Responsible supply chain operations : analytical modeling of emerging issues

Raychaudhuri, Avijit

2017

Raychaudhuri, A. (2017). Responsible supply chain operations : analytical modeling of emerging issues. Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/71545>

<https://doi.org/10.32657/10356/71545>

**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**RESPONSIBLE SUPPLY CHAIN OPERATIONS:
ANALYTICAL MODELING OF EMERGING ISSUES**

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NANYANG BUSINESS SCHOOL**

2017

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Nanyang Business School

A thesis submitted to the Nanyang Technological University in
partial fulfilment of the requirement for the degree of
Doctor of Philosophy

2017

*Dedicated to my parents,
my wife, and our most eagerly awaited cherub*

Acknowledgment

My PhD journey has been one of the most memorable phases of life. I am glad that I have reached the end of the tunnel. As I look forward to the next great adventure in life, I cannot but reminisce about the trying years of my doctoral study and feel indebted to all those who have contributed towards the successful completion of my PhD.

I would like to convey my heartfelt gratitude to my PhD Advisors Dr. S. Viswanathan (Vish) and Dr. Fang Liu for guiding me during my PhD. Both my advisors are two of the most intelligent people that I have ever met. I am extremely glad that I chose both of them as my advisors, as I have been able to imbibe complementary skills from each of them. Words are not enough to express how valuable they have been to me as mentors. With them, I have always had the freedom to speak freely, even it meant presenting counter arguments to their points of view during any discussion. I hope all PhD advisors would be like them in terms of intellectual abilities and providing freedom of expression to their students. I sincerely thank Dr. Vish and his family, and Dr. Fang for giving me the opportunity to become very good friends with them over the years.

I am grateful to the members of my Thesis Advisory Committee and

Thesis Examination Committee Dr. Chen Chien-Ming, Dr. Wang Qinan, Dr. Geoffrey Chua, and Dr. Onur Boyabatli for their deeply insightful comments and suggestions about improving my dissertation. I sincerely thank Dr. Rohit Bhatnagar for his support and interesting discussions with me from time to time. I also thank Dr. Arvind Sainathan, Dr. Wang Jianfu, and Dr. Michael Li for always lending a helping hand, whether in the form of moral or technical advice. Furthermore, I thank Dr. Nilanjan Sen and his family with all my heart for their love and support. I will forever cherish the wonderful conversations and dinners that we have had together.

No amount of gratitude is enough to thank the staff in the PhD Office, who work tirelessly towards supporting the researchers in the Nanyang Business School in producing high quality research. Nevertheless, I sincerely thank Bee Hua, Karen, Nisha, and Tsai Ting for their invaluable support throughout my PhD. Furthermore, I thank Julia, the Secretary to the Division of Information Technology and Operations Management, for her kind assistance in making the PhD experience worthwhile.

I would like to especially thank my dear friend Dr. Mahendra Birhade for making my PhD experience memorable. We bonded through our incessant discussions and debates spanning a gamut of topics including business, politics, philosophy, and many more. He is not only my peer, but also like an elder brother to me. I cannot thank him and his family enough for having become such an important part of my life.

I am thankful to have forged strong bonds with many friends within and outside the Nanyang Business School. My PhD experience would not have been the same without the company of my close friends Jacky, Ben,

Siran, Neha, Zihan, Liu Yan, Zubair, Guangyu, Boqian - from the Nanyang Business School, and Xiaofei from the School of Mechanical and Aerospace Engineering.

My mother Uma Raychaudhuri and father Tridip Raychaudhuri have motivated me to aim for lofty goals since my first memories that I can recall. They have endured a life of struggle to provide me with selfless and unconditional support in all my endeavors. I am proud of my parents and am lucky to be their son. I am glad that I have been able to bring smiles to their faces.

Finally, my wife Deboleena has been the pillar of unrelenting support throughout all the years that we have known each other. Even though we have been married for a few years now, she has always been and will always be my other and better half. I am at a loss for words to express how lucky I was to have spent the toughest years of my PhD with her by my side. Here's to an eternity of togetherness!

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Abstract

Supply chain responsibility is a growing area in the operations management (OM) literature. This dissertation contributes to this literature by analyzing and explaining practical issues which are not well understood but are crucial for comprehending the implications of responsibility in supply chain operations. In this dissertation, a firm's perspective of profit maximization is taken in order to address two corporate decisions which are intrinsic to supply chain responsibility: contracting with suppliers, and investments in improving supply chain responsibility. Throughout the dissertation, non-governmental organizations (NGOs), governments, shareholders, and consumers are collectively referred to as "external entities".

Following the introduction, the second chapter reviews the literature relevant to this dissertation. The extant OM literature on supply chain responsibility is not exhaustive, with most of the work being in the area of responsible sourcing. This is not surprising, considering that challenges to fostering supply chain responsibility are still emerging in practice. As a result, the analytical models used in this dissertation are derived from multiple areas within OM such as responsible sourcing, supplier auditing, and disruption management. The literature review provides a comprehensive discussion

about the literature related to this dissertation.

The third chapter deals with the contract between a firm and her suppliers, when the suppliers differ with respect to their intrinsic degrees of responsibility. Firms generally source from a variety of suppliers, including the potential perpetrators of supply chain responsibility violations. A supply network comprising a profit-maximizing buyer, a supplier who may commit direct responsibility violations (risky supplier), and a supplier who may commit indirect responsibility violations such as unauthorized subcontracting (apparently responsible supplier) is modeled. The firm uses the latter supplier as a backup in case the former becomes unavailable during actual production. The firm's optimal contract terms presented to the suppliers is obtained by solving a mechanism design problem. The analyses of the model suggest that it is indeed optimal for the firm to squeeze her suppliers' margins, in general. However, under certain conditions, the firm might pay a premium to the apparently responsible supplier for reserving backup capacity. Another interesting finding is that through the terms of contract, the firm might cover some of the costs that should have been actually incurred by the apparently responsible supplier.

The fourth chapter addresses the following question: Is external pressure on global firms for improving supply chain responsibility always effective? This chapter uses analytical modeling to analyze a firm's incentives for investing in improving the responsibility of suppliers, when exposed to reputation and financial damage from external entities. In contrast to the existing practice of always criticizing and penalizing the buyers and prevalent academic wisdom (Guo et al. 2015), the analyses of our model suggest: criticizing and

penalizing the firms do not necessarily improve supply chain responsibility when the improvement of supply chain responsibility hinges on investments by the firms. This finding highlights the potential negative effect of criticizing the firms which is observed in practice: after the Rana Plaza disaster in the Bangladesh apparel industry, even though there has been continuous global pressure on firms to invest in improving their suppliers' responsibility, actual investments in supplier improvement have not been significant. Our analyses also find that in general, external entities should restrict the firms' violation costs to a wider (narrower) interval, if the suppliers are more (less) differentiated with respect to their intrinsic riskiness and costliness. Further, it is demonstrated that external entities should directly penalize a supplier if she engages in unauthorized subcontracting. Additionally, unauthorized subcontracting can be avoided if the penalty is higher than a threshold. Finally, a holistic policy for the external entities is devised, which can achieve maximum supply chain improvement within the scope of our model.

Chapter 1

Introduction

“With great power comes great responsibility”

This is possibly the most apt quote that can succinctly put forth the relevance of responsibility in the context of modern supply chains. In modern times, supply chains have evolved into cross-continental behemoths. The sheer span and scope of contemporary supply chains make them powerful players in shaping global economies and societies. By virtue of such power, it behooves supply chains to exercise responsibility in their strategic and recurrent operations. In other words, supply chains should ensure that their existence and operations should not presuppose, endanger, or irremediably damage natural resources, both living and inanimate. Milton Friedman’s interpretation of responsibility might have been different, though. Friedman had said: *“there is one and only one social responsibility of business — to use its resources and engage in activities designed to increase its profits”*. From a supply chain perspective, Friedman’s assertion implies that supply chains should focus exclusively on utilization of resources for survival and financial

prosperity. However, the fallacy of this implication lies in the tacit assumption of resources being inexhaustible. In reality, resources, which comprise both humans and other natural sources, are neither constant nor infinite, and hence not inexhaustible. Hence, optimal utilization of resources is merely a sufficient condition for supply chains to be profitable and sustain profitability. The necessary condition for sustained profitability of supply chains is responsible management of resource availability.

Therefore, I interpret supply chain responsibility as follows: *“The responsibility of a supply chain pertains to prudent management of its living and inanimate resources in order to maintain sustained availability of the resources, and optimal utilization of the resources for ensuring economic sustainability of the supply chain”*. This means that economic sustainability of supply chains is predicated on the sustainability of societies and natural resources. Naturally then, supply chains are responsible for ensuring economic, social and environmental sustainability. If this sounds familiar to the reader, it is simply due to its relation with the concept of the three Ps — *profit, people, and planet* — which form the quintessence of the Triple Bottom Line (TBL) accounting framework. Unfortunately, it was only in 1994 that John Elkington formally conceptualized the interdependence of profit, human, and other natural resources as the basis for the TBL framework for an individual firm (The Economist, 2009). Thus, my interpretation of supply chain responsibility is derived by tempering Friedman’s concept of a firm’s responsibility with the interdependence of the three P’s, and reckoning that ensuring supply chain sustainability is a collective responsibility of the entire supply chain and not of a single firm.

Now that I have stated my interpretation of supply chain responsibility, the natural question to ask is: “Who is (are) responsible for ensuring that a supply chain is responsible?” The answer follows the perception of responsibility being collective instead of being merely individual. That is, the responsibility to ensure that a supply chain operates responsibly does not lie with any single firm, rather it is the shared responsibility of all players related to the supply chain – institutional, governmental, and non-governmental. The primary institutional players in a supply chain are the firms – suppliers, manufacturers, distributors, and retailers. In a vertically integrated supply chain, the central firm has access to information about responsibility challenges and can exert considerable influence throughout its supply chain, and hence is almost exclusively obligated to promote responsibility across its supply chain. However, most supply chains are not vertically integrated, due to the innate complexities posed by vertical integration. In such supply chains, to the extent possible, each firm needs to be aware of responsibility challenges at upstream, on-site, and downstream stages. I say “to the extent possible” because a firm’s actions largely depend upon the behavior/preferences of its shareholders. As such, shareholders of a firm are also responsible for steering the firm towards long-term economic sustainability through responsible management of the resources across its supply chain. Governments are responsible for oversight of the business environment in general, and thereby responsible for enacting and implementing laws which motivate supply chain players to exhibit responsibility. Non-governmental organizations (NGOs) possess the potential to play the crucial role of acting as a bridge between firms and the governments. Due to their ability to focus on specific issues,

social and environmental NGOs are potentially efficient detectors of responsibility challenges in supply chains, which might otherwise remain undetected. They can then influence both firms and governments in order to help/force them remedy the challenges through corporate initiatives, legal action, or enactment and implementation of new laws. Individual consumers should try to become aware of the responsibility challenges that might exist across the supply chains of the products that they use. Although individual consumers rarely have access to adequate information for making responsible choices with regards to their consumables, they should endeavor to acquire as much information as possible, and let their collective voices be heard through social or legal channels. Consumers are also responsible for ensuring responsible disposal of products at the end of product life-cycles. Finally, media serve as the vehicles of information flow across boundaries and barriers. Consequently, they possibly bear the most important role in fostering supply chain responsibility – that of molding corporate responsibility reputation in the eyes of non-business stakeholders.

Unfortunately, common perception is that big firms are the ones who are responsible for ensuring that supply chains operate with responsibility. Anecdotal evidence from recent tragedies in the global apparel industry serves to illustrate the perception. Global corporations sourcing from Bangladeshi apparel factories faced unilateral and harsh criticism for their apparent lack of responsibility in promoting responsibility in their supply chains. Possibly because it is easy to identify and pressure big firms/corporations, external entities like the NGOs and media find it convenient to unilaterally castigate them for the existence of responsibility challenges in their supply chains. It

is interesting to note that with regards to supply chain responsibility, even the academic community has also largely been oblivious to the complexities of modern supply chains. The academic literature primarily explores problems related to identification and sourcing from “responsible suppliers” under various circumstances, from the buyers’ perspective. Naturally, the onus of supply chain responsibility is implicitly assumed to rest with the buyers, which are often modeled after the big firms in real life. But as explained earlier, responsibility of a supply chain does not equate to an individual firm’s responsibility, but is shared among various entities comprising the supply chain. Therefore, the question that arises is: “Does unilateral castigation of big firms always improve supply chain responsibility?” I answer this question through Chapters 3 and 4 of my dissertation. In the former, I evaluate the optimal terms of contract as might be presented by a firm to her suppliers. In the latter, I discuss the efficacy of various mechanisms that are available to external entities for pressuring big firms for improving the responsibility of their supply chains. In accordance with the fact that firms cannot improve supply chain responsibility unless they are profitable, I take the perspective of a profit-maximizing firm for my analyses. The findings from the analyses counter the prevalent practice of unilateral castigation of big firms for all supplier responsibility violations, and thereby have the potential to contribute significantly to our understanding of supply chain responsibility.

The dissertation is organized as follows: Chapter 2 comprises a review of the literature on social and environmental responsibility pertinent to the problems addressed. Chapter 3 addresses the problem of evaluation of the

optimal terms of contracts between a firm and her suppliers. Chapter 4 addresses the extremely important question about the efficacy of external pressure on firms in motivating them to invest in improving supply chain responsibility. Finally, Chapter 5 comprises a summary of the main findings of the dissertation, and a description of ongoing extensions and planned problems for future research.

Chapter 2

Review of Related Literature

The dissertation contributes to the growing literature on responsibility in supply chains. The extant literature focuses on various mechanisms intended for improving supply chain responsibility, from the perspective of various supply chain stakeholders. However, even though buyer investment in supplier improvement is often the key to improving supply chain responsibility, the extant literature on supply chain responsibility does not shed light on how external entities can influence such investment by buyers. This is the first study of the efficacy of external pressure on buyers in improving supply chain responsibility by motivating buyer investment in supplier improvement, and how the external entities can motivate such investment by the buyer.

Guo et al. (2015) compare the profitability of a buyer's decision to source from a cheap but irresponsible supplier versus an expensive and responsible supplier. They model a single buyer selling a product to a fixed size market, and two suppliers who differ only with respect to their probabilities of committing responsibility violations. The "risky supplier" commits

responsibility violation with a positive probability, which is considered to be exogenous. The “responsible” supplier does not commit any responsibility violation with certainty. The buyer decides which supplier(s) to source from and sourcing quantities at the very beginning and does not change the decisions later. Effectively, this leaves the buyer with no flexibility to switch to a different supplier in case a violation occurs. In an extension, the consumers of the product are considered to be heterogeneous in their willingness to pay for responsibly produced products.

In this dissertation, we consider a buyer selling a product which has random demand. Similar to their Guo et al. (2015), the buyer in this dissertation has two suppliers available, one of whom (“risky” supplier) commits responsibility violations with a fixed exogenous positive probability. Unlike in this dissertation, their paper does not consider investments made by the buyer in improving the responsibility of the risky supplier. This is one of the most crucial differences, which allows us to address the problem of improvement of supply chain responsibility from a more practical perspective instead of the traditional concept of responsible sourcing - which propagates the notion that firms should choose only the most responsible suppliers available in order to improve supply chain responsibility.

In contrast to the risky supplier in Guo et al. (2015), the risky supplier in this dissertation is of two types - more risky and less risky. The type of the risky supplier in this dissertation is the supplier’s private information, which means that the buyer is unaware of the supplier’s actual type at the beginning. This difference allows us to analyze the buyer’s decisions pertaining to the terms of contract between her and her suppliers. The other supplier (“ap-

parently responsible” supplier) does not commit any responsibility violation at her own facility. But in contrast to Guo et al. (2015), this supplier is not perfectly responsible in the sense that under capacity constraints, she might engage in indirect violations such as unauthorized subcontracting. Moreover, in contrast to their paper, we allow the buyer to have flexibility to switch between suppliers and change order quantities at a later stage in the sequence of events. In order to accommodate this flexibility, we establish three scenarios which might arise as a result of a combination of the following factors: existence of supplier responsibility violations, occurrence of disruptive events as a result of supplier responsibility violations, and detection of responsibility violations through responsibility audit processes. Finally, in contrast to their model, we account for the possibility of unauthorized subcontracting being committed by the apparently responsible supplier.

Guo et al. (2015) find that supply chain responsibility would improve in a punitive environment for the buyer. This conforms to the traditional view that too much external pressure on the buyers would help improve the responsibility of the supply chain. In contrast, we find that a punitive approach adopted by external entities against the buyers does not necessarily improve supply chain responsibility, as a high level of punishment might not incentivize the buyers to pro-actively invest in supplier improvement.

Plambeck and Taylor (2015) study audit as the buyer’s mechanism to improve responsibility and hiding of compliance violation as the supplier’s mechanism to evade scrutiny. They model one buyer who exerts effort for auditing the responsibility of her supplier. The supplier exerts efforts for responsibility and for hiding of irresponsibility from the supplier responsibility

audit. In contrast to their paper, we model a buyer who has two suppliers available. The buyer can source from either or both suppliers. Furthermore, although we do not consider supplier’s hiding effort, the risky supplier in our model possesses private information about her likelihood of committing responsibility violations. Both these modeling choices give us the opportunity to analyze complex contracting decisions taken by the buyer. Most importantly, Plambeck and Taylor (2015) do not model any effort on the part of the buyer in improving the supplier’s responsibility. In contrast, we present a non-traditional perspective to improving of supply chain responsibility, and model the buyer’s investments in improving her supplier’s responsibility as the key to improving supply chain responsibility.

Plambeck and Taylor (2015) demonstrate that in a single buyer-single supplier setting, too much auditing can backfire when suppliers exert effort to hide evidence of non-compliance from the auditors. Albeit in a different model setting, we also demonstrate that too much auditing does not help improve supply chain responsibility, and hence we advocate against forcing buyers to conduct too much audits.

Kim (2015) analyzes inspection policies followed by environmental regulators and their impact on firm compliance. Consider the audit in their model. In their paper, a firm experiences random breakdowns of an equipment which controls the emission of harmful pollutants from the production process into the atmosphere. An external auditor inspects the firm’s equipment with a fixed frequency and penalizes the firm with a fixed penalty if equipment malfunction is detected. Although the auditor fixes the frequency of audits, she can choose to conduct the audits periodically after fixed time-intervals, or at

random.

Note that while in their paper the equipment breaks down voluntarily, responsibility violations committed by the risky supplier in this dissertation are the supplier's conscious act. However, the risky supplier is comparable to the firm in Kim (2015) in the sense that the risky supplier commits responsibility violations with an exogenous probability. We attribute the exogenous nature of committing violations to the workplace culture fostered by the supplier's management personnel.

The most important comparison between Kim (2015) and our model is with respect to the implications of the audit process. In their paper, the audit process is modeled in terms of frequency of inspections; whereas in our model, the audit process is modeled in terms of investment in audit made by the buyer, or equivalently in terms of the probability that the audit detects supplier responsibility violations, if any. While this difference is attributed to the difference in the research questions, both the audit processes have some sort of similarity.

In both their paper as well as in this dissertation, some decision with respect to the audit process is determined upfront and is not changed later. Whereas in their paper the audit frequency is fixed upfront, in our paper the buyer's investment in audit is fixed upfront. However, unlike in this dissertation, the auditor in their paper can choose the exact method of audit after the inspection frequency has been fixed. This allows them to explore an operational aspect of auditing in their paper. In this dissertation, we do not consider such operational aspects of auditing. Rather, we consider audit as a strategic-level decision for the buyer. In reality, the precise way that audits

are conducted can be decided after the strategic decisions. However, such operational decisions regarding audit are outside the scope of this dissertation, and would require modeling on the lines of Kim (2015).

Özdemir et al. (2016) analyze vertical integration as the buyer's mechanism to promote responsibility in its supply chain. It should be noted that although vertical integration can be considered to be special type of investment in improving supplier responsibility. However, vertical integration is an operationally challenging endeavor due to which few firms might be able to promote supply chain responsibility through vertical integration.

Other papers have studied various problems related to the analysis and management of supply chain responsibility. Chen and Lee (2016) analyze a supplier's decisions to exert responsibility effort under various contracts instituted by a buyer. Kraft et al. (2013) study an NGO's choices between going after firms or the regulatory body in order to eliminate remove potentially hazardous substances being used in products. Fang and Cho (2015) analyze supplier audit as the mechanism employed by coalitions of manufacturers for inducing responsibility among suppliers. Lewis et al. (2015) discuss a multi-period contract mechanism whereby the buyer makes investments to alleviate the harmful effects of non-conducive production conditions. Kalkan et al. (2013) analyze how voluntary and mandatory disclosures affect the social and environmental impacts of a company's operations. Huang et al. (2016) analyze a firm's management of responsibility in a three-tier supply chain. In contrast to this dissertation, none of these papers model buyer investment in improving the responsibility of the supplier(s).

We derive inspiration from models in the supply chain disruption literature. In the context of supply chain responsibility violations, disasters (analogous to supply disruptions) are not necessarily low-frequency events as in (Simchi-Levi et al., 2014), but might be high-frequency, high-impact events. Natural disasters like hurricanes, floods, earthquakes or epidemics are not frequent and hence have low likelihood of occurrence. Responsibility violations are however man-made, and are rampant in numerous supply chains. For example, due to the unsafe working conditions of the apparel factories in developing countries, there is a huge potential for damage to both property and human life. Due to rampant negligence of safety standards and recommendations, serious disasters are simply waiting to occur (Hasan, 2012). Certainly the likelihood of fatal disruptions (which we incorporate in our model) taking place at such factories is very high. Hence, unlike in models wherein the likelihood of disruption might be considered to be negligible, the likelihood of a disruption in our model is not necessarily low.

In general, papers in this area consider the reliability of a supplier in terms of not experiencing production disruptions by modeling the supplier as being of two kinds - reliable and unreliable. The reliability of a supplier is generally defined in terms of the propensity of the supplier to experience disruptions (Yang et al., 2009; Ang et al., 2014; Hu and Kostamis, 2015), or in terms of the probability with which the production system is “up” (Tomlin, 2006; Lim et al., 2010). Our modeling of the risky supplier closely follows that of the unreliable supplier in this literature. Similar to the unreliable supplier, the risky supplier in our model commits responsibility violations with a fixed probability. We model the risky supplier as having two types

- high type (more risky) and low type (less risky), and define a mechanism design problem for the buyer to decide optimal terms of contracts presented to the risky supplier. This is similar to Yang et al. (2009), wherein the manufacturer presents a menu of two contracts to the unreliable supplier. The apparently responsible supplier in our model resembles the reliable supplier in this literature, albeit with a critical difference. The resemblance is attributed to our definition that with certainty, the apparently responsible supplier does not commit any responsibility violation at her own facility. However, under capacity constraints, she might engage in indirect violations such as unauthorized subcontracting and hence is not a perfectly responsible supplier.

As we will explain in §3.2, by virtue of her risky nature, the risky supplier might experience disruptions at her factory. When this happens, the buyer switches to the other supplier for procurement. This dynamic is similar in spirit to the one exhibited in Tomlin (2006), the seminal paper on optimal sourcing strategies under supply chain disruption. Tomlin (2006) models contingent capacity provided by the backup supplier (“reliable supplier”) as costless but time-dependent. In contrast, we model the capacity provided to the buyer by the backup supplier as a costly decision taken by the supplier, based on the contract terms designed by the buyer. This helps us in analyzing the supplier’s incentives for engaging in unauthorized subcontracting.

Supply chain responsibility can be semantically compared to product quality. However, problems related to the former are different than those related to the latter (Chen and Lee, 2016). The quality literature comprises

papers which model quality as an outcome of the supplier’s effort. Kaya and Özer (2009) consider that the supplier determines the quality of the product. Babich and Tang (2012) use deferred payment by a buyer as a mechanism to motivate the supplier to produce goods of desired quality. However, product quality is not affected by supplier responsibility violations. In fact, high product quality is one of the key drivers for global buyers sourcing from the Bangladesh ready-made garments industry which is riddled with egregious violations of supply chain responsibility (*Dawn*, 2013). We consider supplier responsibility as an exogenous factor. We refer the reader to Chen and Lee (2016) for a concise discussion on the basic similarities and differences between supplier responsibility problems and product quality problems, and why quality enforcement mechanisms are not applicable in the context of supplier responsibility problems.

The general assumption in the quality literature is that a sub-par quality product will certainly be detected somewhere during its life-cycle (Mayer et al., 2004; Babich and Tang, 2012; Rui and Lai, 2015; Karaer et al., 2016). In contrast, supplier responsibility violations might remain undetected despite monitoring of production facilities by the buyer.

A number of papers model the inspection of the supplier’s products as a decision (Reyniers and Tapiero, 1995a,b; Starbird, 2001; Hsieh and Liu, 2010; Babich and Tang, 2012). However, the buyer’s decision to choose whether or not to conduct responsibility audits on her suppliers is not relevant to problems involving responsibility violations. This is because conducting such audits is a mandatory requirement for the buyer. The buyer’s investments in conducting supplier responsibility audits considered in our

model are in line with the strategy instituted by international buyer groups such as the Bangladesh Accord on Fire and Building Safety and the Alliance for Bangladesh Worker Safety in order to promote responsibility in supply chains. Hwang et al. (2006) study a model wherein the supplier performs inspection of her own products. This is not applicable in the context of this dissertation because responsibility audits of supplier facilities are performed by the buyers and not by the suppliers themselves.

Mechanisms for sharing of costs arising due to low product quality have been widely discussed in the quality literature (Reyniers and Tapiero 1995a,b, Baiman et al. 2000, Balachandran and Radhakrishnan 2005, Zhu et al. 2007). However, in supplier responsibility problems, the buyer bears the damage to her reputation and finances caused by public revelation of compliance violation by an irresponsible supplier. In our model, the buyer solely bears the costs of violations.

Gurnani et al. (2007) discuss the effect that the sequence of investment and pricing decisions by buyers and suppliers have on supply chain co-competition, wherein the supplier sets the wholesale price. Unlike in their paper, we design an optimal set of contracts consisting of a wholesale price per unit and a transfer price, presented by the buyer to the suppliers. Hsieh and Liu (2010) discuss the effects that varying extents of information available to a supplier and a manufacturer have on their quality and inspection policies. In their paper, the manufacturer produces goods after sourcing components from the supplier. Both the supplier and manufacturer invest in quality, albeit at their own production facilities. In contrast, we consider that the buyer simply procures finished goods from the supplier and sells it to the

end consumers. Also, we model the buyer's investment in improving the supplier's responsibility.

In Balachandran and Radhakrishnan (2005), the buyer inspects the components procured from the supplier before using them for manufacturing the final product. Chao et al. (2009) focus on the supplier's and buyer's effort to improve quality by minimizing design flaws. Both these papers consider supplier investment in product quality. However, the focus of our analyses is buyer investment in improvement of supply chain responsibility. We capture the suppliers' responsibility exogenously through their unit production costs. We assume that the more risky the supplier, the lower is her unit cost of production. Fang and Cho (2015) consider competitive and cooperative interactions among buyers with respect to audits for product safety. Such a setting might be useful while modeling buyers' decisions in the context of the Accord and the Alliance, for example. In order to focus on a single buyer's profit maximization, we do not include competition or cooperation among buyers in our model.

Chapter 3

Buyer-Supplier Dynamics in a Diversified Sourcing Regime: Terms of Contracts

3.1 Supply Chain Responsibility, Violations, and External Pressure on Buyers

The traditional notion of responsible sourcing eschews suppliers with high likelihood of following irresponsible practices in their production and manufacturing operations. This notion stems from the belief that sourcing from only responsible suppliers decreases the risk of supply chain responsibility violations. However, capacity and cost constraints have increasingly forced a plethora of firms (i.e. the “*buyers*”) to source from a wide variety of suppliers, including the potential perpetrators of supply chain responsibility violations.

That is, suppliers differ in the degree to which they are responsible (Berg et al., 2012). We observe such diversified sourcing in the Bangladesh apparel industry (*Dawn*, 2013; Mirdha, 2014), and further evidence of buyers sourcing from diverse suppliers in multiple industries and geographies (Myers, 2013; *O’Keefe*, 2016). Indeed, the prevalent practice of sourcing from suppliers of varying degrees of risk is likely to prevail in the foreseeable future (A.T.Kearney, 2015). Thereby, the traditional notion of responsible sourcing is being rendered moot and impractical.

Supplier responsibility violations can be largely categorized into two kinds, depending on where the violations actually occur: *direct violations* occur at a buyer’s primary supplier’s facility, for example blocking of fire exits of a factory; and *indirect violations* occur at a secondary supplier’s facility but can be linked to the primary supplier, for example unauthorized subcontracting. According to Clifford and Greenhouse (2013); Lahiri and Al-Mahmood (2012), suppliers usually resort to unauthorized subcontracting when their production capacity is insufficient to meet the buyers’ demands. Due to the diversified nature of sourcing, incidents relating to supplier responsibility violations have become common occurrence in industries such as ready-made garments (RMG). For instance, on November 24, 2012, at least 112 people were killed in a fire at the Tazreen factory in Bangladesh (Allchin, 2014). Barely five months later, in one of the world’s worst industrial tragedies, the Rana Plaza building (which housed several garment factories) in Bangladesh collapsed, killing more than 1,130 people and injuring 2,500 others (Ayres, 2014). More than 300 workers were killed in a deadly fire at a textile factory complex in Pakistan in 2012 (ur-Rehman et al., 2012). Child labor is rampant

in coffee and chocolate plantations in Africa, which supply to global corporations (Nall, 2012). An explosion at the Zhongrong metal parts factory in China killed at least 75 workers and injured at least 186 (Young, 2014). Land acquisition practices at the sugarcane plantations in Cambodia have resulted in human rights abuses and forced evictions of inhabitants (Blomberg and Sothear, 2015).

When such incidents are identified and publicized, the buyers are lambasted by external entities such as the media, non-governmental organizations (NGOs), consumers and other stakeholders for doing business with risky suppliers (Biraj, 2013; Pasick, 2013; Cruise, 2011). These external pressures expose the buyers to reputation damage, which in turn might lead to significant direct and indirect costs including mitigation costs, loss of demand or consumer boycott (Guo et al., 2015).

In the above context, we aim to evaluate the optimal contract terms between a buyer and her suppliers in this chapter. We model a supply network comprising a profit-maximizing buyer and two suppliers who may commit different responsibility violations. The buyer contracts with the suppliers for the supply of a single product over one period. Among the two suppliers, one is prone to direct violations (*“risky supplier”*), and the other commits only indirect violations such as unauthorized subcontracting (*“apparently responsible supplier”*) when her production capacity is insufficient to meet the buyer’s demand. Since the two-part tariff contract is one of the most commonly used contract used in such industries as mentioned above, we evaluate two-part tariff contracts in this chapter. Each contract comprises two terms: an upfront transfer payment from the buyer to a supplier, and a unit whole-

sale price paid by the buyer for each unit of product ordered from a supplier. The transfer payment serves as a fixed cost incurred by the buyer for establishing business contract with either supplier. The unit wholesale price on the other hand is incurred by the buyer only when she actually procures from either supplier.

The rest of the chapter is organized as follows: §3.2 describes the model and the sequence of events which span three stages. In the first stage, the buyer designs optimal contracts for her suppliers. In the second stage, the apparently responsible supplier reserves production capacity for the buyer. Finally, in the third stage, the outcome of external perception about the buyer's sourcing practice is realized, and the buyer accordingly procures from either the risky or the apparently responsible supplier. §3.3 discusses the preliminary results derived from solving the second and third stages in reverse order. §3.4 discusses the results and insights from the first stage in which the buyer decides the optimal contract terms to be presented to the suppliers. Finally, §3.5 presents the detailed proofs of the lemmas and theorems in this chapter.

3.2 Model

3.2.1 Model Overview

A single buyer B sells a product to the end market with random demand at a unit retail price p over a single selling season. The supplier base of B

comprises a “*risky supplier*” S_1 and an “*apparently responsible supplier*” S_2 .

The supplier S_1 is risky in the sense that she is prone to committing all kinds of responsibility violations, but her probability of committing on-site responsibility violations is unknown to B . We assume that S_1 can be of two types denoted by $i \in \{H, L\}$ which indicate that S_1 is either *more risky* (S_1^H : *High type H*) or *less risky* (S_1^L : *Low type L*). We denote the probability of violations being committed by S_1 as θ_i , where $\theta_H > \theta_L > 0$. That is, S_1^H is more likely to commit responsibility violations than S_1^L . We denote the probability that S_1 is of type H as $\rho \in [0, 1]$. The probability that S_1 is of type L is thus given by $1 - \rho$. The supplier S_1^i ’s unit production cost is c_1^i . We assume that $c_1^L > c_1^H$, that is the more risky supplier has a lower unit production cost than the less risky supplier.

The supplier S_2 is known to B as a responsible one (through reputation and/or certifications). Unlike S_1 , S_2 does not commit any direct responsibility violation. However, when faced with capacity constraints, S_2 subcontracts production to other unregulated suppliers (who are likely to commit responsibility violations) without authorization from B . Since S_2 does not commit any direct violation but might commit indirect violations, we refer to S_2 as *apparently responsible* instead of *responsible*. This supplier’s unit production cost is c_2 . We assume that $c_2 > c_1^L > c_1^H$, that is the apparently responsible supplier has the highest unit production cost among all the types of suppliers.

Responsibility violations committed by the two suppliers S_1 and S_2 can be detected either by external entities such as NGOs and media, or by formal audit processes carried out by third party auditors. Audits have their own

drawbacks by virtue of the sheer complexity of gleaning evidence of violations at supplier factories (Clifford and Greenhouse, 2013). For example, prior to the Rana Plaza disaster in Bangladesh, Brussels-based auditors Business Social Compliance Initiative had incorrectly approved two of the factories operating in Rana Plaza building (Chao, 2013). We refer to the probability that responsibility violations committed by a supplier is detected as the *audit efficiency of the system*. The audit efficiency of the system is denoted by $\alpha \in [0, 1]$. It follows from our definition of α and θ_i that the probability of the audit detecting a violation that has been committed by S_1 is given by the product $\alpha\theta_i$. We assume that the auditors are capable enough so that the audit process does not flag a violation when there is none. Basically, this relates to a subtle difference with auditing product quality. Quality audits can give false positives since there is a possibility that a good batch might be rejected due to a bad sample. However, in responsibility audits, detection of even a single violation means that the supplier is risky – it is not possible that a supplier who has been flagged as non-compliant is actually responsible. We model supplier responsibility audit as a continuous process. Hence a supplier responsibility violation can be detected at any point within the single period considered in the model. The outcomes of the audits are available to the external entities without any delay. We will describe in more details the implications of our modeling of the audit process in the following section.

3.2.2 Sequence of Events

We divide the sequence of events into three stages: *contracting* (Stage 1), *capacity reservation* (Stage 2), and *external perception and execution* (Stage 3). As mentioned in the previous section, supplier responsibility audit commences simultaneously during the contracting stage and continues until the end of the external perception and execution stage. Figure 3.1 summarizes the sequence of events.

Figure 3.1: Sequence of Events

Contracting stage: The buyer B makes operational level decisions on contracting in this stage. She offers S_1 a menu of two-part tariff contracts $\langle t_1^i, w_1^i \rangle (i \in \{H, L\})$ comprising two components: a transfer payment t_1^i and a unit wholesale price w_1^i . Although B knows the values of θ_H and θ_L , she does not know the type of the S_1 at this stage. That is, B does not know whether S_1 's actual probability of committing responsibility violations is θ_H or θ_L .

On the other hand, S_1 precisely knows her own probability of committing a violation. Therefore, B needs to devise a contracting mechanism so that her lack of information about S_1 's type does not lead to any financial loss for B . Such a loss would happen if S_1^H were to pretend to be S_1^L or vice versa. To avert such a loss, B presents two different contracts to S_1 . Basically, B gives two choices of contracts to S_1 , all the while knowing that S_1 would not earn reservation profit if she were to select the wrong type of contract. Since S_1 is rational, she would never select the wrong type of contract. Thereby, when S_1 selects the contract relevant to her type (out of the two contracts provided by B), B identifies the type to which S_1 belongs. The buyer B 's problem of designing optimal contracts for S_1 is solved using mechanism design. For comprehension of the application of mechanism design to solving similar contracts, we refer the reader to Yang et al. (2009).

The buyer B also offers S_2 a single two-part tariff contract $\langle t_2, w_2 \rangle$ comprising a transfer payment t_2 and a unit wholesale price w_2 . We assume that B 's unit selling price p is higher than her procurement prices w_1^i ($i \in \{H, L\}$) and w_2 .

Capacity Reservation stage: Supplier S_2 reserves production capacity of k units for B and incurs a reservation cost $y(k)$. We assume that $y(k)$ is convex and increasing in k .

External Perception and Execution stage: In this stage, the buyer B takes the actual procurement decisions. Recall that S_2 does not commit any violation at her facility. Hence, at this point in the sequence of events, the auditors do not detect any violation at S_2 's facility. However, since S_1 is prone to committing violations, her actions might affect B 's procurement

decisions at this point in the sequence of events. Whether B decides to procure from S_1 or switch to S_2 for procurement depends on three factors: (i) existence or absence of any responsibility violation committed by S_1 , (ii) occurrence or absence of any disruptive event resulting from responsibility violations committed by S_1 , and (iii) detection of responsibility violations through the audit process.

Given that S_1 commits a violation at her own facility, $\gamma \in [0, 1]$ denotes the probability that a disruptive event (e.g. fire) at her facility will actually occur. Unlike natural disasters which have low likelihood of occurrence, disasters caused due to supplier responsibility violations are much more likely to occur. For example, due to rampant negligence of responsibility by suppliers in the garment industry, serious disasters are simply waiting to occur (Hasan, 2012). At the very least, such disasters disrupt production at supplier facilities.

We assume that γ is independent of i , in order to differentiate among the risks inherent in the production processes in various industries. For instance, consider a supplier of chemicals and another supplier of wooden furniture, both having the same type i . Given that both suppliers are of the same type, the nature of products manufactured by each implies that the γ for the former supplier is greater than that for the latter.

Hence, the probability that a disruptive event occurs at S'_1 's facility when she commits a violation is given by the product $\theta_i\gamma$. Consequently, the probability of the audit detecting a violation committed by S_1 which results in the occurrence of a disruptive event is given by the product $\theta_i\alpha\gamma$. A crucial point should be noted here: the detection of violation might occur

Table 3.1: Sourcing Dynamics: External Perceptions

S_1 Com- mits Vio- lation	Audit Detects Violation	Disruptive Event Occurs	External Perception	Does S_1 Incur z ?	Does S_1 Remain Available?
✓	✓	×	ISB	Yes	No
✓	✓	✓	ISB	Yes	No
✓	×	✓	RD	No	No
✓	×	×	ND	No	Yes
×	—	—	ND	No	Yes

either before or after the occurrence of a disruptive event.¹

Considered together, occurrence (or absence) of responsibility violations and disruptive events at supplier facilities, and the outcome of the audit process affect the way that external entities might perceive B 's sourcing. We denote this *external perception* as τ and classify it into three scenarios: *Irresponsible Sourcing by Buyer* (ISB), *Random Disruption* (RD), and *No Disruption* (ND). The scenarios ISB, RD, and ND relate to how the external entities perceive the supply chain, and hence the terminology external perceptions. Table 3.1 exhibits the various scenarios which lead to the different external perceptions. The external perceptions are explained as follows:

1. *Irresponsible Sourcing by Buyer* (ISB): If S_1 is found to have committed any violation at her facility, then external entities view B 's sourcing negatively and hold her responsible for violations committed by S_1 . Then B incurs a cost of violation $z \geq 0$. In this case, B disrupts procurement from S_1 and switches over to S_2 instead. Although not directly inflicted by external entities, this loss of business serves as an

¹As will be explained later, in case violation is detected before a disruptive event occurs, we assume that B suspends procurement from S_1 . In this case, detection of violation preempts a possible disruptive event.

extreme penalty for S_1 for committing responsibility violations. This punishment is thus a manifestation of the external pressure on buyers to dissociate themselves from risky suppliers.

We have seen earlier that the probability that S_1 is found to have committed any violation at her facility is given by $\theta_i\alpha$. Since the scenario ISB is independent of the occurrence (or absence) of an actual disruptive event (e.g. a factory fire), the probability of ISB is given as: $\mathcal{P}_{ISB}^i = \theta_i\alpha$.

Note that irrespective of whether or not a disruptive event occurs, if a violation committed by S_1 gets detected, then the external perception is ISB. Because of the continuous nature of the audit, the violation can also get detected after a disruptive event instead of before a disruptive event. In this case, the combination of events would be still: violation exists, violation gets detected, disruptive event occurs, which conforms to ROW 1 of the Table 3.1.

2. *Random Disruption* (RD): Violations committed by S_1 at her facility might remain undetected by the system throughout the single period considered in our model. Then, any disruptive event (e.g. a fire) at S_1 's facility is perceived by external entities merely as a random occurrence. The disruption renders S_1 unavailable for sourcing and B switches over to S_2 for procurement. B does not incur any cost of violation. We have seen earlier that $\theta_i\alpha\gamma$ represents the probability that a violation committed by S_1 leads to a disruption and the violation is detected by the audit. However, the scenario RD pertains to the event when a

violation committed by S_1 leads to a disruptive event, but the violation is undetected by the audit. Hence, the probability of RD is given as: $\mathcal{P}_{\mathcal{RD}}^i = \theta_i \gamma (1 - \alpha)$. Note that we do not consider truly random disruptions in our model. The reason is that random disruptions which occur due to reasons other than supplier responsibility violations have no bearing on the cost of violation z . Since we are interested primarily in how z affects B 's decisions, we ignore the possibility of truly random disruptions.

In contrast to ISB, RD occurs when: violation exists, a disruptive event occurs, and violation is not detected (only ROW 3 of Table 3.1). The term RD does *not* imply that a disruption (which has occurred) is actually a random occurrence but rather implies that the disruption is *perceived* to have been a random occurrence. When violation gets detected post disruption, the disruption is *not* perceived as a random occurrence, and hence does not classify as RD. For instance, the Bangladesh Rana Plaza disaster would classify as ISB, even though the existence of egregious violations was mostly detected after the disaster.

3. *No Disruption* (ND): If S_1 *either* does not indulge in any compliance violation *or* the violations remain undetected and no disruptive event occurs, then S_1 remains available for sourcing. In this case, B sources from S_1 and does not switch to S_2 for procurement. Given that S_1 's type i is known, we define the probability that B would switch over from S_1 to S_2 for procurement as $P_S^i = \mathcal{P}_{\mathcal{ISB}}^i + \mathcal{P}_{\mathcal{RD}}^i = \theta_i(\alpha(1 - \gamma) + \gamma)$. Thus the probability of ND is given as: $\mathcal{P}_{\mathcal{ND}}^i = 1 - P_S^i$.

We denote the expected probability of B switching from S_1 to S_2 as $P_S \triangleq E_i[P_S^i]$. If B switches over to S_2 and orders more quantity than the capacity reserved for B (in the *capacity reservation stage*), then S_2 subcontracts production to other risky suppliers without authorization from B . Note the implicit assumption that S_2 's unit subcontracting cost is the same as her own unit production cost c_2 . In reality, subcontracting production without authorization from the buyer B would entail not only unit production cost but also transportation costs and possibly other hiding costs, which of course cannot exceed S_2 's own production cost. Hence the implicit assumption of same costs. Also clearly, if the cost of subcontracting is lesser than the cost of production, then S_2 would always subcontract. This makes the unauthorized subcontracting issue trivial and thereby not interesting to address in the dissertation.

Note that S_1 is B 's primary source of procurement. Hence, S_1 can always compute the quantity that would be ordered by B and therefore does not need to engage in unauthorized subcontracting. However, S_2 acts as a backup in case S_1 becomes unavailable. Hence even though S_2 can compute the quantity that would be ordered by B , she cannot predict whether B would actually place an order with her. Hence if S_2 does not reserve enough capacity, then she engages in unauthorized subcontracting.

If the unauthorized subcontracting by S_2 is detected by the system (with a probability α), then S_2 incurs a penalty $\xi \geq 0$. We have assumed that the penalty ξ is fixed and not dependent on the subcontracting volume. Unauthorized subcontracting is not easy to detect. Determining the exact subcontracting volume is even more difficult. Hence penalty dependent on

the subcontracting volume might not be quite practical. In §5.2, we discuss the implication of relaxing this assumption. Penalties on suppliers for engaging in unauthorized subcontracting might be enforced in multiple ways: contractual agreements with buyers, governmental pressure, cancellation of certification of responsibility etc. For example, Walmart’s policy against unauthorized subcontracting stipulates that a supplier found to be engaging in unauthorized subcontracting may face “*consequences up to and including termination of the supplier’s business with Walmart globally*” (Walmart, 2016). It is logical to assume that if the penalty is designed by B , it would be revenue-neutral in the sense that B does not incur any profit by penalizing S_2 . We will focus on analyzing the relationship between B ’s optimal investment policies and the penalty ξ when it is devised by external entities. In practice, suppliers are generally not directly penalized by external entities even if they engage in unauthorized subcontracting. Our third research question pertains to whether or not they should be directly penalized instead. The penalty ξ helps us to answer this question, as we will analyze the effect of instituting a direct supplier penalty for unauthorized subcontracting on the buyer’s motivation for improving supply chain responsibility.

Note a few important points about the implication of unauthorized subcontracting in our model. First, although B can anticipate how much capacity S_2 will reserve, B cannot anticipate whether unauthorized subcontracting will happen or not. This is because B cannot anticipate the outcome of external perception at the onset. Second, our modeling of unauthorized subcontracting implies that after the realization of external perception, if B were to actually source from S_2 (if τ is either ISB or RD), she would already know

whether S_2 would engage in unauthorized subcontracting or not. However, the auditors would not know about that, and neither would B have to voluntarily report it to the auditors. Hence B 's knowledge about the possibility of unauthorized subcontracting by S_2 does not prevent the occurrence of unauthorized subcontracting. Third, even if we were to assume that B also incurs a penalty if S_2 commits unauthorized subcontracting, the fundamental results derived in §4.3 would not change in principle. Further, as we will show in §4.4.2, B actually pays an indirect penalty if S_2 is caught doing unauthorized subcontracting. Therefore, even without assuming a penalty being incurred by B when unauthorized subcontracting by S_2 is detected, the fundamental results continue to hold in principle.

Finally, the actual demand for the product is realized. We denote the demand as D having positive support $[0, \bar{d}]$ on $\mathbf{R}^+$, where $\bar{d} > 0$ is the maximum possible demand. The probability density function, cumulative distribution and complementary cumulative distribution function of demand are respectively denoted as $f_D(\cdot)$, $F_D(\cdot)$ and $\bar{F}_D(\cdot) = 1 - F_D(\cdot)$.

3.3 Preliminary Decisions for External Perception and Execution & Capacity Reservation Stages

Given B 's investment policy, we analyze the optimal decisions (whenever applicable) and profits for B , S_1 and S_2 in the external perception and execution and capacity reservation stages. In the following analyses, we define

$a \wedge b = \min(a, b)$. Also, $\mathbf{1}_{\{\mathcal{X}\}}$ denotes the indicator function which equals 1 when the event $\mathcal{X}$ occurs, and 0 otherwise.

3.3.1 Stage 3: External Perception and Execution

Ceteris paribus, the newsvendor profit earned by B by procuring goods at a unit wholesale price w_1^i ($i \in \{H, L\}$) from supplier S_1^i and selling them at a unit retail price p is denoted as $\pi^\circ(w_1^i) = pE[q(w_1^i) \wedge D] - w_1^i q(w_1^i)$, where $q(w_1^i)$ denotes the optimal order quantity that maximizes B 's profit from sourcing from S_1^i . From standard results for the newsvendor model, we know that $q(w_1^i) = \bar{F}_D^{-1}\left(\frac{w_1^i}{p}\right)$ ($i \in \{H, L\}$).

Similarly, the newsvendor profit earned by B by procuring goods at a unit wholesale price w_2 from supplier S_2 and selling them at a unit retail price p is denoted as $\pi^\circ(w_2) = pE[q(w_2) \wedge D] - w_2 q(w_2)$, where $q(w_2)$ denotes the optimal order quantity that maximizes B 's profit from sourcing from S_2 . From standard results for the newsvendor model, we know that $q(w_2) = \bar{F}_D^{-1}\left(\frac{w_2}{p}\right)$.

B 's Profit:

The buyer B decides the optimal quantity to be procured from either S_1^i ($i \in \{H, L\}$) (if $\tau = ND$) or S_2 (if $\tau = ISB, RD$). Given the wholesale price w_1^i , the profit earned by B by procuring from S_1^i is positive only when no disruption occurs at S_1^i 's facility. Therefore, the expected profit earned

by B in this stage by procuring from S_1^i is given by:

$$\begin{aligned}\pi_{3,s_1}^b(w_1^i) &= \pi^\circ(w_1^i)\mathcal{P}_{\mathcal{ND}}^i \\ &= \pi^\circ(w_1^i)\{1 - \theta_i(\alpha(1 - \gamma) + \gamma)\}.\end{aligned}$$

Similarly, given the wholesale price w_2 , the profit earned by B by procuring from S_2 is positive only when disruption occurs at $S_1's$ facility. Therefore, given the wholesale price w_2 , the expected profit earned by B by procuring from S_2 is given by:

$$\begin{aligned}\pi_{3,s_2}^b(w_2) &= \pi^\circ(w_2)(1 - \mathcal{P}_{\mathcal{ND}}^i) - z\mathcal{P}_{\mathcal{LSB}}^i \\ &= \pi^\circ(w_2)\{\theta_i(\alpha(1 - \gamma) + \gamma)\} - z\theta_i\alpha.\end{aligned}$$

$S_1's$ Profit:

The risky supplier S_1 is able to sell to B only if there is no disruption at her facility. Given $S_1's$ type $i \in \{H, L\}$ and the wholesale price w_1^i , $S_1's$ expected profit from this stage is defined as:

$$\begin{aligned}\pi_3^{s_1}(w_1^i|i) &= (w_1^i - c_1^i)q(w_1^i)\mathcal{P}_{\mathcal{ND}}^i \\ &= (w_1^i - c_1^i)q(w_1^i)\{1 - \theta_i(\alpha(1 - \gamma) + \gamma)\}.\end{aligned}$$

$S_2's$ Profit:

The apparently responsible supplier S_2 sells to B only when disruption occurs at $S_1's$ facility. If $\tau \neq ND$ and B orders more from S_2 than the capacity reserved k , then S_2 resorts to unauthorized subcontracting and incurs a penalty

ξ with probability α . Therefore, given the wholesale price w_2 and capacity reserved k , S_2 's expected profit from this stage is defined as:

$$\begin{aligned}\pi_3^{s_2}(w_2, k) &= P_S \left((w_2 - c_2)q(w_2) - \xi \alpha \mathbf{1}_{\{q(w_2) > k\}} \right) \\ &= \{ \rho \theta_H + (1 - \rho) \theta_L \} \{ \alpha(1 - \gamma) + \gamma \} \left((w_2 - c_2)q(w_2) - \xi \alpha \mathbf{1}_{\{q(w_2) > k\}} \right).\end{aligned}$$

3.3.2 Stage 2: Capacity Reservation Stage

Given the unit wholesale price w_2 , S_2 decides the optimal reservation capacity k^* by solving the following maximization problem:

$$\pi_2^{s_2}(w_2) = \max_{k \geq 0} \left[\pi_3^{s_2}(w_2, k) - y(k) \right].$$

The optimal capacity reservation decision for S_2 depends on a trade-off between two costs: cost of reservation of capacity (i.e. $y(k)$) and expected penalty due to unauthorized subcontracting, i.e. $P_S \xi \alpha$. The following theorem characterizes S_2 's optimal capacity reservation policy. We assume that when S_2 is indifferent between reserving and not reserving capacity, then she opts for the former.

Theorem 3.3.1. *Optimal Capacity Reservation Policy for S_2*

Given wholesale price w_2 , the optimal capacity reserved by S_2 is:

$$k^* = \begin{cases} q(w_2) & \text{if and only if } y(q(w_2)) \leq P_S \xi \alpha, \\ 0 & \text{if and only if } y(q(w_2)) > P_S \xi \alpha. \end{cases}$$

The supplier S_2 's expected profit from the Capacity Reservation Stage is given

$$\text{by: } \pi_2^{s_2}(w_2) = P_S(w_2 - c_2)q(w_2) - \left[y(q(w_2)) \wedge P_S \xi \alpha \right].$$

The driving factor behind the “all or nothing” capacity reservation policy is the fact that the actual penalty ξ incurred by S_2 is a lump sum which is independent of the subcontracting volume. The quantity ordered from S_2 is either 0 or $q(w_2)$, depending on whether B procures from S_1 or S_2 . If S_2 were to reserve a positive capacity less than $q(w_2)$ and B were to procure from her, then S_2 would still be lacking enough capacity and would therefore subcontract the deficit quantity to other suppliers without authorization from B . However, since the penalty ξ is independent of the subcontracting volume, S_2 would still incur the same penalty ξ . Thus, S_2 can avoid paying the penalty ξ only if she reserves capacity of at least $q(w_2)$ units.

We refer to the two conditions in Theorem 3.3.1 as: *low penalty scenario* (when $y(q(w_2)) > P_S \xi \alpha$) and *high penalty scenario* (when $y(q(w_2)) \leq P_S \xi \alpha$). The low penalty scenario poses a responsibility risk to the supply chain in the form of *possible* unauthorized subcontracting by S_2 . However, the high penalty scenario precludes S_2 from resorting to unauthorized subcontracting *with certainty*. That is, the high penalty scenario has a more profound effect on the occurrence (or absence) of unauthorized subcontracting.

3.4 Stage 1: Contracting Stage

The buyer B offers a menu of two-part tariff contracts $\langle t_1^i, w_1^i \rangle$ ($i \in \{H, L\}$) to S_1 and a single two-part tariff contract $\langle t_2, w_2 \rangle$ to S_2 . We discuss the optimal contracts offered to S_1 in §3.4.1 and the optimal contract offered to S_2 in §3.4.2.

3.4.1 Optimal Contracts Offered to the Risky Supplier

Denote S'_1 's reported type as $\hat{i} \in \{H, L\}$. Then, given S'_1 's true type i and the contract terms t_1^i and w_1^i , the expected profit earned by S_1 in this stage is defined as:

$$\pi_1^{s_1}(t_1^{\hat{i}}, w_1^{\hat{i}} | i) = \pi_3^{s_1}(w_1^{\hat{i}} | i) + t_1^{\hat{i}}, \quad i, \hat{i} \in \{H, L\}.$$

The buyer B decides the optimal contract terms $t_1^{H*}, w_1^{H*}, t_1^{L*}$ and w_1^{L*} to be offered to S_1 for maximizing B 's profit. We restrict our mechanism to truth-telling, incentive compatibility, and individual rationality for supplier participation. Without loss of generality, we assume that the outside option for S_1 is zero. The buyer B 's expected profit from contracting with S_1 in this stage is given by the following mechanism design problem:

$$\pi_{1,s_1}^b = \max_{t_1^H, w_1^H, t_1^L, w_1^L} E_i \left[\pi_{3,s_1}^b(w_1^i) - t_1^i \right] \quad (3.1a)$$

s.t.

$$(I.C.S_1 : i = H) \quad \pi_1^{s_1}(t_1^H, w_1^H | H) \geq \pi_1^{s_1}(t_1^L, w_1^L | H), \quad (3.1b)$$

$$(I.C.S_1 : i = L) \quad \pi_1^{s_1}(t_1^L, w_1^L | L) \geq \pi_1^{s_1}(t_1^H, w_1^H | L), \quad (3.1c)$$

$$(I.R.S_1 : i = H) \quad \pi_1^{s_1}(t_1^H, w_1^H | H) \geq 0, \quad (3.1d)$$

$$(I.R.S_1 : i = L) \quad \pi_1^{s_1}(t_1^L, w_1^L | L) \geq 0, \quad (3.1e)$$

$$w_1^i \geq c_1^i, \quad t_1^i \geq 0, \quad i \in \{H, L\}. \quad (3.1f)$$

Constraints (3.1b) and (3.1c) ensure that S_1 has no incentive to choose

the contract terms which are not meant for her type. Constraints (3.1d) and (3.1e) ensure that S_1 would participate in the contract only if her reservation profit is met.

Theorem 3.4.1. *Optimal Menu of Contracts Offered to S_1*

The optimal menu of contracts offered by the buyer B to the risky supplier S_1 is:

$$\begin{aligned}\langle t_1^{H*}, w_1^{H*} \rangle &= \langle (1 - P_S^H)(c_1^L - c_1^H)q(c_1^L), c_1^H \rangle, \\ \langle t_1^{L*}, w_1^{L*} \rangle &= \langle 0, c_1^L \rangle.\end{aligned}$$

Expected profit earned by B in the Contracting Stage from contracting with S_1 is given by:

$$\pi_{1,s_1}^b = \rho(1 - P_S^H) \left(\pi^\circ(c_1^H) - (c_1^L - c_1^H)q(c_1^L) \right) + (1 - \rho)(1 - P_S^L) \pi^\circ(c_1^L).$$

Theorem 3.4.1 implies that the optimal unit wholesale prices offered by B to S_1 are equal to the unit production costs for the respective types of S_1 . In order to lock down the lower unit wholesale price c_1^H offered to S_1^H , B pays her a positive information rent $(1 - P_S^H)(c_1^L - c_1^H)q(c_1^L)$. Whereas S_1^H earns positive profit equal to the information rent, S_1^L earns just the reservation profit 0. The term $(1 - P_S^H)$ in the information rent gives the probability that B would not switch from S_1^H to S_2 for procurement. Thus, the information rent earned by S_1^H increases in the probability of B not switching over to S_2 from S_1^H . The other term $(c_1^L - c_1^H)q(c_1^L)$ in the information rent is the cost premium incurred by B for procuring from S_1^L instead of S_1^H . For a given c_1^L ,

the premium increases with decrease in c_1^H . Thus, the more differentiated the two types of S_1 are in terms of their unit production costs (costliness), the higher is the cost premium incurred by B .

Numerical Illustration of Contracts Offered to the Risky Supplier

The mechanism design ensures that S_1^H has no incentive to pretend that she is S_1^L , and vice versa. We will illustrate S_1 's lack of financial incentive to pretend to be of the other type with a numerical example. For performing the numerical computations, the following common parameters were used across all sets of input:

$$c_1^H = 0.3, c_1^L = 0.8, c_2 = 1, p = 1.5, \alpha_{min} = 0.6, \gamma = 0.5, h = 0.01.$$

We varied the values of θ_H and θ_L between 0.1 and 0.99 such that $\theta_H > \theta_L$. For each pair of θ_H and θ_L , we computed the profits earned by S_1^H and S_1^L by selecting the contracts relevant to their types, and also the profits earned by them if S_1^H were to select the contract relevant to S_1^L and vice versa. The results are presented in Table 3.2.

The profits earned by S_1^H and S_1^L by selecting the contracts relevant to their types are $\pi_1^{s_1}(t_1^H, w_1^H|H)$ and $\pi_1^{s_1}(t_1^L, w_1^L|L)$ respectively. The profits earned by S_1^H and S_1^L by selecting the contracts not relevant to their types are $\pi_1^{s_1}(t_1^L, w_1^L|H)$ and $\pi_1^{s_1}(t_1^H, w_1^H|L)$ respectively. We see from Table 3.2 that $\pi_1^{s_1}(t_1^H, w_1^H|L)$ is always negative, which is less than the reservation profit 0. Therefore, S_1^L would incur a loss if she were to select the contract relevant to S_1^H . On the other hand, the profit earned by S_1^H by selecting the contract relevant to S_1^L is exactly the same as her profit earned by selecting the contract relevant to her own type. In turn, this is exactly equal to the

Table 3.2: Profits Earned by S_1 With and Without Truth-Telling

θ_H	θ_L	$\pi_1^{s_1}(t_1^H, w_1^H H)$	$\pi_1^{s_1}(t_1^L, w_1^L L)$	$\pi_1^{s_1}(t_1^L, w_1^L H)$	$\pi_1^{s_1}(t_1^H, w_1^H L)$
0.1	0.01	0.21	0	0.21	-0.18
0.1	0.05	0.21	0	0.21	-0.17
0.2	0.01	0.2	0	0.2	-0.2
0.2	0.1	0.2	0	0.2	-0.17
0.4	0.01	0.16	0	0.16	-0.24
0.4	0.1	0.16	0	0.16	-0.21
0.4	0.2	0.16	0	0.16	-0.18
0.4	0.3	0.16	0	0.16	-0.15
0.7	0.1	0.1	0	0.1	-0.27
0.7	0.3	0.1	0	0.1	-0.2
0.7	0.4	0.1	0	0.1	-0.17
0.7	0.6	0.1	0	0.1	-0.11
0.7	0.65	0.1	0	0.1	-0.09
0.9	0.01	0.07	0	0.07	-0.33
0.9	0.1	0.07	0	0.07	-0.3
0.9	0.2	0.07	0	0.07	-0.27
0.9	0.3	0.07	0	0.07	-0.24
0.9	0.4	0.07	0	0.07	-0.21
0.9	0.5	0.07	0	0.07	-0.17
0.9	0.6	0.07	0	0.07	-0.14
0.9	0.8	0.07	0	0.07	-0.08
0.99	0.01	0.05	0	0.05	-0.35
0.99	0.5	0.05	0	0.05	-0.19
0.99	0.98	0.05	0	0.05	-0.04

information rent paid by B to S_1^H . Hence S_1^H also has no motivation to pretend to be S_1^L . Therefore, neither S_1^H nor S_1^L has any financial incentive to select the contract not relevant to her own type, and instead always resorts to truth-telling.

3.4.2 Optimal Contracts Offered to the Apparently Responsible Supplier

Given the contract terms t_2 and w_2 , the expected profit earned by S_2 in this stage is defined as:

$$\pi_1^{s_2}(t_2, w_2) = \pi_2^{s_2}(w_2) + t_2.$$

The buyer B decides the optimal contract terms t_2^* and w_2^* to be offered to S_2 for maximizing B 's profit. The expected profit earned by B from contracting with S_2 in this stage is given by the following maximization problem:

$$\begin{aligned} \pi_{1,s_2}^b(w_2^*) &= \max_{t_2, w_2} E_i \left[\pi_{3,s_2}^b(w_2) - t_2 \right] \\ \text{s.t.} \quad &\pi_1^{s_2}(t_2, w_2) \geq 0, \end{aligned} \tag{3.2a}$$

$$w_2 \geq c_2, \quad t_2 \geq 0. \tag{3.2b}$$

The value of w_2^* offered by B affects S_2 's capacity reservation decision. Hence B considers S_2 's capacity reservation decision when deciding w_2^* . Define the value of w_2 which maximizes the supply chain profit if S_2 reserves capacity for B as $l \triangleq \underset{w_2}{\operatorname{argmax}} [P_S(pE[q(w_2) \wedge D] - c_2 q(w_2)) - y(q(w_2))]$. Further, define the value of w_2 which maximizes the supply chain profit when S_2 does not reserve capacity as $l' \triangleq \underset{w_2}{\operatorname{argmax}} [P_S(pE[q(w_2) \wedge D] - c_2 q(w_2) - \xi \alpha)]$. The following lemma states the relationship between l and l' .

Lemma 3.4.2. $l' = c_2$. Furthermore, $l > c_2$, $q(l) < q(c_2)$, and $y(q(l)) <$

$y(q(c_2))$.

From Lemma 3.4.2, we can infer that if S_2 reserves capacity for B , then B offers a premium on c_2 to S_2 . Intuitively, we can comprehend that B 's decision to offer c_2 or l to S_2 as the optimal unit wholesale price should depend on a trade-off between two costs: the cost of capacity reservation (i.e. $y(q(l))$), and the increase in expected supply chain cost due to ordering $q(c_2)$ instead of $q(l)$. We denote this net rise in expected supply chain cost due to increasing the order quantity from $q(l)$ to $q(c_2)$ as $\mathcal{C}$. Define the difference between the order quantities based on the two optimal unit wholesale prices as $\delta \triangleq q(c_2) - q(l) \geq 0$; the excess expected demand that is satisfied when B orders $q(c_2)$ units instead of ordering $q(l)$ units as $\psi \triangleq E[q(c_2) \wedge D] - E[q(l) \wedge D]$. Then, we have: $\mathcal{C} = P_S(\xi\alpha + c_2\delta - p\psi)$. The following theorem exhibits the intuition about the trade-off between the two costs $y(q(l))$ and $\mathcal{C}$ when B decides the optimal contract terms to be presented to S_2 .

Theorem 3.4.3. *Optimal Contract Offered to S_2*

The optimal contract offered by the buyer B to the apparently responsible supplier S_2 is:

$$\langle t_2^*, w_2^* \rangle = \begin{cases} \langle y(q(l)) - P_S(l - c_2)q(l), & l \rangle & \text{if } y(q(l)) \leq \mathcal{C}, \\ \langle y(q(c_2)) \wedge P_S\xi\alpha, & c_2 \rangle & \text{if } y(q(l)) > \mathcal{C}. \end{cases}$$

Expected profit earned by B in the Contracting Stage from contracting with

S_2 is given by:

$$\pi_{1,s_2}^b(w_2^*) = P_S \left(pE[q(w_2^*) \wedge D] - c_2 q(w_2^*) \right) - y(q(w_2^*)) \wedge P_S \xi \alpha - \alpha \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} z.$$

When B offers the wholesale price l to S_2 , i.e. when $y(q(l)) \leq \mathcal{C}$, the high penalty scenario of Theorem 3.3.1 occurs, thereby prompting S_2 to reserve capacity for B . We know that $\mathcal{C}$ increases in the supplier penalty ξ . This indicates that higher the value of ξ , greater is the possibility that S_2 would reserve capacity for B . When $y(q(l)) > \mathcal{C}$, even though B offers the lower unit wholesale price c_2 , the transfer price is equal to the minimum of the cost of reservation of $q(c_2)$ units of capacity and the expected penalty $P_S \xi \alpha$ incurred by S_2 due to unauthorized subcontracting. Interestingly, cost of reservation of capacity and expected penalty are supposed to be incurred by S_2 . However, we see that when $y(q(l)) > \mathcal{C}$, then B actually covers these costs for S_2 . This indicates that when ξ is not very high, then the costs incurred by S_2 are covered by B . The implications of these observations are demonstrated in greater detail in §4.4.2.

3.5 Proofs

For all the proofs, we use the notation $\leftrightarrow$ to denote necessary and sufficient conditions.

Proof of Theorem 3.3.1.

The supplier S_2 's expected profit from Stage 3 is given by:

$$\pi_3^{s_2}(w_2, k) = P_S \left\{ (w_2 - c_2)q(w_2) - \xi\alpha \mathbf{1}_{\{q(w_2) > k\}} \right\}$$

Therefore, S_2 's expected profit from Stage 2 is given by:

$$\begin{aligned} \pi_2^{s_2}(w_2) &= \max_{k \geq 0} \left[\pi_3^{s_2}(w_2, k) - y(k) \right] \\ &= \max_{k \geq 0} \left[P_S \left\{ (w_2 - c_2)q(w_2) - \xi\alpha \mathbf{1}_{\{q(w_2) > k\}} \right\} - y(k) \right]. \end{aligned}$$

For $k \in [0, q(w_2))$, the profit function is given by: $P_S(w_2 - c_2)q(w_2) - \xi\alpha - y(k)$, which is decreasing and concave in k . Therefore, for $k \in [0, q(w_2))$, the profit function is maximized at $k = 0$. For $k \in [q(w_2), +\infty)$, the profit function is given by: $P_S(w_2 - c_2)q(w_2) - y(k)$, which is decreasing and concave in k . Therefore, for $k \in [q(w_2), +\infty)$, the profit function is maximized at $k = q(w_2)$. Hence, S_2 's maximization problem is essentially to choose either $k = 0$ or $k = q(w_2)$, depending on which of the two values of k globally

maximizes S_2 's profit. Therefore:

$$\begin{aligned}
\pi_2^{s_2}(w_2) &= \max \left[\pi_3^{s_2}(w_2, 0) - y(0), \right. \\
&\quad \left. \pi_3^{s_2}(w_2, q(w_2)) - y(q(w_2)) \right] \\
&= \max \left[P_S \{ (w_2 - c_2)q(w_2) - \xi\alpha \}, \right. \\
&\quad \left. \{ P_S(w_2 - c_2)q(w_2) - y(q(w_2)) \} \right] \quad (3.3) \\
&= P_S(w_2 - c_2)q(w_2) + \max \left[-P_S\xi\alpha, -y(q(w_2)) \right] \\
&= P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S\xi\alpha \right\}.
\end{aligned}$$

In order to find the conditions under which $k^* = 0$ and $k^* = q(w_2)$, we compare the two expressions in equation (3.3). We have:

$$\begin{aligned}
k^* = 0 &\leftrightarrow y(q(w_2)) > P_S\xi\alpha, \\
k^* = q(w_2) &\leftrightarrow y(q(w_2)) \leq P_S\xi\alpha.
\end{aligned}$$

Supplier S_2 's expected profit from Stage 2 is:

$$\pi_2^{s_2}(w_2) = P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S\xi\alpha \right\}. \quad \blacksquare$$

Proof of Theorem 3.4.1.

We know: $\mathcal{P}_{ND}^H = 1 - P_S^H(\alpha, \gamma)$ and $\mathcal{P}_{ND}^L = 1 - P_S^L(\alpha, \gamma)$. Note that $\mathcal{P}_{ND}^H <$

$\mathcal{P}_{\mathcal{N}\mathcal{D}}^L$ because $\theta_H > \theta_L$. Then:

$$\begin{aligned} E_i \left[\pi_{3,s_1}^b(w_1^i) - t_1^i \right] &= \rho \left\{ \left\{ -w_1^H q(w_1^H) + pE \left[q(w_1^H) \wedge D \right] \right\} \mathcal{P}_{\mathcal{N}\mathcal{D}}^H - t_1^H \right\} \\ &\quad + (1 - \rho) \left\{ \left\{ -w_1^L q(w_1^L) + pE \left[q(w_1^L) \wedge D \right] \right\} \mathcal{P}_{\mathcal{N}\mathcal{D}}^L \right. \\ &\quad \left. - t_1^L \right\}. \quad (3.4) \end{aligned}$$

Rewrite the constraints (3.1b)-(3.1e) as:

$$(I.C.S_1 : type - H) \quad (w_1^H - c_1^H)q(w_1^H)\mathcal{P}_{\mathcal{N}\mathcal{D}}^H + t_1^H \geq (w_1^L - c_1^H)q(w_1^L)\mathcal{P}_{\mathcal{N}\mathcal{D}}^H + t_1^L, \quad (3.5a)$$

$$(I.C.S_1 : type - L) \quad (w_1^L - c_1^L)q(w_1^L)\mathcal{P}_{\mathcal{N}\mathcal{D}}^L + t_1^L \geq (w_1^H - c_1^L)q(w_1^H)\mathcal{P}_{\mathcal{N}\mathcal{D}}^L + t_1^H, \quad (3.5b)$$

$$(I.R.S_1 : type - H) \quad (w_1^H - c_1^H)q(w_1^H)\mathcal{P}_{\mathcal{N}\mathcal{D}}^H + t_1^H \geq 0, \quad (3.5c)$$

$$(I.R.S_1 : type - L) \quad (w_1^L - c_1^L)q(w_1^L)\mathcal{P}_{\mathcal{N}\mathcal{D}}^L + t_1^L \geq 0. \quad (3.5d)$$

Therefore, B 's mechanism design problem is maximization of the R.H.S. of equation (3.4) over t_1^i, w_1^i ($i \in \{H, L\}$) subject to constraints (3.5a)-(3.5d) and (3.1f).

First, since $w_1^L > c_1^H$, t_1^H can be reduced to make constraint (3.5a) binding and constraint (3.5c) non-binding. Next, if we reduce t_1^L , then t_1^H can be reduced by an equal amount so that constraints (3.5a) and (3.5b) are unaffected, and constraint (3.5d) becomes binding. Hence, constraint (3.5b) can be binding only if *either* $w_1^H = c_1^L$ and $t_1^H = 0$, *or* $w_1^H < c_1^L$ and $t_1^H = -(w_1^H - c_1^L)q(w_1^H)\mathcal{P}_{\mathcal{N}\mathcal{D}}^L$. In the two cases, w_1^H and t_1^H can *respectively* be

reduced a little to make constraint (3.5b) non-binding without affecting the other constraints. Hence, constraint (3.5b) is non-binding. Now, we observe that constraint (3.5d) can be binding if and only if $t_1^L = 0$ and $w_1^L = c_1^L$. Therefore:

$$t_1^{L*} = 0 \quad \text{and} \quad w_1^{L*} = c_1^L.$$

Substituting the values of t_1^{L*} and w_1^{L*} in the binding constraint (3.5a), we have: $-t_1^H = \left\{ (w_1^H - c_1^H)q(w_1^H) - (c_1^L - c_1^H)q(c_1^L) \right\} \mathcal{P}_{\mathcal{ND}}^H$. Substituting $-t_1^H$, t_1^{L*} and w_1^{L*} in equation (3.4), we have:

$$\begin{aligned} E_i \left\{ \pi_{3,s_1}^b(w_1^i) - t_1^i \right\} &= \rho \left\{ \left\{ -w_1^H q(w_1^H) + pE[q(w_1^H) \wedge D] \right\} \mathcal{P}_{\mathcal{ND}}^H \right. \\ &\quad \left. + \left\{ (w_1^H - c_1^H)q(w_1^H) - (c_1^L - c_1^H)q(c_1^L) \right\} \mathcal{P}_{\mathcal{ND}}^H \right\} \\ &\quad + (1 - \rho) \left\{ \left\{ -c_1^L q(c_1^L) + pE[q(c_1^L) \wedge D] \right\} \mathcal{P}_{\mathcal{ND}}^L \right\} \\ &= \rho \mathcal{P}_{\mathcal{ND}}^H \left\{ -c_1^H q(w_1^H) + pE[q(w_1^H) \wedge D] - (c_1^L - c_1^H)q(c_1^L) \right\} \\ &\quad + (1 - \rho) \mathcal{P}_{\mathcal{ND}}^L \left\{ -c_1^L q(c_1^L) + pE[q(c_1^L) \wedge D] \right\}, \end{aligned}$$

which is concave in $q(w_1^H)$. Taking derivative with respect to $q(w_1^H)$, we have:

$$p\bar{F}\left(q(w_1^{H*})\right) - c_1^H = 0. \quad \text{This implies: } q(w_1^{H*}) = \bar{F}_D^{-1}\left(\frac{w_1^{H*}}{p}\right) = \bar{F}_D^{-1}\left(\frac{c_1^H}{p}\right).$$

Therefore $w_1^{H*} = c_1^H$ and $t_1^{H*} = (c_1^L - c_1^H)q(c_1^L)\mathcal{P}_{\mathcal{ND}}^H$. ■

Proof of Lemma 3.4.2.

Differentiating $P_S\{pE[q(w_2) \wedge D] - c_2q(w_2)\} - y(q(w_2))$ w.r.t. $q(w_2)$, we

derive the FOC as:

$$P_S\{-c_2 + p\bar{F}_D(q(w_2))\} - y'(q(w_2)) = 0.$$

Therefore, at $w_2 = l$, we have:

$$y'(q(l)) = P_S\{-c_2 + p\bar{F}_D(q(l))\}. \quad (3.6)$$

Differentiating $P_S\{pE[q(w_2) \wedge D] - c_2q(w_2) - \xi\alpha\}$ w.r.t. $q(w_2)$, we derive the FOC as:

$$P_S\{-c_2 + p\bar{F}_D(q(w_2))\} = 0.$$

Therefore, at $w_2 = l'$, we have:

$$q(l') = \bar{F}_D^{-1}\left(\frac{c_2}{p}\right).$$

But we know that $q(l') = \bar{F}_D^{-1}\left(\frac{l'}{p}\right)$. Therefore, we have:

$$l' = c_2.$$

Since $y(q(w_2))$ increases in $q(w_2)$, $y'(q(w_2)) > 0 \forall q(w_2) \geq 0$. Hence, $y'(q(l)) > 0$. Using $y'(q(l)) > 0$ in equation (3.6), we have:

$$P_S\{-c_2 + p\bar{F}_D(q(l))\} > 0.$$

Rearranging the terms, we get:

$$q(l) < \bar{F}_D^{-1} \left(\frac{c_2}{p} \right).$$

But we know that: $\bar{F}_D^{-1} \left(\frac{c_2}{p} \right) = q(c_2)$. Therefore, we get:

$$q(l) < q(c_2).$$

We know that $q(w_2)$ decreases in w_2 . Therefore, since $q(l) < q(c_2)$, we have:

$$l > c_2.$$

Finally, since $y(q(w_2))$ increases in $q(w_2)$, we have:

$$y(q(l)) < y(q(c_2)). \quad \blacksquare$$

Proof of Theorem 3.4.3.

$$E_i \left[\pi_{3,s_2}^b(w_2) - t_2 \right] = P_S \pi^\circ(w_2) - t_2 - \alpha \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} z.$$

Also, we see that:

$$\pi_1^{s_2}(t_2, w_2) = P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S \xi \alpha \right\} + t_2.$$

In order to find the optimal contract terms offered by B to S_2 , we maximize

$$E_i \left[\pi_{3,s_2}^b(w_2) - t_2 \right] \text{ over } t_2 \text{ and } w_2, \text{ subject to the constraints (3.2a) and (3.2b).}$$

Therefore, B 's maximization problem for deciding t_2^* and w_2^* can be rewritten

as:

$$\begin{aligned} \pi_{1,s_2}^b(w_2^*) &= \max_{t_2, w_2} P_S \pi^\circ(w_2) - t_2 - \alpha \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} z \\ \text{s.t.} \quad &P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S \xi \alpha \right\} + t_2 \geq 0, \end{aligned} \quad (3.7a)$$

$$w_2 \geq c_2, \quad t_2 \geq 0. \quad (3.7b)$$

Observe that the third term in the objective function, i.e. $-\alpha \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} z$, is independent of both t_2 and w_2 . Hence we will focus on maximizing $P_S \pi^\circ(w_2) - t_2$ subject to the constraints (3.7a) and (3.7b). Optimal profit for B can be improved by reducing the value of w_2 and/or t_2 so that constraint (3.7a) binds. This implies:

$$-t_2 = P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S \xi \alpha \right\}. \quad (3.8)$$

Substituting $-t_2$ in $P_S \pi^\circ(w_2) - t_2$, we get:

$$\begin{aligned} P_S \pi^\circ(w_2) - t_2 &= P_S \pi^\circ(w_2) + P_S(w_2 - c_2)q(w_2) - \left\{ y(q(w_2)) \wedge P_S \xi \alpha \right\} \\ &= P_S \left\{ -c_2 q(w_2) + pE[q(w_2) \wedge D] \right\} - \left\{ y(q(w_2)) \wedge P_S \xi \alpha \right\} \\ &= \max \left[P_S \left\{ -c_2 q(w_2) + pE[q(w_2) \wedge D] \right\} - y(q(w_2)), \right. \\ &\quad \left. P_S \left\{ -c_2 q(w_2) + pE[q(w_2) \wedge D] - \xi \alpha \right\} \right]. \end{aligned} \quad (3.9)$$

That is, at each w_2 , the functional value of the curve $P_S \pi^\circ(w_2) - t_2$ is given by the larger of the two curves $P_S \left\{ -c_2 q(w_2) + pE[q(w_2) \wedge D] \right\} - y(q(w_2))$ and

$P_S\left\{-c_2q(w_2)+pE[q(w_2)\wedge D]-\xi\alpha\right\}$, both of which are concave. Note that $P_S\pi^\circ(w_2)-t_2$ is continuous, since there exists a w_2 at which $y(q(w_2))=P_S\xi\alpha$.

Therefore, the curve $P_S\pi^\circ(w_2)-t_2$ has two local maxima, one at each peak attained by the curves $P_S\left\{-c_2q(w_2)+pE[q(w_2)\wedge D]\right\}-y(q(w_2))$ and $P_S\left\{-c_2q(w_2)+pE[q(w_2)\wedge D]-\xi\alpha\right\}$. The global maximum is given by the larger of these two peaks. Hence, w_2^* is given by the value of w_2 which gives the larger of the two peaks. Recall from §3.4.2 that: $l = \underset{w_2}{\operatorname{argmax}} [P_S\{pE[q(w_2)\wedge D]-c_2q(w_2)\}-y(q(w_2))]$, and $c_2 = \underset{w_2}{\operatorname{argmax}} [P_S\{pE[q(w_2)\wedge D]-c_2q(w_2)-\xi\alpha\}]$. Therefore, w_2^* is equal to either l or c_2 , whichever gives the larger of the two peaks $\pi_{1,s_2}^b(l)$ and $\pi_{1,s_2}^b(c_2)$.

In order to find the conditions under which $w_2^*=l$ and $w_2^*=c_2$, we compare the values of the two peaks $\pi_{1,s_2}^b(l)$ and $\pi_{1,s_2}^b(c_2)$ as follows:

$$\begin{aligned}
& \pi_{1,s_2}^b(l) > \pi_{1,s_2}^b(c_2) \\
& \Leftrightarrow P_S\left\{-c_2q(l)+pE[q(l)\wedge D]\right\}-y(q(l)) > P_S\left\{-c_2q(c_2)\right. \\
& \qquad \qquad \qquad \left.+pE[q(c_2)\wedge D]-\xi\alpha\right\} \\
& \Leftrightarrow P_S\left\{c_2[q(c_2)-q(l)]+pE[q(l)\wedge D]\right\}-y(q(l)) > P_S\left\{pE[q(c_2)\wedge D]-\xi\alpha\right\} \\
& \Leftrightarrow P_S\left\{c_2\delta-p\left[E[q(c_2)\wedge D]-E[q(l)\wedge D]\right]+\xi\alpha\right\} > y(q(l)) \\
& \qquad \qquad \Leftrightarrow P_S\left\{c_2\delta-p\psi+\xi\alpha\right\} > y(q(l)) \\
& \qquad \qquad \Leftrightarrow \mathcal{C} > y(q(l)).
\end{aligned}$$

Therefore, $w_2^*=l$ if $y(q(l))<\mathcal{C}$. Now, observe that: $p\psi-c_2\delta$ must be posi-

tive. Hence, $y(q(l)) < \mathcal{C}$ implies that $y(q(l)) \wedge P_S \xi \alpha = y(q(l))$. Therefore when $w_2^* = l$, from equation (3.8), we have: $t_2^* = y(q(l)) - P_S(l - c_2)q(l)$.

Similarly, it can be proved that $w_2^* = c_2$ if $y(q(l)) > \mathcal{C}$. From equation (3.8), we have: $t_2^* = y(q(c_2)) \wedge P_S \xi \alpha$. If $y(q(l)) = \mathcal{C}$, then B is indifferent between offering $w_2^* = l$ and $w_2^* = c_2$, and we assume that B offers the higher unit wholesale price l .

The expected profit earned by B is given by:

$$P_S \pi^\circ(w_2^*) - t_2^* = P_S \left\{ -c_2 q(w_2^*) + p E[q(w_2^*) \wedge D] \right\} - \left\{ y(q(w_2^*)) \wedge P_S \xi \alpha \right\} - \alpha \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} z. \blacksquare$$

Chapter 4

Improving Supply Chain Responsibility Through Investments by Buyers in Supply Chains: Role of External Entities

4.1 Buyer Investments in Improving Supply Chain Responsibility

In response to external pressure as described in the previous chapter, buyers can invest in supplier audits for detecting responsibility violations, and supplier improvement for decreasing the inherent risk of the risky suppliers.

The survey of 1,400 business leaders by Lee et al. (2012) shows that buyers are primarily motivated to invest in supply chain responsibility in order to avoid damage to customer image and brand equity. H&M invests in initiatives which are intended to improve the responsibility of suppliers in the Bangladesh RMG industry (Rayapura, 2014). In January 2013, Walmart pledged \$1.8m to provide training on fire safety to suppliers in Bangladesh (Montopoli, 2013). As witnessed in the aftermath of the Rana Plaza tragedy in Bangladesh in 2013, the Bangladesh Accord on Fire and Building Safety (*Accord*) and the Alliance for Bangladesh Worker Safety (*Alliance*), comprising primarily North American and European buyers, were set up. Both the Accord and the Alliance were founded with the aim of (i) ensuring that buyers would invest in conducting responsibility audits of their suppliers, and (ii) motivating buyers to invest in improving the safety of the suppliers' working environments. Although signatories to both the consortia have displayed conformance to investing in auditing of suppliers, many have little to show for in terms of investment in supplier improvement (Labowitz, 2014). Thus a clear disconnect exists between the expectation that the buyers would invest in supplier improvement and the buyers' actual efforts towards realizing that expectation.

Considering the above observations, we address the following research questions in this chapter:

Question 1. Does external pressure on the buyers always create incentives for them to invest in improving responsibility of suppliers?

Question 2. How do intrinsic supplier characteristics such as riskiness (probabilities of committing responsibility violations) and costliness (pro-

duction costs) of suppliers affect buyers' optimal decisions?

Question 3. Should suppliers be directly penalized by external entities for committing indirect violations like engaging in unauthorized subcontracting?

We extend the model in Chapter 3 to incorporate investments made by the buyer in supplier responsibility audit and supplier improvement. This allows us to use the results obtained by solving the optimal buyer-supplier contracts into the buyer's optimal contracting and investment policies.

The rest of the chapter is organized as follows: §4.2 discusses the model. §4.3 discusses the stage in which the buyer B decides her optimal contracting and investment strategies. This section addresses the Research Question 1. §4.4 addresses the Research Questions 2 and 3. §4.6 presents a summary of the main results along with their significance in practice. Finally, §4.7 presents the detailed proofs of the lemmas and theorems in this chapter.

4.2 Model

4.2.1 Model Overview

Investment decisions by the buyer B are made ahead of the selling season. Hence, in this chapter, we utilize the model for Chapter 3, and incorporate the buyer's investments in (i) improving the audit efficiency of the system and (ii) decreasing the expected risk of S_1 committing a violation. If S_1 becomes unavailable for any reason, then B sources the product from S_2 .

We refer to this stage as the *Contracting Strategy and Investments stage*.

In this stage, B makes strategic level decisions on contracting and investments. She selects the supplier(s) with whom she would establish a contract. Three contracting strategies are possible: dual contract ($\widehat{\mathcal{D}}$), under which B contracts with both S_1 and S_2 ; irresponsible contract ($\widehat{\mathcal{I}}$), under which B contracts with only S_1 ; and apparently responsible contract ($\widehat{\mathcal{R}}$), under which B contracts with only S_2 . If none of the contracting strategies is profitable, then B leaves the market.

Furthermore, B decides the investments $I_A \geq 0$ in responsibility audit conducted on S_1 and S_2 , and $I_D \geq 0$ in direct improvement of S'_1 's responsibility (supplier improvement). The audit efficiency of the system is given by $\alpha(I_A) \in [0, 1]$, which is a strictly increasing function in B 's investment I_A . We model the buyer's decision on investment in audit as being taken taken upfront and not changed upon gaining further information about the suppliers. That is, the value of $\alpha(I_A)$ is fixed upfront in our model, and B does not revise her audit investment decision later to change the frequency (or equivalently efficiency) of the audit. There are several reasons behind this modeling choice.

First, the dissertation considers investments made by the buyer at the strategic level. The way these audit investments would be done is that the buyer would ask some non-governmental organization (NGO) or external auditor to audit her suppliers on her behalf. The auditing entity would then start auditing all suppliers in the system. Since S_2 does not commit any violation at her own facility, the auditor is unable to detect any violation by

S_2 unless S_2 commits unauthorized subcontracting. The auditor conducts the audits following standard procedures depending on her agreement with the buyer B at the beginning of the selling season. That is, how much B wants to spend on such audits is really a strategic-level decision. Specific auditing decisions like auditing frequency are more operational in nature. Kim (2015) already has a paper in OR which considers operational-level auditing decisions.

Second, $\alpha(I_A)$ is the probability of detection of violation given that a violation exists. That is, after a violation has already been committed by a supplier, the probability of the violation getting detected is $\alpha(I_A)$. Only the probability of committing a violation at S'_1 's factory changes with her type (i.e. H or L) changes. The probability of detection of violations through audit does not change. So irrespective of the nature of the supplier and the exact mechanism of audit, the audit efficiency of the system α remains fixed.

Third, even if the buyer B were to entirely control the process of auditing herself instead of collaborating up with an auditor, investment in audit could not be assumed to necessarily share a positive correlation with the S'_1 's probability of violation. It might be argued that because a High-type supplier is more prone to responsibility violations, it might also be easier to detect violations committed by her. As a consequence to the argument, the investment in auditing a High-type supplier might not always be higher than the investment in auditing a Low-type supplier. As mentioned above, this problem is operational in nature and might be addressed using analytical models different from the ones which have been used in this dissertation. That is to say, such operational decisions are beyond the scope of this dissertation and

need special treatment as in Kim (2015).

We assume that B is required to help the system audit efficiency attain a minimum value $\alpha_{min} \in [0, 1]$ through her investment in audit. She achieves improvement of S'_1 's responsibility by investing in initiatives or processes which can decrease the probability that S_1 is of type H . Hence the probability that S_1 is of type H is given by $\rho(I_D) \in [0, 1]$, which is a strictly decreasing function in B 's investment I_D . Note that investment in supplier improvement decreases expected risk of violation in a single period. Since the probability of violation follows a two-point distribution, we model the decrease in expected risk through reduction of the prior probability that supplier S_1 is more risky. The probability that S_1 is of type L is thus given by $1 - \rho(I_D)$.

Note that in general, papers on sustainability consider some sort of consumer or social welfare function as the objective for maximization. However, since we take a different approach to improving supply chain responsibility in this dissertation, we also take a different approach to analyzing the conditions conducive for improving supply chain responsibility. After all, buyers are rational economic agents. Hence we consider B 's profit maximization problem. But our ultimate goal is to identify the conditions under which it is optimal for B to invest in the supplier S'_1 's improvement. We suggest that the external entities should use the levers available to them in order to promote these conditions which lead to buyer investment in supplier improvement.

4.2.2 Contracting Strategies Available to the Buyer

The three contracting strategies as defined above are $\widehat{\mathcal{D}}$, $\widehat{\mathcal{I}}$ and $\widehat{\mathcal{R}}$. If none of the contracting strategies is profitable, then B contracts with no supplier and leaves the market. The three contracting strategies are described below.

Dual Contract ($\widehat{\mathcal{D}}$): The buyer B contracts with both S_1 and S_2 . The optimal menu of contracts presented to S_1 are $\langle t_1^*, w_1^* \rangle$ ($i \in \{H, L\}$), as derived in Theorem 3.4.1. The optimal contract presented to S_2 is $\langle t_2^*, w_2^* \rangle$, as derived in Theorem 3.4.3. Denote l offered to S_2 under $\widehat{\mathcal{D}}$ as $l_{\widehat{\mathcal{D}}}$. We define B 's expected profit from under $\widehat{\mathcal{D}}$ strategy as: $\pi^{b,\widehat{\mathcal{D}}} \triangleq \pi_{1,s_1}^b + \pi_{1,s_2}^b(w_2^*) - I_A - I_D$.

Irresponsible Contract ($\widehat{\mathcal{I}}$): The buyer B contracts with only S_1 . The optimal menu of contracts presented to S_1 are the same as under $\widehat{\mathcal{D}}$ strategy. We define B 's expected profit from $\widehat{\mathcal{I}}$ strategy as: $\pi^{b,\widehat{\mathcal{I}}} \triangleq \pi_{1,s_1}^b - I_A - I_D$.

Apparently Responsible Contract ($\widehat{\mathcal{R}}$): The buyer B contracts with only S_2 . Hence the expected probability that B would need switch from S_1 to S_2 , i.e. P_S , is equal to 1. The optimal contract presented to S_2 can be derived by simply substituting $P_S = 1$ in the expressions for $\langle t_2^*, w_2^* \rangle$ in Theorem 3.4.3. Denote l offered to S_2 under $\widehat{\mathcal{R}}$ strategy as $l_{\widehat{\mathcal{R}}}$. The following corollary to Theorem 3.4.3 states the optimal contract presented by B to S_2 under $\widehat{\mathcal{R}}$ strategy:

Corollary 4.2.1. *Optimal Contract Offered to S_2 Under $\widehat{\mathcal{R}}$ Strategy*

The optimal contract offered by the buyer B to the apparently responsible

supplier S_2 under the apparently responsible contract ($\widehat{\mathcal{R}}$) strategy is:

$$\langle t_2^*, w_2^* \rangle = \begin{cases} \langle y(q(l_{\widehat{\mathcal{R}}})) - (l_{\widehat{\mathcal{R}}} - c_2)q(l_{\widehat{\mathcal{R}}}), l_{\widehat{\mathcal{R}}} \rangle & \text{if } y(q(l_{\widehat{\mathcal{R}}})) \leq \mathcal{C}' , \\ \langle y(q(c_2)) \wedge \xi\alpha(I_A), c_2 \rangle & \text{if } y(q(l_{\widehat{\mathcal{R}}})) > \mathcal{C}' , \end{cases}$$

where $\mathcal{C}' = \xi\alpha(I_A) + c_2\delta - p\psi$. Also, $l_{\widehat{\mathcal{D}}} > l_{\widehat{\mathcal{R}}}$, $q(l_{\widehat{\mathcal{D}}}) < q(l_{\widehat{\mathcal{R}}})$, and $y(q(l_{\widehat{\mathcal{D}}})) < y(q(l_{\widehat{\mathcal{R}}}))$.

Expected profit earned by B in the Contracting Strategy and Investments Stage under $\widehat{\mathcal{R}}$ strategy is given by:

$$\pi^{b,\widehat{\mathcal{R}}} = pE[q(w_2^*) \wedge D] - c_2q(w_2^*) - (y(q(w_2^*)) \wedge \xi\alpha(I_A)) - I_A - I_D .$$

Let η denote a particular contracting strategy. That is $\eta \in \{\widehat{\mathcal{D}}, \widehat{\mathcal{I}}, \widehat{\mathcal{R}}\}$.

The buyer B 's overall expected profit is defined as: $\pi^b(I_A, I_D|\eta) \triangleq \pi^{b,\eta}$.

4.2.3 Assumptions

We assume that demand follows uniform distribution within the interval $[0, 1]$. In addition, for the sake of simplicity, we assume that $\alpha(I_A)$ and $\rho(I_D)$ are linear in I_A and I_D , respectively. Specifically, we assume: $\alpha(I_A) = \frac{1}{h}I_A$ and $\rho(I_D) = 1 - \frac{1}{h}I_D$, where $h > 0$ and $\frac{1}{h}$ is the coefficient of efficiency of investments. Assuming different coefficients for I_A and I_D instead of a single coefficient $\frac{1}{h}$ result in the same managerial insights. If the investment is not efficient, B would leave the market, i.e. contract with neither S_1 or S_2 . This case is neither interesting nor relevant to our central focus on B 's investment in supplier improvement. Hence, we assume that the investment is efficient

so that B always contracts with at least one of S_1 and S_2 . We also assume that $y(k) = \frac{1}{2}k^2$.

4.3 Buyer's Optimal Contracting and Investment Policies

The optimal policy for B 's investment in audit is derived by comparing the behaviors of the functions for expected profits earned by B under different contracting strategies, with respect to the investment I_A . Although it is fairly straightforward to determine that the expected profit earned by B under the $\widehat{\mathcal{I}}$ and $\widehat{\mathcal{R}}$ strategies decrease in I_A , the analysis of the expected profit function under the $\widehat{\mathcal{D}}$ strategy with respect to I_A is a non-trivial exercise.

Theorem 4.3.1. $I_A^* = \alpha^{-1}(\alpha_{min}) \quad \forall \quad 0 \leq \alpha_{min} \leq 1.$

If a supply chain is riddled with potential violations, then exposing such violations tarnishes the image of B 's sourcing practices. Higher the system audit efficiency, higher are the chances of potential violations being found out, thereby increasing B 's risk of sourcing from S_1 . Hence, when B 's investment in audit helps the system audit efficiency attain α_{min} , she does not expend more resources which would help more potential violations be unraveled and publicized. In other words, B does not have any incentive to invest any more in supplier responsibility audit than what is required. The following discussions focus on B 's optimal policy for contracting strategies and investment in supplier improvement.

Theorem 4.3.2. *Dual contracting strategy ($\widehat{\mathcal{D}}$) always dominates the Irresponsible contracting strategy ($\widehat{\mathcal{I}}$).*

Theorem 4.3.2 implies that B either contracts with both S_1 and S_2 or with only S_2 , but never with S_1 alone. Contracting with only S_1 is too risky because B is exposed to not only the cost of violation z but also total loss of sales if S_1 's facility becomes unavailable for procurement. Hence, if B contracts with S_1 , then she also contracts with S_2 , thereby hedging against the risk of losing her entire sales. The optimal policy for B 's investment in supplier improvement is based on the comparison between the values of the function $\pi^b(I_A^*, I_D|\eta)$ under the $\widehat{\mathcal{D}}$ and $\widehat{\mathcal{R}}$ strategies. Based on the magnitude of z , B chooses either $\widehat{\mathcal{D}}$ or $\widehat{\mathcal{R}}$, and the optimal unit wholesale price(s) ($l_{\widehat{\mathcal{D}}}$, $l_{\widehat{\mathcal{R}}}$ or c_2) to be offered to the supplier(s).

We will first evaluate the case when $\alpha_{min} = 0$. In this case, B is not required to invest any minimum amount in supplier responsibility audit. Then, according to Theorem 4.3.1, B 's optimal policy for investment in supplier audit is to invest zero, that is $I_A^* = 0$. The following theorem states B 's optimal policy for investment in supplier improvement when $\alpha_{min} = 0$.

Theorem 4.3.3. *If the minimum required system audit efficiency $\alpha_{min} = 0$, then the buyer B follows $\widehat{\mathcal{D}}$ strategy. There exists a threshold disruption probability $\hat{\gamma} \in (0, 1]$ such that $I_D^* = 0$ if and only if $\gamma \leq \hat{\gamma}$. Otherwise, $I_D^* = h$.*

The key finding of Theorem 4.3.3 is that B might invest in supplier responsibility even when external pressure for auditing might be absent (i.e. $\alpha_{min} = 0$). This defies the notion that buyers would invest in supplier im-

provement only if they are highly pressured. In this case, B 's optimal policy is oriented towards decreasing the supply risk arising due to the probability γ that a disruptive event might occur at S_1 's facility. Since it is optimal for B to follow $\widehat{\mathcal{D}}$ strategy, B tries to decrease the risk of supply disruption arising from doing business with S_1 . Thereby, we infer that when a minimum audit requirement is not enforced on B , then B invests in supplier improvement only when $\gamma > \hat{\gamma}$, i.e. when the risk of supply disruption at S_1 's facility is significant.

Henceforth, we will present the results and discussions relevant for $\alpha_{min} > 0$, when the buyer exhibits positive investment in auditing effort. The rest of the chapter focuses on its primary goal, which is to analyze the circumstances and conditions which motivate B to invest in S_1 's improvement. For ease of exposition, we define the following ratio:

$$\beta \triangleq \frac{\pi^b(I_A^*, h|\widehat{\mathcal{D}}) + \alpha_{min}\theta_L z - \pi^b(I_A^*, 0|\widehat{\mathcal{R}})}{\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) + \alpha_{min}\theta_H z - \pi^b(I_A^*, 0|\widehat{\mathcal{R}})}.$$

By adding $h\alpha_{min}$ to both the minuend and subtrahend in the numerator, we see that the numerator of β is the excess net revenue that B earns under $\widehat{\mathcal{D}}$ and *maximum investment* h in S_1 as compared to what she earns under $\widehat{\mathcal{R}}$. Similarly, by adding $h\alpha_{min}$ to both the minuend and subtrahend in the denominator, we see that the denominator of β is the excess net revenue that B earns under $\widehat{\mathcal{D}}$ and *zero investment* in S_1 as compared to what she earns under $\widehat{\mathcal{R}}$. Therefore, the ratio β represents the benefit of maximum investment under $\widehat{\mathcal{D}}$ relative to zero investment under $\widehat{\mathcal{D}}$.

Theorem 4.3.4. *For a given minimum required system audit efficiency $\alpha_{min} > 0$, there exist z_1 , z_2 and z_3 which lead to the two scenarios:*

$\Phi 1$: *When $z_1 < z_2$, the following cases are possible:*

C1: If $z \leq z_1$, then B follows $\widehat{\mathcal{D}}$ strategy, and $I_D^ = 0$.*

C2: If $z_1 < z < z_2$, then B follows $\widehat{\mathcal{D}}$ strategy, and $I_D^ = h$.*

C3: If $z \geq z_2$, then B follows $\widehat{\mathcal{R}}$ strategy, and $I_D^ = 0$.*

$\Phi 2$: *When $z_1 \geq z_2$, the following cases are possible:*

C1: If $z \leq z_3$, then B follows $\widehat{\mathcal{D}}$ strategy, and $I_D^ = 0$.*

C2: If $z > z_3$, then B follows $\widehat{\mathcal{R}}$ strategy, and $I_D^ = 0$.*

Further, $z_1 < z_2$ if and only if $\beta > \theta_L/\theta_H$, and $z_1 \geq z_2$ otherwise.

Theorem 4.3.4 establishes the main result of this chapter. As we see in cases $\Phi 1:C3$ and $\Phi 2:C2$, if z is greater than some threshold, then B contracts with only S_2 . This reinforces the common belief that a high extent of punishment for the buyer would motivate her to exclude the risky suppliers from her supply base. Hence, under the traditional notion of responsible sourcing, punishing the buyers as suggested by Guo et al. (2015) is indeed the correct approach to improving supply chain responsibility. *However*, B does not invest in improvement of S'_1 's responsibility in either of the two cases. *Instead*, as we see from case $\Phi 1:C2$, B invests in S'_1 's improvement if and only if $z_1 < z < z_2$. As we have mentioned in §3.1, improvement of supply chain responsibility happens through buyer's investment in supplier improvement,

rather than through sourcing exclusively from the apparently responsible supplier. From Theorem 4.3.4, we can infer that improvement of supply chain responsibility happens only in case $\Phi 1:C2$. Therefore, our result counters the efficacy of a punishment-based approach taken by external entities against the buyers, thereby dispelling the notion that pressuring the buyers with high costs of violation leads to improvement of supply chain responsibility. External entities should exert an intermediate amount of pressure on B so that the cost of violation is restricted to within the interval (z_1, z_2) , in order to motivate B to invest in supplier improvement.

On the other hand, we see from cases $\Phi 1:C1$ and $\Phi 2:C1$ that if z is less than some threshold, then B contracts with both S_1 and S_2 but does not invest in supplier improvement. As discussed above, lack of investment in supplier improvement impedes the improvement of supply chain responsibility. The results from Theorem 4.3.4 might provide some explanations for what we observe in the Bangladeshi RMG industry. While Disney withdrew from Bangladesh in the aftermath of the Tazreen factory fire, many buyers continued to source apparels from Bangladeshi factories even after the Rana Plaza disaster. However, most of the buyers which stayed put did not pro-actively invest in supplier improvement. In the context of Theorem 4.3.4, this is expected to happen if z is low for a buyer. The implication of our finding for the Bangladeshi RMG industry also resonates with the main finding of Jacobs and Singhal (2015). In their paper, the authors find no significant evidence of negative stock market reaction over a period of time following the Rana Plaza disaster, and subsequently articulate that retailers have limited economic incentive to terminate business with Bangladeshi sup-

pliers. Comparing z in our model to the significance of the negative stock market reaction in Jacobs and Singhal (2015), we see that both our result and their finding suggest that if the financial consequences of a disaster are low or negligible, then buyers are not likely to change their supplier pools.

Finally, the theorem demonstrates that the investment-friendly interval (z_1, z_2) exists only if the ratio β is greater than θ_L/θ_H . That is, $\beta > \theta_L/\theta_H$ is a necessary condition for the external pressure to be effective. This shows that possibility of buyer investment in supplier improvement depends on a trade-off between the relative benefit of maximum investment as represented by β , and the relative inherent riskiness of the two types of S_1 as represented by the ratio θ_L/θ_H . Therefore, it is important for the external entities to comprehend how to influence this trade-off so as to increase the chances of B investing in S_1 's improvement. We see that although external entities cannot directly control the parameters θ_L and θ_H , they can influence the value of β in order to ensure that $\beta > \theta_L/\theta_H$. In a market where the risky suppliers are widely differentiated with respect to their probabilities of committing violation, the ratio θ_L/θ_H is small, due to which β is more likely to be greater than θ_L/θ_H . But in a market where θ_L/θ_H is large, β is more likely to be lesser than θ_L/θ_H .

The term β is independent of the cost of investment in supplier audit, i.e. $h\alpha_{min}$, but is dependent on the expected cost of violation committed by S_1^i ($i \in \{H, L\}$), i.e. $\alpha_{min}\theta_i z$. The term $\alpha_{min}\theta_i z$ depends on both the auditing pressure on B as well the external pressure on B for improving the responsibility of her supply base. The intuition behind this observation is that both the probability of supplier violations being detected (i.e. auditing efficiency

α_{min}) and the cost of violation z act as external levers for motivating B to invest in S_1 , thereby decreasing the expected risk of sourcing from S_1 . However, while the value of α_{min} is important for ensuring that investment is feasible (i.e. $\beta > \theta_L/\theta_H$), the value of z determines whether B would actually invest in S'_1 s improvement (i.e. whether $z_1 < z < z_2$ or not). Thus, controlling the magnitude of z is of utmost importance in order to motivate B to invest in S'_1 s improvement. In order to facilitate B 's investment in S'_1 s improvement, external entities need to ensure that z is just high enough to make the two conditions hold: (i) $\beta > \theta_L/\theta_H$ and (ii) $z_1 < z < z_2$.

The above discussions amply demonstrate that in order to facilitate buyer investment in supplier improvement, external entities should analyze various parameters that influence the buyer's investment decisions. In the following section, we discuss such parameters and their effects on the buyer's decision to invest in supplier improvement.

4.4 Motivating Buyer Investment in Supplier Improvement

In the previous section, we have seen that B invests pro-actively in improving S'_1 s responsibility if z is within the interval (z_1, z_2) , provided $z_1 < z_2$. We refer to this interval as B 's *tolerance interval*. In this section, our goal is to analyze the effect of various parameters on the bounds of the tolerance interval. Specifically, we are interested to find factors and mechanisms which either decrease z_1 or increase z_2 or both. This is because lower the value of

z_1 and higher the value of z_2 , greater is the span of costs of violation which B would tolerate and still invest in supplier improvement. Hence, for the following discussions, we assume that $z_1 < z_2$. In §4.4.1, we analyze the effect of the intrinsic supplier characteristics on the bounds of the tolerance interval. Following that, in §4.4.2, we analyze the sensitivity of the bounds with respect to the levers of influence available to the external entities.

4.4.1 Intrinsic Supplier Characteristics

Intrinsic supplier characteristics influence B 's investment in supplier improvement. In our model, three parameters represent the intrinsic supplier characteristics: the costliness of both S_1 and S_2 , which are indicated by the unit wholesale prices c_1^i ($i \in \{H, L\}$) and c_2 ; the probability θ_i that S_1 might commit a violation; and, the probability γ that a disruptive event might occur at S_1 's facility, given that S_1 commits a violation at her own facility.

Theorem 4.4.1. *The bounds of B 's tolerance interval (z_1, z_2) exhibit the following behaviors with respect to the intrinsic supplier characteristics:*

- i. If $P_S^H(I_A^*, \gamma) > \frac{\theta_L}{p(\theta_H - \theta_L)}$, then z_1 strictly decreases in c_2 . Otherwise, if $P_S^H(I_A^*, \gamma) \leq \frac{\theta_L}{p(\theta_H - \theta_L)}$, then there exist $\underline{c}_2'$ and $\overline{c}_2$ such that z_1 strictly decreases in c_2 if $0 \leq c_2 \leq \underline{c}_2'$ or $\overline{c}_2 \leq c_2 \leq p$, but increases in c_2 if $\underline{c}_2' \leq c_2 \leq \overline{c}_2$.*

Further, z_1 strictly increases in c_1^L and strictly decreases in c_1^H .

- ii. If $P_S^L(I_A^*, \gamma) < \frac{p}{p+1}$, then z_2 strictly increases in c_2 . Otherwise, if $P_S^L(I_A^*, \gamma) \geq \frac{p}{p+1}$, then there exist $\underline{c}_2$ and $\overline{c}_2$ such that z_2 strictly increases in c_2 if $0 \leq c_2 \leq \underline{c}_2$ or $\overline{c}_2 \leq c_2 \leq p$, but decreases in c_2 if*

$$\underline{c}_2 \leq c_2 \leq \overline{c}_2.$$

Further, z_2 strictly decreases in c_1^L and is independent of c_1^H .

iii. z_1 strictly decreases in θ_H and strictly increases in θ_L . z_2 is independent of θ_H and strictly decreases in θ_L .

iv. z_2 strictly decreases in γ .

Intuitively, it can be expected that as c_2 increases, z_1 would decrease and z_2 would increase. However, parts (i) and (ii) of Theorem 4.4.1 show that the intuition does not necessarily hold under certain supply chain conditions.

The condition $P_S^H(I_A^*, \gamma) \leq \frac{\theta_L}{p(\theta_H - \theta_L)}$, as given by part (i), represents a class of supply chains wherein θ_H and θ_L are nearly equal, i.e. the risky suppliers are nearly equally risky. In this scenario, investing in S_1 does not lead to a significant reduction in B 's risk of incurring costs of responsibility violations. When $c_2 \leq c_2'$, i.e. c_2 is relatively low, then the benefit from investing in S_1 's improvement, even though not very significant, offsets the cost of switching to S_2 . But when $c_2' \leq c_2 \leq \overline{c}_2$, i.e. c_2 is sufficiently high, then the benefit from investment becomes small enough that B does not invest in S_1 's improvement. Instead, B reduces the total risk of sourcing by offering the higher unit wholesale price l to S_2 , thereby motivating her to reserve capacity and not engage in unauthorized subcontracting. This leads to the counter-intuitive result in part (i) of the theorem. When $c_2 \geq \overline{c}_2$, i.e. c_2 is very high, then there is no scope of reducing cost of switching to S_2 , which is already quite high. Hence investing in S_1 's improvement is attractive in this case, since even a nominal benefit earned by investment serves the purpose of balancing the high operational cost of switching to S_2 .

The condition $P_S^L(I_A^*, \gamma) \geq \frac{p}{p+1}$, as given by part (ii), represents another class of supply chains wherein the expected probability of switching from S_1^L to S_2 (consequently also from S_1^H to S_2) is very high (because $\frac{p}{p+1}$ would be very close to 1). Similar to part (i), investing in S_1 does not lead to a significant reduction in B 's risk of incurring costs of responsibility violations. When $c_2 \leq \underline{c}_2$, i.e. c_2 is small, then even a small benefit from investing in S_1 's improvement offsets the cost of switching to S_2 . But when $\underline{c}_2 \leq c_2 \leq \overline{c}_2$, i.e. c_2 is sufficiently high, then the cost of switching to S_2 becomes very high. The expected probability of switching from S_1 to S_2 being very high itself, the benefit from investment in S_1 's improvement is not enough to offset the cost of switching to S_2 . Hence, B prefers to contract with only S_2 (under $\widehat{\mathcal{R}}$) and pay her the higher unit wholesale price, thereby eliminating the risk of unauthorized subcontracting. This leads to the counter-intuitive result in part (ii) of the theorem. When c_2 is very high ($\geq \overline{c}_2$), then the cost of sourcing from only S_2 (under $\widehat{\mathcal{R}}$) is so high that B prefers to engage in dual contract ($\widehat{\mathcal{D}}$) instead and invest in S_1 , thereby endeavoring to balance the cost of switching to S_2 .

As opposed to the two special classes of supply chains discussed above, general supply chains would conform to the conditions $P_S^H(I_A^*, \gamma) > \frac{\theta_L}{p(\theta_H - \theta_L)}$ and $P_S^H(I_A^*, \gamma) < \frac{\theta_L}{p(\theta_H - \theta_L)}$. Hence for general supply chains, we can glean the following insights from parts (i) and (ii) of Theorem 4.4.1. First, we see that the tolerance interval widens in the difference between S_2 and S_1^L with respect to their costliness. Counter-intuitively, the tolerance interval shrinks in the differentiation between S_1^L and S_1^H with respect to their costliness. The reason for this counter-intuition can be explained by the fact that investment

in supplier improvement decreases the probability $\rho(I_D)$ that S_1 is of type H , and increases the probability $1 - \rho(I_D)$ that S_1 is of type L . If S_1^L is very profitable for B , then B invests in supplier improvement in order to increase the chances of doing business with S_1^L . However, if S_1^H is very profitable for B , then B does not invest in supplier improvement, in order to increase the chances of doing business with S_1^H .

Second, we see that given the costliness of the two types of S_1 , the tolerance interval widens in the difference between S_2 and S_1 with respect to their costliness. In other words, higher the unit cost of production for S_2 , wider the range of costs of violation within which B invests in supplier improvement. This is because the cheaper S_1 is as compared to S_2 , the more profitable she is for B . Hence, B invests in supplier improvement in order to reduce the risk of switching to S_2 .

Furthermore, part (iii) of Theorem 4.4.1 shows that the tolerance interval widens in the difference between the two types of S_1 with respect to their inherent riskiness. This is because higher the difference in riskiness between S_1^H and S_1^L , lower is the riskiness of S_1^L as compared to that of S_1^H . Consequently, the scope of reducing B 's risk of incurring costs of violation through investment in supplier improvement is greater. Finally, part (iv) implies that the maximum cost of violation z_2 which B would tolerate and still invest in supplier improvement is low (high) in an industry or a sourcing base wherein γ is high (low), i.e. when responsibility violations by S_1 are very likely (not much likely) to result in supply disruptions.

We know that B invests in supplier improvement only when $z_1 < z < z_2$. In order to motivate B to invest in supplier improvement, external entities

should restrict B 's cost of violation to within the tolerance interval (z_1, z_2) . From the discussions in this section, we can infer that in general, if the suppliers are highly differentiated with respect to their intrinsic characteristics, then the external entities can penalize B within a wider tolerance interval. Otherwise, external entities should restrict B 's cost of violation to a narrower tolerance interval.

4.4.2 Levers of External Influence

Our goal in this subsection is to find ways in which external entities can influence the buyer's decision to invest directly in supplier improvement. In our model, three parameters represent possible levers of external influence: the minimum required system audit efficiency α_{min} that needs to be attained through B 's investment in audit; the penalty ξ incurred by S_2 if unauthorized subcontracting committed by her is detected; and the cost of violation z which is incurred by B due to external pressure.

Theorem 4.4.2.

i. The upper bound z_2 decreases in the minimum required system audit efficiency α_{min} .

ii. For a given $\alpha_{min} > 0$, there exist positive thresholds of ξ : $\xi_0 < \xi_1 < \xi_2$ such that the following cases hold:

Case 1: z_2 increases and z_1 decreases in $\xi \in [0, \xi_0]$.

Case 2: both z_2 and z_1 decrease in $\xi \in [\xi_0, \xi_1]$.

Case 3: z_2 decreases and z_1 increases in $\xi \in [\xi_1, \xi_2]$.

Case 4: both z_2 and z_1 remain unchanged in $\xi \in [\xi_2, \infty)$.

Furthermore, if $z_1 < z < z_2$ in Cases 1, 2 and 3, then B offers the expected penalty $P_S(I_A^, I_D)\xi\alpha_{min}$ incurred by S_2 as a transfer price to S_2 . If $z_1 < z < z_2$ in Case 4, then B always offers $l_{\widehat{D}}$ as unit wholesale price to S_2 , and in turn S_2 reserves capacity for B .*

Part (i) of the theorem implies that higher the pressure on B to invest in audit, lesser will be the value of her maximum tolerance z_2 . This can be intuitively gauged from Theorem 4.3.1 itself.

Several insights can be derived from part (ii) of the theorem. *First*, even though we assumed that unauthorized subcontracting by S_2 does not lead to any cost of violation for B , yet, penalizing S_2 clearly affects B . As long as $\xi \leq \xi_2$, even though the penalty is supposed to be incurred by S_2 , it is actually incurred by B through the transfer price which she offers to S_2 .¹ Interestingly, this implies that an appropriately designed ξ acts as an indirect penalty for the buyer. Simultaneously, if z is within the tolerance interval, B also invests in supplier improvement. *Second*, the tolerance interval widens for $\xi \in [0, \xi_0]$. This implies that setting ξ to an appropriate positive value (for instance ξ_0) increases the chances that B would invest in supplier improvement. *Third*, if z is within the tolerance interval and $\xi > \xi_2$, i.e. if ξ is sufficiently high, then B invests in supplier improvement as well S_2 reserves capacity for B . This eliminates the possibility of any unauthorized subcontracting.

Using our observations, we formulate a holistic policy which can greatly improve supply chain responsibility.

¹Recall the discussion in Theorem 3.4.3

A Policy for Improving Supply Chain Responsibility

Within the scope of our model, maximum improvement of supply chain responsibility occurs when the following occur simultaneously: (i) B follows $\widehat{D}$ strategy, (ii) B invests in improvement of S_1 's responsibility, and (iii) S_2 does not resort to unauthorized subcontracting. In order to achieve this, we derive a policy which involves all the three levers of influence available to external entities, i.e. α_{min} , ξ , and z .

Step 1. First, the external entities should set the α_{min} . In practice, improvement of supply chain responsibility initiates with responsibility audits. As seen in the Bangladesh RMG industry, the Accord and Alliance have well-defined requirements for audits to be carried out within specified time limits.

Step 2. Following the choice of α_{min} , the magnitude of punishment ξ to be incurred by S_2 can be set to any value exceeding ξ_2 . We denote the tolerance interval at $\xi = \xi_2$ as $(z_1^{\xi_2}, z_2^{\xi_2})$. The bounds of the tolerance interval are independent of any $\xi > \xi_2$, as stated in the theorem.

Step 3. Given α_{min} and some $\xi > \xi_2$, B invests in supplier improvement only if $z_1^{\xi_2} < z < z_2^{\xi_2}$. Therefore, following the choices of α_{min} and ξ , the external entities should attempt to restrict the cost of violation z incurred by B within the tolerance interval $(z_1^{\xi_2}, z_2^{\xi_2})$.

We summarize the relevant practical implications for external entities obtained from this section as follows. *First*, we note that external entities should institute strategic mechanisms for monitoring and penalizing S_2 for any responsibility violations committed by her. Even in the aftermath of

the Bangladesh disaster, suppliers were not penalized by external entities even when they were found to have engaged in unauthorized subcontracting. Our findings endorse the intent of the Cambodian government's policy of instituting penalties for suppliers found engaging in unauthorized subcontracting. *Second*, external entities should diligently endeavor to estimate the cost of violation z that might be incurred by B . They should devise effective strategies for pressuring B so that z is restricted within the interval $(z_1^{\xi_2}, z_2^{\xi_2})$. The cost of violation z comprises direct and indirect financial damages incurred by B due to responsibility violations committed by her supplier, which makes measuring it very difficult. Reasonable estimates of z can be derived only through increased and transparent collaboration between the external entities and B .

4.5 Numerical Illustration

Here we will exhibit a compact illustration of the links between all the main results from the Chapters 3 and 4. The goal of our illustration is to show how the buyer B 's contracting strategy and investment in supplier improvement, B 's cost of violation z , supplier S_2 's penalty for unauthorized subcontracting ξ , and S_2 's capacity reservation decision are related. For performing the numerical computations, the following common parameters were used across all sets of input:

$$\theta_H = 0.8, \theta_L = 0.4, c_1^H = 0.3, c_1^L = 0.8, c_2 = 1, p = 1.5, \alpha_{min} = 0.6, \gamma = 0.5, h = 0.01.$$

We varied the value of ξ from 0 to 0.12. For each value of ξ , we computed

the values of z_1 and z_2 , which are the two thresholds of the cost of violation z . A set of representative values of ξ , z_1 and z_2 are shown in Table 4.1. The entire set of results have been made available in Appendix II Table A.6.

Table 4.1: Threshold Values z_1 And z_2

ξ	z_1	z_2
0	0.0144	0.0444
0.01	0.0125	0.0485
0.02	0.0106	0.0526
0.04	0.0067	0.0607
0.055556	0.0037	0.0671
0.059	0.0031	0.0664
0.06	0.0029	0.0662
0.07	0.001	0.0643
0.070862	0.0008	0.0641
0.072	0.001	0.0639
0.08	0.0025	0.0624
0.09	0.0045	0.0605
0.092	0.0049	0.0601
0.093844	0.0052	0.0597
0.097	0.0052	0.0597
0.1	0.0052	0.0597
0.11	0.0052	0.0597
0.12	0.0052	0.0597

Thereafter, we analyzed the buyer B 's optimal contracting strategy, optimal investment in supplier S_1 's improvement, and optimal capacity reserved by supplier S_2 . The analysis is summarized in Figure 4.1.

Figure 4.1 exhibits the $z - \xi$ plane divided into twelve regions denoted by R1 through R12. As stated in Theorem 4.3.4, the buyer B : (i) contracts with both S_1 and S_2 but does not invest in S_1 's improvement when $z < z_1$, (ii) contracts with both S_1 and S_2 and invests in S_1 's improvement when $z_1 < z < z_2$, and (iii) contracts with only S_2 and does not invest in S_1 's

Figure 4.1: Interplay Between B 's Contracting Strategy, Investment in Supplier Improvement, z , ξ , and S_2 's Capacity Reservation

improvement when $z > z_2$. The values of z_1 and z_2 are shown in Table 4.1. Furthermore, as stated in Theorem 4.4.2, the following can be observed: (i) z_2 increases and z_1 decreases in $\xi \in [0, 0.055556]$, (ii) both z_2 and z_1 decrease in $\xi \in [0.055556, 0.070862]$, (iii) z_2 decreases and z_1 increases in $\xi \in [0.070862, 0.093844]$, and (iv) both z_2 and z_1 remain unchanged in $\xi \in [0.093844, \infty)$.

The traditional notion of responsible sourcing suggests that z should be high and B should contract only with S_2 . Furthermore, the existing practice for external entities is not to penalize suppliers for unauthorized subcontracting. That is in general $\xi = 0$. These two conditions together appear together in the line $\xi = 0$ in the Region R1. We see that even though R1 conforms to the traditional notion of responsible sourcing, it is still vulnerable to the risk

of unauthorized subcontracting, because S_2 does not reserve capacity within this area. In fact, unauthorized subcontracting prevails in all three regions R1, R2, and R3.

It is clear that if the penalty for unauthorized subcontracting $\xi < 0.055556$, then the practice of unauthorized subcontracting cannot be terminated. The best outcome for the supply chain under traditional notion of responsible sourcing occurs when B sources from only S_2 and unauthorized subcontracting does not exist. Therefore ξ should be at least 0.055556 in order for the supply chain to achieve the best outcome under the traditional notion of responsible sourcing. This pertains to either R4, R7, or R10.

As opposed to the traditional notion of responsible sourcing, in this dissertation, we have articulated that sourcing from only S_2 is not practicable for B . Hence, real improvement of supply chain responsibility would happen only if B were to contract with both S_1 and S_2 , and invested in S'_1 's improvement. This pertains to the regions R2, R5, R8, and R11. However, not all of these regions produce the best outcome for the supply chain under the notion of supply chain responsibility improvement as defined in the dissertation. The best outcome occurs when B contracts with both S_1 and S_2 , B invests in S'_1 's improvement, and unauthorized subcontracting does not occur. This pertains to only Region R11 in Figure 4.1. Therefore ξ should be at least 0.093844 in order for the supply chain to achieve the best outcome under the non-traditional perspective of improvement of supply chain responsibility as defined in this dissertation. Finally, the Regions R3, R6, R9, and R12 do not produce the desired outcomes under either the traditional notion of responsible sourcing or the non-traditional definition expressed in

this dissertation.

Figure 4.2: The $z - \xi$ Plane: Summary of Decisions Taken by B and S_2

To summarize, we see that in the $z - \xi$ plane, as we move from low z to high z , there is an improvement in supply chain responsibility under the traditional notion of responsible sourcing. However, as discussed earlier, the traditional notion of responsible sourcing does not account for the fact that buyers source from suppliers with diverse irresponsibility risks. Hence, as proposed in this dissertation, real improvement of supply chain responsibility occurs when the buyer contracts with both the risky and apparently responsible suppliers, and also invests in reducing the risk of sourcing from the former. This is observed only in the middle region in the $z - \xi$ plane. However, although B 's investment in S_1 's improvement is a positive outcome for the supply chain, it still does not always refrain S_2 from engaging in unauthorized subcontracting. In the $z - \xi$ plane, as we move from low ξ to

high ξ , the responsibility of the supply chain improves in the sense that S_2 refrains from engaging in unauthorized subcontracting. Hence in the $z - \xi$ plane, the maximum improvement of supply chain responsibility occurs in the middle-right region. The observations are summarized in Figure 4.2.

4.6 Our Contributions

Our first research objective is to understand the efficacy of external pressure on the buyer towards improving supply chain responsibility. We analyze the optimal terms of contract between the buyer and the suppliers, and use the results to solve for the buyer's optimal policies for investment in supplier responsibility audit and supplier improvement. Prevalent wisdom dictates that in order to improve supply chain responsibility, external entities should always punish the buyer with higher costs of violation. This is also indicated by Guo et al. (2015). In contrast, we find that high cost of violation does not necessarily improve supply chain responsibility, because the buyer might not have incentive to invest in supplier improvement. Instead, the buyer is motivated to invest in supplier improvement only when the cost of violation is moderate. This is because for deciding whether or not to invest, the buyer trades off between the risk of incurring cost of violations, and the risk of incurring cost of switching to the apparently responsible supplier. Only when the former risk is moderate with respect to the latter, the buyer has incentive to decrease the former risk by investing in supplier improvement. Therefore, high external pressure of threat to the buyer's reputation and finances due to supply chain violations might be detrimental to the improvement of supply

chain responsibility.

Our second research objective is to analyze how the intrinsic supplier characteristics affect the buyer's chances of investing in supplier improvement. We find that in general, chances of buyer investment in supplier improvement are high if the risky suppliers are widely differentiated with respect to their riskiness and costliness. Hence in a market wherein suppliers are highly differentiated with respect to their intrinsic characteristics, the external entities can incentivize the buyers to invest in supplier improvement by limiting the buyer's cost of violations within a wider interval. But if the difference between suppliers is not pronounced, then the external entities should restrict the buyer's cost of violation to a narrower interval.

Our final research objective is to examine whether or not external entities should directly penalize suppliers who engage in unauthorized subcontracting. We find that instead of holding the buyer unilaterally responsible for all violations in their supply chains, external entities should directly penalize the apparently responsible supplier if she engages in unauthorized subcontracting. Further, we find that the risk of unauthorized subcontracting can be avoided if the penalty exceeds a threshold. Using our results, we formulate a holistic policy which induces buyer investment in supplier improvement as well as eliminates unauthorized subcontracting, thereby maximizing the improvement of supply chain responsibility.

4.7 Proofs

For all the proofs, we use the notation $\leftrightarrow$ to denote necessary and sufficient conditions. Furthermore, unless the explicit representations are necessary, we will refer to $\alpha(I_A)$ as α and $\rho(I_D)$ as ρ .

Proof of Corollary 4.2.1.

By substituting $P_S(\alpha, \rho) = 1$ in the expression for $\mathcal{C}$, we get $\mathcal{C}' = \xi\alpha + c_2\delta - p\psi$. The optimal contract terms t_2^* and w_2^* can be derived by simply substituting $P_S(\alpha, \rho) = 1$ and $l = l_{\widehat{\mathcal{R}}}$ in the expressions for t_2^* and w_2^* respectively, as stated in Theorem 3.4.3. We will now prove that $l_{\widehat{\mathcal{D}}} \geq l_{\widehat{\mathcal{R}}}$.

From equation (3.6), we have: $y'(q(l)) = P_S \left\{ -c_2 + p\overline{F}_D(q(l)) \right\}$.

For dual contract $\widehat{\mathcal{D}}$, substituting $l = l_{\widehat{\mathcal{D}}}$ in $y'(q(l))$, we have:

$$\frac{y'(q(l_{\widehat{\mathcal{D}}}))}{P_S} - p\overline{F}_D(q(l_{\widehat{\mathcal{D}}})) = -c_2.$$

Since $P_S < 1$, we have:

$$y'(q(l_{\widehat{\mathcal{D}}})) \left\{ 1 + n \right\} - p\overline{F}_D(q(l_{\widehat{\mathcal{D}}})) = -c_2, \quad (4.1)$$

where n is some positive number.

For apparently responsible contract $\widehat{\mathcal{R}}$, substituting $l = l_{\widehat{\mathcal{R}}}$ and $P_S = 1$

in $y'(q(l))$, we have:

$$y'(q(l_{\widehat{\mathcal{R}}})) - p\overline{F}_D(q(l_{\widehat{\mathcal{R}}})) = -c_2. \quad (4.2)$$

Subtracting equation (4.2) from equation (4.1) gives:

$$\left[y'(q(l_{\widehat{\mathcal{R}}})) - p\overline{F}_D(q(l_{\widehat{\mathcal{R}}})) \right] - \left[y'(q(l_{\widehat{\mathcal{D}}})) - p\overline{F}_D(q(l_{\widehat{\mathcal{D}}})) \right] = ny'(q(l_{\widehat{\mathcal{D}}})) \quad (4.3)$$

Since $y(q(l))$ is increasing in $q(l)$, the R.H.S. of equation (4.3) is positive.

Therefore:

$$y'(q(l_{\widehat{\mathcal{R}}})) - p\overline{F}_D(q(l_{\widehat{\mathcal{R}}})) > y'(q(l_{\widehat{\mathcal{D}}})) - p\overline{F}_D(q(l_{\widehat{\mathcal{D}}})) \quad (4.4)$$

Now observe that both $y'(q(l))$ and $-p\overline{F}_D(q(l))$ are increasing in $q(l)$. Hence, the function $y'(q(l)) - p\overline{F}_D(q(l))$ is increasing in $q(l)$. Therefore, from inequality (4.4), we can infer that:

$$q(l_{\widehat{\mathcal{R}}}) > q(l_{\widehat{\mathcal{D}}}).$$

Since $q(l)$ is decreasing in l , we have:

$$l_{\widehat{\mathcal{D}}} > l_{\widehat{\mathcal{R}}}.$$

Further, since $y(q(l))$ increases in $q(l)$, we have:

$$y(q(l_{\widehat{\mathcal{R}}})) > y(q(l_{\widehat{\mathcal{D}}})) \quad \blacksquare$$

In the Contracting Strategy and Investments stage, demand is assumed to follow uniform distribution $U[0, 1]$. Also, $y(k) = \frac{1}{2}k^2 \forall k \geq 0$.

Lemma 4.7.1.

$$\begin{aligned} (i) \quad q(l) &= \frac{P_S(p - c_2)}{P_S p + 1}, \\ (ii) \quad q(c_2) &= 1 - \frac{c_2}{p}. \end{aligned}$$

Proof. From equation (3.6), we have:

$$\frac{d}{d(q(l))} \left\{ y(q(l)) \right\} = P_S \left\{ -c_2 + p \overline{F}_D(q(l)) \right\}.$$

Using uniform distribution and substituting $y(q(l)) = \frac{1}{2}q(l)^2$, we get:

$$\begin{aligned} q(l) &= P_S \left\{ -c_2 + p \left[1 - F_D(q(l)) \right] \right\} \\ &= P_S \left\{ -c_2 + p \left[1 - q(l) \right] \right\} \end{aligned}$$

Rearranging the terms, we get:

$$\begin{aligned} q(l) \left\{ P_S p + 1 \right\} &= P_S(p - c_2) \\ \therefore q(l) &= \frac{P_S(p - c_2)}{P_S p + 1}. \end{aligned}$$

Further, since $q(c_2) = \overline{F}_D^{-1} \left(\frac{c_2}{p} \right)$, we have:

$$\begin{aligned} \overline{F}_D \left(q(c_2) \right) &= \frac{c_2}{p} \\ \leftrightarrow 1 - F_D \left(q(c_2) \right) &= \frac{c_2}{p}. \end{aligned}$$

Using uniform distribution, we get:

$$\begin{aligned} 1 - q(c_2) &= \frac{c_2}{p} \\ \therefore q(c_2) &= 1 - \frac{c_2}{p}. \quad \blacksquare \end{aligned}$$

Lemma 4.7.2.

$$\pi^\circ(c_2) = \frac{(p - c_2)^2}{2p}.$$

Proof.

We know: $\pi^\circ(c_2) = pE[q(c_2) \wedge D] - c_2 q(c_2)$. Using uniform distribution and

Lemma (4.7.1), we have:

$$\begin{aligned}
\pi^\circ(c_2) &= pE[q(c_2) \wedge D] - c_2q(c_2) \\
&= (p - c_2)q(c_2) - \frac{1}{2}p(q(c_2))^2 \\
&= (p - c_2)\frac{(p - c_2)}{p} - \frac{1}{2}p\left\{\frac{(p - c_2)}{p}\right\}^2 \\
&= \frac{(p - c_2)^2}{p} - \frac{1}{2}\frac{(p - c_2)^2}{p} \\
&= \frac{(p - c_2)^2}{2p}. \quad \blacksquare
\end{aligned}$$

Lemma 4.7.3.

$$P_S\left\{-c_2q(l) + pE[q(l) \wedge D]\right\} - y(q(l)) = \frac{(p - c_2)^2}{2} \frac{\{P_S\}^2}{P_Sp + 1}.$$

Proof.

$$\begin{aligned}
&P_S\left\{-c_2q(l) + pE[q(l) \wedge D]\right\} - y(q(l)) \\
&= P_S\left\{-c_2q(l) + pE[q(l) \wedge D]\right\} - \frac{1}{2}(q(l))^2 \\
&= P_S\left\{-c_2q(l) + pq(l) - \frac{1}{2}p(q(l))^2\right\} - \frac{1}{2}(q(l))^2 \\
&= P_Sq(l)(p - c_2) - \frac{1}{2}(P_Sp + 1)(q(l))^2 \\
&= P_S\frac{P_S(p - c_2)}{P_Sp + 1}(p - c_2) - \frac{1}{2}(P_Sp + 1)\left\{\frac{P_S(p - c_2)}{P_Sp + 1}\right\}^2 \\
&= \frac{\left\{P_S(p - c_2)\right\}^2}{P_Sp + 1} - \frac{1}{2}\frac{\left\{P_S(p - c_2)\right\}^2}{P_Sp + 1} \\
&= \frac{(p - c_2)^2}{2} \frac{\{P_S\}^2}{P_Sp + 1}. \quad \blacksquare
\end{aligned}$$

Lemma 4.7.4. *If $y(q(l)) \geq \mathcal{C}$, then $y(q(c_2)) > P_S \xi \alpha$.*

Proof.

Suppose: $y(q(l)) \geq \mathcal{C}$. Then from the proof of Theorem 3.4.3, we have:

$$P_S \left\{ -c_2 q(l) + pE[q(l) \wedge D] \right\} - y(q(l)) \leq P_S \left\{ -c_2 q(c_2) + pE[q(c_2) \wedge D] - \xi \alpha \right\} \quad (4.5)$$

Recall from Lemma 4.7.2 that $-c_2 q(c_2) + pE[q(c_2) \wedge D] = \frac{(p-c_2)^2}{2p}$.

Also, recall from Lemma 4.7.3 that $P_S \left\{ -c_2 q(l) + pE[q(l) \wedge D] \right\} - y(q(l)) = \frac{(p-c_2)^2}{2} \frac{\left\{ P_S \right\}^2}{P_S p + 1}$.

Using these results in (4.5), we get:

$$\frac{(p-c_2)^2}{2} \frac{\left\{ P_S \right\}^2}{P_S p + 1} \leq \frac{P_S (p-c_2)^2}{2p} - P_S \xi \alpha$$

Rearranging the terms, we get:

$$\begin{aligned} \frac{P_S p (p-c_2)^2}{2p^2} - \frac{(p-c_2)^2}{2p^2} \frac{\left\{ P_S \right\}^2 p^2}{P_S p + 1} &\geq P_S \xi \alpha \\ \Leftrightarrow \frac{(p-c_2)^2}{2p^2} \left[P_S p - \frac{\left\{ P_S \right\}^2 p^2}{P_S p + 1} \right] &\geq P_S \xi \alpha \\ \Leftrightarrow \frac{(p-c_2)^2}{2p^2} \frac{P_S p}{P_S p + 1} &\geq P_S \xi \alpha \\ \Leftrightarrow \frac{(p-c_2)^2}{2p^2} &\geq P_S \xi \alpha \frac{P_S p + 1}{P_S p} \end{aligned}$$

Since $\frac{P_S p + 1}{P_S p} > 1$, we have:

$$\frac{(p - c_2)^2}{2p^2} > P_S \xi \alpha$$

Therefore:

$$y(q(c_2)) > P_S \xi \alpha . \quad \blacksquare$$

For the proofs of Theorem 4.3.1 through Theorem 4.3.3, we will denote the functions of I_A and I_D as functions of α and ρ , for the sake of convenience.

Define B 's expected profit after selling the goods procured from S_1^H as:

$$\tilde{\pi}(c_1^H) \triangleq \pi^\circ(c_1^H) - (c_1^L - c_1^H)q(c_1^L)$$

The following lemma states the relation between the functions $\tilde{\pi}(c_1^H)$, $\pi^\circ(c_1^L)$, and $\pi^\circ(c_2)$.

Lemma 4.7.5. $\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L) > \pi^\circ(c_2) > 0$.

Proof. We know that:

$$\pi^\circ(c_2) = \frac{(p - c_2)^2}{2p}.$$

Similarly:

$$\pi^\circ(c_1^L) = \frac{(p - c_1^L)^2}{2p},$$

and:

$$\pi^\circ(c_1^H) = \frac{(p - c_1^H)^2}{2p}.$$

Hence:

$$\begin{aligned}\pi^\circ(c_1^L) - \pi^\circ(c_2) &= \frac{(p - c_1^L)^2}{2p} - \frac{(p - c_2)^2}{2p} \\ &= \frac{(p - c_1^L)^2 - (p - c_2)^2}{2p}.\end{aligned}$$

Since $c_1^L < c_2 < p$, hence $(p - c_1^L) > (p - c_2)$.

Therefore we have:

$$\pi^\circ(c_1^L) > \pi^\circ(c_2). \tag{4.6}$$

Also:

$$\begin{aligned}
\tilde{\pi}(c_1^H) - \pi^\circ(c_1^L) &= \pi^\circ(c_1^H) - (c_1^L - c_1^H)q(c_1^L) - \frac{(p - c_1^L)^2}{2p} \\
&= \frac{(p - c_1^H)^2}{2p} - (c_1^L - c_1^H)\frac{(p - c_1^L)}{p} - \frac{(p - c_1^L)^2}{2p} \\
&= \frac{(p - c_1^H)^2 - 2(c_1^L - c_1^H)(p - c_1^L) - (p - c_1^L)^2}{2p} \\
&= \frac{\left[p^2 + (c_1^H)^2 - 2pc_1^H\right] - 2\left[pc_1^L - (c_1^L)^2 - pc_1^H + c_1^H c_1^L\right]}{2p} \\
&\quad - \frac{\left[p^2 + (c_1^L)^2 - 2pc_1^L\right]}{2p} \\
&= \frac{p^2 - p^2 + (c_1^H)^2 - 2pc_1^H + 2pc_1^H - 2pc_1^L + 2pc_1^L}{2p} \\
&\quad + \frac{2(c_1^L)^2 - (c_1^L)^2 - 2c_1^H c_1^L}{2p} \\
&= \frac{(c_1^H)^2 + (c_1^L)^2 - 2c_1^H c_1^L}{2p} \\
&= \frac{\left(c_1^L - c_1^H\right)^2}{2p} \\
&> 0.
\end{aligned}$$

Therefore we have:

$$\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L). \quad (4.7)$$

Since $c_1^H < c_1^L < c_2 < p$, it is obvious that $\tilde{\pi}(c_1^H)$, $\pi^\circ(c_1^L)$, and $\pi^\circ(c_2)$ are all positive.

Therefore from equations (4.6) and (4.7), we have:

$$\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L) > \pi^\circ(c_2) > 0. \quad \blacksquare$$

Proof of Theorem 4.3.1.

First, we analyze $\pi^b(\alpha, \rho|\widehat{\mathcal{D}})$ w.r.t. α .

Define the component of $\pi^b(\alpha, \rho|\widehat{\mathcal{D}})$ which gives the contribution from doing business with S_1 as:

$$\mathcal{U}(\alpha, \rho) \triangleq \rho \tilde{\pi}(c_1^H) \left\{ 1 - P_S^H(\alpha, \gamma) \right\} + (1 - \rho) \pi^\circ(c_1^L) \left\{ 1 - P_S^L(\alpha, \gamma) \right\} - \alpha \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} z$$

The partial derivative of $\mathcal{U}(\alpha, \rho)$ with respect to α is given as:

$$\frac{\delta \mathcal{U}(\alpha, \rho)}{\delta \alpha} = -\rho \theta_H (1 - \gamma) \tilde{\pi}(c_1^H) - (1 - \rho) \theta_L (1 - \gamma) \pi^\circ(c_1^L) - \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} z \quad .$$

Case 1: $y(q(l_{\widehat{\mathcal{D}}})) \leq \mathcal{C}(\alpha, \rho)$

In this case, $w_2^* = l_{\widehat{\mathcal{D}}}$. Also, $y(q(l_{\widehat{\mathcal{D}}})) \wedge P_S(\alpha, \rho) \xi \alpha = y(q(l_{\widehat{\mathcal{D}}})) = \frac{1}{2}(q(l_{\widehat{\mathcal{D}}}))^2$.

Let: $\hat{\pi}^1(\alpha, \rho) = \pi^b(\alpha, \rho|\widehat{\mathcal{D}})$ when $w_2^* = l_{\widehat{\mathcal{D}}}$.

Define the component of $\hat{\pi}^1(\alpha, \rho)$ which gives the contribution from doing business with S_2 as:

$$\begin{aligned} \mathcal{T}_1(\alpha, \rho) &\triangleq P_S(\alpha, \rho) \left\{ -c_2 q(l_{\widehat{\mathcal{D}}}) + p E[q(l_{\widehat{\mathcal{D}}}) \wedge D] \right\} - y(q(l_{\widehat{\mathcal{D}}})) \\ &= \frac{(p - c_2)^2}{2} \frac{\{P_S(\alpha, \rho)\}^2}{P_S(\alpha, \rho)p + 1} \quad (\text{from Lemma 4.7.3}) \end{aligned}$$

Taking partial derivative with respect to α , we get:

$$\begin{aligned}
\frac{\delta \mathcal{T}_1(\alpha, \rho)}{\delta \alpha} &= \frac{\delta \mathcal{T}_1(\alpha, \rho)}{\delta P_S(\alpha, \rho)} \frac{\delta P_S(\alpha, \rho)}{\delta \alpha} \\
&= \frac{(p - c_2)^2}{2} \left\{ \frac{2P_S(\alpha, \rho)}{P_S(\alpha, \rho)p + 1} - \frac{\{P_S(\alpha, \rho)\}^2 p}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} \\
&\quad \times (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} \\
&= \frac{(p - c_2)^2}{2} \left\{ \frac{2\{P_S(\alpha, \rho)\}^2 p + 2P_S(\alpha, \rho) - \{P_S(\alpha, \rho)\}^2 p}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} \\
&\quad \times (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} \\
&= \frac{(p - c_2)^2}{2} \left\{ \frac{\{P_S(\alpha, \rho)\}^2 p + 2P_S(\alpha, \rho)}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} \\
&= \frac{(p - c_2)^2}{2p} \left\{ \frac{P_S(\alpha, \rho)^2 p^2 + 2P_S(\alpha, \rho)p}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} \\
&= \pi^\circ(c_2) \left\{ \frac{\{P_S(\alpha, \rho)p + 1\}^2 - 1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} \\
&= \pi^\circ(c_2) \left\{ 1 - \frac{1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} .
\end{aligned}$$

We see that:

$$\hat{\pi}^1(\alpha, \rho) = \mathcal{U}(\alpha, \rho) + \mathcal{T}_1(\alpha, \rho) - h\alpha - h(1 - \rho)$$

Differentiating $\hat{\pi}^1(\alpha, \rho)$ with respect to α , we get:

$$\begin{aligned}
\frac{\delta \hat{\pi}^1(\alpha, \rho)}{\delta \alpha} &= \frac{\delta \mathcal{U}(\alpha, \rho)}{\delta \alpha} + \frac{\delta \mathcal{T}_1(\alpha, \rho)}{\delta \alpha} - h \\
&= -\rho \theta_H (1 - \gamma) \tilde{\pi}(c_1^H) - (1 - \rho) \theta_L (1 - \gamma) \pi^\circ(c_1^L) - \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} z \\
&\quad + \pi^\circ(c_2) \left\{ 1 - \frac{1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} (1 - \gamma) \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} - h \\
&= -\rho \theta_H \left[(1 - \gamma) \left\{ \tilde{\pi}(c_1^H) - \left\{ 1 - \frac{1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} \pi^\circ(c_2) \right\} + z \right] \\
&\quad - (1 - \rho) \theta_L \left[(1 - \gamma) \left\{ \pi^\circ(c_1^L) - \left\{ 1 - \frac{1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} \pi^\circ(c_2) \right\} + z \right] \\
&\quad - h \quad .
\end{aligned}$$

We know: $\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L) > \pi^\circ(c_2)$ and $\left\{ 1 - \frac{1}{\{P_S(\alpha, \rho)p + 1\}^2} \right\} < 1$. Therefore:

$$\frac{\delta \hat{\pi}^1(\alpha, \rho)}{\delta \alpha} < 0 \quad .$$

Case 2: $y(q(l_{\widehat{\mathcal{D}}})) > \mathcal{C}(\alpha, \rho)$

In this case, $w_2^* = c_2$; $y(q(c_2)) \wedge P_S(\alpha, \rho) \xi \alpha = P_S(\alpha, \rho) \xi \alpha$ (from Lemma 4.7.4).

Let: $\hat{\pi}^2(\alpha, \rho) = \pi^b(\alpha, \rho | \widehat{\mathcal{D}})$ when $w_2^* = c_2$.

Define the component of $\hat{\pi}^2(\alpha, \rho)$ which gives the contribution from doing business with S_2 as:

$$\begin{aligned}
\mathcal{T}_2(\alpha, \rho) &\triangleq P_S(\alpha, \rho) \left\{ \pi^\circ(c_2) - \xi \alpha \right\} \\
&= \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ \alpha(1 - \gamma) + \gamma \right\} \left\{ \pi^\circ(c_2) - \xi \alpha \right\}
\end{aligned}$$

Taking partial derivative with respect to α , we get:

$$\begin{aligned}
\frac{\delta \mathcal{T}_2(\alpha, \rho)}{\delta \alpha} &= \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ (1 - \gamma) (\pi^\circ(c_2) - \xi \alpha) - \xi \left\{ \alpha(1 - \gamma) + \gamma \right\} \right\} \\
&= \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ (1 - \gamma) \pi^\circ(c_2) - \xi \alpha(1 - \gamma) - \xi \alpha(1 - \gamma) - \xi \gamma \right\} \\
&= \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ (1 - \gamma) \pi^\circ(c_2) - 2\xi \alpha(1 - \gamma) - \xi \gamma \right\} .
\end{aligned}$$

We see that:

$$\hat{\pi}^2(\alpha, \rho) = \mathcal{U}(\alpha, \rho) + \mathcal{T}_2(\alpha, \rho) - h\alpha - h(1 - \rho)$$

Differentiating $\hat{\pi}^2(\alpha, \rho)$ with respect to α , we get:

$$\begin{aligned}
\frac{\delta \hat{\pi}^2(\alpha, \rho)}{\delta \alpha} &= \frac{\delta \mathcal{U}(\alpha, \rho)}{\delta \alpha} + \frac{\delta \mathcal{T}_2(\alpha, \rho)}{\delta \alpha} - h \\
&= -\rho \theta_H (1 - \gamma) \tilde{\pi}(c_1^H) - (1 - \rho) \theta_L (1 - \gamma) \pi^\circ(c_1^L) - \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} z \\
&\quad + \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ (1 - \gamma) \pi^\circ(c_2) - 2\xi \alpha(1 - \gamma) - \xi \gamma \right\} - h \\
&= -\rho \theta_H (1 - \gamma) (\tilde{\pi}(c_1^H) - \pi^\circ(c_2)) - (1 - \rho) \theta_L (1 - \gamma) (\pi^\circ(c_1^L) - \pi^\circ(c_2)) \\
&\quad - \left\{ \rho(\theta_H - \theta_L) + \theta_L \right\} z - \left\{ \rho \theta_H + (1 - \rho) \theta_L \right\} \left\{ 2\xi \alpha(1 - \gamma) + \xi \gamma \right\} \\
&\quad - h .
\end{aligned}$$

Therefore:

$$\frac{\delta \hat{\pi}^2(\alpha, \rho)}{\delta \alpha} < 0 .$$

Therefore from the two cases, we can say that $\pi^b(\alpha, \rho | \widehat{\mathcal{D}})$ is decreasing in $\alpha \in [0, 1]$. Furthermore, it is easy to see that both $\pi^b(\alpha, \rho | \widehat{\mathcal{I}})$ and $\pi^b(\alpha, \rho | \widehat{\mathcal{R}})$ are

decreasing in $\alpha \in [0, 1]$. Therefore, B 's profit always decreases in α , which implies: $\alpha^* = \alpha_{min}$. Consequently, $I_A^* = \alpha^{-1}(\alpha_{min}) \forall 0 \leq \alpha_{min} \leq 1$. ■

Proof of Theorem 4.3.2.

Refer to the proof of Theorem 4.3.1. The components of B 's profit which give the contribution from doing business with S_2 are $\mathcal{T}_1(\alpha, \rho)$ and $\mathcal{T}_2(\alpha, \rho)$ in cases (1) and (2), respectively. In Case (1), $\mathcal{T}_1(\alpha, \rho) > 0$. This implies that B benefits from contracting with S_2 (in addition to her gains/losses from contracting with S_1).

Now consider the term $\mathcal{T}_2(\alpha, \rho)$ in Case (2). From the proof of Lemma 4.7.4, we have:

$$\begin{aligned} \frac{(p - c_2)^2}{2p^2} \frac{P_S(\alpha, \rho)p}{P_S(\alpha, \rho)p + 1} &\geq P_S(\alpha, \rho)\xi\alpha. \\ \Leftrightarrow \frac{(p - c_2)^2}{2p} \frac{P_S(\alpha, \rho)}{P_S(\alpha, \rho)p + 1} &\geq P_S(\alpha, \rho)\xi\alpha \\ \Leftrightarrow \pi^\circ(c_2) \frac{P_S(\alpha, \rho)}{P_S(\alpha, \rho)p + 1} &\geq P_S(\alpha, \rho)\xi\alpha \\ \Leftrightarrow \pi^\circ(c_2) &\geq \xi\alpha\{P_S(\alpha, \rho)p + 1\} \quad . \end{aligned}$$

Since $P_S(\alpha, \rho)p + 1 > 1$, therefore $\pi^\circ(c_2) > \xi\alpha$. That is, $\mathcal{T}_2 > 0$.

This implies that even in Case (2), B benefits from contracting with S_2 (in addition to her gains/losses from contracting with S_1). Since $\mathcal{T}_1(\alpha, \rho) > 0$ and $\mathcal{T}_2(\alpha, \rho) > 0$, the dual contract strategy $\widehat{\mathcal{D}}$ always dominates the irresponsible contract strategy $\widehat{\mathcal{I}}$. ■

Proof of Theorem 4.3.3.

Given, $\alpha_{min} = 0$. Therefore, $\alpha^* = 0$.

Under $\widehat{\mathcal{D}}$ strategy, $\mathcal{C}(\alpha^*, \rho) = -P_S(0, \rho)(p\psi - c_2\delta)$, which is negative. Hence, $y(q(l_{\widehat{\mathcal{D}}})) > \mathcal{C}(\alpha^*, \rho)$, which relates to Case (2) in the proof of Theorem 4.3.1.

Then B 's expected profit under $\widehat{\mathcal{D}}$ strategy is given by $\pi^b(\alpha^*, \rho|\widehat{\mathcal{D}}) = \hat{\pi}^2(\alpha^*, \rho)$.

Therefore:

$$\begin{aligned}
\pi^b(\alpha^*, \rho|\widehat{\mathcal{D}}) &= \mathcal{U}(\alpha^*, \rho) + \mathcal{T}_2(\alpha^*, \rho) - h\alpha^* - h(1 - \rho) \\
&= \mathcal{U}(0, \rho) + \mathcal{T}_2(0, \rho) - h \cdot 0 - h(1 - \rho) \\
&= \rho\tilde{\pi}(c_1^H)\left\{1 - P_S^H(0, \gamma)\right\} + (1 - \rho)\pi^\circ(c_1^L)\left\{1 - P_S^L(0, \gamma)\right\} \\
&\quad + \left\{\rho\theta_H + (1 - \rho)\theta_L\right\}\left\{0 \cdot (1 - \gamma) + \gamma\right\}\left\{\pi^\circ(c_2) - \xi \cdot 0\right\} - h \cdot 0 - h(1 - \rho) \\
&= \rho\tilde{\pi}(c_1^H)\left\{1 - \theta_H\gamma\right\} + (1 - \rho)\pi^\circ(c_1^L)\left\{1 - \theta_L\gamma\right\} \\
&\quad + \left\{\rho\theta_H + (1 - \rho)\theta_L\right\}\gamma\pi^\circ(c_2) - h(1 - \rho) .
\end{aligned}$$

Observe that since $\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L) > \pi^\circ(c_2)$, we have:

$$\begin{aligned}
\pi^b(\alpha^*, \rho|\widehat{\mathcal{D}}) &> \rho\pi^\circ(c_2)\left\{1 - \theta_H\gamma\right\} + (1 - \rho)\pi^\circ(c_2)\left\{1 - \theta_L\gamma\right\} \\
&\quad + \left\{\rho\theta_H + (1 - \rho)\theta_L\right\}\gamma\pi^\circ(c_2) - h(1 - \rho) \\
&= \pi^\circ(c_2) - h(1 - \rho) .
\end{aligned}$$

Define $\underline{\pi} \triangleq \pi^\circ(c_2) - h(1 - \rho)$. Then, $\underline{\pi} < \pi^b(\alpha^*, \rho|\widehat{\mathcal{D}})$.

Under $\widehat{\mathcal{R}}$ strategy, $\mathcal{C}'(\alpha^*, \rho) = -(p\psi - c_2\delta)$, which is negative. Hence, $y(q(l_{\widehat{\mathcal{R}}})) > \mathcal{C}'(\alpha^*, \rho)$, which again relates to Case (2) in the proof of The-

orem 4.3.1. Note that since $P_S = 1$, we have: $\mathcal{T}_2(\alpha, \rho) = \pi^\circ(c_2) - \xi\alpha$. Also, since B does not contract with S_1 under $\widehat{\mathcal{R}}$ strategy, the expected profit does not contain the term $\mathcal{U}(\alpha^*, \rho)$.

Therefore, B 's expected profit under $\widehat{\mathcal{R}}$ is given as:

$$\begin{aligned}
\pi^b(\alpha^*, \rho | \widehat{\mathcal{R}}) &= \mathcal{T}_2(\alpha^*, \rho) - h\alpha^* - h(1 - \rho) \\
&= \mathcal{T}_2(0, \rho) - h \cdot 0 - h(1 - \rho) \\
&= \pi^\circ(c_2) - \xi \cdot 0 - h \cdot 0 - h(1 - \rho) \\
&= \pi^\circ(c_2) - h(1 - \rho) \\
&= \underline{\pi} \\
&< \pi^b(\alpha^*, \rho | \widehat{\mathcal{D}}) \quad .
\end{aligned}$$

Therefore, we have: $\pi^b(0, \rho | \widehat{\mathcal{R}}) < \pi^b(0, \rho | \widehat{\mathcal{D}})$. This implies that if $\alpha_{min} = 0$, then $\widehat{\mathcal{D}}$ strategy always dominates $\widehat{\mathcal{R}}$ strategy. Hence, when $\alpha_{min} = 0$, B always follows $\widehat{\mathcal{D}}$ strategy.

The function $\pi^b(0, \rho | \widehat{\mathcal{D}})$ is linear in ρ . Hence, ρ^* is either equal to 0 or 1. In order to find the condition under which $\rho^* = 0$ and $\rho^* = 1$, we need to compare $\pi^b(0, 0 | \widehat{\mathcal{D}})$ and $\pi^b(0, 1 | \widehat{\mathcal{D}})$.

$$\begin{aligned}
\pi^b(0, 0 | \widehat{\mathcal{D}}) &= \pi^\circ(c_1^L) \left\{ 1 - \theta_L \gamma \right\} + \theta_L \gamma \pi^\circ(c_2) - h \quad , \\
\pi^b(0, 1 | \widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H) \left\{ 1 - \theta_H \gamma \right\} + \theta_H \gamma \pi^\circ(c_2) \quad .
\end{aligned}$$

We see that $\rho^* = 0$ if and only if $\pi^b(0, 0|\widehat{\mathcal{D}}) > \pi^b(0, 1|\widehat{\mathcal{D}})$.

$$\begin{aligned}
\pi^b(0, 0|\widehat{\mathcal{D}}) - \pi^b(0, 1|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L) \left\{ 1 - \theta_L \gamma \right\} + \theta_L \gamma \pi^\circ(c_2) - h \\
&\quad - \left[\tilde{\pi}(c_1^H) \left\{ 1 - \theta_H \gamma \right\} + \theta_H \gamma \pi^\circ(c_2) \right] \\
&= \pi^\circ(c_1^L) \left\{ 1 - \theta_L \gamma \right\} + \theta_L \gamma \pi^\circ(c_2) - h \\
&\quad - \tilde{\pi}(c_1^H) \left\{ 1 - \theta_H \gamma \right\} - \theta_H \gamma \pi^\circ(c_2) \\
&= \gamma \left[\theta_H \left\{ \tilde{\pi}(c_1^H) - \pi^\circ(c_2) \right\} - \theta_L \left\{ \pi^\circ(c_1^L) - \pi^\circ(c_2) \right\} \right] \\
&\quad - \left[\tilde{\pi}(c_1^H) - \pi^\circ(c_1^L) + h \right] .
\end{aligned}$$

Note that since $\tilde{\pi}(c_1^H) > \pi^\circ(c_1^L) > \pi^\circ(c_2)$ and $\theta_H > \theta_L$, the coefficient of γ in the above expression is positive. Therefore:

$$\begin{aligned}
\pi^b(0, 0|\widehat{\mathcal{D}}) &> \pi^b(0, 1|\widehat{\mathcal{D}}) \\
\Leftrightarrow \gamma &> \frac{\tilde{\pi}(c_1^H) - \pi^\circ(c_1^L) + h}{\theta_H \left\{ \tilde{\pi}(c_1^H) - \pi^\circ(c_2) \right\} - \theta_L \left\{ \pi^\circ(c_1^L) - \pi^\circ(c_2) \right\}} .
\end{aligned}$$

Define: $\hat{\gamma} \triangleq \frac{\tilde{\pi}(c_1^H) - \pi^\circ(c_1^L) + h}{\theta_H \left\{ \tilde{\pi}(c_1^H) - \pi^\circ(c_2) \right\} - \theta_L \left\{ \pi^\circ(c_1^L) - \pi^\circ(c_2) \right\}} .$

We see that $\hat{\gamma} > 0$.

From the above analysis, we can say that $\rho^* = 0$ if and only if $\gamma > \hat{\gamma}$. Therefore, $I_D^* = h$ if and only if $\gamma > \hat{\gamma}$. Similarly, it can be proved that $\rho^* = 1$ if and only if $\gamma \leq \hat{\gamma}$, or equivalently, $I_D^* = 0$ if and only if $\gamma \leq \hat{\gamma}$. ■

Preliminaries for the Proof of Theorem 4.3.4.

From the proofs of Lemma 4.7.4 and Theorem 4.3.2, we see that the curve $y(q(l_{\widehat{D}})) = \mathcal{C}(I_A, I_D)$ is equivalent to $\pi^\circ(c_2) = \xi\alpha(I_A)\{P_S(I_A, I_D)p+1\}$. This is a quadratic equation in $\alpha(I_A)$. Let $\alpha(\underline{I_A})$ and $\alpha(\overline{I_A})$ be the solutions to the equation when $I_D = 0$ and $I_D = h$, respectively. We also see that the curve $y(q(l_{\widehat{R}})) = \mathcal{C}'(I_A, I_D)$ is equivalent to $\pi^\circ(c_2) = \xi\alpha(I_A)(p+1)$, which is a linear equation in I_A . Let $\alpha(I'_A)$ be the solution to this equation. Then, provided $0 < \alpha(\underline{I_A}) \leq 1$, $0 < \alpha(\overline{I_A}) \leq 1$ and $0 < \alpha(I'_A) \leq 1$, the order of $\underline{I_A}$, $\overline{I_A}$ and I'_A is given by the following Lemma 4.7.6. If one of $\alpha(\underline{I_A})$, $\alpha(\overline{I_A})$ and $\alpha(I'_A)$ is 0 or exceeds 1, then the result given by the lemma is neither necessarily valid or relevant.

Lemma 4.7.6. $I'_A < \underline{I_A} < \overline{I_A}$.

Proof. Define: $\mathcal{P}_V(I_D) \triangleq \left\{ \rho(I_D)\theta_H + (1 - \rho(I_D))\theta_L \right\}$.

The equation $\pi^\circ(c_2) = \xi\alpha(I_A^*)\{P_S(I_A, I_D)p+1\}$ can be re-written as:

$$\begin{aligned} \xi\alpha(I_A^*)\{P_S(I_A^*, I_D)p+1\} &= \pi^\circ(c_2) \\ \Leftrightarrow p\xi\alpha(I_A^*)P_S(I_A^*, I_D) + \xi\alpha(I_A^*) &= \pi^\circ(c_2) \\ \Leftrightarrow p\xi\alpha(I_A^*)\left\{\alpha(I_A^*)(1 - \gamma) + \gamma\right\}\mathcal{P}_V(I_D) + \xi\alpha(I_A^*) - \pi^\circ(c_2) &= 0 \\ \Leftrightarrow \alpha(I_A^*)^2[p\xi(1 - \gamma)\mathcal{P}_V(I_D)] + \alpha(I_A^*)[p\xi\gamma\mathcal{P}_V(I_D)] + \alpha(I_A^*)\xi - \pi^\circ(c_2) &= 0 \\ \Leftrightarrow \alpha(I_A^*)^2[p\xi(1 - \gamma)\mathcal{P}_V(I_D)] + \alpha(I_A^*)[p\xi\gamma\mathcal{P}_V(I_D) + \xi] - \pi^\circ(c_2) &= 0 . \end{aligned}$$

This is a quadratic equation in $\alpha(I_A^*)$.

Let $t = p\xi(1 - \gamma)\mathcal{P}_V(I_D)$, $u = p\xi\gamma\mathcal{P}_V(I_D) + \xi$ and $v = -\pi^\circ(c_2)$.

$$\begin{aligned}\therefore \frac{u}{t} &= \frac{p\xi\gamma\mathcal{P}_V(I_D) + \xi}{p\xi(1 - \gamma)\mathcal{P}_V(I_D)} \\ &= \frac{\gamma}{(1 - \gamma)} + \frac{1}{p(1 - \gamma)\mathcal{P}_V(I_D)}, \\ \frac{v}{t} &= \frac{-\pi^\circ(c_2)}{p\xi(1 - \gamma)\mathcal{P}_V(I_D)}.\end{aligned}$$

We know that $\alpha(I_A^*) = \frac{-u + \sqrt{u^2 - 4tv}}{2t}$ and $\alpha(I_A^*) = \frac{-u - \sqrt{u^2 - 4tv}}{2t}$ are possible solutions to the quadratic equation. But we see that $\frac{-u - \sqrt{u^2 - 4tv}}{2t} < 0$ whereas $\alpha(I_A^*) > 0$. Hence, solving the quadratic equation for $\alpha(I_A^*)$, we see that $\alpha(I_A^*) = \frac{-u + \sqrt{u^2 - 4tv}}{2t}$ is the only feasible solution. Therefore:

$$\begin{aligned}\alpha^{(I_D)} &= \frac{1}{2} \left[-\frac{u}{t} + \sqrt{\left(\frac{u}{t}\right)^2 - \frac{4v}{t}} \right] \\ &= \frac{1}{2} \left[-\left\{ \frac{\gamma}{(1 - \gamma)} + \frac{1}{p(1 - \gamma)\mathcal{P}_V(I_D)} \right\} \right. \\ &\quad \left. + \sqrt{\left\{ \frac{\gamma}{(1 - \gamma)} + \frac{1}{p(1 - \gamma)\mathcal{P}_V(I_D)} \right\}^2 + \frac{4\pi^\circ(c_2)}{p\xi(1 - \gamma)\mathcal{P}_V(I_D)}} \right], \quad (4.8)\end{aligned}$$

which decreases in $\mathcal{P}_V(I_D)$, and thus increases in I_D .

We know that $I_D \in [0, h]$. Since $\alpha^{(I_D)}$ increases in I_D , therefore, we can say that $\alpha^{(0)} < \alpha^{(h)}$.

Then, $\underline{I}_A = \alpha^{-1}(\alpha^{(0)})$ and $\overline{I}_A = \alpha^{-1}(\alpha^{(h)})$. Since $\alpha(I_A)$ is increasing in I_A and $\alpha^{(0)} < \alpha^{(h)}$, we have: $\underline{I}_A < \overline{I}_A$.

Now, from the equation $\pi^\circ(c_2) = \xi\alpha(I_A)(p + 1)$, we have: $\alpha = \frac{\pi^\circ(c_2)}{\xi(p+1)}$, which is independent of I_D , and hence is a straight line parallel to the I_D axis in the $I_A^* - I_D$ plane. Therefore: $I_A' = \alpha^{-1}\left(\frac{\pi^\circ(c_2)}{\xi(p+1)}\right)$. Since $\mathcal{C}'(I_A^*, I_D) >$

$\mathcal{C}(I_A^*, I_D)$, clearly either $I_A' < \underline{I_A}$ or $I_A' > \overline{I_A}$. Also, since $y(q(l_{\widehat{\mathcal{R}}})) \geq y(q(l_{\widehat{\mathcal{D}}}))$, the size of the region in the $I_A^* - I_D$ plane within which B offers $l_{\widehat{\mathcal{R}}}$ to S_2 is larger under than that within which B offers $l_{\widehat{\mathcal{D}}}$ to S_2 . Therefore the straight line $y(q(l_{\widehat{\mathcal{R}}})) = \mathcal{C}'(I_A^*, I_D)$ lies entirely to the left of the curve $y(q(l_{\widehat{\mathcal{D}}})) = \mathcal{C}(I_A^*, I_D)$ in the $I_A^* - I_D$ plane. Therefore: $I_A' < \underline{I_A} < \overline{I_A}$.

■

Division of the $I_A^* - I_D$ Plane :

The functions for B 's expected profits under $\widehat{\mathcal{D}}$ and $\widehat{\mathcal{R}}$ strategies are:

$$\begin{aligned} \pi^b(I_A^*, I_D | \widehat{\mathcal{D}}) &= \rho(I_D) \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha(I_A^*) \theta_H z \right] \\ &\quad + (1 - \rho(I_D)) \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha(I_A^*) \theta_L z \right] \\ &\quad + P_S(I_A^*, I_D) \pi^\circ(w_2^*) - t_2^* - I_A^* - I_D \end{aligned}$$

and
$$\pi^b(I_A^*, I_D | \widehat{\mathcal{R}}) = \pi^\circ(w_2^*) - t_2^* - I_A^* - I_D.$$

Both $\pi^b(I_A^*, I_D | \widehat{\mathcal{D}})$ and $\pi^b(I_A^*, I_D | \widehat{\mathcal{R}})$ are linear in I_D . Clearly, $\pi^b(I_A^*, I_D | \widehat{\mathcal{R}})$ is maximized at $I_D = 0$. That is, $\pi^b(I_A^*, I_D | \widehat{\mathcal{R}}) = \pi^b(I_A^*, 0 | \widehat{\mathcal{R}})$. Using Lemma 4.7.6, we divide the $I_A^* - I_D$ plane into four rectangular areas. The straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ will lie completely within one the four areas. Hence, given α_{min} , the optimal investment in supplier improvement I_D^* and the optimal contracting strategy ($\widehat{\mathcal{D}}$ or $\widehat{\mathcal{R}}$) is determined by determining the maximum among the expected profit functions $\pi^b(I_A^*, I_D | \widehat{\mathcal{D}})$ and $\pi^b(I_A^*, I_D | \widehat{\mathcal{R}})$ for $w_2^* \in \{l_{\widehat{\mathcal{D}}}, l_{\widehat{\mathcal{R}}}, c_2\}$. Figure 4.3 depicts the four areas. The curve $y(q(l_{\widehat{\mathcal{D}}})) = \mathcal{C}(I_A^*, I_D)$ is the indifference line between B offering $w_2^* = l_{\widehat{\mathcal{D}}}$ or $w_2^* = c_2$ to S_2 under

$\widehat{\mathcal{D}}$. Similarly, the curve $y(q(l_{\widehat{\mathcal{R}}})) = \mathcal{C}'(I_A^*, I_D)$ is the indifference line between B offering $w_2^* = l_{\widehat{\mathcal{R}}}$ or $w_2^* = c_2$ to S_2 under $\widehat{\mathcal{R}}$. Note that under certain conditions, one or more of the areas might not exist. For example, if $\xi = 0$, then only the Area IV exists, and therefore for a given α_{min} , the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ will lie completely within Area IV.

Figure 4.3: The $I_A^*-I_D$ Plane Divided Into Four Areas

- Area I: $\overline{I}_A \leq I_A^* \leq h$: $w_2^* = l_{\widehat{\mathcal{D}}}$ or $l_{\widehat{\mathcal{R}}}$.
- Area II: $\underline{I}_A \leq I_A^* \leq \overline{I}_A$: Including and below the curve $y(q(l_{\widehat{\mathcal{D}}})) = \mathcal{C}(I_A^*, I_D)$, $w_2^* = l_{\widehat{\mathcal{D}}}$ or $l_{\widehat{\mathcal{R}}}$. Above the curve, $w_2^* = c_2$ (under $\widehat{\mathcal{D}}$) or $l_{\widehat{\mathcal{R}}}$. If the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ does not coincide with one of the two boundaries of Area II, i.e. if the line lies completely within the interior of Area II, then it intersects the curve $y(q(l_{\widehat{\mathcal{D}}})) = \mathcal{C}(I_A^*, I_D)$ at a point

where $0 < I_D < h$. Denote this value of I_D as I'_D .

- Area III: $I'_A \leq I_A^* \leq \underline{I_A}$: $w_2^* = c_2$ (under $\widehat{\mathcal{D}}$) or $l_{\widehat{\mathcal{R}}}$.
- Area IV: $0 < I_A^* \leq I'_A$: $w_2^* = c_2$.

The following lemma demonstrates that within the interior of Area II, the intermediate value of I_D , i.e. I'_D , is never optimal.

Lemma 4.7.7. *Within the interior of Area II, $I_D^* \neq I'_D$.*

Proof. Let $\hat{\pi}^1(I_A^*, I_D) = \pi^b(I_A^*, I_D | \widehat{\mathcal{D}})$ when $w_2^* = l_{\widehat{\mathcal{D}}}$. Also let $\hat{\pi}^2(I_A^*, I_D) = \pi^b(I_A^*, I_D | \widehat{\mathcal{D}})$ when $w_2^* = c_2$. $I_D^* = I'_D$ if and only if $\hat{\pi}^1(I_A^*, I_D)$ is strictly increasing in I_D and $\hat{\pi}^2(I_A^*, I_D)$ is strictly decreasing in I_D . Then $\hat{\pi}^1(I_A^*, h) > \hat{\pi}^1(I_A^*, 0)$ and $\hat{\pi}^2(I_A^*, 0) > \hat{\pi}^2(I_A^*, h)$. We will show that these two inequalities cannot hold simultaneously.

Define the following:

$$\begin{aligned}
g(\theta_i) &\triangleq P_S^i(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} , \\
v(\theta_i) &\triangleq \frac{\left\{ P_S^i(I_A^*, \gamma) \right\}^2 p}{P_S^i(I_A^*, \gamma) p + 1} , \\
\mathcal{M} &\triangleq \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) - \alpha_{min} \theta_L z - \left\{ 1 - P_S^H(I_A^*, \gamma) \right\} \tilde{\pi}(c_1^H) + \alpha_{min} \theta_H z - h , \\
\mathcal{G}_1 &\triangleq \left\{ v(\theta_H) - v(\theta_L) \right\} \pi^\circ(c_2) , \\
\mathcal{G}_2 &\triangleq g(\theta_H) - g(\theta_L) .
\end{aligned}$$

Both $g(\theta_i)$ and $v(\theta_i)$ are increasing in θ_i . Hence, $\mathcal{G}_1 > 0$ and $\mathcal{G}_2 > 0$.

Then we have:

$$\begin{aligned}\hat{\pi}^1(I_A^*, h) - \hat{\pi}^1(I_A^*, 0) &= \mathcal{M} - \mathcal{G}_1, \\ \hat{\pi}^2(I_A^*, 0) - \hat{\pi}^2(I_A^*, h) &= -\mathcal{M} + \mathcal{G}_2.\end{aligned}$$

Since $\hat{\pi}^1(I_A^*, h) > \hat{\pi}^1(I_A^*, 0)$ and $\hat{\pi}^2(I_A^*, 0) > \hat{\pi}^2(I_A^*, h)$, we have:

$$\mathcal{G}_1 < \mathcal{M} < \mathcal{G}_2. \quad (4.9)$$

We will prove that the above inequalities do not hold.

Within the interior of Area II, $y(q(l)) < \mathcal{C}(I_A^*, 0)$. From Lemma 4.7.4, we know that if $y(q(l)) \geq \mathcal{C}(I_A, I_D)$, then:

$$\frac{(p - c_2)^2}{2p^2} \frac{P_S(I_A, I_D)p}{P_S(I_A, I_D)p + 1} \geq P_S(I_A, I_D)\xi\alpha(I_A)$$

Similarly, we can derive that if $y(q(l)) < \mathcal{C}(I_A^*, 0)$, then:

$$\begin{aligned}\frac{(p - c_2)^2}{2p^2} \frac{P_S(I_A^*, 0)p}{P_S(I_A^*, 0)p + 1} &< P_S(I_A^*, 0)\xi\alpha_{min} \\ \Leftrightarrow \frac{(p - c_2)^2}{2p} &< \xi\alpha_{min} \left\{ P_S(I_A^*, 0)p + 1 \right\} \\ \Leftrightarrow \pi^\circ(c_2) &< \xi\alpha_{min} \left\{ P_S(I_A^*, 0)p + 1 \right\}.\end{aligned} \quad (4.10)$$

That is, $\pi^\circ(c_2) < \xi\alpha_{min} \left\{ P_S^H(I_A^*, \gamma)p + 1 \right\}$.

This implies that:

$$\begin{aligned}
\frac{\pi^\circ(c_2)}{P_S^H(I_A^*, \gamma)p + 1} &< \xi\alpha_{min} \\
\Leftrightarrow \left[1 - \frac{P_S^H(I_A^*, \gamma)p}{P_S^H(I_A^*, \gamma)p + 1} \right] \pi^\circ(c_2) &< \xi\alpha_{min} \\
\Leftrightarrow \left[\frac{P_S^H(I_A^*, \gamma)p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} \right] &> 0.
\end{aligned}$$

Therefore, since $\theta_H > \theta_L$, we can infer that within the interior of Area II:

$$\begin{aligned}
\left[\frac{P_S^H(I_A^*, \gamma)p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} \right] &> \left[\frac{P_S^L(I_A^*, \gamma)p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \right. \\
&\quad \left. - \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} \right]. \quad (4.11)
\end{aligned}$$

Now, for relation (4.9) to hold, $\mathcal{G}_1$ must be less than $\mathcal{G}_2$. We see that:

$$\begin{aligned}
\mathcal{G}_1 - \mathcal{G}_2 &= P_S^H(I_A^*, \gamma) \left[\frac{P_S^H(I_A^*, \gamma)p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} \right] \\
&\quad - P_S^L(I_A^*, \gamma) \left[\frac{P_S^L(I_A^*, \gamma)p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} \right].
\end{aligned}$$

Since $P_S^H(I_A^*, \gamma) > P_S^L(I_A^*, \gamma)$, using (4.11), we get:

$$\mathcal{G}_1 - \mathcal{G}_2 > 0 \quad .$$

But this means that $\mathcal{G}_1$ is greater than $\mathcal{G}_2$, which contradicts the requirement that $\mathcal{G}_1$ must be less than $\mathcal{G}_2$. Hence, the relation (4.9) does not hold. This im-

plies that the two inequalities $\hat{\pi}^1(I_A^*, h) > \hat{\pi}^1(I_A^*, 0)$ and $\hat{\pi}^2(I_A^*, 0) > \hat{\pi}^2(I_A^*, h)$ cannot be simultaneously true. Therefore within the interior of Area II, $I_D^* \neq I_D'$. ■

Using Lemma 4.7.7, we see that whichever Area the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ falls within, the optimal investment in supplier improvement I_D^* is either equal to 0 or h . Therefore, in each area, B 's optimal contracting and investment strategy is given by the following function:

$$\pi^{b*} = \max \left\{ \pi^b(I_A^*, 0 | \widehat{\mathcal{D}}), \pi^b(I_A^*, h | \widehat{\mathcal{D}}), \pi^b(I_A^*, 0 | \widehat{\mathcal{R}}) \right\},$$

where the optimal unit wholesale prices to be offered within each area are as summarized in Table 4.2.

Table 4.2: Optimal Unit Wholesale Prices Within Four Areas

Area	w_2^* under $\widehat{\mathcal{D}}$ and $I_D^* = 0$	w_2^* under $\widehat{\mathcal{D}}$ and $I_D^* = h$	w_2^* under $\widehat{\mathcal{R}}$
I	$l_{\widehat{\mathcal{D}}}$	$l_{\widehat{\mathcal{D}}}$	$l_{\widehat{\mathcal{R}}}$
II	$l_{\widehat{\mathcal{D}}}$	c_2	$l_{\widehat{\mathcal{R}}}$
III	c_2	c_2	$l_{\widehat{\mathcal{R}}}$
IV	c_2	c_2	c_2

The expressions for all the expected profit functions for B are stated in Table 4.3.

Proof of Theorem 4.3.4.

Because α_{min} is given, we can identify the area within which the line $I_A^* = \alpha^{-1}(\alpha_{min})$ lies. In order to solve for B 's optimal contracting strategy and I_D^* , we will explore each of the four areas separately. Within each area, we will find the maximum among the three functions $\pi^b(I_A^*, 0 | \widehat{\mathcal{D}})$, $\pi^b(I_A^*, h | \widehat{\mathcal{D}})$

Table 4.3: Functions for $\mathbf{B}'\mathbf{s}$ Expected Profits

w_2^*	Profit Function	Simplified Expression
$l_{\widehat{\mathcal{D}}}$	$\pi^b(I_A^*, 0 \widehat{\mathcal{D}})$	$\left\{1 - P_S^H(I_A^*, \gamma)\right\}\tilde{\pi}(c_1^H) - \alpha_{min}\theta_H z + \frac{\left\{P_S^H(I_A^*, \gamma)\right\}^2}{P_S^H(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - h\alpha_{min}$
$l_{\widehat{\mathcal{D}}}$	$\pi^b(I_A^*, h \widehat{\mathcal{D}})$	$\left\{1 - P_S^L(I_A^*, \gamma)\right\}\pi^\circ(c_1^L) - \alpha_{min}\theta_L z + \frac{\left\{P_S^L(I_A^*, \gamma)\right\}^2}{P_S^L(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - h\alpha_{min} - h$
c_2	$\pi^b(I_A^*, 0 \widehat{\mathcal{D}})$	$\left\{1 - P_S^H(I_A^*, \gamma)\right\}\tilde{\pi}(c_1^H) - \alpha_{min}\theta_H z + P_S^H(I_A^*, \gamma)\left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\} - h\alpha_{min}$
c_2	$\pi^b(I_A^*, h \widehat{\mathcal{D}})$	$\left\{1 - P_S^L(I_A^*, \gamma)\right\}\pi^\circ(c_1^L) - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\} - h\alpha_{min} - h$
$l_{\widehat{\mathcal{R}}}$	$\pi^b(I_A^*, 0 \widehat{\mathcal{R}})$	$\frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min}$
c_2	$\pi^b(I_A^*, 0 \widehat{\mathcal{R}})$	$\pi^\circ(c_2) - \xi\alpha_{min} - h\alpha_{min}$

and $\pi^b(I_A^*, 0|\widehat{\mathcal{R}})$, as stated above.

From Table 4.3, we see that the profit functions are of the following forms:

$$\begin{aligned}
 \pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= a - h - \alpha_{min}\theta_L z - h\alpha_{min} , \\
 \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= b - \alpha_{min}\theta_H z - h\alpha_{min} , \\
 \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) &= c - h\alpha_{min} ,
 \end{aligned}$$

where a , b and c are the terms devoid of z and h , in the respective functions. Note that the exact expressions for a , b and c change depending on the specific area within which the line $I_A^* = \alpha^{-1}(\alpha_{min})$ lies. The terms $a - h$ and b are the net revenues earned by B under maximum and zero investments, respectively, under $\widehat{\mathcal{D}}$. Further, the term c is the net revenue earned by B under $\widehat{\mathcal{R}}$.

For each area, we calculate the pairwise differences between the three profit functions as follows:

$$\begin{aligned} d_1 &\triangleq \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) \quad , \\ d_2 &\triangleq \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \quad , \\ d_3 &\triangleq \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \quad . \end{aligned}$$

Substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we get three thresholds for z which we denote as z_1 , z_2 , and z_3 , respectively. Then, the thresholds z_1 , z_2 , and z_3 are of the following forms:

$$\begin{aligned} z_1 &= (-a + b + h)/(\theta_H - \theta_L)\alpha_{min} \quad , \\ z_2 &= (a - c - h)/\theta_L\alpha_{min} \quad , \\ z_3 &= (b - c)/\theta_H\alpha_{min} \quad . \end{aligned} \tag{4.12}$$

The closed-form expressions for z_1 , z_2 and z_3 are stated in Tables 4.4, 4.5 and 4.6 respectively.

Within each area, the relation between z and the optimal profit is as follows:

$$\begin{aligned}
\pi^b(I_A^*, 0 | \widehat{\mathcal{D}}) &\leftrightarrow d_1 \leq 0, d_3 > 0 \leftrightarrow z \leq \min(z_1, z_3), \\
\pi^b(I_A^*, h | \widehat{\mathcal{D}}) &\leftrightarrow d_1 > 0, d_2 > 0 \leftrightarrow \min(z_1, z_3) < z < \max(z_2, z_3), \\
\pi^b(I_A^*, 0 | \widehat{\mathcal{R}}) &\leftrightarrow d_2 \leq 0, d_3 \leq 0 \leftrightarrow z \geq \max(z_2, z_3).
\end{aligned} \tag{4.13}$$

From Equation (4.12), we can derive the following relations:

$$\begin{aligned}
z_2 - z_1 &= (a - c - h)/\theta_L \alpha_{min} - (-a + b + h)/(\theta_H - \theta_L) \alpha_{min} , \\
z_2 - z_3 &= (a - c - h)/\theta_L \alpha_{min} - (b - c)/\theta_H \alpha_{min} , \\
z_3 - z_1 &= (b - c)/\theta_H \alpha_{min} - (-a + b + h)/(\theta_H - \theta_L) \alpha_{min} .
\end{aligned} \tag{4.14}$$

Consider the following two scenarios:

Scenario $\Phi 1$: Suppose $z_2 > z_1$. Then from Equation (4.14), we have:

$$\begin{aligned}
(a - c - h)/\theta_L \alpha_{min} - (-a + b + h)/(\theta_H - \theta_L) \alpha_{min} &> 0 \\
\leftrightarrow (a - c - h)(\theta_H - \theta_L) - (-a + b + h)\theta_L &> 0 \\
\leftrightarrow (a - c - h)\theta_H - (b - c)\theta_L &> 0 .
\end{aligned} \tag{4.15}$$

Dividing (4.15) throughout by $\theta_H \theta_L \alpha_{min}$, we get:

$$(a - c - h)/\theta_L \alpha_{min} - (b - c)/\theta_H \alpha_{min} > 0$$

From (4.14), we know that the above relation implies that:

$$z_2 - z_3 > 0.$$

Therefore, we can say that $z_2 > z_1$ implies that $z_2 > z_3$.

Rearranging the terms of the L.H.S. of (4.15), we get:

$$\begin{aligned} (a - c - h)\theta_H - (b - c)\theta_L &= -(-a + h)\theta_H - c\theta_H - (b - c)\theta_L + b\theta_H - b\theta_H \\ &= -(-a + b + h)\theta_H + b(\theta_H - \theta_L) - c(\theta_H - \theta_L) \\ &= (b - c)(\theta_H - \theta_L) - (-a + b + h)\theta_H \quad . \end{aligned}$$

Hence, from (4.15), we have:

$$(b - c)(\theta_H - \theta_L) - (-a + b + h)\theta_H > 0 \quad . \quad (4.16)$$

Dividing (4.16) throughout by $\theta_H(\theta_H - \theta_L)\alpha_{min}$, we get:

$$(b - c)/\theta_H\alpha_{min} - (-a + b + h)/(\theta_H - \theta_L)\alpha_{min} > 0 \quad .$$

From (4.14), we know that the above relation implies that:

$$z_3 - z_1 > 0.$$

Therefore, we can say that $z_2 > z_1$ implies that $z_3 > z_1$.

Therefore, if $z_2 > z_1$, then $z_1 < z_3 < z_2$. Consequently, $\min(z_1, z_3) = z_1$ and $\max(z_2, z_3) = z_2$.

Scenario $\Phi 2$: Suppose $z_2 \leq z_1$. Then we can similarly prove that $z_2 \leq z_3 \leq z_1$. Consequently, $\min(z_1, z_3) = \max(z_2, z_3) = z_3$.

Therefore, using the two scenarios and the relation between z and B 's optimal profits (as given by (4.13)), we can write B 's optimal contracting strategy and optimal policy for investment in supplier improvement as:

$\Phi 1$: When $z_1 < z_2$:

C1: $z \leq z_1 \leftrightarrow \widehat{\mathcal{D}}$ is optimal; $I_D^* = 0$.

C2: $z_1 < z < z_2 \leftrightarrow \widehat{\mathcal{D}}$ is optimal; $I_D^* = h$.

C3: $z \geq z_2 \leftrightarrow \widehat{\mathcal{R}}$ is optimal; $I_D^* = 0$.

$\Phi 2$: When $z_1 \geq z_2$:

C1: $z \leq z_3 \leftrightarrow \widehat{\mathcal{D}}$ is optimal; $I_D^* = 0$.

C2: $z > z_3 \leftrightarrow \widehat{\mathcal{R}}$ is optimal; $I_D^* = 0$.

Finally:

$$\begin{aligned} \beta &= \frac{\pi^b(I_A^*, h | \widehat{\mathcal{D}}) + \alpha_{\min} \theta_L z - \pi^b(I_A^*, 0 | \widehat{\mathcal{R}})}{\pi^b(I_A^*, 0 | \widehat{\mathcal{D}}) + \alpha_{\min} \theta_H z - \pi^b(I_A^*, 0 | \widehat{\mathcal{R}})} \\ &= \frac{a - h - c}{b - c} . \end{aligned}$$

Therefore it follows from (4.15) that $z_1 < z_2$ if and only if $\beta > \theta_L / \theta_H$.

Similarly, we can prove that $z_1 \geq z_2$ if and only if $\beta \leq \theta_L / \theta_H$. ■

Define: $m \triangleq \gamma(1 - \alpha_{\min}) + \alpha_{\min}$, and $r_1(\theta_i, m) \triangleq \theta_i m \hat{\pi} - \frac{\{\theta_i m\}^2 p}{\theta_i m p + 1} \pi^\circ(c_2)$, where $\hat{\pi} > \pi^\circ(c_2)$, $i \in \{H, L\}$.

That is, $r_1(\theta_i, m) = P_S^i(I_A^*, \gamma) \hat{\pi} - \frac{\left\{P_S^i(I_A^*, \gamma)\right\}^2 p}{P_S^i(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2)$.

Lemma 4.7.8. $\frac{\delta r_1(\theta_i, m)}{\delta \theta_i} > 0$ and $\frac{\delta r_1(\theta_i, m)}{\delta m} > 0$.

Proof. Let: $\mathcal{Q} = \frac{\{\theta_i m\}^2 p}{\theta_i m p + 1}$.

Taking derivative w.r.t. θ_i , we have:

$$\begin{aligned}
\frac{\delta \mathcal{Q}}{\delta \theta_i} &= \frac{2\theta_i m^2 p}{\theta_i m p + 1} - \frac{\left\{\theta_i m\right\}^2 p}{\left\{\theta_i m p + 1\right\}^2} \cdot m p \\
&= \frac{2\theta_i^2 m^3 p^2 + 2\theta_i m^2 p - \theta_i^2 m^3 p^2}{\left\{\theta_i m p + 1\right\}^2} \\
&= \frac{\theta_i^2 m^3 p^2 + 2\theta_i m^2 p}{\left\{\theta_i m p + 1\right\}^2} \\
&= m \cdot \frac{\theta_i^2 m^2 p^2 + 2\theta_i m p}{\left\{\theta_i m p + 1\right\}^2} \\
&= m \cdot \frac{\left\{\theta_i m p + 1\right\}^2 - 1}{\left\{\theta_i m p + 1\right\}^2} \\
&= m \cdot \left[1 - \frac{1}{\left\{\theta_i m p + 1\right\}^2} \right] \\
&< m .
\end{aligned}$$

Therefore:

$$\frac{\delta r_1(\theta_i)}{\delta \theta_i} = m \hat{\pi} - \frac{\delta \mathcal{Q}}{\delta \theta_i} \pi^\circ(c_2)$$

Since $\frac{\delta Q}{\delta \theta_i} < m$, we have: $\frac{\delta Q}{\delta \theta_i} \pi^\circ(c_2) < m \pi^\circ(c_2)$. Therefore:

$$\frac{\delta r_1(\theta_i)}{\delta \theta_i} > m \left\{ \hat{\pi} - \pi^\circ(c_2) \right\}$$

Since $\hat{\pi} > \pi^\circ(c_2)$, we have: $\frac{\delta r_1(\theta_i, m)}{\delta \theta_i} > 0$.

The analysis for $\frac{\delta r_1(\theta_i, m)}{\delta m}$ is similar. ■

Refer to Tables 4.4, 4.5 and 4.6 for the closed-form expressions for z_1 , z_2 , and z_3 , respectively.

Proof of Theorem 4.4.1.

For the proofs of parts (i) and (ii), we define additional notation as follows. In the $I_A^* - I_D$ plane, the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ lies in any one of the four Areas, whose boundaries are defined by the lines $I_A^* = I_A'$, $I_A^* = \underline{I_A}$ and $I_A^* = \overline{I_A}$. As we increase c_2 , the values of I_A' , $\underline{I_A}$ and $\overline{I_A}$ decrease (note that the order does not change). Hence due to increasing c_2 , the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ gradually progresses from Area IV to Area III to Area II to Area I. Obviously then, there exist thresholds of c_2 at which the transitions between consecutive Areas occur. Denote the threshold values of c_2 at which the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ transitions from: (a) Area IV to Area III as $\underline{c_2}$, (b) Area III to Area II as $\underline{c_2}'$, and (c) Area II to Area I as $\overline{c_2}$. Then $\underline{c_2} < \underline{c_2}' < \overline{c_2}$.

Following the construction of the four Areas, we see that $\underline{c_2}$, $\underline{c_2}'$, and $\overline{c_2}$ are the values of c_2 which satisfy the equations $\pi^\circ(c_2) = \xi \alpha(I_A^*)(p+1)$, $\pi^\circ(c_2) = \xi \alpha(I_A^*) \left\{ P_S(I_A^*, 0)p + 1 \right\}$, and $\pi^\circ(c_2) = \xi \alpha(I_A^*) \left\{ P_S(I_A^*, h)p + 1 \right\}$,

respectively. The values of $\underline{c}_2$, c_2' , and $\overline{c}_2$ can be derived as follows:

$$\begin{aligned}
\pi^\circ(\underline{c}_2) &= \xi\alpha(I_A^*)(p+1) \\
\leftrightarrow \frac{(p-\underline{c}_2)^2}{2p} &= \xi\alpha_{min}(p+1) \\
\leftrightarrow (p-\underline{c}_2)^2 &= 2\xi\alpha_{min}p(p+1) \\
\leftrightarrow \underline{c}_2 &= p - \sqrt{2\xi\alpha_{min}p(p+1)} . \tag{4.17}
\end{aligned}$$

$$\begin{aligned}
\pi^\circ(c_2') &= \xi\alpha(I_A^*)\left\{P_S(I_A^*, 0)p+1\right\} \\
\leftrightarrow \frac{(p-c_2')^2}{2p} &= \xi\alpha_{min}\left\{P_S(I_A^*, 0)p+1\right\} \\
\leftrightarrow (p-c_2')^2 &= 2\xi\alpha_{min}p\left\{P_S(I_A^*, 0)p+1\right\} \\
\leftrightarrow c_2' &= p - \sqrt{2\xi\alpha_{min}p\left\{P_S(I_A^*, 0)p+1\right\}} \\
&= p - \sqrt{2\xi\alpha_{min}p\left[\theta_H\{\alpha_{min}(1-\gamma)+\gamma\}p+1\right]} . \tag{4.18}
\end{aligned}$$

$$\begin{aligned}
\pi^\circ(\overline{c}_2) &= \xi\alpha(I_A^*)\left\{P_S(I_A^*, h)p+1\right\} \\
\leftrightarrow \frac{(p-\overline{c}_2)^2}{2p} &= \xi\alpha_{min}\left\{P_S(I_A^*, h)p+1\right\} \\
\leftrightarrow (p-\overline{c}_2)^2 &= 2\xi\alpha_{min}p\left\{P_S(I_A^*, h)p+1\right\} \\
\leftrightarrow \overline{c}_2 &= p - \sqrt{2\xi\alpha_{min}p\left\{P_S(I_A^*, h)p+1\right\}} \\
&= p - \sqrt{2\xi\alpha_{min}p\left[\theta_L\{\alpha_{min}(1-\gamma)+\gamma\}p+1\right]} . \tag{4.19}
\end{aligned}$$

Proof of (i). The coefficient of $\pi^\circ(c_2)$ in the numerators of z_1 for Area I, Area III and Area IV are positive. However, the coefficient for Area II

presents two possibilities. Denote this coefficient of $\pi^\circ(c_2)$ as:

$$\begin{aligned}
\underline{x} &\triangleq \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} - P_S^L(I_A^*, \gamma) \\
&= \frac{\{\theta_H m\}^2 p}{\theta_H m p + 1} - \theta_L m \\
&= \frac{\theta_H^2 m^2 p - \theta_H \theta_L m^2 p - \theta_L m}{\theta_H m p + 1} .
\end{aligned}$$

Therefore:

$$\begin{aligned}
\underline{x} > 0 &\Leftrightarrow \theta_H^2 m^2 p - \theta_H \theta_L m^2 p - \theta_L m > 0 \\
&\Leftrightarrow \theta_H^2 m p - \theta_H \theta_L m p - \theta_L > 0 \\
&\Leftrightarrow \theta_H m p (\theta_H - \theta_L) > \theta_L \\
&\Leftrightarrow \theta_H m > \frac{\theta_L}{p(\theta_H - \theta_L)} \\
&\Leftrightarrow P_S^H(I_A^*, \gamma) > \frac{\theta_L}{p(\theta_H - \theta_L)} .
\end{aligned}$$

Therefore, we see that if $P_S^H(I_A^*, \gamma) > \frac{\theta_L}{p(\theta_H - \theta_L)}$, then $\underline{x} > 0$, and as a result, z_1 decreases in c_2 throughout all four Areas. But if $P_S^H(I_A^*, \gamma) \leq \frac{\theta_L}{p(\theta_H - \theta_L)}$, then $\underline{x} \leq 0$, and as a result z_1 does not decrease in c_2 within Area II, but decreases in c_2 within Areas I, III and IV. Therefore, z_1 strictly decreases in c_2 if $0 \leq c_2 \leq c_2'$ or $\overline{c_2} \leq c_2 \leq p$, but increases in c_2 if $c_2' \leq c_2 \leq \overline{c_2}$.

The analysis for sensitivity of z_1 w.r.t. c_1^L and c_1^H are straightforward.

Proof of (ii). The coefficient of $\pi^\circ(c_2)$ in the numerators of z_2 for Area I and Area IV are negative. However, the coefficient for Area II (same for Area III) presents two possibilities. Denote this coefficient of $\pi^\circ(c_2)$ as $\overline{x} \triangleq$

$P_S^L(I_A^*, \gamma) - \frac{p}{p+1}$. We see that if $P_S^L(I_A^*, \gamma) < \frac{p}{p+1}$, then $\bar{x} < 0$, and as a result, z_2 increases in c_2 throughout all four Areas. But if $P_S^L(I_A^*, \gamma) \geq \frac{p}{p+1}$, then $\bar{x} \geq 0$, and as a result z_2 does not increase in c_2 within Areas II and III, but increases in c_2 within Areas I and IV. Therefore, z_2 strictly increases in c_2 if $0 \leq c_2 \leq \underline{c_2}$ or $\bar{c_2} \leq c_2 \leq p$, but decreases in c_2 if $\underline{c_2} \leq c_2 \leq \bar{c_2}$.

The analysis for sensitivity of z_2 w.r.t. c_1^L and c_1^H are straightforward.

Proof of (iii). The component of the numerator of z_1 in Area I and Area II containing θ_H is:

$$-P_S^H(I_A^*, \gamma)\tilde{\pi}(c_1^H) + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) .$$

Using $\theta_i = \theta_H$ and $\hat{\pi} = \tilde{\pi}(c_1^H)$ in Lemma 4.7.8, we see that the above component decreases in θ_H . That is, the numerator decreases in θ_H .

The component of the numerator of z_1 in Area III and Area IV containing θ_H is:

$$-P_S^H(I_A^*, \gamma)\left[\tilde{\pi}(c_1^H) - \left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\}\right] .$$

Since $\tilde{\pi}(c_1^H) > \pi^\circ(c_2)$, the above component decreases in θ_H . That is, the numerator decreases in θ_H .

Thus, in all the four areas, the numerator of z_1 decreases in θ_H . The denominator for all four areas is $(\theta_H - \theta_L)\alpha_{min}$, which increases in θ_H . Therefore, in all four areas, since the numerator of z_1 decreases in θ_H and the denominator increases in θ_H , we can infer that z_1 decreases in θ_H . The sensitivity of z_1 w.r.t θ_L , and that of z_2 w.r.t θ_H and θ_L can be similarly proved.

Proof of (iv). Observe that the sensitivity of z_2 w.r.t. γ is the same as the sensitivity of z_2 w.r.t. $P_S^L(I_A^*, \gamma)$, because $P_S^L(I_A^*, \gamma)$ increases in γ . Using this observation, it is easy to see that the partial derivative of z_2 w.r.t. γ is negative throughout Areas II, III and IV. For Area I, that the component of z_2 containing $P_S^L(I_A^*, \gamma)$ is: $-P_S^L(I_A^*, \gamma)\pi^\circ(c_1^L) + \frac{\{P_S^L(I_A^*, \gamma)\}^2}{P_S^L(I_A^*, \gamma)^{p+1}}\pi^\circ(c_2)$. Using $\theta_i = \theta_L$ and $\hat{\pi} = \pi^\circ(c_1^L)$ in Lemma 4.7.8, we see that this component decreases in $P_S^L(I_A^*, \gamma)$. Hence this component also decreases in γ . Therefore, we can infer that z_2 decreases in γ throughout all four Areas. ■

Proof of Theorem 4.4.2.

Proof of (i). Using Lemma 4.7.8, we see that for each area, the denominator of z_2 increases and the numerator decreases in α_{min} . Therefore, z_2 decreases in α_{min} .

Proof of (ii). In the $I_A^* - I_D$ plane, the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ lies in any one of the four Areas, whose boundaries are defined by the lines $I_A^* = I_A'$, $I_A^* = \underline{I_A}$ and $I_A^* = \overline{I_A}$. When $\xi = 0$, only the Area IV exists. As we increase ξ , the values of I_A' , $\underline{I_A}$ and $\overline{I_A}$ decrease (note that the order does not change). Hence due to increasing ξ , the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ gradually progresses from Area IV to Area III to Area II to Area I. Denote the threshold values of ξ at which the straight line $I_A^* = \alpha^{-1}(\alpha_{min})$ transitions from: (a) Area IV to Area III as ξ_0 , (b) Area III to Area II as ξ_1 , and (c) Area II to Area I as ξ_2 . Then $\xi_0 < \xi_1 < \xi_2$. The values of ξ_0 , ξ_1 , and ξ_2 can

be derived to be the following:

$$\begin{aligned}\xi_0 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}(p+1)}, \\ \xi_1 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}[\theta_H\{\alpha_{\min}(1-\gamma) + \gamma\}p+1]}, \\ \xi_2 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}[\theta_L\{\alpha_{\min}(1-\gamma) + \gamma\}p+1]}.\end{aligned}$$

Denote $z_1^{(j)}(\xi)$ as the value of z_1 in Area j ($j \in \{I, II, III, IV\}$), for a given ξ . Also denote $z_2^{(j)}$ as the value of z_2 in Area j ($j \in \{I, II, III, IV\}$), for a given ξ . We observe the following:

$$z_1^{(IV)}(\xi) = z_1^{(III)}(\xi) \quad \forall \xi \in [0, \xi_0] \text{ and } z_2^{(IV)}(\xi_0) = z_2^{(III)}(\xi_0).$$

$$z_1^{(III)}(\xi_1) = z_1^{(II)}(\xi_1) \text{ and } z_2^{(III)}(\xi) = z_2^{(II)}(\xi) \quad \forall \xi \in [\xi_0, \xi_1].$$

$$z_1^{(II)}(\xi_2) = z_1^{(I)}(\xi_2) \text{ and } z_2^{(II)}(\xi_2) = z_2^{(I)}(\xi_2).$$

Since $z_1^{(I)}(\xi)$ and $z_2^{(I)}(\xi)$ are independent of ξ , therefore, $z_1^{(I)}(\xi) = z_1^{(I)}(\xi_2)$ and $z_2^{(I)}(\xi) = z_2^{(I)}(\xi_2) \quad \forall \xi \in [\xi_2, \infty)$.

It is easy to see the following: $z_1^{(IV)}(\xi)$ decreases and $z_2^{(IV)}(\xi)$ increases in $\xi \in [0, \xi_0]$; both $z_1^{(III)}(\xi)$ and $z_2^{(III)}(\xi)$ decrease in $\xi \in [\xi_0, \xi_1]$; $z_1^{(II)}(\xi)$ increases and $z_2^{(II)}(\xi)$ decreases in $\xi \in [\xi_1, \xi_2]$. We have already seen that $z_1^{(I)}(\xi) = z_1^{(I)}(\xi_2)$ and $z_2^{(I)}(\xi) = z_2^{(I)}(\xi_2) \quad \forall \xi \in [\xi_2, \infty)$.

Now suppose that $z_1^{(j)}(\xi) < z < z_2^{(j)}(\xi)$, $j \in \{II, III, IV\}$. Then $w_2^* = c_2$ under $\widehat{\mathcal{D}}$. This corresponds to the case $y(q(l_{\widehat{\mathcal{D}}})) > \mathcal{C}(I_A^*, I_D)$. From Lemma 4.7.4 we know: $y(q(c_2)) > P_S(I_A^*, I_D)\xi\alpha(I_A^*)$, therefore $t_2^* = y(q(c_2)) \wedge P_S(I_A^*, I_D)\xi\alpha(I_A^*) = P_S(I_A^*, I_D)\xi\alpha_{\min}$.

Finally, suppose that $z_1^{(I)}(\xi) < z < z_2^{(I)}(\xi)$. Then $w_2^* = l_{\widehat{\mathcal{D}}}$. This corresponds to the case $y(q(l_{\widehat{\mathcal{D}}})) \leq \mathcal{C}(I_A^*, I_D)$. In this case, S_2 reserves capacity

for B . ■

Table 4.4: Expressions for z_1

Area	Numerator of z_1	Denominator of z_1
I	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L)$ $+ \left[\frac{\left\{P_S^H(I_A^*, \gamma)\right\}^2}{P_S^H(I_A^*, \gamma)^{p+1}} - \frac{\left\{P_S^L(I_A^*, \gamma)\right\}^2}{P_S^L(I_A^*, \gamma)^{p+1}} \right] \pi^\circ(c_2) + h$	$(\theta_H - \theta_L) \alpha_{min}$
II	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L)$ $+ \frac{\left\{P_S^H(I_A^*, \gamma)\right\}^2}{P_S^H(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h$	
III	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L)$ $+ \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h$	
IV	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L)$ $+ \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h$	

Table 4.5: Expressions for z_2

Area	Numerator of z_2	Denominator of z_2
I	$\left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + \frac{\left\{P_S^L(I_A^*, \gamma)\right\}^2}{P_S^L(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - \frac{p}{p+1} \pi^\circ(c_2) - h$	$\theta_L \alpha_{min}$
II	$\left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + P_S^L(I_A^*, \gamma) \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - \frac{p}{p+1} \pi^\circ(c_2) - h$	
III	$\left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + P_S^L(I_A^*, \gamma) \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - \frac{p}{p+1} \pi^\circ(c_2) - h$	
IV	$\left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - h$	

Table 4.6: Expressions for z_3

Area	Numerator of z_3	Denominator of z_3
I	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) + \frac{\left\{P_S^H(I_A^*, \gamma)\right\}^2}{P_S^H(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - \frac{p}{p+1} \pi^\circ(c_2)$	$\theta_H \alpha_{min}$
II	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) + \frac{\left\{P_S^H(I_A^*, \gamma)\right\}^2}{P_S^H(I_A^*, \gamma)^{p+1}} \pi^\circ(c_2) - \frac{p}{p+1} \pi^\circ(c_2)$	
III	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) + P_S^H(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} - \frac{p}{p+1} \pi^\circ(c_2)$	
IV	$\left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^H(I_A^*, \gamma)\right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\}$	

Chapter 5

Concluding Observations and Remarks

5.1 Summary of Main Findings

This dissertation contributes to the growing literature on supply chain responsibility by analyzing the impact of external pressure on the buyer's investment in supplier improvement. Although buyer investment in supplier improvement is often crucial towards improving supply chain responsibility, the extant literature on supply chain responsibility does not shed light on how external entities can influence buyers to make such investments. This is the first treatise which incorporates buyer's investment in supplier improvement under the threat of financial and reputation damage due to pressure from external entities as the key to improving supply chain responsibility. We summarize the answers to our research questions consecutively in the following discussion.

Should external entities always highly penalize buyers for violations in their supply chains? Analytical evidence in our exposition counters the prevalent wisdom that supply chain responsibility improves if the buyers face severe castigation and censure from external entities. We find that if the cost of violation (which is a direct manifestation of external pressure) is either low or high, then the buyer does not invest in supplier improvement, and thereby does not help in improving supply chain responsibility. Rather, we find that supply chain responsibility improves only when the cost of violation is moderate, because only under this condition the buyer resorts to positive investment in supplier improvement. This result is explained by the buyer's trade-off between two risks which arise due to the potential of violations being committed by the risky supplier: the risk of incurring cost of violations, and the risk of incurring cost of switching to the apparently responsible supplier. High cost of violation exacerbates the former risk so much that the buyer contracts with only the apparently responsible supplier, thereby hedging against the risk of switching. Consequently, she does not invest in the supplier's improvement. On the other hand, low cost of violation makes the former risk low enough that the buyer contracts with the risky supplier too, but does not invest in her improvement. Only when the cost of violation is moderate, the former risk is moderate enough that the buyer attempts to decrease the risk by contracting with the risky supplier and investing in her improvement. Consider the Rana Plaza disaster in Bangladesh. In the aftermath of the disaster, *The Economist* (2013) stated that the buyers sourcing from Bangladesh would espouse one of the three strategies: continue to source from Bangladesh and proactively invest in supplier improvement,

continue to source from Bangladesh without making proactive investments in supplier improvement, or leave Bangladesh altogether and settle for less risky locations. Our exposition provides analytical evidence and explanation of this prediction in the context of supply chains in general.

How are the buyer's decisions impacted by the intrinsic supplier characteristics? We find that two factors derived from the supplier's intrinsic characteristics affect the buyer's contracting and investment decisions: difference in riskiness (i.e. difference between the probabilities of committing responsibility violations of the two types of the risky supplier), and difference in costliness (i.e. difference between the unit production costs of the risky and apparently responsible suppliers).

Effect of the difference in riskiness. We find that in general, the chances of buyer investment in supplier improvement are higher if the risky suppliers are widely differentiated with respect to their riskiness, than if they are not widely differentiated. Hence in order to motivate the buyer to invest in supplier improvement under the former scenario, external entities can penalize the buyer within a wider interval of the cost of violation. But under the latter scenario, they should restrict the buyer's cost of violation to a narrower interval.

Effect of the difference in costliness. We find that in general, the chances of buyer investment in supplier improvement are higher if the risky and the apparently responsible suppliers differ widely with respect to their costliness, than if they are not widely differentiated. Consider the Bangladeshi RMG industry for illustration of a subtle implication of this result. In order

to improve the responsibility of garment supply chains in Bangladesh, the Bangladesh government raised the minimum wage for garment workers by 77% in 2013 (Devnath, 2013). However, this only led to further squeezing of the Bangladeshi suppliers' margins by the buyers (Quadir, 2014). Our result provides some explanation for why the increase in minimum wages, although well intended, might not be effective in motivating buyers to invest in improving the responsibility of Bangladeshi garment supply chains. An increase in minimum wages is equivalent to an equal increase in costliness of all suppliers, and therefore does not increase the difference in costliness between the risky and apparently responsible suppliers. Hence the buyer has no incentive to invest in supplier improvement.

Should suppliers engaging in unauthorized subcontracting be penalized? We find that in order to motivate the buyer to invest in supplier improvement, external entities should directly penalize the apparently responsible supplier if she engages in unauthorized subcontracting. Further, we find that if the penalty exceeds a threshold, then the risk of unauthorized subcontracting can be avoided. It is interesting to note that while our results favor reduced focus on punishing the buyer, they favor increased focus on punishing the supplier. This is because such a penalty for suppliers indirectly pressures the buyer. The buyer either covers the supplier's expected penalty, or pays her a high unit wholesale price so that the supplier reserves capacity. Our result provides yet another explanation for why the responsibility of the Bangladeshi RMG industry has not improved much even three years after the Rana Plaza disaster. Until 2014, the Bangladeshi government

had never punished the owner of any garment factory where responsibility violations have been committed (Allchin, 2014). Rather, the RMG industry is plagued by a chronic lack of legal enforcement against violators of labor laws (Motlagh, 2016). Our results suggest that the Bangladeshi government should institute regulations and mechanisms for enforcing penalties for unauthorized subcontracting. Such regulations exist for the Cambodian garment and textile industry (Kashyap, 2015). In order to motivate buyers to invest in supplier improvement, the Bangladeshi media and NGOs should work together with the government and buyers to identify and penalize suppliers who engage in unauthorized subcontracting.

Using the above results, we suggest that in order to maximize the improvement of supply chain responsibility, external entities should first set a minimum requirement for system audit efficiency, and then set the supplier's penalty for unauthorized subcontracting to at least the threshold value as explained earlier. Following that, they should determine how much to pressure the buyer with costs of violation. Estimating the buyer's costs of violation is the biggest challenge for the external entities and buyers alike. To that end, extensive long-term collaboration is required between buyers and external entities in the future.

5.2 Future Research

We conclude our exposition by stating some interesting topics for future research.

Research Topic 1: Contract Renegotiation and Long-Run Invest-

ments. In a multi-period setting, the buyer can make multiple investments in improving the responsibility of her supplier pool (or specific suppliers). Also, investments in supplier improvement made at an earlier stage gives the buyer leverage to procure from her suppliers at cheaper rates in later stages. As such, the terms of contract between the buyer and her suppliers might be renegotiated. In this paper, a multi-period problem with contract renegotiation could be studied in order to derive the buyer's long-run investment and contracting strategies.

Research Topic 2: Buyer Competition and Investments. Instead of one buyer, a competition between multiple buyers could be studied in order to address the piggybacking problem which arises when buyers source from the same set of suppliers.

Research Topic 3: Unauthorized Subcontracting. Unauthorized subcontracting in the global garment industry is a menacing issue for global buyers. The garment industry thrives on an extensive subcontracting system, which is often not authorized by the brands and retailers (buyers). Almost always, the subcontractors are also hotbeds of egregious supply chain responsibility violations, thereby exposing the buyers to risks of reputation and financial damage. This is further exacerbated by the lack of visibility that buyers have into their extended supply chains. In this paper, the relation between various types of buyer-supplier contracts and the phenomenon of unauthorized subcontracting can be analyzed. The primary research problem that is to be addressed in this paper is the ways in which different types of contracts affect the incidence of unauthorized subcontracting under various circumstances. The eventual aim is to formulate a mechanism which can

help buyers and external supply chain stakeholders alleviate the menace of unauthorized subcontracting.

Two assumptions were made in §3.2: (i) the apparently responsible supplier does not have the option to fulfill the buyer's orders partially, and (ii) the penalty for unauthorized subcontracting is independent of the subcontracting volume. For this research, the two assumptions might be relaxed in order to facilitate a more complex analysis of the issue of unauthorized subcontracting. Relaxing the first assumption will allow more complicated buyer-supplier contracts to be explored, wherein the apparently responsible supplier might have the option of staggering the order fulfillment. This will give rise to the possibility that even under capacity constraints, she might not engage in unauthorized subcontracting. Relaxing the second assumption might also affect on the supplier's incentive to engage in unauthorized subcontracting. Although it is practically quite difficult to detect the exact subcontracting volume, it might be possible to estimate the volume through increased oversight of the movement of raw materials and partially finished goods. Then, an appropriately designed quantity-dependent subcontracting penalty levied on the supplier might highlight new conditions under which the supplier would be deterred from engaging in unauthorized subcontracting. Therefore, relaxing both the assumptions will potentially generate interesting insights as to how unauthorized subcontracting can possibly be controlled using more complicated contracts and penalties as those discussed in this dissertation.

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Appendix I

This appendix provides a list of all the notations that occur in the paper, within the main exposition as well as within the proofs. The following abbreviations have been used in the list:

pr.: abbreviation for “proof of”.

Lem.: abbreviation for “Lemma”.

Thm.: abbreviation for “Theorem”.

Cor.: abbreviation for “Corollary”.

Tables A.1 and A.2 provide lists of all notations beginning with English and Greek letters, respectively, sorted according to alphabetical order. If multiple notations begin with the same letter, then they are sorted from lowercase to uppercase. If there are multiple notations beginning with the same letter and same case, then they are listed in their order of appearance in the dissertation.

Table A.1: List of All Notations Beginning With English Letters

Symbol	Defined In	Meaning
a	pr. Thm. 4.3.4	the term in $\pi^b(I_A^*, h \widehat{\mathcal{D}})$ which is devoid of z and h
b	pr. Thm. 4.3.4	the term in $\pi^b(I_A^*, 0 \widehat{\mathcal{D}})$ which is devoid of z and h
B	§3.2.1	buyer
c	pr. Thm. 4.3.4	the term in $\pi^b(I_A^*, 0 \widehat{\mathcal{R}})$ which is devoid of z and h
c_1^i	§3.2.1	supplier S_1^i 's unit production cost; $i \in \{H, L\}$
c_2	§3.2.1	supplier S_2 's unit production cost
$c_2', \underline{c_2}, \overline{c_2}$	Thm. 4.4.1	thresholds of c_2
$\mathcal{C}$	§3.4.2	net rise in expected supply chain cost due to increasing the order quantity from $q(l)$ to $q(c_2)$ under $\widehat{\mathcal{D}}$ contracting strategy
$\mathcal{C}'$	Cor. 4.2.1	net rise in expected supply chain cost due to increasing the order quantity from $q(l)$ to $q(c_2)$ under $\widehat{\mathcal{D}}$ contracting strategy

Symbol	Defined In	Meaning
$\bar{d}$	§3.2.2	upper limit of support for demand
D	§3.2.2	random demand
$\hat{\mathcal{D}}$	§4.2.1	dual contracting strategy
$f_D(\cdot)$	§3.2.2	pdf of demand D
$F_D(\cdot)$	§3.2.2	cdf of demand D
$\overline{F}_D(\cdot)$	§3.2.2	ccdf of demand D
$g(\theta_i), \mathcal{G}_1, \mathcal{G}_2$	pr.: Lem. 4.7.7	functions defined for intermediate computations
h	§4.2.3	inverse of the coefficient of investment efficiency
H	§3.2.1	High-type (i.e. more risky) supplier S_1
i	§3.2.1	true types of the supplier S_1 ; $i \in \{H, L\}$
$\hat{i}$	§3.4.1	type reported by supplier S_1 in Stage 1; $\hat{i} \in \{H, L\}$
ISB	§3.2.2	external perception: Irresponsible Sourcing by Buyer
I_A	§4.2.1	buyer's investment in supplier responsibility audit

Symbol	Defined In	Meaning
I_D	§4.2.1	buyer's investment in supplier S_1 's improvement
$\widehat{\mathcal{I}}$	§4.2.1	irresponsible contracting strategy
j	pr. Thm. 4.4.2 (ii)	the four Areas of the $I_A^* - I_D$ plane; $j \in \{I, II, III, IV\}$
k	§3.2.2	capacity reserved by S_2 for B in Stage 3
l	§3.4.2	value of w_2 which maximizes the supply chain profit if S_2 reserves capacity for B
$l_{\widehat{\mathcal{D}}}$	§4.2.2	l offered by B to S_2 under $\widehat{\mathcal{D}}$ strategy
$l_{\widehat{\mathcal{R}}}$	§4.2.2	l offered by B to S_2 under $\widehat{\mathcal{R}}$ strategy
l'	§3.4.2	value of w_2 which maximizes the supply chain profit if S_2 does not reserve capacity for B
L	§3.2.1	Low-type (i.e. less risky) supplier S_1
m	pr. Lem. 4.7.8	function defined for use in Theorems 4.4.1 and 4.4.2
$\mathcal{M}$	pr. Lem. 4.7.7	function defined for intermediate computation
n	pr. Cor. 4.2.1	some positive number

Symbol	Defined In	Meaning
ND	§3.2.2	external perception: No Disruption
p	§3.2.1	unit retail price of product sold by B to consumer
$P_S^i(I_A, \gamma)$	§3.2.2	probability of B switching from S_1^i to S_2 ; $i \in \{H, L\}$
$P_S(I_A, I_D)$	§3.2.2	expected probability of B switching from S_1 to S_2
$\mathcal{P}_\tau^i$	§3.2.2	probability of external perception τ ($\tau \in \{ISB, RD, ND\}$) when B is sourcing from S_1^i ($i \in \{H, L\}$)
$\mathcal{P}_V(I_D)$	pr. Lem. 4.7.6	expected probability of violation being committed by S_1
$q(w)$	§3.3.1	optimal order quantity that maximizes B 's Stage 4 profit
$\mathcal{Q}$	Lem. 4.7.8	function defined for use in Theorems 4.4.1 and 4.4.2
$r_1(\theta_i, m)$	Lem. 4.7.8	function defined for use in Theorems 4.4.1 and 4.4.2
RD	§3.2.2	external perception: Random Disruption
$\hat{\mathcal{R}}$	§4.2.1	apparently responsible contract
S_1^i	§3.2.1	risky supplier of type i ; $i \in \{H, L\}$
S_2	§3.2.1	apparently responsible supplier
t	pr. Lem. 4.7.6	coefficient of $\alpha(I_A^*)^2$ in the quadratic equation

Symbol	Defined In	Meaning
t_1^i	§3.2.2	transfer payments offered by B to S_1 in Stage 2; $i \in \{H, L\}$
$\hat{t}_1^i$	§3.4.1	transfer payment for reported type $\hat{i}$ chosen by S_1 in Stage 2; $i \in \{H, L\}$
t_2	§3.2.2	transfer payment offered by B to S_2 in Stage 2
$\mathcal{T}_1(\alpha, \rho)$	pr. Thm. 4.3.1	component of B 's expected profit containing terms relevant to only S_2 when $y(q(l_{\widehat{\mathcal{D}}})) \leq \mathcal{C}(\alpha, \rho)$
$\mathcal{T}_2(\alpha, \rho)$	pr. Thm. 4.3.1	component of B 's expected profit containing terms relevant to only S_2 when $y(q(l_{\widehat{\mathcal{D}}})) > \mathcal{C}(\alpha, \rho)$
u	pr. Lem. 4.7.6	coefficient of $\alpha(I_A^*)$ in the quadratic equation
$\mathcal{U}(\alpha, \rho)$	pr. Thm. 4.3.1	component of B 's expected profit containing terms relevant to only S_1
v	pr. Lem. 4.7.6	constant term in the quadratic equation
w_1^i	§3.2.2	unit wholesale prices offered by B to S_1 in Stage 2; $i \in \{H, L\}$
w_2	§3.2.2	unit wholesale price offered by B to S_2 in Stage 2

Symbol	Defined In	Meaning
$\underline{x}$	pr. Thm. 4.4.1 (i)	function defined for intermediate computation
$\bar{x}$	pr. Thm. 4.4.1 (ii)	function defined for intermediate computation
$y(k)$	§3.2.2	reservation cost incurred by S_2 for reserving k units of capacity for B in Stage 2
z	§3.2.2	cost of violation incurred by B due to violations committed by S_1
z_1, z_3, z_2	Thm. 4.3.4	thresholds of z
$z_1^{\xi_2}, z_2^{\xi_2}$	§4.4.2	values of z_1 and z_2 at $\xi = \xi_2$
$z_1^{(j)}(\xi)$	pr. Thm. 4.4.2 (ii)	value of z_1 in Area j ; $j \in \{I, II, III, IV\}$
$z_2^{(j)}(\xi)$	pr. Thm. 4.4.2 (ii)	value of z_2 in Area j ; $j \in \{I, II, III, IV\}$

Table A.2: List of Notations Beginning With Greek Letters

Symbol	Defined In	Meaning
α	§3.2.1	audit efficiency of the system
α_{min}	§4.2.1	minimum audit efficiency required to be attained through buyer investment in audit
β	§4.3	benefit of maximum investment under $\widehat{\mathcal{D}}$ relative to zero investment under $\widehat{\mathcal{D}}$
γ	§3.2.2	probability of a disruption at S'_1 's facility, given that she has committed violation
$\hat{\gamma}$	Thm. 4.3.3	threshold of γ in the scenario $\alpha_{min} = 0$
δ	§3.4.2	difference between the order quantities based on the two optimal unit wholesale prices c_2 and l offered by B to S_2

Symbol	Defined In	Meaning
η	§4.2.2	buyer's contracting strategies $\widehat{\mathcal{D}}$, $\widehat{\mathcal{I}}$, and $\widehat{\mathcal{R}}$
θ_i	§3.2.1	probability that S_1^i ($i \in \{H, L\}$) will commit violation
ξ	§3.2.2	penalty incurred by S_2 if she engages in unauthorized subcontracting and it is detected by auditors
ξ_0, ξ_1, ξ_2	Thm. 4.4.2	thresholds of ξ
$\pi^\circ(w_1^i), \pi^\circ(w_2)$	§3.3.1	newsvendor profit earned by B by procuring goods at a unit wholesale price w_1^i and w_2 , respectively
$\pi_{3,s_1}^b(w_1^i)$	§3.3.1	B 's expected profit in Stage 3 by procuring from S_1^i
$\pi_{3,s_2}^b(w_2)$	§3.3.1	B 's expected profit in Stage 3 by procuring from S_2
$\pi_3^{s_1}(w_1^i i)$	§3.3.1	S_1 's expected profit from Stage 3
$\pi_3^{s_2}(w_2, k)$	§3.3.1	S_2 's expected profit from Stage 3
$\pi_2^{s_2}(w_2)$	§3.3.2	S_2 's expected profit from Stage 2
$\pi_1^{s_1}(t_1^{\hat{i}}, w_1^{\hat{i}} i)$	§3.4.1	S_1 's expected profit from Stage 1, given true type i and reported type $\hat{i}$

Symbol	Defined In	Meaning
π_{1,s_1}^b	§3.4.1	B' 's expected profit in Stage 1 from contracting with S_1
$\pi_1^{s_2}(t_2, w_2)$	§3.4.2	S_2 's expected profit from Stage 1
$\pi_{1,s_2}^b(w_2^*)$	§3.4.2	B' 's expected profit in Stage 1 from contracting with S_2
$\pi^{b,\eta}$	§4.2.2	overall expected profit for B under contracting strategy η ($\eta \in \{\widehat{\mathcal{D}}, \widehat{\mathcal{I}}, \widehat{\mathcal{R}}\}$)
$\pi^b(I_A, I_D \eta)$	§4.2.2	expanded form of $\pi^{b,\eta}$
$\tilde{\pi}(c_1^H)$	Lem. 4.7.5	given that B does not have to switch from S_1^H to S_2 , $\tilde{\pi}(c_1^H)$ is B' 's expected profit from contracting with S_1^H in Stage 1
$\hat{\pi}^1(\alpha, \rho)$	pr. Thm. 4.3.1	equal to $\pi^b(\alpha, \rho \widehat{\mathcal{D}})$ when $y(q(l_{\widehat{\mathcal{D}}})) \leq \mathcal{C}(\alpha, \rho)$
$\hat{\pi}^2(\alpha, \rho)$	pr. Thm. 4.3.1	equal to $\pi^b(\alpha, \rho \widehat{\mathcal{D}})$ when $y(q(l_{\widehat{\mathcal{D}}})) > \mathcal{C}(\alpha, \rho)$
$\underline{\pi}$	pr. Thm. 4.3.3	$\pi^\circ(c_2)$ minus B' 's investment in supplier improvement
$\hat{\pi}$	Lem. 4.7.8	a variable whose value is greater than $\pi^\circ(c_2)$
ρ	§3.2.1	probability that S_1 is of type H (i.e. <i>more risky</i> type)

Symbol	Defined In	Meaning
τ	§3.2.2	external perceptions; $\tau \in \{ISB, RD, ND\}$
$\Phi 1$	Thm. 4.3.4	B 's optimal policy for investment in supplier improvement when $z_1 < z_2$
$\Phi 2$	Thm. 4.3.4	B 's optimal policy for investment in supplier improvement when $z_1 \geq z_2$
ψ	§3.4.2	excess expected demand that is satisfied when B orders $q(c_2)$ units instead of ordering $q(l)$ units

Appendix II

Following is a diagrammatic representation of S'_2 's revenues and costs in the Capacity Reservation stage, as plotted against the capacity k :

Figure A.1: Pictorial representation of Theorem 3.3.1

Refer to the proof of Theorem 3.3.1. ■

In §3.2, we introduced the notations α and γ . Depending on whether or S_1 actually commits some responsibility violations, the audit process can have different outcomes. Also, a disruption at S'_1 's facility might or might not occur. The following two tables summarize the probabilities audit outcomes, and the probabilities of disruption occurring at S'_1 's facility, respectively.

Table A.3: Probabilities of Outcomes of Audit

Outcomes of Audit	S_1 Commits Violation	S_1 Commits No Violation
Violation detected	α	0
Violation not detected	$1 - \alpha$	1

Table A.4: Probabilities of Disruption

Disruption Possibilities	S_1 Commits Violation	S_1 Commits No Violation
Disruption likely	γ	0
Disruption unlikely	$1 - \gamma$	1

Refer to §3.2.

In the table below, we summarize the probabilities of the various external perceptions given by $\tau \in \{ISB, RD, ND\}$.

Table A.5: Probabilities of External Perceptions τ : $\mathcal{P}_{\tau}^i$, $i \in \{H, L\}$

τ	$\mathcal{P}_{\tau}^i$
<i>ISB</i>	$\theta_i \alpha (1 - \gamma) + \theta_i \alpha \gamma = \theta_i \alpha$
<i>RD</i>	$\theta_i (1 - \alpha) \gamma$
<i>ND</i>	$(1 - \theta_i) + \theta_i (1 - \alpha) (1 - \gamma) = 1 - \theta_i \alpha (1 - \gamma)$

Refer to §3.2.

In the Contract stage, we solve for the optimal contract terms presented by B to S_2 . As seen in Theorem 3.4.3, B 's expected profit is given by the curve $P_S\pi^\circ(w_2) - t_2$:

$$P_S\pi^\circ(w_2) - t_2 = \max \left[P_S \left\{ -c_2q(w_2) + pE[q(w_2) \wedge D] \right\} - y(q(w_2)), \right. \\ \left. P_S \left\{ -c_2q(w_2) + pE[q(w_2) \wedge D] - \xi\alpha \right\} \right],$$

which is essentially given by the maximum of two concave curves in $q(w_2)$ which meet at the point where $y(q(w_2)) = P_S\xi\alpha$. Recall from Lemma 3.4.2 that $y(q(l)) < y(q(c_2))$.

Using a set of figures, various possible orientations of the curve $P_S\pi^\circ(w_2) - t_2$ are demonstrated below. For the sake of brevity, we define some additional notations:

$$Y_1(q(w_2)) \triangleq P_S \left\{ -c_2q(w_2) + pE[q(w_2) \wedge D] \right\} - y(q(w_2)), \\ Y_2(q(w_2)) \triangleq P_S \left\{ -c_2q(w_2) + pE[q(w_2) \wedge D] - \xi\alpha \right\}.$$

Also, let $\bar{w}_2$ be the value of w_2 such that $y(q(\bar{w}_2)) = P_S\xi\alpha$.

In each of the diagrams, the curve $P_S\pi^\circ(w_2) - t_2$ is the set of points given by whichever of $Y_1(q(w_2))$ and $Y_2(q(w_2))$ is greater for all values of $q(w_2)$.

Refer to the proof of Theorem 3.4.3. ■

Figure A.2: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 1

Figure A.3: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 2

Figure A.4: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 3

Figure A.5: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 4

Figure A.6: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 5

Figure A.7: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 6

Figure A.8: Orientation of $P_S \pi^\circ(w_2) - t_2$: Case 7

Table A.6 below presents the unabridged version of the Table 4.1.

Table A.6: Threshold Values z_1 And z_2 : Unabridged Version

ξ	z_1	z_2
0	0.0144	0.0444
0.002	0.014	0.0452
0.004	0.0136	0.046
0.006	0.0132	0.0468
0.008	0.0129	0.0477
0.01	0.0125	0.0485
0.012	0.0121	0.0493
0.014	0.0117	0.0501
0.016	0.0113	0.0509
0.018	0.0109	0.0517
0.02	0.0106	0.0526
0.022	0.0102	0.0534
0.024	0.0098	0.0542
0.026	0.0094	0.055
0.028	0.009	0.0558
0.03	0.0086	0.0566
0.032	0.0083	0.0575
0.034	0.0079	0.0583
0.036	0.0075	0.0591

ξ	z_1	z_2
0.038	0.0071	0.0599
0.04	0.0067	0.0607
0.042	0.0063	0.0615
0.044	0.006	0.0624
0.046	0.0056	0.0632
0.048	0.0052	0.064
0.05	0.0048	0.0648
0.052	0.0044	0.0656
0.054	0.004	0.0664
0.055556	0.0037	0.0671
0.056	0.0036	0.067
0.057	0.0035	0.0668
0.058	0.0033	0.0666
0.059	0.0031	0.0664
0.06	0.0029	0.0662
0.061	0.0027	0.066
0.062	0.0025	0.0658
0.063	0.0023	0.0656
0.064	0.0021	0.0654
0.065	0.0019	0.0653
0.066	0.0017	0.0651
0.067	0.0015	0.0649
0.068	0.0013	0.0647
0.069	0.0012	0.0645

ξ	z_1	z_2
0.07	0.001	0.0643
0.070862	0.0008	0.0641
0.071	0.0008	0.0641
0.072	0.001	0.0639
0.073	0.0012	0.0637
0.074	0.0014	0.0635
0.075	0.0016	0.0633
0.076	0.0018	0.0631
0.077	0.002	0.0629
0.078	0.0022	0.0628
0.079	0.0024	0.0626
0.08	0.0025	0.0624
0.081	0.0027	0.0622
0.082	0.0029	0.062
0.083	0.0031	0.0618
0.084	0.0033	0.0616
0.085	0.0035	0.0614
0.086	0.0037	0.0612
0.087	0.0039	0.061
0.088	0.0041	0.0608
0.089	0.0043	0.0606
0.09	0.0045	0.0605
0.091	0.0047	0.0603
0.092	0.0049	0.0601
0.093	0.005	0.0599
0.093844	0.0052	0.0597
0.094	0.0052	0.0597

ξ	z_1	z_2
0.095	0.0052	0.0597
0.096	0.0052	0.0597
0.097	0.0052	0.0597
0.098	0.0052	0.0597
0.099	0.0052	0.0597
0.1	0.0052	0.0597
0.101	0.0052	0.0597
0.102	0.0052	0.0597
0.103	0.0052	0.0597
0.104	0.0052	0.0597
0.105	0.0052	0.0597
0.106	0.0052	0.0597
0.107	0.0052	0.0597
0.108	0.0052	0.0597
0.109	0.0052	0.0597
0.11	0.0052	0.0597
0.111	0.0052	0.0597
0.112	0.0052	0.0597
0.113	0.0052	0.0597
0.114	0.0052	0.0597
0.115	0.0052	0.0597
0.116	0.0052	0.0597
0.117	0.0052	0.0597
0.118	0.0052	0.0597
0.119	0.0052	0.0597
0.12	0.0052	0.0597

The Regions exhibited in Figure 4.1 are summarized in Table A.7 below.

Table A.7: Regions R1 Through R12 In Figure 4.1

Region	B contracts with	Does B invest?	Does S_2 reserve capacity?
R1	S_2	No	No
R2	S_1 and S_2	Yes	No
R3	S_1 and S_2	No	No
R4	S_2	No	Yes
R5	S_1 and S_2	Yes	No
R6	S_1 and S_2	No	No
R7	S_2	No	Yes
R8	S_1 and S_2	Yes	No
R9	S_1 and S_2	No	Yes
R10	S_2	No	Yes
R11	S_1 and S_2	Yes	Yes
R12	S_1 and S_2	No	Yes

Appendix III

In this appendix, we will work out in details the derivations for the expressions given in Tables 4.4, 4.5, and 4.6.

Recall that the functions for B' 's expected profits under $\widehat{\mathcal{D}}$ and $\widehat{\mathcal{R}}$ strategies are:

$$\begin{aligned}\pi^b(I_A^*, I_D | \widehat{\mathcal{D}}) &= \rho(I_D) \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha(I_A^*) \theta_H z \right] \\ &\quad + (1 - \rho(I_D)) \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha(I_A^*) \theta_L z \right] \\ &\quad + P_S(I_A^*, I_D) \pi^\circ(w_2^*) - t_2^* - I_A^* - I_D\end{aligned}$$

and
$$\pi^b(I_A^*, I_D | \widehat{\mathcal{R}}) = \pi^\circ(w_2^*) - t_2^* - I_A^* - I_D.$$

This can be re-written as: The functions for B' 's expected profits under $\widehat{\mathcal{D}}$ and $\widehat{\mathcal{R}}$ strategies are:

$$\begin{aligned}\pi^b(I_A^*, I_D | \widehat{\mathcal{D}}) &= \rho(I_D) \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z \right] \\ &\quad + (1 - \rho(I_D)) \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z \right] \\ &\quad + \pi_{1,s_2}^b(w_2^*) - I_A^* - I_D\end{aligned}$$

and $\pi^b(I_A^*, I_D | \widehat{\mathcal{R}}) = \left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} - I_A^* - I_D$,
 where $\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1}$ is simply derived by substituting $P_S = 1$ in the value
 of $\pi_{1,s_2}^b(w_2^*)$.

Depending on which Area is being analyzed, the expressions for $\pi_{1,s_2}^b(w_2^*)$
 and $\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1}$ will be different. For example, in Area I, $y(q(l_{\widehat{\mathcal{D}}})) \leq$
 $\mathcal{C}(\alpha(I_A^*), \rho(I_D))$ and $y(q(l_{\widehat{\mathcal{R}}})) \leq \mathcal{C}'(\alpha(I_A^*), \rho(I_D))$. We know from Table 4.2
 that either $w_2^* = l_{\widehat{\mathcal{D}}}$ or $w_2^* = l_{\widehat{\mathcal{R}}}$.

Hence $\pi_{1,s_2}^b(w_2^*) = \mathcal{T}_1(\alpha(I_A^*), \rho(I_D))$ and $\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} = \left\{ \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1}$,
 where $\left\{ \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1}$ is derived by substituting $P_S = 1$ in the value
 of $\mathcal{T}_1(\alpha(I_A^*), \rho(I_D))$.

Recall from the proof of Theorem 4.3.4 that because α_{min} is given, we can
 identify the area within which the line $I_A^* = \alpha^{-1}(\alpha_{min})$ lies. Therefore, I_D^*
 is simply the value of I_D which generates the maximum expected profit for
 B along the line $I_A^* = \alpha^{-1}(\alpha_{min})$. For each individual area, this maximum
 expected profit is derived by finding the maximum of three expected profit
 functions. Recall that for each area, the three relevant expected profit func-
 tions are given by $\pi^b(I_A^*, 0 | \widehat{\mathcal{D}})$, $\pi^b(I_A^*, h | \widehat{\mathcal{D}})$, and $\pi^b(I_A^*, 0 | \widehat{\mathcal{D}})$. After stating the
 explicit expressions of the relevant functions for each area, we will compute
 the following, as mentioned in the proof of Theorem 4.3.4:

$$\begin{aligned} d_1 &= \pi^b(I_A^*, h | \widehat{\mathcal{D}}) - \pi^b(I_A^*, 0 | \widehat{\mathcal{D}}) \quad , \\ d_2 &= \pi^b(I_A^*, h | \widehat{\mathcal{D}}) - \pi^b(I_A^*, 0 | \widehat{\mathcal{R}}) \quad , \\ d_3 &= \pi^b(I_A^*, 0 | \widehat{\mathcal{D}}) - \pi^b(I_A^*, 0 | \widehat{\mathcal{R}}) \quad . \end{aligned}$$

Finally, substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we obtain the values of the three thresholds z_1 , z_2 , and z_3 for each area.

Note that since $\alpha(I_A) = \frac{1}{h}I_A$ and $\rho(I_D) = 1 - \frac{1}{h}I_D$, therefore $I_A^* = h\alpha(I_A^*) = h\alpha_{min}$ and $I_D = h(1 - \rho(I_D))$. Further, $\rho(0) = 1$ and $\rho(h) = 0$.

Hence, $P_S(I_A^*, 0) = P_S^H(I_A^*, \gamma)$ and $P_S(I_A^*, h) = P_S^L(I_A^*, \gamma)$.

Area I:

From Table 4.2, we know: $w_2^* = l_{\widehat{D}}$ or $w_2^* = l_{\widehat{R}}$.

Hence:

$$\begin{aligned}
\pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) = \frac{(p - c_2)^2}{2} \frac{\{P_S(I_A^*, I_D)\}^2}{P_S(I_A^*, I_D)p + 1} \\
&= \frac{(p - c_2)^2}{2p} \frac{\{P_S(I_A^*, I_D)\}^2 p}{P_S(I_A^*, I_D)p + 1} \\
&= \frac{\{P_S(I_A^*, I_D)\}^2 p}{P_S(I_A^*, I_D)p + 1} \pi^\circ(c_2), \quad \text{and} \\
\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} &= \left\{ \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1} = \frac{(p - c_2)^2}{2} \frac{1}{p + 1} \\
&= \frac{(p - c_2)^2}{2p} \frac{p}{p + 1} \\
&= \frac{p}{p + 1} \pi^\circ(c_2) \quad .
\end{aligned}$$

Therefore the expressions of the three expected profit functions for Area

I are:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= 1 \cdot \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z \right] \\
&\quad + 0 \cdot \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z \right] \\
&\quad + \pi_{1,s_2}^b(w_2^*) - h\alpha_{min} - h \cdot 0 \\
&= \tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z + \frac{\{P_S(I_A^*, 0)\}^2 p}{P_S(I_A^*, 0)p + 1} \pi^\circ(c_2) - h\alpha_{min} \\
&= \tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - h\alpha_{min} . \\
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= 0 \cdot \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z \right] \\
&\quad + 1 \cdot \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z \right] \\
&\quad + \pi_{1,s_2}^b(w_2^*) - h\alpha_{min} - h \cdot 1 \\
&= \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z + \frac{\{P_S(I_A^*, h)\}^2 p}{P_S(I_A^*, h)p + 1} \pi^\circ(c_2) \\
&\quad - h\alpha_{min} - h \\
&= \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
&\quad - h\alpha_{min} - h . \\
\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) &= \left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} - h\alpha_{min} \\
&= \frac{p}{p+1} \pi^\circ(c_2) - h\alpha_{min}
\end{aligned}$$

Therefore, *within Area I*, when $\mathbf{w}_2^* = \mathbf{l}_{\widehat{\mathcal{D}}}$, then:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \\
&\quad h\alpha_{min}, \text{ and} \\
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \\
&\quad h\alpha_{min} - h.
\end{aligned}$$

Within Area I, when $\mathbf{w}_2^* = \mathbf{l}_{\widehat{\mathcal{R}}}$, then:

$$\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) = \frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min}.$$

The above expressions can be verified from rows 1, 2 and 5 in Table 4.3.

Therefore:

$$\begin{aligned} \mathbf{d}_1 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) \quad , \\ &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} - h \\ &\quad - \left[\tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} \right] \\ &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} \\ &\quad + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h + (\theta_H - \theta_L)\alpha_{min}z \quad . \\ \mathbf{d}_2 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \quad , \\ &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} - h \\ &\quad - \left[\frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min} \right] \\ &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - \frac{p}{p+1}\pi^\circ(c_2) \\ &\quad - h - \alpha_{min}\theta_L z \quad . \\ \mathbf{d}_3 &= \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \\ &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} \\ &\quad - \left[\frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min} \right] \\ &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - \frac{p}{p+1}\pi^\circ(c_2) - \alpha_{min}\theta_H z \end{aligned}$$

Substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we obtain the values of the three thresholds z_1 , z_2 , and z_3 for Area I as:

$$\begin{aligned}
\mathbf{z}_1 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} \right. \\
&\quad \left. + \left\{ \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} - \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right\} \pi^\circ(c_2) + h \right] / (\theta_H - \theta_L) \alpha_{min} , \\
\mathbf{z}_2 &= \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} + \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \right. \\
&\quad \left. - \frac{p}{p + 1} \pi^\circ(c_2) - h \right] / \alpha_{min} \theta_L , \\
\mathbf{z}_3 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \frac{p}{p + 1} \pi^\circ(c_2) \right] / \alpha_{min} \theta_H .
\end{aligned}$$

The expressions of $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ can be verified from rows 1 in Tables 4.4, 4.5, and 4.6 respectively.

Area II:

From Table 4.2, we know that if $I_D^* = 0$ under $\widehat{\mathcal{D}}$, then: $w_2^* = l_{\widehat{\mathcal{D}}}$; if $I_D^* = h$ under $\widehat{\mathcal{D}}$, then: $w_2^* = c_2$; and under $\widehat{\mathcal{R}}$, $w_2^* = l_{\widehat{\mathcal{R}}}$.

Hence if $I_D^* = 0$ under $\widehat{\mathcal{D}}$:

$$\pi_{1,s_2}^b(w_2^*) = \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) = \frac{\{P_S(I_A^*, I_D)\}^2 p}{P_S(I_A^*, I_D)p + 1} \pi^\circ(c_2). \quad (5.1)$$

If $I_D^* = h$ under $\widehat{\mathcal{D}}$:

$$\begin{aligned} \pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) = P_S(I_A^*, h) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \\ &= P_S^L(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \}. \end{aligned}$$

Under $\widehat{\mathcal{R}}$, we have:

$$\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} = \left\{ \mathcal{T}_1(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1} = \frac{p}{p+1} \pi^\circ(c_2).$$

We see that $\pi^b(I_A^*, 0|\widehat{\mathcal{D}})$ and $\pi^b(I_A^*, 0|\widehat{\mathcal{R}})$ are the same as in Area I. The expression for $\pi^b(I_A^*, h|\widehat{\mathcal{D}})$ is however different from that of Area I, on account of $\pi_{1,s_2}^b(w_2^*)$ being equal to $\mathcal{T}_2(\alpha(I_A^*), \rho(I_D))$ when $w_2^* = c_2$ under $\widehat{\mathcal{D}}$.

Therefore the expressions of the three expected profit functions for Area

II are:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} . \\
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= 0 \cdot \left[\tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z \right] \\
&\quad + 1 \cdot \left[\pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z \right] \\
&\quad + \pi_{1,s_2}^b(w_2^*) - h\alpha_{min} - h \cdot 1 \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\
&\quad - h\alpha_{min} - h . \\
\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) &= \frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min}
\end{aligned}$$

Therefore, *within Area II*, when $\mathbf{w}_2^* = \mathbf{l}_{\widehat{\mathcal{D}}}$, then:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - \\
&\quad h\alpha_{min}.
\end{aligned}$$

Within Area II, when $\mathbf{w}_2^* = \mathbf{c}_2$, then:

$$\begin{aligned}
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \\
&\quad \xi\alpha_{min}\} - h\alpha_{min} - h.
\end{aligned}$$

Within Area II, when $\mathbf{w}_2^* = \mathbf{l}_{\widehat{\mathcal{R}}}$, then:

$$\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) = \frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min}.$$

The expression for $\pi^b(I_A^*, h|\widehat{\mathcal{D}})$ can be verified from row 4 in Table 4.3. The other two expressions are the same as in Area I.

Therefore:

$$\begin{aligned}
\mathbf{d}_1 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) \quad , \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h\alpha_{min} - h \\
&\quad - \left[\tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} \right] \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} \\
&\quad + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h \\
&\quad + (\theta_H - \theta_L)\alpha_{min}z \quad . \\
\mathbf{d}_2 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \quad , \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h\alpha_{min} - h \\
&\quad - \left[\frac{p}{p + 1}\pi^\circ(c_2) - h\alpha_{min} \right] \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - \frac{p}{p + 1}\pi^\circ(c_2) \\
&\quad - h - \alpha_{min}\theta_L z \quad . \\
\mathbf{d}_3 &= \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - h\alpha_{min} \\
&\quad - \left[\frac{p}{p + 1}\pi^\circ(c_2) - h\alpha_{min} \right] \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1}\pi^\circ(c_2) - \frac{p}{p + 1}\pi^\circ(c_2) - \alpha_{min}\theta_H z
\end{aligned}$$

Substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we obtain the

values of the three thresholds z_1 , z_2 , and z_3 for Area II as:

$$\begin{aligned}
\mathbf{z}_1 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} \right. \\
&\quad \left. + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - P_S^L(I_A^*, \gamma) \{\pi^\circ(c_2) - \xi \alpha_{min}\} + h \right] \Bigg/ (\theta_H - \theta_L) \alpha_{min} , \\
\mathbf{z}_2 &= \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} + P_S^L(I_A^*, \gamma) \{\pi^\circ(c_2) - \xi \alpha_{min}\} \right. \\
&\quad \left. - \frac{p}{p + 1} \pi^\circ(c_2) - h \right] \Bigg/ \alpha_{min} \theta_L , \\
\mathbf{z}_3 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \right. \\
&\quad \left. - \frac{p}{p + 1} \pi^\circ(c_2) \right] \Bigg/ \alpha_{min} \theta_H .
\end{aligned}$$

The expressions of $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ can be verified from rows 2 in Tables 4.4, 4.5, and 4.6 respectively.

Area III:

From Table 4.2, we know that $w_2^* = c_2$ under $\widehat{\mathcal{D}}$; and $w_2^* = l_{\widehat{\mathcal{R}}}$ under $\widehat{\mathcal{R}}$.

Hence if $I_D^* = 0$ under $\widehat{\mathcal{D}}$:

$$\begin{aligned}\pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) = P_S(I_A^*, 0)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\ &= P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} .\end{aligned}$$

If $I_D^* = h$ under $\widehat{\mathcal{D}}$:

$$\begin{aligned}\pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) = P_S(I_A^*, h)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\ &= P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} .\end{aligned}$$

Under $\widehat{\mathcal{R}}$, we have:

$$\left\{\pi_{1,s_2}^b(w_2^*)\right\}_{P_S=1} = \left\{\mathcal{T}_1(\alpha(I_A^*), \rho(I_D))\right\}_{P_S=1} = \frac{p}{p+1}\pi^\circ(c_2) .$$

We see that $\pi^b(I_A^*, h|\widehat{\mathcal{D}})$ and $\pi^b(I_A^*, 0|\widehat{\mathcal{R}})$ are the same as in Area II. The expression for $\pi^b(I_A^*, 0|\widehat{\mathcal{D}})$ is however different from that of Area II, on account of $\pi_{1,s_2}^b(w_2^*)$ being equal to $\mathcal{T}_2(\alpha(I_A^*), \rho(I_D))$ when $w_2^* = c_2$ under $\widehat{\mathcal{D}}$.

Therefore the expressions of the three expected profit functions for Area

III are:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= 1 \cdot \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z \right] \\
&\quad + 0 \cdot \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z \right] \\
&\quad + \pi_{1,s_2}^b(w_2^*) - h\alpha_{min} - h \cdot 0 \\
&= \tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z + P_S^H(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \\
&\quad - h\alpha_{min} \\
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z + P_S^L(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \\
&\quad - h\alpha_{min} - h . \\
\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) &= \frac{p}{p+1} \pi^\circ(c_2) - h\alpha_{min}
\end{aligned}$$

Therefore, *within Area III*, when $\mathbf{w}_2^* = \mathbf{c}_2$, then:

$$\begin{aligned}
\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min} \theta_H z + P_S^H(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} - h\alpha_{min}. \\
\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min} \theta_L z + P_S^L(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} - h\alpha_{min} - h.
\end{aligned}$$

Within Area III, when $\mathbf{w}_2^* = \mathbf{l}_{\widehat{\mathcal{R}}}$, then:

$$\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) = \frac{p}{p+1} \pi^\circ(c_2) - h\alpha_{min}.$$

The expression for $\pi^b(I_A^*, 0|\widehat{\mathcal{D}})$ can be verified from row 3 in Table 4.3. The other two expressions are the same as in Area II.

Therefore:

$$\begin{aligned}
\mathbf{d}_1 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) \quad , \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h\alpha_{min} - h \\
&\quad - \left[\tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h\alpha_{min} \right] \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} \\
&\quad + \left\{ P_S^L(I_A^*, \gamma) - P_S^H(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi\alpha_{min} \right\} - h + (\theta_H - \theta_L)\alpha_{min}z \quad . \\
\mathbf{d}_2 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \quad , \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - \frac{p}{p+1}\pi^\circ(c_2) \\
&\quad - h - \alpha_{min}\theta_L z \quad . \\
\mathbf{d}_3 &= \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h\alpha_{min} \\
&\quad - \left[\frac{p}{p+1}\pi^\circ(c_2) - h\alpha_{min} \right] \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} - \frac{p}{p+1}\pi^\circ(c_2) - \alpha_{min}\theta_H z
\end{aligned}$$

Substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we obtain the

values of the three thresholds z_1 , z_2 , and z_3 for Area III as:

$$\begin{aligned}
\mathbf{z}_1 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} \right. \\
&\quad \left. + \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h \right] / (\theta_H - \theta_L) \alpha_{min} , \\
\mathbf{z}_2 &= \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} + P_S^L(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \right. \\
&\quad \left. - \frac{p}{p+1} \pi^\circ(c_2) - h \right] / \alpha_{min} \theta_L , \\
\mathbf{z}_3 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} + P_S^H(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \right. \\
&\quad \left. - \frac{p}{p+1} \pi^\circ(c_2) \right] / \alpha_{min} \theta_H .
\end{aligned}$$

The expressions of $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ can be verified from rows 3 in Tables 4.4, 4.5, and 4.6 respectively.

Area IV:

From Table 4.2, we know that $w_2^* = c_2$ under both $\widehat{\mathcal{D}}$ and $\widehat{\mathcal{R}}$.

Hence if $I_D^* = 0$ under $\widehat{\mathcal{D}}$:

$$\begin{aligned}\pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) = P_S(I_A^*, 0) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \\ &= P_S^H(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} .\end{aligned}$$

If $I_D^* = h$ under $\widehat{\mathcal{D}}$:

$$\begin{aligned}\pi_{1,s_2}^b(w_2^*) &= \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) = P_S(I_A^*, h) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \\ &= P_S^L(I_A^*, \gamma) \{ \pi^\circ(c_2) - \xi \alpha_{min} \} .\end{aligned}$$

Under $\widehat{\mathcal{R}}$, we have:

$$\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1} = \left\{ \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1} = \pi^\circ(c_2) - \xi \alpha_{min} .$$

We see that $\pi^b(I_A^*, 0 | \widehat{\mathcal{D}})$ and $\pi^b(I_A^*, 0 | \widehat{\mathcal{R}})$ are the same as in Area III. The expression for $\pi^b(I_A^*, 0 | \widehat{\mathcal{R}})$ is however different from that of Area III, on account of $\left\{ \pi_{1,s_2}^b(w_2^*) \right\}_{P_S=1}$ being equal to $\left\{ \mathcal{T}_2(\alpha(I_A^*), \rho(I_D)) \right\}_{P_S=1}$ when $w_2^* = c_2$ under $\widehat{\mathcal{R}}$.

Therefore the expressions of the three expected profit functions for Area

IV are:

$$\begin{aligned}\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\ &\quad - h\alpha_{min}\end{aligned}$$

$$\begin{aligned}\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\ &\quad - h\alpha_{min} - h.\end{aligned}$$

$$\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) = \pi^\circ(c_2) - \xi\alpha_{min} - h\alpha_{min}$$

Therefore, *within Area IV*:

$$\begin{aligned}\pi^b(I_A^*, 0|\widehat{\mathcal{D}}) &= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \alpha_{min}\theta_H z + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \\ &\quad \xi\alpha_{min}\} - h\alpha_{min}.\end{aligned}$$

$$\begin{aligned}\pi^b(I_A^*, h|\widehat{\mathcal{D}}) &= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \alpha_{min}\theta_L z + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \\ &\quad \xi\alpha_{min}\} - h\alpha_{min} - h.\end{aligned}$$

$$\pi^b(I_A^*, 0|\widehat{\mathcal{R}}) = \pi^\circ(c_2) - \xi\alpha_{min} - h\alpha_{min}.$$

The expression for $\pi^b(I_A^*, 0|\widehat{\mathcal{R}})$ can be verified from row 6 in Table 4.3. The other two expressions are the same as in Area III.

Therefore:

$$\begin{aligned}
\mathbf{d}_1 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} \\
&\quad + \left\{P_S^L(I_A^*, \gamma) - P_S^H(I_A^*, \gamma)\right\}\left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\} - h + (\theta_H - \theta_L)\alpha_{min}z \ . \\
\mathbf{d}_2 &= \pi^b(I_A^*, h|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} + P_S^L(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\
&\quad - \left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\} - h - \alpha_{min}\theta_L z \\
&= \pi^\circ(c_1^L)\{1 - P_S^L(I_A^*, \gamma)\} - \{1 - P_S^L(I_A^*, \gamma)\}\{\pi^\circ(c_2) - \xi\alpha_{min}\} - h - \alpha_{min}\theta_L z \ . \\
\mathbf{d}_3 &= \pi^b(I_A^*, 0|\widehat{\mathcal{D}}) - \pi^b(I_A^*, 0|\widehat{\mathcal{R}}) \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} + P_S^H(I_A^*, \gamma)\{\pi^\circ(c_2) - \xi\alpha_{min}\} \\
&\quad - \left\{\pi^\circ(c_2) - \xi\alpha_{min}\right\} - \alpha_{min}\theta_H z \\
&= \tilde{\pi}(c_1^H)\{1 - P_S^H(I_A^*, \gamma)\} - \{1 - P_S^H(I_A^*, \gamma)\}\{\pi^\circ(c_2) - \xi\alpha_{min}\} - \alpha_{min}\theta_H z
\end{aligned}$$

Substituting $d_1 = 0$, $d_2 = 0$, and $d_3 = 0$ and solving for z , we obtain the

values of the three thresholds z_1 , z_2 , and z_3 for Area IV as:

$$\begin{aligned}
\mathbf{z}_1 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} \right. \\
&\quad \left. + \{P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma)\} \{ \pi^\circ(c_2) - \xi \alpha_{min} \} + h \right] / (\theta_H - \theta_L) \alpha_{min} , \\
\mathbf{z}_2 &= \left[\pi^\circ(c_1^L) \{1 - P_S^L(I_A^*, \gamma)\} - \{1 - P_S^L(I_A^*, \gamma)\} \{ \pi^\circ(c_2) - \xi \alpha_{min} \} - h \right] / \alpha_{min} \theta_L , \\
\mathbf{z}_3 &= \left[\tilde{\pi}(c_1^H) \{1 - P_S^H(I_A^*, \gamma)\} - \{1 - P_S^H(I_A^*, \gamma)\} \{ \pi^\circ(c_2) - \xi \alpha_{min} \} \right] / \alpha_{min} \theta_H .
\end{aligned}$$

The expressions of $\mathbf{z}_1$, $\mathbf{z}_2$, and $\mathbf{z}_3$ can be verified from rows 4 in Tables 4.4, 4.5, and 4.6 respectively. ■

Appendix IV

This Appendix refers to the proof of Theorem 4.4.2 part (ii). In the theorem, we do not explicitly demonstrate the continuity of z_1 and z_2 across all four Areas. Here, we will state the detailed workings which demonstrate the continuity.

The thresholds ξ_0 , ξ_1 , and ξ_2 are the values of ξ which form the boundaries between Area IV & Area III, Area III & Area II, and Area II & Area I respectively. The expressions for the thresholds are:

$$\begin{aligned}\xi_0 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}(p+1)}, \\ \xi_1 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}[\theta_H\{\alpha_{\min}(1-\gamma) + \gamma\}p+1]}, \\ \xi_2 &= \frac{\pi^\circ(c_2)}{\alpha_{\min}[\theta_L\{\alpha_{\min}(1-\gamma) + \gamma\}p+1]}.\end{aligned}$$

Recall from the proposition that for a given ξ , $z_1^{(j)}(\xi)$ is the value of z_1 and $z_2^{(j)}$ as the value of z_2 in Area j ($j \in \{I, II, III, IV\}$).

First, let us establish the continuity of $z_1^{(j)}(\xi)$ across all areas. For $z_1^{(j)}(\xi)$

to be continuous in ξ across all four Areas, the following two conditions must hold: **(i)** $z_1^{(IV)}(\xi)$ should be continuous within $[0, \xi_0)$, $z_1^{(III)}(\xi)$ should be continuous within (ξ_0, ξ_1) , $z_1^{(II)}(\xi)$ should be continuous within (ξ_1, ξ_2) , and $z_1^{(I)}(\xi)$ should be continuous within $(\xi_2, 1]$; and **(ii)** $z_1^{(IV)}(\xi_0) = z_1^{(III)}(\xi_0)$, $z_1^{(III)}(\xi_1) = z_1^{(II)}(\xi_1)$, and $z_1^{(II)}(\xi_2) = z_1^{(I)}(\xi_2)$.

From Table 4.4, we see that the expressions for $z_1^{(j)}(\xi)$ are:

$$\begin{aligned}
z_1^{(I)}(\xi) &= \left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) \\
&\quad + \left[\frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} - \frac{\{P_S^L(I_A^*, \gamma)\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right] \pi^\circ(c_2) + h \\
z_1^{(II)}(\xi) &= \left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) \\
&\quad + \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h \\
z_1^{(III)}(\xi) &= \left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) \\
&\quad + \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h \\
z_1^{(IV)}(\xi) &= \left\{1 - P_S^H(I_A^*, \gamma)\right\} \tilde{\pi}(c_1^H) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) \\
&\quad + \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h
\end{aligned}$$

It can be easily seen that $z_1^{(IV)}(\xi)$ is continuous within $[0, \xi_0)$, $z_1^{(III)}(\xi)$ is continuous within (ξ_0, ξ_1) , $z_1^{(II)}(\xi)$ is continuous within (ξ_1, ξ_2) , and $z_1^{(I)}(\xi)$ is continuous within $(\xi_2, 1]$. Therefore condition **(i)** for $z_1^{(j)}(\xi)$ holds.

We see that: $z_1^{(IV)}(\xi) = z_1^{(III)}(\xi) \forall \xi$. Therefore, $z_1^{(IV)}(\xi_0) = z_1^{(III)}(\xi_0)$.

Now we need to test whether $z_1^{(III)}(\xi_1) = z_1^{(II)}(\xi_1)$.

$$\begin{aligned}
z_1^{(III)}(\xi) &= z_1^{(II)}(\xi) \\
\leftrightarrow \left\{ 1 - P_S^H(I_A^*, \gamma) \right\} \tilde{\pi}(c_1^H) &= \left\{ 1 - P_S^H(I_A^*, \gamma) \right\} \tilde{\pi}(c_1^H) \\
- \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) &- \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) \\
+ \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} &+ \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\times \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h &- P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h \\
\leftrightarrow \left\{ P_S^H(I_A^*, \gamma) - P_S^L(I_A^*, \gamma) \right\} &= \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} \\
\times \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} & \\
\leftrightarrow P_S^H(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} &= \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\leftrightarrow P_S^H(I_A^*, \gamma) \xi \alpha_{min} &= P_S^H(I_A^*, \gamma) \pi^\circ(c_2) - \frac{\{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\leftrightarrow P_S^H(I_A^*, \gamma) \xi \alpha_{min} &= \left\{ \frac{\{P_S^H(I_A^*, \gamma)\}^2 p + P_S^H(I_A^*, \gamma) - \{P_S^H(I_A^*, \gamma)\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \right\} \\
&\quad \times \pi^\circ(c_2) \\
\leftrightarrow P_S^H(I_A^*, \gamma) \xi \alpha_{min} &= \frac{P_S^H(I_A^*, \gamma)}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\leftrightarrow \xi \alpha_{min} &= \frac{\pi^\circ(c_2)}{P_S^H(I_A^*, \gamma)p + 1} \\
\leftrightarrow \xi &= \frac{\pi^\circ(c_2)}{\alpha_{min} [P_S^H(I_A^*, \gamma)p + 1]} \\
&= \xi_1 .
\end{aligned}$$

Therefore, $z_1^{(III)}(\xi_1) = z_1^{(II)}(\xi_1)$.

Finally, we need to test whether $z_1^{(II)}(\xi_2) = z_1^{(I)}(\xi_2)$.

$$\begin{aligned}
z_1^{(II)}(\xi) &= z_1^{(I)}(\xi) \\
\leftrightarrow \left\{ 1 - P_S^H(I_A^*, \gamma) \right\} \tilde{\pi}(c_1^H) &= \left\{ 1 - P_S^H(I_A^*, \gamma) \right\} \tilde{\pi}(c_1^H) \\
- \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) &- \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) \\
+ \frac{\left\{ P_S^H(I_A^*, \gamma) \right\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) &+ \left[\frac{\left\{ P_S^H(I_A^*, \gamma) \right\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} - \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right] \pi^\circ(c_2) + h \\
- P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h & \\
\leftrightarrow \frac{\left\{ P_S^H(I_A^*, \gamma) \right\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} \pi^\circ(c_2) &= \left[\frac{\left\{ P_S^H(I_A^*, \gamma) \right\}^2 p}{P_S^H(I_A^*, \gamma)p + 1} - \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right] \pi^\circ(c_2) + h \\
- P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} + h & \\
\leftrightarrow P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} &= \left[\frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right] \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= P_S^L(I_A^*, \gamma) \pi^\circ(c_2) - \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= \left\{ \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p + P_S^L(I_A^*, \gamma) - \left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \right\} \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= \frac{P_S^L(I_A^*, \gamma)}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) \\
\leftrightarrow \xi \alpha_{min} &= \frac{\pi^\circ(c_2)}{P_S^L(I_A^*, \gamma)p + 1} \\
\leftrightarrow \xi &= \frac{\pi^\circ(c_2)}{\alpha_{min} \left[P_S^L(I_A^*, \gamma)p + 1 \right]} \\
&= \xi_2 .
\end{aligned}$$

Therefore, $z_1^{(II)}(\xi_2) = z_1^{(I)}(\xi_2)$.

Hence condition **(ii)** for $z_1^{(j)}(\xi)$ also holds.

Therefore, $z_1^{(j)}(\xi)$ is continuous across all four areas.

Now let us establish the continuity of $z_2^{(j)}(\xi)$ across all areas. For $z_2^{(j)}(\xi)$ to be continuous in ξ across all four Areas, the following two conditions must hold: **(i)** $z_2^{(IV)}(\xi)$ should be continuous within $[0, \xi_0)$, $z_2^{(III)}(\xi)$ should be continuous within (ξ_0, ξ_1) , $z_2^{(II)}(\xi)$ should be continuous within (ξ_1, ξ_2) , and $z_2^{(I)}(\xi)$ should be continuous within $(\xi_2, 1]$; and **(ii)** $z_2^{(IV)}(\xi_0) = z_2^{(III)}(\xi_0)$, $z_2^{(III)}(\xi_1) = z_2^{(II)}(\xi_1)$, and $z_2^{(II)}(\xi_2) = z_2^{(I)}(\xi_2)$.

From Table 4.5, we see that the expressions for $z_2^{(j)}(\xi)$ are:

$$\begin{aligned}
z_2^{(j)}(I) &= \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + \frac{\left\{P_S^L(I_A^*, \gamma)\right\}^2 p}{P_S^L(I_A^*, \gamma)p + 1} \pi^\circ(c_2) - \frac{p}{p + 1} \pi^\circ(c_2) - h \\
z_2^{(j)}(II) &= \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + P_S^L(I_A^*, \gamma) \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - \frac{p}{p + 1} \pi^\circ(c_2) - h \\
z_2^{(j)}(III) &= \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) + P_S^L(I_A^*, \gamma) \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - \frac{p}{p + 1} \pi^\circ(c_2) - h \\
z_2^{(j)}(IV) &= \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) - \left\{1 - P_S^L(I_A^*, \gamma)\right\} \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} - h
\end{aligned}$$

It can be easily seen that $z_2^{(IV)}(\xi)$ is continuous within $[0, \xi_0)$, $z_2^{(III)}(\xi)$ is continuous within (ξ_0, ξ_1) , $z_2^{(II)}(\xi)$ is continuous within (ξ_1, ξ_2) , and $z_2^{(I)}(\xi)$ is continuous within $(\xi_2, 1]$. Therefore condition **(i)** for $z_2^{(j)}(\xi)$ holds.

First, we need to test whether $z_2^{(IV)}(\xi_0) = z_2^{(III)}(\xi_0)$.

$$\begin{aligned}
z_2^{(IV)}(\xi) &= z_2^{(III)}(\xi) \\
&\leftrightarrow \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) = \left\{1 - P_S^L(I_A^*, \gamma)\right\} \pi^\circ(c_1^L) \\
&- \left\{1 - P_S^L(I_A^*, \gamma)\right\} \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} + P_S^L(I_A^*, \gamma) \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} \\
&\quad - h \quad - \frac{p}{p+1} \pi^\circ(c_2) - h \\
&\leftrightarrow - \left\{\pi^\circ(c_2) - \xi \alpha_{min}\right\} = - \frac{p}{p+1} \pi^\circ(c_2) \\
&\quad \leftrightarrow \pi^\circ(c_2) - \xi \alpha_{min} = \frac{p}{p+1} \pi^\circ(c_2) \\
&\quad \leftrightarrow \xi \alpha_{min} = \pi^\circ(c_2) - \frac{p}{p+1} \pi^\circ(c_2) \\
&\quad \leftrightarrow \xi \alpha_{min} = \frac{1}{p+1} \pi^\circ(c_2) \\
&\quad \leftrightarrow \xi = \frac{\pi^\circ(c_2)}{\alpha_{min}(p+1)} \\
&\quad = \xi_0 .
\end{aligned}$$

Therefore, $z_2^{(IV)}(\xi_0) = z_2^{(III)}(\xi_0)$.

Now we need to test whether $z_2^{(III)}(\xi_1) = z_2^{(II)}(\xi_1)$. We see that: $z_2^{(III)}(\xi) = z_2^{(II)}(\xi) \forall \xi$. Therefore, $z_2^{(III)}(\xi_1) = z_2^{(II)}(\xi_1)$.

Finally, we need to test whether $z_2^{(II)}(\xi_2) = z_2^{(I)}(\xi_2)$.

$$\begin{aligned}
z_2^{(II)}(\xi) &= z_2^{(I)}(\xi) \\
\leftrightarrow \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) &= \left\{ 1 - P_S^L(I_A^*, \gamma) \right\} \pi^\circ(c_1^L) \\
+ P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} &+ \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma) p + 1} \pi^\circ(c_2) \\
- \frac{p}{p+1} \pi^\circ(c_2) - h &- \frac{p}{p+1} \pi^\circ(c_2) - h \\
\leftrightarrow P_S^L(I_A^*, \gamma) \left\{ \pi^\circ(c_2) - \xi \alpha_{min} \right\} &= \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma) p + 1} \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= P_S^L(I_A^*, \gamma) \pi^\circ(c_2) - \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma) p + 1} \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= \left\{ \frac{\left\{ P_S^L(I_A^*, \gamma) \right\}^2 p + P_S^L(I_A^*, \gamma) - \left\{ P_S^L(I_A^*, \gamma) \right\}^2 p}{P_S^L(I_A^*, \gamma) p + 1} \right\} \pi^\circ(c_2) \\
\leftrightarrow P_S^L(I_A^*, \gamma) \xi \alpha_{min} &= \frac{P_S^L(I_A^*, \gamma)}{P_S^L(I_A^*, \gamma) p + 1} \pi^\circ(c_2) \\
\leftrightarrow \xi \alpha_{min} &= \frac{\pi^\circ(c_2)}{P_S^L(I_A^*, \gamma) p + 1} \\
\leftrightarrow \xi &= \frac{\pi^\circ(c_2)}{\alpha_{min} \left[P_S^L(I_A^*, \gamma) p + 1 \right]} \\
&= \xi_2 .
\end{aligned}$$

Therefore, $z_2^{(II)}(\xi_2) = z_2^{(I)}(\xi_2)$.

Hence condition **(ii)** for $z_2^{(j)}(\xi)$ also holds.

Therefore, $z_2^{(j)}(\xi)$ is continuous across all four areas. ■