Developments of fundamental locally one-dimensional finite-difference time-domain methods for transmission lines and lumped elements

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Developments of Fundamental
Locally One-dimensional
Finite-Difference Time-domain
Methods for Transmission Lines
and Lumped Elements

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Zaifeng Yang

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Abstract

This thesis presents the developments of fundamental locally one-dimensional finite-difference time-domain (FLOD-FDTD) methods for transmission lines and lumped elements. Unlike explicit FDTD method, the FLOD-FDTD method can use time step larger than Courant-Friedrichs-Lewy (CFL) limit. It also achieves higher efficiency and simplicity with matrix-operator-free right-hand sides (RHS) compared with the conventional LOD-FDTD method. The updating equations of the FLOD-FDTD method incorporating resistors, inductors and capacitors are derived. In addition to the three commonly used lumped elements, explicit FDTD method and FLOD-FDTD method incorporated with memristor are also developed. Memristor is regarded as the fourth lumped element which links charge and flux. The update equations are derived based on Maxwell's equations and the physical model given by Hewlett-Packard (HP) lab. Numerical examples including transmission lines and various lumped elements are provided to show the trade-off between efficiency and accuracy of the proposed 3-D FLOD-FDTD method.

Since the conventional LOD-FDTD method is of first-order accuracy only, second-order temporal-accurate scheme for 3-D FLOD-FDTD method is also introduced to improve the accuracy for LOD-FDTD method with three split matrices. The main iterations of the proposed scheme comprise only one more procedure in addition to

the existing three procedures. Using proper (initial only) input and (often infrequent, field-point only) output processings, the second-order temporal accuracy is achieved and verified analytically. Stability analysis is provided to ascertain the unconditional stability of the proposed scheme and numerical results are demonstrated to justify its higher accuracy. Moreover, an alternative reversed second-order temporal-accurate FLOD-FDTD scheme is also proposed. Numerical examples such as reverberation chamber and transmission line circuit are demonstrated to show the higher accuracy of the proposed FLOD-FDTD method.

Although full 3-D FDTD methods can cater for most general structures and 3-D FLOD-FDTD method can use time step larger than CFL limit for higher efficiency, the computational domain is still large and full 3-D methods are not well-suited to run on mobile devices such as smart phones/pads due to the requirement for large memory storage and long simulation time. To circumvent this problem, novel interconnected multi-one-dimensional (IM1-D) fundamental alternating-direction-implicit (FADI)- and FLOD-FDTD methods for transmission lines with interjunctions are introduced to further enhance the computational efficiency. The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit. Fundamental scheme-based IM1-D FADI- and FLOD-FDTD methods are both derived to enhance the efficiency with matrix-operator-free RHS. The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device. Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems. A microstrip line loaded with stubs and a branch-line coupler

are simulated to show the accuracy and efficiency of the proposed methods. To extend the applicability for handling the couplings between two transmission lines via gaps, a microstrip circuit with gaps is simulated using the proposed methods incorporated with equivalent circuit models involving capacitances. Real-time simulations of these numerical examples provide much intuitional insight for one to observe the electromagnetic waves propagation in time domain on computer or mobile device.

Finally, a novel non-uniform time step (NUTS) FLOD-FDTD method for multi-conductor transmission lines (MTLs) including lumped elements. Unlike conventional and the other LOD-FDTD methods, the NUTS scheme adopts different (non-uniform) time steps for different periods during simulation, in order to reduce the large errors caused by unconditionally stable FDTD methods with uniform time step larger than CFL limit. It is found that NUTS scheme is potentially unstable for ADI-FDTD method, but it is stable for LOD-FDTD method or its implementation based on fundamental scheme. The NUTS FLOD-FDTD method is useful to simulate MTLs with lumped elements in series and parallel connections. Moreover, the method based on multiple 1-D approach is also extended to incorporate lumped elements in cross connection to model near-field coupling between the MTLs. While the electric current density is commonly used for field-circuit coupling of parallel connected lumped elements, the magnetic current density will be utilized for field-circuit coupling of series and cross-connected lumped elements. Numerical results for MTLs with lumped elements in series, parallel and cross connections are provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method. Compared with the FLOD-FDTD method with uniform time step, the NUTS FLOD-FDTD method may achieve higher accuracy by using smaller time step for certain periods of simulation.

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List of Abbreviations

ADI	Alternating-Direction-Implicit
CFL	Courant–Friedrichs–Lewy
CFLN	Courant–Friedrichs–Lewy Number
CPW	Cells Per Wavelength
FADI	Fundamental Alternating-Direction-Implicit
FDTD	Finite-Difference Time-Domain
FLOD	Fundamental Locally One-Dimensional
GPU	Graphics Processing Units
IM1-D	Interconnected Multiple One-Dimensional
LHS	Left-Hand Side
LOD	Locally One-Dimensional
MTL	Multiconductor Transmission Line
M1-D	Multiple One-Dimensional
NUTS	Non-uniform Time Step
PEC	Perfect Electric Conductor
PML	Perfectly Matched Layer
PMC	Perfect Magnetic Conductor
RHS	Right-Hand Side
TEM	Transverse Electromagnetic

List of Notations

Summary of Key notations, in order of their appearances in each Chapter.

E_x, E_y, E_z	Electric Fields Components
H_x, H_y, H_z	Magnetic Fields Components
e_x, e_y, e_z	Auxiliary Electric Fields Components
h_x, h_y, h_z	Auxiliary Magnetic Fields Components
J_x, J_y, J_z	Electric Current Density
M_x, M_y, M_z	Magnetic Current Density
t, f, ω	Time, Frequency, Angular Frequency
Δt	Time Step Size
$\Delta x, \Delta y, \Delta z$	Spatial Step Size
c	Speed of Light in Free Space
ϵ, ϵ_r	Permittivity, Relative Permittivity
μ, μ_r	Permeability, Relative Permeability
σ	Electric Conductivity

Chapter 1

Introduction

1.1 Motivation

Transmission lines are widely used in various microwave circuit and components. They connect various circuit components including lumped elements, shape filters, form matching circuit at input/output ports, etc [1,2]. Electromagnetic waves propagating in a transmission line may interfere with the other lines, circuit components, in the radio frequency (RF) front end. Thus, understanding how the invisible electromagnetic waves propagates in the transmission lines and electronic devices such as lumped elements would help the researchers and engineers to design and improve the existing communication system.

To observe the electromagnetic waves, time-domain electromagnetic numerical methods can be used to facilitate the understanding of microwave field and visually elucidate the underlying physical principles. The conventional finite-difference time-domain (FDTD) method with explicit update equations is widely used for the analysis of electromagnetic waves propagation, which is based on direct solution to Maxwell's

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curl equations [3]. The FDTD algorithm solves both electric and magnetic fields by sampling field components in spatial and temporal domain which represents the differential and integral forms of Maxwell's equations.

Nevertheless, the conventional FDTD method is constrained by the Courant-Friedrichs-Lewy (CFL) condition, which is conditionally stable [4]. The time step depends on the minimum discretized mesh size of the Yee's cell. If the minimum discretized mesh size is very small for the model to be simulated, it would result in a very low computational efficiency overall due to more computational time would be cost and more iterations need to be executed.

Fortunately, unconditionally stable FDTD methods are developed to improve the computational efficiency by using time step larger than the CFL limit [5–10]. Such methods involving implicit update equations are capable of reducing the numbers of iterations compared with the conventional FDTD method when larger time step is used. Choosing the appropriate time step (larger than CFL limit), ones can make a trade-off between efficiency and accuracy. The unconditionally stable FDTD methods can also be extended to solve the electromagnetic problems involving circuit components such as lumped elements. Therefore, the research on unconditionally stable FDTD methods attracts world-wide attentions since the increasing demand of higher efficiency for time-domain electromagnetic numerical methods.

However, some of the unconditionally stable FDTD methods are not fully extended to solve problems including lumped elements [11], especially for some new developed circuit component such as memristor [12]. The memristor characterized by a relationship between the electric charge and flux was recently found, which has not been investigated to be incorporated with the current popular unconditionally stable FDTD methods. Additionally, when larger time step is used for higher computational

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efficiency, the accuracy of the unconditionally stable FDTD method will be degraded. Thus, how to obtain a simulated results with higher accuracy for some unconditionally stable FDTD methods with acceptable additional procedures still remains a question. Although 3-D full-wave FDTD simulation can cater for most general structure such as transmission lines, the computational domain is still large for efficient simulation on some equipment without high computational performance, especially for mobile devices. Therefore, how to improve the simulation for interconnected transmission lines demands a further simplification of the transmission line model and more efficient unconditionally stable FDTD method. Furthermore, larger error will be caused as larger time step is used. When a source are excited or when the electromagnetic waves propagates to some circuit components such as lumped elements, more accurate simulation (smaller time step) are desired for these periods compared with the others. However, almost all the reported unconditionally stable FDTD methods use a constant time step for each iteration. So far, there is no investigation on non-uniform time step unconditionally stable FDTD methods with different time steps in different simulation periods. The stability and applications for the current unconditionally stable FDTD methods have not ever been exploited as well.

In summary, the recently developed unconditionally stable FDTD method needs to be extended to incorporate the lumped elements including the newly found memristor. Moreover, the existing unconditionally stable FDTD method should be further developed for higher accuracy and efficiency. Furthermore, more efficient unconditionally stable FDTD method and more simplified modeling need to be developed for simulations on mobile devices. At last, the non-uniform time step unconditionally stable FDTD method should be investigated and developed.

1.2 Objectives

Based on the motivation highlighted in the previous section, the objectives of the thesis are listed as follows.

- Development of FLOD-FDTD methods incorporated with lumped elements including resistor, capacitor, inductor and a recent found memristor. The FLOD-FDTD method incorporated with lumped elements, including the recent found memristor, can be derived based on current density and fundamental scheme.
- Development of second-order temporal accurate unconditionally stable FDTD methods with three split matrices. Based on the first-order accurate LOD-FDTD method with three split matrices, the corresponding efficient fundamental scheme and higher-order method can be derived with input and output processing. The stability condition can be determined based on the Von Neumann method.
- Development of more efficient FLOD-FDTD method for interconnected multiple transmission lines. More simplified modeling for transmission line circuit can be derived based on the synthesis/analysis method. The corresponding unconditionally FDTD methods can be derived with special treatments at the interjunctions for a more efficient simulation even on mobile devices.
- Investigation of the non-uniform time step scheme for unconditionally stable FDTD methods and its applications for microwave circuit with lumped elements. The stability can be analyzed numerically for both ADI- and LOD-FDTD methods. With smaller time step used for certain periods of simulation, e.g. during source excitations and when large signals pass through lumped elements, the more accurate results may be obtained.

1.3 Major Contributions of the Thesis

The major contributions of the thesis with regard to the developments of FLOD-FDTD methods for transmission line and lumped elements are as follows.

To simulate lumped elements incorporated circuit, 3-D FLOD-FDTD method incorporated with various lumped elements including resistor, capacitor and inductor have been derived. The fundamental scheme achieves higher efficiency and simplicity with matrix-operator-free right-hand sides (RHS), which improve the computational efficiency. In addition to the three commonly used lumped elements, memristor is regarded as the fourth lumped element which links charge and flux. Explicit FDTD method and FLOD-FDTD method incorporated with memristor are also developed. The update equations are derived based on Maxwell's equations and the physical model given by Hewlett-Packard (HP) lab. Numerical examples including transmission line and various lumped elements are provided to show the trade-off between efficiency and accuracy of the proposed 3-D FLOD-FDTD method. For the memristor, the simulation results obtained from the explicit FDTD method is found to be in good agreement with those obtained from SPICE model in Advanced Design System (ADS). The results using FLOD-FDTD method justify the unconditionally stability and show that larger time step will increase the computational efficiency with acceptable discrepancy from explicit FDTD method.

To improve the accuracy of the conventional LOD-FDTD method, the efficient second-order temporal-accurate scheme for 3-D FLOD-FDTD methods with three split matrices has been developed. The main iterations of the proposed scheme comprise only one more procedure in addition to the existing three procedures. Using proper (initial only) input and (often infrequent, field-point only) output processings, the second-order temporal accuracy has been achieved and verified analytically.

1.3. MAJOR CONTRIBUTIONS OF THE THESIS

An alternative reversed second-order temporal-accurate FLOD-FDTD scheme is also proposed. Stability analysis has been provided to ascertain the unconditional stability of the proposed scheme and numerical examples including 3-D transmission lines are demonstrated to justify its higher accuracy. Compared with a recent developed second-order accurate LOD-FDTD method with three-split matrices, the proposed fundamental scheme is 1.84 times faster while maintaining second-order temporal accuracy due to one substep fewer and matrix-operator-free RHS.

To improve the computational efficiency for solving transmission lines circuit, novel interconnected multi-one-dimensional (IM1-D) fundamental alternating-direction-implicit (FADI)- and FLOD-FDTD methods for transmission lines with interjunctions have been introduced. The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit. The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device. Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems. Numerical examples such as stub filter, branch-line coupler are provided to show the effectiveness of the proposed methods even on mobile devices. Compared with conventional FDTD method, the efficiency gain of the proposed FADI and FLOD methods when CFLN=10 are 6.43 and 6.84 for simulation on PC, and 5.45 and 5.57 on iPad 3.

To reduce the error caused by the larger time step during some simulation periods, a novel non-uniform time step (NUTS) LOD-FDTD method has been further developed. Unlike conventional and the other LOD-FDTD methods, NUTS LOD-FDTD

method employs non-uniform time step for different periods during the simulation. The update equation of NUTS LOD-FDTD method with arbitrary time step is given. It is proven from the numerical stability analysis that NUTS scheme is not applicable to ADI-FDTD, but it is stable for LOD-FDTD method. The NUTS FLOD-FDTD method incorporated with lumped elements including resistor, capacitor and memristor for IM1-D applications are derived. To incorporate lumped elements in IM1-D application where the electric field is not defined such as series connected circuit components, a novel method using magnetic current density for incorporating the lumped elements is proposed. The magnetic current density is introduced to solve the lumped elements in series connection, while the electric current density is used as the field-circuit coupling for parallel connected lumped elements. Numerical results for transmission line circuit with various lumped elements have been provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method compare with the conventional uniform time step LOD-FDTD. It is proven that the NUTS FLOD-FDTD can achieve higher accuracy with acceptable more iterations compared with the corresponding uniform time step FLOD-FDTD method.

1.4 Organizations of the Thesis

The rest of the thesis is organized as follows.

Chapter 2 introduces the background and literature review for the related research in unconditionally stable FDTD methods.

In Chapter 3, an efficient FLOD-FDTD method incorporated with various lumped elements is introduced. The updating equations of the FLOD-FDTD method incorporating resistors, inductors and capacitors are derived. In addition to the three

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commonly used lumped elements, explicit FDTD method and FLOD-FDTD method incorporated with memristor are also developed. Memristor is regarded as the fourth lumped element which links charge and flux. Numerical examples including transmission line and various lumped elements are provided to show the trade-off between efficiency and accuracy of the proposed 3-D FLOD-FDTD method.

Chapter 4 presents a second-order temporal-accurate scheme for 3-D FLOD-FDTD method with three split matrices. Stability analysis is provided to ascertain the unconditional stability of the proposed scheme and numerical results are demonstrated to justify its higher accuracy. In addition, an alternative reversed second-order temporal-accurate FLOD-FDTD scheme is also proposed. Numerical examples such as reverberation chamber and transmission line circuit are demonstrated to show the higher accuracy of the proposed FLOD-FDTD method.

Chapter 5 presents the interconnected multi-one-dimensional (IM1-D) FADI- and FLOD-FDTD methods for transmission lines with interjunctions. The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit. The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device. Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems. A microstrip line loaded with stubs and a branch-line coupler are simulated to show the accuracy and efficiency of the proposed methods. To extend the applicability for handling the couplings between two transmission lines via gaps, a microstrip circuit with gaps is simulated using the proposed methods incorporated with equivalent circuit models involving capacitances. Real-

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time simulations of these numerical examples provide much intuitional insight for one to observe the electromagnetic waves propagation in time-domain on computer or mobile device.

Chapter 6 presents a novel non-uniform time step (NUTS) FLOD-FDTD method. Unlike conventional and the other LOD-FDTD methods, NUTS FLOD-FDTD method employs non-uniform time step for different periods during the simulation. The update equation of NUTS FLOD-FDTD method with arbitrary time step is given. Numerical stability analysis shows that NUTS scheme is not applicable to ADI-FDTD, but it is stable for LOD-FDTD method. The NUTS FLOD-FDTD method incorporated with lumped elements including resistor, capacitor and memristor for IM1-D applications are derived. A novel magnetic current density is introduced to solve the lumped elements in series connection, while the electric current density is used as the field-circuit coupling for parallel connected lumped elements. Numerical results for transmission line circuit with various lumped elements are provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method compare with the conventional uniform time step LOD-FDTD. It is proven that the NUTS FLOD-FDTD can achieve higher accuracy with acceptable more iterations compared with the corresponding uniform time step FLOD-FDTD method.

The thesis will be concluded in Chapter 7. Promising research directions are recommended later in this chapter.

Chapter 2

Literature Review

2.1 Overview

In this chapter, a brief literature review for some popularly used unconditionally stable FDTD methods such as ADI-FDTD, LOD-FDTD, fundamental schemes and their extension to lumped elements, non-uniform time step schemes will be introduced.

2.2 Conventional FDTD method

The conventional FDTD method updates the field components in time domain based on the direct solution to Maxwell's curl equations [3, 13], and it is widely used in solving various electromagnetic problems [14, 15]. The Maxwell's curl equations can be expressed as

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t} \mathbf{D} + \mathbf{J} \quad (2.1a)$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t} \mathbf{B} - \mathbf{M}. \quad (2.1b)$$

2.2. CONVENTIONAL FDTD METHOD

Figure 2.1: Positions of electric field and magnetic field components in two Yee's space lattices.

where $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, $\mathbf{B}$, $\mathbf{J}$ and $\mathbf{M}$ are the vector arrays for electric field density, magnetic field density, electric flux density, magnetic flux density, electric current density and magnetic current density, respectively. The constitutive relations are given by

$$\mathbf{D} = \varepsilon \mathbf{E} \quad (2.2a)$$

$$\mathbf{B} = \mu \mathbf{H}. \quad (2.2b)$$

where ε and μ are the electric permittivity and magnetic permeability respectively. According to the Yee algorithm [3], the positions of electric field E_x , E_y , E_z and magnetic field components H_x , H_y , H_z in two Yee's space lattices are shown in Fig. 2.1. Using the second order accurate central difference approximation for space and time derivatives in conjunction with the staggered Yee's cell, the explicit finite difference approximation of six scalar Maxwell's equations can be obtained, which form the update equations for the electric fields and magnetic fields. For brevity, only the update equation of E_x is shown below, the remaining update equations for five the

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other fields components are very straightforward.

$$E_x|_{i+\frac{1}{2},j,k}^{n+1} = E_x|_{i+\frac{1}{2},j,k}^n + \frac{\Delta t}{\varepsilon} \left[\frac{H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}}{\Delta y} \right] - \frac{\Delta t}{\varepsilon} \left[\frac{H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} \right] \quad (2.3)$$

Due to its explicit nature, the conventional FDTD method is subjected to the CFL condition which imposes a maximum constraint on its time step size [4]. The CFL condition is given by

$$v_{\max} \Delta t \leq \left[\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2} \right]^{-1/2} \quad (2.4)$$

where $v_{\max}$ is the maximum phase velocity of the wave propagation in the region. It can be seen that the maximum allowed time step is restricted by the smallest cell size used. In some structures where there are combinations of fine and coarse grids, the time step is always restricted to the size of the fine grids, which degrades the efficiency significantly.

In the reference [16], it is reported that the modification of the 3-D electric field time-stepping algorithm is used to solve the electromagnetics-field problem extended to lumped element circuits. The basis of this formulation, reported originally in [17] for two-dimensional problems, is that circuit elements can be accounted for in Maxwell's equations by adding a lumped electric current density term J_L to the conduction and displacement currents on the RHS:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} + \mathbf{J}_{LE} \quad (2.5)$$

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where $\mathbf{J}_{LE}$ are the electric current density produced by the lumped elements.

Because the FDTD method including the lumped elements has to cope with the electric current densities for resistors, capacitors and inductors, the computational time of the conventional explicit FDTD method is definitely to be increased. Besides, the time step is also constrained by the CFL limit, which leads to a much lower efficiency. Hence, it is necessary to use unconditionally stable FDTD methods with larger time step to increase the computational efficiency.

2.3 Unconditionally Stable FDTD methods

The conventional explicit FDTD is constrained by the spatial step and time step to follow the CFL condition. It results in a low efficiency owing to the finer meshes and longer computational time are required to keep the algorithm stable. To circumvent this issue, the implicit FDTD methods have been widely researched in recent years to keep the algorithm unconditionally stable even larger time steps are used. The earliest reported unconditionally stable FDTD method is the Crank-Nicolson FDTD method in 1947 [5]. However, it cannot be solved using efficient mathematic method such as tridiagonal matrix algorithm. Inspired by the mentioned unconditionally stable FDTD method, several popularly used unconditionally stable FDTD methods will be introduced in the following subsections.

2.3.1 ADI-FDTD method

One popular unconditionally stable FDTD method is the alternating-direction-implicit finite-difference time-domain (ADI-FDTD) method [6, 7, 18]. The idea of ADI-FDTD method can be traced back to the early classic works by Peaceman and Rachford [19], which deals with various parabolic, elliptic and hyperbolic partial differential sys-

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tems. Considering that the electromagnetic waves propagating in a medium with permittivity ϵ and permeability μ , the conventional ADI-FDTD method involving two procedures in matrix form can be written as

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^n + \frac{\Delta t}{2}\mathbf{s}^{n+\frac{1}{2}} \quad (2.6)$$

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^{n+\frac{1}{2}} + \frac{\Delta t}{2}\mathbf{s}^{n+\frac{1}{2}}. \quad (2.7)$$

where

$$\mathbf{u} = [E_x, E_y, E_z, H_x, H_y, H_z]^T, \quad (2.8)$$

$$\mathbf{s} = \left[-\frac{1}{\epsilon}J_x, -\frac{1}{\epsilon}J_y, -\frac{1}{\epsilon}J_z, -\frac{1}{\mu}M_x, -\frac{1}{\mu}M_y, -\frac{1}{\mu}M_z \right]^T. \quad (2.9)$$

and Δt is the time step, $\mathbf{I}$ is the 6 x 6 identity matrix, $\mathbf{A}$ and $\mathbf{B}$ are the two split matrices given by

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_y \\ 0 & 0 & 0 & \frac{1}{\epsilon}\partial_z & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\epsilon}\partial_x & 0 \\ 0 & \frac{1}{\mu}\partial_z & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\mu}\partial_x & 0 & 0 & 0 \\ \frac{1}{\mu}\partial_y & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.10)$$

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$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{1}{\varepsilon}\partial z & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{\varepsilon}\partial x \\ 0 & 0 & 0 & -\frac{1}{\varepsilon}\partial y & 0 & 0 \\ 0 & 0 & -\frac{1}{\mu}\partial y & 0 & 0 & 0 \\ -\frac{1}{\mu}\partial z & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\mu}\partial x & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.11)$$

where ∂x , ∂y and ∂z are the spatial difference operators along x -, y - and z - directions, respectively. Note that the symmetrical source are implemented according to [20, 21], and the field components are updated in alternating direction for the two procedures. The stability and dispersion analysis has been presented in [22, 23].

Recently, a leapfrog ADI-FDTD method is further developed to enhance the computational efficiency with one-step update equation for the field components [24–26]. The development of the of the leapfrog ADI-FDTD method with numerical examples including the stability, dispersive media, boundary conditions have been presented in [27–30]. Nevertheless, the leapfrog ADI-FDTD method does not retain the second-order accuracy.

2.3.2 LOD-FDTD method

Another popular unconditionally stable FDTD method is LOD-FDTD method [8, 31]. Unlike ADI-FDTD method, the field components are updated along one dimension only for each procedure, which allows parallel processing for acceleration [32]. The

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LOD-FDTD method expressed in the matrix is given as

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^n \quad (2.12a)$$

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^{n+\frac{1}{2}}. \quad (2.12b)$$

LOD-FDTD method is more computationally efficient compared with ADI-FDTD method due to less arithmetic operations are involved. However, the conventional LOD-FDTD method is not second-order accurate in temporal like ADI-FDTD method. Its temporal order accuracy is of first order and can be improved to second order when the noncommutative terms $\mathbf{AB}$ and $\mathbf{BA}$ are omitted efficiently as in [9, 33].

Recently, a leapfrog LOD-FDTD method is proposed in [34]. This leapfrog LOD-FDTD method is proved that this algorithm requires less amount of memory and less CPU time while it keeps the same numerical properties in comparisons with the original LOD-FDTD method. However, this so called leapfrog LOD-FDTD method is similar to the fundamental LOD-FDTD method with introduced auxiliary variables according to [35].

2.3.3 LOD-FDTD Method with Three Split Matrices

Unlike the conventional ADI- and LOD-FDTD methods using the two split matrices in (2.10) and (2.11), an alternative scheme of LOD-FDTD has been developed in [11, 36–39] and involves three split matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, whereby each split matrix contains only spatial operators along one direction. The three split matrices $\mathbf{A}$, $\mathbf{B}$

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and $\mathbf{C}$ are listed as follows.

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\partial}{\varepsilon \partial x} \\ 0 & 0 & 0 & 0 & \frac{\partial}{\varepsilon \partial x} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial}{\mu \partial x} & 0 & 0 & 0 \\ 0 & -\frac{\partial}{\mu \partial x} & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.13)$$

$$\mathbf{B} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{\partial}{\varepsilon \partial y} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial}{\varepsilon \partial y} & 0 & 0 \\ 0 & 0 & -\frac{\partial}{\mu \partial y} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\partial}{\mu \partial y} & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.14)$$

$$\mathbf{C} = \begin{pmatrix} 0 & 0 & 0 & 0 & -\frac{\partial}{\varepsilon \partial z} & 0 \\ 0 & 0 & 0 & \frac{\partial}{\varepsilon \partial z} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\partial}{\mu \partial z} & 0 & 0 & 0 & 0 \\ -\frac{\partial}{\mu \partial x} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.15)$$

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The LOD-FDTD method with the above three split matrices in matrix form can be expressed as

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^{n+\frac{1}{3}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^n \quad (2.16a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right)\mathbf{u}^{n+\frac{2}{3}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right)\mathbf{u}^{n+\frac{1}{3}} \quad (2.16b)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n+\frac{2}{3}} \quad (2.16c)$$

From (4.1), three procedures make up one main iteration and each procedure contains only spatial operators along x , y , or z directions. The above scheme is only of first-order temporal accuracy. Higher-order temporal accurate method can be obtained by introducing additional procedures [40,41], which may take more computational time.

2.3.4 Fundamental Schemes for ADI- and FLOD-FDTD Methods

Unlike some efficiency-improved unconditionally FDTD methods such as leapfrog ADI-FDTD, fundamental ADI-FDTD (FADI-FDTD) and fundamental LOD-FDTD (FLOD-FDTD) methods [42] are further developed to enhance the computational efficiency with matrix-operator-free right-hand sides (RHS), while the accuracy is not degraded. The FADI-FDTD method in matrix form read

$$\mathbf{v}^n = \tilde{\mathbf{u}}^n - \mathbf{v}^{n-\frac{1}{2}} \quad (2.17a)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\tilde{\mathbf{u}}^{n+\frac{1}{2}} = \mathbf{v}^n \quad (2.17b)$$

$$\mathbf{v}^{n+\frac{1}{2}} = \tilde{\mathbf{u}}^{n+\frac{1}{2}} - \mathbf{v}^n \quad (2.17c)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right)\tilde{\mathbf{u}}^{n+1} = \mathbf{v}^{n+\frac{1}{2}} \quad (2.17d)$$

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where

$$\mathbf{v} = [e_x, e_y, e_z, h_x, h_y, h_z]^T \quad (2.18)$$

is auxiliary variable array and $\tilde{\mathbf{u}}^n = 2\mathbf{u}^n$, $\tilde{\mathbf{u}}^{n+\frac{1}{2}} = 2\mathbf{u}^{n+\frac{1}{2}}$ are the two assumptions to simplified the formulation and achieve a higher efficiency.

Similarly, the FLOD-FDTD method in matrix form read

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A})\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n \quad (2.19a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (2.19b)$$

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B})\mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} \quad (2.19c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (2.19d)$$

Note that the introduced auxiliary variable array $\mathbf{v}$ does not occupy additional memory due to the reuse of the field components. The fundamental schemes have been widely adopted for unconditionally stable FDTD methods in many applications to realize a higher computational efficiency [43–52]. To be specific, this fundamental scheme can be further applied to graphics processing units (GPU) accelerated simulation [43], body-of-revolution for rotationally symmetric geometry [44], frequency-dependent material [45, 46, 48], dispersive media [50, 52], ADI beam-propagation method [47], absorbing boundary condition such as convolutional perfectly matched layer (PML) [49], perfect electrical conductor (PEC) and perfect magnetic conductor (PMC) [51].

2.3.5 ADI- and LOD-FDTD Methods Extended to Lumped Elements

In the explicit FDTD method extended to lumped elements [13, 16, 17, 53, 54], the circuit elements can be accounted for in Maxwell's equations by adding a lumped

2.3. UNCONDITIONALLY STABLE FDTD METHODS

electric current density term $\mathbf{J}_{LE}$ to the conduction and displacement current as

$$\nabla \times \mathbf{H} = \mathbf{J}_c + \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}_{LE}. \quad (2.20)$$

where

$$\mathbf{J}_{LE} = \left[\frac{I_{LEx}}{\Delta y \Delta z}, \frac{I_{LEy}}{\Delta x \Delta z}, \frac{I_{LEz}}{\Delta x \Delta y}, 0, 0, 0 \right]^T \quad (2.21)$$

Δx , Δy and Δz are the spatial steps along the x-,y- and z-directions, respectively; $\mathbf{J}_{LE}$ are the electric current density through the lumped elements; I_{LEx} , I_{LEy} and I_{LEz} are the corresponding current along the x-,y- and z-directions.

Similarly, the unconditionally stable FDTD methods are also extended to lumped elements [55–59]. The stability is analyzed for ADI- and LOD-FDTD methods are given in [60]. These methods overcome the CFL limitation so that larger time step is able to be used to reduce the computational time and enhance the efficiency. However, such implicit unconditionally stable FDTD methods are still arithmetically complicated especially when the passive lumped elements are incorporated owing to additional computation in the implicit updating equations for resistors, capacitors and inductors. This complexity leads to much more computational time and results in lower efficiency if a large amount of lumped elements are simulated. Furthermore, there is a new circuit element found [12,61], called as the fourth lumped element, and it is seldom reported to incorporate memristor to FDTD and unconditionally stable FDTD so far.

2.3.6 Non-uniform Time Step FLOD-FDTD Method

To the best of the authors' knowledge, almost all the unconditionally stable FDTD methods employ uniform time step during the whole simulation. There is no inves-

2.4. CONCLUSION

tigation on LOD-FDTD method using non-uniform time step to trade off between efficiency and accuracy before. Recently, a non-uniform time step (NUTS) LOD-FDTD method is developed [62].

Given that Δt_n is the time step from the n th time index to the $(n + 1)$ th time index, the update equation of the NUTS LOD-FDTD method can be written as

$$(\mathbf{I} - \frac{\Delta t_n}{2} \mathbf{A}) \mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t_n}{2} \mathbf{A}) \mathbf{u}^n \quad (2.22a)$$

$$(\mathbf{I} - \frac{\Delta t_n}{2} \mathbf{B}) \mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t_n}{2} \mathbf{B}) \mathbf{u}^{n+\frac{1}{2}}, \quad (2.22b)$$

where $\mathbf{A}$ and $\mathbf{B}$ are the two split matrices for LOD-FDTD method. Note that Δt_n is usually larger than the time step for CFL limit Δt_{CFL} . Δt_n can be arbitrary and different for each n . To update the field components using NUTS LOD-FDTD method from $\mathbf{u}^0$ to $\mathbf{u}^{n_{\text{max}}}$, the total time T_{total} used for computation is

$$\begin{aligned} T_{\text{total}} &= \Delta t_1 + \Delta t_2 + \dots + \Delta t_{n_{\text{max}}} = \sum_{n=1}^{n_{\text{max}}} \Delta t_n \\ &= \text{CFLN}_1 \Delta t_{\text{CFL}} + \dots + \text{CFLN}_{n_{\text{max}}} \Delta t_{\text{CFL}} = \Delta t_{\text{CFL}} \sum_{n=1}^{n_{\text{max}}} \text{CFLN}_n \end{aligned} \quad (2.23)$$

Unlike the previous LOD-FDTD methods that constant time step is applied and the $\text{CFLN} = \Delta t / \Delta t_{\text{CFL}}$ remains the same during the whole simulation, NUTS LOD-FDTD method can use smaller time step for certain periods, e.g., when the magnitudes of the field components are quite large or change dramatically during source excitation period.

2.4 Conclusion

Finally, a brief conclusion with table of the comparison of main features of various

2.4. CONCLUSION

FDTD methods is given as follows.

Table 2.1: The comparison of main features of various FDTD methods.

Methods	FDTD	ADI	LOD/SS	FADI	FLOD	LOD (Three Split Matrices)
Unconditionally Stable	No	Yes	Yes	Yes	Yes	Yes
Field Array	6	9	9	9	9	9
Total Flops (RHS) M/D + A/S	30	102	72	42	42	72
Efficiency Gain (RHS)	-	1	1.42	2.43	2.43	1.42
Temporal Order	2 nd	2 nd	1 st	2 nd	1 st	1 st
Complexity	Low	Moderate High	Moderate Low	Low	Low	Moderate Low
Non-uniform Time Step Larger than CFL	Not Stable	Not Stable	Stable	Not Stable	Stable	Stable

Chapter 3

3-D FLOD-FDTD Method Incorporated with Lumped Elements

3.1 Introduction

The FDTD method has been one of the most widely used computational methods to solve various electromagnetic problems such as RF circuits incorporated with lumped elements [13, 16, 17, 53, 54]. In recent research, implicit unconditionally stable FDTD methods are developed to incorporate with lumped elements using time step larger than CFL limit [6, 7, 18, 55–57]. However, such implicit unconditionally stable FDTD methods are still arithmetically complicated especially when the lumped elements are incorporated owing to extra computation in the implicit updating equations for resistors, capacitors and inductors. This complexity leads to much more computational time and results in lower efficiency if a large amount of lumped elements are

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simulated.

Furthermore, the memristor characterized by a relationship between the electric charge and flux was predicted by Chua in 1971 [63]. It is called the forth lumped element in addition to resistor, capacitor and inductor, but it did not cause a sensation at that time until 2008. Strukov *et al.* in Hewlett-Packard (HP) lab successfully fabricated a memristor using a thin film of titanium dioxide and a coupled variable-resistor physical model is given to describe its characteristics [12, 61]. Subsequently, analyses and applications of memristor have attracted global attention from various research areas including reconfigurable electromagnetic switching devices, microwave circuits, non-volatile random access memory, etc. [64–73] The analyses and applications of memristor have attracted much interest recently due to its widespread potentials in nanoelectronics as memory devices, computer logics and other nanoscale devices. Circuit analyses and simulations involving memristor have been relied mainly on its SPICE model [74–79], while full-wave electromagnetic modeling of memristor is still generally lacking.

In this chapter, we present an efficient FLOD-FDTD method incorporated with lumped elements. The FLOD-FDTD achieves higher efficiency and simplicity with the omission of matrix operators in the RHS. The updating equations of the FLOD-FDTD method incorporating resistors, inductors and capacitors will be derived. Numerical results including various lumped elements will be provided to show the trade-off between efficiency and accuracy along with the application of the proposed method. Moreover, the explicit and FLOD-FDTD methods incorporated with memristor are also proposed, whereby the update equations are derived based on Maxwell's equations along with the physical model of memristor given HP lab. According to the physical model, memristor is considered as variable resistor with its resistance (mem-

3.2. FLOD-FDTD METHODS INCORPORATED WITH LUMPED ELEMENTS

distance) varying with electric charge. Numerical results on the accuracy, efficiency and stability of the explicit FDTD method will be given. The simulation results obtained from the explicit FDTD method is found to be in good agreement with those obtained from SPICE model in Advanced Design System (ADS). In addition to explicit FDTD method, the FLOD-FDTD method incorporated with memristor is also proposed to circumvent the CFL stability criterion. The implicit update equations are judiciously formulated in two procedures using modified splitting matrices for memristor-incorporated FLOD-FDTD method. Numerical results will be provided to demonstrate the stability of the method for time step larger than CFL limit, as well as trade-off between efficiency and accuracy.

3.2 FLOD-FDTD Methods Incorporated with Lumped Elements

The conventional 3-D LOD-FDTD method with lumped elements can be expressed in its matrix form with two procedures as

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{A})\mathbf{u}^n + \mathbf{s}_{LE}^{n+\frac{1}{4}} \quad (3.1a)$$

$$(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t}{2}\mathbf{B})\mathbf{u}^{n+\frac{1}{2}} + \mathbf{s}_{LE}^{n+\frac{3}{4}} \quad (3.1b)$$

where $\mathbf{s}_{LE}$ is the source vector caused by the lumped elements related to the current density $\mathbf{J}_{LE}$ in (2.21) and it can be written as

$$\mathbf{s}_{LE} = \left[-\frac{\Delta t}{2\epsilon}J_{LEx}, -\frac{\Delta t}{2\epsilon}J_{LEy}, -\frac{\Delta t}{2\epsilon}J_{LEz}, 0, 0, 0 \right]^T \quad (3.2)$$

3.2.1 FLOD-FDTD Method Incorporated with Resistor R

The updating equation for the current across a resistor (with resistance R) for two procedures read

$$I_{R\xi}^{n+\frac{1}{4}} = (\Delta\xi/2R)(E_\xi^{n+\frac{1}{2}} + E_\xi^n) \quad (3.3a)$$

$$I_{R\xi}^{n+\frac{3}{4}} = (\Delta\xi/2R)(E_\xi^{n+1} + E_\xi^{n+\frac{1}{2}}) \quad (3.3b)$$

$$\xi = x, y, z \quad (3.3c)$$

To enhance the efficiency, the updating equations of FLOD-FDTD with resistors can be derived, free of matrix operator, by introducing an auxiliary variable $\mathbf{v}$ in (2.18). The FLOD-FDTD incorporated with resistors is still separated into two time steps, but with another two simple explicit equations as

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}_R\right)\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n \quad (3.4a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (3.4b)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}_R\right)\mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} \quad (3.4c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (3.4d)$$

$\mathbf{A}_R$ and $\mathbf{B}_R$ are the modified splitting matrices for resistor-incorporated FLOD-FDTD method given as

$$\mathbf{A}_R = \mathbf{A} - (1/2\varepsilon R)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.5a)$$

$$\mathbf{B}_R = \mathbf{B} - (1/2\varepsilon R)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.5b)$$

3.2. FLOD-FDTD METHODS INCORPORATED WITH LUMPED ELEMENTS

where

$$d_{11} = \frac{\Delta x}{\Delta y \Delta z}, d_{22} = \frac{\Delta y}{\Delta x \Delta z}, d_{33} = \frac{\Delta z}{\Delta x \Delta y} \quad (3.6)$$

The modified matrices have been used to obtain the corresponding fundamental form. It can be seen from (3.4a) and (3.4c) that the RHS is still free of matrix operators. Thus, the efficiency is improved compared to the conventional LOD-FDTD with resistor.

3.2.2 FLOD-FDTD method Incorporated with Inductor L

The updating equation for the current through an inductor can be written as

$$I_{L\xi}^{n+\frac{1}{4}} = I_{L\xi}^n + \Delta t \Delta \xi (E_\xi^{n+\frac{1}{2}} + E_\xi^n) / 8L \quad (3.7a)$$

$$I_{L\xi}^{n+\frac{3}{4}} = I_{L\xi}^n + \Delta t \Delta \xi (E_\xi^{n+\frac{1}{2}} + E_\xi^n) / 8L \quad (3.7b)$$

$$\xi = x, y, z. \quad (3.7c)$$

Similarly, the updating equations of the proposed FLOD-FDTD method incorporated with inductor in matrix form can be derived as

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}_L)\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n - \frac{\Delta t}{4\varepsilon}\mathbf{J}_L^n \quad (3.8a)$$

$$\mathbf{J}_L^{n+\frac{1}{2}} = \mathbf{J}_L^n + \mathbf{D}_J \cdot \mathbf{v}^{n+\frac{1}{2}} \quad (3.8b)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (3.8c)$$

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}_L)\mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} - \frac{\Delta t}{4\varepsilon}\mathbf{J}_L^{n+\frac{1}{2}} \quad (3.8d)$$

$$\mathbf{J}_L^{n+1} = \mathbf{J}_L^{n+\frac{1}{2}} + \mathbf{D}_J \cdot \mathbf{v}_E^{n+1} \quad (3.8e)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (3.8f)$$

3.2. FLOD-FDTD METHODS INCORPORATED WITH LUMPED ELEMENTS

where $\mathbf{A}_L$, $\mathbf{B}_L$, $\mathbf{D}_J$ and $\mathbf{J}_L$ are defined in the following:

$$\mathbf{A}_L = \mathbf{A} - (\Delta t/8\varepsilon L)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.9a)$$

$$\mathbf{B}_L = \mathbf{B} - (\Delta t/8\varepsilon L)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.9b)$$

$$\mathbf{J}_L = [J_{Lx}, J_{Ly}, J_{Lz}, 0, 0, 0] \quad (3.9c)$$

$$\mathbf{D}_J = (\Delta t/4L)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.9d)$$

Just like above, $\mathbf{A}_L$ and $\mathbf{B}_L$ are the modified splitting matrices in order to derive the fundamental form. In the RHS of (3.8a) and (3.8d), only the currents through the inductor are added without other matrix operators. It is noted that (3.8b) and (3.8e) are the updating equations for these current which are also necessary for the conventional LOD-FDTD method. However, the corresponding fundamental form is simpler since only the auxiliary variable $\mathbf{v}$ are employed in (3.8b) and (3.8e).

3.2.3 FLOD-FDTD method Incorporated with Capacitor C

The current through the capacitor in the first time step is

$$I_{C\xi}^{n+\frac{1}{4}} = 2C\Delta\xi/(E_\xi^{n+\frac{1}{2}} - E_\xi^n) \quad (3.10a)$$

$$I_{C\xi}^{n+\frac{3}{4}} = 2C\Delta\xi/(E_\xi^{n+1} - E_\xi^{n+\frac{1}{2}}) \quad (3.10b)$$

$$\xi = x, y, z \quad (3.10c)$$

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The FLOD-FDTD updating equations with capacitor are:

$$\left[\frac{1}{2}(\mathbf{I} + \mathbf{D}_C) - \frac{\Delta t}{4}\mathbf{A}\right]\mathbf{v}^{n+\frac{1}{2}} = (\mathbf{I} + \mathbf{D}_C)\mathbf{u}^n \quad (3.11a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (3.11b)$$

$$\left[\frac{1}{2}(\mathbf{I} + \mathbf{D}_C) - \frac{\Delta t}{4}\mathbf{B}\right]\mathbf{v}^{n+1} = (\mathbf{I} + \mathbf{D}_C)\mathbf{u}^{n+\frac{1}{2}} \quad (3.11c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}} \quad (3.11d)$$

Unlike the FLOD-FDTD method with resistor and inductor, the fundamental scheme with capacitor keeps the matrices $\mathbf{A}$ and $\mathbf{B}$ unchanged without any modification, but another auxiliary matrix $\mathbf{D}_C$ is introduced to realize the fundamental form. The auxiliary matrix $\mathbf{D}_C$ is also a diagonal matrix:

$$\mathbf{D}_C = (C/\varepsilon)\mathbf{diag}[d_{11}, d_{22}, d_{33}, 0, 0, 0] \quad (3.12)$$

Thus, the fundamental scheme is still simple and free of matrix operator in the RHS.

3.2.4 Numerical Results

To show the efficiency of the proposed FLOD-FDTD method with lumped elements, the flops count and efficiency gain for the main iterations of the conventional LOD-FDTD and FLOD-FDTD with resistor, capacitor and inductor are illustrated in Table 3.1. The number of multiplications/divisions (M/D) and additions/subtractions (A/S) are both counted in one main iteration with the assumption that all the multiplicative factors are pre-computed. From the table, it is obvious that the flops count has been reduced to a great extent for FLOD-FDTD method. Moreover, the efficiency gains in terms of CPU run time of the proposed FLOD-FDTD method over the conventional LOD-FDTD are obtained to justify the improvement of the computational

Figure 3.1: Time-domain E_z field component in the observation point (52,26,10) using explicit FDTD and FLOD-FDTD when CFLN=1,4 and 8.

efficiency for all the lumped elements.

To verify the accuracy, a $100 \times 48 \times 20$ mm³ PEC cavity filled with capacitive medium of 0.1pF is simulated. Every cell in the cavity is a cube with the length of 2mm. A differentiated Gaussian pulse source along the z-direction is used to excite the EM field in the center of the cavity as

$$(t - t_0)e^{[-(t-t_0)^2/2\tau^2]}, \tau = 50\text{ps}. \quad (3.13)$$

Fig. 3.1 depicts the time-domain E_z field component in the observation point (52,26,10) using explicit FDTD and FLOD-FDTD when CFLN=1, 4 and 8. The time step for explicit FDTD is set to the CFL limit Δt_{CFL} and CFLN is $\Delta t/\Delta t_{CFL}$. It is observed that the result of the FLOD-FDTD with CFLN=1 agrees well with that of the explicit FDTD. Considering the trade-off between efficiency and accuracy,

3.2. FLOD-FDTD METHODS INCORPORATED WITH LUMPED ELEMENTS

Table 3.1: Flops count and efficiency gain of LOD-FDTD, FLOD-FDTD with Resistor, Capacitor and Inductor in one main iteration

Scheme		LOD			FLOD		
Lumped Elements		R	C	L	R	C	L
Implicit RHS	M/D	18	18	24	6	6	12
	A/S	30	30	30	18	24	24
Explicit RHS	M/D	6	6	12	6	6	12
	A/S	24	24	36	12	12	18
Total Flops (RHS)		78	78	102	42	48	66
Total Flops (Overall)		108	108	132	72	78	96
Efficiency Gain (Flops)		/	/	/	1.5	1.38	1.375
Efficiency Gain (Run Time)		/	/	/	1.44	1.37	1.2

when the CFLN increases, there is larger discrepancy, but this may still be tolerable for some applications.

Next, an example is given to demonstrate the application of the proposed FLOD-FDTD method. A shielded microstrip line with resistor and inductor is simulated, where the geometry with side view and 3-D view are shown in Fig.3.2. Each cell is a cube with the size $\Delta x = \Delta y = \Delta z = 1\text{mm}$. The computational domain is $100 \times 80 \times 30$. The lumped elements occupy one cell each and are connected to the microstrip line and ground by PEC wire. A voltage line source is added as

$$V_s = \sin(2\pi ft), f = 0.5\text{GHz}. \quad (3.14)$$

The relative dielectric constant of the microstrip substrate is 10 and the thickness is 10mm. The value of the lumped elements are $R = 5000 \Omega$ and $L = 10\text{nH}$. Figs. 3.3 and 3.4 show the electric fields E_z across the resistor and inductor using explicit

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Figure 3.2: Geometry of a shielded microstrip line with resistor and inductor with (a) side view (b) 3-D view.

3.2. FLOD-FDTD METHODS INCORPORATED WITH LUMPED ELEMENTS

Figure 3.3: Electric field E_z across the resistor using explicit FDTD, FLOD-FDTD when CFLN=1, 2, 4 and 8.

Figure 3.4: Electric field E_z across the inductor using explicit FDTD, FLOD-FDTD when CFLN=1, 2, 4 and 8.

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FDTD and FLOD-FDTD when CFLN=1, 2, 4 and 8. The time step is set to CFL limit for explicit FDTD method. Again, the FLOD-FDTD results agree quite well with those of the explicit FDTD. With FLOD-FDTD at larger CFLN, the circuit simulation including passive lumped elements can run more efficiently.

3.3 Explicit FDTD Method Incorporated with Memristor

Memristor is regarded as the fourth basic circuit element along with the widely used lumped elements including resistor, capacitor and inductor. To the best of the authors' knowledge, there are no full-wave electromagnetic update equations derived for memristor based on Maxwell's equations so far. Meanwhile, full-wave FDTD method incorporated with resistor, capacitor and inductor have been derived and widely utilized in circuit simulation [13,16,17]. In this section, two FDTD methods incorporated with memristor are proposed. The update equations are derived based on Maxwell's equations and the physical model given by HP lab. The first method is derived by calculating the memristance directly while the second method is derived by the relationship between electric charge and flux. Numerical results are given to discuss the accuracy, efficiency and stability of both proposed methods.

3.3.1 Modeling of Memristor

The memristor links electric charge $q(t)$ and flux $\varphi(t)$. The memristance is expressed as

$$M[q(t)] = \frac{d\varphi[q(t)]}{dq(t)}. \quad (3.15)$$

According to the coupled variable-resistor physical model proposed by HP lab in [12], for a linear ionic drift in a uniform field with average ion mobility μ_v , the dopant

Figure 3.5: The coupled variable-resistor model for memristor.

width w (state variable) changes with the electric charge $q(t)$ as follows:

$$\frac{dw(t)}{dt} = \mu_v \frac{R_{ON}}{d} i(t) \quad (3.16)$$

$$w(t) = w(t=0) + \mu_v \frac{R_{ON}}{d} q(t), \quad (3.17)$$

where R_{ON} is the lower bound of the memristance and d is the thickness of the thin semiconductor film. The dopant width w , as shown in Fig. 3.5, determines the memristance $M[q(t)]$, which can be expressed as

$$M[q(t)] = R_{ON} \frac{w(t)}{d} + R_{OFF} \left[1 - \frac{w(t)}{d}\right]. \quad (3.18)$$

The initial memristance $M[q(t=0)]$ can be calculated according to

$$M[q(t=0)] = R_{ON} \frac{w(t=0)}{d} + R_{OFF} \left[1 - \frac{w(t=0)}{d}\right], \quad (3.19)$$

where R_{OFF} is the upper bound of the memristance. The memristance at a any time t can be derived as

$$M[q(t)] = M[q(t=0)] - (R_{OFF} - R_{ON}) \mu_v \frac{R_{ON}}{d^2} q(t). \quad (3.20)$$

3.3. EXPLICIT FDTD METHOD INCORPORATED WITH MEMRISTOR

The above mathematical formulations are all based on the definition of memristance and the physical model in [12]. It can be seen that the memristance M is varied when the dopant width w or the electric charge q is changed.

3.3.2 Explicit FDTD Methods Incorporated with Memristor

In this section, two memristor-incorporated FDTD methods will be presented. The update equations for these two different methods will be derived accordingly.

The first method is derived by calculating the memristance directly according to the physical model in [12]. Assuming a z -directed memristor located along the field component $E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}$, the corresponding Maxwell's differential equation can be discretized as

$$E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} = E_z|_{i,j,k+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{\Delta t}{\varepsilon \Delta x} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^n) - \frac{\Delta t}{\varepsilon \Delta y} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^n) - \frac{\Delta t}{\varepsilon} J_{mem}|_{i,j,k+\frac{1}{2}}^n. \quad (3.21)$$

J_{mem} is the electric current density across the memristor and it can be expressed in terms of the electric fields (in a semi-implicit manner) and the memristance M_{mem} as

$$J_{mem}|_{i,j,k+\frac{1}{2}}^n = \frac{I_{mem}|_{i,j,k+\frac{1}{2}}^n}{\Delta x \Delta y} = \frac{V|_{i,j,k+\frac{1}{2}}^n}{M_{mem}|^n \Delta x \Delta y} = \frac{1}{2} \frac{\Delta z (E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} + E_z|_{i,j,k+\frac{1}{2}}^{n-\frac{1}{2}})}{\Delta x \Delta y M_{mem}|^n} \quad (3.22)$$

After substituting (3.22) into (3.21), we obtain the FDTD update equation:

$$E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{(1 - \frac{\Delta t \Delta z}{2\varepsilon \Delta x \Delta y M_{mem}|^n})}{(1 + \frac{\Delta t \Delta z}{2\varepsilon \Delta x \Delta y M_{mem}|^n})} E_z|_{i,j,k+\frac{1}{2}}^{n-\frac{1}{2}} + \frac{\frac{\Delta t}{\varepsilon \Delta x} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^n)}{(1 + \frac{\Delta t \Delta z}{2\varepsilon \Delta x \Delta y M_{mem}|^n})} - \frac{\frac{\Delta t}{\varepsilon \Delta y} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^n)}{(1 + \frac{\Delta t \Delta z}{2\varepsilon \Delta x \Delta y M_{mem}|^n})} \quad (3.23)$$

When the electric field E of the memristor is changed, the memristance will also be changed due to the different dopant width w . According to (3.16), the state of dopant width w at time step $n+1$ can be calculated using the following differential equation:

$$w^{n+1} = w^n + \Delta t \mu_v \frac{R_{ON}}{d} i^{n+\frac{1}{2}} = w^n + \Delta t \mu_v \frac{R_{ON}}{d} \frac{\Delta z E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{M_{mem}|^n} \quad (3.24)$$

Next, the memristance can be calculated according to the dopant width w in (3.24).

$$M_{mem}|^{n+1} = R_{ON} \frac{w^{n+1}}{d} + R_{OFF} [1 - \frac{w^{n+1}}{d}] \quad (3.25)$$

Based on the above formulations, the time-stepping update equations for the proposed memristor-incorporated FDTD method 1 are summarized as:

- Update the $E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}$ field component according to (3.23).
- Update the dopant width w^{n+1} using (3.24).
- Update the memristance $M_{mem}|^{n+1}$ according to (3.25).

The second method is derived by using the relationship between electric charge $q(t)$ and flux $\varphi(t)$ based on the definition of the memristance and the physical model in [12]. When the memristance M in (3.15) is replaced by (3.20), the relationship between the electric charge $q(t)$ and flux $\varphi(t)$ subject to the initial condition $\varphi[q(0)] = 0$ can

3.3. EXPLICIT FDTD METHOD INCORPORATED WITH MEMRISTOR

be expressed as

$$d\varphi(t) = d\{q(t)R_0 - \frac{R_0}{2C_M}q(t)^2\}, \quad (3.26)$$

$$\varphi(t) = R_0[q(t) - \frac{1}{2C_M}q(t)^2] \quad (3.27)$$

where

$$C_M = \frac{R_0 d^2}{\mu_v R_{ON}(R_{OFF} - R_{ON})} \quad (3.28)$$

and R_0 is the initial memristance $M[q(t=0)]$ which can be calculated from (3.19).

The electric charge $q(t)$ can be expressed in terms of flux $\varphi(t)$ based on (3.27) as

$$q(t) = C_M[1 - \sqrt{1 - \frac{2}{C_M R_0}\varphi(t)}]. \quad (3.29)$$

Thus, the current density across the memristor J_{mem} can be written as

$$J_{mem}|_{i,j,k+\frac{1}{2}}^n = \frac{1}{\Delta x \Delta y} \frac{dq|_{i,j,k+\frac{1}{2}}^n}{dt} = \frac{\Delta z}{2R_0 \Delta x \Delta y} \frac{(E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} + E_z|_{i,j,k+\frac{1}{2}}^{n-\frac{1}{2}})}{\sqrt{1 - \frac{2}{C_M R_0}\varphi|_{i,j,k+\frac{1}{2}}^n}}. \quad (3.30)$$

After substituting the above equation into (3.21), the time-stepping update equations for the proposed memristor-incorporated FDTD method 2 are summarized as the following two procedures:

$$\begin{aligned} (1 + \frac{\Delta t}{\varepsilon} \frac{\Delta z}{2R_0 \Delta x \Delta y} \frac{1}{\sqrt{1 - \frac{2\varphi|_{i,j,k+\frac{1}{2}}^n}{C_M R_0}}}) E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} &= (1 - \frac{\Delta t}{\varepsilon} \frac{\Delta z}{2R_0 \Delta x \Delta y} \frac{1}{\sqrt{1 - \frac{2\varphi|_{i,j,k+\frac{1}{2}}^n}{C_M R_0}}}) E_z|_{i,j,k+\frac{1}{2}}^{n-\frac{1}{2}} \\ &+ \frac{\Delta t}{\varepsilon \Delta x} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^n) - \frac{\Delta t}{\varepsilon \Delta y} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^n) \end{aligned} \quad (3.31)$$

$$\varphi|_{i,j,k+\frac{1}{2}}^{n+1} = \varphi|_{i,j,k+\frac{1}{2}}^n + \Delta z \Delta t E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}. \quad (3.32)$$

Different from the previous memristor-incorporated FDTD method, the memris-

Figure 3.6: Configuration of the microstrip circuit.

tance in the second method is not involved in the FDTD update scheme. If the memristance is required, it still could be calculated separately according to the definition of memristance in (3.15) as well as the relationship between electric charge $q(t)$ and flux $\varphi(t)$ in (3.26). The equation for memristance is derived as

$$M_{mem}|_{i,j,k+\frac{1}{2}}^n = R_0 - \frac{R_0}{C_M} q|_{i,j,k+\frac{1}{2}}^n, \quad (3.33)$$

where the electric charge can be calculated according to (3.29).

Based on the above formulations derived directly based on the relationship between electric charge and flux, the time-stepping update equations for the proposed memristor-incorporated FDTD method 2 are summarized as:

- Update the $E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}$ field component according to (3.31).
- Update the flux $\varphi(n+1)$ using (3.32).

3.3.3 Numerical Results

As shown in Fig.3.6, a microstrip line terminated with a memristor is simulated to verify the accuracy of the proposed two methods. The relative dielectric constant ε_r of the circuit substrate is 9.4 and the thickness is 0.8 mm. A sinusoidal voltage source

Figure 3.7: Simulated results of the microstrip circuit with memristor.

along the z-direction: $V_s = \sin(2\pi 1000)$ V is used to excite the electromagnetic field at the beginning of the microstrip circuit. The time step is set to the CFL condition:

$$\Delta t = \Delta t_{CFL} = \frac{1}{c_{\max} \sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2} + \frac{1}{(\Delta z)^2}}} \quad (3.34)$$

According to the physical model of memristor [12], the average ion mobility is $\mu_v = 1e^{-14}$ m/s. The same mesh is utilized for both two proposed memristor-incorporated FDTD methods. The initial memristance is 500 ohm and the range of the memristance $M \in \{R_{ON} = 100, R_{OFF} = 16e^3\}$ ohm. The thickness of the thin semiconductor film is $d = 1e^{-11}$ m.

The simulated results are shown in terms of the memristance, as in Fig. 3.7. The memristance changes with time as the sinusoidal voltage source provides a varying flux across the memristor. The same microstrip circuit is simulated by a commercial software Advanced Design System (ADS) with the SPICE model in [65] to verify the accuracy of the two proposed methods. In Fig. 3.7, FDTD method 1 is the

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first proposed method by calculating the memristance and FDTD method 2 is the second method using the relationship between electric charge $q(t)$ and flux $\varphi(t)$. It is observed that both proposed simulation methods agree well with the simulated result using SPICE model in ADS.

We run the programs using Matlab on Windows 7 platform with 3.1 GHz Intel i5 processor and 4 GB RAM. After 50000 main iterations by both FDTD methods, the CPU computational time for the first method is 1.82s, and 3.55s for the second method. More computational time is taken for the second FDTD method, although the second method could achieve a more accurate result.

Since memristor is a non-linear component involving many parameters according to the physical model proposed by HP, commonly used stability analysis method such as Von-Neumann is not applicable for memristor-incorporated FDTD method. Nevertheless, the stability of FDTD methods incorporated with memristor can be verified numerically. We have executed both FDTD methods for $1e^6$ iterations and both results of the electric fields across the memristors show that the algorithms are stable when the time step is set to the CFL limitation in (3.34).

To show the robustness of the FDTD method, another example with Gaussian excitation is simulated. The initial memristance is 500 ohm and the range of the memristance $M \in \{R_{ON} = 100, R_{OFF} = 16e^3\}$ ohm. The thickness of the thin semiconductor film is $d = 1e^{-8}$ m. The source is changed to a Gaussian pulse: $\frac{1}{2} \exp(-\frac{t-3\tau}{\tau})^2$ mV/m, $\tau = 1$ ms. The electric field E_z of the memristor and its corresponding memristance are shown in Fig. 3.8. It could be observed that the memristance remains a stable value after the electric field E_z becomes zero.

3.4. EFFICIENT FLOD-FDTD METHOD INCORPORATED WITH MEMRISTOR

Figure 3.8: Electric field and memristance of the memristor excited by Gaussian pulse.

3.4 Efficient FLOD-FDTD Method Incorporated with Memristor

The unconditionally stable FDTD methods incorporated with lumped elements/devices have been developed to simulate microwave circuits including resistors, capacitors, inductors [55], [80]. Nevertheless, to the best knowledge of the authors, there is no unconditionally stable method derived for memristor so far. In this section, an efficient 3-D fundamental LOD-FDTD (FLOD-FDTD) method incorporated with memristor is proposed. The FLOD-FDTD method achieves higher efficiency and simplicity with matrix-operator-free RHS. The updating equations of the FLOD-FDTD method incorporated with memristor are derived. Numerical results will be provided to show the trade-off between efficiency and accuracy.

According to the coupled variable-resistor model proposed in [12], as shown in Fig. 3.5, the memristor can be considered as a resistor whose resistance is varied by the dopant width w . Because there exists an boundary for the change of the

3.4. EFFICIENT FLOD-FDTD METHOD INCORPORATED WITH MEMRISTOR

memristance, the relationship between the dopant state x_{mem} and the current $i(t)$ can be further expressed as

$$\frac{dx_{mem}(t)}{dt} = \frac{1}{d} \frac{dw(t)}{dt} = \mu_v \frac{R_{ON}}{d^2} i(t) f(x_{mem}), \quad (3.35)$$

where μ_v is the average ion mobility and $f(x_{mem}) = 1 - (2x_{mem} - 1)^{2p}$. $f(x_{mem})$ is a non-linear window function [12] [81] that can make the state $x_{mem}(t)$ bounded within the interval $[0, 1]$, and p is the nonlinear exponent constant.

3.4.1 LOD-FDTD Method Incorporated with Memristor

The conventional 3-D LOD-FDTD method incorporated with memristor can be expressed in terms of two procedures as

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}) \mathbf{u}^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}) \mathbf{u}^n + \frac{\Delta t}{2} \mathbf{s}_{mem}^{n+\frac{1}{4}} \quad (3.36a)$$

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}) \mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}) \mathbf{u}^{n+\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{s}_{mem}^{n+\frac{3}{4}}, \quad (3.36b)$$

where

$$\mathbf{u} = [E_x, E_y, E_z, H_x, H_y, H_z]^T, \quad (3.37a)$$

$$\mathbf{s}_{mem} = \left[-\frac{1}{\varepsilon} J_{memx}, -\frac{1}{\varepsilon} J_{memy}, -\frac{1}{\varepsilon} J_{memz}, 0, 0, 0 \right]^T. \quad (3.37b)$$

Δx , Δy and Δz are the spatial steps along the x-,y- and z-directions, respectively; J_{memx} , J_{memy} and J_{memz} are the corresponding current density along the x-,y- and z-directions; $\mathbf{A}$ and $\mathbf{B}$ are the splitting matrices which are defined in [42].

The current density through a memristor in the first procedure from n to $n + \frac{1}{2}$ can be

3.4. EFFICIENT FLOD-FDTD METHOD INCORPORATED WITH MEMRISTOR

derived as (taking $+z$ oriented memristor as an example)

$$J_{memz}|_{i+\frac{1}{2},j,k}^{n+\frac{1}{4}} = \frac{V|_{i+\frac{1}{2},j,k}^{n+\frac{1}{4}}}{\Delta x \Delta y M_{memz}|_{i+\frac{1}{2},j,k}^{n+\frac{1}{4}}} = \frac{\Delta z (E_z|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} + E_z|_{i+\frac{1}{2},j,k}^n)}{2 \Delta x \Delta y M_{memz}|_{i+\frac{1}{2},j,k}^{n+\frac{1}{4}}}. \quad (3.38)$$

where M_{memz} is the corresponding memristance along the z -direction. Similar equation can be written for the second procedure from $n + \frac{1}{2}$ to $n + 1$ with a change in the time index.

3.4.2 FLOD-FDTD Method Incorporated with Memristor

To enhance the efficiency, the updating equations of FLOD-FDTD incorporated with memristor can be derived, free of matrix operator in RHS, by introducing an auxiliary variable v in (2.18). By manipulating (3.36) and (3.38), the FLOD-FDTD method incorporated with memristor can be written as:

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}_{mem})\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n \quad (3.39a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n \quad (3.39b)$$

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}_{mem})\mathbf{v}^{n+1} = \mathbf{u}^{n+\frac{1}{2}} \quad (3.39c)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^{n+\frac{1}{2}}. \quad (3.39d)$$

$\mathbf{A}_{mem}$ and $\mathbf{B}_{mem}$ are the modified splitting matrices for the memristor-incorporated FLOD-FDTD method:

$$\mathbf{A}_{mem} = \mathbf{A} - \frac{1}{2}\mathbf{diag}[\frac{d_{11}}{\varepsilon M_{memx}}, \frac{d_{22}}{\varepsilon M_{memy}}, \frac{d_{33}}{\varepsilon M_{memz}}, 0, 0, 0] \quad (3.40a)$$

$$\mathbf{B}_{mem} = \mathbf{B} - \frac{1}{2}\mathbf{diag}[\frac{d_{11}}{\varepsilon M_{memx}}, \frac{d_{22}}{\varepsilon M_{memy}}, \frac{d_{33}}{\varepsilon M_{memz}}, 0, 0, 0], \quad (3.40b)$$

where

$$d_{11} = \frac{\Delta x}{\Delta y \Delta z}, d_{22} = \frac{\Delta y}{\Delta x \Delta z}, d_{33} = \frac{\Delta z}{\Delta x \Delta y}. \quad (3.41)$$

It can be seen from (3.39a) and (3.39c) that their RHS are free of matrix operators. Thus,

3.4. EFFICIENT FLOD-FDTD METHOD INCORPORATED WITH MEMRISTOR

the efficiency is improved compared to the conventional LOD-FDTD with memristor.

3.4.3 Efficient Implementation of 3-D FLOD-FDTD Method Incorporated with Memristor

The efficient implementation of the 3-D FLOD-FDTD method incorporated with memristor for two procedures according to (3.39) are given as

First Procedure:

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta y^2} + \frac{\Delta t\Delta x/\Delta y\Delta z}{8\varepsilon M_{memx}|_{i+\frac{1}{2},j,k}^{n+1/4}} \right) e_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\varepsilon\mu\Delta y^2} (e_x|_{i+\frac{1}{2},j+1,k}^{n+\frac{1}{2}} + e_x|_{i+\frac{1}{2},j-1,k}^{n+\frac{1}{2}}) \\ &= E_x|_{i+\frac{1}{2},j,k}^n + \frac{\Delta t}{2\varepsilon\Delta y} (H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^n - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^n) \end{aligned} \quad (3.42a)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta z^2} + \frac{\Delta t\Delta y/\Delta x\Delta z}{8\varepsilon M_{memy}|_{i,j+\frac{1}{2},k}^{n+1/4}} \right) e_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\varepsilon\mu\Delta z^2} (e_y|_{i,j+\frac{1}{2},k+1}^{n+\frac{1}{2}} + e_y|_{i,j+\frac{1}{2},k-1}^{n+\frac{1}{2}}) \\ &= E_y|_{i,j+\frac{1}{2},k}^n + \frac{\Delta t}{2\varepsilon\Delta z} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n - H_x|_{i,j+\frac{1}{2},k-\frac{1}{2}}^n) \end{aligned} \quad (3.42b)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta x^2} + \frac{\Delta t d_{33}}{8\varepsilon M_{memz}|_{i,j,k+\frac{1}{2}}^{n+1/4}} \right) e_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\Delta t^2}{8\varepsilon\mu\Delta x^2} (e_z|_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} + e_z|_{i-1,j,k+\frac{1}{2}}^{n+\frac{1}{2}}) \\ &= E_z|_{i,j,k+\frac{1}{2}}^n + \frac{\Delta t}{2\varepsilon\Delta x} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^n) \end{aligned} \quad (3.42c)$$

$$E_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} = e_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} - E_x|_{i+\frac{1}{2},j,k}^n \quad (3.42d)$$

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^n + \frac{\Delta t}{2\mu\Delta z} (e_y|_{i,j+\frac{1}{2},k+1}^{n+\frac{1}{2}} - e_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}}) \quad (3.42e)$$

$$E_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} = e_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} - E_y|_{i,j+\frac{1}{2},k}^n \quad (3.42f)$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^n + \frac{\Delta t}{2\mu\Delta x} (e_z|_{i+1,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - e_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}) \quad (3.42g)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} = e_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} - E_z|_{i,j,k+\frac{1}{2}}^n \quad (3.42h)$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^n + \frac{\Delta t}{2\mu\Delta y} (e_x|_{i+\frac{1}{2},j+1,k}^{n+\frac{1}{2}} - e_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}}) \quad (3.42i)$$

3.4. EFFICIENT FLOD-FDTD METHOD INCORPORATED WITH MEMRISTOR

Second Procedure:

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta y^2} + \frac{\Delta t\Delta x/\Delta y\Delta z}{8\varepsilon M_{memx}|_{i+\frac{1}{2},j,k}^{n+3/4}} \right) e_x|_{i+\frac{1}{2},j,k}^{n+1} - \frac{\Delta t^2}{8\varepsilon\mu\Delta z^2} (e_x|_{i+\frac{1}{2},j,k+1}^{n+1} + e_x|_{i+\frac{1}{2},j,k-1}^{n+1}) \\ &= E_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} + \frac{\Delta t}{2\varepsilon\Delta z} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}) \end{aligned} \quad (3.43a)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta z^2} + \frac{\Delta t\Delta y/\Delta x\Delta z}{8\varepsilon M_{memy}|_{i,j+\frac{1}{2},k}^{n+3/4}} \right) e_y|_{i,j+\frac{1}{2},k}^{n+1} - \frac{\Delta t^2}{8\varepsilon\mu\Delta x^2} (e_y|_{i+1,j+\frac{1}{2},k}^{n+1} + e_y|_{i-1,j+\frac{1}{2},k}^{n+1}) \\ &= E_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} + \frac{\Delta t}{2\varepsilon\Delta x} (H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}) \end{aligned} \quad (3.43b)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{4\varepsilon\mu\Delta x^2} + \frac{\Delta td_{33}}{8\varepsilon M_{memz}|_{i,j,k+\frac{1}{2}}^{n+3/4}} \right) e_z|_{i,j,k+\frac{1}{2}}^{n+1} - \frac{\Delta t^2}{8\varepsilon\mu\Delta y^2} (e_z|_{i,j+1,k+\frac{1}{2}}^{n+1} + e_z|_{i,j-1,k+\frac{1}{2}}^{n+1}) \\ &= E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} + \frac{\Delta t}{2\varepsilon\Delta y} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i+\frac{1}{2},j+\frac{1}{2},k-\frac{1}{2}}^{n+\frac{1}{2}}) \end{aligned} \quad (3.43c)$$

$$E_x|_{i+\frac{1}{2},j,k}^{n+1} = e_x|_{i+\frac{1}{2},j,k}^{n+1} - E_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} \quad (3.43d)$$

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+1} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu\Delta y} (e_z|_{i,j+1,k+\frac{1}{2}}^{n+1} - e_z|_{i,j,k+\frac{1}{2}}^{n+1}) \quad (3.43e)$$

$$E_y|_{i,j+\frac{1}{2},k}^{n+1} = e_y|_{i,j+\frac{1}{2},k}^{n+1} - E_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} \quad (3.43f)$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+1} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu\Delta z} (e_x|_{i+\frac{1}{2},j,k}^{n+1} - e_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}}) \quad (3.43g)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n+1} = e_z|_{i,j,k+\frac{1}{2}}^{n+1} - E_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \quad (3.43h)$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+1} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - \frac{\Delta t}{2\mu\Delta x} (e_y|_{i+1,j+\frac{1}{2},k}^{n+1} - e_y|_{i,j+\frac{1}{2},k}^{n+1}) \quad (3.43i)$$

After the implementation of the first procedure according to (3.42c) - (3.42h), the current through memristor can be calculated using Ampere's Law as

$$\begin{aligned} I_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} &= (H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}})\Delta x \\ &\quad + (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}})\Delta y. \end{aligned} \quad (3.44)$$

Since memristance will change according to the dopant state x_{mem} , x_{mem} will be updated for each sub time step. The dopant state of $+z$ oriented memristor $x_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}}$ can be derived

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from (3.35) and (3.44) as

$$x_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}} = x_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{4}} + \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}^{n+\frac{1}{2}}) I_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}}. \quad (3.45)$$

Subsequently, the update equation of the memristor $M_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}}$ can be written following as

$$M_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}} = R_{ON} x_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}} + R_{OFF} [1 - x_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}}]. \quad (3.46)$$

For implementation of the second procedure, similar update equation can be derived by adding $\frac{1}{2}$ to the time indices in (3.45) and (3.46). Note that the conventional FDTD coefficients for linear lumped elements are constant in time while the coefficients for the proposed FLOD-FDTD method with memristor will vary for different time step according to (3.46).

Because the memristor can be considered as a varying resistor within the interval $[R_{ON}, R_{OFF}]$, the lower bound of the doped state is 0 and the upper bound of the doped state is 1. The following bounding condition should be executed when (3.45) is updated:

$$x_{mem} = \begin{cases} 0, & x_{mem} < 0 \\ 1, & x_{mem} > 1. \end{cases} \quad (3.47)$$

Based on the above formulations, the time-stepping update equations for the proposed memristor-incorporated FLOD-FDTD method are summarized as

- Update the electromagnetic field components for the first procedure of FLOD-FDTD method from n to $n + \frac{1}{2}$ according to (3.39a) and (3.39b), e.g. (3.42c) and (3.42h).
- Calculate the current through memristor using (3.44), update the dopant state $x_{mem}^{n+\frac{3}{4}}$ using (3.45), and check the bounding condition of dopant state in (3.47).
- Update the memristance $M_{memz}|_{i,j,k+\frac{1}{2}}^{n+\frac{3}{4}}$ using (3.46).
- Update the electromagnetic field components for the second procedure of FLOD-FDTD method from $n + \frac{1}{2}$ to $n + 1$ according to (3.39c) and (3.39d), e.g. (3.43c) and (3.43h).

3.5. NUMERICAL RESULTS

Table 3.2: Comparison of arithmetic operation counts in the RHS of one main iteration between 3-D LOD-FDTD and FLOD-FDTD.

Arithmetic operations (RHS)	LOD-FDTD with memristor	FLOD-FDTD with memristor
Implicit (M/D, A/S)	18, 24	6, 18
Explicit (M/D, A/S)	18, 36	18, 24
Total (M/D+A/S)	96	66

- Calculate the current through memristor using (3.44), update the dopant state $x_{mem}^{n+1\frac{1}{4}}$ using (3.45), and check the bounding condition of dopant state in (3.47).
- Update the memristance $M_{memz}|_{i,j,k+\frac{1}{2}}^{n+1\frac{1}{4}}$ using (3.46). (The time indices in (3.44)-(3.46) should be changed accordingly for the second procedure.)

3.5 Numerical Results

Table 3.2 shows the comparison of arithmetic operation counts in the RHS of one main iteration between 3-D LOD-FDTD and FLOD-FDTD. The number of multiplications/divisions (M/D) and additions/subtractions (A/S) are both counted in one main iteration with the assumption that all the multiplicative factors are pre-computed. It can be seen that the total arithmetic operation counts have been decreased for FLOD-FDTD method with memristor and the efficiency gain based on the total arithmetic operation counts in RHS is 1.45.

A numerical example is given to demonstrate the application of the proposed memristor-incorporated FLOD-FDTD method. A shielded microstrip line with a resistor and a memristor is simulated, where the geometry is shown in Fig. 3.9. Each cell is a cube with the side length of 1 mm. The computational domain is $100 \times 80 \times 30$. The resistor and memristor occupy one cell each and are connected to the microstrip line and ground by PEC wires. A current source is added at the center of the microstrip line: $J_z = 5000 \sin(2\pi ft) \text{A/m}^2$ with $f = 1 \text{GHz}$ flowing along z -axis from bottom to the microstrip line. The relative dielectric constant of the microstrip substrate is 4.4 and the thickness is 10 mm. The thickness

3.5. NUMERICAL RESULTS

Figure 3.9: Geometry of a shielded microstrip line with a resistor and a memristor.

of the microstrip is set to 1 mm. The value of the resistor is $R = 5\text{k}\Omega$ and the initial value of the memristor is $5\text{k}\Omega$. The other parameters are $R_{ON} = 10\Omega$, $R_{OFF} = 10\text{k}\Omega$, $\mu_v = 1 \times 10^{-12}\text{m/s}$ and $d = 1 \times 10^{-12}\text{m}$. Fig. 3.10 shows the electric field E_z across the resistor and memristor using explicit FDTD method and FLOD-FDTD method when $\text{CFLN}=1$, where $\text{CFLN} = \Delta t / \Delta t_{\text{CFL}}$. The time step is set to the CFL limit for the explicit FDTD. It can be observed that the electric field across the memristor deviates from the one of resistor. In addition, the results of FLOD-FDTD method when $\text{CFLN}=1$ agree well with the ones of explicit FDTD method.

Fig. 3.11 shows the electric field E_z across the memristor using explicit FDTD method and FLOD-FDTD method when $\text{CFLN}=1, 2, 4$ and 8. It can be seen that the simulated results of FLOD-FDTD method when $\text{CFLN}=1, 2$ and 4 agree well with the result of FDTD method, but the result of FLOD-FDTD method for $\text{CFLN}=8$ deviates from the other results to some extent. The larger time step will cause larger error of the update of memristance for each time step since the update of memristance is based on the calculated value from the previous time step.

Fig. 3.12 shows the simulated memristances across the memristor using explicit FDTD method and FLOD-FDTD method when $\text{CFLN}=1, 2, 4$ and 8. Again, the simulated memristances of FLOD-FDTD method when $\text{CFLN}=1, 2$ and 4 agree well with the result of FDTD method, but the memristances of FLOD-FDTD method for $\text{CFLN}=8$ deviates from

3.5. NUMERICAL RESULTS

Figure 3.10: Electric field E_z across the resistor and memristor using explicit FDTD method and FLOD-FDTD method when CFLN=1.

Figure 3.11: Electric field E_z across the memristor using explicit FDTD method and FLOD-FDTD method when CFLN = 1, 2, 4 and 8.

3.5. NUMERICAL RESULTS

Figure 3.12: Memristance across the memristor using explicit FDTD method and FLOD-FDTD method when $CFLN = 1, 2, 4$ and 8 .

the other results to some degree. The trade-off between efficiency and accuracy has been illustrated here. We have quantified the errors and calculation time for different CFLNs for FLOD-FDTD method with memristor. The relative errors of FLOD-FDTD method with memristor compared with explicit FDTD method are 0.0071, 0.0393 and 0.2103 when CFLN is 2, 4 and 8, respectively. As the CFLN becomes larger, the operator splitting error [82] and the discretization error of the dopant states in (3.45) with larger time step would be the main reasons to cause more error. In addition, the calculated CPU time of the 3-D circuit in Fig. 3.9 from 0 to $1000\Delta t_{CFL}$ using the FLOD-FDTD method with memristor are 130.8 s and 65.3 s when CFLN are 4 and 8, while the explicit FDTD takes 147.8 s. Furthermore, through the simulation of the circuit in Fig.3.9 for a long duration up to 100000 iterations (using CFLN=8), we have validated numerically the stability of our FLOD-FDTD method

3.6. CONCLUSION

incorporated with memristor.

3.6 Conclusion

This section has presented an efficient FLOD-FDTD method incorporated with lumped elements. The FLOD-FDTD achieves higher efficiency and simplicity with the omission of matrix operators in the RHS. The updating equations of the FLOD-FDTD method incorporating resistors, inductors and capacitors have been derived. Numerical results including various lumped elements have been provided to show the trade-off between efficiency and accuracy along with the application of the proposed method. The proposed efficient scheme including lumped elements may be extended to other methods, e.g. LOD2, SS2, etc. Moreover, the explicit and FLOD-FDTD methods incorporated with memristor are also proposed, whereby the update equations have been derived based on Maxwell's equations along with the physical model of memristor given HP lab. According to the physical model, memristor is considered as variable resistor with its resistance (memristance) varying with electric charge. Numerical results on the accuracy, efficiency and stability of the explicit FDTD method have been given. The simulation results obtained from the explicit FDTD method is found to be in good agreement with those obtained from SPICE model in ADS. In addition to explicit FDTD method, the FLOD-FDTD method incorporated with memristor is also proposed to circumvent the CFL stability criterion. The implicit update equations have been judiciously formulated in two procedures using modified splitting matrices for memristor-incorporated FLOD-FDTD method. Numerical results have been provided to demonstrate the stability of the method for time step larger than CFL limit, as well as trade-off between efficiency and accuracy.

Chapter 4

Second-order Accurate FLOD-FDTD Methods with Three Split Matrices

4.1 Introduction

LOD-FDTD method is an unconditionally stable method that is readily amenable to higher number of split steps or procedures [83]. The conventional scheme of LOD-FDTD has two split matrices denoted as $\mathbf{A}$ and $\mathbf{B}$ of LOD1 in [42]. Its temporal order accuracy is of first order and can be improved to second order when the noncommutative terms $\mathbf{AB}$ and $\mathbf{BA}$ are omitted efficiently as in [9] and [33]. An alternative scheme of LOD-FDTD has been developed in [36] and involves three split matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, whereby each split matrix contains only spatial operators along one direction. Here, $\mathbf{A}$ and $\mathbf{B}$ in [36] are different from (more sparse than) those in [42]. While the temporal accuracy of LOD-FDTD method with three split matrices is also of first order, it tends to be much more costly to realize second-order temporal accuracy in the traditional (Strang) way due to many more noncommutative

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terms $\mathbf{AC}$, $\mathbf{CA}$, $\mathbf{BC}$ and $\mathbf{CB}$ [40]. The associated noncommutative error may also be more significant compared to those of LOD-FDTD with two split matrices $\mathbf{A}$ and $\mathbf{B}$ [8], [84].

In this chapter, we propose the second-order temporal-accurate scheme for 3-D FLOD-FDTD method with three split matrices. The main iterations of the proposed scheme comprise only one more procedure in addition to the existing three procedures in [36]. Using proper (initial only) input and (often infrequent, field-point only) output processings, the second-order temporal accuracy is achieved which is to be verified analytically. To further enhance the efficiency, the proposed scheme is implemented based on the principle of fundamental schemes in [42], which features a matrix-operator-free RHS. Moreover, an alternative reversed second-order temporal-accurate FLOD-FDTD scheme is also proposed. Stability analysis is provided to ascertain the unconditional stability of the proposed scheme and numerical results are demonstrated to justify its higher accuracy.

4.2 3-D LOD-FDTD Method with Three Split Matrices

4.2.1 First-order Temporal-accurate Scheme

The conventional LOD-FDTD with three split matrices can be expressed as [36]

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^{n+\frac{1}{3}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^n \quad (4.1a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{B}\right)\mathbf{u}^{n+\frac{2}{3}} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{B}\right)\mathbf{u}^{n+\frac{1}{3}} \quad (4.1b)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n+\frac{2}{3}} \quad (4.1c)$$

where

$$\mathbf{u} = [E_x, E_y, E_z, H_x, H_y, H_z]^T. \quad (4.2)$$

and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are the three split matrices defined therein [36, (1)]. From (4.1), three procedures make up one main iteration and each procedure contains only spatial operators along x , y , or z directions. The above scheme is only of first-order temporal accuracy.

4.2.2 Second-order Temporal-accurate Scheme

To realize second-order temporal accuracy, (4.1b) is split into two procedures and executed before and after (4.1c), resulting in the following four update equations:

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^{n,1} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{A}\right)\mathbf{u}^{n,0} \quad (4.3a)$$

$$\left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right)\mathbf{u}^{n,2} = \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{B}\right)\mathbf{u}^{n,1} \quad (4.3b)$$

$$\left(\mathbf{I} - \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n,3} = \left(\mathbf{I} + \frac{\Delta t}{2}\mathbf{C}\right)\mathbf{u}^{n,2} \quad (4.3c)$$

$$\left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}\right)\mathbf{u}^{n+1,0} = \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{B}\right)\mathbf{u}^{n,3} \quad (4.3d)$$

with the following input and output processings:

$$\text{Input: } \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{1,0} = \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^0 \quad (4.4a)$$

$$\text{Output: } \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{n+1} = \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{n+1,0} \quad (4.4b)$$

Note that the main iterations of the proposed scheme comprise only one more procedure in addition to the three procedures in (4.1). Furthermore, the input processing is only executed for once when the initial fields are (seldom) nonzero. The output processing also needs to be executed only when the result of a particular component at certain time step (often infrequent) and certain location (field-point only) is needed. The overall isotropy of numerical dispersion will not be much degraded if the input and output processings are invoked properly.

4.2.3 Verification of Second-order Temporal Accuracy

To verify the second-order temporal accuracy, we first write down the amplification matrix after n main iterations (denoted as G^n):

$$G^n = \{ (\mathbf{I} - (\Delta t/4)\mathbf{B})^{-1} (\mathbf{I} + (\Delta t/4)\mathbf{B}) (\mathbf{I} - (\Delta t/2)\mathbf{C})^{-1} (\mathbf{I} + (\Delta t/2)\mathbf{C}) (\mathbf{I} - (\Delta t/4)\mathbf{B})^{-1} (\mathbf{I} + (\Delta t/4)\mathbf{B}) (\mathbf{I} - (\Delta t/2)\mathbf{A})^{-1} (\mathbf{I} + (\Delta t/2)\mathbf{A}) \}^n \quad (4.5)$$

Taylor series expansion up to second-order is used as follows:

$$(\mathbf{I} - (\Delta t/4)\mathbf{A})^{-1} = \mathbf{I} + (\Delta t/4)\mathbf{A} + (\Delta t/4)^2 \mathbf{A}^2 + O(\Delta t^3) \quad (4.6a)$$

$$(\mathbf{I} - (\Delta t/4)\mathbf{B})^{-1} = \mathbf{I} + (\Delta t/4)\mathbf{B} + (\Delta t/4)^2 \mathbf{B}^2 + O(\Delta t^3) \quad (4.6b)$$

$$(\mathbf{I} - (\Delta t/2)\mathbf{C})^{-1} = \mathbf{I} + (\Delta t/2)\mathbf{C} + (\Delta t/2)^2 \mathbf{C}^2 + O(\Delta t^3) \quad (4.6c)$$

Next, the above Taylor series expansions are applied to (4.5), we obtain

$$G^n = \mathbf{I} + n\Delta t(\mathbf{A} + \mathbf{B} + \mathbf{C}) + (1/2)n^2\Delta t^2[(\mathbf{A} + \mathbf{B} + \mathbf{C})^2 + (\mathbf{BA} + \mathbf{CA} - \mathbf{AB} - \mathbf{AC})] + O(\Delta t^3). \quad (4.7)$$

After input and output processings are performed, the final amplification matrix is derived as

$$\begin{aligned} & \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A} \right)^{-1} \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A} \right) G^n \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A} \right)^{-1} \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A} \right) \\ &= \mathbf{I} + n\Delta t(\mathbf{A} + \mathbf{B} + \mathbf{C}) + \frac{n^2\Delta t^2}{2}(\mathbf{A} + \mathbf{B} + \mathbf{C})^2 + O(\Delta t^3). \end{aligned} \quad (4.8)$$

It can be seen that the final amplification matrix (4.8) has the same temporal accuracy as the analytical solution up to second-order. Thus, the limitation related to first order temporal accuracy in [85] would be eliminated. Note that in (4.8), more noncommutative terms $\mathbf{AC}$, $\mathbf{CA}$, $\mathbf{BC}$, $\mathbf{CB}$ are involved compared to only $\mathbf{AB}$, $\mathbf{BA}$ in [4, (26)]. The iterative

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method in [84], [86] could be attempted to make the proposed second-order accurate method more accurate by approaching the Crank-Nicolson scheme, but the procedures could be more complicated due to more noncommutative terms.

4.2.4 Efficient Second-order Temporal-accurate FLOD-FDTD Scheme ABCB

To further enhance the efficiency, the proposed scheme is implemented based on the principle of fundamental schemes in [42], which features a matrix-operator-free RHS. We first introduce an auxiliary vector $\mathbf{v}$ in (2.18). Following [42], the fundamental scheme of (4.3) can be denoted as FLOD-FDTD and given by

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{v}^{n,1} = \mathbf{u}^{n,0} \quad (4.9a)$$

$$\mathbf{u}^{n,1} = \mathbf{v}^{n,1} - \mathbf{u}^{n,0} \quad (4.9b)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{8}\mathbf{B}\right)\mathbf{v}^{n,2} = \mathbf{u}^{n,1} \quad (4.9c)$$

$$\mathbf{u}^{n,2} = \mathbf{v}^{n,2} - \mathbf{u}^{n,1} \quad (4.9d)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{C}\right)\mathbf{v}^{n,3} = \mathbf{u}^{n,2} \quad (4.9e)$$

$$\mathbf{u}^{n,3} = \mathbf{v}^{n,3} - \mathbf{u}^{n,2} \quad (4.9f)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{8}\mathbf{B}\right)\mathbf{v}^{n+1,0} = \mathbf{u}^{n,3} \quad (4.9g)$$

$$\mathbf{u}^{n+1,0} = \mathbf{v}^{n+1,0} - \mathbf{u}^{n,3}. \quad (4.9h)$$

where the RHS of (4.9) is now matrix-operator-free. Note that $\mathbf{u}$ and $\mathbf{v}$ are used to store the fields whereas for conventional method in Section II-A, two $\mathbf{u}$'s are needed. Hence, the fields memory arrays are the same.

The differential update equations of (4.9a) and (4.9b) read: 1) Implicit updating for $e_y^{n,1}$

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and $e_z^{n,1}$

$$\frac{1}{2}e_y^{n,1} - \frac{\Delta t^2}{8\varepsilon\mu}\partial_x^2 e_y^{n,1} = E_y^{n,0} - \frac{\Delta t}{2\varepsilon}\partial_x H_z^{n,0} \quad (4.10a)$$

$$\frac{1}{2}e_z^{n,1} - \frac{\Delta t^2}{8\varepsilon\mu}\partial_x^2 e_z^{n,1} = E_z^{n,0} + \frac{\Delta t}{2\varepsilon}\partial_x H_y^{n,0} \quad (4.10b)$$

2) Explicit updating for $E_y^{n,1}$, $E_z^{n,1}$, $H_y^{n,1}$ and $H_z^{n,1}$

$$E_y^{n,1} = e_y^{n,1} - E_y^{n,0} \quad (4.11a)$$

$$E_z^{n,1} = e_z^{n,1} - E_z^{n,0} \quad (4.11b)$$

$$H_y^{n,1} = H_y^{n,0} + \frac{\Delta t}{2\mu}\partial_x e_z^{n,1} \quad (4.11c)$$

$$H_z^{n,1} = H_z^{n,0} - \frac{\Delta t}{2\mu}\partial_x e_y^{n,1} \quad (4.11d)$$

Note that it is unnecessary to update E_x and H_x in (4.9a) and (4.9b) as they are unchanged using the fundamental scheme for this procedure. Other differential update equations can be similarly obtained by expanding and manipulating (4.9c)-(4.9g).

Moreover, the implementation of the implicit update equations in the first procedure (4.9a) and (4.9b) for the proposed FLOD-FDTD scheme ABCB can be written as

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta x^2} + \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i-\frac{1}{2},j+\frac{1}{2},k}\Delta x^2} \right) e_y|_{i,j+\frac{1}{2},k}^{n,1} \\ & - \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta x^2} e_y|_{i+1,j+\frac{1}{2},k}^{n,1} - \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i-\frac{1}{2},j+\frac{1}{2},k}\Delta x^2} e_y|_{i-1,j+\frac{1}{2},k}^{n,1} \\ & = E_y|_{i,j+\frac{1}{2},k}^{n,0} - \frac{\Delta t}{2\varepsilon_{i,j+\frac{1}{2},k}\Delta x} (H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,0} - H_z|_{i-\frac{1}{2},j+\frac{1}{2},k}^{n,0}) \end{aligned} \quad (4.12a)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta x^2} + \frac{\Delta t^2}{8\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i-\frac{1}{2},j,k+\frac{1}{2}}\Delta x^2} \right) e_z|_{i,j,k+\frac{1}{2}}^{n,1} \\ & - \frac{\Delta t^2}{8\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta x^2} e_z|_{i+1,j,k+\frac{1}{2}}^{n,1} - \frac{\Delta t^2}{8\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i-\frac{1}{2},j,k+\frac{1}{2}}\Delta x^2} e_z|_{i-1,j,k+\frac{1}{2}}^{n,1} \\ & = E_z|_{i,j,k+\frac{1}{2}}^{n,0} + \frac{\Delta t}{2\varepsilon_{i,j,k+\frac{1}{2}}\Delta x} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,0} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n,0}) \end{aligned} \quad (4.12b)$$

The implementation of the explicit update equations in the first procedure (4.9a) and (4.9b)

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for the proposed FLOD-FDTD scheme ABCB can be written as

$$E_y|_{i,j+\frac{1}{2},k}^{n,1} = e_y|_{i,j+\frac{1}{2},k}^{n,1} - E_y|_{i,j+\frac{1}{2},k}^{n,0} \quad (4.13a)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n,1} = e_z|_{i,j,k+\frac{1}{2}}^{n,1} - E_z|_{i,j,k+\frac{1}{2}}^{n,0} \quad (4.13b)$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,1} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,0} + \frac{\Delta t}{2\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta x}(e_z|_{i+1,j,k+\frac{1}{2}}^{n,1} - e_z|_{i,j,k+\frac{1}{2}}^{n,1}) \quad (4.13c)$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,1} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,0} - \frac{\Delta t}{2\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta x}(e_y|_{i+1,j+\frac{1}{2},k}^{n,1} - e_y|_{i,j+\frac{1}{2},k}^{n,1}) \quad (4.13d)$$

The implementation of the implicit update equations in the second procedure (4.9c) and (4.9d) for the proposed FLOD-FDTD scheme ABCB can be written as

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y^2} + \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j-\frac{1}{2},k}\Delta y^2} \right) e_x|_{i+\frac{1}{2},j,k}^{n,2} \\ & - \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y^2} e_x|_{i,j+\frac{3}{2},k}^{n,2} - \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j-\frac{1}{2},k}\Delta y^2} e_x|_{i,j-\frac{1}{2},k}^{n,2} \\ & = E_x|_{i+\frac{1}{2},j,k}^{n,1} + \frac{\Delta t}{4\varepsilon_{i+\frac{1}{2},j,k}\Delta y}(H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,1} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n,1}) \end{aligned} \quad (4.14a)$$

$$\begin{aligned} & \left(\frac{1}{2} + \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y^2} + \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j-\frac{1}{2},k+\frac{1}{2}}\Delta y^2} \right) e_z|_{i,j,k+\frac{1}{2}}^{n,2} \\ & - \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y^2} e_z|_{i,j+1,k+\frac{1}{2}}^{n,2} - \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j-\frac{1}{2},k+\frac{1}{2}}\Delta y^2} e_z|_{i,j-1,k+\frac{1}{2}}^{n,2} \\ & = E_z|_{i,j,k+\frac{1}{2}}^{n,1} - \frac{\Delta t}{4\varepsilon_{i,j,k+\frac{1}{2}}\Delta y}(H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,1} - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n,1}) \end{aligned} \quad (4.14b)$$

The implementation of the explicit update equations in the first procedure (4.9c) and (4.9d) for the proposed FLOD-FDTD scheme ABCB can be written as

$$E_x|_{i+\frac{1}{2},j,k}^{n,2} = e_x|_{i+\frac{1}{2},j,k}^{n,2} - E_x|_{i+\frac{1}{2},j,k}^{n,1} \quad (4.15a)$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n,2} = e_z|_{i,j,k+\frac{1}{2}}^{n,2} - e_z|_{i,j,k+\frac{1}{2}}^{n,1} \quad (4.15b)$$

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,2} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,1} - \frac{\Delta t}{4\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y}(e_z|_{i,j+1,k+\frac{1}{2}}^{n,2} - e_z|_{i,j,k+\frac{1}{2}}^{n,2}) \quad (4.15c)$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,2} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,1} + \frac{\Delta t}{4\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y}(e_x|_{i,j+\frac{3}{2},k}^{n,2} - e_x|_{i,j+\frac{1}{2},k}^{n,2}) \quad (4.15d)$$

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The implementation of the implicit update equations in the third procedure (4.9e) and (4.9f) for the proposed FLOD-FDTD scheme ABCB can be written as

$$\begin{aligned}
& \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta z^2} + \frac{\Delta t^2}{8\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j,k-\frac{1}{2}}\Delta z^2} \right) e_x|_{i+\frac{1}{2},j,k}^{n,3} \\
& - \frac{\Delta t^2}{8\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta z^2} e_x|_{i+\frac{1}{2},j,k+1}^{n,3} + \frac{\Delta t^2}{8\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j,k-\frac{1}{2}}\Delta z^2} e_x|_{i+\frac{1}{2},j,k-1}^{n,3} \\
& = E_x|_{i+\frac{1}{2},j,k}^{n,2} - \frac{\Delta t}{2\varepsilon_{i+\frac{1}{2},j,k}\Delta z} (H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,2} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n,2}) \tag{4.16a}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta z^2} + \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i,j+\frac{1}{2},k-\frac{1}{2}}\Delta z^2} \right) e_y|_{i,j+\frac{1}{2},k}^{n,3} \\
& - \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta z^2} e_y|_{i,j+\frac{1}{2},k+1}^{n,3} - \frac{\Delta t^2}{8\varepsilon_{i,j+\frac{1}{2},k}\mu_{i,j+\frac{1}{2},k-\frac{1}{2}}\Delta z^2} e_y|_{i,j+\frac{1}{2},k-1}^{n,3} \\
& = E_y|_{i,j+\frac{1}{2},k}^{n,2} + \frac{\Delta t}{2\varepsilon_{i,j+\frac{1}{2},k}\Delta z} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,2} - H_x|_{i,j+\frac{1}{2},k-\frac{1}{2}}^{n,2}) \tag{4.16b}
\end{aligned}$$

The implementation of the explicit update equations in the third procedure (4.9e) and (4.9f) for the proposed FLOD-FDTD scheme ABCB can be written as

$$E_x|_{i+\frac{1}{2},j,k}^{n,3} = e_x|_{i+\frac{1}{2},j,k}^{n,3} - E_x|_{i+\frac{1}{2},j,k}^{n,2} \tag{4.17a}$$

$$E_y|_{i,j+\frac{1}{2},k}^{n,3} = e_y|_{i,j+\frac{1}{2},k}^{n,3} - E_y|_{i,j+\frac{1}{2},k}^{n,2} \tag{4.17b}$$

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,3} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,2} + \frac{\Delta t}{2\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta z} (e_y|_{i,j+\frac{1}{2},k+1}^{n,3} - e_y|_{i,j+\frac{1}{2},k}^{n,3}) \tag{4.17c}$$

$$H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,3} = H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n,2} - \frac{\Delta t}{2\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}\Delta z} (e_x|_{i+\frac{1}{2},j,k+1}^{n,3} - e_x|_{i+\frac{1}{2},j,k}^{n,3}) \tag{4.17d}$$

The implementation of the implicit update equations in the fourth procedure (4.9g) and

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(4.9h) for the proposed FLOD-FDTD scheme ABCB can be written as

$$\begin{aligned}
& \left(\frac{1}{2} + \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y^2} + \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j-\frac{1}{2},k}\Delta y^2} \right) e_x|_{i+\frac{1}{2},j,k}^{n,4} \\
& - \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y^2} e_x|_{i,j+\frac{3}{2},k}^{n,4} - \frac{\Delta t^2}{32\varepsilon_{i+\frac{1}{2},j,k}\mu_{i+\frac{1}{2},j-\frac{1}{2},k}\Delta y^2} e_x|_{i,j-\frac{1}{2},k}^{n,4} \\
& = E_x|_{i+\frac{1}{2},j,k}^{n,3} + \frac{\Delta t}{4\varepsilon_{i+\frac{1}{2},j,k}\Delta y} (H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,3} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n,3}) \tag{4.18a}
\end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{2} + \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y^2} + \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j-\frac{1}{2},k+\frac{1}{2}}\Delta y^2} \right) e_z|_{i,j,k+\frac{1}{2}}^{n,4} \\
& - \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y^2} e_z|_{i,j+1,k+\frac{1}{2}}^{n,4} - \frac{\Delta t^2}{32\varepsilon_{i,j,k+\frac{1}{2}}\mu_{i,j-\frac{1}{2},k+\frac{1}{2}}\Delta y^2} e_z|_{i,j-1,k+\frac{1}{2}}^{n,4} \\
& = E_z|_{i,j,k+\frac{1}{2}}^{n,3} - \frac{\Delta t}{4\varepsilon_{i,j,k+\frac{1}{2}}\Delta y} (H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,3} - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n,3}) \tag{4.18b}
\end{aligned}$$

The implementation of the explicit update equations in the fourth procedure (4.9g) and (4.9h) for the proposed FLOD-FDTD scheme ABCB can be written as

$$E_x|_{i+\frac{1}{2},j,k}^{n,4} = e_x|_{i+\frac{1}{2},j,k}^{n,4} - E_x|_{i+\frac{1}{2},j,k}^{n,3} \tag{4.19a}$$

$$E_z|_{i,j,k+\frac{1}{2}}^{n,4} = e_z|_{i,j,k+\frac{1}{2}}^{n,4} - E_z|_{i,j,k+\frac{1}{2}}^{n,3} \tag{4.19b}$$

$$H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,4} = H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n,3} - \frac{\Delta t}{4\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}\Delta y} (e_z|_{i,j+1,k+\frac{1}{2}}^{n,4} - e_z|_{i,j,k+\frac{1}{2}}^{n,4}) \tag{4.19c}$$

$$H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,4} = H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n,3} + \frac{\Delta t}{4\mu_{i+\frac{1}{2},j+\frac{1}{2},k}\Delta y} (e_x|_{i,j+\frac{3}{2},k}^{n,4} - e_x|_{i,j+\frac{1}{2},k}^{n,4}) \tag{4.19d}$$

4.2.5 Stability Analysis

The von Neumann method is used to verify the unconditional stability of the proposed scheme.

The eigenvalues of amplification matrix in the main iteration can be solved as:

$$\lambda_1 = \lambda_2 = 1, \tag{4.20a}$$

$$\lambda_3 = \lambda_4 = \frac{X + jY}{Z} \tag{4.20b}$$

$$\lambda_5 = \lambda_6 = \lambda_3^* = \lambda_4^* \tag{4.20c}$$

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where

$$\begin{aligned}
X = & 4096\varepsilon^4\mu^4 - 512\Delta t^2\varepsilon^3\mu^3(2K_x^2 + 3K_y^2 + 2K_z^2) \\
& + 16\Delta t^4\varepsilon^2\mu^2(K_y^4 - 8K_x^2K_y^2 - 16K_x^2K_z^2 - 8K_y^2K_z^2) \\
& - 4\varepsilon\mu\Delta t^6(K_x^2K_y^4 + 8K_x^2K_y^2K_z^2 + K_y^4K_z^2) \\
& - K_x^2K_y^4K_z^2\Delta t^8
\end{aligned} \tag{4.21}$$

$$\begin{aligned}
Y = & 8\{\Delta t^2\varepsilon^2\mu^2(16\varepsilon\mu - K_y^2\Delta t^2)^2[K_x^2K_y^4K_z^2\Delta t^6 \\
& + 32K_x^2K_y^2K_z^2\Delta t^4\varepsilon\mu + 4\Delta t^4\varepsilon\mu K_y^4(K_x^2 + K_z^2) \\
& + 256K_x^2K_z^2\Delta t^2\varepsilon^2\mu^2 + 1024\varepsilon^3\mu^3(K_x^2 + K_y^2 + K_z^2) \\
& + 128K_y^2\Delta t^2\varepsilon^2\mu^2(K_z^2 + K_x^2)]\}^{1/2} \\
K_\xi = & 2\sin(k_\xi\Delta\xi/2)/\Delta\xi, \quad \xi = x, y, z
\end{aligned} \tag{4.22}$$

and k_ξ is the wavenumber along ξ direction. Since X , Y , Z are real, it can be shown that

$$\frac{X^2 + Y^2}{Z^2} = 1. \tag{4.23}$$

Hence, all eigenvalues have magnitude of one and the scheme is unconditionally stable.

4.2.6 Reversed Second-order Temporal-accurate FLOD-FDTD Scheme (BCBA)

When the main iteration in (4.9) is performed in a reversed manner with different input and output processing, another FLOD-FDTD scheme with second-order temporal accuracy

4.2. 3-D LOD-FDTD METHOD WITH THREE SPLIT MATRICES

is derived:

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{8}\mathbf{B}\right)\mathbf{v}^{n,1} = \mathbf{u}^{n,0}, \quad (4.24a)$$

$$\mathbf{u}^{n,1} = \mathbf{v}^{n,1} - \mathbf{u}^{n,0} \quad (4.24b)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{C}\right)\mathbf{v}^{n,2} = \mathbf{u}^{n,1}, \quad (4.24c)$$

$$\mathbf{u}^{n,2} = \mathbf{v}^{n,2} - \mathbf{u}^{n,1} \quad (4.24d)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{8}\mathbf{B}\right)\mathbf{v}^{n,3} = \mathbf{u}^{n,2}, \quad (4.24e)$$

$$\mathbf{u}^{n,3} = \mathbf{v}^{n,3} - \mathbf{u}^{n,2} \quad (4.24f)$$

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{v}^{n+1,0} = \mathbf{u}^{n,3}, \quad (4.24g)$$

$$\mathbf{u}^{n+1,0} = \mathbf{v}^{n+1,0} - \mathbf{u}^{n,3} \quad (4.24h)$$

with the following input and output processings:

$$\text{Input: } \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{1,0} = \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^0 \quad (4.25a)$$

$$\text{Output: } \left(\mathbf{I} + \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{n+1} = \left(\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}\right)\mathbf{u}^{n+1,0} \quad (4.25b)$$

Note that the analytical proof of second-order temporal accuracy is the same as Section 4.2.3 and shall not be repeated.

4.2.7 Numerical Results

An air-filled $100 \times 60 \times 60$ mm³ cavity meshed with uniform cells of 2 mm is simulated to demonstrate the accuracy of the proposed fundamental LOD-FDTD method. A differentiated Gaussian pulse source along the z -direction: $(t - t_0)e^{[-(t-t_0)^2/\tau^2]}$ ($\tau = 350ps$, $t_0 = 3\tau$) is used to excite the EM field at the center of the cavity. Table 4.1 illustrates the normalized norm error of LOD-FDTD in [36], the proposed second-order temporal-accurate FLOD-FDTD scheme and the equivalent FLOD-FDTD scheme (BCBA) for various CFLN = $\Delta t/\Delta t_{CFL}$.

4.2. 3-D LOD-FDTD METHOD WITH THREE SPLIT MATRICES

Table 4.1: Normalized norm error of LOD-FDTD with three split matrices, the proposed second-order temporal-accurate FLOD-FDTD scheme and the reversed FLOD-FDTD scheme (BCBA).

CFLN	1	2	4	8
LOD-FDTD with three split matrices	0.0103	0.0165	0.0400	0.1874
The proposed FLOD-FDTD scheme	0.0066	0.0099	0.0294	0.0547
The proposed FLOD-FDTD scheme (BCBA)	0.0066	0.0099	0.0294	0.0547

The normalized norm error is taken with respect to explicit FDTD results in the whole computational domain at $t = 2440\Delta t_{CFL}$. Note that the output processing for FLOD-FDTD is only required once at $t = 2440\Delta t_{CFL}$. It is observed that both the proposed FLOD-FDTD schemes have lower normalized norm error than LOD-FDTD in [36] due to higher-order temporal accuracy. The Normalized norm errors of the FLOD-FDTD scheme (ABCB) and (BCBA) are the same, which also indicates that the reversed scheme (BCBA) is equivalent to scheme (ABCB). The flops count (RHS) of multiplication/division (M/D) and addition/subtraction (A/S) for the two methods are given in Table 4.2. It is shown that while featuring higher order temporal accuracy, the proposed FLOD-FDTD scheme still achieve comparable (slightly higher) efficiency with respect to LOD-FDTD method in [36]. We further implement both schemes in an air-filled cavity with $120 \times 120 \times 120$ computational domain and the relative CPU time and memory requirement are also shown in Table 4.2. The programs are run using Matlab on Windows 7 platform with 3.1 GHz Intel i5 processor and 4 GB RAM. Although having one additional procedure with extra cost of solving tridiagonal systems, the CPU time incurred by the second-order FLOD-FDTD scheme is still around 0.93 that of first-order LOD-FDTD scheme in [36] due to RHS flops count reduction. Compared with the method in [40], the proposed fundamental scheme is almost two times faster while maintaining second-order temporal accuracy due to one substep fewer and matrix-operator-free RHS. The memory requirements of all the three schemes are also the same.

Next, we simulate a reverberation chamber with two stirrers. The geometry of the chamber with two stirrers is shown in Fig. 4.1, along with the source and observation point. The two stirrers have the same dimension. One is placed on the top of the reverberation chamber

4.2. 3-D LOD-FDTD METHOD WITH THREE SPLIT MATRICES

Table 4.2: Flops count (RHS), relative CPU time and memory for three-step and five-step LOD-FDTD and proposed second-order temporal-accurate FLOD-FDTD scheme.

Arithmetic Operations		LOD-FDTD		Proposed Scheme
		Three-step [36]	Five-step [40]	
Implicit	M/D	12	20	8
	A/S	30	50	16
Explicit	M/D	6	10	8
	A/S	24	40	24
Total (RHS)		72	120	56
Temporal Accuracy		First-order	Second-Order	Second-Order
Relative CPU Time		1	1.71	0.93
Memory		158MBytes	158MBytes	158MBytes

Figure 4.1: Geometry of reverberation chamber with two stirrers.

and the other is one the left. Both two stirrers are modeled as PEC and staircasing is used to approximate the slanted geometries. Each stirrer has four segments which are perpendicular to each other. Each segment has length of 10 cells, width of 10 cells and thickness of 1 cell. The computational domain is $100 \times 60 \times 60$ and cell size is 2 mm. A differentiated Gaussian pulse source is excited along the z -direction at the center of the chamber: $(t - t_0) \exp[-(t - t_0)^2 / \tau^2]$, $\tau = 350 \text{ ps}$, $t_0 = 3\tau$. Fig. 4.2 shows the electric field E_z computed using explicit FDTD method, the proposed FLOD-FDTD scheme and first-order LOD-FDTD scheme in [36] for various CFLNs when the angles of the stirrers are 0 and 90 degrees respectively. To display Fig. 4.2, the output processing is executed for each time step. The

(a)

(b)

Figure 4.2: Electric field E_z at field point computed using explicit FDTD and FLOD-FDTD method for various CFLNs when the angles of the stirrers are (a) 0° and (b) 90° .

Figure 4.3: Geometry of a shielded microstrip line with two open stubs.

spatial grids employed for the explicit FDTD and first-order LOD-FDTD methods are the same as the proposed FLOD-FDTD method. At CFLN=1, the normalized norm error with respect to time at the observation point (60,30,15) for LOD-FDTD [36] is 0.0124 compared with explicit FDTD method, while the error of the proposed FLOD-FDTD method is 0.0081. For CFLN = 2, 4 and 8, the normalized norm errors for LOD-FDTD [12] are 0.0233, 0.0795 and 0.3012 respectively and 0.0128, 0.0390, 0.1423 for the proposed LOD-FDTD method. The computation time of the proposed method (including the output processing only for the observation point) when CFLN=8 is 3.7 times faster than the explicit FDTD method. The results of FLOD-FDTD scheme agree well with that of explicit FDTD method, which further validates our proposed scheme.

Next, we simulate a shielded microstrip line circuit with two open stubs. The geometry of the circuit, along with source and observation point are shown in Fig. 4.3. The thickness of the substrate is 10 mm, cell size is 1 mm and ϵ_r is 4.4. Fig. 4.4 shows the electric field E_z at observation point (33,30,5) computed using explicit FDTD method and FLOD-FDTD schemes for various CFLNs. Since the schemes are comparable, we only show the results using scheme **ABCB**. It is observed that the results of FLOD-FDTD schemes agree well with that of explicit FDTD method, which further validates our proposed schemes. With increased CFLN, larger (but acceptable) discrepancies can be observed.

Figure 4.4: Electric field E_z using explicit FDTD method and FLOD-FDTD schemes for various CFLNs.

4.3 Conclusion

In this chapter, we have proposed second-order temporal-accurate scheme for 3-D LOD-FDTD method with three split matrices. The main iterations of the proposed scheme comprise only one more procedure in addition to the existing three procedures. Using proper (initial only) input and (often infrequent, field-point only) output processings, the second-order temporal accuracy is achieved which has been verified analytically. To further enhance the efficiency, the proposed scheme have been implemented based on the principle of fundamental schemes, which features a matrix-operator-free RHS. Stability analysis has been provided to ascertain the unconditional stability of the proposed scheme and numerical results have been demonstrated to justify its higher accuracy. It is anticipated that the scheme might be extended to other types of partial differential equations (e.g. thermal diffusion equation).

Chapter 5

Interconnected

Multi-One-Dimensional FADI- and

FLOD-FDTD Methods for

Transmission Lines with

Interjunctions

5.1 Introduction

Although full 3-D FDTD methods can cater for most general structures, the computational domain is still large and full 3-D methods are not well-suited to run on mobile devices such as smart phones/pads due to the requirement for large memory storage and long simulation time. To reduce memory and simulation time requirement, one needs to resort to 1-D FDTD methods. However, traditional simplistic 1-D FDTD methods are not sufficient to treat

5.1. INTRODUCTION

electromagnetic waves propagation on multiple main transmission lines and stubs, not to mention if they are interconnected among themselves at various interjunctions. Recently, an explicit FDTD method is developed for efficient simulation and real-time visualization of electromagnetic waves propagation along transmission line circuit [87]. The method finds immediate applications for quick design toolkit with real-time simulation on handy mobile device. However, so far it is applicable for only one main transmission line between a two-port and it cannot deal with more main transmission lines and stubs possibly interconnected among themselves. Moreover, the computational time step is also constrained by CFL condition.

To use time step larger than CFL limit, unconditionally stable FDTD methods have been developed. The arithmetic operations of FADI-FDTD and fundamental FLOD-FDTD are reduced with matrix-operator-free RHS. So far, all the aforementioned unconditionally stable FDTD methods are still not well-suited to run on mobile device due to large memory storage and long simulation time if 3-D or 2-D full-wave problems are simulated. Furthermore, the FADI- and FLOD-FDTD methods for 1-D problems have not been fully exploited because pure simplistic 1-D modelling usually cannot be applied for solving practical problems.

In this chapter, we present the interconnected multi-one-dimensional (IM1-D) FADI- and FLOD-FDTD methods for transmission lines with interjunctions. The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit. Fundamental scheme-based IM1-D FADI- and FLOD-FDTD methods will be derived to enhance the efficiency with matrix-operator-free RHS. The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device. Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems. A microstrip line loaded with stubs and a branch-line coupler will be simulated to show the accuracy and efficiency of the proposed methods. To extend the applicability for handling the couplings between two transmission lines via gaps, a microstrip circuit with gaps will be simulated using the proposed methods incorporated with equivalent circuit models involving capacitances. Real-time simulations of these numerical

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

Figure 5.1: Alignment of field components E_x , H_y and H_z for the m th and $(m+1)$ th main transmission lines possibly interconnected with the s th and $(s+1)$ th stubs.

examples will provide much intuitional insight for one to observe the electromagnetic waves propagation in time-domain on computer or mobile device.

5.2 IM1-D FADI-FDTD Method with Treatments at Interjunctions

Without loss of generality, we consider that there may be $m_{\max}$ main transmission lines and $s_{\max}$ stubs forming the transmission line network rather than only one main transmission line in [87]. Fig. 5.1 shows the alignment of field components E_x , H_y and H_z for the m th and $(m+1)$ th main transmission lines possibly interconnected with the s th and $(s+1)$ th stubs, where $m \in [1, m_{\max}]$ and $s \in [1, s_{\max}]$. Unlike conventional FDTD methods that are explicit subjected to CFL stability constraints, we shall resort to unconditionally stable FDTD methods to optimize the algorithms for running on mobile device using time step larger than CFL limit. The update equations for IM1-D FADI-FDTD method involving one-step update procedure for both main transmission lines and stubs are derived. The source treatments for various interconnection conditions using IM1-D FADI-FDTD method are discussed and the implementation of the proposed method is given.

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

5.2.1 One-step IM1-D FADI-FDTD Method for Stub

The IM1-D ADI-FDTD method for stub can be written in matrix form as

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n+\frac{1}{2}} = \mathbf{u}_s^n + \frac{\Delta t}{2} \mathbf{s}_s^n \quad (5.1a)$$

$$\mathbf{u}_s^{n+1} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n+\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{s}_s^{n+1} \quad (5.1b)$$

where $\mathbf{A}_s$ is the split matrix given by

$$\mathbf{A}_s = \begin{pmatrix} 0 & \frac{1}{\varepsilon} \partial_y \\ \frac{1}{\mu} \partial_y & 0 \end{pmatrix} \quad (5.2)$$

$\mathbf{u}_s^n = [E_x^{(s)}|_n, H_z^{(s)}|_n]^T$ comprises the electromagnetic fields in the s th stub at time step n . $\mathbf{s}_s^n$ is the source treatment vector for the interjunctions of the s th stub at time step n :

$$\mathbf{s}_s^n = [-\frac{1}{\varepsilon} J_x^{(s \leftarrow m)}|_n, 0]. \quad (5.3)$$

where $J_x^{(s \leftarrow m)}$ is the current density for the s th stub that comes from the m th main transmission line. Note that this source treatment vector should have time index n in the first procedure and $n+1$ in the second procedure, instead of common symmetrical source index $n + \frac{1}{2}$ in both procedures. The detailed source treatment using IM1-D ADI-FDTD method will be discussed in Section 5.2.3.

Based on (5.1b) at one time step backward, it can be rewritten as

$$\mathbf{u}_s^n = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n-\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{s}_s^n. \quad (5.4)$$

We combine (5.1b) and (5.4) within time step Δt to yield a compact expression. The one-step update procedure of IM1-D ADI-FDTD method for stub reads

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n-\frac{1}{2}} + \Delta t \mathbf{s}_s^n. \quad (5.5)$$

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

Unlike the conventional ADI-FDTD method that commonly has two update procedures, the proposed IM1-D ADI-FDTD method involves updating in one-step procedure for stub. Note that the electromagnetic fields are updated from $n - \frac{1}{2}$ to $n + \frac{1}{2}$, while the time index for source is n . Such one-step update procedure is different from the one-step leapfrog ADI-FDTD where the E and H updates are leapfrog in time [25]- [88].

To enhance the computational efficiency, we derive matrix-operator-free RHS based on fundamental implicit scheme to arrive at IM1-D FADI-FDTD. Here, ‘F’ stands for “fundamental” which implies the basic form that is not reducible further without any more RHS matrix operator. The update equations of IM1-D FADI-FDTD for the s th stub can be derived as

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{A}_s)\mathbf{v}_s^{n+\frac{1}{2}} = \mathbf{u}_s^{n-\frac{1}{2}} + \frac{\Delta t}{2}\mathbf{s}_s^n \quad (5.6a)$$

$$\mathbf{u}_s^{n+\frac{1}{2}} = \mathbf{v}_s^{n+\frac{1}{2}} - \mathbf{u}_s^{n-\frac{1}{2}} \quad (5.6b)$$

where the auxiliary variable $\mathbf{v}_s^{n+\frac{1}{2}}$ is

$$\mathbf{v}_s^{n+\frac{1}{2}} = [e_x^{(s)}|^{n+\frac{1}{2}}, h_z^{(s)}|^{n+\frac{1}{2}}]^T. \quad (5.7)$$

The proposed method in (5.6) involves efficient fundamental unconditionally stable scheme with matrix-operator-free RHS and simplified one-step update procedure, which is optimized for simulation on mobile device. It should be noted that the fundamental scheme does not degrade the accuracy and no additional memory is needed.

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

5.2.2 One-step IM1-D FADI-FDTD Method for Main Transmission Line

The IM1-D ADI-FDTD method for the m th main transmission line can be expressed in matrix form as

$$\mathbf{u}_m^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^n + \frac{\Delta t}{2} \mathbf{s}_m^{n+\frac{1}{2}} \quad (5.8a)$$

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^{n+1} = \mathbf{u}_m^{n+\frac{1}{2}} + \frac{\Delta t}{2} \mathbf{s}_m^{n+\frac{1}{2}} \quad (5.8b)$$

where $\mathbf{B}_m$ is the split matrix given by

$$\mathbf{B}_m = \begin{pmatrix} 0 & -\frac{1}{\epsilon} \partial z \\ -\frac{1}{\mu} \partial z & 0 \end{pmatrix}. \quad (5.9)$$

$\mathbf{u}_m^n = [E_x^{(m)}|_n, H_y^{(m)}|_n]^T$ comprises the electromagnetic fields in the m th main transmission line at time step n . $\mathbf{s}_m^{n+\frac{1}{2}}$ is the source treatment vector for the interjunctions of the m th main transmission line at time step $n + \frac{1}{2}$:

$$\mathbf{s}_m^{n+\frac{1}{2}} = [-\frac{1}{\epsilon} J_x^{(m \leftarrow s)}|^{n+\frac{1}{2}}, 0] \quad (5.10)$$

where $J_x^{(m \leftarrow s)}$ is the current density for the m th main transmission that comes from the s th stub. The detailed source treatment using IM1-D ADI-FDTD method will be discussed in Section 5.2.3. We combine the two procedures in (5.8) within one time step Δt to yield

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^{n+1} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^n + \Delta t \mathbf{s}_m^{n+\frac{1}{2}}. \quad (5.11)$$

As before, the proposed IM1-D ADI-FDTD method for each main transmission line involves updating in one-step procedure only. Unlike the time marching scheme in (5.5) for stub, the electromagnetic fields are updated from n to $n + 1$, while the time index for source is $n + \frac{1}{2}$. Thus, the update of field components for main transmission line and stub can be

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

Figure 5.2: The treatment at the interjunction and the alignment of field components in the m th main transmission line and the s th stub using IM1-D ADI-FDTD method.

implemented alternately using the proposed IM1-D ADI-FDTD method.

To simplify the above for higher efficiency, the update equations of IM1-D FADI-FDTD for the m th main transmission line are derived as

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t}{4}\mathbf{B}_m\right)\mathbf{v}_m^{n+1} = \mathbf{u}_m^n + \frac{\Delta t}{2}\mathbf{s}_m^{n+\frac{1}{2}} \quad (5.12a)$$

$$\mathbf{u}_m^{n+1} = \mathbf{v}_m^{n+1} - \mathbf{u}_m^n \quad (5.12b)$$

where the auxiliary variable $\mathbf{v}_m^{n+1}$ is given as

$$\mathbf{v}_m^{n+1} = [e_x^{(m)}|^{n+1}, h_y^{(m)}|^{n+1}]^T. \quad (5.13)$$

Again, the IM1-D FADI-FDTD method for the m th main transmission line is also optimized for simulation on mobile device using the efficient fundamental unconditionally stable scheme with matrix-operator-free RHS and simplified one-step update procedure.

5.2.3 Source Treatments at Interjunctions using IM1-D FADI-FDTD Method for Various Interconnection Conditions

To make all the IM1-D FADI-FDTD update equations work cooperatively for solving practical transmission line network with various interjunctions, the source treatments in (5.6)

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

Figure 5.3: Various interconnection conditions using IM1-D FADI-FDTD method.

and (5.12) at the interjunctions are the key procedures. Fig. 5.2 shows the source treatment at the interjunction and the alignment of field components in the m th main transmission line and the s th stub using IM1-D FADI-FDTD method. As shown in the figure, the main transmission line and the stub are modeled as two individual 1-D transmission lines. The (upper) m th main transmission line contains field components $E_x^{(m)}$ and $H_y^{(m)}$. The (lower) s th stub contains field components $E_x^{(s)}$ and $H_z^{(s)}$. $J_x^{(m \leftarrow s)}$ and $J_x^{(s \leftarrow m)}$ are the current densities at the interjunction for the source treatment.

Fig. 5.3 shows various interconnection conditions using IM1-D FADI-FDTD method. Assume that the spatial index of the interjunction for the m th main transmission line is k , and the spatial index of the interjunction for the s th stub is j . Using Ampere's Law, the current densities $J_x^{(m \leftarrow s)}|_k^{n+\frac{1}{2}}$ and $J_x^{(s \leftarrow m)}|_j^n$ at the interjunction can be calculated as

$$J_x^{(m \leftarrow s)}|_k^{n+\frac{1}{2}} = \begin{cases} -\frac{1}{\Delta y} H_z^{(s)}|_{\frac{1}{2}}^{n+\frac{1}{2}} & \text{Top} \\ -\frac{1}{\Delta y} (H_z^{(s)}|_{j+\frac{1}{2}}^{n+\frac{1}{2}} - H_z^{(s)}|_{j-\frac{1}{2}}^{n+\frac{1}{2}}) & \text{Intermediate} \\ \frac{1}{\Delta y} H_z^{(s)}|_{je^{(s)}-\frac{1}{2}}^{n+\frac{1}{2}} & \text{Bottom} \end{cases} \quad (5.14)$$

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

$$J_x^{(s \leftarrow m)}|_j^n = \begin{cases} \frac{1}{\Delta z} H_y^{(m)}|_{\frac{1}{2}}^n & \text{Left} \\ \frac{1}{\Delta z} (H_y^{(m)}|_{k+\frac{1}{2}}^n - H_y^{(m)}|_{k-\frac{1}{2}}^n) & \text{Intermediate} \\ -\frac{1}{\Delta z} H_y^{(m)}|_{ke^{(m)}-\frac{1}{2}}^n & \text{Right} \end{cases} \quad (5.15)$$

where $je^{(s)}$ and $ke^{(m)}$ are the spatial indices of the last Yee's cells for the s th stub and the m th main transmission line, respectively. The current density $J_x^{(m \leftarrow s)}$ for the m th main transmission is contributed by the magnetic fields in the s th stub at the interjunction, while the current density $J_x^{(s \leftarrow m)}$ for the s th stub is contributed by the magnetic fields in the m th main transmission line at the interjunction. Improper source treatments at interjunctions would make the IM1-D FADI-FDTD method unstable. Using the above proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems.

5.2. IM1-D FADI-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

5.2.4 Implementation for IM1-D FADI-FDTD Method

The detailed implementation of the IM1-D FADI-FDTD method is given as

Main TL :

$$\begin{aligned}
& - \frac{\Delta t^2}{8\varepsilon_k^{(m)} \mu_{k-\frac{1}{2}}^{(m)} \Delta z^2} e_x^{(m)}|_{k-1}^{n+1} + \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_k^{(m)} \mu_{k-\frac{1}{2}}^{(m)} \Delta z^2} + \frac{\Delta t^2}{8\varepsilon_k^{(m)} \mu_{k+\frac{1}{2}}^{(m)} \Delta z^2} \right) e_x^{(m)}|_k^{n+1} \\
& - \frac{\Delta t^2}{8\varepsilon_k^{(m)} \mu_{k+\frac{1}{2}}^{(m)} \Delta z^2} e_x^{(m)}|_{k+1}^{n+1} = E_x^{(m)}|_k^n - \frac{\Delta t}{2\varepsilon_k^{(m)} \Delta z} (H_y^{(m)}|_{k+\frac{1}{2}}^n - H_y^{(m)}|_{k-\frac{1}{2}}^n) + \frac{\Delta t}{2\varepsilon_k^{(m)}} J_x^{(m \leftarrow s)}|_k^{n+\frac{1}{2}}
\end{aligned} \tag{5.17a}$$

$$E_x^{(m)}|_k^{n+1} = e_x^{(m)}|_k^{n+1} - E_x^{(m)}|_k^n \tag{5.17b}$$

$$H_y^{(m)}|_{k+\frac{1}{2}}^{n+1} = H_y^{(m)}|_{k+\frac{1}{2}}^n - \frac{\Delta t}{2\mu_{k+\frac{1}{2}}^{(m)} \Delta z} (e_x^{(m)}|_{k+1}^{n+1} - e_x^{(m)}|_k^{n+1}) \tag{5.17c}$$

Stub :

$$\begin{aligned}
& - \frac{\Delta t^2}{8\varepsilon_j^{(s)} \mu_{j-\frac{1}{2}}^{(s)} \Delta y^2} e_x^{(s)}|_{j-1}^{n+\frac{1}{2}} + \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_j^{(s)} \mu_{j-\frac{1}{2}}^{(s)} \Delta y^2} + \frac{\Delta t^2}{8\varepsilon_j^{(s)} \mu_{j+\frac{1}{2}}^{(s)} \Delta y^2} \right) e_x^{(s)}|_j^{n+\frac{1}{2}} \\
& - \frac{\Delta t^2}{8\varepsilon_j^{(s)} \mu_{j+\frac{1}{2}}^{(s)} \Delta y^2} e_x^{(s)}|_{j+1}^{n+\frac{1}{2}} = E_x^{(s)}|_j^{n-\frac{1}{2}} + \frac{\Delta t}{2\varepsilon_j^{(s)} \Delta y} (H_z^{(s)}|_{j+\frac{1}{2}}^{n-\frac{1}{2}} - H_z^{(s)}|_{j-\frac{1}{2}}^{n-\frac{1}{2}}) + \frac{\Delta t}{2\varepsilon_k^{(s)}} J_x^{(s \leftarrow m)}|_j^n
\end{aligned} \tag{5.17d}$$

$$E_x^{(s)}|_j^{n+\frac{1}{2}} = e_x^{(s)}|_j^{n+\frac{1}{2}} - E_x^{(s)}|_j^{n-\frac{1}{2}} \tag{5.17e}$$

$$H_z^{(s)}|_{j+\frac{1}{2}}^{n+\frac{1}{2}} = H_z^{(s)}|_{j+\frac{1}{2}}^{n-\frac{1}{2}} - \frac{\Delta t}{2\mu_{k+\frac{1}{2}}^{(s)} \Delta z} (e_x^{(s)}|_{j+1}^{n+\frac{1}{2}} - e_x^{(s)}|_j^{n+\frac{1}{2}}) \tag{5.17f}$$

Unlike the IM1-D ADI-FDTD method that has more elaborated RHS, the proposed IM1-D FADI-FDTD method features matrix operator-free RHS that is simple, concise and more efficient.

For comparison, the RHS of implicit update in the IM1-D ADI-FDTD method can be

5.3. IM1-D FLOD-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

expressed as (for the m th main transmission line as an example)

$$\begin{aligned} \dots = & \frac{\Delta t^2}{4\varepsilon_k^{(m)} \mu_{k-\frac{1}{2}}^{(m)} \Delta z^2} E_x^{(m)}|_{k-1}^n + \left(1 - \frac{\Delta t^2}{4\varepsilon_k^{(m)} \mu_{k-\frac{1}{2}}^{(m)} \Delta z^2} - \frac{\Delta t^2}{4\varepsilon_k^{(m)} \mu_{k+\frac{1}{2}}^{(m)} \Delta z^2}\right) E_x^{(m)}|_k^n \\ & + \frac{\Delta t^2}{4\varepsilon_k^{(m)} \mu_{k+\frac{1}{2}}^{(m)} \Delta z^2} E_x^{(m)}|_{k+1}^n - \frac{\Delta t}{\varepsilon_k^{(m)} \Delta z} (H_y^{(m)}|_{k+\frac{1}{2}}^n - H_y^{(m)}|_{k-\frac{1}{2}}^n) + \frac{\Delta t}{\varepsilon_k^{(m)}} J_x^{(m)}|_k^{n+\frac{1}{2}}. \end{aligned} \quad (5.18)$$

It can be observed that this RHS has been simplified into (5.17a) which does not include field components $E_x^{(m)}|_{k-1}^n$ and $E_x^{(m)}|_{k+1}^n$. More detailed comparison will be given in Section 5.4.

In (5.17), the permittivity ε and permeability μ for each cell using IM1-D FADI-FDTD method are given by [87]

$$\varepsilon_\xi^{(\zeta)} = \frac{1}{v Z_0|_\xi^{(\zeta)}} = \frac{\sqrt{\varepsilon_{eff}|_\xi^{(\zeta)}}}{c Z_0|_\xi^{(\zeta)}}, \quad (5.19a)$$

$$\mu_\xi^{(\zeta)} = \frac{Z_0|_\xi^{(\zeta)}}{v} = \frac{Z_0|_\xi^{(\zeta)} \sqrt{\varepsilon_{eff}|_\xi^{(\zeta)}}}{c}. \quad (5.19b)$$

Here, ξ and ζ refer to the k th cell for the m th main transmission line or the j th cell for the s th stub. Z_0 is the characteristic impedance, c is the speed of light in free space, v is the phase velocity and ε_{eff} is the effective permittivity of the single transmission line.

5.3 IM1-D FLOD-FDTD Method with Treatments at Interjunctions

Different from ADI-FDTD method, LOD-FDTD method update the field components with different RHS and the field components are updated one-dimensionally for each procedure. Accordingly, the update equations of IM1-D FLOD-FDTD method and the treatments for the interjunctions are different from IM1-D FADI-FDTD method. Moreover, IM1-D FLOD-FDTD is more efficient than IM1-D FADI-FDTD method, but the temporal accuracy is degraded to first order.

5.3. IM1-D FLOD-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

5.3.1 One-step IM1-D FLOD-FDTD Method for Stub

The IM1-D LOD-FDTD method for the s th stub can be written in matrix form as

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^n \quad (5.20a)$$

$$\mathbf{u}_s^{n+1} = \mathbf{u}_s^{n+\frac{1}{2}}. \quad (5.20b)$$

We can combine the above two procedures directly as

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n+\frac{1}{2}} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{A}_s) \mathbf{u}_s^{n-\frac{1}{2}}. \quad (5.21)$$

With the auxiliary variable in (5.7), the update equations of IM1-D FLOD-FDTD for the s th stub can be derived as

$$(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{A}_s) \mathbf{v}_s^{n+\frac{1}{2}} = \mathbf{u}_s^{n-\frac{1}{2}} \quad (5.22a)$$

$$\mathbf{u}_s^{n+\frac{1}{2}} = \mathbf{v}_s^{n+\frac{1}{2}} - \mathbf{u}_s^{n-\frac{1}{2}}. \quad (5.22b)$$

The fundamental scheme with matrix-operator-free RHS and the one-step update procedure are both exploited to optimize for real-time simulation on mobile device. Compared with (5.6) of the IM1-D FADI-FDTD method, the source treatment in terms of current density is not used in (5.22) of the IM1-D FLOD-FDTD method for stub.

5.3.2 One-step IM1-D FLOD-FDTD Method for Main Transmission Line

The IM1-D LOD-FDTD method for the m th main transmission line can be written in matrix form as

$$\mathbf{u}_m^{n+\frac{1}{2}} = \mathbf{u}_m^n \quad (5.23a)$$

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^{n+\frac{1}{2}}. \quad (5.23b)$$

5.3. IM1-D FLOD-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

Figure 5.4: The treatment at the interjunction and the alignment of field components $E_x^{(m)}$, $H_y^{(m)}$ in the m th main transmission line and $E_x^{(s)}$, $H_z^{(s)}$ in the s th stub for IM1-D FLOD-FDTD method.

We can combine the above two procedures directly as

$$(\mathbf{I} - \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^{n+1} = (\mathbf{I} + \frac{\Delta t}{2} \mathbf{B}_m) \mathbf{u}_m^n. \quad (5.24)$$

Next, IM1-D FLOD-FDTD method is derived to enhance the efficiency with matrix-operator-free RHS using the auxiliary variable (5.13). The update equations of IM1-D FLOD-FDTD for the m th main transmission line can be derived as

$$(\frac{1}{2} \mathbf{I} - \frac{\Delta t}{4} \mathbf{B}_m) \mathbf{v}_m^{n+1} = \mathbf{u}_m^n \quad (5.25a)$$

$$\mathbf{u}_m^{n+1} = \mathbf{v}_m^{n+1} - \mathbf{u}_m^n. \quad (5.25b)$$

Compared with (5.12) of the IM1-D FADI-FDTD method, the source treatment in terms of current density is also not used in (5.25) of the IM1-D FLOD-FDTD method for main transmission line.

5.3.3 Treatments at Interjunctions using IM1-D FLOD-FDTD Method

Unlike the IM1-D FADI-FDTD method that employs the treatment of current densities to make the main transmission lines and stubs work cooperatively, the IM1-D FLOD-FDTD

5.3. IM1-D FLOD-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

method treats the electric fields directly for various interjunctions. Fig. 5.4 shows the treatment at the interjunction and the alignment of field components $E_x^{(m)}$, $H_y^{(m)}$ in the m th main transmission line and $E_x^{(s)}$, $H_z^{(s)}$ in the s th stub for IM1-D FLOD-FDTD method. The electric field components will be replaced alternatively at the interjunction between the m th main transmission line and the s th stub as

$$E_x^{(s)}|_{j(\text{interjunction})}^{n-\frac{1}{2}} = E_x^{(m)}|_{k(\text{interjunction})}^{n-1} \quad (5.26a)$$

$$E_x^{(m)}|_{k(\text{interjunction})}^n = E_x^{(s)}|_{j(\text{interjunction})}^{n-\frac{1}{2}}. \quad (5.26b)$$

To be specific, treatments (5.26a) for stubs at the interjunctions will be executed before (5.22). After the field components in stubs are all updated according to (5.22), the treatments (5.26b) for main transmission lines at the interjunctions will be executed before (5.25), and so forth.

The implementation of IM1-D FLOD-FDTD is similar to the one of IM1-D FADI-FDTD method. It should be noted that the proper treatments at the interjunctions for the IM1-D FLOD-FDTD method via field replacements in (5.26) do not include the current density terms like (5.14) and (5.15) in the IM1-D FADI-FDTD method. More detailed comparison will be given in Section 5.4.

5.3. IM1-D FLOD-FDTD METHOD WITH TREATMENTS AT INTERJUNCTIONS

5.3.4 Implementation of IM1-D FLOD-FDTD Method

The detailed implementation of the IM1-D FLOD-FDTD method is given as

Main TL :

$$\begin{aligned} & -\frac{\Delta t^2}{8\varepsilon_k^{(m)}\mu_{k-\frac{1}{2}}^{(m)}\Delta z^2}e_x^{(m)}|_{k-1}^{n+1} + \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_k^{(m)}\mu_{k-\frac{1}{2}}^{(m)}\Delta z^2} + \frac{\Delta t^2}{8\varepsilon_k^{(m)}\mu_{k+\frac{1}{2}}^{(m)}\Delta z^2}\right)e_x^{(m)}|_k^{n+1} \\ & -\frac{\Delta t^2}{8\varepsilon_k^{(m)}\mu_{k+\frac{1}{2}}^{(m)}\Delta z^2}e_x^{(m)}|_{k+1}^{n+1} = E_x^{(m)}|_k^n - \frac{\Delta t}{\varepsilon_k^{(m)}\Delta z}(H_y^{(m)}|_{k+\frac{1}{2}}^n - H_y^{(m)}|_{k-\frac{1}{2}}^n) \end{aligned} \quad (5.27a)$$

$$E_x^{(m)}|_k^{n+1} = e_x^{(m)}|_k^{n+1} - E_x^{(m)}|_k^n \quad (5.27b)$$

$$H_y^{(m)}|_{k+\frac{1}{2}}^{n+1} = H_y^{(m)}|_{k+\frac{1}{2}}^n - \frac{\Delta t}{2\mu_{k+\frac{1}{2}}^{(m)}\Delta z}(e_x^{(m)}|_{k+1}^{n+1} - e_x^{(m)}|_k^{n+1}) \quad (5.27c)$$

$$\text{Stub :} \quad (5.27d)$$

$$\begin{aligned} & -\frac{\Delta t^2}{8\varepsilon_j^{(s)}\mu_{j-\frac{1}{2}}^{(s)}\Delta y^2}e_x^{(s)}|_{j-1}^{n+\frac{1}{2}} + \left(\frac{1}{2} + \frac{\Delta t^2}{8\varepsilon_j^{(s)}\mu_{j-\frac{1}{2}}^{(s)}\Delta y^2} + \frac{\Delta t^2}{8\varepsilon_j^{(s)}\mu_{j+\frac{1}{2}}^{(s)}\Delta y^2}\right)e_x^{(s)}|_j^{n+\frac{1}{2}} \\ & -\frac{\Delta t^2}{8\varepsilon_j^{(s)}\mu_{j+\frac{1}{2}}^{(s)}\Delta y^2}e_x^{(s)}|_{j+1}^{n+\frac{1}{2}} = E_x^{(s)}|_j^{n-\frac{1}{2}} + \frac{\Delta t}{2\varepsilon_j^{(s)}\Delta y}(H_z^{(s)}|_{j+\frac{1}{2}}^{n-\frac{1}{2}} - H_z^{(s)}|_{j-\frac{1}{2}}^{n-\frac{1}{2}}) \end{aligned} \quad (5.27e)$$

$$E_x^{(s)}|_j^{n+\frac{1}{2}} = e_x^{(s)}|_j^{n+\frac{1}{2}} - E_x^{(s)}|_j^{n-\frac{1}{2}} \quad (5.27f)$$

$$H_z^{(s)}|_{j+\frac{1}{2}}^{n+\frac{1}{2}} = H_z^{(s)}|_{j+\frac{1}{2}}^{n-\frac{1}{2}} - \frac{\Delta t}{2\mu_{k+\frac{1}{2}}^{(s)}\Delta z}(e_x^{(s)}|_{j+1}^{n+\frac{1}{2}} - e_x^{(s)}|_j^{n+\frac{1}{2}}). \quad (5.27g)$$

The permittivity ε and permeability μ for each cell related to the characteristic impedance Z_0 and effective permittivity ε_{eff} are the same as (5.19). Compared with the RHS of implicit update in the IM1-D LOD-FDTD method (for the m th main transmission line as an example)

$$\begin{aligned} \dots & = \frac{\Delta t^2}{4\varepsilon_k^{(m)}\mu_{k-\frac{1}{2}}^{(m)}\Delta z^2}E_x^{(m)}|_{k-1}^n + \left(1 - \frac{\Delta t^2}{4\varepsilon_k^{(m)}\mu_{k-\frac{1}{2}}^{(m)}\Delta z^2} - \frac{\Delta t^2}{4\varepsilon_k^{(m)}\mu_{k+\frac{1}{2}}^{(m)}\Delta z^2}\right)E_x^{(m)}|_k^n \\ & + \frac{\Delta t^2}{4\varepsilon_k^{(m)}\mu_{k+\frac{1}{2}}^{(m)}\Delta z^2}E_x^{(m)}|_{k+1}^n - \frac{\Delta t}{\varepsilon_k^{(m)}\Delta z}(H_y^{(m)}|_{k+\frac{1}{2}}^n - H_y^{(m)}|_{k-\frac{1}{2}}^n), \end{aligned} \quad (5.27)$$

it can be observed that the RHS of (5.27a) in IM1-D FLOD-FDTD method has been simplified without field components $E_x^{(m)}|_{k-1}^n$ and $E_x^{(m)}|_{k+1}^n$, and there is no current density treatment

5.4. NUMERICAL RESULTS

like (5.17a). More detailed comparison will be given in Section 5.4.

5.4 Numerical Results

To show the efficiency of the fundamental scheme, the flops count (RHS) of the IM1-D ADI-, FADI-, LOD- and FLOD-FDTD methods in one full iteration and memory are given in Table 5.1. The number of multiplications/divisions (M/D) and additions/subtractions (A/S) are counted with the assumption that all the multiplicative factors are pre-computed. From the table, it is obvious that the flops counts have been reduced for IM1-D FADI- and FLOD-FDTD methods. Note that there is no arithmetic operation for IM1-D LOD-/FLOD-FDTD methods at the interjunctions. Although the efficiency has been improved, the required memory for IM1-D FADI- and FLOD-FDTD methods does not increase. The memory for field arrays in the proposed methods is also given in Table 5.1. By properly reusing the field arrays within the same curly brackets in this table, it can be seen that the required memory for each method is the same. Note that judicious reuse of memory space is important, especially for simulation on mobile device with limited memory.

To verify the proposed IM1-D FADI- and FLOD-FDTD methods, a microstrip line loaded with stubs is simulated as an example, whose schematic is shown in Fig. 5.5. The dielectric constant of the microstrip substrate is 3.8 and the thickness is 0.6 mm. The length $L_1 = 438$ mm, $L_2 = 219$ mm and the width of each microstrip line is $W = 1.27$ mm. Note that the specific 3-D dimensions such as the width of microstrip line and the thickness of the substrate in this example are incorporated in the proposed IM1-D FADI- and FLOD-FDTD methods via permittivity and permeability, which are expressed in terms of the characteristic impedance Z_0 and the effective permittivity ϵ_{eff} according to (5.19). The equations for Z_0 and ϵ_{eff} of microstrip line can be found in [2, 89]. Thus, the IM1-D FADI- and FLOD-FDTD methods can be used to solve practical 3-D transmission line problems. A Gaussian pulse is used to excite the electromagnetic fields at port 1:

$$e^{-(t-t_0)^2/\tau^2}, \quad t_0 = 3\tau \quad (5.28)$$

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Table 5.1: Flops count (RHS) and memory for IM1-D ADI-, FADI-, LOD- and FLOD-FDTD methods.

Methods		IM1-D	IM1-D	IM1-D		IM1-D	IM1-D	IM1-D LOD-
		ADI-FDTD	FADI-FDTD	ADI-/FADI-FDTD		LOD-FDTD	FLOD-FDTD	/FLOD-FDTD
		Main TL / Stub	Main TL / Stub	Interjunction $(s \leftarrow m) / (m \leftarrow s)$		Main TL / Stub	Main TL / Stub	Interjunction
				Top / Bottom /Left / Right	Intermediate			
Implicit (RHS)	M/D	4	1	1	1	4	1	-
	A/S	4	2	1	1	4	2	-
	M/D	2	1	-	-	2	1	-
	A/S	4	3	-	-	4	3	-
Total (RHS)	M/D	6	2	1	1	6	2	-
	A/S	8	5	1	2	8	5	-
	M/D+A/S	14	7	2	3	14	7	-
Memory		$\{E_x^{(m)} ^{n+1}\};$ $\{E_x^{(m)} ^{n}\};$ $\{H_y^{(m)} ^{n+1},$ $H_y^{(m)} ^{n}\};$ $\{E_x^{(s)} ^{n+\frac{1}{2}},$ $E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{H_z^{(s)} ^{n+\frac{1}{2}},$ $H_z^{(s)} ^{n-\frac{1}{2}}\};$ $H_y^{(s)} ^{n-\frac{1}{2}}\}$	$\{E_x^{(m)} ^{n+1},$ $E_x^{(m)} ^{n}\};$ $\{H_y^{(m)} ^{n+1},$ $H_y^{(m)} ^{n}\};$ $\{E_x^{(s)} ^{n+\frac{1}{2}},$ $E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{H_z^{(s)} ^{n+\frac{1}{2}},$ $H_z^{(s)} ^{n-\frac{1}{2}}\};$ $\{e_x^{(m)} ^{n+1}\};$ $\{e_x^{(s)} ^{n+\frac{1}{2}}\}$	-	-	$\{E_x^{(m)} ^{n+1}\};$ $\{E_x^{(m)} ^{n}\};$ $\{H_y^{(m)} ^{n+1},$ $H_y^{(m)} ^{n}\};$ $\{E_x^{(s)} ^{n+\frac{1}{2}},$ $E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{H_z^{(s)} ^{n+\frac{1}{2}},$ $H_z^{(s)} ^{n-\frac{1}{2}}\};$ $H_y^{(s)} ^{n-\frac{1}{2}}\}$	$\{E_x^{(m)} ^{n+1},$ $E_x^{(m)} ^{n}\};$ $\{H_y^{(m)} ^{n+1},$ $H_y^{(m)} ^{n}\};$ $\{E_x^{(s)} ^{n+\frac{1}{2}},$ $E_x^{(s)} ^{n-\frac{1}{2}}\};$ $\{H_z^{(s)} ^{n+\frac{1}{2}},$ $H_z^{(s)} ^{n-\frac{1}{2}}\};$ $\{e_x^{(m)} ^{n+1}\};$ $\{e_x^{(s)} ^{n+\frac{1}{2}}\}$	-

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Figure 5.5: Schematic of a microstrip line with stubs and the real-time visualization of the electric fields E_x at different time instants using IM1-D FADI-FDTD method for CFLN=10 on mobile device.

with $\tau = 1/3$ ns. The mesh sizes Δy and Δz are set to 0.583 mm. The computational time step of the explicit FDTD method is set to the CFL limit

$$\Delta t_{CFL} = \frac{\min(\Delta y, \Delta z)}{c\sqrt{2}}. \quad (5.29)$$

We also define $CFLN = \Delta t / \Delta t_{CFL}$, where Δt is the time step used.

Fig. 5.5 also shows the real-time visualization of the electric fields E_x in the microstrip lines at different time instants using IM1-D FADI-FDTD method for CFLN=10. Apple iPad 3 (Model: MD317 B/A) is used to illustrate the real-time visualization of the electric fields on mobile device. A screen capture of the real-time simulation on iPad 3 is given in Fig. 5.5. Such real-time visualization should be useful for microwave engineers to simulate the initial design of transmission line networks quickly on handy mobile devices.

Fig. 5.6 shows the frequency responses (a) S11 and (b) S21 of microstrip line with stubs simulated using the proposed IM1-D FADI- and FLOD-FDTD methods (CFLN = 1, 10), compared with explicit FDTD, Agilent ADS and Computer Simulation Technology (CST). The ADS results are obtained based on the microstrip (MLIN and MSUB) components in-

5.4. NUMERICAL RESULTS

Figure 5.6: Frequency responses (a) S_{11} and (b) S_{21} of microstrip line with stubs simulated using the proposed IM1-D FADI- and FLOD-FDTD methods (CFLN=1, 10), compared with explicit FDTD, ADS and CST.

volving the physical structure dimensions in the schematic, while the results from CST are based on full 3-D geometrical model of the physical structure. From Fig. 5.6, it can be

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Table 5.2: L^2 norm error of the proposed IM1-D ADI-, FADI-, LOD- and FLOD-FDTD methods compared with explicit FDTD method.

	L^2 Norm Error				
CFLN	1	6	10	12	14
Time step (ps)	2.357	14.142	23.570	28.284	32.998
IM1-D ADI-FDTD	0.0300	0.0501	0.2000	0.2977	0.3994
IM1-D FADI-FDTD	0.0300	0.0501	0.2000	0.2977	0.3994
IM1-D LOD-FDTD	0.0319	0.0528	0.2087	0.3138	0.4258
IM1-D FLOD-FDTD	0.0319	0.0528	0.2087	0.3138	0.4258

Table 5.3: CPU time and efficiency gain for the proposed IM1-D ADI-, LOD-, FADI- and FLOD-FDTD methods (CFLN = 10) compared with explicit FDTD method.

IM1-D Methods	FDTD	ADI	LOD	FADI	FLOD
CPU time on PC (s)	1.704	0.343	0.313	0.265	0.249
Efficiency Gain	-	4.96	5.44	6.43	6.84
CPU time on iPad 3 (s)	21.171	4.670	4.553	3.886	3.803
Efficiency Gain	-	4.533	4.670	5.45	5.57

observed that the S-parameters results using the proposed IM1-D FADI- and FLOD-FDTD methods with CFLN = 1 agree well with the explicit FDTD, ADS and CST (full 3-D model) results. When the time step is set to 10 times the CFL limit, i.e. CFLN = 10, there is small discrepancy especially at higher frequencies but it is still acceptable considering the large CFLN being used for trade-off between efficiency and accuracy.

Moreover, we also fabricated the microstrip line with stubs on a FR4 substrate with relative permittivity of 4.4 and thickness of 1.6 mm. The corresponding dimensions in Fig. 5.5 are $w = 3.1$ mm, $l_1 = 30$ mm and $l_2 = 15$ mm. Fig. 5.7 shows the S-parameters of the microstrip line with stubs simulated using the proposed IM1-D FLOD-FDTD method for CFLN = 10 and the measured results of the fabricated circuit. It can be observed that the simulated results agree well with the measured results even when CFLN = 10.

Subsequently, we further assess the accuracy of the proposed unconditionally stable methods including IM1-D ADI-, FADI-, LOD- and FLOD-FDTD methods in time domain. Using the explicit FDTD method as the benchmark reference, we determine the degradation of accuracy in terms of L^2 norm error as the time step or CFLN increases. Note that the ex-

Figure 5.7: S-parameters of the microstrip line with stubs simulated using the proposed IM1-D FLOD-FDTD method for CFLN = 10 and the measured results of the fabricated circuit.

implicit FDTD is consistent (solving multiple lines simultaneously) and second-order accurate in temporal and spatial. Table 5.2 shows the L^2 norm error of the proposed unconditionally stable FDTD methods compared with explicit FDTD method. Five electric field components E_x located at each line segment (p_1 , p_2 , p_3 , p_4 and p_5 shown in Fig. 5.5) are recorded for time step from 0 to 50000 to calculate the L^2 norm errors in average. From Table 5.2, it can be observed that the L^2 norm errors using the proposed unconditionally stable FDTD methods increase as the CFLN becomes larger. Note that the IM1-D FADI- and FLOD-FDTD methods have the same degree of L^2 norm error as IM1-D ADI- and LOD-FDTD methods, which justifies that the fundamental schemes for IM1-D ADI- and LOD-FDTD methods do not deteriorate the accuracy.

Additionally, we calculate the CPU time for the proposed unconditionally stable FDTD methods to show the efficiency compared with explicit FDTD methods on both computer and mobile device. The example in Fig. 5.5 using the proposed methods for CFLN=10 from 0 to

Figure 5.8: Schematic of a branch-line coupler and the real-time visualization of the electric fields E_x at different time instants using IM1-D FLOD-FDTD method.

$50000\Delta t_{\text{CFL}}$ is simulated. To test the efficiency on personal computer (PC), the programs coded using C++ are run on Windows 7 platform with 3.1 GHz Intel i5 processor and 4 GB RAM. To test the efficiency on mobile device (iPad 3), the programs coded using objective C are run on iOS 6.1 platform with Apple A5X SoC and 1 GB RAM. The CPU time and efficiency gain for the proposed unconditionally stable FDTD methods (CFLN=10) compared with explicit FDTD method are given in Table 5.3. It can be observed that the computational time using explicit FDTD method is increased from 1.704s on PC to 21.171s on iPad 3. This shows that the mobile device would generally have less computing power than PC, and it is important to devise novel algorithms that exploit the available resources (not over-demanding 2D/3D, not too simplistic 1-D lacking interconnections in practice). The proposed efficient unconditionally stable IM1-D FADI- and FLOD-FDTD methods are indeed more suitable to optimize the efficiency for real-time simulation on mobile device.

Next, an example of branch-line coupler is given to show how IM1-D FADI- and FLOD-FDTD methods solve multi-transmission lines and multi-stubs problem. The dimensions and schematic of a microstrip branch-line coupler are given in Fig. 5.8. The lengths are $L_1=43.62$ mm, $L_2=42.66$ mm and the widths are $W_1=1.28$ mm, $W_2=2.15$ mm. The substrate

Figure 5.9: Frequency responses of branch-line coupler simulated using the proposed IM1-D FADI-, FLOD-FDTD methods (CFLN=1, 10) and CST.

is the same as the previous microstrip stub. The mesh sizes Δy and Δz are set to 0.088 mm. A Gaussian pulse with $\tau = 0.1$ ns in (5.28) is used to excite the electromagnetic fields at port 1 of the coupler. Fig. 5.9 shows the frequency responses of branch-line coupler simulated using the proposed IM1-D FADI-, FLOD-FDTD methods (CFLN=1, 10) and CST. From the figure, the S-parameters results using the proposed IM1-D FADI- and FLOD-FDTD methods with CFLN=1 agree well with the CST (full 3-D model) results. The discrepancy between the proposed methods with CFLN=10 and CST is also acceptable.

Fig. 5.8 also shows the real-time visualization of the electric fields E_x in the branch-line coupler at different time instants using IM1-D FLOD-FDTD method. To show the electromagnetic waves propagation of the branch-line coupler at the second working frequency (3GHz), a sinusoidal source is excited at port 1: $\sin(2\pi f_0 t)$, $f_0 = 3$ GHz. It can be observed that the magnitude of E_x near the isolated port (the main transmission line connected to port 4) is close to zero, while the magnitudes of E_x near the through and coupled ports (the main transmission line connected to port 2 and 3) are same. Using the proposed IM1-D FADI- and

Figure 5.10: Schematic of a microstrip circuit with two gaps and equivalent circuit models for the gaps.

FLOD-FDTD methods, one can observe how the amplitude of E_x at port 4 approaches zero in real time. As shown in Fig. 5.8, as time progresses, the electromagnetic waves propagate to port 4 along two main (green and yellow arrows) paths. It can be visualized in real time that the electromagnetic waves from both paths cancel at port 4 due to their opposite phase. This has great educational value and facilitates the understanding of microwave field theory and phenomena in a visual way that elucidates the underlying physics or principles. Meanwhile, frequency-domain methods usually cannot provide such transient observation in real time.

To demonstrate the applicability of the proposed methods for handling the couplings between two transmission lines via gaps and compare with the frequency-domain method, we provide an additional example of a microstrip circuit with two gaps. Fig. 5.10 shows the schematic of this microstrip circuit, along with the equivalent circuit models involving capacitances for characterizing the gap couplings [89, 90]. The dielectric constant of the microstrip substrate is 10.8 and the thickness is 0.7 mm. The lengths of transmission lines and stubs are $L_1=40$ mm, $L_2=10.2$ mm, $L_3=4$ mm, the line width is $W=0.616$ mm, and the width of the gap is $S=0.2$ mm. Referring to [89, 90], one can derive the capacitances $C_g=0.0384$ pF and $C_p=0.0145$ pF in the equivalent circuit models, which may be used to replace the gaps effectively. To extend the proposed IM1-D FADI- and FLOD-FDTD methods for including such capacitances, we resort to the ADI/LOD FDTD methods incorporated with lumped elements as in [10, 55, 57]. A Gaussian pulse with $\tau = 1/30$ ns in (5.28) is used to

Figure 5.11: Frequency responses of the microstrip circuit with two gaps simulated using the proposed IM1-D FADI- and FLOD-FDTD methods with CFLN=10 compared with ADS and HFSS.

excite the electromagnetic fields at port 1. The mesh sizes Δy and Δz are set to 1.86×10^{-5} m. The computational time step of the IM1-D FADI- and FLOD-FDTD methods are set to 10 times the CFL limit (CFLN=10) according to (5.29).

Fig. 10 shows the frequency responses of the microstrip circuit with two gaps simulated using the proposed IM1-D FADI- and FLOD-FDTD methods with CFLN=10, compared with ADS and High Frequency Structural Simulator (HFSS). The ADS results are simulated using the equivalent circuit models involving capacitances, while the results from HFSS are based on full 3-D model in frequency domain. It can be observed that even with CFLN=10, the proposed methods incorporated with capacitances can still effectively characterize the couplings in the gaps, and the results agree with those using ADS and HFSS.

5.5 Conclusion

In this chapter, the IM1-D FADI- and FLOD-FDTD methods for transmission lines with interjunctions have been presented. The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit. Fundamental scheme-based IM1-D FADI- and FLOD-FDTD methods have been derived to enhance the efficiency with matrix-operator-free RHS. The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device. Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems. A microstrip line loaded with stubs and a branch-line coupler have been simulated to show the accuracy and efficiency of the proposed methods. To extend the applicability for handling the couplings between two transmission lines via gaps, a microstrip circuit with gaps has also been simulated using the proposed methods incorporated with equivalent circuit models involving capacitances. Real-time simulations of these numerical examples provide much intuitional insight for one to observe the electromagnetic waves propagation in time-domain on computer or mobile device.

Chapter 6

Non-uniform Time Step

FLOD-FDTD Method

6.1 Introduction

Unconditionally FDTD methods such as ADI-FDTD [6], [18], leapfrog ADI-FDTD [25] and SS/LOD-FDTD [8, 9, 91] methods can use time step larger than CFL limit and thus improve the computational efficiency compared with explicit FDTD method. The LOD-FDTD method is more computationally efficient than ADI-FDTD method. Nevertheless, more numerical errors would be introduced especially when larger time step is used for higher efficiency [85]. To reduce the numerical error for LOD-FDTD method, higher-order methods such as [9], [33] are further developed, but additional treatment procedures may be required. Therefore, one has to make a compromise between efficiency (larger time step) and accuracy (less numerical errors). So far, most unconditionally stable FDTD methods adopt only one and the same (uniform) time step during the whole simulation. Recently, a non-uniform time-step (NUTS) LOD-FDTD method is proposed that adopts different (non-uniform) time steps for different periods during simulation to improve the accuracy [62]. NUTS LOD-FDTD method may be utilized to soothe the computational errors caused by larger time step, espe-

6.1. INTRODUCTION

cially when the magnitudes of certain field components are quite large or change dramatically. Additionally, NUTS scheme is stable and applicable for LOD-FDTD method but it is unstable and cannot be applied for ADI-FDTD method. Moreover, instead of resorting to full 3-D methods that are computationally exhaustive, it would be more efficient to exploit multiple 1-D (M1-D) approach that is better suited for EMC analysis of MTLs. The approach should be extended to deal with MTL circuits that consist of many lumped elements connected in various ways including series, parallel and even cross connections.

Moreover, unconditionally stable FDTD methods incorporated with lumped elements including resistor, capacitor, inductor and memristor) [55–57, 80, 92] have been investigated for higher computational efficiency with larger time step. Generally, electric current density J is commonly used as the field-circuit coupling if the lumped elements are incorporated. To the best of the authors' knowledge, the magnetic current density M has not been used for the coupling between electromagnetic waves and circuit components such as lumped elements. On the other hand, the electric current density J can only be used for the parallel connected lumped elements in IM1-D applications as presented in Chapter 5, and the series connected lumped elements cannot be solved due to the lack of the corresponding electric field component.

This chapter presents a novel non-uniform time-step (NUTS) FLOD-FDTD method for multiconductor transmission lines (MTLs) including lumped elements. The NUTS scheme adopts different (non-uniform) time steps for different periods during simulation, in order to reduce the large errors caused by unconditionally stable FDTD methods with uniform time step larger than CFL limit. It is found that NUTS scheme is potentially unstable for ADI-FDTD method, but it is stable for LOD-FDTD method or its implementation based on fundamental scheme. The NUTS FLOD-FDTD method is useful to simulate MTLs with lumped elements in series and parallel connections. Moreover, the method based on multiple 1-D approach is also extended to incorporate lumped elements in cross connection to model near-field coupling between the MTLs. While the electric current density is commonly used for field-circuit coupling of parallel connected lumped elements, the magnetic current density will be utilized for field-circuit coupling of series and cross-connected lumped elements.

Figure 6.1: Parallel and series connected lumped elements in IM1-D FDTD mesh.

Numerical results for MTLs with lumped elements in series, parallel and cross connections are provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method. Compared with the FLOD-FDTD method with uniform time step, the NUTS FLOD-FDTD method may achieve higher accuracy by using smaller time step for certain periods of simulation.

6.2 Incorporating Lumped Elements in FDTD using Magnetic Current Density M

In IM1-D mesh, there are only E_x and H_y components for electromagnetic waves propagating along the main transmission line. Referring to Fig. 6.1, it is feasible to compute the current density J through the parallel connected lumped element readily [13, 16, 17, 53]. This is because the current density J is along the x-axis and it can be expressed in terms of E_x component directly in the electrical field E_x update equation. However, this is not applicable for series connected lumped element owing to the lack of E_z component in IM1-D FDTD

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mesh and there is no E_z in the update equation.

To resolve the above problem, we propose a novel method by introducing magnetic current density M (instead of J) to be used in the magnetic field H_y (instead of E_x) update equation. The magnetic current density M is defined as

$$\iint \vec{M}_{LE} \cdot d\vec{S} = \iint [-\nabla \times \vec{E}_{LE}] \cdot d\vec{S} = -\oint \vec{E}_{LE} \cdot d\vec{l}. \quad (6.1)$$

Since there is no E_z component in IM1-D FDTD mesh, the values of $E_{z,i+1}$ and $E_{z,i}$ are equal to zero as indicated in Fig. 6.1. The closed loop integral of the four E components then becomes simply the corresponding voltage V_{LE} across the lumped elements in terms of E_x component (without E_z):

$$-\oint \vec{E}_{LE} \cdot d\vec{l} = (E_{x,k} - E_{x,k+1})\Delta x = V_{x,k} - V_{x,k+1} = V_{LE}. \quad (6.2)$$

In this way, the voltages of series connected lumped elements in IM1-D FDTD mesh can be determined.

6.3 NUTS FLOD-FDTD Method Including Lumped Elements in Series and Parallel Connections

In this section, NUTS FLOD-FDTD method for MTLs including lumped elements in series and parallel connections will be presented. Fig. 6.2 shows the schematic of a transmission line circuit with lumped elements including resistor, capacitor and inductor in (a) series and (b) parallel connections. The electromagnetic waves on the transmission lines are denoted in terms of field variables E_x and H_y , which are propagating along z -direction. To incorporate the series and parallel connected lumped elements, magnetic current density M and electric current density J are used for field-circuit coupling respectively.

Based on M1-D approach, the NUTS LOD-FDTD method including lumped elements

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(a) Lumped elements in series connection

(b) Lumped elements in parallel connection

Figure 6.2: Schematic of a transmission line circuit with lumped elements including resistor, capacitor and inductor in (a) series and (b) parallel connections.

can be written in matrix form as

$$(\mathbf{I} - \frac{\Delta t_n}{2} \mathbf{B}) \mathbf{u}^{n+1} = (\mathbf{I} + \frac{\Delta t_n}{2} \mathbf{B}) \mathbf{u}^n + \mathbf{s}_{LE}^{n+\frac{1}{2}}, \quad (6.3)$$

where $\mathbf{u}^n = [E_x^n, H_y^n]^T$, $\mathbf{B}$ is the split matrix given by

$$\mathbf{B} = \begin{pmatrix} 0 & -\frac{1}{\epsilon} \frac{\partial}{\partial z} \\ -\frac{1}{\mu} \frac{\partial}{\partial z} & 0 \end{pmatrix}. \quad (6.4)$$

The NUTS scheme adopts different (non-uniform) time steps for different periods during simulation, e.g. time step Δt_n for time index n , in order to reduce the large errors caused by unconditionally stable FDTD methods with uniform time step larger than CFL limit. In (6.3), $\mathbf{s}_{LE}$ is the source vector due to the lumped elements (LE) in series and parallel

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connections:

$$\mathbf{s}_{LE}^{n+\frac{1}{2}} = [s_{LE_x}^{n+\frac{1}{2}}, s_{LE_y}^{n+\frac{1}{2}}]^T = [-\frac{\Delta t_n}{\varepsilon} J_{LE_x}^{n+\frac{1}{2}}, -\frac{\Delta t_n}{\mu} M_{LE_y}^{n+\frac{1}{2}}]^T. \quad (6.5)$$

J_{LE_x} and M_{LE_y} are the electric current density and the magnetic current density for field-circuit coupling, respectively. The electric current density is commonly used for field-circuit coupling of parallel connected lumped elements. In M1-D approach, due to the lack of electric field along z -direction E_z , the series connected lumped elements cannot be incorporated using J . Thus, the magnetic current density M_{LE_y} is utilized for their field-circuit coupling according to the approach in [87].

To enhance the computational efficiency, we resort to the implementation based on fundamental scheme [42] to arrive at NUTS FLOD-FDTD method. The method can be expressed in matrix form as

$$(\frac{1}{2}\mathbf{I} - \frac{\Delta t_n}{4}\mathbf{B})\mathbf{v}^{n+1} = \mathbf{u}^n + \frac{1}{2}\mathbf{s}_{LE}^{n+\frac{1}{2}} \quad (6.6a)$$

$$\mathbf{u}^{n+1} = \mathbf{v}^{n+1} - \mathbf{u}^n, \quad (6.6b)$$

where $\mathbf{v}$ is the auxiliary variable given by

$$\mathbf{v}^n = [e_x^n, h_y^n]^T. \quad (6.7)$$

It can be seen that the fundamental scheme (6.6) has simple and concise matrix-operator-free RHS (with no more $\mathbf{B}$). The differential equations of NUTS FLOD-FDTD method including lumped elements are

$$\frac{1}{2}e_x^{n+1} + \frac{\Delta t_n}{4\varepsilon} \frac{\partial}{\partial z} h_y^{n+1} = E_x^n - \frac{\Delta t_n}{2\varepsilon} J_{LE_x}^{n+\frac{1}{2}} \quad (6.8a)$$

$$\frac{1}{2}h_y^{n+1} + \frac{\Delta t_n}{4\mu} \frac{\partial}{\partial z} e_x^{n+1} = H_y^n - \frac{\Delta t_n}{2\mu} M_{LE_y}^{n+\frac{1}{2}} \quad (6.8b)$$

$$E_x^{n+1} = e_x^{n+1} - E_x^n, \quad H_y^{n+1} = h_y^{n+1} - H_y^n. \quad (6.8c)$$

Based on (6.8), the implicit (tri-diagonal) and explicit update equations of NUTS FLOD-

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FDTD method including resistor, capacitor and inductor in series and parallel connections are given in the following subsections.

6.3.1 Resistor in Series and Parallel Connections

For resistor in series connection at cell index $k + \frac{1}{2}$, the source can be written as

$$s_{LE_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\mu_{k+\frac{1}{2}}} M_{R_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{\Delta t_n \Delta y R_{k+\frac{1}{2}}}{2\Delta x \Delta z \mu_{k+\frac{1}{2}}} (H_y|_{k+\frac{1}{2}}^{n+1} + H_y|_{k+\frac{1}{2}}^n). \quad (6.9)$$

M_{R_y} is the magnetic current density along y -direction due to the series connected resistor with resistance $R_{k+\frac{1}{2}}$. By substituting (6.9) into (6.8), the update equations of NUTS FLOD-FDTD method including series connected resistor at time index n with time step Δt_n are given as

$$\begin{aligned} & -\frac{c_{R_{k-\frac{1}{2}}}}{8\varepsilon_k \mu_{k-\frac{1}{2}}} \frac{\Delta t_n^2 e_x|_{k-1}^{n+1}}{\Delta z^2} + \left(\frac{1}{2} + \frac{c_{R_{k-\frac{1}{2}}}}{8\varepsilon_k \mu_{k-\frac{1}{2}}} \frac{\Delta t_n^2}{\Delta z^2} + \frac{c_{R_{k+\frac{1}{2}}}}{8\varepsilon_k \mu_{k+\frac{1}{2}}} \frac{\Delta t_n^2}{\Delta z^2} \right) e_x|_k^{n+1} - \frac{c_{R_{k+\frac{1}{2}}}}{8\varepsilon_k \mu_{k+\frac{1}{2}}} \frac{\Delta t_n^2 e_x|_{k+1}^{n+1}}{\Delta z^2} \\ & = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (c_{R_{k+\frac{1}{2}}} H_y|_{k+\frac{1}{2}}^n - c_{R_{k-\frac{1}{2}}} H_y|_{k-\frac{1}{2}}^n) \end{aligned} \quad (6.10a)$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \quad (6.10b)$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = d_{R_{k+\frac{1}{2}}} H_y|_{k+\frac{1}{2}}^n - \frac{c_{R_{k+\frac{1}{2}}}}{2\mu_{k+\frac{1}{2}}} \frac{\Delta t_n}{\Delta z} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}), \quad (6.10c)$$

where

$$c_{R_{k+\frac{1}{2}}} = \left(1 + \frac{\Delta t_n}{\mu_{k+\frac{1}{2}}} \frac{\Delta y R_{k+\frac{1}{2}}}{2\Delta x \Delta z} \right)^{-1} \quad (6.11a)$$

$$d_{R_{k+\frac{1}{2}}} = 2c_{R_{k+\frac{1}{2}}} - 1. \quad (6.11b)$$

For resistor in parallel connection at cell index k , the source can be expressed as

$$s_{LE_x}|_k^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\varepsilon_k} J_{R_x}|_k^{n+\frac{1}{2}} = -\frac{\Delta t_n \Delta x (E_x|_k^{n+1} + E_x|_k^n)}{2\varepsilon_k \Delta y \Delta z R_k}. \quad (6.12)$$

J_{R_x} is the electric current density along x -direction due to the parallel connected resistor

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with resistance R_k . By substituting (6.12) into (6.8), the update equations of NUTS FLOD-FDTD method including parallel connected resistor at time index n with time step Δt_n read

$$\begin{aligned}
& -\frac{\Delta t_n^2 e_x|_{k-1}^{n+1}}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n \Delta x}{4\varepsilon_k \Delta y \Delta z R_k}\right) e_x|_k^{n+1} - \frac{\Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} \\
& = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (H_y|_{k+\frac{1}{2}}^n - H_y|_{k-\frac{1}{2}}^n)
\end{aligned} \tag{6.13a}$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \tag{6.13b}$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = H_y|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{k+\frac{1}{2}}} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}). \tag{6.13c}$$

6.3.2 Capacitor in Series and Parallel Connections

For capacitor in series connection at cell index $k + \frac{1}{2}$, the source can be written as

$$s_{LE_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\mu_{k+\frac{1}{2}}} M_{C_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} \tag{6.14a}$$

$$M_{C_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{M_{C_y}|_{k+\frac{1}{2}}^{n+1} + M_{C_y}|_{k+\frac{1}{2}}^n}{2}. \tag{6.14b}$$

M_{C_y} is the magnetic current density along y -direction due to the series connected capacitor with capacitance $C_{k+\frac{1}{2}}$. By substituting (6.14) into (6.8), the update equations of NUTS FLOD-FDTD method including series connected capacitor at time index n with time step

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Δt_n can be derived as

$$\begin{aligned}
& -\frac{c_{C_{k-\frac{1}{2}}}\Delta t_n^2 e_x|_{k-1}^{n+1}}{4\varepsilon_k\mu_{k-\frac{1}{2}}\Delta z^2} + \left(\frac{1}{2} + \frac{c_{C_{k-\frac{1}{2}}}\Delta t_n^2}{8\varepsilon_k\mu_{k-\frac{1}{2}}\Delta z^2} + \frac{c_{C_{k+\frac{1}{2}}}\Delta t_n^2}{8\varepsilon_k\mu_{k+\frac{1}{2}}\Delta z^2}\right)e_x|_k^{n+1} - \frac{c_{C_{k+\frac{1}{2}}}\Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k\mu_{k+\frac{1}{2}}\Delta z^2} \\
& = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k\Delta z}(c_{C_{k+\frac{1}{2}}}H_y|_{k+\frac{1}{2}}^n - c_{C_{k-\frac{1}{2}}}H_y|_{k-\frac{1}{2}}^n) \\
& - \frac{c_{C_{k-\frac{1}{2}}}\Delta t_n^2}{4\varepsilon_k\mu_{k+\frac{1}{2}}\Delta z}M_{C_y}|_{k-\frac{1}{2}}^n + \frac{c_{C_{k+\frac{1}{2}}}\Delta t_n^2}{4\varepsilon_k\mu_{k+\frac{1}{2}}\Delta z}M_{C_y}|_{k+\frac{1}{2}}^n
\end{aligned} \tag{6.15a}$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \tag{6.15b}$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = d_{C_{k+\frac{1}{2}}}H_y|_{k+\frac{1}{2}}^n - \frac{c_{C_{k+\frac{1}{2}}}\Delta t_n}{2\mu_{k+\frac{1}{2}}\Delta z}(e_x|_{k+1}^{n+1} - e_x|_k^{n+1}) - \frac{c_{C_{k+\frac{1}{2}}}\Delta t_n}{\mu_{k+\frac{1}{2}}}M_{C_y}|_{k+\frac{1}{2}}^n \tag{6.15c}$$

$$M_{C_y}|_{k+\frac{1}{2}}^{n+1} = M_{C_y}|_{k+\frac{1}{2}}^n + \frac{\Delta y\Delta t_n(H_y|_{k+\frac{1}{2}}^{n+1} + H_y|_{k+\frac{1}{2}}^n)}{2C_{k+\frac{1}{2}}\Delta x\Delta z}, \tag{6.15d}$$

where

$$c_{C_{k+\frac{1}{2}}} = \left(1 + \frac{\Delta t_n^2\Delta y}{4\mu_{k+\frac{1}{2}}C_{k+\frac{1}{2}}\Delta x\Delta z}\right)^{-1} \tag{6.16a}$$

$$d_{C_{k+\frac{1}{2}}} = 2c_{C_{k+\frac{1}{2}}} - 1. \tag{6.16b}$$

For capacitor in parallel connection at cell index k , the source can be expressed as

$$s_{LE_x}|_k^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\varepsilon_k}J_{C_x}|_k^{n+\frac{1}{2}} = -\frac{C_k\Delta x\Delta t_n}{2\varepsilon_k\Delta y\Delta z}(E_x|_k^{n+1} - E_x|_k^n). \tag{6.17}$$

J_{C_x} is the electric current density along x -direction due to the parallel connected capacitor with capacitance C_k . By substituting (6.17) into (6.8), the update equations of NUTS FLOD-FDTD method including parallel connected capacitor at time index n with time step Δt_n are

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given by

$$\begin{aligned}
& -\frac{\Delta t_n^2 e_x|_{k-1}^{n+1}}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \frac{\Delta x C_k}{2\varepsilon_k \Delta y \Delta z}\right) e_x|_k^{n+1} - \frac{\Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} \\
& = \left(1 + \frac{\Delta x C_k}{2\varepsilon_k \Delta y \Delta z}\right) E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (H_y|_{k+\frac{1}{2}}^n - H_y|_{k-\frac{1}{2}}^n)
\end{aligned} \tag{6.18a}$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \tag{6.18b}$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = H_y|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{k+\frac{1}{2}}} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}). \tag{6.18c}$$

6.3.3 Inductor in Series and Parallel Connections

For inductor in series connection at cell index $k + \frac{1}{2}$, the source can be written as

$$s_{LE_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\mu_{k+\frac{1}{2}}} M_{L_y}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = -\frac{L_{k+\frac{1}{2}} \Delta x}{\mu_{k+\frac{1}{2}} \Delta y \Delta z} (H_y|_{k+\frac{1}{2}}^{n+1} + H_y|_{k+\frac{1}{2}}^n). \tag{6.19}$$

M_{L_y} is the magnetic current density along y -direction due to the series connected inductor with inductance $L_{k+\frac{1}{2}}$. By substituting (6.19) into (6.8), the update equations of NUTS FLOD-FDTD method including series connected inductor at time index n with time step Δt_n are given as

$$\begin{aligned}
& -\frac{c_{L_{k-\frac{1}{2}}} \Delta t_n^2 e_x|_{k-1}^{n+1}}{4\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{c_{L_{k-\frac{1}{2}}} \Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \frac{c_{L_{k+\frac{1}{2}}} \Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2}\right) e_x|_k^{n+1} - \frac{c_{L_{k+\frac{1}{2}}} \Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} \\
& = E_x|_k^n - \frac{\Delta t_n}{\varepsilon_k \Delta z} (H_y|_{k+\frac{1}{2}}^n - H_y|_{k-\frac{1}{2}}^n)
\end{aligned} \tag{6.20a}$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = H_y|_{k+\frac{1}{2}}^n - \frac{c_{L_{k+\frac{1}{2}}} \Delta t_n}{2\mu_{k+\frac{1}{2}} \Delta z} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}), \tag{6.20b}$$

where

$$c_{L_{k+\frac{1}{2}}} = \left(1 + \frac{L_{k+\frac{1}{2}} \Delta x}{\mu_{k+\frac{1}{2}} \Delta y \Delta z}\right)^{-1}. \tag{6.21}$$

6.3. NUTS FLOD-FDTD METHOD INCLUDING LUMPED ELEMENTS IN SERIES AND PARALLEL CONNECTIONS

For inductor in parallel connection at cell index k , the source can be expressed as

$$s_{LE_x}|_k^{n+\frac{1}{2}} = -\frac{\Delta t_n}{\varepsilon_k} J_{L_x}|_k^{n+\frac{1}{2}} \quad (6.22a)$$

$$J_{L_x}|_k^{n+\frac{1}{2}} = \frac{J_{L_x}|_k^{n+1} + J_{L_x}|_k^n}{2}. \quad (6.22b)$$

J_{L_x} is the electric current density along x -direction due to the parallel connected inductor with inductance L_k . By substituting (6.22) into (6.8), the update equations of NUTS FLOD-FDTD method including parallel connected inductor at time index n with time step Δt_n read

$$\begin{aligned} & -\frac{\Delta t_n^2 e_x|_{k-1}^{n+1}}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \frac{\Delta x \Delta t_n^2}{8\varepsilon_k \Delta y \Delta z L_k}\right) e_x|_k^{n+1} - \frac{\Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} \\ & = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (H_y|_{k+\frac{1}{2}}^n - H_y|_{k-\frac{1}{2}}^n) - \frac{\Delta t_n}{2\varepsilon_k} J_{L_x}|_k^n \end{aligned} \quad (6.23a)$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \quad (6.23b)$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = H_y|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{k+\frac{1}{2}}} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}) \quad (6.23c)$$

$$J_{L_x}|_k^{n+1} = J_{L_x}|_k^n + \frac{\Delta x \Delta t_n}{2L_k \Delta y \Delta z} (E_x|_k^{n+1} - E_x|_k^n). \quad (6.23d)$$

6.3.4 Memristor in Parallel and Series Connection at Time Step n

For memristor with parallel connection along x - direction, the source in (6.6) can be expressed as

$$\begin{aligned} & -\frac{\Delta t_n^2 e_x|_{k-1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} + \frac{\Delta t_n \Delta x}{4\varepsilon_k \Delta y \Delta z Mem|_k^{n+\frac{1}{2}}}\right) e_x|_k^{n+1} \\ & - \frac{\Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (H_y|_{k+\frac{1}{2}}^n - H_y|_{k-\frac{1}{2}}^n) \end{aligned} \quad (6.24a)$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \quad (6.24b)$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = H_y|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{k+\frac{1}{2}}} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}) \quad (6.24c)$$

6.3. NUTS FLOD-FDTD METHOD INCLUDING LUMPED ELEMENTS IN SERIES AND PARALLEL CONNECTIONS

The current across the memristor can be derived by

$$I_{mem}|_k^{n+\frac{1}{2}} = -(H_y|_{k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{k-\frac{1}{2}}^{n+\frac{1}{2}})\Delta y \quad (6.25)$$

After deriving the current across the memristor, the update equations of the memristance are given as

$$\begin{aligned} M_{mem}|_k^{n+1} &= R_{ON}x_{mem}|_k^{n+1} + R_{OFF}[1 - x_{mem}|_k^{n+1}] \\ &= R_{ON}[x_{mem}|_k^n + \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_k^{n+\frac{1}{2}}) I_{mem}|_k^{n+\frac{1}{2}}] \\ &\quad + R_{OFF}[1 - x_{mem}|_k^n - \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_k^{n+\frac{1}{2}}) I_{mem}|_k^{n+\frac{1}{2}}] \\ &= R_{OFF} + x_{mem}|_k^n (R_{ON} - R_{OFF}) + \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_k^{n+\frac{1}{2}}) I_{mem}|_k^{n+\frac{1}{2}} (R_{ON} - R_{OFF}) \end{aligned} \quad (6.26a)$$

where $f(x_{mem}|_k^{n+\frac{1}{2}}) = 1 - (2x_{mem}|_k^{n+\frac{1}{2}} - 1)^{2p}$ is a non-linear window function according to [12, 81] and described in Chapter 3.

For memristor with series connection along z - direction, the source in (6.6) can be expressed as

$$\begin{aligned} & - \frac{c_{M_{k+\frac{1}{2}}} \Delta t_n^2 e_x|_{k+1}^{n+1}}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \left(\frac{1}{2} + \frac{c_{M_{k+\frac{1}{2}}} \Delta t_n^2}{8\varepsilon_k \mu_{k+\frac{1}{2}} \Delta z^2} + \frac{c_{M_{k-\frac{1}{2}}} \Delta t_n^2}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} \right) e_x|_k^{n+1} \\ & - \frac{c_{M_{k-\frac{1}{2}}} \Delta t_n^2 e_x|_{k-1}^{n+1}}{8\varepsilon_k \mu_{k-\frac{1}{2}} \Delta z^2} = E_x|_k^n - \frac{\Delta t_n}{2\varepsilon_k \Delta z} (c_{M_{k+\frac{1}{2}}} H_y|_{k+\frac{1}{2}}^n - c_{M_{k-\frac{1}{2}}} H_y|_{k-\frac{1}{2}}^n) \end{aligned} \quad (6.27a)$$

$$E_x|_k^{n+1} = e_x|_k^{n+1} - E_x|_k^n \quad (6.27b)$$

$$H_y|_{k+\frac{1}{2}}^{n+1} = d_{M_{k+\frac{1}{2}}} H_y|_{k+\frac{1}{2}}^n - \frac{c_{M_{k+\frac{1}{2}}} \Delta t_n}{2\mu_{k+\frac{1}{2}} \Delta z} (e_x|_{k+1}^{n+1} - e_x|_k^{n+1}) \quad (6.27c)$$

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where

$$c_{M_{k+\frac{1}{2}}} = (1 + \frac{\Delta t_n}{\mu_{k+\frac{1}{2}}} \frac{\Delta y Mem|_{k+\frac{1}{2}}^n}{2\Delta x \Delta z})^{-1} \quad (6.28a)$$

$$d_{M_{k+\frac{1}{2}}} = 2c_{M_{k+\frac{1}{2}}} - 1 \quad (6.28b)$$

The current across the memristor can be derived by

$$I_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} = H_y|_{k+\frac{1}{2}}^{n+\frac{1}{2}} \Delta y \quad (6.29)$$

After deriving the current across the memristor, the update equations of the memristance are given as

$$\begin{aligned} M_{mem}|_{k+\frac{1}{2}}^{n+1} &= R_{ON} x_{mem}|_{k+\frac{1}{2}}^{n+1} + R_{OFF} [1 - x_{mem}|_{k+\frac{1}{2}}^{n+1}] \\ &= R_{ON} [x_{mem}|_{k+\frac{1}{2}}^n + \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}}) I_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}}] \\ &\quad + R_{OFF} [1 - x_{mem}|_{k+\frac{1}{2}}^n - \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}}) I_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}}] \\ &= R_{OFF} + x_{mem}|_{k+\frac{1}{2}}^n (R_{ON} - R_{OFF}) + \frac{\Delta t \mu_v R_{ON}}{2d^2} f(x_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}}) I_{mem}|_{k+\frac{1}{2}}^{n+\frac{1}{2}} (R_{ON} - R_{OFF}) \end{aligned} \quad (6.30a)$$

6.4 NUTS FLOD-FDTD Method including Lumped Elements in Cross Connection

In this section, the NUTS FLOD-FDTD method, which is essentially based on M1-D approach, is extended to incorporate lumped elements in cross connection to model near-field coupling between the MTLs. Fig. 6.3 shows the schematic of transmission line circuit with lumped elements in cross connection. For simplicity, we shall consider only two transmission lines below, although the method may be extended readily to deal with couplings among multiple lines. Since the lumped elements connect both transmission lines perpendicularly

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Figure 6.3: Schematic of transmission line circuit with lumped elements in cross connection.

along y -direction, the field components need to be updated based on

$$\left(\frac{1}{2}\mathbf{I} - \frac{\Delta t_n}{4}\mathbf{A}\right)\mathbf{v}^{n+\frac{1}{2}} = \mathbf{u}^n + \frac{1}{2}\mathbf{s}_{LE_c}^{n+\frac{1}{4}} \quad (6.31a)$$

$$\mathbf{u}^{n+\frac{1}{2}} = \mathbf{v}^{n+\frac{1}{2}} - \mathbf{u}^n, \quad (6.31b)$$

where $\mathbf{s}_{LE_c}^{n+\frac{1}{4}}$ is the source vector due to the lumped elements in cross connection as

$$\mathbf{s}_{LE_c}^{n+\frac{1}{4}} = [0, s_{LE_z}^{n+\frac{1}{4}}]^T = [0, -\frac{\Delta t_n}{\mu}M_{LE_z}^{n+\frac{1}{4}}]^T. \quad (6.32)$$

$\mathbf{u}^n = [E_x^n, H_z^n]^T$, $\mathbf{v}^{n+\frac{1}{2}} = [e_x^{n+\frac{1}{2}}, h_z^{n+\frac{1}{2}}]^T$ is the auxiliary variable and $\mathbf{A}$ is the split matrix given by

$$\mathbf{A} = \begin{pmatrix} 0 & \frac{1}{\varepsilon} \frac{\partial}{\partial y} \\ \frac{1}{\mu} \frac{\partial}{\partial y} & 0 \end{pmatrix}. \quad (6.33)$$

To incorporate the cross-connected lumped elements in (6.31), the magnetic current density M will be utilized for their field-circuit coupling as in Section 6.3. In the following, we assume that E_{x_1} , H_{y_1} , E_{x_2} and H_{y_2} are the field components on transmission line 1 (TL_1)

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and transmission line 2 (TL₂), respectively.

6.4.1 Resistor in Cross Connection

For resistor in cross connection at cell index k , the source can be written as

$$s_{LE_z}|_k^{n+\frac{1}{4}} = -\frac{\Delta t_n}{\mu_{R_k}} M_{R_z}|_k^{n+\frac{1}{4}} = -\frac{\Delta t_n \Delta z R_k}{2\Delta x \Delta y \mu_{R_k}} (H_{z,R_k}^{n+\frac{1}{2}} + H_{z,R_k}^n). \quad (6.34)$$

M_{R_z} is the magnetic current density along z -direction due to the cross-connected resistor with resistance R_k . H_{z,R_k} is the magnetic field along z -direction at the cross-connected resistor R_k . The update equations of NUTS FLOD-FDTD method including cross-connected resistor at time index n with time step Δt_n are given by

$$\left(\frac{1}{2} + \frac{c_{R_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{R_k} \Delta y^2}\right) e_{x1}|_k^{n+\frac{1}{2}} - \frac{c_{R_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{R_k} \Delta y^2} e_{x2}|_k^{n+\frac{1}{2}} = E_{x1}|_k^n + \frac{c_{R_k} \Delta t_n}{2\varepsilon_{1,k} \Delta y} H_{z,R_k}^n \quad (6.35a)$$

$$-\frac{c_{R_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{R_k} \Delta y^2} e_{x1}|_k^{n+\frac{1}{2}} + \left(\frac{1}{2} + \frac{c_{R_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{R_k} \Delta y^2}\right) e_{x2}|_k^{n+\frac{1}{2}} = E_{x2}|_k^n - \frac{c_{R_k} \Delta t_n}{2\varepsilon_{2,k} \Delta y} H_{z,R_k}^n \quad (6.35b)$$

$$E_{x1}|_k^{n+\frac{1}{2}} = e_{x1}|_k^{n+\frac{1}{2}} - E_{x1}|_k^n \quad (6.35c)$$

$$E_{x2}|_k^{n+\frac{1}{2}} = e_{x2}|_k^{n+\frac{1}{2}} - E_{x2}|_k^n \quad (6.35d)$$

$$H_{z,R_k}^{n+1} = H_{z,R_k}^{n+\frac{1}{2}} = d_{R_k} H_{z,R_k}^n + \frac{c_{R_k} \Delta t_n}{2\mu_{R_k} \Delta y} (e_{x2}|_k^{n+\frac{1}{2}} - e_{x1}|_k^{n+\frac{1}{2}}), \quad (6.35e)$$

where

$$c_{R_k} = \left(1 + \frac{\Delta t_n}{\mu_{R_k}} \frac{\Delta z R_k}{2\Delta x \Delta y}\right)^{-1} \quad (6.36a)$$

$$d_{R_k} = 2c_{R_k} - 1. \quad (6.36b)$$

Note that (6.35) is to be executed only where there is a resistor in cross connection.

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6.4.2 Capacitor in Cross Connection

For capacitor in cross connection at cell index k , the source can be written as

$$\begin{aligned} s_{LE_z}|_k^{n+\frac{1}{4}} &= -\frac{\Delta t_n}{\mu_{C_k}} M_{C_z}|_k^{n+\frac{1}{4}} \\ M_{C_z}|_k^{n+\frac{1}{4}} &= \frac{M_{C_z}|_k^{n+\frac{1}{2}} + M_{C_z}|_k^n}{2}. \end{aligned} \quad (6.37)$$

M_{C_z} is the magnetic current density along z -direction due to the cross-connected capacitor with capacitance C_k . The update equations of NUTS FLOD-FDTD method including cross-connected capacitor at time index n with time step Δt_n can be derived as

$$\begin{aligned} &(\frac{1}{2} + \frac{c_{C_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{C_k} \Delta y^2}) e_{x1}|_k^{n+\frac{1}{2}} - \frac{c_{C_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{C_k} \Delta y^2} e_{x2}|_k^{n+\frac{1}{2}} = E_{x1}|_k^n + \frac{c_{C_k} \Delta t_n}{2\varepsilon_{1,k} \Delta y} H_{z,C_k}^n \\ &- \frac{c_{C_k} \Delta t_n^2}{4\varepsilon_{1,k} \mu_{C_k} \Delta y} M_{C_z}|_k^n \end{aligned} \quad (6.38a)$$

$$\begin{aligned} &- \frac{c_{C_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{C_k} \Delta y^2} e_{x1}|_k^{n+\frac{1}{2}} + (\frac{1}{2} + \frac{c_{C_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{C_k} \Delta y^2}) e_{x2}|_k^{n+\frac{1}{2}} = E_{x2}|_k^n - \frac{c_{C_k} \Delta t_n}{2\varepsilon_{2,k} \Delta y} H_{z,C_k}^n \\ &+ \frac{c_{C_k} \Delta t_n^2}{4\varepsilon_{2,k} \mu_{C_k} \Delta y} M_{C_z}|_k^n \end{aligned} \quad (6.38b)$$

$$E_{x1}|_k^{n+1} = e_{x1}|_k^{n+1} - E_{x1}|_k^n \quad (6.38c)$$

$$E_{x2}|_k^{n+1} = e_{x2}|_k^{n+1} - E_{x2}|_k^n \quad (6.38d)$$

$$H_{z,C_k}^{n+1} = H_{z,C_k}^{n+\frac{1}{2}} = d_{C_k} H_{z,C_k}^n + \frac{c_{C_k} \Delta t_n}{2\mu_{C_k} \Delta y} (e_{x2}|_k^{n+\frac{1}{2}} - e_{x1}|_k^{n+\frac{1}{2}}) - \frac{c_{C_k} \Delta t_n}{\mu_{C_k}} M_{C_z}|_k^n \quad (6.38e)$$

$$M_{C_z}|_k^{n+1} = M_{C_z}|_k^n + \frac{\Delta t_n \Delta z}{2C_k \Delta x \Delta y} (H_{z,C_k}^n + H_{z,C_k}^{n+1}), \quad (6.38f)$$

where

$$c_{C_k} = (1 + \frac{\Delta t_n^2 \Delta z}{4\mu_{C_k} C_k \Delta x \Delta y})^{-1} \quad (6.39a)$$

$$d_{C_k} = 2c_{C_k} - 1. \quad (6.39b)$$

H_{z,C_k} is the magnetic field along z -direction at the cross-connected capacitor C_k . Note that M_{C_z} needs to be updated iteratively according to (6.38).

6.4. NUTS FLOD-FDTD METHOD INCLUDING LUMPED ELEMENTS IN CROSS CONNECTION

6.4.3 Inductor in Cross Connection

For inductor in cross connection at cell index k , the source can be written as

$$s_{LE_z}|_k^{n+\frac{1}{4}} = -\frac{\Delta t_n}{\mu_{L_k}} M_{L_z}|_k^{n+\frac{1}{4}} = -\frac{L_k \Delta z}{\mu_{L_k} \Delta x \Delta y} (H_{z,L_k}^{n+\frac{1}{2}} + H_{z,L_k}^n). \quad (6.40)$$

M_{L_z} is the magnetic current density along z -direction due to the cross-connected inductor with inductance L_k . H_{z,L_k} is the magnetic field along z -direction at the cross-connected inductor L_k . The update equations of NUTS FLOD-FDTD method including cross-connected inductor at time index n with time step Δt_n are given as

$$\left(\frac{1}{2} + \frac{c_{L_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{L_k} \Delta y^2}\right) e_{x_1}|_k^{n+\frac{1}{2}} - \frac{c_{L_k} \Delta t_n^2}{8\varepsilon_{1,k} \mu_{L_k} \Delta y^2} e_{x_2}|_k^{n+\frac{1}{2}} = E_{x_1}|_k^n + \frac{c_{L_k} \Delta t_n}{2\varepsilon_{1,k} \Delta z} H_{z,L_k}^n \quad (6.41a)$$

$$-\frac{c_{L_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{L_k} \Delta z^2} e_{x_1}|_k^{n+\frac{1}{2}} + \left(\frac{1}{2} + \frac{c_{L_k} \Delta t_n^2}{8\varepsilon_{2,k} \mu_{L_k} \Delta z^2}\right) e_{x_2}|_k^{n+\frac{1}{2}} = E_{x_2}|_k^n - \frac{c_{L_k} \Delta t_n}{2\varepsilon_{2,k} \Delta z} H_{z,L_k}^n \quad (6.41b)$$

$$E_{x_1}|_k^{n+\frac{1}{2}} = e_{x_1}|_k^{n+\frac{1}{2}} - E_{x_1}|_k^n \quad (6.41c)$$

$$E_{x_2}|_k^{n+\frac{1}{2}} = e_{x_2}|_k^{n+\frac{1}{2}} - E_{x_2}|_k^n \quad (6.41d)$$

$$H_{z,L_k}^{n+1} = H_{z,L_k}^{n+\frac{1}{2}} = H_{z,L_k}^{n-\frac{1}{2}} + \frac{c_{L_k} \Delta t_n}{2\mu_{L_k} \Delta z} (e_{x_2}|_k^{n+\frac{1}{2}} - e_{x_1}|_k^{n+\frac{1}{2}}), \quad (6.41e)$$

where

$$c_{L_k} = \left(1 + \frac{L_k \Delta z}{\mu_{L_k} \Delta x \Delta y}\right)^{-1}. \quad (6.42)$$

6.4.4 Further Update Equations for Transmission Lines

From the previous subsections, we have obtained the electric field components on TL_1 and TL_2 , i.e. $E_{x_1}^{n+\frac{1}{2}}$ and $E_{x_2}^{n+\frac{1}{2}}$ for the lumped elements in cross connection. They will be used to further update (from $n + \frac{1}{2}$ to $n + 1$) the field components on the transmission lines at cell index k where there is a cross-connected lumped element as follows:

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TL₁:

$$\begin{aligned}
& -\frac{\Delta t_n^2 e_{x_1}|_{k-1}^{n+1}}{8\varepsilon_{1,k}\mu_{1,k-\frac{1}{2}}\Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_{1,k}\mu_{1,k-\frac{1}{2}}\Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_{1,k}\mu_{1,k+\frac{1}{2}}\Delta z^2}\right)e_{x_1}|_k^{n+1} - \frac{\Delta t_n^2 e_{x_1}|_{k+1}^{n+1}}{8\varepsilon_{1,k}\mu_{1,k+\frac{1}{2}}\Delta z^2} \\
& = E_{x_1}|_k^{n+\frac{1}{2}} - \frac{\Delta t_n}{2\varepsilon_{1,k}\Delta z}(H_{y_1}|_{k+\frac{1}{2}}^n - H_{y_1}|_{k-\frac{1}{2}}^n)
\end{aligned} \tag{6.43a}$$

$$E_{x_1}|_k^{n+1} = e_{x_1}|_k^{n+1} - E_{x_1}|_k^{n+\frac{1}{2}} \tag{6.43b}$$

$$H_{y_1}|_{k+\frac{1}{2}}^{n+1} = H_{y_1}|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{k+\frac{1}{2}}}(e_{x_1}|_{k+1}^{n+1} - e_{x_1}|_k^{n+1}), \tag{6.43c}$$

TL₂:

$$\begin{aligned}
& -\frac{\Delta t_n^2 e_{x_2}|_{k-1}^{n+1}}{8\varepsilon_{2,k}\mu_{2,k-\frac{1}{2}}\Delta z^2} + \left(\frac{1}{2} + \frac{\Delta t_n^2}{8\varepsilon_{2,k}\mu_{2,k-\frac{1}{2}}\Delta z^2} + \frac{\Delta t_n^2}{8\varepsilon_{2,k}\mu_{2,k+\frac{1}{2}}\Delta z^2}\right)e_{x_2}|_k^{n+1} - \frac{\Delta t_n^2 e_{x_2}|_{k+1}^{n+1}}{8\varepsilon_{2,k}\mu_{2,k+\frac{1}{2}}\Delta z^2} \\
& = E_{x_2}|_k^{n+\frac{1}{2}} - \frac{\Delta t_n}{2\varepsilon_{2,k}\Delta z}(H_{y_2}|_{k+\frac{1}{2}}^n - H_{y_2}|_{k-\frac{1}{2}}^n)
\end{aligned} \tag{6.44a}$$

$$E_{x_2}|_k^{n+1} = e_{x_2}|_k^{n+1} - E_{x_2}|_k^{n+\frac{1}{2}} \tag{6.44b}$$

$$H_{y_2}|_{k+\frac{1}{2}}^{n+1} = H_{y_2}|_{k+\frac{1}{2}}^n - \frac{\Delta t_n}{2\mu_{2,k+\frac{1}{2}}}(e_{x_2}|_{k+1}^{n+1} - e_{x_2}|_k^{n+1}). \tag{6.44c}$$

Here, e_{x_1} and e_{x_2} are the auxiliary variables introduced to implement the fundamental scheme. Note that the above update equations are to be executed only for the cells with cross-connected lumped elements on the transmission lines. The field components for other cells without cross-connected lumped elements can be updated by referring to Section II.

6.5 Numerical Results

First, we utilize Fourier method to investigate the eigenvalues of the Fourier amplification matrix using NUTS scheme for both ADI- and LOD-FDTD methods numerically. Fig. 6.4 shows the scatter plot of the eigenvalues of Fourier amplification matrix for different NUTS schemes. In Fig. 6.4 (a) and (c), the uniform time step is used for ADI and LOD when CFLN=1 and 8. The magnitudes of the eigenvalues using both results for uniform time

Figure 6.4: Scatter plot of the eigenvalues of Fourier amplification matrix for different NUTS schemes (a) NUTS [1, 1] (b) NUTS [2, 1] (c) NUTS [8, 8] (d) NUTS [2, 8].

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step are unity as expected. However, when NUTS schemes [2, 1] and [2, 8] are applied, as illustrated in Fig. 6.4 (a) and (c). The magnitudes of the eigenvalues using ADI-FDTD method can be larger than one for certain Fourier wavenumbers, which indicates NUTS scheme for ADI-FDTD method would yield unstable results. Since leapfrog ADI-FDTD is derived based on ADI-FDTD method, the NUTS scheme for leapfrog ADI-FDTD is also not unconditionally stable as well. On the other hand, NUTS LOD-FDTD with scheme [2, 1] and [2, 8] are still stable. The magnitudes of the eigenvalues are still distributed around the circumference of the circle with radius equals to one. Therefore, NUTS scheme is only applicable to LOD-FDTD, which is another advantage over ADI-FDTD in addition to the computational efficiency based on the numerical stability analyses.

Next, an air-filled $80 \times 40 \times 40$ mm³ cavity meshed with uniform cells of 1 mm is simulated to demonstrate the accuracy and efficiency of the proposed NUTS LOD-FDTD method for various non-uniform time steps compared with conventional LOD-FDTD method for various CFLN. The L^2 norm error of the electric field E_z is averaged at 10 observation points with respect to explicit FDTD results from 0 to $T_{\text{total}} = 3.85$ ns. The time step of the explicit FDTD method is set to Δt_{CFL} . The locations of the 10 observation points are: (45,25,20), (35,15,20), (50,30,20), (30,10,20), (45,25,25), (35,15,15), (50,20,25), (35,10,30), (40,20,30), (40,20,25). A line current source of differentiated Gaussian pulse along z -direction: $(t - t_0)e^{[-(t-t_0)^2/\tau^2]}$ ($\tau = 200$ ps, $t_0 = 3\tau$) is used to excite the electromagnetic fields through the center of the cavity.

Table 6.1 shows the comparison between conventional LOD-FDTD method for various CFLN and NUTS LOD-FDTD method for various CFLN_n with different non-uniform time steps. Different time steps for NUTS LOD-FDTD method are expressed in terms of CFLN_n according to (2.23). The L^2 norm errors compared with explicit FDTD method and the iterations counts are calculated for both methods. For NUTS LOD-FDTD method, CFLN_n with non-uniform time step for two different periods $\{\xi, \zeta\}$ are used to simulate the example. ξ is the CFLN used for source excitation period from 0 to 1.155 ns, while ζ is the CFLN used for the rest of simulation until $T_{\text{total}} = 3.85$ ns.

It can be observed from Table 6.1 that the L^2 norm errors of NUTS LOD-FDTD method

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Table 6.1: Comparison between conventional LOD-FDTD method for various CFLN and NUTS LOD-FDTD method for various CFLN_n with different non-uniform time steps.

Method	Conventional LOD-FDTD				
CFLN	1	2	4	6	8
L^2 norm error	0.023	0.054	0.123	0.206	0.317
Iterations	2000	1000	500	333	250
Method	NUTS LOD-FDTD				
CFLN _n	{2, 4}	{2, 6}	{2, 8}	{4, 6}	{4, 8}
L^2 norm error	0.057	0.063	0.072	0.129	0.139
Iterations	650	533	475	383	325
Efficiency Gain Compared with CFLN=1	3.077	3.752	4.211	5.222	6.154

with CFLN_n = { ξ , ζ } lie between the ones of conventional LOD-FDTD method for CFLN = ξ and ζ (mostly close to CFLN = ξ which bears higher accuracy). Furthermore, the efficiency gains are calculated by the counts of iterations for different schemes compared with conventional LOD-FDTD method for CFLN = 1. Take NUTS LOD-FDTD with CFLN_n = {4, 8} as an example, the executed iterations is 325 with an efficiency gain of 6.154 and the L^2 norm error is 0.139. The L^2 norm error is just a little more than the one for CFLN = 4 (i.e. 0.123) and much less than the one for CFLN = 8 (i.e. 0.317) using conventional LOD-FDTD method. With only slight increase of 0.016 for the L^2 norm error with CFLN_n = {4, 8} compared with CFLN = 4, the iteration count is decreased from 500 to 325, which is 1.538 times faster. Similarly, the iteration is 475 for CFLN_n = {2, 8} using NUTS LOD-FDTD method. It is less than 500 for CFLN = 4 using conventional LOD-FDTD method, which means that CFLN_n = {2, 8} is more efficient than CFLN = 4. Moreover, the L^2 norm error with CFLN_n = {2, 8} is also less than the one for CFLN = 4. This result shows that the proposed NUTS LOD-FDTD method with CFLN_n = {2, 8} is not only more computationally efficient but also more accurate than the conventional LOD-FDTD method for CFLN = 4.

Fig. 6.5 shows the electric field E_z at observation point (40,20,25) with respect to time using explicit FDTD, FLOD-FDTD for CFLN = 4, 8 and NUTS FLOD-FDTD with CFLN_n

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Figure 6.5: Electric field E_z at observation point (40,20,25) with respect to time using explicit FDTD, FLOD-FDTD for $\text{CFLN} = 4, 8$ and NUTS FLOD-FDTD with $\text{CFLN}_n = \{4, 8\}$ method.

$= \{4, 8\}$ method. It can be seen that the magnitude of the electric field E_z varies largely when the source is excited, so finer time step used for this period based on NUTS LOD-FDTD can soothe the errors which may otherwise be caused by larger time step.

Next, we utilize Fourier method to numerically investigate the eigenvalues of the Fourier amplification matrix for both NUTS ADI- and LOD-FDTD methods from n to $n + 2$ assuming two different time steps Δt_n and Δt_{n+1} . The eigenvalues correspond to some (100) random sets of Fourier wavenumbers along x -, y - and z -directions. It is found that the magnitudes of the eigenvalues using NUTS ADI-FDTD method may be larger than one for some Fourier wavenumbers. On the other hand, for NUTS LOD-FDTD method, the magnitudes of the eigenvalues are all equal to one. This means that NUTS scheme is potentially unstable for ADI-FDTD method, but it is stable for LOD-FDTD method or its implementation based on fundamental (FLOD) scheme [93]. Such “higher” stability is another advantage of LOD-FDTD over ADI-FDTD in addition to the higher computational efficiency.

Next, we simulate the microstrip line circuit with a resistor at the center as illustrated in Fig. 6.3. The dielectric constant of the microstrip substrate is 3.8 and the thickness is 0.6

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mm. The width and length of both microstrip lines are 1.27 mm and 0.5 m respectively. A Gaussian-modulated sinusoidal pulse is used to excite the electromagnetic waves as given by

$$\sin(2\pi ft)e^{-\left(\frac{t-t_0}{\tau}\right)^2} \quad (6.45)$$

where $f = 1.5$ GHz, $t_0 = 3\tau$, $\tau = 0.5$ ns. The mesh size is 0.3 mm and the time step at CFL limit Δt_{CFL} is set according to [87]. The number of cells per wavelength (CPW) is 200 at 3 GHz ($k\Delta z = 0.0314$, where k is the effective wavenumber). To verify the proposed NUTS FLOD-FDTD method, the L^2 norm errors for the time-domain fields at the observation points connecting the lumped elements are calculated upon comparison with the explicit FDTD method [87]. Table 6.2 shows the L^2 norm errors of the FLOD-FDTD method including lumped elements in parallel and series connections for various CFLNs (with uniform time step). The values of the lumped elements in parallel connection are $R = 50 \Omega$, $C = 10$ pF, $L = 1$ nH, while those in series connection are $R = 75 \Omega$, $C = 1$ pF, $L = 10$ nH. Their normalized (dimensionless) impedances with reference to $Z_0 = 50 \Omega$ at 1.5 GHz are $R/Z_0 = 1$, $X_C/Z_0 = -10.610$, $X_L/Z_0 = 9.425$ for parallel connection and $R/Z_0 = 1.5$, $X_C/Z_0 = -106.103$, $X_L/Z_0 = 94.248$ for series connection. Here and henceforth, CFLN is defined as $\Delta t/\Delta t_{\text{CFL}}$, which may be larger than one for unconditionally stable FLOD-FDTD method. For NUTS FLOD-FDTD method, $\text{CFLN}_n = \{\xi, \zeta\}$ with different CFLNs (i.e. non-uniform time steps) are adopted for different periods (depending on time index n) during simulation: ξ is the CFLN used for the periods from 0 to 3.653 ns and from 6.246 to 9.428 ns, while ζ is the CFLN used for the period from 3.653 to 6.246 ns. Table 6.3 shows the L^2 norm errors of the proposed NUTS FLOD-FDTD method including lumped elements in parallel and series connections for various CFLN_n schemes. From Table 6.2 and 6.3, we see that the proposed NUTS schemes can be applied for FLOD-FDTD method including lumped elements, which exploit another perspective for trade off between efficiency and accuracy.

Fig. 6.6 shows the frequency responses of the microstrip line with lumped elements using explicit FDTD method, FLOD-FDTD method with $\text{CFLN}=20$ and NUTS FLOD-FDTD method with $\text{CFLN}_n=\{20, 10\}$ compared with ADS results. It can be observed that the S-

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Table 6.2: L^2 norm errors of the FLOD-FDTD method including lumped elements in parallel and series connections for various CFLNs

CFLN	1	5	10	14	16	20
Parallel R	0.0016	0.0127	0.0187	0.0333	0.0477	0.0898
Series R	0.0016	0.0127	0.0187	0.0333	0.0477	0.0898
Parallel C	0.0015	0.0132	0.0195	0.0277	0.0369	0.0670
Series C	0.0016	0.0126	0.0186	0.0344	0.0496	0.0934
Parallel L	0.0019	0.0120	0.0187	0.0416	0.0613	0.1156
Series L	0.0016	0.0127	0.0187	0.0329	0.0471	0.0844
Iterations	8000	1600	800	571	500	400

Table 6.3: L^2 norm errors of the proposed NUTS FLOD-FDTD method including lumped elements in parallel and series connections for various CFLN_n schemes

NUTS CFLN _n	{14, 10}	{16, 10}	{20, 10}	{20, 14}	{20, 16}
Parallel R	0.0269	0.0324	0.0488	0.0597	0.0679
Series R	0.0269	0.0324	0.0488	0.0598	0.0679
Parallel C	0.0265	0.0302	0.0403	0.0461	0.0513
Series C	0.0271	0.0329	0.0500	0.0618	0.0704
Parallel L	0.0293	0.0372	0.0604	0.0764	0.0874
Series L	0.0268	0.0321	0.0479	0.0586	0.0667
Iterations	635	583	511	448	428

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Figure 6.6: Frequency responses of the microstrip line with lumped elements using explicit FDTD method, FLOD-FDTD method with CFLN=20 and NUTS FLOD-FDTD method with CFLN_n= $\{20, 10\}$ compared with ADS results: (a) parallel resistor, (b) series resistor, (c) parallel capacitor, (d) series capacitor, (e) parallel inductor, (f) series inductor.

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Table 6.4: L^2 norm errors of the FLOD-FDTD methods incorporated with memristor in parallel and series connections for various CFLNs compared with the corresponding explicit FDTD method in time domain.

CFLN	1	5	10	14	16	20
Parallel M	0.0017	0.0127	0.0185	0.0327	0.0470	0.0888
Series M	0.0016	0.0128	0.0193	0.0342	0.0488	0.0912
Iterations	8000	1600	800	571	500	400

Table 6.5: L^2 norm errors of the proposed NUTS FLOD-FDTD methods incorporated with memrsitor in parallel and series connections with different CFLN_n schemes compared with the corresponding explicit FDTD method in time domain.

NUTS CFLN _n	{14, 10}	{16, 10}	{20, 10}	{20, 14}	{20, 16}
Parallel M	0.0265	0.0320	0.0484	0.0591	0.0672
Series M	0.0275	0.0329	0.0493	0.0605	0.0689
Iterations	635	583	511	448	428

parameters of the microstrip line with lumped elements in series and parallel connections using FLOD-FDTD method with CFLN=20 and NUTS FLOD-FDTD method with CFLN_n={20, 10} agree well with the ones using explicit FDTD method and ADS.

Furthermore, we investigate the NUTS IM1-D FLOD-FDTD method incorporated with memristor in parallel and series connections for the microstrip line circuit shown in Fig. 6.2. The same source in (6.45) and mesh size are used. Table 6.4 shows the L^2 norm errors of the FLOD-FDTD methods incorporated with memristor in parallel and series connections for various CFLNs compared with the corresponding explicit FDTD method in time domain. Table 6.5 shows the L^2 norm errors of the proposed NUTS FLOD-FDTD methods incorporated with memrsitor in parallel and series connections with different CFLN_n schemes compared with the corresponding explicit FDTD method in time domain. It is shown that the NUTS FLOD-FDTD method may achieve higher accuracy by using smaller time step for certain periods of simulation when large signals pass through lumped elements.

Fig. 6.7 and 6.8 show the change of the memristance in parallel and series connections

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Figure 6.7: Memristance in parallel connection with respect to time using explicit FDTD, FLOD-FDTD for CFLN=1, 10 and NUTS FLOD-FDTD with $CFLN_n=\{20, 10\}$.

Figure 6.8: Memristance in series connection with respect to time using explicit FDTD, FLOD-FDTD for CFLN=1, 10 and NUTS FLOD-FDTD with $CFLN_n=\{20, 10\}$.

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Table 6.6: L^2 norm errors of the FLOD-FDTD method including lumped elements in cross connection for various CFLNs

CFLN	1	5	10	14	16	20
Cross R	0.0012	0.0253	0.0999	0.1890	0.2394	0.3423
Cross C	0.0068	0.0325	0.1567	0.3929	0.5707	0.9600
Cross L	0.0015	0.0237	0.0871	0.1608	0.2012	0.2840
Iterations	10000	2000	1000	714	625	500

Table 6.7: L^2 norm errors of the proposed NUTS FLOD-FDTD method including lumped elements in cross connection for various CFLN_n schemes

NUTS CFLN _n	{14, 10}	{16, 10}	{20, 10}	{20, 14}	{20, 16}
Cross R	0.1012	0.1022	0.1057	0.1929	0.2423
Cross C	0.1576	0.1596	0.1666	0.4008	0.5762
Cross L	0.0860	0.0868	0.0906	0.1633	0.2036
Iterations	786	719	626	554	532

with respect to time using explicit FDTD, FLOD-FDTD method for CFLN=1, 10 and NUTS FLOD-FDTD method with CFLN_n={20, 10}. From the figures, it can be seen that the result simulated by FLOD-FDTD method for CFLN=1 agrees well with the explicit FDTD method. In addition, NUTS FLOD-FDTD method with scheme CFLN_n={20, 10} can effectively improve the accuracy compared with FLOD-FDTD method for CFLN=20.

Subsequently, two microstrip lines with lumped elements in cross connection as shown in Fig. 6.3 are simulated. The dielectric constant of the microstrip substrate is 3.8 and the thickness is 0.6 mm. The width and length of both microstrip lines are 1.27 mm and 1 m respectively. The values of the lumped elements in cross connection are the same as those in series connection in the previous example. The cross-connected lumped elements are at the center of both microstrip lines. The same Gaussian-modulated sinusoidal pulse in (6.45) is used to excite the electromagnetic waves. The same mesh size and time step are used as in the previous example.

Table 6.6 shows the L^2 norm errors of the FLOD-FDTD method including lumped el-

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(a) Cross-connected resistor

(b) Cross-connected capacitor

(c) Cross-connected inductor

Figure 6.9: L^2 norm errors for (a) cross-connected resistor, (b) cross-connected capacitor, (c) cross-connected inductor, with $\xi = 12, 16, 20$ and ζ from 6 to 13.

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elements in cross connection for various CFLNs. For each CFLN, uniform time step is used throughout the whole simulation period. For NUTS FLOD-FDTD method with $\text{CFLN}_n = \{\xi, \zeta\}$, ξ is the CFLN used for the periods from 0 to 2.946 ns and from 5.893 to 11.875 ns, while ζ is the CFLN used for the period from 2.946 to 5.893 ns. Table 6.7 shows the L^2 norm errors of the proposed NUTS FLOD-FDTD method including lumped elements in cross connection for various CFLN_n schemes. From Table 6.6 and 6.7, it can be observed that the L^2 norm errors are reduced and the total iterations are fewer for NUTS FLOD-FDTD method compared with the FLOD-FDTD method with uniform time step. Observe that the L^2 norm errors in Table 6.7 are close to those in Table 6.6 for smaller time steps. This is mainly because the errors are dominated by the period with ζ and large magnitudes of Gaussian-modulated sinusoidal pulse passing through the lumped elements. Fig. 6.9 shows the L^2 norm errors for (a) cross-connected resistor, (b) cross-connected capacitor, (c) cross-connected inductor, with $\xi = 12, 16, 20$ and ζ from 6 to 13. It can be seen that the errors would increase as ζ becomes larger for each ξ or ξ becomes larger for each ζ .

Fig. 6.10 shows the frequency responses of two microstrip lines with lumped elements in cross connection using explicit FDTD method, FLOD-FDTD method with $\text{CFLN}=20$ and NUTS FLOD-FDTD method with $\text{CFLN}_n = \{20, 10\}$ compared with ADS results. It can be observed that the S-parameters of the cross-connected lumped elements using larger $\text{CFLN}=20$ will deviate from the results of explicit FDTD method and ADS for higher frequencies, but the deviations can be reduced to a great extent after using NUTS FLOD-FDTD method with $\text{CFLN}_n = \{20, 10\}$. Furthermore, we execute all the numerical examples above for long time simulations up to $10^7 \Delta t_{\text{CFL}}$ using NUTS FLOD-FDTD method and continuous sinusoidal source: $\sin(2\pi ft)$, $f = 1.5$ GHz. The results show that the NUTS FLOD-FDTD method remains stable, thus justifying its unconditional stability.

The computational efficiency of the NUTS FLOD-FDTD method compared with FLOD-FDTD method can be determined based on the number of the iterations. For the example in Fig. 6.3 and according to Table 6.6 and 6.7, the NUTS FLOD-FDTD method with $\text{CFLN}_n = \{20, 10\}$ takes 626 iterations while the FLOD-FDTD method with $\text{CFLN}=10$ takes 1000 iterations. Thus, the computational efficiency gain of the NUTS FLOD-FDTD method

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(a) Cross-connected resistor

(b) Cross-connected capacitor

(c) Cross-connected inductor

Figure 6.10: Frequency responses of two microstrip lines with lumped elements in cross connection using explicit FDTD method, FLOD-FDTD method with $CFLN=20$ and NUTS FLOD-FDTD method with $CFLN_n=\{20, 10\}$ compared with ADS results: (a) cross-connected resistor, (b) cross-connected capacitor, (c) cross-connected inductor.

Figure 6.11: Schematic of three transmission lines with capacitors in cross connection.

compared with FLOD-FDTD method is 1.597 for this example. Additionally, we execute the NUTS FLOD-FDTD, FLOD-FDTD (with uniform time step) and explicit FDTD methods for the circuit in Fig. 6.3 from 0 to $1000 \Delta t_{\text{CFL}}$. The Matlab programs are run under Windows 7 OS on CPU with 3.1 GHz Intel i5 processor and 4 GB RAM. The CPU time using the explicit FDTD method takes 3.345 s, while it takes 1.173 s, 0.724 s and 0.552 s using FLOD-FDTD method when the CFLNs are 10, 15 and 20, respectively. For NUTS FLOD-FDTD method with $\text{CFLN}_n = \{15, 10\}$ and $\text{CFLN}_n = \{20, 10\}$, it takes 0.997 s and 0.712 s, respectively. The occupied memories of the field arrays are 0.157 MB for NUTS FLOD-FDTD method and FLOD-FDTD method with uniform time step, while it is 0.105 MB for explicit FDTD method.

Furthermore, the proposed method can be extended for more transmission lines with lumped elements in cross connection. Fig. 6.11 shows the schematic of three transmission lines with capacitors in cross connection. The values of the capacitors are $C_{12} = C_{23} = 1 \text{ pF}$, $C_{13} = 0.3 \text{ pF}$. The same substrate and mesh as before are used to simulate the microstrip lines. Fig. 6.12 shows the frequency responses of three microstrip lines with capacitors in cross connection using FLOD-FDTD method with $\text{CFLN} = 20$ and NUTS FLOD-FDTD method with $\text{CFLN}_n = \{20, 10\}$ compared with ADS results. From the figure, it can be observed that

Figure 6.12: Frequency responses of three microstrip lines with capacitors in cross connection using FLOD-FDTD method with $CFLN=20$ and NUTS FLOD-FDTD method with $CFLN_n=\{20, 10\}$ compared with ADS results.

the NUTS FLOD-FDTD method with $CFLN_n=\{20, 10\}$ can effectively reduce the errors caused by the FLOD-FDTD method with $CFLN=20$.

6.6 Conclusion

This chapter has presented a novel non-uniform time step NUTS FLOD-FDTD method is introduced and developed. Unlike conventional and the other LOD-FDTD methods, the NUTS scheme adopts different (non-uniform) time steps for different periods during simulation, in order to reduce the large errors caused by unconditionally stable FDTD methods with uniform time step larger than CFL limit. It has been found that NUTS scheme is potentially unstable for ADI-FDTD method, but it is stable for LOD-FDTD method or its implementation based on fundamental (FLOD) scheme. The NUTS FLOD-FDTD method is useful to simulate MTLs with lumped elements in series and parallel connections. Moreover, the method based on M1-D approach has also been extended to incorporate lumped elements in cross connection to model near-field coupling between the MTLs. While the elec-

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tric current density is commonly used for field-circuit coupling of parallel connected lumped elements, the magnetic current density has been utilized for field-circuit coupling of series and cross-connected lumped elements. Numerical results for MTLs with lumped elements in series, parallel and cross connections have been provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method. Compared with the FLOD-FDTD method with uniform time step, the NUTS FLOD-FDTD method may achieve higher accuracy by using smaller time step for certain periods of simulation.

Chapter 7

Conclusion and Future Work

7.1 Conclusion

This thesis has focused on the developments of FLOD-FDTD methods for transmission lines and lumped elements. Firstly, the FLOD-FDTD method has been extended to incorporate the lumped elements including the newly found memristor. Secondly, the FLOD-FDTD method with three split matrices has been developed for second-order accuracy and higher efficiency. Moreover, novel IM1-D FADI- and FLOD-FDTD methods have been developed to simulate transmission lines with interjunctions efficiently even on mobile devices. At last, novel NUTS FLOD-FDTD method has been developed to achieve higher accuracy by using smaller time step for certain periods of simulation.

In Chapter 3, 3-D FLOD-FDTD method incorporated with various lumped elements have been developed as follows.

- The updating equations of the FLOD-FDTD method incorporating resistors, inductors and capacitors have been derived.
- In addition to the three commonly used lumped elements, explicit FDTD method and FLOD-FDTD method incorporated with memristor have also been developed.
- The update equations have been derived based on Maxwell's equations and the physical

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model given by HP lab.

- Numerical examples including transmission line and various lumped elements have been provided to show the trade-off between efficiency and accuracy of the proposed 3-D FLOD-FDTD method.
- For the memristor, the simulation results obtained from the explicit FDTD method is found to be in good agreement with those obtained from SPICE model in ADS. The results using FLOD-FDTD method justify the unconditionally stability and show that larger time step will increase the computational efficiency with acceptable discrepancy from explicit FDTD method.

Since the conventional LOD-FDTD method is of first-order accuracy only, second-order temporal-accurate scheme for 3-D FLOD-FDTD method has been also introduced to improve the accuracy for a recent developed LOD-FDTD method with three split matrices in Chapter 4 as follows.

- The main iterations of the proposed scheme comprise only one more procedure in addition to the existing three procedures. Using proper (initial only) input and (often infrequent, field-point only) output processings, the second-order temporal accuracy has been achieved and verified analytically.
- Stability analysis has also been provided to ascertain the unconditional stability of the proposed scheme and numerical results are demonstrated to justify its higher accuracy.
- To further enhance the efficiency, the proposed scheme have been implemented based on the principle of fundamental schemes, which features a matrix-operator-free RHS.
- It is anticipated that the scheme might be extended to other types of partial differential equations (e.g. thermal diffusion equation).

Although full 3-D FDTD methods can cater for most general structures and 3-D LOD-FDTD method can use time step larger than CFL limit for higher efficiency, the computational domain is still large and full 3-D methods are not well-suited to run on mobile devices such

7.1. CONCLUSION

as smart phones/pads due to the requirement for large memory storage and long simulation time. To circumvent this problem, novel IM1-D FADI- and FLOD-FDTD methods for transmission lines with interjunctions have been developed to further enhance the computational efficiency in Chapter 5 as follows.

- The proposed methods are unconditionally stable and capable of treating multiple main transmission lines and stubs interconnected at various interjunctions using time step larger than CFL limit.
- Fundamental scheme-based IM1-D FADI- and FLOD-FDTD methods have been derived to enhance the efficiency with matrix-operator-free RHS.
- The methods involve one-step update procedure optimized for simulation of main transmission lines and stubs on mobile device.
- Using proper treatments at the interjunctions for various interconnection conditions, the electromagnetic fields in all interconnected main transmission lines and stubs can be updated cooperatively and efficiently to solve practical problems.
- A microstrip line loaded with stubs and a branch-line coupler have been simulated to show the accuracy and efficiency of the proposed methods.
- To extend the applicability for handling the couplings between two transmission lines via gaps, a microstrip circuit with gaps has been simulated using the proposed methods incorporated with equivalent circuit models involving capacitances.
- Real-time simulations of these numerical examples provide much intuitional insight for one to observe the electromagnetic waves propagation in time-domain on computer or mobile device.

At last, a novel NUTS FLOD-FDTD method has also been developed to employ non-uniform time step for different periods during the simulation in Chapter 6 as follows.

- The NUTS scheme adopts different (non-uniform) time steps for different periods during simulation, in order to reduce the large errors caused by unconditionally stable

7.2. FUTURE WORK

FDTD methods with uniform time step larger than CFL limit.

- It has been found that NUTS scheme is potentially unstable for ADI-FDTD method, but it is stable for LOD-FDTD method or its implementation based on fundamental (FLOD) scheme.
- The NUTS FLOD-FDTD method is useful to simulate MTLs with lumped elements in series and parallel connections.
- The method based on M1-D approach has also been extended to incorporate lumped elements in cross connection to model near-field coupling between the MTLs.
- While the electric current density is commonly used for field-circuit coupling of parallel connected lumped elements, the magnetic current density has been utilized for field-circuit coupling of series and cross-connected lumped elements.
- Numerical results for MTLs with lumped elements in series, parallel and cross connections have been provided to show the trade-off between efficiency and accuracy of the proposed NUTS FLOD-FDTD method.
- Compared with the FLOD-FDTD method with uniform time step, the NUTS FLOD-FDTD method may achieve higher accuracy by using smaller time step for certain periods of simulation.

7.2 Future Work

The IM1-D FLOD-FDTD method in Chapter 5 can solve microwave circuit with transmission lines efficiently even on mobile devices, while the accuracy of the simulated results is comparable to the full 3-D methods. However, if the circuit with transmission lines is operating at very high frequencies, the IM1-D FLOD-FDTD method in Chapter 5 would be not enough. The media with effective permittivity and permeability would be varied with frequency especially at high frequencies. Thus, the IM1-D FLOD-FDTD for dispersive media is needed. The unconditionally stable FDTD methods related to dispersive media have been reported

7.2. FUTURE WORK

in [94–98]. Nevertheless, the application of these methods for dispersive media cannot be directly applied in IM1-D FLOD-FDTD method. A more accurate model to describe the characteristics of the transmission lines are needed.

Additionally, the IM1-D FLOD-FDTD method can be extended to coupled lines in the future research. The explicit M1-D FDTD method for coupled lines are already developed by the author and his supervisor and will be published in *IEEE Microw. Compon. Lett.* in 2016. The coupled transmission lines parameters based on even- and odd-mode approach in [90, 99, 100] are incorporated directly into the M1-D FDTD method. Using the M1-D FDTD method, the fine mesh required for the line width and spacing of coupled transmission lines can be omitted. Although the explicit M1-D FDTD method has been realized, the corresponding unconditionally stable method may not be derived according to the IM1-D FADI- and FLOD-FDTD method in Chapter 5.

Furthermore, unconditionally stable FDTD methods related to heat transfer [101, 102] and quantum related problems based on Schrödinger equation are developed in [103–105]. When the FLOD-FDTD methods are used to solve the propagation problems of electromagnetic waves, there may be also related to heat transfer and quantum computation. Thus, the multi-physics techniques based on unconditionally stable FDTD method are required. The unconditionally stable methods proposed in this thesis could also be extended to the mentioned multi-physics techniques.

At last, the NUTS FLOD-FDTD method can be further developed to be applied in various simulations with an adaptive CFLN_n. So far, the proposed NUTS FLOD-FDTD method is investigated with predicted CFLN_n. An adaptive NUTS scheme is required to make the proposed method practical. To achieve an adaptive NUTS scheme, one possible way is to place two monitors before and after the lumped elements to measure the magnitudes of the electric fields. Thus the CFLN_n can be determined adaptively according to whether the monitored magnitudes of the field components are quite large or change dramatically.

Publication List

Journal Publications

1. T. H. Gan, Z. Yang, E. L. Tan, J. X. Lim, M. Q. Huynh and M. Mayank, "A polarization-reconfigurable filtering antenna system," *IEEE Antennas Propag. Mag.*, vol.55, no.6, pp.197-235, Dec. 2013.
2. Z. Yang, E. L. Tan and D. Y. Heh, "Second-order temporal-accurate scheme for 3-D LOD-FDTD method with three split matrices," *IEEE Antennas Wireless Propag. Lett.*, vol.14, pp.1105-1108, Jan. 2015.
3. Z. Yang and E. L. Tan, "A microstrip circuit tool kit app with FDTD analysis including lumped elements," *IEEE Microw. Mag.*, vol.16, no.1, pp.74-80, Feb. 2015.
4. Z. Yang and E. L. Tan, "Two finite-difference time-domain methods incorporated with memristor," *Progress In Electromagnetics Research M*, vol. 42, pp. 153-158, Jul. 2015.
5. Z. Yang and E. L. Tan, "Efficient 3-D fundamental LOD-FDTD method incorporated with memristor," *IEICE Trans. Electron.* vol. E99-C, no.7, Jul. 2016.
6. Z. Yang and E. L. Tan, "3-D non-uniform time step locally one-dimensional FDTD method," *Electron. Lett.*, vol. 52, no. 12, pp. 993 - 994, Jun. 2016.
7. Z. Yang and E. L. Tan, "Multiple one-dimensional FDTD method for coupled transmission lines and stability condition, " *IEEE Microw. Compon. Lett.*, vol. 26, no. 11, pp. 864 – 866, Nov. 2016.

8. Z. Yang and E. L. Tan, "Interconnected multi-one-dimensional FADI- and FLOD-FDTD methods," IEEE Trans. Microw. Theory Techn., vol. 65, no. 3, pp. 684 - 692, Jan. 2017.
9. E. L. Tan and Z. Yang, "Non-uniform time-step FLOD-FDTD method for multiconductor transmission lines including lumped elements", IEEE Trans. Electromagn. Compat., 2017. (to be published)

Conference Publications

1. Z. Yang and E. L. Tan, "Efficient 3-D fundamental LOD-FDTD method with lumped elements," IEEE MTT-S Int. Microwave Symp. Dig., June 2014.
2. Z. Yang and E. L. Tan, "A microwave transmission line courseware based on multiple 1D-FDTD method on mobile devices," IEEE Asia-Pacific Conference on Antennas and Propag., 2015.
3. Z. Yang and E. L. Tan, "A fundamental ADI-FDTD method with implicit update for magnetic fields in the second procedure," IEEE Asia-Pacific Conference on Antennas and Propag., 2015.
4. Z. Yang, E. L. Tan and L. L. Wang, "Upgrading LOD-FDTD to efficient method with second-order accuracy," 2015 Asia-Pacific Microwave Conference (APMC), Nanjing, 2015
5. Z. Yang and E. L. Tan, "Variants of second-order temporal-accurate 3-D FLOD-FDTD schemes with three split matrices," IEEE International Conference on Computational Electromagnetics (ICCEM), Mar. 2016.
6. Z. Yang and E. L. Tan, "A de-embedding technique for diode-incorporated reconfigurable antenna simulation," IEEE APS Int. Antennas Propag. Jul. 2016
7. Z. Yang, E. L. Tan and D. Y. Heh, "Explicit and locally one dimensional finite-difference time domain methods incorporated with memristor," PIER Aug. 2016

8. E. L. Tan and Z. Yang, "Multiple One-Dimensional FDTD Method for Edge/Broadside Coupled Transmission Lines", Asia-Pacific Engineering Research Forum on Microwaves and Electromagnetic Theory, Oct. 2016.
9. Z. Yang and E. L. Tan, "Stability analyses of non-uniform time-step schemes for ADI- and LOD-FDTD methods", IEEE International Conference on Computational Electromagnetics (ICCEM), Mar. 2017.
10. E. L. Tan, D. Y. Heh, and Z. Yang, "Interactive electromagnetic and microwave transmission line educational courseware on iPad", Progress In Electromagnetics Research Symposium, May 2017.
11. E. L. Tan, D. Y. Heh, and Z. Yang, "Recent developments of implicit finite-difference time-domain schemes", Progress In Electromagnetics Research Symposium, May 2017.
12. Z. Yang and E. L. Tan, "3-D Unified FLOD-FDTD Method Incorporated with Lumped Elements", IEEE APS Int. Antennas Propag. Jul. 2017.

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