Modeling, analysis and control of high-order switching power converters

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**MODELING, ANALYSIS AND CONTROL
OF HIGH-ORDER SWITCHING
POWER CONVERTERS**

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	i
TABLE OF CONTENTS	ii
SUMMARY	v
LIST OF ACRONYMS	vii
CHAPTER 1 INTRODUCTION	1
1.1 Background and Motivation	1
1.2 Objectives of This Research Work	6
1.3 Contributions of the Thesis	7
1.4 Organization of This Thesis	7
CHAPTER 2 LITERATURE REVIEW	10
2.1 Review of the State-of-the-art High-order Dc-Dc Boost Converter Topologies	10
2.2 Modeling of High-order Boost-type Dc-Dc Converters	15
2.2.1 Overview of the State-Space Averaging Method	15
2.2.2 Averaged Model of the Hybrid-Boost Converter	18
2.2.3 Averaged Model of the 2-stage Cascade Boost Converter	22
2.3 State-of-the-art Controllers for High-Order Dc-Dc Boost Converters	24
2.3.1 Current-Mode Controller	24
2.3.2 Sliding-Mode Controllers	25
2.3.3 The Output Feedback Controller	27
2.4 Summary	28
CHAPTER 3 COMPARATIVE STUDY OF FIXED CURRENT-MODE CONTROLLERS FOR HIGH-ORDER DC-DC CONVERTERS	30

3.1	Motivation	30
3.2	A Comparative Study of Current-mode Controllers for the Positive Output Elementary Luo (POEL) Converter	31
3.2.1	Comparison Using State-Space Approach	32
3.2.2	Comparison Using Frequency-Response Approach	37
3.3	A Comparative Study of Current-mode Controllers for the 2-Stage Cascade Boost Converter	47
3.3.1	Controller Design Using Root-locus Method	50
3.3.2	Controller design using classical frequency-domain approach	57
3.4	Experimental Results	64
3.4.1	Current-mode Controlled POEL Converter	64
3.4.2	Current-mode controlled 2-stage cascade boost converter	67
3.5	Summary	74

CHAPTER 4 SLIDING-MODE CONTROL OF HIGH-ORDER DC-DC CONVERTERS

4.1	Motivation	75
4.2	Design of Fixed-frequency Pulse-width-modulation (PWM)-Based SM Controllers for the Quadratic Boost Converter	78
4.2.1	Design of PWM-based SM Controller Using Input-inductor Current	80
4.2.2	Design of PWM-based SM Controller Using Output-inductor Current	84
4.3	A Comparative Study of Hysteresis-Modulation (HM)-Based SM Controllers for the Hybrid-type dc-dc Boost Converter	86
4.3.1	Design of HM-based SM Controller Using Output-inductor Current	88
4.3.2	Design of HM-based SM Controller Using Input-inductor Current	93
4.4	Simulation and Experimental Results	98
4.4.1	Implementation of the PWM-based SM controller for the quadratic boost converter	98

4.4.2	HM-based SM for the hybrid boost converter	103
4.5	Summary	110
CHAPTER 5 MODELING AND ADAPTIVE CURRENT-MODE CONTROL OF A HIGH STEP-UP DC-DC CONVERTER		111
5.1	Modeling of a High Step-up dc-dc Converter	112
5.1.1	Averaged State-space Model of the Converter	114
5.1.2	Model Validation	117
5.2	Adaptive Current-Mode Controller Design	119
5.2.1	Proposed control law	120
5.2.2	Stability Analysis	121
5.3	Experimental Results	125
5.4	Summary	129
CHAPTER 6 CONCLUSIONS AND RE-COMMENDATIONS FOR FUTURE WORK		131
6.1	Conclusions	131
6.2	Recommendations for Future Research	133
AUTHOR'S PUBLICATIONS		142

SUMMARY

The output voltage gain of the conventional dc-dc boost converter is often limited as operating this converter at some extremely high values of the duty ratio may lead to a reverse recovery problem of the diode and little room for control when dealing with load and line disturbances. To solve this problem, many transformer-based and transformer-less topologies have been proposed over the past few years to achieve a sufficiently high gain at some lower values of the duty ratio. However, if the industrial application does not demand for dc isolation, the use of a transformer-based dc-dc converter would only increase the overall cost and size of the system. Also, the efficiency degrades due to the losses associated with the secondary winding of the transformer. This makes the transformer-less topologies a more interesting choice and to date, a number of such topologies have been reported in the relevant literature. The objective of this thesis is to study the various modeling and control related aspects of such high-order non-isolated dc-dc boost converters. Towards this end, several control schemes are studied.

As compared to the conventional dc-dc boost converter wherein there are only two state variables available for feedback purposes, the newly developed high-order topologies present certain challenges. Since they contain more inductors and capacitors to achieve the high gain, they present more state variables which can be used for feedback purposes. Therefore, it is necessary to select the most suitable state variables when designing the controllers for these converters. To address this issue, a detailed comparative study of two current-mode controllers (using the input and output inductor currents of the converter) is presented for some high-order dc-dc converters such as the positive output elementary Luo (POEL) converter and the 2-stage cascade boost converter. The objective is to find the most suitable inductor current for the controller design. The analyses of the current-controlled systems are carried out using the state-space and frequency-response approaches.

Sliding-mode (SM) controller is another widely used control methodology for dc-dc converters and it offers several advantages such as excellent large-signal

handling capability, guaranteed stability and robustness against load, line and other parametric variations and ease of implementation. The major concern when implementing the pulse-width-modulation (PWM)-based SM controller is the difficulty in achieving a good steady-state regulation using a single integral term acting on the output voltage error. Even though this problem can be alleviated via the use of a double-integral term in the sliding surface, this approach not only increases the order of the controller but may also require more state variables such as two currents for feedback purposes. Ideally, the controller should be of a lower-order to reduce the cost and for ease of implementation. To solve this problem, a fixed-frequency PWM-based SM controller for the quadratic boost converter using a reduced number of state variables is proposed. The proposed controller requires only one current for its implementation while enjoying the advantages offered by both fixed frequency and double-integral approaches. Two PWM-based SM controllers using the input and output inductor currents of the converter are separately designed to find the most suitable inductor current for the controller design. Moreover, a systematic approach to find the most suitable inductor current for designing the hysteresis-modulation (HM)-based SM controller of a hybrid high-order dc-dc boost converter is also provided.

Lastly, a new adaptive current-mode controller is proposed to regulate the high step-up dc-dc converter. The proposed controller uses the estimate of the load resistance to compute the control signal. This estimate is calculated using the adaptive algorithm and the structure of this adaptive algorithm is such that it leads to an optimized derivative of the estimate which is bounded by a user specified constant. The approximate stability analysis of the controlled system is provided and some tuning guidelines are given to select the appropriate values of the controller gains. Finally, some experimental results are provided to demonstrate better regulation properties of the proposed adaptive current-mode controller compared to existing current-mode controllers.

LIST OF ACRONYMS

POEL	-	Positive output elementary Luo
SM	-	Sliding-mode
PWM	-	Pulse-width-modulation
HM	-	Hysteresis-modulation
PV	-	Photovoltaic
EMI	-	Electromagnetic-interference
MOSFET	-	Metal–oxide–semiconductor field-effect transistor
ZCS	-	Zero-current-switching
KVL	-	Kirchoff's voltage Law
KCL	-	Kirchoff's current Law
CCM	-	Continuous conduction mode
PI	-	Proportional-integral
PD-PBC	-	Parallel-damped passivity-based controller
SEPIC	-	Single-ended primary-inductor converter
V _{p-p}	-	Peak to peak voltage
SMC	-	Sliding-mode control
ESR	-	Equivalent series resistance
F_s	-	Switching frequency
TF	-	Transfer function
CMC	-	Current-mode controller
HF	-	High-frequency
IEEE	-	Institute of Electrical and Electronics Engineers
IET	-	The Institution of Engineering and Technology

CHAPTER 1 INTRODUCTION

This chapter presents an introduction to the high-order boost-type dc-dc converters. It covers their need, the drawbacks of the conventional dc-dc boost converter, the major challenges in designing the controllers for the high-order topologies and a brief overview of some controllers used in this thesis for regulating the high-order dc-dc converters. The research objectives, contributions and organization of this thesis are also presented in this chapter.

1.1 Background and Motivation

The last few decades have seen a continuous growing interest in the development of “green technology”. It has become increasingly popular considering the world-wide concern of reducing global warming and interest in sustainable energy productions to eliminate the pollution caused by the fossil fuel burnings. The four main sources of sustainable energy systems are photovoltaic (PV) modules, batteries of various chemistries, fuel cells and wind energy [1-3]. However, since the output of these energy sources is very low (such as less than 1 V for the photovoltaic cell and 3 or 4 V for the Li-Ion cell), they are unable to interface with the higher power systems of few hundred Watts. To solve this problem, several photovoltaic panels (each having a series connection of number of individual cells) can be connected in a cascade arrangement to increase the voltage levels. However, it may lead to some implementation complexities. As such, a dc-dc converter with a sufficient amount of gain can be reasonably employed before interfacing with a grid [1].

Fig. 1.1 shows the schematic diagram of a micro-inverter consisting of a high step-up dc-dc converter and a dc-ac inverter which are used in tandem to design a PV-powered ac module. The output of the photovoltaic panel is connected to the dc-dc converter which produces an output voltage upto 400 V. This DC voltage is then converted to an AC voltage by an inverter [4]. Apart from this, the necessity of designing some ad-hoc electronic devices in applications such as electric vehicles which are supplied by the renewable energy resources, is prompting engineers to use a 400-V dc bus as a core of the distribution [5]. Also, it is required to convert the 48 V output of the dc battery plant to 380 V required for the intermediate dc bus used in telecom equipment devices for providing the internet services. Moreover, the 12 V output voltage of the car battery is required to be converted to 100 V to power the high intensity discharge lamps used in automobiles [2]. In many of such applications dc-dc converters are required to provide a good output voltage regulation against the load and line variations as well as the circuit parameter uncertainties.

Fig. 1.1 Schmetic of a photovoltaic-powered AC-module [4].

Theoretically, a conventional boost converter can produce an infinite gain with its duty ratio approaching unity. However, in practice, the gain and efficiency of the conventional dc-dc boost converter is often constrained due to the reverse recovery problem of the diode, switching losses, as well as the equivalent series resistance of its capacitor and the parasitic resistance of its inductor [6].

Therefore, many transformer-based and transformer-less topologies have been proposed over the past few years to achieve a sufficiently high gain at lower values of the duty ratio [2, 7-11]. However, if an industrial application does not demand for dc isolation, the use of transformer-based isolated dc-dc converters would only increase the overall cost and size of the system [2]. Also, the efficiency degrades due to the losses associated with the secondary winding of the transformer. This makes transformer-less topologies a more interesting choice and to date, a number of such topologies has been reported in the relevant literature [2, 8-11]. The main objective of thesis is to study the various modeling and control related aspects of such high-order boost-type dc-dc converters.

The development of newly proposed high-order non-isolated dc-dc boost converters is often accompanied by the need to design their corresponding controllers. The voltage-mode control and current-mode control are two widely used methodologies for regulating the output voltage in dc-dc converters. However, most of the high-order boost-type dc-dc converters feature a dynamic behavior similar to that of the conventional boost converter. That is, the non-minimum phase nature of their control to output voltage transfer function makes it quite difficult to design the controller using a single voltage-loop. To overcome this problem occurring in boost-derived topologies, the multi-loop current-mode controller can be suitably employed and it offers several advantages such as over-current protection and a faster transient response [12].

However, even though the current-mode controller has been successfully applied to many higher-order boost-type dc-dc converters in the past [12-17], there are certain issues which are not extensively addressed in these works. In contrast to

the conventional dc-dc boost converter wherein there is only one inductor current available for the feedback purposes, the newly invented higher-order converters present certain challenges. For example, in an N-stage cascade boost converter, there are ' $2N$ ' number of state variables which can be used for feedback purposes (N -inductor currents and N -capacitor voltages). First, as there are now more variables to consider, what are the most suitable state variables for feedback purposes? Is there any condition under which the current-mode control using the specific inductor current fails to work? If there is, then how do we overcome it? All these questions demand further research. These questions will be addressed in chapter 3 of the thesis.

Sliding-mode (SM) control is one of the widely used non-linear control methodologies for dc-dc converters considering their highly non-linear dynamic characteristics. It offers several advantages like excellent large-signal handling capability, guaranteed stability, robustness against load, line and other parametric variations, and ease of implementation [18]. The pulse-width modulation (PWM) and hysteresis-modulation (HM) are two-widely used modulation techniques for implementing SM controllers for the dc-dc converters [5, 18-22]. Some earlier works on developing SM controllers for the higher-order boost converters like the quadratic boost converter are based on the HM-based approach and it offers several advantages like ease of implementation and reduced risk of saturation when operating at high values of duty cycle [5]. However, this solution uses a variable switching frequency which may cause excessive power losses like inductor and switching losses as well as electromagnetic-interference (EMI) generation [19].

To overcome these problems occurring in hysteresis-based modulation, a fixed-frequency, PWM-approach is usually employed [20, 21]. However, despite the successful application of this approach to dc-dc converters, there remain several areas which demand further investigation. Literature review suggests that even though a good dynamic response over a wide range of operating conditions is achieved using this approach, the adoption of single integral term can only alleviate but not fully reduce the steady state error occurring in such type of the controllers [20]. To solve this problem, a sliding surface with an additional integral term was employed in [20] and [21]. However, the sliding surface used in [20] and [21] not only increases the order of the controller but also demands the use of more state variables like two currents in feedback. Ideally the controller should be of low-order for cost and ease of implementation considerations. Also, the least number of currents should be used in feedback as they demand a complex current sensing circuitry. Thus, the challenge is to design a suitable fixed-frequency PWM-based SM controller using a reduced number of state variables without compromising the transient and steady-state specifications of the system. Recently, a sliding-surface composed of state variables and the integral and double integral terms of the output voltage state-variable error has been presented in [22] for regulating the high-order dc-dc converter. Apart from this, it is interesting to note that there has been not much study on the aspect of selecting the most suitable state variables for designing the SM controllers for the high-order boost converters. The appropriate states for feedback purposes may not only determine the overall performance of the controlled system but may also affect the cost and ease of implementation. These issues will be addressed in chapter of 4 of the thesis.

Finally, the modeling and control of a high step-up dc-dc converter will be addressed. This converter gives a very high voltage gain without using a transformer and maintains a low voltage stress across its diodes which allows the use of Schottkey rectifiers for reducing the reverse-recovery current problem. [23]. Also, this converter requires a lower voltage rated MOSFET switch which improves its efficiency by reducing the switching and conduction losses. Despite such various advantages offered by this converter, there is a scarcity of works addressing its regulation problem. To bridge this gap, a new adaptive current-mode control of a high step-up dc-dc converter will be presented in chapter 6.

1.2 Objectives of This Research Work

The objectives of this work can be classified into three categories as follows:

1. To propose a suitable approach to find the most suitable state variables for designing the current-mode controller for the high-order dc-dc boost converters.
2. To design the PWM-based SM controller for the high-order dc-dc boost converter using a reduced number of state variables for feedback purposes and to propose a suitable approach to select the most appropriate state variables to implement the SM controllers for the high-order boost-type dc-dc converters.
3. To improve the structure of the state-of-the-art controllers so that they become more robust to the load variation.

1.3 Contributions of the Thesis

The main contributions of this thesis can be summarized as follows:

- i. Two approaches, namely a state-space based approach and a frequency-response based approach have been proposed to find the most suitable inductor current for designing the current-mode controllers and SM controllers for some high-order dc-dc converters like the POEL converter, the 2-stage cascade boost converter and the hybrid high-order dc-dc boost converter.
- ii. A fixed-frequency, PWM-based SM controller has been designed for the quadratic boost converter such that the controller requires only one current sensor for its implementation while enjoying the advantages offered by the both fixed-frequency and double-integral approaches. Also, a suitable approach has been proposed to select the most appropriate state variables to design the hysteresis-modulation based and PWM-based SM controllers for the high-order boost-type dc-dc boost converters.
- iii. A new adaptive current-mode controller has been proposed to regulate the high step-up dc-dc converter which gives an improved load change response as compared to that obtained using the traditional current-mode controller.

1.4 Organization of This Thesis

The organization of the thesis is as follows:

Chapter 2 presents the advantages of the non-isolated high-order boost-type dc-dc converters over the traditional dc-dc boost converter as well as the isolated dc-dc boost converters. Also, the various types of high-order boost-type dc-dc

converters are given and their corresponding advantages are mentioned. Lastly, some previous research works on the modeling and control aspects of the high-order dc-dc boost converters are presented.

In chapter 3, a comparative study of two current-mode controllers (using the input and output inductor currents of the converter) is carried out for some high-order dc-dc converters like the POEL converter and the 2-stage cascade boost converter. The objective is to find the most suitable inductor current for feedback purposes. To obtain a better insight in the study, both state-space and frequency response approaches are employed.

In chapter 4, the use of a fixed-frequency, PWM-based SM controller for regulating the quadratic boost converter is investigated. The proposed controller requires only one current sensor for its implementation while enjoying the advantages offered by the both fixed-frequency and double-integral approaches. Also, two PWM-based SM controllers using the input and output inductor currents of the converter are separately designed to find the most suitable inductor current for the regulation of the converter. Apart from this, a systematic approach to find the most appropriate inductor current needed to implement the HM-based SM controller for a hybrid high-order dc-dc boost converter is presented.

In chapter 5, the modeling and design of a new adaptive current-mode controller for a high step-up dc-dc converter are addressed. A detailed modeling of the converter with parasitic elements is carried out to obtain its averaged state-space model. An adaptive current-mode controller with an optimized derivative of the estimate of the inverse of the approximate value of the load resistance is proposed

for the regulation of the converter. An approximate stability analysis is provided to gain an insight into the adaptive-controlled system. Finally, some experimental results are provided to compare the performance of the proposed adaptive current-mode controller with that of the existing current-mode controller.

In chapter 6, an overall assessment of the research work is discussed and areas for further research are proposed as well.

CHAPTER 2 LITERATURE REVIEW

In this chapter, the advantages of the non-isolated high-order boost-type dc-dc converters over the traditional dc-dc boost converter as well as the isolated dc-dc boost converters are given. Also, the various types of high-order boost-type dc-dc converters are discussed and their corresponding advantages are mentioned. Lastly, some previous research works done on the modeling and control aspects of the high-order dc-dc boost converters are presented.

2.1 Review of the State-of-the-art High-order Dc-Dc Boost Converter Topologies

Power electronic converters can be broadly classified into four categories, namely, (a) ac-dc converters (rectifiers), (b) dc-dc converters, (c) dc-ac converters (inverters), and (d) ac-ac converters (transformers) [11] . Among these, the dc-dc converters find their applications in different areas like automobiles such as electric vehicles and hybrid electric vehicles, power supply design for LCDs, aerospace and defense applications, etc. [24-27]. Depending on their conversion ratios, the dc-dc converters can be broadly categorized into 3 types, namely, (a) the buck-type dc-dc converter which can produce a dc output voltage less than its dc input voltage, (b) the boost-type dc-dc converter which produce an output voltage greater than their dc input voltage, and (c) the buck-boost-type dc-dc converter which can give an output voltage either less or greater than their dc input voltage [28]. This thesis deals with the boost-type of dc-dc converters.

Fig. 2.1 shows the circuit diagram of the conventional dc-dc boost converter [29]. The parameters R , L , C represent the load resistance, inductance, and capacitance of the converter respectively.

Fig. 2.1 Boost converter circuit [29].

The transfer function M of the converter is given by:

$$M = \frac{V_o}{V_s} = \frac{1}{1 - U}$$

where V_s is the input source voltage and V_o is the output voltage of the converter. Also, U represents the duty ratio of the square waveform applied to the switch S of the converter. It can be seen that, as the duty ratio U approaches unity, the gain of the converter approaches infinity. However, in practice, the maximum attainable duty ratio is often constrained by various factors like the finite turn-on and turn-off times of the switching devices [12]. Also, the gain and efficiency of the conventional dc-dc boost converter is limited due to the reverse recovery problem of the diode, switching losses, as well as by the equivalent series resistance of its capacitor and the parasitic resistance of its inductor [6]. However, dc-dc converters with wide conversion ratios are required in various industrial and commercial areas like telecommunication equipment, portable equipment, uninterrupted power supplies, embedded systems, and renewable energy

systems, etc. [2, 15-17]. For example, when employing the internet services using the telecom standard equipment, the 48-V of the dc battery plant is required to be boosted to about 380-V intermediate DC bus. Also, the high intensity discharge lamps used in automobiles require a start-up voltage of more than 100V using the battery's 12V output [2]. To solve these problems, there can be two possible solutions: (a) use the transformer-based isolated dc-dc converters to get the desired high voltage conversion ratio; or (b) use the high-order transformer-less dc-dc converters which use more number of inductors and capacitors to produce the high voltage gains at some lower values of the duty ratio.

However, there are some drawbacks associated with the transformer-based isolated dc-dc converters. For instance, in isolated converters, the leakage inductance effects may lead to considerable switching surges which can damage the switching devices and may also make the controller design difficult [12]. Also, the transformer leakage inductance could lead to a high voltage stress across switching devices and reduce the converter efficiency [6, 9]. Apart from this, the use of a transformers may lead to electromagnetic interference (EMI) generation problems, increases the switching losses and also increases the size, cost, and volume of the overall converter system [2, 9]. This makes transformer-less topologies a more interesting choice and to date, a number of such topologies have been reported in the relevant literature [2, 8-11]. A brief overview of some of the state-of-the-art non-isolated high-order dc-dc boost converters is provided next.

One of the ways to achieve a high conversion ratio is to connect ‘N’ boost converters in cascade using ‘N’ active switches without using a transformer [30-32]. However, even though the high gain is achieved using such cascade arrangement of the converters, the efficiency degrades as the number of stages in cascade increases. The converter losses are mainly attributed to the N switching devices. To overcome this problem, an interesting topology is proposed [11, 33]. It consists of N-conventional dc-dc boost converters connected in cascade using a single active switch. For N converters connected in cascade, it requires only one active switch and $2N-1$ diodes. As a result, a much higher efficiency is achieved.

In [2], some switched-capacitor/switched-inductor structures are proposed for getting the transformerless hybrid-type dc-dc boost converters. They mainly consist of the switching capacitor/inductor structures combined with the conventional converters like the boost, buck–boost, etc. This new class of converters offer several advantages such as lower energy in the magnetic elements which results in reduced weight, size and cost of the inductors, and thus of the overall power supply. Also, the use of these converters lead to the reduced conduction losses which consequently lead to a better efficiency [2].

In [9], the voltage multiplier technique has been applied to the classical dc-dc converters such as the conventional boost converter to achieve a high step-up voltage gain using a single active switch. A small resonant inductor ($\sim 1\mu\text{H}$ to $4\mu\text{H}$) is included in the voltage multiplier cell which offers the advantages like the operation of the power switch with zero-current-switching (ZCS) and reduction in the reverse recovery current of the diode. Such characteristics will

ultimately lead to the reduced converter commutation losses, allow the converter to operate with a high switching frequency and also increase the efficiency of the converter.

Apart from these converters, an ultra-step up dc-dc boost converter with a high static gain and reduced switch stress is proposed in [23]. This converter uses the hybrid switched-capacitor technique to achieve a high voltage transfer gain at the lower values of the duty cycle. The converter also enables the use of a switch having a low voltage rating which reduces the cost and the conduction losses of the converter. Also, the converter allows the use of Schottky rectifiers due to the reduced voltage stress across the diodes which further reduces the switching and conduction losses in the converter.

Accompanying the introduction of these different kinds of high-order dc-dc boost converters is the development of their corresponding controllers. The controllers are required to regulate the output voltage of the converters in the presence of some load and supply voltage variations as well as the circuit parameter uncertainties. However, a suitable mathematical model of the dc-dc converter is usually required to achieve the theoretical design of the controller as well as to perform the stability analysis of the circuit. In this thesis, a state-space averaging method of the modeling is employed. A brief overview of this modeling approach as well as the state-space averaged models of some of the state-of-the-art high-order dc-dc converters are given in the next section.

2.2 Modeling of High-order Boost-type Dc-Dc Converters

In this section, a brief overview of the averaged state-space modeling method is presented. The averaged state-space models of some of the state-of-the-art high-order boost-type dc-dc converters like the hybrid-type dc-dc boost converter and the 2-stage cascade boost converter are also derived.

2.2.1 Overview of the State-Space Averaging Method

State-space averaging method is one of the most widely used modeling technique for the pulse-width-modulation (PWM)-based dc-dc converters. It requires that the output filter corner frequency to be much smaller than the switching frequency. The following presents the detailed procedure to obtain the averaged state-space models of dc-dc converters operating in the continuous conduction mode [23, 34, 35].

Step 1:

Draw the circuit diagram for each sub-interval of the dc-dc converter. For a dc-dc converter which uses a single active switch, there are two-subintervals of the circuit. The interval '1' occurs when the switch 'S' is in the 'ON' state and the interval '2' is when the switch 'S' is in the 'OFF' state. Fig. 2.2(a) shows the circuit diagram of the conventional dc-dc boost converter and its operating modes.

(a)

(b)

(c)

Fig. 2.2 Conventional dc-dc boost converter and its operating modes: (a) Circuit diagram; (b) ‘Mode 1’ of the operation when switch ‘S’ is ON; and (c) ‘Mode 2’ of the operation when switch ‘S’ is OFF.

Step 2:

Find the state variables of the dc-dc converter. Usually the inductor currents and the capacitor voltages are used as the state variables to model the converter. Write the differential equations representing each sub-interval using the Kirchoff’s voltage and current laws (KVL and KCL respectively).

Applying KVL and KCL to Fig. 2.2(b), the following differential equations can be obtained:

$$\frac{di_L}{dt} = \frac{v_{in}}{L} \quad (2.1)$$

$$\frac{dv_o}{dt} = -\frac{v_o}{RC} \quad (2.2)$$

where i_L and v_o are the average current of inductor L and the average voltages of capacitor C respectively. Also v_{in} and R represent the input voltage and the nominal load resistance of the converter respectively.

Similarly, using KVL and KCL in Fig. 2.2(c) leads to:

$$\frac{di_L}{dt} = \frac{v_{in}}{L} - \frac{v_o}{L} \quad (2.3)$$

$$\frac{dv_o}{dt} = \frac{i_L}{C} - \frac{v_o}{RC} \quad (2.4)$$

Step 3:

The differential or state-space equations obtained for each sub-interval of the dc-dc converter can now be combined using the duty cycle as a weighting factor and the averaged state-space model of the converter can be obtained as follows:

If u denotes the duty ratio of the switch, combining (2.1) - (2.4), gives

$$\frac{di_L}{dt} = u \frac{v_{in}}{L} + (1-u) \left[\frac{v_{in}}{L} - \frac{v_o}{L} \right] = \frac{v_{in}}{L} - \frac{(1-u)v_o}{L} \quad (2.5)$$

$$\frac{dv_o}{dt} = u \left[-\frac{v_o}{RC} \right] + (1-u) \left[\frac{i_L}{C} - \frac{v_o}{RC} \right] = \frac{(1-u)i_L}{C} - \frac{v_o}{RC} \quad (2.6)$$

In matrix form, (2.5) – (2.6) can be written as:

$$\dot{x} = Ax + Bk \quad (2.7)$$

$$y = Cx + Dk \quad (2.8)$$

where $x = [i_L \ v_o]^T$ represents the state variables of the system, $k = E$ is the system input and $y = v_o$ is the system output. Also, matrices A , B , C , and D are given by:

$$A = \begin{bmatrix} 0 & -\frac{(1-u)}{L} \\ \frac{(1-u)}{C} & -\frac{1}{RC} \end{bmatrix}, B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}, C = [0 \quad 1] \text{ and } D = [0] \text{ respectively.}$$

In this way, the averaged state-space model of the dc-dc converter can be obtained. Note that the control variable is the duty ratio. The state-space models of some of the state-of-the-art high-order dc-dc converters are derived in the next section.

2.2.2 Averaged Model of the Hybrid-Boost Converter

Fig. 2.3(a) shows the circuit diagram of the hybrid-type dc-dc boost converter. As compared to the conventional boost converter, it has an additional step-up switching structure which mainly comprises of two capacitors (C_1, C_2) and two diodes (D_1, D_2). The load-side inductor L_2 is added to reduce the output current ripple content [2].

(a)

(b)

(c)

Fig. 2.3 Hybrid-type high-order dc-dc boost converter and its operating modes: (a) Circuit diagram; (b) 'Mode 1' of the operation when $u = 1$; and (c) 'Mode 2' of the operation when $u = 0$.

To simplify the circuit analysis and derive the state-space model of the converter, the following conditions are assumed:

1. The turn-on and turn-off transitions of all the diodes are synchronous with the switching transitions of the power MOSFET Switch 'S'.
2. The converter is operating in the continuous conduction mode (CCM).

-
-
3. The power MOSFET and diodes are treated as ideal with a negligible parasitic resistance.

The converter has two operating modes of conduction as described in the following [36], [37].

‘Mode 1’: During this mode of operation, the power switch ‘S’ is on and two diodes D_1, D_2 are reversed biased. The inductor L_1 is charged from the input source E and the capacitors C_1 and C_2 are discharged in series to supply the energy to the load. From Fig. 2.3(b), the following first-order differential equations can be obtained:

$$\frac{di_{L1}}{dt} = \frac{E}{L_1} \quad (2.9)$$

$$\frac{di_{L2}}{dt} = \frac{v_{C1}}{L_2} + \frac{v_{C2}}{L_2} - \frac{v_o}{L_2} \quad (2.10)$$

$$\frac{dv_{C1}}{dt} = -\frac{i_{L2}}{C_1} \quad (2.11)$$

$$\frac{dv_{C2}}{dt} = -\frac{i_{L2}}{C_2} \quad (2.12)$$

$$\frac{dv_o}{dt} = \frac{i_{L2}}{C_o} - \frac{v_o}{RC_o} \quad (2.13)$$

where $i_{L1}, i_{L2}, v_{C1}, v_{C2}, v_o$ and E are the average current of inductor L_1 , average current of inductor L_2 , average voltage of capacitor C_1 , average voltage of capacitor C_2 , average voltage of capacitor C_o , and the nominal input voltage of the converter respectively. Also R represent the nominal load resistance.

‘Mode 2’: During this mode of operation, the power switch ‘S’ is turned off and two diodes D_1, D_2 become forward biased to provide a path for the input inductor current i_{L1} to flow. The input energy and energy stored in the inductor thus get

transferred to the capacitors C_1 and C_2 as well as to the output capacitor and the load. From Fig. 2.3(c), the following differential equations can be obtained:

$$\frac{di_{L_1}}{dt} = -\frac{v_{C_1}}{L_1} + \frac{E}{L_1} \quad (2.14)$$

$$\frac{di_{L_2}}{dt} = \frac{v_{C_1}}{L_2} - \frac{v_o}{L_2} \quad (2.15)$$

$$\frac{dv_{C_1}}{dt} = \frac{i_{L_1}}{C_1+C_2} - \frac{i_{L_2}}{C_1+C_2} \quad (2.16)$$

$$\frac{dv_{C_2}}{dt} = \frac{i_{L_1}}{C_1+C_2} - \frac{i_{L_2}}{C_1+C_2} \quad (2.17)$$

$$\frac{dv_o}{dt} = \frac{1}{C_o} i_{L_2} - \frac{1}{RC_o} v_o \quad (2.18)$$

By applying the averaging concept and setting $C_1 = C_2 = C$ and $v_{C_1} = v_{C_2} = v_c$, the reduced-order averaged state-space model of the converter can be obtained as [38]:

$$\frac{di_{L_1}}{dt} = -\frac{(1-u)}{L_1} v_c + \frac{E}{L_1} \quad (2.19)$$

$$\frac{di_{L_2}}{dt} = \frac{1+u}{L_2} v_c - \frac{1}{L_2} v_o \quad (2.20)$$

$$\frac{dv_c}{dt} = \frac{(1-u)}{2C} i_{L_1} - \frac{1+u}{2C} i_{L_2} \quad (2.21)$$

$$\frac{dv_o}{dt} = \frac{1}{C_o} i_{L_2} - \frac{1}{RC_o} v_o \quad (2.22)$$

The symbol u denotes the control signal of the MOSFET switch ‘S’ which is given as $u \in [0,1]$. The equilibrium point of (2.19) – (2.22) is given by:

$$I_{L_1} = \frac{V_d^2}{RE}, \quad I_{L_2} = \frac{V_d}{R}, \quad V_C = \frac{V_d+E}{2}, \quad U = \frac{V_d-E}{V_d+E} \quad (2.23)$$

where V_d is the desired value of the output voltage v_o and I_{L_1} , I_{L_2} , V_C , and U represents the steady-state values of the i_{L_1} , i_{L_2} , v_c , and u respectively.

2.2.3 Averaged Model of the 2-stage Cascade Boost Converter

Fig. 2.4 shows the circuit diagram of the 2-stage cascade boost converter. It has two operating modes of conduction. When switch ‘S’ is on, the diode D_1 is conducting which provides a path for the inductor current i_{L_1} to flow and diodes D_2 and D_3 are in off state. The energy is stored in the inductors L_1 and L_2 in the form of a magnetic field.

Fig. 2.4 Circuit diagram of the 2-stage cascade boost converter.

The state-space equations representing the ‘ON’ state of the circuit can be obtained as:

$$\frac{di_{L_1}}{dt} = \frac{E}{L_1} \quad (2.24)$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2} v_{C_1} \quad (2.25)$$

$$\frac{dv_{C_1}}{dt} = -\frac{1}{C_1} i_{L_2} \quad (2.26)$$

$$\frac{dv_{C_2}}{dt} = -\frac{1}{RC_2} v_{C_2} \quad (2.27)$$

where i_{L_1} , i_{L_2} , v_{C_1} , $v_{C_2} = v_o$ and R are the average current of inductor L_1 , average current of inductor L_2 , average voltage of capacitor C_1 , average voltage of capacitor C_2 , and load resistance of the converter respectively.

When switch ‘S’ is OFF, the diode D_1 is off and diodes D_2 and D_3 are now conducting to enable the flow of the inductor current through them in order to charge the output capacitor C_2 . The state-space equations representing the ‘OFF’ state of the circuit can be obtained as:

$$\frac{di_{L_1}}{dt} = -\frac{1}{L_1}v_{C_1} + \frac{E}{L_1} \quad (2.28)$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2}v_{C_1} - \frac{1}{L_2}v_{C_2} \quad (2.29)$$

$$\frac{dv_{C_1}}{dt} = \frac{1}{C_1}i_{L_1} - \frac{1}{C_1}i_{L_2} \quad (2.30)$$

$$\frac{dv_{C_2}}{dt} = \frac{1}{C_2}i_{L_2} - \frac{1}{RC_2}v_{C_2} \quad (2.31)$$

By applying the averaging concept, the state-space model of the converter can be obtained as [39]:

$$\frac{di_{L_1}}{dt} = -\frac{(1-u)}{L_1}v_{C_1} + \frac{E}{L_1} \quad (2.32)$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2}v_{C_1} - \frac{(1-u)}{L_2}v_{C_2} \quad (2.33)$$

$$\frac{dv_{C_1}}{dt} = \frac{(1-u)}{C_1}i_{L_1} - \frac{1}{C_1}i_{L_2} \quad (2.34)$$

$$\frac{dv_{C_2}}{dt} = \frac{(1-u)}{C_2}i_{L_2} - \frac{1}{RC_2}v_{C_2} \quad (2.35)$$

Here, the symbol u denotes the control signal of the MOSFET switch which is given as $u \in [0, 1]$.

2.3 State-of-the-art Controllers for High-Order Dc-Dc Boost Converters

In this subsection, a brief overview of some of the state-of-the-art controllers used for the regulation of high-order boost-type dc-dc converters are given along with their advantages and drawbacks.

2.3.1 Current-Mode Controller

The current-mode control is a widely used regulation method for the dc-dc converters and it offers several advantages such as over-current protection and a faster transient response [12]. The equation of the state-of-the-art current-mode controller is given by:

$$u = U - K_{Pt}(i_L - I_{La}) - K_{It} \int_0^t (v_o(\tau) - V_d) d\tau \quad (2.36)$$

where u is the instantaneous value of the duty ratio, U is the steady-state value of the duty ratio, $K_{Pt}(> 0)$ and $K_{It}(> 0)$ are the controller gains, i_L is the inductor current of the converter and I_{La} is the reference inductor current. This is just one form of the current-mode control in which the inductor current and the output voltage are used for feedback purposes (see [38, 40-42]). The integral action is mainly added to reduce the steady-state error in the output voltage during the actual hardware implementation of the controlled system [38]. In (2.36), the reference inductor current is usually the function of the load resistance of the converter (see (2.23)). Thus, for implementation purposes, I_{La} is calculated using the nominal value of the load resistance R . However, in a practical system, R may change. Also, in applications where R is unknown, (2.36) may not be used. This is the main drawback of the state-of-the-art current-mode controller. To

overcome this problem, an adaptive current-mode controller is proposed in Chapter 6.

2.3.2 *Sliding-Mode Controllers*

Sliding-mode (SM) control is one of the well-established control strategies for the variable structure systems due to its excellent stability properties and ease of implementation [18]. Since the dc-dc converters are characterized by switching, they are inherently variable-structured systems. The pulse-width-modulation (PWM) and the hysteresis-modulation (HM) are two widely used modulation techniques for implementing the SM controller for dc-dc converters. Some earlier works on developing SM controllers for the high-order dc-dc boost converters are based on the hysteresis-based modulation technique as it offers several advantages such as ease of implementation and reduced risk of saturation while operating at high values of the duty cycle [5, 43]

In the traditional HM-based SM control, the expression of the sliding-surface is given by [5]:

$$s(x) = i_L - I_{RT}(t) \quad (2.37)$$

where i_L is the inductor current of the converter and $I_{RT}(t)$ is the reference inductor current trajectory which is generated using a conventional proportional-integral (PI) controller given by:

$$I_{RT}(t) = K_{P_t}(V_d - v_o(t)) + K_{I_t} \int (V_d - v_o(\tau)) d\tau \quad (2.38)$$

Here, K_{P_t} and K_{I_t} are the gains of the PI controller, v_o is the output voltage of the converter and V_d is the reference value of the output voltage. The switching function u_t for the SM controller implementation can then be written as:

$$u_t = \begin{cases} 0 & \text{when } s(x) > 0 \\ 1 & \text{when } s(x) < 0 \end{cases} \quad (2.39)$$

It should be noted that in (2.38), the integral action is mainly added to reduce the steady-state output voltage error in the actual hardware implementation of the controlled system. Setting the derivative of the sliding-surface $s(x)$ to zero, the equivalent control signal can be obtained and it is used in the stability analysis of the SM controlled system [5]. However, the implementation of the HM-based SM controller demands a variable switching frequency which can cause excessive power losses like inductor and switching losses as well as electromagnetic-interference (EMI) generation [19].

To solve this problem, the fixed frequency, PWM-based SM controllers of some high-order dc-dc converters have been proposed in the recent literature [19-21]. These controllers are based on the principle that, as the switching frequency approaches infinity, the averaged dynamics of the SM controlled system can be treated to be equivalent to the averaged dynamics of the PWM system [19]. This allows the equivalent control signal obtained under the SM control operation to be used as the duty cycle control input of the converter. In the state-of-the-art PWM-based SM control, the sliding-surface is of the form given by [44]:

$$s_t = i_L - I_{ref}(t) \quad (2.40)$$

where i_L is the inductor current of the converter and I_{ref} is the reference inductor current trajectory given by:

$$I_{ref}(t) = -mv_o(t) - n \int (v_o(t) - V_d)dt - p \iint (v_o(t) - V_d)dt dt \quad (2.41)$$

Here, m , n , p are positive coefficients and $(v_o(t) - V_d)$ is the output voltage error. A double integral term is used to reduce the output steady-state error to a negligible value. This is usually required for the PWM-based controllers because if only a single integral term is used, it is not explicitly reflected in the control signal used for the controller implementation [20, 21]. Again, setting the derivative of the sliding-surface (2.40) to zero, an equivalent control signal can be obtained which is used for both the stability analysis of the SM controlled system as well as the PWM-based implementation of the controller [20, 21].

2.3.3 The Output Feedback Controller

Recently, some passivity-based controllers (PBCs) have been applied to regulate the output voltage of the conventional dc-dc boost converter [45-47]. Among them, the simplified parallel-damped passivity-based controller (PD-PBC) proposed in [45] has a simple architecture and the corresponding control law can also be extended to regulate the high-order dc-dc converters [39, 48, 49]. The main advantage of this controller is that it eliminates the need of the inductor current sensor for its implementation. The structure of the output feedback control of the traditional dc-dc boost converter is given by [45]:

$$u = \frac{x_{2d} - E}{x_{2d}} \quad (2.42)$$

where u is the control signal of the converter, E is the input voltage of the converter and x_{2d} is the function such that,

$$\dot{x}_{2d} = -\frac{K_1+K_2}{c}x_{2d} + \frac{K_2}{c}v_o + \frac{K_1}{c}V_d \quad (2.43)$$

Here, v_o is the output voltage of the converter, V_d is the desired reference output voltage, and K_1 and K_2 are the controller gains. Additional proportional and integral actions acting on the output voltage error can also be added to the control signal (2.42) to get an improved transient and steady-state response of the controlled converter system [39, 48]. The modified control signal can then be written as:

$$u = \frac{x_{2d} - E - K_p(v_o - V_d) - K_I \int (v_o(t) - V_d)dt}{x_{2d}} \quad (2.44)$$

where K_p and K_I are the proportional and integral gains of the converter respectively.

This far, these are some of the state-of-the-art controllers used for the regulation of the high-order boost-type dc-dc converters. In chapters 3-5, some design issues as well as the improved structures of the controllers of the high-order boost-type dc-dc converters are discussed.

2.4 Summary

In this chapter, a brief overview of some of the state-of-the-art non-isolated high-order boost-type dc-dc converters is provided. Their advantages over the conventional boost converter and the transformer-based dc-dc converters are given and the specific advantages of various topologies are discussed. Also, a

detailed procedure to obtain the averaged state-space model of these converters is described and the averaged models of the hybrid-boost converter and the 2-stage cascade boost converter are derived. Moreover, a brief overview of some of the state-of-the-art controllers used for the regulation of high-order boost-type dc-dc converters is provided.

In chapter 3, various design issues of the current mode controlled high-order boost-type dc-dc converters are discussed.

CHAPTER 3 COMPARATIVE STUDY OF FIXED CURRENT-MODE CONTROLLERS FOR HIGH-ORDER DC- DC CONVERTERS

3.1 Motivation

Current-mode control is a widely used regulation method for dc-dc converters considering its inherent characteristics such as over-current protection and fast transient response [50]. Even though the current-mode control has been successfully applied in the regulation of many high-order boost-type dc-dc converters in the past [12-17], there are certain issues which are not extensively addressed in these works. In contrast to the conventional dc-dc boost converter wherein there is only one inductor current available for feedback purposes, the newly invented higher-order converters present certain challenges. For example, in an N-stage cascade boost converter [12], there are ‘2N’ number of state variables which can be used for feedback purposes (N-inductor currents and N-capacitor voltages). Firstly, as there are now more variables to consider, what are the most suitable state variables for feedback purposes? Is there any condition under which current-mode control using the input inductor current fails to work? If there is, then how do we overcome it? All these questions demand further research. Some works in these aspects have been done recently for the high-order boost converters like the fifth-order boost converter [38].

In this chapter, some detailed comparative studies of two current-mode controllers (designed using the input and output inductor currents of the

converter) are presented for some high-order dc-dc converters like the positive output elementary Luo (POEL) converter and the 2-stage cascade boost converter. To obtain a better insight in these studies, both state-space and frequency response approaches are used. It has been shown that the current-mode controller using the input inductor current is most suitable for regulating the POEL converter with the given set of circuit parameter values. Also, for the 2-stage cascade boost converter with a given set of circuit parameter values, the output inductor current is found to be more suitable for feedback purposes. The conclusions are borne out by theoretical as well as some experimental results.

3.2 A Comparative Study of Current-mode Controllers for the Positive Output Elementary Luo (POEL) Converter

Fig. 3.1 shows the circuit diagram of the POEL converter. This converter uses a single active switch and it has two operating modes of conduction. In ‘Mode 1’ of conduction, the power switch ‘Sw’ is on and in ‘Mode 2’ of conduction, the power switch ‘Sw’ is off.

Fig. 3.1 Circuit diagram of the POEL converter.

The averaged state-space model of this converter operating in continuous conduction mode is described as follows [51, 52]:

$$\frac{di_{L_1}}{dt} = -\frac{1-u}{L_1}v_{C_1} + \frac{uE}{L_1} \quad (3.1)$$

$$\frac{dv_{C_1}}{dt} = \frac{1-u}{C_1}i_{L_1} - \frac{u}{C_1}i_{L_2} \quad (3.2)$$

$$\frac{di_{L_2}}{dt} = \frac{u}{L_2}v_{C_1} - \frac{1}{L_2}v_{C_2} + \frac{uE}{L_2} \quad (3.3)$$

$$\frac{dv_o}{dt} = \frac{1}{C_2}i_{L_2} - \frac{1}{RC_2}v_o \quad (3.4)$$

where i_{L_1} , i_{L_2} , v_{C_1} and $v_{C_2} = v_o$ are the average current of inductor L_1 , average current of inductor L_2 , average voltage of capacitor C_1 and average voltage of capacitor C_2 respectively. The scalar u denotes the duty ratio, where $0 < u < 1$.

From (3.1) – (3.4), the following equilibrium values are obtained:

$$I_{L_1} = \frac{V_o^2}{RE}, \quad I_{L_2} = \frac{V_o}{R}, \quad V_{C_1} = V_{C_2} = V_o, \quad \frac{V_o}{E} = \frac{K}{1-K}, \quad U = \frac{V_o}{E+V_o} \quad (3.5)$$

where I_{L_1} , I_{L_2} , V_{C_1} , $V_{C_2} = V_o$ and U are the equilibrium values of i_{L_1} , i_{L_2} , v_{C_1} , v_o and u , respectively. If the desired value of v_o is V_d , then

$$I_{L_1} = \frac{V_d^2}{RE}, \quad I_{L_2} = \frac{V_d}{R}, \quad V_{C_1} = V_d, \quad \frac{V_d}{E} = \frac{K}{1-U}, \quad U = \frac{V_d}{E+V_d} \quad (3.6)$$

The problem at hand is to find a suitable current mode controller to regulate the output voltage of the POEL converter in the presence of an uncertain load.

3.2.1 Comparison Using State-Space Approach

In this section, the two current mode controllers designed using the input and output inductor currents of the converter are compared using the state-space

approach. The controller using the input inductor current will be studied first, followed by the controller using the output inductor current.

The current mode controller using the input inductor current is described as:

$$u = U - K_{P1}(i_{L_1} - I_{L_1}) - K_{I1} \int (v_o(\tau) - V_d) d\tau \quad (3.7)$$

where $K_{P1}, K_{I1} > 0$. Define the errors as follows:

$$e_1 = i_{L_1} - \frac{V_d^2}{RE}, \quad e_2 = v_{C_1} - V_d, \quad e_3 = i_{L_2} - \frac{V_d}{R}, \quad e_4 = v_o - V_d \quad (3.8)$$

Substituting (3.7) – (3.8) into (3.1) – (3.4) and using (3.6) yield the following set of equations:

$$\dot{e}_1 = -\frac{1}{L_1} \left(\frac{E}{V_d+E} + K_{P1}e_1 + \sigma \right) (e_2 + V_d) + \frac{E}{L_1} \left(\frac{V_d}{V_d+E} - K_{P1}e_1 - \sigma \right) \quad (3.9)$$

$$\dot{e}_2 = \frac{1}{C_1} \left(\frac{E}{V_d+E} + K_{P1}e_1 + \sigma \right) \left(e_1 + \frac{V_d^2}{RE} \right) - \frac{1}{C_1} \left(\frac{V_d}{V_d+E} - K_{P1}e_1 - \sigma \right) \left(e_3 + \frac{V_d}{R} \right) \quad (3.10)$$

$$\dot{e}_3 = \frac{1}{L_2} \left(\frac{V_d}{V_d+E} - K_{P1}e_1 - \sigma \right) (e_2 + V_d + E) - \frac{1}{L_2} (e_4 + V_d) \quad (3.11)$$

$$\dot{e}_4 = \frac{1}{C_2} \left(e_3 + \frac{V_d}{R} \right) - \frac{1}{RC_2} (e_4 + V_d) \quad (3.12)$$

$$\dot{\sigma} = K_{I1}e_4 \quad (3.13)$$

The unique equilibrium point of (3.9) – (3.13) is given by:

$$(e_{1\infty}, e_{2\infty}, e_{e\infty}, e_{4\infty}, \sigma_{\infty}) = (0, 0, 0, 0, 0) \quad (3.14)$$

Linearizing (3.9) – (3.13) about the equilibrium point gives the following system:

$$\dot{z} = Mz \quad (3.15)$$

where $z = [z_1 \ z_2 \ z_3 \ z_4 \ z_5]^T$, $z_1 = e_1 - e_{1\infty}$, $z_2 = e_2 - e_{2\infty}$, $z_3 = e_3 - e_{3\infty}$, $z_4 = e_4 - e_{4\infty}$, $z_5 = \sigma - \sigma_\infty$ and matrix M is given by:

$M =$

$$\begin{bmatrix} -\frac{K_{P1}(V_d+E)}{L_1} & -\frac{E}{L_1(V_d+E)} & 0 & 0 & -\frac{V_d+E}{L_1} \\ \frac{E}{C_1(V_d+E)} + \frac{K_{P1}V_d}{RC_1} \left(\frac{V_d}{E} + 1 \right) & 0 & -\frac{V_d}{C_1(V_d+E)} & 0 & \frac{V_d}{RC_1} \left(\frac{V_d}{E} + 1 \right) \\ -\frac{K_{P1}(V_d+E)}{L_2} & \frac{V_d}{L_2(V_d+E)} & 0 & -\frac{1}{L_2} & -\frac{(V_d+E)}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{RC_2} & 0 \\ 0 & 0 & 0 & K_{I1} & 0 \end{bmatrix}$$

The system (3.15) will be stable if all the eigenvalues of M , i.e., the roots of $|sI - M| = 0$, where s is a complex variable, lie in the open left-half of the complex s -plane. The root locus method can be used to analyze the system stability. This is illustrated as follows.

Consider the POEL converter with following circuit parameter values:

$$E = 12V, V_d = 18V, L_1 = 1mH, L_2 = 10mH, C_1 = 47\mu F, C_2 = 100\mu F, R = 22 \Omega$$

The characteristic polynomial $|sI - M|$ is thus given by:

$$\begin{aligned} m(s) = |sI - M| = & s^5 + (30000K_{P1} + 454.54) s^4 + (3.1 \times 10^7 K_{P1} + \\ & 5.16 \times 10^6) s^3 + (7.62 \times 10^{10} K_{P1} + 3 \times 10^7 K_{I1} + 1.89 \times 10^9) s^2 + \\ & (3.48 \times 10^{13} K_{P1} - 2.61 \times 10^{10} K_{I1} + 3.4 \times 10^{12}) s + 2.55 \times 10^{14} K_{I1} \end{aligned}$$

Fig. 3.2(a) shows the root locus plot of $m(s)$ for $K_{P1} = 0.08$, $0 < K_{I1} < 12$.

The arrows show the poles are moving from $K_{I1} = 0$. The system is stable for all values of K_{I1} in this range.

Fig. 3.2 Root locus plot for $K_{P1} = 0.08$, $0 < K_{I1} < 12$: (a) using input inductor current feedback; and (b) using output inductor current feedback.

Next, the implementation of the controller using the output inductor current is investigated. The controller using the output inductor current assumes the following form:

$$u = U - K_{P2}(i_{L2} - I_{L2}) - K_{I2} \int (v_0(\tau) - V_d) d\tau \quad (3.16)$$

where $K_{P2}, K_{I2} > 0$. Substituting (3.8) and (3.16) into (3.1) - (3.4) and using (3.6) yields the following error dynamics:

$$\dot{e}_1 = -\frac{1}{L_1} \left(\frac{E}{V_d + E} + K_{P2}e_3 + \sigma \right) (e_2 + V_d) + \frac{E}{L_1} \left(\frac{V_d}{V_d + E} - K_{P2}e_3 - \sigma \right) \quad (3.17)$$

$$\dot{e}_2 = \frac{1}{C_1} \left(\frac{E}{V_d + E} + K_{P2}e_3 + \sigma \right) \left(e_1 + \frac{V_d^2}{RE} \right) - \frac{1}{C_1} \left(\frac{V_d}{V_d + E} - K_{P2}e_3 - \sigma \right) \left(e_3 + \frac{V_d}{R} \right) \quad (3.18)$$

$$\dot{e}_3 = \frac{1}{L_2} \left(\frac{V_d}{V_d + E} - K_{P2}e_3 - \sigma \right) (e_2 + V_d + E) - \frac{1}{L_2} (e_4 + V_d) \quad (3.19)$$

$$\dot{e}_4 = \frac{1}{C_2} \left(e_3 + \frac{V_d}{R} \right) - \frac{1}{RC_2} (e_4 + V_d) \quad (3.20)$$

$$\dot{\sigma} = K_{I2}e_4 \quad (3.21)$$

The unique equilibrium point of (3.17) – (3.21) is $(e_{1\infty}, e_{2\infty}, e_{e\infty}, e_{4\infty}, \sigma_{\infty}) = (0, 0, 0, 0, 0)$. Linearization of (3.17) – (3.21) about this equilibrium point yields the system of the form:

$$\dot{z} = Nz \quad (3.22)$$

where

$$N =$$

$$\begin{bmatrix} 0 & -\frac{E}{L_1(V_d+E)} & -\frac{K_{P2}(V_d+E)}{L_1} & 0 & -\frac{V_d+E}{L_1} \\ \frac{E}{C_1(V_d+E)} & 0 & -\frac{V_d}{C_1(V_d+E)} + \frac{K_{P2}V_d}{RC_1}\left(\frac{V_d}{E} + 1\right) & 0 & \frac{V_d}{RC_1}\left(\frac{V_d}{E} + 1\right) \\ 0 & \frac{V_d}{L_2(V_d+E)} & -\frac{K_{P2}(V_d+E)}{L_2} & -\frac{1}{L_2} & -\frac{(V_d+E)}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{RC_2} & 0 \\ 0 & 0 & 0 & K_{I2} & 0 \end{bmatrix}$$

For the same set of circuit parameter values that were used for the controller based on the input inductor current, the characteristic polynomial $|sI - N|$ is given by:

$$\begin{aligned} n(s) = |sI - N| = & s^5 + (3000K_{P2} + 454.54)s^4 + (5.17 \times 10^6 - \\ & 1.24 \times 10^6 K_{P2})s^3 + (2.43 \times 10^{10} K_{P2} + 3 \times 10^7 K_{I2} + 1.89 \times \\ & 10^9)s^2 + (1.16 \times 10^{13} K_{P2} - 2.61 \times 10^{10} K_{I2} + 3.40 \times 10^{12})s + 2.55 \times \\ & 10^{14} K_{I2} \end{aligned}$$

Fig. 3.2(b) shows the root locus plot of $n(s)$ for $K_{P2} = 0.08$, $0 < K_{I2} < 12$. It is seen that even for small values of K_{I2} , there are poles in the right-half complex plane, i.e., the system is unstable. As such, the controller designed using the input inductor current should be used. This confirms why the input inductor current is the preferred variable for feedback purposes.

3.2.2 Comparison Using Frequency-Response Approach

In previous sub-section, the current-mode controllers using the input and output inductor currents of the converter were studied using the state-space approach. In this sub-section, the controllers are studied using another approach, the frequency-response approach, to help give a better insight into the comparative study. To facilitate the comparative study using this approach, a small signal model is first derived for the POEL converter by linearizing (3.1) – (3.4) around the equilibrium values (3.6) and then introducing small ac perturbations. The model obtained is described as:

$$\dot{\tilde{x}} = a\tilde{x} + b\tilde{k} + m\tilde{u} \quad (3.23a)$$

$$\tilde{y} = c\tilde{x} \quad (3.23b)$$

where $\tilde{u}$ represents the small signal variations in the duty cycle u i.e. $u = U + \tilde{u}$ and $\tilde{x}$ represents the small signal variations in the state variables such that $\tilde{x} = [\tilde{i}_{L1} \ \tilde{v}_{c1} \ \tilde{i}_{L2} \ \tilde{v}_{c2}]$ and $\tilde{k} = [\tilde{v}_{in}]$. Also, matrices a, b, m and c are given by:

$$a = \begin{bmatrix} 0 & -\frac{U'}{L_1} & 0 & 0 \\ \frac{U'}{C_1} & 0 & -\frac{U}{C_1} & 0 \\ 0 & \frac{U}{L_2} & 0 & -\frac{1}{L_2} \\ 0 & 0 & \frac{1}{C_2} & -\frac{1}{RC_2} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{U}{L_1} \\ 0 \\ \frac{U}{L_2} \\ 0 \end{bmatrix}, \quad m = \begin{bmatrix} \frac{V_{C1} + V_{in}}{L_1} \\ -\frac{I_{L1} + I_{L2}}{C_1} \\ \frac{V_{C1} + V_{in}}{L_2} \\ 0 \end{bmatrix},$$

$$c = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where $U' = 1 - U$ and V_{C1} , I_{L1} and I_{L2} represent the steady-state value of the capacitor voltage v_{C1} and the steady-state values of the currents flowing through inductors L_1 and L_2 respectively.

The required transfer functions can now be easily obtained from the state-space model (3.23) and are given as follows:

$$G_1(s) = \frac{\tilde{i}_{L1}(s)}{\tilde{u}(s)} = \frac{q_1(s)}{p(s)} = \frac{a_0 s^3 + a_1 s^2 + a_2 s + a_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.24)$$

$$G_2(s) = \frac{\tilde{i}_{L2}(s)}{\tilde{u}(s)} = \frac{q_1'(s)}{p(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.25)$$

$$G_3(s) = \frac{\tilde{v}_0(s)}{\tilde{u}(s)} = \frac{q_2(s)}{p(s)} = \frac{c_1 s^2 + c_2 s + c_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.26)$$

where the small ac perturbations are represented by “ $\sim$ ” and

$$\begin{aligned} a_0 &= \frac{(E+V_d)}{L_1}, \\ a_1 &= \frac{(1-U)(I_{L1}+I_{L2})}{C_1 L_1} + \frac{(E+V_d)}{C_2 L_1 R}, \\ a_2 &= \frac{(1-U)(I_{L1}+I_{L2})}{C_1 C_2 L_1 R} + \frac{(E+V_d)}{L_1 L_2} \left(\frac{1}{C_2} + \frac{U}{C_1} \right), \\ a_3 &= \frac{(1-U)(I_{L1}+I_{L2})}{C_1 C_2 L_1 L_2} + \frac{U(E+V_d)}{C_1 C_2 L_1 L_2 R}, \\ b_0 &= \frac{(E+V_d)}{L_2}, \quad b_1 = -\frac{U(I_{L1}+I_{L2})}{C_1 L_2} + \frac{(E+V_d)}{C_2 L_2 R}, \\ b_2 &= -\frac{U(I_{L1}+I_{L2})}{C_1 C_2 L_2 R} + \frac{(E+V_d)(1-U)}{C_1 L_1 L_2}, \\ b_3 &= \frac{(E+V_d)(1-U)}{C_1 C_2 L_1 L_2 R}, \\ c_1 &= \frac{(E+V_d)}{C_2 L_2}, \\ c_2 &= -\frac{U(I_{L1}+I_{L2})}{C_1 C_2 L_2}, \end{aligned} \quad (3.27)$$

$$c_3 = \frac{(E+V_d)(1-U)}{C_1 C_2 L_1 L_2} ,$$

$$d_0 = \frac{1}{C_2 R} ,$$

$$d_1 = \frac{(1-U)^2}{C_1 L_1} + \frac{1}{L_2} \left(\frac{1}{C_2} + \frac{U^2}{C_1} \right),$$

$$d_2 = \frac{(1-U)^2}{C_1 C_2 L_1 R} + \frac{U^2}{C_1 C_2 L_2 R} .$$

$$d_3 = \frac{(1-U)^2}{C_1 C_2 L_1 L_2}$$

Using the same set of circuit parameter values that were used in Section 3.2.1, the following transfer functions are obtained:

$$G_1(s) = \frac{\tilde{i}_{L_1}(s)}{\tilde{u}(s)} = \frac{q_1(s)}{p(s)} = \frac{3 \times 10^4 s^3 + 3.104 \times 10^7 s^2 + 7.621 \times 10^{10} s + 3.482 \times 10^{13}}{s^4 + 454.5 s^3 + 5.17 \times 10^6 s^2 + 1.896 \times 10^9 s + 3.4042 \times 10^{12}} \quad (3.28)$$

$$G_2(s) = \frac{\tilde{i}_{L_2}(s)}{\tilde{u}(s)} = \frac{q_1'(s)}{p(s)} = \frac{3000 s^3 - 1.248 \times 10^6 s^2 + 2.434 \times 10^{10} s + 1.161 \times 10^{13}}{s^4 + 454.5 s^3 + 5.17 \times 10^6 s^2 + 1.896 \times 10^9 s + 3.404 \times 10^{12}} \quad (3.29)$$

$$G_3(s) = \frac{\tilde{v}_0(s)}{\tilde{u}(s)} = \frac{q_2(s)}{p(s)} = \frac{3 \times 10^7 s^2 - 2.611 \times 10^{10} s + 2.553 \times 10^{14}}{s^4 + 454.5 s^3 + 5.17 \times 10^6 s^2 + 1.896 \times 10^9 s + 3.404 \times 10^{12}} \quad (3.30)$$

It is seen that the transfer function $G_3(s)$ has non-minimum phase characteristics due to the presence of zeros in the right-half complex plane (negative coefficient in the numerator). Thus, it becomes very difficult to design the controller for the regulation of the output voltage using a single voltage loop since the resonance peak presents abrupt changes in the phase (see Fig. 3.3(a)).

(a)

(b)

Fig. 3.3 Bode plots using input inductor current feedback: (a) Bode plots of $G_3(s)$ (dotted) and $G_{ic}(s)$ (Solid); and (b) Bode plot of loop gain $G_L(s)$.

The use of a two-loop control scheme thus becomes necessary in order to minimize the effects of the resonance peak and to increase the bandwidth [15].

This control scheme is shown in Fig. 3.4.

Fig. 3.4 A two-loop control scheme.

As mentioned earlier, the current loop is mainly designed to minimize the resonance peak present in $G_3(s) = \tilde{v}_0(s)/\tilde{u}(s)$. After introducing the current controller $G_c(s) = K_P$ in the controller scheme shown in Fig. 3.4, the transfer function $G_{ic}(s) = \tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ is given by (see Fig. 3.4)

$$G_{ic}(s) = \frac{\tilde{v}_0(s)}{\tilde{i}_{L1ref}(s)} = \frac{\frac{1}{V_p} * G_c(s) * G_1(s)}{1 + \frac{1}{V_p} * G_c(s) * G_1(s) * N} * \frac{G_3(s)}{G_1(s)} \quad (3.31)$$

From (3.24) and (3.26) and Fig. 3.4, $G_{ic}(s)$ can be obtained as:

$$G_{ic}(s) = \frac{\tilde{v}_0(s)}{\tilde{i}_{L1ref}(s)} = \frac{1}{V_p} \frac{K_P q_2(s)}{p(s) + K_P (N/V_p) q_1(s)} \quad (3.32)$$

Here, $1/V_p$ is the transfer function of the PWM modulation circuit such that V_p represents the peak value of the triangular waveform used to generate the PWM waveform for regulating the dc-dc converter. The corresponding characteristic equation is given by:

$$p(s) + K_P (N/V_p) q_1(s) = 0 \quad (3.33)$$

The current loop controller is designed to localize a low-frequency dominant pole

of the characteristic polynomial at a typical value of $s_{1f} = -3/(RC_2)$. Thus, the required value of N can now be obtained using:

$$N = \left. \frac{p(s)V_p}{K_P q_1(s)} \right|_{s=-\frac{3}{RC_2}} \quad (3.34)$$

For $K_P = 0.055$ and $V_p = 1.5 \text{ V}$, the value of N obtained is 4. The resulting transfer function $G_{ic}(s) = \tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ is given by:

$$G_{ic}(s) = \frac{\tilde{v}_0(s)}{\tilde{i}_{L1ref}(s)} = \frac{1.1 \times 10^6 (s^2 - 870.4s + 8.51 \times 10^6)}{(s+1339)(s+2526)(s^2 + 989.7s + 2.517 \times 10^6)} \quad (3.35)$$

The Bode plots of $G_3(s)$ and $G_{ic}(s)$ are shown in Fig. 3.3(a). It should be noted that in the design the controller for the voltage loop (see Fig. 3.4), $G_{ic}(s)$ is nothing but the open-loop plant. Thus, when a current loop using the input inductor current for feedback purposes is introduced, a resonance peak in the low frequency region is not present in the Bode plot of $\tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$. As such, $\tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ can be suitably used as an open-loop transfer function for the design of the voltage loop controller. This simplifies the design of the voltage loop.

Next, the conventional PI controller for the voltage loop is considered:

$$G_v(s) = K_{pv} + \frac{K_{Iv}}{s} \quad (3.36)$$

It is designed with an objective of getting a high gain in the low frequency region to improve the steady-state error performance and to get an overall positive phase margin. For $K_{pv} = 0.09$, $K_{Iv} = 300$ and a feedback gain of $K_h = 0.5$, the PI controller is given by:

$$G_v(s) = \frac{0.09s + 300}{s} \quad (3.37)$$

Here, K_{pv} is designed together with the voltage divider network of gain K_h and the value of $K_{pv}K_h$ is chosen in such a way that the resonance peak in the high frequency region is below 0 dB. The Bode plot of the resulting loop gain $G_L(s)$ (PI controller, voltage divider network and $G_{ic}(s)$) is shown in Fig. 3.3(b). The phase margin achieved is about 77° .

Next, the current-mode controller is designed using the output inductor current for feedback purposes. As in the case of input inductor current feedback, the value of K_p is fixed at 0.055 and then the value of N is obtained to localize the low-frequency dominant pole of the characteristic polynomial at a typical value of $s_{1f} = -3/(RC_2)$. Thus, N can be obtained using (3.34) as

$$N = \left. \frac{p(s)V_p}{K_p q_1'(s)} \right|_{s=-\frac{3}{RC_2}} \quad (3.38)$$

For $K_p = 0.055$ and $V_p = 1.5$ V, the value of N obtained is 11.

Now, the resulting transfer function $G_{ic1}(s) = \tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$ in the case of the output inductor current feedback is given as:

$$\begin{aligned} G_{ic1}(s) &= \frac{\tilde{v}_0(s)}{\tilde{i}_{L2ref}(s)} = \frac{\frac{1}{V_p} * G_c(s) * G_2(s)}{1 + \frac{1}{V_p} * G_c(s) * G_2(s) * N} * \frac{G_3(s)}{G_2(s)} = \frac{1}{V_p} \frac{K_p q_2(s)}{p(s) + K_p(N/V_p)q_1'(s)} \\ &= \frac{1.1 \times 10^6 (s^2 - 870.4s + 8.511 \times 10^6)}{(s+1349)(s+1152)(s^2 - 836.1s + 5.204 \times 10^6)} \end{aligned} \quad (3.39)$$

Note that one of the denominator polynomials of the transfer function, viz., $s^2 - 836.1s + 5.204 \times 10^6$ has a negative coefficient. Thus, the addition of the

current loop using the output inductor current leads to an unstable transfer function $G_{ic1}(s)$. However, it is interesting to note that this instability is not easily observed from the Bode plot as the roots (non-minimum phase zeroes) of the numerator polynomial $s^2 - 870.4s + 8.511 \times 10^6$ are almost cancelled by the unstable roots of the denominator polynomial $s^2 - 836.1s + 5.204 \times 10^6$ (see Fig. 3.5). But the system is still unstable due to the inexact cancellation of the unstable poles from the denominator polynomial. Changing the values of N and K_p does not change the situation and as such it is very difficult to use the output inductor current for feedback purposes to get a stable first-order like single dominant pole transfer function $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$.

Fig. 3.5 Bode plot of $G_{ic1}(s)$ (using output inductor current feedback).

The next thing is to investigate whether the current integral control given by $K(s) = K_p + K_I/s$ will change the situation. Previously, for the output inductor current feedback case, $G_c(s) = K_p$ has been used and it resulted in an unstable transfer function. Now, for the input inductor current feedback case, when a

current integral action of the form $K(s) = K_p + K_I/s$ is introduced instead of $G_c(s) = K_p$, the transfer function $G_{ic2}(s)$ is obtained as given below:

$$G_{ic2}(s) = \frac{\tilde{v}_0(s)}{\tilde{i}_{L1ref}(s)} = \frac{1}{V_p} \frac{(K_p s + K_I) q_2(s)}{p(s)s + (K_p s + K_I)(N/V_p) q_1(s)} \quad (3.40)$$

The value of N to localize the low-frequency dominant pole of the characteristic polynomial at a typical value of $s_{1f} = -3/(RC_2)$ is obtained as:

$$N = \left. \frac{p(s) * s * V_p}{(K_p s + K_I) q_1(s)} \right|_{s = -\frac{3}{RC_2}} \quad (3.41)$$

For $K_p = 0.05$ and $K_I = 10$, the value of N obtained is 4.5 and the resulting transfer function $G_{ic2}(s) = \tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ is given by:

$$G_{ic2}(s) = \frac{1 \times 10^6 (s+200) (s^2 - 870.3s + 8.51 \times 10^6)}{(s+110.4) (s^2 + 3846s + 3.9 \times 10^6) (s^2 + 998.3s + 2.38 \times 10^6)} \quad (3.42)$$

The Bode plots of $G_{ic2}(s)$ are shown in Fig. 3.6. The stable transfer function with no resonance peak in the low frequency region is obtained which simplifies the design of the voltage loop. Thus, for both type of the controllers (i.e., with and without an integral action), the input inductor current can be suitably used for feedback purposes to achieve the advantages offered by the current-mode control scheme.

Fig. 3.6 Bode plot of loop gain $G_{ic2}(s)$ (using input inductor current feedback).

However, for the case of the output inductor current feedback, when a PI controller of the form $K(s) = K_p + K_I/s$ is introduced, the transfer function $G_{ic3}(s)$ is given by:

$$G_{ic3}(s) = \frac{\tilde{v}_0(s)}{\tilde{i}_{L2ref}(s)} = \frac{1}{V_p} \frac{(K_p s + K_I) q_2(s)}{p(s)s + (K_p s + K_I)(N/V_p) q_1'(s)} \quad (3.43)$$

The value of N to localize the low-frequency dominant pole of the characteristic polynomial at a typical value of $s_{1f} = -3/(RC_2)$ is obtained as:

$$N = \left. \frac{p(s)sV_p}{(K_p s + K_I) q_1'(s)} \right|_{s = -\frac{3}{RC_2}} \quad (3.44)$$

For $K_p = 0.05$ and $K_I = 10$, the value of N obtained using (3.44) is 12, and the resulting transfer function $G_{ic3}(s) = \tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$ is given by:

$$G_{ic3}(s) = \frac{1 \times 10^7 (s+200) (s^2 - 870.3s + 8.51 \times 10^6)}{(s+105.1) (s^2 + 2426s + 1.7 \times 10^6)(s^2 - 876.9s + 5.16 \times 10^6)} \quad (3.45)$$

It should be noted that the inexact unstable pole-zero cancellation occurs in (3.45) also and this again results in an unstable system. Such inexact pole-zero cancellation occurs for a wide range of values of K_p and K_I and as such, it is very difficult to design the required controller using the output inductor current.

Remarks: A similar comparison of the current mode controllers can be performed for other high-order converters like the cascade boost, Cuk, SEPIC and zeta converters to find the most suitable inductor current for feedback purposes. The main point to note when developing these controllers is to use only the output voltage and one inductor current for feedback purposes. Further procedure of comparison of the controllers follows the approach described in section 3.2. To avoid repetition, this will not be covered in this thesis.

3.3 A Comparative Study of Current-mode Controllers for the 2-Stage Cascade Boost Converter

Fig. 3.7 shows the circuit diagram of the 2-stage cascade boost converter. As compared to the traditional N-stage cascade boost converter which requires N-active switches for its implementation [30-32], the converter shown in Fig. 3.7 requires only one switch which leads to the decreased switching losses. This converter has two stages of operation. In ‘stage 1’ of operation, the power switch ‘S’ is on and in ‘stage 2’ of operation, the power switch ‘S’ is off.

Fig. 3.7 Circuit diagram of the 2-stage cascade boost converter.

The averaged state-space model of this converter operating in continuous conduction mode is described as follows [10]:

$$\dot{x} = Ax + Bk \quad (3.46)$$

where $x = [i_{L_1}, i_{L_2}, v_{C_1}, v_{C_2}]^T$ represents the state vector of the averaged model, $k = [E]$ is the input voltage, and A and B represent the system matrix and the input vector, respectively. Also, i_{L_1} , i_{L_2} , v_{C_1} , and v_{C_2} are the currents flowing through inductors L_1 and L_2 and the average voltages of capacitors C_1 and C_2 , respectively. The matrix A and the vector B are described as:

$$A = \begin{bmatrix} 0 & 0 & -\frac{1-u}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{1-u}{L_2} \\ \frac{1-u}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1-u}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (3.47)$$

where scalar u denotes the duty ratio, with $0 \leq u \leq 1$. From (3.46) and (3.47), the following equilibrium values are obtained:

$$I_{L_1} = \frac{V_{C_2}^2}{RE}, \quad I_{L_2} = \sqrt{\frac{V_{C_2}^3}{R^2E}}, \quad V_{C_1} = \sqrt{EV_{C_2}}, \quad U = 1 - \sqrt{\frac{E}{V_{C_2}}} \quad (3.48)$$

where U represents the steady state value of the duty ratio and V_{C_2} represents the steady-state value of the output voltage. Setting V_{C_2} at the desired voltage value equal to V_d (with $E < V_d < \infty$) gives the following desired constant values:

$$I_{L_1} = \frac{V_d^2}{RE}, \quad I_{L_2} = \sqrt{\frac{V_d^3}{R^2E}}, \quad V_{C_1} = \sqrt{EV_d}, \quad U = 1 - \sqrt{\frac{E}{V_d}} \quad (3.49)$$

It should be noted that the averaged state-space model, given by (3.46) and (3.47), is non-linear in nature due to the presence of a time varying term, duty ratio u . Thus, to facilitate a comparative study, the small-signal model is first derived for the 2-stage cascade boost converter by linearizing (3.46) and (3.47) around the equilibrium point given by (3.49) and then introducing the small signal ac perturbations. The small-signal model is described by:

$$\dot{\tilde{x}} = a\tilde{x} + b\tilde{k} + m\tilde{u} \quad (3.50a)$$

$$\tilde{y} = c\tilde{x} \quad (3.50b)$$

where $\tilde{u}$ represents the small signal variations in the duty cycle u , i.e., $u = U + \tilde{u}$ and $\tilde{x}$ represents the small signal variations in the state variables such that $\tilde{x} = [\tilde{I}_{L_1} \ \tilde{I}_{L_2} \ \tilde{V}_{C_1} \ \tilde{V}_{C_2}]^T$ and $\tilde{k} = [\tilde{E}]$. Matrices a, b, m and c are given by:

$$a = \begin{bmatrix} 0 & 0 & -\frac{(1-U)}{L_1} & 0 \\ 0 & 0 & \frac{1}{L_2} & -\frac{(1-U)}{L_2} \\ \frac{(1-U)}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{(1-U)}{C_2} & 0 & -\frac{1}{RC_2} \end{bmatrix}, \quad b = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad m = \begin{bmatrix} \frac{E}{(1-U)L_1} \\ \frac{E}{(1-U)^2L_2} \\ \frac{E}{(1-U)^4RC_1} \\ \frac{E}{(1-U)^3RC_2} \end{bmatrix},$$

$$c^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.51)$$

3.3.1 Controller Design Using Root-locus Method

In this section, the two current-mode controllers using the input and output inductor currents of the converter are compared using the state-space approach. The controller using the input inductor current will be considered first to demonstrate its shortcomings for a certain set of circuit parameter values, followed by the controller using the output inductor current.

As can be seen from the averaged state-space model of the 2-stage cascade boost converter given by (3.46) and (3.47), there are four state variables which can be used for feedback purposes. However, not all will be used for the controller design. Only the input inductor current and output voltage state variables are used for feedback purposes initially. The controller structure given by (3.52) is just one form of the current-mode control in which the integral action is employed to ensure a good steady-state regulation against the circuit parameter uncertainties like the load variations [40, 41]. The proposed controller using the input inductor current is given by:

$$u = U - K_{P1}(i_{L_1} - I_{L_1}) - K_{I1} \int (v_o(\tau) - V_d) d\tau \quad (3.52)$$

where $K_{P1}, K_{I1} > 0$. Define the errors as follows:

$$e_1 = i_{L_1} - \frac{V_d^2}{RE}, e_2 = i_{L_2} - \frac{V_d}{R} \sqrt{\frac{V_d}{E}}, e_3 = v_{C_1} - \sqrt{E \cdot V_d}, e_4 = v_o - V_d \quad (3.53)$$

Substituting (3.52) – (3.53) into (3.46) – (3.47) and using (3.49) yields the following set of equations:

$$\dot{e}_1 = -\frac{1}{L_1} \left(\sqrt{\frac{E}{V_d}} + K_{P1}e_1 + \sigma \right) (e_3 + \sqrt{E \cdot V_d}) + \frac{E}{L_1} \quad (3.54)$$

$$\dot{e}_2 = \frac{1}{L_2} \left(e_2 + \frac{V_d}{R} \sqrt{\frac{V_d}{E}} \right) - \frac{1}{L_2} \left(\sqrt{\frac{E}{V_d}} + K_{P1}e_1 + \sigma \right) (e_4 + V_d) \quad (3.55)$$

$$\dot{e}_3 = \frac{1}{C_1} \left(\sqrt{\frac{E}{V_d}} + K_{P1}e_1 + \sigma \right) \left(e_1 + \frac{V_d^2}{RE} \right) - \frac{1}{C_1} \left(e_2 + \frac{V_d}{R} \sqrt{\frac{V_d}{E}} \right) \quad (3.56)$$

$$\dot{e}_4 = \frac{1}{C_2} \left(\sqrt{\frac{E}{V_d}} + K_{P1}e_1 + \sigma \right) \left(e_2 + \frac{V_d}{R} \sqrt{\frac{V_d}{E}} \right) - \frac{1}{RC_2} (e_4 + V_d) \quad (3.57)$$

$$\dot{\sigma} = K_{I1}e_4 \quad (3.58)$$

The unique equilibrium point of (3.54) – (3.58) is given by:

$$(e_{1\infty}, e_{2\infty}, e_{3\infty}, e_{4\infty}, \sigma_{\infty}) = (0, 0, 0, 0, 0) \quad (3.59)$$

Linearizing (3.54) – (3.58) about equilibrium point (3.59) gives the following system:

$$\dot{\beta} = M\beta \quad (3.60)$$

where $\beta = [\beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \beta_5]^T$,

$\beta_1 = e_1 - e_{1\infty}$, $\beta_2 = e_2 - e_{2\infty}$, $\beta_3 = e_3 - e_{3\infty}$, $\beta_4 = e_4 - e_{4\infty}$, $\beta_5 = \sigma - \sigma_{\infty}$ and

$$M = \begin{bmatrix} -\frac{K_{P1}\sqrt{E \cdot V_d}}{L_1} & 0 & -\frac{1}{L_1}\sqrt{\frac{E}{V_d}} & 0 & -\frac{\sqrt{E \cdot V_d}}{L_1} \\ -\frac{K_{P1}V_d}{L_2} & 0 & \frac{1}{L_2} & -\frac{1}{L_2}\sqrt{\frac{E}{V_d}} & -\frac{V_d}{L_2} \\ \frac{1}{C_1}\sqrt{\frac{E}{V_d}} + \frac{K_{P1}V_d^2}{RC_1E} & -\frac{1}{C_1} & 0 & 0 & \frac{V_d^2}{RC_1E} \\ \frac{K_{P1}}{C_2}\frac{V_d}{R}\sqrt{\frac{V_d}{E}} & \frac{1}{C_2}\sqrt{\frac{E}{V_d}} & 0 & -\frac{1}{RC_2} & \frac{1}{C_2}\frac{V_d}{R}\sqrt{\frac{V_d}{E}} \\ 0 & 0 & 0 & K_{I1} & 0 \end{bmatrix}$$

The system (3.60) will be stable if all the eigenvalues of M , i.e., the roots of $|sI - M| = 0$, where s is a complex variable, lie in the open left-half of the complex s -plane. The characteristic polynomial $|sI - M|$ is given by:

$$m(s) = |sI - M| = s^5 + m_4 s^4 + m_3 s^3 + m_2 s^2 + m_1 s + m_0 \quad (3.61)$$

where the coefficients m_i 's are given by:

$$\begin{aligned} m_4 &= \frac{1}{RC_2} + \frac{K_{P1}\sqrt{E.V_d}}{L_1}, \\ m_3 &= \frac{1}{L_2 C_1} + \frac{E}{L_1 C_1 V_d} + \frac{E}{L_2 C_2 V_d} + \frac{K_{P1}\sqrt{E.V_d}}{RL_1 C_2} + \frac{K_{P1}}{L_1 C_1} \frac{V_d}{R} \sqrt{\frac{V_d}{E}} - \frac{K_{I1}}{C_2} \frac{V_d}{R} \sqrt{\frac{V_d}{E}}, \\ m_2 &= \frac{1}{RL_2 C_1 C_2} + \frac{E}{RL_1 C_1 C_2 V_d} + \frac{2K_{P1}\sqrt{E.V_d}}{L_1 C_1 C_2} + \frac{K_{I1}\sqrt{E.V_d}}{L_2 C_2} - \frac{K_{I1}K_{P1}V_d(V_d-1)}{RL_1 C_2} + \frac{K_{P1}}{L_1 L_2 C_2} \sqrt{\frac{E^3}{V_d}} \\ &\quad + \frac{K_{P1}V_d}{R^2 L_1 C_1 C_2} \sqrt{\frac{V_d}{E}}, \\ m_1 &= \frac{E^2}{L_1 L_2 C_1 C_2 V_d^2} - \frac{2K_{I1}V_d}{RL_2 C_1 C_2} \sqrt{\frac{V_d}{E}} - \frac{K_{I1}K_{P1}(E+V_d)}{L_1 L_2 C_2} + \frac{4K_{P1}\sqrt{E.V_d}}{RL_1 L_2 C_1 C_2} - \frac{K_{I1}\sqrt{E.V_d}}{RL_1 C_1 C_2} + \\ &\quad \frac{K_{P1}V_d}{R^2 L_1 C_1 C_2} \sqrt{\frac{V_d}{E}}, m_0 = \frac{K_{I1}}{L_1 L_2 C_1 C_2} \sqrt{\frac{E^3}{V_d^3}} + \frac{K_{I1}}{RL_2 C_1 C_2} \sqrt{\frac{E^3}{V_d}} - \frac{2K_{I1}K_{P1}V_d(V_d-1)}{RL_1 L_2 C_1 C_2} \end{aligned}$$

Next, the root locus method is used to analyze the system stability. This is illustrated as follows. Consider the cascade boost converter with following circuit parameter values:

$$\begin{aligned} E &= 5V, \quad V_d = 15V, \quad L_1 = L_2 = 3.3 \text{ mH}, \quad C_1 = 470 \mu F, \quad C_2 = \\ 330 \mu F, \quad R &= 352 \Omega \end{aligned} \quad (3.62)$$

Fig. 3.8(a) shows the root locus plot of $m(s)$ for $K_{P1} = 0.055$, $0 < K_{I1} < 2$ and Fig. 3.8(b) shows the root locus plot for $K_{I1} = 0.13$, $0 < K_{P1} < 0.02$. The arrows show how the poles are moving from $K_{I1} = 0$ and $K_{P1} = 0$,

respectively. It is seen that even for small values of K_{I1} and K_{P1} in this range, there are poles in the right-half complex plane, i.e., the system is unstable.

Fig. 3.8 Root locus plot using input inductor current feedback: (a) root locus plot for $K_{P1} = 0.055$ $0 < K_{I1} < 2$; and (b) root locus plot for $K_{I1} = 0.13$, $0 < K_{P1} < 0.02$.

To show that the current-mode controller using the input inductor current also does not work for another set of parameters, consider a different set of circuit parameter values given by:

$$\begin{aligned}
 E &= 10V, \quad V_d = 40V, \quad L_1 = 800 \mu H, \quad L_2 = 600 \mu H, \quad C_1 = 100 \mu F, \\
 C_2 &= 50 \mu F, \quad R = 200 \Omega
 \end{aligned} \tag{3.63}$$

Fig. 3.9(a) shows the root locus plot of $m(s)$ for $K_{P1} = 0.07$, $0 < K_{I1} < 7.8$ and Fig. 3.9(b) shows the root locus plot for $K_{I1} = 0.01$, $0 < K_{P1} < 0.2$. Again, it can be seen that the system is unstable even for small values of K_{I1} and K_{P1} in this range. As such, the controller using the input inductor current is not suitable for the regulation of the converter. Note that these root locus plots were

obtained under ideal conditions. If parasitic elements are present in the circuit, then the root locus plots might differ from that obtained earlier.

Fig. 3.9 Root locus plot using input inductor current feedback: (a) Root locus plot for $K_{P1} = 0.07$ $0 < K_{I1} < 7.8$; and (b) Root locus plot for $K_{I1} = 0.01$, $0 < K_{P1} < 0.2$.

The question now is this: since the controller using the input inductor current is not suitable for the regulation of the 2-stage cascade boost converter with the given set of circuit parameter values, then can the controller using the output inductor current be used for this purpose? The question is addressed in this section. When the output inductor current is used for feedback purposes, the structure of the control law is similar to that of (3.52) except that the output inductor current is used for the feedback purposes. It is described by:

$$u = U - K_{P2}(i_{L_2} - I_{L_2}) - K_{I2} \int (v_o(\tau) - V_d) d\tau \quad (3.64)$$

where K_{P2} , $K_{I2} > 0$. Like what was done previously, substituting (3.53) and (3.64) into (3.46) – (3.47) and using (3.49) yields a set of equations with a unique equilibrium point (3.59). The linearized model has a system matrix P given by:

$$P = \begin{bmatrix} 0 & -\frac{K_{P2}\sqrt{E.V_d}}{L_1} & -\frac{1}{L_1}\sqrt{\frac{E}{V_d}} & 0 & -\frac{\sqrt{E.V_d}}{L_1} \\ 0 & -\frac{K_{P2}V_d}{L_2} & \frac{1}{L_2} & -\frac{1}{L_2}\sqrt{\frac{E}{V_d}} & -\frac{V_d}{L_2} \\ \frac{1}{C_1}\sqrt{\frac{E}{V_d}} & -\frac{1}{C_1} + \frac{K_{P2}V_d^2}{RC_1E} & 0 & 0 & \frac{V_d^2}{RC_1E} \\ 0 & \frac{1}{C_2}\sqrt{\frac{E}{V_d}} + \frac{K_{P2}}{C_2}\frac{V_d}{R}\sqrt{\frac{V_d}{E}} & 0 & -\frac{1}{RC_2} & \frac{1}{C_2}\frac{V_d^{3/2}}{RE} \\ 0 & 0 & 0 & K_{I2} & 0 \end{bmatrix}$$

The characteristic polynomial $|sI - P|$ is:

$$p(s) = |sI - P| = s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 \quad (3.65)$$

where the coefficients p_i 's are given by:

$$p_4 = \frac{1}{RC_2} + \frac{K_{P2}V_d}{L_2},$$

$$p_3 = \frac{K_{I2}V_d}{L_2} + \frac{1}{L_2C_1} \left(1 - \frac{K_{P2}V_d^2}{RE} \right) - \frac{K_{I2}}{C_2} \frac{V_d}{R} \sqrt{\frac{V_d}{E}} + \frac{E}{L_2C_2V_d} + \frac{E}{L_1C_1V_d} + \frac{2K_{P2}}{L_2C_2} \frac{V_d}{R},$$

$$p_2 = \frac{2K_{P2}E}{L_1L_2C_1} + \frac{K_{I2}\sqrt{E.V_d}}{L_2C_2} - \frac{1}{RL_2C_1C_2} \left(1 - \frac{K_{P2}V_d^2}{RE} \right) + \frac{E}{RL_1L_2C_2V_d},$$

$$p_1 = \frac{E^2}{L_1L_2C_1C_2V_d^2} - \frac{K_{I2}V_d}{L_2C_1C_2} \sqrt{\frac{V_d}{E}} \left(1 - \frac{K_{P2}V_d^2}{RE} \right) + \frac{3K_{P2}E}{RL_1L_2C_1C_2} + \frac{K_{I2}\sqrt{E.V_d}}{RL_1C_1C_2} -$$

$$\frac{K_{I2}.V_d\sqrt{E.V_d}}{RL_2C_1C_2E} - \frac{K_{P2}K_{I2}V_d^3}{R^2L_2C_1C_2E} \sqrt{\frac{V_d}{E}},$$

$$p_0 = \frac{2K_{I2}E}{L_1L_2C_1C_2} \sqrt{\frac{E}{V_d}}$$

As usual, the linearized system will be stable if all the eigenvalues of matrix P lie in the open left-half complex plane.

Again for the purpose of illustration, the 2-stage cascade boost converter with the circuit parameters given by (3.62) is considered. Fig. 3.10(a) shows the root locus plot of $p(s)$ for $K_{P2} = 0.055$, $0 < K_{I2} < 2$ and Fig. 3.10 (b) shows the root

locus plot for $K_{I2} = 0.13$, $0 < K_{P2} < 0.02$. It is clear that system is stable for the entire range of controller gains as poles always lie in the left-half of the complex s -plane.

Fig. 3.10 Root locus plot using output inductor current feedback: (a) Root locus plot for $K_{P2} = 0.055$, $0 < K_{I2} < 2$; and (b) Root locus plot for $K_{I2} = 0.13$, $0 < K_{P2} < 0.02$.

Also, to show that the controller designed using output inductor current also works for another set of parameters, consider the circuit parameter values given by (3.63). Fig. 3.11(a) shows the root locus plot of $p(s)$ for $K_{P2} = 0.07$, $0 < K_{I2} < 7.8$ and Fig. 3.11(b) shows the root locus plot for $K_{I2} = 0.01$, $0 < K_{P2} < 0.2$. It can be seen that the system is again stable for the entire range of K_{I2} and K_{P2} in this range. All these results confirm the suitability of output inductor current for the feedback purposes.

Fig. 3.11 Root locus plot using output inductor current feedback: (a) Root locus plot for $K_{p2} = 0.07$, $0 < K_{I2} < 7.8$; (b) Root locus plot for $K_{I2} = 0.01$, $0 < K_{p2} < 0.2$.

3.3.2 Controller design using classical frequency-domain approach

In Section 3.3.1, it was demonstrated that for the case of the current-mode controller designed using the input inductor current for feedback purposes, the poles of the closed-loop system will move to the right-half of the complex plane even for small controller gains. In this section, a different method of controller design using a frequency-response approach is adopted to compare the performance of the system using different inductor currents for feedback purposes.

The generalized transfer functions required for designing the controller can be easily obtained from the small-signal model, given by (3.50) and (3.51) and are given as:

$$G_1(s) = \frac{\tilde{i}_{L1}(s)}{\tilde{u}(s)} = \frac{q_1(s)}{p(s)} = \frac{a_0 s^3 + a_1 s^2 + a_2 s + a_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.66)$$

$$G_2(s) = \frac{\tilde{i}_{L_2}(s)}{\tilde{u}(s)} = \frac{q_1'(s)}{p(s)} = \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.67)$$

$$G_3(s) = \frac{\tilde{v}_0(s)}{\tilde{u}(s)} = \frac{q_2(s)}{p(s)} = \frac{c_0 s^3 + c_1 s^2 + c_2 s + c_3}{s^4 + d_0 s^3 + d_1 s^2 + d_2 s + d_3} \quad (3.68)$$

where the small ac perturbations are represented by “ $\sim$ ” and

$$a_0 = \frac{V_{C_1}}{L_1},$$

$$a_1 = \frac{(1-U)I_{L_1}}{C_1 L_1} + \frac{V_{C_1}}{C_2 L_1 R},$$

$$a_2 = \frac{(1-U)I_{L_1}}{C_1 C_2 L_1 R} + \frac{1}{L_1 L_2} \left(\frac{V_{C_1} + (1-U)V_{C_2}}{C_1} + \frac{(1-U)^2 V_{C_1}}{C_2} \right),$$

$$a_3 = \frac{(1-U)^3 I_{L_1} + (1-U)^2 I_{L_2}}{C_1 C_2 L_1 L_2} + \frac{V_{C_1} + (1-U)V_{C_2}}{C_1 C_2 L_1 L_2 R},$$

$$b_0 = \frac{V_{C_2}}{L_2},$$

$$b_1 = -\frac{I_{L_1}}{C_1 L_2} + \frac{(1-U)I_{L_2} + (V_{C_2}/R)}{C_2 L_2},$$

$$b_2 = -\frac{I_{L_1}}{C_1 C_2 L_2 R} + \frac{(1-U)^2 V_{C_2} + (1-U)V_{C_1}}{C_1 L_1 L_2},$$

$$b_3 = \frac{(1-U)^3 I_{L_2} + (1-U)^2 I_{L_2}}{C_1 C_2 L_1 L_2} + \frac{(1-U)^2 V_{C_2} + (1-U)V_{C_1}}{C_1 C_2 L_1 L_2 R},$$

$$c_0 = -\frac{I_{L_2}}{C_2},$$

$$c_1 = \frac{(1-U)V_{C_2}}{C_2 L_2},$$

$$c_2 = \frac{1}{C_1 C_2} \left(\frac{-(1-U)^2 I_{L_2}}{L_1} - \frac{(1-U)I_{L_2} + I_{L_1}}{L_2} \right),$$

$$c_3 = \frac{(1-U)^3 V_{C_2} + (1-U)^2 V_{C_1}}{C_1 C_2 L_1 L_2},$$

$$d_0 = \frac{1}{C_2 R},$$

$$d_1 = \frac{(1-U)^2}{C_1 L_1} + \frac{1}{L_2} \left(\frac{1}{C_1} + \frac{(1-U)^2}{C_2} \right),$$

$$d_2 = \frac{(1-U)^2}{C_1 C_2 L_1 R} + \frac{1}{C_1 C_2 L_2 R},$$

$$d_3 = \frac{(1-U)^4}{C_1 C_2 L_1 L_2} .$$

Again, for the purpose of illustration, consider a 2-stage cascade boost converter with following circuit parameter values:

$$\begin{aligned} E &= 5V, \quad V_d = 15V, \quad L_1 = L_2 = 3.3 \text{ mH}, \quad C_1 = 470 \text{ } \mu\text{F}, \quad C_2 = 330 \text{ } \mu\text{F}, \\ R &= 470 \text{ } \Omega \end{aligned} \tag{3.69}$$

The corresponding transfer functions $\tilde{v}_0(s)/\tilde{u}(s)$ can be obtained as:

$$G_3(s) = \frac{\tilde{v}_0(s)}{\tilde{u}(s)} = \frac{q_2(s)}{p(s)} = \frac{-0.0167 \times 10^4 (s - 4.74 \times 10^4)(s^2 - 22.64s + 4.3 \times 10^5)}{(s^2 + 4.928s + 5.94 \times 10^4)(s^2 + 1.159s + 1.10 \times 10^6)} \tag{3.70}$$

It is clear from (3.70) that $G_3(s)$ has non-minimum phase characteristics due to the presence of zeros in the right-half complex plane which present abrupt changes in the low frequency region of the Bode plot as shown in Fig. 3.12. Thus it is difficult to design the controller using a single voltage loop using this transfer function. The use of a multi-loop control scheme is thus introduced to minimize the effects of the resonance peak and to increase the bandwidth. The multi-loop scheme is shown in Fig. 3.13.

Fig. 3.12 Bode plots of $G_3(s)$ (dotted line) and $\tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ (solid line).

Fig. 3.13 Multi-loop controller scheme.

The first inner loop controller $G_{c1}(s)$ when the input inductor current is used for feedback purposes is designed using the frequency domain approach. It is designed to achieve positive phase and gain margins at a crossover frequency of up to a decade below the switching frequency (below 1000 Hz). Such a criterion is chosen to allow adequate stability margins when the system is subjected to parametric variations and un-modelled parasitic resistances which were neglected in modeling of the converter. A PI controller is designed for inner loop

to meet this criterion. For $V_p = 1\text{ V}$ and $N = 1$, the PI controller of the form $G_{c1}(s) = K_{pf1} + \frac{K_{if1}}{s}$ is chosen. For $K_{pf1} = 2$, $K_{if1} = 70$, $G_{c1}(s)$ is given by :

$$G_{c1}(s) = \frac{2(s+35)}{s} \quad (3.71)$$

Fig. 3.14 shows the Bode plot of the current loop gain using the controller (3.71). The phase margin obtained is 83° and the gain margin obtained is approximately 19 dB. In Fig. 3.12, the Bode plots of $\tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$ and $G_3(s)$ are compared. The former is the open-loop plant in the voltage loop (see Fig. 3.13). It is worth noting that even though the current loop controller is designed to meet the phase margin and gain margin criteria, the resonance peak is still present in the low frequency region of the Bode plot of $\tilde{v}_0(s)/\tilde{i}_{L1ref}(s)$. This makes it difficult to design the voltage loop controller using this transfer function. This resonance peak is removed when the output inductor current is used as will be shown next.

Fig. 3.14 Bode plot of current loop gain using input inductor current feedback.

Now, the design of an inner loop controller $G_{c2}(s)$ using the output inductor current is considered. Again, for $V_p = 1\text{ V}$, and $N = 1$, the PI controller of the

form $G_{c2}(s) = K_{pf2} + \frac{K_{if2}}{s}$ for $K_{pf2} = 2$ and $K_{if2} = 70$ is obtained as:

$$G_{c2}(s) = \frac{2(s+35)}{s} \quad (3.72)$$

Fig. 3.15(a) shows the Bode plot of the current loop gain using the controller (3.72). The phase margin obtained is 77° and the gain margin obtained is approximately 15 dB.

Fig. 3.15 Bode plots when using the output inductor current in feedback of the current loop: (a) Bode plot of the current loop gain; and (b) Bode plots of $G_3(s)$ (dotted line) and $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$ (solid line).

The Bode plots of $G_3(s)$ and $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$ are shown in Fig. 3.15(b) for the case of the output inductor current feedback. It is noted that $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$ is the open-loop plant for the voltage loop (see Fig. 3.13). When the output inductor current feedback is introduced, the resonance peak in the low frequency region is now not present in $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$. This simplifies the voltage loop design.

The PI controller of the form $G_v(s) = K_{pv} + \frac{K_{Iv}}{s}$ for the voltage loop is now considered with objectives of getting a high gain in the low frequency region and to improve the steady state error performance as well as to get an overall positive phase margin. For $K_{pv} = 2$, $K_{Iv} = 200$ and feedback gain $K_h = 0.1$, the PI controller is given by:

$$G_v(s) = \frac{2s + 200}{s} \quad (3.73)$$

K_{pv} is designed together with the voltage divider network with gain K_h and the value of $K_{pv}K_h$ is chosen such that the resonance peak at the high frequency region is below 0 dB. The Bode plot of the resulting voltage loop gain $G_L(s)$ (product of PI controller $G_v(s)$, voltage divider network K_h and $\tilde{v}_0(s)/\tilde{i}_{L2ref}(s)$) is shown in Fig. 3.16. The phase margin obtained is about 70°.

Fig. 3.16 Bode plot of voltage loop gain $G_L(s)$ using output inductor current feedback.

Remark: This work can be extended to the N -stage cascade boost converters for $N = 3, 4$, etc. However, it should be noted that the results of the 2-stage cascade boost converter might differ from that of the other higher-order converters, e.g., the 3-stage cascade boost converter. It might be possible that both the input and the output inductor currents could be used for feedback purposes. Therefore it is necessary to analyze the cascade converter for every N .

3.4 Experimental Results

In this section, some experimental results are provided to verify the theoretical conclusions obtained in sections 3.2 and 3.3.

3.4.1 Current-mode Controlled POEL Converter

A laboratory prototype of the POEL converter was built to validate the results for the current-mode controllers obtained in section 3.2. Eq. (3.7) was implemented

using simple analog components. The following circuit parameter values were used for an experimental work:

$$E = 5V, L_1 = 1mH, L_2 = 10mH, C_1 = 47\mu F, C_2 = 100\mu F$$

The switching frequency used was 20 kHz and $V_p = 1 V$. No filtering of signals was carried out.

Fig. 3.17(a) shows the output response of the converter when a step reference voltage $V_d = 6 V$ was applied. Here, $K_{p1} = 0.07$ and $K_{I1} = 3.3$ were used. Fig. 3.17(b) shows the output response when the reference was changed back to 0 V. Fig. 3.17(c) shows the output response in the presence of a load change from $R = 1 k\Omega$ to $R = 666 \Omega$ (33.3 % reduction) and then back to $R = 1 k\Omega$. An overshoot of approximately 0.4 V and a settling time of around 400 ms was observed. Fig. 3.17(d) shows the output response in the presence of a load change from $R = 1 k\Omega$ to $R = 500 \Omega$ (50 % reduction) and then back to $R = 1 k\Omega$. A comparatively high overshoot (of approx. 0.8 V) was observed with small steady state error of approximately 0.2 V. Such a deviation from the ideal behavior is expected since the current-mode controller is designed using the linearized model of the POEL converter which is valid only in the neighborhood of a specific operating point. Next, the variations in the output response for various values of the controller gains was studied.

Fig. 3.17 System response using $K_{p1} = 0.07$ and $K_{I1} = 3.3$: (a) Transient Output response (0 to 6 V); (b) Transient Output response (6 to 0 V); (c) Output response in the presence of load change ($R = 1 \text{ k}\Omega$ to $R = 666 \Omega$); and (d) Output response in the presence of load change ($R = 1 \text{ k}\Omega$ to $R = 500 \Omega$).

Fig. 3.18(a) shows the transient output response of the converter using $K_{p1} = 0.03$ and $K_{I1} = 0.7$. An overshoot of approximately 0.6 V was observed and the settling of 200 ms was obtained. Figs. 3.18(b)-(d) show the output voltage response for a load change from $R = 1 \text{ k}\Omega$ to $R = 666 \Omega$ and then back to $R = 1 \text{ k}\Omega$ for $K_{I1} = 4$, 3.32 and 2.3 respectively and using $K_{p1} = 0.07$. All these results show that the controller using the input inductor current is suitable for the regulation of the POEL converter.

Fig. 3.18 System response using different values of the controller gains: (a) Transient Output response ($K_{p1} = 0.03$, $K_{I1} = 0.7$); (b) Output response using load change ($R = 1\text{k}\Omega$ to $R = 666\ \Omega$ for $K_{I1} = 4$, $K_{p1} = 0.07$); (c) Output response using load change ($R = 1\text{k}\Omega$ to $R = 666\ \Omega$ for $K_{I1} = 3.3$, $K_{p1} = 0.07$); and (d) Output response using load change ($R = 1\text{k}\Omega$ to $R = 666\ \Omega$ for $K_{I1} = 2.3$, $K_{p1} = 0.07$).

3.4.2 Current-mode controlled 2-stage cascade boost converter

In order to check the effectiveness of the proposed controller using the output inductor current given by (3.64), a prototype of the 2-stage cascade boost converter was designed, built and tested. The set of converter parameter values was the same as used in section 3.3.1, given by (3.62) and no filtering of any signal was carried out. The gate signal for MOSFET switch was generated using

a comparator IC LM311 and a triangular waveform of the magnitude $1V_{p-p}$. For implementation purposes, the output voltage sensor was realized using a simple voltage divider network with a gain of 0.1, whereas the inductor current was measured using the Hall-effect sensor LTS-6NP from LEM.

Fig. 3.19(a) shows the output response and corresponding control signal of the converter when a step reference voltage $V_d = 15 V$ was applied. Here, $K_{p2} = 0.0056$, $K_{I2} = 2.9$ and $K = 0.32 V$ is used. It can be seen that the output voltage has a little or no overshoot and a very small settling time. Fig. 3.19(b) shows the output response in the presence of a load change from $R = 352 \Omega$ to $R = 528 \Omega$ (a 50 % increase) and then back to $R = 352 \Omega$. The response is again faster with small overshoot. Fig. 3.19(c) shows the output response in the presence of a load change from $R = 352 \Omega$ to $R = 822 \Omega$ (a 133 % increase) and then back to $R = 352 \Omega$. The overshoot and settling times were found to have increased slightly as compared to the previous load change. As the current-mode controller was designed using the linearized small-signal model of the 2-stage cascade boost converter, such a deviation from an ideal behavior is expected when large variations about the quiescent operating point are introduced. Fig. 3.19(d) shows the output response and the control signal in the presence of a reference voltage change from $V_d = 15 V$ to $V_d = 10 V$ and then back to $V_d = 15 V$.

Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div

(a)

Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div

(b)

Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div

(c)

Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div

(d)

Fig. 3.19 System response using the output inductor current feedback for $K_{p2} = 0.0056$, $K_{I2} = 2.9$: (a) Transient output response and control signal for a step change in the reference voltage (0-15V); (b) Output response and control signal in the presence of a load change ($R = 352 \Omega$ to $R = 528 \Omega$, vice-versa); (c) Output response and control signal in the presence of a load change ($R = 352 \Omega$ to $R = 822 \Omega$, vice-versa); and (d) Output response and control signal for a step change in the reference voltage (15-10-15V).

Next, for the same experimental setup, the input inductor current was used for feedback purposes instead. Fig. 3.20(a) shows the output transient response and corresponding control signal when a step change in the reference voltage from $V_d = 0 \text{ V}$ to $V_d = 15 \text{ V}$ was applied for $K_{p1} = 0.0056$ and $K_{I1} = 2.9$. There was some steady-state error and noise observed in the output voltage. Next, the ability of the controller to handle the load disturbances was evaluated. Fig. 3.20(b) shows the output response and control signal in the presence of a load change

from $R = 352 \Omega$ to $R = 822 \Omega$ (a 133 % increase) and then back to $R = 352 \Omega$.

The response had a large overshoot and a longer settling time as compared to that obtained using the output inductor current feedback.

Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div Ch1 : 5 V/div, 1 s/div, Ch2 : 2 V/div, 1 s/div

(a)

(b)

Fig. 3.20 System response using input inductor current feedback for $K_{p1} = 0.0056$, $K_{I1} = 2.9$; (a) Transient output response and control signal for a step change in reference voltage (0-15V); and (b) Output response and control signal in the presence of a load change ($R = 352 \Omega$ to $R = 822 \Omega$, vice-versa).

Remark: It can be seen that, in a practical setting where parasitic elements are usually present, both input and output inductor currents could be used for feedback purposes. However, the experimental results also show that the controller using the output inductor current gives an improved transient response as well as more accurate steady-state response as compared to that obtained using the controller with input inductor current feedback.

Next, in order to verify the accuracy of the analytical results obtained using the frequency response approach of design, the multi-loop control scheme was implemented with PSIM version 9.0.3. The set of converter parameter values used was the same as those used in section 3.3.2, given by (3.69) and the Bode plots were obtained using the ‘AC SWEEP’ block in PSIM. According to the

principle of AC analysis, a small excitation signal is injected into the system at an input side and the signal at the same frequency is extracted at the output [53]. Fig. 3.21(a) shows the Bode plot of $v_0(s)/i_{L1ref}(s)$ obtained using the input inductor current for feedback purposes. Here $K_{pf1} = 2$, $K_{if1} = 70$, $K_{pv} = 0.05$ and $K_{Iv} = 5$ were used. As predicted analytically, the Bode plot has a large resonance peak, which makes this transfer function unsuitable to design the voltage loop controller. However, when an output inductor current is used for the feedback purposes, this resonance peak is removed as shown in Fig. 3.21(b). It should be noted that these results in Fig. 3.21 were obtained under ideal conditions in PSIM, i.e., without considering the effect of parasitic elements of the passive components. To obtain the results under non-ideal conditions, when a small parasitic resistance r_{c1} was added in series with the capacitor C_1 , the resonance peak was eliminated from the transfer function $v_0(s)/i_{L1ref}(s)$, as shown in Fig. 3.21(c).

Next, in order to verify the accuracy of the analytical results obtained using the frequency-response approach of design, the multi-loop control scheme was implemented with PSIM version 9.0.3. The set of converter parameter values used was the same as those used in Section 3.3.2, given by (3.69) but using $V_d = 20$ with $N = 1$ and $K_h = 0.1$.

The Bode plots were obtained using the ‘AC SWEEP’ block in PSIM. According to the principle of AC analysis, a small excitation signal is injected into the system at an input side and the signal at the same frequency is extracted at the output [19]. Fig. 3.21(a) shows the Bode plot of $v_0(s)/i_{L1ref}(s)$ obtained using the input inductor current for feedback purposes. Here $K_{pf1} = 2$, $K_{if1} = 70$,

$K_{pv} = 0.05$ and $K_{Iv} = 5$ were used. As predicted analytically, the Bode plot has a large resonance peak, which makes this transfer function unsuitable to design the voltage-loop controller. However, for the same simulation circuit, when an output inductor current is used for the feedback purposes, this resonance peak is removed as shown in Fig. 3.21(b). It should be noted that these results in Figs. 3.21(a) and 3.21(b) were obtained under ideal conditions in PSIM, i.e. without considering the effect of parasitic elements of the passive components. To obtain the results under non-ideal conditions, when a small parasitic resistance $r_{c1} = 5 \Omega$ was added in series with the capacitor C_1 , the resonance peak was eliminated from the transfer function $v_0(s)/i_{L1ref}(s)$, as shown in Fig. 3.21(c).

(a)

(b)

(c)

Fig. 3.21 Bode plots: (a) Bode plot of $v_0(s)/i_{L1ref}(s)$ obtained using the input inductor current for feedback purposes; (b) Bode plot of $v_0(s)/i_{L2ref}(s)$ obtained using the output inductor current for feedback purposes; and (c) Bode plot of $v_0(s)/i_{L1ref}(s)$ obtained using the input inductor current for feedback purposes and considering the effect of parasitic elements in the circuit.

3.5 Summary

In this chapter, some detailed comparative studies of two current-mode controllers (using the input and output inductor currents of the converter) were provided for some high-order boost-type dc-dc converters like the POEL converter and the 2-stage cascade boost converter. The main objective was to propose a suitable approach for selecting the most suitable inductor current for feedback purposes. The results show that this selection is quite specific to the high-order dc-dc converter topology and as such a separate analysis may be required for a specific converter to find the most suitable inductor current to design the current-mode controller for its regulation.

In chapter 4, various design issues of the sliding-mode controlled high-order boost-type dc-dc converters are discussed.

CHAPTER 4 SLIDING-MODE CONTROL OF HIGH-ORDER DC-DC CONVERTERS

4.1 Motivation

In Chapter 3, some detailed comparative studies of the current-mode controllers for high-order dc-dc converters were presented to find the most suitable inductor current for feedback purposes. In this chapter, sliding-mode control (SMC) of the high-order boost-type dc-dc converters is considered. A brief introduction on the state-of-the-art sliding-mode (SM) controllers is first presented, including their advantages and challenges in implementing them for the regulation of high-order dc-dc converters. This is followed by a detailed design of the pulse-width-modulation (PWM)-based SM controller for a quadratic boost converter which uses a reduced number of state variables as compared to some of the state-of-the-art PWM-based SM controllers. Also, some detailed comparative studies of the SM controllers are presented for some high-order dc-dc converters like the quadratic boost converters and the hybrid-type dc-dc boost converters to find the most suitable inductor current for the controller design.

SMC is one of the widely used non-linear control strategies for dc-dc converters as it offers several advantages like a fast transient response, ability to handle the large deviations introduced about the steady-state equilibrium point as well as guaranteed stability and robustness against load and line variations [18], [54]. Some earlier works on developing the SM controllers for the boost-type dc-dc converters are based on the PWM approach and it offers several advantages like

reduced power losses due to the use of a constant switching frequency and a good dynamic response over a wide range of operating conditions [18-21]. Despite the successful application of this approach to dc-dc converters, there remain several areas which demand further investigations. It is well known that the state-of-the-art PWM-based SM controllers which adopt a single integral term to construct their sliding surfaces can only reduce, but not fully alleviate the converter's steady-state regulation error [20]. To solve this problem, a sliding surface with an additional integral term can be employed [20], [21]. However, the sliding surfaces used in [20] and [21] not only increase the order of the controller but also demand the use of more states like two currents for their implementation.

Ideally, the controller should be of a lower-order to reduce the cost and for ease of implementation. Also, the least number of currents should be used in feedback control as this will avoid a complex current sensing circuitry. Thus, the challenge is to design a suitable fixed-frequency PWM-based SM controller using a reduced number of state variables without compromising the transient and steady-state specifications of the system. In [22], a sliding-surface composed of state variables and integral terms of the output error is presented for the regulation of the high-order dc-dc converter.

Apart from this, as compared to the conventional boost converter wherein there is only one inductor current available for feedback purposes, the newer higher-order converters like the quadratic boost converter present certain challenges. As there are now more variables to consider, it is important to find the most suitable state variables to be used in feedback control. Some works on these aspects were

presented when the current-mode controlled higher-order boost-type dc-dc converters were discussed Chapter 3.

Considering all this, the main research aspect of this chapter is to investigate the use of a fixed-frequency, pulse-width-modulation PWM-based SM controller for regulating the quadratic boost converter which can be implemented using a reduced number of state variables. The proposed controller requires only one current sensor for its implementation while enjoying the advantages offered by both the fixed-frequency and double-integral approaches. Also, two PWM-based SM controllers using the input and output inductor currents of the converter are separately designed to find the most suitable inductor current for the regulation of the converter. It is shown that the PWM-based SM controller designed using the input inductor current is most appropriate for regulating the quadratic boost converter. Some simulation and experimental results are also provided to validate these theoretical conclusions.

Next, a SM controller based on the hysteresis-modulation (HM) based approach is considered for the regulation of high-order dc-dc converters. This approach offers several advantages like no risk of saturation while operating at high values of the duty ratio and ease of implementation [5, 43, 55]. Even though the HM-based SM controllers have been applied to the regulation of dc-dc converters, there are certain issues which need to be addressed when the control approach is applied to high-order dc-dc converters like the hybrid-type dc-dc boost converter.

Firstly, as there are now more state variables available for feedback purposes in high-order dc-dc converters, which are the most suitable state variables for designing the sliding surface? Is there any condition under which the SM

controller designed using a specific inductor current fails to work? If there is, then how do we overcome it? All these questions demand further research. To the best of our knowledge, there is no systematic procedure available for selecting the most appropriate state variables for designing the widely used HM-based SM controllers for the high-order boost-type dc-dc converters. In view of this, another objective of this chapter is to propose a systematic approach to find the most suitable inductor current to design the hysteresis-based SM controller for the hybrid-type high-order dc-dc boost converter. To this end, a detailed comparative study of two hysteresis-based SM controllers is carried out using the input and output inductor currents of the converter. Analyses are carried out using the classical frequency response approach. The theoretical results show that the HM-based SM controller using the input inductor current is more suitable for regulating the hybrid-type high-order dc-dc boost converter as compared to the HM-based SM controller designed using the output inductor current. Some simulation and experimental results are also provided to validate these theoretical conclusions.

4.2 Design of Fixed-frequency Pulse-width-modulation (PWM)-Based SM Controllers for the Quadratic Boost Converter

In this section, two fixed-frequency SM controllers based on the PWM-approach are presented together with their derivations and stability analyses. The controller using the input inductor current will be studied first and then followed by the controller using the output inductor current. Considering the non-minimum phase nature of the converter's control to output voltage transfer function, the control schemes are designed to achieve an indirect control of the output voltage via

inductor current tracking. The mathematical model of the quadratic boost converter, theoretical derivation of an equivalent control as well as the procedure to obtain the existence and stability conditions are given.

Fig. 4.1 shows the circuit diagram of the quadratic boost converter. It has two operating modes of conduction. When switch ‘S’ is on, diode D_1 is conducting which provides a path for the inductor current i_{L_1} to flow and diodes D_2 and D_3 are in off state. The energy is stored in the inductors L_1 and L_2 in the form of a magnetic field. When switch ‘S’ is off, diode D_1 is off and diodes D_2 and D_3 are now conducting to enable the flow of inductor current through them in order to charge the output capacitor C_2 .

Fig. 4.1 Circuit diagram of the quadratic boost converter.

The average state-space model of this converter operating in continuous conduction mode is described as follows [12]:

$$\frac{di_{L_1}}{dt} = -\frac{(1-u)}{L_1} v_{C_1} + \frac{E}{L_1} \quad (4.1)$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2} v_{C_1} - \frac{(1-u)}{L_2} v_{C_2} \quad (4.2)$$

$$\frac{dv_{C_1}}{dt} = \frac{(1-u)}{C_1} i_{L_1} - \frac{1}{C_1} i_{L_2} \quad (4.3)$$

$$\frac{dv_{C_2}}{dt} = \frac{(1-u)}{C_2} i_{L_2} - \frac{1}{RC_2} v_{C_2} \quad (4.4)$$

where $i_{L_1}, i_{L_2}, v_{C_1}, v_{C_2} = v_o$ and R are the average current of inductor L_1 , average current of inductor L_2 , average voltage of capacitor C_1 , average voltage of capacitor C_2 , and load resistance, respectively. The symbol u denotes the switch control signal which is given as $u = (0, 1)$. It should be noted that the model is obtained under ideal conditions, i.e., the parasitic resistances $R_{C_1}, R_{C_2}, R_{L_1}$, and R_{L_2} of C_1, C_2, L_1 , and L_2 respectively are assumed to be negligible in the circuit description.

4.2.1 Design of PWM-based SM Controller Using Input-inductor Current

The sliding-surface using the input inductor current is given by:

$$s_1 = i_{L_1} - I_{ref1} \quad (4.5a)$$

In practical systems, the load resistance ‘ R ’ and the input voltage ‘ E ’ of the dc-dc converter can vary and such variations can also change the value of an inductor current reference. Therefore, in order to design the controller which is robust against load disturbances, a dynamic inductor current reference (I_{ref1}) is used.

The inductor current reference is given by

$$I_{ref1} = -mv_{C_2}(t) - n \int (\beta v_{C_2}(t) - V_{ref})dt - p \iint (\beta v_{C_2}(t) - V_{ref})dt dt \quad (4.5b)$$

where m, n, p are positive coefficients, $0 < \beta < 1$ is the scaling factor, and $V_{ref} = \beta V_d$ with V_d being the desired output voltage. The integral action is mainly used to reduce the steady-state output error.

Setting the derivative of the sliding surface s_1 to zero which satisfies the condition for the steady-state operation and using (4.1), (4.4), and (4.5), the equivalent control law can be obtained as:

$$u_{eq1} = \frac{G_s(v_{C1}-E) - K_1(\beta v_{C2} - V_{ref}) - K_2 \int (\beta v_{C2}(t) - V_{ref}) dt - K_3 i_{C2}}{G_s v_{C1}} \quad (4.6)$$

where $K_1 = nL_1$, $K_2 = pL_1$, and $K_3 = (mL_1/C_2)$. Apart from this, $0 < G_s < 1$ is added to conform to the standard of the chip level implementation by scaling the equation downwards. Also, u_{eq1} is continuous and $0 < u_{eq1} < 1$ should be satisfied. It should be noted that (4.6) can be rewritten in terms of the initially chosen state variables like v_{C1} and v_{C2} . As such, it can be implemented using only two state variables. However, the resulting complex form of an equivalent control further complicates its implementation and the use of an instantaneous state variable i_{C2} minimizes such complexity. Here, i_{C2} is the current flowing through the capacitor C_2 .

The fulfillment of the existence condition confirms that the state trajectory in the neighbourhood of the sliding surface will always be directed towards the sliding surface. Since satisfying the local reachability condition ensures the fulfillment of the existence condition, evaluating $\lim_{s \rightarrow 0} s_1 \dot{s}_1 < 0$, and using (4.5) and its derivative gives the necessary existence condition as:

$$\frac{E(\min)}{L_1} + m \frac{i_{C2}(\min)}{C_2} + n(\beta v_{C2}(\min)(t) - V_{ref}) + p \int (\beta v_{C2}(\min)(t) - V_{ref}) dt > 0 \quad (4.7)$$

$$\frac{E_{(max)} - v_{C_1(min)}}{L_1} + m \frac{i_{C_2(max)}}{C_2} + n(\beta v_{C_2(max)}(t) - V_{ref}) + p \int (\beta v_{C_2(max)}(t) - V_{ref}) dt < 0 \quad (4.8)$$

where $v_{C_1(min)}$ represents the minimum value of the voltage of capacitor C_1 , $i_{C_2(min)}$ and $i_{C_2(max)}$ are the minimum and maximum currents flowing through the output capacitor C_2 , $v_{C_2(min)}$ and $v_{C_2(max)}$ denote the minimum and maximum values of the output voltage, and $E_{(min)}$ and $E_{(max)}$ represent the minimum and maximum values of the input voltage of the converter.

Substituting the equivalent control (4.6) into (4.1) – (4.4) gives the following ideal sliding dynamics of the converter:

$$\frac{di_{L_1}}{dt} = -\frac{v_{C_1}}{L_1} \left(\frac{G_s E + K_1 (\beta v_{C_2} - V_{ref}) + K_2 \sigma \beta + K_3 i_{C_2}}{G_s v_{C_1}} \right) + \frac{E}{L_1} \quad (4.9)$$

$$\frac{di_{L_2}}{dt} = \frac{1}{L_2} v_{C_1} - \frac{v_{C_2}}{L_2} \left(\frac{G_s E + K_1 (\beta v_{C_2} - V_{ref}) + K_2 \sigma \beta + K_3 i_{C_2}}{G_s v_{C_1}} \right) \quad (4.10)$$

$$\frac{dv_{C_1}}{dt} = \frac{i_{L_1}}{C_1} \left(\frac{G_s E + K_1 (\beta v_{C_2} - V_{ref}) + K_2 \sigma \beta + K_3 i_{C_2}}{G_s v_{C_1}} \right) - \frac{1}{C_1} i_{L_2} \quad (4.11)$$

$$\frac{dv_{C_2}}{dt} = \frac{i_{L_2}}{C_2} \left(\frac{G_s E + K_1 (\beta v_{C_2} - V_{ref}) + K_2 \sigma \beta + K_3 i_{C_2}}{G_s v_{C_1}} \right) - \frac{1}{RC_2} v_{C_2} \quad (4.12)$$

$$\frac{d\sigma}{dt} = (v_{C_2} - V_d) \quad (4.13)$$

The equilibrium point of (4.9) – (4.13) can be obtained as:

$$I_{L_1} = \frac{V_d^2}{RE}, \quad I_{L_2} = \sqrt{\frac{V_d^3}{R^2 E}}, \quad V_{C_1} = \sqrt{EV_d}, \quad V_{C_2} = V_d, \quad \Sigma = 0 \quad (4.14)$$

where I_{L_1} , I_{L_2} , V_{C_1} , V_d and Σ are the equilibrium values of i_{L_1} , i_{L_2} , v_{C_1} , v_{C_2} and

σ , respectively. Also, $U = 1 - \sqrt{\frac{E}{V_d}}$ represents the steady-state value of the

control signal u .

Now, assuming the static equilibrium conditions given by $V_{C_1} \gg \tilde{v}_{C_1}$ (where superscript ‘ $\sim$ ’ represents small signal variations around the steady-state equilibrium point), $\beta V_{C_2} - V_{ref} = 0$, $i_{C_2} = C_2(d\tilde{v}_{C_2}/dt) = (1 - U)\tilde{i}_{L_2} - \tilde{v}_{C_2}/R$ and considering only the ac terms, the linearization of the ideal sliding dynamics (4.9) – (4.13) around the equilibrium point given by (4.14) lead to the following system:

$$\dot{z} = Mz \quad (4.15)$$

where, $z = [\tilde{i}_{L_1} \ \tilde{i}_{L_2} \ \tilde{v}_{C_1} \ \tilde{v}_{C_2} \ \tilde{\sigma}]^T$, $\tilde{i}_{L_1} = i_{L_1} - I_{L_1}$, $\tilde{i}_{L_2} = i_{L_2} - I_{L_2}$, $\tilde{v}_{C_1} = v_{C_1} - V_{C_1}$, $\tilde{v}_{C_2} = v_{C_2} - V_{C_2}$, $\tilde{\sigma} = \sigma - \Sigma$, and

$M =$

$$\begin{bmatrix} 0 & -\frac{K_3}{G_s L_1} \sqrt{\frac{E}{V_d}} & 0 & -\frac{K_1 \beta}{G_s L_1} + \frac{K_3}{R G_s L_1} & -\frac{K_2 \beta}{G_s L_1} \\ 0 & -\frac{K_3 (\sqrt{E/V_d}) V_d}{G_s L_2 \sqrt{E V_d}} & \frac{1}{L_2} & \sqrt{\frac{V_d}{E}} \left(-\frac{E}{L_2 V_d} - \frac{K_1 \beta}{G_s L_2} + \frac{K_3}{R G_s L_2} \right) & -\frac{K_2 \beta V_d}{G_s L_2 \sqrt{E V_d}} \\ \frac{1}{C_1} \sqrt{\frac{E}{V_d}} & \frac{K_3 V_d}{R G_s C_1 E} - \frac{1}{C_1} & \frac{V_d}{R E C_1} & \frac{V_d \sqrt{V_d/E}}{R E G_s C_1} \left(K_1 \beta - \frac{K_3}{R} \right) & \frac{V_d K_2 \beta}{R E G_s C_1} \\ 0 & \frac{1}{C_2} \left(\frac{K_3 \sqrt{V_d/E}}{R G_s} + \sqrt{\frac{E}{V_d}} \right) & 0 & \frac{V_d}{R E G_s C_2} \left(K_1 \beta - \frac{K_3}{R} \right) - \frac{1}{R C_2} & \frac{K_2 \beta V_d}{G_s C_2 R E} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here, M is the system matrix of the linearized system and for the sake of system stability analysis, $\tilde{v}_{ref}$ is assumed to be zero. The linearized system will be stable if all the eigenvalues of M , i.e., the roots of $|sI - M| = 0$, where s is a complex variable, lie in the open left-half of the complex s -plane. Thus, for a given set of converter parameter values, the controller gains K_i ($i = 1, 2$ and 3) can be chosen to ensure that the real parts of the eigenvalues of M are negative. The root locus method can be used to analyze system stability.

For the purpose of an illustration, consider the quadratic boost converter with the following circuit parameter values:

$$E = 5 \text{ V}, \quad V_d = 10 \text{ V}, \quad L_1 = 3.3 \text{ mH}, \quad L_2 = 3.3 \text{ mH}, \quad C_1 = 470 \text{ } \mu\text{F}, \quad C_2 = 330 \text{ } \mu\text{F}, \quad R = 470 \text{ } \Omega \quad (4.16)$$

Fig. 4.2(a) shows the root locus plot of $m(s) = |sI - M|$ for $K_2 = 20$, $K_3 = 0.8$, $1 \leq K_1 \leq 4$ and using $\beta = 1/15$ and $G_s = 1/6$. The arrows show how the poles are moving for increasing values of the controller gain K_1 . The system is stable for all values of K_1 in this range.

Fig. 4.2 Root locus plots of the closed-loop poles using $K_2 = 20$, $K_3 = 0.8$, $1 \leq K_1 \leq 4$ and $\beta = 1/15$ and $G_s = 1/6$: (a) using input inductor current based sliding-mode controller; and (b) using output inductor current based sliding-mode controller.

4.2.2 Design of PWM-based SM Controller Using Output-inductor Current

When the output inductor current is used in the sliding-surface, (4.5a) is modified to give:

$$s_2 = i_{L_2} - I_{ref2} \quad (4.17a)$$

The structure of (4.17a) is similar to that of (4.5a) except that the output inductor current is used in the sliding surface. Also, I_{ref2} is the dynamic output inductor current reference given by:

$$I_{ref2} = -mv_{C_2}(t) - n \int (\beta v_{C_2}(t) - V_{ref}) dt - p \iint (\beta v_{C_2}(t) - V_{ref}) dt d\lambda \quad (4.17b)$$

Using $\dot{s}_2 = 0$ which satisfies the steady-state condition and using (4.2), (4.4), and (4.17), an equivalent control law can now be obtained as:

$$u_{eq2} = \frac{G_s(v_{C_2} - v_{C_1}) - K_1(\beta v_{C_2} - V_{ref}) - K_2 \int (\beta v_{C_2}(t) - V_{ref}) dt - K_3 i_{C_2}}{G_s v_{C_2}} \quad (4.18)$$

where $K_1 = nL_2$, $K_2 = pL_2$, and $K_3 = (mL_2/C_2)$. Also, u_{eq2} is continuous and $0 < u_{eq2} < 1$ should be satisfied. Like what was done previously for the case of the controller designed using the input inductor current, substituting (4.18) into (4.1) and (4.4) yields a set of equations with a unique equilibrium point (4.14).

The linearized model has a system matrix N given by:

$$N =$$

$$\begin{bmatrix} 0 & -\frac{K_3 E}{G_s L_1 V_d} & -\frac{2}{L_1} \sqrt{\frac{E}{V_d}} & -\sqrt{\frac{E}{V_d}} \left(\frac{K_1 \beta}{G_s L_1} - \frac{K_3}{R G_s L_1} \right) & -\frac{K_2 \beta}{G_s L_1} \sqrt{\frac{E}{V_d}} \\ 0 & -\frac{K_3 (\sqrt{E/V_d})}{G_s L_2} & 0 & -\frac{K_1 \beta}{G_s L_2} + \frac{K_3}{R G_s L_2} & -\frac{K_2 \beta}{G_s L_2} \\ \frac{\sqrt{E}}{C_1 \sqrt{V_d}} & \frac{K_3 \sqrt{V_d}}{R G_s C_1 \sqrt{E}} - \frac{1}{C_1} & \frac{V_d}{R E C_1} & \frac{V_d K_1 \beta}{R E G_s C_1} - \frac{K_3 V_d}{R^2 E G_s C_1} & \frac{V_d K_2 \beta}{R E G_s C_1} \\ 0 & \frac{K_3}{G_s C_2 R} + \frac{1}{C_2} \sqrt{\frac{E}{V_d}} & \frac{\sqrt{V_d}}{C_1 R \sqrt{E}} & \sqrt{\frac{V_d}{E}} \left(\frac{K_1 \beta}{G_s C_2 R} - \frac{K_3}{G_s C_2 R^2} \right) - \frac{1}{R C_2} & \frac{K_2 \beta}{G_s C_2 R} \sqrt{\frac{V_d}{E}} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here, it has been assumed that $V_{C_2} \gg \tilde{v}_{C_2}$, $\beta V_{C_2} - V_{ref} = 0$, $i_{C_2} = C_2(d\tilde{v}_{C_2}/dt) = (1 - U)\tilde{i}_{L_2} - \tilde{v}_{C_2}/R$.

Using the same set of circuit parameter values given by (4.16), the root locus plots can be easily obtained. Fig. 4.2(b) shows the root locus plot of $n(s) = |sI - N|$ for $K_2 = 20$, $K_3 = 0.8$, $\beta = 1/15$, $G_s = 1/6$ and $1 \leq K_1 \leq 4$. It can be seen that the location of the poles have been changed as compared to that in Fig. 4.2(a). Now, there are some poles in the RHS of the s-plane. Even though we assume that due to the parasitic resistance effects, the positions of the unstable poles could slightly vary as compared to that shown in Fig. 4.2 (b) and the poles eventually end-up lying in the LHS of the s-plane, it is evident that the less dominant poles in Fig. 4.2(a) have been moved closer to the imaginary axis in Fig. 4.2(b). This could affect the overall transient response of the system. The settling time of the response is inversely proportional to the real part of the poles [56]. Thus, the settling time of the transient response of the system could increase when SM controller based on the output inductor current is used. As such, the SM controller based on the input inductor current is more suitable as compared to the one designed using the output inductor current.

4.3 A Comparative Study of Hysteresis-Modulation (HM)-Based SM Controllers for the Hybrid-type dc-dc Boost Converter

In this section, two SM controllers using the input and output inductor currents of the converter are analysed using the frequency-response approach. The controller using the output inductor current will be considered first and its shortcomings are demonstrated. Fig. 4.3(a) shows the circuit diagram of the

hybrid high-order dc-dc boost converter. As compared to the conventional boost converter, it has an additional step-up switching structure which mainly consists of two capacitors (C_1, C_2) and two diodes (D_1, D_2). The load-side inductor L_2 is added to reduce the output current ripple content [2].

Fig. 4.3 Hybrid-type high-order dc-dc boost converter and its operating modes: (a) Circuit diagram; (b) 'Mode 1' of operation when $u = 1$; and (c) 'Mode 2' of operation when $u = 0$.

The reduced-order averaged state-space model of the converter given by [48]:

$$\frac{di_{L_1}}{dt} = -\frac{(1-u)}{L_1}v_c + \frac{E}{L_1} \quad (4.19)$$

$$\frac{di_{L_2}}{dt} = \frac{1+u}{L_2}v_c - \frac{1}{L_2}v_o \quad (4.20)$$

$$\frac{dv_c}{dt} = \frac{(1-u)}{2C}i_{L_1} - \frac{1+u}{2C}i_{L_2} \quad (4.21)$$

$$\frac{dv_o}{dt} = \frac{1}{C_o}i_{L_2} - \frac{1}{RC_o}v_o \quad (4.22)$$

The symbol u denotes the control signal of the MOSFET switch such that $u = (0,1)$. The equilibrium point of (4.19) – (4.22) can be obtained as:

$$I_{L_1} = \frac{V_d^2}{RE}, \quad I_{L_2} = \frac{V_d}{R}, \quad V_C = \frac{V_d+E}{2}, \quad U = \frac{V_d-E}{V_d+E} \quad (4.23)$$

where V_d is the desired value of the output voltage v_o and I_{L_1} , I_{L_2} , V_C , and U represents the steady-state values of the i_{L_1} , i_{L_2} , v_c , and u respectively. The problem at hand is to find a suitable hysteresis-based SM controller to regulate the output voltage of the hybrid high-order dc-dc boost converter in the presence of an uncertain load.

4.3.1 Design of HM-based SM Controller Using Output-inductor Current

The dual-loop control scheme used in this chapter is just one form of an indirect controller which consists of an inner-loop controller controlling the output inductor current and the reference signal for the inner-loop is generated at the output of a proportional-integral (PI) compensator processing the output voltage error (see Fig.4.4) [5].

Fig. 4.4 A multi-loop control scheme [5].

The control scheme does not require any external current reference signal and as such it gives a robust performance against load and input voltage disturbances.

The sliding surface using the output inductor current is given by:

$$s(x) = i_{L_2} - I_{RO}(t) \quad (4.24)$$

where $I_{RO}(t)$ is the reference output inductor current trajectory. This trajectory is generated using a conventional PI controller of the form:

$$I_{RO}(t) = K_{P1}(V_d - v_o(t)) + K_{I1} \int (V_d - v_o(\tau)) d\tau \quad (4.25)$$

where K_{P1} and K_{I1} are the gains of the PI controller.

The necessary switching function u_1 for the SM controller implementation can be written as:

$$u_1 = \begin{cases} 0 & \text{when } s(x) > 0 \\ 1 & \text{when } s(x) < 0 \end{cases} \quad (4.26)$$

For the successful operation of the SM control action to occur, an existence condition must be satisfied. The fulfillment of the existence condition confirms that the state trajectory in the neighbourhood of the sliding surface will always be directed towards the sliding surface. Evaluating $\lim_{s \rightarrow 0} s(x)\dot{s}(x) < 0$, and using (4.20) and (4.24) gives the necessary existence condition as:

$$0 < v_o - v_c + L_2 \frac{dI_{RO}}{dt} < v_c \quad (4.27)$$

Next, the expression for an equivalent control law of the SM controller is derived by equating $\dot{s} = 0$. Using (4.20) and (4.24) in $\dot{s} = 0$ yields:

$$u_{eq1} = \frac{v_o - v_c}{v_c} + \frac{1}{v_c} L_2 \frac{dI_{RO}(t)}{dt} = -1 + \frac{1}{v_c} \left(v_o + L_2 \frac{dI_{RO}(t)}{dt} \right) \quad (4.28)$$

where u_{eq1} is continuous and $0 < u_{eq1} < 1$ should be satisfied. Substituting (4.28) into the converter model given by (4.19) and (4.21)-(4.22), the resulting ideal sliding dynamics of the converter is obtained as:

$$\frac{di_{L1}}{dt} = -\frac{1}{L_1} \left(2v_c - v_o - L_2 \frac{dI_{RO}(t)}{dt} \right) + \frac{E}{L_2} \quad (4.29)$$

$$\frac{dv_c}{dt} = \frac{i_{L1}}{2C} \left(2 - \frac{1}{v_c} \left(v_o + L_2 \frac{dI_{RO}(t)}{dt} \right) \right) - \frac{i_{L2}}{2C} \left(\frac{1}{v_c} \left(v_o + L_2 \frac{dI_{RO}(t)}{dt} \right) \right) \quad (4.30)$$

$$\frac{dv_o}{dt} = \frac{i_{L2}}{C_o} - \frac{1}{RC_o} v_o \quad (4.31)$$

The equilibrium point of (4.29) – (4.31) is given by:

$$\overline{i_{L1}} = I_{RO}^2 R/E, \quad \overline{i_{L2}} = I_{RO}, \quad \overline{v_c} = ((I_{RO} \cdot R) + E)/2, \quad \overline{v_o} = I_{RO} \cdot R \quad (4.32)$$

Linearizing the ideal sliding dynamics about the equilibrium point (4.32) and using (4.23), we can obtain the state-space representation of the linearized system described by:

$$\dot{\tilde{z}}_1 = M_1 \tilde{z}_1 + N_1 \tilde{u}_1 + P_1 \frac{d\tilde{u}_1}{dt} \quad (4.33)$$

$$\tilde{y} = Q_1 \tilde{z}_1 \quad (4.34)$$

where the small signal ac perturbations are represented by “ $\sim$ ” and $\tilde{u}_1 = \tilde{I}_{RO}$ is the linearized system input. Also, $\tilde{z}_1$ represents the small signal variations in the state variables, i.e., $\tilde{z}_1 = [\tilde{I}_{L1} \tilde{V}_c \tilde{V}_o]^T$ and $\tilde{y} = [\tilde{V}_o]$ is the corresponding system output. Matrices M_1, N_1, P_1 and Q_1 are given by:

$$M_1 = \begin{bmatrix} 0 & -\frac{2}{L_1} & \frac{1}{L_1} \\ -\frac{E}{C(V_d+E)} & \frac{2V_d^2}{RCE(V_d+E)} & -\frac{V_d}{RCE} \\ 0 & 0 & -\frac{1}{RC_o} \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0 \\ -\frac{V_d}{C(V_d+E)} \\ \frac{1}{C_o} \end{bmatrix}, \quad P_1 = \begin{bmatrix} \frac{L_2}{L_1} \\ -\frac{L_2 V_d}{RCE} \\ 0 \end{bmatrix},$$

$$Q_1 = [0 \quad 0 \quad 1] \quad (4.35)$$

The stability of system is now analysed in the frequency-domain. The necessary current-to-output transfer function is obtained by applying the Laplace transform to (4.33) – (4.35) and is given by:

$$\tilde{G}_{iO}(s) = \frac{\tilde{V}_o(s)}{\tilde{I}_{RO}(s)} = \frac{\left(s^2 - \frac{2V_d^2}{RCE(E+V_d)}s + \frac{2E}{CL_1(E+V_d)}\right)}{C_o\left(s + \frac{1}{RC_o}\right)\left(s^2 - \frac{2V_d^2}{RCE(E+V_d)}s + \frac{2E}{CL_1(E+V_d)}\right)} \quad (4.36)$$

As can be seen from (4.36), the current-to-output transfer function is unstable due to the presence of a negative coefficient in its denominator polynomial. The denominator polynomial has unstable poles which are located in the right half of the complex s-plane. Even though the roots of the numerator polynomial

$\left(s^2 - \frac{2V_d^2}{RCE(E+V_d)} s + \frac{2E}{CL_1(E+V_d)}\right)$ are almost exactly cancelled by the roots of the denominator polynomial $\left(s^2 - \frac{2V_d^2}{RCE(E+V_d)} s + \frac{2E}{CL_1(E+V_d)}\right)$, the stability of a transfer function must be checked without cancelling any common poles and zeros from the transfer function. In particular, systems with unstable pole-zero cancellations are unstable [57]. This is because, even if the exact pole-zero cancellation is theoretically possible, it may not be practically obtainable [58]. Also, if some external disturbances are introduced in such system with the unstable pole-zero cancellations, the output of the system could go unbounded [59]. Also, from (4.36) it is clear that the unstable pole-zero cancellation occurs for all circuit parameter values and thus the SM controlled system designing using the output inductor current for feedback purposes is unstable irrespective of the circuit parameter values.

For the purpose of illustration, consider the hybrid high-order dc-dc boost converter with the following circuit parameter values:

$$E = 5V, \quad V_d = 21.8V, \quad L_1 = L_2 = 680 \mu H, \quad C = C_0 = 220 \mu F, \quad R = 220 \Omega \quad (4.37)$$

Using (4.36) - (4.37), the corresponding current-to-output transfer function $\tilde{v}_0(s)/\tilde{I}_{RO1}(s)$ is obtained as:

$$\tilde{G}_{iO1}(s) = \frac{\tilde{v}_o(s)}{\tilde{I}_{RO1}(s)} = \frac{0.4545 \times 10^4 (s^2 - 146.6s + 2.49 \times 10^6)}{(s + 20.66)(s^2 - 146.6s + 2.49 \times 10^6)} \quad (4.38)$$

It can be seen that a negative polynomial coefficient in the denominator polynomial $(s^2 - 146.6s + 2.49 \times 10^6)$ leads to an unstable transfer function $\tilde{v}_0(s)/\tilde{I}_{RO1}(s)$. Also such instability is not easily observed from the

Bode plot as the roots of the numerator polynomial ($s^2 - 146.6s + 2.49 \times 10^6$) are almost exactly cancelled by the roots (unstable poles) of the denominator polynomial ($s^2 - 146.6s + 2.49 \times 10^6$) (see Fig. 4.5). However, the system is still unstable due to the cancellation of unstable poles from the denominator polynomial.

Fig.4.5 Bode plot of the current-to-output transfer function $\tilde{v}_0(s)/\tilde{I}_{RO1}(s)$ using the output inductor current feedback.

4.3.2 Design of HM-based SM Controller Using Input-inductor Current

The question now is this: Since the SM controller using the output inductor current is not suitable for the regulation of the hybrid high-order boost converter, then can the controller using the input inductor current be used for this purpose? The question is addressed in this section. When the control system is designed using the input inductor current for feedback purposes, the modified sliding-surface is given by:

$$s(x) = i_{L_1} - I_{RI}(t) \quad (4.39)$$

where $I_{RI}(t)$ is the reference input inductor current trajectory given by:

$$I_{RI}(t) = K_{P2}(V_d - v_o(t)) + K_{I2} \int (V_d - v_o(\tau)) d\tau \quad (4.40)$$

Here, K_{P2} and K_{I2} are the gains of PI controller. The structure of (4.39) and (4.40) is similar to that of (4.24) and (4.25) except that the input inductor current is used for feedback purposes. The necessary switching function for the SM controller implementation can be written as:

$$u_2 = \begin{cases} 0 & \text{when } s(x) > 0 \\ 1 & \text{when } s(x) < 0 \end{cases} \quad (4.41)$$

In order to ensure the successful operation of the SM controller to occur, the required existence conditions is obtained by evaluating $\lim_{s \rightarrow 0} s(x)\dot{s}(x) < 0$. Using

(4.19) and (4.39) gives the necessary existence condition as:

$$0 < E - L_1 \frac{dI_{RI}}{dt} < v_c \quad (4.42)$$

The expression for an equivalent control law of the proposed SM controller can be derived by equating $\dot{\sigma}_I = 0$. Using (4.19) and (4.39) in $\dot{s} = 0$, we get,

$$u_{eq2} = \frac{v_c - E}{v_c} + \frac{1}{v_c} L_1 \frac{dI_{RI}(t)}{dt} = 1 - \frac{1}{v_c} \left(E - L_1 \frac{dI_{RI}(t)}{dt} \right) \quad (4.43)$$

where u_{eq2} is continuous and $0 < u_{eq2} < 1$ should be satisfied. Substituting (4.43) into (4.20) – (4.22), the resulting ideal sliding dynamics of the modified system is obtained as:

$$\frac{di_{L2}}{dt} = \frac{1}{L_2} \left(2v_c - E + L_1 \frac{dI_{RI}(t)}{dt} \right) - \frac{1}{L_2} v_o \quad (4.44)$$

$$\frac{dv_{C1}}{dt} = \frac{i_{L1}}{2C} \left(\frac{E}{v_c} - \frac{L_1}{v_c} \frac{dI_{RI}(t)}{dt} \right) - \frac{i_{L2}}{2C} \left(2 - \frac{E}{v_c} + \frac{L_1}{v_c} \frac{dI_{RI}(t)}{dt} \right) \quad (4.45)$$

$$\frac{dv_o}{dt} = \frac{i_{L_2}}{C_o} - \frac{1}{RC_o} v_o \quad (4.46)$$

The equilibrium point of (4.44) – (4.46) is given by:

$$\begin{aligned} \bar{i}_{L_1} &= I_{RI}, \quad \bar{i}_{L_2} = (I_{RI} \cdot E / R)^{1/2}, \quad \bar{v}_c = ((I_{RI} \cdot E \cdot R)^{1/2} + E) / 2, \\ \bar{v}_o &= (I_{RI} \cdot E \cdot R)^{1/2} \end{aligned} \quad (4.47)$$

Linearizing (4.44) – (4.46) about equilibrium point (4.47) leads to the linearized system given by:

$$\dot{\tilde{z}}_2 = M_2 \tilde{z}_2 + N_2 \tilde{u}_2 + P_2 \frac{d\tilde{u}_2}{dt} \quad (4.48)$$

$$\tilde{y} = Q_2 \tilde{z}_2 \quad (4.49)$$

where $\tilde{u}_2 = \tilde{I}_{RI}$ is the linearized system input, $\tilde{z}_2 = [\tilde{i}_{L_2} \tilde{v}_c \tilde{v}_o]^T$ represents the new set of state variables and $\tilde{y} = [\tilde{v}_o]$ is the system output. Matrices M_2, N_2, P_2 and Q_2 are given by:

$$M_2 = \begin{bmatrix} 0 & \frac{2}{L_2} & -\frac{1}{L_2} \\ -\frac{V_d}{C(V_d+E)} & -\frac{2V_d}{RC(V_d+E)} & 0 \\ \frac{1}{C_o} & 0 & -\frac{1}{RC_o} \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 \\ \frac{E}{C(V_d+E)} \\ 0 \end{bmatrix}, \quad P_2 = \begin{bmatrix} \frac{L_1}{L_2} \\ -\frac{L_1 V_d}{RCE} \\ 0 \end{bmatrix},$$

$$Q_2 = [0 \quad 0 \quad 1] \quad (4.50)$$

By applying the Laplace transform to (4.48) – (4.49), the new current-to-output transfer function is obtained as:

$$\tilde{G}_{iI}(s) = \frac{\tilde{v}_o(s)}{\tilde{I}_{RI}(s)} = \frac{d(s)}{e(s)} = \frac{d_2 s^2 + d_1 s + d_0}{e_3 s^3 + e_2 s^2 + e_1 s + e_0} \quad (4.51)$$

where the coefficients $d_0 - d_2$ and $e_0 - e_3$ are given by:

$$\begin{aligned}
d_2 &= \frac{L_1}{C_o L_2} , \\
d_1 &= \frac{2V_d}{R C C_o} \left(\frac{L_1}{L_2(E+V_d)} - \frac{L_1}{L_2 E} \right) , \\
d_0 &= \frac{2E}{C C_o L_2 (E+V_d)} , \\
e_3 &= 1 , \\
e_2 &= \frac{1}{R C_o} + \frac{2V_d}{R C (E+V_d)} , \\
e_1 &= \frac{1}{C_o L_2} + \frac{2V_d}{C C_o R^2 (E+V_d)} + \frac{2V_d}{C L_2 (E+V_d)} , \\
e_0 &= \frac{4V_d}{C C_o L_2 R (E+V_d)} .
\end{aligned} \tag{4.52}$$

Using the same set of circuit parameters given by (4.37), the current-to-output transfer function $\tilde{v}_0(s)/\tilde{I}_{RI1}(s)$ is obtained as:

$$\tilde{G}_{iI1}(s) = \frac{\tilde{v}_o(s)}{\tilde{I}_{RI1}(s)} = \frac{0.4545 \times 10^4 (s^2 - 146.6s + 2.49 \times 10^6)}{(s+25.59)(s^2 + 28.68s + 1.75 \times 10^7)} \tag{4.53}$$

It is worth noting that, in contrast to (4.38), a stable transfer function $\tilde{v}_0(s)/\tilde{I}_{RI1}(s)$ is obtained when the SM controller is designed using the input inductor current.

Next, the voltage-loop is analyzed. The addition of an outer-loop compensator can drastically change the dynamics of the system and thus a separate analysis is required to ensure the overall closed-loop stability. Here, $\tilde{v}_0(s)/\tilde{I}_{RI1}(s)$, given by (4.53), is used as an open-loop transfer function while designing the controller for the outer voltage-loop (see Fig. 4.6).

Fig. 4.6 A closed-loop control scheme.

A simple proportional-integral (PI) controller of the form $G_{C1}(s) = K_{P2} + \frac{K_{I2}}{s}$ is considered for the voltage-loop. An integral action is mainly used to reduce the steady-state error in the output voltage. The controller parameters K_{P2} and K_{I2} are selected with the objectives of obtaining a high gain in the low-frequency region, to reduce the steady-state output error and obtaining an overall positive phase margin to ensure the closed-loop system stability. For $K_{P2} = 0.1$, $K_{I2} = 2$, the PI controller is given by:

$$G_{C1}(s) = \frac{0.1s + 2}{s} \quad (4.54)$$

Using a voltage feedback factor $\beta = 1/5$, the resulting overall closed-loop transfer function $\tilde{v}_o(s)/\tilde{V}_d(s)$ is obtained as:

$$\tilde{G}_v(s) = \frac{\tilde{v}_o(s)}{\tilde{V}_d(s)} = \frac{454.55(s+20)(s^2 - 146.6s + 2.494 \times 10^6)}{(s+8.655)(s+29.87)(s^2 + 106.7s + 1.754 \times 10^7)} \quad (4.55)$$

It should be noted that the polynomial $(s^2 - 146.6s + 2.49 \times 10^6)$ that is present in the transfer function $\frac{\tilde{v}_o(s)}{\tilde{I}_{R1}(s)}$ also appears in the closed-loop transfer function $\frac{\tilde{v}_o(s)}{\tilde{V}_d(s)}$. This polynomial, with non-minimum phase zeros, should not be cancelled when the loop is closed. It should be kept in the closed-loop transfer function to avoid unbounded control actions [60]. The transient response of the

system is mainly determined by the location of the zeroes which are closer to the dominant poles [56]. The controller is designed in such a way that a new zero ($s = -20$) in the closed-loop transfer function is located closer to the dominant pole ($s = -8.655$) as compared to the non-minimum phase zeros which have large real and imaginary parts and are located far away from the dominant pole.

4.4 Simulation and Experimental Results

In this section, some simulation and experimental results are presented to validate the theoretical conclusions obtained in Sections 4.2 and 4.3.

4.4.1 *Implementation of the PWM-based SM controller for the quadratic boost converter*

Some simulation and experimental results are provided to illustrate the features of the proposed PWM-based sliding mode (SM) controllers for the quadratic boost converter discussed in Section 4.2. The two SM controllers designed using the input and output inductor currents of the converter (given by (4.6) and (4.18) respectively) are referred to as ‘controller 1’ and ‘controller 2’ respectively. Fig. 4.7 shows the block diagram of the controlled system implemented using the ‘controller 1’. The implementation using the ‘controller 2’ is exactly similar to that using the ‘controller 1’, except that the feedback variables v_{C_1} and E are replaced by v_{C_2} and v_{C_1} respectively (see (4.18)). The same set of converter parameter values in section 4.2.1, given by (4.16), were used to obtain the results. For the sake of implementation, $\beta = 1/15$ and $G_s = 1/6$ were used.

was rapidly restored to the desired output voltage after the onset of load disturbances. However, both the overshoot as well as the settling time of the response were found to be have considerably increased when the ‘controller 2’ was employed.

Fig. 4.8 System simulation responses: (a) transient output response using ‘controller 1’; (b) transient output response using ‘controller 2’; (c) load change response for $R = 470 \Omega$ to $R = 235 \Omega$ (and vice-versa) using ‘controller 1’; and (d) load change response for $R = 470 \Omega$ to $R = 235 \Omega$ (and vice-versa) using ‘controller 2’.

Next, in order to obtain the experimental validation of the proposed PWM-based SM controllers, a prototype of the quadratic boost converter was built in the laboratory. Equations (4.6) and (4.18) were implemented using the simple analog components. The division function was implemented using the AD633 chip and LTS-6NP current sensor from LEM was used to sense the output capacitor current. In order to implement the PWM-based controller, the control signal for the switch was generated using a comparator IC LM311 and a triangular waveform of magnitude $1 V_{\text{peak-peak}}$ and frequency 20 kHz. The controller gains K_1 , K_2 , and K_3 used were 3.77, 8, and 0.8 respectively. Also, the measured equivalent series resistances (ESRs) of the components were 0.15Ω , 0.12Ω , 2.11Ω , and 2.08Ω for C_1 , C_2 , L_1 and L_2 respectively.

Figs. 4.9(a) and 4.9(b) show the output voltage transient response and corresponding control signal of the converter using ‘controller 1’ and ‘controller 2’ respectively. It can be seen that when ‘controller 1’ was employed, the response had a settling time of ~ 0.2 seconds with a little or no overshoot. However, as predicted theoretically, the transient response had a long settling time when ‘controller 2’ was employed. The ability of the controller to handle the load variations was then measured. Figs. 4.9(c) and 4.9(d) shows the output response and corresponding control signal of the converter when the load resistance was changed from $R = 470 \Omega$ to $R = 235 \Omega$ (and vice versa) using ‘controller 1’ and ‘controller 2’ respectively. All these results show that the proposed PWM-based SM controller can be suitably employed to regulate the output voltage of the quadratic boost converter.

Remark: Note that the controller parameters used to obtain experimental results are slightly different from those used to obtain simulations results. The controller parameters are required to tune a bit differently due to the presence of unmodelled dynamics as well as parasitic resistances in the experimental set-up.

Fig. 4.9 System experimental responses: (a) transient output response using ‘controller 1’; (b) transient output response using ‘controller 2’; (c) load change response for $R = 470 \, \Omega$ to $R = 235 \, \Omega$ (and vice-versa) using ‘controller 1’; and (d) load change response for $R = 470 \, \Omega$ to $R = 235 \, \Omega$ (and vice-versa) using ‘controller 2’.

Remark: As can be seen from Eqs. 4.6 and 4.18, the expressions of an equivalent control signal contains instantaneous values of the capacitor voltages v_{C_1} and v_{C_2} in the denominator respectively. During start-up, v_{C_1} and v_{C_2} are zero. Thus, the control signal may become very large initially. Therefore, there could be a spike in the control signal as observed in Fig. 4.9. When control signal increases, v_{C_1} and v_{C_2} will also increase. This will make the control signal to decrease again and finally the control signal will reach its steady-state value when the output voltage becomes equal to the desired reference voltage.

4.4.2 HM-based SM for the hybrid boost converter

In this sub-section, some simulation and experimental results are provided to validate the theoretical conclusions obtained in Section 4.3. The same set of converter parameter values as used in Section 4.3, given by (4.37), were used to obtain the results. The controller gains used were $K_{P1} = K_{P2} = 0.1$, and $K_{I1} = K_{I2} = 2$. Also, for implementation purposes, a voltage feedback factor $\beta = 1/5$ was used and the switching function given by (4.26) and (4.41) was implemented with the following modification:

$$u_{1s} = \begin{cases} 0 & \text{when } \sigma_o(x) > \delta \\ 1 & \text{when } \sigma_o(x) < -\delta \end{cases} \quad (4.56)$$

$$u_{2s} = \begin{cases} 0 & \text{when } \sigma_I(x) > \delta \\ 1 & \text{when } \sigma_I(x) < -\delta \end{cases} \quad (4.57)$$

Such a modification is required because an implementation of an ideal comparator demands the use of an infinite switching frequency which is quite

impossible considering the inherent limitation of the power devices to operate at such a high frequency. Also, an ideal comparator may produce false switching signals in case noisy input signals are present [5]. Considering this, the limits of the hysteresis band were set as $I_{RI}(\text{or } I_{RO}) + \delta$ and $I_{RI}(\text{or } I_{RO}) - \delta$ where a value of $\delta = 0.1 \text{ V}$ was selected. Fig. 4.10 shows the detailed circuit diagram of the SM controller implementation.

Fig. 4.10 Circuit diagram of the SM controller implementation.

In order to verify the accuracy of the theoretical results obtained in Section 4.3, the hysteresis-based control scheme was implemented in PSIM version 9.0.3. Initially, the output inductor current was used for feedback purposes and eqns. (4.24) - (4.26) were implemented. Figs. 4.11(a) and 4.11(b) show the output voltage response and corresponding output inductor current waveform of the converter in the presence of a change in the reference voltage from $V_d = 0 \text{ V}$ to

$V_d = 21.85 \text{ V}$ at $t = 1 \text{ sec}$. As predicted analytically, the response is unstable and this makes the output inductor current less suitable for feedback purposes.

Fig. 4.11 System simulation response using the hysteresis-based sliding mode controller based on the output inductor current: (a) Output voltage for load and reference-voltage variations; and (b) corresponding output inductor current waveform.

Fig. 4.12(a) shows the output response in the presence of some load and reference voltage variations when the SM controller was implemented using the input inductor current for feedback purposes (see (4.39) - (4.41)). Initially, the reference voltage was changed from $V_d = 0 \text{ V}$ to $V_d = 21.85 \text{ V}$ at $t = 1 \text{ sec}$. Then, the reference voltage was again changed to $V_d = 26.85 \text{ V}$ at $t = 2.5 \text{ sec}$, and finally back to $V_d = 21.85 \text{ V}$ at $t = 3.5 \text{ sec}$. It can be seen that the output voltage was able to track the changes in the reference voltage with a little or no overshoot and a very small settling time. Next, the load resistance was changed from $R = 220 \Omega$ to $R = 110 \Omega$ (a 50 % decrease) at $t = 4.5 \text{ sec}$ and then back to $R = 220 \Omega$ at $t = 6 \text{ sec}$. It can be seen that the output capacitor voltage was rapidly restored to the desired output voltage after the onset of load disturbances. Fig. 4.12(b) shows the corresponding input inductor current waveform for these reference and load variations.

Fig. 4.12 System simulation response using the hysteresis-based sliding mode controller based on the input inductor current: (a) Output voltage for load and reference-voltage variations; and (b) corresponding input inductor current waveform.

Next, a comparative study of the analytical and simulated Bode plots of the current-to-output transfer function was carried out to verify the accuracy of the analytical design of the proposed controller given in Section 4.3. The input inductor current was used for feedback purposes and Fig. 4.13 shows the corresponding Bode plot. The solid line shows the Bode plot of the current-to-output transfer function $\tilde{v}_0(s)/\tilde{I}_{RI1}(s)$ given by eq. (4.53). This transfer function is obtained using the analytical method of the controller design given in Section 4.3. The dotted line shows the Bode plot obtained using the actual SM controlled dc-dc converter circuit implemented in PSIM software. Here, the Bode plots were obtained using the ‘AC SWEEP’ block in PSIM. According to the principle of AC analysis, a small excitation signal is injected into the system at an input side and the signal at the same frequency is extracted at the output [53]. A good correspondence between the analytical Bode plot and the Bode plot obtained using the actual converter circuit validates the analytical design of the proposed controller to regulate the hybrid-type high-order dc-dc boost converter.

Fig. 4.13 Bode plot of the current-to-output transfer function (solid line: Bode plot obtained using the analytical method of the controller design, dotted line: Bode plot obtained using the actual SM controlled converter circuit implemented in PSIM software).

Finally, to obtain the experimental validation of the hysteresis-based SM controller, a prototype of the hybrid high-order boost converter was designed, built and tested in the laboratory. The controller of the form given by (4.24) - (4.26) was realized using the output inductor current for feedback purposes.

Fig. 4.14 shows the output voltage transient response and corresponding output inductor current of the converter using $V_d = 21.85 \text{ V}$. The output voltage was found to be unable to track the reference voltage. The controller was also implemented using the input inductor current for feedback purposes (eqns. (4.39) - (4.41)).

Ch.MATH : 10 V/div, 1 s/div, Ch.4 : 1 V/div, 1 s/div

Fig. 4.14 Output voltage response (red) and corresponding output inductor current (green) waveforms using the output inductor current for feedback purposes ($V_d = 21.85 \text{ V}$).

Fig. 4.15(a) shows the output voltage transient response and corresponding input inductor current of the converter for $V_d = 21.85 \text{ V}$. An almost critically damped output response was obtained with a negligible overshoot. Next, the ability of controller to handle the load disturbances was evaluated. Figs. 4.15(b) and 4.15(c) show the output voltage response and corresponding input inductor current waveform of the converter when the load resistance was changed from $R = 220 \Omega$ to $R = 440 \Omega$ (vice-versa) and from $R = 220 \Omega$ to $R = 720 \Omega$ (vice-versa) respectively. Again, it was found that the output capacitor voltage was rapidly restored to the desired output voltage after the onset of the load disturbances. Fig. 4.15(d) shows the output response in the presence of a reference voltage change from $V_d = 21.85 \text{ V}$ to $V_d = 10.35 \text{ V}$ and then back to $V_d = 21.85 \text{ V}$. All these results show that the SM controller designed using the input inductor current for feedback purposes is more suitable as compared to the one realized using the output inductor current for feedback purposes to regulate the hybrid high-order dc-dc boost converter.

Ch.MATH : 10 V/div, 1 s/div (Output voltage)

Ch.4 : 1 V/div, 1 s/div (input inductor current)

(a)

Ch.MATH : 10 V/div, 1 s/div (Output voltage)

Ch.4 : 1 V/div, 1 s/div (input inductor current)

(b)

Ch.MATH : 10 V/div, 1 s/div (Output voltage)

Ch.4 : 1 V/div, 1 s/div (input inductor current)

(c)

Ch.MATH : 10 V/div, 1 s/div (Output voltage)

Ch.4 : 1 V/div, 1 s/div (input inductor current)

(d)

Fig 4.15 System response using the input inductor current for feedback purposes: (a) Transient output voltage and input inductor current response for $V_d = 21.85V$; (b) Output voltage and input inductor current response for change in the load resistance from $R = 220\ \Omega$ to $R = 440\ \Omega$ and vice-versa; (c) Output voltage and input inductor current response for change in the load resistance from $R = 220\ \Omega$ to $R = 720\ \Omega$ and vice-versa; and (d) Output voltage and input inductor current response for change in the reference voltage from $V_d = 21.85V$ to $V_d = 10.35V$ and then back to $V_d = 21.85V$.

4.5 Summary

Initially, a fixed-frequency, PWM-based SM controller was proposed to achieve the output voltage regulation in the quadratic boost converter. The main advantage of the proposed controller is that as contrast to some of the state-of-the-art PWM-based SM controllers like in [20, 21], it requires only one current sensor for its implementation while enjoying the advantages offered of both the fixed-frequency and double integral approaches. Apart from that, two separate PWM-based SM controllers were designed for the quadratic boost converter to find the most suitable inductor current to design the sliding surface.

Next, a detailed comparative studies of two HM-based SM controllers (using the input and output inductor currents of the converter) was carried out for the hybrid-type high-order dc-dc boost converter. The results show that HM-based SM controller based on the input inductor current should be preferred over the controller based on the output inductor current. Finally, some simulation and experimental results were provided to validate the practical use of the proposed SM controllers to regulate the high-order boost-type dc-dc converters.

In next chapter, the design of a new adaptive current-mode controller for a high step-up dc-dc converter is presented.

CHAPTER 5 MODELING AND ADAPTIVE CURRENT-MODE CONTROL OF A HIGH STEP-UP DC-DC CONVERTER

In Chapter 3, some comparative studies of the state-of-the-art current-mode controllers were presented with an objective of finding the most suitable inductor current for feedback purposes. However, there are some issues associated with the controller structure of the state-of-the-art current-mode controllers used in Chapter 3 as well as those used in [38, 50]. In these type of the controllers, the output error due to the load disturbances is mainly compensated by an integral action acting on the output error. As such, a large value of an integral gain is required to achieve a good steady-state regulation under load variations. However, such a large value of the integral gain may increase the overshoot of the output response. If a small value of the integral gain is used to solve this problem, it may increase the settling time of the output response. Moreover, the control signals in such type of the controllers can have a very large magnitude. To solve these problems, an adaptive current-mode control of the high step-up dc-dc converter is presented in this chapter.

The high step-up dc-dc converter is a sixth-order dc-dc converter which can provide a high gain at some moderate values of the duty ratio. This converter has several other advantages. It maintains a low voltage stress across its diodes which allows the use of Schottkey rectifiers for reducing the reverse-recovery current problem. Also, this converter requires a lower voltage rated MOSFET switch which in turn improves the converter efficiency by reducing the switching, and conduction losses [23]. Despite such various advantages offered by the converter,

there is a scarcity of works addressing the regulation problem of this converter. Moreover, this converter offers an interesting case study on the modeling of dc-dc converters as our investigation suggests that, unlike many other dc-dc converter topologies, the parasitic resistance of the capacitors cannot be ignored while obtaining the average state-space model of the high step-up dc-dc converter. Considering this, a detailed modeling of the converter with parasitic elements is carried out and then an adaptive current-mode controller with an optimized $\frac{d\hat{\theta}}{dt}$ is proposed for the regulation of the converter. Here, $\hat{\theta}$ is the estimate of the inverse of the load resistance. An approximate stability analysis is also provided to gain an insight into the adaptive-controlled system. Finally, some experimental results comparing the performance of the proposed adaptive current-mode controller with that of the existing current-mode controller are presented.

5.1 Modeling of a High Step-up dc-dc Converter

Fig. 5.1 shows the circuit of the high step-up dc-dc converter with some parasitic resistances.

Fig. 5.1 A high-step up dc-dc converter and its operating modes: (a) Circuit diagram; (b) 'Stage 1' of operation when switch S is off; and (c) 'Stage 2' of operation when switch S is on.

Assuming that the converter is operating in the continuous conduction mode, the circuit operation can be divided into two stages, namely, 'Stage 1' of the operation when switch ' S ' is off (see Fig. 5.1(b)) and 'stage 2' of the operation

when switch ‘S’ is on (see Fig. 5.1(c)). Here, i_{L_1} , i_{L_2} , v_C , v_{C_1} , v_{C_2} and v_o represent the average currents of inductors L_1 and L_2 and the average voltages of capacitors C , C_1 , C_2 and C_0 respectively. Also, r_C , r_{C_1} and r_{C_2} represent the parasitic resistances of the capacitors C , C_1 , and C_2 , respectively. It should be noted that, without considering the parasitic resistances of the capacitors, it is not possible to obtain the derivatives of v_C , v_{C_1} , and v_o in terms of the state variables i_{L_1} , v_C , v_{C_1} , and v_o in ‘stage 2’, i.e., it is not possible to obtain the average state-space model of the converter.

5.1.1 Averaged State-space Model of the Converter

To obtain the average state-space model of the converter, the following assumptions have been adopted:

- 1) Inductors L_1 and L_2 have the same value ($L_1 = L_2$) and capacitors C_1 and C_2 have the same value and equal parasitic resistance value ($C_1 = C_2$ and $r_{C_1} = r_{C_2}$).
- 2) The turn-on and turn-off transitions of all diodes are synchronous with the switching transitions of the power MOSFET Switch ‘S’.
- 3) The switching frequency (F_s) is much larger than the natural frequency of the converter dynamics.

‘Stage 1’: During this stage of the operation, switch S is turned off (Fig. 5.1(b)) and the two diodes D_{o1} and D_{o2} are forward biased. This provides a path for inductor currents i_{L_1} and i_{L_2} to flow and the energy stored in inductors L_1 and L_2 is transferred to the capacitors C_1 and C_2 to charge them. On the other hand, the capacitor C_0 is discharged to provide the energy to the load R . Since the inductor currents flowing through the two inductors L_1 and L_2 are same (i.e., $i_{L_1} = i_{L_2}$),

their dynamics can be represented by the same differential equation. Similarly, the two capacitors C_1 and C_2 are effectively in parallel and thus the dynamics of v_{C_1} and v_{C_2} can be combined to form a single differential equation. The differential equations for this stage of the operation are given by:

$$\frac{di_{L_1}}{dt} = -\frac{\beta}{2L_1}i_{L_1} + \frac{1}{2L_1}v_C - \frac{1}{2L_1}v_{C_1} \quad (5.1)$$

$$\frac{dv_C}{dt} = -\frac{1}{C}i_{L_1} \quad (5.2)$$

$$\frac{dv_{C_1}}{dt} = \frac{1}{2C_1}i_{L_1} \quad (5.3)$$

$$\frac{dv_o}{dt} = -\frac{1}{RC_o}v_o \quad (5.4)$$

where $\beta = r_C + r_{C_1}/2$.

‘Stage 2’: In this stage, the switch ‘S’ is turned on and the equivalent circuit diagram is shown in Fig. 5.1(c). The input voltage E is charging the capacitor C whereas the two capacitors C_1 and C_2 provide energy to the load. The differential equations representing this stage of operation can be written as:

$$\frac{di_{L_1}}{dt} = \frac{1}{L_1}E \quad (5.5)$$

$$\frac{dv_C}{dt} = -\frac{1}{r_C C}v_C + \frac{1}{r_C C}E \quad (5.6)$$

$$\frac{dv_{C_1}}{dt} = -\frac{1}{r_{C_1} C_1}v_{C_1} + \frac{1}{2r_{C_1} C_1}v_{C_o} - \frac{1}{2r_{C_1} C_1}E \quad (5.7)$$

$$\frac{dv_o}{dt} = \frac{1}{r_{C_1} C_o}v_{C_1} - \left(\frac{1}{RC_o} + \frac{1}{2r_{C_1} C_o}\right)v_o + \frac{1}{2r_{C_1} C_o}E \quad (5.8)$$

Applying the generalized state-space averaging technique [10] to (5.1) – (5.8) yields the following generalized averaged state-space model of the converter given by:

$$\frac{di_{L_1}}{dt} = -\frac{\beta(1-u)}{2L_1}i_{L_1} + \frac{1-u}{2L_1}v_C - \frac{1-u}{2L_1}v_{C_1} + \frac{u}{L_1}E \quad (5.9)$$

$$\frac{dv_C}{dt} = -\frac{1-u}{C}i_{L_1} - \frac{u}{r_C C}v_C + \frac{u}{r_C C}E \quad (5.10)$$

$$\frac{dv_{C_1}}{dt} = \frac{1-u}{2C_1}i_{L_1} - \frac{u}{r_{C_1}C_1}v_{C_1} + \frac{u}{2r_{C_1}C_1}v_{C_o} - \frac{u}{2r_{C_1}C_1}E \quad (5.11)$$

$$\frac{dv_o}{dt} = \frac{u}{r_{C_1}C_o}v_{C_1} - \frac{1}{C_o}\left(\frac{1}{R} + \frac{u}{2r_{C_1}}\right)v_o + \frac{u}{2r_{C_1}C_o}E \quad (5.12)$$

where the symbol u denotes the control signal of the switch ‘S’ such that $u \in (0,1)$.

By setting (5.9) - (5.12) to zero, the following equilibrium values are obtained:

$$I_{L_1} = \frac{2EU(U+3)}{(1-U)(-RU^2+RU+4r_C+2r_{C_1})}, \quad V_C = \frac{E(2r_{C_1}-2r_C+RU-2r_CU-RU^2)}{(-RU^2+RU+4r_C+2r_{C_1})},$$

$$V_{C_1} = \frac{E(2r_{C_1}-2r_C+RU+r_{C_1}U+RU^2)}{(-RU^2+RU+4r_C+2r_{C_1})}, \quad V_o = \frac{ERU(U+3)}{(-RU^2+RU+4r_C+2r_{C_1})} \quad (5.13)$$

Here, U, I_{L_1}, V_C, V_{C_1} and V_o represent the equilibrium values of u, i_{L_1}, v_C, v_{C_1} and v_o respectively. A simplification of the expressions of the equilibrium values can be done if it is assumed that $\frac{r_C}{R} \approx 0$ and $\frac{r_{C_1}}{R} \approx 0$. Thus, from the expression of V_o in (5.13), setting V_o at the desired value V_d , the approximate value of U can be written as:

$$U_a = \frac{V_d-3E}{V_d+E} \quad (5.14)$$

Using (5.14) in (5.13) and using $\frac{r_C}{R} \approx 0$ and $\frac{r_{C_1}}{R} \approx 0$ gives:

$$I_{L_{1a}} = \frac{V_d(V_d+E)}{2RE}, \quad V_{Ca} = E, \quad V_{C_{1a}} = \frac{V_d-E}{2} \quad (5.15)$$

where $I_{L_{1a}}$, V_{Ca} and $V_{C_{1a}}$ are the approximate values of I_{L_1} , V_C and V_{C_1} respectively.

5.1.2 Model Validation

To verify the accuracy of the average state-space model of the converter using the frequency-domain technique, the generalized small-signal model of the converter is derived by linearizing (5.9) - (5.12) around the equilibrium point given by (5.14) and (5.15). If some small ac perturbations are introduced in the system about the steady-state equilibrium point, the resulting output response can be decomposed into two parts, the ac terms and the dc terms. Neglecting the dc terms (since their derivatives are zero), and focusing on the ac behavior, a small-signal linear model of the high step-up dc-dc converter can be obtained as:

$$\dot{\tilde{x}} = a\tilde{x} + b\tilde{u} \quad (5.16)$$

$$\tilde{y} = c\tilde{x} \quad (5.17)$$

where $\tilde{u}$ represents the small signal variations in the duty cycle u i.e., $u = U_a + \tilde{u}$ and $\tilde{x}$ represents the small signal variations in the state variables such that $\tilde{x} = [\tilde{i}_{L_1} \ \tilde{v}_C \ \tilde{v}_{C_1} \ \tilde{v}_o]^T$. Also, $\tilde{y} = [\tilde{v}_o]$ denotes the small signal variations in the output voltage. Matrices a , b , and c are given by:

$$a = \begin{bmatrix} -\frac{\beta(1-U_a)}{2L_1} & \frac{1-U_a}{2L_1} & -\frac{1-U_a}{2L_1} & 0 \\ -\frac{1-U_a}{C} & -\frac{U_a}{r_C C} & 0 & 0 \\ \frac{1-U_a}{2C_1} & 0 & -\frac{U_a}{r_{C_1} C_1} & \frac{U_a}{2r_{C_1} C_1} \\ 0 & 0 & \frac{U_a}{r_{C_1} C_o} & -\frac{U_a}{2r_{C_1} C_o} - \frac{1}{RC} \end{bmatrix}$$

$$b = \begin{bmatrix} \frac{1}{2L_1}(\beta I_{L_{1a}} - V_{Ca} + V_{C_{1a}} + 2E) \\ \frac{1}{r_C C} (r_C I_{L_{1a}} - V_{Ca} + E) \\ \frac{1}{2r_{C_1} C_1} (-r_{C_1} I_{L_{1a}} - 2V_{C_{1a}} + V_d - E) \\ \frac{1}{2r_{C_1} C_o} (2V_{C_{1a}} - V_d + E) \end{bmatrix}, \quad c^T = [0 \quad 0 \quad 0 \quad 1] \quad (5.18)$$

The control-to-output transfer function (TF) of the converter can now be easily obtained by applying the Laplace transform to (5.16) and (5.17) and using the zero initial conditions. The generalized formula to obtain this transfer function is given by:

$$\frac{\tilde{v}_0(s)}{\tilde{u}(s)} = c(sI - a)^{-1}b \quad (5.19)$$

where matrices a , b and c are given by (5.18).

Now, the Bode diagram obtained from the control-to-output transfer function (5.19) was compared to the Bode diagram obtained from the actual converter circuit implemented in PSIM software, version 9.0.3. The following circuit parameter values were used:

$$E = 3.3 \text{ V}, \quad V_d = 25 \text{ V}, \quad L_1 = L_2 = 1 \text{ mH}, \quad C = C_1 = C_2 = C_o = 68 \text{ } \mu\text{F}, \quad R = 2 \text{ k}\Omega, \quad r_C = r_{C_1} = r_{C_2} = 0.5 \text{ } \Omega, \quad F_s = 50 \text{ kHz} \quad (5.20)$$

The corresponding control signal-to-output transfer function $\tilde{v}_0(s)/\tilde{u}(s)$ is given by:

$$G(s) = \frac{\tilde{v}_0(s)}{\tilde{u}(s)} = \frac{-1.5462 \times 10^7 (s - 6.157 \times 10^4) (s + 3.923 \times 10^4)}{(s + 5.884 \times 10^4) (s + 3.919 \times 10^4) (s^2 + 101.5 s + 2.675 \times 10^4)} \quad (5.21)$$

where the superscript $\sim$ represents the small signal variations in the steady-state values such that $v_o = V_d + \tilde{v}_0$ and $u = U_a + \tilde{u}$. In Fig. 5.2, the curves in solid

are obtained from the transfer function (5.21). On the other hand, the curves in dotted line are obtained from the actual converter circuit implemented in PSIM software. A good correspondence between the analytical Bode plot obtained using the state-space model of the converter and the Bode plot obtained using the actual open-loop converter circuit validates the accuracy of the average state-space model of the converter at intermediate frequencies.

Fig. 5.2 Bode plot of the control-to-output transfer function $\tilde{v}_o(s)/\tilde{u}(s)$ of the converter (solid line: Bode plot obtained from the transfer function (5.21), dotted line: Bode plot obtained from the actual open-loop converter circuit implemented in PSIM software).

5.2 Adaptive Current-Mode Controller Design

In this section, the adaptive current-mode control of the high step-up dc-dc converter is addressed. The state-of-the-art current-mode controller [38, 40-42] is given first to demonstrate its shortcoming to regulate the systems with unknown loads.

The expression of the state-of-the-art current-mode controller is given by [38]:

$$u = U_a - K_{Pt}(i_{L_1} - I_{L_{1a}}) - K_{It} \int_0^t (v_o(\tau) - V_d) d\tau \quad (5.22)$$

where $K_{Pt}(> 0)$ and $K_{It}(> 0)$ are the controller gains and $I_{L_{1a}}$ is given by (5.15). This is just one form of a current-mode control in which the inductor current and output voltage are used for feedback purposes [38, 40-42]. The voltage integral action is mainly added to reduce the steady-state error during the hardware implementation of the controlled system [38]. In (5.22), $I_{L_{1a}}$ is given by $I_{L_{1a}} = \frac{V_d(V_d+E)}{2RE}$ (see (5.15)). For implementation purposes, $I_{L_{1a}}$ is calculated using the nominal value of the load resistance R . Therefore, in applications where R is unknown, (5.22) may not be used. Apart from that, since the output voltage error in the state-of-the-art current-mode controlled converter system is mainly compensated by the term $K_{It} \int_0^t (v_o(\tau) - V_d) d\tau$, there is a trade-off between an overshoot and a settling time of an output response. That is, if a large value of K_{It} is used, it may increase the overshoot of a transient response. Also, a small value of K_{It} could increase the settling time of the output response. Moreover, there is no guarantee that the control signal in (5.22) will be “small” for large values of an output voltage error. To overcome these problems, an adaptive current-mode controller is proposed in the following subsection.

5.2.1 Proposed control law

When the load and parasitic resistances are unknown, the following control law is used:

$$u = U_a - K_P(i_{L_1} - \hat{I}_{L_{1a}}) \quad (5.23)$$

where

$$\hat{I}_{L_{1a}} = \frac{V_d(V_d+E)}{2\hat{R}_a E} = \frac{V_d(V_d+E)}{2E} \hat{\theta} \quad (5.24)$$

In (5.24), $\hat{\theta}$ is the estimate of $\theta = \frac{1}{R_a} \approx \frac{1}{R}$ for $\frac{r_c}{R} \approx 0$ and $\frac{r_{c1}}{R} \approx 0$. Note that, R_a is just the approximation of R . The proposed adaptation algorithm is described by:

$$\frac{d\hat{\theta}}{dt} = -\frac{2\alpha f_m e_4}{1+\alpha^2 e_4^2} \quad (5.25)$$

where $e_4 = v_o - V_d$ and α and f_m are user-specified positive constants. Let

$$f = -\frac{2\alpha f_m e_4}{1+\alpha^2 e_4^2} \quad (5.26)$$

Then

$$\frac{df}{de_4} = -\frac{2\alpha f_m (1-\alpha^2 e_4^2)}{(1+\alpha^2 e_4^2)^2} \quad (5.27)$$

Setting (5.27) to zero yields $e_4 = \pm \frac{1}{\alpha}$ and substituting this in (5.25) gives $\frac{d\hat{\theta}}{dt} = \pm f_m$, and hence $|\frac{d\hat{\theta}}{dt}| \leq f_m$ for all t . This shows that the adaptation algorithm (5.25) has an optimized $\frac{d\hat{\theta}}{dt}$ and it is bounded by f_m . Moreover, (5.25) affords integral action, which is necessary for the adequate operation of a practical control system.

5.2.2 Stability Analysis

With $\frac{r_c}{R} \approx 0$, $\frac{r_{c1}}{R} \approx 0$ and $R_a \approx R$, an approximate stability analysis is performed to gain an insight into the adaptive current-mode controlled system. The following errors are defined:

$$e_1 = i_{L_1} - I_{L_1}, \quad e_2 = v_C - V_C, \quad e_3 = v_{C_1} - V_{C_1}, \quad e_4 = v_o - V_d, \quad \tilde{\theta} = \hat{\theta} - \theta \quad (5.28)$$

Now, substituting (5.28), (5.23) and (5.24) into (5.9) – (5.12) yields the following error dynamics:

$$\begin{aligned} \frac{de_1}{dt} \approx & -\frac{\beta(1-U_a+K_P e_1-\sigma\tilde{\theta})}{2L_1}(e_1 + I_{L_1}) + \frac{1-U_a+K_P e_1-\sigma\tilde{\theta}}{2L_1}(e_2 + V_C) - \\ & \frac{1-U_a+K_P e_1-\sigma\tilde{\theta}}{2L_1}(e_3 + V_{C_1}) + \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{L_1}E \end{aligned} \quad (5.29)$$

$$\frac{de_2}{dt} \approx -\frac{1-U_a+K_P e_1-\sigma\tilde{\theta}}{C}(e_1 + I_{L_1}) - \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{r_C C}(e_2 + V_C) + \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{r_C C}E \quad (5.30)$$

$$\begin{aligned} \frac{de_3}{dt} \approx & \frac{1-U_a+K_P e_1-\sigma\tilde{\theta}}{2C_1}(e_1 + I_{L_1}) - \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{r_{C_1} C_1}(e_3 + V_{C_1}) + \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{2r_{C_1} C_1}(e_4 + \\ & V_d) - \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{2r_{C_1} C_1}E \end{aligned} \quad (5.31)$$

$$\frac{de_4}{dt} \approx \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{r_{C_1} C_o}(e_3 + V_{C_1}) - \frac{1}{C_o}\left(\frac{1}{R} + \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{2r_{C_1}}\right)(e_4 + V_d) + \frac{U_a-K_P e_1+\sigma\tilde{\theta}}{2r_{C_1} C_o}E \quad (5.32)$$

$$\frac{d\tilde{\theta}}{dt} = -\frac{2\alpha f_m e_4}{1+\alpha^2 e_4^2} \quad (5.33)$$

where

$$\sigma = K_P \frac{V_d(V_d+E)}{2E} \quad (5.34)$$

The desired equilibrium point of (5.29) – (5.33) is given by $(e_{1\infty}, e_{2\infty}, e_{3\infty}, e_{4\infty}, \tilde{\theta}_\infty) = (0,0,0,0,0)$. Considering the difficulty of proving the stability of the converter controlled by the adaptive controller, an analysis based on the Lypunov indirect method is carried out [5]. Linearization of (5.29) – (5.33) about the equilibrium point given by $(e_{1\infty}, e_{2\infty}, e_{3\infty}, e_{4\infty}, \tilde{\theta}_\infty)$ and using

the approximate values of I_{L_1} , V_C and V_{C_1} yields the following system of the form [61]:

$$\frac{dy}{dt} \approx Ny \quad (5.35)$$

where $y = [y_1 \ y_2 \ y_3 \ y_4 \ y_5]^T$, $y_1 = e_1 - e_{1\infty}$, $y_2 = e_2 - e_{2\infty}$, $y_3 = e_3 - e_{3\infty}$, $y_4 = e_4 - e_{4\infty}$, $y_5 = \tilde{\theta} - \tilde{\theta}_\infty$, and the matrix N is given by:

$$N = \begin{bmatrix} \frac{-\beta(1-U_a) - \beta K_p I_{L1a} + K_p V_{Ca} - K_p V_{C1a} - 2K_p E}{2L_1} & \frac{(1-U_a)}{2L_1} & -\frac{(1-U_a)}{2L_1} & 0 & \frac{\beta \sigma I_{L1a} - \sigma V_{Ca} + \sigma V_{C1a} + 2\sigma E}{2L_1} \\ \frac{-(1-U_a)r_C - K_p r_C I_{L1a} + K_p V_{Ca} - K_p E}{r_C C} & \frac{-U_a}{r_C C} & 0 & 0 & \frac{\sigma I_{L1a} r_C - \sigma V_{Ca} + \sigma E}{r_C C} \\ \frac{(1-U_a)r_{C1} + K_p r_{C1} I_{L1a} + 2K_p V_{C1a} - K_p V_d + K_p E}{2r_{C1} C_1} & 0 & \frac{-U_a}{r_{C1} C_1} & \frac{U_a}{2r_{C1} C_1} & \frac{-\sigma I_{L1a} r_{C1} - 2\sigma V_{C1a} + \sigma V_d - \sigma E}{2r_{C1} C_1} \\ \frac{-2K_p V_{C1a} + K_p V_d - K_p E}{2r_{C1} C_o} & 0 & \frac{U_a}{r_{C1} C_o} & \frac{-U_a}{2r_{C1} C_o} - \frac{1}{RC_o} & \frac{2\sigma V_{C1a} - \sigma V_d + \sigma E}{2r_{C1} C_o} \\ 0 & 0 & 0 & -2\alpha f_m & 0 \end{bmatrix}$$

The linearized system will be stable if all the eigenvalues of N , i.e., the roots of $p_5(s) = |sI - N| = 0$, where s is a complex variable, lie in the open left-half of the complex s -plane. Thus, for a given set of converter parameter values, the controller gains K_p , f_m and α can be chosen to ensure that the real parts of the eigenvalues of N are negative. As the polynomial $p_5(s)$ is of high order, the root locus method [62] can be used to analyze the stability of the closed-loop system. In this method, two controller parameters, say, α and f_m are kept constant while the other controller parameter K_p is varied until the eigenvalues of N appear in the right-half complex plane. This is done to determine the range of K_p to ensure system stability. The same procedure can be repeated to find the ranges of f_m and α to ensure stability.

For illustration purposes, consider the values of the circuit parameters given in (5.20). The corresponding polynomial $p_5(s)$ is given by:

$$\begin{aligned}
p_5(s) = & s^5 + (7083.04K_p + 98159.84)s^4 + (6.95 \times 10^8 K_p + 2.32 \times 10^9)s^3 + \\
& (1.64 \times 10^{13} K_p - 2.315 \times 10^9 \alpha f_m K_p + 3.14 \times 10^{11})s^2 + (1.51 \times 10^{14} K_p + \\
& 7.4 \times 10^{14} \alpha f_m K_p + 6.17 \times 10^{14})s + 8 \times 10^{18} \alpha f_m K_p
\end{aligned} \tag{5.36}$$

Fig. 5.3(a) shows the root locus plot of $p_5(s)$ for $\alpha = 0.25$, $f_m = 1$ and $0 < K_p \leq 5$. The arrows in the figure show the movement of the poles for increasing values of K_p .

Fig. 5.3 Root locus plots: (a) for $\alpha = 0.25$, $f_m = 1$ and $0 < K_p \leq 5$; (b) for $\alpha = 0.25$, $K_p = 2$ and $0 < f_m \leq 2$; and (c) for $f_m = 1$, $K_p = 2$ and $0 < \alpha \leq 0.5$.

Similarly, Fig. 5.3(b) shows the root locus plot for $\alpha = 0.25$, $K_p = 2$ and $0 < f_m \leq 2$ and Fig. 5.3(c) shows the root locus plot for $f_m = 1$, $K_p = 2$ and $0 < \alpha \leq 0.5$. It is clear that there are limits to the values of K_p , f_m , and α to ensure the stability of the closed-loop system. As such, the appropriate values of the controller parameters are to be chosen to ensure system stability.

Remark: If the system (5.35) is stable, then $y \rightarrow 0$. However, it is worth pointing out that since R_a is just the approximation of R , $\hat{\theta}$ will not tend to $\frac{1}{R}$, but instead it will tend to $\frac{1}{R_a}$.

5.3 Experimental Results

In this section, some experimental results are provided to validate the effectiveness of the proposed adaptive current-mode controller for the high step-up dc-dc converter. Also, the performance of the proposed controller is compared with the current-mode controller of the form used in [38]. The same set of converter parameter values as used in section 5.1.2, given by (5.20), was used to obtain the results. Also, the switching frequency used was 10 kHz. The control objective is to design a controlled system with overshoot of the transient response less than 10% of the nominal output voltage and settling time of the transient response less than 0.7 s.

Initially, the current-mode controller (5.22) was implemented using simple analog components. For implementation purposes, a voltage feedback factor $\beta = 1/10$ was used in (5.22) and the modified control law is given as follows:

$$u = U_{as} - K_{Pt}(i_{L_1} - I_{L_{1a}}) - K_{It} \int_0^t (v_{os}(\tau) - V_{ds}) d\tau \quad (5.37)$$

where $U_{as} = (V_{ds} - 3E_s)/(V_{ds} + E_s)$, $v_{os} = \beta v_o$, $V_{ds} = \beta V_d$, and $E_s = \beta E$.

The inductor current was measured using the Hall-effect sensor LTS-6NP from LEM. Also, in order to implement the PWM-based current-mode controller, the control signal for the switch was generated using a comparator IC LM311 and a triangular waveform of magnitude 1 V_{peak-peak}.

Fig. 5.4(a) shows the output response of the converter and the corresponding control signal for a step reference $V_d = 25$ V and a change in the load resistance R from 2 k Ω to 0.667 k Ω and then back to 2 k Ω . Here, $K_{Pt} = 2$ and $K_{It} = 5$

were used. Even though the output voltage was rapidly restored to V_d after the onset of load disturbances, the response had a considerably large overshoot when $K_{It} = 5$ was used.

Fig. 5.4 Output voltage response and control signal of the current-mode controlled high step-up dc-dc converter for a step change in the reference voltage from $V_d = 0\text{ V}$ to $V_d = 25\text{ V}$ and a change in the load resistance from $R = 2\text{ k}\Omega$ to $R = 0.667\text{ k}\Omega$ (vice-versa) (a) Using $K_{Pt} = 2$ and $K_{It} = 5$; and (b) Using $K_{Pt} = 2$ and $K_{It} = 0.5$.

Fig. 5.4(b) shows the transient output response and the load change response obtained under similar conditions but using $K_{It} = 0.5$. Again, it is evident that, as the value of the integral gain K_{It} was decreased, the overshoot of the transient response was reduced, but the settling time of the load change response was considerably increased. There is a tradeoff between the overshoot of the response and the settling time of the load change response.

Next, the adaptive current-mode controller of the form (5.23) - (5.25) was implemented. Similar to the case of the state-of-the-art current-mode controller discussed earlier, a voltage feedback factor β was introduced in (5.23) to give:

$$u = U_{as} - K_P(i_{L1} - \hat{I}_{L1as}) \quad (5.38)$$

where

$$\hat{I}_{L1as} = \frac{V_{ds}(V_{ds} + E_s)}{2E_s} \hat{\theta}_s, \quad \frac{d\hat{\theta}_s}{dt} = -\frac{2\alpha_c f_{mc} e_{4s}}{1 + \alpha_c^2 e_{4s}^2}, \quad \alpha_c = \frac{\alpha}{\beta}, \quad f_{mc} = \frac{f_m}{\beta}$$

Here, $\hat{\theta}_s = \hat{\theta}/\beta$, and $e_{4s} = \beta e_4$ represent the scaled values of $\hat{\theta}$ and e_4 , respectively and the scaling factor was set as $\beta = 1/10$. The division and multiplication functions were realized using the AD 633 Chip. Also, the values of K_P , α_c and f_{mc} used were 2, 1 and 1, respectively, to achieve satisfactory responses. Fig. 5.5 shows the detailed circuit diagram of the controller implementation.

Fig. 5.5 Circuit diagram of the adaptive current-mode controller implementation.

Fig. 5.6(a) shows the transient output response and corresponding control signal of the converter for a reference input voltage $V_d = 25\text{ V}$ and a change in the load resistance R from $2\text{ k}\Omega$ to $0.667\text{ k}\Omega$ and then back to $2\text{ k}\Omega$. It can be seen that, as compared to Fig. 5.4, an almost critically damped transient output response was obtained with a negligible overshoot. Also, the load change response has a comparatively less worst-case settling time of $\sim 0.4\text{ s}$ as compared to the worst-case settling time of $\sim 1.2\text{ s}$ obtained using the state-of-the-art current-mode controller (5.22). Fig. 5.6(b) shows the output voltage response and corresponding control signal of the converter when the load resistance R was changed from $2\text{ k}\Omega$ to $4\text{ k}\Omega$ and vice-versa. Fig. 5.6(c) shows the output voltage response for a change in the input voltage from $E = 3.3\text{ V}$ to $E = 2.5\text{ V}$ and vice-versa. The output voltage was rapidly restored to V_d after the onset of the input voltage disturbances. Fig. 5.6(d) shows the output voltage response and the control signal when V_d was changed from 25 V to 35 V and then back to 25 V . These results verify the effectiveness of the adaptive current-mode controller for the voltage control of the converter.

Fig. 5.6 Output voltage response and control signal: (a) for a step reference voltage $V_d = 25\text{ V}$ and a change in load resistance R from $2\text{ k}\Omega$ to $0.667\text{ k}\Omega$ (vice-versa); (b) for change in the load resistance R from $2\text{ k}\Omega$ to $4\text{ k}\Omega$ (vice-versa); (c) for change in the input voltage E from 3.3 V to 2.5 V (and vice-versa); and (d) for change in the reference voltage V_d from 25 V to 35 V (and vice-versa).

5.4 Summary

In this chapter, an adaptive current-mode control design for the high step-up dc-dc converter was presented. The main objective was to address the use of the traditional current-mode controllers like those given in [38, 40-42] in the

presence of unknown loads. The experimental results clearly demonstrate the better regulation properties of the proposed adaptive current-mode controller as compared to that obtained using the state-of-the-art current-mode controller.

CHAPTER 6 CONCLUSIONS AND RE- COMMENDATIONS FOR FUTURE WORK

6.1 Conclusions

This thesis presents the research done on the modeling and control aspects of some high-order boost-type dc-dc power converters. The background and motivation of the research work is provided in Chapter 1. In Chapter 2, some high-order dc-dc converter topologies are reviewed as well as the basics of the modeling and control of the dc-dc converters are given. The detailed research work is given in Chapters 3 - 6.

In Chapter 3, a systematic approach to find the most suitable inductor current for implementing the current-mode controller (CMC) of the high-order boost-type dc-dc converters was presented. To this end, some detailed comparative studies of two current-mode controllers (using the input and output inductor currents of the converter) were carried out for the positive output elementary Luo (POEL) converter and the 2-stage cascade boost converter. Both the state-space and frequency response approaches were used to gain a better insight into the comparative studies. For the case of the current-mode controlled POEL converter, it was found that the closed-loop system becomes unstable when the output inductor current was used for feedback purposes. On the other hand, the current-mode controller implemented using the input current was found to be able to successfully regulate the POEL converter. This is in contrast to the case of the 2-stage cascade boost converter wherein it was found that for a given set of circuit parameter values, using the input inductor current for feedback purposes fails to

give some theoretically stable results. To solve this problem the output inductor current was employed for feedback purposes and it was found to give stable theoretical results. In summary, there is no specific current (like either the input inductor current or the output inductor current) which can be generally used for designing the current-mode controller for any high-order dc-dc converter. A separate analysis may be required for every specific converter to find the most suitable inductor current to implement the current mode controller for its regulation.

In Chapter 4, some sliding-mode (SM) controllers for the regulation of high-order boost-type dc-dc converters were considered. Initially, the pulse-width-modulation (PWM)-based SM controller using a reduced number of state-variables was proposed for the 2-stage cascade boost converter. The proposed controller requires only one current for its implementation while enjoying the advantages offered by both fixed-frequency and double-integral approaches. Also, a suitable approach to find the most suitable state variables for implementing the PWM-based SM controller was proposed. It was shown that the controller using the input inductor current is more suitable as based on some sound theoretical foundation and its better regulation properties as compared to the controller based on the output inductor current. Apart from this, a detailed comparative study of two hysteresis-modulation (HM)-based SM controllers (using the input and output inductor currents of the converter) was carried out for the hybrid high-order dc-dc boost converter. For this converter, the theoretical results show that when the HM-based SM controller was designed using the output inductor current for feedback purposes, an unstable pole-zero cancellation occurs in the current-to-output transfer function of the controlled converter and

the resulting instability may not be easily seen from the Bode plot. However, this problem did not appear when the input inductor current was used for feedback purposes. Some simulation and experimental results were also provided to validate the theoretical conclusions.

In Chapter 5, the modeling of a high step-up dc-dc converter together with its control using an adaptive current-mode approach was addressed. The modeling took into account of the presence of some parasitic resistances in the converter circuit in order to obtain the averaged converter dynamics. The accuracy of the derived model of the converter was verified via the frequency-domain technique. In addition, an adaptive current-mode controller of the converter was proposed. This was to address the problem of handling unknown loads when using the state-of-the-art current-mode controllers. As compared to the existing current-mode controller, the proposed controller has better regulation properties as confirmed by the experimental results. Also, the structure of the proposed controller is quite generic and as such, it can be used to regulate any other high-order dc-dc converter. Moreover, the proposed adaptation algorithm can also be combined with other state-of-the-art controllers like the sliding-mode controllers to improve their regulation properties.

6.2 Recommendations for Future Research

In this thesis, the modeling and control of the high-order boost-type dc-dc converters were presented. The recommendations for possible future work are as follows:

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- (1) The implementation of various controllers were carried out using simple analog components. More advanced digital controllers (e.g. [63-66]) for the high-order dc-dc converters with high voltage and power ratings can be considered for future work.
 - (2) The main focus of this thesis is on the high-order boost-type dc-dc converters. The modeling, analysis and control of the high-order buck-type dc-dc converters (e.g. [67]) can be considered for future work.
 - (3) An adaptive algorithm can be combined with some advanced controllers like the SM control technique to improve their regulation properties.
 - (4) The proposed controllers in this thesis are used to achieve output voltage regulation in non-isolated high-order dc-dc converters. Some high-frequency (HF) transformer-based dc-dc converters such as the Dual Active Bridge (DAB) dc/dc converter offer several advantages such as galvanic isolation and high step up/down capabilities. The system power density can be significantly improved by achieving the isolation using a HF transformer [68]. Also, some significantly improved conversion ratios can be achieved by employing the transformers since the voltage conversion ratio depends not only on the control of the power switches, but also on the transformer turns ratio. In future, the non-linear control of such HF transformer-based dc-dc converters [7, 68] can be addressed.

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