints.
- Explicit use of a model.
- Well understood tuning parameters: prediction horizon and optimization problem setup.
- Development time much shorter than for competing advanced control methods.
- Easier to maintain: changing model or specs does not require complete redesign, sometimes can be done on the fly.

It is well known that MPC can be applied for the control of nonlinear system. However, the computational requirement would be greatly increased to obtain the optimum performance. Due to the speed and power consumption constraint of microcontroller used in the attitude control system of the satellite, the MPC is designed based on the linearized model of the satellite.

3.3.1 Kinematic model of the system

To represent the rigid body rotation of a satellite, let the quaternion vector, $\mathbf{q} \in \mathbb{R}^4$ that consists of a vector component ($\boldsymbol{\varepsilon} \in \mathbb{R}^3$) and a scalar component (η) be represented as [27, 42]

$$\mathbf{q} = \begin{bmatrix} \boldsymbol{\varepsilon} \\ \eta \end{bmatrix} \quad (3.1)$$

The first order derivative of $\mathbf{q}$, known as the kinematics equation is given by [43, 44]

$$\dot{\mathbf{q}} = \begin{bmatrix} \dot{\boldsymbol{\varepsilon}} \\ \dot{\eta} \end{bmatrix} = \begin{bmatrix} -0.5\boldsymbol{\omega} \times \boldsymbol{\varepsilon} + 0.5\boldsymbol{\omega} \eta \\ -0.5\boldsymbol{\omega}^T \boldsymbol{\varepsilon} \end{bmatrix} \quad (3.2)$$

where $\boldsymbol{\omega} \in \mathbb{R}^3$ is the body rate of the satellite and superscript “ $\times$ ” represent the vector cross product operation.

Let $\mathbf{q}_s$ represents the satellite orientation (i.e. rotation from satellite body frame to inertial frame), and $\mathbf{q}_T$ as the target quaternion (i.e. rotation from target frame to inertial frame). The error quaternion vector, $\mathbf{q}_E$ between the quaternions $\mathbf{q}_s$ and the inverse of target quaternion, $\mathbf{q}_T^{-1}$ (i.e. rotation from inertial frame to target frame) is given by:

$$\mathbf{q}_E = \begin{bmatrix} \boldsymbol{\varepsilon}_e \\ \eta_e \end{bmatrix} = \mathbf{q}_s \otimes \mathbf{q}_T^{-1} \quad (3.3)$$

where $\otimes$ denotes the quaternion multiplication operation [45].

3.3.2 Dynamic model of the system

While (3.1) and (3.2) describe the orientation of a satellite, it does not include the dynamic motion. To describe the dynamic motion of the satellite, the following Euler's dynamic equation is employed [27, 42, 43]:

$$(\dot{\mathbf{h}} + \dot{\mathbf{h}}_{\text{rw}}) + \boldsymbol{\omega}^\times (\mathbf{h} + \mathbf{h}_{\text{rw}}) = \mathbf{T}_{\text{ext}} \quad (3.4)$$

where $\mathbf{h} \in \mathbb{R}^3$ denotes the angular momentum of the satellite, $\mathbf{h}_{\text{RW}} \in \mathbb{R}^3$ denotes the angular momentum of the reaction wheel and $\mathbf{T}_{\text{ext}}$ is the sum of the total external torque and is given as:

$$\mathbf{T}_{\text{ext}} = \mathbf{T}_{\text{mt}} + \mathbf{T}_{\text{dist}} \quad (3.5)$$

where $\mathbf{T}_{\text{mt}}$ and $\mathbf{T}_{\text{dist}}$ denote the control torque of magnetic actuators and disturbance torque experienced by the satellite respectively.

The angular momentum of the satellite is defined as the product of the body's moment of inertia and its angular velocity:

$$\mathbf{h} = \mathbf{J}\boldsymbol{\omega} \quad (3.6)$$

where $\mathbf{J} \in \mathbb{R}^{3 \times 3}$ is the moment of inertia of the satellite. For VELOX-II, $\mathbf{J}$ is obtained from Computer Aided Design model and is given as:

$$\mathbf{J} = \begin{bmatrix} 0.02982 & 0.00000 & -0.00104 \\ 0.00000 & 0.03303 & 0.00003 \\ -0.00104 & 0.00003 & 0.01507 \end{bmatrix} \text{kgm}^2 \quad (3.7)$$

The angular momentum of the reaction wheel is the product of the moment of inertia of the reaction wheels and its angular velocity:

$$\mathbf{h}_{rw} = J_{rw} \boldsymbol{\omega}_{rw} \quad (3.8)$$

where J_{rw} is the moment of inertia of the reaction wheel and $\boldsymbol{\omega}_{rw}$ is the angular velocity of the reaction wheel.

Substituting (3.6)-(3.8) into (3.4) and rearranging the equation yields:

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega} = -J_{rw} \dot{\boldsymbol{\omega}}_{rw} - \boldsymbol{\omega}^\times J_{rw} \boldsymbol{\omega}_{rw} + \mathbf{T}_{ext} \quad (3.9)$$

where the reaction wheel torque is represented as:

$$\mathbf{T}_{rw} = -J_{rw} \dot{\boldsymbol{\omega}}_{rw} \quad (3.10)$$

and the less dominant gyroscopic torque is neglected:

$$\mathbf{T}_{gyroscopic} = -\boldsymbol{\omega}^\times J_{rw} \boldsymbol{\omega}_{rw} \approx \mathbf{0} \quad (3.11)$$

Substituting (3.10)-(3.11) into (3.9) and rearranging the equation yields:

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}(-\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega}) + \mathbf{J}^{-1}\mathbf{T}_{ext} + \mathbf{J}^{-1}\mathbf{T}_{rw} \quad (3.12)$$

For attitude controller design using only reaction wheels, the total external torque is removed:

$$\dot{\boldsymbol{\omega}} = \mathbf{J}^{-1}(-\boldsymbol{\omega}^\times \mathbf{J}\boldsymbol{\omega}) + \mathbf{J}^{-1}\mathbf{T}_{rw} \quad (3.13)$$

Substituting (3.10) into (3.8) yield:

$$\mathbf{T}_{rw} = -\dot{\mathbf{h}}_{rw} \quad (3.14)$$

3.3.3 State space formulation

Linearizing (3.3) and (3.13) (i.e. using second order Taylor series expansion) about the system equilibrium points of

$$\boldsymbol{\varepsilon}_e = [0 \ 0 \ 0]^T, \eta_e = 1, \boldsymbol{\omega} = [0 \ 0 \ 0]^T, \mathbf{T}_{rw} = [0 \ 0 \ 0]^T \quad (3.15)$$

Then the linearized state space model is obtained as:

$$\begin{bmatrix} \dot{\boldsymbol{\varepsilon}}_e \\ \dot{\boldsymbol{\omega}} \end{bmatrix} = \mathbf{A}_{1c} \begin{bmatrix} \boldsymbol{\varepsilon}_e \\ \boldsymbol{\omega} \end{bmatrix} + \mathbf{B}_{1c} \mathbf{T}_{rw}$$

$$\boldsymbol{\varepsilon}_e = [\mathbf{C}_{1c}] \begin{bmatrix} \boldsymbol{\varepsilon}_e \\ \boldsymbol{\omega} \end{bmatrix} \quad (3.16)$$

$$\mathbf{A}_{1c} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & 0.5\mathbf{I}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}, \mathbf{B}_{1c} = \begin{bmatrix} \mathbf{0}_{3 \times 3} \\ \mathbf{J}^{-1} \end{bmatrix}, \mathbf{C}_{1c} = [\mathbf{I}_{3 \times 3} \ \mathbf{0}_{3 \times 3}]$$

where $\mathbf{I}_{3 \times 3}$ is a 3×3 identity matrix.

From (3.16), it is observed that η_e is absent from the state vector as it is decoupled from the system in the linearization process. Moreover, the moment of inertia of the satellite $\mathbf{J}$ determines the amount of cross coupling within the system.

State space formulation in (3.16) is used for the formulation of unconstrained MPC for satellite attitude. For the formulation of constrained MPC, the controller needs to take in to account of the following actuator constraints:

- $\mathbf{h}_{rw}$ limit: occurs when the driving motor of the reaction wheel reaches its maximum rotational speed $\mathbf{h}_{rw-\max}$.

- $\mathbf{T}_{\text{rw}}$ limit: occurs when the torque to be produced by the reaction wheel reaches a predefined maximum value, $\mathbf{T}_{\text{rw-max}}$. For VELOX-II, this constraint was added as a safety feature in the firmware to prevent overdraw of current from the power sub system.

In order for the MPC to handle the first constraint, $\mathbf{h}_{\text{rw}}$ needs to be added in to the state space formulation. Therefore, by combining (3.14) with (3.16), we get:

$$\begin{bmatrix} \dot{\boldsymbol{\epsilon}}_e \\ \dot{\boldsymbol{\omega}} \\ \dot{\mathbf{h}}_{\text{rw}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{1c} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 6} & \mathbf{0}_{3 \times 3} \end{bmatrix} \begin{bmatrix} \boldsymbol{\epsilon}_e \\ \boldsymbol{\omega} \\ \mathbf{h}_{\text{rw}} \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{1c} \\ -\mathbf{I}_{3 \times 3} \end{bmatrix} \mathbf{T}_{\text{rw}}$$

$$\boldsymbol{\epsilon}_e = [\mathbf{C}_{2c}] \begin{bmatrix} \boldsymbol{\epsilon}_e \\ \boldsymbol{\omega} \\ \mathbf{h}_{\text{rw}} \end{bmatrix} \quad (3.17)$$

$$\mathbf{A}_{2c} = \begin{bmatrix} \mathbf{A}_{1c} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 6} & \mathbf{0}_{3 \times 3} \end{bmatrix}, \mathbf{B}_{2c} = \begin{bmatrix} \mathbf{B}_{1c} \\ -\mathbf{I}_{3 \times 3} \end{bmatrix}, \mathbf{C}_{2c} = [\mathbf{I}_{3 \times 3} \quad \mathbf{0}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}]$$

3.3.4 Formulation of unconstrained MPC

For the formulation of unconstrained MPC control law, the state space model in (3.16) is discretized [46] at a sampling time of T_{s1} to give

$$\begin{bmatrix} \boldsymbol{\epsilon}_e(k_1 + 1) \\ \boldsymbol{\omega}(k_1 + 1) \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} \boldsymbol{\epsilon}_e(k_1) \\ \boldsymbol{\omega}(k_1) \end{bmatrix} + [\mathbf{B}] \mathbf{T}_{\text{rw}}(k_1) \quad (3.18)$$

$$\boldsymbol{\epsilon}_e(k_1) = [\mathbf{C}] \begin{bmatrix} \boldsymbol{\epsilon}_e(k_1) \\ \boldsymbol{\omega}(k_1) \end{bmatrix}$$

where $\mathbf{A} = e^{\mathbf{A}_{1c}T_{s1}}$, $\mathbf{B} = \int_0^{T_{s1}} e^{\mathbf{A}_{1c}\lambda} d\lambda \mathbf{B}_{1c}$, $\mathbf{X} = [\boldsymbol{\epsilon}_e^T \quad \boldsymbol{\omega}^T]^T$, $\mathbf{C} = [\mathbf{I}_{3 \times 3} \quad \mathbf{0}_{3 \times 3}]$

MPC uses the discretized state space model to predict the future state based on the current state of the system. The predicted states are then used to calculate an optimal control action. In the case of unconstrained MPC, the actuator constraints will override the optimal control action.

Using (3.18) to perform successive multiplication, the predicted state vectors in the prediction horizon are expressed as:

$$\hat{\mathbf{\epsilon}}_e = \Phi \begin{bmatrix} \mathbf{\epsilon}_e(k_1)^T & \boldsymbol{\omega}(k_1)^T \end{bmatrix}^T + \mathbf{R} \hat{\mathbf{T}}_{\text{rw}} \quad (3.19)$$

where

$$\hat{\mathbf{\epsilon}}_e = \begin{bmatrix} \hat{\mathbf{\epsilon}}_e(k_1+1|k_1) \\ \hat{\mathbf{\epsilon}}_e(k_1+2|k_1) \\ \vdots \\ \hat{\mathbf{\epsilon}}_e(k_1+N_p|k_1) \end{bmatrix}, \Phi = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \vdots \\ \mathbf{CA}^{N_p} \end{bmatrix}, \mathbf{R} = \begin{bmatrix} \mathbf{CB} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{CAB} & \mathbf{CB} & \mathbf{0}_{3 \times 3} \\ \vdots & \ddots & \vdots \\ \mathbf{CA}^{N_p-1}\mathbf{B} & \dots & \mathbf{CA}^{N_p-N_c}\mathbf{B} \end{bmatrix}$$

$$\hat{\mathbf{T}}_{\text{rw}} = \begin{bmatrix} \hat{\mathbf{T}}_{\text{rw}}(k_1+0|k_1) \\ \hat{\mathbf{T}}_{\text{rw}}(k_1+1|k_1) \\ \vdots \\ \hat{\mathbf{T}}_{\text{rw}}(k_1+N_c-1|k_1) \end{bmatrix}$$

In (3.19), $\hat{\mathbf{\epsilon}}_e$ contains the predicted $\mathbf{\epsilon}_e$ in the prediction horizon N_p and $\hat{\mathbf{T}}_{\text{rw}}$ is the control torque in the control horizon N_c .

In MPC strategy, the optimal control torque $\hat{\mathbf{T}}_{\text{rw}}$ is obtained by minimizing the following quadratic cost function:

$$J_{UC-MPC} = (\hat{\mathbf{\epsilon}}_e - \mathbf{\epsilon}_e^*)^T (\hat{\mathbf{\epsilon}}_e - \mathbf{\epsilon}_e^*) + \hat{\mathbf{T}}_{\text{rw}}^T \hat{\mathbf{T}}_{\text{rw}} \quad (3.20)$$

where $\mathbf{\epsilon}_e^*$ is the set point outputs in the prediction horizon N_p defined as follows:

$$\mathbf{Y}^* = \mathbf{\epsilon}_e^* = \begin{bmatrix} \mathbf{\epsilon}_e^*(k_1 + 1 | k_1) \\ \mathbf{\epsilon}_e^*(k_1 + 2 | k_1) \\ \vdots \\ \mathbf{\epsilon}_e^*(k_1 + N_p | k_1) \end{bmatrix}$$

It can be noted that equal weightage between the error and the control term is used in the cost function (3.20). The weightage can be adjusted if the performance requirements stipulated in Table 3-2 is not met for the chosen T_{s1} , N_p and N_c in section 3.5.

Expanding (3.20)

$$J_{UC-MPC} = (\mathbf{Y}^* - \Phi \mathbf{X}(k_1))^T (\mathbf{Y}^* - \Phi \mathbf{X}(k_1)) - 2\hat{\mathbf{T}}_{rw}^T \mathbf{R}^T (\mathbf{Y}^* - \Phi \mathbf{X}(k_1)) + \hat{\mathbf{T}}_{rw}^T (\mathbf{R}^T \mathbf{R} + \mathbf{I}) \hat{\mathbf{T}}_{rw} \quad (3.21)$$

Differentiating (3.21) w.r.t. $\hat{\mathbf{T}}_{rw}$, the first derivative of the cost function is obtained:

$$\frac{dJ_{UC-MPC}}{d\hat{\mathbf{T}}_{rw}} = -2\mathbf{R}^T (\mathbf{Y}^* - \Phi \mathbf{X}(k)) + 2(\mathbf{R}^T \mathbf{R} + \mathbf{I}) \hat{\mathbf{T}}_{rw} \quad (3.22)$$

To obtain the minimum of J_{UC-MPC} , equate (3.22) to 0,

$$-2\mathbf{R}^T (\mathbf{Y}^* - \Phi \mathbf{X}(k)) + 2(\mathbf{R}^T \mathbf{R} + \mathbf{I}) \hat{\mathbf{T}}_{rw} = 0 \quad (3.23)$$

Rearranging (3.23), the optimal control action can be calculated:

$$\hat{\mathbf{T}}_{rw} = (\mathbf{R}^T \mathbf{R} + \mathbf{I})^{-1} \mathbf{R}^T (\mathbf{Y}^* - \Phi \mathbf{X}(k_1)) \quad (3.24)$$

As the control law uses the receding-horizon scheme [47], the control torque only uses the first element of (3.24):

$$\mathbf{T}_{\text{rw}}(k_1) = [\mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}] (\mathbf{R}^T \mathbf{R} + \mathbf{I})^{-1} \mathbf{R}^T (\mathbf{Y}^* - \Phi \mathbf{X}(k_1)) \quad (3.25)$$

As the set point refers to the desired vector component of error quaternion, $\boldsymbol{\varepsilon}_e^*$ is defined as $[0 \quad 0 \quad 0]^T$. Therefore, (3.25) can be reduced to:

$$\mathbf{T}_{\text{rw}}(k_1) = -[\mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}] (\mathbf{R}^T \mathbf{R} + \mathbf{I})^{-1} \mathbf{R}^T \Phi \mathbf{X}(k_1) \quad (3.26)$$

which simplifies to a fixed gain state feedback controller:

$$\mathbf{T}_{\text{rw}}(k_1) = \mathbf{K} [\boldsymbol{\varepsilon}_e^T(k_1) \quad \boldsymbol{\omega}^T(k_1)]^T = -\mathbf{G} [\boldsymbol{\varepsilon}_e^T(k_1) \quad \boldsymbol{\omega}^T(k_1)]^T \quad (3.27)$$

where

$$\mathbf{G} = [\mathbf{I} \quad \mathbf{0} \quad \cdots \quad \mathbf{0}] (\mathbf{R}^T \mathbf{R} + \mathbf{I})^{-1} \mathbf{R}^T \Phi$$

Using (3.18), the closed-loop poles of (3.26) can be obtained by computing the eigenvalues of the matrix $[\mathbf{A} - \mathbf{B}\mathbf{G}]$.

3.3.5 Formulation of constrained MPC

For the formulation of constrained MPC control law, the state space model in (3.17) is discretized [46] at a sampling time of T_{s1} to give

$$\begin{bmatrix} \boldsymbol{\varepsilon}_e(k_1 + 1) \\ \boldsymbol{\omega}(k_1 + 1) \\ \mathbf{h}_{\text{rw}}(k_1 + 1) \end{bmatrix} = [\mathbf{A}] \begin{bmatrix} \boldsymbol{\varepsilon}_e(k_1) \\ \boldsymbol{\omega}(k_1) \\ \mathbf{h}_{\text{rw}}(k_1) \end{bmatrix} + [\mathbf{B}] \mathbf{T}_{\text{rw}}(k_1) \quad (3.28)$$

$$\boldsymbol{\varepsilon}_e(k_1) = [\mathbf{C}] \begin{bmatrix} \boldsymbol{\varepsilon}_e(k_1) \\ \boldsymbol{\omega}(k_1) \\ \mathbf{h}_{\text{rw}}(k_1) \end{bmatrix}$$

where

$$\mathbf{A} = e^{\mathbf{A}_{2c} T_{S1}}, \mathbf{B} = \int_0^{T_{S1}} e^{\mathbf{A}_{2c} \lambda} d\lambda \mathbf{B}_{2c}, \mathbf{X} = \begin{bmatrix} \boldsymbol{\varepsilon}_e^T & \boldsymbol{\omega}^T & \mathbf{h}_{rw}^T \end{bmatrix}^T, \mathbf{C} = \begin{bmatrix} \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}$$

Using the discrete time state space model in (3.28) and following the steps shown from (3.19) to (3.21), the following cost function for constrained MPC control law can be obtained:

$$J_{\text{C-MPC}} = (\mathbf{Y}^* - \boldsymbol{\Phi} \mathbf{X}(k_1))^T (\mathbf{Y}^* - \boldsymbol{\Phi} \mathbf{X}(k_1)) - 2 \hat{\mathbf{T}}_{rw}^T \mathbf{R}^T (\mathbf{Y}^* - \boldsymbol{\Phi} \mathbf{X}(k_1)) + \hat{\mathbf{T}}_{rw}^T (\mathbf{R}^T \mathbf{R} + \mathbf{I}) \hat{\mathbf{T}}_{rw} \quad (3.29)$$

Unlike unconstrained MPC control law derived in section 3.3.4, the constrained MPC control law would calculate the optimal control actions to be taken subjected to any active constraints. For constrained MPC, Quadratic Programming (QP) minimizes the following generic cost function:

$$J_{\text{QP}} = \frac{1}{2} \hat{\mathbf{T}}_{rw}^T \mathbf{E} \hat{\mathbf{T}}_{rw} + \hat{\mathbf{T}}_{rw}^T \mathbf{F} \quad (3.30)$$

subjected to the active constraints:

$$\mathbf{M} \mathbf{x} \leq \boldsymbol{\gamma} \quad (3.31)$$

where $\mathbf{E}$ and $\mathbf{F}$ are obtained by comparing (3.30) with (3.29):

$$\mathbf{E} = 2(\mathbf{R}^T \mathbf{R} + \mathbf{I}) \quad (3.32)$$

$$\mathbf{F} = -2\mathbf{R}^T (\mathbf{Y}^* - \boldsymbol{\Phi} \mathbf{X}(k_1)) \quad (3.33)$$

Quadratic programming calculates $\hat{\mathbf{T}}_{\text{rw}}$ that minimizes J_{QP} in (3.30) subjected to any active constraints. From (3.29), the term $(\mathbf{Y}^* - \Phi\mathbf{X}(k_1))^T(\mathbf{Y}^* - \Phi\mathbf{X}(k_1))$ is independent of $\hat{\mathbf{T}}_{\text{rw}}$ and can therefore be ignored.

$\mathbf{M}$ and γ describe the active system constraints (as per section 3.3.3). From (3.28) and rearranging the equations, we obtain:

Table 3-3. Formulation of actuator constraints

$\mathbf{h}_{\text{rw-max}}$ limit	$\mathbf{T}_{\text{rw-max}}$ limit
$-\hat{\mathbf{T}}_{\text{rw}}(k_1 + i k_1) \leq \mathbf{h}_{\text{rw-max}} - \mathbf{h}_{\text{rw}}(k_1) \quad (3.34)$	$-\hat{\mathbf{T}}_{\text{rw}}(k_1 + i k_1) \leq \mathbf{T}_{\text{rw-max}} \quad (3.36)$
$\hat{\mathbf{T}}_{\text{rw}}(k_1 + i k_1) \leq \mathbf{h}_{\text{rw-max}} + \mathbf{h}_{\text{rw}}(k_1) \quad (3.35)$	$\hat{\mathbf{T}}_{\text{rw}}(k_1 + i k_1) \leq \mathbf{T}_{\text{rw-max}} \quad (3.37)$

where $\mathbf{h}_{\text{rw-max}} = [\mathbf{h}_{\text{rw-max}} \quad \mathbf{h}_{\text{rw-max}} \quad \mathbf{h}_{\text{rw-max}}]^T$, $\mathbf{T}_{\text{rw-max}} = [\mathbf{T}_{\text{rw-max}} \quad \mathbf{T}_{\text{rw-max}} \quad \mathbf{T}_{\text{rw-max}}]^T$. The corresponding values of $\mathbf{h}_{\text{rw-max}}$ and $\mathbf{T}_{\text{rw-max}}$ can be found in Table 3-1.

Using (3.34) to (3.37), the actuator constraints can be expressed as:

$$\begin{bmatrix}
 -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \vdots & & \ddots & \vdots \\
 -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \cdots & -\mathbf{I}_{3 \times 3} \\
 -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \vdots & & \ddots & \vdots \\
 -\mathbf{I}_{3 \times 3} & -\mathbf{I}_{3 \times 3} & \cdots & -\mathbf{I}_{3 \times 3} \\
 \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \vdots & & \ddots & \vdots \\
 \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \cdots & \mathbf{I}_{3 \times 3} \\
 \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\
 \vdots & & \ddots & \vdots \\
 \mathbf{I}_{3 \times 3} & \mathbf{I}_{3 \times 3} & \cdots & \mathbf{I}_{3 \times 3}
 \end{bmatrix}
 \begin{bmatrix}
 \hat{\mathbf{T}}_{\text{rw}}(k_1 + 0 | k_1) \\
 \hat{\mathbf{T}}_{\text{rw}}(k_1 + 1 | k_1) \\
 \vdots \\
 \hat{\mathbf{T}}_{\text{rw}}(k_1 + N_c - 1 | k_1)
 \end{bmatrix}
 \leq
 \begin{bmatrix}
 \mathbf{h}_{\text{rw-max}} - \mathbf{h}_{\text{rw}}(k_1) \\
 \mathbf{h}_{\text{rw-max}} - \mathbf{h}_{\text{rw}}(k_1) \\
 \vdots \\
 \mathbf{h}_{\text{rw-max}} - \mathbf{h}_{\text{rw}}(k_1) \\
 \mathbf{T}_{\text{rw-max}} \\
 \mathbf{T}_{\text{rw-max}} \\
 \vdots \\
 \mathbf{T}_{\text{rw-max}} \\
 \mathbf{h}_{\text{rw-max}} + \mathbf{h}_{\text{rw}}(k_1) \\
 \mathbf{h}_{\text{rw-max}} + \mathbf{h}_{\text{rw}}(k_1) \\
 \vdots \\
 \mathbf{h}_{\text{rw-max}} + \mathbf{h}_{\text{rw}}(k_1) \\
 \mathbf{T}_{\text{rw-max}} \\
 \mathbf{T}_{\text{rw-max}} \\
 \vdots \\
 \mathbf{T}_{\text{rw-max}}
 \end{bmatrix}
 \quad (3.38)$$

The controller developed in section 3.3.4 and in section 3.3.5 have been implemented in the ADCS microcontroller of VELOX-II. To have an insight of implementation challenges using a microcontroller for both unconstrained MPC and constrained MPC, the complexity of the algorithms needs to be evaluated.

In section 3.3.4, the unconstrained MPC was derived to obtain a simple fixed gain state feedback controller (3.27) with a state matrix, $\mathbf{X} \in \mathbb{R}^{6 \times 1}$. This means that the controller gain, $\mathbf{G} \in \mathbb{R}^{3 \times 6}$ can be computed offline and its size is fixed regardless of chosen N_p and N_c .

On the other hand, the derivation of constrained MPC leads to an increase in the size of the state matrix, $\mathbf{X} \in \mathbb{R}^{9 \times 1}$ to handle constraints through QP. In addition, the variable $\mathbf{F}$ in (3.33) needs to be calculated online as an input for QP. The size of the variable Φ in (3.33) with definition given in (3.19) is a function of N_p and therefore

the complexity of matrix multiplication involved will be greatly increased for very high N_p .

Furthermore, dynamic memory allocation is required for constrained MPC to be implemented for any N_p and N_c where the user has the freedom to change the said tuning parameters without altering the software algorithm. Since dynamic memory allocation may cause memory leaks, it is classified as dangerous programming in microcontrollers. Therefore, implementing QP becomes challenging for microcontrollers. However, it will not be an issue with PC/microprocessors having very big memory size.

3.3.6 Sun-pointing algorithm of VELOX-II

As explained in section 3.2, Sun-pointing will be the default (or house-keeping) pointing mode of VELOX-II.

Typically, when magnetic torquers are activated for momentum dumping of the reaction wheels, they distort the magnetic field measured by the magnetometer. Several methods are presented in the literature [48, 49] to filter out the correct magnetometer reading with magnetic torquers switched on. However, such a method was not implemented in VELOX-II. Therefore, $\mathbf{q}_s$ computed using attitude determination algorithms [50] will be invalid to calculate $\mathbf{q}_E$ (3.3) for attitude control. As a result, the typical Sun-pointing method illustrated in Fig. 2-2 for a near equatorial orbit satellite was not feasible for VELOX-II. However, momentum dumping was not performed during target tracking and nadir pointing modes, making $\mathbf{q}_s$ available for attitude control.

In the sequel, ACS of VELOX-II employs a Sun-pointing mode that does not require $\mathbf{q}_s$. Under this mode, the ACS re-orientates the satellite such that $-\mathbf{Z}_b$ (i.e. the solar panel normal vector) is constantly aligned with the sun vector measured by the sun sensors through-out the non-eclipse period with constant momentum dumping by the magnetic torquers. The quaternion error for re-orientation can be obtained by

$$-\mathbf{Z}_b = \mathbf{A}_E \mathbf{S}_b \quad (3.39)$$

where $-\mathbf{Z}_b$ is the solar panel normal vector (see Fig. 3-1) and is considered in the target frame, $\mathbf{S}_b$ is in the body frame and $\mathbf{q}_E$ will be the quaternion equivalent form of $\mathbf{A}_E$ [27, 42, 51].

One drawback of (3.39) is that, it results in single-axis attitude control and therefore $\mathbf{X}_b$ and $\mathbf{Y}_b$ of the satellite in the ECI frame will be arbitrary assigned. The algorithm in (3.39) will be the default sun-pointing strategy. On the other hand, the efficient momentum dumping pointing method that will be presented in Chapter 4 is considered as the safe-hold pointing mode in the event where one or more reaction wheels suffer from momentum saturation.

3.4 Simulation results and analysis of MPC control law.

This section will analyse both constrained and unconstrained MPC subjected to different levels of gyroscope noise and bounded errors. Their performance were evaluated under different performance criteria such as:

- Steady state error.
- Settling time.
- Maximum overshoot.

3.4.1 Simulation model

The sampling time (i.e. T_{s1}) for both unconstrained and constrained MPC will be fixed at 1s which is limited by the hardware design of VELOX-II ADCS. A fourth order Runge-Kutta integrator was used to propagate the kinematics and dynamic behaviour of the satellite in the simulation. SGP4 orbit propagator was used to simulate the position and velocity vectors of the satellite using the orbital elements (see Table 3-4). An initial disturbance of 180° was given for all simulations with a fixed target tracking vector in the body frame of the satellite. For both unconstrained and constrained MPC, simulations were conducted for varying N_p and N_c . The

pseudorandom function in MATLAB was used to simulate the random noise as per datasheet of both Gyroscopes. Table 3-4. Orbital elements of VELOX-II

Orbital elements	Value
Epoch year & Julian day fraction	15354.74
Inclination	14.986°
Right ascension of the ascending	179.099°
Eccentricity	0.0104
Argument of perigee	35.6°
Mean Anomaly	58.236°
Mean Motion	15.07

3.4.2 Performance criteria: mean error at steady state

Fig. 3-2. MPC performance measurement using mean error at steady state as an index

Fig. 3-2 uses the steady state mean error as an index to analyse how the control algorithm performs when subjected to different MPC algorithms (i.e. unconstrained and constrained MPC) for two types of gyroscope noise.

From Fig. 3-2, both MPC algorithms perform fairly the same when subjected to the same gyroscope noise. The main reason for this same behaviour is that the actuator

constraints are not active in the steady state and unconstrained MPC algorithm will perform in a similar manner to the constrained MPC algorithm.

In addition, higher gyroscope noise leads to higher steady state error for both controllers. This is because ω is a state variable in the control equation as per (3.18) and (3.28) for unconstrained and constrained MPC algorithms respectively.

Both Fig. 3-2 (a) and (b) shows the same relationship between steady state error, N_c and N_p . From the results, it can be observed that the steady state error increases with increasing N_p for $N_c=1$ and $N_p > 4$. This is because the gyroscope noise accumulates along N_p but is no longer the case for $N_c > 1$ where the control action is adjusted to take into account of the accumulation of the gyroscope noise.

However, Fig. 3-3 (with $N_c=1$ filtered out) shows that the steady state error is higher for $N_p < 2$. This behaviour can be explained by using the finite horizon optimal control problem of MPC. Having lower N_p means that the predicted state can differ considerably from the actual state and thus the control system is not necessarily be stable or optimal [40].

The complexity of the MPC algorithm increases exponentially with increasing horizon N (i.e. N_p and N_c) [40]. For constrained MPC, small N means fewer variables or faster computation time when QP is used to minimize (3.30) subjected to (3.31) at each control level. However, complexity of the control algorithm is not a disadvantage for unconstrained MPC as it employs a fixed gain controller (3.27) unlike the constrained MPC that has an online controller gain due to QP.

Fig. 3-3. Closer look at Fig. 3-2 (i.e. $N_c > 1$) for gyroscope noise of 0.9 deg/s.

3.4.3 Performance criteria: settling time

Fig. 3-4. MPC performance measurement using settling time as an index for gyroscope noise of 0.27 deg/s

Fig. 3-5. MPC performance measurement using settling time as an index for gyroscope noise of 0.9 deg/s

Fig. 3-4 and Fig. 3-5 use the settling time as an index to analyse how the control algorithm performs when subjected to different MPC algorithms for two types of gyroscope noise.

It can be clearly observed that the settling time for the controller employing constraint MPC algorithm is lesser than the unconstrained MPC algorithm. The observation made is irrespective of the gyroscope noise level. This clearly shows that a controller which is aware of its actuator constraints can produce an optimum controller sequence in the horizon N_c . In addition, a similar observation as in section 3.4.2 can be made where the settling time increases with increasing N_p for $N_c=1$. The said observation exists for both unconstrained and constrained MPC.

Fig. 3-4 (b) and Fig. 3-5 (b) show that for $N_c > 1$ and $N_p > 6$ the settling time increases linearly with increasing N_p and is independent of N_c . The controller predicts for more future samples (i.e.) and minimizes the cost function to achieve the steady state within the predicted samples. Therefore, settling time increases with

increasing predicted samples or N_p . In other words, as the N_p increases, the controller becomes less aggressive (i.e. the controller gain reduces) and thus causes an increase in the settling time. Yet, for $N_c=1$ and $N_p < 6$, the settling time increases as N_p reduces which can once again be explained using the finite horizon optimal control problem. However, for unconstrained MPC (with $N_c > 1$) results shown in Fig. 3-4 (a) and Fig. 3-5 (a), such observation does not appear. This is because actuator constraints will be active during the transient phase of the controller that is not taken care of by the unconstrained MPC algorithm.

Finally, the lowest settling time across all was found at $N_c=1$ and $N_c=2$ for unconstrained and constrained MPC respectively.

3.4.4 Performance criteria: overshoot

Fig. 3-6. MPC performance measurement using max overshoot as an index for gyroscope noise of 0.27 deg/s

Fig. 3-7. MPC performance measurement using max overshoot as an index for gyroscope noise of 0.9 deg/s

Fig. 3-6 and Fig. 3-7 use overshoot as an index to study how the control algorithm performs when subjected to different MPC algorithms for two types of gyroscope noise. It can be clearly seen that the observations made for both MPC algorithms are independent of gyroscope noise and therefore can be ignored.

Fig. 3-6 (b) and Fig. 3-7 (b) show that for the constrained MPC with $N_p \geq 6$ and $N_c > 1$, there is no overshoot observed unlike unconstrained MPC algorithm, Fig. 3-6 (a) and Fig. 3-7 (a). Therefore using a controller which is aware of its actuator

constraints can produce an optimum controller sequence in the horizon. For $N_p < 6$, the maximum overshoot increases as N_p reduced and can once again be explained using the finite horizon control problem.

However, the behaviour is different for unconstrained MPC where zero maximum overshoot exists only for $N_p > 3$ and $N_c = 1$ (i.e. the actuator constraints are inactive). For $N_p > 3$ and $N_c > 1$, the actuator constraints are not taken care of by the unconstrained MPC algorithm and thus results in an overshoot during the transient phase of the controller.

3.4.5 MPC subjected to gyroscope bounded errors

Fig. 3-8. MPC performance measurement using mean error at steady state as an index for gyroscope noise of 0.9 deg/s subjected to different gyroscope bounded errors.

Fig. 3-9. Closer view of Fig. 3-8 (i.e. $N_c > 1$)

Fig. 3-8 and Fig. 3-9 use the mean error at steady state as an index to study how the control algorithm performs when subjected to different gyroscope bounded errors for different MPC algorithms.

Similar to the case in section 3.4.2, Fig. 3-8 shows that both MPC algorithms perform fairly the same when subjected to the same gyroscope noise. The main reason for this same behaviour is that the actuator constraints are not active in the steady state and unconstrained MPC algorithm will perform in a similar manner to the constrained MPC algorithm.

Fig. 3-8 also shows that, the mean error at steady state increases with increasing N_p for $N_c=1$ as the gyroscope bounded error accumulates along the N_p . As expected, the steady state error is larger when the gyroscope bounded error is large. This is because the error accumulation increases with increasing bounded error at each prediction along N_p . For $N_c > 1$ as the control sequence in the horizon N_c is adjusted to account for the gyroscope bounded errors and the steady state error is greatly reduced and is independent of N_p .

Fig. 3-9 gives a closer view of Fig. 3-8 for $N_c > 1$. It is noted that the bounded gyroscope still affects the steady state error and increases with increasing gyroscope bounded error. Fig. 3-9 also shows that unconstrained MPC is less sensitive to gyroscope bounded errors as compared to constrained MPC. Augmented version of discrete time state space (3.40) was used to overcome the observed steady state error due to gyroscope bounded error across all N_p and N_c . The implemented controller resulted in excessive and undesirable control action.

Using (3.28), augmented MPC can be formulated as:

$$\begin{bmatrix} \Delta \boldsymbol{\varepsilon}_e(k_1+1) \\ \Delta \boldsymbol{\omega}(k_1+1) \\ \Delta \mathbf{h}_{rw}(k_1+1) \\ \boldsymbol{\varepsilon}_e(k_1+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{3 \times 3} \\ \mathbf{C} & \mathbf{I}_{3 \times 3} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\varepsilon}_e(k_1) \\ \Delta \boldsymbol{\omega}(k_1) \\ \Delta \mathbf{h}_{rw}(k_1) \\ \boldsymbol{\varepsilon}_e(k_1) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0}_{3 \times 3} \end{bmatrix} [\Delta \mathbf{T}_{rw}(k_1)] \quad (3.40)$$

$$\boldsymbol{\varepsilon}_e(k) = \begin{bmatrix} \mathbf{C} & \mathbf{I}_{3 \times 3} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{\varepsilon}_e(k_1) \\ \Delta \boldsymbol{\omega}(k_1) \\ \Delta \mathbf{h}_{rw}(k_1) \\ \boldsymbol{\varepsilon}_e(k_1-1) \end{bmatrix}$$

3.5 Selection of MPC controller parameters.

3.5.1 Selection based on ACS requirements.

From all the results presented from sections 3.4.2 to 3.4.5, the following conclusions can be made:

- Very low N_p should be avoided as the predicted state can differ considerably from the actual state and thus the control system is not necessarily be stable or optimal [40].
- For $N_c=1$ and very high N_p , it results in the accumulation of any present gyroscope noise or bounded errors affecting the steady state performance.
- $N_c > 1$ result in overshoot for unconstrained MPC.
- The complexity of MPC algorithm increases exponentially with increasing N_p and N_c . therefore lower N is recommended for constrained MPC [47].

Table 3-5 presents selected N_p and N_c values for both unconstrained and constrained MPC algorithms. The selected N values satisfy VELOX-II ACS requirements when mean error at steady state, settling time and overshoot are taken as performance indexes (see Table 3-2).

Table 3-5. Summary: final selection of N for both constrained and unconstrained MPC

MPC controller	Tuning result of N_p	Tuning result of N_c
	Value	Value
Unconstrained	$N_p = 4$	$N_c = 1$
Constrained MPC	$N_p = 6$	$N_c = 2$

Table 3-6 and Fig. 3-10 present performance of both unconstrained and constrained MPC for selected N values subjected to different gyroscope noise levels. From the results, both the controllers satisfy VELOX-II ACS requirements. However, constrained MPC has a 1.3% and 8.8% better accuracy as compared to the unconstrained MPC for gyroscope noise of 0.27 deg/s and 0.9 deg/s respectively.

Table 3-6. Performance results of unconstrained and constrained MPC for selected tuning parameter N .

MPC controller	Gyroscope noise	Performance criteria			
		Error at steady state		Settling time	Overshoot
		Mean error	STD (σ)		
Unconstrained MPC	0.27 °/s	0.3918 °	0.1789 °	20s-24s	Nil
	0.9 °/s	0.9144 °	0.4218 °		
Constrained MPC	0.27 °/s	0.3868 °	0.1827 °		
	0.9 °/s	0.8406 °	0.4002 °

Fig. 3-10. Step responses of unconstrained and constrained MPC for selected tuning parameter N .

Though N_p for constrained MPC have been minimized to reduce the online computation time, implementing constrained MPC with QP requires additional firmware code size and was a limiting factor for VELOX-II. This is because both ACS

firmware and Attitude Determination System firmware share the same microcontroller (i.e. C8051F120).

It was identified in section 3.4.5 that choosing $N_c > 1$ for both constrained and unconstrained MPC is more desirable as doing so makes both the controllers less sensitive to gyroscope bounded errors. However, choosing $N_c > 1$ and unconstrained MPC results in an overshoot and increase in settling time. Therefore, if gyroscope bounded errors can be minimized through proper calibration of the gyroscope and testing (see section 5.5), the unconstrained MPC will be the most suitable for VELOX-II ACS. However, if the actuator constraints becomes more stringent for future VELOX series of nanosatellites, then constrained MPC would be a suitable choice for implementation.

3.5.2 Selection based on ACS requirements and $\mathbf{h}_{rw}$ usage.

In sections 2.2.2 to 2.4, it was highlighted that satellites in NEOs (such as VELOX-II) are more prone to loss of three axis stabilization as a result of $\mathbf{h}_{rw}$ saturation.

Fig. 3-11. Steady state normalised angular momentum for $N_c = 1$ and changing N_p .

Fig. 3-11 show the steady state $\|\bar{\mathbf{h}}_{rw}\|$ needed for constant Sun-pointing reduces as N_p increases and becomes a near constant for $N_p \geq 12$. As explained in section 3.4.3, when the controller predicts for more future samples (i.e. N_p), the cost function is minimized to achieve the steady state within the predicted samples. This results in a reduction of controller gain (i.e. lower steady state $\|\bar{\mathbf{h}}_{rw}\|$ is needed) and an increase in settling time.

Table 3-7 and Fig. 3-12 present performance of unconstrained MPC algorithm with a N_p of 12 and N_c of 1 for different gyroscope noise levels. Comparing Table 3-6 with Table 3-7, we can notice an increase in both mean error and settling time and are expected as per explanations given in sections 3.4.2 and section 3.4.3. Also from section 3.4.3, we know that a further increase in N_p results in a settling time of more than 60s.

Table 3-7. Performance results of unconstrained MPC for N_p of 12 and N_c of 1.

MPC controller	Gyroscope noise	Performance criteria			
		Error at steady state		Settling time	Overshoot
		Mean error	STD (σ)		
Unconstrained	0.27 °/s	0.5 °	0.24 °	60s	Nil
MPC	0.9 °/s	1.2 °	0.64 °

Fig. 3-12. Step responses of unconstrained MPC for $N_c = 1$ and $N_p=12$.

Therefore, a lower steady state $\|\bar{\mathbf{h}}_{\text{rw}}\|$ (i.e. higher safety margin for momentum saturation) has been attained by compromising both steady state error and settling time performances. However, compromised performances are still within the ACS requirements stipulate in Table 3-2. As a result, unconstrained MPC with a N_p of 12 and a N_c of 1 should be used as the default tuning controller parameters for VELOX-II and the corresponding controller gain matrix $\mathbf{K}$, as per definition in (3.27) is:

$$\mathbf{K}_{\frac{N_c:1}{N_p:12}} = \begin{bmatrix} -7.4679 & -0.0000 & 0.2604 & -31.6868 & -0.0000 & 1.1051 \\ -0.0000 & -8.2718 & -0.0075 & -0.0000 & -35.0977 & -0.0319 \\ 0.2604 & -0.0075 & -3.7740 & 1.1051 & -0.0319 & -16.0135 \end{bmatrix} \quad (3.41)$$

However, if higher performance is demanded for nadir and target tracking, then the corresponding controller gain matrix $\mathbf{K}$ for N_p of 4 and a N_c of 1 can be uploaded to VELOX-II by ground command:

$$\mathbf{K}_{\frac{N_c:1}{N_p:4}} = \begin{bmatrix} -22.7161 & -0.0000 & 0.7921 & -35.4940 & -0.0000 & 1.2377 \\ -0.0000 & -25.1605 & -0.0228 & -0.0000 & -39.3133 & -0.0357 \\ 0.7921 & -0.0228 & -11.4814 & 1.2377 & -0.0357 & -17.9397 \end{bmatrix} \quad (3.42)$$

The unit of $\mathbf{T}_{rw}$ for $\mathbf{K}$ in (3.41) and (3.42) as per definition in (3.26) is in m.Nm.

In addition, (3.41) and (3.42) can be generalized in to the following form:

$$\mathbf{K}_{\frac{N_c:i}{N_p:j}} = \begin{bmatrix} \mathbf{k}_{\epsilon_e \frac{N_c:i}{N_p:j}} \end{bmatrix} \begin{bmatrix} \mathbf{k}_{\omega \frac{N_c:i}{N_p:j}} \end{bmatrix} \quad (3.43)$$

where $\mathbf{k}_{\epsilon_e \frac{N_c:i}{N_p:j}} \in \mathbb{R}^{3 \times 3}$ and $\mathbf{k}_{\omega \frac{N_c:i}{N_p:j}} \in \mathbb{R}^{3 \times 3}$ are the gain corresponding to ϵ_e and ω

respectively at $N_c = i$ and $N_p = j$.

From (3.41) and (3.42), one can notice that an increase in N_p (with N_c unchanged) results in a significant reduction in the gain, $\mathbf{k}_{\epsilon_e \frac{N_c:i}{N_p:j}}$. This analysis compliments the explanation given in section 3.4.3 with regard to the relationship between N_p and controller settling time.

3.6 Controllability and close-loop stability analysis

Having selected a suitable MPC algorithm and its tuning parameters, it is in-deed necessary to analyse the controllability and closed-loop stability of the control algorithm.

3.6.1 Controllability

In order to showcase that the controller designed in section 3.3.4 is completely controllable, we use the “controllability rank condition” theorem [52].

With the discrete time state space model given in (3.18) and denote the dimension of the $\mathbf{B}$ matrix as $n \times m$, then a linear time-invariant system is completely controllable if and only if the $n \times m.n$ controllability matrix:

$$\mathbf{W}_c = [\mathbf{B} \quad \mathbf{AB} \quad \mathbf{A}^2\mathbf{B} \quad \dots \quad \mathbf{A}^{n-1}\mathbf{B}] \quad (3.44)$$

has rank n .

$\mathbf{A}$ and $\mathbf{B}$ matrices of (3.18) is calculated to be:

$$\mathbf{A}_{6 \times 6} = \begin{bmatrix} 1 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B}_{6 \times 3} = \begin{bmatrix} 8.4039 & -0.0005 & 0.5800 \\ -0.0005 & 7.5689 & -0.0151 \\ 0.5800 & -0.0151 & 16.6293 \\ 33.6154 & -0.0021 & 2.3198 \\ -0.0021 & 30.2756 & -0.0604 \\ 2.3198 & -0.0604 & 66.5172 \end{bmatrix} \quad (3.45)$$

where $n=6$ and $m=3$.

Substituting (3.45) in to (3.44), the corresponding rank of matrix $\mathbf{W}_c$ can be calculated to be n where $n=6$ (i.e. full rank). Therefore, the controller designed in section 3.3.4 is completely controllable (i.e. pair $(\mathbf{A}, \mathbf{B})$ is controllable).

3.6.2 Close-loop stability

Referring to (3.27), the closed-loop state matrix is given by [52]:

$$\mathbf{A}_{cl} = [\mathbf{A} - \mathbf{B}\mathbf{G}] \quad (3.46)$$

where $\mathbf{G} = -\mathbf{K}$.

The corresponding eigenvalues (i.e. λ_i) of $\mathbf{A}_{cl}$ determines the closed-loop poles of the system.

By substituting (3.41), (3.42) and (3.45) into (3.46), the eigenvalues of closed-loop state matrix are computed and are presented in Table 3-8 and Fig. 3-13.

Table 3-8. Eigenvalues of closed-loop state matrix for $N_p=4$ and $N_p=12$.

	$N_c=1; N_p=4$	$N_c=1; N_p=12$
Closed-loop poles	$\lambda_1 = 6.9201e^{-5} + 0i$ $\lambda_2 = 2.7492e^{-4} + 0i$ $\lambda_3 = 3.3564e^{-4} + 0i$ $\lambda_4 = 0.619 + 0i$ $\lambda_5 = 0.619 + 0i$ $\lambda_6 = 0.619 + 0i$	$\lambda_1 = 1.7885e^{-6} + 0i$ $\lambda_2 = 7.1060e^{-6} + 0i$ $\lambda_3 = 8.6758e^{-6} + 0i$ $\lambda_4 = 0.8748 + 0i$ $\lambda_5 = 0.8748 + 0i$ $\lambda_6 = 0.8748 + 0i$

Fig. 3-13. Plot of eigenvalues of closed-loop state matrix for $N_p=4$ and $N_p=12$.

As all calculated eigenvalue lies within an open unit circle, the derived controller using unconstrained MPC with $N_c=1$ and $N_p=4$ or $N_p=12$ is closed-loop stable.

3.7 In-orbit results

From section 3.5.2, the default tuning parameters for the unconstrained MPC algorithm has a N_c of 1 and a N_p of 12. The default gyroscope has a noise of $0.9^\circ/\text{s}$ (see Table 3-1). In order to analyse the Sun-pointing performance of VELOX-II, the angular error between the sun vector and $-\mathbf{Z}_b$ axis was calculated. Fig. 3-14 presents the in-orbit Sun-pointing performance of VELOX-II with data recorded at an interval of 1 minute. The mean error is 1.166° with a standard deviation of 0.578° .

Fig. 3-14. . Sun-pointing performance recorded from 16th Dec. 2015 to 1st Feb. 2016.

Table 3-9 summarizes both simulated and in-orbit performance of VELOX-II ACS during the default Sun-pointing mode. It is noted that the in-orbit performance matches closely with the simulated performance. It should also be noted that the gyroscope noise of $0.9^\circ/\text{s}$ refers to the root mean square noise experienced at the maximum dynamic range of $300^\circ/\text{s}$ [53]. Therefore the in-orbit performance is in-deed

subjected to a lower gyroscope noise levels and thus results in a lower mean error and standard deviation when compared with the simulated results.

In addition, it was also found that the in-orbit settling time of the controller is less than 60s. As the in-orbit data is recorded at an interval of 1 minute, the in-orbit performance of controller settling time will also be 1 minute. In chapter 4, the settling time of the controller will be validated experimentally using a hardware-in-the-loop spacecraft simulator.

Table 3-9. In-orbit vs simulated performance of unconstrained MPC for N_p of 12 and N_c of 1.

	N_p	N_c	Gyroscope noise	Performance criteria			
				Error at steady state		Settling time	Overshoot
				Mean error	STD (σ)		
Simulations	12	1	0.9 °/s	1.2 °	0.64 °	60s	Nil
In-orbit				1.166 °	0.578°	<60s	Nil after 60s

3.8 Summary

This chapter presented unconstrained and constrained MPC algorithms. The performance of both the controllers were studied using mean error at steady state, settling time and overshoot as performance indexes for varying prediction horizon N_p and control horizon N_c .

Results and analysis revealed that the constrained MPC has 1.3% and 8.8% better accuracy as compared to the unconstrained MPC for gyroscope noise of 0.27 deg/s and 0.9 deg/s respectively. However, performance of both the controllers with selected

tuning parameters (i.e. $N_c=1$ and $N_p=4$ for unconstrained MPC and $N_c=2$ and $N_p=6$ for constrained MPC) are well beyond the required performance of VELOX-II ACS. In addition, QP of constrained MPC required additional firmware code memory and was a limiting factor for VELOX-II. Therefore, unconstrained MPC was selected as the most suitable attitude acquisition controller for VELOX-II. However, if the actuator constraints becomes more stringent for future VELOX series of nanosatellites, then the constrained MPC would be a suitable choice for implementation.

It was noted that having a N_c of 1 makes the controller more sensitive to gyroscope bounded errors and therefore proper calibration and testing of the gyroscope was required. In addition, having a N_p of 12 with compromised pointing error (but still within the requirements) is more desirable for the satellite during Sun-pointing as it provides a higher safety margin for momentum saturation. However, N_p of 4 shall be used if higher pointing accuracy is demanded.

Finally, the simulated performance of the unconstrained MPC control law was compared to its in-orbit performance. It was found that the in-orbit performance matches closely with the simulated performance.

CHAPTER 4

AN EFFICIENT MOMENTUM DUMPING METHOD FOR NEAR EQUATORIAL ORBIT SATELLITES

4.1 Overview

It was presented in chapter 2 that the torque produced by the magnetic torquers is limited to a plane that is only perpendicular to the geomagnetic field across the satellite body frame. Nevertheless, if the orbit inclination angle with respect to the geomagnetic equator is large, the magnetic field vector periodically changes its direction with respect to the Earth's inertial reference frame. As a result, on average over the orbital period, the magnetic torques can be applied in all directions within a desired time period. On the other hand, the Earth magnetic field vector for satellites in the near equatorial orbit (NEO) is at a much lower variation. This results in the satellite to lose its three-axis stabilization capability.

In this chapter, a new Sun-pointing strategy for the improvement of reaction wheel angular momentum dumping strategy for a near equatorial orbit nanosatellite will be presented. The proposed method aligns both the solar panel of the satellite towards the Sun as well as a secondary body frame vector with the Earth's geomagnetic field vector. To describe the kinematics needed for the proposed method, a cost function will be derived using QUEST (QUaternion ESTimation) method to obtain the error quaternion for re-orientation of the satellite. To produce an optimal control torque, the calculated error quaternion will then be used by the model

predictive controller developed in chapter 3. Finally, simulation results will be presented to benchmark the proposed method against the typical Sun-pointing strategy introduced in chapter 2.

4.2 Momentum dumping controller

In this research, the magnetic torquers are used to assist reaction wheel to unload the saturated momentum. The torque generated by a magnetic torquer, $\mathbf{T}_{mt}$ is given by (2.2). The momentum dumping control law is formulated as

$$\mathbf{T}_{ideal} = -g(\mathbf{h}_{rw} - \mathbf{h}_{Nr w}) = -g\Delta\mathbf{h}_{rw} \quad (4.1)$$

where g is the gain of the momentum dumping controller, $\mathbf{h}_{rw} \in \mathbb{R}^3$ is the reaction wheel stored angular momentum, $\mathbf{h}_{Nr w} \in \mathbb{R}^3$ is the nominal reaction wheel angular momentum and $\Delta\mathbf{h}_{rw} = (\mathbf{h}_{rw} - \mathbf{h}_{Nr w})$.

To determine the required $\mathbf{M}$ to achieve $\mathbf{T}_{ideal}$, an orthogonal projector is used to project $\mathbf{T}_{ideal}$ onto the plane perpendicular to $\mathbf{B}_b$ [54, 55]. A discrete bang-bang control law is then applied with $g = 1$

$$\mathbf{M} = \mathbf{M}_{max} \text{sign}\left(\frac{\mathbf{B}_b \times \mathbf{T}_{ideal}}{\|\mathbf{B}_b\|^2}\right) \quad (4.2)$$

where $\mathbf{M}_{max} \in \mathbb{R}^3$ is the tri-axial maximum dipole strength of the magnetic torquers, $\|\mathbf{B}_b\|$ is the norm of the vector $\mathbf{B}_b$ and $\text{sgn}(\cdot)$ is the signum function.

The control laws given in (3.27) and (4.2) form a distributed control system with two controllers running in parallel. While performing momentum dumping, the control law in (4.2) also produces a disturbance torque leading to a pointing error. In parallel,

the control law in (3.27) attempts to correct the pointing error by using the reaction wheels to absorb the produced disturbance torque. In this process, the reaction wheels' stored angular momentum is reduced.

4.3 Proposed efficient momentum dumping controller

To overcome the limitations that the magnetic torquers are unable to perform efficient momentum dumping along certain axis in NEO, a strategy to align $-\mathbf{Z}_b$ to $\mathbf{S}_b$ and a secondary body frame vector, $\mathbf{V}_b^{\text{opt}} \in \mathbb{R}^3$ to $\mathbf{B}_b$ as shown in Fig. 4-1 is proposed.

Fig. 4-1 Proposed efficient momentum dumping strategy

For optimal momentum dumping of the reaction wheel, the direction of the ideal torque to be applied and actual torque produced by the magnetic actuators should be similar, i.e.

$$\frac{\mathbf{T}_{\text{ideal}}}{\|\mathbf{T}_{\text{ideal}}\|} = \frac{\mathbf{T}_{\text{mt}}}{\|\mathbf{T}_{\text{mt}}\|} \quad (4.3)$$

To achieve the condition (4.3), $\mathbf{B}_b$ must be orthogonal to both $\mathbf{T}_{\text{ideal}}$ and $\mathbf{M}$. Thus the optimal $\mathbf{V}_b^{\text{opt}}$ at any instance is

$$\mathbf{V}_b^{\text{opt}} = \frac{\mathbf{T}_{\text{ideal}}}{\|\mathbf{T}_{\text{ideal}}\|} \times \frac{\mathbf{M}}{\|\mathbf{M}\|} \quad (4.4)$$

To calculate $\mathbf{q}_E$ in (3.27), let the Sun vector in the satellite body frame be $\mathbf{S}_b$. It should be noted that $\mathbf{B}_b$ and $\mathbf{S}_b$ can be obtained from the satellite's magnetometers and Sun sensors respectively. Let $\mathbf{A}_E$ be the rotation matrix corresponding to $\mathbf{q}_E$, then the cost function to be minimized can be defined as

$$J_{EMD} = \left\| \mathbf{B}_b - \mathbf{A}_E \mathbf{V}_b^{\text{opt}} \right\|^2 + w \left\| \mathbf{S}_b - \mathbf{A}_E (-\mathbf{Z}_b) \right\|^2 \quad (4.5)$$

where w is the weight penalty between the vector pairs.

$\mathbf{A}_E$ can be solved by applying an optimization technique to minimize the cost function in (4.5). As a perfect optimal solution i.e. $J_{EMD} = 0$ will never happen, an angular error of the obtained solution always exists between the vector pairs $(\mathbf{B}_b, \mathbf{V}_b^{\text{opt}})$, and $(\mathbf{S}_b, -\mathbf{Z}_b)$. A lower angular error between $\mathbf{B}_b$ and $\mathbf{V}_b^{\text{opt}}$ compliments the condition in (4.3) while a lower angular error between $\mathbf{S}_b$ and $-\mathbf{Z}_b$ implies more solar power can be harvested. By adjusting the weight penalty w in (4.5), the resulting angular errors can be tuned in favour of either vector pairs. This implies that a better momentum dumping efficiency can be achieved at the expense of lowering the solar power harvesting efficiency.

In this research, we make use of the QUEST method [56] to solve for the $\mathbf{A}_E$ in (4.5). The QUEST method was originally developed for attitude determination by using a set of known inertial frame vectors and the measured body frame vectors. The optimal attitude matrix of the satellite, $\mathbf{A} \in \mathbb{R}^{3 \times 3}$ is estimated based on the following cost function

$$J_{QUEST} = \sum_{i=1}^n w_i \left\| \mathbf{U}_b^i - \mathbf{A} \mathbf{U}_I^i \right\|^2 \quad (4.6)$$

where n is the number of vector pairs, $\mathbf{U}_b^i \in \mathbb{R}^3$ is the i^{th} measured body vector, $\mathbf{U}_I^i \in \mathbb{R}^3$ is the i^{th} inertial frame vector and w_i is the weighting factor for the i^{th} vector pair. Replacing inertial and body frame in the QUEST method with body and target frame respectively, results in $\mathbf{A} = \mathbf{A}_E$ and a new cost function

$$J_{QUEST} = \sum_{i=1}^n w_i \left\| \mathbf{U}_t^i - \mathbf{A}_E \mathbf{U}_b^i \right\|^2 \quad (4.7)$$

where $\mathbf{U}_t^i \in \mathbb{R}^3$ is the i^{th} measured target vector.

By comparing (4.5) to (4.7), we have $w_1 = 1$, $\mathbf{U}_t^1 = \mathbf{B}_b$, $\mathbf{U}_b^1 = \mathbf{V}_b^{\text{opt}}$, $w_2 = w$, $\mathbf{U}_t^2 = \mathbf{S}_b$ and $\mathbf{U}_b^2 = -\mathbf{Z}_b$. If $\mathbf{V}_b^{\text{opt}}$ is known, then the QUEST method can be directly applied to solve $\mathbf{A}_E$ which can then be used to obtain the control law input, $\boldsymbol{\varepsilon}_e$.

During the satellite operation, $\mathbf{h}_{\text{rw}}$ varies over time. Therefore, $\mathbf{V}_b^{\text{opt}}$ has to be updated periodically by a supervisor controller. During the steady state when the satellite is in Sun-pointing, the attitude controller (3.27) will constantly adjust $\mathbf{h}_{\text{rw}}$ largely due to the presence of gyroscopic noise. As a result, the momentum dumping

controller (4.2) will cause $\mathbf{V}_b^{\text{opt}}$ and $\boldsymbol{\varepsilon}_e$ to be updated periodically and results in chattering of the reaction wheels [57]. To overcome this problem, $\mathbf{V}_b^{\text{opt}}$ was averaged over n samples in (4.8). Ideally, this will deteriorate the performance of the controller since the ideal torque, $\mathbf{T}_{\text{ideal}}$ has now deviated from the condition in (4.3). However, chattering effect of reaction wheels should always be avoided to reduce the power consumption and to increase the operating life span of the reaction wheel.

$$\mathbf{V}_b^{\text{opt}}(k_2) = \frac{\frac{\mathbf{T}_{\text{ideal}}(k_2)}{\|\mathbf{T}_{\text{ideal}}(k_2)\|} \times \frac{\mathbf{M}(k_2)}{\|\mathbf{M}(k_2)\|} + \sum_{i=1}^n \mathbf{V}_b^{\text{opt}}(k_2 - i)}{n + 1} \quad (4.8)$$

The overall control block diagram connecting Sun-pointing, momentum unloading and optimization supervisor controllers for the proposed efficient momentum dumping (EMD) method is illustrated in Fig. 4-2.

As shown in Fig. 4-2, the proposed EMD strategy can be viewed as two parallel running controllers consisting of a momentum dumping controller together with a Sun-pointing controller and a real time $\mathbf{V}_b^{\text{opt}}$ optimization supervisor controller. The time index k_2 is computed at sampling time of T_{s2} . The reasoning behind the use of a different sampling time for (4.8) will be explained in section 4.4.1.

not affected by the magnetic torquers. Therefore, the sampling time of the supervisor controller (i.e. T_{s2}) was defined to be 5s. It was determined that $n=10$ provided sufficient samples to eliminate the chattering effect of the reaction wheel [57] through averaging of $\mathbf{V}_b^{\text{opt}}$.

Finally, $h_{\text{threshold}}$ was defined to be 0.5 m.Nms for the momentum dumping performance index.

4.4.2 Selection of tuning parameters

To study the effect of N_p and w on the Sun-pointing performance of the EMD method, a simulation study was carried out for different combination of N_p and w . Let α be the Sun-pointing error which can be calculated using $\cos^{-1}(-\mathbf{Z}_b \bullet \mathbf{S}_b)$. From the results presented in Fig. 4-3, it can be observed that an increase in w upto 100 results in a lower α but becomes a near constant as w reaches 1000. Thus, w of 1000 is chosen to maximize Sun-pointing performance. A closer examination at Fig. 4-3 reveals that α increases slightly with an increase in N_p . The Sun-pointing error at 3σ is 0.9875° and 1.805° for N_p of 5 and 30 respectively.

Fig. 4-3. Sun-pointing performance for different w and N_p for the EMD method.

In section 3.5.2, the final selection of N_p was based on $\mathbf{h}_{rw}$ usage at steady state for the Sun-pointing strategy in section 3.3.6. Since the performance of the EMD method is compared against the TSP method, the simulation results of section 3.5.2 was repeated for the TSP method and are shown in Fig. 4-4.

Fig. 4-4 Steady state average normalized reaction wheel angular momentum recorded for changing N_p .

From Fig. 4-4, it is noticed that the steady state $\|\bar{\mathbf{h}}_{rw}\|$ needed for constant Sun-pointing reduces as N_p increases and becomes a near constant for $N_p \geq 15$ unlike in

section 3.5.2 which became a near constant for $N_p \geq 12$. This shows the control action needed for the TSP method is indeed greater than the Sun-pointing strategy in section 3.3.6. As a result, for the purpose of comparison, $N_p = 15$ shall be used to benchmark the momentum dumping efficiency of EMD method against the TSP method. A further increase in N_p will cause an increase in both steady state error and settling time. This explanation justifies the selection of $h_{threshold}$ (i.e. the performance index) in section 4.4.1 with an appropriate margin.

As N_p of the MPC controller has changed, the new controller gain matrix, $\mathbf{K}$ of (3.27) is given below where the unit of $\mathbf{T}_{rw}$ is in m.Nm:

$$\mathbf{K}_{\frac{N_c:1}{N_p:15}} = \begin{bmatrix} -5.9706 & -0.0000 & 0.2082 & -31.3126 & -0.0000 & 1.0921 \\ -0.0000 & -6.6133 & -0.0060 & -0.0000 & -34.6832 & -0.0315 \\ 0.2082 & -0.0060 & -3.0174 & 1.0921 & -0.0315 & -15.8243 \end{bmatrix} \quad (4.9)$$

In addition, closed-loop poles can be calculated following the procedure in section 3.6.2 and are presented in Table 4-1 and Fig. 4-5.

Table 4-1. Eigenvalues of closed-loop state matrix for $N_p=15$.

	$N_c=1; N_p=15$
Closed-loop poles	$\lambda_1 = 8.8963e^{-7} + 0i$ $\lambda_2 = 3.5346e^{-6} + 0i$ $\lambda_3 = 4.3154e^{-6} + 0i$ $\lambda_4 = 0.8999 + 0i$ $\lambda_5 = 0.8999 + 0i$ $\lambda_6 = 0.8999 + 0i$

Fig. 4-5. Plot of eigenvalues of closed-loop state matrix for $N_p=15$.

From Fig. 4-5, all calculated eigenvalue lies within a unit circle, the derived controller using unconstrained MPC with $N_c=1$ and $N_p=15$ is closed-loop stable.

Table 4-2 gives the pseudocode of the Sun-pointing and momentum dumping controller while Table 4-3 gives the pseudocode of the optimization supervisor controller.

Table 4-2: Pseudocode of Sun-pointing and momentum dumping controller**Step 1: Initialization.**

Initialize secondary body frame vector, $\mathbf{V}_b^{\text{opt}}$ for momentum dumping of all three reaction wheels.

$$\mathbf{V}_b^{\text{opt}} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix}^T;$$

Step 2: Obtain error quaternion for re-orientation of the satellite.

Obtain $\mathbf{B}_b$ and $\mathbf{S}_b$ from magnetometer and gyroscope respectively.

Cost function:

$$J_{EMD}(k_1) = \left| \mathbf{B}_b(k_2) - \mathbf{A}_E(k_1) \mathbf{V}_b^{\text{opt}}(k_2) \right|^2 + w \left| \mathbf{S}_b(k_1) - \mathbf{A}_E(k_1) (-\mathbf{Z}_b) \right|^2; w = 1000;$$

Note: time indexes k_1 and k_2 is computed at a sampling time of T_{s1} and T_{s2} respectively.

$$T_{s1} = 1\text{s}; T_{s2} = 5\text{s};$$

Solve for attitude error matrix, $\mathbf{A}_E$ using QUEST method.

Convert $\mathbf{A}_E$ to quaternion equivalent form, $\mathbf{q}_E = \begin{bmatrix} \boldsymbol{\varepsilon}_e \\ \eta_e \end{bmatrix}$.

Step 3: Torque calculation of reaction wheels.

Apply linearized attitude control law:

$$\mathbf{T}_{rw}(k_1) = \mathbf{K} \begin{bmatrix} \boldsymbol{\varepsilon}_e^T(k_1) & \boldsymbol{\omega}^T(k_1) \end{bmatrix}^T;$$

$\mathbf{K}$ is obtained through model predictive control algorithm with a prediction horizon (N_p) of 15, control horizon (N_c) of 1 and a sampling time (T_{s1}) of 1s.

Step 4: Dipole moment calculation of magnetic torquers.

Apply discrete bang-bang momentum unloading control law:

$$\mathbf{M}(k_2) = \mathbf{M}_{\max} \text{sign} \left(\frac{\mathbf{B}_b(k_2) \times \mathbf{T}_{\text{ideal}}(k_2)}{\|\mathbf{B}_b(k_2)\|^2} \right)$$

$$\text{where } \mathbf{T}_{\text{ideal}}(k_2) = g \Delta \mathbf{h}_{rw}(k_2) = g (\mathbf{h}_{rw}(k_2) - \mathbf{h}_{Nrw}).$$

The nominal reaction wheel angular momentum, $\mathbf{h}_{Nrw}$ is defined as a zero vector $\in \mathbb{R}^3$ and $g = 1$.

The dipole moment, $\mathbf{M}(k_2)$ is applied for a duration of 1s.

Process repeats from Step 2.

Table 4-3: Pseudocode of supervisor controller

Step 1: Calculation of secondary body frame vector.

$$\mathbf{V}_b^{\text{opt}}(k_2) = \frac{\mathbf{T}_{\text{ideal}}(k_2)}{\|\mathbf{T}_{\text{ideal}}(k_2)\|} \times \frac{\mathbf{M}(k_2)}{\|\mathbf{M}(k_2)\|}$$

Note: the time index k_2 is computed at a sampling time of $T_{s2} = 5\text{s}$.

Step 2: Averaging of secondary body frame vector.

Average $\mathbf{V}_b^{\text{opt}}$ over $n = 10$ samples to avoid chattering of the reaction wheels.

If $\|\mathbf{h}_{\text{rw}}\| \geq h_{\text{threshold}}$:

$$\mathbf{V}_b^{\text{opt}}(k_2) = \frac{\frac{\mathbf{T}_{\text{ideal}}(k_2)}{\|\mathbf{T}_{\text{ideal}}(k_2)\|} \times \frac{\mathbf{M}(k_2)}{\|\mathbf{M}(k_2)\|} + \sum_{i=1}^n \mathbf{V}_b^{\text{opt}}(k_2 - i)}{n + 1}$$

Else:

$$\mathbf{V}_b^{\text{opt}}(k_2) = \mathbf{V}_b^{\text{opt}}(k_2 - 1)$$

Process repeats from Step 1.

4.4.3 Momentum dumping efficiency

To investigate the performance of proposed method, simulations have been conducted using a time step of 5 days that last for a year starting from 23-Dec-2015 for orbit inclinations ranging from 0° to $+95^\circ$. The sun incident angle on the equatorial plane during the simulation period is shown in Fig. 4-6.

Fig. 4-6. Sun angle observed on the equatorial plane.

For the simulation, the reaction wheels were initiated with 75% saturated $\mathbf{h}_{\text{rw-max}}$ (i.e. -3.87 m.Nms), as defined in Table 3-1.

Fig. 4-7 to Fig. 4-9 show the simulation results for the first equinox season of the year 2016 [58] for 0° orbit inclination angle. Both the results of the TSP and EMD methods are shown for comparison. Fig. 4-7 shows the variation of reaction wheels' angular momentum during the momentum dumping process.

Fig. 4-7. Momentum dumping performance between the TSP and EMD methods for an equatorial orbit during the equinox season.

It can be observed from Fig. 4-7 that the EMD method takes 67.2 minutes to completely unload the stored angular momentum while the TSP method requires 513 minutes. Thus the proposed method is more efficient by a factor of 7.6. To analyse the results in Fig. 4-7, $\mathbf{X}_b$, $\mathbf{Y}_b$ and $\mathbf{Z}_b$ components of $\mathbf{h}_{\text{rw}}$ and $\mathbf{B}_b$ are presented in Fig. 4-8 and Fig. 4-9 respectively.

Fig. 4-8. Variation in h_{rw} for an equatorial orbit during the equinox season: (a) TSP and (b) EMD

Fig. 4-9. Variation B_b for an equatorial orbit during the equinox season: (a) TSP and (b) EMD

Fig. 4-8 (a) shows that the h_{rw} of X_b and Z_b reaction wheels reach the steady state at a faster rate than Y_b . In this case, a low magnetic field along X_b and Z_b results in a very limited magnetic torque along Y_b . As a result, momentum dumping performance of Y_b reaction wheel is reduced. From Fig. 4-9 (a), we can notice that the magnetic field B_b along X_b and Z_b fluctuates about zero, while that of Y_b fluctuates about -25 μ T.

Using the proposed EMD approach, Fig. 4-8 (b) shows that h_{rw} of X_b , Y_b and Z_b reaction wheels reach the steady state nearly at the same time while Fig. 4-9 (b) shows that B_b experienced by the satellite is constantly adjusted by the supervisor controller.

In addition, Fig. 4-9 (b) also shows that the satellite is rotated about $\mathbf{Z}_b$ such that $\mathbf{B}_b$ has a component in both $\mathbf{X}_b$ and $\mathbf{Y}_b$ allowing the momentum dumping of all the three reaction wheels according to (2.2) under the EMD method.

It is also noticed from Fig. 4-8 (a) that the reaction wheels of $\mathbf{X}_b$ and $\mathbf{Z}_b$ continue to fluctuate even after the momentum is completely unloaded. This is because when (4.2) attempts to unsaturate the reaction wheel along $\mathbf{Y}_b$, it also produces a disturbance torque along $\mathbf{X}_b$ and $\mathbf{Z}_b$. For the proposed EMD method, Fig. 4-8 (b) shows a gradual change in $\mathbf{h}_{rw}$ till the complete unsaturation of all the reaction wheels. Thus, the TSP method demands a higher $\mathbf{T}_{rw}$ and consequently more consumed power as compared to the EMD method during the momentum dumping phase.

Next, the simulation study was conducted to cover all seasons (i.e. equinox, summer solstice and winter solstice) for orbit inclinations ranging from 0° to $+95^\circ$. The results of time to fully unsaturate the reaction wheels are presented in Fig. 4-10. Similar to the earlier simulation, the reaction wheels were initiated with 75% of the maximum momentum.

Fig. 4-10 shows that the proposed EMD method outperforms the conventional approach for orbit inclinations that are less than 40° (i.e. typical NEOs). However, the performance of the TSP matches the EMD method as the orbit inclination increases towards the polar orbit. This is expected as the Earth magnetic field variation with respect to the Earth's inertial reference frame increases as the orbit inclination increases. This in turns allow for a higher momentum dumping efficiency.

Fig. 4-10. Comparison of reaction wheel momentum dumping performance between TSP and EMD methods.

A close up view of Fig. 4-10 for the EMD method is shown in Fig. 4-11. It is noted that the efficiency of the EMD method also increases with orbit inclination angles. This is because higher magnetic field variation with respect to the Earth's inertial reference frame will aid to the performance of the EMD method.

Fig. 4-11. Reaction wheels momentum dumping performance of EMD method

4.4.4 Momentum dumping efficiency of fully saturated reaction wheel

Next, we evaluate how the proposed EMD method performs when the reaction wheels are initiated with different saturation levels (i.e. 25%, 50%, 75% and 100%) prior to the momentum dumping process. The worst case is when the saturation level of the reaction wheels are at 100% (i.e. $\mathbf{h}_{rw} = \mathbf{h}_{rw-max}$). This scenario is most likely to happen when the speeds of reaction wheels are controlled to effectively absorb the angular momentum of a tumbling spacecraft [31].

The corresponding results are presented in Fig. 4-12 for an equatorial orbit satellite with 0° orbit inclination angle during the first equinox season of the year 2016 [58].

Fig. 4-12. Momentum dumping performance with different saturation levels (i.e. 25%, 50%, 75% and 100%)

From Fig. 4-12, it is noticed that for simulations with 25%, 50% and 75% initial saturation levels, there is no difference in performance between the EMD methods with and without $\mathbf{h}_{rw-max}$ constraint. This is because the controller in (3.27) still has sufficient unsaturated $\mathbf{h}_{rw}$ to reorientate the satellite to achieve the EMD attitude. But, it is not the case for the simulation initiated with 100% saturation level. In this case,

the controller in (3.27) waits for the magnetic torquers to dump $\mathbf{h}_{\text{rw}}$ as per TSP method and then re-orientates the satellite when there is sufficient allowance of $\mathbf{h}_{\text{rw}}$ to produce $\mathbf{T}_{\text{rw}}$. As a result, the momentum dumping performance of constrained EMD method lags behind unconstrained EMD method by less than 2 mins. However, the lag is negligible when compared with the unsatisfactory performance of the TSP method.

4.4.5 Sun-pointing performance

The Sun-pointing performance of both TSP and EMD methods across all orbit inclinations and seasons are shown in Fig. 4-13. It shows that the Sun-pointing performance of both TSP and EMD methods are nearly a constant and is irrespective of the seasons and orbit inclinations. It is also noticed that the Sun-pointing performance of the TSP method is always superior than the EMD in terms of pointing accuracy across all seasons and orbit inclinations. This observation compliments the explanation provided in section 4.3 that an optimal solution will never yield a perfect scenario (i.e. $J_{\text{EMD}} = 0$ for the EMD method) to have a similar Sun-pointing performance as the TSP method. The results presented in Fig. 4-13 are summarized in Table 4-4, from which two observations can be made:

- In terms of the Sun-pointing efficiency, the EMD method underperforms the TSP method by a negligible angular difference of 0.192° .
- The loss of solar power harvesting performance (i.e. cosine of α) by implementing the EMD method is 0.0013% which is also negligible.

Fig. 4-13. Sun-pointing performance comparison between the TSP and the EMD methods.

Table 4-4. Sun-pointing performance analysis between the TSP and the EMD methods.

Sun-pointing strategy	Sun pointing performance ($\bar{\alpha} + 3\sigma$)	Sun cosine factor / $\cos(\bar{\alpha})$
TSP	1.172° (3 s.f)	0.999968 (6 s.f)
EMD	1.364° (3 s.f)	0.999955 (6 s.f)

4.5 Summary

The improvement of the momentum dumping strategy for a near equatorial orbit satellite through a new Sun-pointing strategy has been presented. The results show that the new approach could unload the angular momentum in each reaction wheel to a desired steady state threshold within 80 minutes across all seasons at low inclination angles. On the other hand, the typical Sun-pointing method may take upto 700 minutes. The proposed method has shown to outperform in terms of momentum dumping efficiency and demanded lesser power requirement as compared to the typical Sun-pointing method. This improvement is at the expense of a negligible loss of harvested solar power of less than 0.0013%.

In the next chapter, the experimental results to validate both the developed satellite attitude controller in chapter 3 and the proposed new Sun-pointing strategy will be provided.

CHAPTER 5

EXPERIMENTAL RESULTS

5.1 Overview

This chapter presents the experimental results to validate both the developed satellite attitude controller in chapter 3 and the proposed new Sun-pointing strategy in chapter 4.

A brief description of an in-house designed laboratory platform which is in the form of an air bearing spacecraft simulator to attain a near microgravity environment will first be presented. The components that form a hardware-in-the-loop (HIL) spacecraft simulator will be introduced. Sun-pointing experiments have been conducted to validate the performance of the MPC controller derived in chapter 3. The results have demonstrated that the mean error is 0.3° which is similarly to the simulation study conducted in chapter 3. Moreover, HIL was also used to validate the performance of the momentum dumping controller and the proposed new Sun-pointing strategy presented in chapter 4.

5.2 Introduction to an air bearing simulator

The ACS is vital to the successful operation of any three axis stabilized spacecraft. Therefore it is extremely important to test the attitude control system prior to the launch of the spacecraft. Air bearing spacecraft simulator has been used as a test bed for verification of spacecraft attitude control system hardware and software development for nearly 57 years [59], virtually coincident with the beginnings of the space race.

Though there are alternative ways of simulating the microgravity space environment such as neutral buoyancy, magnetic suspension, aircraft flying in an arc and drop tower, air bearings are found to be the most common choice [60].

As shown in Fig 5-1, an air bearing consists of a rotor and a stator. In an air bearing spacecraft simulator, a specially designed platform is placed on the moving rotor of the air bearing. Pressurized air passes through orifices of the grounded stator of the bearing creating a thin film of air that supports the entire weight of the platform and the rotor. This slow moving pressurized air imparts virtually no shear between the two bearings making the air film an effective lubricant. In this manner, near microgravity is attained to conduct attitude control experiments in the laboratory.

Fig 5-1. Spherical air bearing for spacecraft simulators

Fig.5-2 shows the detailed block diagram of the air bearing spacecraft simulator. The ACS and the ADS hardware are actual hardware of the nanosatellite VELOX-II. The ACS consists of three reaction wheels and three magnetic torquers. The ADS consists of a 3-axis inertial measurement unit and Sun sensors. A microcontroller is used by both the ACS and ADS.

The entire air bearing spacecraft simulator is powered up using lithium ion battery. In addition, the communication between the moving simulator and the mission control software of VELOX-II is attained via XBee wireless connection.

From the experiment, it was found that the use of an air bearing spacecraft simulator poses a challenge as any misalignment of the centre of gravity (i.e. CG) from the centre of rotation (CoR) results in a disturbance torque that could potentially destabilize the entire system.

Fig.5-2 CAD of VELOX II air bearing spacecraft simulator.

To overcome this problem, an integrated counter weight tuning mechanism has been designed and developed. A closer view of the tuning mechanism design in Fig.5-2 is shown in Fig.5-3.

Coarse tuning is performed with the help of simulator's computer aided design that has a 90% model accuracy. For fine tuning, the mechanism is capable of translating the CG of the air bearing simulator by about $14\mu\text{m}$. The design is also modular which can accommodate counterweights of different masses for tuning of larger systems as illustrated in Fig.5-3 (c).

Fig.5-3 Integrated coarse and fine counterweight tuning mechanism. (a) Coarse tuning, (b) fine tuning, (c).Modular design to accommodate counterweights of different masses

Air bearing spacecraft simulator is typically used to perform qualitative tests in all three directions (i.e. roll, pitch and yaw for $\mathbf{X}_b$, $\mathbf{Y}_b$ and $\mathbf{Z}_b$ directions respectively). However, a quantitative test can only be performed for rotation along the $\mathbf{X}_b$ direction (i.e. roll). If quantitative tests are required for pitch and yaw directions, then extensive fine tuning of counterweights are needed to match the CG of the simulator with its CoR. From the system design of VELOX-II, it was determined that a quantitative test along one direction and qualitative tests on all directions are sufficient.

In the next section, the full experimental setup that includes the in-house developed air bearing spacecraft simulator together with various other components that make up the hardware-in-the-loop spacecraft simulator will be presented.

5.3 Hardware-in-the-loop spacecraft simulator.

The air bearing spacecraft simulator shown in Fig.5-2 [61] was integrated with a two axes high precision rate table, a Helmholtz cage, and a sun simulator. The integrated hardware-in-the-loop (HIL) spacecraft attitude determination and control simulator is shown in Fig 5-4. Experimental results carried out using HIL simulator are presented from section 5.4 to section 5.7.

Fig 5-4. Hardware-in-the-loop spacecraft simulator: 1- Helmholtz cage; 2- air bearing spacecraft simulator; 3-Two axis rate table; 4- sun simulator; 5-PC.

In Fig. 5-4, the two-axis high precision rate table with a 30 arc second accuracy was used to calibrate the magnetometer and Sun sensors of the air bearing spacecraft simulator prior to experiments. The solar panel direction was defined to be along $-\mathbf{Z}_b$

of the air bearing spacecraft simulator. The Sun simulator was positioned in such a way that $\mathbf{S}_b$ lies on the $\mathbf{Y}_b$ - $\mathbf{Z}_b$ plane of the air bearing spacecraft simulator. The Helmholtz cage was used to generate $\mathbf{B}_b$ such that it also lies on the $\mathbf{Y}_b$ - $\mathbf{Z}_b$ plane of the air bearing spacecraft simulator.

Due to the rotor of the air bearing and other hardware, $\mathbf{J}$ in (3.7) of the linearized state space model (3.16) is replaced with the following updated moment of inertia

$$\mathbf{J} = \begin{bmatrix} 0.0197 & 8.6505e^{-5} & -1.6537e^{-5} \\ 8.6505e^{-5} & 0.0205 & 1.1284e^{-4} \\ -1.6537e^{-5} & 1.1284e^{-4} & 0.0266 \end{bmatrix} \text{kgm}^2 \quad (5.1)$$

In the following section, the experimental results to validate the model predictive controller in (3.26) are presented in section 5.4. The experimental results of the model predictive controller in (3.26) subjected to guroscope bounded error will be then presented in section 5.5 to validate simulation results in section 3.4.5. Finally, experimental results to validate momentum dumping controller in (4.2) and the proposed Sun-pointing strategy illustrated in Fig. 4-1 will be presented in section 5.6 and section 5.7 respectively.

5.4 Experimental results of MPC controller

To experimentally validate the unconstrained MPC algorithm in (3.27), the simulated transient responses were compared with results obtained through Sun-pointing experiments conducted with the HIL spacecraft simulator. As the inertia matrix, $\mathbf{J}$ of the air bearing spacecraft simulator is different from VELOX-II, computer simulations were re-conducted with different MPC tuning parameters (i.e.

different N_p while $N_c=1$). The comparison between simulated and experimental results is presented from Fig. 5-5 to Fig. 5-7.

For the experiments, an initial Sun-pointing error α defined as $\cos^{-1}(-\mathbf{Z}_b \bullet \mathbf{S}_b)$ of 45° to 50° was introduced prior to the experiment. The experiment was repeated for different prediction horizons, N_p while keeping the control horizon, $N_c=1$.

The experimental results obtained using the HIL spacecraft simulator were then compared with the simulated step responses. In Fig. 5-5, a shorter N_p of 2 results in an overshoot and can be explained by the finite horizon optimal control problem of MPC. On the other hand, both the simulated and experimental results lack the overshoot for $N_p > 2$ as shown in Fig. 5-6 and Fig. 5-7 for $N_p=4$ and $N_p=6$ respectively. In addition, Fig. 5-6 and Fig. 5-7 show that an increase in N_p results in an increase in the settling time as observed in section 3.4.3. As the experimental results closely match with simulated step responses in Fig. 5-5 to Fig. 5-7, the transient response of the simulation model is validated experimentally.

Fig. 5-5. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=2$.

Fig. 5-6. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=4$.

Fig. 5-7. Experimental transient response vs simulated (with 1 mN.m torque limit) transient response during Sun-pointing by air bearing spacecraft simulator for $N_p=6$.

Fig. 5-8 shows the steady state response of Fig. 5-5. It is observed that the mean Sun-pointing error (without taking sun sensor accuracy into consideration) obtained experimentally is 0.78° while the simulation is 0.21° . As explained in section 5.2, the air bearing simulator is not tuned to operate in the pitch and yaw directions. Therefore, a correction was made to calculate the Sun-pointing error that lies on the $\mathbf{Z}_b - \mathbf{Y}_b$ plane and is presented in Fig. 5-8. The re-calculated Sun-pointing performance from Fig. 5-8 has an error of less than 0.3° giving an error deviation of just 0.09° as compared to the simulated results.

Fig. 5-8 Experimental steady state response vs simulated (with 1 mN.m torque limit) steady state response during Sun-pointing by air bearing spacecraft simulator for $N_p=2$

Thus both the transient and steady state performance of the unconstrained MPC algorithm in (3.26) have been validated.

5.5 Experimental results of MPC controller subjected to gyroscope bounded error.

In section 3.4.5, it was explained that a gyroscope bounded error results in an increased in the steady state error for $N_c=1$. Table 5-1 shows the gyroscope bounded error obtained after calibration using the two-axis rate table which has a 30 arc second accuracy.

Table 5-1. Gyroscope calibration of air bearing spacecraft simulator.

Gyroscope axis	Calibration constant
$\mathbf{X}_b$	0 rad
$\mathbf{Y}_b$	0.016 rad
$\mathbf{Z}_b$	0.024 rad

A Sun-pointing experiment was conducted using both the uncalibrated and calibrated gyroscope. Mission Control Software of VELOX-II was used to input the gyroscope calibration parameters at the start of section (b) of Fig. 5-9. It can be observed from Fig. 5-9 section (a) that the mean Sun-pointing error using an uncalibrated gyroscope is 3.14° and a calibrated gyroscope is 0.29° , validating the observations of section 3.4.5.

Fig. 5-9 Sun pointing performance. Section (a), using uncalibrated gyroscope. Section (b), using calibrated gyroscope.

In the following section, the experimental results to validate the momentum dumping controller in (4.2) and the proposed Sun-pointing strategy illustrated in Fig. 4-1 will be presented.

5.6 Experimental results of momentum dumping controller

For the demonstration of the momentum dumping controller, the $\mathbf{X}_b$ reaction wheel was initiated with 50% saturated $\mathbf{h}_{rw-max}$ (i.e. 2.58 m.Nms) and $-\mathbf{Z}_b$ was aligned with the Sun simulator. The air bearing spacecraft simulator was then commanded to enter into the Sun-pointing mode. As $\mathbf{B}_b$ is inertially fixed by the Helmholtz cage, the magnetic torquers were operated continuously unlike in VELOX-II where the magnetic torquers were only switched on for 1s and switched off for the next 4s.

Two experiments had been conducted to validate the the momentum dumping controller in (4.2). The results are presented in Fig. 5-10. It is observed that the use of a correct controller gain (i.e. $g = +1$) in one of the experiments leads to a complete unsaturation of the reaction wheel (i.e. $\mathbf{h}_{rw}$ fluctuates about zero). On the other hand, the controller with a reversed controller gain (i.e. $g = -1$) in the second experiment results in a full saturation of the reaction wheels (i.e. $\mathbf{h}_{rw} = \mathbf{h}_{rw-max}$). Thus, the results of the two experiments validated the momentum dumping controller.

Fig. 5-10 Experimental results of the momentum dumping controller.

5.7 Experimental results of new Sun-pointing strategy

Fig. 5-11 shows the top view of the experimental setup to demonstrate the new Sun-pointing strategy. $\mathbf{V}_b^{\text{opt}}$ or $\mathbf{U}_b^1$ was defined to be aligned with $\mathbf{Y}_b$.

Fig. 5-11 Experimental results of the momentum dumping controller.

Let β be the magnetic field pointing error which is calculated as $\cos^{-1}(\mathbf{V}_b^{\text{opt}} \bullet \mathbf{B}_b)$ and α be the Sun-pointing error. Moreover, let the weighting factors w_1 and w_2 in (4.7) be equal. Then the QUEST method will re-orientate the air bearing spacecraft simulator such that α and β are equal. For the experiment, it is assumed that α is zero initially (i.e. $-\mathbf{Z}_b$ of air bearing spacecraft simulator is aligned with the Sun simulator) and the initial β is selected to ensure that the Sun simulator is still within the field of view of the 60° Sun sensor in the steady state. The experimental results are shown in Fig. 5-12 which demonstrate that the pointing errors of α and β in the steady state are nearly the same. This illustrates the new Sun-pointing method for efficient momentum dumping of the reaction wheels.

Fig. 5-12 Experimental demonstration of the new Sun-pointing strategy,

5.8 Summary

This chapter presented the design and development of a novel air bearing spacecraft simulator to measure and validate the performance of the ACS of VELOX-II nanosatellite. The air bearing spacecraft simulator was then integrated with a two axes high precision rate table, a Helmholtz cage, and a Sun simulator to form a hardware-in-the-loop (HIL) spacecraft simulator.

The HIL spacecraft simulator was used to validate the MPC controller developed in chapter 3. From the experiment, the mean Sun-pointing error was calculated to be 0.3° while computer simulation gave an error of 0.21° . Moreover, experiments have validated the momentum dumping controller and the proposed new Sun-pointing strategy presented in chapter 4.

CHAPTER 6

CONCLUSION

6.1 Summary of the research

In this research, a new momentum dumping strategy for a NEO satellite has been presented. The improvement is achieved through a new Sun-pointing strategy and shown to outperform the typical Sun-pointing approach.

The proposed Sun-pointing strategy aligns both the solar panel of the satellite towards the Sun as well as a secondary body frame vector with the Earth's geomagnetic field vector. To describe the kinematics needed for the proposed method, a cost function was derived using QUEST (QUaternion ESTimation) method to obtain the error quaternion for re-orientation of the satellite.

To produce an optimal control torque for attitude control, both the unconstrained and constrained model predictive control (MPC) algorithms have been formulated. The MPC controller parameters were then tuned to meet the ACS requirements of VELOX-II. From the performance analysis, the constrained MPC was found to have a 1.3% and 8.8% better accuracy as compared to the unconstrained MPC for gyroscope noise of 0.27 deg/s and 0.9 deg/s respectively. As the implementation of the constrained MPC algorithm requires quadratic programming to handle active actuator constraints, the unconstrained MPC was eventually selected.

The produced torque by the unconstrained MPC re-orientates the satellite while overcoming the disturbance torque produced by the magnetic torquer for momentum

dumping of the reaction wheels. Simulation studies have been conducted to analyse the momentum dumping performance of the proposed method. The results show that the new approach could unload the angular momentum in each reaction wheel to a desired steady state threshold within 80 minutes across all seasons at low inclination angles. On the other hand, the typical Sun-pointing method may take upto 700 minutes. The proposed method has shown to outperform in terms of momentum dumping efficiency and demanded lesser power requirement as compared to the typical Sun-pointing method. This improvement is at the expense of a negligible loss of harvested solar power of less than 0.0013%.

Finally, experiments were conducted using a hardware-in-the-loop spacecraft simulator to validate the unconstrained MPC controller. Moreover, Sun-pointing experiments have validated both the momentum dumping controller and the proposed new Sun-pointing strategy.

6.2 Future works

The efficiency of the new Sun-pointing strategy strongly depends on the performance of the attitude acquisition controller. Following methods can be used in future to improve the performance of the controller:

- It was identified that gyroscope bounded error results in an increased steady state error. Gyroscope bounded error may occur due to inaccurate gyroscope calibration. Therefore developing a control algorithm that is robust against gyroscope bounded errors is indeed an advantage. Model Predictive Control with augmented state vectors will give an integral action to the control algorithm to avoid any bounded steady state errors.
- The noise of the gyroscope affects the overall steady state performance of the controller. This can be achieved by implementing a Kalman filter [62] to reduce the MEMS based gyroscope noise.

LIST OF PUBLICATIONS

M. S. C. Tissera, Y. T. Xing, K. S. Low, and S. T. Goh, "An Efficient Momentum Dumping Method Through an Alternative Sun Pointing Strategy for Small Near Equatorial Orbit Satellite.," presented at the 66th International Astronautical Congress, Jerusalem, Israel. , 2015.

M. S. C. Tissera, J. W. Chia, K. S. Low, and Y. T. Xing, "A Novel Simulator for Measuring the Performance of Nanosatellite's Attitude Control System," presented at the 2016 IEEE Aerospace Conference, Big Sky, Montana, U.S.A, 2016.

J. W. Chia, **M. S. C. Tissera**, K. S. Low, S. T. Goh and Y. T. Xing, "A Low Complexity Kalman Filter for Improving MEMS Based Gyroscope Performance," presented at the 2016 IEEE Aerospace Conference, Big Sky, Montana, U.S.A, 2016.

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