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NANYANG TECHNOLOGICAL UNIVERSITY

Radio Engineering of Relay-Based Cellular Networks

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Summary

Data traffic demand in today's cellular networks is growing very rapidly, and operators are looking for cost-effective solutions to increase the capacity and coverage of their networks. Relay nodes have been proposed as one of the candidate solutions to achieve this target. Briefly, a relay node is a network device which is in charge of forwarding the data received from its reference base stations to a number of controlled mobile users, and forwarding the data transmitted by these mobile users to the base station. In this way, the base station can be partially offloaded, and the spatial density of network access point is increased. A similar result could be obtained by deploying more base stations. However, relays may be preferred by network operators, because they are smaller and cheaper than base stations, and they should hence allow for increasing the density of network stations without an exorbitant increment of the costs related to the deployment of new nodes and the related site rentals. Moreover, relay nodes do not require any backhaul cable, as they are connected to their reference base station via a wireless backhaul link.

However, the deployment of relays entails the creation of a two-hop wireless network. This means, that interference management is more complex compared to traditional cellular networks. Moreover, transmissions on the wireless backhaul link may use a significant quota of the available radio resources, thus reducing the resources available for transmissions to users.

Starting from the expected advantages and disadvantages of relays deployment, this thesis is focused on the performance evaluation and the optimization of the radio layer of relay-enhanced cellular networks. In particular, numerous aspects have been treated, including the application of the fluid network

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model to the modeling of relay networks, the comparison of different candidate scheduling schemes and their impact on performance, the optimization of relays placement in the cell for capacity maximization and the assessment of the energy savings that users can achieve in relay networks.

This thesis analyzes the important tradeoff, between the performance gain due to the higher number of network station, and the performance loss due to the reduced amount of radio resources available at each station for transmissions to/from users. The analysis is carried out in both a static environment, i.e., focusing on a simpler model and ignoring stations buffers queues length and users arrivals and departures, and in a dynamic environment, i.e., considering queuing and arrivals/departures.

The performance of two schedulers, namely the proportional fair and the round robin scheduler, is assessed in the context of relay-enhanced networks, showing that the more complex proportional fair scheduler does not necessarily perform significantly better than a round robin scheduler. This depends on the propagation environment, on the network setup and on the number of relays and users. The study is performed via a novel statistical framework for performance evaluation of relay networks, which allows to take into account fast fading without explicitly simulating it.

Regarding network modeling, a part of this thesis is devoted to the application of the fluid network model to relay networks. The fluid model is a mathematical tool for cellular networks performance analysis, based on approximating a discrete network of transmitters with a continuum. A novel framework for the analysis of relay networks, based on the fluid model, is proposed, and its accuracy is tested in several scenarios. In particular, regular and irregular relay networks are considered, and the fluid model is applied either to the whole network of relays, or only to those relays which are far from the cell of interest. The obtained results allow to assess the accuracy of the fluid model. In particular, they show that the fluid model is accurate when the network of relays is regular, or when it is used to model far relay nodes. In this thesis, the fluid model is also applied to find the optimized relays transmit power. Indeed, the application of

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the fluid model allows to find the borders of the areas where relays are capable of providing a better downlink rate, compared to base stations. This area is then matched as closely as possible with the area where relays provide the highest downlink received power (under the assumption that users are served by the station providing the highest received power), by tuning relays transmit power appropriately. The problem of matching the two areas is solved via a geometrical approach, and an approximate solution is proposed.

A significant part of the thesis is dedicated to relay networks optimization, both on the downlink and on the uplink. For the downlink, a study on the optimal placement of relays for capacity increase is proposed. Indeed, relays placement has a significant impact on network performance, although most of the literature tends to adopt network models where relays are arbitrarily placed. In this thesis, the optimal placement is found under a dynamic system model, where stations buffers queues loads and their impact on interference are taken into account. The adoption of this model improves the accuracy of results, as in a dynamic model interference is not overestimated as, e.g., when all stations are assumed to always have data to transmit. Hence, a more precise evaluation of the mutual interference between relays, and between relays and base stations can be performed. A novel optimization algorithm for relays placement is introduced, based on a multiscale implementation of the simulated annealing algorithm with a number of ad-hoc enhancements, which are described in the text. The obtained results provide useful guidelines for relays placement, and show that the optimal placement takes into account both the need for enjoying a good signal quality on the backhaul link, and that of covering the regions close to the edge of the cell, where users experience the lowest downlink rates.

For the network uplink, this thesis proposes an optimization of the network setup, in order to maximize the energy efficiency of users. Indeed, the deployment of relays shortens the average distance between users and their serving stations. This means that users can transmit to the stations with a lower power, thus reducing the consumption of batteries. Several studies have shown that a pure optimization of network energy efficiency can lead to optimal network

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setups which perform poorly in terms of throughput. Hence, in this work the optimization of energy efficiency is performed under a constraint on the minimum acceptable network performance. A novel optimization framework for uplink energy efficiency is introduced. The search of the best network setup over a very fragmented and difficult configuration space is tackled by means of an exterior search approach, tailored to the studied case. A customized penalty function is proposed and tested, with the aim of increasing the effectiveness of the exterior search. Results show that an optimized relays deployment allows for great users energy savings. These savings are given for different values of the traffic input and number of relays, in several deployment scenarios.

A summary of the main contributions of this thesis is given in Section 1.3.

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List of Main Abbreviations Used

AF or A&F	Amplify and Forward
AL	Access Link
BL	Backhaul Link
BS	Base Station
CDF	Cumulative Distribution Function
CRE	Cell Range Expansion
CSI	Channel State Information
DF or D&F	Decode and Forward
DL	Direct Link
ICIC	Inter-Cell Interference Coordination
ISD	Inter-Site Distance
LOS	Line Of Sight
MCS	Modulation and Coding Scheme
MQS	Maximum Quantile Scheduling
NLOS	Non Line Of Sight
PC	Power Control
PDF	Probability Distribution Function
PF	Proportional Fair
QoS	Quality of Service
RB	Resource Block
RN	Relay Node
RP	Resource Partitioning
RR	Round Robin
RV	Random Variable
SA	Simulated Annealing
SINR	Signal to Interference and Noise Ratio
SIR	Signal to Interference Ratio
SPPP	Spatial Poisson Point Process
UE	User Equipment

Chapter 1

Introduction

1.1 Introduction

The ever-growing demand for broadband mobile internet and on-line wireless multimedia applications, together with the prescriptions of the IMT-Advanced body [1] concerning the performance of fourth-Generation (4G) mobile networks have motivated the development of technical solutions to enhance the throughput and coverage of cellular networks.

According to [1], 4G networks are expected to guarantee high data rates to mobile users (up to several hundreds of Mbit/s) everywhere in the covered area, so as to provide a ubiquitous and uniform service. This target could be achieved by increasing the spatial density of Base Stations (BSs) in the network, thus increasing the quantity of available radio resources per square meter. Nevertheless, in this case network operators would incur a significant increase of their expenses, as BSs installation requires the rental of a dedicated site and the connection via a backhaul link to the network backbone.

For these reasons, the deployment of Relay Nodes (RNs) in cellular networks has been proposed. Essentially, a RN is a network device, which is in charge of forwarding the data received from a BS to one or more users on the network downlink (i.e., transmissions from the BS to the users), while performing the

inverse operation on the network uplink (i.e., transmissions from the users to the BS) (see Figure 1.1). The advantage of RNs compared to BSs is that they

Figure 1.1: a) A traditional cellular network b) A relay-enhanced cellular network.

are functionally simpler. Moreover, they transmit at a lower power, they have a wireless backhaul link and they are smaller. Potentially, RNs can be installed, e.g., on a lamppost, thus reducing operators' site rental costs. Thus, they are deemed as a feasible solution for achieving the performance targets in [1] without incurring into excessive expenses.

However, the deployment of RNs in cellular networks implies the creation of multi-hop wireless networks. Hence, network complexity increases, and the management of radio resources and interference becomes more critical compared to traditional single-hop networks.

The 3GPP [2] standardization body defines RNs as Decode-and-Forward (D&F) devices, i.e., RNs decode and re-encode the received data before forwarding it. However, the scientific literature considers the use of Amplify-and-Forward (A&F) RNs as well. On the one hand, a D&F retransmission scheme is more complex, compared to an A&F retransmission. On the other hand, it is capable of eliminating the effect of noise and interference on the first radio hop, thanks to the full regeneration of the signal by the RN.

Relay nodes are categorized into *in-band RNs*, i.e., part of the available radio resources are dedicated to transmission/reception on the backhaul, and *out-of-band RNs*, i.e., no radio resources are dedicated to the backhaul. These definitions comply with those provided by the 3GPP [2]. Out-of-band relaying

is possible if, e.g., a dedicated narrow-beam antenna is deployed by the RN to communicate to the BS.

The 3GPP standardization body adopts the expression *Evolved Node B* (eNB) to refer to the base station and the expression *User Equipment* (UE) to refer to the mobile user/handset. This thesis adopts the acronym BS for the base station, while the acronym UE denotes the mobile user.

The expression *Backhaul Link* (BL) is used to refer to the link (or set of links) between one or more RNs and the BS. Similarly, the expression *Direct Link* (DL) is used to refer to the link (or set of links) between the BS and the users, and the expression *Access Link* (AL) is used to refer to the link (or set of links) between the RNs and the users (see Figure 3.1, page 39).

1.1.1 Modulation and Frame Structure

Orthogonal Frequency Division Multiple Access (OFDMA) is proposed by 3GPP as the modulation scheme for 4G networks adopting RNs (see [2]). This choice is justified by the fact that the OFDMA modulation is particularly fit for dealing with frequency-selective fading, and by its flexibility in the allocation of resources allowed by OFDMA. Data transmission in OFDMA is carried out by allocating the symbols on multiple subcarriers of narrow band. On each subcarrier, the achieved data rate depends on the conditions of the radio channel at the subcarrier frequency. Good channel quality allows for high data rate transmissions, while a bad channel forces the system to transmit at a low rate on the considered subcarrier. The advantages of using narrow-band subcarriers are manifold, as the corresponding long symbol duration implies a low impact of inter-symbol interference on performance. Moreover, channel fading on one subcarrier can be assumed to be *flat*, thus easing data decoding operations. Figure 1.2 shows a scheme of cellular network based on OFDMA, where two stations A and B are broadcasting their frame on a shared frequency band, and two UEs labeled with 1 and 2 are receiving data from A and B respectively, over the same radio resources. Hence, transmissions towards 1 and 2 are interfering with each other.

Figure 1.2: Downlink of an OFDMA-based cellular networks

Figure 1.3: Generic frame structure for simultaneous mode of operations in in-band relay-enhanced networks

Frame structure design for relay-enhanced networks must take into account the half-duplex nature of RNs, i.e., transmission/reception on the AL and on the BL must be performed on orthogonal radio resources. Hence, the BL and the AL/DL are dedicated two separate parts of the frame. The resulting frame structure is depicted in Figure 1.3. The literature considers both *simultaneous mode of operation*, i.e., RNs and BSs transmit to their users on the same radio resources, and *division mode of operation*, i.e., the transmissions of RNs are performed on orthogonal radio resources with respect to those of BSs.

Resource Partitioning

The Resource Partitioning (RP) policy is the strategy to distribute frame radio resources to the BL and to the AL/DL. Resource partitioning represents a key issue in Radio Resource Management (RRM), and it profoundly affects network performance.

Resource partitioning algorithms for relay-enhanced networks can be classified into two categories: static and dynamic RP algorithms. If a static RP policy is adopted, the quotas of frame resources dedicated to the BL and to

the AL/DL are fixed over the frames (see Figure 1.4). This approach is not optimal in terms of network performance (see [3]), but processing requirements for the BS can be considerably reduced, and interference is less variable [4]. Moreover, transmission overhead for RP-related communications is minimized. If a dynamic RP policy is implemented, the quotas of resources dedicated to BL and to AL/DL change from frame to frame (see again Figure 1.4), according to UEs distribution, radio channel state, etc. In this case, interference is less predictable, but network performance can benefit from the higher flexibility in resources assignment.

Figure 1.4: Static and dynamic RP comparison.

1.1.2 Expected Benefits of Relaying in Cellular Networks

Deployment of RNs in cellular networks is expected to improve network performance in terms of coverage, capacity and energy efficiency.

Relay nodes increase the spatial density of transmitting stations. This leads to an increase of the interference experienced by either the UEs on the downlink or the BSs on the uplink. Moreover, a part of the frame radio resources cannot be used for transmissions to/from UEs, as it is reserved to the BL. However, the increased number of transmitting/receiving stations on the uplink/downlink

decreases the size of cells, and thus leads to a capacity increase. If the channel on the BL is *good enough*, the benefit of a higher number of transmitting stations overcomes the effects of a higher interference and less available radio resources per frame.

Relays can also have a meaningful impact on the energy efficiency of the network, both on the downlink and on the uplink. This is another effect of the higher spatial density of stations, which reduces the average distance between UEs and their serving stations. Hence, BSs and RNs can transmit at a lower power to reach their served UEs with the desired signal strength, and the same holds for UEs on the UL.

1.2 Motivation

The deployment of RNs in relay-enhanced cellular networks is capable of improving network performance, in terms of capacity, coverage and energy efficiency.

However, the complexity of relay networks implies a number of issues for network designers, related to, e.g., the tradeoff between the radio resources dedicated to the backhaul link and those dedicated to the access link, the management of the interference, the tradeoff between energy efficiency and delay, the optimization of network parameters and RNs positions, etc.

This section analyzes the main motivations driving the research presented in this thesis.

1.2.1 Performance of Scheduling Schemes in Relay-Enhanced Networks

The choice between different available schedulers for cellular network nodes is determined by a compromise between the scheduler complexity and its performance. The available literature proposes several studies on the performance of relay networks. However, it does not exhaustively focus on the impact of

scheduling, on the comparison between different scheduling schemes in terms of related performance, and on the impact of propagation parameters on the BL/AL/DL on the performance of different schedulers. This type of study is of great importance in network design, as it allows to define the context where a scheduler performs better than another, and to assess the extent of the performance gain yielded by a complex scheduler, compared to a simpler one. In fact, if the complex scheduler brings little or no performance gain, network designers may opt for a simpler scheduler, e.g., a round-robin scheduler, and reduce the expenses related to network implementation.

Hence, the need arises for a comparative study on scheduling in relay networks, which assesses the relative performance gain of different scheduling schemes, according to a number of BL/DL/AL propagation scenarios, and analyzes the influence of propagation on scheduling gain.

1.2.2 Fast Approximation of Interference and SINR in Relay-Enhanced Networks

Interference and Signal to Interference and Noise Ratio (SINR) computations in relay networks are more complex, compared to the traditional BSs-only networks, due to the increased number of network stations and to the different features of propagation between RNs and UEs, compared to that between BSs and UEs. These computations arise very frequently in studies devoted to, e.g., measuring network performance and searching for the optimal network configuration. Hence, they have a consistent impact on the computational complexity of numerical simulations.

For the BSs-only networks, the literature presents a number of studies on the approximation of interference by means of the application of a fluid network model. The advantage of this approach is that it allows, under some specific assumptions, to compute the average overall interference sum or the average SINR experienced by a UE at a given distance from its serving station with one closed-form formula, without a consistent loss of accuracy.

The application of a fluid model to relay networks may entail a consistent reduction of the computational complexity of network modeling and simulation, because it provides a closed-form formula for approximating the sum of the interference produced by multiple interferers. However, the literature does not cover this possibility, to the best of the author knowledge. Thus, it is necessary to derive a fluid model of relay-enhanced networks and to test its accuracy in approximating the interference sum and the SINR, according to network topology and setup, and to the extension of the network area modeled as a fluid, i.e., the whole network or only a part of it.

1.2.3 RNs Placement

Placing RNs optimally increases downlink network throughput, compared to arbitrary RNs placements or random RNs placements. This is because the average network performance is strongly affected by the mutual interference between network nodes, by the number of cell-edge UEs that can be served by RNs and by RNs backhaul link channel conditions, which all depend on the locations of relays.

The problem of the optimal placement of RNs has already been treated in the literature. However, the existing works mostly focus on a simplified network model, where either the placement is forced to respect a pre-defined pattern, e.g., RNs can be placed only on a circle centered at the BS, or positions of UEs are assumed to be known, or interference is completely neglected or inaccurately estimated by, e.g., adopting a single-interferer model or assuming that all stations transmit at every time instant. This latter assumption may lead the optimization to a strongly suboptimal placement, as it overestimates interference and biases the results.

In view of the existing literature on the optimization of the placement of RNs, it is important to provide an exhaustive study, which analyzes the placement taking into account buffer queue loads at network nodes. In this way it is possible to model network nodes that do not always transmit, i.e., do not always produce interference on the downlink. Hence, the obtained placement is

more accurate, and it provides a deeper insight on the parameters that influence RNs placement.

1.2.4 Uplink Energy Efficiency

Relay deployment produces an increase of UEs energy efficiency, due to the shorter average distance to their BSs, which allows them to transmit at a lower power. However, this may come at the price of a worse performance, because of the additional transmission delay related to the BL of RNs.

Few studies have been devoted so far to the analysis of the uplink energy efficiency of RNs-enhanced networks (which is a different case compared to femtocells deployment, because of the BL). Moreover, these studies present a series of drawbacks, e.g., they neglect the decisive impact of uplink co-channel interference on the uplink transmit power. Moreover, these works perform the energy efficiency maximization without any constraint on UEs performance, while the literature shows that there is a close interaction between performance and energy efficiency, and that a non-constrained maximization of energy efficiency tends to lead to solutions that do not correspond to a good UEs performance.

Hence the need for a study on uplink energy efficiency in relay networks, that analyzes the tradeoff between UEs energy efficiency and UEs performance, and takes into account the impact of interference on performance and network nodes load.

1.3 Summary of Contributions

In this section, the main contributions of the thesis are outlined, according to the related topics.

1.3.1 Scheduling Scheme Comparison

- PF and RR schedulers in relay-enhanced LTE-A networks are compared, by relying on a non oversimplified network model. Effects of fast fading,

multiple interferers, cell range expansion bias, backhaul link quality, relay transmit power and traffic load are taken into account. Moreover, design bounds for the multi-hop network frame structure are proposed. In order to obtain quick results fast fading is not explicitly simulated but its statistical effect is taken into account in performance evaluation. A more realistic interference model is also considered, which brings new insights on RN networks performance and convenience.

- The role of backhaul quality on RN cellular networks performance is studied, by analyzing different backhaul links scenarios. In the out-of-band case, deploying relays is similar to a network densification and therefore leads to a capacity increase. In the in-band case, it is shown that the impact of the backhaul link capacity is not only crucial for the network performance, but also for assessing the convenience to deploy relay nodes.
- In the case of RN networks, the study shows that using PF at relays is more demanding in terms of backhaul resources for in-band relaying. Thus, there are conditions, where the scheduling gain is really small or even negative. This type of study for RN cellular networks is novel, to the best of author knowledge.

1.3.2 Application of the Fluid Model to RNs-Enhanced Networks

- The fluid model is applied for modeling a RNs-enhanced network. Different approaches for the application of the fluid model are proposed, in the context of both regular and non-regular RNs networks.
- A novel approach for the application of the fluid model to a part of the network is introduced. This method allows to overcome some of the drawbacks of the fluid modeling of the whole network, i.e., it accounts for the possibility that a UE be served by a node which is not the nearest one.

- The accuracy of the fluid model in relay networks is assessed by means of network simulations, for the case of regular and irregular networks and for the modeling of a part or the whole network.

1.3.3 Placement of RNs

- Differently from most of the existing literature, a novel RN placement heuristic is proposed, in order to take into account the impact of RNs positioning on performance. Though sub-optimal, this heuristic depends on a single variable, making the optimization process quick compared to traditional meta-heuristic optimization techniques and allowing the analysis of a great variety of scenarios.
- A novel dynamic framework for the evaluation of the performance of RNs placements is introduced, justified by the need to overcome the defaults of widely used static models, included the assumption that users positions are fixed and known. Instead, in this model users positions are unknown, and the analysis is rather based on the spatial distribution of users arrivals. Network stations are modeled as queuing nodes, and their loads are computed with fixed point iterations.
- The fluid model [5] is applied to the network region containing the stations which are far from the cell of interest. This choice is particularly beneficial for the implementation of the RNs placement optimization algorithm, as interference calculations are quicker compared to a traditional discrete network model. Results show that this mixed fluid-discrete network model is valid, as the obtained SINR distribution curves closely match 3GPP LTE-A guidelines.
- A customized simulated annealing algorithm is proposed as a means to find the RNs positions that maximize cell capacity. Indeed, it is necessary to use a stochastic optimization technique, because the number of possible RNs configurations to be tested grows very rapidly with the number of

RNs. The basic SA is enhanced with a number of methods that have been previously used in image processing. The application of these methods to wireless network is however novel, to the best of the author's knowledge.

1.3.4 Uplink Energy Efficiency Maximization

- A framework for the analysis of energy efficiency in uplink relay networks is introduced. This framework considers the dynamic nature of cellular traffic, by means of a model of UEs arrivals and departures at network nodes. Moreover, the loads of BSs and RNs are accounted in the estimation of interference and transmission delays. The propagation model includes shadowing and fast fading, and UEs power control is considered. Overall, this dynamic model is more comprehensive, compared to the existing literature on uplink energy efficiency in heterogeneous networks.
- A non-trivial scheduling scheme is adopted, which represents an approximation of the proportional fair scheduler, and the corresponding probability density function of the SINR of a scheduled UE is derived. This is a novel contribution, to the best of the author knowledge.
- A constrained optimization problem is formulated, with the objective of placing the relays and tuning network parameters so as to maximize the energy efficiency under delay constraints. To solve this problem, an SA algorithm enhanced with exterior search and penalty functions is designed. A discussion on the effectiveness of this approach is proposed.
- The tradeoff between energy consumption and users experienced delay in uplink relays cellular networks is analyzed. This is the first study of this type, to the best of the author knowledge. In particular, the focus of the analysis is on finding the maximum energy saving which can be achieved for a given maximum allowed average delay. Results show that in certain circumstances, relays can help saving energy and reducing delays.

1.4 Organization of the Thesis

This thesis is organized as follows. Chapter 2 contains an extensive literature review about RNs in cellular networks. Chapter 3 introduces a study on the comparison of different scheduling schemes in the context of relay networks, while Chapter 4 is devoted to the application of the fluid model. Chapter 5 presents the optimization of RNs placement with the aim of maximizing cell capacity, and Chapter 6 deals with the maximization of uplink energy efficiency under a performance constraint. Finally, Chapter 7 gives the conclusions of this thesis, and outlines some possible future developments.

Chapter 2

Literature Review

This Chapter contains an extensive literature review about RNs deployment in cellular networks. The papers are grouped according to the topic they address, for the sake of clarity.

2.1 Performance Evaluation of Relay Nodes

Among the early studies on the performance of LTE-Advanced cellular networks with RNs, [6] represents a noteworthy example. In this work, the authors evaluated the extension of cell coverage obtained by deploying RNs close to the cell edge, via numerical simulations. They consider the opportunity of *biasing* the serving station assignment towards RNs, so as to expand relay cell range (this technique is also known as *Cell Range Expansion* (CRE) [7]) and increase the number of UEs served by RNs. Results show that RNs deployment with CRE brings consistent performance improvements. However, the authors have considered only out-of-band relaying, and RNs locations are arbitrarily chosen. [8] focuses on both network coverage and capacity. In this work, RNs are transparent to the user, to which they appear as an BS. They act as a UE when communicating to the BS, which is responsible for the coordination of the multihop transmission and the resource reservation process. Throughput

is computed by means of a closed formula, yielding the highest achievable rate corresponding to a given value of the SINR, and results show that spatial SINR and throughput rate distribution drastically improve, when relays are used, in both coverage and capacity-oriented deployments. The use of RNs for coverage and capacity extension is also analyzed in [9], which proposes for each case a specific arbitrary RNs placement scenario, and derives rate spatial distribution curves.

Capacity and coverage gains due to RNs deployment can be exploited by network designers to decrease BSs spatial density without affecting network throughput. This possibility is studied in [10], which defines the throughput as the 10%-tile of users throughput Cumulative Distribution Function (CDF). The iso-performance curves obtained by the authors highlight a correlation of RNs transmit power with the feasible reduction of BSs density. Indeed, deploying RNs instead of BSs may be economically convenient for network operators, as the cost of RNs is lower than that of BSs. This is confirmed in [11], where the Total Cost of Ownership (TCO) of several RNs deployment solutions is derived. The paper defines a minimum required performance level on the whole cell surface, and compares several iso-performance scenarios. Each scenario represents a combination of number, typology and transmit power of RNs. Capital expenditure (CAPEX), implementation expenditure (IMPEX) and operational expenditure (OPEX) are taken into account in cost evaluation. The highest savings (about 30 %) are achieved using RNs with high and medium power.

[12] addresses the planning and optimization of relay-enhanced cellular networks. It proposes a simplified analytical model for cell capacity evaluation, based on approximating UEs SINR with the ratio of the serving station received power and the dominant interferer received power. Using this approximation, the authors derive the expressions of, e.g., the ideal RNs-BS distance, the mean cell capacity, the cell edge capacity and the optimal number of RNs. Another study based on a dominant interferer SINR model is presented in [13], which analyzes the impact of different relays sites planning approaches on users rate and cell coverage. Analytical formulas for the Probability Distribution Function

(PDF) and CDF of UEs and backhaul rates are derived. Results in both [12] and [13] confirm that RNs deployment yields a consistent performance improvement. In particular, a 60% and 38 % capacity improvement for outdoor and indoor users, respectively, is obtained in [12], while [13] shows a positive linear dependency of coverage on the number of deployed relays. The authors of [12] assume that each RN serves one UE, and no UEs are served by the BS. Hence, the effect of scheduling and cell load on performance is not captured. Moreover, the single-interferer model in [12, 13] may yield inaccurate results, especially, e.g, in an urban scenario, where several interferers may impair useful signal reception. Another possible drawback of [12, 13] is that the authors arbitrarily set RNs positions in the cell. This is a common assumption in papers related to relay-based cellular networks, e.g., [3, 8, 14, 15], [16], [17] and [18], where RNs are uniformly located on a circle centered at the BS. While this assumption may be reasonable when, e.g., few RNs are deployed, it may lead to a strongly non-optimal RN arrangement when the number of RNs grows.

The comprehensive work [16] studies the self-optimization of RN networks, and contains a section on RNs performance. The authors keep the access link propagation parameters constant, while they vary the backhaul link propagation constant, so as to study the impact of backhaul link quality on performance. Results show that relaying is convenient only if the difference between the two propagation constants is higher than a threshold. However, optimization does not consider RNs positioning and CRE. Moreover, performance evaluation is here focused on average cell spectral efficiency, as well as in [12, 19, 13]. Hence, it does not account for the effect of key parameters such as, e.g., cell load and scheduling. The latter is the focus of [20], which introduces a Proportional Fair (PF) scheduling algorithm for multihop OFDMA systems and analyzes its performance. The authors derive the NP-hard PF metric maximization problem. Then, they propose a sub-optimal solution based on separating user routing (i.e. assignment of every UE to a serving node) and resources scheduling, so as to reduce the algorithm complexity. The proposed approach requires the scheduler to know about the SINR of all UEs, as well as in [21], [22], [23]. Such

an approach may be more complex to implement in a real scheduler, compared to the evaluation of the useful received power. Moreover, the paper does not analyze the scheduling gain and its dependency on network parameters (such as cell dimension and number of RNs) and load. Instead, this issue is tackled in [21], which focuses on the impact of PF scheduling on multihop cooperative relay networks, and reports derivations of exact and asymptotic performance measures for AF RNs. These measures are obtained from the symbol error rate and outage probability, which are in turn derived from the analytic expression of the SINR CDF. Results show that the performance gain is proportional to the number of served UEs.

A study on the outage probability in MR networks under PF scheduling is presented in [22]. Exact formulas and upper bounds for outage probability are obtained using the SINR CDF on the backhaul and access link. A similar approach is adopted in [23]. As a result, outage as a function of interferers and users number is plotted. In this work, as well as in [21] and [23], RNs serve the UEs with the best instantaneous SINR or the best instantaneous received signal. This strategy aims at maximizing the overall throughput, but it suffers from a lack of fairness in UEs resources scheduling: UEs with a low average SINR or low useful received signal power, e.g., those far away from their serving station, may be assigned a consistently smaller share of radio resources, compared to users experiencing better average radio conditions.

Non-regenerative relaying techniques are considered in [24], where amplify-and-forward and compress-and-forward relaying schemes are proposed for the network uplink, under the assumption that each UE has a dedicated RN. The authors compare the performance of a system based on the traditional Single-Cell Processing (SCP) at the BS with that of a Multi-Cell Processing (MCP) system. In a MCP system multiple BSs jointly decode the received signals from a given UE, by means of an antenna array at the receiver side. The used network model does not take shadowing into account, and is based on the linear cellular uplink channel described in [25]. The authors derive the sum rate in a 1-tier network, and model the whole system as a 2D linear time-invariant

system, deriving its block impulse response. Starting from this data, the per-cell sum rate of RNs for the MCP and SCP case is derived. A novel distributed compress-and-forward scheme is then proposed for both schemes. Results show that MCP outperforms SCP in all the analyzed scenarios, and its performance is close to the corresponding upper bound. Non-regenerative techniques perform better in MCP system, because the regeneration of the signal at the RN does not allow to fully exploit the potential of MCP.

A number of works in the area of relay-enhanced networks are characterized by a more theoretical approach, and aim at finding general laws for performance evaluation, under simplified assumptions. [26] is an extensive study about the AWGN channel with multiple parallel relays. The paper focuses on the achievable coding rates using decode-and forward, partial decode-and-forward, compress-and-forward and linear relaying. Partial decode-and-forward relaying is a generalization of the well known decode-and-forward strategy. According to this approach, the source splits the message into two independent messages, and then encodes them with a superposition code. The first message is directly decoded at the destination, while the second message is regenerated at the RN before being forwarded. The authors also treat the linear relaying approach, which is an extension of the traditional amplify-and-forward relaying. In the case of linear relaying, the RN forwards a linear combination of the sequence received from the source, exploiting beamforming gains. The adopted network model requires the use of scalar signals in a flat-fading time-invariant channel, with known channel state information at every source and destination. Relay nodes are full duplex, and they transmit and receive in the same band. The channel is represented by a complex scalar following a Rayleigh distribution, and the total transmit power of network stations is assumed to be fixed. The authors derive the upper bound of the capacity for the AWGN channel with parallel relays, by applying the max-flow min-cut theorem. Then, the achievable coding rates and their asymptotic values for an infinite number of relays are obtained for all relaying schemes. Numerical results show that decode-and-forward performs well when the number of RNs or the source-RNs distance

is small. Partial decode-and-forward exhibits a better performance compared to decode-and-forward only when few RNs are deployed and the source-RN distance is high. If there are many RNs, linear relaying performs well, because beamforming gains overcome the effects of noise amplification. Finally, compress-and-forward relaying is convenient when the distance between source and RN is large.

2.2 Radio Resources and Network Parameters Management

The complexity of MR networks and the high number of parameters and variables to be taken into account have motivated the research community to issue a large number of publications devoted to radio resources management and network optimization.

[3] proposes an interesting approach to downlink MR networks optimization. The aim of this work is to set up the network in order to ensure fairness among users, while keeping network throughput as high as possible. The authors assume that every UE can be served by multiple nodes, and every RN can transmit to more than one UE. The fairness is here guaranteed by acting on the length of users buffers queues at each network node. Indeed, stabilization of users buffers queues guarantees a uniform quality of service to every user, as users with poor channel will be dedicated a larger share of resources (more subcarriers or more links). To this purpose, a demand metric for every node-UE link is defined, taking into account the achievable rate and the users buffer queue length of the considered link, and the same measure is defined for the BS-UEs links. Then, the resource allocation problem is formalized as a binary integer linear programming problem. The computational complexity of such problem is prohibitive for high density of UEs or RNs, or for a high number of subchannels. Thus, the authors propose a low-complexity algorithm based on iterations. Results show that the proposed approach guarantees a uniform

throughput with respect to the BS-UE distance. Moreover, fairness grows with the number of RNs.

Another meaningful work on network optimization is presented in [16]. The framework of this paper is based on a queuing model of the network, considering users arrivals and departures. The authors treat both the infinite and the finite BS and RNs buffers cases, and study buffers stability and blocking rate in the former and in the latter case respectively. The paper derives the optimal resource allocation between access and backhaul link under static and dynamic RP. For the case of non-stationary traffic, the authors propose a load balancing algorithm to dynamically adapt resource sharing and zones covered by the RNs to traffic conditions, based on network measurements. To this purpose, they model the dynamic resource allocation as a control problem, and find the optimal controller by means of a tailored Markov Decision Process (MDP) framework, of which they prove the convergence via stochastic approximation techniques. Moreover, in a dedicated section they drop the assumption about the knowledge of traffic distribution, and propose a model-free approach to resource allocation policy design, based on a Partially Observable Markov Decision Process (POMDP), where the controller bases its decisions on the observation of the traffic during a learning phase. It is proved that the proposed policy converges to a local optimum.

2.2.1 The Power Allocation Problem

In [27], the authors optimize bandwidth and power allocation in a cellular system deploying either out-of-band or in-band RNs. The objective is to minimize a weighted power sum functional, under the constraint of providing a minimum uplink and downlink rate on all links (convex minimization problem). The authors show that using bandwidth allocation as an optimization variable consistently improves results. Similarly, [28] proposes a resource and power allocation scheme, with the aim of maximizing system capacity, i.e., users sum rate, while preserving a desired level of fairness in resources allocation. The resulting mixed nonlinear integer programming problem is then transformed into

a nonlinear real programming problem, through relaxation of some constraints. In this way, the authors come up with a simpler concave object function and convex constraints set. Moreover, they propose a fast heuristic to find an approximate solution. Also in [29] the authors study the optimization of power allocation, together with users rate and sub-band allocation; however, this paper differs from [27, 28], as it takes into account imperfect channel estimation at the relay. The target is to maximize the average weighted system goodput, under the constraint of respecting a target outage probability. Similarly to [28], the authors propose a customized algorithm to find the optimal set of parameters. Results show the dependency of system goodput on base stations and relay transmit power, users number and feedback overhead.

2.2.2 The Relay Placement Problem

The placement of RNs has a great impact on network performance, and several papers are dedicated to it. Before arising in the context of cellular networks (and in particular the 4G networks), this problem has been studied in the context of wireless sensors networks [30], WLAN and WiMAX. To a certain extent, RNs optimal placement is similar to the NP-hard [31] facility location problem. However, the mutual dependency of stations loads and the peculiar objective functions represent, among others, two meaningful differences between these two problems.

In [32], the placement of both BSs and RNs is optimized under a capacity constraint. The target is to minimize a given cost function. UEs serving stations are determined so as to minimize the overall cost of the route to the BS. Cost function for every radio hop is proportional to the achievable rate. The authors propose two methods for RNs placement and they compare them.

A simple approach is considered in [33], where RNs are assumed to be placed on a circle centered at the BS. Thus, the only variable to be optimized is the ray of the circle. A similar framework is used in [34] for the cooperative relaying case. Other examples of simplified placement optimization can be found in [35, 36], where the network is composed of one BS that serves a given number of

RNs. The authors of [35] aim at maximizing cell capacity via optimally placing RNs, under the constraint of guaranteeing a minimum bandwidth to each user. Also [37] is based on one network cell, in the context of WLAN networks. The objective function adopted in this paper is the packet transmission time, which is supposed to be minimized. RNs candidate locations are on a discrete grid, and the optimization is carried out by means of a lagrangian relaxation and an algorithm based on iterations. These simplified frameworks can accurately model, e.g., a rural area. However, they may not be effective in dense cellular networks.

The network model in [38, 39, 40, 41, 13] is more complex, and consists of a number of BSs, each of them assisted by multiple RNs. [38] addresses the RNs placement problem via a hierarchical optimization, in the context of WiMAX networks. The authors model the call admission control with a Markov Decision Process, and the network planning by means of a binary integer linear programming. Then, they solve the network planning problem with standard methods. An integer linear programming problem is also solved in [39], which aims at jointly placing BSs and RNs in order to maximize the capacity of the network under a budget constraint. A suboptimal solution is proposed, based on a two stage deployment of RNs and BSs, similarly to [40]. Joint placement of BSs and RNs is also derived in [41], together with transmit power trimming and allocation of users. Optimization is driven by an objective function taking into account both network capacity and installation costs. In this way, the authors come up with a mixed integer non linear program, for which suboptimal solutions based on iterations are proposed.

The presented papers share two consistent drawbacks, as they all model interference inaccurately and they start from the assumption that UEs positions are known by network designers. In particular, interference is not considered at all in [38, 39, 40, 41], while [13] works in a single-interferer framework. Indeed, interference and users positions have a meaningful impact on the optimal locations of RNs, and it is thus desirable to take them into account in the system model.

A more accurate modeling of interference is adopted in [42, 43, 44]. In [42] a fixed point algorithm for RNs placement is introduced, while [43] is based on numerical simulations. Finally, in [44] the authors propose a Simulated Annealing (SA) algorithm. A possible drawback of these papers, as well as [12], is that they assume that all stations transmit at every instant. Hence, they tend to overestimate interference. In fact, this can sensibly affect performance evaluation, and lead to a RNs placement which is not optimal.

2.2.3 Static Frame Resources Partitioning: an Example

[45] proposes a static partitioning policy, which is summarized in the following.

The authors of this study assume that the overall available band W is split into three sub-bands W_1 , W_{21} and W_{22} of bandwidth B_1 , B_{21} and B_{22} , which are dedicated to the AL, BL and DL respectively. The values of B_1 , B_{21} and B_{22} are derived by imposing data rate fairness among UEs served by RNs and BSs, and by imposing that the BL must be able to provide the data rate demanded on the AL.

Indeed, it is desirable that the average rate $\bar{R}_{A,2}$ enjoyed by UEs served by RNs equal that of UEs served by BSs, denoted with $\bar{R}_{A,1}$:

$$\frac{\bar{R}_{A,1}}{\bar{R}_{A,2}} = \frac{\bar{N}_1}{\bar{N}_2}, \quad (2.1)$$

where $\bar{N}_1$ and $\bar{N}_2$ are the average number of UEs attached to BS and RNs respectively.

Moreover, equating the average BL and AL overall rate yields

$$B_1 E[\log(1 + SIR_{AL})] = B_{21} E[\log(1 + SIR_{BL})], \quad (2.2)$$

where SIR_{AL} is the average value of the SIR on the AL, while SIR_{BL} is the corresponding parameter on the BL.

Note that Equation (2.2) is based on the assumptions that the BS has an omnidirectional antenna, and that the path-loss between the BS and each RN is the same.

Finally, the resource partitioning under constraints (2.1,2.2) is given as:

$$\begin{aligned} B_1 &= \frac{1}{(E_{AL}/E_{DL})(f(d_{RN}/D))+(E_{AL}/E_{RL})+1} B, \\ B_{21} &= \frac{E_{DL}/E_{RL}}{(E_{AL}/E_{DL})(f(d_{RN}/D))+(E_{AL}/E_{RL})+1} B, \\ B_{22} &= \frac{(E_{DL}/E_{RL})f(d_{RN}/D)}{(E_{AL}/E_{DL})(f(d_{RN}/D))+(E_{AL}/E_{RL})+1} B, \end{aligned}$$

where $E_{DL} = E[\log(1 + SIR_{DL})]$, $E_{RL} = E[\log(1 + SIR_{RL})]$ and $E_{AL} = E[\log(1 + SIR_{DAL})]$.

2.2.4 Dynamic Frame Resources Partitioning: Two Examples

The authors of [4] propose two examples of dynamic RP. The first RP scheme (which is similar in concept to [45]) is based on estimating the instantaneous channel gain on the DL, BL and AL. The target is to provide the same instantaneous data rate to all UEs at every frame:

$$\frac{C_k^A N_k^A}{N} = \frac{C_k^B N_k^B}{N} = \frac{C^D N^D}{N}, \forall k \quad (2.3)$$

where N_k^A , N_k^B and N^D are the number of Resource Blocks (RBs) dedicated to the access link of RN k , the related backhaul link and the direct link respectively, C is the average RB capacity on a given link and N is the overall number of RBs. In particular, N is given by $N = N^D + \sum_{k=1}^{n_{RN}} (N_k^A + N_k^B)$, leading to the following RP policy:

$$\begin{aligned} N_k^B &= \frac{C_k^A}{C_k^B} N_k^A, \quad k = 1 \dots K \\ N^D &= N - \sum_{k=1}^K \left(1 + \frac{C_k^A}{C_k^B}\right) N_k^A. \end{aligned} \quad (2.4)$$

This approach is fairly simple. However, it may result in a high variability of RP from frame to frame, according to channel conditions, which would make the interference unpredictable and negatively affect the performance. Hence, the authors propose a dynamic RP scheme, based on computing the time average of a given *satisfaction metric*. In this study, the satisfaction metric of a UE is defined as the ratio of its data rate and a *guaranteed bit rate*. The average satisfaction metric at frame t indicated with $Z[t]$. RP is based on maximizing the exponential time-average $\bar{Z}[t] = \frac{T-1}{T} \bar{Z}[t-1] + \frac{1}{T} Z[t-1]$ of the satisfaction metric.

The authors describe the RP computation as a control system, where the control variables are represented by N_k^A and N_k^B , while the system feedback is given by $\bar{Z}[t]$. Control variables depend on one another (see Equation (2.4)). Hence, the authors perform a re-parameterization in order to decouple them, obtaining the following relations:

$$\begin{aligned} N_k^A &= \frac{w_k}{w_0 + \sum_{i=1}^K w_i} N \\ N^D + \sum_{k=1}^K N_k^B &= \frac{w_0}{w_0 + \sum_{i=1}^K w_i} N, \end{aligned} \quad (2.5)$$

where w_k is a RP weight related to RN K , while w_0 is related to the BS.

Then, Equation (2.5) is linearized at $(\tilde{w}_1, \dots, \tilde{w}_K)$, where $\tilde{Z}^D = \tilde{Z}_k^A$, $k = 1, \dots, K$, and the following elements are defined:

$$\delta \mathbf{w}[t] = (w_1[t] - \tilde{w}_1, \dots, w_K[t] - \tilde{w}_K)^T \quad (2.6)$$

$$\begin{aligned} \mathbf{A}_2 = & -\frac{NC^D \tilde{w}_0}{TS(\tilde{w}_0 + \sum_{i=1}^K \tilde{w}_i)^2} \cdot \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix} + \frac{N}{TS(\tilde{w}_0 + \sum_{i=1}^K \tilde{w}_i)^2} \cdot \\ & \cdot \begin{bmatrix} C_1^A (\tilde{w}_1 - \tilde{w}_0 - \sum_{i=1}^K \tilde{w}_i) & C_1^A \tilde{w}_1 & \cdots & C_1^A \tilde{w}_1 \\ \vdots & & \ddots & \vdots \\ C_K^A \tilde{w}_K & \cdots & C_K^A \tilde{w}_K & C_K^A (\tilde{w}_K - \tilde{w}_0 - \sum_{i=1}^K \tilde{w}_i) \end{bmatrix}. \end{aligned}$$

Now, the system can be described as

$$\Delta \bar{\mathbf{Z}}[t] = \mathbf{A}_1 \cdot \Delta \bar{\mathbf{Z}}[t-1] + \mathbf{A}_2 \cdot \delta \mathbf{w}[t-1], \quad (2.7)$$

where $\Delta \bar{\mathbf{Z}}[t] = [\bar{Z}^D[t] - \bar{Z}_1^A[t], \dots, \bar{Z}^D[t] - \bar{Z}_K^A[t]]^T$ and $\mathbf{A}_1 = (T - 1/T)\mathbf{I}$. In order to stabilize UEs satisfaction and achieve fairness among them, $\Delta \bar{\mathbf{Z}}[t]$ and its integral $\Delta \bar{\mathbf{Z}}_I[t]$ should be kept as low as possible. Moreover, resource partitioning variability can be reduced by reducing the time derivative of the weights w_i . This is achieved by setting the weights time derivative as a system input, and updating $\delta \mathbf{w}[t]$ according to the following relation:

$$\delta \mathbf{w}[t] = \delta \mathbf{w}[t-1] + \gamma \mathbf{I} \cdot \mathbf{u}[t-1]. \quad (2.8)$$

Finally, the system can be rewritten as

$$\mathbf{x}[t] = \mathbf{A}\mathbf{x}[t-1] + \mathbf{B}\mathbf{u}[t-1], \quad (2.9)$$

where

$$\mathbf{x}[t] = \begin{bmatrix} \Delta \bar{\mathbf{Z}}[t] \\ \Delta \bar{\mathbf{Z}}_I[t] \\ \mathbf{w}[t] \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0} & \mathbf{A}_2 \\ \gamma \mathbf{I} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \gamma \mathbf{I} \end{bmatrix}. \quad (2.10)$$

The authors adopt a linear quadratic-optimal controller for minimizing

$$J = \sum_{t=1}^{\infty} \left(\sum_{k=1}^K \Delta \bar{Z}_k[t]^2 + \sum_{k=1}^K \Delta \bar{Z}_{I,k}[t]^2 + \rho \sum_{k=1}^K u_k[t]^2 \right), \quad (2.11)$$

while a linear state-feedback regulator is adopted as a controller for (2.9). The proposed framework is capable of stabilizing UEs throughput and mitigating its dependency on the distance from the BS.

2.2.5 Criteria to Assign UEs to Serving Stations

The criterion for the assignment of UEs to RNs or BSs has a non-negligible impact on network performance. In a number of papers, the assignment is independent of RBs scheduling. However, other studies have pointed out that this approach is suboptimal, and they have proposed algorithms for joint UEs assignment and scheduling [3]. The drawback of these algorithms is that they are computationally more complex, compared to independent UEs assignment and scheduling. Joint UEs assignment and scheduling approaches are treated in 2.2.6, while the following literature review is devoted to serving station selection criteria.

In [46] the assignment of UEs to their serving stations is based on maximizing the number of UEs which experience a given Quality of Service (QoS). Both the downlink transmission rate and bit error rate are taken into account in the calculation of the QoS metric. The authors divide the cell into two regions, based on the proximity to the BS. The UEs nearby the BS are served by it, while those which are far away are assigned according to the QoS metric.

Several papers assume that UEs are served by the nearest network node, e.g., [15]. In this case, the areas covered by each node are obtained by means of a Voronoi diagram of the network. This approach simplifies network simulations, relay selection algorithms and theoretical derivations. On the other hand, it is strongly suboptimal, as propagation in a real cellular network is affected by the shadowing phenomenon.

An alternative approach is to associate UEs with the station that provides them with the highest SINR, and it is adopted, e.g., in [47], under the name of *best server policy*. If the best server policy is adopted, part of the UEs in a given Voronoi cell are served by a station which is not the nearest one. Results obtained in [47] show that network performance under best server policy improves with shadowing standard deviation, as stronger shadowing implies greater chances for cell-edge UEs to find a serving station that provides them with a high SINR.

[8] proposes a slightly different policy, where UEs are served by the station providing them with the best SINR, but RNs can be selected only when the 2-hop transmission requires a lower overall amount of radio resources, compared to a single-hop transmission. Similarly, [32] assigns a weight to each UE-RN, UE-BS and RN-BS link, proportional to the inverse of the achievable rate. Then, UEs are assigned to the device corresponding to the smallest cumulative weight on the path between the UE and the BS.

2.2.6 Joint UEs Assignment and Radio Resources Scheduling

The assignment of UEs to network nodes independently of radio resources scheduling results in a simpler network management, compared to joint assignment and scheduling approaches. However, joint assignment and scheduling is the optimal solution, in terms of network performance.

Most joint assignment and scheduling algorithms are based on the constrained maximization of a given quality measure. Their main drawback is represented by the increased computational complexity, due to the fact that the BS must frequently recompute the optimal UEs and resources assignment, and communicate this information to the UEs and the RNs.

A joint routing and scheduling strategy for OFDMA-based networks is proposed in [14]. This work is based on a *dynamic* traffic model, where each UE appears in the network at a given time instant, downloads a file, and then leaves the network. The authors assign a demand metric D to each subcarrier and to each UE-node link, which depends on both the achievable rate and the buffer queue length Q at the considered node:

$$D_{n,m \rightarrow k} = R_{m,k,n} Q_k^m, \quad (2.12)$$

where m , n and k are the indices of the node, subcarrier and the UE respectively, and $R_{m,k,n}$ is the rate that node m can provide to UE k on subcarrier n . The

demand metric is also defined on the BL:

$$D_{n,0 \rightarrow m} = R_{0,m,n} \max_k (Q_k^0 - Q_k^m)^+, m \neq 0, \quad (2.13)$$

where index 0 indicates the BS. This formulation of the demand metric favors the assignment of radio resources to those links that can provide UEs with a high rate or they have a long buffer queue, so as to achieve load balancing and guarantee fairness among UEs.

Resource assignment is derived by maximizing the sum demand D_s , defined as

$$D_s = \max_{\rho, \gamma} \left\{ \sum_{n=1}^N \sum_{m=0}^M \sum_{k=1}^K \rho_{m,k,n} R_{m,k,n} Q_k^m + \sum_{n=1}^N \sum_{m=1}^M \gamma_{0,m,n} R_{0,m,n} \max_k \left\{ (Q_k^0 - Q_k^m)^+ \right\} \right\}, \quad (2.14)$$

where $\rho_{m,k,n}$ is a binary variable indicating the assignment of the n -th subcarrier to the link between UE k and node m , and $\gamma_{0,m,n}$ is another binary variable, which is equal to one when subcarrier n is assigned the the BL between RN m and the BS, and it is equal to zero otherwise.

Simulations results show that the proposed scheme for joint routing and scheduling algorithm performs better, compared to a proportional fair scheduler, and it reduces the dependency of UEs throughput on their position with respect to the server.

2.2.7 Relays for Energy Efficiency

Heterogeneous network deployments are regarded as a promising solution to improve network energy efficiency [48, 49], due to the reduced distance between UEs and serving nodes [50], compared to traditional single-hop networks. In the following, some meaningful papers about network optimization for energy efficiency maximization are described, for both the network downlink and up-link.

Downlink energy efficiency

[51] proposes a joint optimization of power and subcarriers allocation, so as to maximize the energy efficiency of the downlink of a multihop cellular network with AF RNs. In this work the objective function is the ratio between the weighted average spectral efficiency and total network power consumption. Optimization is performed under a total network transmission power constraint. The authors prove that the chosen objective function is quasi-concave. Then, they apply the Dinkelbach's method to find its maximum, thus splitting the problem into a number of simpler parameterized concave programming problems. Each subproblem is then solved via a dual decomposition approach. Results are displayed for a number of test scenarios, and the authors find out that increasing the number of RNs brings an energy efficiency reduction, while placing them close to the BS is energy efficient. Other examples of constrained optimizations are given in [52, 53]. In [52], the authors aim at minimizing downlink power consumption, while guaranteeing a minimum rate to all UEs. The paper derives the expressions of the lower bound of users achievable rates corresponding to a given stations transmit power, and optimization is performed by acting on users subchannels assignment and stations power distribution over the available subchannels. The problem is non-convex, and it is tackled by means of an iterative algorithm. Moreover, the joint subcarrier and power assignment problem is split into two separate subproblems, reducing complexity and allowing for a more effective optimization.

An original approach to network optimization is proposed in [54], which is based on the idea of putting BSs which are lightly loaded into sleeping mode, and transferring their traffic to neighboring cells, exploiting a dynamic RNs-BSs association. In this way, consistent energy savings can be achieved. The paper introduces a model for measuring active and sleeping BSs power consumption. Then, it states the problem of minimizing the overall energy consumption via an optimal choice of BSs-RNs association, under a total available bandwidth constraint. The authors prove that the problem is NP-hard, and propose a heuristic algorithm involving a linear relaxation. Finally, the solution is found

via taboo search, and results show that the proposed scheme can achieve a 40 % reduction of BSs consumption, when the traffic is not high.

Uplink energy Efficiency

So far, the discussion has addressed the downlink energy savings allowed by RNs deployment. However, relays can also be a means to save energy on the cellular uplink (see, e.g., [55, 56]). Indeed, the potential decrease in battery energy consumption for users can be consistent: [57, 58] indicate that uplink transmit power has a consistent impact on UEs overall energy demand.

Hence, heterogeneous networks can be considered as an effective means to extend mobile users batteries duration, and several research works are dedicated to this topic. The authors of [55] and [59] propose a game theory-based framework for the maximization of femtocells uplink energy efficiency, through users transmit powers tuning [55] or radio resources assignment and power control parameters optimization [59]. In [60], closed-form expressions of the Signal to Interference Ratio (SIR) and of the outage probability are derived and used to measure the impact of femtocells coverage and users density on energy efficiency. Both [55] and [59] apply Quality of Service (QoS)-related constraints to the optimization problem, so as to avoid solutions with high energy efficiency but poor performance. Other works in the area of cellular networks, e.g., [61], address this issue by jointly optimizing power and user experienced QoS.

The close interaction between energy consumption and performance [61] makes the problem of energy efficiency maximization in relay-enhanced networks different from that addressed in [55, 59] et al. for the femtocells deployment case; this is because RNs communicate to their serving node through a wireless backhaul link [62], which uses a part of the radio resources and represents a well-known performance bottleneck [63]. Nevertheless, uplink RNs networks energy consumption has received limited attention in the literature.

The authors of [56] address the problem of energy efficiency maximization via optimal assignment of subcarriers and transmit power. They derive the minimum power that UEs have to receive to reach the desired bit rate, and

state the problem in terms of maximization of the sum of UEs energy efficiency, defined as the ratio between users rate and the power received on all subcarriers. To address this complex problem the paper proposes, similarly to [52], a decomposition approach, based on separating the joint power and subcarriers allocation problem into two subproblems. The authors have adopted the Signal to Noise Ratio (SNR) as UEs performance metric, neglecting the effect of uplink interference.

[64] focuses on relays uplink users performance, proposing a joint optimization of their Power Control (PC) parameters. However, it does not consider their energy consumption. The authors of [65] treat the maximization of uplink energy efficiency in relay-enhanced cellular networks, via optimal radio resources allocation [65]. Results obtained in [65] show that RNs deployment allows for a reduction in users power consumption of more than one order of magnitude.

However, users performance in [56] is based on their SNR, thus neglecting interference and its decisive impact. Also, the paper assumes that users are fixed in number and position, and always have data to transmit. Hence, the influence of the traffic intensity and the limitations due to possible overload of nodes are not considered, while a number of studies (see e.g. [66, 61]) show the importance of the traffic load for energy efficiency evaluation.

2.2.8 Inter-Cell Interference Coordination (ICIC)

Interference management in RNs-enhanced network is more complex compared to single-hop networks, as the density of network stations is higher. Hence the need arises for effective interference mitigation methods.

Traditional techniques for interference mitigation are usually based on one of these three approaches: interference randomization, ICIC and interference cancellation. The problem with interference cancellation is that it is capable of reducing the impact of the interference from only one specific interferer, while interference randomization is shown to be of little effectiveness [67]. For these reasons, ICIC is deemed as a suitable technique for mitigating interference in relay networks. In particular, the Fractional Frequency Reuse (FFR) approach

appears to be promising. According to the FFR approach, radio resources are divided between cell-center and cell-edge UEs. Resources for cell-edge UEs are assigned with a higher reuse factor compared to those dedicated to cell-center UEs, thus achieving interference mitigation.

[68] proposes an FFR-based ICIC scheme for multihop networks. According to this scheme, UEs are classified as cell-center or cell-edge UEs based on the preamble received on the uplink. Interference is reduced by applying a different reuse factor to cell-center and cell-edge-dedicated resources, and by transmitting to cell-center UEs with a lower power. Neighboring network sectors are given orthogonal subchannels. Moreover, this work introduces a score-based dynamic scheduling strategy, where the score of a UE on a given subcarrier depends on the required rate, the experienced interference and the path loss. The weight of these parameters in the score function can be changed by the BS according to the desired tradeoff between the overall throughput and the desired QoS. Numerical results show that the proposed scheme outperforms a reuse-1 cellular network, both in terms of throughput for cell-edge UEs and in terms of QoS, especially when there is a high number of UEs in every cell.

The authors of [67] propose another ICIC method for relay networks, based on a Soft Frequency Reuse (SFR) algorithm. The overall band is split into sub-bands, as in [68], and each cell sector is assigned one of the three available bands for cell-edge UEs transmissions. Transmit power is higher towards cell-edge UEs, and scheduling is performed by each station according to a weighted proportional fairness criterion, designed so as to balance resources assignment between DL and BL, and to prevent the buffers on the AL to empty. Numerical results show that this scheme achieves a higher overall throughput and a higher cell-edge throughput, compared to that achieved by means of standard algorithms.

2.3 MAC Protocols for Relay Networks

The deployment of RNs adds complexity to the network MAC layer, and it has thus motivated the research community to propose ad-hoc MAC protocols.

An early work on this subject is represented by [69], where a MAC protocol for cellular multihop transmission is proposed, starting from the HYPERLAN2 (H2) framework. The authors argue that this standard, as well as others, already contains the PHY and MAC functions which are necessary to implement relaying.

However, the literature provides numerous examples of customized relaying protocols for cellular networks. Among them, it is worth mentioning [70], where a novel cooperative relaying protocol to improve UEs data reception is proposed. In this paper, the transmission of messages on the downlink is divided into two phases. During the first phase the BS transmits a message m_1 to the RNs, and the destination UE listens. During the second phase, the RN forwards m_1 to the UE, while the BS synchronously transmits to the UE a second message m_2 . Reception at the UE is performed via a Successive Interference Cancellation (SIC)-based receiver. This is theoretically viable, as multi-user detection is regarded as a candidate technology for receivers design in LTE. The proposed concatenation of direct transmission and multi-hop transmission is shown to perform similarly or even outperform the traditional cooperative transmission and coding.

Two novel MAC protocols, for one-way and two-way relaying respectively, are proposed in [71], and their performance is derived. The one-way protocol is an intuitive extension of the traditional MAC protocols to the case of multihop transmission, and it is based on transmitting data from a source node to a destination node in two phases. During the first phase, the source transmits to the RN using HARQ with Incremental Redundancy (IR). Then, in the second phase the RN uses again HARQ-IR to forward the data to the destination. The two-way relaying protocol is also based on a two-phases transmission. However, in the case of two-way relaying two source nodes can exchange data in both directions with the aid of the RN. In the first phase, source 1 and source 2 transmit their messages to the RN. Hence, the RN receives two superimposed messages, one from source 1 and the other from source 2. Then, in the second phase the RN broadcasts the two superimposed messages, which will be received

by both source 1 and source 2. Each source knows its transmitted message, which can then be canceled from the received signal to obtain the message transmitted by the other source.

The two protocols are compared by analyzing their performance in terms of energy consumption, while considering a transmission delay constraint. Results show that relay networks can be less energy efficient than a single-hop network, if the delay constraint is tight. However, if the delay constraint is loose or the overall transmit power is low, RNs strongly improve network energy efficiency.

[72] introduces two further MAC protocols, in the context of cooperative RNs. The paper addresses the problem of a source node transmitting to one destination node via multiple relays. According to the first protocol, based on HARQ and cooperative beamforming, the source node broadcasts its message to all RNs, and the RNs that decode the message correctly are elected as candidates for transmission on the second hop. The candidates send pilots signals to the destination node, that uses them to measure the quality of the channel towards them. Only RNs with a good channel are selected for transmission to the destination, which is performed by means of cooperative beamforming. In case of incorrect reception, a NACK is sent back together with an updated Channel State Information (CSI), and retransmission is requested. Decoding at the destination is performed by combining the signals received at each retransmission. This protocol has some drawbacks, as it is sensitive to phase noise, incorrect estimation of CSI, synchronization of RNs, etc.

Hence, the authors of [72] propose another protocol based on selective random cyclic delay diversity. In this protocol, the source message is retransmitted to the destination by all RNs that have correctly decoded it. Retransmission is sent together with the pilot signals, and it is performed via random cyclic delay diversity. In case of incorrect reception, destination asks for retransmission only to selected RNs. Again, the destination combines the signals received during successive retransmissions to decode the message. This protocol has several desirable features, e.g., the pilot signals are transmitted together with the message during the first transmission, selective random cyclic delay diversity is less

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sensitive to incorrect CSI estimation and there is no need for synchronization between RNs. Moreover, the second protocol outperforms the first, in terms of both throughput and transmission delay.

Chapter 3

Scheduler Performance

Comparison

This chapter proposes a study of the impact of two traditional schedulers, Proportional Fair (PF) and Round Robin (RR), on the feasibility and on the performance of relay-enhanced LTE-A networks. These two schedulers are indeed natural candidates for implementation in relay nodes (RN) and, following the results obtained in single-hop networks, mobile operators could be tempted to adopt PF because of the good trade-off it offers between cell capacity and fairness. Based on a statistical throughput evaluation model, it is shown that this is not necessarily the best option. The number of RNs, their locations in the cell, and the backhaul link quality have a decisive influence on the scheduler choice. In some scenarios, it is even not convenient to deploy relays as they degrade the network performance compared to the no relay case. The proposed performance evaluation model takes into account fast fading and relies on a sub-optimal RN placement scheme that allows the analysis of many scenarios at a click speed.

The analysis is carried on in a *static* scenario, i.e., under the assumption that all RBs are loaded with data (literature often names this assumption *full loading of subcarriers*). In fact, a large number of papers are based on a static

scenario, because it allows for a deeper analytical modeling of wireless networks. The drawback of this approach is that it does not account for, e.g., the burstiness of cellular traffic, UEs arrival/departure dynamics and station buffer queues length. Moreover, interference tends to be overestimated, because all stations are assumed to be transmitting on every RB. However, the static analysis can represent a good tradeoff between simplicity and accuracy in preliminary network performance studies, where traffic burstiness may not need to be considered.

The chapter is divided into three sections. In Section 3.1 the system model is given. Then, in Section 3.2 a novel static performance evaluation framework for relay networks is introduced. Finally, results are given in Section 3.3.

3.1 Downlink RNs Network Static Model

This section includes a description of the network and propagation models, downlink frame structure and SINR computation. Moreover, the relay placement scheme that will be used to optimize the performance and obtain the results is introduced.

3.1.1 Network Model

This study focuses on the downlink of a hexagonal tri-sectorized cellular network, where each network sector contains one BS sector and n_{RN} RNs. One network sector is considered, and an index $k = 0$ labels its related BS sector, while the RNs are assigned indices $\mathcal{S}_{RN} \triangleq \{1, \dots, n_{RN}\}$. A parameter $\mathcal{S}$ is defined as $\mathcal{S} \triangleq \mathcal{S}_{RN} \cup \{0\}$. Let $\mathcal{E}$ and $\mathcal{R}$ be respectively the set of all BS sectors and all RNs *in the whole network*, and denote with $\mathcal{U}$ the set of all UEs, of cardinality $N_U = |\mathcal{U}|$. Finally, $\mathcal{U}_k$ denotes the set of UEs controlled by k , of cardinality $N_k = |\mathcal{U}_k|$, so that $\mathcal{U} = \cup_{k \in \mathcal{S}} \mathcal{U}_k$. The network model is depicted in Figure 3.1.

Each UE is served by the network node providing the highest received (pilot) power; fast fading effect is not considered in the serving station selection process,

because the received power is assumed to be averaged on a large number of frames. Surface $\mathcal{A}_k$, of area $A_k \triangleq |\mathcal{A}_k|$, is defined as the network region where k is the best server. Cell range expansion CRE is considered, and $\mathcal{B}$ indicates the corresponding *bias factor*.

Figure 3.1: Network model: RNs are served by a sector of an BS sector via the BL; UEs are served either by the BS sector via the DL or by a RN via the AL.

3.1.2 Frame Structure

Available downlink radio resources are organized in frames of duration T_F . A frame is a block of contiguous radio resources limited in frequency and time, and divided into several *resource blocks* (RBs). All stations transmit their own downlink frames over a shared block of radio resources, and use a portion of frame resource blocks to serve their RNs and/or UEs.

Static RP is adopted. Frame resources dedicated to the BL represent a quota τ of the overall resources. The parameter τ is decomposed into several terms τ_k , each referred to the BS sector-RN k link: $\tau = \sum_{k=1}^{n_{RN}} \tau_k$, and define $\underline{\tau} \triangleq (\tau_1, \dots, \tau_{n_{RN}})$. Resulting frame structure is depicted in Figure 5.2 on

page 85, where a sample scenario with one BS sector serving two RNs, i.e., $\mathcal{S}_{RN} = \{1, 2\}$, is depicted.

Figure 3.2: Frame structure

3.1.3 Access and Direct Link Model

UEs are uniformly distributed in the network and their positions are fixed during a frame transmission ¹.

All BS sectors and all RNs use the same power per RB: $P_{tx,0} = P_{BS}$ and $P_{tx,k} = P_{RN}$, $\forall k \in \mathcal{S}_{RN}$, where $P_{tx,k}$ denotes the transmit power of k . The power $P_{rx,i,k}(b, n)$ received by UE i from station $k \in \mathcal{S}$ on RB b of frame n is computed as:

$$P_{rx,i,k}(b, n) = P_{tx,k} g_{i,k}(n) \theta_{i,k}^2(b, n), \quad (3.1)$$

where $g_{i,k}(n)$ is the path gain between i and k , which includes the shadowing and the propagation gain, while $\theta_{i,k}^2(b, n)$ is a term accounting for the fast fading effect. In particular, $g_{i,k}(n)$ is expressed as: $g_{i,k}(n) = \frac{K_{i,k}}{r_{i,k}(n)^{\eta_{i,k}}} A_{i,k}(n) \Omega_{i,k}(n)$, where $K_{i,k} \in \{K_{AL}, K_{DL}\}$ is a propagation constant, $\eta_{i,k} \in \{\eta_{AL}, \eta_{DL}\}$ is the

¹This assumption is made for the sake of simplicity but it is not a strict requirement for the proposed model. In case of non uniform distribution, results in Sections 3.2.2 and 3.2.3 should be modified accordingly.

path-loss exponent, $r_{i,k}(n)$ is the distance between i and k , $A_{i,k}(n)$ is the antenna gain in the direction of i , and $\Omega_{i,k} = 10^{\frac{\xi_{i,k}}{10}}$ is a lognormal Random Variable (RV), where $\xi_{i,k}$ is a normal zero-mean RV of standard deviation $\sigma_{i,k}$. It is assumed that $\forall(i, k) \in \mathcal{U} \times \mathcal{E}$, $\sigma_{i,k} = \sigma_{DL}$, and $\forall(i, k) \in \mathcal{U} \times \mathcal{R}$, $\sigma_{i,k} = \sigma_{AL}$. The parameter $g_{i,k}(n)$ is supposed to be constant over a frame duration and across RBs.

The parameter $\theta_{i,k}(b, n)$ is a Rayleigh-distributed RV. Hence, $\theta_{i,k}^2(b, n)$ is exponentially-distributed, and its mean value is 1. Fast fading on all RBs is drawn according to a Block Rayleigh fading model, i.e., fading $\theta_{i,k}(b, n)$ is fixed on RB b , and fading realizations on different blocks are independent². Let denote with $\bar{P}_{rx,i,k}(n)$ the mean power received by i from k , over all RBs of frame n :

$$\bar{P}_{rx,i,k}(n) = \mathbb{E}_{\theta_{i,k}} [P_{tx,k} g_{i,k}(n) \theta_{i,k}^2(b, n)] = P_{tx,k} g_{i,k}(n). \quad (3.2)$$

Now, the SINR experienced by i on RB b of frame n can be written as

$$\gamma_{i,k}(b, n) = \frac{\bar{P}_{rx,i,k} \theta_{i,k}^2(b, n)}{\sum_{j \in \mathcal{E} \cup \mathcal{R}, j \neq k} \bar{P}_{rx,i,j} \theta_{i,j}^2(b, n) + N}, \quad (3.3)$$

where $N = N_0 B$ is the background noise power, with N_0 the thermal noise spectral density and B the RB bandwidth. This study makes use of the approximation introduced in [74, 75, 76], which is a consequence of the law of large numbers, to simplify the computation of the interferers powers in (3.3):

$$\sum_{j \in \mathcal{E} \cup \mathcal{R}, j \neq k} \bar{P}_{rx,i,j} \theta_{i,j}^2(b, n) \approx \sum_{j \in \mathcal{E} \cup \mathcal{R}, j \neq k} \bar{P}_{rx,i,j}. \quad (3.4)$$

The SINR can thus be rewritten as $\gamma_{i,k}(b, n) = \Gamma_{i,k}(n) \theta_{i,k}^2(b, n)$, where $\Gamma_{i,k}(n)$ is equal to:

$$\Gamma_{i,k}(n) = \frac{\bar{P}_{rx,i,k}}{\sum_{j \in \mathcal{E} \cup \mathcal{R}, j \neq k} \bar{P}_{rx,i,j} + N}, \quad (3.5)$$

and can be seen as the average SINR experienced by i on frame n .

²This is a typical assumption in the literature on relays, in physical layer studies, see e.g. [73], as well as in relay planning studies, see e.g. [13].

The *instantaneous data rate* $\rho_{i,k}(b, n)$ for any user i served by k on RB b of frame n is computed according to [77]:

$$\rho_{i,k}(b, n) = \min\{\alpha \log_2(1 + \beta \Gamma_{i,k}(n) \theta_{i,k}^2(b, n)), \rho_{\max}\}, \quad (3.6)$$

where α is a *bandwidth efficiency factor*, β is an *SINR efficiency factor* and $\rho_{\max}$ represents the maximum achievable spectral efficiency, so as to better model the behavior of a real cellular network, where UEs spectral efficiency is saturated. The parameters α , β and $\rho_{\max}$ depend on the chosen Modulation and Coding Scheme (MCS). Finally, $R_{i,k}(n)$ denotes the mean of $\rho_{i,k}(b, n)$ over fast fading:

$$R_{i,k}(n) \triangleq \mathbb{E}_{\theta_{i,k}}[\rho_{i,k}(b, n)]. \quad (3.7)$$

3.1.4 Backhaul Link Model

As in [13], only the effect of path-loss and shadowing on signal propagation on the BL are considered. This is justified by the fact that RNs do not move, and network planning is performed on the basis of long-term performance indicators. Hence, the signal power received by RN k from BS sector h at distance r on a given RB of frame n is equal to

$$\bar{P}_{rx,k,h}(n) = P_{tx,h} g_{k,h}(n), \quad (3.8)$$

where the shadowing standard deviation on all links between an BS sector and a RN is denoted σ_{BL} . For performance evaluation of in-band relays, two backhaul link scenarios are considered: a favorable case called 'LOS BL' and an unfavorable case called '3GPP BL' (see Section 3.3.2). The average SINR $\Gamma_{k,h}(n)$ during frame n can be approximated by:

$$\Gamma_{k,h}(n) = \frac{\bar{P}_{rx,k,h}}{\sum_{j \in \mathcal{E}, j \neq i} \bar{P}_{rx,k,j} + N}, \quad (3.9)$$

using assumption (3.4) for the interference term. Note that interference on the relay link is uniquely generated by BSs. Now, the average rate $R_{BL,k}(n)$ achieved by RN k on the BL is approximated as:

$$R_{BL,k}(n) = \min\{\alpha \log_2(1 + \beta \Gamma_{k,h}(n)), \rho_{\max}\}. \quad (3.10)$$

3.1.5 Relay Placement Algorithm

When evaluating the performance of a relay-based cellular network, the way relays are placed in the cell plays a crucial role. This subsection thus intends to find a 'good' relay placements in terms of sector throughput for a given pair $(\mathcal{B}, N_{RN})$ of bias and number of relays. The adopted objective function excludes standard methods in convex and non-convex optimization theory [78, 79]. The RN location problem includes the capacitated facility location problem as a special case, so that it is NP-hard [31]. Exhaustive search becomes rapidly unfeasible as the number of RNs and RN candidate sites increase. Popular meta-heuristics, such as SA or taboo search, have been adopted in e.g. [44, 80] for relay placement but they also require a high number of computations.

In this work a RN placement heuristic algorithm is proposed, which yields sub-optimal but very quick results. The basic idea is to perform an exhaustive search over a *well chosen* subset of 'all possible RNs placements. From the results of [80], it can be observed that out-of-band relays tends to be located near the cell border, where radio conditions are worst. In-band relays tend to be closer to the BS in order to benefit from a better backhaul link quality. At last, as the number of RNs or their transmit power increase, there is a repulsion effect that tends to create several rings of relays around the BS.

Relying on these observations, a class of relay topologies is defined. Each topology is defined by the pair $(N_{RN}, \bar{d})$, where $\bar{d}$ is the topology reference distance and is defined hereafter. Relays in a given sector are arranged on one or more tiers, which are outlined so as to keep constant the distance between RNs on the same tier, and the distance between a tier and the cell edge (see

Figure 3.3). The parameter $\bar{d}$ represents the distance between two neighboring tiers, the distance between neighboring RNs on the same tier, and the minimum allowed distance between any two RNs in the network. The number of tiers and the number of RNs on each tier is determined by $\bar{d}$, N_{RN} and the cell dimension.

As shown in Algorithm 1, the method proceeds by steps, and takes as inputs N_{RN} and $\bar{d}$. At step $g > 0$, a certain number of relays n_r remain to be placed. A new tier at a distance $d_g = (1/2 + (k - 1))\bar{d}$ from cell edge is built and $p_g = \min\{n_g; n_r\}$ RNs are placed on it, where n_g is the maximum number of RNs which can be allocated to tier g . This is determined as³ $n_g = \left\lfloor \frac{l_g}{\bar{d}} + 1 \right\rfloor$, where l_g is the length of tier g . The p_g relays assigned to tier g are arranged so that the distance between one RN and its neighbor is $\bar{d}$, and RN locations are symmetric with respect to the sector antenna boresight direction. The algorithm stops when either all RNs have been placed, i.e., configuration $(N_{RN}, \bar{d})$ is feasible, or $d_g \geq R\sqrt{3}/2$, i.e., configuration $(N_{RN}, \bar{d})$ is not feasible because RNs do not fit into their cell sector, with the given $\bar{d}$.

The idea behind this method is to place RNs on a regular pattern, organized in tiers around BSs. Small values of $\bar{d}$ lead to RN patterns, in which all RNs are close to each other, and close to the cell border. Large values of $\bar{d}$ produce placements where RNs are more regularly distributed in the sector. Although the proposed algorithm is suboptimal, its advantage lies in the fact that location of all RNs solely depends on one parameter, allowing the analysis of a large set of situations.

³The function $\lfloor \cdot \rfloor$ yields the highest integer which is lower or equal to the function argument.

Algorithm 1 Relay placement algorithm

```

1: Input parameters:  $N_{RN}$ ,  $d$ 
2: Initialization:  $n_r \leftarrow N_{RN}$ ,  $g \leftarrow 1$ 
3: repeat
4:    $d_g \leftarrow (\frac{1}{2} + (g - 1)) d$ ,
5:   if  $n_r > 0$  then
6:     Outline tier  $g$  and compute  $l_g$ 
7:     Compute  $n_g = \lfloor \frac{l_g}{d} + 1 \rfloor$ 
8:     Compute  $p_g = \min\{n_g, n_r\}$ 
9:     Place  $p_g$  relays on tier  $g$ , distanced by  $d$  and symmetric wrt antenna
       boresight direction
10:     $n_r \leftarrow n_r - p_g$ ,  $g \leftarrow g + 1$ 
11:   end if
12: until  $n_r == 0$  OR  $d_g \geq R\frac{\sqrt{3}}{2}$ 
13: Return: relay placement ( $n_r == 0$ ) OR 'Configuration not feasible' ( $d_g \geq$ 
     $R\frac{\sqrt{3}}{2}$ )

```

Figure 3.3: Proposed RN placement with reference distance $\bar{d}$ and $N_{RN} = 6$.

Complexity of the Proposed Algorithm

Hereafter the complexity of the proposed heuristic is derived and compared to that of a brute force method and a simulated annealing, under the assumption that the number of elementary computations to be performed to evaluate the performance is the same in the three considered optimization techniques. Finally, results obtained with each method are displayed and compared. It is worth recalling that optimization is carried out over the values of P_{RN} , $\mathcal{B}$ and the RN locations.

Suppose that P_{RN} can take P possible values, $\mathcal{B}$ can take B possible values and that there are N_{RN} relays. In the considered study, it is assumed that $P = 6$, $B = 6$ and N_{RN} is between 1 and 7. With the proposed heuristic, the RN location is obtained by varying $\bar{d}$ in an interval $[d_{min}ISD, d_{max}ISD]$ with a step of $d_{min}ISD$, resulting in D values to test. In this work, $\bar{d}$ varies in $[0 : 01ISD; 0 : 25ISD]$ with steps of $0 : 01ISD$, i.e., $D = 26$. The overall number of iterations is thus PBD , independently of N_{RN} . In particular, 936 iterations are performed in this work.

Now, consider a brute force approach, i.e., testing all possible configurations. A brute force algorithm achieves the same accuracy of the proposed suboptimal algorithm if the sector area is discretized so that each surface element has an area $(d_{min}ISD)^2$. The overall sector area is $ISD^2/2\sqrt{3}$. Hence, there are $1/d_{min}^2\sqrt{3}$ possible locations for a single relay. This results in $PB/(d_{min}^2\sqrt{3})N_{RN}$ configurations to test. With the values adopted in this work, 30000 iterations are approximately required for $N_{RN} = 3$. Hence, the complexity of a brute force approach is more than one order of magnitude above that of the proposed algorithm.

3.2 Network Performance and Frame Optimization

This section is devoted to the analysis of the network performance in terms of sector sum throughput under PF and RR scheduling policies. Moreover, the

frame structure optimization problem is tackled. For the sake of simplicity, let Ψ denote the set of network parameters $\{n_{RN}, \bar{d}, \mathcal{B}, P_{RN}\}$. This section focuses on a given Ψ .

3.2.1 Stations and Sector Throughput

The *average sector throughput* is the chosen objective function for comparing different network configurations. Its formulation is given after some intermediate definitions.

Definition 1 (Average sum physical data rate). *For a set of network parameters Ψ and a station $k \in \mathcal{S}$, the average sum physical data rate is defined as the expectation, with respect to frames, of the sum of all data rates delivered to its served users:*

$$H_k \triangleq \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} R_{i,k}(n) \right]. \quad (3.11)$$

In this definition, $\mathbb{E}_n[f(n)] = \lim_{N_F \rightarrow \infty} 1/N_F \sum_{n=1}^{N_F} f(n)$ indicates the average over time frames of a function f . Over time, randomness is due to the number of served UEs, UE locations, shadowing and fast fading. In particular, note that $\mathcal{U}_k(n)$ is the random set that collects all UEs served by k in frame n .

Definition 2 (Average station throughput). *For a set Ψ , a station $k \in \mathcal{S}$ and a given frame structure $\underline{\tau} = (\tau_1, \dots, \tau_{n_{RN}})$, the average station throughput is defined as the expectation, with respect to frames, of the sum of throughputs delivered to its served users:*

$$\bar{T}_k(\underline{\tau}) = \begin{cases} \min \left\{ t_a \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} R_{i,k}(n) \right] ; \tau_k \mathbb{E}_n [R_{BL,k}(n)] \right\}, & \text{for } k \in \mathcal{S}_{RN}, \\ t_a \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} R_{i,k}(n) \right], & \text{for } k = 0, \end{cases} \quad (3.12)$$

Contrary to the definition of the average sum physical throughput, the average station throughput takes into account the bottleneck represented by

the BL for relay stations. This definition is still valid for out-of-band relays, provided that the convention $\tau_k \mathbb{E}_n[R_{BL,k}(n)] = \infty$ is adopted. Note that $\bar{T}_k(\underline{\tau}) = \min\{t_a H_k; \tau_k \mathbb{E}_n[R_{BL,k}(n)]\}$ for $k \in \mathcal{S}_{RN}$ and $\bar{T}_0(\underline{\tau}) = t_a H_0$. In case of out-of-band relays $t_a = 1$ and $\bar{T}_k(\underline{\tau}) = H_k$ for all k .

Definition 3 (Average sector throughput). *For a set Ψ , a sector consisting of one BS sector and n_{RN} relays, and a frame structure $\underline{\tau}$, the average sector throughput is defined as the sum of sector stations throughputs:*

$$C(\underline{\tau}) = \sum_{k \in \mathcal{S}} \bar{T}_k(\underline{\tau}) = t_a H_0 + \sum_{k \in \mathcal{S}_{RN}} \min\{t_a H_k; \tau_k \mathbb{E}_n[R_{BL,k}(n)]\}. \quad (3.13)$$

Again, the convention $\tau_k \mathbb{E}_n[R_{BL,k}(n)] = \infty$ for all k for out-of-band relays is adopted. Readers are reminded that $R_{BL,k}(n)$ is given by (3.10).

Fast fading is a phenomenon that considerably increases the simulation time. This is due to the fact that it varies on a much faster time-scale (few ms) than shadowing or UEs locations. The following sections propose a statistical method for taking into account fast fading without explicitly simulating it. In particular, H_k is analyzed under PF and RR scheduling assumptions.

3.2.2 Station Throughput Under RR Scheduling

With RR scheduling, every station provides in every frame all its served UEs with an equal share of RBs, uniformly distributed in the frame for each UE, and selected regardlessly of the fast fading realization. Under these assumptions, the PDF $\Phi_{\theta^2}(x)$ of θ^2 experienced by the UE scheduled on a given RB is given by $\Phi_{\theta^2}(x) = e^{-x}$. The following result is given:

Proposition 1 (Average sum physical data rate under RR scheduling). Under the assumption that all stations in $\mathcal{S}$ adopt the RR scheduling, the average sum physical data rate of station $k \in \mathcal{S}$ can be written as:

$$H_k = \sum_{v=0}^{N_U} \binom{N_U}{v} p_k^v (1 - p_k)^{N_U - v} v \mathbb{E}_\Omega \left[\frac{1}{A_k} \int_{\mathcal{A}_k} \frac{\mathcal{L}_\rho\left(\frac{1}{\beta \Gamma_k(s)}\right)}{\beta \Gamma_k(s)} ds \right], \quad (3.14)$$

with

$$\mathcal{L}_\rho(s) = \frac{\alpha e^s}{s \ln(2)} \left[-\ln(\bar{t} + 1) e^{-s(\bar{t}+1)} + E_1(s) - E_1(s(\bar{t} + 1)) \right] + \frac{\rho_{\min} e^{-s\bar{t}}}{s}, \quad (3.15)$$

where $\bar{t} = 2^{\frac{\rho_{\min}}{\alpha}} - 1$, $E_1(x) = \int_x^\infty \frac{e^{-u}}{u} du$, $\Gamma_k(s)$ is the SINR of a UE located in s and served by k , p_k is the probability for a UE to be served by k , and expectation is taken over shadowing variations.

Proof. see Appendix A.1. □

Equation (3.14) is used in simulations, with several shadowing values. The parameter p_k is obtained as a ratio of the serving surface to the network surface (assuming uniform UE distribution) and the integral is numerically obtained over the surface of station k (which is constant for a realization of shadowing random variables). Hence, fast fading is not explicitly simulated, but its effect is taken into account.

3.2.3 Station Throughput Under PF Scheduling

If stations have perfect CSI regarding their UEs, a PF scheduler can be used to assign resource blocks to all UEs served by a given station during a frame. Adopting the PF scheduler definition of [81], the PF scheduler assigns each slot to the UE enjoying the best fast fading conditions. Thus:

$$\begin{aligned} \Phi_{\theta^2}(x) &= v e^{-x} (1 - e^{-x})^{v-1}, \\ &= v e^{-x} \sum_{r=0}^{v-1} \binom{v-1}{r} (-1)^r e^{-rx}, \end{aligned} \quad (3.16)$$

where v is the number of UEs served by k .

Proposition 2 (Average sum physical data rate under PF scheduling). Under the assumption that all stations in $\mathcal{S}$ adopt the PF scheduling, the average sum

physical data rate of station $k \in \mathcal{S}$ can be written as (using the notations of Proposition 1):

$$H_k = \sum_{v=0}^{N_U} \sum_{r=0}^{v-1} p_k^v (1 - p_k)^{N_U-v} v^2 \binom{v-1}{r} (-1)^r \mathbb{E}_\Omega \left[\frac{1}{A_k} \int_{\mathcal{A}_k} \frac{\mathcal{L}_\rho\left(\frac{r+1}{\beta\Gamma_k(s)}\right)}{\beta\Gamma_k(s)} ds \right]. \quad (3.17)$$

Proof. see Appendix A.2. $\square$

Again, the obtained expression for H_k is based on average SINR, avoiding fast fading explicit simulation.

3.2.4 Relay-Aware Frame Structure Optimization

The values of t_a and τ are derived by imposing constraints to the frame structure (see 2.2.3).

Frame Structure Optimization

The aim is to find the frame structure $\underline{\tau}^*$ that maximizes the sector throughput $C(\underline{\tau})$ for a given network configuration Ψ .

Lemma 1 (Upper bound for τ_k^*). *The optimal frame structure $\underline{\tau}^*$ is such that $\tau_k^* \in [0; \tau_{k,\max}]$ where:*

$$\tau_{k,\max} = \frac{t_a H_k}{\mathbb{E}_n [R_{BL,k}(n)]}. \quad (3.18)$$

Proof. Consider RN k . As it can be seen from (3.12), BL k data rate linearly increases with τ_k , while AL k data rate linearly decreases with τ_k . As a consequence, the maximum data rate is obtained for $\tau_k = \tau_{k,\max}$, where $\tau_{k,\max}$ is the value of τ_k for which BL k and AL k data rates are equal. If $\tau_k > \tau_{k,\max}$, then the BL can deliver to the RN more data than those that the RN can deliver to its UE. Thus, a part of BL-dedicated RBs do not carry data and they are wasted. Hence, $\tau_{k,\max}$ represents an upper bound for τ_k^* . $\square$

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In the following, it is assumed that $\tau_k \leq \tau_{k,\max}$, $\forall k \in \mathcal{S}_{RN}$. Under this constraint, (3.12) can be rewritten as:

$$\bar{T}_k(\underline{\tau}) = \begin{cases} \tau_k \mathbb{E}_n [R_{BL,k}(n)], & \text{for } k \in \mathcal{S}_{RN}, \\ t_a H_k, & \text{for } k = 0. \end{cases} \quad (3.19)$$

Corollary 1 (Optimal frame structure). *For a given network configuration Ψ , define $\Delta_k(\Psi) = \mathbb{E}_n [R_{BL,k}(\Psi, n)] - H_0(\Psi)$, $k \in \mathcal{S}_{RN}$. If $\Delta_k(\Psi) > 0$ for all $k \in \mathcal{S}_{RN}$, then the optimal frame structure is $\underline{\tau}^* = (\tau_{1,\max}, \dots, \tau_{n_{RN},\max})$ and the optimal sector throughput is given by:*

$$C_{\max}(\Psi) = H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi) \Delta_k(\Psi). \quad (3.20)$$

Proof. In the following it is explicitly recalled that the considered performance parameters depend on the network configuration Ψ . Considering (3.19), (3.13) can be expressed as

$$\begin{aligned} C(\underline{\tau}, \Psi) &= t_a H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} \mathbb{E}_n [R_{BL,k}(\Psi, n)] \tau_k \\ &= H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} \tau_k \Delta_k(\Psi), \end{aligned} \quad (3.21)$$

where $\tau_k \in [0, \tau_{k,\max}(\Psi)]$. The parameters $H_0(\Psi)$ and $\Delta_k(\Psi)$ are completely determined by network setup Ψ , while τ_k , $k \in \mathcal{S}_{RN}$ can be tuned in order to maximize $C(\underline{\tau}, \Psi)$. Now, sector throughput $C(\underline{\tau}, \Psi)$ is a multilinear function of the τ_k , which is non decreasing in every τ_k since it is assumed that $\Delta_k(\Psi) > 0$. As a consequence, it achieves its maximum for $\tau_k = \tau_{k,\max}(\Psi)$. $\square$

Note that the condition $\Delta_k(\Psi) > 0$ for all k is not very restrictive. Indeed, assume that for some k_0 , $\Delta_{k_0}(\Psi) \leq 0$. Then, $C(\underline{\tau}, \Psi)$ is optimized by setting $\tau_{k_0} = 0$ because it is a non increasing function of τ_{k_0} . This means that no resources are dedicated on the BL to k_0 when Δ_{k_0} is negative, leaving k_0 inactive.

This case does not make sense in a practical context, where RNs are deployed in order to be used and to obtain a benefit in terms of, e.g., throughput, coverage, etc. Hence, such configurations are not considered in this work, which focuses on configurations Ψ for which condition $\Delta_k(\Psi) > 0, \forall k \in \mathcal{S}_{RN}$ is fulfilled.

3.2.5 Convenience of Relaying

This section derives a condition to be checked, in order to know whether a network configuration Ψ is *convenient*, i.e., whether it is worth deploying relays with Ψ rather than not deploying relays.

Proposition 3. A network configuration Ψ is convenient w.r.t. the case where no relays are deployed iff:

$$\left(H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} H_k(\Psi) \right) \left(1 - \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi) \right) > \tilde{H}_0, \quad (3.22)$$

where $\tilde{H}_0$ is the average sum physical data rate of the BS sector when no relays are deployed.

Proof. Starting from (3.20), it is possible to state that relaying is convenient in terms of sector throughput if and only if

$$\begin{aligned} \tilde{H}_0 &< H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi) \Delta_k(\Psi), \\ &< H_0(\Psi) + (1 - \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi)) \sum_{k \in \mathcal{S}_{RN}} \frac{H_k(\Psi)}{\mathbb{E}_n[R_{BL,k}(\Psi, n)]} (\mathbb{E}_n[R_{BL,k}(\Psi, n)] - H_0(\Psi)), \\ &< (H_0(\Psi) + \sum_{k \in \mathcal{S}_{RN}} H_k(\Psi)) (1 - \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi)). \end{aligned} \quad (3.23)$$

□

Condition (3.22) highlights the contribution of the proportion of resources dedicated to the backhaul (the factor: $1 - \sum_{k \in \mathcal{S}_{RN}} \tau_{k,\max}(\Psi)$) and of the increase

of the number of network stations (with the sum: $\sum_{k \in S_{RN}} H_k(\Psi)$) to the convenience of RNs deployment. From (3.18), $\tau_{k,max}$ increases with the station k 's delivered data rate and decreases with the backhaul quality. For out-of-band relays, the factor $1 - \sum_{k \in S_{RN}} \tau_{k,max}(\Psi)$ reduces to 1 and there is no influence of the BL. The sum $\sum_{k \in S_{RN}} H_k(\Psi)$ is the result of network densification, which is known to increase system capacity [82].

3.3 Numerical Results and Discussion

In this Section, numerical simulations results are shown and commented.

3.3.1 Simulation Procedure

The following contains a description of how the evaluation of $C_{max}(\Psi)$ is carried out, for any considered set of network parameters $\Psi : \{N_{RN}, \bar{d}, \mathcal{B}, P_{RN}\}$. Monte Carlo simulations are used to obtain numerical results. At each iteration n the following operations are performed:

1. The simulator drops N_U UEs in the network with uniform spatial distribution. Shadowing between any station-UE pair and between BS sectors and associated RNs is calculated.
2. A serving station is associated to each UE, considering the bias factor $\mathcal{B}$.
3. The SINR $\Gamma_{i,k}(n)$ between every UE and its serving station is computed with (3.5) and recorded. In the same way, the SINR $\Gamma_{k,h}(n)$ in (3.9) between the BS sector and every RN is computed and used to compute $R_{BL,k}(n)$ from (3.10).

After 1000 iterations, $\mathbb{E}_n[R_{BL,k}]$ is obtained by averaging all recorded $R_{BL,k}(n)$. Then, H_k is computed according to the chosen scheduling policy with (3.14) or (3.17). In these equations, p_k is obtained with the proportion of UEs served by k along the iterations and the integral is obtained by averaging the integral over

UEs served by k . At last, the optimal frame structure is derived from (3.18). Any Ψ not fulfilling conditions of Corollary 1 or of Proposition 3 is discarded.

Simulations are repeated for several Ψ , in order to find the set yielding the best performance in terms of sector throughput $C_{max}(\Psi)$. In particular, the analyzed range of values for each parameter are indicated in Table 3.1. Values of P_{RN} up to 46 dBm are considered, as high power relay nodes are regarded as a viable solution in particular contexts, e.g., in rural areas [83].

Table 3.1: Analyzed range of values for N_{RN} , P_{RN} , $\mathcal{B}$ and $\bar{d}$.

Parameter	Explored Interval	Step
N_{RN}	[1, 7]	1
P_{RN}	[21, 46] dBm	5 dBm
$\mathcal{B}$	[0, 15] dB	3 dB
d	[0.01 ISD, 0.25 ISD]	0.01 ISD

3.3.2 Simulations Settings

Simulations performed in this paper are compliant with simulation parameters given in [84, case 1], if not otherwise specified (see Table 3.2). The network cluster used to simulate a real network is formed by two cell rings around a central cell. Moreover, the wraparound technique has been employed (six cell clusters are 'wrapped' around the central cluster). All the measurements are carried out on UEs dropped uniformly in every cell of the central cluster, which are assumed to be always in Non Line Of Sight (NLOS) with respect to network stations. Let the term *load* denote the number of UEs dropped in each cell at each simulation snapshot, while the *Inter-Site Distance* (ISD) is defined as the distance between two neighboring BSs.

The case of in-band and out-of-band relaying are considered. For in-band relaying, two possible backhaul link types are studied: 1) *LOS BL*: RNs are always in Line Of Sight (LOS) with respect to their serving BS, and always in NLOS with respect to interfering BSs, as in, e.g., [12]. *LOS BL* can be considered as 'good' backhaul link. 2) *3GPP BL*: RNs can be in LOS or NLOS with respect to their serving BS with a certain probability, depending on the

distance. The same holds for interfering BSs (see [84, case 1]). 3GPP BL can be seen as a 'bad' backhaul link.

Table 3.2: Simulations Settings

Parameter	value
Cell rings	2
P_{BS}	46 dBm
BS antenna gain	14 dBi
RN antenna gain	5 dBi
UE antenna gain	0 dBi
$\rho_{\max}$	4.4 bit/s/Hz
β	1
α	0.6
σ_{AL}	10 dB
σ_{BL}	0 dB (LOS), 6 dB (NLOS)
σ_{DL}	8 dB
UE noise figure	9 dB
RN noise figure	5 dB
η_{AL}	3.75
η_{BL}	2.35 (LOS), 3.63 (NLOS)
η_{DL}	4.28
Antenna Pattern (horizontal)	$-\min[12(\phi/\phi_{3dB})^2, A_m]$, $A_m = 25dB$, $\phi_{3dB} = 70^\circ$

3.3.3 Out-of-Band Relaying

In this section, the out-of-band relaying case is analyzed. Figure 3.4 shows sector throughput $C_{\max}$ vs N_{RN} for variable cell load, considering an ISD of 1km, and for the best network configuration Ψ . $C_{\max}$ steadily increases with N_{RN} , because radio resources available for UEs transmission in the cell increase with N_{RN} , i.e., the network becomes more *dense*. Moreover, it is not needed to use part of radio resources for the BL, as in the case of in-band relaying. The impact of network densification on $C_{\max}$ overcomes that of an increased amount of transmitting RNs, which negatively affects the SINR experienced by UEs. This phenomenon is less accentuated for low cell load. Indeed, in this

case adding more RNs has less positive effect on cell throughput, because there is a non-negligible probability that a RN does not serve any UE.

Note that scheduling gain grows with N_{RN} , when cell load is high (75 UEs per cell). Indeed, due to the increase of relays, interference increases and the average experienced SINR is slightly degraded. The authors of [81] have shown that when the spectral efficiency is upper bounded (as it is the case here with parameter ρ_{max}), a lower average SINR leads to a higher scheduling gain, which explains the observed tendency⁴. Instead, when cell load is lower (e.g., 25 UEs per cell), scheduling gain remains approximately constant with N_{RN} , because RNs serve few UEs (or no UEs at all), and hence there is little UEs diversity gain.

The optimized value of P_{RN} , found through simulations, is 46 dBm (the highest value in the tested range), for all N_{RN} and scheduling policies, when out-of-band relaying is adopted. This is due to the fact that backhaul does not limit performance in the out-of-band case. Thus, RNs can support a high number of UEs without any penalty in terms of capacity. Power $P_{RN} = 46$ dBm tends to equally balance the number of UEs served by BS sectors and RNs.

Figure 3.4: C_{max} vs N_{RN} (out-of-band relaying, ISD=1km).

Figure 3.5: C_{max} vs N_{RN} (in-band relaying, 50 UEs per cell, LOS BL).

⁴Imagine on the contrary that SINRs are so high for all UEs that their data rate is always ρ_{max} , then there is no scheduling gain of PF over RR.

3.3.4 In-Band Relaying

Figure 3.5 plots the value of $C_{\max}$ vs N_{RN} for different ISDs (for a load of 50 UEs per cell), showing the effect of a change in cell dimension on scheduling gain. It can be observed that $C_{\max}$ grows with ISD (see also e.g. [85]), because interference has a lower impact on performance compared to thermal noise. Moreover, backhaul capacity decreases less rapidly with ISD, compared to AL capacity, as η_{BL} is lower on the useful signal in LOS (2.35) than on the interference (3.63 in NLOS). However, RNs serve more UEs (in average) when ISD increases. This is because $\eta_{AL} < \eta_{DL}$. Thus, scheduling gain for UEs served by RNs grows, while the behavior of scheduling gain for UEs served by the BS sector tends to decrease, resulting in no meaningful variations of the overall scheduling gain with respect to ISD.

Figure 3.6 shows the value of $C_{\max}$ and of the scheduling gain vs N_{RN} for different cell loads, when in-band relaying is adopted and a LOS backhaul model is used. The quantity of resources dedicated to backhaul link has a decisive effect on performance. Sector throughput $C_{\max}$ constantly increases with N_{RN} , when RR scheduling is adopted, while, for PF scheduling, $C_{\max}$ stops increasing for $N_{RN} \approx 3$. This is explained by the fact that PF scheduling applied to RNs implies a higher quantity of data to be transmitted to UEs, because of the increased scheduling performance. Hence, there is a higher data demand on the backhaul link, which negatively affects performance. The higher τ overcompensates the benefit of network densification in equation (3.22).

It can be observed that the scheduling gain decreases with N_{RN} . This is again due to the higher backhaul link data rate required by PF scheduler-equipped RNs. Thus, PF scheduling largely outperforms RR when few RNs are deployed, while RR scheduling performance approaches that of PF when N_{RN} grows.

The optimal value of P_{RN} for in-band relaying is 46 dBm for small N_{RN} , while optimal P_{RN} decreases for high N_{RN} . This result can be explained with the fact that, if N_{RN} and P_{RN} are both high, the quantity of UEs served by RNs is such that backhaul link consumption, in terms of radio resources, makes this

Figure 3.6: $C_{\max}$ vs N_{RN} for variable load (in-band relaying, ISD=1km, LOS BL).

Figure 3.7: $C_{\max}(\Psi)$ vs N_{RN} for different BL types (in-band relaying, ISD=1km).

solution not convenient. Hence, P_{RN} must be decreased, in order to balance the number of UEs served by RNs.

3.3.5 Influence of Backhaul Quality

Figure 3.7 plots $C_{\max}$ vs N_{RN} for different backhaul types (for an ISD of 1km and a load of 25 UEs per cell), and shows how the scheduling gain increases with BL quality, and the decisive influence of BL on performance. When the worst performing BL type is adopted (3GPP BL), PF scheduling does not gain much, or even performs worse than RR scheduling. This can be explained by the fact that PF scheduling adopted by RNs implies a higher rate demanded by RNs on the BL. The increase in τ may overcompensate for the increase in UEs instantaneous rates, and network capacity decreases.

If 3GPP BL is adopted, the optimization procedure cannot find any scenario complying with condition of Proposition 3, for all values of N_{RN} . This is the reason for the points missing in one curve of Figure 3.7.

Figure 3.8: Examples of optimized RNs placement (ISD=1km, LOS BL for in-band relays, 25 UEs per cell).

3.3.6 RNs placement

Figure 3.8 shows examples of optimized RNs placement, for $N_{RN} = 3, 4, 7$ and for both in-band and out-of-band cases and optimized Ψ . Reference distance $\bar{d}$ tends to be smaller for in-band relaying, while it grows for out-of-band relaying, as it can be seen for $N_{RN} = 3$. This is due to the fact that BS antenna gain decreases with the distance from BS sector antenna boresight direction. Hence, in-band RNs tend to group close to the boresight direction, in order to have a high backhaul rate. This effect is dominant with respect to the need for RNs to be more uniformly distributed in the cell, in order to increase UEs average experienced SINR. This latter exigence solely determines the RNs placement optimization when out-of-band relaying is adopted, i.e., there is no concern

about backhaul rate.

For the in-band case, although relays are grouped together around antenna boresight direction, it is still convenient to deploy RNs with respect to the no relay case. There is indeed a diversity effect with respect to shadowing: UEs located close to the clustered RNs have a high probability to enjoy a good shadowing on the link towards one of the clustered RNs. This benefits them in terms of serving station received power.

In these experiments, it can be noticed how optimized RNs placement weakly depends on the adopted scheduling policy. This may be due to the fact that a change in RNs placement mainly affects RNs backhaul rate (for in-band relaying) and the cell regions covered by each RN. Both of these parameters do not depend on scheduling.

3.3.7 Coverage

Figure 3.9: Coverage related to optimized RNs placement (ISD=1km, LOS BL for in-band relays, 25 UEs per cell).

Figure 3.9 plots the 5th %ile of UE rate vs N_{RN} for in-band and out-of-band relaying, corresponding to the optimized RNs placement scenarios. Note that in the out-of-band relaying case there is a consistent improvement of cell-edge rate given by RNs deployment, while in the in-band relaying case the coverage

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remains roughly constant. This is again due to the backhaul-related capacity penalty which is related to in-band relaying: the effect of densification and of the improvement of cell-edge UEs SINR is compensated by τ . Backhaul also affects the scheduling gain in terms of coverage, which is lower for the in-band relaying case.

Chapter 4

Fluid Model of Relay-Enhanced Networks

This Chapter introduces the fluid model of cellular networks (Section 4.1) and presents two studies, where the author of this thesis has applied the fluid model to the analysis of relay-enhanced networks. The first study (Section 4.2) studies SINR approximation, while the second (Section 4.3) with RNs long-term transmit power optimization. Another application of the fluid model in relay networks is given in 5.2.5, which introduces an approximation of the interference coming from *far* network stations.

4.1 Introduction to the Fluid Model of Cellular Networks

The fluid model consists in the description of a cellular network by means of a *continuum* of network stations [86], occupying the same surface where the considered network is located.

The fluid model is a powerful tool to simplify interference sum computation in hexagonal [86] and dense Poisson [87] cellular networks, as it allows for

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calculating the sum of interfering powers coming from the considered network surface by means of an integral. In the following, the fluid model is given under the assumption that there is no shadowing. Readers are referred to [76, 88, 80] for the extension to the case with shadowing.

Consider a hexagonal network and assume that P_t denotes the stations transmit power (which is assumed to be the same for all stations), η is the path loss exponent and I is the sum of the interferers powers coming from all the stations located in a given region $\mathcal{S}$. Then the power $P_{rx,i}$ received by a UE from a station i at a distance d_i is given by

$$P_{rx,i} = P_t K d_i^{-\eta}, \quad (4.1)$$

where K is a path loss constant. If $M(\mathcal{S})$ denotes the set of stations in $\mathcal{S}$, the interference sum can be expressed as $I = \sum_{i \in M(\mathcal{S})} P_t K d_i^{-\eta}$. Now, I can be approximated by means of a fluid model of the network. In this case, the discrete stations in $\mathcal{S}$ are substituted with a continuum of stations, having the same *density* ρ_S [stations/m²] of the original network (see Figure 4.1).

Figure 4.1: Traditional discrete network model vs fluid network model.

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Then, the following approximation is made:

$$I = \sum_{i \in M(S)} P_t K d_i^{-\eta} \approx \int_{z \in S} \rho_S P_t K d(z)^{-\eta} dz. \quad (4.2)$$

This approximation is particularly useful when the shape of S presents some regularities. In particular, if the user is at the center of a circular network, the interference coming from all stations outside the central cell is given by [88]

$$I = \int_R^{R_{nw}} 2\pi \rho_S P_t K z^{-\eta} z dz = \frac{2\pi \rho_S P_t K}{2 - \eta} (R_{nw}^{2-\eta} - R^{2-\eta}), \quad (4.3)$$

where R is the inter-site distance and R_{nw} is the network ray. [88] extends the interference sum derivation to the case of a user located anywhere in the central cell, at a distance r_u from the station in the center of the network (which is assumed to be its server). The authors of [88] derive the following expression, by means of an approximation of the integration surface:

$$I(r_u) = \frac{2\pi \rho_S P_t K}{2 - \eta} [(R_{nw} - r_u)^{2-\eta} - (R - r_u)^{2-\eta}]. \quad (4.4)$$

It follows that the Signal to Interference Ratio (SIR) experienced by the mentioned user can be computed as

$$SIR(r_u) = \frac{r_u^{-\eta}(\eta - 2)}{2\pi \rho_S (R - r_u)^{2-\eta}}, \quad (4.5)$$

where it is assumed that the radius of the network is large enough to neglect the term $(R_{nw} - r_u)^{2-\eta}$.

The main advantage of the fluid model is that it allows to obtain approximate closed-form interference and SIR expressions, which solely depend on the distance between the considered user and the network center.

4.2 SINR Approximation in Relay-Enhanced Networks via a Fluid Network Model

In this Section, an analytical expression of the SINR based on a fluid model of the network is derived. The main advantage of this model is that it allows for faster numerical calculations. Results in 4.2.4 show that the proposed method is accurate.

4.2.1 Network Model

The network model is based on a reuse-one hexagonal cellular network with omnidirectional BSs transmitting at power P_{BS} . The half distance between two BSs is named R_{BS} , while ρ_{BS} [BSs/m^2] represents the spatial density of BSs. Network cells are labeled with an index $k = 1 \cdots B$. Every hexagonal cell contains n RNs, evenly distributed on a circle of radius R_R around the BS and transmitting at power P_{RN} . Relays in a given cell are labeled with an index $i = 1 \cdots n$, so that each RNs can be uniquely identified with the pair (i, k) . A RN labeled with a given index i is said to be a *type- i* RN. Suppose that $\varphi + (2\pi/n)i$ denotes the angle of RN i with respect to a horizontal line touching the BS, where φ is an offset parameter, and that the spatial density of RNs is denoted with $\rho_R = n\rho_{BS}$ [RNs/m^2]. Note that the subnetwork composed by all type- i RNs is a hexagonal network of spatial density ρ_{BS} and half distance between neighboring nodes equal to R_{BS} , as in the case of the BSs network. For a given UE at distance r_b from BS b , parameter $r_{i,k}$ measures its distance from RN (i, k) , while $r_{i,k^*(i)} = \min_k \{r_{i,k}\}$ denotes its distance with respect to the closest type- i RN. Users are served according to a best server policy, i.e., they connect to the node providing the highest downlink received power.

4.2.2 SINR Expression

The target of this work is to measure the SINR $\gamma(r_0)$ experienced by a UE in the network, as a function of its distance r_0 from the central BS. For the sake of

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Figure 4.2: Network model representation.

simplicity, the substitution $r = r_0$ is performed. Assume that g represents the path-gain on the channel between the considered UE and the BS, while g_R is similarly defined for the closest RN. The parameter N_{th} represents the thermal noise power.

Case 1: UE Served by the BS

If the UE is served by the BS, the interference comes from all the BSs out of the central cell and all the RNs. Hence

$$\begin{aligned}
 \gamma(r) &= \frac{P_{BS}g(r)}{\sum_{b=1}^B P_{BS}g(r_b) + \sum_{k=0}^B \sum_{i=1}^n P_{RN}g_R(r_{i,k}) + N_{th}} \\
 &= \frac{\frac{P_{BS}g(r)}{\sum_{b=1}^B P_{BS}g(r_b)}}{1 + \sum_{i=1}^n \sum_{k=0}^B \frac{P_{RN}g_R(r_{i,k})}{\sum_{b=1}^B P_{BS}g(r_b)} + \frac{N_{th}}{\sum_{b=1}^B P_{BS}g(r_b)}} \\
 &= \frac{\gamma_0}{1 + I_1 + I_2}, \tag{4.6}
 \end{aligned}$$

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where

$$\gamma_0 = \frac{P_{BSg}(r)}{\sum_{b=1}^B P_{BSg}(r_b)}, \quad (4.7)$$

$$\begin{aligned} I_1 &= \frac{\sum_{i=1}^n \sum_{k=0}^B P_{RNgR}(r_{i,k})}{\sum_{b=1}^B P_{BSg}(r_b)} \\ &= \frac{\sum_{i=1}^n \left(\sum_{k \neq k^*(i)} P_{RNgR}(r_{i,k}) + P_{RNgR}(r_{i,k^*(i)}) \right)}{\sum_{b=1}^B P_{BSg}(r_b)} \\ &= \sum_{i=1}^n \Omega_i (1 + \gamma_{i,k^*(i)}), \end{aligned} \quad (4.8)$$

$$\gamma_{i,k^*(i)} = \frac{P_{RNgR}(r_{i,k^*(i)})}{\sum_{k=0, k \neq k^*(i)}^B P_{RNgR}(r_{i,k})}, \quad (4.9)$$

$$\Omega_i = \frac{\sum_{k \neq k^*(i)} P_{RNgR}(r_{i,k})}{\sum_{b=1}^B P_{BSg}(r_b)}, \quad (4.10)$$

$$I_2 = \frac{N_{th}}{\sum_{b=1}^B P_{BSg}(r_b)}. \quad (4.11)$$

In 4.7, γ_0 can be seen as the SIR experienced by the UE if there are no RNs in the network, while $\gamma_{i,k}$ in (4.9) represents the SIR experienced by the UE if the

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network is composed by type- i RNs only, and the UE is served by RN (i, k) . Moreover, Ω_i in (4.10) is the ratio between the interference from all type- i RNs (except the closest one) and the overall power received by all BSs.

Case 2: UE Served by a Relay Node

If the UE is served by a RN, it is interfered with by all BSs and by all RNs except the server. Hence

$$\begin{aligned} \gamma(r_{j,k^*(j)}) &= \\ &= \frac{P_{RNGR}(r_{j,k^*(j)})}{\sum_{b=0}^B P_{BSg}(r_b) + \sum_{(i,k) \neq (j,k^*(j))} P_{RNGR}(r_{i,k}) + N_{th}} \\ &= \frac{\gamma_{j,k^*(j)}}{1 + \frac{1+\gamma_0}{\Omega_j} + \sum_{i \neq j} (1 + \gamma_{i,k^*(i)}) \Omega_{i,j} + I_3}, \end{aligned} \quad (4.12)$$

where

$$\Omega_{i,j} = \frac{\sum_{k \neq k^*(i)} P_{RNGR}(r_{i,k})}{\sum_{k \neq k^*(j)} P_{RNGR}(r_{j,k})}, \quad (4.13)$$

$$I_3 = \frac{N_{th}}{\sum_{k \neq k^*(j)} P_{RNGR}(r_{j,k})}. \quad (4.14)$$

In (4.13) $\Omega_{i,j}$ can be seen as the ratio of the interference from type- i and that from type- j RNs.

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4.2.3 Approximation of the SINR via Fluid Network Model

In contrast with the classical fluid model assumptions [5], the analyzed network is not homogeneous, as the distances between neighboring nodes are not constant and the transmit power is not the same for all nodes. However, the network can be decomposed into a regular BSs subnetwork and n regular RNs subnetworks (see Figure 4.2). Then, the fluid model can be applied to analyze each of these subnetworks.

The path gain $g_R(r)$ can be expressed as $g_R(r) = K_R/r^{\eta_R}$, where K_R and η_R are the path loss constant and the path loss exponent respectively. The same model is adopted for $g(r)$. Now, (4.7, 4.9, 4.10, 4.11, 4.13, 4.14) can be approximated by means of the fluid model, obtaining

$$\gamma_0 = \frac{(\eta - 2)r^{-\eta}}{2\pi\rho_{BS}(2R_{BS} - r)^{2-\eta}} \quad (4.15)$$

$$\gamma_{i,k^*(i)} = \frac{r_{i,k^*(i)}^{-\eta_R}(\eta_R - 2)}{2\pi\rho_{BS}(2R_{BS} - r_{i,k^*(i)})^{2-\eta_R}} \quad (4.16)$$

$$\Omega_i = \frac{P_{RN}K_R(2R_{BS} - r_{i,k^*(i)})^{2-\eta_R}(\eta - 2)}{P_{BS}K(2R_{BS} - r)^{2-\eta}(\eta_R - 2)} \quad (4.17)$$

$$I_2 = \frac{N_{th}(\eta - 2)}{2\pi\rho_{BS}P_{BS}K(2R_{BS} - r)^{2-\eta}} \quad (4.18)$$

$$\Omega_{i,j} = \frac{(2R_{BS} - r_{i,k^*(i)})^{2-\eta_R}}{(2R_{BS} - r_{j,k^*(j)})^{2-\eta_R}} \quad (4.19)$$

$$I_3 = \frac{N_{th}(\eta_R - 2)}{2\pi\rho_{BS}P_{RN}K_R(2R_{BS} - r_{j,k^*(j)})^{2-\eta_R}}. \quad (4.20)$$

Equations (4.15)–(4.20), together with (4.6), (4.8) and (4.12) allow to calculate the SINR experienced by a UE by only knowing r and the distance of the UE from the nearest RN of each type.

4.2.4 Validation of the Proposed Approach

Numerical Monte-Carlo simulations have been carried on to validate the proposed method for the derivation of SINR. Simulations results are compared with analytical results obtained via the fluid model-based approach. The network

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Figure 4.3: SINR CDF of a UE served by a BS ($R_R = 0.7R_{BS}$).

Figure 4.4: SINR CDF of a UE served by a RN ($R_R = 0.7R_{BS}$).

modeled in the simulations is composed by a central cell and 10 rings of cells around it, with $n = 3$ RNs per cell. Transmit powers are equal to 31 dBm for RNs and 43 dBm for BSs. Users are uniformly distributed in the central cell.

Figures 4.3 and 4.4 plot the UEs SINR CDF, comparing the results obtained with the fluid model with those of simulations. The comparison is performed for the case of UE served by BS (Figure 4.3) and for that of UE served by RN (Figure 4.4). Note that the fluid model approach closely matches numerical results.

Figure 4.5: Average SINR experienced by a UE served by a RN for $R_R = 0.4R_{BS}$, $R_R = 0.7R_{BS}$ and $R_R = R_{BS}$.

Figure 4.5 shows the SINR experienced by a UE served by a RN versus its

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distance from the server. Results are plotted for different values of the RNs-BS distance R_R . Again, results obtained via the fluid model are very close to simulations. As expected, the SINR decreases with the distance to the server, because the power received from the server decreases, while that received from the interferers increases.

In conclusion, the fluid model can be deemed a valid tool to obtain a quick approximation of the SINR experienced by UEs on the downlink, under a simplified path-loss-based propagation model.

4.3 A Geometrical Approach for RNs Optimal Transmit Power Derivation Using a Fluid Network Model

This Section introduces a geometrical method to calculate the optimal transmit power of relays in a heterogeneous network, with the aim of maximizing the average spectral efficiency. The method is based on matching as closely as possible the areas where RNs provide the highest power (and thus they serve the UEs) with the areas where RNs provide the best SIR. The latter areas can be derived by means of a fluid model of the network. Results show that the proposed method is capable of closely approximate the optimal RNs transmit power under a wide set of propagation parameters and RNs location scenarios.

4.3.1 System Model

This study analyzes a reuse-one hexagonal cellular network, enhanced by means of n RNs deployed in each cell. Both BSs and RNs transmit with omnidirectional antennae at power P_{BS} and P_{RN} respectively, and it is assumed that $P_{RN} < P_{BS}$ [2]. Relays are evenly located on a circle centered at the BS, at a distance of R_R from it (see, e.g., [27] [14]). The angle offset of the first RN of each cell with respect to a horizontal line touching the BS is denoted with

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Figure 4.6: Left: Cell scheme. Right: adopted frame structure.

ϕ (see Fig. 4.6, left). UEs are uniformly distributed over the cell of interest, according to a Spatial Poisson Point Process (SPPP), and they are served by the best server (i.e., the node providing the highest received power [89]).

Set $\mathcal{S} = \mathcal{S}_R \cup \mathcal{S}_B$ contains the indices of all network nodes, while sets $\mathcal{S}_R$ and $\mathcal{S}_B$ contain the indices of all BSs and RNs respectively. Transmissions on the downlink are performed under a division mode of operations policy, i.e., they take place in different frame sections [4]. Figure 4.6 (right) shows the resulting frame structure.

Serving Area and SIR Area Definitions

The power received by a user located in z from a node i is expressed as

$$P_i(z) = P_{tx,i} K d_{i,z}^{-\eta}, \quad (4.21)$$

where $P_{tx,i} \in \{P_{BS}; P_{RN}\}$ denotes the transmission power of i , K and η are the path loss constant and the path loss exponent respectively and $d_{i,z}$ is the distance of node i to the user. The area where node i provides the highest downlink received power to UEs, as compared to all other nodes, is named *serving area*.

In an urban environment, thermal noise is often considered to be negligible compared to the sum of the interferers received powers. This assumption is adopted in this work, in accordance with [86]. Thus, the SIR is adopted as channel quality measure. For a user located in z and served by i , the SIR is

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given by

$$\Gamma_i(z) = \frac{P_{tx,i} K d_{i,z}^{-\eta}}{\sum_{k \in \mathcal{S}_i \setminus \{i\}} P_{tx,k} K d_{k,z}^{-\eta}} = \frac{d_{i,z}^{-\eta}}{\sum_{k \in \mathcal{S}_i \setminus \{i\}} d_{k,z}^{-\eta}}, \quad (4.22)$$

where $\mathcal{S}_i$ contains all the nodes of the same type of i (i.e., base stations or relays). The area where i provides the highest SIR to UEs is named the *SIR area* of i . An illustration of the serving and SIR areas of a RN is provided in Figure 4.7, which introduces a number of notations, for the sake of further developments. In particular, $\mathcal{A}_c$ denotes the surface of the cell of interest, $\mathcal{A}_{ij}^P(P_{RN})$ (not in figure) indicates the region where the power received by i is higher than that received by BS j , $\pi_{ij}(P_{RN})$ denotes the border of $\mathcal{A}_{ij}^P(P_{RN})$ inside the cell, $\mathcal{A}_{ij}^S$ is the area where the SIR provided by i is higher than that provided by j and $\gamma_{ij}(P_{RN})$ is the border of $\mathcal{A}_{ij}^S$ inside the cell. The overall SIR area can be obtained as the union of the areas $\mathcal{A}_{ij}^S$ of all RNs i in the cell.

The shape of the SIR area does not depend on the RNs transmit power, as transmit powers are crossed out in (4.22). On the contrary, the serving area depends on P_{RN} . Hence, by varying P_{RN} its shape can be adjusted to match as closely as possible the shape of the SIR area. This concept is developed in the following.

In this study, $\mathcal{D}_{ij}(P_{RN})$ denotes the symmetric difference¹ of $\mathcal{A}_{ij}^S$ and $\mathcal{A}_{ij}^P$ inside the cell, i.e.,

$$\mathcal{D}_{ij}(P_{RN}) \triangleq \mathcal{A}_c \cap (\mathcal{A}_{ij}^S \Delta \mathcal{A}_{ij}^P). \quad (4.23)$$

Moreover, $\mathcal{D}_i \triangleq \cup_{j \in \mathcal{S}_R} \mathcal{D}_{ij}$. $\mathcal{D}_i$ denotes the region inside the cell, where UEs are served by a station different than the one providing the highest SIR.

Fluid Network Model

The SIR in the regular subnetwork represented by the BSs can be approximated using a fluid network model, Hence, for a station $i \in \mathcal{S}_B$,

$$\Gamma_i(z) \approx \frac{\eta - 2}{2\pi \rho_{BS} d_{i,z}^\eta (2R_{BS} - d_{i,z})^{2-\eta}}, \quad (4.24)$$

¹The symmetric difference Δ of two sets A and B is defined as $A \Delta B \triangleq (A \setminus B) \cup (B \setminus A)$.

Figure 4.7: Schematic representation of several definitions given in the main text.

where ρ_{BS} is the spatial density of BSs R_{BS} indicates the half of the distance between two neighboring BSs. When it comes to the non-regular network of RNs, the fluid model under the classical assumptions (i.e., network must be regular and hexagonal) is not applicable. However, this work shows that the fluid model-based approximation of the SINR for the non-regular network case can be still fairly accurate. As a replacing value for the half inter-node distance adopted in regular networks, this study uses the minimum half inter-RN distance R_{RN} . The spatial density of RNs is indicated with ρ_{RN} .

Spectral Efficiency

Spectral efficiency, achieved by a user located in z , is given by

$$C(z, P_{RN}, P_{BS}) = \log_2(1 + \Gamma_{b(z, P_{RN}, P_{BS})}(z)), \quad (4.25)$$

where $b(z, P_{RN}, P_{BS})$ denotes the serving node of a UE in z^2 . Moreover, the notation $C_H(z)$ indicates the highest spectral efficiency achievable by a user in z , i.e., the spectral efficiency experienced by the user if it is served by the

²For the sake of simplicity, the dependency of parameters on P_{BS} is omitted in the following.

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station providing the highest SIR. It follows that

$$\begin{cases} C(z, P_{RN}) = C_H(z), & \text{if } z \notin \mathcal{D}_i \\ C(z, P_{RN}) \neq C_H(z), & \text{if } z \in \mathcal{D}_i \end{cases} \quad (4.26)$$

4.3.2 Formulation of the Problem

Consider a cell $\mathcal{A}_c$ of area A_c . The problem is to maximize the average spectral efficiency in $\mathcal{A}_c$ by acting on P_{RN} . This statement is formalized as

$$P_{RN}^* = \operatorname{argmax}_{P_{RN}} \{\bar{C}(P_{RN})\}, \quad (4.27)$$

under the constraint:

$$0 < P_{RN} < P_{BS}, \quad (4.28)$$

where $\bar{C}(P_{RN}) = \frac{1}{A_c} \int_{\mathcal{A}_c} C(z, P_{RN}) dz$ denotes the spatial average of spectral efficiency over $\mathcal{A}_c$.

The target parameter $\bar{C}(P_{RN})$ cannot be higher than the value corresponding to $\mathcal{D}_i \equiv \emptyset$, as in that case all the UEs are served by the station providing the highest SIR. Nevertheless, there is normally no P_{RN} for which $\mathcal{D}_i \equiv \emptyset$, i.e., $\mathcal{A}_{ij}^P \equiv \mathcal{A}_{ij}^S, \forall i$.

Equation (4.27) can be developed into

$$\begin{aligned} P_{RN}^* &= \operatorname{argmin}_{P_{RN}} \{-\bar{C}(P_{RN})\} \\ &= \operatorname{argmin}_{P_{RN}} \{T - \bar{C}(P_{RN})\} \\ &= \operatorname{argmin}_{P_{RN}} \left\{ \frac{1}{A_c} \int_{\mathcal{A}_c} (C_H(z) - C(z, P_{RN})) dz \right\}, \end{aligned} \quad (4.29)$$

where the value of $T \in \mathbb{R}$ can be chosen arbitrarily. The last formulation is correct because $\frac{1}{A_c} \int_{\mathcal{A}_c} C_H(z) dz$ does not depend on P_{RN} . From a geometrical point of view, the integrand function is equal to zero outside $\mathcal{D}_i$. Thus, (4.29) can be

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rewritten as

$$P_{RN}^* = \underset{P_{RN}}{\operatorname{argmin}} \left\{ \frac{1}{A_c} \int_{\mathcal{D}_i} (C_H(z) - C(z, P_{RN})) dz \right\}. \quad (4.30)$$

Equation (4.30) can be geometrically solved, by locating the border of $\mathcal{D}_i$ and testing several values of P_{RN} . However, this study shows that the range of P_{RN} to be tested can be considerably reduced. In fact, under a wide number of network scenarios and propagation parameters, the range to be explored is tight enough to avoid the evaluation of (4.30).

For reasons of clarity, the geometrical solution to find P_{RN}^* is shown only for the simple network model of this Section. However, the same approach can be applied with little adjustments to more complex models.

4.3.3 Solution of the Problem

In the following, the border of $\mathcal{D}_{ij}$ is derived and the value of the spectral efficiency as a function of P_{RN} is studied. This is a necessary step to set up the algorithm for finding P_{RN}^*

Derivation of $\pi_{ij}(P_{RN})$ and γ_{ij}

The border of the serving area $\pi_{ij}(P_{RN})$ can be derived by imposing the equality between powers received from i and j , obtaining

$$\pi_{ij}(P_{RN}) : (x - x_\delta)^2 + (y - y_\delta)^2 = r^2, \quad (4.31)$$

where the axis in Figure 4.7 are adopted as a reference system, $(x, y) \in \mathcal{A}_c$ are the coordinates of the border, $r = \sqrt{\Omega(x_{RN}(j)^2 + y_{RN}(j)^2)/(\Omega - 1)^2}$, $\Omega = (P_{RN}/P_{BS})^{2/\eta}$ and $(x_{RN}(j), y_{RN}(j))$ are the coordinates of RN j . Thus, $\pi_{ij}(P_{RN})$ represents a section of a circle centered at $(x_\delta, y_\delta) = (x_{RN}/(1 - \Omega), y_{RN}/(1 - \Omega))$, on the straight line joining i and j . The radius r of the circle increases with P_{RN} .

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Following the same reasoning used to find $\pi_{ij}(P_{RN})$, γ_{ij} can be derived by imposing the equality between the expressions of the SIR provided by i and j . Under a fluid network model, this leads to

$$\gamma_{ij} : \frac{\rho_{BS}}{\rho_{RN}} \left(\frac{d_i(x, y)}{d_j(x, y)} \right)^\eta = \left(\frac{2R_{RN} - d_j}{2R_{BS} - d_i} \right)^{2-\eta}, \quad (4.32)$$

where $d_i(x, y) = \sqrt{x^2 + y^2}$ and $d_j(x, y) = \sqrt{(x - x_{RN}(j))^2 + (y - y_{RN}(j))^2}$. The SIR can also be computed by means of (4.22), if the fluid model approximation accuracy is not satisfactory. Again, the expression of γ_{ij} does not depend on P_{RN} .

Bounds on the Feasible Range of P_{RN}

Figure 4.7 intuitively shows that an increasing value of P_{RN} tends to shrink $\mathcal{D}_{ij}$ up to an extent (if the starting P_{RN} is low enough), while a further increase produces the opposite effect.

The following lemma formalizes this intuitive finding.

Lemma 2. (i) Assume that in the cell of interest $\mathcal{A}_{ij}^S \subseteq \mathcal{A}_{ij}^P(P_{RN})$ for all RNs j , then $\bar{C}(P_{RN})$ is an increasing function of P_{RN} . (ii) If $\mathcal{A}_{ij}^P(P_{RN}) \subseteq \mathcal{A}_{ij}^S$ for all RNs in the cell of interest, then $\bar{C}(P_{RN})$ is a decreasing function of P_{RN} .

Proof. For (i) take two sample values of RN power $P_2 > P_1$ that respect the condition. From (4.31), one can find that $\mathcal{A}_{ij}^P(P_2) \subseteq \mathcal{A}_{ij}^P(P_1)$. By joining the expression of $\mathcal{D}_{ij}$ in (4.23) and the first condition of this lemma, it follows that $\mathcal{D}_{ij}(P_2) \subseteq \mathcal{D}_{ij}(P_1)$ for all j . If $P_{RN} = P_2$ expression (4.30) is calculated on a subset of the area where it is calculated for P_1 . Given that the integrand is positive, it follows that $\bar{C}(P_2) \geq \bar{C}(P_1)$. For (ii) the same reasoning can be applied to prove the statement, with the difference that $\mathcal{D}_{ij}(P_1) \subseteq \mathcal{D}_{ij}(P_2)$. $\square$

Lemmas 2 imply that P_{RN}^* has a lower and an upper bound: $P_{RN}^L \leq P_{RN}^* \leq P_{RN}^U$, where P_{RN}^L is the highest P_{RN} respecting the condition $\mathcal{A}_{ij}^S \subseteq \mathcal{A}_{ij}^P(P_{RN})$,

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Algorithm 2 The algorithm designed to derive the approximate P_{RN}^*

```

1: Initialization:  $\mathcal{P}_i = \emptyset$ 
2: Set up a grid  $\mathcal{G}$  of  $N$  points paving the cell area
3: for each relay  $j$  in the cell do
4:   Find a set of points  $p_n^{ij} \in \gamma_{ij}$  using (4.22) or (4.24) on  $\mathcal{G}$ 
5:   for each  $p_n^{ij} = (x_n^{ij}, y_n^{ij}) \in \gamma_{ij}$  do
6:     Compute  $\tilde{P}_{RN}(p_n^{ij})$  using (4.33)
7:      $\mathcal{P}_i \leftarrow \mathcal{P}_i \cup \{\tilde{P}_{RN}(p_n^{ij})\}$ 
8:   end for
9: end for
10:  $(P_{RN}^L, P_{RN}^U) \leftarrow (\min \mathcal{P}_i, \max \mathcal{P}_i)$ 
11: Return  $P_{RN}^* = (P_{RN}^L + P_{RN}^U)/2$ 

```

while P_{RN}^U is the lowest P_{RN} for which condition $\mathcal{A}_{ij}^P(P_{RN}) \subseteq \mathcal{A}_{ij}^S$ holds. However, no guess can be made about the spectral efficiency when $P_{RN}^L \leq P_{RN} \leq P_{RN}^H$.

Another geometrical interpretation of the bounds is provided in the following list:

- For $P_{RN} = P_{RN}^L$ condition $\mathcal{A}_{ij}^S \subseteq \mathcal{A}_{ij}^P(P_{RN})$ is respected, but $\pi_{ij}(P_{RN})$ is tangent to γ_{ij} in at least one point.
- For $P_{RN} = P_{RN}^U$ condition $\mathcal{A}_{ij}^P(P_{RN}) \subseteq \mathcal{A}_{ij}^S$ is respected, but $\pi_{ij}(P_{RN})$ is tangent to γ_{ij} in at least one point.

Equation (4.32) cannot be expressed in closed-form. Hence, the bounds must be numerically computed. However, this study proposes a three-steps procedure for finding an approximate value of the bounds.

1. An N -points grid is defined on the cell surface, and SIR is derived on it via (4.22) or (4.24). Then, a set $\{p_n^{ij}\}_{n \in [1, N_{ij}]} \in \gamma_{ij}$ of border points is derived by interpolation.
2. The parameter $\tilde{P}_{RN}(p_n^{ij})$ denotes the value of P_{RN} for which x_n^{ij} and y_n^{ij} lie on $\pi_{ij}(P_{RN})$. This value can be found using (4.31), yielding

$$\tilde{P}_{RN}(p_n^{ij}) = P_{BS}(l_{nj}^2/l_{n0}^2)^{\frac{\eta}{2}}, \quad (4.33)$$

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Figure 4.8: Effect of n ($d = 3/8D$, $\phi = 0$, $\eta = 3.5$)

Figure 4.9: Effect of ϕ ($n = 3$, $\eta = 3.5$ and $d = 9/20D$)

where $l_{nj}^2 = (x_n^{ij} - x_{RN}(j))^2 + (y_n^{ij} - y_{RN}(j))^2$ and $l_{n0}^2 = x_n^{ij2} + y_n^{ij2}$. Now, compute $\tilde{P}_{RN}(p_n^{ij})$ for all $p_n^{ij} = (x_n^{ij}, y_n^{ij})$, and collect the values in a set $\mathcal{P}_i = \cup_j \{\tilde{P}_{RN}(p_n^{ij})\}$.

3. Calculate $P_{RN}^L = \min \mathcal{P}_i$, $P_{RN}^U = \max \mathcal{P}_i$.

The number N of grid points determines the accuracy of the approximation. In view of the fact that the bounds found with the proposed method are normally tight, the following approximation for P_{RN}^* is introduced: $P_{RN}^* \approx (P_{RN}^L + P_{RN}^U)/2$.

The approximation approach does not require the calculation of integral (4.30) on a large set of candidate values of P_{RN} . Algorithm 2 summarizes the proposed method.

4.3.4 Numerical Results

The algorithm for finding an approximation of the optimal RN power has been tested using either the approach based on a fluid network model ((4.24) and (4.32)) or the exact computation of the SIR (4.22). Performance is assessed on a network with four cells rings around the central cell. The distance between

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two neighboring BSs is set to 500 m, $P_{BS} = 46$ dBm [2]. Two alternatives are compared for our method, which differ at step 4 of the algorithm: use either the fluid model ((4.24) and (4.32)) or the exact SIR formulation (4.22). Measurements are taken on a grid over the central cell. The distance between two points of the grid is 10 m.

Performance of the proposed approach is compared to that of a brute-force optimization approach, i.e., all the values of P_{RN} between 25 and 40 dBm are tested, with a step of 0.1 dB.

Analysis of the Results

Figure 4.8 plots P_{RN}^* as a function of the number of RNs n . The optimal RNs power decreases with n . This result is expected, as more RNs imply that their SIR areas get smaller. Consequently, P_{RN} must also decrease to fit it. The proposed approximation algorithm is accurate when Equation (4.22) is used to compute the SIR, as the error compared to brute-force optimization is less than 1 dB, and normally below 0.5 dB. Larger errors are yielded by the use of a fluid model. Figures 4.9 and 4.10 show that the impact of ϕ and R_R on the accuracy of the approximate P_{RN}^* is limited. The behavior of the curves in Figure 4.10 is due to the fact that when RNs are very close, they have to lower their transmit power in order to reduce the number of served UEs. In this way, RNs do not serve UEs which are in between two RNs, and they are hence strongly interfered. Relay nodes are very close when d/D is small, and when d/D is high (and hence RNs are close to other RNs in neighboring cells).

A number of simulations with log-normal shadowing have been performed, to test the accuracy of the proposed method in this context, finding out that the algorithm still yields results which are less than 0.3 dBm away from the optimal P_{RN} , even for heavy shadowing (10 dB of shadowing standard deviation).

The main advantage of the proposed algorithm is that it requires less computations than a brute-force optimization. While a brute-force-based approach entails $O(N_P N B n)$ computations, algorithm 2 using (4.22) involves only $O(N B n)$

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Figure 4.10: Effect of R_R ($n = 3$, $\eta = 3$, $\phi = 30$)

Figure 4.11: Effect of η ($n = 6$, $d = 3/8D$, $\phi = 0$)

computations. If the fluid model is used, the number of computations drops to $O(Bn + N)$.

In conclusion, this study shows that fluid model accuracy is reduced when considering irregular networks, as highlighted in the results. The accuracy improves when n grows, when RNs are far from the BS and when ϕ grows, as these network setups make the RNs network *more regular* compared to the case where few RNs are located close to the BS.

Chapter 5

Relay Nodes Placement Optimization

This Chapter presents a study on the optimal placement of RNs in cellular networks, with the aim of maximizing cell capacity. The capacity is defined as the highest input traffic that can be carried by the network without any station being overloaded. Station loads are derived by means of a fixed point equation, because of the effect of interference from each station on the loads of all other stations. Optimization of RNs placement is carried out via an ad-hoc SA algorithm, enhanced with an adaptive temperature scheme, a multiscale implementation and *restricted image spaces* [90]. Simulations provide guidelines for RNs placement, for both the case of uniform and non-uniform traffic spatial distribution, and clarify the impact of the backhaul link on the placement of in-band RNs. This study also proposes a method to compute the interference coming from stations which are far away from the cell of interest, based on a fluid network model.

The analysis is based on a *dynamic* model of the system, i.e., UEs buffer queues at network stations and UEs arrivals/departures in the network are taken into account. On the one hand, this type of model usually leads to a more challenging analysis, compared to a static model. On the other hand, it

allows for a deeper understanding of the problem, e.g., to study the impact of interference on the load of stations, etc.

The Chapter is divided into four Sections. Downlink system model is described in Section 5.1. Then, cell capacity is derived in Section 5.2. The optimization problem and its solution are discussed in Section 5.3, while results are shown and commented in Section 5.4.

5.1 System Model

This Section is devoted to the description of the system model. The discussion includes the network model, the frame structure, the propagation model and the traffic model.

5.1.1 Network Model and Topology

The model is focused on the downlink of a reuse-one network. Cells are hexagonal and tri-sectorized, and each network sector is covered by a sector BS. The target is to deploy n RNs in every cell. Stations, i.e., BSs or RNs, are said to be *active* when they are transmitting, while they are labeled as *idle* when they do not transmit. Transmit power of sector BSs and RNs is denoted with P and P_R respectively. Set $\mathcal{B}$ contains all sector BSs, while set $\mathcal{R}$ all RNs. Fig. 5.1 shows an example of hexagonal network with RNs. In this work it is assumed that RNs location pattern in every cell is identical to that in the central cell¹, i.e., relative positions of RNs with respect to the BS do not change from cell to cell. Relays in each cell are labeled with an index $1 \cdots n$. All RNs labeled with a given index i are said to be *type i* relays. Note that the network formed by all type i RNs is a regular hexagonal network (see also Fig. 5.1).

Best server policy (see 2.2.5) is adopted to determine UEs serving stations. The region where station i provides the highest average downlink received power

¹A pattern which changes from cell to cell may improve network performance. However, this possibility is out of the scope of this study.

Figure 5.1: Sample RNs deployment with $n = 4$ relays. R denotes the cell range, and arrows point to the boresight directions of antennae.

is denoted with $\mathcal{S}_i$, and its surface is named S_i . The set $\mathcal{K}_c$ contains all the stations of cell c , while $\mathcal{A}_c \triangleq \bigcup_{i \in \mathcal{K}_c} \mathcal{S}_i$ is the region where the highest average downlink received power is provided by a station of c .

5.1.2 Frame Resource Partitioning

Transmissions on the BL take place in a dedicated part of the frame, of duration τ , while those on the AL and RL share the same resources (see Fig. 5.2). For the sake of simplicity, the total frame duration is set to 1 (this choice does not affect the results), so that transmissions on the AL and RL are dedicated a fraction $1 - \tau$ of the frame duration. For the out-of-band RNs case, τ is set to zero.

5.1.3 Propagation Model

In the following, propagation parameters corresponding to LOS propagation are marked with a superscript L , while superscript N is used for NLOS propagation. Similarly, a subscript $l \in \{B, R, D\}$ denotes a parameter referred to the BL, AL

Figure 5.2: Schematic frame representation for in-band relays (case (a)) and out-of-band relays (case (b)). The BS sector serves $p \leq n$ RNs.

and DL respectively. Consider a receiver u (which may be either a UE or a RN on the BL) and a station j transmitting to it. Parameter θ_j denotes the angle between the u - j line and the boresight direction of the antenna of j . If j is equipped with an omnidirectional antenna (i.e., j is a RN), θ_j is set to 0. Moreover, r_j represents the distance between u and j . The location of u with respect to j is then univoquely defined by the pair r_j, θ_j . Following the 3GPP assumptions for network simulations [91], the propagation gain between j and u is computed as

$$g_l(r_j, \theta_j) = \delta_l(r_j) h_l^L(r_j, \theta_j) + (1 - \delta_l(r_j)) h_l^N(r_j, \theta_j) \quad (5.1)$$

$$= \delta_l(r_j) A_l(\theta_j) \frac{K_l^L}{r_j^{\eta_l^L}} + (1 - \delta_l(r_j)) A_l(\theta_j) \frac{K_l^N}{r_j^{\eta_l^N}} X_l^{(u,j)}, \quad (5.2)$$

where $K_l^L; K_l^N$ and $\eta_l^L; \eta_l^N$ are the path-loss constants and exponents respectively, RV $\delta_l(r_j)$ is Bernoulli distributed and takes value 1 when the corresponding link is LOS, $A_l(\theta)$ is the antenna angular gain pattern (which is constant and equal to 1 for RNs, while $A_B = A_D \triangleq A$ because BS sectors use the same antenna on the BL and DL) and $X_l^{(u,j)}$ is the shadowing, which is modeled as a log-normal RV of variance σ_l^2 . LOS and NLOS probabilities are given in [91]. For any given deployment scenario, this study assumes that LOS and NLOS are fixed in time at any given location, while shadowing varies *slowly* in time (see the following).

5.1.4 Traffic Model

The arrival of UEs in the network is modeled according to a SPPP. Arrival intensity is denoted with $\lambda(s)ds$ [flows/s] for any position s , while the average flow size is indicated with π [bit/flow] (flow sizes are supposed to be independent and identically distributed). For the sake of simplicity, it is assumed that $\lambda(s)$ is the same in all cells. The average flow calls intensity in the cell of interest is indicated with $\bar{\lambda}$ and can be computed as $\bar{\lambda} = \int_{\mathcal{A}_c} \lambda(s)ds/A_c$ [flows/s/m²]. Now, a parameter $\phi(s) = \lambda(s)/\bar{\lambda}$ named *normalized flow intensity* is defined, so that it is possible to decompose $\lambda(s) \triangleq \bar{\lambda}\phi(s)$. Parameter $\phi(s)$ represents the local variation with respect to the average flow calls intensity in the cell. Now, $\omega(s) = \lambda(s)\pi$ [bit/s/m²] represents the *traffic density* in s . Similarly to $\bar{\lambda}$, $\bar{\omega} = \bar{\lambda}\pi$ [bit/s/m²] is the average traffic density in the cell, which can be rewritten in order to highlight the contribution of each cell station: $\bar{\omega} = \sum_{i=1}^{n+3} \bar{\omega}_i \frac{S_i}{A_c}$, where $\bar{\omega}_i$ is the average traffic density on the surface controlled by i : $\bar{\omega}_i \triangleq \frac{\pi}{S_i} \int_{S_i} \lambda(s)ds$.

Network stations are modeled as M/G/1/PS queues. Note that the adopted processor sharing policy corresponds to a fair frame radio resources scheduling. The *load* of the queue associated with the downlink buffer of station i is denoted with $\bar{\rho}_i \geq 0$. Following basic queuing theory, $\bar{\rho}_i$ also represents the probability for station i to be active, assumed that i is not *saturated*, i.e., $\bar{\rho}_i > 1$. This probability is equal to 1 for a saturated station. In order to formalize this concept, the Bernoulli RV α_i is introduced so that $\mathbb{P}[\alpha_i = 1] = \mathbb{E}[\alpha_i] = \min\{\bar{\rho}_i, 1\}$. All stations of the same type are assumed to have the same load. This follows from the RNs and sector BSs placement symmetry (see 5.1.1).

The spectral efficiency $C(s) > 0$ [bit/s/Hz], achieved by a user located in s and served by i at a given time instant, is a function of the experienced SINR, which depends in turn on the received interference, determined by the α_j , $j \neq i$. This dependency can be formalized by writing $C(s, \boldsymbol{\alpha}_{-i}(t))$, where $\boldsymbol{\alpha}_{-i}(t) = [\alpha_1 \cdots \alpha_{i-1} \alpha_{i+1} \cdots]$.

5.2 Performance Evaluation

The performance of a given RNs placement configuration is measured with its corresponding cell capacity, which is derived in this Section.

To this purpose, the BL dimension is first derived in 5.2.1. Then, users spectral efficiency (5.2.4) and the resulting station loads (5.2.2) are obtained, so as to assess whether the considered RNs placement can stand a given traffic intensity. In 5.2.5 a method to quicken interference computation is introduced. This method is based on approximating the interference received from *far* stations by means of a fluid network model. This is useful for both SINR and spectral efficiency evaluation.

5.2.1 Backhaul Link Planning

In the analysis of relay networks, it is desirable to determine the optimal value of the backhaul link-dedicated period of the frame, of duration τ . This derivation is carried out under the assumption that no station is saturated, and focusing on a sector b controlling p_b relays. Moreover, the parameter $\tau_{i,b}$ denotes the quota of frame resources that the sector BS needs to dedicate to transmissions towards RN i , and $\tau_b = \sum_{i=1}^{p_b} \tau_{i,b}$. The average traffic that any RN i needs to serve to its UEs can be written as $\int_{S_i} \omega(s) ds = \bar{\omega}_i S_i$. Now, if $C_{BL}(i, b)$ is the average rate of the backhaul link between b and i , the condition for this link not to be overloaded is $C_{BL}(i, b)\tau_{i,b} > \bar{\omega}_i S_i$. It follows that τ_b has the following lower bound:

$$\tau_b > \sum_{i=1}^{p_b} \frac{\bar{\omega}_i S_i}{C_{BL}(i, b)}. \quad (5.3)$$

The parameter τ_b can be also seen as the load of sector BS b backhaul link.

For the sake of synchronization between network cells, the value of τ must be equal to the highest τ_b over all network sectors: $\tau = \max_b \{\tau_b\}$. The optimal choice is to adopt the smallest τ_b that respects (5.3), so as to maximize UEs throughput. Thus,

$$\tau \triangleq \max_b \sum_{i=1}^{p_b} \frac{\bar{\omega}_i S_i}{C_{BL}(i, b)}. \quad (5.4)$$

If the spatial traffic profile is uniform, i.e., $\omega(s) = \bar{\omega} \forall s$, Equation (5.4) can be simplified into

$$\tau(\bar{\omega}) \triangleq \bar{\omega} \max_b \sum_{i=1}^{p_b} \frac{S_i}{C_{BL}(i, b)}. \quad (5.5)$$

The notation $\tau(\bar{\omega})$ is adopted for the rest of this work, so as to highlight that τ depends on the traffic input. In the case of out-of-band relaying, τ is fixed to zero.

5.2.2 Derivation of Station Loads

Lemma 3. *Station i load can be written as:*

$$\bar{\rho}_i = \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{\mathcal{S}_i} \phi(s) \mathbb{E}_{\alpha} \left[\frac{1}{C(s, \alpha_{-i})} \middle| \alpha_i = 1 \right] ds. \quad (5.6)$$

Proof.

$$\begin{aligned} \bar{\rho}_i &\stackrel{(1)}{=} \lim_{T \rightarrow \infty} \frac{1}{1 - \tau(\bar{\omega})} \frac{\int_0^T \int_{\mathcal{S}_i} \frac{\omega(s)}{C(s, \alpha_{-i}(t))} \mathbb{1}_{\alpha_i(t)=1} ds dt}{\int_0^T \mathbb{1}_{\alpha_i(t)=1} dt} \\ &\stackrel{(2)}{=} \lim_{T \rightarrow \infty} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \frac{\frac{1}{T} \int_0^T \int_{\mathcal{S}_i} \frac{\phi(s)}{C(s, \alpha_{-i}(t))} \mathbb{1}_{\alpha_i(t)=1} ds dt}{\frac{1}{T} \int_0^T \mathbb{1}_{\alpha_i(t)=1} dt} \\ &\stackrel{(3)}{=} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \frac{\mathbb{E}_{\alpha} \left[\int_{\mathcal{S}_i} \frac{\phi(s)}{C(s, \alpha_{-i})} \mathbb{1}_{\alpha_i=1} ds \right]}{\mathbb{P}_{\alpha}(\alpha_i = 1)} \\ &\stackrel{(4)}{=} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \mathbb{E}_{\alpha} \left[\int_{\mathcal{S}_i} \frac{\phi(s)}{C(s, \alpha_{-i})} ds \middle| \alpha_i = 1 \right] \\ &\stackrel{(5)}{=} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{\mathcal{S}_i} \mathbb{E}_{\alpha} \left[\frac{\phi(s)}{C(s, \alpha_{-i})} \middle| \alpha_i = 1 \right] ds. \end{aligned}$$

In (1) the average over the periods of activity of i is rewritten according to the definition, and the load is expressed as the integral over $\mathcal{S}_i$ of the local

contributions $\frac{\omega(s)}{C(s, \alpha_{-i}(t))} ds$ to the overall load. (2) has been obtained by simply decomposing $\omega(s) = \bar{\omega} \phi(s)$. (3) comes again from the definition of average, supposing that $\alpha(t)$ has a time-stationary limit α . (4) is obtained by applying the definition of conditional expectation. Finally, in (5) the integral over $\mathcal{S}_i$ has been pulled out of the average. This is justified with the fact that in the considered propagation model the shadowing at a fixed location varies *slowly* in time, compared to $\alpha(t)$. This is confirmed by, e.g., the outdoor measurements in [92], which claims that the result is to be attributed to the static nature of most of the elements causing shadowing (e.g., buildings, trees, etc.). To summarize, the average is computed over a time interval of sufficient duration for the M/G/1/PS queue to reach the steady-state. However, shadowing at a given position s is considered fixed. Hence, the integral over $\mathcal{S}_i$ is pulled out from the average over $\alpha(t)$. $\square$

Now, a vector ρ is defined, which collects the loads of the n RNs and 3 sector BSs of the central cell, i.e., $\rho \triangleq (\bar{\rho}_1, \dots, \bar{\rho}_{n+3})$, and an operator $F(\rho, \bar{\omega})$ is also defined so that $F(\rho, \bar{\omega}) = (F_1(\rho, \bar{\omega}), \dots, F_{n+3}(\rho, \bar{\omega}))$, where

$$F_i(\rho, \bar{\omega}) = \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{\mathcal{S}_i} \phi(s) \mathbb{E}_{\alpha} \left[\frac{1}{C(s, \alpha_{-i})} \middle| \alpha_i = 1 \right] ds, \quad (5.7)$$

and $\forall i, \mathbb{P}_{\alpha}[\alpha_i = 1] = \mathbb{E}_{\alpha}[\alpha_i] = \min\{\bar{\rho}_i, 1\}$. Moreover, a parameter $\bar{\omega}_i^{max}$ is introduced so that $\bar{\omega}_i^{max} \triangleq \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{\mathcal{S}_i} \frac{\phi(s) ds}{C(s, 1)}$.

Theorem 1. *If $\alpha \mapsto C(s, \alpha)$ is a continuous mapping and $1/C(s, \alpha)$ is non-decreasing in α , $F : \prod_{i=1}^{n+3} [0; \bar{\omega}_i^{max}] \rightarrow \prod_{i=1}^{n+3} [0; \bar{\omega}_i^{max}]$ has at least one fixed point.*

Proof. Consider a station i and a load vector $\rho \in \prod_{i=1}^{n+3} [0; \bar{\omega}_i^{max}]$. The following

holds:

$$\begin{aligned}
 0 \leq F_i(\boldsymbol{\rho}, \bar{\omega}) &\stackrel{(1)}{=} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{S_i} \phi(s) \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{h=1}^N \left(\frac{1}{C(s, \boldsymbol{\alpha}_{-i}(h))} \right) ds, \quad (5.8) \\
 &\stackrel{(2)}{\leq} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{S_i} \phi(s) \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{h=1}^N \left(\frac{1}{C(s, \mathbf{1})} \right) ds, \\
 &\stackrel{(3)}{=} \frac{\bar{\omega}}{1 - \tau(\bar{\omega})} \int_{S_i} \frac{\phi(s)}{C(s, \mathbf{1})} ds, \\
 &\stackrel{(4)}{=} \bar{\omega}_i^{max},
 \end{aligned}$$

where $\boldsymbol{\alpha}_{-i}(h)$ denotes the h -th realization of $\boldsymbol{\alpha}_{-i}$. In (1) the definition of expected value has been applied. (2) comes from the argument that $1/C(s, \boldsymbol{\alpha}_{-i}(h)) \leq 1/C(s, \mathbf{1})$, for any $\boldsymbol{\alpha}_{-i}$. (3) is the result of the average of several terms $(1/C(s, \mathbf{1}))$ with equal value, while in (4) the definition of $\bar{\omega}_i^{max}$ is used. From (4), the fixed point theorem of Brouwer can be applied to state the existence of at least one fixed point. $\square$

The authors of [93, Corollary 1] follow a similar reasoning, in the context of a single-hop network. This work, as well as [93], cannot state the uniqueness of the fixed point, but only its existence (which may not be in the interval $[0, 1)^{n+3}$). In order to model a network with increasing traffic, the choice adopted here and in [93] is to start fixed point iterations from an isolated cell with no interference.

Summarizing, the fixed point iterations are used to get the loads of all stations, given the RNs locations, a traffic spatial pattern $\phi(s)$ and an input traffic $\bar{\omega}$.

5.2.3 Cell Capacity

In this study, the *capacity* of the considered cell is defined as the highest traffic density that can be supported by the cell, provided that no station is saturated:

$$C_{cell} \triangleq \bar{\omega}^{max} A_c [\text{bit/sec/Hz/cell}], \quad (5.9)$$

where

$$\bar{\omega}^{max} = \max\{\bar{\omega} \in \mathbb{R}_+ : F(\boldsymbol{\rho}, \bar{\omega}) = \boldsymbol{\rho} \text{ and } \boldsymbol{\rho} \in [0, 1]^{n+3}\}. \quad (5.10)$$

Given $\phi(s)$ and the RNs locations, the value of $\bar{\omega}^{max}$ can be found by solving (5.7) for several values of $\bar{\omega}$, and finding the highest value that results in no saturated station (e.g., via a dichotomous search).

5.2.4 Derivation of SINR and definition of Spectral Efficiency

Spectral efficiency corresponding to a given location and a given $\boldsymbol{\alpha}_{-i}$ is obtained via a capped Shannon formula:

$$C(s, \boldsymbol{\alpha}_{-i}) = \min \left\{ \log_2(1 + \gamma_s(\boldsymbol{\alpha}_{-i})), \tilde{C} \right\}. \quad (5.11)$$

In (5.11), $\gamma_s(\boldsymbol{\alpha}_{-i})$ represents the SINR in s and $\tilde{C}$ the highest spectral efficiency that can be achieved by the adopted MCS. This function respects the conditions of theorem 1.

The SINR $\gamma_s(\boldsymbol{\alpha}_{-i})$ experienced by a user at location s for a given realization of $\boldsymbol{\alpha}_{-i}$ is expressed as

$$\gamma_s(\boldsymbol{\alpha}_{-i}) = \frac{P_i(s)}{I_s(\boldsymbol{\alpha}_{-i}) + N_{th}}, \quad (5.12)$$

where $P_i(s) = \max_{j \in \mathcal{B} \cup \mathcal{R}} P_j(s)$, N_{th} is the thermal noise power and $I_s(\boldsymbol{\alpha}_{-i}) = \sum_{j \in \mathcal{B} \cup \mathcal{R}, j \neq i} \alpha_j P_j(s)$.

5.2.5 Interference sum Approximation via Fluid Model of Far-Away Network Stations

The computation of SINR via (5.12) must be performed frequently in order to derive cell capacity, for any given RNs placement. Indeed, SINR is calculated at

each cell location for each α , and the computation is repeated for each iteration of the fixed point and each tested $\bar{\omega}$. Thus, a quick SINR evaluation can sensibly speed up the model implementation. At the same time, result accuracy has to be ensured. For this reason, interference computation must consider the contribution of a large amount of stations outside the central cell. Otherwise, edge effects [94] may lead to significantly wrong results. However, considering many stations outside the central cell dramatically increases the computational burden, and the wraparound method [95, 96] is of no help in this context, as it does not improve interference estimation accuracy in the central cell (which is here the focus of capacity estimation).

This work proposes a novel method to compute the interference, based on decomposing the overall interference sum into two contributions, based on the distance of stations from the central cell. The contribution of nearby stations is computed exactly, while that of far-away stations is approximated by means of a customized fluid network model for the case of RNs networks.

To this purpose, the *nearby BSs* are defined as those BSs belonging to the central cell and to the nearest six hegaonal cells, while all other BSs are considered to be *far-away BSs*. Sets $\mathcal{B}_c$ and $\mathcal{B}_f$ denote the set of nearby and far away BSs respectively, so that $\mathcal{B} = \mathcal{B}_c \cup \mathcal{B}_f$. Sets $\mathcal{R}_c$ and $\mathcal{R}_f$ are defined likewise for RNs. All definitions are illustrated in Figure 5.3.

Now, the contribution of nearby stations to the overall interference is named $I_{s,c}(\alpha_{-i})$, while $I_{s,f}(\alpha_{-i})$ denotes that of far-away interference, so that

$$I_s(\alpha_{-i}) = I_{s,c}(\alpha_{-i}) + I_{s,f}(\alpha_{-i}) \quad (5.13)$$

where $I_{s,c}(\alpha_{-i}) = \sum_{j \in \mathcal{B}_c \cup \mathcal{R}_c, j \neq i} \alpha_j P_j(s)$ and $I_{s,f}(\alpha_{-i}) = \sum_{j \in \mathcal{B}_f \cup \mathcal{R}_f, j \neq i} \alpha_j P_j(s)$.

Nearby stations interference $I_{s,c}(\alpha_{-i})$ is calculated as in its definition. Instead, interference $I_{s,f}(\alpha_{-i})$ from far-away stations is approximated with its mean with respect to α and shadowing:

$$I_{s,f}(\alpha_{-i}) \approx \mathbb{E}_{\alpha, X} [I_{s,f}(\alpha_{-i})] = \sum_{j \in \mathcal{B}_f} \mathbb{E}_{\alpha, X} [\alpha_j P_j(s)] + \sum_{k \in \mathcal{R}_f} \mathbb{E}_{\alpha, X} [\alpha_k P_k(s)], \quad (5.14)$$

Figure 5.3: Fluid model of far-away stations network.

where the state (i.e., active or idle) of far-away stations is assumed to be independent of α_i .

In view of the typical parameters of shadowing and LOS probability (e.g., [91, 2]), it is assumed that a UE located in the central cell receives the highest downlink power from a nearby station. Simulations performed for this study and based on [91] have shown that the percentage of UEs served by a far-away station is less than 1%, corroborating this assumption. Moreover, all far away stations are assumed to be in NLOS with respect to central cell UEs. This is also reasonable, considering the relation of LOS probability with the distance (see again [91, 2]).

Thus, far-away interference $I_{s,f}(\alpha_{-i})$ is approximated as

$$\begin{aligned}
 I_{s,f}(\alpha_{-i}) &\approx \sum_{j \in \mathcal{B}_f} \exp(a\sigma_D^2/2) \mathbb{E}_\alpha \left[\alpha_j P K_D^N r_{s,j}^{-\eta_D^N} A(\theta) \right] + \sum_{k \in \mathcal{R}_f} \exp(a\sigma_R^2/2) \mathbb{E}_\alpha \left[\alpha_k P_R K_R^N r_{s,k}^{-\eta_R^N} \right] \\
 &= \exp(a\sigma_D^2/2) \sum_{k=1}^3 \min\{1, \bar{\rho}_k\} \sum_{j \in \mathcal{B}_{f,k}} P K_D^N r_{s,j}^{-\eta_D^N} A(\theta) \\
 &\quad + \exp(a\sigma_R^2/2) \sum_{h=1}^n \min\{1, \bar{\rho}_h\} \sum_{k \in \mathcal{R}_{f,h}} P_R K_R^N r_{s,k}^{-\eta_R^N}, \tag{5.15}
 \end{aligned}$$

where $r_{s,j}$ represents the distance between j and the considered location s , $\mathcal{B}_{f,k}$ includes all type- k far-away BSs, $\mathcal{R}_{f,h}$ is similarly defined for type- h RNs and $a = \ln(10)/10$. The values of K_D^N , η_D^N , K_R^N and η_R^N in (5.15) correspond to NLOS propagation.

The values of $\sum_{j \in \mathcal{B}_{f,k}} P K_D^N r_{s,j}^{-\eta_D^N} A(\theta)$ and $\sum_{k \in \mathcal{R}_{f,h}} P_R K_R^N r_{s,k}^{-\eta_R^N}$ are approximated by means of a fluid network model. For a detailed explanation of the fluid model for cellular networks, readers can refer to 4.1. The fluid model is here applied to each network k of type- k sector BSs $\mathcal{B}_{f,k}$, $k \in \{1 \cdots 3\}$ and to each network h of type- h RNs $\mathcal{R}_{f,h}$, $h \in \{1 \cdots n\}$. Consider, e.g., the sum $\sum_{j \in \mathcal{B}_{f,k}} P K_D^N r_{s,j}^{-\eta_D^N} A(\theta)$, $k \in \{1 \cdots 3\}$. Following [97], the type- k far-away BSs sectors network is replaced by a ring-shaped continuum of sector BS, centered at s (see Figure 5.3). This leads to approximate

$$\begin{aligned}
 \sum_{j \in \mathcal{B}_{f,k}} P K_D^N r_{s,j}^{-\eta_D^N} A(\theta) &\approx \rho_{BS} \int_{R_c - r_0}^{\infty} \int_0^{2\pi} \left(P K_D^N r^{-\eta_D^N} A(\theta) \right) r dr d\theta, \\
 &= \rho_{BS} b \frac{P K_D^N}{\eta_D^N - 2} (R_c - r_0)^{2 - \eta_D^N}, \tag{5.16}
 \end{aligned}$$

where ρ_{BS} denotes the density of BSs sites (which is one third of the density of BS sectors), $b = \int_0^{2\pi} A(\theta) d\theta$, r_0 represents the distance between cell center and s , and R_c the distance between the cell center and the closest far-away BS.

Similarly, it is possible to approximate $\sum_{k \in \mathcal{R}_{f,h}} P_R K_R^N r_{s,k}^{-\eta_R^N}$, $h \in \{1 \cdots n\}$ by

applying the fluid model, yielding

$$\begin{aligned} \sum_{k \in \mathcal{R}_{f,h}} P_R K_R^N r_{s,k}^{-\eta_R^N} &\approx \rho_{BS} \int_{R_c - r_h}^{\infty} \int_0^{2\pi} \left(P_R K_R^N r^{-\eta_R^N} \right) r dr d\theta, \\ &= 2\pi \rho_{BS} \frac{P_R K_R^N}{\eta_R^N - 2} (R_c - r_h)^{2-\eta_R^N}. \end{aligned} \quad (5.17)$$

In (5.17), the parameter r_h measured the distance between the central cell type- h RN and s .

By joining the derivations in (5.16) and (5.17), the overall interference $I_s(\alpha_{-i})$ can be approximated as

$$\begin{aligned} I_s(\alpha_{-i}) &\approx \sum_{j \in \mathcal{B}_c \cup \mathcal{R}_c, j \neq i} \alpha_j P_j(s) + \exp(a\sigma_D^2/2) \sum_{k=1}^3 \min\{1, \bar{\rho}_k\} \rho_{BS} b \frac{P K_D^N}{\eta_D^N - 2} (R_c - r_0)^{2-\eta_D^N} \\ &\quad + \exp(a\sigma_R^2/2) \sum_{h=1}^n \min\{1, \bar{\rho}_h\} 2\pi \rho_{BS} \frac{P_R K_R^N}{\eta_R^N - 2} (R_c - r_h)^{2-\eta_R^N}. \end{aligned} \quad (5.18)$$

The adoption of this approximation technique reduces the computational burden introduced by interference calculation. In fact, only $n + 1$ closed form expressions need to be used to obtain the interference from far-away stations. In the discrete case, assuming a network of N_R rings of cells around the central cell, it would be necessary to calculate the power received from $(n + 3) \sum_{i=N_f}^{N_R} 6i$ far-away stations, where N_f is the smallest ring of far-away stations.

5.3 RNs Placement-Based Optimization for Down-link Capacity Maximization

This section addresses the problem of finding the optimal relay locations. A customized Metropolis-Hastings SA algorithm is adopted to search for the configuration of RNs which performs best, in terms of its related cell capacity C_{cell} (see Section 5.2 and (5.9)). The section begins with the description of the

variables involved in the optimization. The, the basic SA algorithm is briefly recalled, and the adopted enhancements are described.

5.3.1 Problem Formalization

The network *configuration* x is defined by the position of all RNs in the central cell, while the *configuration space* Ω is the set of all feasible network configurations. Relays are assumed to lay on a hexagonal grid covering the whole central cell (see Fig. 5.4). The hexagonal grid is chosen so as to better fit the cell shape. The problem of optimally placing RNs can be seen as a generalization of the capacitated facility location problem. For this reason, it is NP-hard [31], and the cardinality of the configuration space is prohibitive even for reasonable values of n . Consider, e.g., a grid of $N = 1024$ points and $n = 6$ RNs. In this case, $|\Omega| = C_n^N = C_6^{1024} > (1000)^6/6! > 10^{15}$. Thus, finding the optimal configuration via exhaustive search over Ω is unfeasible, and the search is carried on by means of stochastic optimization techniques.

In this study, the parameter $U(x) : \Omega \mapsto \mathbb{R}$ denotes the energy function, i.e., the function that assigns an energy value to a given network configuration x . The energy function is chosen to be the inverse of cell capacity (see (5.9)):

$$U(x) = -C_{cell}(x). \quad (5.19)$$

The configuration with minimum energy over Ω is found by means of the SA algorithm, which is briefly recalled in the following.

5.3.2 The Generic SA Algorithm

The SA is a metaheuristic aimed at solving large non-convex problems, which has been first proposed by Metropolis [98], rediscovered later [99] and then applied until nowadays on a wide range of optimization problems involving a non-trivial energy landscape (see, e.g., [100, 101, 102]). The literature shows

that the SA is an effective algorithm, if its parameters are appropriately tuned (see, e.g., [90]).

now, consider the problem of minimizing $U(x)$ over the configuration space Ω . The SA explores only a subset $\Psi \subset \Omega$ of the configuration space, where usually $|\Psi| \ll |\Omega|$, and is able to find the optimal configuration by means of an *appropriate* selection of the analyzed configurations.

The algorithm proceeds by assigning to each configuration x an exponential probability, given by

$$\mathbb{P}(x) = \frac{\exp(-U(x))}{\sum_{y \in \Omega} \exp(-U(y))}, \forall x \in \Omega. \quad (5.20)$$

Hence, the solution is the x that maximizes $\mathbb{P}(x)$.

According to the Metropolis-Hastings variant [103] of the SA, the set Ψ of configurations to be analyzed is determined according to the following procedure:

- At step $m = 0$ a configuration $x(0) \in \Omega$ is arbitrarily selected and designated as *current solution* $x^*(0)$ for step 0: $x^*(0) \leftarrow x(0) \in \Omega$.
- At any step $m \geq 0$: assume $x^*(m-1)$ is the current solution for step $m-1$. The SA will first draw a new *candidate* solution $x'(m) \in \Omega$ for step m , which complies with the given *proposal law* $r(x^*(m-1) \rightarrow x'(m))$ (defined by the algorithm designer). Then, $x'(m)$ will be accepted as current solution for step m with probability

$$\begin{aligned} \mathbb{P}(x^*(m) \leftarrow x'(m)) = \\ \min \left\{ 1, \exp \left(-\frac{[U(x'(m)) - U(x^*(m-1))]}{T_m} \right) \frac{r(x^*(m-1) \rightarrow x'(m))}{r(x'(m) \rightarrow x^*(m-1))} \right\}, \end{aligned} \quad (5.21)$$

If the proposal law is symmetric, Equation (5.21) simplifies into

$$\mathbb{P}(x^*(m) \leftarrow x'(m)) = \min \left\{ 1, \exp \left(-\frac{[U(x'(m)) - U(x^*(m-1))]}{T_m} \right) \right\}. \quad (5.22)$$

where T_m is a parameter of the SA, usually called *temperature*, which is supposed to decrease to zero *slowly enough* as $m \rightarrow +\infty$, and respect the relation $T_m \geq \frac{T_0}{1+\log(m+1)}$. If $x'(m)$ is not accepted, then $x^*(m) \leftarrow x^*(m-1)$.

In most practical implementations of the SA, the temperature is updated following a law of the kind $T_m = T_0 \cdot h^m$, $h < 1$, where h is *close* to 1.

The SA algorithm has been adopted in this work because it can be easily tuned to adapt to different energy landscapes, by acting on the temperature schedule. This is a particularly useful feature, as the energy surface characteristics vary consistently when P_{RN} or n are varied. Moreover, the SA effectiveness in the considered optimization scenario can be improved by means of a number of techniques (either ad-hoc or already available in the literature), which are described in the following.

5.3.3 Adopted Enhancements of SA Algorithm

In this work, the basic SA algorithm (see 5.3.2), is enhanced with several extensions and variants. In the following, the most important of them are listed:

"Restricted Image Spaces" [90]

In order to increase the effectiveness of the search, the candidate configuration $x'(m)$ is chosen to be close to the current solution. This is practically achieved in this work by changing the position of only one RN from one iteration to the next, and placing it close to its previous location. In particular, the new location for the considered RN is drawn from a continuous gaussian probability

distribution function, centered at the previous location:

$$r(\xi_i, \xi'_i) = \frac{1}{2\pi} \frac{1}{\zeta^2} \mathcal{N}(\|\xi'_i - \xi_i\|; \zeta^2).$$

where, ζ regulates the *freedom* of movement of RNs, and can be varied throughout the iterations, (Fig. 5.4). Once the new position in the continuous space is drawn, the closest node of the discrete RNs placement grid is selected as the position for the RN in the candidate configuration.

In case a candidate location falls out of the cell, it is *periodized* inside it (Fig. 5.4). In this way, the proposal law in (5.21) cancels down, and the acceptance ratio (5.22) still applies.

Figure 5.4: The hexagonal grid of candidate RN positions covering the whole cell. The parameter ξ_i denotes the present location. In case the proposed new position ξ'_i is outside the cell, then it is periodized inside it.

Adaptive Temperature Scheme

The optimal initial temperature T_0 for the SA, according to n and to the network setup is found via an adaptive algorithm, that works by trying several candidate temperatures and estimating their corresponding acceptance ratio.

An estimation method to find the optimal T_0 has been already proposed in [90]. However, [90] assumes knowledge of the energy landscape, which is not given in the context of this work.

The adaptive algorithm proceeds by steps:

- Search of an appropriate T_0 : The initial temperature is found by testing several candidate values. For each candidate T_0 , the related average acceptance ratio Ξ is estimated through a series of preliminary iterations of the SA (the number of iterations depends on the size of the configuration space). Then, if the estimated Ξ falls into a desired range $[a, b]$ (which is equal to $[0.5, 0.8]$ in this work) the candidate temperature is accepted as the initial temperature. Otherwise, a new candidate T_0 is computed via a dichotomous-like update:

$$T_0 \leftarrow \beta T_0 \quad \text{if } \Xi > b \quad \text{and} \quad T_0 \leftarrow \frac{0.5}{\beta} T_0 \quad \text{if } \Xi < a.$$

- Temperature schedule setup: Once the appropriate T_0 is found the number of temperature steps M^l and the temperature schedule T_M^l [90] are computed for each scale $l = 1, 2$ (see next paragraph):

$$T_m^l = T_0^l \times (T_M^l / T_0^l)^{m/M^l}. \quad (5.23)$$

Multiscale Implementation

The use of multiscale algorithms has been introduced in the past for solving problems in the field of applied mathematics and image processing. In the case of image processing, it is worth mentioning the paper [104], where a number of strategies for multiscale implementation are proposed, and [105], which proposes a fast algorithm for segmentation and detection of sequences in video processing. As an example of multiscale implementation for video processing, the movement of a hand can be decomposed into a large scale translation and rotation of the hand, and the small scale detection of fingers movements.

The same approach is used in this work, where a first optimization of RNs positions is performed on a large hexagonal grid, and the optimal positions of RNs on a finer grid are then found in a second phase. The two optimizations, i.e.,

coarse and fine scale optimization, are carried on with a dedicated SA algorithm for each of them. A multiscale implementation entails two main advantages. On the one hand, transitions between symmetry-invariant configurations at a coarse level are avoided during the second phase. On the other hand, the algorithm is more robust to local energy peaks due to shadowing and LOS conditions.

5.4 Results

This section introduces the assumptions that have been adopted in numerical simulations (see 5.4.1), validates the proposed SINR model (see 5.4.2), shows the obtained results and comments them (see 5.4.3).

5.4.1 Simulation Assumptions

Simulation assumptions are grouped according to simulation modules, for the sake of clarity.

Spectral Efficiency Computation

Simulations in this work are carried on in compliance with [91, Appendix: Simulation Assumptions] (case 3), for what concerns propagation parameters (see (5.2)) and network layout.

Transmit powers of BSs and RNs are set to 46 dBm and 30 dBm respectively, while the cell range is equal to 1 km. Regarding shadowing, standard deviation is set to 10, 8 and 6 dB for AL, DL, and BL respectively (in case of NLOS conditions). Correlation between shadowing coming from different sites is considered [13], and the correlation distance is set to 50 m. The Cholesky decomposition is used to compute correlated shadowing realizations [106].

The integrals over $\mathcal{S}_c$ are approximated by computing SINR over a discrete grid of Measurement Points (MPs). Similarly to the RNs placement grid, this

grid is hexagonal. However, it is finer (spacing between points is equal to 25 m), so as to obtain accurate results.

Equation (5.11), yielding the spectral efficiency for a given SINR, is approximated by means of a more realistic MCS which is mentioned in [107]. By applying a real MCS, the possibility of *outage*, i.e., the SINR experienced by the UE is lower than a given threshold, arises. If this is the case, the MP does not contribute to the load. However, if a given RNs placement yields an outage probability higher than 1%, its associated configuration will not be accepted as the current solution (see [108, 109]).

Fixed Point Iteration

The mean spectral efficiency experienced at each MP is approximated with the average over a hundred realizations of α . Equation $F(\rho, \bar{\omega}) = \rho$ is iterated a number of times so as to achieve convergence (in the considered context, six iterations are normally enough to converge). After convergence to the fixed point is achieved, the existence of saturated stations is verified. If this is the case, then $\bar{\omega} > \bar{\omega}^{max}$. If not, then $\bar{\omega} < \bar{\omega}^{max}$ (see (5.10)). The dichotomous update process implemented to find $\bar{\omega}^{max}$ is set up to ensure that the estimation error for C_{cell} is not more than $\pm 2.3 \times 10^{-3}$ [bit/s], which corresponds, on a bandwidth of 10 MHz, to ± 23 [kbit/(s $\times$ cell)].

Simulated Annealing

RNs are placed on a hexagonal grid which covers the whole central cell, and two different RNs cannot be placed on the same point on the grid. The optimization algorithm explores several candidate placements, in order to elect the best performing one, starting from a coarse grid (see Section 5.3) with a 200 m spacing between MPs. In the second phase, the spacing reduces to 50 m, and each RN is not allowed to move more than 300 m away from its optimal position on the coarse grid. The number of configurations explored by the SA on each grid depends on n , e.g., for $n = 3$, the SA schedule entails $M^1 = 30$ temperature

Figure 5.5: SINR CDF, proposed SINR approximation vs. 3GPP results [91] (static case, $n = 0$ or 3 relays).

steps, and for each of them the algorithm explores 250 configurations. During the second phase, $M^2 = 20$ and 150 placements are evaluated at each temperature step. In case the acceptance rate results to be zero for two consecutive temperature steps, the algorithm stops, and the current best configuration is elected as final solution. Although the SA is properly set up and tested, the obtained solutions are *optimized* rather than *optimal*, as proper optimality is actually a theoretical concept, and can be reached only performing the search over an infinite number of configurations.

5.4.2 Model Validation

The SINR approximation method presented in Section 5.2.5 needs to be validated. Figure 5.5 shows the SINR CDF corresponding to $\alpha(t) = 1, \forall t$, i.e., a static scenario. This is done in order to compare the results with those derived in [91] (which uses the same assumptions). Note that there is a good matching between the SINR CDF computed with the proposed model for interference computation, and that obtained in [91] by means of a purely discrete network model. The result can be explained by the low shadowing standard deviations, the shadowing correlation and the negligible probability for UE in the central cell to be served by a far-away station.

5.4.3 Simulation Results

Figure 5.6 plots the average distance of the RNs (in the optimized locations) from the cell center versus the number of RNs, for in-band and out-of-band relays and different transmit powers. Note that out-of-band RNs tend to be located farther from the cell center. This is because of the absence of a backhaul link-related constraint, that makes relays free to be placed in the cell regions close to the border, where users experience the worst interference conditions. When n increases, the distance to cell center tends to reduce, because of a sort of repulsion effect produced by mutual interference between RNs, that pushes some RNs far-away from the cell border, where other RNs are placed. For a high n , a second ring of RNs around the central BS may appear. A similar repulsion effect seems to be triggered by increasing P_R , for $n \geq 5$, probably for the same reason of mutual interference between RNs. Results for the in-band case highlight a trade-off between the need to serve cell-edge UEs and that of guaranteeing a good backhaul channel quality, by staying close to the BS. A decrease of the average distance to the cell center is observed for growing P_R . This is because an increased RNs power implies that more UEs are served by RNs, especially when only few RNs are deployed. Hence, a higher backhaul capacity is needed to serve these UEs, which implies that RNs have to get closer to the BSs.

In Figure 5.7 the average distance between RNs (optimized location) and the cell center is plotted against the number of RN. The target is to show the effect of shadowing on RNs positioning, for both the in-band and the out-of-band case. Shadowing seems not to meaningfully affect out-of-band RNs positioning. Indeed, the UEs serving station selection criteria is capable of compensating channel quality variations due to shadowing. However, shadowing has a great impact in the in-band case. In fact, RNs are not served by the station providing the highest received power, but their serving station is defined before the deployment.² For this reason, RNs BL can be heavily interfered, especially

²The authors of [13] have hypothesized a RNs network where relays are served by the best BS. However, the standard does not consider this possibility.

Figure 5.6: Average distance between RNs and the center of the cell, versus n (in-band and out-of-band, $P_R = 26$, 30 and 46 dBm).

Figure 5.7: Average distance between RNs and the center of the cell, versus n (in-band and out-of-band, $P_R = 46$ dBm).

when relays are far-away from the central BS. Hence, RNs tend to move very close to it so as to enjoy higher LOS probability and a better MCS.

Several samples of RNs optimized placements are provided in Figure 5.8 and 5.9, for the out-of-band and the in-band case respectively. RNs tend to cover the cell edge in the out-of-band case, and when very few RNs are deployed, they tend to cover the cell corners (this case is not displayed). When n increases, a part of the RNs tend to move towards the cell center, and for bigger values of n these relays form a second ring of RNs around the central BS. Note how RNs in the central cell tend to be placed coherently with those in the neighboring cells (see, e.g. the case with $n = 4$ for the out-of-band case, where RNs in the central cell form a regular pattern with those in neighboring cells). Relays are closer to the BS in the in-band case, and the sector BSs antenna boresight direction seems to play a major role in their positioning, contrary to the out-of-band case. In particular, there is a tradeoff between the need to enjoy a high BL rate and that of covering the regions between two sectors, where UEs experience higher interference.

Figure 5.10 plots the cell capacity corresponding to the optimized RNs placement versus n , for varying P_R and for in-band and out-of-band RNs. In the latter case, any increase in n or P_R corresponds to a cell capacity increase.

Figure 5.8: Relays positions resulting from the proposed optimization algorithm, for the out-of-band case ($P_R = 30$ dBm, $n = 2, 4$ and 6 , empty circles represent RNs, filled circles represent BSs).

Figure 5.9: Relays positions yielded by the proposed optimization algorithm, for the in-band case ($P_R = 30$ dBm, $n = 2, 3$ and 4 , empty circles represent RNs, filled circles represent BSs).

This is due to the absence of a backhaul constraint, that makes the out-of-band case similar to a simple *network densification*, which intuitively increases cell capacity (see, e.g., [82]). An increasing P_R leads to an increasing number of UEs served by RNs. This tends to offload the BS, which is thus able to support a higher traffic density. In fact, sector BSs control bigger regions compared to RNs (because of the more favorable propagation conditions towards UEs), and thus they represent the bottleneck in terms of load. Hence, their offloading increases cell capacity (see (5.10)). Results for the in-band case are the product of two opposite effects. As in the out-of-band case, any increase in P_R tends to offload sector BSs, and thus to increase cell capacity. However, this also

Figure 5.10: C_{cell} versus n (in-band and out-of-band, for several P_R).

Figure 5.11: C_{cell} versus n (in-band and out-of-band, $P_R = 30$ dBm).

increases τ , as RNs need to provide a higher quantity of data to their UEs. Results show that the positive effect of densification slightly overcomes the effect of the increase of τ . However, the displayed trends may change by varying the simulations assumptions, e.g., assuming different propagation parameters for the BL. These results are in contrast with those obtained in [12], where the capacity is shown to drop for $n > 3$. This can be attributed to the fact that [12] is based on a static system model, and assumes that RNs are regularly placed on a circle around the cell center.

Figure 5.11 plots the value of cell capacity versus number of RNs, and focuses on the effect of shadowing. The figure corroborates the findings of Figure 5.7, showing that shadowing impact on out-of-band RNs is limited, while it meaningfully affect in-band RNs networks capacity.

The benefit of deploying out-of band RNs is highlighted by the displayed results, while in-band RNs do not perform as well in the considered scenario. In particular, a smaller capacity gain is observed, and the positioning close to the BS favors UEs which may already benefit of good radio conditions when RNs are not deployed. However, in-band RNs can be more convenient if the capacity of the BL is higher because of good propagation [16] or the adoption of, e.g., RN cell selection [13].

Figure 5.12 shows the effect of a non-uniform traffic spatial density on the optimized RNs positioning. Here $\phi(s)$ is modeled according to a bivariate normal

Figure 5.12: Relays positions yielded by the proposed optimization algorithm, for the out-of-band and in-band case, adopting a non-uniform traffic spatial distribution ($P_R = 30$ dBm, $n = 3$, BS is at the center of the cell, filled circles represent RNs).

distribution centered in $(0.3, 0.4)$ km. The standard deviation of the distribution is equal to 0.3 km. The distribution is highlighted by plotting its iso-level curves on the figure. The colors of the iso-level curves depend on the corresponding value of the bivariate (red denotes the highest values, blue the lowest ones). RNs optimized positions are close to the hotspot in both the in-band and the out-of-band case. However, In the out-of-band case RNs are placed close to the cell border so as to cover the regions with the most unfavorable SINR, while in the in-band case RNs positions reflect a trade-off between the need to cover the hotspot and that of having a backhaul rate high enough to serve their UEs.

The presented results confirm and validate the intuition, for what concerns RNs placement. Moreover, they quantify the achievable downlink capacity gain, which is particularly important in order to assess the benefit of RNs deployment.

It is interesting to evaluate the accuracy of the results obtained with the SA algorithm. However, the literature offers little material about it. Moreover, results are derived assuming that the algorithm designer knows the characteristics of the energy surface to a certain extent, which is not the case here. As an example, it is worth mentioning the study in [110], which is based on the application of large deviation theory in a probabilistic context, and exploits

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some information on the energy landscape in order to assess the distance of the optimized solution with respect to the optimal one. This does not apply to the case of this work, because the complexity of the problem is such that it is prohibitive to make any reasonable guess on the energy surface.

Chapter 6

Uplink Energy Efficiency

In this Chapter, UE uplink energy consumption is minimized by optimally setting up the network, while guaranteeing a minimum average uplink QoS. Indeed, uplink energy consumption is one of the major components of the overall UE energy consumption (see, 2.2.7). Hence, its minimization can consistently reduce UEs batteries consumption and extend their charge.

The Chapter is organized as follows. First, the uplink system model is introduced in Section 6.1. Then, Section 6.2 describes the uplink performance evaluation and energy efficiency evaluation framework. Section 6.3 contains the optimization problem statement, and proposes an approach to solve it. Finally, results are given in Section 6.4.

6.1 Uplink System Model

In this work, the uplink of a cellular network is treated. Transmissions are performed on synchronized frames, and each network *station* (i.e. BS or RN) is associated with an uplink frame. Frames are partitioned into RBs of the same bandwidth and time duration, which are labeled with an index τ . Each RB τ in the frame of station k can be either granted to one of the UEs served by k , or be left unused (if there is no UE to be served).

Figure 6.1: Uplink model.

The network is composed of a set $\mathcal{S}$ of S stations, and divided into cells; $\mathcal{S}_B$ denotes the set of BSs, and $\mathcal{S}_R$ indicates the set of RNs, so that $\mathcal{S} = \mathcal{S}_B \cup \mathcal{S}_R$. The focus is on one given cell c containing one *donor* BS [2] and n RNs, which are connected to the BS by means of a wireless *backhaul link* (see Figure 6.1). The set of stations of cell c is denoted $\mathcal{S}_c$. The surface $\mathcal{A}_c$ of cell c is defined as the region where UEs are served by one of the stations of c , and its area is denoted A_c . Similarly, surface $\mathcal{A}_k$, of area A_k , is the region where UEs are served by station k .

UEs select their serving station (either BS or RN) based on the highest received downlink pilot power, and they are connected to it through a wireless *access link*. The network designer can *bias* the choice of UEs serving station, by adding a coefficient $\mathcal{B}$ to the power received from RNs. The higher $\mathcal{B}$, the more UEs will be attached to RNs. This technique is referred to in literature as *cell range expansion* [111], and it is deemed beneficial for network performance as it allows for load balancing.

6.1.1 Frame Structure

Access link and backhaul link transmissions are dedicated two orthogonal sections of the frame (see Figure 6.2). Hence RNs transmissions on the backhaul

Figure 6.2: Frame structure.

link, which uses a quota β of the RBs, do not interfere with UEs transmissions on the access link. This choice is widely adopted in the literature (see e.g. [65, 64]), as it avoids interference between RNs and UEs on the uplink. For the same reason, it is assumed that β is the same for all network stations. The value of β is supposed to be set by the operator and based on considerations on the overall network performance, while this analysis is focused on the performance related to one given network cell, *given* β .

6.1.2 Traffic Model

UEs are assumed to arrive in the network according to a SPPP of intensity $\lambda(s)$ [arrivals/s/m²], to transmit a flow of average size ξ [bit] to their serving station, and to leave the network. Flows transmitted from UEs to a RN on the access link are then forwarded by the RN to its donor BS on the backhaul link.

The traffic density $\omega(s)$ at a given location s is denoted by $\omega(s) = \lambda(s)\xi$ [bit/s/m²], while the average traffic density $\bar{\omega}$ in the network can be computed as $\bar{\omega} = (1/A_{nw}) \int_{\mathcal{A}_{nw}} \lambda(s)\xi ds$ [bit/s/m²], where $\mathcal{A}_{nw}$ is the overall network surface, of area A_{nw} ¹. Parameter $\phi(s) \triangleq \omega(s)/\bar{\omega}, \forall s \in \mathcal{A}_{nw}$ is defined, to account for the local variation of the traffic density with respect to $\bar{\omega}$. The ratio $\phi(s)/A_{nw}$ can be seen as the spatial PDF of UEs arrivals.

¹ A_{nw} is assumed to be big enough so that interference in cell c is accurately computed.

Figure 6.3: Queuing model.

The granularity of RBs is assumed to be *much smaller* than ξ . Hence, access link buffer queue of all stations $k \in \mathcal{S}$ can be modeled as an M/G/1/PS queue (see Figure 6.3), and its load is denoted with ρ_k . Only *stable* scenarios, i.e., $\rho_k < 1, \forall k \in \mathcal{S}$ are considered in this work. Vectors $\boldsymbol{\rho} = [\rho_1 \cdots \rho_S]$ and $\boldsymbol{\rho}_{-k} = [\rho_1 \cdots \rho_{k-1} \ \rho_{k+1} \cdots \rho_S]$, are defined for the sake of further developments. Let introduce a binary random variable $\Theta_k(\tau)$, which is equal to one when RB τ in station k frame is granted to a UE, and equal to zero otherwise. It is assumed that the probability for any station k to receive data on RB τ depends only on ρ_k , and is equal to $\mathbb{E}_\tau[\Theta_k(\tau)] = \min\{1, \rho_k\}$.

Backhaul link buffer queues are also modeled as M/G/1/PS queues (see Figure 6.3). Each flow in the backhaul queue belongs to a class $k \in \{1 \cdots n\}$, according to the transmitting RN.

6.1.3 SINR model

Consider a UE located in s and a RB τ . The instantaneous SINR $\gamma_k(s, \tau)$ experienced by station k for the considered UE is defined as:

$$\gamma_k(s, \tau) = \frac{P_k(s, \tau)}{\sum_{j \in \mathcal{S}, j \neq k} \Theta_j(\tau) P_k(f_j(\tau)) + N}, \quad (6.1)$$

where $f_j(\tau) \in \mathcal{S}_j$ yields the position of the UE scheduled by station j on RB τ , and N is the thermal noise power. In the following, the dependency of all variables on τ is dropped, for the sake of simplicity.

Remark 1. Let $p_{\gamma,k}(s, z)$, $z \geq 0$ be the PDF of $\gamma_k(s)$. $p_{\gamma,k}(s, z)$ can be approximated by a lognormal distribution, i.e., $\gamma_k(s) \sim \text{Log-}\mathcal{N}(\mu_{\gamma,k}, \sigma_{\gamma,k})$.

Proof. Equation (6.1) can be rewritten as:

$$\gamma_k(s) = \frac{P_k(s)x}{\sum_{j \in \mathcal{S}, j \neq k} \Theta_j P_k(f_j)x + Nx} = P_k(s)R_k, \quad (6.2)$$

where x is a realization of RV $X \sim \text{Log-}\mathcal{N}(\mu_X, \sigma_X^2)$, and $R_k \triangleq x / (\sum_{j \in \mathcal{S}, j \neq k} \Theta_j P_k(f_j)x + Nx)$. Note that R_k does not depend on s .

Starting from the sum of interfering powers $\sum_{j \in \mathcal{S}, j \neq k} \Theta_j P_k(f_j)$, the distribution of R_k is discussed in the following. [112, 113, 114] et al. approximate the distribution of this sum with a lognormal distribution, under the assumption that the number of interferers is fixed (i.e., $\Theta_k(\tau) = 1, \forall(\tau, k)$). This differs with the model considered in this study, where stations can be idle. However, [115, 116] et al. show that the lognormal approximation can be extended to accurately model a variable number of interferers. Thus, it is assumed that $\sum_{j \in \mathcal{S}, j \neq k} \Theta_j P_k(f_j) \sim \text{Log-}\mathcal{N}(\mu_{I,k}, \sigma_{I,k}^2)$, where parameters $\mu_{I,k}$ and $\sigma_{I,k}$ can be found by matching the first and second moment of the lognormal distribution with the corresponding moments of the interference sum (see Appendix A.3).

Product $\sum_{j \neq k} \Theta_j P_k(f_j)x$ is also a lognormal RV, as well as Nx . Hence, the denominator of γ_k is the sum of two lognormal RVs, and it can be approximated with another lognormal RV (see, e.g., [116]). It follows that $R_k \sim \text{Log-}\mathcal{N}(\mu_{R,k}, \sigma_{R,k})$,

where $\mu_{R,k}$ and $\sigma_{R,k}$ can be found again via moment matching. The SINR $\gamma_k(s)$ being the product of two lognormal RVs, the approximation [116, 115]: $\gamma_k(s) \sim \text{Log-}\mathcal{N}(\mu_{\gamma,k}, \sigma_{\gamma,k})$ is finally introduced. $\square$

6.1.4 Scheduling Policy

Let assume that stations have perfect CSI about all served UEs and consider a given frame of station k . Moreover, UEs served by k during the considered frame (which we assume to be fixed) are labeled with an index $i = 1 \cdots U$, while their locations are denoted by $s_1 \cdots s_U$ respectively.

Every station implements a policy of *Maximum Quantile Scheduling* [117, 118, 119] (MQS) among served UEs. The MQS scheduler of station k assigns RB τ to the UE i that maximizes its instantaneous SINR $\gamma_k(s_i, \tau)$, with respect to $\gamma_k(s_i, \tau - h)$, $h = 1 \cdots W - 1$ over a window of W RBs. This policy has the property of being fair in RBs allocation between UEs, while maintaining a good throughput performance [118]. Moreover, [118] shows that the performance degradation introduced by an imperfect estimation of the SINR distribution can be lower than that incurred in practical implementations of other popular scheduling algorithms.

The MQS scheduling differs from the well-known PF scheduling [120], because the latter allocates each radio resource to the user maximizing its scaled SINR $\gamma_k(s_i, \tau) / \bar{\gamma}_k(s_i)$ on τ , where $\bar{\gamma}_k(s_i)$ represents the average SINR experienced by k for user i over the last W RBs. However, for large values of W its behavior approximates that of a PF scheduler (see e.g. [117]). The MQS scheduler has been adopted in this work for its good tradeoff between fairness in radio resources assignment and overall performance, and its relatively low computational complexity, compared to a PF scheduler.

Now, consider the distribution $\pi_{\gamma,k}(s_i, z) \triangleq \mathbb{P}(z \leq \gamma_k(s_i) \leq z + dz \mid f_k = s_i)$, $s_i \in \mathcal{S}_k$, $z \geq 0$ of $\gamma_k(s_i)$ conditioned to $f_k = s_i$, i.e., the PDF of the SINR of a *scheduled* UE. Contrary to $p_{\gamma,k}(s_i, z)$, this distribution is not lognormal.

6.1.5 Derivation of $\pi_{\gamma,k}(s_i, z)$

The MQS scheduler orders the values of SINR $\gamma_k(s, \tau - j), j \in \{0 \cdots W - 1\}$ of each UE in ascending order. The *ranking* $r_i(\tau)$ of UE i located in s_i on RB τ is the ranking of $\gamma_k(s_i, \tau)$ in the ordered vector $\gamma_k(s_i, \tau - j), j \in \{0 \cdots W - 1\}$. The lower the ranking, the better the SINR on τ (wrt the SINR on previous blocks). Station k assigns RB τ to the UE with the lowest $r_i(\tau), i \in \{1 \cdots U\}$, where U denotes the overall number of UEs served by k . W is supposed to be large enough so that the probability for two or more UEs to have the same ranking is negligible.

Hence, $\mathbb{P}(r_i(\tau)=\theta)=1/W, \forall \theta \in \{1 \cdots W\}$ ². Moreover, the probability for any UE served by k to be scheduled is $\mathbb{P}(f_k(\tau)=s_i) = 1/U, \forall i \in \{1 \cdots U\}$ (fairness in RBs allocation). Let $\Pi_{\gamma,k}(s_i, z) = \int_0^z p_{\gamma,k}(s_i, t)dt$ denote the CDF of $\gamma_k(s_i, \tau)$. Now,

$$\mathbb{P}(r_i(\tau)=\theta \mid \gamma_k(s_i, \tau)=z) = \binom{W-1}{\theta-1} \Pi_{\gamma,k}(s_i, z)^{W-\theta} (1 - \Pi_{\gamma,k}(s_i, z))^{\theta-1}. \quad (6.3)$$

This value, which does not depend on U , is denoted with

$$Q_\theta(i, z) \triangleq \binom{W-1}{\theta-1} \Pi_{\gamma,k}(s_i, z)^{W-\theta} (1 - \Pi_{\gamma,k}(s_i, z))^{\theta-1}.$$

Then, $\pi_{\gamma,k}(s, z)$ *given* U is derived by applying the Bayes theorem on $\pi_{\gamma,k}(s, z)$:

$$\begin{aligned} \pi_{\gamma,k}(s, z) &= \frac{\mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z) p_{\gamma,k}(s_i, z)}{\mathbb{P}(f_k(\tau)=s_i)} \\ &= \mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z) p_{\gamma,k}(s_i, z) U. \end{aligned} \quad (6.4)$$

It is possible to work on $\mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z)$, applying the law of total probability and conditioning it with respect to the possible rankings $r_i(\tau)$:

$$\begin{aligned} \mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z) &= \\ \sum_{\theta=1}^W \mathbb{P}(r_i(\tau)=\theta \mid \gamma_k(s_i, \tau)=z) \mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z, r_i(\tau)=\theta). \end{aligned} \quad (6.5)$$

² as in a block Rayleigh fading environment each realization of $\gamma_k(s_i, \tau)$ is independent of the others.

The probability of being scheduled depends on the instantaneous SINR only through the ranking: once $r_i(\tau)$ is known, the knowledge of $\gamma_k(s_i, \tau)$ does not add any additional information regarding the probability of being scheduled by k . Hence,

$$\mathbb{P}(f_k(\tau)=s_i \mid \gamma_k(s_i, \tau)=z, r_i(\tau)=\theta) = \mathbb{P}(f_k(\tau)=s_i \mid r_i(\tau)=\theta) = \left(\frac{W-\theta}{W}\right)^{U-1}, \quad (6.6)$$

and (6.4) becomes

$$\pi_{\gamma,k}(s, z) = p_{\gamma,k}(s_i, z) U \sum_{\theta=1}^W Q_{\theta}(i, z) \left(\frac{W-n}{W}\right)^{U-1}. \quad (6.7)$$

So far, $\pi_{\gamma,k}(s, z)$ given U has been derived. Now, let $\Psi = (W - \theta)/W$, and apply the law of total probability, summing up $\pi_{\gamma,k}(s, z)$ given U for all the possible values of U . The following expression is obtained:

$$\mathbb{P}(\gamma_k(s_i, \tau) = z \mid f_k(\tau)=s_i, U > 0) = p_{\gamma,k}(s_i, z) \sum_{\theta=1}^W Q_{\theta}(i, z) (1-\rho_k) \sum_{U=1}^{\infty} \rho_k^{U-1} U \Psi^{U-1}, \quad (6.8)$$

where

$$(1 - \rho_k) \sum_{U=1}^{\infty} \rho_k^{U-1} U \Psi^{U-1} = \frac{1}{\Psi} \frac{(1 - \rho_k)}{\rho_k} \sum_{U=1}^{\infty} U (\rho_k \Psi)^U = \frac{W^2(1 - \rho_k)}{(W - \rho_k(W - \theta))^2}, \quad (6.9)$$

which does not depend on z and s_i , and it is hence the same in every UE location, and for every point of the SINR PDF. Parameter $T_k(W, n) \triangleq W^2(1 - \rho_k)/(W - \rho_k(W - n))^2$ is defined, so that $\pi_{\gamma,k}(s, z)$ can be finally expressed as

$$\pi_{\gamma,k}(s, z) = p_{\gamma,k}(s_i, z) \sum_{\theta=1}^W Q_{\theta}(i, z) T_k(W, \theta). \quad (6.10)$$

6.1.6 UEs Physical Data Rate

Let define the function $C(\gamma) > 0$, yielding the physical data rate achieved, when the receiver experiences SINR γ (or equivalently the throughput of a user, if it were alone in the serving area of its serving station). Function $C(\gamma)$ is assumed to be non decreasing in γ .

6.2 Uplink Energy Efficiency and Performance Evaluation

This section is devoted to the evaluation of the performance and energy efficiency of uplink RNs networks. Performance is measured by means of the average delay experienced by UEs to transmit a file to the serving station.

The section is structured as follows. First, the load of stations buffers queues is derived for the AL and for the BL (see 6.2.1 and 6.2.2 respectively). Then, the average transmission delay is obtained in 6.2.3 and the UEs energy efficiency is given in 6.2.4.

6.2.1 Access Link Load

Let focus on station k , and let $\varrho_k(s), s \in \mathcal{A}_k$ be the contribution of s to the load ρ_k of station k . The following Lemma is given:

Lemma 4. *The local contribution $\varrho(s), s \in \mathcal{A}_k$ to the load of the access link buffer queue of station k is expressed by:*

$$\varrho_k(s) = \frac{1}{1 - \beta} \mathbb{E}_\tau \left[\frac{\bar{\omega}\phi(s)}{C(\gamma_k(s, \tau))} \middle| f_k(\tau) = s \right]. \quad (6.11)$$

Proof. The contribution to the load is expressed as the ratio of the traffic density $\bar{\omega}\phi(s)$ in s to the uplink rate achieved by a UE in s . The term $(1 - \beta)$ takes into account the quota of the frame RBs dedicated to the backhaul link. $\square$

Theorem 2. *The load ρ_k of the access link of station k can be expressed as:*

$$\rho_k = \frac{\bar{\omega}}{1-\beta} \int_{\mathcal{A}_k} \phi(s) \int_0^\infty \frac{\pi_{\gamma,k}(s, z)}{C(z)} dz ds. \quad (6.12)$$

Proof. The load ρ_k is the sum of the contributions $\varrho_k(s)$ over $\mathcal{A}_k$:

$$\begin{aligned} \rho_k &= \frac{1}{1-\beta} \int_{\mathcal{A}_k} \varrho_k(s) ds \\ &= \frac{\bar{\omega}}{1-\beta} \int_{\mathcal{A}_k} \phi(s) \mathbb{E}_\tau \left[\frac{1}{C(\gamma_k(s, \tau))} \middle| f_k(\tau) = s \right] ds. \end{aligned} \quad (6.13)$$

Now, the term $\mathbb{E}_\tau [1/C(\gamma_k(s, \tau)) | f_k(\tau) = s]$ can be expressed according to the law of total probability, obtaining (6.12). $\square$

If all UEs transmit at power $T(s) < T_{\max}$, $\forall s \in \mathcal{S}$, then $\pi_{\gamma,k}(s, z) = \pi_{\gamma,k}(z)$, $\forall s \in \mathcal{S}$ and equation (6.12) reduces to:

$$\rho_k = \frac{\bar{\omega} \Phi_k}{1-\beta} \mathbb{E}_\gamma \left[\frac{1}{C(\gamma)} \right], \quad (6.14)$$

where $\Phi_k = \int_{\mathcal{A}_k} \phi(s) ds$ can be seen as the *effective area* covered by k , in terms of traffic density.

Note that ρ_k is a function of $\boldsymbol{\rho}_{-k}$, as the load depends on the interference from other stations. Now let introduce the operator $F_k(\boldsymbol{\rho}_{-k}) = \bar{\omega}/(1-\beta) \int_{\mathcal{A}_k} \phi(s) \int_0^\infty \pi_{\gamma,k}(s, z)/C(z) dz ds$, which yields the value of ρ_k corresponding to a given $\boldsymbol{\rho}_{-k}$.

Lemma 5. *The operator $\mathbf{F}(\boldsymbol{\rho}) = (F_1(\boldsymbol{\rho}_{-1}) \cdots F_S(\boldsymbol{\rho}_{-S}))$ maps $\boldsymbol{\rho}$ to a finite S -dimensional interval.*

Proof. For all stations $k \in \mathcal{S}$, the corresponding $F_k(\boldsymbol{\rho}_{-k})$ is increasing w.r.t. every coordinate, as an increase in the load of any station $j \neq k, j \in \mathcal{S}$ produces an increase in the interference received at k . Define $\forall k, \rho_k^{\max} = F_k(1 \cdots 1)$, which represents the maximum load of k . Then, $\forall k, \forall \rho_k, F_k(\boldsymbol{\rho}_{-k}) =$

$F_k(\max\{\mathbf{1}, \boldsymbol{\rho}_{-k}\}) \leq F_k(1 \cdots 1) = \rho_k^{\max}$. Hence,

$$\mathbf{F}(\boldsymbol{\rho}) \in \prod_{k \in \mathcal{S}} [0, \rho_k^{\max}], \quad \rho_k^{\max} \in \mathbb{R}^+, \quad \forall k \in \mathcal{S}. \quad (6.15)$$

□

Theorem 3. *There exists at least one $\boldsymbol{\rho}^*$ such that $\mathbf{F}(\boldsymbol{\rho}^*) = \boldsymbol{\rho}^*$, $\boldsymbol{\rho}^* \in \prod_{k \in \mathcal{S}} [0, \rho_k^{\max}]$.*

Proof. This follows from Lemma 5 and (6.15): if $\mathbf{F}$ is a continuous mapping from an S -dimensional closed interval to itself, then the Brouwer's fixed point theorem guarantees the existence of at least one fixed point. □

It is not feasible, to the best of the author's knowledge, to draw any conclusion about the uniqueness of the fixed point. However, following the approach of [93, 80], the fixed point iterations are assumed to start from a single cell without interference ($\boldsymbol{\rho} = \mathbf{0}$) so as to model a scenario of increasing traffic.

6.2.2 Backhaul Link Load

Now, let focus on the backhaul link queue of the BS $j \in \mathcal{S}_c$ of cell c , and introduce the expression of its load $\rho_{BL,j}$. The probability for a given RN k served by j to be selected for backhaul transmission is equal to $\Phi_k / \sum_{k \in \mathcal{S}_c} \Phi_k$. Hence, $\rho_{BL,j}$ can be written as:

$$\rho_{BL,j} = \frac{\bar{\omega}}{\beta} \sum_{k=1}^n \frac{\Phi_k}{R_{BL,j,k}}, \quad (6.16)$$

where $R_{BL,j,k}$ is the average of the inverse of the rate on the backhaul link between RN k and BS j , which depends on propagation conditions, and is equal to:

$$R_{BL,j,k} = \frac{1}{\mathbb{E}_{\tau} \left[\frac{1}{C(\gamma_{BL,j,k}(\tau))} \right]}, \quad (6.17)$$

where $\gamma_{BL,j,k}(\tau)$ denotes the SINR on the backhaul of RN k served by BS j , during slot τ . A detailed derivation of $R_{BL,j,k}$ is presented in Appendix A.4.

6.2.3 User Average Transmission Delay Computation

The user average transmission delay is an effective parameter to measure network uplink performance. In this section, its formulation is derived. Denote with $D_u(s)$ the average delay experienced by a UE located in $s \in \mathcal{A}_k$ to transmit its file to the BS. Parameter $D_u(s)$ can be decomposed into the contribution $D_a(s)$ referred to the delay on the access link, and $D_{BL,k}$ referred to the delay from k to its donor BS ($D_{BL,k} = 0, \forall k \in \mathcal{S}_B$). Moreover, let $x_k \in \mathbb{N}^+$ denote the number of UEs served by k during the frame in which RB τ is transmitted.

Lemma 6. *Parameter $D_a(s)$ is expressed by:*

$$D_a(s) = \xi \frac{1}{1 - \rho_k} \int_0^\infty \frac{\pi_{\gamma,k}(s, z)}{(1 - \beta)C(z)} dz. \quad (6.18)$$

Proof. The delay to transmit a bit of information is the inverse of the UE rate, multiplied by the number x_k of UEs served by the same station during a frame (because each UE is scheduled on a fraction $1/x_k$ of the RBs with MQS). The average transmission delay for the whole flow is hence given by:

$$D_a(s) = \xi \mathbb{E}_\tau \left[\frac{x_k}{(1 - \beta)C(\gamma_k(s, \tau))} \middle| f_k(\tau) = s \right]. \quad (6.19)$$

Considering that $\mathbb{P}(x_k = U | x_k > 0) = \rho_k^{U-1}(1 - \rho_k)$, the law of total probability can be used to average $D_a(s)$ over $x_k > 0$:

$$D_a(s) = \sum_{U=1}^{\infty} \mathbb{P}(x_k = U) \frac{\xi}{1 - \beta} U \mathbb{E}_\tau \left[\frac{1}{C(\gamma_k(s, \tau))} \middle| f_k(\tau) = s \right], \quad (6.20)$$

obtaining expression (6.18). $\square$

Lemma 7. *Parameter $D_{BL,j,k}$ is expressed by:*

$$D_{BL,j,k} = \frac{\xi}{(1 - \rho_{BL,j})\beta R_{BL,j,k}}, k \in \mathcal{S}_R, j \in \mathcal{S}_B. \quad (6.21)$$

Proof. Similarly to what performed for Lemma 6, the derivation proceeds by expressing $D_{BL,j,k}$, for any RN k controlled by BS j , as:

$$D_{BL,j,k} = \frac{\xi}{\beta R_{BL,j,k}} \mathbb{E}_\tau [x_{BL,j}(\tau) | x_{BL,j}(\tau) > 0] \quad (6.22)$$

and apply the law of total probability, obtaining

$$\begin{aligned} D_{BL,j,k} &= \frac{\xi}{\beta R_{BL,j,k}} \sum_{v=1}^{\infty} P(x_{BL,j}(\tau) = v | x_{BL,j}(\tau) > 0) \mathbb{E}_\tau [x_{BL,j}(\tau) | x_{BL,j}(\tau) = v] \\ &= \xi \frac{1}{(1 - \rho_{BL,j}) \beta R_{BL,j,k}}, \end{aligned} \quad (6.23)$$

where $x_{BL,j}$ is the number of flows in the backhaul queue of j during the considered frame. Conventionally, $D_{BL,j,j} = 0$. $\square$

Corollary 2. *The mean transmission delay of a UE served by $k \in \mathcal{S}_R$ on the access link is:*

$$D_k = \int_{\mathcal{A}_k} \frac{\phi(s)}{\Phi_k} D_a(s) ds. \quad (6.24)$$

Proof. The statement can be verified by means of the Little's law:

$$D_k = \frac{\mathbb{E}_\tau [x_k(\tau)]}{\bar{\omega} \Phi_k} = \frac{\rho_k}{1 - \rho_k} \frac{1}{\bar{\omega} \Phi_k}. \quad (6.25)$$

Now, (6.12) can be substituted to the numerator to conclude the proof. $\square$

Corollary 3. *The average delay $\bar{D}_c$ for an uplink flow transmitted in cell c is equal to:*

$$\bar{D}_c = \sum_{k \in \mathcal{S}_c} \frac{\Phi_k}{A_k} (D_k + D_{BL,j,k}). \quad (6.26)$$

6.2.4 UE Performance Definition

The energy efficiency $\epsilon(s)$ of a UE located in $s \in \mathcal{A}_k$ is measured through its average energy consumption per transmitted bit:

$$\epsilon(s) = \mathbb{E}_\tau \left[\frac{T(s)}{C(\gamma_k(s, \tau))} \middle| f_k(\tau) = s \right] = T(s) \int_0^\infty \frac{\pi_{\gamma,k}(s, z)}{C(\gamma_k(s, z))} dz, s \in \mathcal{A}_k \quad (6.27)$$

which can be also expressed as a function of $D_a(s)$:

$$\epsilon(s) = T(s)D_a(s)(1 - \rho_k)(1 - \beta), s \in \mathcal{A}_k. \quad (6.28)$$

while the average uplink energy consumption per bit Π associated to UEs in cell c is given by:

$$\Pi = \frac{1}{A_c} \int_{\mathcal{A}_c} \phi(s)\epsilon(s)ds = \frac{1 - \beta}{A_c} \sum_{k \in \mathcal{S}_c} (1 - \rho_k) \int_{\mathcal{A}_k} T(s)D_a(s)\phi(s)ds. \quad (6.29)$$

6.3 Relay-Enhanced Network Constrained Optimization for User Energy Efficiency Maximization

This section discusses the optimization of UE uplink energy efficiency. The proposed algorithm aims at minimizing the UEs average energy consumption per transmitted bit, by optimizing $\bar{P}$, $\mathcal{B}$ and the RNs placement in the cell of interest, for a given $\bar{\omega}$, n and traffic spatial profile $\phi(s)$. The optimization is constrained to respect a maximum tolerable value of $\bar{D}_c$, so as to take into account the users experienced quality of service. The final goal is to study the energy efficiency-delay tradeoff, via repeating the optimization process for different values of the delay constraint.

To this purpose, the configuration space is defined and the minimization problem to be addressed is introduced. Then, the proposed approach based on the SA algorithm with penalty functions is described.

6.3.1 Problem Statement

RNs are assumed to be placed on a discrete and finite grid of candidate sites inside cell c ³, while the positions of RNs outside cell c are supposed to be already set by the network operator, and hence they are not modifiable during the optimization. Position of a given RN i is denoted with $z(i)$. Moreover, $\bar{\mathbf{P}} = [\bar{P}_{BS} \ \bar{P}_{RN}]$. Parameters $\bar{P}_{BS}$ and $\bar{P}_{RN}$ can vary between a minimum and a maximum value, denoted with $\bar{P}_{\min}$ and $\bar{P}_{\max}$ respectively. Similarly, it is assumed that $\mathcal{B}_{\min} \leq \mathcal{B} \leq \mathcal{B}_{\max}$ (see Section 6.1). The *configuration* $\mathbf{x}$ of the network is defined as the set of positions of the n RNs in cell c , plus the adopted $\bar{\mathbf{P}}$ and $\mathcal{B}$:

$$\begin{aligned} \mathbf{x} : \{ & z(1), \dots, z(n), \bar{\mathbf{P}}, \mathcal{B} \}, \\ & z(i) \in \mathcal{A}_c, \forall i \in \{1 \dots n\}, \bar{P}_{\min} \leq \bar{P} \leq \bar{P}_{\max}, \mathcal{B}_{\min} \leq \mathcal{B} \leq \mathcal{B}_{\max}. \end{aligned} \quad (6.30)$$

Moreover, the *configuration space* Ω is defined as the set of all configurations.

Now, the problem can be formulated as:

$$\text{minimize} \quad \Pi(\mathbf{x}) \quad (6.31)$$

$$\text{subject to} \quad \bar{D}_c(\mathbf{x}) \leq D_{\max}, \quad \mathbf{x} \in \Omega, D_{\max} \in \mathbb{R}^+, \quad (6.32)$$

where constraint (6.32) restricts the domain of feasible solutions to the subspace:

$$\tilde{\Omega} = \{ \mathbf{x} \in \Omega \text{ s.t. } D_c(\mathbf{x}) \leq D_{\max} \} \quad (6.33)$$

Problem (6.31) is in general non-convex, and the cardinality of Ω , especially for high n , makes it intractable via exhaustive search. Hence, this study relies on stochastic optimization algorithms, and proposes a customized version of the well-known SA algorithm to solve (6.31). Readers can refer to 5.3.2 for a brief description of the basic SA algorithm.

³This assumption is consistent with real deployment scenarios, where normally only a certain number of locations inside the cell are available for RNs installation [121].

6.3.2 Proposed Exterior Search Approach for SA

The target is to find a minimizer of $\Pi(\mathbf{x})$ over $\tilde{\Omega}$, where the cardinality of the feasible configurations space depends on the value of the constraint: $|\tilde{\Omega}| = f(D_{\max})$. In particular, f is an increasing function of $D_{\max}$, because the number of feasible configurations increases as the constraint loosens.

When $D_{\max}$ is small, it is reasonable to expect $|\tilde{\Omega}| \ll |\Omega|$. This may reduce the effectiveness of the SA, as the energy surface that the algorithm explores could be fragmented into many isolated regions, some of which are unreachable for the algorithm. Moreover, the optimal solution is expected to tightly respect the constraint, i.e., to lie close to the border of $\tilde{\Omega}$ [122]. Thus, the standard SA may not appropriately cover the region where the solution is, as many of its configurations are non feasible. This problem can be solved by means of an *exterior search* approach [123, 124]. It consists in extending the search to configurations outside $\tilde{\Omega}$, while adding a *penalty* to the energy of those configurations that violate the constraint. This method presents analogies with the Lagrangian relaxation method (see, among the others, the classic works [125, 126, 127]), and it is regarded as a powerful instrument to address constrained optimization problems (see, e.g., [128, 129]). Thus, an exterior search approach is adopted, and (6.31) is reformulated as

$$\text{minimize } \mathcal{F}(\mathbf{x}) = \Pi(\mathbf{x}) + \mathbb{1}_{D_c(\mathbf{x}) > D_{\max}} V(\mathbf{x}, D_{\max}) \quad , \mathbf{x} \in \Omega, \quad (6.34)$$

where $V(\mathbf{x})$ is a function $\Omega \mapsto \mathbb{R}$ accounting for constraint (6.32) and named *penalty function*, which plays a crucial role in the optimization [124].

Penalty functions are classified into *static*, *dynamic*, and *adaptive*. A static penalty function does not change during the optimization. It can be represented, e.g., by a fixed constant added to the energy of those configurations $\mathbf{x} \notin \tilde{\Omega}$, or by a function proportional to the Euclidean distance of the considered configuration to the feasible region [130]. On the contrary, dynamic penalty functions can be adjusted in accordance to the progress of the optimization. A common approach is to gradually increase the penalty with the number of explored configurations

[131], so as to guarantee the convergence of the optimization towards a feasible solution. Finally, an adaptive penalty function (see, e.g., [129, 132]) considers further aspects of the search, such as steering the algorithm towards regions of the energy surface which are deemed promising for the search.

This study proposes the following exterior penalty function, which is dynamic and adaptive at the same time:

$$V(\mathbf{x}, D_{\max}) = \alpha(m-1) \frac{\Pi(\mathbf{x})}{D_{\max}} (D_c(\mathbf{x}) - D_{\max}), \quad (6.35)$$

where α is a constant and m is the SA step. The benefits of using function (6.35) are manifold. First, the penalty is proportional to the violation of constraint $D_{\max}$, favoring the exploration of configurations which are out of $\tilde{\Omega}$ but *close* to it, while penalizing more those which are *far*. At the same time the penalty is independent of the adopted value of $D_{\max}$, depending rather on the percentage of exceeding delay, wrt. $D_{\max}$. This represent an important feature when dealing with constraints of a different order of magnitude, compared to $\Pi(\mathbf{x})$. Moreover, the penalty is proportional to $\Pi(\mathbf{x})$, so as to favor the exploration of configurations outside $\tilde{\Omega}$ when they appear to be *promising*, i.e., their energy is sensibly lower to that of the current solution (hence the adaptive nature of (6.35)). Finally, acting on α it is possible to tune the percentage of acceptance for configurations outside $\tilde{\Omega}$. Some notes on the convergence of the SA adopting this type of exterior penalty functions are given in Appendix A.5.

6.3.3 Initial Temperature Choice

Following [80], the problem of finding a good value for the initial temperature T_0 in each considered optimization problem is addressed by means of a dichotomous search, to be performed during a series of preliminary runs of the algorithm.

6.4 Results

This section presents the results of numerical simulations. Simulations assumptions are given in 6.4.1, and numerical results are shown in 6.4.2.

6.4.1 Simulation Assumptions

The proposed theoretical framework has been tested using a numerical network simulator. Selected test scenario is drawn according to [62, case 1] for what concerns propagation, shadowing and stations transmit power. The network is composed by one central BS surrounded by 6 BSs regularly distributed around it on a circle of ray $r=500\text{ m}$. RNs placement is optimized in the central cell only, while RNs in the surrounding cells are assumed to be regularly distributed on a circle around their donor BS. All stations have omnidirectional antennas. The same realization of shadowing (drawn according to [62]) is used for all simulations, so as to guarantee the comparability of results.

If not otherwise mentioned, a *uniform* traffic density spatial distribution is adopted, i.e., $\phi(s)=K, \forall s \in \mathcal{A}_{nw}$, in order to draw general conclusions about energy efficiency. A dedicated subsection shows the performance under a *non-uniform* traffic distribution. Parameter β is set to 0.1.

Function C is approximated by means of the MCS indicated in [107], so as to take into account the effect of an upper-bounded capacity function, which is the typical case in real deployments. Also, fast fading on the backhaul link is not considered. This choice is justified by the assumption that RNs do not move, and optimization of their positions is performed on a long-term basis (see, e.g. [13]). It is assumed that $\mathcal{B}_{\min}=0$ and $\mathcal{B}_{\max}=15\text{ dB}$. Any configuration $\mathbf{x}$ resulting in the overload of one or more stations, i.e., $\exists k \in \mathcal{S} : \rho_k \geq 1$, is regarded as non-valid, i.e., $\mathbf{x} \notin \Omega$.

The fixed point of $\mathbf{F}(\boldsymbol{\rho})$ is found by iteratively computing stations loads for each station $k \in \mathcal{S}$. During a given iteration t , probabilities $\pi_{\gamma,k}(z, s)$ are computed according to $\boldsymbol{\rho}(t-1)$, obtained at iteration $t-1$, and then fed into (6.12) to get $\boldsymbol{\rho}(t)=\mathbf{F}(\boldsymbol{\rho}(t-1))$. The value of $\boldsymbol{\rho}(0)$ is set to $\mathbf{0}$, and the evolution of

$\rho(t)$ throughout the iterations is monitored. Iterations are stopped when either $|\rho_k(t) - \rho_k(t-1)| < 0.01, \forall k \in \mathcal{S}$, or $\exists k : \rho_k(t) \geq 1, k \in \mathcal{S}$. In the latter case, the analyzed configuration is labeled as non-valid. Similarly, fixed point iterations are used to compute the loads on the backhaul link, after ρ is determined.

The SA algorithm is implemented as described in Section 6.3. The algorithm yields a solution after 45 temperature steps. For each tested temperature value, the energy of a certain number of network configurations is calculated. This number varies according to n , e.g., for $n = 2$, 400 network configurations are tested at each temperature step. For every optimization, the SA is repeated 4 times, and the best solution among the 4 obtained solutions is elected as a final result. The configuration corresponding to the final result is denoted with $\mathbf{x}^*$.

6.4.2 Numerical Results

Let denote with $\mathbf{x}_0^*$ the configuration with minimum energy when no RNs are deployed in the whole network, and with $\Pi_0(\mathbf{x}_0^*)$ its corresponding energy. Moreover, the ratio $\Pi(\mathbf{x}^*)/\Pi_0(\mathbf{x}_0^*)$ is defined as the *energy efficiency ratio* for the sake of simplicity. Unless otherwise mentioned, results are expressed in terms of the energy efficiency ratio, so as to show the energy efficiency gain resulting from RNs deployment, and they are plotted against the normalized constraint $D_{\max}/D_0$, where D_0 is the value of $\bar{D}_c$ corresponding to $\mathbf{x}_0^*$, i.e., the average delay when no RNs are deployed. Hence, a point in the region $\Pi(\mathbf{x}^*)/\Pi_0(\mathbf{x}_0^*) < 1$ corresponds to an energy efficiency gain with respect to the network with no RNs, whereas a point in the region $D_{\max}/D_0 < 1$ corresponds to an average uplink transmission delay gain. These results, obtained under a wide range of $\bar{\omega}$ and n show that RNs are a viable means to reach meaningful uplink energy efficiency gains, without impairing users performance (see all figures in this section). All the obtained curves exhibit a hyperbolic-like shape (as in, e.g., [61, Figure 1]), which follows from the nature of the constrained optimization. When the constraint is very large, $\tilde{\Omega}$ includes the whole energy landscape. Hence, the obtained solution cannot improve if $D_{\max}$ is further increased, as it can be observed in the right part of the curves. Instead, if the constraint is tight it can be

assumed that $|\tilde{\Omega}| \ll |\Omega|$, and it is unlikely that any configuration that performs well lie in $\tilde{\Omega}$. This explains the trend observed in the left part of the curves.

Energy Efficiency - Delay Trade-off

Figure 6.4 shows the trade-off between energy efficiency and delay, for a varying number of RNs. Note how RNs deployment brings consistent UEs energy savings. Let consider the region where $D_{\max}/D_0 > 1$. The energy efficiency gain increases with n . This is an expected result, as more RNs means that more UEs can transmit to a serving node which is close to them. On the contrary, a lower number of RNs performs better when $D_{\max}/D_0 < 0.5$. This behavior can be attributed to the backhaul delay of flows served by RNs, that limits the search to those configurations with a very low associated backhaul delay. This can be detrimental in terms of energy efficiency (e.g., when RNs are forced to be close to the BS to lower the backhaul delay, thus increasing the interference), and the effects are more visible when n grows, as more UEs tend to be served by RNs, and the component of D_c due to backhaul delay increases.

Figure 6.4: Energy efficiency ratio versus delay: trade-off. Parameter $\bar{\omega} = 5$ sus offered traffic, for uniform traffic profile and $n = 1$.

Effect of Offered Traffic

The effect of $\bar{\omega}$ on energy efficiency is analyzed in Figure 6.5, where $\Pi(\mathbf{x}^*)$ and $D_{\max}$ are scaled with the constants $\Pi_0(\mathbf{x}_0^*(\bar{\omega} = 5))$ and $D_0(\bar{\omega} = 5)$ respectively,

obtained in a network with no RNs and a traffic density $\bar{\omega} = 5$ [bit/s/m]². This choice has been made in order to keep the same scaling constants for all curves. Hence, in Figure 6.5, the energy efficiency ratio is given by $\Pi(\mathbf{x}^*)/\Pi_0(\mathbf{x}_0^*(\bar{\omega} = 5))$ for all curves. Note that the deployment of 1 RN is sufficient to reduce the average energy consumption of UEs to less than a half, compared to the BSs-only network. Now, let focus on the region $D_{\max}/D_0(\bar{\omega} = 5) > 1$. It can be observed that the network with 1 RN is more energy efficient than that without RNs, even if $\bar{\omega}$ increases to 6 times its reference value. Nonetheless, users performance is negatively affected by the increase of $\bar{\omega}$. In particular, for $\bar{\omega} \approx 25$ and for higher traffic densities, an energy efficiency gain is observed only if the average delay is greater than that of the reference case.

Energy Efficiency of RNs vs Small Cells

Figure 6.6 compares the results obtained with the proposed system model, with those obtained using small cells instead of RNs (i.e., $\beta = 0$, $D_{BL,j,k} = 0$, $\forall j \in \mathcal{S}_B$, $\forall k \in \mathcal{S}_R$), highlighting the difference of this work with respect to those dedicated to the deployment of wired backhaul devices. As expected RNs allow for a smaller energy efficiency gain wrt small cells, for a fixed $D_{\max}$. This is due to the transmission delay on the backhaul link, to the increased delay on the access link (caused by the fact that $\beta < 1$ when RNs are deployed) and to the constraints on RNs placement related to backhaul path loss and shadowing. Performance of RNs is more impaired when the traffic density or the number of devices increase. However, RNs deployment still yields consistent uplink energy efficiency gains.

Effectiveness of Exterior Search with Penalty Function

Figure 6.7 compares the results of the optimization using penalty function and exterior search, with those obtained by means of an interior search. Both the interior and the exterior search have been carried on with the same number of iterations, for an unbiased comparison. The exterior search proves to be more

Figure 6.6: Comparison between wireless and wired RNs/femtocells, for several values of $\bar{\omega}$ and n .

Figure 6.7: Comparison between exterior and interior search results, for $\bar{\omega} = 5 \text{ bit/s/m}^2, \beta = 0.1, n = 1$.

effective when the delay constraint is tight, as $\tilde{\Omega}$ is expected to be fragmented and $|\tilde{\Omega}| \ll |\Omega|$. No meaningful gain in terms of search effectiveness is observed when the constraint is loose.

Effect of Offered Traffic Spatial Distribution

Figure 6.8: Energy efficiency vs delay tradeoff: $\bar{\omega} = 5 \text{ bit/s/m}^2, \beta = 0.1$, comparison of uniform and non-uniform traffic profile $\phi(s)$ for varying n .

Figure 6.8 compares the results obtained using a uniform $\phi(s)$ with those obtained when $\phi(s)$ is the sum of a uniform traffic profile and a bi-dimensional gaussian function, centered at $s : \{x = 115m; y = 143m\}$. This is done to see the effect of a sample non-uniform traffic density spatial distribution on

results. We can see that RNs allow for larger energy efficiency gains when traffic is non-uniform. This is because in the latter case the BS-only network is less energy-efficient compared to the uniform traffic profile case. Hence, it enjoys the highest energy efficiency gain from RNs deployment.

Chapter 7

Conclusions and Future Work

In this thesis, several aspects of RNs deployment in 4G cellular networks have been treated, i.e., modeling of RNs networks via fluid network model, scheduling schemes performance comparison, RNs placement for performance optimization and energy efficiency of RNs-enhanced networks. Taking into account the results of these studies, the main conclusions of the thesis are summarized in the following, and a number of possible future developments of the presented works are outlined.

7.1 Conclusions

Results in Chapter 3 have highlighted that PF scheduling applied to RNs-enhanced networks stations does not always bring a consistent performance improvement with respect to RR scheduling, e.g., when 1) many RNs are deployed and in-band relaying is adopted, or 2) backhaul rate is low, or 3) the network load is low and out-of-band relaying is adopted. Moreover, Figure 3.7 shows the key role of backhaul link in determining the performance of RNs-enhanced networks. Indeed, the choice of the backhaul link propagation model has a decisive impact on simulations results.

In 4.2, it is shown that the fluid model of cellular networks [88] is an effective tool for interference and SINR approximation in relay-enhanced networks,

provided that the network of RNs is hexagonal, or it can be decomposed into several hexagonal networks, and that UEs are served by the nearest station. In this case the accuracy of the fluid model is high, and the computational burden of static simulations can be considerably reduced. Results in 4.3.4 indicate that the fluid model is less effective in non-regular RNs networks, except for some particular RNs placements. Under a dynamic network model, the method proposed in 5.2.5 for modeling the interference coming from far network stations is accurate. The high variability of interference in the dynamic case and the high probability for a UE to be served by a near station suggest that a fluid network model may not work properly if it is applied on the whole network. However, its application on a part of the network allows for great computational savings without an accuracy loss.

Chapter 5 has analyzed the optimization of RNs placement, and the cell capacity corresponding to optimized placements. Results in 5.4 provide useful guidelines for the placement of RNs, highlighting the tradeoff that arises in the in-band RNs deployment scenario, between the exigence of enjoying a high backhaul rate, and that of covering the edge of the cells, where users with the worst channel conditions are. Out-of-band RNs do not have to satisfy a backhaul rate constraint, and they are hence free to place on the cell edge, unless many RNs are deployed. In that case, the best choice is to place a part of the RNs on the edge, and a part inside the cell, so as to mitigate the interference between RNs. According to Figure 5.10, RNs deployment is capable of increasing cell capacity, both in the in-band and in the out-of-band case. For out-of-band RNs, the increase is consistent, and cell capacity can be doubled by adding few RNs. This is an expected result, as the deployment of out-of-band RNs is similar in principle to a network densification, which is known to increase cell capacity. For in-band RNs, a smaller increase in cell capacity is observed, compared to out-of-band RNs, and several RNs must be deployed to yield a consistent capacity boost. This is due to the frame radio resources that must be dedicated to the backhaul link, which are not available for transmission/reception towards UEs.

The economic advantage of deploying in-band RNs may be too small to justify their development, in view of the smaller capacity gain yielded by this solution, compared to network densification by means of, e.g., out-of-band RNs or additional BSs deployment. In particular, the latter alternative comes at no additional development price for network operators, as BSs have been already developed, tested and deployed in the previous years. However, [16] and Figure 3.7 show that in-band RNs performance is strongly dependent on the backhaul link quality. In Figure 5.10, backhaul link has been modeled according to [91]. It is not unrealistic to assume that more favorable backhaul propagation conditions may be observed in a number of real deployment scenarios. In that case, in-band RNs performance would strongly improve, as the BL represents the main performance bottleneck [63].

Chapter 6 has analyzed the potential of RNs deployment as a means to achieve energy efficiency gains on the uplink. In 6.4 it is shown that RNs deployment has the capability of reducing the consumption of UEs batteries, thanks to the reduced average distance of UEs with respect to their serving stations. This does not imply a performance loss, even though uplink transmissions via an in-band RN have to experience an additional delay due to the backhaul. In fact, RNs allow the network to support a higher uplink traffic input, compared to the case without RNs, while increasing the energy efficiency (see Figure 6.5).

7.2 Future Work

This Section outlines a number of possible future developments of the work presented in this thesis.

7.2.1 Performance Assessment

Chapter 4 and Section 5.2.5 have studied the application of the fluid network model to regular and irregular RNs networks. In both scenarios, the fluid model

has been applied to the whole network. However, the application to far stations interference computation is limited so far to the regular RNs network scenario (see 5.2.5). It would be of interest to study the accuracy of the fluid model for the case of irregular network of far RNs, so as to see whether the computational complexity gains allowed by the fluid model can be extended to a dynamic network of far stations with generic topology.

Few studies have been devoted to the coverage gains that can be achieved via RNs deployment in a dynamic framework and in a wide variety of scenarios, to the best of the author knowledge. Indeed, a dynamic model would allow to properly evaluate the achievable gains, as in the static case interference tends to be overestimated, and results are consequently biased. The aspect of coverage is strategic to determine the advisability of deploying RNs, hence a proper evaluation of the achievable gains would be of help to assess the importance of RNs in 4G networks.

It has been mentioned that backhaul propagation parameters have a major impact on the performance of in-band RNs. Reference [16] tries to quantify this claim, under a very simple network model, while Figure 3.7 plots network performance for two scheduling schemes and three sample backhaul propagation scenarios. However, the need arises for an exhaustive study about the backhaul of RNs, clarifying the conditions for in-band RNs to achieve a performance gain sufficient to justify their development and deployment, compared to the out-of-band RNs or traditional BSs.

7.2.2 Network Optimization

Several works have been devoted to network optimization (see 2.2). In most of them, the BS is assumed to have perfect knowledge of the CSI related to all UEs in the cell (also those controlled by RNs), and it performs the optimization of resources assignment or the joint optimization of transmit power and resources assignment, in the whole cell. Most likely, the optimization is computationally heavy, and has to be carried out in real time. Hence the need to study the tradeoff between computational complexity required at the BS and

the performance gain obtained by means of various optimization schemes. In fact, it may happen that the computational complexity is high enough to make a given optimization scheme unfeasible for a network operator.

Moreover, centralized optimization of resources assignment implies a consistent increase of transmission overheads. This is because the BS needs to get the CSI of all UEs (also those controlled by RNs), and needs to send them back the information about resources assignment. This uses a part of the frame resources, which may be important when there are many UEs or when most of the UEs are controlled by RNs (as in this case all the information need to pass also on the BL). Hence, future work should consider the amount of transmission overheads implied by the use of a centralized optimization scheme in RNs-enhanced networks, focusing on the tradeoff between the quantity of overhead and the performance gain.

Another topic which has not been exhaustively treated in the literature is the effect of an incorrect estimation of CSI (as it may happen in real scenarios) on the performance of optimal resources assignment algorithms. Indeed, the performance may meaningfully change when the CSI is not correctly estimated, and the extent of this change contributes to determine whether it is worth implementing a given optimization scheme or not. In relay network, the CSI of a UE may take a longer time to reach the BS, compared to a traditional network, as it may have to pass through the BL, and hence there is a higher chance that the channel may have sensibly changed in the meantime. Thus, this type of study in the context of RNs deployment is of a high importance.

Finally, most of the studies on network optimization are focused on either the downlink or the uplink of the network, and the proposed solutions and network setups are valid for the downlink or the uplink only. This approach may present some drawbacks, as the optimal network setup on the downlink may be non-optimal on the uplink. As an example, one may think about the optimal placement of RNs on the downlink, which does not necessarily correspond to that on the uplink. The need arises for a study that jointly optimizes network setup on both the uplink and the downlink, choosing an appropriate joint performance metric and applying it to several deployment scenarios, measuring

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the performance and comparing the optimal performance to that obtained by optimizing the uplink and downlink separately.

Appendices

Appendix A

A.1 Proof of Proposition 1

Let start from the definition of H_k in (3.11) and of $R_{i,k}(n)$ in (3.7), assuming that fast fading follows the PDF $\Phi_{\theta^2}(x) = e^{-x}$:

$$\begin{aligned}
 H_k &= \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} \int_0^\infty \min\{\alpha \log_2(1 + \beta \Gamma_{i,k}(n)x), \rho_{\min}\} e^{-x} dx \right] \\
 &= \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} \int_0^\infty \min\{\alpha \log_2(1 + t), \rho_{\min}\} e^{-\frac{t}{\beta \Gamma_{i,k}(n)}} \frac{dt}{\beta \Gamma_{i,k}(n)} \right] \\
 &= \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k(n)} \frac{1}{\beta \Gamma_{i,k}(n)} \mathcal{L}(\min\{\alpha \log_2(1 + t), \rho_{\min}\}) \left(\frac{1}{\beta \Gamma_{i,k}(n)} \right) \right] \quad (\text{A.1})
 \end{aligned}$$

where $\mathcal{L}(f(t))(s) = \int_0^\infty e^{-st} f(t) dt$ is the Laplace transform (a similar approach is used in [74]). Let denote $\mathcal{L}_\rho(s) \triangleq \mathcal{L}(\min\{\alpha \log_2(1 + t), \rho_{\min}\})(s)$ and let $\bar{t}$ be the value for which $\alpha \log_2(1 + t) = \rho_{\min}$, i.e., $\bar{t} = 2^{\frac{\rho_{\min}}{\alpha}} - 1$. Hence:

$$\begin{aligned}
 \mathcal{L}_\rho(s) &= \int_0^{\bar{t}} e^{-st} \alpha \log_2(1 + t) dt + \rho_{\min} \int_{\bar{t}}^\infty e^{-st} dt \\
 &= \frac{\alpha e^s}{\ln(2)} \int_1^{\bar{t}+1} e^{-sh} \ln(h) dh + \frac{\rho_{\min} e^{-s\bar{t}}}{s} \\
 &= \frac{\alpha e^s}{s \ln(2)} \left[-\ln(\bar{t} + 1) e^{-s(\bar{t}+1)} + E_1(s) - E_1(s(\bar{t} + 1)) \right] + \frac{\rho_{\min} e^{-s\bar{t}}}{s} \quad (\text{A.2})
 \end{aligned}$$

where the identity $\int e^{ax} \ln(x) dx = \frac{e^{ax} \ln(x)}{a} - \frac{1}{a} \int \frac{e^{ax}}{x} dx$ and where $E_1(x) = \int_x^\infty \frac{e^{-u}}{u} du$ is used. Now, when there are N_U users in the network, and denoting p_k the probability for a UE to be served by k , it can be derived from (A.1) that

$$\begin{aligned} H_k &= \sum_{v=0}^{N_U} \binom{N_U}{v} p_k^v (1-p_k)^{N_U-v} \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k} \frac{\mathcal{L}_\rho(\frac{1}{\beta \Gamma_{i,k}(n)})}{\beta \Gamma_{i,k}(n)} \middle| |\mathcal{U}_k| = v \right] \\ &= \sum_{v=0}^{N_U} \binom{N_U}{v} p_k^v (1-p_k)^{N_U-v} v \mathbb{E}_\Omega \left[\frac{1}{A_k} \int_{\mathcal{A}_k} \frac{\mathcal{L}_\rho(\frac{1}{\beta \Gamma_k(s)})}{\beta \Gamma_k(s)} ds \right], \quad (\text{A.3}) \end{aligned}$$

where $\Gamma_k(s)$ is the SINR of a UE located in s and served by k . Here, the fact that UE spatial distribution is uniform is exploited, and so the expectation over n knowing v is v times the average value for a UE uniformly distributed over the serving area. The only remaining source of randomness is shadowing.

A.2 Proof of Proposition 2

This proof starts again from the the definition of H_k in (3.11) and of $R_{i,k}(n)$ in (3.7), assuming that fast fading follows the PDF $\Phi_{\theta^2}(x) = v e^{-x} (1 - e^{-x})^{v-1}$ (a similar approach is followed in [16, 74, 75]) when k serves v users:

$$\begin{aligned} H_k &= \sum_{v=0}^{N_U} \binom{N_U}{v} p_k^v (1-p_k)^{N_U-v} \times \\ &\quad \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k} \int_0^\infty \min\{\alpha \log_2(1 + \beta \Gamma_{i,k}(n)x), \rho_{\min}\} \times \right. \\ &\quad \left. v e^{-x} \sum_{r=0}^{v-1} \binom{v-1}{r} (-1)^r e^{-rx} dx \middle| |\mathcal{U}_k| = v \right] \\ &= \sum_{v=0}^{N_U} \binom{N_U}{v} p_k^v (1-p_k)^{N_U-v} \times \\ &\quad \mathbb{E}_n \left[\sum_{i \in \mathcal{U}_k} \sum_{r=0}^{v-1} v \binom{v-1}{r} (-1)^r \frac{\mathcal{L}_\rho(\frac{r+1}{\beta \Gamma_{i,k}(n)})}{\beta \Gamma_{i,k}(n)} \middle| |\mathcal{U}_k| = v \right] \\ &= \sum_{v=0}^{N_U} \sum_{r=0}^{v-1} p_k^v (1-p_k)^{N_U-v} v^2 \binom{v-1}{r} (-1)^r \mathbb{E}_\Omega \left[\frac{1}{A_k} \int_{\mathcal{A}_k} \frac{\mathcal{L}_\rho(\frac{r+1}{\beta \Gamma_k(s)})}{\beta \Gamma_k(s)} ds \right] \quad (\text{A.4}) \end{aligned}$$

A.3 Finding parameters $\mu_{I,k}$ and $\sigma_{I,k}$

The mean $M_{1,Ik}$ of $\sum_{j \neq k} \Theta_j P_k(f_j)$ is equal to:

$$M_{1,Ik} = \mathbb{E}_\tau \left[\sum_{j \neq k} \Theta_j P_k(f_j) \right] = \sum_{j \neq k} \rho_j \mathbb{E}_\tau [P_k(f_j)] = \sum_{j \neq k} \rho_j \int_{\mathcal{A}_j} \frac{\varrho(s)}{\rho_j} T(s) G_k(s) ds = \sum_{j \neq k} \rho_j Y_{j,k}, \quad (\text{A.5})$$

where $Y_{j,k} = \int_{\mathcal{A}_j} \frac{\varrho(s)}{\rho_j} T(s) G_k(s) ds$ depends on the physical environment. In view of the assumption that UEs power limitations have a less pronounced impact on performance in an urban scenario (see, e.g., [64]), it is assumed, for the calculation of $Y_{j,k}$, that $T(s) < T_{\max}, \forall s \in \mathcal{A}_j$. Hence, the following approximation is introduced: $\varrho(s)/\rho_j \approx \phi(s)/\Phi_j$.

The variance $M_{2,Ik}$ of I_k can be found considering that interfering signals from other stations are independent from one another:

$$\begin{aligned} M_{2,Ik} &= \mathbb{E}_\tau \left[\left(\sum_{j \neq k} \Theta_j P_k(f_j) - \rho_j Y_{j,k} \right)^2 \right] = \sum_{j \neq k} \mathbb{E}_\tau [\Theta_j^2 T^2(f_j) G_k^2(f_j) v^2(f_j) + \rho_j^2 Y_{j,k}^2 \\ &\quad - 2\Theta_j T(f_j) G_k(f_j) v(f_j) \rho_j Y_{j,k}] = \sum_{j \neq k} 2\rho_j H_{j,k} - \rho_j^2 Y_{j,k}^2, \end{aligned} \quad (\text{A.6})$$

where $H_{j,k} = \int_{\mathcal{A}_j} \frac{\varrho(s)}{\rho_j} T^2(s) G_k^2(s) ds$ depends on the physical environment. Again, the approximation $\varrho(s)/\rho_j \approx \phi(s)/\Phi_j$ is used.

Finally, $\mu_{I,k}$ and $\sigma_{I,k}$ are found by matching $M_{1,Ik}$ and $M_{2,Ik}$ with the mean and variance of the approximating lognormal, yielding $\mu_{I,k} = \ln(M_{1,Ik}) - \sigma_{I,k}^2/2$ and $\sigma_{I,k}^2 = \ln(1 + M_{2,Ik}/(M_{1,Ik})^2)$.

A.4 Backhaul Rate Derivation

Starting from Equation (6.17), $R_{BL,j,k}$ is defined as

$$R_{BL,j,k} = \frac{1}{\mathbb{E}_\tau \left[\frac{1}{C(\gamma_{BL,j,k}(\tau))} \right]}, \quad (\text{A.7})$$

where $\gamma_{BL,j,k}(\tau)$ is given by

$$\gamma_{BL,j,k}(\tau) = \frac{P_{BL,j}(s_{RN,j,k})}{\sum_{h \in \mathcal{S}_B, h \neq j} \Theta_{BL,h}(\tau) P_{BL,j}(f_{BL,h}(\tau)) + N}, \quad (\text{A.8})$$

Parameter $\Theta_{BL,h}$ in (A.8) is a binary random variable indicating whether the backhaul of h is active on τ , i.e., $P(\Theta_{BL,h} = 1) = \rho_{BL,h}$, while $P_{BL,j}(s)$ is the power received by BS j from a RN located in s , $s_{RN,j,k}$ indicates the location of RN k served by BS j and $f_{BL,h}(\tau)$ yields the location of the RN scheduled by h for backhaul transmission on τ .

During each slot τ a RN k in cell j can be scheduled for uplink transmission with probability

$$p_s(j, k) = \frac{\tilde{\rho}_{BL,j,k}}{\sum_t \tilde{\rho}_{BL,j,t}}, \quad (\text{A.9})$$

where $\tilde{\rho}_{BL,j,t} = (\bar{\omega}\Phi_t/(\beta R_{BL,j,t}))$. Let $T = |\mathcal{S}_B|$ denote the total number of cells in the network, and $i_j(\tau), j \in \mathcal{S}_B, i_j(\tau) \in \{0 \cdots n_{RN}\}$ denote the index of the RN scheduled on the backhaul of BS j on RB τ , where $i_j(\tau) = 0$ means that no RN has been scheduled for transmission. Now, assuming that backhaul scheduling decisions in a given cell do not depend on those taken in other cells, the probability of having a given set $\{i_1(\tau), \dots, i_T(\tau)\}$ of scheduled RNs on RB τ is equal to

$$\mathbb{P}(f_{BL,1}(\tau) = s_{RN,1,i_1}, \dots, f_{BL,T}(\tau) = s_{RN,T,i_T}) = \prod_j p_s(j, i_j(\tau)). \quad (\text{A.10})$$

There are $(n_{RN} + 1)^T$ possible scheduling combinations in all cells. An index $\delta, \delta = \{1 \cdots (n_{RN} + 1)^T\}$ is assigned to each combination, and the substitution $\mathcal{V}(\delta) = \prod_j p_s(j, i_j(\delta))$ is performed. Finally, the law of total probability can be applied to obtain

$$R_{BL,j,k} = \frac{1}{\sum_{\delta=1}^{(n_{RN}+1)^T} \mathcal{V}(\delta) \frac{1}{C\left(\frac{P_{BL,j}(s_{RN,j,k})}{\sum_{h \in \mathcal{S}_B, h \neq j} P_{BL,j}(s_{RN,h,i_h(\delta)}) + N}\right)}}, \quad (\text{A.11})$$

where it is assumed that $P_{BL,j}(s_{RN,h,0}) = 0$.

A.5 Some properties of Gibbs distributions for penalized energies

The purpose of this Appendix is to give an hint of the convergence of a SA process when an increasing exterior penalty such as (6.35) is added in the global energy, coupled with an adequate SA temperature scheme. Consider such an augmented energy:

$$\mathcal{F}(\mathbf{x}) = U(\mathbf{x}) + \mu \Phi(\mathbf{x}) \quad , \quad \text{for instance } \mu = \mu(m) = \alpha(m-1) \geq 0 \quad \text{and}$$

$$\Phi(\mathbf{x}) = \mathbb{1}_{D_c(\mathbf{x}) > D_{\max}} \frac{\Pi(\mathbf{x})}{D_{\max}} (D_c(\mathbf{x}) - D_{\max}) \quad (\text{see Eq. (33)}) \quad .$$

It is possible to say that Φ is a **penalty** if the set of its minimizers

$$\Omega^* = \{ x \in \Omega \text{ s.t. } \Phi(\mathbf{x}) = \Phi^* = \min_{\mathbf{y} \in \Omega} \Phi(\mathbf{y}) \} \quad (\text{A.12})$$

is exactly the feasible subspace $\tilde{\Omega}$ (this holds for (33)).

Now let us in general define the following set of "iso-constrained" subspaces:

$$\epsilon \in \mathbb{R} \mapsto \Omega_\epsilon = \{ \mathbf{x} \in \Omega \text{ s.t. } \Phi(\mathbf{x}) = \epsilon \} \quad ,$$

and consider the exponential distribution given in (30) (see also [133]).

For any value of ϵ such that $\Omega_\epsilon \neq \emptyset$, one has :

$$\forall \mathbf{x}, \mathbf{y} \in \Omega_\epsilon \quad , \quad \frac{P(X = \mathbf{y})}{P(X = \mathbf{x})} = \frac{\exp - \mathcal{F}(\mathbf{y})}{\exp - \mathcal{F}(\mathbf{x})} = \frac{\exp - [U(\mathbf{y}) + \mu \Phi(\mathbf{y})]}{\exp - [U(\mathbf{x}) + \mu \Phi(\mathbf{x})]} = \frac{\exp - U(\mathbf{y})}{\exp - U(\mathbf{x})}$$

So one can safely write in such an iso-constrained subspace:

$$P(X = \mathbf{x} \mid \mathbf{x} \in \Omega_\epsilon) = \frac{\exp - U(\mathbf{x})}{Z_\epsilon = \sum_{\mathbf{y} \in \Omega_\epsilon} \exp - U(\mathbf{y})} \quad (\text{also an exponential distribution})$$

(A.13)

Two key points are now at stake in view of simulated annealing purposes [133, 134] :

- first, as $\mu \rightarrow +\infty$ (μ is similar to an inverse temperature associated to the constraint $\Phi(\mathbf{x})$), the global distribution (30) becomes concentrated on the subspace Ω^* defined in formula (A.12) while keeping the exponential form (A.13) . The proof is classical: one can re-write (30) as

$$P(X = \mathbf{x}) = \frac{\exp - [U(\mathbf{x}) + \mu \Phi(\mathbf{x})]}{Z = \sum_{\mathbf{y} \in \Omega} \exp - [U(\mathbf{y}) + \mu \Phi(\mathbf{y})]} = \frac{\exp - [U(\mathbf{x}) + \mu (\Phi(\mathbf{x}) - \Phi^*)]}{\sum_{\mathbf{y} \in \Omega} \exp - [U(\mathbf{y}) + \mu (\Phi(\mathbf{y}) - \Phi^*)]}$$

$$\text{Now, } \left\{ \begin{array}{ll} \text{either } \Phi(\mathbf{x}) > \Phi^* & \text{and } \exp - \mu (\Phi(\mathbf{x}) - \Phi^*) \xrightarrow{\mu \rightarrow +\infty} 0 \\ \text{or } \Phi(\mathbf{x}) = \Phi^* & \text{and } \exp - \mu (\Phi(\mathbf{x}) - \Phi^*) = 1 \quad \forall \mu \end{array} \right. \quad \square$$

- then, consider distribution (30) with a temperature parameter $T > 0$. It can be written as:

$$\begin{aligned} P_T(X = \mathbf{x}) &= \frac{\exp - [\frac{U(\mathbf{x}) + \mu \Phi(\mathbf{x})}{T}]}{Z_T = \sum_{\mathbf{y} \in \Omega} \exp - [\frac{U(\mathbf{y}) + \mu \Phi(\mathbf{y})}{T}]} \\ &= \frac{\exp - [\frac{(U(\mathbf{x}) - U^*)}{T} + \frac{\mu}{T} (\Phi(\mathbf{x}) - \Phi^*)]}{\sum_{\mathbf{y} \in \Omega} \exp - [\frac{(U(\mathbf{y}) - U^*)}{T} + \frac{\mu}{T} (\Phi(\mathbf{y}) - \Phi^*)]} , \end{aligned}$$

where U^* is the minimum of value of objective function $U(\cdot)$ on Ω^* . Now, if both

$$T \rightarrow 0^+ \text{ and } \mu \rightarrow +\infty \text{ s.t. } \mu' = \frac{\mu}{T} \rightarrow +\infty \text{ (new inverse temperature) ,}$$

a similar analysis as before, now in two steps shows that for the penalty case where $\Omega^* = \tilde{\Omega}$, the distribution of interest $P_T(\cdot)$ concentrates on those configurations with minimal energy U^* on feasible subspace $\tilde{\Omega}$. See rigorous proof in [133, 134].

Appendix B

List of Publications

This appendix lists the publications related to this thesis.

- *Journal paper*: M. Minelli, M. Ma, M. Coupechoux, J.-M. Kelif, M. Sigelle, P. Godlewski, Optimal Relay Placement in Cellular Networks, *IEEE Transactions on Wireless Communications*, Feb. 2014.
- *Journal paper*: M. Minelli, M. Ma, M. Coupechoux and Ph. Godlewski, Scheduling Impact on the Performance of Relay-Enhanced LTE-A Networks, *IEEE Transactions on Vehicular Technology*, Apr. 2015.
- *Journal paper*: M. Minelli, M. Ma, M. Coupechoux, J.-M. Kelif, M. Sigelle and Ph. Godlewski, Energy-Delay Trade-Off in Relay-Based Cellular Networks, undergoing minor revision for *IEEE Transactions on Mobile Computing*.
- *Journal paper*: M. Minelli, M. Coupechoux, J.-M. Kelif, M. Ma and Ph. Godlewski, SIR Estimation in Hexagonal Cellular Networks with Best Server Policy, *Springer Wireless Personal Communications*, Mar. 2012.
- *Conference paper*: M. Minelli, M. Ma, J.-M. Kelif, M. Coupechoux and M. Sigelle, A fluid Approach for Performance Analysis of LTE-A Networks with Relays, *IEEE PIMRC Workshop*, Sept. 2013.

APPENDIX B. LIST OF PUBLICATIONS

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- *Conference paper*: M. Minelli, M. Ma, M. Coupechoux and Ph. Godlewski, A Geometrical Approach for Power Optimization in Relay-Based Cellular Networks, *IEEE NCC*, Feb. 2014.
- *Conference paper*: M. Minelli, M. Coupechoux, J.-M. Kelif, M. Ma and Ph. Godlewski, Relays-Enhanced LTE-Advanced Networks Performance Studies, *IEEE Sarnoff Symposium*, May 2011.

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