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INVESTIGATION ON REGIME SHIFTS IN COUPLED SOCIO-ECOLOGICAL SYSTEM

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To my Parents

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List of Abbreviations

SES		Socio-Ecological System
CPR		Common Pool Resource
TSL		Tavoni-Schülter-Levin
PGG		Public Goods Game
ABM		Agent Based Model
EGT		Evolutionary Game Theory
EWS		Early Warning Signals
CSD		Critical Slowing Down
MFA		Mean Field Approximation
PA		Pair Approximation
PCA		Principal Component Analysis
SVD		Singular Value Decomposition
ER		Erdős-Rényi
BA		Barabasi-Albert
CL		Chung-Lu
LFR		Lancichinetti-Fortunato-Radicchi
XS		Xulvi-Brunet - Sokolov

Abstract

The social and economic progress of mankind essentially relies on the services of ecosystems that come in the form of hospitable climate, clean air, fresh water, and also nutritious food. As our civilization advances, the coupling between human and ecology becomes more significant, but the enhanced coupling has so far served to deteriorate the state of the ecosystem. Although many studies have found that rational individuals cooperate to preserve their ecological resources, some fundamental questions such as how the interaction structure affects their cooperation remains unanswered.

This thesis presents an integrative approach that aims to answer these questions. We use common pool resource (CPR) framework from TSL model, with complex network structure. This framework assumes resource utilization through Cobb-Douglas production function and social control by neighboring individual through ostracism mechanism. By using this model, we can study the influence of different type of social network on dynamical stability of the whole system. This analysis provides useful insights for studying socio-ecological system with different social settings. Since it based on CPR framework, the results from this model can be applied to general coupled socio-ecological system with any specific social interaction and common resource sharing. In particular, we focus our investigation on regime shifts in coupled socio-ecological system since such a critical transition typically bears catastrophic effects. We performed our studies based on theoretical modeling, with computational simulations and analytical calculations used to express the results of our model.

We have shown from numerical simulations that alternate stable states and hysteresis results from the intrinsic nature of the social interaction network structure. Furthermore, we have validated these results by analytical calculations based on the approach of statistical physics and non-linear dynamics. We have used mean field theory and pair approximation to reveal the existence of multiple stable regimes in the system. We have found strong association between network structure and dynamical behavior in the system with the presence or absence of hysteresis behavior dependent on the social network degree. In this respect, our research has shown that social interaction network structure can lead to a stronger social inertia against change within the SES when average network degree increases. Under this scenario, critical transitions are delayed which brings about the effect of hysteresis. In addition, network structure also plays a role in influencing the accuracy of generic early warning signals in predicting the upcoming regime shifts. We also found strong association between hysteresis behavior and network topology such as the scale-free and modular structure. Interestingly, the existence of community structure in the social interaction network gives rise to additional stable regimes between the cooperative and the defective regime. The community structure in the social network interaction corresponds to social segregation in which the new intermediate stable regime corresponds to communities of defectors free riding on communities of cooperators. This generates disharmony between the two sets of communities. Significantly, the intermediate stable regime possesses characteristics that relate closely to those observed in the real-world data of a traditional SES: the Balinese subaks. In addition, we have also explored the effects of different social mechanisms such as the ostracism and voluntary enforcement mechanism on the emergence of social cooperation. We catalogued the social dynamical behavior due to these mechanisms in the form of a phase diagram. We compare the dynamical similarity and dissimilarity between them and found the interesting phase of bi-stability and disequilibrium dynamical behavior, in addition to the cooperation

phase, the defection phase, and mixture of cooperation and defection phase, in specific parameter region.

In summary, by providing a more generic investigation on the complex network of social interactions and new forms of social mechanisms, this thesis has brought progress to the study of SES regime shift as well as the early warning prediction of its possible occurrence.

1

Introduction

1.1 Coupled Socio-Ecological System

In the twenty-first century, human domination on Earth has created tremendous impact towards its ecosystem. Our world is now threatened by the potential occurrence of massive climate change (86, 142), pollution, and resource scarcity (74). In addition, significant deterioration of forests (13), water and hydrologies (151), as well as biodiversity (117) has been observed. Based on a lot of empirical evidences, many scientists believe history has reached the era of Anthropocene. Anthropocene is the period when the cumulation of human activities have created significant and profound impact towards earth's ecology(174). In this era, social and ecological dynamics cannot be separated and should be considered as one single coupled socio-ecological system (SES)(139).

Coupled socio-ecological system (SES) describes the dynamical interdependencies between humans and the Earth's ecosystem. This system is not only ubiquitous in the current era, but also can be found in traditional system. One of the real world coupled socio-ecological systems is the subak system in Bali, Indonesia(93). Subak represents the water management system for the paddy fields in the island of Bali. In this system, human institution interacts with the natural ecosystem of waterfall and rivers to optimize the use of the water resources to increase the yield of rice. After thousands of years of evolution, there arises a strong coupling between the human institution and the ecosystem with the emergence of a subak culture that ensures the stability of the two entities(50, 94, 96).

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Different type of socio-ecological system can also be found in other part of the earth.

One specific coupled SES is based on the availability of a common pool resource (CPR). In CPR system, all related individuals have open access to the resource such that it can be depleted by all the users. In this scenario, everyone will be better off if they exercise restraint on their usage on the resource. However, such a behavior is not followed by a rational and selfish person who is motivated by self-interest to maximize individual benefits. This situation is known as tragedy of the commons as described by Hardin (67). The occurrence of green house effect, deforestation, industrial pollution, traffic congestion, or even individual littering can be considered as the effect of this social dilemma. Moreover, many systems also exhibit similar properties as the common pool resource system, for instance Balinese subaks where the farmers shared common source of water from a spring or irrigation canal.

Common pool resource problem is very relevant to the current society where competition over limited resources is ubiquitous. Research indicates that we consume more resources than what the ecological system can regenerate (191). The survival of our civilization thus depends on our ability to cooperate. Although preservation of common pool resource is impossible from economic perspective, empirical study shows the opposite conclusion. Ostrom observed local norm and self-organized interaction can initiate collective and cooperative actions for sustainable practices without the involvement of any centralized governance in many levels of societies (138). Several efforts have been made to investigate the common pool resource problems, whether by using specific model (130, 165, 169) or empirical data and experiment (69, 84, 140, 150). Despite all these studies, the effective CPR society remains unexplained, because our understanding on the relation between social structure and cooperative actions is very limited.

In contrast to previous studies on social or ecological phenomena, there is no formal framework or even any proper integrative foundation to study the science of socio-ecological phenomena (193). Recent attempts have been made to model coupled socio-ecological system by taking into account both social and ecological factors (57, 99). In the studies of coupled socio-ecological system (SES), the social agents act not only as external factors that influence the ecological system,

1.1 Coupled Socio-Ecological System

they can also adjust their behavior in response to any ecological alteration. Since the interaction between social and ecological components can be very complex, researches usually employ agent based modeling to simulate its interaction and uncover the emerging behavior of the system.

Most agent based models try to describe the situation as realistic as possible by involving all related components(56, 58, 59, 110, 167) Therefore, the proper theoretical analysis of such model is almost impossible. On the other hand, many abstract theoretical models are not empirically grounded and detach from the reality(57, 169, 182). Moreover, many studies just collect the empirical data from real world socio-ecological system and use statistical analysis to describe the behavior without any theoretical framework/model to explain the data(90, 95, 112, 175). In order to bridge this chasm, we try to relate the empirical data with the numerical result with solid analytical framework from statistical physics and nonlinear dynamics. In fact, this is one of the main contribution of this thesis in this field. The development of theoretical framework for coupled socio-ecological system should be driven by real-world systems. Hence, a careful choice of the relevant social and ecological variables is essential to formulate the theoretical model, in addition to being constrained by reasonable assumptions. The correctness of the model is then assessed by its ability to predict the trend of the social and ecological dynamics.

The study of coupled socio-ecological system should serve to benefit our environment, since social and economic progress essentially depend on ecological services such as hospitable climate, clean air, water, food, and other resources. Therefore, a comprehensive study of socio-ecological interaction is definitely a predominant key to a sustainable future. Moreover, from a practical point of view, a proper understanding of coupled socio-ecological system is relevant in the guidance of public policy decisions. In fact, ill-conceived governmental policies have led to: pest outbreak in Balinese padi rice fields (94); failure of water management by state institution (114); and poor forest protection by the government in comparison to forest that are locally managed (31).

1. INTRODUCTION

1.2 Alternate Stable State and Regime Shift

In this thesis, we specifically study the socio-ecological system in its relation with the concept of multiple stable regimes and regime shifts. Regime shift describes the abrupt structural change from one stable regime to another alternative stable regime with a totally different behavior(155, 159). It is widely known that ecosystems such as forest, grassland, desert, and lakes undergo catastrophic bifurcation and move into new stability through small gradual changes(29). The transition of a lake from clear to turbid(158), and coral reef collapse(66) are a few common cases of regime shift in ecology. Perhaps the most striking example is the birth of the Sahara desert 6000 years ago from an abundant vegetated area into a dry bare land(46). Abrupt transition can also be found in our society such as extinction of ancient civilizations, breakdown of modern governments, and economic crisis(47, 147).

Such abrupt shift can be explained by critical transition between two regimes. Complex nonlinear interaction between elements in the system may yield the existence of multiple stable regimes. Stable regimes are associated with local basin of attractors that dynamically pull the system towards them. Any alteration of the variable or parameter would reshape the stability landscape which may shift the system from one regime to another. This concept is not really new and had been expressed in the form of catastrophe theory by Rene Thom(183). This idea becomes widely accepted in ecology after Crawford Holling linked it to practical ecology(79). On the other hand, the development in statistical physics has uncovered the relation between phase transition and its underlying microscopic interaction. Recently, there is a resurgence of this idea in many system through the work of Marten Scheffer(155). This concept is also popularized to a wider audience by Malcolm Gladwell through his book, *Tipping point*(62). We will explore more about this concept in the next chapter.

A comprehensive review on this concept will provide a lot of practical implications for the policy makers. A good understanding on the mechanism behind regime shift can help us provide good solution to escape from undesirable environmental policies or to device preventive strategies from dangerous transitions. For instance, good policies can serve to prevent the extinction of fisheries and

forestry, and international cooperation to mitigate climate change. The concept learnt from this study can also provide better strategies to promote good transition. For instance, nutrient alteration for ecosystem recovery, or even microcredit injection to escape from poverty trap and mitigate societal inequality(155). The understanding gained can also help the development of detection method to anticipate any abrupt and dangerous change which serve to indicate the vulnerability of the current situation. The insight from the concept can also help the development of sustainable and resilience system.

1.3 Overview of the Thesis

Chapter 2 provides a comprehensive overview of the related conceptual and theoretical framework to construct and analyze coupled socio-ecological system in this thesis. The framework in this chapter consists of game theory, network theory, statistical mechanics, and non-linear dynamics.

Chapter 3 introduces the basic concept of socio-ecological model. Specifically, this chapter explains the construction of model, its socio-ecological elements, and updating rules.

Chapter 4 presents the behavior of socio-ecological system and its behavior in the setting of complex social interaction and ostracism mechanism. The connection between alternate stable states and social structure is explained through numerical simulation and analytical approximation. Its implication on the phenomena of hysteresis and regime shifts is also discussed. The result in this chapter is important as a foundational concept to understand the rest of the thesis. The results in this chapter has been published in: SUGIARTO, H.S., CHUNG, N.N., LAI, C.H. & CHEW, L.Y. (2015). Socioecological regime shifts in the setting of complex social interactions. *Physical Review E*, **91**, 062804 (178)

Chapter 5 presents the prediction of generic temporal and spatial early warning signals in anticipating the upcoming regime shift of socio-ecological system. Several generic early warning signals are compared with null model to measure its accuracy against false warning. The results in this chapter has been published in: SUGIARTO, H.S., CHEW, L.Y., CHUNG, N.N. & LAI, C.H. (2015).

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Complex Social Network Interactions in Coupled Socio-Ecological System: Multiple Regime Shifts and Early Warning Detection. *Proceedings of the International Conference on Applied Physics, Simulation and Computers*, **28**, 196–204 (177)

Chapter 6 presents the behavior of socio-ecological system with several communities. The connection between the existence of multiple regimes and segregated social structure is explained through numerical simulation and analytical approximation. The analysis in this chapter includes the phenomena of local regime shift and multiple hysteresis. Relation between social disharmony and community segregation is also discussed. Several possible community settings are also considered. The results in this chapter is currently under review for publication.

Chapter 7 presents the empirical data analysis from anthropological survey (courtesy of Prof John Stephen Lansing). The analysis involves dimensionality reduction by partial hierarchical clustering and principal component analysis. The result reveals the correlation structure among descriptors and the existence of several stable regimes with different socio-ecological characteristics that correspond to the socio-ecological model. The result in this chapter is currently under preparation for publication.

Chapter 8 presents the dynamical phase of socio-ecological system under different social settings. Social mechanism with more strategies is discussed and compared with ostracism mechanism by using phase diagrams. The emergence of more complex dynamics such as stable disequilibrium is revealed through numerical results. The result in this chapter is also under preparation for publication.

Finally, Chapter 9 presents the summary, conclusion, and outlook of the thesis.

2

Conceptual and Theoretical Framework

2.1 Game Theory

2.1.1 Classical Game Theory

The most difficult task in the study of socio-ecological system is to understand human decision with respect to the available resources. One common approach to model human decision is through a game theoretical framework. Game theory is a mathematical framework to study decision making among several rational agents. The early work in game theory can be traced back to the mathematician, John von Neumann(124). By using applied mathematics, he tried to study the rational economic interaction of several individual during the decision making process. The fundamental assumption in game theory is that the agents possess complete knowledge and make rational decision to adopt optimal strategy based on a common utility function.

The decision making procedure in game theory is commonly represented in a matrix which consist of players, strategies, and utilities. Figure 2.1 below shows the most famous scenario in game theory known as prisoner's dilemma. In this setting, two prisoners are isolated without any possibility of communicating with each other. If both of them defect each other, each of them serves longer years in prison (let say 2 years in prison). If one of them defects the other prisoner

2. CONCEPTUAL AND THEORETICAL FRAMEWORK

while the other prisoner cooperate, the defector will be free while his friend will be punished heavily (let say 4 years in prison). If both of them cooperate, they will be punished by a lesser charge (let say 1 year in prison). This scenario is illustrated as a utility matrix below. Both players will be better off if they cooperate. However, no matter what is the other player's strategy, any player will always have a larger payoff by defecting.

		Prisoner II	
		Cooperate	Defect
Prisoner I	Cooperate	1 year, 1 year	Free, 4 years
	Defect	4 years, Free	2 years, 2 years

Figure 2.1: (Color online). Utility matrix of 2-player prisoner's dilemma situation. Both players have two strategies: cooperate and defect. If both players cooperate, both only serve 1 year in prison. If both players defect, both serve 2 years in prison. If one player choose cooperate and another player choose defect, the cooperator will serve 4 years in prison and the defector will be free.

The Nash equilibrium captures this final outcome when every agent play their best response strategy with all the other agents(120). Consequently, the Nash equilibrium of prisoner's dilemma is when both of them betray each other. Game theory provides a rational solution for individual decision on certain situation. However, the solution becomes impossible when we face very complicated situation with a large number of players and multiple strategies. In its further development, the rationality assumption is considered to be unrealistic and has been replaced by the concept of bounded rationality, where every individual can only make decision based on the limited information available to them.

2.1.2 Evolutionary Game Theory

Game theory progresses after John Maynard Smith proposed the idea of evolutionary game theory by applying game theory to solve biological interaction in the population level(171). By identifying the utility function to be proportional to reproductive fitness, the population of better strategies increases and dominates other strategies. Evolutionary game theory describes the evolution of agent's strategy every time step through an updating mechanism which is depicted as births-deaths in the biological context or imitation/learning in the social context. The game starts with certain proportion of strategy in the population. According to certain game rules, some strategies are increased while others decreased. The subsequent proportion of strategy is then used as a basis for the next game. The game is continued until equilibrium is reached.

This framework has introduced dynamics and multiple players interactions into the classical game theory. Evolutionary game theory approach has been widely adopted by biologist, psychologist, sociologist, and anthropologist to explain the widespread of cooperating behavior in biological and social context(131). Moreover, this approach has been applied to a wide range of discipline including economics, transportation, computer science, politics, and even physics.

Under the deterministic limit of infinite population and well mixed interaction among individuals, the approximate evolution of the system will naturally lead to a set of nonlinear differential equation called replicator equations. Replicator equations capture the time evolution of the fraction of each strategy in the population(75). The replicator differential equation can be written as

$$\dot{x}_i = x_i \left(f_i(x) - \sum_j^n x_j f_j(x) \right), \quad (2.1)$$

where x_i is the fraction of individual with strategy i from total population, and $f_i(x)$ is the fitness of strategy i .

In finite population the deterministic replicator equation no longer holds because internal stochasticity dominates the system depending on the updating mechanism. The situation become more complicated when we introduce internal structure into the system(132, 133).

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2.1.3 Spatial Evolutionary Game Theory

In real situation, the interaction between individuals is limited as it depends on the internal spatial structure of the society. In general, direct interactions among individuals are limited to their nearest neighbors only. Therefore, a more realistic approach to study social interaction is to introduce spatial structure and local interaction into the evolutionary game dynamics(101). In this framework, the strategy is updated individually under local interaction instead of updating it as a whole in the population level(170). The example of strategy configuration snapshot in spatial game theory is shown in figure 2.2. The network does not solely represent geographical connection, it can also represent social, cultural, language, or even economic interactions depending on the real system that we are dealing with. Therefore, the topology may vary from the most simple such as a lattice network to the more complex structure such as scale-free network and modular network(136).

Spatial structure and local interaction has been proven to be the major element for the explanation of diverse significant phenomena in various systems. In physics, the local interaction is important key to understand phase transition and critical phenomena in magnetic system(83) and superconductivity(104). In biology, the local chemical interaction between cells is fundamental to the emergence of different skin pattern(189). Moreover, local interaction can also throw light on the specific pattern of animal aggregation behavior such as swarming, schooling, and shoaling among fishes and birds(148). In economics, Hotelling discussed how individual market put their focus into spatial regions to enjoy local monopolistic status(80). In sociology, Schelling explained the emergence of racial segregation by introducing spatial pattern in his analysis(162). Local interaction was also used by Axelrod to highlight the coexistence of different cultural traits(4).

2.1.4 Application of Evolutionary Game Theory on Coupled Socio-Ecological Dynamics

In socio-ecological system, every individual makes decision while the ecological variables change concomitantly with the social behavior. In this context, evolutionary game theory can be used to predict the evolution of societal behavior

Figure 2.2: (Color online). Typical snapshot of strategy configuration in Spatial Game Theory. The colors represent different strategy of the node. Specifically, color blue represent strategy 1, color red represent strategy 2, color green represent strategy 3. We can keep adding different colors to represent different strategies depending on what kind of spatial game we use. In spatial game every players only interact and compare their strategy with their nearest neighbor only. Based on specific updating mechanism, for each time step, each player updates their strategy accordingly.

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based on individual decision and interaction. Since the payoff is closely related to the social and ecological variable, every time each individual makes a decision, his/her action affects the ecological variable and change the payoff. Therefore, in contrast to the normal form of game theory, the payoff changes with time and the decision taken in the system.

Since the amount of resource depends on each individual decision, the cooperation among the users is essential to protect the ecology. Although everyone understands that they are better off if they exercise restraint on the use of the resource, such a behavior is not followed by a rational and selfish person who is motivated by self-interest to maximize individual benefits. In the end, everyone will deplete the ecological resource and put the whole society in a bad state. As mentioned in chapter 1, this situation is known as the tragedy of the commons(67). The tragedy of the commons is very similar with one class of game theoretic problem called the prisoner's dilemma. This similarity make game theoretical approach advantageous in dealing with socio-ecological situation such as the tragedy of the commons.

2.2 Network Theory

2.2.1 Network Introduction

In section 2.1.3, we briefly mention about the existence of spatial structure in the system. In general, any social interaction can be represented as a network. Network is a representation of a set of interconnected objects. Network is represented as vertices/nodes and edges/links. The vertices-edges relation can describe a lot of interaction such as protein-reaction in biological network, neuron-synapse in neural network, person-friendship in social network, web-hyperlink in internet network, airport-flight in transportation network, and many more. Because of its simplicity and generality, network analysis has been applied in a lot of different disciplines.

The earliest work on network can be found in Euler solution of the Konigsberg bridge problem(54). However, the first rigorous analysis of network is studied by

Erdős(53). After that, Milgram did the famous empirical work on social interaction and found the six degree of separation(116). Duncan Watts later introduced the small world network to explain this phenomenon(194). The empirical observations of real network show the power law properties in a lot of different network. Barabasi later developed an algorithm to show that the power law properties can arise naturally from simple interaction(6). After that Mark Newman introduced the concept of modularity to reveal the existence of community structure in the network(125).

2.2.2 Network Properties

A network can be represented in the form of an adjacency matrix and adjacency list. Adjacency matrix is a matrix with elements $e_{ij} = 0$ if there is no link between i and j and $e_{ij} = 1$ if there is a link between i and j . The network in figure 2.2 is an example of an undirected network in which two connected nodes can interact in both direction and the adjacency matrix is symmetric. Throughout the whole thesis, we only consider this undirected interaction. In some networks such as the World Wide Web, the interaction between nodes is only running in a certain direction. This network is known as directed network where the adjacency matrix is no longer symmetric. In certain network, the connection strengths among the links are unequal. This network is known as weighted network and the adjacency elements of this network is no longer 0 and 1 but can be any real number. There are other type of more complex network which cannot be represented by using ordinary adjacency matrix such as hypergraph, bipartite network, and multiplex network.

Every network has several structural properties that can be captured by certain measures. For instance, we can calculate how many links that are connected to one particular node. This measure is known as degree, in which the degree of a node i can be written as $k_i = \sum_j e_{ij}$. This concept can be generalized by counting the degree of all nodes to obtain its statistical information. In general, the statistical property of network degree is captured by the degree distribution, $P(k)$. Degree distribution also contains certain topological features of a

2. CONCEPTUAL AND THEORETICAL FRAMEWORK

network, for instance the degree distribution of a lattice follows delta distribution, the degree distribution of Erdős-Rényi (random) network follows a Poisson distribution(53), and the degree distribution of Barabasi-Albert (scale-free) network follows a power law distribution(6). The average degree of a network is written as $\langle k \rangle = \sum_k kP(k)$. Average degree is an useful measure because it captures how dense the network connection is. High degree network indicates very connected network while low degree network indicates less connected network.

Some nodes can be more important than other nodes. In order to classify the degree of importance between different nodes, many centrality measures have been developed. Different centrality measure describes different possible definition of classification. The most common measure is degree centrality which ranks the node based on individual degree. Degree centrality gives us the sense that the one with more connection is better than the one with less connection. In reality, not all connections are equally important. To capture this effect, researchers develop eigenvector centrality. An extension of eigenvector centrality is the Katz centrality and PageRank which has been used by google to rank different website in their search engine. Other centrality measures such as closeness and betweenness centrality use path distance to classify the important node. These measures are very useful in determining which nodes are central in controlling information flow. The centrality measures indicate the existence of hubs or central actors in the network. In some topology such as scale-free network, the hubs are very essential to regulate the overall network activity.

Research shows that people from the same language, race, religion, and other common traits interact more strongly and tend to segregate themselves from different people(113). The preference to interact with similar others is known as homophily. This behavior creates several communities of densely clustered nodes in the interaction network which is known as community structure. Community structure and segregated interaction often affect the outcome of overall social behavior(153). This segregation property is usually detected by classifying the modular network into several communities(61). The strength of the community structure is evaluated by the modularity measure(128). The network that consist of several communities is known as a modular network. This network is ubiquitous in real world and there are a lot of community detection algorithms to discover

such network. There is a specific interaction preference based on degree similarity which is known as assortativity(125). The opposite of this behavior is called disassortativity which is the tendency to interact with different degree node. The network with this topological properties is known as assortative and disassortative network.

2.2.3 Network Topology

In this thesis we consider several network topologies to study the social behavior under different interaction structure. There are several network models with unique topology, however in this thesis we only consider random network, scale-free network, and modular network.

2.2.3.1 Random Network

Random network is a network model in which each node is randomly connected to another node without any preference. This is the oldest proposed network to model some features in real world networks. A random network is constructed through Erdős-Rényi (ER) algorithm(53) (Figure 2.3). The mathematical property of random network has been studied mathematically(129). Random network topology is characterized by a Poissonian degree distribution. In this distribution, some nodes have higher and lower degree than the average degree. Many nodes' degree are close to the average degree, while nodes with very high and very low degree are rare. Therefore, the hubs are not significantly dominant in this network. Random network can explain the connectedness and short path distance in real world network. However, this network cannot explain high clustering feature, the presence of dominant hubs, and multiple communities in real network. Small-world network is an extension of random network with very similar network properties such as poisson degree distribution, no hubs, and no communities. The only difference is that small world network has high clustering property(194). Small-world network property is not considered in this thesis because of its similar topological property with the random network.

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Figure 2.3: (Color online). Random (Erdős-Rényi) Network. The figure above shows a population $N = 200$ and average degree $\langle k \rangle = 5$. The size of each node represents the degree (number of links connected to the node).

Figure 2.4: (Color online). Scale-free (Chung-Lu) Network. The figure above shows a population $N = 200$ and average degree $\langle k \rangle = 5$. The size of each node represents the degree (number of links connected to the node).

2. CONCEPTUAL AND THEORETICAL FRAMEWORK

2.2.3.2 Scale-free Network

The tremendous development of computational processing capabilities facilitate rigorous characterization of real network. Empirical data shows the presence of several strong hubs in the network. This property is associated with power law degree distribution. In this distribution, there are several hubs with very high degree and a lot of nodes with very low degree. This type of network is known as scale-free network. The most prominent model to mimic this feature is described by the Barabasi-Albert (BA) algorithm, which is also known as preferential attachment(6). In this algorithm, high degree nodes have higher probability to connect with other nodes. This algorithm begins with an initial graph of N_0 nodes, where N_0 must be equal or larger than 2 and each node must have at least degree 1 in the initial graph (to make sure every nodes are connected). Next, a new node is added to the graph, one at a time. Each new node is then randomly connected to any of the N existing nodes with the probability proportional to the existing nodes' degree. This algorithm can model the growth of network with scale-free distribution. Because of this deep insight and powerful explanatory power, Barabasi-Albert model is the most cited network model ever. However, this algorithm has some limitation in controlling the average degree, which makes it difficult to compare directly with random network.

There is an algorithm that is able to produce power law distribution from specific average degree random network known as the Chung-Lu (CL) algorithm(33). This algorithm can create power law degree distribution for any particular average degree (figure 2.4). This algorithm assigns each node with specific weight, i.e. $w_1, w_2, \dots, w_n$, where $w_i w_j / \sum_k^N w_k$ represents the probability that node i and j is linked. Two random nodes are then chosen and two random numbers (one for each node) are drawn. These nodes are linked if the first random number is smaller than the weight of the first node and the second random number smaller than the weight of the second node. This process is repeated until the expected average degree is achieved. There are also several proposed algorithms that bridge the random and scale-free network. Despite the success of scale-free network, this network still lack many important properties of real network such as modularity and assortative feature.

2.2.3.3 Modular Network

Recent studies in empirical network reveal the existence of community structure in the network. Community structure is associated with grouping of network into several modules with denser connection between nodes from the same community rather than connection between nodes from different community. This structure is usually caused by homophily phenomena in which each nodes prefer connecting with other similar nodes(113). Many algorithms have been proposed to detect the hidden community structure in the network such as Girvan-Newman algorithm(61), hierarchical method(1), modularity maximization(35, 127), and many others. To address this unique real network feature, several network model with community structures have been proposed. Many of these model make use of modularity maximization to construct the model. However, there are more intuitive way to construct the effect of community structure using mixing parameter.

Mixing parameter measures the average fraction of links from other communities over the average total links of a node. For example Fig. 2.5 shows a random network, $\langle k \rangle = 15$ with (a) 2 communities and (b) 4 communities with mixing parameter, $\mu = 0.2$. In the case of a 2 community society, $\mu = 0.5$ represents perfectly integrated society because every individuals interact without any community preference. Reducing μ makes the society more segregated because a lower μ indicates that every individuals prefer to interact with their own community. Based on this definition, there are $(1 - \mu)\langle k \rangle$ intra-community neighbors and $\mu\langle k \rangle$ inter-community neighbors on average. Community structure and segregation effect are established through rewiring original undirected random network with N nodes and $\langle k \rangle$ average degree according to mixing parameter (μ). Each node is classified into several community groups and each link is rewired accordingly by using Lancichinetti-Fortunato-Radicchi (LFR) benchmark algorithm(91). The natural extension of similarity preference in the network structure is the assortative and disassortative behavior in which each node prefers to connect with other nodes with similar or dissimilar degree. There are several interesting algorithms that produce assortative network such as assortativity maximization(126)

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or Xulvi-Brunet - Sokolov (XS) algorithm(199). Assortative network is not discussed in this thesis and will be considered as future work.

Figure 2.5: (Color online). Network with community structure. The figure above shows a population $N = 100$ and average degree $\langle k \rangle = 15$ for the case of (a) 2 communities and (b) 4 communities with same population size. Different color represents different community. The graph is constructed by rewiring original random network by using mixing parameter $\mu = 0.2$. The size of each node represents the degree (number of links connected to the node).

2.2.4 Dynamics on Network

The purpose in examining several types of network topological structure is to provide a more comprehensive analysis on the behavior of system that is represented by such structure, for instance disease spreading(143, 190) or the evolution

of social behavior in interaction network(180, 181). In this framework, the state of each node can evolve with time. Since we want to study the evolution of social behavior, the state of each node represents the strategy in the game theoretic framework. The strategy of each node/individual will change depending on interactions with other strategies.

In general the state of node i changes according to its previous state and its interaction with other node. This process can be written as

$$s_i(t+1) = f\left(s_i(t), \vec{s}_j(t)\right), \quad (2.2)$$

where s_i represents the state of node i , whereas j represents any nodes adjacent to i .

The dynamics on network is more intuitively studied through computational simulation. In this framework, the dynamical process is simulated through specific related algorithm. By simulating the process, interesting macroscopic behavior of the system may emerge from the microscopic interaction. This concept is known as agent based modeling. However, many interesting results from computational simulation cannot be understood without an analytical framework. Therefore, let us move on to the topic of statistical physics.

2.3 Statistical Physics

2.3.1 Microscopic Interaction and Macroscopic Behavior

Statistical physics is the study of behavior of physical system with a large populations by implementing the methods from statistics and probability theory. The approach of statistical physics can be traced back to Maxwell in his modeling of physical system in a probabilistic fashion(145). Statistical physics develops the link between the microscopic interactions among individual parts with the macroscopic behavior of the global system. In this sense, there is a close association between social interaction in network and statistical physics. In the microscopic level, the interaction between two individual is described by the updating mechanism. This procedure is similar to Monte Carlo simulation in statistical physics

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such as the Ising model(83). The simple interaction rule in the microscopic level may create complex macroscopic behavior such as critical transition.

In statistical physics, critical transition is generally associated with the order parameter that linked to the external control parameter. In general, the system changes continuously by varying the external control parameter. However, there can exist a specific value of control parameter called critical point that can separate two different phases in the system(172). First-order transition (discontinuous transition) is associated with a discontinuous derivative of the order parameter. Therefore, the state of the system jumps from one phase to a different phase as the control parameter is varied beyond a certain point. The transition from random movement of particles (liquid) to ordered crystalline (solid) is an example of discontinuous transition. Second order transition is attributed with a continuous derivative of the order parameter. Since there is no explicit distinction between two different phases at the critical point, this transition is associated with a continuous transition. Second order critical transition is also related to scaling behavior and length correlation. The critical transition that arises from implicit connection between simple microscopic interaction and complex macroscopic behavior in statistical physics is an essential foundation for the understanding of complex behavior in other systems such as the biological and social systems.

2.3.2 Stochastic Approach in Spatial Game

In social system, local interaction and strategy updating are governed by updating mechanism based on evolutionary game theory. The updating mechanism of the individual decision is similar to the metropolis algorithm in statistical physics by exploiting the same iterative procedure of Monte Carlo simulation(115). In this algorithm, we choose a random agent and a random neighbor. Then we generate from a uniform distribution, a random number in between 0 and 1. We compare this number with normalized utility difference to decide whether to copy the neighbor's strategy or not. This procedure is repeated until final equilibrium is reached.

The updating mechanism above can be represented in terms of stochastic differential equations. Before we do that, let us introduce some probability nota-

tions. Let $P(x)$ be the probability of finding a random player with strategy x . Let $P(x|y)$ be the conditional probability of finding a player with strategy x given that the adjacent node is occupied by another player with strategy y . Bayesian identity dictate that, $P(x|y)P(y) = P(y|x)P(x) = P(xy)$, where $P(xy)$ is the probability of finding xy pairs in the system. Similarly, we can write any conditional probability $P(x|R)$ and expand previous notation to describe any structural motif R . The symbol R represents any specific strategy configuration. For instance when R is just a node with strategy y , $P(x|R)$ is written as $P(x|y)$. Similarly, when R is a strategy configuration with structure $y <_y^x$, $P(x|R)$ is written as $P(x|y <_y^x)$.

For simplicity, the short hand notation, P_x and $q_{x|y}$ will be used instead of $P(x)$ and $P(x|y)$. Since we have N individuals in total, therefore the number of player with strategy x can be approximated as $N_x = NP_x$. Furthermore, let $T^+(N_x)$ be the transition probability that N_x players are increased by 1 and $T^-(N_x)$ be the transition probability that N_x players decreased by 1. The exact formulation of transition probability depends on the updating mechanism of the model. In the later chapters, we will calculate the specific transition probability for each specific problem.

From the transition probability we can construct the master equation of the system.

$$\begin{aligned} P(N_x, \tau + 1) - P(N_x, \tau) = & \\ & P(N_x - 1, \tau)T^+(N_x - 1) - P(N_x, \tau)T^-(N_x) + \\ & P(N_x + 1, \tau)T^-(N_x + 1) - P(N_x, \tau)T^+(N_x). \end{aligned} \quad (2.3)$$

where τ represents simulation time steps.

We then rescale the master equation with the population size N , such that $P_x = \frac{N_x}{N}$ and $t = \frac{\tau}{N}$ and we will obtain

$$\begin{aligned} P(P_x, t + 1/N) - P(P_x, t) = & \\ & P(P_x - 1/N, t)T^+(P_x - 1/N) - P(P_x)T^-(P) \\ & + P(P_x + 1/N)T^-(P_x + 1/N) - P(P_x)T^+(P_x). \end{aligned} \quad (2.4)$$

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Next, we perform Taylor's expansion with respect to t until $\frac{1}{N}$ term and P_x until the $\frac{1}{N^2}$ term and neglect the higher order terms(88):

$$\begin{aligned}
P(P_x, t) + \frac{dP(P_x, t)}{dt} \frac{1}{N} - P(P_x, t) = \\
\left(P(P_x, t) - \frac{dP(P_x, t)}{dP_x} \frac{1}{N} - \frac{1}{2} \frac{d^2 P(P_x, t)}{dP_x^2} \frac{1}{N^2} \right) \left(T^+(P_x) - \frac{dT^+(P_x, t)}{dP_x} \frac{1}{N} - \frac{1}{2} \frac{d^2 T^+(P_x, t)}{dP_x^2} \frac{1}{N^2} \right) \\
+ \left(P(P_x, t) + \frac{dP(P_x, t)}{dP_x} \frac{1}{N} + \frac{1}{2} \frac{d^2 P(P_x, t)}{dP_x^2} \frac{1}{N^2} \right) \left(T^-(P_x) + \frac{dT^-(P_x, t)}{dP_x} \frac{1}{N} + \frac{1}{2} \frac{d^2 T^-(P_x, t)}{dP_x^2} \frac{1}{N^2} \right) \\
- P(P_x) T^-(P_x) - P(P_x) T^+(P_x)
\end{aligned} \tag{2.5}$$

which becomes

$$\begin{aligned}
\frac{dP(P_x, t)}{dt} \frac{1}{N} = \\
- \frac{1}{N} \left(\frac{dP(P_x, t)}{dP_x} T^+(P_x) + P(P_x, t) \frac{dT^+(P_x, t)}{dP_x} \right) + \frac{1}{N} \left(\frac{dP(P_x, t)}{dP_x} T^-(P_x) + P(P_x, t) \frac{dT^-(P_x, t)}{dP_x} \right) \\
+ \frac{1}{N^2} \frac{dP(P_x, t)}{dP_x} \left(\frac{dT^+(P_x, t)}{dP_x} + \frac{dT^-(P_x, t)}{dP_x} \right) + \frac{1}{N^2} \frac{1}{2} \frac{d^2 P(P_x, t)}{dP_x^2} (T^+(P_x) + T^-(P_x)) \\
+ O\left(\frac{1}{N^3}, \frac{1}{N^4}\right).
\end{aligned} \tag{2.6}$$

By simplifying equation 2.6 and ignoring higher order terms, we obtain the Fokker-Planck equation:

$$\frac{d}{dt} P(P_x, t) = - \frac{d}{dP_x} \left[P(P_x, t) (T^+(P_x) - T^-(P_x)) \right] + \frac{1}{2} \frac{d^2}{dP_x^2} \left[P(P_x, t) \frac{T^+(P_x) + T^-(P_x)}{N} \right], \tag{2.7}$$

which is equivalent to the associated nonlinear Langevin equation, whose noise term $\eta(t)$ is linked to the distribution $P(P_x, t)$ through

$$\frac{dP_x}{dt} = (T^+(P_x) - T^-(P_x)) + \sqrt{\frac{T^+(P_x) + T^-(P_x)}{N}} \eta(t). \tag{2.8}$$

The variable $\eta(t)$ is a random fluctuating term with zero average $\langle \eta(t) \rangle = 0$ and time correlation given by $\langle \eta(t) \eta(t') \rangle = \delta(t - t')$.

In the thermodynamic limit where $N \rightarrow \infty$, the diffusion terms vanish, leading to

$$\frac{dP_x}{dt} = (T^+(P_x) - T^-(P_x)). \tag{2.9}$$

Before we move to the next section, we discuss the validity of stochastic approach above in analyzing our computational simulation. The transition probability $T^\pm(P_x, t)$ depends on the two variables: the fraction of strategies P_x and time t . The Taylor expansion works accurately when this function is smooth. In general, numerical simulation is not smooth because of its discrete nature. However, when the related variables evolve gradually, we can assume the simulation is smooth. In our system, we can approximate this discrete updating function as a smooth function when the global strategy proportion is changing slowly. This approximation is fulfilled under the condition when the population is large and the strategy is updated one at the time ($1/N$ fraction for every time step). This requirement is satisfied in our simulation because we consider asynchronous updating mechanism in which only single node is updated for every time step. Furthermore, when the population size is large, we can neglect higher order terms. However, larger population size means longer simulation time and larger memory required for the simulation. In this thesis, we use population size in the range between 50 to 200 depending on our purposes. Below this range, it may cause incompatibility with our analytical approximation. Beyond this range, it is considered numerically impractical and may give rise to out of memory situation.

The solution of this stochastic differential equation depends on the exact form of the transition probability which is too complex in most cases. Therefore, we shall discuss on certain approximation that can help us obtain the analytical solution.

2.3.3 Mean Field and Higher-Order Approximation

Although the updating mechanism and social interaction can be very simple, the exact analytical solution of the global behavior of such system is almost impossible to obtain. In most cases, the state probability of the system (P_x) is a function of the pair probability ($q_{x|y}$). The simplest way to approximate the macroscopic behavior of the system is to use mean field approximation (MFA), which ignores local interaction in the system(72). The main assumption of mean field approximation is that state of any individual can be approximated as the average state of the system ($q_{x|y} \approx P_x$), regardless of the local structure of that

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particular individual. For instance, by using this approximation, the probability to find xy pairs in the system can be approximated as the probability of x meet y in the well-mixed situation ($P_{x|y} = q_{x|y}P_y \approx P_xP_y$). This solution often fails to predict the essential characteristic of the system because complex behavior often originates from the local interaction.

The next simplest extension of mean field approximation is pair approximation (PA) which tries to capture the state correlation between the nearest neighbor(5). Practically, pair probability ($q_{x|y}$) is an unknown quantity and we need another equation to find this quantity. Unfortunately, the equation of pair probability ($q_{x|y}$) depends on the triple probability ($q_{x|yz}$) and sometimes the probability of more complex motifs such as ($q_{x|<^y_z}$). The main assumption of pair approximation is the state and local structure of any individual can be approximated as the state of its nearest neighbors ($q_{x|R} \approx q_{x|y}$). Therefore, we can reduce all the complex motifs into pair relation and solve this term.

Similarly, we can also work on the equation for triple probability, however this equation also requires the probability of more complex structural motifs (triangle, square, star, etc)(152). This procedural sequence will never stop, and in the end we need to stop at the i^{th} -order approximation. For instance, in triple approximation, we may reduce all related structural motifs into triple relation, ($q_{x|R} \approx q_{x|yz}$). Higher order approximation would definitely produce more accurate solution, however the equations may become intractable as more motifs are involved.

2.3.4 Equilibrium and Non-Equilibrium

Some systems exhibit the thermodynamic property of being in equilibrium. Under this situation, if we let the system evolves with time long enough, the state of the system would converge to the global equilibrium. In equilibrium, the analytical complexity can be simplified because the principle of detailed balance is obeyed by the system. The principle of detailed balance dictates that the transition probability from one state to another is equal to the reverse transition. However, most real systems are far from equilibrium which is impossible to trace analytically through all the interaction in the system. One way to analyze it is

through dynamical system theory and the framework of bifurcation framework since non-equilibrium phenomenon is one of the trademark of nonlinearity. Such general treatment of equilibrium and non-equilibrium system will be discussed through the performance of dynamical system analysis in section 2.3.4.

2.4 Non-linear Dynamics

2.4.1 Dynamical System

The formulation of dynamical system is pioneered by Newton as he tried to explain the movement of celestial objects. It turns out that this framework is very useful in engineering for the development of radio, laser, non-linear oscillator, and many other devices with interesting behavior(176). In fact, the invention of computer allows numerical study of the system without the need of analytical solution. In fact, this is the main approach of non-linear dynamics. For example, Edward Lorenz discovered the chaotic dynamics through computer simulation(106). Feigenbaum later uncovered the universal phenomenon in the transition towards chaotic dynamics through the renormalization method, which serves to link non-linear dynamics to statistical physics(55). Finally, Mandelbrot developed the concept of fractal to explain the non-integer structure of strange attractor, and this concept has been applied to a vast variety of complex system(111).

The dynamics of a system can be described by the time evolution of the states of that system. Dynamical system is often used in physics and mathematics to depict all kinds of processes. Basically any complex physical, chemical, biological, and social interactions can be simplified by using several sets of differential equation. The general expression of the dynamics of the system can be written as:

$$\begin{aligned}\frac{dx_1}{dt} &= f_1(x_1, \dots, x_n) \\ &\vdots \\ \frac{dx_n}{dt} &= f_n(x_1, \dots, x_n)\end{aligned}\tag{2.10}$$

where $x = (x_1, \dots, x_n)$ is the n -dimensional states of the system, and $f = (f_1, \dots, f_n)$ is a set of smooth time independent functions that govern the time evolution of

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the system. The formulation can also be expressed in the more compact vector form as follows:

$$\frac{d\vec{x}}{dt} = f(\vec{x}). \quad (2.11)$$

In the context of socio-ecological system, there are at least two coupled dynamical equations. One equation governs the social dynamics, while the other equation depicts the ecological dynamics. In section 2.3.2 we have discussed how the social dynamical equation can be approximated through several frameworks from statistical physics. This approximation reduces all microscopic interactions into several equations that govern specific strategy in the macroscopic social behavior. If there are two strategies available, we would only need a single equation to describe their behavior provided we have a constant population, since the equation for the other strategy can be obtained through the probability identity: $1 = P_{s1} + P_{s2}$. P_{si} is the probability of finding a person with strategy i . Furthermore, the dynamical equation for ecological system can be approximated through an inflow-outflow mechanism by ignoring the local behavior of the resource. We can safely assume that all individuals have free access to any part of the resource based on the definition of common pool resource.

The solutions of these differential equations are portrayed as a unique trajectory via the n -dimensional state coordinates $(x_1, \dots, x_n)$, which depends on the initial conditions. The space where this trajectory moves is called phase space. Fixed points are the points in phase space where $\frac{dx}{dt} = 0$. These points can be stable and unstable depending on the trajectories near the fixed points. Fixed points are called stable equilibrium or attractor if nearby trajectories approach this fixed point. On the other hand, the fixed points are called unstable equilibrium or repeller if nearby trajectories avert this fixed point. The solution can be solved exactly when the dynamical functions $(f_1, \dots, f_n)$ are linear. The eigenvectors represent the direction of the trajectories while the eigenvalues represent the rate of change towards equilibrium. However, in most system, the dynamical functions are in the form of nonlinear equations and have no exact solution. Nevertheless, the dynamical behavior near equilibrium can be linearly approximated by using only the first order Taylor expansion. Nowadays, the solutions and trajectories of nonlinear dynamical functions can be solved numerically through computer.

There is also another type of dynamical formulation which is known as difference equations or iterated maps. In contrast to differential equations, we are dealing with situation where time is discrete as in difference equations. The dynamics between these two types of dynamical system become very similar when the time increment is very small and the system evolves in a much longer time. In general, the behavior of the system can be stable, periodic, or even chaotic depending on the stability regime of the current system.

2.4.2 Attractors and stability

The dynamical behavior and stability of any system depends on the types of attractors in the system. To illustrate the stability of the system, consider one dimensional dynamical equation $\frac{dx}{dt} = f(x)$. The intersection between the curve $y = \frac{dx}{dt} = f(x)$ and the line $y = 0$ gives the fixed points. If the curve is above this line, the dynamical flow is towards the positive direction, and if the curve is below this line, the dynamical flow goes towards the negative direction. If the dynamical flow of the region after fixed point moves towards negative direction and the dynamical flow of the region before fixed point moves towards positive direction, we call this point as the stable fixed points. This is because the dynamical flow near the fixed point is always attracted towards it. On the other hand, we call this point an unstable fixed point if the dynamical flow near it is divergent.

The stability of the system can also be understood through the concept of potential landscape. In dynamical system theory, the potential is defined as(176):

$$\frac{dx}{dt} = -\frac{dV(x)}{dx} \tag{2.12}$$

In this representation, the stable fixed point is located at the bottom of the valley and unstable fixed point is located at the peak of the hill. The width and depth of the valley is related to the resilience of the stable state(155). The system is not resilient when a small perturbation is able to push the system over the boundary of attraction. This potential representation give us a more intuitive way to understand the dynamics since the system behavior can be likened to a ball moving from higher potential to the lower potential as it slides from the top of a hill to the bottom of a valley.

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Stable and unstable points are the simplest attractors in dynamical system. They are point attractors. In 2-dimensional system, there can also exist other types of stability regime related to the limit cycle. Limit cycle is associated with close orbit attractors. There are several mechanisms that generate this dynamical behavior. Limit cycle typically appears when stability analysis of the dynamical system yields complex eigenvalues. It may also be caused by self-repeating regime shift between fast and slow dynamics. When the dimension of the system goes beyond 4, the oscillation can behave quasi-periodically, i.e. almost periodic. Quasi-periodic regime is associated with the torus attractor. In higher dimensional system, the dynamical behavior can become more complex. There exist other stability regime known as chaotic dynamics which is associated with random looking dynamics, and usually possesses non-integer or fractal structure attractor known as the strange attractor.

2.4.3 Bifurcation

In many cases, dynamical system displays new stable and unstable solutions as a parameter is varied and when this happens, the system is said to bifurcate(176). Bifurcation is associated with a major shift in dynamical structure under the variation of control parameter. Beyond the bifurcation point, a new fixed point can be created or destroyed and the system enters a new and different stability regime. Bifurcation provides a framework to understand the phenomena of critical transition because it explains the significant changes of the dynamical structure if the control parameter is altered beyond the critical point. Symmetry breaking and catastrophe phenomena are famous example of the application of bifurcations theory in real world.

There are different types of bifurcation that represent different transition from one particular dynamical regime to another. Saddle-node bifurcation occurs via the disappearance of fixed point. Transcritical bifurcation explains the exchange of stability between stable and unstable fix points. Pitchfork bifurcation emerges through the appearance of symmetrical pair of stable or unstable fixed points. Hopf bifurcation illustrates the appearance of limit cycle. There are other more complex bifurcations that generate limit cycles such as infinite period bifurcation,

homoclinic bifurcation, and heteroclinic bifurcation. There are also several types of bifurcation that are related to chaotic regime such as the flip bifurcation, tangent bifurcation, and period doubling bifurcation.

In correspondence to statistical physics, the supercritical pitchfork bifurcation is related to second order phase transition, while the fold bifurcation to first order phase transition. Fold bifurcation is a type of local bifurcation that occurs when stable and unstable points collide and annihilate. Since there is no stable solution beyond this point, the system is forced to make a sudden jump to a different stable point. Fold bifurcation is often used in catastrophic theory to explain sudden and dramatic changes. The parallel analogy between bifurcation and critical transition offers a link between dynamical system theory and statistical physics. From statistical physics, we can study the emergence of macroscopic behavior of the whole system from microscopic interaction between its parts. From dynamical system theory, we can study the dynamics, bifurcation, and stability of the system under alteration of external parameters.

Specifically, discontinuous transition can be explained through a change in potential landscape via fold bifurcation under a continuous alteration of external control parameter. This transition occurs when there are alternate valleys (stable) separated by a hill (unstable) in a certain range of control parameter between two distinct critical points. Beyond the bifurcation point, the unstable hill suddenly disappears and the system can move freely from one stable state to another stable state. This potential landscape representation is also known in statistical physics as the Landau's theory of critical transition. This concept is employed to explain the phenomena of regime shift in chapter 4. Once the transition occurs, the system cannot be reversed back to its previous state unless we reverse the control parameter beyond another bifurcation point. Therefore, there exists two different critical transition in two different directions, forward and backward. This property of irreversibility is associated with the hysteresis effect, in which the state of the system depends not merely on the variables and parameters but also on the history of the system. In chapter 1, we have briefly discussed about the phenomena of regime shift and its implication. In this section, we have conceptually explained these phenomena as a consequence of fold bifurcation and the existence of alternate stable states.

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2.4.4 Critical Slowing Down and Early Warning Signals

There is a special dynamical property when the system is close to a critical point. This property is known as critical slowing down. Mathematically, the eigenvalue approaches zero when the control parameter is very near to the bifurcation point. Eigenvalue is related to the speed at which the system approaches equilibrium. There are several implications of this phenomenon that can be used as early-warning indicator for any dynamical movement approaching a bifurcation point. Since the rate of change around equilibrium approaches zero during critical slowing down, the recovery process from any small perturbations becomes slower than before(161). In real situation, the dynamics is always fluctuating since there is a lot of external noise from the surrounding environment. The dynamical fluctuation plus the slowing down in the time evolution is reflected in the increasing autocorrelation and variance in the time series. The autocorrelation increases because the current state becomes more and more correlated with the previous state. The variance increases because the attracting force from the stable fixed point becomes weaker to counter any perturbations as the system gets near to the bifurcation point. Similar indicator can also be observed in spatial dimension such as an increasing spatial correlation and spatial variance across connected parts(38).

3

Model

3.1 Introduction

Model is a representation of reality that we are trying to understand, but not reality itself. George Box famously said, "*All model are wrong but some are useful*". Although model is just representation, model is essential to the understanding of any phenomena. Lord Kelvin once said, "*I am never content until I have constructed a mechanical model of the subject I am studying. If I succeed in making one, I understand; otherwise I do not*". The correctness of the model depends on its basic assumptions. Good assumptions may not always be completely true but they must be realistic enough to capture the essential elements and are able to explain the phenomena. The variety of models ranges from detailed to versions that are simple. Detailed model is necessary to perform accurate prediction and it usually considers the contribution from all related components. Simple model is often used to explain the key phenomenon and it usually only considers selected important components that captures the most essential mechanism.

Mathematics is a very useful tool for the construction of a model. Based on several simple assumptions, many phenomena can be modeled by a few lines of equations. However, when the interaction is too complicated, the mathematics may not able to trace the complete details of the interaction. The rapid development of computer technologies has provided a useful tool to trace all the detailed processes in the interaction through numerical simulation. Basically, we ask the computer to mimic the interactions and generate the behavior within an artificial

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society(52, 92). Very often, we use computer simulation to study such society and validate the results of the simulation through analytical approximation.

3.2 Modeling Complex System

Nowadays, computer simulations have been used in both natural and social sciences(19, 70, 109). The application of simulation modeling ranges from physical system such as condensed matter to social problem such as economics. The widespread use of computer simulation in many fields is influenced by recent paradigm to promote interdisciplinary research. This paradigm is known as complexity theory, which is a collection of interrelated concepts developed from several disciplines(2, 3).

In the past centuries, people were interested in the problem of simplicity(195). Many problems were reduced into the most essential variables. The detailed interaction between components was often ignored. The problem of simplicity usually can be solved easily through top down approach by introducing a set of differential equations. This solution can be very useful in many physical system for instance when we want to analyze the movement of an object. However, most real world problems in biological and social system is not simple because they consist of many components with complex interactions in the form of organic wholes.

In recent decades, people changed their direction to investigate the problem of complexity(2, 195). A wide range of complex problems in biological and social system cannot be explained by reducing it into only several variables. Many phenomena in complex systems often grow from interaction of individual behavior. Therefore, bottom up approach is more useful to analyze complex system. By simulating the evolution and interaction for each component, we can investigate the emergence of phenomena in the population level.

Complex system has several unique properties that cannot be found in simple system. Complex system by nature is nonlinear. In linear system, the outcome is proportional to the cause. On the other hand, the outcome of nonlinear system is usually unpredictable. Another consequence of nonlinearity is the existence of

multiple attractors and path dependency such as hysteresis behavior(100). Complex system is also an emergent system, in which the macro phenomena emerges from local interactions and activities among its components(36). Therefore, the behavior of different levels can be very different from each other. Complex system also involves learning and self-organization(173, 188). It does not follow preordained pattern but learn and evolve independently based on current environment.

Coupled Socio-Ecological System consists of complex interacting components, therefore it should be modeled using complexity paradigm(11). SES consists of social and ecological system which inter-relate through feedback mechanism to regulate the flow of resources. It is complex and it involves many components with nonlinear interactions. It is also adaptive and evolves continuously to adjust towards the current condition. Ecological components consist of natural dynamics that regulate the flow of resources. Social component consists of many individuals that want to exploit or preserve the resource. Moreover, the social components will interact, learn, and even punish each other through social mechanism. The outcome and robustness of this system depends on the interaction between its components. SES theory usually integrates ideas from many concepts of resilience and sustainability(11, 60).

3.3 Nature of Model

Based on the complex nature of coupled socio-ecological system as discussed above, the model must have several building blocks:

Agent. Agent is the smallest independent unit element in the model. In SES system, the agents are individuals who utilize the resources. Agents base their action on certain utility. It can be for personal profit or even different consideration such as social norms or moral considerations(45).

Social Interaction. As social being, each agent will interact in certain manner. Realistically, the interaction between individuals is constrained by the internal societal network. Every agents are characterized by bounded rationality because they are not fully aware of the global situation and only aware of the nearest neighbor situation. Social interaction also includes any form of norm enforcement and social control(138).

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Learning and Adaptation. Every agent should be able to learn from other agent and update their strategy over time. They should also be able to adapt from current environment. All these features are important in temporal evolution of the system, whether an update in agent states or even environmental properties(10).

Co-evolution. The social and ecological components regularly interact as an integrated wholes. Both social and ecological systems are coupled through feedback mechanism which gives rise to their dynamical behavior where a change in one system affects the other system and vice versa(10, 87).

Emergent System. The overall behavior of the system is an aggregate from all complex interactions between all components. This process generates macroscopic behavior which cannot be envisaged based on the individual component. Therefore the behavior at the emergent level can be different from the behavior at the individual level(36).

3.4 Common Pool Resource Framework

Sustainable coupled socio-ecological system requires a balanced interaction between social and ecological system. For instance, imagine a society of farmers who rely on the common water resource to irrigate their crops. Since the harvest yields depend on adequate irrigation, each field would consume a certain amount of water. This process will deplete the water resources in the long run if left unchecked. Therefore every farmers should restrain their water usage in order to maintain the amount of resources. However, a farmer can choose to overuse the water resource for his own benefit. Hence, is it possible for a farmer to cooperate with the rest if he can just free ride and maximize his own harvest? This dilemma is often discussed in common pool resource problem.

The model on common pool resource is motivated by several empirical observations from Ostrom which described the sustainable resource management practices through social norms(138). The earliest framework on common pool problem is described in the paper by Dasgupta and Heal(44), and improved by Chichilnisky(32), in which they consider a diminishing return production function. They presented the emergence of suboptimal outcome caused by excessive

exploitation of the common resource. After that, Sethi and Somanathan proposed costly punishment as a social mechanism to promote the cooperation in resource sharing(169). Noailly synthesized these several frameworks to construct a bottom up evolutionary game theoretical model(130), which is later used by Tavoni as a basis for his social mechanism. Tavoni proposed the equity-driven ostracism as a social mechanism in the context of renewable resource management, which is known as the TSL (Tavoni-Schülter-Levin) model(182). Ostracism mechanism enables the avoidance of the situation of total defection (tragedies of the commons) through the existence of mixed equilibrium.

In this thesis, we investigate the common pool dilemma within the framework and assumptions used in previous models, especially those of the TSL model. The TSL model takes the form of nonlinear equations that couple the dynamics of social cooperation to the dynamics of a renewable common pool resource. The model shows that cooperative behavior can be promoted through social ostracism against defectors who overuse the common pool resources. However, the TSL model assumes that the population interact in a well-mixed manner, i.e. everyone interacts with everyone else. Therefore, we incorporate network structure to provide a more realistic representation of complex social interactions. In the context of such a social network, an individual is represented by a node with the edges correspond to social interactions. Different social structures thus correspond to different network degree and topology (34), and we expect different social interaction patterns to have dissimilar impact on the promotion of cooperative behavior.

3.5 Coupled Socio-Ecological System

Here, we review the common pool dilemma and ostracism mechanism described by the TSL model (182). In the TSL paradigm, there exists coupling between the ecological and social system. In this thesis, we have modified the model by incorporating local discrete updates so that the social and ecological variables are updated in every time step.

Time step here is not related to the real clock time but time step represents updating procedure in numerical simulation model. Time step is required because

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we use discrete modeling in which the state of the system changes at discrete interval. It means for each time increment in the simulation, all necessary variables are computed and the state of the system is updated. After we finish updating the state of the system, we will increase the time step by 1 and the whole updating process is repeated. The physical meaning of this timescale is that of a period between two consecutive harvests. For computational convenience, the simulation time step is fixed to 1 for both social and ecological dynamics, but the result is also robust under different timestep. In chapter 4 section 4.6.1 we shall discuss the possibility when the social and ecological dynamics evolve at different rate.

3.5.1 Ecological System

We define the ecological state to be the amount of natural resource (for instance, fresh water) available for human exploitation. The amount of available resource R is described by an explicit inflow and outflow mechanism against time t . Specifically, we assume a constant resource inflow which increases the available resource, while the outflow causes resource depletion through natural depreciation and human extraction. This leads to the following equation for resource dynamics:

$$\frac{\Delta R}{\Delta t} = c - d \left[\frac{R}{R_{max}} \right]^2 - qER. \quad (3.1)$$

The first term of the equation is the constant resource inflow c , the second term is the natural depreciation which is parametrized by the constant d , and the last term is the loss due to human extractive effort. Note that q is the technological parameter associated with the efficiency of resource utilization, while E represents the total extractive effort which is defined in Equation 3.2 below. In the absence of human extraction, the resource would equilibrate to its maximum capacity R_{max} for $c = d$. Since the time step is assumed to be $\Delta t = 1$ in this thesis, the equation of resource dynamics has been rewritten in the discrete form as follows:

$$R_{t+1} = R_t + c - d \left[\frac{R_t}{R_{max}} \right]^2 - qE_t R_t. \quad (3.2)$$

3.5 Coupled Socio-Ecological System

In the equation above we apply quadratic dependency in resource depreciation term. This quadratic dependency portrays the situation that the depreciation is very slow when the resource level is low, however the depreciation is very fast when the resource is beyond the storage capacity. The form of this ecological equation is adapted from TSL nonlinear model(182). Furthermore, we can also found this quadratic dependency in other CPR model(130). This natural depreciation is not only caused by evaporation but mainly from hydrological discharge. In hydrological model, this quadratic relation is known as hyperbolic recession curve. Based on the estimated flow from hydrological statistic, this quadratic relation is closely reproduced the behavior of the river(146).

3.5.2 Social System

The human population is assumed fixed which implies the absence of movement of people into and out of our society. The total population N is divided into two categories: the co-operator N_c , and the defector N_d . The co-operator would extract the available resource by putting in effort e_c , while the defector would extract more resource by putting in a larger effort e_d . Therefore, the total effort of our society is given by

$$E = N_c e_c + N_d e_d = N [f_c e_c + [1 - f_c] e_d] , \quad (3.3)$$

where f_c is the fraction of co-operators in the society.

The co-operators agree to abide by the norm and stick to the community efficient effort ($e_c = e_{eff}$). On the other hand, the defectors violate the rule by maximizing their individual payoff through putting in effort $e_d = e_{nash}$. Here, e_d is the Nash equilibrium of individual effort. The whole society and ecosystem would be better off if all the individuals cooperate. However, at the individual level, it is more rational to defect because of the enhanced individual incentive. Each individual would face this dilemma.

The payoff of each individual is given by the difference between income and cost. The income is derived from a fraction of the total production yield F , which is assumed to relate positively to the availability of natural resource and the total

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effort of the society. F takes the form of the Cobb-Douglas production function as follows:

$$F = \gamma E^\alpha R^\beta, \quad (3.4)$$

where γ is the production constant. While the parameters α and β represent the output elasticity of labor and resource respectively, they obey the relation $\alpha + \beta < 1$ to ensure that F has the characteristic of diminishing returns. On the other hand, we assume that the cost depend linearly on the individual effort, with the proportionality constant being w signifying the opportunity cost of labor. With this, the payoff for the co-operator is written as

$$\pi_c = \frac{e_c}{E} F(E, R) - w e_c, \quad (3.5)$$

while the payoff for the defector is

$$\pi_d = \frac{e_d}{E} F(E, R) - w e_d. \quad (3.6)$$

However, there is an additional social cost to be paid by norm violators. In this thesis we consider two types of social mechanism. In first mechanism, we follow the work of Tavoni, who proposed the concept of equity driven ostracism(182). This situation will be discussed in most of the chapters. In second mechanism, we follow the well known social mechanism in evolutionary game theory which is known as voluntary punishment/enforcement. This mechanism is also used by Noailly in his common pool framework(130). This situation will be addressed in comparison with ostracism mechanism in chapter 8. The second mechanism is used to help us understand the effect of different social mechanism on coupled socio-ecological system.

3.5.2.1 Equity Driven Ostracism Mechanism

Under ostracism mechanism, each defector would be socially ostracized from the community of neighboring co-operators (n_c) via a denial of service or in the form of social disapproval. Since the interaction between individuals is constraint by the underlying network structure, the neighbor of a particular agent is defined as

3.5 Coupled Socio-Ecological System

its nearest neighbor in the network. In other words, ostracism towards a particular defector is mediated through these connected neighbors. Quantitatively, we model social ostracism through the Gompertz function:

$$O(n_c) = he^{\tau e^{gn_c}}. \quad (3.7)$$

where the parameter h represents the amplitude of ostracism. The parameters τ and g govern the shape and effective threshold of the ostracism function. Ostracism mechanism is often used as social sanction in many societies. Study has also found several empirical evidences of ostracism mechanism in CPR system(102).

In the case of equity driven ostracism, the strength of the social sanction not only depends on the number of co-operator but also on the payoff difference between defector and co-operator. The consequence of equity driven ostracism leads to the following utility for defectors:

$$u_d(n_c) = \pi_d - O(n_c) \frac{\pi_d - \pi_c}{\pi_d}, \quad (3.8)$$

whereas the utility of the co-operator remains the same without any payoff reduction from the social pressure:

$$u_c = \pi_c. \quad (3.9)$$

3.5.2.2 Voluntary Enforcement Mechanism

In the case where cooperation is established through the mechanism of voluntary enforcement there are three types of strategy: cooperator, defector, and enforcer. Analogous to the previous section, the cooperator and enforcer extract the natural resource according to the agreement to avoid overexploitation, while defector puts in more effort to maximize their profit. The moderation on extraction is enforced through local monitoring. The defectors are punished by their neighboring enforcers, while cooperators ignore the exploitative behavior of their neighboring defectors.

This mechanism can be found in many real systems. For instance, In the biological system: a honey bee worker can sting (punish) larger insect by sacrificing its life(119). In the social system, we also found the local monitoring and

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enforcing of free-riders in communal forests in Honduras(185). A person can voluntarily put effort to bring the violators to the local authorities who impose fines, compulsory labor, and even jail. There are other communities which also have similar mechanism(138). Moreover, many researches already used punishment and enforcement as common mechanisms to promote cooperation(73, 85, 169).

To punish the defectors, each enforcer has to bear a cost of h_c for each neighboring defectors while incurring a sanction damage h towards every neighboring defectors. Thus, the punishment is performed individually rather than collectively, and the enforcers need to sacrifice themselves voluntary in order to punish the defectors. This social mechanism leaves no effect on the cooperators:

$$u_c = \pi_c. \quad (3.10)$$

However it will reduce defector utility in proportion to the number of neighboring enforcers:

$$u_d = \pi_d - h_s k_e. \quad (3.11)$$

Moreover, it also reduces the enforcer utility in proportion to the number of neighboring defectors:

$$u_e = \pi_c - h_c k_d. \quad (3.12)$$

These expressions then allow us to calculate the utility of every agent and to determine whether they choose the cooperative, enforcing, or defective strategy. The average utility of the agent would then serve as a measure of success. The evolution of the agent's strategy is based on a comparison of its utility by means of an updating mechanism discussed in the next subsection.

3.6 Updating Mechanism

There are numerous updating rules in evolutionary games (170). The basic mechanism in these games is the adoption by an agent on the more profitable strategy after making a comparison with their neighbor's utility through an iterative process. Examples of evolutionary games are the birth-death, death-birth (201), imitation (184), link dynamics (179), pairwise comparison (154), and global updating (192). The birth-death and death-birth updating is more suitable in the

biological context while the imitation, link dynamics, and pairwise comparison is usually employed in the social context.

In our case, we employ asynchronous pairwise comparison such that at each time step a random individual is selected to update his strategy by comparing against a randomly matched neighbor. This mechanism is shown in Fig. 3.1a. If the utility of his matched neighbor is higher than his utility, he will adopt his neighbor's strategy with a certain probability. This probability is proportional to the utility difference between him and his matched neighbor. This local interaction adds additional complexity to the socio-ecological system.

There are several reasons why asynchronous updating is used rather than synchronous updating. First, asynchronous updating is more appropriate in simulating real process in most system(81, 166). In biological system, sometimes there is biological clocks that create seasonal pattern or synchronization effect in several biological processes. However, in most social interactions, the agents decide, interact, and act at different time. Since the purpose of our simulation is imitating real world process, asynchronous updating will be the best choice. Second, asynchronous updating can avoid the generation of pattern artifact(51). This effect is famously described when Huberman(81) points out that cooperation pattern on Nowak's spatial game(132) is just an artifact. Third, since asynchronous mechanism update one agent at the time, the dynamics is gradual and continuous. Therefore, the asynchronous dynamics can be expressed and approximated by differential equation(81).

The updating mechanism described above involves selection without mutation. In this case, no change in strategy is to be expected when all the agents have the same strategy. In particular, the system will not be able to evolve to another stable region if it is in either the all co-operator or all defector state. To study socio-ecological regime shift, we consider periodic mutation in our simulation. This flips the strategy of a randomly chosen player after a certain period. Specifically, we have $m_p = N^2$. In other words, a player will be chosen randomly to switch its strategy into the alternate strategy after every N^2 selection processes. This mechanism is illustrated in Fig. 3.1b.

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Figure 3.1: (Color online). Updating mechanism. Here, cooperators are represented by circles marked with C while defectors are represented by circles marked with D . The updating mechanism in (a) represents the process of selection. At each time step, a random individual compares his utility with that of a random neighbor. The probability of an individual changing his strategy to the opposite strategy is proportional to the utility difference. The updating mechanism in (b) represents the process of random mutation. At a certain mutation period, a random individual is selected to flip its strategy (for example, from cooperative strategy to defective strategy, and vice versa).

3.7 Model Parameter

This model includes many variables to mimic the social interaction as realistic as possible. Although the model seems complicated, the basic mechanism is simple and the parameter values in this model are relative to this mechanism. Basically parameters: c, d, R_{max} are related to the input rate, output rate, and the storage capacity of the resource. Moreover, parameters: $\alpha, \beta, \gamma, e_c, e_d, w$ are responsible for the adjustment of the payoff difference between co-operators and defectors. Parameters: h, t, g are for the adjustment of the shape of ostracism function. A combination between payoff difference and ostracism function can affect the strategy imitation. Finally, the network parameters define neighbor distribution which affects the imitation dynamics and ostracism mechanism. In addition, the parameters are chosen such that $w \sim F/E$, $h \sim \pi_d$ and $O(1) > \pi_d$. Here, $A \sim B$ means the variable A is of the same order of size as the parameter B . For example, if the parameter γ is to be multiplied by a factor of κ to give realistic amount of production, both w and h will need to be multiplied by the same factor. Variables π_c and π_d depend on parameters e_c and e_d which are related to system Nash equilibrium(44, 182). Therefore when the system size is changed, the parameter e_c and e_d are scaled and parameter h should be scaled accordingly. This is to guarantee the existence of multi-stable states, including the defector, cooperator and mixed equilibrium. With these, similar results on hysteresis will be obtained (all these phenomena will be discussed in chapter 4).

3.8 Comparison with other Socio-Ecological Model

Literature on human-nature interaction provides various comparison and direction on modeling such system. Generally we can find many independent models on both social system and ecological system. Many of these use simple equation to describe social or ecological phenomena.

Recent advances on network science have accelerated the study of human dynamics. Many social network models have been developed to accommodate this purpose. We can find study of social phase transition(192), cultural history(163), cooperation(135, 179), and many more by using social network approach. In this

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sense, those models have similar structural interaction features as our model. However they did not consider any ecological factor in their analysis.

At the same time, the development of network science also accelerates the study of ecological network. In the past, people study ecological components in separation without any connection. Many ecological features are reduced into functional response of dynamical equations(77, 78, 107). Nowadays we can find ecological analysis by using network approaches(8, 164). Similarly, these models did not consider any social factor in their analysis.

Constructing coupled model can be very challenging because both social and ecological system have their own academic traditions with different approaches. Many models focus on one particular system by sacrificing the substantial aspects of the other system. For instance there are ecological models that treat the social system as mere external factors(158). On the other hand, there are also social models that treat ecological system as mere external factors(108).

Cultural evolution has tried to couple the interaction between human and nature from Darwinian paradigm. One of the earliest attempt is by modeling gene-culture co-evolution. The models from this framework have tried to explain many cultural phenomena(20). This approach is too simplistic because it reduces the cultural aspects into genetic variation. The extension of this cultural evolution paradigm is niche construction. This paradigm focuses on the study of how humans alter their environmental condition. It is better than previous approach because it considers environmental dynamics(134).

Generalized ecological theory is another perspective in coupled socio-ecological model. It incorporates human population into ecological dynamics which reduces human interaction into a part of ecology. Usually it uses standard logistic and Lotka-Volterra equation to describe the social system so that it can be directly coupled with ecological equation. This approach has been used to describe the behavior of geopolitical organization(186, 187).

Some models were developed to tackle very specific socio-ecological problems. For instance the analysis of farmland abandonment(57), collapse of ancient civilization(21), inequality and resources(118), tourist and biodiversity conservation (97). Some models employed similar approaches with our model by analyzing general common pool resource problem. For instance the analysis on survival of

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norm abiding individuals in CPR society(182), human adaptive behavior in CPR system(23), CPR with life cycle(25), resource collapse(22), and many more. All these models are considered as top down modeling (in the form of dynamical equations). This modeling approach is very limited and it reduces many crucial interactions into simplistic formula.

In recent years, we can find more realistic model through the incorporation of stochastic elements(26) or even using system dynamics approach(30). Moreover, recent trends in interdisciplinary research have encouraged more realistic modeling through bottom up simulation. In the literature, we can find several bottom up simulations such as agent based models to simulate specific coupled socio-ecological system(56, 58, 59, 110, 165, 167). However in contrast with top down models, bottom up models are very difficult to be investigated analytically because of their complexity. In this context, our model is useful because although it is a bottom up model, the phenomena in this model can be explained theoretically through analytical approximation.

Furthermore, our model is also incorporated with complex network structure. This feature make this model more realistic in comparison to other nonlinear(57, 182) or agent based(165) coupled socio-ecological models. In contrast with many simplistic socio-ecological interaction, this model can simulate different type of social network and its effect on dynamical stability of the coupled socio-ecological system. Therefore, the analysis and insight from this model can be applied to general coupled socio-ecological system with any specific social interaction and common resource sharing.

3.9 Application and Limitation of the Model to Socio-Ecological System

Before we discuss about the simulation results and insights from this model, let us discuss about possible application and limitation of this model. In general, it can be applied to investigate any common pool society. For instance, it can be used to analyze the irrigation system, fisheries, forestry, pollution, or even climate change. Furthermore, in chapter 7 we compare the results from this model with

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coupled socio-ecological irrigation data. The detail of resource dynamics can be different for other ecological system, however the overall interactions should be similar. For instance, the resource dynamics in this model is more appropriate for irrigation system (linear renewable resource) rather than fisheries or forestry (logistic resource). Similarly, other socio-ecological system requires different adjustment on the model, either on the ecological aspects or the social aspects.

One main concern in any socio-ecological system is the topic of sustainability and resilience. Ecological preservation will be impossible in the midst of self-centered individuals. Therefore specific social mechanism is necessary to create sustainable society. Good SES model should be able to show the impact of specific social norm and social structure towards its resilience. One strength of this model is it can be applied to any social structure. In later chapter we will study the effect of different interaction network on the stability of SES system. This model can also be used to study different social mechanisms. Therefore, it helps us to predict the dynamical behavior of such system under different structure and mechanism. This model also helps us to understand the cause of systemic failure so that we may anticipate disastrous event.

The complexity of this model can be both a strength and a weakness. This model involves many components and parameters that are not easily analyzed. However by involving all essential components in coupled socio-ecological system, we can understand it holistically. With this current form of linear renewable resource, this model is most suitable with irrigation system and other similar system. However, the detail of the model can also be modified to suit other real world systems. By modifying some parts of the model we can also apply it to other system such as fisheries system in which the network of fisherman should self-organize to preserve the number of fishes. We can also use it to study the climate change in which every country should self-organize through specific social mechanism to prevent global warming.

Analytical tractability is also another strength of this model. Although it is very complicated, we can nonetheless provide mathematical description for any interesting phenomena by using specific approximation. This aspect is very important especially in managing complex system. In highly nonlinear system,

3.10 Application of the Model to Balinese Subak

altering one component may create unpredictable ripple effect to the entire system. Therefore theoretical description is essential for understanding specific phenomena accurately so that we can maintain or reverse certain condition without creating a side effect that damage the whole system.

However, there are several limitations of this model. First, we should realize this model is not developed for exhaustive predictive purpose. In order to do real time prediction, we must adjust its parameters value with specific real world parameters. Most of the time we also need to incorporate additional components that exist in particular system.

This model also assumes homogeneous agent with similar responsibility. In real social system, we may find social interaction with level of hierarchy with different roles. For instance, local government, manager, village leader, ordinary farmer, etc. This model also assumes static networked interaction. In the long term, individual may change the interaction or even the arrival of new member and the departure of old member.

This model also assumes simple ecological model. In reality, we may have multiple interconnected resources that allow the resource flowing from certain spatial location to the other. This model also did not consider spatial movement from one location to another. For instance, both fisherman and fish can move from one place to another place. Development and investigation of such models can be an interesting direction for future research on coupled SES.

3.10 Application of the Model to Balinese Subak

In previous section we have discussed about several possible applications of this model to coupled socio-ecological system. In this section, we discuss about the direct application of this model to specific real world coupled socio-ecological system in Bali Indonesia. The comparison between the model and the SES data is analyzed in chapter 7, in which we examine the results from this model with the empirical data from a traditional water management system which is known as Balinese subak. Subak represents a coupled socio-ecological unit in which human institution interacts with the natural environment such as waterfall and rivers to manage the use of water resources to increase the yield of rice.

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This model is compatible with subak and other irrigation system for several reasons. First, there is strong coupling between human institution and natural resources in subak society. Therefore subak can be studied through coupled SES model. Next, the water management in irrigation system can be described by common pool resource which is compatible with the framework of this model. Moreover, the linear renewable resource is also suitable to water dynamics. The use of Cobb-Douglas production function in the model has implied that ecological resource can be viewed as production input (such as water in agriculture and industry) instead of the value of the resource itself (such as fish in fisheries). Furthermore, the individual behavior conformation through social pressure and norm such as ostracism mechanism is more effective in traditional society rather than monetary punishment such as fine. The social interaction is usually not uniform because every society has their own social network structure. Subak culture is segregated by caste and class, therefore the society forms community structure which can be studied through our model. Finally, subak has evolved for a very long period of time such that each subak resides in different stable regime with different socio-ecological characteristics. The model can help us to identify the behavior of different socio-ecological regimes.

4

Social Interaction, Regime Shift, and Hysteresis

4.1 Introduction

As briefly mention in the chapter 1, the phenomenon of regime shift is observed in many systems. Ecological regime shifts such as species extinction and massive biodiversity reduction, or the population explosion of certain species, for instance eutrophication, have been studied extensively (122, 158). Dramatic ecosystem regime shifts where lakes turn from clear to turbid (158) and Sahara regions collapsed suddenly into deserts (46) have been widely documented. However, the phenomena of social-ecological regime shift, where a breakdown of social cooperation coupled with a sudden drop in ecological resources, have yet to be thoroughly investigated. The focus in this chapter is to investigate the occurrence of social-ecological regime shift triggered by various social and ecological drivers.

In general, regime shift happens suddenly by forcing the system into a new stability region (157). Specifically, the system undergoes a transition from one stable regime to another stable regime. Due to the presence of feedback mechanism, the system may resist the transition between stable states unless the perturbation is large enough. The state of the system depends not merely on the variables and parameters but also on the history of the system. This path dependency gives rise to the effect of hysteresis. Hysteresis usually emerges via a change on certain driving parameter. When the state of the system is near the boundary of the

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stable and unstable fixed points, any tiny variation would push the system from one basin of attraction to another. The impact or regime shift can be disastrous because reversing the parameter may not bring the system back to its previous state (159). Note that the effect of hysteresis has also been studied in other ecological models (37, 123) and game theoretical models (192, 197, 200). In this chapter, we investigate how social network structure influences the phenomenon of hysteresis. While recent research has suggested that connectivity of a network structure is usually associated with a resistance to change (161), the study of how complex social-ecological interactions can alter bifurcation characteristics is still unexplored, and thus is the subject of investigation in this chapter.

4.2 Regime Shift and Hysteresis

The SES under study can adopt either a cooperative or a defective regime. For our studies, initial conditions are chosen such that the system resides in a cooperative regime at the beginning. To ensure that the system is in the steady state, we evolve the system for a sufficiently long time before altering the control parameter. For the first half of a hysteresis cycle, we increase the control parameter continuously and quasi-statically, driving the system gradually along the steady state values within a particular regime. The state of the system is then recorded when a new equilibrium is reached after each alteration. This process is repeated until a critical parameter is exceeded and the system undergoes a regime shift. We then reverse the process to evolve the system towards its initial state. Note that each complete cycle of parameter alteration gives a hysteresis curve.

Figure 4.1 shows the cooperation hysteresis curve in a SES with well-mixed population which is driven by variation in the amount of resource inflow. Results are averaged over 100 simulations. In this simulation, social interaction among the population is represented by a complete graph (where all individuals are connected to all individuals). The cooperativeness is measured by the fraction of cooperators in the population, P_c . Here, increments of resource inflow have gradually led the society to behave less and less cooperative until a critical point is reached. When the system is near the first transition point c_1 , a further increment in the amount of resource triggers a critical transition towards the defector

equilibrium. Instead of a gradual change, the result is a drastic change (a breakdown in cooperation and a collapse of available resource level) in the state of the system. Once the transition takes place, previous states of the system cannot be restored through reversing the same path. During the second half of the hysteresis cycle, cooperativeness of the population does not increase sharply back to its previous values at c_1 as we decrease the amount of resource inflow. Instead, it increases gradually by following a different path before a second transition point c_2 is reached. Again, a further decrease in the amount of resource inflow triggers another transition, this time from the defective regime to the cooperative regime.

4.3 The Effect of Network Degree

Next, we simulate hysteresis cycles for populations that interact in a complex way. Here, the Erdos-Renyi graph with a size of $N = 50$ is used as the social network and we vary the average degree $\langle k \rangle$ of the network to model a society with different average number of social connection. The set of hysteresis curves obtained are shown in Fig. 4.2. We observe that as the average degree $\langle k \rangle$ decreases, the width ($\Delta c = |c_1 - c_2|$) of the hysteresis curve reduces. As shown in Fig. 4.2, critical transitions happen around $c_1 = 50$ and $c_2 = 22$ for population with $\langle k \rangle = 45$. For social network with a lower degree (for example $\langle k \rangle = 25$), the regime shift towards defective regime happens earlier (at $c_1 = 40$) while the regime shift towards cooperative regime occurs at slightly larger values of resource inflow ($c_2 = 25$). Interestingly, hysteresis curve is no longer observed for a population with very low number of social connection (i.e. $\langle k \rangle = 5$). In this society, the fraction of co-operators decreases faster as the control parameter increases. During the second half of the cycle, the system regains its original state following the same path as we reverse the process. We have also compared our numerical results with analytical approximation where we have found good agreement (see Fig. 4.11). The details of our analytical assumptions and approximation can be found in section 4.6

A critical transition affects not only social variables but also ecological variables of a coupled SES. The counterpart to the aforementioned social regime shift is the ecological regime shift. While the resource inflow basically increases the

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Figure 4.1: (Color online). Simulation results for hysteresis driven by variation of resource inflow ($0 \leq c \leq 60$). Here, state variable of the hysteresis is represented by fraction of co-operators. Social interaction among the population of size $N = 50$ is represented by a complete graph. Parameters used are: $d = 50$, $R_{max} = 200$, $q = 1$, $\alpha = 0.6$, $\beta = 0.2$, $\gamma = 10$, $w = 15$, $h = 0.34$, $g = -10$, $\tau = -150$. Each point is run for a period of 10^4 time steps for the system to reach equilibrium. After the system reaches equilibrium, the parameter c is increased (or decreased) by an amount of 0.1 and the process is repeated. Results are averaged over 100 ensembles.

Figure 4.2: (Color online). Simulation results for hysteresis in social cooperation driven by variation of resource inflow ($0 \leq c \leq 60$) for populations connected through Erdős-Rényi random graphs with $N = 50$ and $\langle k \rangle = 45$ (squares), 25 (triangles), 15 (circles), or 5 (stars). Note that the value of the other parameters employed are the same as those used in Fig. 4.1. Results are averaged over 100 ensembles.

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amount of resource level, it also induces defective behaviors among the population. During a critical transition, breakdown of cooperation leads to a collapse of the resource level. The collapse of resource level triggered by a minute increase in resource inflow is prevented by hysteresis to be restored back to its original level through reducing the resource inflow by the same amount. Figure 4.3 shows a set of hysteresis curves in terms of the amount of available resource. Similar dependence on the number of social ties is observed. Analogous to the social regime shift, as the average social connection decreases, the width of the hysteresis curve decreases. The occurrence of transitions to the defective regime is observed to move to the left as the network degree decreases. On the other hand, the occurrence of transitions to the cooperative regime during the second half of the cycle is observed to move slightly to the right as the network degree decreases. In addition, the result shows that ecological regime shift occurs at the same critical location as the social regime shift. Again, in the case of a society with sparse connection, hysteresis is no longer observed.

4.4 Regime Shift under Social Control Parameter

Next, let us study regime shift driven by social factors. Here, we present two different social controlling parameters: ostracism strength and opportunity cost. In the case of ostracism strength, the result does not exhibit any hysteresis as shown in Fig. 4.4. In fact, when the regime shift occurs from the co-operative to the defective state, it is impossible to re-establish the cooperative state. This is exhibited by the curves continuing to maintain in the defective state beyond the critical transition points without flipping as shown in Fig. 4.4. It results from the fact that there are too few cooperators since it is in the defective state. This implies that the social cost of defection lies at the low end of the ostracism function, which is nearly 0. Hence, no matter how large the ostracism strength h is, the social cost is still nearly 0. Furthermore, because π_d is always greater than π_c , the utility of the co-operator can never exceed that of the defector. There is thus no way for the defector to switch to the co-operative state. Finally, we again

Figure 4.3: (Color online). Simulation results for hysteresis driven by variation of resource inflow ($0 \leq c \leq 60$) for populations connected through Erdős-Rényi random graphs with $N = 50$ and $\langle k \rangle = 45$ (squares), 25 (triangles), 15 (circles), or 5 (stars). Here, state variable of the hysteresis is represented by the amount of available resource R^* . Again, the value of the parameters used are the same as those of Fig. 4.1. Results are averaged over 100 ensembles.

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notice a delay in the transition from the cooperative to the defective state as the network degree increases, which is consistent with our prior results.

Figure 4.5 shows hysteresis curve with the opportunity cost as controlling parameter. The opportunity cost depends on the economics and the market situation, such as the existence of alternative jobs or opportunities. Here, the resource inflow is fixed at $c = 50$. As in the ecological-driven hysteresis, we observe a similar set of hysteresis curves driven by socio-economical factor. The transition from cooperative regime to defective regime is achieved by a decrease in the opportunity cost. Reversing the process causes a second transition from defective regime to cooperative regime. Again, our result shows that the bifurcation characteristic of a SES is fragile against the decrement in the number of social ties. For populations with sparse social connections, we observe an absence of the irreversible hysteresis behavior.

4.5 The Effect of Topology and Network Size

Before we discuss about more complex topology, we will discuss about the hysteresis results on a degree-regular random graph. Here we use pairing model to construct artificial random regular graph(198). This algorithm ensures that each node has the same number of connection. However, this algorithm is only limited for constructing low degree regular random graph. For high degree network, we will use different approach to produce such network by reversing the algorithm (we start with complete network and remove the link via the pairing algorithm).

From the numerical simulation we observe similar trend: the hysteresis reduces as we lower the network degree. This result is similar to the hysteresis behavior in Erdős-Rényi graph. The main difference between hysteresis phenomenon between regular random graph and Erdős-Rényi random graph is mainly on its critical points. For the case of regular random graph, the backward critical points c_2 are very close to each other regardless of its average degree. Moreover, the forward critical point c_1 in low degree-regular network occurs earlier in comparison to the same degree Erdős-Rényi graph. On the other hand, the forward critical point c_1 in high degree-regular network occurs later in comparison to the same degree Erdős-Rényi graph.

Figure 4.4: (Color online). Simulation results for hysteresis driven by variation of ostracism strength ($0 \leq h \leq 1$). Here, state variable of the hysteresis is represented by fraction of co-operators. Social interactions among the population are represented by Erdős-Rényi random graphs with $N = 50$ and $\langle k \rangle = 45$ (squares), 25 (triangles), 15 (circles), or 5 (stars). Parameters used are: $c = d = 50$, $R_{max} = 200$, $q = 1$, $\alpha = 0.6$, $\beta = 0.2$, $\gamma = 10$, $w = 15$, $g = -10$ and $\tau = -150$. Each point is run for a period of 10^4 time steps for the system to reach equilibrium. After the system reaches equilibrium, the parameter c is increased (or decreased) by an amount of 0.1 and the process is repeated. Results are averaged over 100 ensembles.

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Figure 4.5: (Color online). Simulation results for hysteresis driven by variation of opportunity cost ($14 \leq w \leq 20$). Here, state variable of the hysteresis is represented by fraction of co-operators. Social interactions among the population are represented by Erdős-Rényi random graphs with $N = 50$ and $\langle k \rangle = 45$ (squares), 25 (triangles), 15 (circles), or 5 (stars). Parameters used are: $c = d = 50$, $R_{max} = 200$, $q = 1$, $\alpha = 0.6$, $\beta = 0.2$, $\gamma = 10$, $h = 0.34$, $g = -10$ and $\tau = -150$. Each point is run for a period of 10^4 time steps for the system to reach equilibrium. After the system reaches equilibrium, the parameter c is increased (or decreased) by an amount of 0.1 and the process is repeated. Results are averaged over 100 ensembles.

Figure 4.6: (Color online). Simulation results for hysteresis in social cooperation driven by variation of resource inflow ($0 \leq c \leq 60$) for populations connected through pairing random regular graphs with $N = 50$ and $\langle k \rangle = 45$ (squares), 25 (triangles), 15 (circles), or 5 (stars). Note that the value of the other parameters employed are the same as those used in Fig. 4.1. Results are averaged over 100 ensembles.

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Most real world social networks are not random graphs. Here, we study the effect of different topology on regime shift in SES. Specifically, we compare the results obtained for two different network topologies: Erdős-Rényi network and the scale-free network generated by using the Chung-Lu algorithm (33). In addition, we consider a larger network size of $N = 200$. Simulation results are shown in Fig. 4.7. There is a slight difference between the two hysteresis curves shown in Fig. 4.7. Reduction in the hysteresis width is observed to be smaller for populations with the scale-free interaction networks.

4.6 Analytical Approximation

In this section, we perform analytical estimation on the hysteresis curves that were obtained by numerical simulations. From the updating mechanism in previous chapter (section 3.6), we can construct a master equation for the number of co-operator. Let $P_c(P_d)$ be the probability that a co-operator (a defector) is being selected to update its strategy respectively. $q_{d|c}$ is the probability that a co-operator compares his payoff with a neighboring defector, while $q_{c|d}$ is the probability that a defector compares his payoff with a neighboring co-operator. Then, the probability that the total number of co-operator would increase by one is given by:

$$T^+(P_c) = P_d q_{c|d} (u_c - u_d) + P_d \frac{1}{m_p}. \quad (4.1)$$

Note that the first term on the right relates to the replicator dynamics, where $P_d q_{c|d}$ corresponds to the probability that a defector would compare against a co-operator in his neighborhood as a result of random selection and random matching as discussed above. The comparison to be made is on their utility. In the algorithm, we implement the rule that the chosen defector will switch and become a co-operator if a number computed from a random number generator (with a uniform distribution) is less than $[u_c - u_d]/u_{max}$. Otherwise, the defector will not change. Thus, this gives rise to the factor $[u_c - u_d]/u_{max}$ within the first term on the right of the equation. Moreover, the second term on the right of Eq. 4.1 gives the probability that a defector will flip to a co-operator due to the mutation mechanism, since if a defector is being chosen, its chance of being flip is

Figure 4.7: (Color online). Simulation results for hysteresis driven by variation of resource inflow ($0 \leq c \leq 40$). Here, state variable of the hysteresis is represented by fraction of co-operators. Social interactions among the population are represented by either Erdős-Rényi random graphs (squares) or scale-free networks (circles) with size $N = 200$ and average degree $\langle k \rangle = 10$. Note that the scale-free topology is generated by the Chung-Lu algorithm. The value of the parameters employed are the same as those of Fig. 4.1. Results are averaged over 100 ensembles.

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$1/m_p$. Similarly, the probability that the number of co-operator would decrease by one is given as follows:

$$T^-(P_c) = P_c q_{d|c} (u_d - u_c) / u_{max} + P_c \frac{1}{m_p}. \quad (4.2)$$

By using the stochastic process in chapter 2 (section 2.3.2), we can form and expand master equation by using the transition probability. We then obtain the following approximate differential equation of the strategy dynamics:

$$\frac{d}{dt} P(P_c, t) = - \left(\frac{d}{dP_c} [P(P_c, t) [T^+(P_c) - T^-(P_c)]] \right). \quad (4.3)$$

The condition of equilibrium is satisfied when $T^+(P_c) - T^-(P_c) = 0$, i.e.,

$$\begin{aligned} 0 &= \left(P_d q_{c|d} (u_c - u_d) / u_{max} + P_d \frac{1}{m_p} \right) \\ &\quad - \left(P_c q_{d|c} (u_d - u_c) / u_{max} + P_c \frac{1}{m_p} \right). \end{aligned} \quad (4.4)$$

By using the probability identity: $P_c q_{d|c} = P_d q_{c|d} = P_{cd}$, we can rewrite the previous equation in the following form:

$$0 = (P_{cd} (u_c - u_d) / u_{max}) + \frac{1}{m_p} (P_d - P_c). \quad (4.5)$$

Here, P_{cd} denotes the probability for a randomly selected pair to be a cooperator-defector pair. In the case when mutation is rare, we can approximate $\frac{1}{m_p} \cong 0$ and drop the mutation term. Moreover, P_{cd} is strictly larger than zero since the mutation mechanism ensures the existence of at least one co-operator and defector pair. The above equation reduces to $u_c = u_d$, which is then further simplified to the following form:

$$\sum_i \rho_i^* O(n_{c_i}) = \pi_d(e_d, R^*), \quad (4.6)$$

where ρ_i gives the occurrence probability of the fraction of n_{c_i} co-operators in the defector neighborhood. Moreover, R^* gives the amount of ecological resource at equilibrium, $dR/dt = 0$ (or set $R_t = R_{(t+1)}$ in Eq. 3.2) which respectively leads to

$$R^* = -E + \left(\sqrt{E^2 + 4c \frac{d}{R_{max}}} \right) \frac{R_{max}^2}{2d}. \quad (4.7)$$

In fact, Eqs. 4.6 and 4.7 give the stable and unstable fixed points of our coupled SES, in addition to those given by $P_c = 0$ and $P_c = 1$.

From another perspective, Eqs. 4.7 and 4.6 are the consequence of Eq. 3.1 and the following differential equation (Langevin version of master equation):

$$\frac{dP_c}{dt} = P_c [1 - P_c] \frac{\pi_d - \pi_c}{\pi_d} \left[\sum_i \rho_i O(n_{c_i}) - \pi_d \right] \quad (4.8)$$

respectively.

Equation (4.8) gives the mixed equilibrium when cooperators and defectors coexist. The phase diagram of this situation will be discussed in chapter 8 (refer to figure 8.1 for more discussion on phase diagram). For the existence of a mixed equilibrium, the absolute magnitude of the production yield per effort (F/E) have to be of the same order as the parameter for the opportunity cost w , while the maximum ostracism h have to be of the same order as the defector's payoff π_d . If the ostracism $O(1) > \pi_d$, there exists a co-operator equilibrium which is stable. On the other hand, a stable mixed equilibrium exists when $O(1) < \pi_d$. Similarly, a stable defector equilibrium exist if $O(0) < \pi_d$, while a stable mixed equilibrium exist when $O(0) > \pi_d$. The parameters used in our study are chosen such that bistable state exists for $0 < c < 60$. The values of the parameters can however be multiplied by any scaling factor to model realistic empirical production of a particular system of common pool resource. For example, if the parameter γ is to be multiplied by a factor κ to give a realistic amount of production, both w and h have to be multiplied by the same factor in order to ensure the existence of the hysteresis phenomenon.

4.6.1 Time Scale Difference

In our model, availability of the resource is updated after every update of the agent's strategy. One can include more number of strategy updates before updating the amount of available resource to create time scale difference between the social and ecological variables. Alternatively, one can also scale the update of the resource by a small number after every update of the agent's strategy to observe the effect of time scale difference. We had investigated these two situations

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and do not find any difference in the results. Essentially, this is a consequence of the following results from Ref (149). In Ref. (149), the authors explain that slow-fast systems can be presented in a form where time is scaled by a small positive constant $\varepsilon \ll 1$ such that the dynamical equation can be re-expressed in the following form:

$$\frac{dP_c}{dt} = F(P_c(t), R(t)), \quad (4.9)$$

$$\frac{dR}{dt} = \varepsilon G(P_c(t), R(t)). \quad (4.10)$$

Our model can be placed precisely in this form and hence the argument in Ref. (149) applies to our case. Specifically, if we were to change our system to one with a faster time scale for social variable than ecological variable, we would expect the social system to reach equilibrium eventually while the amount of available resource remains constant. This would give a value of P_c at a quasi-steady state which would then influence the slow dynamics of the ecosystem. Hence, the dynamics of the coupled system will be different if different time scales are considered. However, our interest in this chapter is in the equilibrium states of the coupled system, i.e. when $\dot{P}_c = 0$ and $\dot{R} = 0$. From the equations above, we can easily observe that time scale difference does not affect the solution of the equilibrium state (i.e. when $\dot{P}_c = 0$ and $\dot{R} = 0$, the solutions are the same independent of the value of ε). Thus, the results on hysteresis characteristics are robust against time scale difference.

4.6.2 Mean Field Approximation

4.6.2.1 Random Assumption

To solve equation 4.6, we first assume that connections among co-operators and defectors are random. Mean field assumption indicates the probability of a defector connecting to a co-operator in the population is the same with the probability of average cooperator, $q_{c|d} = P_c$. The probability of connecting to another defector in the population is then equal to $q_{d|d} = 1 - P_c$. From this assumption, we can write the form of the cooperative probability distribution with respect to the

direct neighbor of a defector as

$$\rho_i = \binom{\langle k \rangle}{i} P_c^i (1 - P_c)^{\langle k \rangle - i}. \quad (4.11)$$

Here, ρ_i denotes the probability that i neighbours of a randomly selected defector with degree $\langle k \rangle$ are co-operators.

By using this assumption, we plot in Fig. 4.8 the set of stable and unstable fixed points as a function of the control parameter c for various values of $\langle k \rangle$. The solid line represents the set of stable fixed points and the dashed line represents the set of unstable fixed points. Regime shift occurs at the point where the stable and unstable fixed points intersect. This point is associated with the fold bifurcation which is the source of the phenomenon of hysteresis. Our analytical approximation affirms the existence of alternate stable states. In addition, as the network degree decreases, the width of the hysteresis curve decreases. As expected, the assumption of random connection underestimates the reduction in hysteresis width especially when the degree is small.

Mean field approximation reduces the neighboring strategy into the average system strategy. Therefore this solution is not only more appropriate to a complete network but also to an infinite population. Consequently, we will expect the mean field approximation to be more accurate as the population size increases. The hysteresis phenomenon itself is guaranteed for any system size. However, proper scaling is needed to obtain the exact same critical point (refer to section 3.7). Note that the social dynamical equations (eq 4.8) is a function of population size N because both the cooperator and defector payoffs are functions of population size. As a result, all the analytical predictions in this thesis work for higher number of population. For instance, in chapter 6 we can predict the transition for $N = 100$ analytically.

In figure 4.9 a & b below, we have shown that ordinary mean field approximation for complete network is indeed more accurate for a larger population size. Moreover, we can approximate the infinite population situation analytically. Basically, we assume binomial distribution become more similar with delta function in the infinite population limit $\left(\binom{\langle k \rangle}{i} P_c^i (1 - P_c)^{\langle k \rangle - i} \sim \delta(P_c - i/k) \right)$. Therefore we can approximate the ostracism contribution into simple relation

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Figure 4.8: (Color online). Analytical estimation of resource inflow-driven social hysteresis with population size $N = 50$. We have taken the graph degrees to range from $\langle k \rangle = 5$ (magenta "innermost" line) to $\langle k \rangle = 45$ (red "outermost" line) based on the assumption that the agents connect randomly among each other. Note that the solid line represents the stable fixed points and the dashed line represents the unstable fixed points. The contact point between stable and unstable fixed point is associated with the fold bifurcation which results in the hysteresis effect. Parameters used are the same as those of Fig. 4.1.

($O(n_{ci}) \sim O(P_c)$). By using this approximation, the solution for infinite population can be solved. In figure 4.9 c, we have compared mean field approximation for several different population sizes with infinite population. From the figure we can see the mean field approximation is approaching the infinite population solution as the population size increases.

4.6.2.2 Clustered Assumption

The variation of our analytical results with that of the numerical results as shown in Fig. 4.8 indicates that the connection between the two types of agents is not purely random. Instead, individuals with the same strategy tend to cluster together under the replicator dynamics. In consequence, we next consider the possibility of clustering. In fact, the occurrence of grouping of the same strategy into several clusters is quite common in evolutionary games (63, 135), since it serves to enhance the survival of the clustered strategy. Here, we assume an extreme case with the formation of only a single defector cluster and a single co-operator cluster throughout the replicator dynamics. In particular, we assume that each defector connects to all other defectors when the degree is larger or equal to the number of defectors in the system. This assumption indicates that a defector connects to $N[1 - P_c] - 1$ other defectors and $\langle k \rangle - N[1 - P_c] + 1$ cooperators. Hence, the probability distribution of a co-operator being the direct neighbor of the defector takes the following form:

$$\rho = \begin{cases} 1, & \text{for } n_c = \frac{\langle k \rangle + 1 - N[1 - P_c]}{\langle k \rangle}, \\ 0, & \text{for other } n_c. \end{cases} \quad (4.12)$$

When the number of defector is larger than the degree of the social network, we assume that there exist at least one connection between co-operator and defector as otherwise the network will become two separate networks. We assume at most one co-operator connects to a defector with a probability of $\frac{k_{dc}}{N[1 - P_c]}$, where k_{dc} is the number of connections between co-operators and defectors. Let k_{dd} be the connections within the defector cluster. Therefore, $k_{dd} + k_{dc} = N\langle k \rangle[1 - P_c]$. In this case the form of the probability distribution of the co-operators being a

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Figure 4.9: Single realization of hysteresis behavior with ordinary mean field approximation by altering the resource inflow ($0 \leq c \leq 100$) for complete graph with population size (a) $N=50$, (b) $N=200$, (c) analytical comparison between several different population sizes with infinite population.

direct neighbor of a defector is given by:

$$\rho = \begin{cases} 1 - \frac{r\langle k \rangle}{r+1}, & \text{for } n = 0, \\ \frac{r\langle k \rangle}{r+1}, & \text{for } n_c = \frac{1}{\langle k \rangle}, \\ 0, & \text{for other } n_c, \end{cases} \quad (4.13)$$

and

$$r = k_{dc}k_{dd} = \frac{l}{N[1 - P_c] - 1}. \quad (4.14)$$

With this assumption, we plot in Fig. 4.10 the set of stable and unstable fixed points as a function of the control parameter c for various values of $\langle k \rangle$. For social network with large degree, this assumption gives a similar prediction as the random connection assumption. Again, when network degree is lowered, analytical approximation deviates from the numerical result. In fact, the numerical results lie between the predictions from these two analytical approximations. This results from the fact that actual connection pattern among the co-operators and the defectors lie between purely random and purely clustered.

4.6.2.3 Improved Assumption

Next, let us obtain an improved analytical approximation of Eq. 4.6. This requires us to view the social connections between co-operators and defectors to be of two types: random and clustered. In other words, the connection pattern among the two types of agent is neither purely random nor purely clustered but somewhere in between. In fact, it is easy to perceive a tendency for the defector to cluster together to protect their community from being ostracized. These assumptions imply that ρ_i^* can be decomposed into $\rho_{ran_i}^*$ and $\rho_{clus_i}^*$ for the case of random and clustered interactions respectively. Note that the details of the form of $\rho_{ran_i}^*$ and $\rho_{clus_i}^*$ have been given and derived in previous sections. Then, by letting p_{rn} to be the probability that the interaction is completely random and p_{cl} to be the probability that the interaction is completely clustered with $p_{cl} + p_{rn} = 1$, the following equation is obtained after putting all the information into Eq. 4.6:

$$p_{rn} \sum_i \rho_{ran_i}^* O(n_{c_i}) + p_{cl} \sum_i \rho_{clus_i}^* O(n_{c_i}) = \pi_d(e_d, R^*) \quad (4.15)$$

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Figure 4.10: (Color online). Analytical estimation of resource inflow-driven social hysteresis with population size $N = 50$. We have considered the graph degrees to range from $\langle k \rangle = 5$ (magenta "leftmost" line) to $\langle k \rangle = 45$ (red "rightmost" line) based on the assumption of single cluster interaction between co-operators and defectors. The solid line represents the set of stable fixed points and the dashed line represents the set of unstable fixed points. The contact point between the stable and unstable fixed point is associated with the fold bifurcation which results in the hysteresis effect. Parameters used are the same as those of Fig. 4.1.

where $p_{cd} = P_c(1 - P_c)$. This equation together with Eq. 3.1 would enable us to calculate the stable and unstable fixed points and to compare them against those obtained through numerical computation.

The comparison between numerical results and analytical prediction is shown in figure 4.11. From the comparison, we can observe close agreement between numerical and analytical prediction. The result shows that this approximation can predict the double regimes in large degree and single regime in low degree. The numerical data points represented in this figure show the average value of many realizations. Near the critical point, some realizations have experienced the regime shift, while other realizations have not. By averaging all these points, we may obtain a new point in the middle which is some distance away from analytical curve. In this way, the outlier does not contradict our analytical prediction.

4.6.3 Pair Approximation

The assumption of social connections between co-operators and defectors is not needed if we know the exact distribution of CD links. By using pair approximation we can obtain a rough estimate of the conditional probability, $q_{c|d}$.

From the updating mechanism, we know that a defector may copy the strategy of his/her chosen neighboring cooperator if the utility of his/her neighbor is larger than his/her utility. By simple logical deduction, we can infer that the number of CC pair increases by $[\langle k \rangle - 1] q_{c|dc} + 1$ every time the number of cooperators increases by one. This number is obtained by counting the number of neighboring non-chosen cooperator ($[\langle k \rangle - 1] q_{c|dc}$) plus the chosen cooperator (1). Similarly, the CC pair decreases by $[\langle k \rangle - 1] q_{c|cd}$ every time the number of cooperators decreases by one.

At equilibrium, detailed balance dictates that a cooperator becomes a defector at the same rate as a defector becomes a cooperator:

$$1 + [\langle k \rangle - 1] q_{c|dc} = [\langle k \rangle - 1] q_{c|dc} . \quad (4.16)$$

This equation can also be written as

$$\frac{1}{\langle k \rangle - 1} = q_{c|dc} - q_{c|dc} . \quad (4.17)$$

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Figure 4.11: (Color online). A comparison between hysteresis given by numerical results and analytical estimation for populations with sparse ($\langle k \rangle = 5$) and dense ($\langle k \rangle = 45$) connections. Note that the numerical results are represented as either stars ($\langle k \rangle = 5$) or squares ($\langle k \rangle = 45$). Analytical estimations are represented by lines. The solid line represents the set of stable fixed points and the dashed line represents the set of unstable fixed points. The analytical estimation is obtained by using mean field approximation with assumption that the probability distribution of the co-operator to defector interaction lies between that of a random and clustered configuration (see Eq. 4.15).

By means of pair approximation ($q_{c|dc} \simeq q_{c|d}$ and $q_{c|cd} \simeq q_{c|c}$), Bayesian identity ($q_{c|d}P_d = q_{d|c}P_c$), and also the probability identity ($q_{c|d} + q_{d|d} = 1$ and $q_{c|c} + q_{d|c} = 1$), eq. 4.17 can be solved. This lead to the following relation:

$$q_{c|d} = \frac{\langle k \rangle - 2}{\langle k \rangle - 1} P_c. \quad (4.18)$$

Hence, the form of the probability distribution with respect to a direct neighbor of a defector being a cooperator is given by:

$$\rho_i = \binom{\langle k \rangle}{i} \left(\frac{[\langle k \rangle - 2] P_c}{[\langle k \rangle - 1]} \right)^i \left(1 - \frac{[\langle k \rangle - 2] P_c}{[\langle k \rangle - 1]} \right)^{\langle k \rangle - i}. \quad (4.19)$$

With this approximation, we plot in Fig. 4.12 the set of stable and unstable fixed points. The result shows that pair approximation can predict the double regimes in large degree and single regime in low degree. While the result may not be as close as those obtained based on the improved mean field assumption, it nonetheless yields the main feature of the model without any additional structural assumption.

4.6.4 Stability Analysis

Our numerical and analytical results suggest that there is hysteresis and alternate stable states in high degree network. On the other hand, no hysteresis and alternate stable states are found in the low degree network. This phenomenon is actually caused by degree-driven bifurcation. To illustrate this concept, we plot dP_c/dt (eq. 4.8) against P_c in Fig. 4.13a (high degree network, $\langle k \rangle = 45$) and Fig. 4.13b (low degree network, $\langle k \rangle = 5$) by using pair approximation.

In high degree network (Fig. 4.13a), there are two stable fixed points separated by one unstable fixed point. In this situation, the system either lies on a defective (low P_c) regime or cooperative (high P_c) regime. To move from one regime to the other, the control parameter is changed so that the unstable fixed point disappears which opens up the boundary between the two stable regimes. This process will be illustrated in figure 4.14 by using stability landscape. On the other hand, in a low degree network (Fig. 4.13b), there is only one stable fixed point. Therefore, there is no hysteresis and no regime shift in this case.

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Figure 4.12: (Color online). A comparison between hysteresis given by numerical results and analytical estimation for populations with sparse ($\langle k \rangle = 5$) and dense ($\langle k \rangle = 45$) connections. Note that the numerical results are represented as either stars ($\langle k \rangle = 5$) or squares ($\langle k \rangle = 45$). Analytical estimations are represented by lines. The solid line represents the set of stable fixed points and the dashed line represents the set of unstable fixed points. The analytical estimation is obtained by using pair approximation (see Eq. 4.19).

Figure 4.13: (Color online). Analytical solution of social differential equation when the resource inflow $c = 30$, for (a) high degree network ($\langle k \rangle = 45$) and (b) low degree network ($\langle k \rangle = 5$). The curve represents the analytical equation of $dP_c/dt = f(P_c)$. The fixed points are the intersection between the curve and the horizontal line. The red arrow represents the direction of the dynamical flow.

To illustrate the regime shift phenomenon, we plot in figure 4.14 the stability landscape of the high degree network ($\langle k \rangle = 45$) by using equation 2.12. Initially, there is only one basin of attraction for the cooperative regime. By increasing the control parameter c , the alternate basin of attraction emerges. However, the system still maintains its original state because the cooperative and defective regime are separated by a hill in the landscape. The hill corresponds to the unstable fixed point in the landscape. By further increasing the control parameter c , the cooperative basin of attraction disappears alongside with the hill. This forces the system (represented as a black ball) to change its position from the cooperative regime to the defective regime.

4.7 Discussion and Conclusion

It has been demonstrated in previous studies on social dilemma that cooperation collapses as the resources become more easily available. Such abrupt disruption of cooperation is commonly associated with hysteresis and regime shift (see for

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Figure 4.14: (Color online). Analytical construction of the stability landscape for high degree network ($\langle k \rangle = 45$). Different colors represent the stability landscape for different resource inflow: $c = 20$ (blue), $c = 32.5$ (red), $c = 50$ (yellow). The black ball represents the current position of the system in the stability landscape.

example Ref. (192)). In this chapter, a similar breakdown in cooperation was observed although in a different context, namely social dilemma on the use of natural resource under ecological constraint. In contrast to the previous works, the dynamics of the ecological system is explicitly studied in this chapter. In particular, the available resource level is observed to collapse as social cooperation becomes untenable, giving rise to a socio-ecological regime shift. On the other hand, cooperation within the model of Ref. (192) is promoted by a networked interaction between the players. No cooperation is observed when the population interact in a well-mixed manner. In this case, multi-stable states and hysteresis occur as a result of the tendency for high-degree nodes to preserve their initial strategies. However, in our study, cooperation is promoted through social ostracism and it exists for both well-mixed and networked populations. Notably, we have demonstrated how complex social interactions can alter the occurrence of hysteresis and regime shift in coupled socio-ecological system.

With a small disturbance, the SES under study can transit from one regime to another regime when it is near to the critical point. This results in a drastic and sometimes disastrous change in the socio-ecological state of the system. It is difficult for the system to restore to its previous state after shifting to an alternative state (66, 79, 82). Hence, in the presence of the characteristic of bifurcation, any SES with a state near to the critical point is not resilient. This particular point is known as the tipping point and at this point, a small perturbation is enough to make a large irreversible change. In our model, the width of the hysteresis curve and the position of the tipping point are both dependent on the connectivity of the social networks. In particular, as network connectivity decreases, the value of P_c at which critical transition occurs increases (see Fig 4.2 with respect to the critical point c_1). In this case, while maintaining a certain level of cooperativeness in a community with dense connection ensure resilience of the system, the same level of cooperativeness may not guarantee resilience in another community with sparse social connection. In this context, predictability of tipping point and hysteresis width is important for the management of a common pool resource. It helps in determining the best policy for a particular system.

Here, our analysis has demonstrated how social connectivity affects tipping points and bifurcation characteristics of a SES. The impact of social connec-

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tivity on hysteresis width and hence resilience of a social-ecological system can be understood in what follows. When the socio-ecological system is initially in the cooperative regime, social ostracism is an important mechanism to prevent defection during the first half of the hysteresis cycle. It is less effective in promoting cooperation during the second half of the cycle since cooperators are the minorities under such circumstances. When there are many social connections, the reduction or increment of a single cooperator has relative less weight on the effectiveness of social ostracism. During the first half of the hysteresis cycle, a slight increment in the resource inflow would reduce the number of cooperators by a slight amount. This small decrease in the number of cooperator would not cause much change to the effectiveness of social ostracism. In this case, social sanction decreases gradually with the decrease of the number of cooperators. On the other hand, when the number of social ties is small, a reduction or increment of a single cooperator can have a large impact on the effectiveness of social ostracism within the local cooperator communities. As the resource inflow increases, effectiveness of social sanction decreases sharply with the decrease of the number of cooperators. Hence, the system reaches the critical point very soon when social sanction can no longer balance the extra payoff offered by defective behavior. At this point, cooperation breakdown and the society are dominated by the defecting strategy. Hence, the critical point where the system experiences a breakdown of cooperation is different for social networks with different network degree.

Similarly, critical transition happens earlier for population with a smaller number of social ties during the second half of the hysteresis cycle. Nonetheless, as effectiveness of social ostracism in promoting cooperation is not as strong in the defective regime, the second transition point does not differ much for social networks with different density of connectivity. In fact, the same argument applies to the case of Fig. 4.7 for the scale-free and random graph topologies. Although these graphs have the same average degree, a scale-free graph contains a greater proportion of nodes with a larger degree. This ensures that the ostracism mechanism is more effective for the scale-free interaction network at the cooperative regime. In consequence, critical transition happens later. As before, the transition from defective regime back to the cooperative regime occurs at similar value

of the previous critical point. This results from ostracism exerting negligible influence within a larger population of defectors such that the degree structure of the network has minimal effect. Nonetheless, the transition occurs because the reduction in resource inflow brings π_c closer to π_d , this enhances the ostracism effect as cooperators start to emerge from the defectors such that at a critical moment, we observe a transition from the defective state to the cooperative state. Moreover, when the degree distribution becomes broader than poissonian, we will expect the appearance of nodes with many links and also the appearance of nodes with few links. Therefore, the network structure becomes more similar with scale-free network. Consequently, by using the similar argument in scale-free network, we will expect the hysteresis width become broader.

Lastly, we believe that the socio-ecological model discussed in this chapter can be applied to specific real-world socio-ecological system. Specifically, the use of Cobb-Douglas production function in the model implies that ecological resource can be viewed as production input (such as water in agriculture and industry) instead of the value of the resource itself (such as fish in fisheries). The result in this chapter can be extended to the domain of decision making and management planning in coupled socio-ecological system. While we have discussed on the hysteresis effect and socio-ecological robustness, and how network properties can influence it, we would like to emphasize that a more detailed knowledge on the social network of a particular society will give additional implication for its proper management. By paying more attention to social network properties and also specific socio-ecological parameters, we can avoid undesired regime shift.

4. SOCIAL INTERACTION, REGIME SHIFT, AND HYSTERESIS

5

Early Warning Signals

5.1 Introduction

With the presence of risk in the sudden and persistent collapse of socio-ecological system, it would be very useful if we are able to anticipate regime shifts before the transition occurs(156, 160, 161). For complex systems, a lack of detailed information makes it difficult to determine the exact position of the tipping point. To circumvent this difficulty, there is a rapid growth in the study of early warning signals of critical transition based on the generic behavior in the vicinity of regime shifts(27, 39, 40, 41, 42, 71, 103, 168). Early warning signals can be used as an indicator of a pending regime shift. It gives us the lead time to preempt the regime shift and to start a procedure of evacuation if the regime shift is unavoidable(12, 161). However, early warning signals cannot predict the future transition accurately all the time(15, 16, 68). There are possibility of false positive (when the early warning signal indicates an approaching transition but turns out to be false detection) and false negative (when the early warning signal failed to predict the approaching transition). In this chapter, we shall explore the applicability of conventional early warning signals such as autocorrelation and standard deviation to detect the occurrence of regime shifts accurately for our coupled socio-ecological model.

In general, critical slowing down occurs in any type of system near the transition point. This phenomena also occurs in both continuous and discontinuous transitions(43). In both continuous and discontinuous transition, the stability

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landscape of the system will be flattened when approaching the regime shift (low resilience). The steepness of stability landscape determines the capacity of the system to absorb any perturbations. Therefore, the variance and autocorrelation will increase in flat landscape(155). Similarly, near the region of thermodynamical phase transition, we can observe the occurrence of criticality behavior including critical slowing down(49, 76). This generic early warning signals are also observed in different bifurcation type(18, 64).

Dynamical behavior near tipping point such as critical slowing down and stability changes are commonly captured in conventional early warning signals. Some early warning signals are a direct consequence of critical slowing down, such as increasing variance and autocorrelation (refer to chapter 2, section 2.4.4 for more detail). Other early warning signals are also a direct consequence of stability landscape alteration such as increasing skewness and kurtosis. Early warning signals have shown many positive results in detecting many types of upcoming regime shifts. However, in vast range of dynamical systems, regime shift may also occur without any warning (silent catastrophe)(14, 16, 17, 68). The results in this chapter also show the failure of several indicators in predicting the upcoming regime shift.

There are several reasons why conventional EWSs may fail(43). Problem on the data such as observational error and sparsely sampled time series may obscure the result(28, 144). Observational error is usually caused by inaccurate observation or some defect on the measurement process. Limitation on the measurement resolution may also create big gap between two consecutive measurements and generate sparsely sampled time series. Early warning indicator from sparse input is often perform poorly because it can be easily overshadowed by correlated noise. However this observational problem can be minimized by robust data processing as long as we have enough data(40).

Technical problem is more difficult to be resolved and most regime shift cannot be detected under such condition. Both stochastic and fluctuating environment is the most common problem in capturing the early warning trends(28, 144). This background pattern can cover the actual dynamical trend and change the result. This distortion can be anticipated by comparing the current trend with the null model(15). The early warning signal also cannot be detected when the system is

already too close to the critical threshold or when the system experiences critical transition very fast(12). Under this situation, the observational data is usually not adequate to capture the effect of critical slowing down. Another example is when the system is not in equilibrium (non-stationary)(48). In this situation, the observed trend is influenced by dynamical attraction rather than critical slowing down. Therefore the indicators would perform very poorly under such situation. The existence of specific type of bifurcation(68, 160) is also not followed by critical slowing down. Finally, early warning signals are very difficult to observe, especially when the system is too complex and strongly correlated(24, 39, 121). Local interaction dynamics between components may interfere with global early warning pattern in the system. Recent study also shows conventional EWS works well on independent system but performs poorly on coupled system(9).

All these factors can affect and even reduce the accuracy of early warning signals in coupled socio-ecological system. Let us now identify the specific aspect in this model that may interfere with early warning pattern when the system is approaching regime shift. First the model is inherently stochastic because the selection and mutation algorithm involves random number generator. Therefore, the noise may overshadow the actual dynamics and generate the false signals. Statistical comparison with the null model is needed to avoid false detection. Second this model is in the form of coupled system and also strongly interconnected, especially in high degree network. Therefore we may expect inaccurate prediction on some early warning indicators.

5.2 Early Warning Signals

Typically, early warning signals are obtained by exploiting the generic behavior of the system close to critical transition, such as the phenomenon of critical slowing down. However, these early warning signals normally suffer from false detection, be it false positive or false negative. The false alarm is commonly caused by sensitivity of critical slowing down towards environmental stochasticity. Hence, the indicator trend is driven by stochastic perturbation rather than actual dynamics. Therefore, it is necessary to provide a statistical comparison between the test model and the null model to determine its accuracy(15, 16, 17). The

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test model relates to the case where our control parameter increases very slowly till it reaches the tipping point (10 increments in 10,000 steps), i.e. we select the portion that precedes the potential transition. On the other hand, the null model is the situation without regime shift with the control parameter kept fixed, such that the system is only driven by stochastic fluctuations. The early warning signals can be obtained by means of either temporal patterns or spatial patterns. The accuracy of the early warning signals for different network degrees will be compared.

5.2.1 Temporal Patterns

Temporal early warning signal is often handy because in most cases the time series data is the only information available to us. The data analysis that yields the early warning signals of interest usually requires several steps which include preprocessing, filtering, probing, and significance testing whose details can be found in Dakos et al(40). In empirical observations, we are typically restrained by the frequency of observation (i.e. the time interval between points in the datasets). However, this does not happen in our case since our time series data arise from the model, and our results show that the accuracy of the early warning signals obtained is independent of the frequency of observation. Therefore, we only illustrate situations when the time interval is 50.

In this analysis, we use a rolling window with a size that is half of the whole time series datasets. We also filter the trends by using Gaussian smoothing to avoid spurious indications caused by the presence of strong local correlation structures in the time series (figure 5.1a, panel 1). The de-trended data (figure 5.1a, panel 2) is then analyzed by several conventional indicators such as autocorrelation, standard deviation, skewness and kurtosis. Theoretically, as the system approaches critical transition, we expect a trend of increasing autocorrelation and standard deviation.

In order to account for the statistical errors that appear in the early warning signals, we have quantified the indicator's trends by using the Kendall tau rank correlation (figure 5.1a, panel 3-6). To achieve this, we have replicated 1,000 realizations for each time series measurement from our simulation for both the case

of test model (approaching regime shift) and the null model (non-approaching regime shift). Kendall tau rank correlation for each indicators and from each time series realization is calculated. Different realizations may result in different prediction value and we record all these value in the form of a histogram. This histogram can be treated as a probability distribution of prediction value from specific indicator. In our analysis, we obtain the prediction distribution for the test model (blue distribution) and the null model (red distribution). The distributions for both cases are compared to determine the accuracy of the early warning indicators (figure 5.1b, panel 1-4). The overlapping region between test (blue) and null (red) distribution indicates uncertain prediction region because any early warning signals within this region can be explained by null model (no regime shift). Within this region the range of signals for the real and false signals cannot be distinguished. Note that the vertical lines indicate the $p\text{-value} = 0.05$ of the null model. Any value beyond these lines is considered significant. The accuracy of specific indicator is then quantified by the proportion of significant predictions attained against the total number of predictions attempted.

5.2.2 Spatial Patterns

Spatial early warning signal is only useful if we have complete spatial information of the system. It often provides more accurate predictions in comparison to temporal early warning signal, although it is more difficult to exploit them due to insufficient spatial data in many cases. In our work, the spatial pattern is obtained from the spatial distribution of each strategy (cooperative or defective) within the network structure. In our calculations, we denote the cooperative strategy as 1 and the defective strategy as 0.

The spatial autocorrelation is quantified by the Moran spatial correlation(39, 137). The standard deviation, skewness and kurtosis are modified into spatial measures as the second, third, and fourth moments about the spatial mean respectively(89). Theoretically, when the system gets closer to critical transition, there is an increase in spatial autocorrelation and standard deviation.

We shall quantify the trends exhibited by the spatial indicators by means of the Kendall tau rank correlation (figure 5.2a, panel 1-4). We have generated 1,000

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Figure 5.1: (Color online). Procedure for temporal early warning signals. Figure (a) represents a single realization of early warning detection. Panel 1 gives the time series of the fraction of co-operators. The red curve is obtained after passing the time series through a Gaussian filter. Panel 2 shows the de-trended data that is employed for subsequent analysis. Each indicator is calculated within a rolling window that is half the size of the whole data. Panel 3-6 illustrates the trend for the following indicators: autocorrelation at lag-1, standard deviation, skewness, and kurtosis. The trend is then further analyzed by means of the Kendall tau rank correlation. Figure (b) shows a comparison between the null model and the test model for each temporal indicator. The distribution in red represents the trend from the 1000 realizations of the null model. The vertical lines indicate the positions of the $p\text{-value} = 0.05$. On the other hand, the blue distribution represents the trend from the 1000 realizations of the test model. The indicator trend is considered significant if its value exceeds the $p\text{-value}$ of the null model.

Figure 5.2: (Color online). Procedure for spatial early warning signals. Figure (a) illustrates a single realization of spatial early warning signals. Panel 1-4 give the trend of spatial autocorrelation (Moran coefficient), standard deviation, skewness, and kurtosis respectively. The trend is then analyzed using the Kendall tau rank correlation. Figure (b) shows a comparison between the null model and the test model for different spatial indicators. The distribution in red is obtained from the trend of 1000 realizations of the null model. The vertical lines show the position of the $p\text{-value} = 0.05$. Blue distribution is derived from the trend of 1000 realizations of the test model. The indicator trend is considered significant if its value exceeds the $p\text{-value}$ of the null model.

realizations from time series simulation for the case of test model (approaching regime shift) and also the null model (non-approaching regime shift). These are illustrated in panel 1-4 of figure 5.2a. In similar manner, we record all Kendall tau rank correlation for each indicators into a histogram, then the distributions obtained for these two cases (test and null model) are then compared to determine its accuracy (figure 5.2b, panel 1-4). Then we compare the two distribution to find the prediction region which can be explained by null model (no regime shift). Note that the vertical lines indicate the $p\text{-value} = 0.05$ of the null model. Any value beyond these lines is considered as significant. Again, the accuracy of specific indicator is quantified by the proportion of significant predictions achieved against the total number of predictions attempted.

5.3 The effect of Connectedness on the Accuracy of Early Warning Signals

In this section, we shall show our numerical results and discuss the effects of network connectivity on the accuracy of several conventional early warning signals. Note that all the necessary details regarding the early warning signals have already been discussed in the methods section.

5.3.1 Temporal Patterns

In previous section we have compared test and null distribution to obtain significant prediction. Next we compare the accuracy of specific indicator for different network degree. This comparison will help us know whether social network affects the accuracy of early warning predictions. Figure 5.3 illustrates the accuracy of specific temporal indicators as the network degree varies. Our results here show the percentage of significant correct prediction. We found that most of the temporal early warning signals are not accurate enough to predict the approaching transition. From the figure, we can see that only temporal standard deviation is sensitive enough to detect future regime shift. Other indicators are not able to distinguish between the null model (stable system) and the test model (system approaching critical transition).

We observe that the accuracy of temporal standard deviation increases as the network degree is lowered. This results from the following. For the case of high degree network, the stability of the regime reduces gradually as the control parameter increases. On the other hand, the stability drops faster in the case of low degree network as the control parameter increases. Since the dynamics of the system fluctuates more rapidly (higher variance) when the system is unstable, the temporal standard deviation is able to capture the increasing trend of variance more effectively. This explains why the temporal standard deviation performs better when the network degree is low versus that when the network degree is high.

5.3 The effect of Connectedness on the Accuracy of Early Warning Signals

Figure 5.3: (Color online). Accuracy of temporal early warning signals. A plot on the accuracy of different temporal early warning indicators: autocorrelation at lag-1, standard deviation, skewness, and kurtosis (see legend), versus the network degree. Note that each point is obtained after determining the percentage of significant accurate predictions through a comparison made against the null model.

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5.3.2 Spatial Patterns

Figure 5.4 shows the accuracy of specific spatial indicators as the network degree changes. In comparison to the temporal early warning signals, we found that most of the spatial indicators are sensitive enough to predict the occurrence of an approaching regime shift. In fact, the second, third, and fourth moment of the spatial mean (i.e. the standard deviation, skewness, and kurtosis) are able to predict the approaching regime shift with 100% accuracy. For these indicators, the distribution of the null model and the test model is found to be totally separated without any overlap. On the other hand, the accuracy of the Moran I spatial correlation is found to increase as the degree is lowered, and it can predict with 100% accuracy when the network degree is very low. This can be understood as follows. Near the critical transition, we can perceive that every part of the network becomes spatially more similar to each other. For high degree network, since everyone is almost connected to everyone else, the system is spatially correlated. This makes it difficult to distinguish between the null model and the test model. Such spatial correlation reduces as the network degree decreases, thus enabling the null model to be distinguishable from the test model.

5.4 Conclusion

We have investigated into the influence of complex social network interactions on regime shifts in coupled socio-ecological system and our ability to make accurate prediction on its occurrence. We have based our study with the inclusion of social interactions modeled by different network topologies and employing a discrete choice mechanism to update agent's strategy which involves selection (utility driven strategy selection) and mutation (random strategy updating).

By comparing the results in figure 5.3 and figure 5.4, we learn several insights. First, many early warning signals fail in making accurate prediction. Second, the spatial early warning signals are more accurate than temporal early warning signals. In fact, many spatial indicators can predict with 100% accuracy. Third, in general, early warning indicators predict more accurately in a system with low connection. Moreover, our results also show that intrinsic social network

Figure 5.4: (Color online). Accuracy of spatial early warning signals. A plot on the accuracy of various spatial early warning indicators: Moran coefficient, spatial standard deviation, spatial skewness, and spatial kurtosis (see legend), versus the network degree. Note that each point is obtained after determining the percentage of significant accurate predictions through a comparison made against the null model.

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properties can yield subtle effects on the accuracy of early warning signals. Thus, a more detailed understanding on the social interaction network properties as well as the associated socio-ecological parameters within a society would provide deeper insights that will be important for its proper protection. More importantly, it will enable us to improve our abilities to anticipate or even avoid the unsought for regime shift that can be catastrophic.

6

Community Structure and Multiple Stable Regimes

6.1 Introduction

In a self-organized resource management system, social capital and structure of the communities are of particular importance(105). While individuals within a community form close ties with each other(141, 196), different communities may tend to avoid social contact in order to protect their identities and cultures(113). In network theory, such tendency towards social segregation is described by the network's community structure(61), where the limited cross-community interactions can result in social disharmony and unrest. On the other hand, the presence of weak ties between communities is an avenue of new information and social influence that is not available to those locked entirely into a network of endogenous relationship(65). Such network-based analysis has recently been employed to reveal the community structure of CPR users based on the ethnic diversity in Hawaiian fisheries(7).

An important open question to be addressed in this chapter is the impact of community structure on social cooperation at both the societal and community level as the individuals engaged in economic utilization of the common pool resources. The subject of interest is to uncover the intrinsic dynamical behavior of the system as it undergoes irreversible collapse or revival in both the social and ecological variables. In this respect, we envisage the possible occurrence

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of new stability regimes that could mitigate the drastic critical transition from one extreme regime to another(159). The elucidation of such regime shifts has motivated us to understand how the decline and revival of social cooperation in one community can affect the sustainable and cooperative behavior of another community in terms of inter- and intra-community relationships.

Figure 6.1: (Color online). Network with two community groups. The figure above shows a population $N = 100$ and average degree $\langle k \rangle = 15$ for the case of (a) $\mu = 0.5$ and (b) $\mu = 0.15$ with same population size. The size of each node represents the degree (number of links connected to the node).

To address this question, let us now consider a society where every individual relies on a common pool resource. In our model, each individual is represented by a node in an Erdős-Rényi complex network of mean degree $\langle k \rangle$ with social interaction between agents corresponds to a link in the network. The network consists of two community groups of equal size that are tightly connected internally, while

the groups can have different degree of inter-connection between them depending on their level of inter-relatedness. This relationship between groups is expressed in terms of a mixing parameter μ (91), which quantifies the probability that the link of a node is connected to its external communities. The pictorial representation of two community graph with different μ can be seen in figure 6.1. Similar to previous chapters, we consider in this chapter the socio-ecological system with ostracism mechanism as in chapter 3.

6.2 Multiple Regime Shifts

We begin by considering an integrated society where $\mu = 0.5$. We assume that it is subjected to an external drive which is the resource inflow c . We start with the scenario where all agents are cooperator and $c = 0$. Parameter c is increased slowly and after each increment, allows the system to reach steady state. We call this the forward path or direction. While this process progresses, we observe a gradual increase in the number of defectors, indicating a shift from full social cohesion of cooperation to states of greater disharmony. The latter results from an increasing number of conflicting strategies in the population. At a critical resource inflow, regime shift occurs and the system transits discontinuously from one extreme regime to another in both the social and ecological variables. In the new regime, there is a larger proportion of defectors than cooperators. Cooperation has drastically reduced while social disharmony persists. A further increase in c eventually leads to the state of all defectors, upon which cooperation vanishes although the system has reached a state of harmony since there is no differing strategies. While the state of cooperation can be restored by decreasing c (the backward path or direction), the process now follows a different path. Thus, the system displays irreversibility in the form of a hysteresis loop. This behavior is illustrated in Fig. 6.2 and Fig. 6.3.

By reducing μ , community structure appears. As the society becomes more segregated with a smaller μ , we observe that cooperation collapses earlier in the forward path. For weakly segregated societies ($\mu = 0.3$ and $\mu = 0.25$), the collapse of social cooperation occurs concomitantly in the two communities. When the segregation becomes sufficiently large ($\mu = 0.2$ and $\mu = 0.15$), we

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Figure 6.2: (Color online). The effect of community segregation on social cooperation driven by resource inflow ($0 \leq c \leq 60$) with a network ($N = 100, \langle k \rangle = 45$) of two community groups, where $\mu = 0.5$ (square), 0.3 (circle), 0.2 (triangle), and 0.15 (diamond). Results are averaged over 100 realizations. Inset: single realization ($\mu = 0.2$) on P_c in community 1 (red line), and community 2 (blue line).

Figure 6.3: (Color online). The effect of community segregation on ecological resource driven by resource inflow ($0 \leq c \leq 60$) with a network ($N = 100$, $\langle k \rangle = 45$) of two community groups, where $\mu = 0.5$ (square), 0.3 (circle), 0.2 (triangle), and 0.15 (diamond). Results are averaged over 100 realizations. Inset: single realization ($\mu = 0.2$) of total payoff in community 1 (red line), and community 2 (blue line).

6. COMMUNITY STRUCTURE AND MULTIPLE STABLE REGIMES

observe the interesting phenomenon of a double regime shifts. Specifically, for the more segregated society at $\mu = 0.15$, the first regime shift happens earlier while the second regime shift occurs much later compared to $\mu = 0.2$ as c increases. A closer examination in terms of single realization (see inset in Fig. 6.2) shows that cooperation did not collapse globally but instead at the local community level, e.g. community 2 (the red curve) collapses first. In other words, cooperation is maintained at the community level before it is eventually destroyed at a second critical transition. It is important to note that the order of community collapses is purely random as it depends on the manner in which the defector spreads in each community. Like the case of fully integrated society, the system would not reverse its path as we reduce c from the all defectors state. There is again hysteresis but now double regime shifts back to the cooperative state even for the weakly segregated societies.

6.3 The Variation of Mixing Parameter and Mean Degree in Two Communities

Double regime shift does not occur all the time and it happens beyond a critical μ in both the forward and backward direction. The critical values can be approximated by simple analytical estimation. For the forward direction, the spread of defectors is curtailed by the ostracism mechanism. On average, any node is connected with other node from the same community (different community) with probability $1 - \mu$ (μ). Hence, the probability that a cooperator from community 1 is connected with a defector from community 1 (community 2) is $(1 - \mu)q_{d1|c1}P_c1$ ($\mu q_{d2|c1}P_{c1}$), and defector utility from community 1 (community 2) is reduced by ostracism $\langle O_1(k_c1 + k_c2) \rangle$ ($\langle O_2(k_c2 + k_{c1}) \rangle$). Therefore, we can estimate weighted ostracism to prevent defector spreading in community 1 as $O_1^w = (1 - \mu)q_{d1|c1}P_c1\langle O_1(k_c1 + k_c2) \rangle + \mu q_{d2|c1}P_{c1}\langle O_2(k_c2 + k_{c1}) \rangle$. By using similar reasoning, we can also estimate weighted ostracism to prevent defector spreading in community 2 as $O_2^w = (1 - \mu)q_{d2|c2}P_c2\langle O_2(k_c2 + k_{c1}) \rangle + \mu q_{d1|c2}P_{c2}\langle O_1(k_{c1} + k_{c2}) \rangle$. In general, we can estimate the ostracism strength that deters the change of a

6.3 The Variation of Mixing Parameter and Mean Degree in Two Communities

Figure 6.4: (Color online). The numerical and analytical estimation of critical segregation, which is the transition point between single and double regime shifts. The results are obtained for (a) increasing resource inflow (μ_c^f) , and (b) decreasing resource inflow (μ_c^b) , against mean degree of the complex network. The numerical estimates are shown in blue squares while the analytical estimates in red triangles.

cooperator in community i to a defector as a weighted sum of intra- and inter-community ostracism: $O_i^w = (1 - \mu) q_{di|ci} P_{ci} \langle O(i, j) \rangle_i + \mu q_{dj|ci} P_{ci} \langle O(j, i) \rangle_j$, where $i \neq j$ and shorthand notation $\langle O(i, j) \rangle_i$ represents the evaluation of average ostracism O within community i . A further approximation is made by letting $\langle O(i, j) \rangle_i \approx O([1 - \mu] \langle k \rangle q_{ci|di} + \mu \langle k \rangle q_{cj|di})$.

Without loss of generality, let us assume that $P_{d1} > P_{d2}$. Now, if $O_1^w > O_2^w$, defectors would tend to spread in community 2 and we expect cooperativeness to collapse simultaneously in community 1 and 2. On the other hand, if $O_1^w < O_2^w$, defectors would further increase in community 1 leading to a local collapse of cooperativeness within community 1 and not community 2. Therefore, the critical μ at which single regime shift transits to double regime shift in the forward direction μ_c^f is to be determined based on the condition: $O_1^w = O_2^w$. Analogously, for the backward direction, we would need to ascertain the cost that prevents the spread of cooperators. This cost is proportional to the average number of intra- and inter-CD (cooperator-to-defector) links to the defector in community i : $L_i = (1 - \mu) q_{ci|di} P_{di} + \mu q_{cj|di} P_{di}$. Based on similar reasoning, the critical μ for

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single to double regime shifts in the backward direction μ_c^b is to be obtained by solving $L_i = L_j$. We have determined the critical μ based on these analytical approximations as well as numerical calculation as shown in Figs. 6.4a and 6.4b, where we observe that $\mu_c^f \leq \mu_c^b$. Note that for the numerical estimates, the critical μ is defined as the value that generates half the total number of single and double regime shifts. In summary, we have double regime shifts in both the forward and backward directions when $\mu < \mu_c^f$; single regime shift in the forward direction and double regime shifts in the backward direction when $\mu_c^f \leq \mu < \mu_c^b$; and single regime shift in both directions when $\mu \geq \mu_c^b$.

Figure 6.5: (Color online). The effects of segregation of the two communities on hysteresis in social cooperation for different network degree: (a) $\langle k \rangle = 15$ and (b) $\langle k \rangle = 5$, with mixing parameter $\mu = 0.5$ (square), $\mu = 0.3$ (circle), $\mu = 0.2$ (triangle), and $\mu = 0.15$ (diamond). Note that $N = 100$, and the results were obtained after averaging 100 hysteresis realizations.

By examining the results from numerical calculations in Figs. 6.4a and 6.4b, we observe that $\mu_c^f \approx \mu_c^b$ for small $\langle k \rangle$. In fact, the transition from single to double regime shift becomes less synchronized between the forward and backward directions as $\langle k \rangle$ increases. Another effect of $\langle k \rangle$ is on the width of the hysteresis loop. As $\langle k \rangle$ decreases, the width of the loop reduces (see Figs. 6.5a and 6.5b). At $\langle k \rangle = 5$, the width vanishes while the three regimes persist, and the system now becomes reversible.

6.4 Analytical Approximation

Next, we perform a detailed quantitative analysis on our coupled social-ecological system. First, we solve for the fixed point of the resource dynamics and obtain:

$$R^* = -E + \left(\sqrt{E^2 + 4c \frac{d}{R_{\max}}} \right) \frac{R_{\max}^2}{2d}. \quad (6.1)$$

By using the stochastic process in chapter 2 (section 2.3.2), we can form and expand master equation by using the transition probability. We then obtain the following approximate differential equation of the strategy dynamics:

$$\frac{d}{dt}P(P_C, t) = -\frac{d}{dP_C}[P(P_C, t)(T^+(P_C) - T^-(P_C))].$$

Where $T^+(P_C)$ ($T^-(P_C)$) is the probability that the number of cooperator increases (decreases) by one. The presence of community structure is indicated by P_{ci} (P_{di}), which denotes the probability that a cooperator (defector) is being found in community i (with $i = 1, 2$). Note that $P_{ci} + P_{di} = 1$. We define the conditional probability $q_{ci|dj}$ as the probability that a defector in community j has a neighboring cooperator in community i , where $i, j = 1, 2$.

Moreover, based on the algorithm, an agent will copy the strategy from a random neighbor with the probability proportional to their normalized utility difference. In society with 2 community, the probability that the number of cooperators increase by one is a sum of contribution from community 1, $(1 - \mu) [P_{d1}q_{c1|d1}(u_{c1} - u_{d1})/u_{\max}] + \mu [P_{d1}q_{c2|d1}(u_{c2} - u_{d1})/u_{\max}]$ and community 2, $\mu [P_{d2}q_{c1|d2}(u_{c1} - u_{d2})/u_{\max}] + (1 - \mu) [P_{d2}q_{c2|d2}(u_{c2} - u_{d2})/u_{\max}]$. Therefore the total contribution is,

$$\begin{aligned} T^+(P_{c1}, P_{c2}) = & \\ & + (1 - \mu) [P_{d1}q_{c1|d1}(u_{c1} - u_{d1})/u_{\max} + P_{d2}q_{c2|d2}(u_{c2} - u_{d2})/u_{\max}] \\ & + \mu [P_{d1}q_{c2|d1}(u_{c2} - u_{d1})/u_{\max} + P_{d2}q_{c1|d2}(u_{c1} - u_{d2})/u_{\max}]. \end{aligned} \quad (6.2)$$

Similarly, the probability the number of cooperators decreases by one is

$$\begin{aligned} T^-(P_{c1}, P_{c2}) = & \\ & + (1 - \mu) [P_{d1}q_{c1|d1}(u_{d1} - u_{c1})/u_{\max} + P_{d2}q_{c2|d2}(u_{d2} - u_{c2})/u_{\max}] \\ & + \mu [P_{d1}q_{c2|d1}(u_{d2} - u_{c1})/u_{\max} + P_{d2}q_{c1|d2}(u_{d1} - u_{c2})/u_{\max}]. \end{aligned} \quad (6.3)$$

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Then, by means of the Bayesian identity $P_{ci,dj} = q_{ci|dj}P_{dj} = q_{dj|ci}P_{ci}$ and the condition of equilibrium $T^+(P_{c1}, P_{c2}) - T^-(P_{c1}, P_{c2}) = 0$, we obtain the fixed points of the social dynamics by solving this equation. In which in the generalized form can be written as

$$(1 - \mu) \sum_{i=j} [P_{ci,dj} (u_{ci} - u_{dj}) / u_{max}] + \mu \sum_{i \neq j} [P_{ci,dj} (u_{ci} - u_{dj}) / u_{max}] = 0, \quad (6.4)$$

where $P_{ci,dj}$ is the probability that a cooperator in community i is connected to a defector in community j .

In general, the average number of intra-community links of a node is $(1 - \mu)\langle k \rangle$ while the average inter-community links is $\mu\langle k \rangle$. Let us assume that a defector from community i is connected to l_{ci} neighboring cooperator from the same community and m_{cj} neighboring cooperator from a different community, i.e. $i \neq j$ here. The frequency of such a configuration is given by: $B_{l_{ci}}(q_{ci|di}) B_{m_{cj}}(q_{cj|di})$, which is described by the binomial distribution $B_i(q) = k! q^i (1-q)^{k-i} / [i!(k-i)!]$. By substituting this frequency and the definition of the utility, we obtain:

$$\frac{\pi_d - \pi_c}{\pi_d} \sum_{i \neq j} I_{ij} = 0, \quad (6.5)$$

where $I_{ij} = ((1 - \mu) P_{ci,di} + \mu P_{cj,di}) (\langle O(i, j) \rangle - \pi_d)$ and

$$\langle O(i, j) \rangle = \sum_{l_{ci}=0}^{(1-\mu)\langle k \rangle} \sum_{m_{cj}=0}^{\mu\langle k \rangle} B_{l_{ci}}(q_{ci|di}) B_{m_{cj}}(q_{cj|di}) O(l_{ci} + m_{cj}). \quad (6.6)$$

In order to solve the above equations, we have applied the same pair approximation method as in section 4.6.3. Essentially, the approximation assumes that μ is sufficiently low such that most of the increase in CC (cooperator-to-cooperator) pair arises from intra-community neighbors instead of inter-community neighbors. This implies that the number of CC pair in community i increases by $[(1 - \mu)\langle k \rangle - 1] q_{ci|di} + 1$ and decreases by $[(1 - \mu)\langle k \rangle - 1] q_{ci|ci}$ every time the number of cooperators increases and decreases by one respectively. At equilibrium, detailed balance dictates that a cooperator becomes a defector at the same rate as a defector becomes a cooperator. By solving these relations, we obtain

$q_{ci|di} = [(1 - \mu) \langle k \rangle - 2] P_{ci} / [(1 - \mu) \langle k \rangle - 1]$. On the other hand, the probability of inter-community link is akin to the selection of a random node from the other community in lieu of such connection being assumed rare. Therefore, the conditional probability of inter-community link is equal to the probability of the strategy in the external community, i.e. $q_{ci|dj} = P_{ci}$ with $i \neq j$.

By inputting these conditional probabilities into Eq. (6.5) and using Eq. (6.1), we would be able to solve for P_c and c . To achieve this, we note that $P_c = (P_{c1} + P_{c2}) / 2$ and the two communities are identical. This allows us to make two assumptions. For single regime shift with $\mu \geq \mu_c^b$, we let $P_{c1} = P_{c2}$. For the case of double regime shifts, we take: (i) $0 \leq P_c \leq 0.5$, $P_{cj} = 0$; (ii) $0.5 \leq P_c \leq 1$, $P_{ci} = 1$, with $i \neq j$ to account for each regime, and $\mu < \mu_c^b$. Strictly speaking, there is a single regime shift in the forward direction and a double regime shifts in the backward direction for $\mu_c^f \leq \mu < \mu_c^b$. Nonetheless, our analytical solution turns out to correspond closely to the numerical results even in this latter case.

The solution of these analytical approximations are illustrated in Fig. 6.2 as solid and dotted lines which represent the stable and unstable states respectively. The analytical result illustrates the presence of an additional regime of stable states (moderately cooperative at the societal level) sandwiched between the regimes of cooperator-dominant and defector-dominant stable states. This additional regime accounts for the observation of double regime shifts. If we were to trace the analytical curve by adjusting c , we would encounter critical transitions between the stable regimes at the fold bifurcations (the meeting point of the stable and unstable states). A careful examination indicates the possibility for the system to display a double hysteresis loop. Figure 6.2 illustrates a comparison of our analytical approximation against the numerical results, which shows good correspondence.

6.5 Multiple Regime Shifts for any Number of Communities

Our theoretical model can be generalized to n communities. Numerically, this is achieved by rewiring the original undirected random network such that each node is classified according to their respective community groups with each link being reassigned based on a benchmark algorithm. On the other hand, for the analytical approximation, we shall extend the previous procedure. Let P_{ci} (P_{di}) denotes the probability that a cooperator (defector) is being found in community i ($i = 1, 2, \dots, n$), with $P_{ci} + P_{di} = 1$. As before, $q_{ci|dj}$ is the conditional probability that a defector in community j has a neighboring cooperator in community i . The average number of intra-community links of a node is $(1 - \mu)\langle k \rangle$, and the average number of links to the external communities is $\mu\langle k \rangle$. Thus, the average number of inter-community links is $\mu\langle k \rangle / (n - 1)$.

In general, the probability that the number of cooperators increase by one becomes

$$\begin{aligned} T^+(P_{c1}, \dots, P_{cn}) = & \\ & + (1 - \mu) \sum_{i=j} [P_{di} q_{cj|di} (u_{cj} - u_{di}) / u_{max}] \\ & + [\mu / (n - 1)] \sum_{i \neq j} [P_{di} q_{cj|di} (u_{cj} - u_{di}) / u_{max}] , \end{aligned} \quad (6.7)$$

while the probability that it decreases by one is

$$\begin{aligned} T^-(P_{c1}, \dots, P_{cn}) = & \\ & + (1 - \mu) \sum_{i=j} [P_{ci} q_{dj|ci} (u_{dj} - u_{ci}) / u_{max}] \\ & + [\mu / (n - 1)] \sum_{i \neq j} [P_{ci} q_{dj|ci} (u_{dj} - u_{ci}) / u_{max}] . \end{aligned} \quad (6.8)$$

As in the section 6.4, the need to establish the fixed points of the social dynamics requires the solution of the equation 6.5:

$$\frac{\pi_d - \pi_c}{\pi_d} \sum_{i \neq j} I_{ij} = 0 ,$$

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where generalized I_{ij} now is in the form of

$$I_{ij} = \left((1 - \mu) P_{ci, di} + \frac{\mu}{n - 1} \sum_{j \neq i} P_{cj, di} \right) (\langle O(i, j) \rangle - \pi_d), \quad (6.9)$$

and generalized $\langle O(i, j) \rangle$ now is in the form of

$$\langle O(i, j) \rangle = \sum_{l_{ci}=0}^{(1-\mu)\langle k \rangle} B_{l_{ci}}(q_{ci|di}) \prod_{j \neq i} \sum_{m_{cj}=0}^{\mu\langle k \rangle} B_{m_{cj}}(q_{cj|di}) O\left(l_{ci} + \sum_{j \neq i} m_{cj}\right). \quad (6.10)$$

We next apply this general formulation to $n = 2$ and 4 ($N = 100$, $\langle k \rangle = 15$, and $\mu = 0.2$). In the case of two communities, the equation to solve is:

$$\begin{aligned} 0 = & (P_{c1d1}(1 - \mu) + P_{c2d1}\mu) \left(\sum_{l_{c1}}^{(1-\mu)\langle k \rangle} \sum_{m_{c2}}^{\mu\langle k \rangle} B_{l_{c1}}(q_{c1|d1}) B_{m_{c2}}(q_{c2|d1}) 0(l_{c1} + m_{c2}) - \pi_d \right) \\ & + (P_{c2d2}(1 - \mu) + P_{c1d2}\mu) \left(\sum_{l_{c2}}^{(1-\mu)\langle k \rangle} \sum_{m_{c1}}^{\mu\langle k \rangle} B_{l_{c2}}(q_{c2|d2}) B_{m_{c1}}(q_{c1|d2}) 0(l_{c2} + m_{c1}) - \pi_d \right) \end{aligned} \quad (6.11)$$

For four communities, the equation becomes:

$$\begin{aligned} 0 = & \left(P_{c1d1}(1 - \mu) + (P_{c2d1} + P_{c3d1} + P_{c4d1})\frac{\mu}{3} \right) \\ & \times \left(\sum_{l_{c1}}^{(1-\mu)\langle k \rangle} \sum_{m_{c2}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c3}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c4}}^{\frac{\mu\langle k \rangle}{3}} B_{l_{c1}}(q_{c1|d1}) B_{m_{c2}}(q_{c2|d1}) B_{m_{c3}}(q_{c3|d1}) B_{m_{c4}}(q_{c4|d1}) 0(l_{c1} + m_{c2} + m_{c3} + m_{c4}) - \pi_d \right) \\ & + \left(P_{c2d2}(1 - \mu) + (P_{c1d2} + P_{c3d2} + P_{c4d2})\frac{\mu}{3} \right) \\ & \times \left(\sum_{l_{c2}}^{(1-\mu)\langle k \rangle} \sum_{m_{c1}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c3}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c4}}^{\frac{\mu\langle k \rangle}{3}} B_{l_{c2}}(q_{c2|d2}) B_{m_{c1}}(q_{c1|d2}) B_{m_{c3}}(q_{c3|d2}) B_{m_{c4}}(q_{c4|d2}) 0(l_{c2} + m_{c1} + m_{c3} + m_{c4}) - \pi_d \right) \\ & + \left(P_{c3d3}(1 - \mu) + (P_{c1d3} + P_{c2d3} + P_{c4d3})\frac{\mu}{3} \right) \\ & \times \left(\sum_{l_{c3}}^{(1-\mu)\langle k \rangle} \sum_{m_{c1}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c2}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c4}}^{\frac{\mu\langle k \rangle}{3}} B_{l_{c3}}(q_{c3|d3}) B_{m_{c1}}(q_{c1|d3}) B_{m_{c2}}(q_{c2|d3}) B_{m_{c4}}(q_{c4|d3}) 0(l_{c3} + m_{c1} + m_{c2} + m_{c4}) - \pi_d \right) \\ & + \left(P_{c4d4}(1 - \mu) + (P_{c1d4} + P_{c2d4} + P_{c3d4})\frac{\mu}{3} \right) \\ & \times \left(\sum_{l_{c4}}^{(1-\mu)\langle k \rangle} \sum_{m_{c1}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c2}}^{\frac{\mu\langle k \rangle}{3}} \sum_{m_{c3}}^{\frac{\mu\langle k \rangle}{3}} B_{l_{c4}}(q_{c4|d4}) B_{m_{c1}}(q_{c1|d4}) B_{m_{c2}}(q_{c2|d4}) B_{m_{c3}}(q_{c3|d4}) 0(l_{c4} + m_{c1} + m_{c2} + m_{c3}) - \pi_d \right) \end{aligned} \quad (6.12)$$

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Similarly, we apply the pair approximation to solve several unknown conditional probabilities as previous section 6.4. By inputting these conditional probabilities into Eqs. (6.11) and (6.12), we would be able to solve for P_c and c for each cases. To achieve this, we note that $P_c = \sum_{i=1}^n P_{ci}/n$ and the n communities are identical. This allows us to make two assumptions. The first assumption relates to an integrated society where we expect the proportion of cooperators in each community to be the same. Thus, we have $P_{c1} = P_{c2} = P_{c3} = \dots = P_{cn}$, which corresponds to a single regime shift with $\mu \geq \mu_c^b$. The second assumption goes to the other extreme where the society is segregated, such that some communities are perfectly cooperative ($P_{ci} = 1$), while the rest of the communities are perfectly defective ($P_{cj} = 0$). This corresponds to n -tuple regime shifts with $\mu < \mu_c^f$, and we have (i) $0 \leq P_c \leq 1/n$, $P_{c2} = P_{c3} = P_{c4} = \dots = P_{cn} = 0$; (ii) $m/n \leq P_c \leq (m+1)/n$, $P_{c1} = P_{c2} = \dots = P_{cm} = 1$, $P_{c(m+2)} = P_{c(m+3)} = \dots = P_{cn} = 0$, where $m = 1, 2, \dots, n-2$; and (iii) $(n-1)/n \leq P_c \leq 1$, $P_{c1} = P_{c2} = P_{c3} = \dots = P_{c(n-1)} = 1$. In between these extremes ($\mu_c^f \leq \mu < \mu_c^b$), we observe x -tupe regime shifts in the forward direction and y -tupe regime shifts in the backward direction with $x < y \leq n$. Our application of the second assumption to this latter case still yields close correspondence between analytical solution against numerical results according to our observations.

Specific application of this procedure to the case of 2 communities gives $P_{c1} = P_{c2}$ for integrated mixing parameter ($\mu > \mu_c^b$). When the mixing parameter is segregated ($\mu < \mu_c^f$), we have (i) $0 \leq P_c \leq 0.5$, $P_{c2} = 0$; (ii) $0.5 \leq P_c \leq 1$, $P_{c1} = 1$. Similarly, for 4 communities, the condition $P_{c1} = P_{c2} = P_{c3} = P_{c4}$ applies for integrated mixing parameter ($\mu > \mu_c^b$). On the other hand, for segregated mixing parameter ($\mu < \mu_c^f$), we have (i) $0 \leq P_c \leq 0.25$, $P_{c2} = P_{c3} = P_{c4} = 0$; (ii) $0.25 \leq P_c \leq 0.5$, $P_{c1} = 1$, $P_{c3} = P_{c4} = 0$; (iii) $0.5 \leq P_c \leq 0.75$, $P_{c1} = P_{c2} = 1$, $P_{c4} = 0$; (iv) $0.75 \leq P_c \leq 1$, $P_{c1} = P_{c2} = P_{c3} = 1$. For intermediate segregation level, the second assumption is applicable as stated above. The analytical solutions for 2 and 4 communities with $\mu = 0.2$ obtained based on this approach is illustrated in Figs. 6.6a and 6.6b.

Figure 6.6: (Color online). Analytical prediction of the stable and unstable fixed points for networks with (a) 2 communities ($N_1 = N_2 = 50$), and (b) 4 communities ($N_1 = N_2 = N_3 = N_4 = 25$), where $N = 100$, $\langle k \rangle = 15$, and $\mu = 0.2$. The stable fixed points are plotted as solid line and the unstable fixed points as dashed line. Note that blue line (outer) corresponds to analytical prediction based on mean field approximation while red line (inner) to pair approximation. The black squares are numerical results.

6.6 Regime Shifts for Two Communities of Different Size

Multiple regime shift is also occur for unequal community population. Let consider 2 communities network with community's size, $N_1 = 70, N_2 = 30$, and mixing parameter, $\mu = 0.2$. When the driving parameter is varied in both direction, the regime shift of both community happen separately. The magnitude of regime shift is associated with the size of community that experience cooperation collapse, therefore the system may have two different cooperative level of middle regime depending on which community that experience regime shift. In Fig. 6.7a and Fig. 6.7b, we can see two different possibilities of regime shifts order in both direction.

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Figure 6.7: (Color online). The effect different communities size ($N_1 = 30, N_2 = 70$) to cooperation hysteresis driven by variation of resource inflow ($0 \leq c \leq 50$) with interaction network with 2 communities ($N = 100, \langle k \rangle = 15, \mu = 0.2$). Cooperation of total population is represented by black square, while cooperation of community 1 and 2 are represented by red and blue lines. (a) In this realization, regime shift of community 1 is followed by regime shift of community 2 in forward direction and regime shift of community 2 is followed by regime shift of community 1 in backward direction. (b) In this realization, regime shift of community 2 is followed by regime shift of community 1 in forward direction and regime shift of community 1 is followed by regime shift of community 2 in backward direction.

Figure 6.8: (Color online). The effect different socio-ecological characteristics to cooperation hysteresis driven by variation of resource inow ($0 \leq c \leq 50$) with interaction network with 2 communities ($N = 100$, $\langle k \rangle = 15$, $\mu = 0.2$) of different characteristic: (a) The effect ostracism strength difference to the hysteresis in social cooperation. The ostracism strength difference is varied with parameter $\Delta h = 0$ (black square), $\Delta h = 0.02$ (red circle), $\Delta h = 0.05$ (blue triangle). (b) The effect of discrimination to the hysteresis in social cooperation. The payoff differential is varied with parameter $\Delta s = 0$ (black square), $\Delta s = 0.2$ (red circle), $\Delta s = 0.5$ (blue triangle).

6.7 Regime shifts for Two Communities of Different Characteristics

Often two different communities perceive ostracism more seriously and also receive discrimination of favoring one community over other community. We can model differences in ostracism by introducing new parameter, Δh , where the ostracism strength in community 1 is $h + \Delta h$ while the ostracism strength in community 2 is $h - \Delta h$ to maintain the average ostracism strength as h . Similarly, we can model differences in payoff by introducing parameter, Δs , where the payoff of any individual from community 1 will be increased by $1 + \Delta s$ and the payoff of any individual from community 2 will be reduced by $1 - \Delta s$. Δs is an abstract quantity that can reduce or increase the payoff which can be manifested in many

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forms such as taxation preferences, unequal opportunity or access towards certain social or economical rights, and even difference in skill.

The numerical result of this situation shows that double regime shift and hysteresis continue to occur when the communities have different ostracism strengths: community 1 with $h + \Delta h$ and community 2 with $h - \Delta h$ (see Fig. 6.8a); and with payoff differential: community 1 with $1 + \Delta s$ and community 2 with $1 - \Delta s$ (see Fig. 6.8b).

6.8 Social Disharmony

Let us now analyze the features of this new regime in a strongly segregated society. In this regime, a strongly segregated society self-organizes into communities with different social behavior as it exploits a common pool resource. In our case of two communities, it is a community of cooperator and a community of defector. Notably, the latter community is observed to enjoy a higher payoff by free-riding on the former community which serves to maintain the ecological resource even though its payoff has been dampened by the defective act of the latter community. The defective act has not only significantly reduced the ecological resource, it has also created an inequality of payoff between community 1 and 2 (Inset of Fig. 6.3). We examine this social aspect more closely by quantifying the social disharmony within the society. We differentiate between two types of social disharmony: internal and external. Internal (external) social disharmony is measured by the number of internal (external) CD links divided by the total number of intra- (inter-) community links. It is interesting that internal social disharmony is low in this SC regime where cooperation has collapsed in one community but maintained in the other. While the external social disharmony is high, the low internal social disharmony has enabled this society to sustain cooperation in one community further than that of a fully integrated society as the system is being driven externally. Note that such a disparity in disharmony is absent in a fully integrated society (see Fig. 6.9).

Figure 6.9: (Color online). Variation of internal (red line), and external (blue line) disharmony against source inflow ($0 \leq c \leq 60$) with network ($N = 100, \langle k \rangle = 45$) of two communities with $\mu = 0.2$. Also shown is the social disharmony for $\mu = 0.5$ (black circle)

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6.9 Conclusion

In summary, this chapter has demonstrated the dynamical behavior of social cooperation in an integrated or segregated society that exploits a common pool resource for economic production. In particular, we have discussed our results for a two-community society; for a society with more than two communities; with different mean degree; and even when the communities possess heterogeneities and are non-identical. In these diverse circumstances, we observe the occurrence of multiple step-sized regime shifts in both the forward and backward directions, and also multiple hysteresis loops. Similarly, social cooperation, resource utilization, and societal payoff, which are optimal on average when the society is integrated, are worse off under social segregation in the presence of multiple communities. Nonetheless, the trend is not downwards all the way. Beyond a certain degree of segregation, we again observe the emergence of new stability regimes. These regimes possess social cooperativeness at the community level which serves to boost up these indicators at the societal level, albeit at a social cost of greater inter-community conflicts. Moreover, these regimes provide a cushion against the drastic consequences that may happen during the critical transition between the extreme regimes.

7

Empirical Data Analysis

7.1 Introduction

The result from previous chapter shows the existence of multiple stable regimes in coupled socio-ecological system due to its nonlinear responses to perturbations. However without any empirical evidence, it is difficult to justify whether this model is valid or not. Since common pool framework is general, the result from the model should be observed in any socio-ecological system with renewable resources such as irrigation system. Different system may have differences in detail such as the dynamics of the resources, social norms, etc. However since the general mechanism is similar, we may expect nonlinear feature of the model such as the existence of alternative regimes can be observed in many similar real-world coupled socio-ecological system. In this chapter we will show that the results in previous chapter can be observed in real-world coupled socio-ecological system: the Balinese rice terraces (subak).

A subak is an agrarian society of farmers who exploit water, which is a common pool resource, for irrigation and rice production activities. It is a self-organized society without centralized governance, and its social and ecological context fulfill the basic requirements of our model (93). Our model is applicable to explain Balinese subak data for several reasons. First, this model is well suited for common pool type of socio-ecological system such as irrigation. Traditional Balinese village itself is largely dependent on its social cooperation on irrigation water management for their economic survival. Second, this model is incorporated

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with social network structure. Therefore it can be used to study socially segregated community based on caste or class in subak society. Third, this model can describe the multiple stable regimes along with socio-ecological characteristic for each stable regime. In traditional Balinese subak, all farmers keep adjusting their behavior to social and ecological evolution. Different villages have evolved independently and have their own cultural characteristics. After a long period of socio-ecological evolution, we can expect each subak resides in different stable regime with different socio-ecological characteristics. Therefore our model is suitable to study of Balinese subak.

If we compare with the alternative approaches in chapter 3 section 3.8, our model is found to be more appropriate in relation to Balinese subaks. Subak is a unit of coupled socio-ecological system. Therefore we cannot use the model that reduces everything into mere social interactions(135, 179) or mere ecological interactions(77, 78, 107). Coupled nonlinear models (57, 182) are also inadequate to describe the highly complex interdependent interaction such as subak. Moreover, many coupled models(57, 165, 182) do not incorporating social network structure which make it impossible to analyze the structural effect such as social segregation on socio-ecological stability and characteristic. In this context, our model is more compatible with Balinese subak as a socio-ecological system.

In an earlier pilot study, Lansing *et al* used principal component analysis to analyze survey data from 83 farmers in 8 subaks, which experience similar social and ecological conditions, but respond to them in different ways corresponding to the cooperator-dominant and defector-dominant regimes in the model (95). In this thesis, we use a larger set of data than before. Our data was obtained from farmers of 20 different subaks in the Bali island. It is based on surveys whose queries can be qualitatively reduced to $M = 36$ descriptors. It consists of responses from about 20 to 30 farmers of each subak, giving a total of $N = 493$ farmers surveyed. To analyze and classify the socio-ecological regimes in the data, we employ the method of principal component analysis. Moreover, Hierarchical Clustering method is also used to uncover the important descriptors and reduce the dimensionality of the data.

Before analyzing survey data, we shall describe the nature of this data. The data is obtained by surveying farmers from different subaks. Single subak can

be seen as a unit of a coupled socio-ecological system. In this survey, N farmers were asked about their opinion on M questions regarding specific socio-ecological condition. Therefore, different question captures different socio-ecological variable, for instance: caste, water excess, social conflicts, and many more. Later we describe specific M socio-ecological variable as descriptors. A very detailed description of descriptors and subaks are described in the Appendix.

The data can be written in the $N \times M$ matrix form. There are N rows representing N objects, and M columns representing M descriptors. The data matrix is given as follows:

$$A_{N,M} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,M} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,M} \end{pmatrix}.$$

where $a_{i,j}$ represents the opinion of farmer i regarding descriptor j . For instance $a_{20,23}$ indicates the opinion of farmer 20 regarding descriptor 23 (social conflicts). If this value is high, it means that the farmer perceives a lot of social conflicts in his subak. On the other hand, if this value is low, it means that farmer perceives few social conflicts in his subak.

7.2 Principal Component Analysis

Analyzing multivariate data can be very challenging because of its large dimensionality. Principal Component Analysis (PCA) is a statistical procedure that helps us to analyze multivariate data(98). It utilizes the singular value decomposition (svd) on the z-scores of the data matrix.

In section 7.1, we have discussed that opinion data can be stored in matrix form, A_{ij} . Next, we can perform basic statistical analysis such as calculating the average and standard deviation on this matrix. The mean value of farmer opinion about descriptor j can be written as:

$$\overline{A_j} = \sum_{i=1}^{i=N} A_{ij}. \quad (7.1)$$

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While, the standard deviation of the farmer opinion about descriptor j can be written as:

$$\sigma_j = \sqrt{\sum_{i=1}^{i=N} \frac{(A_{ij} - \overline{A_j})^2}{N}}. \quad (7.2)$$

Before performing PCA, the data matrix is first standardized into its z score:

$$Z_{ij} = \frac{A_{ij} - \overline{A_j}}{\sigma_j}. \quad (7.3)$$

Since we handle various type of data with different dimension, standardization is necessary to avoid skewed result from some wide range descriptors. Z score standardization is a common method in PCA to rescale the data. Next, we calculate the $M \times M$ correlation matrix

$$R = \frac{1}{N} Z^T Z. \quad (7.4)$$

Then we perform eigen decomposition of R by means of singular value decomposition:

$$|R - \lambda_i I| = 0, \quad (7.5)$$

$$R = E \Lambda E^T. \quad (7.6)$$

where E is eigen-vector matrix and Λ is eigen-value matrix. Reduced eigen-vector matrix is then constructed by using the first and second eigen-vector $U = (E_{i1} E_{i2})$ and the reduced matrix represents the first and second principal component.

Note that z_{ij} is the z-score coordinates of data i on the normalized descriptors j and u_{ij} is the principal component j with vector component i . Let f_{i1} be the projection of data i on the first principal axis

$$f_{i1} = z_{i1}u_{i1} + \cdots + z_{iM}u_{M1} = \vec{z}_i \cdot \vec{u}_i. \quad (7.7)$$

Then, the position of all data with respect to the system of principal axes is given by the matrix component of scores F :

$$F = ZU. \quad (7.8)$$

This process is similar to rotating the descriptor's coordinate into the principal component coordinate (The distance between data is preserved during the rotation). In a similar way, we can project the data of each subak into the reduced principal coordinate. The data of a subak itself is defined by the average value of their farmer opinion data in a particular subak:

$$\overline{A_j} = \frac{1}{N} \sum_{i=1}^{N_{subak}} A_{ij} . \quad (7.9)$$

From the projection of subak data on reduced principal coordinates, we can analyze the location of particular subak on principal coordinates. The distance between two different subak data on principal coordinate represents how similar their socio-ecological characteristic. If the distance is very large, it indicates those subaks have very different socio-ecological behavior. More precisely, the data points from the same regime are to be close together because they are socio-ecologically similar, while data points from different regimes are to be far apart because they are socio-ecologically different. By using this analysis, we can investigate the existence of alternative regimes and which subaks are located on particular regime.

Furthermore, other than the data points, we can also project the descriptors (original axes) on the reduced principal coordinates (biplot). We then draw arrows to these points which represents the descriptor axes on the principal coordinates. The angle between two descriptors represent the correlation between them. Therefore we can analyze the socio-ecological characteristic of specific regime and also correlation between socio-ecological variables through the direction of the arrows and the angle between them.

7.3 Data Analysis

By performing the set of procedures on the data matrix, we found that the first and second principal component of the PCA explain only 48.5% of the data, although there are clear clustering of the several subaks at the principal axis (Fig. 7.1). Moreover, there is no clear correlation pattern among descriptor arrows in

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the biplot (Fig. 7.2). From section 7.2 we have discussed that the subaks distribution on the principal axis can be used to identify the socio-ecological regimes and the descriptor arrows can be used to identify socio-ecological characteristic of such regime. By removing descriptors that are relatively insignificant and independent by means of hierarchical clustering, we can improve the PCA result and better understand the socio-ecological behavior of this system. Thus, the strategy is to discriminate the irrelevant descriptors from the more relevant ones.

Figure 7.1: (Color online). The projection of original subak data on the first and second principal component axis. There are 20 subaks descriptors being projected to the principal axes. Here we list 20 subaks on the figure and we color them based on their location on principal components. Blue color: 1. aban, 2. bayad, 4. calo, 5. cebok, 6. jasan, 7. kebon, 8. kedisan kaja, 9. kedisan kelod, 12. pakudui, 13. sebatu, 15. dukuh, 16. tegan, 17. tampuagan hilir, 18. tampuagan hulu, 19. teba, 20. timbul; red color: 10. kulub atas, 11. mantring; green color: 3. betuas, 14. selukat.

Figure 7.2: (Color online). The projection of original descriptors on the first and second principal component axis. There are 36 original descriptors being projected to the principal axes. Here we list 36 descriptors on the figure in counterclockwise direction. From 0 to 90 degree there are: 3. sharecrop, 6. sold, 35. class, 19. self water excess, 18. water excess, 5. purchased, 29. fines frequency, 10. born. From 90 to 180 degree there are: 11. condition of cannal, 9. satisfaction, 12. condition of sawah, 26. cropping patterns, 8. harvest, 36. resilience, 17. follow decision, 13. synchronize, 2. owned. From 180 to 270 degree there are: 28. reading rules, 30. condition of subak, 16. religious participation, 24. subak head election, 27. collective work, 25. fines, 4. inherited, 16. religious participation, 14. attend meeting, 1. number of farmers. From 270 to 0 degree there are: 20. pest frequency, 32. technical problem, 21. self pest frequency, 34. caste, 23. social conflicts, 22. theft frequency, 33. social problems, 7. income from sawah.

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Basically, hierarchical clustering should uncover x distinct clusters from a reduced set of m descriptors. To find the reduced descriptors, we proceed to cluster descriptors by grouping and reordering them by using hierarchical clustering on correlation matrix R . Correlation matrix R is shown in Fig. 7.3 in the form of descriptor correlation map. Descriptor i and j is correlated if most farmers answer both questions with similar answer. Refer to equation 7.4 for the mathematical description of correlation matrix. If two descriptors are strongly correlated, its box will be represented by red color. On the other hand, if two descriptors are not correlated, its box will be represented by blue color. The process begins with a seed cluster consisting of a pair of most strongly correlated descriptors. We then consider complete-linkage distance between this seed cluster and the rest of the descriptors. The descriptor that is closest to the seed cluster based on the complete-linkage distance criterion is incorporated into the cluster. This process is repeated to grow the cluster, until all relevant descriptors have been incorporated. The descriptor correlation map is then reordered to reveal the grouping structure of the descriptors. The reduced set of descriptors are then chosen based on the correlated grouping in the reordered distance correlation map (see Fig. 7.4). Our analysis found 19 reduced descriptors, and by using these descriptors, the first and second principal component can now account for more than 72% of the data.

We then proceed to plot a biplot based on these 19 descriptors (see Fig. 7.6). Instead of plotting all descriptor arrows, we group the descriptor arrow according to its similarity and plot the group mean arrow and standard deviation cone (the spread of arrows from the same group). By plotting in such a way, we can see clearly three clusters of descriptors in the biplot, which we term regime C, D and S. Regime C contains the most numbers of correlated descriptors with many of them relate to cooperation such as synchronize pattern, obey meeting decision, follow meeting, attend meeting, work participation, condition of subak, inherited farmland, collective work, reading rules, fine, choose subak head. Regime D corresponds to defection. The relevant descriptors here are converse to cooperativeness and mainly associated with resource abundance such as water excess, self water excess, sharecrop, class. Regime D is anti-correlated with regime C which implies that regime C is linked to resource shortfall. Regime S is the regime of

Figure 7.3: (Color online). Descriptor correlation map by using opinion data of subak level in the order of original descriptors. The color represents how correlated two descriptors in terms of their correlation matrix.

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Figure 7.4: (Color online). Descriptor correlation map by using opinion data of subak level in the order of descriptors after hierarchical clustering. The color represents how correlated two descriptors in terms of their correlation matrix.

7.4 Projection of Empirical Data to the Model

social disharmony and is observed to be uncorrelated with regimes C and D. It has descriptors in the social domain such as social conflicts, social problem, caste problem, and water stealing frequency.

Similarly, we can match each subak with the correlated descriptors according to their position in the principal axis. Dukuh, Tampuagan Hilir, Tampuagan Hulu, Jasan, Tegan, Aban, Teba, Sebatu, Pakudui, Kedisan Kaja, Kedisan Kelod, Kebon, Calo, Cebok, Bayad and Timbul, are found to locate in the direction of the cooperation descriptors. The subaks Betuas and Selukat are located in the direction of resource descriptor, while Mantring and Kulub Atas in the direction of social descriptor. The grouping of subak points on the principal coordinate can be seen in figure 7.5. Each grouping can be associated with specific descriptors which also represent specific socio-ecological regimes. This finding implies that each subaks can be classified into three different regimes with different socio-ecological characteristics.

Interestingly, these results show clear evidence of a good match between the features of our theoretical model to the key social-ecological characteristics of the subaks. For example, regime C is associated with the cooperative regime in our model since the descriptor analysis indicates the same characteristics of strong cooperation and low resource utilization. On the other hand, regime D is linked to the defective regime in lieu of high resource utilization and low cooperation. Finally, regime S corresponds to the semi-cooperative regime with descriptors analysis indicating frequent social conflicts, moderate cooperation, as well as moderate resource utilization. We find interesting connection here between social conflicts and caste, which illustrates the consequence of the underlying social segregation. Thus, there is strong consistency in the results from the subaks and those from the theoretical model.

7.4 Projection of Empirical Data to the Model

Next, we show specifically our approach in associating each subaks to its regime with respect to our model. To do this properly, we process the data to obtain the frequency of each subak in attaining the particular descriptor. This is achieved by defining an appropriate threshold for each descriptor and then counts the

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Figure 7.5: (Color online). The projection of subak data based on reduced descriptors on the first and second principal component axis of the biplot. Here we list 20 subaks on the figure and we color them based on their location on principal components. Blue color: 1. aban, 2. bayad, 4. calo, 5. cebok, 6. jasan, 7. kebon, 8. kedisan kaja, 9. kedisan kelod, 12. pakudui, 13. sebatu, 15. dukuh, 16. tegan, 17. tampuagan hilir, 18. tampuagan hulu, 19. teba, 20. timbul; red color: 10. kulub atas, 11. mantring; green color: 3. betuas, 14. selukat.

Figure 7.6: (Color online). The projection of reduced descriptors on the first and second principal component axis of the biplot. Here we classify 19 descriptors based on their correlation. The direction of the arrow gives the mean direction of the clustered descriptors, while the colored cone indicates one standard deviation of the angular spread of the corresponding descriptors. Blue (large cone) is for cooperative descriptors (synchronize pattern, obey meeting decision, follow meeting, attend meeting, work participation, condition of subak, inherited, collective work, reading rules, fine, choose subak head); green (small cone) for resource descriptors (water excess, self water excess, sharecrop, class); and red (medium cone) for social disharmony descriptors (social conflicts, social problem, caste problem, and water stealing frequency).

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number of farmers' response that goes beyond the threshold. The fraction of response above the threshold shall give the frequency measure of that descriptor with respect to the subak. After which, we average the frequency measures of the relevant descriptors to represent the three variables of the model: resource inflow, fraction of cooperators, and social disharmony, respectively. This would allow us to map each subak to the regime determined analytically from our theoretical model. For cooperation, the relevant descriptors are synchronized cropping, meeting attendance, and work participation. In the case of resource inflow, it is water-shortage at the subak and individual farmer level, which are related to resource availability in the subaks. Social disharmony is captured by the descriptors: caste, social conflicts, and social problem.

In order for the data to be comparable to the model, we need only to normalize and scale the resource inflow for it to lie within the same range as the model, since fraction of cooperators and social disharmony are already in the same probabilistic form as the model. With this, we are ready to superimpose each subak on the c - P_c - ρ_{CD} plot together with the analytical solution based on a segregated network of $k = 45$ and $\mu = 0.2$ (see Fig. 7.7). The results show a clear clustering of the 20 subaks into three regimes. In particular, 16 subaks: Dukuh, Tampuagan Hilir, Tampuagan Hulu, Jasan, Tegan, Aban, Teba, Sebatu, Pakudui, Kedisan Kaja, Kedisan Kelod, Kebon, Calo, Cebok, Bayad and Timbul, are found to cluster near to the cooperative regime. The subaks Betuas and Selukat cluster at the defective regime, while Mantring and Kulub Atas cluster close to the regime of social disharmony. Thus we are able to map each subaks to their respective regimes based on this analysis.

Figure 7.7: (Color online). Close correspondence of three clusters of subaks to the three regimes indicated by the analytical curves: Cooperation (sphere); Disharmony (pyramid); and Defection (cube). Note that ρ_{CD} here is the fraction of external CD link.

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8

Social Mechanism and Phase Diagram

8.1 Introduction

In previous chapters, we have studied the emergence of cooperation through local interaction by using the equity driven mechanism. The behavior and stability of socio-ecological system is analyzed under the effect of network structure and external control parameter by means of the ostracism driven mechanism. The effectiveness of equity driven ostracism based on the fear of society disapproval towards norm violators has been demonstrated without being inflicted any cost to the norm conformist. Society disapproval may result in exclusion from social protection, help, and other benefits.

While there are several empirical evidences of ostracism mechanism in CPR system, punishment and enforcement are also often used as common mechanisms to promote cooperation. A lot of inappropriate behavior are reduced through punishment and enforcement. However, although enforcer can promote cooperative behavior, there is a cost incurred in adopting such a strategy. In other words, the enforcer needs to pay a price in order to enforce the social norm for the sake of maintaining the ecological resources.

In this chapter we shall study the socio-ecological system based on a different approach. We shall use the phase diagram approach from statistical physics to

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compare the dynamical differences between equity driven ostracism and voluntary enforcement mechanism.

8.2 Ostracism Mechanism

First, we discuss on the establishment of cooperation through the ostracism mechanism, where the defectors will be suppressed by social pressure from neighboring cooperators. The simulation is performed until the whole system has converged to a statistically stationary state. To reveal several possible stable phase, we have studied the equilibrium across different values of the socio-ecological parameters. Each phase is characterized after the composition of the survival strategies have reached the stationary distribution. To simplify the analysis, we have only chosen two different representative parameters, resource inflow and ostracism strength as our ecological and social control parameter. A variation of the resource inflow alters the payoff of every agent as a result of the temptation to defect. A change in the ostracism strength modifies the social pressure to subdue the defector.

The numerical data and analytical approximation suggest the existence of both continuous and discontinuous phase transitions between phases represented by line between phase regions. In both the phase diagrams in Figs. 8.1 and 8.3, we have shown our analytical prediction of phase transitions (solid and dotted lines) on its numerical result (colored region). Solid lines represent continuous phase transitions while dotted lines represent discontinuous phase transitions. The analytical approximation is taken from our solutions based on pair approximation of the socio-ecological stability in chapter 4 (section 4.6.3). The continuous lines indicate the stable solution in the parameter space for $P_c = 0$ and $P_c = 1$ because the intersection between stable solution and line $P_c = 1(P_c = 0)$ represents the continuous transition from purely cooperators (defectors) phase to mixed phase. Whereas, discontinuous lines indicate the intersection between stable and unstable solution in the parameter space which also represents the points where one of its solution disappear. The pair approximation prediction for the phase transition is in good agreement with the numerical result.

We start with the highly connected social interaction ($\langle k \rangle = 45$, $N = 50$). The phase diagram of this situation reveal the existence of 4 phases, purely co-

Figure 8.1: (Color online). Phase diagram of socio-ecological system in high degree network ($\langle k \rangle = 45$) by using ostracism mechanism to promote cooperation. x-axis represents the resource inflow, while y-axis represents the ostracism strength. Solid lines represent continuous phase transitions and dotted lines represent discontinuous phase transitions. The image is smoothed by using spatial median.

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operators, purely defectors, mixture between co-operators and defectors, and bi-stable state (see Fig. 8.1). These 4 phases occupy different region in the phase diagram depending on the value of the resource inflow and ostracism strength. In this figure, the cyan (blue) region represents pure cooperators (defectors) phase. The red region represents mixed phase where the stable solution consists of certain proportion of cooperators and defectors. The yellow region is unique because it represents bistable phase where we can find 2 stable solutions.

In general, we have two dynamical effects as we vary the two control parameters. First, raising the resource inflow would reduce the fraction of co-operators. Second, raising the ostracism strength would prevent defector invasion. Raising (lowering) the resource inflow is similar with tracing the phase diagram horizontally from left (right) to right (left). This horizontal movement is represented in figure 8.2a in which we plot the analytical solution of P_c when parameter c is altered. Furthermore, raising (lowering) the ostracism strength is similar with tracing the phase diagram vertically from bottom (top) to top (bottom). This vertical movement is represented in figure 8.2b in which we plot the analytical solution of P_c when parameter h is altered

Figure 8.2: Analytical stable and unstable solutions along variable c and h . It is similar as crossing the phase diagram (a) horizontally along variable c for $h = 0.1, h = 0.34, h = 0.45, h = 0.7$, (b) vertically along variable h for $c = 10, c = 20, c = 50$

In the case of low ostracism strength, by increasing the resource inflow, the system will experience the continuous phase transition from total cooperation to a coexistence of cooperation and defection before finally become total defection (refer to figure 8.2a for $h = 0.1$). In this situation, the social pressure is too weak to defend against defector invasion, so that the fraction of co-operators is continuously reduced for each increment of c . We have more interesting situation where the ostracism strength is strong enough to create bi-stable effect. In this situation, by increasing resource inflow, the defector invasion will be damped by the neighboring co-operators through the ostracism mechanism. However, the system will experience discontinuous critical transition resulting in a collapse of cooperation beyond the critical point. Once the transition takes place, the system cannot be reversed to the previous state without reducing the resource inflow beyond another critical point since there are not enough co-operators to ostracize the defector. This hysteresis behavior indicate the existence of two stable state in the system between two critical points (This hysteresis phenomena can be seen in figure 8.2a for $h = 0.34$). Both stable states consist of cooperators and defectors mixture, however one stable state has more co-operators than the other and vice versa. In the case of very high ostracism strength, the point of discontinuous transition point lies beyond the point of continuous transition point indicated (see figure 8.2a $h = 0.7$). In this case, the system would collapse directly either to the pure cooperator phase or the pure defector phase.

Now let us discuss about the situation of increasing and decreasing the ostracism strength (vertical movement on the phase diagram). In the case of low resource (e.g. $c = 10$), the cooperativeness increases and decreases continuously by varying the ostracism strength. However, in the case of intermediate resource and beyond (e.g. $c \geq 30$), there is an interesting bistable phase. When the whole society collapses into defectors phase, the cooperativeness cannot be revived by varying the ostracism mechanism (see figure 8.2b for $c = 50$), because no matter how strong the cooperation, there are always not enough cooperators to perform effective ostracism. The best way to escape from defector phase is by reducing the resource inflow (horizontal path in the phase diagram). This phenomena also show that varying one parameter is more effective than varying other parameters in order to escape from the basin of attraction (in this case defector

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regime). Thus understanding which parameter to vary will greatly improve our management on CPR system. Furthermore, since the social system is coupled to the ecological resource, any alteration in cooperation will also affect the amount of resource whether it is gradual reduction or sudden collapse. The change in cooperation and ecological resource will also alter the total payoff in a particular society.

After discussing individual slices on the phase diagram, we now have better understanding about this phase diagram as a whole. We can observe in phase diagram (figure 8.1) that bistable region and monostable region are separated by discontinuous transition (dashed) line. Beyond this line, one stable solution is suddenly disappear and the system jumps into another stable solution. However we may also observe the continuous transition (solid) line intersect this region. Actually, this line represents the boundary between pure and mixed phase in one of its stable solution. Because there are two stable solutions, one solution can be mixed equilibrium while another solution can be pure equilibrium. Therefore within this yellow (bistable) region we can find several possible bistable situations (refer to figure 8.2 to see the stable points) which are: pure cooperator with pure defector ($c = 60, h = 0.7$), pure cooperator with mixed equilibrium ($c = 40, h = 0.7$), pure defector with mixed equilibrium ($c = 60, h = 0.45$), and both solutions are mixed equilibrium ($c = 30, h = 0.34$). Because of this fact, the bistable area spans several regions in the analytical prediction. Every time we cross the continuous transition (solid) line within the bistable area, either one of its stable solutions changes from pure to mixed strategy or mixed to pure strategy. Therefore in specific region, the system may experience regime shift into pure phase, while in other specific region, the system may experience regime shift into mixed phase.

Moreover, altering the network connection will change the configuration of the phase diagram. In the case of lesser social interaction, the size of bi-stable phase becomes smaller which indicate a reduction in the width of hysteresis curve (see Fig. 8.3). This phenomenon is directly related to the ostracism mechanism since network structure affects the configuration of neighboring co-operators. In low degree network ($\langle k \rangle = 5$), any disappearance of a co-operator will greatly reduce the social pressure towards neighboring defectors. Similarly, any appearance of

Figure 8.3: (Color online). Phase diagram of socio-ecological system in low degree network ($\langle k \rangle = 5$) by using ostracism mechanism to promote cooperation. x-axis represents the resource inflow, while y-axis represents the ostracism strength. Solid lines represent continuous phase transitions and dotted lines represent discontinuous phase transitions. The image is smoothened by using spatial median.

a co-operator will greatly strengthen the social pressure towards neighboring defectors. Therefore, whether we decrease or increase the temptation, the system will follow the same path (no hysteresis, no bi-stable state) unless the ostracism strength is very strong to yield bi-stable effect. This explains the reduction in size of the bi-stable phase.

8.3 Voluntary Enforcement

In this section we explore a more complex (3 strategies) system where cooperation is established through voluntary individual enforcement. In this case, we have two social variables (cost and sanction) that determine the effectiveness of individual enforcement. Similarly, we will analyze the system through the phase diagram which is defined by two representative parameters. Our ecological and social parameter in this case are resource inflow and sanction respectively. By varying resource inflow, we alter the temptation to defect. On the other hand, a variation of sanction changes the enforcement effort to subdue the defector invasion. The cost of enforcing is kept constant, $h_c = 0.02$. In our 3 strategies system, there are 8 possible dynamical phases: pure cooperator; pure defector; pure enforcer; mixture between co-operator and defector; mixture between co-operator and enforcer; mixture between defector and enforcer; mixture between co-operator, defector, and enforcer; disequilibrium dynamics.

Similarly, we start with a highly connected network ($\langle k \rangle = 45$). In the case of low sanction and low resource inflow, the defector utility is too low and nobody will become a defector. In this regime, there is a coexistence between co-operators and enforcers. When there is no defector, the utility of co-operators and enforcers is the same which makes both strategy indistinguishable. Therefore, this situation is similar to a pure cooperator regime. Since increasing the resource inflow will increase the defector utility, at a certain critical point some defectors will emerge to challenge the cooperator-enforcer (CE) dominance. Since the sanction is too weak, the appearance of defector cannot be prevented by existing enforcers. In high degree network, it is very likely for the defectors to be connected to the enforcers. Moreover, during the phase transition, each defector gives additional cost to the neighboring enforcers and causes them to be converted to either a cooperator or a defector. Therefore, there were to a coexistence of cooperators and defectors in this new regime. If we were to increase the resource inflow even further, the utility of defector will surpass the cooperators and force all the cooperators to become defectors.

In the case of high sanction, the punishment from enforcer is strong enough to reduce the neighboring defector utility significantly. However, the enforcer must

Figure 8.4: (Color online). Phase diagram of socio-ecological system in high degree network ($\langle k \rangle = 45$) by using voluntary enforcement mechanism to promote cooperation. x-axis represents the resource inflow, while y-axis represents the ostracism strength. Solid lines represent the continuous phase transitions. The image is smoothed by using spatial median.

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Figure 8.5: (Color online). Evolution of strategies caused by voluntary enforcement mechanism on high degree network ($\langle k \rangle = 45$). Blue color represents fraction of cooperators, red color represents fraction of defectors, green color represents fraction of enforcers. Each figure represents the same phase coordinate ($c = 50, h_s = 0.12$) with different initial condition, (a) $P_c(0) = 0.35, P_d(0) = 0, P_e(0) = 0.35$, (b) $P_c(0) = 0, P_d(0) = 1, P_e(0) = 0$.

also pay additional cost for each neighboring defector. If the system is dominated by defector, the enforcer is not able to overcome the dominance of the defector because they need to pay a high cost in enforcement. On the other hand, if the system is dominated by enforcer, the defector temptation also cannot contend with the total punishment from neighboring enforcers. This kind of situation creates bi-stability, in which both defector dominance and enforcer dominance are both stable. The time evolution of these two different stable equilibrium can be seen in figure 8.5. The bi-stability region in parameter space is quite large. However, the size of this region shrinks and the cyclical dynamical phase emerges as we reduce the network degree.

Now let us consider the low degree connection ($\langle k \rangle = 5$). As before, we have phase transition from a total cooperative regime to a total defective regime in the case of low sanction. For high sanction, the situation is more interesting. Increasing the resource inflow will increase the temptation to defect. However, the punishment from enforcer toward neighboring defector is strong enough to

counter this temptation. In the end, we have coexistence phase between co-operator, defector, and enforcer. If we increase the resource inflow even more until the critical point, the system will enter an interesting region on the phase diagram. In this region, the temptation is strong enough to lure the co-operator to become the defector. Therefore the number of defector will be majority in the system. However, the sanction from the enforcer is strong enough to reduce defector utility and the number of defector will be decreased because the enforcers start to gain control of the situation. During this process, cooperators can sneak in to replace the enforcers because the utility of cooperator is larger than enforcer since the enforcer's utility is reduced by the cost to punish the defector. As the number of enforcer reduces, the punishment becomes weaker and the population of defector grows and dominates the system. This cyclical situation is known as dynamical stable disequilibrium. The time series of 6 different phases in low degree situation are displayed in figure 8.7 with cyclical behavior illustrated in figure 8.7f.

In the phase diagram, we have also included our analytical estimation on the points of phase transition (appear as a line in the diagram). The detailed analysis of this approximation is discussed in the next section. Three strategies dynamics is too complex to be solved. However, we can reduced significant amount of complexities by considering only the specific behavior of boundary transition between one phase with another phase. Generally, it will give us the continuous or discontinuous solutions and these solutions are quite accurate only for particular transitions. Similar with the previous section, The continuous lines in both $\langle k \rangle = 45$ and $\langle k \rangle = 5$ indicate the stable solutions in the parameter space for the situation of $P_c = 0, P_d = 1, P_e = 0$, or $P_c = 1, P_d = 0, P_e = 0$, or $P_c = 0, P_d = 0, P_e = 1$ depending on the transition boundary we want to find. The discontinuous lines in both $\langle k \rangle = 45$ and $\langle k \rangle = 5$ indicate the intersection between stable and unstable solution in the parameter space.

8.4 Analytical Approximation

The analytical approximation for 2 strategies using ostracism mechanism has been discussed in chapter 4. In this chapter, we have re-used the equilibrium

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Figure 8.6: (Color online). Phase diagram of socio-ecological system in low degree network ($\langle k \rangle = 5$) by using voluntary enforcement mechanism to promote cooperation. x-axis represents the resource inflow, while y-axis represents the ostracism strength. Solid lines represent the continuous phase transitions. The image is smoothened by using spatial median.

solutions given by equations 3.1 and 4.8. These equations are derived based on the pair approximation relation of equation 4.19. With this approximation, we obtain a set of stable and unstable fixed points. We plot the continuous transition (intersection between the stable solution and $P_c = 0$ ($P_c = 1$)) and discontinuous transition (intersection between stable and unstable point) on the phase diagram to compare the analytical prediction with the numerical results. The intersection between stable solution with $P_c = 0$ ($P_c = 1$) is the transition point between defector (cooperator) phase to mixed phase. The intersection between stable and unstable point represents fold bifurcation point. Beyond this point, that stable and unstable point collides and annihilates therefore the whole system will collapse to another stable point. The abrupt nature of this transition point represents the discontinuous transition.

In this section we shall focus more on the analytical approximation of 3 strategies system. By using equation 2.9, we can obtain the dynamical evolution of each strategies:

$$\frac{dP_c}{dt} = (T^+(P_c) - T^-(P_c)), \quad (8.1)$$

$$\frac{dP_d}{dt} = (T^+(P_d) - T^-(P_d)), \quad (8.2)$$

$$\frac{dP_e}{dt} = (T^+(P_e) - T^-(P_e)). \quad (8.3)$$

The transition probability can be obtained from each specific updating mechanism:

$$\frac{dP_c}{dt} = -P_c \left[q_{d|c}(u_d - u_c)/u_{max} + q_{e|c}(u_e - u_c)/u_{max} \right], \quad (8.4)$$

$$\frac{dP_d}{dt} = -P_d \left[q_{c|d}(u_c - u_d)/u_{max} + q_{e|d}(u_e - u_d)/u_{max} \right], \quad (8.5)$$

$$\frac{dP_e}{dt} = -P_e \left[q_{c|e}(u_c - u_e)/u_{max} + q_{d|e}(u_d - u_e)/u_{max} \right], \quad (8.6)$$

which can be expanded as,

$$\frac{dP_c}{dt} = -P_c \left[q_{d|c} \left(\pi_d - \pi_c - h_s \sum_{k_e}^{(k)} B_{k_e}(q_{e|dc}) \langle k \rangle_e \right) + q_{e|c} \left(-h_c \sum_{k_d}^{(k)} B_{k_d}(q_{e|dc}) k_d \right) \right] / u_{max}, \quad (8.7)$$

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$$\frac{dP_d}{dt} = -P_d \left[q_{c|d} \left(\pi_c - \pi_d + h_s \sum_{k_e}^{\langle k \rangle} B_{k_e}(q_{e|dc}) k_e \right) + q_{e|d} \left(\pi_c - \pi_d - h_c \sum_{k_d}^{\langle k \rangle} B_{k_d}(q_{d|ed}) k_d + h_s \sum_{k_e}^{\langle k \rangle} B_{k_e}(q_{e|de}) k_e \right) \right] / u_{max}, \quad (8.8)$$

$$\frac{dP_e}{dt} = -P_e \left[q_{c|e} \left(h_s \sum_{k_e}^{\langle k \rangle} B_{k_e}(q_{e|de}) k_e \right) + q_{d|e} \left(\pi_d - \pi_e + h_c \sum_{k_d}^{\langle k \rangle} B_{k_d}(q_{d|ed}) k_d - h_s \sum_{k_e}^{\langle k \rangle} B_{k_e}(q_{e|de}) k_e \right) \right] / u_{max}. \quad (8.9)$$

Note that binomial distribution is defined as

$$B_i(q) = \frac{\langle k \rangle!}{i!(\langle k \rangle - i)!} q^i (1 - q)^{\langle k \rangle - i}$$

and the first moment is given by

$$\sum_{i=0}^{\langle k \rangle} \frac{\langle k \rangle!}{i!(\langle k \rangle - i)!} q^i (1 - q)^{\langle k \rangle - i} i = \sum_{i=0}^{\langle k \rangle - 1} \frac{\langle k \rangle(\langle k \rangle - 1)!}{(i - 1)!(\langle k \rangle - i)!} q^{(i-1)} (1 - q)^{(\langle k \rangle - i)} q = \langle k \rangle q$$

In general, it is difficult to obtain the single exhaustive solution that can be applied to any situation. However, we are only interested on the boundary transition between one phase with another phase. Therefore, we can simplify the system by focusing on the specific particular phase transition.

First let us simplify the system by considering the situation where there is no enforcer. In this simplified system, we can analyze the interaction between cooperator and defector only to obtain the boundary between C and CD phase, and also between CD and D phase. This situation is captured by equation, $0 = \pi_d - \pi_c = e_d(F/E - w) - e_c(F/E - w)$. The solution of this equation is represented by two vertical line in the phase diagram as the boundary between C and CD phase, and also between CD and D phase.

After that, we assume there is only few enforcer and few defector in the midst of co-operator majority. This situation represents the boundary between CE phase with CDE phase. Here we assume the enforcer is only connected to one defector and the defector is only connected to one enforcer because they are very few. This situation is captured by equation, $0 = (\pi_c - h_c) - (\pi_d - h_s)$. The solution of this equation is represented in the phase diagram by the curve line as a boundary between CE and CDE phase.

Next, let us introduce small portion of enforcer in the midst of cooperator-defector majority. This situation represents the boundary between CD phase with CDE phase, and also between D phase with CDE phase. The enforcer cannot invade the co-operator because both have same payoff while the enforcer payoff is reduced to pay the cost for the neighboring defector. The enforcer can only invade the defector because defector payoff is reduced by sanction from enforcer. The interaction between these two strategies is represented by equation, $0 = (\pi_c - (k-1)q_{d|e}h_c) - (\pi_d - h_s)$. Here we assume the defector is only connected to one enforcer because they are very few and the enforcer is surrounded by $(k-1)q_{d|e}$ defector. The solution of this equation is represented in the phase diagram by the curve line as a boundary between both CD and D phases with CDE phase.

Finally, we want to estimate the boundary of disequilibrium phase. Near to this phase, the number of defector and cooperator is very few. Therefore we assume the enforcer only connect with one defector and the defector connect with $(k-1)q_{e|d}$ enforcer. This interaction is represented by equation, $0 = (\pi_c - h_c) - (\pi_d - (k-1)q_{e|d}h_s)$. The solution of this equation is represented in the phase diagram by the curve line as the outer boundary of disequilibrium phase. Please note that there is a discontinuous phase transition in the lower boundary.

By using all these estimations for each specific situation, we can approximate the phase transition boundaries in the phase diagram for 3 strategies system. Specifically, we plot the continuous transition (intersection between the stable solution and $P_c = 0$ ($P_c = 1$)) and discontinuous transition (intersection between stable and unstable point) on the phase diagram for each particular phase transition to compare the analytical prediction with the numerical results. This comparison has been discussed in the previous section.

We can also use this analytical approximation to understand the mechanism behind disequilibrium dynamics (refer to figure 8.8). Without lost of generality let us consider specific case with parameters: $c = 50$ and $h_s = 0.12$. We first plot the stable and unstable solution of disequilibrium boundary along parameter c by fixing parameter $h_s = 0.12$ (represented by blue curve). Please note that y axis is P_e , where $1 - P_e = P_d + P_c$. The stable state for $c = 50$ is marked with a circle in position 1 and the system now is in CDE phase. As we know, the cooperator payoff is higher than enforcer because the enforcers pay additional

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cost to punish the surrounding defectors. Therefore, we can expect slowly some enforcers become cooperators. Here we also plot the stable and unstable solution when 20% of enforcer become cooperator (red curve). This new curve shows that previous curve is shrinking as the number of enforcer is reduced. The stable state for $c = 50$ is marked with a circle in position 2. We can see that eventually the system will exceed the bifurcation point and collapse to total defection (marked with position 3). After collapse, the system is now within the range of DE region and will equilibrate to its stable solution (green curve). The stable state for $c = 50$ is marked with position 4 and the system now is in DE phase. Since there is no co-operator, the shape of previous red curve returns to original blue curve. The system is now above the unstable fixed point of blue curve, therefore the system will return back to original CDE phase which is marked with position 1.

8.5 Discussion

By studying the dynamics of two different social mechanisms, we can compare the strengths and weaknesses of these two mechanisms. In high degree network, we observe that both mechanisms have similar effect in preventing the uncooperative behavior by creating bi-stable equilibrium. However bi-stable region comprises hidden systemic risk, in which any external pressure may force the system beyond the unstable point and shift the whole system to the alternative stable state. Once the system experience the regime shift, it cannot be easily revert back to the original situation. This shift can be very abrupt and disastrous which in the literatures is known as a regime shift. Moreover, both mechanisms are ineffective in preventing uncooperative behavior when the ostracism or sanction is not strong enough.

As the degree is reduced, the bi-stable phase region shrinks. Since the bi-stability is the dynamical basis for regime shift to occur, the abrupt transition becomes a continuous transition when the bi-stable region disappears. This phenomenon is observed in the case of ostracism mechanism. For the case of voluntary enforcement, the disappearance of bi-stability is accompanied by the appearance of a new type of dynamical phase. This new phase consists of a coexistence between co-operator, defector, and enforcer, and also constant changes in their

proportion. This dynamical behavior is difficult to control due to its disequilibrium nature.

A comparison between the outcome of different social mechanisms may reveal new insights to manage the socio-ecological system. Different social mechanism or social structure should be treated differently because of their unique effects on the social dynamics. Moreover, understanding several dynamical phases can improve our control on the socio-ecological system. Our analysis here provide awareness on the type of phase transition that can happen under different environmental setting. Harnessing these phase transitions may help us to achieve better system management and control by promoting the good transition and avoiding the bad transition.

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Figure 8.7: (Color online). Evolution of strategies caused by voluntary enforcement mechanism on low degree network ($\langle k \rangle = 5$). Blue color represents fraction of cooperators, red color fraction of defectors, green color fraction of enforcers. Each figure represents different phase coordinate, specifically: (a) $c = 20, h_s = 0.04$ (CD), (b) $c = 100, h_s = 0.02$ (D), (c) $c = 0, h_s = 0.2$ (CE), (d) $c = 80, h_s = 0.1$ (DE), (e) $c = 50, h_s = 0.2$ (CDE), (f) $c = 50, h_s = 0.1$ (CDE-disequilibrium).

Figure 8.8: (Color online). Analytical approximation for the fraction of enforcers as the function of resource inflow in low degree network ($\langle k \rangle = 5$) by using voluntary enforcement mechanism. Blue curve is the solution of the equation, $0 = (\pi_c - h_c) - (\pi_d - (k - 1)q_{e|d}h_s)$. Red curve is the solution of the equation, $0 = (\pi_c - h_c) - (\pi_d - (k - 1)0.8q_{e|d}h_s)$. Green curve is the solution of the equation, $0 = (\pi_c - (k - 1)q_{d|e}h_c) - (\pi_d - h_s)$. The circle represents the state of the system at particular situation.

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Conclusion

In this final chapter, let me summarize all the essential results to give a general overview on the research outcome of this thesis. The main purpose of the thesis is to investigate the phenomenon of regime shifts in coupled socio-ecological systems. The motivation is to find out how the form of social connections among individuals in a society affects their level of cooperation as they depend on a common pool ecological resource. The interaction between social action and ecological resource utilization is interdependent and there is mutual feedback between them. Since there is no centralized government, the whole society needs to cooperate spontaneously to preserve the natural resource. The emergence of cooperation is thus self-organized through local interaction and the underlying social mechanism. To study this problem, we have introduced a coupled socio-ecological model (chapter 3) by extending and modifying the Tavoni-Schüler-Levin model. Inspired by the behaviour of real-world coupled socio-ecological system, we have incorporated specific social interaction network topology to mimic real-world social interactions under reasonable assumption. Our model is realistic as it considers the necessary details and essential components of actual social-ecological systems. While it is an agent-based model, it is simple enough to be analytically tractable. Moreover, it can reproduce the basic stable regimes and social-ecological characteristics of a real-world social-ecological system: the Balinese subaks. Such a correspondence between theoretical model and empirical data has yet to be achieved in the field of coupled socio-ecological systems.

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In order to understand the observed macroscopic socio-ecological phenomena, we need to study the consequence of the underlying microscopic interactions and processes in the model. In this respect, we examine into the effect of network structure on the phenomena of regime shift and hysteresis in chapter 4. Here, we discuss how the average degree of the social interaction network alters the stability structure of the coupled socio-ecological system. The model shows the presence of regime shift as a control parameter is varied beyond a critical value. This regime shift points to the existence of two stable regimes in the system, which is the cooperative and the defective regime. Moreover, simulations also show the path dependency and hysteresis effect between the two regimes. Numerical results indicate that lowering the network degree reduces the hysteresis effect. Moreover, the occurrence of alternate stable states disappears when the interaction degree is very low. These results indicate that dense social interactions lead to a greater social inertia to change while coupled to an ecological resource, for example from the cooperative regime to the defective regime and vice versa. Our results are further validated through analytical approximation by employing reasonable assumptions based on the framework of statistical physics and non-linear dynamics to reveal the regimes of stability and instability within the system. We have extended our study to the effect of scale-free social interactions where hysteresis behavior is continued to be observed in the coupled socio-ecological system.

Regime shift is typically catastrophic and the ability to pre-empt its occurrence would safe guard the sustainability of our natural ecological system. Hence, in chapter 5, we explore into the use of diverse generic early-warning signals to predict the occurrence of upcoming regime shift. To achieve this, we have employed the data generated from our model. By increasing the control parameter slowly, we enable the system to approach a regime shift. The resulting data allows us to test diverse early-warning indicators such as the auto-correlation, variance, skewness, and kurtosis. In order to determine the accuracy of each indicator and to avoid false detection, we compare the trend to a null hypothesis where a control parameter is kept constant. We compare the accuracy of early warning detection of social-ecological regime shifts based on both temporal and spatial approaches. Our results show that spatial early warning indicators are better in

distinguishing between real and false signals. Despite its strengths, the information needed to construct spatial early warning indicator is more challenging in real-world situations.

In chapter 6, we investigate into another form of social interactions among the agents which signify the presence of social segregation. Social segregation is common in society and it could arise due to race, class, caste, language etc. In the context of our modeling, it appears in the form of community structure in the complex social interaction network. It is interesting that such community structure modifies the regime shift and hysteresis in coupled socio-ecological system in an unexpected way. Our numerical results show that in addition to the two regimes of cooperation and defection at the societal level, intermediate stable regime also exists with cooperation and defection at the community level. The new regime occurs midway between the two classic regimes and is associated with free-riding behavior at the community level. Specifically, it demonstrates the phenomenon of communities of defectors free-ride on communities of co-operators while all members of the society exploit the common pool ecological resources. In such a society, there are multiple regime shifts caused by the existence of the additional stable regimes between the cooperative and defective regimes. The number of additional stable regimes as well as the number of possible regime shifts depends on the number of communities in the social interaction networks. We have validated our numerical results via theoretical analysis by means of the pair approximation to construct solutions of the stable regimes. While multiple stable regimes have been discussed in the literature, the demonstration of its existence analytically has yet to be achieved in current research. One of the significant outcome in our research here is the uncovering of social disharmony between communities in the intermediate stable regime, which have important implications within real-world coupled social-ecological systems.

The relation between the intermediate stable regime and its occurrence in real-world socio-ecological system is observed empirically in chapter 6. The empirical data was obtained through surveying the farmers of the Balinese subak. By means of partial hierarchical clustering and principal component analysis (PCA), we have uncovered three groups of correlated descriptors. Interestingly, results from PCA indicate the presence of three clusters which is consistent with the

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three stable regimes obtained from the segregated model in chapter 6 as we apply it to the empirical data. Moreover, the associated descriptors of each cluster from the empirical analysis also show good correspondence to the socio-ecological characteristics of the respective regime in the model. Such an agreement between empirical observation and theoretical modeling narrows the gap in current socio-ecological research since current modeling effort tends to be independent of data analysis with minimal connection between them.

In chapter 8, we have expanded our theoretical model in order to investigate the effect of different social mechanisms on the emergence of cooperativeness in coupled social-ecological system. In particular, we have compared the social dynamics of the systems when the mechanism is either that of ostracism or voluntary enforcement by means of the phase diagram. By varying a social and an ecological parameter, we analyze the pattern of behavior in terms of parameter region based on both numerical and analytical perspectives. From our analysis, we found interesting similarity and dissimilarity between the dynamical behavior of these two social mechanisms. While we found phases of cooperation, defection, mixture of cooperation and defection, and also the bistable phase under these two mechanisms, the voluntary enforcement mechanism gives a new phase of cyclical behavior that is absent in the ostracism mechanism. In addition, there are also interesting transition scenarios between the different phases: the transition could be either continuous or discontinuous. It would be interesting if such phases of social dynamics can be found in real world system that operates based on similar mechanism. The elucidation of such real world system will be left as a topic for future study.

In summary, we have presented an integrative study on coupled social-ecological systems in terms of its stability regimes and the critical transitions between these regimes. These regimes correspond to the social behavior of cooperation and defection, and also the ecological regime of resource abundance or resource shortfall. Depending on the topology of the network of social interactions, the social-ecological system can display multiple stable regimes, where transition between these regimes represents the evolution between stable societal states in the system. In this sense, our model provides a rich platform for the exploration of diverse social-ecological scenarios. While we have performed these explorations

based on computational simulation, analytical calculation and empirical observation, we have also connected the results from our theoretical model to real-world empirical data from Balinese subaks. In consequence, our integrative approach has shown promise in bridging the gap between empirical data analysis and theoretical modeling effort. In this way, I believe the research in this thesis has made substantial contribution to our understanding on multiple regime shifts and the occurrence of multiple stable regimes in socio-ecological system, and has thus served to bring progress to this field.

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Appendix A

Description of the Descriptors

In this appendix, we will describe the detail information about the data. In total, there are 36 descriptors: 'number of farmers', 'owned', 'sharecrop', 'inherited', 'purchased', 'sold', 'income from sawah', 'harvest', 'satisfaction', 'born', 'condition of cannal', 'condition of sawah', 'synchronize', 'attend meeting', 'work participation', 'religious participation', 'follow decision', 'water excess', 'self water excess', 'pest frequency', 'self pest', 'theft frequency', 'social conflicts', 'subak head election', 'fines', 'cropping pattern', 'collective work', 'reading rules', 'fines frequency', 'condition of subak', 'follow meeting', 'technical problems', 'social problems', 'caste', 'class', 'resilience'.

Moreover, there are 20 subaks/villages: 'Aban', 'Bayad', 'Betuas', 'Calo', 'Cebok', 'Jasan', 'Kebon', 'Kedisan kaja', 'Kedisan kelod', 'Kulub atas', 'Mantring', 'Pakudui', 'Sebatu', 'Selukat', 'Dukuh', 'Tegan', 'Tampuagan Hilir', 'Tampuagan Hulu', 'Teba', 'Timbul'.

Here we are listing associate question of each particular descriptors. All questionnaires below are designed by anthropologist Stephen J Lansing.

Number of farmer: Number of subak members

Owned: Amount of sawah you farm now, owned by me ...
hectares

Sharecrop: Amount of sawah you farm now, I sharecrop ... hectares

Inherited: Have you inherited sawah? A=1=yes, B=0=no

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Purchased: Have you purchased sawah? A=1=yes, B=0=no

Sold Have you sold sawah? A=1=yes, B=0=no

Income from sawah: ... Your income from sawah, compared to your total family income, is: A 100%, B 66%, C 50%, D 25%, E 10%

Harvest: Your last harvest of beras (unhulled rice)...kg/ha

Satisfaction: Your level of satisfaction with this last harvest? A very satisfied 4, B satisfied 3, C less satisfied 2, D not satisfied 1

Born: 11. Were you born in this village? A=Yes=1, B=No=0,

Condition of cannal: ... The condition of your subaks irrigation canals now, compared to the usual in Bali: A very good 4, B good 3, C average 2, D not good 1

Condition of sawah: The condition of your subaks sawah now, compared to the usual in Bali: A very good 4, B good 3, C average 2, D not good 1

Synchronize: How many subak members follow (a decision to) plant at the same time? A All 100% coded as 1, B 0.85, C 0.5, D. less than half, coded as 0.4

Attend meeting: How many subak members (regularly) attend every subak meeting? A more than 80% coded as .85, B between 60-80% coded as .70, C less than 60% coded as .5

Work participation: How many subak members regularly participate in each collective work activity such as working on the canals? A more than 80% coded as .85, B between 60-80% coded as .70, C less than 60% coded as .5

Religious participation: How many subak members regularly participate in religious activities carried out by the subak, like offerings? A more than 80% coded as .85, B between 60-80% coded as .70, C less than 60% coded as .5

Follow decision: How many subak members usually follow (obey) the decisions of the subak head and the subak meetings? A All coded as 1, B most coded as .75, C half .50, D less than half .25

Water excess: Are water shortages frequent in the subak during the dry season? A never coded as 4, B seldom coded as 3, C sometimes 2, D Frequent 1

Self water excess: Do you yourself frequently experience water shortages? A never coded as 4, B seldom coded as 3, C sometimes 2, D Frequent 1

Pest frequency: Are there frequent pest infestations? A never coded as 1, B seldom coded as 2, C sometimes 3, D Frequent 4

Self pest: Do you yourself frequently experience pests? A never coded as 1, B seldom coded as 2, C sometimes 3, D Frequent 4

Theft frequency: Is water theft frequent in the subak? A never coded as 1, B seldom coded as 2, C sometimes 3, D Frequent 4

Social conflicts: Are (social/personal) conflicts frequent among subak members? A never coded as 1, B seldom coded as 2, C sometimes 3, D Frequent 4

Subak head election: ... Selection of subak head A=veneer of democracy coded as 0, B=democratic/just (ie fair) coded as 1

Fines: Fines A=veneer of democracy coded as 0, B= democratic/just (ie fair) coded as 1

Cropping pattern: Choice of cropping pattern A=veneer of democracy coded as 0, B= democratic/just (ie fair) coded as 1

Collective work: Organization of collective work A=veneer of democracy coded as 0, B= democratic/just (ie fair) coded as 1

A. DESCRIPTION OF THE DESCRIPTORS

Reading rules: Reading and following written rules of the subak
A=veneer of democracy coded as 0, B= democratic/just (ie fair) coded as 1

Fines frequency: Are fines frequently imposed in your subak? A never
coded as .01, B seldom coded as .5, C often coded as 1

Condition of subak: The condition of your subak now is: A Excellent,
still intact coded as 5, B Good enough coded as 4, C some problems have
begun 3, D not good 2, E Bad 1

Follow meeting: Are the results of subak meeting usually followed
and carried out by subak members? A Always coded as 3, B mostly coded
as 2, C seldom coded as 1

Technical problems: Are there frequent technical problems in the subak,
like water shortages, pests and low production? A seldom coded as 1, B
sometimes coded as 2, C frequently coded as 3

Social problems: Are there frequent social problems in the subak, like
conflict among members, water theft, failure to work together or to care
about religious rites in the subak? A seldom coded as 1, B sometimes
coded as 2, C frequently coded as 3

Caste: In your opinion, is there a connection between the
strength/capability of the subak and caste conflicts within the subak? A
seldom coded as 1, B sometimes coded as 2, C frequently coded as 3

Class: In your opinion, is there a connection between the
strength/capability of the subak and differences in the level of prosperity
or poverty of subak members? (Note: we refer to this as class) A seldom
coded as 1, B sometimes coded as 2, C frequently coded as 3

Resilience: the capability of the subak members to overcome
difficulties, whether technical or social, is A Very capable 3, B Capable 2,
C not very capable 1

Here we are also listing the number of farmers that answering the survey in each particular subak:

Aban:		26 farmers
Bayad:		26 farmers
Betuas:		27 farmers
Calo:		26 farmers
Cebok:		26 farmers
Jasan:		26 farmers
Kebon:		27 farmers
Kedisan Kaja:		26 farmers
Kedisan Kelod:		26 farmers
Kulub Atas:		26 farmers
Mantring:		21 farmers
Pakudui:		26 farmers
Sebatu:		26 farmers
Selukat:		29 farmers
Dukuh:		27 farmers
Tegan:		26 farmers
Tampuagan Hilir:		21 farmers
Tampuagan Hulu:		21 farmers
Teba:		27 farmers
Timbul:		27 farmers

A. DESCRIPTION OF THE DESCRIPTORS

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