Aerodynamics modelling of floating offshore wind turbines

Singapore Wala, Abdulqadir Aziz

2017

Singapore Wala, A. A. (2017). Aerodynamics modelling of floating offshore wind turbines.
Doctoral thesis, Nanyang Technological University, Singapore.

<http://hdl.handle.net/10356/71898>

<https://doi.org/10.32657/10356/71898>

NANYANG
TECHNOLOGICAL
UNIVERSITY

AERODYNAMICS MODELLING OF FLOATING OFFSHORE WIND TURBINES

ABDULQADIR AZIZ SINGAPORE WALA
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING
2017

**AERODYNAMICS MODELLING OF
FLOATING OFFSHORE WIND TURBINES**

By

ABDULQADIR AZIZ SINGAPORE WALA

A THESIS PRESENTED TO THE
NANYANG TECHNOLOGICAL UNIVERSITY
IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

2017

Abstract

An unsteady Blade Element Momentum (uBEM) method is presented for the analysis of Floating Offshore Wind Turbines (FOWTs). FOWTs allow wind turbines to be placed in deeper waters with stronger and more consistent winds. However, the floating platform motions result in motions of the wind turbine not found in fixed-bottom wind turbines, thus the widely-used blade element momentum (BEM) method, a static analysis method, cannot be used to calculate wind turbine blade forces.

Different unsteady aerodynamic phenomena associated with FOWTs are analysed, with corrective models developed for an accurate analysis of FOWT aerodynamics. The NREL 5MW virtual wind turbine was studied in axial flow and sinusoidal surge motions at different frequencies and amplitudes, using full-3D computational fluid dynamics simulations.

The effects studied include the unsteady airfoil effect, stall delay effect, tip-loss, and transient wake states. The unsteady airfoil effect is studied based on experimental data from the Ohio State University, and a modified Beddoes-Leishman model is presented based on airfoil shape parameters and an optimisation-based methodology. The stall-delay effect was accounted for using the Du & Selig model, and a novel methodology for real-time stall delay incorporating the unsteady airfoil effect is presented. A novel correction and modelling methodology is presented for tip-losses, through a correction to Glauert's implementation of the Prandtl tip-loss factor, which incorporates changes in tip speed ratio as well as changes in wake state. Buhl's version of the Glauert correction for the turbulent wake state was used as a basis to model the propeller and propeller brake states of the wind turbine, which are rare for fixed-bottom wind turbines but can be expected to occur periodically for FOWTs. The dynamic wake model of Øye is optimised to take into account the level of unsteadiness found in FOWTs and work in conjunction with other unsteady models.

Finally, these models are incorporated into the uBEM method and a FOWT aerodynamics analysis tool is created with low computational time, and more realistic force predictions. Future studies can extend these methodologies to more motion degrees of freedom.

I dedicate this thesis to
the 52nd Dai al-Mutlaq, Dr Syedna Mohammed Burhanuddin,
and the 53rd Dai al-Mutlaq, Dr Syedna Aali Qadr Mufaddal Saifuddin,
both my spiritual fathers.

My daughter Ruqaiyyah, I hope that one day, this will inspire you to aim for
the highest peak in every endeavour.

ACKNOWLEDGMENTS

The 52nd Dai Al Mutlaq of the Dawoodi Bohra Community, my spiritual father, Dr Syedna Mohammed Burhanuddin (RA), had always taught me that knowledge is the true path to salvation, and urged his followers to excel in their own fields. He has been a constant source of blessings, strength and inspiration. His successor and beloved son, the 53rd Dai Al Mutlaq, Syedna Mufaddal Saifuddin (TUS) has taught me to aim for the highest peak, and to get there through perspiration, and that the highest calling for Man is the pursuit of knowledge and constant innovation. His humility has been a great example and inspiration.

I would like to thank my supervisor Associate Professor Ng Yin Kwee, who still hasn't stopped believing in me and always has an encouraging word to keep me going, my co-supervisor Dr Narasimalu Srikanth who always pushed a me little bit harder to achieve more, and the Energy Research Institute @ NTU (ERI@N) for all the opportunities. I would like to thank the Economic Development Board (EDB), who have given the opportunity for this industrial postgraduate programme. I would like to especially thank the Lloyds Register Global Technology Centre, Singapore, especially Dr Johan Gullman-Strand, Mr Anand Bahuguni and Mr Krishna Sivalingam for their advice and support. Peter Davies has provided a lot of input and advice for the direction of this research.

Janaab Badrul Jamali Bhaisaheb Najmuddin has been a source of guidance and wisdom, and his advice had led me to embark on this PhD.

My family has put up with all my shenanigans. Mum and Dad, always with a word of support or a listening ear, my brother Hozefa, sister-in-law Umme Hani and niece, Insiyah, who always manages to cheer me up after a long day.

My wife, Fatema has only observed my life as a PhD student, since we were married just before I embarked on this journey. She has been a constant rock, doing everything she could to make sure I got through. In my final year of research, she gave me the greatest gift of all: our daughter Ruqaiyyah. Ruqaiyyah, thank you for coming into my life, and making it so special. You are my world.

TABLE OF CONTENTS

	page
ABSTRACT	i
ACKNOWLEDGMENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	ix
LIST OF SYMBOLS	xvii
CHAPTER	
1 Introduction	1
1.1 Significance	1
1.2 Problem Description	2
1.3 Objectives & Scope	5
1.4 Contributions of the Thesis	6
1.5 Organisation of Thesis	6
2 Literature Review	8
2.1 Wind Turbine Aerodynamics	8
2.1.1 Momentum Theory	8
2.1.2 The Blade Element Theory	10
2.1.3 Effects of Rotation on Aerodynamic Forces	11
2.1.4 Tip Losses	14
2.1.5 Turbulent Wake State	15
2.2 FOWT Unsteady Aerodynamics	16
2.2.1 Sources of Unsteadiness in FOWTs	17
2.2.1.1 Rotor and tower structure	21
2.2.2 Unsteady Aerodynamic Phenomena	22
2.2.2.1 Dynamic wake effect	22
2.2.2.2 Unsteady airfoil effects	25
2.2.2.3 Skewed wake effects	31
2.2.2.4 Transient wake states	35
2.3 Review of FOWT Experiments	40
2.4 Computational Modelling of FOWT Aerodynamics	46
2.4.1 Computational Fluid Dynamics	47
2.4.1.1 Direct rotor modelling (DRM)	47
2.4.1.2 Generalised actuator disc methods	51
2.4.2 Vortex Methods	53
2.4.3 Momentum Methods	55
2.4.3.1 Methods of accelerating potential	55
2.4.3.2 The blade element momentum method	57
2.4.3.3 Unsteady blade element momentum (uBEM) method	61

2.4.4	Fatigue, Aerodynamics, Structures, and Turbulence code (FAST)	63
3	Unsteady Blade Element Momentum (uBEM) Method	65
3.1	Coordinate Systems	65
3.1.1	Platform Orientation	66
3.1.2	Rotor Orientation	66
3.1.3	Blade Orientation	68
3.1.4	Coordinate Transformations	69
3.2	Relative Velocity	71
3.2.1	Velocity Component Due to Platform Motion	72
3.3	Induced Velocities	73
3.3.1	Tip-Loss Factor	78
3.3.2	Glauert Correction	79
3.4	Blade Elemental Forces	80
3.5	uBEM Algorithm	81
3.6	Concluding Remarks	82
4	Computational Fluid Dynamics (CFD) Methodology	84
4.1	Simulation conditions	84
4.2	Computational Mesh	86
4.2.1	Rotor Geometry	86
4.2.2	Mesh Setup	88
4.3	CFD Setup	91
4.4	Initial and Boundary Conditions	91
4.4.1	Dynamic Mesh Setup	91
4.5	Mesh Refinement Study	92
4.6	Time step refinement	95
4.7	Concluding Remarks	98
5	Tip-Loss Model	99
5.1	Theory	100
5.1.1	Prandtl tip-loss [22]	100
5.1.2	Empirical corrections for Prandtl tip-loss	102
5.1.2.1	Lindenburg [96]	102
5.1.2.2	Shen et al [93]	103
5.1.2.3	Xu and Sankar [94]	104
5.2	Methodology	104
5.2.1	Simulation matrix	104
5.2.2	Tip-loss factors from CFD	105
5.2.3	Correction to Prandtl tip-loss model	108
5.3	Results and Discussion	114
5.4	Concluding Remarks	121
6	Unsteady Airfoil Model	123
6.1	Methodology	124
6.1.1	Unsteady airfoil experimental results	124

6.1.2	Beddoes-Leishman model [6]	125
6.1.2.1	Unsteady attached flow	125
6.1.2.2	Unsteady trailing edge separation	127
6.1.2.3	Vortex lift	129
6.1.2.4	Drag and moment components	131
6.2	Optimisation	131
6.3	Simplified model	135
6.4	Shape-parameter based model	136
6.5	Results and Discussion	142
6.6	Concluding Remarks	146
7	Combined Stall-Delay and Unsteady Airfoil Model	148
7.1	Model descriptions	149
7.1.1	Du & Selig [16]	149
7.1.2	Combined Stall-Delay and Unsteady Airfoil Model	150
7.1.2.1	Stall-delayed unsteady lift	150
7.1.2.2	Stall-delayed unsteady drag	152
7.2	Results and Discussion	152
7.3	Concluding Remarks	157
8	Dynamic Wake Model	159
8.1	Review of Dynamic Wake Models for BEM	159
8.1.1	Single Differential Equation Model	159
8.1.2	Two Differential Equations Model	160
8.2	Modified two equation model	162
8.3	Results and Discussion	162
8.4	Concluding Remarks	166
9	Transient Wake States	168
9.1	Theory	168
9.1.1	Buhl's Glauert Correction Factor	168
9.1.2	Extended Glauert Correction	169
9.1.2.1	Vortex ring and propeller brake state	169
9.2	Results and Discussion	172
9.3	Concluding Remarks	176
10	Results and Discussion	177
10.1	Concluding Remarks	189
11	Conclusion	191
11.1	Future Work	193
	REFERENCES	196
	APPENDIX: The Polak-Ribière Conjugate Gradients Method	208
	PUBLICATIONS	210

LIST OF TABLES

<u>Table</u>	<u>page</u>
2-1 Degree of flow unsteadiness based on reduced frequency [40]	26
2-2 Percentage of time spent by FOWT rotor segments in propeller state for different conditions and platform types [53]	39
2-3 Computational time for various aerodynamic analysis methods for wind turbines, for 5 rotor rotations under 15ř azimuth steps with 10 blade elements using an Intel Dual Core 2.4 Ghz CPU [71]	54
2-4 Comparison of different iterative methods to obtain a solution for BEM [78]	60
4-1 Wind turbine surge conditions used in dynamic wake study	85
4-2 Airfoils used in NREL 5MW turbine	86
4-3 NREL 5MW turbine blade properties	88
4-4 Percentage change in thrust and shaft torque at each blade segment, compared to medium mesh for static simulation	93
5-1 Rotor RPM, wind speeds simulated in present study, with the tip speed ratios (λ) of simulations displayed in the cells	105
6-1 Experimental conditions and airfoils used in present study	125
6-2 Values for A_1 , b_1 , A_2 and b_2 proposed by [6] and applied in [103], [104] and [105]	126
6-3 Values for T_P , T_f and T_v from various sources	130
6-4 Model constants for reference and optimised cases	132
6-5 Accumulated error for reference and optimised cases for each airfoil	133
6-6 Accumulated error for reference and optimised cases for each Reynolds Number	133
6-7 Accumulated error for reference and optimised cases for each Frequency	133
6-8 Accumulated error for reference and optimised cases for each Amplitude angle of attack	134
6-9 Accumulated error for reference and optimised cases for each mean	134
6-10 Model constants for original optimised and simplified model (equation (6–29)) cases	136
6-11 Values of τ , A and $\tau(A)$ for each airfoil	137

6-12	Accumulated error for each airfoil in generally optimised and airfoil-specific optimised cases	139
6-13	Suction surface shape-dependent values of constants in Beddoes-Leishman model after final optimisation	142
6-14	Accumulated error in each airfoil for generally optimised and airfoil-specific optimised cases	142
10-1	Computational time for CFD and uBEM in present study	177

LIST OF FIGURES

Figure	page
1-1 Different contributing sources to a wind turbine flow field structure [2]	2
1-2 Summary of components related to wind turbine aerodynamics . .	3
2-1 Illustration of the streamlines past the rotor, and axial velocity and pressure distributions upstream and downstream of the rotor (subscript 0 represents freestream values) [8]	8
2-2 Angles related to a blade element	11
2-3 Pressure distributions for post-stall angles of attack, at 30% blade span, compared to 2D distribution (black filled circles) [11]	12
2-4 Coefficients of lift for a wind turbine blade cross sections, at various blade spans, compared to 2D distribution (black line) [13]	13
2-5 Different wake states of a wind turbine. [23]	15
2-6 6 degree-of-freedom motions of a floating platform	17
2-7 Predicted spar-buoy platform motions with an NREL 5MW turbine (generated using FAST). Wind input generated using IEC von Karman turbulence model and mean $V_0 = 10ms^{-1}$, nominally axial flow. Wave kinematics modelled using JONSWAP spectrum with significant incident wave height of 6m and peak spectral period of 10 seconds. [26]	18
2-8 Changes in effective wind speed at blade tips due to predicted motions of NREL 5MW turbine motions of the spar-buoy platform (Based on motions given in Figure 2-7) [26]	19
2-9 Pendulum-like motion of FOWT rotor resulting from pitching of the floating platform	20
2-10 Growth trend for offshore wind turbine rated capacity and rotor diameter [33]	21
2-11 Measured rotor shaft torque on the Tjæreborg wind turbine for a rotor blade step pitch from 0.1° to 3.7° in 0.5s and back [37] . . .	24
2-12 Comparison between steady and dynamic lift coefficients of an S809 airfoil using the Ohio State University experimental data (data from [41])	27
2-13 Reduced frequency distribution along a NREL 5MW rotor blade operating in yaw at 8m/s wind speed at rated RPM	28

2-14	Coefficient of lift for different amplitudes of pitching for an S809 airfoil at $k = 0.077$ plotted using the Ohio State University experimental data (data from [41])	29
2-15	Dynamic stall (leading edge stall) process over an airfoil [40]	30
2-16	Difference between the wake skew angle χ , and the yaw angle γ , of a wind turbine from the top view [45]	32
2-17	Wake velocities, non-dimensionalised with wind velocity, against radial distance from the rotor hub at downstream distances of 1, 3 and 4.78 rotor diameters for yaw angles of (a) 0° and (b) 40° [46]	33
2-18	Coefficient of lift distribution for airfoil segments along a wind turbine blade for rotor yaw angles of 30° and 45° , compared to lift coefficients at zero yaw (triangle pointing left) and static wind tunnel measurements for the airfoil (triangle pointing right) [47]	34
2-19	A helicopter rotor in the vortex ring state [51]	36
2-20	Percentage of time spent by rotor segment in VRS for below (a) rated, (b) rated and (c) above rated conditions [50]	37
2-21	Lift and drag coefficients against angle of attack (α) for the NACA 64-618 airfoil section under model and full scale wind conditions at a span of 0.7 of the rotor radius [56]	42
2-22	Coefficient of thrust and power at model (smooth and rough) and full scale (0° and 6.4° blade tip pitch) against tip speed ratio (TSR). [56]	43
2-23	Wake velocities in the wind direction, normalised against the wind speed at hub height, for (a) fixed and (b) floating cases [59]	44
2-24	Comparison of lift (top) and drag (bottom) coefficients for model (line) and full-scale (+) airfoil [60]	45
2-25	Comparison of coefficient of thrust (top) and torque (bottom) for model (line) and full-scale (+) turbines [60]	46
2-26	Different mesh zones created for the sliding mesh method [62]	48
2-27	Overset mesh with additional mesh structures to ensure mesh overlap [63]	49
2-28	Comparison of aerodynamic torque between a deformable blade and a rigid blade for the NREL 5MW rotor [66]	50
2-29	Instantaneous turbulence vortical structures in the wake of wind turbines above waves, obtained using (a) actuator line method, and (b) actuator disc method [68]	52
2-30	Comparison of BEM and GDW predictions of the Tjæreborg wind turbine undergoing a step pitch from 0.2° to 3.9° and back [45]	61

2-31	Power and thrust predictions for the NREL 5MW turbine undergoing platform pitching motions with amplitude of 1° and frequency of 0.1Hz [64]	63
3-1	The four coordinate systems defined for the FOWT in uBEM	65
3-2	Angles related to a wind turbine rotor	67
3-3	Motions related to a wind turbine rotor	68
3-4	Angles related to a wind turbine blade	68
3-5	Control volume assumption to explain the Glauert equation (adapted from [8])	75
3-6	Flowchart of uBEM algorithm	82
4-1	Effective wind speed experienced by rotor during surging motion	86
4-2	Distribution of twist and chord against non-dimensionalised radius (r/R)	87
4-3	O-grid generated around the rotor blade	89
4-4	y^+ over the blade during static simulation	89
4-5	Front view of computational domain	90
4-6	Side view of computational domain	90
4-7	Displacement of the rotor against time in CFD simulations	92
4-8	Percentage difference in rotor thrust relative to medium mesh for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$	93
4-9	Percentage difference in rotor shaft torque relative to medium mesh for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$	94
4-10	Percentage difference in rotor thrust relative to medium mesh for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$	94
4-11	Percentage difference in rotor shaft torque relative to medium mesh for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$	95
4-12	Percentage difference in rotor thrust relative to medium time step for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$	96
4-13	Percentage difference in rotor shaft torque relative to medium time step for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$	96
4-14	Percentage difference in rotor thrust relative to medium time step for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$	97
4-15	Percentage difference in rotor shaft torque relative to medium time step for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$	97

5-1	Distribution of F against r/R at $\lambda = 13.19$, using different weighting factors	106
5-2	Distribution of F against r/R at $\lambda = 10.72$, using different weighting factors	106
5-3	Distribution of F against r/R at $\lambda = 8.06$, using different weighting factors	107
5-4	Distribution of F against r/R at $\lambda = 5.50$, using different weighting factors	107
5-5	Optimisation process for A and B in equations (5–15) and (5–16)	109
5-6	Distribution of F against r/R at $\lambda = 13.19$, using the windmill state model, compared to F extracted from CFD	110
5-7	Distribution of F against r/R at $\lambda = 10.72$, using the windmill state model, compared to F extracted from CFD	111
5-8	Distribution of F against r/R at $\lambda = 9.90$, using the windmill state model, compared to F extracted from CFD	111
5-9	Distribution of a against r/R at turbulent wake state λ	112
5-10	Distribution of F against r/R at $\lambda = 13.19$	114
5-11	Distribution of F against r/R at $\lambda = 8.06$	115
5-12	Distribution of F against r/R at $\lambda = 4.95$	115
5-13	Total relative F error against λ	116
5-14	Distribution of thrust (kN) against r/R at $\lambda = 13.19$	117
5-15	Distribution of thrust (kN) against r/R at $\lambda = 8.06$	117
5-16	Distribution of thrust (kN) against r/R at $\lambda = 4.95$	118
5-17	Total relative thrust error against λ	118
5-18	Distribution of shaft torque (kN) against r/R at $\lambda = 13.19$	119
5-19	Distribution of shaft torque (kN) against r/R at $\lambda = 8.06$	120
5-20	Distribution of shaft torque (kN) against r/R at $\lambda = 4.95$	120
5-21	Total relative shaft torque error against λ	121
6-1	Best fit curves for LS(1)-0421MOD (top), S809 (middle) and S815 (bottom) airfoil suction surfaces using equation (6–30)	138
6-2	Plot of T_P against τ and quadratic regression	139
6-3	Plot of T_V against A and quadratic regression	140

6-4	Plot of T_f against $\tau(A)$ and quadratic regression	140
6-5	Plot of K_α against $\tau(A)$ and quadratic regression, with anomalous points circled in red	141
6-6	Plot of normalised between the model constants in the shape-based model and general optimisation, and shape-based model and airfoil-specific optimisation against change in error	143
6-7	Coefficient of lift for oscillatory pitching motion of LS(1)-0421MOD airfoil, at $Re=7.5 \times 10^5$, Amplitude= 5° , Frequency= 1.85Hz and Mean $\alpha=14^\circ$	144
6-8	Coefficient of lift for oscillatory pitching motion of NACA 4415 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 10° , Frequency= 1.85Hz and Mean $\alpha=8^\circ$	145
6-9	Coefficient of lift for oscillatory pitching motion of S801 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 5° , Frequency= 1.85Hz and Mean $\alpha=20^\circ$	145
6-10	Coefficient of lift for oscillatory pitching motion of S814 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 10° , Frequency= 1.85Hz and Mean $\alpha=14^\circ$	146
7-1	Lambda against time for different blade segments for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$	153
7-2	C_L against time for $r/R = 0.19$ for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using pre-applied and real-time stall-delay	153
7-3	C_L against time for $r/R = 0.32$ for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using pre-applied and real-time stall-delay	154
7-4	C_L against time for $r/R = 0.45$ for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using pre-applied and real-time stall-delay	154
7-5	Predicted thrust against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using pre-applied and real-time stall-delay	155
7-6	Predicted shaft torque against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using pre-applied and real-time stall-delay	155
7-7	Predicted thrust against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using static airfoil aerodynamics and unsteady aerodynamics model, compared to CFD	156
7-8	Predicted shaft torque against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using static airfoil aerodynamics and unsteady aerodynamics model, compared to CFD	157

8-1	Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD . .	163
8-2	Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD . .	163
8-3	Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD . .	164
8-4	Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD	165
8-5	Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD	165
8-6	Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD	166
9-1	Thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$ extracted from CFD, and effective wind velocity of rotor	170
9-2	Shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$ extracted from CFD, and effective wind velocity of rotor	171
9-3	Thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ extracted from CFD, and effective wind velocity of rotor	171
9-4	Shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ extracted from CFD, and effective wind velocity of rotor	172
9-5	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, compared to CFD	173
9-6	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, compared to CFD	173
9-7	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, compared to CFD	174

9-8	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, compared to CFD	175
9-9	Axial induction factor against time for 5 different blade segments	175
10-1	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, compared to CFD	178
10-2	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, compared to CFD	178
10-3	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, compared to CFD	179
10-4	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, compared to CFD	179
10-5	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, compared to CFD	180
10-6	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, compared to CFD	180
10-7	Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque	181
10-8	Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque	181
10-9	Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque	182
10-10	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, compared to CFD	183
10-11	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, compared to CFD	183
10-12	uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, compared to CFD	184
10-13	uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, compared to CFD	184

10-14uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, compared to CFD	185
10-15uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, compared to CFD	185
10-16Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque	186
10-17Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque	186
10-18Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque	187
10-19Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque	188
10-20Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque	188
10-21Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque	189

LIST OF SYMBOLS

Greek Symbols

α	Angle of attack for an airfoil	$[rad]$
β	Twist angle of blade cross-section	$[rad]$
Γ	Circulation	$[m^2s^{-1}]$
γ	Angle of orientation of floating platform	$[rad]$
Δ	Denotes the change in a variable	
θ	Angle of orientation of rotor or blade	$[rad]$
Λ	Blade element rotational speed ratio	
λ	Rotor tip speed ratio	
ν	Kinematic viscosity	$[m^2s^{-1}]$
ρ	Density of fluid	$[kgm^{-3}]$
τ	Thickness parameter for airfoil shape Time constant for dynamic wake model	$[m^{-1}]/[s^{-1}]$
ϕ	Flow angle	$[rad]$
χ	Wake skew angle	$[rad]$
ω	Angular rotational rate of rotor	$[rads^{-1}]$

Roman Letters

A	Area Constant for corrective model in tip-loss model Airfoil curvature parameter	$[m^2]$.
$\mathbf{A}$	Velocity vector component due to rotor motion	$[ms^{-1}]$
A_i	Circulatory lift time constant, $i = 1, 2$	
a	Axial induction factor	
a'	Rotational induction factor	
B	Constant for corrective model in tip-loss model Number of rotor blades	
$\mathbf{b}$	Transformation matrix	
b_i	Circulatory lift time constant, $i = 1, 2$	
C_D	3-dimensional coefficient of drag	
C_d	2-dimensional coefficient of drag	
C_L	3-dimensional coefficient of lift	
C_l	2-dimensional coefficient of lift	
C_M	Coefficient of moment	
C_n	Force coefficient normal to rotor plane	
C_P	Coefficient of power	
C_T	Coefficient of thrust	
C_t	Force coefficient tangential to rotor plane	
c	Airfoil chord length	$[m]$
D	Decay function	
F	Tip-loss factor	
$\mathbf{f}$	Force vector	$[kgms^{-2}]$
f	Position of flow separation relative to chord length	
f_g	Glauert correction factor, dimensionless	
$\mathbf{G}$	Arbitrary vector	

g	3-dimensional boundary layer correction for lift, dimensionless	
gd	3-dimensional boundary layer correction for drag, dimensionless	
H	Hub height	$[m]$
K_α	Impulsive lift time constant	
k	Reduced frequency	
M	Torque generation force	$[kgms^{-2}]$
$\dot{m}$	Mass flow rate	$[kgs^{-1}]$
$\mathbf{n}$	Vector $(0, 0, -1)$	
p	Pressure	$[kgm^{-1}s^{-2}]$
P_y	Force in y -direction	$[kgms^{-2}]$
P_z	Force in z -direction	$[kgms^{-2}]$
r	Local radius along blade	$[m]$
R	Rotor radius	$[m]$
s	Dimensionless time	
T	Thrust Time constant	$[kgms^{-2}]$
t	time	$[s]$
U	Speed	$[ms^{-1}]$
u	Wind speed at rotor plane	$[ms^{-1}]$
u_1	Wind speed in far wake	$[ms^{-1}]$
V_0	Undisturbed wind speed (scalar)	$[ms^{-1}]$
$\mathbf{V}_0$	Undisturbed wind velocity (vector)	$[ms^{-1}]$
$\mathbf{V}'$	Wind velocity a rotor plane	$[ms^{-1}]$
$\mathbf{W}$	Induced velocity	$[ms^{-1}]$

Abbreviations

ALE	Arbitrary Lagrangian-Eulerian methods
ALM	Actuator line method
BEM	Blade element momentum
DRM	Direct rotor modelling
FOWT	Floating offshore wind turbine
FVW	Free vortex wake
GDW	Generalised dynamic wake
uBEM	Unsteady blade element momentum method

Chapter 1

Introduction

1.1 Significance

The science of extracting energy from wind is not a new one, yet presents many complexities that are yet to be fully understood today, despite significant research being carried out in this field. Wind turbines represent a culmination of this research, converting mechanical wind energy into electrical energy used to supply power grids worldwide. By mid-2013, the total capacity of wind turbines installed worldwide amounted to 3.5% of global electricity demand [1]. With rising global energy demand and increasing calls for renewable energy, it can be expected that this proportion will continue to increase.

The total installed wind capacity mostly comprises fixed-bottom wind turbines, either on land or in shallow waters. In fixed-bottom wind turbines, flow is often assumed axial and steady for modelling simplicity. Unsteadiness in the flow, though existent, may be assumed insignificant for most operating cases, with corrections to the steady-state model for periodic conditions such as skewed wake and tower effects. The sources contributing to a fixed-bottom wind turbine flow field structure are outlined in Figure 1-1.

Figure 1-1. Different contributing sources to a wind turbine flow field structure [2]

The abundance of ocean as compared to land means that Floating Offshore Wind Turbines (FOWTs) are increasingly becoming a viable option to satisfy this demand. It is predicted that the cost of building, installing and maintaining large floating platforms will be offset by the higher quality of offshore winds [3], which are stronger, more consistent and less turbulent. However, the floating structures introduce new and significant sources of unsteadiness to the flow field, due to 6 degree-of-freedom platform motions brought about by wave action.

1.2 Problem Description

Accurately predicting wind turbine aerodynamics is non-trivial, but an important part of wind turbine design, maintenance, failure analysis and wind farm optimisation. Wind turbine aerodynamics may be divided into two inter-related components, the rotor aerodynamic forces and the flow field momentum balance. The airflow around the rotor gives rise to aerodynamic forces that causes it to rotate and extract energy from the wind. By Newton's third law, an equal and opposing force is exerted on the wind, which results in a velocity deficit at the rotor plane and in the wake, affecting the airflow around the rotor. The aerodynamics of a wind turbine can thus be described as the balance between the rotor aerodynamic forces and the flow around it, as displayed in Figure 1-2

Figure 1-2. Summary of components related to wind turbine aerodynamics

The flow field momentum balance is affected by turbulence, wind shear from operation in the atmospheric boundary layer, trailing tip and hub vortices from the rotor, the dynamic wake effect due to the time lag between rotor aerodynamic forces and its effect on the flow and the skewed wake effect, due to the changes in the flow field when the rotor plane is non-orthogonal to the wind.

The rotor aerodynamic forces are a result of the cross-sectional aerodynamics which include effects such as flow separation and unsteady aerodynamic forces, tip and hub losses, which affect any finitely bladed rotor, the distribution of forces along the rotor blade, and rotational effects which result in spanwise flow that delays the onset of stall.

FOWT rotors, subject to movements due to the 6 degree-of-freedom motions of the floating platform, will experience additional velocity components, which will change the rotor aerodynamic forces. Due to these motions, the rotor can be expected to operate in a skewed wake condition, when the rotor

is yawed or tilted with respect to the wind, which can be significant at angles in excess of 10° . The unsteadiness due to the additional velocity components arising from the platform motions also result in the dynamic wake and unsteady airfoil effects. These effects, explained in detail in Chapter 2, need to be modelled appropriately for an accurate computation of the aerodynamic forces on the wind turbine.

The platform motions would also cause significant dynamic structural stresses on the tower and blades, and affect the drive train dynamics, which transfer the mechanical energy from the rotor to the generator. The relationship between these components of the FOWT and the wind and wave input is complex, and requires a multi-physics, fully-coupled simulation to obtain an accurate assessment. Such a simulation is composed of individual components modelling the wave input prediction, floating platform dynamics, mooring line dynamics, tower structural dynamics, drive train dynamics, wind input prediction and rotor blade aerodynamics, communicating with each other to exchange information since the input of some components are the outputs of others. This requires a high degree of accuracy from each component. The varied scenarios and operating conditions of FOWTs would mean that many different simulations must be run for a single FOWT. Thus, components of the fully-coupled simulation must be able to execute at low computational cost.

Various methods of predicting FOWT aerodynamics exist, including highly complex full 3D CFD simulations of FOWTs, actuator disc models, vortex methods, the method of accelerating potential and the blade element momentum method (BEM), a preferred model in the industry, due to its simplicity and robustness. It uses simple momentum theory, by looking at the momentum balance in a streamtube of flow through the rotor. The blade is divided into segments, and each annular streamtube assumed independent. The momentum balance for each segment is used to find the local blade aerodynamic forces,

which in turn is used to find the flow velocity at the rotor. This iterative procedure forms the solution algorithm. However, this is based on very simplistic assumptions, such as a 1-dimensional momentum balance, an infinitely-bladed wind turbine and laminar flow, leading to many shortcomings, which are corrected with specific models, such as tip-loss, Glauert correction for high axial induction, and skewed wake models. These corrections have made BEM into a robust method that can accurately predict the aerodynamics of fixed-bottom wind turbines. However, being a steady-state model, it cannot model unsteady effects, and annulus-averaged forces of the blade elements mean that individual blade force distributions cannot be attained for structural analyses. To overcome these limitations, developments have been made to the unsteady BEM method (uBEM). This deviates from BEM by solving the momentum balance for each segment of each blade (or $1/B^{th}$ of an annulus, where B is the number of blades). This allows for uBEM to solve for flow conditions specific to each segment of each blade. The uBEM method has shown to have accuracy comparable to BEM for static rotor cases. The treatment of each blade separately allows for the inclusion of dynamic wake effects and dynamic local airfoil effects, meaning that uBEM can potentially model the unsteady aerodynamics of FOWTs with a high degree of accuracy, depending on the corrective models.

1.3 Objectives & Scope

The present research can thus be summarised as using experimental data or higher order methods, such as direct rotor modelling to simulate steady and unsteady flow environments for a wind turbine, to generate empirical relations and form a turbine-specific uBEM model which is applicable in wide-ranging flow scenarios.

The flow phenomena studied include tip-loss, the unsteady airfoil effect, stall-delay and transient wake states. Corrective models are developed for these and integrated into uBEM.

The scope of the research is limited to the study of the NREL 5MW virtual wind turbine [4] in steady and unsteady axial flow scenarios.

1.4 Contributions of the Thesis

Within the objectives described in Section 1.3, the contributions of this thesis are as follows:

1. Justification of Glauert's assumption [5] for the axial induced velocity in yaw.
2. Novel tip-loss model valid over a range of tip speed ratios and flow states.
3. Improved Beddoes-Leishman model [6] based on airfoil shape parameters.
4. Novel combined unsteady airfoil and stall-delay model.
5. An extension of the Glauert correction [7] for vortex ring, propeller brake and propeller brake states.
6. An unsteady blade element momentum (uBEM) model incorporating all the corrective models developed within the scope of the present research.

1.5 Organisation of Thesis

The development of corrective models for uBEM requires the different flow aspects to be studied independently. Thus, the study of each flow phenomenon would possess its own methodology specific to the phenomenon.

The layout of the thesis as follows:

Chapter 2: Review of literature associated with FOWT aerodynamics, including the FOWT

motions, associated aerodynamic phenomena, methods of analysis and experimental and computational work.

Chapter 3: Description of the unsteady Blade Element Momentum method.

Chapter 4: CFD methodology, including initial and boundary conditions, mesh setup, mesh

refinement study, and dynamic simulation setup.

- Chapter 5: A survey of tip-loss models and development of a novel methodology for changing tip speed ratios and wake states.
- Chapter 6: An overview of unsteady airfoil models and development of a novel unsteady airfoil model based on airfoil shape parameters.
- Chapter 7: Development of a novel combined stall-delay and unsteady airfoil model for applications in unsteady FOWT environments.
- Chapter 8: Review of dynamic wake models and optimisation of a 2-equation dynamic wake model for FOWT applicability.
- Chapter 9: Development of an extended Glauert model for application in FOWT aerodynamics.
- Chapter 10: Results of uBEM computations against CFD simulations and discussions.
- Chapter 11: Conclusion and future work.

Chapter 2

Literature Review

2.1 Wind Turbine Aerodynamics

Wind turbine aerodynamics is composed of two inter-related and simultaneously working components, the flow field momentum balance, and the rotor aerodynamic forces, as was summarised in Figure 1-2.

2.1.1 Momentum Theory

In the momentum theory, the rotor is treated as a permeable actuator disc, across which there is a discontinuous drop in pressure. This is demonstrated through Figure 2-1.

Figure 2-1. Illustration of the streamlines past the rotor, and axial velocity and pressure distributions upstream and downstream of the rotor (subscript 0 represents freestream values) [8]

With reference to Figure 2-1, the thrust on the rotor is equal to the perpendicular rotor disc area multiplied by the pressure drop across the rotor.

$$T = \Delta p A \quad (2-1)$$

Applying the Bernoulli equation upstream and downstream of the rotor (to avoid the pressure discontinuity), the pressure drop can be found.

$$p_0 + \frac{1}{2}\rho V_0^2 = p + \frac{1}{2}\rho u^2 \quad (2-2)$$

$$p - \Delta p + \frac{1}{2}\rho u^2 = p_0 + \frac{1}{2}\rho u_1^2 \quad (2-3)$$

Subtracting (2-2) from (2-3), the pressure drop, thus the thrust is found.

$$\Delta p = \frac{1}{2}\rho (V_0^2 - u_1^2) \quad (2-4)$$

$$T = \Delta p A = \frac{1}{2}\rho A (V_0^2 - u_1^2) \quad (2-5)$$

If the streamlines in Figure 2-1 form a closed streamtube, then the law of conservation of mass will yield the mass flow rate.

$$\dot{m} = \rho V_0 A_0 = \rho u A = \rho u_1 A_1 \quad (2-6)$$

The thrust on the rotor is found by using the law of conservation of momentum.

$$T = \dot{m} (V_0 - u_1) = \rho u A (V_0 - u_1) \quad (2-7)$$

Equating (2-5) and (2-7), a relationship between the three velocities is found.

$$u = \frac{1}{2} (V_0 + u_1) \quad (2-8)$$

Defining the axial induced velocity as $W_n = V_0 - u$, and substituting into (2-8) yields:

$$2W_n = (V_0 - u_1) \quad (2-9)$$

Substituting equation (2-9) into (2-7) allows for the calculation of the induced velocity.

$$T = 2\rho u A W_n \quad (2-10)$$

$$W_n = \frac{T}{2\rho u A} \quad (2-11)$$

The axial induced velocity can be non-dimensionalized with respect to the wind speed, V_0 . This yields the axial induction factor, a .

$$a = \frac{W_n}{V_0} \quad (2-12)$$

The velocity at the rotor plane can thus be defined using the axial induction factor.

$$u = V_0 (1 - a) \quad (2-13)$$

Substituting (2-12) and (2-13) into (2-10), a coefficient of thrust, C_T , can be defined for the rotor.

$$C_T = \frac{T}{\frac{1}{2}\rho V_0^2 A} = 4a(1 - a) \quad (2-14)$$

2.1.2 The Blade Element Theory

The blade element theory was created for the computation of airplane propeller performance. The theory states that a propeller or a rotor blade can be divided into segments for aerodynamic analysis. Once the aerodynamic forces are determined, these can be integrated over the whole blade and over one revolution to determine the thrust and torque produced.

At the blade element level, the blade cross-section is observed, and its orientation is related to the flow. The angles involved are the flow angle, ϕ , the angle of attack, α , and the twist, β , shown in Figure 2-2.

Figure 2-2. Angles related to a blade element

The flow angle, ϕ , is the angle between the relative wind velocity vector and the rotor plane. This is a summation of the other two angles, the angle of attack, α , which is the angle between the relative wind velocity vector and the airfoil chord, and the twist, β , which is the angle between the airfoil chord and the rotor plane.

Using these angle definitions, the coefficients of lift and drag, and thus the force coefficient in the thrust direction and in the rotational direction can be obtained.

2.1.3 Effects of Rotation on Aerodynamic Forces

One method of obtaining aerodynamic forces is by determining the angle of attack through the flow-field momentum balance while concurrently using the 2-dimensional lift and drag coefficients of the airfoil cross-section of the blade, based on previous experimental results. The effect of rotation on the blade cross-sectional aerodynamics should be accounted for before employing this methodology. At angles of attack when stall occurs, greater cross-sectional aerodynamic forces have been observed. The radial velocity imparted on the

air through the Coriolis effect contributes to the suppression of flow separation, in a phenomenon known as stall delay [9].

Tangler [10] compared the power output from field-testing a 4m-diameter wind turbine and predictions based on the Blade Element Momentum (BEM) method. The predicted power output was lower at lower tip speed ratios, where stall phenomena are expected to be significant.

Butterfield et al [11] observed significant stall-delay at lower blade stations for a Grumman Windstream 33 wind turbine [12]. It was found that the negative pressure coefficient over the suction surface of the airfoil was responsible for the observed stall delay, while the pressure distribution over the pressure surface of the airfoil remained relatively constant, as shown in Figure 2-3. The comparison of the 3-dimensional and 2-dimensional coefficients of lift and drag for rotor cross-sections (S809 airfoil) at different radii are shown in Figure 2-4.

Figure 2-3. Pressure distributions for post-stall angles of attack, at 30% blade span, compared to 2D distribution (black filled circles) [11]

Figure 2-4. Coefficients of lift for a wind turbine blade cross sections, at various blade spans, compared to 2D distribution (black line) [13]

The stall delay effect has been studied extensively, with many stall delay models proposed to predict the stall delay effect.

Snel et al [14], Chaviaropoulos and Hansen [15], Du and Selig [16] and Lindenburg [17] proposed models with a stall delay weightage factor, g , that resulted in a 3-dimensional lift coefficient that was a weighted average of the inviscid lift and the 2-dimensional measured lift.

$$C_l = g \cdot C_{l\alpha} \cdot (\alpha - \alpha_0) + (1 - g) \cdot C_{l,2D} \quad (2-15)$$

$$C_d = g \cdot C_{d0} + (1 - g) \cdot C_{d,2D} \quad (2-16)$$

where C_l and C_d are the 3D coefficients of lift and drag respectively, $C_{l\alpha}$ is the lift curve slope, α_0 is the zero-lift angle of attack, C_{d0} is the drag at $\alpha = 0$ and $C_{l,2D}$ and $C_{d,2D}$ are the 2D coefficients of lift and drag respectively.

Each author has proposed a different form for g , depending on properties such as local chord length, radius and speed ratios. Each of these models is semi-empirical in nature, and was tested against different wind turbines. Breton et al [18] studies the accuracy of these models against the NREL UAE Phase VI [19] data by implementing them in a prescribed wake vortex code. Shortcomings were found in predicting the aerodynamics near the tip, but these

may be due to tip losses instead of stall delay. It was also suggested to include a "cut-off" angle of attack where the stall-delay model is no longer used, since after the flow is fully separated, the radial flow component within the boundary layer will have minimal impact on the aerodynamic forces.

2.1.4 Tip Losses

One assumption of one-dimensional momentum theory is an infinitely bladed rotor. This simplification results in an unrealistic distribution of forces, since there are no trailing vortices from the rotor blades. In reality, trailing vortices from a finitely-bladed rotor reduce the aerodynamic lift produced by the rotor.

Trailing vortices are a result of Helmholtz's second theorem, which states that a vortex filament cannot end in a fluid, thus the vortex is trailed into the wake. The uneven distribution of circulation over the blade span means that trailing vortices occur at all positions along the blade span, creating a vortex sheet. The strength per unit length of the trailing vortex is equal to the gradient of the circulation along the blade. At the blade tip, the circulation must be zero, thus, the strongest trailing vortex is found at the tip [20]. Similarly, strong trailing vortices would be found at the hub of the wind turbine as well. Since it is highly pronounced at the tip, the phenomenon is known as tip-loss. In this thesis, the phenomenon for this form of loss of lift at both the tip and hub will be referred to collectively as tip-loss.

An exact distribution for tip-loss depends on the distribution of circulation along the blade, which, in turn, depends on tip speed ratio, airfoil shape distribution and twist distribution along the blade. Wood et al [21] developed tip-loss models which were successfully applied to the Betz-Goldstein optimised rotors, with helical wakes, which showed that the local flow velocity computed using the momentum equations could be used simply to determine the tip-loss.

Prandtl [22] formulated the optimal distribution of circulation along the blade to minimise these losses, based on which, a general form of tip-loss

model was created. The Prandtl model and other tip-loss models are discussed in further detail in Chapter 5.

2.1.5 Turbulent Wake State

The relationship between the wake flow and the aerodynamic forces on the wind turbine are dependent on the wake flow structure. These can be generalised into several wake states, as illustrated in Figure 2-5.

Figure 2-5. Different wake states of a wind turbine. [23]

During regular wind turbine operation, two wake states are considered. These are based on the extent of the reduction of the wind velocity in the wake, or the ratio of the induced velocity to the ambient wind speed. The first state is the windmill state, when the rotor acts in accordance to idealised momentum theory, detailed in Section 2.1.1, where the coefficient of thrust, C_T , is given in equation (2-10).

The windmill state occurs at higher wind speeds. At lower wind speeds, the wake flow starts to recirculate, generating a turbulent wake state. This occurs due to the difference in flow speeds between the freestream and within the wake being large enough for the free shear layer between the wake and

freestream to become unstable, generating eddies which bring momentum from the freestream into the wake [8]. Here, the idealised momentum theory breaks down, since it would predict a drop in the thrust force, which does not occur in reality, as shown in Figure 2-5.

Since rated wind speeds are different for different wind turbines, it is simpler to describe occurrence of the turbulent wake state by axial induction factor; the wake state usually occurs at $a > 0.4$.

Based on the experimental data shown in Figure 2-5, Glauert proposed an equation to model the relationship between C_T and a in the turbulent wake state [7].

$$C_T = 4aF \left(1 - \frac{1}{4} (5 - 3a) a \right), a > \frac{1}{3} \quad (2-17)$$

where F is the tip-loss factor.

Despite the inclusion of F in 2-17, discontinuities still existed at $a = 0.4$ between the momentum theory and Glauert equation, at values of F other than 1. This was corrected by Buhl [24], in an equation that would ensure the continuity with momentum theory at $a = 0.4$ and have a value of $C_T = 2$ when $a = 1$, for all values of F .

$$C_T = \left(\frac{8}{9} \right) + \left(4F - \frac{40}{9} \right) a + \left(\frac{50}{9} - 4F \right) a^2 \quad (2-18)$$

2.2 FOWT Unsteady Aerodynamics

FOWTs are deployed in deep sea, which afford them significant advantages such as improved wind conditions [25] and reduced visual and noise impact due to their distance from land. This deployment is not viable without using a moored floating platform, which would be subject to 6 degree-of-freedom movements due to the action of the waves. This creates complex flow scenarios with regard to the wind turbine due to its operation in a primarily unsteady environment, which are not significant to fixed-bottom wind turbines.

2.2.1 Sources of Unsteadiness in FOWTs

The movements from the floating platforms and their effects on the FOWTs will be discussed in this section. The 6 degree-of-freedom motion directions may be described as shown in Figure 2-6 with three translational and three rotational directions.

Figure 2-6. 6 degree-of-freedom motions of a floating platform

Figure 2-6 presents that the motions of the platform in the rotational directions create translational and rotational motion components in the FOWT rotor, which would be greatest at the tips of the rotor blades. Thus there are additional velocity components that can be significant when the movement in all 6 directions is accounted for.

A study was performed by Sebastian and Lackner [26] for the motions of the spar-buoy type floating platform using the FAST aero-servo-hydro-elastic

code from NREL [27] to simulate the response of the NREL 5MW baseline turbine [4] with a Hywind spar-buoy platform [28] to generated waves and wind to simulate FOWT operating conditions.

Figure 2-7. Predicted spar-buoy platform motions with an NREL 5MW turbine (generated using FAST). Wind input generated using IEC von Karman turbulence model and mean $V_0 = 10ms^{-1}$, nominally axial flow. Wave kinematics modelled using JONSWAP spectrum with significant incident wave height of $6m$ and peak spectral period of 10 seconds. [26]

Figure 2-7 shows the predicted motions of a spar-buoy platform. The surge direction motion indicates high amplitude, reaching a 23m displacement. In addition, the pitch and yaw angles of the floating platform demonstrate a non-zero mean and motions from 0° to 5° for pitch and a similar range of -1.5° to 3.5° for yaw. This demonstrates that the FOWT is expected to operate almost continuously with its rotor plane non-orthogonal to the wind direction, resulting in a skewed wake, which will be discussed further in Section 2.2.2.3. Furthermore, the motions of the floating platform result in additional wind components on the FOWT, which is seen in Figure 2-8.

Figure 2-8. Changes in effective wind speed at blade tips due to predicted motions of NREL 5MW turbine motions of the spar-buoy platform (Based on motions given in Figure 2-7) [26]

Figure 2-8 reveals that the movements of the platform can result in additional velocity components to the FOWT blades. It displays the largest possible additional wind components that a segment of the FOWT blade can experience due to the movements in Figure 2-7) by computing the movements at the blade tips. Roll motions were not included. Instead, the surge velocity due to platform pitching of a blade facing downwards (leftmost plot, bottom row) and upwards (centre plot, bottom row) are displayed.

The additional velocity component due to the surge motions as well as the pitch (blade upwards) and yaw motions display high amplitudes and frequencies. These components were computed to be as high as 1.5m/s, which is 15% of the ambient wind speed in the simulation [26]. A change of more than 2.5m/s in a time period shorter than 10 seconds was displayed as a result of the platform motions. This shows that the changes in the velocity components faced by the

FOWT rotor blades is large and can occur at high rates, contributing to the high degree of unsteadiness in the flow.

Bagbanci [29] showed that different platform types had different dominant directions of motions. The barge platform showed similarly high displacement amplitudes for the surge, pitch and yaw motions, but very high frequencies in heave, pitch and roll motions. The semi-submersible platform WindFloat [30] showed a very high displacement amplitude for pitch only, high frequencies for surge, pitch and roll and a very high frequency for heave motion.

The frequencies of these motions are an important parameter as they reflect the rate of change of flow conditions faced by the rotor. In addition, the pitching motion was significant in both frequency and amplitude for all the floating platform types. The pitching motion of the platform results in a pendulum-like motion for the rotor, adding to the surge velocity component, as shown in Figure 2-9.

The surge motion was shown to be significant in the spar-buoy and barge platforms, and acts generally in the wind direction. This implies that it will have a direct impact on the velocity and rate of change of velocity faced by the entire rotor. In addition, the motion may result in the rotor moving into its own wake, creating more complex flow structures that require analysis for accurate prediction of the FOWT aerodynamics.

Figure 2-9. Pendulum-like motion of FOWT rotor resulting from pitching of the floating platform

2.2.1.1 Rotor and tower structure

Simple analyses of wind turbine aerodynamics may assume rigid bodies for both the blades and tower structures. For smaller wind turbines, this is a valid assumption, since the deflection of the tower and the blades would be negligible. For FOWTs, which use larger wind turbines, this assumption must be re-evaluated: blade and tower deflections may still be small compared to other effects, but their combined effect may be significant.

The rotor diameters for FOWTs can exceed 160m, as shown in Figure 2-10, meaning a blade of over 80m in length.

The NREL 5MW wind turbine has a blade length of 61.5m [4]. Stølsmark [31] showed that the blade tip deflection for a wind speed for 11.4m/s under waves of 5m height and peak spectral period of 12s could be as high as 9m. In the OC3 Phase 4 code comparison test by Jonkman and Musial [32] significant blade deflection was also reported for the various conditions run, with a deflection of 4.5m for conditions listed in load case 5.1 [32, p. 57].

Figure 2-10. Growth trend for offshore wind turbine rated capacity and rotor diameter [33]

These studies show that the blade deflection is significant. This deflection pushes the blade tips into the wake, resulting in reduced performance. Furthermore, the deflections were non-static, meaning dynamic flow conditions for the wind turbine blade, especially close to the tip. These effects must be accounted for by means of an aeroelastic simulation, coupling the aerodynamics with a structural code to predict the blade deflection. While the structural dynamics of the blade is not within the scope of the present work, the aerodynamics depends on the blade deflection, which depends on an accurate force distribution prediction along the blade. Thus, the aerodynamics tool developed must be able to provide time-accurate predictions for rotor performance as well as specific force distributions on the blades.

Jonkman and Musial [32] also showed that the tower-top deflection under load case 5.1 was minimal, peaking at less than 0.6m. The insignificant tower deflection was also found by Christiansen et al [34]. Compared to a height of 90m [4], this is insignificant, but the motion frequencies may be high and result in high velocities in the rotor. This would require further analysis to understand the wind velocity components due to these motions.

2.2.2 Unsteady Aerodynamic Phenomena

The sources of unsteadiness highlighted in Section 2.2.1, result in highly significant unsteady aerodynamic phenomena in FOWT aerodynamics. Some phenomena have been studied and modelled for specific cases involving bottom-fixed wind turbines, or have been studied for rotorcraft aerodynamics. The phenomena discussed in this section will include dynamic wake, unsteady airfoil effects, skewed wake, wind shear and in some extreme pitching surging motions of the platform, transient wake states.

2.2.2.1 Dynamic wake effect

Dynamic wake effect refers to the time lag in achieving steady wake conditions, brought about by large scale effects affecting the most of or the entire wind turbine rotor, such as collective blade pitching, wind gusts or rotor speed

or sudden changes in rotor thrust [2]. This lag is due to the fact that the flow field cannot immediately change to reflect changes in the rotor aerodynamic forces, as a result of the inertia of the air mass in the wake [35]. Effects such as yawed flow and partial span pitch can also result in dynamic wake. In the case of yawed flow, the aerodynamic forces will change periodically, resulting in a time delay [36].

The wake flow field influences the induced velocity. This affects the angle of attack of the rotor blade. In a steady wind, this angle of attack would be constant, since the rotor aerodynamic forces would be steady, resulting in a steady effect on the flow field, and thus a steady influence on the rotor aerodynamics. In unsteady flow conditions, there are two effects to be considered:

1. The angle of attack experienced by the rotor blade cross-sections, thus aerodynamic force distribution is dependent on the wake velocities.
2. The wake flow is dependent on the force distribution on the rotor blades, which is dependent on the rotor aerodynamic forces.

The first part refers to the unsteady airfoil effects experienced by the blade cross-sections, which will be described in Section 2.2.2.2.

The second part describes the dynamic wake effect. The wake velocities are a result of the time-varying trailing wake vortices, and the response of the trailing vortices to changes in the rotor aerodynamic forces results in the dynamic wake effect.

Experiments measuring the response of a wind turbine to a step increase and decrease in the angle of attack of the blade were sponsored by the European Commission in a joint project, and conducted using the Tjæreborg 2MW wind turbine [37].

Figure 2-11. Measured rotor shaft torque on the Tjæreborg wind turbine for a rotor blade step pitch from 0.1° to 3.7° in 0.5s and back [37]

Figure 2-11 shows the measured rotor shaft torque when the rotor blade was pitched. An overshoot in the rotor shaft torque was observed when the blade was pitched, and the torque then decayed to a fairly steady value. This is due to the fact that when the blade was pitched, the wake velocities did not immediately change to reflect the steady-state wake velocities at the new pitch angle. Instead, there was a time delay that resulted in a higher angle of attack, as the induced velocities had not increased to reflect the change in aerodynamic forces. The angle of attack experienced by the blade then slowly decayed to the steady-state value [36].

Modelling the dynamic wake effect using these step changes in flow conditions is useful, since any arbitrary change in flow conditions can be divided into a series of step changes. Thus, modelling these step changes accurately should provide a good platform to model the dynamic wake effect in more complex flow scenarios.

Dynamic wake effects have been shown to be dependent both on rate of change, for example, the rate of pitching, and on axial induction. At high axial

induction factors, as well as at low pitching rates, dynamic wake effects have been shown to be less significant [36].

Since the rate of change is an important factor in the significance of dynamic wake, a characteristic time scale for this effect is defined as D/U , where D is the rotor diameter and U is the wake velocity [38]. Phenomena with a time scale larger than D/U can be considered quasi-steady [39]. For the 5 MW virtual turbine from NREL, this ranges from 3.5-8.5, based on values with cut-out wind speed at rated RPM and cut-in wind speed at cut-in RPM.

Many methods of modelling the aerodynamics of wind turbines inherently include the dynamic wake effect, and will be discussed in Section 2.4. Methods that compute the steady-state aerodynamics of wind turbines would require a dynamic wake model to account for the time lag in the induced velocity. The different models will be discussed in Chapter 8.

2.2.2.2 Unsteady airfoil effects

Along a large wind turbine blade, different airfoil segments would experience different incoming wind conditions, which change periodically or aperiodically due to wind shear, yaw/pitch of the rotor, tower effects and movement of the rotor relative to the incoming wind and its wake. These changes would have large effects on the aerodynamics of the airfoil cross-sections of the wind turbine blades. Generally, the unsteady airfoil aerodynamics is divided into two phenomena, attached flow phenomenon and post-separation phenomenon.

The degree of unsteadiness of an airfoil is an important measure of whether unsteady airfoil aerodynamics effects would be significant. This can be gauged by means of a reduced frequency analysis. The reduced frequency, k , is the ratio of the product of the angular oscillation frequency, ω , and half-chord length of the airfoil, $c/2$, to the freestream velocity, V :

$$k = \frac{\omega c}{2V} \quad (2-19)$$

Using the reduced frequency, the degree of unsteadiness can be determined based on Table 2-1. As a dimensionless parameter, the use of the half-chord, as compared to simply using the chord length, is completely arbitrary, as the same conclusions and comparisons can be made. However, the concept of reduced frequency did not originate from wind turbine aerodynamics, but from helicopter aerodynamics, thus the use of half-chord. Since this is the accepted formulation for reduced frequency in literature on wind turbine aerodynamics, it is presented here as well.

Table 2-1. Degree of flow unsteadiness based on reduced frequency [40]

Reduced Frequency	Flow Type
$k = 0$	Steady
$0 < k < 0.05$	Quasi-steady
$0.05 < k < 0.2$	Unsteady
$k > 0.2$	Highly Unsteady

Thus, values for k above 0.05 should be treated as unsteady, where unsteady airfoil effects would be significant. Due to the changing chord distribution along a wind turbine blade, the unsteady airfoil phenomenon may not be widespread, and could be localized to certain regions of the blade. The locality and span of the unsteady airfoil phenomenon would thus have an important bearing on the significance of the effect on the overall aerodynamics of the rotor.

Figure 2-12. Comparison between steady and dynamic lift coefficients of an S809 airfoil using the Ohio State University experimental data (data from [41])

The problem with this classification of flow unsteadiness by Leishman [40] is that it was produced for helicopter rotors, which consist of thin airfoils. In wind turbine rotors, however, the airfoils are very thick. From Figure 2-12, significant deviation from steady behaviour can be observed at reduced frequencies lower than 0.05. While the value of $k = 0.05$ is still quoted by some literature, another classification by Leishman [2] places this figure at a more conservative 0.01. Another value published by Pereira et al [42] is 0.02, but generally 0.05 can be considered too high a tolerance for quasi-steady flow behaviour. This classification is important when considering the degree of unsteadiness the cross sections of a rotor blade experience, as this will affect the aerodynamic forces computed. For the NREL 5MW rotor operating in yaw at 8m/s wind and rated RPM, the distribution of k along a blade is shown in Figure 2-13.

Figure 2-13. Reduced frequency distribution along a NREL 5MW rotor blade operating in yaw at 8m/s wind speed at rated RPM

Using the original classification, only half the blade would be operating in an unsteady environment and this may lead to erroneous results. Classifying $k > 0.01$ as unsteady would place the entire rotor blade in an unsteady environment, and the unsteady aerodynamics of the blade would be appropriately modelled.

The distinction between unsteady and highly unsteady behaviour is not clearly defined. It is obvious that in highly unsteady flows, the unsteady effects are more significant, but there are no models specifically for highly unsteady flows. However, one possible difference is that in highly unsteady flows, even when the Mach number is below 0.3, the unsteadiness may result in compressible flow. For wind speeds below $M=0.3$, the frequency of the unsteadiness must be far below the sonic frequency, or $\omega c/a < 0.3$. Frequencies higher than this would result in compressible flows over the airfoil [2]. However, the significance and frequency of such occurrences have not been determined, but are unlikely to be significant. For the largest chord along the NREL 5MW wind turbine, this corresponds to a motion frequency of 3.49Hz. This frequency may occur in a structural component on the FOWT, but may not result in significant

pitching or velocity changes along an airfoil chord. The effect of compressible flow due to the frequency of the motion of the airfoil is not known, and is not modelled in unsteady airfoil models. Instead, these models are usually formulated based on the local flow Mach number. Furthermore, modelling such compressibility effects would increase computational cost due to an increase in complexity of numerical strategy [2, 40].

In addition to the frequency of unsteadiness, the degree of change also plays a role in the degree of deviation from static behaviour. If the amplitude of the motion is small, the aerodynamic forces would likely be close to the static values. The coefficients of lift for different amplitudes of sinusoidal pitching are shown in Figure 2-14.

Figure 2-14. Coefficient of lift for different amplitudes of pitching for an S809 airfoil at $k = 0.077$ plotted using the Ohio State University experimental data (data from [41])

The attached flow phenomenon, also known as the Theodorsen effect, is an inviscid phenomenon that refers to the lag between change in local flow conditions and aerodynamic loads. This phenomenon was modelled using potential flow by Theodorsen [43], for 2-dimensional flat plate, using the half-chord as

the characteristic length, leading to the same characterization today. The attached flow phenomenon is a result of vortex shedding, increasing the circulation in the wake. By Kelvins theorem, the total amount of circulation does not change, thus the circulation around the airfoil must decrease, resulting in a reduction of lift [44].

The post-separation phenomenon can be divided further into two phenomena. The first is similar to the attached-flow phenomenon, but occurs in the non-linear region of the lift curve, where regular trailing edge (TE) stall occurs. The second, called dynamic stall, is a result of shedding of a vortex from the leading edge (LE) of the airfoil. This shed vortex creates a region of low pressure over the suction surface of the airfoil, thereby generating additional lift. As the vortex is shed, the lift increases momentarily and returns to its static stall value.

Figure 2-15. Dynamic stall (leading edge stall) process over an airfoil [40]

Figure 2-15 shows that for an airfoil pitching upwards, after static stall at stage 1, the vortex forms over the leading edge of the airfoil at stage 2, which coincides with the static moment stall of the airfoil. This then propagates over the airfoil resulting in an increase in lift. Once the vortex has passed the trailing edge of the airfoil, there is a sharp decrease in lift between stages 3 and 4. When the airfoil starts pitching downwards, the lift starts returning to the static value, between stages 4 and 5. Once the airfoil angle of attack is low enough, at stage 5, the flow reattaches and the lift is equal to the static lift.

Predicting this mechanism requires understanding when the leading edge vortex forms, and how long it takes to propagate along the airfoil surface. The dynamic stall, the Theodorsen effect and the impulsive lift are transients that will decay to steady-state values, and need to be modelled accordingly. The different modelling techniques for unsteady airfoil effects will be discussed in Chapter 6.

2.2.2.3 Skewed wake effects

When the wind turbine rotor plane is not orthogonal to the wind, meaning that the rotor is yawed or tilted with respect to the incoming wind, several factors must be taken into account:

- A. The difference in wind speed at different blade azimuth (A blade that is moving into the wind, versus a blade that is moving out).
- B. The difference in aerodynamic forces between rotor blades as a result of these differences.
- C. The difference in wake velocities at different rotor azimuths due to the different blade forces (which in turn affect the blade aerodynamic forces).

These factors have two results:

- 1. Cyclic blade loading in different phases for each blade.

2. A wake skewed from the direction normal to the rotor plane, with the skew angle, χ , larger than the angle of yaw or tilt, γ , as shown in Figure 2-16.

Figure 2-16. Difference between the wake skew angle χ , and the yaw angle γ , of a wind turbine from the top view [45]

The first result causes the dynamic wake and unsteady airfoil effects. Since the blade forces are changing over a rotation of the rotor, changing flow field momentum balance exists for the individual rotor blade causing the dynamic wake effect. Wake measurements from 3 downstream locations from a wind turbine rotor in axial flow and yawed flow by Andresen [46] showed that the wake recovery is greater for the wind turbine in yaw, as shown in Figure 2-17. The differences suggest a more complex interaction between the yawed rotor and the flow field, possibly due to the dynamic wake effect, since the effect of the aerodynamic forces is less significant further downstream, suggesting the time delay between the aerodynamic forces and wake velocities might promote a greater wake recovery.

(a) $\gamma = 0^\circ$

(b) $\gamma = 40^\circ$

Figure 2-17. Wake velocities, non-dimensionalised with wind velocity, against radial distance from the rotor hub at downstream distances of 1, 3 and 4.78 rotor diameters for yaw angles of (a) 0° and (b) 40° [46]

However, the lack of force distribution measurements along the wind turbine blade means that a clear relationship between the dynamic wake effect and the rotor yaw could not be established, since the expected steady-state induced velocities and the actual induced velocities cannot be compared. Such a comparison would enable a quantification of the dynamic wake effect, and any differences in the effect at different rotor azimuths.

Due to the azimuthal differences in wind and induced velocities, blade cross-sections periodically experience a change in flow velocity, meaning that the flow over the airfoil is unsteady. At large yaw angles, this can result in significant deviations from steady behaviour. A wind tunnel experiment of a wind turbine rotor in yaw by Haans [47] showed significant hysteresis of the lift coefficients of airfoil segments along the blade, as shown in Figure 2-18. This was even without dynamic stall, since static stall had not occurred.

Figure 2-18. Coefficient of lift distribution for airfoil segments along a wind turbine blade for rotor yaw angles of 30° and 45° , compared to lift coefficients at zero yaw (triangle pointing left) and static wind tunnel measurements for the airfoil (triangle pointing right) [47]

The second result and its implications are more difficult to quantify: the aerodynamics of a wind turbine with a skewed wake effect is complex. The identification of the direction of the wind turbine wake is important in optimization of wind farm layouts [48]. While many methods of modelling wind turbine

aerodynamics inherently model the skewed wake effect, an explicit computation of the wake skew angle would enable wind farm optimization with shorter computational time. Furthermore, analytical methods that compute axisymmetric wind turbine aerodynamic forces require this angle to implement skewed wake corrections.

The complexity of the skewed wake effect has not been fully studied: Generally it is known that the skewed wake phenomenon encompasses the dynamic wake and unsteady airfoil effects, but any additional effect on the flow field momentum balance have not been studied. Instead modelling this effect has centred on corrections for axis-symmetric predictions and whether advanced modelling techniques can adequately predict the rotor forces. A more thorough analysis of the effect would enable more realistic modelling, thus allowing accurate corrections to analytical predictive tools, increasing their effectiveness.

2.2.2.4 Transient wake states

Based on Figure 2-5, there are wake states that may occur in due to transient operations of the wind turbine, beyond the windmill state and turbulent wake state discussed in Section 2.1.5. These are the vortex ring and propeller brake states.

At low wind speeds, it is possible for pitching and surging motions of a FOWT platform to reach frequencies that translate to relative wind velocities on the rotor blades that when combined with the wind speed, are equal to and opposite the rotor wake velocity. When this occurs, the axial induction factor is equal to one, or the effective wake velocity is zero. This means that the trailing vortices at the rotor tip and hub are no longer convected downstream from the rotor [49], and form a toroidal recirculation path around the rotor [50]. This is called the vortex ring state (VRS), and in this state the rotor loses energy to the wind. The recirculation of the flow in a helicopter rotor is shown in Figure 2-19.

In describing the VRS for a helicopter rotor, it was stated by Johnson [49] that VRS would occur whenever the trailing vortices could not be convected away fast enough, and instead collect around the rotor. This implies that a zero wake velocity is not necessary for VRS; instead it is likely that VRS may occur when the wake velocity is sufficiently low such that the pitch of the helical trailing vortex is small enough for the interactions between the vortices of the present rotor rotation and the previous one to be significant. However, the lack of experimental and simulation study on the wind turbine rotor in the vortex ring state makes it difficult to quantify the occurrence of this phenomenon.

Figure 2-19. A helicopter rotor in the vortex ring state [51]

VRS is an unsteady phenomenon, since the vortices generated would build up and break away periodically [49]. Sørensen [52] performed a study of wake states with a constantly-loaded actuator disc, to note that the thrust on the actuator disc at the turbulent wake state corresponds to the Glauert correction discussed in Section 2.1.5. The VRS was found to be a transient and unstable state that eventually settled into the propeller-brake state, where the coefficient of thrust can be expected to be exceptionally high, and the flow recirculates to a direction opposing the wind.

The rotor motions of a FOWT due to platform motions also contribute to the transience of the VRS, since only very specific combinations of wind and rotor RPM would produce the wake state.

Sebastian and Lackner [50] attempted to quantify the time spent by a FOWT rotor in the VRS, by running the FAST code for 3 different conditions for different FOWT platform types. This is presented in Figure 2-20. These conditions are based on the rated conditions of the rotor, where the rotor speed and the wind speed are optimum for the maximum power output from the generator. The rated wind speed is the lowest wind speed where the rotational speed required for the generator to reach its maximum power output is achieved. The conditions used are below-rated, with $V_0=6\text{m/s}$ and rotor $\text{RPM}=8.76$, rated, with $V_0 = 11.2\text{m/s}$ and rotor $\text{RPM}=12.1$, and above-rated, with $V_0 = 18\text{m/s}$ and rotor $\text{RPM}=12.1$.

Figure 2-20. Percentage of time spent by rotor segment in VRS for below (a) rated, (b) rated and (c) above rated conditions [50]

This analysis quantified the occurrence of VRS as:

$$V_0 < 2|W_n| \quad (2-20)$$

Using the momentum theory alone, this would signify the flow recirculating in the wake, as shown in equation (2-9). However, at $V_0 < 2.5|W_n|$, it was shown in Section 2.1.5 that the momentum theory breaks down, thus cannot be used as a basis of describing VRS. The vortex structure around the wind turbine must be analysed before concluding on the occurrence of VRS.

Sebastian and Lackner [53] summarised the time a FOWT rotor spent in the propeller state, through finding the amount of time the axial induction factor was negative during simulations in three different conditions for different platform motions. This is included in Table 2-2.

Table 2-2. Percentage of time spent by FOWT rotor segments in propeller state for different conditions and platform types [53]

Condition	Platform Type	Motion	$a_{root} < 0$ [%]	$a_{mid} < 0$ [%]	$a_{tip} < 0$ [%]
Below-rated	Monopile	-	42.81	0	0
	ITI Energy barge	Surge	42.83	0	0
	ITI Energy barge	Heave	42.83	0	0
	ITI Energy barge	Pitch	36.67	0	0
	ITI Energy barge	Surge+Heave+Pitch	36.07	0	0
	OC3Hywind sparbuoy	Pitch	42.83	0	0
	OC3Hywind sparbuoy	Yaw	42.83	0	0
	OC3Hywind sparbuoy	Pitch+ Yaw	42.83	0	0
	MIT/NREL TLP	Surge	42.83	0	0
Rated	Monopile	-	42.81	0	0
	ITI Energy barge	Pitch	42.83	0	0
Above-rated	Monopile	-	42.9	28.95	100
	ITI Energy barge	Pitch	39.94	31.17	90.74
	OC3Hywind sparbuoy	Pitch	34.58	41.11	86.74
	OC3Hywind sparbuoy	Yaw	41.56	26.44	98.35
	OC3Hywind sparbuoy	Pitch+Yaw	34.46	41.08	85.42

2.3 Review of FOWT Experiments

Experiments involving wind turbines are challenging to conduct, as using full-scale wind turbines are impossible in wind tunnels due to their size and field tests are expensive and require a large amount of time for data reduction [54]. Furthermore, actual site real time conditions cannot be controlled to identify specific trends for aerodynamic phenomena. Using scale models are necessary, but maintaining similarity with full-scale conditions is the biggest challenge.

The flow Reynolds number, Re , is an important parameter in aerodynamics, as it is the ratio of the inertial to viscous forces in a flow. This parameter characterises flow into laminar and turbulent flow, and affects the force coefficient on airfoil segments.

$$Re = \frac{Uc}{\nu} \quad (2-21)$$

where U is the flow velocity, the vector combination of rotational and wind velocity for a wind turbine, c is the airfoil chord and ν is the kinematic viscosity of the air.

If the NREL 5MW rotor was to be scaled by a factor of 10, the velocity would need to be increased by a factor of 10. Since the local blade segment flow is the vector combination of the rotational and wind velocities, the rotor rotational speed would need to be increased by a factor of 10^2 , since the rotor radius would be reduced by a factor of 10 as well. A rated rotor RPM of 12.1 would need to be increased to 1210 for Reynolds similarity. To better match the Reynolds number, experiments could be conducted in a water tunnel as well, as the kinematic viscosity of water is about a tenth that of air. Even so, the rotor diameter of the NREL 5MW rotor, when scaled by a factor of 10, would be about 12m, which is still very large, and would require large and expensive wind tunnels.

Since this is not realistically achievable, the rotor tip speed ratio, λ , is maintained instead. This is defined as the ratio between the rotational velocity at the rotor tip and the wind speed.

$$\lambda = \frac{\Omega R}{V_0} \quad (2-22)$$

where Ω is the rotor rotational speed in radians per second, and R is the rotor radius.

The advantage of maintaining λ is that the flow angle, which is the angle between the total velocity vector and the rotor plane, is maintained. Assuming a low dependence on Reynolds number, the force coefficients and distributions in the rotor would be similar. This allows the wind velocity to be determined by other parameters.

For scale models to be placed in wave tanks, Froude similarity is an important parameter for semi-submerged objects, such as the FOWT platform, to maintain similitude in hydrodynamic forces [55].

$$Fr = \frac{U}{\sqrt{gL}} \quad (2-23)$$

where U is the water velocity, g is the acceleration due to gravity and L is the length of the platform.

To scale the wind for a scale model FOWT, the ratio between the wind velocity and wave celerity is maintained, which does not represent the Froude number, but still ensures the ratio between wind and wave forces [56] like Froude scaling. Indeed, this type Froude scaling was employed in scale model experiments by Martin [56] and Huijs et al [57].

Martin [56] created a 1/50 geometric scale model of the NREL 5MW wind turbine. However, due to Froude scaling of the wind speed, the Reynolds number dropped from 1.2×10^7 to 3.6×10^3 at a span of 0.7 of the rotor radius, resulting in a difference in flow regimes, with turbulent flow at full scale to laminar at model scale. This resulted in earlier flow separation and

stall at the model scale, with thicker boundary layers, causing reduced lift and increased drag coefficients, as shown in Figure 2-21.

To resolve the problem of Reynolds dissimilarity, an attempt was made to match the flow regimes, by introducing some leading edge roughness to the wind turbine blade. This resulted in turbulent flow over the airfoil, suppressing flow separation. The model rotor achieved optimum performance when the blade was orientated at an angle of 6.4° to the rotor plane, as compared to 0° for the full scale rotor. The comparison of the model rotor coefficients of power and thrust to the full scale rotor at both 6.4° and 0° blade tip orientations are shown in Figure 2-22. A clear improvement is seen in matching the coefficient of thrust of the rotor between the model scale and the full scale rotor at 0° blade tip orientation.

Figure 2-21. Lift and drag coefficients against angle of attack (α) for the NACA 64-618 airfoil section under model and full scale wind conditions at a span of 0.7 of the rotor radius [56]

Figure 2-22. Coefficient of thrust and power at model (smooth and rough) and full scale (0° and 6.4° blade tip pitch) against tip speed ratio (TSR). [56]

Huijs et al [57] designed a wind turbine to match the thrust coefficient at each . The coefficient of power could not be matched, but the objective of the experiment was not to study the aerodynamics, but the study the interaction between the forces on the rotor and the forces on the platform. For this, only the coefficient of thrust needed to be matched. The wind turbine design is detailed by Ridder et al [58]. The approach used here to redesign the wind turbine to match the aerodynamic force coefficients at full-scale will reduce the impact of the dissimilarity in Reynolds number. A possible extension to this work could be to redesign the blades to match the distribution of force coefficients as well.

The scale model experiments by Martin [56] and Huijs et al [57] are meant for full simulation of the FOWT, including the structural dynamics of the tower and hydrodynamics of the floating platform. These result in added uncertainty, and the experimental conditions, while matching the full-scale conditions, cannot be adjusted to observe specific aerodynamic phenomena. Instead, an approach involving only a scale model wind turbine actuated to produce specific motions should be adopted.

The approaches by Martin [56] and Huijs et al[57] did attempt to match the aerodynamics of the rotor, with Martin attempting to do so at every blade segment, allowing a matching of the aerodynamic force distribution along the rotor blade. This permits more realistic experiments, as specific aerodynamic phenomena are likely to be matched as well. An improvement to this approach would be to optimise the blade cross-sectional profiles and orientations to better match the full-scale aerodynamic coefficients, an approach adopted by Rockel et al [59]. Here, the rotor was actuated to mimic the motion of a FOWT with a pitching platform. The incoming wind and wake was recorded using Laser-Doppler anemometry, shown in Figure 2-23. However, the rotor was optimised for performance, and did not match any specific wind turbine, so conclusions about a specific wind turbine could not be drawn. However, it enables a thorough understanding of the flow structures related to a FOWT. Thorough analyses of the wake were performed, and further work should include measurements of pressure distributions or forces along the blade, to explore the relationship between the aerodynamic forces and the flow field.

Figure 2-23. Wake velocities in the wind direction, normalised against the wind speed at hub height, for (a) fixed and (b) floating cases [59]

The approach of turbine-specific cross-sectional coefficient matching was adopted by Martin et al [60]. In this methodology, the 1/30 scale rotor of the NREL 5MW turbine was designed for water tunnel or towing tank testing. This helped reduce the Reynolds dissimilarity present. The airfoil cross-sections of the rotor were designed to produce a similar lift-curve at the model Re. However, the Reynolds dissimilarity was still large, and the smaller Reynolds number resulted in thicker boundary layers, thus increased drag, as shown in Figure 2-24.

Figure 2-24. Comparison of lift (top) and drag (bottom) coefficients for model (line) and full-scale (+) airfoil [60]

This approach led to a favourable match in thrust coefficient at several rotor tip speed ratios, but the difference in coefficients of drag at the model and full-scale meant that the torque coefficient of the model was significantly reduced, as illustrated in Figure 2-25.

Figure 2-25. Comparison of coefficient of thrust (top) and torque (bottom) for model (line) and full-scale (+) turbines [60]

Since the coefficient of thrust is correlated to wake phenomena such as the turbulent wake state, this approach is likely to produce the best representation of full-scale unsteady aerodynamic phenomena associated with FOWTs.

2.4 Computational Modelling of FOWT Aerodynamics

Computational modelling of FOWT aerodynamics would reduce the dependency on experiments to predict the FOWT aerodynamics forces, thereby reducing costs. However, the added aerodynamic phenomena require methods presently used, created for fixed-bottom wind turbines, to be extensively validated for the unsteady flow cases of FOWTs.

The most complete computational model for wind turbines uses computational fluid dynamics to resolve the airflow around the wind turbine. Using

spatially discretised Reynolds Averaged Navier-Stokes equations, with turbulence models, requires a computational grid. A geometry of the wind turbine is created, and the airflow resolved in the computational grid built around it in a method called direct rotor modelling. Generalised actuator disc methods simplify this method by imposing the forces generated by the wind turbine on the flow, as compared to simulation of the physical wind turbine in direct rotor modelling. This requires an accurate model to predict the local aerodynamic force coefficients along the rotor blade. The vortex methods resolve the vortices shed from the blade to attain the flow velocities at the rotor plane. These are used to find the forces along the rotor blades, which need to be modelled accurately, similar to the generalised actuator disc models. Finally, the momentum models further simplify the flow field to use a global momentum balance to find the velocities at the rotor blade, and a model is required to predict the local aerodynamic forces along the rotor blade.

2.4.1 Computational Fluid Dynamics

Computational fluid dynamics (CFD) methods are the most computationally intensive methods of modelling wind turbine aerodynamics. Using the Navier-Stokes equations to resolve the airflow around a wind turbine ensures that the model is theoretically accurate. However, complications arise when considering the discretisation of these equations, as a computational grid would be required, and resolved adequately to accurately capture the flow physics.

2.4.1.1 Direct rotor modelling (DRM)

The most accurate way of modelling the FOWT aerodynamics would be developing a 3-dimensional model of the wind turbine rotor and build a computational mesh around it, a direct rotor model. However, this is highly computationally intensive. Accurate simulations would require adequate resolution of boundary layers [61], and the massive amount of data produced will require time-intensive post processing.

To produce the mesh, a sliding mesh method can be utilised. Two or more fluid zones are used, moving relative to each other along the zone interfaces. The grid is updated at every time step and the results across the interfaces are interpolated. However, complex motions would pose a challenge in a sliding mesh method, as a mesh must be created with appropriate zones to account for all motions. The sliding mesh technique was employed by Ren et al [62], with a rotation zone for the rotor blades within a motion zone for the entire FOWT, as shown in Figure 2-26. This was a multi-phase simulation, which simulated the wave hydrodynamics as well, and as such, the focus was the response of the FOWT to the wind and wave forces, rather than observing aerodynamic phenomena.

Figure 2-26. Different mesh zones created for the sliding mesh method [62]

Another meshing method used is the overset or Chimera mesh. In this method, a geometry-specific mesh can be placed over a structured mesh, and the flow properties passed from one mesh to another through interpolation. Interpolation errors can be significant and propagate through the whole flow field, but when the interpolation is robust, the overset mesh method can allow highly complex movements of the wind turbine, in all 6 degrees of freedom.

The overset mesh method has been used in direct rotor simulations of the platform pitching motions of the NREL 5MW FOWT by Bekiropoulos et al [63] and Tran et al [64]. In both these cases, the motion was found using the FAST design code and prescribed in the simulation. This allows for a greater focus on the aerodynamic phenomena.

In [63], to ensure the robustness of the interpolation between grids, a certain degree of overlap had to be guaranteed at every orientation of the FOWT. Thus, additional grid structures had to be constructed around the FOWT to ensure the overlap, so that there is no hole where the background grid is cut out and the overlapping grid moves away. This resulted in a large grid of 30 million cells, shown in Figure 2-27.

Figure 2-27. Overset mesh with additional mesh structures to ensure mesh overlap [63]

In [64], the background grid was created without a cut out region to accommodate the FOWT-fixed grid. Instead, the solver would define the grid cells that become active or inactive during the motion of the FOWT. The rotation of the rotor was not simulated physically, but by a multiple reference frames approach, in which the rotor is static, but computed as rotating with the blades in the same geometric position at every time step. This approach

improved the computational time, but meant that the interactions between the rotor and surrounding fluid were weak. The approach resulted in the lack of accuracy in relative velocities resulting from the platform motion, since these velocities are dependent on the blade positions, as explained in Section 2.2.1.

Another method that has been proposed for FOWT simulations is the Arbitrary Lagrangian-Eulerian (ALE) approach. In the ALE approach, the computational mesh consists of nodes that may either remain stationary or move with the flow field. In this manner, the Navier-Stokes equations are solved in a way that is both Lagrangian and Eulerian. As the nodes move, remeshing is required to reconnect the nodes. This significantly increases computational time. However, the flexibility of this method allows the simulation of highly complex movements, including blade deformation, which cannot be produced using the sliding or overset mesh methods. This allows for a more detailed study of the aeroelastic response of the rotor to various loads. This method was used by [65, 66] to compare the performance of a deformable blade to a rigid blade, an assumption normally used in direct rotor modelling. This comparison is included in Figure 2-28.

Figure 2-28. Comparison of aerodynamic torque between a deformable blade and a rigid blade for the NREL 5MW rotor [66]

While running a DRM simulation once will produce data that can be analysed in numerous different ways both for engineering assessment as well as scientific advancement, the computational cost of running the numerous simulations required would be too great for a fully coupled simulation. However, the usefulness of DRM would lie in analysing aerodynamic flow phenomena to improve analytical methods.

2.4.1.2 Generalised actuator disc methods

In the generalized actuator disc models, the computation of the boundary layers (and by extension, the computational resources required to accurately resolve it) is circumvented by modelling the effects of the turbine itself on the airflow. The wind turbine is represented by a body force, averaged over a disc, line or surface. The turbine is modelled by explicitly adding the force, $\mathbf{f}$, in the Navier-Stokes equations [67].

$$\frac{D\mathbf{u}}{Dt} = \mathbf{f} - \frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} \quad (2-24)$$

Using the Blade Element Theory in Section 2.1.2, the forces can be determined from looking up an aerodynamics table for an airfoil representing a segment of the rotor blade, with the normal and rotational velocity components found at the rotor plane using the Navier-Stokes equations. By Newtons third law, the forces that are applied are equal and opposite to the aerodynamic forces on the rotor, since the effects of the rotor are being introduced on the flow.

This method, when applied in an unsteady simulation, would model the wake effects, including dynamic and skewed wake effects. This means that the flow field momentum balance is handled by the Navier-Stokes equations. However, the aerodynamic forces on the rotor are added explicitly, and need to be modelled accurately. In an unsteady flow, this means an accurate model for the unsteady airfoil aerodynamics.

These forces can be applied in two ways. The first would involve averaging the forces over the entire revolution, and applying them piecewise in concentric discs, called the actuator disc method. This means that forces applied would be axis-symmetric, and dependent on averaged flow conditions, thus the structural dynamics on individual blades cannot be assessed. However, the reduced computational time is an advantage that makes the actuator disc methods suitable for wind farm wake simulation [68].

The second way is the actuator line method, which avoids the assumption of axial symmetry by distributing the loading along lines that represent the blade forces [69]. These lines rotate as the blades would in an unsteady simulation. The individual vortices would form around these lines and be shed, resulting in a more accurate flow simulation.

Both the actuator disc and actuator line methods were applied by Yang et al [68] for FOWT simulations under wind conditions affected by sea waves. The actuator disc model was applied to an entire wind farm to analyse the power output efficiency, while the actuator line model was used on a single rotor to study the wake dynamics in detail. These are shown in Figure 2-29. However, rotor motions due to wave-platform interactions were not introduced in both cases.

Figure 2-29. Instantaneous turbulence vortical structures in the wake of wind turbines above waves, obtained using (a) actuator line method, and (b) actuator disc method [68]

The actuator line model is better suited to understand the detailed wake dynamics of the rotor, and the forces on individual blades. However, it is dependent on the accuracy of the force distribution applied on the blades, using the blade element theory and a suitable model for the unsteady airfoil aerodynamics. A significant advantage is the computational time of the actuator disc or line models compared to direct rotor modelling. Simulations of the NREL Unsteady Aerodynamic Experiment Phase VI turbine [19] by Singapore Wala and Ng [70] were performed using direct rotor modelling with 7.8 million cells in the mesh, and the actuator disc method using just 112, 000 cells. Furthermore, the induced velocities can be directly extracted by analysing the flow velocities at the rotor plane, while the direct rotor model would have a physical rotor, so the induced velocities would have to be computed based on local cross-sectional forces, requiring more post-processing.

The aerodynamic forces are modelled much in the same way as analytical models which will be described later in this section. Thus, the actuator line model would be able to provide a good assessment of the dynamic and skewed wake modelling capabilities of these analytical models, as these effects would be inherent in the actuator line model.

2.4.2 Vortex Methods

Vortex methods use the potential flow theory to compute the flow field momentum balance of the wind turbine. The forces generated by the rotor are represented by bound vortices, which then generated the trailing and shed vortices in the wake. The velocities induced by the vortices are computed using the Biot-Savart law, explained in detail by Leishman [40]. These induced velocities are used to find the velocity at the rotor plane, and the aerodynamic force coefficient is found using a lookup table or unsteady airfoil model, which is discussed in Chapter 6.

Prescribed vortex wake methods convect the vortices by fixing the shape of the wake to a cylindrical or other empirically determined shapes, which is

not physically accurate, since the wake can change based on the degree of unsteadiness in the flow and the skewness of the wake. However, this method can achieve high degree of accuracy in axial flows and low levels of wake skew.

Free vortex wake methods on the other hand allow the vortices to propagate freely, making the wake position part of the solution. The Biot-Savart Law is used to determine both the induced velocities and the position of the vortex filament. This results in a greatly increased computational time. A free vortex wake model developed by Currin and Long [71] was used to determine the aerodynamics of the NREL Unsteady Aerodynamic Experiment Phase VI turbine, and the computational time required was significantly larger than the prescribed wake model, shown in Table 2-3.

Table 2-3. Computational time for various aerodynamic analysis methods for wind turbines, for 5 rotor rotations under 15° azimuth steps with 10 blade elements using an Intel Dual Core 2.4 Ghz CPU [71]

Model	Time(s)
Blade Element Momentum	2.624
Generalized Dynamic Wake	2.620
Dynamic Prescribed Wake	27.97
Free Vortex Wake	1027.5

A free vortex wake model was created by Sebastian and Lackner [72] and compared to data from the MEXICO experiment compiled by Schepers and Snel [73]. Both the rotor thrust and torque were predicted with good accuracy. The model by Sebastian and Lackner [74] was used to determine the aerodynamics of the NREL 5MW FOWT under platform motions determined using FAST (Fatigue, Aerodynamics, Structures, and Turbulence code). However, this was not validated against direct rotor modelling methods or experiments, so the accuracy of the model could not be ascertained.

2.4.3 Momentum Methods

Momentum methods use the momentum theory and blade element theory to determine the aerodynamic forces on a FOWT rotor. They do not resolve the entire flow field, but make assumptions about the momentum balance that enable the determination of the induced velocities, and thus the force distribution over the entire rotor. These are divided into two types: the blade element momentum method (BEM) and the methods of accelerating potential.

2.4.3.1 Methods of accelerating potential

The methods of accelerating potential replace the momentum equations from Section 2.1.1 with momentum balance equations based on the Euler equations.

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V}' \cdot \nabla \right) \mathbf{V}' = -\nabla p \quad (2-25)$$

where $\mathbf{V}'$ is the velocity at the rotor plane, ρ is the air density and p is the pressure.

It is assumed that the induced velocity is negligible, thus the velocity at the rotor plane is assumed to be equal to the wind speed.

$$\rho \left(\frac{\partial}{\partial t} + V_0 \cdot \nabla \right) \mathbf{V}' = -\nabla p \quad (2-26)$$

where V_0 is the wind speed.

This assumption is important to derive the formulation for the methods of accelerating potential, but is not always true. At low wind speeds or highly skewed wakes, the induced velocity would be significant, with the rotor operating in the turbulent wake or vortex ring states. Codes that apply this method, such as AeroDyn [45], the aerodynamics module of FAST, need to switch to other methods in these wake states. This may introduce discontinuities in the solution.

Applying the nabla operator, ∇ , to both sides of equation (2-26) again yields:

$$\rho \left(\frac{\partial}{\partial t} + V_0 \cdot \nabla \right) \nabla \mathbf{V}' = -\nabla^2 p \quad (2-27)$$

Using the continuity equation, where $\nabla \cdot \mathbf{V}' = 0$, the Laplace equation for pressure is attained from (2-27), and the pressure gradient can be related to the rate of change of velocity from (2-26).

$$\nabla^2 p = 0 \quad (2-28)$$

$$\rho \frac{\partial \mathbf{V}'}{\partial t} = -\nabla p \quad (2-29)$$

The solution of the Laplace equation leads to Legendre polynomials after a coordinate transformation. This formulation is provided in detail by Burton et al [75]. The Legendre polynomial series have to be truncated for practical calculations, and form a series of equations that are solved simultaneously to determine the flow parameters, based on force coefficients derived from the blade element theory.

$$[M] \{\dot{a}\} + [L]^{-1} \{a\} = \{C\} \quad (2-30)$$

where $[M]$ is the added mass matrix which accounts for the dynamic wake effect, $[L]$ is the inflow gain matrix which accounts for the radial and azimuthal variation in induced velocities, $\{a\}$ is the induced velocities, and $\{C\}$ is the aerodynamic force coefficients.

The solution requires a set of induced velocities in order to compute the aerodynamic force coefficients, which are used to compute the induced velocities in the next time step, forming a time-marching scheme. This means it needs to be initialised with a set of induced velocities, once again requiring another method to do so.

The Pitt and Peters solution [76] to this problem was averaged over the entire rotor disc. This was assumed to be valid for annular segments of the

rotor, and applied by radially segmenting the rotor and finding the force coefficients for each radial segment in the rotor. This method was used in a model developed by Garrad Hassan within the JOULE 1 programme, described by Snel and Scheepers [37] and implemented by Suzuki [35].

When the Legendre polynomial is truncated with more terms, the method of accelerating potential would incorporate the radial variation in aerodynamic forces in the rotor. This is called the generalised dynamic wake (GDW) model. This was first introduced for wind turbines by Suzuki [35] and has been implemented in the AeroDyn tool by NREL [45], the aerodynamics analysis tool within FAST.

The advantage of the methods of accelerating potential is that they inherently model the skewed wake effect as well as the dynamic wake effect. The generalised dynamic wake model may be considered a more complete model since it applies the solution to directly determine the force distribution on the rotor, while the annular application of the Pitt and Peters solution may be less computationally demanding, since only a 3 by 3 matrix is being solved, though many times over due to the number of radial segments.

However, the solutions require a coordinate transformation that makes the addition of corrective models difficult, thus it would be difficult to overcome the shortcomings of this method with corrective models. This makes the methods of accelerating potential unreliable as an analytical tool for FOWT aerodynamics.

2.4.3.2 The blade element momentum method

The methods of acceleration potential are unable to account for wake rotation, due to the rotation of the rotor blades. In AeroDyn, the blade element momentum method (BEM) is used to compute the rotational induced velocity [45]. BEM is an important tool that is used extensively in the wind turbine industry. It is the combination of the simple momentum theory and the blade element theory detailed in Section 2.1 and is a steady-state, axisymmetric

model, which uses annular-averaged values of velocities and induced velocities. This implies that all the blades would be computed as having the same load distributions, a situation that is not accurate especially when operating with a skewed wake.

Using the blade element theory, the rotor can be divided into segments of length dr . The area of the annulus is hence $2\pi r dr$. Placing this into (2-11) yields:

$$W_n = \frac{dT dr}{2\rho u (2\pi r dr)} = \frac{dT}{4\rho u \pi r} \quad (2-31)$$

Similarly, the tangential, or rotational induced velocity can be postulated to be:

$$W_t = \frac{dM}{4\rho u \pi r} \quad (2-32)$$

where M is the torque-generating force on the rotor, and dM is the elemental force.

Since the induced velocities oppose the wind and rotational directions, we can compute the relative velocity for each element:

$$\mathbf{V}_{rel} = (\Omega r - W_t) \mathbf{t} + (V_0 - W_n) \mathbf{n} \quad (2-33)$$

where $\mathbf{n}$ is the unit vector normal to the rotor plane, and $\mathbf{t}$ is the unit vector opposite to the rotational direction for a blade element.

Using (2-33) and Figure 2-2, the angle of attack, α , and thus the aerodynamic forces for each blade element can be determined via an appropriate look up table or aerodynamic model once the induced velocities are known. These are then used to find the elemental thrust and tangential forces. However, it is these forces that are needed to find the induced velocities in the first place. Thus, BEM begins with an initial guess of the induced velocities (usually $W_n = W_t = 0$), and the blade aerodynamics forces are found, leading to new

induced velocities. These are used again in equations (2-31) and (2-32) in an iterative process, until a converged solution is found.

BEM directly applies 1-dimensional momentum theory, thus the corrections mentioned in Sections 2.1.3 to 2.1.5 are required for a realistic BEM computation of wind turbine blade forces.

The iterative process in BEM, which uses the calculate values of induced velocity in the next iteration, can work well under most conditions, but in some conditions, relaxation may be required to ensure convergence, a method used by Maheri et al [77]. Ning [78] reduced the equations for the induced velocities to be based only on the flow angle, ϕ .

$$a(\phi) = \frac{\kappa(\phi)}{1 + \kappa(\phi)} \quad (2-34)$$

$$a'(\phi) = \frac{W - t}{\Omega r} = \frac{\kappa'(\phi)}{1 + \kappa'(\phi)} \quad (2-35)$$

where κ and κ' are given by:

$$\kappa = \frac{cBC_n(\phi)}{4 \sin^2(\phi)} \quad (2-36)$$

$$\kappa' = \frac{cBC_t(\phi)}{4 \sin(\phi) \cos(\phi)} \quad (2-37)$$

where c is the chord length at the centre of a blade element, B is the number of blades and C_n and C_t are the blade cross-sectional force coefficients in the normal and rotational directions with respect to the rotor plane. The force coefficients are found based the angle of attack, a function of ϕ , as shown in Section 2.1.2. This is explained in further detail in Chapter 3.

The equation reduction technique allowed faster (fewer function calls) and guaranteed convergence (zero failure rate or divergence rate of algorithm) since only the value of ϕ needs to converge, as shown in Table 2-4.

Table 2-4. Comparison of different iterative methods to obtain a solution for BEM [78]

Algorithm	Average Function Calls	Failure Rate (%)
Fixed Point	31.8	12.6
Newton	79	5.8
Steffenson	16.4	16.3
Powell Hybrid	72.3	16.2
Levenberg-Marquardt	92.3	8.8
Ning Method	11.3	0.0

Another important feature of the method by Ning [78] is that a solution could be found even beyond the traditional limits of BEM, when the axial induced velocity exceeds the wind speed, corresponding to the vortex ring and propeller brake states in Figure 2-5. This method was not validated for this flow region, but the solution mechanism can be adapted to include corrections if necessary.

In an unsteady environment, BEM will produce steady-state wakes, such that in a time-marching simulation, it will assume a new steady-state environment at every time step. This neglects the dynamic wake effect, as shown in Figure 2-30, which compares the results of BEM and GDW to measured rotor power of a wind turbine with its blades undergoing a step pitch.

To correct for this, two methods were presented in [37], using differential equations to correct for the dynamic wake effect. These effectively allowed BEM to be a time-marching algorithm, as induced velocities are corrected using these differential equations, then applied directly in the next time step to calculate the rotor forces. The GH Bladed software by Garrad Hassan and Partners [79] used the result from the Pitt and Peters solution [76] to the accelerating potential method in axial flow to correct for the dynamic wake effect, which involves an added mass term. These are discussed in Chapter 8.

Figure 2-30. Comparison of BEM and GDW predictions of the Tjæreborg wind turbine undergoing a step pitch from 0.2° to 3.9° and back [45]

Due to its simplicity, BEM has been used in fully-coupled aero-servo-hydro-structural codes for FOWTs including FAST [27], GH Bladed [79] and HAWC2 [80]. However, BEM does not compute force distributions on individual blades, and computes the forces and induced velocity averaged over all the blades in the rotor. Movements such as the platform pitching motion, which result in different relative velocities experienced by each blade of the rotor (as shown in Figure 2-8), would result in different force distributions on each blade, which would be unaccounted for by BEM. Since these force distributions need to be accurately modelled to predict the structural dynamics of the blade, the formulation of BEM makes it unsuitable for modelling FOWT aerodynamics.

2.4.3.3 Unsteady blade element momentum (uBEM) method

The unsteady blade element momentum method (uBEM), uses the same momentum equations applied in BEM, but applied separately to each element of each blade. Thus, the flow conditions at each blade element of each blade are used to compute the induced velocities and blade forces. This greatly increases the ability of uBEM to model the unsteady aerodynamics associated

with FOWTs, since the differences in flow conditions at each blade can be accounted for.

A concept similar to uBEM was implemented by Sant [81], applying flow conditions specific to each segment of each blade. However, these were averaged over the whole annulus, and the algorithm was iterated to convergence at each time step, signifying a steady-state response.

A formulation for uBEM is introduced and detailed in Hansen [8], and applied by Silva and Donadon [82] to analyse the accuracy of an unsteady airfoil model. It was also applied by Tran et al [83] for a wind turbine facing gust loads, and by Tran et al [64] for the NREL 5MW turbine in platform pitching motion, and compared it to CFD. In [64], the unsteady airfoil model was not used, and rotor hub velocity was used for all blade elements. The power and torque predictions of the uBEM in [64] are compared to CFD in Figure 2-31.

Figure 2-31 shows significant deviation from the CFD for power and thrust predictions. This is likely to be caused by the use of the rotor hub velocity at each blade element, compared to the actual velocity at the blade element, as well as a lack of an unsteady airfoil model, which was shown in Section 2.2.2.2 to be needed for accurate computation of FOWT aerodynamics. Furthermore, while uBEM is less accurate in [64], comparison of individual blade forces are not shown, which are likely to be better reflected in uBEM compared to the other momentum methods.

Due to its ability to model unsteady flows, when dynamic stall, unsteady airfoil effects and skewed wake effects are considered, and its ability to consider flows for each blade separately, uBEM has the potential to be a reliable and accurate tool for modelling the aerodynamics of FOWTs. The use of simple momentum theory means that corrective models can be easily formulated and added to enhance the accuracy and the range of conditions within which uBEM

may be applied. The formulation of uBEM can be found in detail in Chapter 3.

Figure 2-31. Power and thrust predictions for the NREL 5MW turbine undergoing platform pitching motions with amplitude of 1° and frequency of 0.1Hz [64]

2.4.4 Fatigue, Aerodynamics, Structures, and Turbulence code (FAST)

FAST is a time domain, aero-servo-hydro-elastic code from NREL [27] for the simulation of wind turbines. The response of a FOWT can be simulated based on wind and wave inputs, which will generate rotor aerodynamic forces and platform motions respectively. These are used to find the structural responses of the tower and blades, as well as simulate the motion of the rotor, which would affect its aerodynamic loading.

The aerodynamics module of FAST, AeroDyn [45] uses either BEM or GDW to compute the aerodynamic forces on the FOWT rotor. Due to the limitations of GDW, AeroDyn switches to BEM for low wind speeds, and used BEM to compute the rotational induced velocities along the rotor blade. The BEM equations include a skewed wake model, and a tower shadow model is included to account for the presence of the tower, which is especially important in downstream turbines (wind turbines with the nacelle and tower upstream). The Beddoes-Leishman model is used for the unsteady airfoil effect, but this is not applied in the computation of the rotor forces used to find the induced velocities. Instead, the unsteady airfoil model is decoupled from the momentum equations, and only applied after as an input to the structural dynamics computations for the blades.

Since AeroDyn was initially developed for fixed-bottom wind turbines, the changes in axial velocity at the rotor due to blade motion were assumed to be small, since they are due to blade or tower bending. Thus, the induced velocity is assumed to be a component of the wind speed only, and the velocities due to the blade motion are only added to the rotor plane velocities to calculate the flow angle, ϕ . They are not included in the momentum equations. This is an invalid assumption in FOWTs, since it is shown in Section 2.2.1 that the velocities due to the rotor motion can be significant. FAST was used to compute the response of FOWTs in the OC3 [32, 84] test cases for different platform types, in which different wind and wave inputs were used to determine the platform motions as well as aerodynamics and structural responses of the FOWTs.

Chapter 3

Unsteady Blade Element Momentum (uBEM) Method

This chapter will describe the uBEM method in detail, including coordinate system transformations required for blade-specific coordinate systems, application of the momentum theory and blade element theory, corrective models and an algorithmic understanding of uBEM.

3.1 Coordinate Systems

Figure 3-1. The four coordinate systems defined for the FOWT in uBEM

Four coordinate systems are important in uBEM (Figure 3-1), which are:

1. The earth-fixed system that is the inertial system in which the wind vector is defined
2. The platform-fixed system that is attached to the platform, in which the platform motions are defined
3. The rotor-fixed system that is attached to the hub or nacelle of the rotor, with the z-axis in the direction of rotor thrust, in which a degree of yaw or tilt with respect to the platform may be defined
4. The blade-fixed system that is attached to each blade, and the x-axis is in the radial direction, which rotates with the blades

3.1.1 Platform Orientation

The angles and motions for the floating platform are shown in Figure 2-6.

The definition of the angles and motions are based on a right-handed coordinate system, and are consistent with definitions for floating platforms in general. While the x, y and z axes may be defined differently by hydrodynamics codes (z-axis as x-axis, for example), it is simpler to maintain this definition and apply the appropriate transformations to align with the motions.

3.1.2 Rotor Orientation

The angles specific to the rotor are the yaw, θ_y , and tilt, θ_t as shown in Figure 3-2.

In conventional definitions, the yaw and tilt angles are measured relative to the incoming wind vector. However, this creates many complications when the wind changes direction. Instead, the yaw and tilt angles in uBEM are measured relative to the platform, as shown in Figure 3-1. This way, changing wind directions will not require recalculation of θ_y and θ_t . The formulation of uBEM will inherently account for wind direction as well as platform and rotor orientations.

Figure 3-2. Angles related to a wind turbine rotor

An important point to note is the use of the word tilt and the term θ_t , which differs from the term pitch used for describing the same angular orientation of the floating platform. The reason for this is to avoid confusion between the tilt and the blade pitch, which are very different parameters. The term tilt is not a new term, and is used in many texts describing wind turbine aerodynamics, including Hansen [8], upon which much of the uBEM model described here is based.

The motions specific to the wind turbine rotor are surge, heave and sway, as shown in Figure 3-3.

Surge refers to motions in the forward and backward direction, heave is in the upward and downward direction and sway is in the left and right direction. These motions are a result of the platform movements.

Figure 3-3. Motions related to a wind turbine rotor

3.1.3 Blade Orientation

The important angles associated with the wind turbine blades specifically, are the cone, θ_c , and azimuth, θ_a , as shown in Figure 3-4.

Figure 3-4. Angles related to a wind turbine blade

The azimuth is the position of the rotor blades. For the whole rotor, the azimuth is generally computed for the first blade only (the azimuth of the other blades are multiples of $2\pi/B$ from the first, where B is the number of blades). The azimuth is the clockwise angle of the blade from the 12 o'clock position, that is, a blade that is pointing exactly upward for a rotor with no tilt or cone.

The cone angle involves all blades pointing forward or backward, and the convention is that if they bend to form an arrow pointing into the wind, the cone angle is positive, and vice versa.

These are separated from the rotor angles, as they do not affect rotor orientation, but specifically affect blade orientation.

3.1.4 Coordinate Transformations

The wind vector is defined in the earth-fixed system, while the blade elements are defined in the blade fixed system. Thus, to compute the relative velocity at each blade element, a transformation from the earth-fixed to the blade-fixed system is required, and is done by successively transforming the earth-fixed system to the platform-fixed, then rotor-fixed and finally the blade-fixed system. Other transformations required involve transforming the blade-fixed to the platform-fixed system and back, to find the velocity component due to the platform motion.

The transformation between the earth-fixed and platform-fixed systems is the transformations for yaw (subscript y), pitch (subscript p) and roll (subscript r) angles. These defined by the following transformation matrices.

$$\mathbf{b}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma_y) & \sin(\gamma_y) \\ 0 & -\sin(\gamma_y) & \cos(\gamma_y) \end{bmatrix} \quad (3-1)$$

$$\mathbf{b}_2 = \begin{bmatrix} \cos(\gamma_p) & 0 & -\sin(\gamma_p) \\ 0 & 1 & 0 \\ \sin(\gamma_p) & 0 & \cos(\gamma_p) \end{bmatrix} \quad (3-2)$$

$$\mathbf{b}_3 = \begin{bmatrix} \cos(\gamma_r) & \sin(\gamma_r) & 0 \\ -\sin(\gamma_r) & \cos(\gamma_r) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3-3)$$

The transformation of a vector $\mathbf{G}_3$ from an earth-fixed system to $\mathbf{G}_2$, a platform-fixed system would be:

$$\mathbf{G}_2 = \mathbf{b}_3 \mathbf{b}_2 \mathbf{b}_1 \mathbf{G}_3 \quad (3-4)$$

The transformation between the platform-fixed system and the rotor-fixed system are transformation for yaw (rotation in the x-axis, subscript y) and tilt (rotation in the y-axis, subscript t). These are defined by the transformation matrices below.

$$\mathbf{b}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_y) & \sin(\theta_y) \\ 0 & -\sin(\theta_y) & \cos(\theta_y) \end{bmatrix} \quad (3-5)$$

$$\mathbf{b}_5 = \begin{bmatrix} \cos(\theta_t) & 0 & -\sin(\theta_t) \\ 0 & 1 & 0 \\ \sin(\theta_t) & 0 & \cos(\theta_t) \end{bmatrix} \quad (3-6)$$

The transformation of a vector $\mathbf{G}_2$ from a platform-fixed system to $\mathbf{G}_1$, a rotor-fixed system would be:

$$\mathbf{G}_1 = \mathbf{b}_5 \mathbf{b}_4 \mathbf{G}_2 \quad (3-7)$$

To complete the transformation to a blade-fixed system, two more transformation matrices are defined:

$$\mathbf{b}_6 = \begin{bmatrix} \cos(\theta_a) & \sin(\theta_a) & 0 \\ -\sin(\theta_a) & \cos(\theta_a) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3-8)$$

$$\mathbf{b}_7 = \begin{bmatrix} \cos(-\theta_c) & 0 & -\sin(-\theta_c) \\ 0 & 1 & 0 \\ \sin(-\theta_c) & 0 & \cos(-\theta_c) \end{bmatrix} \quad (3-9)$$

The transformation of a vector $\mathbf{G}_3$ from an earth-fixed system to $\mathbf{G}_0$, a blade-fixed system would be:

$$\mathbf{G}_0 = \mathbf{b}_7 \mathbf{b}_6 \mathbf{b}_5 \mathbf{b}_4 \mathbf{b}_3 \mathbf{b}_2 \mathbf{b}_1 \mathbf{G}_3 \quad (3-10)$$

The last transformation needed is transforming the blade-fixed coordinates to a platform-fixed system. This involves transposing the transformation vectors and reversing their order. The transformation of a vector $\mathbf{G}_0$ from

a blade-fixed system, to G_2 , a platform-fixed system to would be found by reversing the order of the matrices and transposing them (superscript T).

$$G_2 = b_4^T b_5^T b_6^T b_7^T G_0 \quad (3-11)$$

3.2 Relative Velocity

The relative velocity, V_{rel} , is the velocity relative to the blade element of the rotor. This is the sum of all the velocity components experienced by a blade element, and is described in this section.

$$V_{rel} = V_0 + V_{rot} + W + A \quad (3-12)$$

where V_0 is the undisturbed wind velocity in blade-fixed coordinates, V_{rot} is the velocity component due to the blade rotation, W is the induced velocity and A is the velocity component due to the platform motion.

Starting with the undisturbed wind velocity in earth-fixed coordinates, V_3 , we use equation (3-10) to find the undisturbed wind velocity V_0 in blade-fixed coordinates.

Next, the component due to the rotation of the blades, V_{rot} , is computed.

$$V_{rot} = \begin{pmatrix} 0 \\ -\Omega r \cos(\theta_c) \\ 0 \end{pmatrix} \quad (3-13)$$

where Ω is the rotational speed of the rotor in $rad s^{-1}$ and r is the elemental radius.

The induced velocity vector, W is taken from the previous time step.

$$W = \begin{pmatrix} 0 \\ W_y \\ W_z \end{pmatrix} = \begin{pmatrix} 0 \\ W_t \\ W_n \end{pmatrix} \quad (3-14)$$

The final component, A , is due to the movement of the rotor brought about by the floating platform. The computation of this is based on the

position and orientation of the platform relative to the previous time step, and is explained in Section 3.2.1.

3.2.1 Velocity Component Due to Platform Motion

The floating platform of a FOWT can move with 6 degrees of freedom. The 3 translational motions are heave, surge and sway, and the 3 rotational motions are yaw, pitch and roll, as shown in Figure 2-6.

The rotational motions will also cause movement of the blades, and so all of these need to be computed. To do this, the coordinates of each element of each blade need to be transformed into platform-fixed coordinates, r_p , using equation (3-11). Since the x-axis runs along the length of the blade in the blade-fixed coordinates, as shown in Figure 3-1, the radius of the element centre can be used to find the platform-fixed coordinate of each blade element. To complete the transformation, the hub height of the rotor must be added.

$$r_p = b_4^T b_5^T b_6^T b_7^T \begin{pmatrix} r \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} H \\ 0 \\ 0 \end{pmatrix} \quad (3-15)$$

where H is the hub height.

The wind velocity vector due to the platform motion can thus be found, given the velocities and angular rate of platform motion [72].

$$A_2 = \begin{pmatrix} -\dot{x} - r_{p,y}\dot{\gamma}_r - r_{p,z}\dot{\gamma}_p \\ -\dot{y} - r_{p,x}\dot{\gamma}_r - r_{p,z}\dot{\gamma}_y \\ -\dot{z} - r_{p,x}\dot{\gamma}_p - r_{p,y}\dot{\gamma}_y \end{pmatrix} \quad (3-16)$$

This is then converted to blade-fixed coordinates:

$$A = b_7 b_6 b_5 b_4 A_2 \quad (3-17)$$

Using the same methodology, other coordinate systems may be added, for example, a tower-fixed system, to account for other sources of relative wind components. In the present study, the tower and blades are assumed rigid.

3.3 Induced Velocities

With the relative velocity, the flow angle, ϕ can be calculated.

$$\phi = \arctan\left(\frac{V_{rel,z}}{-V_{rel,y}}\right) \quad (3-18)$$

The twist of the airfoil, β is the sum of the tip pitch and local geometric twist, and this is used to find the angle of attack, α .

$$\alpha = \phi - \beta \quad (3-19)$$

BEM and uBEM codes usually find the local aerodynamic force coefficients by means of a lookup table, usually pre-treated with a stall-delay model. Due to the unsteady FOWT environment, with changing wind-speeds and speed regulation for the rotor, the amount of stall-delay on a wind turbine can be expected to change. A stall-delay model should thus be applied in real-time. Chapter 7 details the Du & Selig model [16] which is implemented in the present study.

We define $\mathbf{n}$ as a unit vector in the direction of rotor thrust.

$$\mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \quad (3-20)$$

This is used to find the wind velocity at the rotor plane, $\mathbf{V}'$.

$$\mathbf{V}' = (\mathbf{V}_0 + \mathbf{A}) + \mathbf{n}(\mathbf{n} \cdot \mathbf{W}) \quad (3-21)$$

To find the induced velocities, it is assumed that only lift contributes to the momentum balance [8].

$$W_z = W_n = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \cos(\phi) cB}{4\rho|\mathbf{V}'|\pi r} \quad (3-22)$$

$$W_y = W_t = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \sin(\phi) cB}{4\rho|\mathbf{V}'|\pi r} \quad (3-23)$$

since each blade only covers $1/B$ of an annulus.

At zero yaw, this reduces to the classical BEM theory (since it was derived from classical BEM at zero yaw). Equation (3-22) also corresponds to Glauerts solution for a gyrocopter in forward flight [5], which represents a 90° wake skew angle and is equivalent to an elliptically loaded wing. It is thus assumed that equation (3-22) applies for any yaw angle, although this is not rigorous [8].

Using axial momentum theory (as compared to Glauerts solution for gyrocopters), the z-component of $\mathbf{V}'$, V'_z , would be used instead of its vector length. However, at 90° wake skew angle, if the wind turbine continues to produce thrust, it will act as a lifting surface, adding credence to the elliptical wing assumption of Glauert. A fictitious cylindrical control volume has been assumed by several authors, including Hansen [8], Burton et al [75], Sant [81], Bramwell et al [85] and Seddon [86], in order to explain Glauerts assumption that this equation can be extended to any wake skew angle (or yaw angle). This is indicated in Figure 3-5.

The cross-section of the cylindrical control volume is a circle with radius R , the radius of the rotor. This means that the control volume is the same, as if the rotor plane is normal to the wind, regardless of rotor orientation. This control volume explanation does not have a theoretical basis, and cannot be considered a suitable explanation or justification of Glauert's assumption. The method presented here, based on the underlying assumptions, is able to rigorously justify Glauert's equation for all wake skew angles.

Figure 3-5. Control volume assumption to explain the Glauert equation (adapted from [8])

Glauert's equation assumes a uniform actuator disc. This is a permeable disc through which a force is exerted on the air, resulting in the induced velocity, which is the velocity deficit in the wake (this can also be seen the other way around, as the velocity deficit resulting in a force on the rotor). The force exerted by this disc is assumed uniform, which is a simplifying assumption to reduce the dimensionality of the problem. In reality, using the solution to the Euler equations presented in Section 2.4.3.1, or using simple vortex methods, it can be shown that there must be an azimuthal variation in rotor loads for any wake skew angle. However, this assumption is the basis for blade element momentum methods, and is applied to obtain a value for the induced velocity.

The next assumption is that the actuator disc is permeable; therefore the 1-dimensional axial momentum theory applies. This means that the mass flow rate through the rotor disc is:

$$\dot{m} = \rho A V'_z \quad (3-24)$$

The momentum thrust force would be given by:

$$T_{mom} = \dot{m} \Delta V = \rho A V'_z \cdot 2W_n \quad (3-25)$$

since the induced velocity in the wake is twice the induced velocity at the rotor plane.

It is also assumed that when the skew angle is non-zero, the total thrust force has a component resulting in the rotor behaving like an elliptic wing, which is termed the circulation thrust.

$$T_{cir} = \rho V'_y \pi \frac{R}{2} \Gamma \quad (3-26)$$

where the circulation for the circular wing, Γ , is related to the induced velocity by:

$$W_n = \frac{\Gamma}{2D} = \frac{\Gamma}{4R} \quad (3-27)$$

This has been proven for the limiting case of a 90° skew angle using the Euler equations by Bramwell [87] and the ring vortex method by Shaidakov [88]. It is therefore reasonable to assume that any flow parallel to the rotor plane would result in the circulation thrust. Using equations (3-26) and (3-27), and noting that the induced velocity in the wake is twice the induced velocity in the rotor plane, the circulation thrust is defined as:

$$T_{cir} = \rho \pi R^2 V'_y \cdot 2W_n = \rho A V'_y \cdot 2W_n \quad (3-28)$$

As a permeable lifting surface, the momentum thrust and circulation thrust of the rotor cannot be taken as independent components of the total thrust. There is a degree of overlap between these components for any skew angle between 0° and 90° . Thus, the vector sum of the two velocity components are used to find the total thrust.

$$T = \rho A \sqrt{V_z'^2 + V_y'^2} \cdot 2W_n = 2\rho A |\mathbf{V}'| W_n \quad (3-29)$$

Applying the area for an annular section, we get:

$$T = 4\rho|\mathbf{V}'|\pi rW_n \quad (3-30)$$

which is equivalent to equation (3-22).

The proof shown here shows how the thrust force from axial momentum theory and circulation of an elliptical wing are combined logically to compute thrust based on the rate of change of momentum. Each of these components has been proven individually, and would hold true using the axial and tangential components of $\mathbf{V}'$, the velocity at the rotor plane. Thus, this provides a more rigorous explanation of the applicability of the Glauert equation at any wake skew angle.

The problem with Glauert's relation is that an axial induction factor is not clearly defined. From the thrust perspective, it is clear that the axial induction factor should be:

$$a = \frac{-W_n}{|\mathbf{V}_0 + \mathbf{A}|} \quad (3-31)$$

This would lead to the same relation between coefficient of thrust and axial induction factor shown in equation (2-14).

The power extracted by the wind turbine is:

$$P = \frac{1}{2}\dot{m} (|\mathbf{V}_0 + \mathbf{A}|^2 - |\mathbf{V}'|^2) \quad (3-32)$$

Since the velocity component tangential to the rotor plane does not change across the rotor ($V_{0,y} + A_y = V'_y$),

$$P = \frac{1}{2}\dot{m} (|\mathbf{V}_0 + \mathbf{A}|^2 - |\mathbf{V}'|^2) = \frac{1}{2}\dot{m} ((V_{0,z} + A_z)^2 - V_z'^2) = \frac{1}{2}\rho A|\mathbf{V}'|W_n (V_{0,z} + A_z - W_n) \quad (3-33)$$

In the 1-dimensional momentum theory for a wind turbine in axial flow, the coefficient of power, C_P , is defined as:

$$C_P = \frac{P}{\frac{1}{2}\rho V_0^3 A} = 4a(1-a)^2 \quad (3-34)$$

It is clear that the same definition of axial induction factor cannot be used in determining the coefficient of power. Instead, the aerodynamic power extracted should be calculated explicitly, then the coefficient of power determined. Furthermore, the power is calculated from the result of the computations, while the thrust force and induced velocities are required to advance the computations. Thus, it would be more useful to maintain the definition of axial induction factor presented in (3-31).

3.3.1 Tip-Loss Factor

The fact that the rotor is composed of a finite number of blades means that root and tip vortices are produced. These reduce the thrust produced by the rotor, especially near the blade tip and root, and carry energy away from the rotor. To account for these losses, a tip-loss factor, F , is introduced. The tip-loss factor is in fact a tip and root loss factor, but will be referred to as tip-loss for simplicity.

The most commonly used tip-loss model is the Prandtl tip-loss factor [22], based on the optimum circulation distribution for a wind turbine blade.

Tip-loss is a complex problem as it is dependent on the distribution of circulation over the blade, thus dependent on airfoil shape, chord length and twist distribution over the blade. Since different airfoils have different lift curves, and airfoils with different chord lengths and twist angles will react to different extents to changes in the flow, tip-loss should be turbine-specific and dependent on several factors such as tip speed ratio and wake turbulence. A novel methodology for developing a tip-loss model that possesses these properties is described in Chapter 5 with a description of other tip-loss models found in literature.

Once the tip-loss factor, F , is obtained, it can be introduced into equations (3-22) and (3-23) to determine the induced velocities.

$$W_z = W_n = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \cos(\phi) cB}{4\rho F |\mathbf{V}'| \pi r} \quad (3-35)$$

$$W_y = W_t = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \sin(\phi) cB}{4\rho F |\mathbf{V}'| \pi r} \quad (3-36)$$

3.3.2 Glauert Correction

The Glauert correction is used to correct for the deviation from momentum theory at high axial induction factors. The correction factors of Glauert [7] and Buhl [24] were presented in Section 2.1.5. The Buhl equation was formulated to ensure no discontinuities arise from changing tip-loss factors:

$$C_T = \left(\frac{8}{9}\right) + \left(4F - \frac{40}{9}\right) a + \left(\frac{50}{9} - 4F\right) a^2 \quad (3-37)$$

For application in uBEM, equation (3-37) must be put in the form:

$$C_T = 4a(1 - f_g a) \quad (3-38)$$

where f_g is the Glauert correction.

Rearranging, gives Glauert correction to be applied:

$$f_g = \frac{2}{9aF} \left(5 - \frac{1}{a}\right) - 1 + \frac{50}{36F} \quad (3-39)$$

This is then included in $\mathbf{V}'$, and in equations (3-35) and (3-36).

$$W_z = W_n = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \cos(\phi) cB}{4\rho F |(\mathbf{V}_0 + \mathbf{A}) + f_g \mathbf{n}(\mathbf{n} \cdot \mathbf{W})| \pi r} \quad (3-40)$$

$$W_y = W_t = \frac{-\frac{1}{2}|\mathbf{V}_{rel}|^2 C_l \sin(\phi) cB}{4\rho F |(\mathbf{V}_0 + \mathbf{A}) + f_g \mathbf{n}(\mathbf{n} \cdot \mathbf{W})| \pi r} \quad (3-41)$$

The Glauert correction presented here is only applicable for the turbulent wake state, and increases the range of uBEM to $0 < a < 1$. In the unsteady FOWT environment, transient operating points may move beyond this range of axial induction factors. Thus, an extended Glauert model is needed to model the relationship between induced velocities and forces in the vortex ring,

propeller brake, and propeller wake states. This extended model is described in Chapter 9.

These values of induced velocities can be used in the next time step of uBEM, for each blade element of each blade. However, these are still based on steady state momentum theory, and need to be corrected for the time lag between forces on the rotor and wake velocities, called the dynamic wake effect. The dynamic wake model is discussed in Chapter 8

3.4 Blade Elemental Forces

The corrected induced velocities are then used in equation (3-12) to determine the relative velocity, and the flow angle (equation (3-18)) and angle of attack (equation (3-19)) determined again. Using these, the airfoil force coefficients are determined.

In the unsteady aerodynamic environment of a FOWT, an unsteady airfoil model is required to account for airfoil-level unsteadiness. Unsteady airfoil models usually have empirically-determined constants which are generally optimised over many airfoil shapes, thus have lower levels of accuracy for most airfoil shapes. Chapter 6 will present a shape-specific modified Beddoes-Leishman model developed using an optimisation-based methodology. The unsteady airfoil model is used only after the determination of the induced velocities, since the time scales at airfoil-level unsteadiness are much smaller than rotor-level unsteadiness [36], thus would not have an effect on the wake momentum balance.

A real-time stall-delay model is also needed to account for changes in speed ratios along the blade. An effective approach would be to implement stall-delay in real-time. However, this is difficult, since the stall-delay effect would change aerodynamic properties needed for the unsteady airfoil model to function. Chapter 7 will present a novel combined stall-delay and unsteady airfoil method, based on the Du & Selig [16] and modified Beddoes-Leishman model from Chapter 6.

Having obtained the local cross-sectional coefficients of lift (C_l) and drag (C_d), the coefficients of forces normal and tangential to the rotor plane can be found.

$$C_n = C_l \cos \phi + C_d \sin \phi \quad (3-42)$$

$$C_t = C_l \sin \phi - C_d \cos \phi \quad (3-43)$$

These are then used to compute the forces P_z and P_y , in the z and y directions respectively

$$P_z = \frac{1}{2} |\mathbf{V}_{rel}|^2 C_n c \quad (3-44)$$

$$P_y = \frac{1}{2} |\mathbf{V}_{rel}|^2 C_t c \quad (3-45)$$

where ρ is the density of the air and c is the chord length of the mid-section of the element.

The mid-section of the element is used for simplicity, although the chord length at the volumetric centre of the element should be used instead, which would not be far from the mid-point of the element, thus, there would be a negligible difference in chord lengths.

3.5 uBEM Algorithm

The uBEM algorithm is summarised in Figure 3-6.

It begins with reading the blade geometry and flow parameters, followed by an initialisation of the blade positions and discretisation of the blades. The induced velocities are then initialised, and the time marching algorithm started. The relative velocities are computed, followed by the blade element forces, used to find the induced velocities, which are then corrected using the dynamic wake model. These are used in the next time step to compute the next relative velocity.

Figure 3-6. Flowchart of uBEM algorithm

3.6 Concluding Remarks

The uBEM method described in this chapter will form the basis of modelling other aerodynamic phenomena described in this thesis. The advantage of uBEM is that it is time-marching and resolves the forces on each blade individually, thus forces due to azimuthally non-uniform flows can be computed more accurately.

A justification for the application of Glauert's gyrocopter equation at any yaw angle was presented, based on a combination of circulation and momentum theory. This is a more logical and rigorous justification compared to the fixed-control volume description provided by other authors.

Later chapters will describe corrective models required for uBEM to be suitable for FOWT aerodynamics.

Chapter 4

Computational Fluid Dynamics (CFD) Methodology

This chapter will describe the computational methodology used for the direct rotor models of the NREL 5MW wind turbine [4]. The direct rotor model involves a full 3-dimensional Navier-Stokes computations of the flow around a rotating wind turbine. The direct rotor model in this study represent the higher order method used to formulate corrective models for uBEM. This requires a computational grid that is suitably fine to resolve the boundary layers and capture important flow features and time steps that are suitably small to capture important transient effects. Thus, this chapter will describe the spatial and time discretisation study for the CFD simulations conducted. The CFD simulations were conducted using ANSYS CFX 16.1.

4.1 Simulation conditions

Numerous simulations of different flow scenarios were conducted in this study, including:

1. A static rotor at $RPM = 11.45$ and $V_0 = 10ms^{-1}$.
2. A static rotor at several different tip speed ratios, as will be described in Chapter 5.
3. A rotor in surging motion with $RPM = 11.45$ and $V_0 = 10ms^{-1}$ at several amplitudes and frequencies , as will be described in Table 4-1.

Table 4-1. Wind turbine surge conditions used in dynamic wake study

Amplitudes (m)	2, 4, 6
Frequencies (Hz)	0.02, 0.1, 0.5

The highest frequency of $0.5Hz$ is not an expected frequency of platform motion. However, it can occur in structural components, and extreme events such as storms. The frequency was chosen to transcend normal operating conditions of wind turbines and reach the vortex ring and propeller brake states.

A mesh refinement and time step refinement study was conducted using the following conditions:

1. A static rotor at $RPM = 11.45$ and $V_0 = 10ms^{-1}$.
2. A rotor at $RPM = 11.45$ and $V_0 = 10ms^{-1}$ in surging motion under two conditions: A frequency of $0.02Hz$ with an amplitude of $2m$ as well as a frequency of $0.1Hz$ with an amplitude of $4m$.

The wind speed and RPM of the rotor are below-rated conditions, but operating in the windmill state. The effective wind speed for the surging rotor will increase to above-rated and reduce to below-rated speeds during the simulations, as shown in Figure 4-1.

Figure 4-1. Effective wind speed experienced by rotor during surging motion

4.2 Computational Mesh

4.2.1 Rotor Geometry

The rotor geometry was been defined in [4]. The 5MW rotor is a 3-bladed rotor composed of 6 different airfoil cross-sections over the length of each blade. The airfoil names in the NREL 5MW document [4] are abbreviated, thus the airfoil names are included in Table 4-2.

Table 4-2. Airfoils used in NREL 5MW turbine

Airfoil	Airfoil Name	Thickness (%)
DU40_A17	DU99-W-405LM	40.50
DU35_A17	DU99-W-350	35.09
DU30_A17	DU97-W-300LM	30.00
DU25_A17	DU91-W2-250	25.00
DU21_A17	DU93-W-210LM	21.00
NA64_A17	NACA 64-681	18.00

The blade is twisted and tapered, with each blade station having an decreasing twist and chord length from root to tip. This is shown in Table 4-3 and in Figure 4-2.

Figure 4-2. Distribution of twist and chord against non-dimensionalised radius (r/R)

The document only lists the profiles at the element centres, and how these are merged into one blade depends on the method of modelling used. The tip chord is 0, but this results in some modelling difficulty. Thus, the tip was given a very small, finite chord of 0.129 m with the NA64_A17 profile. No hub geometry was created. Instead, the cylinder in Node 1 was extended to zero radius.

ANSYS ICEM CFD 16.1 was used to construct the blade geometry. The airfoil profile coordinates are imported and the blade surface created by lofting over the suction and pressure surface curves.

Table 4-3. NREL 5MW turbine blade properties

Node	Element centre radius(m)	Twist($^{\circ}$)	Element length(m)	Element centre chord(m)	Cross- sectional profile
1	2.8667	13.308	2.7333	3.542	Cylinder
2	5.6000	13.308	2.7333	3.854	Cylinder
3	8.3333	13.308	2.7333	4.167	Cylinder
4	11.7500	13.308	4.1000	4.557	DU40_A17
5	15.8500	11.480	4.1000	4.652	DU35_A17
6	19.9500	10.162	4.1000	4.458	DU35_A17
7	24.0500	9.011	4.1000	4.249	DU30_A17
8	28.1500	7.795	4.1000	4.007	DU25_A17
9	32.2500	6.544	4.1000	3.748	DU25_A17
10	36.3500	5.361	4.1000	3.502	DU21_A17
11	40.4500	4.188	4.1000	3.256	DU21_A17
12	44.5500	3.125	4.1000	3.010	NA64_A17
13	48.6500	2.319	4.1000	2.764	NA64_A17
14	52.7500	1.526	4.1000	2.518	NA64_A17
15	56.1667	0.863	2.7333	2.313	NA64_A17
16	58.9000	0.370	2.7333	2.086	NA64_A17
17	61.6333	0.106	2.7333	1.419	NA64_A17

4.2.2 Mesh Setup

A the computational domain was a one-third cylinder, so that a periodic simulation could be conducted to reduce computational time. As such, only one blade was used in the computational mesh. The mesh was divided into 3 principal regions: Blade, Near-wake and Outer.

The blade region is the mesh directly around the blade. An o-grid was generated around the blade to ensure the boundary layer could be resolved accurately. The height of the first layer is between $6.6\mu\text{m}$ to $8.2\mu\text{m}$, depending on the position on the blade. This is shown in Figure 4-3.

Figure 4-3. O-grid generated around the rotor blade

The mesh was refined to ensure that $y^+ < 1$ over most of the blade, as shown in Figure 4-4. The y^+ only exceeds 1 near the tip, where it is still low at $y^+ = 3$.

Figure 4-4. y^+ over the blade during static simulation

The near-wake region is a cylindrical region of refined mesh to ensure that the flow features in the near-wake are satisfactorily resolved, as the resolution of

the near-wake region is needed for the accurate determination of blade forces. The near-wake extends $1R$ upstream and $3.33R$ downstream of the rotor. The radius of the region is $1.17R$.

The outer region extends $4.8R$ upstream and $11R$ downstream of the rotor. Its radius is $6R$. The near-wake and outer regions are shown in Figures 4-5 and 4-6.

Figure 4-5. Front view of computational domain

Figure 4-6. Side view of computational domain

4.3 CFD Setup

4.4 Initial and Boundary Conditions

The present study involved flows axial to the rotor, thus computational time was reduced by using only a single blade in a 120° periodic simulation. The rotation of the blade was simulated using a multiple reference frames or frozen rotor method, and rotation is simulated in the near-wake and blade regions.

The $k-\omega$ SST [89] turbulence model was used, without transition modelling. The fluid was air at 25°C, with a turbulence intensity of 5%. A high resolution scheme was used for advection and turbulence numerics, while for transient simulations, a second order backward Euler scheme was used. Pressure-velocity coupling was solved using a fourth order Rhie-Chow interpolation.

The inlet boundary was set as a velocity inlet with uniform velocity, while the outlet was set as a pressure outlet. The curved boundary of the outer region was set as an opening. The wind turbine blade was set to a no-slip wall.

4.4.1 Dynamic Mesh Setup

The rotor surge motion was simulated using a deforming mesh. The deforming mesh was setup up such that the mesh in the blade and near-wake regions were given a high rigidity, so that deformation is limited to the outer region, where there are bigger cells, so negative volumes would not occur.

The surge motion is defined by the equation:

$$-A + A \cos(2\pi f) \quad (4-1)$$

where A is the amplitude of motion and f is the frequency

The entire blade and near-wake regions were set to move with this equation. The equation assumes the mesh starts at the forward-most position, since there are more cells downstream of the rotor, thus it would facilitate smoother deformation. The surge motion is shown in Figure 4-7.

Figure 4-7. Displacement of the rotor against time in CFD simulations

4.5 Mesh Refinement Study

A mesh refinement study was conducted with 3 meshes: coarse (3.3×10^6 cells), medium (3.7×10^6 cells), and fine (6.0×10^6 cells).

The mesh refinement was first carried out on the static case, and the percentage difference in thrust and torque forces at each blade element from the medium mesh recorded in Table 4-4 for the coarse and fine meshes. The forces at each blade element for the fine mesh were found to be within 1.05% of the medium mesh, thus the medium mesh was deemed sufficiently refined for the study.

The dynamic simulations were run with timestep $\Delta T = 1/(200f)$. This means that there were 200 timesteps per period of oscillatory motion. The mesh refinement was performed for the two cases of rotor surging motion. The total thrust and torque of the coarse and fine meshes were compared to the medium mesh. The percentage differences are plotted against time in Figures 4-8 to 4-11.

Table 4-4. Percentage change in thrust and shaft torque at each blade segment, compared to medium mesh for static simulation

r/R	Thrust (%)		Torque (%)	
	coarse	fine	coarse	fine
0.19	3.02	1.05	2.18	0.81
0.25	0.29	0.14	0.28	0.15
0.32	0.01	0.00	0.02	0.05
0.38	0.03	0.06	0.04	0.08
0.45	0.00	0.09	0.03	0.10
0.51	0.00	0.00	0.01	0.10
0.58	0.00	0.07	0.00	0.08
0.64	0.00	0.00	0.01	0.07
0.71	0.00	0.00	0.01	0.09
0.77	0.00	0.00	0.00	0.10
0.84	0.00	0.05	0.00	0.09
0.89	0.00	0.00	0.04	0.06
0.93	0.00	0.00	0.02	0.08
0.98	0.01	0.02	0.02	0.07

Figure 4-8. Percentage difference in rotor thrust relative to medium mesh for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$

Figure 4-9. Percentage difference in rotor shaft torque relative to medium mesh for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$

Figure 4-10. Percentage difference in rotor thrust relative to medium mesh for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$

Figure 4-11. Percentage difference in rotor shaft torque relative to medium mesh for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$

All differences were found to be less than 1% from the medium mesh, thus, the medium mesh could be used. Furthermore, the differences were not proportional to the surge motion of the rotor, thus the error was found to be acceptable.

4.6 Time step refinement

Besides spatial discretisation, temporal discretisation of the simulations need to be studied to ensure that the time step used is adequately small to capture the transient flow phenomena. In the coarse temporal discretisation, the time step was twice the size of the medium time step, thus the coarse temporal discretisation had 100 time steps per motion period. In the fine temporal discretisation, the time step was half that of the medium time step, thus had 400 time steps per motion period. The total thrust and torque of the coarse and fine time steps were compared to the medium time step. The percentage differences are plotted against time in Figures 4-12 to 4-15.

Figure 4-12. Percentage difference in rotor thrust relative to medium time step for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$

Figure 4-13. Percentage difference in rotor shaft torque relative to medium time step for surge motion with Amplitude= $2m$ and Frequency= $0.02Hz$

Figure 4-14. Percentage difference in rotor thrust relative to medium time step for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$

Figure 4-15. Percentage difference in rotor shaft torque relative to medium time step for surge motion with Amplitude= $4m$ and Frequency= $0.1Hz$

The results from Figures 4-12 to 4-15 show that the coarse time step had more than 1% difference from the medium time step, for the surge motion with an amplitude of $4m$ and frequency of $0.1Hz$. The fine time step always

maintained a difference of less than 1%. This suggests that the medium time step is refined enough to capture transient flow phenomena.

4.7 Concluding Remarks

The CFD methodology explained in this chapter is the basis for all CFD work done during the course of the present research. The future chapters will detail specific flow conditions simulated for the development of the models detailed within those chapters, based on refinement studies performed both spatially and temporally within this chapter. It was shown that the spatial and temporal discretisation was sufficient to capture static and transient flow phenomena of FOWTs.

Chapter 5

Tip-Loss Model

The most fundamental correction to the 1-dimensional momentum theory is that of tip-loss, which corrects for the assumption of an infinitely-bladed rotor. This phenomenon exists at any rotor loading or flow condition, and results in a loss in circulation closer to the blade tip and root regions. Prandtl [22] showed that this circulation tends to zero at the tip (and it can be similarly conjectured to be zero at the root). Glauert [90] implemented Prandtl's theory into BEM, by correcting the induced velocity at the rotor plane, but not the mass flow.

Wilson and Lissaman [91] later applied the tip-loss correction to both the induced velocity and the mass flow, and de Vries [92] updated this to correct the mass flow in the tangential momentum equation as well so as to ensure orthogonality between induced velocity and relative velocity.

Shen et al [93] provided an empirical correction for the de Vries model for better tip-loss prediction near the tip region in the NREL Phase VI and the Swedish WG 500 rotors. Xu and Shankar [94] provided an empirical model based on Glauert's implementation of Prandtl's model for the NREL Phase VI turbine, without enforcing the zero-circulation condition at the tip.

The difficulty with modelling the tip-losses stems from the transformation of airfoils, pitch and chord over the blade. This makes each wind turbine unique in its distribution of circulation over the blade, thus having unique distributions of tip losses. A constant in the theoretical models is the dependence on tip speed ratio. Thus, to minimise modelling error from attempting to create a general correction for tip-losses in wind turbines, an empirical methodology

is proposed using simulations or experimental data to determine the tip-loss factor, and modelling these as a correction to the Prandtl tip-loss factor based on non-dimensionalised rotor radius. The correction is designed to maintain the zero-tip loss condition at the rotor tip, to ensure consistency with theory [45].

This chapter will first describe the Prandtl tip loss factor with the Glauert implementation, followed by a description of the proposed methodology, finally describing the results through CFD analysis of the NREL 5MW wind turbine [4] used in this study.

5.1 Theory

5.1.1 Prandtl tip-loss [22]

The Prandtl tip-loss factor [22] was originally developed to account for tip-losses in propellers, and used as a means of determining the optimum design for a propeller. The tip loss represents the ratio of the circulation in a finitely-bladed rotor to that of an infinitely-bladed one. Glauert [90] later described the use of the Prandtl tip loss factor, and implemented this in the blade element momentum method. The Prandtl tip-loss factor takes the following form:

$$F_{Prandtl} = \frac{2}{\pi} \cdot \arccos(e^{-f}) \quad (5-1)$$

where f is a function dependent on local radius, r , tip radius, R and the distance between two vortex lines, s :

$$f = \pi \frac{(R - r)}{s} \quad (5-2)$$

Prandtl assumes a cylindrical wake, an assumption valid at higher tip speed ratios and number of blades. $F_{Prandtl}$ approximates the ratio between the circulation at the blade segment and the circulation of an infinitely-bladed rotor to the ratio between the average fluid velocity within an infinite array

of semi-infinite plates, and the velocity of these plates, which are moving in a direction normal to themselves [95].

The distance s can then be described by the geometry of a screw:

$$s = \frac{2\pi R \sin \phi_R}{B} \quad (5-3)$$

where ϕ_R is the flow angle relative to the normal of the rotor plane, at the tip and B is the number of blades (thus number of vortex lines).

Substituting equation (5-3) into equation (5-2) gives the expression given by Glauert [90]:

$$f = \frac{B (R - r)}{2 R \sin \phi_R} \quad (5-4)$$

Noting the difficulty of finding the tip flow angle, Glauert simplified this further for implementation in the blade element momentum method [90]:

$$f_{Gl} = \frac{B (R - r)}{2 r \sin \phi_r} \quad (5-5)$$

This gives a tip loss factor of:

$$F_{Gl} = \frac{2}{\pi} \cdot \arccos(e^{-f_{Gl}}) \quad (5-6)$$

Equation (5-5) assumes that the flow angle, for small flow angles, is negatively linearly correlated to local blade segment radius. For completeness, losses at the blade hub should be considered, and to find the hub-loss, $(R - r)$ in equation (5-5) should be replaced with $(r - R_{hub})$, where R_{hub} is the radius of the hub [45]. R_{hub} should taken to be the point where the cylindrical section of the blade meets the airfoil cross-section. The total losses at the tip and hub are then the product of the tip-loss and hub-loss factors.

In order to apply the tip loss factor, Glauert [90] corrected the change in momentum in the coefficient of thrust, but not the mass flux through the rotor plane:

$$CT = 4aF(1 - a) \quad (5-7)$$

where a is axial induction factor, the ratio between the velocity change in axial flow and the axial wind speed.

Wilson and Lissaman applied the tip loss correction for mass flux in the axial direction as well [91]:

$$CT = 4aF(1 - aF) \quad (5-8)$$

To ensure the orthogonality between the axial and tangential induced velocities, de Vries extended this to the tangential mass flux [92].

Considering the local speed ratio at blade stations closer to the hub are smaller, the induced velocities at those blade stations will be more significant than those closer to the tip. However, closer to the hub, the tip-loss factor also tends to unity, thus the difference between Glauert's and Wilson and Lissaman's as well as de Vries' formulations is minimal closer to the root. Closer to the tip, while the tip-loss is more significant, the induced velocities are smaller and more insignificant, thus, the two formulations remain similar.

5.1.2 Empirical corrections for Prandtl tip-loss

5.1.2.1 Lindenburg [96]

Lindenburg presents an equation for f based on average induced velocities, suggesting that the tip vortex is spread over the blade, not concentrated at the tip [96]. The trailing vortex is assumed to move at a velocity averaged between freestream and the wake velocity. Thus, Lindenburg's correction, f_{Li} , is given by [96]:

$$f_{Li} = \frac{B}{2} \left(\frac{R - r}{r} \right) \sqrt{1 + \lambda_r^2 \left(\frac{1 + \sqrt{F_{Li}} a'}{1 - \sqrt{F_{Li}} a'/2} \right)^2} \quad (5-9)$$

where λ is the tip speed ratio and F_{Li} is given by:

$$F_{Li} = \frac{2}{\pi} \cdot \arccos \left(e^{-f_{Li}} \right) \quad (5-10)$$

The iterative mechanism for determining F_{Li} suggests that in addition to the axial and tangential induced velocities, the tip-loss factor would need to converge as well. The Lindenburg model addresses issues with assuming individual blade elements when determining the trailing vortex line geometry. However, it does not address changes in the circulation distribution over the blade based on rotor geometry, and still uses the optimal distribution assumption used by Prandtl.

5.1.2.2 Shen et al [93]

Shen et al applied a corrected Prandtl tip loss factor to the aerodynamic force coefficients, in addition to the tip-loss correction applied by de Vries [93]. The correction takes the form of a function g multiplied by f [93]:

$$f_{Sh} = g_{Sh} \cdot f_{Gl} \quad (5-11)$$

where g is related to λ by:

$$g_{Sh} = e^{-c_1(B\lambda - c_2)} \quad (5-12)$$

where c_1 and c_2 are empirically determined constants.

The model proposed by Shen et al, by acting directly on the aerodynamic force coefficients, acts as both a correction to the Prandtl tip-loss model and as a stall-delay model. This approach thus cannot distinguish between these effects, and may not be universally applicable. The function g_{Sh} used here depends solely on tip speed ratio, which suggests no adjustments to the circulation distribution over the rotor based on shape. However, the approach of correcting f with a function g is a good engineering method for attaining a tip-loss model closer to reality.

5.1.2.3 Xu and Sankar [94]

The Xu and Sankar model [94] was developed using CFD simulations of the NREL Phase VI experiment [19]. The tip loss factor from CFD was computed as the ratio of the cross-sectional lift coefficient from CFD to the coefficient of lift from a stall-delayed 2-dimensional coefficient on lift table, then empirically modeled as [94]:

$$F_{XS} = \begin{cases} (F_{Gl}^{0.85} + 0.5) / 2, & \text{for } r/R \geq 0.7 \\ 1 - (r/R) (1 - F_{XS,r/R=0.7}) / 0.7, & \text{for } r/R < 0.7 \end{cases} \quad (5-13)$$

The stall delay imposed was a simple 1° stall delay, not one based on the CFD data. Furthermore, to ensure that the tip loss distribution is continuous, the tip loss at $r/R = 0.7$ is needed, and must be computed, even if it does not fall on the centroid of any of the blade segments used.

The use of a non-zero trailing edge tip loss factor has been debated, and found to be theoretically inaccurate [45]. However, engineering applications of the model in BEM generally compute blade segment forces at the centroid of blade segments, thus an accurate reflection of the tip condition is unnecessary. The engineering model should be assessed based on its applicability and accuracy when applied correctly. The Xu and Sankar model, being fitted to the tip loss factors at r/R of 0.8 to 1, may not accurately reflect the losses at blade segments closer to the hub.

5.2 Methodology

5.2.1 Simulation matrix

In order to cover a range of tip speed ratios, a total of 11 simulations were run. The wind and rotor RPM were staggered, in order to account for changes in wind speed and RPM separately from the tip speed ratio. The simulation matrix is shown in Table 5-1. CFD simulations of the NREL 5MW virtual wind turbine, as described in Chapter 4 were run at these values of tip speed ratios,

λ , and compared to BEM calculations to determine the true tip-loss factors of the wind turbine.

Table 5-1. Rotor RPM, wind speeds simulated in present study, with the tip speed ratios (λ) of simulations displayed in the cells

		Wind Speed (ms^{-1})					
		7	8	9	10	11	12
RPM (min^{-1})	14	13.19					
	13	12.25	10.72				
	12			9.90	8.80		
	11				8.06	7.26	6.60
	10					6.00	5.50
	9						4.95

5.2.2 Tip-loss factors from CFD

The tip-loss factors for each blade segment were extracted from CFD based on an optimisation of the tip-loss factor in the BEM at each blade segment to match the CFD-generated thrust and torque values. The objective function of the n^{th} segment of the blade is a combination of the relative error between CFD-generated blade sectional thrust and torque and BEM-generated values, and this is minimised:

$$E_n = q_w \frac{\text{abs}(T_{CFD} - T_{BEM})}{T_{CFD}} + (1 - q_w) \frac{\text{abs}(M_{CFD} - M_{BEM})}{M_{CFD}} \quad (5-14)$$

where q_w is a weighting factor, and the subscripts CFD and BEM represent thrust and shaft torque values derived from CFD and BEM respectively.

E_{seg} is optimised at each blade segment using a simple golden section search to determine the value of F . Since hub-loss is also found using this method, the models proposed in this paper will also include hub-loss.

Figures 5-1 to 5-4 show the distributions of F at different tip speed ratios.

Figure 5-1. Distribution of F against r/R at $\lambda = 13.19$, using different weighting factors

Figure 5-2. Distribution of F against r/R at $\lambda = 10.72$, using different weighting factors

Figure 5-3. Distribution of F against r/R at $\lambda = 8.06$, using different weighting factors

Figure 5-4. Distribution of F against r/R at $\lambda = 5.50$, using different weighting factors

An observation is that the distribution of F for $q_w = 0$ and $q_w = 1$ are almost the same. This is because matching the thrust results in a very large error in torque, while matching the torque results in a significantly smaller error in thrust.

5.2.3 Correction to Prandtl tip-loss model

The proposed correction to the Prandtl tip-loss model takes a form similar to the correction proposed by Shen et al [93], as shown in equation (5-11) a function g is proposed, which changes with radius at each TSR:

$$g_{tip} = \frac{A}{(2 - r/R)^B} \quad (5-15)$$

$$g_{hub} = \frac{A}{(1 + (r - R_{hub}) / (R - R_{hub}))^B} \quad (5-16)$$

where A and B depend on tip speed ratio and wake turbulence.

The complex nature of the relationship between tip-loss, tip speed ratio and wake turbulence means that the relationships between A and B , and flow conditions cannot be defined simply.

g is then multiplied with f_{Gl} , and F_{new} is determined:

$$F_{new,tip} = \frac{2}{\pi} \cdot \arccos \left(e^{-g_{tip} f_{Gl,tip}} \right) \quad (5-17)$$

$$F_{new,hub} = \frac{2}{\pi} \cdot \arccos \left(e^{-g_{hub} f_{Gl,hub}} \right) \quad (5-18)$$

$$F_{new} = F_{new,tip} \cdot F_{new,hub} \quad (5-19)$$

Since specific tip-loss and hub-loss factors cannot be extracted from the CFD computations, instead, yielding a combined loss factor, it is more useful to focus on A and B as corrective coefficients for the entire loss factor, without treating tip and hub-losses separately. A and B can be expected to depend on tip speed ratio and wake turbulence, and the determination of the distribution of A and B will follow the optimisation process shown in Figure 5-5.

Figure 5-5. Optimisation process for A and B in equations (5-15) and (5-16)

The conjugate gradient optimisations in steps (5) and (7) in Figure 5-5 are the Polak-Ribière non-linear conjugate gradients method [97] described in the Appendix. At step (5), the optimisation involves the minimisation of E_{blade} , the sum of errors in thrust and torque from equation (5-14) for all k blade segments at each tip speed ratio:

$$E_{blade} = \sum_{n=1}^k E_n \quad (5-20)$$

At step (7) in Figure 5-5, the optimisation is the minimisation of the the sum of errors in thrust and torque from equation (5-20) for all tip speed ratios:

$$E_{total} = \sum_{TSR} E_{blade} \quad (5-21)$$

where TSR represents each individual tip speed ratio. Note that this is different from λ , which represents the value of the tip speed ratio. The weighting factor, q_w for equations (5-20) and (5-21), which utilise equation (5-14) was maintained at 0.5.

First, the optimisation process was applied for the tip speed ratios at the windmill wake state, comprising tip speed ratios from 4.95 to 8.06.

The distribution of values of A and B against tip speed ratio in the windmill wake state are:

$$A = \left(1 - \frac{1}{\lambda + 1}\right)^{2.4} \quad (5-22)$$

$$B = 1.3 \quad (5-23)$$

The distribution was then extended to the turbulent wake state to observe the deviations from the windmill state. These are presented in Figures 5-6 to 5-8.

Figure 5-6. Distribution of F against r/R at $\lambda = 13.19$, using the windmill state model, compared to F extracted from CFD

Figure 5-7. Distribution of F against r/R at $\lambda = 10.72$, using the windmill state model, compared to F extracted from CFD

Figure 5-8. Distribution of F against r/R at $\lambda = 9.90$, using the windmill state model, compared to F extracted from CFD

These figures reveal that the tip and hub-losses are affected by the wake turbulence. Two observations were made:

1. The hub losses are affected by wake turbulence despite the axial induction factor at radii closer to the hub being lower, as shown in Figure 5-9. This means that a metric to define overall wake turbulence is required.

2. The tip losses increase (reduced values of F) near the tip and the point of this increase appears to change with tip speed ratio. It is likely that the increase in tip-loss due to wake turbulence occurs beyond a fixed induction factor.

Figure 5-9. Distribution of a against r/R at turbulent wake state λ

By modelling the values of A and B in equations (5-15) and (5-16) appropriately, while defining the conditions for these changes in the model, the tip and hub losses in the turbulent wake state can be modelled.

Since the wake turbulence affects tip and hub losses separately, separate models can be developed and optimised to account for these changes.

For hub-loss, it was found that the axial induction factors near the hub were low, despite the observed increase in hub-losses. It could be postulated that the entire wake could be considered as being turbulent when more than half of the rotor disc area has an axial induction factor of more than 0.4, the induction factor where Buhl's correction for the relationship between the axial induction factor and coefficient of thrust is enforced [24]. Half the rotor disc area correlates to $r = (1/\sqrt{2}) R$. The induced velocity at the blade segment

including this point thus can be used as a basis to determine the wake state of the rotor.

As shown in Figure 5-9, the axial induction factor at the blade segment which includes $r = 0.707R$ is above 0.4 at tip speed ratios where the hub-loss increases (decrease in value of F) to more than windmill-state values. This is then used as a condition for F at $r/R < 0.707$

The distribution of A and B at $r/R < 0.707$ was modelled at tip speed ratios in the turbulent wake state, 9.90 to 13.19. Near the hub, A and B can be defined for the turbulent wake state, when $a_{(r/R=0.707)} > 0.4$:

$$A = 5.59 \exp \left(-5.32 a_{(r/R=0.707)} \right) \quad (5-24)$$

$$B = 1.3 \quad (5-25)$$

For the losses near the tip in the turbulent wake state, Figures 5-6 to 5-8 show that the axial induction factor at each blade segment can be used to determine when the amount of losses near the tip increases. For $r/R > 0.707$, it was found to be when the axial induction factor is above 0.54.

The distribution of A and B at $r/R > 0.707$ was modelled at tip speed ratios in the turbulent wake state, 9.90 to 13.19. Near the tip, A and B can be defined for the turbulent wake state, when $a > 0.54$:

$$A = -0.0606 \exp (0.2077\lambda) + 1 \quad (5-26)$$

$$B = -0.071\lambda + 6.0 \quad (5-27)$$

The constants in equation (5-26) are defined to 4 decimal places due to the sensitivity of the factor A for these flow conditions.

Finally, the entire model can be defined:

$$A = \begin{cases} 5.59 \exp(-5.32a_{(r/R=0.707)}), & \text{for } r/R < 0.707, a_{r/R=0.707} > 0.4 \\ -0.0606 \exp(0.2077\lambda) + 1, & \text{for } r/R > 0.707, a > 0.54 \\ \left(1 - \frac{1}{\lambda+1}\right)^{2.4}, & \text{otherwise} \end{cases} \quad (5-28)$$

$$B = \begin{cases} -0.071\lambda + 6.0, & \text{for } r/R > 0.707, a > 0.54 \\ 1.3, & \text{otherwise} \end{cases} \quad (5-29)$$

5.3 Results and Discussion

The distribution of F from CFD, and calculated loss factors using the Glauert (equation (5-6)), Lindenburg (equation (5-10)) and present models are compared in Figures 5-10 to 5-12.

Figure 5-10. Distribution of F against r/R at $\lambda = 13.19$

Figure 5-11. Distribution of F against r/R at $\lambda = 8.06$

Figure 5-12. Distribution of F against r/R at $\lambda = 4.95$

The distribution of F from the present method shows a significant improvement in accuracy, which is quantified in Figure 5-13, which shows the sum of absolute error for each blade segment relative to the CFD-attained F against tip speed ratio for the Glauert, Lindenburg and present models.

Figure 5-13. Total relative F error against λ

The relative error for the distribution of F over the blade is consistently lower for the present method as compared to the Glauert and Lindenburg models. The relative error for the present model also has a small variation across the range of tip speed ratios.

There is a slight increase in error observed at higher and lower tip speed ratios. At the higher tip speed ratio, shown in Figure 5-10, the values of F in the turbulent wake state are difficult to predict, since the wake turbulence results in a distribution that cannot be described precisely with a smooth function. However, the present model is still significantly more accurate than the Glauert and Lindenburg models.

At the lower tip speed ratio, shown in Figure 5-12, a significant portion of the blade is in stall, or experiencing stall delay. Thus, the accuracy of the present model at lower tip speed ratios is limited by the accuracy of the stall delay model.

The capability of any aerodynamic model is ultimately quantified based on its ability to predict the aerodynamic forces. Figures 5-14 to 5-16 compare the thrust distribution of the Glauert, Lindenburg and present models in comparison to CFD-attained blade-sectional thrust values.

Figure 5-14. Distribution of thrust (kN) against r/R at $\lambda = 13.19$

Figure 5-15. Distribution of thrust (kN) against r/R at $\lambda = 8.06$

Figure 5-16. Distribution of thrust (kN) against r/R at $\lambda = 4.95$

The sum of absolute error for thrust in each blade segment relative to the CFD-attained thrust against tip speed ratio for the Glauert, Lindenburg and present models is shown in Figure 5-17.

Figure 5-17. Total relative thrust error against λ

Thrust prediction is improved consistently from the Glauert model. Against the Lindenburg model, thrust prediction is more accurate at higher and lower tip speed ratios, due to better prediction in the turbulent wake state and more

tip speed ratio-specific model. However, for $\lambda = 6.00$ to 8.80 , the Lindenburg model predicts thrust more accurately. This is because the Lindenburg model appears to be more suited to thrust prediction, which as mentioned in section 5.2.2, would be at the cost of accuracy in predicting shaft torque. By using $q_w = 1$ in equation (5-14), the present model can be optimised for thrust prediction as well.

Figures 5-18 to 5-20 compare the shaft torque distribution of the Glauert, Lindenburg and present models in comparison to CFD-attained blade-sectional shaft torque values.

Figure 5-18. Distribution of shaft torque (kN) against r/R at $\lambda = 13.19$

Figure 5-19. Distribution of shaft torque (kN) against r/R at $\lambda = 8.06$

Figure 5-20. Distribution of shaft torque (kN) against r/R at $\lambda = 4.95$

The sum of absolute error for shaft torque in each blade segment relative to the CFD-attained shaft torque against tip speed ratio for the Glauert, Lindenburg and present models is shown in Figure 5-21.

Figure 5-21. Total relative shaft torque error against λ

The accuracy in prediction of blade sectional shaft torque is significantly greater in the present model, and the total relative error at each tip speed ratio is consistently small. The advantage of separately modelling F in the turbulent wake state is exemplified by the high relative error in prediction of blade sectional shaft torque for the Glauert and Lindenburg models, while the error remains low for the present model.

5.4 Concluding Remarks

A new tip loss modeling methodology is presented, which consists of a correction to the Glauert implementation of Prandtl's tip loss model. The methodology consists of using CFD or experimental data to obtain blade sectional thrust, shaft torque and tip loss factors at several tip speed ratios. Based on the blade sectional data, the present model is optimised to match the CFD or experimentally obtained blade sectional thrust and shaft torque. The error between predicted and obtained values of the thrust and shaft torque are minimised to obtain the optimum tip loss distribution. The thrust and shaft torque are combined by means of a weighting factor, as shown in equation (5-14), to attain a desired level of accuracy in either thrust or shaft torque prediction.

In the present study, the methodology was applied to the NREL 5MW virtual wind turbine, using CFD to obtain blade sectional aerodynamic forces. The range of tip speed ratios used covered both windmill state and the turbulent wake state. This enabled the formulation of distributions of the model coefficients based on observations of flow conditions.

The present model was compared to the Glauert and Lindenburg models to show a significant improvement in accuracy in predicting the tip-loss factor, thrust, and shaft torque. The present model also performed consistently over all tip speed ratios. This ensures the present model can be used in different operating conditions, and can be applied in a dynamic simulation towards more accurate predictions of dynamic thrust and shaft torque.

Further studies may be conducted to understand the dynamic behaviour of tip-loss, through unsteady simulations with oscillating wind conditions. It is hoped that future work may include this methodology to model the tip-loss at other wake states such as the propeller, vortex ring and propeller brake states.

Chapter 6

Unsteady Airfoil Model

Unsteady airfoil aerodynamics comprises the attached flow phenomenon, or Theodorsen effect [43] and the post-separation phenomena. The attached flow phenomenon is an inviscid flow phenomenon caused by vortex shedding, which increases circulation in the wake. By Kelvin's theorem, the total amount of circulation does not change, thus the circulation around the airfoil must decrease, resulting in a reduction of lift. Since the effect of this change in circulation is not instantaneous, there is a time lag between the change in flow conditions such as flow velocity or angle of attack, and the change in lift coefficient of the airfoil. The post-separation phenomena consist of the change in coefficient of lift due to the change in circulation by Kelvin's theorem and the vortex lift due to the propagation of a leading edge vortex over the airfoil suction surface, in a phenomenon called dynamic stall.

Numerous models have been formulated to predict unsteady airfoil effects, including the Øye model [98], Beddoes-Leishman model [6], Onera model [99] and Snel model [100].

The Øye and Beddoes-Leishman models were compared by Haans and Mikkelsen [47] by applying them in an actuator line model and comparing them to experimental results. The conclusion was that the Beddoes-Leishman was a more complete model, since leading-edge flow separation effects were modelled. In the work by Holierhoek et al [101], the Onera, Beddoes-Leishman, and Snel models were compared to experimental studies by the Ohio State University [41]. The results could not conclusively show superiority of any of the models. However, it was noted that the Onera and Beddoes-Leishman

models had parameters that were airfoil-specific, thus could be calibrated for specific airfoils to produce more accurate results. The Beddoes-Leishman model thus remains a preferred model for wind turbine aerodynamics, and has been applied in the AeroDyn [45] and GH Bladed [79] tools.

The Beddoes-Leishman model predicts the unsteady attached flow lift using an indicial function with 4 constants. In addition, the impulsive lift component due to the fluid displacement around the moving airfoil is modeled with a time decay. For the separated flow-circulation lift, the position of flow separation is used in the equation for lift on a flat plate in a potential Kirchhoff flow. This is modeled based on the leading edge pressure and a time lag in change of position of flow separation. Finally, the vortex lift is modeled based on an increase in lift while the vortex propagates over the airfoil, and a time decay when it leaves the trailing edge.

The time constants present in the Beddoes-Leishman model are dependent on airfoil shape, however, few studies have correlated these to shape parameters. The problem is that these correlations are complex, and difficult to form, since interactions between the time constants are highly significant, with the result of an equation involving one time constant being used in the next equation. This brings into question the term ‘time-constant’, since they are dependent on several parameters, but were presented as constants initially. Beginning with the values of the constants generally optimised over all experimental conditions, it is possible to methodically develop correlations between the ‘constants’ and airfoil shape parameters. In this chapter, a study is presented in which the shape parameters for the suction surface are used, based on a general equation for a round-nosed, sharp trailing edged airfoil [102].

6.1 Methodology

6.1.1 Unsteady airfoil experimental results

The unsteady airfoil experiments were conducted by the Ohio State University, and the database of experimental results were available from NREL

[41]. Sinusoidal pitching motions were introduced to a range of wind turbine airfoils, each at several Reynolds numbers, 3 different frequencies, 2 different amplitudes and 3 mean angles of attack. In the present study, experimental data for 10 airfoils were used, each under 54 combinations of Reynolds numbers, frequencies, amplitudes and mean angles of attack, with a total of 540 data sets used. The airfoils and the experimental conditions chosen for the present study are summarised in Table 6-1.

Table 6-1. Experimental conditions and airfoils used in present study

Parameter	Conditions
Airfoil	LS(1)-0417MOD, LS(1)-0421MOD, NACA 4415, S801, S809, S810, S812, S814, S815, S825
Reynolds Number	Low ($\sim 0.75 \times 10^6$), Medium ($\sim 1.00 \times 10^6$), High($\sim 1.25 \times 10^6$)
Frequency (Hz)	Low (~ 0.60), Medium (~ 1.18), High (~ 1.85)
Mean-Peak Amplitude ($^\circ$)	5.5, 10
Mean α ($^\circ$)	8, 14, 20

6.1.2 Beddoes-Leishman model [6]

A non-dimensionalised time, s is defined for time step n .

$$s_{(n)} = \frac{2V_\infty \cdot t_{(n)}}{c} \quad (6-1)$$

where V_∞ is the freestream velocity, $t_{(n)}$ is the time at time step n and c is the airfoil chord length.

6.1.2.1 Unsteady attached flow

The unsteady attached flow is computed by defining an effective angle of attack for the unsteady airfoil [6].

$$\alpha_E = \alpha_n - X(s) - Y(s) \quad (6-2)$$

where $X(s)$ and $Y(s)$ are indicial functions of non-dimensional time, s , defined as:

$$X(s_{(n)}) = X(s_{(n-1)}) \exp(b_1(\Delta s_{(n)})) + A_1 \cdot \Delta \alpha_{(n)} \cdot \exp\left(\frac{-b_1 \cdot \Delta s_{(n)}}{2}\right) \quad (6-3)$$

$$Y(s_{(n)}) = Y(s_{(n-1)}) \exp(b_2(\Delta s_{(n)})) + A_2 \cdot \Delta \alpha_{(n)} \cdot \exp\left(\frac{-b_2 \cdot \Delta s_{(n)}}{2}\right) \quad (6-4)$$

Δ refers to the change in the parameter from the previous time step. A_1 , b_1 , A_2 and b_2 are dimensionless, empirical constants. These were proposed by [6] and since applied in [103], [104] and [105] as shown in Table 6-2.

Table 6-2. Values for A_1 , b_1 , A_2 and b_2 proposed by [6] and applied in [103], [104] and [105]

A_1	b_1	A_2	b_2
0.3	0.14	0.7	0.53

The circulatory attached flow lift is then computed using:

$$C_L^C = C_{L\alpha} \cdot (\alpha_E - \alpha_0) \quad (6-5)$$

where $C_{L\alpha}$ is the lift curve slope in the linear region of the lift curve and α_0 is the angle of attack of zero lift.

The impulsive lift is given by:

$$C_{L(n)}^I = \frac{4 \cdot K_\alpha \cdot c}{V_\infty^2} \cdot \left(\frac{\Delta u_{p(n)} - \Delta u_{p(n-1)}}{\Delta t} - D_{(n)} \right) \quad (6-6)$$

where K_α is a constant related to Mach number, set to 0.846 for a Mach number of 0.15, as described in [106]. u_p is the plunge velocity given by [106]:

$$u_p = V_\infty(\Delta \alpha) + \alpha(\Delta V_{\infty(n)}) \quad (6-7)$$

where $\Delta V_{\infty(n)}$ is the change in freestream velocity from the previous time step and $D_{(n)}$ is the deficiency function given by:

$$D_{(n)} = D_{(n-1)} \exp\left(\frac{-\Delta t}{K_{\alpha} \cdot T_I}\right) + \left(\frac{\Delta u_{p(n)} - \Delta u_{p(n-1)}}{\Delta t}\right) \exp\left(\frac{-\Delta t}{2K_{\alpha} \cdot T_I}\right) \quad (6-8)$$

The total unsteady attached flow lift, or potential flow lift is given by:

$$C_L^P = C_L^C + C_L^I \quad (6-9)$$

6.1.2.2 Unsteady trailing edge separation

The unsteady attached flow lift, as described by [44] requires defining the point of flow separation under steady state conditions. This is done using the equation for lift on a flat plate in a potential Kirchhoff flow [107]. The point of flow separation for an airfoil in steady-state flow is defined at each angle of attack:

$$f^{st} = \left(2\sqrt{\frac{C_L^{st}(\alpha)}{C_{L\alpha} \cdot (\alpha - \alpha_0)}} - 1\right)^2 \quad (6-10)$$

where C_L^{st} is the steady-state lift.

f^{st} is set to zero (fully separated flow) at angles of attack where the value of f^{st} derived from equation (6-10) does not retrieve the static lift coefficient:

$$C_L^{st} \neq \left(\frac{1 + \sqrt{f^{st}(\alpha)}}{2}\right)^2 \cdot (\alpha - \alpha_0) \quad (6-11)$$

Using experimental lift curves, $C_{L\alpha}$ is taken to be:

$$C_{L\alpha} = \max\left(\frac{C_L^{st}(\alpha)}{\alpha - \alpha_0}\right) \quad (6-12)$$

The flow separation over an airfoil is dependent on pressure distribution, which is related to lift [44]. There is a time lag between this pressure response and the lift, and this time lag is modelled through a further time lag to the unsteady attached flow lift:

$$C_L'^P = C_L^P - D_P \quad (6-13)$$

D_P is a deficiency function given by:

$$D_{P(n)} = D_{P(n-1)} \exp\left(\frac{-\Delta s_{(n)}}{T_P}\right) + (C_{L(n)}^P - C_{L(n-1)}^P) \exp\left(\frac{-\Delta s_{(n)}}{2 \cdot T_P}\right) \quad (6-14)$$

where T_P is a dimensionless time constant.

This time-lagged unsteady attached flow lift is used to find a new effective angle of attack:

$$\alpha_f = \frac{C_L'^P}{C_{L\alpha}} - \alpha_0 \quad (6-15)$$

The position of flow separation based on the new effective angle of attack is looked up from the static data table:

$$f' = f(\alpha_f) \quad (6-16)$$

The time lag between the unsteady pressure response and the position of flow separation is modelled to find a dynamic position of flow separation.

$$f^{dyn} = f' - D_f \quad (6-17)$$

D_f is the deficiency function given by:

$$D_{f(n)} = D_{f(n-1)} \exp\left(\frac{-\Delta s_{(n)}}{T_f}\right) + (f'_{(n)} - f'_{(n-1)}) \exp\left(\frac{-\Delta s_{(n)}}{2 \cdot T_f}\right) \quad (6-18)$$

where T_f is a dimensionless constant.

In equation (6-17), the dynamic position of flow separation (with respect to the airfoil chord) is modelled with the assumption of a time lag between dynamic flow separation and the flow separation based on the unsteady pressure response on the airfoil. The flow separation based on the unsteady pressure response on the airfoil is found using a look-up table of static positions of

flow separation with the effective angle of attack derived from the time-lagged unsteady attached flow lift. The time lag is modelled using the deficiency function in equation (6-18) with a time constant of T_f .

The total dynamic lift with trailing edge separation is then found using the equation for lift on a flat plate in a potential Kirchoff flow [107]:

$$C_L^f = \left(\frac{1 + \sqrt{f^{dyn}}}{2} \right)^2 \cdot (\alpha_E - \alpha_0) \quad (6-19)$$

When $f^{dyn} = 0$, this is set to:

$$C_L^f = C_L^{st}(\alpha_E) \quad (6-20)$$

The circulation of an airfoil is linearly related to the lift produced, not angle of attack, since the lift and angle of attack do not share a linear relationship when there is flow separation. Thus, equations (6) to (25) are used as an initial estimate of C_L^f , to find the change in circulatory angle of attack, $\Delta\alpha_c$.

$$\Delta\alpha_c = \frac{C_{L(n)}^f - C_{L(n-1)}^f}{C_{L\alpha}} \quad (6-21)$$

$\Delta\alpha_c$ then replaces $\Delta\alpha$ in equations (6-3) and (6-4), and equations (6) to (25) are repeated to find C_L^f .

6.1.2.3 Vortex lift

Vortex lift refers to the dynamic stall phenomenon, where leading edge separation causes the propagation of a vortex over the suction surface of the airfoil, resulting in a momentary increase in lift. While the leading edge separation onset criterion has been discussed, and has been based on the potential flow lift exceeding a certain value [103], this criterion does not need to be applied explicitly, and can instead be applied when the feed to the vortex lift exceeds zero [106]. The feed for the vortex lift is modelled using the difference between circulatory attached flow lift and dynamic lift with trailing edge separation:

$$\Delta c_v = c_{v(n)} - c_{v(n-1)} \quad (6-22)$$

$$c_v = C_L^C - C_L^f \quad (6-23)$$

The vortex lift then is:

$$C_{L(n)}^v = C_{L(n-1)}^v \cdot \exp\left(\frac{-\Delta s_{(n)}}{T_v}\right) + \Delta c_v \cdot \exp\left(\frac{-\Delta s_{(n)}}{2 \cdot T_v}\right) \quad (6-24)$$

where T_v is a dimensionless constant. This is set to zero if the feed to the vortex lift is negative, the angle of attack is increasing, or exceeds 50° [106].

The total unsteady lift is then:

$$C_L^{dyn} = C_L^I + C_L^f + C_L^v \quad (6-25)$$

The constants T_P , T_f and T_v have been proposed by [6] for thin airfoils at different Mach numbers, starting from 0.3. In addition, [105] performed an optimisation of the constants against experimental data and suggested a set of values. [103] proposed values based on an analysis of both these sets of constants. These are included in Table 6-3.

Table 6-3. Values for T_P , T_f and T_v from various sources

Source	T_P	T_f	T_v
Leishman & Beddoes [6] (M=0.3)	1.7	3	6
Mert [105]	0.8	5	2
Pereira [103]	1.5	5	6

The values cited in [105] were based on optimisations to reduce the error between experimental and model-predicted coefficients of lift and drag. The optimisations were performed using the method of moving asymptotes, and involved experimental data both from [41] and other sources. Due to the similarity in approach in performing an optimisation to reduce the error between

predicted and experimental coefficients, these values were used as the start point for the optimisation described in Section 6.2.

6.1.2.4 Drag and moment components

The unsteady drag is modelled as an additional unsteady component to static drag, based on parameters predicted during the computation of dynamic lift [44].

$$C_D^{dyn} = C_D^{st}(\alpha_E) + C_L^{dyn} \cdot (\alpha - \alpha_E) + \left(C_D^{st}(\alpha_E) \cdot \left(\left(\frac{1 - \sqrt{f^{dyn}}}{2} \right)^2 - \left(\frac{1 - \sqrt{f^{st}(\alpha_E)}}{2} \right)^2 \right) \right) \quad (6-26)$$

The dynamic moment includes additional components based on impulsive lift [106].

$$C_M^{dyn} = C_M^{st}(\alpha_E) - \frac{C_L^I}{4} \quad (6-27)$$

The equations for both coefficients of dynamic drag and dynamic moment consist of static coefficients and components found during derivation of the dynamic lift coefficient. Thus, analysis of the error produced from predicting the dynamic lift coefficient would be sufficient in determining the accuracy of the model.

6.2 Optimisation

The optimisation of the Beddoes-Leishman model constants were performed using the Polak-Ribière non-linear conjugate gradients method [97]. This is a local optimisation algorithm, which is used in place of a global optimisation to save computational time. The start point of the optimisation used values from [105], as shown in Tables 6-2 and 6-3, and the objective function was minimisation of the error:

$$error = \sum_{n=1}^k \left(C_L^{sim}(t_{(n)}) - C_L^{exp}(t_{(n)}) \right)^2 \quad (6-28)$$

where the superscript *sim* refers to simulation results, *exp* refers to experimental results, and k is the total number of time steps in all cases.

The parameters optimised were A_1 , b_1 , A_2 , b_2 , K_α , T_P , T_f and T_v . The constants proposed in [105] were first used to compute a reference error, and compared to the error from the optimised constants, presented in Table 6-4.

Table 6-4. Model constants for reference and optimised cases

	A_1	b_1	A_2	b_2	K_α	T_P	T_f	T_v	Error
Mert [105]	0.300	0.140	0.700	0.530	0.846	0.800	5.000	2.000	9.35
Optimised	-0.200	0.504	0.736	0.431	0.314	0.555	5.747	2.867	8.55

The optimisation resulted in an 8.5% reduction in total error computed, with the raw error shown in Table 6-4. The drawback of this form of optimisation, over 540 sets of experimental data, is the lack of completeness in the efficacy of the model. The optimisation cannot be guaranteed to produce an improvement in all 540 sets of data, and indeed, the optimised model shows an increase in error for 124 cases compared to the baseline model. This means that the optimal model constants are dependent on each case. Furthermore, noise in the experimental results also affected the effectiveness of the optimisation, since the optimisation was based on the accuracy of prediction at each data point.

The accumulated error for each experimental condition under each parameter shown in Table 6-1 was compared, and shown in Tables 6-5 to 6-9.

Table 6-5. Accumulated error for reference and optimised cases for each airfoil

Airfoil	Mert [105]	Optimised	Reduction (%)
LS(1)-0417MOD	1.28	1.12	12.5
LS(1)-0421MOD	1.04	0.90	14.0
NACA 4415	0.49	0.49	1.8
S801	0.98	0.86	12.3
S809	0.83	0.77	7.0
S810	1.48	1.33	9.8
S812	0.70	0.69	0.3
S814	0.60	0.59	3.0
S815	0.78	0.75	3.8
S825	1.18	1.07	9.6
Total	9.35	8.55	8.5

Table 6-6. Accumulated error for reference and optimised cases for each Reynolds Number

Re	Mert [105]	Optimised	Reduction (%)
Low	4.21	3.83	9.2
Medium	2.91	2.67	8.1
High	2.24	2.06	8.0
Total	9.35	8.55	8.5

Table 6-7. Accumulated error for reference and optimised cases for each Frequency

Frequency	Mert [105]	Optimised	Reduction (%)
Low	2.81	2.49	11.1
Medium	2.88	2.59	10.2
High	3.67	3.48	5.2
Total	9.35	8.55	8.5

Table 6-8. Accumulated error for reference and optimised cases for each Amplitude angle of attack

Amplitude (°)	Mert [105]	Optimised	Reduction (%)
5.5	4.37	3.95	9.7
10	4.98	4.60	7.6
Total	9.35	8.55	8.5

Table 6-9. Accumulated error for reference and optimised cases for each mean

Mean (°)	Mert [105]	Optimised	Reduction (%)
8	1.02	0.94	8.0
14	2.83	2.51	11.5
20	5.50	5.11	7.1
Total	9.35	8.55	8.5

The accumulated error for each of the conditions at each experimental parameter reveals that the optimisation registered a reduction in error for all conditions, thus the optimisation is not skewed away from any particular condition, and any distribution of cases where the optimised constants performed worse than the reference ones would be likely random. It is notable in Table 6-5 that some airfoils such as LS(1)-0417MOD, LS(1)-0421MOD, and S801 registered a reduction in error of greater than 10% while others such as NACA 4415, S812, S814 and S815 registered less than 4% reduction in error. These differences suggest that the optimisation might have been more favourable for some airfoils, however the airfoils with the lowest reductions in error already had the lowest error as well. This implies that the Beddoes-Leishman model itself is better suited to those airfoils. These findings were further explored with an optimisation of the simplified model described in Section 6.3.

6.3 Simplified model

The result of the optimisation presented in Table 6-4 shows a small difference between the indicial constants b_1 and b_2 of 0.07, compared to the original values, which have a difference of 0.39. This small difference possibly indicates that one time-scale is sufficient to simulate the indicial response of the effective angle of attack due to Kelvin's theorem, as shown in equations (6-2) to (6-4). Equations (6-2) to (6-4) can thus be replaced with:

$$\alpha_E = \alpha_n - X(s_{(n-1)}) \exp(b_1 (\Delta s_{(n)})) + A_1 \cdot \Delta \alpha_{(n)} \cdot \exp\left(\frac{-b_1 \cdot \Delta s_{(n)}}{2}\right) \quad (6-29)$$

The constants for the simplified model were optimised, with the optimised constants from the original model summarised in Table 6-4 used as the start point. The start point for the value A_1 in the simplified model was the sum of A_1 and A_2 in the optimised original model, and the start point for b_1 in the simplified model was the weighted average of b_1 and b_2 from the optimised original model, using A_1 and A_2 as weighting parameters.

The results of the optimisation of the simplified model are tabulated in Table 6-10.

There was no change in error, and there was minimal variation in the constants K_α , T_P , T_f and T_v . This indicates that the simplified model is in fact similar to the original model. Furthermore, the start point of the optimisation of the simplified model, using the weighted average of the optimised original model constants, was close to the final optimised result. This shows that the weighted average approach was an accurate method of determining the constant, with differences due to the fact that the weighted average is not an exact method in combining exponents of an indicial response. When comparing each experimental case, the most significant difference in error between the two models was 1.2%. This implies that the simplified model behaves in a similar manner as the original model across all cases.

Table 6-10. Model constants for original optimised and simplified model
(equation (6-29)) cases

	A_1	b_1	A_2	b_2	K_α	T_P	T_f	T_v	Error
Original									
optimised model	-0.200	0.504	0.736	0.431	0.314	0.555	5.747	2.867	8.55
Simplified									
optimised model	0.570	0.430	-	-	0.310	0.554	5.741	2.868	8.55
Weighted average start point	0.536	0.404	-	-	0.314	0.555	5.747	2.867	-

6.4 Shape-parameter based model

Most of the unsteady effects occur over the suction surface of the airfoil, since the suction effect results in most of the lift generated by an airfoil. It would thus be a logical conclusion that the values of the various constants would depend primarily on the shape of the suction surface. for most round-nosed, sharp trailing-edged airfoils, the suction surface shape can be described by (for x (chord-wise) and z -coordinates) [102]:

$$z = \tau \cdot \sqrt{\frac{x}{c}} \cdot \left(1 - \frac{x}{c}\right) \cdot \left(1 - \sum_{i=1}^N A_i \cdot \left(\frac{x}{c}\right)^i\right) \quad (6-30)$$

where c is the chord, τ is a thickness parameter, and A_i is the i^{th} coefficient of the N -polynomial.

Using $N=1$, the equation is simplified to:

$$z = \tau \cdot \sqrt{\frac{x}{c}} \cdot \left(1 - \frac{x}{c}\right) \cdot \left(1 - A \cdot \left(\frac{x}{c}\right)\right) \quad (6-31)$$

In equation (6-31), A is a curvature parameter that determines the general shape of the curve, while τ is a thickness parameter that determines the

thickness of the airfoil. Expanding the equation would show that $\tau(A)$ would also be an important parameter for the airfoil shape, which takes into account both curvature and thickness. Unless the airfoils were designed using equation (6-31), their profile would not precisely match the equation. Instead, the parameters τ and A were chosen to obtain the best fitting curves to each airfoil's suction surface. These are summarised in Table 6-11.

Table 6-11. Values of τ , A and $\tau(A)$ for each airfoil

Airfoil	τ	A	$\tau(A)$
LS(1)-0417MOD	0.299	0.205	0.061
LS(1)-0421MOD	0.350	0.201	0.070
NACA 4415	0.274	-0.182	-0.050
S801	0.222	-0.500	-0.111
S809	0.230	-0.164	-0.038
S810	0.191	-0.576	-0.110
S812	0.232	-0.400	-0.093
S814	0.259	-0.341	-0.089
S815	0.268	-0.326	-0.087
S825	0.227	-0.446	-0.101

The best-fit curves using equation (6-31) for the airfoil suction surfaces are shown in Figure 6-1 for three of the airfoils.

Each airfoil in the experiment was placed under 54 different conditions. The constants K_α , T_P , T_f and T_V in the simplified Beddoes-Leishman model were optimised, with the objective function as the error in equation (6-28), using all of the experimental conditions for each airfoil. The values of A_1 and b_1 were kept constant, while the values of K_α , T_P , T_f and T_V were optimised for each airfoil. Then, the values for A_1 and b_1 were then optimised generally for all airfoils, in a successive optimisation approach.

Figure 6-1. Best fit curves for LS(1)-0421MOD (top), S809 (middle) and S815 (bottom) airfoil suction surfaces using equation (6-30)

This process was repeated until convergence, when the general optimisation step produced an error within 0.0001 of the total error from the airfoil-specific optimisation step. This method was used over a more accurate and logical nested optimisation technique where the airfoil-specific optimisation is performed at each iteration of the general optimisation due to reduce computational time. The assumption is that the airfoil-specific optimisation step would produce a much more significant reduction in error as compared to the general optimisation step. The error for optimisation of the constants for each airfoil is compared to the error from the general optimisation of constants for all the airfoils at once, and included in Table 6-12.

Table 6-12. Accumulated error for each airfoil in generally optimised and airfoil-specific optimised cases

Airfoil	General	Airfoil-specific	Reduction(%)
LS(1)-0417MOD	1.12	1.05	6.0
LS(1)-0421MOD	0.90	0.83	7.3
NACA 4415	0.49	0.45	8.2
S801	0.86	0.85	0.8
S809	0.77	0.72	5.8
S810	1.33	1.12	16.1
S812	0.69	0.65	6.1
S814	0.59	0.51	12.3
S815	0.75	0.73	2.8
S825	1.07	1.04	2.4
Total	8.55	7.96	7.0

These comparisons showed that τ has a strong quadratic correlation to the optimised value of the constant T_P and A to T_V . The best-fit quadratic relationship between T_P and τ was found to have a negative minimum point, which was adjusted to be positive to ensure numerical stability. These are illustrated in Figures 6-2 and 6-3.

Figure 6-2. Plot of T_P against τ and quadratic regression

Figure 6-3. Plot of T_V against A and quadratic regression

The values of K_α and T_f were then optimised again for each airfoil, with the values of A_1 , b_1 from the previous optimisation, and the quadratic correlations between A and T_P and τ and T_V implemented in the general optimisation step, in the successive optimisation approach described previously. The new optimised values of T_f had a good quadratic correlation to $\tau(A)$ as shown in Figure 6-4.

Figure 6-4. Plot of T_f against $\tau(A)$ and quadratic regression

The values of K_α appeared to have a weak quadratic correlation to $\tau(A)$, with an R^2 value of 0.60 (before correction). However, 3 points, from the LS(1)-0417MOD, S814 and S825 airfoils appeared to deviate much more from the best-fit curve than the rest. These points were removed from the K_α against $\tau(A)$ and the quadratic relation showed a closer fit to the remaining data points. This is presented in Figure 6-5, with the three points removed for the best fit circled in red.

The quadratic relations for K_α and T_f to $\tau(A)$ were added to those for T_P to τ and T_V to A , to form a suction surface shape-based model. This model, together with the values for A_1 and b_1 were optimised again, generally for all airfoils, to obtain the values and correlations shown in Table 6-13.

Figure 6-5. Plot of K_α against $\tau(A)$ and quadratic regression, with anomalous points circled in red

It is likely that this is not a true physical representation of the relationship, since the quadratic relationship would result in negative values of K_α and T_f , and extremely large values of T_P and T_V as A , τ and $\tau(A)$ extend beyond the presently utilized ranges. It can be conjectured that these constants should asymptotically tend to certain threshold values as the shape parameter for the airfoil increases or decreases, possibly in a Gaussian-type function as a better

representation of this relationship. At present, however, a quadratic relationship is used, since the range of shape parameters is small.

Table 6-13. Suction surface shape-dependent values of constants in Beddoes-Leishman model after final optimisation

Constant	Value
A_1	0.93
b_1	0.52
K_α	$-64.4(\tau(A))^2 - 2.7\tau(A) + 0.8$
T_P	$72.8\tau^2 - 45.1\tau + 7.0$
T_f	$-260.7(\tau(A))^2 - 21.3\tau(A) + 6.5$
T_V	$22.5A^2 + 9.3A + 2.3$

6.5 Results and Discussion

The error associated with the suction surface is compared to the error from the fixed-constant model seen in Table 6-10, for each airfoil as well as in total, as shown in Table 6-14.

Table 6-14. Accumulated error in each airfoil for generally optimised and airfoil-specific optimised cases

Airfoil	General	Shape-based	Reduction(%)
LS(1)-0417MOD	1.12	1.07	4.8
LS(1)-0421MOD	0.90	0.85	5.8
NACA 4415	0.49	0.45	6.8
S801	0.86	0.86	0.03
S809	0.77	0.74	3.8
S810	1.33	1.14	14.3
S812	0.69	0.66	4.4
S814	0.59	0.56	5.3
S815	0.75	0.74	1.3
S825	1.07	1.07	0.01
Total	8.55	8.12	5.0

An improvement is noticeable in all cases, supporting the hypothesis of airfoil shape-dependence of the model constants. The different changes in model accuracy can be explained by the linear distance between the model constants in the shape-based model and general optimisation, and shape-based model and airfoil-specific optimisation. The linear distances are normalised against the general-optimisation parameter values, and compared to the reduction in error from the general optimisation to the shape-based model, and the increase in error from the airfoil-specific optimisation to the shape-based model in Figure 6-6.

Figure 6-6. Plot of normalised between the model constants in the shape-based model and general optimisation, and shape-based model and airfoil-specific optimisation against change in error

It is generally observed that the increased distance that the model has from the optimised value, the greater the change in error. The airfoils whose parameters were closest to the optimum after the general optimisation showed very little reduction in model error. Similarly, parameters from the shape-based model that deviated the most from the airfoil-specific optimisation showed the greatest increase in error comparatively.

The differences between the shape-based model approach and the airfoil-specific optimisations can be attributed to the fact the shape-dependence of

A_1 and b_1 were not included in this study, thus these were generally-optimised. In some airfoils, differences between suction surface shape and best-fit using equation (6-31) were significant and the fact that the pressure surface shapes were not considered in this study also contributed to the differences. An approach using both the suction and pressure surface shapes would be ideal, but would not be possible with just 10 airfoils as reference. For thin airfoils, this study can be performed using the airfoil camber shape instead.

The shape-based model is compared to the general optimisation for several experimental cases in Figures 6-7 to 6-10.

Figure 6-7. Coefficient of lift for oscillatory pitching motion of LS(1)-0421MOD airfoil, at $Re=7.5 \times 10^5$, Amplitude= 5° , Frequency=1.85Hz and Mean $\alpha=14^\circ$

Figure 6-8. Coefficient of lift for oscillatory pitching motion of NACA 4415 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 10° , Frequency=1.85Hz and Mean $\alpha=8^\circ$

Figure 6-9. Coefficient of lift for oscillatory pitching motion of S801 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 5° , Frequency=1.85Hz and Mean $\alpha=20^\circ$

Figure 6-10. Coefficient of lift for oscillatory pitching motion of S814 airfoil, at $Re=7.5 \times 10^5$, Amplitude= 10° , Frequency= 1.85Hz and Mean $\alpha=14^\circ$

The shape-based model is able to better predict the lift hysteresis due to the oscillatory pitching motion of the airfoils. However, from the lift predictions of the NACA 4415 in figure 6-8 and S801 airfoil in figure 6-9, there is still some significant difference between the experimentally measured and predicted lift, thus the Beddoes-Leishman model needs to be improved further for wind turbine applications.

6.6 Concluding Remarks

A simplified Beddoes-Leishman model has been presented, using a 1-equation model for the unsteady attached-flow circulation around the airfoil compared to the traditional 2-equation model. It is found that the 1-equation model works with a similar level of accuracy as the 2-equation model, thus the number of equations, computational time and run-time memory in large simulations employing the Beddoes-Leishman model may be reduced.

The constants of the simplified Beddoes-Leishman model were optimised against the measured coefficients of lift for 10 airfoils in 54 different unsteady conditions each. The error between predicted and experimental values for this

model was used as a baseline for comparison to airfoil-specific optimisation of the model and the shape-based model proposed. A 7% total reduction in average error for all 10 airfoils was observed when the constants K_α , T_P , T_f and T_V were optimised for each airfoil.

The optimised constants for each airfoil were compared to airfoil suction surface shape parameters, found by using the best fit of equation (6–31) to the airfoil suction surface. Quadratic relations between K_α and $\tau(A)$, T_P and τ , T_f and $\tau(A)$, and T_V and A were found and implemented in a suction surface shape-based Beddoes-Leishman model. This model had a 5% reduction in error compared to the general-optimisation case, and showed improvements in accuracy for all airfoils, although these varied generally based on how much the shape-based model constants changed from the general optimisation.

The present study explored the shape dependencies of 4 constants in the models, and only used the suction-surface shape as reference. Quadratic relationships were found and used due to the limited number of airfoils in this study, although a Gaussian-type distribution is preferred for the given shape parameters. Future attempts at improving this model should strive to include the pressure surface shape as well.

Chapter 7

Combined Stall-Delay and Unsteady Airfoil Model

All computational methods in wind turbine aerodynamics, short of a full 3D computational fluid dynamics simulation by creation of a suitably fine computational grid around the rotor geometry, would require the determination of the local cross-sectional aerodynamics based on present flow conditions at the particular rotor station. This is done through a simple lookup table for the generally steady aerodynamics found in fixed-bottom rotors, while it is modelled using an appropriate unsteady aerodynamics model for the unsteady aerodynamics generally found in floating offshore wind turbines (FOWTs) due to the 6 degree of freedom platform motions. However, using 2-dimensional airfoil data in these applications would be erroneous, given that the radial flows due to the rotation of the rotor result in significant stall delay in the rotor blades.

The stall delay effect is generally accounted for by means of correction to the static aerodynamics table before performing a simulation or analysis, since for most wind turbines, the operating tip speed ratio (TSR) is known beforehand. For FOWTs however, the TSR is constantly changing as the relative wind changes periodically based on platform motions. This means that the stall delay effect would constantly be changing over the rotor blade during operation. Thus, the stall delay model needs to be constantly called during operation and combined with the unsteady aerodynamics model for an improved description of FOWT blade cross-sectional aerodynamics.

This chapter will present combined stall-delay and unsteady airfoil model, by implementing the Du & Selig [16] model for stall-delay and modified Beddoes-Leishman model discussed in Chapter 6 for unsteady airfoil aerodynamics in a manner that will take into account the changing local speed ratio of the FOWT blade cross-section.

7.1 Model descriptions

7.1.1 Du & Selig [16]

The Du & Selig model [16] is based on the Coriolis effect on 3D boundary layers over the blade. The equations correct the 2D coefficients of lift and drag to account for the stall delay.

$$C_l^{sd}(\alpha) = g \cdot C_{l\alpha} \cdot (\alpha - \alpha_0) + (1 - g) \cdot C_{l,2D} \quad (7-1)$$

$$C_d^{sd}(\alpha) = gd \cdot C_{d0} + (1 - gd) \cdot C_{d,2D} \quad (7-2)$$

where $C_{l\alpha}$ is the lift curve slope from 2D data, $C_{l,2D}$ is the 2D coefficient of lift, α_0 is the angle of attack at zero lift, $C_{d,2D}$ is the 2D coefficient of drag and C_{d0} is the coefficient of drag at zero lift. g and gd are flow separation point functions given by:

$$g = \frac{1}{2\pi} \cdot \left(\frac{1.6(c/r) - (c/r)^\Lambda}{0.1267 + (c/r)^\Lambda} - 1 \right) \quad (7-3)$$

$$gd = \frac{1}{2\pi} \cdot \left(\frac{1.6(c/r) - (c/r)^{\Lambda/2}}{0.1267 + (c/r)^{\Lambda/2}} - 1 \right) \quad (7-4)$$

where c is the chord length at the local blade station, r is the local radius, and Λ is the inverse of the local speed ratio given by:

$$\Lambda = \frac{V_{rel,wind}}{V_{rot}} \quad (7-5)$$

where $V_{rel,wind}$ is the total relative wind velocity at the blade section, and V_{rot} is the rotational wind velocity at the blade section.

g and gd are set to 0 when the angle of attack exceeds 30° [108] or is below -20° .

Since Λ is expected to change with time in a FOWT, the Du & Selig model cannot be pre-applied to 2D aerodynamic data for airfoils.

7.1.2 Combined Stall-Delay and Unsteady Airfoil Model

A combined model for application in fully-coupled models for FOWTs would require several properties. Firstly, the periodically changing local wind conditions at each blade station due to platform motions, and the possible addition of a control module to simulate changes in the rotor RPM, would result in changing local wind conditions as well as TSR, resulting in constantly changing Reynolds numbers, unsteady airfoil effects and stall delay. The combined model must thus be able to interpolate between data of several Reynolds numbers as well as account for the changing stall delay. Secondly, the model must use 2D static airfoil lift and drag data, so that the information input into wind turbine aerodynamics models at present would be enough for the current model.

An important feature of the Du & Selig stall delay model is that the lift curve slope of the linear portion of the lift curve does not change. This is an input parameter for the Beddoes-Leishman model, and can be predetermined from the 2D airfoil data. This will facilitate the interpolation of the data from datasets of different flow Reynolds numbers.

7.1.2.1 Stall-delayed unsteady lift

At a given flow condition, the flow velocity is used to determine the Reynolds number, based on which the $C_{l\alpha}$, α_0 and C_{d0} are interpolated from the 2 closest Re datasets available.

$$C_{l\alpha} = C_{l\alpha}(Re) \quad (7-6)$$

$$\alpha_0 = \alpha_0(Re) \quad (7-7)$$

$$C_{d0} = C_{d0}(Re) \quad (7-8)$$

α_f , the new effective angle of attack to based on time-lagged unsteady attached flow lift, is found based on equation (6-15). Equation (7-1) is then applied at α_f , and the stall-delayed static coefficient of lift, $C_L^{sd}(\alpha_f)$ is found, which is used to find the position of flow separation at α_f , f' .

$$f' = \left(2 \sqrt{\frac{C_L^{sd}(\alpha_f)}{C_{L\alpha} \cdot (\alpha_f - \alpha_0)}} - 1 \right)^2 \quad (7-9)$$

If the derived value of f' does not retrieve $C_L^{sd}(\alpha_f)$, it is set to 0.

$$f' = \begin{cases} f', & \text{if } C_L^{sd}(\alpha_f) = C_{L\alpha} \left(\frac{1 + \sqrt{f'(\alpha_f)}}{2} \right)^2 (\alpha_f - \alpha_0) \\ 0, & \text{otherwise} \end{cases} \quad (7-10)$$

Equation (7-10) is used in (6-17) to (6-19) are then applied to find the dynamic position of flow separation, f^{dyn} , using which the dynamic coefficient of lift with trailing edge separation, C_L^f is found. if $f^{dyn} = 0$, then C_L^f is set to the stall-delayed coefficient of lift based on the effective angle of attack:

$$C_L^f = \begin{cases} C_{L\alpha} \left(\frac{1 + \sqrt{f^{dyn}}}{2} \right)^2 (\alpha_E - \alpha_0), & \text{if } f^{dyn} > 0 \\ C_L^{sd}(\alpha_E), & \text{otherwise} \end{cases} \quad (7-11)$$

Vortex lift, caused by leading-edge flow separation is also affected by stall-delay, since the separation at the leading-edge is also delayed by the radial flows over the blade. As shown in equations (6-22) and (6-23), the feed of the vortex lift is based on the difference between circulatory lift and stall-delayed dynamic coefficient of lift with trailing edge separation. The vortex lift in equation (6-24) should also be limited to an equivalent circulatory lift at 2° angle of attack.

$$\max(C_L^v) = C_{L\alpha} (0.035 - \alpha_0) \quad (7-12)$$

7.1.2.2 Stall-delayed unsteady drag

The stall-delayed static coefficient of drag, $C_D^{sd}(\alpha_E)$, can be determined by finding the coefficient of drag from 2D data and applying equation (7-2). After which, the static position of flow separation at α_E , $f^{st}(\alpha_E)$, may be found using:

$$f^{st} = \left(2 \sqrt{\frac{C_L^{sd}(\alpha_E)}{C_{L\alpha} \cdot (\alpha_E - \alpha_0)}} - 1 \right)^2 \quad (7-13)$$

The equation (7-10) is reapplied for f^{st} to determine if the flow has fully separated:

$$f^{st} = \begin{cases} f^{st}, & \text{if } C_L^{sd}(\alpha_E) = 0.25 \cdot C_{L\alpha} \cdot \left(\frac{1 + \sqrt{f^{st}(\alpha_E)}}{2} \right)^2 \cdot (\alpha_E - \alpha_0) \\ 0, & \text{otherwise} \end{cases} \quad (7-14)$$

The stall-delayed unsteady coefficient of drag, $C_D^{sd,dyn}$ is then found using:

$$C_D^{sd,dyn} = C_D^{sd}(\alpha_E) + C_L^{dyn} \cdot (\alpha - \alpha_E) + \left(C_D^{sd}(\alpha_E) \cdot \left(\left(\frac{1 - \sqrt{f^{dyn}}}{2} \right)^2 - \left(\frac{1 - \sqrt{f^{st}(\alpha_E)}}{2} \right)^2 \right) \right) \quad (7-15)$$

The Du & Selig model does not correct the coefficient of moment of the airfoil, thus, the unsteady moment from the unsteady airfoil model, equation (6-27) would be used instead.

7.2 Results and Discussion

The changes in Λ against time for the NREL 5MW wind turbine are shown in Figure 7-1 for the wind turbine in a sinusoidal surge motion with and amplitude of 6m and frequency of 0.5Hz.

Figure 7-1. Lambda against time for different blade segments for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$

Figure 7-1 shows periodic changes in Λ of 20 to 40%. This significant change in Λ will affect the degree of stall-delay on the local blade segment.

The coefficient of lift is shown for elements at $r/R = 0.19$, $r/R = 0.32$ and $r/R = 0.45$ on the rotor, with a pre-treated stall delay and with a real-time stall delay, in Figures 7-2 to 7-4.

Figure 7-2. C_L against time for $r/R = 0.19$ for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using pre-applied and real-time stall-delay

Figure 7-3. C_L against time for $r/R = 0.32$ for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using pre-applied and real-time stall-delay

Figure 7-4. C_L against time for $r/R = 0.45$ for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using pre-applied and real-time stall-delay

Some highly significant differences in coefficient of lift are observed over the different blade segments. This implies that the prediction in real-time force distribution over the blade for structural analysis would be highly uncertain without a real-time stall delay implemented.

The rotor thrust and shaft torque are compared, with a pre-treated stall delay and with a real-time stall delay, in Figures 7-5 and 7-6.

Figure 7-5. Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using pre-applied and real-time stall-delay

Figure 7-6. Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using pre-applied and real-time stall-delay

A 12% difference in peak predicted shaft torque between the pre-applied stall-delay and real-time stall-delay is observed. This would affect fatigue and power-prediction analyses for a FOWT.

The rotor thrust and torque are compared, with the combined stall-delay and unsteady airfoil model and with a real-time stall delay on static airfoil aerodynamic data, in Figures 7-7 and 7-8.

The thrust prediction of the unsteady airfoil model appears to be less accurate than the static airfoil model. However, the tip-loss model described in Chapter 5 was shown to overpredict thrust. Furthermore, the unsteady airfoil model is able to predict the sharper peaks more accurately, which are due to impulsive lift and vortex lift generated from the rapid motion. This is also reflected in the increased accuracy of the unsteady airfoil model in predicting the shaft torque of the rotor, highlighting the importance of the model in uBEM.

Figure 7-7. Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ using static airfoil aerodynamics and unsteady aerodynamics model, compared to CFD

Figure 7-8. Predicted shaft torque against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = $6m$ using static airfoil aerodynamics and unsteady aerodynamics model, compared to CFD

7.3 Concluding Remarks

The stall-delay model of Du & Selig was combined with the airfoil-shape dependent modified Beddoes-Leishman model from Chapter 6 to produce a model which simulates the motion of the boundary layer both due to changes in angle of attack and Coriolis effect with blade rotation.

The changes in speed ratio during the surging motion of a FOWT were demonstrated, which resulted in variation for coefficient of lift, and finally changes in rotor thrust and torque. It was shown that pre-application of a stall-delay model to airfoil aerodynamic data would not model the changes in stall-delay, thus, a real-time stall delay model is required that works in combination with an unsteady airfoil model for an accurate assessment of FOWT aerodynamics. In addition, a comparison of uBEM with the unsteady airfoil model versus uBEM with static aerodynamics tables shows increased accuracy in predicting the shaft torque, and better reflection of the distribution of thrust and shaft torque against time.

Future work may involve careful turbine-specific calibration of the stall-delay model to produce a greater degree of accuracy when assessing FOWT aerodynamics. This can be done through CFD studies using a wide-ranging simulation matrix, although the separation of stall-delay and tip-loss remains a problem.

Chapter 8

Dynamic Wake Model

The unsteady flow environment of a FOWT means that aerodynamic forces acting on the FOWT are constantly changing. By Newton's third law, the forces imposed by the flow on the rotor are also imposed by the rotor on the flow, with an equal magnitude and opposite direction. This results in the change in momentum, upon which the momentum theory is built. However, the momentum theory used in BEM defines only the steady-state momentum balance. In a transient flow, the momentum balance is constantly changing. The forces imposed by the rotor on the flow will take time to affect the far wake, causing a time lag in the global momentum balance. This time lag is called the dynamic wake effect.

The dynamic wake effect may be modelled using the unsteady Euler equations, by the method of accelerating potential described in Chapter 2. When the 1-dimensional momentum equations are used, like in BEM and uBEM, a dynamic wake model is required to correct the steady-state induced velocities.

8.1 Review of Dynamic Wake Models for BEM

8.1.1 Single Differential Equation Model

Snel and Scheppers [37] presented several models to account for the dynamic wake effect. One of these, the ECN (Energy research Centre of the Netherlands) differential equation model, used a single differential equation for the axial induced velocity.

$$\tau \frac{dW_n}{dt} = C_T |\mathbf{V}_0 + \mathbf{A}|^2 - 4W_n F |\mathbf{V}_0 + \mathbf{A} + f_g \mathbf{n} (\mathbf{n} \cdot \mathbf{W})| \quad (8-1)$$

where τ is given by:

$$\tau = 4Rf_a(r/R) \quad (8-2)$$

where f_a is a function of the radial position of the element, which reduces to 0 at the tip.

Using this equation, a calculation scheme was proposed by Øye [109], where an intermediate induced velocity, W_{int} , is calculated using equation (3-40), but using the value of V_0 from the previous time step. For the n th time step, this is applied to:

$$W_{n(n)} = W_{n(n-1)} + \left(W_{int} - W_{n(n-1)} \right) \frac{|V_0 + A|dt}{\tau} \quad (8-3)$$

Another was developed by Garrad Hassan and Partners, presented by Snel and Scheppers [37], based on the Pitt and Peters solution to an actuator disc [76], and is used in the GH Bladed tool [79]. The equation is the same as the ECN model, except that the time constant, τ , is given by:

$$\tau = \frac{16}{3\pi} \frac{r_2^3 - r_1^3}{r_2^3 - r_1^2} \quad (8-4)$$

where r_2 and r_1 are the upper and lower radii of the blade element, respectively.

8.1.2 Two Differential Equations Model

The Øye model [109] uses 2 differential equations to account for the dynamic wake effect. It was validated and described in Snel and Schepers [37], and showed a good match with experimental data, while remaining simple to implement. This model is based on a simple actuator-disc vortex method, and models both the time lag in the near wake as well as the far wake.

$$W_{int} + \tau_1 \frac{dW_{int}}{dt} = W_{qs} + q\tau_1 \frac{dW_{qs}}{dt} \quad (8-5)$$

$$W + \tau_2 \frac{dW}{dt} = W_{int} \quad (8-6)$$

where W_{qs} is the quasi-static value of the induced velocity vector found using equations (3-40) and (3-41), W_{int} is an intermediate value of the induced

velocity, $\mathbf{W}$ is the corrected value of the induced velocity, $q = 0.6$ and τ_1 and τ_2 are time constants given by:

$$\tau_1 = \frac{1.1}{1 - 1.3a} \frac{R}{|\mathbf{V}_0 + \mathbf{A}|} \quad (8-7)$$

$$\tau_2 = \left(0.39 - 0.26 \left(\frac{r}{R} \right)^2 \right) \tau_1 \quad (8-8)$$

where a is the axial induction factor described in (3-31) and a cannot exceed 0.5.

To solve the differential equations, Hansen [8] suggests using a backward difference scheme for the right hand side of equation (8-5) and solving the rest analytically:

1. Solve $\mathbf{W}_{qs}^i$ using equations (3-40) and (3-41).
2. Estimate the right hand side of equation (8-5) using a backward difference scheme:

$$\mathbf{H} = \mathbf{W}_{qs}^i + q\tau_1 \frac{\mathbf{W}_{qs}^i - \mathbf{W}_{qs}^{i-1}}{\Delta t} \quad (8-9)$$

3. Solve equations (8-5) and (8-6) analytically:

$$\mathbf{W}_{int}^i = \mathbf{H} + (\mathbf{W}_{int}^{i-1} - \mathbf{H}) \exp \left(\frac{\Delta t}{\tau_1} \right) \quad (8-10)$$

$$\mathbf{W}^i = \mathbf{W}_{int}^i + (\mathbf{W}^{i-1} - \mathbf{W}_{int}^i) \exp \left(\frac{\Delta t}{\tau_2} \right) \quad (8-11)$$

While this model has shown good agreement with experimental results, the model is based on a very simple vortex method, and the experiments are limited to a few cases involving pitching of the rotor blades.

The present study will apply the Øye model to uBEM, and τ_1 and τ_2 will be optimised to predict the forces on the NREL 5MW wind turbine rotor in sinusoidal surging motion. The model will be compared to CFD data and optimised to match the data.

8.2 Modified two equation model

The Øye model was based on vortex code results, and modelled the dynamic wake effect without the dynamic airfoil effect. This means that the model may cover some of the unsteady airfoil effects, including impulsive and vortex lift. Since a modified Beddoes-Leishman model is used in the present application, the model needs to be updated to ensure that it remains compatible with the inclusion of the unsteady airfoil model.

To correct the Øye model, a single corrective factor is introduced, A_{dw} , such that:

$$\tau_1 = A_{dw} \frac{1.1}{1 - 1.3a} \frac{R}{|\mathbf{V}_0 + \mathbf{A}|} \quad (8-12)$$

$$\tau_2 = A_{dw} \left(0.39 - 0.26 \left(\frac{r}{R} \right)^2 \right) \tau_1 \quad (8-13)$$

Based on comparisons with CFD-generated rotor thrust and shaft torque, A_{dw} is given a value of 0.025. While this means that the optimised time constants have a magnitude of only 2.5% of the original, the model is still significant in FOWT aerodynamics.

8.3 Results and Discussion

The rotor thrust of the NREL 5MW wind turbine in sinusoidal surging motions with a frequency of $0.5Hz$ are predicted with the uBEM code including the Øye model, present model and no dynamic wake model, and compared to CFD in Figures 8-1 to 8-3.

Figure 8-1. Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

Figure 8-2. Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

Figure 8-3. Predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

The effect of the dynamic wake model on thrust prediction appears to be more significant for motions with amplitude of $2m$ and $6m$.

The rotor shaft torque for the same motions are predicted using the Øye model, present model and no dynamic wake model, and compared to CFD in Figures 8-4 to 8-6.

Figure 8-4. Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

Figure 8-5. Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

Figure 8-6. Predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, for uBEM the Øye model, present model, and no dynamic wake model, compared to CFD

The dynamic wake effect, as would be expected, appears most significant in the motion with $6m$ amplitude. This represents the motion with the highest peak velocities, thus the greatest differences between the models is visible at the peaks and troughs. In the rotor thrust and shaft torque distributions shown, the error between predicted and CFD-simulated thrust and shaft torque do not appear to be largely due to the dynamic wake model, and is likely due to the reduced accuracy in unsteady airfoil effect and tip-loss model in deep stall and extreme wake turbulence.

8.4 Concluding Remarks

A study of the dynamic wake effect was conducted, and the two-differential-equation Øye model was optimised for application in the present uBEM model for FOWT aerodynamics. Due to the modelling of unsteady airfoil aerodynamics by the modified Beddoes-Leishman model discussed in Chapter 6, it was found that the time constants from the original Øye model were too large. The time constants were optimised using a simple scaling factor, found to reduce the constants to 2.5% of the original value.

A comparison of uBEM computations with and without the dynamic wake model to rotor thrust and shaft torque from CFD of the NREL 5MW wind turbine in sinusoidal surging motions showed that the dynamic wake effect was highly significant, especially at higher motion frequencies. It was found that the dynamic wake model was needed to transition between wake states, a phenomenon which will be discussed in Chapter 9.

Future work on dynamic wake modelling can extend the study to more motion frequencies. The raw values of τ_1 and τ_2 can be optimised at each blade station, to determine a model that accurately varies with rotor radius.

Chapter 9

Transient Wake States

During the operation of a fixed-bottom wind turbine, there are two expected wake-states: Windmill state and the turbulent wake state, which occurs at high axial induction, when the wake flow recirculates and generates wake turbulence. During the motion of a floating offshore wind turbine (FOWT), the rotor relative wind speed can be expected to change, resulting in the vortex ring state, propeller brake state and propeller states. These are shown in Figure 2-5.

The momentum theory breaks down outside of the windmill state, as flow recirculation becomes significant, thus the far-wake velocities do not reflect a 1-dimensional momentum balance in the wake. In order to correct for the relationship between axial induction factor and coefficient of thrust in the turbulent wake state, Glauert [7] developed a correction for the coefficient of thrust. Buhl [24] later improved on this correction to ensure continuity at different values of tip-loss. Since normal operating conditions of fixed-bottom wind turbines would result in axial induction factor between 0 and 1, representing the windmill and turbulent wake states, no corrections have been developed for wake states outside this range. This chapter will present a correction based on the Buhl-correction and correlated to simulations of the NREL 5MW wind turbine in high-frequency sinusoidal surge motion.

9.1 Theory

9.1.1 Buhls Glauert Correction Factor

Buhl's [24] equation for the Glauert correction factor was presented in Section 2.1.5. The Buhl equation was formulated to ensure no discontinuities

arise from changing tip-loss factors (equation (2-18 reproduced for ease of reference):

$$C_T = \left(\frac{8}{9}\right) + \left(4F - \frac{40}{9}\right)a + \left(\frac{50}{9} - 4F\right)a^2 \quad (9-1)$$

For application in uBEM, equation (9-1) must be put in the form:

$$C_T = 4a(1 - f_g a) \quad (9-2)$$

where f_g is the Glauert correction.

Rearranging, gives Glauert correction to be applied:

$$f_g = \frac{2}{9aF} \left(5 - \frac{1}{a}\right) - \frac{25}{18F} + 1 \quad (9-3)$$

9.1.2 Extended Glauert Correction

The extended Glauert correction is based on Buhl's equation extended to the vortex ring, propeller brake and propeller wake states. Since data is limited, the specific relationships between coefficient of thrust and axial induction factor are not used. Instead, f_g is designed to match CFD thrust and torque data.

In order to account for the propeller wake state, the axial induction factor from equation (3-31) must account for the effective wind direction:

$$a = \frac{-W_n}{|\mathbf{V}_0 + \mathbf{A}|} \frac{V_{0,z} + A_z}{|V_{0,z} + A_z|} \quad (9-4)$$

9.1.2.1 Vortex ring and propeller brake state

The vortex ring state is a transient state that decays to the propeller brake state. Therefore, the correction for both states do not need to be modelled separately. Buhl's equation was extended into the propeller brake state, as it is expected that coefficient of thrust should continue to increase as per Buhl's relation in equation (9-1).

As the wind velocity becomes negative due to the rotor motion, the rotor briefly enters the propeller state. The propeller state follows from the propeller brake state, where the axial induced velocity is extremely high, thus a high coefficient of thrust. The effective wind velocity, as well as thrust and shaft torque extracted from CFD for the two most extreme rotor motions are demonstrated in Figures 9-1 to 9-4.

Figure 9-1. Thrust against time for rotor in surging motion with Frequency = 0.5Hz and Amplitude = 4m extracted from CFD, and effective wind velocity of rotor

Figure 9-2. Shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$ extracted from CFD, and effective wind velocity of rotor

Figure 9-3. Thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ extracted from CFD, and effective wind velocity of rotor

Figure 9-4. Shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$ extracted from CFD, and effective wind velocity of rotor

Theoretically, at the point of zero velocity, when the effective wind velocity changes direction, the axial induction factor is infinite. This results in an infinite coefficient of thrust. When the velocity changes direction, the thrust does not. At this point, the rotor is briefly in the propeller state. As the flow-field momentum balance responds to this change, the rotor moves out of the propeller state. This implies a transient propeller state, that does not continuously lose torque, as demonstrated by the short period of negative shaft torque when effective velocity is negative in Figures 9-2 and 9-4. Since the propeller state is expected to be similar to the windmill state and follow momentum theory, there is no Glauert correction at the propeller wake state.

9.2 Results and Discussion

The propeller brake and propeller wake states only occurred at a motion frequency of $0.5Hz$. The rotor thrust predicted by CFD is compared to uBEM with the present extended Glauert correction in Figures 9-5 and 9-6.

Figure 9-5. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, compared to CFD

In both Figures 9-5 and 9-6, the peak thrust is observed to differ by 30%. This is due to the tip-loss model from Chapter 5 showing a larger error for thrust, as compared to shaft torque.

Figure 9-6. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, compared to CFD

The rotor shaft torque predicted by CFD is compared to uBEM with the present extended Glauert correction in Figures 9-7 and 9-8.

Figure 9-7. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, compared to CFD

In Figures 9-7 and 9-8, it is demonstrated that the extended Glauert model predicts the rise in torque after the loss of torque in the propeller state. However, in Figure 9-7, uBEM can be seen to overpredict the small rise in shaft torque after the propeller state. This implies that the transition between wake states is important and needs to be modelled as well.

Figure 9-8. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, compared to CFD

The axial induction factor at several blade stations are compared against time for surge frequency of $0.5Hz$ and amplitude of $6m$ in Figure 9-9.

Figure 9-9. Axial induction factor against time for 5 different blade segments

From Figure 9-9, it is observed that the peak axial induction factor reaches a value of 3 and -20. The negative axial induction factor indicates the propeller state, and induction factor greater than 1 indicates the propeller brake state.

Reference to the negative torque observed in Figure 9-8 indicates that the propeller brake and propeller states are both transient, and result in power loss.

9.3 Concluding Remarks

The Glauert correction of Buhl was extended to the propeller brake state, and the momentum assumptions were used in the propeller state of the wind turbine.

The proposed model shows a good correlation with CFD and matches the rotor thrust and shaft torque against time well. While there was significant peak error, the thrust and shaft torque distribution against time was largely followed. The transition to the propeller state was captured by the model, and showed the negative torque found in CFD. The extreme velocity changes resulted in the rotor blade operating in stall at high velocities, and extreme wake turbulence at low velocities. Since the accuracy of prediction of tip-loss and airfoil aerodynamic forces are lower in these flow regimes, some significant differences could be observed between predicted and simulated peak thrust and torque. This will be discussed in Chapter 10.

Future work should extend the present study to a greater range of rotor motions. An effective method would be to use actuator disc models with blade element theory-based force distributions, to reduce computational time, and observe the relationship between coefficient of thrust and the induced velocities.

Chapter 10

Results and Discussion

The computational time for the CFD simulation versus uBEM computation for the NREL 5MW turbine in sinusoidal surge motion with a frequency of $0.1Hz$ and amplitude of $2m$ is shown in Table 10-1

Table 10-1. Computational time for CFD and uBEM in present study

Method	CPU Time (s)
CFD	2317000
uBEM	750

The thrust and shaft torque predictions of the uBEM code are compared to the CFD simulations for the low frequency ($0.02Hz$) surge motions in Figures 10-1 to 10-6.

Figure 10-1. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, compared to CFD

Figure 10-2. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, compared to CFD

Figure 10-3. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, compared to CFD

Figure 10-4. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, compared to CFD

Figure 10-5. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, compared to CFD

Figure 10-6. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, compared to CFD

At the low frequency, the uBEM prediction of thrust and shaft torque has a good level of accuracy compared to CFD. The error is quantified by taking the difference between uBEM and CFD predicted thrust and shaft torque as a percentage of steady-state thrust and shaft torque respectively, at the wind

speed and RPM of the simulations. These are displayed in Figures 10-7 to 10-9.

Figure 10-7. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque

Figure 10-8. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque

Figure 10-9. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.02Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque

A peak error of less than 9% was recorded for thrust and less than 5% for shaft torque.

The thrust and shaft torque predictions of the uBEM code are compared to the CFD simulations for the medium frequency ($0.1Hz$) surge motions in Figures [10-10](#) to [10-15](#).

Figure 10-10. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, compared to CFD

Figure 10-11. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, compared to CFD

Figure 10-12. uBEM predicted thrust against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, compared to CFD

Figure 10-13. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, compared to CFD

Figure 10-14. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, compared to CFD

Figure 10-15. uBEM predicted shaft torque against time for rotor in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, compared to CFD

Once again, a strong match between CFD and uBEM predicted thrust and shaft torque is observed. The relative errors for medium frequency surge motions are shown in Figures [10-16](#) to [10-18](#).

Figure 10-16. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque

Figure 10-17. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque

Figure 10-18. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.1Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque

For medium frequencies, the peak error reaches 15%. However, this is shown to be periodic, and occurs at peak forces, which are much higher than the steady-state force. This means that the error relative to the real-time forces is lower.

For the high frequency ($0.5Hz$) surge motions, the uBEM predicted thrust and shaft torque has been compared to CFD previously in Chapters 7 to 9. Thus, only the error as a percentage of the steady state thrust and shaft torque are shown in Figures 10-19 to 10-21.

Figure 10-19. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $2m$, as a percentage of steady-state thrust and shaft torque

Figure 10-20. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $4m$, as a percentage of steady-state thrust and shaft torque

Figure 10-21. Error between uBEM prediction of thrust and shaft torque in surging motion with Frequency = $0.5Hz$ and Amplitude = $6m$, as a percentage of steady-state thrust and shaft torque

Here, peak errors reach as high as 330%. This is again due to the vast difference in peak forces compared to the steady-state forces. Another observation is that at $6m$ amplitude, the peak error increases at every period. This suggests that the high levels of wake turbulence generated result in a decay at a timescale similar to the motion period. It is also likely that the wake turbulence persists from instantaneous effective velocities that result in high turbulence wake states, when the induced velocities are very high, into instantaneous effective wind speeds that would result in the windmill state in steady operation. This means that the Glauert correction may require an unsteady model to account for the persistence of wake turbulence into the windmill state.

10.1 Concluding Remarks

The uBEM method is shown to be able to accurately predict FOWT aerodynamics at the low and medium frequencies, which are within the FOWT operational window. At high frequencies, uBEM tends to over-predict the thrust and shaft torque, although the aerodynamic trends are predicted, and

the shape of the uBEM-predicted curves of thrust and shaft torque against time correspond to CFD-generated values.

Chapter 11

Conclusion

An unsteady blade element momentum method (uBEM) is presented for floating offshore wind turbine (FOWT) aerodynamics analysis. uBEM applies the BEM equations to each individual rotor blade, to better account for azimuth-specific changes in rotor aerodynamics. The momentum balance at any angle of yaw was achieved through an equation which reduces to classical BEM at zero yaw, and Glauert's equation for a gyrocopter in forward motion at 90° yaw. The equation was then justified for all angles of yaw, using a combination of momentum and circulation theory, providing a stronger argument than the often-quoted fixed control volume theory.

The present study involves development of corrective models for uBEM, based on experimental and direct rotor modelling (DRM), or full 3D computational fluid dynamics (CFD) data. The effects studied include the unsteady airfoil effect, stall delay effect, tip-loss, and transient wake states.

The tip-loss effect was studied using CFD simulations several over tip speed ratios. A significant finding was that the tip-loss was dependent on tip speed ratio and wake turbulence. A novel correction and modelling methodology was presented for tip-losses, through a correction to Glauert's implementation of the Prandtl tip-loss factor optimised against CFD-computer thrust and rotor shaft torque. The model incorporated changes in tip speed ratio as well as changes in wake state, making it better suited to model tip-loss in the constantly-changing flow environments of FOWTs.

The unsteady airfoil effect was studied based on experimental data from the Ohio State University, and unsteady circulation based on Kelvin's theorem

was modelled with one equation instead of two. An optimisation study of the time constants found that they were in fact parameters dependent on airfoil shape. These parameters were modelled against suction surface shape parameters based on a general shape equation for airfoils. This modified Beddoes-Leishman model showed increase accuracy for all airfoils modelled, and showed that a shape-based model is more suited to modelling the unsteady airfoil effect for a wind turbine with changing cross-sectional shapes over the blade.

The stall-delay effect was accounted for using the Du & Selig model, and a novel methodology for real-time stall delay incorporating the unsteady airfoil effect was presented. The differences in predicted thrust and local airfoil lift coefficients showed that pre-application of stall-delay on 2D aerodynamic data is not suitable for FOWTs, since the speed ratio would constantly change due to the changing flow environment as well as speed regulation on the wind turbines.

The dynamic wake effect was studied, and it was found that previously determined time constants were too large. It is postulated that the previous time constants also accounted somewhat for the unsteady airfoil effect. In the present study, the optimised time constant was found to be only 2.5% of the original values. However, the dynamic wake effect at high frequencies was shown to be a significant phenomena, and the transition of the rotor between wake states would not have been predicted without it.

Buhl's version of the Glauert correction for the turbulent wake state was extended to the propeller and propeller brake states of the wind turbine, which are rare for fixed-bottom wind turbines but can be expected to occur periodically for FOWTs. The correction factor of Buhl was extended to the propeller brake state as the Glauert correction for that state. The propeller state follows momentum theory, thus no Glauert correction is needed. The wake induced velocities were analysed, and it was found that at high motion frequencies and peak velocities, the effective wind speed faced by the rotor becomes negative.

An analysis of the induced velocities from uBEM calculations showed that the propeller brake and propeller states are transient states that decay to the turbulent wake state, such that power-loss in backwards wind is not a persistent phenomenon.

Finally, the above mentioned corrective models were implemented in the uBEM framework. The uBEM computations were compared to CFD simulation data. Comparisons of thrust and shaft torque were shown, and it was found that high levels of accuracy, with uBEM-predicted shaft torque within 15% of CFD values for the low and medium frequencies. At high frequencies, the effective wind velocities resulted in transient wake states and stall. Corrective models within both these scenarios are limited in their accuracy, accounting for significant differences uBEM and CFD for peak thrust and shaft torque. However, it was shown that the unsteady airfoil model and extended Glauert correction were able to ensure that the distribution of forces against time was maintained. Sharp peaks in thrust and shaft torque were observed in high frequency motion, which were captured by uBEM.

The models have been developed based on CFD results of the NREL 5MW virtual wind turbine, but the methodologies presented are based on sound engineering principles of correlation and optimisation. These methodologies, grounded in a theoretical understanding of wind turbine aerodynamics, may be extended to any data set available from experiments or superior CFD modelling. The established methodology is capable of forming generalised models from a large set of data or specialised models from specific wind turbines.

11.1 Future Work

For each component of this thesis, future work is suggested:

1. CFD studies extended to yaw and tilt degrees of freedom, then extended to multiple degrees of freedom to observe and study more combinations of unsteady effects such as skewed wake. This has been performed in

literature using an overset grid, but the studies conducted should look at yaw or pitch individually first, then add more degrees of freedom.

2. Skewed wake corrections should be developed for uBEM based on CFD in point 1 above. A dynamic skewed wake model, accounting for time-changing wake skew angles should be developed. This could be achieved with a single-differential model with a radius-dependent time factor.
3. Extension of the tip-loss model to the propeller, vortex ring, and propeller brake wake states. A study may be conducted on how tip-loss changes in a dynamic flow scenario as compared to a static one, with the outcome of a dynamic tip-loss model.
4. Further development of the modified Beddoes-Leishman model to include a greater variety of airfoil shapes, and include pressure surface shape as part of the model. 2D unsteady CFD studies based on the discussed shape parameters could enhance the understanding of the model.
5. Calibration of the Du & Selig model to improve stall-delay prediction. It is likely that turbine-specific calibration is required, though this effect must be separated from tip-loss.
6. Extension of the study of the dynamic wake effect to more motion frequencies. The raw values of τ_1 and τ_2 can be optimised at each blade station, to determine a model that accurately varies with rotor radius.
7. Improvements to the extended Glauert correction. The Glauert correction should be studied further in the propeller brake state using an actuator disc model coupled with the blade element theory. The actuator disc model will have a shorter computational time and will allow for more simulations and facilitate easier observation of the relationship between coefficient of thrust and axial induction factor.

It is the hope of the author that the general methodology of this thesis, using higher order or experimental data to develop model corrections for lower

order models will provide a basis for a greater understanding of wind turbine aerodynamics.

REFERENCES

- [1] 2013 Half-Year Report. Technical report, World Wind Energy Association, Bonn, Germany, 2013.
- [2] J. G. Leishman. Challenges in modelling the unsteady aerodynamics of wind turbines. *Wind Energy*, 5(2-3):85–132, 2002. ISSN 1099-1824. doi: 10.1002/we.62.
- [3] W. Musial, S. Butterfield, and A. Boone. Feasibility of Floating Platform Systems for Wind Turbines. Technical Report NREL/CP-500-34874, National Renewable Energy Laboratory, Golden, CO, USA, 2003.
- [4] J. Jonkman, S. Butterfield, W. Musial, and J. Scott. Definition of a 5-MW reference wind turbine for offshore system development. Technical Report NREL/TP-500-38060, National Renewable Energy Laboratory, Golden, CO, USA, 2009.
- [5] H. Glauert. *A general theory of the autogyro*, volume 1111. HM Stationery Office, 1926.
- [6] J. G. Leishman and T. S. Beddoes. A generalised model for airfoil unsteady aerodynamic behaviour and dynamic stall using the indicial method. In *Proceedings of the 42nd Annual Forum of the American Helicopter Society*, Washington DC, USA, 1986.
- [7] H. Glauert. The analysis of experimental results in the windmill brake and vortex ring states of an airscrew. *Aeronautical Research Committee Reports and Memoranda*, (Rept. 1026), 1926.
- [8] M. O. L. Hansen. *Aerodynamics of Wind Turbines*. Earthscan, London, UK, 2nd edition, 2008. ISBN ISBN-13: 978-1-84407-438-9.
- [9] I. Herráez, B. Stoevesandt, and J. Peinke. Insight into Rotational Effects on a Wind Turbine Blade Using NavierStokes Computations. *Energies*, 7(10):6798–6822, Oct 2014. ISSN 1996-1073.
- [10] J. Tangler. Comparison of wind turbine performance prediction and measurement. *Journal of Solar Energy Engineering*, 104(2):84–88, 1982.

- [11] C. P. Butterfield, D. Simms, W. Musial, and G. Scott. Spanwise aerodynamic loads on a rotating wind turbine blade. Technical Report SERI/TP-257-3983, Solar Energy Research Institute , Golden, CO, USA, 1990.
- [12] C. Butterfield and E. Nelsen. Aerodynamic testing of a rotating wind turbine blade. Technical Report SERI/TP-257-3490, Solar Energy Research Institute , Golden, CO, USA, 1990.
- [13] C. Bak and J. Johansen. Three dimensional corrections of airfoil characteristics based on pressure distributions. In *European Wind Energy Conference (EWEK 2006)*, Athens, Greece, 2006.
- [14] H. Snel, R. Houwink, and J. Bosschers. Sectional prediction of lift coefficients on rotating wind turbine blades in stall. Technical Report ECN-C-93-052, Netherlands Energy Research Foundation (ECN), Petten, Netherlands, 1994.
- [15] P. Chaviaropoulos and M. O. Hansen. Investigating three-dimensional and rotational effects on wind turbine blades by means of a quasi-3D Navier-Stokes solver. *Journal of Fluids Engineering*, 122(2):330–336, 2000.
- [16] Z. Du and M. Selig. A 3-D Stall-Delay Model for Horizontal Axis Wind Turbine Performance Prediction. In *1998 ASME Wind Energy Symposium*, pages 9–19, Reno, NV, USA, 1998.
- [17] C. Lindenburg. Modelling of rotational augmentation based on engineering considerations and measurements. In *European Wind Energy Conference (EWEK 2004)*, London, UK, 2004.
- [18] S.-P. Breton, F. N. Coton, and G. Moe. A study on rotational effects and different stall delay models using a prescribed wake vortex scheme and NREL phase VI experiment data. *Wind Energy*, 11(5):459–482, 2008.
- [19] M. M. Hand, D. A. Simms, L. J. Fingersh, D. W. Jager, J. R. Cotrell, S. Schreck, and S. M. Larwood. Unsteady Aerodynamics Experiment

- Phase VI : Wind Tunnel Test Configurations and Available Data Campaigns. Technical Report NREL/TP-500-29955, National Renewable Energy Laboratory, Golden CO, USA, 2001.
- [20] E. Branlard. Wind turbine tip-loss corrections. Master's thesis, Technical University of Denmark, 2011.
 - [21] D. Wood, V. Okulov, and D. Bhattacharjee. Direct calculation of wind turbine tip loss. *Renewable Energy*, 95:269–276, 2016.
 - [22] L. Prandtl and A. Betz. *Vier Abhandlungen zur Hydrodynamik und Aerodynamik: mit einer Literaturübersicht als Anhang*. Kaiser Wilhelm-Instituts für Strömungsforschung, 1927.
 - [23] D. M. Eggleston and F. S. Stoddard. *Wind turbine engineering design*. Van Nostrand Reinhold Company, New York, New York, USA, 1987.
 - [24] M. L. Buhl. A New Empirical Relationship between Thrust Coefficient and Induction Factor for the Turbulent Windmill State. Technical Report NREL/TP-500-36834, National Renewable Energy Laboratory, Golden, CO, USA, 2005.
 - [25] Renewable energy technologies: Cost analysis series - Wind power. Technical Report Volume 1: Power Sector, Issue 5/5, International Renewable Energy Agency, Bonn, Germany, 2012.
 - [26] T. Sebastian and M. A. Lackner. A Comparison of First-Order Aerodynamic Analysis Methods for Floating Offshore Wind Turbines. In *48th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*, Orlando, FL, USA, 2010. AIAA.
 - [27] NWTC Computer-Aided Engineering Tools (FAST by Jason Jonkman, Ph.D), July 2014. URL <http://wind.nrel.gov/designcodes/simulators/fast/>.
 - [28] F. G. Nielsen, T. D. Hanson, and B. Skaare. Integrated dynamic analysis of floating offshore wind turbines. In *25th International Conference on Offshore Mechanics and Arctic Engineering*, pages 671–679, Hamburg,

Germany, 2006.

- [29] H. Bagbanci. Dynamic analysis of offshore floating wind turbines. Masters thesis, Instituto Superior Tecnico, Universidade Tecnica de Lisboa, Lisbon, Portugal, 2011.
- [30] D. Roddier, C. Cermelli, A. Aubault, and A. Weinstein. WindFloat: A floating foundation for offshore wind turbines. *Journal of Renewable and Sustainable Energy*, 3:033104–1 – 033104–34, May 2010.
- [31] R. Stølsmark. Dynamic response of a floating offshore wind turbine. Masters thesis, University of Stavanger, Stavanger, Norway, 2010.
- [32] J. Jonkman and W. Musial. Offshore Code Comparison (OC3) for IEA Wind Task 23 Offshore Wind Technology and Deployment. Technical Report NREL/TP-5000-48191, National Renewable Energy Laboratory, Golden, CO, USA, 2010.
- [33] Wind in our Sails - The coming of Europe's offshore wind energy industry. Technical report, European Wind Energy Association, Brussels, Belgium, 2011.
- [34] S. Christiansen, T. Bak, and T. Knudsen. Damping wind and wave loads on a floating wind turbine. *Energies*, 6:4097–4116, August 2013.
- [35] A. Suzuki. *Application of dynamic inflow theory to wind turbine rotors*. PhD thesis, The University of Utah, Salt Lake City, UT, USA, October 2000.
- [36] H. Snel and J. Schepers. Investigation and modeling of dynamic inflow effects. In *Proceedings of the European Community Wind Energy Conference and Exhibition*, Travemünde, Germany, 1993. ECN.
- [37] H. Snel and J. G. Scheppers. Joint investigation of dynamic inflow effects and implementation of an engineering method. Technical Report ECN-C-94-107, Netherlands Energy Research Foundation (ECN), Petten, The Netherlands, 1995.

- [38] J. Scheepers. IEA Annex XX: Dynamic Inflow effects at fast pitching steps on a wind turbine placed in the NASA-Ames wind tunnel. Technical Report ECN-E-07-085, Netherlands Energy Research Foundation (ECN), Petten, The Netherlands, 2007.
- [39] H. Imamura. Aerodynamics of wind turbines. Technical report, Yokohama National University, Yokohama, Japan, 2001.
- [40] J. G. Leishman. *Principles of Helicopter Aerodynamics*. Cambridge University Press, 2001.
- [41] OSU Wind-Tunnel Test Data, July 2012. URL http://wind.nrel.gov/airfoils/OSU_data/data/.
- [42] R. Pereira, G. Schepers, and M. D. Pavel. Validation of the Beddoes-Leishman Dynamic Stall Model for Horizontal Axis Wind Turbines using MEXICO data. In *49th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition*, Orlando, FL, USA, 2011. AIAA.
- [43] T. Theodorsen. General theory of aerodynamic instability and the mechanism of flutter. Technical Report NACA-TR-496, National Advisory Committee for Aeronautics, 1979.
- [44] M. H. Hansen, M. Gaunaa, and H. A. Madsen. A Beddoes-Leishman type dynamic stall model in state-space and indicial formulations. Technical Report Risø-R-1354(EN), Risø National Laboratory, Roskilde, Denmark, 2004.
- [45] P. J. Moriarty and A. C. Hansen. Aerodyn Theory Manual. Technical Report NREL/TP-500-36881, National Renewable Energy Laboratory, Boulder, CO, USA, 2005.
- [46] B. Andresen. Wake behind a wind turbine operating in yaw. Master's thesis, Norwegian University of Science and Technology (NTNU), Trondheim, Norway, 2013.

- [47] W. Haans and R. Mikkelsen. Airfoil models in the actuator line code assessed with near-wake measurements on a yawed rotor. In *45th AIAA Aerospace Sciences Meeting and Exhibit*, Reno, NV, USA, 2007.
- [48] T. Valentine. Aerodynamic effects and load fluctuation of yawed wind turbines. Master's thesis, Rensselaer Polytechnic Institute, Hartford, CT, USA, 2010.
- [49] W. Johnson. Model for vortex ring state influence on rotorcraft flight dynamics. Technical Report NASA TP 2005-213477, National Aeronautics and Space Administration (NASA), Moffett Field, CA, USA, 2005.
- [50] T. Sebastian and M. A. Lackner. Offshore floating wind turbines - an aerodynamic perspective. In *49th AIAA Aerospace Sciences Meeting*, Orlando, FL, USA, 2011.
- [51] J. Drees and W. P. Hendal. Airflow patterns in the neighbourhood of helicopter rotors: A description of some smoke tests carried out in a windtunnel at Amsterdam. *Aircraft Engineering and Aerospace Technology*, 23(4):107–111, April 1951.
- [52] J. N. Sørensen, W. Z. Shen, and X. Munduate. Analysis of wake states by a full-field actuator disc model. *Wind Energy*, 1(2):73–88, 1998. ISSN 1099-1824.
- [53] T. Sebastian and M. A. Lackner. Unsteady near-wake of offshore floating wind turbines. In *50th AIAA Aerospace Sciences Meeting*, Nashville, TN, USA, 2012.
- [54] T. Wang. A brief review on wind turbine aerodynamics. *Theoretical & Applied Mechanics Letters*, 2(6):062001–1 – 062001–8, September 2012.
- [55] J. Azcona, F. Boucotrouch, M. González, J. Garciandía, M. Xabier, F. Kelberlau, and T. A. Nyaagard. Aerodynamic thrust modelling in wave tank tests of offshore floating wind turbines using a ducted fan. *Journal of Physics: Conference Series*, 524(1):1–11, June 2014.

- [56] H. R. Martin. Developments of a scale model wind turbine for testing of offshore floating wind turbine systems. Master's thesis, The University of Maine, Orono, ME, USA, 2011.
- [57] F. Huijs, E.-J. d. Ridder, and F. Savenije. Comparison of model tests and coupled simulations for a semi-submersible floating wind turbine. In *Proceedings of the ASME 2014 33rd International Conference on Ocean, Offshore and Arctic Engineering (OMAE2014)*, San Francisco, CA, USA, 2014.
- [58] E.-J. d. Ridder, W. Otto, G.-J. Zondervan, F. Savenije, and F. Huijs. State of the art model testing techniques for floating wind turbines. In *European Wind Energy Association EWEA OFFSHORE*, Frankfurt, Germany, 2013.
- [59] S. Rockel, E. Camp, J. Schmidt, J. Peinke, R. B. Cal, and M. Hölling. Experimental study on influence of pitch motion on the wake of a floating wind turbine model. *Energies*, 7(4):1954–1985, March 2014.
- [60] S. Martin, S. Day, and C. B. Gilmour. Rotor scaling methodologies for small scale testing of floating wind turbine systems. In *Proceedings of the ASME 2015 34rd International Conference on Ocean, Offshore and Arctic Engineering (OMAE2015)*, St John's, NL, Canada, 2015.
- [61] B. Sanderse, S. van der Pijl, and B. Koren. Review of CFD for wind-turbine wake aerodynamics. *Wind Energy*, 14:799–819, 2011.
- [62] N. Ren, Y. Li, and J. Ou. Coupled wind-wave time domain analysis of floating offshore wind turbine based on Computational Fluid Dynamics method. *Journal of Renewable and Sustainable Energy*, 6:023106–1 – 023106–13, 2014 2014.
- [63] D. Bekiropoulos, R. M. Rieß, T. Lutz, E. Krämer, D. Matha, M. Werner, and P. V. Cheng. Simulation of unsteady aerodynamic effects on floating offshore wind turbines. In *11th German Wind Energy Conference*, Bremen, Germany, 2012.

- [64] T. Tran, D. Kim, and J. Song. Computational fluid dynamic analysis of a floating offshore wind turbine experiencing platform pitching motion. *Energies*, 7:5011–5026, August 2014.
- [65] Y. Bazilevs, M.-C. Hsu, J. Kiendl, R. Wüchner, and K. U. Bletzinger. 3D simulation of wind turbine rotors at full scale. Part II: Fluid–structure interaction modeling with composite blades. *International Journal for Numerical Methods in Fluids*, 65:236–253, January 2011.
- [66] Y. Bazilevs, M.-C. Hsu, I. Akkerman, S. Wright, K. Takizawa, B. Henicke, T. Spielman, and T. E. Tezduyar. 3D simulation of wind turbine rotors at full scale. Part I: Geometry modeling and aerodynamics. *International Journal for Numerical Methods in Fluids*, 65:207–235, January 2011.
- [67] B. Sanderse. Aerodynamics of wind turbine wakes. Technical Report ECN-E-09-016, Netherlands Energy Research Foundation (ECN), Petten, The Netherlands, 2009.
- [68] D. Yang, L. Liu, and L. Shen. Coupled wind LES and ocean wave simulation with actuator disk or line models for offshore wind farm study. In *11th World Congress on Computational Mechanics (WCCM XI)*, pages 1–2, Barcelona, Spain, 2014.
- [69] R. Mikkelsen. *Actuator disc methods applied to wind turbines*. PhD thesis, University of Denmark, Lyngby, Denmark, 2003.
- [70] A. A. Singapore Wala and E. Y.-K. Ng. Actuator disc modeling based on aerodynamic data extraction from direct rotor modeling of the NREL Phase VI turbine. *Progress in Computational Fluid Dynamics*, 14:352–362, 2014.
- [71] H. Currin and J. Long. Horizontal axis wind turbine free wake model for AeroDyn. Research final report, Oregon State University, Klamath Falls, OR, USA, 2009.

- [72] T. Sebastian and M. A. Lackner. Development of a free vortex wake method code for offshore floating wind turbines. *Renewable Energy*, 46: 269–275, 2012.
- [73] J. G. Schepers and H. Snel. Model rotor experiemnts in controlled conditions. Technical Report ECN-E-07-042, Netherlands Energy Research Foundation (ECN), Petten, The Netherlands, 2007.
- [74] T. Sebastian and M. A. Lackner. Characterization of the unsteady aerodynamics of offshore floating wind turbines. *Wind Energy*, 16:399–352, April 2013.
- [75] T. Burton, D. Sharpe, N. Jenkins, and E. Bossanyi. *Wind Energy Handbook*. John Wiley & Sons, Ltd, Chichester, United Kingdom, 1st edition, 2001.
- [76] D. Pitt and D. Peters. Theoretical prediction of dynamic inflow derivatives. *Vertica*, 5:21–34, 1981.
- [77] A. Maheri, S. Noroozi, C. Toomer, and J. Vinney. Damping the fluctuating behaviour and improving the convergence rate of the axial induction factor in the BEMT-based rotor aerodynamic codes. In *European Wind Energy COnference (EWEC 2006)*, Athens, Greece, 2006.
- [78] S. A. Ning. A simple solution method for the blade element momentum equations with guaranteed convergence. *Wind Energy*, 17:1327–1345, September 2014.
- [79] E. A. Bossanyi. GH Bladed theory manual. Technical report, DNV-GL, 2003.
- [80] HAWC2 DTU Wind Energy, 2003. URL <http://www.hawc2.dk>.
- [81] T. Sant. *Improving BEM-based aerodynamic models in wind turbine design codes*. PhD thesis, TU Delft, Delft University of Technology, Delft, The Netherlands, 2007.

- [82] C. T. Silva and M. V. Donadon. Unsteady blade element-momentum method including returning wake effects. *Journal of Aerospace Technology and Management*, 5:27–42, January 2013.
- [83] T.-T. Tran, D.-H. Kim, and K.-S. Bae. Extreme load estimation for a large wind turbine using CFD and unsteady BEM. In *Computational Science and Its Applications ICCSA 2013*, pages 127–142, Ho Chi Minh City, Vietnam, 2014.
- [84] J. Jonkman, T. Larsen, A. Hansen, T. Nygaard, K. Maus, M. Karimirad, Z. Gao, T. Moan, I. Fylling, J. Nichols, M. Kohlmeier, J. P. Vergara, D. Merino, W. Shi, and H. Park. Offshore Code Comparison Collaboration within IEA Wind Task 23: Phase IV Results Regarding Floating Wind Turbine Modeling. In *European Wind Energy Conference (EWECC 2010)*, Warsaw, Poland, 2010.
- [85] A. R. S. Bramswell, G. Done, and D. Balmford. *Bramswell's Helicopter Dynamics*. Edward Arnold (Publishers) Ltd, Oxford, United Kingdom, 2nd edition, 1976.
- [86] J. Seddon. *Basic Helicopter Aerodynamics*. Blackwell Scientific Publishers Ltd, Oxford, United Kingdom, 1st edition, 1990.
- [87] A. R. S. Bramswell. Some remarks on the induced velocity field of a lifting rotor and Glauert's formula. Technical report, HM Stationery Office, London, UK, 1971.
- [88] V. I. Shaidakov. Application of the ring vortex method to aerodynamic design of lifting rotor systems. Technical report, DTIC Document, Charlottesville, VA, USA, 1971.
- [89] F. R. Menter. Improved two-equation k-omega turbulence models for aerodynamic flows. Technical Report NASA-TM-103975, NASA Ames Research Center, Moffett Field, California, USA, 1992.
- [90] H. Glauert. *Airplane propellers*, pages 169–360. Springer Berlin Heidelberg, Berlin, Germany, 1935.

- [91] R. E. Wilson and P. B. S. Lissaman. Applied aerodynamics of wind machines. Technical report, Oregon State University, Corvallis, Oregon, USA, 1974.
- [92] O. de Vries. Fluid dynamic aspects of wind energy conversion. Technical Report AGARD-AG-243, Advisory Group for Aerospace Research and Development, Neuilly-sur-Seine, France, 1979.
- [93] W. Z. Shen, R. Mikkelsen, J. N. Sørensen, and C. Bak. Tip loss corrections for wind turbine computations. *Wind Energy*, 8(4):457–475, 2005.
- [94] G. Xu and L. N. Sankar. Application of a viscous flow methodology to the NREL Phase VI rotor. In *ASME 2002 Wind Energy Symposium*, pages 83–93. American Society of Mechanical Engineers, 2002.
- [95] E. E. Larrabe and S. E. French. Maximum Induced Loss Windmills and Propellers. *Journal of Wind Engineering and Industrial Aerodynamics*, 15:317–327, 1983.
- [96] C. Lindenburg. Investigation into Rotor Blade Aerodynamics Analysis of the stationary measurements on the UAE phase-VI rotor in the NASA-Ames wind tunnel. Technical Report ECN-C-03-025, Energy research Centre of the Netherlands, July 2003.
- [97] E. Polak and G. Ribière. Note sur la convergence de méthodes de directions conjuguées. *ESAIM: Mathematical Modelling and Numerical Analysis - Modélisation Mathématique et Analyse Numérique*, 3(R1): 35–43, 1969.
- [98] S. Øye. Dynamic stall, simulated as a time lag of separation. In *Proceedings of the 4th IEA Symposium on the Aerodynamics of Wind Turbines*, Rome, Italy, 1991.
- [99] C. Tran and D. Petot. Semi-empirical model for the dynamic stall of airfoils in view of the application to the calculation of responses of a helicopter in forward flight. *Vertica*, 5:35–53, 1981.

- [100] H. Snel. Heuristic modelling of dynamic stall characteristics. In *European Wind Energy Conference (EWECE 97)*, Dublin, Ireland, 1997.
- [101] J. G. Holierhoek, J. B. de Vaal, A. H. van Zuijlen, and H. Bijl. Comparing different dynamic stall models. *Wind Energy*, 16:139–158, 2013.
- [102] B. M. Kulfan and J. E. Bussoletti. "Fundamental" Parametric Geometry Representations for Aircraft Component Shapes. In *11th AIAA/ISSMO Multidisciplinary Analysis and Optimisation Conference*, Portsmouth, Virginia, USA, 2006.
- [103] R. B. S. Pereira. Validating the Beddoes-Leishman Dynamic Stall Model in the Horizontal Axis Wind Turbine Environment. Master's thesis, Universidade Tecnica de Lisboa, 2010.
- [104] J. E. Minnema. Pitching moment predictions on wind turbine blades using the Beddoes-Leishman model for unsteady aerodynamics and dynamic stall. Master's thesis, The University of Utah, 1998.
- [105] M. Mert. Optimisation of semi-empirical parameters in the FFA-Beddoes dynamic stall model. Technical Report FFA TN 1999-37, Flygtekniska Frsöksanstalten, 1999.
- [106] A. Björck. DYNSTALL: Subroutine and Package with a and Dynamic stall model. Technical Report FFAP-V-110, Flygtekniska Frsöksanstalten, 2000.
- [107] B. E. Thwaites. *Incompressible Aerodynamics*. Oxford University Press, 1960.
- [108] D. Hu, O. Hua, and Z. Du. A study on stall-delay for horizontal axis wind turbine. *Renewable Energy*, 31:821–836, May 2006.
- [109] S. Øye. Unsteady wake effects caused by pitch angle changes. In *Proceedings of the first IEA Symposium on the aerodynamics of wind turbines*, London, UK, 1986. IEA.

APPENDIX

The Polak-Ribière Conjugate Gradients Method

The conjugate gradients method is used to find the solution for a system of linear equations by finding the steepest gradient in the direction of an initial vector. The non-linear conjugate gradients method is a method of optimisation of non-linear problems, which uses the steepest gradient approach to find an optimum (where the gradient is 0). This approach finds the local optimum based on gradients alone.

The general non-linear conjugate gradients method is as follows:

1. Determine a suitable start point
2. Compute the steepest gradient at the start point

$$\mathbf{s}_0 = \Delta \mathbf{x}_0 = -\nabla_x f(\mathbf{x}_0) \quad (\text{A-1})$$

3. Perform a line search to find the optimum value of α_0 when:

$$\mathbf{x}_1 = \min (f(\mathbf{x}_0 + \alpha_0 \mathbf{s}_0)) \quad (\text{A-2})$$

4. At every subsequent step, compute the steepest gradient

$$\Delta \mathbf{x}_n = -\nabla_x f(\mathbf{x}_n) \quad (\text{A-3})$$

5. Find β_n using the Polak-Ribière method:

$$\beta_n = \frac{\Delta \mathbf{x}_n^T (\Delta \mathbf{x}_n - \Delta \mathbf{x}_{n-1})}{\Delta \mathbf{x}_{n-1}^T \Delta \mathbf{x}_{n-1}} \quad (\text{A-4})$$

6. Find the new conjugate direction

$$\mathbf{s}_n = \Delta \mathbf{x}_n + \beta_n \mathbf{s}_{n-1} \quad (\text{A-5})$$

7. Perform a line search to find the optimum value of α_n when:

$$\mathbf{x}_{n+1} = \min (f(\mathbf{x}_n + \alpha_n \mathbf{s}_n)) \quad (\text{A-6})$$

8. Update the solution with $\mathbf{x}_{n+1}$, and repeat the process at step 4 until an optimum is reached. The optimum is reached when the gradient is 0, or set at a certain tolerance.

The advantage of the Polak-Ribière conjugate gradients method is that a 'hard' solution for the line search in steps 3 and 7 are not required, so lower tolerances can be used to achieve the optimum point. Thus, the solution mechanism may be sped up by setting lower tolerances in the line search.

PUBLICATIONS

- [1] Abdulqadir Aziz Singapore Wala and Eddie Yin-Kwee Ng. "Actuator disc modeling based on aerodynamic data extraction from direct rotor modeling of the NREL Phase VI turbine". In: *Progress in Computational Fluid Dynamics* 14 (2014), pp. 352–362.
- [2] Abdulqadir Aziz Singapore Wala, Eddie Yin-Kwee Ng, Anand Bahaguni, and Narasimalu Srikanth. "A novel methodology for wind turbine tip-loss corrections in different wake-states". In: *Renewable Energy* (Under Review, Submitted: Feb 2017).
- [3] Abdulqadir Aziz Singapore Wala, Eddie Yin-Kwee Ng, Anand Bahaguni, and Narasimalu Srikanth. "Quantification and Modelling of the Dynamic Wake Effect for Floating Offshore Wind Turbines". In: *Offshore Technology Conference*. Houston, TX, USA, May 2017.
- [4] Abdulqadir Aziz Singapore Wala, Eddie Yin-Kwee Ng, and Narasimalu Srikanth. "A Beddoes-Leishman-type model with an optimisation-based methodology and airfoil shape parameters". In: *Wind Energy* (Under Review, Submitted: Aug 2016).